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F.M. Broadcasting

THE use of frequency modulation at v.h.f. for broadcasting has for long been planned as the answer to the serious interference from continental transmissions which is experienced on the medium and long wavebands in many districts. For some years, comparative tests between a.m. and f.m. were carried out with transmitters at Wrotham and it has been known for some time that the choice for the v.h.f. service would be frequency modulation.

The plan is to cover most of the country with the three programmes, Light, Home and Third, radiated from co-sited transmitters using frequency modulation with carrier frequencies in Band II (88–100 Mc/s). Some of these stations will be sited with the B.B.C. television stations and provision for them was made when they were erected. The masts, for instance, have included the Band II aerials, which are in the form of vertical slots and hence radiate with horizontal polarization.

At each main station there are to be six 10-kW transmitters. Three transmitters will be modulated by the three programmes and the outputs combined and fed to one-half of the aerial system. The other three transmitters will also be modulated by the three programmes and their outputs combined and fed to the other half of the aerial system. Each programme will thus, in effect, be handled by two transmitters so that, in the event of a breakdown in any one, the result will be merely a reduction of 3 db in signal strength of the corresponding programme. Similarly, a fault in one-half of the aerial will cause only a 3-db drop in the signals corresponding to all three programmes. Since the transmitters can be switched to either half of the aerial, a simultaneous aerial and transmitter fault can cause up to 6-db drop in one signal and a 3-db drop in the other two. The chances of a fault causing a complete cessation of service are thus greatly reduced.

The v.h.f. service was inaugurated on 2nd May, when Wrotham ceased experimental transmission and started its regular service. This is the first of the f.m. stations and, because it has grown out of an experimental period, it differs in its equipment from the future ones. The two older 25-kW transmitters (with two stand-by 4·5-kW transmitters) carry two programmes. Their outputs are combined and then split to feed the two halves of the aerials. The outputs of two new 10-kW transmitters are fed in one to each half of the aerial and carry another programme. The final result is much the same, but is differently achieved.

The transmissions have a deviation of 75 kc/s with 50 μsec pre-emphasis time constant, and the effective radiated power is 120 kW with horizontal polarization. The Wrotham frequencies are 89·1 Mc/s (Light), 91·3 Mc/s (Third) and 93·5 Mc/s (Home). The programmes are 2·2 Mc/s apart and this is to be a standard spacing between the three frequencies of all stations.

The transmitting plan is definite and straightforward and should eventually result in the greater part of the country being covered by the three programmes with a minimum of interference. The receiving side is much less definite and must remain so until the public knows what it wants. The present medium- and long-wave services are to remain. In some areas they are quite satisfactory and in some remote areas they will be better than Band II. The need for purely a.m. medium- and long-wave receivers will persist, therefore.

In very many areas, Band II will give a much better service and there may be a demand for purely Band II sets, as the B.B.C. programmes will in most cases be the same. A three-band type of set, covering medium and long waves as well as Band II, is, however, the kind which is at present being usually provided. It would appear, therefore, that listeners wish to be able to receive continental programmes as well as those of the B.B.C.

The Band II receiver can be combined with the television receiver and some examples are already on the market. Technically, Band II reception fits in with the television set better than with the ordinary sound broadcast receiver. The 'front-end' is already of a v.h.f. pattern and the sound i.f. bandwidth is usually adequate. The main changes to a Band III television set would thus seem to be merely the provision of Band II coils in vacant positions of the turret, a switch to replace the a.m. detector by a ratio detector and a switch to render the vision circuits inoperative.

In addition, the Band II stations are to be, in the main, co-sited with the Band I B.B.C. television stations, thus offering possibilities of combined receiving aerials for Bands I and II and, in some cases, perhaps for Band III also.

From an engineering standpoint, there is a lot to be said for combining the Band II f.m. receiver with the television set. Hitherto, however, the public has generally preferred to keep television and sound broadcasting separate. Although television sets combined with medium- and long-wave sets are available, and have been since the beginning of television, the bulk are television only.

If this public preference continues, then the Band II set must be combined with the medium- and long-wave set and, indeed, this is essential for those who do not want television. The method of combination is at present fairly complex. Basically, the sets are conventional broadcast ones with a triode-heptode mixer, one i.f. stage at a frequency around 465 kc/s, a diode detector and an a.f. amplifier. For Band II, the mixer is switched to act as an extra i.f. stage, 10·7-Mc/s i.f. transformers are brought into circuit and the a.f. amplifier is switched from the a.m. detector to a ratio detector. The set, as thus modified, is preceded by a Band II tuner, which often includes a double-triode of which one half acts as an earthed-grid r.f. stage and the other as a mixer-oscillator.

We referred above to the possibility of combining aerials for Bands I, II and III. This is a matter which might be convenient where f.m. and television sets are combined, but it is not very probable that it will be commonly done. Good f.m. reception is possible with lower field strengths than are necessary for good television pictures and so the need for an efficient aerial system is less. In the primary service area an indoor aerial will probably be quite adequate and is in accord with the public's taste in aerials for sound broadcasting. It is significant that several f.m. sets which we have seen have contained a built-in Band-II aerial.

The use of frequency modulation at v.h.f. for broadcasting is a new thing in this country but it has, of course, been used elsewhere for a long time. From the technical point of view, the matters of chief interest are the forms adopted for the transmitters and how the receiving side will develop. Although the latter is bound by technical and economic limits, it is one in which public taste will affect the matter considerably.

W. T. C.

RECTIFIER-FILTER CHARACTERISTICS

Approximate Methods of Calculation

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SUMMARY.—By making certain simplifying assumptions it is possible to arrive at reasonably simple expressions for the calculation of the performance of rectifier-filter circuits which employ thermionic rectifiers. The agreement with measured values is within about 5%, which can be regarded as satisfactory since allowance must be made for variations in components which may easily amount to more than 5% in some cases.

THE exact calculation of the performance of rectifiers and their associated filters involves complicated mathematical expressions and in most cases graphical methods of solution are necessary. Roberts[1], Schade[2] and Waidelich[3] have produced sets of curves from which the performance may be predicted, provided the conduction resistance of the rectifier and the load resistance are known. Rectifiers may also be designed by using curves given by rectifier manufacturers.

However, it is more satisfactory to use a mathematical approach and in the following treatment some approximations are made in order to simplify the theory. The final results given by the approximate methods are sufficiently close to experimental values to warrant their use.

The first simplifying assumption made is that in the expansion of the function $\cos x$ in terms of x, the first two terms give a good approximation, which holds reasonably well provided x does not exceed approximately one radian.

In what follows, half-wave rectifiers with capacitor-input filters and full-wave rectifiers with either capacitor-input or choke-input filters will be considered.

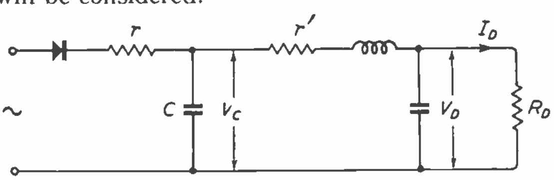

Fig. 1. *Equivalent circuit of half-wave rectifier with capacitor-input filter.*

1. Rectifier with Capacitor-Input Filter

The problem is simplified as follows:—

(i) It is assumed that the rectifier acts as a linear device while conducting. This is not strictly true since a 3/2 power law is generally followed but usually quite an amount of resistance is added in series with the rectifier so that the overall characteristic approaches linearity. A reasonable value of average rectifier resistance for a capacitor-input filter may be found by determining the forward resistance at about twice rated output current for a full-wave rectifier and four times rated output current for a half-wave rectifier (see appendix).

(ii) The assumption is made that the reservoir capacitor has a very large capacitance so that the direct voltage across it may be considered constant during the cycle, for the purpose of calculating the fall of voltage with increase in load current. From the curves of Schade[2], it appears that this is a reasonable approximation provided that $2\pi fCR$ is greater than 25 in the case of a half-wave rectifier and greater than 10 in the case of a full-wave rectifier; where f is the supply frequency, C is the capacitance of the reservoir capacitor and R is the effective d.c. load resistance.

If the capacitor is smaller than given by the above relation the output voltage will be less than predicted by this approximate method.

Half-Wave Rectification

The equivalent circuit is shown in Fig. 1. Current flows through the rectifier when the alternating voltage exceeds the voltage V_c across the capacitor. If V_c is assumed to be constant, the current pulse will be symmetrical as shown in Fig. 2.

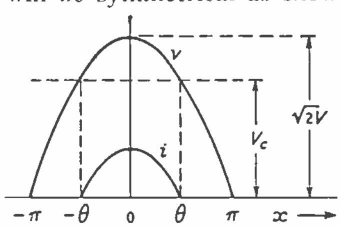

Fig. 2. *Rectifier voltage and current waveforms for the circuit of Fig. 1.*

Conduction commences at a phase angle θ before the alternating voltage reaches its peak value and cut-off occurs at a phase angle θ after the peak value.

During conduction, the difference between the alternating voltage and the capacitor voltage appears across the effective rectifier resistance

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r and hence the instantaneous current through the rectifier may be found. It is shown in the appendix that if the approximation for cos x is used, this current is given by

$$i = \frac{V}{\sqrt{2}r}\left(\theta^2 - x^2\right)$$

The output current, which is the average value of the instantaneous current, may be found by integrating over a full cycle and dividing by 2π (see appendix).

This yields the following expression:

$$I_D = 0{\cdot}15\,\frac{V}{r}\,\theta^3 \quad .. \quad .. \quad .. \quad (1)$$

The maximum instantaneous current occurs when $x = 0$ and hence the ratio of peak to average current is

$$\frac{\text{Peak Current}}{\text{Average Current}} = \frac{4{\cdot}71}{\theta} \quad .. \quad (2)$$

The voltage across the reservoir capacitor is:

$$V_c \approx \sqrt{2}\,V\,(1 - 0{\cdot}5\,\theta^2)$$

and this may be expressed in terms of the output current as

$$V_c \approx \sqrt{2}\,V\left\{1 - 1{\cdot}78\left(\frac{rI_D}{V}\right)^{2/3}\right\} \quad (3)$$

The output voltage V_D will be less than V_c due to the resistance of the choke included in the filter section which follows the reservoir capacitor,

$$V_D = V_c - r'I_D$$

The ripple voltage across the reservoir capacitor may be found by determining the alternating components of the rectifier current. This may be accomplished by means of Fourier analysis employing the approximation for cos nx as shown in the appendix. Due to symmetry about $x = 0$, the Fourier series will contain no sine terms.

Only the fundamental component of current will be considered since it has the greatest amplitude and is least reduced by subsequent filtering.

This fundamental component is shown to be (see appendix)

$$I_1 = \sqrt{2}\,I_D\,(1 - 0{\cdot}1\,\theta^2) \quad .. \quad (4)$$

If it is assumed that all this current passes through the reservoir capacitor, the ripple voltage across it may be found at frequency f.

$$V_1 = \frac{I_1}{2\pi fC} = \frac{0{\cdot}225\,I_D}{fC}\left(1 - 0{\cdot}1\,\theta^2\right) .. \quad (5)$$

This will be correct provided a filter with high inductance follows the reservoir capacitor. If the inductive reactance is comparable with the reactance of the reservoir capacitor it will increase the alternating current through this capacitor since the inductive and capacitive currents are in phase opposition, and hence the ripple voltage across the reservoir capacitor will be increased.

The efficiency of the rectifier may be found since the input power at the supply frequency is VI_1 watts, and the d.c. output is V_DI_D watts.

$$\text{Efficiency} = \frac{V_DI_D}{VI_1} \times 100\%$$

$$= \frac{V_D}{\sqrt{2}\,V\,(1 - 0{\cdot}1\theta^2)} \times 100\% \quad (6)$$

Full-Wave Rectification

With full-wave rectification the current pulse through each rectifier will be the same as for a half-wave rectifier if the angle of flow, 2θ, is the same.

Since there are now two rectifiers working alternately, the output current will be doubled, each rectifier contributing one-half of the total average current.

In the case of a rectifier which employs a centre-tapped transformer, each rectifier has a resistance r and the current expressed in terms of the angle θ will be

$$I_D = 0{\cdot}3\,\frac{V}{r}\,\theta^3 \text{ (centre-tapped transformer)} \quad (7)$$

The reservoir capacitor voltage will be

$$V_c = \sqrt{2}\,V\left\{1 - 1{\cdot}11\left(\frac{r}{V}I_D\right)^{2/3}\right\} .. \quad (8)$$

When a bridge circuit is employed there are always two rectifiers in series so that the effective resistance is $2r$.

$$I_D = 0{\cdot}3\,\frac{V}{2r}\,\theta^3 \text{ (bridge connection)} \quad .. \quad (9)$$

The ratio of peak current to average current per rectifier is the same as for the half-wave case. In terms of the total current, the peak rectifier current is halved.

$$\frac{\text{Peak rectifier current}}{\text{Total output current}} = \frac{2{\cdot}36}{\theta} \quad .. \quad (10)$$

No supply-frequency component of current flows through the reservoir capacitor since the fundamental component of the combined current pulses has a frequency which is twice that of the supply. Hence only even harmonics of the supply frequency occur.

The double supply-frequency current through the reservoir capacitor corresponds to twice the second-harmonic component of the current in each rectifier. Since the total output current is also twice the average current per rectifier, the double-frequency component will be:

$$I_2 = \sqrt{2}\,I_D\,(1 - 0{\cdot}4\,\theta^2) \quad .. \quad (11)$$

As in the half-wave case, higher harmonics are neglected and hence the r.m.s. ripple voltage at double supply-frequency may be found by assuming that all the alternating current passes through the reservoir capacitor.

$$V_2 \approx \frac{I_2}{4\pi f C}$$

$$= 0{\cdot}112 \frac{I_D}{fC}\left(1 - 0{\cdot}4\,\theta^2\right) \quad .. \quad (12)$$

The efficiency expression will be the same as for the half-wave rectifier since the supply-frequency component of current for each rectifier is the same as in the half-wave case

$$\text{Efficiency} = \frac{V_D}{\sqrt{2}\,V\,(1 - 0{\cdot}1\,\theta^2)} \times 100\% \quad (13)$$

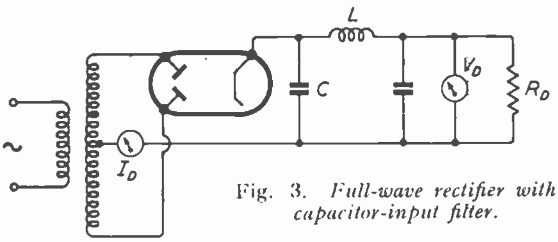

Fig. 3. *Full-wave rectifier with capacitor-input filter.*

Practical Example

In order to illustrate the usefulness of the theory, the circuit in Fig. 3 was used.

A 5Y3-GT rectifier was supplied by a 400-0-400 V centre-tapped transformer. Each half of the transformer secondary had a resistance of 97 ohms and the 230-V primary had a resistance of 12 ohms. Thus the effective resistance added in series with each rectifier is $97 + (400/230)^2 \times 12 = 134$ ohms. The leakage inductance was only 15 millihenrys and was neglected. The rated maximum current of the rectifier was 125 milliamperes and the frequency of the supply was 50 c/s.

The characteristics may be calculated as follows: From the table in the appendix the effective rectifier resistance is found to be 370 ohms to which must be added the 134 ohms of the transformer. Hence $r = 504$ ohms.

Angle θ

Acording to equation (7):

$$\theta^3 = \frac{rI_D}{0{\cdot}3\,V} = \frac{504}{0{\cdot}3 \times 400} I_D = 4{\cdot}2\,I_D$$

$\therefore$ When $I_D = 125$ mA, $\theta = 0{\cdot}808$ radian.

Reservoir Capacitor Voltage

This is given by equation (8):

$$V_c = \sqrt{2}\,V\,(1 - 0{\cdot}5\,\theta^2) = 566\,(1 - 0{\cdot}5\,\theta^2) \text{ volts}$$

$$\text{or } V_c = 566\left\{1 - 1{\cdot}11\left(\frac{504}{400} I_D\right)^{2/3}\right\}$$

$$= 566\left\{1 - 1{\cdot}3\,I_D^{2/3}\right\} \text{ volts}$$

When $I_D = 125$ mA, $V_c = 382$ volts.

Minimum Size of C

The size of reservoir capacitor should be calculated at the maximum output current since then the effective load resistance, R, is a minimum.

$$R \approx V_c/I_D$$

The maximum current is 125mA

$$\therefore R = \frac{382}{0{\cdot}125} = 3{,}060 \text{ ohms.}$$

For reasonable prediction

$$2\pi fCR > 10$$

$$\therefore C > \frac{10}{2\pi \times 50 \times 3060}$$

$$> 10{\cdot}5\,\mu\text{F}$$

Thus the minimum value of capacitance was used in the experiment.

Output Voltage

The volt drop in the filter choke must be subtracted from V_c.

$$V_D = V_c - 200\,I_D$$

When $I_D = 125$ mA, $V_D = 357$ volts.

