# *Commentary*

A BILL dealing with the future of television in this country is shortly to come before Parliament and without doubt the main interest will be centred on the controversial question of sponsored television. We feel that there is a good case for an alternative system to the BBC although we are by no means certain that sponsored television is a satisfactory solution.

The BBC has earned for itself a reputation for accuracy and good taste which is the envy of the world, and no praise is too high for its superb handling of the Coronation, particularly with regard to the scenes in the Abbey, but to suggest that these recent performances are typical examples of BBC entertainment, and to compare them with the alleged mishandling of the subsequent American rebroadcasts is, in our view, quite misleading.

Unquestionably the BBC achieved its finest hour at Westminster, but this in itself is not a reason for appointing the BBC as the sole arbiter of what is good for us in television.

Television programmes of the scope and magnitude of the Coronation do not occur every week and there are many hours of broadcasting—both in sound and television—which the BBC has to fill up with whatever material it can afford and has to hand. Many of these programmes are mere "pot-boilers" and no one pretends—certainly the BBC does not—that these reach any great cultural or intellectual heights. In the business of providing ordinary every-day entertainment there is no reason to suppose that the sponsoring organization will do any worse than the BBC—they may even do better—nor need it be assumed that British sponsoring will follow the American pattern.

Behind all the opposition to sponsored television there is the implication that the viewing public, who are surely no different to any other section of the community, possess no sense of taste or power of discrimination and must therefore be guarded against anything which is not considered good for it. There are already adequate safeguards for the films and press to prevent the libelous or the obscene, and no doubt similar arrangements can be made for sponsored television so that it does not offend the accepted standards of good taste. Thereafter if sponsored television fails to provide programmes that do not enlighten or entertain the viewer has only to switch off or change over to the BBC:

On the other hand it should not be assumed that the BBC has failed to realize the full possibilities of this new form of entertainment. Left to itself, the BBC could, we are sure, work out its own destiny and develop a range of programmes comprising all that is best in the way of plays, music, and so on, with outside broadcasts of current events that would satisfy most of the different public tastes for the greatest possible time within the framework of the existing single network.

What is so alarming about the BBC monopoly is the possible State intervention in and control of the monopoly, and it is for this reason alone that an alternative system becomes so highly desirable.

But once the general pattern of sponsored television has been agreed upon and the necessary safeguards arranged it will require more than a mere nod of the head on the part of the Government to bring sponsored television into being and we await with interest the publication of the Technical Advisory Committee's report, which is expected shortly, to learn how the various technical difficulties have been overcome.

In the first place, the BBC is unlikely to sell any of its "dead" time to the sponsoring organizations, so that the only outlet to the latter is a separate network of stations operating mostly in the u.h.f. band. To provide a coverage equal to that of the BBC's television network some 50 transmitters would be required, of which six to eight would be in the London area. To start with only the more populous areas of the country would be dealt with, but even so the connecting together of a large number of stations for the simultaneous transmission of a live programme is likely to be an extremely costly undertaking. The cost of 50 such stations with all their mobile apparatus and ancillary equipment might well exceed £1 000 000, and as a similar sum might be required to join them together we doubt if the necessary capital can be raised by a sponsoring group. Moreover, trained technical staff in sufficient numbers to man the stations are not easy to come by at a moment's notice. Live programmes, except for local transmission or from a few closely grouped stations can therefore be ruled out and the only choice is the film recorded programme.

Secondly, the viewer himself figures prominent in the discussions on sponsoring. Even when the standards of programmes have been fixed, the stations erected, staffed, and transmitting, means have still to be provided at the receiving end in the form of an adaptor and—depending on the frequency bands allocated—an additional aerial system as well. The cost of such aerial and adaptor would be about £15, an additional outlay which many viewers may not wish to incur.

These technical and economic aspects are formidable and may well delay the start for several years.

# Flight Simulators

By K. H. Simpkin*, A.M.I.E.E., A.M.Brit.I.R.E., and E. T. Emms*, B.Sc., A.R.C.S.

*This introductory article is intended to bring to the notice of readers, engaged in other spheres of activity, the new and expanding field of flight simulation. The history of flying training devices is briefly covered and leads to a consideration of the increasing demands of training to meet which the modern electronic flight simulator has been evolved. The technical and economic implications of these simulators are examined and the authors conclude with a description of a modern flight simulator, in which the fundamental design requirements are treated in general terms and methods of computation are indicated.*

IN recent years, a new term has been coined in aviation circles. The term " flight simulator " is used to describe one of the most remarkable applications of electronic art today, that of reproducing the true flight characteristics of an aircraft in an equipment which is permanently fixed to the ground.

The story of aircraft flying training devices is a long and interesting one. For many years scientists and engineers have attempted to produce a machine which will enable aircrew to practice flying without the attendant danger and wastage entailed in the use of a real aeroplane.

The well-known " Link " trainer, used extensively during the war, enabled pilots to practice the technique of using radio navigation and landing aids while flying " blind " on instruments and released many aircraft flying hours for operational use, which might otherwise have been used for training purposes. This trainer consisted of an enclosed



*The type D4 mechanically and pneumatically operated blind flying trainer, a forerunner of the flight simulator*

cabin, fitted with aircraft type controls and instruments, which was pivoted in such a manner that it was free to move to a limited extent in response to the actions of the pilot. Built-in mechanical and pneumatic mechanisms produced motions of the cabin in accordance with the position of the controls, and contact was maintained between the pilot and an instructor who was seated at the control desk, upon which the computed motion of the trainer over the ground was recorded and from which radio signals were given.

This type of trainer fulfilled a valuable purpose, but suffered from the disadvantage that the cabin and the flying characteristics produced were generalized and not those of any specific type of aircraft. Experienced pilots tended therefore to be aware of an artificiality and the training value of such devices was accordingly somewhat restricted.

The new flight simulator, however, sets out to reproduce the characteristics of a particular aircraft as accurately as possible and endeavours to maintain a high degree of realism in order to take full advantage of the additional training value provided by " atmosphere ".

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* Air Trainers Ltd.

Briefly, the simulator may be divided into several distinct units. The cabin, in which the training crew are housed, is a mock-up version of the crew section of the aircraft fuselage and is often that section of an actual aeroplane. This unit, which is fixed to the ground and does not move, contains every item with which the real aircraft is normally equipped. The windows are, of course, misted, but realism is carried to the extent that it is impossible for the crew to tell, once in the simulator, whether they are in a real aircraft or not.

By the side of this dummy cabin is a computor rack, the function of which is to calculate, from knowledge of the crew's actions from the start of the flight onwards, what will be the reading of every instrument in the cabin and what will be the reaction force on every control. This data is fed back into the fuselage and is used there to reproduce the instrument readings and to actuate the forces on the controls. In addition, this computor calculates the position of the aircraft with reference to the ground and supplies such data to the control desk.

The control desk, at which the instructor is seated, is a centralized point from which external conditions can be applied to the simulator. It is here that the position of the aircraft is plotted on a map from data supplied by the computor. From his control desk the instructor can reproduce, by the operation of switches, a large variety of emergency conditions for the crew to deal with. He may produce the effects of weather changes and may also select suitable radio navigational aids for use by the crew, at such times as they would have come within range.

The mention of navigational aids brings us to the third unit, which is a separate electronic computor, the function of which is to produce a realistic simulation of the radio aids normally used on the type of aircraft simulated, in order to enable the crew in the fuselage to make use of these aids while training.

The whole simulator is normally housed in a special building on the aerodrome. It usually incorporates several hundreds of radio valves in its make-up, together with many miles of wire and complex electro-mechanical com-

ponents. As may well be imagined a careful study of the probable uses of such a machine is necessary before one is purchased by an airline company.

Let us, therefore, examine the uses and economics of a flight simulator. Unfortunately few figures of costs are available in the United Kingdom, so American figures will be quoted as indicative of the general trend.

An average multi-engined flight simulator costs only slightly less than the aircraft which it simulates, that is approximately £100 000, depending upon the complexity involved. Yet, according to the Curtiss-Wright company, this first cost can be reclaimed within five years of use. For example from figures quoted by that company, on flight simulators they have constructed in America, the operating cost of a flight simulator is of the order of

which does arise produces such a violent press reaction. Investigation into these accidents often shows that the circumstances were such that the pilot was unable to take effective action because of lack of training in dealing with such an emergency.

This fact cannot, however, be blamed on the crew or the company, when it is realized that many of these emergency conditions are not reproducible at will in an aircraft without unjustified hazard to aircraft and crew. It is to the credit of the crews that all emergency conditions which are amenable to practice are practised consistently. It rarely happens nowadays, for example, that a twin engined aircraft crashes due to failure of one engine.

The flight simulator, however, being a true representation of the aircraft, is able to fill the breach. Severe emergency
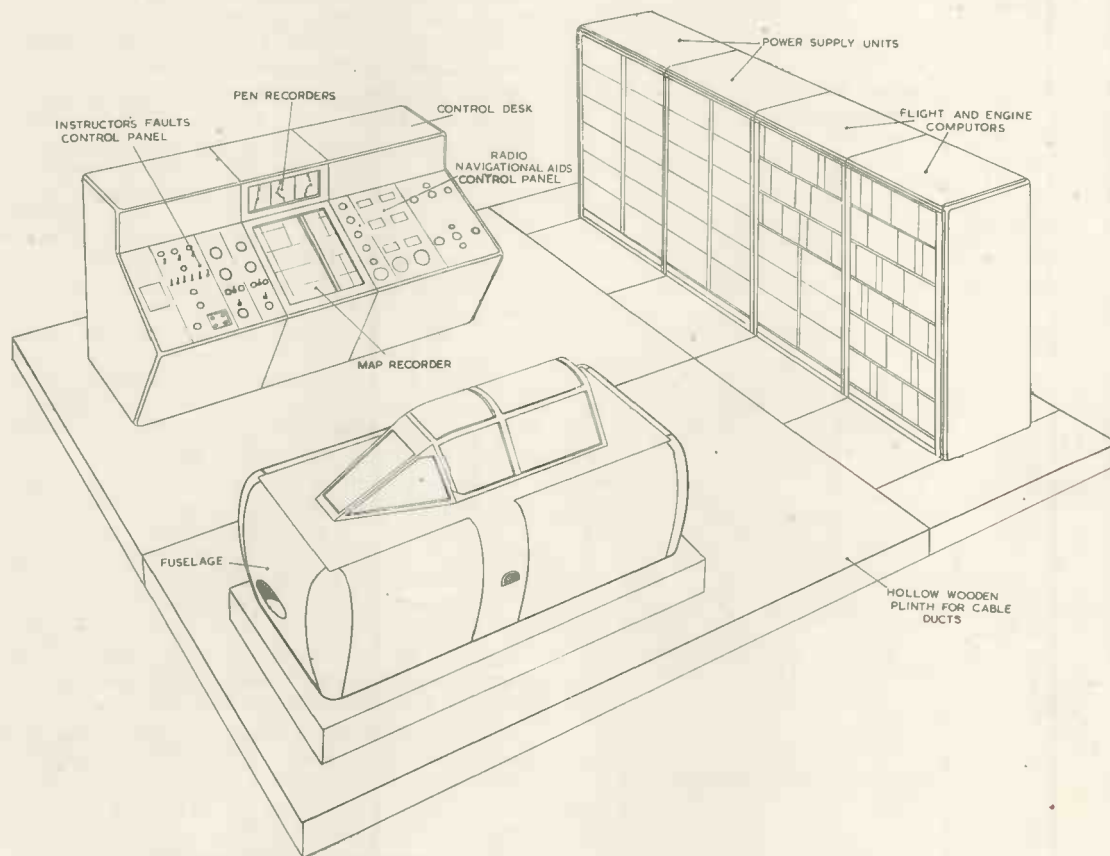


*The layout of a typical flight simulator*

$30 to $40 per hour, whereas the cost of flying a multi-engined aircraft such as a D.C.6 is about ten times this.

If, by the use of a simulator, an aircraft can be released for passenger service, the revenue potential then gained is quoted as of the order of $1 500 per hour. That the flight simulator is effective in training personnel can be seen when Pan-American Airways quote a reduction of flying time on conversion courses to new types from 21 hours flying to 8 hours by the use of a simulator. This, in itself, using the simulator for 13 hours a day would result in a saving of over a million dollars a year.

Apart from the economic aspect, however, there is the further viewpoint of safety. It is unfortunate that, in these days of safe air travel, when the major airline companies are publishing consistently low figures of accident rate per passenger mile of operation, the occasional accident

conditions, such as engine cut on take-off can be simulated and crews may thereby be given hours of practice in dealing with situations which would result in unjustifiable risk if attempted in the real aircraft.

Furthermore, it is a proven fact, that crews, after undergoing a period of simulator training, do work together far more efficiently and calmly under emergency conditions.

This advantage of the simulator does not apply only to the flying technique. Navigational training can be effectively carried out using the equipment normally carried, and no longer is the radio operator or navigator segregated in his training, but can carry it out, at all times, as an integral member of the crew.

We have, so far, discussed the flight simulator itself, as a number of large units, each having a specific function. Let us now examine, briefly, how the simulator works.

An aeroplane is a physical body, and like all other physical bodies, it obeys the laws of nature. When flying in the air, the aeroplane will remain stationary or in steady motion, that is in equilibrium, unless acted upon by external forces. For example, it would accelerate towards earth under the influence of its own weight if the force of gravity were not exactly balanced out by the lift force on the wings.

The forces which act upon an aeroplane are produced by its weight, by the thrust of its engines and propellers or jets, and by the flow of the air over its wings and tail-surfaces. A change in any one of these results in a change of motion of the aircraft until the new force is balanced out and equilibrium is restored. The instantaneous motion of the aircraft is therefore determined by the instantaneous configuration of the forces acting upon it.

In order to deal with these forces and the resultant
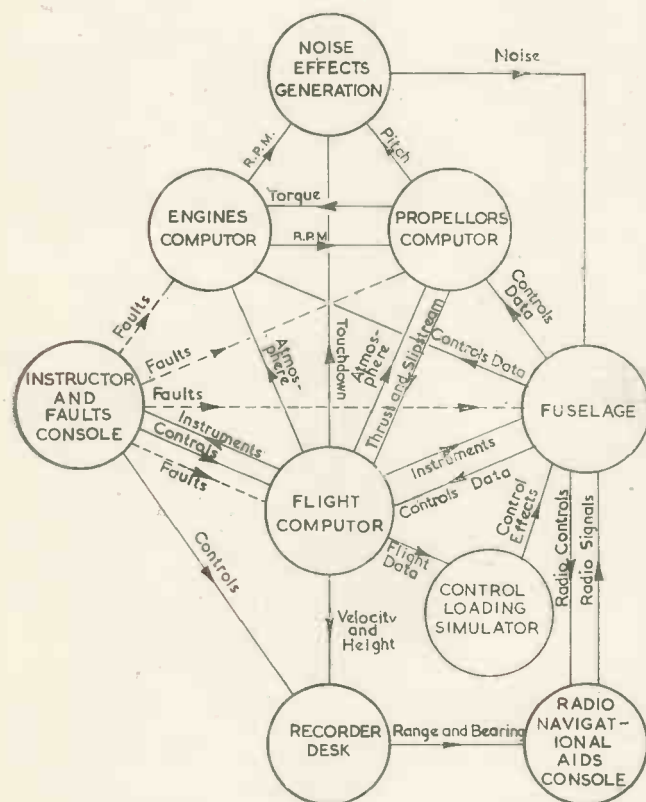


*Routing the information in a flight simulator*

motions of the aircraft in a logical manner, it is convenient to consider all motions of the aircraft as being the results of linear forces acting in directions fore and aft, athwartships, and vertical to the aircraft and of angular moments about those axes.

From these forces and moments, and the linear and angular accelerations arising from them, may be computed a great deal of information about the flight of the aircraft.

For example, if any one of the linear accelerations be integrated with respect to time, the component velocity of the aircraft along that axis is obtained. The component velocities fore and aft and athwartships determine the angle of yaw of the aircraft, and those fore and aft and vertical decide the angle of incidence. These angles, in conjunction with the velocity components themselves, give the aerodynamic forces and moments referred to previously.

If a similar set of axes is set up with reference to a point on the surface of the earth, it is possible, by the use of co-ordinate geometry, to resolve the motions of the aircraft with respect to its own axes into motions with respect to these earth axes. Integration of the resultant vertical earth velocity will then record height above the earth, and integration of the north and east component velocities, with due allowance for wind, will record position on the earth with reference to the starting point.

It is clear, however, that the problem involves the calculation of forces and moments on the aircraft from a knowledge of motions of the aircraft which are themselves dependent upon these forces and moments. The aircraft, itself, is a complex feedback system which seeks equilibrium by counterbalancing the forces acting upon it by those produced by its resultant motion. It is not surprising, therefore, that the simulator which computes the motion of the aircraft is an analogue of the aircraft and comprises a complex servo system. The section which does this operation is known as the flight computor.

The flight computor is then a complex servo system which operates in this manner. From an initial knowledge of the forces and moments about the aircraft axes, it computes the linear and angular accelerations involved and, by integration, and knowledge of the previous attitude of the aircraft with respect to the air stream, it computes angles of incidence and yaw. From these, and from knowledge of the control positions in the fuselage and the aircraft flying characteristics, it can then compute the forces and moments arising from these conditions. These forces and moments are the initial knowledge referred to just previously and are returned as inputs to the computor, which being a complex servo mechanism then seeks equilibrium in the same way as the real aircraft. The flight computor, therefore, supplies continuous information about the motion of the aircraft with respect to its own axes.

Another section of the flight computor is composed of a number of servo-mechanisms which continuously record the position of the aircraft axes with respect to the reference set of earth axes. From this section can be obtained the motion of the aircraft with respect to earth and hence, by integration, height and position.

In order to provide further realism, the flight computor computes the reaction forces which arise on the pilot's controls when they are moved, and these forces are reproduced at the controls in the fuselage.

The flight computor needs to know the position and motion of the controls, which data is supplied from the fuselage and is then able to compute from the motion of the aircraft, the readings of the flying instruments which are simulated in the cockpit.

Further sources of data are the engine and propeller computors. The engine computor is a complex electronic calculator, which, from a knowledge of the controls in the cockpit, of predetermined engine data, and of the atmospheric conditions, as supplied by the flight computor, is able to give continuous information of the engine revolutions per minute, torque, oil temperature and pressure and other relevant data. This data is used to operate the engine instruments in the fuselage and as input information for the propeller computor.

The propeller computor is another electronic calculator which, from a knowledge of propeller controls, engine speed and other data, is able to supply continuous thrust and slipstream data to the flight computor.

This team of computors interact considerably, and together can predict the whole flight data of the aeroplane.

Two major ancillaries then make use of this information. A recorder unit plots the instantaneous flight track and height of the aircraft for the benefit of the instructor. A radio aids computor utilizes the knowledge of the air-

craft position relative to fixed transmitters and beacons to supply correct bearings and radio signals to the radio equipment in the fuselage.

Since there is a high degree of control over all aspects of the aircraft's flight, it is possible to reproduce faults in the various parts of the aeroplane and its ancillaries and to introduce emergency conditions at will. The controls for producing these faults are grouped into a faults panel close to the instructor's desk, so that at the turn of a few switches, the instructor can provide the sternest test of aircrew skill wihout danger to himself or the crew.

This complex collection of electronic calculators is then the flight simulator. Its uses in the present are too obvious almost to be mentioned. One might well wonder, however, whether such a machine is limited to training alone. A higher order of accuracy is obtainable than is necessary for training purposes, and with such accuracy, a flight simulator can be a powerful aid to the design of aircraft. It is not idle thinking to imagine the system in which flying prototypes are neces- sary to the earlier processes of designing an aeroplane, being replaced by one in which such early design in- vestigations are carried out on flight simulator, with the attendant advantages of more controllable test conditions, more complete instrumenta- tion, and the elimination of risk to valuable test pilots.

The flight simulator is but a particular breed of an exten- sive species. Such computors can be produced to simulate any physical motion, the laws of which are known. Simula- tors have been produced in this country for training motor-car drivers and the extension of the principle to the navigation of ships, or the flight of rockets is imme- diately apparent.

(5) The initial cost of the simulator is less than that of the aircraft being simulated.

(6) The running cost is appreciably smaller than that of the real aircraft and in particular a smaller number of specialists are required for training crews.

(7) The crews work as an integral team in the simulator from the start of training, thereby building up mutual confidence.

(8) New crew combinations or control layout can be tried out on a simulator by operating companies without using aircraft hours.
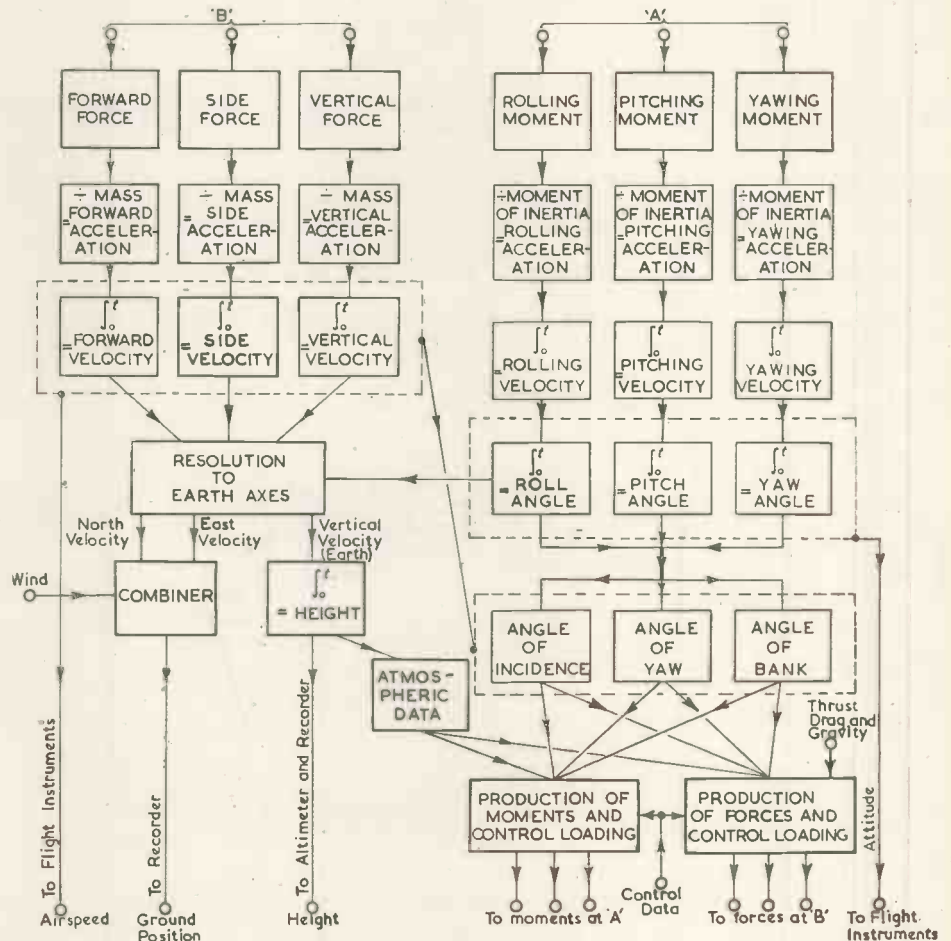


*Arrangement of the computors*

(9) Experienced crews can benefit from regular practice and refresher courses during bad weather periods.

(10) The simulator is entirely independent of weather conditions.

**Summary**

It is of advantage to list the advantages of the training obtained by the use of flight simulators as compared with that obtained by the use of actual aircraft.

(1) The crew may be subjected (in the flight simulator) to conditions and faults which may be difficult and hazardous to attempt in a real aircraft.

(2) The real aircraft is released for operational duties.

(3) The training period may be considerably shorter in the flight simulator than in the real aircraft due to intensive training methods. (A greater degree of con- fidence may be imbued into the crew, by the more extensive training curriculum).

(4) In the event of a fault, the flight simulator may be repaired in a shorter time than it takes to repair a real aircraft—" write off " is impossible.

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# An Electronic Helmsman for a Motor Yacht

By W. H. Alexander*, B.Sc., A.Inst.P., and D. M. McCallum*, B.Sc., A.M.I.E.E.

*An automatic helmsman, suitable for controlling a small motor yacht, is described. It is constructed largely from Government surplus equipment. The performance is such that when the desired course was suddenly altered by 90° the boat settled down on the new heading within 15 seconds after one small overshoot of 3°.*

STEERING a motor yacht at sea is tedious. Because of its poor natural stability in yaw it requires constant attention to course and correction to the helm. This is the type of unrewarding occupation particularly suited to automatic control. However, the expense and weight of commercially available automatic helmsmen prohibit their use in small boats. The helmsman described uses, in the main, Government surplus equipment and makes no pretence to sophistication.

As in all servo systems there are four main components. In this application these are:—

As in most servo systems there is also a stabilizing signal to prevent hunting. In this case this is a voltage proportional to the rate of turn of the boat. This has the action of reducing the applied rudder when the boat is swinging back on to the correct heading before it has actually reached it and even, if the rate of turn is fast, reversing the rudder so as to check the swing of the boat. The need for this is readily seen and the action is normally applied by the human helmsman without difficulty, although he may well be unaware of the equations in the Appendix.

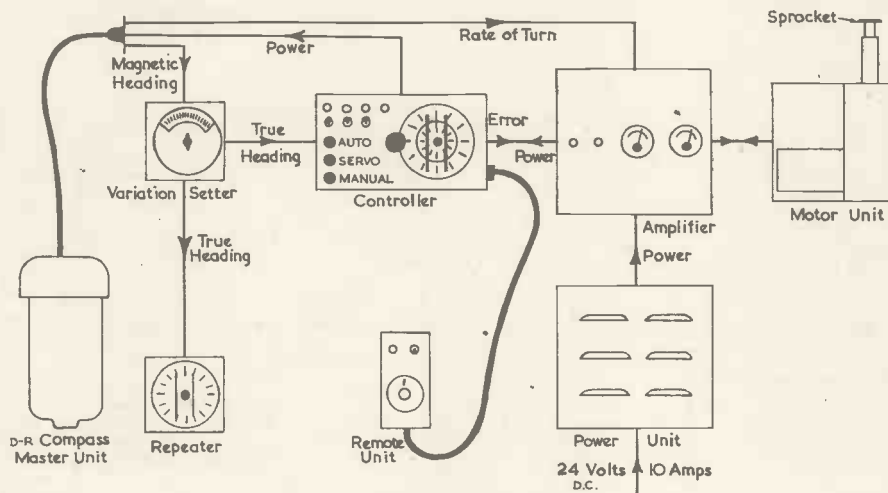The layout of the units may be seen from Fig. 1 and



Fig. 1. The interconnexion of the units

1. *The Input Unit* which demands the required heading. As far as the yachtsman sees this is a compass ring which is set manually to the desired heading.

2. *The Output Unit* which indicates the actual heading. This information is obtained from an ex-R.A.F. Distant Reading Master Compass Unit which relays its information by an M-type data transmission system to any required number of repeaters.

   One of these repeaters in the Controller turns the shaft of a potentiometer whose case is turned to represent the input heading.

3. *The Error Detecting Unit* is really this potentiometer, the output heading being equal to the input heading when the potentiometer slider is midway between the diametrically opposed feeding points on the winding.

4. *The Motor* which produces the corrective force required to reduce the error to zero. This is a velodyne motor generator, type 74, turning the wheel through a chain drive.

their disposition in the boat from Fig. 2. These are mounted in the best available places for convenience of operation and access and to keep the interconnecting leads short except in the case of the master compass unit. As this depends on the earth's magnetic field for its information it is mounted in the forecastle where there is the minimum disturbance by machinery.

It is necessary to understand, at least in outline, the operation of the master compass unit so that the derivation of the rate of turn signal may be clear. A small electrically driven gyro has its rotor axis lined up with the magnetic north. The gyro acts as a filter on the information from a bar magnetic compass. As the magnetic compass is disturbed and swings violently due to the ship's movement only its mean position over a few minutes can be taken to have any meaning, and it is used to apply a low precessing torque to the gyro which filters out the random oscillations of the magnetic compass. As only light torques can be taken from the gyro, a torque amplifier is necessary to drive the transmitter of the M-type step-by-step data transmission link. The torque amplification is done by

* Ferranti Ltd., Edinburgh.

Fig. 2. The disposition of the units



Fig. 2. The disposition of the units
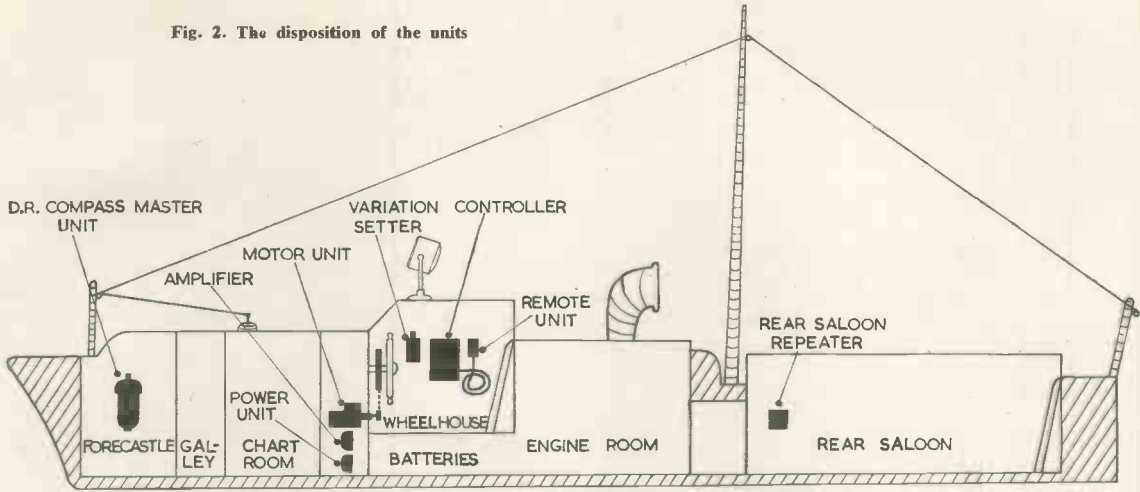
having a contact carried from the gyro on an arm. This contact slides on a piece of quartz one-half of which has a conducting surface. The quartz is carried on a frame turning round the gyro gimbal frame and driven by a motor. When the contact is made, a relay causes the motor to drive in one direction and when it is broken to drive in the opposite direction. The relay thus makes and breaks regularly at a frequency determined by the frame inertia, motor gearing and so on, and the frame hunts with an amplitude of about $\frac{1}{2}°$. As the gyro is fixed in space the frame must turn with respect to the boat when the boat turns, so that it will remain in a constant position relative to the gyro. To do this the relay will be made for longer intervals than it is open when the boat is turning in one direction and open for longer intervals than it is closed when the boat is turning in the opposite direction. To derive the voltage proportional to the rate of turn a slave relay is driven from the relay in the master compass unit and the fixed terminals of a changeover contact set in it are connected to the positive and negative supplies feeding

the bearing potentiometer (Fig. 3, relay $C$ and contacts $C_1$). Smoothing the voltage on the moving contact with a $1M\Omega$ resistor and a $4\mu F$ capacitor we obtain a voltage on the capacitor which varies about a mean value with the rate of turn of the boat.

