WIRELESS ENGINEER

Vol. 32 MAY 1955 No. 5

The Electrostatic Loudspeaker

ALTHOUGH the electrostatic loudspeaker has been known for many years, it has not so far found much practical use. One of its main attractions is the possibility of applying the driving force uniformly over the whole area of the diaphragm and so achieving for it a piston-like motion. Against this, however, must be set the fact that, until recently, this force has been a non-linear function of the e.m.f. applied to the loudspeaker. As a result, in order to secure reasonably-small amplitude distortion, it has been necessary so to restrict the amplitude of vibration of the diaphragm that the loudspeaker has not been a useful device at other than high frequencies.

We said above "until recently". We do not mean to imply by this that there has been any recent change in the laws of nature. What has occurred has been the discovery of how to construct the electrostatic loudspeaker so that what has always been regarded as its inherent limitation disappears. Its response can now be made almost completely linear. Its efficiency, too, turns out to be very high, although it is not yet possible fully to realize this.

In its simplest and oldest form, the electrostatic loudspeaker comprises two conducting plates mounted parallel to each other and close together as shown diagrammatically in Fig. 1. One plate A is rigid, the other B forms the diaphragm and is made as thin and light as possible and is free to move within the constraint imposed by its mechanical mounting.

The signal voltage v is applied, in series with a polarizing voltage V, between the two plates. If the plates are of area A they form a capacitor $C = \kappa_0 A/d$ where d is their spacing and κ_0 is the permittivity of free space. The polarizing voltage produces a uniform electric field V/d between the plates, neglecting fringe effects, and there is a force of attraction between them of $\frac{1}{2}QV/d$. But $Q = CV$, hence this force is $\frac{1}{2}\kappa_0 A V^2/d^2$.

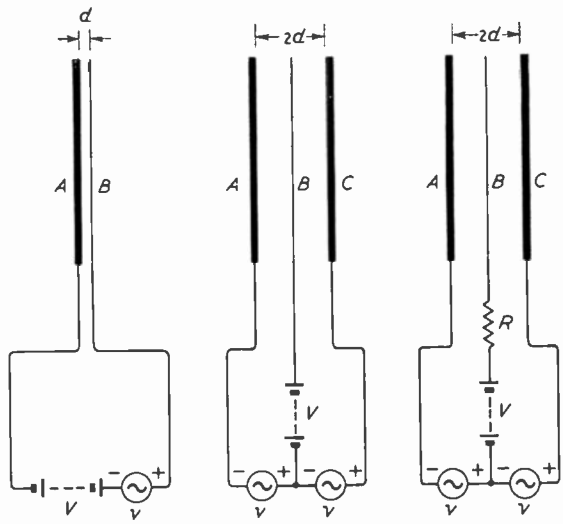

Fig. 1 Fig. 2 Fig. 3

This force is balanced by the constraint of the diaphragm mounting and, with no signal, the diaphragm takes up a position such that it is separated from the fixed plate by the distance d. Now let a signal voltage v be applied. When this is positive, it assists V and the force of attraction is increased and, under the constraint of its mounting, the diaphragm moves nearer to the fixed plate by some distance x. When v is negative it opposes V, the force of attraction is reduced and the diaphragm moves further from the fixed plate.

Under these conditions, the force is

$$F = \tfrac{1}{2}\kappa_0 A\left(\frac{V + v}{d - x}\right)^2 \quad \ldots \quad (1)$$

The relation between force and voltage is obviously far from a linear one.

One step in the development of the loudspeaker was the introduction of push-pull operation[1]. This is sketched in Fig. 2. The diaphragm B is placed midway between two fixed plates A and C and is spaced by d from each. If x is now regarded as a displacement from this mid-position, it is plain that equation (1) can be applied to each side of the diaphragm if we remember that v and x change sign when we pass from one side to the other. For the push-pull loudspeaker, therefore,

$$F = \tfrac{1}{2}\kappa_0 A\left[\left(\frac{V + v}{d - x}\right)^2 - \left(\frac{V - v}{d + x}\right)^2\right]$$
$$= 2\kappa_0 A\,\frac{Vv(x^2 + d^2) + xd(V^2 + v^2)}{(d^2 - x^2)^2} \quad (2)$$

The law of variation of F with v is different, but it is still not a linear one.

The lack of linearity arises out of the fundamental fact that the force is proportional to the square of the voltage. If we turn back to the fundamental relations, however, we find that this is so because voltage enters into the force equation in two ways. Basically, the force is $\tfrac{1}{2}QV/d$, which is a linear relation. It is only because $Q = CV$ in the arrangements that we have so far discussed that the square-law relation enters. If we can devise some way of keeping the charge constant we can achieve linearity.

This is what has now been done to a substantial degree[2, 3]. Consider the arrangement shown in Fig. 3. The time constant of R and the total capacitance of B to A and C is made large compared with the time occupied by one half-cycle of the lowest frequency to be reproduced. When the diaphragm vibrates, there is then no time for any appreciable current to develop in R and the diaphragm carries substantially constant charge. For purely static conditions, of course, the presence of R does not affect matters and equation (2) still applies.

It is interesting to note that in the old circuit of Fig. 2 a resistance was usually inserted in series with V in order to limit the current in the event of a flashover. It was made as low as possible, however, so as not to affect the operation of the loudspeaker. It is rather ironical that this was just the thing that should not have been done.

An exact analysis of Fig. 3 under dynamic conditions is rather difficult. We can, however, see what happens if we consider the diaphragm to be initially charged to the voltage V and, in doing so, to acquire the total charge $2Q$, and then to be completely disconnected from the battery.

With the diaphragm in its mid-position and $v = 0$ there is charge $Q = \kappa_0 AV/d$ on each of its sides. Now let the diaphragm be displaced by x. On one side, the capacitance becomes $C_1 = \kappa_0 A/(d - x)$; on the other $C_2 = \kappa_0 A/(d + x)$. If the diaphragm is a conductor both sides of it must be at the same potential but, as the capacitances on the two sides are unlike, the charges on the two sides must be unlike. Let the potential be V_1; then the charge on one side will be $Q_1 = \kappa_0 AV_1/(d - x)$ and on the other $Q_2 = \kappa_0 AV_1/(d + x)$. As there can be no loss of charge from the diaphragm, $Q_1 + Q_2 = 2Q$. Therefore $V_1 = V(1 - x^2/d^2)$.

The force on the diaphragm thus becomes

$$F = \tfrac{1}{2}\kappa_0 AV^2(1 - x^2/d^2)^2\left[\frac{1}{(d - x)^2} - \frac{1}{(d + x)^2}\right]$$
$$= \frac{2\kappa_0 AV^2}{d^2}\cdot\frac{x}{d} \quad \ldots \quad (3)$$

This is the static force on an insulated diaphragm carrying a constant total charge. It is zero when the diaphragm is midway between the fixed plates, but only then. If the charge on each side of the diaphragm were constant, this force would be zero for any position of the diaphragm.

However, this is not the whole story. In one practical case, at least, the diaphragm is not a conducting sheet fed from a battery through a high resistance. The diaphragm and the resistance are one, for the diaphragm is made from material of very high resistivity. This may affect matters considerably, for it may well result in the diaphragm having an internal time constant which tends to prevent appreciable redistribution of charge on rapid displacement. Such a time constant would have to be large compared with one half-cycle of the lowest frequency to be reproduced. The diaphragm would then be operating under conditions of substantially constant charge on each side and the polarizing voltage would cause negligible force upon it at any amplitude of vibration.

It is interesting to see whether such conditions are possible in the diaphragm. Suppose that the diaphragm itself can be regarded as a sheet of near insulating material of thickness t. Assume that the charges on its surface can be regarded as capacitor plates. Then the capacitance between its two sides is $\kappa A/t$. Let the material be of resistivity ρ, then the resistance between its faces will be $\rho t/A$ and the time constant CR will be $\kappa\rho$. For reproduction down to 50 c/s, this should be large compared with 0·01 second, say $\kappa\rho = 0{\cdot}05$ sec. We might assume $\kappa_r = 2{\cdot}5$

making $\kappa = 2{\cdot}5 \times 8{\cdot}854 \times 10^{12}$ F/m $= 22{\cdot}1 \times 10^{-12}$ F/m. Therefore we require ρ to be about $5 \times 10^{-2}/22{\cdot}1 \times 10^{-12} = 2{\cdot}26 \times 10^{9}$ ohms-metre $= 2{\cdot}26 \times 10^{11}$ ohms-cm.

This is certainly a feasible value and it does appear that with it the charge redistribution will not take place across the diaphragm under operating conditions.

The diaphragm, however, is clearly much more like an insulator than a conductor and it may be asked how it can be charged from a polarizing supply. This is perfectly possible, however, if the support is a circumferential conductor to which the polarizing voltage is applied. The insulation of this conductor from the fixed plates will then form a shunt leakage path, and so will not tend to prevent the polarizing voltage from appearing on the diaphragm.

It is thus not unreasonable to assume that the diaphragm operates with substantially constant charge Q on each of its faces. Whatever the position of the diaphragm, the force on it due to the polarizing supply is then negligible.

Now consider the signal conditions. If the diaphragm is absent, the signal $2v$ sets up an electric field v/d in the space between A and C, Fig. 3. We now interpose the diaphragm with total charge $2Q$. The total field is then v/d on each side plus the fields on each side due to the charge. The forces are $\frac{1}{2}Q$ times these totals and the resultant force is their difference. Since the relation is clearly a linear one, we can consider the two fields separately. We have already found that the force due to the charge by itself is substantially zero, so the force due to the signal is simply

$$F_s = Qv/d \qquad \ldots \quad (4)$$

but $Q = \kappa_0 A V/d$,

so $F_s = \kappa_0 A V v/d^2$

The force does not vary with the position of the diaphragm. Provided that the charge on each side of the diaphragm can be kept substantially constant, therefore, the loudspeaker will be substantially linear in its response, so far as the electrical side is concerned.

We have here touched only on the fringe of some of the interesting electrical aspects of the electrostatic loudspeaker in its new guise. Measurements have shown that amplitude distortion of under 0·5% is practicable and that the new method of operation with a large time constant reduces amplitude distortion by some 25 db as compared with the old push-pull arrangement. With modern acoustic technique the frequency response is good and the even drive over the whole surface of the diaphragm prevents some of the effects known as 'cone break-up'.

The true efficiency is very high indeed. However, the signal voltage is developed across a large capacitance, $\kappa_0 A/2d$, and this makes it hard to supply power to the loudspeaker efficiently at low frequencies. The volt-amperes needed are higher than the watts and as a result there is a power loss in the resistance of the supply circuit. The overall efficiency of the loudspeaker and its supply system is, therefore, not remarkable, but this may be improved.

The polarizing voltage required is high, several kilovolts, but at virtually no current, so that it can be obtained fairly cheaply from an r.f. oscillator in the manner sometimes adopted for cathode-ray tube e.h.t. supplies.

Time will show whether or not the electrostatic loudspeaker will be a serious rival of the moving-coil speaker. There is one place, however, where it fits as though it were made for it; that is, in the television receiver. The e.h.t. system of the tube is there to supply the polarizing voltage and there is no stray magnetic field to affect the c.r. tube.

W.T.C.

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JUNCTION-TRANSISTOR TRIGGER CIRCUITS

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SUMMARY.—The paper describes an experimental investigation of some simple trigger circuits using junction transistors. Because of the similarity between the junction transistor with earthed emitter and the thermionic valve with earthed cathode, the transistors are used in conventional trigger circuits of the types used with thermionic valves. Details are given of bistable and monostable trigger circuits, multivibrators, emitter-coupled trigger circuits, a scale-of-two circuit and a blocking oscillator designed on this principle. The circuits can generate pulses with a minimum length of about 10 microseconds and have a power consumption of the order of one milliwatt per stage.

1. Introduction

MOST published work on the use of transistors in pulse applications relates to circuits using point-contact transistors[1-7]. Relatively little work seems to have been done on the use of junction transistors in this field*, although some pulse circuits using junction transistors have been described by P. G. Sulzer[8] and E. W. Sard[9]. One reason for this is that point-contact transistors have been available for a longer time and in greater numbers. Another reason is that the point-contact transistor has greater than unity current gain without a phase reversal, so that a bistable trigger circuit needs only one transistor. The junction transistor has either less than unity current gain (when the emitter is the input terminal) or a phase reversal (when the base is the input terminal). Consequently, a bistable trigger circuit requires two junction transistors. Another reason for the use of point-contact transistors is that their more rapid transient response permits shorter pulses and higher operating speeds than are obtainable with currently-available junction transistors.

Techniques have been evolved for designing point-contact transistor trigger circuits which are reliable despite variations in the characteristics of the transistors themselves. These techniques[5,6,7] usually involve the use of fairly large supply voltages (e.g., 15 to 50 V) in order to stabilize operating currents, and diodes are used in order to define voltage excursions. Junction transistor circuits appear able to operate satisfactorily with much smaller supply voltages and usually without needing diodes. Typical point-contact type circuits require two diodes, so that if the price of crystal triodes can be brought down to twice that of crystal diodes the objection to junction-type circuits on the score of the number of valves required will disappear. This will be especially true if junction transistors ultimately become cheaper than point-contact ones, as seems likely, because they have no catswhiskers and do not require forming.

This paper describes an experimental investigation of some simple trigger circuits using junction transistors of a type now available in this country and operating from low voltage supplies (e.g., $\pm$ 1·5 V). Because of the similarity between the junction transistor with earthed emitter and the thermionic valve with earthed cathode, the transistors are used in conventional trigger circuits of the types used with thermionic valves.

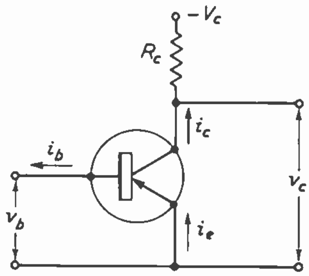


Fig. 1. *Earthed-emitter circuit using p-n-p junction transistor.*

2. Junction Transistor Characteristics

The most useful circuit configuration for use with the junction transistor in trigger circuits appears to be the earthed-emitter circuit in which the base is used as the input terminal as shown in Fig. 1. This arrangement is closely analogous to the thermionic valve with earthed cathode and can be used in similar circuits. If the transistor is a *p-n-p* junction type, it will be conducting if both base and collector are biased negative with respect to the emitter. A positive-going input signal reduces the p.d. between base and emitter, so reducing the emitter current; this reduces the collector current, so the p.d. across the load resistance R_c falls and the collector becomes more negative. The circuit thus provides a phase reversal like the thermionic valve.

In the earthed-emitter circuit, the current from the base is equal to the difference between the emitter and collector currents ($i_b = i_e - i_c$). In a junction transistor the collector and emitter

*Since this paper was written, further work has been described by J. B. Oakes[15], F. C. Alexander[16], and T. A. Pugh[17].

MS accepted by the Editor, September 1954

currents are nearly equal, so the base current is much smaller than either. Consequently, the input resistance is relatively high and there is a large current gain between base and collector. If the current gain between emitter and collector is α, then the current gain between base and collector is approximately

$$\alpha' = \frac{\alpha}{1 - \alpha}$$

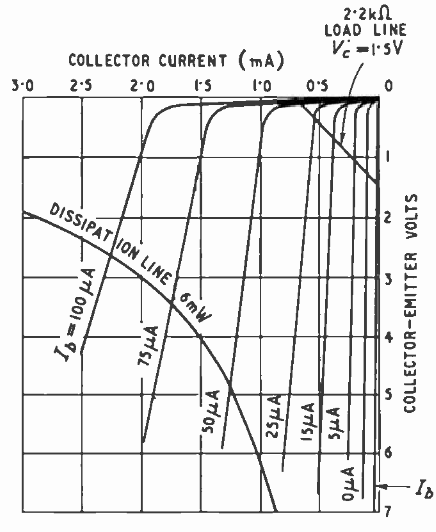
If the transistor has a base resistance r_b and emitter resistance r_e then the input resistance is approximately

$$r_i = r_b + \frac{r_e}{1 - \alpha}$$

Thus if $\alpha = 0{\cdot}95$, for example, the earthed-emitter circuit provides a current gain of 19 and if $r_e = 10$ ohms and $r_b = 500$ ohms the input resistance is 700 ohms. The input resistance is still low compared with that of a thermionic valve so it is still convenient to regard the transistor as a current-controlled device.

