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Rotating Bar Magnet from Different Points of View

On p. 28 we publish a letter from Professor Cullwick dealing with his contribution to the discussion on Mr. Hammond's paper, which formed the basis of our October Editorial. The subject is the familiar problem of the cylindrical bar magnet rotated about its axis, and the question whether the e.m.f. which causes a current to flow through the external circuit and galvanometer is induced in the magnet or in the external circuit. Two or three contributors to the discussion, including Professor Cullwick, appeared to support the latter view.

Before discussing his letter we will once again state briefly the views which we have always supported. If there is no external circuit the magnet is rotating in its own magnetic field, which is undergoing no change in either magnitude or direction either outside or, macroscopically speaking, inside the magnet. The magnetic force H at any point is produced by the myriads of spinning or whirling electrons throughout the magnet. When the magnet is rotated, the free conducting electrons, being moved in the magnetic field, experience a force which moves them either towards or away from the surface, which thus becomes charged negatively or positively and sets up an electric field in the space around the magnet. The distribution of charge on the surface and throughout the steel adjusts itself until its field exactly counterbalances at every point in the steel the force on the free electrons due to their motion in the magnetic field.

If, with any d.c. dynamo on open circuit, one examines the space between the terminals, one finds an electric field, and any electron in the space experiences a force; also, of course, a current is set up in a piece of wire connected between the terminals, but no one would maintain that the e.m.f. is induced in this space. The *fons et origo* of the external electric field is the e.m.f. induced in the armature by the rotation of the conductors in the magnetic field. Similarly with the rotating bar magnet; when an external circuit is connected between brushes rubbing on the middle and one end of the magnet, a current is set up, and its immediate cause is the electric field in which the wire is situated, but, as in the ordinary dynamo, this external field is primarily due to the e.m.f. induced in the rotating armature; it is this e.m.f. that is setting up a p.d. between the terminals and producing an external electric field.

There is the point of view of an observer fastened to the magnet and therefore rotating with it. To him everything in the magnet is at rest—except the electronic movements within the molecules—and the external circuit is rotating in the external magnetic field, and the e.m.f. is induced in the wire; this e.m.f. sets up a p.d. between the brushes and causes a current to flow through the magnet. As we have previously pointed out, his mechanical notions would be equally strange, for he would endow the external circuit, and not the magnet, with centrifugal force. One of his observations, however, is worth considering, in view of what follows. To him the free-conduction electrons in the magnet are at rest, as are also the molecules which are producing the magnetic field; there is no relative rotation. This agrees, of course, with the observation of the ordinary stationary observer, that all magnetizing molecules and free electrons are rotating at the same rate, but unless one endows the lines of magnetic flux with physical reality, or, in other words, adopts the bristle theory, there is no meaning in the question as to whether the magnetic field is rotating or not.

In the discussion, Dr. K. J. R. Wilkinson described magnetism as "a very convenient but unessential concept"; one can fix one's attention on the cause, viz. the molecules containing the whirling electrons, and, leaving the unessential magnetic concept out of consideration, ask if this cause is rotating relatively to the conduction electrons, and the answer is "No". Then why should they exercise any force on them?

To this question Professor Cullwick's answer is that they do not, or rather that they exercise two equal and opposite forces. We need not go into the matter fully here, as it is set out in detail in Professor Cullwick's letter. We both come to the conclusion that there are two forces acting on the free electron, one $q(v \times B)$ due to its movement in the magnetic field, and the other qE equal and opposite to it, due to an electric field. Whereas we have ascribed the electric field to the distribution of charge and electric potential set up by the displaced free electrons, Professor Cullwick ascribes it to some want of electric balance among the whirling electrons in the magnetizing atoms, due to the movement of the atoms. This has the doubtful advantage of removing any force on the free electrons *ab initio*, whereas from our point of view equilibrium is only attained by the redistribution of the free electrons throughout the magnet.

As Professor Cullwick says "the electric field acting on the external leads is the same by either theory"; hence the p.d. is the same, but whereas the surface charges in one case are displaced free electrons, in the other they are whirling electrons, and, assuming open-circuit, the free electrons are undisturbed. In both cases the external field is due to the displacement of electrons in the magnet but, whereas in one case the displacement is due to the electromagnetic force, in the other it is due to relativity. In the latter case, Professor Cullwick apparently does not like the setting up of an electromagnetic force throughout the material being called the induction of the e.m.f. When the external circuit is connected there is a current of free electrons through it and through the magnet. The departure of electrons from the negative terminal of the magnet into the external circuit tends to charge the steel there positively; similarly the arrival of electrons at the positive terminal tends to charge that part negatively. Thus there is an electric force throughout the steel in opposition to the relativistic electric force, so that the electromagnetic force on the free electrons is no longer exactly counterbalanced by the electric force on them, and they consequently flow through the magnet. If the terminals are connected by a wire of negligible resistance, the free electrons will distribute themselves in such a way as to cancel the effect of the relativity electrons thus reducing the terminal p.d. to zero. In the magnet the line integral of the electromagnetic force drives the current through the resistance of the magnet.

From both points of view the electromagnetic force at every point in the magnet is, except under short-circuit conditions, opposed by an electrostatic force. From our point of view this latter is due solely to the displacement of free electrons, but from the other it is the resultant of tw electrostatic forces, one due to the relativist displacement of the magnetizing electrons, and the other to the displacement, if any, of the free electrons. In both cases the terminal p.d. and external current, and also the resultant electric field inside and outside the magnet are the same. Whichever point of view one adopts, the location of the induction or production of the e.m.f. is surely within the magnet. As in the ordinary series dynamo, an electric field is set up outside, but to find the source of this field one must look inside the magnet.

There are two observers who would agree with Professor Cullwick that the e.m.f. is induced outside the magnet; one, to whom we have already referred, being fastened to the magnet, and rotating with it, would maintain that the magnet was at rest, and the e.m.f. induced in the rotating external circuit; the other, a believer in the 'bristle' theory, picturing the lines of force or flux as physical entities rotating with the magnet, would maintain that the e.m.f. was due to these lines cutting the external circuit.

All this discussion arose out of the reading of a paper on the teaching of electromagnetism, and one cannot help wondering at what stage, if any, in the teaching of this subject to students of electrical engineering, these abstruse complexities —if we dare so call them—should be introduced.

G. W. O. H.

PULSE RESPONSE OF SIGNAL RECTIFIERS

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SUMMARY.—There appears to be little information available on the response to a pulse or step input for a diode signal rectifier fed from a tuned circuit, as in the usual wideband amplifiers for television, radar, etc.

The loading imposed on a tuned circuit by a wideband rectifier is first considered. Then the response time for circuit plus rectifier is examined, both directly and via the modulation-frequency response characteristic. Finally, a series of measurements confirms the theory for the simplest case (a single circuit centrally tuned) and gives some information on the response with the more complex amplifiers usually employed in practice. A triode detector is used for comparison tests.

1. Introduction

THE response to step waves or pulses of r.f. and v.f. circuits has been the subject of numerous papers in recent years owing, no doubt, to the increasing practical usage of non-sinusoidal waveforms, as in television and radar. The third major element in any receiver—the signal rectifier (or detector)—however, has received little attention. This may well be due to the greater theoretical difficulties involved in dealing with non-linear elements. As an illustration of the intractable nature of these problems, we may recall that the action of a rectifier supplied with a sinusoidally-modulated wave from a tuned circuit was not even approximately clarified until about 15 years ago!

In the practical design of receivers for picture or pulse-modulated transmissions, it appears to be usual to design the r.f./i.f. amplifier, the video amplifier, and the rectifier separately for adequate bandwidth or time constant: the time constant of the rectifier is usually taken as that of the rectifier output (load) circuit. In the course of testing vision receivers on a pulse input, phenomena were frequently observed which could not be accounted for on the above basis, and other similar effects were found when tests were made specifically on the rectifiers:—

(1) If a wideband i.f. amplifier employing staggered circuits (as commonly used for vestigial-sideband reception) is tested, complete with its diode rectifier, on a pulse input, the response is found to be decidedly different according as the carrier is set above or below the midband frequency, even if the c.w. response is perfectly symmetrical about that frequency.

(2) The response of such a circuit to a positive-going r.f. step modulation is often found to be decidedly different from its response to a negative step. This is, of course, very undesirable since the waveform cannot then be corrected by any video amplitude or phase corrector. In some cases this effect may originate in the r.f. amplifier, but in others it can be definitely traced to the detector.

The existing theory of diode rectifiers shows that such phenomena must be explained on the basis that the performance of the rectifier is dependent upon the impedance of the circuit from which it is fed, or that the response of the last tuned circuit depends upon the damping imposed by the rectifier which will vary with video frequency and waveform.

It also appears that the overall pulse response of the amplifier plus rectifier is not obtainable directly by calculation or by measurement of either:—

(a) the c.w. response of the r.f. amplifier, plus the v.f. response of the detector and v.f. amplifier measured separately, or

(b) the response to an r.f. pulse of the r.f. amplifier, plus the response to a v.f. pulse of the detector load circuit and v.f. amplifier.

It is not necessarily even obtainable from:—

(c) the overall sine-wave video-modulation response characteristic for the complete receiver.

In this paper we are interested only in the case where the diode is fed from a tuned circuit, the theory for a low-impedance source being sufficiently well known. We first (Sect. 2) note certain corrections to present-day rectifier theory which are necessary when dealing with wideband circuits. We then (Sect. 3) attempt a direct attack upon the problem of the rectification of an r.f. pulse by a diode detector. The general case is found to be outside the scope of simple analysis, but results are obtained for several special cases. The alternative approach, representing the pulse as a band-frequency spectrum, and using the known theory for rectification of a carrier with sidebands, is then considered (Sect. 4).

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This approach yields more useful information; in the simplest case, where the rectifier is fed from a single symmetrically-tuned circuit, the effective time constant is calculated, but even here, some approximation is unavoidable.

Resort is therefore made to actual pulse-response measurements, some of the tests being made upon a complete amplifier of the type used in television receivers, and some upon circuits chosen to illustrate the theory.

A triode detector (so called infinite-impedance circuit) is used for comparison tests, and is found to have certain advantages owing to its high input impedance. This eliminates the interaction between circuit and detector which is so troublesome in the case of the diode, and enables the responses of the r.f. amplifier and the detector to be separately examined. The triode detector is not, however, ideal in all respects since its linearity is poorer than that of a diode.

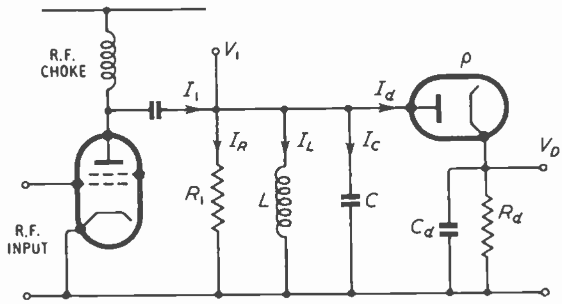


Fig. 1. *Diode rectifier fed from tuned circuit.*

2. Basic Diode Theory

2.0. The theory of diode rectification (other than that for pulses) has been treated by numerous writers, notably Wheeler[1], and we are here only concerned to show up certain limitations arising when this theory is applied to wideband circuits.

A diode fed from a valve and tuned circuit is shown in Fig. 1. The valve is taken as a current generator: the diode is operating as a linear envelope detector, and the input frequency f_0 is equal to the circuit resonant frequency. R_1 represents the total circuit loss; for the calculations with a pulse input, the series loss must be assumed negligible relative to the parallel losses, as is usual when dealing with wideband circuits. We shall first consider an unmodulated (c.w.) input and will assume C_d is large so that V_d is constant (i.e., negligible ripple).

The diode current consists of a series of pulses, and may be represented by:—

$$I_d = I_{d0} + I_{d1} \sin \omega_0 t + I_{d2} \sin 2\omega_0 t + \text{etc.})$$
$$= I_{d0} (1 + a_1 \sin \omega_0 t + a_2 \sin 2\omega_0 t + \text{etc.}).$$

The fundamental and harmonic components of V_1 may be similarly written as V_{11}, V_{12}, V_{13}, etc. We shall put V_r for the r.m.s. value of V_1.

Wheeler[1] assumes in his analysis that the impedance of the tuned circuit is zero at frequencies $2f_0$, $3f_0$, etc., as well as at zero frequency. Under these conditions V_1 must be sinusoidal, and it is easily shown that the detector input impedance $R_{in} = R_d/2\eta$, where η is the voltage efficiency, $V_d/1.4V_r$. This is the conventional result quoted in the textbooks and may also be obtained by equating input and output power to the rectifier, again assuming a sinusoidal input.

For wideband working, we cannot make the above assumptions, and we have, providing $Q>2$, the following circuit equations:—

$$V_{11} = (I_1 - I_{d1}) R_1.$$
$$V_{12} = I_{d2} \cdot 0{\cdot}67/j\omega C.$$
$$V_{13} = I_{d3} \cdot 0{\cdot}37/j\omega C.$$
$$V_{14} = I_{d4} \cdot 0{\cdot}27/j\omega C.$$
$$V_{1n} = I_{dn} \cdot 1/nj\omega C \text{ for } n \text{ large.}$$

The departure of V_1 from a sine wave is most simply given by the r.m.s. harmonic voltage relative to total r.m.s. voltage—

$$p = \frac{\sqrt{(V_{12}^2 + V_{13}^2 + \text{etc.})}}{1{\cdot}4\, V_r}$$
$$= \frac{I_{d0}\sqrt{(0{\cdot}47a_2^2 + 0{\cdot}14a_3^2 + 0{\cdot}07a_4^2 + \text{etc.})}}{1{\cdot}4\, \omega_0 C V_r}$$
$$= \frac{V_d}{1{\cdot}4V_r} \cdot \frac{A}{\omega_0 C R_d}.$$

We have also $V_{11}^2 = 2V_r^2 (1 - p^2)$.

Here $A = \sqrt{(0{\cdot}45a_2^2 + 0{\cdot}14a_3^2 + \text{etc.})}$ For small conduction angles, the first few coefficients a_1, a_2, a_3, a_4, etc., will each be equal to 2 from Fourier's theory and thus A will approximate to 2. As the conduction angle is increased, A will slowly fall, its exact value depending upon the waveform of I_d, which is not easily evaluated.

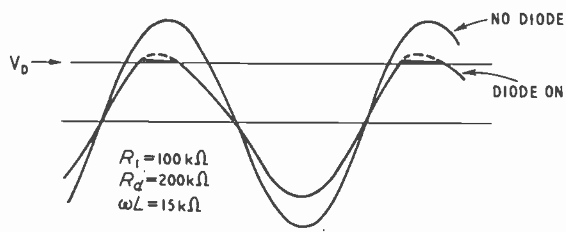


Fig. 2. *Waveform of voltage across tuned-circuit feeding diode. The dashed line is for 2kΩ added in series with diode.*

Fig. 2 shows a typical waveform encountered during a brief experimental investigation undertaken to confirm these findings. The distortion of the peak was found to be broadly proportional to the ratio of R_d to $1/\omega_0 C$ and to be independent of R_1 over wide limits, as indicated by the above formula.

The distortion of the input waveform may also be approached directly by means of the differential equations for the circuit. This was attempted, but was found to lead to formidable equations. Qualitatively we can see that, when the diode conducts, a part of the current I_L is diverted from C into the diode: thus the positive voltage peak across C will be truncated by an amount proportional to the ratio I_d/I_L which is in turn dependent upon $\omega_0 L/R_d$ or $1/\omega_0 C R_d$.

Our primary interest is in the *input resistance*, R_{in} which must be strictly defined as V_{11}/I_{d1}.

$$R_{in} = \frac{V_{11}}{I_{d1}} = \frac{V_{11}}{V_d} \cdot \frac{R_d}{a_1} = \frac{1{\cdot}4 V_r}{V_d} \cdot \frac{R_d}{2} \cdot \frac{2\sqrt{(1-p^2)}}{a_1}.$$

Our final result for R_{in} is thus the conventional value $R_d/2\eta$ multiplied by a correction factor $2\sqrt{(1-p^2)}/a_1$. The order of magnitude of this correction may be seen by assuming a small conduction angle: then $V_d = 1{\cdot}4 V_r$, and the correction factor reduces to $\sqrt{(1 - 4/\omega^2 C^2 R_d^2)}$, which would $= 0{\cdot}75$ in the practical case of $C = 15$ pF at 16 Mc/s with $R_d = 2$ kΩ.

2.1. We have hitherto assumed, with the conventional theory, that C_d is large, the difference between V_d and $1{\cdot}4 V_r$ being solely due to the resistance ρ of the diode. In wideband working, C_d must be kept small (in order to avoid video cut off), and this reduces the detector efficiency in two ways:—

(a) If $C_d R_d$ were not much greater than the time of one r.f. cycle, the mean value of V_d would be less than $1{\cdot}4 V_r$ even if ρ were negligible. The loss due to this cause is given by $V_d = (1 - 1/2 f_0 C_d R_d) \times 1{\cdot}4 V_r$ provided that ρ/R_d and $1/f_0 C_d R_d$ are both less than, say, 0·1.

(b) If the capacitance across the diode is not much less than C_d the valve is in effect tapped on to the circuit.

In a typical television receiver with an i.f. of 16 Mc/s, the loss of output voltage was about 30% due to rectifier resistance, 20% for (a) and 10% for (b): η was thus 0·5 and the input resistance was about $1{\cdot}5\ R_d$ or 50% greater than that estimated from the textbook formula.

Both these effects cause a loss in voltage efficiency η, but do not affect the power lost. Thus if the input has to be increased in a ratio η to compensate for one or both of these effects, the input resistance must be increased in the ratio η^2, and will no longer comply with the formula $R_{in} = R_d/2\eta$. If an i.f. choke is included in the diode load circuit, that part of the total capacitance which follows this choke must not be included in C_d for the purpose of this paragraph.

2.2. It should be noted that neither of these corrections will alter the *relative* value of the input impedance to the different components of a modulated wave. The formulae for variation in input impedance with modulation frequency given by Wheeler and elaborated by Terman ('Radio Engineers Handbook,' 1943, p. 558) will therefore still be valid provided that we write them in terms of these impedances only and do not assume that $R_d/2R_{in}$ is equal to η. For our calculations in Sect. 4 we will use the symbol k to represent $R_d/2R_{in}$ and will bear in mind that, though k is dependent primarily upon η, the two quantities should not be equated. We shall also write R_2 for the parallel impedance of the tuned circuit with detector load on, so that we have

$$R_2 = \frac{R_1 R_d}{2kR_1 + R_d}$$

2.3. The above theoretical considerations suggest that measurements of diode efficiency and input resistance are not quite so simple to make or to interpret as is usually assumed. It is worth noting here a few typical pitfalls which are especially noticeable in tests on detectors intended for television receivers with a relatively low intermediate frequency (from 10 to 25 Mc/s).