We now have all the signals necessary to control the boat and these are fed into the velodyne amplifier as shown in Fig. 3. This is a straightforward push-pull amplifier feeding the fields of the motor of a motor generator type 74. The armature is fed with current through a resistor from the 24V supply. The motor produces a rudder position indicated by the rudder potentiometer which is proportional to bearing error minus rate of turn of the boat. The diagram is complicated by an additional facility which enables the boat to be steered from a remote point as explained later. Local velocity feedback round the motor reduces the effect of varying rudder loads.

The final drive to the wheel is through a 7 : 1 chain reduction drive, the final sprocket being mounted on the

Fig. 3. The velodyne amplifier
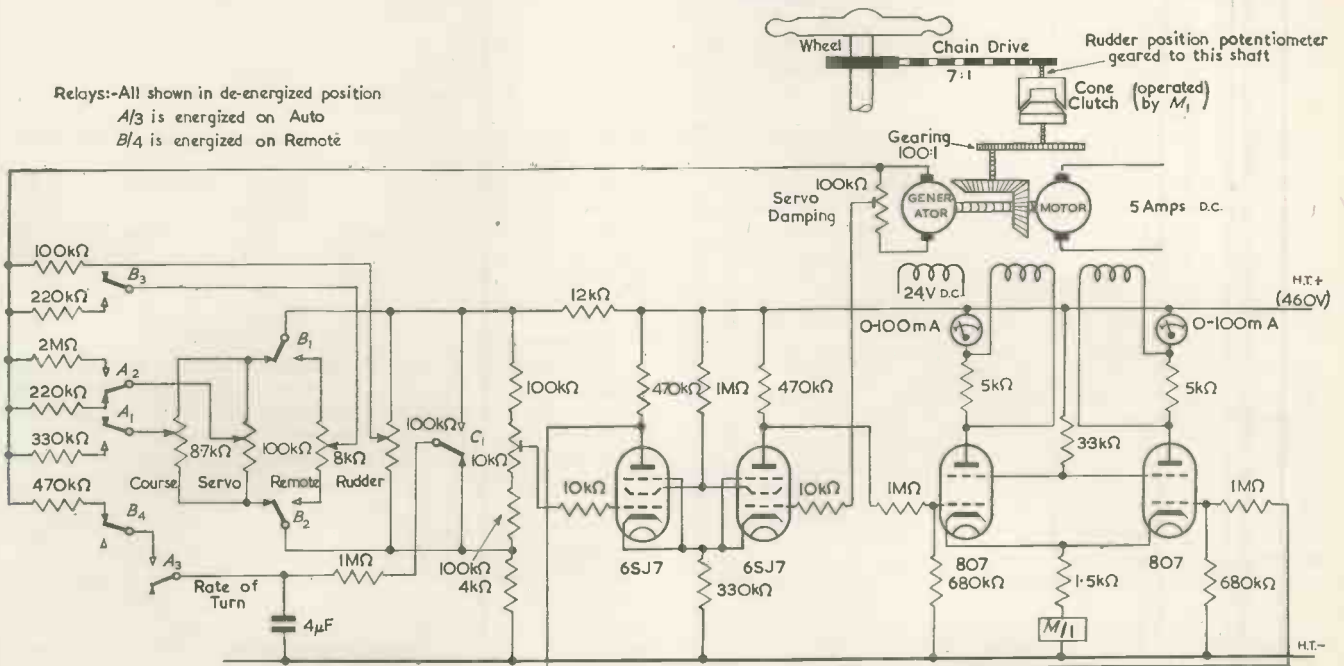
back of the wheel. The driving sprocket is on the output shaft from the motor unit which projects through the bulkhead from the chartroom (Fig. 2). There is a gearbox on the motor output with a stepdown of 100 : 1 so that the total stepdown is 700 : 1. This means there is about 20lb ft of torque available at the wheel, and as the normal torque required on the wheel is about 4lb ft this is adequate for all conditions except possibly very heavy following seas.

Switches on the controller allow the gyro and heaters to be started preparatory to using the helmsman, or the gyro compass to be used by itself. When auto-control is engaged the H.T. supply is switched on. The power supply is an ex-I.F.F. motor generator running off 24V through a carbon pile regulator and generating 460V H.T. for the valves. The clutch in the drive to the steering wheel is operated by the relay in the cathode circuit of the output valves of the amplifier so that the drive will be released if there is an H.T. or heater fault. When servo or remote control is engaged the compass unit is not used in the helmsman, but the rudder position may be controlled either from a potentiometer in the controller or from a potentio-



*The controller and variation setter installed in the wheelhouse. The chain drive from the motor unit goes to the sprocket on the back of the wheel.*

meter in a small control box on a flexible cable so that the boat may be controlled from the deck. In these cases the system is operating as a simple rudder position servo.

To test the helmsman in the laboratory a small simulator was built using a gramophone turntable driven by a velodyne machine. The rudder position voltage was fed into the velodyne to produce rate of turn of the table through a network having a time-constant of about 5 seconds to represent the yacht's lag in answering the helm. The rate of turn of the turntable was adjusted to be about one revolution in 70 seconds for half rudder, a typical value for the boat in question. In this way the range of feedback was tested and found to be adequate.

After installation a number of trials were successfully carried out. When the desired course was suddenly altered by 90° the boat settled down on the new heading within 15 seconds after one small overshoot of 3°. This performance was considered to be quite satisfactory and the success of the helmsman was considered proved when an experienced yachtsman said of her "She luffs beautifully".

**Acknowledgment**

## APPENDIX

### TRANSFER FUNCTIONS

A block diagram of the servo system on auto-control is given in Fig. 4(a) and on servo or remote control in Fig. 4(b). The symbols used are listed at the end of the Appendix. For the auto-control system the equations of motion are:—

$$\theta = \theta_i - \theta_o \quad \dots\dots\dots\dots\dots\dots\dots\dots (1)$$
$$K_2(K_1\theta - \phi_o - K_3\theta'_o) = \phi'_o + T\phi''_o \quad \dots\dots (2)$$
$$K_4 = \theta'_o + T_2\theta''_o \quad \dots\dots\dots\dots\dots\dots\dots (3)$$

Taking the Laplace transform (assuming all the variables to be zero initially) of the equation which results on eliminating $\theta$ and $\phi_o$ gives an overall transfer function

$$\frac{\theta_o}{\theta_i} = \cfrac{1}{1 + \left(\cfrac{1 + K_3K_4}{K_1K_4}\right) p + \left(\cfrac{1 + K_2T_2}{K_1K_2K_4}\right) p^2 + \left(\cfrac{T_1 + T_2}{K_1K_2K_4}\right) p^3 + \left(\cfrac{T_1T_2}{K_1K_2K_4}\right) p^4}$$

The corresponding transfer function for servo or remote control is

$$\frac{\phi_o}{\phi_i} = \cfrac{1}{1 + (1/K_2) p + (T_1/K_2) p^2}$$

### CHOICE OF PARAMETERS

The values of $K_4$ and $T_2$ are more or less fixed and outside the control of the designer of the helmsman. They vary with the speed of the yacht but not to any appreciable extent over the normal range of speeds. The amplifier gain constant $K_2$ was chosen to give the required accuracy in
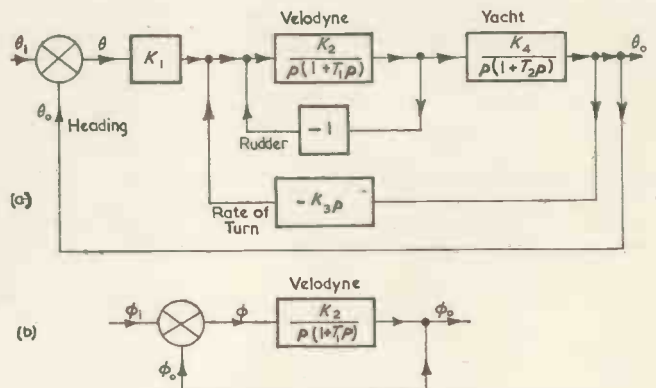


Fig. 4. (a) Auto control; (b) servo or remote control

positioning the rudder, and the time-constant $T_1$ adjusted by means of the " servo damping " control (Fig. 3) to give a satisfactory response on servo and remote control. $K_1$ was then chosen to give an sufficient overall gain constant for the auto-control system and the response was finally adjusted by choice of $K_3$, the rate term constant. Without the rate term, that is with $K_3$ zero, the system was found to be unstable.

### NON-LINEARITIES

The equations in this Appendix determine the stability of the system with respect to small disturbances and its response to small inputs. For large inputs such as the 90° step function mentioned earlier they are not strictly accurate. Equation (3) representing the response of the yacht to the rudder is at best only a reasonable approximation to the actual performance, assuming as it does that the steady state rate of turn is strictly proportional to the rudder angle. It is very nearly correct, however, for small rudder angles. Equation (2) is also true only for small inputs, for if the left-hand side is large a speed is demanded of the motor in excess of its maximum, which is reached with the amplifier in a limiting condition. After a sudden 90° change in desired heading the motor soon

reaches its maximum speed and maintains it for a considerable part of the time taken to reach the new heading. When the correct heading is neared the behaviour returns to that deduced from the linear theory. It was to study this phenomenon that the simulator was constructed.

LIST OF SYMBOLS

$\theta_i$ desired heading.

$\theta_0$ actual heading.

$\theta$ error in heading.

$\phi_i$ desired rudder angle.

$\phi_0$ actual rudder angle.

$\phi$ error in rudder angle.

$T_1$ time-constant of the velodyne.

$T_2$ time-constant of the yacht.

$K_1$ the ratio of input voltages for unit angles of error and rudder (dimensionless).

$K_2$ the rate of turn of rudder for unit angle of rudder (dimensions $T^{-1}$).

$K_3$ the ratio of input voltages for unit rate of turn of the yacht and unit angle of error (dimensions $T$).

$K_4$ the rate of turn of the yacht for unit angle of rudder (dimensions $T^{-1}$).

$p$ the Laplace parameter.

A bar over a variable denotes the Laplace transform of the variable.

# Television Equipment for Oceanographic Research

Some details have been given on underwater television equipment developed by the Admiralty for oceanographic studies in the Royal Research Ship *Discovery II*. It is lighter and more easily handled than the equipment used for salvage work in H.M.S. *Reclaim*. The television camera has been fitted with a stereoscopic attachment, but little experience has as yet been obtained with this technique.

Although the first concrete proposal for an underwater television system in this country was for marine biological research, the course of events decreed that the first few cameras to come into use should be applied to more utilitarian purposes, namely to diving and salvage operations. Early in 1952, however, the National Institute of Oceanography suggested that a camera should be installed in R.R.S. *Discovery II* for a preliminary investigation into the applications of underwater television to oceanographic research. An underwater casing to house a Pye image orthicon camera was accordingly designed by the Admiralty Research Laboratory, Teddington, with these trials in mind.

The camera was first installed in the *Discovery II* late in the summer of last year.

The casing constructed for the camera installed in H.M.S. *Reclaim* was designed for a depth of 1 000ft. This design, although capable of being handled by the equipment on H.M.S. *Reclaim,* was too unwieldy for normal use.

The internal dimensions of the new camera, which was also designed for a similar depth, allowed considerably reduced dimensions for this casing. The total weight of this camera is now only 7cwt.

As in previous designs, the camera window and cable gland were all mounted on the square baseplate of the casing, short legs being added to protect the cable and window when landing on the deck.

Special modifications for underwater working were made at the laboratory to provide for remote alignment control for the camera pick-up tube, and remote image section heater control, and a " dampometer " or moisture indicator is included, this consisting of a piece of blotting paper, the resistance of which is continuously monitored.

The camera was equipped with $2\frac{1}{2}$in. f/1·9 and $1\frac{1}{2}$in. f/2 lenses, either of which could be selected underwater by the remotely controlled turret. An auxiliary lens system was used to achieve approximately the same angle of view in water as would normally be obtained in air with these lenses. A split-field stereoscopic attachment using surface aluminized mirrors could be fitted to the $1\frac{1}{2}$in. lens and used through the plane window, i.e., without the auxiliary lenses. This provides right and left eye pictures side by side on the screen, each one covering $25°$ by $180°$ underwater.

A modified English Electric T/V receiver was placed in the ward-room as the main monitor. Pictures displayed on the camera control unit monitor were photographed when required by a camera mounted on a bracket attached to the unit.

The Institute of Oceanography reports that trials with underwater television have provided new information about the sea floor, and show that the technique can develop into a valuable oceanographical instrument, especially if used in conjunction with other gear. In its present form it is regarded as cumbersome, and its scope is limited by the weight of the camera in its steel casing, which makes it unsafe to use except in calm water, and by the difficulty of lowering it to any great depth. Both these difficulties are likely to be reduced in due course.

Observations on the sea floor were made in about 33 fathoms off Falmouth, in 60 fathoms near the mouth of the English Channel, and in about 80 fathoms near the edge of the continental slope. Further observations were made on the continental shelf off Portugal on the Gettysburg Bank, on the rocky sea floor round the Azores and on the Princess Alice Bank south of the Azores.

## PPI Display of Naval Review
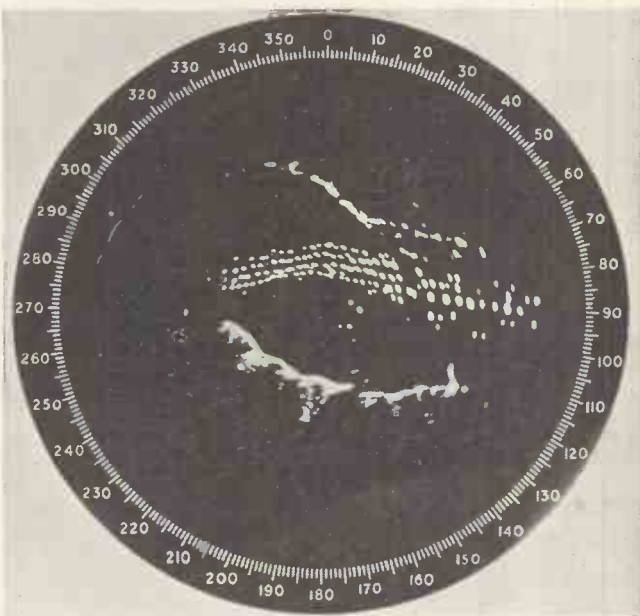


*The above PPI display represents the assembled ships at Spithead during the Coronation Naval Review as they appeared on the screen of the Radiolocator IV radar on board the Marconi yacht " Electra II "*

# Electrical Synthesis of Musical Tones

By Alan Douglas

(Part 1)

## Music, Noise, Characteristics of tones singly and in Combination

*The apparent ease with which it is possible to produce sustained tones from valve oscillators has led to the publication and marketing of some circuits which fall short of even the minimum requirements for æsthetic tolerance. These articles outline some of the fundamentals of tone production and draw attention to certain limitations which are often conveniently overlooked, to the detriment of the art. Extremely satisfying results are realizable by proper means.*

"MUSIC was forced to shape for itself the material on which it works. Painting and sculpture find the fundamental character of their materials, form and colour, in nature itself, which they strive to imitate. Poetry finds its material ready formed in the words of language. Music alone finds an infinitely rich but totally shapeless plastic material in the tones of musical instruments. There is a greater and more absolute freedom in the use of material for music than for any other of the arts; but certainly it is more difficult to make a proper use of absolute freedom."

So wrote von Helmholtz in 1880. It is indeed true that the licence implied by artistic freedom has in many cases exaggerated the value of musical sounds produced by electrical means. The appraisal of tonal values is an individual thing, changing with temperament, surroundings and other contexts. In the pipe organ world, vitriolic comments alternate with lush approval on examples of various makers; yet each and all of these instruments fulfil their designed purpose, whatever personal opinion may be, because the tones produced are based on immutable physical laws, and, unless purposely made grotesque, are acceptable from the start.

Helmholtz, Koënig and Rayleigh laid the foundations of tonal research on a massive scale; economics did not then enter into the realm of musical instruments. Today, both science and economics loom very large in the picture, and where the more complex musical instruments are concerned, alternative means by which expense may be reduced are eagerly sought.

The purpose of these articles is to survey the present position of electrically synthesized or produced musical sounds, and if in parts they seem elementary, this is only because it is so easy to overlook some commonplace factor which may not at first sight appear important. This section is devoted to the "material for music".

Any textbook on physics will explain how sound waves are propagated and received. We can call them sound waves because the sense which detects them is that of hearing, though they are really waves of air pressure and rarefaction. The distinguishing characteristic of a musical sound is that it is periodic and complex; also, it must be maintained for a sufficient time to allow of identification. The only other parameter by which we can identify a sound as musical is that of training and usage. We can tell a trumpet because we have always known it as a trumpet, and for no other reason. Most musical sounds are known, though some are infrequently met with, but it is possible to produce new sounds. At first, some of these appear strange, but time is required to get used to departures from accepted standards; this is particularly true when a number of sources contribute to the tonal spectrum. Today, the intricate texture of complex music such as Wagner's "Tannhäuser" is assessed as magnificent; yet, when first performed in public, it was heralded as "an extravagance of noise". Indeed, one eminent critic writing in 1846 said "it seems to me that a man who will not only write such a thing, but actually have it engraved (printed), has little call for an artistic career".

We cannot build up a complex spectrum for electrical synthesis without appreciating the properties of the individual simple waves or curves from which all musical sounds are compounded.

Ohm, in 1843, stated: "All musical tones are periodic; all varieties of tone quality are due to particular combinations of a larger or smaller number of simple tones; every motion of the air which corresponds to a complex musical tone or a composite mass of musical tones is capable of being analysed into a sum of simple pendular vibrations, and to each simple vibration corresponds a simple tone which the ear may hear." The elaborate investigations of
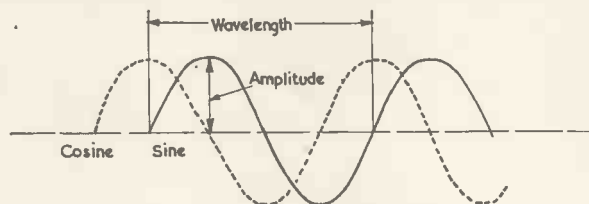


Fig. 1. Simple Sine Wave

Helmholtz could only add, "the quality of a musical tone depends solely on the number and relative strength of its simple partial tones and in no respect on their difference of phase".

Both of these investigators, and many at a later date, are of course referring to a tone which has actually started and is regarded as being in a steady state; but it will be shown later that the starting characteristics and noise of generation have a profound bearing on the value and identification of a particular tone.

Firstly, then let us note the properties of simple waves. A sine curve is an instantaneous representation of the condition of motion in a simple wave, Fig. 1. A curve of the same form, but differing in phase by 90° is a cosine curve. Amplitude is the height of a crest above the axis, therefore half the total displacement. A period is the time required to trace one wavelength, i.e., a crest and a trough. The frequency or pitch is the number of periods, or wavelengths, per second. The phase varies along the axis, passing through a complete cycle in one wavelength. The velocity is equal to the wavelength multiplied by the number of waves per second. The energy, or intensity, of a sine wave varies as the square of the amplitude if the frequency remains constant, or vice versa. When both amplitude and frequency vary, the intensity varies as the square of the product of amplitude and frequency, or

$$I = f^2 A^2$$

Simple examples of the foregoing are to be seen in Fig. 2. Here curve B has a frequency the same as A, but its amplitude is twice as great, hence it represents a sound four times as loud. Curve C has an amplitude the same as A, but its frequency is twice as great, so again its loudness is four times that of A. Taking an uneven example, curve D has a frequency of 3·3 and an amplitude of 0·3, so its loudness is equal to A.

Even in such simple waveforms we must be on our guard against assessing loudness from the geometry of the curve. This is better illustrated by a complex waveform, Fig. 3. Curves A and B have loudnesses represented by 1 and 9. Curve C contains both A and B and its loudness is therefore 10. If it were assumed that the loudness of A and C were represented by the squares of their measured widths, the value for C would be 1·6 as compared with A, which is only one-sixth of its real loudness. From these examples can be seen the necessity for analysing a complex wave into its simple components, to find the intensity for each component, and then to sum these intensities; the figures show that a simple measurement of the amplitudes and frequencies of the complex curve will not give its loudness[1].
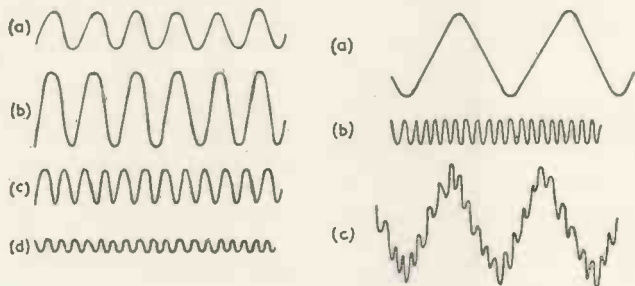


Fig. 2. Illustrating the relationship between frequency, amplitude and intensity

Fig. 3. Example of compound wave
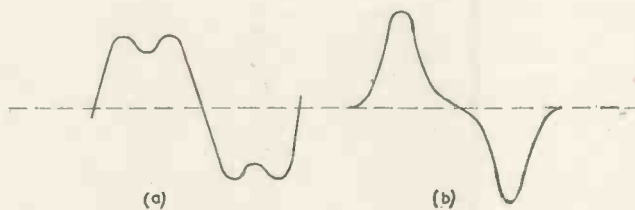


Fig. 4. Effect of phase on wave shape

This is further borne out by Fig. 4, which shows two curves A and B of equal loudness. Each is composed of fundamental and second harmonic, of loudness 1 and 4 respectively; only the phases of the components are different in the two curves, which at first sight might appear to be of different intensities.

Now the separate components of a musical tone must be in some relationship to sound musical. Such components are variously known as harmonics, overtones or partials. Strictly, harmonics must be components having frequencies which are an exact muliple of the fundamental wave; inharmonic components are in existence in many tones and these are best called overtones to avoid confusion; the expression partial, while correct for any component, is not sufficiently precise and will not be used.

The ear can, of course, only hear components over some range; the upper limit is very variable, for age, temperament, inclination and other factors operate differently in different cases; see, for example, the effect of age alone on tonal perception. (Fig. 5.) It is desirable to suggest we should be able to hear a tone of, say, 10kc/s to establish a reference; but the operation of higher frequencies as combination tones does influence the fidelity in a marked
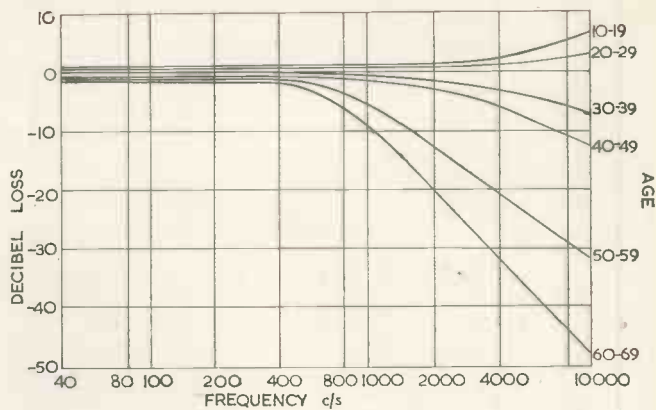


Fig. 5. Hearing loss with increasing age

fashion and the extreme pitches up to about 15kc/s cannot be dismissed as insignificant by any means.

The lower limit is about 16c/s, for although the ear can hear a much lower vibration rate, the individual waves are heard as such and the sound is not continuous. Much lower tones can be heard if they have an appreciable harmonic content, the fundamental appearing as a beat tone.

The frequency range brings us to the question of acceptable pitch standards. All attempts to generate musical sounds would be abortive unless they could be used with other sound sources or the voice. For this reason, a standard of pitch must be available. Such a standard is now happily established at A = 440c/s. Many instruments are still to be found tuned to another pitch, but new instruments are adjusted to this international standard. It is interesting to note that A = 440c/s was first put forward by Schiebler in 1834, and considerably adopted in Germany at that time. The intolerable range of pitches in use, as compiled by Ellis, showing over 242 standards ranging from A = 370 to 567c/s, would have made musical synthesis impossible in many cases.

It is helpful to look at the range of notes to be used for complete complex tones; this is not the same as the frequency range mentioned above for the range of independent waves which may be needed for synthesis, but represents what coverage may be expected from a musical instrument in terms of octaves. The musical staff may be said to have eleven lines, as in Fig. 6. The middle line is omitted unless required for a note, this note being middle C. Short lines are used as shown to extend the compass. The pitch range of the actual notes lies within the lines shown, except for the organ, which may extend upwards or even downwards. Therefore the range of notes in which we would be interested lies between C = 32·7 to C = 4186·0c/s, with a possible extension to C = 8372·0c/s. For the utmost fidelity, frequency range for the sources will require an upper limit of 16kc/s. The author cannot find

Fig. 6. Octaves required for range of useful notes
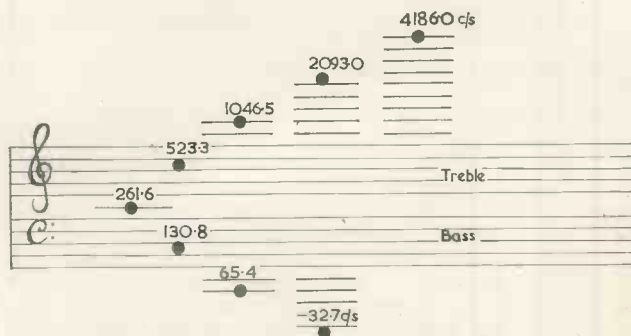
any evidence either published or borne out by experiments over many years that this limit is not adequate.

At the moment we are only considering a single tone, whether simple or complex. It has already been said that measurements are made on such tones when in a steady state. However, all tones must be initiated. Experience has shown that the starting characteristics of such tones have an almost equal importance whether the sound is quite simple, or very complex. Helmholz has stated that the sound from a tuning fork, blowing across the neck of an open bottle, or softly played on a flute, is identical; these are extremely simple tones. The part played by association of ideas enters so largely into the interpretation of musical sounds that it would seem laughable to state that an eminent musician could not tell the difference between the above sounds; yet this is only true when the source can be seen; a number of musicians were unable to tell which of these sources produced the sound when the source was not visible and the tone was heard *after* it had started. When the starting sounds were added, identification was easily possible, so clearly the properties of the initiation can be of great importance.

Instruments which can be effectively synthesized include the pipe organ; indeed, it is perhaps the instrument most widely imitated. Since the organ must contain a number of stops simulating orchestral instruments, and since the stopped class of pipe features prominently in its tonal make-up, long observation has assessed the fidelity value of a particular kind of pipe on a tone which includes a complex starting tone in the case of nearly every kind of pipe except the diapason. It is generally agreed that the constituents of these starting tones are inharmonic, but in some cases they are definitely a musical note distinct from the steady state note. In other cases, the starting sound is a noise and the mechanism by which organ flue pipe tones (that is flutes, strings, diapasons) are generated is such that noise must inevitably accompany the production of a steady note[2]. This will be referred to in more detail when we come to consider the simulation of organ tone by electrical means.

So it is clear that while the steady tone state of any instrument is to be considered musical, the starting of such tones is, in nearly every case, unmusical; in other words, a form of noise. To understand the complete sound spectrum required for synthesis, we must pay some attention to noise.

It has often been put forward that noise and tone are two separate and distinct states of vibration, but in fact the statement that noise is non-periodic is not always true, and perhaps it is more correct to regard noise and tone as terms of contrast. There is a transition from noise to tone in most cases except in the case of sudden or isolated noises; further, many waveforms of short duration are not really those of noise, but are too short to be recognized by the ear. 25 to 40 milliseconds is required by an expert observer to really identify a sound, and probably much longer if the attention is not acutely drawn to the incidence of the sound. Here again we can detect the personal element, for example the drawing of a cork from an empty bottle is regarded as a noise, but it is really quite a musical tone; however, the senses tend to concentrate on a possible noise and, when the waveform commences, the mind automatically thinks of the process as producing a noise and does not trouble to wait and listen for the short musical tone following. For the same reason, the electrical reproduction of instruments having a pronounced initial noise of generation, such as the string family, saxophone, piccolo, etc.[3], often acquires quite a different character when heard on a loudspeaker, being smoother in general than the original; this is because the very high frequencies composing the generating noises of the tone are not within the range of the amplifier or loudspeaker.

Noise in the sense of a different wave spectrum from the main tonal spectrum is an essential ingredient of most musical sounds, and the physical means by which this noise is produced are, of course, absent from electrical tone generators. This is one of the reasons why such synthetic tones are not, and cannot be, the same as those produced in the instruments simulated. Such a statement brings us at once to the question of tonal degradation, and this is coupled with some very complex phenomena in instruments having many independent tone sources. This subject is of great importance and space will be devoted to it in Part 2 of this series. There are one or two further characteristics of the ingredients of complex sounds which we should now note.

It is commonly held that the seventh and ninth harmonics of a tone source are inharmonic and objectionable. If the waveform of a sound is periodic, then every harmonic must be in tune, and such a sound is musical. Fig. 7 shows an analysis of a clarinet tone, in which strong 7th and 9th harmonics are present, representing 8 per cent and 15 per cent of the total loudness respectively; no one will dispute the beauty of clarinet tone.
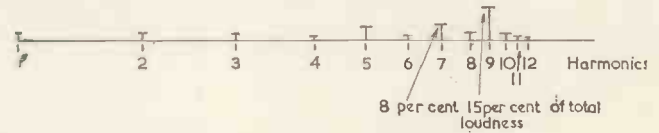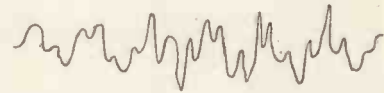
Fig. 7. Analysis of clarinet tone

Fig. 8. Inharmonic waveform of bell tone

Fig. 9. Change in phase at instant of reversal of violin bow

The foregoing implies that the whole harmonic series required for the formation of a tone at any instant is simultaneously produced in some form of generator. Should tonal synthesis be attempted from the mixing of separate waves produced by independent generators tuned to the intervals of the equally tempered scale, the position is quite different, since in this case none of the intervals are exactly in tune except octave intervals. Such synthesis raises special problems which are examined in due course.

Sounds of a musical nature can be composed of an inharmonic waveform, a good example of which is found in a bell. Fig. 8 shows a curve of a bell with no apparent waveform; an analysis of any part of it would probably show an infinite number of terms, but the real sound is undoubtedly composed of a finite number of overtones, which, being inharmonic, are incapable of analysis. Naturally such a sound cannot be synthesized electrically. Imitations are produced, with a certain likeness, but an instant's comparison with the original will show the dissimilarity.

It has been stated that differences in phase have no effect on a sound as interpreted by the ear. This refers, of course, to differences in a single sound source. For example, the change in phase during bowing a violin string is clearly shown in Fig. 9. The ear cannot detect any change in tone. It is often said that the effects of phase contribute to the effect of tonal richness in instruments having many tone sources, such as pipe organs. No evidence has been adduced that there is any such effect, and it is much more probable that the subtle coloration which the many sources do undoubtedly produce, is produced by beats.

Beat tones do, in fact, contribute much more to music than is generally supposed. For example, the ear detects a fundamental tone in the lower notes of a violin. The real fundamental is extremely weak, because it lies below the

fundamental resonance of the violin body and any sound output lower in pitch than the resonance is very small. Adjacent strong upper harmonics produce the beat tone which sounds to the ear like the actual fundamental or pitch note of the sound being produced.