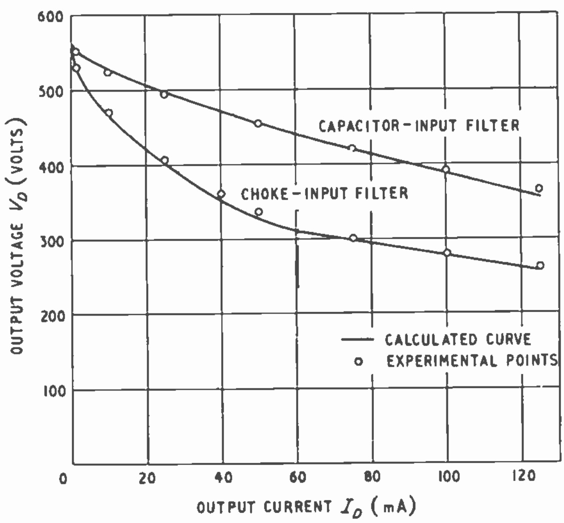

Fig. 4. *Calculated and measured performance curves of rectifier circuit.*

Ripple Voltage

The 100-c/s ripple is the most important component.

According to equation (12):

$$V_2 = \frac{0{\cdot}112 \times 10^6}{50 \times 10{\cdot}5} I_D\,(1 - 0{\cdot}4\,\theta^2)$$

$$= 214\,I_D\,(1 - 0{\cdot}4\,\theta^2)$$

$$\text{or} \quad V_2 = 214\,I_D\,\{1 - 1{\cdot}04\,I_D^{2/3}\}$$

When $I_D = 125$ mA, $V_2 = 20$ volts.

Efficiency

By equation (13)

$$\text{Efficiency} = \frac{V_D}{\sqrt{2}V(1 - 0{\cdot}1\,\theta^2)} \times 100\%$$

$$= \frac{V_D}{566\,(1 - 0{\cdot}1\,\theta^2)} \times 100\%$$

or

$$\text{Efficiency} = \frac{V_D}{566\,(1 - 0{\cdot}26\,I_D{}^{2/3})} \times 100\%$$

When $I_D = 125$ mA, Efficiency $= 67{\cdot}4\%$.

Table 1 shows the calculated quantities for the rectifier and also the measured values.

It is evident that there is good agreement between predicted and measured values. In the case of the output-voltage curves shown in Fig. 4, the discrepancy is less than 3% but in the case of the ripple voltage it amounts to about 15%. Roughly 4% of this discrepancy is due to the fact that the choke of the filter has an inductance of only 6·5 henrys and thus the ripple voltage is increased.

This means that the corrected ripple voltage is about 10% lower than the measured value. As an approximation this is reasonable, especially since circuit components may differ from their nominal values by 10–20%.

In order to check further the validity of the theory, a 5Z4G rectifier was substituted for the 5Y3GT valve and output voltage measured with varying output current. This rectifier has an effective resistance of 130 ohms at twice rated current and it was found that again the curve could be predicted to within 3%.

2. Full-Wave Rectifier with Choke-Input Filter

The half-wave rectifier is not considered here because it has very poor regulation and is not often used in practice.

In order to simplify the theory it is necessary to assume that the effect of the combined rectifier and choke resistance on instantaneous current may be neglected in comparison with the inductive reactance of the choke. It is also assumed that a capacitor of large capacitance follows the choke so that the output voltage is substantially constant during the whole cycle. In practice this is approximately true, especially if the inductance of the choke is large.

The combined resistance r of rectifier and choke cannot be neglected completely and for simplicity it is assumed that it causes a volt drop rI_D in the output voltage. Thus it is equivalent to inserting a resistance r after the capacitor, in series with the load resistance.

TABLE 1

Calculated Values						Measured Values	
Output Current I_D (mA)	θ Radian	Capacitor Voltage V_c (volts)	Output Voltage V_D (volts)	Ripple Voltage V_2 (volts)	Effi-ciency %	Output Voltage V_D (volts)	100-c/s Ripple V_2 (volts)
0	0	566	566	0	—	—	—
1·8	0·196	556	555	0·4	98·3	551	0·4
10	0·348	531	529	2·0	94·5	523	2·2
25	0·471	504	499	4·9	90·0	494	5·3
50	0·593	466	456	9·2	83·5	455	10·3
75	0·680	435	420	13	77·7	421	15
100	0·749	408	388	17	72·7	392	19
125	0·808	382	357	20	67·4	367	23

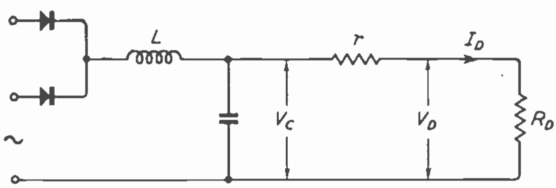

Fig. 5. *Equivalent circuit of full-wave rectifier with choke-input filter.*

Hence the simplified equivalent circuit will appear as in Fig. 5. The effective rectifier resistance is in this case determined at rated output current.

It is necessary to consider two conditions of operation, viz:

(i) When the combined current from the two rectifiers is not continuous but is interrupted for part of each half-cycle, the average current is said to be below the critical value.

(ii) When the combined current is continuous and never falls to zero during any part of the cycle, the average current is above the critical value which marks the transition between the two conditions.

(i) *Average Current below the Critical Value*

As shown in Fig. 6, the current in each rectifier commences at a phase angle θ before the alternating voltage peak, at the instant when the alternating voltage equals the voltage across the capacitor. During conduction the difference between the alternating supply voltage and the constant capacitor voltage appears across the inductance and therefore the variation of current with time may be found. In the appendix the instantaneous current is shown to be

$$i = \frac{V}{6\sqrt{2}\,\pi fL}\left(3\theta^2 x - x^3 + 2\theta^3\right) \quad .. \quad (14)$$

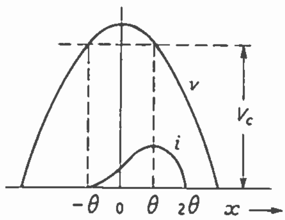

Fig. 6. *Waveforms for Fig. 5 with the average current below the critical value.*

Provided θ is less than $\pi/4$ radian, the current commences at phase angle $-\theta$, increases while the alternating voltage is greater than the capacitor voltage and reaches a maximum when the two voltages are equal at a phase angle θ. Then the alternating voltage falls below the capacitor voltage and consequently the current falls until it cuts off at a phase angle 2θ.

The average current under these conditions will be

$$I_D = 0{\cdot}0805\,\frac{V}{fL}\,\theta^4 \quad .. \quad .. \quad .. \quad (15)$$

If θ exceeds $\pi/4$ and the average current is still below the critical value, the current will cut off only after the second rectifier has taken over. Since this complicates the expression for current, the region between $\theta = \pi/4$ and the critical current will not be dealt with here. This region is small because critical current is reached soon after θ exceeds $\pi/4$, so that an adequate curve may be drawn through predicted points without considering this region.

Thus θ may be found in terms of the average current for values of θ less than $\pi/4$, and hence the equivalent capacitor voltage may be calculated.

$$V_c \approx \sqrt{2}\,V\,(1 - 0{\cdot}5\,\theta^2)$$

$$= \sqrt{2}\,V\left\{1 - 1{\cdot}76\sqrt{\left(\frac{fL}{V}\,I_D\right)}\right\} \quad .. \quad (16)$$

According to the simplified equivalent diagram the output voltage will now be

$$V_D = V_c - rI_D$$

where r represents the combined resistance of the rectifier and the choke.

The ripple voltage is not calculated for currents below the critical value since the analysis is not simple and since it is usual to operate rectifiers with choke-input filters at currents which exceed the critical current.

(ii) *Average Current above the Critical Value*

When the combined rectifier current becomes continuous the voltage appearing at the input to the choke consists of consecutive half waves as shown in Fig. 7.

If the rectifier resistance is neglected, the average value of these half waves will be the voltage across the capacitor and it can be shown that the Fourier series for the full-wave rectified curve is

$$\sqrt{2}\,V\left(\frac{2}{\pi} + \frac{4}{3\pi}\cos 2x - \frac{4}{15\pi}\cos 4x + \ldots\right)$$

Thus the average value is $\frac{2\sqrt{2}}{\pi}V$ and is independent of current, provided it flows continuously.

The lowest frequency component is at double the frequency of the supply and also does not depend on current. Its magnitude is $V_2 = \frac{4}{3\pi}V$ (r.m.s.)

The alternating current component at this frequency is determined by the choke impedance and will be

$$I_2 \approx \frac{V_2}{4\pi fL} = 0{\cdot}0337\,\frac{V}{fL} \quad .. \quad .. \quad (17)$$

The component currents at higher frequencies are much smaller and may be neglected.

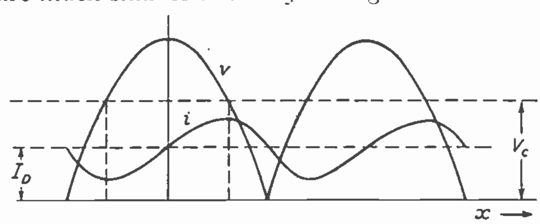

Fig. 7. *Waveforms for Fig. 5 with the average current above the critical value.*

When the peak value of the alternating component of the combined rectifier current is equal to the output current, the instantaneous current will just reach zero twice per cycle. Thus the critical current is equal to the peak value of the alternating component

$$\therefore \text{Critical current} = \sqrt{2}\,I_2 = 0{\cdot}0477\,\frac{V}{fL} \quad (18)$$

Beyond this critical current, therefore, the equivalent capacitor voltage is constant at

$$V_c = \frac{2\sqrt{2}}{\pi}V = 0{\cdot}9\,V$$

and the ripple voltage is constant at

$$V_2 = \frac{4}{3\pi} V = 0{\cdot}424\ V \text{ (r.m.s.)} \quad .. \quad (19)$$

The output voltage will not be constant but will be approximately given by:

$$V_D \approx V_c - rI_D$$
$$= 0{\cdot}9\ V - rI_D \quad .. \quad .. \quad (20)$$

From the simplified equivalent circuit it is evident that the efficiency is approximately given by:

$$\text{Efficiency} \approx \frac{V_D}{V_c} \times 100\% \quad (21)$$

Practical Example

An experiment was conducted in which the same circuit as before was used but the reservoir capacitor was omitted. The choke had an inductance of 6·5 henrys (nominally 8 henrys). The resistance r consists of the equivalent transformer resistance of 134 ohms, the effective rectifier resistance of 480 ohms (found at rated current) and the choke resistance of 200 ohms.

Thus r = 814 ohms.

(i) *Current below the Critical Value*

The angle θ is given by equation (15)

$$\theta^4 = \frac{fL}{0{\cdot}0805\ V} I_D = \frac{50 \times 6{\cdot}5}{0{\cdot}0805 \times 400} I_D$$
$$= 10{\cdot}1\ I_D$$

θ must not exceed $\pi/4$ if calculations are to be reasonably correct in this region.

The limiting current to which the theory applies is $\left(\frac{\pi}{4}\right)^4 \times \frac{1}{10{\cdot}1} = 37{\cdot}6$ mA. Thus currents up to about 40 mA may be considered.

For a current of 25 mA, $\theta = 0{\cdot}71$ radian. The equivalent capacitor voltage will be

$$V_c = \sqrt{2}\, V (1 - 0{\cdot}5\ \theta^2)$$
$$= 566\ (1 - 0{\cdot}5\ \theta^2) \text{ volts}$$
$$= 566\ (1 - 1{\cdot}59 \sqrt{I_D}) \text{ volts.}$$

The output voltage is

$$V_D = V_c - 814\ I_D \text{ volts}$$

When I_D = 25 mA, V_D = 405 volts.

(ii) *Current above the Critical Value*

By equation (17)

$$\text{Critical current} = 0{\cdot}0477 \frac{V}{fL} = \frac{0{\cdot}0477 \times 400}{50 \times 6{\cdot}5}$$
$$= 58{\cdot}7 \text{ mA}$$

The equivalent capacitor voltage is constant for currents exceeding the critical value

$V_c = 0{\cdot}9\ V = 360$ volts

The ripple voltage is: $V_2 = 0{\cdot}424\ V = 170$ volts

The output voltage is: $V_D = 360 - 814\ I_D$

When I_D = 125 mA, V_D = 258 volts

Calculated and measured values are compared in Table 2.

TABLE 2

Calculated Values				Measured Values	
Output Current I_D (mA)	Output Voltage V_D (volts)	Efficiency %	θ Radian	Output Voltage V_D (volts)	Ripple Voltage V_2 (volts)
0	566	—	—	—	—
1·7	529	99·5	0·35	530	7·7
10	468	98·3	0·56	470	42
25	405	95·3	0·71	406	91
40	355	91·5	0·80	362	132
50	—	—	—	338	152
60	311	86·5	—	—	—
75	299	83·0	—	300	171
100	279	77·5	—	281	172
125	258	71·6	—	263	172

Output voltage curves are shown in Fig. 4. Again the agreement is good, the greatest error being less than 3%. The ripple-voltage measurements also bear out the theory very well at currents which exceed the critical value (Calculated V_2 = 170 V).

By means of an oscilloscope it was found that the critical current was about 62 mA which agrees well with the calculated value of about 59 mA.

REFERENCES

[1] N. H. Roberts, *Wireless Engineer*, 1936, Vol. 13, pp. 351, 423.
[2] O. H. Schade, *Proc. Inst. Radio Engrs*, 1943, Vol. 31, p. 341.
[3] D. L. Waidelich, *Trans. Amer. Inst. elect. Engrs*, 1941, Vol. 60, p.1161.

APPENDIX

The following symbols are used:—

x = $2\pi ft$
f = frequency of the supply
t = variable time
V = effective (r.m.s.) alternating voltage applied to a rectifier
V_c = average voltage across the first capacitor
V_D = direct output voltage
I_D = direct output current
i = instantaneous supply current
v = instantaneous supply voltage
I_1 = effective value of supply-frequency current component
I_2 = effective value of current component with double the frequency of the supply
I_n = effective value of current component with frequency nf
θ = phase angle at which rectifier current flow commences
$\cos x \approx 1 - 0{\cdot}5x^2$
r = effective rectifier resistance

Theory

1. *The Capacitor-Input Filter*

(a) Half-wave rectifier:

The instantaneous supply current will be

$$i = \frac{v - V_c}{r} = \frac{\sqrt{2}\,V}{r}\left(\cos x - \cos\theta\right)$$

$$\approx \frac{\sqrt{2}\,V}{r}\left(1 - 0{\cdot}5\,x^2 - 1 + 0{\cdot}5\,\theta^2\right)$$

$$= \frac{V}{\sqrt{2}\,r}\left(\theta^2 - x^2\right)$$

The average value of this current is given by

$$I_D = \frac{1}{2\pi}\int_{-\pi}^{\pi} i.dx = \frac{1}{\pi}\int_0^{\theta}\frac{V}{\sqrt{2}\,r}\left(\theta^2 - x^2\right)dx$$

$$= \frac{V}{\sqrt{2}\,\pi r}\left|\theta^2 x - \frac{1}{3}x^3\right|_0^{\theta}$$

$$= \frac{\sqrt{2}\,V\theta^3}{3\,\pi r} = 0{\cdot}15\,\frac{V}{r}\,\theta^3$$

The instantaneous current reaches its maximum value when $x = 0$

$$I_P = \frac{V}{\sqrt{2}\,r}\,\theta^2$$

Thus the ratio of peak to average current is

$$\frac{I_P}{I_D} = \frac{3\,\pi}{2\,\theta} = \frac{4{\cdot}71}{\theta}$$

Since $V_c = \sqrt{2}\,V\cos\theta$ the voltage across the capacitor is approximately given by

$$V_c \approx \sqrt{2}\,V\,(1 - 0{\cdot}5\,\theta^2)$$

$$= \sqrt{2}\,V\left\{1 - 0{\cdot}5\left(\frac{r}{0{\cdot}15\,V}\,I_D\right)^{2/3}\right\}$$

$$= \sqrt{2}\,V\left\{1 - 1{\cdot}78\left(\frac{r}{V}I_D\right)^{2/3}\right\}$$

The magnitude of the nth harmonic component may also be found

$$\sqrt{2}\,I_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(i.\cos nx)\,dx \approx \frac{2}{\pi}\int_0^{\theta} i\,(1 - 0{\cdot}5\,n^2x^2)\,dx$$

$$= \frac{2}{\pi}\left\{\int_0^{\theta} i.\,dx - 0{\cdot}5\,n^2\int_0^{\theta} ix^2.\,dx\right\}$$

$$= 2\,I_D - \frac{n^2}{\pi}\int_0^{\theta}\frac{V}{\sqrt{2}\,r}\,(\theta^2 - x^2)\,x^2.\,dx$$

$$= 2\,I_D - \frac{n^2.\sqrt{2}\,V}{15\pi r}\,\theta^5$$

$$= 2\,I_D\,(1 - 0{\cdot}1\,n^2\,\theta^2)$$

The first two components will be:

At frequency f: $I_1 = \sqrt{2}\,I_D\,(1 - 0{\cdot}1\,\theta^2)$

At frequency $2f$: $I_2 = \sqrt{2}\,I_D\,(1 - 0{\cdot}4\,\theta^2)$

(b) Full-wave rectifier:

I_D and I_2 are doubled, but I_1 becomes zero.

$$\therefore \quad I_D = 0{\cdot}3\,\frac{V}{r}\,\theta^3$$

and $$I_2 = \sqrt{2}\,I_D\,(1 - 0{\cdot}4\,\theta^2)$$

The ratio of peak current per rectifier to total average current is half that of the half-wave rectifier

$$\therefore \quad \frac{I_P}{I_D} = \frac{2{\cdot}36}{\theta}$$

Expressed in terms of I_D, the capacitor voltage is

$$V_c = \sqrt{2}\,V\left\{1 - 0{\cdot}5\left(\frac{r}{0{\cdot}3\,V}\,I_D\right)^{2/3}\right\}$$

$$= \sqrt{2}\,V\left\{1 - 1{\cdot}11\left(\frac{r}{V}\,I_D\right)^{2/3}\right\}$$

2. *The Choke-Input Filter*

(a) Current below the critical value:

The voltage across the choke will be $v - V_c = L\,\frac{di}{dt}$

$$\therefore\ 2\,\pi fL.\,di = (v - V_c).\,d(2\,\pi ft) = \frac{V}{\sqrt{2}}(\theta^2 - x^2).\,dx$$

Thus instantaneous current is given by:

$$i = \frac{V}{2\sqrt{2}\,\pi fL}\int(\theta^2 - x^2).\,dx$$

$$= \frac{V}{2\sqrt{2}\,\pi fL}\,(\theta^2 x - \tfrac{1}{3}\,x^3 + k)$$

$$= \frac{V}{6\sqrt{2}\,\pi fL}\,(3\,\theta^2 x - x^3 + 3k)$$

Current commences when $x = -\theta$.