(a) If the diode (or crystal) is fed from a source of very low impedance, the matter is fairly simple, with the exception that the capacitance C_d will have a large effect upon voltage efficiency, and this may also be affected by the i.f. choke.

(b) If the diode is fed from a high impedance source, such as a tuned circuit in the anode lead of an i.f. amplifier valve, matters are much more confusing. First, reversing the diode will alter the apparent efficiency, if the input V_1 to the diode is measured by the usual type of peak-reading valve voltmeter: this, of course, is due to the distortion of the input waveform. Secondly, any alteration in either the internal impedance of the diode, or in C_d, will alter the ratio of V_1 to V_d as in (a), but will change the ratio of the input at the grid of the valve to V_d in quite a different way, owing to the change in damping.

Note also that none of the alternative definitions for voltage efficiency is really satisfactory:—

(a) V_d/(peak input). Usually quoted but obviously unsatisfactory.

(b) $V_d/1{\cdot}4 V_r$, as used above. Still affected by impedance of circuit feeding diode, though to a less degree.

(c) V_d/V_{11}. This is difficult to measure and results will still be dependent upon the harmonics present and hence upon the source impedance.

3. Diode with Pulse Input

3.0. We will assume a rectangular pulse or step input, and will first examine the limiting case where $C_dR_d \ll 2CR_2$, the diode being fed from a single circuit tuned to f_0. (See Fig. 1.)

Here V_d will evidently follow the form of any change in the envelope of V_r exactly and R_{in} will be the same as on c.w. Thus the overall step-response time (10% to 90%) for V_d will be that calculated for the circuit shunted by a resistance R_{in}; i.e., $T_r = 4{\cdot}4CR_2$. Further, the time will be the same on positive and on negative steps and will be unaffected by the modulation factor. The waveform of V_d will be exponential. (*N.B.* These results are only accurately true to the extent that the rectifier is exactly linear.)

The condition $C_dR_d \ll 2CR_2$ may also be written $C_d \ll C/(k + R_d/2R_1)$, which reduces to $C_d \ll C/k$ in the particular case where $2R_1 \gg R_d$.

As C_d is increased to the point where C_dR_d becomes of the same order as $2CR_2$, the time of rise and of fall of V_d will start to increase. Also R_{in} will exceed its c.w. value when V_r is falling and will be lower when V_r is rising, the concept of an input resistance becoming somewhat misleading under these circumstances. This case is analysed in Sect. 3.2.

3.1. Consider next the other limiting case where $C_dR_d \gg 2CR_1$.

Here the action at the time when the r.f. input is switched on (positive step) is quite different from that when it is switched off (negative step).

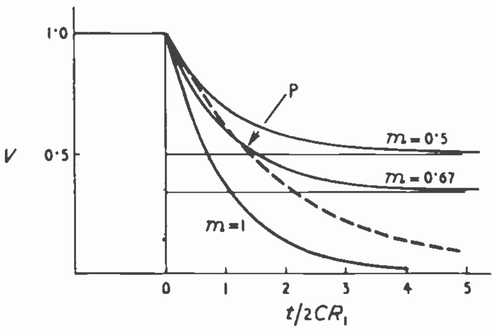


Fig. 3. *Fall of r.f. envelope (solid lines), and voltage across diode load (dashed line for $C_dR_d = 4CR_1$), on negative step input.*

(a) If the input is switched off, V_d will be unable to follow the fall in V_r and the diode will cease to conduct. Assuming for the moment a case where V_d substantially $= 1{\cdot}4V_r$, the problem is quite straightforward. C_d will discharge exponentially through R_d and response time $T_r = 2{\cdot}2C_dR_d$ if the input falls to zero. If the modulation factor m is less than unity, the envelope only falls to $(1 - m)$ of the initial value and the initial rate of fall of V_r is proportionally less, though that of V_d is unaffected while the diode is non-conducting. In Fig. 3 is shown the exponential fall of the r.f. envelope for three values of m: the dashed line represents the fall of V_d for $C_dR_d = 4CR_1$ and $m = 1$. The initial rate of fall of V_r, becomes equal to that of V_d when $mC_dR_d = 2CR_1$, corresponding to the curve for $m = 0{\cdot}5$ in the case shown in Fig. 3. If m is below this critical value, no non-following occurs, and the response time cannot be worse than $4{\cdot}4CR_1$.

Moreover, even when non-following does occur, the response time falls rapidly as m is reduced below unity; it is clear that the time during which non-tracking occurs is given by

$$(1 - e^{-t/C_dR_d}) = m\,(1 - e^{-t/2CR_1})$$

Also, provided $mC_dR_d > 4CR_1$, the time of fall is given by $T_r = C_dR_d\,[\log(1 - 0{\cdot}9m) - \log(1 - 0.1m)]$.

For $m = 0{\cdot}67$ in the figure, for example, V_d will follow the dashed line up to the point P and will lie between this and the solid line thereafter; the fall time will be only half of that for $m = 1{\cdot}0$.

If V_d is substantially less than $1{\cdot}4V_r$, the diode will continue to conduct somewhat until the envelope has fallen below V_d. This period should be added to the periods calculated in the last paragraph, but the correction will not usually be large since the envelope falls rapidly initially (time constant $= CR_2$).

(b) *On a positive-going step* the diode must abstract extra current from the circuit in order to charge C_d, and so R_{in} is here low. An exact analysis for this case does not appear to be at all easy; some idea of the probable result may perhaps be obtained from an analysis of a diode fed from a purely resistive source given by Burgess[4]. The response shown is not exponential, but rises fast at first (initial time constant $= \pi R_3C_d$) and then more and more slowly, so that time of rise to 90% of full amplitude is 14 R_3C_d: here R_3 is equal to $(R_1 + \rho)$ in parallel with R_d, where ρ is the diode impedance. The input damping is very heavy (R_{in} here $= 2\rho$) initially, but falls to its static value ($R_{in} = R_d/2k$) eventually.

This heavy initial loading corresponds to the charging current (V_1/ρ) taken by C_d during the whole positive half cycle, and must occur equally when the diode is fed from a tuned circuit, provided only $C_dR_d > 2CR_1$.

If the modulation factor of the step is less than unity, the initial loading will be less: the initial voltage due to the step across the diode is directly proportional to m, and so the initial $R_{in} = 2\rho/m$ (or rather higher when m is low since I_d will then flow for less than half the cycle).

3.2. *Where C_dR_d is neither very large nor very small,* the following analysis gives a partial solution.

Assuming $f_0 \gg 1/C_dR_d$, and neglecting diode capacitance, we apply the method of Sect. 2 to a step-modulated input.

We have:—

$$I_{d_1} = a_1 I_{d_0} = a_1\left(\frac{V_d}{R_d} + C_d\frac{dV_d}{dt}\right)$$

Then the value R'_{in} of the input resistance as it varies during the transition period is given by

$$\frac{1}{R'_{in}} = \frac{I_{d_1}}{V_{11}} = \frac{a_1 V_d}{R_d V_{11}} \cdot \left(1 + \frac{C_dR_d}{V_d} \cdot \frac{dV_d}{dt}\right)$$

Unfortunately, in order to make any further progress, we must now assume that the conduction angle is small but finite, as on a negative step with $\rho/R_d \ll 1$ and $C_dR_d < 2CR_1$: then a_1 will always be near 2 and V_d/V_{11} close to unity, so that we may put R_dV_{11}/a_1V_d equal to $R_d/2$, and $k = 1$.

Under these conditions the response must be exponential, and putting $V_d = V_{d_0}\ e^{-t/T_d}$, we have

$$\frac{1}{R'_{in}} = \frac{2}{R_d}\left(1 - \frac{C_dR_d}{T_d}\right)$$

The overall time constant of the circuit shunted by R'_{in} is then $2CR'_2 = 2C/(1/R_1 + 1/R'_{in})$

$$= 2C\Big/\left\{1/R_1 + \left(1 - \frac{C_dR_d}{T_d}\right)\Big/R_d\right\}$$

and this must be equal to T_d.

Thus we obtain

$$T_d = \frac{2(C + C_d)}{1/R_1 + 2/R_d} = 2CR_2\,(1 + C_d/C)$$

This result is limited in application as stated above: when applied to a positive step, the analysis gives $T_d = 2CR_2 \times (1 - C_d/C)$, but this result is not valid since the conduction angle is here initially very large. It agrees with the formula obtained in Sect. 4.2 by quite different methods, and on quite different assumptions, and its implications are more fully discussed there. The criterion for no distortion which emerges from this formula is $C_d \ll C$ for $k = 1$: this is easier to satisfy than the simple criterion $C_dR_d \ll 2CR_2$, except where $R_1 \gg R_d$ when the two criteria merge.

3.3. We have hitherto assumed that the detector is fed from a single circuit tuned to the incoming frequency: if this is not the case, as with staggered circuits, or bandpass coupling, or vestigial-sideband working, the problem becomes exceedingly complex. Under these conditions the pulse-envelope response of the circuit taken alone is oscillatory and cannot be said to have any definable time constant: moreover, the impedance from which the diode is fed is no longer resistive even at carrier frequency.

The criterion for avoiding distortion is not at all clear, though no doubt we should be quite safe if we could keep $2{\cdot}2C_dR_d$ much less than the response time of the r.f. amplifier.

The problem of the pulse response in the general case, where C_dR_d is not negligible, awaits the attention of those versed in the application of operational methods to non-linear circuits. The only point which is certain is that the increase in overall time constant as C_dR_d is increased does not correspond to that given by a linear amplifier stage with time constant C_dR_d.

4. Diode with Sinusoidally Modulated Input

4.0. The following formula is given by Terman[2] and is based upon a paper by Wheeler[1]: it shows the modification to the modulated envelope across a tuned circuit which occurs when a diode detector is connected to it. The formula has been slightly simplified by assuming that the diode efficiency is the same for carrier and sidebands. We have also altered the definition of k, as discussed in Sect. 2.2, in order to widen the application of the formula.

$$\frac{m'}{m} = \frac{1 + 2kZ_0/R_d}{1 + 2kZ_d/Z_s}$$

Where m'/m is actual modulation depth at the diode input terminals relative to the modulation depth m with diode inoperative.

Z_0 and Z_s are impedances of the tuned circuit (or transformer), as looked at from the diode, at the carrier f_0 and at the upper sideband frequency ($f_0 + p/2\pi$) respectively.

R_d is diode load impedance to d.c.

Z_d is diode impedance to the modulation frequency.

k is the ratio $R_d/2R_{in}$, where R_{in} is the diode input resistance to c.w.

This formula assumes that the time constants are such that the diode output voltage V_d can follow the envelope of V_1 exactly, and, with this same proviso, we can extend the analysis as follows to give us the modulation-frequency output of the diode:—

$$\frac{V_{dp}}{V_{d0}} = \frac{m'_p}{m_p} \cdot \frac{m_0}{m'_0} \cdot \frac{m_p}{m_0} = \frac{1 + 2kZ_0/R_d}{1 + 2kZ_s/Z_d} \cdot \frac{Z_s}{Z_0} = \frac{Z'_d}{R'_d}.$$

Here the suffixes p and 0 indicate the values of diode output V_d (and of m' and m) at a modulation frequency $p/2\pi$, and at a very low modulation frequency respectively. Z'_d is Z_d shunted by $2kZ_s$ and R'_d is R_d shunted by $2kZ_0$.

We have assumed here that (as is usual with wideband amplifiers) the diode load is the same at a low modulation frequency as at d.c. so that $m'_0 = m_0$.

Thus it appears that the overall modulation-frequency response is that of the diode load shunted by the low-pass analogue of the r.f. circuit, but with all the impedances in the latter scaled up in the ratio $2k/1$ or R_d/R_{in}.

Further, the c.w. selectivity characteristic of the circuit, or ratio of volts (E_s) across circuit at $(\omega_0 + p)$ to that (E_0) at ω_0 with diode on, is given by

$$\frac{E_s}{E_0} = \frac{Z_s \times R_d/2k}{Z_s + R_d/2k} \cdot \frac{Z_0 + R_d/2k}{Z_0 \times R_d/2k}$$
$$= \frac{Z_s}{Z_0} \cdot \frac{1 + 2kZ_0/R_d}{1 + 2kZ_s/R_d}$$

Whence we obtain the following relation between the overall modulation-frequency response and the selectivity measured in the ordinary way:—

$$\frac{V_{dp}}{V_{d0}} = \frac{1 + 2kZ_s/R_d}{1 + 2kZ_s/Z_d} \cdot \frac{E_s}{E_0} = \frac{Z'_d}{R''_d} \cdot \frac{E_s}{E_0}$$

where Z'_d and R''_d are equal to Z_d and R_d shunted by $2kZ_s$.

If the single tuned circuit and detector are preceded by any amplifier with selectivity definable by e_s/e_0, then

$$\frac{V_{dp}}{V_{d0}} = \frac{Z'_d}{R''_d} \cdot \frac{E_s}{E_0} \cdot \frac{e_s}{e_0}$$

and the ratio Z'_d/R''_d still gives us the overall modulation-frequency characteristic from the overall c.w. characteristic.

If the diode is fed from a network of coupled circuits, then the impedances Z_0, etc., are those looking into the network from the diode. These formulae are not restricted to cases where the circuit feeding the diode is tuned to f_0 (e.g., Z_0 is not necessarily $= R_1$), provided that Z_s is the same (except in sign) for each sideband.

4.1. When attempting to apply the above analysis to the case of a pulse or step input, we must note its limitations:—

(a) The analysis assumes that V_d can follow the envelope of V_1, and thus will cease to hold if the rate of fall of the envelope at the diode exceeds V_1/C_dR_d volts/sec. Wheeler (loc. cit.) considers the effects of non-following, but his analysis applies only to sinusoidal modulation, and we must employ the direct approach of Sect. 3.3 under non-following conditions.

(b) We have assumed k to be constant except where non-following occurs. Actually, as discussed in Sect. 3, k will tend to exceed its c.w. value on positive steps (i.e., R_{in} is here low): on negative steps it will be less than its c.w. value, especially when the condition for non-following is approached. The input must also be large enough for strictly linear detection.

(c) Terman[2] assumes that $R_d/2R_{in}$ is equal to $V_d/1{\cdot}4V_r$. This is not necessarily true, but we have eliminated this source of error by writing $k = R_d/2R_{in}$ as discussed in Sect. 2.2.

(d) Our assumption of equal efficiency on carrier and sidebands will introduce errors only where the a.c. impedance of the diode load is substantially lower than the d.c. value: since the load at low frequencies is normally the same as at d.c. on a wideband detector, such errors would only arise where pC_d is so high that the applicability of the formula would in any case be doubtful on the grounds discussed in (b).

(e) The sidebands are assumed equal, and in phase, but in cases where the tuning is unsymmetrical, rough estimates may no doubt be made by evaluating V_{dp}/V_{d0} separately for each sideband, adding the results, and dividing by two. Phase differences are neglected in this procedure, as in the common method of estimating the effect of detuning which is found in practice to lead to useful approximate results.

Subject to these limitations, the pulse response can, in theory, be calculated from V_{dp}/V_{d0}.

In the general case, however, it is not usually practicable to advance beyond the mainly qualitative results given in Sect. 4.3.

4.2. In the case where the *diode is fed from a single circuit*, of parallel resistance R_1, tuned to the carrier and where the diode load comprises only R_d shunted by C_d, (see Fig. 1) we can reduce the equations to a more useful form.

We have $\begin{cases} Z_0 = R_1 \\ Z_s = R_1/(1 + 2jpCR_1) \\ Z_d = R_d/(1 + jpC_dR_d) \end{cases}$

Inserting these values in the first equation for V_{dp}/V_{d0} in 4.0. above, we find

$V_{dp}/V_{d0} = 1/[1 + jp(C + kC_d)\, 2R_2]$

where R_2 is R_1 and $R_d/2k$ in parallel as before.

Thus the overall effect of tuned circuit plus detector is the same as that of a single circuit having a time constant $T_c = 2R_2(C + kC_d)$.

This result may be interpreted in several ways:—

(a) Since $T_c = 2\,CR_2\,(1 + kC_d/C)$, the overall result may be obtained from the time constant of the loaded circuit (as obtained from a c.w. test), multiplied by $(1 + kC_d/C)$, just as though the circuit capacitance had been increased to $(C + kC_d)$.

(b) Rearranging terms, we have

$$T_c = 2CR_2 + \frac{2kR_1R_d}{2kR_1 + R_d} \cdot C_d$$

This shows that the detector may be regarded as adding to the time constant of the loaded circuit a time equal to that for the diode load shunted by a resistance $2kR_1$.

(c) Rearranging again, we have

$$T_c = (C_d + C/k) \times \frac{2kR_1R_d}{2kR_1 + R_d}$$

Thus we may say that the overall time constant of a detector fed from a tuned circuit is equal to that for the detector load shunted by a capacitance C/k and a resistance $2kR_1$.

This overall time constant will often be less than that of the detector load taken alone.

(d) From the form of the expression for V_{dp}/V_{d_0} it is clear that the detector should never be considered as a separate stage (e.g., a video stage with time constant C_dR_d) in cascade with the tuned circuit.

The form of these results clearly enables them to be applied directly to the case of a pulse input, subject, of course, to the limitations referred to in (a) and (b) of Sect. 4.1.

The conclusions can be most conveniently summarized by means of a diagram (Fig. 4) showing the variation in response time T_r with the ratio of C_d to C. Response time is used instead of time constant since it is the only convenient criterion in those cases (e.g., where non-following occurs with $m < 1$) where the response is not exponential: where it is exponential, we merely use the relation $T_r = 2{\cdot}2\,T_c$. When the modulation takes the form of a positive step, T_r should follow the line $4{\cdot}4CR_2\,(1 + kC_d/C)$ as given by the formulae above. On a negative step, results should follow the same line when the modulation factor is small; when m exceeds a certain critical value, non-following will occur as discussed in Sect. 3.1: T_r will then be greater than for a positive step and will rise with further increase in modulation factor up to a maximum of $T_r = C_dR_d$, as given by the dashed line, when $m = -1$. The point where the full and dashed lines intersect corresponds, of course, to the condition for commencement of non-following for $m = -1$, viz. that $C_dR_d = 2CR_1$.

