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CONTENTS

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SOME METHODS OF LOCATING THE OPTIC AXIS IN QUARTZ

Radio communication technique to-day embraces many arts and sciences, and requires the services of the physicist as often as the electrical engineer. Particularly is this applicable to the quartz oscillator, where recent developments have required the application of a very specialised branch of optics—the study of interference patterns resulting from the passage of polarised light through doubly-refracting substances.

IT may frequently be necessary to determine the angle between the optic axis and the faces of a section of quartz, either for the purpose of cutting a piezo-electric oscillator at the desired angle, or in order to determine the nature of an existing oscillator. There are two methods which may be employed in the location of the direction of the optic axis by means of polarised light :—

(A) Viewing at right angles to the optic axis, and obtaining extinction.

(B) Viewing along the axis, or along a direction at an acute angle to the axis, and measuring the interference pattern.

The piezo-electric oscillators commonly used for wireless purposes may be divided into two classes, " simple " cuts in which the optic axis lies in the plane of the crystal, and is usually parallel to one pair of edges, and " inclined " and " skew " cuts in which the optic axis is not in the plane of the crystal. As usually prepared, crystals have a smooth matt surface, which prevents the formation of clear images during optical examination ; but special polishing of the surfaces (which is the ideal) can be avoided by using a liquid whose refractive index is equal to that of quartz (methyl salicylate). This liquid fills in the irregularities in the surface of the quartz, and owing to the equality of the refractive indices there is no deviation of the rays in passing into it from the quartz ; on leaving the surface of the liquid, the light is refracted exactly as if it were leaving quartz.

The optical apparatus must include polarising and analysing Nicol prisms,* an eyepiece with calibrated scale, means of focussing converging light on the specimen (the polarising Nicol is usually arranged to give a parallel beam), and a rotating table calibrated in degrees. All of these are to be found in a petrographic microscope, and the description of the methods will be given with reference to such a microscope, as illustrated diagrammatically in Fig. 1.

* A new alternative to the Nicol prism for polariser or analyser is the " Polaroid " screen (marketed by Polaroid Products Ltd., 39, Lombard Street, E.C. 3) ; this consists of a sheet made up of sub-microscopic polarising crystals, all similarly oriented, cemented between glass plates. The efficiency of polarisation is said to be better than 99½ per cent., but the residual light is the extreme red and blue constituents of the spectrum, giving a faint purple light in place of extinction. The advantages over Nicols are lower cost and the possibility of obtaining a much larger area.

Method (a).

If the specimen is fairly thick (say, 2 mm. upwards), it may be stood on edge on the rotating table of the microscope. After removing the converging lens, so as to give a parallel beam, the Nicol prisms should be crossed to give a dark field, and the microscope may be focussed on the top edge of the crystal. (The Bertrand lens will not be required with parallel light.) In general the edge of the crystal will then appear light against the dark field, but in a complete revolution of the turntable four positions will be found, at intervals of 90 degrees, at which the crystal also appears dark. In these four positions the optic axis of the specimen is parallel to the plane of polarisation of one or other of the crossed Nicols. The axis of the calibrated scale in the eyepiece should be set up parallel with the plane of one of the Nicols; then by setting the turntable so that the edge of the specimen is parallel to the axis of the scale, and comparing this setting with the angles at which light was extinguished, two alternative values for the angle between the optic axis and the edge of the specimen are obtained (angles α and β in Fig. 2). The ambiguity between these two values, the sum of the two being 90°, cannot be avoided in this method. It is therefore to be recommended only when the direction of the optic axis is known roughly, within 10° or 15°, and is not too near 45°.

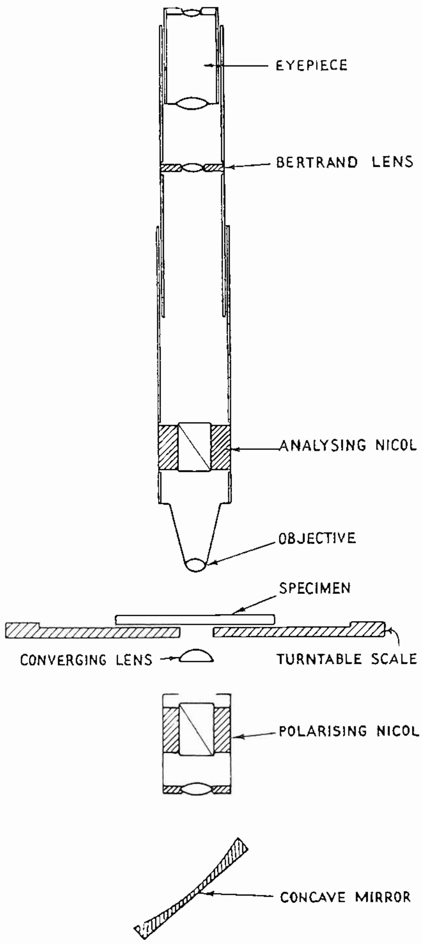

FIG. 1.

Method (b).

The alternative method involves considerably more calculation, but is free from the ambiguity of the method described above. It depends upon the fact that if a beam of convergent light shines through a slice of a doubly-refracting crystal, with the axis of the beam parallel to the optic axis of the specimen, there is an interference pattern produced consisting of concentric coloured rings (if white light is used—or light and dark rings with monochromatic light) and a black cross passing through the centre (see Fig. 3). The centre of this figure is called the Melatope, and the arms of the black cross the Isogyres. The isogyres are parallel to the planes of the Nicols, and occur because, as seen in the first method, a specimen of quartz placed between crossed Nicols cuts off the light wherever a plane containing the optic axis of the quartz coincides with the plane of either of the Nicols. The centre of the pattern, the melatope, represents those rays of the beam of light which converged upon the optic axis of the specimen.*

* The name "melatope" means "black spot"; for in crystals which are merely doubly refracting (e.g., mica) the ordinary and extraordinary rays travel with equal velocity along the

If a section of quartz cut at right angles to the optic axis is placed flat on the microscope stage, so that the optic axis is parallel to the axis of the microscope, the pattern of Fig. 3 will be obtained centrally in the field of view. But if the section be inclined to the optic axis, the pattern formed will still be centred

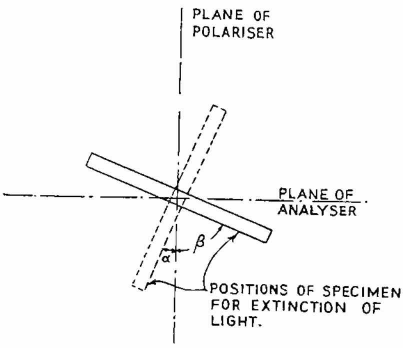

FIG. 2.

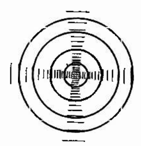

FIG. 3.

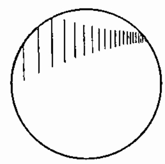

FIG. 4A.

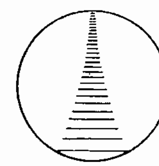

FIG. 4B.

about the optic axis of the specimens and so will fall off the centre of the field of view. If the inclination is large, only the edge of the pattern, perhaps only one of the isogyres, will appear in the field of view, as in Fig. 4A or 4B. The isogyre always points towards the centre of the pattern, and by rotating the specimen each isogyre in turn can be made to pass through the field of view. It is clear that the distance of the melatope from the centre of the field of view depends upon the inclination of the optic axis, so that by geometry and the constants of the microscope the one can be calculated from the other. If the melatope is distant D from the centre of the field of view, the angle V between the optic axis of the specimen and the axis of the microscope is given by the formula,

$$\sin V = \frac{D}{1.55 \times K} \quad \ldots \quad (1)$$

where K is a constant of the given microscope.*

If the constant K is not known—it will be different for each objective in a given microscope—it can be determined by observing specimens having a known inclination to the optic axis. For this purpose a piece of mica may be used ; for mica has *two* optic axes, and so produces a twin interference pattern, with the axes at equal angles to the cleavage planes. If D_0 be the distance from the centre of the field of view to either of the pair of melatopes seen with mica,

$$K = \frac{D_0}{0.5825} \quad \ldots \quad (2)$$

As yet nothing has been said of the method of setting up the microscope. Referring to Fig. 1, the converging lens between the polarising Nicol and the stage

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optic axis, so that in this direction there is complete extinction of light between crossed Nicols. But quartz has also the power of optical rotation, in addition to double refraction, so that the centre of the pattern with a quartz specimen will not be a black spot, but one whose colour depends upon the thickness, just the same as when viewing in the ordinary polariscope.

* The value 1.55 has been taken for the mean refractive index of quartz. The values given by Kaye and Laby for sodium light are 1.5443 and 1.5544 for the ordinary and extraordinary rays respectively ; the variations over the whole range of the visible spectrum are about $\pm \frac{1}{2}$ per cent. in each case, so that the figure of 1.55 is sufficiently accurate for our purpose.

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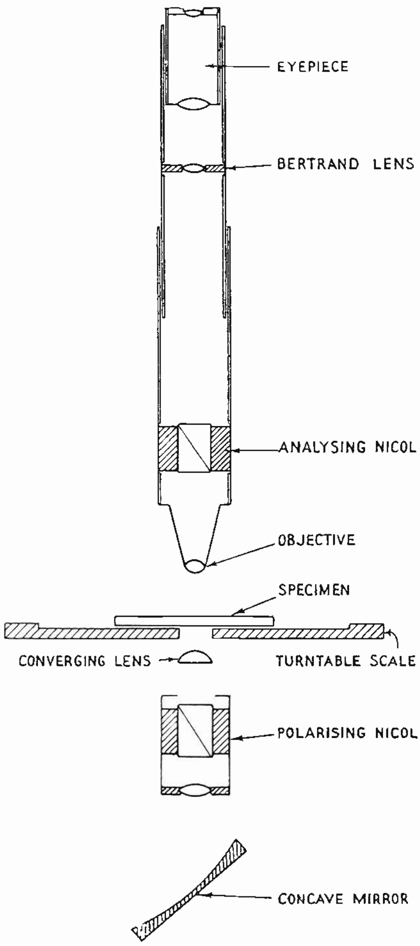
If the specimen is fairly thick (say, 2 mm. upwards), it may be stood on edge on the rotating table of the microscope. After removing the converging lens, so as to give a parallel beam, the Nicol prisms should be crossed to give a dark field, and the microscope may be focussed on the top edge of the crystal. (The Bertrand lens will not be required with parallel light.) In general the edge of the crystal will then appear light against the dark field, but in a complete revolution of the turntable four positions will be found, at intervals of 90 degrees, at which the crystal also appears dark. In these four positions the optic axis of the specimen is parallel to the plane of polarisation of one or other of the crossed Nicols. The axis of the calibrated scale in the eyepiece should be set up parallel with the plane of one of the Nicols; then by setting the turntable so that the edge of the specimen is parallel to the axis of the scale, and comparing this setting with the angles at which light was extinguished, two alternative values for the angle between the optic axis and the edge of the specimen are obtained (angles α and β in Fig. 2). The ambiguity between these two values, the sum of the two being 90°, cannot be avoided in this method. It is therefore to be recommended only when the direction of the optic axis is known roughly, within 10° or 15°, and is not too near 45°.



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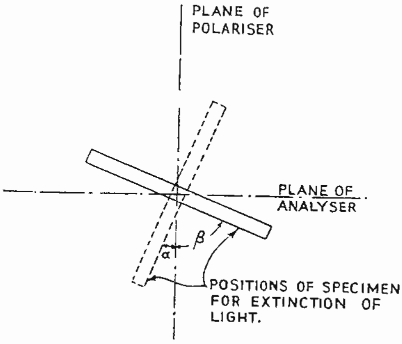

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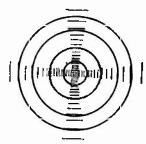
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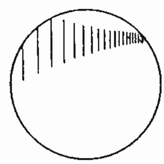
FIG. 4A.

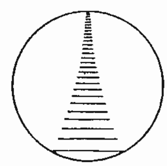
FIG. 4B.