The collector-voltage–collector-current characteristics for a typical junction transistor with earthed emitter are shown in Fig. 2(a). Apart from the orientation of the axes, there is an obvious similarity between these characteristics and those of a thermionic pentode. However, the transistor characteristics have been plotted for several values of input current whereas thermionic valve characteristics are plotted for various values of input (i.e., grid) voltage. The relation between input voltage and input current for the same transistor is given by the characteristics shown in Fig. 2(b). Fig. 2(a) shows that reduction of the base current to zero only reduces the collector current (above the knee of the curve) to about 0·1 mA. Further reduction in collector current can be produced by making the base positive so that the base current is reversed. For all base voltages above 0·1 V positive I_c is approximately constant at 10 μA*.

The transistor is clearly capable of working with a very low collector supply voltage, as can be seen from the load line drawn on Fig. 2(a) which represents a 2,200-ohm collector load with a supply voltage of $-$ 1·5 V. When the transistor is turned fully 'on' the collector voltage is less than 0·15 V and when it is switched right 'off' the voltage is nearly 1·5 V. The transistor thus provides as large a signal relative to its supply voltage as a thermionic pentode which bottoms at 50 V and has a supply voltage of 500 V: equally reliable circuit operation should result. A technique for designing 'on-off' circuits using the junction transistor is first to choose a collector voltage and load resistance that provide adequate bottoming conditions. Thus with $V_c = 1{\cdot}5$ V and $R_c = 2{\cdot}2\ \text{k}\Omega$ the collector current is approximately 0·6 mA provided that the base current exceeds 0·05 mA. It is then necessary to arrange the base voltage excursion to ensure that the base is at least 0·1 V positive when the transistor is required to be off and that the base current



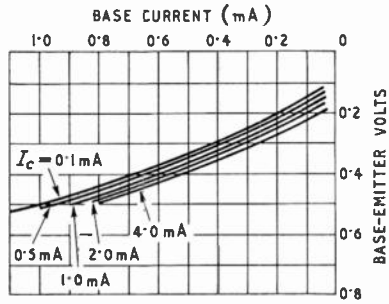


Fig. 2. *Characteristics of OC10 transistor in the earthed-emitter connection. (a) collector voltage—current curves, (b) base voltage—current curves. (For potentials ≥ 0·1 V positive on base, I_b remained almost constant at 9μA and produced 10μA I_c.)*

exceeds 0·05 mA when the transistor is required to be on. Provided that these requirements can be met, the trigger circuit configurations which are used with thermionic valves should also be suitable for junction transistors having earthed emitters. A simple specification for transistors to use in these circuits is that with $V_c = -\ 1{\cdot}5$ V and $R_c = 2{,}200\ \Omega$, then at $i_b = 0{\cdot}05$ mA, $-\ v_c \leqslant 0{\cdot}15$ V and at $v_b = +\ 0{\cdot}1$ V, $i_c \leqslant 25\ \mu$A. These conditions were found suitable for all transistors of types OC10, OC11, OC12, OC70, OC71, 2N34 and 2N36 so far tested. Tests on a few samples of transistors 2N38 and CK722 indicate that a base current of 0·1 mA is required to ensure reliable bottoming of these types. The circuits described in the following sections were constructed using Mullard transistors type OC10.

* The minimum value of collector current (usually designated by I_{co}) is obtained by reducing the emitter current to zero. This condition is approached when the emitter is biased negative with respect to base, so the earthed-emitter circuit of Fig. 1 will have its base positive with respect to earth and current approximately I_{co} will flow into the base and out of the collector.

3. Bistable Trigger Circuit

Fig. 3 shows a junction transistor circuit similar to the well-known Eccles-Jordan trigger circuit using thermionic valves. The polarities of the supply voltages are the reverse of those used with thermionic valves as a result of using *p-n-p* transistors. When the circuit is in stable equilibrium, one transistor, say V_1, is bottomed and the potential divider connected to its collector holds the base of V_2 positive, so this transistor is cut off. The collector of V_2 is therefore nearly at − 1·5 V and the potential divider holds the base of V_1 sufficiently negative to ensure that it is bottomed. A negative pulse applied to the base of V_2 or the collector of V_1 or a positive pulse applied to the base of V_1 or the collector of V_2 causes the circuit to change over to the other stable state with V_2 on and V_1 off.

The power consumption of the circuit shown in Fig. 3 is 1·3 mW. The resistances used in the divider chains are such that, with $\pm$ 10% tolerance on resistance values and $\pm$ 5% tolerance on supply voltages, the base of the 'off' transistor is at least 0·1 V positive and the base of the 'on' transistor has at least 0·05 mA current flowing. These tolerances permit all values to be adverse together, including one supply having a high voltage when the other is low. If the positive and negative supply voltages are always equal, then they can have a tolerance of $\pm$ 20%. A positive bias equal to the negative supply can be derived by using a single 3-V supply with a

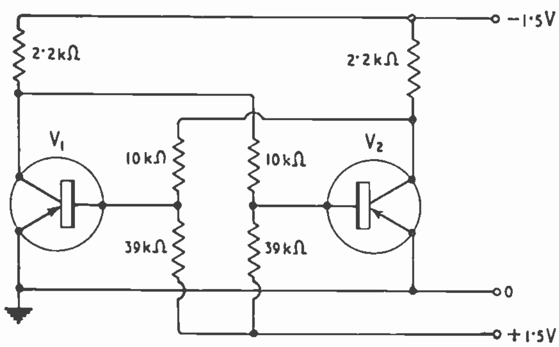


Fig. 3. *Eccles-Jordan trigger circuit.*

2·2-kΩ resistor common to the two emitters as shown in Fig. 10, but allowance must then be made for the tolerance on this resistance and the power consumption of the circuit is also increased. If wider tolerances are required, then higher supply voltages are necessary (e.g., $\pm$ 3 V). The circuit may still work if the supply voltages or component values are outside limits, but the collector of the 'on' transistor will then be above the bottoming potential or that of the 'off' transistor below the supply voltage, with the result that the amplitude of the output signal is not exactly defined.

The circuit of Fig. 3 was found to trigger when a 10-μsec positive pulse of 1 V was applied via a 0·001-μF capacitor to the base of the 'on' transistor. When 0·005-μF capacitors were connected across the 10-kΩ coupling resistors, the circuit could be triggered by a 10-μsec positive pulse of 0·25 V applied to the base or a 1·2-V pulse applied to the collector, but when negative pulses were applied to the base 1·2 V was required to trigger the circuit.

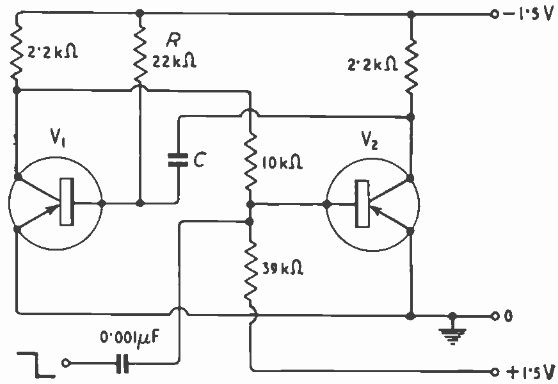


Fig. 4. *Flip-flop circuit.*

4. Monostable Trigger Circuit

Fig. 4 shows a monostable trigger circuit or 'flip-flop'. In the position of stable equilibrium, transistor V_1 is bottomed because its base is taking current from the negative supply and the potential divider connected to its collector holds V_2 cut off. When the circuit is triggered into the unstable state by applying a negative pulse to the base of V_2, V_2 bottoms and the rise in its collector potential is transferred to the base of V_1 by the capacitor C. This cuts off V_1, whose collector potential falls and holds the base of V_2 sufficiently negative to ensure that it remains bottomed. The circuit remains in this state until the capacitance C has discharged sufficiently through R for V_1 to be turned on again. Positive feedback round the loop comprising V_1, the potential divider, V_2 and C then ensures that the circuit rapidly reverts to its original state with V_1 bottomed and V_2 turned off. The circuit thus produces a negative pulse at the collector of V_1 and a positive pulse at the collector of V_2, each with a duration determined by the time constant CR.

In the circuit of Fig. 4 the resistance values in the d.c. coupling are chosen as described in Section 3 to provide a minimum base current of 0·05 mA to V_2 when it is on. Similar considerations fix the maximum value of resistance R at 27,000 ohms.

Fig. 5 shows the waveforms obtained at the collectors and emitters when the circuit is triggered by a square-wave input signal, which is 'differentiated' by the input coupling capacitor. The length of the pulse at the collector of V_1 [Fig. 5(a)] is accurately determined by the saw-tooth waveform at its base [Fig. 5(b)] and agrees with the value calculated from the time-constant. The length of the pulse at the collector of V_2 [Fig. 5(c)] is less accurately determined and is a little longer than that at the collector of V_1. The rise and fall times of the collector pulses are 2 μsec and 7 μsec approximately. A pulse of 9 μsec duration was obtained from the collector of V_1 when C was 600 pF [Fig. 5(e)]. If C is reduced further, V_1 fails to cut off completely and the pulse amplitude falls. No improvement in waveform could be obtained by connecting a capacitor across the 10,000-ohm coupling resistor.

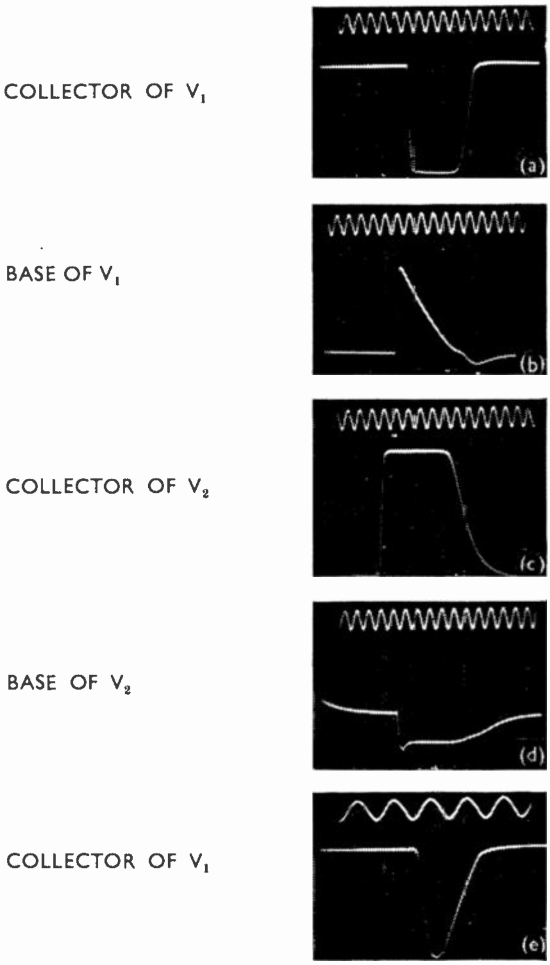


Fig. 5. *Oscillograms of waveforms in a flip-flop circuit. (a)-(d) refer to a 30-μsec pulse at the collector of V_1 with C = 2,000 pF; (e) is for a 9-μsec pulse with C = 600 pF. The timing waveforms are all 200 kc/s.*

5. Multivibrator Circuit

Fig. 6 shows a multivibrator circuit based on the familiar circuit using thermionic valves. If V_2 is initially bottomed and if V_1 is cut off by a positive potential on its base, this will decay exponentially towards the negative supply voltage until V_1 is turned on. The rise in collector potential of V_1 is applied to the base of V_2 by means of the capacitor C_2 and turns V_2 off. The

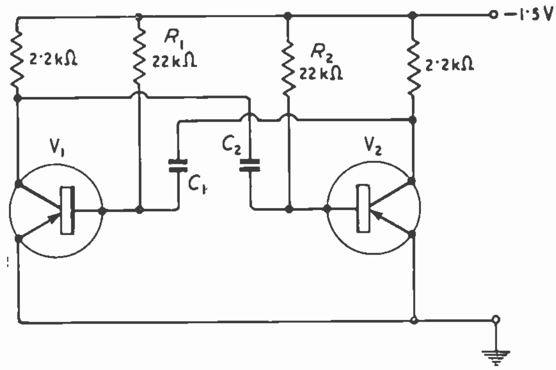


Fig. 6. *Multivibrator circuit.*

collector potential of V_2 falls to the negative supply with a time constant determined by its collector load and capacitance C_1, since the base of V_1 is conducting to earth (V_1 is bottomed). Meanwhile, the base potential of V_2 (which is positive) falls exponentially toward the negative supply voltage until V_2 is turned on. The circuit then reverts to the initial condition with V_1 cut off and V_2 bottomed and the cycle of operations recommences. In order to ensure reliable bottoming, the valve which is 'on' requires a minimum base current of 0·05 mA and this fixes the maximum value of resistances R_1 and R_2 at 27,000 ohms in order to permit a tolerance of ± 10% on their values.

The circuit of Fig. 6 was found to operate satisfactorily over a wide range of frequencies. Fig. 7(a) to (d) shows oscillograms of the waveforms on the bases and collectors of the two transistors when the circuit was oscillating at 1·5 kc/s approximately ($R_1 = R_2 = 22$ kΩ, $C_1 = C_2 = 0{\cdot}025$ μF). As was expected, the waveforms are similar to those obtained in the corresponding thermionic valve circuit. The maximum frequency at which the circuit would oscillate was approximately 60 kc/s ($R_1 = R_2 = 22$ kΩ, $C_1 = C_2 = 820$ pF). At this frequency the waveforms were poor [see Fig. 7(e) to (h)]. Operation up to 150 kc/s could be obtained by increasing the supply voltage to 6 V, but at these frequencies the circuit was not self-starting: it proved necessary to apply a positive pulse to the base of one transistor in order to make oscillation commence.

6. Scale-of-Two Circuit

Fig. 8 shows a binary counter circuit based on the similar circuit using thermionic valves. If V_2 is initially bottomed, V_1 will be cut off. Consequently, the p.d. across C_2 will be greater than that across C_1. A short positive input pulse (applied via C_3 and diode D_1) will bring the collector of V_1 to a higher potential and its change in voltage will be transmitted by C_2 to the base of V_2.

This brings the circuit to a state in which both transistors are cut off but both collectors are at a small potential. When the input pulse disappears, each collector tends to fall towards the negative supply voltage and so lower the potential of the other base. Because the p.d. across C_1 is less than that across C_2, the base of V_1 reaches first the point at which the transistor begins to conduct. This prevents the collector of V_1 rising further and keeps the base of V_2 biased positive. The collector of V_2 therefore falls to the full negative supply voltage and ensures that V_1 is turned fully on at its base. Application of the pulse has thus caused the circuit to assume the opposite state from that in which it was initially.

