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Commentary

IT has become increasingly obvious ever since the BBC first announced its Five Year Plan for television in this country that a single network could not be expected to satisfy all the people all the time, and that sooner or later plans to provide an alternative programme would have to be evolved. As television extended northwards and westwards from London the different tastes and habits of the various sections of the population made this need more apparent than ever and there is no doubt that sponsored television has contributed much to the public interest in these matters.

But such demands cannot be satisfied overnight and many problems have to be solved. Apart from such difficulties as Government restriction on capital expenditure, one of the foremost is the availability of suitable television channels in the rapidly overcrowding wavebands.

There have been several Government statements recently on the future both of television and sound broadcasting but no firm decision could be taken until the technical aspects were first settled.

A Television Advisory Committee was therefore set up in October of last year under the chairmanship of Admiral Sir Charles Daniel "to advise the Postmaster General on the development of television and sound broadcasting at frequencies above 30Mc/s and related matters, including competitive television services and television for public showing in cinemas and elsewhere."

These are very wide terms of reference, and to advise the P.M.G. on all these aspects would take an appreciable time; consequently the first report of the Television Advisory Committee which has just been published deals only with the development of television in the home both by the BBC and competitive organizations, leaving sound broadcasting and cinema television to be discussed in later reports.

In the range of frequencies for television and sound broadcasting surveyed by the Committee—namely from 30Mc/s to 3 000Mc/s—there are, in fact, no more than three V.H.F. and two U.H.F. bands which have been assigned by international agreement for this purpose.

Designated bands I to V, their frequency ranges are as follows:

Band I	41—68Mc/s	V.H.F.
Band II	87·5—100Mc/s	V.H.F.
Band III	174—216Mc/s	V.H.F.
Band IV	470—585Mc/s	U.H.F.
Band V	610—960Mc/s	U.H.F.

A technical subcommittee under Dr. W. G. Radley, was brought into being to examine the rather weighty technical considerations and it is largely on the work of this subcommittee that this report is based.

The report is therefore technical in nature and it must be left for a higher authority to shape the future of television by the BBC and the competitive organizations for the report does little more than disclose the present state of affairs.

Dealing with the bands in turn the Committee finds that all the available channels in Band I will be required to complete the BBC's present network some details of which were outlined recently by Sir Ian Jacob, the Director-General of the BBC. As is known, this network will comprise the existing five high power stations—including a new transmitter for the London area—and the proposed five medium power stations which have been held up on account of capital expenditure restrictions.

These ten stations will be supplemented by low power stations at the Isle of Man and the Channel Islands and it has been put forward that a further six low power stations will be required for those "pockets" not served by the higher powered stations. Thus the network will comprise some eighteen stations all but the six low power stations being accommodated in Band I.

This network forms the very foundation of British television broadcasting and it must be agreed almost as a matter of national policy that this network should be completed as soon as possible, leaving the second or alternative BBC network and the claims of the competitive organizations to be met from the remaining bands.

Band II is regarded as too narrow to be of much use for television but it will come into prominence in the future for the Committee suggests that this band be reserved for sound broadcasting. This will do much to alleviate the unsatisfactory reception now existing on the medium and long wavebands.

Band III would appear to be the most desirable for television from several points of view but it is largely occupied by the fixed and mobile services and by the highly important radio navigational aids, and none of these can be transferred to another part of the frequency spectrum without incurring considerable cost.

One of the principal advantages of Band III is the much greater service area it can provide—it is estimated that at least twice as many transmitters would be required to provide the same coverage by bands IV and V—but no less important is the fact that television development plans in many European countries are based on the use of this band and it is argued, with some justification, that if the British radio industry is to compete successfully in the export markets there must be a thriving home market for television transmitters and receivers in this band.

But the present situation is that there are only two channels which can be made immediately available and these would allow only a part of the alternaive network the BBC proposes. The competitive organizations and the remaining section of the BBC alternative network would thus be transferred to Bands IV and V.

Here there are however no less than 93 channels available on a basis of 5Mc/s bandwidth, fifty of which could be made available immediately, but the report emphasizes that costs both at the transmitting and receiving ends would be high.

It is perhaps fortunate as far as the Television Advisory Committee is concerned that its terms of reference do not compel it to settle these controversial problems.

As we see it, there can be no argument against the claims of the BBC to complete the present network; but for the rest it appears to be a straight fight between the alternative network and the competitive organizations for the available channels in Band III. Whoever obtains these channels will gain on the score of wider service area and reduced operating costs, but the viewer, unless he restricts his viewing to the present network, faces additional costs all round—a considerably more expensive receiver than at present and a veritable forest of aerials from his roof.

A Linear Sweep

CATHODE-RAY POLAROGRAPH

By H. M. Davis*, B.Sc., A.R.I.C., and Joyce E. Seaborn*, B.Sc.

The circuit of a linear sweep cathode-ray polarograph, which has been designed primarily for use by operators with little knowledge of electronic equipment, is presented.

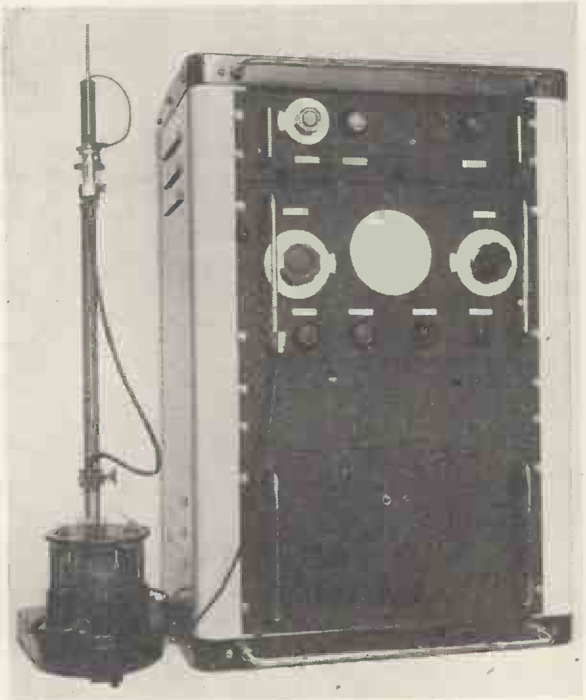
The sensitivity of the instrument is considerably greater than that of instruments hitherto commercially available, and it offers advantages both in rapidity of operation and in the resolution obtained between adjacent steps. A number of refinements, not previously described in instruments of this type, have been incorporated. These include compensation for the capacitive current flowing in the cell and means for the presentation of a derivative trace.

THERE is a considerable field of application for electronics in the chemical laboratory, particularly in instruments for the measurement of small voltages and currents. One such instrument is the polarograph, which is used to determine the concentration of reducible ions in a solution, by measurement of the current that flows when the ions are electrolytically reduced.

Only a brief description of the polarographic method of analysis is given here, full details may be found in any of the standard works on the subject, such as that by Kolthoff and Lingane[1]. The technique consists essentially of electrolysing a solution of the material to be analysed, in a cell having a mercury anode and a dropping mercury cathode. The voltage applied to the cell is varied by means of a potentiometer, usually over the range 0-2 volts, and the cell current is recorded on a sensitive galvanometer.

There are several reasons for the choice of a dropping mercury cathode, one of which is that, if effects due to contamination by the reduced ions are to be eliminated, the cathode surface must be continually renewed. This is achieved quite simply in practice by causing mercury to flow from a reservoir of adjustable height, through a fine glass capillary which has one end immersed in the solution. The drops which form at the immersed end fall through the solution after a life of a few seconds and are collected in the anode pool. Electrical connexions are made to the mercury in the reservoir and to the anodic pool in the bottom of the cell by means of platinum tipped wires.

The current through the galvanometer fluctuates continually with the growth and fall of each drop at the cathode and it is therefore necessary to use a heavily damped instrument of long period to ensure that the mean cell current is indicated.



The complete equipment.

The substance to be analysed is dissolved in a solution containing an excess of an electrolyte, such as potassium chloride, which is not reduced within the range of cell voltage normally used in analysis. Under these conditions, as the voltage across the cell is increased from zero, the current is at first very small, but when the voltage reaches the value at which one of the ionic species present in the solution is reduced, the current rises sharply with voltage until it reaches a limit imposed by the rate of diffusion of the ions towards the cathode. If other reducible ionic species are present, similar step-wise increases in the current occur as their reduction potentials are attained.

The potential at the mid-point of each step, where the current is equal to one-half of its limiting value, is known as the "half-wave potential" and it is characteristic of the ionic species being reduced. The total change of current is proportional to the concentration of the ionic species in the solution. Both these quantities are obtained from the curve of mean cell current against voltage, which may be plotted either manually or automatically. A typical polarogram is shown in Fig. 1. The time taken to produce the curve with a photographic polarograph is about ten minutes.

The response of the galvanometer to the current change at each step must be sufficiently fast relative to the rate of change of cell voltage, to avoid serious distortion of the curve. This sets a limit to the damping which can be imposed on the instrument and it is usually necessary to adopt a compromise value, which does not always reduce the drop wave sufficiently at high instrumental sensitivities, for easy and accurate measurement of step height to be possible.

The Linear Sweep Cathode-Ray Polarograph

An alternative method of polarography due to Randles[2,3,4] is well suited to electronic treatment and offers

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advantages in sensitivity, speed and resolution between adjacent steps. In this system, a linear potential sweep is developed across the cell once in the life of each drop, instead of being spread over the lives of many drops, as in instruments of the type described above. A resistor is connected in series with the cell and the voltage appearing across it, which is a measure of the cell current, is amplified by a D.C. differential amplifier and applied to the Y deflexion plates of a long after-glow cathode-ray tube. The voltage appearing across the cell is also amplified and applied to the X deflexion plates of the tube, so that the resulting display is a graph of cell current as a function of cell voltage. A special circuit is used to maintain a constant rate of change of potential across the cell, irrespective of the ohmic drop occurring in the series resistor.

It is necessary to delay the onset of the potential sweep until late in the life of each drop, when the rate of change of drop surface is small, since any major change of surface would cause undesired variations in the cell current. This delay period is made constant for every drop by locking the sweep generator in synchronism with the dropping

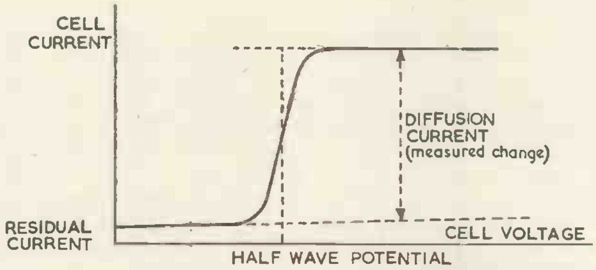

Fig. 1. Trace obtained with conventional polarograph when examining solutions containing cations due to one element

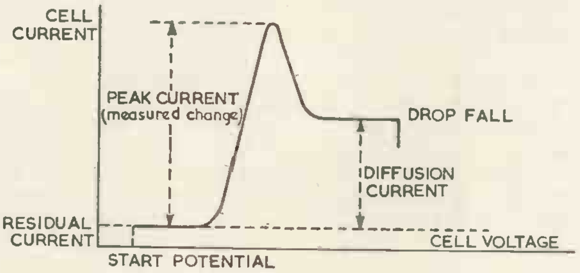

Fig. 2. Trace obtained on linear sweep C.R. polarograph when examining a solution containing cations due to one element

electrode, thus ensuring a reproducible trace.

The effect of applying a rapid potential sweep is to produce a current waveform which exhibits a peak, because of the rapid stripping of ions from the solution immediately surrounding the drop when their reduction potential is attained. As the potential continues to rise, the current falls below the peak value until it reaches a level governed by the rate at which ions can diffuse through the solution towards the drop (Fig. 2). It was shown by Randles[3,4] both theoretically and experimentally, that the rise in current occurring at the peak is linearly related to the concentration of the ionic species undergoing reduction and it is this rise which is measured in analysis.

It is an advantage of the linear sweep polarograph that, since the current rises to a maximum value which is always greater than the diffusion current level, it is inherently more sensitive than instruments of the conventional type.

Instruments have been designed, operating on this principle, by Randles and Airey[3,4] by Weidmann[5], and by Snowden and Page[6].

The instrument constructed by Randles and Airey, in the laboratories of the Chemical Inspectorate, was successful in that it clearly demonstrated the analytical value of the method, but because of the complexity of its controls, it was unsuitable for use by operators without considerable knowledge of electronic equipment.

The apparatus has therefore been completely redesigned with the requirements imposed by routine analytical use in mind and, in particular, the panel controls requiring manipulation by the operator have been reduced to the minimum number consistent with full flexibility. Some of the quantities which were variable in the earlier instrument have been fixed, where this could be done without impairing performance. These are, the rate of change of cell potential, which has been set at 0·3 volt per second; the delay period before the onset of the sweep, which has been set at five seconds; and the voltage gain factors of the X and Y deflexion amplifiers. In addition, the overall sensitivity of the instrument has been increased, and a method has been devised for the neutralization of effects due to the cell capacitance[7]. Provision has also been made for an alternative trace, closely corresponding to the time derivative of the cell current waveform, to be displayed on the cathode-ray tube. This trace is more sharply peaked than the parent waveform, and the resolution of the instrument is accordingly increased. Under favourable conditions, peaks corresponding to reductions occurring at potentials only 30mV apart may readily be separated.

Instruments based on this design have been in use in the Chemical Inspectorate for the past three years and, although developments have from time to time been carried out, no basic modifications have been found necessary.

Description of Circuits

Details of the circuits used (Figs. 4, 5 and 6) are now described.

Sweep Generator and Compensation Circuits

These circuits are the most important in the apparatus. They are required:

(1) To generate a potential waveform which
  (a) has a steady value during a fixed period commencing with the start of growth of each drop;
  (b) thereafter changes linearly with time;
  (c) returns to the initial steady state at the fall of each drop.

(2) To reproduce this potential waveform at the cell anode, irrespective of the ohmic drop occurring in the cell series resistor.

(3) To compensate as far as possible for effects arising from the capacitive nature of the cell impedance.

The circuits performing these three functions are as follows.

1. *Generation of the potential sweep waveform* ($V_1V_2V_7V_8$)

The timing of the sweep is accomplished by a normally free running multivibrator (V_1V_2) having metastable states lasting for five and two seconds. One valve of the pair (V_2) is, however, a pentode and the synchronizing pulse derived from the fall of the drop is applied to its suppressor grid by means of C_5 and R_7, so as to drive this grid negative. When this happens, anode current in V_2 is cut off, the grid of V_1 is driven positive and the potential of its anode falls, thereby maintaining V_2 cut off. This condition corresponds to the quiescent period of the sweep generator and continues until the potential of the grid of V_2 has risen sufficiently for the valve again to conduct. By this time the negative going pulse applied to the suppressor grid of V_2 has decayed to zero and the anode circuit of this valve can again pass current. Cumulative action follows and V_2 is then in full current and V_1 is cut off. The voltage sweep, which is controlled by the multivibrator, then starts and is continued until the next pulse is received at the suppressor grid of V_2, when the cycle repeats. Provided that a sufficiently high value of cell series resistor is used, no action is required by the operator to establish synchronization, other than to make the dropping time of the mercury cathode slightly less than the total period of the multivibrator, i.e. seven seconds. If, however, due to incorrect conditions, no synchronizing pulse is applied to V_2, the sweep is always terminated at the end of the normal period of the multivibrator.

The potential sweep is generated at a high level by means of a circuit of the bootstrap type. V_{8a}, which is controlled

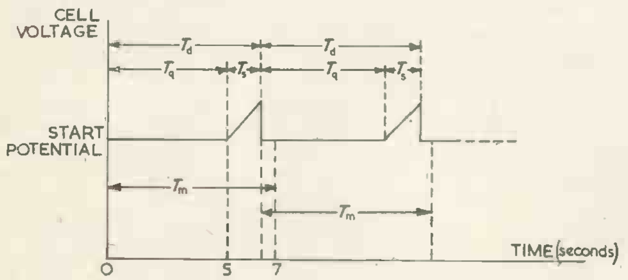

Fig. 3. The voltage waveform at the cell anode

T_d = drop time, T_q = quiescent period, T_s = duration of linear sweep, T_m = normal period of multivibrator.

by the multivibrator, acts as a switch across the timing capacitor C_{10} and allows the sweep to start when its grid is driven negative. The cathode-follower V_{8b} produces a nearly linear rise in potential across C_{10}, by feedback action, and the output of the generator is applied to the network R_{25}-R_{29}. The voltage applied by this network to the grid of V_9 consists of a steady voltage, which may be varied by means of R_{29}, on which is superimposed, late in the life of each drop, a component rising linearly with time. The rate of change of this latter component is approximately 0·3V/sec and is constant for all settings of R_{29} (Fig. 3).

2. *Compensation for ohmic drop in the cell series resistor* (V_9-V_{12})

The form of compensation circuit described by Randles and Airey is efficient, but suffers from the disadvantage that both the sweep generator and the source defining the sweep start potential must be floating with respect to earth

The circuit arrangement used in the authors' instrument[7] was selected as the result of experiment with a number of different designs, aimed at avoiding the difficulty referred to above.

V_9 and V_{10} together serve as a comparator stage. The potential appearing at the slider of R_{29} is applied to the control grid of V_9 and that appearing at the cell anode is applied to the grid of V_{10}.

If, due to an increase in cell current, the potential of the control grid of V_{10} tends to fall relative to that of the control grid of V_9, the effect is that current in the common cathode resistor R_{33} is decreased, with a consequent fall in potential of the anode of V_9. This change increases the voltage at the anode of V_{11a} and hence the voltages at the cathode of V_{12} and at the cell anode, i.e. the action is such as to maintain a constant voltage difference between the cell anode and the slider of the potentiometer R_{29}. The potential existing across the cell during the period before the sweep starts, can thus be controlled by the potentiometer through the intermediary of the compensation circuit.

The basic voltage relation between the cell anode and the slider of the potentiometer may be varied over a small range by adjustment of R_{30}, while retaining satisfactory operation of the amplifier. In the circuit described, R_{30} is adjusted so that with the slider of R_{29} at its minimum potential, the cathode of V_{12} is 0·5V negative with respect to earth, i.e. the electrode that is normally the cell anode is negative relative to the earthed cell cathode. This step is taken to produce the required range of sweep start potentials (+0·5V to −2·0V, European Sign Convention), on varying R_{29}.

It should be noted that, if the use of start potentials between 0 and +0·5V is not contemplated, the performance of the compensation amplifier may be somewhat improved by setting R_{30} so that the cathode of V_{12} is at zero potential under the conditions stated above.

With the circuit as described the feedback has the effect of reducing the voltage drop at the cell terminals, due to current flow in the series resistor, to about one two-hundredth of what it would be in the absence of the compensation. It follows that a variable series resistor (S_1) may be used to control the overall sensitivity of the instrument without materially affecting the voltage existing across the cell.

Since with the highest value of series resistor a current change of one millimicroamp is easily detected, it is desirable that the grid currents of all the valves whose input grids are connected to the cell, should be both small and stable under the conditions of operation. For this reason, the valves V_9 and V_{10} are of the EF37A type which have the described characteristics, particularly when the anodes, screens and heaters are operated at reduced ratings, as is the case in the present design. A similar reason applies to the choice of this valve type for the input stages of the X and Y amplifiers.

It is clearly desirable that the amplifier should be free of any appreciable drift in the period likely to be required for the analysis of a solution. The most important source of drift in an amplifier of this type, assuming reasonably well stabilized H.T. supplies, is variation in the operating temperature of the cathode of the input valves, due to changes in the heater supply voltage. This has been minimized in the present instrument by supplying the heaters of V_9 and V_{10} in series from the main stabilized power supply, and the drift rate of the output voltage does not exceed one millivolt per hour after an initial warming up period of one hour.

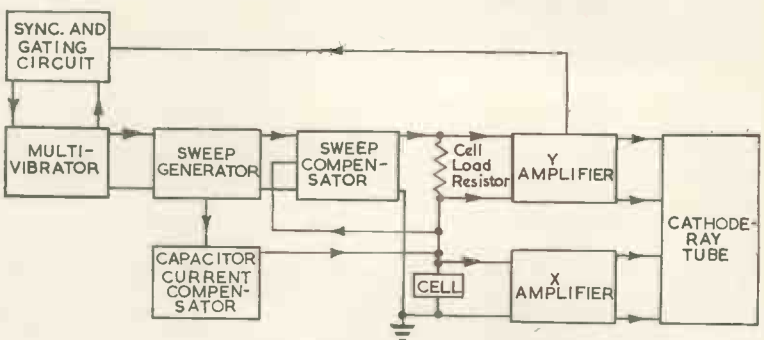

Fig. 4. Block diagram of complete equipment

3. *Compensation for the effect of cell capacitance*

Very approximately, the polarographic cell may be regarded as equivalent to a high resistance in parallel with a capacitance of about one microfarad. It will be seen that when the potential sweep is applied, an almost constant current will flow into the capacitive part of the cell impedance, governed by the relation $i = C\,dv/dt$ (with the usual notation). When high values of cell series resistor are in use, in order to achieve great sensitivity, this current causes

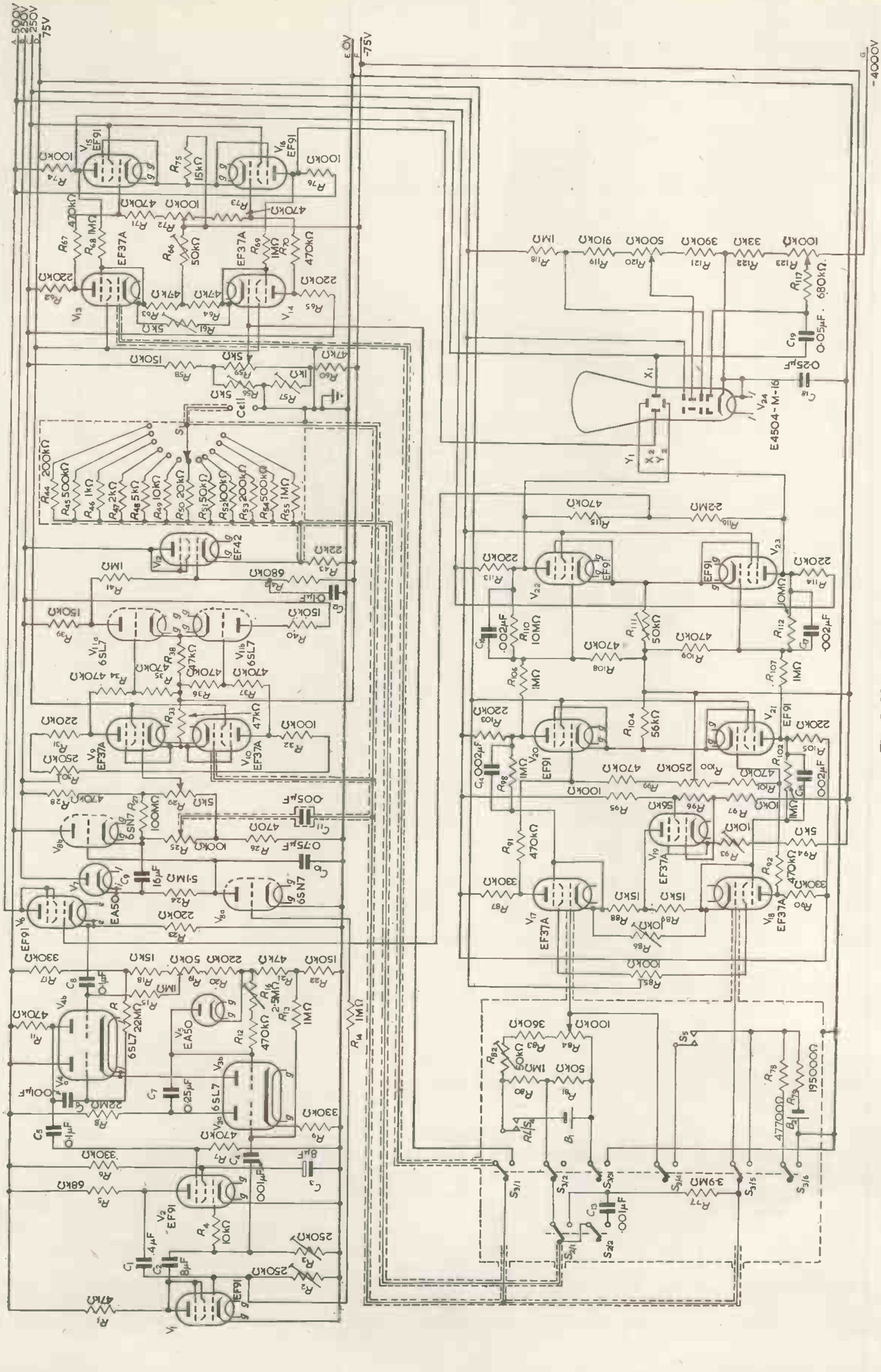

Fig. 5. Measuring circuit

a voltage step to be injected into the Y amplifier of sufficient magnitude to deflect the cathode-ray tube spot from the screen. For example, in the apparatus now described, the maximum cell load resistance is 1MΩ and $dv/dt = 0{\cdot}3$V/sec. Hence, the voltage injected into the Y amplifier during the sweep may be as much as 0·3V, while the input required for full screen deflexion is only about 30mV.

The method used to overcome this effect depends on the injection of a constant current into the cell circuit during the sweep period, of sufficient magnitude to produce the required rate of change of voltage across the capacitive part of the cell impedance[7]. This may be readily

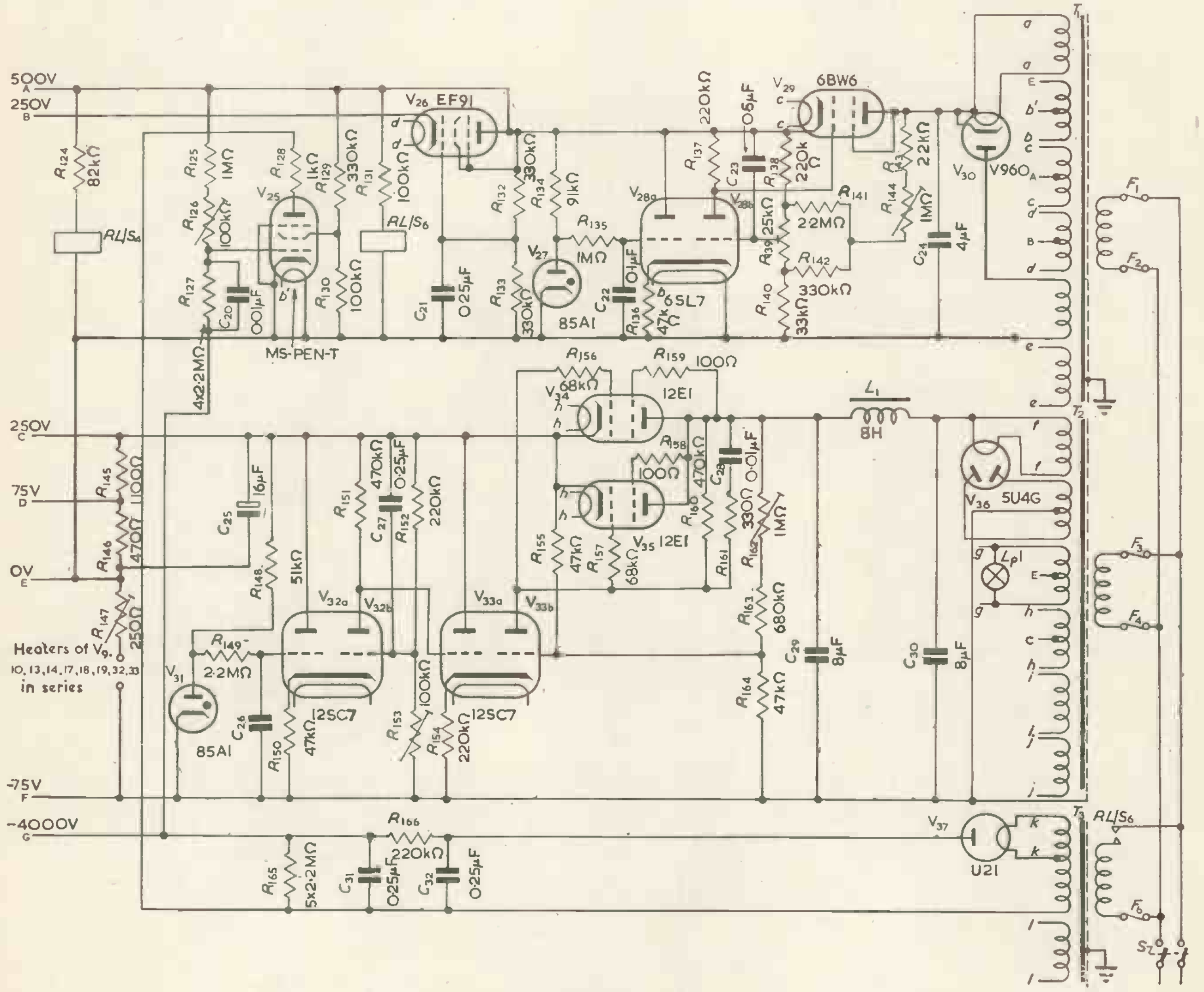

Fig. 6. Stabilized power supplies

accomplished by connecting a capacitor C_{11} of suitable value between the anode of the cell and a potential divider in the cathode circuit of V_{8b}. In the circuit described, the rate of change of voltage at the cathode of V_{8b} is 60V/sec, i.e., 200 times that applied to the cell, during the sweep period. With the cell capacitance about 1μF it follows that C_{11} should be 0·005μF. Fine setting of the current is accomplished by varying the control R_{25}.

The method has the advantage that the whole of the available shift voltage can be applied to backing off the voltage across the cell series resistor arising from diffusion current flow in the cell, and to measurement of the magnitude of peaks occurring on the trace due to ionic deposition.

Synchronization and Paralysis Circuits (V_3-V_6)

When a mercury drop falls from the capillary, a sharp fall in current occurs (Fig. 2), from which is derived the pulse used to trigger the multivibrator to the state corresponding to the quiescent period of the sweep generator. The synchronization circuit (V_4 and V_6) producing this pulse must be sufficiently sensitive to respond to a very small input, such as is received when strong solutions are being investigated and the cell series resistor is low. Because of this high sensitivity, the synchronizing circuit may also respond to the fairly rapid fall in current occurring immediately after the peak on the trace and terminate the sweep prematurely. For this reason, the operation of the synchronizing circuit is paralysed for a period beginning when the sweep commences and ending when the cell current becomes diffusion controlled and is hence stable (see Fig. 2).

The paralysis circuit[8] consists of a double triode V_3 and a diode V_5. The double triode is connected as a cathode-coupled flip-flop and the normally-on section V_{3b}

forms part of the cathode circuit of the synchronization valve V_4. At the start of the sweep V_3 is switched to the unstable condition with V_{3b} cut off, rendering the synchronization circuit inoperative. After a period determined by the time-constant of its grid circuit, V_{3b} again conducts and turns on V_4. A positive pulse of sufficient amplitude at the grid of V_{4b} will then trigger V_4, which is also connected as a cathode-coupled flip-flop, and cause a negative pulse of fixed amplitude to be delivered to the suppressor circuit of V_2, thus resetting the multivibrator.