On substituting for x, the expression becomes

$$-\,3\theta^3 + \theta^3 + 3k = 0$$

$$\therefore \quad 3k = 2\theta^3$$

$$\therefore \quad i = \frac{V}{6\sqrt{2}\,\pi fL}(2\theta^3 + 3\theta^2 x - x^3)$$

The expression in brackets may be factorized:

$$2\theta^3 + 3\theta^2 x - x^3 = (\theta + x)^2.\,(2\theta - x)$$

This means that current will cut off when $x = 2\theta$.

It is thus possible to find the average current:

$$I_D = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} i.\,dx$$

$$= \frac{1}{\pi}\int_{-\theta}^{2\theta}\frac{V}{6\sqrt{2}\,\pi fL}\,(2\theta^3 + 3\theta^2 x - x^3)$$

$$= \frac{V}{6\sqrt{2}\,\pi^2 fL}\left|2\theta^3 x + \frac{3}{2}\,\theta^2 x^2 - \frac{1}{4}\,x^4\right|_{-\theta}^{2\theta}$$

$$= \frac{V}{6\sqrt{2}\,\pi^2 fL}\left(4\theta^4 + 6\theta^4 - 4\theta^4 + 2\theta^4 - \frac{3}{2}\theta^4 + \frac{1}{4}\,\theta^4\right)$$

$$= \frac{9V}{8\sqrt{2}\,\pi^2 fL}\,\theta^4$$

$$= 0{\cdot}0805\,\frac{V}{fL}\,\theta^4$$

The equivalent capacitor voltage will be:

$$V_c \approx \sqrt{2}\,V\,(1 - 0{\cdot}5\theta^2)$$

$$= \sqrt{2}\,V\left(1 - 0{\cdot}5\sqrt{\frac{fL\,I_D}{0{\cdot}0805\,V}}\right)$$

$$= \sqrt{2}\,V\left(1 - 1{\cdot}76\sqrt{\frac{fL}{V}\,I_D}\right)$$

Typical Values of r

In the following table some values of equivalent resistances of high-vacuum rectifiers are given, being taken from manufacturers' curves.

Full-Wave Rectifier	Rated current (mA)	r at rated current (ohms)	r at twice rated current (ohms)	Half-Wave Rectifier	Rated current (mA)	r at twice rated current (ohms)	r at 4 × rated current (ohms)
5U4	225	250	200	35Z5	100	90	70
5V4	175	130	100	117Z3	90	125	100
5Y3	125	480	370	81	85	570	460
5Z4	125	160	130	25Z5	75	150	120
6X5	70	310	240	117Z6	60	130	100
6ZY5	40	450	350	12Z3	55	155	120
				1-V	45	220	180

TRANSIENT RESPONSES WITH LIMITED OVERSHOOT

Approximation by Principal Mode

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SUMMARY.—The problem is considered of designing a system transfer function such that, in the response to a step-function input, the overshoot shall be limited and consist largely of the principal mode of oscillation. A method is stated by which the size of any real term in the response may be calculated relative to the maximum value of the principal mode. Graphical conditions are given restricting the permissible pole and zero positions in certain cases when this ratio is specified. The procedure is illustrated by examples.

1. Introduction

THE problem of synthesizing a system transfer function such that the overshoot in the step-function response shall be limited and consist largely of the principal mode of oscillation has been considered by Mulligan[1]. In his paper, which deals with general transfer functions, the conditions are investigated under which the locations of the maxima and minima of the step-function response may be approximated by the times at which the principal mode term by itself passes through maximum and minimum values. General requirements are also given in order that the contributions to the overshoot due to terms other than the principal mode shall be small.

The purpose of this paper is to state a method of obtaining the magnitude of any real term relative to the principal mode at the time the principal mode passes through its maximum value, for a transfer function having, in addition to the principal pair of poles, any number of real poles and zeros. The restriction to real subsidiary poles and zeros is made in order to simplify the graphical work necessary for the application of the method. The theory given below takes complex subsidiary poles and zeros into account. Again, although the curves given refer only to zeros in the left half of the complex plane, which will be the case unless 'non-minimum phase' elements are admitted, see Bode[2], the theory does not require this restriction. The poles of the transfer function (defined by response function/driving function) are, for stability, however, confined to the left half-plane.

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2. First Maximum of Principal Mode

Let there be a general transfer function having g zeros and f poles. Then the transfer function is determined by*

$$\left(B\prod_{i=1}^{g}(p-z_i)\right)\bigg/\left(\prod_{h=1}^{f}(p-p_h)\right) \quad .. \quad (1)$$

*The notation $\prod_{i=1}^{g}(p-z_i)$ is used for the continued product $(p-z_1)\,(p-z_2)\,(p-z_3)\,......\,(p-z_g)$

The z_i are the zeros, real or complex and situated in any part of the complex plane except at the origin; the p_h are the poles, real or complex, all of which are assumed to be distinct and confined to the left half-plane. In order that the response at infinite frequency should be finite or zero, $g \leqslant f$. The factor B is a constant which determines the zero-frequency gain of the system. In the work that follows, B will be given that value which makes the response of the network unity at infinite values of time when the input is a unit-step function. If the response to a unit-step function is $A(t)$ and its Laplace Transform is $A(p)$, then for the above network

$$A(p) = \frac{B \prod_{i=1}^{g} (p - z_i)}{\prod_{h=1}^{f} (p - p_h)\, p} \quad \ldots \quad (2)$$

Since $A(t)$ is to be unity at $t = \infty$, then, by the Final Value theorem[3] of transform theory, $\lim\ pA(p) \to 1$ as $p \to 0$;

i.e., $B = (-1)^q \left(\prod_{h=1}^{f} |p_h|\right) \Big/ \left(\prod_{i=1}^{g} |z_i|\right)$, where q is the number of zeros in the right half-plane.

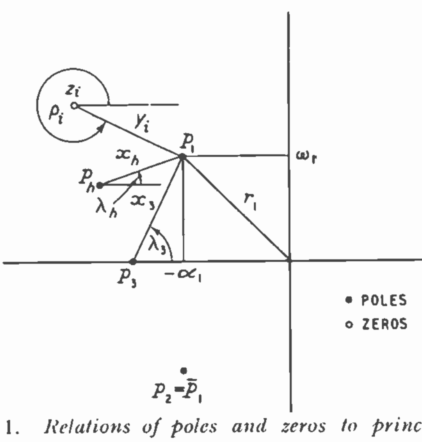

Fig. 1. *Relations of poles and zeros to principal pole.*

The pair of complex poles which give rise to the principal oscillatory mode is that pair of conjugate poles which have the least damping and also, normally, the greatest frequency. They will be denoted by p_1 and $\bar{p}_1$ and occupy the positions $-\alpha_1 + j\omega_1$ and $-\alpha_1 - j\omega_1$ as shown in Fig. 1. The position of any other pole or zero with respect to p_1 may be specified by the distance from that pole or zero to p_1, and by the angle which the vector from the pole or zero to p_1 makes with the real axis. The angles here are measured in the positive or anticlockwise sense. Thus for any zero z_i, the vector from z_i to p_1 is $p_1 - z_i$, which may be written $y_i \epsilon^{j\rho_i}$ and for any pole p_h, $p_1 - p_h = x_h \epsilon^{j\lambda_h}$. The same forms apply for real zeros and poles.

The term in $A(t)$ due to the pole at $p = p_1$ is

$$\frac{B \prod_{i=1}^{g} (p_1 - z_i)}{\prod_{h=3}^{f} (p_1 - p_h)\, \omega_1 p_1} \cdot \frac{\epsilon^{p_1 t}}{2j}$$

If $p_1 = r_1 \epsilon^{j\phi_1}$, this may be written

$$\frac{B \prod_{i=1}^{g} y_i\, \epsilon^{-\alpha_1 t}}{\prod_{h=3}^{f} x_h\, \omega_1 r_1} \cdot \frac{\epsilon^{j(\omega_1 t - \phi_1 - P + Z)}}{2j}$$

where $P = \lambda_3 + \lambda_4 + \ldots + \lambda_f$ and $Z = \rho_1 + \rho_2 + \ldots + \rho_g$. Here it is taken that the pole p_2 is the conjugate of p_1; i.e., $p_2 = \bar{p}_1$. The pole at $p = \bar{p}_1$ gives the conjugate of the above expression and the result for the principal pair of poles is therefore the oscillatory term $F_1(t)$, where

$$F_1(t) = \frac{B \prod_{i=1}^{g} y_i}{\omega_1 r_1 \prod_{h=3}^{f} x_h} \epsilon^{-\alpha_1 t} \sin(\omega_1 t - \phi_1 - P + Z) \quad \ldots \quad (3)$$

This has either maximum or minimum values where $F_1'(t) = 0$; i.e., when $\omega_1 t - P + Z = 0, \pi, 2\pi \ldots$ etc., the maximum values being positive and the minimum values being negative. Let the first zero of $F_1'(t)$ occurring for a positive value of t, take place at

$$\omega_1 t = n\pi + P - Z \quad \ldots \quad (4)$$

If this value of t when substituted in equation (3) makes $F_1(t)$ positive, a maximum occurs, of value

$$\frac{\prod_{h=1}^{f} |p_h|}{\prod_{i=1}^{g} |z_i|} \cdot \frac{\prod_{i=1}^{g} y_i}{\prod_{h=3}^{f} x_h\, r_1^2} \cdot \exp.\{-\alpha_1 (n\pi + P - Z)/\omega_1\}$$

This may be rearranged to give

$$\epsilon^{-\alpha_1 n\pi/\omega_1} \left(\prod_{i=1}^{g} \delta_i\right) \Big/ \left(\prod_{h=3}^{f} \delta_h\right) \quad \ldots \quad (5)$$

where $\delta_i = (y_i / |z_i|)\, \epsilon^{\alpha_1 \rho_i / \omega_1}$

and $\delta_h = (x_h / |p_h|)\, \epsilon^{\alpha_1 \lambda_h / \omega_1}$.

If the value of t in (4) makes $F_1(t)$ negative, the first maximum of $F_1(t)$ occurs π/ω_1 seconds later, and is given by the expression (5) multiplied by $\epsilon^{-\alpha_1 \pi/\omega_1}$. Examination of (5) shows it to be composed of factors of the same form, one for each pole or zero. In the case of real poles or zeros it is easy to calculate how this factor varies as the pole or zero moves along the negative real axis to minus infinity, where it has a value of unity. Fig. 2 shows the common logarithm of δ, which is termed the overshoot factor. This is done for two values of the principal pole-pair giving fairly wide differences of damping, namely $\alpha_1/\omega_1 = 0{\cdot}5$ and $\alpha_1/\omega_1 = 1{\cdot}0$. Further as the factors δ involve only ratios of distances or angles, altering the sizes of the poles and zeros without alteration of their angular relationships leaves δ unchanged. For this reason, all pole and zero distances are expressed in terms of ω_1. This process is termed normalizing. The normalized positions of the principal pole-pair are, therefore, $-0{\cdot}5 \pm j1$, $-1 \pm j1$ for the curves 1 and 2 respectively. Fig. 2 also shows the angle λ or ρ in degrees.

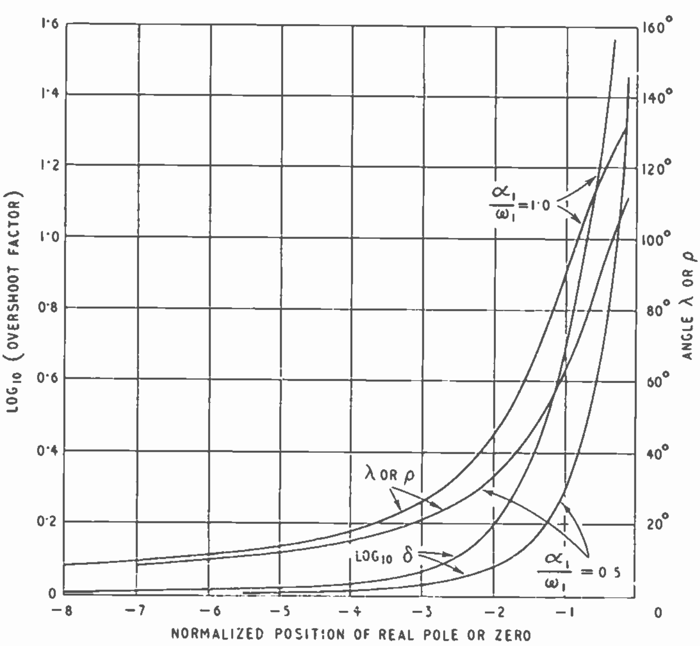

Fig. 2. *Logarithm of overshoot factor.*

In the case of a pair of conjugate complex poles or zeros the overshoot factor for the pair reduces to a form $\delta\delta'$ where the prime denotes quantities for the conjugate pole or zero with negative imaginary part. Clearly to show how this factor varies over the entire plane, or at least over that area of the complex plane in the neighbourhood of p_1, a number of curves are required similar to those of Fig. 2, each curve relating to a fixed value of the imaginary part of the pole or zero. A chart of this type is required for each value of α_1/ω_1. Alternatively, a chart may be prepared showing lines along which the overshoot factor is a contant, the abscissae and ordinates being the real and imaginary parts of any pole or zero. Reference may be made to Mulligan's paper for charts of this nature.

Assuming, meanwhile, that the values of all terms other than the principal mode are negligible by the time the principal mode reaches its first maximum, the overshoot, which is the response $A(t)$ less the constant value unity, is given by the expression (5) either by itself or multiplied by $\epsilon^{-\alpha_1\pi/\omega_1}$ according to whether the first turning point of the principal mode is a maximum or a minimum. Or, taking the logarithm of expression (5)

$$\log_{10}(\text{overshoot}) = -(n\pi\alpha_1/\omega_1)\log_{10}\epsilon + \Sigma\Delta_z - \Sigma\Delta_p \quad \ldots \quad \ldots \quad (6)$$

where $\Delta = \log_{10}\delta$ and $\Sigma\Delta_z$, $\Sigma\Delta_p$ are the sums of the logarithmic contributions due to the zeros and poles respectively. The time at which maximum overshoot occurs, neglecting other terms, is given by equation (4), or π/ω_1 seconds later as already explained. The application of these results is given in Sections 3 and 4.

3. Relative Magnitude of Nearest Real Term

If the poles and zeros are numbered according to the magnitudes of their real parts (i.e., their horizontal distances from the origin), the poles p_1 and p_2 constituting the principal pole-pair, the next least-damped pole is p_3. Let it be taken that p_3 is a real pole, then writing $p_3 = -a_3$, $a_3 - \alpha_1$ is positive (or zero in the limiting case of $a_3 = \alpha_1$). It is recalled also, that although the poles are confined to the left half-plane, no such restriction limits the zeros. The numbering convention for zeros is to hold irrespective of whether they are to the left or right of the origin.

