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INFORMATION EXPLOSION

In 1963 more than a million individual papers on science and technology were published throughout the world in some 50,000 periodicals. This was in addition to the technical information released in about 90,000 books, 100,000 research reports and 250,000 patents. The continuing rapid expansion of published information has been referred to in the U.S.A. as the 'information explosion'.

This 'explosion', if uncontrolled, gives rise to the twin evils of inefficient working and the unwitting duplication of work already done: controlled, it can put at the disposal of the research worker knowledge which no group of workers, however large, could hope to discover for themselves.

The problem is probably nowhere more acute than in electronic and radio science and engineering. If, last year, an electronic engineer had wished *to make sure* he saw everything of importance he would have had to read some 6,000 papers in the issues of more than ninety English-language periodicals alone!

Abstract journals are useful when searching through past literature but, as they cannot easily be produced as 'up-to-date' publications, they are less satisfactory as a means of keeping abreast of *current* information. Moreover, the electronic engineer has suffered the loss of the classified abstracts formerly published in *Electronic Technology*.

How best to provide the research worker and engineer with a means of quickly and conveniently reviewing new information in the field of electronic and radio engineering has been under consideration by the Institution's Research Committee for some time. One proposal is that the Institution should produce a monthly publication giving the titles of all the important papers, notes and communications on electronics and radio engineering as they appear in the current issues of periodicals. Only the title, author and periodical reference would be given for each item; these would be listed under a small number of subject headings.

Such a listing of titles, without abstracts, is by no means ideal. One has only to examine the 885 items in the Institution's "Abstracts of Papers Published in the Journal of the Brit.I.R.E. 1952–63" to realize that the title can rarely describe adequately the scope of a paper. However, the title gives some indication, even if imperfectly, and a list of current titles arranged under a small number of broad subject headings (duplicating entries where they refer to more than one subject) does satisfy two of the main criteria for a 'current-awareness' publication; it can be produced with a minimum of delay and it is easy to scan.

The Research Committee do not look upon the proposed title list as more than a first step since, initially at least, it is proposed to include only items from English-language periodicals. For a more comprehensive and, of necessity, long-term solution to the information problem in our field we must look to the National Electronics Research Council, who have recognized the importance of the problem by appointing their first Working Party to examine the whole problem and make recommendations for its solution.

T. M. A.

INSTITUTION NOTICES

Birthday Honours

The Council congratulates the following members of the Institution whose appointments appear in Her Majesty's Birthday Honours List:

Graham Douglas Clifford (*Member*) to be a Companion of the Most Distinguished Order of St. Michael and St. George. (Mr. Clifford has held the position of Secretary of the Institution since 1938; he has travelled widely throughout the Commonwealth.)

Commander John Archibald Bedford, R.N. (*Member*), to be an Ordinary Officer of the Most Excellent Order of the British Empire. (Commander Bedford currently holds an appointment of Assistant Director in the Directorate of Weapons Radio (Naval) of the Ministry of Defence at Bath.)

The Fifth Clerk Maxwell Memorial Lecture

On 26th May last, Sir Gordon Radley, K.C.B., C.B.E., Ph.D.(Eng.), M.I.E.E., gave the fifth of the Institution Lectures, which were established in 1951 to honour James Clerk Maxwell, the eminent physicist on whose work much of contemporary radio engineering is based. Sir Gordon's lecture, which was given in London, took as its main theme the future developments which may be expected in microelectronics, particularly with reference to computer engineering. The lecture was received with great applause and its publication in the July issue of *The Radio and Electronic Engineer* will be awaited with interest by members who are aware of the high standard of the Clerk Maxwell Lectures.

Honorary Member

At the conclusion of the Clerk Maxwell Memorial Lecture, on 26th May, the President of the Institution, Mr. J. L. Thompson, announced that the Council had that afternoon decided to elect Sir Gordon Radley to Honorary Membership in recognition of his outstanding work in the development of international communications. The Council's decision was unanimously approved by all the members present and a formal presentation at which Sir Gordon will sign the Roll of Honorary Members will be held in London during the coming session.

Award of the Churchill Gold Medal

The Institution's Council has been invited to submit a nomination for the Churchill Gold Medal award, which has been instituted by the Society of Engineers with the agreement of Sir Winston S. Churchill, K.G. The purpose of the Medal is to commemorate the achievements of engineers during the war years under the inspiration of Sir Winston.

The terms of award are that it shall be made once in every two years to the engineer responsible for the most noteworthy development during that period, or for an important contribution to contemporary engineering. It is confined to candidates in the British Commonwealth.

Members of the Institution are invited to submit nominations, in confidence, to the Council for consideration at its meeting on 15th July.

Previous Medallists include Sir Frank Whittle, Sir William Wallace, Sir Christopher Hinton, Sir John Cockcroft, Lord Hives and Sir Geoffrey de Havilland.

The Christopher Columbus Prize for Telecommunications

The Christopher Columbus International Prize for Communications was created by the City of Genoa in 1955 to honour the name and memory of Christopher Columbus. The Prize is awarded "for the most distinguished contribution to the advancement of communication between men: a discovery, an achievement, research successfully completed or action boldly launched—a contribution both of technical or scientific and of social significance, such as might promote closer neighbourliness and combined effort between the nations".

For the purpose of making the award the vast field of communications has been divided into five sections: communications by land, communications by sea, communications by air, postal and telecommunications, and space communications. The award, which consists of a gold medal and 5 million lire (about £2,865), is made each year for one of the five sections. It can be given to an individual, a corporate body or, collectively, to several persons.

In the present year, 1964, the Prize will be awarded for a contribution to Telecommunications, and the President of the Institution has been invited by the President of the National Research Council of Italy to make a nomination for this award. In order that the fullest consideration can be given to suitable recipients, members are invited to forward proposals to the Secretary for discussion at the meeting of the Council of the Institution on 15th July.

Members will recall that the Institution was asked to submit similar nominations in 1955 and 1959. It is particularly interesting to note, in view of the announcement that Sir Gordon Radley is to be elected an Honorary Member, that Sir Gordon and Dr. Mervin J. Kelly received the Prize in 1955 in recognition of their combined direction of the development and laying of the first Transatlantic telephone cable. Another Honorary Member and Clerk Maxwell Memorial Lecturer, Dr. Vladimir K. Zworykin, was a joint recipient in 1959.

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Stereophonic Broadcasting and Reception

By

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AND

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Presented at a meeting of the Electro-Acoustics Group in London on 11th March 1964.

Summary: A brief survey of various systems for stereophonic broadcasting which have been proposed in recent years is presented. The pilot-tone (Zenith-G.E.) system, which has been adopted for stereophonic broadcasting on v.h.f. in the U.S.A., is considered in more detail.

The design of receivers for this system is discussed, with particular reference to the requirements for minimizing susceptibility to interference, and a decoder which meets these requirements is described.

1. Introduction

The present paper does not attempt to consider various applications of stereophonic techniques to sound and television broadcasting in the future. It restricts its consideration to stereophonic systems which are suitable for sound broadcasting on v.h.f. in Band II, and which enable a stereophonic receiver to reproduce two audio-frequency signals. This development must be regarded as the first serious application of stereophony to broadcasting and has, in fact, been in operation since June 1961 in the United States. Although the applications of stereophony to medium-frequency (m.f.) sound broadcasting[1] and to television[2] are being considered in America, they are of rather less interest in the European area, in the first case because of the congested state of the m.f. band and, in the case of television, because of the greater attention at present being paid to other lines of development.

Although experiments in broadcast stereophony, using two transmitters, have taken place from time to time, broadcasting authorities have generally felt that they should wait until reproducers for stereophonic recordings had found their place in the home before giving serious consideration to corresponding developments in broadcasting. The present wide sale of stereophonic gramophone records not only means that there is a good case for considering broadcast stereophony: it also means that the system that will make maximum use of existing facilities and experience is one that accepts the same compromise regarding complexity. In other words, the system should provide two independent a.f. channels, each covering the full range of audio frequencies, and intended for reproduction on a left- and right-hand loudspeaker.

A stereophonic system described as a 'multiplex' system is understood as one which provides the two channels by suitable modulation of a single transmitter. If a multiplex system is to be used with existing transmitter networks, it is also essential that it should not require an increase of channel width in the radio-frequency spectrum. To overcome the difficulty encountered with multiplex systems of attaining a good signal/noise ratio, alternative systems have been proposed which employ a single a.f. channel together with a narrow-band 'steering' signal.[3, 4] The output of the single a.f. signal at the receiver is apportioned between the left-hand and right-hand loudspeakers in a ratio determined by the steering signal. While such systems are entirely successful from the signal/noise ratio point of view it has been found that, with a reasonably simple receiver, there are certain types of programme which are reproduced with unnatural stereophonic effects; this would apply to all listeners irrespective of their proximity to the transmitter. It is reasonable therefore to consider only two-channel multiplex systems, which provide for the broadcaster the same basic facility that is available in disk recording.

After a discussion of the principles of a multiplex stereophonic system, various systems that have been proposed in the past few years will be briefly described and their theoretical performance considered. The remainder of the paper will be devoted to a more detailed discussion of the pilot-tone system, and to consideration of requirements in receiver design with this system. Initially known as the Zenith-G.E. system, it is the one adopted in the U.S.A. and also considered by most European countries as the most suitable of the systems that have been proposed. The detailed characteristics and related receiver design details would naturally differ if a different multiplex system were to be adopted in the European area, but many of the basic considerations would apply to any of the multiplex systems under serious consideration.

2. System Principles

2.1. *Requirements of a Stereophonic System*

The main characteristics required of a stereophonic broadcasting system may be summarized as follows assuming that, on economic grounds, transmitters of the existing f.m. sound broadcasting service are to be used.

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(1) It should be compatible; i.e. listeners with ordinary (monophonic) receivers should be able to obtain satisfactory reproduction of the transmitted stereophonic programme.

(2) The service area and standard of reception for the monophonic listener—as determined by noise level, interference, etc.—should be substantially unaffected.

(3) The service area for stereophonic reception should approach the monophonic service area as closely as possible, taking into account the use of an improved but not too elaborate receiving aerial if necessary.

(4) The quality of stereophonic reproduction obtainable should be good; this requires two full-bandwidth a.f. channels with reasonably low distortion and cross-talk so that under good reception conditions there should be no noticeable deterioration that can be attributed to the transmission system.

(5) The effect of propagation factors (particularly multiple paths) on stereophonic reproduction should be as small as possible, preferably no more serious than with monophonic transmission and reception.

2.2. *Essential Features of Systems Meeting the Requirements*

Throughout this paper we shall denote the stereophonic a.f. signals intended for the left-hand and right-hand loudspeakers by A and B respectively and where convenient we shall use M to denote $\frac{1}{2}(A+B)$ and S to denote $\frac{1}{2}(A-B)$.

Concerning the first requirement, it is generally agreed, as a result of subjective listening tests, that the sum of the two stereophonic signals gives an entirely acceptable version of the programme from a single loudspeaker. Compatibility is therefore obtained if the a.f. component of modulation of the transmitter corresponds to the M signal, and if the remaining information, usually as the S signal, is used to modulate the transmitter in a way that does not affect ordinary receivers. The additional signal can, however, be detected by stereophonic receivers; the stereophonic signals are then obtainable as $A = M+S$ and $B = M-S$.

The simple idea of transmitting the S signal by amplitude modulation of the transmitter while the M signal frequency-modulates the transmitter is ruled out (quite apart from the difficulty of increased transmitter costs) because, in practice, the received S signal would be seriously affected by distorted breakthrough of the M signal. This would result from the spurious amplitude modulation which is produced by frequency modulation in the presence of slight receiver misalignments or under multipath propagation conditions.

In all multiplex systems that are serious contenders the S signal modulates an ultrasonic sub-carrier; the resulting signal is then added to the M signal to form the so-called multiplex signal. This composite signal is finally used to frequency-modulate the transmitter. The only essential difference between the various systems lies in the form of modulation which is applied to the sub-carrier. It follows from the principle common to all the systems that the stereophonic receiver is required to carry out an additional detection process to demodulate the S signal and in so doing it will also demodulate those noise components in the discriminator output which lie in the region of the sub-carrier frequency and which, in a monophonic system, would not produce any audio-frequency output. Bearing in mind the 'triangular' noise spectrum, with higher noise levels occurring at the higher frequencies, which is characteristic of f.m. systems, it can be appreciated that, where receiver first-circuit noise is the limiting factor, the signal/noise ratio with stereophonic reception is markedly lower than with a monophonic system. This, of course, is the price which is paid for increasing the bandwidth of the transmitted information without correspondingly increasing the channel width used for transmission. The reduction of signal/noise ratio applies not only to first-circuit noise but also to impulsive interference, co- or adjacent-channel interference and many other disturbances of the receiver input signal, though the extent of the degradation in these other cases is often reduced by other factors.

One further point of importance is the audio bandwidth required for the S signal. Assuming that the M signal transmitted includes audio frequencies from 30 c/s to 15 kc/s, it has been found from subjective experiments[6] that the S signal requires a minimum bandwidth of approximately 100 c/s to 8 kc/s in order to avoid a just-perceptible deterioration—on certain types of programme at least—as compared with the case in which the full a.f. range is transmitted on both channels. Thus little saving of bandwidth of the S channel is possible without compromising stereophonic reproduction. It will, accordingly, be assumed that receivers will not be designed for a reduced S-channel band and in the theoretical discussion of system performance an upper-frequency limit of 15 kc/s will be assumed for all a.f. signals.

The requirement for the cross-talk level between A and B is that it should not exceed about -26 dB up to 5 kc/s, with some easement at higher frequencies as discussed later in the paper. Amplitude and phase inequality of the overall response of the M and S channels is an important cause of cross-talk between

A and *B*, the relationship being given in Appendix 2, Section 10.3. In particular, care is needed in the parts of the transmitter and receiver where the relative delay (and hence phase) of the *M* and *S* signals may be affected, not only in audio-frequency circuits but also in circuits handling the multiplex or complete f.m. signal.

3. Recent Proposals for Stereophonic Multiplex Systems

The parameters of a number of systems that have been proposed in the last few years are given in Tables 1 and 2. In some cases closely similar systems have been proposed on both sides of the Atlantic, largely independently; such systems are mentioned in the latest form in which they are known in Europe. For example, the Siemens System[7] is similar to a proposal by the Philco Corporation[8] in the U.S.A., and the pilot-tone system is similar to an earlier system proposed by the Grundig Company in Germany. The German Loewe system is also very similar to a proposal of Philips[9] in the Netherlands.

An important (but not necessarily the most important) property of a system is the signal/noise ratio performance on weak signals when first-stage receiver noise tends to set the limit; this property can be readily evaluated theoretically as discussed in Appendix 1. The results for the *M* signal (affecting monophonic listeners) and for the *A* signal (affecting stereophonic listeners) are given in Tables 1 and 2 in terms of the signal/noise ratio degradation as compared with monophonic transmission and reception, assuming well-designed receivers. In judging the importance of these degradations, one must bear in mind that there is usually some signal/noise ratio in hand in the monophonic case. As discussed in Appendix 1, Section 9.2, the theoretical ratio is, for example, 77 dB in a monophonic system (reference signal ±47 kc/s deviation at 400 c/s) for an input signal level of −67 dB (mW) taking a receiver noise figure of 10 dB. This input level corresponds to that obtained from a half-wave dipole at a height of 30 ft in an ambient field of 250 μV/m, the nominal monophonic service limit.

Table 1

Stereophonic systems for f.m. broadcasting using f.m. subcarrier

System No.	System Name	Sub-carrier frequency	Peak deviation of main carrier (% of ± 75 kc/s) By *M* signal	By sub-carrier	Peak deviation of sub-carrier by *S* signal	Degradation of signal/noise ratio (dB) relative to monophonic system *M* signal	*A* or *B* signal
1	Crosby	50 kc/s	50%	50%	±25 kc/s	6·0	12·5
2	R.A.I.	50 kc/s	67%	33%	±20 kc/s	3·5	17·6
3	E.B.U.	50 kc/s	80%	20%	±20 kc/s	1·95	22·0

Table 2

Stereophonic systems for f.m. broadcasting using a.m. subcarrier

System No.	System Name	Sub-carrier frequency	Peak deviation of main carrier (% of ±75 kc/s) By *M* signal when $A = B$	By sub-carrier sidebands only when $A = -B$	By sub-carrier or pilot-tone when $A = B = O$	Degradation of signal/noise ratio (dB) relative to monophonic system *M* signal	*A* or *B* signal
4	Loewe (full sub-carrier)	35 kc/s	75%	25%	25%	3·75	30·25
5	Pilot-tone (suppressed sub-carrier)	38 kc/s +19 kc/s pilot	90%	90%	8–10% (19 kc/s)	3·9	21·5
6	Mullard[5]	32·5 kc/s	52·5%	82·5%	5·3% (32·5 kc/s) 47·5% (65 kc/s)	8·6	23·3
7	Siemens (lower sideband)	30 kc/s	94%	94%	6%	3·5	15·0
8	U.S.S.R. (partially suppressed sub-carrier)	31·25 kc/s	80%	80%	20%	4·9	24·5

In comparing the various systems, we first consider the disturbance of the service to existing monophonic listeners. Systems 2, 3, 4, 5 and 7 appear satisfactory as far as the degradation of the theoretical M signal/noise ratio is concerned. This figure, as given in Tables 1 and 2, represents the reduced deviation effective for a monophonic receiver and therefore also gives the approximate increase in interference from other sources such as motor-cars.

Of these systems, the Loewe system shows a rather severe increase in noise for stereophonic reception, while the Siemens system requires a rather complex receiver if the full theoretical performance in stereophony is to be achieved. The U.S.S.R. system, as put forward at the C.C.I.R. Plenary Assembly in Geneva in 1963, is in many ways similar to the pilot-tone system. A normal amplitude-modulated sub-carrier is passed through a notch filter centred on the sub-carrier frequency, and a compensating peaking filter is used in the receiver. In comparison, the pilot-tone system transmits the sidebands fully, but completely suppresses the sub-carrier, a tone of one-half of the sub-carrier frequency being transmitted as a reference signal from which the sub-carrier can be regenerated in the receiver.

In considering the results of field trials in the U.S.A., the F.C.C. narrowed the choice to systems including those represented in the E.B.U., felt that the pilot-tone system was the best of those proposed. The present position is that international agreement on a system must probably await the meeting of C.C.I.R. Study Group XI (Broadcasting) meeting in Vienna in 1965. Meanwhile, experimental transmissions, mostly with the pilot-tone system, are being made in various European countries from time to time. In the U.K., high-power experimental transmissions of the pilot-tone system were started in the London area from the 91·3 Mc/s Wrotham transmitter in June 1962.† In the remainder of the paper we will therefore consider the characteristics and instrumentation of the pilot-tone system.

4. The Pilot-tone System

In the pilot-tone system the multiplex signal (i.e. the signal which controls the frequency modulation of the transmitter) consists of three parts:

(1) the M signal, which is in the audio-frequency range;

(2) a sub-carrier signal consisting of the result of suppressed-carrier amplitude modulation of a 38-kc/s signal by the S signal; and

(3) a steady 19-kc/s pilot tone used for synchronizing as discussed later.

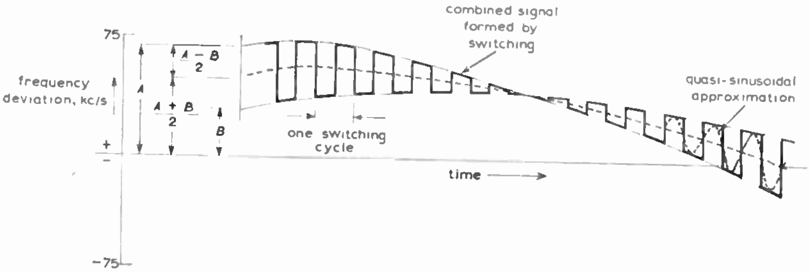


Fig. 1. Idealized time-division multiplex system.

1 and 5 in Tables 1 and 2 before the latter was finally selected.[10] Other field and laboratory tests in Europe co-ordinated by the Stereophonic Working Party of the European Broadcasting Union (E.B.U.) suggested that the most promising systems were 3 and 5. The f.m. sub-carrier systems 1 and 2 with the larger amplitude of sub-carrier had tended to show, in the case of monophonic reception, too great a sensitivity to certain kinds of interference—especially that from other transmissions spaced by 50 kc/s and 100 kc/s—and this is one of the reasons why system 3 rather than system 1 was carefully examined in Europe and assessed in comparison with system 5.

While no firm recommendation for a system was made by the C.C.I.R. in 1963, a number of countries, It is nevertheless helpful to forget for the moment this description, and to make a completely different approach to the system. We consider the transmission of two audio-frequency signals A and B by 'time division' multiplex (Fig. 1). This means we switch at regular intervals between A and B; the resultant waveform alternates between following the A signal and following the B signal, and we may refer to the process as 'square-wave switching'.

It is seen from the figure that the low-frequency component consists of the mean signal $\frac{1}{2}(A+B) = M$

† At the time of writing, experimental transmissions take place on Tuesday, Wednesday and Thursday mornings, 10.30 to 11.00 a.m. (tone tests) and 11.15 to 11.45 a.m. (programme tests).

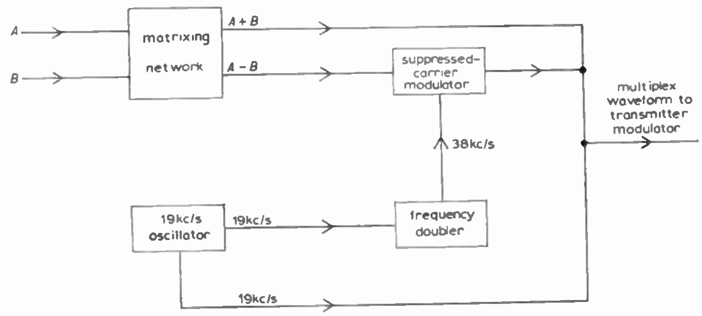


Fig. 2. Stereophonic coder, block schematic.

and that the rapid alternations have an amplitude of $\frac{1}{2}(A-B) = S$. In order to avoid the wide spectrum of frequencies which the square-wave type of multiplex waveform would entail, we approximate by a quasi-sinusoidal waveform as shown by the dotted curve on the right-hand side. The multiplex signal still alternates between A and B but we now have, for the high-frequency component, a waveform representing suppressed-carrier amplitude modulation. Neglecting the 19-kc/s tone, this corresponds to our original description. We could therefore generate the multiplex signal equally well either by a square-wave switch followed by a low-pass filter removing harmonics of the sub-carrier frequency,‡ or else by direct modulation, using the S signal, in a suppressed-carrier modulator, and addition of the M signal. For transmitting purposes in the B.B.C. tests it was considered more convenient to use the latter method, and the block schematic of the transmitting arrangement is shown in Fig. 2.

A similar situation exists in receiver design, that is to say, it is possible either to demodulate the sub-carrier signal separately to obtain the S signal and, by sum and difference combination with the M signal, derive A and B, or else to employ a switching technique which gives the stereophonic signals directly. This latter technique is discussed in detail in Section 5, where it is shown that some anti-cross-talk correction is needed to allow for square-wave switching with a sinusoidal sub-carrier.

Figure 2 also shows the addition of the 19-kc/s pilot signal mentioned earlier; this is of constant amplitude and the F.C.C. specification[10] sets the level to correspond to between 8 and 10% of the total frequency deviation. There is a convention regarding the phase which must be carefully observed in order that the regenerated sub-carrier in the receiver obtained by frequency doubling shall be of the correct phase. At this stage it is simpler to define the complete system by means of an equation. The instantaneous deviation of the transmitter, expressed as a fraction of 75 kc/s, is

$$0{\cdot}9[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t]$$
$$= 0{\cdot}9[M+S\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t]$$

where $\omega_s/2\pi$ is the sub-carrier frequency (38 kc/s) and A and B represent the pre-emphasized audio signals which vary within the range ± 1. The pilot amplitude has been set at 9% of the system deviation of ± 75 kc/s. It will be seen from the expression that the phase convention is that, when the pilot signal passes through zero, the sub-carrier signal also makes a zero crossing and that, at times when A is more positive than B, this crossing should be in the positive-going direction. The advantage of a pilot-tone frequency equal to one-half of the sub-carrier frequency is apparent from the spectrum of the multiplex signal shown in Fig. 3. The receiver can more easily filter a 19-kc/s signal than a 38-kc/s signal because the former is well spaced from interfering sidebands or audio-frequency signals.

5. The Receiver

5.1. *General Requirements for Stereophonic Reception*

It is convenient to consider a receiver for the pilot-tone system in two sections: the tuner, i.e. that part up to and including the discriminator, which selects, amplifies and demodulates the incoming frequency-modulated signal to produce the multiplex waveform, and the subsequent circuits which are concerned with decoding the multiplex signal to recover the separate A and B audio-frequency outputs.

The vector diagrams in Fig. 4 show the signals at various stages through the system with two separate

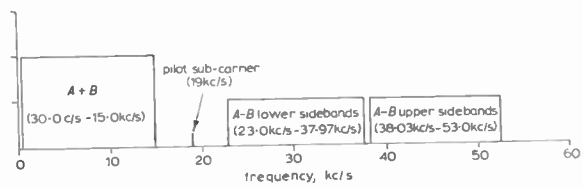
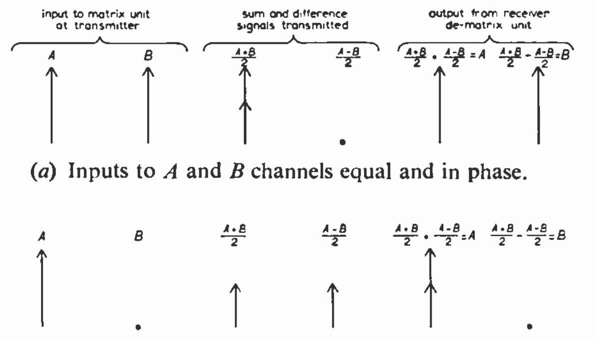


Fig. 3. Spectrum of pilot-tone system multiplex waveform.

‡ It is also necessary to reduce in level the difference-sideband signal by a factor of $\pi/4$ to obtain a sinusoidal amplitude equal to the original square-wave amplitude.



(*a*) Inputs to A and B channels equal and in phase.

(*b*) Input to A channel only.

Fig. 4. Vector relationships of signals in an ideal pilot-tone stereophonic system.

conditions of modulation. From this it can be seen that, for correct reconstitution of the A and B signals or 'de-matrixing', the relative phases and amplitudes of the sum and difference signals must be preserved. In Fig. 4(*b*), for example, which represents a modulation input applied to the A channel only, if the difference vector $\frac{1}{2}(A-B)$ were shifted in phase or changed in amplitude relative to the sum vector $\frac{1}{2}(A+B)$, the resultant B signal, ideally $\frac{1}{2}(A+B)-\frac{1}{2}(A-B)$, would no longer be zero and there would be crosstalk from the A channel into the B channel output.

