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The Fundamentals of Electromagnetism

IT is a sign of the times and an indication of the change that has taken place in the approach to the study of electromagnetism, to open a book with the above title and find, on turning over the pages, that, except for a footnote on p. 76, the magnetic force H is first mentioned on p. 160, but this is actually the case in Professor Cullwick's new edition of his book,* as it was also in the first edition in 1939. A new edition of this thought-provoking book is very welcome, although, as the author explains, owing to his prolonged absence from academic work, changes in the present edition are few, except for the correction of inaccuracies, the clarification of some points, and removal of 'various appendages of doubtful value.' We trust that in his new position as Professor at University College, Dundee, he will be able to devote himself more whole-heartedly to such academic pursuits.

When the author speaks of lines of force, he means lines of magnetic induction, and says that he is adhering to Faraday's original use of the term, and that " the viewpoint taken in this book is that H is merely a certain measure of the flux-density called the m.m.f. gradient." We fear that a student will find this very confusing. Surely one can only have a gradient of a scalar quantity that has a definite value at any point. One cannot define the m.m.f. at a point, and therefore one cannot measure its gradient. The author treated the electric field in the same way in the first edition, but, although it still figures in the index as 'e.m.f. gradient', we are pleased to see that it has been deleted from the text.

* "The Fundamentals of Electromagnetism", by E. G. Cullwick. Pp. 327 + xxvi with 139 illustrations. Cambridge University Press. Price 18s.

The confusion of H and B is very unfortunate. Although mathematicians of a certain type may dislike analogies, they are very helpful in enabling a student to visualize abstract phenomena and magnitudes. He has been familiar from childhood with mass, weight, force and movement, and has no difficulty in understanding the extension of a test specimen when subjected to a force or load, nor the obvious next step, the distribution of the total applied force over the cross-section to obtain the localized cause or stress which is pictured as producing the strain at any point. The student to whom this is familiar will find no difficulty in following a similar procedure in the case of the magnetic circuit. The prime cause is a number of ampere-turns, which produces a magnetic flux around the circuit. Just as the force was divided over the area, to find the stress at any point in pounds per square inch, so the ampere-turns or m.m.f. can be divided over the length of the path, to find the localized cause or magnetic force at any point in ampere-turns per metre or per centimetre. This we designate by H and regard as the localized cause of the magnetic induction B at the point, just as the stress is regarded as the localized cause of the strain. Similarly the e.m.f., which is the prime cause of the current or displacement in an electric circuit, can be divided over the length of the electric circuit or path to obtain the localized cause or electric force $\mathcal{E}$ at any point. One may question the real existence of the stress or of H or $\mathcal{E}$, but it is in each case the quotient of two very real things.

We do not suggest that there is any physical analogy between the various phenomena, but we do maintain that the parallel methods of approach and similarity of treatment are very helpful. To insist on referring to lines of magnetic induction as lines of force because Faraday in those far-off days did not distinguish clearly between them, is a strange procedure, which can only cause confusion or uncertainty in the mind of the student.

Clerk Maxwell is not treated quite so respectfully, for when he introduced and defined electric displacement he wrote "the displacement through a given surface is the quantity which passes through it" and "At the beginning of each unit tube of displacement there is a unit of positive electricity, and at the end of the tube there is a unit of negative electricity." This is, however, far too simple and 'rational' for Professor Cullwick, who introduces his own unit tube and on p. 10 states that "A tube of unit displacement thus has its origin on a charge of value $1/4\pi$ and terminates (as will be proved later) on a charge of equal value but of opposite sign." In a footnote is is stated that in rationalized units, the total displacement from a charge q is equal to q, but this was true of the concept as introduced and defined by Maxwell before rationalized units had been thought of. The author says "The part played by the vector D in the romantic development of electrical science is one of the first magnitude, but whether it is a concept which must be retained, or whether it will come to be regarded in the same light as a distinguished servant who, after work of supreme value, is now honourably retired from active service, is an interesting question." We feel that the author would have treated him more honourably if he had not distorted him with the irrational $1/4\pi$. The author's object in derationalizing D was apparently to be able to write $D = \kappa_0\mathcal{E}$, in keeping with $B = \mu_0 H$, instead of the Maxwellian $4\pi D = \kappa_0\mathcal{E}$. This treatment of displacement is also contrary to the B.S.I. Glossary.

On the cover of the book it states that it is unorthodox and original in method, designed to encourage its readers to think freshly. After a foreword on units and formulae there are five chapters; the first deals with the electrostatic field and the electric current, the second with the magnetic field and electromagnetic induction, the third with the magnetic field of the electric current, the fourth with ferromagnetism and permanent magnets, and the fifth with electromagnetic waves and vector potential. After each chapter there is a number of exercises, some of which are worked out and discussed at length and form a valuable addition to the text. There is a prologue and an epilogue, both well worth reading. In the former the author says "It appears to me that a problem of supreme importance now faces all teachers of electrical engineering (and of the preparatory physics courses), a problem whose solution necessitates a radical change of viewpoint, and methods of presenting the fundamentals of electricity and magnetism." . . . "This book has been written with the aim of making some contribution, however imperfect, to the solution of this problem. The concluding sentence of the epilogue is "If we lose a comfortable but blinding faith we gain, one may hope, a humble open-mindedness which should prepare us to welcome, as they are uncovered, new fragments of the truth."

G. W. O. H.

The Intrinsic Impedance of Space

Professor Constantinescu of Bucharest has called our attention to an ambiguity in the September Editorial. In referring to a spherical surface we used the terms latitudinal distance and longitudinal distance; by the former we meant distance measured along a line of latitude, and by the latter, distance measured along a line of longitude. Since, however, latitude is measured from the equator towards the poles, that is, along a line of longitude, the terms are certainly somewhat ambiguous, and it would be better to say more definitely that the distances are measured along the lines of latitude or longitude.

G. W. O. H.

LOW-DISTORTION POWER VALVES

By G. Diemer and J. L. H. Jonker

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SUMMARY.—A survey is given of various low-distortion valve constructions. Two new constructions are described by means of which the second harmonic of a single-stage class-A pentode amplifier can be considerably reduced, resulting in a reduction of the total distortion by a factor of 2 up to an output of about 25% of the static anode dissipation. For large outputs (up to 50% of the static anode dissipation) the distortion is the same as that of a normal valve. The new valves have an I_a–V_g characteristic that is practically linear in the neighbourhood of the normal operating point.

1. Introduction

IN the course of valve development many attempts have been made to lower the distortion which is especially liable to arise in the last stage of audio-frequency amplifiers.

In 1937 Kleen[1] gave a survey of the results that had been obtained up to that date by various methods. The aim of this article is to give a somewhat more extensive survey including some new results obtained since 1937, while two different methods that have been studied in more detail by the present authors will be discussed.

Negative feedback is one well-known measure for reducing distortion, but negative feedback causes a serious decrease of the effective mutual conductance. In the following we shall confine ourselves, therefore, to special valve constructions which give low distortion in the valve itself. We, therefore, start with an analysis of the different causes contributing to the non-linearity of the valve characteristic. Only tetrodes and pentodes will be dealt with because these valves have the highest efficiency.

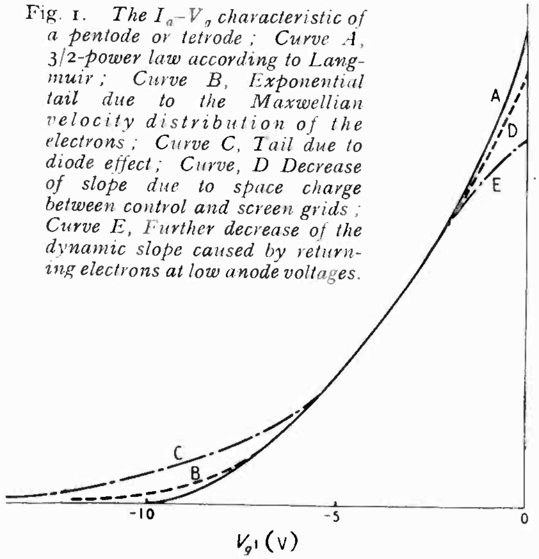

Fig. 1. *The I_a–V_g characteristic of a pentode or tetrode; Curve A, 3/2-power law according to Langmuir; Curve B, Exponential tail due to the Maxwellian velocity distribution of the electrons; Curve C, Tail due to diode effect; Curve, D Decrease of slope due to space charge between control and screen grids; Curve E, Further decrease of the dynamic slope caused by returning electrons at low anode voltages.*

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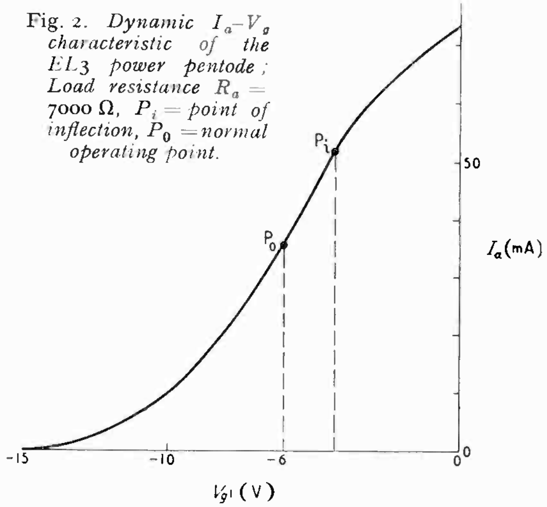


Fig. 2. *Dynamic I_a–V_g characteristic of the EL3 power pentode; Load resistance R_a = 7000 Ω, P_i = point of inflection, P_0 = normal operating point.*

2. The I_a-V_g Characteristic

The I_a–V_g characteristic of a tetrode or a pentode is governed mainly by Langmuir's 3/2-power law (curve A in Fig. 1). This law is valid when

1. The initial velocities of the electrons are negligible.

2. The electric field at the cathode surface is homogeneous.

3. Space charge between the control grid and the screen grid plays no important part, and

4. The ratio of screen-grid current to anode current is independent of the anode current.

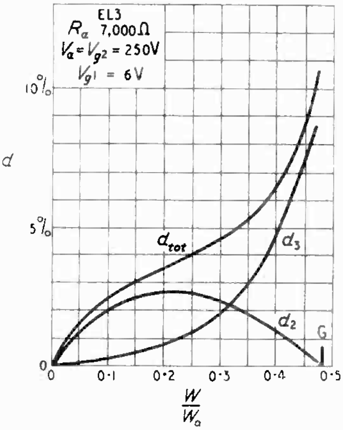
Since these four conditions are not fulfilled in practice, deviations from the 3/2-power law occur due to the following causes:

1. Because of the Maxwellian velocity distribution of the emitted electrons, for small values of I_a the I_a–V_g characteristic is exponential (curve B).

2. In high-slope power valves the latter deviation, however, is negligible in comparison to that caused by the inhomogeneity of the field strength at the cathode surface (i.e., the so-called diode effect, curve C). To get a high value of the anode

current as well as of the mutual conductance the distance between the cathode and the control grid is made equal to the pitch of that grid or even smaller. For such dimensions the electron emission in the region between the wires of the grid can only be suppressed by very large negative grid voltages.

3. In some cases, especially with large values of anode current and distance between control and screen grids the space charge in this region causes a decrease of the slope (curve D).



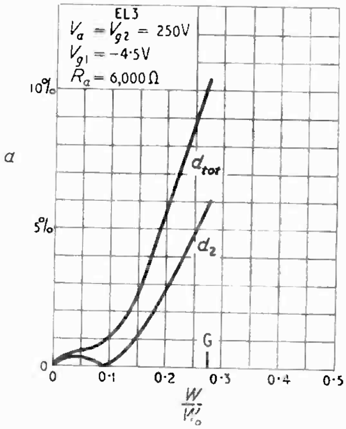


Fig. 3 (left). Distortion of the EL3 as a function of the efficiency W/W_0: W = dynamic output, $W_0 = I_a V_a$ = static anode dissipation, d_2 = second harmonic distortion, d_3 = third-harmonic distortion, d_{tot} = total harmonic distortion, G = limit set by grid current.

Fig. 4 (right). Distortion of the EL3 at $V_{g1} = -4.5$ V; load resistance $R_a = 6000\ \Omega$, G = limit set by grid current.

4. The ratio of screen-grid current to anode current may vary with varying I_a due to three causes:

(a) The anode circuit of the valve usually contains a load resistance, and at low anode voltages many of the electrons deflected by the grid wires cannot reach the anode and return to the screen grid, so that another deviation from the dynamic characteristic occurs (curve E).

(b) The influence of the space charge between the screen grid and the anode may result in a deviation similar to curve E.

(c) Valves in which the control grid and screen grid are 'lined up' may show a variation in the relative number of electrons intercepted by the screen grid, because of the variation in electron focusing by the control grid.[13] This effect will be discussed in detail in Section 4, as it has provided us with a possibility of reducing the distortion.

It is not our aim to give a full list of all the effects contributing to valve distortion, and we have only mentioned the phenomena that are important in our investigations. The influence of secondary emission at the anode has already

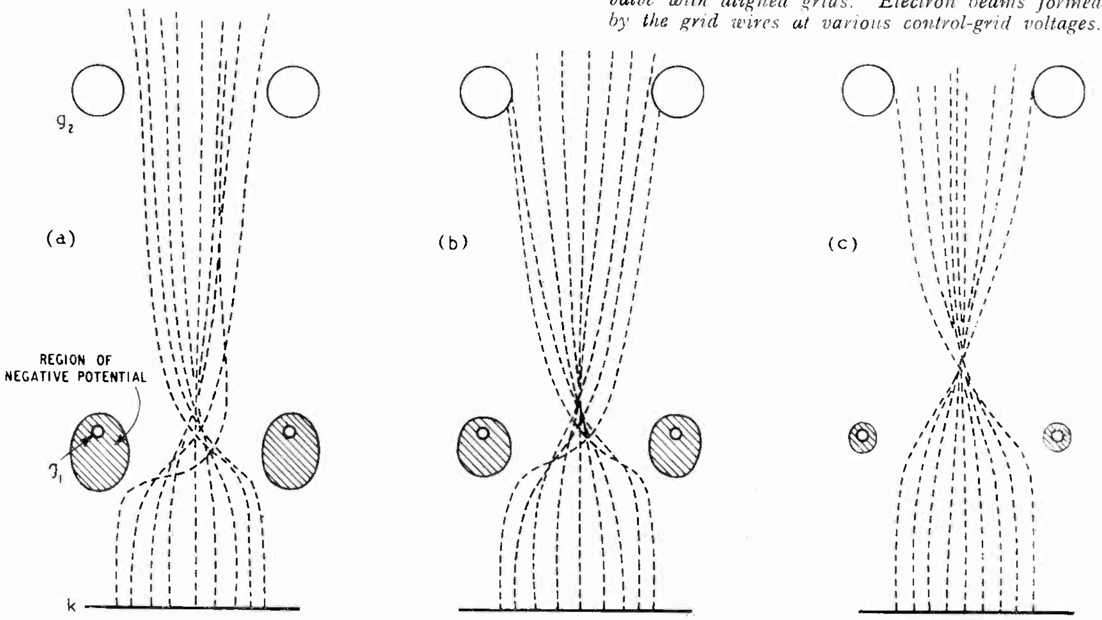


Fig. 5. Cross section of a low-distortion power valve with aligned grids. Electron beams formed by the grid wires at various control-grid voltages.

been discussed in detail by Jonker and Heins van der Ven,[2,14] so it will not be discussed here.

As an example of a normal dynamic pentode characteristic, in which most of the effects mentioned above are present, Fig. 2 shows the dynamic I_a–V_g characteristic of the EL3 pentode. As may be seen, the point of inflection caused by the load resistance lies at about $V_{g1} = -4.5$ V, while the normal grid bias for this valve is $V_{g1} = -6$ V. Thus for normal grid bias we shall have, when the signal voltages are not too large, a dynamic characteristic showing on the average a curvature which results in considerable second-harmonic distortion in the output.* This may also be seen from Fig. 3, where for the EL3 the distortion is given as a function of the efficiency W/W_0 under normal operating conditions (W = output of the valve, $W_0 = I_a V_a$ = static anode dissipation). With strong signals ($W/W_0 \approx 0.5$) the third harmonic begins to play the most important part, because of the S-shape of the characteristic due to the load resistance.

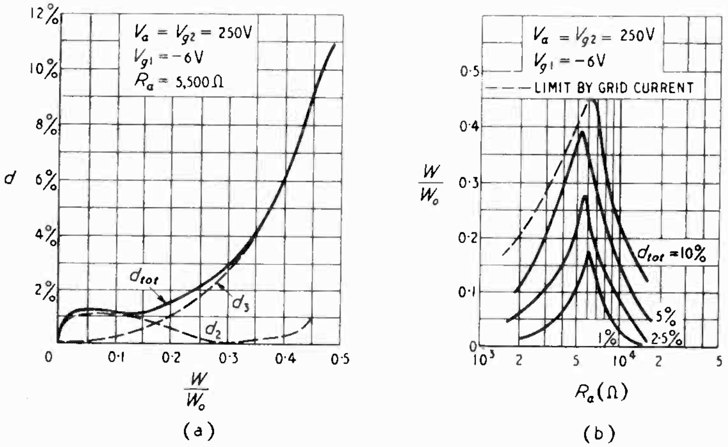


Fig. 6 (left). *Distortion of a valve having an electrode arrangement like that of Fig. 5. (a) Distortion as a function of efficiency; (b) Efficiency W/W_0 as a function of R_a at various constant values of d_{tot}.*

Now it is obvious that when signals are not too strong the distortion caused by the second harmonic can be reduced by adjusting the grid bias to the point of inflection or by using a suitably chosen value of the load resistance. Adjusting both the grid bias and the load resistance to their optimum values with regard to low second-harmonic distortion for the EL3 gives a distortion curve as illustrated in Fig. 4. We see that up to an efficiency of 15% the distortion is indeed improved, but the maximum output is limited by the point where grid-current distortion appears; for this pentode the maximum efficiency has thus been reduced from 50% to 25%.