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$$\sin V = \frac{D}{1.55 \times K} \qquad (1)$$

where K is a constant of the given microscope.*

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θ_2 on the turntable scale. (N.B.—The first position must be the exact centre of the scale; the second position should not be at the extreme edge, since the various lenses tend to introduce distortion at the edge of the field of view.) Then the differences $\theta_1 - \theta_2$ is an angle which will be denoted by a. A number of readings should be taken of both θ_1 and θ_2, and rather higher accuracy can be obtained by reading θ for two positions of the isogyre equally spaced from the centre of the scale, giving the angle $2a$ and distance $2d$; the reading of the angle for the central position is, of course, not required in this case. We then find an angle β which is given by

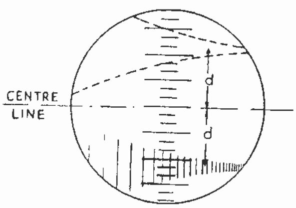

FIG. 6.

$$\sin \beta = \frac{d}{1.55 \times K} \qquad \cdots \qquad \cdots \qquad (3)$$

where K is the same constant as in equation (1). It is then necessary to find $\tan \beta$, which can be done either by looking up tables (find β from sine table and (3), then use tangent table for $\tan \beta$) or from the relation

$$\tan \beta = \frac{\sin \beta}{\cos \beta} = \frac{\sin \beta}{\sqrt{(1 - \sin^2 \beta)}} = \frac{d/1.55 \text{ K}}{\sqrt{[1 - (d/1.55 \text{ K})^2]}} \qquad \cdots \qquad (4)$$

If V is again the angle between the optic axis of the specimen and the axis of the microscope,

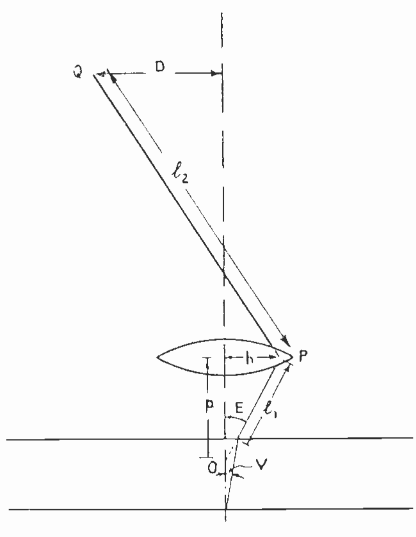

FIG. 7.

$$\tan V = \frac{\tan \beta}{\sin a} \qquad \cdots \qquad \cdots \qquad \cdots \qquad (5)$$

The chief limitation in this method is the difficulty of an accurate determination of the angle a. If $2a$ is 30°, and both θ_1 and θ_2 are liable to an error of $\frac{1}{4}$°, the accuracy of a is only 1 in 60, and these represent favourable figures for 6 readings each of θ_1 and θ_2 and a specimen of 30°-40° inclination.

The following example illustrates the method using the displacement of the isogyres. In examining a particular low-temperature-coefficient piezo-electric oscillator, a value of 0.5 on the eyepiece scale was taken for d, and making 6 observations of the turntable angles giving displacement d on either side of centre, the mean value found for a was 13.9°. The microscope constant being 1.614, the angle β was 11° 32′, and $\tan \beta = 0.2041$. But $\sin a = 0.2402$, so that V = 40.3°. From the published information available, it is highly probable that the specimen was cut by the makers to a nominal angle of 41°, in quite good agreement with the measured value.

Method (c).

Where mechanism is available for tilting the specimen through a measurable angle while it is under observation (e.g., a microscope with universal stage), a more

must be brought into action, and illumination should be obtained from a source of large area, using the concave mirror on the microscope. Light from the sky is ideal, but if artificial light must be used, the source should present a uniformly illuminated surface of about 1 ft. diameter at 3 ft. 6 ins. distance from the microscope. The focal length of the objective to be used depends upon the thickness of the specimen—a 4 mm. objective is suitable for specimens up to 5 mm. thick, 16 mm. objective from 5 mm. to 20 mm. thick, and so in proportion for thicker sections. All the upper part of the microscope should now be cleared—the eyepiece removed, and the analysing Nicol and the Bertrand lens moved out on their slides. The objective is then focussed so that the whole field of view is filled by the incident light; this usually requires the objective to be set quite close to the specimen. On inserting the analysing Nicol (set at right angles to the polariser)* the interference pattern is seen, and on adding the Bertrand lens and eye-piece with graduated scale a magnified image is obtained of both interference pattern and scale; this must be focussed by drawing out the tube of the microscope. The accuracy of the measurement is not appreciably affected by variations in the setting of objective and eyepiece positions.

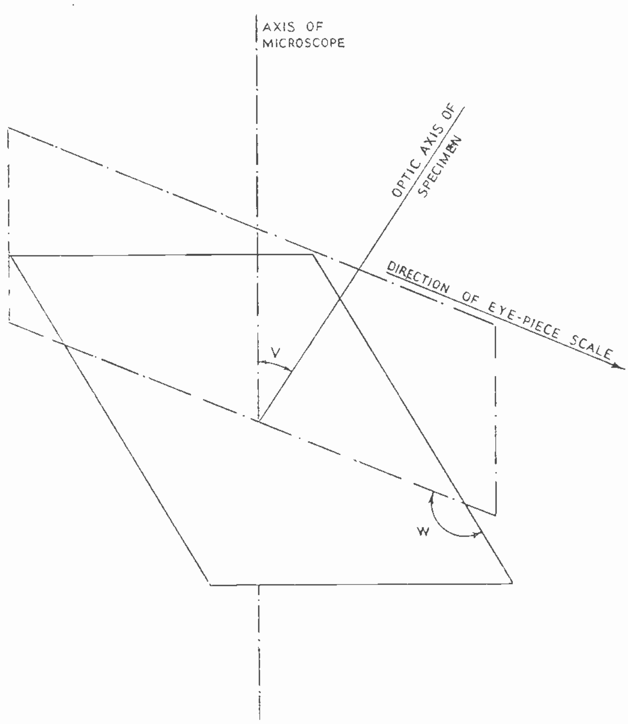

Fig. 5.

If the melatope falls within the field of view, the angle V between the optic axis of the specimen and the axis of the microscope is found at once from equation (1). The eyepiece scale should be set to coincide with the 0°-180° line of the turntable scale, so that the direction of the plane which contains the optic axis and is at right angles to the surface of the specimen is known. Thus in Fig. 5 the angle V between the optic axis and the normal to the surface of the specimen is found by equation (1), and the angle W between the principal plane through the optic axis and the edge of the crystal is found directly from the readings of the turntable scale.

If the inclination of the optic axis of the specimen is so great that the melatope falls outside the field of view, its distance must be deduced from the positions of the isogyres. First rotate the specimen till one of the isogyres lies across the centre of the eyepiece scale, and read the turn-table scale, θ_1. Then rotate the specimen till the isogyre has moved a distance d across the eyepiece scale (see Fig. 6), giving a reading

* If, by mistake, the analyser is parallel to the polariser, the pattern will be seen with a *white* cross; in case of doubt, set for darkness with specimen removed.

θ_2 on the turntable scale. (N.B.—The first position must be the exact centre of the scale; the second position should not be at the extreme edge, since the various lenses tend to introduce distortion at the edge of the field of view.) Then the differences $\theta_1 \sim \theta_2$ is an angle which will be denoted by α. A number of readings should be taken of both θ_1 and θ_2, and rather higher accuracy can be obtained by reading θ for two positions of the isogyre equally spaced from the centre of the scale, giving the angle 2α and distance $2d$; the reading of the angle for the central position is, of course, not required in this case. We then find an angle β which is given by

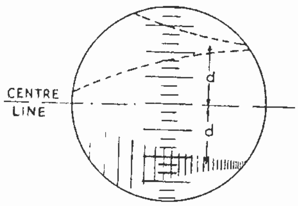

FIG. 6.

$$\sin \beta = \frac{d}{1.55 \times K} \quad \ldots \quad \ldots \quad (3)$$

where K is the same constant as in equation (1). It is then necessary to find $\tan \beta$, which can be done either by looking up tables (find β from sine table and (3), then use tangent table for $\tan \beta$) or from the relation

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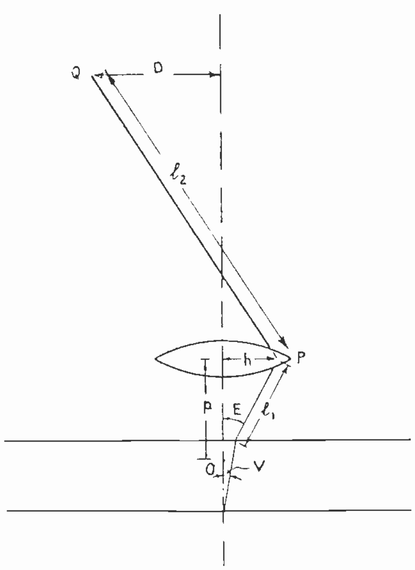

FIG. 7.

The chief limitation in this method is the difficulty of an accurate determination of the angle α. If 2α is 30°, and both θ_1 and θ_2 are liable to an error of $\frac{1}{4}$°, the accuracy of α is only 1 in 60, and these represent favourable figures for 6 readings each of θ_1 and θ_2 and a specimen of 30°-40° inclination.

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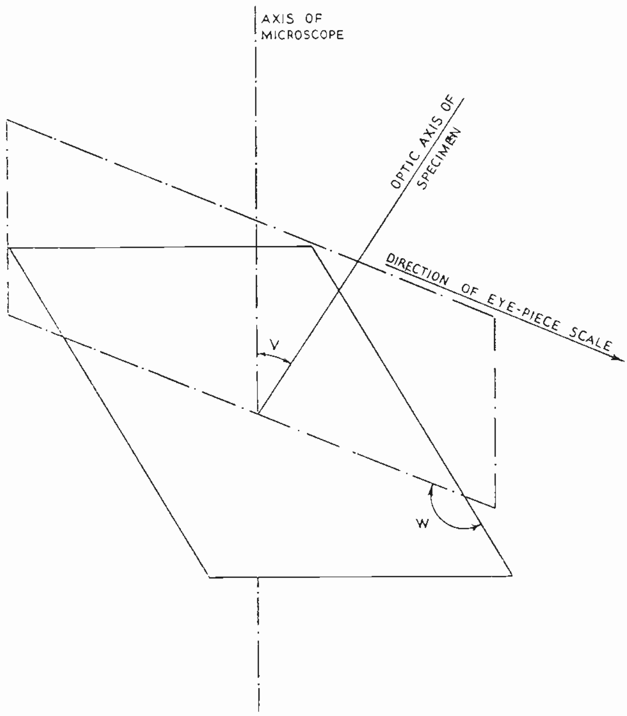

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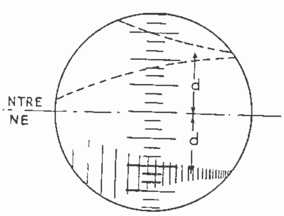

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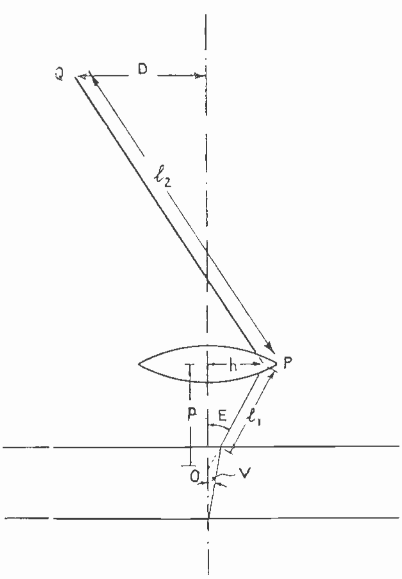

FIG. 7.

The chief limitation in this method is the difficulty of an accurate determination of the angle α. If 2α is $30°$, and both θ_1 and θ_2 are liable to an error of $\frac{1}{4}°$, the accuracy of α is only 1 in 60, and these represent favourable figures for 6 readings each of θ_1 and θ_2 and a specimen of $30°$-$40°$ inclination.

The following example illustrates the method using the displacement of the isogyres. In examining a particular low-temperature-coefficient piezo-electric oscillator, a value of 0.5 on the eyepiece scale was taken for d, and making 6 observations of the turntable angles giving displacement d on either side of centre, the mean value found for α was $13.9°$. The microscope constant being 1.614, the angle β was $11° 32'$, and tan $\beta = 0.2041$. But sin $\alpha =$ 0.240, so that $V = 40.3°$. From the published information available, it is highly probable that the specimen was cut by the makers to a nominal angle of $41°$, in quite good agreement with the measured value.

Method (c).

Where mechanism is available for tilting the specimen through a measurable angle while it is under observation (e.g., a microscope with universal stage), a more

direct method is to tilt the specimen until the melatope falls in the centre of the field of view. Owing to refraction, the angle V of the optic axis to the normal to the surface of the specimen is not equal to the angle δ of tilt, but is given by

$$\sin V = \frac{\sin \delta}{1.55} \qquad (6)$$

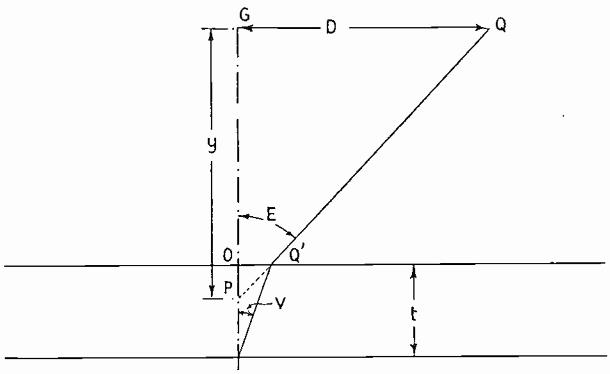

FIG. 8A.