The term in the response due to the real pole p_3 is now evaluated. From (2), this is in fact

$$\frac{B.\prod_{i=1}^{g}(p_3 - z_i)\,\epsilon^{p_3 t}}{\prod_{h=1,\,h\neq 3}^{f}(p_3 - p_h)p_3} \quad \ldots \quad \ldots \quad (7)$$

At the first turning point of the principal mode, taking meanwhile the case when this gives a maximum, equation (4) gives the required value of t, and substituting in (7) gives the value of the real term, namely

$$\frac{B.\prod_{i=1}^{g}(p_3 - z_i)}{\prod_{h=1,\,h\neq 3}^{f}(p_3 - p_h)p_3} \cdot \exp\{p_3(n\pi + P - Z)/\omega_1\} \quad (8)$$

Now $p_3 - z_i$ is the vector from z_i to p_3; i.e., from z_i to $-a_3$. For real zeros, $|p_3 - z_i|$ is the distance from z_i to $-a_3$, and for conjugate complex zeros z_i and $\bar{z}_i$, $(p_3 - z_i)(p_3 - \bar{z}_i)$ is the square of the distance from z_i to $-a_3$. Furthermore $(p_3 - p_1)(p_3 - p_2) = (p_3 - p_1)(p_3 - \bar{p}_1) = x_3^2$. The magnitude of the real term, irrespective of its sign is therefore

$$\frac{|B| \prod_{i=1}^{g} |p_3 - z_i|}{a_3 x_3^2 \prod_{h=4}^{f} |p_3 - p_h|} \cdot \exp\{-a_3 (n\pi + P - Z)/\omega_1\} \quad (9)$$

The magnitude of the principal mode maximum is, from (3),

$$\frac{|B| . \prod_{i=1}^{g} y_i}{r_1^2 \prod_{h=3}^{f} x_h} . \exp\{-\alpha_1 (n\pi + P - Z)/\omega_1\}$$

The ratio of real term to principal mode is therefore

$$\frac{\prod_{i=1}^{g} |p_3 - z_i|}{a_3 x_3^2 \prod_{h=4}^{f} |p_3 - p_h|} \cdot \frac{r_1^2 \prod_{h=3}^{f} x_h}{\prod_{i=1}^{g} y_i}$$
$$\exp\{-(a_3 - \alpha_1)(n\pi + P - Z)/\omega_1\}$$

This may be rearranged to give

$$\beta \left(\prod_{i=1}^{g} \gamma_i \right) \Big/ \left(\prod_{h=4}^{f} \gamma_h \right) \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

where $\beta = \dfrac{r_1^2}{a_3 x_3} \exp\{-(a_3 - \alpha_1)(n\pi + \lambda_3)/\omega_1\}$

$$\gamma_i = \frac{|p_3 - z_i|}{y_i} \exp\{(a_3 - \alpha_1)\, \rho_i/\omega_1\}$$

$$\gamma_h = \frac{|p_3 - p_h|}{x_h} \exp\{(a_3 - \alpha_1)\, \lambda_h/\omega_1\}$$

In the event of $\omega_1 t = n\pi + P - Z$ resulting in a minimum for the principal mode, the expression (10) for the ratio of real term to principal mode should be multiplied by $\exp\{-(a_3 - \alpha_1)\,\pi/\omega_1\}$.

The expression (10) is composed of the factor β multiplied by a quantity γ_i for each zero present and divided by a similar quantity γ_h for each pole present other than the principal poles and the pole at $-a_3$. The factors γ_i, in the case of real zeros, may also be written

$$\epsilon^{\rho_i \cot \lambda_3} . |\sin(\lambda_3 - \rho_i)| / \sin \lambda_3$$

and similarly for γ_h; thus γ_i may be expressed wholly in terms of λ_3 and the angle ρ_i for the particular zero z_i. Since the ratio given by (10) is a quotient of factors γ_i and γ_h, it is convenient to calculate the common logarithm of the factor as λ_3 and ρ_i vary. This is shown in Fig. 3 which gives lines of constant $\log_{10}\gamma$. From this it can be seen that as a real zero approaches the pole $-a_3$ (i.e., as ρ_i tends to λ_3) the real term due to that pole becomes vanishingly small. As the zero moves from a position slightly to the left of $-a_3$ to an infinite distance to the left, its effect on the real term becomes less and less, being zero at $\rho_i = 0$. To the right of $-a_3$, there is, however, a point on the real axis at which a zero will have no effect on the ratio (10). This is shown by the line having zero logarithm. The signs shown on the graphs of Fig. 3 are correct for zeros. For subsidiary real poles $\log_{10}\gamma$ must be given the opposite sign.

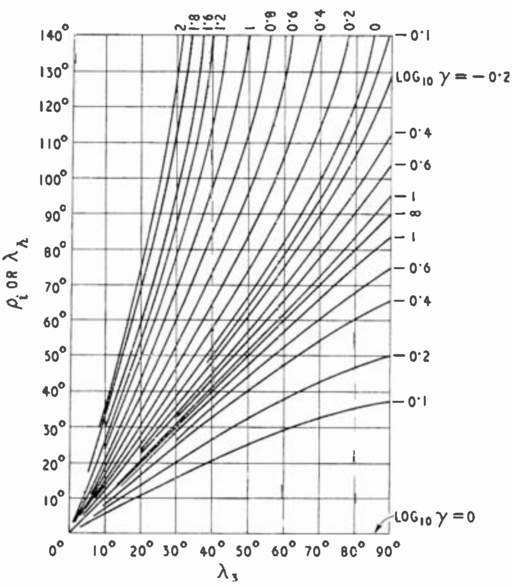

Fig. 3. *Contours of constant* $\log_{10} \gamma$ *for real poles and zeros.*

In the case of a pair of complex poles and zeros the factors are of the form $\gamma\gamma'$, the prime again referring to the pole or zero with the negative imaginary part. As it is clear that a series of figures similar to Fig. 3 would be required to show the variation of the factor for all positions of the complex pole or zero and for all values of λ_3, a practical difficulty arises. It is doubtful in this case if the added complexity brought about by using such a series of charts is worth the trouble, quite apart from the computation required.

The preliminary factor β is shown in logarithmic form in Fig. 4. The variation of $\log_{10}\beta$ with λ_3 is given for the two values of α_1/ω_1 and for the case $n = 1$. This case covers a great many practical cases which occur with both the poles and the zeros confined to the left half-plane.

The application of the above results is now illustrated by an example. Let us take the case of a transfer function having four poles and two zeros. Let it also be specified that in the overshoot of the response to a unit step, the value of the principal mode is to be 0·1; i.e., 10%, with the nearest real term contributing not more than ± 2%. In addition, no zeros are allowed in the right half-plane; i.e., non-minimum phase elements are excluded. Choosing one pair of complex poles to act as the principal pair, and taking the remaining poles and zeros real, let the normalized positions of $-\alpha_1 \pm j\omega_1$ be $-1 \pm j1$. Primed quantities will be used to denote the normalized positions, $a_3' = a_3/\omega_1$, etc. This gives the transfer function

$$\frac{r_1'^2 a_3' a_4' (p + b_1')(p + b_2')}{b_1' b_2' (p + a_3')(p + a_4')(p + 1 - j1)(p + 1 + j1)}$$

where $z_1 = -b_1$, $z_2 = -b_2$ and $r_1'^2 = 2$.

It remains to choose a_3', a_4', b_1', b_2' so that $\log_{10}$ (overshoot), given by equation (6), is equal to -1; also that $\log_{10}$ (ratio of real term/principal mode), given by $\log_{10}\beta + \Sigma \log_{10}\gamma_z - \Sigma \log_{10}\gamma_p$ according to (10), should not exceed $\log_{10} 0{\cdot}2$; i.e., $-0{\cdot}7$.

In the absence of any other restrictions the positions may be chosen fairly widely. A few trials and references to Figs. 2, 3 and 4 indicate the general trends which the pole and zero positions must follow. The principal mode restriction requires that $\Sigma\Delta_z - \Sigma\Delta_p = 0{\cdot}365$ if, in equation (6), $n = 1$. In the positions finally chosen, $\lambda_3 = 75°, a_3' = 1{\cdot}27$; $\lambda_4 = 40°, a_4' = 2{\cdot}2$; $\rho_1 = 95°$, $b_1' = 0{\cdot}91$; $\rho_2 = 50°$, $b_2' = 1{\cdot}83$. The logarithmic contributions to the overshoot, from Fig. 2 are 0·48, 0·153, 0·76, 0·238, all respectively, which gives $\Sigma\Delta_z - \Sigma\Delta_p = 0{\cdot}365$. To check the ratio (real term due to $-a_3$)/(principal mode), we have $\log_{10}\beta = -0{\cdot}34$, from Fig. 4 and, from Fig. 3, $\log_{10}\gamma$ is equal to $-0{\cdot}15$, $-0{\cdot}28$, $-0{\cdot}28$ for λ_4, ρ_1 and ρ_2 respectively. The logarithm of the ratio is therefore $-0{\cdot}34 - (-0{\cdot}15) - 0{\cdot}28 - 0{\cdot}28 = -0{\cdot}75$, which will satisfy the design objective. The resulting transfer function is therefore

$$\frac{B'(p + 0{\cdot}91)(p + 1{\cdot}83)}{(p + 1{\cdot}27)(p + 2{\cdot}2)(p + 1 - j1)(p + 1 + j1)}$$

with $B' = 3{\cdot}355$.

Since B' is positive, odd values of n in equation (4) give maxima of the principal mode. The first of these occurs at $\omega_1 t = \pi + \lambda_3 + \lambda_4 - (\rho_1 + \rho_2) = \pi - 30°$, i.e., at $t' = \omega_1 t = 2{\cdot}615$ sec. The response to unit step is

$$1 + 0{\cdot}533\epsilon^{-1{\cdot}27t'} + 0{\cdot}321\epsilon^{-2{\cdot}2t'} + 1{\cdot}917\epsilon^{-t'} \sin(57{\cdot}3t' - 104{\cdot}8)°$$

giving a first maximum of value 0·099, as against the required 0·1. At $t' = 2{\cdot}615$ the real terms have values 0·0193 and 0·00103 respectively. As the first term is 0·195 of the principal mode, the design objectives have been realized. Note that the value of the second real term may also be

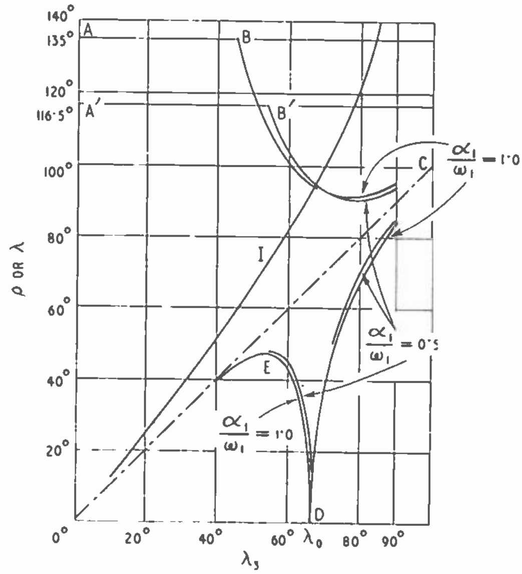

Fig. 5. *Permissible locations for ratio* $\left(\frac{\textit{real term}}{\textit{principal mode}}\right) \leqslant 0{\cdot}2$.

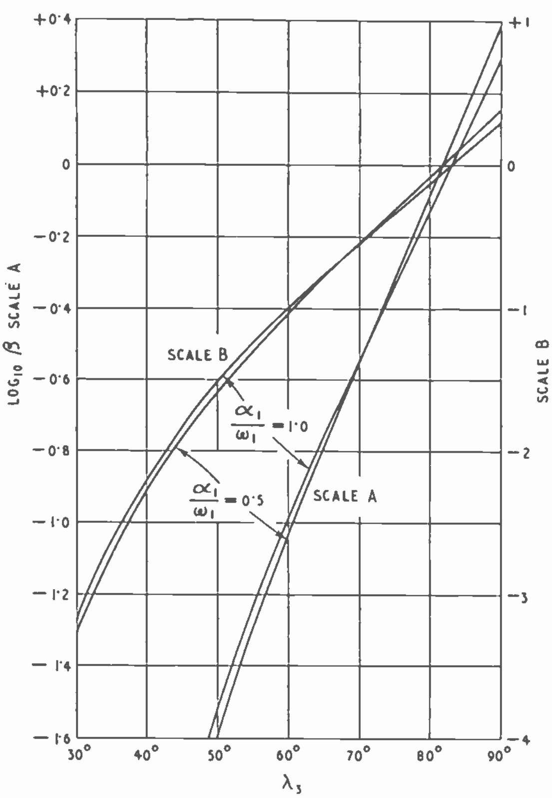

Fig. 4. Log_{10} $\left(\frac{\textit{real term}}{\textit{principal mode}}\right)$ *for one additional real pole.*

predicted from the curves if desired. The ratio in this case is

$$\text{antilog}\ (\log_{10}\beta + \Sigma\log_{10}\gamma_z - \Sigma\log_{10}\gamma_p)$$
$$= \text{antilog}\ (-2{\cdot}23 + 0{\cdot}95 - 0{\cdot}1 - 0{\cdot}6) = 0{\cdot}0105,$$

which gives the magnitude of the second real term equal to 0·00104. This, however, is quite negligible. The normalized response is shown by curve 1 of Fig. 6.

The actual time, as against the 'normalized' time, at which it is desired to achieve the maximum overshoot may be given. Suppose it is t_m. Then according to equation (4), ω_1 need only be chosen so that

$$\omega_1 = (n\pi + P - Z)/t_m$$

In this example $\omega_1 = (\pi + P - Z)/t_m$; i.e., $(\pi - \pi/6)/t_m$. If this ratio is k say, then the required pole and zero positions are simply k times the ones given above.

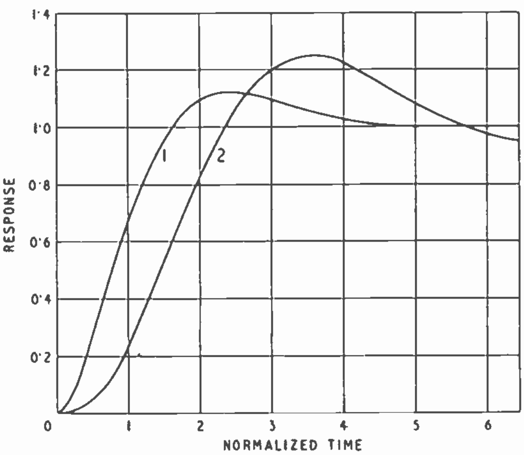

Fig. 6. *Response of examples worked out in the text.*

4. Permissible Locations of Poles and Zeros

The above method of estimating the magnitudes of real terms in the case of a general transfer function can be used in certain simple cases to define the area within which the subsidiary real poles and zeros may lie, in order that the relative magnitude of the least-damped term may not exceed a prescribed value. If, for example, the ratio of real term/principal mode at the time of principal mode maximum, is not to exceed 0·2, then in the case of one real pole and one real zero, $\log_{10}\beta + \log_{10}\gamma = -0{\cdot}7$, so that for a given position of the principal pole-pair, a relation between the pole and zero is established. Such a relation is shown by the curves BCD in Fig. 5, where the pole and zero positions are specified by the angles λ and ρ respectively. The curves are composed of two branches BC and DC which become asymptotic along the line $\rho = \lambda$. The point $\lambda_3 = \lambda_0$ is that given by $\log_{10}\beta = -0{\cdot}7$; for the case $\alpha_1/\omega_1 = 1$ this gives $\lambda_0 = 66{\cdot}5°$ and since $\log_{10}\gamma = 0$, then $\rho = 0$ or 94° from Fig. 3. Since $\log_{10}\gamma$ is negative for values of λ_3 greater than λ_0, two values of ρ are obtained, given by the two branches BC and DC of the curves. Values of λ_3 less than λ_0 give a positive $\log_{10}\gamma$ and hence only one value of ρ. Admitting only zeros in the left half-plane, restricts ρ to less than 116·5° and 135° in the cases $\alpha_1/\omega_1 = 0{\cdot}5$ and 1·0 respectively. Any value of ρ within and on the boundary ABCDO therefore, may be chosen for a given value of λ_3, and this will result in the ratio of the real term to principal mode being less than or equal to 0·2. A series of such curves corresponding to different ratios could be drawn and in this way a fairly large amount of information about the response could be obtained if desired.

Similar relations for the case of two real subsidiary poles and no zeros are given by the curves OED. Since λ_3 refers to the least damped pole, λ_4 must be less than λ_3 and reference to Fig. 3 will show that the condition curve can have only one branch in this case, namely that given by OED. Within the area OED any value of λ_4 corresponding to a second real pole position may be obtained.

To show similar conditions in the case of three subsidiary poles and zeros would require a family of curves for a particular ratio of the real term to the principal mode and it is doubtful if this is worth while in view of the fact that the general case can easily be calculated from Figs. 3 and 4, as in the illustrative example. A simple addition to Fig. 5, however, shows one possibility for the arrangement of the poles and zeros. This is given by the curve I which corresponds to $\log_{10}\gamma = 0$; i.e., the addition of a real pole or zero will leave the ratio real term/principal mode unaltered. The curve I is in fact the line marked zero in Fig. 3 redrawn on Fig. 5. This is a useful location to be remembered in synthesizing a transfer function.

For example, given two real poles and one real zero in addition to the principal pair of poles, let it be desired to design a transfer function giving about 25% overshoot in the step response and with the least damped real term contributing about a fifth of the overshoot. The least damped real term will in fact increase the overshoot (i.e., be positive) if an odd number of zeros lies to the right of this pole and decrease it if the number of zeros to the right of the pole is even. In the first place, therefore, let about 20% overshoot be allowed to the principal mode. The overshoot being greater than in the first example, the less damped principal pole pair position will do; i.e., take $-\alpha_1 \pm j\omega_1 = -0{\cdot}5 \pm j1$ in the normalized plane.

The procedure is much the same as in the first example except that the subsidiary pole and zero positions will be chosen to lie on the curves of Fig. 5; i.e., we shall be assured a ratio of real term/ principal mode of 0·2. Then with $\lambda_3 = 60°$, $\lambda_4 = 45°$ and $\rho_1 = 82{\cdot}5°$ according to the curves of Fig. 5, the increments Δ_p and Δ_z, from Fig. 2, are 0·26, 0·147 and 0·53 respectively, giving an overshoot of about 27·5%. This may be reduced by moving the zero to the left, say with $\lambda_3 = 55°$, $\lambda_4 = 48°$ and $\rho_1 = 74{\cdot}5°$. According to Fig. 2, this should give about 23% overshoot, which will be deemed near enough.