The resistance values shown in Fig. 8 are chosen as explained in Section 3. Capacitance C_3 ensures that a square wave applied to the input terminal is 'differentiated' to produce short pulses for driving the counter. The values of the capacitances across the coupling resistors were chosen empirically to ensure that the change in p.d. across each resistor during the input pulse was sufficiently small to ensure reliable operation. The circuit was found to operate satisfactorily at any frequency up to 20-kc/s. Fig. 9(a) to (e) shows oscillograms of two binary counters

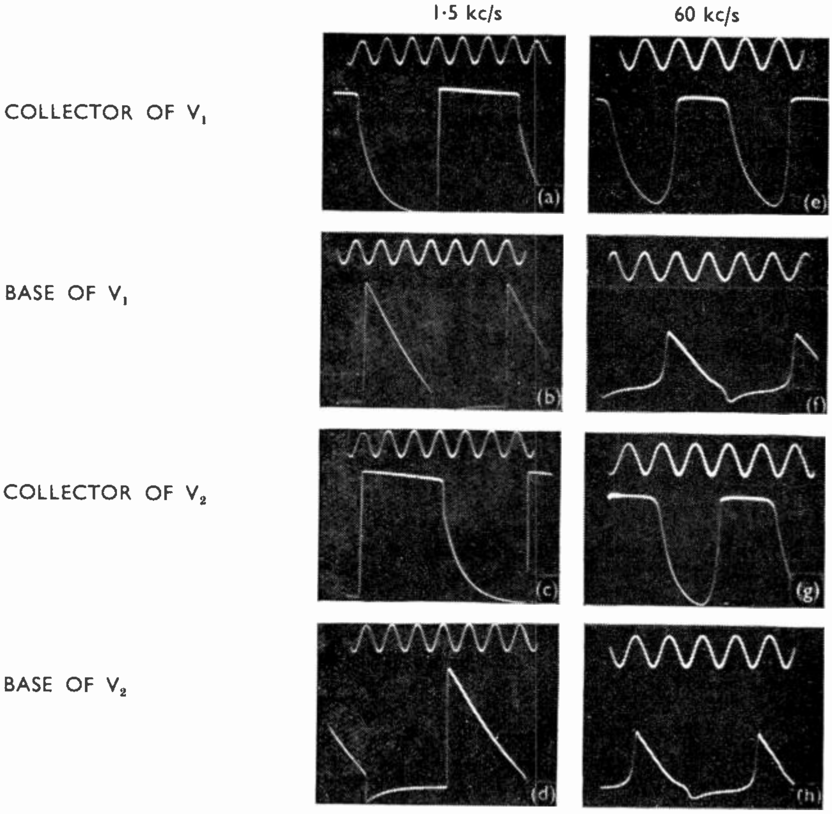


Fig. 7 (above). *Multivibrator waveforms. Waveforms (a)-(d) are for operation at 1·5 kc/s with $C = 0{\cdot}025\ \mu F$ (the timing waveform is 10 kc/s). Waveforms (e)-(h) are for 60 kc/s with $C = 820\ pF$ (the timing waveform is 200 kc/s).*

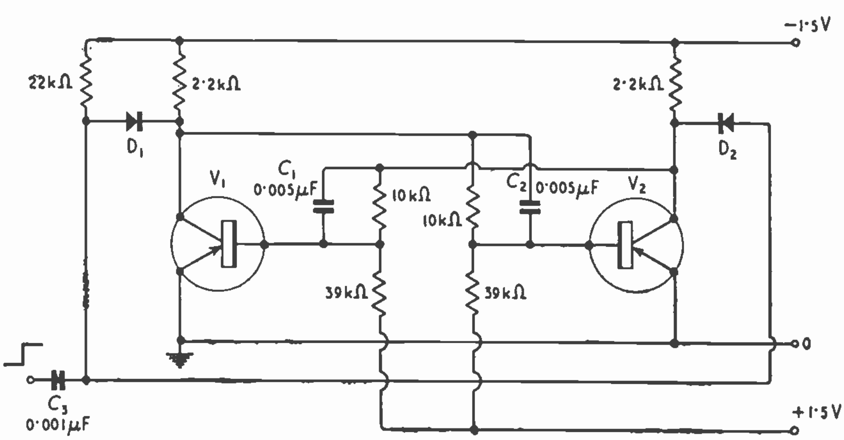


Fig. 8 (right). *Scale-of-two circuit. The diodes D_1 and D_2 are type CV425 (B.T.-H. type CG1C point-contact germanium diode).*

operating in cascade from a 1·5-kc/s input signal, while (f) to (k) show the same circuits with a 20-kc/s input signal. Above 20 kc/s the shape of the waveforms is so poor that the first stage is nearly failing. Waveforms (l) to (n) show the first stage misoperating when the input frequency has been increased to 25 kc/s.

positive with respect to the emitter it is cut off and the collector current of V_2 remains constant at its bottoming value (over region AB on the characteristic).

When the base becomes sufficiently negative for V_1 to take a small current, the increase in its collector potential is applied to the base of V_2 by

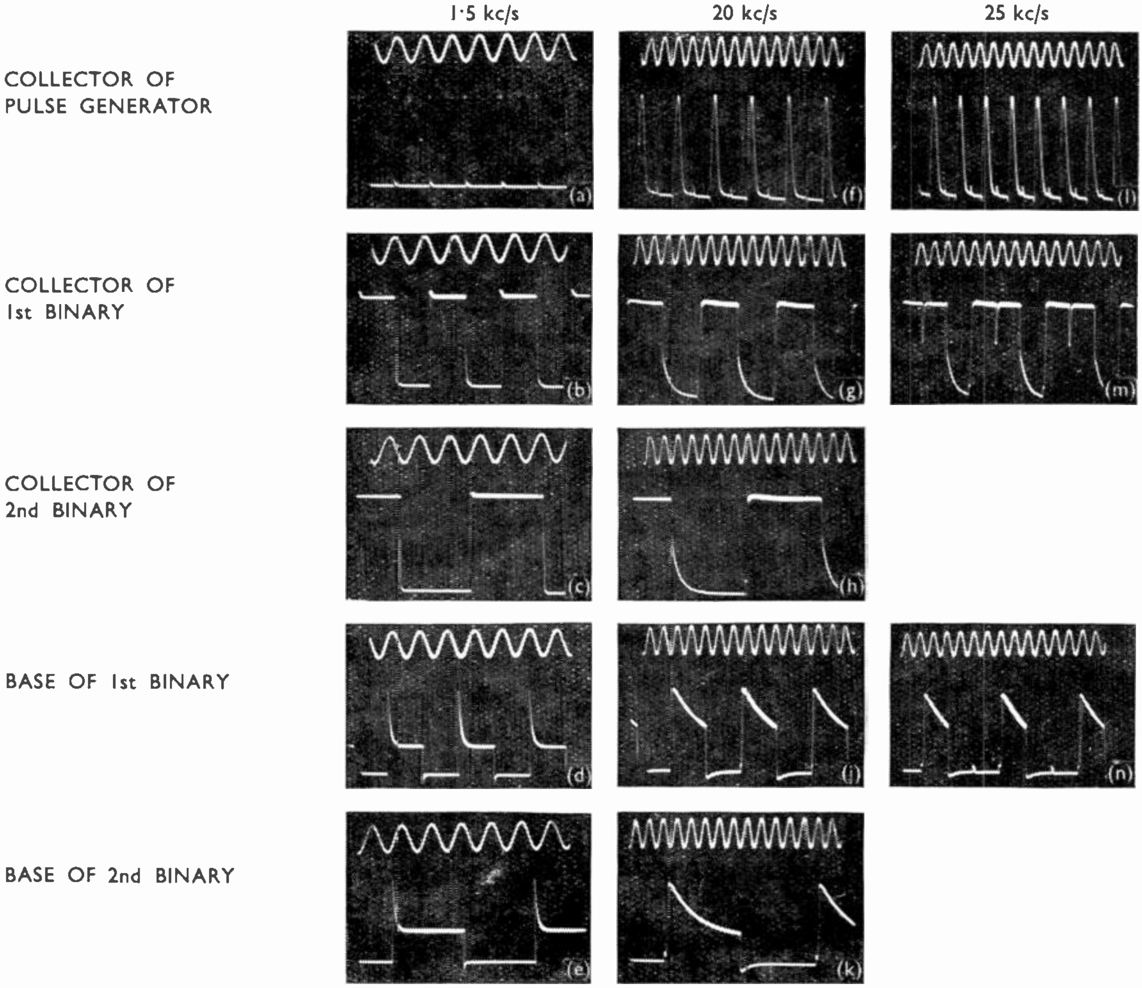


Fig. 9. *Waveforms of binary counter. Two binary stages in cascade fed from pulse generator. Waveforms (a)-(e) pulse repetition frequency 1·5 kc/s (timing frequency 2 kc/s); (f)-(k) pulse repetition frequency 20 kc/s (timing frequency 50 kc/s); (l)-(n) pulse repetition frequency 25 kc/s, 1st binary just failing (timing frequency 50 kc/s).*

7. Emitter-Coupled Circuits

Fig. 10 shows a circuit in which the emitters are coupled by a common resistor. This circuit is thus analogous to the Schmitt cathode-coupled trigger circuit using thermionic valves and has similar properties. If the base of V_1 in initially at earth potential, this transistor is biased off by the p.d. across R_e due to the current flowing in V_2 which is bottomed. If the base potential of V_1 is gradually lowered, then the circuit behaves as shown in Fig. 11. As long as V_1 has its base

the potential divider. The collector current of V_2 thus begins to decrease as the base potential of V_1 continues to fall (along BC). The decrease in p.d. across R_e caused by the fall in current of V_2 makes the emitter of V_1 more positive, causing its current to increase. When the current of V_1 has become large enough to provide sufficient gain for this action to be cumulative, the circuit snaps over into the state in which V_1 is on and V_2 is off (along CD). Transistor V_1 is now bottomed and the potential divider biases V_2 beyond cut off. The collector current of V_2 is then very small and does not change when the base of V_1 is made more negative (along DE).

If the base of V_1 is now brought back towards earth potential, V_2 will remain cut off for as long as the base of V_1 is sufficiently negative to keep its collector bottomed (over

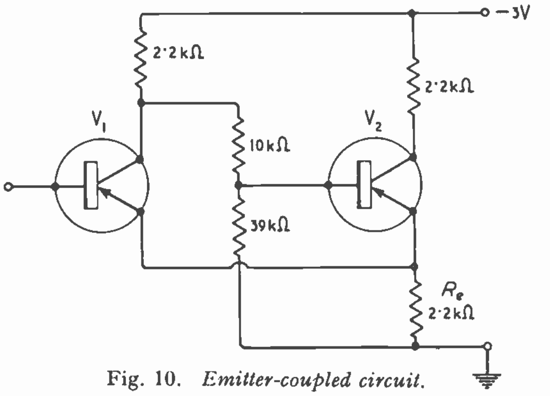


Fig. 10. *Emitter-coupled circuit.*

region ED). When the collector potential of V_1 begins to decrease, the base potential of V_2 is lowered by means of the potential divider and this transistor begins to take a small current (region DF). When the current in V_2 becomes sufficient to provide enough gain for triggering, the circuit snaps back into the state with V_2 on and V_1 off (along FB). V_2 is now bottomed and its collector current does not change any more as the base of V_1 is brought back to earth potential (over region BA). The circuit thus exhibits the backlash which is characteristic of the 'negative grid base' associated with the Schmitt circuit[10].

The d.c. coupling between the collector of V_1 and base of V_2 was replaced by a resistance-

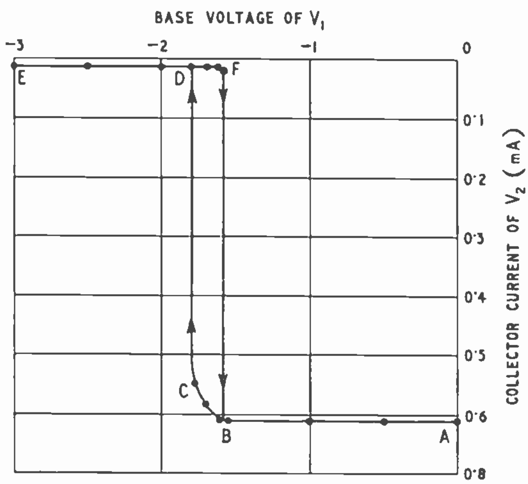


Fig. 11. *Measured characteristic curve for an emitter-coupled circuit using transistors type OC10.*

capacitance coupling of the form shown in Fig. 4 and a flip-flop circuit was obtained. The general performance of this circuit was similar to that of the circuit described in Section 4.

8. Blocking Oscillator

Fig. 12 shows a blocking oscillator which is triggered by a negative pulse applied via C_2. In the quiescent state, the transistor is 'off' because resistor R is connected to a positive bias voltage. The negative pulse applied to the base causes the transistor to conduct and the collector current, which flows through the primary of the transformer, induces a voltage in the secondary which is fed back to the base in such a phase as to increase the current further.

This positive feedback causes the first half cycle of an oscillation for which the transformer provides the tuned circuit. During this half cycle the collector voltage has risen and the diode D_1 is cut off. The oscillation is of considerable

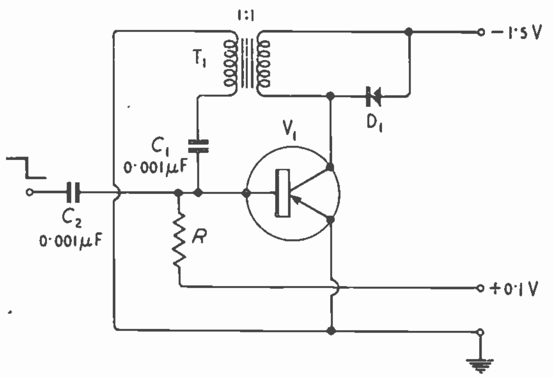


Fig. 12. *Blocking oscillator (triggered). The diode is a B.T.-H. germanium junction type GJ3D.*

amplitude and a heavy base current flows through the coupling capacitor C_1. At the beginning of the second half cycle, the base swings positive and the transistor is cut off. There is thus no positive feedback and the tuned circuit is heavily damped by the diode D_1 which conducts when the collector potential falls below the supply voltage. The third half cycle of oscillation is therefore of negligible amplitude and fails to drive the base sufficiently negative to make the transistor conduct. The circuit thus produces only one half cycle of appreciable amplitude, after which the base is at a large negative voltage due to the charge on C_1. This capacitor discharges through resistor R and the base voltage decays towards + 0·1 V. While that capacitor is holding the base positive by an amount greater than the amplitude of the input pulses these are unable to trigger the circuit, whose further action is thus blocked for a time depending on the time constant C_1R.

If the resistor R is connected to a negative instead of a positive voltage, the circuit is made free running. After each pulse of base current has charged C_1, the base potential falls towards a negative potential so that, after an interval, the transistor conducts again and another half cycle of oscillation occurs. The pulse repetition frequency can easily be varied by adjusting the value of C_1 or R or the negative bias voltage.

The circuit of Fig. 12 was constructed using a transformer wound on a ferrite ring core with 200 turns in each winding. The core had a rectangular cross-section, its outside diameter was 1·0 in., its inside diameter 0·76 in. and its depth 0·19 in. This produced a pulse with a duration of 10 μsec and rise and fall times of 2 and 1 μsec approximately. The waveforms obtained are shown in Fig. 13. Fig. 13(a) and (b) show the collector and base waveforms when the circuit is free running at 7 kc/s. Fig. 13(c) and (d) show the collector and base waveforms when the junction diode is replaced by a point-contact type; the damping on the transformer winding is less efficient. It was found that a load resistance of down to 680 ohms could be connected across the collector winding before the circuit failed to bottom [see Fig. 13(e)]. Fig. 13(f) and (g) show the circuit used as a frequency divider with a ratio of 5:1; the input frequency is 25 kc/s.

