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The *Q*-factor of Single-layer Coils

AN interesting article on this subject by Professor Friedrich Benz appeared in a recent number of the Austrian Journal *Elektrotechnik und Maschinenbau* (January 1949, p. 7). When calculating the inductance of single-layer coils one usually consults tables or curves giving Nagaoka's calculated values for coils of different ratios of diameter to length. These are based on the assumption that the coil is equivalent to a thin cylinder. This method is not very convenient when calculating the *Q*-factor of coils and its dependence on the shape of the coil. If the coil of T turns is very long the formula

$$L = \pi^2 T^2 D^2 / l \times 10^{-9} \text{ H}$$

is approximately correct where D is the diameter and l the length in centimetres, although even if $l = 10D$ the error is still $+ 4.5$ per cent. For very short coils the formula for a narrow band, viz.

$$L = 2\pi T^2 D(\log_e 4D/l - 0.5)10^{-9} \text{ H}$$

can be used. If $l = 0.3D$ the error is $- 1.5$ per cent.

Benz finds that by an empirical modification of the theoretical formula for a long coil, a formula is obtained which gives accurate results for coils of the dimensions usually employed.

The formula is

$$L = \frac{\pi^2 T^2 D^2 \, 10^{-9}}{l(1 + 0.45 \, D/l)} \text{ H}$$

For values of l/D between 0.4 and infinity the error is less than 1 per cent; for $l/D = 0.3$ the error is $- 1.3$ per cent and for $l/D = 0.2$ it is $- 4$ per cent. For most practical purposes this makes Nagaoka's tables and curves unnecessary. This formula is not new, however, but comes of age this year, for it was given by H. Wheeler in a slightly different form in *Proc. Inst. Radio Engrs*, October 1928, p. 1398. According to Wheeler, Professor Hazeltine had, some years before, established the empirical formula

$$L = \frac{0.8 \, a^2 T^2}{6a + 9l + 10c} \mu\text{H}$$

for a coil of the type shown in Fig. 1 (all dimensions in inches). This is approximately correct if the shape does not depart too much from that shown in the figure. This prompted Wheeler to find a similar empirical formula for a thin cylindrical coil, and his formula was

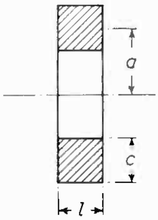

Fig. 1

$$L = \frac{a^2 T^2}{9a + 10l} \mu\text{H}$$

Here again the dimensions are in inches, and Wheeler stated that the formula was correct to 1 per cent if $l > 0.8a$. He also stated that for values of l between $0.2a$ and a it was more accurate to use the formula

$$L = \frac{a^2 T^2}{8a + 11l} \mu\text{H}$$

For $a = l$ both formulae obviously give the same result; if $a/l = 2.5$ the latter gives a result 5 per cent greater than the former and therefore within 1 per cent of the correct value.

If the dimensions are expressed in centimetres and D substituted for $2a$ the former formula becomes

$$L = \frac{D^2 T^2}{2.54 \times 40l(1 + 0.45D/l)} \mu\text{H}$$

which, since $\pi^2 \times 2.54 \times 40$ is almost exactly 1000, is the same formula as that used by Benz. It is also given by Sturley on p. 129 of Part I of his "Radio Receiver Design".

Benz draws attention to the letter from Callendar[1] discussing the experimental results obtained by Medhurst.[2] Callender pointed out that if $l > D/2$ Medhurst's results could be expressed within a few per cent by the formula

$$Q = 0.15\sqrt{f}\Big/\left(\frac{2}{D} + \frac{1}{l}\right)$$

which can be written

$$Q = 0.075\,\frac{D\sqrt{f}}{1 + 0.5D/l} = 0.075\,\frac{(Dl)\sqrt{f}}{l + 0.5D}$$

For a given value of Dl this gives a maximum value of Q when $l = 0.5D$. This assumes that the frequency is high and that the ratio of wire diameter to wire spacing approximates to the optimum value.

For straight round wire carrying current at such a frequency that the penetration is small, the ratio of the high-frequency resistance R_f to the steady-current resistance R_0 is[3]

$$R_f/R_0 = \pi r\sqrt{\frac{f\mu}{10^9\rho}} + 0.25$$

At very high frequencies the second term can be neglected and for copper wire of d-mm diameter we have

$$R_f/R_0 = 3.8 \cdot 10^{-3}\, d\sqrt{f}$$

from which

$$R_f = 2.57 \cdot 10^{-6}\, TD\sqrt{f}/d$$

By substituting these values of L and R_f in the formula $Q = \omega L/R_f$ and putting F_w for the winding factor d/a where a is the spacing of the turns we obtain the formula

$$Q = 0.24\,F_w\,\frac{D\sqrt{f}}{1 + 0.45\,D/l}$$

but a further factor F_s must be introduced to allow for the fact that the current is now confined to the inner surface of the wire. The value of F_s will depend on the winding factor F_w. If the turns are nearly touching, so that F_w is nearly unity, the current may be assumed to be confined to about a third of the circumference of the wire, whereas if $F_w = 0.7$ it will be confined to about a half of the circumference. We thus obtain the formula

$$Q = 0.24\,F_w\,F_s\,\frac{D\sqrt{f}}{1 + 0.45\,D/l}$$

Putting $F_s = 0.3$ for $F_w = 1$ and $F_s = 0.45$ for $F_w = 0.7$, an approximation can be made for other values of F_w. For given values of D, l and f the quality of the coil depends on the product $F_w\,F_s$, and the maximum value of this occurs when F_w is about 0.7 Medhurst found that for long coils the best value of F_w was between 0.6 and 0.8. Putting $F_w = 0.7$ and $F_s = 0.45$ in the above formula gives

$$Q = 0.075\,\frac{D\sqrt{f}}{1 + 0.45\,D/l}$$

which is almost exactly the same as Callendar's formula based on Medhurst's measurements.

Another matter discussed by Benz is the effect on the Q-factor of dielectric losses and the capacitance of the coil. He points out that the dielectric losses increase as the third power of the frequency, and that as the resonant frequency of the coil is approached Q decreases, even in the absence of dielectric losses, due to the coil-capacitance charging current. He recommends that the operating frequency should not exceed a fifth of the resonant frequency of the coil if it is desired to maintain a high Q value.

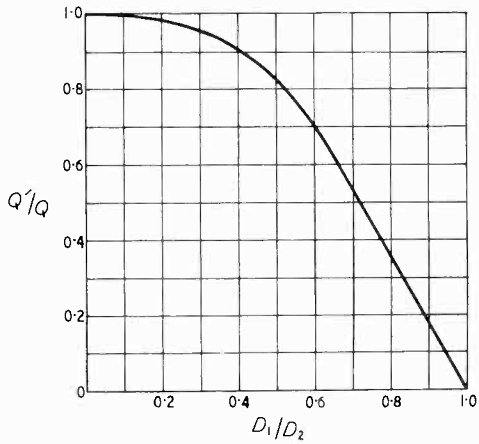

Fig. 2

The effect of screening cans on the Q-factors of coils is also discussed. This is a subject to which we have in the past devoted several Editorial articles.[4] One cannot expect to obtain more than a rough approximation by considering the screening can as a short-circuited secondary coupled to the coil. The apparent impedance of a coil $L_1\,R_1$ coupled to a short-circuited coil $L_2\,R_2$ is $R_1 + R_2k^2L_1/L_2 + j\omega L_1(1 - k^2)$ where k is the coupling coefficient and $R_2 \ll \omega L_2$. Hence the resistance is increased in the ratio $\left(1 + k^2\frac{R_2L_1}{R_1L_2}\right)$ and the inductance decreased in the ratio $(1 - k^2)$; this will cause Q to be decreased to Q' where

$$Q' = Q(1 - k^2)\Big/\left(1 + k^2\frac{R_2L_1}{R_1L_2}\right)$$

If the coil be regarded as a closely-wound helix of flat strip, and the can as a single short-circuited turn of the same material and therefore with the same penetration,

$$R_1/R_2 = D_1l_2T^2/D_2l_1$$

Assuming the coil and can to have the same ratio D/l

$$L_1/L_2 = \frac{D_1^2 l_2 T^2}{D_2^2 l_1} = \frac{D_1 T^2}{D_2}$$

and $Q' = Q(1 - k^2)/(1 + k^2 D_1/D_2)$

If a steady current be assumed to flow around the can the ratio M/L_2 of the linkages will be $D_1^2 T/D_2^2$. Hence

$$k^2 = M^2/L_1 L_2 = \left(\frac{M}{L_2}\right)^2 \times \frac{L_2}{L_1} = \left(\frac{D_1}{D_2}\right)^3$$

and $Q' = Q\{1 - (D_1/D_2)^3\}/\{1 + (D_1/D_2)^4\}$

This gives the result plotted in Fig. 2, which shows the reduction of Q for different values of the ratio of the diameter of the coil to that of the screening can.

G. W. O. H.

REFERENCES

[1] *Wireless Engineer*, June 1947, Vol. 24, p.185.
[2] *Wireless Engineer*, February and March 1947, Vol. 21, pp. 35, 80.
[3] Howe. *J. Instn elect. Engrs*, 1916, Vol. 54, p.473.
[4] *Wireless Engineer*, March and July 1934, also December 1940.

Some Radio Unpleasantries in the U.S.A.

In the *Proceedings of the Radio Club of America*[1] there is a report of a lecture by Dr. E. H. Armstrong of f.m. fame entitled "A Study of the Operating Characteristics of the Ratio Detector and its Place in Radio History." Before coming to the ratio detector he describes the radio world of thirty-five years ago with its spark transmitters and crystal detectors, Poulsen arcs and tikkers. He then mentions that the 'three element vacuum tube (audion)' had been invented six years before but lay idle and neglected, and that of the 3,000 pages of seven leading text books less than a single page was devoted to the audion. The reason for this 'almost unbelievable situation' lay in the fact that 'the inventor of the audion had never understood the operation of his device'.

Even after Armstrong had fully explained the operation of the audion in 1914 and read a paper on the subject before the I.R.E. in 1915, his theory was challenged by de Forest who mentioned that he had frequently proved that the plate current was reduced by either a positive or negative charge on the grid. Armstrong then describes how he invented the regenerative circuit in 1912 and demonstrated it in 1913 and 1914 to various leading telephone engineers. In 1914 Lee de Forest described his ultraudion which he claimed did not involve the regenerative principle. Armstrong reproduces de Forest's 'fantastic explanation' and explains how in 1916 by redrawing de Forest's diagram he showed the operation in its true light. To understand the acerbity of Armstrong's criticisms one has to remember that he was one of four claimants (the others being de Forest, Meissner and Langmuir) and that after years of litigation the American courts awarded the feedback patent to de Forest.

All this is, however, introductory to the real subject of his lecture, which is the 'ratio detector' a device described in a bulletin issued by the R.C.A. License Laboratory and in a paper read before the Institute of Radio Engineers, but apparently withdrawn from publication.* This ratio detector replaces Armstrong's discriminator in the receiver of frequency-modulated transmissions. Armstrong says 'general realization of the commercial value of f.m. was delayed for many years. But when its commercial value became apparent to everyone, history repeated itself and the modern counterpart of the ultraudion appeared on the scene in the form of the ratio detector. Let us now examine this seemingly technical innovation.' He then shows by a number of diagrams that 'the aura of mystery hung over this seemingly unfamiliar device' was due to the way it was drawn. He admits, however, that it is so designed that it acts both as a suppressor of amplitude modulation and as a discriminator-detector. One gathers that from Armstrong's point of view, the part of the villain originally played by de Forest had been taken over by the Radio Corporation of America.

He concludes his lecture with a very entertaining prophecy, viz., 'that the day would surely arrive when the direction of engineering by the members of the legal profession would come to an end, because the unholy mess that they had made of radio would soon be apparent to everyone. The writer predicted that engineering would again be directed by engineers, and he even ventured to think that the day might arrive when some highly successful executives would come to believe that there was something after all to the text of the Eighth and Ninth Commandments, stating that—in case the audience could not immediately place them by number—"Thou shalt not bear false witness against thy neighbour," and "Thou shalt not steal."'

G. W. O. H.

[1] Vol. 25, No. 3, 1948.

*Although Armstrong does not mention the fact, it was also described in the *R.C.A. Review*, June 1947.

EARTHED-GRID POWER AMPLIFIERS

V.H.F. Sound and Vision Transmitters

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1. General

In a previous article[1] the author has referred briefly to the advantages of the earthed-grid cathode-excited power amplifier over the conventional earthed-cathode grid-excited amplifier for the output stage of high-power v.h.f. broadcast transmitters. He suggested, for example, that it should find particular application in wideband television transmitters operating in the 41 to 66-Mc/s band with power outputs in the region of 50 kW. When large transmitting valves are used in an earthed-cathode balanced (push-pull) amplifier, the relatively high minimum value of the output tank-circuit shunt-capacitance, formed by the anode-to-grid capacitance in parallel with the equal anode-to-grid feedback neutralizing capacitances, seriously restricts the maximum power output of the valves for a given modulation bandwidth response of the output circuit and leads to an abnormally low conversion efficiency. This invariably implies operating the valves at the limit of their grid and anode dissipations in an effort to obtain a reasonable performance from a given valve type.

Furthermore, at operating frequencies above about 50 Mc/s the low value of the output-circuit inductance necessary to tune the high shunt capacitance of the earthed-cathode balanced circuit introduces physical difficulties in circuit construction and output coupling arrangements in 'wire-connected' or even parallel-line amplifiers. There is also an increasing tendency towards instability because the resonant frequency of the relatively high capacitance neutralizing capacitors with their inductive leads [See Fig. (a)] more closely approaches the operating frequency. They then become more frequency selective and fail to provide adequate neutralization and freedom from phase modulation even over the modulation sideband range.

These difficulties are appreciably reduced in the earthed-grid amplifier because, while the anode-grid and neutralizing capacitances (if any) still form the minimum shunt capacitance of the output tank circuit, the neutralizing capacitances are very much smaller. The valve feedback capacitance to be neutralized is now that of the anode to cathode which with valves of the CAT21 type, for example, is only about 1/30th of the anode-to-grid capacitance. Neglecting the effect of stray capacitances, therefore, the minimum shunt-capacitance of the output tank circuit of the earthed-grid balanced amplifier is virtually only half that of the equivalent neutralized grounded-cathode amplifier. When the effect of stray shunt capacitances between anodes is included, there is no reason why, with a good circuit layout, the total effective anode-to-anode capacitance should be greater than 0.7 times that of the earthed-cathode amplifier. This means that, for a given r.f. bandwidth response of the output circuit, the maximum value of the load resistance referred to the anodes of the valves can be raised by approximately 1.4 to 1.

Thus, for the same peak anode current the anode voltage swing will be increased in the same ratio, provided the direct anode supply voltage is raised to permit this, so that the peak power output is proportionally increased and the valves operate at a higher efficiency. Conversely, a corresponding increase in the r.f. bandwidth response may be obtained for the same power output and efficiency. The curves of Fig. 1 show, for example, that a pair of valves of the CAT21 class, as used in the 405-line vision transmitters under construction for the Sutton Coldfield and Northern Television Stations, are capable of an estimated maximum power output at full picture white of 55 kW when used in a grid-modulated grounded-grid balanced linear amplifier output stage as compared with 38 kW for an equivalent earthed-cathode amplifier designed for symmetrical double-sideband operation with an r.f. bandwidth response 0.75 db down at ± 2.75 Mc/s on a 50-Mc/s carrier. The curves also illustrate the way in which the possible peak power output from valves of a given filament and effective capacitance between anodes depends on the r.f. bandwidth response of the output circuit.

It is of interest to note here that with an asymmetric sideband system the necessary overall bandwidth of the amplifier output circuit can be considerably reduced, as a flat response to the wanted sideband can still be obtained by offsetting the carrier frequency to one side of the bandpass centre frequency. Higher output powers are therefore possible as compared with double-sideband transmission. Fig. 2 illustrates the CAT21 valve and its characteristics are given in Table I. It is a water-cooled triode of the ring grid seal type particularly suitable for v.h.f. grounded-grid operation and may be adequately driven by the air-cooled type ACT26 valve shown in the same photograph.

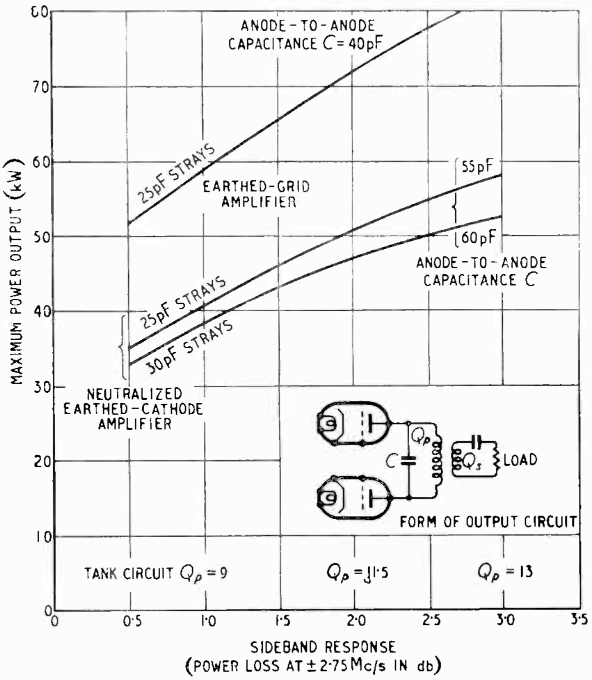

Fig. 1. *Curves showing relation between estimated maximum r.f. power output and sideband response at 2.75 Mc/s for a pair of valves of the CAT 21 class, assuming a maximum anode current of 25-A peak, 180° angle of flow, a critically-coupled output circuit (loaded Q_p = 2 loaded Q_s) and double-sideband generation. The curves ignore the limitation imposed by the rated grid and anode dissipation and the effect of the valve damping on the effective loaded Q of the primary.*

2. Driving Power

The curve in Fig. 1 for the earthed-grid amplifier ignores the power transferred to the output load from the driver stage. This is an important fundamental feature of cathode-driven earthed-grid operation and can readily be seen from a consideration of Fig. 3, illustrating the basic form and equivalent load circuit of this type of amplifier. If the grid is earthed through zero impedance, and the cathode potential is driven, say, positive with respect to earth, and therefore with respect to grid, the anode potential also becomes more positive than the cathode. Thus, the anode-to-earth e.m.f. E_a is developed in series and in phase with the cathode-to-earth driving voltage E_k and the total voltage applied across the output load resistance R_L is $E_L = E_a + E_k$ so that the driver and amplifier operate in series to supply the fundamental component I of the load current to the output load.

The total power W_L delivered to the output load is thus

$$W_L = I(E_a + E_k) \text{ watts (using r.m.s. values).}$$

The power W_t delivered by the driver to the output load is

$$W_t = I\,E_k = W_L \times \frac{E_k}{E_a} \text{ watts}$$

and the total output power W_d from the driver is

$$W_d = W_t + W_g + W_b \text{ watts}$$

where W_g is the driving power absorbed in the amplifier grid circuit and is approximately equal to E_k(peak) × I_g(mean) and W_b is the power absorbed in any bias supply and is equal to E_b(mean) × I_g(mean).

The driving power required for an earthed-grid amplifier is thus greater than that of an equivalent earthed-cathode amplifier by the amount W_t which is several times larger than $W_g + W_b$. Furthermore, when the amplifier is amplitude modulated, I, and hence W_t, vary proportionally with the level of modulation and so constitute a large variable load on the driver. This power, however, is delivered to the output load and when increased conversion efficiency of the earthed-grid stage for the wideband television case is taken into account, the overall conversion efficiency of the combination of earthed-grid amplifier and driver is still significantly higher than that of an equivalent earthed-cathode amplifier and driver giving the same power output.

In amplitude-modulated vision transmitters the need for a wideband response characteristic inevitably means operating the valves of the modulated amplifier, and any subsequent linear power amplifiers, with a relatively low referred anode-load resistance thereby limiting the amplitude of the anode voltage swing and useful direct anode supply voltage. This, in turn, imposes a limitation on the amplitude of the peak positive cathode-to-grid voltage swing permissible at full modulation (picture white), in order to keep the peak grid current and maximum grid dissipation within the rated limits of the valves. It follows that the permissible cathode-to-grid driving voltage is in general somewhat lower than would be the case where a wideband response is not involved. In any case, however, in high-power modulated vision transmitters, where amplitude modulation is applied to grid of the final output stage, it is an asset from the point of view of the rating of the video modulator to obtain full r.f. output power with the lowest possible video-modulating voltage in the peak-white condition, which means that the power gain of the final stage valves should be exploited to the limit. If the final stage is an earthed-grid amplifier this implies that the power supplied by the driver to the output load will be a minimum.