4.3. In the general case where the detector is preceded by a multi-circuit r.f. amplifier, the condition for non-following on negative step modulation is not expressible in terms of the circuit constants, and we have to fall back upon the less useful criterion of Sect. 4.1(a). This criterion suggests that the ratio R_1/R_d should still be kept as high as possible, since the rise time (with diode load off) of the r.f. envelope at the detector will increase with R_1, while a higher value for R_1 will enable R_d to be reduced for any given R_2 (i.e., bandwidth).

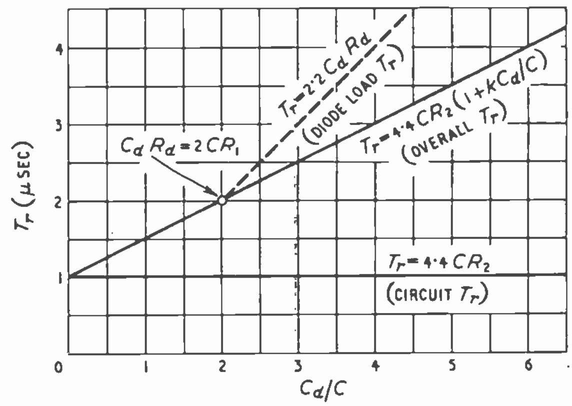


Fig. 4. *Theoretical variation of response time T_r with the capacitance ratio C_d/C.*

For positive modulation (or low negative modulation) the reliability of the general expressions of (4.0) as an indicator of pulse response is clearly as good as that of the simpler expressions in (4.2) and the following deductions should be valid for double-sideband working. The last equation of (4.0) shows the modulation-frequency output V_{dp}/V_{d_0} as the product of three factors—

$$\frac{V_{dp}}{V_{d_0}} = \frac{1 + 2kZ_s/R_d}{1 + 2kZ_s/Z_d} \times \frac{E_s}{E_0} \times \frac{e_s}{e_0}.$$

(a) When the last circuit is tuned to the carrier the analysis of (4.2) shows that the first two factors taken together are numerically equal to the selectivity factor E_s/E_0 for the last circuit with time constant (or Q) increased in the proportion $(1 + kC_d/C)$.

We can, therefore, obtain an actual modulation-frequency response equal to that of the original amplifier plus a perfect detector, by decreasing the Q of the last circuit in the ratio $(1 + kC_d/C)$ by a suitable decrease in R_d.

(b) When the last circuit is not tuned to the carrier, Z_s cannot be represented by the simple expression $R_1/(1 + 2jpCR_1)$. Since the pulse response time is roughly proportional to the bandwidth for 3-db loss, however, an estimate may be made by ascertaining Z_s for the sideband frequency $(f_0 + \Delta f)$, where Δf is the frequency at which the overall modulation frequency characteristic, as estimated from the selectivity alone, is 3 db down. The effect of the detector will here not be exactly equivalent to that of an increase in Q of the last circuit.

In the case of vestigial-sideband reception a staggered amplifier is frequently used, with the last circuit tuned to near the carrier frequency. In this case the remarks in (a) above should afford at least a useful approximation except for a large negative step, where non-following may occur. It should be noted that the envelope response of the r.f. amplifier taken alone tends to be 'rounded off' when m approaches -1.

this permitted a direct measurement of item (b) and a check on (d).

This triode detector may be regarded as an over-biased cathode follower or as an anode-bend detector with feedback, but no serious attempt has been made to work out the theory of its action in detail. Such an attempt has been made by Squierer and Goundry[3], but their analysis neglects all but first-order terms in calculating the input and output impedances, and does not lead to any very useful results. It is evident that the output load R_d will be shunted by the valve output impedance, which is equal to the effective value of $1/g_m$ (order of 1,000 ohms), and that this will greatly improve the response time. The value of this output impedance will rise with increase in bias resistance and with increase in r.f. input: it will also vary with time after the start of the transition when a step is applied. The 'bottom bend' in the d.c. characteristic is much longer and more rounded than for a diode, and this gives rise to a lower efficiency and poorer linearity (see Fig. 6), but reduces greatly the possibility of 'non-following' on a negative r.f. input step.

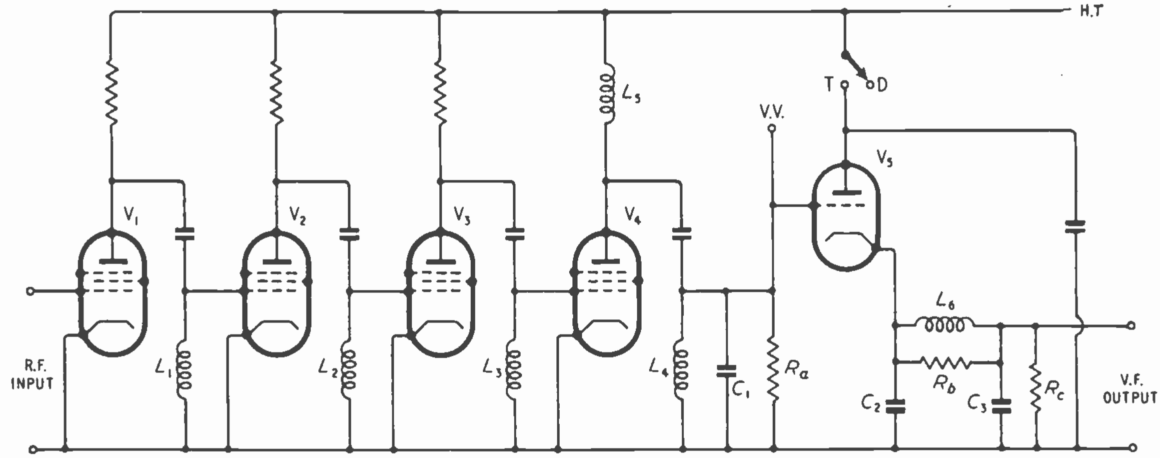


Fig. 5. *Circuit used for tests (V.V. = valve voltmeter). Switch to D for diode detector; for triode, switch to T and add loading resistor across R_a.*

5. Experimental Results

5.0. A difficulty which presented itself at the outset was that of separating the effects of the various parts of the circuit:—

(a) the detector considered alone.

(b) the tuned circuit or circuits preceding the detector.

(c) the frequency-variable loading imposed by the detector on the circuit feeding it.

(d) the test apparatus chain.

With this in view, all the tests with the diode were repeated with a triode detector, using the so-called infinite-impedance circuit. (See Fig. 5.) The input impedance of the triode is high enough (actually a negative resistance of some 50 kΩ under our conditions) effectively to eliminate item (c) in the above list, since the triode is fed from a circuit of parallel resistance less than 5 kΩ. The response time is also much shorter than that for the diode with a given C_dR_d, and

5.1. Tests were made with modulation factors of 1·0 and 0·5, and with positive and negative steps, giving four measurements of response time for each condition, designated as $m = 1$, $m = -1$, $m = 0{\cdot}5$ and $m = -0{\cdot}5$.

The pulse test apparatus used was that described in a previous article[5] and was capable of a discrimination of 0·01 μsec and an accuracy of 5% in time measurement. The rise time (10% to 90%) for the test apparatus (including the pre-amplifier used in Sect. 5.2) was checked by taking a measurement on the triode rectifier using C_dR_d = 0·10 μsec and $2CR_1$ = 0·04 μsec: this gave a figure of 0·07 μsec, indicating that the rise time for the test apparatus may be neglected in comparison with most of the figures shown in the following sections, having regard to the fact that rise times normally add quadratically. Rise time for the video waveform before modulation was better than 0·05 μsec.

Input to the rectifier was normally kept at 4 V r.m.s., but special tests were made at other inputs.

For convenience, the same valve (half an ECC91) was used for both diode and triode detector. As a diode, the voltage was removed from the anode (see Fig. 5) and the grid then showed a capacitance to cathode (1·5 pF) and a V_g–I_g characteristic closely similar to that of a typical diode (7 mA at + 2·0 V). When the valve was switched as a triode, a resistance (representing the diode input impedance to c.w.) was connected between grid and earth of such a value as to keep the voltage at the input (i.e., the circuit R_2) the same as with the diode.

Rectification characteristics are given in Fig. 6 for the triode and the diode with two values of load. The non-linearity of these characteristics will affect the measured output response time: the magnitude of the effects to be expected in the worst case can be estimated from the figures in Table 1 which have been calculated for two shapes of pulse after passing through an amplifier having a true square-law characteristic and zero time constant.

Since the actual detector characteristics are nearly linear above 0·5 V (for diode) or 2 V (for triode), the effects of non-linearity upon T_r will be small with 4 V input and m = 0·5. The only really large effect will occur with an exponential input and m = −1, where the 'tail' will be considerably shortened since it will fall in the square-law region of the characteristic.

The circuit used in the tests is shown, shorn of inessentials, in Fig. 5. For the tests (Sect. 5.2) on a single circuit feeding the detector, the first three stages (L_1, L_2 and L_3) were heavily damped to form a very flat bandpass about 8-Mc/s wide, and were merely used to raise the level of the input signals to the order of a volt at the grid of V_4. The capacitance of the circuit feeding the diode is C_1 plus strays (24 pF), and its parallel resistance is R_a shunted by stray losses (about 40 kΩ). The diode load capacitance C_d is $(C_2 + C_3)$ plus strays, including the input capacitance of the probe feeding the wideband amplifier for the pulse viewer: likewise R_d is R_c slightly shunted by the probe (40 kΩ). L_5 is an r.f. choke, and L_6 another choke (40 μH) shunted by R_b (3·3 kΩ) to obtain over-critical damping: the reactance of L_6 is considerably less than R_d at the highest video frequency concerned, but a check was made by further reducing R_b to make sure that L_6 did not appreciably affect the response. C_2 was 10 pF, plus 6 pF from valve and strays: C_3 was switched to make up the total C_d of 30, 50, 80, 130 or 174 pF.* The input frequency was 17·3 Mc/s. A valve voltmeter (V.V.) was permanently attached across L_4, and the diode current was also measured.

TABLE 1

Input	Output T_r rel. to Input T_r Rise (+)	Fall (−)
Exponential Rise	1·20	0·50
Linear Rise	0·80	0·80

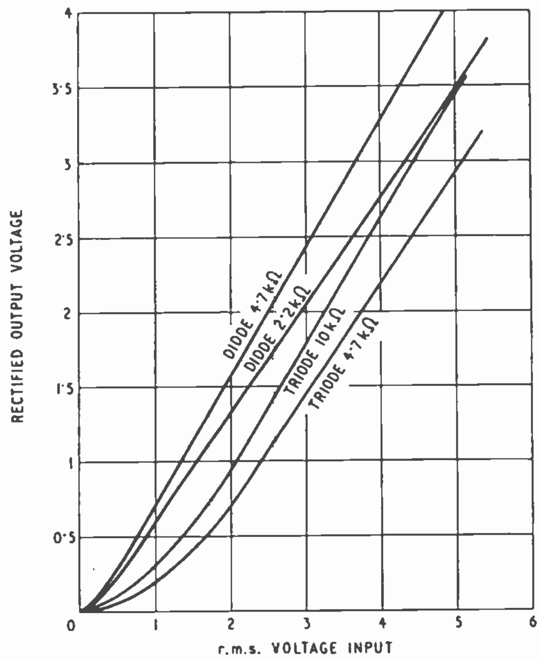


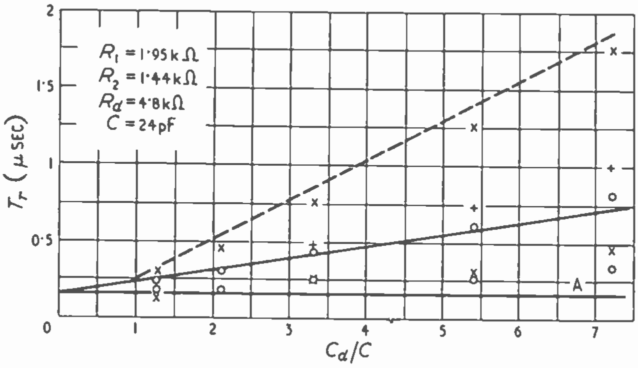
Fig. 6. *Output from rectifiers with various loads.*

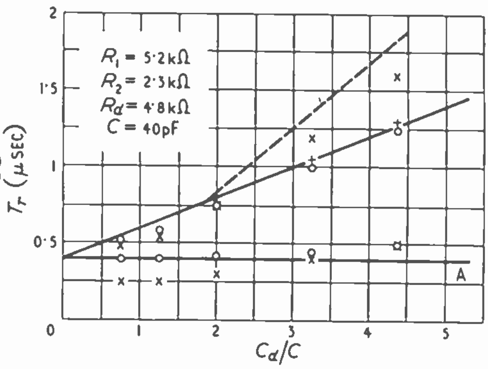
5.2. The first series of tests were made in order to check the theory for a single tuned circuit feeding the rectifier, and results are plotted in Figs. 7, 8, and 9. As in Fig. 4, the horizontal line is the response time ($4·4CR_2$) for the circuit alone, with rectifier loading on. The sloping dashed line ($2·2C_dR_d$) represents T_r for the rectifier output load, and the sloping solid line is the theoretical overall T_r assuming no 'non-following'.

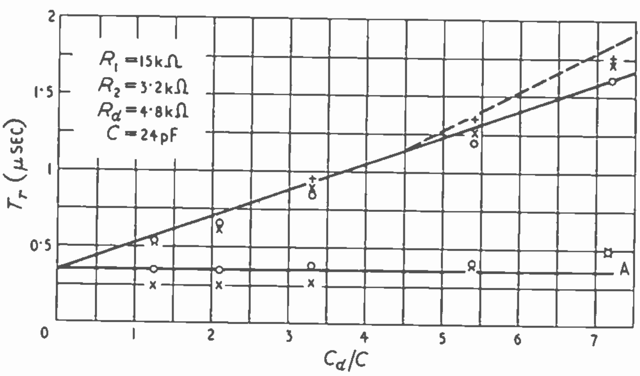
The output load resistance R_d was kept at

* This arrangement made it possible to change the v.f. time constant without appreciably altering the rectification efficiency.

4·8 kΩ throughout, and the three diagrams show the change in measured overall response time with C_d for three values of the constants in the tuned circuit. Measurements under the four conditions of step modulation (i.e., for $m = \pm 1$ and $m = \pm 0{\cdot}5$) are plotted but in the cases of $m = \pm 0{\cdot}5$ points have only been inserted when they differed to a significant extent from those for $m = +1$.







Figs. 7, 8 and 9. *Performance of a detector fed from a single tuned circuit. The lines correspond to those in Fig. 4. Measured results are indicated by the points:— × for m = — 1,* $\overset{+}{\circ}$ *m = + 1, + m = — 0·5. The lower row of points, near line A, are for the triode detector, and the rest are for the diode.*

In all figures, the upper rows of points, roughly following the theory, are for the diode, and the lower row of points, keeping quite close to the line for $T = 4{\cdot}4\ CR_2$, are those for the triode.

The input resistance to the diode was found to be 4·7 kΩ and the efficiency $\eta = V_d/1{\cdot}4V_r$ was 0·68 at 4 V input. Thus in this case $k = R_d\sqrt{2}R_{in} = 0{\cdot}51$, being considerably less than η for the reasons discussed in Sect. 2. The fact that V_r was measured by an ordinary (diode type) valve voltmeter may also have contributed to the difference between k and η.

Additional tests were made to determine the effect of the input to the rectifier. At small inputs (about 1 V r.m.s.) the figures for $m = -1$ were reduced to those obtained for $m = +1$, or even slightly below them for C_d small. At a larger input (10 V), the figures for $m = -1$ increased somewhat; for the diode they approximated closely to $2{\cdot}2C_dR_d$, while for the triode they rose by as much as 50% in the worst cases (T_r was 0·70 for $2{\cdot}2C_dR_d = 1{\cdot}85$). Figures for $m = +1$, and for $m = \pm 0{\cdot}5$ were little affected by input within these limits.

The measured results for $m = +1$, and $+0{\cdot}5$ agree quite well with theory in the case of the diode. For the triode, they indicate an effective output resistance of around 800 ohms which renders its response much faster than that of the diode for a given value of C_dR_d. Figures for $m = -0{\cdot}5$ do not differ widely from those for $m = +1$.

The response time for $m = -1$ is usually somewhat less than indicated by the theory of Sect. (4.2), but the difference can be accounted for as follows:—

(a) T_r for the diode is always less than $2{\cdot}2C_dR_d$ under non-following conditions, especially at low inputs. This must be due to the fact that the diode conducts at zero applied volts, thus hastening the discharge of C_d once V_d becomes small, and shortening the 'tail' which accounts for a large part of the total response time.

(b) In all other cases, the non-linear input/output characteristics will shorten the output response time. This is most noticeable with the triode, where the measured T_r is sometimes actually less than that for the circuit alone: since the characteristic is roughly square law below 2 V input, the 'tail' of the exponential will fall in this region, and the output T_r may be reduced by some 30–40%. (The 'tail' will fall in a relatively linear region on a positive step, and the effect upon T_r is unlikely to exceed 10%.)

The effective output resistance (as estimated from T_r) of the triode rises, as expected, with input for $m = -1$, starting at 800 ohms for 1 V input and reaching some 2,000 ohms at 10 V r.m.s. input.

In many of the tests, including those giving values of T_r in agreement with theory, an appreciable deviation from exponential form was observed. This was considered to be due to the factors mentioned in the last paragraph and to the time-variable loading of the circuit by the diode. (See Sect. 3.1.)

5.3. Apart from the tests designed to check the theory, two special sets of tests were made with the single tuned circuit feeding the diode detector:—

(a) The effect of a *video-compensator inductance* added in series with R_d was examined.

When no non-following was occurring, T_r could be reduced to 0·6 of its value without compensation, without introducing overshoot or dissymmetry.

In a typical case showing non-following, (C = 24 pF, $R_1 = 2\,k\Omega$, C_d = 50 pF, $R_d = 4{\cdot}8\,k\Omega$), results for T_r were as shown in Table 2.

The compensator thus actually improved the symmetry of the pulse: this is because L is adjusted for just no overshoot at $m = -1$, and does not give the full reduction in T_r at $m = +1$ owing to the extra damping which is then present.

