THE MARCONI REVIEW

Volume XXII *Number* 134 *Third Quarter* 1959

The Marconi Review is published four times a year

Annual subscription £1. Single copies 5s. Postage paid to any part of the world

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Recent Filter Developments

…ctronic devices often include electrical filters to separate oscillations …one frequency from those of another frequency. Such selection is …ieved in the simplest cases by using a single resonant circuit, to accept …reject a single frequency or a narrow band of frequencies; Campbell and …bel, in the theory of wave filters, showed how a network of electrical …ctances could be used to produce filters for the effective selection of a …nd of frequencies; the design techniques have been further improved by …rlington and others.

These complex filters have found their greatest application in electrical …nmunications; indeed the economic success of many kinds of radio and …e communication depends upon ability to realize the necessary …formance in the filter networks. Early filter theory assumed that the …ctances in the electrical network possessed no resistance; the presence … resistance in all practical components means that the calculated …ponse curve and low insertion loss cannot always be achieved.

Coils with dust-iron or ferrite cores and plastic film condensers have led to …proved performance of electrical filters and decreased size. It is, …wever, not always possible to achieve the desired response curve at the …osen frequency with coil and condenser filters even with the best …ssible components.

Mechanical resonators sometimes display properties which would …d application in a filter structure; the tuning fork is an example of a …sonant system of low decrement and high frequency stability. In …artz crystal resonators, the mechanical oscillations are utilized through …e piezo-electric effect to produce a two-terminal electrical network, the equivalent of which cannot be approached with practical coils an condensers. W. P. Mason studied the properties of quartz crystals a circuit elements and showed how they could be used in the range 50 t 500 kc/s in band-pass filter networks having extremely steep edges to th response characteristic. This work is now being extended to the mega cycle frequency band, where small filters using crystals are being applie in mobile communication equipments.

The quartz crystal is essentially a two-terminal device which behav like a series tuned circuit of very high Q, with some parallel capacitanc The filter is made of several units of this kind, connected together to giv the desired response. Each crystal represents not only the mechanica resonator, but also a " built in " piezo-electric transducer.

Electro-mechanical filters described in this issue work in a differen way from the crystal filter; in these filters the electrical oscillations a transformed to mechanical oscillation in a magneto-striction transduce The mechanical energy is then filtered in an assembly of mechanic resonators and coupling elements, which produce a band-pass response the mechanical structure. After filtering the mechanical oscillations a transformed back into electrical energy in a second transducer syster

The electro-mechanical filter can fulfil many, but not all, of the functio of present designs of crystal and coil/condenser band-pass filters in th frequency range 100 to 500 kc/s. Its great merit is its compactness ar low temperature coefficient, coupled with a reasonable price if made sufficient numbers to justify expensive machinery. It should have usef applications where space and weight are valuable, and in which th extremely high electrical performance of the best crystal filters is n justified.

G. L. GRISDAL

A THEORETICAL ANALYSIS OF THE TORSIONAL ELECTRO-MECHANICAL FILTERS*

By W. STRUSZYNSKI, Dipl. Ing. (Warsaw)

what follows, mainly torsional vibrations of cylindrical rods are con- lered, since such resonators have been adopted for electro-mechanical filter sign. Other modes of vibration are analysed briefly only in order to identify em in some spurious responses.

There is a close equivalence between mechanical vibrations and electrical cillations. In fact the differential wave equations for the torsional vibrations one dimension are identical with those of electrical transmission line stems.

The theory of electrical networks is now a highly developed science on which e based modern methods of filter design. It is, therefore, the object of this ticle to translate all the mechanical properties of the system into their ctrical equivalents so that an electrical filter network can be designed and elements converted into a corresponding mechanical equivalent.

OMPARISON OF TORSIONAL VIBRATION OF A ROD WITH LECTRICAL OSCILLATIONS IN A TRANSMISSION LINE

he analysis is based on the theory of torsional vibration[1, 2, 3] in the incipal mode of an infinite rod. For rods of a finite length some approxi- ation is involved. For practical purposes, however, only small correction ctors have to be introduced for accurate determination of the resonant ngths.

Direct equivalence is adopted since it is more consistent, although in me applications inverse equivalence[1] may be more convenient. In g. 1 is shown a small element of a rod and in Fig. 2 a similar element of

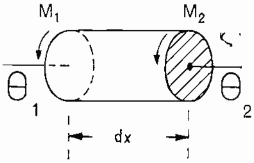

Fig. 1. Element of rod

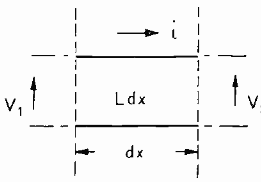

Fig. 2. Element of line

*First printed by Marconi's Wireless Telegraph Co. Ltd. as an internal Technical Report 7th November 1957.

a twin transmission line. A concentric line could be used equally well fo the purpose.

The differential equations for the two cases are of identical form:

A. MECHANICAL

$$\frac{\partial^2 \dot{\theta}}{\partial x^2} = JK \frac{\partial^2 \dot{\theta}}{\partial t^2} \quad \ldots\ldots\ldots\ldots (1a)$$

$$\frac{\partial^2 M}{\partial x^2} = JK \frac{\partial^2 M}{\partial t^2} \quad \ldots\ldots\ldots\ldots (2a)$$

Where:

$\dot{\theta}$	$= \frac{\partial \theta}{\partial t} =$	angular velocity of the section dx at x
θ	$=$	angle of twist
M	$=$	moment
J	$=$	polar moment of inertia per unit length
K	$=$	moment of compliance per unit length

B. ELECTRICAL

$$\frac{\partial^2 \dot{q}}{\partial x^2} = LC \frac{\partial^2 \dot{q}}{\partial t^2} \quad \ldots\ldots\ldots\ldots (1b$$

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \quad \ldots\ldots\ldots\ldots (2b$$

Where:

$\dot{q}$	$= \frac{\partial q}{\partial t} = i =$	current in th section at x
q	$=$	charge
v	$=$	voltage
L	$=$	inductance pe unit length
C	$=$	capacitance pe unit length

$x =$ distance $\quad t =$ time

These are the wave equations, and the corresponding velocities c propagation are:

$$u_m = \frac{1}{\sqrt{JK}} \quad \ldots\ldots\ldots\ldots (3a) \qquad u_e = \frac{1}{\sqrt{LC}} \quad \ldots\ldots\ldots\ldots (3b$$

and the characteristic impedances are:

$$N_0 = \sqrt{\frac{J}{K}} \quad \ldots\ldots\ldots\ldots (4a) \qquad Z_0 = \sqrt{\frac{L}{C}} \quad \ldots\ldots\ldots\ldots (4b$$

The solution of these wave equations is well known from transmissio line theory. In establishing equivalence, the response of the system t excitation of zero frequency is of some interest, and leads to the followin relations:

A. MECHANICAL

$$\theta - \theta_0 = M_0 Kx \quad \ldots\ldots\ldots\ldots (5a)$$

This expresses the angular twist of the rod of length x produced by a constant moment M_0. Kx is the total moment of compliance.

B. ELECTRICAL

$$q - q_0 = v_0 Cx \quad \ldots\ldots\ldots\ldots (5b$$

This expresses the d.c. charge o the line of length x produced b the voltage v_0. Cx is the tot capacity of the line.

(For the definitions of J and K see Appendix 1)

DIMENSIONS

These considerations show a complete equivalence of mechanical and electrical quantities. It is, however, not possible directly to use electrical quantities instead of mechanical because of the difference in dimensions. This can be seen clearly from comparison of equations (5a) and (5b) where the angle of twist $[\theta - \theta_0]$ is dimensionless whilst the equivalent quantity, the electrical charge, has the dimension of charge:

$$[q - q_0] = Q$$

This difficulty can be overcome by introducing a factor δ such that:

$$[\delta] = Q$$

Hence:

$$(\theta - \theta_0)\,\delta = q - q_0 \qquad (6)$$

This factor will be referred to as a transducer transfer ratio since the transducer is an element which converts electrical energy into mechanical or vice-versa, and its properties determine the angle of twist in relation to the charge.

EQUIVALENCE OF MECHANICAL AND ELECTRICAL QUANTITIES

Using a transducer transfer ratio δ the relationship between the mechanical and the electrical quantities can be obtained. The relationship for θ and q is given by eqn. (6). The same equation determines the relation between $\dot{\theta}$ and i.

It is assumed that the equivalent electric line has the same velocity as the velocity of propagation in the rod.

$$u_m = u_e \qquad (7)$$

It follows that the length of the line l_e is the same as that of the rod l_m.

$$l_m = l_e \qquad (8)$$

This condition could be satisfied if the electric line were filled with a medium which has a very high dielectric constant (the permeability of the medium can be assumed to be equal to unity). The required magnitude of the dielectric constant may be not realizable in practice, but this does not invalidate the concept of equivalence.

From these premises a consistent set of relations can be established which are shown in Table I. The last relation in the table gives another definition of the transducer transfer ratio

$$\delta = \sqrt{\frac{N}{Z}} \qquad (9)$$

Where:

N = mechanical impedance on one side of the transducer

Z = electrical impedance on the other side of the transducer.

In the practical computation of the mechanical part of the filter, i.e., of the chain of resonators and couplers, the magnitude of the transducer transfer ratio δ can be arbitrarily chosen provided the transducer can be matched to the mechanical load presented by the filter. The relevant formulae for the calculation of the parameters of the mechanical system are recorded in Appendix 2.

TABLE I

EQUIVALENCE OF MECHANICAL AND ELECTRICAL QUANTITIES

(direct relation)

δ = transducer transfer ratio (dimension $[\delta] = Q$)

Mechanical	Electrical	Relation	Dimension
1. Angle of twist θ	Charge q	$\theta\delta = q$	$[q] = Q$
2. Angular velocity $\dot{\theta}$	Current $i = \dot{q}$	$\dot{\theta}\delta = i$	$[i] = T^{-1}Q$
3. Velocity of propagation u_m	Velocity of propagation u_e	$u_m = u_e$	$[u_e] = LT^{-1}$
4. Length of the bar l_m	Length of the line l_e	$l_m = l_e$	$[l_e] = L$
5. Moment M	Voltage v	$M\frac{1}{\delta} = v$	$[v] = ML^2T^{-2}Q^{-1}$
6. Moment of compliance per unit length K	Capacitance per unit length C	$K\delta^2 = C$	$[C] = M^{-1}L^{-3}T^2Q^2$
7. Polar moment of inertia per unit length J	Inductance per unit length L	$J\frac{1}{\delta^2} = L$	$[L] = MLQ^{-2}$
8. Mechanical characteristic impedance N_0	Electrical characteristic impedance Z_0	$N_0\frac{1}{\delta^2} = Z_0$	$[Z_0] = ML^2T^{-1}Q^{-2}$

EQUIVALENT CIRCUIT FOR THE TORSIONAL FILTER

The torsional filter is composed of a number of half wave resonators, which will be referred to as slugs. These are linked by quarter wave couplers, which will be referred to as necks. A part of such a filter rod is shown in Fig. 3a, and the equivalent concentric transmission line is shown in Fig. 3b.

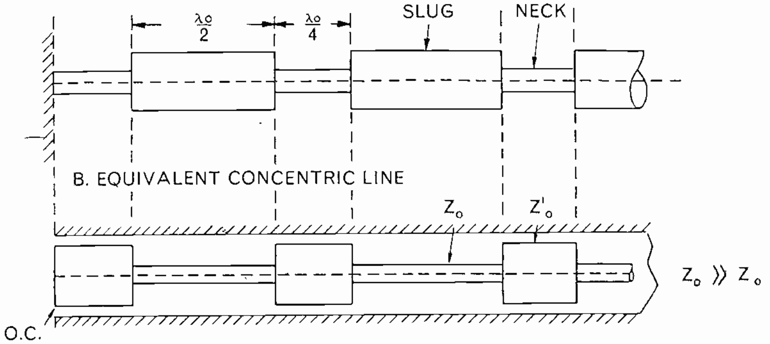

Fig. 3. Torsional mechanical filter

The solid clamping at the two ends of the rod corresponds to an open ircuit in a line. This is slightly confusing, but if it is borne in mind that he angular velocity $\dot{\theta} = 0$ at the clamps, which corresponds to $i = 0$ n the electrical equivalent, it becomes clear that the line is open circuit t those points.

The characteristic impedance of the slugs is much higher than that of he necks. Their ratio is in fact proportional to the fourth power of the atio of their radii.

The effect of discontinuities at the junction of different sections of the ne is neglected and a sudden transition from one value of the charac-eristic impedance to the other is assumed.

In Fig. 4 is shown a transducer system composed of two magnetostric-ive ferrite rods biased with a permanent magnet (not shown in the

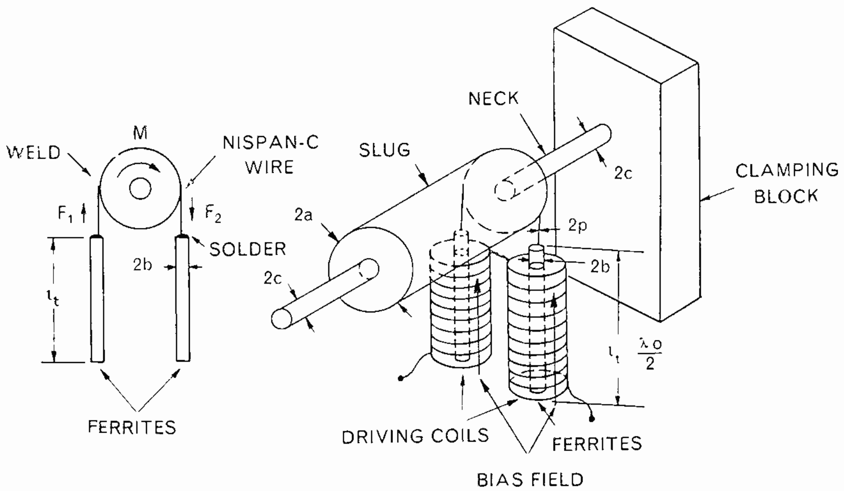

Fig. 4. Transducer system

diagram) and driven by two coils whose fields are in antiphase to produc the push-pull action required for the torsional excitation.

For the electrical equivalent it is necessary to decide whether th equivalent e.m.f. is acting in series or in parallel with the line. For th purpose a rod composed of two sections and its electrical equivalent ar considered in Fig. 5. A constant moment is applied, i.e. d.c. voltage i the electrical equivalent. It can be seen that only a series connection o e.m.f. gives the voltage and charge distribution consistent with that o the angle of twist and the moment. It helps in reasoning to remember tha at the section where the moment is applied, the angle of twist is commo to the two sections of the rod. This corresponds to a common current i the electric case which is true only when the source is connected in series

The transducers should be connected to the outer ends of the extrem slugs (see Fig. 6). Then the quarter wave open line in series with the sourc behaves as a short circuit at the resonance frequency and has only a smal reactance in the pass band by virtue of a low characteristic impedance o the line. Similarly at the receiving end of the line, the load resistance i connected in series with the quarter wave open circuit line.