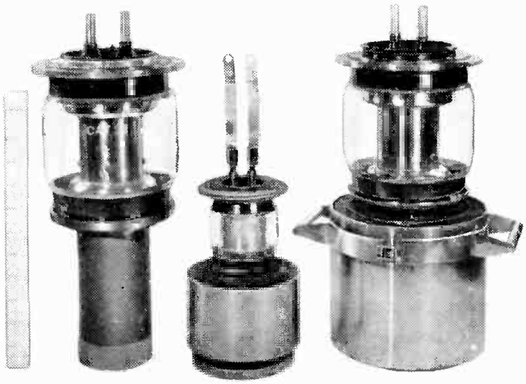

Fig. 2. *Three typical valve types for high-power v.h.f. earthed-grid operation, water-cooled CAT21 without jacket (left), air-cooled ACT26 (centre), air-cooled ACT21 (right): size compared with a foot rule. Their characteristics are given in Table I. (Reproduction by permission of the M.O. Valve Co. Ltd.)*

TABLE I

Characteristics	CAT21	ACT26	ACT21
Filament voltage ..	12.2	6.5	12.0
Filament current (A)	325	105	320
Anode voltage (Max.)* .. (kV)	10	5	10
Peak emission current (90% saturation) .. (A)	30	15	25
Maximum anode dissipation .. (kW)	20	5	10
Amplification factor	20	22	20
Capacitance: (pF)			
anode-to-grid ..	32	18	30
grid-to-cathode ..	42	22	39
anode-to-cathode	1.1	0.4	1.1
	Mc/s V_a	Mc/s V_a	Mc/s V_a
* Relation between anode voltage (V_a) and frequency.	50 100%	100 100%	50 100%
	75 70%	150 80%	75 70%
	100 60%	200 70%	100 60%
		250 60%	
		300 50%	

It is of interest to note, however, that where such limiting factors do not apply it is practicable in suitable cases to proportion the characteristics of an earthed-grid amplifier so that larger amounts of power are provided by its driving stage until, in the limit, the total power delivered to the output load could be contributed equally by the driver and amplifier and identical valves used in both stages. This is because the power gain of an earthed-grid balanced amplifier can be controlled by the use of anode-to-cathode neutralizing capacitances larger than the anode-to-cathode feedback capacitances of the valves themselves, together with an appropriate adjustment of the grid-to-grid reactance, by which means varying amounts of negative-feedback voltage may be introduced between grid and cathode. Such an arrangement might, at high frequencies, be preferable to using two valves in parallel on each side of a balanced output-stage linear amplifier used to amplify modulated signals. Alternatively, in the case of an anode-modulated class C earthed-grid amplifier the power gain may be adjusted over wide limits simply by varying the value of the grid bias so that larger or smaller values of driver voltage E_k and hence driving power $E_k I$ are required to drive it fully to saturation in the carrier condition.

This latter method, however, is not applicable to a grid-modulated linear amplifier because in this case the value of the grid bias is fixed by other considerations. These features are further discussed in Section 6.

3. Modulation Requirements

It follows as a direct consequence of the fact that the driver and earthed-grid amplifier feed power in series to the output load that if the amplifier is anode modulated, as would normally be the case for an a.m. sound transmitter, it is necessary also to modulate the driver. This is because the total voltage developed across the output load is $E_a + E_k$ and obviously both components must be subjected to the modulating voltage simultaneously if 100% modulation of the output carrier is to be obtained with a substantially linear characteristic over the complete modulation cycle. Ideally, the driving voltage should be 100% modulated and the linearity characteristic of the driver should be good since any distortion components generated by the driver will appear in the output load. In most conventional cases, however, the power provided by the driver will be a relatively low proportion of the total output power (roughly between 10% to 20%) and, where this applies, a modulation characteristic linear up to 80% for the driver is regarded as satisfactory in practice. Since the driving voltage is modulated, the amplifier should be operated with self-bias rather than fixed bias in order to prevent too sharp a cut-off in anode current at the troughs of modulation. The output power W_m of the modulator will be greater than that of an equivalent earthed-cathode amplifier by the amount of power required to modulate the driver and if 100% modulation of both the driver and amplifier is assumed, is given by

$$W_m = \frac{1}{2}\frac{W_a}{\eta_a} + \frac{W_d}{\eta_d}$$

where η_a and η_d are the conversion efficiencies of the amplifier and driver respectively.

Anode modulation of both the amplifier and driver presents little practical difficulty, apart from the complication that the modulating voltage applied to the driver will in general be lower than for the final amplifier but, if the driver is itself an earthed-grid amplifier, it may be necessary to modulate its driver, and so on if very low distortion values are required. A transmitter constructed from a number of anode-modulated earthed-grid triode amplifiers in cascade is by no means an improbable arrangement, and would prove convenient for v.h.f. a.m. sound transmitters operating in the 88 to 108-Mc/s band or at higher frequencies at which the high minimum value of the anode circuit shunt capacitance of the normal neutralized grounded-cathode amplifier becomes unwieldy irrespective of considerations of bandwidth.

When amplitude modulation is applied to the grid of the final earthed-grid amplifier (grid-bias modulation), as is usually the case in a high-power modulated television transmitter, there is no need to modulate the driver stage. Indeed the main criterion here is that, for reasons stated later, the amplitude of the cathode-to-grid r.f. driving voltage should ideally remain constant over the video modulation cycle if a conventional video-modulating signal is assumed.

Furthermore, it is important from the point of view of the video modulator that the effective capacitance presented to it by the amplifier input circuit should be a minimum in order to limit the volt-ampere output required from the modulator to provide the required signal rise times and video-frequency response.

With modulation applied between grid and earth, therefore, it is not practicable actually to earth the grid for r.f. by a high-capacitance by-pass capacitor as is permissible and usual with an anode-modulated or frequency-modulated earthed-grid amplifier operated at a fixed grid-bias voltage.

With a balanced push-pull amplifier circuit, however, the earthing of the grid-circuit centre point is unnecessary because it is already at zero r.f. potential with respect to earth (for the fundamental frequency). The effective r.f. circuit capacitance which appears across the modulator is thus limited to that of the grid-to-cathode capacitance of both amplifier valves plus grid-circuit strays (see Fig. 5).

This is approximately half the capacitance which would be presented by an equivalent earthed-cathode amplifier having cross-connected anode-to-grid neutralizing capacitors, since for video frequency the neutralizing capacitors are effectively in parallel with the grid-to-cathode capacitance.

In the case of f.m. sound or vision transmitters with carrier frequencies in the 41 to 68 Mc/s or 88 to 108-Mc/s bands or higher, a cascade arrangement of single-sided earthed-grid triode amplifiers is eminently suitable. Here again no problem of multiple modulation arises because the frequency-modulated carrier is generated at low power and all the succeeding earthed-grid stages operate as high-efficiency hard-driven class C constant-voltage r.f. amplifiers. Each stage can be arranged to pass on a high proportion of its output power to the succeeding stage and, depending on the available valve types, the best compromise between the power gain per stage and power transferred per stage can be arrived at either by suitably proportioning the neutralizing capacitors and grid-to-grid reactance or by adjustment of the grid bias as described later. Control of power gain by the first method would be unlikely for transmitters operating with carrier frequencies in the 88 to 108-Mc/s band or higher, because at these frequencies single-sided coaxial-type circuits as distinct from 'wired' or parallel-line circuits would be preferred in all but the very low power stages and in a coaxial circuit the anode-to-cathode feedback capacitance is neutralized by adjustment of the grid-to-earth reactance alone. Since, however, in an f.m. transmitter the valves are class C operated, adjustment of the power gain by choice of grid bias can conveniently be used.

The characteristics of earthed-grid coaxial-type circuits are dealt with in Section 8.

4. Negative Feedback Due to Driver

The earthed-grid amplifier has an inherent degree of negative feedback between output and input which is absent in the earthed-cathode amplifier. This is because, as shown in Figs. 3 and 7, the fundamental component I of the output-load current flows through the effective internal resistance R_d of the driver and thereby produces across that resistance a voltage R_dI in phase opposition to the open-circuit output voltage of the driver and which with amplitude modulation, varies with the level of modulation. It might be expected that in a linear r.f. amplifier amplifying amplitude-modulated signals this feedback could be used to reduce distortion, but this would require the degree of feedback to remain constant over the modulating cycle. The effective resistance of the driver would thus have to be constant, a requirement not easy to achieve in practice. In the case of a grid-modulated linear amplifier the amplitude of the cathode-to-grid r.f. driving voltage must be substantially constant over the modulation cycle, if a linear modulation characteristic is to be obtained. If, however, the negative feedback is significant and its degree varies over the modulating cycle, the amplitude of E_k will vary in a non-linear manner and this, when added to the inevitable regulation of the driver output voltage, produced by the variable grid-current loading of the amplifier for different levels of modulation, can produce a serious falling off in the amplitude of the driving voltage at modulation voltages approaching the peak positive value.

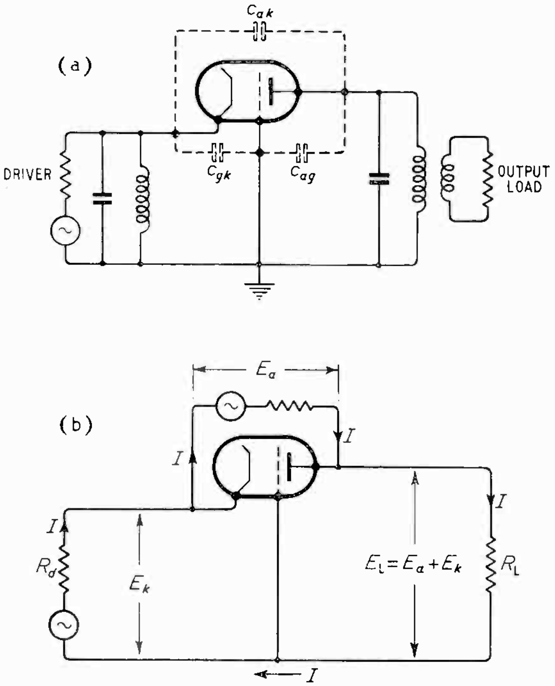

Fig. 3. *Basic (a) and equivalent (b) circuits of earthed-grid amplifier and driver.*

In the video case such a reduction in the amplitude of the driving voltage produces 'flattening' of the picture whites and, furthermore, causes difficulties in accurately adjusting the required 70/30 picture/sync pulse ratio of the radiated signal. It also makes the correct driver and modulator output voltages somewhat difficult to determine and a fairly precise knowledge of the r.f. peak voltage swing effective between grid and cathode is essential in order to obtain a true class B operating condition for the modulated amplifier at peak white. It will be appreciated that, at a peak white modulating signal, the amplifier should ideally generate anode current r.f. pulses of the maximum allowable amplitude and of just 180° duration, in order to obtain the best compromise between peak-power output and conversion efficiency. At the same time, at the trough of a synchronizing signal the anode current should be just cut off, while a black level signal should generate anode-current pulses of less than 180° duration and of amplitude sufficient to give a fundamental frequency component of 30% of the peak-white amplitude. This operating condition is illustrated in Fig. 4(a) and it will be evident that the three conditions mentioned cannot be satisfied unless the amplitude of the r.f. driving voltage is equal to the overall amplitude of the radio modulating signal and remains substantially constant over the modulating cycle.

The other forms of r.f. feedback which affect the amplitude of the cathode-to-grid driving voltage are those produced by the circuit reactive currents flowing through common impedances and are separately dealt with in the section on neutralizing. Fig. 4(b) illustrates two of the simpler types of non-linear modulation characteristic which can arise from these causes.

The best way of avoiding the uncertain effects of the type of non-linear negative feedback discussed above is to make the effective resistance of the driver very low, so that the degree of feedback is small. This is reasonably easy to achieve if the driver is a very hard-driven class C amplifier or an r.f. cathode follower, both of which have a low internal impedance at the carrier frequency. Drivers having effective internal resistances less than 10 ohms can be obtained in this way.

In an anode modulated hard-driven class C operated earthed-grid amplifier the effect of negative feedback is less significant because in class C operation relatively large non-linear variations in the amplitude of the driving voltage can be tolerated without seriously upsetting the linearity of the output modulation characteristic and in any practical case the driver is also modulated.

Even so, with either type of amplitude-modulated earthed-grid amplifier the form of coupling between the driver output and the amplifier cathode circuit is important. A transmission-line coupling, unless half a wavelength long, may be unsuitable because the terminating load resistance is not constant, so that the variation in the load impedance referred to the driver valves at different modulation levels may be appreciably exaggerated by the appearance of a significant reactive component. Inductively-coupled resonant circuits having a high kVA/kW ratio and giving a good voltage step-down (or the electrical equivalent using cathode driving connections of short length and low reactance tapped well down the driver output circuit) provide the most practical way of minimizing variations in the amplitude and phase angle of the driver output voltage due to this cause.

It is considered that in vision transmission the overall modulation characteristic of the radiated

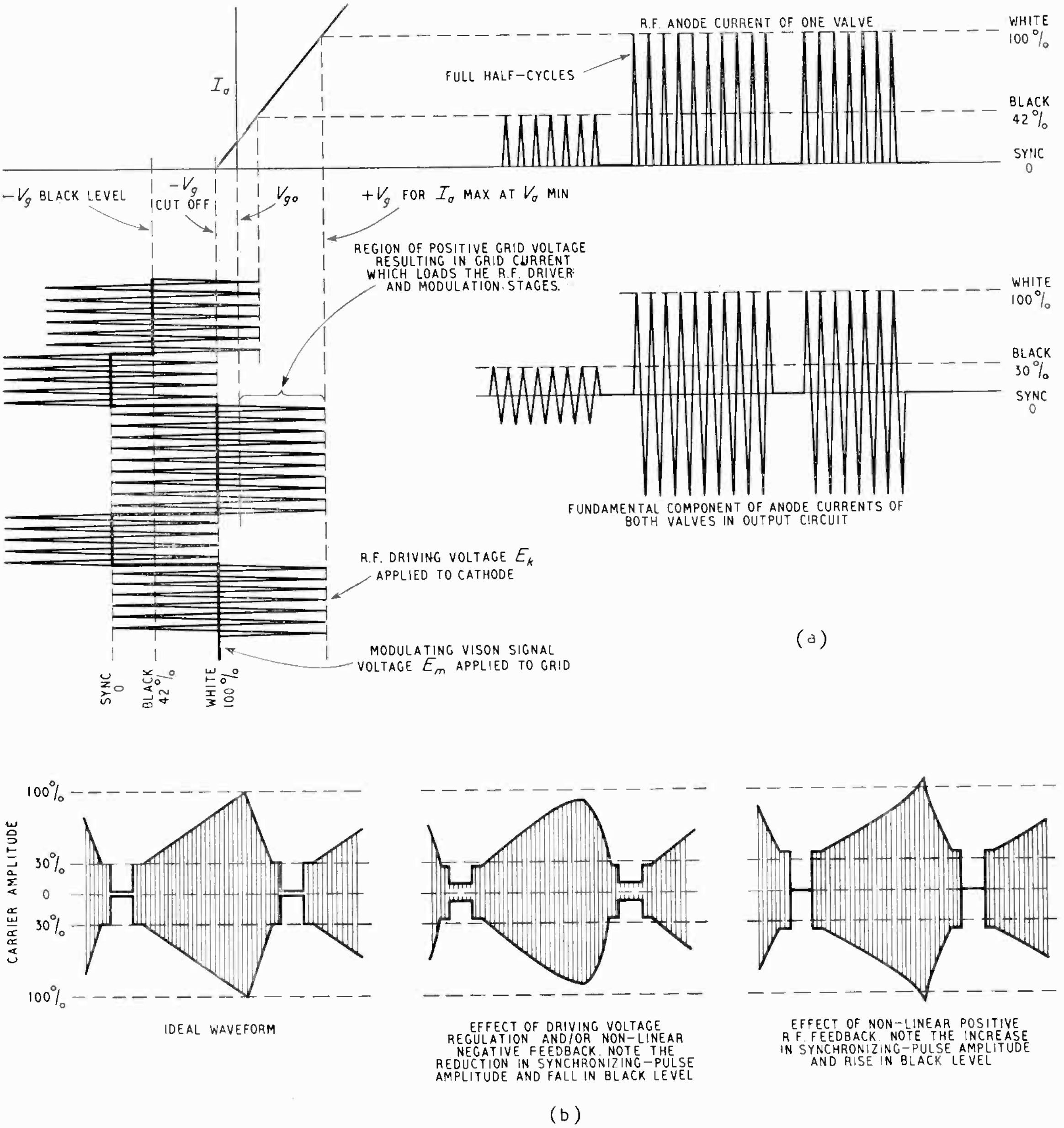

Fig. 4. (*a*) *Illustration of the idealized operating conditions for a balanced (push-pull) r.f. power amplifier, grid-modulated by a vision signal. The short intervals of black level which, in practice, occur before the start and at completion of each line of picture signal are omitted. (Reprinted from author's previous article*[1] *by permission of 'Electronic Engineering'.)* (*b*) *Output waveforms of modulated amplifier, grid-modulated by a vision test signal of sync, suppression and line saw-tooth illustrating the non-linear modulation characteristic produced by factors which cause a change in the amplitude of the r.f. driving voltage.*

signal should be such that within the picture signal range it does not depart from linearity by more than about 4% due to all causes, namely, the regulation of the r.f. driver, spurious r.f. feedback, the impedance of the power supplies and the operation of the d.c. restoration and black level stabilizing devices. Serious attempts to reduce the regulation of the driver stage to, say, below 2% are therefore worth-while.

resonant line can be constructed vertically between the two valves. This allows very short horizontal driving connections between it and the cathodes of the CAT21 valves in the earthed-grid amplifier in the adjacent screened compartment.

A suitable voltage step-down between the driver high-kVA cathode line and the cathodes of the final amplifier can be arranged by con-

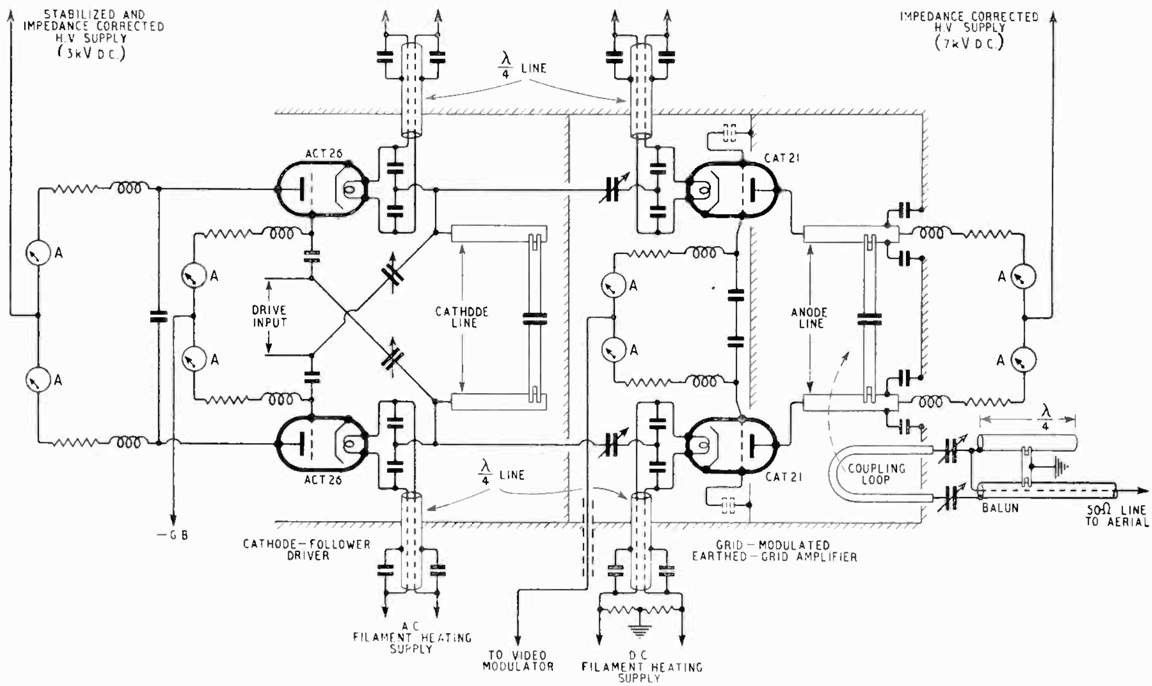

Fig. 5. *Television transmitter. Suggested practical circuit for a grid-modulated earthed-grid amplifier driven by a r.f. cathode follower. Frequency range* 41—66 *Mc/s, output power* 50 *kW (peak white), r.f. bandwidth response flat to within* −0.5 *db at* ±2.8 *Mc/s. The cathode and anode parallel lines are tuned either by varying their length by means of the adjustable shorting bar or by varying their effective Z by adjusting the spacing between the conductors.*

A robust cathode-follower driver, with its low output impedance, offers the greatest possibility of providing a constant driving voltage over the wide load range imposed upon it by the earthed-grid modulated amplifier. It may also be expedient to load the driver anode circuit with resistance to prevent a possible sharp rise in its output voltage at the bottom of the sync pulses, during which the load on the driver is very small.

The push-pull cathode-follower driver tends to give a very compact physical circuit layout for use with a high-power earthed-grid push-pull final amplifier because with driver valves of the ACT6 type, for example, the cathode parallel-

structing the driving connections as a capacitance potential divider. This also provides a relatively high capacitance between the cathodes of the earthed-grid amplifier and hence a path of low impedance for the return to the cathodes of the harmonics of the amplifier anode-current r.f. pulses. This point is further discussed in the section on neutralizing.

Fig. 5 illustrates a practical circuit for a grid-modulated earthed-grid amplifier, driven by a r.f. cathode follower suitable for a television transmitter having a maximum output of approximately 50 kW (peak white) with a bandwidth substantially flat up to ± 3 Mc/s.

5. Neutralizing

The complete neutralization and stabilization of any driven radio-frequency amplifier may be defined as the suppression of all forms of mutual coupling by common impedances between the input and output circuits of the amplifier so that (a) the input and output circuits are independent of each other with respect to reactive currents and (b) the driving voltage occurring between grid and cathode is unaffected in both amplitude and phase by the voltage developed across the output circuit. If this condition is not fulfilled, there will exist in the amplifier a number of feedback voltages between the output and input circuits in the form of negative and/or positive feedback together with quadrature components. Positive feedback leads in the limit to self-oscillation of the amplifier, negative feedback to loss in grid-to-cathode drive. If the amplifier is amplitude modulated, the combination of positive and negative feedback, together with any quadrature feedback, results in a non-linear modulation characteristic and phase modulation.

