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National Radio Exhibition

ELSEWHERE in this issue, we describe some of the developments and trends in broadcast-receiver practice as exemplified by apparatus exhibited at Earls Court. The emphasis still lies on television but the advent of sound broadcasting at v.h.f. has given a new stimulus to sound receivers.

The purely v.h.f. receiver is rather rare. The facility is usually combined with something else, and generally, with the a.m. short-, medium- and long-wave sound receiver. However, there are many cases in which v.h.f. sound is combined with television. We pointed out in our June Editorial that "Technically, Band II reception fits in with the television set better than with the ordinary sound broadcast receiver". Designers have obviously fully realized this, for the number of sets containing television and Band II f.m. is much greater than we expected to see.

On the purely television side, the most obvious change is the general increase of picture size. The 9-in. tube has disappeared from the normal domestic set and there are only a few 12-in. types. The 14-in. is still common, but it is significant that there are relatively few among the newer models. The vast majority of the new receiver designs have a 17-in. tube and nearly every manufacturer has at least one model with a 21-in.

With the advent of these big tubes, the use of projection is decreasing except for the really big picture of some 5-ft diagonal. It is the small back-projection set of about 2-ft diagonal which is a direct competitor of the big tubes and which seems to be in danger of displacement.

In one way or another, television sets all now cover Bands I and III and, almost without exception, station selection is by a rotary multi-position knob which feels to the user like a switch, even if it does not actually control a switch! Some form of vision channel a.g.c. is often fitted and is sometimes called automatic picture (or contrast) control. It is not always a true gated a.g.c. system, but is quite often a simple form which affects the d.c. component of the picture. Technically, some people consider this to be undesirable, but it is a very debatable point and one which is dependent in some degree on subjective matters. It is significant that in most sets the d.c. component is deliberately reduced, although not always in the same way.

From the user's point of view, the advent of Band III television has added controls. In addition to the station selector knob, there is invariably a fine tuning control. The purpose of this is to counter oscillator drift. It is to be hoped that, in the course of time, development will enable this to be dispensed with, for we feel that, ideally, the user control should be only a switch-like station selector. Even if adequate oscillator stability can be obtained on the test bench, however, manufacturers are wise not to omit the fine control until considerable experience has been gained with the operation of sets under domestic conditions.

The exhibition did cover some other aspects of radio than the purely broadcast. The Services had displays; some industrial apparatus was shown; and some test equipment and component manufacturers exhibited. While of considerable individual interest, these sections in no way fully covered their fields. It was only in 'domestic radio and television' that Earls Court could be considered as fully representative of British practice.

W. T. C.

NEW FILTER THEORY OF PERIODIC STRUCTURES

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SUMMARY.—Characteristic reflection and transmission factors which depend on a parameter called reference impedance are specified for linear and passive four-terminal networks. These characteristic factors constitute a convenient set of functions in terms of which a theory of four-terminal networks (e.g., filters) of certain periodic structures can be derived. The relationships between the characteristic factors and the conventional sets of circuit constants, especially of the image functions of symmetrical circuits, are investigated. The method is illustrated by some examples of multi-sectional periodic structures which are a combination of cascade-connected four-terminal circuits and loss-free transmission lines. It is shown that major simplifications of the theory occur in loss-free and symmetrical cases where the general use of diagrams is possible and helpful.

1. Introduction

THE propagation of waves in periodic structures has many practical applications in various fields of physics[1,2]. Fortunately, the behaviour of waves in periodic structures of various kinds is often similar so that the same theory can be applied in different cases. The simplest, one-dimensional, case which will be investigated here is usually treated by analogy to the propagation of electromagnetic waves on electrical circuits and transmission lines.

It is common practice to specify an electrical four-terminal circuit by a particular set of circuit constants. Examples are the impedance or admittance matrix, the 'general' circuit constants, the image or iterative parameters or the short-circuit and open-circuit impedances. Which of the various sets of circuit constants is the most suitable depends upon the particular problem.

A particular set of functions, the 'characteristic reflection and transmission factors', has been found useful for describing the wave propagation in acoustical filters[3]. This method, which is considered to be novel, is easily applicable to electrical techniques and will be outlined and extended in the present paper. As the characteristic factors are to some extent more general than the usual circuit constants it will be necessary to investigate the relationships between those factors and constants more closely. Another point concerning the way the argument will be presented may be mentioned at the beginning. Some of the experimental procedures which will be described (e.g., the specification of the characteristic factors in terms of certain characteristics of the standing-wave pattern on transmission lines connected to the input and output of a four-terminal network) may be applied more frequently in certain analogies than in electrical technique itself, but they are presented here in detail because they are considered to be particularly useful for a clear specification of important features of the characteristic factors.

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2. Characteristic Factors of a Four-Terminal Circuit

In Fig. 1(a) a transmission line which has the characteristic impedance Z_0 is connected at the input (1,1′) and output (2,2′) terminals of a four-terminal circuit. The lines may be loss-free so that Z_0 has a real value, and the output line is assumed to be non-reflecting. If an electrical wave (which may be taken as a voltage wave throughout this paper) is propagated along the input line toward the four-terminal circuit (incident wave), there will be a reflected wave at the input terminals forming a standing wave on the input line and there will be a transmitted wave at the output terminals forming a progressive wave on the output line.

The complex ratio of the voltages of the reflected and incident waves at the input terminals is called the characteristic reflection factor $R_I = A_I \exp. j\alpha_I$ and the complex ratio of the voltages of the transmitted wave at the output terminals and of the incident wave at the input terminals is called the characteristic transmission factor $T_I = B_I \exp. j\beta_I$.

The characteristic factors R_I and T_I can be easily represented in terms of certain characteristics of the standing and progressive waves[4]. A position on the input line may be denoted by a value y and on the output line by a value x. As an example, in Fig. 1(b), the magnitude and phase values of the standing wave (unbroken lines) with the incident and reflected waves separately (broken lines) and of the transmitted wave are plotted against y and x respectively. Let the voltages of the maxima and minima on the input line be E_{Imax} and E_{Imin}, the voltage of the transmitted wave E_{Itr}, the distance between the input terminals and the first minimum of the standing wave $\Delta_I y$ and the distance between the output terminals and that position on the output line where the phase of the transmitted wave is π different [indicated by + and − signs in Fig. 1(b)] from the phase of the maximum between the first and second minimum be Δ_{Ix}. With these quantities, the characteristic factors can easily be found by elementary wave theory to be:

$$A_I = \frac{1 - E_{Imin}/E_{Imax}}{1 + E_{Imin}/E_{Imax}}$$

$$\alpha_I = 4\pi \frac{\Delta_{Iy}}{\lambda} + \pi \qquad \ldots \quad \ldots \quad (1)$$

$$B_I = \frac{2\, E_{Itr}/E_{Imax}}{1 + E_{Imin}/E_{Imax}}$$

$$\beta_I = 2\pi \frac{\Delta_{Ix} + \Delta_{Iy}}{\lambda} - \frac{\pi}{2} \qquad \ldots \quad \ldots \quad (2)$$

where λ is the wavelength which can be determined from the double distance of two successive minima of the standing wave.

In the general asymmetrical case, the factors R_I and T_I are not sufficient to define a linear and passive four-terminal circuit completely. More measurements are required where the wave is incident on the terminals of the four-terminal circuit previously at the output, with the new output terminated by the non-reflecting transmission line; i.e., the four-terminal circuit is reversed. Let the new measurements be E_{IImin}/E_{IImax}, E_{IItr}/E_{IImax}, Δ_{IIy} and Δ_{IIx} and the new characteristic factors R_{II} ($= A_{II}$ exp. $j\alpha_{II}$) and T_{II} ($= B_{II}$ exp. $j\beta_2$). These will be related by equations identical in form with Equs. (1) and (2). It can be shown[4] that

$$T_I = T_{II} = T \equiv B \text{ exp. } j\beta \qquad \ldots \quad \ldots \quad (3)$$

or $B_I = B_{II} = B$ and $\beta_I = \beta_{II} = \beta$

The characteristic factors R_I, R_{II} and T are complex dimensionless numbers and their frequency characteristics are used in this paper to specify a linear and passive four-terminal circuit. It is obvious that these factors depend on the characteristic impedance Z_0 of the input and output transmission lines. This value of Z_0, which is a real quantity throughout this paper, may be called the reference impedance of the characteristic factors and in general will be denoted by the term Z_R.

Unlike the usual circuit constants which are completely determined by the data of the four-terminal circuit, the characteristic factors depend on Z_R which represents a quantity outside and independent of the four-terminal network. Z_R forms a link between the four-terminal circuit and certain external operating conditions, and there is a family of characteristic factors if Z_R is taken as parameter. This means that the characteristic factors are slightly more general than the usual circuit constants. The possibility of choosing the characteristic factors according to the

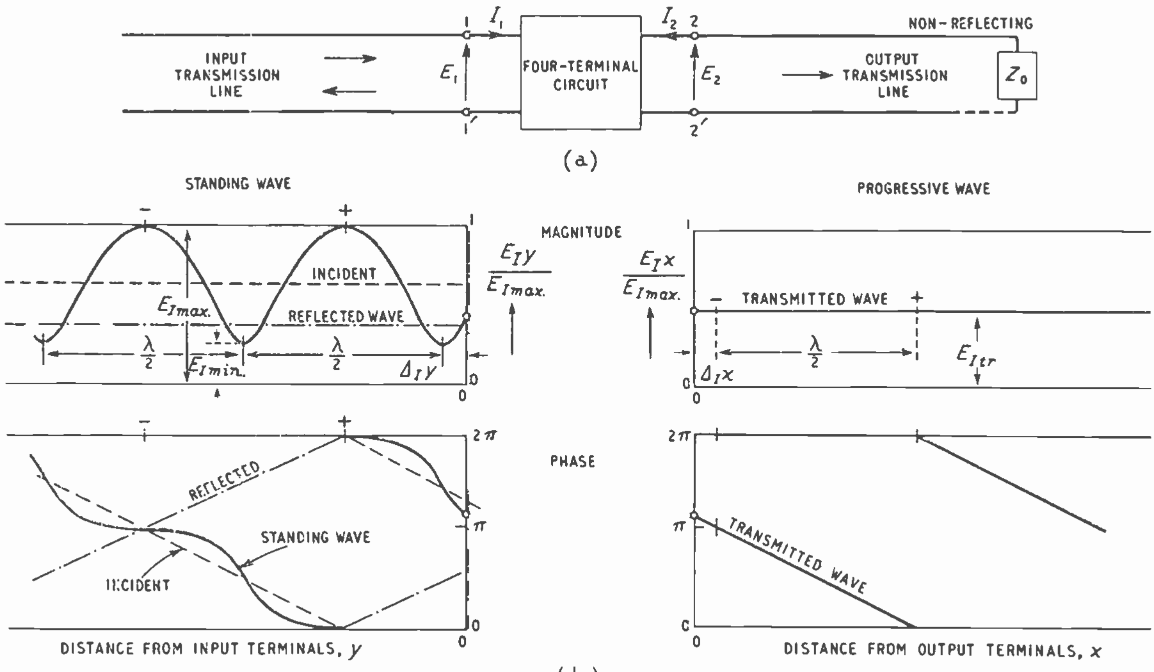

Fig. 1. *(a) Four-terminal circuit with an input and non-reflecting output-transmission line; (b) Example showing the magnitude and phase of the standing and progressive (voltage) waves plotted over the distance y from the input terminals (1,1′) or over the distance x from the output terminals (2,2′). (The broken lines belong to the incident and reflected wave.)*

appropriate value of Z_R and some general useful features of these factors as will be described later, are helpful in simplifying some filter problems which are rather complex if treated in the conventional way.

One feature of the characteristic factors is particularly useful for a discussion of periodic structures as mentioned in the introduction; viz., these factors are changed in a simple way if the input and output terminals are changed by adding a portion of the transmission line (with the characteristic impedance $Z_0 = Z_R$) to the four-terminal circuit at the input and output. Let the added length of the transmission line at the input be Dy and that at the output be Dx, then the characteristic factors R'_I, R'_{II} and T' related to the new input and output terminals are given by:

$$A'_I = A_I$$
$$A'_{II} = A_{II} \quad \ldots \quad (4)$$
and $B' = B$

$$\alpha'_I = \alpha_I - 4\pi \frac{Dy}{\lambda}$$
$$\alpha'_{II} = \alpha_{II} - 4\pi \frac{Dx}{\lambda} \quad \ldots \quad (5)$$
$$\beta' = \beta - 2\pi \frac{Dx + Dy}{\lambda}$$

There is no change of the magnitudes and the change of the phase values is simply proportional to the reciprocal value of the wavelength.

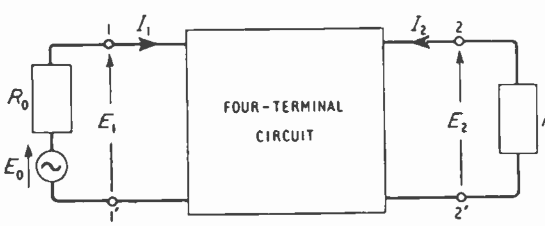

Fig. 2. *Four-terminal circuit with resistance R_0 connected at the input and output.*

3. Four-Terminal Circuit with Resistance R_0 connected to the Input and Output

The characteristic factors have been derived above by considering the wave transmission and reflection at a four-terminal circuit between transmission lines. There are, of course, other ways of obtaining these factors, and it seems to be useful for electrical measuring technique to show this in the case where the same resistance R_0 is connected at the input and output of the four-terminal circuit (cf. Fig. 2). The subscripts $_I$ or $_{II}$ may be used as before for distinguishing the connections shown in Fig. 2 from those with reversed input and output terminals respectively. It is obvious that in Fig. 1, where the network has transmission lines as terminations, the characteristic reflection factors depend only on Z_0 and the input impedance according to the 'matching' formula:

$$R_{I,II} = \frac{(E_1/I_1)_{I,II} - Z_0}{(E_1/I_1)_{I,II} + Z_0} \quad \ldots \quad (6)$$

and from Fig. 2 it can be seen immediately that

$$\left(\frac{E_1}{E_0}\right)_{I,II} = \frac{(E_1/I_1)_{I,II}}{(E_1/I_1)_{I,II} + R_0} \quad \ldots \quad (7)$$

If the reference impedance is $Z_R = Z_0 = R_0$ Equs. (6) and (7) yield:

$$\left(\frac{E_1}{E_0}\right)_{I,II} = \frac{1}{2} + \frac{R_{I,II}}{2} \quad \ldots \quad (8)$$

The ratio of output and input voltage can be represented by the characteristic factors.

$$\left(\frac{E_2}{E_1}\right)_{I,II} = \frac{T}{1 + R_{I,II}} \quad \ldots \quad (9)$$

which yields with Equ. (8):

$$\frac{E_2}{E_0} = \frac{T}{2} \quad \ldots \quad (10)$$

or

$$\frac{E_2}{E'_2} = T \quad \ldots \quad (11)$$

where E'_2 is the voltage across the load R_0 with the four-terminal circuit in Fig. 2 replaced by direct connection between the terminals 1 and 2 and similarly 1′ and 2′. Equs. (8) and (10) show that the characteristic factors with the reference impedance $Z_R = R_0$ can be specified by determining the complex voltage ratios $(E_1/E_0)_{I,II}$ and E_2/E_0 of the voltages shown in Fig. 2. Equ. (11) shows that the characteristic transmission factor T is equal to the reciprocal value of the so-called insertion ratio of a four-terminal circuit with resistance R_0 connected to the input and output.

These results show that the use of the characteristic factors is profitable if the transmission characteristics of a four-terminal circuit have to be found in terms of known constant-impedance terminations. Such a situation occurs frequently in practical filter problems. If both the characteristic impedance Z_0 of the transmission lines (Fig. 1) and the resistance R_0 connected to the input and output (Fig. 2) are the same and equal to the reference impedance $Z_R = Z_0 = R_0$, the characteristic factors of the same four-terminal circuit specified in this and in the preceding section are obviously identical, provided that the relation between the frequency and the wavelength is known and that a change from Z_0 to R_0 does not change the wave form (mode of wave) at the input and output terminals, as can occur in certain analogies to an electrical transmission line [cf. Equ. (53)].

4. Characteristic Factors and Impedance Matrix

The relationships between the characteristic factors and the conventional circuit constants are investigated for two reasons. One is that experimental and theoretical methods for determining the usual circuit constants of a four-terminal circuit are well known so that they can be used in connection with such relationships to derive the characteristic factors as well, and the other that these relationships will reveal further features of the characteristic factors which are useful for simplifying practical filter problems.