A further requirement which concerns both sections of the receiver is that of signal/noise ratio. As discussed in Sections 2 and 3, the pilot-tone system in common with other multiplex systems is inherently more susceptible to noise and interference of all types than is a monophonic system, and it is important that the signal/noise ratio should not be further degraded by avoidable noise arising from instrumental deficiencies in the receiver.

In designing the tuner for a stereophonic receiver, two characteristics of the pilot-tone system should be borne in mind:

(i) the increase in the transmitted modulation-frequency bandwidth from 15 kc/s to 53 kc/s;

(ii) the increased susceptibility to noise and interference.

Ideally, the overall response at the discriminator output should be uniform in amplitude and linear in phase up to 53 kc/s. Any distortion of the multiplex spectrum occurring here will have precisely the same effect as the distortion caused by the decoder low-pass filter discussed in Section 5.2.1, and the resulting crosstalk can be evaluated by the formulae derived in Appendix 2. One factor affecting the overall response is the amplitude characteristic of the intermediate frequency amplifier. It is not always appreciated that at the higher frequencies for which the modulation index is of the order of unity or less, the modulation-frequency response of a f.m. receiver depends upon the i.f. response over the bandwidth occupied by the first-order modulation sidebands in a manner very similar to that of an a.m. receiver.[11] Attenuation of the higher frequency components of the multiplex waveform due to this cause could be removed by increasing the i.f. bandwidth but this expedient is to be regarded with caution since it increases the possibility of adjacent-channel interference, an aspect of performance in which the pilot-tone system is in any case only marginally acceptable.

Generally speaking, quite large departures from the ideal modulation-frequency response at the discriminator output can be tolerated since the mean difference in level between the sum and difference sideband signals can be compensated, as discussed in Section 5.2.1.

In view of the reduction in signal/random-noise ratio with this system, the tuner noise factor is important in determining the minimum signal level at which satisfactory reception is possible. Owing to the presence of cosmic noise the effective aerial temperature in Band II is some 1200°–1500° K and it is therefore not possible to realize the full benefit of a very low receiver noise factor.[12] For example, a true receiver noise factor of 6 dB would give an effective noise factor of between 8·5 dB and 9 dB when allowance for cosmic noise is made.

Efficient amplitude limiting is essential in a stereophonic tuner in order to remove the avoidable amplitude modulation component of any interference which may be present. It is also necessary to ensure that the tuner output level remains within the range in which satisfactory operation of the decoder is obtained for all usable r.f. signal levels. This presents no great difficulty; r.f. signals below one or two hundred microvolts would generally give an unacceptable signal/noise ratio, and with inputs above this level the receiver a.g.c. is normally fully operative.

It may be said that, in general, the requirements of the tuner are similar in type to those applicable for monophonic receivers, although more stringent in degree. The novel part of the receiver is that concerned with decoding the multiplex signal.

5.2. *The Decoder*

The functions of the decoder are:

(i) To regenerate from the 19-kc/s pilot tone the 38-kc/s sub-carrier which is suppressed in transmission.

(ii) To demodulate the difference signal.

(iii) To dematrix the sum and difference signals, i.e. recombine them to recover the original A and B channel modulation signals.

Noise and interference accompanying the radio-frequency input signal to the receiver will produce unwanted components in the multiplex input applied to the decoder. As far as components lying within the frequency bands occupied by the sum signal and by the difference-sideband signal are concerned, they will be demodulated in precisely the same way as the wanted signal to produce an audible output; the design of the decoder can have no effect on this. Decoder design does, however, affect the response of the receiver to those components of interference and noise which occur above 53 kc/s and in the region of the pilot-tone frequency. Unless appropriate precautions are taken, frequencies above the limit of the normal multiplex spectrum can intermodulate with harmonics of the sub-carrier to produce an audio-frequency output in the difference channel. Similarly, components of interference close to 19 kc/s can enter the sub-carrier regenerating circuits and produce unwanted modulation of the sub-carrier; this also can give rise to an audio-frequency output.

Although the sub-carrier regenerator might appear to be the logical starting point in a discussion of decoders it will be more convenient to deal first with the basic operation of demodulating the multiplex signal.

5.2.1. The switching decoder

It is possible to combine operations (ii) and (iii) and produce the A and B signals directly from the multiplex signal without a separate dematrixing process. The type of decoder which operates on this principle is somewhat simpler than that in which the sum and difference signals are extracted separately, and will be described first.

Figure 5 shows the decoder in block schematic form. The multiplex input is fed to two demodulators, together with demodulating signals f_1, f_2 at the sub-carrier frequency, and the A and B outputs are produced directly. The general expression for the multiplex waveform as given in Section 4, is

$$\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t$$

where A and B represent the audio-frequency input signals to the left- and right-hand channels respectively and $\omega_s/2\pi$ is the sub-carrier frequency, 38 kc/s.

If the demodulators were linear multiplying devices and

$$f_1(t)=\tfrac{1}{2}+\sin\omega_s t$$
$$f_2(t)=\tfrac{1}{2}-\sin\omega_s t$$

then the outputs of the demodulators would be

$$E\ =[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t]\times\times[\tfrac{1}{2}+\sin\omega_s t]$$

$$E_B=[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t]\times\times[\tfrac{1}{2}-\sin\omega_s t]$$

Considering only the audio-frequency terms in the products

$$E_A=\tfrac{1}{4}(A+B)+\tfrac{1}{4}(A-B)=\tfrac{1}{2}A$$
$$E_B=\tfrac{1}{4}(A+B)-\tfrac{1}{4}(A-B)=\tfrac{1}{2}B.$$

The coefficients of the terms representing the contributions of the sum and difference signals in each channel are numerically equal. Perfect stereophonic demodulation is thus achieved with the assumed form of $f_1(t)$ and $f_2(t)$, as has been pointed out by De Vries.[13]

This multiplication process is a precise operation which, by domestic receiver standards, would probably be expensive to realize in practice. An attractive alternative, however, is to use two simple diode gating circuits operating as on/off switches with a 1/1 mark/space ratio. This is equivalent to multiplication by a square wave, for which case the output signals are:[13]

$$E_A=[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t]\times\times\left[\tfrac{1}{2}+\frac{2}{\pi}\sin\omega_s t+\frac{2}{3\pi}\sin 3\omega_s t\ldots\right]$$

$$E_B=[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\sin\omega_s t+0{\cdot}1\sin\tfrac{1}{2}\omega_s t]\times\times\left[\tfrac{1}{2}-\frac{2}{\pi}\sin\omega_s t-\frac{2}{3\pi}\sin 3\omega_s t\ldots\right]$$

Again considering only the audio-frequency terms in the products,

$$E_A=\frac{A+B}{4}+\frac{A-B}{2\pi}=\frac{\pi+2}{4\pi}A+\frac{\pi-2}{4\pi}B$$

$$E_B=\frac{A+B}{4}-\frac{A-B}{2\pi}=\frac{\pi+2}{4\pi}B+\frac{\pi-2}{4\pi}A$$

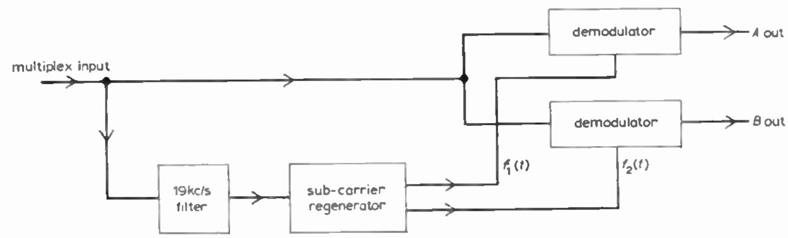


Fig. 5. Switching decoder, block schematic.

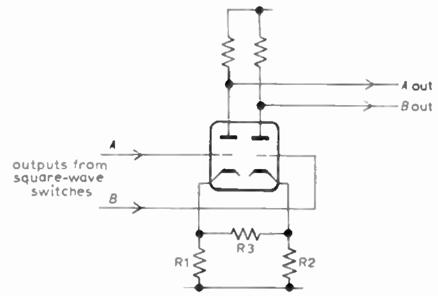


Fig. 6. Audio-frequency amplifier with partial common-mode suppression.

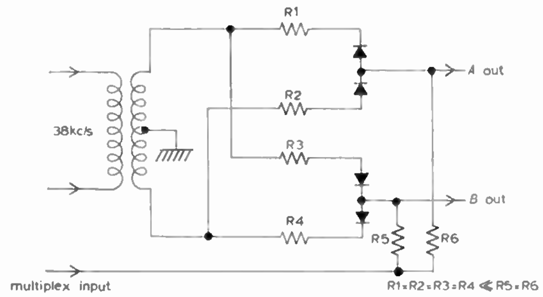


Fig. 8. Balanced switching demodulator.

Here, the coefficients of the sum and difference terms are not equal, with the result that part of the *A* channel input is reproduced from the *B* channel output and vice versa. This is a fundamental result of applying the square-wave switching to the signal, but the cross-talk can be compensated by passing the multiplex signal through a network which attenuates the sum signal by a factor of $2/\pi$ relative to the difference-sideband signal. Alternatively, this compensation can be applied after demodulation and one method of achieving this is by the use of an audio-frequency amplifier with partial common-mode suppression as shown in Fig. 6. With inputs to the two grids in-phase, as would arise from the sum signal, R3 has no effect on the gain of the stage. With anti-phase inputs, however, the negative feedback is reduced by the presence of R3. Thus the gain is greater for the difference-signal component than for the sum-signal component and this gain differential can be adjusted within quite wide limits by suitably proportioning R1, R2 and R3.

The application of this type of circuit is, of course, not limited to correcting the deficiencies of the square-wave switching system If R3 is made variable it forms a convenient control to compensate for unbalance between the sum and difference signal amplitudes arising elsewhere in the receiver.

The simplest form of switching demodulator is shown in Fig. 7. The waveform of the switching sub-carrier input is normally sinusoidal but, provided that it is sufficiently large compared with the multiplex input, the operation of the circuit will approximate closely to the square wave switching condition. This results, however, in a large sub-carrier frequency component appearing in the output, which, even after de-emphasis, can be an embarrassment in the following a.f. stages. Even more important, any audio-frequency modulation of the regenerated sub-carrier will also be reproduced. Both of these effects can be minimized by the use of a balanced switching circuit, such as that shown in Fig. 8, in which the output at the sub-carrier frequency is suppressed to an extent limited only by the degree of balance achieved in the diode bridges. This form of demodulator is also much less susceptible to amplitude modulation of the subcarrier switching input. Nevertheless, if the sub-carrier input waveform is sinusoidal, the switching waveform is in fact not a true square wave but a clipped sine wave and amplitude modulation of the sub-carrier can produce pulse-width modulation of the switching waveform and hence can still have some effect on the demodulated audio output.

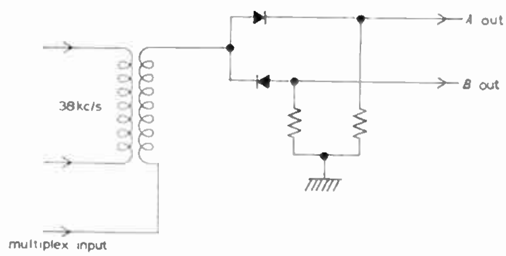


Fig. 7. Unbalanced switching demodulator.

In view of the degradation of signal/noise ratio which can be caused by interference components above 53 kc/s in the multiplex waveform, it is highly desirable that the demodulators be preceded by a low-pass filter. If this filter is not to introduce differential phase or amplitude errors in the demodulated sum and difference signals, which would result in cross-talk between the *A* and *B* channels, certain requirements must be satisfied. If $r(f)$ is the amplitude response and $\phi(f)$ is the phase response at a frequency f, while f_a and f_s are respectively the audio modulation frequency and the sub-carrier frequency, the requirements may be stated as follows:—

(i) The phase characteristic must be such that $\frac{d}{df}\phi(f)$ has the same value at $f = f_a$, $f = f_s - f_a$ and $f = f_s + f_a$ for all values of f_a.

(ii) The amplitude characteristic must be such that the ratio between $r(f_a)$ and $\frac{r(f_s - f_a) + r(f_s + f_a)}{2}$ is constant for all values of f_a.

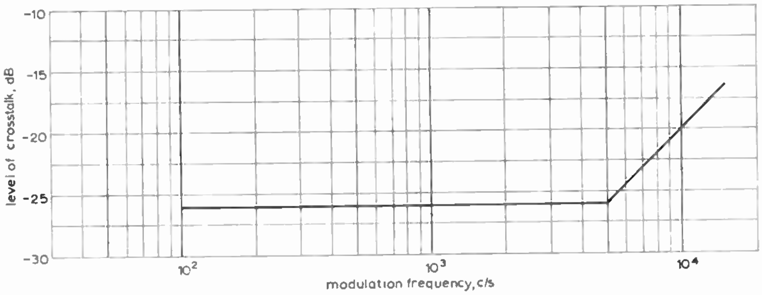


Fig. 9. Minimum perceptible level of crosstalk between *A* and *B* channels.

Note that this ratio need not necessarily be unity since any constant inequality between the sum and difference signals can be compensated in the same way as the difference in detection efficiency of the square-wave switch demodulator.

If the filter has constant attenuation and a linear phase characteristic between 30 c/s and 53 kc/s, these requirements are of course fulfilled. A practical simple filter with a cut-off frequency in the region of 55 kc/s would approximate quite closely to the required characteristic except in the immediate vicinity of the cut-off frequency. This would produce a phase shift, and possibly some attenuation, of the upper sideband of the difference-sideband signal at high modulation frequencies. Formulae are derived in Appendix 2 for calculating the resulting phase shift and attenuation of the demodulated difference signal and also the crosstalk which this would produce.

Figure 9 gives the level of crosstalk, as a function of frequency, which is given in C.C.I.R. Report No. 293 (Geneva, 1963) as just perceptible in a stereophonic broadcasting system. It shows that the permissible crosstalk increases at 6 dB per octave at frequencies above 5 kc/s, hence there is considerable tolerance of departures from the ideal characteristic in the decoder multiplex signal filter.

5.2.2. The sub-carrier regenerator

The requirement to be satisfied by the sub-carrier regenerator is that the 38 kc/s output should be in the correct phase to produce maximum output from the demodulators, free from both long-term and audio-frequency phase variations and from audio-frequency amplitude modulation. The output of the difference signal component from the demodulators is proportional to cos ψ, where ψ is the phase error of the switching sub-carrier (see Appendix 2). The output of the sum signal component is, however, independent of ψ. Long-term phase drift will thus produce crosstalk between the *A* and *B* channels, and phase modulation at an audible frequency will produce amplitude modulation of the difference signal.

In view of the fact that the difference signal amplitude follows a cosine law against the sub-carrier phase error, it is important that the initial phase error is small in order to minimize the disturbance to the output arising from any subsequent drift or phase modulation. It is therefore undesirable to use variation of sub-carrier phase as a means of making adjustments to the difference signal amplitude.

To minimize disturbances of the 38-kc/s output by interference, some means of amplitude limiting should be incorporated in the sub-carrier regenerating channel and the bandwidth of the filter circuits should be as small as is practicable. This second parameter is a matter for compromise since the requirement for minimum susceptibility to interference conflicts with that for good long-term phase stability.

Either of two methods can be adopted for regenerating the 38-kc/s sub-carrier. The pilot-tone can be used to lock a local oscillator at the pilot or sub-carrier frequency or it can be applied to some form of frequency doubler. In principle, it is probably easier to obtain good amplitude-modulation suppression with the locked-oscillator but this type of circuit is more prone to produce phase modulation of the output as a result of amplitude modulation of the locking signal. It is difficult to generalize, since the performance is determined more by the details of design than by the type of circuit adopted.

5.2.3. The sum-and-difference separation decoder

Figure 10 shows the block schematic of the type of decoder in which demodulation of the difference signal and de-matrixing are carried out separately. The difference-sideband signal is extracted from the multiplex input by the high-pass filter, added to the regenerated 38-kc/s sub-carrier, and the $A-B$ audio-frequency signal obtained by rectification in a mean or envelope detector. The delay network in the $A+B$ channel is required to compensate for the delay introduced by the high-pass filter. In the de-matrix network the sum and difference signals are combined to form the *A* and *B* outputs.

This form of decoder embodies some additional circuit features as compared with the switching type.

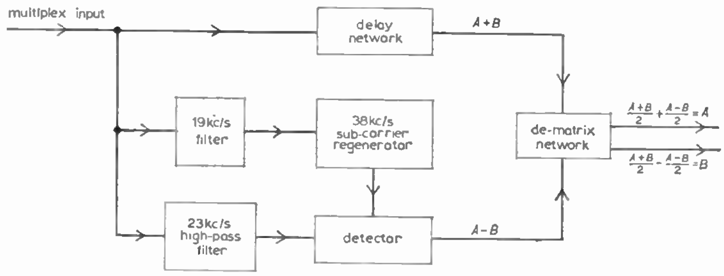


Fig. 10. Sum-and-difference separation decoder.

It requires additional filtering to separate the sum and difference signals, and the resultant differential delay in the sum and difference channels has to be corrected. If a simple diode detector is used a higher degree of amplitude modulation suppression is required in the sub-carrier regenerating channel, since the difference detector will treat a.m. sidebands accompanying the sub-carrier in precisely the same way as components of the wanted difference-signal modulation.

As in the switching decoder, a low-pass filter with a cut-off frequency of about 55 kc/s is highly desirable to reduce the effects of interference. The effects of phase shift of the sub-carrier and distortion of the spectrum of the difference-sideband signal are sensibly identical with both types of decoder circuit, provided that the reintroduced sub-carrier in the sum-and-difference decoder is large relative to the peak difference-sideband signal. If the sub-carrier amplitude is less than two or three times that of the peak double-sideband signal the forms of distortion which are associated with asymmetrical sideband reception of a.m. may become appreciable.

5.3. *A Practical Decoder Design*

Having discussed the general requirements of decoders, the details can best be illustrated with reference to a specific design. Figure 11 shows the circuit diagram of a switching decoder.

The 19-kc/s pilot tone is extracted from the multiplex signal in the cathode circuit of V1A by the single-circuit filter L1,C3. V2 is a linear amplifier, the anode load being a second 19-kc/s tuned circuit L2,C9. Diodes MR1 and MR2 form the frequency doubler and V1B functions as a saturated grid limiter and 38-kc/s amplifier. The resonant primary circuit of the transformer T1 gives further filtering of the regenerated sub-carrier and a sinusoidal switching voltage of 40 V peak-to-peak is developed across the secondary of T1. Figure 12 shows the static limiting characteristic of the sub-carrier regenerating channel.

The multiplex signal is taken from the anode of V1A through the low-pass filter C5,L3,C6 and the feed resistors R15 and R16 to the switching bridges. The 19-kc/s tuned circuit L1, C3 in the cathode

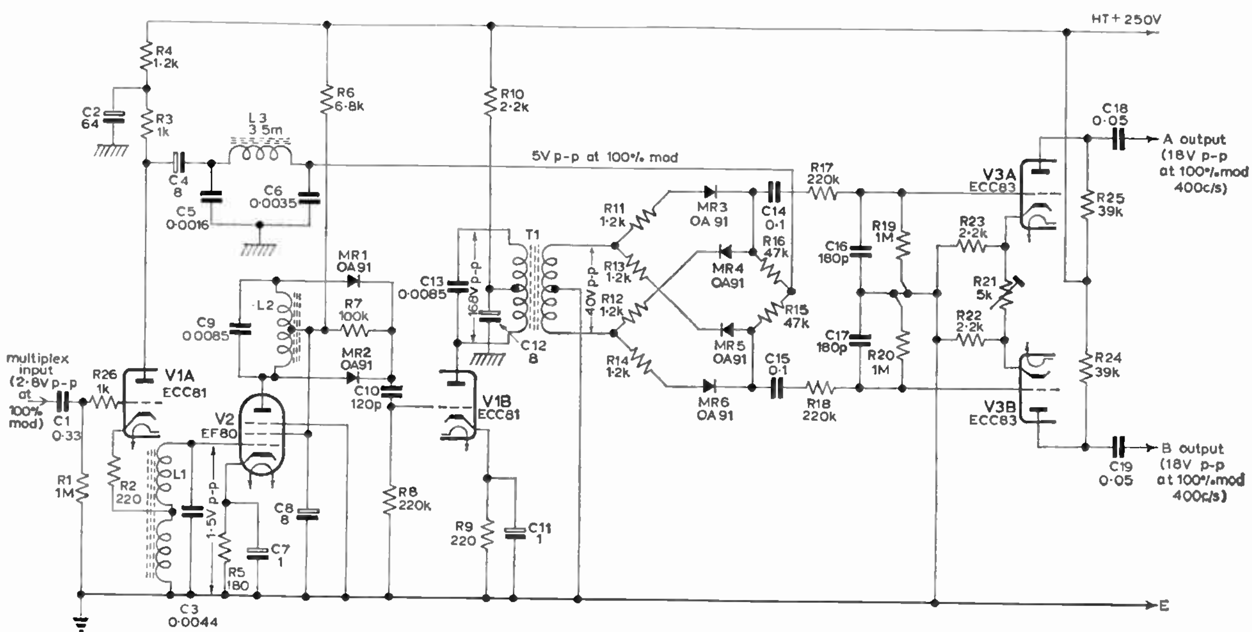


Fig. 11. Switching decoder, circuit diagram.

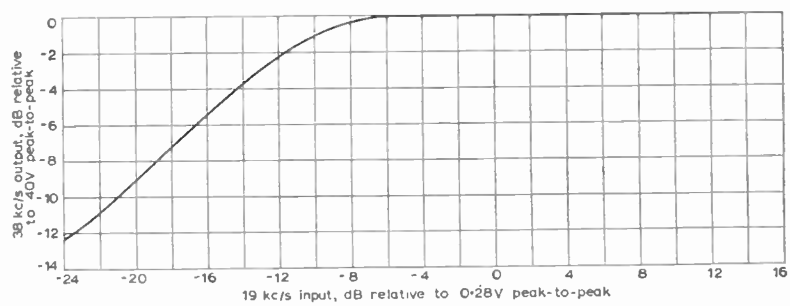


Fig. 12. Static limiting characteristic of sub-carrier regenerating channel.

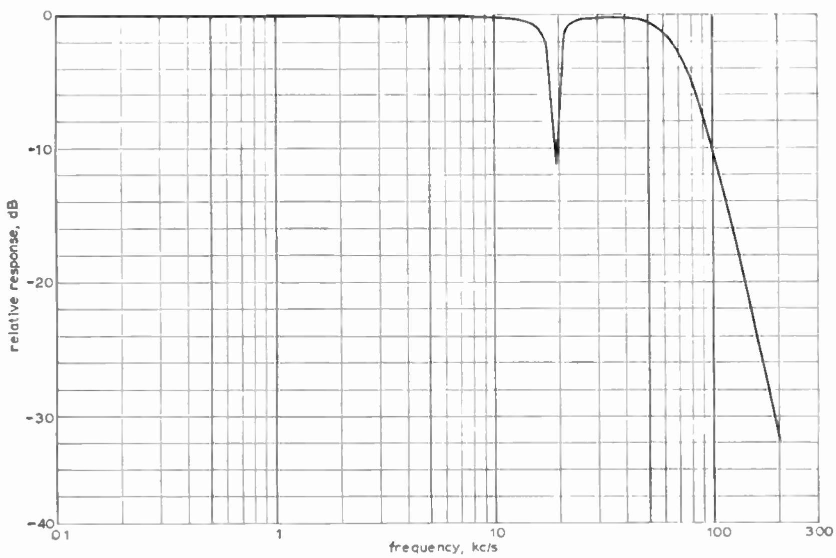


Fig. 13. Amplitude response of multiplex signal channel.

circuit of V1A also functions as a notch filter and reduces the amplitude of the pilot-tone component in the multiplex waveform fed to the switching circuits. This, in turn, reduces the amplitude of any unwanted beat tones arising from intermodulation between the pilot tone and the modulation components of the multiplex signal due to non-linearities in the demodulators or subsequent stages. The measured overall amplitude and phase characteristics of the multiplex signal channel are shown in Fig. 13 and in Fig. 14, curve (a).

Considering the phase response, a linear phase characteristic merely represents a constant time-delay to all components of the signal and hence has no effect on the process of stereophonic demodulation. In evaluating the crosstalk which will result from imperfections in the response of the multiplex channel, the phase errors to be taken into account are those which represent departures from the linear phase condition.

In this case the maximum errors occur at the highest modulation frequency 15 kc/s. By putting the appropriate values from Figs. 13 and 14 into the expression given in Appendix 2, Section 10.1, the phase shift and attenuation of the demodulated difference signal may be determined; we have

phase error at 23 kc/s, $\theta_1 = 14°$

phase error at 53 kc/s, $\theta_2 = -7°$

amplitude factor at 23 kc/s,† $k_1 = 0{\cdot}99$

amplitude factor at 53 kc/s,† $k_2 = 0{\cdot}97$.

From these values we find that the demodulated difference signal at 15 kc/s will be reduced by 0·2 dB and phase-retarded by 10·6 deg.

However, we see from Figs. 13 and 14 that, at 15 kc/s, the sum signal is reduced by 0·4 dB and retarded by 9 deg. Thus the differential amplitude and phase errors are 0·2 dB and 1·6 deg which, if they were the only imperfections in the circuit, would produce a cross-talk level of −36 dB between the *A* and *B* channels (see Appendix 2, Section 10.3).

The source impedance presented to the switching circuits by the transformer T1 is 1250Ω; the series resistors R11 to R14 prevent an undue drop in the secondary voltage due to loading by the short-circuited pair of diodes. The presence of these

† The constant attenuation of 0·2 dB over the difference-sideband signal portion of the spectrum has been ignored since it can be compensated in the audio-frequency stage.

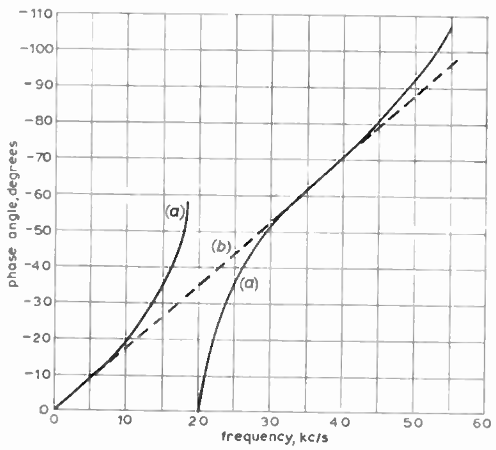


Fig. 14. Phase characteristic of multiplex signal channel.
(*a*) Measured characteristic.
(*b*) Ideal linear phase characteristic.

resistors, together with the finite resistance of the switching diodes in the nominally short-circuited condition also produces slight in-phase cross talk between the *A* and *B* channels but this can be corrected in the following stage V3.