9. Speed of Operation

The circuits described in the preceding sections can produce pulses with a minimum length of about 10 μsec using junction transistors commercially available in this country. A very similar performance was also obtained from these circuits using a few samples of junction transistors made in the U.S.A. (types CK722, 2N34, 2N36 and

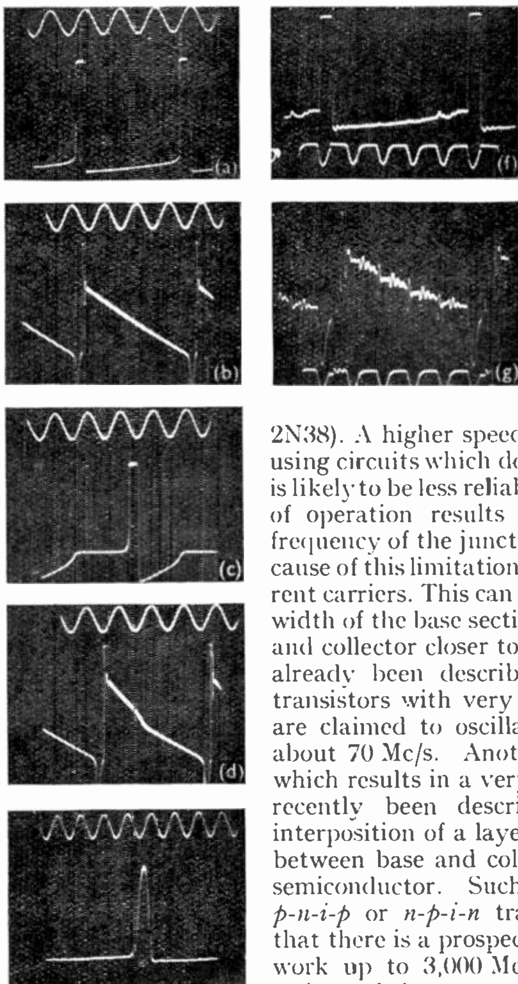


Fig. 13. *Blocking oscillator waveforms. (a)-(d) free running at 7-kc/s repetition frequency (C = 0·025 μF, R = 1·5MΩ returned to −1·5V) and timing frequency of 20 kc/s. (e) collector loaded with 680 Ω, timing frequency 100 kc/s. (f) and (g) operation as a divider (C = 0·01 μF, R = 10 kΩ); the input pulses, which are also shown, have a repetition frequency of 25 kc/s.*

2N38). A higher speed may possibly be obtained using circuits which do not bottom, but the action is likely to be less reliable. The relatively slow speed of operation results from the low alpha cut-off frequency of the junction transistor. The physical cause of this limitation is the transit time of the current carriers. This can be made less by reducing the width of the base section so as to bring the emitter and collector closer together. Two methods have already been described for producing junction transistors with very thin base layers[11,12] which are claimed to oscillate at frequencies of up to about 70 Mc/s. Another method of construction which results in a very high cut-off frequency has recently been described[13] which involves the interposition of a layer of high-purity germanium between base and collector to act as an intrinsic semiconductor. Such transistors are known as *p-n-i-p* or *n-p-i-n* transistors and it is claimed that there is a prospect that they may be made to work up to 3,000 Mc/s. However, it has been estimated that a cut-off frequency of 10 Mc/s can be obtained by improvement of conventional triode structures[13]: this should be adequate for most trigger circuit applications.

It should be possible to make some increase in the alpha cut-off frequency without reducing the electrode spacing or introducing an intrinsic semiconductor layer if it is permissible to reduce the value of α. This is because the high-frequency response is influenced by the life-time of holes in the base region as well as by the width of the base region itself. It has been shown[14] that, for earthed-emitter circuits, the carrier life-time τ is the predominant influence on the cut-off frequency so that an n-fold reduction in carrier life-time should produce very nearly an n-fold increase in cut-off frequency. However, reducing τ will also reduce α. Fortunately, a smaller value of α is acceptable for trigger circuits than for amplifiers, because triggering will occur provided that the gain round the loop is greater than unity. If, for example, one transistor has $\alpha = 0{\cdot}98$ and another has $\alpha = 0{\cdot}89$, solely because of a difference in carrier life-time, the second transistor should have a cut-off frequency six times that of the first. An improvement in operating speed of this order would greatly extend the usefulness of junction transistor trigger circuits and transistors made of material with lower carrier life-time might be cheaper than those produced by methods which achieve a high cut-off frequency and still preserve a high current gain.

10. Conclusions

The junction transistor with earthed emitter is sufficiently similar to the thermionic valve with earthed cathode to enable it to be used in conventional trigger circuits of the types used with thermionic valves. The circuits require different component values, which usually result in the transistor circuits having lower impedances than thermionic valve circuits. The junction transistor circuits operate at lower currents than valve circuits and operate from very much lower voltage supplies. Consequently, the power consumption is very greatly reduced and is small even compared with that used by point-contact transistor circuits. In typical circuits described in this paper the power consumption is about one milliwatt per stage.

As a consequence of the small supply voltages used, the output voltages obtainable are also small. These voltages are of course ample for operating similar circuits, but may be too small to drive other types of circuit or to operate indicators. In a counter, an indication is usually required of the state of each stage, and it is a disadvantage not to be able to use simple indicators, such as the neon lamps often used with thermionic valve scalers. The most convenient indicator seems to be a moving-coil voltmeter which can be switched to read the collector voltages of the different stages. In digital computing apparatus there is usually a large number of circuits but external outputs are required from relatively few of them, so thermionic valves or point-contact transistors can be used to amplify the output signals without greatly increasing the total power consumption. The low output voltage obtainable from junction transistor trigger circuits should therefore not be a disadvantage in this application.

The chief disadvantage of junction transistor circuits, at present, seems to be the relatively low operating speed. The circuits described in the paper produced pulses of lengths down to 10 μsec and operated up to about 50 kc/s. This is much slower than is obtainable from trigger circuits using point-contact transistors or thermionic valves, but compares favourably with trigger circuits using cold-cathode gas-filled valves or saturated magnetic cores. There should thus be quite a wide range of applications for junction transistor trigger circuits, particularly in circumstances where their reliability and low power consumption are advantages. If future developments increase their operating speed and reduce their cost, the usefulness of junction transistors in trigger circuits will be still further extended.

Acknowledgments

Acknowledgment is made to Siemens Brothers & Co. Limited for permission to publish this paper. Thanks are also due to Mr. B. D. Simmons for his assistance in the experimental work.

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INPUT RESISTANCE OF L.F. UNIPOLE AERIALS

With Radial Wire Earth Systems

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SUMMARY.—The input resistance of a low-frequency unipole aerial is calculated. The earth system consists of a number of radial conductors buried just below the surface of the soil. The integrals involved in the solution are evaluated, in part, by graphical methods. The final results are plotted in a convenient form to illustrate the dependence of the input resistance on number and length of radial conductors for a specified frequency and earth conductivity. The curves should be useful in the design of earth systems for low-frequency transmitting aerials. It is pointed out that increasing the radius of the earth system beyond a certain limit gives only a slight improvement in radiation efficiency.

LIST OF SYMBOLS

(m.k.s. units are employed throughout)

(ρ, ϕ, z)	= cylindrical polar co-ordinates
h	= height of an ideally top-loaded aerial or the equivalent height of an unloaded aerial
a	= length of radial wires
σ	= conductivity of ground
ϵ	= permittivity of ground
ϵ_0	= permittivity of air ($= 8{\cdot}85 \times 10^{-12}$)
μ_0	= permeability of free space ($= 4\pi \times 10^{-7}$)
γ	= intrinsic propagation constant of ground
η	= characteristic impedance of ground
γ_0	= propagation constant of air
η_0	= characteristic impedance of air ($= 120\pi$)
β	= wave number in air ($= -j\gamma_0$)
ω	= angular frequency
λ	= wavelength in air
Z_t	= self-impedance at terminals of aerial
Z_0	= self-impedance of the aerial for a perfectly-conducting earth plane
ΔZ_t	= self-impedance increment ($= Z_t - Z_0$)
ΔR_t	= real part of ΔZ_t
ΔX_t	= imaginary part of ΔZ_t
I_0	= current at terminals of aerial ($= \sqrt{2} \times$ r.m.s. current)
$H^{\infty}_{\phi}(\rho, 0)$	= the tangential magnetic field of the aerial on a perfectly-conducting earth plane of infinite extent
$E_\rho(\rho, 0)$	= the tangential electric field of the aerial on the imperfect ground
$I(z)$	= current along the aerial
$H_\phi(\rho, 0)$	= the tangential magnetic field of the aerial on the imperfect ground
η_c	= the surface impedance of the air-ground interface
η_s	= the surface impedance of the radial wire grid
d	= the spacing between the radial wires
c	= radius of the wires of the grid
γ_e	= the effective propagation constant of a wire in the air-ground interface
N	= number of radial wires in the earth system
ΔZ	= self-impedance increment for an ideal circular ground screen of radius a
ΔZ_s	= correction to ΔZ to account for the losses within the ground screen
b	= limit of integration for equations (12) and (13)
$Ei(-x)$	$= -\int_x^{\infty}(e^{-t}/t)dt$
$O(y)$	= a quantity whose order of magnitude is equal to y
V and θ	= magnitude and phase of the complex number $Ve^{j\theta}$ defined in equation [15(b)]
P	= variable of integration ($= \rho/\lambda$)
H_1	= height of aerial in wavelengths ($= h/\lambda$)
F and ψ	= magnitude and phase of the complex number $Fe^{j\psi}$ defined in equation (15a)
p, q, δ, A, B, C_1	= dimensionless quantities defined in the text following equation [15(a)]
R_0	= the input resistance of the aerial for a perfectly-conducting earth plane
C	= Euler's number ($= 0{\cdot}5772\ldots$)
h_0	= actual height of the unloaded aerial
$\mathrm{Si}(x)$	$= \int_0^x \frac{\sin t}{t}\, dt$
$\mathrm{Ci}(x)$	$= -\int_x^{\infty} \frac{\cos t}{t}\, dt$
δ	$= \left(\frac{\epsilon_0\omega}{\sigma}\right)^{\frac{1}{2}}$ = ground conductivity parameter

MS accepted by the Editor, May 1954

Introduction

AERIAL systems for low-frequencies are designed, usually, to work in conjunction with a radial-wire earth system buried just below the surface of the earth. The purpose of this wire grid is to provide a low-loss path for the aerial base current and consequently to improve the radiation efficiency.

The rules for earth-system design are usually empirical and based on the results of experiments on existing installations. The first systematic study of this problem was carried out by Brown[1,2] and his associates who were mainly concerned with the operation of half-wave aerials for the broadcast band. Sometime later Abbott[3] developed a procedure for selecting the optimum number of radial conductors, given the values of the electrical constants of the ground. An important related problem is the actual change of input impedance of the aerial with different sizes and types of earth systems. This analysis has been carried out by Leitner and Spence[4] and more recently by Storer[5], for a vertical aerial situated over a perfectly-conducting disc. However, they only considered the case where the surrounding medium was free space.

In a previous paper[6] the electrical characteristics of a vertical aerial with a radial conductor system was studied. Employing an approximate method the input impedance was calculated. To illustrate the nature of the problem only a quarter-wave unipole was considered in detail since it was the case most easily computed. Curves were plotted showing the dependence of the input base resistance on number and length of radial conductors for a specified frequency and ground conductivity. It is the purpose of this paper to extend the solution and calculations for shorter aerials with top-loading.

With reference to a cylindrical polar co-ordinate system (ρ, ϕ, z) the aerial of height h is coincident with the positive z axis as indicated in Fig. 1. The earth screen is of radius a and lies in the plane $z = 0$ which is also the surface of the ground. The conductivity and permittivity of the ground are denoted by σ and ϵ respectively, and the permittivity of the air is denoted by ϵ_0. The permeability of the whole space is taken as μ_0 which is taken to be that of free space. The intrinsic propagation constant γ and characteristic impedance η of the earth medium are defined by

$$\gamma = [j\mu_0\omega(\sigma + j\omega\epsilon)]^{1/2}$$

and $\eta = [j\mu_0\omega/(\sigma + j\omega\epsilon)]^{1/2}$

where ω is the angular frequency. The propagation constant γ_0 and the characteristic impedance η_0 of the air are then defined by

$$\gamma_0 = j(\epsilon_0\mu_0)^{1/2}\omega = j2\pi/\lambda = j\beta$$

and $\eta_0 = (\mu_0/\epsilon_0)^{1/2} \approx 377$ ohms

where λ is the wavelength in air.

The self-impedance at the terminals of the aerial is now denoted by Z_t which can be broken into two parts by setting, $Z_t = Z_0 + \Delta Z_t$ where Z_0 is the self-impedance of the same aerial if the earth plane were perfectly conducting and infinite in extent. Thus ΔZ_t is the difference between the self-impedance of the aerial over the imperfect and the perfect earth plane. It is called the self-impedance increment and can be written in terms of a real and imaginary part as follows

$$\Delta Z_t = \Delta R_t + j\Delta X_t \quad \ldots \quad (1)$$

where ΔR_t and ΔX_t represent the resistance and reactance increments. If the current amplitude at the terminals of the aerial is I_0, the power required to maintain this current is $I_0^2R_t/2$ watts. If the ground were perfectly conducting, the input power would be $I_0^2R_0/2$ where R_0 is the real part of Z_0. The additional power required to maintain the same r.m.s. current $I_0/\sqrt{2}$ at the terminals is $I_0^2\Delta R_t/2$.

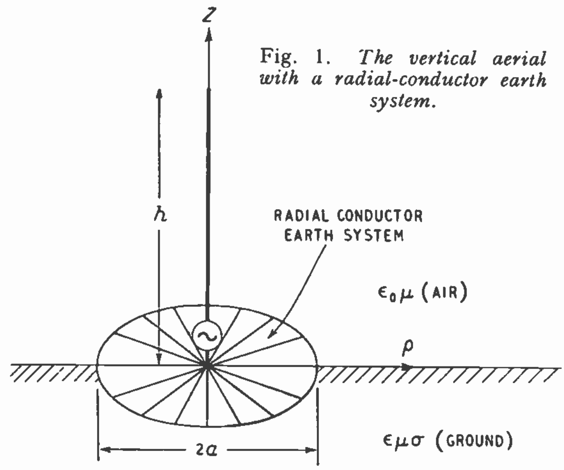

Fig. 1. *The vertical aerial with a radial-conductor earth system.*

General Theory

It was shown in the previous paper[6] that the self-impedance increment ΔZ_t could be written in the following form:

$$\Delta Z_t = -\frac{1}{I_0^2}\int_0^\infty H_\phi^\infty(\rho, 0)\, E_\rho(\rho, 0)\, 2\pi\rho\, d\rho \quad \ldots \quad (2)$$

where $H_\phi^\infty(\rho, 0)$ is the magnetic field of the aerial tangential to a perfectly conducting earth plane of infinite extent and $E_\rho(\rho, 0)$ is the tangential electric field on the imperfect earth.* This formula also follows immediately from the work of Monteath[7]. If the current on the aerial is $I(z)$ amps it follows that

$$H_\phi^\infty(\rho, 0) = -\frac{1}{2\pi}\frac{\delta}{\delta\rho}\int_0^h \frac{\exp.[-j\beta(z^2+\rho^2)^{1/2}]}{(z^2+\rho^2)^{1/2}} I(z)\, dz \quad \ldots \quad (3)$$

The electric field $E_\rho(\rho, 0)$ is essentially an unknown quantity. However, since $|\gamma| \gg |\gamma_0|$ an approximate boundary condition can be employed expressed by,

$$E_\rho(\rho, 0) \approx -\eta_c H_\phi(\rho, 0) \quad \ldots \quad (4)$$

where $H_\phi(\rho, 0)$ is the tangential magnetic field for the imperfect earth and η_c is the surface impedance of the air-ground interface.