The transfer function is thus

$$\frac{B'(p + 0{\cdot}76)}{(p + 1{\cdot}2)(p + 1{\cdot}41)(p + 0{\cdot}5 - j1)(p + 0{\cdot}5 + j1)}$$

where $B' = r_1'^2 a_3' a_4'/b_1' = 2{\cdot}78$.

The normalized response is

$$1 + 3{\cdot}26\epsilon^{-1{\cdot}2t'} - 3{\cdot}34\epsilon^{-1{\cdot}41t'} + 1{\cdot}557\epsilon^{-0{\cdot}5t'}\sin(57{\cdot}3t' - 143{\cdot}8)°$$

The principal mode has a maximum value of 0·229; i.e., 22·9% when $t' = 3{\cdot}61$ seconds. The value of the real terms are then 0·043 and 0·021, or as fractions of the principal mode 0·19 and 0·092 respectively. Since the first is very nearly one fifth, the design restriction has been satisfied. The relative magnitude of the second real term can, if desired, be checked also by means of Figs. 3 and 4. The total overshoot is about 25%, as can be seen from Fig. 6, curve 2.

5. Application

The application of the foregoing theory to the synthesis of networks or more general structures with given transient performance is now illustrated by considering two simple examples. The first of these relates to the recovery characteristic of an automatic speed regulator, and the second to a network sometimes used as an interstage coupling in video amplifiers. It is in fact in the field of servo-mechanisms and pulse-reproducing circuits generally that the theory will be of some interest.

The procedure is to form the transfer function relating output to input, normalized in amplitude, if necessary, so that the zero-frequency gain is unity. This gives in general a quotient of the form

$$\frac{(d_0/c_0)(p^g + c_{g-1}p^{g-1} + \ldots + c_1 p + c_0)}{p^f + d_{f-1}p^{f-1} + \ldots + d_1 p + d_0} \quad \ldots \quad (11)$$

where the coefficients are made up of sums of products of the system constants. In terms of the normalized pole and zero positions, $-a_3'$, $-a_4'$, ... $-b_1'$, $-b_2'$ ... etc., the expression (11) is

$$\frac{B(p + \omega_1 b_1')(p + \omega_1 b_2') \ldots (p + \omega_1 b_g')}{(p + \omega_1 a_3')(p + \omega_1 a_4') \ldots (p + \omega_1 \alpha_1' - j\omega_1)(p + \omega_1 \alpha_1' + j\omega_1)} \quad \ldots \quad (12)$$

where $B = \omega_1^{f-g}(r_1'^2 a_3' a_4' \ldots a'_{f-2})/(b_1' b_2' \ldots b_g')$

The normalized pole and zero positions are now selected to give the desired transient performance in the manner already indicated and the transfer function (12) is then expanded. By equating coefficients of the same powers of p in (11) and (12), a set of relations is given from which the desired system parameters may be obtained.

Consider then Fig. 7, which shows a speed-regulating system. The recovery of the system, subsequent to the application of a disturbing torque to its output shaft, may be adequately investigated, as far as the oscillatory behaviour of the regulator is concerned, by analysing its performance when the 'demanded speed' is given a sudden small increase in its value. The system transfer function relating to output speed to the 'demanded speed' is therefore formulated. The immediate synthesis problem is to find out what proportions of voltage and current feedback should be used and the value of the time-constant of the feedback mesh in order that the response of the output should contain only one overshoot of limited size, say about 10%. This problem may be solved using Whiteley's[4] 'standard form' technique and as far as derivation of the unknown parameters is concerned, the process demonstrated below is much the same. The essential difference however is that, here, throughout the design process of manipulating the poles and

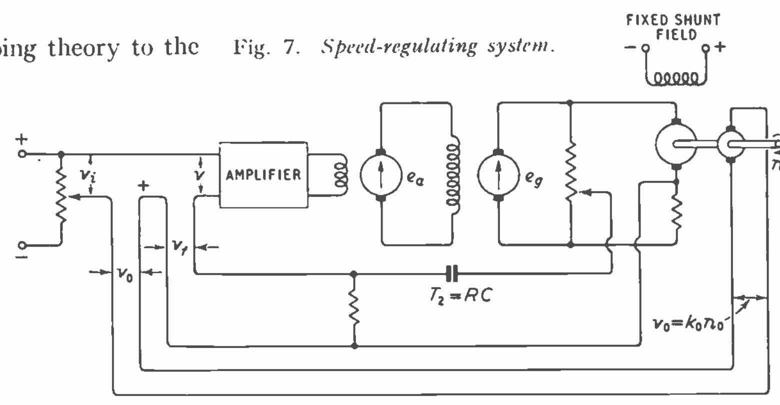

Fig. 7. *Speed-regulating system.*

zeros of the transfer function, the overshoot is known and may be arranged to take any magnitude from negligible values upwards.

If the relatively small time-constant of the amplifier output stage be neglected, there are two remaining time constants in the forward sequence. These occur in the exciter circuit and in the electro-mechanical relationship between the output speed and the generator e.m.f. Denoting these by T_1 and T_0, and ignoring the auxiliary feedback loop at the moment, we have the following relations:

$$v/k_0 = n_i - n_0$$

$$\frac{e_g}{v} = \frac{k_1}{1 + pT_1}$$

$$\frac{n_0}{e_g} = \frac{1}{k'(1 + pT_0)}$$

where n_i = demanded speed = v_i/k_0
n_0 = output speed
T_0 = $JR_a/k'k''$ = output time constant
J = total inertia
R_a = armature circuit resistance
k', k'' = back e.m.f. and torque constants of motor.

The constants J, k', k'', k_0 all refer, of course, to the same shaft. The transfer function relating n_0 to v is therefore

$$\frac{K/k_0}{(1 + pT_1)(1 + pT_0)}$$

where $K = k_0 k_1/k'$. K is in fact the static gain of the system and requires to be fixed from considerations of steady-state accuracy. The value of K so determined is usually large enough to cause a very poorly damped train of oscillations in the recovery characteristic subsequent to a disturbance. To remove this defect the further feedback loop shown is necessary. The input voltage to the feedback loop consists of a part proportional to the output speed and a part proportional to the output acceleration. It is thus of the form $k_2(1 + \alpha p)\, n_0$ where α is the ratio of acceleration feedback to velocity feedback. The transfer function of the CR network is $pT_2/(1 + pT_2)$, where $T_2 = CR$; the transfer function of the added feedback loop is therefore

$$\frac{v_f}{n_0} = \frac{pT_2 k_2 (1 + p\alpha)}{(1 + pT_2)}$$

The resulting transfer function of the complete system is then

$$\frac{n_0}{n_i} = \frac{x(p + 1/T_2)}{p^3 + x\alpha p^2 k_2/k_0 + x(1 + k_2/k_0)\,p + x/T_2} \quad \ldots \ldots \ldots (13)$$

where $x = K/T_1 T_0$. In deriving the above result several straightforward algebraic steps are required and those terms not containing the gain factor K have been neglected in comparison with those containing K. This is quite permissible for large values of K. Here the following values are taken for the system constants:—$T_1 = 0{\cdot}5$ sec, $T_0 = 1$ sec, $K = 500$, $k_0 = 1{\cdot}6$; hence $x = 1{,}000$.

The transfer function (13) may also be written

$$\frac{n_0}{n_i} = \frac{\omega_1{}^2 r_1'^2 a_3'}{b_1'} \cdot \frac{(p + \omega_1 b_1')}{(p + \omega_1 a_3')(p + \omega_1 \alpha_1' - j\omega_1)(p + \omega_1 \alpha_1' + j\omega_1)} \quad (14)$$

and it remains to choose the normalized positions of the poles and of the zero so that the overshoot is limited to 10%. This aspect has been dealt with in Sections 3 and 4 of the paper. The normalized principal pole-pair position at $-1 \pm j1$ will do quite well and give sufficient damping. No conditions restrict the real pole and zero other than the overshoot requirement and the fact that the real term should not contribute much to the overshoot. If we take $a_3' = 2$, then since $\Delta_z - \Delta_p = 0{\cdot}365$ (for $\log_{10}$ overshoot $= -1$), $b_1' = 1{\cdot}14$ from Fig. 2. Also, since $\rho_1 = 82°$ and $\lambda_3 = 45°$, their intersection in Fig. 5 lies within ABCDO and certainly the real term will be less than 0·2 of the principal mode.

Expanding (14) therefore

$$\frac{n_0}{n_i} = \frac{3{\cdot}51\,\omega_1{}^2 (p + 1{\cdot}14\,\omega_1)}{(p_3 + 4\omega_1 p^2 + 6\omega_1{}^2 p + 4\omega_1{}^3)} \quad \ldots \quad (15)$$

and equating the coefficients in the above to those in (13),

$$x = 3{\cdot}51\omega_1{}^2$$

$$x/T_2 = 4\omega_1{}^3$$

$$x(1 + k_2/k_0\lambda) = 6\omega_1{}^2$$

$$x\alpha\, k_2/k_0 = 4\omega_1$$

Hence $\omega_1 = 16{\cdot}9$ and the required values of k_2, T_2 and α are 1·137, 0·052 sec, and 0·095 respectively. This time of maximum overshoot, from equation (4), with $n = 1$, is 0·148 sec and the output is

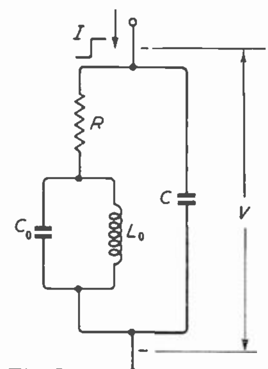

Fig. 8.
Video-coupling network.

$$1 + 0{\cdot}754\,\epsilon^{-33{\cdot}8t} + 1{\cdot}767\,\epsilon^{-16{\cdot}9t} \cdot \sin(16{\cdot}9t - 1{\cdot}71)$$

This is shown in terms of the normalized time $t' = \omega_1 t$ by the curve 1 of Fig. 9.

The second example, which concerns the response of the circuit in Fig. 8 to a unit step of current, has been chosen to display certain other points of interest in the application of this method of synthesis. In this case the transfer function relating the output to the input is the impedance function of the network. If this is amplitude-normalized to give a zero-frequency transmission equal to unity, we have $V_0 = V/R$ and the resulting transfer function is

$$\frac{V_0}{I} = \frac{1}{T} \cdot \frac{p^2 + p/RC_0 + \omega_0{}^2}{p^3 + (1/RC_0 + 1/T)\,p^2 + \omega_0{}^2 p + \omega_0{}^2/T} \quad \ldots \ldots \ldots (16)$$

where $T = RC$ and $\omega_0^2 = 1/L_0C_0$. This may also be written

$$\frac{V_0}{I} = \frac{\omega_1 r_1'^2 a_3'}{b_1'b_2'} \cdot \frac{(p + \omega_1 b_1')(p + \omega_1 b_2')}{(p + \omega_1 a_3')(p + \omega_1 \alpha_1' - j\omega_1)(p + \omega_1 \alpha_1' + j\omega_1)} \quad (17)$$

the primed quantities denoting the normalized pole and zero positions as before. It is desired to evaluate L_0 and C_0 of the compensating reactance in order that the step-response should have as small an overshoot as possible. In particular for multi-stage video amplifier applications only about 1% overshoot might be permitted, but slightly more would be possible for a single-stage amplifier. For small overshoots the normalized principal pole-pair position at $-1 \pm j1$ requires to be taken. From equation (6) and Fig. 2 it is concluded that a real pole position near to the principal pole would be necessary for small overshoots. Inspection of the transfer function (16), however, indicates that certain other conditions require to be fulfilled in selecting the values of a_3', b_1' and b_2', due to relations between the coefficients of the numerator and denominator of (16).

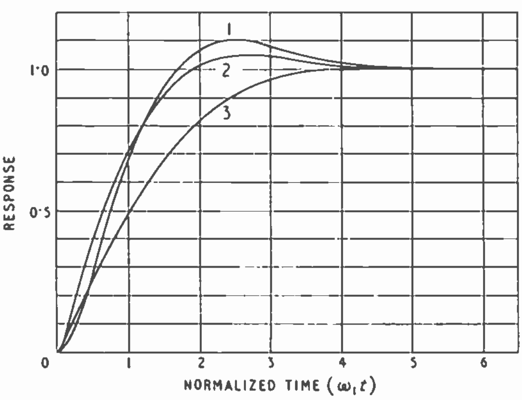

Fig. 9. *Response curves of examples considered in the text. Curve 1 is for the speed regulator of Fig. 7, curves 2 and 3 refer to a video coupling under different conditions.*

Expanding (17), the conditions are

$$2 + a_3' - (b_1' + b_2') = \frac{2a_3'}{b_1'b_2'} \quad .. \quad (18)$$

$$2 + 2a_3' = b_1'b_2' \quad .. \quad .. \quad (19)$$

The simplest procedure is to assign the magnitude of one of the zeros and derive the remaining values. Taking $b_2' = 4$ will not greatly increase the overshoot and this results in $b_1' = 3{\cdot}87$, $a_3' = 6{\cdot}74$. From equation (6) and Fig. 2, the logarithmic overshoot will be $-1{\cdot}365 + (2 \times 0{\cdot}08) - 0{\cdot}01 = \bar{2}{\cdot}705$; i.e., about 5%. The required values of L_0 and C_0 are obtained from the transfer function

$$\frac{0{\cdot}87\omega_1 (p^2 + 7{\cdot}87\omega_1 p + 15{\cdot}48\omega_1^2)}{p^3 + 8{\cdot}74\omega_1 p^2 + 15{\cdot}48\omega_1^2 p + 13{\cdot}48\omega_1^3}$$

by equating the coefficients with those of (16). In terms of R and C which are the nominally fixed parameters, this gives $L_0 = 0{\cdot}442\,CR^2$, $C_0 = 0{\cdot}11\,C$.

This example differs from the previous in two main respects. First, additional restrictions such as (18) and (19) appear, and secondly, the overshoot requires to be much smaller, in fact practically speaking, non-existent. Concerning the first point, one or more additional conditions may enter the problem depending on the relationship of the adjustable parameters of the system to those that are not adjustable. Although the conditions are not difficult to handle, the question now arises as to what form of structure should be added to a network or system (for the purposes of compensation of transient reponse) in order to have freedom of adjustment of a sufficient number of coefficients. Conversely there is no reason to add components to a system if an acceptable performance can be obtained with fewer components. This problem is obviously closely connected with the relative numbers of energy-storage and energy-dissipative elements in the system. Although outside the scope of the present paper there is no doubt that this would form an interesting and profitable investigation.

Concerning the second point, space precludes a full discussion of the procedures which are available to reduce the overshoot to smaller values. Three approaches are indicated, however, one of these being adopted for illustrative purposes with reference to the second example.

In the first place, the principal pole-pair may be moved to the left, say to a normalized position at $-2 \pm j1$. Additional curves for this pole-pair position and others may be entered on Fig. 2. Equation (6) indicates that this more damped position will certainly give smaller overshoots.

Secondly, a design may be worked out in which the nearest real term is arranged to oppose the principal mode, the actual overshoot being the difference of the two. From equation (7), taking the case when there are no zeros in the right half-plane (i.e., B is positive) the term due to the pole at $-a_3$ will be negative if there are only complex zeros or an even number of real zeros to the right of $-a_3$. This is the required distribution if the least damped real term is to oppose the principal mode. Under these conditions we can see, in fact, from Fig. 4, neglecting other poles and zeros, that values of λ_3 approximately equal to 80° (i.e., a_3' approximately equal to 1·2 and 0·65 for $\alpha_1' = 1{\cdot}0$ and 0·5 respectively) will give a real term equal in magnitude to the principal mode. This is a valuable and widely applicable method of reducing the overshoot to negligible proportions.

Thirdly, the use of complex zeros having imaginary parts somewhat greater than the imaginary part of the principal pole provides overshoot factors which result in greatly reduced overshoots. This is a fairly general result as long as the real parts of the zeros are of the same order as the real part of the principal pole. A chart covering the region of the complex plane in the vicinity of the principal pole is undoubtedly desirable although additional complexity results. If we revert to the second example, from which conditions (18) and (19) resulted, and assume a value of 1·3 for a_3', b_1' and b_2' take the complex values $-1{\cdot}368 \pm j1{\cdot}65$. The overshoot factors defined by equation (5) may be calculated for each of these zeros. $\text{Log}_{10}\, \delta_i$ is in fact 1·858 and 0·717 for b_1' and b_2' respectively, while $\rho_1 + \rho_2 = 6{\cdot}662$ radians. Hence the logarithmic overshoot is, from (6),

$$-3\pi\,(\alpha_1/\omega_1)\log_{10}\epsilon + \Sigma\Delta_z - \Sigma\Delta_p$$

That is, $-4{\cdot}095 + 2{\cdot}575 - 0{\cdot}46 = \bar{2}{\cdot}02$; i.e., 1·05% overshoot. The relative magnitude of the real term may be found according to the method of Section 4, but as it is negative in any case it is bound to decrease the overshoot. Application of the theory, however, shows it to be about 0·63 of the magnitude of the principal mode. The total predicted overshoot should thus be 1·05% − 0·6% = 0·4% which is, practically speaking, non-existent. The required values of L_0 and C_0 are obtained from

$$\frac{0{\cdot}565\omega_1\,(p^2 + 2{\cdot}735\omega_1 p + 4{\cdot}6\omega_1^2)}{p^3 + 3{\cdot}3\omega_1 p^2 + 4{\cdot}6\omega_1^2 p + 2{\cdot}6\omega_1^3}$$

by equating the coefficients with those of (16). This gives $L_0 = 0{\cdot}336\,CR^2$ and $C_0 = 0{\cdot}206\,C$. The normalized response is $1 - 1{\cdot}086\epsilon^{-1{\cdot}3t'} + 0{\cdot}762\epsilon^{-t'}\sin(t' + 3{\cdot}025)$ shown by curve 3 in Fig. 9. Curve 2 shows the normalized response for the previous design.