3. Older Low-Distortion Valve Constructions

In the course of time many proposals have been made for reducing the distortion by special electrode constructions. Attempts have been made to improve the tail of the characteristic by the electron-optical focusing of surplus current on an auxiliary electrode.[3] It has been proposed to reduce the curvature for large values of I_a by using simultaneously two different control grids,[1, 4] by using a deflection electrode together with a special form of the output anode,[5] by using the partition of the cathode current by a specially shaped grid and a plate[6] or by the influence of space charge.[7] Recently

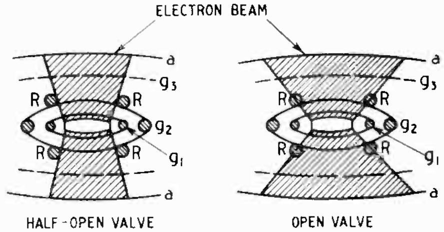
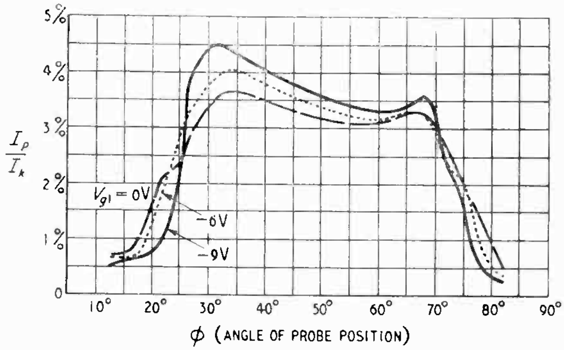


Fig. 7 (above). *Electron beams formed by the rods of the control grid at different values of the control-grid voltage.*

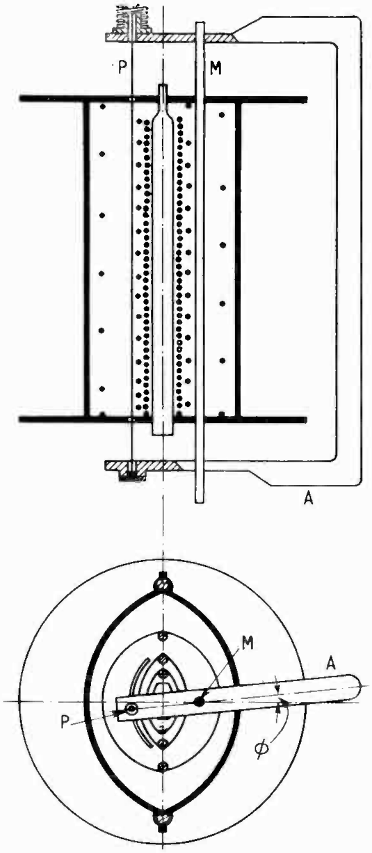
Fig. 8 (below). *Width of the electron beams shown in Fig. 7, as measured by means of a movable probe (see Fig. 9).*



Brian[8] and Pickering[9] have published results obtained when using a tetrode with a positive first grid (space-charge grid). Although their

* For a good comparison we used experimental EL3 pentodes that were made at the same time as the low-distortion pentodes to be discussed in Section 4. The EL3 characteristics given here, therefore, show small deviations from the EL3 characteristics normally published.

construction seems to us the most useful hitherto published, we wish to make the following remark. The maximum efficiency of their valve is much lower than that of a pentode, especially when the



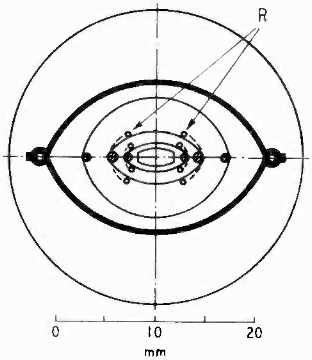


Fig. 9 (left). *Construction of an experimental valve containing a movable probe P for measuring the width of the electron beam.*

Fig. 10 (right). *Electrode arrangement of a low-distortion power valve using additional focusing rods.*

valves are not used in push-pull (for many applications one prefers to use a class-A amplifier with one single valve). Now, as was shown in Section 2, it is also possible to reduce the distortion of a pentode by adjusting R_a and V_{g1} to a suitable value. In this case the maximum efficiency for the pentode is even higher than that of the space-charge-grid valve.

However, none of the special valve constructions have up to now been produced on a large scale. The reasons for this may be :

1. Some of the constructions are too expensive, because they are rather intricate.

2. The other properties of the valve (e.g. maximum efficiency, mutual conductance, internal resistance) are sometimes worse than those of the normal valve.

3. In some cases the decrease of distortion depends too much upon the value and phase angle of the load impedance.

Still a low-distortion power valve with other characteristics normal is always attractive, because a smaller amount of feedback is required, resulting in a higher value of the mutual conductance, and difficulties resulting from the frequency-dependence of the feedback are less serious.

We have, therefore, investigated the problem from a different angle, trying only to avoid curvature of the dynamic characteristic in the neighbourhood of the normal operating point leaving the tail, caused by diode effect, and the upper curvature, caused by the presence of the load resistance, as they are. It is our conviction that it is not possible to improve the latter appreciably when only simple electrode constructions are used.

We have succeeded in making the region of the I_a–V_g characteristic around the normal operating point fairly linear by two different methods, in both of which use is made of the variation in screen current with varying control-grid voltage, namely by

1. Lining up the first and second grids of the pentode, and

2. Using the influence that the rods of the control grid have upon the electron flow.

These two methods were already indicated by Kleen,[1] but as far as we know they have not been studied extensively before. The results of our experiments are given in Section 4.

4. Low-Distortion Power Pentodes

It is well known that the wires of the control grid in a normal valve divide the electron current into several flat beams which cross over somewhere beyond the grid plane (see e.g., Knoll and others[10, 11], and Jonker [2, 12]).

In Fig. 5 a cross-section is given of a valve in which the wires of the control and screen grids are lined up. The shape of the electron beams formed by the wires of the control grid, as photographed from large-scale models on a rubber sheet, are also drawn for three different control-grid voltages ; viz, strongly negative, near the operating point, and slightly negative. From extensive experiments on the rubber sheet with various grid dimensions we concluded that for suitable dimensions of the screen grid it is possible to intercept a varying part of the beams by the combined action of the focus displacement and the variation in aperture of the electron beams at different control-grid voltages.

For our purpose the dimensions were chosen so that the interception was the maximum at a grid voltage somewhat less negative than the normal operating point.

These experiments with the help of models on the rubber sheet have led to a pentode construction very closely resembling the EL3, and showing much less distortion than the corresponding normal valve (see Fig. 6). Of course with this method there are limits in the dimensioning of the valve, because the grids have to be lined up, and for a given construction of the control grid and a given distance between the control and screen grids the diameter of the screen-grid wires is fixed, on account of the conditions set by the interception effect.

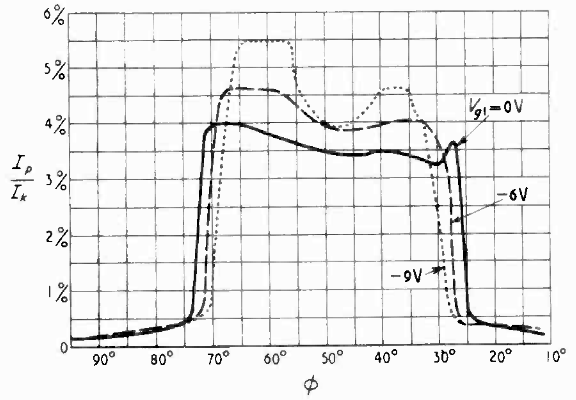
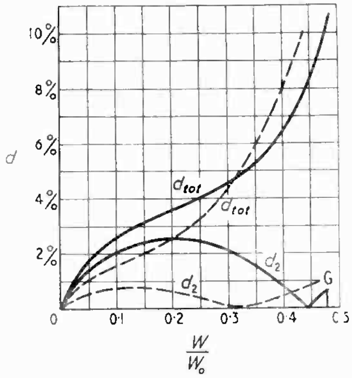


Fig. 11. *The same as Fig. 8, but using additional focusing rods connected to the control grid.*

The experiments indicated that good results can be obtained with various distances between the two grids provided the wire diameters are suitably chosen.



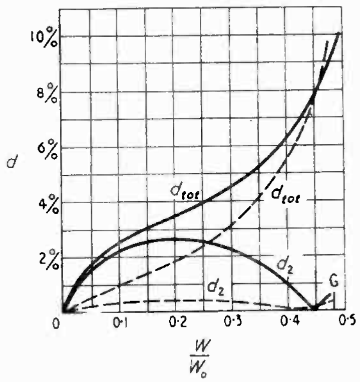


Fig. 12 (left). *Distortion with a valve construction like that of Fig. 7 as a function of the efficiency W/W_0; G = limit set by grid current. Solid line curves for EL3, dotted for low-distortion valve of Fig. 10;* $V_a = V_{g2} = 250$ V, $V_{g1} = -6$ V, $R_a = 7{,}000\ \Omega$.

Fig. 13 (right). *The same as Fig. 12 but with additional focusing rods.*

The second method we developed, which makes use of the variation in aperture of the electron beams, seems, however, much more attractive, because it can be applied to any normal valve without any variation of grid dimensioning and with only a slight variation of the valve characteristics. The idea may be easily understood from Fig. 7. The electron stream is divided by the negatively-charged rods of the control grid into two beams, the width of which decrease as a function of the control-grid voltage.

Fig. 8 shows the intensity of the beam at various places as measured by means of a probe (experimental valve, see Fig. 9). Now a variable amount of the anode current can be intercepted by placing auxiliary positive electrodes of a suitable form near the edge of the electron beams. For our purpose it proved to be sufficient to use cylindrical rods (R) of 0.3 mm diameter (see Fig. 7). The best position for these rods was determined from the measurements with the probe valves.

The edges of the beams can be made better defined by using additional focusing rods connected to the control grid (see Figs. 10 and 11).

Figs. 12 and 13 give the results of the output measurements taken on the latter valves (Fig. 13 with additional focusing rods). The best results were obtained with the valves fitted with additional rods; the curves give the mean values for five valves. The spread in the distortion of the individual valves was not very great (of the order of 20%). The tolerances in the positions and the shape of the intercepting rods are of the same order of magnitude as the tolerances in normal cathode-grid constructions of power pentodes. We see that for an output up to about half of the maximum useful output the distortion is reduced by about a factor of 2 this improvement being due to the absence of the second harmonic. This is more than was obtained by varying the grid bias of a normal EL3 valve (see Fig. 4). Moreover, the new valves have the same maximum output as that of the normal valve under normal operating conditions ($W/W_0 \approx 50\%$). This elimination of the second harmonic again proved to be not very sensitive to variation of the load resistance R_a, just as was the case with the valves having lined-up grids.

Finally Fig. 14 gives the slope of the dynamic

I_a–V_g characteristic for a normal pentode EL3 and for a low-distortion valve according to Fig. 10. We see that for the low-distortion valve the slope is almost constant within a large voltage region around the normal operating point. The same may be concluded from Fig. 15, where the second-and third-harmonic distortion for small signal ($V \approx 0.2$Vr.m.s.) as a function of grid bias are given for the same valves. From these

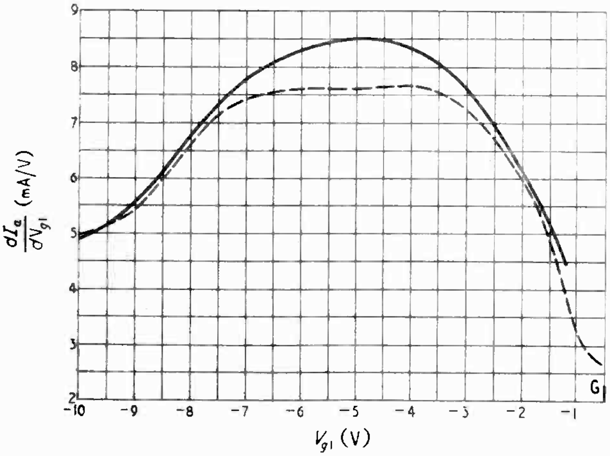


Fig. 14. *Slope of the dynamic I_a–V_g characteristic as a function of V_{g1} for a normal pentode EL3 and for a low-distortion pentode like that of Fig. 10. G = limit set by grid current. Solid line curves for EL3, dotted for low-distortion valve of Fig. 10; $V_a = V_{g2} = 250$ V, $V_{g1} = -6$ V, $R_a = 7{,}000\ \Omega$.*

measurements we may also safely conclude that as long as the signal is not too large the *intermodulation* of the new valves will be considerably smaller than that of the non-corrected pentode. The characteristics given in Figs. 14 and 15 provide us with a measure for the quality with regard to both distortion and intermodulation independently of the frequencies for which the valve may be used.

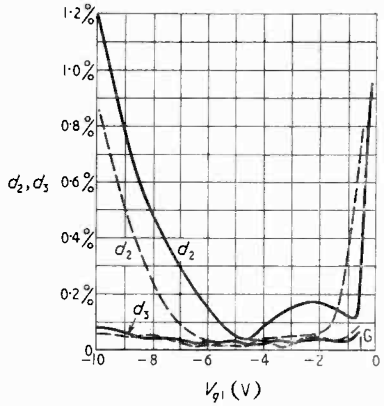


Fig. 15. *Second harmonic for small input voltage ($V \approx 0.2$ V r.m.s.) as a function of V_{g1} for the EL3 and for a low-distortion pentode. Solid line curves for EL3, dotted for low-distortion valve of Fig. 10; $V_a = V_{g2} = 250$ V, $V_{g1} = -6$ V, $R_a = 7.000\ \Omega$.*

REFERENCES

[1] W. Kleen, *Telefunkenröhre*, Vol. 10, pp. 147–156, 1937.
[2] J. L. H. Jonker, *Wireless Engineer*, Vol. 16, pp. 274–286, 1939; A. J. Heins van der Ven, *Wireless Engineer*, Vol. 16, pp. 383–390, 1939.
[3] German patent 732727, French patents 923791 and 808004.
[4] French patent 824521.
[5] French patent 774703.
[6] British patent 451662.
[7] Dutch patent 53142, French patent 819632.
[8] W. S. Brian, *Electronics*, Vol. 20, pp. 121–123, Aug. 1947.
[9] N. Pickering, *Electronics*, Vol. 21, pp. 96–100, March 1948.
[10] M. Knoll. *Z. techn. Physik*, Vol. 15, pp. 584-591, 1934.
[11] M. Knoll, and J. Schloemilch, *Arch. f. Elektrotechnik*, Vol. 28, pp. 507-516, 1934.
[12] J. L. H. Jonker, *Thesis Delft*, 1942.
[13] J. L. H. Jonker, *Philips Res. Rep*, Vol. 4, pp. 355 365, 1949.
[14] J. L. H. Jonker, *Philips tech. Rev.*, Vol. 10, pp. 346-351, 1949.

PHASE-SHIFT OSCILLATOR

By W. C. Vaughan, M.B.E., B.Sc., Ph.D., A.M.I.E.E.

THE type of oscillator of which examples are shown in Figs. 1 (a) and (b) is in widespread use as an audio-frequency source. It depends for its action upon the exact reversal of phase of the voltage developed across AB, introduced by the network between JK and CD. This occurs at one particular frequency determined by the values of the resistors and capacitors forming the network. Provided that the gain due to the valve is at least equal to the loss introduced between JK and CD by the phase-shifting network, self-oscillation will occur at this frequency. The popularity enjoyed by this class of oscillator in the audio-frequency range is due not only to its extreme simplicity, but also to an ability to give output waveforms of better shape than those from conventional LC circuits, and the fact that its frequency stability is at least as good as that of the beat-frequency oscillator.

In some instances, four-mesh networks are used for phase shifting, and a fair range of frequency variation is possible with both these and the three-mesh arrangements merely by making one of the components of the network variable. The phase of the voltage across AB is advanced 180° by the type of network shown in Fig. 1 (a), while that employed in Fig. 1 (b) introduces an equal phase retardation. For fixed-frequency working, it is usual to employ uniform networks in which all the resistors and all the capacitors have equal values. Despite their comparatively simple construction, it has been the writer's experience that the precise behaviour of the phase-shifting network is rarely completely understood and that, as a result, serious misconceptions regarding the design of these types of oscillator are by no means uncommon.

In elementary descriptions of the phase-shift oscillator, it is often stated that each of the elements of the three-mesh network advances or retards the phase of the applied voltage by 60°. It is also frequently assumed that at the frequency at which the network causes exact phase reversal, the input impedance of the arrangement is a pure resistance, and that the possibilities of frequency variation are restricted because only a uniform network can bring about exact phase reversal. Furthermore, in the case of uniform networks, the frequency of oscillation is sometimes quoted as the reciprocal of the time constant of the CR element. All these views are fallacious and militate against the design of phase-shift oscillators for specific purposes. Clear understanding of the behaviour of the arrangement in any of its forms can only come from a detailed study of the circumstances under which the phase-shifting network can introduce exact voltage reversal, and it is the purpose of this article to examine this problem in the case of three- and four-mesh networks

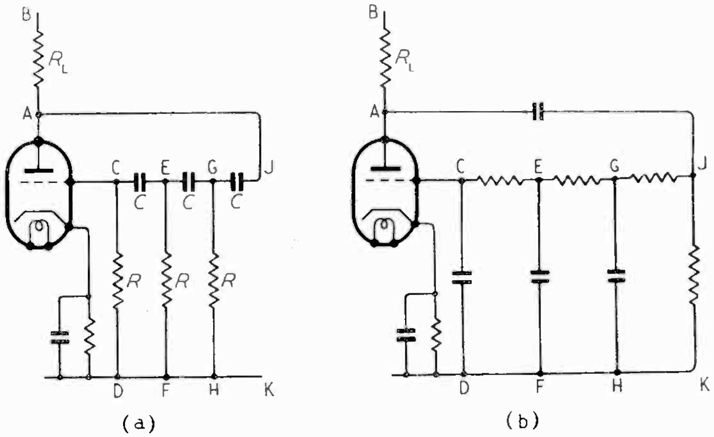


Fig. 1. *Typical RC-oscillator circuits.*

of the phase advancing and retarding types. General treatment is not difficult although it is somewhat tedious, but the results which emerge are clearly of considerable value in devising systems to meet individual requirements.