The period of paralysis is controlled by R_{16} and the threshold level for triggering V_4 is varied by means of R_{19}.

X Amplifier (V_{13}-V_{16})

This part of the circuit amplifies the voltages appearing across the cell to a sufficiently high level for use on the X deflector plates of the cathode-ray tube. The voltage gain of the amplifier is controlled by negative feedback and is adjusted by means of R_{61} so that a deflexion of 1cm in the X direction corresponds to an input voltage of 0·05V.

One of the input grids is connected to the cell anode, the other to a shift network R_{56}-R_{60}. The balance control R_{72} in the amplifier is so adjusted that, when the potentials of the two input grids are the same, the cathode-ray tube spot lies on a particular line near the left-hand side of the graticule. This line is thereafter taken as the reference for start potential settings.

Any required start potential for the sweep may then be selected by applying a known potential to the input grid of V_{14} from the network R_{56}-R_{60} and adjusting R_{29} until the spot returns to the reference line. The control R_{59} is calibrated over a range of +2·0V to −0·5V relative to earth. The signs of the voltages are reversed on the control in the authors' instrument, so as to conform with the European Sign Convention.

The start potential is determined by this procedure with sufficient accuracy for all normal requirements. In the event of greater accuracy being required for a special purpose the quiescent cell potential may be found by connecting a suitable high impedance meter directly to the cell terminals. The potential at which any peak occurs may be calculated from a knowledge of the start potential and the displacement of the peak in the X direction.

All power supplies for the amplifier, with the exception of that for the heaters of the output stage, are stabilized and the drift rate referred to the input does not exceed one millivolt per hour after an initial warming up period of one hour.

Y Amplifier (V_{17}-V_{23})

The voltage appearing across the cell series resistor is presented to a conventional three-stage balanced amplifier to which negative feedback is applied in two loops. The high frequency response of the amplifier is restricted by means of $C_{14}C_{15}$ and $C_{16}C_{17}$ in order to remove the necessity for screening the cell from A.C. pick up.

It is necessary that the discrimination of the amplifier against in-phase inputs should be sufficiently high to ensure that, with the cell circuit open, the trace does not deviate appreciably from the horizontal at any start potential setting. This implies that the input valves must be carefully matched, but provision is made in the design for eliminating effects due to small variations in valve characteristics. The method of matching is described later.

The adjustment of the amplifier is carried out as follows. After setting the mean anode potentials of V_{17} and V_{18} to 75V by means of R_{93}, the gain of the amplifier is adjusted to 10 000 using R_{86}. The calibrated shift control R_{84} is then set to 50mV and the trace, with zero input and the sensitivity switch S_1 set to its minimum value, is adjusted to the centre line of the graticule by means of R_{100}.

The trace is then examined at different start potential settings. If it has a slope, the differential screen control R_{85} is adjusted in the required direction to compensate this effect, and R_{100} is again adjusted until the spot sweeps across the centre of the graticule. This examination and adjustment is repeated, until the slope of the trace is negligible at all start potential settings. If this condition cannot be achieved the input pair are replaced by more closely matched valves.

The shift voltage for the amplifier is produced by the network R_{80}-R_{84}. The potentiometer R_{84} is a high grade Colvern type, fitted externally with mechanical stops to ensure that the slider does not run off the track. The control knob for the potentiometer carries an engraved dial, reading from 0-200mV in steps of 1mV for an angular rotation of 300 degrees. This control may be used merely as a beam shifting device or alternatively, for the measurement of a peak height in terms of millivolts appearing across the cell load resistor. In the latter case, the peak current can be calculated as the cell series resistor is known.

The voltage appearing across the shift potentiometer may be calibrated against a standard cell, which provides a reference potential of 200mV at the junction of the resistors R_{78} and R_{79}. For this purpose the shift control is set to

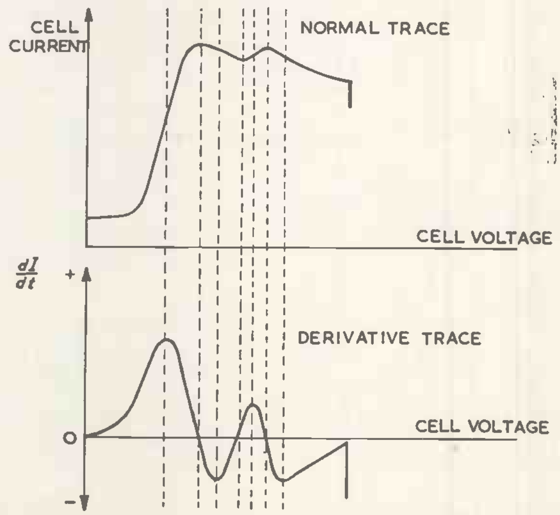

Fig. 7. Traces obtained when examining a solution containing cations due to two elements with adjacent reduction potentials

the 200mV mark and S_3 is set to the "Calibrate" position. The amplifier and cathode-ray tube then function as a null indicator and R_{82} is adjusted until no deflexion of the cathode-ray tube spot occurs when the input grids of the amplifier are shorted together by means of S_5.

Differentiating Circuit

The switch S_2 enables the cell current waveform to be fed to the Y amplifier through the coupling C_{13} R_{77}. The time-constant of this coupling is only five milliseconds and the trace then corresponds approximately to the time derivative of the current waveform. Definite maxima and minima are exhibited by the derivative waveform, even where the peaks on the normal trace have become merged because the difference between their reduction potentials is small (Fig. 7).

It is not claimed that the derivative trace has more than qualitative significance but it is useful in the preliminary examination of solutions. Sensitivity is, of course, considerably reduced and it is necessary to increase the value of cell load resistor to about twenty times that required for examination of the normal trace.

Cathode-Ray Tube Circuit (V_{24})

This is of the conventional type and embodies the usual controls for brilliance (R_{123}) and for focus (R_{120}). A brightening waveform is derived from the X amplifier output and applied to the grid by means of the differentiating network C_{19} R_{117} thus enabling satisfactory beam brightness to be obtained during the sweep, without risk of burning the cathode-ray tube phosphor during the quiescent period.

Astigmatism in the tube can be controlled by adjustment of R_{66} and R_{111}, which vary the mean voltage levels of the output valve anodes in the X and Y amplifiers respectively.

The graticule for the cathode-ray tube measures 10cm by 10cm and is divided into millimetre squares.

Stabilized Power Packs

Three main supplies are used:

(a) Main 325V, 150mA Supply

This circuit provides output voltages of 250, 75, 0 and −75V, relative to chassis, for the D.C. amplifiers. These voltages are derived from tappings on the resistance chain R_{145} R_{146} R_{147}, which also controls the stabilized current supply to the heaters of V_9, V_{10}, V_{13}, V_{14}, V_{17}, V_{18}, V_{19}, V_{32} and V_{33}.

The circuit is based on the design due to Miller[9] suitably modified to provide the required overall output voltage and to enable advantage to be taken of the high stability reference voltage provided by the Mullard type 85A1. The output, which is controlled by R_{153}, is set to 325V on load and R_{147} is then adjusted until the voltage between the negative supply rail and chassis is 75V. It only remains to find the correct setting of R_{162} to achieve maximum stabilization of the output voltage when the mains input fluctuates by ±10 per cent about the normal level.

The primary of the transformer for this supply must be adequately screened from the secondaries. It is also preferable to connect the mains to the primary in such a way that the neutral side of the winding is adjacent to the screen.

(b) Subsidiary 500V and 250V Supply

This circuit supplies the necessary power at the 500V level for the anodes of the final stages of the deflexion amplifiers and also provides a 250V line for the sweep generator. This circuit draws a varying current, which cannot be taken from the potential divider across the main supply unit as the voltage distribution would be upset. The 500V line is also used as a reference potential for the E.H.T. stabilizer.

The output of the series stabilizer is adjusted to 500V by means of R_{139}, and R_{144} is set to give maximum stabilization for mains variations of ±10 per cent. The 250V output is derived from a cathode-follower V_{26}.

(c) E.H.T. Supply

The E.H.T. is obtained from a transformer fed half-wave rectifier. V_{25} defines the potential of the positive side of the supply, relative to chassis, and its grid is controlled by a potential divider R_{125} R_{126} R_{127}, connected between the negative E.H.T. rail and the stabilized 500V line. The valve thus functions as a series stabilizer and R_{126} is so adjusted that, with the normal mains supply voltage, the E.H.T. output is about 4 000V negative, relative to chassis.

The stabilizer valve must have a high amplification factor and, as it is required to absorb the fluctuations of the unstabilized E.H.T. voltage, must also have excellent anode insulation. These requirements are met by the MS/Pen-T, which has a top cap anode.

It is essential that the E.H.T. voltage shall not be switched on until the 500V reference is available, as the full E.H.T. voltage will otherwise appear across the stabilizer, because of the large negative potential applied to its grid. To prevent this, the mains supply to the E.H.T. transformer is switched in by a relay RL/S_6, which is energized by means of the 500V line.

Constructional Details

The general layout of the apparatus can be seen from the photograph. The sweep generator, synchronization circuit and compensation circuits are grouped on the top chassis, the C.R.T. and the deflexion amplifiers are on the middle chassis, and the power supplies are housed in the bottom of the rack.

The panel controls are as follows, reading from left to right:

Top Chassis.

X shift R_{59}. Start potential R_{29}. Capacitor current compensation R_{25}.

Middle Chassis.

Sensitivity S_1. Normal/derivative S_2. Brilliance R_{123}. Focus R_{120}. Measure/calibrate S_3. Y shift R_{84}. Calibration key S_5.

Paralysis time (R_{16}) and sync. sensitivity (R_{19}) controls are mounted behind the panel of the sweep generator chassis and holes are provided so that they can be adjusted by screwdriver without removing the chassis from the rack. The variable resistor R_{82} controlling the voltage across the Y shift potentiometer is similarly mounted behind the panel of the amplifier chassis.

The usual precautions which are desirable in the construction of D.C. amplifiers[10] as regards matching of valves, layout of components, shielding from draughts, etc., have been taken throughout the apparatus in order to reduce drift.

The matched EF37As for the input stages of the amplifiers are selected by measuring the anode currents of a number of well aged valves at grid voltages of −1·6, 1·8 and 2·0V, with anode and screen at 75V and heater current 150mA. Pairs are chosen for equality of anode current and mutual conductance.

The cell is connected to the instrument by means of a coaxial cable. The inner conductor is taken to the cell anode and the braiding is connected both to the cell cathode, via the mercury reservoir, and to the electrode stand. The apparatus is provided with an earth line, independent of the mains, which is connected directly to the cell cathode terminal.

Operating Procedure

With the prepared cell connected to the apparatus, the rate of the dropping mercury cathode is adjusted until the sweep generator is synchronized.

If the composition of the sample to be analysed is unknown, the required range of cell potential is swept in sections by progressive adjustment of the start potential control, e.g. the range 0-2V can be completely covered with adequate overlap by setting the start potential at 0, 0·4, 0·8, 1·2 and 1·6V, since the amplitude of the voltage sweep is about 0·5V.

The instrument has the great merit that the sensitivity can be varied, if desired, for each successive sweep, i.e., every seven seconds, whereas the sensitivity of the conventional instrument cannot normally be altered during the time taken to record one trace, which is about ten minutes.

A qualitative examination of the solution can thus be carried out very speedily. This examination enables the potentials at which peaks occur to be determined as already described in the section on the X amplifier, and forms the basis for subsequent quantitative analysis.

The isolation and measurement of the peak due to a particular ionic species is carried out by setting the start potential at a slightly lower value than that corresponding to the peak. All ionic species having lower reduction potentials than the required species consequently undergo reduction during the quiescent period and the currents due to these reductions are thus diffusion controlled. Because of this the first portion of the trace obtained when the sweep starts is horizontal and forms a convenient datum for measurement of the ensuing peak.

The peak height is adjusted to a suitable value, by means of the sensitivity switch, and it is then measured either by using the cathode-ray tube graticule or by means of the calibrated Y shift control. The corresponding ionic concentrations are determined by comparing the results with those obtained from standards under the same conditions, or they may be calculated from Randles' equation.

Advantages

The advantages of the instrument may be summed up as follows:—

(a) The maximum sensitivity is about one hundred times that of the photographic type of polarograph. It does not follow, however, that such extreme sensitivities can always be employed, as this will depend on the nature of the solution to be analysed.

(b) The analysis of a solution containing a number of ions can be carried through in a few minutes. This is the case, even when their concentrations differ widely, as a new setting of the sensitivity control may be employed for each ion.

(c) The peaks due to ions reducing at voltages only 30-40 millivolts apart can be resolved.

(d) Staff can readily be trained in its use because the steps are under continuous observation and the effects of changing a control setting are immediately obvious.

(e) Where permanent records are required, these may be secured by photographing a single trace with a suitable oscilloscope camera. A succession of such records may be made on a solution containing several ions with the same advantages as regards speed that were mentioned in (b) above.

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The Characteristics of Some Large-Current Glow-Discharge Tubes

By F. A. Benson*, M.Eng., Ph.D., A.M.I.E.E., M.I.R.E.

The characteristics of certain glow-discharge voltage-regulator tubes have been studied and described by several independent investigators including Kirkpatrick[1], Titterton[2], Cain[3], Clucas and the author[4-7]. Most of the previous work has, however, been concerned with the relatively-low-current tubes, although some 75mA types have received attention. It does not seem to be generally realized that there are some useful glow-discharge tubes available which cover a wide range of current and for which the maximum current is about 200mA. The tube CV1199 (NS2) is one of this type, its specified current range being 30/180mA and its normal running voltage 97·5V. It might be expected that, in view of the large maximum current, the stability of such a tube would be rather poor, but no information appeared to be available to check this.

The author has recently studied the short-time behaviour of 12 new CV1199 tubes and the results obtained are briefly reported below. It will be seen that the stability of this type of tube is reasonably good.

Striking Voltage

Limits: 111-125V
Average value: 116·5V
Specified value: 140V (maximum)
Maximum variation observed for a single tube when striking thirty times at ½ minute intervals: 2·5V.

Running Voltage (for complete current range)

Variations from tube to tube: 88·2-105·7V
Variations for a single tube: 3·4V maximum
2·2V average

Temperature Coefficient of Running Voltage (at tube current of 50mA)

Maximum: $-0{\cdot}28$mV/V/°C
Average: $-0{\cdot}17$mV/V/°C

Initial Drifts (at tube current of 180mA)

Maximum positive drift (percentage of running voltage):	0·18
„ negative „ „ „ „ „	4·18
Average positive „ „ „ „ „	0·06
„ negative „ „ „ „ „	3·48

Time for running voltage to become steady: 21min maximum
18min mean.

One tube was still drifting rapidly at random after 20 minutes' operation.

The value of 4·18 is greater than any previously published figure[4], but might have been expected because of the large current and consequently temperature rise of the tube.

Extinction Voltage

Minimum 88·5V
Maximum 96·0V
Average 91·2V

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A Range of 400c/s and 1 600c/s Transductors for Service Use

By A. G. Milnes*, M.Sc., and C. S. Hudson*, B.Sc., Ph.D., Whit. Sch.

Standardization of transductors for Service use is discussed and details are given of a range of auto-self-excited transductors. These designs are for supply voltages of 13, 25, 50, 115 and 200 volts R.M.S. *and cover the power range up to 85 volt-amperes as single units. In three-phase connexion the power range is extended by a factor of up to three.*

Current amplifications of between 20 and 180 are obtained with the recommended load resistances and each transductor has three equal control windings. The core materials are Mumetal or Permalloy C for the small input designs and H.C.R. or Permalloy F for the power designs.

MAGNETIC amplifiers are being applied increasingly in servo- and regulating-systems in the Services because of their inherent robustness, reliability and circuit convenience. The basis of a magnetic amplifier is a saturable-core reactor, known as a transductor, that corresponds in some respects to the thermionic valve of an electronic amplifier.

A transductor consists essentially of two or more magnetic cores with numerous windings and therefore its construction is not difficult provided the necessary materials and winding machines are available. This need to produce the basic device has, however, tended to confine the use of magnetic amplifiers to a comparatively few people who have acquired the necessary design knowledge and experience. There is no doubt that if transductors were as readily available in the laboratory as are thermionic valves, more engineers would consider their use and would gain some first-hand knowledge of their advantages and limitations.

General agreement is now possible on what constitutes good design practice, although a few years ago this was not so. Therefore differences between designs tend not to be fundamental and some measure of standardization has become practicable.

Details are given later of designs that it is proposed to standardize and make readily available to Service users. It would clearly not be practicable to confine the transductors used in Service applications of magnetic amplifiers to a few standard types, and this is not the intention of the proposed standardization action. But it is hoped that these available designs will be suitable for bread-board versions of amplifiers in trial applications. At a more advanced stage in any project it may be essential to tailor-make the transductor to suit the particular problem. In many instances it is expected that different control input winding arrangements will be required on otherwise standard transductors.

The great majority of magnetic amplifiers in Service applications are designed for supply frequencies of either 400c/s or 1 600c/s. At low supply frequencies, tranductor sizes and time-constants are large and this limits their application at 50c/s.

A Survey of Existing Transductor Designs

An analysis has been made of the transductor designs used in a jet-pipe temperature control amplifier; a voltage and frequency controller for an invertor; a mixing amplifier for a blind landing apparatus; a thermocouple amplifier with a pen-recorder output; and two auto-pilot amplifiers. These magnetic amplifiers contained 14 different transductor designs—neglecting minor differences between input winding ratios.

Of these transductors, six were of auto-self-excited type, seven were of series type with separate self-excitation, and one was of bias type with overall feedback. The present trend in new designs is towards auto-self-excited connexions since the advantages of these types, namely reduced copper loss and lower internal resistance, are now more fully realized than a few years ago.

Nine of the transductor designs were input or early-stage types required for sensitivity and not for an appreciable power output. The supply voltages of these transductors ranged from 10 to 22 volts and all except one had cores of Mumetal type material. Almost without exception, these input transductors were used as pairs in push-pull circuits.

The remaining five transductors were output types designed to work up to their limiting thermal ratings. The supply voltages were 90, 100, 115, 200 and 230 volts and the core materials were Mumetal, H.C.R. and Crystalloy

Of the 14 designs, eight had three or more control windings and in addition bias windings were present in the majority of the designs for trimming purposes. The ratios of control winding to main-winding turns tended to be low and values of between a half and unity were common. This represents good design practice since the time-constant is proportional to this ratio for a transductor of given voltage amplification.

For a series transductor the time-constant is related to the voltage amplification by the equation:

$$\tau = 1/4f\left(\frac{dV_{av}}{dV_c}\right)N_c/N_a \qquad (1)$$

where f is the supply frequency

V_{av} is the gross average load voltage

V_c is the control voltage

N_c is the number of control turns

and N_a is the number of main-winding turns per core.

For the parallel auto-self-excited connexion the number of main-winding turns per core, N_a', is twice that for the equivalent series transductor and the corresponding time-constant equation is:

$$\tau = 1/2f\left(\frac{dV_{av}}{dV_c}\right)N_c/N_a' \qquad (2)$$

The significance of these equations is that for a low time-constant, the voltage amplification of a transductor

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should be obtained, as far as practicable, by positive feed-back action (self-excitation) and not by the use of large numbers of control turns occupying a considerable part of the total winding space.

In the designs examined, the control winding resistances were distributed in value between 8 and 860 ohms with a median value of 40 ohms. This wide variation of input resistance can hardly be provided in a limited range of standard designs.

Toroids versus Laminations

A range of special laminations, E-shaped with double-width yoke for over-lap assembly, has been developed for transductor cores of high permeability. These laminations have saturation characteristics that are only a little inferior to those of strip toroidal cores. However toroidal construction makes more effective use of the material and is usually somewhat less in weight than the equivalent transductor with lamination cores, since the toroidal construction is a compact shape well suited for canning. Care and experience are needed for toroidal winding if the results are to be satisfactory.

Toroidal core sizes for the transductors have been taken from the range shown in Table 1.

TABLE 1
A Range of Toroidal Core Sizes suitable for Transductors

CORE NO.	INTERNAL DIAMETER (inch)	EXTERNAL DIAMETER (inch)	STRIP WIDTH (inch)	MAGNETIC CROSS-SECTL. AREA (sq. cm.)
13/2	13/16	1 1/16	1/8	0·10
/4			1/4	0·20
/6			3/8	0·30
15/4	15/16	1 5/16	1/4	0·30
/6			3/8	0·45
/8			1/2	0·61
16/4	1	1½	1/4	0·40
/6			3/8	0·61
/8			1/2	0·81
20/3	1¼	1⅞	3/16	0·38
/6			3/8	0·76
/9			9/16	1·13
24/3	1½	2¼	3/16	0·45
/6			3/8	0·91
/9			9/16	1·37
28/3	1¾	2⅝	3/16	0·53
/6			3/8	1·06
/9			9/16	1·58

If the power into the load is to be large for a given volume of transductor, analysis shows that tall transductors with cores of large strip width are preferable. A further advantage of these proportions is that the plan-area is small and this is usually convenient. On the other hand, the maximum height of transductor that can be wound is limited (say to 2in.) by the dimensions of the winding machine shuttle. The ratio of height to plan dimensions is also limited by the difficulty of press-drawing exceptionally tall cans in steel.

Auto-self-excited Transductors

Output stage transductors should be designed with ratings corresponding to the thermal limits so that the maximum possible powers are obtained from given core sizes. At the full output condition copper losses are responsible for the temperature rise in a transductor. (The iron losses are usually small and become almost zero at the full output condition, since the flux changes are then small.) In this respect the auto-self-excited transductor connexion is advantageous, since its copper losses are only one-half of those of the equivalent series transductor, because the load current flows in alternate parallel paths in each half-cycle instead of continuously in each winding.

A further advantage of the auto-self-excited connexion is that the internal resistance is lower, because the main windings are not all in series, and this tends to permit a slightly increased output voltage. The amplifications of the auto- and separately-self-excited connexions are identical in theory and usually nearly so in practice. It follows that for the power transductors of the standard range, the auto-self-excited connexion is the natural choice.

With input type transductors the thermal limits are rarely reached because the currents are small and therefore either auto- or separately-self-excited transductors may be used. In the standard range it was originally intended to have

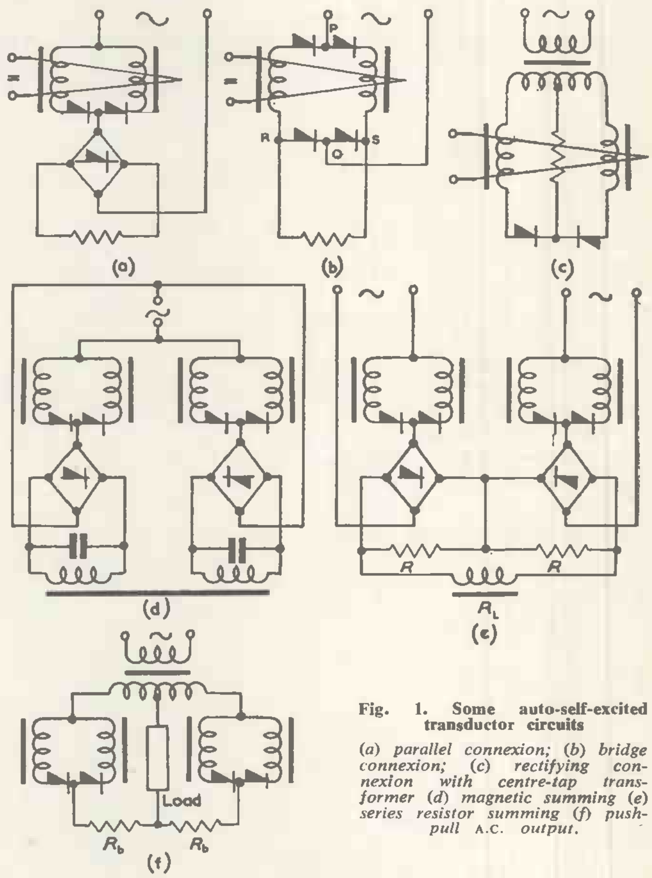

Fig. 1. Some auto-self-excited transductor circuits

(a) parallel connexion; (b) bridge connexion; (c) rectifying connexion with centre-tap transformer (d) magnetic summing (e) series resistor summing (f) push-pull A.C. *output.*

the main element windings in two parts so that either of these connexions could be used at choice. However, this proposal was dropped in favour of simple auto-self-excited types because of the practical difficulty in the provision of the necessary numbers of terminal seals on cans of small plan area. For the same reason it was decided not to provide special gain adjustment windings on the auto-self-excited designs. It is intended that any gain adjustment of these types shall be by change of load resistance; by shunt resistor across the auto-self-excitation rectifiers; or by use of a spare control winding for overall positive or negative feedback.

Some auto-self-excited transductor circuits are shown in Fig. 1 for single units and for push-pull pairs. Of the single-unit circuits (a), (b) and (c), the parallel connexion has the advantage that the reverse voltages across the auto-self-excitation rectifiers are low. For this circuit the mean reverse voltage depends on the transductor current and its winding and rectifier forward resistances, whereas in the bridge and rectifying connexions the reverse voltages are more directly related to the supply voltage. The reverse voltages should not be greater than 10 volts peak for a double-voltage (25V R.M.S.) selenium rectifier disk if the leakage currents of the auto-self-excitation rectifiers are to be small and to have no influence on the transductor sensitivity under all operating conditions of temperature. With perfect rectifiers, and ideal core properties, the theoretical sensitivities of the three arrangements (a), (b) and (c) are identical.

Magnetic summing, as shown in Fig. 1(d), may be used for a push-pull output if the load apparatus has twin input coils. Smoothing capacitors across the rectifier bridge outputs may be desirable to prevent interaction effects. Where such capacitors are used a series damping resistance may be necessary to prevent resonance. If the load has a centre-tapped input winding, the transductors must be supplied from a transformer with separate secondary windings. For a single input winding to the load, the series resistor summing circuit shown in Fig. 1(e) may be used, although the presence of the resistors R causes loss of sensitivity and of power. The load resistance to summing resistor ratio should be between 0·5 and unity. An alternative push-pull arangement, not illustrated, is known as the "T" summing circuit.

If an alternating current output is required to depend in magnitude on the control current and to reverse in phase with change in signal direction, the push-pull arrangement of Fig. 1(f) may be used. To prevent an excessive current circulation between the two transductors it may be necessary to bias the cores to a low standing-current condition and to insert ballast resistors R_b. From an A.C. output that is magnitude and phase sensitive, it is, of course, possible to obtain a push-pull D.C. output by the use of a phase-sensitive rectifier or demodulator arrangement. This is sometimes found instead of the more usual summing arrangements.

General Notes on the Designs

The transductors of the range are divided into input and power types. The input types are designed for supply voltages of 13, 25 and 50 volts R.M.S. and have Mumetal or Permalloy C cores worked at a flux density of 5 000 gauss peak. For the power transductors the voltages are 115 and 200V and the cores are H.C.R. or Permalloy F with a design flux density of 11 000 gauss peak. For 400c/s operation the strip thickness of the toroidal cores is 0·004in., and this is acceptable for 1 600c/s operation, although a thickness of 0·002in. would be preferable.

Each transductor has three control windings with turns usually equal to one-half the number of main-winding turns per core. In addition there is a bias winding with a quarter of the turns of a control winding.

In the design work care was taken that the main-winding resistance was suitably small compared with the load resistance in order to maintain reasonable efficiency and a high power output from the given core size. The proportion of the winding space allocated to the main-winding was as much as 2/3rd of the total: the design procedure was to assess the requirements of the control windings and then use the rest of the available space for the main-windings.

Current amplifications of between 20 and 100 were aimed at by designing the transductors to have suitable reactances in comparison with their load resistances. For an auto-self-excited transductor the theoretical equation for the current amplification is:

$$dI_a/dI_c = 1/\pi\ (X/R)\ N_c/N_a' \quad \ldots\ldots\ (3)$$

where X is the unsaturated reactance of one element and R is the total main circuit resistance including rectifier and load resistances.

The transductor power amplification, neglecting internal and rectifier resistances, is:

$$P = \left(\frac{dI_a}{dI_c}\right)^2 R/R_c \quad \ldots\ldots\ (4)$$

Therefore from Equations (2), (3) and (4), the ratio of power amplification to time-constant is:

$$\frac{P}{\tau} = \frac{2}{\pi} f \frac{X}{R} \quad \ldots\ldots\ (5)$$

Thus the requirement of a high reactance to resistance ratio for large current amplification (see Equation (3)) is also a criterion for the ratio of power amplification to time-constant.

The reactance for a given transductor voltage may be increased by a reduction of the core area and a corresponding increase in the number of main winding turns. Such a change may require an increase in the proportion of the bobbin space allocated to the main windings. Reduction of the load resistance, as a means of increasing the X/R ratio, involves an increase in current and an increase in the transductor copper loss unless the winding resistance is reduced by a further allocation of space to the main windings.

One consequence of these design proportions is that with normal control winding resistances the ratio of control turns to main winding turns tends to be small, as discussed previously, although this ratio in itself is not an important criterion of transductor performance. For a given space allocated to the control winding it is possible to increase the number of control turns with corresponding change of control resistance, to match the signal source impedance, without any first order influence on the power amplification or time-constant.

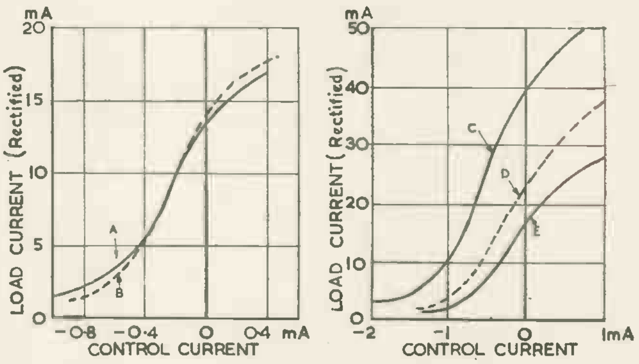

Fig. 2. Control characteristics for some input type designs

In the design calculations for the power types, the copper losses were balanced against an estimated safe rate of dissipation per square inch of surface area for a 40°C average temperature rise of the main-windings. Some attempt was made to adjust the designs so that there was no serious mis-matching between the rated transductor output currents and the normal current ratings of standard rectifier disks.

Input Type Designs

Details of the input type designs are given in Table 2 and some characteristics for certain of these designs are shown in Fig. 2.

Transductor A is a 13 volt 400c/s design which may also be used at 50 volts 1 600c/s. Thus designs E and A are identical. The load resistance for E is large because this design may be used at 1 600c/s with germanium diode rectifiers which are of large forward resistance. The characteristic for E in Fig. 2 is for a D.C. load resistance of 1 000 ohms and selenium rectifiers. Design D has half the core area of E and is suitable for 25 volts 1 600c/s operation with a load of 330 ohms.