Beats are formed when two simple tones sound together; the number of beats is equal to the differences of the frequencies. If there are only a few beats per second, they are easily detectable as separate pulsations. If, however, there are many beats, the ear hears a third tone equal to the frequency difference of the two sources. Fig. 10 shows the appearance of a beat tone. When the two waves are in phase as at A, they combine and produce a loud sound. At C, however, when the motion of one wave is in one direction and that of the other wave in the opposite direction, they tend to cancel, reducing the sound output. The beat tone at A and B sounds as if it were a real tone, but in fact it is only subjective, since the whole of the wave is composed of the



Fig. 10. Formation of beats

components due to the two sources and nothing else; but these subjective tones influence the ear in the same way as real tones.

This example of beats is derived from simple waves, but the effect from complex waves is equally real and highly complicated; it is impossible of analysis but it serves to underline the difficulty of re-creating a simulation of complex tones from many separate sources through the medium of one tone outlet only. It is, in fact, impossible.

Beats are commonly introduced into otherwise steady musical tones to form a vibrato or tremolo. Such beats are very slow in number and confined to a frequency range (for the notes) above 64c/s. The frequency difference being great, the separate pulsations modulate the main tone as an easily distinguishable wavering of the tone and are a customary ingredient of vocal music. If, on the other hand, a means is provided for injecting a beat tone into each separate note of a frequency bearing some fixed relation to each note, then the effect is much more subtle and rich; such an effect is to be found in the pipe organ and is known as a celeste. It also obtains in the orchestra, where players of instruments covering a limited pitch range each introduce a vibrato of their own choosing at rates related to the range of notes of the particular instrument they are playing, e.g., the vibrato rate of a violin is faster than that of a violoncello; the combination of these many main tones so modulated is responsible for the extraordinary richness of a string orchestra. It can be reproduced by electrical means in multi-note electrical tone generators and adds to their appeal in an æsthetic sense.

So far we have considered sustained sounds only, but one characteristic class of sounds is percussive in nature; the piano, harpsichord, banjo, guitar, etc. In such instruments there is a highly complex vibrational state because the separate tone sources (the strings) are commonly coupled through a soundboard or resonator. Such sounds are all characterized by a rapid attack or rise in initial amplitude, almost of a transient nature; followed by a gradual decay curve which is due to the absorption of energy by the resonator. There is mutual coupling between adjacent strings especially those harmonically related to the one being excited, and the waveform is constantly changing during the period of audibility. Electrically, the initiation of the strike tone and the decay characteristic can be closely simulated, but the change in harmonic content and the beat tones during decay (and there are many) cannot be simulated. Hence such electrical simulations have a curiously clear and pure tone, as would the physical vibrators of the guitar, etc., if there was not the mutual interaction between the strings due to the coupling by the common soundboard and the proximity of other strings tuned to the harmonics of the string being used.

The above suggests that it would not be profitable to synthesize some kinds of musical instruments and this forms part of the subject matter of the next section.

*(To be continued)*

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# A Sound Delay System

In 1949 Haas discovered that for some as yet inadequately explained reason the human hearing mechanism locates the apparent source of sound from a multiplicity of loudspeakers as being at the nearest loudspeaker. This apparently simple finding has had a considerable effect on sound reinforcement techniques where suitable control of the time delays of the signal to each loudspeaker makes it possible to obtain a remarkable improvement in intimacy and realism.

The time delay required for the average large hall usually falls in the range of 30 to 120 milliseconds and has generally been obtained by the use of a continuously rotating drum of magnetically coated material.

A simpler solution, using no rotating equipment, is to insert an acoustic delay line in a suitable part of the amplifier chain. Preliminary work has shown this to be a perfectly feasible solution and equipment of this type is now in production by the B.T.H. Co., Rugby. Several equipments are now being installed in large auditoria where a preliminary experimental equipment has demonstrated its value. The complete equipment is assembled in the unit shown in the accompanying illustration, having overall dimensions of 24in. by 20in. by 32in. high. The delay is provided by a length of rubber hose cut to the appropriate length.
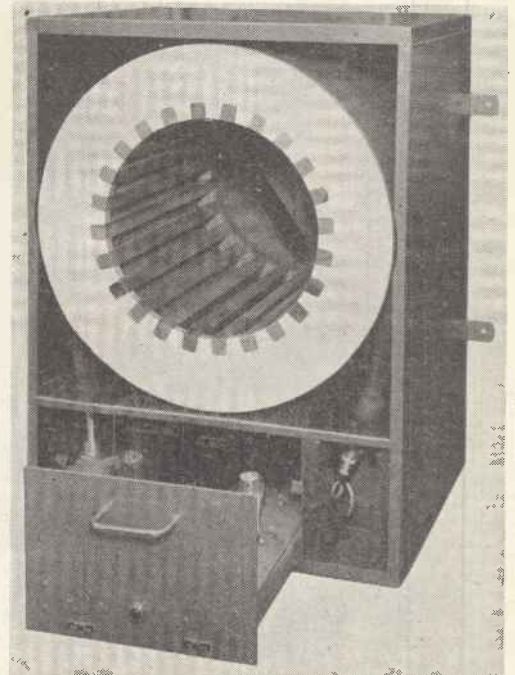
# A Recurrent Sweep Chronograph

## for the Measurement of Small Time Intervals especially in Ballistics

Hilary Moss, Ph.D., M.Brit.I.R.E., M.I.E.E.

*An equipment suitable for the precise determination of projectile velocities in ballistic investigations is described. It has an advantage over the more normal counter chronograph in that the pulses are displayed on a C.R.T. and, being of invariant characteristics, they cannot be confused with random noise signals.*

*The instrument has a time resolution of $\pm 15\mu sec$, while the maximum elapsed time measurable is $10^5 \mu sec$.*

THE precise determination of projectile velocities is fundamental in ballistic investigations. The history of the art contains many ingenious methods which, in the early days, were usually mechanical. More recently, the application of electronics to this field has resulted in substantial advances in accuracy and convenience.

A preferred modern method fires the projectile through two large coils separated by a distance of the order of one hundred feet. The magnetic disturbance which results generates small E.M.F. in each coil. Electronic methods are then used to amplify these disturbances and to accurately measure the time interval between them, so that the mean projectile velocity over the intervening path may be precisely calculated.

Probably the most accurate time interval measuring devices are "counter chronographs." These are electronic time scaling networks which measure the passage of time by counting the number of "time increments" which elapse between any two phenomena. These time increments are the duration of each cycle of some internal oscillator. The latter is usually a highly accurate crystal controlled standard often having a frequency of the order of $10^5$ c/s and thus yielding a time increment and a resolution of $10\mu sec$. Decade dividing stages operating suitable glow tubes, then permit the total number of cycles of oscillation during the interval being measured, to be read off.

All such counter chronographs are provided with "gate" circuits which start the counting network on receipt of the first pulse and stop it on receipt of the second one. The application of such counters to the ballistic problem discussed above, requires merely that the pulses from the two coils are suitably amplified and clipped, so that they contain some sharply defined "edge" to act as a precise triggering signal. Provided the two coils are identical the phase relationship between the triggering pulses and the position of the projectile is unimportant.

Experience has shown that this method is probably the most accurate of all, and it is widely used. However, it suffers from the drawback that the counter chronograph may be triggered by spurious pulses which the operator has no means of distinguishing from the genuine signals. Often this problem does not arise, since the noise level on short transmission lines may be made sufficiently low, but with some gun ranges this is a serious problem since for operational reasons the metering station has to be a very long way from the firing path.

The recurrent sweep chronograph to be described overcomes this drawback, since the pulses are displayed on a C.R.T. Being of invariant, characteristic shape they cannot be confused with random noise signals. This particular instrument has a time resolution of about $\pm 15\mu sec$, which is somewhat lower than the average counter instrument but quite adequate for most purposes. The maximum elapsed time measurable is $10^5$ sec, so that the maximum fractional accuracy is just under one part in seven thousand.

## Principles of the Recurrent Sweep Chronograph

This is a display type instrument, which traces the gun signals on a C.R.T. surface raster, and superimposes a time measuring matrix.

The logical starting point is the shape of the signals from the gun-range coils. This is sketched in Fig. 1. It closely resembles a differentiated Gaussian function. Typically its duration is of the order of $1\,000\mu sec$, and its amplitude may be about 10mV. In order to define accurately a reference edge, the signal is amplified, and then to avoid confusion of the record, is clipped. The full-line curve of Fig. 1 then indicates the signal form, AB being the reference edge.



Fig. 1. Detail of signal

These pulses are then deflexionally superimposed on a ten-line surface raster described on the C.R.T. as in Fig. 2a. The surface, as distinct from a simple linear, raster is used merely to increase the time resolution. Each
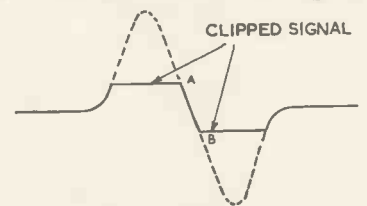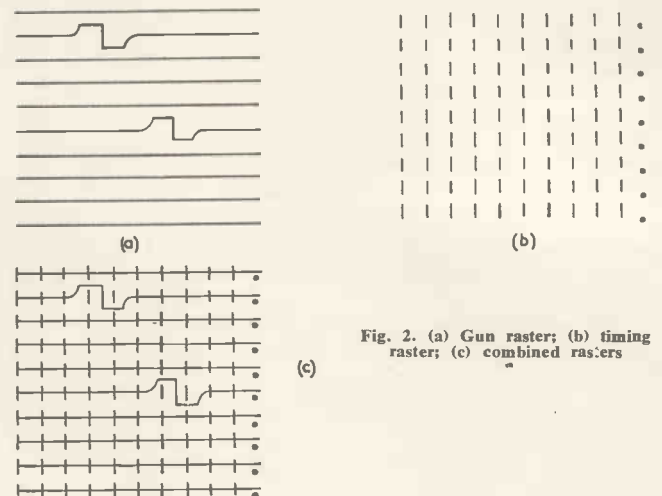


Fig. 2. (a) Gun raster; (b) timing raster; (c) combined rasters

line is scanned in $10^4 \mu sec$, and the whole frame is scanned in $10^5 \mu sec$. The raster is scanned continuously, and special circuits ensure that the flyback time in both line and frame is less than $10\mu sec$.

Normally the display is blanked out, but on receipt of the *leading* edge of the first gun signal (not the part AB), a brightening circuit operates, and switches on the C.R.T. for 1/10sec, i.e., until one complete frame has been scanned. Thus the second pulse is automatically displayed provided it arrives within the frame period, which is the maximum time interval measurable.

The raster is controlled from a precise 1 000c/s tuning fork, followed by two series connected decade dividers, which drive the line and frame generators. The linearity of this raster is very high, but errors in time measurements on it are reduced by superimposing sharp "pips," 1 000μsec, apart. These are derived directly from the tuning fork and a shaping network. Thus the time interval between the two pulses under study is measured to the nearest 1 000μsec, by a pure *counting* process (limited in accuracy *only* by the fork accuracy), and it is only in the interpolation between the "pips" that the accuracy is limited by such matters as trace resolution and linearity.

The timing raster (Fig. 2b) is photographically super-imposed on the gun signal record. Shortly before the gun is fired, the shutter of the recording camera is opened: the leading edge of the first pulse switches on the C.R.T. in time to receive the reference edge AB, and the camera records both this and the second pulse; a switch is then moved which lays down the timing raster: the shutter is closed and the record is complete (Fig. 2c). A Polaroid-Land camera is used, and this embodies a very ingenious and satisfactory self-contained development process. So far this is virtually unknown in this country, but it has found wide application in the U.S.A. especially for oscillo-record work.

This camera employs a special film, incorporating its own developer which is contained in a capsule. The latter may be broken by a lever after each frame is exposed, and a system of rollers then spreads the developer over the film. Development time is one minute, and a permanent record is obtained.

Experience seems to show that it is very diffi-cult to attempt to lay down both gun signal and timing rasters simultaneously. No C.R.T. yet developed has two guns capable of achieving the high degree of coincidence between the two rasters which is essential. If on the other hand the timing pips are mixed with the gun signal and applied together to the same set of plates, then trouble arises when the reference edge of the gun signal is too near a time pip. The successive raster method used has proved entirely satisfactory—it requires merely that the circuits are stable over the brief interval of time between writing the two frames.

**Details of the Circuits**

Fig. 3 shows a block schematic indicating in more detail the functions so far described. In the left-hand top corner is the master-timing tuning fork, which is a tem-perature controlled packaged unit made by American Time Products Corporation. This is followed by a squarer stage and then an Eccles-Jordan bi-stable multivibrator giving sharp pulses at 1 000 P.R.F. This again is followed by a decade frequency divider (also a packaged unit made by the Potter Instrument Co.) The output consists of sharp pulses at 100 P.R.F. and these drive the line time-base which in turn feeds the horizontal deflectors through a push-pull amplifier. A second decade divider fed from the first, yields pulses at 10 P.R.F. and these drive the frame time-base. Thus the raster is continuously scanned. The signals, either from the gun coils or time mark generator,

are introduced into the grid of one of the cathode coupled frame time-base amplifier valves.

To understand the remainder of Fig. 3, suppose that the "function switch" is in the right-hand position, i.e., the instrument is ready to receive the gun signals. These are introduced via the left-hand bottom corner, are ampli-fied and clipped, and then applied along with the frame time-base signal to the frame time-base amplifier. The switch S is also in the upper position so that the amplified gun signals trigger the Eccles-Jordan multivibrator M. This fires the "one shot" trigger which generates a square brightening pulse lasting approximately 1/10sec. The Eccles-Jordon multivibrator M acts as a reset relay. and prevents any further signals from firing the "one shot" until it has been reset.

The function switch is then turned slowly through the vertical position (No. 2) to the left-hand position (No. 1). In the vertical position a number of circuit re-adjustments are automatically performed. Moving across the switch from left to right—

Bank 1. The multivibrator M is re-set.

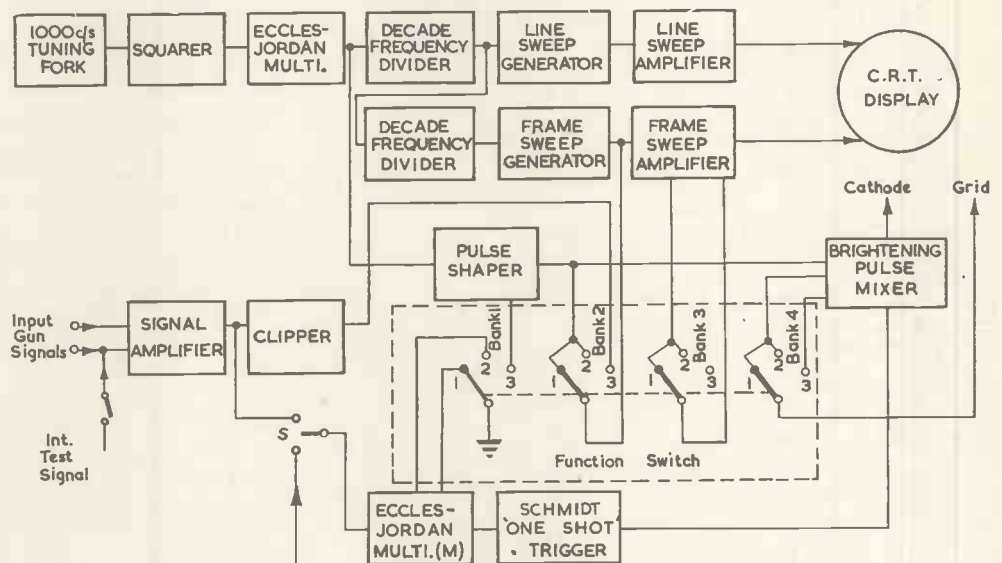Bank 2. The frame sweep amplifier is connected to the



Fig. 3. Basic block diagram (without power supplies)

pulse shaper driven by the fork, and is dis-connected from the gun signal line.

Bank 3. A slight change is made in the D.C. level of the frame sweep amplifier, to ensure that the timing pips write at the correct level with regard to the gun signal reference edge AB.

Bank 4. The grid bias on the C.R.T. is changed so that the relative writing intensities on the two rasters are appropriate.

Finally the function switch is moved to the left-hand position (No. 1). This changes nothing except Bank 1 which fires the Eccles-Jordan and thus lays down the timing raster. In that way the record is completed.

Some of the circuit elements shown in Fig. 3 will now be treated in more detail.

(1) THE TIME-BASE GENERATORS

Both line and frame generators are essentially identical apart from slight changes in component values in the charg-ing network. It suffices therefore to discuss one only, which will be the line time-base. This is shown in Fig. 4.

The whole problem is to obtain a very fast flyback, so that there is virtually no chance of the reference edge AB occurring during this period. The specification calls for
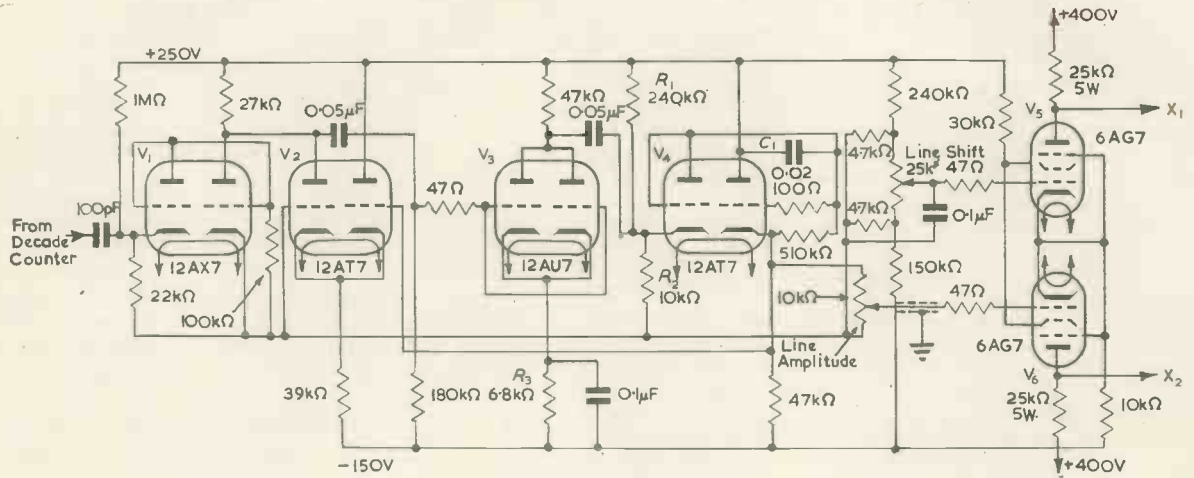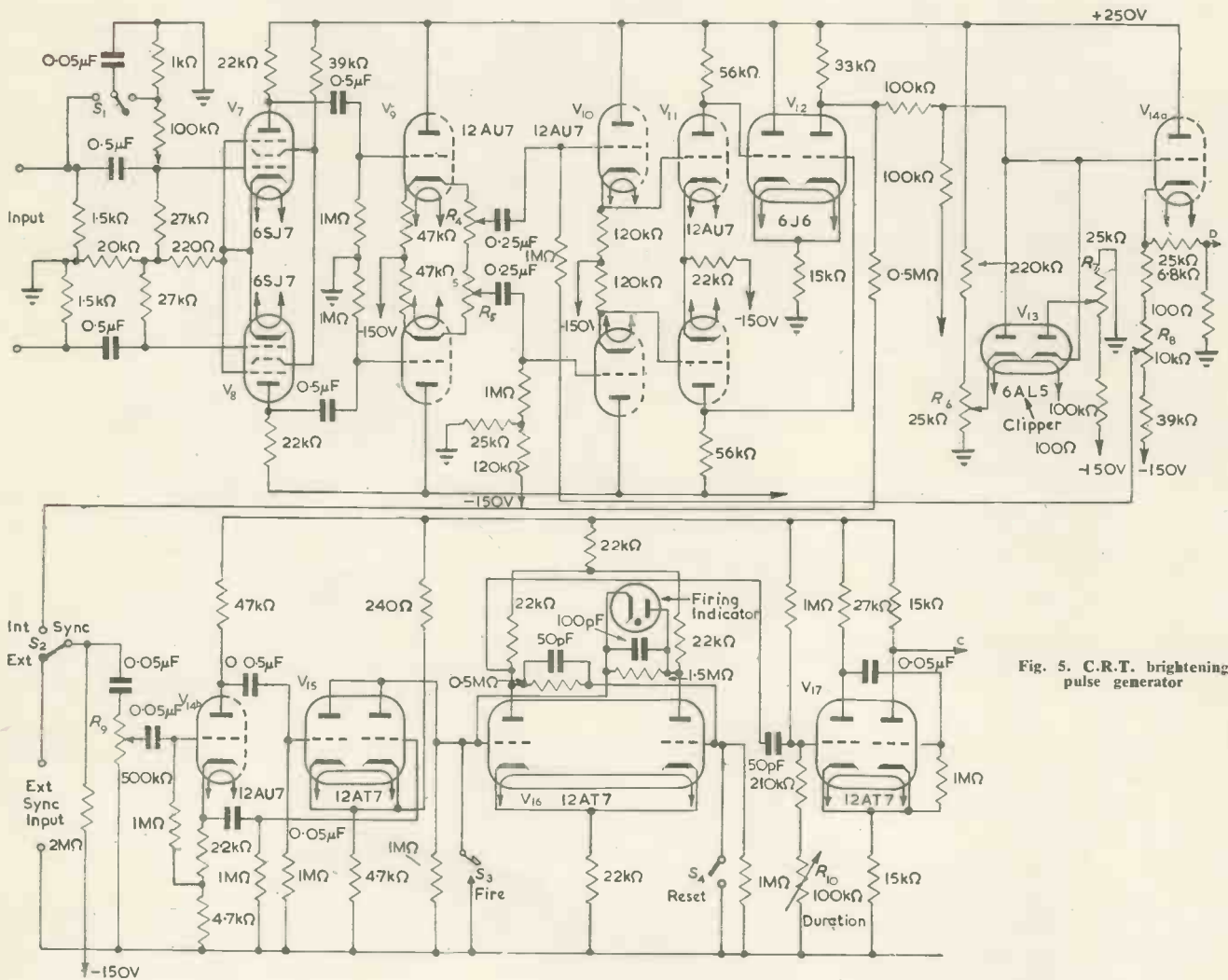
Fig. 4. Line sweep generator

Fig. 5. C.R.T. brightening pulse generator

less than 10μsec on both line and frame. This is a flyback ratio of 1 000 and 10 000 respectively, which is quite outside normal circuit performance.

Referring to Fig. 4, the output amplifier stages $V_5$ and $V_6$ are conventionally cathode coupled. The right-hand side of $V_4$ is also conventional, being a " bootstrap " circuit to achieve a high degree of linearity. The discharge circuit yielding the flyback is new.

Normally the left-hand portion of $V_4$, which is diode connected, is cut-off by the positive potential on its cathode developed by the potentiometer $R_1$ and $R_2$. This permits the time-base to develop the forward working stroke, during which time the cathode of the right-hand side of $V_4$ travels in a positive direction. Presently a pulse from the decade counter is applied via the limiting stage $V_1$ to the paralleled grids of $V_3$. This valve is normally near cut-off by virtue of the bias developed across the cathode load $R_3$. The large amplitude positive pulse applied from $V_1$ causes the instantaneous anode potential of $V_3$ to fall through several hundred volts towards the negative rail potential. This pulls down the cathode of the left-hand side of $V_4$ which immediately becomes highly conductive, and so very rapidly discharges $C_1$. During this flyback period the cathode of the right-hand side of $V_4$ moves very rapidly negative. This potential change is transmitted to the right-hand grid of $V_2$, which is a cathode coupled amplifier. At a defined point in the flyback period, and considerably before the potential across the diode portion of $V_4$ is low, the left-hand side of $V_2$ thus becomes highly conductive, and this shunts the output circuit of $V_1$. This shuts off the positive pulse being applied to the grids of $V_3$, and so terminates the flyback.

This technique avoids the necessity of paying much attention to the trailing edge of the flyback controlling pulses coming from the decade divider.

The frame time-base amplifier includes a switch (Bank 2 of function switch—Fig. 3) in the grid circuit of one of the output valves, which feeds either the gun signals or time marks through appropriate time-constants on to the frame axis. Another switch (Bank 3 of function switch) operates in the shift potentiometer network of the other valve to bring about a new level setting, so that the time pips write at the correct height.

(2) THE SIGNAL AMPLIFIER AND BRIGHTENING PULSE GENERATOR

These units are shown in Fig. 5. The input to the amplifier which is of the different type, is in the left-hand top corner. A differential input system is essential to give high rejection of in-phase signals and so to permit working on long transmission lines. The network around the switch $S_1$ allows a synthetic signal to be injected for test purposes. The gain control comes after the second stage, and consists of $R_4$ and $R_5$ which are mechanically ganged. Clipping is performed by the double diode $V_{13}$ which is a 6AL5. $R_6$ and $R_7$ control the clipping level. Symmetry of clipping requires that the grid of the cathode-follower stage $V_{14a}$ should normally reside at approximately zero volts to ground. This adjustment is made by means of the connexion between the slider of the potentiometer $R_8$ and the grid of the top half of $V_{10}$. The output at D goes via Bank 2 of the function switch and a suitable time-constant to the grid of one of the cathode coupled output valves of the frame time-base.

From the anode of $V_{12}$ a connexion is made via a $0.5M\Omega$ resistor to the input of the brightening pulse generator. This take-off occurs before the clipping stage, and ensures that the Eccles-Jordan fires on the first rise of the gun

signal, considerably before the reference portion AB arrives. This gun signal is fed via $S_2$ and attenuator $R_9$ to one half of a 12AU7 ($V_{14b}$). This is loaded in both anode and cathode, and when followed by $V_{15}$ in the manner shown, acts so as to yield a negative going pulse at the left-hand grid of $V_{16}$, regardless of the sign of the input pulse to the generator.

This pulse cuts-off the left-hand side of $V_{13}$, which is a normal Eccles-Jordan trigger. The sharp resultant positive pulse developed in the left-hand anode then fires the " one shot " $V_{17}$, and a positive pulse is developed at C. The duration of this can be controlled by $R_{10}$ and for the values shown is about 1/10sec.

Once the left-hand side of $V_{16}$ is cut-off, further negative going pulses have no effect, and the trigger is " jammed ". This prevents any further brightening pulses from spoiling

Fig. 6. C.R.T. feed networks
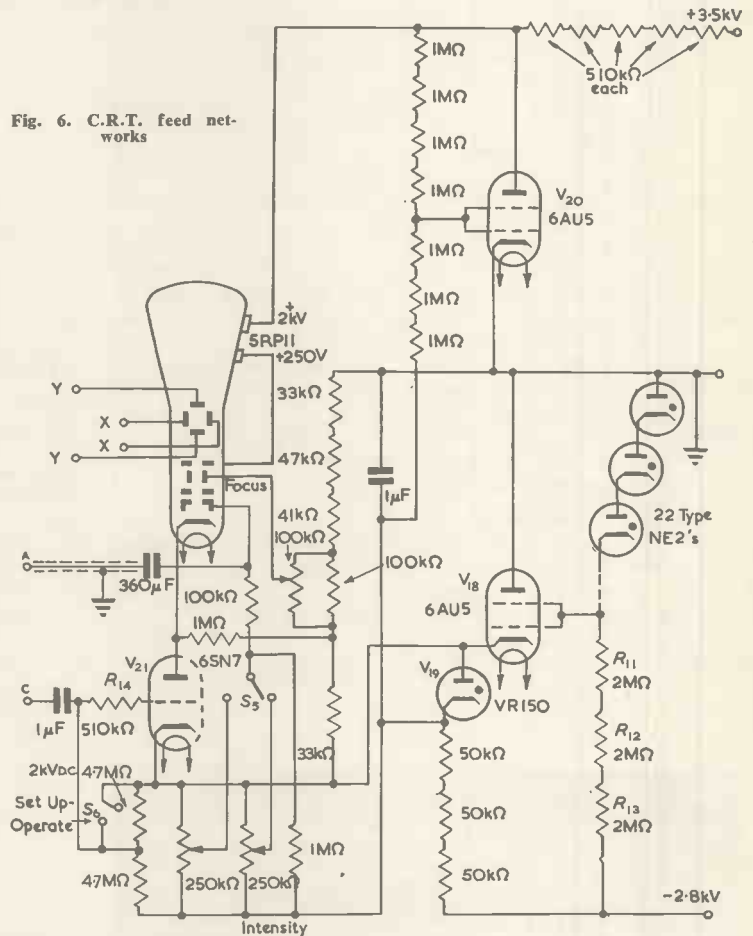
the record. Resetting of the trigger is done by closing $S_4$, which again renders the left-hand side conducting. This operation is automatically done on Bank 1, position 2 of the function switch. The Eccles-Jordan is fired manually by closing $S_3$, and this is done automatically on position 1, Bank 1.

(3) CATHODE-RAY TUBE FEED CIRCUITS

These networks are drawn in Fig. 6. The actual high-tension source is a conventional voltage doubling network and is not shown. The centre point of the two voltage doubling capacitors is grounded, so yielding a supply giving about $+3.5kV$ and $-2.8kV$ to ground. The differences in ground potential are due to unequal current drain. This general arrangement is made necessary by the use of a post-accelerator type of C.R.T.

Both positive and negative supplies are electronically stabilized. The stabilizing reference potential is generated by a series connexion of some twenty-two type NE2 glow tubes in series with $R_{11}$, $R_{12}$, and $R_{13}$. The voltage across the glow tubes is about 1·9kV. $V_{18}$ thus acts as a cathode follower shunt regulator, and produces a stabilized line 1·9kV below ground. A further stabilized rail about 150V more negative is provided by the glow-tube $V_{19}$.

Positive rail stabilization for the post accelerator is given by the simple shunt regulator tube $V_{20}$, which derives its control potential from the negative line.

Arrangements for the supply of the focusing anode and grid potentials are quite conventional. The switch $S_5$ is Bank 4 of the function switch, and serves to re-adjust the mean brightness of the C.R.T. when the raster is changed from gun signal to timing pips.

Short brightening pulses synchronized with the timing pips are introduced at A. The timing pips applied to the frame amplifier are differentiated by a short time-constant, so as to produce pulses almost proportional to the writing speed of the pip. In this manner the trace brightness over the whole pip is held nearly constant.

The frame brightening pulse from the "one shot" multivibrator is fed in at C. $V_{21}$ which is one-half of a 6SN7 serves to limit the pulse by reason of grid current flow in the 510kΩ grid resistor, so that "shading" of the rastar is avoided. The positive going pulse makes $V_{21}$ conduct, and this pulls the C.R.T. cathode negatively, so brightening the trace. The switch $S_6$, when closed, permanently opens $V_{21}$, and this permits the rasters to be set up in focus and brightness while the instrument is being adjusted. $S_6$ is then opened, and the display becomes blanked out until the appropriate pulse arrives.

### The Complete Instrument

The remainder of the circuit although fairly extensive, is quite conventional and calls for no special comment.

Fig. 7 shows the complete instrument, including the Polaroid-Land camera in situ. A viewing port in the top of the camera tube permits the rasters to be set, without actually removing the body of the camera.