The mode of operation of these arrangements is quite unlike that of the LC type of oscillator in which there is a periodic interchange of energy between the electric and magnetic fields. It is also essentially different from the behaviour of the relaxation oscillator in which the steady charging of a capacitor through a resistor is interrupted by the instability which occurs when the potential across one of these components attains a particular value. The mechanism of initiating the oscillation in the phase-shifting circuit has been described from the transient standpoint in a paper by P. G. M. Dawe[1], which

MS accepted by the Editor, March 1949

[1] P. G. M. Dawe, *Engineering*, Oct. 1947, Vol. 164, pp. 429-432.

shows that the initial charging and subsequent discharging of the capacitors as the result of the application of a voltage step, produces overswing effects which give, at the output of the network, voltage reversals similar to a heavily damped oscillation. If a steadily rising voltage is applied to the system, the output voltage falls to zero, changes its sign, and again returns to zero.

This account, which gave a qualitative and quantitative treatment of the transient condition before steady oscillations ensue, pointed out that if the h.t. supply to the circuit shown in Fig. 1 (a) is a voltage step, the anode-cathode potential difference is unable to rise immediately to its final value because of the heavy positive feedback due to the capacitors. The static value is not, in fact, reached until the grid capacitor is fully charged, but this at once commences to discharge and produces a further rise in the anode potential. The rate of increase of anode voltage is, therefore, finite, and as observed earlier, the output voltage of the network between C and D will pass through zero, change its sign and again return to zero. The anode voltage must, therefore, reach a maximum, after which it will pass through the equilibrium value and then fall to a minimum. In this way the oscillation commences.

The present article deals with the subsequent condition after the oscillation becomes steady, and is based on the assumptions (a) that oscillations can be initiated, and (b) that they will be of sinusoidal waveform. The first of these is a matter of common experience, but it remains to determine the conditions under which oscillations can be sustained. Since the network can give phase reversal for one frequency only, the second assumption is plausible.

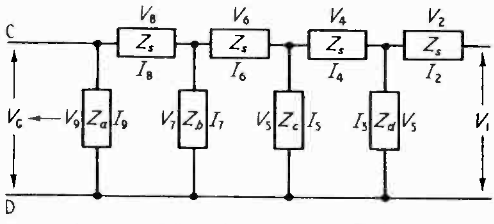


Fig. 2. *Generalized four-mesh network for shunt variable element.*

For the purposes of this investigation, it will be assumed that the load resistance R_L connected between A and B in Figs. 1 (a) and (b) is substantially smaller than the input impedance of the network measured between J and K. As a result, the currents set up in the components of the various meshes may be regarded as due to an alternator of voltage V_L, frequency $f = \omega/2\pi$, and negligibly low internal impedance. A single general solution for the voltage and current distributions about a four-mesh network involves very unwieldy expressions, so that two forms of generalized network will be considered. In the first instance, the four-mesh network shown in Fig. 2, having identical series elements Z_s, but different shunt impedances Z_a, Z_b, Z_c, Z_d, will be examined. The impedance expressions appearing in the following analysis may, of course, be real, imaginary, or complex, and it is the aim to determine the voltages $V_9, V_8, \ldots V_1$ across the components, and the currents $I_9, I_8, \ldots I_2$ through these, in terms of the applied voltage V_L. The procedure is somewhat more straightforward if these quantities are determined in terms of the output voltage V_G which the network applies between the grid and cathode of the valve.

If no current is taken from the network at CD, then the current I_8 is equal to I_9, so that

$$I_8 = I_9 = \frac{V_9}{Z_a} = \frac{V_G}{Z_a} \quad \ldots \quad (1)$$

The voltage V_8 is the product of I_8 and Z_s, so that,

$$V_8 = I_8 Z_s = V_G Z_s / Z_a \quad \ldots \quad (2)$$

V_7 is the sum of V_8 and V_9, hence,

$$V_7 = V_G [1 + Z_s/Z_a] \quad \ldots \quad (3)$$

and

$$I_7 = V_7/Z_b = V_G \left[\frac{Z_s}{Z_a Z_b} + \frac{1}{Z_b}\right] \quad \ldots \quad (4)$$

Furthermore, the current I_6 is the sum of I_7 and I_8, so that,

$$I_6 = V_G \left[\frac{Z_s}{Z_a Z_b} + \frac{1}{Z_a} + \frac{1}{Z_b}\right] \quad \ldots \quad (5)$$

Proceeding thus, it is possible to write down successively the values of voltage and current for each component in the network. Thus,

$$V_6 = I_6 Z_s = V_G \left[\frac{Z_s^2}{Z_a Z_b} + Z_s\left(\frac{1}{Z_a} + \frac{1}{Z_b}\right)\right] \quad \ldots \quad (6)$$

$$V_5 = V_6 + V_7 = V_G \left[\frac{Z_s^2}{Z_a Z_b} + Z_s\left(\frac{2}{Z_a} + \frac{1}{Z_b}\right) + 1\right] \quad \ldots \quad (7)$$

$$I_5 = \frac{V_5}{Z_c} = V_G \left[\frac{Z_s^2}{Z_a Z_b Z_c} + Z_s\left(\frac{2}{Z_a Z_c} + \frac{1}{Z_b Z_c}\right) + \frac{1}{Z_c}\right] \quad \ldots \quad (8)$$

$$I_4 = I_5 + I_6$$
$$= V_G\left[\frac{Z_s^2}{Z_aZ_bZ_c} + Z_s\left(\frac{1}{Z_aZ_b} + \frac{2}{Z_aZ_c} + \frac{1}{Z_bZ_c}\right) + \frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c}\right] \quad \ldots \quad (9)$$

$$V_4 = I_4 Z_s$$
$$= V_G\left[\frac{Z_s^3}{Z_aZ_bZ_c} + Z_s^2\left(\frac{1}{Z_aZ_b} + \frac{2}{Z_aZ_c} + \frac{1}{Z_bZ_c}\right) + Z_s\left(\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c}\right)\right] \quad \ldots \quad (10)$$

$$V_3 = V_4 + V_5$$
$$= V_G\left[\frac{Z_s^3}{Z_aZ_bZ_c} + Z_s^2\left(\frac{2}{Z_aZ_b} + \frac{2}{Z_aZ_c} + \frac{}{Z_bZ_c}\right) + Z_s\left(\frac{3}{Z_a} + \frac{2}{Z_b} + \frac{1}{Z_c}\right) + 1\right] (11)$$

$$I_3 = \frac{V_3}{Z_d}$$
$$= V_G\left[\frac{Z_s^3}{Z_aZ_bZ_cZ_d} + Z_s^2\left(\frac{2}{Z_aZ_bZ_d} + \frac{2}{Z_aZ_cZ_d} + \frac{1}{Z_bZ_cZ_d}\right) + Z_s\left(\frac{3}{Z_aZ_d} + \frac{2}{Z_bZ_d} + \frac{1}{Z_cZ_d}\right) + \frac{1}{Z_d}\right] \ldots \quad (12)$$

$$I_2 = I_3 + I_4$$
$$= V_G\left[\frac{Z_s^3}{Z_aZ_bZ_cZ_d} + Z_s^2\left(\frac{1}{Z_aZ_bZ_c} + \frac{2}{Z_aZ_bZ_d} + \frac{2}{Z_aZ_cZ_d} + \frac{1}{Z_bZ_cZ_d}\right) + Z_s\left(\frac{1}{Z_aZ_b} + \frac{2}{Z_aZ_c} + \frac{3}{Z_aZ_d} + \frac{1}{Z_bZ_c} + \frac{2}{Z_bZ_d} + \frac{1}{Z_cZ_d}\right) + \frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c} + \frac{1}{Z_d}\right] \quad \ldots \quad \ldots \quad (13)$$

$$V_2 = I_2 Z_s$$
$$= V_G\left[\frac{Z_s^4}{Z_aZ_bZ_cZ_d} + Z_s^3\left(\frac{1}{Z_aZ_bZ_c} + \frac{2}{Z_aZ_bZ_d} + \frac{2}{Z_aZ_cZ_d} + \frac{1}{Z_bZ_cZ_d}\right) + Z_s^2\left(\frac{1}{Z_aZ_b} + \frac{2}{Z_aZ_c} + \frac{3}{Z_aZ_d} + \frac{1}{Z_bZ_c} + \frac{2}{Z_bZ_d} + \frac{1}{Z_cZ_d}\right) + Z_s\left(\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c} + \frac{1}{Z_d}\right)\right] \quad (14$$

$$V_L = V_1 = V_2 + V_3$$
$$= V_G\left[\frac{Z_s^4}{Z_aZ_bZ_cZ_d} + Z_s^3\left(\frac{2}{Z_aZ_bZ_c} + \frac{2}{Z_aZ_bZ_d} + \frac{2}{Z_aZ_cZ_d} + \frac{1}{Z_bZ_cZ_d}\right) + Z_s^2\left(\frac{3}{Z_aZ_b} + \frac{4}{Z_aZ_c} + \frac{3}{Z_aZ_d} + \frac{2}{Z_bZ_c} + \frac{2}{Z_bZ_d} + \frac{1}{Z_cZ_d}\right) + Z_s\left(\frac{4}{Z_a} + \frac{3}{Z_b} + \frac{2}{Z_c} + \frac{1}{Z_d}\right) + 1\right] \quad (15)$$

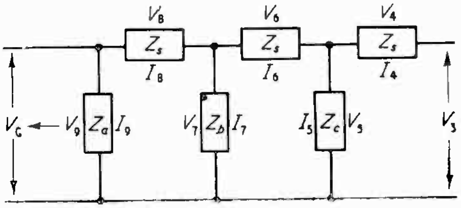


Fig. 3. *Generalized three-mesh network for shunt variable element.*

The corresponding relation for a three-mesh network of the type shown in Fig. 3 is obtained by putting $V_L' = V_3$ in equation (11). Equations (1) to (15) now determine the circumstances under which three- and four-mesh networks can produce the required phase reversal, and also define the variation of the frequency at which this occurs, as well as the voltage loss due to the network as each of its shunt impedances is varied in turn.

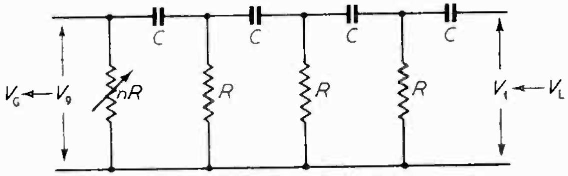


Fig. 4. *Four-mesh network with series capacitance elements.*

Four-mesh Network with Shunt Variable

Let us now examine the case of a four-mesh network using series capacitors and shunt resistors as shown in Fig. 4. If the former each have a capacitance C, three of the shunt elements a resistance R, and the fourth a resistance nR, then by substituting $Z_s = 1/j\omega C$, $Z_b = Z_c = Z_d = R$, and $Z_a = nR$ in equation (15) we obtain,

$$V_L = V_G\left\{\left[1 + \frac{1}{\omega^4C^4nR^4} - \frac{1}{\omega^2C^2R^2}\left(\frac{10}{n} + 5\right)\right] + j\left[\frac{1}{\omega^3C^3R^3}\left(\frac{6}{n} + 1\right) - \frac{1}{\omega CR}\left(\frac{4}{n} + 1\right)\right]\right\} \quad (16)$$

If V_G' is to be in exact antiphase with V_L, the imaginary part of equation (16) must vanish, so that the frequency at which this occurs is given by,

$$\frac{1}{\omega^3C^3R^3}\left(\frac{6}{n}+1\right)=\frac{1}{\omega CR}\left(\frac{4}{n}+6\right)$$

whence,

$$f=\frac{1}{2\pi CR}\sqrt{\frac{6+n}{4+6n}} \quad \ldots \quad (17)$$

Substituting this result in equation (16) we obtain,

$$\frac{V_L}{V_G}=1+\frac{1}{n}\left[\left(\frac{4+6n}{6+n}\right)^2-\frac{4+6n}{4+n}(10+5n)\right] \quad \ldots \quad (18)$$

The formulae giving the frequency at which phase reversal occurs, and the corresponding relations which define the voltage loss as Z_b, Z_c, and Z_d are varied in turn are shown in Table I. In the case of a uniform network, n is unity, so that in this instance, the frequency at which the phase is advanced by 180° is given by $f=\frac{1}{2\pi CR}\sqrt{\frac{7}{10}}$ and the ratio of input to output voltage is $V_L/V_G=-18.4$. The negative sign indicates, of course, that the phase has been advanced by 180° and not 360°. Provided therefore that the valve is able to ensure a voltage gain of at least 18.4, self-oscillation will occur at a frequency $f=0.1331/CR$. It is worthy of note that in all cases, the voltage loss introduced by the network is independent of the individual values of C and R, although it must be remembered that unless R is considerably greater than the a.c. resistance r_a of the valve, it may be impossible to choose a sufficiently large value of R_L to secure the required amplification.

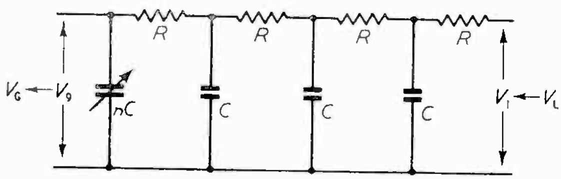


Fig. 5. *Four-mesh network with shunt-capacitance elements.*

The alternative case which we shall now consider is that in which the series components of the network are resistors of equal value R, and the shunt arms are capacitors as shown in Fig. 5. In this instance, suppose that $Z_a=1/j\omega nC$, $Z_b=Z_c=Z_d=1/j\omega C$, and $Z_s=R$. Equation (15) then gives the relation between input and output voltages as,

$$V_L=V_G[\omega^4nC^4R^4-j\omega^3R^3(2nC^3+2nC^3+2nC^3+C^3)-\omega^2R^2(3nC^2+4nC^2+3nC^2+2C^2+2C^2+C^2)+j\omega R(4nC+3C+2C+C)+1]$$

$$V_L=V_G\{[1+\omega^4nC^4R^4-\omega^2C^2R^2(10n+5)]-j[\omega^3C^3R^3(6n+1)-\omega CR(4n+6)]\} \quad \ldots \quad (19)$$

Equating the imaginary part of this expression to zero, we find that exact phase reversal occurs at a frequency defined by,

$$f=\frac{1}{2\pi CR}\sqrt{\frac{4n+6}{6n+1}} \quad \ldots \quad (20)$$

and that the voltage loss due to the network is

$$\frac{V_L}{V_G}=1+n\left(\frac{4n+6}{6n+1}\right)^2-\frac{4n+6}{6n+1}(10n+5) \quad (21)$$

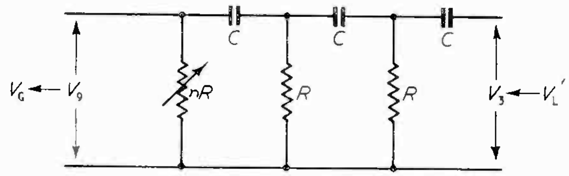


Fig. 6. *Three-mesh network with series-capacitance elements.*

For a uniform four-mesh network with resistive series components and capacitive shunt arms a phase retardation of 180° occurs at a frequency $f=\frac{1}{2\pi CR}\sqrt{\frac{10}{7}}=\frac{0.1902}{CR}$, the ratio of input to output voltage being 18.4 as before.

Three-mesh Network with Shunt Variable

Corresponding relations may be obtained for the three-mesh network shown in Fig. 6 by substituting $Z_s=1/j\omega C$, $Z_a=nR$, and $Z_b==Z_c=R$ in equation (11). It is then evident that,

$$V_L'=V_G\left\{\left[1-\frac{1}{\omega^2C^2R^2}\left(\frac{4}{n}+1\right)\right]+j\left[\frac{1}{\omega^3C^3nR^3}-\frac{1}{\omega CR}\left(\frac{3}{n}+3\right)\right]\right\} \quad (22)$$

The frequency at which the network provides exact phase reversal is obtained by equating the imaginary part of (22) to zero, whence,

$$f=\frac{1}{2\pi CR}\sqrt{\frac{1}{3+3n}} \quad \ldots \quad (23)$$

If this result is introduced into equation (22), the voltage loss due to the network is seen to be,

$$\frac{V_L'}{V_G}=-\left[14+\frac{12}{n}+3n\right] \quad \ldots \quad (24)$$

For a uniform network, all the shunt components have identical resistances, so that the frequency at which a phase advance of 180° takes place is $f = \frac{1}{2\pi CR}\sqrt{\frac{1}{6}} = \frac{0.0649}{CR}$, the ratio of the input to output voltage then being $V_L'/V_G = -29$.