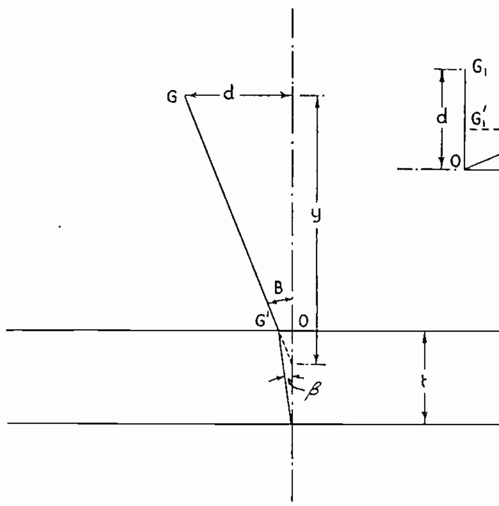

FIG. 8B.

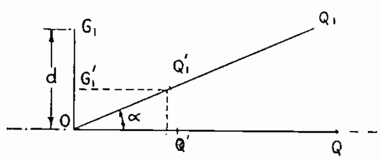

FIG. 8C.

If the specimen can be rotated after it has been tilted, to ensure correct centring, this is probably the most accurate method ; but it requires a mechanical system which is not commonly available.

The author wishes to thank Mr. Myers, of the Marconi Company, for information concerning the petrographic microscope.

APPENDIX. I.

Proof of the Formulae employed.

Since the Bertrand lens and eyepiece serve only to form a magnified image, we may consider the image formed by the objective, as shown diagrammatically in Fig. 7 for the case of the melatope falling within the field of view. The ray O P Q drawn in the diagram is supposed to be the one which travels parallel to the optic axis in the specimen, and forms the melatope in the image, so that D is the distance of the melatope from the axis of the microscope. It is clear that if, for various angles E of the emergent ray, the lengths l_1 and l_2 of ray from specimen to lens and lens to image are constant, then $D/l_2 = h/l_1 = \sin E$. If the focussing of the microscope is left unaltered, it might be thought that the distance p rather than l_1 would be constant ; but this would imply that the centre and outside of the field of view could not be simultaneously in focus. Actually the lenses of the objective are so designed as to correct this effect, and we may assume that $D = K \sin E$ (Abbe's sine law is fundamental in the design of optical instruments). But if E is the angle of the emergent ray and V the angle of the ray within the quartz, $\sin E = n \sin V$ where n is the refractive index of quartz, and may be taken as 1.55. Hence $D = K \sin E = n K \sin V$, giving equation (1) of the text.

When the melatope falls outside the field of view, the sine law no longer holds, since the microscope is not focussed for the melatope, but for the portion of an isogyre which falls within the field. Thus in Fig. 8A it is the axial distance y that will remain constant, not the ray length Q′ Q. Consequently we have $D = y \tan E$ (instead of $D = PQ \sin E$). The movement of the isogyre within the field of view, however (see Fig. 8B), must still follow a sine law, so that $d = y \sin B = n\, y \sin \beta$ where B and β are the angle of emergence and angle within the quartz of the ray forming the isogyre. Turning to the plan view in Fig. 8C, let Q′, Q'_2 and G′ be the points of emergence from the quartz of the rays forming Q, Q_1 and G_1 in the image ; the points of emergence from the quartz are to be used in preference to the points of the image, since G_1 falls within and Q outside the field of view. We have already

$$D = y \tan E \quad \text{(I)}$$

$$d = y \sin B = n\, y \sin \beta \quad \text{(II)}$$

From Fig. 8C

$$\frac{O\,G'_1}{O\,Q'_1} = \sin \alpha \quad \text{(III)}$$

and if t is the thickness of the quartz

$$\left.\begin{aligned} O\,Q'_1 &= t \tan V \\ \text{and} \quad O\,G'_1 &= t \tan \beta \end{aligned}\right\} \quad \text{(IV)}$$

By combining (III) and (IV)

$$\tan \beta = \tan V \,.\, \sin \alpha \quad \text{(V)}$$

The relation between the distance d in the image actually formed by the eye-piece of the microscope and the angle β is, of course, governed by the same constant as relates D and V when the melatope falls within the field of view, so that we may write

$$d = n\, K \sin \beta \quad \text{(IIA)}$$

$$\left.\begin{aligned} \text{Hence} \quad \tan V &= \frac{\tan \beta}{\sin \alpha} \\ \text{where} \quad \beta &= \arc \sin \frac{d}{n\, K} \end{aligned}\right| \quad \text{(VI)}$$

which is equation (5) of the text.

APPENDIX II.

Error Due to Taper in the Specimen.

If the upper and lower faces of the specimen are not parallel, but are inclined to each other at a small angle, this will cause an error whose magnitude depends upon the relation between the directions of the taper of the specimen and the inclination of its optic axis. Let δ be the small angle between the faces, measured in the plane which contains the inclination of the optic axis ; then if E is the angle which the ray emerging from the melatope makes with the normal to the upper surface, $E + \delta$ is its angle to the axis of the microscope (assuming the specimen is resting squarely on its lower surface). Then in the derivation of equation (1) of the text we shall have $D = K \sin (E + \delta)$ in place of $D = K \sin E$, or in deriving equation (5) we shall replace equation (I) of Appendix I by $D = y \tan (E + \delta)$. Thus a correct value of V could be calculated for a specimen tapered at a known angle in the plane containing the inclination of its optic axis.

CATHODE RAY OSCILLOGRAPH ELLIPSES

These notes are written for the benefit of engineers who have to use a cathode ray oscillograph to investigate the phase and amplitude relations between two sinusoidal voltages of the same frequency. The problem is a well known one and there is nothing original in these notes, but there seemed to be a need for a simple statement of the rules which are used to interpret the ellipses obtained. It is assumed that in any given experiment the signal voltages of the same frequency have constant though in general different amplitudes with a constant phase difference between them. The analysis can easily be extended to the case where the amplitudes or phase are slowly varying by applying it in turn to the succession of instantaneous ellipses obtained, but no attempt is made here to deal with the problem of harmonics in either of the signals. It is also assumed that the oscillograph is perfect in the sense that the pairs of plates are strictly at right angles and that there is no origin distortion or any other source of non-linearity present.

Definitions and General Analysis.

THE two directions of the displacement for the pairs of plates taken separately can be taken to define a set of rectangular axes X′OX, Y′OY, see Fig. 1. The x displacement alone can be represented by

$$x = a \sin pt \qquad (1)$$

where $p = 2\pi f$ and f is the frequency, while the amplitude a is shown by OA and OA′.

The y displacement alone can be represented by

$$y = b \sin (pt+\delta) \qquad (2)$$

where the amplitude b is shown by OB and OB′ and δ is the constant phase difference between the two signals.

Eliminating pt from equations (1) and (2) gives

$$b^2x^2 - 2\,ab \cos \delta\, xy + a^2y^2 = a^2b^2 \sin^2\delta \qquad (3)$$

which is the equation of the ellipse obtained when both signals are put on together. The major and minor axes of this ellipse are not in general along X′OX and Y′OY. The phase angle δ has to be determined, by measurements on the ellipse. The spot of the oscillograph will trace out this ellipse with a frequency of repetition f. Unless f is small, persistence of vision will make the ellipse appear as a steady pattern and it will be impossible to tell in which direction it is being traced.

If the position P of the spot at any time t is represented in terms of its polar co-ordinates r and θ see Fig. 1,

then $$x = r \cos \theta = a \sin pt \qquad (4)$$

and $$y = r \sin \theta = b \sin (pt + \delta) \qquad (5)$$

and it can be seen that if $\sin \delta$ is positive the ellipse is traced out in a clockwise direction and in a counterclockwise direction when $\sin \delta$ is negative (*cf.* Appendix).

The amplitudes a and b can be measured directly by measuring the lengths of the straight lines $2a$ and $2b$ when each signal is put on alone, and the problem becomes the determination of the phase angle δ from the values of a and b and the various measurements that can be made on the ellipse. The latter are set out in a series of propositions and in practice the method chosen will depend upon the accuracy with which the ellipse can be measured up and the goodness of the ellipse obtained experimentally.

Propositions.

A. It is obvious that the ellipse must touch the rectangle described on A′oA, B′oB and if the points of contact are M,M′ and N,N′ (see Fig. 2), then AM = A′M′ and BN = B′N′ are obtained by putting $x = a$ and $y = b$ respectively in equation (3).

Thus AM = A′M′ = $b \mid \cos \delta \mid$ .. .. (6)

and BN = B′N′ = $a \mid \cos \delta \mid$ .. .. (7)

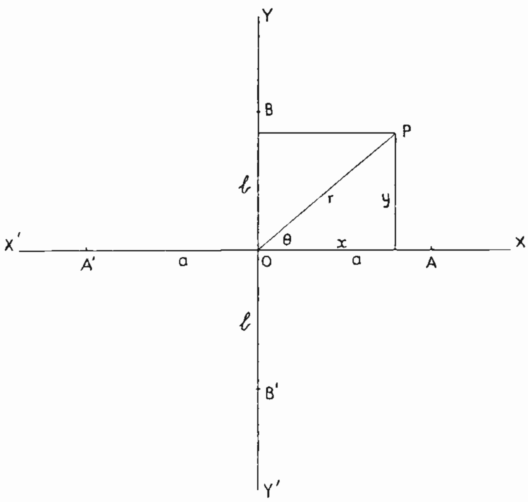

FIG. 1.

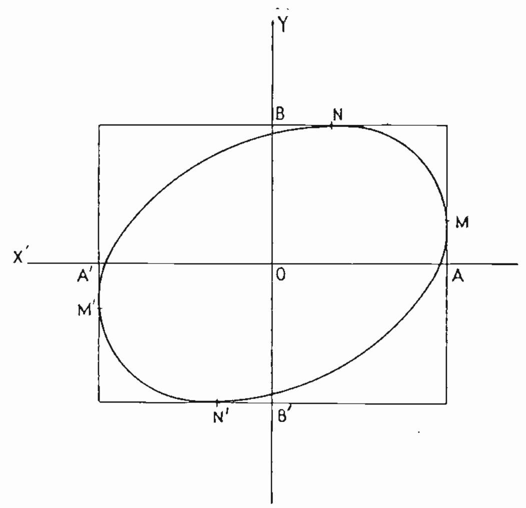

FIG. 2.

where as usual the straight brackets represent the modulus or the numerical value of the enclosed function. If therefore we can measure AM or BN we can use the known values of a and b to deduce the numerical value of cos δ.

B. We can also measure the intercepts on X′OX and Y′OY where the ellipse cuts the axes. These are shown in Fig. 3 as OR, OR′ and OQ, OQ′. We have by putting $y = 0$ and $x = 0$ respectively in equation (3)

OR = OR′ = $a \mid \sin \delta \mid$ .. .. (8)

OQ = OQ′ = $b \mid \sin \delta \mid$ .. .. (9)

Thus we have a method of determining the numerical value of sin δ. These two propositions, A and B, give theoretically two very simple methods of determining δ, but they are not usually applied in practice, because they imply an ellipse which can be measured up accurately and is free from even the small amount of distortion which is obtained normally on an oscillograph. The ambiguity of the quadrant in which δ lies can be resolved partly by considering the orientation of the ellipse, while there remains the uncertainty due to our ignorance of the direction in which the ellipse is being traced. This aspect of the problem is discussed later.

C. In general a more accurate measurement of δ can be made by considering the length of the major and minor axes of the ellipse and their orientation with respect to the axes X′OX and Y′OY.

In Fig. 4 the major and minor axes are shown as S′OS and T′OT in which $OS = OS' = a_0$ and $OT = OT' = b_0$ and $\angle SOX = \theta_0$.

Now if A is the area of the ellipse it can be shown that

$$A = \pi a_0 b_0 = \pi ab \;|\sin\delta|$$

so that $$|\sin\delta| = \frac{A}{\pi ab} \qquad (10)$$

or $$|\sin\delta| = \frac{a_0 b_0}{ab} \qquad (11)$$

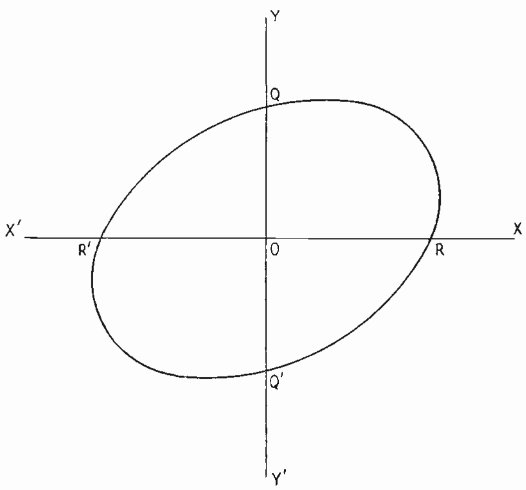

FIG. 3.