A linear, passive four-terminal circuit may be specified in the usual way by its impedance matrix defined by linear relationships between the input and output voltages (E_1 and E_2) and currents (I_1 and I_2) of Fig. 1(a).

$$E_1 = Z_{11}I_1 + Z_{12}I_2$$
$$E_2 = Z_{12}I_1 + Z_{22}I_2 \quad \ldots \quad \ldots \quad (12)$$

With the ratios of the elements of the impedance matrix

$$z_{11} = \frac{Z_{11}}{Z_R},\ z_{22} = \frac{Z_{22}}{Z_R},\ z_{12} = \frac{Z_{12}}{Z_R} \quad \ldots \quad (13)$$

(where Z_R is the reference impedance) it can be shown that[4]

$$R_I = 1 - \frac{2(1 + z_{22})}{(1 + z_{11})(1 + z_{22}) - z^2_{12}}$$
$$R_{II} = 1 - \frac{2(1 + z_{11})}{(1 + z_{11})(1 + z_{22}) - z^2_{12}}$$
$$T = \frac{2z_{12}}{(1 - z_{11})(1 + z_{22}) - z^2_{12}} \quad (14)$$

From Equ. (14) the general formulae for representing the impedance matrix of a four-terminal circuit can be derived:

$$z_{11} = \frac{(1 + R_I)(1 - R_{II}) + T^2}{(1 - R_I)(1 - R_{II}) - T^2}$$
$$z_{22} = \frac{(1 - R_I)(1 + R_{II}) + T^2}{(1 - R_I)(1 - R_{II}) - T^2}$$
$$z_{12} = \frac{2T}{(1 - R_I)(1 - R_{II}) - T^2} \quad \ldots \quad (15)$$

[It so happens that Equs. (15) can be obtained from Equs. (14) simply by replacing z_{11} by $-R_I$, z_{22} by $-R_{II}$ and z_{12} by T and that the reverse procedure can be applied for obtaining Equs. (14) from Equs. (15).] For a loss-free four-terminal circuit the real parts of the elements of the impedance matrix must all be zero and from Equs. (14) or (15) it can be shown that in such a case the following relation between the characteristic factors holds valid:

$$A^2_I = A^2_{II} = 1 - B^2$$
$$\frac{\alpha_I + \alpha_{II}}{2} - \beta = \mp\frac{\pi}{2} \quad \ldots \quad \ldots \quad (16)$$

A further simplification of Equs. (15) as well as of Equ. (16) is possible for symmetrical circuits ($z_{11} = z_{22}$) where a change of the input and output terminals does not change the characteristic reflection factors. In this case the subscripts can be omitted:

$$R_I = R_{II} = R = A \exp. j\alpha \quad \ldots \quad (17)$$

and Equs. (16) are simplified to:

$$A^2 + B^2 = 1$$
$$\alpha - \beta = \mp\frac{\pi}{2} \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

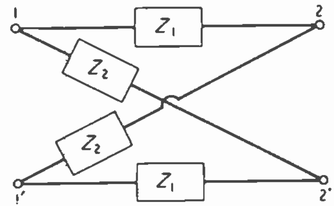

Fig. 3. *Lattice type circuit.*

The theory of symmetrical filters, which is of particular practical interest, can be usually brought to a simpler form by representing a four-terminal circuit by its equivalent lattice type circuit (Fig. 3) which is determined by the impedances Z_1 and Z_2 of the two arms

$$Z_1 = Z_{11} - Z_{12} \quad \ldots \quad \ldots \quad \ldots \quad (19)$$
$$Z_2 = Z_{11} + Z_{12}$$

From these formulae and Equs. (13), (15) and (17) we have

$$\frac{Z_1}{Z_R} = \frac{1 + (R - T)}{1 - (R - T)}$$
$$\frac{Z_2}{Z_R} = \frac{1 + (R + T)}{1 - (R + T)} \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

5. Relation between the Characteristic Factors and the Short-Circuit and Open-Circuit Impedances

Sometimes the termination of a four-terminal circuit by either a non-reflecting transmission line or a constant impedance as has been suggested may not be practicable in electrical measuring technique. In such cases, however, it is usual to measure the input-impedance with the four-terminal circuit short or open circuited at the output. It will be shown briefly how such impedance values can be used to represent the characteristic factors.

The short-circuit and open-circuit (input) impedances may be called Z_{SCI} and Z_{OCI} respectively, and those with reversed input and output terminals may be called Z_{SCII} and Z_{OCII}. These four impedance values are not independent and it can be shown from Equ. (12) that

$$\frac{Z_{SCI}}{Z_{SCII}} = \frac{Z_{OCI}}{Z_{OCII}} \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

The ratios of the short-circuit and open-circuit impedances and the reference impedance Z_R can be written thus:

$$\left.\begin{aligned} z_{SCI} &= \frac{Z_{SCI}}{Z_R},\ z_{SCII} = \frac{Z_{SCII}}{Z_R}, \\ z_{OCI} &= \frac{Z_{OCI}}{Z_R},\ z_{OCII} = \frac{Z_{OCI}}{Z_R} \end{aligned}\right\} \quad \ldots \quad (22)$$

and the characteristic factors can be derived from the definition of these impedances and Equs. (12) to (14).

$$R_I = \frac{z_{OCI}\,(z_{SCII} + 1) - z_{OCII} - 1}{z_{OCI}\,(z_{SCII} + 1) + z_{OCII} + 1}$$

$$R_{II} = \frac{z_{OCI}\,(z_{SCII} - 1) + z_{OCII} - 1}{z_{OCI}\,(z_{SCII} + 1) + z_{OCII} + 1}$$

$$T = \frac{2\sqrt{z_{OCI}\,(z_{OCII} - z_{SCII})}}{z_{OCI}\,(z_{SCII} + 1) + z_{OCII} + 1} \quad (23)$$

For a symmetrical network $z_{SCI} = z_{SCII} = z_{SC}$ and $z_{OCI} = z_{OCII} = z_{OC}$ which yields with Equ. (23)

$$R = \frac{z_{OC}\,z_{SC} - 1}{z_{OC}\,z_{SC} + 2\,z_{OC} + 1}$$

$$T = \frac{2\sqrt{z_{OC}\,(z_{OC} - z_{SC})}}{z_{OC}\,z_{SC} + 2\,z_{OC} + 1} \quad \ldots \quad (24)$$

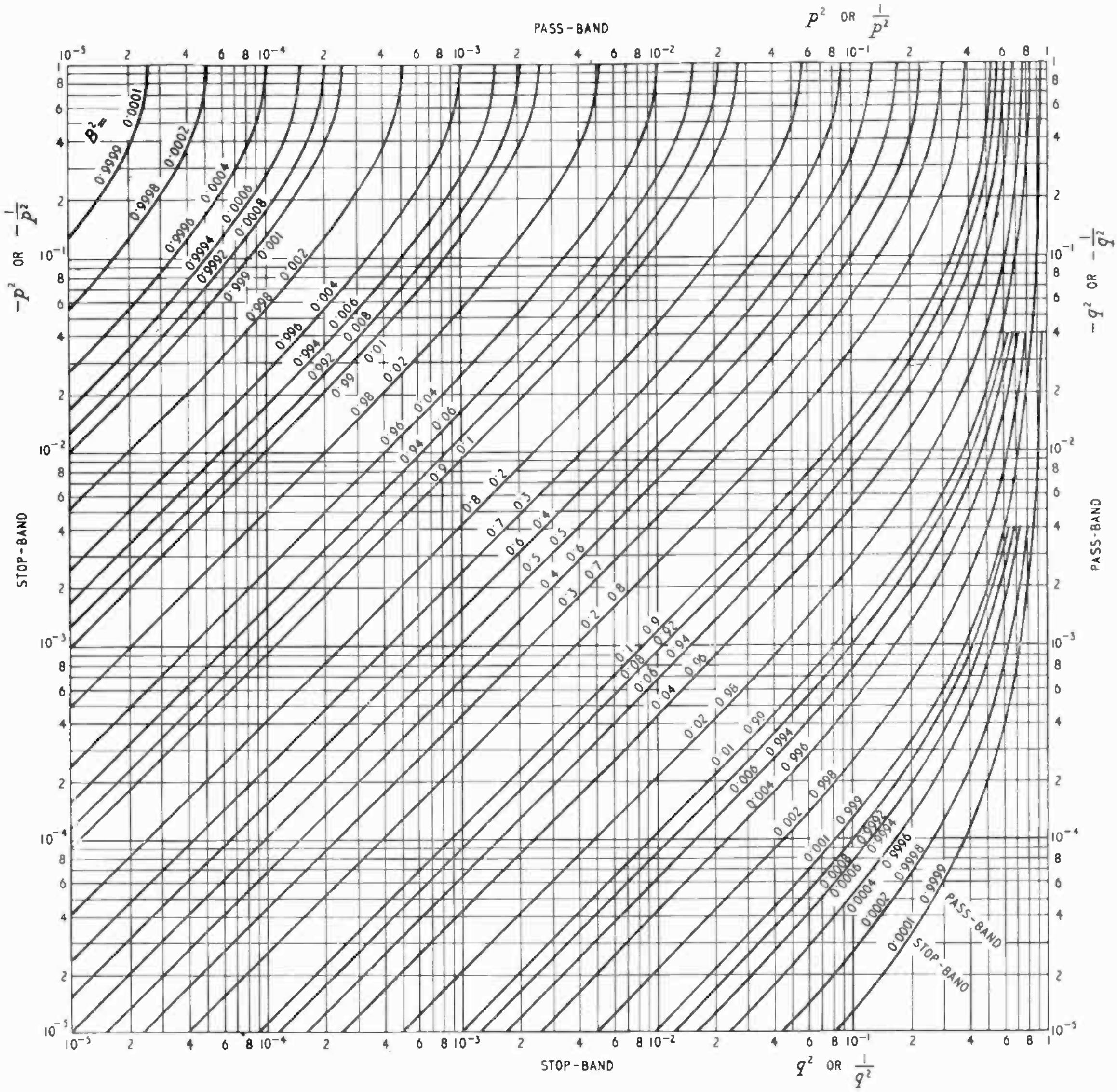

Fig. 4. *Relation between the image functions p and q of a loss-free symmetrical four-terminal circuit for various values of B^2 in a pass-band and in a stop-band.*

Methods for determining short-circuit and open-circuit impedances are well developed in electrical measuring technique so that Equs. (23) or (24) will be helpful for representing the characteristic factors.

6. Relation between the Characteristic Factors and the Image Parameters

It is well known that the image parameters of a four-terminal network play a major part in the theory of periodical structures of which certain cases, as will be shown later, can be profitably represented with the aid of the characteristic factors. The relationships between the characteristic factors and the image parameters are therefore of fundamental importance. For simplicity only the symmetrical case will be discussed. Two specific functions which are related to the lattice type circuit of Fig. 3 in a simple way are often used for describing a symmetrical four-terminal circuit[5], one is the image impedance function $Z_i = \sqrt{Z_1 Z_2}$ and the other is the image attenuation function $Q = \sqrt{Z_2/Z_1}$. The ratio of Z_i and Z_R may be called P and with Equ. (20)

$$P = \frac{\sqrt{Z_1 Z_2}}{Z_R} = \sqrt{\frac{(1+R)^2 - T^2}{(1-R)^2 - T^2}}$$

and

$$Q = \sqrt{\frac{Z_2}{Z_1}} = \sqrt{\frac{(1+T)^2 - R^2}{(1-T)^2 - R^2}} \quad \ldots \quad (25)$$

With these formulae and Equs. (20) the values of R and T are found to be

$$R = \frac{Q\,(P^2 - 1)}{(P+Q)\,(1+PQ)}$$

and

$$T = \frac{P\,(Q^2 - 1)}{(P+Q)\,(1+PQ)} \quad \ldots \quad \ldots \quad (26)$$

For loss-free and symmetrical four-terminal circuits P and Q will be called p and q. From Equs. (25) and (18) it can be shown that:

$$p = \sqrt{\frac{\cos\alpha + A}{\cos\alpha - A}} = \sqrt{\frac{\pm\sin\beta + \sqrt{1-B^2}}{\pm\sin\beta - \sqrt{1-B^2}}} \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

and

$$q = \sqrt{\frac{\cos\beta + B}{\cos\beta - B}}$$

These formulae can be brought to a more suitable form for representing the dimensionless functions p and q by B and β or in the reverse case for representing B and β by p and q. One particularly useful form for later discussion is:

$$B^2\left[\left(\frac{1+q^2}{1-q^2}\right)^2 - \left(\frac{1+p^2}{1-p^2}\right)^2\right] + \left(\frac{1+p^2}{1-p^2}\right)^2 = 1 \quad \ldots \quad \ldots \quad (28)$$

Another shorter form is

$$B^2 = \frac{p^2\,(1-q^2)^2}{(p^2-q^2)\,(1-p^2q^2)} \quad \ldots \quad \ldots \quad (29)$$

and

$$\cos^2\beta = B^2\left(\frac{1-q^2}{1-q^2}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

From these formulae it can be seen that a change from p to $1/p$ (reciprocal or dual four-terminal circuit)[5] or from q to $1/q$ (change of Z_1 and Z_2 and vice versa in Fig. 2) does not alter the values of B^2 or $\cos^2\beta$, where on the other side the latter values belong to four different four-terminal circuits which can be specified (besides the value of Z_R) by p and q, by p and $1/q$, by $1/p$ and q or by $1/p$ and $1/q$. (This latter ambiguity is, however, trifling and will not be elaborated here.) A graphical representation of Equs. (28) or (29) can be made easily because it can be brought to a similar form for the pass-bands and for the stop-bands. In a pass-band the relations $q^2 \leqslant 0$ and $p^2 \geqslant 0$ are valid and from Equ. (28)

$$p^2 = \frac{\sqrt{1 - B^2\left(\frac{1+q^2}{1-q^2}\right)^2} + \sqrt{1-B^2}}{\sqrt{1 - B^2\left(\frac{1+q^2}{1-q^2}\right)^2} - \sqrt{1-B^2}} \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

where it is permissible to replace p^2 by $1/p^2$. In a stop-band the relations $q^2 \geqslant 0$ and $p^2 \leqslant 0$ are valid and from Equ. (28)

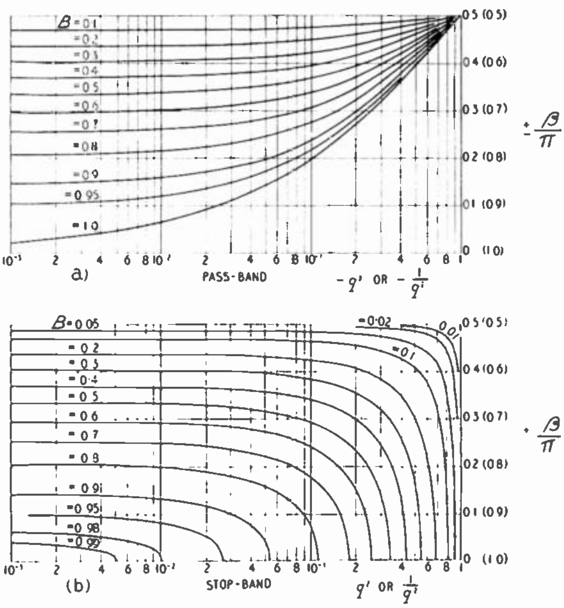

Fig. 5. *The phase value β/π of the characteristic transmission factor of a loss-free symmetrical four-terminal circuit plotted against q^2 (or $1/q^2$); (a) in a pass-band, (b) in a stop-band.*

$$q^2 = \frac{\sqrt{1 - (1 - B^2)\left(\frac{1 + p^2}{1 - p^2}\right)^2} + B}{\sqrt{1 - (1 - B^2)\left(\frac{1 + p^2}{1 - p^2}\right)^2} - B} \qquad (32)$$

where it is again permissible to replace q^2 by $1/q^2$.

Equs. (31) and (32) are indeed of similar form and can be represented in one diagram as has been done in Fig. 4, B^2 being taken as the parameter. The graphical representation of Equ. (30) is easy and is shown in Fig. 5(a) for the pass-band and in Fig. 5(b) for the stop-band, B being taken as the parameter. It can be seen that in a pass-band $\cos^2\beta \leqslant B^2$, in a stop-band $\cos^2\beta > B^2$ and at a cut-off frequency (where $q^2 = 0$ or ∞) $\cos^2\beta = B^2$. With the aid of the formulae and diagrams derived in this section a change from the image functions to the characteristic factors of a symmetrical four-terminal circuit and vice versa can easily be made.

(To be concluded)

TRANSMISSION-LINE TERMINATION

Measurement Through a Mismatched Junction

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SUMMARY.—In a previous article[1] certain theorems of plane geometry were applied to the analysis of the properties of a waveguide junction. These concepts are further extended in the present article and applied to a particular problem of common occurrence, namely, the determination of the impedance of an unknown termination when measured through a junction between two dissimilar transmission lines. The method avoids the necessity of ensuring a good match at the junction.

Principal Symbols

l = length of a section of transmission line
λ = wavelength in free space
λ_g = wavelength in the transmission line
Γ = reflection coefficient (complex)
$|\Gamma|$ = modulus of the reflection coefficient
$\arg \Gamma$ = argument (phase) of Γ
ϕ = a geometrical angle
Γ' = Image of Γ by the junction
α = attenuation of a length of transmission line in decibels

1. Introduction

A PREVIOUS article[1] has pointed out that the input–output relationship of a passive reciprocal two-terminal-pair network leads to transformations on the Smith chart which are bilinear and hence conformal. A number of useful geometrical relationships were established, most of which depend on the important property of a bilinear transformation that circles are transformed into circles. It was shown that these properties could be used as the basis of an experimental technique for determining the properties of a section of waveguide when measured through a mismatched junction. A simple graphical method of obtaining the scattering coefficients of the junction was also described.