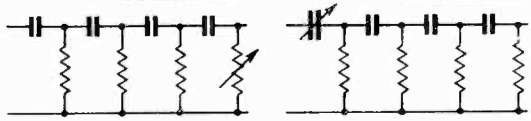
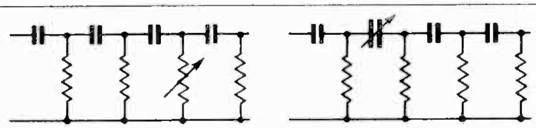
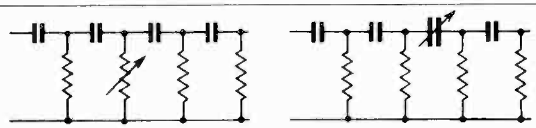
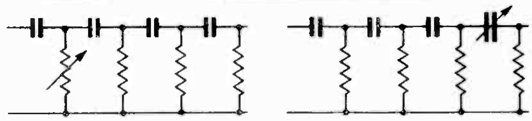
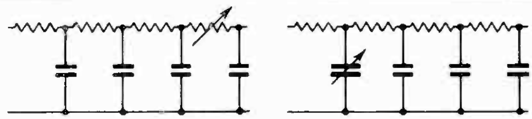
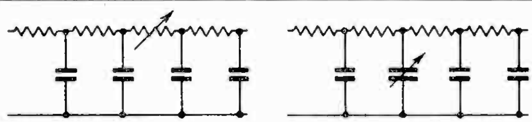
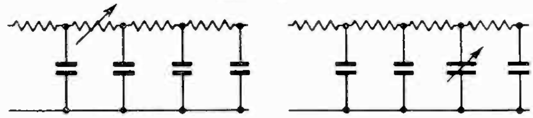
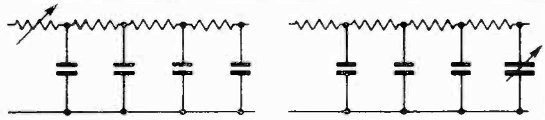
If the series arms of the three-mesh network are resistors of equal value, and the shunt arms are capacitors as in Fig. 7, then by inserting $Z_s = R$, $Z_a = 1/j\omega nC$, and $Z_b = Z_c = 1/j\omega C$ in equation (11) we obtain,

$$V_L' = V_G\{[1 - \omega^2 C^2 R^2 (4n + 1)] - j\,[\omega^3 n C^3 R^3 - \omega CR\,(3n + 3)]\} \quad (25)$$

Phase retardation of 180° will occur when the imaginary part of equation (25) is zero; i.e., when,

$$f = \frac{1}{2\pi CR}\sqrt{3 + \frac{3}{n}} \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

TABLE I

CASE	$V_L \rightarrow$ V_G $V_L \rightarrow$ V_G	f	$\frac{V_L}{V_G}$
(a)		$\frac{1}{2\pi CR}\sqrt{\frac{n+6}{6n+4}}$	$\left\{1 + \frac{1}{n}\left[\left(\frac{6n+4}{n+6}\right)^2 - \left(\frac{6n+4}{n+6}\right)(5n+10)\right]\right\}$
(b)		$\frac{1}{2\pi CR}\sqrt{\frac{2n+5}{7n+3}}$	$\left\{1 + \frac{1}{n}\left[\left(\frac{7n+3}{2n+5}\right)^2 - \left(\frac{7n+3}{2n+5}\right)(8n+7)\right]\right\}$
(c)		$\frac{1}{2\pi CR}\sqrt{\frac{2n+5}{8n+2}}$	$\left\{1 + \frac{1}{n}\left[\left(\frac{8n+2}{2n+5}\right)^2 - \left(\frac{8n+2}{2n+5}\right)(8n+7)\right]\right\}$
(d)		$\frac{1}{2\pi CR}\sqrt{\frac{2n+5}{9n+1}}$	$\left\{1 + \frac{1}{n}\left[\left(\frac{9n+1}{2n+5}\right)^2 - \left(\frac{9n+1}{2n+5}\right)(9n+6)\right]\right\}$
(e)		$\frac{1}{2\pi CR}\sqrt{\frac{n+9}{5n+2}}$	$\left\{1 + n\ \left(\frac{n+9}{5n+2}\right)^2 - \left(\frac{n+9}{5n+2}\right)(6n+9)\right\}$
(f)		$\frac{1}{2\pi CR}\sqrt{\frac{2n+8}{5n+2}}$	$\left\{1 + n\ \left(\frac{2n+8}{5n+2}\right)^2 - \left(\frac{2n+8}{5n+2}\right)(7n+8)\right\}$
(g)		$\frac{1}{2\pi CR}\sqrt{\frac{3n+7}{5n+2}}$	$\left\{1 + n\ \left(\frac{3n+7}{5n+2}\right)^2 - \left(\frac{3n+7}{5n+2}\right)(7n+8)\right\}$
(h)		$\frac{1}{2\pi CR}\sqrt{\frac{4n+6}{6n+1}}$	$\left\{1 + n\ \left(\frac{4n+6}{6n+1}\right)^2 - \left(\frac{4n+6}{6n+1}\right)(10n+5)\right\}$

The voltage ratio in this case is, of course,

$$\frac{V_L'}{V_G} = -\left[14 + 12n + \frac{3}{n}\right] \quad \ldots \quad \ldots \quad (27)$$

By putting n equal to unity, we obtain the frequency at which the three-mesh network can give a 180° phase retardation as $f = \frac{\sqrt{6}}{2\pi CR} = \frac{0.3898}{CR}$. Under these circumstances $V_L'/V_G = -29$.

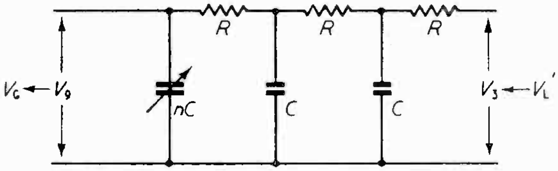


Fig. 7. *Three-mesh network with shunt-capacitance elements.*

Summaries of the results pertaining to the various forms of three- and four-mesh phase advancing and retarding networks are shown in Tables I and II.

If the variable component of the phase-shifting network is one of the series elements, then the generalized arrangement takes the form of Fig. 8. Proceeding as before, by assuming that no current is taken from the network, the relations for the voltage and current distribution are as follows,

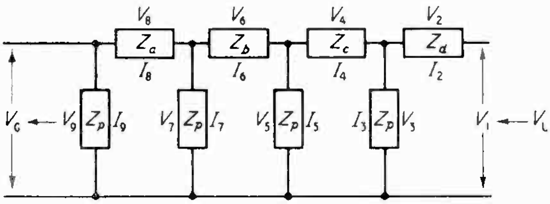


Fig. 8. *Generalized four-mesh network for series variable element.*

$$I_9 = I_8 = \frac{V_G}{Z_p} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

$$V_8 = V_G \frac{Z_a}{Z_p} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

$$V_7 = V_G\left[\frac{Z_a}{Z_p} + 1\right] \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

$$I_7 = V_G\left[\frac{Z_a}{Z_p^2} + \frac{1}{Z_p}\right] \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$I_6 = V_G\left[\frac{Z_a}{Z_p^2} + \frac{2}{Z_p}\right] \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

$$V_6 = V_G\left[\frac{Z_aZ_b}{Z_p^2} + \frac{2Z_b}{Z_p}\right] \quad \ldots \quad \ldots \quad (33)$$

$$V_5 = V_G\left[\frac{Z_aZ_b}{Z_p^2} + \frac{1}{Z_p}(Z_a + 2Z_b) + 1\right] \quad (34)$$

$$I_5 = V_G\left[\frac{Z_aZ_b}{Z_p^3} + \frac{1}{Z_p^2}(Z_a + 2Z_b) + \frac{1}{Z_p}\right] \quad (35)$$

$$I_4 = V_G\left[\frac{Z_aZ_b}{Z_p^3} + \frac{1}{Z_p^2}(2Z_a + 2Z_b) + \frac{3}{Z_p}\right] \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

$$V_4 = V_G\left[\frac{Z_aZ_bZ_c}{Z_p^3} + \frac{1}{Z_p^2}(2Z_aZ_c + 2Z_bZ_c) + \frac{3Z_c}{Z_p}\right] \quad (37)$$

$$V_3 = V_G\left[\frac{Z_aZ_bZ_c}{Z_p^3} + \frac{1}{Z_p^2}(Z_aZ_b + 2Z_aZ_c + 2Z_bZ_c) + \frac{1}{Z_p}(Z_a + 2Z_b + 3Z_c) + 1\right] \quad (38)$$

$$I_3 = V_G\left[\frac{Z_aZ_bZ_c}{Z_p^4} + \frac{1}{Z_p^3}(Z_aZ_b + 2Z_aZ_c + 2Z_bZ_c) + \frac{1}{Z_p^3}(Z_a + 2Z_b + 3Z_c) + \frac{1}{Z_p}\right] \quad (39)$$

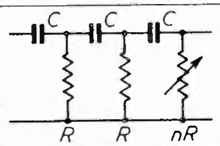
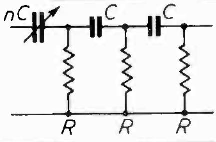
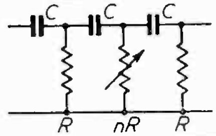
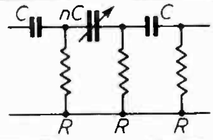
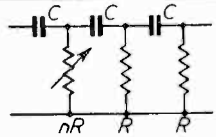
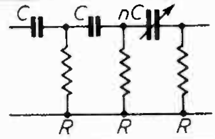
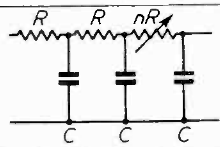
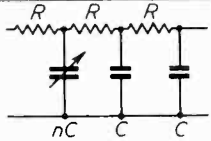
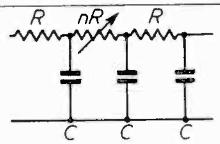
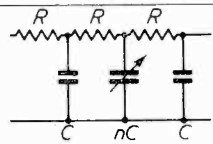
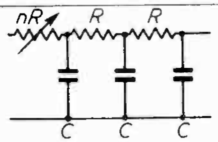
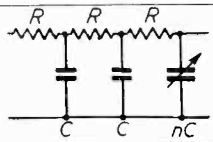
$$I_2 = V_G\left[\frac{Z_aZ_bZ_c}{Z_p^4} + \frac{1}{Z_p^3}(2Z_aZ_b + 2Z_aZ_c + 2Z_bZ_c) + \frac{1}{Z_p^2}(3Z_a + 4Z_b + 3Z_c) + \frac{4}{Z_p}\right] \quad (40)$$

$$V_2 = V_G\left[\frac{Z_aZ_bZ_cZ_d}{Z_p^4} + \frac{1}{Z_p^3}(2Z_aZ_bZ_d + 2Z_aZ_cZ_d + 2Z_bZ_cZ_d) + \frac{1}{Z_p^2}(3Z_aZ_d + 4Z_bZ_d + 3Z_cZ_d) + \frac{4Z_d}{Z_p}\right] \quad (41)$$

$$V_L = V_1 = V_G\left[\frac{Z_aZ_bZ_cZ_d}{Z_p^4} + \frac{1}{Z_p^3}(Z_aZ_bZ_c + 2Z_aZ_bZ_d + 2Z_aZ_cZ_d + 2Z_bZ_cZ_d) + \frac{1}{Z_p^2}(Z_aZ_b + 2Z_aZ_c + 3Z_aZ_d + 2Z_bZ_c + 4Z_bZ_d + 3Z_cZ_d) + \frac{1}{Z_p}(Z_a + 2Z_b + 3Z_c + 4Z_d) + 1\right] \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

The foregoing equations now enable us to

TABLE II

CASE	$V_L' \rightarrow$ V_G	$V_L' \rightarrow$ V_G	f	$\frac{V_L}{V_G}$
A			$\frac{1}{2\pi CR\sqrt{3n+3}}$	$-\left[14 + 3n + \frac{12}{n}\right]$
B			$\frac{1}{2\pi CR\sqrt{4n+2}}$	$-\left[15 + 8n + \frac{6}{n}\right]$
C			$\frac{1}{2\pi CR\sqrt{5n+1}}$	$-\left[16 + 10n + \frac{3}{n}\right]$
D			$\frac{\sqrt{\frac{5}{n}+1}}{2\pi CR}$	$-\left[16 + 3n + \frac{10}{n}\right]$
E			$\frac{\sqrt{\frac{4}{n}+2}}{2\pi CR}$	$-\left[15 + 6n + \frac{8}{n}\right]$
F			$\frac{\sqrt{\frac{3}{n}+3}}{2\pi CR}$	$-\left[14 + 12n + \frac{3}{n}\right]$

examine the circumstances under which any type of three- or four-mesh network can bring about phase reversal of the applied voltage.

Four-mesh Network with Series Variable

As a typical example of a four-mesh network with one series component variable, let us examine the case shown in Fig. 9. In this instance, $Z_p = R$, $Z_a = 1/j\omega nC$, and $Z_b = Z_c = Z_d = 1/j\omega C$ and if these values are inserted in equation (42), we obtain

$$V_L = V_G\left\{\left[1 + \frac{1}{\omega^4 R^4 n C^4} - \frac{1}{\omega^2 R^2 C^2}\left(\frac{6}{n} + 9\right)\right] + j\left[\frac{1}{\omega^3 R^3 C^3}\left(\frac{5}{n} + 2\right) - \frac{1}{\omega RC}\left(\frac{1}{n} + 9\right)\right]\right\} \quad \ldots \quad (43)$$

This form of network will therefore give a phase advance of 180° when

$$f = \frac{1}{2\pi RC}\sqrt{\frac{5 + 2n}{1 + 9n}} \quad \ldots \quad (44)$$

Under these circumstances

$$\frac{V_L}{V_G} = 1 + \frac{1}{n}\left[\left(\frac{9n + 1}{2n + 5}\right)^2 - \frac{9n + 1}{2n + 5}(6 + 9n)\right] \quad \ldots \quad (45)$$

Three-mesh Network with Series Variable

The general solution for a three-mesh network with a variable series component, as in Fig. 10, is obtained from equation (38) as

$$V_L' = V_G\left[\frac{Z_a Z_b Z_c}{Z_p^3} + \frac{1}{Z_p^2}(Z_a Z_b + 2 Z_a Z_c + 2 Z_b Z_c) + \frac{1}{Z_p}(Z_a + 2Z_b + 3Z_c) + 1\right] \quad (46)$$

The formulae for the frequency at which exact phase reversal takes place with this form of network, and the voltage loss appropriate to the differing forms of the arrangement are shown in Table II.

General Conclusions

It is not easy to draw any general conclusions from the results of Tables I and II, but it is a simple matter to illustrate in graphical form the effects of altering the value of one of the variable

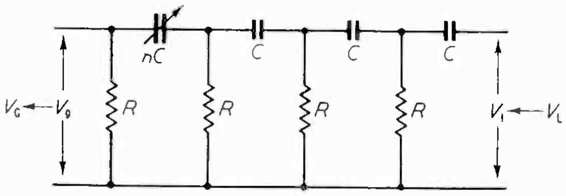


Fig. 9. *Series-capacitance four-mesh network.*

components in any particular case. Typical examples showing the variation in frequency and the degree of amplification which must be provided to sustain self-oscillation are shown in

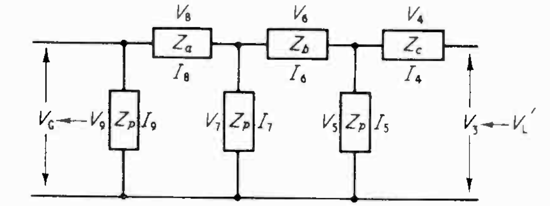


Fig. 10. *Generalized three-mesh network for variable series element.*

Figs. 11 and 12. It is interesting to note that the voltage loss is not a minimum when the network has a uniform configuration, although small deviations from uniformity do not make substantial alterations in the ratio of input to output voltage. It must be obvious, however, that adjustment of any single component in the network modifies the input impedance so that there is no justification for the view that the latter remains reasonably constant if this modification has little effect on the voltage loss. The input impedance, which is in all cases complex, forms part of the anode load of the valve, so that its changes will seriously affect the stage gain unless R_L is small by comparison. Values of input impedance for any form of network may be computed from the foregoing expressions for input voltage and current.

The frequency stability of the phase-shift oscillator is clearly dependent on the constancy of the component values, and the effects of variation in any of these can be estimated from the frequency formulae which have been derived. One other factor affecting frequency stability should however be mentioned. The formulae for the frequency at which exact phase reversal occurs have all been based on the assumption that the grid-cathode input impedance of the valve imposes no load on the network. It will be evident from the above analysis that in so far as the existence of grid current implies a modification in the effective shunt impedance Z_a, the frequency of oscillation may differ appreciably from the formulae given, unless the amplitude of grid swing is small. The use of an unbypassed cathode resistor to provide negative feedback is usually recommended for the purpose of preserving a good waveform from this type of oscillator. Because it also helps to minimize changes in grid current which might result from variations in supply voltages, negative feedback contributes to improved frequency stability.

In view of the many misconceptions in regard

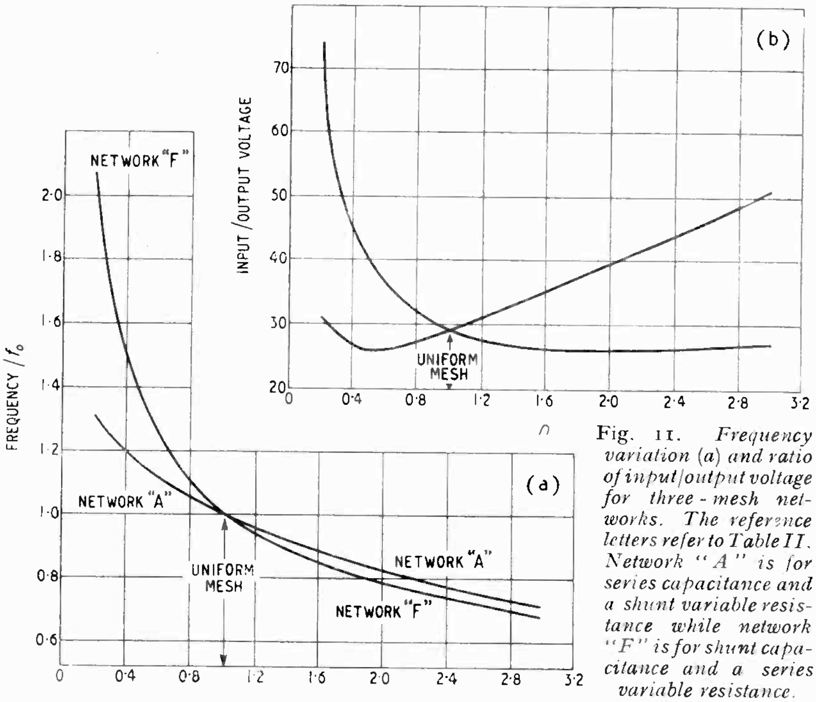


Fig. 11. *Frequency variation (a) and ratio of input/output voltage for three-mesh networks. The reference letters refer to Table II. Network "A" is for series capacitance and a shunt variable resistance while network "F" is for shunt capacitance and a series variable resistance.*

to the functioning of the phase-shifting network, it is perhaps worthwhile, in conclusion, to evaluate the voltage and current distribution for a uniform three-mesh phase advancing network such as that shown in Fig. 1 (a) The frequency at which this network provides exact phase reversal is $f = \dfrac{0.0649}{CR}$ and the voltages and currents in its different parts are given in Table III.