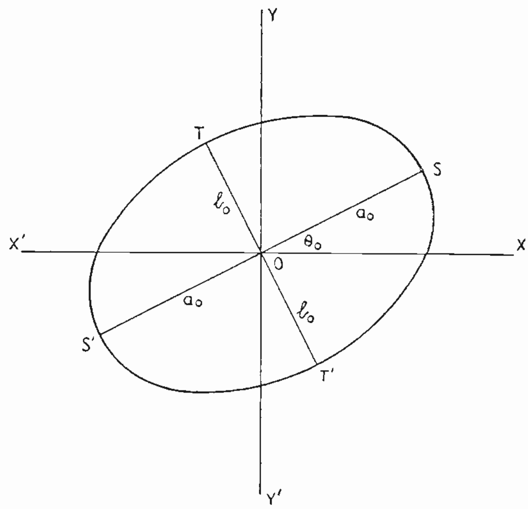

FIG. 4.

If the ellipse is well shaped so that a_0 and b_0 can be measured with reasonable accuracy equation (11) may be useful to determine δ.

If a_0 and b_0 are rather uncertain it may be useful to measure the area of the ellipse, e.g., by tracing it on to a piece of paper and using a planimeter or counting squares, and then to use equation (10).

It can also be shown that

$$\tan 2\theta_0 = \frac{2ab\cos\delta}{a^2 - b^2} \qquad (12)$$

so that if θ_0 can be estimated with reasonable accuracy $\cos\delta$ can be found provided a is not nearly equal to b, when θ_0 approaches $\frac{\pi}{4}$ or becomes ill-defined.

In practice it is often useful to get a cross check between these three methods given in equations (10), (11) and (12).

The Ambiguity in the Value of δ.

By studying in greater detail the orientation of the axes of the ellipse some light is thrown on the question of the quadrant in which δ lies. Ultimately the ambiguity due to our ignorance of the direction in which the ellipse is being traced can only be resolved by appealing to some further experimental procedure, e.g., being able to control the phase and watching the effect on the shape and orientation of the ellipse. In problems in which the phase difference may be greater than 2π the method presupposes that there is other experimental evidence available for deciding the phase angle to the nearest multiple of 2π.

The expression for tan $2\theta_0$ given in equation (12) gives two values of θ_0 which are $\frac{\pi}{2}$ apart, corresponding to the major and minor axes of the ellipse. In order to find out which is which we will make the convention that the signal with the greater amplitude is made to give the x displacement, i.e., in our notation $a > b$.

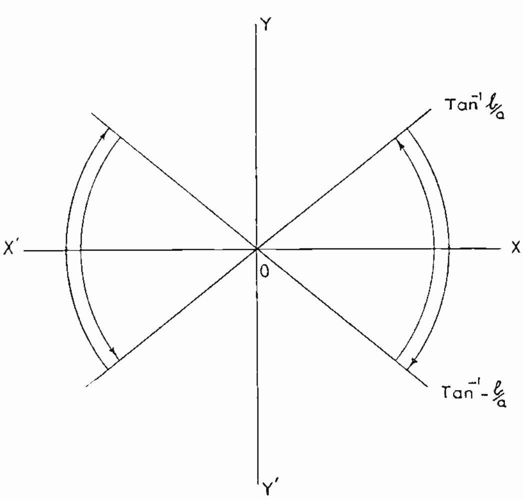

FIG. 5.

Now when $\delta = 0$ it is obvious that $a_0 = \sqrt{a^2+b^2}$ $b_0 = 0$ and $\theta_0 = \tan^{-1}\frac{b}{a}$ in the 1st and 3rd quadrants, and the ellipse degenerates into a straight line.

When $\delta = \frac{\pi}{2}$ $a_0 = a$ along the x axis $b_0 = b$ along the y axis and $\theta_0 = 0$, the major axis of the ellipse lying along the x axis.

When $\delta = \pi$ $a_0 = \sqrt{a^2+b^2}$ $b_0 = 0$ and $\theta_0 = \tan^{-1}\left(\frac{-b}{a}\right)$ in the 2nd and 4th quadrants, the ellipse again degenerating into a straight line.

When $\delta = \frac{3\pi}{2}$ $a_0 = a$ along the x axis.

$b_0 = b$ along the y axis

and $\theta_0 = 0$, the major axis of the ellipse again lying along the x axis. It can easily be seen that for negative values of δ the picture is the same as for the equivalent positive value of δ except that the direction in which it is traced is reversed, which, as we have seen, cannot normally be detected by eye.

It follows that as δ increases from 0 to 2π the major axis oscillates from the line $\tan^{-1}\frac{b}{a}$ through the x axis to the line $\tan^{-1}\left(\frac{-b}{a}\right)$ and back again as indicated by the arrows in Fig. 5.

All we can say without further experimental evidence is that when θ_0 lies in the 1st and 3rd quadrants $\delta = \pm \alpha \pm 2n\pi \quad n = 0, 1, 2,$ etc., and that when θ_0 lies in the 2nd and 4th quadrants

$$\delta = \pm \alpha \pm (2n + 1)\pi \qquad n = 0, 1, 2, \text{ etc.}$$

where $0 < \alpha < \frac{\pi}{2}$

and $\alpha = \cos^{-1}\left[\frac{a^2 - b^2}{2ab} \mid \tan 2\theta_0 \mid\right]$

The condition for quadrature is always given by the major axis lying along the x axis and the minor axis along the y axis.

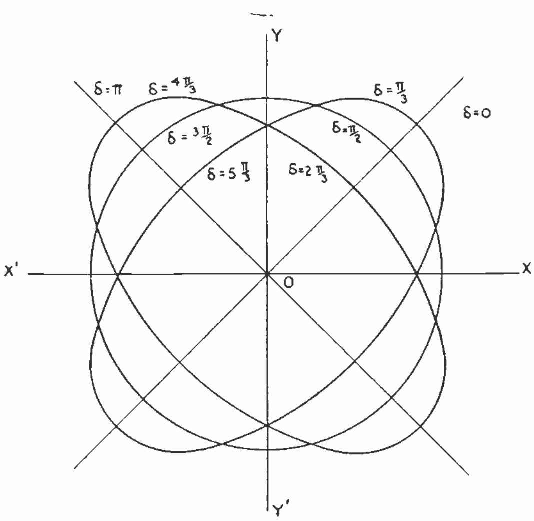

FIG. 6.

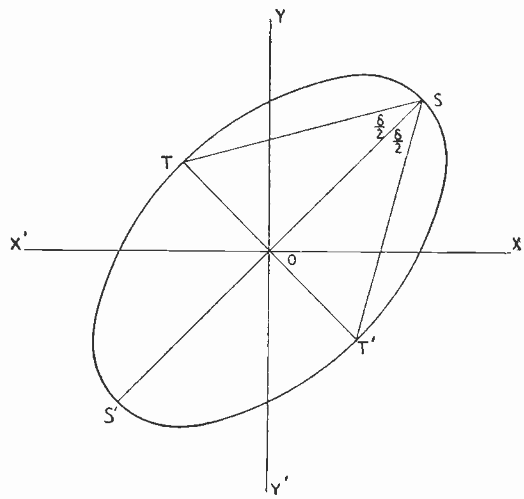

FIG. 7.

The Special Case of Equal Amplitudes.

When $a = b$ equation (12) gives $\tan 2\theta_0 = \infty$ independent of the value of δ except when $\delta = \frac{\pi}{2}$ and θ_0 is indeterminate corresponding to a circular pattern. In general $2\theta_0 = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ and $\theta_0 = \frac{\pi}{4}$ or $\frac{3\pi}{4}$, i.e., the axes of the ellipse lie along the 45 deg. positions.

In this special case the axes no longer oscillate, the straight line along the $\theta_0 = \frac{\pi}{4}$ line for $\delta = 0$ opens out into an ellipse, the major axis along this line gradually reducing from $a\sqrt{2}$ to a as the minor axis increases from 0 to a when the ellipse becomes a circle for $\delta = \frac{\pi}{2}$, and then it closes in as the major and minor axes exchange places until it degenerates to a straight line along the $\theta = \frac{3\pi}{4}$ line when $\delta = \pi$. The process is retraced as δ increases from π to 2π. The cycle of operations is shown in Fig. 6. In this case quadrature is always shown by a circle.

There is one other useful property in this special case which replaces the $\tan 2\theta_0$ relation of equation (12). The axes of the ellipse can be shown to be

$$a_0 = \sqrt{2a} \left| \cos \frac{\delta}{2} \right| \quad \ldots \quad (13)$$

$$b_0 = \sqrt{2a} \left| \sin \frac{\delta}{2} \right| \quad \ldots \quad (14)$$

which yield values of δ from measurements of a_0 and b_0. But we also have

$$\frac{a_0}{b_0} = \cot \frac{\delta}{2}$$

Now in Fig. 7 it will be seen that $\cot \text{OST} = \frac{a_0}{b_0}$

so that $\text{OST} = \frac{\delta}{2}$ or $\angle \text{TST}' = \delta$

so that the minor axis subtends an angle δ at the extremities of the major axis. This is a most useful property since it gives a direct measure of δ, and in many experiments it is possible to control the amplitudes and make them equal without altering the phase angle between the signals.

Strictly speaking in this argument we have considered the particular case of $0 < \delta < \frac{\pi}{2}$. In the general case we ought to replace δ by α where $0 < \alpha < \frac{\pi}{2}$

and then $\delta = \pm \alpha \pm 2n\pi$ Major axis in 1st and 3rd quadrants.
or $\delta = \pm \alpha \pm (2n+1)\pi$ Major axis in 2nd and 4th quadrants.

Appendix.

The statement made earlier that the direction of tracing of the ellipse is clockwise when $\sin \delta$ is positive, and counterclockwise when $\sin \delta$ is negative can be seen graphically or analytically as follows :—

From equations (4) and (5)

$$\tan \theta = \frac{b}{a} \frac{\sin (pt + \delta)}{\sin pt} = \frac{b}{a} \cos \delta + \frac{b}{a} \cot pt \sin \delta.$$

$$\therefore \sec^2 \theta \frac{d\theta}{dt} = -\frac{bp}{a} \operatorname{cosec}^2 pt \sin \delta.$$

$$\therefore \frac{d\theta}{dt} = -\left\{\frac{bp}{a} \frac{\operatorname{cosec}^2 pt}{\sec^2\theta}\right\} \sin \delta.$$

The expression in the brackets is positive so that the sign of $\frac{d\theta}{dt}$ is that of $-\sin \delta$.

But clockwise traverse corresponds to $\frac{d\theta}{dt}$ negative and therefore to $\sin \delta$ positive.

Similarly counterclockwise traverse corresponds to $\frac{d\theta}{dt}$ positive and $\sin \delta$ negative.

G. Millington.

OSCILLATORS STABILISED BY RESONANT LINES

Communication services are now making use of frequencies too high for the convenient application of quartz control to the oscillators required. A new type of stabilisation by "resonant line" is commonly used on these frequencies. The results claimed by various workers for this type of oscillator are summarised and a review of the theory underlying this type of stabilisation indicates the limits within which it may usefully be applied.

IN the literature of the subject there appears to be an indiscriminate use of the expressions "long line" and "resonant line" in describing generators employing oscillatory circuits with distributed inductance and capacity. Although "long line" was the term first used, due to the fact that the theory of propagation of waves on lines several wavelengths or more long is commonly known as "long line theory," the phrase "resonant line" is more appropriate to the application to oscillators, since in practice the line is normally fairly short—a quarter or half wavelength. It is now the term most commonly used.

The manner in which the frequency stability of a valve oscillator is influenced by the characteristics of a resonant system connected to it (for example, a quartz crystal or a highly selective tuned circuit made up with the aid of a resonant line or any other suitable components) has been outlined in a previous paper.*

At normal frequencies we may neglect the time of transit of electrons through the valve in comparison with the period of oscillation; both the current and the dissipation of energy within the valve are then in phase with the applied voltages, and the valve must behave as a pure resistance, having zero phase-angle between total applied voltage and anode current.

If for any reason the grid voltage Vg is not exactly in phase with the anode voltage Va, it follows that the anode current Ia, which is always in phase with $Va + \mu Vg$, must differ in phase from Va alone by an amount depending on μ and the phase difference between Va and Vg. But the phase difference between Va and Ia must also be appropriate to the nature of the anode circuit, which means in practice that the frequency of oscillation must therefore differ from the resonant frequency of the anode tuned circuit by such an amount that this circuit will have the necessary phase angle. Hence the more rapid the change of phase angle of the tuned circuit with frequency, the less the change of frequency that will be produced by a given set of conditions.

The phase difference between Va and Vg referred to above may be influenced by change of load on the oscillator, change of μ resulting from variation of operating potentials on the valve (though μ is by far the most constant of the valve characteristics) or change of grid current. A similar process operates in regard to the change of frequency resulting from non-linearity of the valve characteristic, for Llewellyn† has shown that a non-linear valve resistance is equivalent to a linear resistance and a reactance in series; the valve thus has a phase-angle of its own, whose magnitude is de-

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* "Frequency Stability of Valve Oscillators," Wireless Engineer, Oct., 1936, Vol. 13, p. 539.