TABLE 2

	$m = +1$	$m = -1$
No compensation	0·29 μs	0·48 μs
With L for just no overshoot . .	0·23 μs	0·30 μs

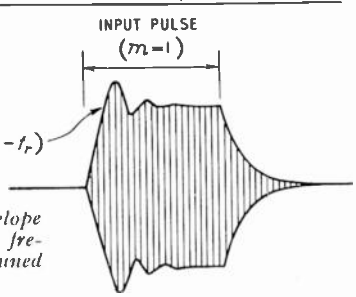


Fig. 10. *Form of envelope response to pulse at frequency f_0 for circuit tuned to f_r.*

(b) A few tests were made with the *circuit tuned away from the input frequency f_0*. Under these conditions the r.f. envelope for unity modulation factor shows a fast rise ($m = +1$) with a tendency to oscillate in amplitude, while the fall ($m = -1$) is an exponential identical with that obtained when the circuit is on tune. (See Fig. 10.)

The peculiarities of the diode (see Sect. 3.1) enable it to follow an envelope of this type remarkably well: in a typical case a diode with $2{\cdot}2C_dR_d = 0{\cdot}32\,\mu$sec fed from a circuit detuned by 6 db showed $T_r = 0{\cdot}09\,\mu$sec at $m = +1$ and $0{\cdot}30\,\mu$sec at $m = -1$.

When the modulation factor is reduced, T_r for the envelope on a negative step falls, approaching that for a positive step: here, too, the diode can follow the envelope well for the reasons given in Sect. 3.1, and T_r was about $0.10\,\mu$sec for both $m = +0.5$ and $m = -0{\cdot}5$ in the circuit quoted above.

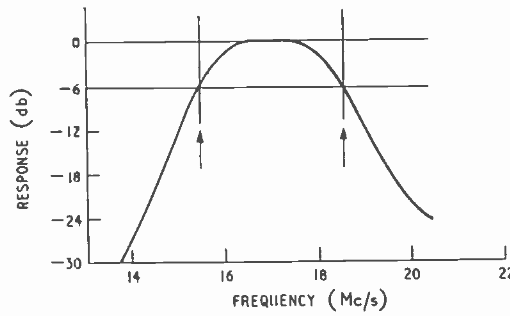


Fig. 11. *Frequency characteristic for the 4-circuit staggered amplifier; arrows indicate the alternative carrier frequencies for vestigial-sideband operation.*

5.4. A series of tests was finally made with the amplifier shown in Fig. 5 adjusted to comprise two exactly similar staggered pairs in cascade, the degree of stagger approximating to the usual 'critical' or optimal-flatness value. The first and third circuits were tuned to 16 Mc/s and the second and fourth (with detector connected) to 18 Mc/s. The overall r.f. curve was adjusted for symmetry at the −6-db points, which were at 15·5 and 18·5 Mc/s: the curve (see Fig. 11) was then symmetrical throughout the normal pass-band, but the cut-off was somewhat sharper on the low-frequency side (−26 db at 14 Mc/s) than on the high-frequency side (−20 db at 20 Mc/s). This was presumably due to the inherent asymmetry of circuits having a bandwidth which is a considerable fraction of their centre frequency.

Some preliminary measurements on overall pulse response for $m = \pm 0{\cdot}5$ showed:—

(a) With the triode detector, the response with carrier at 15·5 M/cs was not quite identical with that at 18·5 Mc/s. Overshoot was greater (15% instead of 5%) at 18·5 Mc/s: this could be accounted for by adding sidebands, which showed a slightly rising modulation-frequency characteristic at 18·5 Mc/s, owing to the slower cut-off above this frequency. A pre-shoot (overshoot before the transition) appeared at 15·5 Mc/s, presumably due to the phase shift associated with the more rapid cut-off below the carrier frequency.

T_r was 0·15 μsec at both frequencies, and was virtually unaffected by varying C_d to give values from 0·19 to 0·50 for $2{\cdot}2C_dR_d$.

(b) With the diode detector the response with carrier at 15·5 Mc/s approximated closely to that with the triode, and was hardly affected (for $m = \pm 0{\cdot}5$) by variation in C_d within the limits mentioned.

At 18·5 M/cs, however, the response was slightly slower for $2{\cdot}2C_dR_d = 0{\cdot}19$ μsec, and was greatly affected both in waveform and value of T_r when C_d was increased. This is evidently due to the high value of Z_0 (see Sect. 4.0) at 18·5 Mc/s as compared with that at 15·5 Mc/s.

A more systematic examination was then made of the diode response under typical vestigial-sideband conditions. The carrier was set to 18·5 Mc/s, since it is usually essential in practice to have the circuit feeding the diode tuned near the carrier in order to obtain adequate output. Table 3 shows the measured response times for two values of C_d. A resistive shunt was then placed across the tuned circuit, R_d was increased to bring R_2 (and therefore the voltage input to the detector) back to the same value, and the measurements repeated. Figures were also taken on the triode for comparison.

TABLE 3

R_1 (kΩ)	R_d (kΩ)	C_d (pF)	$2{\cdot}2C_dR_d$ (μsec)	T_r for four values of m: −0·5	+0·5	−1	+1
30	2·9	30	0·19	0·18	0·22	0·26	0·20
		50	0·31	0·21	0·24	0·34	0·23
8·5	5·5	30	0·36	0·21	0·22	0·37	0·21
		50	0·60	0·40	0·29	0·50	0·26
Triode		50	0·60	0·15	0·17	0·21	0·16

The waveform did not show any appreciable overshoot with the diode.

It is seen that, for a given i.f. bandwidth, T_r is worse with the resistive shunt across the tuned circuit, especially for $m = -1$, where non-following occurs. It is also evident that the overall T_r for the amplifier can be much less than the rise time $2{\cdot}2C_dR_d$ for the diode load taken alone (except in the case of $m = -1$). This is partly owing to the effective shunt reflected from the tuned circuit (see Sect. 4.0) and partly owing to the overshoot which would occur with a perfect detector (see results on triode above).

6. Practical Conclusions

The theory presented in previous sections cannot, unfortunately, be reduced to a series of rules for the practical design of signal rectifier circuits in wideband receivers. To explore fully the practical problems involved would entail an examination of the relative merits of different circuits and rectifiers (the use of a push-pull circuit, and/or a crystal rectifier, for example, may enable C_d to be reduced), and such questions are outside the scope of this paper. In this connection, the triode detector used in Sect. 5 may have some applications, but it certainly cannot be regarded as a panacea owing to its poor linearity.

In general, it may be said that any given practical design must be a compromise between (a) speed of response (or bandwidth) and (b) gain and available output from i.f. stage plus detector. To maximize gain and output we are required to maximize R_2 and η: this means making R_d and R_1 as high as possible subject to an adequate speed of response being obtained. It may, of course, be advantageous to tap or couple the diode into the tuned circuit; this will enable a high effective value for C/C_d to be obtained with minimum loss of gain, but will unfortunately lower the effective ratio of R_1 to R_d.

The simple rough criterion for negligible distortion of pulses by the detector is (see Sect. 3) that $2{\cdot}2C_dR_d$ should be less than, say, half the response time of the previous amplifier. This is often not practicable: for example, for a 5-Mc/s overall passband, T_r is about 0·08 μsec: thus we should have to make $R_d < 900$ ohms if $C_d = 20$ pF, and this would give poor output and efficiency. The results given in earlier sections indicate the conditions under which distortion may occur when a higher value of R_d is used.

Perhaps the most important practical conclusion to be drawn from the theory is that there is no advantage to be gained by using a shunt resistor across the circuit feeding the diode to provide a part of the required damping. From a common-sense viewpoint, one might have expected that the addition of such a shunt would increase linearity and uniformity of response to different types of modulation, in view of the variable damping provided by the diode. In fact, however, the shunt increases the tendency to non-following, with consequent deterioration of the response time on a large negative step relative to that under other conditions.

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TWO-SECTION TRANSMISSION-LINE TRANSFORMER

By M. S. Wheeler

(Electronics Engineering Dept., Westinghouse Electric Corporation, Bloomfield, New Jersey, U.S.A.)

SUMMARY.—A solution for the two-section transmission-line transformer is given in a manner suitable for magnetron to waveguide coupling. The special but important case of equal-length sections is first considered. This is then generalized with the approximation that only small inequalities in length exist.

Introduction

A WAVEGUIDE-TYPE transformer is found very convenient at microwave frequencies for impedance transformation between magnetrons and the relatively high impedance of standard waveguide. For most applications a simple quarter-wave transformer is sufficient, but at times more is required in the way of bandwidth. A first approximation to a tapered line (and definitely easier to fabricate) is a two-section transformer[1] which will be discussed from the point of view of magnetron loading. Two lengths of transmission line are considered between magnetron cavity and waveguide. The comparatively small and frequency-insensitive discontinuity capacitances[2] are neglected, the practice being to correct the designed transformer length for end effects after analysis. Correction is made for guide wavelength at the same time.

Transformer Requirements

At the principal magnetron mode the transformer is primarily required to present a certain resistance R_0 and negligible reactance X_0 to the magnetron cavity from a matched transmission line of characteristic impedance R_3. If the magnetron is tunable this requirement is made over a band of frequencies. The transformation ratio R_3/R_0 may be determined by the guide impedance and the required loading. The variation of R_0 within the frequency band determines the output power variation, other valve properties remaining constant. The maximum allowable value of X_0 is fixed by the magnetron external Q and the extent to which a matched line is permitted to affect the valve operating frequency. If other magnetron modes are found to compete strongly with the principal mode, a large value of R_0 and low X_0 may be required at certain other discrete frequencies.

Two-Step Transformer

Considering these requirements one may derive an expression for the transformer input impedance Z_0 with a matched line R_3 coupled to its output as in Fig. 1. If ϕ is the electrical length of the first section and R_1 its characteristic impedance, the matrix for the first section of lossless line from four terminal network theory is

$$\begin{Vmatrix} \cos\phi & jR_1 \sin\phi \\ \frac{j}{R_1}\sin\phi & \cos\phi \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & D \end{Vmatrix} \quad \ldots \quad (1)$$

where A, B, C and D are defined by means of

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

in four-terminal network terminology.

Cascading the two networks one obtains

$$\begin{Vmatrix} \cos\phi & jR_1 \sin\phi \\ \frac{j}{R_1}\sin\phi & \cos\phi \end{Vmatrix} \times \begin{Vmatrix} \cos\psi & jR_2 \sin\psi \\ \frac{j}{R_2}\sin\psi & \cos\psi \end{Vmatrix} = \begin{Vmatrix} a & b \\ c & d \end{Vmatrix} \quad (3)$$

Z_0 is given in terms of a, b, c and d.

$$Z_0 = \frac{V_0}{I_0} = \frac{aV_3 + bI_3}{cV_3 + dI_3} = \frac{aR_3 + b}{cR_3 + d} \quad \ldots \quad (4)$$

Upon evaluating a, b, c and d, Z_0 may be written

$$Z_0 = \frac{R_3 + j\left[m\left(R_2 \cos\phi \sin\psi + R_1 \sin\phi \cos\psi\right) - n\left(\cos\phi\cos\psi - \frac{R_1}{R_2}\sin\phi\sin\psi\right)\frac{R_3^2}{R_2}\right]}{m^2 + n^2 R_3^2/R_2^2} \quad \ldots \quad (5)$$

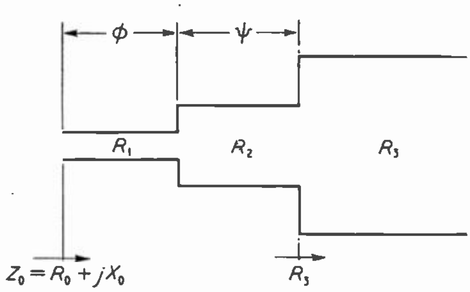


Fig. 1. *Two-section transmission-line transformer.*

MS accepted by the Editor, January 1954

where $m = \cos\phi\cos\psi - \frac{R_2}{R_1}\sin\phi\sin\psi$

$n = \cos\phi\sin\psi + \frac{R_2}{R_1}\sin\phi\cos\psi$

It is found convenient to change the independent variables ϕ and ψ to electrical line lengths α (representing the total transformer length) and β (representing the difference in lengths of the two sections).

$$\phi + \psi = \alpha \qquad \phi - \psi = \beta \qquad \ldots \qquad \ldots \qquad (6)$$

With this substitution one obtains

$$\frac{Z_0 R_3}{R_1 R_2} = \frac{4\frac{R_3^2}{R_1 R_2} + j\left[p\left\{(1 + R_2/R_1)\sin\alpha + (1 - R_2/R_1)\sin\beta\right\} - \frac{R_3^2}{R_1 R_2}q\left\{(1 + R_1/R_2)\cos\alpha + (1 - R_1/R_2)\cos\beta\right\}\right]\sqrt{\left(\frac{R_3^2}{R_1 R_2}\cdot\frac{R_1}{R_2}\right)}}{p^2 R_2/R_1 + q^2 R_3^2/R_1 R_2} \times \frac{R_2}{R_1} \quad \ldots \quad (7)$$

where

$p = (1 + R_2/R_1)\cos\alpha + (1 - R_2/R_1)\cos\beta$

$q = (1 + R_2/R_1)\sin\alpha - (1 - R_2/R_1)\sin\beta$

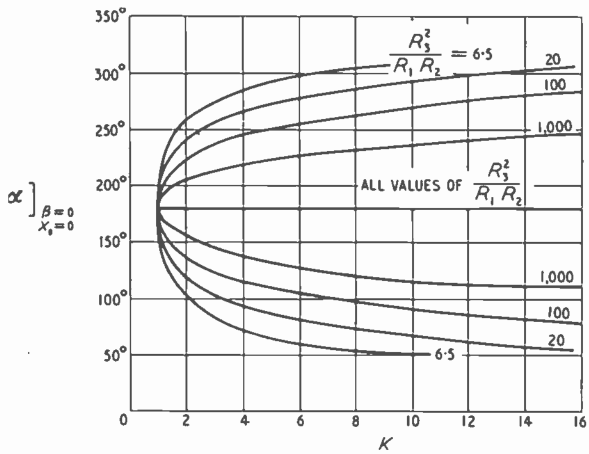


Fig. 2. *Roots of X_0 for equal-length sections.*

Equal Step Length

An understanding of the general case of unequal step lengths is obtained by first viewing the symmetrical case where the steps are of equal length; i.e., $\beta = 0$. With this requirement (7) reduces to

$$\left.\frac{Z_0 R_3}{R_1 R_2}\right]_{\beta=0} = \frac{4\frac{R_3^2}{R_1 R_2} + j\left[(1 + R_2/R_1)\cos\alpha + 1 - R_2/R_1 - \frac{R_3^2}{R_1 R_2}\left\{(1 + R_1/R_2)\cos\alpha + 1 - R_1/R_2\right\}\right](1 + R_2/R_1)\sin\alpha\sqrt{\left(\frac{R_3^2}{R_1 R_2}\cdot\frac{R_1}{R_2}\right)}}{\frac{R_2}{R_1}\left\{(1 + R_2/R_1)\cos\alpha + 1 - R_2/R_1\right\}^2 + \frac{R_3^2}{R_1 R_2}(1 + R_2/R_1)^2\sin^2\alpha} \times \frac{R_2}{R_1}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Of great interest are the zeros of the imaginary part of Z_0, as a primary requirement of the transformer at the principal magnetron mode is the vanishing of X_0. At the roots of $X_0(\alpha) = 0$, then, the section impedances may be adjusted to give the desired transformer ratio. One root is immediately obvious. For $\alpha = 180°$ (8) reduces to

$$\left.\frac{Z_0 R_3}{R_1 R_2}\right]_{\substack{\beta = 0\\ \alpha = 180°}} = \frac{R_3^2}{R_1 R_2}\bigg/\left(\frac{R_2}{R_1}\right)^2 = K^2 \qquad (9)$$

This defines K^2 which will be found a useful design constant. The reactive term vanishes at two other important points symmetrically spaced about 180° where

$$\left.\cos\alpha\right]_{\substack{\beta=0\\ X_0=0}} = -\frac{\left(1 - \frac{1}{K}\sqrt{\frac{R_3^2}{R_1 R_2}}\right)\left(1 + K\sqrt{\frac{R_3^2}{R_1 R_2}}\right)}{\left(1 + \frac{1}{K}\sqrt{\frac{R_3^2}{R_1 R_2}}\right)\left(1 - K\sqrt{\frac{R_3^2}{R_1 R_2}}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

At these roots it is very interesting to learn that

$$\left.\frac{Z_0 R_3}{R_1 R_2}\right]_{\substack{\beta=0\\ X_0=0}} = 1 \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Thus if a transformer were designed with centre frequency at $\alpha = 180°$, the symmetrical roots of $X_0 = 0$ might be used as a measure of the bandwidth. These roots exist only for $K > 1$ as shown in Fig. 2, analogous to the case of overcoupling in the analysis of two tuned circuits. At the same time K^2 is the fractional resistive change (limited by magnetron performance) over the band. Design significances may now be attached to R_2/R_1 and $R_3^2/R_1 R_2$ for it may be seen from (11) and the definition of K that at the frequency extremes

$$\frac{R_3^2}{R_1 R_2} = \frac{R_3}{R_0} \quad \text{and} \quad \frac{R_2}{R_1} = \frac{1}{K}\sqrt{\frac{R_3}{R_0}} \quad \ldots \quad (12)$$

Choice between (9) and (11) as the design equation is almost arbitrary. In this case (11) was chosen which is equivalent to designing from the edge of the frequency band in order to preserve the significance of $R_3^2/R_1 R_2$ and R_2/R_1 in the work which is to follow.

The transformer bandwidth from (10) can now be seen to increase with permissible K^2 and to vary inversely with the required transformation ratio. A family of such curves is shown in Fig. 3, plotted for a given transformation ratio of 1,000. This represents about the highest ratio that might be met and thus the minimum typical bandwidth. It is now readily seen that the proposed definition of bandwidth is not entirely ideal. For example, zero bandwidth is implied at $K = 1$, although it can be seen from Fig. 2 that this represents fairly wideband operation. For certain applications other definitions may be more descriptive. These will not be explored here in the interests of brevity.