Two designs at 25 volts 400c/s are given. Design B has twice the reactance of C and is intended for operation with a larger load resistance. The recommended load resistances are given as guides only and smaller values may be used without overheating of the transductors. The recommended values quoted are high in order that variation of rectifier forward resistance and of main-winding resistance shall not seriously disturb the balance of push-pull pairs. The current amplifications quoted in column (vi) of Table 2 are for single transductors with the recommended load resistances. If the three control windings are connected in series the gain will be correspondingly increased. The resistances in column (viii) are for single control windings and must be doubled for push-pull transductor pairs. For matching to low impedance control sources the three control windings may be connected in parallel.

The transductors are canned in push-pull pairs and the cans oil filled. Eighteen terminals are needed since there will be four main-windings, three control windings, and two bias windings for a push-pull pair.

Power Type Designs

Details of the proposed range of power transductors are given in Table 3. There are five designs for 400c/s operation—two of them being three-phase six-core transductors—and three designs for use at 1 600c/s. The three-phase designs are included because in aircraft equipment the magnetic amplifier supply is usually a three-phase 400c/s invertor and a ruling exists that the phase loadings should be balanced. Of the various possible three-phase transductor connexions, the auto-self-excited parallel arrangement shown in Fig. 3 is one of the most effective. Weight for weight its performance is as good as that of a single-phase connexion.

The two-core designs in Table 3 may be used either as single-phase transductors or as the above three-phase arrangement by the use of a two-core unit in each line. The upper limit of power handled by the proposed range in this way will be of the order of $\frac{1}{4}$ kilowatt. The estimates of performance given in Table 3 are tentative and may require adjustment when the performance of each design has been fully evaluated in practice.

Because of the difficult supply position of thin strip nickel-iron materials, some consideration was given to the possible use of grain-oriented silicon iron in place of H.C.R. or Permalloy F. A readily available grain-

TABLE 2

Proposed Input Type Auto-Self-Excited Transductors

DESIGN	SUPPLY		MAIN-WINDING RESISTANCE APPROX. (ohms)	RECOMMENDED LOAD RESISTOR (ohms)	MINIMUM CURRENT AMPLIFICATION PER CONTROL WINDING	CONTROL WINDING FOR SINGLE TRANSDUCTOR (3 per transductor)		CAN DIMENSIONS FOR A PAIR OF TRANSDUCTORS	
	VOLTAGE R.M.S.	FREQUENCY (c/s)				TURNS	RESISTANCE APPROX. (ohms)	PLAN (inches)	HEIGHT (inches)
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
A	13	400	40	330	20	450	40	1 $\frac{23}{32}$ square	3$\frac{1}{4}$
B	25	400	60	680	20	600	50	1 $\frac{31}{32}$,,	3$\frac{1}{2}$
C	25	400	20	220	30	370	50	2 $\frac{9}{32}$,,	3$\frac{3}{4}$
D	25	1 600	30	330	20	450	40	1 $\frac{23}{32}$,,	2$\frac{7}{8}$
E	50	1 600	40	680/1 000	30/20	450	40	1 $\frac{23}{32}$,,	3$\frac{1}{4}$
F	50	400	40	680	30	600	70	2 $\frac{9}{32}$,,	3$\frac{3}{4}$

TABLE 3

Proposed Power Type Auto-Self-Excited Transductors

DESIGN	SUPPLY		MAIN WINDING RESISTANCE (APPROX.) (ohms)	CONTROL WINDING (3 per transductor)		PERFORMANCE ESTIMATE				CAN DIMENSIONS FOR SINGLE TRANSDUCTOR		
	VOLTAGE R.M.S.	FREQUENCY (c/s)		TURNS	RESISTANCE APPROX. (ohms)	RECOMMENDED LOAD RESISTANCE (ohms)	LINEAR OUTPUT CURRENT (amps)	MEAN VOLT-AMPS OUTPUT LINEAR	MINIMUM CURRENT AMPLIFICATION PER CONTROL WINDING	PLAN (inches)	HEIGHT (inches)	TERMINALS
*G**	115V line	400	16	580	100	220	0·44	40	120	2 $\frac{9}{16}$ square	3 $\frac{9}{16}$	20
H	115	400	13	440	45	150	0·41	25	90	2 $\frac{9}{32}$,,	3$\frac{1}{2}$	12
I	200	400	14	600	70	220	0·48	50	110	2 $\frac{9}{16}$,,	3 $\frac{9}{16}$	12
J	200	400	8	500	100	150	0·75	85	120	3 $\frac{5}{16}$,,	4 $\frac{1}{16}$	12
K	115	1 600	10	430	50	150	0·32	15	50	1 $\frac{23}{32}$,,	2$\frac{7}{8}$	12
L	200	1 600	11	430	40	270	0·4	40	60	1 $\frac{31}{32}$,,	3 $\frac{5}{16}$	12
M	200	1 600	5	420	60	150	0·66	70	90	2 $\frac{9}{32}$,,	3$\frac{1}{2}$	12
*N**	200V line	400	11	700	110	250	0·70	120	180	3 $\frac{5}{16}$,,	4 $\frac{1}{16}$	20

** Three-phase transductors with six cores.*

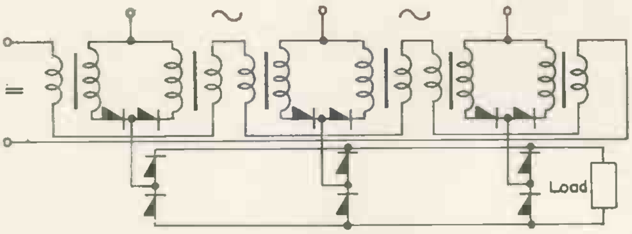

Fig. 3. Three phase auto self-excited parallel transductor with separate load rectifier bridge

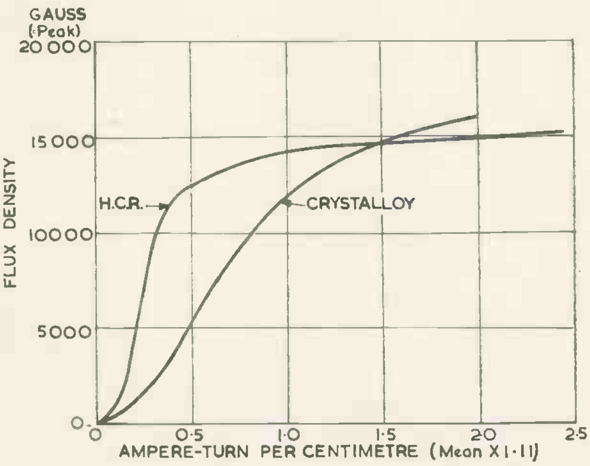

Fig. 4. Magnetization curves for H.C.R. and Crystalloy at 400c/s

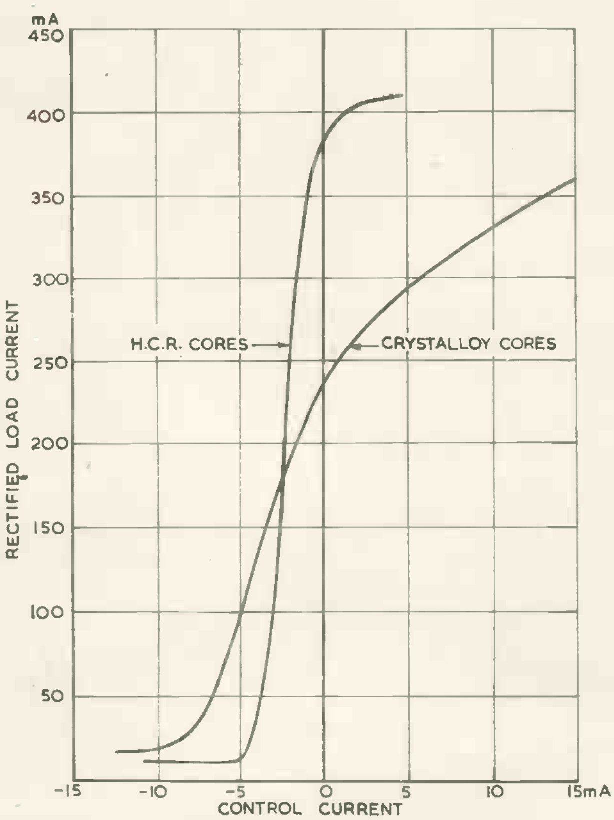

Fig. 5. Characteristics for design H with H.C.R. and Crystalloy cores
115V 400c/s; load 150 Ω

oriented silicon iron produced in this country is Crystalloy and this was used for comparative tests with H.C.R. The 400c/s magnetization curves of the two materials (H.C.R. sample 0·002in. thick, and Crystalloy 0·005in. thick) are shown in Fig. 4 and the corresponding transductor characteristics are given in Fig. 5. The useful power output for the silicon iron cores is about half of that for the H.C.R. cores and the current amplification about a third. This rules out the use of grain-oriented silicon iron except in applications where the highest standards of performance are not required.

The magnetization properties shown in Fig. 4 do not represent the best that can be obtained with these materials, which are under continuous development. For example, grain-oriented silicon iron has been produced in the U.S.A. with properties at least as good as the best curve in Fig. 4. There have, of course, been counterbalancing improvements in the nickel-iron materials. In such high grade materials there are inevitably certain batch to batch variations in properties, and the transductor designer must take care to specify the magnetization properties required.

The can sizes (based on standard C-core type cans) for the power transductors are given in Table 3. This type of canning provides excellent humidity and tropical exposure protection, but almost doubles the weight of each transductor. Consideration is to be given to other, possibly lighter, methods of protection in the near future.

Conclusion

The transductors that have been described are to be made available with fairly complete performance information and it is believed that they will be found convenient in a wide range of magnetic amplifier applications in the Services and in industry. Some further work is in hand on a range of 50c/s transductors.

Acknowledgments

The authors have had the valued advice and assistance of Elliott Bros. (London) Ltd. and Parmeko Ltd. during the preparation of the range.

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SARAH

SARAH—Search And Rescue And Homing—is a miniature radio beacon designed and produced by Ultra Electric Ltd. It weighs 20oz and measures 7in. × 1in. × 1in. This, together with a battery unit weighing 32oz is attached to the crash survivor's life jacket. The peak output power of the beacon is 16 watts which gives a maximum range of 66 miles to a searching aircraft at 10 000 feet. The signal can thus be picked up by an aircraft within an area of 10 000 square miles. Surface ships fitted with the 24lb Sarah receiving equipment can receive the beacon's signals at a range of 6 miles. In the searching craft the signals are displayed on a cathode-ray tube. A two-way speech unit (illustrated) weighing 12 oz can also be incorporated to facilitate the final stages of a rescue. To initiate transmission the survivor has only to pull a small ring which releases a 31in flexible aerial and switches on.

An Electronic Trigger Mechanism as a Diagnostic Aid in Electro-Encephalography

By M. G. T. Hewlett*, M.B.E.

When patients are undergoing electro-encephalographic examination, the application of photic stimuli is often of great value in encouraging the brain to produce electrical discharges which may be characteristic or diagnostic of a particular disorder.

A method is described of locking such stimuli to selected frequency components present in the complex electrical signals from the brain.

This process often results in diagnostic patterns in the electro-encephalogram where other methods have failed or have produced only indefinite changes.

PRESENT-DAY electro-encephalographic technique usually involves the simultaneous recording of the potential differences arising between six or eight pairs of electrodes spaced on the surface of the subject's scalp. The complex electrical signals arising in the brain are normally displayed as six or eight "time/voltage" graphs on a continuously moving paper strip. The increasingly available data upon the departures from normal of these tracings has enabled great advances to be made in the diagnosis and treatment of a variety of neurological and psychiatric disorders.

It is known, however, that recordings taken under passive resting conditions may fail to show up specific patterns in the E.E.G. tracings which are associated with some of these disorders, and in almost all E.E.G. centres various stimulating procedures are applied to patients undergoing examination.

One of the most valuable procedures is known as photic stimulation[1], where a brilliant flashing light is placed close to the patient's eyes. The flashing rate is varied both at random and in various harmonic sequences. Frequently, abnormalities are evoked in the E.E.G. record which would not otherwise have been seen and which may aid the diagnosis of the particular disorder under investigation. An extension of this method, often called "triggering"[2,3], is where the lamp flashes are locked either to dominant peaks in the complex E.E.G. signal from certain parts of the brain, or to specific frequency components selected from the complex signal.

If arrangements are made to vary the instant of flash with respect to the start of each cycle of a selected frequency and, in some cases, if flashes are evoked at harmonics or sub-harmonics of the selected component, abnormal E.E.G. patterns may be produced which would not have been seen using free or "unlocked" photic stimulation.

A flexible and accurately controlled trigger unit was built in 1950 and was given extended clinical trials[4,5]. Its success as a diagnostic aid in routine use in a busy electro-encephalographic department was such that a new and improved version has now been produced. The ensuing paper deals with the design of this second or MKII version. Its main advantages over the original instrument lie in its greater flexibility, compactness and stability.

Description of System

A block diagram is shown in Fig. 1 and a full circuit diagram in Fig. 2.

The complex output signal from any selected E.E.G. channel is fed, via a pre-amplifier, into a resonant amplifier.

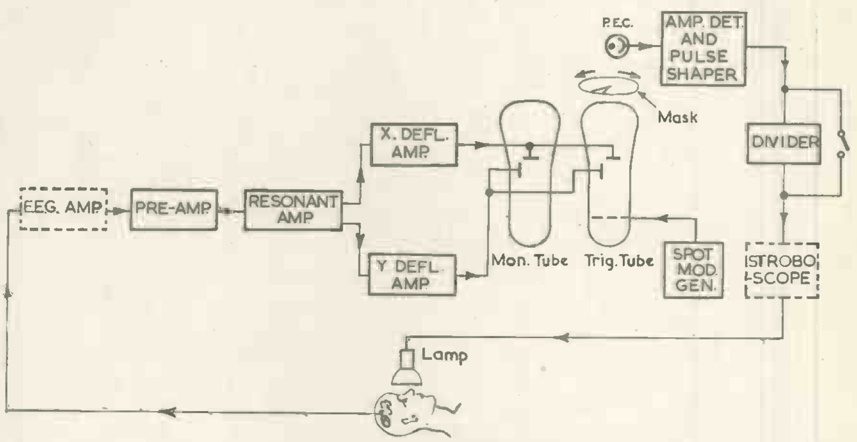

Fig. 1. Block diagram of system.
Units shown dotted are external to the trigger mechanism described.

The latter is tuneable between 2c/s and 40c/s in one range. Two sinusoidal outputs, in quadrature at the selected frequency, are available from the resonant amplifier and these are fed to the X and Y deflector plates of both a monitor and a trigger cathode-ray tube. The resulting circular trace on the trigger C.R.T. screen is viewed by a multiplier photocell mounted coaxially with the tube. An interposed circular mask is pierced by any suitable combination of slits (usually one or more equi-spaced radial slits) and a trigger pulse is generated in the photocell each time the "spot" on the C.R.T. screen passes a slit in the mask. Thus trigger pulses at the fundamental or at any harmonic of the selected frequency can be generated, and phase adjustment between the selected frequency and the pulse can be made by rotating the mask.

The trigger pulses are amplified, shaped and passed either directly to a Scophony high-power stroboscope or via a divider circuit which selects either every alternate, third or fourth pulse. This provides flashes at the 2^{nd}, 3^{rd} or 4^{th} sub-harmonic of the selected frequency if required.

* *Formerly Bristol Mental Hospitals, now E.M.I. Engineering Development Ltd.*

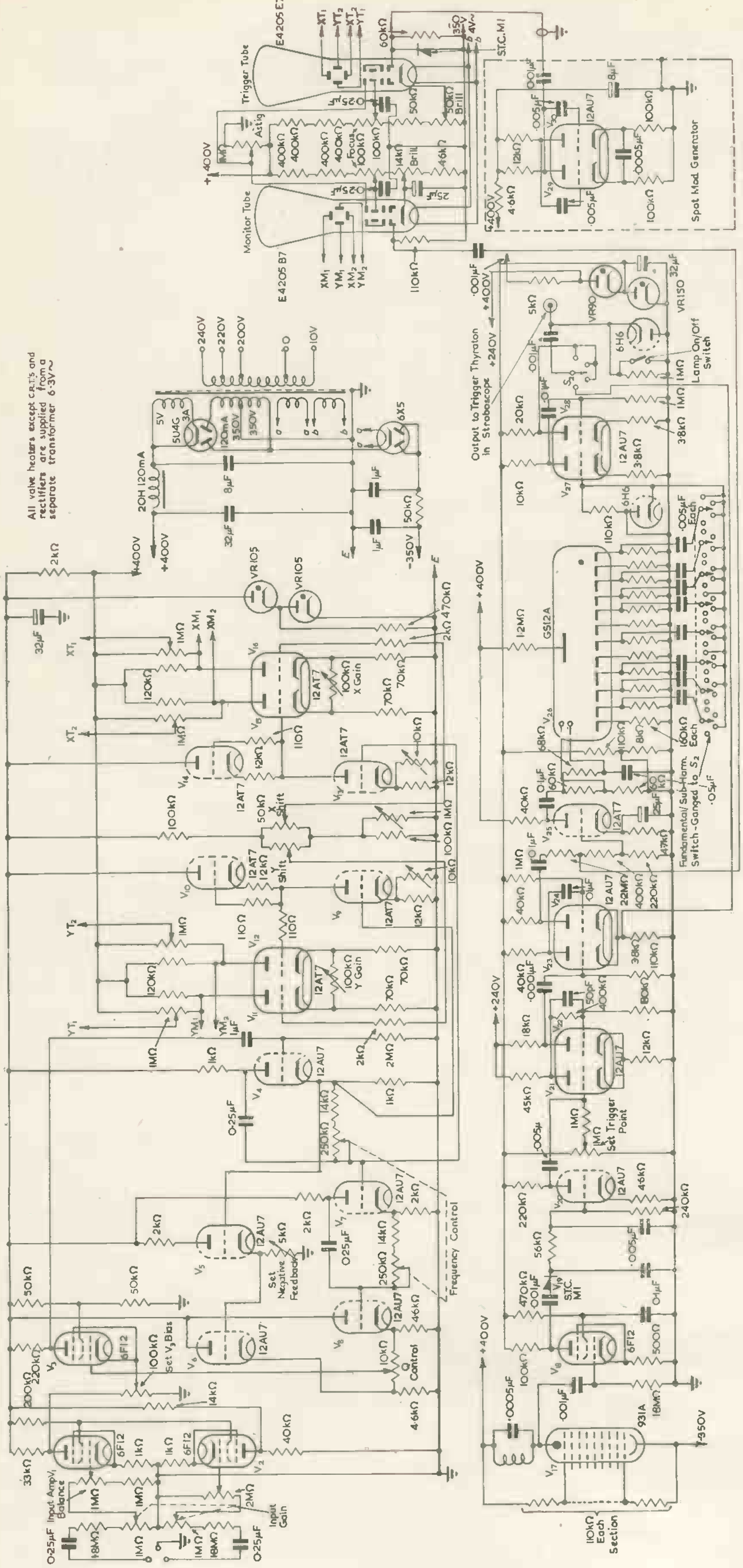

Fig. 2. Complete circuit diagram.

The Circuit

Pre-amplifier and Resonant Amplifier

The output from any selected E.E.G. channel is fed into a pre-amplifier, V_1V_2, designed for push-pull input and single-ended output. The balance of this amplifier can be adjusted to give a discrimination of at least 500:1 against in-phase signals at its input. The pre-amplifier is directly coupled into the cathode circuit of an amplifier V_3, which forms the feedback loop for a phase shift system comprising V_4, V_5, V_6, V_7, V_8[6]. The direct coupling eliminates phase shifts throughout the operating frequency band (2-40c/s) and provides bias for the loop amplifier V_3, itself directly coupled to the phase shift system, without resort to too high a value of cathode bias resistor and its attendant negative current feedback. The phase shift system, V_4, V_5, V_6, V_7, V_8, is arranged to give a phase shift of 180° at any selected frequency between 2c/s and 40c/s. The phase shift is in two stages of 90° each and is performed by the resistance-capacitance network connected between the anodes and cathodes of both V_4 and V_7. In each of the stages V_4 and V_7 a 90° phase shift occurs at a frequency where the capacitative reactance and the resistance of the networks are equal. The loop amplifier V_3 has an aperiodic phase shift of 180° and is connected between the output and input of the phase shift system. Thus a feedback system is formed and a loop gain sufficient to overcome the network losses would cause the circuit to oscillate at the frequency which satisfied the condition for a total phase shift of 360°. In this application the loop gain is reduced below the oscillation point and the circuit behaves as a resonant amplifier. Tuning in one band between 2 and 40c/s is effected by using an accurately ganged twin potentiometer for the resistive elements of the two-phase shift networks. The output from the phase shifter is fed to the aperiodic loop amplifier via a cathode-followed stage V_8, this increases the load impedance presented to the preceding stage and provides an output at a low D.C. level with respect to ground so that direct coupling can conveniently be used between V_8 and V_3. The two stages V_5 and V_6 are connected in parallel with the phase shift chain but have an aperiodic phase shift of zero degrees and thus form a negative feedback system at the resonant frequency. This use of negative feedback stabilizes the gain of the whole system throughout the frequency band and also provides a method of varying the loop gain by changing the ratio of positive to negative feedback at the potentiometer marked "Q" control.

An idea of the stability of the system may be gained by setting this potentiometer ("Q" control) so that the circuit just oscillates. Despite the absence of non-linear stabilizing elements and of any great degree of H.T. stabilization, the circuit will oscillate stably at a sub-maximal level for considerable periods at frequencies as low as 2c/s. As has been stated, the normal adjustment of the circuit is such that oscillation does not occur and the circuit behaves as a resonant amplifier. Any frequency component between 2 and 40c/s may thus be selected from the complex E.E.G. signal which is fed into the resonant amplifier via the pre-amplifier. Since the selected component undergoes a phase shift which includes two changes of 90° each, there are two outputs in quadrature available. These are taken from the cathode of V_4 and the grid of V_7 respectively, and are fed via direct couplings to two separate D.C. deflexion amplifiers V_{9-12} and V_{13-16}. The two outputs from these amplifiers are fed to the X and Y deflector plates of a 2½ inch monitor C.R.T. and a 2½ inch trigger C.R.T.

The Deflexion Amplifiers and Cathode-Ray Tubes

The deflexion amplifiers each consist of a triode cascode stage followed by a long tailed pair of triodes. Their gains can be individually controlled by variable cathode coupling resistors in their output stages and X and Y shifts for the cathode-ray tube beam are obtained by varying the D.C. level of the "free" grid of each long tailed pair.

The bias resistors in the cathode leads of V_9 and V_{13} were adjusted experimentally to ensure minimum amplifier drift under conditions of varying heater voltage[7].

A sinusoidal signal from the resonant amplifier produces a circular trace on both cathode-ray tube screens, the diameter of the trace being proportional to the peak amplitude of the signal. In normal use, since the selectivity of the resonant amplifier is finite, the traces on the cathode-ray tubes can never be perfectly circular when the input is a complex signal. However, the departure from circularity is not usually serious and where difficulty due to a high amplitude component close to the resonant frequency is experienced, the selectivity of the resonant amplifier may be increased. Increased selectivity, of course, increases the fly-wheel effect of the resonant amplifier.

The trigger cathode-ray tube has auxiliary pre-set shift controls connected across the anode loads of the deflexion amplifier output valves in order to compensate for differences in beam alignment between the two tubes. Since these controls also alter the signal level to the X and Y plates of the trigger tube independently, correct adjustment ensures good beam centering and allows for differences in the deflexion sensitivities between the two tubes. This method of correction results in a slightly smaller deflexion sensitivity on the trigger tube, but this is of no consequence. When adjusting the trace centering and circularity on the trigger tube, the auxiliary shift potentiometers are gradually reduced in turn from the maximum gain position until the correct result is secured; with monitor and trigger tubes using identical electron guns very little adjustment is necessary. The monitor tube has a green medium persistence screen and the trigger tube a short persistence blue screen.

The Trigger System, Photocell Amplifier and Pulse-Forming Circuit

A circular opaque mask clipped into a perspex carrier and a multiplier photocell are mounted in a light-tight housing in front of the trigger tube. The mask is between the tube and the cell. The mask can be rotated by an external knob and is normally pierced by one or more equi-spaced radial slits approximately 1mm wide and extending from a radius of 5mm to the edge of the mask.

When a component selected from the complex E.E.G. signal is causing rotation of the "spot" on the cathode-ray tube screen, a pulse is generated in the photocell each time the spot passes a slit in the mask. Where one radial slit is used the pulses occur at the selected frequency, with two slits spaced at 180° the pulses are at the 2nd harmonic of the selected frequency and so on. Continuous phase adjustment between these pulses and the selected component may be secured by rotation of the mask.

The mask carrier and mounting are arranged so that the mask and carrier may be ejected by raising the rotation control knob.

In order to amplify and shape the pulses from the photocell, it has been found convenient to modulate the beam intensity of the trigger tube with a comparatively high frequency square wave signal (33kc/s) generated by a cathode coupled multivibrator V_{29-30}. The photocell anode load is a resonant circuit tuned to the modulation frequency. Thus a great deal of trouble associated with the high inherent noise level of the cell and hum injected into the amplifier from the photocell supply circuits is avoided. The high frequency signal from the photocell is amplified by V_{18} and rectified by V_{19}. The carrier frequency is filtered out and further amplification of the pulse is effected by V_{20}. Negative pulses appearing at the anode of V_{20} are used to trigger a Schmidt trigger circuit which is normally biased so that V_{21} is conducting and V_{22} cut off. The Schmidt trigger circuit is adjusted so that residual noise fluctuations never reach a high enough level to trigger it and the square wave output at the anode of V_{22} is free from noise and of a pulse width determined by the rate at which the cathode-ray tube spot traverses the mask slit. This pulse is differentiated and applied to a delay multivibrator V_{23}, V_{24}. The multivibrator triggers only on positive pulses and therefore responds only to the leading edge of the pulse from the Schmidt trigger. When trigger pulses at the fundamental selected frequency are in use, the pulses are taken from the cathodes of the delay multivibrator and applied directly to the lamp firing circuit. The pulses are clipped by a diode to remove overswing due to *RC* couplings in the circuit.

When trigger pulses are required at frequencies which are sub-harmonically related to the selected frequency component, the output of the delay multivibrator is routed to an inverting amplifier V_{25} and is applied via a suitable network to the guide electrodes of a 12 cathode Dekatron counter tube. The network provides a direct and a delayed rise pulse to this tube to ensure correct glow transfer from one cathode to the next[8].

A switch selects either alternate, every third or every fourth cathode of the counter tube and connects them in parallel to the input of the isolating amplifier V_{27}, V_{28}. Thus, in addition to fundamental working, output pulses at the 2nd, 3rd or 4th sub-harmonic of the selected frequency component are available. As before, phase adjustment by mask rotation may be made. Output pulses at harmonics of the selected component require a mask with more than one slit.

Whatever combination is in use, the output pulses from V_{28} are positive and these are taken to the grid of the lamp triggering thyratron in a Scophony high power stroboscope. The "contactor" jack on this instrument was rewired and used for the pulse input from the trigger unit.

Power Supply Circuits

The cathode-ray tube supply circuits are conventional except that the high voltage supply which feeds both cathode-ray tubes and the multiplier photocell is half above and half below ground. This simplifies insulation problems and the supply which is negative to ground is obtained from a half-wave rectifier connected to one end of the H.T. winding on the mains transformer. The unbalance on this winding may be neglected since the negative supply current is less than 2·5mA. The H.T. and L.T. supplies for the other circuits are conventional. The H.T. output from the power pack is approximately 115mA at 380V. Two VR105 stabilizers are used to give a supply at 210V for the pre-amplifier, resonant amplifier and deflexion amplifiers. Two further stabilizers are used to give supplies at 240V and 150V for auxiliary circuits.

AUXILIARY CIRCUITS

The spot modulating square wave for the trigger cathode-ray tube is obtained from a cathode coupled multivibrator V_{29}, V_{30}. The whole of this circuit is enclosed in a screening box to prevent interaction with other circuits. An output square wave of 1/1 mark/space ratio with a peak amplitude of 50V and a frequency of 33kc/s is applied to the grid of the trigger cathode-ray tube. An M1 rectifier is used at the cathode-ray tube grid to give D.C. level restoration.

In order to maintain visual monitoring of the circuit conditions, the lamp flash is indicated on the trace on the monitor cathode-ray tube as a short gap. This gap is obtained by feeding a negative pulse from the cathode circuit of V_{23-4} to the grid of the monitor cathode-ray tube.

Operation

The instrument is at present used in conjunction with an Ediswan MkII 8 channel electroencephalograph and an Ediswan MkII low frequency wave analyser. The trigger mechanism is automatically switched to whichever channel is being analysed. Usually a dominant frequency component in the alpha band (8-13c/s) is selected and the trigger gain and Q are adjusted to give a satisfactory circular trace. Triggering is started and phase adjustment is made as required to build up cerebral activity at the frequency selected. In turn, other frequencies are tried and rapid frequency changes are made up and down the band in various harmonic series. Harmonic and sub-harmonic flashes may be tried, particularly if the action of triggering produces interesting changes in the E.E.G. In many instances, patterns diagnostic of epilepsy may be evoked by this method where other procedures have failed or have produced only indefinite E.E.G. changes.

Acknowledgments

Thanks are due to Dr. E. C. Turton, Mrs. A. B. Spear, Miss J. P. Rockett and Miss V. D. Heaven for their patience in using the instrument as a routine on all patients undergoing E.E.G. examination, and to Dr. R. E. Hemphill for permission to publish this paper.

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A Very Wide Band Dummy Load

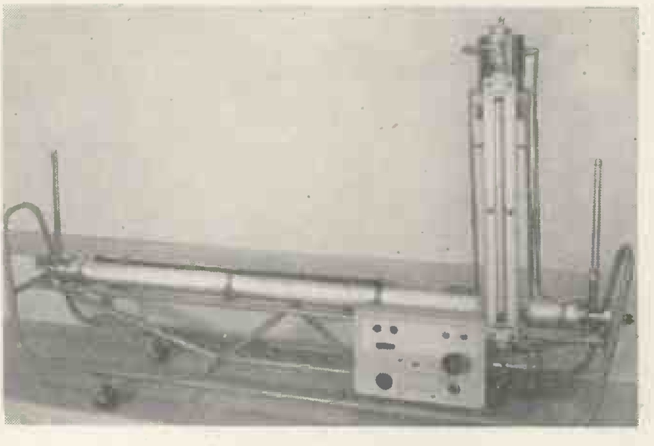
IN the development of U.H.F. communications and other electronic equipment, there is an increasing need for improved methods of measuring power over a wide frequency range. This form of measurement presents a difficult problem since the method adopted depends not only on the magnitude of the power but also on the particular frequency at which it is to be measured. In such measurements it is also necessary to ensure correct matching to the source and at the same time to avoid mismatches due to the introduction of the power measuring device.