The de-emphasis circuits precede the first audio amplifier V3 and reduce the amplitude of the ultrasonic components in the input to this stage. Precise matching of the time constants is required in order to preserve a constant ratio of common-mode to differential-mode gain over the entire modulation-frequency range.

5.4. *Performance*

The performance figures given are for the decoder when fed with a 2·8 V peak-to-peak multiplex signal from a stereophonic coder. If the decoder were embodied in a complete receiver some deterioration in overall performance might well result from imperfections in the tuner.

Figures 15 to 18 and Table 3 show the performance test results. The modulation frequency response curve shown in Fig. 15 is the average of both *A* and *B* channels, since the maximum difference between them was 0·2 dB. The crosstalk shown in Fig. 16 is rather worse at the extreme low and high modulation frequencies than can be accounted for by the characteristics of the multiplex channel alone. This result represents the sum of various instrumentalities in both coder and decoder but, since it is well within the subjective tolerances shown in Fig. 9, it was not investigated further.

Susceptibility to interference components in the multiplex signal in the region of 19 kc/s is shown in Fig. 17, and the method of measurement requires some explanation. Any interference sidebands accompanying the regenerated sub-carrier can produce

Table 3

Harmonic distortion with 400 c/s modulation

Modulation, relative to maximum	Channel to which modulation applied	Distortion at output of channel *A*	*B*
100%	*A*	0·7%	—
,,	*B*	—	0·8%
,,	*M*	1·6%	1·5%
,,	*S*	0·7%	1·0%
40%	*A*	0·7%	—
,,	*B*	—	0·7%
,,	*M*	0·7%	0·7%
,,	*S*	0·6%	0·6%

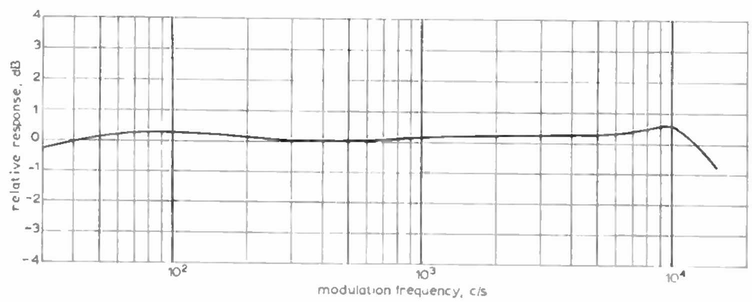


Fig. 15. Modulation-frequency response of *A* and *B* outputs.

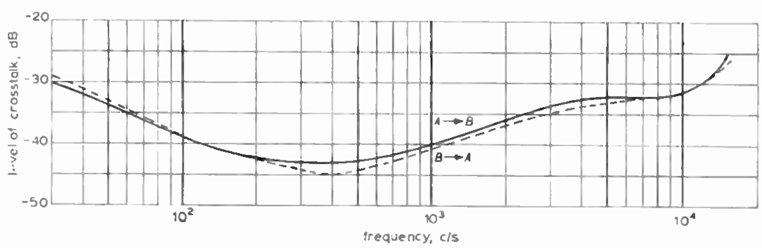


Fig. 16. Crosstalk characteristic between *A* and *B* channels.

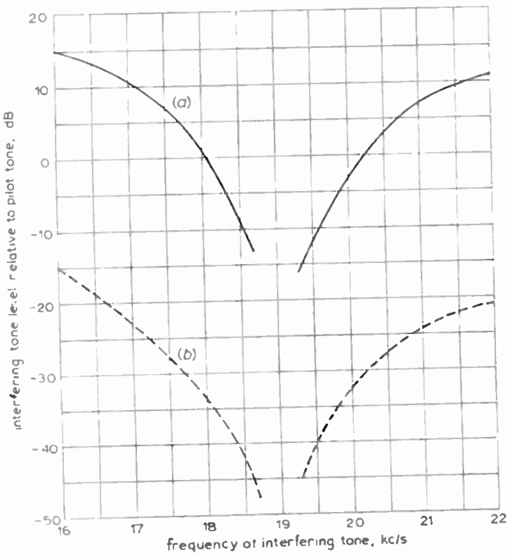


Fig. 17. Susceptibility to interference in sub-carrier regenerating channel.

(*a*) With adequate sub-carrier limiting.

(*b*) With no sub-carrier limiting.

a disturbance of the audio-frequency output of the receiver either by direct demodulation (as, for example, with an amplitude modulated sub-carrier in a sum and difference separation decoder) or by intermodulation with the wanted signal (as, for example, with a phase modulated sub-carrier). In order to reveal the presence of both mechanisms a test signal having 40% modulation at 640 c/s in the *A* channel alone was applied together with an interfering sinusoidal signal at a frequency f_I, f_I being varied in steps from 16 kc/s to 22 kc/s. At each step the level of interference was increased from zero until the largest unwanted component of the receiver output (ignoring components at harmonics of the wanted modulation frequency) was -40 dB relative to the output level of the wanted 640 c/s modulation. The level of the interfering signal f_I was then recorded. Figure 17 shows the results of the test with, for comparison, results of a similar test on a switching decoder with little or no limiting in the sub-carrier channel and unbalanced single diode switching circuits.

Susceptibility to unwanted components of the multiplex signal at frequencies above 53 kc/s was tested by measuring the 1-kc/s output obtained with an input signal of constant amplitude set, in turn, at frequencies of f_s+1 kc/s, $2f_s+1$ kc/s. .. nf_s+1 kc/s, where f_s is the sub-carrier frequency (38 kc/s). The results are shown in Fig. 18 with, for comparison, results of a similar test on the same decoder but with the low-pass filter C5, L3, C6 removed.

The results for total harmonic distortion are given in Table 3.

The performance is regarded as generally satisfactory. One way in which it could probably be improved is by redesigning the multiplex channel low-pass filter with a somewhat lower cut-off frequency. This would give increased rejection of interference in the region of 70 to 150 kc/s, at the expense of some increase in crosstalk at the higher modulation frequencies which could well be tolerated.

6. Interference

Frequent references have been made in the foregoing discussion to the importance of those aspects of stereophonic receiver design which affect susceptibility to interference and it is of interest to consider some types of interference in more detail.

6.1. *Adjacent-Channel Interference*

The presence of an interfering transmission spaced in frequency by Δf from that to which the receiver is tuned will give rise to a spectrum of interference, in the output of the discriminator, centred on the frequency Δf and extending outwards to an extent determined by the modulation of the two transmissions. Where $\Delta f = 200$ kc/s, almost all of this interference spectrum lies outside the bandwidth occupied by the wanted multiplex signal. Nevertheless, an audible output will be produced if any components of the interference are demodulated by beating with harmonics of the sub-carrier frequency. In such a case, the provision of a low-pass filter in the multiplex signal channel would be expected to give a very substantial improvement. Where $\Delta f = 100$ kc/s the improvement, though significant, would be smaller since the interference spectrum will now overlap that of the wanted multiplex signal and cannot be entirely eliminated without disturbance of the wanted modula-

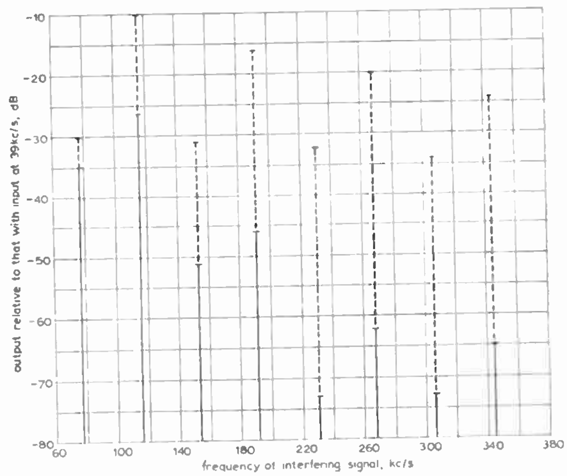


Fig. 18. Susceptibility to interference from multiplex signal components above 53 kc/s.

——— With low-pass filter. - - - - - Without low-pass filter.

tion. Subjective tests with a very simple switching decoder, fed from the discriminator of a typical medium-priced domestic f.m. receiver, showed an improvement of 15 dB where $\Delta f = 200$ kc/s and 7 dB where $\Delta f = 100$ kc/s by the use of a filter with a response above 40 kc/s similar to that shown in Fig. 13. The improvement quoted represents the increase in the level of the interfering signal required to produce an output interference subjectively rated as 'perceptible'.

6.2. *Multipath Propagation*

The distortion arising from multipath propagation,[14] that is when the received signal consists of a number of components which have travelled by paths of different lengths and hence suffered different time delays, has been found rather more severe with the pilot-tone system than with the normal monophonic system. This is particularly so where the reflected signals involved have time delays corresponding to comparatively small path differences, 5 miles or less.

One effect of the presence of signals with path differences close to 5 miles is to produce a form of intermodulation in which the pilot tone is amplitude modulated by the low-frequency components of the sum signal. If this modulation is not adequately suppressed by amplitude limiting in the sub-carrier regenerating channel it produces distortion of a particularly unpleasant character. Objective measurements have been made to determine the extent of the improvement which sub-carrier amplitude limiting can provide and the results of one test are given in Fig. 19. They show the amplitude of harmonics produced by two decoders when fed from the discriminator output of a medium-priced f.m. receiver. The r.f. signal input to the receiver contained one delayed component with an amplitude of 10% and a delay corresponding to a path difference of 5 miles relative to the main signal; the signal was modulated 100% at a frequency of 120 c/s in the *A* channel alone. Both decoders were switching types, one with a high degree of limiting, and the other with little or no limiting, in the sub-carrier regenerating channel. (Figure 17 gave the results of tests on the same two decoders.) It can be seen that, although some distortion remains even with the better decoder (this due largely to unavoidable distortion of the sum- and difference-signal information), the total distortion has been very substantially reduced. Two points must be emphasized with regard to the results given in Fig. 19. The first is that the condition measured, with a low modulation frequency and a single delayed signal having a delay corresponding to a path difference of 5 miles, is that for which the difference between adequate and inadequate sub-carrier limiting is most marked. The second is that it presents a somewhat over-pessimistic picture of the total effect of multipath propagation. The amplitudes of individual harmonics vary very rapidly with small changes in the relative phase of the reflected and direct signals. For the purposes of the test, this phase was adjusted to produce the maximum amplitude of each harmonic in turn and that maximum value is shown in the diagram. In a practical situation, therefore, the harmonics will not all reach the indicated values simultaneously.

6.3. *Impulsive Interference*

This form of interference would be quite serious to listeners to a stereophonic broadcasting service at the fringe of the transmitter service area. Subjective tests indicate that, with a high level of interference, the difference between adequate sub-carrier limiting and no limiting in the decoder corresponds to a difference in the subjective rating of the interference of about one grade in the scale 'imperceptible', 'just perceptible', 'perceptible', 'slightly disturbing' and 'disturbing'.

6.4. *Comparison of Monophonic and Pilot-tone Systems for Susceptibility to Interference*

It is not possible to give exact figures for the comparison of performance of a stereophonic with a monophonic system in the presence of the forms of interference considered above. The effects are determined partly by the characteristics of the interference and partly by the design of the receiver. However, in the authors' experience, the following represents

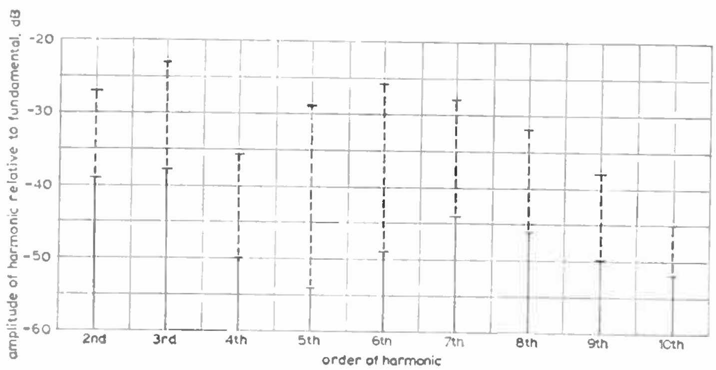


Fig. 19. Effect of sub-carrier limiting on multipath propagation distortion.
——— With adequate sub-carrier limiting.
------ With no sub-carrier limiting.

the average performance attainable, assuming a well-designed receiver; the importance of an adequate design cannot be too heavily stressed.

(i) *Adjacent-channel Interference.* In terms of the wanted/unwanted signal ratio at the receiver input for 'perceptible' interference, the pilot-tone system requires an additional protection of about 15 dB when the interfering transmission is offset by 100 kc/s but negligible extra protection, not more than 2 to 3 dB, with frequency offsets of 200 kc/s or more.

(ii) *Multipath propagation.* The effect of multipath propagation varies rapidly with the relative delay of the reflected signal. When this delay is comparatively long, corresponding to a path difference greater than about 8 miles, the subjective assessment of the resulting distortion averages some ½ grade worse, in the subjective scale referred to earlier, with the pilot-tone system. As the path difference is reduced, the difference between the stereophonic and monophonic systems increases from 1 grade at 5 miles to 3 grades at 2 miles. In the case of a 2-mile path difference, this does not necessarily mean that distortion will be widespread, since reflected signals with such a short path difference are rarely large enough to cause audible distortion in the monophonic case. The greatly increased sensitivity in stereophony means, however, that the shorter path differences will become important in a number of cases, but there is insufficient experience to make a practical comparison with monophony.

(iii) *Impulsive Interference.* With impulsive interference at a low level the output signal/noise ratio is some 20 dB worse with stereophony, as might be expected since the effect is similar to that of random noise. This corresponds approximately to two grades on the subjective scale. As the impulse level is increased this disparity reduces considerably and, in the practical situation with impulses covering a wide range of amplitudes, the overall subjective difference is about one grade.

All of the comparisons given above apply to stereophonic reception. Compatible reception of the stereophonic transmission on a normal monophonic receiver would be degraded only to a negligible extent.

With the present monophonic v.h.f. broadcasting service, 98% of the population of the U.K. can obtain satisfactory reception. If a stereophonic service were instituted, using the pilot-tone system and the existing v.h.f. transmitter network, the corresponding figure for stereophonic reception is estimated at approximately 93%, this reduction being the result of the increased susceptibility to interference of the stereophonic system. The latter figure assumes an adequate standard of receiver performance and the use of directional aerials where necessary in fringe areas. Special measures, such as the provision of additional transmitters and the revision of certain frequency allocations would improve the position to some extent but would not wholly make good the loss. In any case, it would be essential to ensure that shortcomings in receiver performance did not add to the inherent limitations of the system and thus produce a further reduction of the effective stereophonic service area.

7. Acknowledgments

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9. Appendix 1

This appendix gives the basic formulae for the theoretical signal/noise ratios for multiplex systems. They apply on the assumptions that the output noise level arises from a continuous source of noise at the receiver input or first amplifying stages and that the signal/noise ratio is high.

9.1. *List of Symbols*

A, B = audio-frequency signals in left and right stereophonic channels.

M, S = sum, $\frac{1}{2}(A+B)$, and difference, $\frac{1}{2}(A-B)$, audio-frequency signals.

$\pm H_0$ = peak frequency-deviation in standard monophonic broadcasts (± 75 kc/s).

$\pm H_m$ = peak frequency-deviation of main carrier by M signal (f.m. sub-carrier systems).

f_s = frequency of sub-carrier.

$\pm H_s$ = peak frequency-deviation of main carrier by sub-carrier.

$\pm h_s$ = peak frequency-deviation of sub-carrier by S signal (f.m. sub-carrier systems).

$\pm H'_m$ = maximum deviation of main carrier by the M signal possible when $A = B$ (no S signal).

$\pm H''_m$ = maximum deviation of main carrier by the M signal possible with a finite A signal input and zero B signal input.

$_a = \omega_a/2\pi$ = maximum audio frequency.

τ = pre-emphasis time constant.

a_1 = r.m.s. signal/noise ratio improvement factor† for pre-emphasis in a.m.

$$= \left[\frac{\arctan \omega_a \tau}{\omega_a \tau}\right]^{-\frac{1}{2}}$$

a_2 = r.m.s. signal/noise ratio improvement factor† for pre-emphasis in f.m.

$$= \frac{\omega_a \tau}{3}\left[1 - \frac{\arctan \omega_a \tau}{\omega_a \tau}\right]^{-\frac{1}{2}}$$

† These factors do not include correction for reduced modulation at the lower audio frequencies, i.e. they assume that the modulation level at low audio frequencies remains constant when pre-emphasis is applied.

b = factor of reduction of modulation at the lower audio frequencies applied in conjunction with pre-emphasis, compared with the modulation in a system without pre-emphasis. (Expressed as voltage ratio <1.)

N = noise factor of receiver (power ratio).

V_0 = receiver input signal voltage corresponding to an available power of 1 mW.

V_r = actual receiver input signal voltage.

9.2. *Performance on Monophonic Transmissions*

The r.m.s. signal/noise ratio for a 100% modulated double-sideband amplitude-modulated transmission, no pre-emphasis being used, is

$$\left(\frac{10^{-3}}{2kT_0}\right)^{\frac{1}{2}} \cdot \frac{V_r}{V_0} \cdot \frac{1}{(Nf_a)^{\frac{1}{2}}} \qquad \ldots\ldots(1)$$

where $(10^{-3}/2kT_0)^{\frac{1}{2}} = 3{\cdot}55 \times 10^8$, taking kT_0 as the available thermal noise power per unit bandwidth at room temperature ($T_0 = 290°$ K).

The f.m./a.m. improvement factor for r.m.s. signal/noise ratio, for a standard monophonic f.m. transmission, is

$$\frac{\sqrt{3}H_0}{f_a} \cdot a_2 b \qquad \ldots\ldots(2)$$

The r.m.s. signal/noise ratio for a monophonic f.m. transmission may be derived from the product of expressions (1) and (2). Thus, for the U.K. standards, $H_0 = 75$ kc/s, $\tau = 50\ \mu$s and $b = 0{\cdot}63$ (-4 dB); taking $f_a = 15$ kc/s and $N = 10$ (noise factor of 10 dB), a theoretical signal/noise ratio of 77·2 dB is obtained with an input carrier level of -67 dB (mW). The reference signal in this case corresponds to a deviation of ± 47 kc/s (0·63 times ± 75 kc/s) at a low audio frequency such as 400 c/s.

9.3. *Degradation Factors for Stereophonic Transmission*

The theoretical noise performance of multiplex stereophonic systems is summarized in Table 4 in the form of degradation factors for the r.m.s. signal/noise ratio relative to monophonic f.m. transmission performance.

The calculations in the case of monophonic reception have been based on the reduction in deviation of the main carrier by the M signal as compared with the ± 75 kc/s deviation for a monophonic transmission. In the case of the f.m. sub-carrier system there is a definite limit to the amplitude of the M and of the S signal that can be transmitted, while in the a.m. sub-carrier systems the limit is more complex. However, for an a.m. sub-carrier system with 50% deviation allocated to the sub-carrier, or when the sub-carrier is suppressed leaving the upper and lower sidebands, it turns out that there is a definite limit to the amplitude of the A and B signals that may be transmitted. Other elements in the chain (audio-frequency amplifiers, recording machines, land lines, etc.) do not generally place a very rigid limit on the signal amplitudes in either the M and S or the A and B form. Thus, with the f.m. sub-carrier system, it is assumed that the programme level may be adjusted in all cases so that the peaks of the M signal give the full deviation of the main carrier allocated to it. Only in very exceptional types of programme would this procedure overload the f.m. sub-carrier channel carrying the S signal. In the case of a.m. sub-carrier systems a different criterion is necessary in achieving the optimum modulation level. Thus when the A and B signals are subject to a definite limit, the programme level would be adjusted so that these signals take up the permitted peak levels. The M modulation provided for the monophonic listener then depends on the degree of correlation between the A and B signal waveforms. The result for uncorrelated A and B signals of equal mean power corresponds most closely with the average result obtained in practice.[15]

When stereophonic reception is considered, the signal/noise ratio of the sub-carrier channel relative to a monophonic transmission channel is first evaluated. For high signal/noise ratios, it may be shown that modulation of a sub-carrier which deviates the main carrier by $\pm\sqrt{2}$ radians is equivalent in performance to the same type of modulation applied directly to the main carrier instead. Since monophonic f.m. transmission is used as a reference in Table 4 the expression for the S signal is straightforward for f.m. sub-carrier systems but, in the case of a.m. sub-carrier systems, the effects of pre-emphasis in a.m. and of losing the f.m./a.m. improvement factor must be included.

Finally, for evaluating the performance of the stereophonic channels, allowance must be made for occupation of both the A and B channels. The results given in Table 4 for the A signal assume that modulation levels are set up as discussed above, and that the A and B signals are of equal mean power but uncorrelated. The noise from the main signal channel has been neglected for simplicity since it is usually small compared with the noise at the output arising from the sub-carrier channel.

Table 4

Theoretical r.m.s. signal/noise ratio degradation factors

	F.M. sub-carrier system	A.M. sub-carrier system
Monophonic reception (M signal) (i) for $A = B$, (ii) for uncorrelated A and B signals of equal power	(i) and (ii) $\dfrac{H_m}{H_0}$	(i) $\dfrac{H'_m}{H_0}$ (ii) $\dfrac{(H'_m H''_m)^{\frac{1}{2}}}{H_0}$ †
Sub-carrier channel (S signal) at maximum modulation	$\dfrac{H_s h_s}{\sqrt{2} H_0 f_s}$	$\dfrac{H_s f_a a_1}{\sqrt{6} H_0 f_s a_2}$
Stereophonic channel (A signal) for uncorrelated A and B signals of equal power	$\dfrac{H_s h_s}{H_0 f_s}$	$\dfrac{H_s f_a a_1}{\sqrt{6} H_0 f_s a_2} \cdot \left(\dfrac{2H''_m}{H'_m}\right)^{\frac{1}{2}}$

† This is an approximation assuming $1 \leqslant H'_m/H''_m \leqslant 2$.

Notes: (1) In the f.m. sub-carrier system, $H_m + H_s$ normally equals H_0 in order to give a total deviation equal to that used in monophonic transmissions.

(2) In the a.m. sub-carrier system employing the full sub-carrier component with double-sideband modulation, it may be shown that the conditions for a total deviation of $\pm H_0$ and no overmodulation of the sub-carrier are respectively $H'_m = H_0 - H_s$ and $H''_m = (H_0 - H_s)^2/H_0$.

(3) In the a.m. sub-carrier system (double sideband) with partial or complete suppression of the sub-carrier component, H_s must be taken as the deviation of the main carrier by the sub-carrier that would be needed supposing that the sub-carrier were fully restored before transmission.

(4) For the Siemens system (single sideband a.m. sub-carrier) and Mullard system (two effective sub-carriers) the appropriate corrections have been made in Table 2, but the details are not given here.

10. Appendix 2

10.1. *Effect of Distortion of Difference-Sideband Spectrum*

$\frac{\omega_s}{2\pi}$ = sub-carrier frequency.

$\frac{p}{2\pi}$ = modulation frequency.

$\omega_1 = \omega_s - p$.

$\omega_2 = \omega_s + p$.

θ_1 = phase advance of lower sideband.

θ_2 = phase advance of upper sideband.

k_1 = factor by which lower sideband amplitude is changed.

k_2 = factor by which upper sideband amplitude is changed.

If the correct difference-sideband signal is

$$\sin pt \sin \omega_s t = \tfrac{1}{2}\cos \omega_1 t - \tfrac{1}{2}\cos \omega_2 t$$

the effect of the distortion is to change this to

$$\frac{k_1}{2}\cos(\omega_1 t + \theta_1) - \frac{k_2}{2}\cos(\omega_2 t + \theta_2)$$

The output of a square-wave switching demodulator is then

$$\left[\frac{k_1}{2}\cos(\omega_1 t + \theta_1) - \frac{k_2}{2}\cos(\omega_2 t + \theta_2)\right] \times$$
$$\times \left[\tfrac{1}{2} + \frac{2}{\pi}\sin \omega_s t + \frac{2}{3\pi}\sin 3\omega_s t \ldots\right]$$

Considering only the product terms containing audio-frequency components we have

$$\frac{1}{\pi}[k_1 \sin \omega_s t \cos(\omega_1 t + \theta_1) - k_2 \sin \omega_s t \cos(\omega_2 t + \theta_2)]$$

or, again taking only the audio-frequency terms,

$$\frac{1}{2\pi}[k_1 \sin(pt - \theta_1) + k_2 \sin(pt + \theta_2)]$$
$$= \frac{1}{2\pi}[k_1(\sin pt \cos \theta_1 - \cos pt \sin \theta_1) +$$
$$+ k_2(\sin pt \cos \theta_2 + \cos pt \sin \theta_2)]$$
$$= \frac{1}{2\pi}[\sin pt(k_1 \cos \theta_1 + k_2 \cos \theta_2) -$$
$$- \cos pt(k_1 \sin \theta_1 - k_2 \sin \theta_2)] \quad \ldots\ldots(3)$$

Let $k_1 \cos \theta_1 + k_2 \cos \theta_2 = a$

and $k_1 \sin \theta_1 - k_2 \sin \theta_2 = b$.

Expression (3) then becomes

$$\frac{1}{2\pi}(a \sin pt - b \cos pt) = \frac{1}{2\pi}(a^2 + b^2)^{\frac{1}{2}} \sin(pt + \phi)$$

where $\phi = \arctan\left(-\frac{b}{a}\right)$

Thus the demodulated difference signal is advanced in phase by $\phi = \arctan\left(-\frac{b}{a}\right)$ and changed in amplitude by a factor $\frac{1}{2}(a^2 + b^2)^{\frac{1}{2}}$.

Where only the amplitudes of the sidebands are changed, the expression for the demodulated signal reduces to $\frac{1}{2\pi}(k_1 + k_2)\sin pt$. This gives zero phase error and an amplitude change of $\frac{1}{2}(k_1 + k_2)$ in the demodulated difference signal.

Where only the phases of the sidebands are changed, the expression for the demodulated difference signal reduces to $\frac{1}{\pi}\cos\frac{\theta_1 + \theta_2}{2}\sin\left(pt - \frac{\theta_1 - \theta_2}{2}\right)$, i.e. a phase error of $\frac{\theta_1 - \theta_2}{2}$ and an amplitude change of $\cos\frac{\theta_1 + \theta_2}{2}$.