If ρ is greater than a (the radius of the earth screen) η_c can be replaced by η. If ρ is less than a, η_c is the intrinsic impedance η_s of the earth screen in parallel with the impedance η of the earth. In a previous investigation[8,9] of this problem it was assumed that η_s was zero, so that the earth system was equivalent to a perfectly-

*N.B., $H_\phi^\infty(\rho,0) = [H_\phi^\infty(\rho, z)]_{z\to 0}$

conducting disc of radius a. The limits of the integration in equation (2) are then from $\rho = a$ to $\rho = \infty$.

A more general case is when η_s is comparable in magnitude to η, in which if follows that

$$\eta_c = \frac{\eta\eta_s}{\eta + \eta_s} \text{ for } 0 \leqslant \rho \leqslant a$$

$$\text{where } \eta_s = j\eta_0 \frac{d}{\lambda} \log_e \frac{d}{2\pi c}$$

and where d is the spacing between the radial conductors and c is the radius of the wire. The expression for η_s has been derived[10] for a wire grid in free space where it was necessary to assume that $|\gamma_0 d| \ll 1$. Since the grid is lying on the earth plane, this restriction must be replaced by $|\gamma_e d| \ll 1$ where γ_e is the effective propagation constant for propagation along a thin wire in the interface and is given by[11]

$$\gamma_e = \left(\frac{\gamma_0^2 + \gamma^2}{2}\right)^{1/2}$$

If there are N radial conductors it can be seen that d can be replaced by $2\pi\rho/N$ since N is usually of the order of 100. It is assumed also that $H_\phi(\rho, 0)$ is not very different from $H_\phi^\infty(\rho, 0)$ in the region of the ground plane where the losses are significant. This approximation has also been discussed previously[8,9] and it certainly appears to be valid if $|\gamma| \gg |\gamma_0|$.

The impedance increment ΔZ_t is then written in the following form

$$\Delta Z_t = \Delta Z + \Delta Z_s \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$\text{where } I_0^2 \Delta Z \approx \eta \int_a^\infty [H_\phi^\infty(\rho, 0)]^2 \, 2\pi\rho \, d\rho \quad \ldots \quad (6)$$

$$\text{and } I_0^2 \Delta Z_s \approx \int_0^a \frac{\eta\eta_s}{\eta + \eta_s} [H_\phi^\infty(\rho, 0)]^2 \, 2\pi\rho \, d\rho. \quad (7)$$

The first term ΔZ corresponds to the self-impedance of the unipole over a perfectly conducting discoid, whereas the second term ΔZ_s accounts for the finite surface impedance of the radial-conductor earth system.

Assuming a sinusoidal current distribution for $I(z)$ the magnetic field $H_\phi^\infty(\rho, 0)$ can be expressed in closed form. The integration indicated by equation (6) can then be carried out and the result expressed in terms of sine and cosine integrals[9]. Curves of the function ΔZ for unterminated aerials have been computed from this formula[6]. The integrations indicated in equation (7), however, cannot be carried out analytically. It is necessary to resort to a numerical procedure for this case.

Impedance Calculation

From equation (3) it follows that

$$H_\phi^\infty(\rho, 0) = \frac{\rho}{2\pi} \int_{z=0}^{h} \left[\frac{\exp.[-j\beta(\rho^2 + z^2)^{1/2}]}{(\rho^2 + z^2)^{3/2}} + j\beta \frac{\exp.[-j\beta(\rho^2 + z^2)^{1/2}]}{\rho^2 + z^2}\right] I(z)\, dz. \quad \ldots \quad \ldots \quad (8)$$

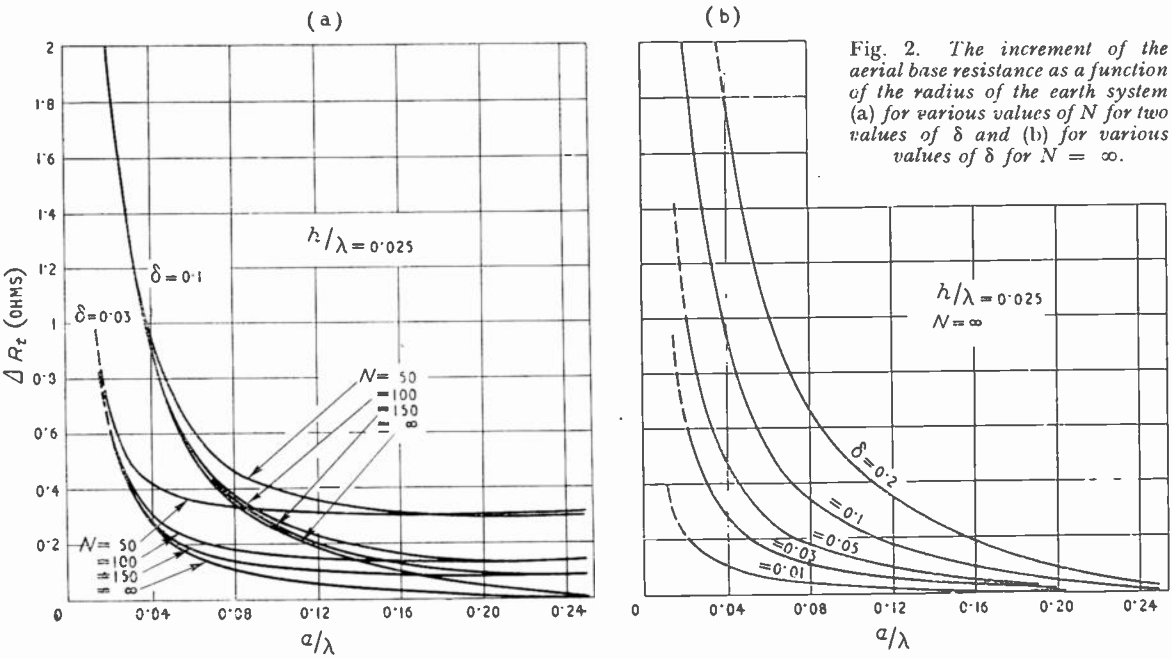

Fig. 2. *The increment of the aerial base resistance as a function of the radius of the earth system* (a) *for various values of N for two values of δ and* (b) *for various values of δ for N = ∞.*

For top-loaded aerials that are short compared with a wavelength a reasonable approximation is $I(z) \approx I_0$, over the length of the aerial. Also, exp. $[-j\beta(\rho^2+z^2)^{1/2}]$ is given to sufficient accuracy by exp. $(-j\beta\rho)$ so that

$$H_\phi^\infty(\rho, 0) \approx \frac{e^{-j\beta\rho}\rho I_0}{2\pi}\int_0^h [(\rho^2+z^2)^{-3/2} + j\beta(\rho^2+z^2)^{-1}]\,dz \approx \frac{e^{-j\beta\rho}\rho I_0}{2\pi}\left[\frac{h}{\rho(\rho^2+h^2)^{1/2}} + \frac{j\beta}{\rho}\tan^{-1}\frac{h}{\rho}\right] \quad \ldots \quad (9)$$

This can be expanded in a power series in (h/ρ) as follows:

$$H_\phi^\infty(\rho, 0) \approx \frac{e^{-j\beta\rho}I_0}{2\pi\rho}\left[\left(\frac{h}{\rho}\right)\left(1+j\beta\rho\right) - \left(\frac{h}{\rho}\right)^3\left(\frac{1}{2}+\frac{j}{3}\beta\rho\right)+\ldots\right] \quad \ldots \quad (10)$$

and correspondingly,

$$\left[H_\phi^\infty\left(\rho,0\right)\right]^2 \approx \frac{I_0^2 e^{-2j\beta\rho}}{4\pi^2\rho^2}\left[\left(\frac{h}{\rho}\right)^2\left(1+j\beta\rho\right)^2 - \left(\frac{h}{\rho}\right)^4\left(1+j\beta\rho\right)\left(1+2j\beta\rho/3\right)+\ldots\right] \quad \ldots \quad (11)$$

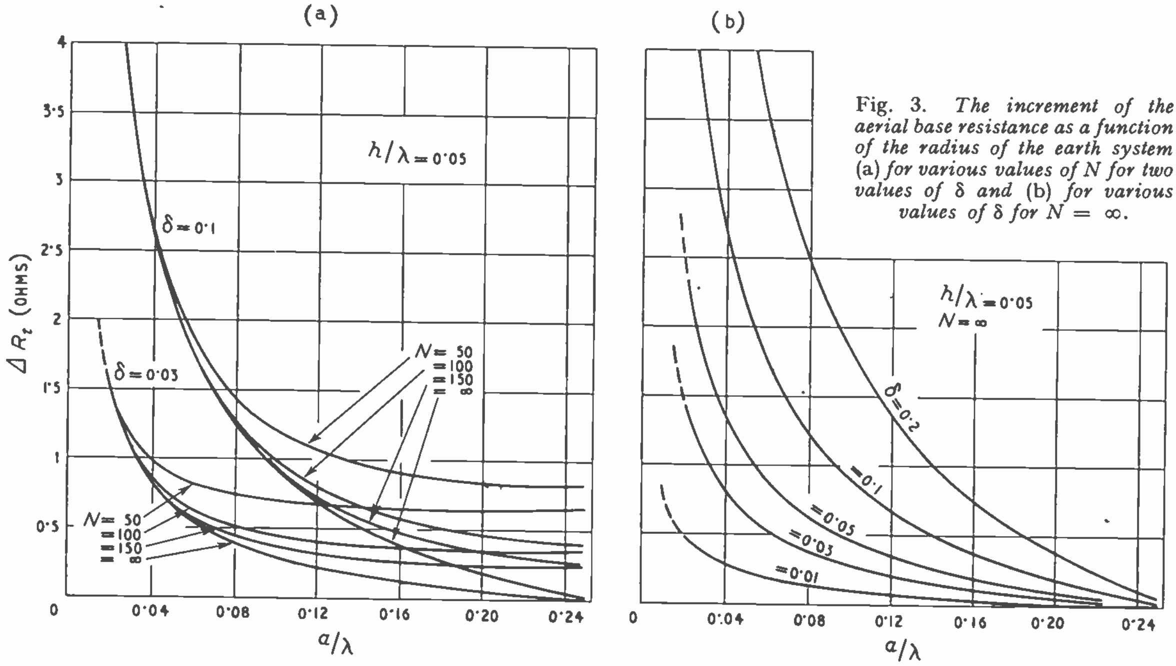

Fig. 3. *The increment of the aerial base resistance as a function of the radius of the earth system* (a) *for various values of N for two values of* δ *and* (b) *for various values of* δ *for* $N = \infty$.

The self-impedance ΔZ is then given by a series of integrals as follows:

$$\Delta Z = \frac{\eta}{I_0^2}\int_a^b [H_\phi^\infty(\rho,0)]^2\, 2\pi\rho\, d\rho + \frac{\eta h^2}{2\pi}\int_b^\infty e^{-2j\beta\rho}(1+2j\beta\rho-\beta^2\rho^2)\rho^{-3}\,d\rho$$

$$-\frac{\eta h^4}{2\pi}\int_b^\infty e^{-2j\beta\rho}(1+5j\beta\rho/3-2\beta^2\rho^2/3)\rho^{-5}\,d\rho + \ldots \quad \ldots \quad (12)$$

The integration of ρ has been broken conveniently into two ranges, from a to b and b to ∞. The distance b is chosen sufficiently large so that the series of integrals converges rapidly. The integral with limits a to b is integrated by graphical means. Using the exponential integral defined by,

$$Ei(-2j\beta b) = -\int_b^\infty e^{-2j\beta\rho}\rho^{-1}\,d\rho$$

the impedance ΔZ is written

$$\Delta Z = \frac{\eta}{I_0^2}\int_a^b [H_\phi^\infty(\rho,0)]^2\, 2\pi\rho\, d\rho + \frac{\eta h^2\beta^2}{2\pi}\left[\frac{1}{\beta b}\left(\frac{1}{2\beta b}+j\right)e^{-2j\beta b} - Ei\left(-2j\beta b\right)\right]$$

$$-\frac{\eta h^2\beta^2}{2\pi}\left(\frac{h^2}{b^2}\right)\left\{\left[\left(\frac{1}{4\beta^2b^2}+\frac{1}{18}\right)+j\left(\frac{7}{18\beta b}-\frac{\beta b}{9}\right)\right]e^{-2j\beta b} + \frac{2}{9}\beta^2b^2\,Ei\left(-2j\beta b\right)\right\} + \frac{\eta h^2\beta^2}{2\pi}\times O\left(\frac{h^4}{b^4}\right) \quad \ldots \quad (13)$$

When $h^2/b^2 \ll 1$, the terms containing higher powers of h^2/b^2 can be neglected.

The increment of the input resistance ΔR_t for purposes of calculation is now written

$$\Delta R_t = \text{Real part of } (\Delta Z_s + \Delta Z)$$

$$= \frac{\sqrt{2}}{2\pi}\int_0^A \frac{pq\,F^2(H_1,P)}{[p^2+(p+q)^2]^{1/2}\,P}\cos\left(2\psi - 4\pi P + \frac{3\pi}{4} - \tan^{-1}\frac{p+q}{p}\right)dP$$

$$+ \frac{\sqrt{2}}{2\pi}\int_A^B F^2(H_1,P)\,P^{-1}\cos(2\psi - 4\pi P + \pi/4)\,dP + \sqrt{2}\,\pi\,p\,H_1^2\,V\cos(\theta + \pi/4) \quad \ldots \quad \ldots \quad (14)$$

where $F(H_1, P)\,e^{j\psi} = \dfrac{H_1/P}{[1+(H_1/P)^2]^{1/2}} + j\,2\pi P\tan^{-1}(H_1/P)$, ... (15a)

$$p = 120\pi\delta/\sqrt{2}, \qquad q = \frac{240\pi^2 P}{N}\log_e\frac{P}{NC_1}, \qquad \delta = (\epsilon_0\omega/\sigma)^{1/2},$$

$$H_1 = h/\lambda, \qquad P = \rho/\lambda, \qquad A = a/\lambda, \qquad B_1 = b/\lambda \text{ and } C_1 = c/\lambda,$$

and

$$Ve^{j\theta} = 2\left[\frac{j}{\beta b}\left(1 - \frac{j}{2\beta b}\right)e^{-j2\beta b} - Ei\left(-2j\beta b\right)\right] \quad \ldots \quad \ldots \quad (15b)$$

Summarizing, this formula for ΔR_t should be accurate to within a few per cent under the restrictions that $H_1 \leqslant 0{\cdot}1$ (electrically short aerial), $\epsilon\omega/\sigma \ll 1$ (negligible displacement current in soil), and $(B_1/H_1)^2 = (b/h)^2 = 25$.

Presentation of Results

It hardly needs to be mentioned that the major part of this work has to do with the evaluation of the integrals in equation (14). A graphical procedure was adopted employing a conventional area planimeter. The resulting values of the integrals, so obtained, are believed to be accurate to within 1%.