Many of the limitations set by previous methods of R.F. power measurement have been overcome in the development, at the Mullard Research Laboratories, of a very wide band dummy load. This device has been specifically designed as a high-precision measuring instrument for use in conjunction with a development project in the field of U.H.F. communications. Essentially it takes the form of a short coaxial line, filled with carbon tetrachloride, the inner conductor consisting of a very thin gold film on a glass rod. This presents a purely resistive input impedance of 75 ohms at frequencies from 100Mc/s upwards. The power absorbed by the dummy load is measured in terms of the temperature rise in the carbon tetrachloride cooling liquid between inlet and outlet of the dummy load. The rate of circulation of the cooling liquid is regulated with the aid of a flowmeter incorporated in the instrument.

The maximum power handling capacity of this new dummy load is 600 watts, and the power measurement accuracy is $\pm 1{\cdot}5$ watts or ± 2 per cent whichever is the greater. These figures refer to specific conditions set by the particular application in question. The principle on which the dummy load is based, however, is fundamental and it could be readily applied to other conditions, should the occasion arise.

In the design of this form of dummy load, the comparatively high resistance of the inner conductor results not so much from the specific resistance of the metallic film but from its extreme thinness. This results in a very small cross sectional area. The actual surface area available for cooling however, remains quite large, and the power handling capacity of a thin film resistor under these conditions is unusually high. In order to obtain uniform power dissipation throughout the whole length of the inner conductor, the resistance of the gold film is graded. This is achieved by constructing the inner conductor from a number of carefully prepared sections of varying resistance.

It is essential in a wide band dummy load to avoid any kind of discontinuity, either physically, due to a change in diameter; or electrically, due to a change in the properties of the dielectric media—the cooling liquid—filling the coaxial line. In this development it was therefore necessary to find a cooling liquid and a solid dielectric having as far as possible identical dielectric properties. Carbon tetrachloride and polyethylene were found to satisfy these conditions. Carbon tetrachloride as a coolant offers additional advantages in that its specific heat is 0·2, or 1/5th of that of water. This means that any dummy load using carbon tetrachloride will be five times more sensitive than a load using water as the cooling medium.



The wide band dummy load

Simplified Solution of Phase-Shift Networks

By R. D. Trigg*, A.M.Brit.I.R.E.

It is intended to show a method of solving complex networks with particular application to phase-shift circuits employed in RC oscillators and selective amplifiers.

With certain provisions, the operator j is eliminated from the treatment until the final stages of the analysis is reached. The method considerably reduces the complexity normally met with in such analyses, and leads to greater accuracy.

Three examples are given relating to the single-phase pentode oscillator, Wien bridge oscillator, and the parallel-T null network.

IN the solution of *CR* ladder, Wien bridge, parallel-*T*, and other networks for phase-shift oscillators and selective amplifiers, the employment of the conventional j-notation throughout the treatment, can involve considerable labour and complexity. It is possible, however, and perhaps not generally realized, that the operator j can be omitted from almost the entire solution, and need only be employed at the final conclusion, where phase and modulus components are solved.

The method is to assume arbitrarily that all the reactive elements in the phase-shift network possess only the properties of pure resistances, and temporarily suspend their complexity from the algebraic treatment until the final stages are reached. As will be seen, this considerably simplifies the analysis, and it is to be noted, that the method can be extended to the solution of any complex network.

A comprehensive treatment of *CR* ladder networks for phase-shift oscillators has been given by Vaughan[1], and it is proposed to illustrate here the simplified method in the analysis of these and other networks.

Single-Stage Pentode Oscillators

Fig. 1 shows the equivalent circuit of a 3-mesh phase-

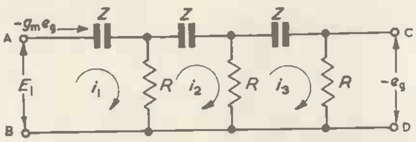


Fig. 1. 3-mesh phase-shift network

shift network which operates in conjunction with a pentode; AB being connected to its anode circuit, and CD to its input. The anode resistor of the valve is assumed large in comparison with *R*.

The criteria of oscillation are:

(1) The network shall produce a phase lead of 180° in voltage at CD with respect to AB.

(2) The valve shall have a mutual conductance g_m which produces a voltage $-e_g$ at the output of the transmission chain, when (1) is fulfilled, and when a current $-g_m e_g$ flows into it.

From the criteria we know the attenuation constant ($-E_1/e_g = \alpha$) *is* real and negative; and from this can determine the ratio of the input and output currents given in the figure, viz:

$$i_1/i_3 = g_m R = \beta \quad \ldots\ldots\ldots\ldots\ (1)$$

Putting $Z = \pm jX$, $K = X/R$

where *X* is the reactance of each capacitor, we can write:

$$Z = jKR \quad \ldots\ldots\ldots\ldots\ (2)$$

and agree temporarily to regard *Z* as a scalar quantity, with the proviso that though it can be manipulated algebraically with respect to *R*, while divorced from its complex function, it can never be related numerically; i.e., it is not a simple numeral.

* Redifon Ltd.

With this in mind, the mesh equations of Fig. 1 can be written down in the "resistive" form, as follows:

$$i_1(R + Z) - i_2R = E_1$$
$$-i_1R + i_2(2R + Z) - i_3R = 0$$
$$-i_2R + i_3(2R + Z) = 0$$

and i_3 solved from the determinants

$$i_3 = \frac{\begin{vmatrix} R+Z & -R & E_1 \\ -R & 2R+Z & 0 \\ 0 & -R & 0 \end{vmatrix}}{\begin{vmatrix} R+Z & -R & 0 \\ -R & 2R+Z & -R \\ 0 & -R & 2R+Z \end{vmatrix}} = \frac{E_1R^2}{R^3+6R^2Z+5RZ^2+Z^3} \quad \ldots\ldots\ldots\ (3)$$

whence

$$\alpha = E_1/i_3R = 1 + 6Z/R + 5Z^2/R^2 + Z^3/R^3$$

Substituting Equation (2):

$$\alpha = 1 + 6jK - 5K^2 - jK^3 \quad \ldots\ldots\ldots\ (4)$$

Since α is real, the odd powers of *K* add to zero

$$6jK - jK^3 = 0$$
$$\therefore K = \sqrt{6}$$

and Equation (4) reduces to:

$$|\alpha| = 1 - 5K^2$$
$$= -29$$

This result leads to the solution of Equation (1).

Expressing i_1 in the determinantal form from the mesh equations:

$$i_1 = \frac{\begin{vmatrix} E_1 & -R & 0 \\ 0 & 2R+Z & -R \\ 0 & -R & 2R+Z \end{vmatrix}}{D} = E_1\left(\frac{3R^2+4RZ+Z^2}{D}\right) \quad \ldots\ldots\ldots\ (5)$$

Where *D* is the major determinant given in Equation (3)

Dividing (3) into (5):

$$i_1/i_3 = \frac{3R^2 + 4RZ + Z^2}{R^2}$$

this yields:

$$\beta = 3 + 4Z/R + Z^2/R^2$$
$$= 3\ 4jK - K^2$$
$$|\beta| = \sqrt{(3 - K^2)^2 + 16K^2}$$
$$= g_m R$$

and since:

$$K = \sqrt{6}$$
$$R = \frac{10{\cdot}24}{g_m}$$
$$X = \frac{10{\cdot}24\sqrt{6}}{g_m}$$

Wien Bridge Oscillator

In Fig. 2 is shown the frequency-selective network of the conventional Wien bridge oscillator, which employs two valves in cascade between AB and CD.

Since two valves are used E_1 and e_g must be in the same phase to produce oscillation, hence α is real and positive.

The impedance between A and B is:

$$Z_i = R + Z + \frac{RZ}{R+Z}$$
$$= \frac{R^2 + 3RZ + Z^2}{R+Z}$$

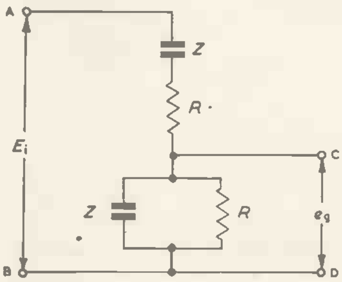


Fig. 2. Frequency selective network of Wien bridge oscillator

therefore the voltage e_g is equal to:

$$E_i \cdot \frac{(R+Z)}{R^2 + 3RZ + Z^2}\left(\frac{RZ}{R+Z}\right) = e_g$$

whence

$$\alpha = \frac{R^2 + 3RZ + Z^2}{RZ}$$
$$= R/Z + 3 + Z/R \quad \ldots\ldots\ldots\ldots\ (6)$$

Since α is real

$$R/Z + Z/R = 0$$

or

$$1/jK = -jK$$

thus

$$K = 1$$

and

$$X = R$$

and Equation (6) reduces to:

$$|\alpha| = 3$$

Parallel-T Null Network

Fig. 3 shows the parallel-*T* sections used in the feedback loop of selective amplifiers.

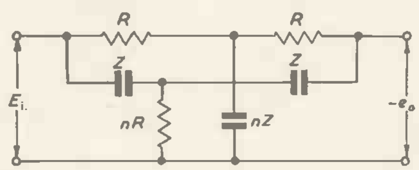


Fig. 3. Parallel-T sections used in feedback loop

Zero feedback occurs in the loop, when the voltage $-e_o$ is zero, and the network then produces a phase-lead of 180°.

The analysis is simplified when each *T*-section in Fig. 3 is replaced by its equivalent π-section from the following relations.

$$Z_1 = R/nZ\ (R + 2nZ)$$
$$Z_2 = R + 2nZ$$
$$Z_3 = Z/nR\ (2nR + Z)$$
$$Z_4 = 2nR + Z$$

applying these to the network in Fig. 4,

let $Z_{13} = Z_1$ in parallel with Z_3

„ $Z_{24} = Z_2$ „ „ „ Z_4

then

$$Z_{13} = \frac{RZ(R + 2nZ)(2nR + Z)}{nR^2(R + 2nZ) + nZ^2(2nR + Z)}$$

$$Z_{24} = \frac{(R + 2nZ)(2nR + Z)}{(2n + 1)(R + Z)}$$

and

$$-e_o/E_i = \frac{Z_{24}}{Z_{13} + Z_{24}}$$
$$= \frac{nR^2(R+2nZ)+nZ^2(2nR+Z)}{RZ(1+2n)(R+Z)+nR^2(R+2nZ)+nZ^2(2nR+Z)}$$

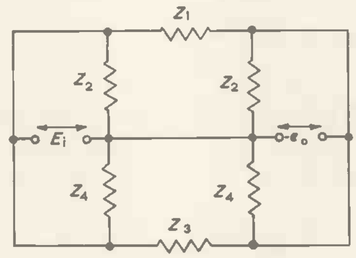


Fig. 4. Simplification of Fig. 3

Thus when $e_o = 0$

$$nR^2(R + 2nZ) = -nZ^2(Z + 2nR)$$

or

$$nR^2(R + 2jnRK) = nR^2K^2(jRK + 2nR)$$

Equating real components:

$$nR^3 = 2n^2R^3K^2$$

whence

$$K = \sqrt{1/2n}$$

Equating imaginaries:

$$2jn^2R^3K = jnR^3K^3$$

whence

$$K = \sqrt{2n}$$
$$\therefore K = 1;\ n = 1/2.$$

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Encapsulated Reference Resistors

By F. L. Valder*, M.I.R.E.

This article describes a method of producing reliable high-voltage resistors of a reasonable order of stability for general purposes. Their robust nature lends itself well to situations where rough handling is inevitable.

OWING to comparatively high manufacturing costs, the demand for reference resistors for use with high voltages of the order of 25-50 kilovolts has, until recently, been restricted to their use in laboratory measuring equipment. This high cost has been an embarrassment when making use of these resistors as potential dividing networks in stabilized R.F. E.H.T. equipment which merely forms part of complete apparatus, the cost of which must be pared to a minimum. It is proposed, therefore, to describe herein the construction of a reference resistor suitable for use in E.H.T. supplies and fulfilling the necessary requirements of reasonable long-term stability, low risk of open circuit conditions, high mechanical strength and economical manufacture.

The general construction can be seen in Figs. 1 and 2. Fig. 1 shows the mounting plate for carrying the individual resistor units. The plate may be of any insulating material capable of withstanding the end to end voltage stress and Perspex has been found to be excellent for the purpose.

* *Formerly Cinema Television, Ltd.*

It was also found, however, inadvisable to use synthetic resin-bonded materials, as these interfere with the setting of the polyester resins in which the unit is finally encapsulated.

Referring again to Fig. 1, the mounting strip should be as narrow as practicable, the space between the resistor mounting holes and the edge of the strip being governed only by the question of mechanical strength during the mounting operation. It will be understood that, in order to make a satisfactory crack-free polyester resin casting, the distance between the edge of the resistor mounting strip (which runs the whole length of the finished casting) and the outside surface of the casting should be not less than a certain figure depending on the stress pattern set up while the casting is forming. Thus, the narrower the mounting plate, the smaller may be the diameter of the finished casting.

A wide variety of individual resistor units have been used in experiments, the most satisfactory which the author has found being the Erie Type 8. In addition a number of tests were conducted to determine how the long term stability varied with resistance and wattage rating.

As the total value of reference resistor was to be of the order of 340MΩ, initial experiments were commenced with individual units of 10MΩ each. It was soon discovered that, with a P.D. of 25 000V across thirty-five of this value, large resistance changes occurred over a test period of some weeks, two such chains eventually becoming open-circuited. Further experiments were made increasing the

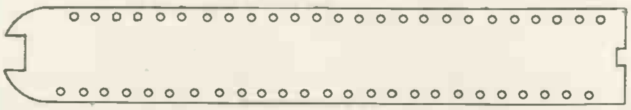

Fig. 1. The mounting strip

Fig. 2. A unit ready for casting

number of resistors until the stress across each resistor was as low as 500V with very little improvement in results. Various alternative unit values were tried down to 4·7MΩ, but tests proved that the compromise value of 6·8MΩ admirably met the requirements. Accordingly, a number of reference resistors were fabricated utilizing this value. Tests carried out under working conditions over a period of twelve months have shown that the long-term drift in resistance value is less than 2 per cent, the short-term stability being such that an E.H.T. unit regulated by a resistor of this type remained stable to better than 1 per cent for ambient temperature changes of 25°C.

Fig. 2 shows a unit mounted ready for casting. In this case fifty 6·8MΩ Type 8 resistors were used and it will be noted that no particular attention has been paid to stressing the resistor ends. They have been wired in series and soldered in the usual way. The high potential end terminates in a soldering tag held between the head of a bolt and a nut, while the other end, being considered "earthy", has been accorded no special treatment.

In order to construct a satisfactory mould in which to make the castings, a number of materials were tried and both Polythene and glass were found to be satisfactory. In the case of the former, the method employed was to centre-bore rod of a suitable size to the required length, diameter and shape for the finished casting, allowances being made for inevitable shrinkage. Polythene has the dual advantage that no mould release agent is required and also that it is easily machined to the desired shape. The glass moulds were formed to a shape similar to large test tubes with a small central hole in the domed end. Through this hole passes the bolt at the high potential end of the resistor unit, a liquid-tight seal being made by means of a rubber washer and a nut outside the glass mould. The use of glass as a mould produces a high gloss surface on the finished casting and a mould release agent need not be used, but has the disadvantages that mould shaping is limited and that it is much more susceptible to damage from careless handling than Polythene.

The moulding material used was Marco-Resin Type SB28C; the following formulation was found to be very satisfactory regarding setting time, freedom from cracking and crazing:

SB28C Resin	.. 200	grams
Monomer "C"	.. 4	"
Accelerator "E"	.. 4	"
Catalyst "H"	.. 4	"
Mica Powder	.. 30	"

This formulation will set in approximately 30 minutes at an ambient temperature of 20°C and will not crack if the minimum distance between the edge of the resistance mounting plate and the outside surface of the casting is not less than ¼ inch.

Due to the poor thermal conductivity of the casting resin, no attempt should be made to dissipate the normal rated wattage of the unit resistors used under the described conditions. For a chain consisting of fifty Type 8 resistors in a casting measuring 7 inches long by 1 inch

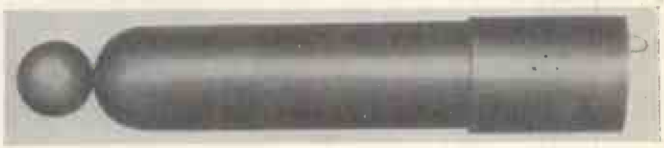

Fig. 3. The completed resistor

diameter, the maximum permissible dissipation was found to be 2 watts, and with this dissipation the required degree of stability was maintained indefinitely. The question of power rating is, however, relatively unimportant, since the applications of resistors of this type do not require a high power dissipation. Furthermore, the maximum permitted voltage across the unit resistors recommended by the manufacturers is reached before the full wattage rating is attained within the chain.

Fig. 3 shows a complete resistor with stress ball at the high potential end. The ball is also used as a terminal connector, being internally threaded to take the bolt at the top of the resistor. The physiological effect of accidental shock is minimized, to say nothing of the reduction in cost if this stress ball is made from a plastic rather than from a metal, the anti-corona properties being comparable in either case. This fact does not appear to be universally appreciated.

Finally, it should be emphasized that the reference resistors which have been described are not intended to replace expensive high-stability types designed for highly accurate measurement work. Rather do they fulfil the need for reliable, cheap and easy-to-fabricate resistors which may be included in commercial equipment with confidence if used under the conditions described.

Acknowledgment

The author thanks Messrs. Cinema Television Limited for their kind permission to publish this article.

Radio Equipment on the British 1953 Everest Expedition

By Brig. E. J. H. Moppett†, M.I.E.E., and R. W. Lewis*, B.Sc.

IN planning this year's successful expedition Col. Sir John Hunt decided that radio contact with the outside world would have slight effect upon the team's chances of success. Equipment for this purpose would necessarily be of considerable weight and its elimination permitted other more useful stores to be carried. He therefore restricted the radio equipment to that required for internal communication. This explains the much criticized delay in news getting out from the expedition. This decision enabled the radio equipment taken to be on an all-dry battery basis.

The most vital factor in timing the final assault on the summit was knowledge of the progress of the monsoon, so as to take advantage of the quiet period just before it breaks over the Himalayas. All India Radio and the BBC Overseas Service agreed to broadcast special monsoon weather forecasts for the expedition and the first radio requirement was short wave broadcast receivers to receive these bulletins.

The other radio requirement was for radio phone between climbers and between the camps on the way up the mountain, so as to be able to check and regulate the flow of stores and personnel during the build-up to get the climbers poised for the final assault. It was never the intention to use radio at the highest camps, for the assault was planned on a time-table and the weight carried had to be reduced as much as possible to allow every ounce of the vital oxygen to be carried.

† *Pye, Ltd.* * *Vidor, Ltd.*

The communication requirement was met by specially produced V.H.F. walkie-phones for use by the climbers, that could also be used as camp-to-camp transceivers. The sets had to impede the climbers as little as possible by weight or bulk. The battery consumption had to be low to give long use without change of batteries, and to reduce the total weight carried. Both sets and batteries had to be able to stand up without deterioration under these extreme conditions, and in tropical heat on the way to the Himalayas.

The Pye walkie-phone sets were arranged to be worn on the chest, above the oxygen apparatus, if in use, with a microphone on the top of the set, in a fixed mounting, ready for the climber to speak into at any time. A midget earphone was worn on one ear to fit easily under the head-dress. There is a press "send/receive" switch on top of the set for operation by the left thumb.

The walkie-phones operate on 72Mc/s, employing crystal control for both sending and receiving. They are set up for single frequency simplex working to allow a number of sets to work as a net. Battery consumption is so low, that with little over 2lb weight of batteries, carried under the climbers clothes, the set will operate for 90 hours. The weight of the walkie-phone itself is just under 5lb.

When in use worn by the climber, the walkie-phone employs a steel tape aerial, attached to the top of the set, sloping forward from the user. For operation from inside the tents, a tripod form of aerial was devised. This was desirable because the ground is hard rock or ice and it is not easy to drive in pegs for guy ropes. Rocks or blocks of ice can be piled round the tripod legs, which can also be opened out horizontally to give greater stability under really severe conditions, with some loss of height. This same tripod was arranged to hold either the V.H.F. unipole, then constituting a form of ground plane aerial, or the vertical whip aerial for short wave broadcast reception. Twelve and thirty feet feeders allowed the tents to be in sheltered positions with the aerials sited more favourably from the radio point of view. With this equipment the climbers were able to talk from camp to camp while they and their sets were warmly tucked away in their sleeping bags.

The broadcast receivers supplied were slightly modified Pye PE70B export receivers. These are dry battery operated and have band-spread reception in the standard short wave broadcast bands, up to 18Mc/s. They were re-boxed in wood instead of bakelite to stand up to rough handling and the batteries were fitted inside the case. The same H.T. batteries as those employed for the walkie-phones were used. The boxed sets with batteries weighed 15lb.

Dry batteries give very reduced output below 0°C. It was decided to overcome this major difficulty by fitting the batteries in a waistcoat worn under the climbers clothing to maintain the batteries near body heat. Carrying the batteries like this, allowed the size of the walkie-phones to be reduced, giving a better weight distribution. The weight could be shared between two climbers by one wearing the batteries and another the set, and plugging together to use the radio.

The dry batteries for the H.T. and L.T. supplies of the V.H.F. walkie-phones and the H.F. radio receivers, had to withstand transportation through zones of relatively high temperature, and to operate in use at temperatures around zero fahrenheit, as well as being robust enough to withstand rough handling. The batteries for the walkie-phones were considered the most difficult problem. The electrical performance and make-up of the batteries finally chosen was as follows:

90V Battery for the V.H.F. Transceiver (Vidor G1106)

The walkie-phones required 90V at 11mA for H.T. The H.T. comprised two blocks, each containing 35 Kalium cells (Vidor KV1 Cell), housed in an aluminium container and potted in a polyester resin, weighing 3¼lb in all. Tested to an end point of 65V, the life was 90 hours at 20°C, 66 hours at 0°C and 41·5 hours at 10°C.

1·5V L.T. Battery for the V.H.F. Transceiver

The L.T. required was 300mA at 1·5V.

This battery consisted of two U.1 size dry cells 2 15/16in. high, 1¼in. diameter, connected in parallel, housed in a cardboard container waterproofed with a microcrystalline wax. This battery, except for the wax coating, is identical to the Vidor commercial type.

Unlike the Kalium cell, the frequency of use of a dry cell when loaded heavily affects its duration.

On life test to an end point of 0·1V this battery gave on continuous test 18 hours at 20°C, 11·5 hours at 0°C and 8·5 hours at −10°C, while on intermittent test it gave 40 hours at 20°C, 20 hours at 0°C and 14 hours at −10°C. The battery weighs 8oz.

Batteries for the H.F. Receiver

The same type of H.T. battery as used for the walkie-phone was used for this set. The L.T. battery was the standard Vidor L5050 1·5V type. This battery gave 75 hours life even at continuous test at −10°C.

All equipment was tested at −50°C. For this temperature it was found necessary to replace PVC leads and microphone housings by rubber. The sets all worked normally at this low temperature. Considerable thought was given to the standardization of coaxial connectors and battery plugs throughout the equipment to avoid the risk of embarrassment by loss of odd parts. All plugs and connectors had to be suitable for use when wearing several pairs of gloves. The walkie-phones were fully waterproof.

The expedition took eight walkie-phones with full spares, and two broadcast receivers. The equipment was shipped out by air to reduce handling and lessen unnecessary exposure to tropical conditions on the way. Two walkie-phones and a broadcast receiver were also supplied to the Special Correspondent of *The Times* who followed the expedition out and joined Col. Sir John Hunt's party at the mountain.

On the return of the expedition to this country, Sir John spoke very highly of the performance of the radio equipment, which contributed very significantly, he said, to the well organized build-up for the final assault. The equipment and the batteries proved most reliable. The highest point at which the walkie-phone was used was 24 000ft. A set was taken to 28 000ft but sustained mechanical damage, not the fault of the equipment, and was not used there. This set remained on the South

G. Lowe, a member of the Everest team using a Pye walkie-phone with Sir Edmund Hillary looking on

Col and must be the highest piece of radio equipment on earth. Ranges of up to six miles were obtained from good sites and non-optical communication was sometimes obtained, no doubt due to a combination of the comparatively low frequency used and the reflexions from other mountains.

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Note.—It is worth recalling that the British 1936 Everest Expedition was equipped with radio communication apparatus for the first time. At that time it was "decided to omit the usual field telephone sets in favour of ultra-short wave transmitting and receiving apparatus, for in previous expeditions the weight of wire for a telephone service was too great for one man to carry in the rarefied atmosphere."

Accordingly a 5 metre transmitter and receiver which, together with the batteries, weighed just over 28lb, was designed by Messrs. Eddystone, and sets were supplied to the Everest team. In his subsequent address to the Royal Geographical Society Brigadier W. R. Smijth-Windham indicated that over 550 messages were handled by the equipment in under six weeks.

A brief technical description of the equipment was given in our April 1937 issue.—Ed. E.E.

The Electrical Synthesis of Musical Tones

By Alan Douglas

(Part 2)

Tonal Intervals ; Total Degradation ; Tonal Characteristics of Physical Instruments ; Electronic Tone Generators

BEFORE the production of complex musical sounds is examined, the properties of the musical scale should be noted. No purpose can be served by comparing the natural or just intonation scale with the equally tempered scale, since all keyboard and most other instruments must use the latter, in order to be able to play with other instruments in any key.

The equally tempered (E.T.) scale is an adjustment of the natural scale by equalizing the ratios of each adjacent semitone to the value of $\sqrt[12]{2}$, a ratio of 1·05946. This means that the equal temperament frequency will not correspond with the true harmonic frequency for many intervals, and this makes it impossible to synthesize really complex tones from any electrical generators in which each source supplies a separate simple wave corresponding to the intervals of the E.T. scale. For instance, in a generator where the sources are geared or immutably coupled by positive mechanical means, the ratios of such coupling means would correspond to the intervals of the E.T. scale. A glance at Table 1 will show that while harmonics 2, 3, 4, 6, 8 and 12 might be combined, the useful 5, 7, 9 and 11 would result in severe distortion. Even so, the use of 6 and 12 cause the tone to assume a somewhat hard nature, due to the beating of the small inharmonic content of these intervals. Thus total synthesis from a generator of this kind must be simple, although a certain richness may be achieved by adding octaves to the main tone. This is not true synthesis, but does give variety of effect. Only one generator of this type is now made commercially.

TABLE 1. Harmonic and Equal Temperament Frequencies.

HARMONIC	HARMONIC FREQUENCY	EQUAL TEMPERAMENT FREQUENCY	FREQUENCY DIFFERENCE	PERCENTAGE DIFFERENCE
1	440	440	0	0
2	880	880	0	0
3	1320	1318·51	− 1·49	−0·113
4	1760	1760	0	0
5	2200	2217·46	+ 17·46	+0·795
6	2640	2637·02	− 2·98	−0·113
7	3080	3135·96	+ 55·96	+1·81
8	3520	3520	0	0
9	3960	3951·06	− 8·94	−0·225
10	4400	4434·92	+ 34·92	+0·795
11	4840	4698·64	−141·36	−2·91
12	5280	5274·04	− 5·96	−0·113

It is therefore clear that the best way to achieve real simulation is to use a generator which will, for any one fundamental pitch, also supply as extensive a harmonic series as may be thought desirable. The series will then be truly related to the pitch note and synthesis will be possible. One example of such a generator using rotating mechanically-coupled sources is in production. In this, the true harmonic series is engraved on the rotating elements, so each element is the equivalent of a series of independent sine wave generators correctly pitched.

All other generators fulfilling this condition accurately use valves as the source of oscillations.

It is apparent that, even if the generated harmonic series is exactly accurate, the discrepancy in the frequency ratios due to E.T. tuning must give rise to many beating or combination effects at various intervals of the compass, this again depending on the strength of adjacent harmonics; if these are few and widely separated the effect will be small; if many, close together, the effect will be marked, particularly if loud. The poor resolving powers of a single diaphragm loudspeaker will add further difference tones. Thus the problem of synthesizing complex sounds becomes involved, especially at high loudness levels.

The foregoing suggests that some distortion will be inevitable; but it is necessary first to consider tonal degradation. This is not the same as distortion, but implies a modification of the harmonic content of a simulated sound as compared with the original sound.

Perhaps no word in communications is so overworked as "fidelity". The question of fidelity is so conjectural that it is not possible to define the limits either way. Electrical and acoustic coloration play a large part in the tonal spectrum heard by the observer and the question of the final result is impossible of analysis; nor would this be of value, since listening conditions vary so enormously. The procedure for designing electrical circuits of low distortion and wide frequency response is well established, but that does not mean that the sound reaching the ear is undistorted or properly balanced. The deficiencies of loudspeakers, standing waves set up in rooms, resonances and damping due to sections of the room operating in a different way at different loudness levels, all tend to alter the effect of the waveform entering the amplifier output circuits. Many of these factors disappear in large buildings, but some may become worse. Generally the observer has little or no control over these aspects of reproduced sound. What he can control, however, is the waveform before amplification.

What we have to consider is, can acceptable synthesis of the kinds of tone desired be carried out by reasonable and reproducible means, and, if not, what can be cut out of any tone colour without any appreciable difference being detected?

It is common knowledge that the reproduction of musical instruments through the average radio receiver is mediocre. The tone control fetish introduces further mutilation; yet does the average listener find any difficulty in identifying the various kinds of instruments being played? The better informed musically may experience some irritation at times, but this is reasonably balanced by the fact that their sense of hearing does its best to fill in such tones as are missing. The ear always attempts to find a similarity between a sound as heard and what it knows that sound should be like.

It is further true that in some classes of musical instrument the tonal specification is not fixed or exact; an oboe always sounds like an oboe but one organ does not sound like another except in a general way. Even the violin is capable of an extraordinarily wide range of tonal texture, yet, except for the lower notes, it cannot be mistaken for anything else. So clearly there must be something which fixes the main tonal characteristics of most musical instruments, and variations can take place around this kind of "core".

All single tone producers generate a fixed band of characteristic frequencies which is known as a formant. This band exists independently of the main tone, though with it. The formant group owes it origin to the configuration and substance of the resonating part of the instrument. Fig. 11 shows a group of formant frequencies measured in 1949 for several instruments; not all of these are true harmonics. A certain minimum energy is required to excite the resonator fully, and if this is not available the tone may change; further, the formant band will not usually operate over a range of more than about three octaves. It may disappear if this range is exceeded, or it may re-appear an octave higher. In multiple tone producers like the pipe organ, each pipe has its own formant group. In the piano, formants do not really occur, owing to the flatness of response of the resonator, i.e., the soundboard.