10.2. *Effect of Phase Shift of Regenerated Sub-carrier*

The output of the difference signal from the demodulator, if the switching sub-carrier is shifted from the correct phase by an angle ψ, will be

$$(\sin pt \sin \omega_s t)\left[\frac{2}{\pi}\sin(\omega_s t + \psi)\right]$$
$$= \frac{1}{\pi}\sin pt[2 \sin \omega_s t \sin(\omega_s t + \psi)]$$

Taking only the audio-frequency term we have

$$\frac{1}{\pi}\sin pt \cos \psi$$

10.3. *Effect of Phase Shift and Amplitude Change of Demodulated Sum and Difference Signals*

Let the sum signal amplitude be changed by factor r_m and the difference signal amplitude be changed by factor r_s; also let the sum signal phase error be α_m, and the difference signal phase error be α_s.

In the case where modulation input of unit amplitude is applied to the *A* channel alone, it follows from a geometrical construction of the type given in Fig. 4 that the *A* and *B* channel outputs of the receiver are

$$A = \tfrac{1}{2}\sqrt{r_m^2 + r_s^2 + 2r_m r_s \cos|\alpha_m - \alpha_s|}$$
$$B = \tfrac{1}{2}\sqrt{r_m^2 + r_s^2 - 2r_m r_s \cos|\alpha_m - \alpha_s|}$$

and the resulting crosstalk level, expressed in decibels is

$$-20\log_{10}\sqrt{\frac{r_m^2 + r_s^2 + 2r_m r_s \cos|\alpha_m - \alpha_s|}{r_m^2 + r_s^2 - 2r_m r_s \cos|\alpha_m - \alpha_s|}}$$

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Double-sideband Parametric Conversion Using Non-linear Resistance and Capacitance

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Summary: The power gain of a double-sideband up-converter using both non-linear resistance and capacitance is found. With the assumption that the second-harmonic components of resistance and capacitance are zero, it is shown that if the elements are pumped in quadrature the forward gain can be made arbitrarily high with 'cold' tuning whilst the reverse gain is zero.

Furthermore it is shown that unidirectional operation with arbitrarily high forward gain can be achieved for any up-conversion frequency ratio.

Expressions are also derived for the bandwidth and noise figure of the device.

Finally, it is also shown that arbitrarily high forward gain and unidirectionality appear possible when the second-harmonic components of resistance and capacitance are non-zero.

List of Symbols

B_0	Susceptance of ω_q circuit.
B_{+1}	Susceptance of $(\omega_q+\omega_p)$ circuit.
B_{-1}	Susceptance of $(\omega_q-\omega_p)$ circuit.
$C(t)$	Time-varying capacitance.
$C_0, 2C_1, 2C_n$	Fourier coefficients of $C(t)$.
$g(t)$	Time-varying conductance.
$g_0, 2g_1, 2g_n$	Fourier coefficients of $g(t)$.
G_0	Total effective conductance of ω_q circuit.
G_{+1}	Total effective conductance of $(\omega_q+\omega_p)$ circuit.
G_{-1}	Total effective conductance of $(\omega_q-\omega_p)$ circuit.
g_S	Source conductance.
g_L	Load conductance.
g_e	Equivalent shot-noise conductance of resistance diode.
I_0	Current at frequency ω_q.
I_{+1}	Current at frequency $(\omega_q+\omega_p)$.
I_{-1}	Current at frequency $(\omega_q-\omega_p)$.
I_e	Equivalent saturated current of resistance diode.
N_o	Noise output power.
v_0	Voltage at frequency ω_q.
v_{+1}	Voltage at frequency $(\omega_q+\omega_p)$.
v_{-1}	Voltage at frequency $(\omega_q-\omega_p)$.
Y_0	Admittance of ω_q circuit.
Y_{+1}	Admittance of $(\omega_q+\omega_p)$ circuit.
Y_{-1}	Admittance of $(\omega_q-\omega_p)$ circuit.
ω_p	Pump frequency.
ω_q	Signal frequency.
$\Delta\omega$	Frequency deviation about ω_q.
ω_q'	Resonant frequency of ω_q circuit.
2δ	Fractional bandwidth.

1. Introduction

It is well known that up-conversion from a frequency ω_q to a frequency $(\omega_q+\omega_p)$ can be achieved by means of non-linear capacitors.[1] In the single-sideband (or three-frequency) parametric up-converter, the non-linear capacitor is 'pumped' at frequency ω_p and tuned circuits allow either currents to flow or voltages to exist only at the signal and upper-sideband frequencies. This type of converter has a maximum power gain of $(\omega_q+\omega_p)/\omega_q$. In the double-sideband (or four-frequency) converter where the output is taken at the upper-sideband, but the lower-sideband is allowed to exist, the gain can be increased to $4(\omega_q+\omega_p)/\omega_q$ with 'cold' tuning.‡ With suitable tuning the gain can be increased to any desired value.[2]

Edwards[3] has shown that a non-linear resistor pumped in phase with a non-linear capacitor merely

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‡ 'Cold' tuning is where the mean reactances of the individual circuits are arranged to cancel with the mean reactance of the non-linear capacitor.

reduces the gain obtainable by the pure capacitor. However, Engelbrecht[4] has shown that if the non-linear resistor is pumped in quadrature with the capacitor then, for large up-conversion ratios, gain in excess of $(\omega_q+\omega_p)/\omega_q$ can be obtained with a single-sideband circuit. Furthermore, it is possible to make such a circuit unidirectional, i.e. finite gain from ω_q to $(\omega_q+\omega_p)$ and zero reverse gain from $(\omega_q+\omega_p)$ to ω_q. Szerlip and Howson[5,6] have shown, for the single-sideband converter, that it is possible to obtain arbitrarily high gain with either parallel or series R-C circuits. However, under these circumstances, the reverse gain is non-zero and increases as the forward gain increases.

The work presented here is a general analysis of a double-sideband parametric converter using non-linear resistance and capacitance. The resistance and capacitance are in parallel and pumped in quadrature. (See Appendix 1.) It is shown that it is possible to obtain arbitrarily high forward gain with 'cold' tuning while maintaining the desirable unidirectional property obtainable in the single-sideband converter. General expressions for gain, bandwidth and noise figure are also derived.

2. General Analysis

The notation used here is identical in almost all respects to that used in a previous paper by Tucker.[7] The essential circuit of the device is shown in Fig. 1. It is assumed that the pump voltage is much greater than any signal voltages present, so that the resistance and capacitance elements can be considered as time-varying with a fundamental frequency ω_p. The tuned circuits are considered to be ideal, so that only a voltage at the signal frequency (ω_q) exists across the source admittance, Y_0; only a voltage at the upper-sideband $(\omega_q+\omega_p)$ exists across the load, Y_{+1}; and only a voltage at the lower-sideband $(\omega_q-\omega_p)$ exists across the 'idler' admittance, Y_{-1}. The time-varying capacitance may be represented by the Fourier series

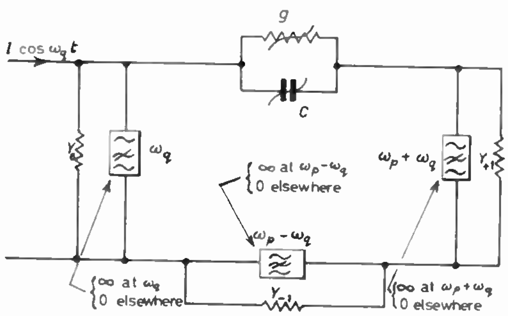

Fig. 1. Double-sideband converter using parallel resistance and capacitance elements.

$$C(t) = C_0 + \sum_{n=1}^{\infty} 2C_n \cos n\omega_p t \qquad \ldots\ldots(1)$$

It is convenient to analyse the non-linear resistance as a time-varying conductance so that,

$$g(t) = g_0 + \sum_{n=1}^{\infty} 2g_n \cos n(\omega_p t+\pi/2) \qquad \ldots\ldots(2)$$

hence

$$C(t) = C_0+2C_1\cos\omega_p t + 2C_2\cos 2\omega_p t + \ldots \qquad \ldots\ldots(3)$$

$$g(t) = g_0-2g_1\sin\omega_p t - 2g_2\cos 2\omega_p t + \ldots \qquad \ldots\ldots(4)$$

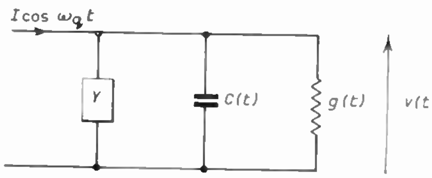

Fig. 2. Equivalent circuit of double-sideband converter.

The equivalent circuit of the device is shown in Fig. 2 and hence the small-signal currents, $i(t)$, and the small-signal voltages $v(t)$ are related by

$$i(t) = g(t)v(t) + \frac{\mathrm{d}}{\mathrm{d}t}[C(t)v(t)] + Yv(t) \qquad \ldots\ldots(5)$$

In general the voltage $v(t)$ will contain components at all possible modulation frequencies $n\omega_p\pm\omega_q$ and can be written as

$$v(t) = \sum_{m=-\infty}^{\infty} v_m[\cos(\omega_q+m\omega_p)t+\theta_m] \qquad \ldots\ldots(6)$$

Furthermore, it has been shown elsewhere[7] that the phase angle always carries the same subscript as the voltage magnitude, so that if the voltages v_m are considered to be vector quantities the phase angle can be dropped.

In this particular case voltages exist only at ω_q, $(\omega_q+\omega_p)$ and $(\omega_q-\omega_p)$ so that by manipulating eqns. (3), (4), (5) and (6), we arrive at equations for currents at the three frequencies.

At frequency ω_q:

$$I_0 = v_0\{Y_0+g_0+\mathrm{j}\omega_q C_0\}+v_{+1}\{-\mathrm{j}g_1+\mathrm{j}\omega_q C_1\}+ \\ +v_{-1}\{\mathrm{j}g_1+\mathrm{j}\omega_q C_1\} \qquad \ldots\ldots(7a)$$

At frequency $(\omega_q+\omega_p)$:

$$0 = v_0\{\mathrm{j}g_1+\mathrm{j}(\omega_q+\omega_p)C_1\}+ \\ +v_{+1}\{Y_{+1}+g_0+\mathrm{j}(\omega_q+\omega_p)C_0\}+ \\ +v_{-1}\{-g_2+\mathrm{j}(\omega_q+\omega_p)C_2\} \qquad \ldots\ldots(7b)$$

At frequency $(\omega_q-\omega_p)$:

$$0 = v_0\{-\mathrm{j}g_1+\mathrm{j}(\omega_q-\omega_p C_1\}+ \\ +v_{+1}\{-g_2+\mathrm{j}(\omega_q-\omega_p)C_2\}+ \\ +v_{-1}\{Y_{-1}+g_0+\mathrm{j}(\omega_q-\omega_p)C_0\} \qquad \ldots\ldots(7c)$$

For convenience it will be assumed that the second-harmonic components of capacitance and conductance are negligible so that $C_2 = 0$ and $g_2 = 0$. If

$$Y_0 = G_0 + jB_0 \qquad Y_{+1} = G_{+1} + jB_{+1}$$

and $\quad Y_{-1} = G_{-1} + jB_{-1}$

(where G_0 and G_{+1} include the transformed source and load conductances g_S and g_L respectively) and we arrange that the mean susceptances of the individual circuits are tuned out such that

$$B_0 + \omega_q C_0 = 0 \qquad B_{+1} + (\omega_q + \omega_p) C_0 = 0$$

and $\quad B_{-1} + (\omega_q - \omega_p) C_0 = 0.$

Equations (7a–c) then reduce to

$$I_0 = v_0\{g_0 + G_0\} + v_{+1}\{-jg_1 + j\omega_q C_1\} + + v_{-1}\{jg_1 + j\omega_q C_1\} \qquad \text{......(8a)}$$

$$0 = v_0\{jg_1 + j(\omega_q + \omega_p) C_1\} + v_{+1}\{g_0 + G_{+1}\} + + v_{-1}\{0\} \qquad \text{......(8b)}$$

$$0 = v_0\{-jg_1 + j(\omega_q - \omega_p) C_1\} + v_{+1}\{0\} + + v_{-1}\{g_0 + G_{-1}\} \qquad \text{......(8c)}$$

Under these circumstances the input admittances of the three circuits at ω_q, $(\omega_q + \omega_p)$ and $(\omega_q - \omega_p)$ are respectively

$$Y_{i,0} = g_0 - \frac{[g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1]}{[g_0 + G_{-1}]} + \frac{[-g_1 + \omega_q C_1][g_1 + (\omega_q + \omega_p) C_1]}{[g_0 + G_{+1}]} \qquad \text{......(9a)}$$

$$Y_{i,+1} = g_0 + \frac{[-g_1 + \omega_q C_1][g_1 + (\omega_q + \omega_p) C_1][g_0 + G_{-1}]}{[g_0 + G_0][g_0 + G_{-1}] - [g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1]} \qquad \text{......(9b)}$$

$$Y_{i,-1} = g_0 - \frac{[g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1][g_0 + G_{+1}]}{[g_0 + G_0][g_0 + G_{+1}] + [g_1 + (\omega_q + \omega_p) C_1][-g_1 + \omega_q C_1]} \qquad \text{......(9c)}$$

Clearly the three input admittances have no susceptance terms (it should be remembered that the mean susceptances of the individual circuits were tuned out), so that if $\omega_q C_1 \geqslant g_1$ the condition for stability $G_m + \mathrm{Re}(Y_{i,m}) > 0$ ($m = 0$, $+1$ or -1) is that

$$[g_0 + G_0][g_0 + G_{-1}] > [g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1] \qquad \text{......(10)}$$

2.1. *Calculation of Forward Gain*

If we now consider the double-sideband up-converter with output at frequency $(\omega_q + \omega_p)$ then it is easily seen from eqn. (8) that the power in the load, $|v_{+1}|^2 g_L$ is given by

$$|v_{+1}|^2 g_L = \frac{[g_1 + (\omega_q + \omega_p) C_1]^2 [g_0 + G_{-1}]^2 |I_0|^2 g_L}{\{[g_0 + G_{+1}]([g_0 + G_0][g_0 + G_{-1}] - [g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1]) + [-g_1 + \omega_q C_1][g_0 + G_{-1}][g_1 + (\omega_q + \omega_p) C_1]\}^2} \qquad \text{......(11)}$$

The maximum available input power is $|I_0|^2/4g_S$, and hence the forward power gain of the system is

$$P_f = 4 g_L g_S \left| \frac{v_{+1}}{I_0} \right|^2 \qquad \text{......(12)}$$

Therefore, if we arrange that $g_1 = \omega_q C_1$ the forward gain becomes

$$P_f = \frac{4 g_L g_S [g_1 + (\omega_q + \omega_p) C_1]^2 [g_0 + G_{-1}]^2}{[g_0 + G_{+1}]^2 \{[g_0 + G_0][g_0 + G_{-1}] - [g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1]\}^2} \qquad \text{......(13)}$$

Clearly the gain becomes arbitrarily high as $[g_0 + G_0][g_0 + G_{-1}] \rightarrow [g_1 + \omega_q C_1][g_1 - (\omega_q - \omega_p) C_1]$ ......(14)

Thus the condition for stability given by the inequality (10) will be obeyed for practical values of gain.

2.2. *Calculation of Reverse Gain*

The reverse gain will be given by $4g_L g_S\left|\frac{v_0}{I_{+1}}\right|^2$ where I_{+1} is the input current at frequency $(\omega_q+\omega_p)$. From eqn. (8) this becomes

$$P_r = \frac{4g_L g_S[-g_1+\omega_q C_1]^2[g_0+G_{-1}]^2}{\{[g_0+G_{+1}]([g_0+G_0][g_0+G_{-1}]-[g_1+\omega_q C_1][g_1-(\omega_q-\omega_p)C_1])+ +[-g_1+\omega_q C_1][g_0+G_{-1}][g_1+(\omega_q+\omega_p)C_1]\}^2} \qquad \text{......(15)}$$

Hence if $g_1 = \omega_q C_1$ the reverse gain is zero.† When the input frequency deviates from ω_q the reverse gain will not be zero, but for a wide range of ω_q it will be considerably smaller than the reverse gain of other parametric converters with comparable forward gains.

In the single-sideband converter analysed by Engelbrecht,[4] for unidirectional operation, the maximum forward gain was $\frac{1}{4}(\omega_q+\omega_p)^2/\omega_q^2$ for an ideal passive variable resistance. Again, the reverse gain is non-zero when the input frequency deviates from ω_q.

3. Bandwidth of Double-Sideband Converter

To determine the bandwidth of the converter it is necessary to consider in more detail the input, output and idler circuits. For simplicity it is assumed that all are simple parallel-tuned circuits, although this type of circuit does not normally give the maximum bandwidth obtainable. Using a high-Q approximation, the circuit admittances can be rewritten as

$$Y_0+j\omega_q C_0 = G_0[1+2jQ_0\delta] \qquad \text{......(16)}$$

$$Y_{+1}+j(\omega_q+\omega_p)C_0 = G_{+1}\left[1+2jQ_{+1}\delta\frac{\omega_q'}{(\omega_q'+\omega_p)}\right] \qquad \text{......(17)}$$

$$Y_{-1}+j(\omega_q-\omega_p)C_0 = G_{-1}\left[1-2jQ_{-1}\delta\frac{\omega_q'}{(\omega_q'-\omega_p)}\right] \qquad \text{......(18)}$$

where ω_q', $(\omega_q'+\omega_p)$ and $(\omega_q'-\omega_p)$ are the resonant frequencies of the three circuits, and

Q_K is the Q factor of circuit $K(K = 0, +1, \text{or } -1)$

$$\omega_q = \omega_q'+\Delta\omega \qquad \text{......(19)}$$

$$(\omega_q+\omega_p) = (\omega_q'+\omega_p)+\Delta\omega \qquad \text{......(20)}$$

$$(\omega_q-\omega_p) = (\omega_q'-\omega_p)+\Delta\omega \qquad \text{......(21)}$$

and

$$\delta = \frac{\Delta\omega}{\omega_q} \qquad \text{......(22)}$$

If in addition we make the substitutions

$$c = \frac{Q_{+1}}{Q_0}\frac{\omega_q'}{(\omega_q'+\omega_p)} \qquad \text{......(23)}$$

$$d = -\frac{Q_{-1}}{Q_0}\frac{\omega_q'}{(\omega_q'-\omega_p)} \qquad \text{......(24)}$$

and

$$s = (2Q_0\delta)^2 \qquad \text{......(25)}$$

the forward gain given by eqn. (13) becomes

$$P_f = \frac{4g_L g_S[g_1+(\omega_q+\omega_p)C_1]^2[(g_0+G_{-1})^2+sd^2+G_{-1}^2]}{\alpha^2+s\beta^2} \qquad \text{......(26)}$$

where

$$\alpha = (g_0+G_{+1})\{(g_0+G_0)(g_0+G_{-1})+[-g_1+(\omega_q-\omega_p)C_1][g_1+\omega_q C_1]-sdG_0G_{-1}\}- -scG_{+1}\{G_0(g_0+G_{-1})+G_{-1}d(g_0+G_0)\} \qquad \text{......(27)}$$

and

$$\beta = cG_{+1}\{(g_0+G_0)(g_0+G_{-1})+[-g_1+(\omega_q-\omega_p)C_1][g_1+\omega_q C_1]-sdG_0G_{-1}\}+ +(g_0+G_{+1})\{G_0(g_0+G_{-1})+G_{-1}d(g_0+G_0)\} \qquad \text{......(28)}$$

To determine the bandwidth we now have to equate the general gain expression given by eqn. (26) to half the gain at resonance given by eqn. (13). It is necessary to make further assumptions to simplify the calculation of bandwidth, namely that the power gain is high so that the fractional bandwidth of the converter is small. Hence powers of s greater than 1 can be neglected. Furthermore, the high-gain condition means that condition (14) applies and most of the terms in s can be neglected. Hence we have

† See Appendix 2 for reverse gain when C_2 and G_2 are finite.

$$s \simeq \frac{\{[g_0+G_0][g_0+G_{-1}]-[g_1-(\omega_q-\omega_p)C_1][g_1+\omega_q C_1]\}^2}{\{G_0[g_0+G_{-1}]+dG_{-1}[g_0+G_0]\}^2} \qquad \text{......(29)}$$

Hence the gain-(bandwidth)^2 product becomes

$$(2\delta)^2 \times \text{gain} = \frac{4g_L g_S[g_1+(\omega_q+\omega_p)C_1]^2[g_0+G_{-1}]^2}{[g_0+G_{+1}]^2\{G_0[g_0+G_{-1}]+dG_{-1}[g_0+G_0]\}^2} \times \frac{1}{Q_0^2} \qquad \text{......(30)}$$

If $G_0 = G_{+1} = G_{-1} = G$, eqn. (30) reduces to

$$(2\delta)^2 \times \text{gain} = \frac{4g_L g_S C_1^2[\omega_q+(\omega_q+\omega_p)]^2}{G^2[g_0+G]^2\left[Q_0+Q_{-1}\dfrac{\omega_q'}{(\omega_p-\omega_q')}\right]} \qquad \text{......(31)}$$

4. Noise Figure of Converter

It is not practicable here to calculate the effect of all possible noise sources. Only the more important sources will be considered, namely the thermal noise due to the conductances of the three tuned circuits and the shot-noise of the variable-resistance diode. Thus the equivalent noise currents $\mathscr{I}_K(K = 0, +1$ or $-1)$ of the three circuits are given by

$$|\mathscr{I}_0|^2 = 4kT\Delta f(G_0'+g_S+g_e) \qquad \text{......(32)}$$

$$|\mathscr{I}_{+1}|^2 = 4kT\Delta f(G_{+1}+g_e) \qquad \text{......(33)}$$

$$|\mathscr{I}_{-1}|^2 = 4kT\Delta f(G_{-1}+g_e) \qquad \text{......(34)}$$

where $G_0 = G_0'+g_S$ and g_e is the equivalent shot-noise conductance of the resistance diode. For a junction diode, if $\bar{i}$ is the shot-noise current we have[8]

$$\bar{i}^2 = 2eI_e\Delta f = 4kT\Delta f g_e \qquad \text{......(35)}$$

where I_e is the equivalent saturated current.

Hence $$g_e = \frac{eI_e}{2kT} \qquad \text{......(36)}$$

The currents on the left-hand side of eqns. (8) can now be replaced by $\mathscr{I}_0, \mathscr{I}_{+1}$ and $\mathscr{I}_{-1}$ so that the mid-band noise output power becomes

$$N_o = \frac{4kT\Delta f g_L}{|A|^2}\{[G_0'+g_S+g_e][g_0+G_{-1}]^2[g_1+(\omega_q+\omega_p)C_1]^2+(G_{+1}+g_e)\Big([g_0+G_0][g_0+G_{-1}]- -[g_1+\omega_q C_1][g_1-(\omega_q-\omega_p)C_1]\Big)^2+(G_{-1}+g_e)[g_1+\omega_q C_1]^2[g_1+(\omega_q+\omega_p)C_1]^2\} \qquad \text{......(37)}$$

where $|A|^2$ is the denominator of eqn. (11).

Noise figure, F, is defined as

$$F = \frac{N_o}{kT\Delta f \times \text{gain}} \qquad \text{......(38)}$$

Therefore

$$F = 1 + \frac{G_0'+g_e}{g_S} + \frac{(G_{-1}+g_e)[g_1+\omega_q C_1]^2}{g_S[g_0+G_{-1}]^2} + + \frac{[G_{+1}+g_e]\{[g_0+G_0][g_0+G_{-1}]-[g_1+\omega_q C_1][g_1-(\omega_q-\omega_p)C_1]\}^2}{g_S[g_1+(\omega_q+\omega_p)C_1]^2[g_0+G_{-1}]^2} \qquad \text{......(39)}$$

Under high gain conditions the last term in the equation is small compared with the other terms so that the noise figure reduces to

$$F = 1 + \frac{G_0'+g_e}{g_S} + \frac{(G_{-1}+g_e)[g_1+\omega_q C_1]^2}{g_S[g_0+G_{-1}]^2} \qquad \text{......(40)}$$

5. Comparison of Double-sideband R-C Converter with other Parametric Converters

5.1. *Unidirectional Devices*

The only other similar unidirectional device is the single-sideband variable resistance and capacitance converter discussed by Engelbrecht. When this is operated under unidirectional conditions its gain is approximately $\frac{1}{4}[1+(\omega_q+\omega_p)/\omega_q]^2$ and only exceeds the gain of the ordinary single-sideband parametric up-converter when $(\omega_q+\omega_p) \gg \omega_q$. Consequently, the double-sideband R-C device is superior to its single-sideband counterpart, since its gain can be made arbitrarily large at any ratio of output to input frequency.

The noise figure of the double-sideband converter must of necessity be inferior to that of the single-sideband one under equal gain conditions, since further sources of noise are introduced. These sources are the parallel conductance of the tuned circuit and the equivalent shot-noise conductance of the resistance diode at frequency $(\omega_p - \omega_q)$. However, under high gain conditions in the double-sideband case, the noise contribution from the upper-sideband circuit is substantially reduced as seen from eqn. (39) so that the noise figures of the two devices will not be greatly dissimilar.

It is difficult to compare bandwidths since no published figures are available for the single-sideband device. Furthermore, the bandwidth of the single-sideband converter cannot be deduced directly from that of the double-sideband one since the high-gain assumption used in the latter case does not apply to the former.

As stated previously, the usefulness of the single-sideband R-C converter is critically dependent upon the up-conversion ratio. From eqn. (13), by suitable choice of the circuit conductance, it is clear that arbitrarily high gain can be achieved for all up-conversion ratios for the double-sideband case. However, the gain-(bandwidth)^2 product of eqn. (31) shows that the performance of the converter is not independent of the frequency conversion ratio. For fixed gain, the bandwidth of the converter reduces with reduced conversion ratio.

5.2. *Non-unidirectional Devices*

The double-sideband variable capacitance converter has its gain restricted to $4(\omega_q + \omega_p)/\omega_q$ when its individual circuits are 'cold' tuned. This gain can be increased by more arbitrary tuning techniques to any desired value, but may be difficult to set up. The converter discussed in this paper is capable of high gains with 'cold' tuning.

The single-sideband negative-resistance converter[9] is also capable of high gains with cold tuning; it is interesting therefore to compare it with the converter discussed here. Using similar notation to that used above, the gain, gain-(bandwidth)^2 product and noise figure expressions for the negative resistance converter can be written as:

$$\text{gain} = \frac{4 g_L g_S C_1^2 (\omega_q - \omega_p)^2}{G_{-1}^2 [G_0 G_{-1} - C_1^2 \omega_q (\omega_p - \omega_q)]^2} \qquad \ldots\ldots(41)$$

$$(2\delta)^2 \times \text{gain} = \frac{4 g_L g_S C_1^2 (\omega_q - \omega_p)^2}{G_{-1}^2 G_0^2} \times \frac{1}{\left[Q_0 + Q_{+1} \frac{\omega_q'}{(\omega_p - \omega_q')}\right]} \qquad \ldots\ldots(42)$$

$$\text{noise figure } F = 1 + \frac{G_0}{g_S} + \frac{G_{-1} G_0^2}{g_S C_1^2 (\omega_q - \omega_p)^2} \qquad \ldots\ldots(43)$$

Comparing these with eqns. (13), (31) and (40), the gains and bandwidths are very similar. If $g_0 = G$, then the ratio of the bandwidths of the two devices is $(\omega_q + (\omega_q + \omega_p))/2(\omega_p - \omega_q)$ for equal gains. So that when $\omega_p = 4\omega_q$ the ratio is unity and the gains and bandwidths correspond exactly. It is difficult to make a comparison of the noise figure expressions since they contain different terms, but the noise figure of the double-sideband R-C converter will certainly be inferior to that of the single-sideband converter.