The operation of the filter can be explained in simple terms in th following way. If the system on the receiving end is loaded with resistanc equal or almost equal to the characteristic impedance of the necks then for the resonant frequency of the slugs, the impedance presented at th transmitting end is equal to the load resistance.

In the slugs high torsional moments are generated because of a con- siderable difference between their characteristic impedances and those of the necks. No transformation of impedances occurs, however, at the resonant frequency of the slugs, since they behave as half wave lines. Thus the system as a whole remains matched.

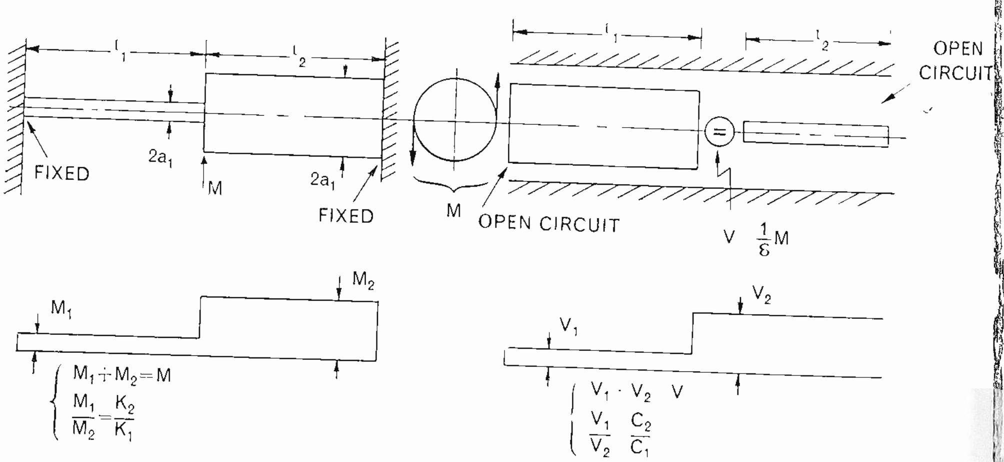

Fig. 5. Location of equivalent e.m.f. in the line

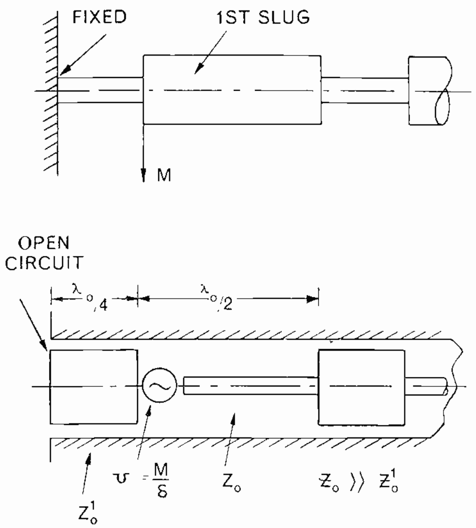

Fig. 6. Positioning of transducer

A slight deviation from the resonant frequency results in reflections in ie systems and thus produces the desired filter attenuation. This crude cture helps in understanding the mechanism of operation of the filter.

It is more convenient to present the equivalent circuit of the slug as a rcuit with lumped constants, viz. as a Π section as in Fig. 7. The shunt ti-resonant circuits of a high L to C ratio, can be omitted, since it has finite impedance at the resonance frequency, and off resonance it is eavily shunted by the low capacitive impedance of the necks.

Thus the slug is represented by a series L_1, C_1 circuit and the neck as a ur terminal network which has the inverter[4] properties of a quarter ave line. The equivalent circuit is, therefore, as shown in Fig. 8, where $_{12}$, Z_{23}, ... are the characteristic impedances of the quarter wave verters.

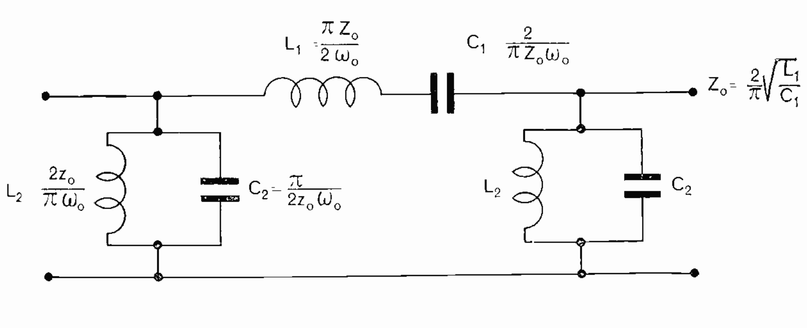

Fig. 7. Lumped equivalent of a slug

diagram) and driven by two coils whose fields are in antiphase to produc the push-pull action required for the torsional excitation.

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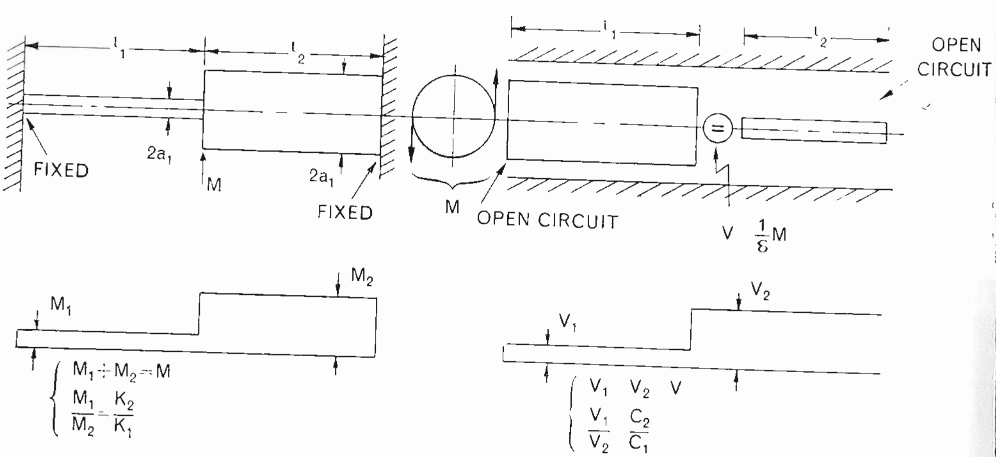

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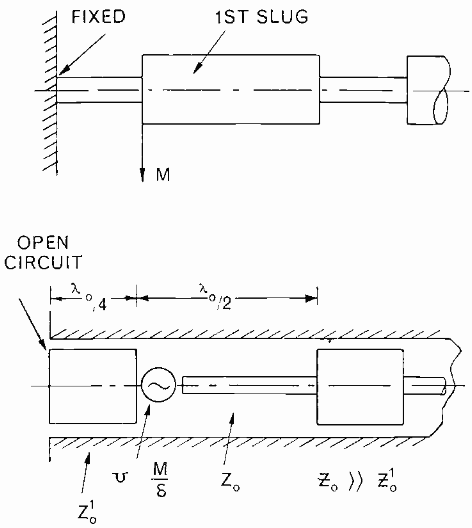

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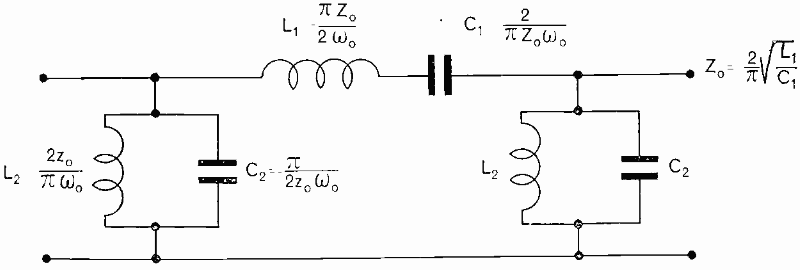

Fig. 7. Lumped equivalent of a slug

DESIGN OF MECHANICAL FILTER FOR THE REQUIRE RIPPLE IN THE PASS-BAND

TRANSFORMATION INTO INVERSE-ARM FILTER

The circuit of Fig. 8 can be transformed into a form commonly used in th filter design. In Fig. 9a is shown one section of the filters. Its series L_1, C circuit is transformed by the quarter wave inverter into an equivalent parallel circuit L_1', C_1' as shown in Fig. 9b. The operation is repeated several times, each time one series circuit with its appropriate four termina inverter being added. In Fig. 10 is shown the second stage of the trans formation. The parallel circuit again becomes a series one and the added

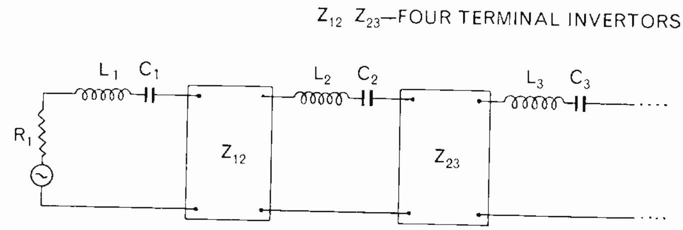

Fig. 8. Equivalent circuit of a filter

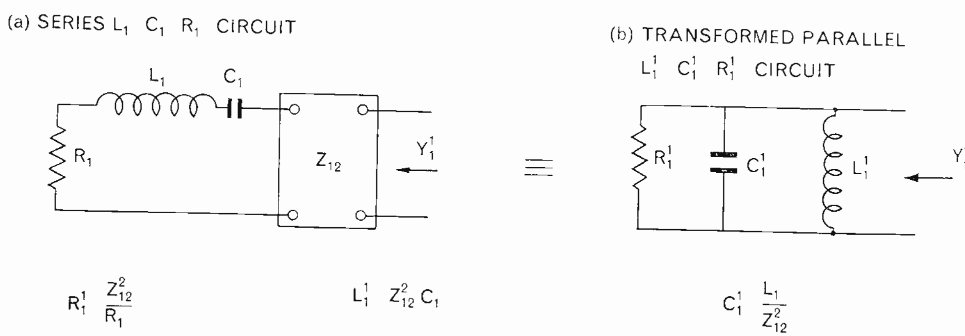

Fig. 9. Quarter wave inverter

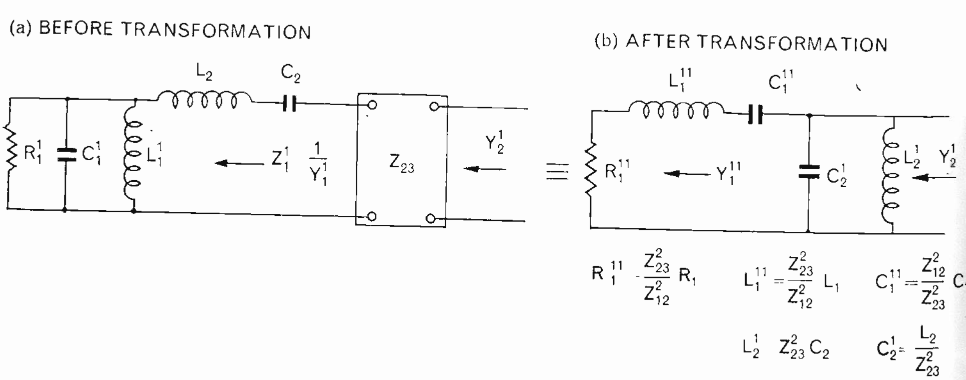

Fig. 10. Second stage of transformation

series circuit is converted into a parallel one. The transformation is carried out until the whole filter is converted into a chain of series and parallel L, C circuits, which is often referred to as an inverse-arm form. Since the most common design for mechanical filters is one composed of nine mechanical resonators, this particular case will be considered.

The system is completely symmetrical about the central slug, which is the fifth in succession. Thus the transformation has to be carried out as far as the fifth resonator and the remaining part of the filter is the mirror image of the first. This is very important because the two transducers are then identical. This, of course, is true for any odd number of resonators. It should be noted that in the transformation the resistance of the source and the load are also transformed to the value

$$\frac{Z^2_{23}\, Z^2_{45}}{Z^2_{12}\, Z^2_{34}}\, R_1$$

In the last stage of transformation it is possible to return to the original element values in the first circuit R_1, L_1, C_1 by dividing all the values of the resistances and the inductances and by multiplying the values of all the capacities by the above coefficient.

The final circuit is shown in Fig. 11 with all the transformation factors τ_1, τ_2, ... expressed in terms of the characteristic impedances Z_{12}, Z_{23}, ... of the quarter wave coupling lines (necks). The parameters for the band pass filter in the inverse-arm form can be derived directly from the low pass prototype(5).