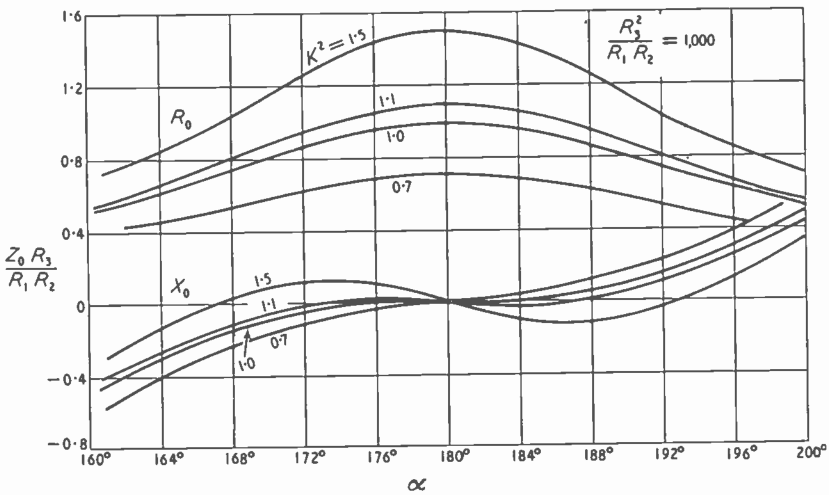


Fig. 3 (above). *Frequency characteristics of two-step transformer for a given transformer ratio.*

Fig. 4 (right). *Derivative of X_0 at its zeros with the associated line length.*

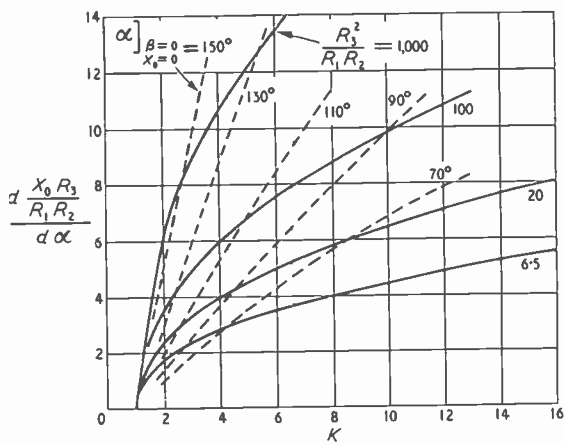
For purposes of selective loading or to obtain a shorter transformer length, one of the other roots of $X_0 = 0$ may be set at the magnetron centre frequency. When this is done it is found easier to define frequency sensitivity in terms of the derivative of X_0 at its root, because X_0 is generally linear over its range of interest. Evaluation of this derivative gives

$$\left.\frac{d\dfrac{X_0 R_3}{R_1 R_2}}{d\alpha}\right]_{\substack{\beta=0\\ X_0=0}} = \frac{\left(\dfrac{R_3^2}{R_1 R_2}-1\right)\left(K^2-1\right)}{\sqrt{K}\left(\dfrac{R_3^2}{R_1 R_2}\right)^{1/4}\left(\sqrt{\dfrac{R_3^2}{R_1 R_2}}K-1\right)} \quad \ldots \quad \ldots \quad (13)$$

where $R_3^2/R_1 R_2$ and R_2/R_1 have the same significance as given in (12). This information is plotted in Fig. 4, limiting the transformer length for a given transformer ratio and maximum allowable X_0 in the frequency band. The familiar quarter-wave transformer is one such example where $R_2/R_1 = 1$ and $\alpha = 90°$ at centre frequency.

Unequal Step Lengths

Asymmetries in the roots of $X_0 = 0$, caused by unequal section lengths are generally



only useful for selective loading. From the previous discussion it might be expected that the greatest interest will lie in the region of small inequalities. To get a useful solution let β (the electrical difference in length) be so restricted that

$$\sin\beta \approx \beta; \qquad \cos\beta \approx 1. \quad \ldots \quad \ldots \quad (14)$$

The roots are then found to be given by

$$\beta\Big]_{X_0=0} = \frac{\left\{\dfrac{R_3^2}{R_1 R_2}\left[\cos\alpha\left(\dfrac{R_1}{R_2}+1\right)+\left(1-\dfrac{R_1}{R_2}\right)\right]-\left[\cos\alpha\left(\dfrac{R_2}{R_1}+1\right)+\left(1-\dfrac{R_2}{R_1}\right)\right]\right\}\left(\dfrac{R_2}{R_1}+1\right)\sin\alpha}{\left\{\dfrac{R_3^2}{R_1 R_2}\left[\cos\alpha\left(\dfrac{R_1}{R_2}+1\right)+\left(1-\dfrac{R_1}{R_2}\right)\right]+\left[\cos\alpha\left(\dfrac{R_2}{R_1}+1\right)+\left(1-\dfrac{R_2}{R_1}\right)\right]\right\}\left(1-\dfrac{R_2}{R_1}\right)} \quad (15)$$

which are plotted in the $\alpha\beta$ plane in Fig. 5 in terms of R_2/R_1 and R_3^2/R_1R_2. It should be noticed that for a given transformer with physical length y_1 and y_2, the parameters α and β are related by

$$\alpha = \beta \frac{y_1 + y_2}{y_1 - y_2} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

That is, the specific locus in the $\alpha\beta$ plane for a given transformer is a straight line which is traversed as frequency is varied. The actual zeros of X_0 are given by the intersection of this line with the appropriate curve of constant R_2/R_1. In general this may be triple-valued as shown by the locus on Fig. 5 for $\alpha = -0{\cdot}02\,\beta$. The roots of $X_0 = 0$ are thus easily derivable by this graphical method. One additional bit of information available is the value of R_0 at these roots. It is given by

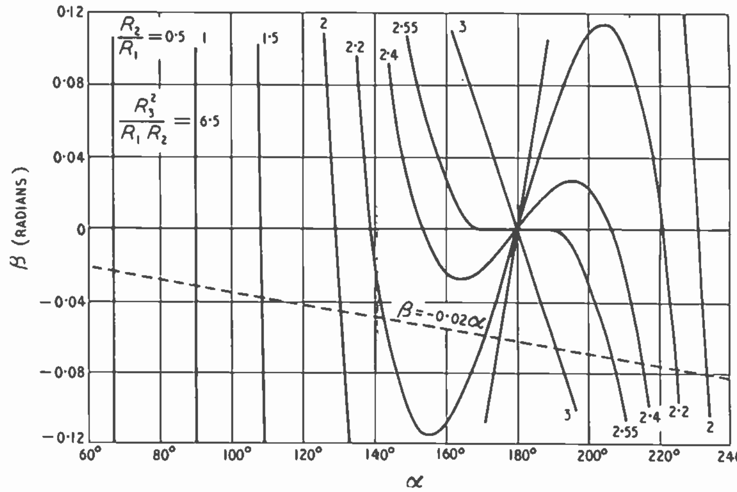


Fig. 5. *Roots of X_0 for a given transformer ratio.*

$$\left.\frac{Z_0 R_3}{R_1 R_2}\right]_{X_0 = 0} = \frac{\left(\frac{R_2}{R_1} + 1\right)\sin\alpha - \left(\frac{R_2}{R_1} - 1\right)\sin\beta}{\left(\frac{R_2}{R_1} + 1\right)\sin\alpha + \left(\frac{R_2}{R_1} - 1\right)\sin\beta} \quad \ldots \quad \ldots \quad (17)$$

derivable from (7).

Conclusions

Two regions of general interest are seen to exist for a two-step transmission-line transformer, both having steps of equal or nearly equal length. The first region of interest centres at a total transformer length of 180 electrical degrees. The transformation ratio, R_3/R_0, and uniformity of this ratio, K, are seen to be design factors inverse to bandwidth. A second region of interest is centred at line lengths of less than 180° giving real transformation ratios. Here, again, bandwidth is inverse to the transformation ratio; and for a given ratio, the greater the line length the greater the bandwidth. Compromising these requirements, information has been given which would make it possible to design a transformer for a given application.

Acknowledgment

I wish to thank Miss S. Hamalian and Mr. B. A. Perry for assistance in computation.

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INDEPENDENT TELEVISION AUTHORITY

P. A. T. Bevan, B.Sc., M.I.E.E., has been appointed chief engineer of the Independent Television Authority. He has been with the Engineering Division of the B.B.C. since 1934 and recently in the Planning and Installation Department, where he has been associated with the planning of the post-war B.B.C. television service.

AWARDS

The Institute of Radio Engineers has awarded Fellowships of the Institute to Professor G. W. O. Howe, D.Sc., LL.D., M.I.E.E., "for his pioneering work in radio and his outstanding contributions to engineering education"; and to T. E. Goldup, C.B.E., M.I.E.E., "for his pioneering achievements in the design and development of thermionic tubes and his contributions to the technical and administrative counsels of the British radio industry".

R.E.C.M.F. EXHIBITION

The exhibition of components organized by the Radio and Electrical Component Manufacturers' Federation of 22 Surrey Street, Strand, London, W.C.2, will be held this year from 19th–21st April at Grosvenor House, Park Lane, London, W.1. Application cards for tickets can be obtained from the Federation.

ABSTRACTS AND REFERENCES INDEX

The Index to the Abstracts and References throughout 1954 is in course of preparation and will be supplied with the March 1955 issue. Subscribers will receive the Index automatically. As usual, a selected list of the journals scanned for abstracting, with publishers' addresses, will be included.

HIGH-FREQUENCY PHENOMENA

Communication and Power Engineering Concepts

By William A. Tripp

THE scientific foundations of communication engineering rest largely on the wave analysis of the electromagnetic field, while those of the power branch of electrical engineering rest more specifically on what might be called ordinary electricity and magnetism. The two groundworks have much in common, yet have grown in somewhat divergent directions. The workers in the two branches talk a different language and think in terms of different concepts. To the communications engineer the language and activities of space vectors are of prime importance, while to the power engineer the currents and voltages of the circuit are the important things.

This is not as it should be. The phenomena of nature are the only true facts, and these must apply to both. While a particular phenomenon may appear in exaggerated form in the one branch of engineering, there is no reason why it cannot be dealt with in a manner equally familiar to workers in both branches. The following comments are offered with the idea of pointing the way to a reunification of divergent trends and a clarification of common features of the two branches. Following this, application will be made to specific cases, showing how the ordinary current and voltage concepts can be applied to high-frequency transmission lines, including waveguides. By these means the operation of these lines will assume a degree of reality in terms familiar to ordinary engineering.

One of the useful forms of electricity and magnetism is the particle electrodynamics. The phenomena important to power engineering can be explained in terms of particle electrodynamics, although this is not often done in texts written especially for the power engineer. In wave analysis, on the other hand, it commonly is recognized that the activities of charged particles are the causative agents. Although the particle electrodynamics does not, in itself, yield a wave field, it often is used to determine the magnitudes of the space vectors used in wave field analysis.

The activities or states of charged particles which are of importance are: (1) the positions of charges; (2) their time rate of change of position, or velocity; and (3) the time rate of change of their velocity, or acceleration. The intensities of the effects due to (1) and (2) fall off very rapidly with distance from the source, being inversely proportional to the square of the distance. However, (3) falls off much less rapidly, being inversely proportional to the first power of the distance. It is usual to assume that, where radiation effects are concerned, the distances are such that (1) and (2) have become negligible and that only (3), the acceleration effects, need be considered.

MS accepted by the Editor, February 1954

Acceleration Effects

There is another very important reason why the acceleration effects predominate in radiation phenomena, upon which sufficient emphasis is not often placed. This is the fact that the acceleration magnitudes which can be attained in practice are much greater in proportion than the attainable magnitudes of the other states. It is not possible with known insulations, and especially when air is the dielectric, to attain high concentrations of charges at small separations between points of different potential, without encountering breakdown. In the case of velocity phenomena the rate of movement of the charge is extremely slow and nothing much can be done about it, except in such machinery as electron accelerators and the like, which are primarily useful in unallied applications. Ordinary electrical engineering is, and long has been, pressing the frontiers of man's ingenuity with respect to both charge concentration and current flow, but in high-frequency phenomena the acceleration, which is a direct function of the frequency for a given current magnitude, can reach very high values with even small values of current and charge concentration.

For this reason acceleration effects are important at high frequencies not only in cases of far actions, but also in arrangements where the conducting regions are close together. This class includes the newly-developed conductor designs such as waveguides and coaxial cables. The other phenomena, especially those involving charge concentration, are important in these cases also, due to the closeness of areas of different potential. But due to the high frequencies used with these arrangements the acceleration effects play a dominant role. It is, in fact, the exaggerated magnitude of certain acceleration phenomena, not noticeable at low frequencies, that makes possible the peculiar usefulness of these arrangements at high frequencies.

The acceleration effect is encountered in ordinary electrical engineering in the voltage induced as a result of varying current. Commonly called the 'transformer effect', the mathematical

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formulation of this for a pair of coupled circuits is:

$$V_2 = -M \, di_1/dt$$

where V_2 is the voltage induced in the second circuit,

M is the mutual inductance of the two circuits, and

i_1 is the current in the first circuit.

In the particle electrodynamics it is recognized that:

$$i_1 = n_1 e u_1$$

and $di_1/dt = n_1 e a_1$

where n_1 is the number of moving charges in unit length of the first circuit,

e is the value of charge on each charge carrier,

u_1 is the average velocity of moving charges in the first circuit, and

a_1 is du_1/dt.

Now also $M = \int_{s_1} \int_{s_2} (1/r_{12}) \, \mathbf{ds}_1 \, \mathbf{ds}_2$

is a fundamental definition of mutual inductance, where the integrals represent summation along the complete lengths of the two circuits of which $\mathbf{ds}_1$ and $\mathbf{ds}_2$ are elementary (vector) lengths, and r_{12} is the distance of separation between any two elementary lengths. This formula integrates into the mutual-inductance formulae for any configuration (manner and degree of coupling) of circuits and also into the self-inductance formulae when self-inductance is recognized for what it really is, the coupling of a circuit with itself.

Substituting these values of di_1/dt and M in the above formula for V_2 yields:

$$V_2 = -n_1 e \int_{s_1} \int_{s_2} (1/r_{12}) \, a_1 \, \mathbf{ds}_1 \, \mathbf{ds}_2$$

in which a_1 is correctly located under the integrals because it partakes of the directional relationship of $\mathbf{ds}_1$.

Since this formula expresses the summation of voltage in the total of circuit (2) due to the variation of current in circuit (1), we deduce:

$$\mathcal{E}_2 = -n_1 e \int_{s_1} (1/r_{12}) \, a_1 \, \mathbf{ds}_1$$

for the electric intensity at $\mathbf{ds}_2$ due to all of circuit (1) and

$$d\mathcal{E}_2 = -n_1 e (1/r_{12}) \boldsymbol{a}_1$$

for the electric intensity at $\mathbf{ds}_2$ due to $\mathbf{ds}_1$ and

$$\mathcal{E} = -\boldsymbol{a}/r$$

for the electric intensity at any point in space at distance r due to acceleration of unit charge.

In radiation phenomena, the electric intensity due to an accelerated charge is taken as the component of the above value of $\mathcal{E}$ perpendicular to the radius vector; that is, the above value multiplied by the sine of the angle between the direction of the acceleration $\boldsymbol{a}$ and the radius vector r. This seems to be a carry-over from the formula for the magnetic field intensity due to current in a conductor of finite length according to Ampère's formula, often incorrectly called the Biot-Savart formula.

The usual wireless transmission arrangement lends some credence to this idea because the receiving circuit usually is so far removed from the transmitter that we are dealing with a very small segment of the radiated pulse, and the direction of travel is more apt than not to be transverse to the acceleration of the initiating charges. In such cases the angle is $\pi/2$, the value of the sine is unity, and the two expressions yield the same value. These circumstances have led to the prevailing concept that there is no radiation in the direction of the acceleration.

When this divergence of ideas regarding directional radiation was mentioned to a group of atomic scientists some years ago, they stated that in electron accelerators a very intense beam had been observed tangential to the electron path; that is, in the line of the major acceleration. This beam has since been the subject of some research and has been reported upon in the scientific Press.

Coupled Circuits

In ordinary electrical engineering it is necessary to use the full value of $-\boldsymbol{a}/r$ in order to treat the circuits of that branch of engineering. These circuits are, of course, what are commonly considered close-coupled. However, there is nothing in circuit analysis or in electrodynamics which sets a dividing line between close-coupled circuits and loosely-coupled circuits. Nor is there any mathematical delimitation to the degree of coupling where the ordinary formulae can be used. The logical conclusion is that $\mathcal{E} = -\boldsymbol{a}/r$ represents a basic fact of nature. In some texts where this is discussed, it is pointed out that $-\boldsymbol{a}/r$ might also be accompanied by a constant or other term which cancels out around a circuit, because our methods of measurement must use complete circuits and could not detect such a term. However, there is no known reason to include such a term, and certainly it would have no bearing on the use of only a part of $-\boldsymbol{a}/r$ in radiation analysis.

Loose coupling is not a matter of distance alone. When the wavelength is so short that it is in the range of magnitude of some of the circuit dimensions or the separation between circuit components, the coupling will be relatively loose even though the distances are relatively short. In these cases the time of transit of the pulse must be taken into account as it influences the manner in which the several circuit components phase in with each other. This condition is not commonly encountered in ordinary electrical engineering. In the particle analysis it is recognized and taken care of by means of the retarded potentials. But in the wave analysis of the electromagnetic field it does not seem to be adequately accounted for. When the voltage in a particular direction is derived from the expression for curl $\mathcal{E}$, there is nothing which makes explicit allowance for the time of transit of the wave throughout the circuit components. Some consider that Maxwell's displacement current in the expression for curl $\boldsymbol{H}$ serves this purpose, but this result does not follow from the nature of the displacement, nor from the way it is employed in the usual field analysis. In fact, it usually is neglected when determining the magnetic intensity due to the currents causing the inductive effects.

Induced voltages usually are calculated. even in high-frequency analysis, by determining the magnetic flux linkages either by calculating the flux from the usual formulae for flux density due to current flow, or from the inductance formulae for common geometrical configurations, which is the same thing. But these formulae are based on uniform current throughout the originating circuit and make no allowance for difference in time of arrival of the inductive effect in different parts of the receptor circuit. The flux linkages, therefore, are determined only for the condition that assumes a lapse of time sufficient for the flux density to reach its final value throughout all space. While it is possible to determine the value of the flux density at any point in space after the 'news' as to the value of the originating current has reached that point, it would be necessary, in the general case, to take into account the entire past history of the current in order to determine the simultaneous value of flux density throughout all space for variable current. In dealing with high-frequency phenomena in circuits of even moderate size, it is clear that the instantaneous value of total flux in a given area would depend upon the past history over several cycles, and that the magnetic intensity at different points of the circuit would correspond to different phases of the cycle. Thus the usual formulae cannot apply.