To avoid risk of magnetic interference, the C.R.T. is enclosed in a mumetal screen, and in addition most of the power supplies are contained in the separate box.
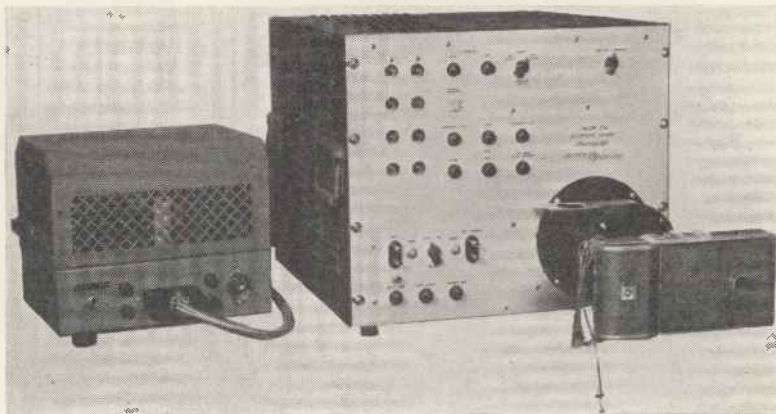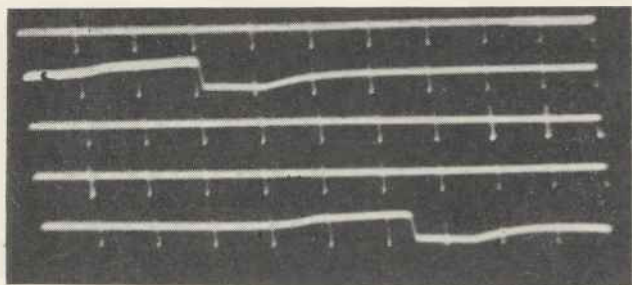


Fig. 7. The complete instrument



Fig. 8. Portion of a typical record

Fig. 8 is a portion of a typical record. Only five of the ten lines are shown. The time interval between the crossover portions of the waves is 33 400μsec, and the estimated uncertainty is about ±15μsec. The fractional accuracy is thus about one part in two thousand five hundred.

## Television and Sound Broadcast of The Royal Review

Details have just been announced of the technical arrangements by which the BBC broadcast both in television and sound the Coronation Review of the Fleet by Her Majesty the Queen from Spithead on June 15th.

For the television broadcast three cameras were located in the dockyard at Portsmouth and two cameras on board both H.M. Aircraft Carrier *Eagle* and Recovery Ship *Reclaim*.

The new feature in this television broadcast was that for the first time cameras were placed on board ship and that in consequence a number of special circuit arrangements were necessary. To avoid hum-bars, a 50c/s locking signal derived from the electricity grid system was fed by a radio link to the ship-borne cameras operating from the ships' electricity supply. The vision signals from all seven cameras and accompanying commentaries afloat and ashore were conveyed by centimetric links to a BBC mobile Control Room at the Admiralty Signal and Radar Research Establishment at Portsdown near Portsmouth. Further centimetric relay equipment installed at Butser Hill and Gibbet Hill carried the programme to the BBC receiving site at Highgate, where it was fed to the main television network.

The television transmitters and associated equipment on board H.M.S. *Eagle* and H.M.S. *Reclaim* were designed by Pye Ltd. and have an output of 500 watts giving a range of some 15 to 20 miles.

The directional aerial arrays on board H.M.S. *Eagle* and H.M.S. *Reclaim* were beamed on Portsdown Hill and were stabilized in azimuth to compensate for any swinging or turning movements of the ships.

Equipment used for the broadcast included cameras of the image-orthicon type made by Marconi's Wireless Telegraph Co., Ltd., and Pye Ltd., and radio links manufactured by these firms as well as by Standard Telephones and Cables Ltd., the General Electric Co. and Mullard Ltd.

The sound broadcast commentaries from the ships were conveyed to the main reception point at Southsea Castle by radio link and thence, together with those from the shore commentary points, to London and the BBC transmitters throughout the country by G.P.O. Lines.

For the European Services and European visiting broadcasters one live commentary point and eight tape recording channels were provided in H.M.S. *Indomitable*. Overseas services and visiting broadcasters were provided with one live commentary point and eight sets of recording gear in H.M.S. *Indefatigable*.

# Synthesis of Uniform Transmission Line

By D. W. C. Shen*, Ph.D.

*The uniform transmission line is synthesized by an infinite number of four-terminal networks; the limiting values of the associated infinite matrices are evaluated mathematically as well as by physical reasoning.*

It has been shown in the matrix theory of four-terminal networks[1,2] that any linear, passive, bilateral quadripole structure is characterized by a square matrix of the second order, given by the equation:

$$[M] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \dots \dots \dots \dots \dots (1)$$

The elements of $[M]$, known as the general circuit parameters, are related by the equation:

$$AD - BC = 1 \dots \dots \dots \dots \dots (2)$$

and hence the determinant of the matrix is unity. The usefulness of the associated matrix of the four-terminal network is best illustrated by considering the cascade connexion of two circuits of Fig. 1, in which the first network with parameters, $A_1$, $B_1$, $C_1$, $D_1$, is terminated in a second network with parameters, $A_2$, $B_2$, $C_2$, $D_2$. Denoting the voltage and the current at the junction by $E$ and $I$ respectively and applying the fundamental matrix equations



Fig. 1. Cascade connexion of two circuits

connecting the input and output quantities to both networks, we have:

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} E \\ I \end{bmatrix} \dots \dots \dots \dots (3)$$

and

$$\begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix} \dots \dots \dots \dots (4)$$

The associated matrix of the whole network may be obtained by eliminating $E$ and $I$ between Equations (3) and (4). Thus:

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\begin{bmatrix} E_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} E_2 \\ I_2 \end{bmatrix} \dots (5)$$

and we have the following matrix relation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \dots \dots \dots (6)$$

whose elements are the general circuit parameters of the whole circuit and that of the component circuits.

For every given network, the associated matrix has a fixed set of elements characteristic of the network; thus, for the uniform transmission line, whose total series impedance is $Z$ and whose total shunt admittance is $Y$, the associated matrix is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \sqrt{(ZY)} & \sqrt{(Z/Y)} \sinh \sqrt{(ZY)} \\ \sqrt{(Y/Z)} \sinh \sqrt{(ZY)} & \cosh \sqrt{(ZY)} \end{bmatrix} \dots \dots \dots (7)$$

Now, physically the uniform line may be approximated by dividing the line into $n$ identical sections, each of which is replaced by either an equivalent $T$-network or an equiva-



Fig. 2. Equivalent T-network

lent $\pi$-network. The associated matrix of the $T$-network of Fig. 2 is:

$$[M] = \begin{bmatrix} 1 + \dfrac{ZY}{2n^2} & Z/n\left(1 + \dfrac{ZY}{4n^2}\right) \\ Y/n & 1 + \dfrac{ZY}{2n^2} \end{bmatrix} \dots (8)$$
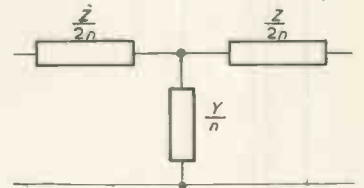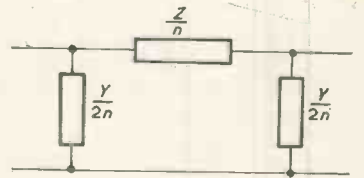


Fig. 3. Equivalent π-network

and for the $\pi$-network of Fig. 3, it is:

$$[M] = \begin{bmatrix} 1 + \dfrac{ZY}{2n^2} & Z/n \\ Y/n\left(1 + \dfrac{ZY}{4n^2}\right) & 1 + \dfrac{ZY}{2n^2} \end{bmatrix} \dots \dots (9)$$

Besides $T$- and $\pi$-networks, the sections of a uniform line may take on many other forms of identical networks or of mixed networks. Thus the $T$- and the $\pi$-sections may be replaced by the structures shown in Fig. 4(a) and Fig. 4(b), whose associated matrices are given respectively as:
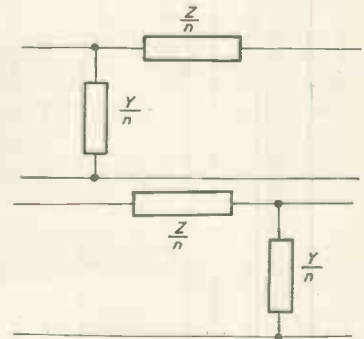


Fig. 4. Alternative structures

$$[M] = \begin{bmatrix} 1 & Z/n \\ Y/n & 1 + \dfrac{ZY}{n^2} \end{bmatrix} \dots \dots (10a)$$

* The University of Adelaide.

$$[M] = \begin{bmatrix} 1 + \dfrac{ZY}{n^2} & Z/n \\ Y/n & 1 \end{bmatrix} \quad \ldots \ldots (10b)$$

Evidently the uniform line may be regarded as the limiting case of either of the above approximation networks as the number of sections, $n$, becomes infinite, while $Z$ and $Y$ remain constant. Therefore, from physical reasoning, we are led to the relations:

$$\underset{n \to \infty}{\text{Lim}} \begin{bmatrix} 1 + \dfrac{ZY}{2n^2} & Z/n\left(1 + \dfrac{ZY}{4n^2}\right) \\ Y/n & 1 + \dfrac{ZY}{2n^2} \end{bmatrix}^n$$

$$= \underset{n \to \infty}{\text{Lim}} \begin{bmatrix} 1 + \dfrac{ZY}{2n^2} & Z/n \\ Y/n\left(1 + \dfrac{ZY}{4n^2}\right) & 1 + \dfrac{ZY}{2n^2} \end{bmatrix}^n$$

$$= \underset{n \to \infty}{\text{Lim}} \begin{bmatrix} 1 & Z/n \\ Y/n & 1 + \dfrac{ZY}{n^2} \end{bmatrix}^n = \underset{n \to \infty}{\text{Lim}} \begin{bmatrix} 1 + \dfrac{ZY}{n^2} & Z/n \\ Y/n & 1 \end{bmatrix}^n$$

$$= \begin{bmatrix} \cosh \sqrt{ZY} & \sqrt{Z/Y}\,\sinh \sqrt{ZY} \\ \sqrt{Y/Z}\,\sinh \sqrt{ZY} & \cosh \sqrt{ZY} \end{bmatrix} \quad \ldots \ldots (11)$$

To demonstrate the above relations mathematically, start with the fundamental theorem of a square matrix. Since every square matrix must satisfy its characteristic equation[3], we have:

$$[M]^2 - (\lambda_1 + \lambda_2)[M] + \lambda_1\lambda_2[I] = 0 \quad \ldots \ldots (12)$$

or

$$[M]^2 = (\lambda_1 + \lambda_2)[M] - \lambda_1\lambda_2[I] \quad \ldots \ldots (13)$$

where $[M]$ is a square matrix of order 2, $\lambda_1$ and $\lambda_2$ are the two unequal latent roots, and $[I]$ is the identity matrix. Multiply Equation (13) by $[M]$ and then eliminate $[M]^2$ by Equation (13); we get:

$$[M]^3 = \frac{1}{\lambda_2 - \lambda_1}\left\{(\lambda_2^3 - \lambda_1^3)[M] - \lambda_1\lambda_2(\lambda_2^2 - \lambda_1^2)[I]\right\} \quad (14)$$

Repeating the process, we finally obtain the following identity:

$$[M]^n = \frac{1}{\lambda_2 - \lambda_1}\left\{(\lambda_2^n - \lambda_1^n)[M] - \lambda_1\lambda_2(\lambda_2^{n-1} - \lambda_1^{n-1})[I]\right\} \quad \ldots \ldots (15)$$

The characteristic equation of the associated matrix $[M]$ given in Equation (1) is:

$$\lambda^2 - (A + D)\lambda + 1 = 0 \quad \ldots \ldots (16)$$

For any of the four basic structures given above, we note that $A + D = 2(1 + ZY/2n^2)$, and hence the characteristic equation is the same for all the relations in Equation (11). The two roots of Equation (16) are:

$$\lambda_1 = 1 + \frac{ZY}{2n^2} - \sqrt{\left(\frac{ZY}{n^2} + \frac{Z^2Y^2}{4n^4}\right)}$$

$$\lambda_2 = 1 + \frac{ZY}{2n^2} + \sqrt{\left(\frac{ZY}{n^2} + \frac{Z^2Y^2}{4n^4}\right)} \quad \ldots (17)$$

According to Equation (15), we have:

$$\underset{n \to \infty}{\text{Lim}}[M]^n = \underset{n \to \infty}{\text{Lim}} \begin{bmatrix} A\left(\dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}\right) - \dfrac{\lambda_2^{n-1} - \lambda_1^{n-1}}{\lambda_2 - \lambda_1} & B\left(\dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}\right) \\ C\left(\dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}\right) & D\left(\dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}\right) - \dfrac{\lambda_2^{n-1} - \lambda_1^{n-1}}{\lambda_2 - \lambda_1} \end{bmatrix} \quad (18)$$

which will be used to prove the relations obtained previously by physical reasoning.

It is interesting to note that Equation (18) can also be

obtained by a collineatory transformation[4,5] of the matrix $[M]$. In order to diagonalize the matrix $[M]$, the following matrix is introduced:

$$[L] = C \begin{bmatrix} \dfrac{\lambda_1 - D}{C} & \dfrac{\lambda_2 - D}{C} \\ 1 & 1 \end{bmatrix} \quad \ldots \ldots (19)$$

and its inverse

$$[L]^{-1} = \frac{1}{\lambda_1 - \lambda_2} \begin{bmatrix} 1 & \dfrac{-\lambda_2 + D}{C} \\ -1 & \dfrac{\lambda_1 - D}{C} \end{bmatrix} \quad \ldots \ldots (20)$$

Thus we may write:

$$[L]^{-1}[M][L]$$

$$= \frac{C}{\lambda_1 - \lambda_2} \begin{bmatrix} 1 & \dfrac{-\lambda_2 + D}{C} \\ -1 & \dfrac{\lambda_1 - D}{C} \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \dfrac{\lambda_1 - D}{C} & \dfrac{\lambda_2 - D}{C} \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{bmatrix} \quad \ldots \ldots (21)$$

Conversely, we have:

$$[M] = [L] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [L]^{-1} \quad \ldots \ldots (22)$$

An interesting relationship follows from Equation (22), namely:

$$[M]^2 = [L]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}[L]^{-1}[L]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}[L]^{-1} = [L]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}^2[L]^{-1}$$

$$\ldots \ldots (23)$$

This relationship may readily be generalized with the result that the $n^{\text{th}}$ power of the non-singular square matrix $[M]$ is given by:

$$[M]^n = [L]\begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix}[L]^{-1} \quad \ldots \ldots (24)$$

Now, post-multiplying $\begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix}$ by $[L]^{-1}$ gives:

$$\begin{bmatrix} \dfrac{\lambda_1^n}{\lambda_1 - \lambda_2} & \lambda_1^n\dfrac{D - \lambda_2}{C(\lambda_1 - \lambda_2)} \\ \dfrac{-\lambda_2^n}{\lambda_1 - \lambda_2} & \lambda_2^n\dfrac{\lambda_2 - D}{C(\lambda_1 - \lambda_2)} \end{bmatrix} \quad \ldots \ldots (25)$$

and pre-multiplying Equation (25) by $[L]$ results in the following matrix:

$$\begin{bmatrix} \dfrac{\lambda_1^n(\lambda_1 - D) - \lambda_2^n(\lambda_2 - D)}{\lambda_1 - \lambda_2} & \dfrac{(\lambda_1^n - \lambda_2^n)(\lambda_1 - D)(D - \lambda_2)}{C(\lambda_1 - \lambda_2)} \\ C\dfrac{\lambda_1^n - \lambda_2^n}{\lambda_1 - \lambda_2} & \dfrac{\lambda_1^n(D - \lambda_2) - \lambda_2^n(\lambda_1 - D)}{\lambda_1 - \lambda_2} \end{bmatrix} \quad (26)$$

Remembering that $\lambda_1 + \lambda_2 = +(A + D)$ and $\lambda_1\lambda_2 = 1$, we can reduce Equation (26) to Equation (18) immediately.

To find the limits of Equation (11), let us choose the $\pi$-network as an example. Thus Equation (18) for the $\pi$-network may, after arrangement, be written as:

$$\text{Lim}_{n \to \infty} [M]^n = \begin{bmatrix} \dfrac{(\lambda_2^n - \lambda_2^{n-1}) - (\lambda_1^n - \lambda_1^{n-1})}{\lambda_2 - \lambda_1} + \dfrac{ZY}{2n^2} \dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} & \dfrac{Z}{n} \dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} \\[2ex] \left(1 + \dfrac{ZY}{4n^2}\right) \dfrac{Y}{n} \dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} & \dfrac{(\lambda_2^n - \lambda_2^{n-1}) - (\lambda_1^n - \lambda_1^{n-1})}{\lambda_2 - \lambda_1} + \dfrac{ZY}{2n^2} \dfrac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} \end{bmatrix} \dots (27)$$

The following limits are readily computed:

(1) $\text{Lim}_{n \to \infty} n(\lambda_2 - \lambda_1) = 2\sqrt{(ZY)}$

(2) Let $\lambda_1 = e^{-a}$, $\lambda_2 = e^{+a}$,
then $\lambda_2^n - \lambda_1^n = 2 \sinh na$

Differentiate $\lambda_1^n$ with respect to $n$; we have:

$$\frac{d\lambda_1^n}{dn} = \frac{d(\lambda_1^n)}{d\lambda_1} \frac{d\lambda_1}{dn} = e^{-a_1}(-a)$$

or

$$n^2 \lambda_1^{n-1}\left(-\frac{ZY}{n^3} + \frac{ZY/n^3 + Z^2Y^2/2n^5}{\sqrt{(ZY/n^2 + Z^2Y^2/4n^4)}}\right) = \lambda_1^n(-na)$$

As $n \to \infty$, taking the limits on both sides, we get:

$$\text{Lim}_{n \to \infty}(na) = \sqrt{(ZY)} \text{ or } \lambda_1^n = e^{-\sqrt{ZY}} \text{ and } \lambda_2^n = e^{+\sqrt{ZY}}$$

Hence

$$\text{Lim}_{n \to \infty} Z/n \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} = \frac{Z \sinh \sqrt{(ZY)}}{\sqrt{(ZY)}}$$
$$= \sqrt{(Z/Y)} \sinh \sqrt{(ZY)} \dots (28)$$

and

$$\text{Lim}_{n \to \infty}\left(1 + \frac{ZY}{4n^2}\right)\frac{Y}{n} \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} = \text{Lim}_{n \to \infty} \frac{Y}{n} \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}$$
$$\frac{Y \sinh \sqrt{(ZY)}}{\sqrt{(ZY)}} = \sqrt{(Y/Z)} \sinh \sqrt{(ZY)} \quad (29)$$

(3)

$$\text{Lim}_{n \to \infty}\left\{\frac{(\lambda_2^n - \lambda_1^{n-1}) - (\lambda_1^n - \lambda_1^{n-1})}{\lambda_2 - \lambda_1} + \frac{ZY}{2n^2} \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1}\right\}$$
$$= \text{Lim}_{n \to \infty} \frac{(\lambda_2^n - \lambda_1^{n-1}) - (\lambda_1^n - \lambda_1^{n-1})}{\lambda_2 - \lambda_1} = \text{Lim}_{n \to \infty} \frac{\lambda_2^n(1 - \lambda_1) - \lambda_1^n(1 - \lambda_2)}{\lambda_2 - \lambda_1}$$
$$= \text{Lim}_{n \to \infty} \frac{e^{\sqrt{ZY}}\sqrt{ZY} + e^{-\sqrt{ZY}}\sqrt{ZY}}{} = \cosh \sqrt{(ZY)} \dots (30)$$

From Equations (28), (29) and (30) it follows immediately that:

$$\text{Lim}_{n \to \infty}[M]^n = \begin{bmatrix} \cosh \sqrt{(ZY)} & \sqrt{(Z/Y)} \sinh \sqrt{(ZY)} \\ \sqrt{(Y/Z)} \sinh \sqrt{(ZY)} & \cosh \sqrt{(ZY)} \end{bmatrix}$$

The other equalities given in Equation (11) can be proved with very little modification because the characteristic equation is the same for all cases.

The above approach to the uniform line replaces the conventional method of differential equations and clearly demonstrates the elegance of matrix theory in network synthesis.

It is obvious that the behaviour of artificial lines and other structures of similar type, such as wave filters, is closely allied to that of the transmission line. In fact, the structure known as a constant-$K$, low-pass filter with its filtering properties, was originally discovered as a result of inserting loading-coil inductance in telephone lines[6]. The ideal filter possessing no dissipative elements is equivalent

Fig. 5. Cascaded structure
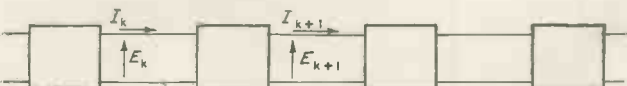


to a transmission line over a band of frequencies which may be made as wide as required; outside that band the resemblance ceases and the filter exhibits its cut-off effect.

The utility of the latent roots of the associated matrix in determining the band-pass characteristic of a quadripole can easily be demonstrated. Consider the cascaded structure of Fig. 5. The fundamental matrix equation of the $k^{th}$ quadripole is:

$$\begin{bmatrix} E_k \\ I_k \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} E_{k+1} \\ I_{k+1} \end{bmatrix} \dots (31a)$$

or

$$\begin{bmatrix} E_{k+1} \\ I_{k+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}\begin{bmatrix} E_k \\ I_k \end{bmatrix} \dots (31b)$$

If the inverse matrix is made equal to a constant, say $S$, times the identity matrix,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = S[I] \dots (32)$$

Obviously $S$ is $1/\lambda$. Now set $\lambda_1 = e^{-a}$, $\lambda_2 = e^{+a}$; we have:
$$\lambda_1 + \lambda_2 = A + D = 2 \cosh a = 2 \cosh (a + jb) \dots (33)$$

For zero attenuation, $a = 0$, Equation (33) reduces to:

$$\frac{\lambda_1 + \lambda_2}{2} = \frac{A + D}{2} = \cos b \dots (34)$$

so two conditions are imposed upon the two unequal latent roots: In terms of $A$ and $D$, they are:

(1) $A + D$ must be a real number. This can always be satisfied if the network is non-dissipative.

(2) $-1 \leqslant \dfrac{A + D}{2} \leqslant 1$. This is true only

for certain frequencies.

Guillemin[2] gives the condition:

$$-1 \leqslant \frac{\sqrt{Z_{11}Z_{22}}}{Z_{12}} \leqslant 1,$$

where $Z_{11} = A/C$, $Z_{12} = 1/C$, $Z_{22} = D/C$. Hence, in terms of $A$ and $D$, the condition is simply:

$$-1 \leqslant \sqrt{AD} \leqslant 1$$

Our condition (2) is not much different from that given by Guillemin, since $\dfrac{A + D}{2} \leqslant \sqrt{AD}$, and for a symmetrical structure, $A = D$; they are identical.

From the viewpoint of synthesis, the artificial line simulates the real line at one particular frequency only, but approximates to the real line over a certain band of frequencies. The greater the number of sections simulating a given length of line, the greater the band of frequencies over which the properties of the artificial and real line correspond until the number of sections becomes infinite it is then identical with the real line.

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# The Sunbury Portable Frequency Standard

## for 100c/s to 100kc/s

By A. H. Morser*

*A portable frequency reference source for calibrating laboratory instruments is described. It has an output in the range 100c/s to 100kc/s and was designed primarily for calibrating Sunbury electronic tachometers. The fundamental frequency standard is a vacuum mounted 100kc/s quartz crystal. Correct operation and an output greater than 25V R.M.S. are maintained with mains variations from 200 to 250V.*

A REFERENCE standard was required for calibrating various laboratory instruments measuring time and speed. Some of these were not portable, so it was desirable to standardize them in their working positions. A single signal in the low audio frequency range, accurate to 0·1 per cent, satisfied most requirements.

For experimental work a resistance-capacitance oscillator with good short-term stability (better than 0·2 per cent per hour) was available. In order to check this throughout its range, the standard had to provide outputs at 1kc/s and 10kc/s.

A large number of fractions and multiples of standard frequency can be recognized by the simultaneous display of sine waves on a double-beam oscilloscope, and by this means interpolation gaps can be reduced to a few per cent of the unknown frequency.

The output may be used to drive non-linear circuits fed to electronic tachometers, or compared with other sinusoidal voltage generators.

## Design

Sunbury electronic tachometers derive their input impulses from either an electromagnetic pick-up facing one or more projections on the engine shaft, or from the ignition system of the engine. At maximum speeds, the tachometers receive 100-200 impulses per second. The lowest output frequency from the standard has therefore to be of this order.

It was decided to use a vacuum-mounted quartz crystal as the master oscillator, as these crystals have high accuracy and stability and low initial cost. The lowest frequency readily available, however, was 100kc/s and several stages of frequency division were required to obtain an output frequency of 100c/s.

Three consecutive divisions by ten were employed, with the intermediate frequencies available at the output terminals. It was undesirable that divider adjustments should be accessible from the panel. In the event of valve or component failure, no output should be available, otherwise a fault leading to division say by 9 and 11 consecutively could produce an output only 1 per cent different from the desired frequency.

The frequency division is obtained by regenerative modulation, and Table 1 details the reasons for preferring this method to the more usual multivibrator.

In the functional block diagram (Fig. 1), the three compartments enclosed by dotted lines represent the three

divider stages, which are electrically similar. The frequency of the input to the first stage is 100kc/s which modulates a 90kc/s input in valve $V_3$ to produce an output at 10kc/s. The 10kc/s signal drives the non-linear valve $V_4$, from

### TABLE 1
#### Frequency Division by Different Methods

| ONE STAGE OF DIVISION BY 10 | TYPE OF DIVIDER CIRCUIT | | | |
|---|---|---|---|---|
| | REGENERATIVE DIVIDER | LOCKED MULTI-VIBRATOR | SCALER | PULSE GENERATOR AND STAIRCASE INTEGRATOR |
| VALVES USED .. | 2 pentodes | 2 triodes | 8 triodes | 1 pentode, 4 diodes |
| VALVE ENVELOPES .. | 2 | 1 | 4 | 3 |
| TUNED CIRCUITS .. | 2 | 0 | 0 | 0 |
| OTHER CRITICAL COMPONENTS .. .. | None | Valves $C$ & $R$ | R | $C$ & $R$ |
| OUTPUT FOR NO INPUT | None | False | None | None |
| OUTPUT FOR WRONG INPUT FREQUENCY $f$ | None | False | $f/10$ | $f/10$ |
| NATURE OF OUTPUT .. | Sinusoidal | Rectangular impulse | Square wave | Rectangular impulse |
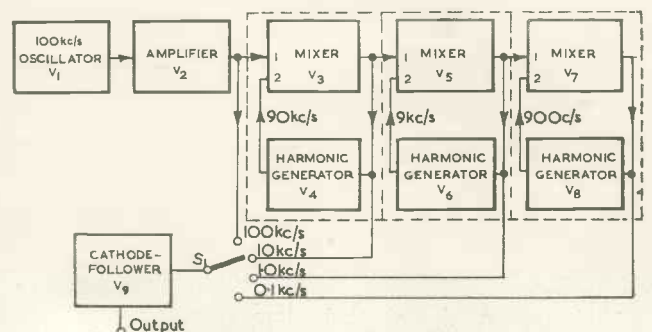


Fig. 1. Functional block diagram

which the ninth harmonic at 90c/s is extracted and fed back to $V_3$. The stability of the output depends on the tuned circuits being sufficiently sharp and stable to prevent division by 9 or 11, a condition easy to satisfy at frequencies above 100c/s. The dividers will only operate when their input frequencies are approximately correct. The wanted frequency is selected by switch $S_1$ and fed through a cathode follower to the output terminals.

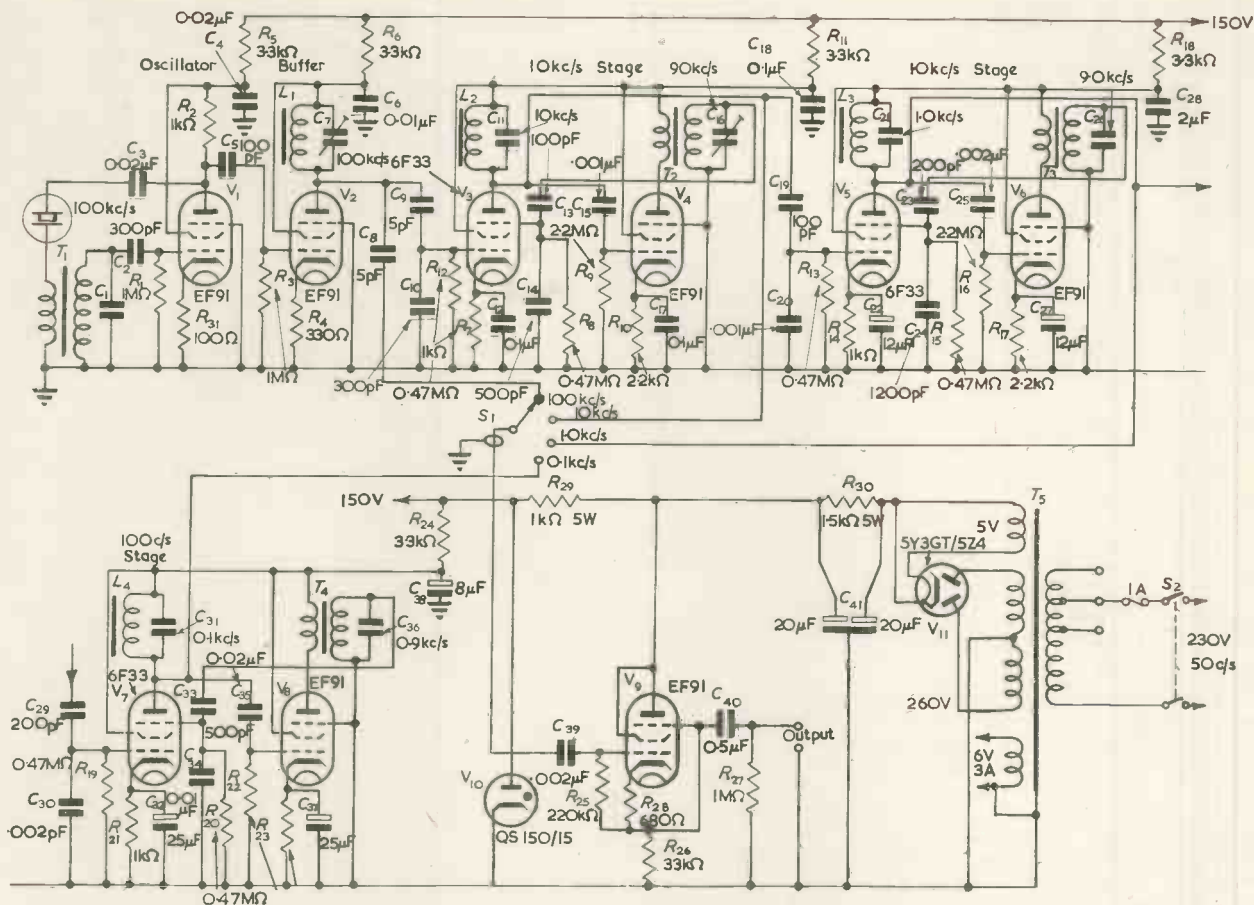The theoretical basis of this type of divider was

Fig. 2. The complete circuit

thoroughly discussed by an American investigator[1], in 1939. He gave a comprehensive list of references.