If, then, means could be made available to produce

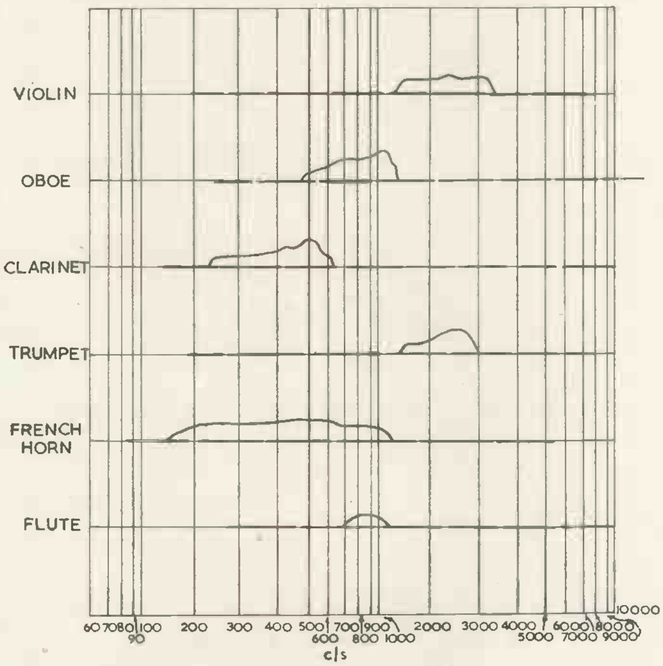


Fig. 11. Formant frequencies

synthetic formant groups, and then add or superimpose other pitches, a high degree of realism should be attainable. This is borne out in practice.

The permissible amount of tonal degradation is directly proportional to the exactitude of the tuning of the constituent harmonics, and the purity of the independent components. Analysis of a string tone (which may be held to embrace the range covered by a violin and a violincello) shows so many variations according to the construction of the instruments that, while in old violins as many as 20 harmonics have been measured, a remarkably good string tone can be generated from four or five. One of the main characteristics of all stringed instruments of the viole class is a weak fundamental. If, then, the 2^{nd}, 3^{rd}, 4^{th} and 6^{th} are present, and suitably adjusted, very passable synthesis is possible. The extreme complexity of a very "rich" string tone is a drawback and no advantage comes from attempting to imitate it; it does not have the definition and clarity of simpler string tone. The above synthesis is characteristic of the *string family*, i.e. it applies over a wide pitch range, so it may be used for the violin, viola and violincello.

Similarly, a compound of fundamental, 2^{nd}, 5^{th}, 6^{th}, 9^{th}, 10^{th} and 12^{th} produces a thin reedy tone which may be called the oboe or cor anglais type. By keeping the fundamental small and using 3^{rd}, 4^{th}, 5^{th}, 6^{th}, 7^{th}, 8^{th}, 11^{th} and 12^{th}, more sonorous reeds of the trumpet type are synthesized. A clarinet waveform has already been shown in Part 1. It should be noted that measured tone spectra of the foregoing instruments contain at least twice as many harmonics as those given; but the synthesized tones are remarkably good, and this is an instance of legitimate tonal degradation; for fully to compound these tones would be most uneconomical.

The ear is less ready to detect changes in the composition of complex tones than in simpler tones. Fig. 12 shows the waveform of the same note on a flute, played softly, of medium strength, and overblown. The analysis of the tones is appended. The overblown condition produces a "harmonic flute", an agreeable sound if maintained over a range of notes. Thus we might deduce that a flute-like tone need only contain few harmonics and is easily

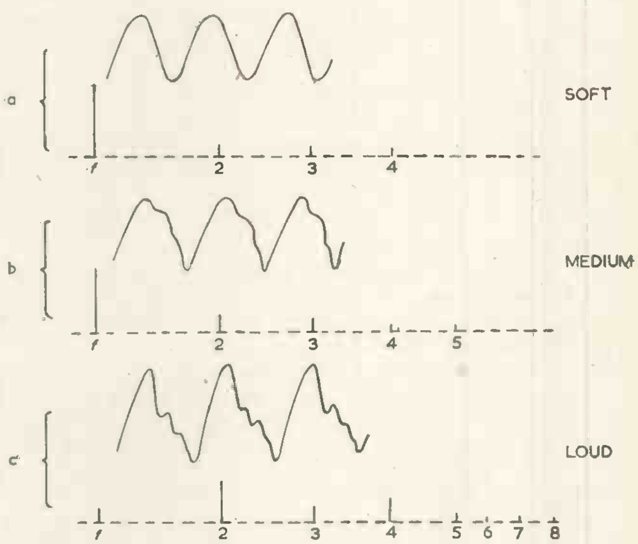


Fig. 12. Analysis of flute tones

synthesized. Of course, the pure wave from a tuning fork is flute-like, but such simple sounds rapidly become tiring and insipid, and are immediately masked and lost if other complex tones are added to them. Simple tones do not exist in any musical instrument. The amount of power required to develop a loud note from a sine wave source at 32c/s (= CCC 16ft pitch) would be phenomenal, and undoubtedly wreck any loudspeaker in a short time; but with quite a small content of 2^{nd} and 3^{rd}, 4^{th} or 5^{th}, good 16ft tones can be produced with small powers.

Some tones have special properties which allow of synthesis in the steady state but will not appear real due to other manipulative effects during playing. Such a sound is that of a saxophone. The technique of lip control and the constant changing of harmonic content due to different wind pressures, coupled with a variable vibrato, cause a synthesized saxophone to sound dull and lifeless.

Piano and other percussive tones are not successfully imitated; all electronic tones having this decay envelope are "new" tones. This is primarily because the harmonics in a normal decay wave train do not die away at the same rate, and because they inter-modulate and beat during the decay part of the cycle. Therefore they are in constantly changing relationship and so cannot be imitated.

Having amassed some details of tonal characteristics, the next step is to see what kind of tones can profitably be synthesized by electrical means. It is almost certain that these tones are such as can be played by means of a keyboard. Most people can play a tune on keys, often because the intervals (or notes) can always be seen, partly because numbers of notes can be played at the same time, and also because the piano is the most common domestic musical instrument. Further, even if the skill exists, what possible advantages would accrue from an electrical violin, flute, or other single tone source? Only that, by means of playing keys, a number of notes could be made to sound together. The simplicity, portability and expressive playing technique of the original instrument would vanish with no commensurate gain. Moreover there would be no relative standards of loudness even if the tonal value was accurate, which would lead to forcing of the tone and prevent a balance with other instruments. This is clearly borne out by listening to electronic instruments simulating the violin; for by no manipulation of the gain control or playing keys can the delicacy of bowing be simulated. Yet for any steady state, the synthesis may be nearly perfect. This is because every movement of the violin bow generates a fresh set of harmonics and their constantly changing resonances. Therefore the tone under dynamic conditions cannot be synthesized by electric circuits and the same is true of any other orchestral instrument. The tonal spectrum of every such instrument undergoes continual change with every alteration in pitch and power.

Thus such synthesis as is effective can only reproduce a steady tone state, equivalent to a set of organ pipes of that tone; or a tone state similar to that of instruments having a restricted power range such as the flute, oboe, clarinet, cor anglais and the like.

We can decide, then, that with the exception of the electric guitar (which is not a tone generator but a tone convertor), only tones playable from a keyboard form a fruitful field for simulation.

Here we can divide sharply into three alternatives (1) instruments having a keyboard but on which only one note at a time can be played; (2) single keyboard instruments for producing percussive sounds, on which combinations of notes can be played; (3) instruments having one or more keyboards which are organesque in tone.

The single keyboard single note instrument forms a solo tone source requiring an accompaniment. It is available commercially for attachment to a piano, this being the most probable source of accompaniment. Means are provided to prevent two adjacent notes from sounding together if accidentally struck. A variety of different tones can be extracted from such devices, singly or in combination. Some simulate known tones, others are "new".

The multi-note single keyboard percussive instrument has a more limited appeal; the piano is so highly developed that unless tones of the harpsichord or spinet type are preferred, the appeal is mostly of a novelty nature. Nevertheless, some very beautiful tones can be produced by such instruments. For reasons to be explained later, no instrument of this kind is now in production.

The multi-note single or multiple keyboard instrument for sustained tones would appear to offer the greatest scope. At the outset one must recognize that all kinds of musical sounds cannot be produced in the one instrument, thus it must fall into one of the three classes above. Setting aside for the moment the domestic appeal of this latter type of instrument it is the multi-keyboard instruments of an organ-like nature which have seen the most intensive development commercially. From the constructor's point of view, such instruments are easier to co-ordinate with limited test equipment than percussive keyboard sources. Many instruments of this class are in current production.

All electronic keyboard instruments have a high degree of similarity of attack or tonal initiation for each note. The only variations possible are those of rapid change in volume by manipulation of some expression device; the introduction of a vibrato; and, in some cases, the introduction of a percussive effect. All of these factors can be introduced into any type of tone generator, so first of all typical generators will be described and, subsequently, tone forming methods. Owing to the complexity of construction, mechanical generators will not be considered except for the vibrating reed type.

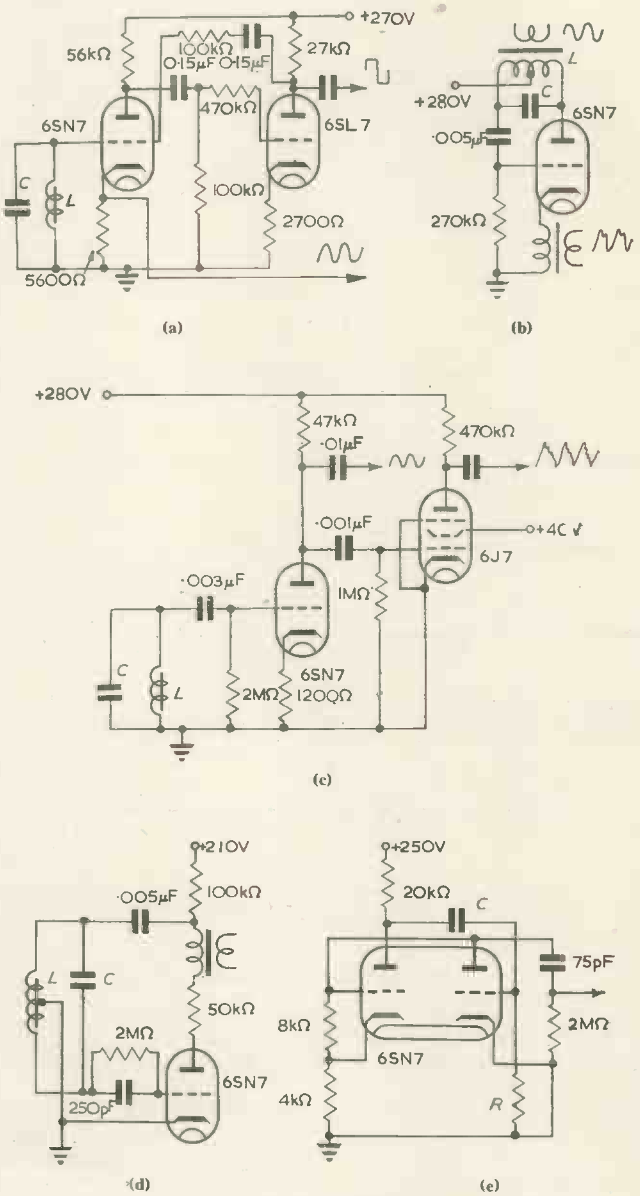


Fig. 13. Tone generators

Fifteen makes of keyboard instruments using valve oscillators are manufactured. Each of these generates a complex waveform, or more than one waveform, for every single note. Some employ one series of generators from which the different pitch ranges are extracted, others have quite separate generators for each pitch range.

Fig. 13(a) shows a very useful generator from which a sine and square wave can be simultaneously obtained. Fig. 13(b) is a diagram of another generator giving a sine wave and also a pulse of high harmonic content. Fig. 13(c) is another way of obtaining a sine wave and one with many harmonics, the pentode being over-driven to produce this condition. Fig. 13(d) is a generator in which the output is harmonically rich. All the foregoing are *LC* oscillators and have the characteristic stability of such circuits. Fig. 13(e) is a multivibrator oscillator producing a wave of high harmonic content.

Fig. 14(a) shows a frequency dividing circuit, the output of which is rich in harmonics. Fig. 14(b) is another way of doing this, and Fig. 14(c) yet another alternative. The advantage of frequency division is that only the upper 12 tone sources need to be tuned, all other notes in octave relationship being obtained by dividers in cascade, so that if the prime source is in tune, the remaining

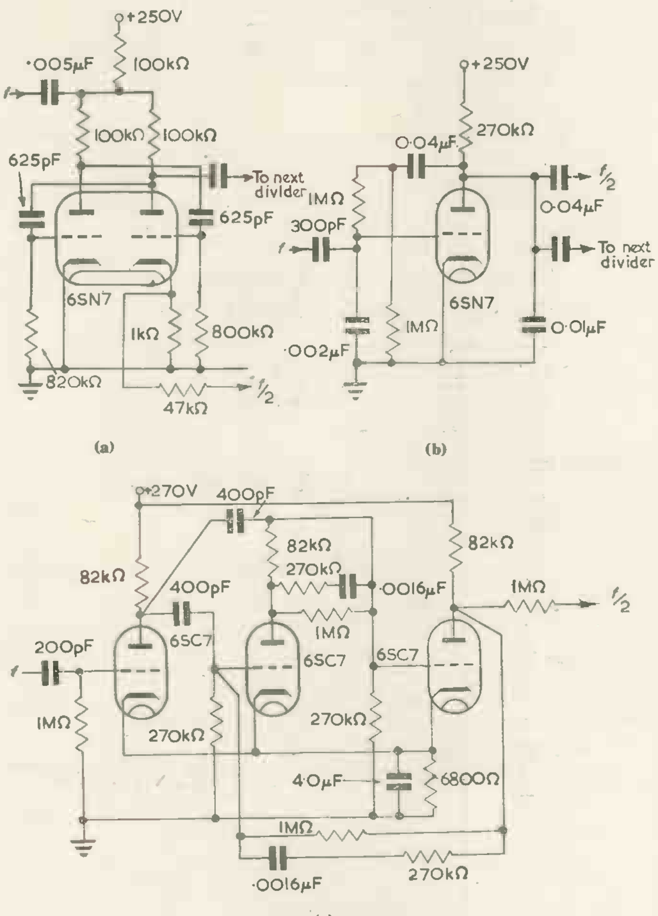


Fig. 14. Frequency dividers

divisions must be so. The disadvantage is that the waveform from many forms of divider is not entirely suitable for tone forming. All the circuits shown here are, however, in actual use in commercial instruments. They vary in complexity, some using simple triode valves, some multi-grid values, and some having more than one valve; their differing configurations being largely due to questions of policy and patents.

Circuits for generating and injecting a vibrato or tremulant usually operate on the oscillator circuits directly, before any tone forming takes place. Fig. 15(a) shows one way of generating the necessary low frequency of 6 to 7c/s, and Fig. 15(b) is another circuit. In this latter arrangement, the second valve acts as a switch to connect and disconnect small capacitors across the main tuning circuits. In the event of the grid bias having to be modulated, the grid leak may be returned to the top of the cathode resistor R_{k_2}; this vibrato voltage is cancelled by earthing the grid leak as shown. It is most important that the vibrato waveform be symmetrical on either side of the axis, or unpleasant cross-modulation results. The above circuits fulfil the required conditions.

Percussive circuits are rather difficult. One of the reasons for this is that one is not used to hearing a percussive start to the tone of, say, a clarinet, so there is no way of judging whether the effect is correct. Fig. 16(a) is one way of introducing a rapid initiation and slower decay for the sound, and Fig. 16(b) shows a more refined method.

So far it has been shown how to produce useful waveforms for musical purposes, the next step is the forming of acceptable tonecolours. The development of filters or resonant circuits for this purpose can be divided into two

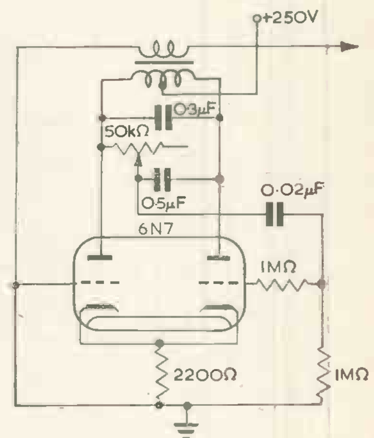
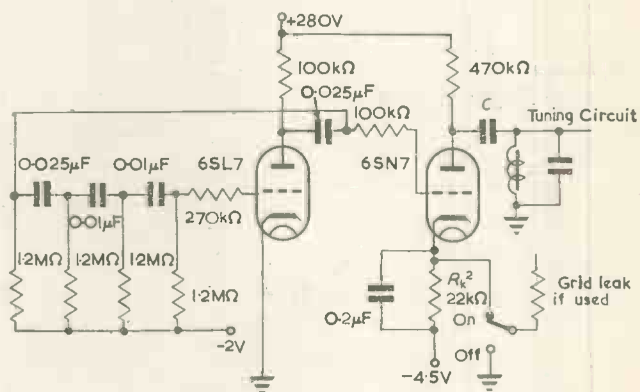


Fig. 15(a) right (b) below Vibrato oscillators



groups, those for melodic or single note instruments, and those for polyphonic or multi-note instruments. Since there is no question of combining notes on melodic instruments, we find the simplest application here in the form of shunt circuits between the signal outlet from the generator and earth.

The most elementary form of control is a capacitor placed across the line. This would be of such a value as to bypass the upper frequencies. Its response can be modified by a resistor in shunt or series with it. Such a control gives a very non-linear response and is only of use over a limited range. A much more effective control is a series-parallel network of the form shown in Fig. 17(a). By modification of the constants most tones of a flute-like nature can be produced, and four or five octaves can be covered.

It is difficult to invert this low-pass filter to a high-pass form when applied in shunt, such a filter being useful to form string tones; a better simulation being obtained from a tuned *LC* filter resonating at the main harmonic frequency of the string formant band. If the level is kept low, the tone is good, but if it is raised too much the tone will become reedy. Such a filter is shown in Fig. 17(b). Reed tones are best simulated by other arrangements of resonant inductances, as in Fig. 17(c). It is merely a matter of placing the major resonance in the correct frequency band. Such filters will not in general cover more than three octaves, but an advantage of the shunt grouping is that several units with overlapping characteristics can be placed in series to extend the range, as in Fig. 17(d). Most of these filters produce a difference in the loudness level when others are added or subtracted. Compensating resistors can be added to equalize this. Such a circuit is shown in Fig. 17(d).

In applying filters for adding together on a large scale, these simple circuits will not suffice, multiple filters being required for electronic organs. There are two approaches to this subject. In one case, all tone-forming circuits are brought to a common impedance value suitable for combination. In the other, each tone-forming circuit feeds the grid of a separate mixing valve, the outputs of these valves being eventually combined. Additionally, more than one filter fed from more than one tone source may work into a common output. For instance, in forming a tone of the open diapason class it may be desirable to add a separate upper harmonic group to the fundamental group. This is achieved as in Fig. 18(a) by feeding the filters from tone sources separated by an octave. This means a sufficient number of contacts on the keys, two sets in this instance. It will be seen that the combination works into a resistive load and seven or eight such filters may be applied in parallel to such an input with little change in relative level.

A further extension of this method is to supply one filter from one wave shape, and another with a different wave. In the filter shown in Fig. 18(b), the very difficult tone of a Vox Humana organ stop is successfully simulated.

In the above types of filter, the level for each tone-colour is set by a regulating resistor *R*. This also largely prevents one filter from reacting on another. It may be,

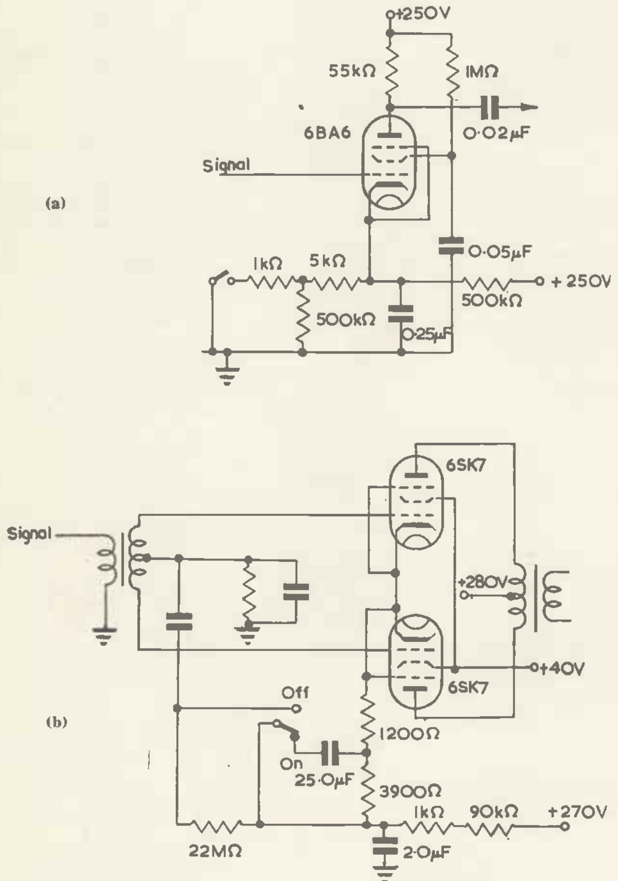


Fig. 16. Percussion circuits

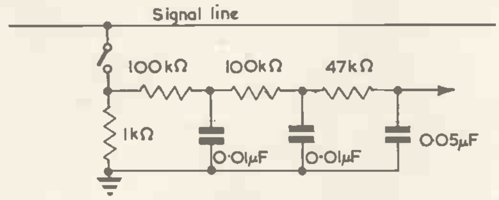


Fig. 17(a). Flute filter

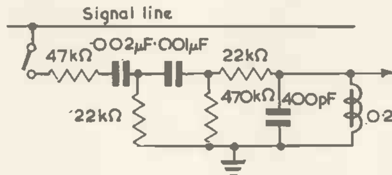


Fig. 17(b). String filter

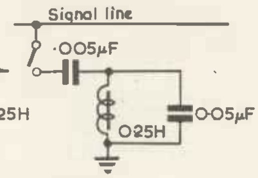


Fig. 17(c). Reed filter

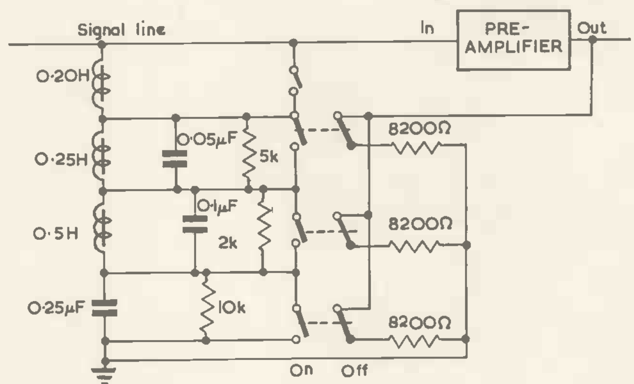


Fig. 17(d). Series filters

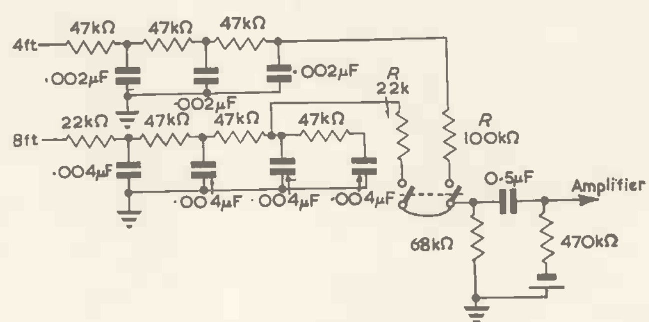


Fig. 18(a). Organ diapason filters

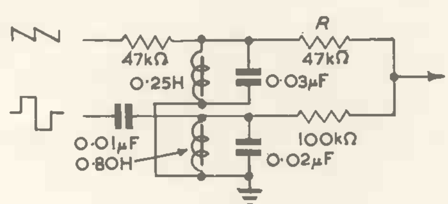
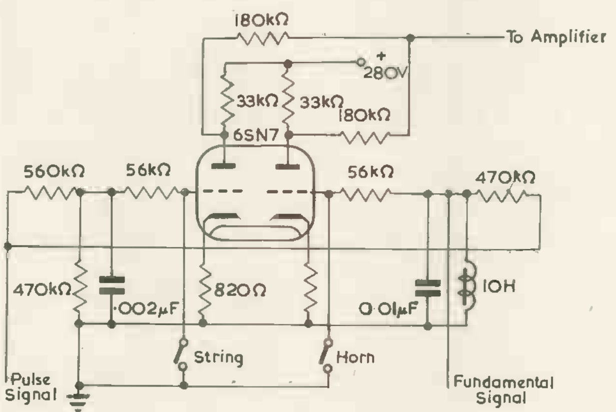


Fig. 18(b). Vox Humana filter

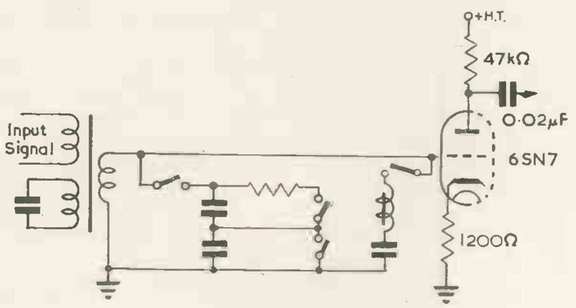
however, that if the filter is complex or requires many elements to produce the correct tonecolour, the attenuation becomes so high that the tone cannot attain the correct value; for, of course, there are quite definite optimum values for the range of loudness levels of different musical sounds. In this case the use of a separate tone-forming valve is desirable.

It can be seen that filter examples given do not always terminate in the correct load; in this case the attenuation is made as high as practicable, for in this case the input impedance becomes almost equal to the characteristic impedance regardless of the termination; and the loss due to mis-match rarely exceeds 2 or 3db. All the filters shown are in actual use in commercial instruments, but it will be appreciated that their effectiveness depends entirely on the shape and frequency range of the waveform supplied to them.

Fig. 19(a) shows a method by which tones are formed from the injection of two signals into a valve. The fundamental is practically free from harmonics, the pulse signal has many. Fig. 19(b) illustrates a way of doing this with



(a)



(b)

Fig. 19. Tone forming circuits

a single input waveform which contains all the desired harmonics. The regulation of the signal level for each tone is easily achieved in either of these circuits.

It will be observed that nearly all of these tone filters are of the bandpass type. This is because greater realism is obtained if circuits accentuating the formant frequency groups are inserted. At one time it was considered that true additive synthesis would give the most accurate results, and if the tone sources are quite pure and perfectly in tune this can still be the case; such exactitude of tuning can only be accomplished by mechanically coupled generators. A recent instrument employs this method using electrostatic generators.

In general, however, it has been found easier to provide complex waves from valve oscillators and modify them by means of complex filters. In a well-designed system this improves the choral effect when many tones are combined by comparison with the additive sine wave system, since there must inevitably be masking of frequencies already in use by addition of further combinations from the same sources. The unavoidable small fluctuations in tune do not appear to affect the result from the complex sources, since the whole harmonic series follows such deviations.

Since the formant groups for most instruments are well defined, the configuration of the tone-forming elements falls into three groups: (1) Resonant inductive filters emphasizing a band of both even and odd harmonic frequencies, for which purpose a triangular wave of sawtooth form is required. To these filters may be added suitably controlled fundamental and octave pitches. (2) Resonant inductive filters emphasizing odd harmonics only, for which purpose a square wave is required. (3) High- and low-pass *RC* filters into which several single frequency sources may feed. From group 1, tones of the viola, oboe, horn, trumpet and clarion type may be extracted. From group 2, clarinet, Vox Humana, stopped diapason, hohl flute, etc., are obtained. In the case of the two latter tones, the resonant peaks are flattened by shunt resistors and a strong second harmonic may have to be introduced. Group 3 provides the bourdon, diapason, flute and tibia class, for which purpose fundamental and second harmonic predominate; but the second harmonic source may be modified by a smaller degree of filtering so as to contain traces of higher harmonics. Good string tones of low intensity can also be simulated by high-pass filters in this group, the fundamental being suppressed and all other harmonics combined at almost the same amplitude.

In one pattern of mechanically coupled rotary generator the groups of harmonics necessary to provide the formant bands are engraved as an integrated series, e.g. 3rd and 5th, all odd, all even, etc.; these can then be "mixed" in various ways to produce the tonecolours.

A useful way to accentuate upper harmonics is to rectify part of the signal and so obtain pulses; by correct circuit constants very high frequencies can be produced. Rectification is also used in one commercial instrument to provide odd harmonics from a sawtooth wave source.

Undoubtedly it would result in greater fidelity over a wider pitch range if means could be found to generate, directly in the oscillators, the waveforms corresponding to the actual tonecolours required. Such a circuit does exist, and while the author is unable to disclose details of it at the moment, it will be brought to notice in due course.

The vibrating reed generator deserves brief mention. This is electrostatic, the moving tongues of the reeds acting as small variable capacitors. The overtones from a free reed are inharmonic and unpleasant, but by suitable disposition of the pick-up elements, most of these can be suppressed. The harmonic content remaining is mostly fundamental and second, but by the exercise of considerable skill coupled with special selection and adjustment of the reeds, other usable harmonics can be made available. Such generators probably owe their richness of tone to the multiplicity of small beats resulting from the slightly imperfect tuning of the many tone sources. Therefore no tone-forming in the sense previously discussed takes place, but some smoothing of the waveform is made use of to form flute-like tones. In such organs as use vibrating reeds we find only flute, diapason and string tone, but of many different pitches from 16ft. to 2ft.

A number of other matters germane to the production of synthetic musical tones will be discussed in the next section.

(To be continued)

A Study of a Second Order Sampling Servo

By S. R. Cooper*, B.Sc.

This article considers from a simple mathematical angle a second order servo working on discontinuous data, the information being sampled at regular intervals of time. The characteristic equation of the servo is obtained and two convenient parameters for studying it are obtained. Using these parameters the necessary conditions for stability are found. The transient response is considered in some detail and it is shown how the nature of it depends upon these parameters. For clarity, a similar continuous servo is treated side by side. Responses to step, velocity and accelerating inputs are considered. The response of a servo to a sinusoidal input is also considered.

THE use of servo-mechanisms in processes of automatic control is well known and the basic principles governing their behaviour are generally familiar. In the more usual servo the input information is continuously available. There are, however, a large number of cases in which the input information is not continuously available. The servo input may then be regarded as samples of some continuous function taken at regular intervals of time. In such cases the servo has to do two things, firstly it must follow exactly the input function at the actual sampling instants, and secondly it should provide smooth interpolation between these sampling instants.

The theory governing the continuous case is well known, the application of the continuous case to the sampling case can only be approximate. If the sampling frequency is high compared with any variation in the quantity to be controlled the continuous theory may be used with fair accuracy. Generally, this is not so. The mathematical theory of such systems has recently been developed[1].

This article describes an electronic system which has been built to demonstrate some of the most important aspects of the behaviour of such control systems.

Theoretical Approach

The word "servo" comes from the Latin word for slave, and this gives the clue to its behaviour. Thus, a servo-mechanism is a system which, fed with an input, will endeavour to copy this input faithfully, usually with power amplification between the input and the output.

In particular, this article deals with what are called "closed-loop" servos. This is a servo where some fraction of the output, very often unity, is fed back and compared with the input. The difference between the two which is called the error, or the correction, is then used to correct the output of the servo. The error may be either positive or negative. Such a mechanism is said to be an error-actuated device. Where precision is required the servo must have a closed loop. Although servos may take a variety of forms, e.g., mechanical, electronic, or electro-mechanical, the theory is quite general.