In the present article some of the more elementary concepts of projective geometry in the plane are introduced. These concepts, which are outlined in Appendix 1, lead to a number of particularly-simple graphical constructions. In order to demonstrate the practical importance of the graphical method, discussion is centred on a particular problem frequently encountered in the laboratory. It is the problem of determining the relative impedance of an unknown termination to one transmission line when the measurements of standing-wave ratio are executed in a second transmission line having properties different from the first. The method has the special advantage of avoiding the need for a good match between the two transmission lines.

Although the discussion is restricted to one particular problem, it will be apparent that the method is capable of much wider application.

MS accepted by the Editor, December 1954

2. Measurement Technique

The termination under investigation will be referred to as the 'unknown termination'. It will be assumed that it is incorporated in some kind of transmission line referred to as 'line B' which may be different physically from the transmission system associated with the measuring gear, which will be referred to as 'line A'.

To connect line A to line B a transducer or junction of some sort will be required. For reasons with which we are not concerned here the match between the lines A and B at the junction is presumed to be rather poor.

Before making measurements with the termination in position, it is necessary to determine the properties of the junction and the line B. This is accomplished in the manner illustrated in Fig. 1. The line B is equipped with a movable short circuit whose position is varied over half a wavelength so as to describe a circle on the Smith Chart. Due to the mismatch at the junction, the circle will not be concentric with the centre of the Smith Chart[1]. The s.c. termination is now replaced by the unknown termination as illustrated in Fig. 2, and *a single measurement is made*.

From the above data it is possible to evaluate the reflection coefficient (and hence relative impedance) of the termination as measured in the line B.

3. Reference Planes

Where the junction consists of a symmetrical discontinuity of negligible thickness in an otherwise continuous guide, the reference plane of the junction is easy to establish, and may for convenience be taken at the centre of the discontinuity.

In other cases, where the junction is large in size or of irregular physical shape, the definition of a satisfactory single reference plane is rather difficult. In this case it is preferable to select two arbitrarily chosen reference planes, one on the input side and the other on the output side.

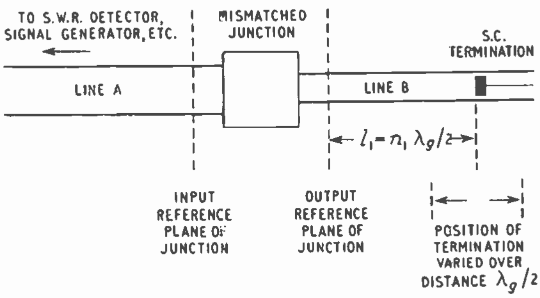


Fig. 1. *Determination of properties of junction and line B; λ_g = wavelength in line B.*

In order that these reference planes may be identified physically without ambiguity it is preferable to choose them so that they lie outside the junction proper, and therefore in the lines to which the junction is connected. This is clearly illustrated in Figs. 1 and 2.

Provided we are interested only in the properties of the termination then the choice of reference planes is entirely arbitrary, and the only important requirement is that the same reference planes be used throughout.

On the other hand, if it is also required to deduce the scattering coefficient of the junction[1-3] then the choice of reference planes is of significance and requires specification.

4. Simplifying Assumptions

The following simplifying assumptions will be made:—

1. The length l_1 in Fig. 1 is an integral number of half wavelengths (i.e., n_1 is an integer).
2. The length l_2 in Fig. 2 is equal to l_1.
3. The loss of the line B over a length l_1 or l_2 is so small as to be negligible.

The effect of relaxing these assumptions is discussed briefly in Section 7.

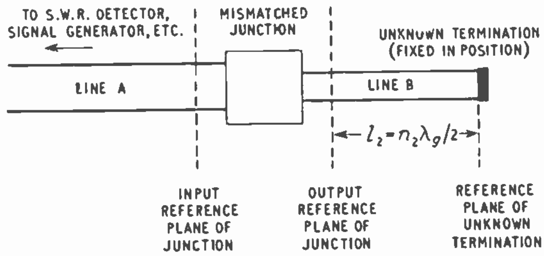


Fig. 2. *Determination of properties of a termination when measured through a mismatched junction; λ_g = wavelength in line B.*

5. Properties of the Line and Junction

The position of the s.c. termination in Fig. 1 is varied over a half wavelength about its mean position. For each position of the s.c. termination the input v.s.w.r., and position of the standing-wave minimum are observed. From the v.s.w.r. the modulus of the reflection coefficient is obtained:—

$$|\Gamma| = \frac{1 - (\text{v.s.w.r.})}{1 + (\text{v.s.w.r.})} \qquad \ldots \quad (1)$$

where v.s.w.r. is defined as minimum over maximum.

Arg. Γ is given by the position of the standing-wave minimum measured relative to the arbitrarily chosen input reference plane of the junction.

The values of Γ so obtained are plotted on the Smith Chart as illustrated in Fig. 3. It will be

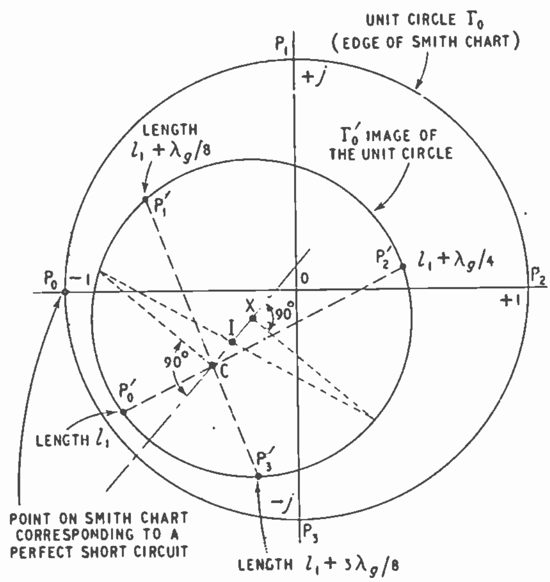


Fig. 3. *Finding the Iconocentre for the junction and the line.*

found that all the points lie on a circle Γ_0'. This circle is the 'image'[1] or transform of the circle that would have been obtained had the measurement been made in the line B. Since we have assumed that the line B is virtually lossless, the circle Γ_0' is the image of the unit circle Γ_0; i.e., the edge of the Smith Chart.

For simplicity, it will be assumed that we have moved the s.c. termination in steps of $\lambda_g/8$ at a time—where λ_g is the wavelength in the line B. Hence the circle Γ_0' will be defined by four points in the manner shown. Since the point P_0' corresponds to a length of line l_1 equal to an integral number of half wavelengths, it must be the image by the junction of the point P_0 on the Smith Chart corresponding to a perfect short-circuit; i.e., the point $\Gamma = -1$.

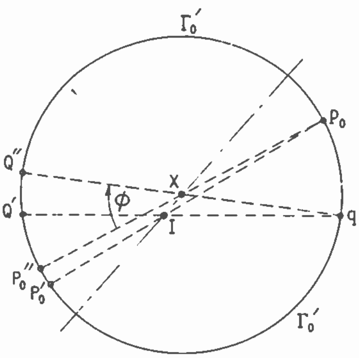


Fig. 4. *Geometrical construction when unknown termination is lossless; φ is the actual angle measured on the chart with an ordinary protractor.*

To define the properties of the junction and the line B, we require to know the position of the Iconocentre. (The Iconocentre[1-3] is by definition the image of the point 0; i.e., it is the value of Γ which would be obtained if the line B were terminated in a matched load.) To obtain the Iconocentre (assuming no matched load is available):—

1. Draw in the chords between opposite pairs of measured points so as to obtain the crossover point C.
2. Find the centre X of the circle Γ_0'.
3. Draw the line CX; i.e., the line of centres.
4. Use the construction shown in dotted lines to obtain the position of the Iconocentre.

If desired we could now evaluate the scattering coefficients of the junction[1,2,3]. However, if we are interested only in the properties of the unknown termination, this is not necessary.

NOTE.—A number of other constructions for obtaining the Iconocentre are possible[1,2,3]. Provided the s.c. termination can be moved accurately in steps of $\lambda_g/8$, the construction given here is the most satisfactory. (A condensed proof is given in Appendix 2.)

6. Properties of an Unknown Termination Assumed Lossless

Taking a simple case first it will be assumed that the termination is lossless. The appropriate construction is illustrated in Fig. 4 where Q' is the value of reflection coefficient obtained when the short-circuit is replaced by the unknown termination (see Fig. 2). Since the termination is lossless, Q' must lie on the circle Γ_0'.

In this case the construction is singularly simple. P_0' is projected through I to 'p_0', and 'p_0' is projected through X to P_0''. This is known as a transvection. (See Appendix 2.) The point Q' is likewise transvected to Q''. The angle ϕ supplies all the data required to specify the properties of the termination.

To construct the point Q, of which Q' is the image by the junction, we proceed as in Fig. 5. In other words the true reflection coefficient of the unknown termination is given by:—

$$\Gamma = e^{+j(\pi - \phi)}$$

the reference plane being that of the termination itself. (The latter follows directly from assumption 2, Section 2.)

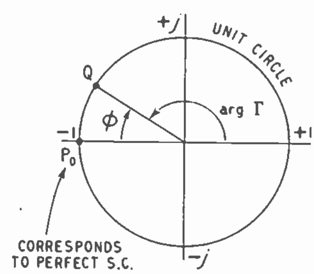
The impedance of the termination (in this case a pure reactance) may either be read directly from the Smith Chart or computed by means of the relationship:—

$$Z/Z_0 = \frac{1 + \Gamma}{1 - \Gamma} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where Z_0 is the characteristic impedance of transmission line B.

It should be noted that in proceeding from Fig. 4 to Fig. 5 the sense of the angle ϕ as well as its magnitude must be maintained.

Fig. 5. *Smith Chart representation of unknown termination for case when termination is lossless. The reference plane is that of the termination itself.*



7. Properties of an Unknown Termination, General Case

In Fig. 6 are shown the constructions to be used when the termination is both dissipative and reactive. To obtain the phase of the reflection coefficient we use the construction shown in Fig. 6(a):—

1. Draw a straight line through X_0 and I. This line is known as the line of centres[1,5,6].
2. Draw IH_0 at right angles to the above line.
3. Draw the tangent at H_0 to the circle Γ_0', so as to intercept the line of centres at

L. (The points I and L are the limiting points[1,5,6] of the family of circles of which both the Γ_0' and the Γ_1' circles are members.)

4. Construct the perpendicular bisector of the line IL as shown. This line is known as the radical axis[1,5,6].
5. On the radical axis find S_Q the centre of the circle through Q′ and I. (i.e., draw the perpendicular bisector of Q′I.)
6. Draw the tangent at Q′ to this circle. The point of intersection X_1 of this tangent with the line of centres is the centre of the circle Γ_1' through Q′.
7. On the radical axis find S_P the centre of the circle through P_0' and I. (i.e., draw the perpendicular bisector of P_0'I.)
8. Measure the angle ϕ between the two circular arcs at I, centres S_P and S_Q.
   The angle ϕ determines the phase of the reflection coefficient. (i.e., $\arg \Gamma = \pi - \phi$.)
   To determine the modulus of the reflection coefficient we proceed as follows [Fig. 6(b)]:—
9. Take *any* point K′ on the circumference of circle Γ_0'. Transvect K′ to K″ as shown.
10. Project K′ through A to 'a'.
11. Join 'a' to K″ thus cutting the line of centres at B.

The modulus of the desired reflection coefficient is given by:—

$$|\Gamma| = BX_0/CX_0 \qquad \ldots \quad \ldots \quad \ldots \qquad (3)$$

The corresponding point Q may now be plotted on the Smith Chart as shown in Fig. 6(c). Q is accordingly the reflection coefficient of the unknown termination when measured in the line B, the reference plane being that of the unknown termination.

The impedance of the termination may be obtained either from the Smith Chart for the point Q or by application of Equ. (2).

8. Modifications when the Line B is Lossy

If the losses in the line B are not negligible, the circle Γ_0' will be the image of some circle of smaller radius than the unit circle Γ_0. The constructions of Figs. 3, 4 and 6 are unaltered, but the interpretation of the results must be modified.

Thus in Fig. 5 the point Q will lie on a circle of radius:—

$$\Gamma = \text{anti-log}_{10}\alpha/10 \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

where α is the attenuation in dB of the length of line $l_1 = l_2$.

Similarly in Fig. 6(c) the expression for $|\Gamma|$ given by Equ. (3) is modified to:—

$$|\Gamma| = \frac{B\,X_0}{C\,X_0}\,\text{anti-log}_{10}\alpha/10 \quad \ldots \quad \ldots \qquad (5)$$

In both the above expressions it is assumed that $l_1 = l_2$. However, if this is not so, and line B is lossy, then certain complications arise. It is not proposed to describe the appropriate constructions in this case because in practice it is usually possible to ensure either that the lengths are nearly the same, *or* alternatively use a very short length of line which is virtually lossless.

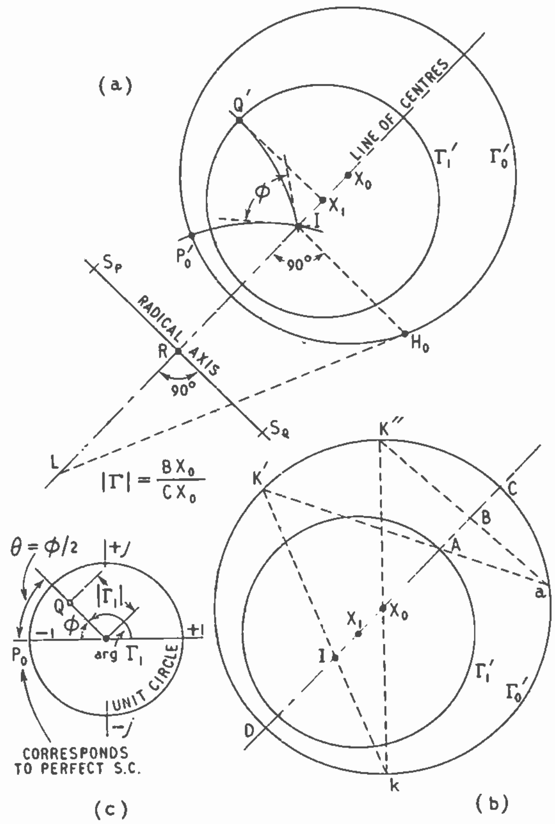


Fig. 6. *Geometrical construction for lossy termination; (a) finding ϕ; (b) finding $|\Gamma|$; and (c) Smith Chart representation of lossy termination. The reference plane is that of the termination itself.*

9. Further Note on Reference Planes

If $l_2 = l_1$ but n_1 and n_2 are not integers then P_0' is the image of some point other than P_0. Otherwise, however, this does not affect the computation in any way, because it is equivalent to an arbitrary shift in the position of the output reference plane. As will have been observed, the position of this reference plane only enters into the calculation quite incidentally.

On the other hand, if $l_2 \neq l_1$ then an adjustment must be made accordingly. This may be allowed for quite easily in the final stages of the calculation once the angle ϕ has been obtained. Thus the angle marked off on the Smith Chart in Fig. 6(c) and Fig. 5 will be equal to:—

$$\phi - (l_2 - l_1)\,4\pi/\lambda_g$$

10. Variation with Frequency

In the general case, where both the junction and the termination are frequency sensitive, there is no alternative but to repeat the whole procedure from beginning to end, including the determination of the Iconocentre, for each frequency. There are, however, a number of special cases in which it is possible to simplify the routine considerably. The most important example occurs when the junction is frequency insensitive (i.e., it can be represented by a perfect transformer and a length of transmission line). Fig. 3 remains basically unchanged, except for a rotation of I about O, and a rotation of the points P_0', P_1', etc., around the circumference of the circle Γ_1'. The first rotation depends on the total length of line to the left of the junction, and the second on the total length on the right of the junction, the equivalent length of line within the junction being included in an appropriate manner. Thus the same diagram may be used throughout. Additional simplifications will occur according to whether the transmission line B and/or the termination are dissipative or not. Space precludes a detailed discussion of these topics, which in any case are best understood through practical experience with the technique.

11. Junctions in Cascade

The result of two successive bilinear transformations is itself a bilinear transformation. Thus in determining the relative impedance of the unknown termination a series of junctions in cascade may be regarded as a single junction. However, it is not possible to determine the scattering coefficients[1] of any individual junction unless measured either separately, or in combination with other junctions whose parameters are already known. Although suitable graphical constructions have been devised for determining the properties of one junction through another junction they are rather complex, and the analysis is sometimes best carried out with the aid of the so-called "projective chart"[4], a description of which is beyond the scope of the present article.

Acknowledgments

The author wishes to thank Standard Telecommunication Laboratories for permission to publish this article. The article is based to a large extent on ideas originally proposed by G. A. Deschamps to whom special acknowledgment is accordingly due.

APPENDIX 1

General Theorems

The following well-known propositions in the fields of plane and projective geometry form the basis of the constructions described in the texts. Full proof will be found in a number of standard textbooks[5,6].

Proposition 1. Cross-ratio of a range of four points

Let A, B, C, and D be any four points on a line 'p' as in Fig. 7. Let (AB) denote the distance between A and B measured in the sense A→B. As a necessary consequence of sign;

$$(AB) = -(BA)$$

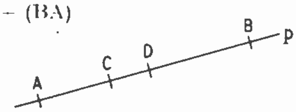


Fig. 7.