The circuits are laid out in sequence from the oscillator at one end to the 100c/s stage at the end remote from the power supply. The latter is placed, together with the stabilizer and cathode follower, in one corner at the back of the chassis.

## Description of the Circuits
### DIVIDERS

The circuit diagram (Fig. 2) shows that the divider stages are all similar in static electrode voltages, current consumption and type of valves employed, etc. The alternating voltages are of the same order on each stage, and only the tuned circuits, coupling and by-pass capacitors vary to suit the particular frequencies at that stage. The values for the coupling and by-pass capacitors are not critical, the smallest values to give reliable working being employed. All components except those in the tuned circuits are ± 20 per cent of nominal value.

Table 2 gives the data on the tuned circuits. The resonant capacitances are lower than at first to be expected, because the capacitive potentiometers feeding the following valves are in parallel with them. Coils with suitable dust cores are used to give a high Q at their resonant frequencies.

Each divider consists of a 6F33 triple grid control valve used as a mixer frequency changer, and an EF91 as

harmonic generator. The 100kc/s signal, for example, is fed into grid 1 of the mixer valve $V_3$ at an amplitude of 1·5-3 volts. Less signal gives a lower output, but larger inputs give no more output with the risk of operating at some unwanted harmonic. The mixer takes in a 90kc/s signal of the same amplitude on grid 3, and the resultant signal operates a 10kc/s tuned circuit in the anode supply. The 10kc/s signal is used to overdrive $V_4$, an EF91 from which the ninth harmonic is extracted by a tuned circuit and fed back. The system starts itself above a critical low amplitude of input. The Q of the 10kc/s circuit determines the extent to which the system will not work on adjacent modes, because a change to dividing by 9 or 11, changes this frequency by 10 per cent to 9·091 or

**TABLE 2**
Tuned Circuit Data

| COIL | FREQUENCY | TRANS-FORMER RATIO | $L$ | $Q^*$ | $C\dagger$ | WINDING | CORE |
|------|-----------|------|------|------|------|---------|------|
| $T_1$ | 100kc/s osc. | 1 : 10 | 6mH | >100 | 450–600pF | 30t. 36 s.w.g./ 300t. 36 s.w.g. | C54/V |
| $L_1$ | 100kc/s | — | 38mH | >100 | 20–50pF | 750t. 38 s.w.g. | ,, |
| $T_2$ | 90kc/s | 1 : 5·5 | 15mH | 100 | 120–150pF | 500t 36 s.w.g./ 90t. 38 s.w.g. | ,, |
| $L_2$ | 10kc/s | — | 90mH | 60 | 2 700–3 000pF | 1 250t. 40 s.w.g. | ,, |
| $T_3$ | 9kc/s | 1 : 5·5 | 170mH | 50 | 1 600–1 750pF | 1 665t. 42 s.w.g./ 280t. 42 s.w.g. | ,, |
| $L_3$ | 1·0kc/s | — | 1·0H | >100 | 0·024–0·035$\mu$F | Toroidal coil S274 | |
| $T_4$ | 0·9kc/s | 1 : 5·5 | 1·0H | >100 | 0·029–0·031$\mu$F | ,, ,, S275 | |
| $L_4$ | 0·1kc/s | — | 12·5H | 30 | 0·2$\mu$F | ,, ,, S276 | |

\* At working frequency.
† All capacitors are silver mica except the 100c/s capacitor which is a 750V wkg. tubular paper shunted to value.
The cores and coils were supplied by Salford Electrical Instruments.

11·111kc/s, while a 90kc/s signal is only changed by just over 1 per cent.

The 6F33 anode amplitude is restricted to about 50V peak, because the anode impedance falls with decreasing anode voltage. The circuit values are arranged not to overdrive this circuit and so cause reduction of selectivity. In the 100c/s stage it was found necessary to reduce the amplitude of the 100c/s signal by reducing the feedback from the 900c/s circuit to get stable operation. This was done by increasing $C_{34}$. The harmonic generator is operated with a grid base of about 4 volts. With a peak anode signal of 30-50V, conditions are at the optimum for producing ninth and adjacent harmonics.

The 90kc/s tuned circuit is used as a transformer whose primary impedance extracts maximum power from the pulse of anode current in the harmonic generator. The resulting high voltage swing on the secondary is divided down to feed the mixer. This complete circuit has a very poor loop gain at 10kc/s, something like 1/100th of its gain at 90kc/s, so that 10kc/s oscillation can be avoided. Similarly 90kc/s oscillation is prevented by the low impedance at that frequency of the 10kc/s tuned circuit.

The harmonic generator is self biased to near cut-off. This means that signals of low amplitude will produce harmonics in approximately the same phase relation as large signals, and there is no sudden transition of operating mode while starting, or with changing amplitude of input.

OSCILLATOR

The oscillator uses a commercial vacuum-mounted crystal oscillating at its series resonance. The anode waveform of $V_1$ is of low amplitude and far from sinusoidal, so a buffer amplifier is used to feed the divider.

OUTPUT STAGE

The output is taken from a cathode-follower, for which the power supply and bias arrangements are adequate to produce a 35V sinusoidal output at the selected frequency. The output impedance is about 1 000 ohms.

POWER SUPPLY

A high tension supply of 150V was chosen as the highest voltage obtainable from a single glow discharge tube ($V_{10}$). This results in very low component dissipations and working voltages, while stage gain is not much less than that obtained by working at 250V. In this circuit, the low value of output voltage is unimportant.

The H.T. supply for each divider stage is taken to both anodes and screens from a common by-pass capacitor.

Resistance-capacitance smoothing is adequate, and at the same time provides the voltage drop for the gas tube. The cathode-follower output stage is fed from an intermediate point at 200V in this chain, in order to prevent it overloading. The total power consumption of 50 watts consists of a heater consumption of 20 watts, an H.T. consumption of 13 watts (including rectifier drop), a rectifier filament consumption of 10 watts and transformer losses of 7 watts. For battery operation the 5Y3 rectifier would be changed to selenium, resulting in a power economy of 10 watts in heater consumption, and with a suitable transformer, another 3 watts in reduced rectifier and transformer losses. The overall battery consumption would be 6 to 6·5A at 6V.

Initial Adjustment

When the instrument had been constructed, it was put into operation by tuning each circuit in turn to its nominal frequency using a resistance capacitance oscillator. A valve voltmeter was joined between earth and the appropriate frequency changer grid, in which position the added capacitance of the meter had little effect on the tuning. The oscillator circuit was then tuned for maximum voltage at $V_1$ anode. The 100kc/s buffer circuit $L_2C_1$ was excited by the oscillator and the output at grid 1 of $V_3$ was adjusted for maximum value. The 90kc/s tuned circuit was driven

from $V_4$ grid and tuned for maximum output at grid 3 of $V_3$.

The adjustment of these circuits was not critical. When the inductances of $L_2$, $L_3$, $T_3$, $L_4$ and $T_4$ were measured on a 1 000c/s bridge, the calculated value of each tuning capacitance, including the capacitance of the appropriate grid feed potentiometer, was found to be within 1 per cent of the experimentally determined best value. It is thus possible to construct the whole frequency standard without using an external oscillator, since $T_2$ can be tuned to the centre of the range of capacitance variation over which a 10kc/s output is obtained. The "tens" circuits give maximum output and stability when tuned exactly to resonance, and their capacitances are large enough to permit variations in valve and wiring capacitances to be ignored. Care must be taken to use high grade silver mica capacitors in the tuned circuits, although a tropical quality neoprene sealed wax-paper capacitor is suitable for tuning $L_4$ to 100c/s.

In approximately one year's operation, the only occasions of failure were traced to mechanical drift of the 90kc/s tuning capacitor, and deterioration of a non-tropical paper capacitor used to resonate $L_4$.

Correct operation and an output greater than 25V R.M.S. are maintained with mains supply variations from 200 to 250V, and using valves at the high and low limits of transconductance.
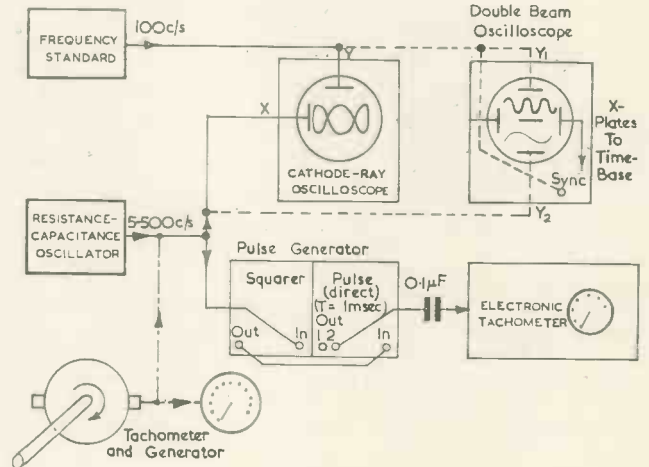


Fig. 3. Tachometer calibration

Applications

To obtain single point calibrations of electronic tachometers at dial readings corresponding to 100 impulses per second, a non-linear circuit is used to feed 100 impulses per second from the frequency standard to the tachometer input terminals.

For checking the scaling of electronic tachometers, the arrangement of Fig. 3 is used. A variable frequency oscillator drives the tachometer by means of an impulse generator. At the same time, the oscillator output voltage is fed to the Y-plates of an oscilloscope whose X-plates are fed from the frequency standard. A large number of multiples and fractions of 100c/s can be recognized by means of their Lissajous figures formed on the screen. For frequencies much lower than 100c/s, the alternative arrangement displaying the 100c/s standard on one beam of a double-beam oscillosocpe enables frequencies such as 15c/s to be set up easily by means of the standard.

When the scaling of an electric tachometer is to be checked, it is driven by a small variable-speed motor, and the tachometer generator output replaces the variable frequency oscillator in Fig. 3. The impulse generator and electronic tachometer are then omitted. The oscilloscope is usually used with a double-beam display in which small

changes of speed do not give such rapid changes of diagram as in Lissajous' method.

Mains frequency can be checked using an oscilloscope and a stop watch to measure the time for Lissajous figure to regain its original shape. This does not indicate whether the mains are above or below nominal frequency, which must be determined separately by the less accurate double-beam method.

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# NOISE-FREE INSTRUMENT CABLE

## By M. Lorant

AN instrument cable free of spurious electrical signals due to mechanical shock and vibration has recently been developed by the U.S. National Bureau of Standards. The noise-free cable is the result of a recent study of spurious cable signals which was carried out as part of a programme of basic instrumentation. In the course of the investigation, a detailed theoretical explanation was formulated for the generation of the spurious signals.

In connexion with the application of a piezo-electric accelerometer recently developed at the Bureau, it became necessary to reduce as much as possible the spurious signals due to motion of the cable. Experiments were therefore set up to compare the performance under dynamic stress of various experimental and commercial coaxial cables consisting of an inner conductor, an insulating dielectric, and a conducting shield. The cables were connected between a small piezo-electric accelerometer and a cathode follower, and the output of the cathode follower was recorded through a D.C. amplifier on a direct-inking oscillograph. The standard test for cable noise consisted in grasping a short section of the cable and subjecting it to severe strains by twisting and bending, while making sure that no strains or motion were transmitted to the accelerometer itself. Another test consisted in alternately compressing and releasing a short section of the cable with a pair of pliers.

It was found that standard microphone or phonograph pick-up cable gave noise signals under these conditions as high as 500 millivolts peak-to-peak. However, the noise was definitely lowered (to about 60mV peak-to-peak) by tightening the shield over the dielectric, thus improving the contact between braided shield over the dielectric, thus improving the contact between the dielectric and the shield.

A colloidal suspension of graphite in benzene was then applied, first to the outside of the dielectric only and then to the inside of the dielectric only. While no further reduction in noise was obtained by coating the outside only, the cable signal due to mechanical flexing was markedly reduced by coating the inside of the dielectric, provided the shield remained tight over the dielectric. Under these conditions, the residual noise was reduced to less than 3mV peak-to-peak. By coating both inside and outside of the dielectric with the graphite suspension, the noise was almost completely eliminated.

In another series of tests, an experimental cable was made up from an outer braided shield, the same rubber like dielectric as before, and an insulated central conductor. This cable was found to be quite noisy and was not materially improved by applying the graphite suspension to either the inside or outside of the dielectric.

Analysis of these results, in combination with other reported data on cable noise, led to the formulation of a theory which describes completely the mechanism of noise generation in instrument cables. According to this theory, the noise signals are due to currents set up in the cable when static charges are separated at the surfaces of the dielectric. To free the cable of these unwanted signals, it is only necessary that the inside and outside surfaces of the dielectric be entirely covered by a well-bonded conductive coating.

Whenever a metal makes intermittent contact with a solid dielectric, a separation of electrical charges takes place, just as when two different dielectrics are rubbed together. Friction is not necessary for this effect, but it increases the amount of separated charge, probably by insuring better mechanical contact before separation. Thus, as a small area of the cable shield separates from the dielectric, a surface charge is left on the dielectric, and an opposite "bound" charge is left on the shield. As the air gap increases, the charge on the shield becomes increasingly "free", and a portion of this charge is redistributed by flowing through the terminal impedance of the cable, so that it ultimately appears on the central conductor, at a point opposite the separated charge on the dielectric. It is this flow of current through the terminal impedance of the cable which develops the noise voltage. It is apparent that the process can also take place when the inner conductor, or both conductors, are free to separate from the dielectric.

This theory of cable noise production seems to be confirmed by experimental results. For example, signals generated in the manner described are in theory limited by the maximum density of surface charge possible without breakdown of the gas (usually air) between the cable dielectric and the separated conductor. When the noise voltage generated by this maximum theoretical surface charge is calculated from the electrical constants of the system, it is found to be not greatly in excess of some of the experimental data. Several other theoretical predictions have been verified by simple experiments. Thus, as predicted by the theory, the voltage generated by a sudden separation of one of the cable conductors from the dielectric rises suddenly to a peak followed by an exponential decay. Moreover, both the theory and experimental observations indicate that the amplitude of the record peak is proportional to the area of cable affected, inversely proportional to the total cable capacitance (including any parallel capacitances), and is reduced for a given surface charge if the allowed relative motion between the movable conductor and the dielectric is reduced. It is also found, both theoretically and in practice, that the initial amplitude of the peak is independent of the terminal resistance of the cable, provided
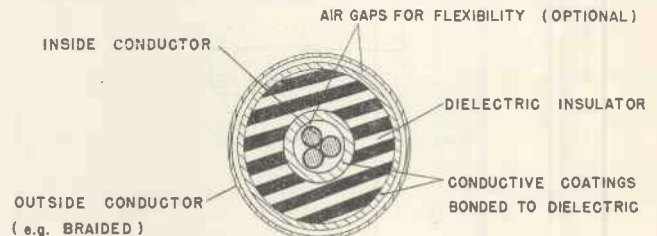


*A cross section of the cable*

this resistance is high enough to produce noise pulses of duration sufficient with respect to both the time required for separation and the high-frequency response of the electrical system.

Experiments at the Bureau have shown that the resistance of the conductive coating need not be especially low. In fact, it has been found that a conductive coating having a resistance of the order of 1 to 50 megohms per linear inch of cable reduces the unwanted cable signals by a factor of 500 or more.

Various types of conductive paints, suspensions of graphite, and conductive rubbers that may be used for the conductive coating are available commercially, and several patents have been issued on methods for bonding such coatings to a cable insulator. One method for making " noise-free " cable which appears to have commercial possibilities consists in extruding concentric layers of conductive rubber, non-conductive dielectric, and more conductive rubber over a metallic conductor, preferably stranded for flexibility. All conductors of a multi-conductor cable could be treated in this manner. Neither the inside conductor nor the outside shield need fit tightly; in a flexible cable they should preferably be somewhat loose. If desired, a protective insulating outside jacket may be added; this jacket would not require a conductive coating. There appear to be no limitations on how small such a noise-free cable can be made.

# The Equivalent Q of RC Networks

By D. A. H. Brown*, M.A., A.Inst.P.

*The effective Q of an RC network used as a frequency determining element in a phase shift oscillator is defined as $\frac{1}{2}|(d\phi/d\alpha)_{\alpha=1}|$ where $\phi$ is the phase difference between input and output, $\omega$ is the angular frequency, $\omega_0$ the operating angular frequency of the network, and $\alpha = \omega/\omega_0$. This definition is obtained by analogy with a parallel tuned LC circuit. An equivalent Q is then obtained for all the RC networks commonly employed as frequency determining elements. The Q so obtained is usually about unity, but in the case of the capacitance bridged-T may rise to 10.*

IT is widely known that the Q factor of the *LC* circuit used in an R.F. oscillator determines its frequency stability but much less attention has been paid to the equivalent Q of the *RC* network used as the frequency determining element in an *RC* oscillator. This is probably because the Q in the latter is much lower (about two orders of magnitude) and therefore other factors, such as temperature and supply voltage variation, have relatively much greater influence. Nevertheless it seems profitable to determine the Q factor of each network so that, other things being equal, the one of highest Q, giving the greatest stability, may be chosen.

## The Equivalent Q

If a parallel tuned circuit of the type shown in Fig. 1 is fed from a constant current source the phase-angle $\phi$
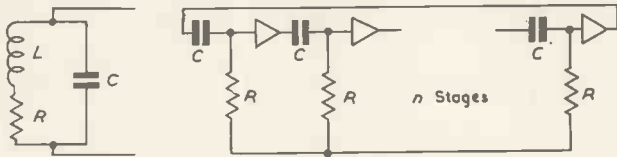


Fig. 1. Parallel tuned circuit

Fig. 2. Isolated high-pass *CR* networks

between the input current and output voltage developed across the circuit is:

$$\phi = \tan^{-1}\frac{\omega L(1 - \omega^2 LC) - \omega CR^2}{R} \quad \dots\dots\dots (1)$$

Writing now $Q = \omega_0 L/R$, $\omega_0^2 = 1/LC$, and $\alpha = \omega/\omega_0$ then:

$$\phi = \tan^{-1}[Q(1 - \alpha^2) - \alpha/Q] \quad \dots\dots\dots (2)$$

The operating frequency is normally at or very close to $\alpha = 1$, so that $\phi$ is small provided Q is high, and we may approximate by replacing the $\tan^{-1}$ of the angle by the angle itself. Then:

$$\phi \approx Q(1 - \alpha^2) - \alpha/Q$$

and $d\phi/d\alpha \approx -2\alpha Q - 1/Q$

At the operating frequency $\alpha = 1$ and, always provided Q is large ($> 10$ say).

$$(d\phi/d\alpha)_{\alpha=1} = -2Q \quad \dots\dots\dots\dots (3)$$

Neglecting the negative sign, which is here not so important as the numerical value,

$$Q = \frac{1}{2}|(d\phi/d\alpha)_{\alpha=1}| \quad \dots\dots\dots\dots (4)$$

We see, therefore, that the Q factor is equal to one-half the rate of change of phase with fractional change of frequency at the operating point. Physically this is enlightening, for it is easy to see that the more rapid the change of phase about the operating frequency the more closely will the frequency be controlled, whereas the mere

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statement that the Q should be high is not directly related to the operating conditions.

Equation (4) can now be applied to the networks commonly used in *RC* oscillators. This has previously been done by Whale[1] for the Wien bridge network, but no systematic derivation and collation of results seems to have been attempted for the other networks commonly used.

## Isolated CR Networks

One form of phase shift oscillator uses *CR* networks isolated by amplifying valve stages as shown in Fig. 2. Consider the case of three stages. Then each of the three *CR* coupling networks is operating at 60° phase shift and we have to determine the rate of change of phase with frequency at the operating frequency producing 60° phase shift per section.

The phase lead produced by a single *CR* coupling is

$$\phi = \tan^{-1}\frac{1}{\omega CR} \quad \dots\dots\dots\dots (5)$$

At the operating frequency $\alpha = 1$ and $\omega = \omega_0$, and then, since the phase shift is 60° per section,

$$\omega_0 CR = \sqrt{3}$$

$$\therefore \phi = \tan^{-1}\frac{\omega_0}{\omega\sqrt{3}}$$

$$= \tan^{-1}\frac{1}{\alpha\sqrt{3}}$$

$$\therefore \frac{d\phi}{d\alpha} = \frac{\sqrt{3}}{3 + 1/\alpha^2}\left(-\frac{1}{\alpha^2}\right)$$

$$\therefore \left(\frac{d\phi}{d\alpha}\right)_{\alpha=1} = -\frac{\sqrt{3}}{4} \quad \dots\dots\dots\dots (6)$$

Since there must be three networks to give 180° phase shift the overall rate of change of phase with fractional frequency deviation is three times this amount and the final Q is:

$$Q = \frac{1}{2} \times 3|(d\phi/d\alpha)_{\alpha=1}| = \frac{3\sqrt{3}}{8} = 0.65$$

In an exactly similar way the effective Q of four such networks operating at 45° phase shift or of five networks giving 36° phase shift each may be calculated. In practice with an even number of networks each separated by a phase inverting amplifier stage, a further unity gain phase inverting stage will have to be added to produce positive feedback between output and input after the coupling networks have produced 180° phase shift.

It can easily be shown that the effective Q is the same whether the low-pass type of *RC* network (Fig. 3) or the high-pass *CR* type (Fig. 2) is used.

The results of these calculations for 3, 4, 5 and 6 section couplings as well as the asymptotic value for an infinite number are summarized in Table 1. For completeness the loss through all the networks is also given for the various
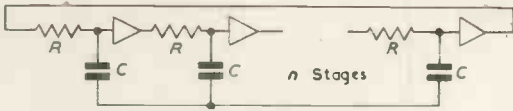
Fig. 3. Isolated low-pass $RC$ networks

cases; the reciprocal of this figure is the minimum gain required to produce oscillations.

## RC or CR Ladder Network

In practical phase shift oscillators it is not always convenient to use isolated networks requiring buffer amplifier stages between them; it is considerably easier to use a single amplifying valve and a ladder network of the types shown in Figs. 4 and 5. The Q of these networks may be found starting from the complex attenuation ratio $E_o/E_i$, where $E_o$ is the output voltage and $E_i$ the corresponding input voltage at angular frequency $\omega$; several authors[2,3,4,5] have indicated how this may be derived.

For a three stage $RC$ network:

$$E_o/E_i = \frac{1}{(1 - 5X^2) + jX(6 - X^2)} \cdots \cdots (7)$$

where $X = \omega CR$.

For 180° phase change through the network the imaginary part must be zero which gives the operating frequency $\omega_o$ as:

$$\omega_o^2 C^2 R^2 = X_o^2 = 6 \cdots \cdots \cdots (8)$$

or

$$\omega_o = \sqrt{6}/CR \cdots \cdots \cdots (9)$$

The phase-angle between $E_o$ and $E_i$ at any angular frequency $\omega$ is given by:

$$\phi = \tan^{-1}\left[-\frac{X(6 - X^2)}{1 - 5X^2}\right]$$

In terms of $a$ this becomes:

$$\phi = \tan^{-1}\left[-\frac{6\sqrt{6}\,a(1 - a^2)}{1 - 30a^2}\right] \cdots \cdots (10)$$

Since $\phi$ is near 180° and tan $\phi$ is small this becomes approximately:

$$\phi \approx \pi - \frac{6\sqrt{6}\,a(1 - a^2)}{1 - 30a^2}$$

$$d\phi/da = -\frac{6\sqrt{6}\,(1 - 3a^2)}{1 - 30a^2}$$

and

$$(d\phi/da)_{a=1} = -6\sqrt{6} \times 2/29 \cdots \cdots \cdots (11)$$

$$= -1·014$$

$$\therefore Q = 0·507 \cdots \cdots \cdots (12)$$

Similar calculations may be made for four, five or six
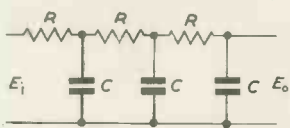


Fig. 4. Three stage $RC$ ladder network
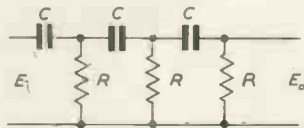


Fig. 5. Three stage $CR$ ladder network

stage networks and the corresponding equations are given below.

For a four stage network:

$$E_o/E_i = \frac{1}{(1 - 15X^2 + X^4) + jX(10 - 7X^2)} \cdots (13)$$

$$\omega_o CR = X_o = \sqrt{\frac{10}{7}} \cdots \cdots \cdots (14)$$

$$\phi = \tan^{-1}\left[-\frac{10\sqrt{(10/7)}\,a\,(1 - a^2)}{1 - 150/7\,a^2 + 100/49\,a^4}\right] \cdots (15)$$

$$d\phi/da = \frac{10\sqrt{(10/7)}\,(1 - 3a^2)}{1 - 150/7\,a^2 + 100/49\,a^4} \cdots \cdots (16)$$

$$(d\phi/da)_{a=1} = -1·30$$

$$Q = 0·65 \cdots \cdots \cdots (17)$$

For a five stage network:

$$E_o/E_i = \frac{1}{(1 - 35X^2 + 9X^4) + jX(15 - 28X^2 + X^4)} \cdots (18)$$

$$\omega_o CR = X_o = 0·7392 \cdots \cdots \cdots (19)$$

$$\phi = \tan^{-1}\left[-\frac{X_o a\,(15 - 28X_o^2 a^2 + X_o^4 a^4)}{1 - 35\,X_o^2 a^2 + 9\,X_o^4 a^4}\right] \cdots (20)$$

$$(d\phi/da)_{a=1} = 1·41$$

$$Q = 0·705 \cdots \cdots \cdots (21)$$

For six stages:

$$E_o/E_i = \frac{1}{(1 - 70X^2 + 45X^4 - X^6) + jX(21 - 84X^2 + 11X^4)} \cdots \cdots \cdots (22)$$

$$\omega_o CR = X_o = \sqrt{0·2588} \cdots \cdots \cdots (23)$$

$$\phi = \tan^{-1}\left[-\frac{X_o a\,(21 - 84X_o^2 a^2 + 11\,X_o^4 a^4)}{1 - 70\,X_o^2 a^2 + 45\,X_o^4 a^4 - X_o^6 a^6}\right] \cdots (24)$$

$$(d\phi/da)_{a=1} = -1·46$$

$$Q = 0·730 \cdots \cdots \cdots (25)$$

The results of this section are summarized in Table 2.

These results hold equally for the high-pass $CR$ (Fig. 5) or low-pass $RC$ type (Fig. 4) of ladder network.

### The Tapered Phase Shift Oscillator

It will be seen from the results obtained that the ladder type of network is inferior to the isolated type of networks in having, for the same number of stages, both a lower Q and greater loss. Sulzer[6] has shown that the loss of a ladder network may be considerably reduced by gradually increasing the impedance of each section of the ladder type network. The resulting network, in the case of three sections only is shown in Fig. 6. It is of interest to find the variation of Q with the parameter $a$. Proceeding as before it can be shown that the complex attenuation ratio is:

$$E_o/E_i = \frac{1}{1 - (3 + 2/a)X^2 + jX\{(3 + 2/a + 1/a^2) - X^2\}} \cdots \cdots \cdots (26)$$

where
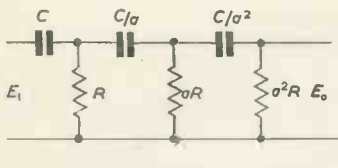
$$X = \omega CR \cdots \cdots \cdots (27)$$

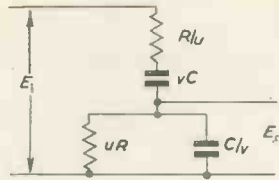Fig. 6. Three stage tapered $CR$ ladder network



Fig. 7. Wien bridge

and

$$\omega_o CR = X_o = \sqrt{(3 + 2/a + 1/a^2)} \ \dots\dots (28)$$

$$\phi = \tan^{-1}\left[ -\frac{X_o{}^3 \, a(1 - a^2)}{1 - BX_o{}^2 a^2} \right] \dots\dots\dots (29)$$

where $B = 3 + 2/a$

$$Q = \frac{X_o{}^3}{1 - BX_o{}^2} \ \dots\dots\dots\dots (30)$$

Table 3 gives Q as a function of $a$. The rise in Q towards the asymptotic value of $a = \infty$ is not rapid enough to be of practical importance.

TABLE 3

| $a$ | 1 | 2 | 3 | 4 | $\infty$ |
|------|------|------|------|------|------|
| Q | 0·507 | 0·548 | 0·571 | 0·586 | 0·650 |
| LOSS | 1/29 | 1/16 | 1/12·85 | 1/11·47 | 1/8 |

### The Wien Bridge Network

This has been considered by Whale[1] for the case of the network shown in Fig. 7. His results may be summarized as follows; the complex attenuation ratio is:

$$E_o/E_i = \frac{ya^2 + ja(1 - a^2)}{(1 - a^2)^2 + a^2 y^2} \ uv \ \dots\dots\dots (31)$$

where

$$y = \frac{u^2 + v^2 + u^2 v^2}{uv} \ \dots\dots\dots\dots (32)$$

$$a = \omega/\omega_o \ \dots\dots\dots\dots (33)$$

and

$$\omega_o = 1/CR \ \dots\dots\dots\dots (34)$$

The Q tends to a maximum value of 0·5 when $u$ and $v$ both tend to zero; however, the price to be paid is an attenuation increasing to infinity at $u = v = 0$. Therefore the network is usually used in the symmetrical form of $u = v = 1$, and then

$$Q = 1/3$$

and attenuation $= 1/3$.

### Null Transmission Networks

The bridged-$T$ (Fig. 8) and the twin-$T$ (Fig. 9) network have the property that at one particular frequency they have infinite rejection between input and output. An account of the properties of these networks has been given by Fleisher[7] with particular reference to their use in selective amplifiers, while Gamertsfelder and Holdam[8] emphasize their use in oscillators. For the latter application they are used in the negative feedback path of an amplifier to reduce the gain below that necessary for oscillation, except at the frequency of zero transmission, when there will be no negative feedback.



Fig. 8. Bridged-$T$ network
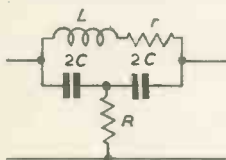


Fig. 9. Twin-$T$ network
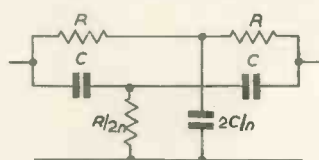
### The Bridge-T

The complex attenuation ratio for this network is:

$$E_o/E_i = \frac{1}{1 - j\dfrac{a}{a^2-1}\dfrac{2}{Q_o}} \ \dots\dots\dots (35)$$

where $Q_o$ is the Q of the inductance used at the operating frequency $\omega_o$; for no transmission $\omega_o$ must satisfy the two conditions:

$$Q_o = 4\omega_o RC \qquad \omega_o{}^2 LC = 1$$

The phase-angle $\phi$ introduced is:

$$\phi = \tan^{-1}\frac{2}{Q_o} \cdot \frac{a}{a^2 - 1}$$

whence

$$Q = Q_o/2 \ \dots\dots\dots\dots (36)$$

The Q factor of the network is one-half that of the inductance used alone. This result has been derived by Fleisher[7] by considering the frequencies at which the network response falls to $1/\sqrt{2}$ of its maximum value (unity). This latter definition, however, gives a different result from that obtained by applying the rate of change of phase criterion when applied to a tuned feedback amplifier; this point will be considered after discussing the twin-$T$ circuits.