To sum up, the double-sideband device has the disadvantage of being complex, but it is unidirectional and gives arbitrarily large gain without inverting the signal spectrum.

6. Conclusions

A general analysis of a double-sideband converter using non-linear resistance and capacitance pumped in quadrature has been presented, with the assumption that the variation in the element impedances is wholly due to the pump voltage. It has been shown possible to produce a converter by this means which is unidirectional and has arbitrarily high gain for any ratio of output to input frequency. Furthermore, this gain can be achieved with 'cold' tuning.

The converter has been compared with both the single-sideband R-C converter and the single-sideband negative-resistance converter. In many respects its response is similar to that of the negative resistance converter, although its noise figure is inferior. It also differs in that it is unidirectional and does not invert the signal spectrum.

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8. References

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9. Appendix 1: Determination of the Optimum Pumping Angle for the Double-sideband R-C Converter

In recent papers[10, 11, 12] it has been shown that the maximum gain of a single-sideband non-inverting R-C converter is achieved when the resistance and capacitance are pumped in quadrature. Here the optimum pumping angle is deduced for the double-sideband non-inverting R-C converter.

For convenience it will be assumed that C_2 and g_2 are negligible, so that the capacitance and conductance variations can be written as

$$C(t) = C_0 + 2C_1 \cos(\omega_p t + \theta) \qquad \ldots\ldots(44)$$

$$g(t) = g_0 + 2g_1 \cos \omega_p t \qquad \ldots\ldots(45)$$

where θ is the relative phase angle between the pumps. Manipulating eqns. (5), (6), (44) and (45) the following equations can be obtained.

At frequency ω_q:

$$I_0 = v_0\{Y_0 + g_0 + j\omega_q C_0\} + v_{+1}\{g_1 + j\omega_q C_1 e^{-j\theta}\} + v_{-1}\{g_1 + j\omega_q C_1 e^{j\theta}\} \qquad \ldots\ldots(46a)$$

At frequency $(\omega_q + \omega_p)$:

$$0 = v_0\{g_1 + j(\omega_q + \omega_p)C_1 e^{j\theta}\} + v_{+1}\{Y_{+1} + g_0 + j(\omega_q + \omega_p)C_0\} + v_{-1}\{0\} \qquad \ldots\ldots(46b)$$

At frequency $(\omega_q - \omega_p)$:

$$0 = v_0\{g_1 + j(\omega_q - \omega_p)C_1 e^{-j\theta}\} + v_{+1}\{0\} + v_{-1}\{Y_{-1} + g_0 + j(\omega_q - \omega_p)C_0\} \qquad \ldots\ldots(46c)$$

Hence for an input at frequency ω_q and an output at frequency $(\omega_q + \omega_p)$ and assuming the individual circuits are 'cold' tuned, we have

$$v_{+1} = \frac{\{g_1 + j(\omega_q + \omega_p)C_1 e^{j\theta}\}[G_{-1} + g_0]I_0}{\{(G_0 + g_0)(G_{+1} + g_0)(G_{-1} + g_0) - (G_{-1} + g_0)[g_1 + j\omega_q C_1 e^{-j\theta}][g_1 + j(\omega_q + \omega_p)C_1 e^{j\theta}] - (G_{+1} + g_0)[g_1 + j\omega_q C_1 e^{j\theta}][g_1 + j(\omega_q - \omega_p)C_1 e^{-j\theta}]\}} \qquad \ldots\ldots(47)$$

We can now write

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad \ldots\ldots(48)$$

$$e^{-j\theta} = \cos\theta - j\sin\theta \qquad \ldots\ldots(49)$$

So that the forward gain, given by $4g_L g_S \left|\frac{v_{+1}}{I_0}\right|^2$, becomes

$$P_f = \frac{4g_L g_S (G_{-1} + g_0)^2 \{(g_1 - (\omega_q + \omega_p)C_1 \sin\theta)^2 + (\omega_q + \omega_p)^2 C_1^2 \cos^2\theta\}}{\left(\{(G_0 + g_0)(G_{+1} + g_0)(G_{-1} + g_0) - (G_{-1} + g_0)[g_1^2 - \omega_q(\omega_q + \omega_p)C_1^2 + \omega_p C_1 g_1 \sin\theta] - (G_{+1} + g_0)[g_1^2 - \omega_q(\omega_q - \omega_p)C_1^2 - \omega_p C_1 g_1 \sin\theta]\}^2 + \{(G_{-1} + g_0)(\omega_p + 2\omega_q)C_1 g_1 \cos\theta + (G_{+1} + g_0)(2\omega_q - \omega_p)C_1 g_1 \cos\theta\}^2\right)} \qquad \ldots\ldots(50)$$

Thus P_f can be written as

$$P_f = \frac{4g_L g_S [A - B\sin\theta]}{[H + D\sin\theta]^2 + E^2 \cos^2\theta} \qquad \ldots\ldots(51)$$

where

$$A = (G_{-1} + g_0)^2 [g_1^2 + (\omega_q + \omega_p)^2 C_1^2] \qquad \ldots\ldots(52)$$

$$B = 2(\omega_q + \omega_p)C_1 g_1 (G_{-1} + g_0)^2 \qquad \ldots\ldots(53)$$

$$D = -(G_{-1} + g_0)\omega_p C_1 g_1 + (G_{+1} + g_0)\omega_p C_1 g_1 \qquad \ldots\ldots(54)$$

$$E = C_1 g_1 [(G_{-1} + g_0)(2\omega_q + \omega_p) + (G_{+1} + g_0)(2\omega_q - \omega_p)] \qquad \ldots\ldots(55)$$

$$H = (G_0 + g_0)(G_{+1} + g_0)(G_{-1} + g_0) - (G_{-1} + g_0)[g_1^2 - \omega_q(\omega_q + \omega_p)C_1^2] - (G_{+1} + g_0)[g_1^2 - \omega_q(\omega_q - \omega_p)C_1^2] \qquad \ldots\ldots(56)$$

Differentiating eqn. (54) to find the maximum gain we get

$$\Delta^2 \frac{dP_f}{d\theta} = 4g_L g_S[\{[H+D\sin\theta]^2+E^2\cos^2\theta\}\times \times(-B\cos\theta)-(A-B\sin\theta)\times \times\{2[H+D\sin\theta]D\cos\theta - 2E^2\times \times\cos\theta\sin\theta\}] \quad \text{......(57)}$$

where Δ is the denominator of eqn. (51). The condition for a maximum or minimum is $dP_f/d\theta = 0$.

Therefore $\cos\theta = 0$ is a solution ......(58)

leaving

$$\sin^2\theta[B(E^2-D^2)] + \sin\theta[2A(D^2-E^2)] + +BC^2+BE^2+2ACD = 0 \quad \text{......(59)}$$

The roots of $\sin\theta$ in eqn. (59) can be shown to be imaginary, so that we are left with eqn. (58), i.e.

$$\theta = \frac{\pi}{2} \quad \text{or} \quad -\frac{\pi}{2} \quad \text{......(60)}$$

By differentiating eqn. (51) a second time it can be shown that a maximum occurs at $\theta = -\pi/2$ and a minimum at $\theta = \pi/2$.

10. Appendix 2: Forward and Reverse Gain Conditions when C_2 and G_2 are Finite

Utilizing eqn. (7) and assuming that only the circuits at ω_q and $(\omega_q+\omega_p)$ are 'cold' tuned we have

$$0 = v_0\{G_0+g_0\}+v_{+1}\{-jg_1+j\omega_q C_1\}+ +v_{-1}\{jg_1+j\omega_q C_1\} \quad \text{......(61a)}$$

$$I_{+1} = v_0\{jg_1+j(\omega_q+\omega_p)C_1\}+v_{+1}\{G_{+1}+g_0\}+ +v_{-1}\{-g_2+j(\omega_q+\omega_p)C_2\} \quad \text{......(61b)}$$

$$0 = v_0\{-jg_1+j(\omega_q-\omega_p)C_1\}+ +v_{+1}\{-g_2+j(\omega_q-\omega_p)C_2\}+ +v_{-1}\{G_{-1}+g_0+jB_{-1}\} \quad \text{......(61c)}$$

where B'_{-1} is the mean susceptance of the circuit at $(\omega_q-\omega_p)$.

For an input I_{+1} at frequency $(\omega_q+\omega_p)$ the output voltage v_0 at frequency ω_q is

$$v_0 = \frac{[-jg_1+j\omega_q C_1][G_{-1}+g_0+jB'_{-1}]- -[jg_1+j\omega_q C_1][-g_2+j(\omega_q-\omega_p)C_2]}{\Delta_1} \quad \text{......(62)}$$

where Δ_1 is the determinant formed by the coefficients of eqns. (61).

Therefore the conditions for the reverse gain to be zero are

$$B'_{-1} = \frac{(\omega_p-\omega_q)C_2(g_1+\omega_q C_1)}{(g_1-\omega_q C_1)} \quad \text{......(63)}$$

and

$$(G_{-1}+g_0) = \frac{g_2(g_1+\omega_q C_1)}{(g_1-\omega_q C_1)} \quad \text{......(64)}$$

Thus the reverse gain can be made zero for all values of C_2 and g_2 by detuning the idler circuit.

Detuning the idler circuit alone does not allow arbitrarily high forward gain to be achieved. However, if the output circuit at $(\omega_q+\omega_p)$ is detuned it now becomes possible to get arbitrarily high gain.

In addition to the restrictions placed on the idler circuit in eqns. (63) and (64), the output terminations must be such that

$$B'_{+1} = \frac{-(\omega_p+\omega_q)C_2(g_1-\omega_q C_1)}{(g_1+\omega_q C_1)} \quad \text{......(65)}$$

$$(G_{+1}+g_0) = \frac{g_2(g_1-\omega_q C_1)}{(g_1+\omega_q C_1)} \quad \text{......(66)}$$

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The Discussion on this paper starts on page 439

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Parametric Up-conversion by the Use of Non-linear Resistance and Capacitance

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Summary: The power gain of parametric up-converters using both non-linear resistance and capacitance is found for the condition of conjugate matching at the output termination only. It is shown that for the elements pumped in quadrature it is possible to obtain input impedances with a negative real part, and that under these conditions the gain may be arbitrarily large in principle for any value of pump frequency greater than zero. It is also shown that the device may be used as a negative-resistance parametric amplifier in which the pump frequency is lower than the signal frequency.

Previous results showing that the up-converters may be made unidirectional in operation are confirmed, and it is shown that the gain under these conditions may be optimized by suitable adjustment of the mark/space ratio of the resistance variation. Attempts are made on physical grounds to explain both the unidirectional operation and also the high gain possible with R-C converters. Finally, some experimental confirmation of the theory is given.

List of Symbols

$C(t)$	Time-varying capacitance.
$C_0, 2C_1, 2C_r$	Fourier coefficients of $C(t)$.
$\frac{1}{C}(t)$	Time-varying elastance.
$\left(\frac{1}{C}\right)_0, \left(\frac{2}{C}\right)_1, \left(\frac{2}{C}\right)_r$	Fourier coefficients of $\frac{1}{C}(t)$.
$C'_0, \frac{1}{2}C'_1, \frac{1}{2}C'_r$	The reciprocal of the Fourier coefficients of $\frac{1}{C}(t)$.
$g(t)$	Time-varying conductance.
$g_0, 2g_1, 2g_r$	Fourier coefficients of $g(t)$.
g_f	Forward conductance.
g_b	Reverse (or back) conductance.
G'_0	Input conductance.
G_{+1}	Load conductance.
i_0	Current at frequency ω_q.
i_{+1}	Current at frequency $\omega_p+\omega_q$.
I_0	Source current (at frequency ω_q).
I_{+1}	Source current (at frequency $\omega_p+\omega_q$).
I'_0	Equivalent source current (at frequency ω_q).
I'_{+1}	Equivalent source current (at frequency $\omega_p+\omega_q$).
n	$\sqrt{g_f/g_b}$
P_f	Forward power gain.
P_r	Reverse power gain.
$r(t)$	Time-varying resistance.
$r_0, 2r_1, 2r_r$	Fourier coefficients of $r(t)$.
R_0	Source resistance.
R_{+1}	Load resistance.
R'_{+1}	Output resistance.
s	Ratio of 'mark' to 'mark plus space'.
v_0	Voltage at frequency ω_q.
v_{+1}	Voltage at frequency $\omega_p+\omega_q$.
V_0	Source voltage (at frequency ω_q).
V'_{+1}	Equivalent source voltage (at frequency $\omega_p+\omega_q$).
Y_0	Source admittance.
Y'_0	Input admittance.
Y_{+1}	Load admittance.
Y'_{+1}	Output admittance.
Z_0	Source impedance.
Z_{+1}	Load impedance.
Z'_{+1}	Output impedance.
θ_0	Phase angle of v_0.
θ_{+1}	Phase angle of v_{+1}.
τ	Pulse width.
ω_p	Pump frequency.
ω_q	Signal frequency.
$\omega_p+\omega_q$	Output frequency.

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1. Introduction

Parametric up-converters using back-biased semi-conductor diodes as non-linear capacitors are well-known,[1] and have found useful application for high-frequency, low-noise amplification. Usually such devices are of the three frequency, non-inverting type —that is, only currents or voltages at the signal frequency, the diode pump frequency, and the sum of these frequencies occur in the circuit. The output is taken at the sum frequency, so that the wanted side-band is not inverted.

The equations of Manley and Rowe[2] establish that the maximum power gain of a parametric converter such as has been discussed, in which the non-linear element is a pure capacitor, is the ratio of the output to the input frequency. Edwards[3] has shown that the addition of a non-linear resistor to such a circuit if pumped in phase with the capacitor merely results in a degradation of this performance. There have been other papers in this field[4,5,6] and in particular Engelbrecht[5] has established recently that if the non-linear resistor is pumped in quadrature with the capacitor it is possible to obtain greater power gain than with the capacitor alone, for large ratios of output to input frequency. It was also established that it is possible to make such a circuit unidirectional —that is, to have a finite gain from input to output, and zero gain in the reverse direction. This is advantageous, since under these conditions noise from a following stage cannot be transferred to the converter input. The work reported in this paper is in the main a more general analysis of the case of the resistance and capacitance pumped in quadrature than has hitherto been made. The condition of conjugate matching of source impedance and input impedance has been removed, and it is shown that without that limitation the gain of either series or parallel R,C circuits may become infinite for certain element values and pump phasing. Attempts are made to explain both the gain and directional properties of the devices in simple terms. The corresponding down-converters are briefly mentioned. No consideration is given to lower-sideband (inverting) up-converters in this paper, since it is felt that the important properties of these devices are already adequately recorded elsewhere.[5,6,11]

2. The Parametric Up-Converter with Parallel R and C Elements

The circuit to be analysed is shown in Fig. 1. It will be assumed throughout this work that the pump voltages across the resistor and capacitor are much larger than the corresponding signal voltages, so that the value of resistance or capacitance at any time is controlled entirely by the pump(s)†. That said, there is no theoretical need to consider the pump voltages

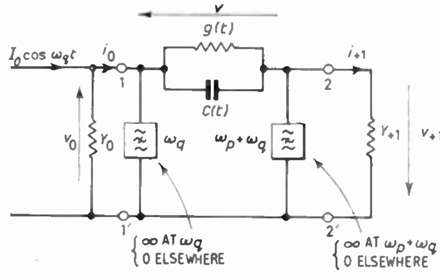

Fig. 1. Parametric converter with parallel R and C elements.

further, since the circuit analysis may be undertaken from a consideration of the *time*-variation of the two elements. Hence in Fig. 1 no pump generators are shown, the capacitance and resistance being merely represented as functions of time. The band-stop filters in the circuit are considered to be ideal, so that only a voltage at signal frequency (ω_q) appears across the source admittance (Y_0), and a corresponding voltage at upper sideband frequency ($\omega_p+\omega_q$) appears across the load (Y_{+1}). The nomenclature used for currents and voltages at various frequencies is the same as that used in a previous paper[7,8] The time-varying capacitance may in general be represented by a Fourier series, i.e.

$$C(t) = C_0 + \sum_{r=1}^{\infty} 2C_r \cos r\omega_p t \qquad \ldots\ldots(1)$$

For convenience the resistor will be considered as time-varying conductance. Since this is pumped so that it is in quadrature with the capacitor, if the conductance variation is $\pi/2$ *ahead* of the capacitance,

$$g(t) = g_0 + \sum_{r=1}^{\infty} 2g_r \cos r\left(\omega_p t - \frac{\pi}{2}\right) \qquad \ldots\ldots(2)$$

$$= g_0 + 2g_1 \sin \omega_p t - 2g_2 \cos 2\omega_p t \ldots \qquad \ldots\ldots(3)$$

On the other hand, if the conductance variation is $\pi/2$ *behind* that of the capacitance,

$$g(t) = g_0 + \sum_{r=1}^{\infty} 2g_r \cos r\left(\omega_p t + \frac{\pi}{2}\right) \qquad \ldots\ldots(4)$$

$$= g_0 - 2g_1 \sin \omega_p t - 2g_2 \cos 2\omega_p t \ldots \qquad \ldots\ldots(5)$$

In our analysis we shall consider the latter case, and be able to deal with the former when necessary by considering g_1 to be negative. No higher coefficients will enter into the equations. The current through the two elements is

$$\frac{d}{dt}\{C(t).v\} \qquad \ldots\ldots(6)$$

and

$$g(t).v \qquad \ldots\ldots(7)$$

† It is also assumed that if there are separate pumps for the resistor and capacitor, these do not interact. In practice this will mean balanced resistor and capacitor networks.

where v is the voltage across them.

$$v = v_0\cos(\omega_q t+\theta_0)+v_{+1}\cos(\overline{\omega_p+\omega_q}t+\theta_{+1}) \quad \ldots\ldots(8)$$

The equations for current at signal and sideband frequencies in the circuit may now be formulated. If v_0 and v_{+1} are considered to be complex in order to include the effect of the phase angles, these are

$$I_0\cos\omega_q t = Y_0 v_0\cos\omega_q t + \left[g(t)(v_0\cos\omega_q t+v_{+1}\cos\overline{\omega_p+\omega_q}t) + \frac{d}{dt}\{C(t)(v_0\cos\omega_q t+v_{+1}\cos\overline{\omega_p+\omega_q}t)\}\right]_0 \quad \ldots\ldots(9)$$

and

$$0 = Y_{+1}v_{+1}\cos\overline{\omega_p+\omega_q}t + \left[g(t)(v_0\cos\omega_q t+v_{+1}\cos\overline{\omega_p+\omega_q}t) + \frac{d}{dt}\{C(t)(v_0\cos\omega_q t+v_{+1}\cos\overline{\omega_p+\omega_q}t)\}\right]_{+1} \quad \ldots\ldots(10)$$

where the suffices after the brackets indicate the frequencies at which they are to be evaluated. Evaluating the equations, replacing 'sin' by 'j cos' where necessary, the vector equations are

$$I_0 = (Y_0+g_0+j\omega_q C_0)v_0+(-jg_1+j\omega_q C_1)v_{+1} \quad \ldots\ldots(11)$$

$$0 = (jg_1+j\overline{\omega_p+\omega_q}C_1)v_0+(Y_{+1}+g_0+j\overline{\omega_p+\omega_q}C_0)v_{+1} \quad \ldots\ldots(12)$$

Let the terminating admittances be

$$Y_0 = G_0-j\omega_q C_0 \quad \ldots\ldots(13)$$

$$Y_{+1} = G_{+1}-j\overline{\omega_p+\omega_q}C_0 \quad \ldots\ldots(14)$$

Then the equations (11) and (12) become

$$I_0 = (g_0+G_0)v_0+j(\omega_q C_1-g_1)v_{+1} \quad \ldots\ldots(15)$$

$$0 = j(\overline{\omega_p+\omega_q}C_1+g_1)v_0+(G_{+1}+g_0)v_{+1} \quad \ldots\ldots(16)$$

The circuit to the left of the terminals 22′ may be regarded as a current generator I'_{+1} at sideband frequency in parallel with an admittance Y'_{+1} also defined at that frequency. From the circuit it is clear that

$$I'_{+1}+Y'_{+1}v_{+1} = -Y_{+1}v_{+1} \quad \ldots\ldots(17)$$

From (12), $I'_{+1}+Y'_{+1}v_{+1}$

$$= (jg_1+j\overline{\omega_p+\omega_q}C_1)v_0+(g_0+j\overline{\omega_p+\omega_q}C_0)v_{+1} \quad \ldots(18)$$

From (15), $I'_{+1}+Y'_{+1}v_{+1}$

$$= j(g_1+\overline{\omega_p+\omega_q}C_1)\cdot\left\{\frac{I_0-j(\omega_q C_1-g_1)v_{+1}}{g_0+G_0}\right\} + (g_0+j\overline{\omega_p+\omega_q}C_0)v_{+1} \quad \ldots\ldots(19)$$

Therefore

$$I'_{+1}+Y'_{+1}v_{+1} = \frac{j(g_1+\overline{\omega_p+\omega_q}C_1)}{G_0+g_0}I_0 + \left\{\frac{(\overline{\omega_p+\omega_q}C_1+g_1)(\omega_q C_1-g_1)+(g_0+G_0)(j\overline{\omega_p+\omega_q}C_0+g_0)}{G_0+g_0}\right\}v_{+1} \quad \ldots\ldots(20)$$

Hence

$$I'_{+1} = \frac{j(g_1+\overline{\omega_p+\omega_q}C_1)}{G_0+g_0}I_0 \quad \ldots\ldots(21)$$

and

$$Y'_{+1} = \frac{(\overline{\omega_p+\omega_q}C_1+g_1)(\omega_q C_1-g_1)+g_0(g_0+G_0)}{G_0+g_0}+j\overline{\omega_p+\omega_q}C_0 \quad \ldots\ldots(22)$$

For a maximum power output, the output admittance must be the conjugate of the load admittance.

Therefore $\quad (Y'_{+1})^* = Y_{+1} \quad \ldots\ldots(23)$

From (14) and (22), it is clear that the imaginary part of Y_{+1} has been chosen correctly, and that

$$G_{+1} = g_0 - \frac{(g_1-\omega_q C_1)(g_1+\overline{\omega_p+\omega_q}C_1)}{G_0+g_0} \quad \ldots\ldots(24)$$

$$\text{The power in the load} = \frac{|I'_{+1}|^2}{8G_{+1}} \quad \ldots\ldots(25)$$

$$= \frac{(g_1+\overline{\omega_p+\omega_q}C_1)^2 I_0^2}{8(G_0+g_0)\{g_0(G_0+g_0)-(g_1-\omega_q C_1)(g_1+\overline{\omega_p+\omega_q}C_1)\}} \quad \ldots\ldots(26)$$

The maximum available power from the source is

$$\frac{I_0^2}{8G_0} \qquad \ldots\ldots(27)$$

so that the power gain from input to output of the circuit is

$$P_f = \frac{G_0(g_1+\overline{\omega_p+\omega_q}\,C_1)^2}{(G_0+g_0)\{g_0(G_0+g_0)-(g_1-\omega_q C_1)(g_1+\overline{\omega_p+\omega_q}\,C_1)\}} \qquad \ldots\ldots(28)$$

P_f will be termed from henceforth the 'forward' gain. For a passive resistance element (which excludes devices which have a negative resistance over some part of their characteristic, such as the tunnel diode) $|g_1| \leqslant g_0$.

The power gain of the converter becomes infinite when

$$g_0(G_0+g_0) = (g_1-\omega_q C_1)(g_1+\overline{\omega_p+\omega_q}\,C_1) \qquad \ldots\ldots(29)$$

For a diode for which $g_0 \simeq g_1$, this means that

$$g_0 G_0+\omega_q(\omega_p+\omega_q)C_1^2 \simeq \omega_p C_1 \cdot g_0 \qquad \ldots\ldots(30)$$

The equation cannot be satisfied for all values of G_0. If $G_0 = kg_0$, for instance, and $\omega_p = n\omega_q$, the resultant quadratic for g_0 will only have real roots if

$$k < \tfrac{1}{4}n^2/(n+1).$$

Therefore for small step-up ratios $(\omega_p+\omega_q)/\omega_q$ in the converter, G_0 will have to be correspondingly small to allow high gain to be obtained. From (24) it is clear that G_{+1} is always small for high gain.

If G_0 is taken to be very much less than g_0, (30) reduces to

$$g_0 \simeq \left(1+\frac{\omega_q}{\omega_p}\right) . \ \omega_q C_1 \qquad \ldots\ldots(31)$$

It can be seen from a consideration of a later equation (35) in conjunction with the condition for infinite gain that the converter so acts because the input admittance has a negative real part of $(-G_0)$. Thus the converter cannot be conjugately matched at the input end with passive terminations, and in fact shows maximum gain when the input admittance is equal to $(-Y_0)$.† For the conductance pumped $\pi/2$ rad *ahead* of the capacitance, in contrast with the case just considered, the forward gain is always finite for terminations with positive real parts. This can be seen by substituting $(-g_1)$ for g_1 in (28). Using the same approximation for g_1 as above, and if $g_0 \ll G_0 \ll \overline{\omega_p+\omega_q}\,C_1$ then the gain approaches $(\omega_p+\omega_q)/\omega_q$, so that the addition of the non-linear resistance to the circuit brings no benefits if pumped with this phase angle.

Previous work[6,11] has shown that the same conclusions about the optimum pumping angles apply when the maximum 'unconditionally-stable' power gain is being calculated; 'unconditionally-stable', that is, for any terminations at either pair of terminals 11′ and 22′.