If we take A and B as reference points then the ratio (AC)/(CB) is unique for C whatever its position on the line 'p'. Hence this ratio may be used as the co-ordinate of C. The co-ordinate of D relative to A and B may likewise be defined at (AD)/(DB).

The ratio of the co-ordinates of C and D is called the cross-ratio. If the points are specified in the order A, B, C, D (independently of their order on the line 'p') then the cross-ratio may be written:—

$$(A\ B\ C\ D) = \frac{(AC)}{(CB)} \frac{(DB)}{(AD)} = \frac{(AC)}{(AD)} \frac{(BD)}{(BC)}$$

A and B are called a pair of conjugate points as also C and D. (The logarithm of the cross-ratio is known as the 'hyperbolic distance' between C and D. For further details of this concept and its uses see reference 4.)

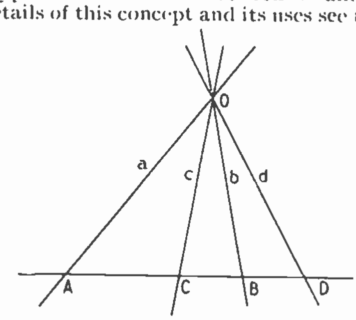


Fig. 8.

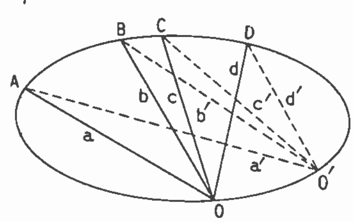


Fig. 9.

Proposition 2. Cross-ratio of a pencil of four lines

If 'a, b, c, d' be a pencil of four lines (Fig. 8) passing through a common vertex O then the cross-ratio is similarly defined as:—

$$(a\ b\ c\ d) = \frac{\sin A\hat{O}C \sin B\hat{O}D}{\sin A\hat{O}D \sin B\hat{O}C}$$

It is evident that the above expression is independent of the position and orientation of the line 'p'. Hence it follows that:—

(1) The section of a pencil of four lines by one line gives a range of the same cross-ratio.

(2) The projection of a range of four points through any centre, gives a pencil of the same cross-ratio.

Proposition 3. Invariance of the cross-ratio[1,7]

The cross-ratio is invariant under a bilinear transformation. (The input–output relationship of a reciprocal passive network is always bilinear.) Thus if 'p' is a line on the Smith Chart and if the effect of the junction mismatch is to transform 'p' into another line 'p′', and the points A, B, C, D into A′, B′, C′, D′, then:

$$(A'\ B'\ C'\ D') = (A\ B\ C\ D)$$

Proposition 4. Charles theorem

Four fixed points on a conic subtend a pencil of constant cross-ratio at any point on the conic. Thus in Fig. 9: —

(a′ b′ c′ d′) = (a b c d)

For the special case where the conic is a circle the proof of this theorem is quite trivial.

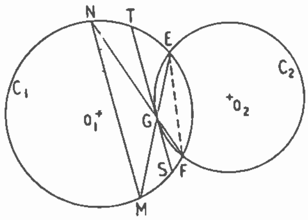


Fig. 10.

Proposition 5. Intersecting circles

Let the two circles C_1 and C_2 in Fig. 10 intersect at E and F. Let G be an arbitrary point on arc EF (on circle C_2 say). Project EG to M and FG to N as shown, then MN is parallel to the tangent ST. (Since MNF = MEF and since MEF = SGF it follows that MNF = SGF and hence MN is parallel to ST.)

If the circles intersect orthogonally then MN is a diameter of C_1. (Since the circles cut orthogonally, O_1E and O_1F are tangents at E and F to the circle C_2. Hence O_1EM = NFE; but O_1ME = O_1EM and NME = NFE, so that NME = O_1ME. Hence O_1 must be a point on MN; i.e., MN must be a diameter.)

APPENDIX 2

Validity of the Constructions

The validity of the constructions follows from the propositions of Appendix 1 and reference 1.

Construction of Fig. 3 for finding the Iconocentre

In Fig. 11 is shown a chord P_1' P_3' (image of the diameter P_1P_3 in Fig. 3), the Iconocentre, and the cross-over point C. Project P_1' through I to M, and P_3' to N. Now it is shown in reference 1 that the circle drawn through the points P_1', I, and P_3' intersects the circle Γ_0' orthogonally. (This follows generally from the fact that the arc $P_1'IP_3'$ is the image of the diameter P_1IP_3 in Fig. 3, and from the angle preserving properties of a bilinear transformation.) But from proposition 5 in Appendix 1 MN must be a diameter of the circle Γ_0'. From propositions 2 and 4 it follows that if I and X are fixed points then C must also be a fixed point independent of the orientation of the diameter MN. Thus all chords joining points corresponding to transmission line lengths differing by $\lambda_g/4$ pass through the cross-over point C. The actual construction shown in dotted lines in Fig. 3 is a particular case of Fig. 11 with AB and MN intersecting at right angles.

Construction of Fig. 4. The Transvection

Consider Fig. 10 in relation to proposition 5. Assuming the circles intersect orthogonally we can identify circle C_1 with the circle Γ_0' and the point L with the Iconocentre I. Thus in Fig. 4 the diameter $P_0''p_0$ is parallel to the tangent at I to the circle which passes through P_0' and is orthogonal to the circle Γ_0'. Similarly the diameter $Q''q$ is parallel to the tangent at I to the orthogonal circle through Q' and I. The angle ϕ in Fig. 4 may thus be identified with the angle ϕ in Figs. 7(a) and (b) of reference 1. It follows from the above that the angle ϕ in Fig. 4 provides the information required. It should be stressed that the transvection is a useful and important construction. It provides a very rapid method of relating all points P on the circumference of the circle Γ_0 to all points P″ on the circumference of the circle Γ_0'. It may also be used for points lying on different member circles of the family (see next para.).

Construction of Fig. 6(a)

This follows directly from Fig. 8 of reference 1 on the assumption that the unknown termination can always be replaced by a section of lossy transmission line of appropriate length terminated by a short circuit. (It is assumed that the characteristic impedance of this extra length of line is identical to that of the line B in Figs. 1 and 2.) A variation in position of this short circuit would delineate the circle Γ_1' in Fig. 6(a).

Note that the transvection of Fig. 4 may be used as alternate method of finding ϕ. In this case P_0' is transvected through I and X_0, whereas Q' is transvected through I and X_1.

Construction of Fig. 6(b)

This may be regarded as a special extension of the transvection technique to points interior to the circles. Applying propositions 2 and 5 of Appendix 1, it follows that:

$$(D\ C\ X_0\ B) = (D\ C\ I\ A)$$

But in proposition 3 it is stated that the cross-ratio is invariant under a bilinear transformation. Hence if I were the image of X_0 then A would be the image of B, and the circle Γ_1' would be the image of a circle through B centre X_0. Now the result of two successive bilinear transformations is itself a bilinear transformation[1,7]. Thus we may introduce a second transformation involving only linear displacement and uniform enlargement which shifts X_0 to O and enlarges Γ_0' to Γ_0. Since the enlargement is uniform it follows that the ratio X_0B/X_0C is a constant under this second transformation. In other words the radius of the circle Γ_1 through Q in Fig. 6(c) = BX_0/CX_0.

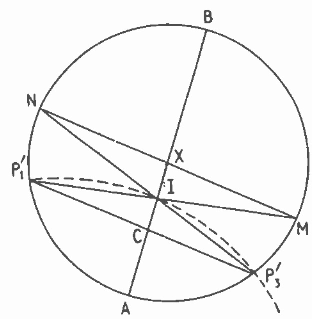


Fig. 11.

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STABILITY OF OSCILLATION IN VALVE GENERATORS

By A. S. Gladwin, Ph.D., D.Sc.
(*University of Sheffield*)

(*Continued from p. 253, September issue*)

11. Amplitude Stability—Squegging

A FORM of instability consisting in a periodic modulation of the oscillation amplitude is well known by the colloquial name of 'squegging'. In the simplest example, periodic instability is represented by the double encirclement shown in Fig. 9(c). This implies that the equation $A(p) = 0$ is at least of the second degree and, since the measure of p is the reciprocal of time, there must exist at least two time constants in the feedback network. These may be associated with the low-frequency impedances Z_i^0, etc., with the high-frequency impedances Z_i^+, etc., or with both. In what follows it is assumed that the oscillation has aperiodic stability, for otherwise there would be no point in discussing periodic stability.

Instability due entirely to low-frequency time constants would mean that the oscillator had been constructed to generate both low- and high-frequency waves simultaneously. Instability due entirely to high-frequency time constants may be examined by putting $Z_i^0 = Z_0^0 = Z_t^0 = 0$. This ensures that no low-frequency voltages exist in the network. The grid-bias voltage is then fixed, and it is sufficient to consider the case of no grid current, so that $S_0 = S_1 = S_2 = 0$ and $r_g = \infty$. Then from (6.3) and (8.2)

$$A(p) = (1 - bZ_E^+/R_E) \quad \ldots \quad (11.1)$$

where $b = (G_0 + G_2)/(G_0 - G_2)$

Since aperiodic stability is assumed, $b < 1$.

Now when $r_g = \infty$ the frequency stability function $F(p)$ given by (9.1) differs from (11.1) only in that $b = 1$. Consideration of Figs. 8(b) and 9(c) then shows that if $A(p)$ encircles the origin $F(p)$ must also do so. Periodic amplitude instability is therefore accompanied by aperiodic frequency instability, and it can be seen that when the network parameters are adjusted towards critical values frequency instability always precedes amplitude instability. Since this form of amplitude instability can have no separate existence it is unnecessary to consider it further.

There remains the third possibility, which is the only one of practical or theoretical importance, that instability is due to the existence of suitable time constants associated with both low- and high-frequency impedances. The coupling between the two is provided by the frequency-changing property of the amplifier. In the example shown in Fig. 12 the feedback network is an anti-resonant circuit with a coupling coil, and the grid-bias is obtained by a grid-leak and capacitor. A decoupling resistance and capacitor are also included in the anode circuit. This network has one high-frequency and two low-frequency time constants.

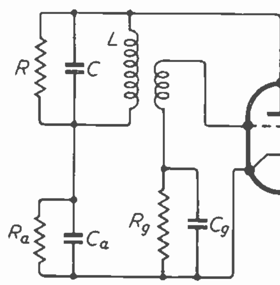

Fig. 12. *Oscillator with three time constants.*

In order that the network should be symmetrical the Q-factor must be large, the coefficient of coupling between the two coils must be 1, and the impedances of C_a and C_g must be small at the oscillation frequency. It can be shown by the more general methods of Section 15 that departures from these conditions produce only second-order errors.

Since the coefficient of coupling is 1, Z_i^+ and Z_0^+ are proportional to Z_t^+, and $Z_n^+ = 0$. If the Q-factor is very large $Z_t^0 = 0$ and Z_i^0 and Z_0^0 are the impedances of the combinations R_aC_a and R_gC_g. Then from (6.3) and (8.2)

$$A(p) = (1 + G_0Z_i^0/\mu)\{(1 + S_0Z_0)(1 + BZ_t^+/R_t) - DZ_0^0Z_t^+/R_gR_t\} - (E + FZ_0^0/R_g)\, Z_i^0Z_t^+/R_aR_t \quad \ldots \quad (11.2)$$

where

$$\left.\begin{aligned} B &= (G_0 + G_2)(R_t + R_i/\mu) + (S_0 + S_2)(R_0 - kR_t) \\ D &= 2S_1R_g\{G_1R_t + S_1(R_0 - kR_t)\} \\ E &= 2\{G_1(R_t + R_i/\mu) - kS_1R_t\}G_1R_a/\mu \\ F &= 2\{G_1S_0(R_t + R_i/\mu) - G_0S_1R_t\}G_1R_aR_g/\mu \end{aligned}\right\} \quad \ldots \quad (11.3)$$

Also

$$\left.\begin{aligned} R_a/Z_i^0 &= 1 + pT_a \\ R_g/Z_0^0 &= 1 + pT_g \\ R_t/Z_t^+ &= 1 + pT_t \end{aligned}\right\} \quad \ldots \quad (11.4)$$

where $T_a = R_aC_a$, $T_g = R_gC_g$, $T_t = 2Q/\omega_0 = 2R/\omega_0^2L$.

The expression for R_t/Z_t^+ is an approximation valid for large values of Q.

Substituting (11.4), into (11.2), the equation $A(p) = 0$ becomes

$$a_0p^3 + a_1p^2 + a_2p + a_3 = 0 \qquad \ldots \quad (11.5)$$

with

$$\left.\begin{aligned} a_0 &= T_aT_gT_t \\ a_1 &= (1 + G_0R_a/\mu)T_gT_t + MT_aT_g \\ a_2 &= LT_a + M(1 + G_0R_a/\mu)T_g \quad ET_g \\ a_3 &= L(1 + G_0R_a/\mu) - E - F \\ \text{and} \quad L &= (1 + S_0R_g)(1 + B) - D \\ M &= (1 + S_0R_g)T_t/T_g + 1 + B \end{aligned}\right\} \qquad \ldots \quad (11.6)$$

Since a_0 is positive the Routh-Hurwitz stability criteria are

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0,\ a_1a_2 - a_0a_3 > 0 \qquad (11.7)$$

Now the l.h.s. of (11.5) is in fact the numerator of $A(p)$, the denominator being $(1 + pT_a)(1 + pT_g)(1 + pT_t)$. Hence $A(0) = a_3$. The third inequality in (11.7) is therefore the condition for aperiodic stability which has been assumed to be satisfied. Inspection of (11.7) shows that only three of the inequalities are independent for, if the last three are satisfied, the first is automatically satisfied, and similarly the second is satisfied if the other three are satisfied. It is therefore sufficient to consider the last and one or other of the first two.

The general problem with three time constants is dealt with in the following Section: two simpler cases are considered here. It is first supposed that the decoupling resistance $R_a = 0$. This is a situation often approximated in practice. Then $E = F = a_0 = 0$. The characteristic equation (11.5) reduces to a quadratic, and, since $a_1 = T_gT_t$ is now always positive, the criterion for periodic stability is simply $a_2 > 0$, which, from (11.6) is

$$(1 + S_0R_g)T_t/R_g > -(1 + B)$$

$$\text{or} \quad (1 + S_0R_g)QX_g/R_g > -\tfrac{1}{2}(1 + B) \quad \ldots \quad (11.8)$$

where $X_g = 1/\omega_0C_g$, and from (11.3), (4.11), (5.5), and (5.14)

$$\tfrac{1}{2}(1 + B) = G_2R_E + (S_2r_g + G_2R_E)(R_0 - kR_t)/r_g \quad \ldots \quad \ldots \quad \ldots \quad (11.9)$$

Stability is aided by large values of Q, S_0R_g, and X_g/R_g. Regarding R_g and X_g as the disposable parameters this requires R_g and X_g/R_g to be large. However, a large value of X_g/R_g leads to a decrease in grid-bias voltage, to the production of harmonic voltages and to the appearance of a reactive component in the grid-input impedance[19]. To avoid these undesirable effects X_g/R_g must not exceed a certain maximum value which can be shown to be a function of b_gR_g. Taking this maximum as the value[19] corresponding to a reduction on grid-bias voltage of 1%, it is found that $(1 + S_0R_g)X_g/R_g$ varies from 0·2 to 0·62 when b_gR_g varies from 10 to 1,000. It would appear that stability is aided by making R_g as large as possible, the value of C_g being chosen to suit. A lower limit to C_g is set by the shunting effect of the valve and wiring capacitance, and an upper limit to R_g because of grid emission and ionization. Any further improvement can be obtained only by increasing the grid–cathode conductance b_g.

A second way of improving stability is to make $-\frac{1}{2}(1 + B)$ small. From (11.9) this means that $-G_2R_E$ should be small and $S_2(R_0 - kR_t)$ large. Fig. 6 shows that $-G_2R_E$ is small when H is large and this requires a large value of K and a small value of Y; i.e., a small oscillation amplitude. From the formulae of Appendix 2 it can be shown that S_2 is large when R_g is small. This conflicts with the previous requirement that R_g should be large. Hence, depending on the relative values of the two terms in $-\frac{1}{2}(1 + B)$, stability may be improved by increasing or decreasing R_g (with corresponding adjustment of C_g). If C_g is fixed stability is always improved by decreasing R_g.

When $\frac{1}{2}(1 + B) > 0$ the stability criterion (11.8) is satisfied whatever the values of Q, R_g, and C_g. On comparing (11.9) and (10.15) it is seen that this is identical with the condition for aperiodic stability with fixed grid-bias voltage and with grid current flowing. The physical explanation of this correspondence is obvious. If periodic stability is independent of T_g then T_g can be made infinite. The grid-bias voltage could then change only infinitely slowly, and so the grid leak and capacitor could be replaced by a generator of e.m.f. V_g. Thus there exists a critical amplitude below which the oscillation is unconditionally stable. If $r_g \gg R_0 - kR_t$ the critical amplitude is that which makes $G_2 = 0$; i.e., for which $H = 1{\cdot}42$ (Fig. 6). Hence, and using (4.18), the critical amplitude is

$$V_{g1} = -V_{ca}/(K + 0{\cdot}42)(1 + R_l/\mu R_T) \quad (11.10)$$

For a given oscillation amplitude, stability is therefore improved by using a valve for which V_{ca} is large and μ small. Inspection of (11.9) shows that when r_g is not very large its effect is to strengthen the inequality (because $S_2r_g + G_2R_E > 0$ in most practical arrangements) and thus to increase the maximum amplitude of absolute stability.