The input impedance of the network under these circumstances is $(0.83 - j2.7)R$. In relation to the applied voltage V_L, the voltage V_5 is advanced 55.8°, while V_7 is further advanced by 56.4°. Though these differ little from the value of 60° sometimes quoted, the latter figure often results from a totally mistaken view of the behaviour of the network.

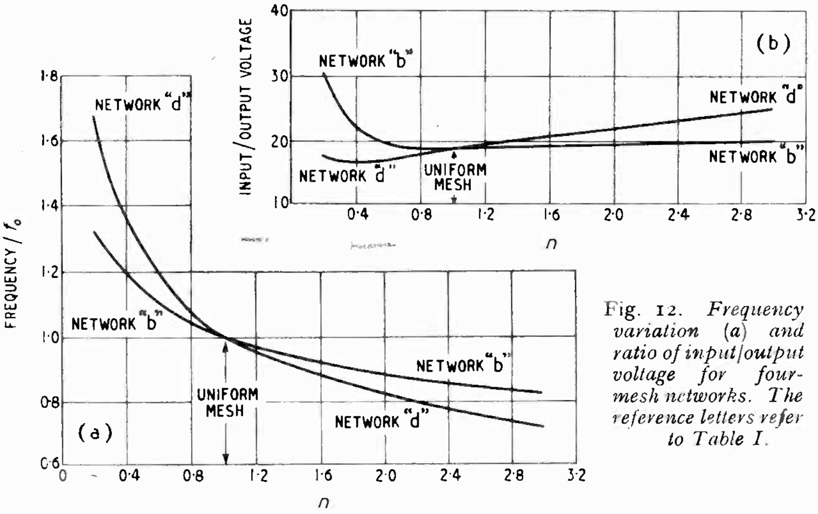


Fig. 12. Frequency variation (a) and ratio of input/output voltage for four-mesh networks. The reference letters refer to Table I.

TABLE III

$V_3 = V'_L$	$I_4 = \dfrac{V'_L}{29R}[3 + j4\sqrt{6}]$
$V_4 = \dfrac{V'_L}{29}[24 - j3\sqrt{6}]$ $= 0.865\, V'_L \cos(\omega t - 17.0°)$	$= 0.354\dfrac{V'_L}{R}\cos(\omega t + 73.0°)$
$V_5 = \dfrac{V'_L}{29}[5 + j3\sqrt{6}]$ $= 0.307\, V'_L \cos(\omega t + 55.8°)$	$I_5 = \dfrac{V'_L}{29R}[5 + j3\sqrt{6}]$ $= 0.307\dfrac{V'_L}{R}\cos(\omega t + 55.8°)$
$V_6 = \dfrac{V'_L}{29}[6 + j2\sqrt{6}]$ $= 0.267\, V'_L \cos(\omega t + 39.2°)$	$I_6 = \dfrac{V'_L}{29R}[-2 + j\sqrt{6}]$ $= 0.109\dfrac{V'_L}{R}\cos(\omega t + 129.2°)$
$V_7 = \dfrac{V'_L}{29}[-1 + j\sqrt{6}]$ $= 0.091\, V'_L \cos(\omega t + 112.2°)$	$I_7 = \dfrac{V'_L}{29R}[-1 + j\sqrt{6}]$ $= 0.091\dfrac{V'_L}{R}\cos(\omega t + 112.2°)$
$V_8 = \dfrac{V'_L}{29}[j\sqrt{6}]$ $= 0.084\, V'_L \cos(\omega t + 90°)$	$I_8 = I_9 = -\dfrac{V'_L}{29R}$
$V_9 = -\dfrac{V'_L}{29}$ $= 0.0345\, V'_L \cos(\omega t + 180°)$	$= 0.0345\dfrac{V'_L}{R}\cos(\omega t + 108°)$

GRAPHICAL STATISTICAL METHODS

By W. R. Hinton, A.M.I.E.E., M.I.R.E.

SUMMARY.—Two things are attempted in this paper; first, to provide the beginner with powerful statistical tools and techniques for examining data graphically, and secondly, to direct the attention of those already using Probability Paper to the quality of fit required before an assumption of normality is justified. Graphical methods are described which require no theoretical knowledge of statistics at all, and the natural development of a conception of a 'Normal Population,' and the evolution of Probability Paper, is described in most simple terms; however, sufficient information is given to prevent errors due to incorrect plotting, which have been seen in some previously published works. The examples have been chosen deliberately to be diverse, to emphasize the wide range of problems to which statistical methods can be applied, and the main purpose of the article is to encourage interest in this fascinating and most useful subject.

Introduction

BROADLY speaking, we are often concerned with examining the properties of similar things, and with forming an idea of what could be classified as a typical object. A typical object could be defined as one which is like the majority of objects in a fairly large sample. No less important is some idea of the way individuals differ from the typical one, and what proportion of the total have reasonably close properties to the typical one.

Generally these ideas are formed intuitively, and comprise the 'experience' of the observer, and quite often such ideas are difficult to define and communicate to anyone else.

Statistics is a subject which is primarily concerned with classifying, grouping and examining data, and offers a language by which the above ideas can be conveyed. For example, the most probable value of an observed quantity (i.e., the typical value) is called the 'Mode,' and a constant which is descriptive of the way in which individuals cluster round the typical value is called the 'Standard Deviation.' Likewise there are recognized graphical methods of presenting and analysing data, such as the Histogram and Ogive, which have been designed to convey particular ideas and allow certain conclusions to be drawn with a minimum of labour.

Ideas have been developed of 'populations' of individuals and of the distribution of individuals in the population; of samples of individuals from a population and of the distribution of individuals in the sample. It is not surprising to find, therefore, that an ideal parent population has been conceived; ideal, that is, in the amount of information which can be inferred from the way individuals are distributed in the population, and in the simplicity with which it can be described and defined. This ideal distribution is known as the 'Normal Distribution' or the 'Gaussian Distribution,' and is found to approximate closely to many distributions of observable quantities in nature.

The main point of testing data to see whether individuals follow a Normal Distribution, is to know whether the extensive predictions, proper to the ideal population, can be applied with any confidence to the problem in hand. This simply means that if the data is normally distributed, much labour is saved because the various predictions have been tabulated and published by theoretical workers in the field. If the data is not normal, one has to make one's own calculations and predictions.

The correct attitude is to decide what information is required from the data, and whether a graphical solution is adequate; if not, whether one has sufficient data to justify a normality test or a curve fitting analysis. Often one finds that there are so few individuals in the sample at one's disposal, that a curve fitting analysis would be rather absurd, and obviously, if this is so, any labour-saving system of curve fitting (like probability paper) is equally unreliable. For this reason probability paper is a snare for the unwary; as the temptation to use it for very small samples from unknown populations is very great. Of course, it is true that, if the data follows a normal distribution, the graph on the probability paper will be a straight line, but it may not be generally realized how little is the deviation from linearity which can be tolerated in a practical case for an assumption of normality to be justified, as will be shown later.

The novelty and fascination of probability paper tends to eclipse the usefulness of the common Ogive (from which it is derived), and an imperfect understanding of the evolution of probability paper can lead to errors due to incorrect plotting, or to a wrong interpretation of the curve. Some attempt is made, therefore, in the following notes, to provide a background of simple, graphical, statistical methods, and to demonstrate some inherent limitations of probability paper.

The Histogram

In surveying a mass of numerical results one would intuitively group identical results together, and perhaps arrange the resulting groups in ascending order of magnitude. A logical extension of this idea would be to group results which fell between definite boundaries, and to arrange these groups in ascending order of magnitude. In this way the vast amount of detail would be made more comprehensible, and any significant difference between one group and another would become apparent. For example, suppose that a Government Department was required to requisition suits for demobilized armed forces, and it was essential to conserve raw materials and labour. The problem would be to discover how many sizes of suits would be required, and how many of each should be manufactured.

The first step might be to take a sample of men at random, and measure their individual heights to the nearest inch, as shown in columns (1) and (2) in Table I. (Note, therefore, that the height of a man recorded as 58 in may actually be anywhere between $57\frac{1}{2}$ in and $58\frac{1}{2}$ in.)

TABLE I

(1) Height to the nearest Inch	(2) Number of Men	(3) Number of Men in Group	(4) % of Total Men in Group	(5) Cumulative % Men
58 . .	1			
59 . .	1	5	0.8	
60 . .	3			0.8
61 . .	7			
62 . .	13	52	8.4	
63 . .	32			9.2
64 . .	52			
65 . .	74	214	34.6	
66 . .	88			43.8
67 . .	92			
68 . .	89	256	41.3	
69 . .	75			85.1
70 . .	47			
71 . .	25	84	13.6	
72 . .	12			98.7
73 . .	5			
74 . .	2	8	1.3	
75 . .	1			100
Totals . .	619	619	100	

Suppose that the data is grouped into broader classes, for example, into men with heights between $57\frac{1}{2}$ in and $60\frac{1}{2}$ in and between $60\frac{1}{2}$ in and $63\frac{1}{2}$ in, and so on, as shown in column (3). This can be plotted as shown in Fig. 1 (a), where each pillar represents a group of data by its height, being proportional to the number of men within the group, and the edges of the pillar define the group boundaries. Such a figure is known as a Histogram, and is valuable for showing the 'Mode' or most frequent value, which in this case is about 68 in, and the dispersion, or scatter, either side of the Mode. In a word, it shows the distribution of individuals in the sample. [In some problems it may be more convenient to plot the height of Histogram pillars proportional to the percentage of total observations, as given in Table 1, column (4) and shown in Fig. 3.]

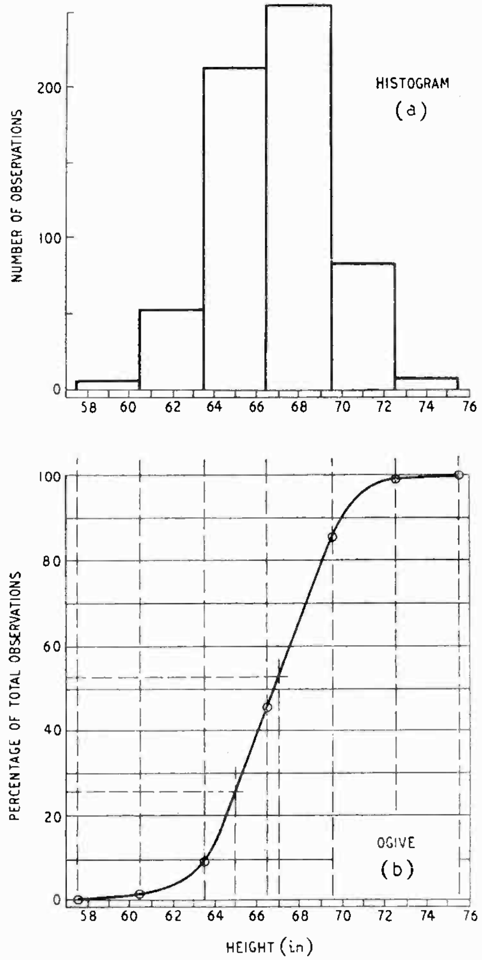


Fig. 1. *Histogram and Ogive showing the distribution of heights of 619 men, and the method of determining the proportion of men with heights between, say, 65 in and 67 in.*

The Ogive

In practice, the group boundaries of the Histogram are chosen arbitrarily so as to make a presentable figure, and whereas it is possible to see what proportion of men have heights between, say, $66\frac{1}{2}$ in and $69\frac{1}{2}$ in, it is not easy to see what proportion would have heights between 65 in and 67 in (say), or what proportion exceed 70 in in height.

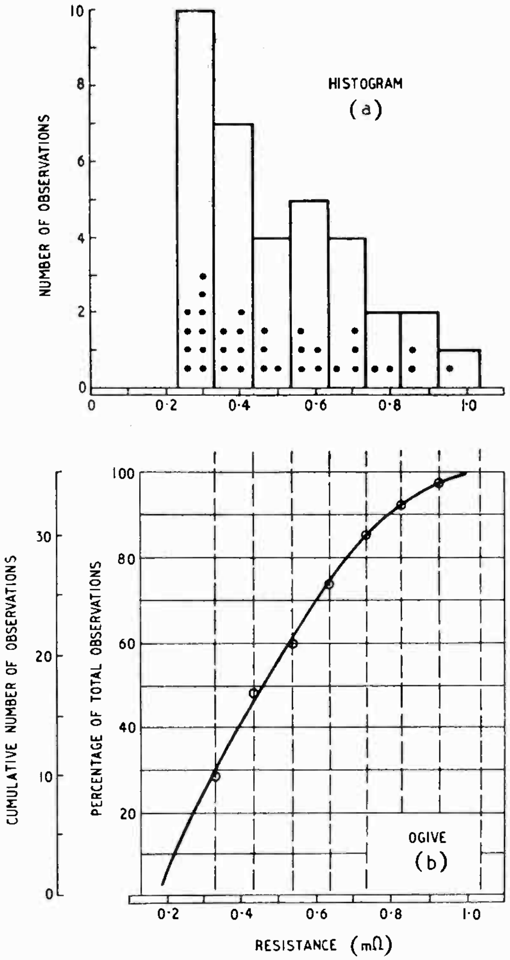


Fig. 2. *Dot Diagram, Histogram, and Ogive of the measured resistance of a heavy-current switch under full-load conditions, showing extrapolation of the extremes.*

To solve this kind of problem it is better to compute the cumulative percentage of men, as shown in Table 1, column (5). For example, 0.8% of the men had heights between $57\frac{1}{2}$ in and $60\frac{1}{2}$ in and 9.2% between $57\frac{1}{2}$ in and $63\frac{1}{2}$ in, and so on. Notice that no man is recorded with height less than $57\frac{1}{2}$ in, and that 0.8% of the men were encountered over the range $57\frac{1}{2}$ in to $60\frac{1}{2}$ in, therefore the cumulative percentages must be plotted at the edges of the class intervals, as shown in Fig. 1 (b). This curve is an "Ogive," and provides different information from the Histogram. For example, the proportion of men having heights less than 67 in is 52% and that less than 65 in is 26%, therefore the proportion of men between heights 65 in and 67 in is the difference :

$$52 - 26 = 26\%.$$

This means that if one standard design of suit is intended to fit men between 65 in and 67 in in height, the number to be requisitioned should be 26% of the total, and so on. Likewise it seems hardly worth while making suits in bulk for statures less than about 60 in or over 73 in and these men could be fitted individually.

One valuable property of the Ogive is that it is fairly insensitive to the choice of class intervals, (histogram groups) and therefore smooths out the data which was coarsened by grouping for the histogram.

This example has shown what valuable information can be obtained from the application of very elementary statistical tools and a little common sense, and there has been no need to test the data for normality, or confuse oneself with predictions based on the Normal Distribution.

The Ogive is particularly useful for analysing data whose typical value (Mode) tends to be close to an extreme value. For example, in inspecting electrical switches, the switch resistance can never be less than the resistance of the conductors forming the switch parts, but always more, depending upon the condition of the contact surfaces. In such a case the Histogram is unsymmetrical or "skew" [Fig. 2 (a)] and the Ogive shows the minimum resistance quite clearly, which is of course, a useful parameter for judging the quality of the design.

If a technique is employed so that the extreme points of zero and 100% are not plotted, the Ogive gives an extrapolated value for the minimum resistance which is insensitive to the choice of histogram class intervals, (pillar widths).

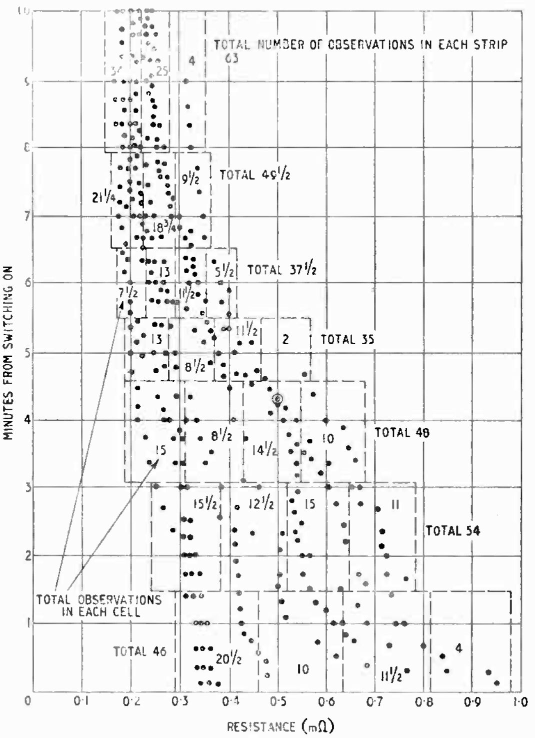
This is a reasonable technique to adopt, because otherwise the arbitrary choice of class interval would arbitrarily fix the point of zero cumulative observations at the edge of the first pillar ; whereas it is much better to let the entire data weight the choice of this point by extrapolation. Similar remarks apply to the 100% point. It is very instructive to experiment with grouping data differently, and observe how slightly the Ogive is affected.

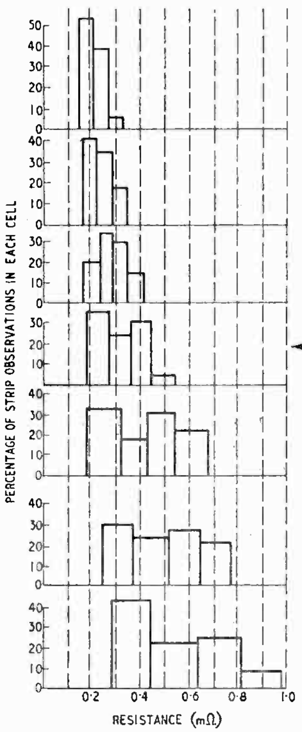
Graphical Grouping of Data

It saves a great deal of time and labour to record each observation directly on the graph paper to be used for the Histogram, by making a bold dot opposite the appropriate point on the abscissa, thus forming rudimentary columns of dots as observations are repeated. This method has the great advantage that one can see when sufficient data has been collected, as the distribution of dots begins to suggest a definite form. It is then easy to mark the dots off into suitable groups and construct a Histogram based on the number of dots falling into each group, as shown in Fig. 2. (Where a dot falls on a group boundary, one half of an observation should count in each group.) This is called a Dot Diagram, and the intermediate step of drawing the Histogram is not necessary for constructing an Ogive, as the total number of dots encountered up to a given boundary can be seen directly. Likewise, by observing quantities to the nearest convenient unit, the dots form clean columns and have the appearance of histogram pillars, which therefore, need not be drawn.