What would have to be added to the flux concept would be some term which indicates the rate at which flux 'streams by' any given point in space due to current change. This might be considered the converse of the motion of a conductor in a magnetic field. But the formula $\mathcal{E} = -\boldsymbol{a}/r$ serves this purpose mathematically, and gives the result directly, without the need for the intermediate instrumentality of the magnetic field. There is good reason to believe that the case of the moving conductor and the effect of changing current involve two different phenomena of nature. The former is a case of changing geometry with constant electromagnetic conditions, the latter a case of constant geometry and changing electromagnetic conditions. The fact that they both produce forces on electrical charges which integrate into a voltage along a conductor has led to their being considered the same phenomenon. But in the particle electrodynamics the former must be evaluated from the force equations involving velocity of charge, while the latter is evaluated from the force equations involving acceleration of charge.

Electric Space Vector

The subsequent comments on inductive effects will, therefore, be developed around the direct application of an electric space vector, of value $\mathcal{E} = -\boldsymbol{a}/r$, without use of the flux concept. There is no particular difficulty about this, except perhaps the concept of what the electromagnetic wave actually is. But here we get into the realm of metaphysics instead of the mere evaluation of physical effects. In true wave propagation the energy alternates between potential and kinetic forms by reason of the interplay of certain characteristics of the medium. $\mathcal{E}$ and $\boldsymbol{H}$ have long been viewed as the vehicles for this action in the electromagnetic wave. The alternative is to consider the wave as an $\mathcal{E}$ pulse, of value $-\boldsymbol{a}/r$, and determine parameters necessary to its propagation, much as we do in dealing with waves in physical media, such as the wave height on the surface of a liquid, or the fluctuations of density in the body of a medium.

It is not proposed to pursue these general considerations further in this discussion, but to turn our attention to actual conditions. Before considering specific cases, it will be helpful to discuss certain effects which, although based on ordinary induction action, are not evident at low frequencies but appear in very pronounced degree at high frequencies. These effects are brought out the more clearly by direct application of the interpretation of $\mathcal{E} = -\boldsymbol{a}/r$ just discussed.

(A) Consider first the self-inductance L per unit length of a straight conductor. Here L is presumed to be a constant, independent of frequency f, the influence of frequency being accounted for in the reactance, $X = 2\pi fL$. This is substantially correct for low frequencies, where the wavelength is so long that any difference of current phase occurs in parts of the conductor so remote from each other that the effect is negligible. The inductance is obtained by a multiple integration of $-di/dt$ (or $-\boldsymbol{a}/r$) along the conductor, the di/dt being assumed as unit change of current in unit time, and tacitly being assumed to have the same time phase throughout the conductor.

When, however, we are dealing with high frequencies, where several wavelengths may exist along a relatively short length of the conductor, this assumption no longer applies. The regions of different current phase may be so close to each other that their inductive effects will not combine in the ordinary manner assumed when integrating *di/dt*, but must be used in their proper time phase. This is a different proposition from combining the effects of different phases of multi-phase circuits in power engineering, as time of transit is not then considered. But when the wavelength is short, the inductive effect at any point of a conductor must be obtained by using the *di/dt* for other points, of either the same conductor or other conductors, for such time previous to allow for the time of transit to the point under consideration. The speed of transmission is the velocity of light. Consider the case of sinusoidal current in the conductor. The rate of sinusoidal pulse transmission along the conductor will also be approximately the speed of light. Since we are dealing for the moment with self-inductance we shall disregard other nearby conductors. A survey of the time phase of *di/dt* along the conductor will reveal the following interesting facts:

(A1) Since the rate of transmission is the same for both the inductive effect and the current phase, all the portions of the conductor behind (nearer the source than) the point under consideration will contribute to the inductive effect at that point as though they were in the same time phase. For it will be seen that each point will have passed through a given phase at some time previously, but it will take just that length of time for the effect to reach the point under consideration.

(A2) For the same reason, the portions of the conductor ahead of (farther from the source than) the point under consideration will contribute inductive effects as though they were at progressively later time phase. Thus a point one-quarter wavelength ahead will phase in one-half cycle later, because it actually is one-quarter cycle in time phase later and the effect has to travel one-quarter wavelength. A point one-half wavelength ahead will phase in a full cycle later, and so on. It will therefore be seen that the portion of the conductor between one-eighth and three-eighths wavelength ahead will contribute in such phase as to act like a negative inductance. Portions still farther ahead will contribute in alternately positive and negative phase, at progressively reduced strength because of their increasing distance. At sufficiently high frequencies the portion of the conductor ahead contributes very little to the self-inductance.

(A3) As a result of (A2) the self-inductance of the conductor will decrease with rising frequency, tending to approach one-half its low frequency value due to this effect alone.

(A4) As a result of the combination of (A1) and (A2), the inductance of the conductor is not uniform along its length. In portions near the source, where there is not much length behind to contribute according to (A1), the inductance will be very low. In portions far from the source, where there is considerable length behind, the inductance will range between the normal low-frequency value and that implied by (A3), other effects being disregarded. This is not to be confused with the ordinary 'end effect' condition of low-frequency analysis.

(B) When a cross-sectional dimension of the conductor is in the same range of magnitude as the wavelength, the time of transmission of induction effects has a further influence on the self-inductance. Consider two points in the same cross-section of such a conductor, one-half wavelength apart. With longitudinal transmission the currents will be in time phase, but due to the time required for the induction effect due to one to reach the other, they will act as though they were in opposite time phase. In other words, they will act like return conductors to each other, reducing the effective inductance of the conductor. The higher the frequency the closer the 'return conductor' and the lower the effective inductance. At sufficiently high frequencies a conductor of considerable cross-sectional size carrying longitudinal current can act as though it were a single strip of about one-half wavelength width, with less than normal self-inductance because of the closely adjacent 'return conductor'. This effect prevails over a short but vital distance along the conductor for each cross-section, gradually changing to the equivalent of a filamentary conductor.

(C) Mutual inductance also is affected by the time of transmission of the inductive effect. Consider two parallel conductors, the one being the return to the other. Here the longitudinal currents in directly opposite portions of the pair of conductors will be in opposite time phase. However, if the conductors are one-half wavelength apart the currents will phase in with each other as though they were in time phase, increasing the effective inductance of the complete circuit. On the other hand, if the conductors are a full wavelength apart, they will phase in according to their true time phase. As with the self-inductance effect of (B) above, this effect prevails for some distance lengthwise with respect to each cross-section. It should also be noted that the effect of (A1) takes over for these more distant portions of the conductor lengths with respect to mutual inductance, but in the opposite sense.

Transmission Lines

We are now in position to review the operation of high-frequency transmission lines such as waveguides, coaxial cables and paired conductors. These may be presumed to act as they do because some of the effects just discussed occur under optimum conditions. The wave does not follow the guide because of some inexplicable predilection. The wave energy is concentrated in the region of the guide first because the guide responds to the exciting influence in such a way as to make it act as a channel which can draw power from the source in very superior amounts and, secondly, in some cases it reacts in such a way as to help sustain the transmission of this power. The following comments indicate how these optimum conditions can be interpreted in the light of the familiar inductive effect of ordinary electrical engineering, occurring under some of the special circumstances discussed above.

One of the commonest waveguide applications is a hollow rectangular conductor operating near its critical wavelength in the first transverse-electric mode. If the cross-sectional dimensions of the tube are a and b for the long and short sides respectively, this will be characterized by an alternating electric field between the a sides with a wavelength of $\lambda_c = 2a$ approximately. The guide will transmit this and shorter wavelengths, but not longer wavelengths, except for highly damped evanescent waves.

Considering a representative cross-section of the guide, the electric field vector will lie in the plane of the cross-section, perpendicular to the a sides. According to ordinary electromagnetic concepts, this electric field vector may be due to accumulation of charge of opposite signs on the a sides, or acceleration of charge in the b sides, or both. Accepting both premises, we see that the condition is satisfied by the passage of current along the b sides, with the proper phase sequence in relation to the frequency, so as to cause charge to leave one a side and accumulate on the other. This process reverses and repeats in time with the frequency at which the waveguide is energized. Thus in this mode of operation the cross-section of the guide acts like a two-branch circuit of length b (and possibly extending a little over into the a sides), carrying an alternating current and terminated in a capacitor, the plates of which are the a sides fed at two points.

In order for the guide to act effectively, the phase relations in the various parts of the circuit must tend to sustain each other. We note now that the two branches of the cross-section circuit, the b sides, are in effect two portions of the same conductor separated by a distance a or one-half wavelength. The currents must be in time phase, otherwise the 'capacitor' would be short-circuited. According to (B) above this is a condition of minimum inductance, the currents in the two b sides tending to sustain each other. Thus one of the necessary conditions of this mode of operation is met. This is the first wavelength at which this condition will prevail. It cannot prevail at longer wavelengths. Thus, the occurrence of a critical wavelength for this mode of operation is satisfied and evaluated.

There will be current flow in the a sides also, which sometimes has been considered the important current. Actually it is not closely related to the optimum condition of operation of the waveguide, as a review of the current pattern will show. Therefore, it will not be considered further here.

Phase Velocity

We consider now the action of each b side as a whole; that is, lengthwise of the guide. Here we turn to a known circumstance which heretofore has not been assigned any great significance in the operation of the waveguide. This is the so-called phase velocity, which approaches infinity at the critical wavelength. This infinite value has a very definite, although simple meaning. The phase velocity has been described as the speed at which cross-sections of like phase travel along the length of the guide. For this speed to be infinite requires merely that all cross-sections be always at the same phase as one another, all the way along the guide. But this means simply that a steady-state condition of alternating current has been established in the guide, such that it is acting in unison throughout its entire length. More specifically, each b side is acting like a circuit of length b and conductor width S, where S is the length of the guide.

The ability of a conductor of such great width as S to operate effectively without the appearance of an extreme skin effect is found again in (B) above, from which it will be seen that the *effective* width of the b side is only of the order of magnitude of a or one-half a wavelength, instead of S. Also it must be remembered that the effective self-inductance of these a-width strips is less than normal because the next adjacent strips on each side bear a negative inductive relationship to the one between. More distant strips will be successively positive and negative, in decreasing strength with increasing distance.

When a waveguide operates in a longitudinal-electric mode, there will be found inductive relationships which are influential in governing the operation. This mode is characterized by an electric field vector longitudinal in the guide, the direction alternating with each half wavelength distance along the guide. According to ordinary circuit theory, this would indicate longitudinal current flow in the tube walls, with charge accumulation at the node points of the current. From (A3) above, it will be seen that this is a possibility, as the inductance of the circuit will decrease with increasing frequency. More important, in accordance with (B) the inductance will be very greatly reduced when the wavelength is so short that much of the cross-section of the conductor acts as the 'return conductor' to itself.

This latter observation suggests that there should be some wavelength at which this condition first becomes effective. Since all portions of a given cross-section will have current in the same time phase, and since negative inductance effect occurs between currents separated by one-quarter to three-quarters of a wavelength, the longest wavelength at which this condition would be realized would be when as much of the cross-section as possible is in this range of distance from the remainder of the cross-section. For a waveguide of rectangular cross-section, an approximation to this condition would be when the maximum dimension of the cross-section (that is, a diagonal) is three-quarters of a wavelength. Then the adverse fourth quarter wavelength has not begun to appear in the cross-section. Then $\lambda_c = (4/3)\sqrt{(a^2 + b^2)}$. For a typical proportion of rectangular guide with b a little less than $a/2$, say $4a/9$, this gives $\lambda_c = 1{\cdot}46a$. This is a very close check on the commonly accepted value of $1{\cdot}414a$ for this mode of operation. It is commonly understood that this value is independent of b. This does not appear from the above analysis, but b is not critical. The critical wavelength would range from $1{\cdot}33a$ to $1{\cdot}88a$ for $b = 0$ to $b = a$. If the waveguide has a circular cross-section, of radius r, the corresponding condition would be met at $2r = (3/4)\lambda_c$. From this we get $\lambda_c = 2{\cdot}67r$. This is a very close check on the commonly accepted value of $2{\cdot}61r$ for the critical wavelength of a circular guide in the first longitudinal-electric mode of operation. An interesting observation is that 282 degrees of each complete cross-section is acting like the return conductor to the other 78 degrees.

Other transmission lines, such as the coaxial cable and the paired two-conductor line, are subject to unique inductive conditions, although they conform more nearly with ordinary circuits. They are readily treated by ordinary circuit analysis, and are so treated even in analyses given over rather exclusively to wave analysis. The wave analysis considers these lines as transmitting in the transverse electromagnetic mode because there is a major potential difference directed transversely between the two conductors, and the line currents are represented by a magnetic vector perpendicular to the plane of the conductors. However, there are strong longitudinal electric inductive effects resulting from the di/dt of the line currents. These lines are inherently low inductance lines due to the close spacing of the conductors, but the inductance is further reduced at high frequencies in accordance with (A3) above. There is nothing in the geometry of their arrangement which indicates that there would be a critical wavelength, and it is a fact that these lines will transmit all wavelengths. However, (C) does indicate that there should be a region of minimum effectiveness when the conductor spacing is about one-half wavelength, and a region of optimum effectiveness when the conductor spacing is about a full wavelength. These facts do not at present have much practical significance because, with the wavelengths and voltages presently at our disposal, it still is easier to obtain minimum inductance by close spacing of the conductors. That is, practical wavelengths are substantially larger than permissible spacings. It is probable that as operating ranges are extended, technical ingenuity will tend to maintain this balance in favour of close spacing. For instance, as voltages are raised, advantage can be taken of the benefit of pressurizing the cable, as has been done at commercial frequencies.

Conclusions

The foregoing comments hardly more than scratch the surface of this approach to high-frequency transmission. No extensive mathematical analysis or formulation of laws has been attempted. The main purpose has been to demonstrate the important role played by the currents, as distinguished from their apparent role according to the wave analysis. In the wave analysis the presence of currents would seem to be more detrimental than otherwise, such as by causing power loss due to I^2R in the conductors. Also it is known that current flow affects the locations where slots may be cut in waveguides so that they either do, or do not, interfere with operation. For instance, for the rectangular guide in the first transverse electric mode, the only place where a slot can be located without disturbing operation is anywhere and of any length down the middle of one or both of the a sides. But this would be expected from the above analysis of the current pattern, as this is the neutral axis of the 'capacitor' plates, across which there is no current flow. It should be clear, from what has been said here, that the currents in the conductors play a far more important role in the operation of these devices, by assisting in the conditions required for their operation, and, indeed, making it at all possible under some circumstances. Thus the prevailing idea that all the important action takes place in the space adjacent to the conductor, which has a rather extensive and authoritative background, is seen to be somewhat short of being entirely correct.

It is hoped that these comments will aid in the resolution of some of these matters. Specifically, it is felt that the following points emerge from the foregoing: (1) Evidence of the sterility of the flux concept as a vehicle of mutual intelligence between workers in the communication and power branches of electrical engineering. (2) Demonstration of certain errors which have developed around the usual application of the flux concept. (3) Clarification of certain phenomena by a strict adherence to the principles of electrodynamics. (4) The possibility of developing more powerful analytical tools for the study of electricity and magnetism. (5) The prospect that new tools and methods may lead, as they so frequently do, to broadened horizons and increased benefits.

NOMOGRAM FOR Q OF A CAVITY

Derivation from Standing-Wave Measurements

By J. D. Harmer, B.Sc., Graduate I.E.E.

SUMMARY.—Cavity Q factors can be derived from measurements, at and around resonant frequency, on the standing waves set up in a uniform feeder coupled to the cavity. The Q factor is usually obtained from the frequency bandwidth of the voltage standing-wave ratio curve at a certain height. The method is extended here so that the Q factor can be estimated from a few experimental points only, without plotting a complete curve, and a nomogram is provided to facilitate the calculation.

This shortened procedure for measuring Q factors is satisfactory provided it is known that the shape of the resonance curve is undisturbed by adjacent resonances.

Introduction

THE properties of a cavity resonant in the centimetre or millimetre wavebands can be studied at one of its couplings by the well-known method[1,2,3,4] of coupling the cavity to a uniform matched transmission system through which power is fed to the cavity over the range of its resonant frequencies. The impedance presented by the cavity at its coupling can be expressed[2]:

$$Z = \frac{\frac{1}{Q_x} \cdot Z_0}{j\left(\frac{\lambda_r}{\lambda} - \frac{\lambda}{\lambda_r}\right) + \frac{1}{Q_0}} \quad , \quad .. \quad (1)$$

where Z_0 is the transmission system characteristic impedance,

Q_x is a factor representing coupling between cavity and feeder,

Q_0 is the internal resonance factor of the cavity,

λ is wavelength (λ_r at resonance).

This impedance causes reflection and gives rise to standing waves in the transmission system. These standing waves can be measured and the cavity impedance and hence Q factors derived therefrom.

This account describes a method for deriving Q factors from a minimum number of experimental measurements of the standing waves.