In this oscillator the grid-bias voltage varies with V_{g1} but the 'anode bias', or mean anode voltage, remains fixed. This suggests that a new type of periodic instability may exist in an oscillator with a fixed grid-bias voltage if the mean anode voltage varies with V_{g1}; i.e., if R_a is finite. It will be assumed that the grid-bias voltage is sufficient to stop grid current flow. Then $S_0 = 0$ and $r_g = \infty$.

Since there is no grid current the time constant T_g has no influence on the result and may therefore be given any convenient value. Putting $T_g = 0$ reduces the characteristic equation (11.5) to a quadratic. (The same result is obtained by dividing through by T_g and then putting $T_g = \infty$.) As before, the criterion for periodic stability is $a_2 > 0$. From (11.3), (11.6) and (11.9) this is

$$\left.\begin{array}{ll} & (1 + G_0R_a/\mu)T_t/T_a > -(1 + B) \\ \text{or} & (1 + G_0R_a/\mu)QX_a/R_a > -G_2R_E \end{array}\right\} \quad (11.11)$$

where $X_a = 1/\omega_0C_a$.

If $G_2 < 0$ the amplitude is stable for all values of T_t and T_a. This also guarantees aperiodic stability (10.11). If $G_2 > 0$ periodic instability will exist for suitable values of T_t and T_a, but a restriction (10.10), must now be imposed on R_a to ensure aperiodic stability.

When grid current flows and V_g is constant, S_0 and S_2 are finite but $Z_0^0 = 0$. The stability criterion is the same as the first form of (11.11) but the full expression (11.9) for $1 + B$ must now be used.

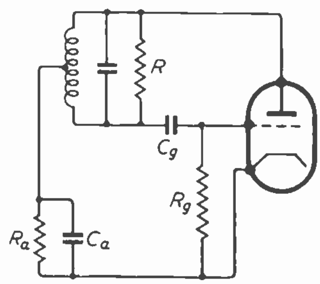

Fig. 13. *Hartley oscillator.*

12. Amplitude Stability—Three Time Constants

When all three time constants in the circuit of Fig. 12 are finite the increase in the complexity of the analysis makes a general discussion much more difficult. One method of approach is to compare the oscillator with another which differs from it only in that $R_a = 0$. For such an oscillator the stability criterion is simply $a_2 > 0$, or from (11.6), $M > 0$. The problem is therefore to find whether this inequality is strengthened or weakened by making R_a finite.

Of the stability criteria (11.7) the last and one of the first two must be satisfied. Using (11.6) the first can be written

$$M > -(1 + G_0R_a/\mu)T_t/T_a \quad .. \quad (12.1)$$

Since G_0 is always positive this inequality is strengthened for all values of R_a. The last inequality in (11.7) is more complicated, but a simplification can be made by assuming that R_a is small and neglecting terms in R_a^2 and T_a^2. Using (11.3) and (11.6) this leads to

$$M^2T_a/T_t + M(1 + 2G_0R_a/\mu) - E > 0.$$

If $E < 0$ stability is enhanced by increasing R_a from 0. Inspection of (11.3) shows that $E < 0$ if

$$G_1(1 + R_i/\mu R_t) > kS_1$$

For a three-halves-law amplifier G_1 is positive and is usually much greater than S_1. Only for very small amplitudes of oscillation is $E > 0$ and such amplitudes are in any case absolutely stable. The effect of R_a is therefore to improve stability.

When R_a is not very small no simplification is possible. The criterion $a_1a_2 - a_0a_3 > 0$ is a quadratic in M with coefficients depending on T_a, T_t and T_g. By substituting for M according to (11.6) this can be made a quadratic in T_g from which the critical value of T_g can be calculated. The expressions for the coefficients in this inequality are rather long and in numerical calculations it is more convenient to deal directly with the expressions for a_0, etc., given by (11.6). The labour of computation is slightly reduced by expressing L, E and F in the following forms. It is assumed that V_g is proportional to V_{g1}.

$$L = -2(1 + S_0R_g)(1 + R_0/r_g)(1 - kR_T/r_g)(KG_1 - G_2)R_E$$
$$E = 2(1 + R_0/r_g)\{G_1R_E - (S_1r_g + G_1R_E)\,kR_T/r_g\}\,G_1R_a/\mu$$
$$E + F = -2(1 + S_0R_g)(1 + R_0/r_g)\{R_E(KG_0 - G_1)(1 - kR_T/r_g) + (S_2/S_1)\,kR_T/r_g\}G_1R_a/\mu \quad (12.2)$$

The second type of oscillator to be discussed is the Hartley circuit shown in Fig. 13. This differs essentially from Fig. 12 in that the low-frequency transfer impedance Z_t^0 is no longer 0. As before, the characteristic equation is a cubic. The work is straightforward but the algebra is excessively tedious. Some simplification is achieved by assuming that μ is very large. Then with the same assumptions as before

$$A(p) = \{1 + S_0Z_0^0 + (G_0 - kS_0)Z_t^0\}(1 + BZ_t'/R_t) - \{Z_0^0/R_g + (G_1/S_1 - k)Z_t^0 R_g\}DZ_t^+/R_t \quad (12.3)$$

B and D are given by (11.3), and R_t/Z_t^+ by (11.4). Since the impedance of the antiresonant circuit is negligible at low frequencies

$$R_g/Z_0^0 = \{1 + p(T_a + T_g + R_aC_g) + p^2T_aT_g\}/\{1 + p(T_a + R_aC_g)\}$$
$$R_a/Z_t^0 = \{1 + p(T_a + T_g + R_aC_g) + p^2T_aT_g\}\,pT_g$$

Substituting these expressions into (12.3) the coefficients in the cubic equation for p are obtained as

$$a_0 = T_aT_gT_t$$
$$a_1 = MT_aT_g + T_gT_t\{1 + R_a(G_0 + 1/R_g) + S_0R_a(1 - k)\}$$
$$a_2 = MT_g + L(T_a + R_aC_g) + (1 + B)(G_0 - kS_0)R_aT_g - D(G_1S_1 - k)R_aC_g$$
$$a_3 = L \quad .. \quad (12.4)$$

M and L are given by (11.6) and (12.2), and $1 + B$ and D by (11.9) and (11.3).

The stability criteria are again taken as the first and last inequalities in (11.7). The first is

$$M > -\{1 + R_a(G_0 + 1/R_g) + S_0R_a(1 - k)\}T_l/T_a \quad .. \quad (12.5)$$

Since G_0, S_0 and $1 - k$ are positive, the effect of R_a is to strengthen the inequality. The other inequality can be simplified by assuming that R_a is small, that $k = 1$, and that r_g is large compared with R_0 and R_l. This gives

$$M^2T_a/T_l + M(1 + G_0R_a) - 2(G_1^2 - G_0G_2)R_aR_E > 0$$

For a three-halves-law amplifier $G_1^2 - G_0G_2 > 0$, and the effect of R_a is to strengthen this inequality also. The stabilizing influence of R_a is due to the strong degenerative feedback which exists at low frequencies.

When R_a is not very small the full expressions for a_0, etc., must be used. As for the previous oscillator, M can be substituted according to (11.6) to obtain the criterion in the form of a quadratic in T_g.

When $T_g = \infty$ the stability criteria become $a_1 > 0$, $a_2 > 0$. With suitable (fixed) values for the other parameters these inequalities give two values of R_a between which the amplitude is stable, instability existing for all higher and lower values.

13. Asymmetrical Networks

An asymmetrical impedance is one which does not satisfy the symmetry equation $Z(p^* + j\omega_0) = Z^*(p + j\omega_0)$. Since no impedance function is truly symmetrical, a more practical definition of asymmetry is that $|Z(p^* + j\omega_0) - Z^*(p - j\omega_0)|$ should not be negligible compared with the maximum value of $|Z(p + j\omega_0)|$ over the relevant range of p.

When the impedances in a feedback network are asymmetrical the expressions (6.3) for a_1, etc., cannot in general be simplified, and the full expression for $D(p)$ would run to hundreds of terms. One effect of asymmetry is therefore to increase greatly the complexity of the analysis. Only in a few special cases is it possible to make simple statements about stability comparable with those for symmetrical networks. Otherwise, simplifying assumptions must usually be introduced to obtain a manageable solution.

Physically, the complication is due to the fact that when a carrier wave modulated in either amplitude or frequency is applied to a network which is asymmetrical with respect to the carrier frequency, the transmitted wave is modulated in both amplitude and frequency. Hence, if the impedances of an oscillator feedback network are asymmetrical with respect to the oscillation frequency, any change in oscillation amplitude must be accompanied by a change in frequency, and vice versa.

This is the reason why the determinant for $D(p)$ cannot now be factorized to obtain independent criteria for frequency and amplitude stability. For the same reason asymmetrical-circuit oscillators, such as the resistance-capacitance type, cannot be keyed without 'chirping'; i.e., without a change of frequency during the periods of build-up and decay of the oscillation.

Most asymmetrical networks fall into one or other of two classes—those in which asymmetry is a slight and unavoidable imperfection, and those which are inherently highly asymmetrical. Also, a network may have local symmetry near to ω_0 but be quite asymmetrical when considered over the total range of p. In near-asymmetrical networks the coupling between amplitude and frequency changes is small: one type of modulation is dominant and the other simply comcomitant. The stability criteria are only slightly different from those of a symmetrical network. In highly-asymmetrical networks amplitude and frequency modulations may be of comparable magnitude and the distinction between amplitude and frequency stability becomes blurred, though in many cases it will still be possible to say that instability is mainly of one kind or the other.

Using (7.1) the determinant for $D(p)$ can be written as

$$D(p) = \tfrac{1}{2}\{F(p)A(p) + F^*(p^*)A^*(p^*)\} \quad (13.1)$$

where $F(p) = a_1 - c_1 - (a_0 - c_0)b_1/b_0$

$$A(p) = b_0(a_{-1} + c_{-1}) - b_{-1}(a_0 + c_0) \quad .. \quad .. \quad .. \quad (13.2)$$

These expressions are more general forms of the frequency- and amplitude-stability functions used in previous Sections, but they no longer have the same physical significance. $D(p)$ can be written in terms of the symmetrical and anti-symmetrical parts of these functions.

Let $F_s(p) = \tfrac{1}{2}\{F(p) + F^*(p^*)\}$

and $F_a(p) = \tfrac{1}{2}\{F(p) - F^*(p^*)\}$

be the symmetrical and anti-symmetrical parts of $F(p)$, with similar expressions for $A(p)$. Then

$$D(p) = F_s(p)A_s(p) + F_a(p)A_a(p)$$

Let p_1 be a root of $F(p) = 0$ and let $p_1 + d$ be a corresponding root of $D(p) = 0$. Then if d is small

$$d = -F_aA_a/(F_s'A_s + F_a'A_a + F_aA_a')$$

the values of the functions being taken at p_1. If the asymmetry of both $F(p)$ and $A(p)$ is small, then d is of the second order of smallness, except possibly at certain critical points.

Aperiodic stability is first considered. This requires $D'(0) > 0$. From (7.2), $F(0) = 0$, hence

$$D'(0) = \tfrac{1}{2}\{F'(0)A(0) + F'^*(0)A^*(0)\} \quad (13.3)$$

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Although $D'(0)$ is thus known when $F'(0)$ and $A(0)$ are known the individual values of these quantities do not by themselves indicate stability or instability, for they are no longer related to the number of possible encirclements which the loci of $F(p)$ and $A(p)$ can make, and the sum of these encirclements is not necessarily equal to the number of encirclements made by $D(p)$. However, in two special cases, it is possible to express $D'(0)$ as the product of two independent factors which can be interpreted as frequency- and amplitude-stability functions.

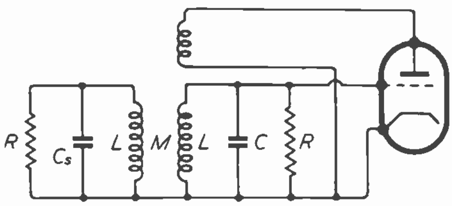


Fig. 14. *Circuit for frequency hysteresis.*

It is observed that $D'(0)$ depends entirely on the behaviour of the oscillator for very small values of p so that, although amplitude and frequency changes may be very rapid, the stability criterion is the same as if these changes were infinitely slow and may therefore be expressed in terms of the steady-state behaviour discussed in Section 4. The amplitude and frequency are defined by $R_E gh(K,Y) = -1$ and $X_E = 0$. Slow changes of amplitude and frequency can take place independently if either:—

(1) The steady-state frequency is independent of small amplitude changes, or

(2) The steady-state amplitude is independent of small frequency changes.

If μ and k are assumed constant the only amplitude-dependent quantity in the expression (4.15) for X_E is r_g. The frequency-dependent quantities in the amplitude equation are R_E and K. Condition (1) or (2) is therefore satisfied if

(a) r_g is constant, or
(b) X_E is independent of r_g, or (13.4)
(c) $R_E'/R_E + 2G_2 R_E K'/K = 0$.

(a) and (b) correspond to condition (1), and (c) to condition (2). (c) is the total derivative of the amplitude equation with respect to frequency, and is obtained by using (4.8), (5.10) and (5.14).

r_g is constant if the grid-bias voltage is fixed and sufficient to stop grid current ($r_g = \infty$), or if the grid-current characteristic has the semi-linear form of (4.19) with $V_{cg} = 0$.

Using (4.11), (4.12), and (4.15), conditions (b) and (c) can be expressed in terms of the primary quantities R_t, X_t, etc., but the general formulae are rather awkward. However, if $\mu = \infty$ the frequency equation (4.15) becomes $X_T = 0$. But, from (4.11) $Z_T = Z_t/(1 + Z_0/r_g)$. If Z_T is to be resistive irrespective of the value of r_g, both Z_t and Z_0 must be resistive, and so also must $Z_\theta = Z_0/(1 + Z_0/r_g)$. Condition (b) then simplifies to $X_\theta = 0$. When $\mu = \infty$ the parameter K is independent of frequency and condition (c) reduces to $R_E' = 0$. If it is assumed that the second derivative of R_E with respect to frequency is not zero at ω_0, the last equation means that the amplitude transmission characteristic of the feedback network has 'local' symmetry in the immediate neighbourhood of ω_0.

Although amplitude and frequency changes are coupled the criteria for aperiodic instability become uncoupled if (13.4) is satisfied. This point is illustrated and developed in the next Section.

14. Aperiodic Stability

Two modes of operation will be considered—first with r_g constant, and second with a very small oscillation amplitude and with $\mu = \infty$. If r_g is constant V_g is proportional to V_{g1}, and from (13.2), (13.3), (6.3), etc.

$$D'(0) = P_0 P_f P_a$$

where

$$P_0 = 2(1 + S_0 R_g) \left| 1 + (Z_0 - kZ_t)/r_g \right|^2$$
$$P_f = X_E'$$
$$P_a = KG_1 - G_2 + (G_1^2 - G_0 G_2 - kKG_1/R_g) R_a/\mu$$

Comparing this with (9.2) and (10.2) it is seen that $D'(0)$ is the same as the product of $F'(0)$ and $A(0)$ for a symmetrical-network oscillator but with $| 1 + (Z_0 - kZ_t)/r_g |$ in place of $1 + (R_0 - kR_t)/r_g$. Since $P_0 > 0$, $D'(0)$ is positive if P_f and P_a have the same sign. The conditions $P_f > 0$, $P_a > 0$ are identical with the criteria (9.3) and (10.3) for frequency and amplitude stability in symmetrical networks. The second possibility that both P_f and P_a are negative must be rejected, since in the limit of vanishing asymmetry the criteria must become those for a symmetrical network.

Hence it seems reasonable to say that if $P_f > 0$ and $P_a < 0$ the instability is essentially of amplitude, the frequency change being merely concomitant, and vice versa if $P_f < 0$ and $P_a > 0$. Independent criteria are obtained because condition (13.4a) is fulfilled. If the grid-bias voltage is fixed and sufficient to stop grid current the same formulae are obtained but with $r_g = \infty$ and $K = 0$.