The applications given have been discussed at some length in order to present a full statement of the problem. With some familiarity with the method, designs may be worked out fairly rapidly. Again, although the examples refer only to cubic and quartic equations, no difficult situation arises for higher-order systems, particularly if the structure can be formed to yield a sufficient number of independently adjustable coefficients.

6. Conclusion

In designing transfer functions such that their step responses give overshoots composed largely of the principal mode of oscillation, a method has been stated for the rapid calculation of the magnitude of any real term. Given the ratio of the real term to the principal mode, it has been shown how to realize a transfer function with this property and having a prescribed overshoot in the step-response. For certain simple cases the permissible locations of the subsidiary poles and zeros have been shown graphically.

The application of the method as a means of improving transient response has been illustrated by several examples and a discussion given of the types of situations that are likely to arise. The method is widely applicable. For design purposes all the information concerning the principal mode of oscillation and any other real term may be condensed into three charts.

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D.S.I.R. IONOSPHERIC ABSORPTION MEASURING EQUIPMENT

By W. R. Piggott, O.B.E., B.Sc.

(Official communication from D.S.I.R. Radio Research Station, Slough)

SUMMARY.—The measurement of ionospheric absorption involves employing very stable and linear r.f. pulse-measuring equipment. A brief description of the design of suitable equipment is given. Similar units have given satisfactory service for more than ten years.

1. Introduction

THE regular measurement of the absorption of radio waves in the ionosphere was started in south-east England by Professor (now Sir Edward) Appleton at the beginning of 1935[1]. During the war the work was transferred to the Radio Research Station, Slough, and the equipment redesigned and duplicated. About fifteen absorption-measuring receivers have been built since 1941 and used for a number of different purposes which demand a flexible and stable pulse or c.w. measuring unit with an abnormally wide and linear dynamic range. The techniques employed have always been very simple and flexible and may, therefore, be useful to others interested in making similar measurements. This article is restricted to a discussion of the main design factors and the difficulties which have arisen during the last twelve years.

2. Principles of Ionospheric-Absorption Measurement

The absorption present in a trajectory through the ionosphere varies with ionospheric structure, frequency and time. For practical purposes it usually lies between the limits of 1 db and 70 db ($0{\cdot}1 - 8\ n$). Small changes in the absorption must often be measured accurately since the corresponding losses are much greater at oblique incidence. This is only possible if the stability of the equipment is very high. It is also necessary to be able to separate and measure the multiple reflections from the ionosphere and desirable to separate the ordinary and extraordinary wave components.

The normal method of measurement (Fig. 1) involves radiating a pulse signal, the amplitude of which is constant with time and does not vary rapidly with frequency. The reflected wave is received on a stable aerial system, which may be designed to select one component, and the resultant signal fed to a special receiver. The output is displayed on a conventional cathode-ray tube display unit which is used as a voltmeter. This enables both the delay time and amplitude of the reflected signals to be measured. The effective amplification between the aerial and the c.r.t. can be altered by means of attenuators in the input of the receiver and between the frequency changer and i.f. amplifier. Thus the amplifiers are always operating under similar conditions when a measurement is being made.

A full description of the method of analysis of absorption measurements has recently been given elsewhere[2]. This also describes the specialized techniques for measuring very small absorptions in the presence of large fluctuations (fading) due to irregularities in the ionosphere.

3. Transmitters

The transmitters have been designed to take advantage of the fact that it is much easier to

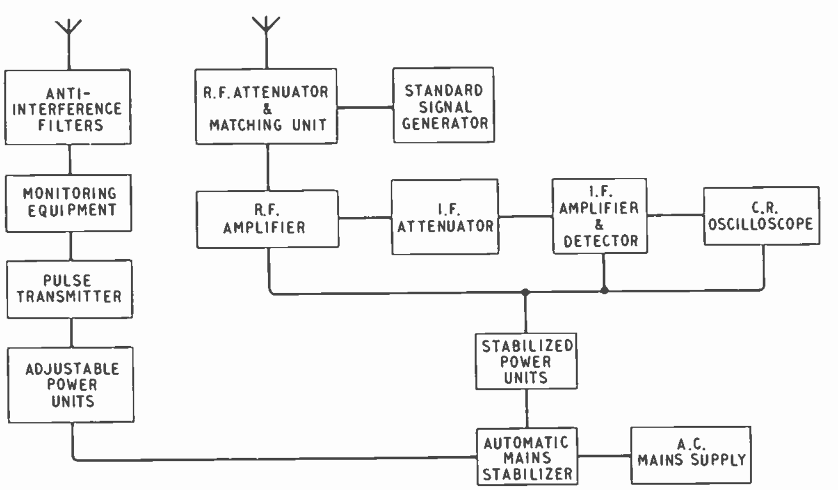

Fig. 1. *Block diagram of typical absorption measuring equipment.*

MS accepted by the Editor, July 1954

maintain a radio-frequency pulse at constant amplitude in a fixed load than to measure the peak power delivered. Very simple mains-operated pulse transmitters have proved to be adequate for routine measurements on fixed frequencies provided that the power supplies can be stabilized or adjusted to give constant outputs. It is very convenient to provide a variable pulse duration adjustment to enable the best compromise between height discrimination and dispersion to be found. This should be variable between about 50 microseconds and 500 microseconds. A peak voltmeter to monitor the output voltages between the aerial transmission lines and earth is also essential.

The main features of the circuit diagram for a simple transmitter are shown in Fig. 2. The frequency range 1·0–10·0 Mc/s can be covered by using four switched coils, the output from which can be fed into one or more aerial systems. A peak power output of about 0·5 kW is adequate for manual observations, though more power is desirable when using automatic methods of measurement.

Equally successful push-pull transmitters have been designed with two type 813 valves used as tetrodes; similar output powers are obtained. The power output can easily be increased by using aperiodic power amplifiers.

4. Transmitting Aerials

Three types of aerials have been used successfully with this type of transmitter:—

(i) two concentric vertical rhombic aerials;
(ii) two delta aerials similarly mounted;
(iii) two horizontal terminated folded-dipole aerials.

The first two systems are particularly useful[3] when the equipment is to be used for many different purposes but they must be constructed with some care to prevent changes in their size and shape or interaction with other aerial systems. In general, both the polar diagram of the aerial and the impedance presented to the transmitter vary slowly with frequency. These systems radiate a significant amount of vertically-polarized energy and therefore set up considerable ground-wave fields which may cause interference locally on frequencies up to the television bands (40–50 Mc/s). This disadvantage can be minimized by using horizontal terminated folded-dipoles, and carefully screening the transmitter.

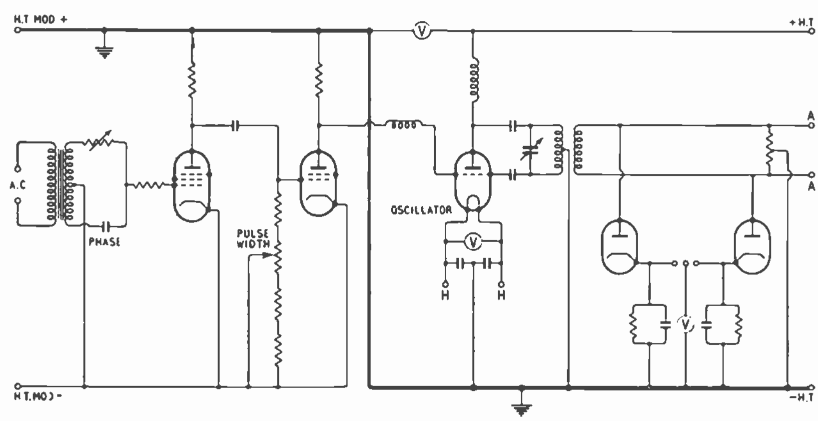

Fig. 2. *Basic form of the transmitter.*

These aerials are also stable and easily coupled to the transmitter but the bandwidth available is less than that for the other types. In practice the optimum choice of aerial is usually controlled by the local conditions at the transmitting station.

5. Receivers

The receivers used have been specially designed for accurate pulse-amplitude measurements. The design is based on that of a conventional superheterodyne receiver without automatic gain control, modified to give an abnormally high overall amplification and dynamic range and to possess great inherent stability. The receiver also includes standard attenuator units for measuring the relative strengths of the input signals.

5.1. *R.F. Amplifier*

For the frequency bands normally employed in absorption measurements, 0·5 Mc/s to 10 Mc/s, the absolute noise sensitivity of the receiver is unimportant: the amplitude of the aerial noise and interference being many times the set noise. Thus it is convenient to sacrifice some signal in order to isolate the tuned circuits of the r.f. amplifier from detuning effects due to changing aerial impedances and also to provide a constant input impedance into which an r.f. attenuator can be matched with reasonable accuracy. Several alternative methods can be used, each of which has its particular advantages and disadvantages. A very simple solution is to provide a 10-db or 20-db resistive pad in the input circuit of the receiver. In our receivers, electronic isolation has been used successfully, the first circuit consisting of one, or more, wideband transformers feeding into an earthed-grid amplifier, the output of which is coupled to the main r.f. amplifier. This enables the signal-to-noise ratio to be maintained up to about 40 Mc/s and thus increases the range of application of the receiver.

Both capacitance and inductance trimming of the r.f. and oscillator circuits are necessary so that accurate tracking can be achieved; otherwise small changes in the circuits can alter the total amplification. In addition, each r.f. circuit is provided with a variable trimmer capacitor controlled from the front panel so that it is always possible to confirm that the alignment is maintained or to correct for any small residual errors. It is most important that the output of the oscillator should be great enough on all frequencies to prevent changes of amplification with oscillator output.

Since the receiver is used as a standard r.f. microvoltmeter, the amplification must remain constant and be independent of the type of signal present. This condition is most easily satisfied when the amplifiers are operated at constant gain and the effective amplification is altered by inserting passive attenuating networks.

Each stage should be designed to have maximum stability and a linear input-to-output curve over the widest range of signal strengths. In particular, it is important that the d.c. grid potentials should be independent of signal modulation and amplitude; otherwise the receiver sensitivity will change with the signal strength of both the wanted signal and any unwanted signals present, and the input to output relation will be non-linear. Provided that the screen-grid potentials are stabilized by potentiometers connected to the cathodes it is possible to adjust the relative potentials so that the cathode current is almost independent of signal amplitude even when the stage is grossly overloaded. This adjustment involves an appreciable loss in the maximum available stage gain. However, distortion due to the time constants of decoupling circuits is minimized, and may be made negligible without danger from instability due to insufficient decoupling even with an i.f. gain of 130 db ($3 \times 10^6 : 1$) at 500 kc/s. Correct adjustment may easily be confirmed by feeding a pulse signal from the transmitter into the receiver in parallel with a weak c.w. signal from a signal generator. No change of sensitivity before and after the pulse should be detectable. This test must be done at several frequencies in all bands using pulse signals at least as large as the largest signal likely to be received in practice. A pulse amplitude to c.w. amplitude ratio above 140 db ($10^7 : 1$) is usually adequate for this purpose. With certain valve types, e.g. EF50, this mode of operation minimizes the change of stage gain with the maximum mutual conductance of the valve so that, in practice, the stage gain is constant throughout the major part of the life of the valve. This greatly simplifies the problem of keeping the receiver sensitivity constant over long periods.

The independence of stage gain and signal strength also simplifies measurement when the signal-to-noise or signal-to-interference ratios are small.

5.2. *Attenuators*

The attenuator system should provide a continuous range of about 160 db ($10^8 : 1$), though, if it is not desired to measure the direct signal from a transmitter in close proximity, 100 db ($10^5 : 1$) will be adequate. In order to avoid screening problems it is desirable to divide the attenuator into two units, one operating at r.f. and one at i.f. The former can also be used to prevent overloading the frequency-changer stage when the mean signal strength is high, thus greatly increasing the range of amplitudes which can be measured conveniently.

In the latest receivers, the r.f. attenuator consists of a lattice of 10 db (3·16 : 1) switched units using non-inductive resistance elements. Since it is necessary to measure the mean amplitudes of fading signals, red and green pilot lights are switched on to identify odd and even steps and the variable i.f. attenuator has two scales so that amplitudes can be measured directly without multiplying by 3·16. This additional complication is justified since the maximum to minimum ratio of a fading signal is often about 10 : 1 and it is desirable that the ratio given by the continuously variable attenuator should not exceed 20 : 1. Otherwise frequent switching of the preset 20-db steps is necessary when the mean amplitude has values near the ends of the range of the continuously adjustable unit.

The i.f. attenuator is in the form of a constant capacitance network coupling the two circuits of the first i.f. transformer. Provided that the maximum coupling is less than critical (this implies an insertion loss), decreasing the coefficient of coupling causes a proportional decrease in the voltage built up across the secondary circuit. The low-impedance elements of the circuit can be designed to work into a screened transmission line joining the r.f. and i.f. units, thus giving a convenient coupling unit. The main reasons for using this technique, however, are that even the best quality wirewound variable potentiometers will not withstand the wear involved in making large numbers of observations, recalibration is slow and tedious, and well-designed variable capacitors last almost indefinitely.

The scale of the continuously-variable attenuator can be arranged to give adequate reading accuracy by cutting some of the capacitor plates, care being taken to avoid warping or mechanical weakness. Suitable padding capacitors are used to restrict the variation to 24 db (16 : 1). The second variable balancing capacitor is then trimmed so that the input capacitance of the network (Fig. 3) remains constant when both are coupled mechanically. The attenuator is completed by switched 20-db and 40-db constant capacitance π networks. These may be made to be both accurate and stable by selecting suitable capacitors and can, therefore, be used as internal reference standards for checking the accuracy and stability of the continuously variable and r.f. attenuator units. This is a valuable feature which greatly reduces the difficulty of maintaining high relative accuracy over a wide range of amplitudes.

frequency signals at the output, the main residual high-frequency components being at twice the intermediate frequency. With suitable construction and proper decoupling of heater and h.t. supplies, it is very easy to make the amplifiers completely stable even when the input and output leads are unscreened. Operation for an initial aging period of about 100 hours is required to enable the components to reach equilibrium before the amplifiers are finally adjusted and calibrated. The overall amplification then remains constant until the valves reach the end of their useful life. It is, of course, essential that high-grade components are used, that their operating conditions are very conservative, and that no attempt is made to restrict the size of the units.

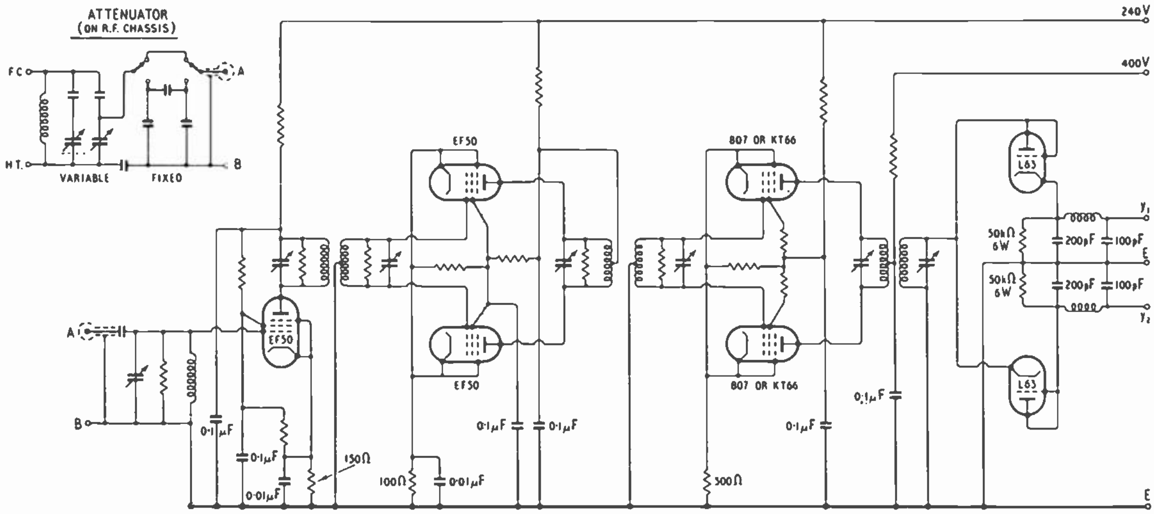

Fig. 3. *Attenuator and i.f. amplifier showing main points of design.*

5.3. *I.F. Amplifier and Detector*

The i.f. amplifier and detector unit is also designed for maximum stability and minimum distortion from decoupling circuits. Long-term stability is maintained by using critically-coupled i.f. transformers. The overall i.f. amplification when operating with a 12-in. c.r.t. should be about 130 db (3×10^6), the last stage being a push-pull power amplifier feeding into a voltage-doubler detector. The load resistance of the detector must be kept small to minimize distortion and hence the large peak-to-peak voltage desirable requires a considerable power output. Excellent results can be obtained using two type 807 valves, up to 1,000-V peak-to-peak undistorted output being available, the exact value depending on the h.t. applied to the anodes of this stage. This system also gives an extensive linear dynamic range without distortion of the pulses. The symmetry of the output stages restricts the amount of unwanted intermediate-

In practice, we find that the majority of faults occur in the waveband switches. Using miniature techniques it would be worth while to replace these with a separate radio-frequency amplifier for each waveband.