The impedances which give rise to the feedback couplings are the interelectrode valve capacitance common to the input and output circuit and any common circuit impedances. The currents which produce the feedback voltages across these impedances are the output circuit reactive circulating current, the active cathode-to-anode load current pulses and, to a far lesser degree, the input driving reactive circulating current and the rectified grid current. In an amplifier of the 'wire connected,' as distinct from the coaxial circuit or resonant cavity type, the inductance of the connecting leads and of the valve electrodes themselves is appreciable as the operating frequency is increased and in short-wave amplifiers constitutes the common circuit impedance across which the feedback voltages are developed. Thus the complete neutralization and stabilization of a high-frequency amplifier either of the earthed-cathode or earthed-grid type ideally requires the cancellation of

(a) The appropriate interelectrode feedback capacitance of the valve.

(b) The inductance of the grid assembly and grid lead.

(c) The inductance of the cathode assembly and cathode lead.

Fully neutralized and stabilized balanced amplifiers of the earthed-cathode and earthed-grid type are shown in Fig. 6.

The interelectrode feedback capacitance is neutralized by the well-known capacitance bridge method using cross-connected capacitors C_n, while the inductances of the grids and cathodes and their connecting leads is cancelled by series tuning them with the capacitors C_c and C_g

All three capacitors should be adjustable if the amplifier is required to work over a range of carrier frequencies but in the earthed-grid case, since c_{ak} and hence C_n are comparatively small, the effect of series inductance in the physical connections to C_n is usually not significant and,

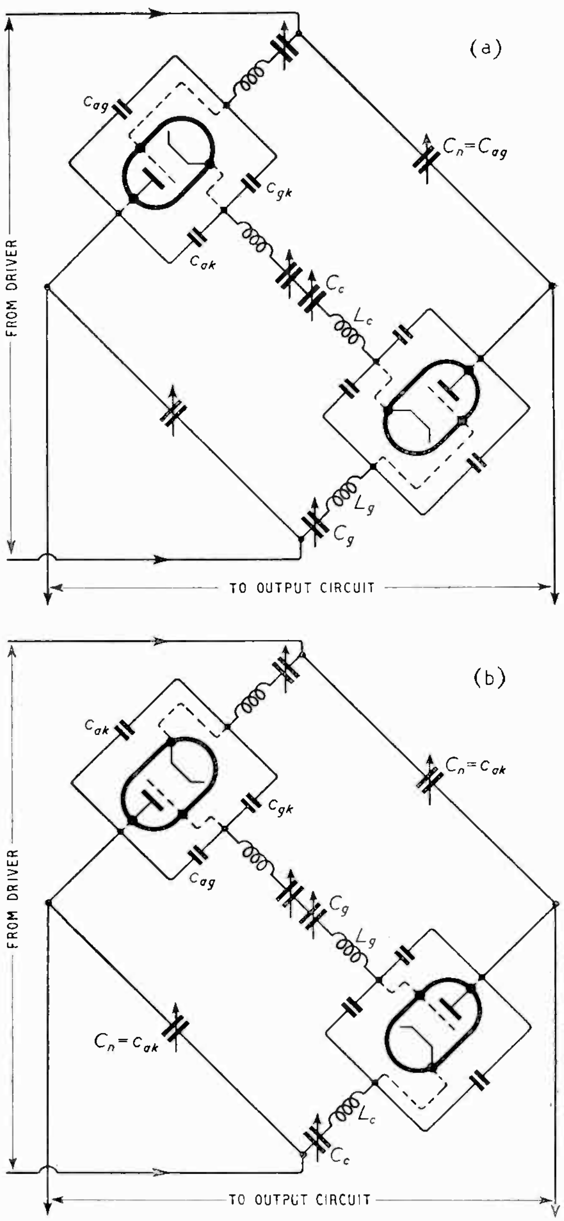

Fig. 6. *Complete neutralization of a driven balanced r.f. amplifier. (a) Earthed-cathode amplifier in which $C_n = c_{ag}$, (b) earthed-grid amplifier in which $C_n = c_{ak}$ and is very much smaller than c_{ag}.*

once set, a fixed value of C_n will serve over a wide frequency range.

The seriousness of imperfect neutralization on the performance of the amplifier is, of course, a question of magnitude but its importance can be realized when it is considered that the inductance of 8 inches of two-inch copper tubing is approximately 0.1 μH, which at 50 Mc/s represents an impedance of 30 ohms. In a high-power amplifier, where reactive circulating currents reach the order of 100 amperes, the voltage developed across this impedance is 3,000 volts which may well be in excess of the input driving voltage.

The effect of impedance between grid and grid, or grid and earth in the single-sided case, has been excellently described by C. E. Strong[2] and the following explanation is mainly his. Fig. 7 shows the equivalent circuit of a single valve earthed-grid amplifier having an impedance Z between grid and ground. Z is made either inductive or capacitive depending upon the adjustment of C_g. If Z is an inductive reactance at the working frequency and if it is considerably less than the reactance of the anode-to-grid capacitance c_{ag}, as is the case in practice, the voltage developed between grid and ground across Z by the reactive circulating current I_1 in the output circuit is 180° out of phase with the anode-to-earth and cathode-to-earth voltages. Thus, when the cathode is driven positive with respect to earth the grid is driven negative by the amount ZI_c and the driving voltage effective between cathode and grid is thus increased by this amount. The action constitutes positive feedback. Similarly, if the grid-to-earth impedance Z is a capacitive reactance the voltage developed between grid and earth across Z is in phase with the anode-to-earth and cathode-to-earth voltages. Thus, when the cathode is driven positive the grid is also driven positive with respect to earth and the driving voltage effective between cathode and grid is reduced by the amount ZI_c. This action constitutes negative feedback.

The effect of the fundamental-frequency feedback voltage between output and input introduced by incomplete cancellation of the small anode-to-cathode capacitance c_{ak}, has now to be considered. This is of practical consequence in an earthed-grid amplifier only if (a) the effective resistance of the driver is not negligibly small or (b) the driving voltage is removed, bearing in mind that if the driver is modulated the driving voltage is in effect removed in the troughs of modulation.

It will be evident from Fig. 7 that if R_d is appreciable but small in comparison with the reactance of c_{ag}, the anode-to-earth voltage acting across c_{ak} and R_d in series produces a current I_3 through them which leads this voltage by almost 90°. The current through R_d produces a voltage R_dI_3 which leads the driver output voltage by almost 90° and thus constitutes a quadrature feedback voltage. If the amplifier is amplitude modulated either on the grid or anode, the amplitude of the quadrature feedback voltage varies with modulation and, if the driver output voltage is constant, phase modulation also results. If, however, the driver is itself modulated, which for reasons previously mentioned would normally be the case for a.m. sound transmitters, the relative amplitudes of the driver and quadrature feedback voltages remain the same and phase modulation is avoided.

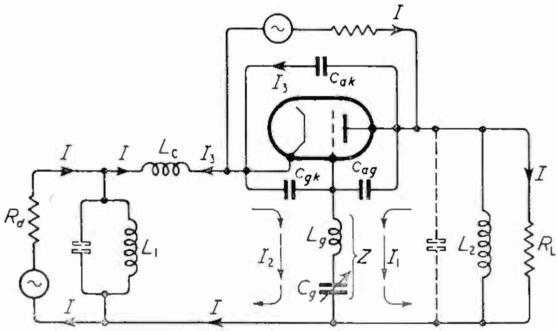

Fig. 7. Circuit illustrating the paths of active and reactive circuit currents and neutralization of valve-feedback capacitance by means of a reactance Z between grid and earth in an earthed-grid amplifier.

If the driving voltage is suddenly removed or modulated to zero, the impedance through which the current I_3 now flows is no longer R_d but the cathode to earth input impedance of the amplifier. This is very high in comparison with R_d but is also a pure resistance if the grid circuit is in tune at the fundamental frequency. A considerably higher feedback voltage is thus developed between cathode and earth, the phase of which is very much less than 90° leading on the anode-to-earth voltage so that a large component of positive feedback exists which if of sufficient magnitude may cause self-oscillation of the amplifier.

It should be noted that incomplete cancellation of c_{ak} also permits feed-through from the input to the output circuits and although this is small in an earthed-grid amplifier because c_{ak} is small, it is nevertheless important in cases where the r.f. driving voltage of the amplifier is constant.

The presence of the driving voltage causes a small constant reactive current to appear in the output circuit so that the output carrier cannot be modulated to zero. Thus, in a grid-modulated video amplifier, for example, the output carrier level is not zero at the bottom of the synchronizing pulses as should ideally be the case. The amplitude of the radiated synchronizing pulse measured from black level is reduced and the picture to sync pulse ratio is incorrect.

In an earthed-grid amplifier the effects of the anode-to-cathode feedback capacitance can be neutralized by making the grid-to-earth impedance an appropriately low inductive reactance, instead of using the more conventional cross-connected capacitors or anode-to-cathode inductance, to introduce a current equal to I_3 but of opposite phase into the cathode-to-earth circuit and thereby eliminate its effects. Considering the amplifier purely as a network, the elements of which are formed by the inter-electrode capacitances (Fig. 6) this can be shown to be true. It will be seen that if the impedance Z between the grid and earth is a low inductive reactance the voltage E_g developed across Z is 180° out of phase with the anode-to-earth voltage. Therefore E_g drives current through the grid-to-cathode capacitance c_{gk} which is 180° out of phase with the current fed through the anode-to-cathode capacitance.

The two currents are equal, and cancel out in the input circuit, if Z is such that

$$Z = -\frac{X_{ag}\,X_{gk}}{X_{ag} + X_{gk} + X_{ak}}$$

where X_{ag} is the reactance of c_{ag} and so on. Nevertheless, as previously described, if the grid-to-earth impedance Z is an inductive reactance positive feedback is introduced as far as the cathode-to-grid drive is concerned and the effect from the point of view of the valve is not quite the same as direct neutralization of the anode-to-cathode capacitance.

It is, however, a most convenient method provided c_{ak} is small. It is easily arranged by suitably adjusting the grid inductance neutralizing capacitors (C_g in Fig. 6). It will be appreciated that to meter independently the grid current of the two valves d.c. bias blocking capacitors will be required in each grid connection so if these are constructed as adjustable capacitors of suitable rating they can serve both purposes. This is the arrangement usually adopted for the coaxial type of circuit.

The elimination of anode-to-cathode neutralizing capacitors reduces the minimum shunt capacitance of the output circuit between anodes and can considerably simplify the physical layout of 'wire-connected' or parallel-line balanced circuits. A good deal of mechanical ingenuity is required to position such capacitors and to achieve cross connections of sufficiently low inductance at operating frequencies of 50 Mc/s and above. Furthermore, a small amount of linear positive feedback may usefully be tolerated even in a grid-modulated amplifier as it can assist in maintaining the cathode-to-grid r.f. driving voltage constant over the modulation cycle by compensating for the inherent regulation of the output voltage of the driver when modulating in the positive direction.

The order of magnitude of Z and the positive feedback voltage to be expected in a practical case can be estimated by inserting in the above expression for Z appropriate values of interelectrode capacitances for a valve of the CAT21 class. Taking $c_{ag} = 30$ pF, $c_{gk} = 40$ pF and $c_{ak} = 1.1$ pF, the approximate reactances of which are -106, -80 and -2900 ohms respectively at 50 Mc/s, the corresponding value for Z is $+2.75$ ohms. In a 50-kW wideband vision amplifier the reactive current I_1 through c_{ag} would be of the order of 75 amperes peak at full white so that the positive-feedback peak voltage produced between grid and earth would be some 200 V. This would more than compensate for the driver regulation which, in a well designed stage, need not exceed about 3%, or some 30 volts, on a normal driving voltage of 1,000 volts peak, over the complete modulating cycle; i.e., from the bottom of the synchronizing pulses to peak white.

A grid-to-grid reactance of $2Z = +5.5$ ohms corresponds at 50 Mc/s to an inductance between grids of only 0.0175 μH which is likely to be considerably less than the minimum inductance of the grid assemblies and grid-to-grid connection achievable in a practical design of 'wire-connected' amplifier. Series compensating capacitors in the grid circuit are thus essential Alternatively, the grid-to-grid connection could be constructed as a parallel line of appropriate length to provide the necessary compensating capacitive reactance, but this would be less physically convenient than capacitors except at very high frequencies and in this case the choice would not arise as a coaxial circuit would be used. Turning to the effects produced by incomplete neutralization of the cathode-lead inductance consideration of Figs. 6 and 7 indicates that the fundamental component I of the active load current returns to the cathode through this inductance and thereby develops in series with the cathode-to-earth driving voltage E_c a voltage IZ_c in quadrature with it. If the amplifier is amplitude modulated either on the anode or grid the amplitude of IZ_c varies with modulation and, if E_c is constant, phase modulation will also result. If the amplifier and driver are both modulated, phase modulation is avoided.

Compensation of the cathode and cathode-lead inductance is not so critical as the grid inductance because the current I is much lower than I_c. Even so, liberties should not be taken because the connections between the driver and cathodes also form part of the cathode inductance concerned and, as pointed out previously, even a few inches of quite large diameter conductor has an appreciable reactance at 50 Mc/s. With a high-power amplifier a quadrature component of several hundred volts may easily occur. Unless, therefore, an unusually compact physical layout can be obtained for the cathode circuit, compensation of the cathode inductance by series capacitors should be included.

Such compensation is effective only for the fundamental frequency component of the load-current r.f. pulses and a relatively high inductive impedance may still be offered to the harmonic components returning to the cathode. This causes harmonic distortion of the cathode-to-grid driving-voltage waveform and may result in a fall in the conversion efficiency of the amplifier. It will be appreciated, however, that only a proportion of the load current harmonic components return via the cathode inductance as a second path is provided by the grid-to-cathode capacitance which, in v.h.f. amplifiers, is of relatively low impedance.

The essential practical criterion for the neutralization of the feedback capacitance to give minimum coupling between the output and input circuits in an earthed-grid amplifier is the same as for the conventional earthed-cathode amplifier, namely, that when the input and output circuits are tuned through resonance, maximum grid current should coincide with minimum direct anode current under normal power conditions.

It should be noted, however, that if preliminary neutralizing adjustments are attempted in the conventional manner with drive applied and the amplifier anode voltage removed but with the filaments still heated, the output power cannot be reduced to zero by adjustment of the anode-to-cathode neutralizing capacitors (or grid-to-grid reactance) as in the case of an earthed-cathode amplifier. This is because, for part of the negative half-cycle of r.f. driving voltage applied to the cathode, the grid and anode both have a positive potential with respect to the cathode, so that a small anode current flows in the output circuit between anode and cathode, even if neutralization is correct.

Further, if a path exists for the d.c. component, as will be the case if the anode-supply rectifier filaments are left heated, the anode-current meters will indicate a small anode current. This means that preliminary neutralizing adjustments for minimum coupling between output and input must be made with the amplifier filaments cold and these will not necessarily be correct when the filaments are heated because of the slight but inevitable change in the valve capacitances.

The appearance of direct anode current when drive is applied, does, however, enable the output circuit of the amplifier to be tuned, and a preliminary adjustment made of the loading, before applying the anode supply voltage.

(To be concluded)

TRANSIT-TIME EFFECTS IN U.H.F. VALVES

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1. Introduction

IT is well known that the performance of the conventional valve falls off very rapidly above some frequency the value of which is determined by the geometry of the electrode structure. Many effects combine to produce this loss of efficiency, among which may be mentioned (a) increased losses due to high-frequency resistance, (b) the increasing importance of lead inductance and interelectrode capacitance, and (c) profound modifications of the valve 'constants' due to transit-time phenomena. It is with category (c) that the present article is primarily concerned, but it should be noted at the outset that a self-consistent theory of the u.h.f. valve can only be achieved by taking into account the influence of each category of effect on the other two. For example, the inductance of the cathode lead would not be of such importance if it were not for the fact that it must carry an ever increasing current as the frequency increases, due to the diminished reactive impedance of the grid-cathode space.

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Attempts to elucidate transit-time phenomena have been greatly hampered by the intrusion into the argument of factors due to (a) and (b), and it is only recently that valves have been made in a form which has allowed the investigator to separate the variables and so to estimate their relative importance. Disc-seal valves, operating in the common-grid condition, undoubtedly offer the most attractive field for further research in this most important field.

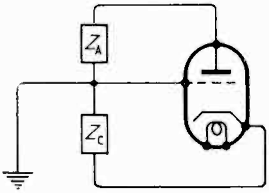

Fig. 1. *Basic common-grid valve circuit.*

Qualitatively, there is nothing difficult to understand about transit-time phenomena. At low frequencies the time taken by the electrons to traverse the distance between two valve electrodes is (on the average) negligibly small compared with a period of oscillation of the associated circuit but, as the frequency increases, this is no longer true. In these circumstances conventional valve theory ceases to apply, for a cloud of electrons repelled from an electrode by an electric field does not reach the vicinity of another electrode until the field has changed appreciably. The first and most obvious effect of 'time of flight' is, therefore, that the thermionic current lags behind the propelling voltage: the inertia of the electron, small though it is, has to be considered as one of the factors in the situation. It is when this physical concept is incorporated in a quantitative analysis that difficulties begin to arise, and the work of Benham, Llewellyn and Muller, while it has certainly shed light on certain aspects of the problem, cannot be said to satisfy the engineer.

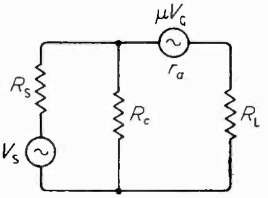

Fig. 2. *Equivalent circuit of Fig. 1.*

2. A Practical Example

To show how difficult the analysis of a valve plus circuit can be at u.h.f., the case of a specific power amplifier, working at 500 Mc/s may be quoted. In practice, if circuits correctly designed for this frequency are fitted to the CV257 valve, an output of 100 watts at an anode efficiency of 65% may be obtained by means of 10-watts drive power. Such performance is very useful, no matter how it is achieved, but when the attempt is made to examine the action theoretically, considerable difficulties are encountered.

The valve is used as shown in Fig. 1, where the tank circuit and its load are denoted by the impedance Z_A, and the drive circuit by the impedance Z_C.

The valve is mounted at the end of a concentric-line circuit so that the anode-grid space forms an integral part of the impedance Z_A. If it is otherwise desirable, another concentric line may be terminated by the grid-cathode space to form the impedance Z_G, although as will be seen in the sequel, it is not usually necessary. Now let us consider this arrangement in more detail. First, the valve is used in the common-grid condition, corresponding to the 'inverted amplifier' of earlier days, and this circuit has some rather distinctive properties. Imagining for a moment that the valve is operating as a linear device, the circuit equivalent of Fig. 1, neglecting all transit-time effects, is as shown in Fig. 2, where the input voltage is considered to be in series with a resistance R_s and applied across a resistive impedance R_c, the valve having an amplification factor μ and a dynamic resistance r_a. R_L is the effective load presented by Z_A.

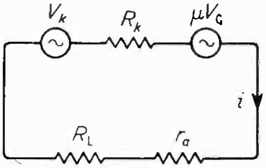

Fig. 3. *Simplest equivalent of Fig. 1.*

This circuit may in turn be re-drawn as in Fig. 3, showing that the source, the valve generator and the load are effectively in series.

Then $V_G = V_k - R_k i$

and $V_k - R_k i + \mu V_G - (r_a + R_L)i = 0$

$$\text{Therefore } i = \frac{V_k(\mu + 1)}{r_a + R_L + (\mu + 1)R_k}$$

$$= \frac{V_k}{R_k + (r_a + R_L)/(\mu + 1)}$$

Clearly, $(r_a + R_L)/(\mu + 1)$ may be regarded as the feedback impedance seen by the source.

It has been shown by P. E. Trier in some work shortly to be published that under these circumstances the following conditions apply for maximum stage gain; viz.,

(1) $R_L = Mr_a$

(2) Power gain $= \left(\dfrac{\mu + 1}{M + 1}\right)^2 \dfrac{R_c}{r_a}$ — See Fig. 2

where (3) $M = 1 + (\mu + 1)\dfrac{R_c}{r_a}$

and (4), the source is matched to the input impedance by a suitable tapping on the (tuned) input circuit.

Still assuming that our valve is operating in a linear manner, let us insert the constants characteristic of the CV257; i.e., $\mu = 22$ and r_a

= 1,100 ohms. Then the following Table of gain and R_L versus R_c is obtained.

TABLE I

R_c (Ω) ..	1	10	100	1,000	10,000	∞
Power gain	0.12	1.0	16.3	14.9	20.0	23
R_L (Ω) ..	1,100	1,210	3,410	5,150	15,950	∞

Now the impedance R_c can be measured for class A operation, and when this is done for the CV257 a value of the order of 200 ohms is obtained at 500 Mc/s, showing that a true gain of more than 9 is impossible.

To make a corresponding analysis of the class C amplifier is a matter of some difficulty, and when account has to be taken of the nature of R_c—whether it is wholly due to high-frequency losses in the electrode structure or to transit-time losses in the electron stream—the problem becomes almost unmanageable. One point of practical importance emerges. Since the shunt impedance R_c is so small, there is little advantage to be gained from the use of a concentric-line input circuit. A low L to C ratio is quite permissible, and a good coil and capacitor will function admirably.

Having indicated how complex the real problems of the u.h.f. valve are, let us now examine some of the more important transit-time effects in an idealized framework.

3. Real Transit-Time Effects

Considering the triode once more, it is best to separate out (at least for the purpose of discussion) the two zones in the valve, (a) the cathode-grid space and (b) the grid-anode space.

The cathode-grid space is the more important, for experience has shown that it is here that transit-time effects first begin to appear as the frequency is increased. Valves which operate very inefficiently as amplifiers at a certain frequency will operate with much greater relative efficiency as frequency multipliers with the same output circuit, and this fact clearly indicates that the limiting factor is the loss in the input space. But the grid-anode space is theoretically more interesting for two reasons. First, it presents a less intractable problem to the mathematician; and secondly, certain theoretical results with regard to inertia effects in the grid-anode space have received rather elegant experimental verification. For these reasons it is proposed to consider first the grid-anode space (or a modification of it). Even in this simple case it will be found that very stringent conditions must be imposed before the theoretical argument can be developed. In what follows the attempt will be made to derive in the simplest possible fashion a result initially suggested by the work of Benham,[1] and developed in his own inimitable fashion by Llewellyn.[2]

4. Damping Without Space Charge

Consider a plane parallel structure, consisting of a thermionic cathode K (Fig. 4) a grid G, and an anode A. Let there be a constant positive potential V_0, on the grid G, the cathode being grounded, and let there be an alternating potential difference $V_A \sin(\omega t + \phi)$ between G and A. Let the distance between G and A be D.