The computed values of ΔR_t for fixed aerial heights are plotted as a function of a/λ for various values of δ and N in Figs. 2 to 4. The wire radius to wavelength ratio, c/λ, is taken to be 10^{-6} which corresponds to No. 8 B. & S. wire at 183 kc/s. The curves in Figs. 2(a), 3(a) and 4(a) are for two fixed values of the ground conductivity parameter δ, whereas the curves in 2(b), 3(b) and 4(b), for a perfect ground screen $N=\infty$, show a wider range of δ. With the results plotted in this form, values of ΔR_t for intermediate values of N and δ can be estimated quickly by interpolation. For purposes

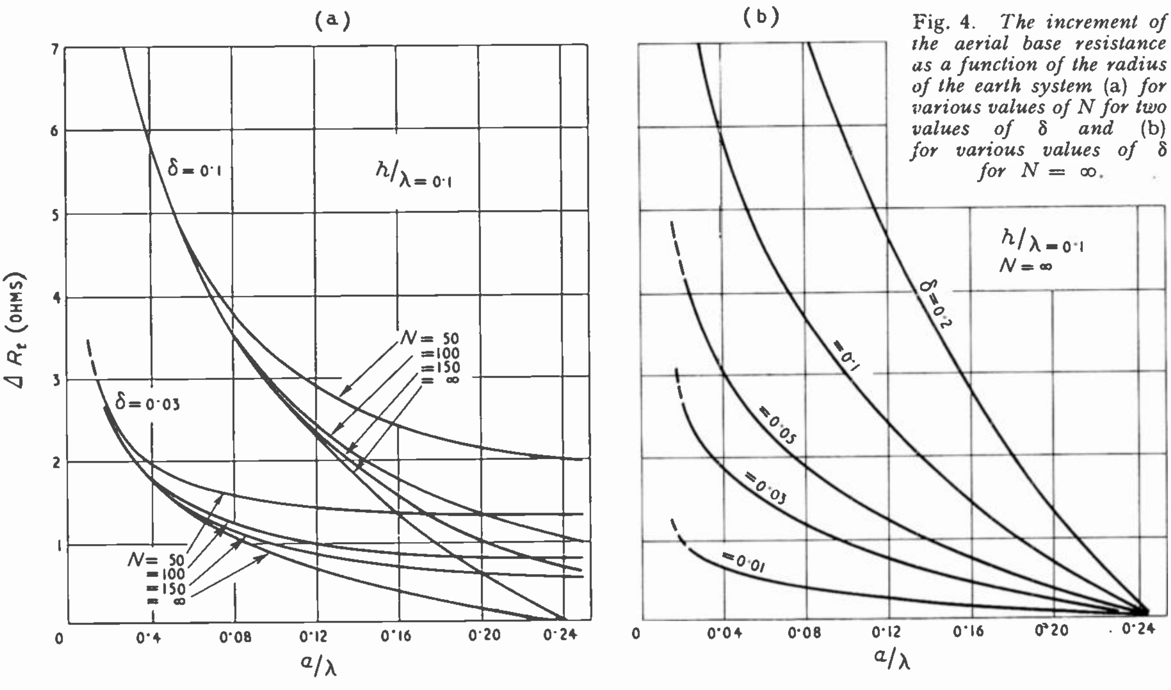

Fig. 4. *The increment of the aerial base resistance as a function of the radius of the earth system* (a) *for various values of N for two values of δ and* (b) *for various values of δ for N = ∞.*

of comparison, the values of ΔR_t for a quarter-wave unipole ($h = \lambda/4$) are shown plotted in Fig. 5(a) and 5(b). The data for these curves are taken from a previous paper[6].

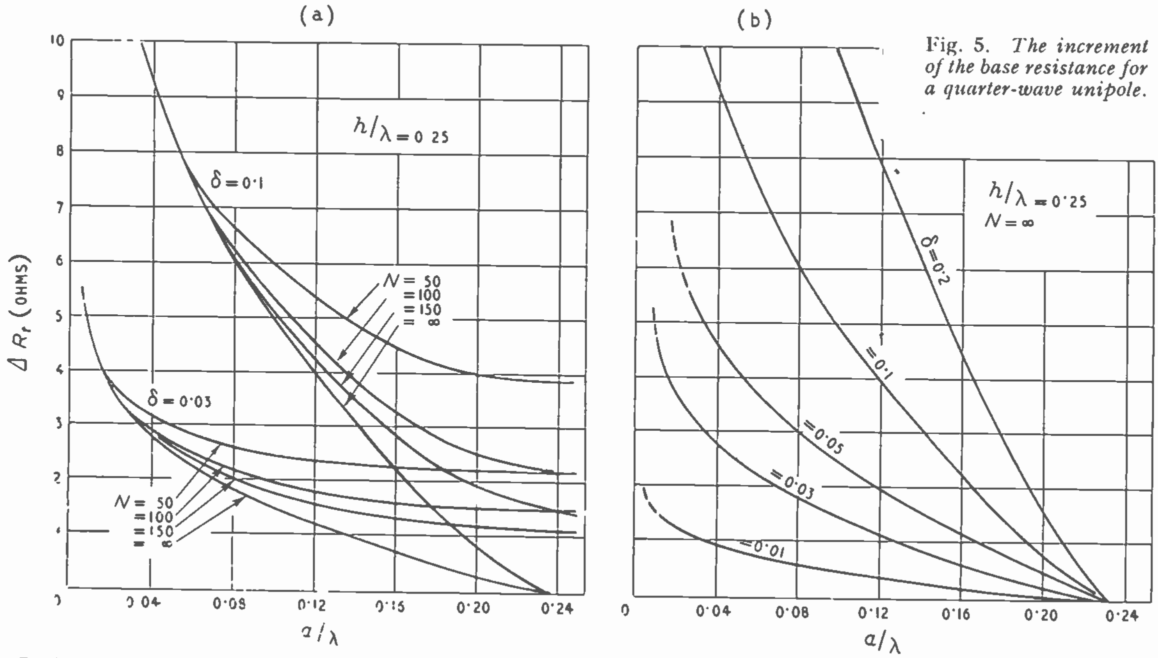

Fig. 5. *The increment of the base resistance for a quarter-wave unipole.*

It is apparent immediately that the increase of ΔR_t with diminishing earth-screen radius is much more rapid for the short aerials than for the quarter-wave unipole. This behaviour is connected with the fact that the induction and static fields of short aerials are more significant than those for a higher aerial, such as a quarter-wave unipole.

The appropriate value of δ to use in connection with these curves is obtained conveniently from Fig. 6, when the ground conductivity and the frequency are specified.

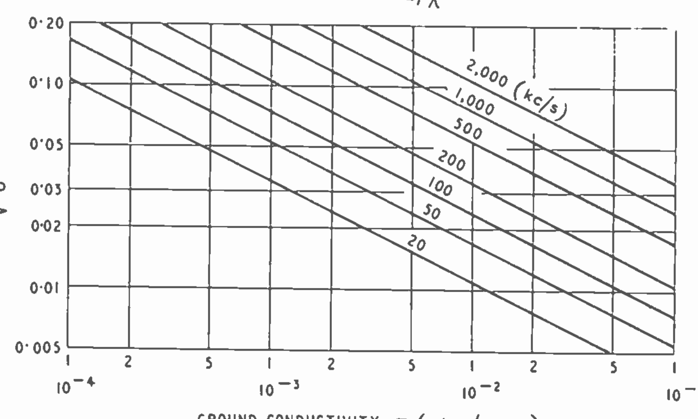

Fig. 6. *The parameter δ as a function of frequency in kc/s and ground conductivity.*

The effect of changes in wire radius is slight. This fact is illustrated in Fig. 7 where ΔR_t is shown plotted as a function of a/λ for various values of the wire radius/wavelength ratio for a quarter-wave unipole.

For earth screens which are small compared with the wavelength ΔR_t varies in a linear manner with δ. That is, it varies directly as the square root of the frequency and inversely as the square root of the ground conductivity. However, for larger values of a/λ as is illustrated in Fig. 8, the values of ΔR_t become somewhat less sensitive to changes in δ.

Up to this point, the discussion has been limited mainly to vertical aerials with ideal top-loading. It is usually justified in these cases to assume that the current distribution is constant from the base of the aerial to its upper end. It has been shown also by Brown[1,2], Smeby[12], and others[13] that the radiation resistance R_0 can be computed by considering only the current on the vertical portion of the aerial. The contributions for the currents flowing on the loading 'umbrella' or 'cone' are usually negligible if the top-loading has been adjusted for maximum radiation resistance. This radiation resistance is given by [14]

$$R_0 \approx 160\,\pi^2\,(h/\lambda)^2$$

It is possible now to apply the above results to aerials without top-loading by defining an 'equivalent' or 'effective' height h. This value is obtained by assuming that the loaded and unloaded unipoles are electrically equivalent if their radiation resistances are equal. For thin unloaded unipoles the current distribution is approximately sinusoidal and if its actual height is denoted by h_0, the radiation resistance, R_0, is given by

$$\sin^2 \beta h_0 \, R_0 = 30\,(C + \log 2\beta h_0 - \mathrm{Ci}\, 2\beta h_0) + 15\,(\mathrm{Si}\, 4\beta h_0 - 2\,\mathrm{Si}\, 2\beta h_0) \sin 2\beta h_0 + 15\,(C + \log \beta h_0 - 2\,\mathrm{Ci}\, 2\beta h_0 + \mathrm{Ci}\, 4\beta h_0) \cos 2\beta h_0 \quad \ldots \quad (16)$$

where $C = 0{\cdot}5772$. Si and Ci are the sine and cosine integral functions. R_0 is just one-half of the self-resistance of a thin, centre-driven, aerial of length $2h_0$ situated in free space[14]. When h_0/λ is small compared with unity, $R_0 \approx 40\,\pi^2\,(h_0/\lambda)^2$.

Using these formulae to calculate the equivalent height h of the unloaded aerials it follows that for $h/\lambda = 0{\cdot}025$, $0{\cdot}050$ and $0{\cdot}10$ $h_0/\lambda = 0{\cdot}050$, $0{\cdot}095$, and $0{\cdot}175$ respectively. In other words, it is probable that the curves in Figs. 2, 3, and 4 for loaded aerials also apply to unloaded aerials of heights $0{\cdot}050\lambda$, $0{\cdot}095\lambda$ and $0{\cdot}175\lambda$ respectively. The accuracy of this procedure can be checked by comparing the results with more exact previous calculations for an unloaded unipole situated over a circular, perfectly conducting, disc laid on the ground. Employing the data in Figs. 2, 3 and 4 the function ΔR_t for the loaded unipole and $N = \infty$ is plotted in Fig. 9 as a function of h/λ for selected values of a/λ. The corresponding curves, for the unloaded unipole are obtained by the above mentioned approximate procedure and are also shown in Fig. 9. The encircled points on the dotted curves are obtained from calculations carried out from equation (16) of reference 6. The points indicated by $\times$ for $h/\lambda = 0{\cdot}25$ can be obtained either from equation (16) of reference

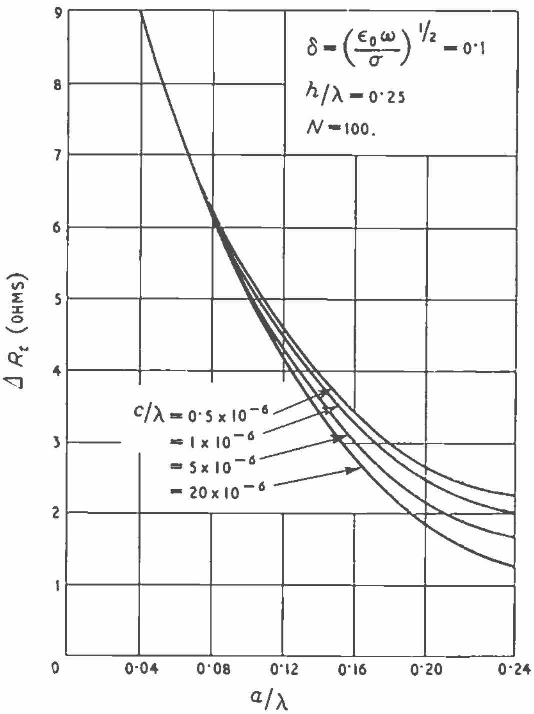

Fig. 7. *The effect of changing wire radius on the increment of input resistance ΔR_t.*

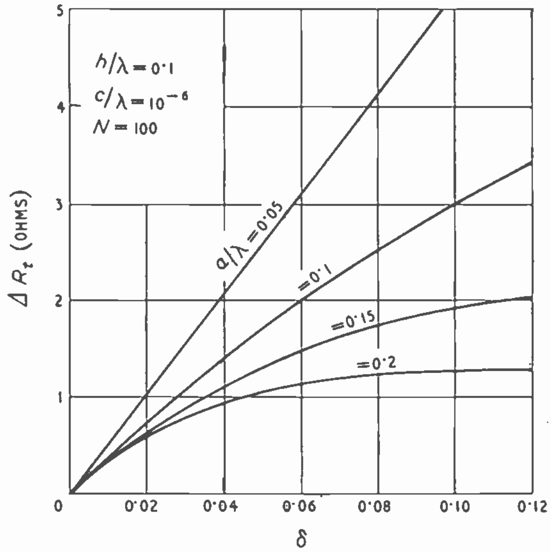

Fig. 8. *An illustration to show how ΔR_t varies with ground conductivity and frequency.*

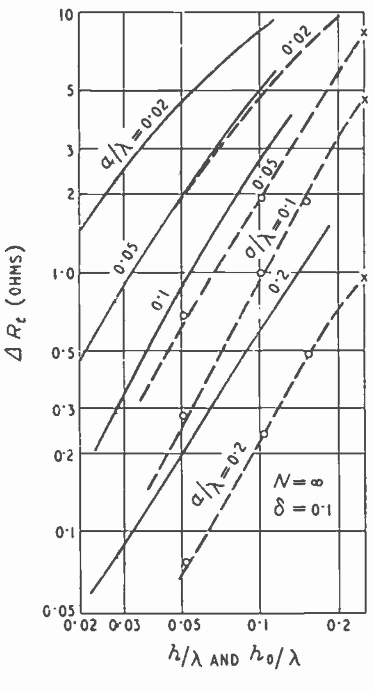

Fig. 9. *The solid curves represent ΔR_t for an ideally loaded unipole of height h, whereas the dashed curves correspond to the estimated values for an unterminated unipole of height h_0. The indicated points are plotted from more exact formulae.*

6 or directly from Fig. 5(b). The good agreement between the two methods of calculation for ΔR_t is reassuring.

It is also instructive to plot ΔR_t as a function

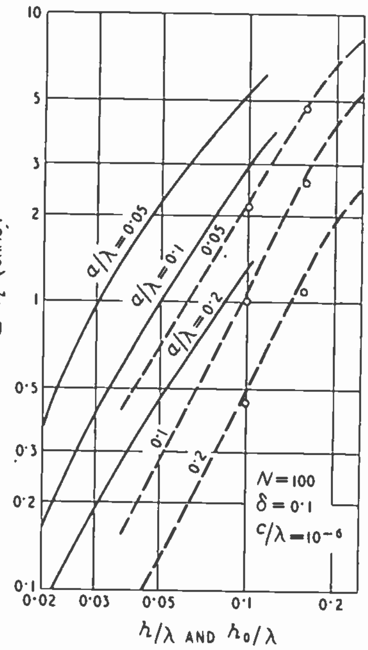

Fig. 10. *The curves for the loaded unipole and the estimated curves for the unloaded unipole are shown by the solid and dashed lines respectively. The encircled points represent values computed by Monteath.*

of h/λ for both loaded and unloaded unipoles for a finite value of N. The results are shown in Fig. 10 for $N = 100$, $\delta = 0{\cdot}1$, $c/\lambda = 10^{-6}$ employing data from Figs. 2, 3, 4 and 5. The encircled points correspond to results, communicated to us privately by Mr. G. D. Monteath of the B.B.C. for unloaded unipoles with heights of $0{\cdot}10\lambda$ and $0{\cdot}167\lambda$. Again the agreement is very satisfactory.

Both the curves in Figs. 9 and 10 illustrate the near linear log-log relationship between ΔR_t and h/λ. This behaviour is also prevalent for other values of N and δ and provides a convenient means of interpolating and extrapolating for values of h/λ other than those shown in Figs. 2 to 6.