In practice almost all control systems are servo systems and in a large proportion of these cases the feedback loop is completed by the eye, brain and hand of a human observer. The nature of a servo-mechanism depends upon the function it is required to perform, thus servo-mechanisms vary greatly in complexity. The one this article is concerned with is given in block diagram form in Fig. 1.

In order to derive an equation determining the behaviour of the servo it is necessary to define some important quantities.

The input is the quantity, which has to be followed, controlled, etc., it is defined as $u(t)$.

The output is the path or function which the servo *actually* follows, this is defined as $v(t)$.

The rate is the derivative of the output, i.e., the input to the final integrator in Fig. 1, it is defined as $w(t)$.

The correction is the difference between the input and the output, it is defined as $c(t)$.

$$c(t) = u(t) - v(t) \quad \dots\dots\dots\dots (1)$$

u, v, w, and c are all functions of time. The particular servo being considered is a simple second order servo (Fig. 1). It will be seen that the input and output are

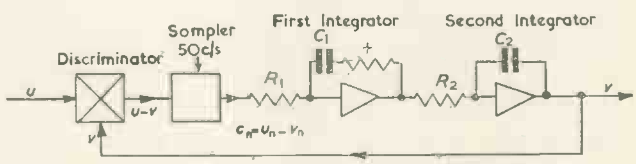


Fig. 1. Second-order sampling servo

compared in a discriminator producing a correction which may be either positive or negative in sign. The correction is then sampled with a sampling frequency of $1/T$ per second. The correction is measured at the sampling intervals and then held constant in a "box car" circuit until the next sampling instant, T-sec later, when it is erased and the new correction measured. It will be seen that by producing a continuous error and then sampling it at regular intervals, the practical case has been conveniently simulated, where it is actually the input which is only known at the sampling time. The sampled correction is integrated by two integrators in series. It will be noted that the first integrator contains a resistor in its local feedback chain. It will be shown later that this is necessary to keep the servo stable. It may be regarded as a pure integrator in parallel with a D.C. amplifier of gain r/R_1.

When it is desired to obtain an equation governing the behaviour of a physical system we generally write down a differential equation using ordinary continuous differential coefficients. Having obtained such an equation the problem is how to solve it, and in many servo problems this may be very difficult. In this case it is only its values at certain instants of time that are of interest, to be exact, at the sampling instants. It is found, therefore, more convenient to handle if it is turned into a difference equation.

To do this differential coefficients are not needed, but instead finite differences. As the only interest now is in the value of $f(t)$ at the sampling instants, i.e., when $f(t) \to f(nT)$ n integral, $f(nT)$ will be written as f_n, divided differences will be used which are defined thus:

$$\Delta f_n = \frac{f_{n+1} - f_n}{T} \quad \dots\dots\dots\dots (2)$$

Hence:

$$f_{n+1} - f_n = T\Delta f_n \quad \dots\dots\dots\dots (3)$$

* *R.R.D.E., Ministry of Supply.*

Therefore:

$$f_{n+1} = (1 + T\Delta)f_n \quad \ldots\ldots\ldots\ldots \quad (4)$$

Δf_n is analogous to the differential coefficient. If f_n is the set of values of a continuous function so that $f_n = f(nT)$ then:

$$L_{T/T\to 0}\, \Delta f_n = df/dt$$

The Characteristic Equation

An equation relating the quantities u_n and c_n will now be derived. It has already been seen that the correction is the quantity actually fed into the servo, or the quantity

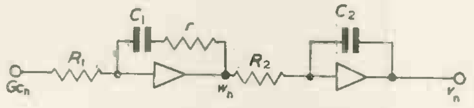


Fig. 2. Simplification of Fig. 1

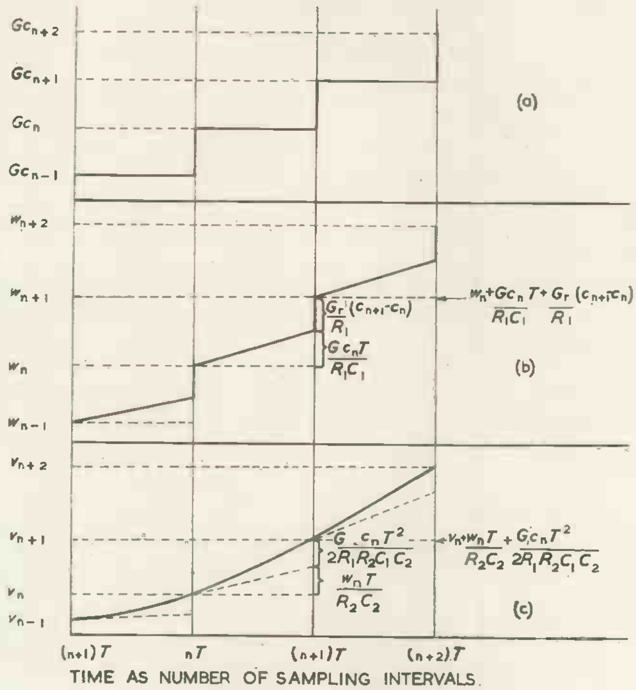


Fig. 3. Integration and amplification of error signal

which drives the servo, so that the circuit of Fig. 1 may be replaced by that of Fig. 2 and consideration given to what happens to the correction on passing through the two integrators. Fig. 3 shows the correction, the output and also the output from the first integrator w (as in Fig. 2) plotted against time as an integral number of sampling intervals.

The correction changes its value instantaneously at the sampling instant and then remains constant for T-sec. This change in the correction is immediately amplified through the first integrator, thus producing a step in w, shown dotted in Fig. 3(b) and this amplified correction is integrated in the final integrator to give the straight line shown dotted in Fig. 3(c). At the same time the new value of the correction is integrated in the first integrator to give the sloping line in Fig. 3(b). This integrated correction is now integrated again in the final integrator thus producing a squared term which gives a parabolic curve to the output.

It may be shown that if an integrator is fed with a step function input of E volts at a time $t = 0$, it will produce an output Et/RC. This applies also to the first integrator which we shall treat as an integrator in parallel with an amplifier of gain r/R_1.

Finally before obtaining the equation the gain in the discriminator circuit will be considered. Any RC circuit has a time-constant, whether it is being used as a differentiating circuit or an integrating circuit. Now if an amplifier be inserted the time-constant is effectively modified. This is equally true of a servo system which has its own natural period. In this case the gain will arise in the discriminator and sampling circuits, and we shall find that it effectively alters the natural period of the servo. This can be allowed for in the following manner.

Let the sampling mechanism have a voltage amplification factor G so that the input in Fig. 2 is Gc.

Let c_n be the correction at the n^{th} sampling instant and similarly u_n, v_n, w_n.

Suppose $f_n = L_T f(nT + \epsilon)/\epsilon \to +O$, so that any changes which take place instantaneously at the sampling intervals are supposed to have occurred before the measurement of f_n.

From Fig. 3(b) we see that:

$$w_{n+1} = w_n + Gr/R_1\,(c_{n+1} - c_n) + \frac{TGc_n}{R_1C_1} \quad \ldots\ldots\ldots\ldots \quad (5)$$

From Fig. 3(c) we see also that:

$$v_{n+1} = v_n + \frac{T}{R_2C_2} + 1/2\,\frac{T^2Gc_n}{R_1R_2C_1C_2}$$

But $v_{n+1} - v_n = T\Delta v_n$ from Equation (3)

Hence:

$$T\Delta v_n = \frac{T}{R_2C_2}w_n + 1/2\,\frac{T^2Gc_n}{R_1R_2C_1C_2} \quad \ldots\ldots \quad (6)$$

Similarly:

$$T\Delta v_{n+1} = \frac{Tw_{n+1}}{R_2C_2} + 1/2\,\frac{T^2Gc_{n+1}}{R_1R_2C_1C_2} \quad \ldots\ldots\ldots \quad (7)$$

Substituting for w_{n+1} in Equation (7) gives

$$T\Delta v_{n+1} = \frac{T}{R_2C_2}\left[w_n + Gr/R_1\,(c_{n+1} - c_n) + \frac{TGc_n}{R_1C_1}\right] + 1/2\,\frac{T^2Gc_{n+1}}{R_1R_2C_1C_2} \quad \ldots \quad (8)$$

Subtracting Equation (6) from Equation (8) gives

$$T(\Delta v_{n+1} - \Delta v_n) = \frac{TrG}{R_1R_2C_2}(c_{n+1} - c_n) + \frac{T^2Gc_n}{R_1R_2C_1C_2} + 1/2\,\frac{T^2G}{R_1R_2C_1C_2}(c_{n+1} - c_n)$$

Remembering $f_{n+1} - f_n = T\Delta f_n$ and $\Delta f_{n+1} - \Delta f_n = T\Delta^2 f_n$ we get

$$T^2\Delta^2 v_n = \frac{T^2rC_1G}{R_1R_2C_1C_2}\Delta c_n + \frac{T^2Gc_n}{R_1R_2C_1C_2} + 1/2\,\frac{T^3G\,\Delta c_n}{R_1R_2C_1C_2}$$

But $c_n = u_n - v_n$ from Equation (1)

$$\Delta^2 u_n = \Delta^2 c_n + \frac{G}{R_1R_2C_1C_2}(rC_1 + T/2)\,\Delta c_n + \frac{Gc_n}{R_1R_2C_1C_2}$$

Hence

$$\left[\Delta^2 + \frac{G}{R_1R_2C_1C_2}(rC_1 + T/2)\,\Delta + \frac{G}{R_1R_2C_1C_2}\right]c_n = \Delta^2 u_n \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

It should also be noted that if $T \to O$, as $n \to \infty$, while $nT \to t$, the corresponding equation for the normal continuous case is:

$$\left[p^2 + \frac{rC_1Gp}{R_1R_2C_1C_2} + \frac{G}{R_1R_2C_1C_2}\right]c(t) = p^2\,u(t) \quad \ldots\ldots \quad (10)$$

Solution of Differential Equation

This very familiar type of equation will be recognized as a simple second order differential equation whose solution is well known. The solution to Equation (10) consists of two parts. Firstly, the complementary function, which is obtained by putting $u(t) = 0$.

$$c(t) = A \exp(\alpha t) + B \exp(\beta t) \quad \ldots\ldots (11)$$

α and β are the roots of the auxiliary equation, i.e.,

$$m^2 + \frac{rC_1G}{R_1R_2C_1C_2}\, m + \frac{G}{R_1R_2C_1C_2} = 0 \quad \ldots. (12)$$

and A and B are arbitrary constants which can only be determined from a knowledge of the boundary conditions. The correctness of this solution may be confirmed by substituting Equation (11) in Equation (10) with $u(t) = 0$. For a servo to be stable, the real parts of the roots must always be negative in which case these terms always damp out. As this solution is derived with $u(t) = 0$, it will be seen that these terms are always present whatever the nature of the function $u(t)$. As they are not permanent they are said to represent the transient behaviour of the servo and it will be shown later that the transient response is exceptionally important in the study of servo-mechanisms.

It will be remembered that α and β may be real, imaginary or complex. If they are complex they will, of course, be conjugate, and Equation (11) may easily be shown to be of the form:

$$c(t) = \exp(-bt)\,(A \cos \omega t + B \sin \omega t) \quad \ldots. (13)$$

where $\omega = \frac{\alpha - \beta}{2}$, and $b = \frac{\alpha + \beta}{2}$. This in turn may be rearranged to give $c(t) = \sqrt{(A^2+B^2)} \exp(-bt) \cos(\omega t + \phi)$ where ϕ is a phase-angle. It can be seen therefore that the transient response may be an exponentially damped sinusoidal oscillation. If the exponential term were ever positive the oscillations would be self sustaining, but as $b = \left[\frac{rC_1G}{2\,R_1R_1C_1C_2}\right]$, physical limitations prevent this.

Returning to Equation (10) the remaining part of the solution, known as the particular integral, is obtained by putting in the appropriate function for $u(t)$ and depends solely on this function. This solution is a permanent one and gives therefore the steady state behaviour of the servo. From this equation we can deduce the magnitudes of any steady-state errors or lags.

In an identical manner we find that the solution of Equation (9) also consists of two parts. This time, however, the complementary function, which is given by solving Equation (9) for $u_n = 0$, is found to be of the form:

$$c_n = A(1 + \alpha T)^n + B(1 + \beta T)^n \quad \ldots\ldots.. (14)$$

where α and β are the roots of the auxiliary equation. Letting $n \to \infty$, $T \to 0$, while $nT \to t$, Equation (14) becomes $c(t) = A \exp(\alpha t) + B \exp(\beta t)$, which is Equation (11).

The particular integral is again a function of the input, but we shall be concerned very little with it here as the general mathematical treatment is rather beyond this article. It will be seen that in the sampling case the values of α and β in Equation (14) again affect the nature of the transient response and this will be dealt with next.

Importance and Analysis of the Characteristic Equation

In the experimental study of servo-mechanisms there are two approaches which are particularly important. These are, the response to a step function and the response to a harmonic input. The latter is of great importance when one is concerned with the frequency characteristics. It has already been shown that whatever the nature of the input, the transient terms are always present in the output. For studying the transient response it is only necessary to provide some initial disturbance as an input. A step input provides this disturbance simply and satisfactorily.

The knowledge of the transient behaviour of a servo is of fundamental importance. Firstly, it tells whether the servo is stable. Secondly, it gives some indications as to how near or how far away from instability the servo is. It is rarely enough to know the servo is stable and will follow the input, it is generally required to know the manner in which it will follow. Thus, once the response to a step function is known, quite a lot can be deduced about the transient behaviour, e.g., if the servo's response to a step function were very lightly damped sine wave with a natural period of the order of seconds, then it would be of little practical use. This is because once the servo had been disturbed with an input it would take much too long to settle down ready to follow a further change in the input.

Complementary Function

The complementary function for both the sampling and continuous cases have been obtained. It will now be considered in rather more detail. As the continuous case is generally more familiar the cases will be considered side by side.

Continuous Case

The complementary function which was given in Equation (11) is:

$$c(t) = A \exp(\alpha t) + B \exp(\beta t) \quad \ldots\ldots.. (11)$$

where α and β are the roots of:

$$m^2 + \frac{rC_1Gm}{R_1R_2C_1C_2} + \frac{G}{R_1R_2C_1C_2} = 0 \quad \ldots\ldots.. (12)$$

This can be rewritten as:

$$m^2 + 2bm + C = 0 \quad \ldots\ldots\ldots.. (15)$$

The complementary function has three possible forms, if $b^2 > C$

$$c(t) = A \exp -(b + \sqrt{b^2 - C})t + B \exp -(b - \sqrt{b^2 - C})t \quad ..(16)$$

if $b^2 = C$, the roots of Equation (12) are coincident and the solution is:

$$c(t) = (A + Bt) \exp(-bt) \quad \ldots\ldots\ldots (17)$$

and finally if $b^2 < C$, the roots are complex and the complementary function may be written:

$$c(t) = \exp(-bt)\,[A \cos \sqrt{C - b^2}t + B \sin \sqrt{C - b^2}t] \quad .. (18)$$

Stability Limits—Continuous Case

It is obviously essential from a practical point of view that a servo should be stable. Therefore, it is necessary to know if there are any limits to the stability of the system, and if so, what they are. Considering Equations (16), (17) and (18) it can be seen that in each case there is a damping term $\exp(-bt)$ present, and it has been shown that b is always positive, i.e., the exponential term is always negative and thus the transient response always dies away exponentially. This article is primarily interested in the mathematical similarities and differences between continuous and sampling servos and it is for this reason that the mathematically stable region has been obtained. It must, however, be emphasized that mathematical and practical stability are not the same thing, and when designing a practically useful servo, other and more stringent criteria have to be used.

Nature of Transient Response—Continuous Case

The transient response as defined by Equations (16), (17) and (18) can now be considered. The roots of the auxiliary equation may be complex and conjugate, real and coincident, or real unequal and negative. They can never be real and positive. The solution given in Equation (18) represents a lightly damped sinusoidal response, which is said to be underdamped. (See Fig. 16(b).) When the roots are equal the response is no longer oscillatory and the servo is said to be critically damped. The initial overshoot is not followed by a further overshoot. Finally the damping is increased, the roots are unequal and the response is given by Equation (11) or (16). The initial response is now more rapid, but the overshoot is followed by a very slow aperiodic recovery. This condition is said to be overdamped. (Figs. 12(b)-13(b).)

To show how the nature of these responses depends upon the values of R_1, R_2 C_1, C_2, and G the two time-constants $\sqrt{[(R_1R_2C_1C_2)/G]}$ and rC_1 have been plotted as x and y, respectively in Fig. 4. As shown previously, all physically real (i.e., positive) values of resistors and capacitors represent stable conditions so that the whole of the region shown in Fig.4 is mathematically stable. The line $rC_1 = 2\sqrt{[(R_1R_2C_1C_2)/G]}$ represents the condition for equal roots, i.e., $b^2 = C$. It will be shown later that plotting the same functions on a different scale gives the h-k plane. This plane presents very simply a great deal of fundamental information.

Sampling Case

In this case, although the treatment and results are broadly similar to those of the continuous case, the stability limits are more stringent and the responses more varied.

Again, starting by considering the auxiliary equation:

$$m^2 + \frac{G}{R_1R_2C_1C_2}(rC_1 + T/2)\,m + \frac{G}{R_1R_2C_1C_2} = 0 \qquad (19)$$

For convenience let $rC_1/T = k$, and $\sqrt{(R_1R_2C_1C_2}/T\sqrt{G} = h$, Equation (19) now becomes:

$$m^2 + 1/h^2T\,(k + 1/2)m + 1/h^2T^2 = 0 \qquad (20)$$

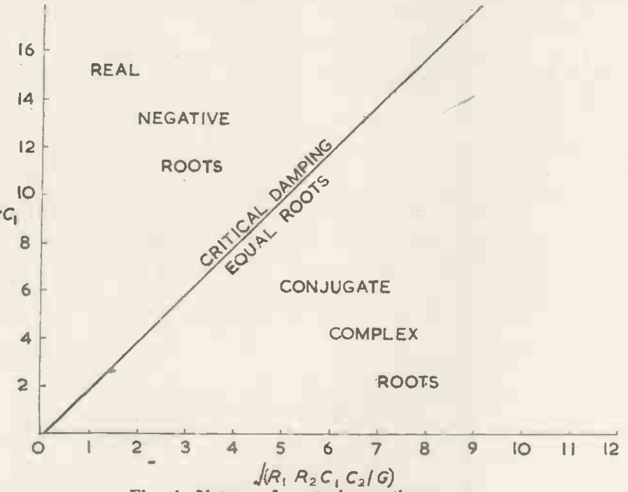


Fig. 4. Nature of roots in continuous case

The complementary function is:

$$c_n = A(1 + \alpha T)^n + B(1 + \beta T)^n \qquad (14)$$

where α and β are the roots of Equation (19) or (20).

It has been seen previously that the transient response depends upon the nature of these roots. The roots in turn depend upon the coefficients of Equation (20). Thus we see that the relationship between h and k determines the roots and the roots in turn determine the nature of the transient response. These parameters turn out to be extremely important in the study of this servo and it is found very convenient to plot k against h. From the position in the h-k plane of any similar second order sampling servo, its transient behaviour and indeed other characteristics such as its ability to deal with a noisy input can be estimated immediately.

Stability Limits. Sampling Case

The roots of the auxiliary equations are α and β. But Equation (14), for example, shows that αT and βT are of less interest $(1 + \alpha T)$ and $(1 + \beta T)$. For clarity of treatment $(1 + \alpha T)$ will be used, not αT. From now on $(1 + \alpha T)$ or $(1 + \beta T)$ will be referred to as the " bracket ".

To determine the stability limits consider the responses of the system. Again the roots may be complex conjugates or real. When $4h^2 > (k + 1/2)^2$ the roots and hence the brackets are complex and the complementary function may be written:

$$c_n = R^n(A \cos n\theta + B \sin n\theta) \qquad (21)$$

If this is to be stable it is necessary that $c_{n+1} < c_n$ (i.e., the terms are dying away). This can only be so if $R^{n+1} < R^n$, which means that the magnitude of $R < 1$. To satisfy this condition we find that $k > 1/2$. This defines one stability limit (see Fig. 5).

When $(k + \frac{1}{2})^2 > 4h^2$ the roots and the brackets are always real, but the brackets are not necessarily of the same sign. This apparently simple statement covers an intrinsic property of sampling servos which is fundamentally and completely different from any behaviour of the continuous case. As this may be somewhat unfamiliar to most readers it will be considered in detail.

It has just been stated that the roots and brackets are always real (in this part of the h-k plane). αT cannot be positive so it is only necessary to consider negative values. Consider $f_n = (1 + \alpha T)^n$. Firstly when αT lies between 0 and -1, e.g. let $\alpha T = -\frac{1}{2}$. If $n = 0,1,2,3, \dots f_n = +1, +\frac{1}{2}, +\frac{1}{4}, +\frac{1}{8}, + \dots$. It is seen immediately that f_n is dying away in a manner closely resembling an exponential decay.

Now consider the same function when αT lies between -1 and -2, e.g. let $\alpha T = -1\frac{1}{2}$. Again, for $n = 0,1,2,3, \dots f_n = +1, -\frac{1}{2}, +\frac{1}{4}, -\frac{1}{8}, + \dots$ (see Fig. 9(a)). Here

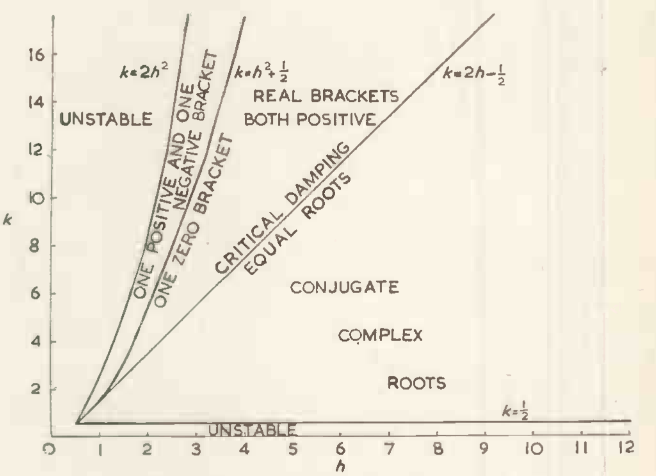


Fig. 5. Map of h-k plane in sampling case

may be seen the difference from the continuous case because the response has become oscillatory—without complex roots. This has no counterpart in the continuous case. Note that the oscillations are damped out and so the servo is still stable.

Finally, consider the case when αT is numerically greater than 2, and negative, e.g. $\alpha T = -2\frac{1}{2}$. The corresponding values for f_n are now 1, $-3/2$, $+9/4$, $-27/8$, $+ \dots$. The response is again oscillatory, but $f_{n+1} > f_n$, or in other words the oscillations are building up and the servo is no longer stable. The condition for stability here is that $k < 2h^2$ so that the h-k plane is now bounded by the line $k = \frac{1}{2}$, and the parabola $k = 2h^2$. Inside this region the servo will be stable, outside it will be unstable.

Nature of Transient Response

Now having found which parts of the h-k plane may be used if the servo is to be stable, the nature of the actual transient responses can be considered.

The first and main division of the h-k plane is that between real and complex roots. This is given by the line $k = 2h - \frac{1}{2}$, when $k < 2h - \frac{1}{2}$, the roots and hence the brackets are complex and the equation may be written as

$$c_n = R^n(A \cos n\theta + B \sin n\theta) \qquad (21)$$

which represents an oscillation which is almost but not quite sinusoidal (Fig. 16(a)). θ, it should be noted, is a function of h and k and hence T. The natural period of oscillation is approximately $2\pi hT$.

k mainly influences the damping term, and the damping steadily increases until $k = 2h - \frac{1}{2}$, when the roots are equal and by analogy with the continuous case, may be described as critically damped.

The equation is now

$$c_n = (A + Bn)\,(1 + \alpha T)^n \quad \ldots\ldots\ldots\ldots \quad (22)$$

$\alpha = \beta$, and both brackets are real, equal and positive. The response now consists of an overshoot followed by an aperiodic recovery. Finally $(1 + \alpha T)$ cannot be greater than unity.

The final division of the h-k plane is the parabola $k = h^2 + \frac{1}{2}$. When $k < h^2 + \frac{1}{2}$ (and $k > 2h - \frac{1}{2}$) the complementary function is

$$c_n = A(1 + \alpha T)^n + B(1 + \beta T)^n \quad \ldots\ldots\ldots\ldots \quad (14)$$

In this case both the brackets are real, positive and less than unity, hence they damp out. The response is overdamped and consists of a slight overshoot followed by a slow aperiodic recovery.

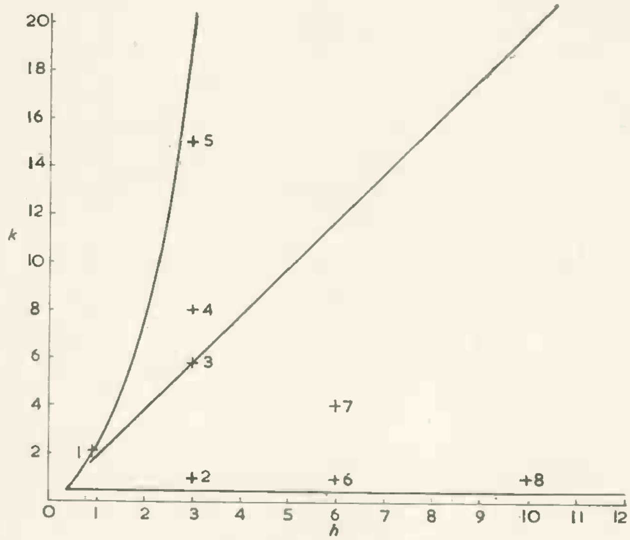


Fig. 6. Position of responses

When $k = h^2 + \frac{1}{2}$, one bracket is zero and the other remains positive. This is in effect the limiting case of the overdamped response.

Finally when $k > h^2 + \frac{1}{2}$ (and less than $2h^2$ for stability) the complementary function is again

$$c_n = A(1 + \alpha T)^n + B(1 + \beta T)^n$$

but here one bracket is negative, the other remaining positive. This produces an oscillatory waveform. This waveform is approximately triangular and has a constant period $= 2T$ (Fig. 9(a)). While the magnitude of $(1 + \alpha T)$ is less than unity, the oscillations die away almost exponentially until finally $k > 2h^2$ and the oscillations are sustained.

The h-k plane is given in Fig. 5 with the divisions marked. For all the foregoing conditions to be true, $h > 1$. When h is less than unity, somewhat different conditions exist as the servo is fringing on two unstable regions with different modes of oscillations.

Physical Interpretation of Responses Demonstrated

In this section responses shown in Figs. 9(a)-16(a) will be considered and an attempt made to interpret them physically. The oscillographs show some typical responses which have been obtained experimentally.

In every case the top left shows the step input, and immediately below it the *error*. The top right shows the resultant output and underneath it is the rate. It should be noted that the bottom left-hand tube displays the error and not the correction. This does not affect the principle involved. However, from the mathematical point of view of the theory it is perhaps unfortunate, e.g., see below where the sinusoidal type or response is discussed. To obtain the correction it is necessary only to imagine the mirror image about the X axis, i.e. anything positive becomes negative and vice versa.

As mentioned above, these are experimental results displayed on four C.R.T. operated by a common single stroke time-base. Thus for any particular response the time scale is the same on all four tubes. The Y-plate sensitivities may vary due to tolerances in the amplifiers feeding the C.R.T. The experimental model worked on the mains frequency, i.e. $T = 1/50$sec, so that by counting the errors it is possible to assess the speed of a particular response except in the sinusoidal case when the separate errors cannot be distinguished.

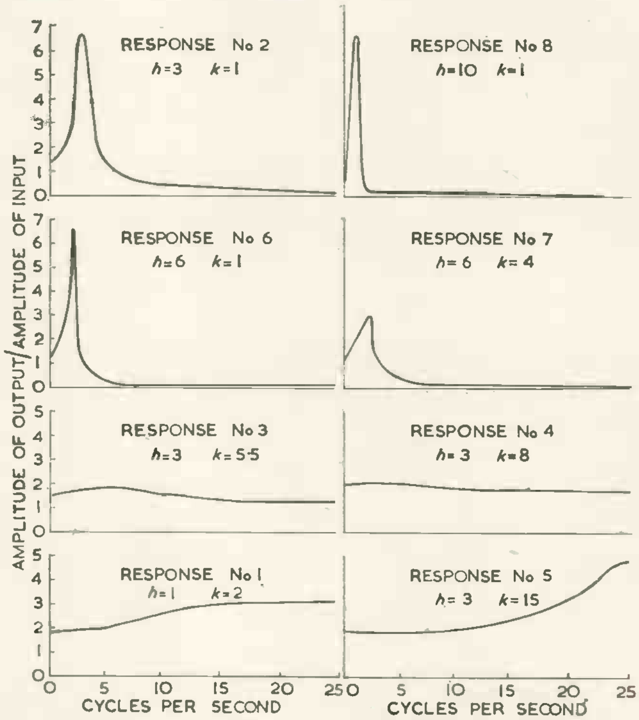


Fig. 7. Frequency characteristics of sampling servo corresponding to step responses in Figs. 9(a) to 16(a)

For interest, Figs. 9(b)-16(b) show the corresponding responses when the sampling mechanism is removed and the servo becomes continuously operated. It will be seen from Figs. 4 and 5 that the response which corresponds to coincident roots in the sampling case, i.e. $k = 2h - \frac{1}{2}$, Fig. 5 will not correspond exactly to coincident roots in the continuous case, i.e. $rC_1 = 2\sqrt{(R_1R_2C_1C_2/G)}$ (Fig. 4). For each pair of cases, however, the circuit parameters are identical, e.g. Figs. 9(a) and 9(b) are absolutely identical except for presence and absence of the sampling mechanisms. The position of these actual experimental results in the h-k plane are shown in Fig. 6.

Responses 2, 6 and 8, Figs. 10(a), 14(a), 16(a) all represent $k = 1$, i.e. they are comfortably inside the stability limit, and are in the complex part of the h-k plane (Fig. 6) and are therefore almost sinusoidal. It will be seen at once that as h increases the damping decreases. At the same time—although not so easily apparent—the period is increasing. This may be seen if the error is closely examined in responses 2 and 8, in response 2 it is easily seen to consist of discontinuities, in response 8 it appears to be continuous owing to the fact that the time-base has been slowed down. Looking at the original servo Fig. 2 for a moment it can be seen that k small means r small and it can therefore be ignored. The servo now consists of two perfect integrators and hence the correction is being doubly integrated and, owing to the absence of a direct path through the first integrator, there are no discontinuities and the output is a smooth second order curve.