The frequency stability criterion $X_E' > 0$ will now be used to examine a hysteresis effect which occurs in the oscillator shown in Fig. 14. When the resonant frequency of the secondary circuit is varied by adjusting the capacitor C_s, the oscillation frequency varies as shown in Fig. 15. For simplicity the grid-bias voltage is fixed and sufficient to stop grid current, but an identical result is obtained with grid-leak bias (constant r_g) provided the resistance of the primary circuit is adjusted to allow for the extra damping due to grid current. It is also assumed that $\mu = \infty$, but the same results are obtained for finite values of μ provided the coefficient of coupling between anode and grid coils is 1. These assumptions make Z_E equal to Z_t and proportional to $- Z_0$ (since a phase reversal is necessary to maintain oscillation).

$$\left.\begin{aligned} &\text{Let } C_s = (1 + s)C,\ \omega_r^2 LC = 1,\ \omega_0 = \omega_r + \omega, \\ &Q = R/\omega_r L \\ &M = bL,\ u = b^2Q^2 - 1,\ x = 2Q\omega/\omega_r + sQ \end{aligned}\right\} \quad \ldots \quad (14.1)$$

If Q is large and b and s are small

$$R/Z_0 = \{2 + u - x(x - sQ) + j(2x - sQ)\}/(1 + jx)$$

The possible steady-state frequencies are those which make Z_E, and so Z_0, real. Equating the imaginary part of Z_0 to 0 gives

$$x^3 - sQx^2 - ux - sQ = 0 \quad \ldots \quad (14.2)$$

The nature of the roots of this equation may be examined by the standard methods[33]. The discriminant is

$$\Delta = \{4s^4Q^4 + s^2Q^2(27 + 18u - u^2) - 4u^3\}/108$$

and the equation has one real root, two equal roots, or three real roots as Δ is positive, zero, or negative. From this it is not difficult to show that if $u < 0$ (i.e., if $bQ < 1$) there is only one real root, and so only one possible steady-state frequency whatever the value of C_s. When $u > 0$ there are three real roots for a certain range of values of sQ, and one real root outside this range.

The criterion for aperiodic frequency stability, $X_E' > 0$, is equivalent to $- X_0' > 0$. Now $- X_0$ can be written as a fraction, of which the numerator is the l.h.s. of equation (14.2) and the denominator is real and positive. For large values of ω, $- X_0$ has the same sign as ω and, since $- X_0$ is also a continuous single-valued function of ω, it follows that when equation (14.2) has three distinct real roots, then $- X_0' > 0$ for the smallest and largest values of ω, and $-X_0' < 0$ for the intermediate value. This is easily seen by considering the graph of $- X_0$ against ω. Hence, of the three possible steady-state frequencies two are stable and the third unstable. The dotted line in Fig. 15 shows the unstable values.

At the points 'a' and 'c' where irreversible jumps take place, the frequency equation has two equal roots. Setting the discriminant equal to 0 gives the critical values of sQ as

$$\left.\begin{aligned} &8(sQ)^2 = y + (y^2 + 64u^3)^{\frac{1}{2}} \\ &\text{where } y = u^2 - 18u - 27 \end{aligned}\right\} \quad \ldots \quad (14.3)$$

The corresponding critical frequencies are

$$\omega/\omega_r = - s\{(1 + 3u/s^2Q^2)^{\frac{1}{2}} + 2\}/6 \quad \ldots \quad (14.4)$$

At the points 'b' and 'd' the stable frequencies (corresponding to the third real root of the equation) are

$$\omega/\omega_r = s\{(1 + 3u/s^2Q^2)^{\frac{1}{2}} - 1\}/3 \quad \ldots \quad (14.5)$$

When $s = 0$ the two circuits are tuned to the same frequency ω_r which is also the oscillation frequency if $u < 0$. If $u > 0$ the two stable frequencies are given by $x^2 = u$, or

$$\omega/\omega_r = \pm \tfrac{1}{2}u^{\frac{1}{2}}/Q \quad \ldots \quad \ldots \quad \ldots \quad (14.6)$$

The maximum and minimum frequencies at 'f' and 'g' are found by differentiating (14.2) with respect to sQ and setting $d\omega/dsQ = 0$. This gives a second equation which in conjunction with (14.2) can be solved to give sQ and x. The results are

$$\begin{aligned} sQ &= \pm \tfrac{1}{2}(1 - u) \\ \omega/\omega_r &= \pm(1 + u)/4Q = \pm b^2Q/4 \end{aligned} \quad \ldots \quad (14.7)$$

This type of hysteresis effect has been studied from a somewhat different point of view by van der Pol[3].

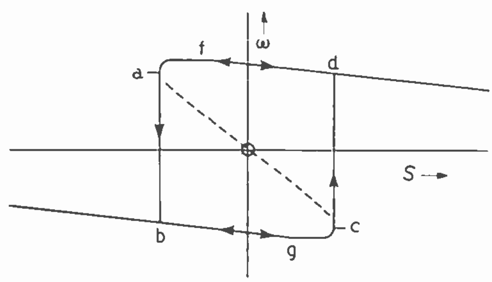


Fig. 15. *Frequency hysteresis.*

Instability at the threshold of oscillation (i.e., for very small values of V_{g1}) is next considered. As for the symmetrical network treated in Section 10, the analysis will be restricted to a three-halves-law amplifier with an exponential grid-current characteristic. It is also assumed that $\mu = \infty$. This makes $q = 1$ according to (4.16) and (5.13), and also $Z_T = R_T$, since equation (4.15) for the steady-state frequency is now $X_T = 0$. Using (13.2), (13.3), etc., the result is

$$D'(0) = P_0(P_f P_a + P_c)$$

$$\left.\begin{aligned} \text{where } P_0 &= 2(V_{g1}/V_g)^2\,(1 + S_0R_g)\,|\,1 + (Z_0 - kZ_t)/r_g\,|^2 \\ P_f &= - X_E'/R_E \\ P_a &= Y^2/32\,(1 - Y)^2 + nY/4\,(1 - Y) + \tfrac{1}{2}m\,(k - R_\theta/R_T)\,R_E/r_g \\ P_c &= - \tfrac{1}{2}mX_\theta R_E'/R_T r_g \end{aligned}\right\} \quad \ldots \quad \ldots \quad (14.8)$$

The values of the parameters are those at the inception of oscillation and m and n are given by (5.8). From (9.2) and (10.7) it is seen that $P_0 P_f P_a$ is the same as $F'(0)A(0)$ for a symmetrical network but with $1 + (Z_0 - kZ_t)/r_g$ in place of $1 + (R_0 - kR_t)/r_g$. Since $P_0 > 0$ the first requirement for stability is that

$$P_f P_a + P_c > 0 \qquad \ldots \ldots \ldots (14.9)$$

In a symmetrical network the criteria for aperiodic stability are $P_f > 0$, $P_a > 0$. The term P_c therefore represents a coupling which is 0 if m or X_θ or R_E' is 0. If $m = 0$ then from (5.8) $dr_g/dV_{g1} = 0$ and condition (13.4a) is satisfied. $X_\theta = 0$ and $R_E' = 0$ are the forms taken by (13.4b) and (13.4c) when $\mu = \infty$.

In an asymmetrical network aperiodic instability may exist even when P_f and P_a are positive. Conversely, the oscillation may be stable when either is negative, but the possibility of stability with both negative is excluded for the same reason as before. A sharp distinction between frequency and amplitude instability is now impossible.

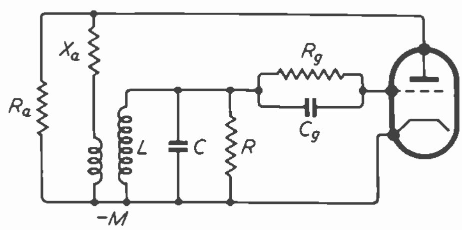


Fig. 16. *Asymmetrical network oscillator.*

Oscillators of this type are rare for, in most feedback networks, the zeros of X_E lie near to maximum or minimum values of R_E, and R_E' is therefore small. The effects are most easily demonstrated by choosing a rather artificial type of circuit, such as that shown in Fig. 16. X_a is a reactance, positive or negative, which is large compared with the impedance of the tuned circuit coupled through the mutual inductance $-M$. It is assumed also that the Q-factor of the circuit is large and that the reactance of C_g is small.

The oscillation frequency differs from the resonant frequency of L and C by an amount depending on the ratio X_a/R_a.

Let $R_1 = Rr_g/(R + r_g)$, $M = bL$ $\qquad \ldots (14.10)$

Straightforward algebra gives the following results

$$R_T = -bR_1/(1 + X_a^2/R_a^2),\ R_T/R_\theta = -b,\ X_\theta/R_\theta = X/R_a \\ X_E'/R_E = 2R_E/\omega_0^2 M,\ R_E' = -X_E'X_a/R_a,\ R_T = R_E/(1 + kR_E/r_g) \qquad \ldots \ldots (14.11)$$

Since $P_f > 0$ the stability criterion (14.9) can be written $P_a > -P_c/P_f$, and using (14.11) and (14.8) this becomes

$$P_a > -\tfrac{1}{2}mR_E(X_a/R_a)^2/br_g$$

$$\text{or } P_a > \tfrac{1}{2}m\{R_E + bR_1(1 + kR_E/r_g)\}/br_g \qquad (14.12)$$

The second form is more convenient for computation and the first shows how the stability is affected by X_a. As for the symmetrical network, in bility is possible only if $m > 0$. The effect of the coupling term P_c is to reduce the stability margin.

15. Periodic Stability

In view of the complexity of any general treatment, only the simplest example will be considered. The oscillator of Fig. 16 is again chosen. As before, it is assumed that μ, Q and X_a are large and, in addition, that r_g is large compared with Z_0 and Z_t and is independent of V_{g1}. Then $R_t = R_T = R_E$. From (13.2)

$$F(p) = bpT_t/(1 + bpT_t)$$

$$A(p) = \frac{(1 + S_0R_g + pT_g)(2G_2R_t + b^*pT_t) - 2G_1R_tS_1R_g}{(1 + pT_g)(1 + b^*pT_t)}$$

where $b = (1 + jX_a/R_a)$, $T_g = R_gC_g$, $T_t = 2Q/\omega_0$

Using (13.1), $D(p)$ can be written as a rational function of p, the numerator of which is

$$pT_t(a_1p^2 + a_2p + a_3)$$

where

$$a_1 = T_gT_t$$
$$a_2 = (1 + S_0R_g)T_t + 2G_2R_tT_g + T_tX_a^2/R_a^2$$
$$a_3 = 2(1 + S_0R_g)\,G_2R_t - 2G_1R_tS_1R_g$$

The Routh-Hurwitz stability rules are that a_1, a_2 and a_3 should have the same sign. The condition $a_3 > 0$ is simply the criterion for aperiodic stability which is assumed to be satisfied. For periodic stability $a_2 > 0$, which can be written as

$$(1 + S_0R_g + X_a^2/R_a^2)\,T_t/T_g > -2G_2R_E \qquad (15.1)$$

This may be compared with (11.8) to which it reduces when $X_a = 0$. When X_a/R_a is small compared with $1 + S_0R_g$ the asymmetry results in only a second-order error when the symmetrical-network criterion is used. The is in agreement with the general result obtained in Section 13. It is clear from the form of the criterion that any instability is primarily of amplitude.

16. Two-Terminal Oscillators

Much attention has been given in the literature to the two-terminal oscillator, for it is the simplest to treat mathematically, and the equation to a four-terminal oscillator (with no grid current and no anode decoupling impedance) can be reduced to the same form. The oscillator consists of a passive linear network of impedance Z connected in parallel with a nonlinear resistance element which has a negative slope over part of its range.

It is assumed that Z has a significant value only near to the oscillation frequency, so that any direct, low-frequency, or harmonic voltage across Z is negligible. The steady-state voltage is then $v = V \cos \omega_0 t$. If the current–voltage relation for the nonlinear resistance is $i = f(v)$, the current of fundamental frequency is $I \cos \omega_0 t$ where

$$I = (2/\pi)\int_0^\pi f(V \cos x) \cos x dx = Vh(V) \qquad (16.1)$$

This must be equal and opposite to the current of fundamental frequency in the linear network. Hence if $Z = R + jX$, the steady-state equations are

$$X = 0 \text{ and } 1 = -Rh(V) \quad .. \quad .. \quad (16.2)$$

A small disturbance v_d is added to v and the sum of the currents due to v_d in $Z(p)$ and in the nonlinear resistance is equated to 0. The analysis follows that of Section 5 and gives

$$\begin{bmatrix} a_1 & c_1 \\ a_{-1} & c_{-1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_{-1} \end{bmatrix} = 0$$

where $a_1 = 1 + G_0 Z^+$, $c_1 = G_2 Z^+$, $a_{-1} = G_2 Z^-$, $c_{-1} = 1 + G_0 Z^-$

and $G_n = (1/\pi)\int_0^\pi f'(V \cos x) \cos nx dx$

$$G_0 - G_2 = -1/R \quad .. \quad .. \quad .. \quad (16.3)$$

The stability function is $D(p) = \begin{vmatrix} a_1 & c_1 \\ a_{-1} & c_{-1} \end{vmatrix}$

Proceeding on the lines of Section 13,

$$D(p) = \tfrac{1}{2}\{F(p)A(p) + F^*(p^*)A^*(p^*)\}$$

where $F(p) = 1 - Z^+/R$, $A(p) = 1 + (G_0 + G_2)Z^-$.

The first criterion for aperiodic stability is $D'(0) = -2G_2X' > 0$. But stability is obtained only if the frequency and amplitude are separately stable, and this requires

$$-X' > 0 \text{ and } G_2 > 0 \quad .. \quad .. \quad (16.4)$$

This may be expressed another way. If an effective conductance for the nonlinear resistance at fundamental frequency is defined as $Y_e = I/V$, then from (16.1), (16.3), and (16.2), $dY_e/dV = 2G_2/V$. Let the admittance of the linear network be $Y = G + jB$. Then $-X' = R^2B'$. Hence the stability criteria can be expressed as $dB/d\omega > 0$ and $dY_e/dV > 0$. It is not sufficient for the product of these quantities to be positive[13].

In the conventional treatments of the problem $f(v)$ is usually represented as a power series $f(v) = \sum_0^\infty g_n v^n$. From (16.1) and (16.2) the steady-state amplitudes are given by

$$1/R + \sum_0^\infty g_{2n+1} (\tfrac{1}{2}V)^{2n} (2n+1)!/n!\,(n-1)! = 0 \quad .. \quad .. \quad (16.5)$$

and from (16.3) and (16.4) the criterion from amplitude stability is

$$\sum_1^\infty g_{2n+1} (\tfrac{1}{2}V)^{2n} (2n+1)!/(n+1)!(n-1)! > 0 \quad .. \quad .. \quad (16.6)$$

Adding (16.5) and (16.6) gives

$$1/R + \sum_0^\infty g_{2n+1} (\tfrac{1}{2}V)^{2n} (2n+1)!/(n!)^2 > 0$$

which agrees with the results obtained by Appleton and van der Pol[1].

It is implicit in the analysis that $f(v)$ is a single-valued function of v; i.e., the nonlinear resistance is the 'voltage-controlled' type. For the 'current-controlled' type of resistance the voltage is a single-valued function of current. The analysis can be carried out in exactly the same way as before by replacing all the circuit elements by their duals.

Periodic instability can be treated in the same way as in previous Sections. Since the low-frequency impedance of the network is assumed to be zero only frequency instability is of interest. The criteria (16.4) are therefore not sufficient to guarantee stability. If the low-frequency impedance is made finite, periodic amplitude instability can also exist.

(To be concluded)

OBITUARY

It is with deep regret that we have to announce the death of Harold Lister Kirke, C.B.E., after a long illness. Born in 1895, he served in World War I as an officer in the Royal Fusiliers, the signals branch of the Royal Engineers and in the Royal Corps of Signals.

In 1924 he joined the B.B.C. becoming, in 1925, head of what later became the research department. From 1950 he was assistant chief engineer until he retired through illness.

COMMUNICATION BY SCATTER TECHNIQUES

A symposium on this subject is being organized by the Professional Group on Antennas and Propagation of the Institute of Radio Engineers, and will be held in Washington, D.C., U.S.A., on 14th and 15th November. There will be four sessions.

The advance registration fee is $2.50 and should be sent to Scatter Symposium, George Washington University, School of Engineering, Washington 6, D.C., U.S.A., prior to 31st October.

NEW APPOINTMENTS

D. G. Tucker, D.Sc., Ph.D., M.I.E.E., M.Brit.I.R.E., has been appointed to the chair of electrical engineering at the University of Birmingham. He joined the Post Office research station, Dollis Hill, in 1934 and, since 1950, he has been in the Royal Naval Scientific Service at H.M. Underwater Detection Establishment. Eleven of his many technical papers have appeared in *Wireless Engineer*.

J. W. R. Griffiths, B.Sc., A.M.I.E.E., is leaving H.M. Underwater Detection Establishment to join the Electrical Engineering Department of the University of Birmingham as a lecturer. He has contributed articles to *Wireless Engineer* as a co-author with Professor Tucker.

22nd NATIONAL RADIO EXHIBITION

THE exhibition was again held at Earls Court, from 26th August to 3rd September and, as usual, it was mainly concerned with broadcast receiving equipment. As has been the case for a good many years now, television formed the major attraction. Sound broadcasting has been given a fresh impetus, however, by the opening during the year of the first of the B.B.C.'s Band II f.m. transmitters at Wrotham, Kent.

Last year, there were quite a number of Band II receivers, but now they are general and most makers have several models. There is a great deal of uniformity in the basic design and it is the usual practice to combine Band II reception with the medium- and long-wave set. For these a.m. bands, and often for one or more short-wave bands also, the receiver has a triode-heptode frequency changer, one i.f. stage at a frequency around 465 kc/s, and a diode detector. There may be one or more further diodes for a.g.c., and there are one or two i.f. stages. This forms the basic broadcast receiver which has remained unchanged in its essential form for many years, however much one set may vary from another in details.