It is more convenient from the mechanical point of view to maintain the diameter of all the slugs the same and since their lengths must be also the same:

$$L_1 = L_2 = \ldots\ldots = L \tag{10}$$
$$C_1 = C_2 = \ldots\ldots = C \tag{11}$$

It is still possible to obtain the required characteristics of the filter by a

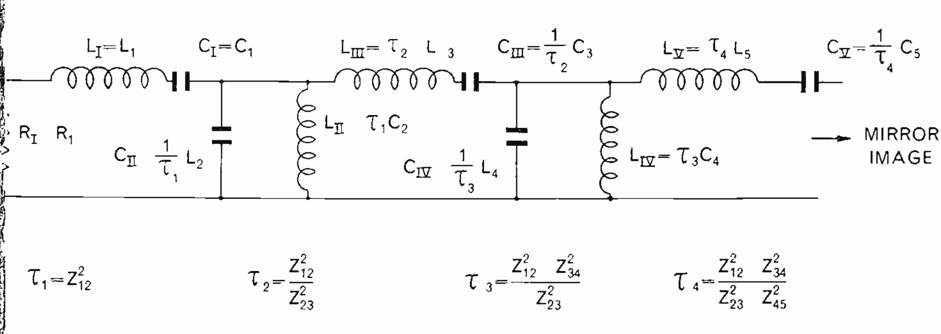

Fig. 11. Final equivalent circuit

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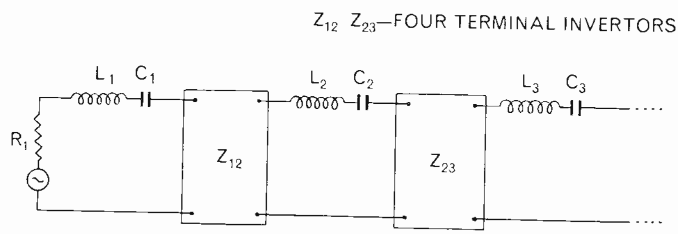


Fig. 8. Equivalent circuit of a filter

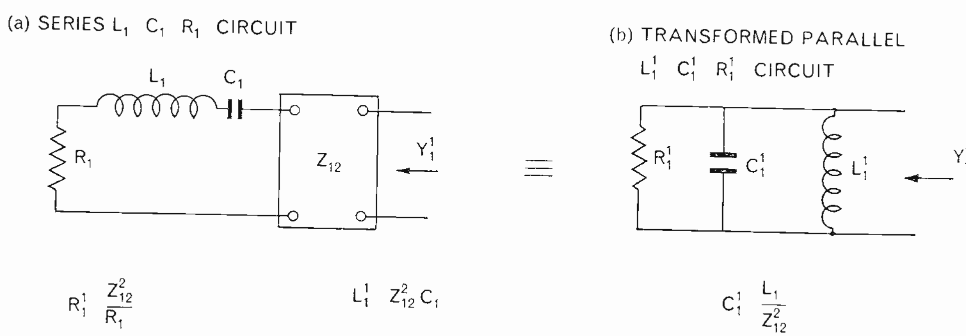
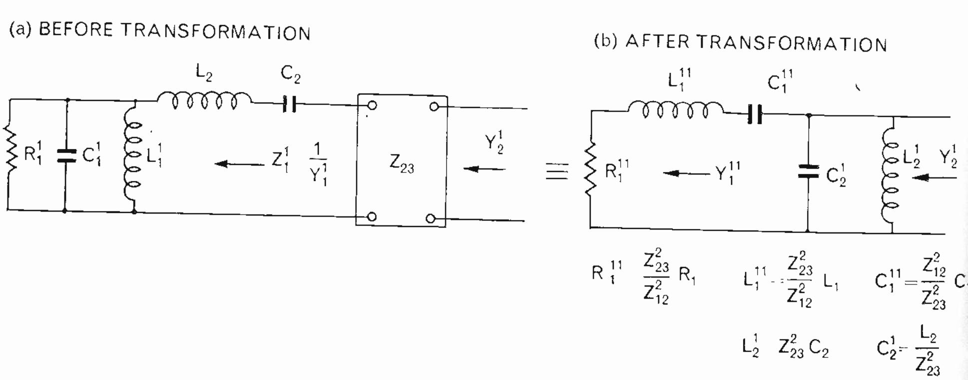


Fig. 9. Quarter wave inverter



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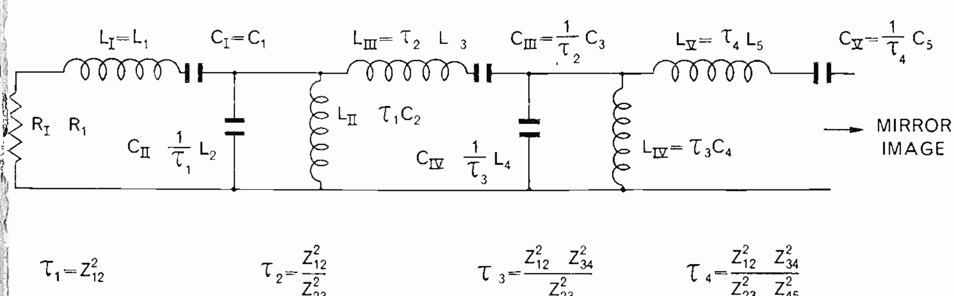

Fig. 11. Final equivalent circuit

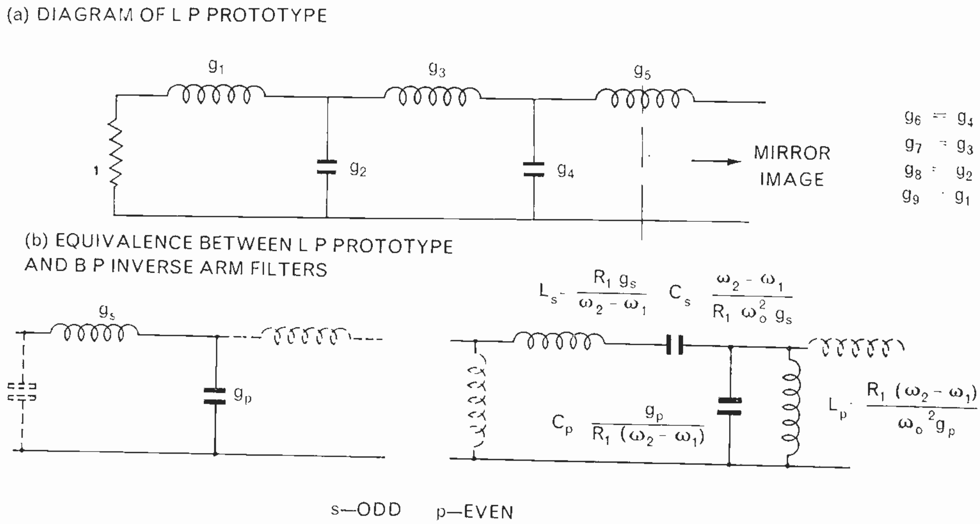

Fig. 12. LP prototype filter

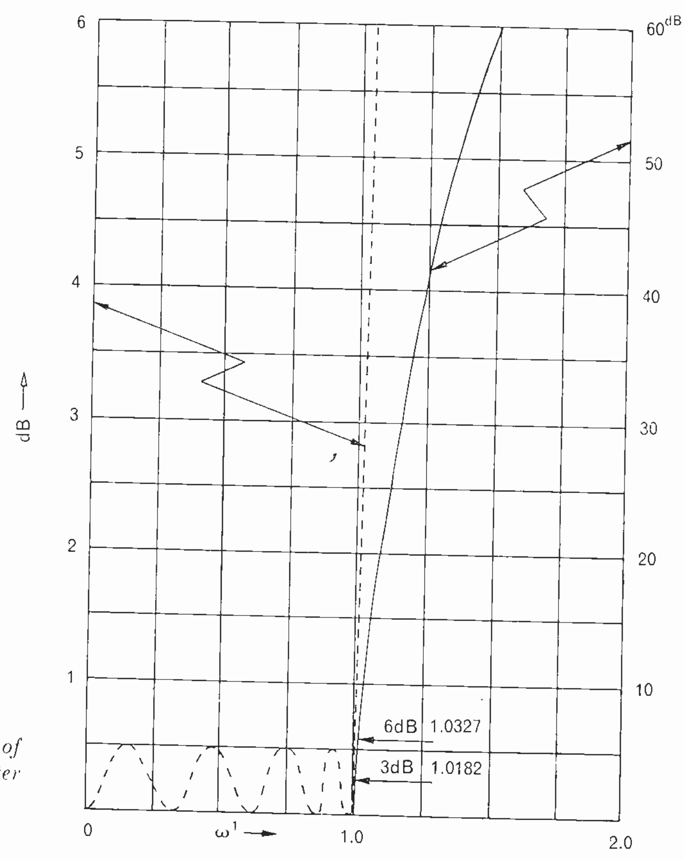

Fig. 13. Attenuation curve of a nine element prototype filter (0.5dB ripple)

suitable choice of factors τ_1, τ_2, ... which depend on the characteristic impedance of the couplers, i.e. the neck diameters.

LOW PASS PROTOTYPE

The low pass prototype is shown in Fig. 12a for nine resonators. The formulae for the calculation of its elements are known([5]). The cut-off frequency of the prototype filter is such that; $\omega'_1 = 1$. The load and source resistance is $R_1 = 1$ ohm.

The values of the filter elements, i.e., the inductances (g_1, g_3, g_5.) and the capacitances (g_2, g_4) depend on the accepted magnitude of the ripple in the pass band. For the Tchebycheff equal ripple response, numerical values for 0·5 dB were computed, these are:

$g_1 = 1{\cdot}74$ H $\qquad g_2 = 1{\cdot}27$ F

$g_3 = 2{\cdot}65$ H $\qquad g_4 = 1{\cdot}37$ F

$g_5 = 2{\cdot}71$ H

The corresponding attenuation curve is shown in Fig. 13.

CALCULATION OF THE CIRCUIT ELEMENTS

The value of the circuit elements in Fig. 11 can now be expressed in terms of the prototype elements:

$$L_{I} = L_1 = \frac{R_1 g_1}{\omega_2 - \omega_1} \quad \ldots\ldots(12) \qquad C_{I} = C_1 = \frac{\omega_2 - \omega_1}{R_1 \omega_0^2 g_1} \quad \ldots\ldots(13)$$

$$L_{II} = \tau_1 C_2 = \frac{R_1 (\omega_2 - \omega_1)}{\omega_0^2 g_2} \quad \ldots(14) \qquad C_{II} = \frac{1}{\tau_1} L_2 = \frac{g_2}{R_1 (\omega_2 - \omega_1)} \quad \ldots(15)$$

$$L_{III} = \tau_2 L_3 = \frac{R_1 g_3}{\omega_2 - \omega_1} \quad \ldots\ldots(16) \qquad C_{III} = \frac{1}{\tau_2} C_3 = \frac{\omega_2 - \omega_1}{R_1 \omega_0^2 g_3} \quad \ldots\ldots(17)$$

etc.

(The expression for the remaining elements can be readily written since the law of repetition can be easily traced.)

Where:

L_I, L_{II}. .... = inductances in the inverse arm equivalent circuit of Fig. 11.

C_I, C_{II}, ... = capacitance in the inverse arm circuit of Fig. 11.

τ_1, τ_2, ... = transformation factors.

R_1 = load and source resistance.

L_1, L_2, ... = inductances of the slugs 1, 2.........., which are assumed to be identical eqn. (10).

C_1, C_2, ... = capacitances of the slugs 1, 2.........., which are assumed to be identical eqn. (11)

ω_2, ω_1 = top and bottom cut-off angular frequencies.

$\omega_0 = \sqrt{\omega_1 \omega_2}$ = resonance angular frequency of the slugs.
$g_1, g_2 \ldots\ldots\ldots$ = values of the elements in the low pass prototype filter.

There is only the apparent inconsistency in the dimensions in eqns. (12) to (17). This results from the fact that R_1 enters as a ratio $R_1/1$ because of the re-normalisation of the load from 1 ohm to R_1 ohm, and the factors g_1, g_2, etc. enter as $\omega_1' g_1$, $\omega_1' g_2$, etc. Here $\omega_1' = 1$ is the cut-off frequency in the prototype.

Taking into account eqns. (10) and (11), the transformation factors: $\tau_1, \tau_2, \ldots\ldots$ can be determined from eqns. (12) to (17):

$$\tau_1 = R_1^2 \frac{g_1}{g_2} = Z_{12}^2 \tag{18}$$

$$\tau_2 = \frac{g_3}{g_1} = \frac{Z_{12}^2}{Z_{23}^2} \tag{19}$$

$$\tau_3 = R_1^2 \frac{g_1}{g_4} = \frac{Z_{12}^2 Z_{34}^2}{Z_{23}^2} \tag{20}$$

$$\tau_4 = \frac{g_5}{g_1} = \frac{Z_{12}^2 Z_{34}^2}{Z_{23}^2 Z_{45}^2} \tag{21}$$

Where:

$Z_{12}, Z_{23}, Z_{34} \ldots\ldots\ldots$ = characteristic impedances of the necks between the slugs 1 and 2, 2 and 3 etc.

It can be seen from eqn. (18) that

$$Z_{12} = g_1 \frac{1}{\sqrt{g_1 g_2}} R_1 \tag{22}$$

Where the coefficient of R_1 differs very little from unity (for 0·5 dB ripple it is equal to 1·17), so that to a first approximation $Z_{12} \doteq R_1$. The remaining characteristic impedances of the necks are expressed as:

$$Z_{23} = g_1 \frac{1}{\sqrt{g_2 g_3}} R_1 \tag{23}$$

$$Z_{34} = g_1 \frac{1}{\sqrt{g_3 g_4}} R_1 \tag{24}$$

etc.

The characteristic impedance of the slug:

$$Z_0 = \frac{2}{\pi} \sqrt{\frac{L}{C}} = \frac{2\, \omega_0\, R_1\, g_1}{\pi\, (\omega_2 - \omega_1)} \tag{25}$$

METHOD OF DESIGN OF MECHANICAL FILTER

All the required electrical quantities are given on page 130. The translation of them into the dimensions of the mechanical elements can be done if the transducer transfer ratio δ is known. In practice, however, usually the starting point for the design is the diameter of the slug, and since the properties of the material also are known, the mechanical characteristic impedance of the slug N_0 can be calculated.

This is given in Appendix 1 as:

$$N_0 = \frac{\pi a^4}{2} \cdot \sqrt{\mu\rho} \qquad (26)$$

Where:

a = radius of the slug
μ = modulus of rigidity
ρ = density

The magnitude of the transducer transfer ratio δ is then immaterial.

In order to avoid confusion with units when computing numerical values use of MKS system is advisable and the relevant quantities are given in Appendix 3.

The mechanical characteristic impedances of the necks can be expressed directly in terms of the slug impedance. From equations (9) and (22-25):

$$N_{12} = \frac{\pi}{2} \frac{\omega_2 - \omega_1}{\omega_0} \frac{1}{\sqrt{g_1 g_2}} N_0 \qquad (27)$$

$$N_{23} = \frac{\pi}{2} \frac{\omega_2 - \omega_1}{\omega_0} \frac{1}{\sqrt{g_2 g_3}} N_0 \qquad (28)$$

etc.

Where

N_0 = mechanical characteristic impedance of the slug
N_{12}, N_{23}, ... = mechanical characteristic impedances of the necks between the slugs 1 and 2, 2 and 3, etc.

It is still more convenient to express the neck radii directly in terms of the slug radii, using equations (27) and (28):

$$c_{12} = \left[\frac{\pi}{2} \frac{\omega_2 - \omega_1}{\omega_0} \frac{1}{\sqrt{g_1 g_2}} \right]^{\frac{1}{4}} a \qquad (29)$$

$$c_{23} = \left[\frac{\pi}{2} \frac{\omega_2 - \omega_1}{\omega_0} \frac{1}{\sqrt{g_2 g_3}} \right]^{\frac{1}{4}} a \qquad (30)$$

etc.

Where

c_{12}, c_{23}, ... = radii of the necks between the slugs 1 and 2, 2 and 3, etc.

TOLERANCES

In order to realize the required characteristic in the pass band, extremely high accuracy in machining the rod would be required. This is because the characteristic impedance of the mechanical elements, i.e. slug or neck, depends on the fourth power of the diameter.

A numerical example of an actual filter would show this point more clearly.