Conclusion

No attempt has been made in this paper to consider the economic factors but rather the emphasis has been on showing the manner in which the impedance varies with the number and length of radial wires, aerial height, and ground conductivity. The curves should be useful in the design of earth systems for low-frequency transmitting aerials. It would appear that many earth systems are probably more extensive than necessary since the benefits gained by employing large radius screens are lost if there are not a sufficient number of radial conductors. This is particularly so if the ground conductivity is relatively high.

The calculations for the base input-resistance are derived on the assumption of uniform current distribution along the vertical portion of the aerial. It has been indicated, however, that the results are also applicable to aerials with non-uniform current distribution if the quantity h is regarded as an effective height.

Acknowledgments

The need for the curves presented in this paper was pointed out by Dr. T. W. Straker. Further advice and suggestions were also given by Dr. F. R. Abbott, Dr. W. J. Surtees, Mr. R. S. Thain and Mr. J. S. Belrose. Appreciation must also be expressed to Mr. G. D. Monteath of the British Broadcasting Corporation who communicated to us unpublished reports of his research.

APPENDIX

Measurements on a 250-ft umbrella top-loaded unipole show good agreement with the theory. The following observed values were supplied to us by Mr. R. S. Thain of this laboratory:

Length of Radials, a = 800 ft.
Number of Radials, N = 120
Ground Conductivity, $\sigma = 2{\cdot}0 \times 10^{-3}$ mhos/metre
Frequency, 97 kc/s
Radiation Resistance as calculated from field strength measurements, $R_0 = 0{\cdot}50$ ohm
Input resistance measured on a bridge, $R_i = 0{\cdot}75$ ohm
Observed resistance increment, $\Delta R_t = 0{\cdot}25$ ohm
Theoretical value of resistance increment (for $h/\lambda = 0{\cdot}025$, $a/\lambda = 0{\cdot}08$, $\delta = 0{\cdot}07$), $\Delta R_t = 0{\cdot}23$ ohm.

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Note: In a recent paper, G. Bekefi, (*Can. Jour. of Phys.*, 1954, Vol. 32, p. 205) has applied a variational technique to obtain a solution which agrees favourably with our equation (13) and the formula of Monteath[7] for the ideal circular ground screen.

TRANSFORMATION FOR CONSTANT-IMPEDANCE NETWORKS

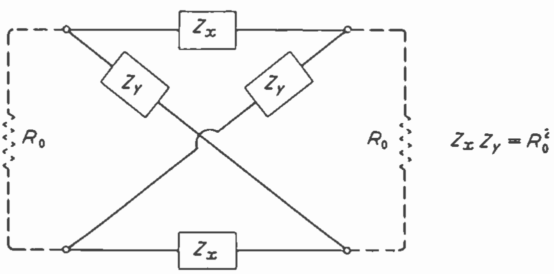
By H. J. Orchard, M.Sc., A.M.I.E.E.

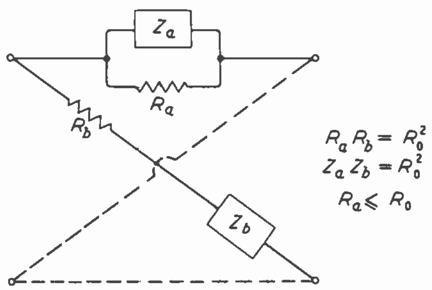
SUMMARY.—Certain constant-impedance networks having a restricted range of variation of loss often contain components which are difficult to manufacture. The transformation which is described overcomes this difficulty at the expense of two extra resistors in the network and a small amount of frequency-independent loss added to the characteristics. Another application is the absorption of inductor dissipation in special cases.

1. Constant-Impedance Networks

IN line-transmission circuits the distortion produced by cables, filters and other related equipment is frequently corrected by the use of constant-impedance networks, the most general form of which is represented by the symmetrical lattice configuration shown in Fig. 1. The name derives from the fact that the image impedance is a constant resistance R_0, achieved by making the two arm impedances Z_x and Z_y satisfy the relation

$$Z_x \cdot Z_y = R_0^2 \quad \ldots \quad (1)$$





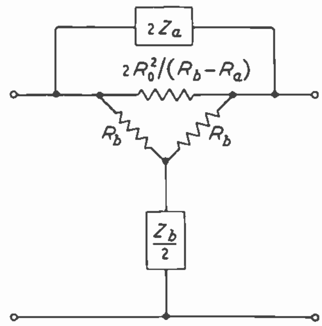


Fig. 1 (above). *General form of lattice network.*

Fig. 2 (right). *Restricted form of lattice network.*

Fig. 3 (extreme right). *Bridged-T equivalent of Fig. 2.*

Such networks were originally proposed by Zobel[1] and have found very wide application because of their simplicity and generality; in fact Bode[2] has proved the remarkable property that any physically-realizable transfer characteristic can be produced by a single constant-impedance network of this type when operating between resistive terminations equal to R_0.

In the interests of easy construction it is often preferable to provide any given characteristic by means of a tandem connection of fairly simple networks. When only the loss (and not the phase) is prescribed these networks are conveniently restricted in complexity to the form shown in Fig. 2, with $R_a \leqslant R_0$ and Z_a a simple reactance of either one or two elements. Although the condition $R_a \leqslant R_0$ confines the network to producing minimum-phase characteristics and restricts the maximum phase shift to $\pm$ 90 degrees, it does, nevertheless, allow it to be constructed in the equivalent bridged-T form shown in Fig. 3, and this is how it is most commonly made. Despite the rather severe restrictions imposed upon Z_a it is still possible, as Bode[2] has proved, to produce any physically-realizable loss characteristic by using a suitable tandem connection of these bridged-T networks.

Maximum loss occurs when Z_a is infinite (and Z_b is consequently zero) and, because the network then behaves like a pad, the value of this maximum loss is known as the pad loss. Denoting this pad loss, in nepers, by A_1 we have the relations

$$R_a = R_0 \tanh \tfrac{1}{2}A_1 \quad \ldots \quad (2a)$$

$$R_b = R_0 \coth \tfrac{1}{2}A_1 \quad \ldots \quad (2b)$$

$$\frac{2R_0^2}{R_b - R_a} = R_0 \sinh A_1 \quad \ldots \quad (2c)$$

When A_1 is small, R_a becomes small compared with R_0 and the result is to lower the impedance level of Z_a which is in parallel with R_a. This not only leads to component values in Z_a and Z_b which are difficult to manufacture but also magnifies the effects of stray inductances and capacitances. The main purpose of the trans-

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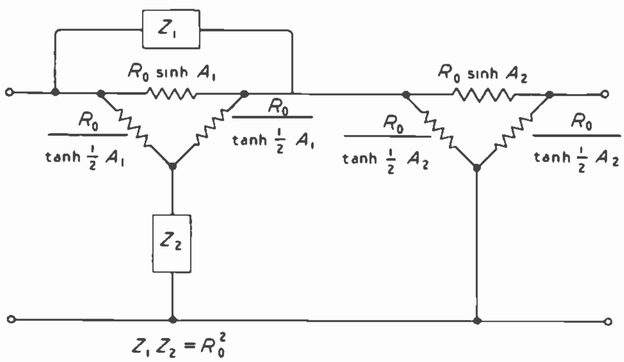
formation to be described in the next section is to raise the impedance level of Z_a so as to reduce these undesirable effects accompanying a small pad loss. A subsidiary application is to permit the absorption of dissipation in the inductors when Z_a is a series-tuned circuit and Z_b a parallel-tuned circuit.



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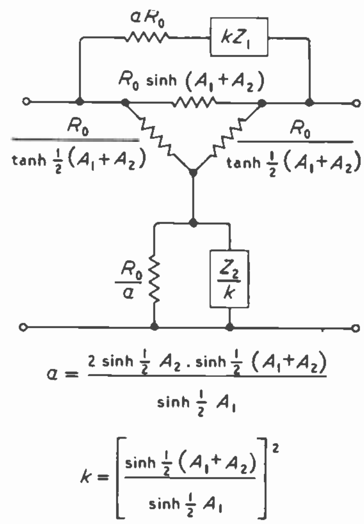

Fig. 4. *The tandem connection of a bridged-T network and a pad is equivalent to a single, more complex bridged-T.*

2. The Transformation

The transformation can be regarded as the rearrangement into one network of a tandem connection of two networks, one being a bridged-T network of the form just described with a pad loss of A_1 nepers and the other a symmetrical pad with a loss of A_2 nepers, both networks having the same image impedance R_0. The result is illustrated in Fig. 4. The reactance arms, which were shown in Fig. 3 as $2Z_a$ and $Z_b/2$, have been rewritten Z_1 and Z_2 respectively while the resistance components have been given in terms of R_0 and A_1 and A_2 rather than the arms of the parent lattice.

The characteristics of the transformed network appearing on the right-hand side of Fig. 4, will be the same as those of the bridged-T network on the left-hand side from which it is derived except for an additional constant loss of A_2 nepers. This additional constant loss represents the sacrifice which must be made for any benefits which the transformation may confer.

The proof of the transformation is fairly simple and will be merely outlined. One method is to put all three networks into lattice form and note that the series-arm impedances are given by $R_0 \tanh \frac{1}{2}\theta$ where θ is the image transfer coefficient of the network concerned. As the tandem-connected networks are matched as regards image impedances their transfer coefficients add directly, so that by applying the addition theorem for the hyperbolic tangent we get an expression for the series-arm impedance of the transformed lattice in terms of those of the other two. This expression can then be broken down into the form indicated in Fig. 4.

A somewhat similar equivalence has been given by Dagnall and Rounds[3] in connection with the absorption of dissipation in lattice all-pass networks.

3. Applications

The main purpose of the transformation as mentioned in Section 1 is to raise the impedance level of Z_1 when the pad loss A_1 is small and it achieves this by virtue of the large value obtained for k, the multiplying constant on Z_1. k is given by the expression

$$k = \left[\frac{\sinh \frac{1}{2} (A_1 + A_2)}{\sinh \frac{1}{2} A_1}\right]^2 \quad \ldots \quad \ldots \quad (3)$$

and for values of A_1 and A_2 less than about 0·25 neper (2·2 db) we may approximate the hyperbolic functions by the first term in their power series expansions and obtain the useful approximation

$$k \approx \left(1 + \frac{A_2}{A_1}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

in which the units of A_1 and A_2 can be taken either in nepers or in decibels.

Thus, for example, when the pad loss in the original network is only 0·2 db say, the addition of 1·0 db of constant loss will cause the impedance level of Z_1 in the transformed network to be increased by a factor of approximately $(1 + 1{\cdot}0/0{\cdot}2)^2 = 36$ times. It is in this region of losses (i.e., less than about 1·0 db) that the use of the transformation will be found of greatest value.

As the amount of added loss A_2 is not critical it is often convenient to choose its precise value so that the shunt resistors $R_0/\tanh \frac{1}{2} (A_1 + A_2)$ in the transformed network assume some preferred value.

Another use for the transformation is in the case where Z_1 is a series-tuned circuit when the effect of dissipation in the inductors may cause a greater change of the characteristic around the frequency of resonance than can be tolerated. In applying the transformation we place a resistor aR_0 in series with kZ_1; reducing this resistor by the amount of the equivalent series resistance of the manufactured inductor in kZ_1 will effectively remove the dissipation. The parallel-tuned circuit in the inverse arm Z_2/k will have a corresponding resistor R_0/a for absorbing the equivalent shunt resistance of its inductor. If both inductors have the same Q and the value of A_2 is carefully chosen the extra resistances required in the network can be provided entirely by inductor dissipation so eliminating the need for extra components.

Acknowledgment

Acknowledgment is made to the Engineer-in-Chief of the G.P.O. for permission to make use of the information contained in this paper.

REFERENCES

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[2] H. W. Bode, "Network Analysis and Feedback Amplifier Design", D. Van Nostrand, New York, 1945.
[3] C. H. Dagnall and P. W. Rounds, "Delay Equalization of Eight-Kilocycle Carrier Program Circuits", *Bell System Technical Journal*, 1949, Vol. 28, p. 181.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

High-Stability Oscillators

SIR,—In spite of the time since it appeared, I would like to draw attention to a point in G. G. Gouriet's paper in *Wireless Engineer* for April 1950. A statement is made which I believe to be incorrect and it is repeated in a later paper in *Proc. Inst. Radio Engrs* by J. K. Clapp. This statement is that the effect upon frequency of a phase-change caused by non-linearity is directly proportional to the L/C ratio. I maintain that the effect upon frequency is independent of the L/C ratio.

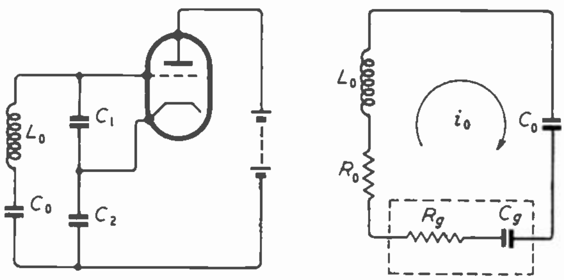

Fig. 1.

In Fig. 1(a), I show the basic circuit and, at (b), its equivalent, these being Figs. 4 and 3 respectively of Gouriet's paper. In this, he regards R_g and C_g as the maintaining circuit and L_0, C_0 and R_0 as the frequency-determining circuit. I regard this division as incorrect, because when C_0 is increased to infinity, and the circuit becomes that of an ordinary Colpitts' oscillator, one is left with only L_0 and R_0 in the 'frequency-determining circuit'.

It is plain from Fig. 1(a) that the circuit is, in reality, that of a Colpitts' oscillator in which, because of C_0, the valve is effectively tapped down the tuned circuit. The resulting transformation ratio is the same for in-phase components of anode current as for quadrature components. The phase relation between the components of the tank current will thus be the same as the phase relation between the components of anode current causing them. Consequently, the degree of tapping down has no effect upon the reaction of the tuned circuit to changes of phase of the anode current of the valve driving it.

I submit, therefore, that the circuit of Fig. 1(a) has no basic advantage over the ordinary Colpitts' oscillator. In practice, of course, its use may be advantageous in that it may permit more suitable values of components to be used.

San Diego, California, U.S.A.
28th February, 1955.

W. B. BERNARD

Multiloop Feedback Amplifiers

SIR,—I was very interested in Mr. Cutteridge's article 'Multiloop Feedback Amplifiers' in your November issue of last year, and also in the comments made by Dr. Cruickshank (letter, February 1955). There are one or two remarks I would like to make regarding the stability problem associated with multiloop feedback amplifiers.

Nyquist's stability criterion, in its original form, was established for the single-loop amplification function. This function is most valuable in design work and can be measured experimentally. However, the function used in Mr. Cutteridge's work is

$$1 - A_1 B_1 - A_1 A_2 B_3 - A_2 B_2 + A_1 A_2 B_1 B_2 \qquad (1)$$

This equated to zero is the characteristic equation of the differential equation of the system. The classical way of determining the stability of the system consists of direct calculation of the roots of this equation. If any root of this equation has a positive real part, this system is unstable, and vice versa.

It should be borne in mind that Nyquist's plot is the conformal mapping of the frequency axis ($j\omega$) and that the area inside the graph represents a map of the right-hand half of the complex-frequency plane. If it does not contain the origin the characteristic equation of the system has no roots with a positive real part, and consequently the system is stable. Function (1) equated to zero, being a polynomial of the complex frequency p, apparently needs only one mapping for deciding whether it has any roots with a positive real part. This use of Nyquist's plotting of the characteristic equation has already been noted by some authors,[1, 2, 3]. Furthermore, as F. E. Bothwell[2] showed, when using the characteristic equation of the system, it is more advantageous, and easier, to use the Routh-Hurwitz determinant instead of the Nyquist plot. He states that it is not difficult by means of the Routh-Hurwitz criterion to find even the boundary conditions for stability. However, it must be admitted that this is only possible in the relatively simple cases.