A cosine is being integrated to give a sine, which in turn is being integrated to give minus a cosine. This means that the output is lagging behind the rate by 90°. Similarly, the rate is lagging behind the correction by 90°, so that the output is lagging behind the correction by 180°. But as mentioned at the beginning of this section, the oscillograph actually shows the error which in turn is 180° out of phase with the correction and is thus in phase with the output. If two photographs are examined, the error and output will be seen to be in phase and the rate 90° out of phase. Response 7 shows h the same as in response 6, i.e. $h = 6$, but k is now 4 as against 1 (Fig. 15(a)). It will be seen that the damping has increased considerably, but the period (allowing for the time-base) is approximately the same. Examination of the rate will show that amplification is still slight.

If we return to response 2, and then on to number 3, we see the effect of increasing k. This has increased the damping so that the response is no longer oscillatory but is "critically damped" (equal roots). It will be seen to consist of a single overshoot followed by a gentle aperiodic recovery. Looking at the rate we see that the correction is being amplified considerably. As a result the output appears to be made up of straight segments, although in fact there must always be a second order effect present. A further increase in k gives response 4, now both brackets are positive and unequal. This response, which is over-damped and very similar to the continuous case, is similar to number 3. The amplification is slightly more pronounced, the output is not quite so smooth, but the transient is slightly quicker.

Finally, increasing k still further gives response 5 which shows the triangular waveform corresponding to one negative root. Referring again to Fig. 2 and bearing in mind that the first integrator is equivalent to an integrator in parallel with a D.C. amplifier, it can be seen that r is now so large that the amplifying properties of the first integrator have largely swamped its integrating properties. In consequence, the servo approximates to a D.C. amplifier followed by an integrator. If the rate is examined this will be clearly seen. It is for this reason that the output looks at a first glance to be made up of linear elements. Looking closely it can be observed that they are still slightly curved. This must be so as a contribution from the integrator is always present. What is happening therefore is that correction is being amplified, and this amplified correction is so large that it over-drives the servo output, thus producing another correction of the opposite sign. This process keeps being repeated. And in fact, when the amplification is sufficiently large, the corrections never die away and the servo goes on oscillating. This over-correction is a characteristic of sampling servos and has no counterpart in the continuous case. Response 1 shows the same case, but slightly nearer the stability boundary showing the near exponential decay.

Velocity and Accelerating Inputs

It has been stated $\Delta f_n = \dfrac{f_{n+1} - f_n}{T}$ .... (2)

from which it follows that

$$\Delta^2 fn = \frac{\Delta f_{n+1} - \Delta f_n}{T}$$

$$= \left(\frac{f_{n+2} - f_{n+1}}{T} - \frac{f_{n+1} - f_n}{T} \right) \Big/ T$$

$$= \frac{(f_{n+2} - 2f_{n+1} + f_n)}{T^2} \quad \text{.............. (23)}$$

And the characteristic equation of the servo was obtained.

$$\left[\Delta^2 + \frac{G}{R_1R_2C_1C_2}(rC_1 + T/2\Delta + \frac{G}{R_1R_2C_1C_2} \right]$$

$$c_n = \Delta^2 u_n \quad \text{............... (9)}$$

With the aid of these two equations the manner in which the servo follow an input which contains first or second derivations of position can be considered which is of considerable importance in servos. It has already been shown that the servo does in fact follow a step function without permanent error. This can be confirmed quite simply by considering the right-hand side of Equation (9).

It is obvious that a step function has no first difference and no second difference (except at $t = 0$). Hence Equation (23) gives zero for the right-hand side, which is the equation we have already met and solved as the complementary function. We have found this contains no permanent terms.

Velocity Input

In a similar manner we consider a velocity input function,

$$u_n = Vt = VnT$$
$$u_n = VnT$$
$$u_{n+1} = V(n + 1)T$$
$$u_{n+2} = V(n + 2)T$$

Substituting in Equation (23) gives

$$\Delta^2 u_n = VT/T^2[n + 2 - 2(n + 1) + n] = 0$$

Again the correction is zero, as $n \to \infty$, $V_n \to u_n$.

Accelerating Inputs

To find the error in the case of the accelerating input let $u_n = \frac{1}{2}at^2 = \frac{1}{2}a(nT)^2$

$$u_n = \tfrac{1}{2}a(nT)^2$$
$$u_{n+1} = \tfrac{1}{2}a(nT + 1)^2$$
$$u_{n+2} = \tfrac{1}{2}a(nT + 2)^2$$

So that $\Delta^2 u_n = a/2T^2 \; [(n + 2)^2 - 2(n + 1)^2 + n^2] \, T^2 = a$

Substituting in Equation (9) we get

$$\left[\Delta^2 + \frac{G}{R_1R_2C_1C_2}(rC_1 + T/2)\Delta + \frac{G}{R_1R_2C_1C_2} \right] c_n = a$$

$$c_n = a \cdot \frac{1}{\Delta^2 + \dfrac{G(rC_1 + T/2)\Delta}{R_1R_2C_1C_2} + \dfrac{G}{R_1R_2C_1C_2}}$$

Noting Δa and $\Delta a^2 = 0$

$$c_n = R_1R_2C_1C_2 \; a/G \quad \text{........... (24)}$$

The correction is now proportional to the acceleration and proportional to the effective time constant of the servo squared. Finally by putting $v_n = u_n - c_n$ as $n \to \infty$

$$v_n = u_n - \frac{R_1R_2C_1C_2}{G} \; a$$

This is now a steady state error as distinct from the transient errors already dealt with. In this case it will be seen that the output will always lag behind the input and these errors are called lags.

This is of course the normal behaviour for a second order servo; position and velocity inputs are followed without error, any inputs containing accelerations or changes of accelerations are only followed with errors.

The Harmonic Response

In the continuous case a sinusoidal oscillation is fed in, the frequency varied and the ratio of the output and input amplitudes measured, in addition the phase lag may be measured. The frequency of the output is the same as that of the input.

In the sampling case the situation is not so straightforward. The servo only sees the input sine wave once every T seconds. Sine waves are of course periodic and hence if any two sine waves have the same values at the instants corresponding to the sampling instants, then the servo will be unable to differentiate between them. Actually all sine waves of the form

$$\exp\left[i\left(\frac{2\pi m \pm \omega_0}{T}\right)t\right]$$

(m-integral) have the same values at the sampling instants, so that the servo presented with a wave of frequency $\left(\frac{2\pi m \pm \omega_0}{T}\right)/2\pi$ will imagine that it is receiving a wave of frequency $\omega_0/2\pi$.

This in turn leads to a second difference, it has been stated that in the continuous case, if the input is sinusoidal then so is the output. In the sampling case this is no longer true. If the input is a sine wave the sampled points lie on a sine wave and the output at the sampling instants also lie on a sine wave but the continuous output wave will not be sinusoidal.

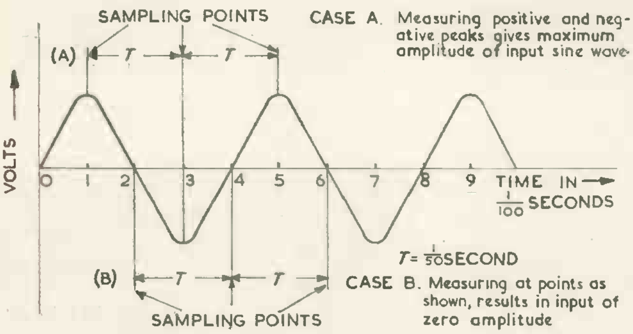


Fig. 8. The importance of the phase in sampling a sine wave

An example may make this clear (Fig. 8). Assume a servo with a sampling frequency of 50 times per second, and an input sine wave of frequency 25c/s. If the servo samples the sine wave when the phase-angle of the latter is 90°, it will next sample 1/50sec later, i.e. corresponding to a phase angle of 270°. This means that the servo is sampling the positive and negative peaks, so that the output will be an almost triangular waveform. Again, should the sampling take place at the intervals corresponding to 0° and 180°, then the output of the servo will be zero. In between these extremes there are, of course, a series of intermediate values for the apparent amplitude of the input sine wave. The foregoing remarks should be sufficient to emphasise that "frequency response" used in the case of the sampling servo is very different from the meaning ascribed to it in the continuous case. Any problem involving frequency response needs very careful consideration therefore.

The frequency amplitude characteristics have been measured experimentally using a low frequency oscillator. This was done using an oscilloscope and measuring the quantity $\frac{\text{amplitude of output.}}{\text{amplitude of input.}}$ The peak-to-peak amplitude of the input, a sine wave, is easily obtained, the amplitude of the output was taken as being the difference between the most positive and the most negative level obtained with a given frequency as the phase at which it was sampled varied. This will be understood if one thinks for the moment of the example of the 25c/s wave given above.

To obtain the complete response it is only necessary to vary the input from 0-25c/s. From 25-50c/s the response curve represents a mirror image and after this it is periodic every 50c/s. The frequency response curves have been measured for the same circuit parameters that have already been considered in the photographs. These are shown in Fig. 7. It will be noted that where the system has a natural "sinusoidal" frequency the usual resonance phenomenon occurs. When the transient response is triangular, the amplitude increases rapidly in the region of the frequency of oscillation ($1/2T = 25$c/s). Where the transient response is aperiodic the amplitude is fairly even.

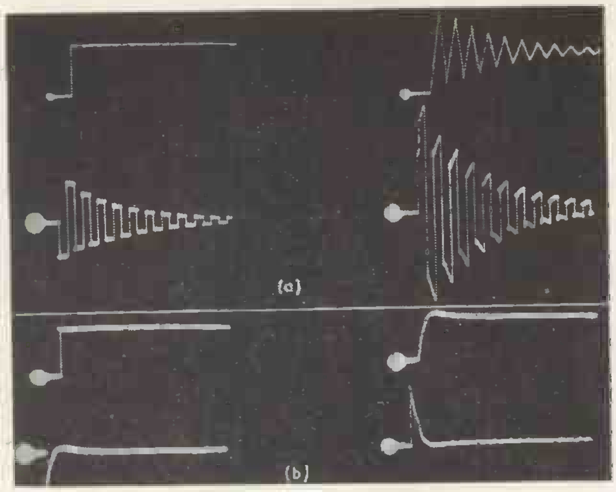


Fig. 9(a). Response No. 1. Sampling case
Fig. 9(b). Response No. 1. Continuous case

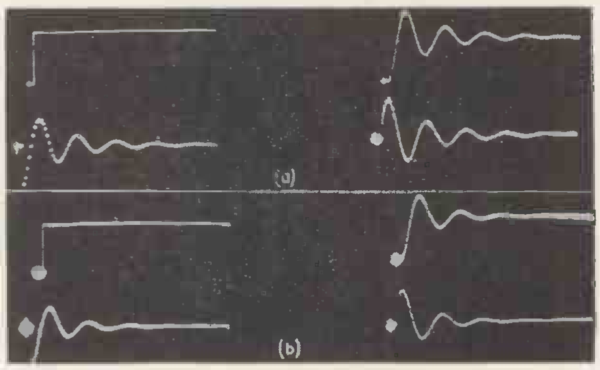


Fig. 10(a). Response No. 2. Sampling case
Fig. 10(b). Response No. 2. Continuous case

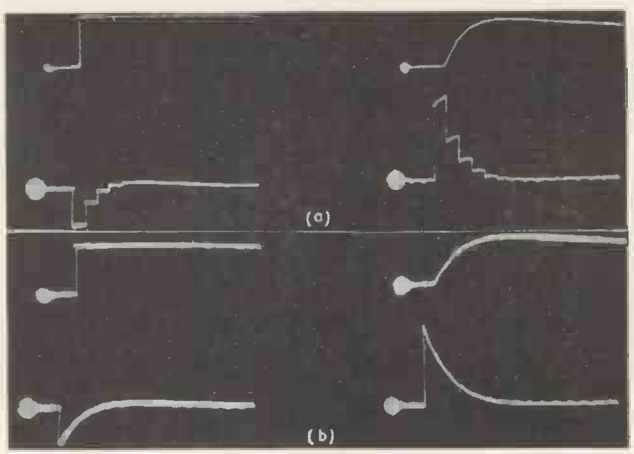


Fig. 11(a). Response No. 3. Sampling case
Fig. 11(b). Response No. 3. Continuous case

Conclusion

A simple second-order electronic servo being fed with sampled data has been considered. This type of servo is of interest because of its increasing use and rather different behaviour from the more usual continuous servo. The characteristic equation has been obtained and solved for a number of inputs. The complementary function has been solved and the transient response examined in some detail. In addition oscillographs of the main responses to a step fuction are given for a sampling servo. Its harmonic response has been experimentally determined.

It has been found that the servo is unstable if there is insufficient damping. Similarly if it is too damped it is

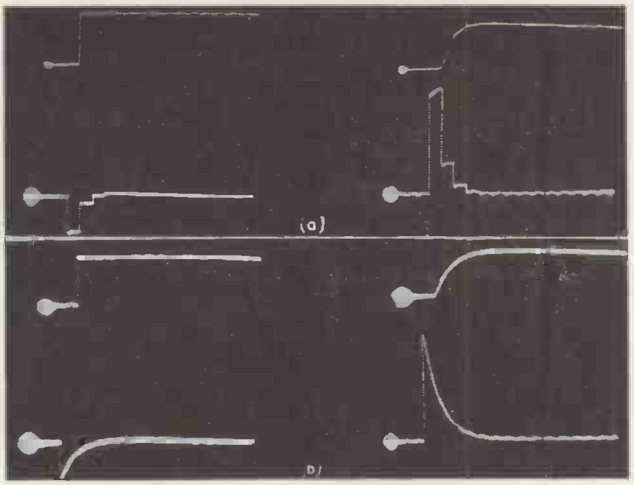


Fig. 12(a). Response No. 4. Sampling case
Fig. 12(b). Response No. 4. Continuous case

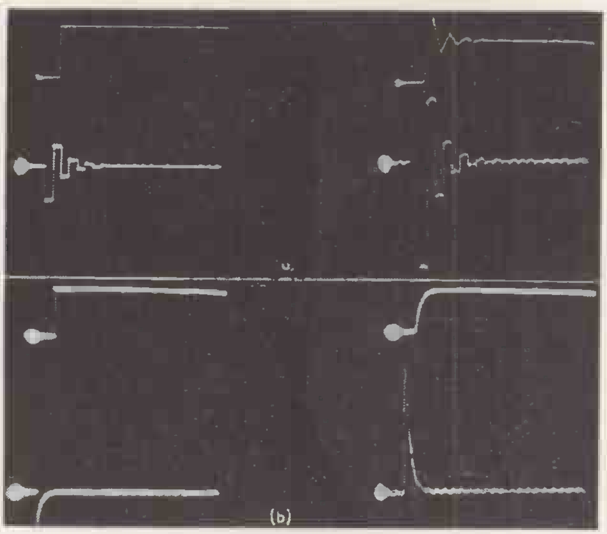


Fig. 13(a). Response No. 5. Sampling case
Fig. 13(b). Response No. 5. Continuous case

again unstable, but this time the oscillation is triangular instead of almost sinusoidal. This in turn leads to more variety in the transient responses. If the sampling frequency is very rapid and the damping is sufficient the behaviour approximates to that of the continuous case.

The servo takes a sampled and therefore discontinuous data and produces an apparently smooth although actually discontinuities in the change of curvature are present. Like its continuous counterpart it follows a step or velocity input without error but lags behind accelerating and higher-order inputs.

When the servo is fed with a sinusoidal input the output is not sinusoidal. The servo is quite unable to differentiate between any waves of pulsatance $\left(\frac{2\pi m}{T} \pm \omega_0\right)$ and its response repeats every $1/T$c/s.

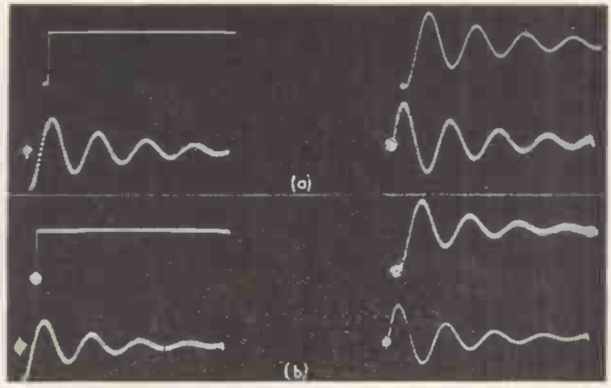


Fig. 14(a). Response No. 6. Sampling case
Fig. 14(b). Response No. 6. Continuous case

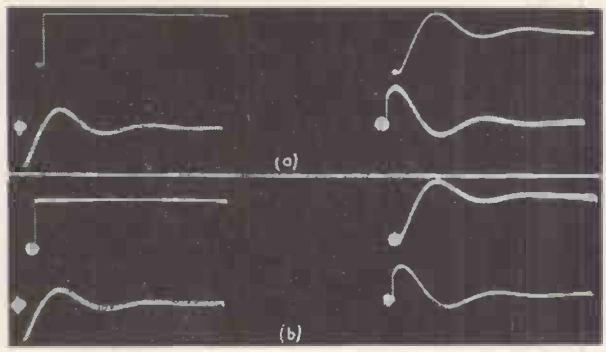


Fig. 15(a). Response No. 7. Sampling case
Fig. 15(b). Response No. 7. Continuous case

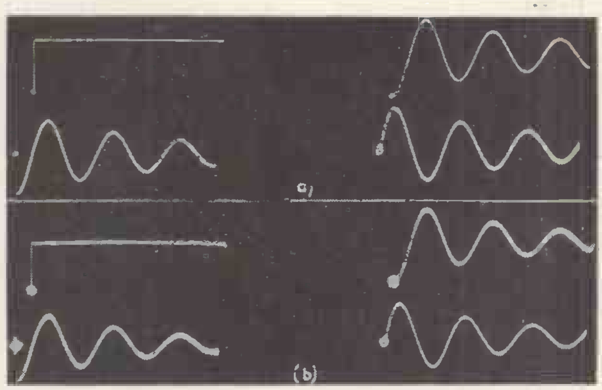


Fig. 16(a). Response No. 8. Sampling case
Fig. 16(b). Response No. 8. Continuous case

Acknowledgments

The author would like to express his thanks to Mr. A. E. Bailey for his help and encouragement and also to the Chief Scientist, Ministry of Supply, for permission to publish this article.

REFERENCE

[1] Holt-Smith, C., Lawden, D., Bailey, A. E. Characteristics of Sampling Servo Systems. Automatic and Manual Control. (Butterworth, 1952.)

LETTERS TO THE EDITOR

(We do not hold ourselves responsible for the opinions of our correspondents)

Measurement of Amplitude Distortion in D.C. Amplifiers

DEAR SIR,—It may not be realized by users of D.C. amplifiers that the well known intermodulation method of testing for amplitude distortion may be modified to yield a quick and simple way of assessing the linearity of D.C. amplifiers.

In place of the two frequencies used in the intermodulation method, a small alternating signal of constant amplitude, and a steady or D.C. signal which may be varied from zero to the full handling capacity of the system are fed together into the amplifier. The method consists of simply measuring the amplitude of the alternating signal at the output of the amplifier for a number of different values of the D.C. signal. The variation of sensitivity of the amplifier is thus determined at points over the working range.

The amplifiers tested in this laboratory were fed with a 1 000c/s signal, and a D.C. signal from a small battery and potentiometer arrangement. The output was measured by means of a rectifier valve-voltmeter designed to deal with both single-sided and push-pull outputs. Fig. 1 shows the circuit of the valve-voltmeter.

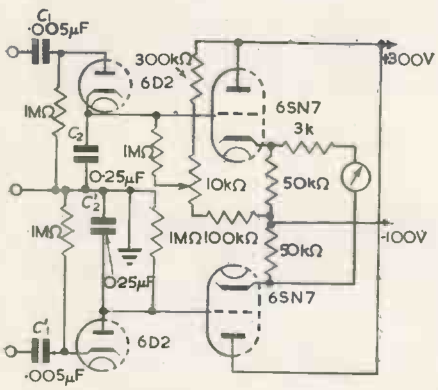


Fig. 1. Circuit of the valve-voltmeter

The 1 000c/s output is rectified by the diodes and fed to the grids of the 6SN7 double triode arranged as a cathode-follower. After adjustment the microammeter gives a reading which is a measure of the sensitivity of the amplifier, and by altering the D.C. input, the variation of sensitivity over the working range may be read off.

Should the amplifier under test have a push-pull output, the capacitors C_1, C_2, C_1' and C_2' must be matched so that

$$\frac{C_1}{C_1+C_2} = \frac{C_1'}{C_1'+C_2'}$$

in order to feed the cathode-followers with inputs which are identical fractions of the two outputs from the amplifier.

The use of the cathode-followers ensures that the effect of any difference in the response of the two halves of the 6SN7 is reduced to a minimum. Such precautions are necessary as the linearity of the two sides of a push-pull amplifier may differ, and for accurate measurement it is important that each should contribute correctly to the reading of the microammeter.

The main advantage of the method lies in its simplicity and ease of operation. The shape of the output-input curve is not obtained directly, but if required may be calculated provided sufficient values of its slope are taken.

Yours faithfully,

W. G. P. LAMB,

Ministry of Supply. A.W.R.E.

Direct Reading Thermistor Bridge

DEAR SIR,—In their excellent article on the design of a direct reading thermistor bridge in the February, 1953, issue of ELECTRONIC ENGINEERING, Messrs. Pearson and Benson mention that the resistance law for bead type thermistors holds over a range from zero frequency to 10 000Mc/s. I believe that I am correct in stating that careful experiments have shown that the error is about 8 per cent at 10 000Mc/s and about 2-3 per cent at 3 000Mc/s, the error being in that direction which would give a low power reading on the bridge. The small amount of power that is lost is presumably dissipated largely in the glass envelope of the thermistor.

On the question of zero compensation and sensitivity compensation, I have adopted the decade resistance box method mentioned but have found it advantageous actually to perform a separate calibration of the chosen disk thermistors, and then to calculate, as shown below, the values of compensating shunt resistors and series resistors from the equation that I derived for this purpose.

I have found that the S.T.C. type KB5251/80 and KB2231/80 are the most suitable for zero and sensitivity compensation respectively and although the method adopted gives only 2-point matching in each case I have observed, for zero compensation at least, an accuracy of better than 0·1 per cent per degree C between 0°C and +40°C. It is difficult to measure the resulting accuracy of sensitivity compensation due to small variations in R.F. power.

Fig. A shows a similar arrangement of

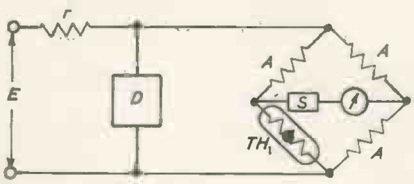


Fig. A. Bridge network arrangement

the bridge network to that in the article. *D* and *S* are decade resistance boxes and for the zero compensation, the R.F. mount containing TH_1 is, enclosed in a suitable container, the temperature of which is varied over the range required. *S* is adjusted to give reasonable sensitivity and *D* is set say every 5°C to give balance. A curve is drawn of *D* against temperature. Resistance *r* is maintained at 3 500 ohms. From the curve obtained, the average slope of the resistance/temperature curve may be read. This is known as the mean required slope = α ohms/°C.

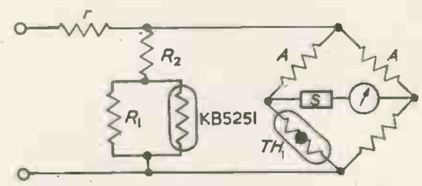


Fig. B. Average difference of resistance between curves over required temperature range

The KB5251 thermistor is calibrated by immersion in transformer oil, varying the temperature over the required range and noting the resistance value, as measured by a Wheatstone Bridge, say every 5°C. A curve of this is drawn and the value of the KB5251 resistance at two suitable points in the temperature range is read. For normal thermistor bridges, temperatures of 10°C and 30°C would be suitable. Let the two resistance values obtained be *a* ohms and *b* ohms with *a* the higher value, and let *t* be the temperature range. It will be noticed that the slope of the KB5251 thermistor in ohms/°C is much higher than the mean required slope, therefore by shunting the KB5251 with a suitable value of resistor, R_1 ohms, the two slopes can be made approximately parallel. From the data above, R_1 can be calculated from the following quadratic equation:—

$$R_1^2\ (a-b-\alpha t) - R_1\ \alpha t\ (a+b) - \alpha tab = 0.$$

Typical values are as follows:—

α = 5·2 ohms/°C.

a at 10°C = 840 ohms.

b at 30°C = 340 ohms

t = 20°C

Then R_1 = 477 ohms.

i.e.

The chosen KB5251 thermistor when shunted with 477 ohms will give the mean required slope of 5·2 ohms/°C at least between the range of 10°C to 30°C. The slope of the combined KB5251 and its shunt resistor can now be plotted on the same graph as the mean required slope. The two curves will be seen to be approximately parallel, the KB5251 plus shunt resistor being at the lower resistance value. The average difference of resistance between the curves over the required temperature range represents the series resistance R_2 that is needed. (See Fig. B).

Wire wound potentiometers may be used and their values set up by means of a Wheatstone Bridge and they can then be locked.

Similar methods can be used for evaluating the sensitivity compensation network S, the values being calculated in the same way. In this case, of course, a stable source of R.F. power is required.

This method has been so successful that test runs, after setting up as above, have shown no chance of obtaining any improvement in temperature compensation by further adjustment of R_1 or R_2.

In regard to bridge sensitivity, I have found the D.C. method of measuring bridge current at balance with no R.F., then bridge current at balance with R.F. and then calculating the value of applied R.F. (which gave some known meter deflexion) to be quite satisfactory. By using a double coil meter of high sensitivity which is shunted to measure initial bridge current, then backed off by current through the second coil, shunt removed and movement reversed (owing to the fact that the second current is lower than the first) it is possible to measure quite accurately the change of bridge current in the presence of the initial standing current.

Yours faithfully,

K. F. TREEN,
Whetstone,
London, N.20.

The authors reply:

DEAR SIR,—We thank Mr. Treen for his interesting letter and we note that using a graphical method of matching he has been successful in temperature compensating a thermistor bridge using KB5251/80 and KB2231/80 disks.

With regard to the first paragraph of his letter the operation of bead-thermistor bridges (balanced or direct-reading) for the measurement of microwave power depends on the premise that P watts of microwave power produces the same change in bead resistance as P watts of D.C. power. In the same way as other workers we have accepted this as being true. Up to the time of designing the bridge (nearly three years ago) and writing the article, the only work tending to disprove this seemed to be that of Reference 2 of the article. This states that a thermistor bridge was checked against the accepted fundamental method of measuring microwave power, namely the water calorimeter. Errors of a few per cent were observed at wavelengths around 3cm, large errors were found in the millimetre range of wavelengths but no figures were given for errors, if any, at wavelengths near 10cm.

It may be that other investigations have been carried out on this topic in the last two years. In fact, Mr. Treen's letter suggests that careful measurements have now been made, possibly by himself. We should be pleased and very interested to have references to, or details of such work.

From Mr. Treen's comments the KB5251/80 disk thermistors seem to be easier to match than the KB420 type for zero-drift compensation. His method of calibrating the disk and using a figure for "mean required slope" should work very well when the resistance/temperature curve to be matched is fairly straight. Much more variation of the shape of the matching-network characteristic is possible when three resistors are used but this is not always necessary.

In connexion with checking the sensitivity compensation practically (paragraph 3 of Mr. Treen's letter) a klystron as a source of microwave power is as stable as its power supplies and a short fixed probe coupled through a crystal rectifier to a microammeter should give a fairly accurate indication as to when the power delivery down the guide is stable. Continuous monitoring of the bridge supply voltage E is also necessary.

A double-coil meter, as suggested, is an ideal means of measuring the change of bridge current in a balanced thermistor bridge.

Yours faithfully,

R. M. PEARSON,
F. A. BENSON,
Department of
Electrical Engineering,
University of Sheffield.

The Fourpole Transmission Matrix

DEAR SIR,—The fourpole transmission matrix derived by Mr. W. R. Hinton in his paper "The Measurement of '*A*' Matrix Elements of Passive Networks" which appeared in the April issue is readily obtained by the elementary operations of fourpole matrix theory.

The six Strecker-Feldtkeller fourpole equations are so related that any transformation matrix may be expressed in terms of the elements from any of the other five. Mr. Hinton's derivative is merely an expression of the transmission matrix in terms of elements selected from the mutually inverse immittance matrices

$$\mathbf{Z} = \begin{pmatrix} z_{11} & z_{12} \\ -z_{12} & z_{22} \end{pmatrix} = \frac{1}{\det \mathbf{Y}} \begin{pmatrix} y_{22} & -y_{12} \\ y_{12} & y_{11} \end{pmatrix} \quad \ldots\ldots (1)$$

$$\text{and } \mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} \\ -y_{12} & y_{22} \end{pmatrix} = \frac{1}{\det \mathbf{Z}} \begin{pmatrix} z_{22} & -z_{12} \\ z_{12} & z_{11} \end{pmatrix} \quad \ldots\ldots (2)$$

where, in his notation,

$$z_{11} \equiv Z_{OCP};\quad z_{22} \equiv -Z_{OCQ};\quad y_{11} \equiv 1/Z_{SCP};\quad y_{22} \equiv -1/Z_{SCQ} \quad \ldots\ldots (3)$$

Equating elements on the principal diagonals in (2) gives

$$\det \mathbf{Z} = z_{11} z_{22} + z_{12}^2 = z_{22}/y_{11} \text{ or } z_{11}/y_{22} \quad \ldots\ldots (4)$$

from which

$$z_{12} = \pm\sqrt{[z_{22}(1/y_{11} - z_{11})]} \text{ or } \pm\sqrt{[z_{11}(1/y_{22} - z_{22})]} \quad \ldots\ldots (5)$$

The transmission matrix in terms of the elements of $\mathbf{Z}$ is

$$\mathbf{A} = -\frac{1}{z_{12}} \begin{pmatrix} z_{11} & -\det \mathbf{Z} \\ 1 & -z_{22} \end{pmatrix} \quad \ldots\ldots (6)$$

which, substituting from (4) and (5), may be written

$$\mathbf{A} = \frac{\pm}{\sqrt{[z_{22}(1/y_{11} - z_{11})]}} \begin{pmatrix} z_{11} & -z_{22}/y_{11} \\ 1 & -z_{22} \end{pmatrix}$$

$$\text{or } \frac{\pm 1}{\sqrt{[z_{11}(1/y_{22} - z_{22})]}} \begin{pmatrix} z_{11} & -z_{11}/y_{22} \\ 1 & -z_{22} \end{pmatrix} \quad \ldots\ldots (7)$$

In view of the definitions (3) the first expression on the right-hand side of equation (7) is identical with that obtained by Mr. Hinton; the second expression is alternative.

Yours faithfully,

S. R. DEARDS,
The College of Aeronautics.

The author replies:

DEAR SIR,—I am indebted to Mr. Deards for drawing attention to the classical analysis of the problem of expressing "*A*" Matrix Elements in terms of the open- and short-circuit impedance measurements. His treatment shows, once again, the power of matrix methods in the derivation of generalities which are rather difficult or tedious to do by less refined methods.

The direct method of making fourpole transformations was not adopted in the article because it requires a greater knowledge of matrix algebra and would therefore appeal to a small minority of readers. If written at the same level, that is, from the defining equations and assuming no prior knowledge of matrix algebra, it may have proved somewhat indigestible as, not only would one have to derive the inverse matrix, but also the transformations of the "*Z*" to the "*A*," and make some explanation of the change in the sign of some matrix elements when the networks are turned end over end so that the direction of power flow is reversed.