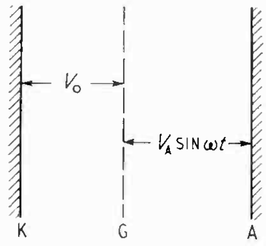

Fig. 4. *Section through plane parallel electrode structure.*

Consider an electron in the space between G and A. Then, since there is no electron emission in the space, and since the electrons are moving with a finite speed due to V_0, space-charge effects are small, and will be neglected. The acceleration of a typical electron is given by

$$\frac{d^2x}{dt^2} = \frac{e}{m}\frac{V_A}{D}\sin(\omega t + \phi) \quad \ldots \quad \ldots \quad (1)$$

whence, integrating twice, and noting that $dx/dt = \sqrt{2V_0 e/m}$ when $x = 0$ and $t = 0$,

$$\frac{dx}{dt} = \sqrt{2V_0 e/m} + \frac{e}{m}\frac{V_A}{\omega D}[\cos\phi - \cos(\omega t + \phi)] \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\text{and } x = \sqrt{2V_0 e/m}\, t + \frac{e}{m}\frac{V_A}{\omega D}\left[t\cos\phi + \frac{1}{\omega}\left\{\sin\phi - \sin(\omega t + \phi)\right\}\right] \quad \ldots \quad (3)$$

Let $x = D$ at time $t = T$, and write

$$T = \frac{D}{\sqrt{2V_0 e/m}} + \delta$$

so that δ is the variation in the transit-time caused by the alternating field. Let another condition be imposed upon the variables. Let V_A/V_0 be small. Then terms in δ^2, δ^3, etc., can be neglected in an expansion of T as a power series. Let $2V_0 e/m = v_0$ and $eV_A/m\omega D = v_A$ = the amplitude of the additional electron velocity due to the alternating field. Then substituting in equation (3),

$$D = D + v_0\delta + v_A\left(\frac{D}{v_0} + \delta\right)\cos\phi + \frac{v_A}{\omega}\sin\phi - \frac{v_A}{\omega}\sin\left(\frac{\omega D}{v_0} + \omega\delta + \phi\right) \quad \ldots \quad (4)$$

The last term on the right-hand side may be written

$$\frac{v_A}{\omega}\left[\sin\left(\frac{\omega D}{v_0}+\phi\right)+\omega\delta\cos\left(\frac{\omega D}{v_0}+\phi\right)\right]$$

to the correct degree of approximation.

Solving equation (4) for δ

$$\delta = \frac{v_A\left[\sin\left(\frac{\omega D}{v_0}+\phi\right)-\sin\phi-\frac{\omega D}{v_0}\cos\phi\right]}{\omega\left[v_0+v_A\left\{\cos\phi-\cos\left(\frac{\omega D}{v_0}+\phi\right)\right\}\right]} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Substituting this value in equation (2), then

$$\left.\frac{dx}{dt}\right]_{t=T} = v_0 + v_A\left[\cos\phi - \cos\left(\frac{\omega D}{v_0}+\phi\right)\right]$$
$$+ v_A^2\sin\left(\frac{\omega D}{v_0}+\phi\right)\left(\frac{\omega\delta}{v_A}\right) \quad \ldots \quad (6)$$

Let this value of dx/dt for $t = T$ be represented by u, the real velocity of an electron which started from the grid under the influence of an alternating field of instantaneous magnitude $V_A \sin \phi/D$. Form the expression u^2/v_0^2 from equations (5) and (6), neglecting $(v_A/v_0)^4$ and higher terms, and writing $\omega D/v_0 = \theta =$ the average transit angle.

Then

$$\left(\frac{u}{v_0}\right)^2 = 1 + 2\frac{v_A}{v_0}[\cos\phi - \cos(\theta+\phi)]$$
$$+\left(\frac{v_A}{v_0}\right)^2[\cos^2\phi + \cos^2(\theta+\phi) - 2\cos\phi\cos(\theta+\phi)$$
$$+ 2\sin(\theta+\phi)\{\sin(\theta+\phi) - \sin\phi - \theta\cos\phi\}]$$

The average value of $(u/v_0)^2$ over one cycle of the alternating field is

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{u}{v_0}\right)^2 d\phi = \xi$$

whence, $\xi = 1 + \left(\frac{v_A}{v_0}\right)^2[2(1-\cos\theta) - \theta\sin\theta]$.

Now write $(v_A/v_0)^2$ in terms of V_A, V_0 and θ.

Then $\xi = 1 + \left(\frac{V_A}{2V_0}\right)^2 \frac{1}{\theta^2}[2(1-\cos\theta) - \theta\sin\theta]$

or $\frac{1}{2}m\bar{u}^2 = V_0 e +$

$$\frac{eV_A^2}{4V_0}\left\{\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2}\right\} \quad \ldots \quad (7)$$

where $\bar{u}^2$ is the mean-square value of the electron velocity. Write equation (7) in the form

$$\frac{1}{2}m\bar{u}^2 - V_0 e = \frac{eV_A^2}{4V_0}F(\theta) \quad \ldots \quad \ldots \quad (8)$$

where $F(\theta)$ represents the function

$$F(\theta) = \frac{2(1-\cos\theta) - \theta\sin\theta}{\theta^2} \quad \ldots \quad (9)$$

Fig. 5 exhibits the variation of $F(\theta)$ with θ.

The left-hand side of equation (8) is the excess kinetic energy possessed by the average electron arriving at the anode, this excess energy having been abstracted from the alternating field. If there are n electrons striking the anode per

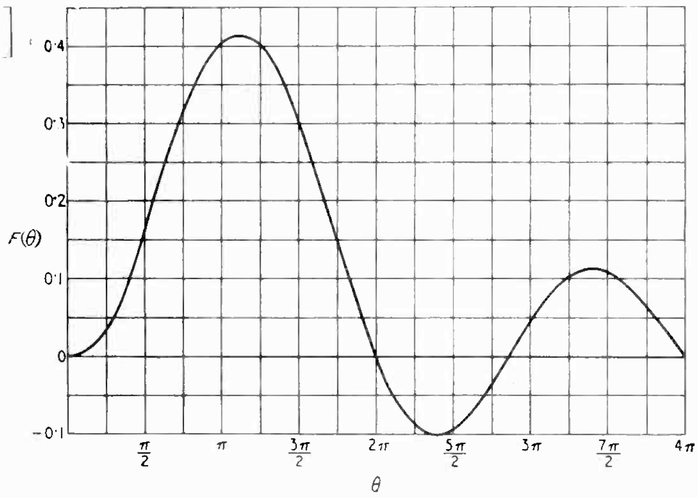

Fig. 5. *Variation of $F(\theta)$ with θ.*

second, then $n(\frac{1}{2}m\bar{u}^2 - V_0 e)$ is a measure of the power taken from the alternating source and can be equated to $V_A^2/2R_1$, where $V_A^2/2$ is the mean-square value of $V_A \cos(\omega t + \phi)$, and R_1 is the damping resistance of the grid-anode space. The current flowing across the space is $ne = I$, and therefore, re-writing equation (8),

$$\frac{V_A^2}{2R_1} = \frac{IV_A^2}{4V_0}F(\theta) \text{ or } R_1 = \frac{2V_0}{I}\cdot\frac{1}{F(\theta)} \quad (10)$$

Bearing in mind the initial assumptions, namely, that the effect of space-charge on the electric field is negligible and that V_A/V_0 is small, equation (10) is an accurate solution, indicating how an electron beam can damp a high-frequency circuit with which it is associated. The form of $F(\theta)$ indicates that when the transit-time is negligible the damping is negligible, and that the damping resistance decreases as the transit-time increases. For larger values of θ the space behaves like a negative resistance, and $F(\theta)$ is a periodic function which asymptotes to zero.

Llewellyn[2] has made an analysis of the space-charge limited diode which shows that the latter behaves like a circuit element of resistance R_2 and reactance X_2 in series, where

$$R_2 = R_0 F(\theta) \quad \ldots \quad (11)$$

and $X_2 = -\frac{1}{\omega C}\left[1 - \frac{\theta^3}{6}\{\theta(1 - \cos\theta) - 2\sin\theta\}\right]$

$F(\theta)$ being given by equation (9) above. In Llewellyn's equation the constant R_0 is the diode static slope, given by

$$R_0 = \frac{\partial V}{\partial i} = \frac{2}{3}\frac{V}{I}$$

where V is the potential at the anode and I is the diode current. The reason for Llewellyn's R_2 being inverted with respect to R_1 is that R_1 is a parallel damping resistance and R_2 is a series damping resistance. In short, if R_1 and X_1 are in parallel, replacing R_2 and X_2, then $R_1R_2 = X_1X_2$.

Now in our derivation of equation (10) we neglected space-charge entirely. Llewellyn appears to take account of it from the beginning. But the difference between the two is only in the constant, independent of θ. As will be shown later this is a very surprising result, and one which leads us to suspect that the effect of space-charge has not been properly incorporated into Llewellyn's theory.

Returning to equation (9), it will be seen that $F(\theta)$ is negative and of finite magnitude over the range $\theta = 2\pi$ to $\theta = 2.86\pi$. In this region, where the damping resistance is negative, a device such as that shown in Fig. 3, with a suitable tuned circuit attached across GA, will tend to become self-oscillatory with a frequency given by f, where

$$f = \frac{\sqrt{2V_0 e/m}}{2\pi D}\theta$$

These oscillations have been obtained by various observers,[2] and to a certain degree they verify the above theory. In early 1940, while searching for a new means of producing microwave oscillations, the writer produced powers of a few microwatts by this method at a frequency of 2,500 Mc/s. No one, so far as the writer is aware, has obtained useful powers from such a device, although the theory sketched above would indicate that the efficiency would be about half that of a klystron. One of the chief limitations is that the minimum negative value of R_1 is determined by the total emission from an oxide-coated cathode, and therefore cannot be less than about 5,000 ohms per square cm.

Equation (10) is of value in the theory of velocity-modulated devices. The space GA corresponds to the bunching gap of a klystron amplifier, and, to a lesser degree, of a klystron oscillator. The beam damping impedance R_1 is sometimes sufficiently small in magnitude to be included with the other losses in the input circuit, although such amplifiers and oscillators usually operate at values of θ less than $\pi/2$.

5. Damping in Grid-Cathode Space

Having made a fairly satisfactory examination of transit-time damping in a space which is free from space-charge effects, let us turn our attention to the much more complex cathode-grid space. It must be said at once that a simple theory such as that given in the previous section is not possible. To satisfy ourselves of the complexity of the problem it is only necessary to examine the derivation of the Langmuir-Child law in a critical manner, remembering that it is to be applied to a transit-time theory. Referring back to Fig. 3, let the distance measured from the cathode K, be represented by x, and let the initial electron velocities be grouped around the velocity $v_0 = \sqrt{2V_0 e/m}$. Let the potential at a plane distant x from K be V, and let the velocity of the electron at that point be v.

Then $\frac{1}{2}mv^2 - V_0 e = Ve$ in the absence of collision or radiation. But by Poisson's equation, $d^2V/dx^2 = 4\pi\rho$, where ρ = the charge density at the plane x, and the current $I = \rho v$.

$$\text{Therefore } \frac{d^2V}{dx^2} = 4\pi I\sqrt{\frac{1}{2(V + V_0)e/m}} = \frac{a}{\sqrt{V + V_0}}$$

and, integrating,

$$\frac{dV}{dx} = 4a\sqrt{V + V_0} + C_1 \quad \ldots \quad (13)$$

where C_1 is a constant.

It is at this point that the development of the Child Law demands an approximation, for it is assumed that $dV/dx = 0$, when $V = V_0$. This makes the constant $C_1 = 0$, and

$$\frac{4}{3}(V + V_0)^{\frac{3}{4}} = \sqrt{4a}\,x + C_2$$

If, now, we assume that $V = -V_0$ at $x = 0$,

$$\text{then} \quad I = \frac{1}{9\pi}\sqrt{\frac{2e}{m}}\frac{V^{\frac{3}{2}}}{x^2} \quad \ldots \quad (14)$$

It must be clearly realized that although it is necessary to assume a finite electron velocity v_0 at the cathode to obtain a potential minimum in the space, the Child Law assumes that the field at the cathode is zero. The more general solution of the differential equation (13)

$$\frac{1}{6a^2}\{2a\sqrt{V + V_0} - C_1\}\{4a\sqrt{V + V_0} + C_1\}^{\frac{1}{2}} = x + C_2$$

is useless for practical work.

But this is not the end of the matter. The electrons emitted by the cathode have velocities distributed about v_0 roughly according to

Maxwell's Law, and this means that another factor further falsifies the Child Law under certain conditions. It is not suggested that equation (14) is a bad approximation for most purposes, but it must be used with caution in transit-time problems, where the nature and position of the potential minimum may be all important.

For example, it is commonly stated that the transit-time of an electron in a fully space-charge limited diode is 1.5 times the transit-time in an identical temperature-limited diode. This is only true if the initial electron velocity is assumed to be zero. Some experiments recently reported by C. N. Smyth[3] point significantly to the importance of the charge cloud in the vicinity of the cathode. Smyth found that when a plane parallel diode of cathode-anode spacing 0.008 cm was placed across a resonant line at a frequency of 3,300 Mc/s, damping occurred even when the negative anode bias made appreciable direct current impossible. The experiment was done with small signal input. The damping could be reduced by cooling the cathode, showing that it was a function of the space-charge or of the cathode surface. The figures given by Smyth are shown in Table II, and for this effect he proposes the name 'total emission damping.'

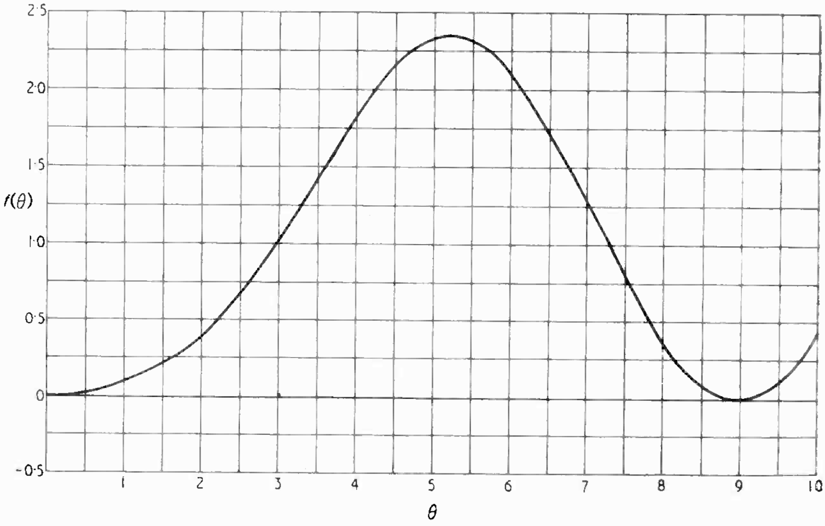

Fig. 6. *Variation of* $f(\theta)$ *with* θ.

TABLE II

Negative bias on anode (V) . .	20	10	5	1
Resistance per sq cm (Ω) . .	∞	Apprec.	200	50

The physical mechanism giving rise to Smyth's effect is not difficult to imagine. The presence of the high-frequency signal disturbs the space charge, and causes some electrons to return to the cathode with velocities in excess of v_0. If, under certain conditions of frequency, this number is a large fraction of the total, serious damping will result. The problem is formally similar to that considered earlier in the present paper, with the important difference that a space-charge cloud is present, and it is interesting to see if a similar analysis can be attempted.

The phenomena occurring in the grid-cathode space can be idealized in one of two physically distinct fashions. It may be done by ignoring space-charge completely, and imagining a space in which electrons, emitted with a finite velocity v_0 from the cathode, move towards the grid in the retarding field due to the effective negative potential of the latter. These electrons would then come to rest and return to the cathode in the manner of a ball thrown vertically into the air. The second way of idealizing the phenomenon is to imagine the electrons emitted by the cathode to be returned to it under the action of a retarding field due to the space-charge itself, the idealization lying in the fact that this retarding field would be regarded as independent of distance from the cathode. Either physical picture leads to the same mathematical conditions, expressed by the equation of motion of a typical electron in the form

$$\frac{d^2x}{dt^2} = \frac{e}{m}\left[-\frac{V_G}{D} + \frac{V_S}{D}\sin(\omega t + \phi)\right]$$

where x is measured away from the cathode surface, V_G/D is the constant retarding field whether due to the grid potential alone or to the grid potential associated with the space-charge, and $V_S \sin(\omega t + \phi)$ is the signal voltage applied between cathode and grid or cathode and space-charge.

Integrating this equation twice, and writing the transit-time of the electron

$$T = T_0 + \delta \text{ where } T_0 = \frac{2v_0 D}{V_G\, e/m}$$

an expression can be found for δ which, substituted in the equation giving the velocity, leads to the equation

$$\frac{\overline{u}^2}{v_0^2} = 1 + \frac{4V_s^2}{V_G^2}\frac{1}{\theta^2}\Big[\Big(1 - \frac{1}{\theta^2}\Big)\{4(1 - \cos\theta) - 4\sin\theta + \theta^2(1 - \cos\theta)\} + \{2(1 - \cos\theta) - \theta\sin\theta\}\Big]$$

$$= 1 + \frac{4V_s^2}{V_G^2} f(\theta) \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

where $\overline{u}^2$ is the mean square velocity of all the electrons and $\theta = \omega T_0$.

The function $f(\theta)$ is exactly comparable with $F(\theta)$ of equation (9) and, like it, vanishes when θ tends to zero. The mode of variation of $f(\theta)$ with θ is shown in Fig. 6.

Rewriting equation (15),

$$\tfrac{1}{2}m\overline{u}^2 - \tfrac{1}{2}mv_0^2 = \frac{4V_s^2(\frac{1}{2}mv_0^2)}{V_G^2} f(\theta)$$

let $\frac{1}{2}mv_0^2 = V_0 e$, and let the cathode emit N electrons per second. Since there is no current to the grid and the condition is stable, all these electrons return to the cathode. As before we equate the energy loss per second to $V_A^2/2R$ where R is a damping resistance. Then

$$\frac{V_s^2}{2R} = \frac{4NV_s^2V_0}{V_G^2} f(\theta)$$

$$\text{or} \quad R = \frac{V_G^2}{8V_0}\cdot\frac{1}{I}\cdot\frac{1}{f(\theta)} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

where I is the total emission of the cathode per second. Van der Ziel[4] and van der Ziel and Versnel[5] have recently reported measurements of total emission damping, using a diode in which D was 0.1 cm and the cathode area was 10 sq cm. Their measurements were made at wavelength of about 5 metres.

Although there are still gaps in the theory, the value of R calculated from equation (16) appears to be of the correct order of magnitude to explain Smyth's and van der Ziel's results. For example, if $V_G = 1$, $V_0 = 1/16$, each being expressed in volts, and $Ne = 100$ mA/sq cm, then R has a minimum value of 10 ohms, which occurs when $\theta \approx 5$. Assuming that the transit-time is wholly defined by the grid voltage, Smyth's $\theta \approx 3$, for which $R = 20$ ohms/sq cm. As van der Ziel points out, the value of $1/R$ is not proportional to the total emission for high values of the latter. This suggests that the penetration of the retarding field into the space-charge is not so complete at high charge densities and that the simple theory given above requires considerable modification.

However, it is noteworthy and encouraging that in equation (16) we have at least an approximation to a quantitative theory of total emission damping. So far as the writer is aware, this analysis of the non-conducting diode has not been made before. The detailed calculation is given in the appendix to the present paper.

6. Conclusion

The object of the article is to examine some of the simpler transit-time phenomena, and to suggest a mathematical technique which appears to be well suited to cases where space-charge distortion of the electrostatic field can be neglected. Attention is drawn to the fact that there is grave doubt regarding some of the results which have been quoted in the past for space-charge limited currents. On the other hand, it is shown that 'total emission damping' in the grid-cathode space of a triode can be explained without recourse to space-charge theory. Whether the explanation is any more than accidentally correct can only be determined by an extensive experimental investigation.

APPENDIX

Transit-Time Damping in Non-Conducting Diode

Let it be assumed that all the electrons are emitted from the cathode with velocity v_0, and that they move under the action of a constant retarding field V_R/D superimposed upon a small alternating field

$$V_s \sin(\omega t + \phi)/D.$$

Then the equation of motion of the typical electron is

$$\frac{d^2x}{dt^2} = \frac{e}{m}\Big[-\frac{V_R}{D} + \frac{V_s}{D}\sin(\omega t + \phi)\Big]$$

$$\text{giving } \frac{dx}{dt} = v_0 - \frac{e}{m}\frac{V_R}{D}\Big[t + \frac{V_s}{\omega V_R}\Big\{\cos(\omega t + \phi) - \cos\phi\Big\}\Big]$$

$$\text{and } x = v_0 t - \frac{e}{m}\frac{V_R}{D}\Big[\tfrac{1}{2}t^2 - \frac{V_s}{\omega V_R}\Big\{t\cos\phi - \frac{1}{\omega}\sin(\omega t + \phi) + \frac{1}{\omega}\sin\phi\Big\}\Big]$$

Consider the time taken by an electron to return to the cathode in the absence of the alternating field. Let this time be T_0.

Then $T_0 = \dfrac{2v_0D}{V_R\, e/m}$ and $\omega T_0 = \theta =$ transit angle.