Mean frequency of the filter =	f_0	=	251·8 kc/s
Bandwidth: $f_2 - f_1$		=	3 kc/s
Ripple should not exceed:	0·5 dB		0·1 dB
Diameter of the slug $2a$	= 0·2500 inch		0·2500 inch
Length of the slug l	= 0·2210 inch		0·2210 inch
Diameter of the necks $2c_{12}$	= 0·0835 inch		0·0854 inch
$2c_{23}$	= 0·0792 inch		0·0795 inch
$2c_{34}$	= 0·0785 inch		0·0781 inch
$2c_{45}$	= 0·0783 inch		0·0781 inch
Length of the necks l_1	= 0·1100 inch		0·1100 inch

It can be seen that the change in the diameter $2c_{12}$ by 0·0019 inch and in the remaining diameters by less than 0·0004 inch results in the ripples being 0·5 dB instead of 0·1 dB. It seems, therefore, that the accuracy required should be of the order of 0·0002 inch which is about $\frac{1}{4}\%$ of the diameter, i.e. about 1% of the characteristic impedance. To maintain such a tolerance in production is expensive. Besides the diameter, the accuracy of the length and radiusing of the neck edges are also critical. It is also of great importance to maintain homogeneity of the material.

LONGITUDINAL (COMPRESSIONAL) VIBRATIONS OF RODS

Considerations of longitudinal waves in this article are required only for the design of transducers of the type shown in Fig. 4, where rods of a small diameter are used. For this application the simplified method is sufficiently accurate and the results can be presented in clearer forms.

The velocity of propagation of longitudinal vibrations is different from that of torsional vibrations and is, for rods of small diameter, given by:

$$u_0 = \sqrt{\frac{E}{\rho}} \tag{31}$$

Where:

u_0 = velocity of propagation of the longitudinal vibrations
E = Young's modulus of elasticity
ρ = density

Numerically this velocity is much higher than that for the torsional vibration, in fact for Ni-Span " C " alloy.

$$u_0 = 1{\cdot}58\ u_2$$

Where:

u_2 = velocity of torsional vibration

The characteristic impedance for longitudinal vibration depends on the second power only of the radius of the rod:

$$(N_o)_l = \pi a^2 \sqrt{E\rho} \tag{32}$$

Thus the longitudinally vibrating system can, to a first order of approximation, be considered as non-dispersive and can also be presented in the form of an equivalent transmission line.

CONDITIONS OF MATCHING THE MECHANICAL IMPEDANCES OF TRANSDUCER RODS WITH SLUGS

Fig. 4 shows the schematic arrangement of a transducer system. The two magnetostrictive ferrite rods operate in the longitudinal mode as well as the two Ni-Span C wires connecting them with the slug.

There are two problems in matching. One is to arrange that the mechanical characteristic impedances of the ferrite rod and Ni-Span C wires are the same and equal to the resistance presented by the filter in the passband. The other is the matching of the electrical circuit to the magnetostrictive rods. Only the first problem will be considered at present.

The mechanical load resistance of the filter can be obtained from equation (22):

$$r = N_{12} \sqrt{\frac{g_2}{g_1}} \tag{33}$$

Where:

$r = \delta^2 R_1$ = mechanical load resistance of the filter

$N_{12} = \delta^2 Z_{12}$ = mechanical characteristic impedance of the neck between slugs 1 and 2

$g_2, g_1,$ = values of the first two elements of the prototype filter

The characteristic impedance of a rod is given in Appendix 1 so that:

$$N_{12} = \frac{\pi c_{12}{}^4}{2} \sqrt{\mu_{Ni}\, \rho_{Ni}} \tag{34}$$

Where:

c_{12} = radius of the slug

μ_{Ni} = modulus of rigidity for Ni-Span C

ρ_{Ni} = density for Ni-Span C

The angular velocity ($\dot{\theta}$) of the section of the neck for a given moment M:

$$\dot{\theta} = \frac{M}{r} \tag{35}$$

This moment is produced by two forces F_1 and F_2 acting in the Ni-Span C wires welded to the slug. In a properly designed transducer these forces must be equal: $F_1 = F_2 = F$ so that:

$$M = 2Fa \tag{36}$$

Where:

F = force produced by the longitudinal vibrations in the wire

a = radius of the slug = arm of the force F

The linear velocity of the wire $\dot{y}$ is:

$$\dot{y} = a\,\dot{\theta} \tag{37}$$

Thus the loading of the wire can be found from equations (33), (35). (36) and (37) as:

$$N_1 = \frac{F}{\dot{y}} = \frac{N_{12}}{2a^2}\sqrt{\frac{g_2}{g_1}} \tag{38}$$

The wire is matched correctly if its load is equal to the characteristic impedance of the wire for the longitudinal vibration $N_1 = (N_0)_{Ni}$.

$$(N_0)_{Ni} = \pi\, p^2 \sqrt{E_{Ni}\,\rho_{Ni}} \tag{39}$$

Where:

$(N_0)_{Ni}$ = characteristic impedance of the Ni-Span C wire for longitudinal vibrations.

p = radius of the wire

E_{Ni} = Young's modulus of elasticity for Ni-Span C

The radius of the wire p can be expressed in terms of the neck radius c_{12} from equations (34), (38) and (39) as:

$$p = \frac{c_{12}^2}{2a}\left[\frac{\mu_{Ni}}{E_{Ni}}\,\frac{g_2}{g_1}\right]^{\frac{1}{4}} \tag{40}$$

The matching of the Ni-Span C wire to the ferrite rod requires the equality of their characteristic impedances $(N_0)_{Ni} = (N_0)_{Fe}$ since from equation (32).

$$(N_0)_{Fe} = \pi\, b^2 \sqrt{E_{Fe}\,\rho_{Fe}} \tag{41}$$

Where:

b = radius of the ferrite rod

E_{Fe} = Young's modulus of elasticity for ferrite

ρ_{Fe} = density for ferrite

Thus the radius of the ferrite rod b can be expressed in terms of the radius p of the Ni-Span wire as:

$$b = p\left[\frac{E_{Ni}}{E_{Fe}}\,\frac{\rho_{Ni}}{\rho_{Fe}}\right]^{\frac{1}{4}} \tag{42}$$

SPURIOUS RESPONSES

Mechanical resonators can be excited to various modes of vibration, whose resonant frequencies differ from that of the desired mode, if suitable forces are applied. These forces can be produced in the transducers themselves or can arise in the chain of resonators by the transformation of forces from those of the desired mode. This transformation can occur due to irregularities in machining, structure, or to inhomogeneity of the material.

In the case of the torsional filter of the type shown in Fig. 3a the excitation of the spurious modes is mainly due to forces produced by the transducers.

The spurious excitation is transferred through the couplers and the following resonators, and reaches the filter output. The couplers and resonators, being of like dimensions, act as elements of the filter operating in a spurious mode. The elements may be slightly out of tune, due to individual trimming for the torsional mode, which may not necessarily improve the tuning of the spurious mode. This may produce a number of sharp responses which are very close to each other.

Only the spurious modes, for which the resonant frequencies are close to the torsional resonance, are of practical importance. For those modes whose resonant frequency differs appreciably from the torsional one, sufficient attenuation is obtained from the selectivity of the electrical circuits and from the input and output transducers.

The lack of balance of the two elements of the transducer (see Fig. 4) results in unequal forces (i.e. F_1 F_2.) and thus a transverse force is applied at the centre of the resonator. This produces a transverse shear mode (see Fig. 14a). The flexural mode([2], [3],) which could be produced by the unbalance force is well below the frequency of the torsional mode, and hence is of little practical interest.

Even when the transducer is perfectly balanced (i.e. when $F_1 = F_2$) the two forces will result in a pure moment M, only if the body is rigid. Since it is elastic a differential transverse shear mode would arise (see Fig. 14b).

According to experimental evidence, the resonant frequency of the two transverse shear modes are close and above that of the torsional mode.

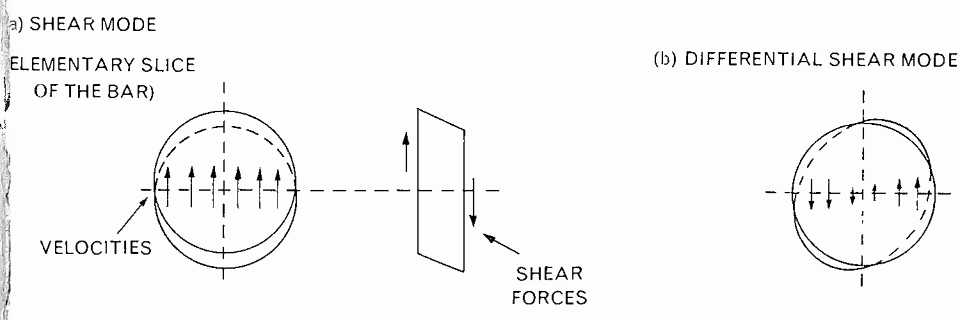

Fig. 14. Transverse shear modes

These two modes appear to be the most troublesome in the type of filter under consideration.

At present no theoretical treatment for the two transverse modes have been evolved and none of the existing theoretical treatments fits the experimental facts. The calculated resonant frequencies for the other known modes are well outside the working frequency band. Table II shows calculated and measured spurious frequencies for a typical filter.

TABLE II

RESONANT FREQUENCIES OF A FREE RESONATOR VIBRATING IN DIFFERENT MODES

(FOR THE RESONATOR USED IN NUMERICAL EXAMPLES ON PAGE 132)

Mode	Resonant frequency (kc/s)		Remarks
	Calculated	Experimental	
Torsional	259	251·8	desired mode
Flexural 1st order	188	approx. 150	
2nd order	445·6		not detected experimentally
Longitudinal	355·8		not detected experimentally
Concentric shear	748·9		not detected experimentally
Coaxial shear	559		not detected experimentally
Transverse shear		approx. 290*	
Differential transverse shear		approx. 320*	strong mode

*These are approximate values of the mid frequencies of the wide spectrum of spurious responses.

SUPPRESSION OF SPURIOUS RESPONSES

There appear to be two approaches to the problem of suppression of spurious responses. One is based on suppression of the unwanted forces in the transducers themselves, whilst using all the resonators of the same type.

The other is based on rejection of the unwanted vibrations by some of the resonators, whose spurious responses differ from the rest. Thus at least two types of resonators are required in the chain. The first method is essentially concerned with the design of a balanced transducer, the second is concerned with the design of a resonator.

In the first method the transducer must satisfy the following requirements,

(i) lack of transverse force

(ii) the tangential forces must be uniformly applied to the circumference of the resonator.

These conditions can be satisfied by the use of a ferrite disc vibrating in a concentric shear mode([6], [7]) (see Fig. 15) cemented on the Ni-Span C

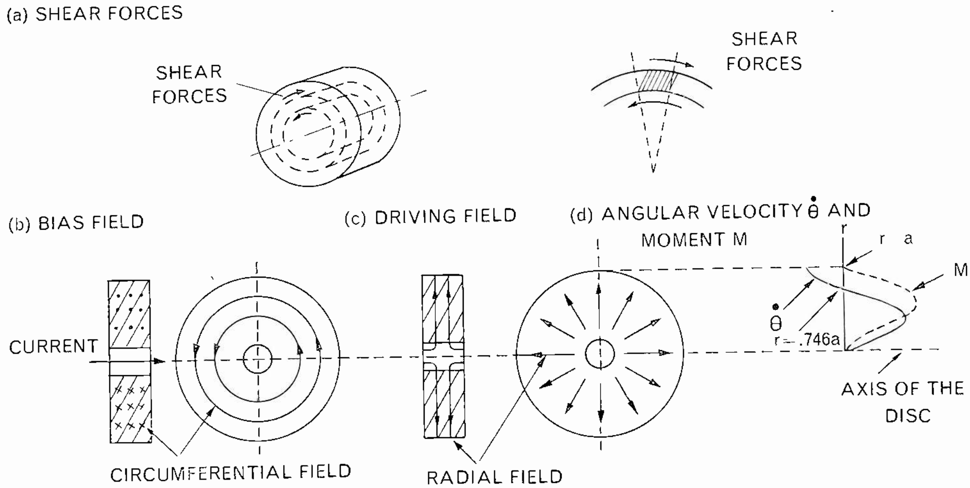

Fig. 15. Concentric shear mode

neck. It is in fact a ferrite toroid, which, once magnetised by an axial current, will retain its magnetisation. Thus a circumferential bias field is obtained. The radial driving field must be produced by suitable transducer coils. The distribution of the moments and of the angular velocities is shown in Fig. 15d.

The second method for suppression of spurious responses is based on the use of dissimilar resonators for which torsional resonance occur on the same frequency while the spurious resonance differ.

The simplest way of achieving this is to alter the diameters of some resonators. The resonance frequency of the torsional vibration depends on the length of resonators only provided that the diameter does not exceed the value at which the second mode of torsional vibration is possible. With the change of diameter, however, the characteristic impedance is altered, hence it is necessary to modify the diameter of the coupling necks suitably to maintain the correct loading of the filter chain.

Another possible solution is to use dumbbell resonators. Then the resonance is obtained with a length which is much smaller than that of the

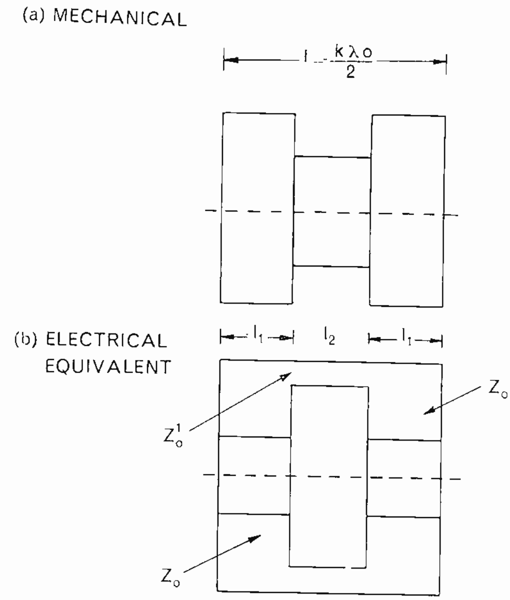

Fig. 16. Dumbbell resonator (slug)

uniform slug (of length $\lambda o/2$). An example of such a resonator is given in Fig. 16a and its electrical equivalent in Fig. 16b. (Some modification of the dumbbell resonators is possible if instead of a sharp step in diameters a gradual transition in diameters is made).

The calculation of the resonant frequency and of the characteristic impedance of the dumbbell slug can be readily carried out for the equivalent transmission line system shown in Fig. 16b. The dumbbell slug has a lower characteristic impedance (Z_0'') than an ordinary slug (Z_0) made out of the same material.

$$Z_0'' = m\, Z_0 \qquad (43)$$

where:

$$m < 1$$

The degree of lowering of the characteristic impedance depends on the shortening factor $k = \frac{2l}{\lambda_0}$

The necks act as impedance transformers. They must thus be modified to suit the new load impedance. The following simple design rule can be formulated. The filter should be designed first with ordinary slugs. Then the ordinary slugs can be replaced by dumbbell slugs, provided that adjacent necks are changed. Their characteristic impedance must be made equal to m times the original value, if the neck is between two dumbbell slugs, or equal to $\sqrt{m}$ times the original value, if it is between a dumbbell and an ordinary slug. Thus the diameters of the necks around the dumbbell slugs are smaller than the corresponding neck diameters for the ordinary slugs.