† "Constant Frequency Oscillators," F. B. Llewellyn, Proc. I.R.E., Vol. 19 (1931), p. 2063.

pendent on the amount of harmonic distortion occurring and the nature of the external circuit. The phase/frequency characteristic of the oscillatory circuit is therefore the criterion of the magnitude of the frequency change resulting from any variation of valve characteristics, supply voltages and loading conditions. A different category of frequency disturbance arises from change of reactance in some part of the oscillatory system. The mechanical and thermal stability of the major components of the tuning circuit—coil and condenser, resonant line, quartz crystal—are outside the scope of this discussion ; but there may be changes in the inter-electrode capacities of the valve, and in some cases a reactive component thrown back from the load circuit. If either the main circuit reactances are small compared with the reactances likely to be shunted across them, or the same effect is obtained by tapping down both valve and load on the tuned circuit, the effect of these stray reactances will be correspondingly reduced. But either method will demand a tuned circuit of low decrement in order to provide sufficient impedance in the valve anode circuit, and an oscillatory circuit of low decrement will have the steep phase/frequency characteristic which was shown above to be desirable. Thus a tuned circuit of low decrement is of assistance in minimising frequency changes of all types.

2. High "Q" Circuits and Resonant Lines as Tank Circuits.

The steepness of the phase/frequency curve of an oscillatory circuit is directly related to the ratio of oscillatory energy to power loss, which latter is commonly denoted by "Q." Electrical tuned circuits of reasonable bulk are comparatively heavily damped, and are not comparable in this respect with mechanical systems such as tuning forks and quartz crystals ; consequently the two latter have been adapted to the electrical generation of stable frequencies. Present day technique normally employs tuning forks operating at audible frequencies and quartz crystals at frequencies from about 10^5 to 5×10^6 c/s, followed by frequency multiplication where necessary.* Both are low power generators requiring to be followed by considerable power amplification.

Now the greater part of the loss in a well designed electrical tuned circuit occurs in the inductance, so that at higher frequencies, where a given reactance results from a smaller inductance using a shorter length of copper, the loss tends to decrease. Against this must be set the increased resistance of the conductors, owing to skin effect ; but as reactance increases as the frequency, but skin-effect resistance increment only as the square root of the frequency, the ratio of reactance to resistance improves with rising frequency. Increasing the cross-section of the conductor forming the inductance decreases the resistance, and at the same time increases the self-capacity, so that ultimately the same large conductor provides both the inductance and the capacity of the oscillatory circuit. On these lines two types of construction have been involved. Kolster (see "Generation and Utilisation of Ultra Short Waves in Radio Communication," Proc. I.R.E., December, 1934, Vol. 22, p. 1335) mounted two hemispherical copper shells on a copper tube, the gap between the rims of the two hemispheres controlling the capacity, and the total length of path from one edge to the other through the hemispheres and the tube fixing the inductance. One of the advantages common to both Kolster's type of construction and resonant lines

* In Germany Tourmaline crystals have been worked experimentally up to 2.5×10^8 c/s : see "Kristallsteurung für Dezimeterwellen," by H. Straubel, *Hochfrequenz und Elektroakustik*, May, 1936, p. 152,

is that their low resistance minimises the power dissipated in the oscillatory circuit, while their large bulk affords good cooling, so that a large amount of oscillatory power can be handled.

The alternative construction employs a resonant line, that is, a transmission line having uniform capacity and inductance per unit length, and of such dimensions that a standing wave system is set up. The length of such a line may be an odd number of quarter wavelengths, if one end is fed from a source of reasonably high impedance and the far end short circuited, or an even number of quarter waves if the line is open at the far end. In view of the length of line required, this system is most useful at the higher frequencies, of about 7 Mc/s upwards; dielectric losses can be made negligible, so that the Q obtained depends on the area of surface of the conductors. The characteristic impedance of a line depends upon the ratio of spacing to diameter of the conductors, so that in order to decrease the resistance only, all dimensions other than the length should be increased in the same ratio; it is well known that for minimum loss the ratio of inside to outside diameters of a concentric tube line should be approximately 3.6, and the optimum ratio of spacing to diameter for an open wire line is very similar.* A resonant line may be expected to have a temperature coefficient of frequency equal to the linear coefficient of expansion of the material of which it is constructed, i.e., 16.7×10^{-6} for copper; but two methods of temperature compensation are suggested by Hansell and Carter (*loc. cit.*). According to one method, a rod of Invar steel is to be placed within the inner tube to retain it at a constant length, part of the tube being corrugated to allow for expansion of the copper; this does not appear to have been used in practice. Alternatively a tube filled with oil or other expansible liquid is placed beside the line, or within it, and the expansion of the oil works a compensating condenser by means of a diaphragm mechanism. This is stated to require very careful initial adjustment, but to be satisfactory when once set up. Provided that a considerable bulk can be tolerated, a large value of Q is obtainable at the higher frequencies; at a frequency of 20 Mc/s a resonant line of length about 50 inches and diameter 10 inches should have a Q of about 5,500. Although these dimensions are fairly large, it must be remembered that a concentric line is practically self-screening, and can therefore be mounted in proximity to other apparatus without special precautions.

It is stated in Terman's paper (*loc. cit.*) that "the impedance of a line varies according to exactly the same law as does the impedance of an ordinary parallel resonant circuit." It is easily verified that the phase/frequency characteristic of a resonant line, which is the factor directly concerned with oscillator stability, follows a law identical with that of a lumped parallel resonant circuit. This characteristic of the line is independent of the number of quarter wavelengths it contains, so that a quarter wave line is the most generally useful; a longer line may be required in order to handle an extra large amount of energy without risk of breaking down the dielectric, an increase of length of the line being somewhat analogous to increasing the

* The derivation of these figures from the specific resistance of copper and the current distributions is fully discussed in the following papers :—

(1) "Resonant Lines in Radio Circuits," F. E. Terman, Elect. Eng., July, 1934, Vol. 53, p. 1046,

(2) "Frequency Control by Low Power Factor Line Circuits," Hansell and Carter, Proc. I.R.E., April, 1936, Vol. 24, p. 597.

(3) "Concentric Tube Lines," B. J. Witt, MARCONI REVIEW, Jan.-Feb., 1936, No. 58, p. 20,

roof capacity of a long wave transmitting aerial for the same purpose. Taking as an example a short circuited quarter wave line, the standard equation for the input impedance is

$$Z = Z_0 \tanh (\alpha l + j\beta l) = Z_0 \cdot \frac{\sinh 2\alpha l + j \sin 2\beta l}{\cosh 2\alpha l + \cos 2\beta l} \quad \ldots \quad (1)$$

where Z_0 is the characteristic impedance of the line and equal to $\sqrt{\frac{L}{C}}$ at high frequencies, and $\alpha + j\beta$ the propagation constant of the line. If R is the resistance per unit length of the line, we have $\alpha = \frac{R}{2Z_0}$, and $\beta = \frac{2\pi f}{C}$ where f is the applied frequency and c is the velocity of propagation of electro-magnetic waves. Taking the ratio of reactive to resistive components of equation (1), the phase-angle of the line is

$$\delta = \text{arc tan} \left\{ \frac{\sin 2\beta l}{\sinh 2\alpha l} \right\}$$

$$= \text{arc tan} \left\{ \frac{\sin (4\pi f l/c)}{\sinh (Rl/Z_0)} \right\} \quad \ldots \quad (2)$$

Putting $Rl = \rho$, the total resistance of the line, and approximating $\sinh (Rl/Z_0) = Rl/Z_0$ since the attenuation of the line will be small, equation (2) becomes

$$\delta = \text{arc tan} \left\{ \frac{Z_0}{\rho} \sin (4\pi f l/c) \right\} \quad \ldots \quad (3)$$

Since we are only concerned with frequencies near resonance, we may put $f = f_0 + \delta f$ where f_0 is the resonant frequency such that $4\pi f_0 l/c = \pi$. The phase angle is then

$$\delta = \text{arc tan} \left\{ \frac{Z_0}{\rho} \sin (\pi + 4\pi l \delta f/c) \right\}$$

$$= \text{arc tan} \left\{ \frac{-Z_0}{\rho} \sin (\pi \frac{\delta f}{f_0}) \right\} \quad \ldots \quad (4)$$

which for small values of $\frac{\delta f}{f_0}$ is equivalent to

$$\delta = \text{arc tan} \left\{ \frac{-\pi Z_0}{\rho} \cdot \frac{\delta f}{f_0} \right\} \quad \ldots \quad (5)$$

For a simple parallel resonant circuit, the phase angle, using similar notation, is

$$\delta = \text{arc tan} \left\{ \frac{2\pi f L}{\rho} \cdot \frac{f^2 - f_0^2}{f^2} \right\}$$

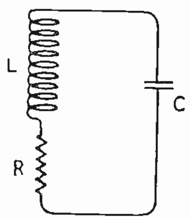
which in the neighbourhood of resonance reduces to

$$\delta = \text{arc tan} \left\{ \frac{4\pi f L}{\rho} \cdot \frac{\delta f}{f_0} \right\} \quad \ldots \quad (6)$$

Since $\frac{2\pi f L}{\rho}$ is the Q of a parallel tuned circuit of resistance ρ, comparison of (5) and (6) shows that the resonant line behaves exactly in the same way as would a parallel tuned circuit of Q equal to $\frac{\pi Z_0}{2\rho}$; this will be taken as the equivalent Q of a resonant line.

An open circuited or short circuited line of length not equal to an integral number of quarter wavelengths behaves as a reactance, as is well known. Terman states that :—

" The selectivity characteristics of circuits involving reactances derived from lines are not the same as when ordinary lumped reactances are used, however. This is because the reactance obtained from lines varies more rapidly with frequency than does the reactance of an ordinary condenser or coil. *As a result, a reactance having a certain magnitude and ' Q ' has a greater effective selectivity than does a lumped reactance having the same magnitude and ' Q '.*"



It is, however, unfair to make a comparison between a line, possessing both inductance and capacity, and a single condenser or coil ; the proper comparison is between a line used as a reactance at a frequency off resonance, and on ordinary parallel tuned circuit similarly employed. The impedance of a short circuited line given in equation (1) can be simplified for frequencies remote from resonance by approximating cosh $2\alpha l = 1$ and sinh $2\alpha l = 2\alpha l$ when the reactive component is found to be

$$X = Z_0 \tan \frac{2\pi f l}{c} \qquad (7)$$

By differentiating, it is found that the rate of change of reactance with frequency is given by

$$\frac{dX}{X} = \left\{ \frac{4\pi l/\lambda}{\sin 4\pi l/\lambda} \right\} \frac{df}{f} \qquad (8)$$

and it is comparison of (8) with the expression for a simple inductance $\frac{dX}{X} = \frac{df}{f}$ which leads Terman to say that

$$\frac{\text{Selective factor of line reactance}}{\text{Selective factor of lumped reactance}} = \frac{4\pi l/\lambda}{\sin (4\pi l/\lambda)} \qquad (9)$$

Now the impedance of the parallel resonant circuit of Fig. 1, in which all loss is assumed to be concentrated in the inductance (to correspond with the line in which all loss is assumed to be in the conductor, and none in the dielectric) is

$$Z = \frac{R + jpL}{jpCR + 1 - p^2 LC} \qquad (10)$$

where p is $2\pi \times$ frequency and p_0 will be used for the resonant value such that $p_0^2 = \frac{1}{LC}$. The reactance is then found to be

$$X = \frac{R^2 + pL\,(1 + p^2/p_0^2}{p^2 C^2 R^2 + (p^2/p_0^2)^2} \qquad (11)$$

For frequencies remote from resonance, we may neglect terms containing the power factor pCR, which should be of the order of 1/100, giving the simplified form

$$X \doteqdot \frac{pL\,(1 + p^2/p_0^2)}{(p^2/p_0^2)^2} \qquad \text{(11A)}$$

$$= L\left\{\frac{p_0^4}{p^3} + \frac{p_0^2}{p}\right\}$$

From this we find

$$\frac{dX}{dp} = L\left\{\frac{-3p_0^4}{p^4} - \frac{p_0^2}{p^2}\right\} \qquad \text{(12)}$$

$$\text{and } \frac{1}{X}\,\frac{dX}{dp} = \frac{-L\left\{3\frac{p_0^4}{p^4} + \frac{p_0^2}{p^2}\right\}\cdot\frac{p^4}{p_0^4}}{p\,L\left\{1 + \frac{p^2}{p_0^2}\right\}}$$

$$\text{whence} \qquad \frac{dX}{X} = -\left\{\frac{\frac{p^2}{p_0^2} + 3}{\frac{p^2}{p_0^2} + 1}\right\}\frac{dp}{p} \qquad \text{(13)}$$

This shows that the resonant circuit used as a reactance can have a " selectivity " between 3 and 4 times greater than that of a simple reactance. But in neither case, tuned circuit or line, is this of much practical value. For the reactance tends to a low numerical value at frequencies far from the fundamental resonance, which necessitates a high-impedance line or tuned circuit to obtain sufficient reactance, and this in turn entails an increase of resistance (unless the bulk can be greatly increased), and these losses counteract the apparent gain from rate of change of reactance.