In short, Mr. Deards' equations (1), (2) and (3) can only be written down on sight (and understood), by people already skilled in the art, but these people are not likely to require the article as they can very well do their own analysis.

A far wider reading public is well acquainted with equivalent network concepts, and this approach does, in my opinion, give some reality to what might otherwise appear to be just a clever manipulation of abstract symbols according to the rules of an equally abstract game.

These remarks are not intended to detract from Mr. Deard's excellent contribution, but to bring out the point that the topic must be worthy of the treatment otherwise an article is reduced to a tedious technical note which few people will bother to read.

The real point of both my articles on this subject, which I sincerely hope has not been missed, was to encourage the use of "*A*" matrices by showing that a great deal of useful work can be done with no more knowledge than the defining equations and the observed rule of "multiplication" corresponding to the connexion of networks in cascade.

Yours faithfully,

W. R. HINTON,
Staines, Middlesex.

BOOK REVIEWS

Photoelectric Multipliers

By S. Rodda. 180 pp., 46 figs. Demy 8vo. Macdonald & Co., Ltd. 1953. Price 22s. 6d.

THIS monograph, by an author well known for his work in the field of secondary emission and photoelectric devices, is intended to give a fairly simple account of photomultiplier tubes for users of these powerful tools who wish to obtain a more complete insight into their underlying physical principles. The emphasis is thus on basic physics, tube characteristics, and tube applications, rather than on methods of tube manufacture. The author has also brought together a great deal of information from the published literature in order to enhance the value of the book to those more skilled in the art. The pursuit of this aim has, however, led to the inclusion of a certain amount of matter of rather ephemeral interest.

Two introductory chapters summarize the relevant facts on secondary emission and photoelectricity. The latter of these includes a clear account of the definition of sensitivity in microamps per lumen, and gives a useful table showing how the sensitivity of Cs-Sb and Cs-Ag surfaces varies with the colour temperature of a tungsten illuminant.

Chapter III is an historical survey of early designs of secondary emission multipliers, and leads on to a most valuable resumé of Rajchman's 1938 thesis on electrostatic multipliers, in which the essential design features of this class of tube are clearly described.

The design and characteristics of commercially available tubes are discussed in Chapters V and VI, which cover types with focused dynode structures, such as the RCA 931 and 5819 and the range of EMI tubes with venetian blind electrodes. The reader should remember that both RCA and EMI have added new tubes to their lists since the data of these chapters were collected.

The statistical origin of noise in the output of photo-tubes and multipliers is explained in Chapter VII, and its magnitude is compared with the noise signals due to resistors and valves. The importance of high photosensitivity is stressed.

Methods of measuring small output currents from multipliers are described, and lead on to a useful discussion of applications in spectroscopy, astronomy and photometry. Chapter X is devoted to television type applications, and the opportunity is taken to discuss the advantage of secondary emission multiplication in camera tubes.

Two chapters on nuclear radiation detectors, and an appendix, close the book. Chapter XII deals with particle multipliers, of Be-Cu and similar materials, which find application in the detection of heavy ions.

Chapter XI, on scintillation counting, has obviously suffered from a delay between collection of the material and publication so that much of the information given tends to be obsolete, having been published at a time of rapid advance in the subject. In particular, the data given on the relative efficiencies of the main organic phosphors are in inverse order to those which are now accepted for these materials. It would have perhaps been more in keeping with the main part of the book if a fundamental treatment of the phosphor-photomultiplier relationship had been given, rather than a snapshot of the subject as it was in 1950 to 1951.

Apart from some indecision on indices in Chapters VII and VIII, and a persistent misspelling of scaler as scalar in Chapter XI, the production is free from misprints. Printing and binding are of good quality, and the illustrations are clear and well proportioned. At 22s. 6d., the monograph is not dear by modern standards, and may confidently be recommended as generally fulfilling the aim of the author.

J. SHARPE.

Die Gestalt Der Elektrischen Freileitung (The Shape of the Electric Overhead Line)

By Dr. Milan Vidmar. 199 pp., 49 figs. Medium 8vo. Verlag Birkhäuser, Basël. 1952. Price S.fr.19·75.

THIS is not a book for the beginner. It is pleasant to note that its author has lost nothing of his very personal and original style which is well known also from his previous books on transformers and electrical machines. Controversial though the treatment may be in some instances there is no doubt that the designer as well as the user of overhead lines will find many thought-inspiring ideas developed in this volume. Economical aspects are given preponderance. When nearly forty years ago Vidmar's first book on transformers appeared, Korndörfer stated in his review: the book " is written by an expert who lives in and with his problems and knows how to describe them in a fascinating manner." The same may be said about the present volume.

In an introductory chapter the general problem of giving the overhead line its proper shape is discussed on the basis of Kelvin's rule requiring the balance between the amortization of prime costs and the running costs of current transmission. A refinement of this rule is proposed for the particular problem of transmission lines and special consideration is given to the problem of service interruptions. In the next chapters the choice of the most suitable span, of the cross-section of the three-phase line, of the diameter of the conductor, of the various states of the conductor and of the material used for it are discussed. A remarkable feature in this last respect is the recommendation of pure aluminium conductors not reinforced by steel wires for certain cases [see also Science Abstracts B 1439 (1953)]. This proposal certainly merits careful investigation by those concerned. In dealing with the most economical span a very simple formula is derived and its validity is confirmed by practical experiences on the transmission line from Boulder Dam to Los Angeles and on the lines of the British grid system. A final chapter deals with shaping the overhead line as regards its electromagnetical properties. An interesting result to which the author has been led by his investigations may be of some value in the future development of high power transmission. He has found that the ohmic voltage drop in a line loaded with its natural power has only half its expected value if the line shunt conductance is negligible. But, as the author admits, the practical saving due to this discovery will not be very considerable. Vector diagrams and circle diagrams for transmission lines are discussed and some examples of their application are given.

That the author is an original thinker is also shown by the fact that only very few literature references are presented. In this respect one is reminded of the books of C. P. Steinmetz although the mathematical demands on the student are less exacting. The book is very well produced.

R. NEUMANN.

Primer of Electronics and Radiant Energy

By Don Caverly. 330 pp., 44 figs. Demy 8vo. 2nd Edition. McGraw-Hill Publishing Co., Ltd. 1952. Price 47s.

THIS excellently written, printed and illustrated book is one of those rare educational volumes which are so interesting to read that the reader absorbs knowledge quite painlessly.

It covers much of the enormous field of electronics from a basic conception of atoms to such things as fluorescent lamps, radar, television, the effect on plant life of various amounts of light, cosmic radiation, nuclear fission, sound recording, diathermy and a great many others.

The treatment of each is necessarily limited, but is sufficient to give the reader some idea of the subject and is discussed in an easy and authoritative manner which is very simple to follow. It is essentially a digest. It follows no real sequence and is practically impossible to review adequately.

This reviewer found it on the whole very satisfactory and the complaints are small ones. One criticism is of the illustrations on page 239 which are intended to show the workings of a triode valve by comparing it to water in a pipe. The triode is drawn on its side and underneath is the drawing of a "T" joint in a water pipe with a valve in the centre limb. The obvious comparison is that the anode current in a triode (or the water flowing out of the right-hand top pipe) is the difference between the cathode current (or the water flowing into the top left-hand pipe) and the grid current (the valve controlled water flowing down the centre pipe).

It is also a pity that so much space should be consumed on electricity and magnetism including Ohms Law, and the effect of resistors in series and parallel.

The book, which costs 47s., contains 330 pages only, and of these approximately one-sixth is absorbed in this way. It might have been better to assume that the reader either knew as much as is covered in this chapter or could obtain knowledge and pleasure from the book without it.

On the whole a pleasant book to read to find out something of what is going on in many of the branches of electronics.

C. H. BANTHORPE.

The Resonant Cavity Magnetron

By R. S. H. Boulding. 147 pp., 80 figs. Demy 8vo. George Newnes Ltd. 1952. Price 21s.

THIS book can be recommended, almost without reservation, to both those who wish to learn and those who need to know about the mode of operation of the modern magnetron.

The single presentation in the first part of the book of the complicated phenomena occurring in a magnetron should provide no difficulties to the newcomer to the subject even though he may be of limited mathematical ability and this treatment will be found refreshing to the expert.

It is thought, however, that the short Chapter VII would have been rendered much more useful had it there been shown how simply the equation of the threshold line is related to its point of intersection with the cut-off parabola and this would have allowed fuller discussion of the possibilities of scaling magnetron designs.

The desires of the magnetron user have obviously been kept in mind as is shown by the treatment of the Rieke diagram and the long line effect but the reader will learn little of the modern techniques of actual construction. These techniques are, however, common to or developed from those used in the transmitting valve industry and it is probably unfair to expect any treatment of them in a volume of this size.

H. A. H. BOOT

Modulators and Frequency-Changers

By D. G. Tucker. 232 pp., 115 figs. Demy 8vo. Macdonald & C., Ltd. 1953. Price 28s.

THE scope of the book is clearly stated in the introduction; it concerns modulators for use in line and radio systems and is restricted to low power level amplitude modulation at frequencies up to 10Mc/s. It deals in detail with the design and performance of modulators and frequency-changers employing rectifier elements or receiver type valves, in particular with series, shunt and ring modulators. The book is the outcome of the author's many years of design and research experience, and gathers together much material otherwise available only in his papers in a variety of journals and in Post Office Research Reports of limited circulation; it should be warmly welcomed by all engineers who have to do with the design or application of modulators. The book should also appeal to communications students by virtue of the realistic approach adopted towards modulation, which is regarded as a deliberate process aimed at an optimum result rather than a phenomenon arising indirectly from the non-linear behaviour of a circuit element.

An extensive treatment of the principles of amplitude modulation and modulating circuits in the early chapters of the book provides a good background for subsequent discussion of detailed design considerations. The process of amplitude modulation is expressed in terms of the action of a modulating function, usually of approximately rectangular waveform, generated by one of the two signals involved; the detection of single and double sideband signals is treated in the same way. An important inclusion is polyphase modulation, though it is not made clear that at least a three-phase system is required to produce a phase sequence. For purposes of analysis, modulator circuits are in most cases regarded as switched linear circuits with idealized rectangular modulating functions.

Later chapters show how the characteristics of practical modulator elements depart from the ideal, causing modification of the modulating function and deterioration of modulator performance; methods of evaluating these effects are given. Methods of analysis for modulators (series, shunt and ring) with frequency dependent rectifier elements and terminations are also described. The book concludes with a discussion of the improvement of modulator performance which can be effected by the application of negative feedback, and an examination of various forms of modulator distortion.

The book is written in an attractive style and the treatment is clear throughout. No advanced mathematics is used, and it is evident that the author has taken pains to avoid unnecessary algebraic manipulation, which could easily have made the presentation of the subject tedious. The text is illustrated by a large number of diagrams and graphs, and a praiseworthy feature is the inclusion of numerical examples and experimental results for illustrative purposes. An extensive bibliogra-' is provided.

A. R. BOOTHROYD.

Wireless and Electrical Trader Year Book 1953

264 pp. Demy 8vo. 24th Edition. Trader Publishing Co., Ltd. Price 10s. 6d.

A NEW feature in this edition is the Mains Voltage Directory which covers all the principal towns in Great Britain. The comprehensive list of the I.F. values of commercial radio receivers which have been marketed during the past five years has been revised and extended. Other time-saving data ranges from specifications of current radio receivers, legal information and a directory of trade associations. The Directory sections are printed on tinted paper for ease of reference.

One of the principal aims of this book is to assist traders to keep abreast of the constant changes in the names, addresses, telephone numbers and products of the firms engaged in the radio and electrical industries.

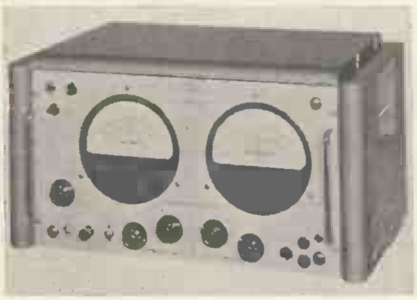
ELECTRONIC EQUIPMENT

A description, compiled from information supplied by the manufacturers, of new components, accessories and test instruments.

Phase Sensitive Voltmeter

(Illustrated below)

COVERING a frequency range of 20c/s to 20kc/s, this instrument indicates on two 6in. centre zero meters the in-phase and quadrature components of an applied test voltage with respect to a given reference voltage. Precision thermo-couple type wattmeters are used to indicate the levels. Their principle advantage lies in the fact that the indicated values of the resolved components are unaffected by the presence of harmonics or spurious frequencies in the signal under test, provided that the reference signal is of sine waveform. The provisional accuracy of indication is better than ± 2 per cent of full-scale on all ranges. The input range for both signal and reference channels is 15mV-15V in seven half decade sensitivity



ranges. Other features include an input impedance for both channels in excess of 50MΩ; provision of internal or external earthing to obviate earth loops, and an internal regulated H.T. supply.

The instrument can be used for measuring the transmission characteristics of amplifiers, networks, transformers and high speed servo systems since direct indication of the resolved components at a series of test frequencies enables their values to be plotted easily on a Nyquist diagram. The instrument is also suitable for use with A.C. strain gauge systems since by switching the quadrature meter out of circuit, the reference meter can be calibrated direct in strain and will provide indication free from errors introduced by phase-shift in the system or by the presence of harmonics.

Solartron Laboratory Instruments, Ltd.,
22 High Street,
Kingston, Surrey.

Plastic Film Capacitors

(Illustrated centre column)

THE "Plastapack" plastic film capacitor is a high performance component for use in instruments where capacitors of the highest possible grade are required, as for example computors and medical and industrial equipment. The dielectric consists of a specially treated form of polystyrene film and the complete capacitors are hermetically sealed.

The outstanding feature is the exceptionally high insulation resistance, this is indicated in the following comparisons:—

Insulation Resistance of a Typical 1μF Capacitor at 20°C

With paper dielectric ...	10 000MΩ
With mica dielectric	20 000 to 30 000MΩ
With plastic film dielectric	500 000 to at least 1 000 000MΩ

It will be readily apparent that this characteristic will be invaluable in any instrument where long time-constants are required. For normal routine testing and production purposes the manufacturers quote an insulation resistance of not less than 250 000MΩ/μF in the higher capacitance values, and in excess of 700 000MΩ absolute in all other values, at 20°C in both cases.

The decrease in insulation resistance with increase in temperature is also greatly superior to a paper dielectric. The temperature coefficient of capacitance for all types of Plastapacks is $(-150 \pm 60) \times 10^{-6}/°C$, while the power factor is of the order of ·0005 over a wide range of frequencies.

The capacitors are available in a range of values from 0·01μF to 4μF in rectangular metal cans and from 100pF to 0·005μF in tinned copper tubes. The cans have glass insulated terminals and the tubes wire terminations brought out via PTFE (polytetrafluorethylene) bungs.

The present working temperature range is −40°C to +60°C but it is hoped to raise this to +70°C in the near future. The working voltage is 350V D.C. and the test voltage 700V D.C. The standard tolerance of capacitance is ±20 per cent but they can be supplied as close as ±2 per cent or better at extra cost.

The Telegraph Condenser Co., Ltd.,
North Acton,
London, W.3.

Instrument Soldering Iron

THE latest addition to the range of Henley soldering irons is the Solon instrument model. This is a 25W iron for use on 220-240V supply. It has an overall length of 9in., a bit diameter of 3/16in., and weighs 3¼oz. This iron is completely demountable and all parts can be easily replaced.

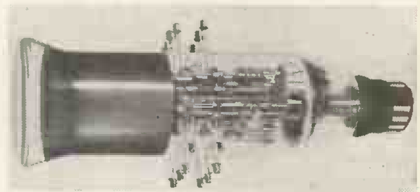
W. T. Henley's Telegraph Works Co., Ltd.,
51-53, Hatton Garden,
London, E.C.1.

Four Gun Cathode-Ray Tube

(Illustrated below)

THE four gun cathode-ray tubes manufactured by 20th Century Electronics Ltd. employ for independent electron guns each with characteristics similar to their type D.6. The four sets of deflector plates are brought out to side connexions arranged on either side of the bulb. The pairs of Y plates are screened both from each other and from the X plates to give freedom from intermodulation.

The four grids and four focus electrodes, together with common A_3, A_1, cathodes and heaters (in pairs) are con-



nected to the standard G.E.C. twelve side-contact cap which has four additional sockets in the base plate.

The sensitivity (V_{a3} × mm/V) of the X plates is 570 and that of the Y plates is 530; the diameter of the screen is approximately 160mm.

20th Century Electronics, Ltd.,
Dunbar Works,
West Norwood,
London, S.E.27.

Pullin 1½in. Meters

(Illustrated top right)

THE new Pullin Series 15—1½in. meter has been designed to satisfy the need for a precision and robust instrument wherever space and weight are limiting factors. It is an ideal meter for use in portable electronic and communications installations.

This meter is made in two types, the non-illuminated and the illuminated. The non-illuminated is available in various self-contained ranges from 50μA up to 500mA, and voltages up to 250V inclusive. The illuminated type can be supplied with self-contained ranges from 50μA up to 10mA. For higher ranges external shunts or multipliers would be required. The illuminated pattern is made possible by a simple yet ingenious method of construction. A pigmy lamp mounted on a specially designed assembly at the rear of the case is inserted within the open area of the D'Arsonval type movement. The light from the lamp passes through the translucent dial and is diffused over the scale. Lamps are

supplied for either 6·3V or 12V, and can be used on A.C. or D.C. The lamp is easily replaced without disturbing the movement or scale.

The case is of moulded bakelite and provided with back of board metal clamp for flush fitting. All meters are calibrated for non-magnetic panels. If steel panels are to be used the thickness of the panel should be stated when ordering.

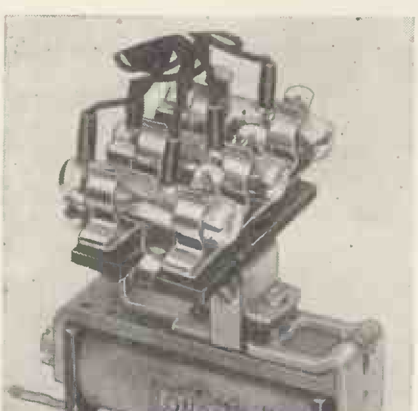
Measuring Instruments (Pullin) Ltd.,
Winchester Street,
London, W.3.

Mercury Switch Relay
(Illustrated below)

THE Londex type KR/QA is a sensitive relay fitted with heavy duty mercury switches and can be used to obviate the use of a heavy contactor in electronic control gear. The use of mercury tubes not only increases the switching capacity of the relay but also widens its scope of application, as this type of enclosed contact ensures safe operation in the presence of inflammable vapours, while control over long pilot wires and the design of intrinsically safe circuits is facilitated by the low consumption of the relay.

The relay is intended for D.C. operation and a range of windings is available up to a resistance of 20kΩ. The power required for operation varies from 0·25 to 1W depending on the number and size of mercury switches fitted. Various switch arrangements can be fitted, from three switches rated at 6A each to two switches rated at 20A each.

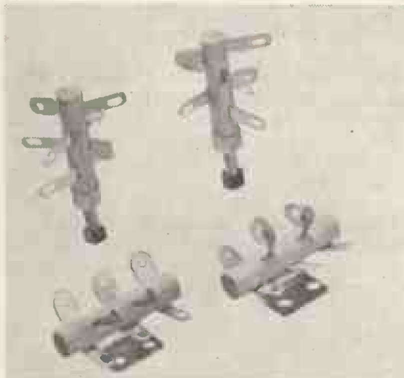
Londex, Ltd.,
207 Anerley Road,
London, S.E.20.



Cactus and Porcupine Tag Mountings
(Illustrated below)

THESE two components have been designed for securing and connecting components in radio and electronic equipment.

In the case of the Cactus four live tags are provided, and the component is suitable for vertical mounting, while the Porcupine has two earthing points and three live tags and is suitable for horizontal mounting.



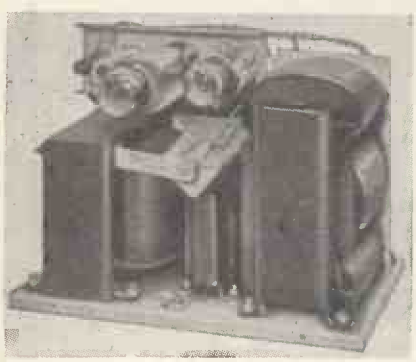
In each case the bodies are of porcelain; all metals parts are silver plated, and a feature of both types is that the tag strips may very simply be removed and reversed, or put into any variety of combinations that is required. The Cactus is 1-1/10in. high when mounted, and the Porcupine is 8/10in. high and 1-1/10in. long when mounted.

The insulation resistance is better than 100kMΩ and they are suitable for working up to 750V with a flash-over of 4·5kV R.M.S.

United Insulator Co., Ltd.,
Oakcroft Road,
Surbiton, Surrey.

A.C. Stabilizers
(Illustrated below)

THE BAVR voltage stabilizers are made in three sizes, 200VA, 500VA,



and 1kVA, and are independent of frequency between the normal commercial limits of 47-52c/s.

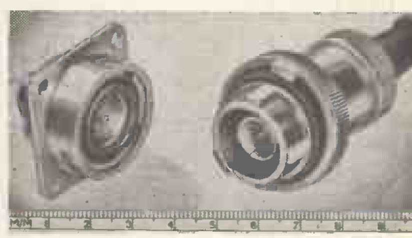
The control unit is a magnetic amplifier the inductance of which varies with the D.C. passed through control coils. Stabilization is achieved by monitoring the output side and regulating automatically the D.C. component, so as to adjust the A.C. output and keep it constant within precise limits. The stability attained is 0·15 per cent for an input of 235V + 6 per cent to −12 per cent.

Claude Lyons, Ltd.,
180-2 Tottenham Court Road,
London, W.1.

Oscilloscope Voltage Calibrator
(Illustrated below)

THE purpose of the model 207 oscilloscope voltage calibrator is to measure the input voltage to the oscilloscope by comparing the deflexion given by the signal with that obtained from a calibrated voltage source. The output from the calibrator is adjustable from 4mV to 100V (peak-to-peak) in 15 ranges and an accuracy of ±4 per cent is claimed.

Electronic Assemblies,
546 Kingsland Road,
London, E.8.



Double Screened Coaxial Fittings
(Illustrated above)

THESE plugs and sockets have similar features to the Belling-Lee "Screenectors" but they are designed for double screened coaxial cable with an overall diameter from 0·156in. to 0·312in. The electrical performance is expected to be well within the requirements of specification R.I.C./322.

The outer housings are constructed of light alloy and are splash proof but not sealed. A quick action retaining ring is fitted to the plug. The dielectric between the centre conductor and the inner braid is polythene, and that between the inner and outer braids nylon filled phenolic material. To facilitate soldering the centre plug and socket contacts can be removed from the insulant.

Belling and Lee, Ltd.,
Cambridge Arterial Road,
Enfield, Middx.

Notes from the Industry

The University of Liverpool is arranging, as part of its annual programme of advanced refresher courses for graduate research workers, engineers, and others in industry or research establishments, a course of eight lectures on the Theory and Applications of the Electric Arc. This course will describe the electrical, optical, thermal and gaseous properties of the electric arc and will then apply this experimental and theoretical data to a study of the behaviour of arcs occurring in engineering practice. The lectures will be held in the University of Liverpool on Tuesdays at 7 p.m., commencing 13 October, 1953, and the fee is £2 2s. Further particulars may be obtained from the Director of Extra-Mural Studies, 9 Abercromby Square, Liverpool, 7.

The College of Aeronautics sixth annual presentation of diplomas and prizes was held on 26 June at the College. The presentations were made by the Rt. Hon. Lord Hives. Among those who received diplomas were the following: Mr. D. J. Mayes for a thesis on "Continuous Wave Distance Measuring Equipment" and Mr. M. A. Perry for a thesis on "A New Method of Measuring Aircraft Speed and Height by Radio".

Radio Industry Training Scheme. The first certificate under the Radio Industry Council scheme for training radio technicians has been awarded to Mr. Bernard Thomas Hill who has received his practical training as an electronic engineer with A. C. Cossor, Ltd. The scheme for training radio technical and laboratory assistants was inaugurated in agreement with the Ministry of Labour and National Service to increase the number of technicians available in the industry. Trainees register with the Radio Industry Council at the age of 16 or 17 and receive a nationally recognized certificate on completion of training at the age of 21.

Metropolitan-Vickers Electrical Co., Ltd., announce that Dr. Willis Jackson has been appointed Director of Research and Education and has also joined the board of the company. Dr. Willis Jackson has been Professor of Electrical Engineering at the Imperial College of Science and Technology, University of London, since 1946.

The Chancellor of the Exchequer, Mr. R. A. Butler, has accepted the invitation of the Radio Industry Council to be guest of honour at the annual banquet at the Savoy Hotel, London, on 18 November next.

Sir George H. Nelson, chairman and managing director of the English Electric Group, recently opened the Nuclear Particle Laboratory at Queen Mary College, University of London. The inauguration of this laboratory is the first occasion on which applied nuclear physics has been officially instituted as part of the undergraduate engineer's education in this country.

The British Thomson-Houston Co., Ltd., announce that Mr. Hugh Jack has relinquished his executive duties as Chief Electrical Engineer. He will be succeeded by his chief assistant, Mr. G. S. C. Lucas.

Electrothermal Engineering Limited announce that they have now commenced production in a unit they have acquired at Hamlet Court Road, Westcliff-on-Sea, which will give the company a considerable increase of production space. The premises at 270 Neville Road, London, E.7, will still function as Head Office.

The BBC announce that Mr. F. M. Dimmock, Head of the Equipment Department, will retire from the Corporation in October this year. Mr. E. C. Drewe, Assistant Head of Research Department, will succeed Mr. Dimmock.

Oryx Electrical Laboratories have appointed as their sole distributors Messrs. A.N.T.E.X. (Anglo Netherland Technical Exchange, Ltd.) of 3 Tower Hill, London, E.C.3. All future orders and inquiries for soldering instruments will in future be dealt with by them.

New BBC Transmitting Station at Towyn. This station is now complete and will take over the service on 341m (881kc/s) from the temporary transmitter which has been operating from a caravan on the site since December, 1952. This new Welsh transmitter has a power of 5kW—the highest so far used in the group of additional low-power stations which the BBC, since 1951, has been bringing into operation in different parts of the United Kingdom to improve the reception of the Home Services.

Errata. In the description of the Mullard Electronic Temperature Controller on p. 266 of the June, 1953, issue, "$\pm 0{\cdot}2°C$" should read "$\pm 0{\cdot}02°C$".

On p. 298 of the July, 1953, issue, the reference at the foot of page 298 in connexion with "Fine Tuning Arrangements" should read "Communication from The Telefunken Co. via E.M.I. Ltd."

The following amendments should be made to Mr. C. Morton's Letter to the Editor on p. 311 of the July issue. The equation at the foot of column 1 should read $E_0/E = 1/[1 - (\alpha_1 + \alpha_2) + (1/\mu)]$. Column 2, line 11, this equation should read $m = v_2/(v_1 + v_2)$. The two line drawings should be transposed, i.e., Fig. C for Fig. D and vice-versa. For "Fig. 6" in the text read "Fig. C". On p. 312, column 1, for "Fig. 7" read "Fig. D".

PUBLICATIONS RECEIVED

ALL-INSULATED INDUSTRIAL WIRING SYSTEM describes the Various BICC wiring systems which are available for many domestic and industrial applications. The C.M.A. standard cables listed in this catalogue are made and tested in accordance with the Standards and Formulæ of the Cable Makers' Association and each type of cable and flexible cord mentioned has an individual reference number common to all members of the C.M.A. British Insulated Callender's Cables Limited, 21 Bloomsbury Street, London, W.C.1.

EQUIVALENT RADIO TUBES VADE-MECUM is the 10th edition in the series and is intended to be a quick reference for the possible international exchanges or substitutions of radio tubes. The tables of this book are devised to give a maximum of practical information in a simple form. P. H. Brans, Ltd., Antwerp.

ELECTRONIC THERMOSTAT, THERMOMETER BRIDGE, MULTIPOINT THERMOMETER and ELECTRONIC RESISTANCE THERMOMETERS are the titles of four brochures produced by Fielden (Electronics) Ltd. explaining the design, purpose and operation of these instruments. Fielden (Electronics) Ltd., Manchester.

PYE SCIENTIFIC INSTRUMENTS is a new catalogue featuring the scientific instruments currently produced by W. G. Pye & Co. Ltd. It includes details of a number of new instruments in addition to information on the products already manufactured by this company. Copies of the catalogue are obtainable free on application to W. G. Pye & Co. Ltd., "Granta Works", Newmarket Road, Cambridge.

BAKELITE LAMINATED GEARS is a booklet which describes the Bakelite laminated silent gear material, types of gears, machining, installation principles, lubrication, etc. There are also tables giving the physical and mechanical properties and limits of thicknesses of this material, together with illustrations. Bakelite Limited, 12-18 Grosvenor Gardens, London, S.W.1.

THE HELVIN PRESSURE BUNG is a brochure describing the Hellermann product which has provided an answer to the problem of passing electrical cables through pressure bulkheads in aircraft. Hellermann Ltd., Tinsley Lane, Crawley, Sussex.

LIST OF BROADCAST RECEIVING APPARATUS APPROVED AS SUITABLE FOR USE IN SCHOOLS is the fourth revised list issued by the School Broadcasting Council for the United Kingdom. All the apparatus mentioned has been tested by the Council under school conditions. Copies of the list and further information may be obtained from the Secretary, School Broadcasting Council for the United Kingdom, 55 Portland Place, London, W.1.

BRITISH TELECOMMUNICATIONS RESEARCH is a brochure of introduction to Taplow Court, home of British Telecommunications Research Ltd. and gives a brief insight into the work of the team of specialists working there. British Telecommunications Research Ltd., Taplow Court, Buckinghamshire.

AERIAL is a new quarterly which was published by Marconi's for the first time in January this year. It is distributed by them to some 3 000 interested people all over the globe and tells the contemporary story of the achievements of Marconi's Wireless Telegraph Company in the building and installation of every type of capital wireless equipment. Marconi's Wireless Telegraph Company Ltd., Marconi House, Chelmsford.

A CATALOGUE OF RECENT PUBLICATIONS has been issued by The Institute of Physics which covers books in the Physics in Industry Series, Journal Supplements, Monthly Journals, books on the Profession, Monographs for Students, Reports and Pamphlets, Charts and Nets for X-Ray Crystallography, etc. The Institute of Physics, 47 Belgrave Square, London, S.W.1.

TELEVISION MANUFACTURERS' RECEIVER TROUBLE CURES VOLUMES 1 AND 2 are the first two books in a series of volumes which deal with specific American television receiver troubles and their cures. These trouble cures are the American manufacturers' answers to some of the problems that may arise in their particular receivers. John F. Rider Publisher, Inc., 480 Canal Street, New York, N.Y., U.S.A.