There are two limitations to the extent of shortening of the length of the dumbbell resonators:

(i) reduction of Q of the resonators

(ii) reduction of L/C ratio of the shunt circuit in the equivalent Π network of the dumbbell slug.

(iii) reduction of neck diameter.

CONCLUSIONS

Mechanical vibrations of the torsional type propagate along the rods in a non-dispersive manner. There exists for such vibrations a strict equivalence with the electrical phenomena in the transmission line. All mechanical quantities can be expressed in terms of electrical quantities, provided a "transducer transfer ratio," having the dimensions of charge, is introduced as a coefficient.

A direct equivalence, (moment→voltage and velocity→current), gives a consistent and useful practical system of presenting mechanical quantities in terms of more familiar electrical quantities.

The torsional filter consisting of resonators (slugs) and couplers (necks) can be presented as an equivalent concentric transmission line composed of $\lambda o/2$ sections of high impedance (corresponding to slugs) and $\lambda o/4$ sections of low impedance (corresponding to necks), all connected in cascade.

The resonators, by a further approximation, can be presented as a series of L and C circuits. The couplers then act as $\lambda o/4$ inverters, transforming series into parallel circuits and vice versa, so that an inverse-arm filter is obtained.

The characteristic impedance of the couplers can be so dimensioned that the arms of the filter conform to the values of the prototype designed for Tchebycheff equal ripple response in the passband, the ripple not exceeding a given limit.

Very stringent mechanical tolerances, especially in relation to the neck radii, must be maintained, since the neck characteristic impedance is proportional to the fourth power of the radius of the neck. A tolerance of 1% in impedance requires $\frac{1}{4}$% in radius, which is of the order of 0·0002 inch.

A starting point in the design of an electro-mechanical filter is the radius of the rod. For this, the constants of the material (the density and the modulus of rigidity) determine the resonant length. The relative bandwidth required and the values of the elements in the prototype filter determine the loading of the filter and the radii of the quarter wave necks.

Since the loading of the filter is fixed by the above parameters, the transducer must be designed to match this load.

Other modes of vibration can cause spurious responses if suitable forces are generated in the system. The system behaves again as a proper filter chain composed of like elements but with a resonance frequency differing from that of the torsional mode. A number of sharp responses can result, since the resonators are not properly tuned for these modes.

Two transverse shear modes are responsible for spurious responses in the present torsional filters. A much wider band of frequencies is obtained due to stronger coupling, since in this case the characteristic impedances are proportional to the second power of radii only.

The excitation of the shear modes can be caused by the lack of balance in the transducer and by a non-uniform distribution of the forces on the circumference of the resonator.

There are two methods of avoiding spurious responses:

(a) suppression of unwanted forces in the transducers

(b) use of at least two types of resonators in the filter chain whose spurious resonances differ appreciably.

A balanced transducer can be designed in a form of a ferrite disc with circumferential bias field and radial driving field. The disc has to be cemented on the filter rod and vibrates in the concentric shear mode, producing a pure torsion on the rod.

Dissimilar resonators can be designed as slugs of different diameter or as dumbbell shaped slugs shorter than the ordinary uniform slug $\lambda o/2$ long. These two types can be combined in the filter chain.

ACKNOWLEDGEMENTS

The author is indebted to Dr. G. L. Grisdale for propounding the problem and for subsequent advice. He also wishes to acknowledge the valuable help of Mr. H. Bache and his continuous interest, both in the preparation of this article and in supplying all experimental data.

APPENDIX 1

EXPRESSIONS FOR THE PARAMETERS OF THE MECHANICAL SYSTEM

The moment of compliance K is defined as:

$$K = \frac{1}{\mu A \kappa^2} \tag{44}$$

Vhere:

K = moment of compliance per unit length
μ = modulus of rigidity (one of the Lamés constants)
A = area of the section
κ = polar radius of gyration

'or a circular section where a = radius of the rod

$$\kappa^2 = \frac{a^2}{2}$$

hus:

$$K = \frac{2}{\pi a^4 \mu}$$

he polar moment of inertia is defined as:

$$J = A\kappa^2\rho \tag{45}$$

here:

J = polar moment of inertia per unit length and
ρ = density

'or a circular section

$$J = \frac{\pi a^4}{2}\rho$$

he characteristic impedance N_0 from equations (4a), (44) and (45) is:

$$N_0 = A\kappa^2\sqrt{\mu\rho}$$

or a circular section

$$N_0 = \frac{\pi a^4}{2}\sqrt{\mu\rho}$$

'he velocity of propagation from equations (3a), (44) and (45) is:

$$u_m = \sqrt{\frac{\mu}{\rho}}$$

his is independent of dimensions of the rod provided the radius is small
n terms of the wavelength. The velocity is the same for all frequencies.
'hus the system is non-dispersive.

APPENDIX 2

EQUIVALENCE OF MECHANICAL AND ELECTRICAL QUANTITIES

Since the velocities and length of the bar and the equivalent line were assumed equal, i.e.,

$$u_m = u_e$$
$$l_m = l_e$$

the relationship between the remaining parameters can now be deduced. The relationship between M and v can be found from equations (5a), (5b) and (6)

$$MKl_m\delta = vCl_e \tag{46}$$

and from the equality of both forms of energy:

$$E = \frac{1}{2}KM^2 = \frac{1}{2}Cv^2 \tag{47}$$

$$M\frac{1}{\delta} = v \tag{48}$$

From equations (47) and (48)

$$K\delta^2 = C \tag{49}$$

Using another form of expression for the energy:

$$E = \frac{1}{2}J\dot{\theta}^2 = \frac{1}{2}Li^2 \tag{50}$$

From equations (6) and (50)

$$J\frac{1}{\delta^2} = L \tag{51}$$

From equations (4), (49) and (51):

$$N_o\frac{1}{\delta^2} = Z_o \tag{52}$$

Table I gives a summary of the results in a clearer form.

APPENDIX 3

NUMERICAL VALUES OF MECHANICAL QUANTITIES

To avoid confusion with units an MKS system is used. Then:

Transducer transfer ratio	δ is expressed in	Coulombs
Moment	M	kg m
Moment of compliance	K	$kg^{-1}\ m^{-2}$
Moment of inertia	J	$kg\ sec^2$
Mechanical impedance	N_o	kg m sec
Modulus of rigidity	μ	$kg\ m^{-2}$
Young's modulus of elasticity	E	$kg\ m^{-2}$
Density	ρ	$kg\ sec^2\ m^{-4}$

ength l and radii a, c m

elocity u m $\sec^{-1}$

The constants for Ni-Span C were taken as:

odulus of rigidity, 10×10^6 lb/sq. inch i.e. $\mu = 7{\cdot}03 \times 10^9$ kg/m²

ne specific gravity is 8·15 which gives the density

$$\rho_{Ni} = 0{\cdot}831 \times 10^3 \text{ kg sec}^2/\text{m}^4$$

oung's modulus of elasticity is 25×10^6 lb/sq. in. i.e.

$$E_{Ni} = 17{\cdot}6 \times 10^9 \text{ kg/m}^2$$

ne velocity of propagation of the torsional mode:

$$u_2 = \sqrt{\frac{u}{\rho}} \text{ giving } u_2 = 2909 \text{ m/sec}$$

ne velocity of propagation of the longitudinal mode

$$u_0' \fallingdotseq 1{\cdot}58\, u_2 \text{ hence } u_0' \fallingdotseq 4599 \text{ m/sec}$$

The corresponding values for the Ferrites used in the transducers are nown only with some approximation. These were assumed to be:

The density $\rho_{Fe} \fallingdotseq 0{\cdot}46 \times 10^3$ kg sec²/m⁴

Young's modulus of elasticity:

$$E_{Fe} \fallingdotseq 14{\cdot}8 \times 10^9 \text{ kg/m}^2$$

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A PRACTICAL ELECTRO-MECHANICAL FILTER

By H. BACHE, M.Eng, A.M.I.E.E.

The following article discusses the manufacture of electro-mechanical filter employing mechanical elements operating in the torsional mode. As an exampl a component used as an upper side-band filter for a carrier frequency o 250 kc/s is described. The importance of having material for both the elemen and the transducer systems in the correct state before manufacture commence is emphasized.

These filters are useful in applications where small size, rigidity, low los and stability (particularly with changes in temperature) are required. In th present design any input or output impedance between 600Ω and 100,000 *may be obtained. Bandwidths up to about* 3% *in the range of* 100 *to* 500 *kc can be accommodated, though changes in these two parameters may involv re-tooling for each filter type.*

The complete assembly is contained in a sealed metal container, n adjustments being necessary in service.

The maximum input level is approximately 1*mW* (*i.e.*, 5·5*V RMS* fo 30*k*Ω) *above which non-linearity between input and output starts.*

INTRODUCTION

Electro-mechanical filters consist of a number of mechanical resonato with appropriate coupling elements. Mechanical vibrations are trans mitted through this system which is frequency selective. A transduce system converts the input electrical energy into the mechanical form an another transducer system at the output fulfils the converse functio

The mechanical vibrations in the resonators and their couplers may b of the torsional, longitudinal, flexural or shear modes. The choice of whic mode to use for a particular resonator design depends chiefly on th resonant frequencies of modes of oscillation of the element other than th required mode, and their relation to the desired frequency band. (A electrical tuned circuit resonates at a unique frequency, whereas mechanic systems have a number of frequencies at which they resonate and eac resonance can be of a different mode from the others.) It was decided t concentrate on the torsional mode, which is relatively free from thes spurious modes and which also gives a filter element which can be mounte in such a way that a rugged assembly is attained.

Using either the design procedure given by Struszynski(1) or the mor empirical approach given by Roberts and Burns(2), electro-mechanic filters may be designed for bandwidths between 0·01% and 3% for use i the frequency range 100 to 500 kc/s. In what follows we shall be more particularly concerned with discussing filters for use in single-sideband applications, accommodating a normal P.O. speech channel, with a 250 kc/s carrier. Other bandwidths and frequencies can be attained, but in general bandwidths up to 3% in the range 200 to 300 kc/s provide a convenient component, both from the application and from the manufacturing aspects.

The filter consists essentially of a mechanical element composed of a number of half wave resonators, vibrating in the torsional mode, coupled by quarter wave sections, the resonators being tuned to the centre frequency of the filter (Fig. 1a). Narrower band filters (e.g., 50 c/s bandwidth at 250 kc/s) require three-quarter wave couplers between each resonator to give the required coupling with reasonable mechanical rigidity (Fig. 1b). The extra quarter wave sections at the ends of the

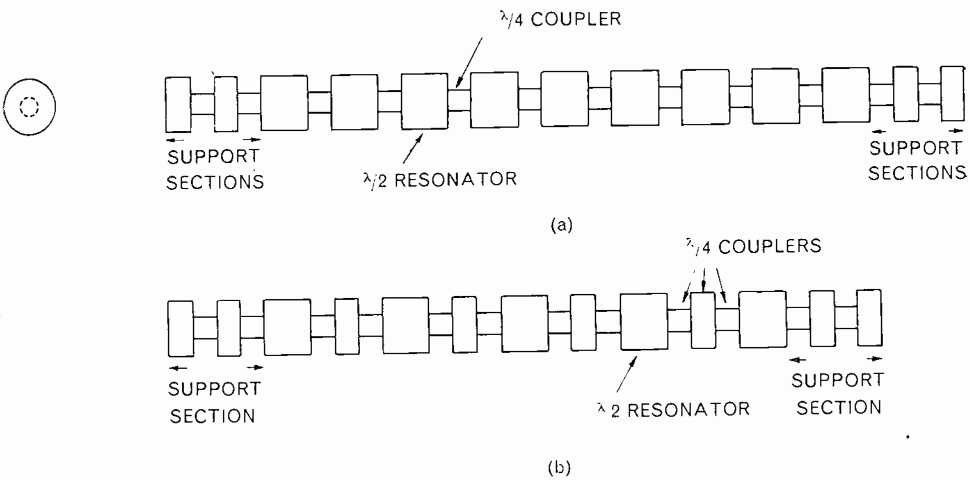


Fig. 1.

element are for mechanical convenience of mounting. The performance in the attenuation band (in particular the bandwidth at − 60 dB relative to the passband) can be altered by adding more resonators. Usually nine are sufficient to give a shape factor (i.e., ratio of the 60 dB bandwidth to the 6 dB bandwidth) of approximately 1·5. The element is driven by a transducer system, in this example two ferrite rods vibrating longitudinally. There is a similar system at the output end to give an electrical output signal (Fig. 2). The ferrite rods are driven by a binocular pair of coils.

The limitations to performance are, in general, associated with the level at which the filters operate, the bandwidth and the frequency range.

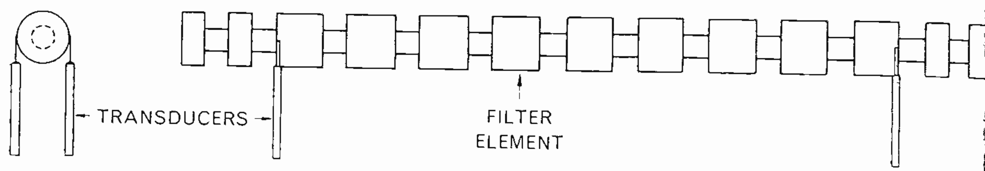


Fig. 2.

The maximum input level is governed by the properties of the transducer system (in this example ferrite rods), because the magnetostrictive effect is sufficiently linear over only a small range of applied magnetic field.

The fractional bandwidth is, for the torsional mode and to a first order, proportional to the fourth power of the ratio of coupler to resonator diameter; hence wider bandwidths give too large a coupler diameter. Other modes, such as flexural or longitudinal, could be used, as in these modes the power of this ratio is less than four.

The physical size of a resonator to operate below 100 kc/s gives a filter which is too large to have any advantage over other systems, while above 500 kc/s the resonators are too small to be handled conveniently.

FILTER ELEMENT

The filter element is the main component of the filter, and provides the frequency selective characteristic.

To achieve the desired performance, the element must have the following properties:—

(1) High mechanical "Q."

(2) Low frequency-temperature coefficient ($< 6 \times 10^{-6}/°C$ is taken as a standard)

(3) Reasonable "machineability"

(4) Stable operation over long periods

To satisfy all these requirements is not easy, but two alloys have the necessary properties, namely "Ni-Span C"(3) and "Vibralloy"(4, 5). The latter has not as yet been tested sufficiently for production use, but shows promise. Ni-Span C alloy is therefore the only material used at present for filter elements.