Now let $T_0 = T + \delta$ and substitute in the equation for x to find δ.

$$0 = -v_0\delta - \frac{e/m.V_R}{2D}\delta^2 + \frac{V_R e/m.}{D}\frac{V_s}{\omega V_R}\Big[T\cos\phi + \delta\cos\phi - \frac{1}{\omega}\Big\{\Big(1 - \frac{\omega^2\delta^2}{2}\Big)\sin(\omega T_0 + \phi) + \omega\delta\cos(\omega T_0 + \phi) - \sin\phi\Big\}\Big]$$

Multiply throughout by $\dfrac{2D\omega^2}{V_R e/m}$ and let $\dfrac{V_s}{V_R} = a$ where a is small.

Then $(\omega\delta)^2[1 - a\sin(\theta + \phi)] + \omega\delta[\theta - 2a\{\cos\phi - \cos(\theta + \phi)\}] - 2a[\theta\cos\phi - \sin(\theta + \phi) + \sin\phi] = 0$

or $A(\omega\delta)^2 + B(\omega\delta) + C = 0$

Then $B^2 - 4AC = \theta^2 - 4a[-\theta\cos\phi - \theta\cos(\theta + \phi) + 2\sin(\theta + \phi) - 2\sin\phi] + 4a^2[2 + \cos^2\phi - \cos^2(\theta + \phi) - 2\cos\theta - 2\theta\sin(\theta + \phi)\cos\phi]$

$$= \theta^2 - 4aX + 4a^2Y = \theta^2\Big[1 - \frac{4a}{\theta^2}(X - aY)\Big]$$

The term in the bracket can be expanded by the Binomial Theorem, since $X/\theta^2 \to$ a finite number when $\theta \to 0$.

$$\text{Therefore } \sqrt{B^2 - 4AC} = \theta\Big[1 - \frac{2a}{\theta^2}X + \frac{2a^2}{\theta^2}\Big(Y - \frac{X^2}{\theta^2}\Big)\Big]$$

neglecting a^3, a^4, . . .

Now let $\cos\phi - \cos(\theta + \phi) = Z$.

$$\omega\delta = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-\theta/2 + aZ + \theta/2 - aX/\theta + a^2(Y - X^2/\theta^2)/\theta}{1 - a\sin(\theta + \phi)}$$

To insert this value of $\omega\delta$ in the expression for dx/dt, rewrite

$$\frac{dx}{dt} = v_0 - \frac{2v_0}{\theta}\omega\delta + \frac{2v_0}{\theta}\frac{V_S}{V_R}\{\cos\phi - \cos(\omega T_0 + \phi + \omega\delta)\}$$

Therefore

$$\frac{dx/dt}{v_0} = -1 + \frac{2a}{\theta}\{\cos\phi - \cos(\theta + \phi)\} - \frac{2\omega\delta}{\theta}\{1 - a\sin(\theta + \phi)\}$$

$$= -1 + \frac{2a}{\theta}\left\{Z - Z + \frac{X}{\theta}\right\} - \frac{2a^2}{\theta^2}\left(Y - \frac{X^2}{\theta^2}\right)$$

$$= -1 + \frac{2aX}{\theta} - \frac{2a^2}{\theta_2}\left(Y - \frac{X^2}{\theta_2}\right)$$

Again writing the mean square value of $dx/dt = \bar{u}^2$,

$$\frac{\bar{u}^2}{v_0^2} = 1 + \frac{4a^2}{\theta^2}\frac{1}{2\pi}\int_0^{2\pi}\left(X^2 + Y - \frac{X^2}{\theta^2}\right)d\phi$$

$$= 1 + \frac{4V_S^2}{V_R^2}\frac{1}{\theta^2}\left[\left(1 - \frac{1}{\theta^2}\right)\left\{4(1 - \cos\theta) - 4\theta\sin\theta + \theta^2(1 + \cos\theta)\right\} + \{2(1 - \cos\theta) - \theta\sin\theta\}\right]$$

REFERENCES

[1] *Phil. Mag.*, February 1931.

[2] "Electron Inertia Effects," Cambridge Physical Tract, 1941.

[3] *Nature*, 1946, Vol. 157, p. 841.

[4] *Nature*, 1947, Vol. 159, p. 677.

[5] *Nature*, 1947, Vol. 159, p. 640.

NEW BOOKS

Pulses and Transients in Communication Circuits

By Colin Cherry. Pp. 317 + xvi with 129 illustrations. Chapman and Hall Ltd., 37, Essex Street, London, W.C.2. Price 32s.

The scope of this book is well described in a phrase of the author's, who says that it is intended to 'bridge the gap between simple alternating-current theory and operational methods.'

The point of departure is the basic set of differential equations describing the behaviour of a linear network and its solution in terms of exponential and, in particular, sine waves; the mathematics is not shirked but here, as throughout the book, appeal is made to physical arguments wherever possible, rather than to formal mathematical reasoning. A discussion of the 'frequency-spectrum' of a waveform and its expression in the form of a Fourier series or Fourier integral brings out the relation between 'frequency-response' and 'time-response'; it is followed by separate studies of the steady-state characteristics and the transient response of networks, the latter including the effects on waveform of amplitude and phase distortion. This leads on to various approximate and idealized characteristics which, although perhaps impossible in real networks, help in estimating the general behaviour of systems in reproducing the waveform of a transmitted signal. The characteristics of networks and the relation between frequency-response and time-response are further discussed from the point of view which regards the output signal as being made up of a large number of echoes of the input signal with different delay and different amplitudes. This approach lends itself to the approximate calculation of transient response, given the steady-state characteristics, and also to methods of synthesizing networks with given characteristics.

Two particular components of telecommunication systems are dealt with in more detail, namely multistage amplifiers and asymmetric-sideband channels. The basic factors affecting the transient response of identical stages in tandem are analysed, including the question of optimum bandwidth for best signal-to-noise ratio. Unequal distortion of sidebands in an amplitude-modulated system leads to non-linear distortion of the envelope, but it is shown how linear technique can be extended to this case; the distortion of pulses in particular practical asymmetric-sideband systems is dealt with. A very valuable feature of the book is the list of nearly 200 references to books and articles giving more detailed discussion of the various theoretical and practical points raised.

There are a few minor errors which should not give trouble to the careful reader. Some of the remarks on phase distortion may be misleading. Thus Fig. 60(c), 'Response typical of phase distortion,' might be assumed to show the effect on a rectangular pulse of a network with only phase distortion, such as the phase equalizers referred to in the text. Such a network would, however, leave unaltered the steep sides of the pulse, apart from a possible change of polarity; the waveform shown is, in fact, typical of combined amplitude and phase distortion. Later on, in section 4.40, the effect of phase distortion on a step wave is discussed; a low-pass system is assumed. The statement is made that if the higher frequencies are delayed more than the lower, the build-up time is spoilt but overswings are reduced, whereas if the higher frequencies are less delayed, the build-up can be steeper with overswings. This is the wrong way round: it does not agree for instance with such common characteristics as those illustrated in Figs. 51, 52, and 82.

To sum up, this book is mainly concerned with the problem of estimating transient response, given amplitude and phase characteristics. It deals with the problem in broad outline and with a practical flavour and is concerned with generally applicable, even though approximate, rules rather than precise calculations for given network configurations. It fills a gap in the literature and will be very welcome, especially to the many engineers who have been brought up on conventional a.c. theory and now have to turn their minds to circuits in which reproduction of the input waveform as faithfully as possible is the object.

W. E. T.

Microwave Antenna Theory and Design

Edited by Samuel Silver. Pp. 623 + xviii, 338 Figs. (Vol. 12, M.I.T. Radiation Laboratory Series). McGraw-Hill Publishing Co., Aldwych House, London, W.C.2. Price 48s. (in U.K.).

This book is one of a series of volumes which have been prepared by the staff of the Radiation Laboratory of the Massachusetts Institute of Technology operating under the supervision of the National Defence Research

Committee. An editorial staff was appointed and authors chosen from among the experts in the various fields. This staff worked at the preparation of these volumes for six months or more after the work of the Radiation Laboratory was completed, the finances being provided by the Office of Scientific Research and Development. The work described, however, is not confined to that of any one laboratory, but is the collective result of work done in many laboratories, in the United States, England, Canada and other Dominions Although this book contains over 600 pages it is explained in the preface that many subjects have had to be omitted completely or treated very briefly. Two chapters on rapid scanning antennas had to be omitted, partly on grounds of military security, but some of the material has been incorporated in other chapters.

The book is divided into sixteen chapters, of which the first gives a general survey of the subject. The second chapter is devoted to circuit relations, four-terminal networks, reciprocity and Thévenin's theorems, transmission-line charts, antennas as impedances and coupled systems. In the chapter on radiation from antennas the reader is referred to Stratton's " Electromagnetic Theory " for more detailed treatment, although the treatment seems fairly complete. The m.k.s. system of units is employed throughout the book, which means that current densities are expressed in amperes per square metre and charge densities in coulombs per cubic metre. It is pointed out that it is really a misnomer to call an aerial of length $\lambda/2$ a half-wave dipole, since the charges are distributed over the whole aerial, but it is retained in the book because of its convenience.

Chapter 4 is entitled ' Wavefronts and Rays ' and deals with Huygens-Fresnel principle and Fermat's principle, while Chapter 5 is devoted to the important but complex subjects of scattering and diffraction, including Babinet's principle and its application to slot aerials. These subjects are continued in the following chapter on ' Aperture illumination and Antenna patterns ' in which the transition from the Fresnel to the Fraunhofer region is discussed. The first third of the book has thus been devoted to general theoretical considerations of microwave antennas. Chapter 7 deals with microwave transmission lines, including the various types of waveguide, and the following chapter with the feeding connection between the line and the antenna. These first eight chapters have been written by the editor, S. Silver, but Chapter 9 on ' Linear-array Antennas and Feeds ' is by J. E. Eaton, L. J. Eyges and G. G. Macfarlane, the last named being of the British Telecommunications Research Establishment, Malvern. This chapter of nearly 80 pages deals with broadside and end-fire arrays, slot radiators in waveguide walls, beacon antenna systems and allied subjects.

Chapter 10 by J. R. Risser discusses electromagnetic horns fed from waveguides ; this is a complex subject and is discussed very fully. The same can be said of the following chapter in which the same author deals with dielectric and metal-plate lenses. Chapter 12 on ' Pencil-beam and simple Fanned-beam Antennas ' is by S. Silver ; it deals with paraboloid reflectors and the problems that arise in connection with them, the so-called pillbox and cheese antennas, and the half-beacon antenna. In the following chapter L. C. van Atta and T. J. Keary discuss ' Shaped-beam Antennas,' the shaping and construction of the various types of reflector, and the use of multiple horn feeds for illuminating the reflector. Chapter 14 by van Atta and R. M. Redheffer is entitled ' Antenna installation Problems ' ; it discusses the location of the antenna on land, on ships and on aeroplanes, the effects due to the radome or dielectric housing, and the shaping of the latter to obtain the best results. The last two chapters are by H. Krutter on measurement techniques and O. A. Tyson on measurement equipment, both of great importance in obtaining experimental confirmation of the theories developed in the earlier chapters. Measurement of impedance, standing-wave ratios, intensity of field and gain are all discussed together with the necessary klystrons, magnetrons, amplifiers and detectors.

Numerous references are given as footnotes throughout to publications giving further details of points referred to.

One cannot but be impressed by the fact that this book of over 600 pages is No. 12 in a series of 27 volumes on radar and allied subjects prepared by the Radiation Laboratory staff. It shows every sign of having been prepared with the greatest care ; the symbols and nomenclature have been carefully chosen and are consistent from beginning to end. In the Editorial of July 1943 we pointed out that the so-called reciprocity theorem of Maxwell was really a variant of Helmholtz's theorem ; we are interested to notice that on p. 19 Silver refers to it as the Rayleigh reciprocity theorem. It is evidently of doubtful parentage.

The volume under review is undoubtedly a very valuable addition to the literature of the subject. Although written by several authors it shows no sign of discontinuity and this is doubtless due to careful editing ; in the preface the editor states that he has made use of his prerogative to revise the chapters written by other authors and that the responsibility for errors of omission or commission is his. He overlooked the fact that one author used capacitive and another capacitative. He thanks Dr. G. G. Macfarlane of T.R.E. for his critical review of several of the theoretical chapters as well as his contribution on the theory of slot radiators. This confirms the impression that no trouble has been spared to make the book as authoritative and at the same time as readable as possible. G. W. O. H.

Mercury Arcs (2nd Edition).

By F. J. Teago and J. F. Gill. Pp. 107 + viii with 50 illustrations. Methuen & Co., Ltd., 36, Essex St., London, W.C.2. Price 6s. 6d.

Outline of Radio

By R. S. Elven, T. J. Fielding, E. Molloy, H. E. Penrose, C. A. Quarrington, M. G. Say, R. C. Walker and G. Windred. Pp. 688 with 503 illustrations. George Newnes Ltd., Tower House, Southampton St., Strand, London, W.C.2. Price 21s.

Basic Technical Electricity

By H. Cotton. Pp. 238 with 80 illustrations. Cleaver-Hume Press Ltd., 42a, South Audley St., London, W.1. Price 8s. 6d.

Electronics in the Factory

Edited by Professor H. F. Trewman. Pp. 188 with 63 illustrations. Sir Isaac Pitman & Sons Ltd., Parker St., Kingsway, London, W.C.2. Price 20s.

Electronic Time Measurements

Edited by Britton Chance, Robert I. Hulziger, Edward F. MacNichol, Jr. and Frederick C. Williams. Pp. 538 + xviii. (Vol. 20, M.I.T. Radiation Laboratory Series). McGraw-Hill Publishing Co., Aldwych House, London, W.C.2. Price 42s. (in U.K.).

THE ENGINEERS' GUILD

A Scottish Branch of the Guild is being established with headquarters in Glasgow and at a meeting held on 6th May a provisional committee was elected to deal with its development.

OSCILLATION AMPLITUDE IN SIMPLE VALVE OSCILLATORS

By A. S. Gladwin, B.Sc., A.M.I.E.E.

(King's College, University of London)

(Concluded from page 170, May issue)

7. Amplitude Instability (Squegging)

In the previous Sections methods were developed for calculating the oscillation amplitude, based on the assumption that the amplitude is stable; but this is not always so.

For instability of any kind to exist it is necessary that a small increase in the oscillation amplitude should cause the amplification round the whole circuit at the oscillation frequency to become greater than 1 and vice versa; i.e., dA/dV_{g1} must be positive at the value of V_{g1} in question. The difference between the two kinds of instability is that in the periodic type the fundamental requirement is satisfied only when certain relations exist between the rate of change of the oscillation amplitude, the time-constant R_gC_g of the grid capacitor and resistor, and the Q factor of the feedback network. This type of instability is essentially dynamical.

In contrast, the aperiodic type of instability does not depend on any of the time factors mentioned above, though these may limit the speed with which the amplitude passes through the unstable values.

Periodic instability or squegging is of greater practical importance and will be treated first. When this occurs the oscillation amplitude becomes modulated or periodically interrupted. As is well known, this kind of instability can be eliminated by reducing the time-constant of the grid resistor and capacitor.

When the value of the grid capacitor is increased just beyond the critical point where instability sets in, the modulation is of small amplitude and sinusoidal waveform. This experimental fact is used as the basis of the following analysis.

Since the amplitude of the modulation is small the modulation may be regarded as a perturbation of the steady state and the effects which it produces may be calculated by quasilinear methods. The high-frequency oscillation may be regarded as a carrier wave whose function is to convey the modulation through the valve and feedback network, both of which alter the amplitude and phase of the modulation in a calculable manner. So far, then, as the modulating wave is concerned the oscillator may be regarded as a low-frequency feedback amplifier with known amplitude-frequency and phase-frequency characteristics, and Nyquist's criterion[3] for the stability of such amplifiers may be applied.*

The physical mechanism of squegging is roughly as follows. If the alternating grid voltage has a small sinusoidal modulation the grid-bias voltage is also modulated sinusoidally, but to a lesser extent and with a lagging phase which depends on the magnitude of the time-constant R_gC_g. The modulation on the alternating anode current is the resultant of the modulations produced by the grid alternating voltage and the grid-bias voltage. Since an increase in the magnitude of the grid-bias voltage reduces the anode current, the resultant modulation on the anode current has a leading phase, and may also, owing to the non-linearity of the valve characteristic, have an increased amplitude.

In its passage through the feedback network the modulation is retarded and reduced in amplitude to an extent depending on the Q factor and the modulation frequency. At some frequency the phase shift produced by the network may be equal and opposite to that produced by the valve, so that the total phase shift is zero. If only one such frequency exists, and if at this frequency the total amplification of the modulation round the circuit is greater than 1, the modulation tends to increase, and the oscillation amplitude is therefore unstable. If the amplification is less than 1, or if no frequency exists at which the total phase shift is zero, the oscillation amplitude is stable.

The operation of the oscillator will now be analysed from this point of view. It will be supposed that R_g/X_g is larger than the values indicated in Fig. 4, and that the modulation frequency is small compared with the oscillation frequency, so that the formulæ developed in the first part of Section 4 can be applied to the instantaneous values of the slowly varying grid-bias voltage and oscillation amplitude. The effect of harmonic voltages is neglected.

*This method of determining the amplitude stability has also been described by Edson[4].

To investigate the stability of any particular amplitude, suppose that the oscillator is actually operating with this amplitude. Then the problem is to find whether this condition represents a state of stable or unstable equilibrium. In low-frequency feedback amplifiers the method of investigating the stability is to interrupt the circuit at some point and find the amplification and phase shift round the circuit over the whole frequency range. The equivalent procedure for the oscillator is as follows.

Suppose that a signal generator of zero impedance having a terminal voltage v_3 equal to that between 5 and 4 (Fig. 1) is connected between 5 and 4, and let an impedor of value equal to the impedance at fundamental frequency between 5 and 4, looking towards the grid, be connected between 3 and 4; then the current flowing between 3 and 5 is zero, and the circuit may be broken at this point without altering the voltages in any part of the oscillator.

Now let the signal-generator voltage be modulated in amplitude to become $V_3\ (1 + a\cos\omega_s t)\cos\omega_0 t$ where $a \ll 1$ and $\omega_s \ll \omega_0$.

The alternating grid voltage becomes

$$V_{g1m}\cos\omega_0 t = V_{g1}(1 + a\cos\omega_s t)\cos\omega_0 t \quad (7.1)$$

The subscript m is used to denote the modulated value of a quantity.

Let the grid-bias voltage be

$$V_{gm} = V_g\{1 + a_1\cos(\omega_s t - \theta_1)\} \quad \ldots \quad (7.2)$$

From (4.3) the mean value of the grid current over a period long compared with $1/\omega_0$ but short compared with $1/\omega_s$ is

$$I_{gm} = \frac{b_g V_{g1m}}{\pi}(\sin\phi_m - \phi_m\cos\phi_m)$$

in which

$$\cos\phi_m = \frac{-(V_{gm} - V_{cg})}{V_{g1m}} = \cos\phi\left\{1 - a\cos\omega_s t + \frac{a_1 V_g}{V_g - V_{cg}}\cos(\omega_s t - \theta_1)\right\}$$

Let $\phi_m = \phi + \gamma$, γ being small, then

$$\cos\phi_m = \cos\phi - \gamma\sin\phi$$

and so

$$\gamma\sin\phi = \cos\phi\left\{a\cos\omega_s t - \frac{a_1 V_g}{V_g - V_{cg}}\cos(\omega_s t - \theta_1)\right\}$$

The expression for I_{gm} becomes

$$I_{gm} = \frac{b_g V_{g1}}{\pi}(1 + a\cos\omega_s t)(\sin\phi - \phi\cos\phi + \gamma\phi\sin\phi).$$

On substituting for $\gamma\sin\phi$ and putting [from (4.3)]

$$V_{g1}\frac{b_g R_g}{\pi}(\sin\phi - \phi\cos\phi) = -V_g$$

$= V_{g1}\cos\phi - V_{cg}$, I_{gm} can be written

$$I_{gm} = \frac{-V_g}{R_g}\left[1 + a\left\{1 + \frac{\phi b_g R_g}{\pi}\left(1 - \frac{V_{cg}}{V_g}\right)\right\}\cos\omega_s t - \frac{a_1\phi b_g R_g}{\pi}\cos(\omega_s t - \theta_1)\right] \quad (7.3)$$

This current is equal to the sum of the steady and low-frequency currents flowing through R_g and C'_g $(= C_g + c_{gk})$, which is

$$I_{gm} = \frac{-V_{gm}}{R_g} - C'_g\frac{d}{dt}V_{gm}$$

$$= \frac{-V_g}{R_g}\Big\{1 + a_1\cos(\omega_s t - \theta_1) - a_1\omega_s C'_g R_g\sin(\omega_s t - \theta_1)\Big\} \quad \ldots \quad (7.4)$$

In (7.3) $\cos\omega_s t$ may be written as

$$\cos\theta_1\cos(\omega_s t - \theta_1) - \sin\theta_1\sin(\omega_s t - \theta_1)$$

Equating the coefficients of $\cos(\omega_s t - \theta_1)$ in (7.3) and (7.4) gives

$$\frac{a_1}{a} = \cos\theta_1\frac{1 + \dfrac{\phi b_g R_g}{\pi}\left(1 - \dfrac{V_{cg}}{V_g}\right)}{1 + \dfrac{\phi b_g R_g}{\pi}}$$

and equating the coefficients of $\sin(\omega_s t - \theta_1)$ gives

$$\frac{a_1}{a} = \sin\theta_1\left\{1 + \frac{\phi b_g R_g}{\pi}\left(1 - \frac{V_{cg}}{V_g}\right)\right\}\Big/\omega_s C'_g R_g.$$

Since $\phi b_g R_g/\pi$ is always large compared with 1 the first equation can be replaced with negligible error by

$$\frac{a_1}{a} = \left(1 - \frac{V_{cg}}{V_g}\right)\cos\theta_1 \quad \ldots \quad \ldots \quad \ldots \quad (7.5)$$

The ratio of the two equations is

$$\tan\theta_1 = \frac{\omega_s C'_g R_g}{1 + \dfrac{\phi b_g R_g}{\pi}}$$

Let $\phi = \phi_0 + \delta$ Then

$$1 + \frac{\phi\, b_g\, R_g}{\pi} = \left(1 + \frac{\phi_0\, b_g\, R_g}{\pi}\right)\left(1 + \frac{\delta}{\phi_0 + \pi/b_g R_g}\right)$$

Substituting for ϕ_0 and δ from (4.4) and (4.5) gives

$$1 + \frac{\phi\, b_g\, R_g}{\pi} = \frac{b_g\, R_g}{\pi}\tan\phi_0\left\{1 - \frac{V_{cg}\,\pi\cos^2\phi_0}{V_{g1}\, b_g\, R_g\sin^3\phi_0}\right\}$$

Over the range of values of $b_g R_g$ shown in Fig. 2 the quantity $\frac{\pi \cos^2 \phi_0}{b_g R_g \sin^3 \phi_0}$ varies from 0.29 to 0.34. Since V_{ca}/V_{g1} is always small it is permissible to put

$$1 + \frac{\phi\, b_g R_g}{\pi} \approx \frac{b_g R_g}{\pi} \tan \phi_0 = S \qquad .. \quad (7.6)$$

This quantity is shown in Fig. 6 as a function of $b_g R_g$.