### The Twin-T

The complex attenuation ratio for this network is:

$$E_o/E_i = \frac{1}{1 - j\dfrac{a}{a^2 - 1} \cdot \dfrac{2(n + 1)}{\sqrt{n}}} \ \dots\dots (37)$$

By comparison with Equation (35) for the bridged-$T$ it can be seen that the twin-$T$ is equivalent to a bridged-$T$ with a coil of:

$$Q_o = \frac{\sqrt{n}}{n + 1} \ \dots\dots\dots\dots (38)$$

and hence the equivalent Q of a twin-$T$ network is:

$$Q_{TT} = \frac{1}{2} \cdot \frac{\sqrt{n}}{n + 1}$$

This has a maximum value of 1/4, where $n = 1$, i.e. for a symmetrical twin-T network. However, as pointed out by Fleisher[7], $Q_{TT}$ is only a very slowly varying function of $n$ and other considerations may make a value of $n$ other than unity desirable.

It is interesting to note that though the phase of the output voltage from a twin-$T$ or bridged-$T$ network jumps discontinuously from 90° lagging just below the null frequency to 90° leading just above the null frequency, the rate of change of phase with frequency is continuous at the null frequency.

### Tuned Feedback Amplifier Using the Bridged-T and Twin-T Networks

The effect of using a bridged-$T$ or twin-$T$ network in the negative feedback path of an amplifier is to multiply the rate of change of phase by the gain of the amplifier A, and the effective Q should therefore be:

$$Q_{eff} = A \cdot (Q \text{ of network}).$$

Hence the effective Q of a bridged-$T$ tuned amplifier should be:

$$Q = A \cdot Q_o/2 \ \dots\dots\dots\dots\dots (39)$$

and of a twin-$T$:

$$Q = A \cdot 1/4 \ \dots\dots\dots\dots\dots (40)$$

Results very similar to these have been derived by Fleisher and by Punnett[9]. However, Fleisher derives the Q by considering the bandwidth between half power points and

obtains for a twin-$T$ network:

$$Q = \frac{\sqrt{[(A+1)^2 - 2]}}{4} \ldots\ldots\ldots\ldots\ldots ,41)$$

which, if $A \gg 1$ approximates to:

$$Q_{TT} = \frac{A+1}{4} \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

This is the result also derived by Punnett. For the bridged-$T$ Fleisher gives, by analogy with the twin-$T$:

$$Q_{BT} = \frac{A+1}{2} \cdot Q_0 \ldots\ldots\ldots\ldots (43)$$

The slight difference between Equations (43) and (39), and between (42) and (40) arises from the different definitions of Q adopted. Though the difference is of little practical account the results (39) and (40) seem to be preferable for two reasons; firstly, they can be derived by strict application of the criterion here adopted, namely $Q = \frac{1}{2}(d\phi/da)_{a=1}$, in the following way. The gain of the feedback amplifier (Fig. 10) is:

$$\dot{G} = \frac{A}{1 + A\beta}$$

Now for a bridged-$T$ network we have:

$$\beta = \frac{1}{1 - i \dfrac{a}{a^2 - 1} \dfrac{2}{Q_0}}$$

and if it is fed back negatively then:

$$G = \frac{A}{1 - A \cdot \dfrac{1}{1 - j \dfrac{a}{a^2 - 1} \dfrac{2}{Q_0}}}$$

If this expression is simplified and the phase-angle extracted it is possible to work out the rate of change of phase with frequency. This gives the result of Equation (39).

The second reason for preferring Equations (39) and (40)
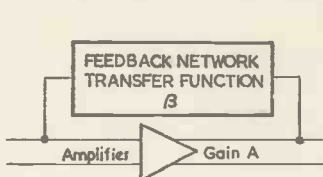
Fig. 10. Feedback amplifier
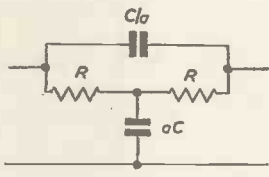
Fig. 11. Capacitance bridged-$T$ network

is that they apply equally to low values of $A$ and tend to zero when $A$ tends to zero. Fleisher's expression (41) becomes imaginary when $A < \sqrt{2} - 1$.

### Minimum Transmission Networks

The transmission functions of the bridged-$T$ and twin-$T$ networks show a theoretical zero transmission at a particular frequency. There are a number of other networks which have a similar transmission characteristic except that the transmission only goes to a minimum, not zero, at the working frequency.

The Capacitance Bridged-$T$

The simplest of these is the bridged-$T$ network of Fig. 1. (No other name seems to have been applied to this network to distinguish it from the bridged-$T$ discussed previously). The complex attenuation ratio for this network is:

$$E_0/E_1 = \frac{a(1 - a^2) + j2a}{a(1 - a^2) + ja(2 + a^2)} \ldots\ldots\ldots (44)$$

$$\phi = \tan^{-1} - \frac{(1 - a^2)a^3a}{a^2(1 - a^2) + 2a^2(2 + a^2)} \ldots\ldots (45)$$

and

$$(d\phi/da)_{a=1} = \frac{a^2}{2 + a^2}$$

If $a$ is large this approximates to:

$$(d\phi/da)_{a=1} = a$$

and

$$Q = a/2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

If now the network is used in the feedback path of an amplifier gain $A$, the effective Q as a tuned amplifier becomes:

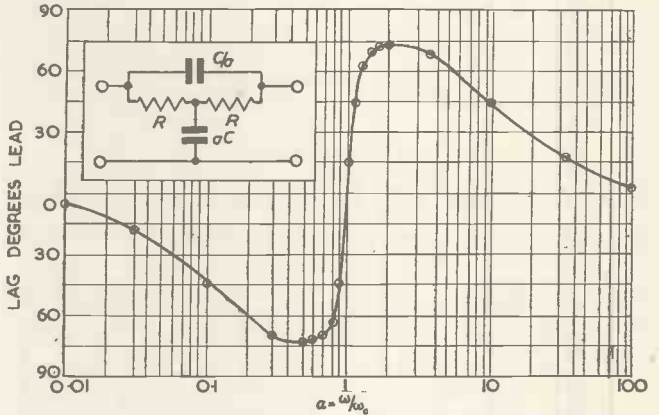$$Q = \frac{a^3}{(2 + a^2)[a^2/A + 2/A + 2]} \ldots\ldots (47)$$

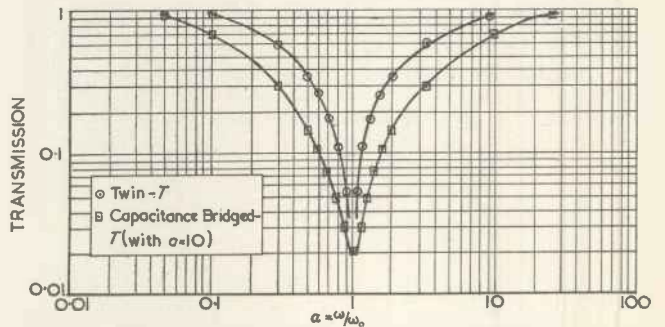Fig. 12. Capacitance bridged-$T$ phase characteristic with $a = 10$

Fig. 13. Comparison of transmission of twin-$T$ and capacitance bridged-$T$ (with $a = 10$)

and as $A$ tends to infinity, this becomes:

$$Q_{A \to \infty} = a/2 \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Thus the Q of the network above used as a rejection network is the same as the Q obtained for the tuned amplifier when the gain $A$ is very high. This statement, however, must be qualified. The phase characteristic of the network is given in Fig. 12 and it can be seen that though the rate of change of phase with frequency near the minimum transmission frequency is very steep, yet the phase has a maximum and a minimum returning to zero phase shift at both high and low extremes of frequency; this phase characteristic is reflected in a poor amplitude rejection characteristic which is worse than that of a twin-$T$. Fig. 13 shows a comparison of the amplitude rejection of the twin-$T$ and capacitance bridged-$T$ with $a = 10$.

The rapid phase change near the resonant frequency should make the network very useful in oscillator circuits where this property leads to good frequency stability, but the poor amplitude response makes the circuit of little value

as a sharp rejection filter, or to provide a narrow band sharply tuned amplifier.

## OTHER MINIMUM TRANSMISSION NETWORKS

Some variants on this type of minimum transmission network have been considered by Sulzer[10], but the essential phase and amplitude relations are not greatly altered and they will not be considered here.

## Conclusion

It has been shown that in general the equivalent Q of $RC$ frequency determining networks is near unity; this is about two orders of magnitude less than that of a good $LC$ circuit as used in R.F. oscillators. Other things being equal, it would therefore appear that an $RC$ oscillator is inherently less stable than an $LC$ oscillator by a factor of one hundred.

The reason for the low Q of $RC$ circuits has not been discussed. It would appear to be due to the dissipative elements in the network. On the other hand, the simple capacitance bridged-$T$ discussed may have a Q of 10 with $a = 20$, which is quite practicable. This is much higher than that of any of the other networks and should make the network particularly useful as the frequency determining element of an oscillator.

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# The Prevention of Interference by a Single Unwanted Frequency in Television Reception*

OCCASIONALLY a television receiver suffers interference from an unwanted signal. This signal may be removed, without appreciable effect on the desired signal, by the following method:—

Two coaxial cable stubs are connected together and their junction is connected to a point in the feeder from the aerial to the receiver. Fig. 1 shows one of a number of possible arrangements for use when the interfering frequency is greater than that of the wanted signal. In the figure, the right-hand portion of the stub is of such a length that it provides approximately zero impedance to the interfering signal, and thus prevents its passage to the receiver. This right-hand stub, however, also has an effect upon the wanted signal, since it will appear as a reactance at that frequency and will introduce attenuation if its effect is not compensated. The compensation



**Fig. 1.** One stub open-circuited and one short-circuited $x$ = an odd integer

**Fig. 2.** Both stubs open-circuited $n$ = any integer

is carried out by the left hand portion of the stub whose reaction at the wanted frequency is equal in value and opposite in sign to that of the right-hand stub.

If one of the stubs is short-circuited and the other is open-circuited, the total length of the two stubs should be $\lambda/4$ at the required signal frequency, the length of the open-circuited portion being $\lambda/4$ at the interfering signal frequency.

There is an infinite variety of alternative arrangements, since either stub can be lengthened in multiples of $\lambda/4$ and used open- or short-circuited, as necessary. Each increase in $\lambda/4$ in any one stub changes the open-circuit or short-circuit requirements at the end of the stub. The combined length of the stubs is always $n$ $\lambda/2$ of the wanted frequency if both ends are either open or short-circuited, and an odd multiple $x$ of $\lambda/4$ if one end is open and the other short-circuited.

In practice it is more convenient to have the ends open-circuited, since it is then more easy to trim them to length.

If the shorter of the two stubs (rejector stub) is made somewhat shorter than $\lambda/4$, its electrical length may be increased by the aid of a variable capacitor attached to the end which is remote from the aerial feeder, and fine tuning for the removal of the unwanted signal can be obtained. In this case, however, the reactance of the rejector stub at the wanted frequency will be reduced and the length of the stub, which is shown short-circuited in Fig. 1, must be lengthened to compensate for the change.

Fig. 2 shows the general case when the interfering frequency is either higher or lower than that of the wanted frequency.
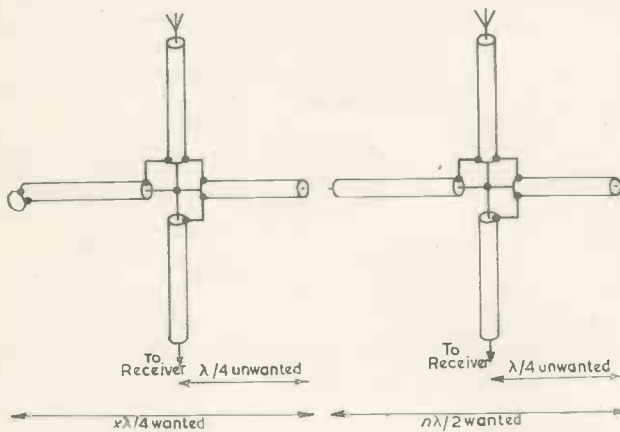
## Fine Tuning Arrangements*

A METHOD of tuning in short wave reception is to employ a capacitor for coarse tuning throughout a range and in addition to include a fine tuning inductor in series with the main tuning inductor so as to give a band spreading action throughout given bands.

Such an arrangement gives a fine tuning range which is proportional to the mean frequency of the fine tuning band. On the other hand all such bands have approximately the same absolute width of 200-300kc/s so that ideally a fine tuning arrangement should be one in which the fine tuning range provided by the adjustment is independent of the adjustment of the principle tuning element.

A solution to this problem is provided by connecting a capacitor of suitable value across the principle tuning inductor. The magnitude of capacitance required is of the order of that provided by trimmer capacitors and consequently it is possible to use the existing trimmer for the purpose, this no longer being connected across the main tuning capacitor but being arranged in shunt with the main tuning inductor.

When the principle tuning action is achieved by the method of adjusting the principle tuning inductor then an alternative circuit may be used in which there is connected in parallel to the fixed tuning capacitor the series connexion of a fine tuning capacitor and an inductor of such dimensions that the series connected circuit acts as a capacitance in the main tuning range.

The arrangements described may be used when other circuit elements are present in the tuning circuit, for instance, when a " padding " capacitor is used for the purpose of ganging in a superheterodyne receiver.

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*\* Communication from E.M.I. Engineering Development Ltd.*

*\* Communication from The Telegraph Co. via E.M.I. Ltd.*

# A Simple Low Frequency Amplifier

By J. R. Beattie*, B.Sc., and G. K. T. Conn*, M.A., Ph.D.

*A simple amplifier of low-frequency signals is described in which a twin resistance-capacitance network is used. There are separate and independent controls of frequency and of the Q-factor or pass-band. With the particular parameters specified, the instrument may be tuned over the range 1c/s to 3c/s and the Q-factor varied up to 20. Alternative specifications may be met by suitable adjustment of the components. The amplifier is stable and has a characteristic which is linear to within 1 per cent.*

THERE are several fields of investigation requiring the amplification of signals of very low frequency. For instance, the time of response of radiation thermocouples is such that the rate of interruption of the light is limited to frequencies of 15c/s or less, in encephalography significant records are obtained at frequencies as low as 0·1c/s. A number of electronic amplifiers suitable for low frequencies have been described[1]. The present report describes a simple arrangement which is, in essence, a phase-shift oscillator used in a condition of "damped resonance". The Q-factor and the resonant frequency may be altered independently simply by varying the value of resistors. A gain of the order of 10[3] is obtained by using two stages. This is adequate for our purpose since the instrument is used in conjunction with a tuned galvanometer-amplifier. Additional stages may be added if required.

## Theory

To be specific, consider an arangement which can be tuned over the range 1c/s to 3c/s and of which the effec-
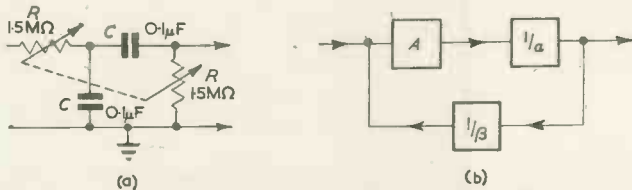


Fig. 1(a) Twin *RC* coupling network. The values of *R* and *C* quoted give a frequency of 1·06c/s. (b) The simple circuit

tive Q-factor may be varied up to 20. Limitation of the pass-band of an amplifier is not uncommon, but the difficulty of using a parallel-*T* network[2] lies in the tuning. The attractive feature of a damped, phase-shift oscillator[3] is that it offers a means of controlling the frequency and the Q-factor independently. Selective amplification is obtained by using a resistance-capacitance network and stability by negative feedback.

The selective resistance-capacitance network, referred to as the "twin *RC* network", is illustrated in Fig. 1(a) It consists of two identical capacitors *C*, and two identical, variable ganged resistors, *R*. Simple analysis of this circuit shows that the complex attenuation given by the ratio of the output voltage to the input voltage is:

$$E_o/E_i = 1/\left\{3 + j(x + 1/x)\right\} \text{ where } x = \omega CR \ .. \ (1)$$

$\omega/2\pi$ is the frequency of the input signal. At a frequency $f$ given by $1/(2\pi RC)$, the attenuation is real, positive and equal to 1/3. If such a network be used in a valve ampli-

fier with regenerative feedback of the output, the amplification will show a resonance peak at the frequency $f$.

A block diagram of the circuit is shown in Fig. 1(b). The amplifier $A$, whose gain is also referred to as $A$, is followed by the twin *RC* network of attenuation $1/a$, $a$ being in general complex. It is assumed for the moment that there is no discrimination of frequency or phase in the amplifier itself. Positive feedback of a fraction of the
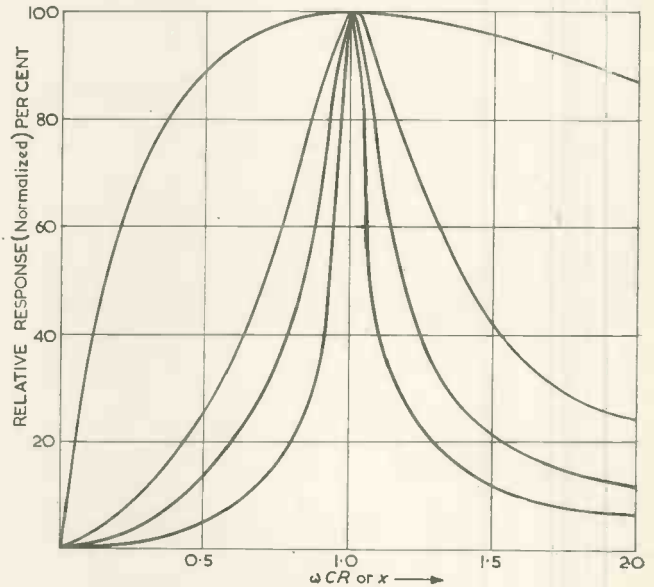


Fig. 2. Normalized response of the simple circuit plotted against *x* which is proportional to frequency for various values of Q

output is obtained by a resistor; in practice this is a cathode bias resistor common to both valves. The gain, $G$, of the system is then:

$$G = \frac{A/a}{1 - A/a\beta} = \frac{A}{a - A/\beta}$$

$1/\beta$ is the feedback ratio.

Substituting the value of $a$ from Equation (1) above gives:

$$G = \frac{A}{(3 - A/\beta) + j(x - 1/x)} \quad \cdots\cdots \quad (2)$$

and

$$|G| = \frac{A}{\sqrt{\left\{(3 - A/\beta)^2 + (x - 1/x)^2\right\}}} \cdots (3)$$

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The normalized response is plotted against $x$ in Fig. 2. For a given value of the product $CR$, $x$ is proportional to the frequency.

The sharpness of the resonance may be specified by a Q-factor, where:

$$Q = x_0 / \delta x$$

$x_0$ is the resonant value ($\omega_0 CR = x_0 = 1$) and $\delta x$ is that departure from resonance which reduces the gain to $1/\sqrt{2}$ of that at resonance.

At resonance:

$$|G|_M = A / (3 - A / \beta)$$

When

$$x - 1/x = 3 - A/\beta, \text{ then } |G| = |G|_M / \sqrt{2}$$

If we substitute $x = x_0 - \delta x$, where $\delta x$ is the departure from resonance, it is easy to show that:

$$Q = 1/\delta x = 2/(3 - A/\beta) \ldots \ldots \ldots (4)$$

Equations (3) and (4) may be combined to obtain the response at resonance in the form:

$$|G_M| = AQ/2 \ldots \ldots \ldots \ldots (5)$$
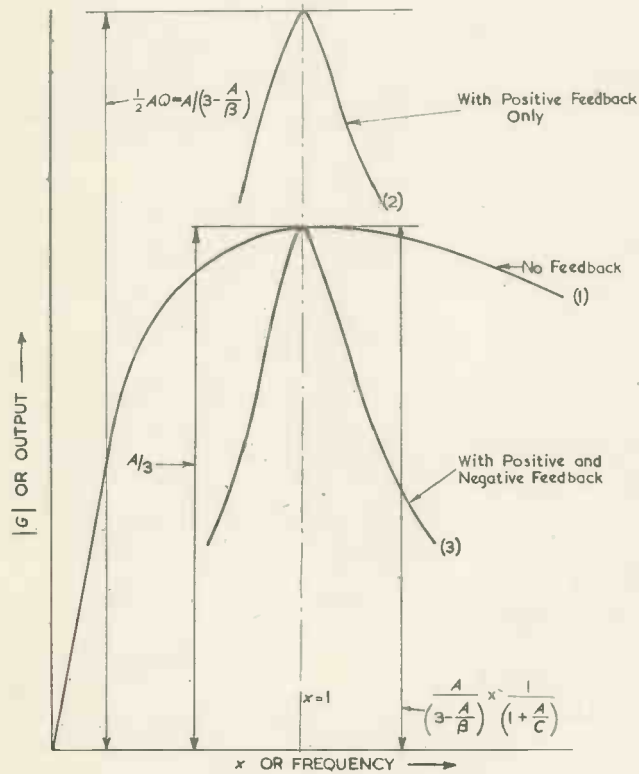


Fig. 3. Output plotted against frequency

(1) with no feedback, (2) with positive feedback (see Fig. 1(b)), (3) with positive feedback and such negative feedback (see Fig. 4) that the output at $x = 1$ is the same as with no feedback at all.

The gain of the amplifier depends directly on the Q-factor as one would expect.

## Stability

Stability is related to linearity of response. If high Q-factors are used instability follows; for instance free oscillation takes place if $A/\beta \geqslant 3$. In the range $2 \leqslant A/\beta \leqslant 3$, the gain varies from $A$ at one extreme to infinity at the other.

The gain of the amplifier with the attenuator alone and no positive feedback is $A/\alpha$ or $A/3$ at resonance, as shown in Fig. 3. The sharp resonance obtained when regenerative feedback is used gives a gain at resonance of $AQ/2$ or

$A/(3 - A/\beta)$. The associated instability may be limited by degenerative or negative feedback. If this be applied by a purely resistive network there will be no discrimination of frequency in such feedback, so that the tuning is unaffected. Thus the ordinates of all points on curve 2 in Fig. 3 are scaled down by $1/(1 + A/C)$, where $1/C$ is the new feedback ratio. If the negative feedback is sufficient to ensure that $|G|_M$ reverts to the original value of $A/3$, the resultant stability and linearity are those of the valves themselves. In practice it is not necessary to apply such a high proportion of negative feedback, though this is the case illustrated by curve (3) in Fig. 3.

These considerations may be set out in a more exact analytical form. The possibility to be avoided is that a small change in the amplification $A$ leads to an inordinate change in $|G|$. An arbitrary factor of instability, $F_I$, may be defined as:

$$F_I = \frac{\delta |G|_M}{|G|_M} \bigg/ \left( \frac{\delta A}{A} \right)$$

or the fractional change in gain per unit fractional change in $A$. It follows readily that:

$$F_I = 3/(3 - A/\beta) = 3Q/2 \ldots \ldots (6)$$

Alternatively:

$$|G|_M = A/3(F_I) \ldots \ldots \ldots \ldots (7)$$

Let the negative feedback, introduced as in Fig. 4 by a resistive link, be of attenuation $1/C$ so that the effective amplification is now:

$$A_n = \frac{1 + A/C}{A} \ldots \ldots \ldots \ldots \ldots (8)$$

If the feedback be large so that $A/C \gg 1$, we have the case commonly considered in which $A_n = C$ and is
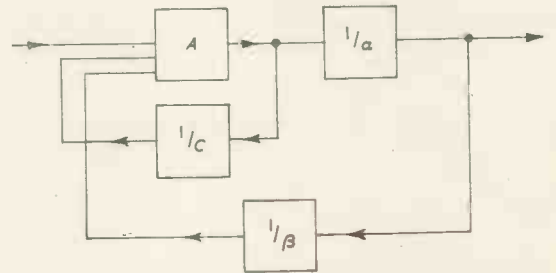


Fig. 4. Final circuit with both positive and negative feedback

independent of the conditions of operation of the valves. These conditions may, however, be unnecessarily rigorous If we define a stability factor, S.F., as:

$$F_s = \frac{\delta A_n}{A_n} \bigg/ \left( \frac{\delta A}{A} \right)$$

using Equation (8) it follows that:

$$F_s = 1/(1 + A/C) = A_n/A.$$

After applying negative feedback the gain at resonance becomes from Equation (8).

$$|G|_M = A_n(F_I) = A/3(F_I) \times (F_s) \ldots \ldots (9)$$

The form of Equation (9) indicates the complementary nature of the two expressions which is implied by calling one the "instability factor" and the other the "stability factor". If the product is unity, Equation (9) shows that the gain at resonance is precisely what it would have been if no feedback of any kind were used. The negative feedback required to fulfil the condition may be expressed in several equivalent ways by making use of Equation (6). Thus:

$$\frac{3Q}{2} \times \frac{A_n}{A} = 1 \text{ or } \frac{3Q}{2} = 1 + A/C \text{ or } \frac{3}{(3 - A/\beta)} = (1 + A/C)$$

As already noted, it has not been found necessary in practice to employ such a high degree of stabilization as that envisaged.

### Experimental Details

The complete circuit is shown in Fig. 5 which emphasizes the simplicity of the arrangement. The amplifier consists of two valves coupled by the twin *RC* network
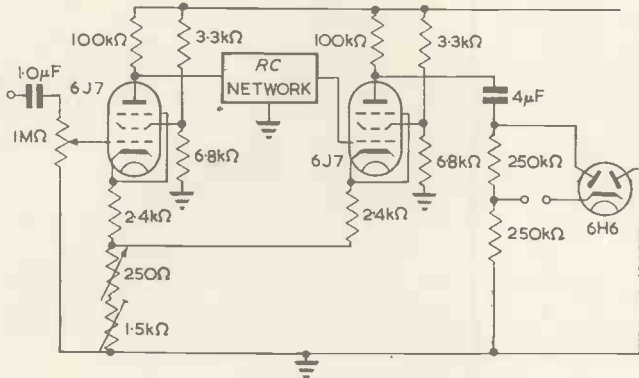


Fig. 5. Final circuit

referred to above. After full-wave rectification by a diode circuit, the output is measured on a microammeter.

The twin *RC* network has a number of attractive experimental features. As already noted, attenuation at resonance is by a factor 1/3, whereas the triple *R.C.* network commonly used in a phase-shifting oscillator[4] attenuates by a factor of 1/29. At resonance there is no phase change in the twin *RC* network and the attenuation is moreover independent of the frequency chosen. The resonant frequency is independent of the Q-factor and these may be controlled independently. Finally the latter
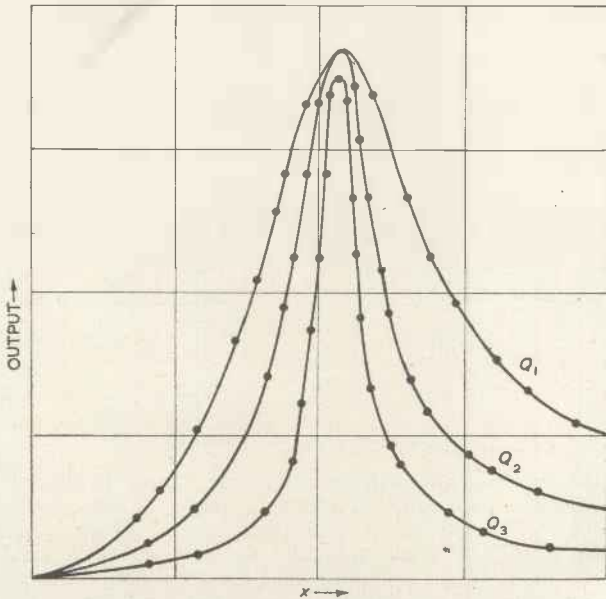


Fig. 6(a). Experimental response curves for various values of Q (tuned to 2·14c/s)

half of the network consists of a blocking capacitor and resistor, so that the attenuator may be connected directly to the grid of the second valve.

Positive feedback is furnished by the common cathode resistors. The range of values of the Q-factors to be used

in practice is determined by pre-setting the 1 500Ω resistor. Addition of a rheostat of 250Ω ensures that the Q-factor may be varied up to 20.

Negative feedback is applied to the first valve by the bias resistor of 2 400Ω which is not shunted by a capacitor for this reason. It should be noted that negative feedback is also applied to the second valve in the same way. The value of the cathode resistor, 2 400Ω, is determined by the bias required by the grid of each valve. It was found in practice that these resistors provided an entirely satisfactory stability with a gain of rather more than $A/3$. The strict conditions envisaged in the theoretical examination above were not therefore adopted and no additional negative feedback was required. No decoupling capacitors are used to minimize troubles associated with phase changes.

In the form illustrated in Fig. 5 the amplifier has a practical gain of the order of $10^3$. Obviously this may be extended if necessary by additional stages. Experimental curves of the output against frequency are shown in
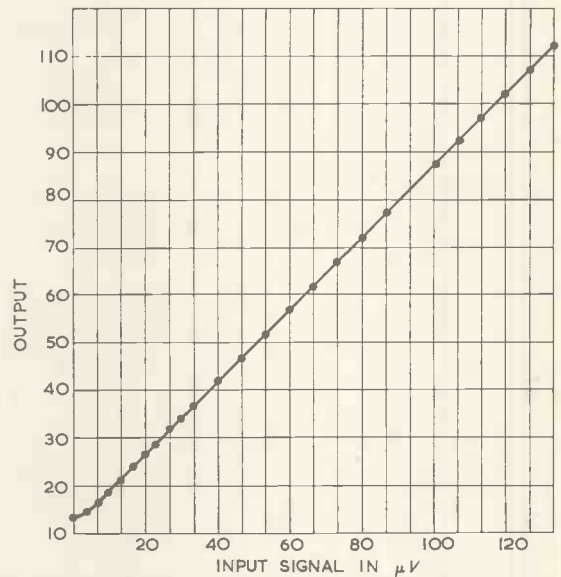


Fig. 6(b). Illustration of linearity—output against input for a frequency of 2·14c/s (Q = 9)

Fig. 6(a) for various values of the Q-factor. With the components shown in Fig. 5 the instrument may be tuned from 1c/s to 3c/s and the Q-factor varied from 5 to 20. The curves shown in Fig. 6(a) were obtained by tuning to a frequency of 2·1c/s. The applied signal, which was sinusoidal in waveform, was given by a simple phase-shift oscillator. Linearity of the response, which is clearly better than 1 per cent over the operational range, is illustrated in Fig. 6(b). The slight curvature near the origin is due to the characteristics of the diode and may be eliminated if necessary either by a discriminator or by backing-off.

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# An Analogue Reciprocal Function Unit
## for Use with Pulsed Signals

P. A. V. Thomas*, B.Sc.

*A summary of existing methods of obtaining a reciprocal function is given before entering on a description of the developed unit.*

*The unit itself consists fundamentally of a monostable multivibrator, for which it is shown in an appendix, that under suitable conditions the width of the pulse generated is approximately inversely proportional to the reference voltage.*

*The pulse width variation is then converted to an amplitude variation using a constant velocity time-base so that the output voltage of the unit is approximately inversely proportional to the input voltage; a graph is included showing the results obtained in practice.*

THERE are a number of occasions on which it is necessary to produce an output voltage which is inversely proportional to a continuously variable amplitude input voltage when the input consists of a series of pulses as obtained in multi-channel equipment. If the pulses are relatively short, i.e. of a few microseconds duration, the function unit must have a high speed of operation and instantaneous response.