The gain from output to input (or the 'reverse' gain) for the same terminating conditions considered for (26) may now be calculated. The circuit to the right of terminals 11′ in Fig. 1 will be regarded as a current generator (I_0') at signal frequency in parallel with an admittance Y_0'. By analogy with (21) and (22) we have that

$$I_0' = \frac{\mathrm{j}(\omega_q C_1-g_1)}{g_0+G_{+1}} I_{+1} \qquad \ldots\ldots(32)$$

where I_{+1} is a current source of frequency $\omega_p+\omega_q$ applied to the output. Similarly

$$Y_0' = G_0'+\mathrm{j}\omega_q C_0 \qquad \ldots\ldots(33)$$

where

$$G_0' = g_0 - \frac{(g_1-\omega_q C_1)(g_1+\overline{\omega_p+\omega_q}\,C_1)}{g_0+G_{+1}} \qquad \ldots\ldots(34)$$

Combining (34) and (24)

$$(G_0'-g_0)(G_{+1}+g_0) = (G_{+1}-g_0)(g_0+G_0) \qquad \ldots\ldots(35)$$

The power in the admittance Y_0 is, from (32) and (33),

$$\frac{1}{2G_0}\left\{\frac{G_0 I_0'}{G_0+G_0'}\right\}^2 \qquad \ldots\ldots(36)$$

so that the reverse gain is, from (32) and (36),

$$P_r = \frac{4G_0 G_{+1}(\omega_q C_1-g_1)^2}{(G_0+G_0')^2(g_0+G_{+1})^2} \qquad \ldots\ldots(37)$$

which reduces, after substitution from (35) for G_0', and (24) for G_{+1}, to

$$P_r = \frac{G_0(\omega_q C_1-g_1)^2}{(G_0+g_0)\{g_0(G_0+g_0)-(g_1-\omega_q C_1)(g_1+\overline{\omega_p+\omega_q}\,C_1)\}} \qquad \ldots\ldots(38)$$

Thus the ratio of forward to reverse gain is given by

$$\frac{P_f}{P_r} = \left\{\frac{g_1+\overline{\omega_p+\omega_q}\,C_1}{g_1-\omega_q C_1}\right\}^2 \qquad \ldots\ldots(39)$$

If $g_1 = \omega_q C_1$, the device is unidirectional with zero reverse gain, under which conditions, from (28) when

† In the Appendix it is shown that it is also possible to operate the device so that the *output* admittance has a negative real part, which means that the converter may be operated as a negative-resistance parametric amplifier in which the pump frequency (ω_p) is lower than the 'signal' frequency $(\omega_p+\omega_q)$.

$g_0 = G_0$ the forward gain attains a maximum value of

$$P_f = \left(\frac{g_1}{2g_0}\right)^2 \cdot \left(1+\frac{\omega_p+\omega_q}{\omega_q}\right)^2 \quad ......(40)$$

To attain higher gain it is necessary to depart from unidirectionality. However, (31) shows that g_1 has only to be increased slightly above $\omega_q C_1$ before the gain becomes arbitrarily-high, particularly when the step-up ratio $(\omega_p+\omega_q)/\omega_q$ is large. Under these conditions, from (39) it is possible to obtain high forward gain whilst still maintaining a large ratio of forward to reverse gain. For example, if $g_1 \simeq g_0$, and $\omega_p = 3\omega_q$, the maximum unidirectional gain is, from (40), 8 dB. But if $2G_0 = g_0 \simeq 1{\cdot}9\,\omega_q C_1$, $P_f = 20{\cdot}6$ dB, $P_r = 4{\cdot}3$ dB.

The gain obtainable is therefore greater than that of a 'cold-tuned' four-frequency parametric converter using non-linear capacitance, which is $4(\omega_p+\omega_q)/\omega_q$. If such a converter is off-tuned, arbitrarily-high gain is obtainable as for the RC converter. Noise figures for both converters will be higher than for the ordinary three-frequency capacitance converter. Both noise and bandwidth properties are being investigated at present, and details will probably be published at a later date.

Note that if the converter is pumped so that the conductance waveform is $\pi/2$ rad *ahead* of that of the capacitance, when g_1 may be considered negative, the device can only be unidirectional in the opposite sense. In other words, the forward gain can be made zero whilst the reverse gain remains finite.

3. The Up-Converter with Series R and C Elements

The circuits shown in Fig. 2 and Fig. 3 are those of up-converters with series R and C elements. They are for all practical purposes identical, as shown previously,[7] and require the same equations for their

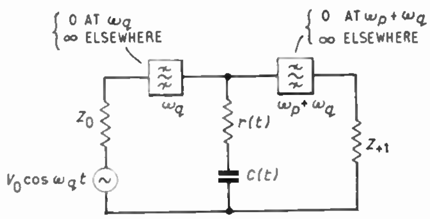

Fig. 3. Shunt form of the parametric converter with series R and C elements.

analysis. A previous analysis of the circuit of Fig. 3 has been made, for conditions of conjugate matching at the input and output terminals of the converter.[5] The terminology adopted in the previous analysis differed in several ways, in particular in using the Fourier coefficients for the conductance and capacitance, rather than those of the resistance and elastance, as will be done here.

The resistance and capacitance will be considered time-varying elements in a similar manner to before, and we will consider initially that

$$\frac{1}{C}(t) = \left(\frac{1}{C_0}\right) + \sum_{r=1}^{\infty} 2\left(\frac{1}{C}\right)_r \cos r\omega_p t \quad ......(41)$$

If the resistance variation is $\pi/2$ rad *behind* this, then

$$r(t) = r_0 - 2r_1 \sin\omega_p t - 2r_2\cos 2\omega_p t ... \quad ......(42)$$

and as before the case of the resistance variation being $\pi/2$ rad *ahead* of (41) will be dealt with by considering r_1 negative.

Using similar conventions to the previous work, if the currents through the terminations are $i_0\cos\omega_q t$ and $i_{+1}\cos\overline{\omega_p+\omega_q}\,t$, then the loop voltage equations at signal and sideband frequencies are

$$V_0\cos\omega_q t = Z_0 i_0 \cos\omega_q t + \left[r(t)(i_0\cos\omega_q t + i_{+1}\cos\overline{\omega_p+\omega_q}\,t) + \frac{1}{C(t)}\int(i_0\cos\omega_q t + i_{+1}\cos\overline{\omega_p+\omega_q}\,t)\,dt\right]_0 \quad ......(43)$$

and

$$0 = Z_{+1} i_{+1}\cos\overline{\omega_p+\omega_q}\,t + \left[r(t)(i_0\cos\omega_q t + i_{+1}\cos\overline{\omega_p+\omega_q}\,t) + \frac{1}{C(t)}\int(i_0\cos\omega_q t + i_{+1}\cos\overline{\omega_p+\omega_q}\,t)\,dt\right]_{+1} \quad ......(44)$$

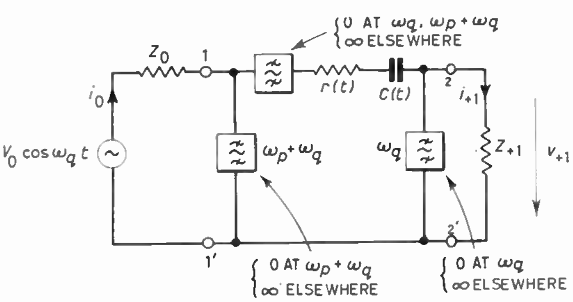

Fig. 2. Parametric converter with series R and C elements.

Evaluating the equations as before,

$$V_0 = \left(Z_0 + r_0 + \frac{1}{j\omega_q C_0'}\right) i_0 + \left(\frac{1}{j\overline{\omega_p+\omega_q}\,C_1'} - jr_1\right) i_{+1} \quad ......(45)$$

$$0 = \left(\frac{1}{j\omega_q C_1'} + jr_1\right) i_0 + \left(Z_{+1} + \frac{1}{j\overline{\omega_p+\omega_q}\,C_1'} + r_0\right) i_{+1} \quad ......(46)$$

It will be assumed that

$$Z_0 = R_0 - \frac{1}{j\omega_q C'_0} \qquad \ldots\ldots(47)$$

and

$$Z_{+1} = R_{+1} - \frac{1}{j\omega_p + \omega_q C'_1} \qquad \ldots\ldots(48)$$

Considering the circuit to the left of terminals 2,2′ to be replaced by an equivalent voltage generator (V'_{+1}) at sideband frequency in series with an impedance Z'_{+1}, after some manipulation we find that

$$V'_{+1} = -\frac{j\left(r_1 - \dfrac{1}{\omega_q C'_1}\right)V_0}{Z_0 + r_0 + \dfrac{1}{j\omega_q C'_0}} \qquad \ldots\ldots(49)$$

and

$$Z'_{+1} = R'_{+1} + \frac{1}{j\omega_p + \omega_q C'_1} \qquad \ldots\ldots(50)$$

where

$$R'_{+1} = \frac{r_0(r_0+R_0) - \left(r_1 + \dfrac{1}{\omega_p + \omega_q C'_1}\right)\left(r_1 - \dfrac{1}{\omega_q C'_1}\right)}{r_0 + R_0} \qquad \ldots\ldots(51)$$

For a conjugate match at the output terminals $R'_{+1} = R_{+1}$, so that the power in the load resistance is $|V'_{+1}|^2/8R_{+1}$. Since the maximum available power from the source is $V_0^2/8R_0$, from (49) and (51) the forward power gain may be calculated to be

$$P_f = \frac{R_0\left(r_1 - \dfrac{1}{\omega_q C'_1}\right)^2}{(r_0+R_0)\left\{r_0(r_0+R_0) - \left(r_1 + \dfrac{1}{\omega_p+\omega_q C'_1}\right)\left(r_1 - \dfrac{1}{\omega_q C'_1}\right)\right\}} \qquad \ldots\ldots(52)$$

Since for a passive resistance element $|r_1| \leqslant r_0$, and assuming also that $\omega_p \gg \omega_q$ the denominator is approximately

$$(r_0+R_0)\left\{r_0 R_0 + \frac{1}{\omega_q(\omega_p+\omega_q)C_1'^2} + \frac{1}{\omega_q C'_1} r_1\right\} \qquad \ldots\ldots(53)$$

It is easy to see that forward gain can never become infinite. If, however, the resistance variation is now considered to be $\pi/2$ rad *ahead* of that of the elastance $1/C(t)$, the sign of r_1 is reversed, and it is clear from (53) that the gain will now become infinite when

$$r_0 R_0 + \frac{1}{\omega_q(\omega_p+\omega_q)C_1'^2} = \frac{r_1}{\omega_q C'_1} \qquad \ldots\ldots(54)$$

From (51), R_{+1} becomes very small as the gain tends to infinity.

Other calculations for this case follow the same pattern as for the parallel R-C converter and will be omitted here. It is sufficient to say that it is readily shown[5] that the device may be unidirectional with finite *forward* gain for the resistance variation $\pi/2$ *ahead* of that of $1/C(t)$, and in that case the forward gain becomes, for $r_0 = R_0$,

$$P_f = \left(\frac{r_1}{2r_0}\right)^2\left(1 + \frac{\omega_p+\omega_q}{\omega_q}\right)^2 \qquad \ldots\ldots(55)$$

4. The Optimization of the Gain of Unidirectional Converters

It has been shown (40), (55) that it is possible to make either form of up-converter unidirectional, but that if this is done the gain is not infinite, but a function of the ratio of output to input frequencies, and of g_1/g_0 or r_1/r_0. For a fixed frequency ratio, therefore, the question remains as to how to optimize the resistance or conductance ratios. In this section we shall consider how these theoretically may be made very close to unity for high-efficiency diodes. In practice it may be difficult to achieve this sort of improvement at frequencies for which the converters are most valuable.

If the diode is switched from forward to reverse conduction by a large sinusoidal voltage, then the variation of incremental resistance approximates to a square wave of equal mark/space ratio.[9] It will be assumed that this square-wave variation of diode conductance switches instantaneously from a small value of conductance (the reverse conductance, g_b) to a much larger value (the forward conductance, g_f), and vice versa. If a d.c. bias is applied in addition to the diode, then this mark/space ratio varies. It will now be shown under what conditions a square-wave variation in diode resistance optimizes the ratio of g_1/g_0, or of r_1/r_0.

It has been shown elsewhere[10] that a square-wave variation of incremental conductance may be represented by the Fourier series

$$g(t) = g_b + s(g_f - g_b) + \frac{2}{\pi}(g_f - g_b) \times \times \sum_{r=1}^{\infty} \frac{\sin(r\pi s)}{r}\cos r\omega_p t \qquad \ldots\ldots(56)$$

where s is the ratio of 'mark' to 'mark plus space' (see Fig. 4) and

$$n^2 = g_f/g_b \gg 1 \quad \text{for all efficient diodes.}$$

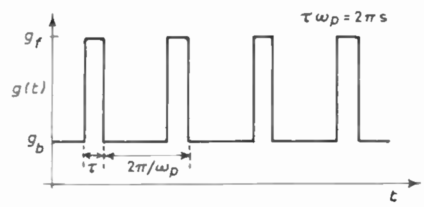

Fig. 4. Square-wave conductance variation.

Thus, using the notation of Section 2,

$$g_0 = g_b + (g_f - g_b)s \qquad \ldots\ldots(57)$$

$$g_1 = \frac{1}{\pi}\sin(\pi s)\,.\,(g_f - g_b) \qquad \ldots\ldots(58)$$

To establish the maximum value of g_1/g_0, the ratio is differentiated as a function of s and the result equated to zero. This leads to the result that g_1/g_0 is a maximum when

$$\tan \pi s = \frac{\pi[1+(n^2-1)s]}{n^2-1} \qquad \ldots\ldots(59)$$

This equation is difficult to solve analytically, but at least it is clear that for large n

$$\tan \pi s \simeq \pi s \qquad \ldots\ldots(60)$$

and therefore $s \to 0$, and the square wave consists of short pulses of high conductance, as shown in Fig. 4. Plots of g_1/g_0 for two values of n are given in Fig. 5.

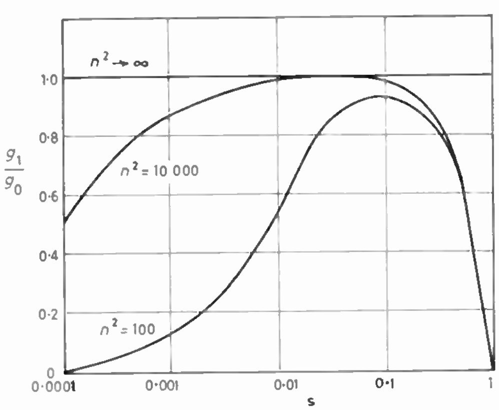

Fig. 5. Effect of variation of mark/space ratio of $g(t)$ upon g_1/g_0.

It can be seen that with a diode having a large ratio of forward to reverse conductance it is possible to have g_1/g_0 approach very closely to unity, and that the value of mark/space ratio necessary need not be controlled very accurately.

The corresponding result for the optimization of r_1/r_0 may readily be deduced. The desired square-wave variation of resistance consists of short pulses of high resistance.

5. Tentative Physical Explanations of Converter Operation

The up-converters described in previous sections have two remarkable properties: (i) the possibility of unidirectional operation; (ii) the increase in gain from that of the corresponding capacitive up-converter for the addition of a non-linear resistance which although varied, always remains positive.

A reasonable explanation for the unidirectional operation may be found in the field of polyphase modulators. For the circuit of Fig. 6 can be said to bear many resemblances to the circuit of section 2, if the non-linear capacitor plus the $\pi/2$ phase shifter in the upper branch be regarded as equivalent to a non-linear resistor (except on an energy-storage basis).

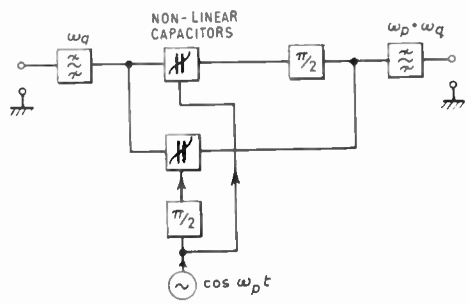

Fig. 6. Block schematic of polyphase upconverter (non-linear capacitor).

If the circuit blocks in Fig. 6 are regarded as being separated by buffer amplifiers which may be connected in whichever is the appropriate direction, a simple analysis may be undertaken.

For a voltage of $\cos \omega_q t$ applied at the left-hand terminals the output will be proportional to

$$\sin \overline{\omega_p + \omega_q}\,t + \sin \overline{\omega_p + \omega_q}\,t \qquad \ldots\ldots(61)$$

On the other hand, a voltage of $\cos \overline{\omega_p + \omega_q}\,t$ applied to the output terminals will give an output at the remote end of the device proportional to

$$\sin \omega_q t + \sin(-\omega_q)t = 0 \qquad \ldots\ldots(62)$$

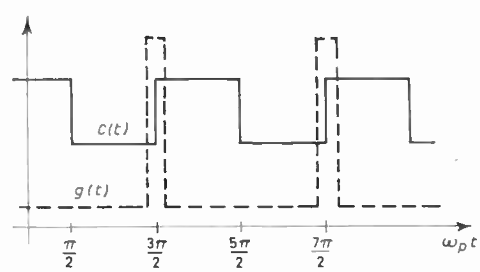

Fig. 7. Idealized capacitance and conductance variations.

so that unidirectionality has been achieved. It is suggested that the up-converters discussed earlier in this paper behave in this sense as approximations to such a polyphase circuit.

With regard to the increased circuit gain of the up-converters over more conventional capacitive types, Fig. 7 proves instructive. In this diagram, which is for the up-converter of section 2, it has been assumed that the capacitor has an equal mark/space square-wave variation of capacitance, and that the conductance variation has been optimized as discussed in the last section. The phasing has been chosen so that the up-converter gain is a maximum. If the resistance is considered to be idealized as a lossless switch, it can be seen that for most of a cycle, the up-converter is identical to a normal capacitive up-converter. Energy will be pumped into the circuit from the pump source when the capacitor decreases in value, since $q = Cv$, q cannot change instantaneously, and energy is proportional to Cv^2. There is no voltage across the capacitor and therefore no energy exchange when the capacitance increases, the switch having closed just before the capacitance rises. Hence energy is continually pumped into the circuit, in a more efficient way than for other conventional parametric amplifiers (except the degenerate amplifier), and thus the increased gain of the R-C circuit over conventional up-converters seems reasonable.

A similar argument can be put forward for the series combination of non-linear capacitance and resistance.

It is also possible to use an equivalent circuit representation of the circuit to obtain an understanding of its mode of operation. This method, although not so fundamental as the above, is capable of wider use.[12]

6. Experimental Work

A two-phase generator has been constructed, capable of supplying two pump voltages at the same frequency (150–500 kc/s) but with a relative phase shift of 0–180 deg between the channels. Figure 8 gives the simplified schematic of a parametric converter with parallel R-C elements which has been built. The R and C elements were constructed as balanced bridges in order to minimize the interaction of the two pump sources used to drive them. The non-linear resistors were G.E.C. silicon diodes, type SX 781SG, and the non-linear capacitances were I.R.C. silicon Zener diodes, type IZ 10. The latter had a zero-bias capacitance of about 1870 pF.

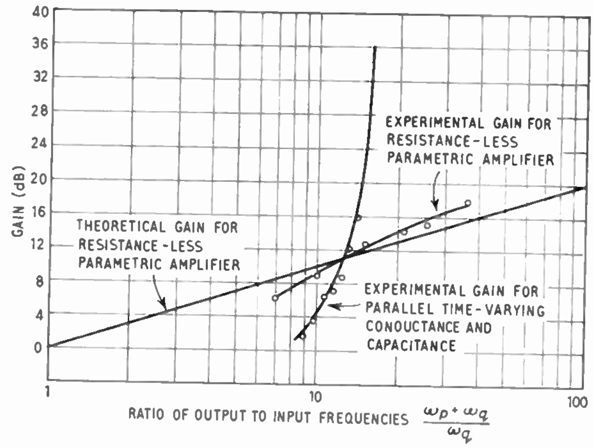

Fig. 9. Gain of parallel R-C converter.

For the experimental work that followed, the pump frequency (ω_p) was chosen to be 300 kc/s, and the signal frequency (ω_q) to be between 15–50 kc/s. The circuit was first used without the non-linear resistor bridge and the power gain as a normal up-converter is recorded on Fig. 9. It will be seen that for high ratios of upper sideband to signal frequency the gain was higher than predicted. This was due to the difficulty found in rejecting adequately the corresponding lower sideband.

When the non-linear resistor bridge was added to the circuit and the two-phase generator was adjusted so that the R and C were pumped in quadrature, it

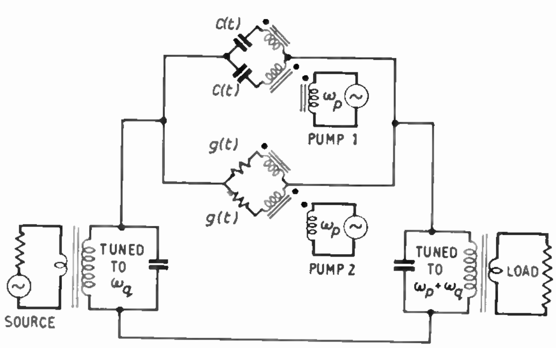

Fig. 8. Experimental parametric converter circuit.

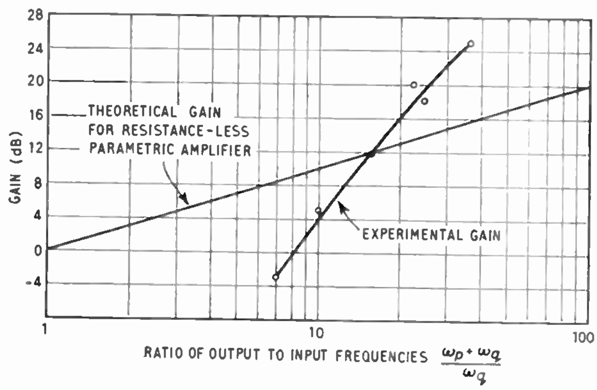

Fig. 10. Gain of series R-C converter.

was found possible to make the converter gain so high that the circuit oscillated for sideband to signal ratios between 8 and 15. It was not considered practicable to work at ratios greater than 15 because of the problem of rejecting the lower sideband. In order to compare the gain of the converter for different sideband frequencies, its performance was degraded by adding 3 kΩ resistors in series with each resistance diode. Under these conditions the near-vertical curve shown in Fig. 9 was obtained, and it can be seen that the gain varies with frequency in a manner that can be predicted from (29).† It was found exceptionally difficult to measure all the necessary circuit parameters, so that it is impossible to claim more than qualitative agreement.

A second experiment was performed in which the R and C of the circuit of Fig. 8 were connected in series rather than in parallel as in the work already described. This circuit is not readily analysed, being considerably more difficult to manipulate than the circuit described in Section 3. However, it was found that once again it was possible to obtain power gains greater than $(\omega_p+\omega_q)/\omega_q$ with the circuit, the results being plotted in Fig. 10.

7. Conclusions

The expression for the power gain of the up-converter using a parallel combination of non-linear capacitance and conductance has been found, for the condition of conjugate matching at the output termination only. It has been shown that when the input admittance has a negative real part the gain tends to infinity, and that this only happens when the conductance is pumped at a phase angle $\pi/2$ rad *behind* the capacitance. Infinite power gain is possible in principle for any value of pump frequency greater than zero. Under these conditions the reverse—i.e. down-conversion—gain also tends to infinity.

Previous results have also been confirmed, showing that the up-converter may be made unidirectional whilst still having a forward gain greater than $(\omega_p+\omega_q)/\omega_q$ for large ratios of output to input frequency. Since this gain is also dependent on the ratio of g_1/g_0, it has been shown how the latter ratio may be optimized by changing the mark/space ratio of the conductance variation so that for most of the pump cycle the diode has low conductance.

A corresponding analysis of the up-converter using a series combination of the same elements has shown similar results. In this case the resistance was pumped at a phase angle $\pi/2$ rad *ahead* of the elastance for optimum performance.

It has been shown that for either type of up-converter, if the resistance is pumped in antiphase to the preferred angle, only a degradation of power gain is recorded when the circuit is compared with a corresponding conventional up-converter without the non-linear resistance.

Attempts have been made to explain on physical grounds the unidirectional properties of the converters, by comparing them with a polyphase parametric converter. An argument for the large gains that are possible using the R-C converters is also given.

Experimental results are given which establish that both parallel and series R-C converters can be constructed which have significantly greater gain than corresponding capacitive converters. The results show that it is easier to obtain high gain in practical R-C converters if the pump frequency is much larger than the signal frequency.

It is shown in the Appendix that it is possible to use the type of converter discussed in this paper as a negative-resistance amplifier with a pump frequency below the signal frequency.

† This follows because the left-hand side of the equation, $g_0(G_0+g_0)$, is easy to make as large as desired. The right-hand side of the equation is much more difficult to increase relative to the left-hand side, since for a passive resistance $|g_1| \leqslant g_0$, and an increase in C_1 will in practice eventually result in an increase in G_0. (If the capacitive diode is considered to be an ideal non-linear capacitor in parallel with a linear resistance, the latter will appear as part of G_0 and G_{+1}.) Hence a progressive reduction of ω_p, lessening the ratio of output to input frequencies in Fig. 9, will eventually make it impossible to achieve arbitrarily-high gain in a given circuit, and the gain will thereafter decrease in the manner shown.

8. Acknowledgments

The authors wish to record their thanks for helpful discussions with Mr. J. G. Gardiner and Mr. K. L. Hughes. Thanks are also due to Professor D. G. Tucker for providing the facilities to enable the experimental work to be undertaken.

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10. Appendix

From equation (22)

Y_{+1} has a negative real part if

$$(\omega_q C_1 - g_1)(\overline{\omega_p + \omega_q} C_1 + g_1) + g_0(g_0 + G_0) < 0 \qquad \ldots\ldots(63)$$

if the conductance is pumped $\pi/2$ rad *behind* the capacitance.

Hence

$$g_1 > \omega_q C_1 \qquad \ldots\ldots(64)$$

and the device is not unidirectional. However, it may be used as a negative-resistance amplifier, considering the signal circuit to be the *output* loop. In this case the pump frequency will be below the signal frequency.

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The discussion on this paper starts on page 439

STANDARD FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Deviations, in parts in 10^{10}, from nominal frequency for **May 1964**

May 1964	GBR 16kc/s 24-hour mean centred on 0300 U.T.	MSF 60 kc/s 1430–1530 U.T.	Droitwich 200 kc/s 1000–1100 U.T.	May 1964	GBR 16 kc/s 24-hour mean centred on 0300 U.T.	MSF 60 kc/s 1430–1530 U.T.	Droitwich 200 kc/s 1000–1100 U.T.
1	− 150·4	− 151·2	+ 1	17	− 150·3	− 151·3	+ 5
2	− 150·8	− 151·0	+ 2	18	− 150·6	− 151·3	+ 4
3	− 151·5	− 151·7	+ 2	19	− 151·3	− 149·5	+ 4
4	− 151·5	—	0	20	− 149·6	− 149·8	+ 4
5	− 151·4	− 150·6	+ 1	21	− 150·5	− 150·0	+ 5
6	− 150·7	—	0	22	− 150·4	− 151·9	+ 6
7	—	− 150·8	+ 1	23	− 150·7	− 151·5	+ 4
8	− 150·2	− 150·4	− 1	24	− 151·4	− 152·6	+ 2
9	− 150·2	− 150·7	—	25	− 151·1	− 150·4	+ 2
10	− 150·0	− 149·1	+ 2	26	—	− 152·6	+ 3
11	− 150·2	− 150·4	+ 2	27	− 150·1	—	+ 4
12	− 150·7	− 150·0	+ 1	28	− 150·0	− 150·3	+ 5
13	− 151·1	− 149·5	+ 1	29	− 150·6	− 151·5	+ 3
14	− 150·9	− 152·6	+ 2	30	− 151·6	− 151·9	+ 5
15	− 150·8	− 151·0	+ 2	31	− 152·0	—	+ 3
16	− 150·0	− 150·6	+ 4				

Nominal frequency corresponds to a value of 9 192 631 770 c/s for the caesium F,m (4,0)–F,m (3,0) transition at zero field.