To this basic receiver are now added extra parts to permit f.m. reception in the 82–100-Mc/s band. The connections are altered by the waveband switch so that the oscillator section of the triode-heptode is rendered inoperative and the heptode section functions as an extra i.f. stage. The intermediate frequency is changed, usually to 10·7 Mc/s, and an extra pair of diodes is brought into circuit to act, with a frequency discriminator, as a ratio detector. In this way, the basic a.m. receiver is converted to form the i.f., detector and audio part of an f.m. receiver. It is preceded by a two-valve tuner.

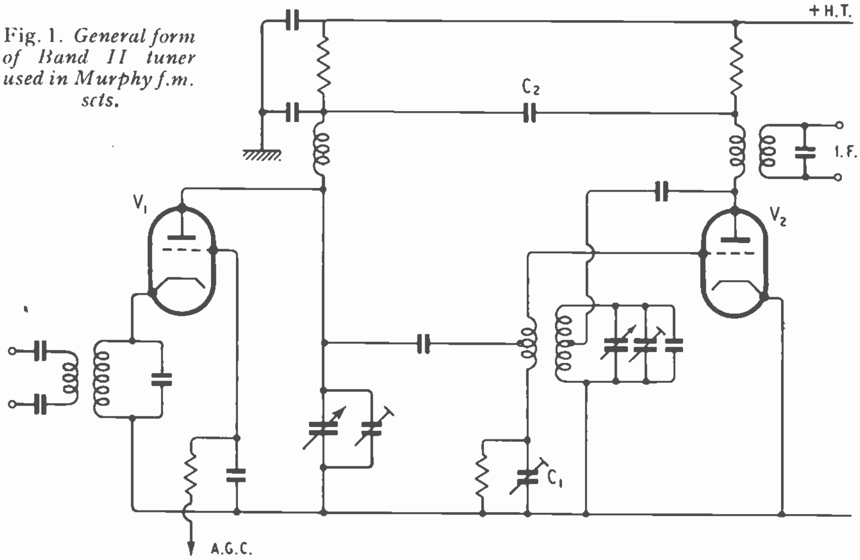


Fig. 1. *General form of Band II tuner used in Murphy f.m. sets.*

Nearly always, this comprises a duo-triode, of which one half acts as an r.f. stage and the other as a detector-oscillator type of frequency-changer. One, and sometimes both, of the r.f. circuits is fixed-tuned and only the oscillator and one r.f. circuit are adjustable for station selection.

There are some exceptions to this general form: both Bush and G.E.C., for instance, use r.f. pentodes for both the amplifier and the frequency changer. However, the double-triode circuit is so widespread that it is almost standard.

In order to minimize oscillator radiation, the r.f. stage is often neutralized and is nearly always connected to the detector-oscillator through a bridge circuit. As an example, the arrangement used by Murphy is shown in Fig. 1. This set is actually rather unusual in that the two triodes are separate instead of being in a common envelope, but that does not affect the circuit. The oscillator is V_2 and is of the tuned-anode type, the primary of the i.f. transformer being used to choke-feed the oscillator tuned circuit.

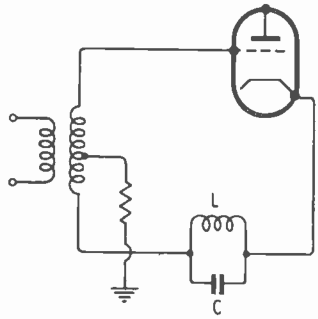


Fig. 2. *Common form of neutralized r.f. stage in Band II tuners.*

The signal is fed in from V_1 to a tapping on the reaction coil. Viewed from this tapping, there are two paths to earth, one via the lower part of the coil and C_1 and the other via the upper part and the grid–cathode capacitance of V_2. These paths form a bridge which can be balanced by the adjustment of C_1, and then the tapping point is at earth potential for the oscillator frequency.

Not only is the interaction between the signal and oscillator circuits reduced by this but the magnitude of oscillator voltage on the anode of V_1 is kept small. This last is necessary to avoid serious radiation from the aerial, for the triode V_1 is not by itself an adequate barrier.

This valve is used as an earthed-grid stage, the cathode circuit being fixed-tuned. The anode circuit is variably-tuned, however, the tuning capacitance being ganged to that of the oscillator.

An interesting feature of this circuit is the use of positive feedback at i.f. Internal negative feedback in V_2 gives this valve a low output resistance which damps the i.f. transformer; this is counteracted by positive feedback via C_2.

The use of a true earthed-grid connection for the r.f. stage is by no means invariable. Neutralizing, as often applied, turns it into a kind of hybrid between the earthed-grid and earthed-cathode circuits and the circuit diagram fails to show upon which the stage is based. Quite a widely-used arrangement has the form sketched in Fig. 2, in which LC is an i.f. trap, and it could clearly be regarded as either earthed-grid or earthed-cathode.

The pentode has an advantage over the triode in not requiring to be neutralized and its use does also somewhat simplify the frequency-changer. With a pentode, it is normal to use a Hartley form of oscillator between the control and screen grids. Bush apply the signal to a null-point tapping on the coil, so that the basic difference between this and the triode circuit lies in the separate electrodes for the oscillator anode and i.f. output. G.E.C., however, apply the signal to the cathode of the frequency-changer.

H.M.V. 1252 f.m. unit for use with existing a.m. sets.

Generally speaking, the a.m./f.m. switching is less complex than one might suppose. Usually, the 465-kc/s and the 10·7-Mc/s i.f. transformers have their windings in series so that the amplifier has two pass-bands. Usually, but not always, switches short-circuit one or more windings of the unwanted i.f. transformers. Some makers short out several windings, but others do none at all. The detectors are duplicated so that it is only necessary to switch over the audio circuits from one detector to the other.

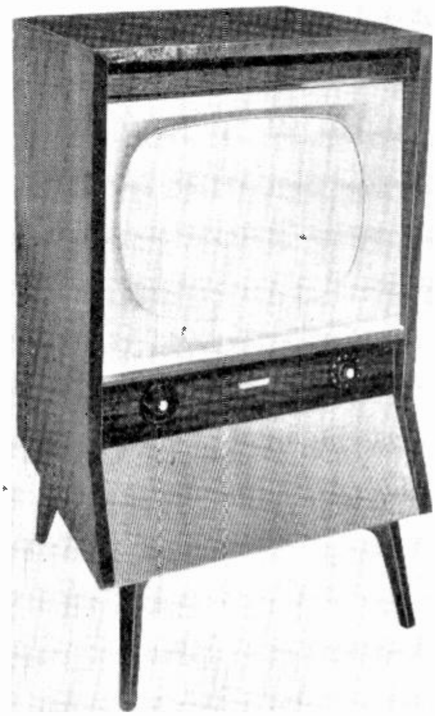
As already stated, most sets covering Band II do so as well as the a.m. bands. There are, however, a few purely f.m. sets and there are some adaptors, which are virtually f.m. sets with only a small amount of a.f. amplification; they are intended to feed into the pickup terminals of an existing set. In quite a lot of sets, f.m. reception of Band II sound is combined with television. All English Electric television sets, for instance, can be obtained with or without f.m. sonnd. The television set does rather lend itself to this, for its front end is already of a v.h.f. type and its sound i.f. is of a suitable frequency and bandwidth to handle an f.m. signal. The provision of Band II reception thus entails merely a few extra v.h.f. coils, a frequency discriminator and ratio detector and a little more switching.

This is actually surprisingly small because, in many cases, nothing extra is needed at signal frequency. It is common to fit television sets with turret tuners having 12 or 13 positions. It is also quite common not to provide the full sets of coils. Only two or three sets are fitted to enable one Band I and one or two Band III channels to be selected, since this is the most that any viewer is likely to want in the foreseeable future. The coils fitted are, of course, the ones appropriate to the viewer's district, and extra ones can always be fitted if ever it should be necessary. This being so, there are plenty of blank spaces in the turret in which f.m. coils can be fitted and still leave enough room for a good deal of future television development.

Band III being on nearly twice the frequency of Band II, rather more complex circuitry becomes necessary. There is now almost complete standardization of the basic circuitry of television 'front ends.' A double-triode cascode r.f. stage is used with a triode-pentode frequency-changer. Usually, there are three signal circuits, one before the r.f. stage and a coupled pair between this and the mixer.

There are, of course, many minor differences between different tuners in the way of obtaining bias voltages and arranging decoupling. The only major one, however, lies in the mechanism of station selection. The true turret is probably in the majority, and has the obvious merit that the coils are brought right up to the job. The so-called incremental-inductance system has its adherents, however, including H.M.V., Invicta and Pye; in this, all coils are connected in series and a wafer switch selects the right combination.

Other methods are used, G.E.C., for instance, have a switch selecting among three sets of coils, one Band I and two Band II. They can be pre-tuned to any one Band I and any two Band II stations, which gives the user all he is likely to want for some time to come. Last year, Bush used a similar arrangement but with a two-way switch and continuous tuning on both bands. This



English Electric C45 FM with 21-in. tube and Band II coverage.

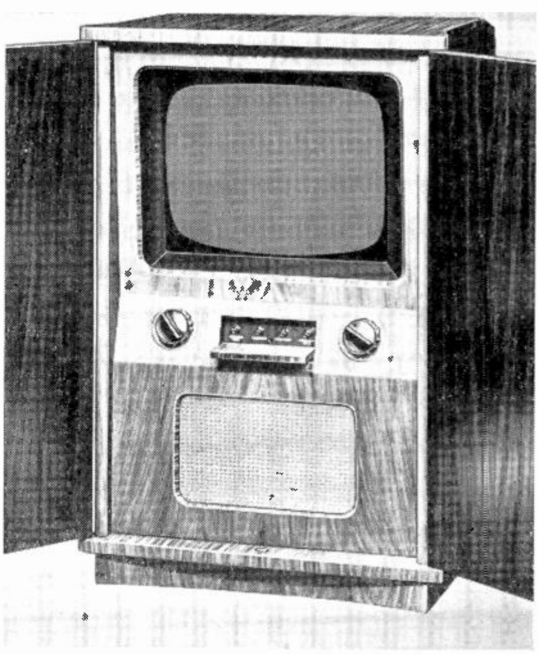
year, they use much the same electrical arrangement but it has been modified mechanically. A control knob with a clicker mechanism is used to give 12 pre-set positions and operates the tuning mechanism through cams, the band-changing switch being operated automatically at the right point. There are two coil assemblies, one for each band, and in each all coils are wound on a common former, end to end and spaced apart, through which the ganged dust-iron cores slide. So far as the user is concerned, this system is the same as a turret or an incremental-inductance control, and it might well be impossible to tell which is fitted without delving into the interior.

In all cases there is a fine tuning control. This commonly operates on the oscillator only. The control is usually concentric with the station-selector knob and operates a very small variable capacitor.

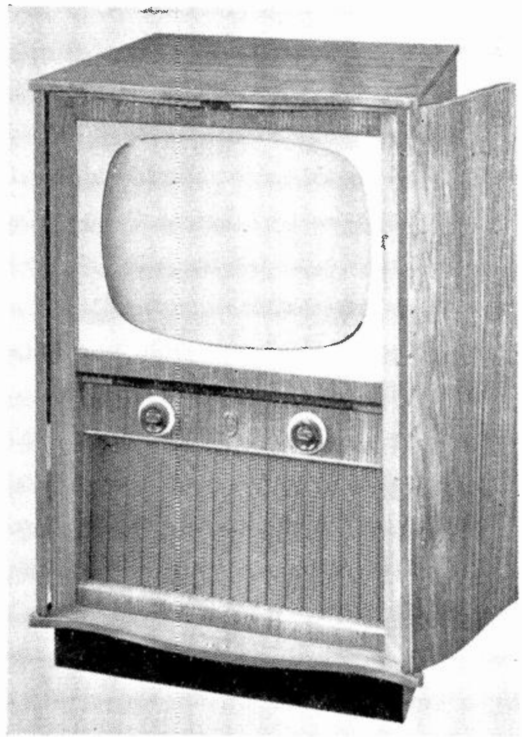
More and more use is being made of the 'standard' intermediate frequencies of 34·65 Mc/s for vision and 38·15 Mc/s for sound. One result of this is an increase in the number of sound-channel rejector circuits. A few years ago only one was common. Now there are generally from three to five. There is a tendency to reduce the number of i.f. stages and many sets now have only two in the vision channel, whereas three were usual only a few years ago.

Detector, ignition interference suppressors, video circuits and sync separators are largely unchanged. Crystal diodes are more often replacing thermionic diodes in these circuits, but one can hardly point to any definite swing over to them. There are, too, some changes in video circuits. More sets now include two video stages, but still not very many.

When two video stages are used, one is generally a pentode stage of normal type and the other a triode cathode-follower, a triode-pentode being used for the two. One advantage is a lower output impedance, the other is higher gain. This seems a little odd at first, for the cathode follower can only provide a gain of less than unity. However, it removes the capacitance of the c.r. tube and sync separator from the video stage and this can then be designed to give so much more gain that it more than compensates for the loss in the cathode-follower stage.



Pye Luxury 17 console; pre-set controls are accessible behind a cover plate (shown open).



Bush TUG 59 with 21-in. tube.

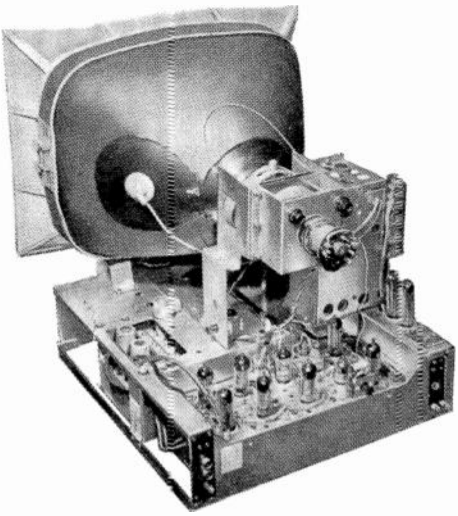
There are, however, a few cases where two truly amplifying stages are used which represents a new development in British practice, although common enough in the U.S.A.

Scanning circuits are in the main unchanged. For the line scan a blocking oscillator or multivibrator is used to drive a pentode with a ferrite-cored auto-transformer, provided with e.h.t. rectifier and boost diode. The use of a ferrite-cored coil with its saturation controlled by a permanent magnet is being increasingly used for a linearity control. Development has been mainly in improved design so that it is now possible to obtain full scan for a wide-angle tube and an e.h.t. supply of 16 kV or more with an h.t. line of only 170 V and an input power of the order of 20 W. Very few years ago, such a performance would have seemed incredible.

Picture sizes are on the increase. The 9-in. tube is definitely out for normal viewing and there are now very few 12-in. Last year, 14-in tubes were in the majority but this year they have given way to the 17-in. Nearly every maker now has 21-in. models, whereas only a few firms showed such large tubes a year ago. One result of this is a decline in the number of projection sets since, for domestic purposes, they now offer little or no advantage over direct-viewing as regards picture size. Projection sets have not disappeared, however, and Ferranti list a number of models, but also make directly-viewed types.

One development is in the portable television set shown by Ekco and weighing only 30 lb. This has a 9-in. tube, a telescopic aerial and covers Band II as well as Bands I and III. It operates from a.c. or from a 12-V car battery.

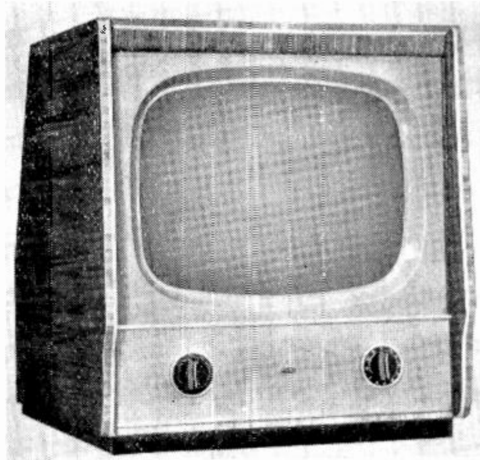
One result of Band III has been an increase in the use of a.g.c. in television receivers. Its main function is, of course, to reduce fading and aircraft flutter and so it finds its main application in fringe-area models. However, it is also useful in reducing the effects of any difference of signal strength between the two bands and so making contrast adjustments less important when changing from one to another. This alone does not justify the use of a.g.c., for it can be dealt with even more effectively by switched gain controls, as in some Murphy models.



Ferranti television receiver chassis.

There are two kinds of a.g.c.—mean level or black level—depending upon whether the tendency is to keep the one or the other constant. A mean-level control is much the simpler and is quite widely used. Generally, the mean voltage on the grid of the sync separator is used after filtering; one or more diodes may be used to introduce delay. Kolster-Brandes adopt quite an elaborate delay system which holds off the a.g.c. voltage from the r.f. stage on weak signals and then controls only the i.f. stages. On medium signals, r.f. and i.f. stages are controlled, while for strong signals the i.f. stages have their bias clamped at a fixed maximum and control takes place only on the r.f. stage. In addition, there is a tie-up with the sound a.g.c. system so that this controls the vision channel if the vision signal fails and so prevents the vision gain from rising too much.