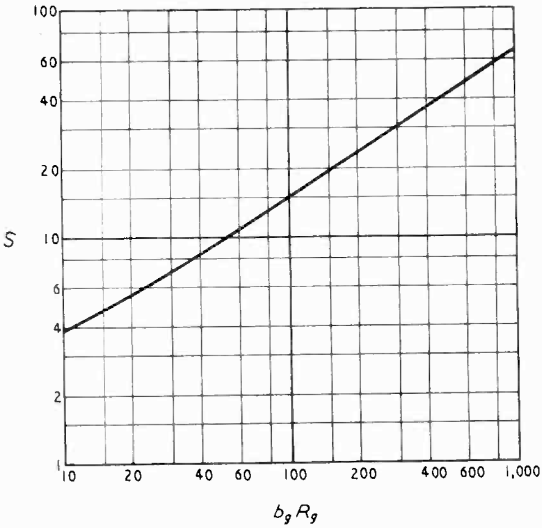

Fig. 6. *Graph of instability constant S.*

The expression for $\tan \theta_1$ can now be written as

$$\tan \theta_1 = \omega_s C'_g R_g / S \qquad .. \qquad .. \qquad .. \quad (7.7)$$

From (4.7) the amplitude of the grid current of fundamental frequency is

$$I_{g1m} = 2 I_{gm}(1 - \phi^2_m/10 + \cdot \cdot)$$
$$\approx 2 I_{gm}(1 - \phi^2/10)$$

From (7.4) and (4.7) this is

$$I_{g1m} = I_{g1}\{1 + a_1 \cos(\omega_s t - \theta_1) - a_1 \omega_s C'_g R_g \sin(\omega_s t - \theta_1)\} \qquad .. \quad (7.8)$$

The grid input resistance is therefore

$$r_{gm} = \frac{V_{g1m}}{I_{g1m}} = r_g \{1 + a \cos \omega_s t - a_1 \cos(\omega_s t - \theta_1) + a_1 \omega_s C_g' R_g \sin(\omega_s t - \theta_1)\} \quad (7.9)$$

To calculate the anode current it is necessary to know first the anode voltage. It was shown in Section 3 that the anode voltage of fundamental frequency could be represented as the sum of two components, one in phase with, and the other in quadrature with the grid voltage. It was further shown in Section 5 that the quadrature component has a negligible effect on the amplitude of the anode current. It may therefore be assumed that the effective part of the anode voltage is

$$- V_{a1}\{1 + a_2 \cos(\omega_s t - \theta_2)\} \cos \omega_0 t$$

It was shown in Section 3 that no change can occur in the amplitude or phase of the modulation as the voltage wave passes through the feedback network from the anode to the grid, and it may be assumed that the same is true for intermediate points in the network. The voltage between 3 and 4 is therefore

$$V_3\{1 + a_2 \cos(\omega_s t - \theta_2)\} \cos \omega_0 t$$

To determine the stability of the oscillation amplitude this voltage is to be compared with the voltage between 5 and 4 produced by the signal generator, which is

$$V_3(1 + a \cos \omega_s t) \cos \omega_0 t$$

One of the conditions for instability is that the two modulations should have the same phase; $\therefore \theta_2 = 0$. Another condition is that the amplification should be greater than 1; $\therefore a_2/a > 1$. It will appear later that there can be only one value of ω_s for which $\theta_2 = 0$. Consequently, by Nyquist's theory, the two conditions above are sufficient to determine the instability of the oscillation amplitude.

The anode current is

$$i_{am} = b_a\{v_{gm} + v_{aam}/\mu - V_{ca}\}^{\frac{3}{2}} - k i_{gm}.$$

Putting $v_{aam} = - V_{a1}(1 + a_2 \cos \omega_s t) \cos \omega_0 t$, substituting for v_{gm} and V_{gm} from (7.1) and (7.2), and expanding the result by Taylor's theorem

$$i_{am} = b_a\{(V_{g1} - V_{a1}/\mu) \cos \omega_0 t + V_g - V_{ca}\}^{\frac{3}{2}} - k\, i_{gm}$$
$$+ \{(a V_{g1} - a_2 V_{a1}/\mu) \cos \omega_s t \cos \omega_0 t + a_1 V_g \cos(\omega_s t - \theta_1)\}$$
$$\times \tfrac{3}{2} b_a\{(V_{g1} - V_{a1}/\mu) \cos \omega_0 t + V_g - V_{ca}\}^{\frac{1}{2}}$$

Let $\tfrac{3}{2} b_a\{(V_{g1} - V_{a1}/\mu) \cos \omega_0 t + V_g - V_{ca}\}^{\frac{1}{2}}$
$= g_0 + 2 g_1 \cos \omega_0 t + 2 g_2 \cos 2\omega_0 t + \ldots$

On substituting for I_{g1m} from (7.8) the amplitude of the anode current of fundamental frequency can be expressed as

$$I_{a1m} = I_{a1} + (g_0 + g_2)(a V_{g1} - a_2 V_{a1}/\mu) \cos \omega_s t + 2 a_1 g_1 V_g \cos(\omega_s t - \theta_1)$$
$$- a_1 k \frac{V_{g1}}{r_g}\{\cos(\omega_s t - \theta_1) - \omega_s C_g' R_g \sin(\omega_s t - \theta_1)\}$$

Putting $V_{a1} = - G_i R_i V_{g1}$ and $V_{g1} = - I_{a1}/G_t$

$$I_{a1m} = I_{a1}\Big[1 - g'_0(a/G_t + a_2 R_i/\mu) \cos \omega_s t - a_1\Big(\frac{2 g_1 V_g}{G_t V_{g1}} - \frac{k}{G_t r_g}\Big) \cos(\omega_s t - \theta_1) - \frac{a_1 k}{G_t r_g} \omega_s C'_g R_g \sin(\omega_s t - \theta_1)\Big] \quad (7.10)$$

where $g_0' = g_0 + g_2 = \frac{3b_a}{2\pi}\int_{-\pi}^{\pi}\{V_{g1}(1 + G_tR_i/\mu)$

$\cos\theta + V_g - V_{ca}\}^{\frac{1}{2}}\cos^2\theta\, d\theta$

$$g_1 = \frac{3b_a}{4\pi}\int_{-\pi}^{\pi}\{V_{g1}(1 + G_t R_i/\mu)\cos\theta + V_g - V_{ca}\}^{\frac{1}{2}}\cos\theta\, d\theta$$

From the integral evaluated in the Appendix the following expressions are obtained for g_0' and g_1

When $V_{g1}(1 + G_tR_i/\mu) < V_g - V_{ca}$; i.e.,

$$\left(\frac{V_{ca}}{V_g} - 1\right) > \frac{1}{K}$$

$$\left.\begin{aligned} g_0' &= \frac{3b_a}{2}(V_g - V_{ca})^{\frac{1}{2}}\left\{1 - \frac{3}{32K^2(1 - V_{ca}/V_g)^2}\right\} \\ g_1 &= \frac{-3b_a}{8K}\cdot\frac{V_g}{(V_g - V_{ca})^{\frac{1}{2}}}\left\{1 + \frac{3}{32K^2(1 - V_{ca}/V_g)^2}\right\}\end{aligned}\right\} \quad \ldots \quad (7.11)$$

and when $\left(\frac{V_{ca}}{V_g} - 1\right) < \frac{1}{K}$

$$\left.\begin{aligned} g'_0 &= \frac{3b_a}{2^{\frac{3}{2}}}\left(\frac{-V_g}{K}\right)^{\frac{1}{2}}H\left(1 - \frac{7H}{16} + \frac{19H^2}{256} + \frac{65H^3}{8192} + \ldots\right) \\ g_1 &= \frac{3b_a}{2^{\frac{5}{2}}}\left(\frac{-V_g}{K}\right)^{\frac{1}{2}}H\left(1 - \frac{3H}{16} - \frac{5H^2}{256} - \frac{35H^3}{8192} + \ldots\right)\end{aligned}\right\} \quad \ldots \quad (7.12)$$

where H is given by (5.8).

Expression (7.10) for the anode current was derived from a consideration of the relations between the currents and voltages in the valve, but the anode current and anode voltage must also satisfy certain relations determined by the properties of the feedback network. Some of these properties were examined in Section 3. Expression (3.13) shows that the amplitude of the anode current of fundamental frequency associated with the anode voltage

$-V_{a1}(1 + a_2\cos\omega_s t)\cos\omega_0 t$ is

$$\frac{V_{a1}}{R_i}\left(1 + a_2\cos\omega_s t - 2a_2 Q\frac{\omega_s}{\omega_0}\sin\omega_s t\right)$$

The derivation of this expression, however, was based on the assumption that R_i is constant, but since the grid input resistance is modulated the impedance between 3 and 4 must also be modulated, and hence R_i is modulated. From (3.10) and (7.9) the modulated value of R_i is

$$R_{im} = R_i\left[1 + \frac{R_0}{R_0 + r_g}\{a\cos\omega_s t - a_1\cos(\omega_s t - \theta_1) + a_1\omega_s C'_g R_g\sin(\omega_s t - \theta_1)\}\right]$$

R_0, it will be recalled, is the resistive component of the network output impedance with the power supplies to the valve disconnected. Substituting R_{im} for R_i in the above expression for the amplitude of the fundamental anode current gives

$$I_{a1m} = I_{a1}\left[1 + a_2\cos\omega_s t - 2a_2Q\frac{\omega_s}{\omega_0}\sin\omega_s t - \frac{R_0}{R_0 + r_g}\{a\cos\omega_s t - a_1\cos(\omega_s t - \theta_1) + a_1\omega_s C'_g R_g\sin(\omega_s t - \theta_1)\}\right]$$

The trigonometrical terms in this and in (7.10) can be expressed in terms of $\cos\omega_s t$ and $\sin\omega_s t$. When this is done the coefficients of $\cos\omega_s t$ and $\sin\omega_s t$ in the two expressions may be equated. Equating the coefficients of $\cos\omega_s t$ and substituting for a_1 according to (7.5), and for $\omega_s C'_g R_g$ according to (7.7)

$$\frac{a_2}{a}\left(1 + \frac{g'_0 R_i}{\mu}\right) = \frac{R_0}{R_0 + r_g} - \frac{g'_0}{G_t} - \left(1 - \frac{V_{cg}}{V_g}\right)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g} + \frac{2g_1 V_g}{G_t V_{g1}}\right)\cos^2\theta_1 - S\left(1 - \frac{V_{cg}}{V_g}\right)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right)\sin^2\theta_1$$

Equating coefficients of $\sin\omega_s t$ and substituting for $\omega_s C'_g R_g$

$$\frac{a_2}{a} = \frac{R_g}{2QSX_g}\left(1 - \frac{V_{cg}}{V_g}\right)\left\{\frac{2g_1 V_g}{G_t V_{g1}} - (S - 1)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right)\right\}\cos^2\theta_1.$$

Solving these two equations for $\cos^2\theta_1$ and a_2/a_1

$$\cos^2\theta_1\left(1 - \frac{V_{cg}}{V_g}\right)\left\{\frac{2g_1 V_g}{G_t V_{g1}} - (S - 1)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right)\right\}\left\{1 + \frac{R_g}{2QSX_g}\left(1 + \frac{g'_0 R_i}{\mu}\right)\right\} = \frac{R_0}{R_0 + r_g} - \frac{g'_0}{G_t} - S\left(1 - \frac{V_{cg}}{V_g}\right)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right) \quad (7.13)$$

and $$\frac{a_2}{a}\left(1 + \frac{g'_0 R_i}{\mu} + \frac{2QSX_g}{R_g}\right) = \frac{R_0}{R_0 + r_g} - \frac{g'_0}{G_t} - S\left(1 - \frac{V_{cg}}{V_g}\right)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right) \quad \ldots \quad (7.14)$$

From (7.7)

$$\frac{\omega_s}{\omega_0} = \frac{SX_g}{R_g}(\cos^{-2}\theta_1 - 1)^{\frac{1}{2}}$$

This equation gives the modulation frequency at which the phase shift of the modulation round the oscillator is zero. If the value of $\cos^2\theta_1$ given by (7.13) is greater than 1 or less than 0 no real value of ω_s exists and the oscillation amplitude is then stable. It is clear that ω_s can have only one real value, as positive and negative frequencies are physically indistinguishable. Using the value of $\cos^2\theta_1$ given by (7.13) the criterion for a real value of ω_s can be expressed as

$$\frac{R_g}{2QSX_g}\left(1 + \frac{g'_0 R_i}{\mu}\right) > \frac{-\frac{g'_0}{G_t} + \frac{V_{cg}}{V_g}\cdot\frac{R_0}{R_0 + r_g} - \left(1 - \frac{V_{cg}}{V_g}\right)\left(\frac{2g_1 V_g}{G_t V_{g1}} - \frac{k}{G_t r_g}\right)}{\left(1 - \frac{V_{cg}}{V_g}\right)\left\{\frac{2g_1 V_g}{G_t V_{g1}} - (S - 1)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right)\right\}} > -1 \quad \ldots \quad (7.15)$$

The second requirement for instability is that the amplification of the modulation a_2/a should be greater than 1. From (7.14) this requirement can be expressed in the form

$$\frac{2QSX_g}{R_g} < -\frac{r_g}{R_0 + r_g} - \frac{g'_0}{G_t}(1 + G_t Ri/\mu) - S\left(1 - \frac{V_{cg}}{V_g}\right)\left(\frac{R_0}{R_0 + r_g} - \frac{k}{G_t r_g}\right) \quad (7.16)$$

Both (7.15) and (7.16) must be satisfied before instability can occur. It is clear that the most important factor is not the time-constant $C'_g R_g$ but the quantity

$$\frac{R_g}{2QSX_g} = \frac{\pi\omega_0 C'_g}{2Qb_g \tan\phi_0}$$

Thus instability may be stopped either by reducing C'_g or by increasing Q in the same ratio, all other parameters remaining constant. Reduction of the grid resistor R_g has the effect of increasing $\tan\phi_0$ but, as can be found from Fig. 6, the rate of change of $\tan\phi_0$ with R_g is small, so that a greater proportional change in R_g would be necessary. The formulae also show that if the frequency of an oscillator is varied over a wide range instability is more likely to occur at the higher frequencies—a fact already well known from experiment. This tendency is offset to some extent by the fact that Q usually increases with frequency.

It is also clear from (7.15) and (7.16) that no matter how large $C'_g R_g$ may be, the oscillation amplitude is stable if the middle term in (7.15) is less than -1 or the right-hand term in (7.16) is negative. But if the time-constant is infinitely large no change can take place in the grid-bias voltage, and so the grid resistor R_g could be replaced by a source of fixed potential of value V_g. It is sometimes claimed that stable operation of oscillators is not possible with a fixed grid-bias voltage, but it is clear from the above that this statement is not generally true.

It has been assumed that the oscillator circuit is arranged in the manner shown in Fig. 1. The preceding formulae may, however, be applied to any circuit arrangement if the value of the reactance X_g is modified as follows. It is obvious that the important capacitance is the effective capacitance, at low frequencies, in parallel with the resistor R_g. This capacitance can easily be determined by inspection of the complete oscillator circuit. If its value is C_{ge}, the effective value of X_g to be inserted in the formulae is $X_{ge} = 1/\omega_0 C_{ge}$.

The criteria for stability which have been developed here are based on the assumption that if a small sinusoidal modulation of the oscillation amplitude tends to die out, the amplitude is stable under all conditions. It is conceivable, however, that the amplitude may be stable with respect to small disturbances but may be unstable when large disturbances such as the switching of the h.t. supply voltage, take place. Experimental evidence on this point is inconclusive, but appears to indicate that if such an effect occurs it makes only a small difference to the critical values of X_g.

As an example of a stability calculation the oscillator, the constants of which are given at the end of Section 6, will be examined. It is required to find the maximum value of R_g/X_g consistent with stable operation when the dynamic resistance of the resonant circuit by itself is 10,000Ω. The Q of the resonant circuit by itself is 10.

From the performance figures worked out in Section 6

$V_{g1} = 24.3$ V, $V_g = -21.4$ V, $K = 0.98$, $R_i = 9580\Omega$, $r_g = 58200\Omega$, $G_t = -0.000209$ mho.

From Fig. 6 $S = 11.5$; from (7.12) and (5.8) $g'_0 = 0.001$ mho, and $g_1 = 0.000555$ mho.

The grid input resistance reduces the Q of the network below that for the resonant circuit alone to the value $\frac{10 \times 9580}{10{,}000} = 9.58$. Since the ratio of the mutual inductance between coils to the inductance of the anode coil is 0.5 the output resistance R_0 is $10{,}000 \times 0.5^2 = 2{,}500$.

Substituting these values into (7.15) gives

$$0.0067 \frac{R_g}{X_g} > 0.016 > -1$$

which is satisfied if $R_g/X_g > 2.4$

Similarly (7.16) is satisfied if $R_g/X_g > 114$.

The oscillation amplitude is therefore stable if $R_g/X_g < 114$. In this particular example there is not a large margin between the value of R_g/X_g which is small enough to satisfy the stability requirement and the value which is large enough to produce a negligible error in the formulae for the grid-bias voltage and the grid input resistance.

8. Oscillation Hysteresis

There are two types of oscillation hysteresis which may occur in a self-biased oscillator. The first and more familiar type is associated with small oscillation amplitudes and with values of coupling between anode and grid circuits just sufficient to start or maintain oscillation. The second type is peculiar to large amplitudes and large values of coupling, and is rarely encountered in practical oscillators. It will be discussed at the end of the Section.

If the coupling (e.g., the mutual inductance) between anode and grid circuits is increased from a small value, a point is reached beyond which oscillation begins spontaneously. It might be expected that when the coupling is just greater than this critical value the oscillation amplitude would be very small, and that the oscillation would cease when the coupling is reduced below this value.

Sometimes, however, it is found that when the coupling is increased beyond the critical value the oscillation suddenly starts with an amplitude of perhaps several volts. If the coupling is then reduced the oscillation persists down to a second critical value of coupling less than that required to initiate oscillation, and at this point the oscillation, still perhaps with a comparatively large amplitude, ceases abruptly. Similar behaviour may result if the coupling is fixed and some other parameter, such as the h.t. voltage, is varied.

The effect obviously indicates a form of amplitude instability affecting all amplitudes below a certain value, but the instability in this case is of quite a different character from that studied in the previous Section. The sole effect of the time-constant and the Q factor is now to limit the speed with which the oscillation amplitude passes from zero through the unstable values to its final stable condition. It will be shown that the effect is caused by the variation of the grid input resistance with oscillation amplitude. At small amplitudes the resistance may be only a fraction of its value at large amplitudes. Consequently the effect is most pronounced in those oscillators where the grid input resistance contributes largely to the total power dissipation in the feedback network.

The hysteresis effect may be explained as follows. Oscillation begins spontaneously, with an initially small amplitude, when the coupling is raised to a value such that the total amplification through the valve and feedback network is just greater than 1. The oscillation causes the grid-bias voltage to become more negative so that the amplification of the valve decreases, but the grid input resistance is increased so that the attenuation of the feedback network is reduced. If the total amplification increases with increasing oscillation amplitude a state of instability exists, and the amplitude increases and continues to increase until a stable position is reached at which any further increase in amplitude would cause the total amplification to become less than 1. It is also obvious that the value of coupling required to maintain an oscillation, once started, at an amplitude corresponding to a high input resistance may be less than that required to initiate oscillation.

To investigate the effect and to find the critical values of coupling and oscillation amplitude the operation of the valve with small values of grid voltage must be examined. In Section 4 it was assumed that the relation between grid current and grid voltage is a linear one. This approximation is justified when most of the grid current flows at a positive grid voltage, but may be seriously in error for negative voltages.

When the grid voltage is more negative than about -0.3 volt the relation between grid current and voltage is approximately exponential

$$i_g = a \exp (dv_g) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (8.1)$$

a and d being constants. d depends chiefly on the cathode temperature, and for oxide-coated cathodes operated at normal rating is between 7 and 10. a depends also on the cathode temperature and on the valve geometry.

It will be assumed as in Section 4 that the time-constant C_gR_g is so large that the voltage across C_g and R_g produced by the flow of grid current is substantially constant.