3. Practical Results Claimed in Published Papers.

All the papers which have been published so far are concerned solely with short and ultra-short waves ; the following are brief accounts of typical transmitters :

(1) A Kolster type circuit (not resonant line, but shell and tube construction) working at 60 Mc/s maintained constant frequency within $\pm$ 100 c/s (i.e. 10 parts in 6×10^6) when the anode voltage was varied between 60 per cent. and 175 per cent. of normal. No figures are given for change of filament current or output load. Temperature coefficient 25×10^{-6} per °C. (" Generation and Utilisation of Ultra Short Waves in Radio Communication," F. A. Kolster, Proc. I.R.E., Dec. 1934, Vol. 22, p. 1335.)

(2) Resonant line transmitters on 6.725 Mc/s and 13.42 Mc/s, with oil expansion working a condenser to compensate for temperature changes : weekly average of maximum variations are 52 and 56 parts in 10^6 respectively. (" Frequency Control by Low Power Factor Line Circuits," Hansell and Carter, Proc. I.R.E., Apr. 1936, Vol. 24, p. 597.)

(3) A telephone circuit has been put into service in the Hawaiian Islands on 220 and 230 Mc/s, using resonant-line stabilised transmitters. No figures are given for frequency stability, but " the weekly checks so far have shown that the control method used

should be entirely adequate." The service is run semi-unattended, so that there is no tuning of receivers to compensate transmitter frequency drift, and H.T. is switched off between calls. ("The Hawaiian Radio Telephone System," Harrington and Hansell, Elect. Eng., Aug. 1935, Vol. 54, p. 822.)

(4) The R.C.A. Victor Company is reported to have built a transmitter consisting of a single valve resonant-line stabilised oscillator, followed by one push-pull stage of amplification, output 40 K.W. at frequencies from 40 to 60 Mc/s. Frequency stability is stated to be 1 in 10^5 for any of the following: 40°C. temperature change, 20 per cent. anode voltage change, 5 per cent. filament voltage change. Some form of temperature-compensated resonant line is used. ("Reported at Cleveland," Electronics, May 1936, Vol. 9, p. 11. This is a notice of the I.R.E. Convention at Cleveland.)

Wwedensky, Michailow and Skibarko claim to have carried out a theoretical and practical investigation ("The Stabilisation of Oscillators by Systems with Distributed Constants," Russian Journal of Technical Physics, Vol. 4, p. 337), but their theory does not appear correct. They find that the measured stability is 2 to 5 times greater than the calculated value (for a given change of anode voltage), but, as explained in the appendix, there are some points in their theory which are not acceptable.

4. Long Lines for Anode—Grid Coupling.

In one of the earlier papers ("New Methods of Frequency Control Employing Long Lines," Conklin, Finch and Hansell, Proc. I.R.E., Nov. 1931, Vol. 10, p. 1918), it was suggested that apart from the use of resonant lines as oscillatory circuits, stability could be attained by the use of a line of an integral number of half-waves in length, used as coupling between anode and grid circuits, and terminated in its characteristic impedance at the grid end. (This was described as "aperiodic" long line frequency control, owing to the absence of reflections in the line). It was stated that if the frequency changed, there would then be a change of phase between anode and grid voltages, due to the inequality between the length of line and the new wavelength, such as would tend to restore the proper frequency. Qualitatively this is correct; if the frequency falls, the phase of Vg will be advanced, Va must lag in order that $Va + \mu Vg$ may remain in phase with Ia, and Va lags behind Ia when the anode circuit is capacitive, i.e. when the frequency rises above the resonant frequency of the anode tuned circuit. The *magnitude* of such phase-change, however, is small compared with the phase-change in a tuned circuit or resonant line. For the phase difference between the ends of a line when the frequency is $f_0 + \delta f$ is $2\pi l\,(f_0 + \delta f)/c$; if the line is a half wavelength at the correct frequency, $2\pi l f_0/c = \pi$ so that the phase shift for a change of frequency δf is

$$\delta = \pi \frac{\delta f}{f_0} \qquad \text{(14)}$$

This is negligible compared with the phase changes for either resonant line or tuned circuit, given in equations (5) and (6), where Q enters into the expression. It may be noted that this "aperiodic line" type of circuit has not been mentioned in more recent publications.

5. Relation to Oscillators of Lower Frequency.

Hansell and Carter ("Frequency Control by Low Power Factor Line Circuits," Proc. I.R.E., Apr. 1936, Vol. 24, p. 597) suggest frequency division as a means of controlling transmitters of lower frequency by means of master oscillators stabilised with resonant lines on higher frequencies. But it is clear from the considerations set out above that the resonant line has no inherent stabilising action peculiar to its construction; it is merely a convenient form of high Q resonant circuit at very high frequencies. The concentric line has, however, an incidental advantage of some importance, in being self-screened so that there is negligible external field. For transmitters of lower frequency it would therefore appear more logical to employ circuits of high Q built in the form most convenient for the working frequency, rather than resonant lines at a higher frequency plus dividing circuits.

In comparing the Q of resonant lines and ordinary tuned circuits, it must be remembered that resonant lines are decidedly bulky; for example, at 20 Mc/s (15 metres) a resonant line with a Q of about 5,500 will have a diameter of 10 inches and a length of about 50 inches. By comparison, at 1 Mc/s a coil of 200 μH, 10 inches diameter but only 5 inches long, is calculated to have a Q of 780; the additional losses in an air condenser should be very small. But difficulties are encountered in attempting to improve on coils of this order.

Formulae for the high-frequency resistance of coils have been given by S. Butterworth ("Wireless World," Dec. 8th and 15th, 1926, Vol. 19, pages 754-759 and 811-815). The optimum gauge of wire is found in terms of a factor P defined by

$$P^2 = \frac{LS^2}{D^3} \qquad (15)$$

where L is the inductance in μH, D the diameter in centimetres, and S a function of the shape of the coil. It follows that for a coil of fixed inductance and shape (the optimum shape is a single-layer solonoid of length equal to half the diameter), P^2 is proportioned to $\frac{1}{D^3}$.

There is then a function connecting $\frac{f}{P^2}$ with Pd, where d is the optimum diameter of wire; but if the coil is to be made of very low loss, $\frac{f}{P^2}$ is large, so that the function takes on a limiting value.

$$d = \frac{0.165}{P} \qquad (16)$$

By combining (15) and (16),

$$d = 0.165\sqrt{\frac{D^3}{LS^2}} \qquad (17)$$

The number of turns required for a given inductance is determined by

$$N = \sqrt{1{,}000\, L/(L_0 D)} \qquad (18)$$

where L_0 is a shape factor, so that the length of wire required for a given inductance is

$$l' = N\pi D = \pi\sqrt{1{,}000\, LD/L_0} \qquad (19)$$

The d.c. resistance of a coil of given inductance and of the gauge of wire for least *high-frequency* resistance is therefore related to inductance and diameter according to the expression,

$$R = x \frac{l'}{d} = \frac{x\pi}{0.165} \sqrt{\frac{1{,}000\, L^2 S^2}{L_0 D^2}} \quad \ldots \quad (20)$$

Since S and L_0 are both shape constants, it follows that R varies simply as L; and since the formula relating H.F. resistance to d.c. resistance does not contain L, the Q of a coil at a given frequency will be independent of its inductance, but depends on size.

The H.F. resistance R_e is related to the d.c. resistance by the formula

$$R_e = R \left\{ 1 + F + \left(\frac{KNd}{2D} \right)^2 G \right\} \quad \ldots \quad (21)$$

where K is a shape factor and F and G functions of wire diameter and frequency, all of which have been tabulated by Butterworth. The following figures have been calculated for coils of 200 μH. made to various diameters, but all having length equal to radius :—

Diameter of coil..	5 inches	10 inches	20 inches	40 inches
Optimum Wire diameter	0.045 ,,	0.12 ,,	0.29 ,,	0.98 ,,
Number of turns	39	27.5	19.3	13.5
H.F. Resistance (ohms)	2.3	1.61	0.57	0.4
Q of Coil	546	780	2,210	3,150

It would not pay to increase the diameter of a coil indefinitely, even if the bulk were no objection; for as the turns become fewer, the length of wire required for a given inductance increases, which tends to offset the decrease of resistance resulting from larger conductor diameter. When the turns become few, the inductance per turn also decreases considerably with increasing wire diameter, again limiting the extent to which resistance can be lowered.*

It appears therefore that at 1 Mc/s a Q of about 2,000 is as high as can be reasonably attained; since this is hardly sufficient to stablise a normal oscillator, some improvement in oscillator circuit is to be sought.

The table above is based on the use of solid (or tubular) conductor for the winding of the inductance. If stranded wire were to be used, a very large number of strands would be necessary; obviously the radius of strand must be of the same order as the depth of penetration of the current at the working frequency for optimum efficiency, and the overall cross-section of the cable must be comparable with that of the solid conductor which it replaces to make the best use of the available space. Apart from the cost of such cable (it must, of course, be Litzendraht stranded, not plain twisted cable), the risk of damage to individual strands is a serious objection to the use of such a conductor; and since the large bulk of the coil still remains, it hardly seems worth using. As mentioned above, the bulk of the coil is more serious than would be the case of a concentric line, owing to the external field and space required for screening.

* To make an inductance of a single turn comparable with the 5 in. coil quoted, the loop would need to be of the order of 35 metres diameter.

6. Conclusions.

1. The frequency-stabilising action of resonant lines is due solely to their high value of Q (ratio of reactance to resistance), and any other circuit of similar Q would be equally effective.

2. The stabilising effect of high Q circuits is primarily concerned with the effects of non-linearity of valve characteristic, grid current, variation of external loading *resistance*, and variation of valve amplification factor. It has a secondary effect in respect of change of external *reactance* connected to the oscillatory system and change of inter-electrode capacity of the valve. The temperature coefficient of the tuned circuit has, of course, its usual effect; for a resonant line, the temperature coefficient of frequency should be equal to the linear coefficient of expansion of the material of which it is made. (Usually copper, 16.7 parts in 10^6 per °C.)

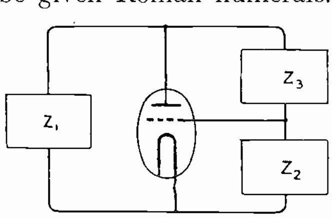
3. It is difficult to construct circuits of very high Q at frequencies below about 5 Mc/s; very fine stranding of the wire used for winding an inductance would be necessary, and accurate design. A resonant line at lower frequencies would be excessively bulky.

APPENDIX.

Discussion of "The Stabilisation of Oscillators by Systems with Distributed Constants," by Wwedensky, Michailow and Skibarko, Russian Journal of Technical Physics, Vol. 4 (1934), No. 2, p. 337.

This is practically the only theoretical paper yet published, and therefore deserves some consideration. The work is very thorough, and employs some admirable mathematical and graphical methods of dealing with oscillator conditions; it is particularly unfortunate that there are two points with which the present author cannot agree—points relating to the theory of valve oscillators in general, not resonant line systems solely.

In the following discussion, equations quoted from the Russian paper will be given their numbers from the original paper (e.g. (2.1)), while new equations will be given Roman numerals.



The symbols used by the Russian authors are: D for $1/\mu$ (= "Durchgriff"), S for the anode slope conductance of the valve, and K apparently for the ratio of grid to anode voltages, V_g/V_a, as determined by the circuit arrangements. The typical circuit is taken to be that shown in Fig. 2, and the fundamental circuit relations are given as

$$K = D + \frac{1}{SZ_a} \qquad (2.1)$$

$$K = -\frac{Z_2}{Z_2 + Z_3} \quad \text{and} \quad \frac{1}{Z_a} = \frac{1}{Z_2 + Z_3} + \frac{1}{Z_1} \qquad (2.2)$$

Equation (2.1), however, appears quite inexplicable. For if we assume $I_a = (1/\rho)(V_a + \mu V_g)$, $V_a = -ZI_a$, and $V_g = KZI_a$, we have

$$I_a = (1/\rho)(-ZI_a + \mu KZI_a)$$

whence $$K = \rho/\mu Z + 1/\mu$$

or in terms of D and S

$$K = D\,(1 + \frac{1}{SZ_a}) \quad \ldots \quad \text{(i)}$$

which is to be contrasted with (2.1) above. This equation (i) may also be derived directly in terms of D and S, using D as the factor by which an anode circuit resistance must be multiplied or a conductance divided to transfer it to an equivalent in the grid circuit. Thus the anode circuit has two conductances S and $1/Z_a$ in series, whose resultant as viewed from the grid is equivalent to $S/[D(SZ_a + 1)]$ so that the anode current is

$$I_a = V_g S/[D(SZ_a + 1)] \quad \ldots \quad \text{(ii)}$$

On writing $V_g = KZ_aI_a$ and simplifying, this again reduces to

$$K = D\,(1 + \frac{1}{SZ_a}) \quad \ldots \quad \text{(i)}$$

Having obtained a general relation between the valve and circuit constants, the next step is to specify the circuit constants thus :

$$\frac{1}{Z_1} = M + jN,\ Z_2 = \xi + j\eta,\ Z_3 = \frac{1}{j\omega C_3} \quad \ldots \quad (2.4)$$

the assumption being that Z_3 is purely capacitive, probably the anode-grid capacity of the valve.