Another advantage of collecting experimental data by dot diagrams, is that it shows up any drift in performance due to warming up of equipment, etc. This becomes apparent when one finds the columns of dots not being filled in a random manner, but that ones hand is gradually moving across the page, as time goes on. This is most noticeable in the resistance measurements mentioned earlier, and the solution is to make several independent experiments, noting the time from the instant of switching on at which the observation is made. (Obviously the switch must be given time to cool down between experiments and strictly speaking, the observations should be made at about the same rate.) The data is recorded as bold dots, or other marks, located at the appropriate time, as shown in Fig. 3, and constitutes a Scatter Diagram which is intended to show whether one variable seems to depend on another. Clearly, in Fig. 3, the value of switch resistance observed, depended, to a great extent on the relative time at which the observation was made.

The method of treating the data is to divide the diagram into suitable vertical strips so that the dots enclosed can be considered to have occurred at roughly the same relative time. Then each strip of data can be treated as a normal dot diagram to form Histograms or Ogives as required. Fig. 3 thus gives a very clear picture of the performance of the switch. For example, about five minutes after switching on, the odds would be even that the switch resistance would not be greater than 0.32 mΩ because the Ogive shows that out of a large number of observations, one can expect 50% to lie below this value and 50% above. Likewise, the chance that the resistance would exceed 0.47 mΩ is about one in twenty, because the Ogive shows that about 95% of a large number of observations could be expected to be below this value.





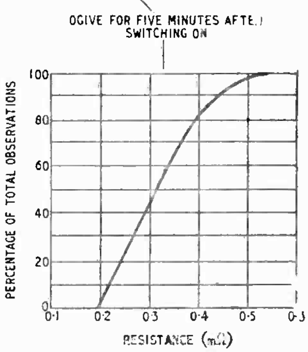


Fig. 3. *Scatter Diagram, Histograms and an Ogive of the measured resistance, of a heavy-current switch under full-load conditions, showing the method of treating data.*

Before leaving the subject of Dot and Scatter Diagrams, there is one disadvantage that should be mentioned, namely that because one can see the diagram taking shape, one is inclined to cheat, or shall we say, be biased in one's judgment, and cast the dots into where one thinks they should go. A great deal of self-discipline has to be exercised.

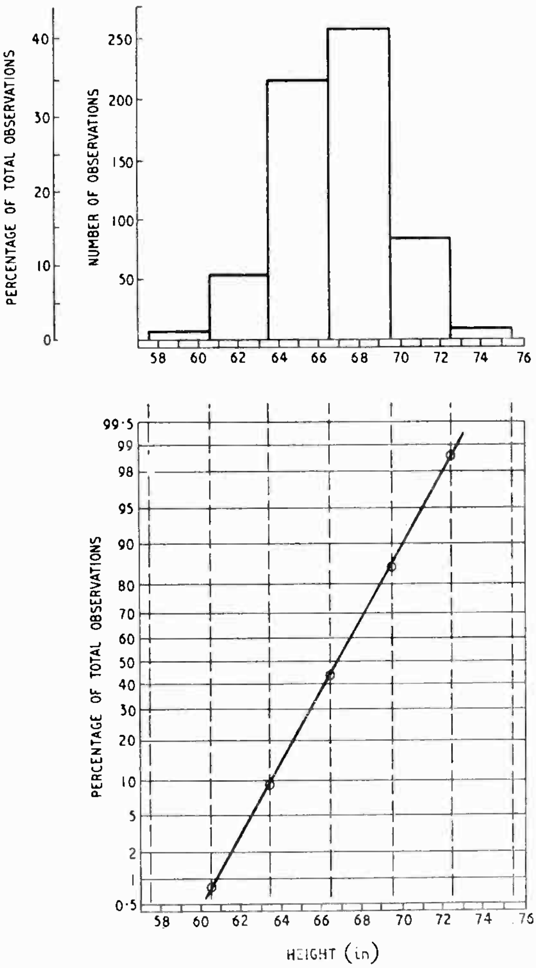


Fig. 4. Histogram with alternative scales and the Probability Paper Ogive of the distribution of heights of 619 men (as for Fig. 1), showing the correct method of plotting points at the edges of the class intervals.

Arithmetical Probability Paper

Probability Paper is simply a special graph paper for constructing Ogives, in which the vertical scale for 'Percentage of Total Observations' is stretched, at the extremities in a particular way. This scale is called the Probability Scale because the Ogive represents a population of a large number of individuals from which one can deduce the odds of an observation falling above or below a particular value. Let it be emphasized at once that it is not essential to use special probability paper to do this; indeed we have seen in the previous section that the common Ogive can be used.

The prefix, 'Arithmetical' means that the scale for the variable is linear, while with Logarithmic Probability Paper, the scale for the variable is logarithmic; the probability scale being the same as for the arithmetical paper. Obviously, one could use a logarithmic scale for the variable, in the construction of the common Ogive, if such a scale offered a clearer picture of the data.

Most Ogives are S-shaped curves, and become difficult to read at the extremes. If the scale is expanded over these sections, the readability of the graph is increased, but this does not mean that the reliability of the data is increased. In exactly the same way, inspecting the position of an ammeter pointer with a powerful magnifying lens does not increase the precision of a current measurement, because the calibration may not be reliable. This means that when Ogives are plotted on probability paper, one must be extremely cautious about making use of extrapolated information at the extremes of probability scales, say, outside the range of 5% to 95%.

At this stage it is convenient to emphasize the correct method of plotting data on probability paper. Since the curve is really an Ogive, the points corresponding to cumulative grouped data must be plotted at the boundaries of the Histogram groups, as shown in Fig. 4, and not at the central values of the groups which, by the way, is a common error. Since there is no zero or 100% on the probability scale, data for these points must necessarily be omitted.

If the plotted points fall exactly on a straight line, the data is normally distributed, because the probability scale has been specially stretched to make this so. Naturally one does not expect experimental points to fit a straight line perfectly, but the limited extent of deviation allowable may not be appreciated, and one should have at least twenty points to plot before drawing any serious conclusions about normality.

This is clearly demonstrated in Fig. 5, which shows data plotted on probability paper, from rectangular and triangular populations. Most people would feel justified in drawing a straight line through the points given by either of the 8 cell Histograms and might, therefore, be led erroneously to believe that both populations were normally distributed.

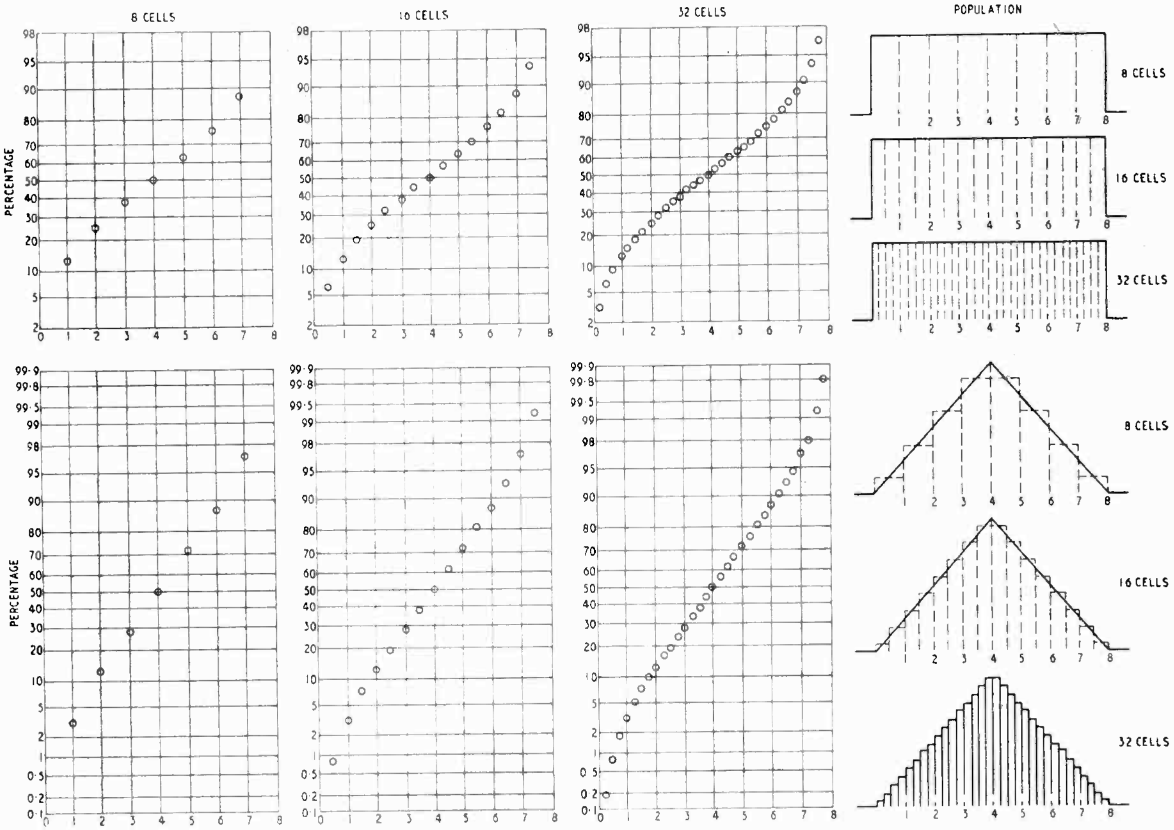


Fig. 5. *Ogives, plotted on Probability Paper, for 8-, 16- and 32-cell Histograms of data arising from rectangular and triangular parent populations, showing the small departure from linearity when few points are available.*

The data for 16 Cells begins to show the curving of the extremities, which is clearly confirmed with 32 Cells, so that in practice it is advisable to have at least twenty points when testing for normality. Even then, the recommended attitude to be adopted, when an approximately straight line is obtained, is that such data *could* arise from a normal population, and not that it *does*.

The examples shown may be considered rather wide deviations from a normal population, and indicate the limited sensitivity of probability paper to differentiate between distributions when relatively few points are available. One feature worth noting is the relatively high probability of the first point for the rectangular distribution compared wlth the more normal triangular one; for example, 12.5% for the rectangular, against 3.1% for the triangular. This reveals that the former points are coming from a rectangular type of distribution because quite a high proportion of the results are encountered for quite a small invasion into the edge of the data.

Ungrouped Data

The popularity of probability paper is due, in no small measure, to the ease with which random individual observations can be handled, although the common Ogive can be used in exactly the same way. (This knowledge may save a lot of time and labour when supplies of the special graph paper are not available.)

Let us consider the case of our first example, the heights of men to be fitted with suits. The Histogram of Fig. 1 has analysed the results of 619 individual observations, and the question is whether a sample of say ten individuals could provide the same information. Obviously not: but a fair idea of the distribution can be obtained if certain assumptions are justified.

The first assumption to be made is that each observation has the same weight. In other words that each of the ten observations represents about the same number of men in the parent population of 619. This means that each individual observation in the sample is assumed to represent 61.9 men or 10% of the total men in the parent population.

The second assumption is that each sample individual is typical of the 10% of the parent population it is assumed to represent; that is, that the majority of the 61.9 heights of men can be considered to be clustered round the sample value.

A third assumption (that, from previous experience or other considerations, the data can be expected to be normally distributed) is valuable, but not essential, as it means that using probability paper, the best straight line can be drawn through the points, and so allows fairly small samples to be used.

The second assumption is the key to the method of plotting, because it means that if the sample individuals are arranged in ascending order of magnitude, each in turn corresponds to the mean value of successive groups of population. In other words, each sample individual is assumed to fall near the centre of the group and therefore in the example chosen, has 5% of the population below it to one boundary of its group, and 5% of the population above it to the other boundary of the group. In approaching the first sample individual then, 5% of the total parent population would be encountered, and in reaching the second, 15% would have been passed, made up of the first group (10%) plus half of the next group (5%), and so on. Thus for ten individuals in the sample, they should be plotted on the Ogive, or on probability paper, at the following percentages of total observations:

5, 15, 25, 35, 45, 55, 65, 75, 85, 95%.

In general, if there are n observations in a sample, they should be spaced at $100/n$% and the first observation should occur at $100/2n$%. Example: Suppose the heights of ten men, taken at random, were as shown in Table II, and that it was known that a normal distribution could be expected. The proportion of men with heights between 65 in and 67 in is required.

TABLE II

	Inches									
Heights of ten men taken at random, and placed in ascending order	53	64	65	66	66	67	68	69	70	71
Plot on Probability Scale at the following percentages	5	15	25	35	45	55	65	75	85	95

The results are shown plotted on probability paper in Fig. 6, and from the best straight line, we have:

Proportion of men having heights less than 67 in = 52%

Proportion of men having heights less than 65 in = 25%

Proportion of men having heights between 67 in and 65 in = 27%

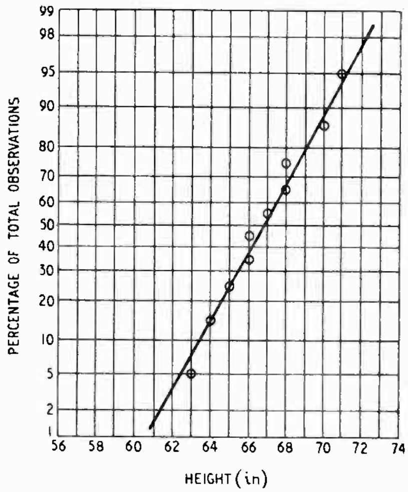


Fig. 6. *Ogive, plotted on Probability Paper, of a small sample of ten individual observations taken at random from the population of* 619 *heights of men shown in Figs.* 1 *and* 4.

The Normal Distribution

So much has been said about testing for normality, and whether or not individuals can be expected to follow a normal distribution, that a short discussion of its properties is appropriate at this stage.

As mentioned previously, the Normal Distribution is a conception of an ideal population of individuals, from the point of view of mathematical analysis. It is based on the assumption that deviations of individuals, from the mean value, obey three laws:

(1) The probability of small deviations from the mean value is greater than the probability of large deviations.

(2) The probability of a certain deviation above the mean value is equal to the probability of an equal deviation below the mean value.

(3) The probability of a "huge" deviation is very small indeed.

If we imagine a 'Normal' Histogram, the first law means that the central pillars tend to peak near the mean; the second law means that the histogram is symmetrical about the mean; and the third, that the columns vanish fairly quickly as the distance from the mean increases. If we now imagine the width of the histogram pillars to be made extremely small, and consequently the number of pillars extremely large, the tops of the pillars would follow a bell-shaped curve, as shown in Fig. 7. The mathematical law of this curve has been deduced from the postulations above, and a knowledge of this allows a great deal to be predicted about the probability of an individual falling between specified boundaries; i.e., into any specified Histogram group.

One of the most important deductions is that two parameters are sufficient to define and describe the distribution; and both parameters can be calculated without drawing the distribution, or conversely, obtained directly from the Histogram or Ogive, without calculation. These parameters are the 'Mean' and the 'Standard Deviation.'

Because the distribution is symmetrical, the Mean corresponds to the peak of the bell-shaped curve. The Standard Deviation is taken to be the distance from the Mean to the point of inflection of the curve, shown as sigma in Fig. 7, and both these points are easy to see on the Histogram.

As far as the Ogive is concerned, it has been calculated that, for a Normal Distribution, 34.2% of the total population can be expected between the mean and the standard deviation and, by symmetry, 50% of the population exists each side of the mean. This means that the standard deviation can be found by taking the difference in abscissa corresponding to ordinates of 50% and 15.8% (34.2% from the mean), or ordinates 50% and 84.2% as shown in Fig. 7.

The Standard Deviation is used as a yardstick for specifying the distribution of individuals in a normal population, and tables are published giving the proportions of population encountered between the mean and various deviations expressed as fractions or multiples of a standard deviation. Table III is a simplified version of one.

TABLE III

Deviation from Mean	Percentage of Total Population: Between Mean and Deviation (%)	Between ± Deviation (%)	Outside range of ± Deviation (%)
$\frac{2}{3}\sigma$ (approx.) ..	25	50	50
σ	34.2	68.3	31.7
2σ	47.7	95.4	4.6
3σ	49.85	99.7	0.3

For example, one can see that the chance of an individual observation falling outside the range $\pm 2\sigma$, is approximately one in twenty (4.6%). Likewise, the odds are about even that an observation would be within the range $\pm \frac{2}{3}\sigma$. (One should be very confident that the data is normally distributed, before venturing to predict for deviations greater than 3σ.)

Perhaps the most important of all deductions made by theoretical workers, concerning Normal Distributions, is that whatever the distribution of individuals in a parent population (rectangular, triangular, double humped, or any other shape), the distribution of the means of random samples drawn from that population, tends to be Normal, provided, of course, the parent population remains stable.