MS accepted by the Editor, January 1954

Derivation of Q Factors

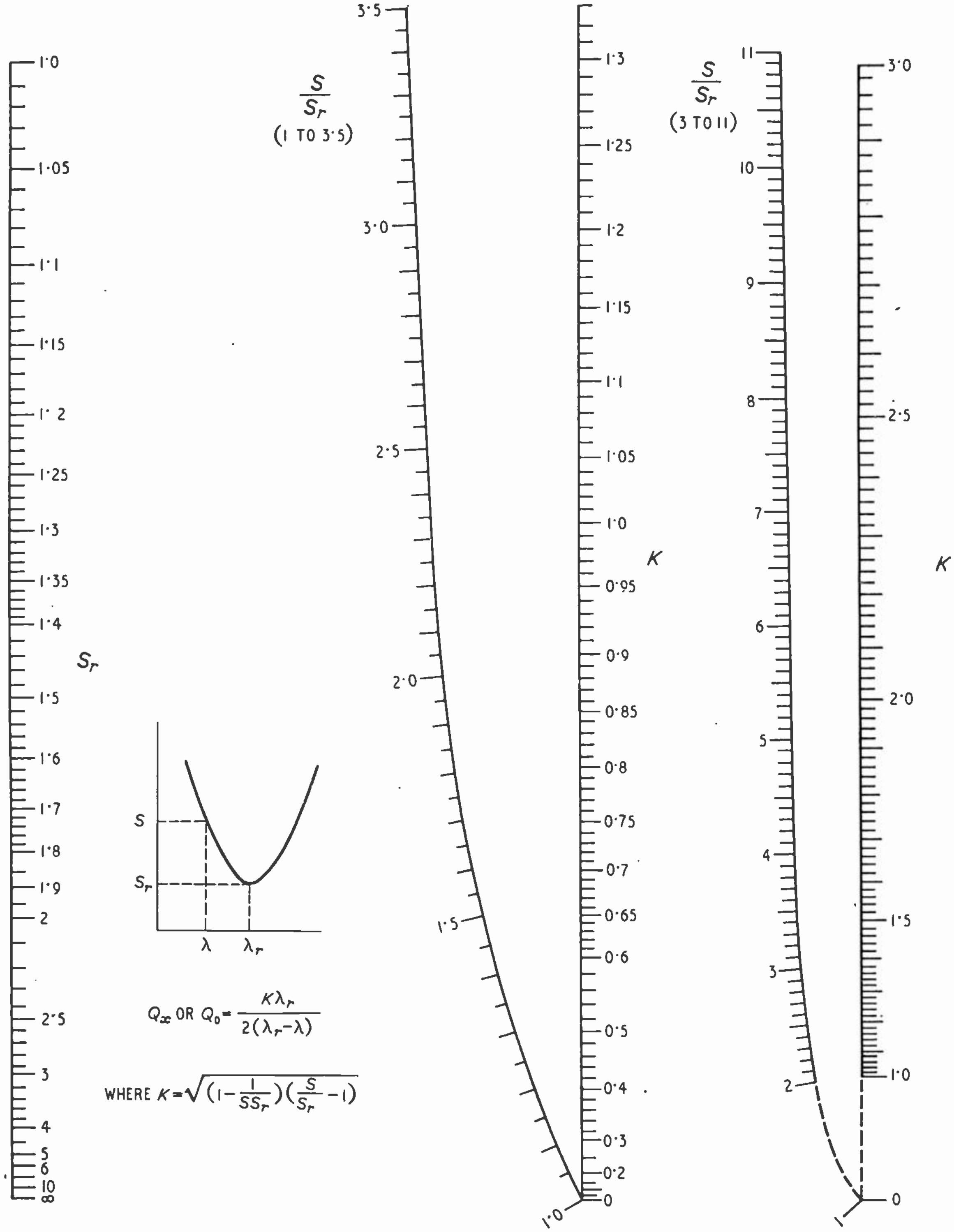
The voltage standing-wave ratio S is defined as the ratio of maximum to minimum voltage along the uniform feeder, so that S is always $\geqslant 1$. It is shown in the appendix that, when plotted as a function of frequency, S exhibits a minimum value S_r at resonance such that

$$S_r = \frac{Q_0}{Q_x} \text{ if } Q_0 > Q_x$$

$$\text{or } S_r = \frac{Q_x}{Q_0} \text{ if } Q_x > Q_0. \qquad \text{(6 from Appendix)}$$

Which of the two factors Q_x and Q_0 is the greater can be determined by observing the way in which the position of the standing-wave minimum changes as the frequency is varied through resonance. (See, for example, Ref. 4.) The smaller of the two factors Q_x and Q_0 can be derived[3] as the resonant wavelength divided by the bandwidth of the resonance curve at the point where S is approximately $2S_r$. The larger factor can then be obtained from the first by multiplying by S_r.

However, with the aid of the nomogram given in this article, the factor Q_x (or Q_0, whichever is smaller) can be derived from the bandwidth of the voltage standing-wave ratio resonance curve at *any* height. The practical advantage of this method is that it is then unnecessary to identify the points on the resonance curve where $S = 2S_r$—which can only be found by measuring at surrounding points and interpolating. (Each point



Cavity 'Q' measurement from standing-wave ratios.

on the curve is found by first setting frequency and then deriving S from two voltage measurements.) One point easily identified experimentally is the minimum value S_r at resonance, and in the proposed method this is used together with values of S and frequency at any other experimental point off resonance to derive Q_x or Q_0.

Use of the Nomogram

The revised procedure is as follows:—

(1) Obtain experimentally the standing-wave ratio and wavelength at resonance, S_r and λ_r.

(2) Obtain S and λ at any other point on the resonance curve.

(3) Calculate the half bandwidth (expressed as a fraction of the wavelength at resonance) at this point $\left|\dfrac{\lambda_r - \lambda}{\lambda_r}\right|$.

(4) Calculate S/S_r.

(5) Choose the appropriate scale in the nomogram for S/S_r and note its associated K scale.

(6) Lay a straight edge through points on the S_r and S/S_r scales and read off the value of K on the associated K scale.

(7) Calculate Q_x (or Q_0 if smaller) which is given by

$$\frac{K}{2\left|\dfrac{\lambda_r - \lambda}{\lambda_r}\right|} \quad \text{.. (9 from Appendix)}$$

(8) Calculate the greater Q factor by multiplying the smaller by S_r.

Although S_r can be found very accurately, a single measurement is insensitive for determining λ_r. It is satisfactory to correct for possible inaccuracy in the measurement of λ_r by measuring S and λ at a third point on the opposite side of resonance from the second point, using this with S_r and λ_r to calculate a second value for Q_x (or Q_0) and averaging this with the first result.

This method of estimating Q factors from three experimental points only—at resonance and at a frequency above and below has been found satisfactory provided it is known that the shape of the resonance curve is undisturbed by adjacent resonances.

Acknowledgments

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APPENDIX

Derivation of Q_x or Q_0 from V.S.W.R.—Frequency Resonance Curve.

The impedance of the cavity Z can be expressed in terms of the feeder characteristic impedance Z_0 as[2]

$$\frac{Z}{Z_0} = \frac{\dfrac{1}{Q_x}}{j\left(\dfrac{\lambda_r}{\lambda} - \dfrac{\lambda}{\lambda_r}\right) + \dfrac{1}{Q_0}} \quad \text{.. .. .. .. (2)}$$

The resultant reflection coefficient is

$$r = \left(\frac{Z}{Z_0} - 1\right) \Big/ \left(\frac{Z}{Z_0} + 1\right)$$

$$\text{whence } r = \frac{\dfrac{1}{Q_x} - \dfrac{1}{Q_0} - j\left(\dfrac{\lambda_r}{\lambda} - \dfrac{\lambda}{\lambda_r}\right)}{\dfrac{1}{Q_x} + \dfrac{1}{Q_0} + j\left(\dfrac{\lambda_r}{\lambda} - \dfrac{\lambda}{\lambda_r}\right)} \quad \text{.. .. (3)}$$

Defining the voltage standing-wave ratio as

$$S = \frac{1 + |r|}{1 - |r|} \quad \text{.. .. .. .. (4)}$$

we deduce from (3) and (4) that

$$S\left(\frac{1}{Q_x} - \frac{1}{SQ_0}\right)\left(\frac{1}{Q_0} - \frac{1}{SQ_x}\right) = \left\{\frac{\lambda_r}{\lambda} - \frac{\lambda}{\lambda_r}\right\}^2 \quad \text{(5)}$$

At resonance, $\lambda = \lambda_r$ and $S = S_r$, so that

$$S_r = \frac{Q_0}{Q_x} \text{ or } \frac{Q_x}{Q_0} \quad \text{.. .. .. .. (6)}$$

The choice of solution depends on whether $Q_0 > Q_x$ or $Q_x > Q_0$, since S is always $\geqslant 1$. We develop the case where $Q_0 > Q_x$, when $S_r = Q_0/Q_x$. Since equation (5) is symmetrical in Q_x and Q_0, the result obtained will apply when $Q_x > Q_0$ if Q_x and Q_0 are interchanged.

Eliminating Q_0 from equations (5) and (6)

$$Q_x = \frac{\sqrt{\left\{1 - \dfrac{1}{SS_r}\right\}\left\{\dfrac{S}{S_r} - 1\right\}}}{\left|\dfrac{\lambda_r}{\lambda} - \dfrac{\lambda}{\lambda_r}\right|} \quad \text{.. (7)}$$

$$\text{Defining } K = \sqrt{\left(1 - \frac{1}{SS_r}\right)\left(\frac{S}{S_r} - 1\right)} \quad \text{.. (8)}$$

and approximating $\dfrac{\lambda_r}{\lambda} - \dfrac{\lambda}{\lambda_r} = \dfrac{(\lambda_r + \lambda)(\lambda_r - \lambda)}{\lambda\lambda_r}$

to $2\dfrac{\lambda_r - \lambda}{\lambda_r}$,

equation (7) becomes

$$Q_x = \frac{K}{2\left|\dfrac{\lambda_r - \lambda}{\lambda_r}\right|} \quad \text{.. .. .. (9)}$$

Then, from equation (6),

$$Q_0 = S_r Q_x. \quad \text{.. .. .. .. (10)}$$

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CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

The Teaching of Electromagnetism

Sir,—In your October 1954 Editorial, Professor Howe, in quoting extracts from the I.E.E. discussion of Mr. Hammond's paper, does me the compliment of directing a heavy bombardment upon my contribution (No. 12 of his list) in terms which by now have a familiar ring. There is, he is convinced, something wrong with my reasoning, and he reads into it conclusions which to him are incredible and fantastic. Since I was allowed, in my written contribution to the discussion, only enough space to give the barest summary of relativistic theory, perhaps I owe it both to Professor Howe and to the readers of his stimulating Editorials to explain in more detail.

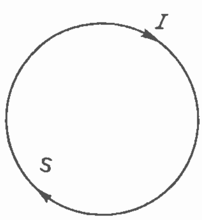

Fig. 1.

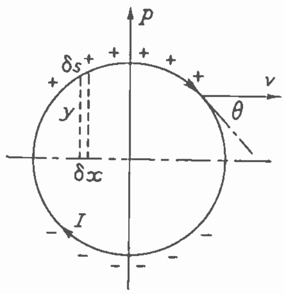

Fig. 2.

Fig. 1 shows a circular conducting loop of area S, carrying a steady current I. It has a magnetic moment $\boldsymbol{m} = \mu_0 I\, \boldsymbol{S}$, where $\boldsymbol{m}$ and $\boldsymbol{S}$ are vectors perpendicular to the plane of the loop and directed into the paper. Fig. 2 shows the same current-loop moving with uniform rectilinear velocity v, which is very small in comparison with c, the velocity of light, from left to right. Now it is well known that if the fundamental laws of electromagnetism conform to the principles of relativity (and it is generally accepted nowadays that they *must* conform), then the loop must assume, for the observer who sees it moving, a *charge distribution* of amount $(vI \cos\theta)/c^2$ per unit length of wire. This charge distribution is therefore positive on the upper half, and negative on the lower half of the loop, as shown, with a cosine variation around the loop.

The explanation of this charge, by relativity theory, is as follows. The current is due to the drifting of conduction electrons through the wire. When the wire is stationary let the drift velocity of the electrons be w, then if the total charge of the moving electrons per unit length is $-\sigma$, the current is given by $I = \sigma w$, I and w being in opposite directions. If the resistance of the loop is negligible, or if the current is due to a uniformly distributed e.m.f., there will be no net charge on any element of the wire and so in each unit length there will be a positive charge, $+\sigma$, due to the stationary atomic nuclei. This charge, being stationary, causes no current.

When the loop is moving, as in Fig. 2, the positive charges move with velocity v, but the conduction electrons do *not* move, according to relativistic kinematics, with a velocity equal to the vector sum of v and w. Taking the simplest case, when w is parallel or anti-parallel to v, as at the bottom and top of the loop, the velocity of the conduction electrons is given by:

at the top $\quad v_e = \dfrac{v - w}{1 - vw/c^2}$,

at the bottom $\quad v_e = \dfrac{v + w}{1 + vw/c^2}$.

Since v is taken to be extremely small in comparison with c, the current I can be taken to be the same as when the loop is stationary. Consider the top of the loop: the current is the resultant of the positive charge σ moving with velocity v, and the conduction electrons moving with an average velocity v_e. Let the charge per unit length of the conduction electrons be $-\sigma'$. Then we must have

$$I = \sigma v - \sigma'(v - w)(1 - vw/c^2) = \sigma w$$

from which we find that

$$\sigma - \sigma' = \sigma v w/c^2 = Iv/c^2. \quad \ldots \quad (1)^*$$

Thus at the top of the loop there appears a surplus of positive over negative charge of amount Iv/c^2.

If the charge per unit length of the conduction electrons at the bottom of the loop is $-\sigma''$, the current (from right to left) is given by

$$I = \sigma''(v + w)/(1 + vw/c^2) - \sigma v = \sigma w$$

so that $\quad \sigma'' - \sigma = \sigma v w/c^2 = Iv/c^2, \quad \ldots \quad (2)$

and there is a surplus of negative charge of amount Iv/c^2.

Now consider the *electric* moment of these charges about the diameter of the loop parallel to v (Fig. 2). The charge on the element δs is

$$\delta q = (vI\, \delta s \cos\theta)/c^2 = vI\, \delta x/c^2.$$

The moment of this about the axis is $vIy\, \delta x/c^2$, so that by integration the electric dipole moment of the whole loop is

$$p = vI\, S/c^2 = \kappa_0 v m \quad \ldots \quad (3)$$

where m is the *magnetic* moment. The direction of the *polarization* of the loop is as shown in Fig. 2. As a vector, therefore,

$$\boldsymbol{p} = \kappa_0(\boldsymbol{v} \times \boldsymbol{m}) \quad \ldots \quad (4)$$

We consider a permanent magnet to consist of myriads of extremely small current loops. These may in fact consist of electrons moving in orbits about positive nuclei, or of spinning electrons: in either case the above relativistic theory applies, for if we take all the atomic positive charges away from the wire loop we do not affect the *electric moment*, and a spinning electron can be considered as being built up of elementary rings of charge. The intensity of magnetization $\boldsymbol{M}$† is equal to the magnetic moment per unit volume, and the polarization $\boldsymbol{P}$ is equal to the electric dipole moment per unit volume. It therefore follows, from (4), that a moving permanent magnet appears to possess an *electric polarization*

$$\boldsymbol{P} = \kappa_0(\boldsymbol{v} \times \boldsymbol{M}) \quad \ldots \quad (5)$$

owing to the apparent redistribution of the electricity in the electron orbits or the spinning electrons in the atoms, and *not* owing to any displacement of *conduction* electrons. That is, equation (5) holds irrespective of whether the magnet is a conductor or an insulator.

The electric field intensity caused by the uniform rectilinear motion of a magnet is well known to be

*If I_0 is the current when the loop is stationary, the rigorous relationship is

$I = \beta\sigma v - \sigma'(v - w)/(1 - vw/c^2) = \beta I_0 = \beta\sigma w$

whence the surplus of positive charge is $(\beta\sigma - \sigma') = Iv/c^2$, where $\beta = (1 - v^2/c^2)^{-1/2}$ and is taken equal to unity in obtaining equation (1).

†Previously written $\boldsymbol{J}$.

$$E = B \times v \quad .. \quad .. \quad .. \quad .. \quad (6)$$

where B is the induction or flux density and v is the velocity of the magnet. Since $B = \mu_0 H + M$ we can write (6) in the form

$$E = \mu_0(H \times v) + M \times v \quad .. \quad .. \quad (7)$$

But for a polarized body the basic relation is

$$D = \kappa_0 E + P \quad .. \quad .. \quad .. \quad .. \quad (8)$$

where D is the electric flux density or displacement, or

$$E = D/\kappa_0 - P/\kappa_0 \quad .. \quad .. \quad .. \quad .. \quad (9)$$

Equating corresponding terms of (7) and (9) we obtain, for the electric state inside a moving magnet,

$$D = \frac{H \times v}{c^2}, \qquad P = \kappa_0(v \times M). \quad .. \quad (10)$$

The first expression is analogous to $B = v \times E/c^2$ for the magnetic flux-density caused by a moving charged body, and the expression for P is identical with (5).

Thus this theory conforms with the Principle of Relativity, that the mathematical formulation of physical laws takes the same form for each of two observers in uniform rectilinear relative motion. Equation (8) holds for a moving magnet as well as for a stationary dielectric. The above equations have a direct counterpart in the theory of the magnetic conditions within a moving dielectric.

It should be noted that not a single conduction electron is displaced from the position it occupies when the magnet is at rest. For the force on a charge, inside and moving with the magnet, is,

$$F = q(E + v \times B) \quad .. \quad .. \quad .. \quad (11)$$
$$= 0, \text{ from (6).}$$

So, since there is no force to move the conduction electron from its original position in the magnet, it stays there.

In my discussion of Mr. Hammond's paper, I stated that equation (7) is applicable, very simply, to the case of a rotating cylindrical magnet if the latter is long enough for H to be ignored. I repeat this statement. The diameter of an atom is of the order of 10^{-8} cm, so if the rotating magnet has a radius of, say, 1 cm, the diameter of an atom near the periphery is about 10^{-8} times the radius. This is about the relationship between a loop two inches in diameter and the radius of the earth.

If a current-carrying loop a few inches in diameter moving with a constant velocity of 900 ft/sec is to be credited with an electrical polarization, is this to be denied if, in addition, it has an angular velocity of $7 \cdot 3 \times 10^{-5}$ radian per sec and a normal acceleration of $6 \cdot 7 \times 10^{-2}$ ft/sec/sec? And if so, why?

When a copper cylinder rotates in an axial magnetic field set up by an external source, there is no intensity of magnetization and conduction electrons are displaced in the cylinder until they take up positions where their electrostatic field just balances the motional force $v \times B$. In such a case Professor Howe is correct in saying that the e.m.f., in the arrangement shown in his figure, is induced in the cylinder and not in the external stationary leads.

The thesis, supported by both Professor Howe and Mr. Hammond, that the same applies to a rotating permanent magnet is consistent with H. A. Lorentz's original non-relativistic electron theory, which dates from 1895. In 1905 Einstein published his famous paper on the 'restricted' Theory of Relativity, entitled "Zur Elektrodynamik bewegter Körper", and the consequences of applying this theory to the electromagnetic conditions in moving media were first worked out by H. Minkowski and M. Born in the period 1908–10. A study of the literature will show that it is well recognized that Lorentz's early equations for a moving medium are consistent (provided $v \ll c$) with the principles of relativity *only* when the relative permeability, μ, is equal to unity.

It seems strange that the teaching of electromagnetism should lag so far behind available and established knowledge. The trouble arises, however, from the fact that the theory of relativity has come to be regarded as a very difficult subject, suitable in the higher education of applied mathematicians alone. Its implications in the elementary theory of electromagnetism have not, therefore, as yet been fully studied and realized.