It is important to have the material in a "standard" state before machining, so that one set of dimensions will give closely similar characteristics for a large batch of filters of a particular type. Due to variations from batch to batch, the solution annealed material is therefore heat treated to give more consistent properties. The treatment is, in general, a precipitation hardening one, carried out in a hydrogen atmosphere at about 500° to 800°C for periods of up to four hours, giving an alloy with tensile strength of the order of 10^5 lbs/in^2, elongation about 40%, and hardness about 300 D.P.N. The treatment could be carried out in a vacuum, the main point being to avoid atmospheres containing gases which, at these temperatures, react with the material, e.g., nitrogen and oxygen. After satisfactory samples are made from each batch, the whole lot has to be treated similarly, and further samples taken from the treated material. Once the batch has been passed, the remaining processes are of relatively routine nature.

The test samples are made by grinding the treated material to the required resonator diameter, and cutting into cylinders of different lengths (0·200 inch to 0·250 inch long by 0·250 inch diameter are typical samples for 250 kc/s filters), care being taken to avoid hot working the material during these processes. The sample cylinders have their resonant frequencies measured by using the magnetostrictive properties of the alloy. If a cylinder is polarized circumferentially and placed coaxially inside a coil, when the cylinder resonates a large impedance is reflected back into the coil. The coil is used as one arm of a bridge, the mechanical resonance of the cylinder being detected by a sharp change in the balance; this is shown in Fig. 3.

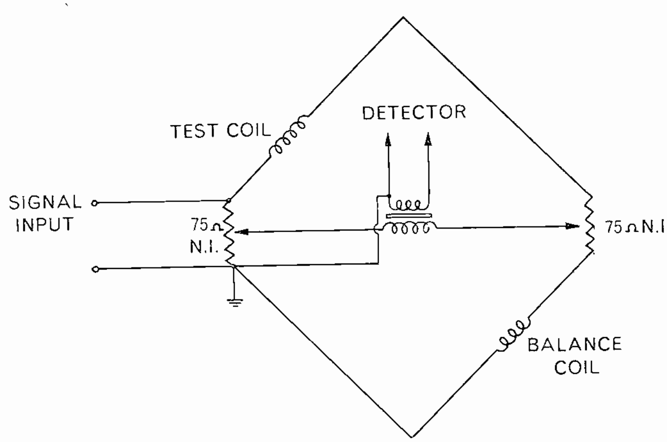


Fig. 3.

The frequency-temperature coefficient is determined by placing the coil and test sample in an oven, and by varying the temperature slowly from — 40° to + 80°C. To preserve the state of the material in machining is as important as to achieve the required dimensions, hence techniques of centreless grinding have been applied to manufacture the elements. Other processes, such as turning, could be used, but the time taken, the dimensional variations attained and the effect on the state of the alloy would be unacceptable.

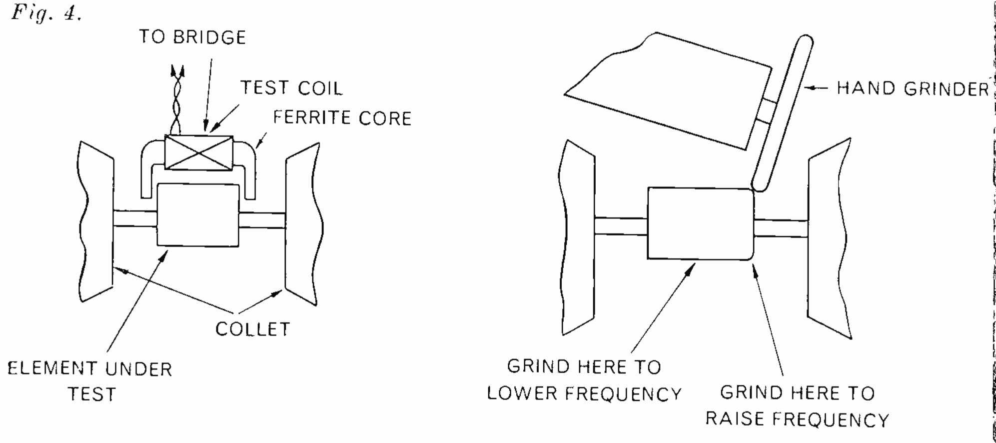
In theory all the quarter wave couplers should have different diameters, but only those adjacent to the end resonators are significantly larger than the centre ones. In practice, therefore, only the end coupling sections are made larger than the centre(1).

After the raw stock has been through ground to the resonator diameter (0·250 inch ± 0·002 inch) the couplers are shaped simultaneously by plunge grinding using a formed grinding wheel. The accuracy and consistency of this process is sufficient to give satisfactory results. It has been discovered that consistency of error from the nominal value from section to section in any one element is at least as important as achieving the correct dimensions in giving the desired filter performance, especially regarding the passband ripple.

After machining, samples of the elements are tested in a jig for frequency-temperature stability. When there is no change in resonant frequency in two successive tests, with an intervening period of twenty-four hours at a temperature of about 100°C, the samples are considered to be sufficiently stable.

The whole batch is similarly treated and then accepted for tuning. Each resonator is tuned to the centre frequency of the filter by removing small amounts of material from its edges to raise the frequency or from its centre to lower it (Fig. 4). For the example taken, the frequency tolerance

Fig. 4.



is ± 10 c/s for this operation. When the adjacent resonators are clamped, the resonant frequency of each may be measured by causing a bridge to go off balance when an RF signal is varied in frequency, in a similar manner to the testing of the cylindrical samples.

When the filter has been tuned, the transducers are spot welded to the appropriate resonators in a jig.

TRANSDUCERS

The transducers used at present are made of either a special binary ferrite or of Ni-Span C wire. The latter is used for narrow bandwidth filters (e.g. 50 c/s bandwidth at 250 kc/s) whilst the ferrite is used for wider bandwidths.

The ferrite is a binary nickel zinc ferrite, with impurities added to damp the mechanical Q. By this means some of the termination for the filter is provided mechanically, the rest being provided by the electrical circuits, including the input and output coil assemblies.

The ferrites are tested and sorted for electro-mechanical coupling coefficient K which is usually expressed as a percentage (see appendix 1) in steps of 1%, then tinned at one end and a $\frac{1}{8}$ inch length of Ni-Span C wire soldered to each so that the wire and ferrite are coaxial. This transducer assembly is then tuned to the correct frequency (within 2 kc/s of the filter centre frequency), after which it is ready for welding on to the element. The value of K for the four transducers for any one element must be within a 1% step. A jig holding four transducers and the Ni-Span C element in the correct relative positions is required for the welding process. When wire transducers are used, these are tuned to the correct frequency, and are then welded on to the element in a similar jig. The biasing magnets used are of Ticonal G alloy, and are needed to ensure that the transducers are operating under optimum magnetic conditions.

FILTER ASSEMBLY

The element, with the transducers welded in place, is placed in an assembly consisting of a casting containing a clamp for each end of the element, and the necessary coil and capacitor assemblies for driving the transducers and for producing an output signal (Fig. 5). A centre web on the casting acts as an electrical screen between the output and input terminals, as well as providing a mechanical support for the element under grave shock and vibration conditions. It also provides extra rigidity for the whole assembly.

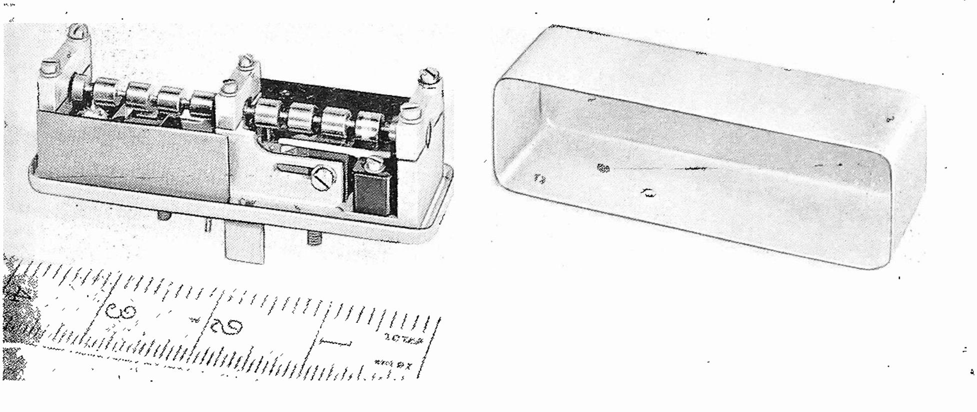


Fig. 5.

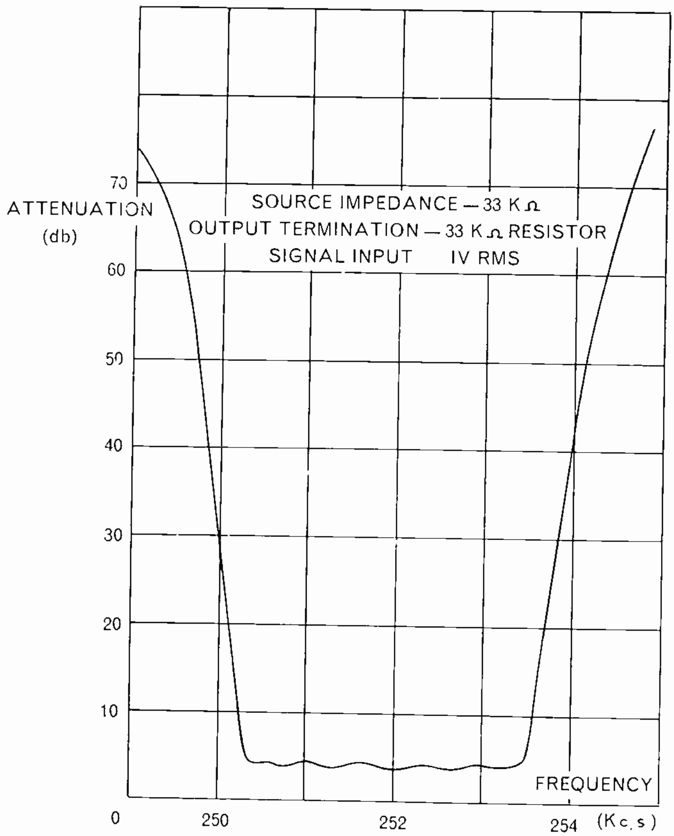


Fig. 6.

When assembled, the position of the magnets is adjusted to give the optimum response curve for a given source impedance and terminating load. If necessary the tuning capacitors are adjusted. This adjustment is made with the aid of an alignment oscilloscope using a slow sweeping speed in order to obtain an accurate response curve. Since the mechanical resonators have a high Q, a sweep of 2 secs. duration is required for displaying the full response. Due to the slow sweep speed, normal beat methods of displaying frequency markers are not accurate enough. Use has, therefore, been made of a circuit which compares the phase of the swept signal with that of another tuneable oscillator, whose frequency can be accurately measured. The output of this comparator triggers a mono-stable multi-vibrator whose output is applied to a "Y" amplifier. By this means, the frequency discrimination is within the limits imposed by the thickness of the C.R.T. trace. (A 10 inches diameter tube, with a long persistence phosphor, has been successfully used for the display). Frequency measurement is carried out by the "counter" technique.

The filter cover is soldered in place, and the whole assembly filled with a dry atmosphere. A further test is carried out after the final processes of painting and labelling.

ONCLUSIONS

ıe major problems to be overcome in any electro-mechanical filter oduction lie in the material used. For the particular filters considered. sting the Ni-Span C alloy and the ferrite are the most important parts the process. Whilst some of the subsequent processes are not easy, the iished product could be made by less precise (though not more economic) ocesses.

It has been shown that a compact, reliable, rugged filter assembly can made economically, though changes in bandwidth and/or centre equency will require different tooling. The system described is aligned r a 30 kΩ source and a 30 kΩ resistive load. A typical response curve is own in Fig. 6, and the full specification in Appendix 2.

Samples of laboratory models have performed very satisfactorily in rious experimental applications, including use in a CR 100 receiver hose I.F. was changed to 250 kc/s), a multichannel telephone system d a wave analyser.

Some laboratory models have been tested for harmonic distortion and cond and third order intermodulation products, which were each better an — 50 dB relative to the test tones.

CKNOWLEDGEMENTS

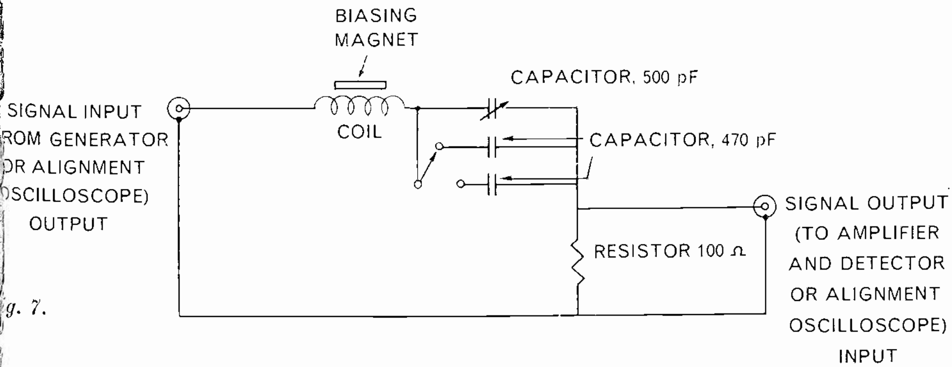
ıe author wishes to acknowledge the practical assistance given by H. Clarke, K. R. Perry and W. C. Mills in the development of the ethods described above.

PPENDIX 1

ERRITE TEST PROCEDURE

ıe procedure is the same irrespective of whether the ferrite rods or the sembled transducer are to be examined.

) The parts are tested on the test set (Fig. 7) for electro-mechanical upling coefficient (K), which is expressed as a percentage. The parts are aced in the coil, the latter being tuned so that the detector indicates



ig. 7.

two peaks which are to be symmetrically placed about a dip when the frequency of a low impedance source is varied.
(2) If necessary, the parts are tuned by grinding one of the ends of the ferrite so that the dip is at the correct frequency (f_o) (290 kc/s ± 2 for the ferrites or 250 to 254 kc/s for the transducers).
(3) The electro-mechanical coupling coefficient (K) is expressed as

$$\frac{(f_1 - f_2)\,100}{f_o}\,\%$$

where f_1 is the frequency of the higher frequency peak
f_2 is the frequency of the lower frequency peak
f_o is the frequency of the dip
(4) The ferrites are sorted in values of K in steps of 1% for the range of 14% to 20%.

APPENDIX 2

ELECTRO-MECHANICAL FILTER TYPE 4835/A
250 kc/s UPPER-SIDEBAND FILTER
CENTRE FREQUENCY—251,850 c/s
Test Level—1 V.RMS at Filter Input Terminals.