U.D.C. 621.375/6

Parametric Amplifiers and Converters with Pumped Inductance and Capacitance

By

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AND

K. L. HUGHES, B.Sc. (*Graduate*) ‡

Presented at a meeting of the Institution in London on 10th March 1964.

Summary: It is shown that when both inductance and capacitance are pumped in a parametric amplifier or converter, a special kind of resonance involving only the variable parts of the L and C may occur and produce null responses. An equivalent circuit has been devised for the up-converter, in which an idealized frequency-converter appears. Double-sideband converters are also briefly considered.

1. Introduction

The use of both inductance and capacitance as variable (or pumped) elements in a parametric amplifier gives rise to some special features in its performance which do not arise when only one element is pumped. Apart from the intrinsic interest of these features, it is conceivable that a practical varactor of some kind might have variable capacitance and inductance (for example a varactor comprising basically a non-linear inductance might also have non-linear self-capacitance), and the effects would then clearly be important and need to be understood.

It is intended that this paper on the behaviour of circuits with pumped inductance and capacitance should be only a brief and introductory one. In what follows the notation is identical with that of a previous paper[1] by one of the authors, and uses the basic work set out therein.

2. Up-converters

The circuits concerned are shown in Figs. 1 and 2 in schematic form. For analysis, using time-varying $L(t)$ and $C(t)$, the circuits may be drawn as in Figs. 3 and 4. Of course, in practice, parallel $L(t)$ and $C(t)$ could be used with Fig. 1 and series $L(t)$ and $C(t)$ with Fig. 2; but these would be very difficult cases to analyse. In Fig. 3, Z has the value of Z_0 at the signal frequency ω_q, and Z_{+1} at the output frequency $\omega_q+\omega_p$, where ω_p is the pump frequency; it is infinite at all other frequencies. Similarly Y, in Fig. 4, takes the values Y_0 and Y_{+1} at the two frequencies concerned, and is infinite elsewhere. If lower-sideband converters are considered, the relevant values are Z_{-1} and Y_{-1} at the output frequency $\omega_q-\omega_p$. When, as is usual in practice, $\omega_p>\omega_q$, the positive frequency $\omega_p-\omega_q$ may be used with Z_{1-} and Y_{1-}, which are then the conjugates of Z_{-1} and Y_{-1}.

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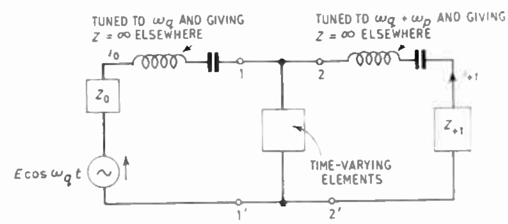

Fig. 1. Time-varying elements in a circuit with only two non-zero currents.

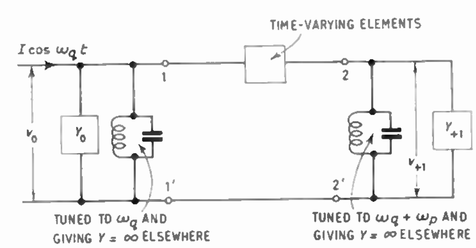

Fig. 2. Time-varying elements in a circuit with only two non-zero voltages.

The time-varying elements in Fig. 3 are represented by Fourier series

$$L(t) = L_0 + L_1 \cos \omega_p t + \ldots \qquad \ldots\ldots(1)$$

$$\frac{1}{C(t)} = \left(\frac{1}{C}\right)_0 + \left(\frac{1}{C}\right)_1 \cos \omega_p t + \ldots \qquad \ldots\ldots(2)$$

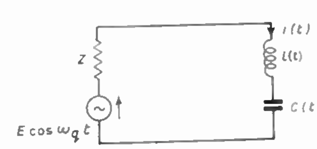

Fig. 3. Circuit equivalent of Fig. 1.

In Fig. 4 they are

$$\frac{1}{L(t)} = \left(\frac{1}{L}\right)_0 + \left(\frac{1}{L}\right)_1 \cos\omega_p t + \ldots \qquad \ldots\ldots(3)$$

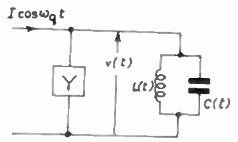

Fig. 4. Circuit equivalent of Fig. 2.

$$C(t) = C_0 + C_1 \cos\omega_p t + \ldots \qquad \ldots\ldots(4)$$

Since the circuit of Fig. 4 is the exact dual of that of Fig. 3, only the latter need be considered here. The behaviour of the other circuit corresponds exactly.

For Fig. 3, the equations of voltage balance are:

For frequency ω_q:

$$E = \left\{Z_0 + \mathrm{j}\left[\omega_q L_0 - \frac{1}{\omega_q}\left(\frac{1}{C}\right)_0\right]\right\} i_0 +$$

$$+ \tfrac{1}{2}\mathrm{j}\left[\omega_q L_1 - \frac{1}{\omega_q + \omega_p}\left(\frac{1}{C}\right)_1\right] i_{+1} \qquad \ldots\ldots(5)$$

For frequency $\omega_q + \omega_p$:

$$0 = \tfrac{1}{2}\mathrm{j}\left[(\omega_q + \omega_p)L_1 - \frac{1}{\omega_q}\left(\frac{1}{C}\right)_1\right] i_0 +$$

$$+ \left\{Z_{+1} + \mathrm{j}\left[(\omega_q + \omega_p)L_0 - \frac{1}{\omega_q + \omega_p}\left(\frac{1}{C}\right)_0\right]\right\} i_{+1} \qquad \ldots\ldots(6)$$

Thus the transfer admittance is

$$\frac{i_{+1}}{E} = \frac{-\tfrac{1}{2}\mathrm{j}\left[(\omega_q+\omega_p)L_1 - \dfrac{1}{\omega_q}\left(\dfrac{1}{C}\right)_1\right]}{\left\{Z_0 + \mathrm{j}\left[\omega_q L_0 - \dfrac{1}{\omega_q}\left(\dfrac{1}{C}\right)_0\right]\right\}\left\{Z_{+1} + \mathrm{j}\left[(\omega_q+\omega_p)L_0 - \dfrac{1}{\omega_q+\omega_p}\left(\dfrac{1}{C}\right)_0\right]\right\} + \dfrac{1}{4}\dfrac{\omega_q}{\omega_q+\omega_p}\left[(\omega_q+\omega_p)L_1 - \dfrac{1}{\omega_q}\left(\dfrac{1}{C}\right)_1\right]^2} \qquad \ldots\ldots(7)$$

which very clearly has a null when

$$\omega_q(\omega_q + \omega_p) = \frac{1}{L_1}\cdot\left(\frac{1}{C}\right)_1 \qquad \ldots\ldots(8)$$

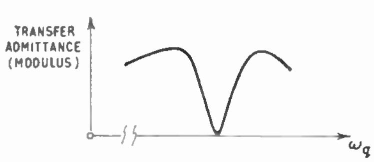

Fig. 5. Showing zero in transfer admittance of up-converter when L and C are pumped in phase.

Thus L_1 and $(1/C)_1$ behave as resonant elements connected across the transmission path. This would be a serious matter if it occurred in the operating frequency band. Figure 5 shows roughly the nature of the response. That maxima occur on each side of the zero is obvious, not only from eqn. (7), but also because any practical system must have a finite bandwidth.

It can easily be shown that a null occurs at the same value of ω_q if the converter is used to transmit backwards, i.e. from $\omega_q + \omega_p$ to ω_q.

For the lower-sideband converter, eqn. (6) is replaced by the corresponding one for frequency $\omega_q - \omega_p$. The numerator of the transfer admittance i_{-1}/E then becomes

$$-\tfrac{1}{2}\mathrm{j}\left[(\omega_q - \omega_p)L_1 - \frac{1}{\omega_q}\left(\frac{1}{C}\right)_1\right] \qquad \ldots\ldots(9)$$

so that if $\omega_p > \omega_q$, there is no null effect.

Now consider that the phase of pumping is reversed for $C(t)$. This means that we now have

$$\frac{1}{C(t)} = \left(\frac{1}{C}\right)_0 - \left(\frac{1}{C}\right)_1 \cos\omega_p t + \qquad \ldots\ldots(10)$$

We see immediately from eqns. (7) and (9) that now the null effect is absent from the upper-sideband converter, but has instead been introduced into the lower-sideband converter.

If the pumping of $C(t)$ is done in quadrature with that of $L(t)$, so that

$$\frac{1}{C(t)} = \left(\frac{1}{C}\right)_0 \pm \left(\frac{1}{C}\right)_1 \sin\omega_p t \pm \qquad \ldots\ldots(11)$$

then we see that the numerators in eqns. (7) and (9) become complex, and evidently then no null effect can occur.

It is of interest that in any of the upper-sideband converter cases, i.e. whatever the relative phases of pumping, if the input and output circuits are properly matched to the converter circuit on the basis of a conjugate match at a particular frequency, for the particular pumping arrangements used, then the power gain is readily shown to be

$$\frac{\text{power in load at } \omega_q + \omega_p}{\text{power available from source at } \omega_q} = \frac{\omega_q + \omega_p}{\omega_q} \qquad \ldots(12)$$

in every case. This is, of course, the same gain as is obtained from a normal single-element parametric converter. At the frequency at which a null occurs (in the case of in-phase pumping) this result still applies in principle, but as the correct matching impedances have then a zero resistance, this result is then not meaningful.

3. Negative-resistance Parametric Amplifier

The lower-sideband converter, as is well known, shows a negative resistance in its input impedance (at terminals 1,1′ in Fig. 1) when $\omega_p > \omega_q$. This forms the basis of the most important class of parametric amplifier.

When $L(t)$ and $C(t)$ are pumped in opposite phase, (i.e. eqns. (1) and (10) apply), the input impedance has a real part

$$R_{i0} = \frac{-\frac{1}{4}\left[\omega_q L_1 - \frac{1}{\omega_p-\omega_q}\left(\frac{1}{C}\right)_1\right]\left[(\omega_p-\omega_q)L_1 - \frac{1}{\omega_q}\left(\frac{1}{C}\right)_1\right] R_{-1}}{R^2_{-1} + \left[X_{-1}-(\omega_p-\omega_q)L_0 + \frac{1}{\omega_p-\omega_q}\left(\frac{1}{C}\right)_0\right]^2} \qquad \ldots\ldots(13)$$

where it has been assumed that $\omega_p > \omega_q$ and that

$$Z_{-1} = R_{-1} + jX_{-1}$$

or

$$Z_{1-} = Z^*_{-1} = R_{-1} - jX_{-1} = R_{1-} + jX_{1-}.$$

It can immediately be seen that this negative resistance diminishes to zero when the condition corresponding to eqn. (8) applies, i.e. $\omega_q(\omega_p-\omega_q) = 1/L_1 \,.\, (1/C)_1$, so that the graph of resistance is of the type shown in Fig. 6.

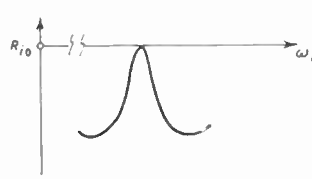

Fig. 6. Showing zero in negative resistance of parametric amplifier when L and C are pumped in opposite phase.

When $L(t)$ and $C(t)$ are pumped in the same phase, or in quadrature, no null occurs.

4. Equivalent Circuits

Equivalent circuits of complex devices are often helpful in understanding circuit behaviour and in exploiting the characteristics of the devices. But in the case of parametric amplifiers and converters, satisfactory equivalent circuits are not easily found. In the present case, for example, the difficulty can be seen when it is appreciated that in eqn. (5) the current i_{+1} is multiplied by the reactance of L_1 at frequency ω_q and by the reactance of $(1/C)_1$ at frequency $\omega_q+\omega_p$; in eqn. (6) the current i_0 is multiplied by the reactance of L_1 at $\omega_q+\omega_p$, and by the reactance of $(1/C)_1$ at ω_q. It would be possible, no doubt, to devise an equivalent circuit with a large number of idealized elements each of which changes the frequency of a current or voltage without changing its magnitude, or which have unilateral characteristics. An exact representation of the equations could then be obtained, but it is hard to see any use for such a complicated arrangement.

A relatively simple equivalent circuit which the authors have devised is shown in Fig. 7, although this has disadvantages too. It includes one idealized (and presumably unrealizable) element which provides a forward frequency change between ω_q and $\omega_q+\omega_p$ and a forward power gain of $(\omega_p+\omega_q)/\omega_q$, and which furthermore is ideally iterative in its impedance

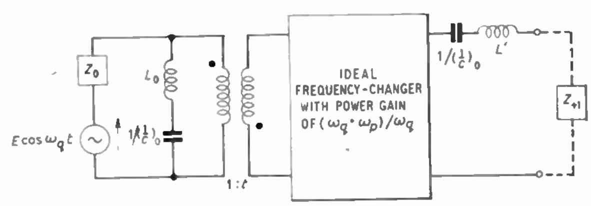

Fig. 7. Equivalent circuit of up-converter, with pumped L and C.

relationships. This means that at one pair of terminals exactly the same impedance is seen as terminates the other pair in spite of the change of frequency. The element transmits both ways, and its backward power gain is $\omega_q/(\omega_q+\omega_p)$.

The turns ratio (t) of the transformer is

$$t = \tfrac{1}{2}\,.\,\frac{\sqrt{\omega_q(\omega_q+\omega_p)}\,.\,L_1 \pm \frac{1}{\sqrt{\omega_q(\omega_q+\omega_p)}}\left(\frac{1}{C}\right)_1}{\omega_q L_0 - \frac{1}{\omega_q}\left(\frac{1}{C}\right)_0} \qquad \ldots\ldots(14)$$

and the inductance L' is

$$L' = \frac{1}{4}\,.\,\frac{\left[\omega_q L_1 \pm \frac{1}{\omega_q+\omega_p}\left(\frac{1}{C}\right)_1\right]^2}{\omega_q\left[\omega_q L_0 - \frac{1}{\omega_q}\left(\frac{1}{C}\right)_0\right]} + L_0 \qquad \ldots\ldots(15)$$

where the plus sign in the numerators is taken if $L(t)$

and $C(t)$ are pumped in opposite phase, and the minus sign if in the same phase.

It will be found that this equivalent circuit gives the eqns. (5) and (6) representing the up-converter. Also it represents correctly the circuit performance for backward transmission. As an equivalent circuit it has the disadvantages that t, L and the power gain of the ideal frequency changer are all a function of ω_q. Moreover, the configuration suggests that there should be a null transmission at the frequency of resonance of L_0 and $1/(1/C)_0$, whereas no such null occurs since t then becomes infinite. The null which actually can occur is indicated only by the formula for t and not by the circuit configuration.

The equivalent circuits of an up-converter with only pumped inductance or only pumped capacitance are easily derived by putting $(1/C)_0 = (1/C)_1 = 0$ or $L_0 = L_1 = 0$ respectively. The expressions are then very greatly simplified.

It is perhaps reasonable to conclude from the above discussion that equivalent circuits are unlikely, in general, to be very helpful in studying the properties of parametric converters, since they are no simpler to use than the circuit equations themselves. But one case in which they have proved useful is in studying the differences in behaviour between circuits with periodically-varying inductance and/or capacitance and those with periodically-varying resistance (e.g. modulators).[2] For this case, the rather remarkable contrast in behaviour is strikingly demonstrated by the equivalent circuits.

5. Double-sideband Converters

In all the above discussion, it has been assumed that power exists only at the signal frequency and at one sideband frequency. If the tuning arrangements are such that both upper and lower sideband power exists, then in respect of many of the performance characteristics of the converter quite different results are obtained from those which apply to a single-sideband converter.[3, 4] Most of these are not particularly associated with the use of pumped inductance *and* capacitance, and apply equally to a double-sideband converter with pumped capacitance alone; but the use of pumped inductance *and* capacitance can give not only some special resonant effects corresponding to those described for the single-sideband converter, but also a significant difference (which may be of practical value) in the conditions for very high gain. The authors have investigated these matters and reported the results in a paper of limited circulation.[5]†

† In view of the great complexity of the mathematical analysis in relation to the limited amount of new results, it is not proposed to publish this paper, but copies can be made available to those interested.

The main conclusions reached are that

(*a*) If the inductance and capacitance variation is confined to the first-order terms (i.e. at frequency ω_p), then a zero response is still obtained in the transfer admittance at the frequency given by eqn. (8) as a kind of resonance of L_1 and $(1/C)_1$ (provided the variations are in suitable phase); but whereas in the single-sideband case the resistive part of the input impedance at the terminals 1,1′ (referring to Fig. 1) also shows a zero under these conditions, in the double sideband case it does not.

(*b*) If second-order terms, with coefficients L_2 and $(1/C)_2$, appear in the inductance and capacitance variation, these cause interaction between upper and lower sidebands, and in consequence no zero response is obtained at the frequency given by eqn. (8), nor at any other frequency. But an arbitrarily high gain (i.e. any value of gain up to infinity) can be obtained when the mean reactances are all tuned out. (This condition is often referred to as 'cold' tuning.) In the case of pumped capacitance (or inductance) alone, arbitrarily high gain can be achieved only by making special non-tuning adjustments of the terminating reactances,[4] which may be much more difficult.

It also appears likely, from recent work by Howson,[6] that especially advantageous results can be obtained from a double-sideband converter using inductance and capacitance pumped in quadrature. This condition permits, by suitable adjustment of all the circuit parameters, arbitrarily high forward gain to be obtained simultaneously with zero backward transmission. The analysis is so complex that full results are not yet available. It is clear, however, that such an amplifier would achieve the same results as the converter with pumped capacitance and resistance, but with a much better noise figure.[3]

6. Conclusions

When both inductance and capacitance are pumped in a parametric amplifier or converter, null responses may be obtained when the relative phases of pumping are suitable. They occur at the same frequency for both directions of transmission through the converter. It is therefore doubtful if such effects could be usefully exploited, but they might need to be avoided if a varactor had non-linear inductance and capacitance. In the case of double-sideband up-converters the use of pumped inductance and capacitance together gives a possible advantage in the conditions for obtaining a very high gain.

The difficulties of devising equivalent circuits for a parametric converter have been discussed, and it is concluded that it would, in general, be as simple to study the properties of a converter directly from the circuit equations as from an equivalent circuit.

7. References

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Manuscript first received by the Institution on 8th January 1964 and in final form on 24th April 1964. (*Paper No. 911.*)

Discussion on

"Some New Possibilities in Parametric Circuits"†

Held in London on 10th March 1964

In the chair: Mr. B. F. GRAY, B.Sc.(Eng.)

Dr. F. J. Hyde: In the papers which have been presented tonight the emphasis has been on ideas. Therefore I should like to start by saying something about the translation of ideas into practice in an actual amplifier.[1]‡ There are four main requirements: (i) gain, (ii) bandwidth, (iii) noise, (iv) unidirectional transmission. In an upconverter the relationship between the output and input frequency spectra is also of importance.

Messrs. Howson and Szerlip and Messrs. Hughes and Howson have concentrated attention on the gain of unidirectional amplifiers in which two dissimilar non-linear elements are pumped in quadrature. Bandwidth and noise have been given less attention. Since the achievement of high gain is not usually a difficult matter, it is pertinent to look in more detail at the factors which affect bandwidth and noise. Let us consider the comparatively simple case of a negative resistance amplifier incorporating a single varactor diode. The small-signal equivalent circuit of such a diode is shown in Fig. A.

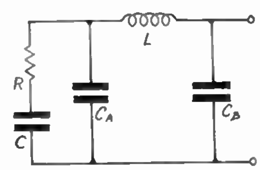

Fig. A. Small-signal equivalent circuit of a varactor diode.

This equivalent circuit is a good approximation when there is no direct current flowing in the diode. Here C represents the capacitance of the junction and R is the resistance of the adjacent semiconductor material plus contact resistance to it. L, C_A and C_B are a lumped circuit representation of the stray reactance in the diode holder. In 'pill-type' diodes, which are becoming common, $C_A \ll C_B$ and we can define a self-resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{......(A)}$$

when the diode terminals are electrically short-circuited.

† The following papers were presented at the meeting:

K. L. Hughes and D. P. Howson, "Parametric up-conversion by the use of non-linear resistance and capacitance", *The Radio and Electronic Engineer*, **27**, No. 6, pp. 417–24, June 1964.

D. P. Howson and A. Szerlip, "Double sideband parametric conversion using non-linear resistance and capacitance" *ibid.*, pp. 425–34.

D. G. Tucker and K. L. Hughes, "Parametric amplifiers and converters with pumped inductance and capacitance", *ibid.*, pp. 435–39.

D. G. Tucker, "Highly-efficient generation of a specified harmonic or sub-harmonic by means of switches" *The Radio and Electronic Engineer*, **28**, No. 1, July 1964 (To be published with separate discussion).

‡ Reference is given on page 441.

It has been shown[2] that if this frequency is chosen as the idler frequency then the maximum power gain-bandwidth product can be achieved for a non-degenerate amplifier. If any other idler frequency is chosen the additional external reactance elements which have to be added to effect tuning at the chosen frequency increase the stored energy of the system. In consequence the Q-factor of the idler circuit is increased and the amplifier bandwidth is decreased. It is therefore important in any analysis concerned with bandwidth to include the stray reactances associated with the diode in the equivalent circuit. In a well-designed amplifier of this type the main source of noise internal to the amplifier is the thermal noise associated with R. This was justifiably neglected by Messrs. Hughes and Howson, since the main source of noise in their amplifier will be associated with the non-linear resistance diode. To obtain high gain they require a large value of g_1, the amplitude of the first Fourier component of diode conductance. This presumably means that the diode will be pumped over a relatively large part of its d.c. characteristic. The $1/f$ noise and shot noise which are associated with the accompanying flow of direct current due to the non-linearity may be quite large and will add to the thermal noise of the spreading resistance.[3] Both of the current-dependent noise components are insignificant in varactor diode amplifiers. They are also relatively insensitive to temperature so that no significant improvement can be obtained by cooling, as is the case with varactor diodes.

Messrs. Hughes and Howson compare their amplifier with the four-frequency varactor amplifier[4] and point to the difficulties of tuning such an amplifier. It should be noted, however, that in a single varactor diode amplifier there is usually no need to couple the pump supply to the varactor in a specific way. In an amplifier involving two pumps it will clearly be necessary to pay careful attention to the design of the pump circuits, since quadrature pumping has to be achieved. There is a further complication since the mean capacitance of the varactor and the mean resistance of a non-linear resistance are dependent on pump level.

I believe that we shall see a number of designs of two-diode amplifier in the near future. It seems to me more likely, however, that these will involve two varactor diodes rather than one varactor and one non-linear resistance. A practical amplifier of this type has been described by Pearson and Lunt.[5] Two similar diodes are chosen so that each acts as an electrical short circuit for the other at its natural resonant frequency, defined by eqn. (A), which is chosen as the idler frequency. This amplifier is a lower-sideband negative-resistance amplifier and has a wide bandwidth, although it is not unidirectional.

If, in the spirit of the papers which we have had presented tonight, we ignore stray reactances and resistances, it is then possible to suggest another type of amplifier incorporating two varactor diodes. This is shown in Fig. B.

If the series resonant frequency $\omega_s = 1/\sqrt{(L_s C_s)}$ is made equal to the shunt resonant frequency $\omega_p = 1/\sqrt{(LC)}$ the filter has a single pass band. High gain and wide bandwidth can be achieved in this degenerate system, but the amplifier is bidirectional. If ω_s is made different from ω_p there are two pass bands separated by a stop band, i.e. the system is non-degenerate. It can be arranged that the sum of the phase shifts, β_s and β_i of the signal and idler frequencies, respectively, from D1 to D2 is $\pi/2$. If the corresponding phase shift β_p at the pump frequency is also $\pi/2$, then for a signal propagating in the forward direction

$$\beta_s + \beta_i = \beta_p \qquad \text{......(B)}$$

This is the condition for the parametric amplification at each diode to be additive.[6] On the other hand, for propagation in the reverse direction

$$\beta_s + \beta_i = \beta_p - \pi \qquad \text{......(C)}$$

Hence the amplification at D2 of a signal emanating from the load end will be cancelled by an equal attenuation at D1 and the reverse gain will be unity. Under high gain conditions such an amplifier would be almost unidirectional. If this condition could be directly realized in practice it would not therefore be necessary to use a circulator to separate the input and output channels. Its noise performance would be good, since only low-noise varactor diodes are used. However, the practical realization of an amplifier such as this in the ultra high frequency or microwave region will be subject to the same difficulties that have already been related to the equivalent circuit representation of Fig. A.

The analysis by Professor Tucker and Mr. Hughes of simultaneous pumping of L and C is of considerable theoretical interest. There may be some unexpected limitation on the noise performance of amplifiers based on this model due to spontaneous fluctuations in the inductance which may account for the noise of parametric amplifiers incorporating non-linear ferrites.[7] Spontaneous fluctuations in the capacitance of varactors has also been postulated,[8] but experimental results would suggest indirectly that this is small.[9]

Mr. J. D. Pearson: The papers read tonight have been in the main theoretical with some experimental verification at low frequencies. Parametric amplifiers are normally regarded as microwave devices and a great deal of work needs to be done to extend the principals in the papers to the microwave region. However, this type of theoretical analysis gives the microwave engineer indications on what he should be looking for in the way of semiconductor devices in order to improve the performance of his parametric amplifiers.

Mr. B. C. Taylor: Has Professor Tucker done any investigation on parametric up-convertors when the signal power (more especially the output power) is a substantial

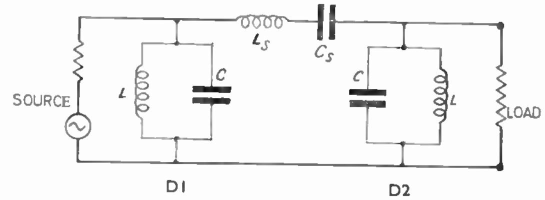

Fig. B. Filter-type amplifier incorporating two varactor diodes.

fraction of the pump power? If so, does he know the effect of the signal output/pump power ratio on the linearity of the signal input/output characteristic? I have seen a brief reference to some work in America where output/pump efficiencies of 50% were claimed for good linearity, though how good it did not say.