In his letter, Dr. Cruickshank suggested using the inverse Nyquist diagram, and hence the following function

$$A_1 A_2 \left(\frac{1}{Y} - B_3\right) = 0 \quad \ldots \quad (2)$$

This does not, however, change the matter, for it is easy to see, if one substitutes the expression for Y in this equation, that it becomes exactly the same as the equation proposed by Mr. Cutteridge [(1) above]; i.e., the characteristic equation of the system differential equation.

On the other hand, if the loop amplification function is used, because of the above mentioned practical value in design work, the function for Nyquist plotting becomes, using the same notation as in Mr. Cutteridge's Fig. 1(c):—

$$\frac{B_3 A_1 A_2}{(1 - A_1 B_1)(1 - A_2 B_2)} \quad \ldots \quad (3)$$

This function, as Dr. Cruickshank has mentioned, may have poles in the right half-plane. By using the so called Principle of Argument of the Meromorphical functions, it is possible to find a very simple analytical criterion to decide the exact number of the poles of the loop amplification function in the right half-plane. (It is worth while to notice that this theorem can be used in a straightforward proof of the Routh-Hurwitz criterion.) This can be done fairly easily by inspection of function (3) on the imaginary axis without plotting any diagram, and then it is sufficient only to plot one Nyquist diagram of the loop amplification function to decide upon the stability of the multiloop system. This procedure seems to be a much easier way than the one proposed by Bode which consists in successive plotting of Nyquist's diagram.

Very much confusion exists about the definition and use of Nyquist's stability criterion. Nyquist's criterion is the conformal mapping of the imaginary axis ($j\omega$) by means of a *particular* function—this function being almost exclusively used in practical design because, as well as showing whether or not a system is stable, it can be helpful in deciding what modifications are necessary in the system to make, say, an unstable system stable. This is the great reason for its common use. It is only when the loop amplification function is used that we have a true Nyquist plot. But if we allow the change of function to say the characteristic equation, which is what Mr. Cutteridge did, it is evident then that the Routh-Hurwitz criterion is much easier to apply and more powerful in use.

B. D. Rakovich

Marconi College,
Chelmsford, Essex.
23rd March 1955.

[1] A. Vazsonyi, "A Generalization of Nyquist's Stability Criteria", *Journal of Applied Physics*, Sept. 1949, Vol. 20, pp. 863–867.
[2] F. E. Bothwell, "Nyquist Diagrams and the Routh-Hurwitz Stability Criterion", *Proc. Inst. Radio Engrs*, Nov. 1950, Vol. 38, p. 1345.
[3] A. Leonhard, "A New Method of Investigating Stability", *Arch. Elektrotech.* Jan.-Feb. 1944, Vol. 38, pp. 17–28.

Coefficient Errors in Analogue Computors

Sir,—In an analogue computor designed to solve linear differential equations it is usually desirable to know the inaccuracy of the solution due to known coefficient errors.

Let the subsidiary equation which the computor is to solve, be

$$F(p)\bar{x} = \sum_0^n A_r p^r x = 0$$

and let the error in A_r be δA_r.

The equation which the computor solves is therefore

$$\sum_0^n (A_r + \delta A_r) p^r \bar{x} = 0$$

$$\text{i.e.,} \sum_0^n A_r p^r + \sum_0^n \delta A_r p^r = 0 \quad \ldots \quad (1)$$

If the error [i.e., the second term in (1)] is small this will produce a displacement of the roots of the original equation by a small amount. Let the k^{th} root of the original equation be S_k. Then the displaced root is $S'_k = S_k + \beta_k$.

Then $F(S'_k) = F(S_k + \beta_k) = F(S_k) + \beta_k F'(S_k) + \ldots$ by Taylor's Theorem.

From (1)

$$\sum_0^n A_r S'^r_k + \sum_0^n \delta A_r S'^r_k = 0$$

Therefore

$$F(S_k) + \beta_k F'(S_k) = -\sum_0^n \delta A_r S_k^r$$

Neglecting terms in β_k greater in power than the first, since β_k is small.

Hence

$$\beta_k = -\frac{\sum_0^n \delta A_r S_k^r}{F'(S_k)}.$$

As an example the error in the roots of a second-order equation will be derived in the case where $\delta A_0 = 0{\cdot}1\%$. The equation is $\ddot{x} + \dot{x} + x = 0$, $F(p) \equiv p^2 + p + 1 = 0$, $F'(p) = 2p + 1$, $A_0 = 1$, $A_1 = 1$, $A_2 = 1$, $\delta A_0 = 0{\cdot}001$,

$$S_1 = \frac{-1 + \sqrt{3}j}{2}, \quad S_2 = \frac{-1 - \sqrt{3}j}{2}$$

$$\beta_1 = \frac{\sqrt{3}j}{3000}, \quad \beta_2 = \frac{-\sqrt{3}j}{3000}.$$

This represents a change in frequency which is clearly seen to be correct by working out the error from the solution of a quadratic.

The method described above for estimating the errors due to inaccurate coefficients should prove useful in assessing the accuracy of an analogue computor in each individual case.

H. Fuchs

Electronics Dept.,
The University,
Southampton.
31st March 1955.

CORRECTION

Three errors occurred in equation (2.2) of Appendix 2 of the paper "Discriminator Circuit Analysis" in the April issue. The bracketted expression on the right-hand side of the equation for $D^n.v$ should be raised to the nth power. The terms D^{-2} and D^{-m} should both be D^{-1}.

NEW BOOKS

Dielectrics and Waves

By Arthur R. von Hippel. Pp. 284 + xii. Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 128s.

Prof. von Hippel, who is Professor of Electrophysics and Director of the Laboratory for Insulation Research at the Massachusett's Institute of Technology, is well known as an authority on dielectrics. His book is more than an excellent account of these materials. It is an attempt to establish intercommunication between the advancing sectors of physics and engineering all along the line. He shows how problems in pure physics are amenable to the methods of electrical engineering, and how the physicist's microscopic approach has developed materials tailored to engineering requirements. He calls the book "a biography of dielectrics", and hopes that the reader may follow it with the attention normally reserved for a detective story. It is indeed biography on the Odyssean scale, and detection shorn of Baker Street irregularity—definitely not "elementary . . .".

Physicists and engineers have devised their own worlds of statistics, quanta and fields. Between the worlds of microscopic concept and macroscopic practice lies the limbo of actual individual materials. The author approaches this from both sides, and the two paths converge happily on their objective. The book is really a treatise on electromagnetism and another on modern physics, both leading up to (or woven into) the study of dielectrics. Two separate books of this standard would do any author credit, but to amalgamate them into a single continuous story is a magnificent achievement.

The ideas of complex permeability and permittivity are first presented, and these lead to a description of dielectric properties in terms of the parameters, attenuation distance, decibel loss, complex refractive index, and conductivity. Several pages of nomograms are given for reference. Optical behaviour is related to electrical properties; it is refreshing to find geometrical and physical optics lucidly summarized in terms of the properties of a loss-free dielectric. Field phenomena are discussed in terms of equivalent circuits, and the lumped circuit equivalents are applied to properties of materials. The molecular world of physics is approached via the macroscopic field quantities polarization and magnetization, the elementary quantities being field-producing dipoles rather than particles. Then follows the historical development of atomic theory from the early Bohr atom to modern quantum mechanics. Returning once more to materials, the process of molecule formation between quantum-mechanical atoms, and the calculations of binding energy and dipole moments, are followed by a detailed account of the solid state, given in relatively simple terms. The author's own studies of the behaviour of barium titanate provide a wealth of illustration of the general properties of ferro-electrics. The close analogy between the organization of electric and magnetic dipole moments leads on to ferro-, ferri- and anti-ferromagnetic materials. Other solid-state phenomena —piezoelectricity, semiconductors, and metallic conduction—are fitted into the picture; and the book ends with an account of conduction and various forms of breakdown in solid dielectrics.

Excellent diagrams, concise writing, clear mathematical statement, and a pretty wit that filters in irrepressibly, make the book intelligible and interesting to read. Whatever his own background, the reader is likely to find here new ideas, and new ways of approaching familiar topics. This does not mean that the material is all intrinsically novel; it is simply that no one person who has left his student days behind will ever have had the opportunity to sort out his more recently acquired parcels of knowledge in their relation to one another. The book does this so well that it should encourage people to make this kind of effort. The examples at the end, though based on well-known results, seem to be of the standard of minor research exercises; but perhaps after a second or third reading of the text one might be fortified to attempt them.

To quote from the preface: "Nobody can leave his problems to others without losing control over his destiny. The electrical engineer has to remember that he is an applied scientist and join his colleagues of physics and chemistry in a co-operative venture of 'molecular electrical engineering'. Seen from this point of view, I would want this book to be a trumpet of Jericho; alas, it may only loosen some bricks that will fall on the author's head". The author's wish may well be fulfilled; his fears are groundless, since the reader who follows him through to the end will accept his aims and presentation with enthusiasm. If he himself has dropped a brick, it is a large one; the kind that people lay as foundation stones.

G. R. N.

Television (2nd Edition)

By V. K. Zworykin, E.E., Ph.D., and G. A. Morton, Ph.D. Pp. 1037 + xv. Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 140s.

This is a book to which the expression 'a weighty tome' may be applied with literal truth; it weighs 3¾ lb! The first edition, which appeared in 1940, contained a modest 646 pages and cost 36s.; this one, with just over 50% more pages, costs nearly four times as much.

The content of the book covers an enormous range; there are few matters connected with television that do not receive some mention. It is divided into four parts: 1, Fundamental physical principles; 2, Principles of television; 3, Component elements of an electronic-television system; and 4, Color television, industrial television and television systems.

Part 1 covers electron physics, fluorescent materials and electron optics, while Part 2 deals with television fundamentals, scanning, resolution and like matters, camera pick-up devices and reproducing methods. Mechanical as well as electronic methods are discussed.

Part 3 has a chapter on the iconoscope and another in which other camera tubes are treated; such as, the orthicon, the image orthicon and the videcon: the reproducing c.r. tube has a chapter and so has the electron gun. Then there are chapters on video amplifiers, scanning and synchronization, the transmitter and the receiver.

In Part 4, a chapter is devoted to colour fundamentals and another to the principles of colour television while the final one on colour, is entitled practical colour television. The section concludes with two chapters on industrial television and practical television systems.

The authors give no indication in their preface of the kind of reader for whom they have written. The treatment is by no means the superficial one that might be expected from the enormous field which is covered, but it is not nearly deep enough to satisfy the specialist in any part of it. The book will enable an engineer to obtain a good outline of the technicalities of almost any aspect of television and the good bibliography which is included will direct his attention to sources of more detailed information on particular subjects.

As might be expected from these authors, the parts of the book which deal with camera tubes or c.r. tubes are by far the best. That is where their interests lie. The circuitry is, relatively, poorly treated and not very well balanced. For example, the chapter on receivers is of only 43 pages. It is true that video amplifiers are separately treated and that scanning and synchronization have a chapter to themselves but, even allowing for this, the space devoted to receiving problems is rather small.

W. T. C.

Fundamentals of Transistors

By LEONARD M. KRUGMAN, B.S., M.S., P.E. Pp. 140. Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 21s.

This is an American book and the author intends it "for the technician and the amateur", but he considers that it will also serve "the initial needs of engineering students and engineers who are confronted with transistors for the first time".

The first two quite-short chapters cover basic semiconductor physics and transistors and their operation. That they are very elementary and give no real understanding of the internal operation of the transistor is clear from the fact that these two chapters together occupy only 18 pages. Thereafter, the transistor is treated as a black box having characteristics which are determinable by external measurements at its terminals.

The third and fourth chapters cover the development of equivalent circuits for transistors in earthed-base, earthed-emitter and earthed-collector connection and the evaluation of input and output resistances. Both point-contact and junction transistors are dealt with. After this, there are chapters on amplifiers, oscillators and transistor high-frequency and other applications. In them, the cascading of transistor stages is treated and such practical matters as obtaining bias supplies.

The book is for those who want to use transistors and to design apparatus which embodies them. The author is, therefore, quite right to omit any serious discussion of the internals of the transistor.

Quite a large part of the treatment is mathematical. Nothing beyond ordinary algebra is needed but, to gain the full benefit from the book, one has to be more familiar with algebraic interpretation than are most amateurs and many technicians. The reader is not helped by the voltage and current convention used, and he must have good eyesight if he is to distinguish clearly between the very small 'o', 'e' and 'c' subscripts which are used to denote output, emitter and collector. There is no list of symbols and so one has to become familiar with the symbolism employed before one can benefit from the book.

In spite of these defects, the book is undoubtedly one of the best introductions to transistor characteristics and circuits that has so far appeared. There is no doubt at all that it will be extremely useful to the budding engineer and the more serious technician, and many qualified engineers will find it very helpful. Few who are interested in transistors can afford to be without it.

W. T. C.

Basic Television, Principles and Servicing (2nd Edition)

By BERNARD GROB. Pp. 660 + xv. McGraw-Hill Publishing Co. Ltd., 95 Farringdon Street, London, E.C.4. Price 64s.

The Radio Amateur's Handbook 1955 (32nd Edition)

Pp. 620. American Radio Relay League, West Hartford 7, Connecticut, U.S.A. Price $4.

Radar Pocket Book

By R. S. H. BOULDING, O.B.E., B.Sc., M.I.E.E., A.M.I.Mech.E., F.Inst.P. Pp. 176 + vii. George Newnes Ltd., Tower House, Southampton Street, London, W.C.2. Price 15s.

MEETINGS

I.E.E.

11th May. "Transistors and other Semi-Conductor Devices"—Group of papers including "Junction Transistor Noise in the Frequency Range 7-50 kc/s", by W. L. Stephenson, B.Sc., and "Noise in Silicon Microwave Diodes", by G. R. Nicoll, B.Sc.

19th May. Annual General Meeting to be followed at 6.30 by "Human Relations in Industry", by the Rt. Hon. Lord Citrine, P.C., K.B.E.

These meetings will commence at 5.30 and will be held at the Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, W.C.2.

BRIT.I.R.E.

18th May. "The Development of the Underwater Television Camera", by D. R. Coleman, D. A. Allanson and B. A. Horlock. To be held at 6.30 at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for March 1955

Date 1955 March	Frequency deviation from nominal: parts in 10^8		Lead of MSF impulses on GBR 1000 G.M.T. time signal in milliseconds
	MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.	
1	−0·5	+2	+49·4
2	−0·5	+2	+48·0
3	−0·5	+2	+47·7
4	−0·6	+2	+47·0
5	NM	+2	NM
6	NM	+1	NM
7	−0·6	+2	+45·3
8	−0·5	+2	+44·8
9	−0·4	+2	+44·3
10	−0·5	+2	+43·5
11	−0·5	+2	+42·7
12	−0·5	+2	NM
13	−0·5	+2	NM
14	−0·5	+2	+41·2
15	−0·5	+3	+40·6
16	−0·6	+2	+39·9
17	−0·6	−2	+39·4
18	−0·4	−1	+38·1
19	−0·4	−1	NM
20	−0·4	−1	NM
21	−0·5	0	+36·1
22	NM	0	NM
23	−0·5	0	+35·7
24	−0·5	0	+35·1
25	−0·4	0	+34·1
26	−0·4	−1	NM
27	−0·4	0	NM
28	−0·5	0	+32·9
29	−0·4	0	+33·8
30	−0·4	0	+31·9
31	−0·5	0	+31·3

The values are based on astronomical data available on 1st April 1955.
The transmitter employed for the 60-kc/s signal is sometimes required for another service.

NM = Not measured.