When properly constructed, receivers of the type described above possess a day-to-day stability in sensitivity comparable with that of a good commercial signal generator. Nevertheless, a programme of regular comparisons with a signal generator is desirable to detect deterioration in the valves or other components. These measurements should be made at least once a week if the equipment is used for routine investigations. Small fluctuations which could be attributed either to the signal generator or to the receiver should be ignored.

5.4. *Display Units*

The display unit is conventional in design except that, as the cathode-ray tube is used as a standard voltmeter, it is necessary to stabilize the high-tension supplies. Simple linear time bases are used giving a limited range of recurrence rates, but a considerable range of velocities and high resetting accuracy. Adjustable delays can be used so as to move the signal to any part of the time-base trace. The range calibration is conventional, the trace being temporarily blacked out at intervals corresponding to height changes of 25 km. Eye strain is minimized by using large tubes, 9-in. and 12-in. diameter being the most convenient sizes.

5.5. *Power Units*

All power units throughout the equipment are roughly stabilized using simple, conventional circuits and, in addition, the main supply to both transmitting and receiving units is controlled by a commercial automatic voltage stabilizer.

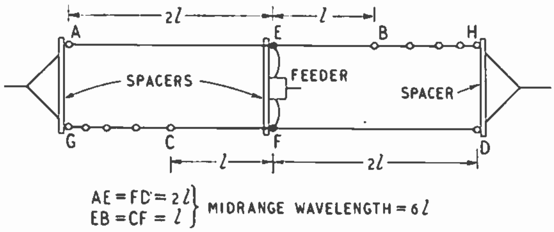

Fig. 4. *Double doublet aerial system. The points EF are joined to either (a) spaced twin feeder about* 300–500-*ohms impedance or (b) wideband screened transformer to screened transmission line. AG, EF and HD are spacers of width about one metre or more. The feeder is suspended from EF.*

6. Receiving Aerials

The signal-to-noise ratio of the receiving-aerial system is usually more important than its pick-up factor. Hence, for the particular conditions found in vertical-incidence sounding, the types of aerial which are best for transmitting are seldom ideal for reception and a considerable gain of sensitivity can be obtained by using systems which suppress interfering signals. Two types of systems have been used for vertical-incidence absorption measurements:

(i) double-doublet systems,

(ii) terminated folded-dipole systems.

Both types can be balanced accurately so as to suppress the vertically-polarized interfering signals which largely control the available sensitivity.

The former system, Fig. 4, has great advantages when applied to multi-frequency observations, since its intrinsic pick-up, apart from the effects of the reflections in the ground, is almost independent of frequency over a three to one range. Terminated folded dipoles can also be used though the sensitivity changes are larger. They are easier to construct and couple efficiently to a transmission line.

Careful adjustment of the balance of either system usually gives a considerable increase in the operational signal-to-noise ratio of the equipment which otherwise could only be obtained by using high-power transmitters.

The aerial systems are most difficult to calibrate and control and should therefore be built carefully and kept away from less stable systems.

Two exactly similar aerials of either type, mounted at right angles to each other, can be used with phase-shifting equipment to give a circularly-polarized receiving system, though folded dipoles give a simpler mechanical construction. Variants of the circuits discussed by Phillips[4] have proved very satisfactory for this purpose. Owing to a mistake in the original reference, Phillips' asymmetrical coupling circuits are incorrect and physically impossible ($C_1/C_2 = S^2$ in his Fig. 6 should be $C_2/C_1 = S^2$).

One of the best practical methods of obtaining an index which is sensitive to receiving aerial changes is to measure the residual noise or, less accurately, interference, present in the middle of the day. With properly-balanced horizontal aerials this noise level is very sensitive to small changes in aerial shape or construction and is therefore a good index of mechanical and electrical stability. It is, of course, necessary to analyse the data obtained on several frequencies and several days before a change in the aerials can be detected with certainty. Fortunately it is easy to construct aerials whose sensitivity at vertical incidence is very insensitive to mechanical alterations so that it is only necessary to detect major faults. Direct measurements of the ground pulse intensity may sometimes be used provided exhaustive experiments in the site have shown that they are reliable. This index may be misleading since the very large signals can cause non-linear effects locally.

The maintenance of a constant r.f. output from a pulse transmitter presents serious difficulties since few of the simple methods of monitoring the output give consistent results over long periods. The best procedure appears to be to stabilize the transmitter power unit as carefully as possible, to operate under conditions for which the output power is least sensitive to changes of applied voltages and to valve aging, and to monitor the output in several ways. The use of a peak voltmeter to measure the voltage between each feeder line and earth, or between the feeders, is valuable provided the equipment is stable. The readings obtained are very sensitive to the exact physical arrangement, however, and, owing to standing waves in the feeder, may change appreciably without significant changes in the true output power. Thus it is always necessary to confirm that any changes found are real before using this type of measurement as an index of the power radiated. Comparisons with other parameters, e.g. anode current, or h.t. voltage, will usually show whether the changes observed are likely to be significant. These problems can be greatly diminished if two independent equipments are available which can be interchanged.

Acknowledgments

The design of this equipment has been greatly influenced by many past and present members of the staff of the Radio Research Station, Slough. The mechanical construction and layout, which is an important element in the success of the units, is largely due to Mr. A. G. Wilson.

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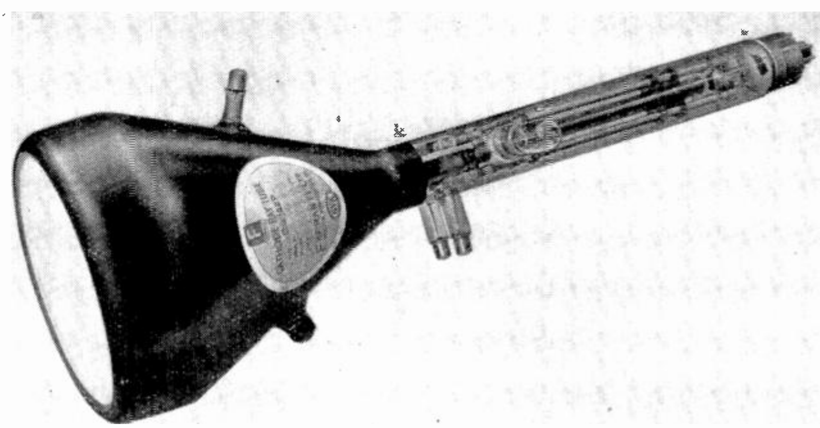
PHYSICAL SOCIETY'S EXHIBITION

THIS year's exhibition marked a break with tradition; for the first time it was not held at Imperial College as the previous thirty-eight have been. In its new home at the New Horticultural Hall, the 39th exhibition lost a good deal of its distinctive character. It was larger this year, with some 35% more exhibitors; but, probably because of this, it seemed even more cramped.

The effect of the lack of space was most evident upon the research exhibits, which do need quite a lot of room for their proper display, and it is just these exhibits which have, in the past, given the Physical Society's Exhibition its unique place. This year, the research items were mainly confined to physically small apparatus; transistors were more appropriate than electron microscopes!

In the review of last year's exhibition, it was said that the character of the exhibition was changing, and then in a way which we rather welcomed, for the electronic aspects seemed to be dwindling somewhat and taking on more their proper proportions in the realm of physics. This change was apparently only a temporary one, for this year electronics predominated once more. Optics, chemistry and nucleonics were there, but only to a small extent.

The field of semiconductors is one in which a great deal of work is being carried out. The emphasis has, this year, shifted somewhat from germanium to silicon and from the transistor to the diode. Considerable development is taking place in applying silicon to power diodes. B.T.-H. were showing types having only 1/1000 of the reverse current of a germanium diode and which will operate up to 200°C. They have a very sharp and controllable breakdown voltage.



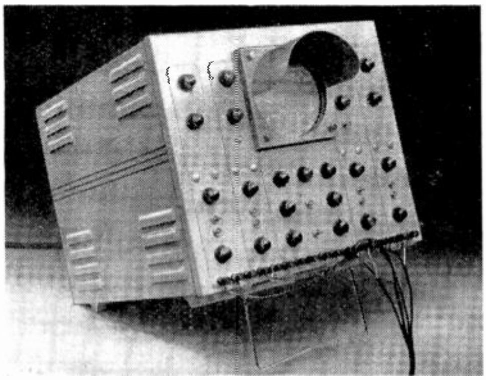
Ferranti 06/20P c.r. tube; the Y-plate 'horns' are on the same side of the tube neck.

S.T.C. demonstrated the Zener effect in silicon diodes by applying an input varying up to 100 V and showing that the diode voltage closely followed the input up to 36 V but, after that, remained substantially constant. The effect may have application in voltage regulation but, in the example shown, was limited by the permissible dissipation of only 50 mW in the diode used.

In general, the exhibits were more devoted to applications than to fundamentals although, on the G.E.C. stand, several internal effects were pointed out, including 'hole-storage'. Mullard demonstrated a field transistor, operating at 100 kc/s, in a Colpitts' oscillator circuit. This transistor appeals in being nearer to the thermionic valve than some of the other kinds, for it has a positive 'anode' and a negative 'grid'. Constructionally, it is a rod of germanium showing electron conductivity, the two ends of which form 'cathode' and 'anode'. The 'grid' is a ring of germanium in its middle which forms a circumferential junction with the rod.

Mullard also showed the application of transistors and selenium rectifiers to small h.t. supply units. The transistor replaces the valve in a circuit which is substantially that of the so-called r.f. h.t. supply units.

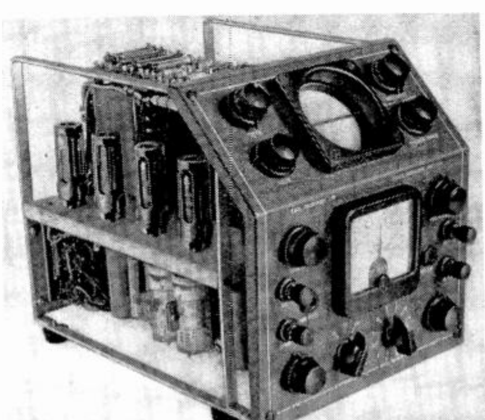
The largest unit gives an output of 2 kV at 0·8 mA and 150 V at 3 mA for an oscilloscope with an efficiency of 70%, the input being at 12 V. A smaller unit, with an input at 4·5 V and the same efficiency, provides 500 V at 50 μA for a Geiger counter.



Southern Instruments M974 four-channel oscilloscope.

Although everyone knows that the transistor is exceedingly small, not everyone realizes the extent to which associated components have been reduced in size. One of the most obvious applications of the transistor is to the hearing aid, where space is at a great premium. Fortiphone showed a range of miniature components for this application, among which a microphone transformer is worthy of mention, because it measures only 0·347 in. by 0·671 in. by 0·25 in. There are two models, of which the DR1 has a 31 : 1 ratio with a primary inductance of 600 H and resistance of 9 kΩ.

The transistor has, up to now, been unique among amplifying valves in requiring no heater supply. It is no longer so, for D.M. Tombs exhibited a cold-cathode amplifier valve requiring neither heater power nor vacuum. The cathode is a needle point facing a blunt anode and the 'grid' is a ring electrode around the needle. The voltage used is sufficient to produce a corona discharge between cathode and anode; and the 'grid', by varying the electric field distribution at the needle, can control the discharge. It is stated that for one model $\mu = 5$ and $r_a = 200$ MΩ.



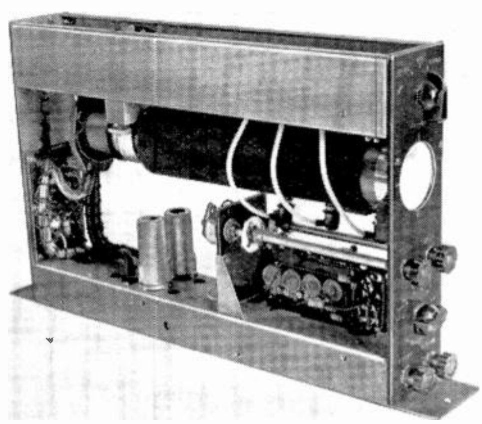
E.M.I. type W.M.1 oscilloscope, without its case.

D.M. Tombs also showed a loudspeaker depending upon corona for its operation. It comprises virtually an assembly of corona triodes. The corona current produces a movement of air and so from a suitable assembly of triodes an air movement over a reasonably large area can be effected.

In the field of cathode-ray tubes, 20th Century Electronics showed their well-known precision flat-face types, which are obtainable with up to four guns. Ferranti showed an interesting range of flat-face tubes in which the deflector-plate connections are brought out directly to horns in order to reduce capacitance and lead length to a minimum. In one, the O6/20P, the Y-deflector 'plates' are merely wires, although the X plates are conventional. This is to minimize the transit time through the plates, for the tube is intended for operation up to 10,000 Mc/s. The Y wires can be matched to a twin-wire transmission line. The tube is designed for operation up to 20 kV and the Y sensitivity is $150/V_a$ mm/V.

The Emitron 4EP1 is a 4-in. tube, in which post-deflection acceleration is used. With 5 kV on the final anode and an extra 5 kV on the accelerator, a writing speed of 100 km/s can be obtained. The deflector plates have their connections brought out to side pins. There is a beam trap on one X plate to permit triggered operation without grid pulsing.

The c.r. oscilloscope was shown in very many forms, ranging from simple to the complex and from the general-purpose to the special-purpose types. Some are so elaborate that it would take several pages to describe their capabilities, let alone their forms of circuitry.



Cinema-Television tube unit without its case; it is one of the 'bricks' of a photographic recording apparatus.

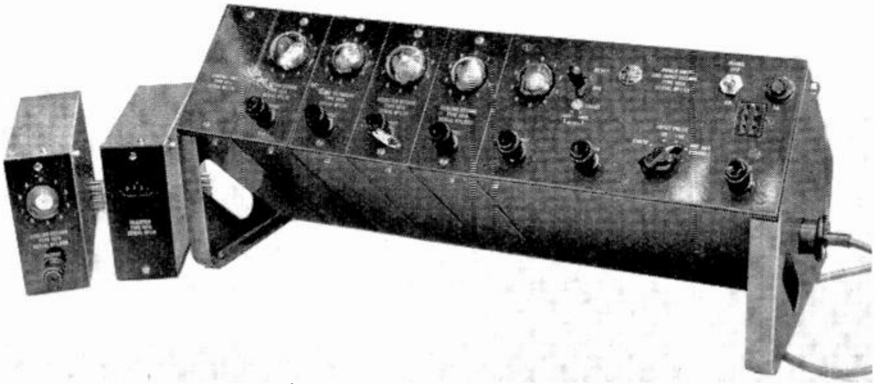
One interesting example is the Southern Instruments M974, which includes a four-gun tube and four Y amplifiers, so that four signals can be displayed together without interaction and to the same time scale. The amplifier response extends from d.c., to 100 kc/s with a gain of 80–1,500 times. The instrument is a development of one shown last year.

Another oscilloscope of quite different type is the E.M.I. type W.M.1. This is a compact and light-weight instrument with a 2½-in. tube; it has a voltmeter in the shift circuit arranged to indicate the shift voltage. As a result, the change of voltage needed to move a displayed waveform across a hairline indicates the amplitude of the waveform directly. The d.c. level of a waveform can also be measured. The amplifier bandwidth varies from 1 Mc/s at full gain to 3 Mc/s at low gain and the deflection sensitivity is 1 cm/V.

The Cossor 1059 is unusual in having a double-beam c.r. tube with post-deflection acceleration. Both of the two Y amplifiers have a response of from 5 c/s to 10 Mc/s for −3 db and the sensitivity is 5 cm/V. The time-base is of the triggered type with a speed range of 0·1 μsec/cm to 50 msec/cm.

ing 10 kc/s to 10 Mc/s in six ranges. An LC oscillator is used and stabilized in amplitude by a thermistor bridge circuit. The output is claimed to be constant within 0·5 db over the entire frequency range. In many of its instruments, this firm has retained the special form of indicator which it produced last year; this is the combination of a conventional dial with a mechanical counter, so that the indication is by the combination of figures appearing in a window with the dial-reading itself.

Pointer-type measuring instruments are, in the main, unchanged, as one would expect, since they have reached



(Left.) *An assembly of Ericsson counting units. The connecting plugs can be seen on the single units shown alongside.*

(Below.) *Avo signal generator with internal amplitude- and frequency-modulation facilities.*

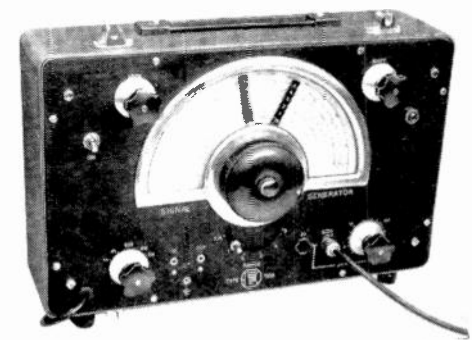
As in previous years, Nagard adopt a unit principle of construction in some of their oscilloscopes, so that the appropriate amplifiers, time-bases, etc., may be employed. This unit principle is now being extended to other forms of apparatus and Cinema-Television have adopted it in their multi-channel photographic-recording apparatus. The various 'bricks' available include tube units, power-supply units, time-trace units and pre-amplifiers.