### Existing Methods

Several methods of obtaining a reciprocal function exist, both explicit and implicit, and a brief review is given below before describing the method developed to overcome the disadvantages mentioned.

1. *Electro-mechanical,* in which a potentiometer is mechanically or hand driven and a servo-mechanism feedback path is used to maintain a balance[1,2]; this method obviously suffers from its long time-constant, but can be extremely accurate.

2. *Logarithmic,* in which use is made of contact type metal rectifiers or diode vacuum valves working at very low currents in the region where the law $V = r \log_e i$ exists so that the operation can be carried out by subtraction of logarithms[3]; though the time response of this arrangement is satisfactory, it suffers from poor stability unless precautions are taken. Metal rectifiers are extremely sensitive to temperature variations, which cause not only a change in level of the output, but also the law[4], and in the case of the diode the output is similarly affected by variations of heater temperature due to changes of the heater voltage[5], though this could be reduced by stabilizing the heater supply; ageing also causes a drift in the output[5].

3. *Valve characteristics* have been used for multiplication and division by varying two electrode potentials over a selected region where the anode current is proportional to the potentials[6]; again the stability is dependent on the power supplies though not to the same extent as the previous method.

4. *Pulsed Attenuator*[7]. This method overcomes the difficulties of the last two by allowing the valve characteristics to be of only secondary importance. Two attenuators, whose gains are equal and controlled by a pulse of variable width and a feedback system, are connected in such a manner that the output is given by the expression $V_{out} = A(v_1/v_2)$, where $A$, $v_1$ and $v_2$ are steady voltages or signals; thus by making $A$ and $v_1$ constant and $v_2$ the input voltage a reciprocal output is obtained. As a definite time is taken for the operation of the unit, being a balancing system, it is unlikely to be suitable for pulse signals unless a very fast multivibrator (which is used to vary the gain of the attenuators) is used and this might cause unstable operation.

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This short review covers the main types of analogue dividers and it was the last method that led to the development of the function unit described below, i.e. it should be independent of power supplies.

### Theory of Operation of Function Unit

It is shown in Appendix 1 that under suitable conditions a monostable multivibrator will give a rectangular pulse, the width of which is approximately inversely proportional
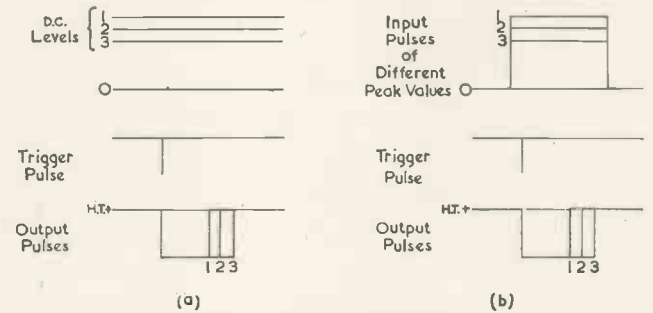


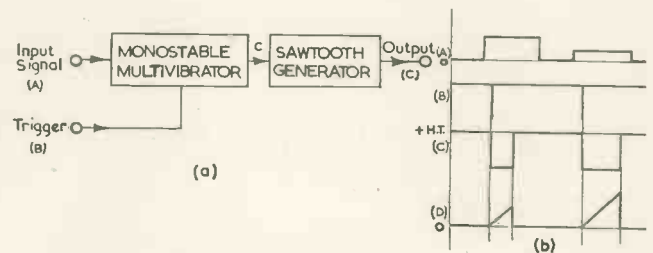Fig. 1. Comparison of (a) steady voltage and (b) pulse input to a monostable multivibrator



Fig. 2. Basic circuit and waveforms

to the D.C. potential applied to a given grid. If the D.C. potential is replaced by a rectangular pulse, an output pulse is obtained whose width is identical to that which would be obtained with a D.C. potential equal to the peak value of the input pulse, providing that the time boundaries of the output pulse are within those of the input pulse; this is shown more clearly in Fig. 1. Thus the width of the output pulse is inversely proportional to the peak value of the input pulse.

### Description of Basic Function Unit

The basic function unit is shown in Fig. 2 and consists of a monostable multivibrator and a linear saw-tooth generator

of constant velocity. The input signal of variable amplitude (A) is converted into a rectangular pulse of variable width and constant amplitude (C), this then being applied to the saw-tooth generator to convert the pulse width variation into a proportional amplitude variation as shown at (D). Thus the amplitude of the output saw-tooth is the reciprocal of the amplitude of the peak value of the input rectangular pulse.

### Description of Complete Function Unit

Certain refinements were added in the complete unit shown in Fig. 3(a), and it will be seen that two memory circuits ($V_1$, $V_2$, $V_{10}$, $V_{11}$) have also been added.

as an anode and the main anode is electron coupled so that the operation of the multivibrator is not influenced by variations of the load following it. The output negative rectangular pulse (E) is then applied to the linear saw-tooth generator $V_8$ and $V_9$, a Bootstrap sweep circuit[8,9], after passing through a cathode-follower $V_7$ to isolate the low input impedance of the normally conducting $V_8$ from the anode of $V_6$. The saw-tooth (F) is then applied to a second memory circuit $V_{10}$ and $V_{11}$ triggered from a third timing pulse (G). Thus the output signal (H), which is approximately rectangular, has a peak value that is the reciprocal of the peak value of the input pulse (A).

The second memory circuit may not always be necessary
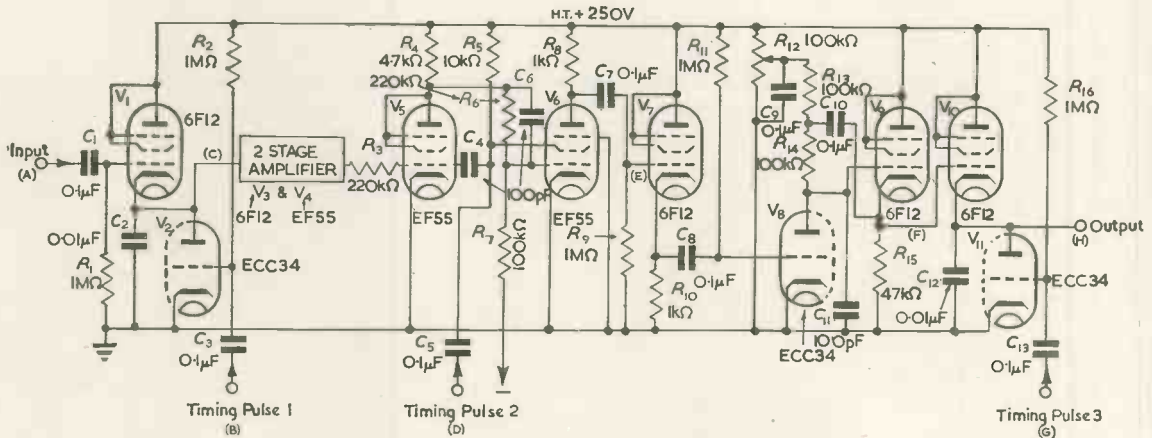


Fig. 3(a). Circuit of complete unit
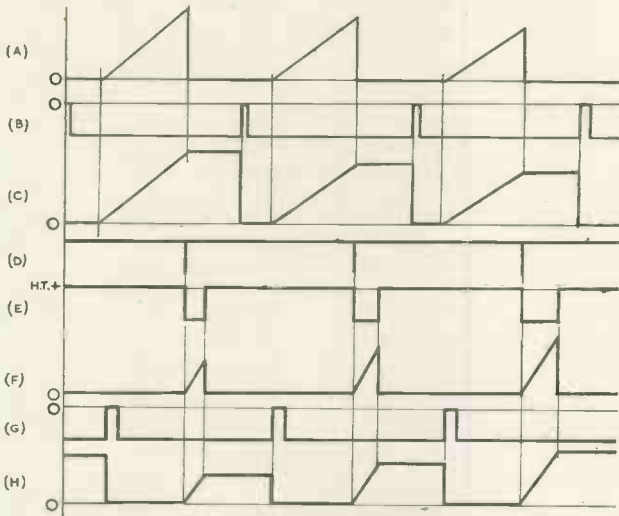


Fig. 3(b). Waveforms of complete unit

As stated above the output signal is dependent on the peak value of the input signal and not on its shape so that, unless the input is a rectangular pulse, it is first necessary to produce a rectangular pulse, the amplitude of which is equal to the peak value of the input signal. In Fig. 3(b) the input is shown as a series of saw-tooth voltages (A) which is applied to the grid of $V_1$ the cathode-follower of a memory circuit (see Appendix 2), a suitable timing pulse (B) being applied to $V_2$, the discharging valve. The output of $V_1$ (C), after amplification in a two-stage amplifier (not shown in detail), is then applied as the reference voltage to the monostable multivibrator, $V_5$ and $V_6$, which is triggered from a second timing pulse (D); it will be noted that the multivibrator is slightly modified from that given in Appendix 1 in that $V_6$ uses the screen grid of a pentode
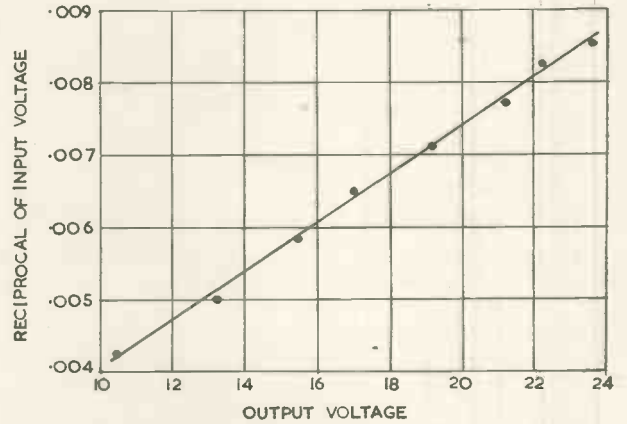


Fig. 4. Graph of output voltage against reciprocal of input voltage

but was used in the original unit as the output signal was displayed on a cathode-ray tube and it is more convenient for brightening up the trace at the peak value as the brightening pulse may be applied any time during the flat top of the pulse (H) instead of being applied at the exact instant of the waveform (F) reaching its peak value.

The source of the timing pulses has not been shown as any conventional method of timing pulse generation may be employed.

### Results

A test was carried out by applying a pulse of variable amplitude and 10μsec width to the input and measuring the amplitude of both input and output pulses using an oscilloscope. The output voltages were then plotted against the reciprocal of the input voltage values and the results are shown in Fig. 4, showing that the unit is quite satisfactory.

## APPENDICES

### 1.—DERIVATION OF CONDITIONS FOR RECIPROCAL FUNCTION OPERATION OF MONOSTABLE MULTIVIBRATORS

Fig. 5 shows the basic circuit of the anode coupled monostable multivibrator together with its waveforms; as the circuit is so well known a description of its operation is unnecessary.

With reference to the $V_{2g}$ waveform in Fig. 5(b), during the time the voltage changes from $v_a$ to $v_b$—these both being measured from the reference voltage $V_r$—along an exponential curve, whose time-constant $T = C_2(R_1 + R_5)$ and would eventually reach the value $V_r$ if it were not for the valve $V_2$.
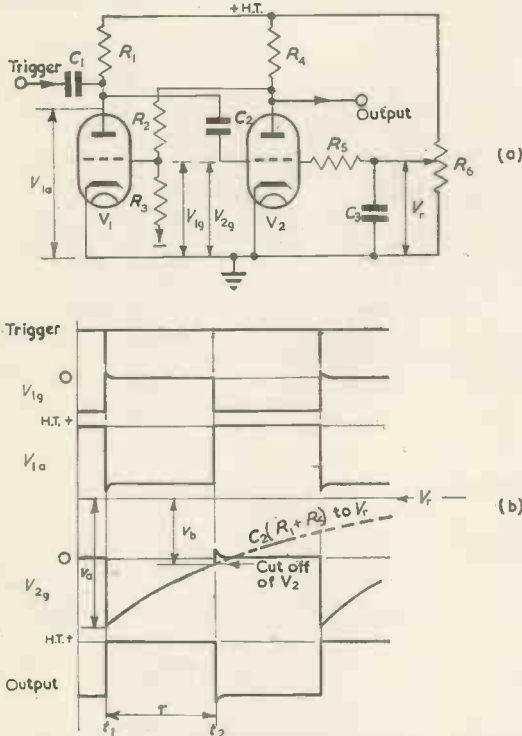


Fig. 5. Basic monostable multivibrator (anode coupled)

Now for an exponential fall in voltage due to the discharge of a capacitor through a resistor:

$$v = V \cdot e^{-t/T}$$

where $v$ is the voltage after time $t$.

$V$ is the fall in voltage after an infinite time.

$e$ is the base of natural logarithms.

$T$ is the time-constant for the circuit.

And in the particular case:

$$V = v_a, \quad v = v_b, \quad t = \tau,$$

$$\therefore \quad v_b = v_a \cdot e^{-\tau/T} \quad \text{or} \quad e^{\tau/T} = v_a/v_b \quad \dots \dots \dots (1)$$

Now the negative voltage swing of $V_{2g}$ at time $t_1$ is equal to the negative voltage swing of $V_{1a}$, assuming perfect coupling.

Thus:

$$v_a = V_r + \delta i_1 \cdot R_1 \quad \dots \dots \dots \dots (2)$$

where $\delta i_1 =$ the change of $V_1$ anode current from cut-off to full conduction at the existing potentials.

and:

$$v_b = V_r + V_{c_2} \quad \dots \dots \dots \dots \dots (3)$$

where $V_{c_2} =$ the cut-off voltage of $V_2$ at the existing conditions. Also by expansion:

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots \text{etc.}$$

and if $x < 0.1$, $x^2$ and higher terms can be neglected, so that:

$$e^x \approx 1 + x \quad \dots \dots \dots \dots \dots (4)$$

Substituting Equations (2), (3) and (4) in (1) we obtain:

$$1 + \tau/T = \frac{V_r + \delta i_1 \cdot R_1}{V_r + V_{c_2}}$$

$$T + \tau = T\left[\frac{V_r + \delta i_1 \cdot R_1}{V_r + V_{c_2}}\right]$$

$$\tau = T\left[\frac{V_r + \delta i_1 \cdot R_1}{V_r + V_{c_2}} - 1\right]$$

$$= T\left[\frac{\delta i_1 R_1 - V_{c_2}}{V_r + V_{c_2}}\right]$$

If $V_{c_2} \ll V_r$ then:

$$\tau \approx T\frac{\delta i_1 - V_{c_2}}{V_r}$$

$$= K/V_r$$

i.e. the width of the pulse generated is inversely proportional to the reference voltage $V_r$, providing $\tau < 0.1\,T$ and $V_{c_2} \ll V_r$.

### 2.—THE MEMORY CIRCUIT

The memory circuit is shown in Fig. 6 and consists of two triodes, a capacitor and coupling circuits.
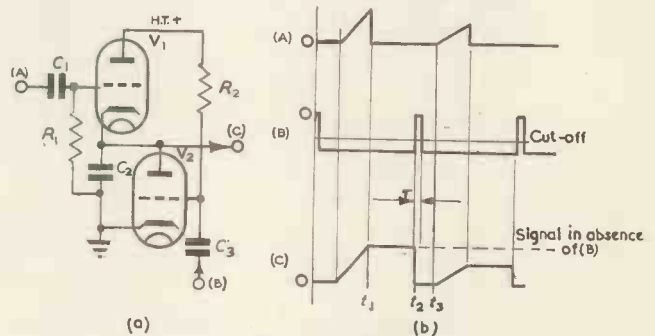


Fig. 6. The memory circuit

Initially $V_2$ is non-conducting due to D.C. restoration of the timing pulses (B), causing the normal level of (B) to be below the cut-off voltage of $V_2$; thus $V_1$ acts as a cathode-follower with a capacitive load[10,11], allowing $C_2$ to charge up to the peak value of the input voltage (A) at time $t_1$; this output voltage would remain at this level indefinitely if we neglect leakage and (A) does not increase in amplitude. But at time $t_2$ the timing pulse (B) momentarily discharges $C_2$ by allowing $V_2$ to conduct for the time $\tau$ and thus allows $C_2$ to charge up to a new lower level at time $t_3$.

It is seen therefore that the circuit allows the peak value of a given input signal to be "remembered" for a longer time than that of the initial peak period.

#### Acknowledgment

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# Notes from the Industry

**Sir George Nelson,** chairman and managing director of the English Electric Company, Ltd., recently bought from Mrs. Mildred Wells, a first cousin of Lady Siemens, the originals of more than 250 letters, written by celebrated men of science, statesmen, artists and noblemen to Sir William Siemens. These have been bound in two handsome volumes and were recently privately exhibited in London. A third volume, containing a short biography of Sir William Siemens, written by Mr. W. H. Kennet, printed copies of all the letters and facsimile reproductions of 22 of them, has been presented by Sir George Nelson to Colonel B. H. Leeson, the President of the Institution of Electrical Engineers. In addition to the three leather bound volumes, one thousand copies of a general edition of the letters have been printed. These are to be presented to universities, libraries, and engineering societies.

**The North-West Branch of the Institution of Electronics** announce that the eighth annual electronics exhibition will be held at the College of Technology, Sackville Street, Manchester, from July 15-21. The exhibition will be officially opened on July 15 at 2.30 p.m. by Sir Geoffrey Jefferson (Emeritus Professor of Neuro-Surgery, University of Manchester). An extensive programme of lectures will be presented during the exhibition, dealing with topics from all branches of electronics, and includes lectures of general interest as well as those of a highly technical nature. An exhibition catalogue is also available, price 1s. 6d. Applications for admission tickets, lecture programmes and lecture admission tickets can be made to the Honorary Exhibition Organizing Secretary, Mr. W. Birtwistle, 17 Blackwater Street, Rochdale, Lancs., or by direct application at the exhibition reception desk during the period of the exhibition.

**Television in Hungary.** First details of preparations for the introduction of television in Hungary have been disclosed by Mr. Antal Katona, Minister for Postal affairs. Transmissions will begin in 1954. At first these will be confined to "documentary programmes" on an experimental basis. A year later the service will switch to studio programmes and on-the-spot broadcasts.

**The Ministry of Supply** announces that Professor Sir Eric Rideal is to be chairman of the Minister's Advisory Council on Scientific Research and Technical Development in succession to Professor Sir John Lennard-Jones. Sir Eric Rideal, who is Professor of Physical Chemistry at Kings College, London, has been associated with research work of the Ministry of Supply since the early part of the last war.

**Industrial Applications of Electronics.** A two-day symposium on the Industrial Applications of Electronics was held on May 4 and 5 at Liege, in Belgium, under the auspices of the Institut Electrotechnique Montefiore. Some 40 papers were presented by speakers from Belgium, France, Germany, Holland, and Sweden, the subjects ranging from non-linear servomechanisms to the practical problems of the application of electronics to cold rolling strip mills and machine tools. The only British contribution was a film of the NRDC Elliott Computor type 401 (described in the May, 1953, issue of Electronic Engineering, pp.213-214), which was shown during the discussion following a paper on Magnetic Recording in Computors.

**The BBC** have acquired a three-acre site at Pages Lane, Bexhill, approximately one mile west of the centre of the town, for the permanent low-power 2kW transmitting station that is being provided to improve reception of the Home Service in the area. This permanent station is expected to be in service before the end of the year.

**Cambridge Summer School in Automatic Computing.** A summer school in programme design for automatic digital computing machines will be held in the University Mathematical Laboratory at Cambridge during the period September 22-October 2, 1953. It will be along the same lines as those held previously. A detailed syllabus and form of application may be obtained from G. F. Hickson, M.A., Secretary of the Board of Extra-Mural Studies, Stuart House, Cambridge. Completed application forms should be returned not later than July 18.

**The Institute of Physics** announce that their thirty-third annual report was adopted at the annual general meeting of the Institute on May 29. 251 new members were elected to grades for which approved academic qualifications in physics are demanded. 651 applications for membership were considered during 1952 and the total membership at the end of the year was 4 347.

**Marconi's Wireless Telegraph Co., Ltd.,** have designed, manufactured and installed a new short-wave broadcasting transmitter which has begun operation at Abu Zaabal, near Cairo. The plan to build this 100kW short-wave transmitter, and a 100kW medium-wave transmitter with aerial system, now under construction by the company at the same site, was originally drawn up some years ago with the intention of giving world-wide and local coverage for the Arabs. The range of this short-wave transmitter is such that people from any part of the world will now be able to listen to Egyptian programmes without any difficulty.

# BOOK REVIEWS

### Television Engineers' Servicing Manual

Edited by E. Molloy. 760 pp., 585 figs. Royal 8vo, Geo. Newnes Ltd. 1952. Price 42s.

THIS book is stated to deal with "the installation, repair and maintenance of television receivers, and while it should be of value to those concerned with these things, it is doubtful if it is in the best form. Service data has to be constantly added to keep it up to date, and some sort of loose-leaf binding is very useful for this purpose. No indication is given that this is to be one of a series of books, each concerned with a particular year, such as the American Rider's Manuals, so it must be considered as a complete volume.

Following a brief history of each of the 16 " Specialist Contributors " there is a short survey of types of television receivers. It contains some odd observations such as " until recently television could only be operated from A.C. mains," and under " Focusing " " . . . .in A.C./D.C. receivers the drop in voltage across the (focus) coil can hardly be afforded. When the H.T. current is limited, therefore, some sort of permanent magnet is desirable," but is interesting and up to date.

Next comes a series of General Notes on Installations. These are very useful and particular stress is placed on obtaining maximum benefit from test card "C" when setting up. Only one with considerable practical experience of commercial receivers would have written " The circle is a critical linearity test over the central area, and if the receiver is correctly adjusted it should be circular in shape."

In the following section on aerials and feeders, many kinds of commercial indoor and outdoor aerials are described and illustrated and attenuators are shown with a table of resistors for various amounts of attenuation. The table might well have shown the attenuation as a voltage ratio as well as in dbs. The resistors given are all " preferred " values, which is a useful practical point.

The next five pages are used to deal with the effect of the geographical position of the receivers, and the field strength maps of the London and Sutton Coldfield transmitters are shown. This is followed by a short description of equipment used in servicing, some notes on fault finding, and the alignment of receivers for single sideband reception. There are a number of excellent John Cura tele-snaps in those early pages, showing various faults on a television screen.

The rest of the book is concerned with very complete servicing details of 55 receivers, mainly 2-3 years old, made by 29 manufacturers. The average treatment for each model is an illustration of the complete receiver in its cabinet, one or more of the chassis with an indication of the position of various components, details of the removal of the chassis from the cabinet, position and adjustments of the controls, a circuit diagram, a description of the circuit, voltage readings at various points, modifications or variations in the circuits, and a list of possible faults.

Most of this material seems to be copied from the Manufacturers' Service Manuals and for this reason is very varied from make to make. In particular, the circuit diagrams are drawn in a number of ways, many are poor, and some are practically unreadable unless much effort and time is spent on tracing wires. The printing of the book is excellent, it is what is printed that is hard to follow. Some manufacturers even print some parts of a circuit upside down, but to prevent these parts being studied by inverting the page, the values are printed right side up.

It is a pity that the circuits were not re-drawn in a uniform and clear manner.

It would also have been helpful if the makes had been arranged alphabetically, with the model numbers listed. As it is, to find a particular make one has to find the list of contents where the makes are given in page order, or find page 51 where the makes are listed in alphabetical order, and then find the appropriate page and finally see if a particular model is described. Such delays are especially annoying.

Several descriptions could also have been written more clearly, e.g. the description of a frame time base on page 170, where the screen potential of a Miller/Transitron type of oscillator is stated to be negative part of the time and the anode current is stated to increase until the anode voltage is " nearly at the same potential as the cathode and anode current is at its minimum."

As already stated, the printing is excellent and on good paper. The illustrations are of equally high standard, and the book is probably good value for the circuit diagrams alone, which work out at approximately 9d. each.

C. H. BANTHORPE.

### High Fidelity Simplified

By Harold D. Weiler. 208 pp., 104 figs. Medium 8vo. John F. Rider Inc., New York. 1952. Price $2·50.

YOU are an " audiophile ", whose desire is to achieve high quality reproduction, but you do not possess the technical qualifications to enable you to assess the relative merits of the vast array of equipment from which you must select your buy ? Then this book has been written for you.

The early part explains the nature of sound and the function of hearing, so the reader can understand the problems

of good reproduction. Probably, to the uninitiated reader, the author's presentation would prove quite enlightening, but to this reviewer the value of its lucidity is somewhat offset by some technical inaccuracies. One gathers, for instance, that the logarithmic law in the sensation of loudness is entirely due to the amplitude of eardrum movement. If this were true, intermodulation products would be colossal, rendering our critical faculties of listening completely impossible. Also we learn that a signal 40db below the one we want to listen to is sufficient to mask it. This may account for the level at which some people operate their radio receivers!

The latter part is concerned with the variety of systems available. Most of the illustrations are of typical examples of various items, from loudspeakers, record-players, tuners and amplifiers, to complete music room installations, by various American manufacturers. The pros and cons of various systems are fairly given, so the reader has a good opportunity to decide which will serve his own objective best. The level, as the author intends, is kept introductory, to enable the reader to grasp points in technical literature that must have been somewhat obscure to him before.

One passage amused the reviewer: American long-playing records are given a frequency range of 50 to 10 000c/s. The British are making them from 40 to 14 000c/s. So the author suggests a new standard of 30 to 15 000c/s. The addition of 1 000 cycles may seem a lot numerically, but the author should realize, as he explains the significance of logarithmic frequency scale elsewhere, that the ratio 15/14 is little over a semitone! It's rather like a piano manufacturer adding A sharp at the top end to excel his rivals.

N. H. Crowhurst.

## Radio and Radar Technique
By A. T. Starr. 812 pp., 300 figs. Medium 8vo. Sir Isaac Pitman & Sons, Ltd. 1953. price 75s.

THIS comprehensive book by an established author gives in a single volume a summary of the techniques used in modern radio and radar. Clearly such a book is only made possible by rigorous selection of material, and Dr. Starr refrains from discussing specific applications, and he excludes those topics usually dealt with in a University Degree Course.

The condensed style, the frequent abbreviations and the excessive use of mathematical explanations make this book much more difficult to read than either the M.I.T. Radiation Laboratory series or the I.E.E. Radiolocation papers from which sources much of the material has been taken. The following sentence is by no means untypical of the sort of difficulty confronting the reader:—
"Thus, P.A.M. having sidebands of the A.M. type will have no improvement but P.W.M., P.T.M. and P.F.M. having sidebands of the P.M. or F.M. type will have an improved S/N" (page 40). However, the book contains a remarkable amount of useful information, especially in the first five chapters, and for these and the extensive bibliography it is well worth having.

The book is divided into seven chapters followed by thirty appendices which are equivalent in length to a further four chapters. Chapter 1 enumerates pulse-methods of location and communication and considers the fundamental limits set by the available bandwidth. Chapter 2 is on propagation followed in Chapter 3 by waveguide-techniques and in Chapter 4 by a description of aerials for both metre and centimetre wavelengths. Chapter 5 describes radar valves, including velocity-modulated and travelling-wave types. These chapters account for approximately one half of the book and the remaining two chapters describe circuit techniques. Here the material is not selected with the same care as in the earlier chapters; for example, stabilized power supplies are not mentioned, but several pages and 19 diagrams refer to pulse counters. The appendices are mainly mathematical and much of this material could have been simplified and incorporated in the main text.

The bibliographies at the end of each chapter are excellent and form a very valuable addition to the book. In this respect the book is much better than the M.I.T. series and, as it has been published more recently, it has the added advantage that the references are more up-to-date.

V. H. Attree.

## Fundamentals of Engineering Electronics
By W. G. Dow. 627 pp., 117 figs. Medium 8vo. John Wiley & Sons, Inc., New York. Chapman & Hall Ltd., London. 1952. Price 68s.

THIS book describes the fundamental processes involved in the internal operation of electron devices. It bridges the gap between the theoretical approach of the physicist and the practical approach of the engineer. The text is appropriate to the final year of a degree course, but the concise mathematical demonstrations and the excellent bibliography will make it useful to engineers working in the field of electronics.

This is a second edition, and when the first edition was published (in 1937) the only electron devices in wide use were vacuum and gasfilled tubes with hot cathodes, and photo-cells. Thus in the earlier edition emphasis was placed on electron trajectories in the triode and on the phenomena associated with gas discharges and photo-electric emission. In recent years many new devices have become of engineering importance; examples of these are magnetrons, V.M. tubes, photo-multipliers, high-intensity light-sources, and transistors. The second edition includes the basic principles underlying the operation of these devices, and to make room for the new material certain topics discussed in the first edition have been omitted.

The first six chapters are concerned with the field distribution and electron behaviour in valves and cathode ray tubes. Of these chapters, the one on cathode ray devices (Chapter III) is undoubtedly below the general standard; post-deflector acceleration and transit-time effects are not mentioned, and the section on television camera-tubes occupies less than a page. Chapter VII discusses

thermionic cathodes, and Chapter VIII introduces the idea of the Fermi energy-level diagram which readily leads to the discussion of electron behaviour in conductors and semi-conductors. This chapter is perhaps the most valuable in the book and it is disappointing to find that the description of actual semi-conductor devices occupies little more than a page. Chapters IX and X deal with the application of conventional valves in amplifiers and oscillators, followed in Chapter XI by an introduction to the principles of microwave tubes. Chapter XIII deals with the ionization and exitation of atoms, which is the basis of modern spectroscopy. Chapter XIV is on photo-

electricity, and the remainder of the book gives the fundamentals of gaseous discharge devices.

The references at the end of the book are up-to-date, given with the full title, and with minor exceptions are from recognised journals. The value of this section would have been enhanced by the inclusion of more European work and by an author index.

In spite of these criticisms there is no doubt that Professor Dow, with the aid of his associates at the University of Michigan, has produced a good book and one which, because it deals with fundamentals, is less ephemeral than the usual electronics text.

V. H. Attree.

# ELECTRONIC EQUIPMENT

A description, compiled from information supplied by the manufacturers, of new components, accessories and test instruments.

### Supersonic Flaw Detector

*(Illustrated below)*

THE Mk5 is the latest of the range of flaw detectors manufactured by Messrs. Kelvin and Hughes Ltd.

It is designed specifically for the inspection of welded pressure vessels where the equipment must be passed through a manhole. The equipment, which is totally self-contained measures 17in. by 9in. by 5in. and has a total weight of less than 30lb.

Among its most notable features are the following. The scale can be expanded so that the width of the screen represents only a small part of the total range, thus facilitating thickness measurement or the individual detection of closely grouped flaws.

A very bright trace even on the shortest range is achieved and is adequate for daylight viewing.

The instrument which uses the new lightweight leads can be operated with the full range of Kelvin Hughes probes. A special combined transceiver probe (illustrated) provides many of the
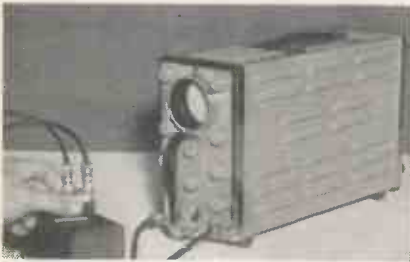


advantages of the dual probe system while allowing the ease of handling inherent in a single probe.

The coaxial sockets are duplicated to allow for use of special filters on a depth and thickness gauge.

The cathode-ray tube and all valves used in this new equipment are easily obtainable standard types.