Then $v_g = V_{g1} \cos \omega_0 t + V_g$

Substituting this into (8.1) on the assumption that v_g is always negative, gives for the grid current

$$i_g = a \exp(dV_g)[I_0(dV_{g1}) + 2I_1(dV_{g1}) \cos \omega_0 t + 2I_2(dV_{g1}) \cos 2\omega_0 t + \cdot \cdot]$$

in which I_0, I_1, and I_2 are the modified Bessel coefficients of the first kind of order 0, 1, and 2.

The mean current is

$$I_g = a \exp(dV_g) I_0(dV_{g1}) = -\frac{V_g}{R_g} \quad .. \quad (8.2)$$

and the amplitude of the component of fundamental frequency is

$$I_{g1} = 2a \exp(dV_g) I_1(dV_{g1})$$

Hence the grid input resistance at fundamental frequency is

$$r_g = \frac{V_{g1}}{I_{g1}} = -\frac{R_g}{2} \cdot \frac{V_{g1}}{V_g} \cdot \frac{I_0(dV_{g1})}{I_1(dV_{g1})} \quad .. \quad (8.3)$$

For any assumed value of V_{g1} (8.2) can be solved graphically to find V_g. This is most conveniently done by plotting the curve $y = x \exp x$. Then for any value of V_{g1} the corresponding value of V_g is given by $V_g = -x/a$ where $y = adR_g I_0(dV_{g1})$. When V_g has been found r_g can be calculated from (8.3).

For small value of dV_{g1} the following approximations are valid

$$I_0(dV_{g1}) = 1 + \left(\frac{dV_{g1}}{2}\right)^2$$

$$I_1(dV_{g1}) = \frac{dV_{g1}}{2} + \frac{1}{2}\left(\frac{dV_{g1}}{2}\right)^3$$

Let V'_g be the value of V_g when $V_{g1} = 0$ and let $V_g = V'_g + \delta V_g$. If V_{g1} is small it follows from (8.2) and the above approximation for I_0 that

$$\delta V_g = \left(\frac{dV_{g1}}{2}\right)^2 \frac{V'_g}{1 - dV'_g} \quad .. \quad .. \quad (8.4)$$

(8.3) becomes, using the approximation for I_1

$$r_g = r'_g (1 + hV^2_{g1}) \quad .. \quad .. \quad (8.5)$$

in which $h = \frac{-d^2}{8} \cdot \frac{1 + dV'_g}{1 - dV'_g}$ and $r'_g = \frac{-R_g}{dV'_g}$

Over the range of values of a, d, and R_g encountered in practice dV'_g usually lies between -2.5 and -6.5. Thus the grid input resistance for very small oscillation amplitudes may be much less than for large amplitudes, at which it is approximately $-\frac{R_g}{2} \cdot \frac{V_{g1}}{V_g}$.

The effective amplification round the oscillator at fundamental frequency may be defined as follows. Suppose that the network is interrupted between 3 and 5, and that a source of zero impedance having a terminal voltage $V_3 \cos \omega_0 t$ is connected between 5 and 4. Between 3 and 4 is connected an impedor of value equal to the impedance between 5 and 4 looking towards the grid. Let the amplitude of the grid fundamental voltage be V_{g1} and let the anode current have a fundamental component of amplitude I_{a1}. This produces a voltage of amplitude V_{3a} between 3 and 4 such that $V_{3a} = -\frac{I_{a1}}{G_t} \cdot \frac{V_3}{V_{g1}}$ The amplification is therefore

$$A = \frac{V_{3a}}{V_3} = -\frac{I_{a1}}{G_t V_{g1}}$$

For any particular oscillation amplitude to be stable A must be 1 and must increase as V_{g1} decreases and decrease as V_{g1} increases.

Since V_{g1} is assumed to be small, expression (5.2) may be used to represent the anode current. This leads to

$$A = -\frac{3}{2}\frac{b_a}{G_t}(1 + G_t R_i/\mu)(V_g - V_{ca})^{\frac{1}{2}} \left\{1 - \frac{V^2_{g1}(1 + G_t R_i/\mu)^2}{32(V_g - V_{ca})^2}\right\} + \frac{k}{G_t r_g} \quad (8.6)$$

Let G'_t and R'_i be the values of G_t and R_i when V_{g1} is zero and oscillation is about to begin. As the oscillation amplitude increases G_t and R_i also change because r_g increases according to (8.5) The effect of changes in r_g on the values of G_t and R_i was examined in Section 3. Using formulae (3.9) and (3.10) and expression (8.5) for r_g the amplification can be written as

$$A = 1 + V^2_{g1}\left[\left(1 - \frac{k}{G'_t r'_g}\right)\left\{\frac{hR_0}{R_0 + r'_g} + \frac{d^2 V'_g}{8(1 - dV'_g)(V'_g - V_{ca})} - \frac{\left(1 + \frac{G'_t R'_i}{\mu}\right)^2}{32(V'_g - V_{ca})^2}\right\} - \frac{hk}{G'_t(R_0 + r'_g)}\right]$$

Small amplitudes will be unstable if A increases with V_{g1}; i.e., if the coefficient of V^2_{g1} is positive. On substituting for h and rearranging the terms, this criterion for the existance of hysteresis becomes

$$\frac{V'_g}{V'_g - V_{ca}} - \frac{1 + dV'_g}{R_0 + r'_g}\left\{R_0 + \frac{kr'_g}{k - G'_t r'_g}\right\} > \frac{(1 - dV'_g)}{4d^2} \cdot \frac{\left(1 + \frac{G'_t R'_i}{\mu}\right)^2}{(V'_g - V_{ca})^2} \quad .. \quad (8.7)$$

The critical values of G_t and R_i are also related by (8.6), with $A = 1$ and $V_{g1} = 0$.

$$G'_t = -3\frac{b_a}{2}\left(1 + \frac{G'_t R'_i}{\mu}\right)(V'_g - V_{ca})^{\frac{1}{2}} + \frac{k}{r'_g} \quad (8.8)$$

When hysteresis exists the initial stable amplitude, if it is of the order of a few volts, may be calculated by the methods described in

Section 6. The minimum stable oscillation amplitude is difficult to calculate, but it can readily be found, if required, by graphical methods. In examples where the oscillation amplitude is neither large nor very small there may be difficulty in deciding whether formulae (4.6) and (4.8) or formulae (8.2) and (8.3) should be used to calculate the grid-bias voltage and grid input resistance. If these quantities are calculated from both sets of formulae it will usually be found that there is a range of values of V_{g1} over which the two sets give substantially the same result. for smaller values of V_{g1} formulae (8.2) and (8.3) would be used, while for larger values (4.6) and (4.8) would be more accurate. This type of hysteresis effect has also been studied experimentally by Zepler[5].

Example: The feedback network of an oscillator consists of a parallel-resonant circuit having a dynamic resistance of 10^5 ohms connected between 3 and 4 with a coupling coil connected between 1 and 2; $R_g = 10^5$ ohms. The oscillation amplitude is controlled by adjusting the mutual inductance between the grid and anode coils. The valve constants are: $b_g = 0.00065$, $V_{cg} = -0.13$ V, $b_a = 0.00056$, $V_{ca} = -9.1$ V, $\mu = 20$, $a = 10^{-4}$, $d = 7$. It is required to find if a hysteresis effect exists and, if so, the initial stable oscillation amplitude which is obtained when the mutual inductance is increased from zero.

Using the method described earlier, equation (8.2) can be solved to find $V'_g = -0.44$ V, then $r'_g = 32500\Omega$. From (8.8) $G'_t = -0.00244$ mho, and so $G'_t R'_i/\mu = -0.00084$. On substituting these values into (8.7) it is found that the inequality is satisfied and a hysteresis effect therefore exists.

From the above figures the ratio of the mutual inductance to the inductance of the grid coil is -0.0167 at the critical point where oscillation begins. Following the methods of Section 6 the stable oscillation amplitude corresponding to this value of coupling is 5.2 volts.

The second and less familiar type of hysteresis effect is peculiar to large oscillation amplitudes, and takes the form of a jump from one stable amplitude to another. The critical value of amplitude at which instability occurs can be found as follows.

For large values of V_{g1} the amplitude of the anode current of fundamental frequency is given by (5.6) and the amplification can be written as

$$A = -\frac{3b_a}{2^{5/2}G_t} V_{g1}^{1/2} (1 + G_tR_i/\mu)^{3/2}$$

$$H^2\left(1 - \frac{H}{8} - \frac{5H^2}{512}\right) + \frac{k}{G_t r_g}$$

If instability exists dA/dV_{g1} must be positive. It may be assumed that when V_{g1} is large the quantities G_t, R_i, r_g, and K are substantially independent of V_{g1}. On carrying out the differentiation, the condition that dA/dV_{g1} should be positive becomes

$$4(1 - K) > 3H\left(1 - \frac{H}{48} - \frac{11H^2}{1536} + \cdots\right)$$

Substituting for H according to (5.8) and reversing the series

$$\frac{V_{ca}}{V_g} < \frac{1 - K}{3K} + \frac{(1 - K)^2}{27K} + \cdots$$

As all terms are small compared with the first this is

$$\frac{V_g}{V_{ca}} > \frac{3K}{1 - K} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8.9)$$

It is obvious from this that instability can occur only if $K < 1$, since negative values of V_g/V_{ca} are inadmissible.

It can be shown by differentiating (5.7) that the condition for dA/dV_{g1} to be zero is the same as that for $d\left(\frac{V_g}{V_{ca}}\right)/dN$ to be infinite. If the graphs of Fig. 5 were continued upwards it would be found that for all values of K less than 1 the slope would ultimately become infinite and then negative. This tendency is most obvious in the graph for $K = 0.7$. All values of V_g/V_{ca} in these graphs greater than the critical values given by (8.9) correspond to unstable amplitudes.

Once the oscillation amplitude has passed the stable point there is nothing in the theory to prevent it from increasing indefinitely. The amplitude is then limited by the fact that when the anode voltage becomes comparable with the grid voltage, or small compared with the screen voltage, the division of current between the electrodes is altered and the anode receives a rapidly decreasing share of the total current. In most oscillators indeed this limitation takes effect before the amplitude reaches the critical unstable value. Instability of this type has in fact been observed only under somewhat artificial conditions.

When K is greater than 1 it follows from (5.7) and (5.8) that as N increases V_g/V_{ca} is asymptotic to the value

$$\frac{K}{K - 1}$$

REFERENCES

1 F. E. Terman: "Radio Engineering" 2nd Edition, p.350. McGraw Hill Book Co., New York, 1937.

2 J. Marique: "Notes on the Theory of Diode Rectification," *Wireless Engineer*, January 1935, Vol. 12, p. 17.

3 H. Nyquist: "Regeneration Theory," *Bell Syst. tech. J.*, January, 1932, Vol. 11, p. 126.

4 W. A. Edson: "Intermittent Behaviour of Oscillators," *Bell Syst. tech. J.*, January 1945, Vol. 24, p. 1.

5 E. E. Zepler: "Oscillation Hysteresis in Grid Detectors." *Wireless Engineer*, August 1946, Vol. 23, p. 222.

APPENDIX

Consider the integral

$$I = \int_{-\pi}^{\pi} (a \cos\theta + b)^n \cos^m \theta \, d\theta$$

in which a and n are positive and m is a positive integer or zero. The integrand is to be taken as zero when $a \cos \theta + b$ is negative. Two cases arise depending on whether $a > b$ or $a < b$.

If $a < b$ the integrand exists for all values of θ and

$$I = b^n \int_{-\pi}^{\pi} \left(1 + \frac{a}{b}\cos\theta\right)^n \cos^m \theta \, d\theta.$$

$$= b^n \int_{-\pi}^{\pi} \left(\cos^m\theta + \frac{na}{b}\cos^{m+1}\theta + \frac{n(n-1)}{2!}\frac{a^2}{b^2}\right.$$

$$\left.\cos^{m+2}\theta + \cdots\right) d\theta.$$

If m is even

$$I = 4b^n \int_0^{\pi/2} \left(\cos^m\theta + \frac{n(n-1)}{2!} \cdot \frac{a^2}{b^2} \cos^{m+2}\theta + \cdots\right) d\theta$$

$$= \frac{2b^n \pi^{\frac{1}{2}} \Gamma\left(\frac{m+1}{2}\right)}{\left(\frac{m}{2}\right)!} \left[1 + \frac{n(n-1)}{2!} \frac{(m+1)}{(m+2)} \frac{a^2}{b^2} + \cdots\right]$$

If m is odd

$$I = 4b^n \int_0^{\pi/2} \left(\frac{na}{b}\cos^{m+1}\theta + \frac{n(n-1)(n-2)}{3!}\frac{a^3}{b^3}\right.$$

$$\left.\cos^{m+3}\theta + \right) d\theta$$

$$= \frac{2\, nab^{n-1}\pi^{\frac{1}{2}}\Gamma\left(\frac{m+2}{2}\right)}{\left(\frac{m+1}{2}\right)!} \left[1 + \frac{(n-1)(n-2)}{3!}\right.$$

$$\left.\frac{(m+2)}{(m+3)}\frac{a^2}{b^2} + \cdots\right]$$

If $a > b$

$$I = \int_{-\beta}^{\beta} (a \cos\theta + b)^n \cos^m\theta \, d\theta.$$

where $\cos\beta = -b/a$

$$\text{then } I = 2^{n+1} a^n \int_0^{\beta} \left(\sin^2\frac{\beta}{2} - \sin^2\frac{\theta}{2}\right)^n \cos^m\theta \, d\theta.$$

$$\text{Let } x = \frac{\sin^2\frac{\theta}{2}}{\sin^2\frac{\beta}{2}}$$

Then

$$I = 2^{n+1} a^n \sin^{2n+1}\frac{\beta}{2} \int_0^1 (1-x)^n x^{-\frac{1}{2}} \left(1 - 2x\sin^2\frac{\beta}{2}\right)^m$$

$$\left(1 - x\sin^2\frac{\beta}{2}\right)^{-\frac{1}{2}} dx.$$

$$= 2^{n+1} a^n \sin^{2n+1}\frac{\beta}{2} \int_0^1 (1-x)^n \left\{x^{-\frac{1}{2}} - \left(2m - \frac{1}{2}\right)x^{\frac{1}{2}}\sin^2\frac{\beta}{2}\right.$$

$$+ x^{\frac{3}{2}}\left(2m^2 - 3m + \frac{3}{8}\right)\sin^4\frac{\beta}{2}$$

$$\left. - x^{\frac{5}{2}}\left(\frac{4}{3}m^3 - 5m^2 + \frac{53}{12}m - \frac{5}{16}\right)\sin^6\frac{\beta}{2} + \cdots\right\} dx$$

But the integral $\int_0^1 x^p (1-x)^n \, dx$ defines the Beta function $B(p+1, n+1)$ which may be expressed in terms of Gamma functions as $\frac{\Gamma(p+1)\,\Gamma(n+1)}{\Gamma(p+n+2)}$ Substituting this in the above series of integrals gives

$$I = 2^{n+1}\pi^{\frac{1}{2}} a^n \frac{\Gamma(n+1)}{\Gamma\left(n+\frac{3}{2}\right)} \sin^{2n+1}\frac{\beta}{2}\left[1 - \frac{(4m-1)}{2(2n+3)}\sin^2\frac{\beta}{2}\right.$$

$$+ \frac{3}{8}\frac{(16m^2 - 24m + 3)}{(2n+3)(2n+5)}\sin^4\frac{\beta}{2}$$

$$\left. - \frac{5}{16}\frac{(64m^3 - 240m^2 + 212m - 15)}{(2n+3)(2n+5)(2n+7)}\sin^6\frac{\beta}{2} + \cdots\right]$$

CORRESPONDENCE

Ignition Interference

SIR,—The practical importance of multiple sparking in motor-car ignition systems to which Mr. Callendar has recently drawn attention (*Wireless Engineer*, March 1949) has, we agree, been given little consideration in previous publications. In his letter Mr. Callendar refers mainly to the interference with the sound channel. In our opinion, however, this feature is more important in relation to the vision channel.

Some ten years ago subjective tests using commercially-available television receivers indicated that suppression of the interference to a level giving satisfactory reception of the sound signal was more than adequate for good reception of the vision signal. This conclusion was substantiated by tests carried out about two years ago provided no noise suppression was employed in the sound channel. The use of certain noise suppressors in the sound channel, however, resulted in the interference in this channel becoming inaudible even when appreciable interference was apparent on the picture. It thus appears that noise suppression in the receiver can deal adequately with sound reception provided that suppression at the source is sufficient to give good reception of the vision transmission.

Recent tests using a generator producing single impulses having a repetition frequency corresponding to normal engine speeds have shown that so few spots appear on the screen that, unless some defocusing of the spot occurs, or the line synchronization is affected, the interference will be negligible. With a motor-car ignition system, however, the interference was severe in the form of broad bands produced by the multiple sparking. In both cases the interference in the sound channel was negligible when a limiter was employed. Thus the elimination of multiple sparking would reduce considerably the effect of ignition interference to television, although we think that some degree of resistance suppression would still be necessary. The cause of multiple sparking is understood, arising as it does from the relation between the leakage inductance and self-capacitance of the ignition coil, the breakdown voltage of the sparking plug and the characteristics of the subsequent discharge. A quantitative investigation of the problem is, however, not easy as many of the parameters of the discharge are extremely difficult to measure and control. In motor vehicles the effects are complicated by engine conditions and by the presence of the distributor spark gap.

Research on the problems of ignition in internal combustion engines is being carried out by the E.R.A. and the ignition-equipment manufacturers although not primarily from the aspect of radio interference. It is hoped that this will throw further light on the interference problem although we are not optimistic about the prospects of eliminating multiple sparking with the conventional ignition system.

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"Are Transit Angle Functions Fourier Transforms?"

SIR,—I must first request indulgence by raising a question which, in the eyes of some, may seem elementary. For example, in the minds of such eminent authorities as Stuart Ballantyne, who based his derivation of mean-square noise current on the Rayleigh-Schuster theorem[1]—also used by Fraser, equation 9, with the same result[2]—the transit-angle function as determined by Fourier transform theory also gives the frequency spectrum of the noise. In a recent paper by Campbell, Francis and James,[3] the reader is led to infer more than this—one is rather given to suppose that the response of thermionic systems to signals other than those due to random fluctuations of emission is also given simply by the Fourier transform of the current waveform appropriate to the geometry of the device and to its space-charge condition.

When the writer began his work in the transit-time field, it did not seem to him to follow that the high-frequency performance of a thermionic system could be conditioned by the Fourier spectrum of any purely direct-current property of the system. While this instinctive conclusion may yet be correct, he himself went some way to undermine it by establishing, in 1937, that the impedance of a thermionic system to the nth harmonic of the applied potential could be derived from knowledge of the $(n-1)$th harmonic of the 'variation time' (which harmonic itself requires knowledge of lower-frequency harmonics), and, in particular, the fundamental impedance involves knowledge of the zero$^{\text{th}}$ harmonic only; i.e., the d.c. transit time.[4]

By a calculation similar to that of Fraser,[2] but which leaves the initial instant arbitrary at t_0 instead of o, one obtains in real notation for a linear saw-tooth wave the slightly different function (after replacing t_0 by $t_1 - \tau$) resulting from phase shift:

$$G_1(\omega, t_0) = \frac{e}{\sqrt{2\pi}} \frac{2}{\omega^2\tau^2} \{(1 - \cos \omega\tau).\cos \omega t_1 + (\sin \omega\tau - \omega\tau).\sin \omega t_1\}$$

The frequency behaviour of this function (in respect of both in-phase and quadrature components) is precisely the same as that of the velocity of a single electron in a temperature-limited plane diode, as determined by the writer and others,[5, 6] and since the current induced by the single electron is proportional to the velocity, there is seen in this elementary example to be complete agreement between Fourier-derived and dynamically-derived frequency spectra. One must, of course, take care that one does not confuse frequency spectra derived from the consideration of a single current pulse (excited by a single electron) with those derived by appropriate, if detailed, dynamical averaging of all pulses, or parts of pulses, coexisting at a given instant due to all electrons then present. It is with this state of affairs that we are concerned in practice. So far as the writer has been able to discover, transit time dynamics—preferably by the Jarvis-Witt-Benham technique[4]—is alone capable of yielding the correct frequency spectra.

This is a fairly surprising conclusion but, in order to illustrate the mathematical impasse which appears to have been reached, one may draw attention to the fact that the Fourier transform $G_2(\omega,t_0)$ of a parabolic pulse differs from any known transit-angle function in connection with the plane diode, whereas in order to be consistent with the previous result it should, in the space-charge case, agree with the known function for the electron velocity, and, in the temperature-limited case with the known function for the distance traversed in a given time.

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REFERENCES

[1] Ballantyne, Stuart, "Schrott Effect in High-frequency Circuits," *J. Franklin Inst.*, Aug. 1928, Vol. 206, p. 159.

[2] D. B. Fraser, "Noise Spectrum of Temperature-limited Diodes," *Wireless Engr*, April 1949, Vol. 26, p. 129.

[3] N. R. Campbell, V. J. Francis and E. G. James, "Valve Noise and Transit Time," *Wireless Engr*, May 1948, Vol. 25, p. 148.

[4] W. E. Benham, "Electron Inertia as a Cause of Harmonics in Valves," *Nature*, April 1937, Vol. 139, p. 591. "A Contribution to Tube and Amplifier Theory," *Proc. Inst. Radio Engrs*, Sept. 1938, Vol. 26, pp. 1093 and 1130.

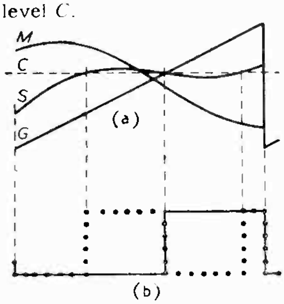
[5] W. E. Benham, "Theory of the Internal Action of Thermionic Systems at Moderately High Frequencies," *Phil. Mag.* Feb. 1931, Vol. 11, pp. 457 and 498.

[6] C. J. Bakker and G. Vries, "On Vacuum Tube Electronics," *Physica*, July 1935, Vol. 2, p. 683.