MAX. PASSBAND RIPPLE	TERMINATING IMPEDANCES
2·5 dB	30 kΩ
1·5 dB	60 kΩ
1·0 dB	100 kΩ
2·0 dB	10 kΩ and 200 kΩ

Max. Transmission Loss 5 dB
Max. DC Volts Between any Terminal and Earth 300

Attenuation	LF		HF	
	Min. Freq. c/s	Max. Freq. c/s	Min. Freq. c/s	Max. Freq. c/s
3 dB	250,250	250,300	253,450	253,500
40 dB	249,850	249,900	253,900	254,000
60 dB	249,450	249,550	254,250	254,350
>28 dB	250,000	250,000		

For a temperature range of − 40°C to 80°C the loss and ripple should not change by more than 1 dB and attenuation at 250,000 c/s should not be less than 25 dB. Overall frequency/temperature coefficient $< 6 \times 10^{-6}$.

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BOOK REVIEWS

INCIPLES OF TRANSISTOR CIRCUITS

S. W. Amos, B.Sc (Hons.), A.M.I.E.E. Iliffe and Sons Ltd. 21s. net

s volume, intended as an introduction to use of transistors for "professional igners, students, and amateur construc- s," requires of the reader only the imum of knowledge of algebraic manipu- on.

Chapter I introduces the subject con- tionally, through a superficial but none- less adequate consideration of the aviour of charge-carriers in semi- ducting materials.

The basic principles of the operation of nt-contact and junction transistors form subject of Chapter 2; current gain, tage gain with a resistive load, power n, and variation of current gain with quency are defined and considered.

The three subsequent chapters deal pectively with Common Base, Common itter, and Common Collector Amplifiers, ating the transistor as an active three minal tee-network. Judged individually, h chapter gives as complete a treatment each configuration as one might reason- ly expect, while typical values are con- ually emphasized both for point-contact d junction types. One is left with the nviction, however, that in treating each nfiguration according to the same strict ttern, as the author has done, a great deal unnecessary duplication has resulted. It uld surely suffice, indeed, be preferable, a later edition, to curtail the two latter pters by ruthless compression of the ntents, and to display the results for the ree configurations side-by-side in a table. further cause for criticism is the treatment of the point-contact transistor, now obsolete, on equal terms with the junction type throughout four chapters.

A chapter on Bias Stabilization follows; this is a wholly admirable treatment in simple terms, with numerical examples based on typical values of transistor para- meters, and should prove valuable to a newcomer to the subject.

The design of small, and large, signal amplifiers, based on the groundwork estab- lished in the earlier part of the book, occupies the next two chapters; a discussion of neutralization and unilateralization of h.f. amplifiers is included.

The last two chapters ("Transistor Superheterodyne Receivers" and "Other Applications of Junction Transistors and Other Types of Transistors") fail to main- tain the standard set elsewhere. A large number of transistor application topics is covered in a sketchy fashion, which might, nevertheless, have been acceptable had a comprehensive bibliography been included for further reading; this is, however, not the case. An example of a particularly inade- quate treatment of a device (". . . Other Types of Transistor") occurs in the final chapter; the pnpn transistor being dis- missed in seven lines, with no mention of its potentialities as a high-speed switch, nor, indeed, any hint that it may have a current gain exceeding unity. These criticisms and a few minor typographical errors notwith- standing, the book can be recommended as a reasonably priced introduction to the design of transistor equipment.

APPARATUS FOR THE MEASUREMENT OF TENSOR PERMEABILITY AND DIELECTRIC PROPERTIES OF FERRITES AT X-BAND FREQUENCIES

By W. S. CARTER, B.Sc, Ph.D.

In investigations into the preparation and applications of ferrites it necessary to express ferrite properties in terms of some quantities m universal than their performance in the configuration of the applicati Suitable quantities would be saturation magnetization, dielectric consta dielectric loss, tensor permeability and magnetic absorption line width. apparatus described in this article has been designed to measure all but first of these. Although it is not possible to derive quantitatively the performa of a ferrite in various applications from these quantities, they at least for basis for unambiguous comparison.

The apparatus described reached its final form in two main steps, the f resulting in equipment which in principle would measure the desired qua tities, but which in practice was subject to errors and inconvenience wh made it necessary partly to re-design it. As the principles of measurem appear more clearly in the original design, this will be described first description of the technical improvements to make it more practically use following.

ORIGINAL APPARATUS

After due consideration of the various methods of measuring the requir quantities, it was decided to use the resonant cavity technique as t allows the direct evaluation of all the magnetic and electric paramet individually, from the measured shift in resonant frequency and chan in Q of the cavity in use. An E_{010} transmission type cavity was construct for the dielectric measurements, and an H_{112} absorption type cavi designed by Dr. W. Jasinski for the magnetic measurements.

As mentioned above, the actual measurements that have to be ma are the shift in resonant frequency and the change in Q of the cavity wh the ferrite is inserted. In the magnetic case the variation of both the quantities with strength of applied DC field is also required. The ıeasurements were originally made using the experimental arrangement ıdicated in Fig. 1.

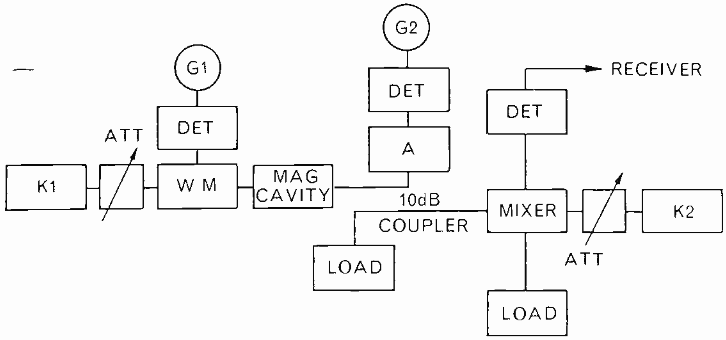


Fig. 1.

With this arrangement the resonant frequency of the magnetic cavity ʼas found by tuning klystron K1 for minimum output on galvanometer ;2, and then with K1 set at the same frequency, tuning the wave meter ›r minimum output on galvanometer G1. The width of the Q curve was ›und as follows: The reference frequency klystron K2 was set to, or nearly ›, the resonant frequency of the cavity. Klystron K1 was then tuned ntil the output on galvanometer G2 indicated that this frequency orresponded to one of the half power points on the Q curve of the cavity. 'he difference in frequency between K1 and K2 was then measured using standard commercial receiver. This procedure was repeated using the ther "half-power-point". In this way the width of the Q curve was ıeasured in terms of the calibration of the receiver.

In order to make the dielectric measurements, the E_{010} cavity was ıserted at "A" and the measurements were made in a manner similar to ıat described above. It was found to be unnecessary to remove the ıagnetic cavity when making the dielectric measurements as the resonant requencies of the two cavities were sufficiently far apart to avoid interərence.

The power supplies required for the klystrons used (K 311's) were 50 V.HT, 6·3 V heaters, and suitable voltage negative with respect to the athode for the reflectors. It was found necessary to supply the heaters ·om an accumulator and the reflector voltage from dry batteries to əduce 50 c/s pick up. The HT was originally supplied by a bank of ccumulators, but as the stability of this supply depended on all the cells eing in good condition, it was changed to a rectified and stabilized voltage erived from the AC mains, which has subsequently proved to be entirely atisfactory in operation.

The chief disadvantages of the system were:

1. With 10 dB of padding attenuation for the klystron the output was not sufficient to allow great accuracy even when using very sensitive galvanometers.

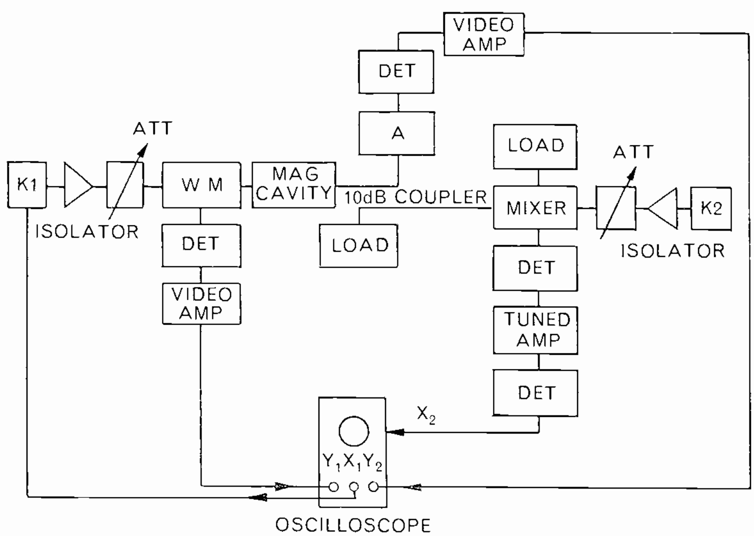
2. This method of measurements requires the reference frequency, which was provided by klystron K2, to remain constant for an appreciable period of time. The drift in frequency of both K1 and K2 during the period of measurement of one value of Q was found to be sufficiently large to make the results very inaccurate.

Though the first drawback could be overcome in part by the use of ferrite isolators in place of the padding attenuators, the second could have been overcome only by constructing a frequency standard operating in the X-Band. Even if this were done the measurement of Q would still rely on the short term stability of the klystron K1, as the tuning of the receiver and of K1, which is indicated by the galvanometer reading, cannot be done instantaneously.

MODIFIED APPARATUS

The deficiencies in the system of measurement have been largely overcome by the modifications described below and indicated in Fig. 2.

Fig. 2.



The arrangement is similar to that shown in Fig. 1, and an illustration of the apparatus is given in Fig. 3. The principal modification is the introduction of frequency modulation to the output of klystron K1, which is achieved by applying to the reflector a saw-tooth wave form, obtained from the time base circuit of an oscilloscope. By employing this frequency modulation, the output of K1 sweeps through the Q curve of the cavity in about fifty milliseconds, and the etected output is displayed on the oscilloscope after video amplification. he frequency difference between the outputs of klystrons K1 and K2 now aries during each cycle, and by setting K2 on the resonant frequency of ie cavity in use, this difference frequency becomes zero at that point. y passing the difference frequence through a frequency sensitive

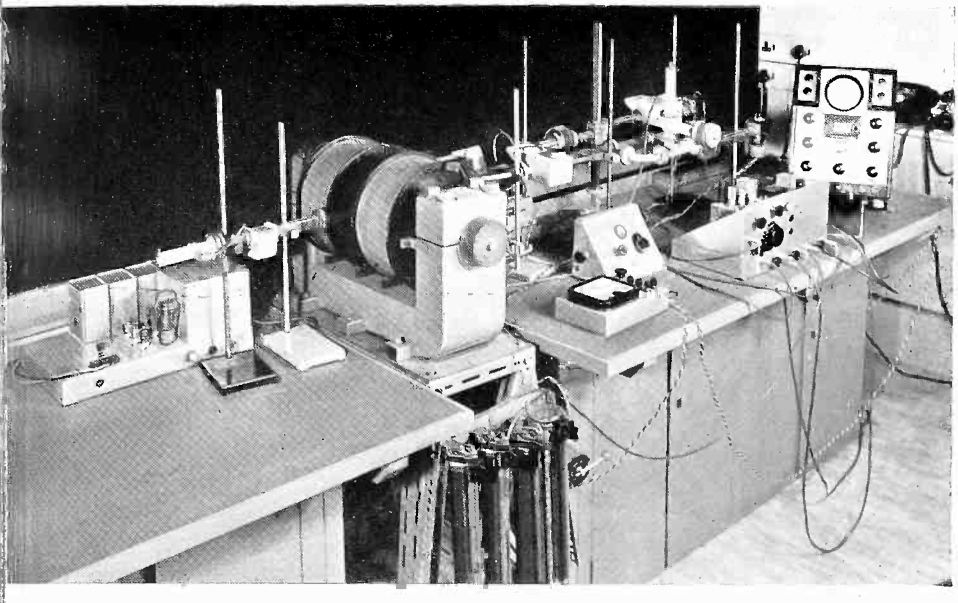


Fig. 3. Apparatus for determining dielectric and magnetic constants of ferrites

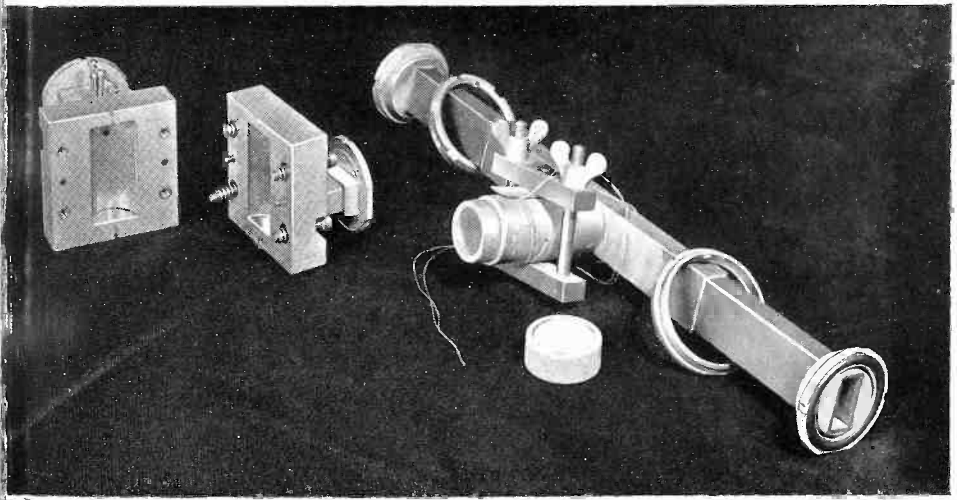


Fig. 4. Specimen waveguide cavities for use in apparatus shown in Fig. 2

amplifier, which is tuneable, and applying the output to the X-plates the oscilloscope after detection, two marker pips are obtained on t Q curve which are separated by twice the frequency to which the selecti amplifier is tuned.

In this arrangement disadvantage (1) has been overcome in the fir place by using isolators in place of the "padding" attenuators and in t second place by amplifying the output after detection. This is now possib because of the modulation which is applied to K1. Disadvantage (2) h been overcome by the application of the modulation, because this mea that the whole Q curve is repetitively traced out within fifty millisecond As the Q measurements are now made by placing the marker pips on t half power points by simultaneous adjustment of the frequency sensiti amplifier and the reflector voltage of klystron K2, any long term dri in the reference frequency is no longer important. Short term deviatio in the output of K2 (which mainly consists of 50 c/s) do, however, cau trouble, as they result in a blurring of the marker pips. Most of this troub has been eliminated by decoupling the reflector of klystron K2.

DIELECTRIC MEASUREMENTS

The original cavity constructed for these measurements was an E transmission type cavity having both length and diameter about 1 inc its volume V_0 being 12·3 c.c.

The microwave field configuration within a cavity operating in th mode is such that along the cavity axis the electric component is maximum, and the magnetic component zero. By using specimens in t form of thin rods placed axially in the cavity, advantage is taken of th microwave field configuration so that the resultant shift in resona frequency and change in Q of the cavity are due only to the dielectr properties of the ferrite specimen.