Using equation (i) in place of the original (2.1), which latter we have shown to be incorrect, we find in place of two expressions numbered (2.6) and (2.7) in the original paper the equations

$$-S\xi - D = DM\xi - \eta N + N/\omega C_3 \quad \ldots \quad \text{(iii)}$$

$$-\eta S = DM\,(\eta - \frac{1}{\omega C_3}) + DN\xi \quad \ldots \quad \text{(iv)}$$

The Russians at this stage proceeded to solve their equations for ξ and η in terms of S, D, and the remaining circuit constants, *implicitly assuming S to be an independent variable.* But S is not an independent variable (see " Frequency Stability of Valve Oscillators," *loc. cit.*) ; it is a factor which automatically adjusts itself, through change of amplitude of oscillation, according to the setting of the circuit constants. For a full solution of this pair of equations, one must take account of the fact that ξ and η are not independent of each other, neither are M and N ; both are functions of the frequency and the circuit constants. One should therefore replace ξ, η, M and N by the appropriate functions of frequency (just as Z_3 was replaced by $1/j\omega C_3$), when the two equations can be solved for the two unknowns, frequency and S, the latter determining the amplitude of oscillation. The valve then appears in the frequency equation only through the factor D, not S, in the linear regime ; but in practice harmonic distortion occurs, and the apparent reactance of the valve mentioned above on p. 23 will enter into the equation. However, ξ and η for a resonant line involve circular functions, so that the frequency equation is not readily solved, even if M and N be known.

Since the calculations of stability in the original paper were based on change of S, which does not enter into the true frequency equation, it is not surprising that the experimental results found by Wwedensky, Michailov and Skibarko were several times better than their predicted values.

D. A. Bell.

AN INDICATOR FOR ULTRA SHORT WAVELENGTHS

With the commercial development of U.S.W. transmitters a need has been felt for a simple frequency meter of robust construction for getting such transmitters.

Because of the inclusion of several stages of frequency multiplication in most U.S.W. transmitters, a frequency indicator for use with them must cover a wide band.

The instrument described herein was made up for use at one particular station, but its performance makes it suitable for general use.

Frequency Band.

ONE of the transmitters for use with which this instrument was made has a master oscillator working at 2,500 kc/s. and this frequency is multiplied, in following stages, by 3 to 7,500, by 3 again to 22,500 and then by 2 to 45,000 kc/s. the radiated frequency.

It was decided, in order to give the instrument a somewhat wider application, to make its range 2 mc/s. to 100 mc/s.

Tune Indicator.

As it is normally to be used for monitoring oscillating circuits, the frequency meter is designed as a grid detector circuit using a miniature triode valve of the H.L.2K. type, and sensitive galvo which shows a deflection only when the instrument is tuned to the frequency of the oscillator under test.

A jack is included to take headphones for listening to modulation where present.

Construction—General.

Diagrams showing plan and elevations of the instrument are shown in Figs. 1, 2 and 3.

For robustness this experimental model is made of $\frac{1}{16}$ in. copper sheet in the form of a box 11 ins. by 10 ins. by 6 ins. high with an inner partition giving two screened compartments for the U.S.W. and S.W. units. All components are carried on the top cover which has two hinged lids giving access to the coils for range changing. The top cover also carries a super structure 5 ins. by 5 ins. by 3 ins. high which houses the valve, batteries, galvo and switches.

In the interests of simplicity and portability the instrument is made entirely self-contained, which means there are no loose coils or external batteries. Moreover, to avoid change of calibration through possible deformation of the main tuning components these are all securely fixed *inside* the box and excitation is via a pick-up coil and flexible screened lead connected to another coil loosely coupled to the tuned circuit inside the box.

Controls.

The only controls are a battery press switch, which is conveniently situated for pressing with the thumb as the left hand holds the box steady, and the right hand operates the tuning control, and two tuning knobs, only one of which is used at a time according to the range required.

The battery press switch avoids discharging batteries through forgetting to switch off, but a lock is provided for holding the button pressed when neuting or other long operation is in process.

or in terms of D and S

$$K = D\left(1 + \frac{1}{SZ_a}\right) \qquad \text{(i)}$$

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$$-S\xi - D = DM\xi - \eta N + N/\omega C_3 \qquad \text{(iii)}$$

$$-\eta S = DM\left(\eta - \frac{1}{\omega C_3}\right) + DN\xi \qquad \text{(iv)}$$

The Russians at this stage proceeded to solve their equations for ξ and η in terms of S, D, and the remaining circuit constants, *implicitly assuming S to be an independent variable.* But S is not an independent variable (see " Frequency Stability of Valve Oscillators," *loc. cit.*) ; it is a factor which automatically adjusts itself, through change of amplitude of oscillation, according to the setting of the circuit constants. For a full solution of this pair of equations, one must take account of the fact that ξ and η are not independent of each other, neither are M and N ; both are functions of the frequency and the circuit constants. One should therefore replace ξ, η, M and N by the appropriate functions of frequency (just as Z_3 was replaced by $1/j\omega C_3$), when the two equations can be solved for the two unknowns, frequency and S, the latter determining the amplitude of oscillation. The valve then appears in the frequency equation only through the factor D, not S, in the linear regime ; but in practice harmonic distortion occurs, and the apparent reactance of the valve mentioned above on p. 23 will enter into the equation. However, ξ and η for a resonant line involve circular functions, so that the frequency equation is not readily solved, even if M and N be known.

Since the calculations of stability in the original paper were based on change of S, which does not enter into the true frequency equation, it is not surprising that the experimental results found by Wwedensky, Michailov and Skibarko were several times better than their predicted values.

D. A. Bell.

AN INDICATOR FOR ULTRA SHORT WAVELENGTHS

With the commercial development of U.S.W. transmitters a need has been felt for a simple frequency meter of robust construction for getting such transmitters.

Because of the inclusion of several stages of frequency multiplication in most U.S.W. transmitters, a frequency indicator for use with them must cover a wide band.

The instrument described herein was made up for use at one particular station, but its performance makes it suitable for general use.

Frequency Band.

ONE of the transmitters for use with which this instrument was made has a master oscillator working at 2,500 kc/s. and this frequency is multiplied, in following stages, by 3 to 7,500, by 3 again to 22,500 and then by 2 to 45,000 kc/s. the radiated frequency.

It was decided, in order to give the instrument a somewhat wider application, to make its range 2 mc/s. to 100 mc/s.

Tune Indicator.

As it is normally to be used for monitoring oscillating circuits, the frequency meter is designed as a grid detector circuit using a miniature triode valve of the H.L.2K. type, and sensitive galvo which shows a deflection only when the instrument is tuned to the frequency of the oscillator under test.

A jack is included to take headphones for listening to modulation where present.

Construction—General.

Diagrams showing plan and elevations of the instrument are shown in Figs. 1, 2 and 3.

For robustness this experimental model is made of $\frac{1}{16}$ in. copper sheet in the form of a box 11 ins. by 10 ins. by 6 ins. high with an inner partition giving two screened compartments for the U.S.W. and S.W. units. All components are carried on the top cover which has two hinged lids giving access to the coils for range changing. The top cover also carries a super structure 5 ins. by 5 ins. by 3 ins. high which houses the valve, batteries, galvo and switches.

In the interests of simplicity and portability the instrument is made entirely self-contained, which means there are no loose coils or external batteries. Moreover, to avoid change of calibration through possible deformation of the main tuning components these are all securely fixed *inside* the box and excitation is via a pick-up coil and flexible screened lead connected to another coil loosely coupled to the tuned circuit inside the box.

Controls.

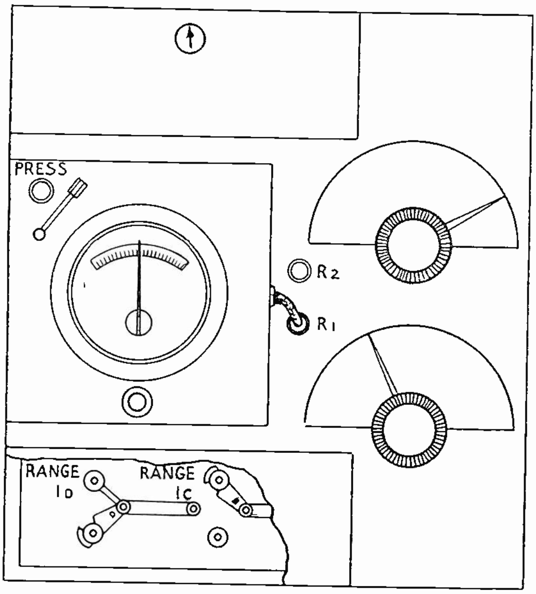
The only controls are a battery press switch, which is conveniently situated for pressing with the thumb as the left hand holds the box steady, and the right hand operates the tuning control, and two tuning knobs, only one of which is used at a time according to the range required.

The battery press switch avoids discharging batteries through forgetting to switch off, but a lock is provided for holding the button pressed when neuting or other long operation is in process.

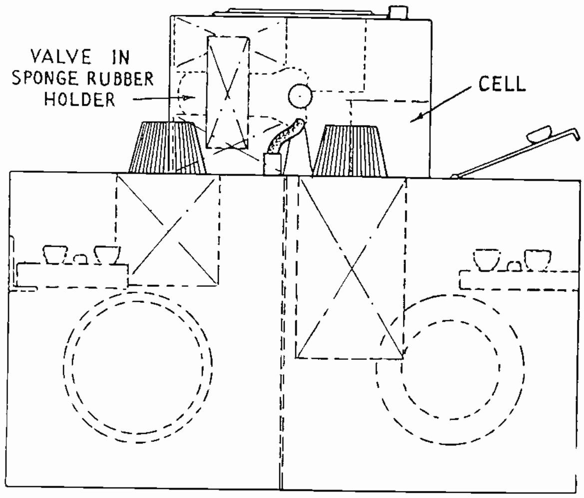
Circuit Design.

The circuit diagram is shown in Fig. 4.

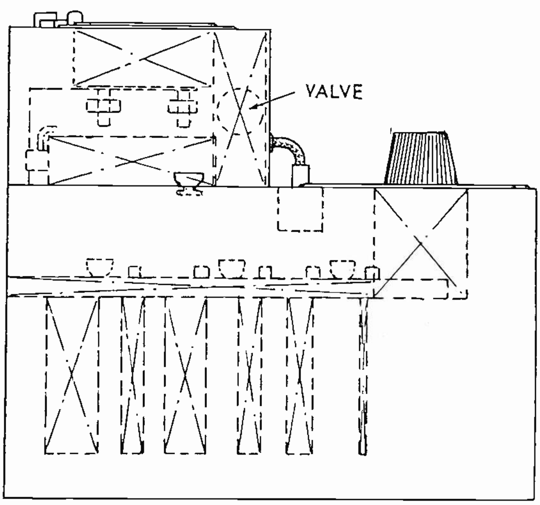
To cover the band of 2 to 100 mc/s. it will be seen there are two major ranges consisting of four and three minor ranges respectively. One or other of the major



PLAN
FIG. 1.



SIDE ELEVATION
FIG. 2.



FRONT ELEVATION
FIG. 3.

ranges of 100 to 36 mc/s. and 39 to 2 mc/s. is selected by means of a plug and socket connection from the valve grid. The minor ranges are obtainable from link positions on panels beneath the two hinged lids.

For instance, range 1A. is composed of a 7 to 45 $\mu\mu F$ variable condenser with its moving vanes earthed and $\frac{3}{4}$ of a single 3 ins. diameter turn which is returned to earth via a link. When this link is moved to its alternative position for range 1B. the $\frac{3}{4}$ turn is connected in series with two more turns spaced away from the first, which two turns complete their circuit to earth via a second link, and so on.

By this method of spacing and sequential connection, parallel capacities are reduced to a minimum and there is no hang on inductance.

To avoid self-resonating acceptor circuits in the unused turns, the "alternative link position" in each case is also permanently wired across, so that the unused turns following a live coil are shorted and earthed via the link.

The excitation coupling winding is split up into two coils fixed between ranges 1A. and 1B., and 1C. and 1D., and wired in series. The pick-up coil is made suitable for both ranges.

These ranges are :—

1A. 100-50 mc/s. (3-6 metres)
1B. 60-27.25 mc/s. (5-11 metres)
1C. 30-13.61 mc/s. (10-22 metres)
1D. 15.39-9.4 mc/s. (19.5-32 metres)

Connection to the valve grid is via a 3 $\mu\mu$F condenser to limit the effect of valve changing and of the semi-flexible plug connection, on the tuned circuit, but if the condenser is increased in capacity the sensitivity of the instrument is improved considerably. Obviously this capacity cannot be changed after calibrating. The de-capped H.L.2K. actually has a grid-fil: capacity of 3.5 $\mu\mu$F and grid-anode-fil: capacity of 6.3 $\mu\mu$F. *Major range* 2 is similar to range 1 except that there are only three sub-ranges using a tuning condenser of 20-180 $\mu\mu$F and a grid condenser of 8 $\mu\mu$F.