Pulse-splitting

SIR,—In the operation of pulse-modulation systems there is a curious effect which appears to have escaped previous notice. This is the phenomenon of pulse-splitting, which is the breaking of a pulse into two or more separate pulses by the process of modulation.

The effect is best demonstrated by taking, as an example, the conventional type of modulator circuit in which a periodic triangular generating wave G plus a modulating wave M are fed into a limiter. In the limiter circuit a pulse begins or ends when the sum S of G and M rises above, or falls below, a certain critical level C.



The figure illustrates the operation of the modulator during the $(n+1)$th modulation cycle from $t = n/f_v$ to $t = (n+1)/f_v$, f_v being the fundamental frequency of G. The critical level C is taken positive and the amplitude of G is, for convenience, taken as 1. It is also supposed that M is continuous.

(*a*) Shows the input to the limiter and (*b*) shows the output. In (*b*) the solid line shows the single unmodulated pulse produced by G in the absence of M, and the dotted line shows the two pulses produced by the sum S of G and M.

An essential restriction is that the minimum value of M should be numerically less than $1 - C$, and the maximum value should be less than $1 + C$, for otherwise, without further restrictions on M, a pulse might be suppressed or two pulses might coalesce.

A point at which $S = C$ and dS/dt is positive marks the beginning of a pulse, and one at which $S = C$ and dS/dt is negative corresponds to the end of a pulse. Since the point $t = (n + 1)/f_g$ always marks the end of a pulse, it is obvious that the necessary and sufficient condition for pulse-splitting to occur is that there should be another point t_1 in the range $n/f_g < t_1 < (n + 1)/f_g$ at which $S = C$ and $dS/dt < 0$.

For example, if $M = a \sin (2\pi f_m t + \phi)$

and a is positive and less than $1 - C$, the criteria are

$a \sin (2\pi f_m t_1 + \phi) + 2 f_g t_{1-2n} - 1 = 0$.

and $a\, 2\pi f_m \cos (2\pi f_m t_1 + \phi) + 2 f_g < 0$.

The inequality cannot be satisfied unless $a > f_g/\pi f_m$. When this requirement is met ranges of values of t_1 can be found to satisfy the inequality, and it can be shown that unless f_m and f_g are co-measurable there is a value of t_1 in at least one of these ranges which satisfies also the first equation for some particular value of n. Thus, in general, splitting does not take place in every modulation cycle and, if f_m and f_g are co-measurable, does not necessarily occur at all.

Similar conditions for more complicated modulating waves can easily be derived.

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Triode Interelectrode Capacitances

Sir,—In the recent paper[1] entitled "Triode Interelectrode Capacitances" the authors appear to have missed a fundamental feature of the g_m/c_{gk} curves.

North[2] derives the following equation for the variation of grid-cathode capacitance in the three-halves region of the characteristic (modified by change of symbols, the removal of the factor $(C_1 + C_2)$ and the change from c_{gk} to Δc_{gk}):

$$\Delta c_{gk} = \tfrac{1}{3} c_0 + \tfrac{2}{3} g_m \tau_2 \left[1 - \frac{1}{k - 1} + \frac{1}{3(k + 1)^2}\right] \quad (1)$$

where c_{gk} = hot capacitance

c_0 = cold capacitance (that part only which is in the electron stream).

g_m = mutual conductance.

τ_2 = anode transit time.

To a first approximation, k in this equation may be neglected, giving

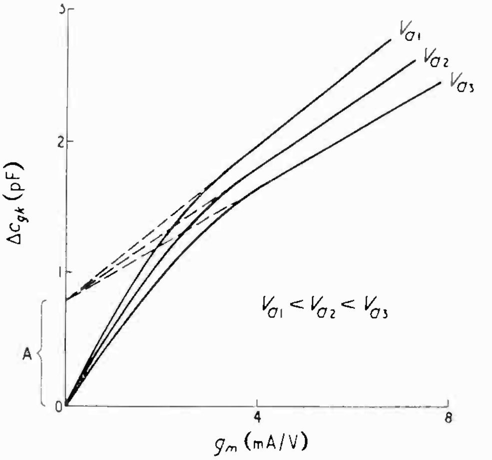
$$\Delta c_{gk} \approx \tfrac{1}{3} c_0 + a g_m \quad \ldots \quad (2)$$

where a is a constant for a given anode voltage.

This equation suggests that Δc_{gk} would be linear with g_m, the curves starting from a capacitance equal to one-third of the cold value c_0. The equation is not valid, however, outside the three-halves region of the characteristic and this statement must be taken as meaning 'outside the region where the *whole* of the cathode area obeys a three-halves law'. Owing to the non-homogeneous nature of the field at the cathode, this region is not reached until an appreciable anode current is attained, often of the order of 5 mA/sq cm.

Below this current, those parts of the cathode lying directly below the grid wires are governed by the retarding-field exponential characteristic, and do not contribute to Δc_{gk} until the potential barrier in front of the cathode approaches zero. As the grid bias is reduced, therefore, a continuous gradation is to be expected between the cold, or cut-off, capacitance and the straight line predicted by the above equation. This behaviour is suggested by some of the author's curves and is shown more strongly by curves taken by the present writer (while at Electronic Tubes, Ltd.) on the CV138 valve connected as a triode. The latter were of the form shown in the figure, where the slope of the upper linear portion of the curves was greater for low values of anode voltage, in agreement with Equ. (1), where τ_2 is approximately proportional to $V_a^{-\frac{1}{2}}$. The intercept of these asymptotes was at about $\Delta c_{gk} = 0.8$ pF, which agrees with the equation since c_0, the true grid-to-cathode capacitance, is about 2.5 pF. The 'projected cut-off increment' A should therefore be $1/3 \times 2.5 = 0.8$ pF. In agreement with the idea above that the departure of the curves from the asymptotes is due to the non-homogeneous field at the cathode, the asymptote is reached at a lower value of g_m for low values of V_a, when the anode penetration and field disturbance are less.

It may be noted that there is a similar change in shape in the V_g/g_m curves, due fundamentally to the same cause. At large negative grid voltages, only those parts of the cathode directly underneath points midway between adjacent grid wires contribute to the anode current; as the grid is made more positive the slope increases rapidly under the influence of an increasing effective cathode area. When the whole cathode is contributing to the electron stream this rapid rise ceases and the three-halves region, with $g_m \propto I_a^{1/3}$ commences. This point, which is at approximately V_a/μ, should be the same as that at which the capacitance curve departs from linearity and this agreement was qualitatively observed. The word 'qualitatively' is used in view of the difficulty in estimating the point precisely from the curves.



The effect observed by Zepler and Hekner that the increase in c_{gk} begins before appreciable I_a flows, may be compared with the analysis of van der Ziel,[3] in Fig. 7 of whose paper the capacitance increment is shown as large even when I_a is only 1/100 of that at the exponential point.

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REFERENCES

1 E. E. Zepler & J. Hekner, *Wireless Engineer*, Feb. 1949, Vol. 26, p. 58.
2 *Proc. Inst. Radio Engrs*, 1936, Vol. 24, p. 108.
3 *Philips Research Reports*, 1946, Vol. 1, p. 97.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationary Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

604 877.—Low-frequency calling-signal arrangement, applicable to radio or other carrier-wave communication systems.

Standard Telephones and Cables Ltd. (assignees of Le Materiel Telephonique. Soc. Anon.). Convention date (France) 26th December, 1944.

606 213.—Switching device for reducing the loop resistance of the transmission lines in a low-frequency system for distributing broadcast programmes to local subscribers.

Communications Patents Ltd., and P. Adorian. Application date 10th January, 1946.

606 291.—Low-frequency transformer-unit with plug-in connections, particularly for coupling low-impedance microphones and other devices to an amplifier.

K. Jones (trading as Electro-Voice Products). Application date 11th January, 1946.

606 297.—Compact construction of amplifier and associated controls, particularly suitable for deaf-aid apparatus.

E. Shipton. Application date 11th January, 1946.

606 299.—Sound-recording amplifier in which a fraction of the input signal is utilized to operate an a.v.c. system capable of maintaining a balanced volume level.

Dictaphone Corp. Convention date (U.S.A.) 29th January, 1945.

607 561.—Contrast-control system for the reproduction of acoustic signals, wherein the control-voltage is applied only at a predetermined level of output volume.

Electrical Fono-Films Co. Akt. Convention date (Denmark), 13th August, 1942.

AERIALS AND AERIAL SYSTEMS

605 344.—Short-wave aerials of the flared waveguide type, in which backwardly-projecting flanges are added to the longer sides of the aperture, in order to broaden the radiated beam.

O. M. Bohm, A. R. G. Owen, L. G. Reynolds and C. S. Wright. Application date 7th November, 1945.

606 157.—Tuning a dipole aerial by altering its effective length through Bowden-wire or like gearing, which passes through the coaxial feed-line, and is associated with a quarter-wave impedance.

R. C. Mildner. Application date 23rd January, 1946.

606 772.—Dipole aerial with limbs which are capable of telescopic adjustment to suit different wavelengths.

L. S. Hargreaves and Aerialite Ltd. Application date 18th January, 1946.

606 926.—Directive aerial, of the parabolic-mirror type, in which an auxiliary deflecting element is provided in order to eliminate undesired lobes of radiation.

Western Electric Co. Inc. Convention date (U.S.A.) 24th, January, 1945.

607 159.—Toroidal coil encircling the wing or fusilage of an aeroplane so as to serve as a streamlined aerial, and method of coupling it to a wireless set.

W. A. Johnson. Application dates 28th January, 1946, and 31st January, 1947.

607 589.—Arrangement of quarter-wave elements for decoupling a dipole aerial from an adjacent conductor, such as a supporting mast or a coaxial feed-line.

Hazeltine Corp. (assignees of H. A. Wheeler). Convention date (U.S.A.), 13th December, 1943.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

604 493.—Navigational system in which primary trains of pulses are radiated in the form of overlapping beams, the median-line being distinguished by secondary pulses which are alternately aligned with each primary beam.

Sadir-Carpentier. Convention date (France) 21st October, 1943.

604 499.—Automatic gain-control system, of the kind which depends upon an initial polarizing voltage, for echo-ranging receivers, particularly for under-water use.

S. A. Byard and C. S. Wright. Application date 21st December, 1945.

604 592.—Presentation system for a radiolocation indicator, wherein range is shown as an intensity modulation of a straight-line time base, which is shifted bodily to indicate the direction and elevation of the target under observation.

W. B. Lewis and R. H. A. Carter. Application date 24th March, 1945.

604 608.—Radiolocation equipment in which a locally-generated strobe is utilized to select, and thereafter automatically to follow, a desired target, say to enable a fighter plane to close-in on a hostile bomber.

F. C. Williams. Application date 3rd October, 1945.

604 672.—Radiolocation apparatus in which ground clutter is eliminated by utilizing the Doppler effect produced by moving targets to derive clear-cut secondary signals on a mosaic screen of the storage type.

W. S. Elliott, C. A. Johnson and R. S. Webley. Application date 7th March, 1945.

604 717.—Super-regenerative circuit, particularly of the responder type, in which the frequency of the pulses used for interrogation serves to stabilize the sensitivity of the receiver.

Ferranti Ltd. and H. Wood. Application date 30th November, 1945.

605 053.—System for constantly indicating his position to the pilot of an aeroplane by comparing the relative phases of pulsed signals radiated in cyclic order from a group of spaced land-beacons.

F. C. Williams. Application date 24th May, 1945.

606 019.—Radiolocation set in which a time-delay circuit is utilized to block-out echoes other than those from the nearest reflecting object, so as to serve as a

crash-warning device, or as an altimeter.
Philco Products Inc. (assignees of W. E. Bradley). Convention date (U.S.A.), 27th January, 1945.

606 549.—Construction of time-delay transmission line, suitable for securing echo or reflection effects, or for generating the pulses used in radiolocation.
Hazeltine Corporation (assignees of M. J. Ditoro). Convention date (U.S.A.) 12th March, 1945.

607 092.—Variable resonance-chamber having an adjustable damping-factor for testing the efficiency of radiolocation equipment.
Western Electric Co. Inc. Convention date (U.S.A.) 31st January, 1945.

607 988.—Motor-driven capacitor-switch for varying the axis of directivity of an array of aerials.
The British Thomson-Houston Co. Ltd. (communicated by The General Electric Co.). Application date 13th February, 1946.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

605 206.—Interference eliminator, for the impulsive type of disturbance having steep flanks, consisting of the combination of one or more diodes with a resistance-capacitance circuit having a predetermined time-constant.
D. Weighton and Pye Ltd. Application date 24th October, 1945.

605 306.—H.F. coil with a tubular slotted former and a screw-threaded tuning core, particularly adapted to form the i.f. transformer of a superheterodyne radio receiver.
Radio and Television Trust Ltd. (formerly Philco Radio and Television Corporation of Gt. Britain Ltd.) and C. P. V. Vaufrouard. Application date 19th December, 1945.

605 377.—Multi-band receiver with a tuning system comprising a pre-set inductance and capacitances, of low cost, and designed to simplify the operation of trimming or lining-up.
A. C. Cossor Ltd. and A. H. A. Wynn. Application date 20th December, 1945.

605 523.—Circuit arrangement for stabilizing the sensitivity of a super-regenerative set of the kind that responds automatically to an interrogating pulsed signal, say for identification purposes.
Ferranti Ltd., H. Wood, R. H. Davies and J. R. Whitehead. Application date 28th December, 1945.

605 715.—Detecting frequency-modulated signals by sweeping the electron beam of a discharge tube over a pair of anodes possessing different coefficients of secondary emission.
Marconi's W.T. Co. Ltd. (assignees of G. C. Sziklai) Convention date (U.S.A.) 20th May, 1944.

605 802.—Radio-gramophone provided with a dual set of control knobs which co-operate with each other so as to be effective whether the lid of the cabinet is open or closed.
E. K. Cole Ltd. and A. W. Martin. Application date 5th January, 1946.

605 808.—Ultra-short-wave receiver arranged to detect either amplitude or frequency-modulated signals by suitably controlling the transit time of electrons passing between the screen grid and anode of a four-electrode valve.
Philips Lamps Ltd. Convention date (Netherlands) 15th January, 1941.

606 007.—Discriminator type of receiver for phase or frequency-modulated signals, in which the standard pair of diodes are connected in series-aiding polarity.
Radio Corporation of America. Convention date (U.S.A.) 7th September, 1945.

606 015.—Portable receiver with a hinged lid which houses a frame aerial and automatically switches on the set, with compact fittings to hold the batteries and a pair of headphones.
H. B. W. Holt and P.A.M. Ltd. Application date 8th January, 1946.

606 599.—Frame aerial with a coupling transformer of the permeability type, the moving core of which serves to tune the set without altering the aerial inductance.
The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 19th January, 1945.

606 817.—Plastic cabinet for a radio receiver, moulded in the form of two open-sided boxes, which are bolted together front and rear to facilitate access to the circuit components.
De La Rue Plastics Ltd., F. E. Middleditch and S. R. Hawkins. Application date 7th December, 1945.

606 821.—Circuit arrangement for reducing the effect of shunt capacitances normally present in the tuned-anode couplings of r.f. amplifiers and oscillators.
K. A. Hartley. Application date 16th January, 1946.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

605 109.—Television receiver adapted to select any one of a number of television programmes, radiated on different carrier frequencies, but with common synchronizing signals, from a given point of advantage.
Standard Telephones and Cables Ltd. (assignees of E. Labin). Convention date (U.S.A.) 25th August, 1944.

605 961.—Television transmitter in which provision is made for stabilizing the percentage of the carrier available for the video signals when utilizing blacker-than-black synchronization.
Farnsworth Television and Radio Corporation. Convention date (U.S.A.) 15th February, 1945.

606 145.—Television system wherein an original video-signal, covering a frequency range of say 20 Mc/s, is converted into a series of ten signals each covering a range of 2 Mc/s.
E. P. Rudkin. Application date 21st November, 1945.

606 324.—Television system in which a stereoscopic effect is secured by presenting the two aspects of a picture in rapid alternation, one being of reversed polarity to the other.
Farnsworth Television and Radio Corporation. Convention date (U.S.A.) 29th March, 1945.

607 221.—Television cabinet fitted with a telescopic upper portion, which is raised by motor-driven mechanism and is fitted with an optical system for viewing the received picture.
Marconi's W.T. Co. Ltd. (assignees of R. V. Beshgetoor). Convention date (U.S.A.) 18th October, 1944.

607 315.—Television transmitter tube, of the low beam-velocity type, with means for generating a d.c. component for regulating the average background illumination of the picture.
Marconi's W.T. Co. Ltd. (assignees of R. D. Kell). Convention date (U.S.A.) 6th July, 1944.

607 779.—Saw-tooth oscillator for generating variable-frequency scanning voltages, suitable for a secret system of facsimile signalling, or for television.

Standard Telephones and Cables Ltd. (assignees of L. A. de Rosa). Convention date (U.S.A.) 4th January, 1943.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

603 883.—Circuit arrangement by which discontinuities in the impedance values of the transformer of a ring modulator are eliminated from the working range of frequencies.

Cie Genérale d'Electricité. Convention date (France) 26th July, 1944.

604 076.—Waveguide of circular cross-section fitted with internal bars for varying the polarization of the transmitted wave.

E. Wild and C. S. Wright. Application date 23rd November, 1945.

604 803.—Modulating circuit of the balanced-bridge type in which provision is made to offset any asymmetry due to the use of screened transformers.

Soc. Anon. de Télécommunications. Convention date (France) 6th August, 1942.

606 780.—Frequency-modulating system in which speech signals are used to generate saw-toothed oscillations of varying slope, which are applied to alter the phase of the carrier-wave.

Standard Telephones and Cables Ltd. (assignees of S. Frankel). Convention date (U.S.A.) 23rd January, 1945.

606 910.—Stabilized frequency-changing circuit, comprising at least three stages, with selecting and interpolating devices, adapted for the transmission or reception of signals, or as a frequency meter.

Soc. Independante de T.S.F. Convention dates (France) 19th July and 14th September, 1944, and 15th March, 1945.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

604 130.—Phase-differentiated synchronizing arrangement for a system of multi-channel signalling by means of modulated pulses.

Standard Telephones and Cables Ltd., P. K. Chatterjea and L. W. Houghton. Application date 3rd January, 1945.

604 203.—Multi-channel system of communication for moving vehicles, particularly aircraft, comprising one or more terminal stations and a chain of intermediate repeater beacons.

Standard Telephones and Cables Ltd. (assignees of E. M. Deloraine). Convention date (U.S.A.) 20th April, 1944.

604 230.—Multi-channel signalling system, in which one message is frequency-modulated, another is amplitude-modulated, and a third pulse-modulated.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 19th August, 1944.

604 413.—Circuit arrangement for filtering or separating-out from a signal-modulated carrier-wave, pulses of greater or less amplitude than a desired amplitude.

Standard Telephones and Cables Ltd. (assignees of D.D. Grieg). Convention date (U.S.A.) 15th May, 1943.

604 438.—Signalling system in which two stations repeat pulses back and forth to each other, in order to limit and select the distance of intercommunication without mutual interference.

Marconi's W.T. Co. Ltd. (assignees of C. W. Hansell). Convention date (U.S.A.) 16th September, 1943.

604 817.—Single-channel receiver, and supervisory circuits, for a multi-channel time-modulated pulsed signalling system, suitable for broadcasting.

Standard Telephones and Cables Ltd. and M. M. Levy. Application date 28th December, 1945.

605 128.—Multi-channel pulsed signalling system in which provision is made to prevent cross-talk due to the flattening of the pulse flanks during transmission.

"Patelhold" Patentverwertungs &c. A.G. Convention date (Switzerland) 23rd December, 1944.

605 178.—Pulsed system of signalling in which modulation is effected solely by varying the number of pulses transmitted within periodic intervals, either by adding to or substracting from a given number that represents zero signal.

Marconi's W.T. Co. Ltd. (assignees of H. O. Peterson). Convention date (U.S.A.) 8th January 1944.

605 681.—Multi-channel signalling system, in which different carrier-waves are frequency-modulated, for communication to and from moving trains and wayside stations.

Westinghouse Brake and Signal Co. Ltd. (assignees of P. N. Bossart). Convention date (U.S.A.) 30th January, 1945.

606 314.—Time-delay device for selecting a desired channel in a multiplex system of signalling by spaced trains of pulses.

Sadir-Carpentier. Convention date (France) 20th March, 1945.

607 098.—Discriminator circuit, for the selective reception of different signals in a pulsed system utilizing a given ratio of pulse-width to pulse-period.

Standard Telephones and Cables Ltd. (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 19th June, 1944.

CONSTRUCTION OF ELECTRON-DISCHARGE DEVICES

604 303.—Electrode assembly of a discharge tube, and associated circuit, for generating a velocity-modulated stream of electrons.

G. Longo. Convention date (France) 8th March, 1945.

605 469.—Construction of an electron-discharge tube with hollow resonators, of the velocity-modulation type, designed to generate oscillations of the order of 20 000 megacycles per second.

N. C. Barford. Application date 14th September, 1945.

SUBSIDIARY APPARATUS AND MATERIALS

604 198.—Cathode-ray tube, with mosaic screen and scanning system, for reproducing X-ray images of the teeth, or of other internal parts of the body.

B. J. Edwards and Pye Ltd. Application date 26th February, 1945.

604 246.—Preventing capacitance-losses in a frequency-doubling circuit for ultra-short waves, comprising at least two diodes which are fed in phase-opposition.

Philips Lamps Ltd. Convention date (Netherlands) 15th August, 1940.

605 734.—Bridge type of wattmeter, utilizing Thermistor or other non-linear resistances, particularly for use with aerial feeders and like short-wave coaxial-line circuits.

J. G. Yates, R. C. Robbins and T. P. Hvinden. Application date 14th December, 1945.