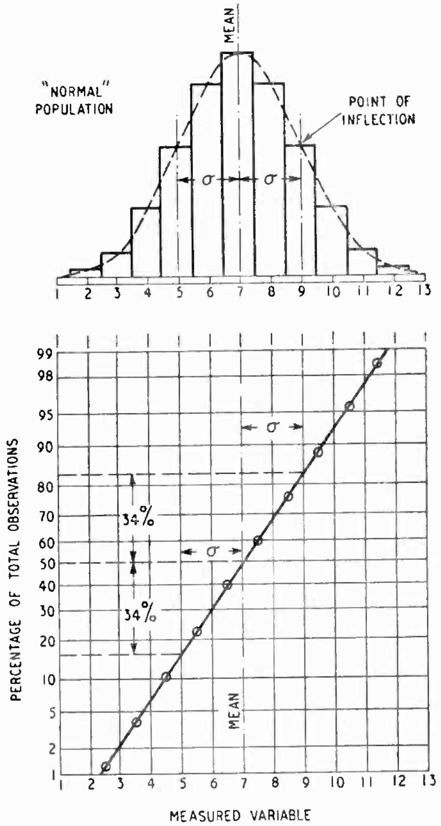


Fig. 7. *Histogram, and Probability Paper Ogive, of a 'Normal Distribution,' showing the method of recognizing the parameters 'Mean' and 'Standard Deviation'.*

This last comment is the key to one important application of Probability Paper, namely to test the stability of populations. Successive batches of twenty or more means of samples, should show roughly the same mean and standard deviation if the population is stable. This is the basis of Quality Control, where instability in the parent population of objects means that rejects will occur unless the drift is arrested. When a sample is obtained with a mean value deviating from the grand mean by a certain amount, one is justified in using Normal Distribution Tables to compute the probability that such a deviation could be expected by pure chance, and act accordingly. Notice, however, that it is not absolutely essential to understand Normal Distributions to operate a Quality Control system; one could conduct a 100% inspection at the beginning of production, when the process was operating satisfactorily, and construct an Ogive from the results. This could then be used to define the limits within which subsequent sample individuals should fall.

Remember that if one intends to use the means of samples of five individuals, the Ogive must be constructed from the means of groups of five individuals. In short, decide on the size of sample, and commence with 100% inspection. This simply means that the samples are successive groups of individuals. Calculate the mean of each sample and plot as a dot diagram. When the dot diagram takes on a definite shape, plot the Ogive. Decide now how many false alarms can be tolerated, say one in ten, and select limit values from the Ogive accordingly. In the example chosen, one is prepared to be given a false alarm once in ten times, in other words 10% of the total population will be allowed to fall outside the Ogive limit lines so that about once in ten times an observed mean will exceed the limits by pure chance. This means that the limits must be chosen to allow 5% of the population to occur below the lower limit, and 5% above the upper limit. Thus draw lines on the Ogive at 5% and 95% and read off the corresponding limits of the variable.

Take samples at reasonable intervals, and investigate as soon as any sample mean exceeds either of the limits. (If the sample means are plotted as a scatter diagram in time, and the limit lines are marked on the scatter diagram, it is easy to see when there is a drift in the process which will eventually cause rejects.)

This elementary system of Quality Control is the basis of the more refined methods, which are designed to economize in labour by taking advantage of other theoretical deductions and properties associated with Normal Distributions.

Conclusions

The author hopes that the reader will agree that the first conclusion is that a great deal of useful work can be done graphically, with Histograms, Ogives and Scatter Diagrams, without any theoretical knowledge of Normal Distributions, and the like. This is important because it encourages the use of efficient statistical methods among those who might be put off by a more theoretical approach, and provides a good background for the appreciation of the more advanced ideas, later on. The danger of a limited theoretical knowledge lies in the temptation to use apparently simple tools, which have been developed from very advanced theory, because then they are imperfectly understood, and erroneous conclusions may be drawn from the results.

Probability Paper is directly concerned with Normal Distributions and this implies that some knowledge of the theory of these distributions is desirable for its correct use. Generally the use of it will be justified only when some application of the properties of normal distributions is sought, for example, when used as a labour-saving device to avoid calculations, and economize in size of sample, when the data is known to be normally distributed. Or again, when testing the stability of populations, making use of the expected normal distribution of sample means.

The actual testing of data for normality should be regarded cautiously, and not seriously attempted with less than twenty points, preferably more. One should also have a clear idea whether a test for normality is justified. For example, in the problem of providing suits for men of various heights, it is absolute nonsense to make elaborate tests and extensive predictions concerning the normality of the heights, in order to avoid waste of materials, when the selection is ultimately influenced by personal preferences in colours, designs and cuts. An Ogive analysis would be adequate to get the proportions approximately right.

Likewise, as the author knows to his cost, it is advisable to check the data for stability of parent population before attempting any serious curve fitting.

The final conclusion drawn, is that Probability Paper is rather an insensitive tool for discriminating between distributions, and a large number of points are required before pronouncing judgment on the probable Parent population.

Acknowledgments

The author wishes to express his thanks to the Chief Scientist of the Ministry of Supply for permission to publish this article, and to those friends and colleagues who first introduced the writer to this most fascinating and useful subject.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Network Theorem

SIR,—Some 15 or 20 years ago, I noticed that the theorem stated here could be derived very simply from the basic equations which define the properties of a quadripole.

The derivation appeared to be so obvious that I presumed it to be well known. Experience over a long period has shown, however, that this is not the case and I feel that it should be published. It will doubtless draw the fire of experts in the field, but it may prove useful to the larger group who employ network theorems to shorten the labour of circuit analysis and design.

The theorem may be stated as follows :—

" In any linear, passive network in which the application of a p.d. E_A to a pair of terminals 'A' leads to the appearance of a p.d. E_B at another pair of terminals 'B,' the injection of current i_B at 'B' will lead to the appearance of a current i_A in a link strapping terminals 'A,' the ratio i_B/i_A being equal to the ratio E_A/E_B."

Put broadly this indicates that the ratio between the output open-circuit p.d. and the input p.d. is identically equal to the ratio of the output short-circuit current to the input current, the direction of transmission of power through the network being reversed in the second case.

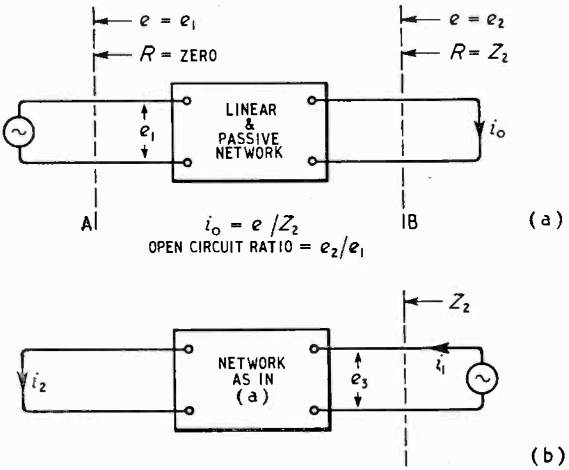


Fig. 1

The proof is simple. Referring to Fig. 1 it should be observed that Z_2, the output impedance in (a) is identical with the input impedance in (b) because terminals A are in both cases short-circuited, the generator in (a) being of zero internal impedance. By reciprocity $i_0/e_1 = i_2/e_3$, but $i_0 = e_2/Z_2$ and $e_3 = i_1 Z_2$; therefore, $e_2/(e_1 Z_2) = i_2/(i_1 Z_2)$, whence $e_2/e_1 = i_2/i_1$.

This theorem is of more than academic interest, for it has been applied directly to the design of new networks from known types. Moreover, it has practical applications. An example will illustrate this :—

Let us suppose it is required to measure, experimentally, the ratio between the input current to one branch of a network and the output current flowing in a second branch of the network. Let us suppose also that, as is often the case, it is impossible to place a current meter (or a resistance across which a p.d. could be measured) in the second branch ; then the theorem can be applied and the desired ratio derived by voltage measurements, power being applied at a break in the second branch and the output p.d. being measured across the first branch.

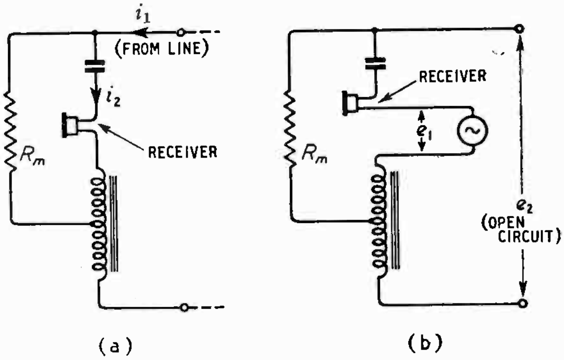


Fig. 2

An example of the experimental application is given in Fig. 2. Here the network on test is the Post Office Standard C.B. Telephone Instrument circuit. It is required to know the current i_2 in the receiver per unit of input line-current, i_1. To measure this by the p.d. across the receiver involves the measurement of the receiver impedance, and correction for changes in this as the frequency of i_1 is altered.

By the arrangement of (b) the voltage ratio e_2/e_1 is measured directly and this, by the theorem, is identical with the ratio i_2/i_1.

Southbourne,
Hants.

E. R. WIGAN.

Gain of Aerial Systems

SIR,—In the September 1949, issue of *Wireless Engineer*, Mr. D. A. Bell states that " the maximum gain of an aerial in which the effective phase varies gradually, by a total amount of approximately π from one edge of the aperture to the opposite one, is twice the gain of a uniformly illuminated aperture." The implication that an aperture of given size is twice as efficient when operating as an end-fire radiator as when operating as a broadside radiator, is of practical importance particularly since, with the development of radio lenses, it is becoming more easy to control both phase and amplitude across an aperture. Unfortunately, as will be shown below, this increase in gain is not realizable in practice and is no more than the normal increase in gain when a radiator is placed at the surface of a perfect reflector.

In calculating the polar diagram of an aperture by the usual Fourier Transform method, it is assumed that the field strength is zero everywhere in the plane of the aperture except actually within the aperture boundary ;

i.e., that the aperture is set in an infinite perfectly conducting plane. When the aperture is uniformly phased, the radiated field will be negligible in the vicinity of the aperture plane, outside the aperture, and it will make little difference to the final result whether the aperture lies in a conducting plane or not. The theoretical result, which assumes a conducting plane may, therefore, be applied to the practical case, when no such plane exists.

When, however, the aperture is so energized that an end-fire beam is radiated, the field strength in the vicinity of the conducting plane is no longer zero, at any rate in the direction of the beam. It is, therefore, not possible to realize the theoretical result from the Fourier transform unless the radiating aperture is assumed to be situated at the surface of a perfectly conducting plane. Since the gain of any aerial system is doubled by placing the system on the surface of a perfectly conducting plane, it follows that there is no reason to suppose that a practical end-fire aperture, which at best, of course, may only be surrounded by a small conducting area, is any more efficient than a broadside aperture. In other words, the factor two arises from the presence of an infinite conducting plane and not because one aperture is an end-fire radiator and the other a broadside radiator.

It will be seen from the above that there is no question of an end-fire aperture transgressing the laws governing optical resolution. Similarly, there is no possibility of obtaining a narrower pulse from a given frequency bandwidth as is suggested in Appendix II (ii).

R.R.D.E., Malvern. J. BROWN.

The Intrinsic Impedance of Space

SIR,—The paradox dealt with in your September Editorial raises an important issue concerning 'rationalization' of a system of units.

As is well known, the relation $\int_0 Hds = 4\pi i$ which is valid for an unrationalized system, is replaced by $\int_0 Hds = i$ in a rationalized system. One example of the result of this is that we have the following two expressions for the magnetic-field intensity in a long solenoid

Rationalized	*Unrationalized*
$H = \frac{ni}{l}$	$H = \frac{4\pi ni}{l}$

For a given physical system (i.e., for given numerical values of n, i and l) we shall thus have two different values for H, according to whether we are using a rationalized or an unrationalized system. Only two explanations of this are possible: first, that what we have called H in a rationalized system is really a different concept from what we have called H in an unrationalized system; or, secondly, that H is the same concept but is expressed in different units in the two systems. The latter explanation is widely accepted, but is it correct?

The laws of physics (e.g., Ohm's Law) were expressed by their discoverers as proportionalities. The units of the quantities involved were then so chosen as to make the constant of proportionality equal to unity or (less often) some other simple number. The algebraic equations thus define and inter-relate the units. Even without reference to any physical system we may legitimately write 1 volt equal to 1 amp times 1 ohm (or, volt = amp × ohm). This justifies the valuable procedure of substituting both figures *and units* into formulae and carrying out the operations of multiplication and division not only with the figures (to determine the numeric part of the result) but also with the units (to determine the units in which the result is expressed).

Now consider a solenoid of length one metre, having 1000 turns and carrying a current of 1 A. With the unrationalized system we have H equal to $4\pi \times 1000$ A/metre while with the rationalized system we have H equal to 1000 A/metre. The units in the two cases are the same as determined by the unit-substitution method. We, therefore, conclude (with distressingly small support from our friends) that the concept of magnetic-field intensity in a rationalized system is different from the concept which goes by the same name in an unrationalized system, though both are expressed in the same units. It is as though two persons were speaking of the 'size' of a circle, one of them meaning the radius and the other the circumference.

In both a rationalized and an unrationalized system the intrinsic impedance of a medium may be defined as the ratio of electric-field intensity to magnetic-field intensity in plane waves travelling through the medium. Electric-field intensity is the same concept in both systems, but magnetic-field intensity is not. Thus the concept of intrinsic impedance in a rationalized system differs from the concept which goes by the same name in an unrationalized system, and it is not in the least surprising that, for free space, one of these concepts is represented by 377 ohms whereas the other is represented by 30 ohms.

EMRYS WILLIAMS.
University College of North Wales, Bangor.

[There is one very satisfactory statement in the above letter, and that is that the writer gets small support from his friends; it would be distressing were it otherwise.

If, after calculating any impedance by juggling with electric- and magnetic-field intensities in rationalized and unrationalized systems of units, results are obtained which, when converted into ohms, give two different values of the same impedance, then something has gone wrong with the juggling. Any differences in the units of H should be ironed out in the final conversion into ohms, since amperes, volts and ohms and the impedance in ohms are the same in all systems.

A solenoid of length 1 metre, having 1000 turns and carrying a current of 1 ampere has a magnetizing force H of 1000 amperes per metre, but if one takes as the unit of H, not one ampere per metre but $1/(4\pi)$ of an ampere per metre, then $H = 4\pi \times 1000$ units but not $4\pi \times 1000$ amperes per metre. If one measured a length in inch units and obtained 1000, and then measured the same length in units of $1/(4\pi)$ inch, one would obtain $4\pi \times 1000$, but one would not be justified in writing $4\pi \times 1000$ inches. Similarly, one cannot write amperes per metre after any number, irrespective of the actual units employed, as the writer appears to do.

The persons giving different values for the size of a circle should be asked to define 'size.' There is no need to define H: that is done in many books, but it is essential to state the units employed. G. W.O. H.]

Secondary Emission Tubes in Wideband Amplifiers

SIR,—Secondary emission tubes such as the Philips EFP60 have obvious application in amplifiers for a wide band of frequencies by virtue of their high mutual conductance and low output capacitance. It seems, however, worth while to draw attention to a further facility, which is not generally realized, although it has been known for many years. This is the possibility of obtaining a push-pull output from a single tube, by having loads in series with both dynode and anode. Since the anode current exceeds the (negative) dynode current by a factor of 1.3, with mutual conductances in the same ratio, if exactly equal and opposite outputs are required then the respective loads must be in the inverse ratio.

A number of such tubes may be used to provide push-pull output from a distributed amplifier.[1] The control grids would be fed from one delay line in the normal manner, while one series of inductances between the anodes and another series between the dynodes would form the two output lines. For exact push-pull the characteristic impedances of the two lines must be in the inverse ratio of the corresponding mutual conductances, and further their velocities of propagation must both be equal to that of the grid line. To satisfy both these requirements the ratio of anode to dynode capacitance must be the same as the mutual conductance ratio; in the case of the EFP60, this will involve the addition of external capacitance to the dynode.

Data are not yet available for the grid input conductance of the EFP60, but from structural considerations it is probably the same as the EF50, namely 1000 μmhos at 100 Mc/s, as compared with 130 μmhos for the 6AK5 at the same frequency. However this higher input conductance will not be a serious disadvantage except at frequencies above 100 Mc/s, particularly since the larger current change available will permit fewer tubes to be used for a given output.

Preliminary results indicate that to obtain a push-pull output of 160 volts excursion with bandwidth of 50 kc/s to 100 Mc/s, 14 6AK5 tubes with a total voltage gain of 30 could be replaced by four EFP60 with a voltage gain of 65. This is for use as a pulse amplifier with low steady current in the EFP60 in order to keep within wattage limitations.

N. F. Moody,
Chalk River Laboratory, G. J. R. McLusky.
National Research Council of Canada.

[1] Ginzton, et al., *Proc. Inst. Radio Engrs*, August 1948, Vol. 36, No. 8.

BOOK REVIEWS

Industrial High Frequency Electric Power

By E. May, B.Sc., M.I.E.E. Pp. 355 + xi with 208 illustrations. Chapman & Hall, 37 Essex St., London, W.C.2. Price 32s.

This book, which is intended to serve as an introduction to the field of industrial high-frequency technology, is based on a course of lectures given at Birmingham Central Technical College by the Author, who is the chief electrical engineer to Birlec Ltd. It starts at the beginning and Chapter 1 is a summary of basic circuit theory, with special reference to alternating currents. The book is largely concerned with the rapid production of heat in the place where it is required by means of high-frequency magnetic and electric fields. Chapter 2 deals with arc and spark oscillators and inverters, Chapter 3 with h.f. alternators, and Chapter 4 with triode valves as h.f. power generators. Chapter 5 deals with class B and C operation of power amplifiers with tuned loads. In Chapter 6 the h.f. current is applied to magnetic induction heating, and in Chapter 7 to dielectric heating; the latter includes such things as stub matching for tuning feeds. Chapter 8 deals with auxiliary equipment and h.f. measurements, and Chapter 9 with industrial applications and operating problems. A very extensive list of references is given at the end.

We note that the author always writes ampère, apparently unaware that the accent was dropped many years ago, like the last two letters of Faraday's name and the last letter of Volta's. He also writes consistently KC/S, although the person who made the diagrams knew better and has kc/s. KVAR in the text becomes kVar in the diagram. There is an American flavour about the chapter on valves with its plate currents and plate voltages, but these are minor details. The description is always very clear, the diagrams are excellent, and the book can be unreservedly recommended to anyone interested in this subject of growing importance.

G. W. O. H.

Permanent Magnets

By F. G. Spreadbury, A.M.Inst.B.E. Pp. 280 + viii. Sir Isaac Pitman & Sons, Ltd., Parker Street, London, W.C.2. Price 35s.