In the case of the rotating *conducting* magnet, the resultant electric field, both inside and outside the magnet, is the same by both relativistic and non-relativistic theory, but these theories differ as to the ultimate reason for the existence of this field. In assimilating relativistic theory to the case where a complete circuit is arranged through stationary leads we are therefore forced to define more precisely what we mean when we say that an e.m.f. is induced in any particular part of the circuit. Since the electric field acting on the external leads is the same by either theory, how can we say that an e.m.f. is induced in the magnet, by the older theory, but in the leads by relativistic theory? This is a question of fundamental interest, and space prevents a full discussion here. I have found, however, that a consistent scheme can be built on the following simple criterion.

Imagine all material parts of the circuit to be turned into insulators of unity dielectric constant, while the magnetization of any part remains unchanged. Then the e.m.f. is induced in those parts in which there exists an electromagnetic force tending to polarize the insulating material.

If, now, in a practical case we distinguish between this *electromagnetic* force and the *electrostatic* field or force which is caused by the charges (conduction electrons or bound charges in an insulator) displaced from their neutral positions *by* the electromagnetic force, we see that the above criterion leads to the definition of the induced e.m.f. as the *line-integral of the electromagnetic force*. This is the same, for a closed circuit, as the line-integral of the resultant force, per unit charge, since the closed line-integral of an electrostatic field is zero, and it provides a precise and consistent definition of the induced e.m.f. in *part* of a circuit.

E. G. Cullwick.

Department of Electrical Engineering,
University of St. Andrews,
Queen's College, Dundee.
20th October 1954.

Networks Attenuation and Input Impedance

Sir,—If a linear, passive, four-terminal network, having image impedances Z_{i1} and Z_{i2}, is terminated in an impedance $Z_T = rZ_{i2}$ then the ratio α_r of input to output voltage and the ratio r' of input impedance to Z_{i1} can be calculated from formulae involving three parameters:—

1. The ratio α of input to output voltage when the network is correctly terminated; i.e., $r = 1$.
2. The ratio α_∞ of input to output voltage when the output terminals are open-circuited.
3. The mismatch ratio r.

All of these parameters can be complex, of course. Frequently they are found to be more useful than formulae involving hyperbolic functions of θ — the propagation coefficient.

The formulae are easily developed by use of the linear parameters of the network. They are as follows:—

1. $$\frac{v_1}{v_2} = \alpha_r = \frac{1}{r}(\alpha - \alpha_\infty) + \alpha_\infty$$

2. $$\frac{Z_m}{Z_{i1}} = r' = \frac{\alpha + (r-1)\alpha_\infty}{\alpha r - (r-1)\alpha_\infty}$$

The formulae can be simplified if the network is symmetrical. For then there is a simple relationship between α and α_∞. In fact $2\alpha_\infty = \alpha + \frac{1}{\alpha}$

Using this expression:

1. becomes $\alpha_r = \dfrac{\alpha^2(r+1)+(r-1)}{2\alpha r}$ and

2. becomes $r' = \dfrac{\alpha^2(r+1)+(r-1)}{\alpha^2(r+1)-(r-1)}$

The formulae were originally developed in connection with work on attenuators, when it was found useful to plot universal graphs of α_r versus r and of r' versus r for various values of α. They have also proved to be useful in performing calculations on filter networks.

R. TALKS.

London, S.E.19.
10th September 1954.

Solution of Cubics and Quartics

SIR,—I have just seen the article by Mr. A. C. Sim in the November issue of *Wireless Engineer*, and he may be interested in the following presentation of the standard solution that I have found the most convenient and free from small difference difficulties.

Writing the cubic equation in the reduced form

$$z^3 + 3pz + 2q = 0$$

define a quantity $r = q^2 + p^3$.

Case 1. When r is positive.

There are two conjugate complex roots and one real root. Put

$$P = [\,|q| + \sqrt{r}\,]^{1/3}$$

and $$Q = \frac{|p|}{P}$$

where $P > Q$ and both are positive.

There are four possibilities:—

(*a*) *For p and q both positive.*

The roots are

$$-(P-Q) \quad \text{and} \quad \frac{1}{2}(P-Q) \pm j\frac{\sqrt{3}}{2}(P+Q)$$

(*b*) *For p positive and q negative.*

Multiply the roots in (*a*) by -1.

In these cases if $q^2 \ll p^3$, so that P and Q are nearly equal, $P - Q$ may be computed to the accuracy of P and Q themselves from the exact expression

$$P - Q = \frac{2\,|q|}{3p + (P-Q)^2}$$

where in the denominator $(P-Q)^2$ is small compared with $3p$ and may be computed directly from P and Q.

(*c*) *For p negative and q positive.*

The roots are

$$-(P+Q) \quad \text{and} \quad \frac{1}{2}(P+Q) \pm j\frac{\sqrt{3}}{2}(P-Q)$$

(*d*) *For p and q both negative.*

Multiply the roots in (*c*) by -1.

In these cases if $|p|^3 \approx q^2$, so that r is small, then use

$$P - Q = \frac{2\sqrt{r}}{3\,|p| + (P-Q)^2}$$

where as before $(P-Q)^2$ in the denominator can be computed directly from P and Q.

Case 2. When r is negative.

There are three real roots, and p must be negative. Put

$$\phi = \tan^{-1}\left[\frac{|q|}{\sqrt{|r|}}\right] \text{ where } 0 < \phi < \pi/2.$$

There are two possibilities:—

(*a*) *For q positive.*

The roots are $2\sqrt{|p|}\sin\dfrac{\phi}{3}$, $\quad 2\sqrt{|p|}\sin\left(\dfrac{\pi}{3}-\dfrac{\phi}{3}\right)$,

and $-2\sqrt{|p|}\sin\left(\dfrac{\pi}{3}+\dfrac{\phi}{3}\right)$

where all the angles are in the range 0 to $\pi/2$.

(*b*) *For q negative.*

Multiply the roots in (*a*) by -1.

When $q^2 \ll |p|^3$, so that ϕ is very small, a convenient exact alternative form for the first root in (*a*) is

$$\frac{2q}{|p|\left[1 + 2\cos\frac{2\phi}{3}\right]} \text{ which} \rightarrow \frac{2q}{3\,|p|} \text{ as } \phi \rightarrow 0.$$

I have pointed out elsewhere (*Quart. Jour. Mech. and Appl. Math.* Vol. 7, 1954, p. 360) that the cubic equation with complex coefficients can be solved directly if one is prepared to extract the cube roots of complex numbers.

For the quartic equation expressed in the reduced form

$$z^4 + az^2 + bz + c = 0$$

it is convenient to write the quadratic factors in the form

$$\left(z^2 + \alpha z + \frac{\beta+\gamma}{2}\right)\left(z^2 - \alpha z + \frac{\beta-\gamma}{2}\right) = 0$$

β can then be found as any one of the roots of the cubic equation

$$\beta^3 - a\beta^2 - 4c\beta + 4ac - b^2 = 0$$

which can be reduced to the standard form and solved as above. When this equation has a complex pair of roots, it is obviously simplest to choose the real root corresponding to the roots of the quartic grouped in conjugate pairs.

α is then found from

$$\alpha^2 = \beta - a$$

where either root may be used, and then γ is given by

$$\gamma = -b/\alpha.$$

The quadratic factors can then be constructed and solved for z. This method is also adaptable to equations with complex coefficients.

G. MILLINGTON.

Research Division,
Marconi's Wireless Telegraph Company Ltd.,
Great Baddow, Essex.
24th November 1954.

SIR,—The textbook method described by Dr. Millington is, of course, reliable and easy. It was the disadvantages of this method, however, which prompted my article. The method I have described can be followed, if desired, entirely using only a slide rule. The textbook method not only requires trigonometrical tables, but has more possible cases, and sometimes involves more operations.

Ultimately, personal taste will determine the method to be employed, but if slide-rule accuracy is all that is required it will be found on close examination that my proposed routine involves less time and trouble than the textbook method.

I would like to take this opportunity of correcting an error in my article. The caption to Fig. 2 should read "The root α for *positive* λ".

A. C. SIM.

Standard Telecommunication Laboratories, Ltd.,
Enfield, Middlesex.
3rd December 1954.

NEW BOOKS

Millimicrosecond Pulse Techniques

By I. A. D. Lewis, M.A.(Cantab.), A.Inst.P., Graduate I.E.E. and F. H. Wells, M.Sc.(Eng.), D.I.C., A.M.I.E.E., S.M.I.R.E. Pp. 310 + xiv. Pergamon Press Ltd., 242 Marylebone Road, London, N.W.1. Price 40s.

The book starts with a theoretical introduction in which the laws and methods of circuit analysis are explained. Transmission lines, both uniform and helical, are then treated, followed by transformers. While a little information on pulse transformers is given, the types mainly discussed are quarter-wave and tapered-line transformers.

There is a chapter on cathode-ray oscilloscopes and the book concludes with two chapters on applications, one being to nuclear physics and the other to miscellaneous matters.

The book is intended mainly for "the physicist who, with perhaps little experience of the electronic art, wishes to call these new techniques to his aid". The authors regard it as a "collation of relevant material taken from known fields of electronic engineering, together with an account of special developments in the millimicrosecond range" and imply that it will be regarded by the electronic engineer as a reference book.

The physicist without a good background of electronic matters will find it rather tough going and the engineer who is interested in millimicrosecond pulse techniques will probably find it more useful than the authors seem to envisage.

The book is clearly written and by no means unduly mathematical. It is nicely printed and produced and contains an extensive bibliography.

W. T. C.

Electronics

By Thomas Benjamin Brown. Pp. 545 + xi. Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 60s.

The author, who is Professor of Physics at the George Washington University, has written this to be a textbook in the old sense, in which the "content, the order of its arrangement, and the mode of its presentation are planned to meet most effectively the needs of the classroom and the laboratory". The "Electronics" of the title is, rather unusually, used correctly, for the book is mainly about valves. It starts with the diode and, with that as a basis, proceeds to the triode and thence to the more complex types.

Circuit matters are discussed to some extent but rarely more than is necessary to bring out the important factors in the use of valves. The mathematics employed is by no means complex, involving little or nothing more than elementary calculus and the 'j' notation.

A great many experiments are described and it is intended that the student should carry them out. Problems are also included. The book should be useful to the near beginner in the study of the valve.

W. T. C.

Automatic Voltage Regulators and Stabilizers

By G. N. Patchett, Ph.D., B.Sc.(Hons.Lond.), A.M.I.E.E., M.I.R.E., A.M.Brit.I.R.E. Pp. 335 + viii. Sir Isaac Pitman & Sons, Ltd., Parker Street, Kingsway, London, W.C.2. Price 50s.

The title of this book may be slightly misleading to the radio engineer, for it does not deal to any great extent with the kind of regulators and stabilizers which naturally come first to his mind; that is, h.t. voltage stabilizers using hard valves and/or gas tubes. These are treated, but only relatively briefly, for the book deals with all kinds, including mechanical types.

After the introductory chapter, principles and classification are dealt with and then two separate chapters deal with regulators embodying measuring and regulating units of the discontinuous and the continuous control types. Chapter VI covers electrical continuous-control measuring units and Chapter VII with regulating and controlling units, while the two final chapters deal with voltage regulators themselves.

There is a very extensive bibliography and throughout the book there are frequent references to it. The book is to a large extent descriptive and it is largely free from mathematics. It is not in itself a designer's book. It does, however, well review the whole field of voltage regulation and it should enable a designer to pick out the most suitable methods for his problem and then the bibliography will lead him to the more detailed literature.

W.T.C.

Acoustics

By Leo L. Beranek. Pp. 481 + x. McGraw-Hill Publishing Co., Ltd., 95 Farringdon Street, London, E.C.4. Price 64s.

The author is an acknowledged authority on acoustic measurements and has a catholic interest in all matters relating to vibration and sound. In the present book he has written down the facts, the formulae and the ideas that 'the engineer or scientist who wishes to practise in the field of acoustics and who does not intend to confine his efforts to theoretical matters must know'.

Primarily it is a teaching book and for this reason one accepts the incongruities of homely explanations, such as the imaginative squeezing of cubes of air in the hand, alongside rigorous derivation and solution of the basic wave equations. The European reader will also have to contend with what may be to him new and unfamiliar symbols and units, the derivation (or condensation) of which is not always at once apparent. Nevertheless, the practical designer will find much of direct use to him, even if he is not able to cope with the basic mathematical material.

There is a particularly good chapter on vented ('bass-reflex') loudspeaker enclosures containing the hitherto unpublished results of an extensive investigation by J. J. Baruch and H. C. Lang of the Acoustics Laboratory, M.I.T.

One of the best features of this book is the continuous interpolation of worked examples—all of an eminently practical flavour. The student who uses this text for his initiation will most certainly continue to carry it with him as a reference book in his subsequent career as a practising acoustical engineer.

F. L. D.

Crystal Rectifiers and Transistors

Compiled by E. Molloy. Pp. 170. George Newnes, Ltd., Tower House, Southampton Street, Strand, London, W.C.2. Price 21s.

Electrical Who's Who (Third Edition)

Compiled by *Electrical Review*. Pp. 354. Published by Electrical Review Publications, Ltd., and distributed by Iliffe & Sons, Ltd., Dorset House, Stamford Street, London, S.E.1. Price 21s.

Development of the Guided Missile (Second Edition)

By Kenneth W. Gatland, F.R.A.S. Pp. 292. Published for *Flight* by Iliffe & Sons, Ltd., Dorset House, Stamford Street, London, S.E.1. Price 15s.

The book deals mainly with the missile itself and its propulsion, but there is a chapter on the guided bomb and an appendix on telemetering.

The Distribution of Radio Brightness on the Solar Disk: Interstellar Hydrogen

International Scientific Radio Union (U.R.S.I.) Special Reports Nos. 4 and 5, in one cover. Pp. 72. General Secretariat of U.R.S.I., 42 rue des Minimes, Brussels, Belgium. Price 14s. 6d. (including postage).

F.B.I. Register of British Manufacturers 1955 (27th Edition)

Pp. 1089. Published for the Federation of British Industries by Kelly's Directories and Iliffe & Sons, Ltd., Dorset House, Stamford Street, London, S.E.1. Price 42s. (including postage).

This new edition includes French, German and Spanish glossaries, among which are translations of every heading used in the buyers' guide. The guide classifies over 6,800 F.B.I. member firms under 5,000 trade headings. There are lists of trade associations, brands and trade names, and trade marks.

Documents of the VIIth Plenary Assembly of the International Radio Consultative Committee, London, 1953

Vol. I. Recommendations made by the Committee, Reports, Resolutions adopted by the Committee, Questions to be studied, and Study Programmes. Pp. 406. Publications Department, International Telecommunications Union, Palais Wilson, Geneva, Switzerland. In English or in French. Price 23.10 Swiss francs.

Zur Theorie der Elektronenstrahlröhren mit periodischem Aufbau

By Dr. Heinrich Derfler. Pp. 54. Verlag Leeman, Zürich, Switzerland. Price 6.25 fr.

What Every Engineer Should Know about Rubber

By W. J. S. Naunton, M.A.(Cantab.), M.Sc., Ph.D. (London), Dip.Chem.(Munich), F.R.I.C. Pp. 128. British Rubber Development Board, Market Buildings, Mark Lane, London, E.C.3. Price 3s. 6d. (including postage).

NATIONAL BUREAU OF STANDARDS:

Protection against Betatron-Synchrotron Radiation up to 100 Million Electron Volts.

National Bureau of Standards Handbook 55. Pp. 52. Price 25 cents.

Formulas for Computing Capacitance and Inductance

By Chester Snow. National Bureau of Standards Circular 544. Pp. 69. Price 40 cents.

Statistical Theory of Extreme Values and Some Practical Applications

By Emil J. Gumbel. Pp. 51 + viii. National Bureau of Standards Applied Mathematics Series 33. Price 40 cents.

Miniature Intermediate-Frequency Amplifiers

By Robert K-F. Scal. Pp. 46 + iv. National Bureau of Standards Circular 548. Price 40 cents.

Government Printing Office, Washington 25, D.C., U.S.A.

MEETINGS

I.E.E.

12th January. "Thermionic Valves of Improved Quality for Government and Industrial Purposes", by E. G. Rowe, M.Sc., P. Welch and W. W. Wright, B.Sc.

18th January. "The Human Operator in Closed Loop Control Systems", discussion to be opened by Professor C. Holt Smith, M.Sc.

24th January. "Radio Aids to Marine Navigation", by Captain F. J. Wylie, R.N.(Retd.).

The above meetings will be held at the Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, S.W.1, and will commence at 5.30.

27th January. "Courier to Carrier in Communications", Faraday Lecture by T. B. D. Terroni, B.Sc., commencing at 6 o'clock at Central Hall, Westminster, S.W.1.

BRIT.I.R.E.

26th January. "A Survey of Tuner Designs for Multi-Channel Television Reception", by D. J. Fewings, B.Sc., and S. L. Fife. Meeting will commence at 6.30 at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1.

TELEVISION SOCIETY

19th January. The Fleming Memorial Lecture—"The Perception of Colour", by Professor W. D. Wright, to be held at the Royal Institution, Albemarle Street, London, W.1, at 7 o'clock.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for November 1954

Date 1954 November	Frequency deviation from nominal: parts in 10^8		Lead of MSF impulses on GBR 1000 G.M.T. time signal in milliseconds
	MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.	
1	−0·5	+1	+2·2
2	−0·5	+1	+0·5
3	−0·6	−1	−0·4
4	−0·6	+2	+0·8
5	−0·6	+1	+1·4
6	−0·5	+2	NM
7	−0·5	+2	NM
8	−0·5	+2	+2·6
9	−0·5	+2	+3·5
10	−0·8	+2	NM
11	−0·8	+2	+4·6
12	−0·6	+2	+5·1
13	−0·6	+3	NM
14	−0·6	+3	NM
15	−0·6	+3	+4·7
16	−0.6	+1	+3·4
17	−0·5	+2	NM
18	−0·5	+2	NM
19	−0·5	+2	+4·7
20	NM	+3	NM
21	−0·5	+3	NM
22	−0·6	+3	+8·1
23	−0·6	+3	+8·7
24	−0·5	+3	+8.9
25	−0·5	+3	NM
26	−0·5	+4	+9·3
27	−0·5	+4	NM
28	−0·5	+5	NM
29	−0·5	+4	+9·0
30	−0·5	+4	+9·4

The values are based on astronomical data available on 1st December 1954.

NM = Not Measured.