The theoretical disadvantage of mean-level a.g.c. is that it reduces the d.c. component of the signal applied to the c.r. tube. It is a theoretical disadvantage, because many designers are doubtful about the advantage of retaining the d.c. component in practice; so much is

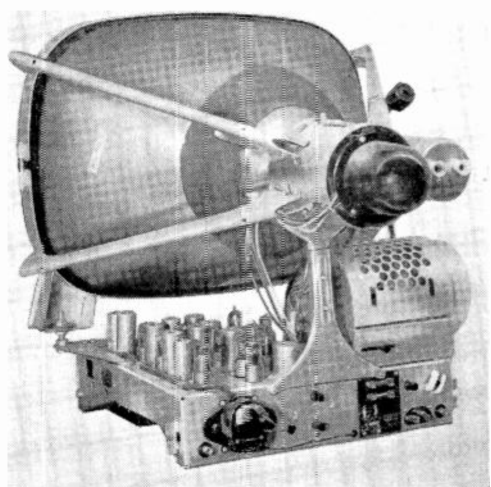


Pilot TV 97 with 17-in. tube.

this the case that, in most receivers, it is in one way or another deliberately reduced.

The black-level control does not affect the d.c. component but necessitates a gated a.g.c. system and, hence, a more complex one. The only suitable parts of the signal occur during the frame sync pulses or the back porch immediately after the line sync pulse. Gating can be at frame or line frequency, therefore, and Ultra retain the frame-gating system used last year. Most designers adopt line gating, however, and Ekco and Pye are among those employing it.

G.E.C. do so also in some models and the arrangement is shown in Fig. 3. V_1 is the ordinary sync separator fed at its grid with the video signal in which the sync pulses are positive-going. At the anode of V_1, only negative-going sync pulses appear. Time constant C_1R_1 is more effective at the end of a pulse, when V_1 becomes non-conductive, than at the start, with the result that the output pulses of V_1 are rather longer than the input



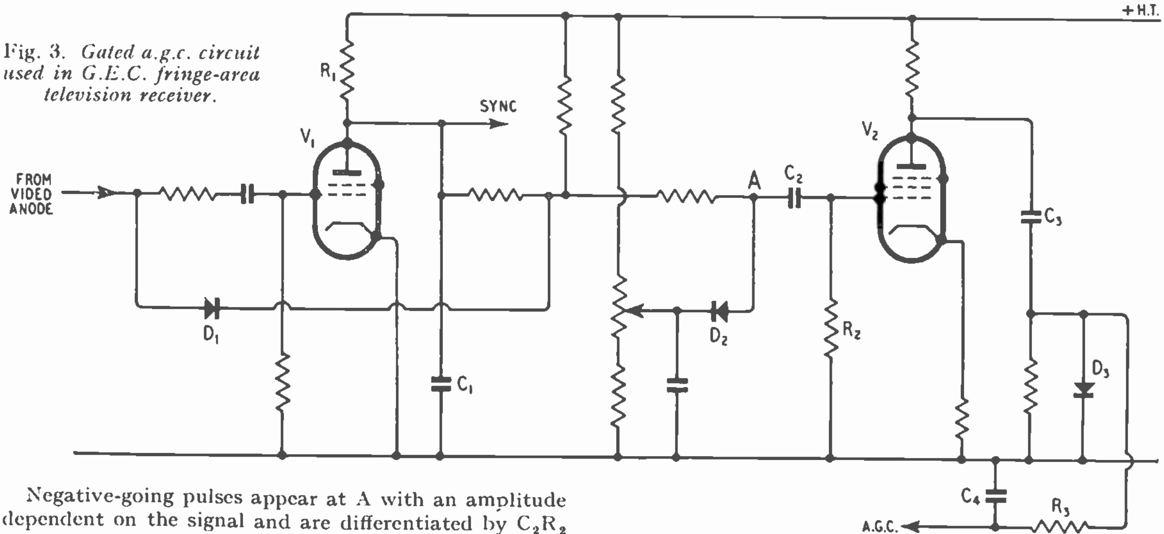
Chassis of Murphy V 291 with 21-in. tube.

pulses and so overlap into the back porch. The pulses at this point are applied to the cathode of D_1, to the anode of which the video signal is applied. D_1 is cut off except when a pulse appears on its cathode. D_1 conducts during the sync pulse and afterwards for a short time in the back-porch region. Since the sync pulses correspond to substantially zero signal, their effect is always the same, but the overlap during the back porch produces an output dependent on signal strength in the form of a narrow pulse. D_2 is cut off by the pulse from V_1 but at other times is conductive to clamp point A to a fixed potential.

The h.t. supply for television sets is almost universally taken straight from the mains through a half-wave valve or metal rectifier with a simple smoothing circuit embodying very high values of capacitance. Valve heaters are usually connected in series. Many sets are thus as suitable for d.c. mains as for a.c., but not all. Quite a few sets incorporate a transformer for supplying certain valve heaters and the primary is then used as an auto-transformer for the h.t. circuit.

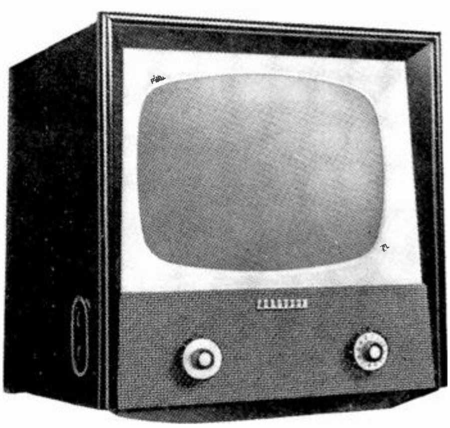
One result of Band III has been to complicate television aerials. The simple X or H Band I aerial no longer suffices. For areas of fairly-high field-strength Band III

Fig. 3. *Gated a.g.c. circuit used in G.E.C. fringe-area television receiver.*



Negative-going pulses appear at A with an amplitude dependent on the signal and are differentiated by C_2R_2 and amplified and clipped by V_2. The positive pulses from V_2 are passed through C_3 to the d.c. restorer D_3 and integrated by C_4R_3 for the a.g.c. voltage.

This particular circuit is an unusual one in that the gating pulse is derived directly from the sync pulse. Ferguson, incidentally, do this too, but in a different way. Generally, however, the gating pulse comes from the line timebase. If flywheel sync is used, the relative timing of sync pulse and gating pulse become variable. This does not preclude the use of gated a.g.c., but makes the permissible tolerances more critical.



Ferguson 206 T table model.

elements can be obtained for attachment to existing Band I aerials. For new installations, combined Band I and III aerials are available and are of a similar, but often rather more elaborate, nature. The tendency is to use a fairly simple form of Band I aerial with adjacent Band III elements, which have little effect on Band I, but which form a Yagi aerial on Band III. In this Yagi, the Band I elements sometimes function in a higher mode than the half-wave. This is when the Band III frequency is a multiple of the Band I. Some or all the elements are then bent forwards so that the array has a herring-bone appearance. This is done to alter the direction of the lobes, the aim being to coalesce two divergent lobes of a straight dipole more than one wavelength long into a single forward-pointing one. When the Band III frequency is not a multiple of the Band I, the array becomes virtually two aerial systems with the driven elements mounted very close together and fed in parallel. For fringe areas, the arrays acquire more elements and the Band I and Band III elements become two distinct aerials of Yagi type, coupled by a matching device.

Band II might have been expected still further to complicate the aerial problem, for here horizontal polarization is used, whereas television is mainly with vertical. However, sets for Band II usually have a built-in loaded dipole with provision for the connection of an external aerial. It seems to be envisaged that most people will employ the internal aerial for f.m. reception, as they already do for medium and long waves. For these bands there is now a great tendency to include ferrite-rod aerials in sets.

'High-fidelity' has, in the past, been a very specialized field rather remote from domestic radio. There are now signs of an expanding interest in it and a certain amount of equipment in this category was shown. The term is applied to apparatus designed for an unusually wide frequency response and extremely low non-linearity distortion.

One aspect which attracted considerable attention is '3 D' reproduction. This is not to be confused with stereophonic, for it is applied to wide-angle (up to 360°) distribution of a single source. Several loudspeakers are used with a cabinet having numerous apertures arranged either to cover the complete area around it, or, by using reflections from corner walls, to fill the space on one side of it completely.

The H.M.V. 'Stereosonic' system utilizes a two-channel tape recording, the two records being made from two microphones mounted at right angles. Reproduction is by spaced loudspeakers.

Specto showed a twin-track reproducer for this system which is designed to work with a pair of Tannoy Dual Concentric loudspeakers. Interest in tape recording and reproduction continues to grow. One practical difficulty always lies in avoiding hum because of the extremely small signal level from the tape at low frequencies. In the Reflectograph reproducer, this is overcome by using a transistor pre-amplifier. This is noteworthy as marking one of the first occasions on which the transistor has entered production equipment intended for general use, apart, of course, from the hearing aid.

Another transistor application, this time for the future, is to power output stages. Mullard demonstrated a development model of an a.f. power transistor, the OC 15. A pair in class B will deliver 10-watts output with a peak current of some 2 A each at 12 V. This transistor is a *p-n-p* type and is sealed into a metal can which is bolted to the chassis to secure good heat transfer. Its equivalent circuit characteristics are:—$r_b = 5\ \Omega$, $r_e = 0{\cdot}25\ \Omega$, and $r_c = 20\ \text{k}\Omega$. In earthed-emitter operation its current amplification factor is about 25.

G.E.C. have developed a point-contact transistor, the EW 51, for h.f. operation; α becomes 0·7 of its l.f. value at 4 Mc/s. In a pulse amplifier, an output rise-time of under 0·15 μsec is obtainable with an input pulse of 0·05-μsec rise-time.

NEW BOOKS

Linear Feedback Analysis

By J. G. THOMASON, B.Sc. Pp. 335 + x. Pergamon Press Ltd., 4-5 Fitzroy Square, London, S.W.1. Price 55s.

The author states in his preface that this book "is intended to provide an analytical background for the young science graduate working in electronics or servo-mechanism research". There are eleven chapters and an appendix.

The book opens with a chapter on steady-state circuit analysis and this is followed by one on the Laplace transformation in circuit analysis. Chapter 3 is illustrative of the technique of the transformation. Feedback does not appear until Chapter 4, in which its fundamentals are discussed. Amplifying stage design is the title of Chapter 5 and it covers, rather briefly, the fundamentals of the subject.

This initial third of the book is essentially of an introductory nature. The heart of the book comprises three chapters: Chapter 6 on the stability of feedback systems, Chapter 7 on gain-phase analysis and Chapter 8 on stabilization techniques. The concluding chapters, illustrative of feedback amplifiers, feedback integrators and differentiators and stabilized power supplies, show practical examples of the use of feedback.

The author was formerly with the Atomic Energy Research Establishment and his treatment of his subject has a coloration which is foreign to the engineer. It is a thing which has been noticed with other authors connected with this establishment and it probably results from adopting a physicist's approach as distinct from an engineer's. As a result, the approach and emphasis sometimes seem strange to an engineer, especially to one brought up on communications. In examples, the stress tends to be on low frequencies and d.c. amplification and, although one illustration is given of the design of an a.f. amplifier, there is nothing about really wideband amplifiers. Some statements strike an engineer as incorrect, as they are, in his sense of the words. When the author refers to the signal-handling capacity of a valve, the context shows that he means the output of which it is capable. For some 30 years or more the engineer has meant the input which it can handle.

This is a rather trivial example of an unfortunate tendency today. The physicist tends to borrow well-known terms from the engineer and to use them with a different meaning.

In another place, there is a change from the current terminology which will amuse the engineer because it is a reversion to his old practice. The author does not like the terms differentiating and integrating circuits and explains in Chapter 3 that he considers leading and lagging (referring to phase) circuits preferable.

Power engineers still use the terms, but communications engineers rather abandoned them, though by no means entirely, in the late thirties when television and radar shifted the emphasis in circuits from the steady-state to the transient response. Since then, they have generally considered the terms differentiating and integrating as preferable, because they more truly represent the action of the circuit. The lead and lag on sine waves in the steady-state comes about because of the differentiating and integrating action, so that these terms are descriptive under all conditions.

These matters of terminology will hold up few readers, many will be much more disturbed by the mathematics. It is necessary to feel at home in the complex *p*-plane with its poles and zeros and contours. It is true that the author explains these things, but the arguments depending upon them are hard to follow until they have become second nature. This applies, of course, not only to this book but to much of modern technical literature and, in his use of this approach, the author is following the modern trend. For those with the necessary mathematical equipment, the book is undoubtedly a good one.

W. T. C.

Television Receiver Servicing: Vol. 2. Receiver and Power Supply Circuits

By E. A. W. Spreadbury, M.Brit.I.R.E. Pp. 308. Published for *Wireless & Electrical Trader* by Trader Publishing Co. Ltd., and distributed by Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 21s.

Vol. 1, which appeared some time ago, covered time-bases and their associated circuits. Vol. 2 deals with r.f., i.f., and video circuits and subsidiary connected matters, such as interference suppressors, a.g.c., multi-channel tuners. Power supplies are also treated.

Guide to Broadcasting Stations, 1955—1956
8th Edition.

Compiled by the staff of *Wireless World*. Pp. 80. Published for *Wireless World* by Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 2s. 6d. (postage 2d.).

An Automatic Counter for the Measurement of Impulsive Interference

By J. Miedzinski. E.R.A. Technical Report M/T114. Pp. 24. The British Electrical & Allied Industries Research Association, Thorncroft Manor, Dorking Road, Leatherhead, Surrey. Price 12s.

Active Networks

By Vincent C. Rideout. Pp. 485 + xvi. Constable & Co. Ltd., 10 Orange Street, London, W.C.2. Price 42s.

Circuits and Networks

By Glenn Koehler. Pp. 349 + x. The Macmillan Co., 60 Fifth Avenue, New York 11, U.S.A. Price $6.50.

CORRECTION

The price of "The Suppressed Frame System of Telerecording" was incorrectly given in the September issue (p. 258) as 15s. It should be 5s.

MEETINGS

I.E.E.

6th October. President's Inaugural Address by Sir George H. Nelson, Bart.

18th October. "Analogues and Equivalent Circuits", discussion to be opened by W. A. Turner, B.Sc.(Eng.), at 6 o'clock.

19th October. Address by the Chairman of the Radio and Telecommunication Section, H. Stanesby.

31st October, at 2.30. "The Technique of Ionospheric Investigation using Ground Back-Scatter" and "A Study of Ionospheric Propagation by means of Ground Back-Scatter", by E. D. R. Shearman, B.Sc.(Eng.); "An Experiment to Test the Reciprocal Radio Transmission Conditions over an Ionospheric Path of 740 km", by R. W. Meadows, B.Sc.(Eng.); "An Experimental Test of Reciprocal Transmission over Two Long-Distance High-Frequency Radio Circuits", by F. J. M. Laver (B.Sc.) and H. Stanesby. At 5.30—"V.H.F. Propagation by Ionospheric Scattering and its Application to Long-Distance Communication", by W. J. Bray, M.Sc.(Eng.), J. A. Saxton, D.Sc., Ph.D., R. W. White, B.Sc. and G. W. Luscombe B.Sc.(Eng.).

3rd November. "The New High-Frequency Transmitting Station at Rugby", by Captain C. F. Booth, O.B.E. and B. N. MacLarty, O.B.E.

These meetings will be held at the Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, W.C.2, and will commence at 5.30, except where otherwise stated.

Brit. I.R.E.

26th October. Annual General Meeting at 6 o'clock, followed by "Recent Advances in Microwave Tubes", by Dr. R. Kompfner at 7, at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1.

Television Society

7th October. "Progress in American Colour Television", by D. C. Birkinshaw, M.B.E.

27th October. "V.H.F. Aerial Problems", by G. J. Lomer, B.A.

These meetings will commence at 7 at the Cinematograph Exhibitors' Association, 164 Shaftesbury Avenue, London, W.C.2.

British Kinematograph Society

26th October. "Special Effects for Television and Electronic Films", by A. M. Spooner, Ph.D., B.Sc., to be held at the Gaumont-British Theatre, Film House, Wardour Street, London, W.1, at 7.15.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for August 1955

Date 1955 August	Frequency deviation from nominal: parts in 10^8	
	MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.
1	+0·3	+4
2	+0·3	+4
3	+0·3	+3
4	+0·4	+4
5	+0·3	+4
6	+0·3	+5
7	+0·3	+4
8	+0·4	+4
9	+0·3	+4
10	+0·3	+4
11	+0·3	+4
12	+0·4	+3
13	+0·4	+5
14	+0·4	+5
15	+0·4	+5
16	+0·3	+5
17	+0·4	+4
18	+0·4	+4
19	+0·3	+4
20	N.M.	+4
21	N.M.	+4
22	+0·4	+4
23	+0·5	+3
24	+0·4	+3
25	+0·4	+4
26	+0·4	+4
27	N.M.	+5
28	N.M.	+5
29	+0·5	+5
30	+0·5	+5
31	+0·5	+5

The values are based on astronomical data available on 1st September 1955.

N.M. = Not Measured.

The publication of the values previously given under the heading "Lead of MSF impulses on GBR 1000 G.M.T. time signal" has been discontinued. For some time past GBR has radiated the same system of provisional uniform time to which MSF is periodically adjusted and a comparison between the two transmissions is no longer considered necessary.