A handle on the front panel for carrying the instruments in a vertical position also gives protection to the controls.

The principle of operation, which is by now well known is as follows. As the C.R.T. spot passes the zero mark on the scale a pulse of ultrasonic energy in the form of a narrow beam is directed into the specimen under test by means of the transmitting crystal mounted in an exploring probe. Energy reflected, for example by the boundary of the specimen, is picked up by the receiver crystal and causes a vertical deflexion of the C.R.T. spot. The presence of a flaw is indicated by any deflexion of the spot between the zero mark and the boundary echo. The position of the flaw in relation to the depth of the specimen can be deduced from the relative positions of the flaw echo and the boundary echo. By making use of the scale expansion feature any part of the trace may be selected and expanded so as to allow detailed examination of the echo.

The operating frequencies are chosen to suit the type of material being tested and are selected by means of a switch. Frequencies available are $\frac{5}{8}$, $1\frac{1}{4}$, $2\frac{1}{4}$, $2\frac{1}{2}$, and 5Mc/s, intermediate frequencies can be obtained, if required, by the use of plug-in filters. The range represented by full deflexion of the time-base can be varied from 2in. to 12ft when using the ordinary vertical probes. These distances are approximately halved when transverse wave probes are used.

Kelvin and Hughes (Industrial) Ltd.,
2 Caxton Street,
London, S.W.1.

### Thermometer Bridge

*(Illustrated below)*

THIS instrument constitutes a simple method of accurate temperature measurement in the form of a robust industrial unit, with no galvanometer or fragile moving parts. It consists of a Wheatstone bridge, designed to operate with standard Weston platinum resistance bulbs. An alternating voltage is applied to the bridge, the out-of-balance voltage of which is amplified and used to operate an electronic indicator (magic eye).

The temperature is read directly from the calibrated dial when this has been rotated by hand until balance of the bridge is indicated by the magic eye.

The electronic equipment is housed in a small diecast aluminium case suitable for wall mounting on any industrial site. The resistance bulb, with its pocket, is inserted at the temperature measuring point and connected by a cable to the electronic unit.

This unit is available in four temperature ranges covering from $-200°C$ to $+500°C$.

The equipment is calibrated for Weston standard platinum resistance bulbs. It can be supplied with a multi-way switch, if required, so that several points can be examined on the same instrument.

Fielden (Electronics) Ltd.,
Wythenshawe,
Manchester.



### Alternating Voltage Stabilizer

*(Illustrated below)*

THIS instrument provides for an automatically regulated and variable alternating output voltage from the normal mains supply. It incorporates a servo-mechanism which has been developed to meet the most exacting laboratory and industrial applications where a completely undistorted waveform and rapid response together with extreme reliability and accuracy are essential. Efficiency on full load is 97 per cent

The unit is assembled on a steel chassis and a 19in. by $8\frac{1}{4}$in. panel. It is protected by a dust cover and is normally contained in a robust steel case. Alternatively, the unit can be withdrawn from the case and placed in a Post Office rack.

The stabilized output voltage is continuously variable between 200 and 240V at a current from zero to 10A, but the input voltage must be within $-17\frac{1}{2}$ per cent to $+6$ per cent of the set output voltage.



The output voltage is maintained constant to within 1 per cent while a frequency variation of 10 per cent may result in a maximum change of 0.2 per cent in the output voltage. The system is unaffected, both in accuracy and response, by large changes in ambient temperature.

The output voltage is monitored by a sensitive comparator circuit, which measures the error from the correct value as set by the control knob. If the error exceeds 1 per cent a small A.C. motor drives a Philips variable autotransformer which restores the voltage to its proper value, and the motor then stops.

The rate of correction, when the mechanism has reached full speed, is 12 volts per second. The bandwidth which determines the speed at which the mechanism can reverse its motion is $2\frac{1}{2}$c/s, corresponding to an effective time-constant for the whole system of approximately 70 milliseconds.

Protective devices are incorporated in the form of limit switches. Thus, if the input voltage goes outside the specified range, and this can occur when the unit is used as a variable voltage supply, the motor will be switched off automatically. For example, if the input is at 180V, nominally 200, and the output is raised to 220, the limit switch will operate and the motor will remain switched off until

the output setting is reduced or the input voltage rises.

In the event of a serious fault occurring, the motor is further protected by a thermal cut-out.

Although the range of the standard unit is 200-240 volts output, provision is made, by the embodiment of a simple modification, to draw maximum power at any required voltage. For example, the unit can be arranged to supply 350A at 6·3V so that the low voltage is fully stabilized against varying load conditions or fluctuating voltage supply. The component parts necessary for the introduction of this modification are supplied according to requirements.

The unit can also be arranged to have two output supplies whereby both are stabilized against input voltage fluctuations and one fully stabilized against load variations.

Servomex Controls Ltd.,
Crowborough Hill,
Jarvis Brook, Sussex.

## Interference Suppressor Components

### (Illustrated below)

THREE new types of suppressors (illustrated) have been added to the range of bushing suppressors manufactured by the Dubilier Condenser Co. The new types, which are available with
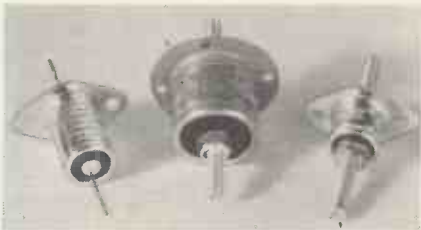
both ceramic and neoprene end seals, are rated at 50V D.C. and are suitable for use on low voltage D.C. generators and motors. They have excellent suppression characteristics up to 600Mc/s.

Also recently introduced is a range of tropical suppressor filter units which are hermetically sealed in a metal container with silastic terminals, and three new types of suppressor chokes, one utilizing a toroidal ferroxcube core, the second a wavewound choke on a dust core, and the third a miniature choke in a moulded bakelite box. These chokes are for suppression of interference on the normal sound broadcasting bands and are not suitable for suppression of interference to television programmes.

Dubilier Condenser Co. (1925), Ltd.,
Victoria Road,
London, W.3.

## P.V.C. Wire Stripper

WOLF ELECTRIC TOOLS, LTD., announce that Wolf solderguns type numbers 31 and 41 and soldering irons numbers 32 and 42, will in future be fitted with a patent wire stripper for stripping plastic covered wires prior to soldering. The stripper is conveniently fitted to the barrel of the tool and in a position for correct working temperature.

Wolf Electric Tools, Ltd.,
Hanger Lane, London, W.5.

## Variable Frequency Vibration Drive Amplifier

### (Illustrated below)

THIS vibration drive equipment has a frequency range from 50c/s to 10kc/s and will provide up to 250 watts (Class A) into any load impedance between 0·5 ohms and 50 ohms. The considerable power available is sufficient for the vibration of complete production assemblies so that design engineers are able to determine component and structural design performance under dynamic conditions and to carry out fatigue tests.

In order that a uniform drive characteristic may be obtained at all frequencies, and under varying operating conditions, an A.V.C. type of control amplifier is used, to ensure continuous amplitude-controlled acceleration and constant velocity. The control action is determined by a signal derived from a pick-up coil mounted on the vibration table and this signal is fed back into the control amplifier. In addition to a continuously variable gain control, the amplifier has a three position switch to select a drive amplitude equivalent to × 1, × 10 or × 100g.

The equipment, which is mounted in a 6ft rack, consists of a beat frequency oscillator, the control amplifier, buffer and output stage and power supply units. A system of safety relays links and fuses prevents damage due to premature application of the H.T.

The equipment is based upon an original design of the General Electric Co., Ltd., Stanmore, in conjunction with the C.I.E.M.E., Ministry of Supply.

The output characteristic is linear ±0·5db from 20c/s to 5kc/s with a slight deterioration from 5kc/s to 10kc/s, the output power being 250 watts in class A or 1 000 watts in class AB2.

In the illustration the drive amplifier is shown in use in conjunction with the panoramic wave analyser model 1950, this latter unit will analyse any complex waveform into its fundamental and harmonics between the range of 20c/s to 64kc/s.

Industrial Electronics,
Derby Road, London, S.W.14.

## Cyldon Teletuner Type TV12

### (Illustrated below)

THE Cyldon Teletuner type TV12 is a two valve unit performing the functions of R.F. amplifier and frequency changer of a television receiver, giving the facility of selection of any one of twelve carrier frequencies by means of a single knob control.

Channel selection is made by a rotating turret containing twelve aerial and twelve R.F. coupling and oscillator coils, each coil being mounted on a low-loss bakelite contact strip and retained on the turret by means of a phosphor bronze clip. All circuit switching is accomplished by means of eleven spring pressure contacts, rotation of the turret bringing the coils for the desired channel into circuit and leaving the remaining eleven completely isolated. The tuner can be supplied to cover television transmissions from any of the systems used throughout the world in the carrier frequency range of 50-220Mc/s.

The signal input to the tuner may be made either at an impedance of 300 ohms balanced with respect to earth, or at 75 ohms unbalanced. The R.F. amplifier uses a double triode valve in a cascode circuit with neutralizing on the input triode. Provision is made for the use of A.G.C. on this valve.

The frequency changer uses a triode-pentode valve with inductive injection of local oscillator voltage. Bandpass coupling is used between the R.F. amplifier and mixer stages, giving good curve-shape characteristics.

Fine adjustment of the local oscillator frequency is provided by a trimmer capacitor, the spindle of which is brought out concentrically with the main channel selector spindle.

Extensive shielding, both internal and external, is provided, together with by-passing of A.G.C., H.T. and heater feeds to reduce oscillator radiation.

Signals at intermediate frequency from the mixer anode may be fed to the I.F. amplifier at either high impedance or at 75 ohms, the coupling transformer being incorporated in the tuner chassis. The intermediate frequency is in the range 40-47Mc/s.

Supplies required to the tuner are 220V H.T. 16V at 0·3A, the heaters being series connected, and an A.G.C. line with a standing voltage of −1·5V.

Sydney S. Bird and Sons, Ltd.,
Cambridge Arterial Road,

# LETTERS TO THE EDITOR

*(We do not hold ourselves responsible for the opinions of our correspondents)*

## A D.C. Amplifier for use in pH Measurements

DEAR SIR,—In his interesting article in the February, 1953, issue Mr. C. Morton devotes the major part of his analysis to a theoretical circuit (Fig. 3) which works in a basically different way from his practical amplifiers (Figs. 4 and 5). He partially recognizes the distinction towards the end of the article, but dismisses it so briefly as to leave the impression that with minor modifications the preceding analysis holds good for the practical circuits, whereas in fact any resemblance between the two systems is purely coincidental.

Consider the coupling between the two halves of the amplifier. In Fig. 3 it is obtained solely by current flowing through the load resistor $r_o$. If $r_o$ were $\infty$, the second half of the amplifier, $A_2$ would be completely disconnected and would not function at all. This behaviour is correctly expressed by equation (16): when $i_o = 0$, $i_2 = 0$. But in the practical amplifier circuits the mode of operation is quite different. The cathode resistor denoted by $R_k$ or $R_{13}$ (which Mr. Morton explains is needed for adjusting the cathode potentials of $V_1$ and $V_2$ but is not deliberately included for phase inversion or coupling, although incidentally contributing to or reinforcing that effect) ensures that the practical amplifiers continue to work as symmetrical coupled pairs even when $r_o = \infty$. This resistor is in fact the sole means of coupling the two halves, not only when the load is open-circuited (when Mr. Morton's explanation of coupling obviously breaks down) but also when it is working normally, with finite $r_o$. For the action via $r_o$ not only does not provide positive coupling: it opposes the actual coupling, being in fact negative feedback. This can be seen by considering Mr. Morton's explanation of how Fig. 3 works: a positive potential to the control grid of $A_1$ impresses a negative potential on the control grid of $A_2$. But in the practical circuits this point is actually driven positive (by the cathode coupling) against the influence of $i_o$.

The introduction of Fig. 3 and associated explanation is therefore not only irrelevant to the practical circuits shown but is liable to confuse.

The same remark applies to the statement that any desired degree of negative feedback between 0 and 100 per cent can be obtained by varying $r_o$. In Fig. 5 the whole available output voltage is fed back, and in the normal use of terms the feedback would be said to be 100 per cent, regardless of the value of load resistance.

A valuable feature of this heavy feedback is that by using this type of circuit with suitable valves and component values\* the output resistance of the

\* SCROGGIE. M. G., *Valve Voltmeter Without Calibration Drift. Wireless World*, 58, 14 (1952).

amplifier can be made as low as 2 ohms and the output voltage is equal to the input voltage, so that any D.C. voltmeter connected to the output terminals reads correctly when the combination is used as a high-input-resistance valve voltmeter.

Incidentally, may I again make a plea against the use of non-standard symbols? Mr. Morton employs $\mu$ to denote amplifier gain, although this symbol has for a generation been adopted universally for valve amplification factor. He is therefore driven to denoting the latter by $\mu'$. For this misuse of $\mu$ he can blame certain authorities on negative feedback, but they are at least consistent in denoting the fractional feedback by $\beta$, whereas he makes confusion worse confounded by substituting $\alpha$ (the appropriate symbol for amplifier gain), using $\beta$ for something else. And it has for some time been agreed to denote internal valve parameters by small letters; so $R_a$ denotes an external anode resistance. The British Standard lists are readily obtainable and it would spare the reader unnecessary difficulty if authors would adhere to them wherever applicable.

Yours faithfully,

M. G. SCROGGIE.
Elstree Hill Laboratory,
Bromley.

DEAR SIR,—It seems to me that a number of points made by Mr. C. Morton in his article published in your February issue are open to question; and I should like to advance the following arguments and comments.

(1) Mr. Morton claims, "phase inversion is effected without the aid of potential dividers, common feedback resistors, or other auxiliary circuit components." But if we look at Fig. 3, "The amplifier described," we find that no phase inversion is effected; the circuit is not a push-pull amplifier, but two single-sided amplifiers placed back-to-back with the indicating meter connected between them (Fig. A). The input signal is applied to only one of the amplifiers, and the other is useful only for drift compensation. The fact that the output resistance of the lower amplifier is made low by feedback is, of course, irrelevant to the argument, however useful it may be for other reasons. Incidentally, unless the output resistance of the lower amplifier is negative, the physical direction of $i'_2$ is opposite to the direction of the arrowhead. All this does not condemn the arrangement, of course, since drift compensation is of great importance in a D.C. amplifier designed to measure potentials of the order of 1 volt.

(2) When it comes to devising a circuit the arrangement of Fig. 3 is difficult to realize, since input terminals independent of earth are demanded. Mr. Morton overcomes this difficulty by connecting his input voltage in series with the feedback connexion of the upper or active amplifier, although this means that either the input or the H.T. supply must float with respect to earth, which may sometimes be a disadvantage and restrict the usefulness of the circuit.

(3) As Mr. Morton points out the common cathode resistors $R_k$ $R_{13}$ in Figs. 4 and 5 couple together the upper and lower amplifiers, and I estimate that because of this the amplifiers do after all operate in pretty well balanced push-pull.

(4) I should like to stress a point not included in Mr. Morton's article, and this is that although it is good practice to begin a D.C. amplifier with a balanced or push-pull stage (for drift compensation), there is no real need to keep to push-pull stages right to the output; and in practice a single-sided output stage is often much more convenient, since one side of the output can then be the earth line. Such amplifiers are well known in

Fig. B (right). High resistance input amplifier (after Valley and Wallman)

Fig. A (below). Back-to-back arrangement of single-sided amplifiers
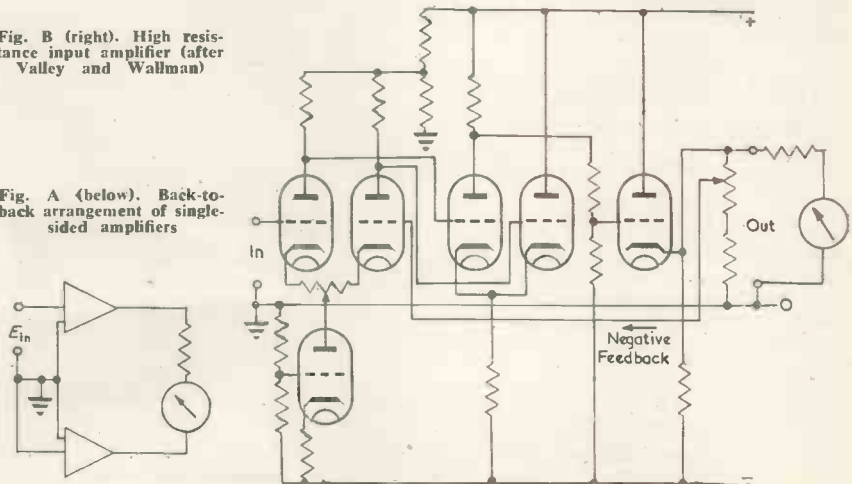
the computing field, Valley and Wallman describe an amplifier for low-voltage very-high-resistance inputs (M.I.T. Radiation Lab. Series, Vol. 18, p. 480, Fig. 11.63), and I enclose a sketch (Fig. B), showing in outline the circuit of a similar amplifier I have used myself.

(5) I am grateful to Mr. Morton for reporting the extremely low grid current he has found with the Marconi-Osram KTW61. I have believed for a long time that low-mu valves are intrinsically the most suitable in applications requiring low grid current, since only low-mu valves (inner mu for a pentode, of course) can be operated at very low voltages (say 10 volts or even less); and I have myself obtained good results with the Mullard EL32. It had not occurred to me, however, that in the common variable-mu pentodes we have types worthy of investigation.

Yours respectfully,

E. F. GOOD,

R.R.D.E.,

Malvern.

DEAR SIR,—I have read with great interest the article on " A D.C. Amplifier for use in *pH* Measurements " by .C. Morton in your February issue, but I am surprised that he makes no mention of the use of " acorn " type valves, such as the 954 as electrometer valves, especially when used with G3 as the input, as described by Nielsen (*Rev. Sci. Instrum., 18,* 78 (1947). This type of valve has also been used in circuits with 100 per cent negative feedback, with either $g_1$ or $g_2$ injection, as described for instance by Harris (*Electronic Engng., 23,* 109 (1951) for *pH* measurements, etc.

Yours faithfully,

PETER S. BENNETT,

Lancaster.

### The Author replies:

DEAR SIR,—I am indebted to Mr. Good and to Mr. Scroggie for their explanations of the manner in which the amplifier functions, and for their comments upon the effect of the common cathode resistors $R_k$ and $R_{13}$ (in the circuits of Figs. 4 and 5 respectively) in producing phase inversion. A similar coupling effect is of course also produced by the common screen resistor $R_8$ of Fig. 4. The writer abandoned the use of these circuits some time ago in favour of a modification, the publication of which has hitherto been prohibited by the Patent Comptroller under national security regulations. When the connexions of Fig. 3 are rearranged as in Fig. C, the degree of phase inversion may be continuously adjusted from zero to 100 per cent, and the gain from unity to infinity, without disturbing the zero setting of the output meter. Both effects are produced in the absence of common cathode or screen resistors, and are independent of the load resistance.

It can be shown that for this modified circuit the gain is $E_o/E = 1[1-(a_1+a_2)+(1/\mu)]$. If $r_2$ is adjusted in such a way as to satisfy the relationship $a_2 = 1/\mu$ (e.g., by varying $r_2$, with $a_1 = 0$, until it is found that a known applied E.M.F. is

accurately reproduced in the output circuit), the calibration of $r_1$ for use as a gain control is readily calculable from the simple expression $E_o/E = 1/(1-a_1)$. The gain is unity for $a_1 = 0$ and theoretically infinite when $a_1 = 1$. Alternatively, $r_2$ may be used as a phase splitting control, the value of $a_2$ required to give any desired degree of phase inversion being calculable from the expression $a_2 = m$ $[1+(1/\mu)]$ where $m - v_2/(v_1 + v_2)$ is the fraction of the output voltage which is generated by $A_2$. The degree of phase inversion is zero $(m=0)$ when $a_2 = 0$, or 100 per cent $(m=0.5)$ for $a_2 = 0.5$ $[1+(1/\mu)]$. The latter condition is realized
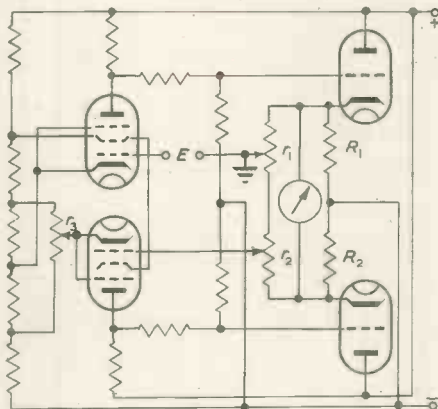


Fig. C. Rearrangement of connexions of Fig. 3

in practice by adjusting $r_2$ until it is found that an applied E.M.F. produces equal and opposite changes in the voltages across the equal resistors $R_1$ and $R_2$, the overall gain being subsequently controlled solely by adjustment of $r_1$. As no current flows through $r_1$ or $r_2$ in the absence of an applied E.M.F., the adjustment of the gain and phase splitting controls does not entail readjustment of the zero setting of the output meter.

An additional advantage of the arrangement is that it permits of the utmost latitude in the choice of the connexions of the feedback loop. It is immaterial, for example, whether the loop extends across an odd or an even number of stages. When the phase relationship between the input and output voltages is the reverse of that denoted by the signs in Fig. 6, the gain is given by the expression $E_o/E = 1/[a_1+a_2+(1/\mu) -1]$ which reduces to $E_o/E = 1/a_1$ when $a_2 = 1-(1/\mu)$. Unit and infinite gain then corresponds to values of $a_1$ of unity and zero respectively. The adjustment of $a_2$ to satisfy the requirement $a_2 = 1 -(1/\mu)$ is effected by varying $r_2$ (with $a_1$



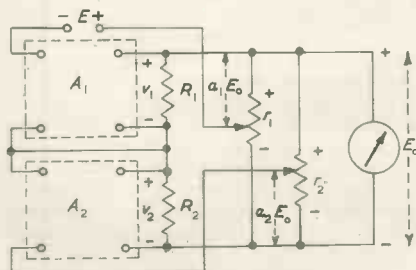Fig. D. Practical circuit diagram

=1) until it is found that a known applied E.M.F. is accurately reproduced in the output circuit. 100 per cent phase inversion is obtained when $a_2 = 0.5[1- (1/\mu)]$.

In view of Mr. Good's remarks concerning the difficulty of realizing circuits of this type in practice and, in particular, of providing sound earth connexions, a practical circuit diagram is given in Fig. D. Normally the input is earthed at the point shown in the diagram. Unless one is prepared to short-circuit $r_1$ and use $r_2$ as the gain control, the output circuit cannot be earthed : as, however, the load consists of a meter, highly insulated from the panel, this is unimportant. The negative pole of the H.T. supply must be insulated from the chassis. The H.T. power consumption is usually less than two watts, and the isolation of the compact mains unit from earth presents no difficulty. The use of a semi-floating H.T. supply is common practice in D.C. instrumentation, a recent example being provided by the D.C. valve voltmeter of Brewer (*J. Sci. Instrum., 30,* 91 (1953)). Circuit arrangements of this type are not, however, suitable for use in massive instruments of high power consumption. The slight additional cost (as compared with a single-ended amplifier) of the balanced output stage is fully justified by the improvement in zero stability which, in a *pH* meter or low range D.C. valve voltmeter, is of paramount importance. The potential divider $r_3$ is used for preliminary zero adjustment.

In reply to the point raised by Mr. Bennett it may be pointed out that the properties of acorn valves were carefully investigated by Hay (*Electronic Engng., 23,* 258 (1951)), who reached the same conclusion as Mr. Good, viz., that modern small output pentodes are more suitable for use as substitutes for electrometer valves. Perhaps the most valuable feature of Mr. Hay's article is his emphasis of the fact that the secret of success in improving the grid current characteristic of the acorn valve consists in reducing the anode current to a minute fraction of that recommended by the manufacturer. This led the writer recently to study the performance of other valves at anode currents in the neighbourhood of $1\mu A$. It was found that the grid current of R.F. pentodes, when operated under these conditions, is very greatly reduced; moreover, provided that the valve is used in the input stage of an efficient cathode-follower circuit, the reduction of the anode current almost to the vanishing point does not entail any loss in stage gain or appreciable reduction in the maximum input voltage which the amplifier is capable of accepting. This somewhat surprising discovery is accounted for by the fact that, if the anode current of the input valve is reduced to $1/n^{th}$ of its normal value, the anode load may be increased $n$-fold without change in the H.T. voltage; thus, despite the deterioration in valve characteristics due to the unfavourable operating conditions, the stage gain is maintained or even increased. Table 3 contains a comparison of the performance of the EF37A valve when operated (a) as a normal A.F. amplifier, (b) under the conditions recommended by the manufacturers for use as an electrometer valve substi-

TABLE 3.

|  |  |  |  | (a) | (b) | (c) |
|---|---|---|---|---|---|---|
| Anode current | .. | .. | .. | ($\mu$A.) | 3,000 | 80 | 1·0 |
| Screen current | .. | .. | .. | ($\mu$A.) | 800 | 20 | 0·25 |
| Anode and screen voltages | .. | — | 100 | 45 | 20 |
| Grid bias .. | .. | .. | .. | — | −2·0 | −2·0 | −1·7 |
| Heater voltage | .. | .. | .. | — | 6.3 | 4·5 | 4·0 |
| H.T. voltage | .. | .. | .. | — | 400 | 125 | 120 |
| Anode load | .. | .. | .. | (M$\Omega$) | 0·1 | 1·0 | 100 |
| Internal anode impedance | .. | (M$\Omega$) | 1·0 | 5 | 200 |
| Mutual conductance | .. | .. | ($\mu$A.V.) | 1,800 | 240 | 9 |
| Amplification factor | .. | .. | — | 1,800 | 1,200 | 1,800 |
| Grid current | .. | .. | .. | ($\mu\mu$A.) | 1,000 app. | 6 | < 0·1 |
| Stage gain .. | .. | .. | .. | — | 160 | 200 | 600 |

tute, (c) at an anode current of 1$\mu$A. It will be seen that by reducing the anode current from 3mA to 1$\mu$A and the H.T. voltage from 400 to 120, the anode load being simultaneously raised from 0·1 to 100M$\Omega$, the grid current may be reduced in the ratio 1 : 10 000 with a fourfold increase in stage gain. If EF37A valves operating under these conditions are used in the circuit of Fig. 7 in conjunction with ECC33 double triode valve, a negative input voltage of 50 is accepted by the amplifier and reproduced in the output circuit when the controls are in the minimum or "cathode-follower" position.

Turning to Mr. Scroggie's comments on the terminology used in my article, a 10$\Omega$ Unipivot galvanometer is frequently used as the load in the circuit of Fig. 5, and the output voltage is then approximately 1 per cent of the applied E.M.F. Admittedly, the degree of negative feedback is, strictly speaking, 100 per cent, but its effect on the performance of the amplifier under these conditions is negligible. Mr. Scroggie describes the type of feedback employed in his amplifier as "heavy"; using this terminology, the feedback applied to the input circuit of Fig. 5 may be defined as "heavy" when the load $r_0$ is of high ohmic resistance, becoming "light" as $r_0$ approaches zero. It is doubtful whether the description would become more precise if these indefinite terms were substituted for those used in the article. The characteristics of the amplifier are fully defined, in terms from ambiguity, by equations (1) to (6).

Using the circuits of Figs. 4 and 5, amplifiers having an internal resistance $\gamma$ of less than 2$\Omega$ were produced several years before the appearance of Mr. Scroggie's article. As emphasized in equations (20) and (23), all that is required for the purpose is an increase in $\mu$. These earlier amplifiers resembled Mr. Scroggie's in that the grid current amounted to about 10⁻¹⁰A, and were useless for $pH$ measurements. The internal resistance of the amplifier referred to above is about 1·5$\Omega$, and may be reduced to zero or rendered negative by adjustment of $r_1$.

As correspondence published in ELEC-TRONIC ENGINEERING and elsewhere has shown, opinion in regard to the desirability of the standardization of symbols is divided. My own view is that an author should be at liberty to use any symbols he thinks fit, provided that these are clearly defined, but I have no desire to revive or to take part in this controversy.

Finally, the opportunity is taken of correcting printers' errors in the following equations:—

$$\beta = (R_a/R + 1)/\mu_{21} \qquad (19)$$

$$\beta = \left\{ 1 + 1/\mu' (R_a/R + 1) \right\}/m \quad (22)$$

Yours faithfully,

C. MORTON,

Chelsea Polytechnic.

## A New Method of Assembling Magnetic Recording Heads

DEAR SIR,—The assembly of conventional magnetic recording or reproducing heads of toroidal form is complicated by the need for a thin gap-spacer of non-magnetic material separating the pole tips of the two half-toroids. Gap-spacers of the order of 0·5mil are common, and as the cross-sectional area of the gap may be only ⅛in. × 1/32in. or less, the placing in, fastening and clamping of the spacer is difficult. The difficulty is increased when narrow track recording is employed, and the gap height may be considerably less than ⅛in. An alternative technique for producing the necessary gap, and which at the same time obviates the need for clamping plates, is to stick the half-toroids together at the pole tips using a suitable adhesive such as one of the synthetic resins now available. The residual interlayer of adhesive then acts as a gap-spacer of predetermined dimensions. For manufacture the two half-toroids are placed in a clamping jig with adhesive between the back and front gaps. By suitable design of the clamping jig it is possible to control the clamping pressure in the sticking operation and the relationship between the resulting gap-thickness and pressure can be determined by experiment.

This technique is particularly suited to the construction of heads of solid mag-netic material, such as one of the ferrites or " dust-core " material, rather than of laminated material.

Yours faithfully,

I. W. MERRY,

Research Department,

BBC Engineering Division.

## A Visual Method for the Determination of Electrolytic Conductivity

DEAR SIR,—I have read with great interest the article by J. K. Grierson in your March issue.

While the circuit described is of considerable interest, it does appear to me to be unduly complicated and expensive for this application, and I am wondering whether Mr. Grierson has tried the simple but ingenious circuit described by H. K. Moneypenney for the measurement of electrolytic conductivities? (*J. Sci. Instruments., 26*, 120 (1949)).

In Moneypenney's circuit, the bridge network forms part of the feedback path of a two-valve oscillator, and changes in the cell-capacitance are automatically corrected for by changes in the frequency of oscillation. Balance can thus be obtained by adjustment of the resistance only.

Yours faithfully,

PETER S. BENNETT,

Lancaster.

## The Author replies:

DEAR SIR,—In answer to Mr. Bennett the following are my comments.

The Moneypenney circuit was very interesting, but, when constructed, was found to suffer from the following three major difficulties.

(1) Non-linearity in iron cored transformers caused a seriously distorted oscillator waveform, which, in the presence of leakage capacitances in the practical bridge, reduced accuracy.

(2) Drift and spurious indications of the balance position.

(3) It is inherent in the circuit design that the fundamental frequency of oscillation varies. As an electrolytic conductivity is, in general, a function of the frequency of the oscillation employed, this leads to a further inaccuracy.

The figure of 0·1 per cent accuracy for the visual method is a conservative one, and changes of conductivity of the order of 0·01 per cent have been easily detected. In practice the apparatus is not complex or expensive to build. It is easy to operate and has been used for accurate routine experiments for a considerable time.

Yours faithfully,

J. K. GRIERSON,

Dunlop Rubber Co., Ltd.,

Birmingham.