The relations between the components of the complex dielectric consta ($\varepsilon' - j\varepsilon''$) and the frequency shift for a rod specimen placed in an E cavity have been calculated[1] and are given by

$$\frac{\delta\omega}{\omega} = -A\,(\varepsilon' - 1)$$

$$\text{and}\quad \left(\frac{1}{Q_1} - \frac{1}{Q_0}\right) = 2A\varepsilon''$$

where ω = the resonant frequency of the cavity

$\delta\omega$ = the shift in resonant frequency on insertion of the specime

V_1 = volume of ferrite specimen placed in cavity.

V_0 = volume of cavity = 12·3 c.c.

Q_0 = Q of empty cavity.

$Q_1 \quad = Q$ of cavity with specimen in place.

$A \quad = 1{\cdot}855 \dfrac{V_1}{V_0}$ (1·855 for this mode of operation of the cavity).

The measurement of ε' presents little difficulty, as the change of onant frequency of the cavity ($\delta\omega$) may be made directly using the wave ter.

The measurement of the loss tangent $\left(\tan\delta = \dfrac{\varepsilon''}{\varepsilon'}\right)$ is rather more com-icated.

From above, $\varepsilon'' = \dfrac{1}{2A}\left(\dfrac{1}{Q_1} - \dfrac{1}{Q_0}\right)$

and so,

$$\tan\delta_e = \frac{\varepsilon''}{\varepsilon'} = \frac{1}{2A\varepsilon'}\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right)$$

$$= \frac{V_0}{3{\cdot}7\,V_1\varepsilon'}\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right)$$

$$\doteqdot \frac{V_0}{3{\cdot}7\,V_1\varepsilon'}\left(\frac{\Delta\omega_0 - \Delta\omega_1}{\omega_0}\right)$$

To increase the sensitivity of measurement of $\tan\delta_e$ an E_{020} cavity was istructed having a diameter of 2·2″ and a length of 3·16″ ($V_0 = 200$ c.c.). is cavity has a Q of 27,000 as compared with 10,500 for the E_{010} cavity. specimen cavity is shown in Fig. 4.

For this cavity the value of ε' and ε'' are given as

$$\frac{\delta\omega}{\omega} = 4{\cdot}319\,(\varepsilon' - 1)\,\frac{V_1}{V_0}$$

d

$$\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right) = 2 \times 4{\cdot}319\,(\varepsilon'')\,\frac{V_1}{V_0}$$

that

$$\tan\delta_e = \frac{\varepsilon''}{\varepsilon'} = \frac{V_0}{8{\cdot}6\,\varepsilon'_1 V_1}\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right)$$

here the symbols have the same meaning as in the case of the E_{010} vity).

MITS AND ACCURACY OF MEASUREMENT OF TAN δ_e

hen considering the limit in accuracy to which measurements can be de it is necessary to examine the factors which contribute to the aluation of $\left(\dfrac{1}{Q_1} - \dfrac{1}{Q_0}\right)$.

This factor can be rewritten as

$$\frac{\Delta\omega_1}{\omega_1} - \frac{\Delta\omega_0}{\omega_0} \doteqdot \frac{\Delta\omega_1 - \Delta\omega_0}{\omega_0}$$

where $\Delta\omega_0$ is the frequency difference between half power points of emp cavity and $\Delta\omega_1$ is the frequency difference between half power points the cavity with specimen.

The limit to which $\Delta\omega$ can be measured is affected on the one hand b the sensitivity of the selective amplifier, and on the other hand by t accuracy to which the marker pips can be set on the half power points the Q curve. In the apparatus considered, the selective amplifier can b read to $\pm$ 5 kc/s whereas the limit of setting the marker pips is probab $\pm$ 5%. For low loss specimens $\Delta\omega_1 - \Delta\omega_0 \ll \Delta\omega_0$. Thus if $\Delta\omega_0$ = 400 kc/s (i.e. the value for the E_{020} cavity) the measurement of $\Delta\omega_1 - \Delta$ can only be made to approximately $\pm$ 50 kc/s.

If these values are inserted in the above expressions for tan δ and 10 taken as a representative value of ε' for the ferrite materials we have a indication of the limits and accuracy of measuring this quantity.

(1) for the E_{010} cavity

$$\tan\delta_e = \frac{V_0}{37\,V_1} \times \frac{5 \times 10^4}{10^{10}} = \frac{V_0}{V_1} \times 1{\cdot}3 \times 10^{-7} \text{ approx.}$$

(2) for the E_{020} cavity

$$\tan\delta_e = \frac{V_0}{86\,V_1} \times \frac{3 \times 10^4}{10^{10}} = \frac{V_0}{V_1} \times 3{\cdot}5 \times 10^{-8} \text{ approx.}$$

This means that, if measurements are made on the same material ar the same value of $\frac{V_0}{V_1}$ is used in the two cavities, the results given by th E_{020} cavity will be more sensitive than those given by the E_{010} cavity by factor of about 4.

If it is assumed that a specimen of 1 mm. radius is used on the E cavity (i.e. $V_1 = 0{\cdot}23$ c.c.) the limiting value of tanδ that can be measur is

$$\tan\delta_e = \frac{200}{0{\cdot}23} \times 3{\cdot}5 \times 10^{-8}$$
$$= 3{\cdot}5 \times 10^{-5}$$

MAGNETIC MEASUREMENTS

The H_{112} cavity constructed for the magnetic measurements has a micr wave field configuration which makes the centre of the cavity a poi where the electric field component has a zero and the magnetic fie component is a maximum. The cavity, which is the absorption type, d by two inputs in phase and space quadrature which makes the gnetic component at the centre of the cavity circularly polarized. The eraction of a ferrite sphere, placed at the centre of the cavity, with this crowave field can be described in terms of an effective permeability of $(\mu \pm \alpha)$ (the sign depending on the sense of circular polarization with pect to an axially applied static magnetic field), or in terms of a onance absorption line width. It is most convenient to express the gnetic loss characteristics in different ways near and far from resonance. ar resonance the absorption line width is most suitable and far from onance the imaginary component of permeability. When considering effective permeability, μ and α are the components of the tensor rmeability given by Polder (1949)(2) by the equation

$$B = \mu_0 \begin{bmatrix} \mu & -j\alpha & 0 \\ j\alpha & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} .H$$

The quantities which have to be measured are the shift of resonant quency and the change in Q of the cavity from the unloaded values at rious values of applied field. Provided that the dimensions of the sphere much less than a wavelength and the perturbing effect is not too large can be assumed that the shift in resonant frequency depends only on permeability.

The relationships between frequency shift and permeability, and change Q and loss(3) are:

$$\frac{\delta\omega}{\omega_0} = A\,R$$

$$\text{and} \quad \frac{1}{2}\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right) = A\,I.$$

here $\delta\omega$ is the change in resonant frequency of the cavity when the ecimen is inserted and a magnetic field is applied.

Q_0 is the Q of the empty cavity

Q_1 is the Q of the cavity when the field is applied

$$A = \frac{-3\,(\omega^2 \mu_0 \varepsilon_0 - k^2)}{2\omega^2 \varepsilon_0 \mu_0 \left(1 - \frac{1}{\tau^2}\right) J_1^2(\tau)} \cdot \frac{V_1}{V_0}$$

R and I are defined by

$$R + jI = \frac{\mu \pm \alpha - 1}{\mu \pm \alpha + 2}$$

now μ and α are written as $\mu = \mu' - j\mu''$ and $\alpha = \alpha' - j\alpha''$

$$\text{Then } R = \frac{\mu' \pm \alpha' - 1}{\mu' \pm \alpha' + 2}$$

and

$$I = \frac{3(\mu'' \pm \alpha'')}{(\mu' \pm \alpha' + 2)^2}$$

From these equations all the components of the tensor permeability c be obtained from the experimentally determined values of $(\delta\omega)$ and However, in a number of applications the quantities required are t more directly obtained effective permeability and the loss tangent circularly polarized radiation.

If the effective permeabilities with respect to the two senses of circul polarization are designated μ_+ and μ_- so that

$$\mu_+ = (\mu - \alpha) \text{ and } \mu_- = (\mu + \alpha)$$

then

$$\mu_+' = (\mu' - \alpha') \text{ and } \mu_+'' = (\mu'' - \alpha'')$$

$$\mu_-' = (\mu' + \alpha') \; \mu_-'' = (\mu'' + \alpha'')$$

then

$$3\mu_\pm'' = I\,(\mu_\pm' + 2)^2$$

ACCURACY AND LIMITING VALUE OF TAN δ_m

When $\mu_\pm' = 0{\cdot}5$,

$$\mu_\pm'' = \frac{I\,6{\cdot}25}{3} \doteqdot 2I$$

$$\text{and } \tan\delta_m = \frac{2I}{\mu'} \doteqdot 4I = \frac{2}{A}\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right)$$

$A = -2{\cdot}34 \times 10^5 \times r^3$ (where r = radius of ferrite sphere in metre

If r is taken as 1 mm., $A = -2{\cdot}3 \times 10^{-4}$. The appropriate value $\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right)$ in this case is 50 kc/s as in the case of the E_{010} cavity, a

$$\tan\delta_m = 5 \times 10^{-2}$$

If r is taken as 2·1 mm., $\tan\delta_m = 5 \times 10^{-3}$

Even using a sphere of 5 mm. diameter, which seems to be rather lar for this cavity, the sensitivity of measuring the loss is a factor of 10 dow on that obtained for the dielectric constant in the E_{020} cavity.

DISCUSSION OF SPECIMEN SHAPE

It has been suggested that dielectric measurements could be made co veniently by using a spherical sample in the H_{112} cavity, thus making t one cavity sufficient for both magnetic and dielectric measurement Although this might be adequate in a particular case, it was decid against imposing this limitation on the apparatus as a large volume specimen can be used in an E_{010} cavity before there is an apprecia departure from the perturbation condition. The E_{010} cavity is also preferre rom the practical point of view, as small rods of square cross-section, hich are suitable for use in this type of cavity, can be prepared more onveniently than spheres.

ONCLUSION

he apparatus described above has been developed to the stage where outine measurements of dielectric constant and loss can be made on each atch of ferrite produced. The measurement of the tensor permeability equires more time to prepare the specimens as well as making the easurements, which means that its application has to be restricted to epresentative samples.

These dielectric measurements, combined with a routine measurement f the value of saturation magnetization, and the selected permeability r line width measurements will provide sufficient information to determine the suitability of the ferrite for any given application.

CKNOWLEDGEMENTS

he author wishes to make acknowledgements to Dr. R. J. Benzie, who itiated and directed the development of the apparatus, and to Mrs. H. ache who was jointly responsible for the work carried out.

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BOOK REVIEWS

HE PRACTICAL HI-FI HANDBOOK

y Gordon J. King, Assoc. Brit.I.R.E. M.I.P.R.E. M.T.S. Odhams Press Ltd. 25s. net

his book sets out to provide "practical nd up-to-date information on the various inds of hi-fi equipment", and in this it cceeds admirably.

The author stresses, in his foreword, the ifficulty, not of deciding what to include ut what to omit in planning a book of mited size, and his choice of contents has een, on the whole, a happy one.

Recording on disc and tape microphones mplification, record playing equipment and udspeakers are dealt with in separate hapters, and although the treatment given o any one subject is not as exhaustive as ne would like, the available space has been lly utilized and the information given sufficient to give the reader a good insight into the fundamental problems and to encourage him to further reading. It is a pity, in this respect, that no bibliography is included and that even references to other works on the subject are almost non-existent.

Misprints are few and far between, but the contractions for micro- and milie- (μ and m) seem to have got mixed up in some places, and the contraction for micro-farad is not μ but μF. Apart from such details, however, the book is attractively produced and can be recommended to those who wish for an introduction to a subject which is becoming increasingly popular.

PHYSICS AND MATHEMATICS IN ELECTRICAL COMMUNICATION

by James Owen Perrine. Chapman and Hall Ltd 50s. net

The only electrical problems dealt with in this book are the simple L, C, R, circuit and the long line with series and shunt resistance.

It therefore scarcely fulfils the hopes raised by the title and by the fact that it contains over 250 large pages. Some light is thrown on the matter by the sub-title which claims it to be a treatise on conic section curves, exponentials, alternating current, electrical oscillations and hyperbolic functions, but it is not a treatise at all, as it contains very little in the way of formal proof and quotes most of the mathematical results used.

It is, in fact, based on the assumption that the average student cannot understand circular and hyperbolic functions as they occur in conventional textbooks, and it might well be a verbatim report of a series of lectures aimed at removing the difficulties by various simplifying explanations. As such, the style leaves much to be desired when it is printed and it is not likely to find much favour on this side of the Atlantic.

The philosophy of the author can best be illustrated from the preface in which he says: "learning is a slow and continuing process. It takes time to acquire knowledge. Learning is not a 'one shot' affair. New ideas are met for the first time, and then need to be meditated on many times. The ideas and concepts to be learned need to be expressed with a wide variety of different words and points of view. This teaching doctrine does not mean verbosity and redundancy. Repetitions and several reviews are necessary. Brevity may be the soul of wit, but not of learning and understanding. Ideas require a long time to sink in. Hence there are more words, drawings, curves and tables per idea than ordinarily found in technical treatises."

There are indeed! At one point in the text, he says "it is possible that the reader may think that the expository and narrative style herein used is repetitious and a bit redundant" and then proceeds to justif himself by a statement that is as verbos and redundant as the above quotation, an which is typical of the book as a whole. I is only necessary to turn the pages to se how little mathematics and how much tal the book contains, and to attempt to us the index to find out how discursive an uneven is the treatment. Over a page i taken up on a specious demonstration o Pythagoras's Theorem after which the youn schoolboy who "cuts 'cater-corner' acros an empty lot" may find that his "one bloc square vacant lot experience now begins t 'make sense'," while in the only seriou piece of mathematical analysis in the boo more than a page of cumbersome algebra manipulation is used to derive a result i line theory that could be obtained in two o three lines.

A large part of the book is taken up i demonstrating that "'*e*' has no magi mysterious, extraordinary, transcendenta imaginary or subtle meaning, and that it a number of a particular numerical value, bu it makes no contribution whatever to unde standing and insight." To establish th remarkable conclusion, there are over fort pages concerned with what the author cal geometrical retrogressions or exponenti equations.

The reader would be well advised to pa lightly over this section of the book with i digressions into many other parts of physi and to seek elsewhere for a simple treatme of the exponential function in its own righ The author found his inspiration in the wel known books "Calculus made easy" an "Exponentials made easy." It is significan however, that entertaining as these book were, they have never become generall used, for there is no easy way to math matical understanding. The present boo with its tedious repetition and verbose sty is even less likely to achieve the end th author so ardently desires.