During recent years technical and commercial developments of permanent magnets have gone ahead so rapidly as to outstrip the literature of the subject, so such an up-to-date and comprehensive work as this is particularly welcome. The author's output having been so considerable and by no means concentrated in one narrow field, one might be prepared for a rather superficial treatment of the present subject. Any such fears are proved by the event to be unjustified, however, and he has gone almost to extreme lengths in leaving no statement, however elementary, without its mathematical demonstration. That is not to say that the book is acceptable only to readers with a mathematical bias; where appropriate the treatment is descriptive and practical, and throughout it is concise.

Chapter 1 is a review of the fundamental principles of magnetism with special attention to those used in the calculation of permanent-magnet circuits. It is perhaps a pity that the author did not take this opportunity to change over to the internationally-adopted m.k.s. system of units. One or two non-standard symbols were noted in this chapter (σ for resistivity and ρ for reluctance), and the use of dl (not even δl) to denote the length of the gap cannot be commended. Data on permanent-magnet materials are given in Chapter 2, including such recent developments as bakelite-moulded magnets. A concise account of the theories of magnetism, past and present, follows in Chapter 3. An important aspect of permanent-magnet circuit design—magnetic leakage—has Chapter 4 to itself. Then follow: Chapter 5 on applications of permanent magnets; Chapter 6 on magnetic design; Chapter 7 on magnetic measurements and instruments; Chapter 8 on magnetization and magnetizers; and Chapter 9 on demagnetization and demagnetizers. The last two include some examples of the author's own design.

In Chapter 5, special interest will no doubt be taken in the section devoted to loudspeakers. The theoretical treatment of the factors involved hardly hints at the complexity wrapped up in the mechanico-acoustical properties of the cone, however.

The book is not only well written but excellently produced, and few misprints were noticed. M. G. S.

Einführung in die Theorie der Spulen und Übertrager mit Eisenblechkernen

By Richard Feldtkeller. S. Hirzel Verlag, Stuttgart. In 3 parts: Part I Spulen. Pp. 190 with 120 illustrations. 10.50 Marks. Part II Übertrager. Pp. 130, with 83 illustrations. 8 Marks. Part III Berechnungsunterlagen. Pp. 65, with 70 illustrations. 4 Marks.

This "Introduction to the Theory of Coils and Transformers with laminated-iron cores" is one of a series of monographs dealing with telegraphy and telephony. This is a second edition, revised and extended; the first edition was published in 1944. It deals with the subject from the point of view of the telecommunication engineer and, although it is very mathematical in parts, it is at the same time very practical, and the problems of design are always kept in view. The first volume deals with coils, the second with transformers, and the third consists of graphs and tables to be used in detailed calculation and design.

The author is a professor at Stuttgart Hochschule and Director of the Institute for 'Nachrichten Technik.'

An iron-cored coil is simple enough if one assumes a constant permeability and neglects hysteresis and eddy currents, but the problem becomes very complex when these things are taken into account. Hysteresis necessitates the introduction of complex permeability, and this is further complicated by the effect of eddy currents in the iron laminations on the flux distribution. Further problems are introduced if the alternating current is superposed on a direct current, and if air-gaps are present. All these points are discussed very fully.

In the second volume these complexities are put aside and the coupling transformer is dealt with on more orthodox lines. Load matching, effect of magnetic leakage, effect of self-capacitance of coils, resonance conditions, width of frequency band and such matters are all discussed and the results set out mathematically and graphically.

The book incorporates a large amount of experimental data obtained by measurements on various types of laminations, and this is used in the design data.

This book is undoubtedly a valuable addition to the available information on this subject.

G. W. O. H.

Tables of Higher Functions

By Jahnke and Emde. Fourth Revised Edition. Pp. 300 + xii, with 177 illustrations. B. G. Teubner Verlagsgesellschaft, Leipzig. Price 12 marks.

The fourth edition of this well-known work has been revised by Professor Emde. It was printed and being bound for issue in 1944, but the whole edition was destroyed by bomb and fire damage, and has had to be reprinted. Among the additional matter in this new edition are an extension of the table of the error integral, a table of functions of the parabolic cylinder, also of Laguerre function, of spherical harmonics of the second kind, and of the incomplete Anger and Weber functions. One of the tables of Bessel functions has been replaced by a more correct table, and formulae and figures are given for the use of Debye's convergently beginning series for the Bessel functions with complex argument and order. A number of corrections have been made and supplements added.

As in the earlier editions, the book is bilingual, everything being given in both German and English.

G. W. O. H.

Tables of Elementary Functions

By Fritz Emde. Second Edition. Pp. 181 + xii with 83 illustrations. B. G. Teubner Verlagsgesellschaft, Leipzig. Price 12 Marks.

Like the fourth edition of the Tables of Higher Functions, this would have been published in 1944, but the whole edition was destroyed at the bookbinders by war damage and has had to be reprinted.

This is almost a reproduction of the first edition of 1940. Before that date all the Tables, both elementary and higher, were in one volume but Professor Emde wisely decided to divide it into two parts, Elementary and Higher, and the first 78 pages of the old Jahnke-Emde volume were removed in 1940, and made the basis of this new volume. It goes as far as hyperbolic functions of a complex variable, Chebyshev's polynomials, and Planck's radiation function, but does not contain such things as elliptic and Bessel functions; they are dealt with in the other volume.

Everything is given in both German and English. In addition to the Tables, there is much descriptive matter showing the development of the formulae, and, wherever they are of any help, graphs and relief diagrams are given.

G. W. O. H.

International Radio Tube Encyclopedia

Edited by Bernard R. Babani. Pp. 410 + lxxxiv, Bernards Ltd., The Grampians, Western Gate, London. W.6. Price 42s.

This book gives characteristics of some 15,000 valves. The data is presented in tabular form and the valves are grouped under the headings: Radio Receiving Tubes; Triode Transmitting Tubes; Transmitting Tetrodes; Pentodes and Other Transmitting Tubes having more than Five Elements; Rectifiers; Thyratrons; Regulator and Control Tubes; Tuning Indicators; Cathode-Ray Tubes; Photo Tubes; and Rare Tubes and Their Equivalents. Under each heading the valves are arranged in alphabetical and numerical order.

Given its type number, it is easy to turn up the characteristics of any valve. The book, however, does not enable the inverse operation to be carried out; one cannot determine from it, without a prohibitively laborious search, what valves exist with given characteristics. The data includes the usual characteristics, such as, typical operating voltages and currents, a.c. resistance, amplification factor, mutual conductance, load resistance, cathode-bias resistance and base connections. In addition to British, American and Continental commercial types, British and American Service valves are included.

The base connections are given in tabular form with the other data and for identification purposes there are several pages of base drawings. These could be improved, for the differences between different types are not always adequately shown. The base designations, too, are not always the standard ones; thus, the UX bases do not appear as such but have names like USG7, USM7B, etc.

W.T.C.

Radio Engineering. (Volume 2).

By E. K. Sandeman, Ph.D., M.I.E.E. Pp. 579+xxi. Chapman & Hall, Ltd., 37, Essex St., London, W.C.2. Price 40s.

This second half of the work retains in full measure the inconsistencies of style and presentation of the first, while the material presented is less easy to commend. The subject matter includes interference and noise, receivers, measuring equipment, equalizer design, feedback, network theory, and filters. There are in addition 104 pages of appendices, including a large collection of formulae and information on a variety of subjects, such as tuning methods, special network problem, Fourier analysis, etc. A bibliography occupies 75 pages. The index to the whole work, given in Vol. 1, is repeated, and so is the whole preface, in which the author claims the book to be suitable not only for experienced engineers but for complete novices. Although a period of nearly two years has elapsed between publication of the volumes, there are no corrections to Vol. 1.

As before, the applicability of the test to radio engineering outside the B.B.C. is purely coincidental. In fact, the section on the testing of receivers has the appearance of being an unedited B.B.C. departmental order. Apart from this section, the treatment of receivers extends to only 40 pages being, as the author explains, largely from the viewpoint of the user. It is a pity he did not adhere more strictly to his intention to keep off receiver design, for it is clearly not his metier. For example, after mentioning the types of aerial coupling primary that resonate respectively above and below the working waveband, he states that a primary resonating within the waveband "has application only to special cases of receivers designed to receive a particular frequency, but is described immediately below because it is the most simple method to apply and one which will probably be of most use to engineers other than designers of receiving sets"! He then proceeds to describe it in relation to a receiver tuning from 200 to 600 metres, remarking finally that because its efficiency is not well maintained over this waveband the other methods (not described in detail) are preferred.

The circuit taken to illustrate simple a.v.c. as distinct from delayed a.v.c. is unfortunate for the purpose, since (apparently unperceived by the author) an initial negative bias is applied to the a.v.c. anode. In describing another circuit the author admits that it is not clear why two of the components cannot be eliminated. Surely, however, he ought to confine his explanations to circuits about which he is quite clear. Elsewhere, in connection with r.f. amplification, one is given the impression that the use of pentodes to render neutralization unnecessary is quite a recent development, and so is the replacement of thermocouples in a.f. meters by copper-oxide rectifiers.

A detailed treatment of diode voltmeters for measuring high r.f. voltages in transmitters is an original feature of the chapter on measuring equipment, and it is a pity that it is not more lucid. The sections on a.f. bridges are poor, and made worse by misprints. Some attention is given to the measurements of harmonics, but intermodulation products are so studiously ignored that even the General Radio Co's Wave Analyzer is called a "Harmonic Analyser."

As in Vol. 1, the author shows most enthusiasm in dealing with circuit calculations. It is doubtful, however, whether his readers will share his enthusiasm for some of his methods. After quite a helpful section on general methods of network analysis, he launches a 'simplified' system of dealing with four-terminal networks. To avoid using hyperbolic functions, of which (he says) no satisfactory tables exist, he invents and defines a new set of functions, denoted by 'sins,' 'coss,' 'tans' etc. The definition is the last one sees of them, until their reappearance among the other formulae in the first appendix. The terms 'unsymmetrical,' 'asymmetrical,' and 'dissymetrical' are used, but their distinctions are not made clear. After an alternative solution of the symmetrical bridged-T by matrices, there follow 33 pages on the method of matrices, which will be of special nterest owing to the scarcity of such information in English.

In contrast to all this theory, and not explicitly related to it, the chapter on filters is a set of design rules, the nucleus being a number of tables reproduced from American books.

The author frankly disclaims that the extensive bibliography is complete or systematic, and one cannot but concur with him. For example, while there are 91 references on aerials and 45 on noise there are 2 on distortion, one of them being "The Buzz Effect in Pentodes" from the June 1938 issue of this journal. There are no references to valve voltmeters, although the treatment in the book itself is confined almost exclusively to the one special type mentioned above.

The lack of care taken to achieve clarity in exposition is revealed also in details such as consistency of symbols. In the chapter on feedback in amplifiers, the symbol u is used not only with its accepted meaning but also with two other meanings. The use of L to denote impedance is another disconcerting peculiarity. When an abbreviation such as 'i.f.' has been adopted, it is difficult to see why it should be considered necessary in addition to write out the word 'frequency' in full, as is consistently done. Within a few pages, 'c/s' 'cycles' and 'p:s' are used apparently to mean the same thing. 'Schottky,' 'Schroteffekt,' and 'Lissajous' are mis-spelt.

Unfortunately these are only a few of the examples that could be quoted, and it is to be hoped that before being reissued the text will be carefully revised.

M. G. S.

Fundamentals of Radio-Valve Technique

By J. DEKETH. Pp. 535 + xxii. Cleaver-Hume Press Ltd., 42a, South Audley St., London, W.1. Price 35s.

This book is the first of a series dealing with valves and their applications and is published by N. V. Philips' Gloeilampenfabrieken, Eindhoven.

It opens with chapters describing the basic principles of the action of the valve, the physical concepts of electrons and electric currents, the behaviour of electrons in electrostatic and magnetic fields, and electron emission, both primary and secondary. The components, construction and manufacture of valves are then treated. All this occupies some 100 pages.

The functions of valves, their symbols in circuit diagrams and their classification are next dealt with. Valve characteristics are then given a chapter and followed by an explanation of how the various grids affect them. Inter-electrode capacitance, the curvature of the characteristics and its representation by an exponential series are covered.

The remainder, rather more than one-half of it, is devoted to valves in circuits. Amplifiers, rectifiers and oscillators are described as well as frequency-changers, a.g.c., negative feedback, noise, s.w. properties of valves, microphony, etc.

The book concludes with Appendices covering units, elementary d.c., a.c. and oscillatory-circuit formulae, universal curves of band-pass filters, skin effect, calculation of inductance and capacitance and various tables and graphs.

An enormous amount of material is included in the book. On the whole the treatment is fairly elementary and involves little more than simple mathematics, and not a great deal of that. It is profusely illustrated with circuits, graphs and valve characteristics and there are many exceedingly good diagrams illustrating equipotential lines in valves.

As might be expected from the origin of the book all the valves quoted as examples are Philips' types. This is not a serious drawback but the book does suffer slightly from the fact that the techniques described are also largely confined to Philips' practice.

Symbols and nomenclature often follow Continental rather than British practice. The translation is exceedingly good, although some peculiarities of expression have crept in.

The book is more suited to the student than the engineer, but the latter will undoubtedly find parts of it useful for reference.

W. T. C.

Laboratory and Workshop Notes

Compiled by RUTH LANG, Ph.D., A.Inst.P. Pp. 272 + xii. Edward Arnold & Co., 41, Maddox St., London, W.1. Price 21s.

Atmospheric Turbulence

By O. G. SUTTON, B.Sc. Pp. 107 + vii. Methuen & Co., Ltd., 36, Essex St., Strand, London, W.C.2. Price 6s.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specificatioes obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

625 291.—A ribbon microphone wherein the pole pieces comprise stacks of laminations shaped to provide air passages to give a directional effect.

The B.T.H. Co., Ltd., and W. A. Bocock. Application date November 7th, 1946.

625 563.—Electromagnetic disc recording or reproduction of sound using a magnetizable needle within a coil with a closed magnetizable casing surrounding them, with a small hole for the needle.

The British Thomson-Houston Company Ltd. Convention date (U.S.A.) January 9th, 1945.

625 744.—A rainproofing screen for a microphone of closely woven material with filaments 0.0075 in or less; e.g., umbrella silk or stainless steel.

Standard Telephones and Cables Ltd., W. D. Cragg and M. L. Gayford. Application date October 2nd, 1946.

626 912.—A sound amplification or loud-speaking telephone using two channels of different phase displacement characteristics, the output to the transmitter being taken therefrom in continually changing manner to promote stability.

Standard Telephones and Cables Ltd., S. S. Hill and W. D. Cragg. Application date January 14th, 1947.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

625 034.—A p.p.i. radar display wherein echo signals beat with a reference oscillation to give a 'striated' display, the slope of which depends on rate of approach by Doppler effect.

W. S. Elliott. Application date May 31st, 1946.

625 734.—The indication on aircraft of the positions of beacons and an obstacle is obtained by the synchronized rotation of slotted discs in front of selectively operable ring-shaped discharge lamps.

Standard Telephones and Cables Ltd. (assignees of H. G. Busignies). Convention date (U.S.A.) June 30th, 1945.

625 891.—Blind-landing system in which the aircraft equipment is automatically adjusted according to distance for guiding the craft on a different course from a transmitted beam.

A. C. Cossor Ltd. and L. H. Bedford. Application date June 28th, 1945.

RECEIVING CIRCUITS AND APPARATUS

(*See also under Television*)

625 121.—Frequency changer for 600-Mc/s receiver comprising a double diode or double triode using two sets of tuned transmission lines to which the signal and the local oscillator are coupled.

Sylvania Electric Products Inc. Convention date (U.S.A.) February 14th, 1942.

625 258.—'Exalted-carrier' receiver having two i.f. channels, one of which provides a frequency-controlling potential for a local oscillator and the other is a demodulation channel.

Marconi's Wireless Telegraph Co. Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) February 14th, 1945.

625 358.—A frequency changer especially for radio-link repeater stations, with two local oscillators to permit amplification at a low i.f. and re-transmission at a slightly different radio frequency from the received frequency.

The General Electric Company Ltd. and R. J. Clayton. Application date June 24th, 1947.

625 436.—Receiver tuning indicator comprising parallel transparent strips each with an end-illuminating lamp at one end.

The General Electric Company Ltd., F. N. Garthwaite, E. Wilson and C. W. M. Read. Application date August 7th, 1946.

625 449.—Receiver for phase or frequency-modulated signals in which false tuning is avoided by a muting circuit operative except when tuned to the mid-band point.

Marconi's Wireless Telegraph Company Ltd. (assignees of A. Wright). Convention date (U.S.A.) November 6th, 1945.

625 908.—Multi-range oscillator for a superheterodyne receiver using a multi-electrode valve to provide two independently operable oscillator circuits, the two circuits having different frequency ranges.

Radio Corporation of America. Convention date (U.S.A.) August 1st, 1945.

625 968.—Permeability-tuned circuit having primary and secondary windings and a movable core, the secondary circuit being tuned by said core and the primary winding having a shunt capacitor.

The British Thomson-Houston Company Limited. Convention date (U.S.A.) January 19th, 1945.

RECEIVING CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

624 896.—A viewing unit for use remotely from a television receiver in which the c.r.t. is positioned face downward and viewed through an adjustable mirror.

H. Ibbotson. Application date February 20th, 1947.

624 983.—Colour television in which four colour component images are sequentially scanned, transmitted, reproduced and recombined by optical means.

G. E. Sleeper Jr. Convention dates (U.S.A.) February 5th and August 30th, 1943.

SUBSIDIARY APPARATUS AND MATERIALS

623 029.—Measuring wave power in a hollow guide by collectors positioned to collect in-phase voltages and anti-phase voltages proportional to electric intensity and to magnetic intensity respectively, and feeding square-law rectifiers.

Standard Telephones and Cables Ltd. (assignees of Le Material Telephonique, Société Anonyme). Convention date (France) June 30th, 1943.

624 257.—Piezo-electric crystal mounting wherein the crystal is in an evacuated bulb and has an electrode coating engaged by curved resilient conducting wires.

Western Electric Co. Inc. Convention date (U.S.A.) June 5th, 1940.