Ericsson also adopt a unit construction, this time for counting equipment. There are scaling and selector decade units, a scale-of-twelve selector, an electronic gate, a register and a control unit, among others. With their aid, a variety of counting instruments can be built up. The units are inter-connected by plugs and sockets on their adjacent faces.

Signal sources for test and measurement were exhibited in a great variety of kinds. Airmec showed an ultra-low-frequency oscillator, type 852, with a range of 0·03–30 c/s, for which it is claimed that the distortion is under 2%. An output of 500 μV to 50 V peak is available. Solartron displayed a square-wave generator, model GO.511, which generates square waves of 50 c/s, 10 kc/s, 100 kc/s and 1 Mc/s with rise and fall times of about 1/40 of the period of the wave. The output is 10 V at 75 Ω. An LC oscillator is used to generate the two highest-frequency signals, an RC oscillator for the 10-kc/s signal and the mains supply for the 50-c/s.

A very different piece of apparatus is the Avo wideband a.m./f.m. signal generator. This is intended chiefly for the maintenance of broadcasting receivers and it covers 5–255 Mc/s. Internal amplitude modulation by a 1,000-c/s sine or square wave is provided and, over the band of 80–100 Mc/s, internal frequency modulation by a 1,000-c/s sine wave, the deviation being 40 kc/s. The frequency scale is adjustable so that it can be set precisely at any point with the aid of a reference signal.

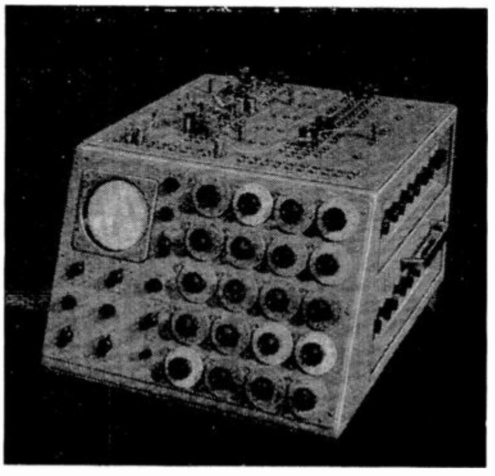
The Wayne Kerr type 0.22B is a video oscillator cover-



a very high state of development. An interesting moving-iron instrument was shown by the Royal Aircraft Establishment, however, in which the dependence of the movement upon springs has been avoided in order to prevent shock and vibration from affecting the accuracy. Basically, a small permanent magnet is arranged to rotate under the influence of two magnetic fields. One is a control field provided either by a fixed permanent magnet or an electromagnet carrying a reference current, the other field is provided by an electromagnet carrying the current to be measured. Screening against external magnetic fields is provided and the instrument can be housed in a standard 2-in. case. The accuracy is ± 1% of full scale.

Considerable development is taking place in materials and the Services Electronics Research Laboratories demonstrated an interesting effect which occurs with magnetic alloys of the kind giving a square hysteresis loop (H.C.R.). When the amplitude of the magnetizing field is reduced from saturation until the loop is only just rectangular, it is found that the loop becomes unstable and slowly changes until saturation is reached in one direction only, the direction of magnetization finally reached being a matter of chance.

Plessey showed magnetostrictive ferrite materials for which they claim the advantages of high resistivity and easy fabrication to any shape. This firm also showed Casonic, a ceramic transducer material, consisting mainly of a polycrystalline form of barium titanate. It is intended for use in ultrasonics and is said to be unaffected by humidity.



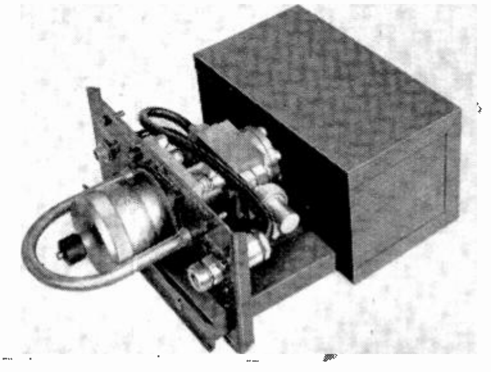
Computor unit of the Saunders-Roe Miniputor analogue computor.

Magnetic materials are finding application as storage devices, since a current pulse through a winding can change the state of residual magnetism. A well-known application of this is to the matrix store and Mullard showed a small version of this in which transistors were used to select the required row and column. Such devices are, perhaps, elements of the computor of the future. Existing computors, however, depend largely upon the valve. There is now a tendency to produce smaller computors than in the past and one example is the Saunders-Roe Miniputer. This is of the analogue type and can solve linear differential equations. Its ten amplifiers each have a sign reverser, and there are 20 coefficient potentiometers and a two-trace c.r. display. Problems are set up by plugging the necessary connecting links and components into the sockets of the patch-panel on top of the instrument.

In the microwave field, development is continuous but no longer spectacular; 'plumbing' has at last become a familiar thing. The travelling-wave tube is now well known; the backward-wave tube, or carcinotron, is relatively new and is characterized by a wide tuning range which is obtainable by varying the beam accelerating potential. Mullard showed a type which operates with the collector at 200 V and a beam current of 25 mA and provides an output ranging from 50 mW at 11 Mc/s to 120 mW at 18 kMc/s.

Measurement was the subject of many of the exhibits in the microwave range, however. An interesting design of frequency meter is the Indicator Unit XT306 developed by Wayne Kerr for Service use. It is an absorption type with a 'magic-eye' indicator and a series of plug-in resonators with micrometer heads. It covers bands S to O and needs an input of less than 0·5 mV.

Attenuators were well in evidence and Elliott showed a continuously-variable X-band attenuator with a total range of 100 db. This firm also demonstrated a method of calibrating such attenuators. Two signals are adjusted for equality and then added to give a known 6·02-db step, from which further steps can be determined.



Resonator unit of Wayne Kerr frequency meter.

For lower frequencies, the Advance turret attenuator gives steps of 10 db up to 50 db with resistance arms. It is of coaxial construction and of 75-Ω impedance and intended for frequencies up to 1,000 Mc/s.

NEW BOOKS

Transistors

By Abraham Coblenz, B.E.E., M.E.E., and Harry L. Owens, E.E. Pp. 313 + xv. McGraw-Hill Publishing Co. Ltd., 95 Farringdon Street, London, E.C.4. Price 42s. 6d.

Of the 15 chapters in this book, 11 have been previously published; these are Chapters 2–12. Of the four new chapters, one is historical, one covers manufacturing processes and the others deal with silicon and special topics. They form interesting reading but contribute little to an understanding of the main topic of the book, which is the theory and applications of transistors. It is Chapters 2–12 which cover this.

These chapters can be divided into two groups 2–7 and 8–12. In the first group, the aim is to provide a simple explanation of the internal operation of the transistor, a thing that is badly wanted. Unfortunately, the attempt does not succeed. The plan upon which these chapters are based is a good one. There is a simple introduction followed by a digression from the main theme to provide the reader with essential background knowledge, and then the main explanation.

The background chapters are quite well done, but they give more information than is used in the following explanation. Quite a bit of space is given, for instance, to picturing the electron as a wave-packet and to the Heisenberg uncertainty principle but, when the explanation of the transistor is reached, neither is referred to.

The treatment of conduction and forbidden energy bands for semiconductors is much clearer than one usually finds it. When one reaches it, the explanation of transistor action fails largely because, after three chapters of background material, it fizzles out in a few pages!

The point-contact transistor has, nominally, 15 pages but 6½ of them deal with diode junctions and one with a summary and bibliography. Of the remaining 7½ pages, only about four pages are devoted to the point-contact transistor itself, the remainder giving information about its characteristics. The junction transistor chapter itself is slightly longer but has only one page on the theory of operation of the *p-n-p* transistor and slightly more on the *n-p-n* type; nearly five pages are devoted to the transistor tetrode, the *p-n* hook transistor and the photo-transistor. The rest of the chapter is given up to methods of making transistors and to statements about typical characteristics. The attempt at explanation thus seems to have been given up just when it started.

In the second group of chapters, the transistor is treated as part of a circuit. In Chapter 8, Electronics of Transistors, the usual 'black-box' outlook is adopted and equivalent circuits are developed for the earthed-base transistor. Chapter 9 covers Small-signal parameters and Chapter 10, the earthed emitter and earthed-collector connections. In Chapter 11, the theory of switching circuits is treated and, in Chapter 12, the cascade connection of transistor stages.

The treatment in these chapters is conventional in its form but it is better carried out than usual and it is pleasant to see that careful attention is paid to the definition of terms. One is very rarely in doubt about the author's meaning.

The book as a whole is thus a very varied one. Chapters 1 and 13–15 are interesting but not important. Chapters 3, 4 and 5 provide a good deal of useful background information about what goes on in semiconductors and will be useful to the newcomer to the subject. Chapters 8–12, dealing with circuit aspects, are good. It should, however, be pointed out that the absence of any transistor characteristic curves will make them unconvincing to some. One thing that does mar this part is the absence of any reference to the effect of temperature on transistor characteristics and, naturally, of how to design transistor circuits to minimize it.

W. T. C.

On Radio Measurements and Standards

Proceedings of the XIth General Assembly of the International Scientific Radio Union, Vol. X, Part 1, Commission 1. Pp. 57. Secrétariat-Général, U.R.S.I., 42 Rue des Minimes, Bruxelles, Belgium. Price 8s. 8d. (including postage).

TV Stations

By WALTER J. DUSCHINSKY. Pp. 136. Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 96s.

This is an American book and has the unusual page size of 11⅝ in. by 8⅝ in. Part 1 covers the planning and organization of a television station, while Part 2 deals with matters such as personnel, aerial towers, studios, audience participation, microwave relays, etc. It is described as a guide for architects, engineers and managements.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Ultra-Low-Frequency Oscillator

SIR,—In the present microwave age, it is refreshing to see that applications are also arising for instruments operating at ultra-low frequencies. The use of frequencies in the range 0·01–1·0 c/s is certainly of interest to teachers of a.c., with centre-zero moving-coil meters phase relations can be strikingly demonstrated, and students can plot the waveforms themselves. With 3-phase currents, the possibility of observing directly the relations between line and phase voltages, etc., is attractive. It is obviously necessary to use R-C circuits at these frequencies, but it is difficult to obtain large enough time constants. A circuit is described here which enables resistors and polystrene capacitors of moderate values to be used, the effective value of the resistor being increased by use of the 'cathode-follower effect'.

One stage of an oscillator making use of this principle is shown in Fig. 1; in general, n such stages connected in a ring will give an n-phase oscillator. (An extra phase-reversing stage is needed if $n = 4$). With three such stages, oscillation occurs if the overall gain of the stage is 2; i.e., if

$$R_a = 2/g_m + 2(R_b + R_c) \qquad (1)$$

(The usual assumption is made that $\mu \gg 1$)

The oscillation frequency is given by

$$1/\omega = \sqrt{3}\,CRM, \qquad (2)$$

Where M, the multiplication factor, is given by

$$M = 1 + \mu R_c/\{r_a + 2/g_m + (\mu + 3)\,R_b + 3R_c\} \qquad (3)$$

Several advantages of this circuit are apparent. The C-R configuration (of the phase-shift network) avoids the need for a direct coupling, and usually gives a lower frequency (for the same component values) than the R-C configuration. There is considerable degeneration making the circuit stable and easy to control. Amplitude limiting can be achieved with the 3-phase rectifier circuit of Smiley*; a block-type thermistor so mounted as to give a long time constant, might also be used.

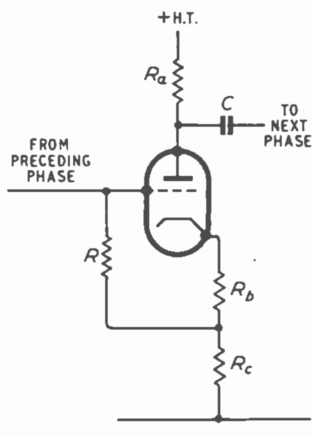


Fig. 1

A trial circuit using three ECC35 valves, with $C = 0{\cdot}1\ \mu$F and $R = 2{\cdot}2$ MΩ, oscillated at about 0·065 c/s.

It can be seen from equations (2) and (3) that, to obtain the lowest frequency, and to reduce the dependence of

*Smiley, G., *Proc. Inst. Radio Engrs*, April 1954, pp 677/680.

frequency on valve parameters, R_c should be made as large as possible.

This can be achieved by using a pentode as cathode load for the triode. It is then necessary to use a separate valve to provide the amplification.

An example of a stage designed in this way is shown in Fig. 2; V_4 is an output cathode-follower. A 1,000-pF capacitor is shunted across the anode load of at least one stage to prevent oscillation at a high audio frequency, in a mode involving the stray capacitances. One section of the limiting circuit is shown, with its reference

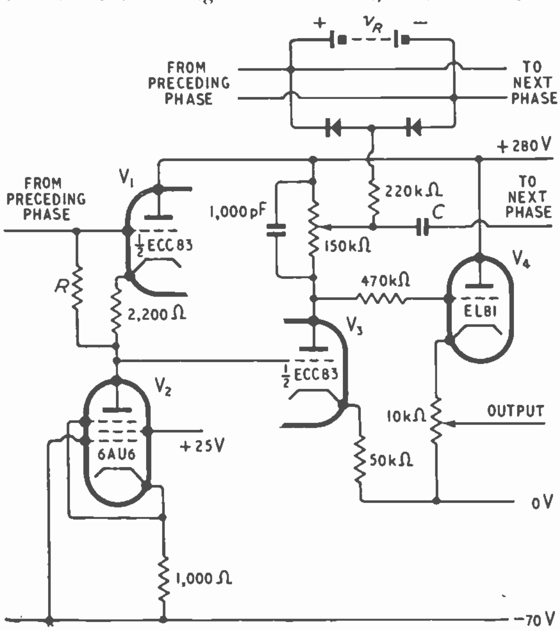


Fig. 2

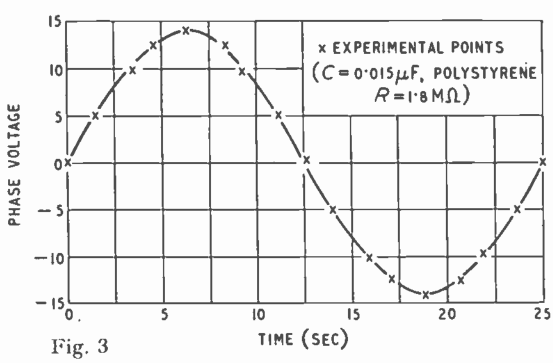


Fig. 3

voltage v_R. Fig. 3 shows a plot (made with voltmeter and stop watch) of one phase voltage, when all three had been set to approximate equality. The experimental points are superimposed on a sine wave of the same amplitude.

Another possible application of the circuit would be to provide a continuously-tunable oscillator or filter for low audio frequencies, using a 3-gang variable capacitor as the tuning element. Frequencies as low as 1 c/s could readily be obtained in this manner.

M. D. Armitage

South-East Essex Technical College,
Dagenham, Essex.

6th April 1955.

PHYSICAL SOCIETY'S SUMMER MEETING

This year's Summer Meeting of the Physical Society will be held in the Lecture Theatre of the George Holt Physics Laboratory at Liverpool University, the subject being "High Energy Nuclear Physics". There will be contributors from the Atomic Energy Research Establishment, the Birmingham, Glasgow, Liverpool, Oxford and Rochester (U.S.A.) Universities and University College, London.

The meeting will be on 5th and 6th July and accommodation can be arranged if application is made before 16th June. Non-members are welcome and application forms are obtainable from the Physical Society, 1 Lowther Gardens, Prince Consort Road, S.W.7.

GERMANIUM

Supplies of single-crystal and polycrystalline germanium of transistor grade are obtainable from G. A. Stanley Palmer, Maxwell House, Arundel Street, Strand, London, W.C.2. The material can be supplied to specified resistivity and with excess minority carrier life-times of 100 μsec, 200 μsec, or more. Sample quantities are available for experimental purposes.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for April 1955

Date 1955 April	Frequency deviation from nominal: parts in 10^8 — MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.	Lead of MSF impulses on GBR 1000 G.M.T. time signal in milliseconds
1	−0·4	−1	+30·8
2	−0·4	0	NM
3	−0·4	NM	NM
4	−0·4	0	+29·0
5	−0·4	0	+28·4
6	−0·5	0	+29·5
7	−0·5	+1	+27·9
8	−0·4	+1	+27·8
9	−0·4	+1	NM
10	−0·4	+1	NM
11	NM	+1	NM
12	−0·4	+1	+26·5
13	−0·4	+1	+26·6
14	−0·4	+1	+26·2
15	−0·4	+1	+26·7
16	NM	+1	NM
17	NM	NM	NM
18	−0·2	+1	+26·4
19	−0·3	+1	+26·2
20	−0·3	+2	+25·9
21	−0·3	+2	+26·6
22	−0·3	+2	+26·3
23	−0·3	+2	NM
24	−0·3	+2	NM
25	−0·3	+2	+26·2
26	−0·3	+3	+25·7
27	−0·2	+2	+25·9
28	−0·2	+2	+25·8
29	−0·2	+3	+25·6
30	−0·2	+2	NM

The values are based on astronomical data available on 1st May 1955.
The transmitter employed for the 60-kc/s signal is sometimes required for another service.

NM = Not measured.