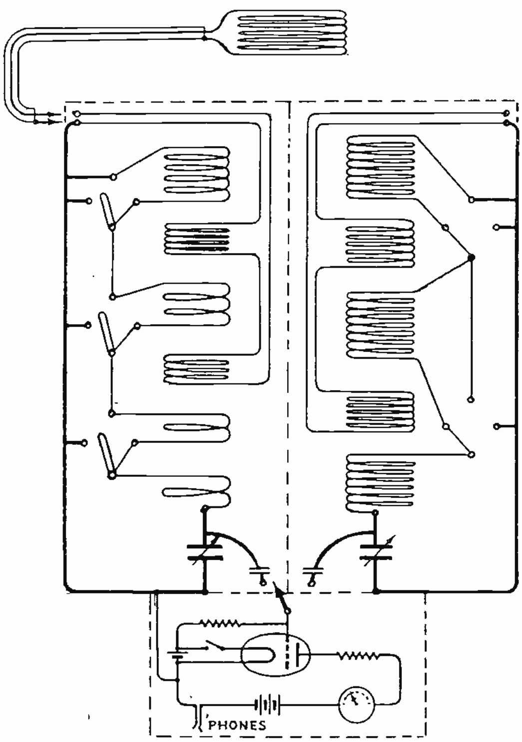


FIG. 4.

The ranges are :—

2A. 10-4.3 mc/s. (30-70 metres)
2B. 6.0-2.72 mc/s. (50-110 metres)
2C. 3.0-1.5 mc/s. (100-200 metres)

The low temperature filament of the valve is supplied by a Siemens Q or Exide X.123 cell, and the anode is made positive by a 3-cell bias battery or flash lamp battery. The grid resistance is returned to the positive end of the filament and this is sufficient in overcoming the space charge to allow the anode current to get away from zero quickly as the input signal to the grid is increased (e.g., by tuning).

Working the valve as a grid rectifier even with such a low anode voltage gives greater sensitivity than when connecting as a simple diode.

L. BOUNDS.

A NEW TELEVISION MONITORING SYSTEM

The following article describes a new monitoring oscillograph arrangement for maintaining the vision and sound radio transmitters of a television station at their correct levels.

FOR a complete television broadcasting service, sound must be transmitted the same time as vision. Where the vision signals are transmitted by a short wave radio transmitter it has been found convenient to transmit the sound intelligence with a separate transmitter working on an adjacent wavelength and of a comparable power output to that of the vision transmitter. In this manner the range of the two transmitters will be approximately equal; this is satisfactory, since neither vision nor sound by itself has much entertainment value.

With a transmitting station of this type it has been found convenient in practice to utilise a common aerial and high frequency amplifier for vision and sound in the receiver. For satisfactory working, however, it has been found necessary to design the receiver for some particular ratio of vision to sound input voltages from which the permissible variation is not very great. It is necessary therefore to monitor carefully the outputs of the two transmitters to make certain that the pre-arranged ratio of powers is adhered to. It is true that the variations of power output of the sound transmitter during transmissions should be infrequent and small if it is properly attended to, variations of supply voltage and the heating up of resistances being the chief causes of changes of power output during transmission. Over long periods, however, valve replacements have to be made, routine adjustments and possibly modifications to circuit arrangements may be found necessary. It is necessary to keep a check on changes of power output during these operations. The vision transmitter is usually more carefully monitored during transmission and variations of level should not be tolerated in any case. It is more the changes from day to day that have to be guarded against.

Description of Monitoring Device.

From the output feeders of each of the two transmitters (Fig. 1) a small amount of high frequency carrier is taken off a condenser by suitable high frequency cables to some convenient point where a cathode ray oscillograph may be mounted. Here the cable from the vision is terminated in a tuned circuit whose one side is earthed, the other side being connected to the anode of a diode valve. There is a resistance between the cathode and earth and across this are connected the deflecting plates of the oscillograph. In this manner the modulation waveshape at the vision transmitter output may be observed, the deflection shown on the tube being proportional to the transmitter power output at any particular instant.

The cable from the sound transmitter feeder is also terminated by a tuned circuit, one end of which is earthed and the other taken through a condenser to the anode of a diode valve. The cathode of this valve is earthed, the anode is connected through three chokes in series to the cathode of the other diode. The connecting points between these chokes are taken to earth through condensers. The first of these chokes—that is the one connected to the anode of the sound diode—is a high frequency choke for preventing the passage of carrier frequency to the oscillograph, the second choke offers a high impedance to all audio frequencies, and the third, which may in practice be made easily of two or three different chokes in series, will offer a high impedance to all television modulation frequencies.

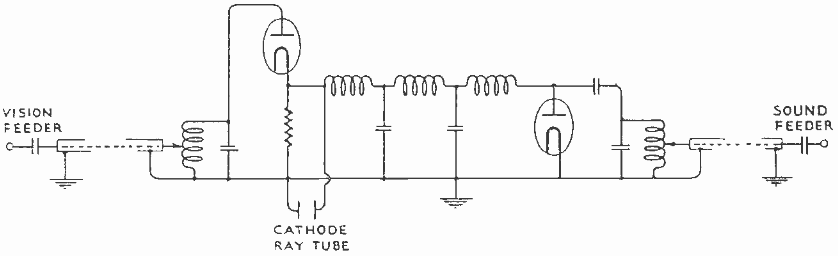


FIG. 1.

It will now be seen that the presence of carrier at the output of the sound transmitter will produce a steady direct voltage across the cathode ray tube proportional to the transmitter output and varying only slightly with modulation. The voltage will be opposite in sense to that produced across the tube by the vision transmitter.

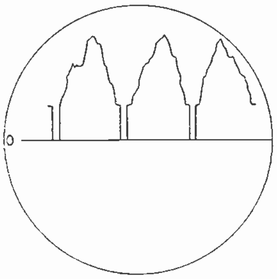


FIG. 2.

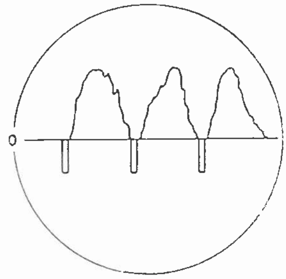


FIG. 3.

If now it is desired to adjust the power output of the sound transmitter to be equal to the power of the vision transmitter at, say, " picture black " level, the sound transmitter's power must be increased until the black level or the output waveform observed on the oscillograph is biased down to the point of zero deflection, which should be marked across the centre of the tube. (See Figs. 2 and 3.) In the same manner, the sound power may be adjusted to be equal to any desired fraction of vision black level. It is assumed—and this is generally the case—that the bottoms of the synchronising pulses represent zero carrier output from the vision transmitter.

A satisfactory alternative arrangement which could be used at a distance from the transmitter, if required, would consist of a common aerial and wide band high frequency amplifier feeding the two diode rectifiers described above. Changes of gain in the amplifier is common to picture and sound and would be of no consequence.

W. S. L. TRINGHAM.

MARCONI NEWS AND NOTES

NEW BROADCASTING STATION FOR YUGOSLAVIA

THE Marconi Company has received an order for the supply of a 20-kilowatt broadcast transmitter to be installed near Belgrade.

This transmitter will replace the existing low-power Marconi station which has given unfailing service since its installation seven years ago. Its reliability is indicated by the fact that there has been no technical breakdown during the whole of this period.

The new 20-kilowatt installation, which will be built entirely at the Marconi Works at Chelmsford, will be crystal driven, and the Marconi " series modulation " system will be used.

All the latest improvements of broadcasting technique will be incorporated in the new station. Quality of the highest order, ease of control, and provision for the extension of the transmitter to higher power—should this be desired at a future date—are some of the features which will make this station one of the most advanced in design.

The aerial will be of the inverted quarter-wave type, and it will be suspended between two 100-metre towers.

Broadcasting Station for Argentina.

THE Marconi Company has also received an order for the installation of a crystal-driven broadcast transmitter for local service in the important Argentine city of Rosario.

Although intended for service over a limited area only, the design of the new station provides a standard of quality and performance in conformity with the latest development of broadcasting technique.

The Empire Flying Boats.

ON October 22nd "Canopus," the first of 28 Empire flying boats which are being built for Imperial Airways at Rochester, left England for the Mediterranean, where it is to be used on the Brindisi-Alexandria air service.

As might be expected, the new flying boats are fitted with the most modern piloting and navigational equipment far in advance of anything hitherto installed in commercial aircraft.

Amongst the many modern aids to navigation in " Canopus," the special Marconi equipment takes an important place. The installation is of the A.D.57a/58a type arranged for transmission and reception on the short and medium wavebands, and includes direction finding and " homing."

Three aerials are used with the equipment :—

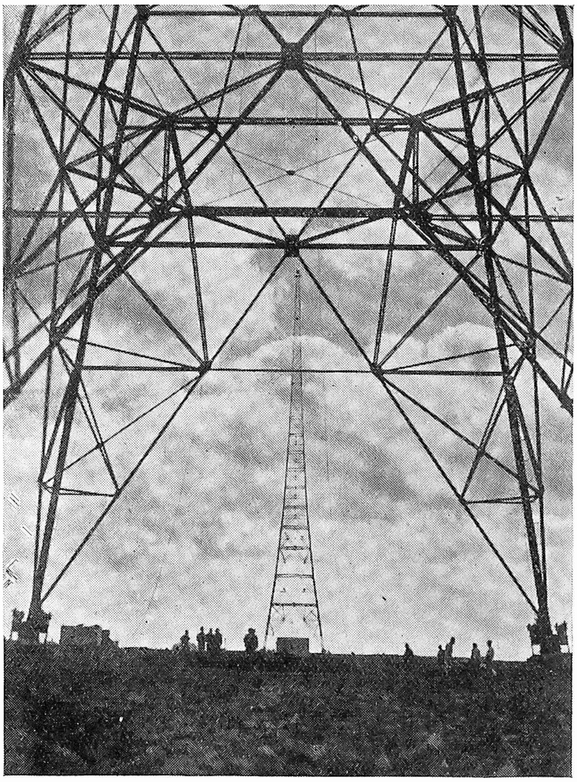
1. A trailing aerial for communication in flight.
2. A second fixed aerial for use when the trailing aerial has been wound-in preparatory to landing or when the flying boat rests on the water.
3. A direction finding loop for obtaining bearings to indicate the position of the flying boat relative to neighbouring stations.

New Marconi Broadcasting Aerial.

A NEW broadcast aerial of improved characteristics has been developed in the Marconi Research Department. This system is known as the Marconi "folded top" broadcast aerial, and is particularly valuable for mast-supported aerials of the inverted quarter-wave type.

The improved results of this type of radiator now in use at the Palestine broadcasting station fully confirm the claims made for it.

In the next issue of THE MARCONI REVIEW a technical description of the aerial will be given.



Jerusalem Broadcasting Station.

Marconi Screening Harness for Aero Engines.

OWING to the increasing use of short and ultra-short waves for aircraft wireless communication, complete screening of the ignition systems of aero engines has become an everyday necessity.

The Marconi Company has recently introduced a highly efficient screening harness which is not only perfectly satisfactory from an electrical point of view but at the same time is of sound engineering construction, proof against petrol, oil and water, and able to withstand the continuous vibration to which it is subjected.

The Company have already received large orders for these screening harnesses, a number of which are being manufactured for the Bristol Aeroplane Company for fitting to Pegasus engines for Imperial Airways flying boats.

Pensions for Marconi Operatives.

THE Marconi Company has established a contributory pension fund for its operatives employed at the Chelmsford Works and other of the Company's production establishments in England.

The main purpose of the scheme is to make provision for retirement through age or invalidity and for life assurance during the years of employment.

A pension fund for the Marconi Company's staff is already in existence and the new fund is intended to benefit the rest of the Company's employees, who are principally engaged at Chelmsford Works.

A seventy-five per cent. acceptance was necessary to put the scheme into effect. Actually over 95 per cent. of the eligible operatives enrolled in the scheme—which came into effect on October 22nd—indicating the enthusiasm with which it was received.

The cost of the contributions is shared by the members and the Marconi Company.

The scheme ensures, among other benefits, an immediate life assurance while the member is in the service of the Company, and a pension for life for members on retirement at the age of 65.

Formation of New Company—Marconi-Ekco Instruments, Limited.

MARCONI'S WIRELESS TELEGRAPH CO., LTD. and E. K. Cole, Ltd. have formed a jointly owned company called Marconi-Ekco Instruments, Ltd. The chairman of the new company is the Rt. Hon. Lord Inverforth, P.C.

The main object of this company is to combine the well known activities of both the Marconi and Ekco companies in the fields of measuring instruments, diathermy and electro-medical apparatus. The manufacturing, technical and research experience of the two companies will enable Marconi-Ekco Instruments, Ltd. to offer a complete range of high-class apparatus at attractive prices. This range will embrace not only highly specialised precision apparatus for radio and other purposes, but also the more popular general requirements within the limits prescribed for the new company.

It is the intention to co-ordinate and intensify research in connection with the above-mentioned fields and considerable progressive developments may be anticipated.

The address of Marconi-Ekco Instruments, Ltd. is Marconi Offices, Electra House, Victoria Embankment, London, W.C.2.