Messrs. Howson, Hughes, Szerlip and Tucker (*in reply*): We are in general agreement with Dr. Hyde's assessment of the noise sources within RC parametric amplifiers, although we have not found it necessary to pump the resistance diodes over a wide range of their d.c. characteristics as he suggests. Inevitably these forms of amplifier will be more noisy than their counterparts using only non-linear capacitance, although to some extent this is offset by their more attractive properties. The practical value of the circuits depends to a great extent on what minimum noise figures are obtainable, however, and as yet neither our experiments nor those of other workers have evaluated this. Engelbrecht has suggested around 3 dB, and Chang's work on high-gain tunnel diode converters[10] also points in this direction.

With regard to the difficulty of setting up the pump circuits for quadrature pumping, we are hopeful that it may prove possible to space the two diodes by $\lambda/4$ in a waveguide fed by a single pump source. It would obviously be more attractive if a single non-linear diode were available which could be fed from one pump source and would give the appropriate resistance and reactance variation. At low frequencies it has been found possible to achieve an effect of this nature with a junction diode having significant minority carrier storage; and it has also been shown[11] that a tunnel diode may be used as both a non-linear resistance and capacitance in a closely-related type of parametric amplifier.

In reply to Mr. Taylor, we must point out that in an ideal varactor circuit the ratio of output power (at frequency $\omega_q + \omega_p$) to pump power (at frequency ω_p) is necessarily fixed at a value $(\omega_s + \omega_p)/\omega_p$ according to the well-known Manley-Rowe relationship. Any value below this obtained in practice must be due to the effect of the loss resistance of the varactor and other parts of the circuit, and *may* thus be connected with the linearity of the signal amplification. It is perhaps worth pointing out that when one talks of 'small-signal' operation (in connection with obtaining good linearity) one means that the signal voltages and/or currents are small compared with the pump voltages and/or currents; this is quite different, in a reactive circuit, from saying that the signal powers are small compared with the pump power. But apart from these general points, we have no numerical data on practical microwave power ratios.

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Joint Symposium on

"SIGNAL PROCESSING IN RADAR AND SONAR DIRECTIONAL SYSTEMS"

with special reference to systems common to Radar, Sonar, Radio Astronomy, Ultrasonics and Seismology

UNIVERSITY OF BIRMINGHAM, 6th–9th JULY 1964

Organized by The Institution and The Department of Electronic and Electrical Engineering, University of Birmingham

PROGRAMME

The papers in the Symposium have been grouped under broad headings in order to bring together those on the application of signal processing in directional systems in the fields of radio astronomy, radar, sonar, seismic detection, and ultrasonics respectively. There is a certain amount of overlapping but it is believed that this will help to point the aim of the Symposium, namely the common features of the techniques in the different disciplines.

Monday, 6th July (11.30 a.m.—12.45 p.m.; 2.15—5.15 p.m.)

RADIO ASTRONOMY

Lecture by Professor Martin Ryle, F.R.S., on 'Arrays and Processing Problems in Radio Astronomy'.

'One-dimensional Aperture Synthesis in Radar Astronomy'—R. S. Roger and J. H. Thomson (*Nuffield Radio Astronomy Laboratories, Jodrell Bank*).

'Stretch: A Time-transformation Technique'—W. J. Caputi (*Cutler-Hammer*).

'Enhancing the Angular Resolution of Incoherent Sources'—A. C. Schell (*U.S.Air Force Cambridge Research Laboratories*)

'Statistical Optimization of Antenna Processing Systems'—G. O. Young (*Hughes Aircraft*).

6.15 p.m.—Buffet Reception in the University Staff House

Tuesday, 7th July (9.30 a.m.—1 p.m.; 2.15—5.15 p.m.)

RADAR

'The Combination of Pulse Compression with Frequency Scanning for Three-dimensional Radars'—K. Milne (*Decca Radar*).

'Radar Receiving Array with I.F. Multiple-beam-forming Matrix'—J. Salomon, S. Pichafroy and J. Hurbin (*Compagnie Française Thomson-Houston*).

'Theoretical and Experimental Studies of the Resolution Performance of Multiplicative and Additive Aerial Arrays'—E. Shaw and D. E. N. Davies (*University of Birmingham*).

'Multiplicative Processing Antennas for Radar Applications'—A. A. Ksienski (*Hughes Aircraft*).

'A Note on Multiplicative Receiving Systems and Radar'—R. Blommendaal (*Nederlandsch Radar Proefstation*).

'Wideband, High-gain, Variable Time Delay Techniques for Array Antennas'—J. B. Payne (*U.S.A.F., Rome Air Development Centre*).

'A New Type of Cross Correlation Radar System'—R. H. MacPhie (*University of Waterloo, Ontario*).

'Phased Array Radar Systems'—K. F. Molz (*Bendix Corporation*).

'An Analogue Polarization Follower for Measuring the Faraday Rotation of Satellite Signals'—G. F. Vogt (*U.S. Army Electronics Research and Development Laboratory*).

Wednesday, 8th July (9.30—11.15 a.m.)

ASPECTS COMMON TO RADAR AND SONAR

'A Side-lobe Suppression System for Primary Radar'—J. Croney and P. R. Wallis (*Admiralty Surface Weapons Establishment*).

'Planar Arrays with Unequally Spaced Elements'—M. I. Skolnik and J. W. Sherman (*Electronic Communications*).

'The Use of an Effective Transmission Pattern to Improve the Angular Resolution of Within-pulse Sector-scanning Radar or Sonar Systems'—D. C. Cooper (*University of Birmingham*).

Wednesday, 8th July (11.45 a.m.—1 p.m.; 2.15—3.30 p.m.)

SEISMIC DETECTION

Lecture by MR. IEUAN MADDOCK, O.B.E., on 'Detection of Underground Nuclear Explosions'.

'Least-squares Array Processing for Signals of Unknown Form'—M. J. LEVIN (*Massachusetts Institute of Technology, Lincoln Laboratory*).

'The Recording and Analysis of Seismic Body Waves using Linear Cross Arrays'—F. E. WHITEWAY (*Atomic Weapons Research Establishment*).

(4—5.15 p.m.)

SONAR

'Theoretical and Experimental Properties of Two-element Multiplicative Multi-frequency Receiving Arrays including Superdirectivity'—B. S. MCCARTNEY (*Formerly University of Birmingham*).

'Theoretical Possibilities of a Digital Sonar System'—D. NAIRN (*University of Birmingham*).

7.15 for 7.45 p.m.—Symposium Dinner in the Avon Room of the University Refectory

Thursday, 9th July (9.30 a.m.—12.45 p.m.)

ULTRASONICS AND SONAR

'The Effect of a Linear Phase Taper on the Near Field of an Ultrasonic Multi-element Array'—L. KAY and M. J. BISHOP (*Formerly University of Birmingham*).

'Optimum Directional Pattern Synthesis of Circular Arrays'—G. ZIEHM (*Atlas-Werke*).

'The Effect of Noise on the Determination of Direction in a Multiplicative Receiving System'—C. R. FRY and PROFESSOR D. G. TUCKER (*University of Birmingham*).

'Optimum Line and Crossed Arrays for the Detection of a Signal on a Noise Background'—PROFESSOR H. S. HEAPS (*Nova Scotia Technical College*) and C. WADDEN (*Naval Research Establishment, Nova Scotia*).

'The Use of Quantizing Techniques in Directional Systems'—PROFESSOR H. S. HEAPS (*Nova Scotia Technical College*) and P. W. WILLCOCK (*Naval Research Establishment, Nova Scotia*).

2—3.20 p.m. Discussion on 'Future Research in Signal Processing for Directional Systems: the possibilities for collaboration among the various fields of application'.

Synopses of some of the Papers to be presented at the Symposium

(The first group of synopses was published in the May *Journal*)

One-Dimensional Aperture Synthesis in Radar Astronomy

R. S. ROGER AND J. H. THOMSON. (*Nuffield Radio Astronomy Laboratories.*)

By utilizing successive positions relative to the moon of the 250-ft Mark I radio telescope as array elements, multiple beams of less than one minute of arc in width have been synthesized at 100 Mc/s. Resultant strip-distributions across the moon's disc clearly show the central bright spot and the lower intensity return out to the limbs. It is shown that the method is equivalent to spectral analysis of the returned energy.

Stretch: A Time-transformation Technique

W. J. CAPUTI. (*Cutler Hammer Airborne Instrumentation Laboratory.*)

This paper describes a passive time-transformation technique that permits an exchange of signal time duration for bandwidth. It is not limited to electrical signals or to a particular portion of the spectrum. It is linear in the sense that the principle of superposition is applicable. Good waveform fidelity is preserved, and signals that are resolvable at the input remain resolvable at the output, even though the bandwidth may be changed by a large factor. Signal/noise ratios are not affected by the transformation. The technique uses elements familiar to those versed in pulse compression radar techniques.

Application of the technique to wide-bandwidth, wide-baseline directional systems will be discussed. Removal of most of the bandwidth takes place at the individual receiving sites. Because the phase and amplitude characteristics of the signals are preserved, remoting, combining, processing, and display of the information may be done at practical bandwidths.

Enhancing the Angular Resolution of Incoherent Sources

ALLAN C. SCHELL. (*U.S. Air Force Cambridge Research Laboratory.*)

The technique under study is a means for the enhancement of angular information available at the output of an antenna system. The technique is applicable to incoherent source maps, and thus relates to radar target distributions and radio astronomy maps. An attempt is made to use the available antenna output to the best advantage consistent with the signal/noise level and the character of the signal and noise. The processing is carried out in the spatial frequency domain and is intended to yield a map that, in most instances, is of higher quality (as regards resolution and ambiguity reduction) than is produced by the antenna alone. Clearly, if the output resolution is finer than that of the antenna, some sort of information must be added by the processor. It is the criterion for extending the data and its subsequent use that is the central issue, and an arbitrary but consistent scheme has been settled upon.

The Combination of Pulse Compression with Frequency Scanning for Three-dimensional Radars

K. MILNE. (*Decca Radar.*)

The paper describes proposals for three-dimensional radar which combine 'within-pulse' frequency scanning in the vertical plane with pulse compression. Long range detection requires high pulse energies and modest pulse repetition frequencies. Detection in precipitation and other forms of distributed clutter demands short pulse lengths. Pulse compressions enable these two requirements to be met within the peak power limitations of currently available transmitter valves. The need for a high data rate implies that all elevation positions should be examined on each pulse. 'Within-pulse' frequency-scanning achieves this and can provide two-way discrimination against returns in side-lobes.

Design principles of the radar are discussed. Vertical scanning is obtained by feeding a beam-squinting array from a variable-frequency transmitter and pulse compression is applied to all received signals. The coverage can be shaped by controlling the transmitted spectrum. Elevation information is obtained from frequency filters. The minimum length to which the received pulse can be compressed is determined by the aerial's group delay. The concept of a dispersive aerial having a group delay varying with frequency is introduced: this provides an additional degree of freedom for controlling the coverage diagram and can be used to minimize the bandwidth required for the radar. An example is given of a three-dimensional S-band radar design suitable for air-traffic control purposes.

Wideband, High-gain, Variable Time Delay Techniques for Array Antennas

LT. JOHN B. PAYNE, U.S.A.F. (*Rome Air Development Centre.*)

The purpose of this paper is to present two improved techniques for obtaining a wideband, high gain, nondispersive, variable time delay device to steer array antennas. The first time delay technique utilizes two wideband helix structures separated by a cylindrical drift tube to obtain a variable delay of 20 or more nanoseconds with a gain of 20 to 30 dB. An electron beam passes through the two helices and drift region. Signal energy coupled on to the beam at the input helix is decoupled at the output helix. A variable delay is obtained by controlling the beam velocity through the drift region by varying the drift tube potential. Gain variation is minimized by maintaining constant beam velocity through the helices. The gain of the device is proportional to output helix length.

The second method for obtaining a variable time delay is realized by switching time delay elements into the signal path. Such a technique is controlled digitally and is referred to as a digital delay line. In this technique each switch is replaced by a wideband amplifier. It is the bandwidth of these amplifiers that determines the system bandwidth (200 to 800 Mc/s). Each bit is capable of producing 12 dB gain. The amplifiers can be easily gated on or off.

The Use of Quantizing Techniques in Directional Systems

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Detection of a weak signal upon a noise background may be dependent upon the ability of a system to distinguish between the correlation functions of the signal and the noise background. The correlation functions are functions of both time and space.

If a receiving array in space is steered by means of time delays between appropriate elements and if the response is processed by digital methods then the computation time determines the time for a complete sweep through the directions of interest. Such time may be reduced by very coarse quantizing before digital processing.

The purpose of the present paper is to present a general theory of the effect of quantizing upon digital computation of Fourier transforms. A similar theory applies in relation to correlation functions.

Theoretical and Experimental Studies of the Resolution Performance of Multiplicative and Additive Aerial Arrays

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Some interim results of a theoretical and experimental study of multiplicative signal processing for receiving aerial arrays, such as those used in centimetric radar systems are described. Theoretical comparison is made between multiplication and other forms of demodulation such as linear and square-law rectification, in terms of the effect of demodulation on the ability of the directional array to resolve two closely spaced signal sources (or targets in a radar system). It is shown that the multiplicative system is superior to the other two systems for targets of approximately the same signal strength. It is also shown that improvements in resolution can result from the use of integration if the signals from the two sources are incoherent or partially incoherent. The effect of noise on resolution is briefly discussed and attention is drawn to the effect of the different directional responses of the different signal and noise products of demodulation.

An experimental eight-element multiplicative array operating in the 3-cm band is described which is also capable of fast electronic scanning. Experimental directional responses of this array are presented for both single target and multiple target excitation, for static measurements and at electronic scanning rates of 2 kc/s.

Least-squares Array Processing for Signals of Unknown Form

MORRIS J. LEVIN. (*Lincoln Laboratory, Massachusetts Institute of Technology.*)

Methods for processing the outputs of an array of sensors to provide estimates of the waveform, velocity, and arrival angle of the incident signal in the presence of noise are derived from a least-squares fit criterion. This simple postulate, involving a minimum of assumptions, is found to imply time-shift and sum processing which is shown to be equivalent to other techniques previously developed on the basis of more specialized models. Approximations are obtained for the variances of the resulting estimates and the method is compared with the more elaborate maximum-likelihood approach. The resulting array pattern is considered and the separation of the various components of an incident signal or noise field by the insertion of band-pass filtering is discussed.

The method is appropriate for applications such as seismology and passive sonar in which the signal waveform is unknown, yet cannot be realistically represented as a stationary random process (as is required, for example, by Weiner filtering theory). The only assumption is that the signal is a uniformly propagating plane wave. The error analysis, however, is based on the assumption of additive, stationary, Gaussian noise.

The Recording and Analysis of Seismic Body Waves using Linear Cross Arrays

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Seismic signals from a single event usually contain a number of components (phases) which have travelled by different propagation paths, or with a different mode of propagation. These may be superimposed and obscure signal components of interest. Seismic background noise may also be of sufficient amplitude to obscure the signal onset, which is often of relatively small amplitude, or even obscure the whole signal.

An array of seismometers, spaced over a distance comparable to the signal wavelength, can be used as a filter to separate and help identify signal components on the basis of azimuth and apparent velocity at the earth's surface. A signal/noise ratio improvement is also obtained for the first arrival, improving the accuracy of locating the hypocentres using triangulation methods from several stations.

Linear cross arrays have been operated during recent years by the United Kingdom Atomic Energy Authority and many events analysed, an example of which is shown. The theoretical performances of symmetrical cross- and L-shaped arrays are given in the form of directivity patterns, and their method of use described. Correlation methods are shown to be necessary for obtaining a good azimuth or velocity response, and their advantages and limitations considered.

Theoretical Possibilities of a Digital Sonar System

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The principles of a new type of sonar system are described; in it the detection and location of echo sources are achieved by performing digital (or logical) operations on suitably encoded phase information from an array of transducers. Some aspects of the expected performance are discussed.

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The Digital Analysis of Electron-optical Systems

By

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AND

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Summary: Very accurate methods which are particularly suitable for use on digital computers are described for the analysis of electron-optical systems having cylindrical symmetry. The electric fields are determined using a highly convergent iterative method, and the magnetic fields are determined from analytic expressions fitted to physical measurements of the fields along the axes of the systems. The equations of electron motion are integrated using predictor-corrector methods with automatic control of the integration interval. The methods are sufficiently accurate to permit the detailed analysis of systems in which high-order imaging properties are important. Results obtained for an image orthicon camera tube and an image converter tube are also described.

1. Introduction

In the analysis of electron-optical systems it is necessary first to determine the electric and magnetic fields to which electrons are subjected and then to integrate the equations of electron motion through the fields. For some simple configurations the fields may be determined analytically, but in general it is necessary for this purpose to use either an analogue device, such as a resistor network[1] or an electrolytic tank,[2] or a numerical finite difference method, such as Southwell's relaxation method.[3] The equations of electron motion have been integrated using analogue devices,[4] although numerical methods are generally considered to be more satisfactory.[5, 6, 7]

The application of these methods has mainly been limited to systems for which only approximate results are required and until recently little progress has been made in the analysis of systems in which high order imaging properties play an important part, such as image tubes. For the analysis of systems of this type extremely high accuracy is essential, and although the numerical integration techniques mentioned above are reasonably satisfactory it has been difficult to obtain adequate field data. The solution of field problems to very high accuracy by means of analogue devices is cumbersome, and Southwell's relaxation method, which is not suitable for use on digital computers, is unsatisfactory for this purpose. However, progress has been made recently in the development of highly convergent iterative methods, particularly suitable for digital computers, for the solution of finite difference analogues to Laplace and Poisson equations, and these have greatly facilitated the accurate analysis of electron-optical systems.

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In this paper a numerical procedure is described for the accurate analysis of cylindrically symmetric electron-optical systems not seriously affected by space charge, in which electric and magnetic fields may be present. With the iterative method used, a set of several thousand finite difference equations representing an electric field problem can easily be solved to an accuracy of one part in 100 000. If a magnetic field is present, this is determined from an analytic expression based on a series of measurements of the field along the axis of the system. The equations of electron motion are integrated using predictor-corrector methods with automatic control of the integration interval, which keeps the integration errors within very small limits. The procedure has been used for the analysis and improvement of several quite different types of imaging devices, and for all calculated trajectories the radial and angular errors have been estimated to be smaller than 10 microns and 2×10^{-4} radians respectively.

2. The Determination of Electrostatic Fields

The determination of the electrostatic potential distribution in an electron-optical system with cylindrical symmetry involves solving Laplace's equation in cylindrical co-ordinates for a half-section through the axis of the system. To solve such a problem numerically, it is first replaced by a finite difference analogue, i.e. the half section is covered with a mesh, and finite difference equations are established which relate the value of the potential at each mesh point to the values of the potential at neighbouring mesh points. In this way the problem of solving Laplace's equation is reduced to one of solving a large set of simultaneous linear algebraic equations. The problems involved in producing adequate finite difference analogues are well known and have been discussed in detail by several authors.[8, 9]

The resulting sets of equations are very large, but each equation contains only a few non-zero coefficients, and for these reasons they are almost invariably solved by iterative methods. Approximate solutions can be obtained by using Southwell's relaxation method,[3] but such solutions cannot be used as a basis for accurate trajectory calculations. The effect of discretization errors (i.e. errors due to replacement of Laplace's equation by a finite difference analogue) on trajectory calculations is cumulative, and to keep them within satisfactory limits in practical problems, which often involve very irregular electrode geometries, it is necessary to use finite difference analogues of several thousand equations. Such problems can only be solved on digital computers, for which Southwell's method is unsuitable both because it is unsystematic and because unless it is used with considerable artifice convergence is very poor.

A method which has been used for solving such problems on digital computers is the Young-Frankel successive over-relaxation method[8, 9] a systematic iterative method in which the rate of convergence is a function of a certain parameter, known as the 'accelerating factor', which for any particular problem has an optimum value. If the optimum value of the accelerating factor is used the rate of convergence can be extremely high, and the number of iterations needed to solve a typical electron-optical field problem by this method may be smaller by a factor of thirty than the number needed to solve it to the same accuracy by systematic ordinary relaxation. Unfortunately the optimum accelerating factor for any particular problem is not usually known accurately prior to solution of the problem, and a small error in the estimation of the optimum accelerating factor can seriously reduce the rate of convergence. (The use of an accelerating factor smaller than the optimum value by 1·5% can double the number of necessary iterations). To overcome this difficulty a technique of automatic optimization of the successive over-relaxation method has recently been developed by one of the authors,[10] in which successively better estimates of the optimum accelerating factor are obtained and used during the solution of a problem. The number of iterations needed to solve a problem by this method only exceeds the minimum possible number by a small fraction, and the method yields accurate estimates of the errors at any stage.

Theoretically, this method of automatic optimization is only applicable to the solution of a set of equations whose matrix of coefficients is symmetric positive definite. The matrix of coefficients of a finite difference analogue of a cylindrically symmetric field problem is not naturally exactly symmetric, because of the presence of the first-order term in Laplace's equation when this is expressed in terms of cylindrical co-ordinates. If a non-uniform mesh is used, this also spoils the symmetry of the set of coefficients. Such sets of equations could be symmetrized, but numerical experiments performed by the authors have indicated that in practice the effects of violation of the symmetry conditions in this way are quite negligible. The method has been applied directly to a great number of electron-optical problems, and always found to be perfectly satisfactory.

The method has been incorporated in general DEUCE computer programs for solving any two-dimensional Laplacian problem. From a description of a mesh and boundary geometry in numerical terms, the computer evaluates the coefficients of all the finite difference equations. The set of equations is stored in the computer in a very compact form, only the non-zero coefficients of dissimilar equations being retained. The computer is then supplied with the values of the fixed boundary potentials and the accuracy to which a solution is required, after which it calculates the required solution. The program can deal with non-regular meshes and curved boundaries, and up to 5000 mesh points may be used. The amount of manual work involved in preparing a problem for computation is largely a function of its geometric complexity, but in general this takes from one to eight hours, and the amount of computing time required to solve a set of 1500 equations to an accuracy of one part in 100 000 is approximately forty minutes. The work involved in obtaining a new solution for a slightly modified boundary geometry is fairly trivial. Examples of fields obtained, for which the finite difference analogues were solved to an accuracy of one part in 50 000 are shown in Fig. 1 and Fig. 2. For the image orthicon image section shown in Fig. 1, the half section was covered with a square mesh of 19×43 points. For the infra-red image converter shown in Fig. 2, a graded mesh of 25×56 points was used, the mesh spacing being indicated in the figure.

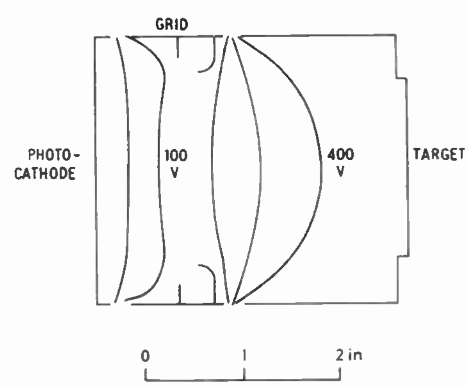

Fig. 1. Image section of image orthicon.

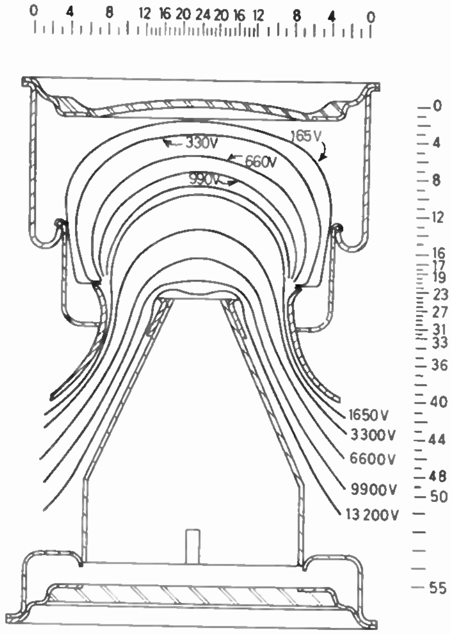

Fig. 2. Infra-red image converter.

It is often desirable to have knowledge of the electric field in a system for several different combinations of electrode potentials, in which case the most efficient way of determining the various fields is to use the method of superposition. For a system having n electrodes, $n-1$ field problems are solved, each solution being for one electrode at unit potential and all the other electrodes at zero potential. Another computer program is then used to form the required linear combinations of these different solutions. Since this only involves multiplying each solution by a specified scalar and adding together the resulting solutions, a large number of potential distributions for different combinations of electrode potentials can be found in a few minutes.

3. The Determination of Magnetic Fields

For trajectory calculations on a digital computer an axially symmetric magnetic field is most conveniently specified by values at the nodes of a mesh of the magnetic vector potential, A_θ, which may be determined from the following expression:[11]

$$A_\theta = \sum_{i=0}^{\infty} \frac{(-1)^i B_a^{(2i)}}{i!(i+1)!}\left(\frac{r}{2}\right)^{2i+1}$$

where r is the radius of a point at which A_θ is required, and $B_a^{(2i)}$ is the $2i$-th axial derivative of the axial magnetic field B_a at the corresponding point on the axis. The axial derivatives of B_a may be determined directly from a series of physical measurements of B_a along the axis of a system, by evaluating a series involving numerical differences of these.[12] Alternatively, since the axial field of magnetic focusing systems usually varies smoothly, it is usually possible to define B_a by an analytic and simple differentiable function of axial position which gives close agreement with the physical measurements. This method is preferable because it effectively involves smoothing of the measurements and it simplifies the calculation of the derivatives. Computer programs using each method have been developed for the calculation of values of A_θ at the mesh points of any specified rectangular mesh.

4. The Determination of Electron Trajectories

4.1. *The Equations of Electron Motion*

The equations of electron motion in an axi-symmetric electric and magnetic field may be expressed in the following form[5]:

$$d^2r/dt^2 = (e/m)\partial V/\partial r - (e/m)^2(A_\theta + C/r)(\partial A_\theta/\partial r - C/r^2) \quad \ldots\ldots(1)$$

$$d^2z/dt^2 = (e/m)\partial V/\partial z - (e/m)^2(A_\theta + C/r)\partial A_\theta/\partial z \quad \ldots\ldots(2)$$

$$d\theta/dt = (e/m)(A_\theta/r + C/r^2) \quad \ldots\ldots(3)$$

where r, z and θ denote respectively the radial, axial and angular co-ordinates of the electron, t denotes time, V and A_θ denote the electrostatic and magnetic vector potentials, (e/m) is the charge/mass ratio of the electron, and C is a constant of integration which can be found from the initial conditions for a trajectory, using eqn. (3):

$$(e/m)C = r^2\, d\theta/dt - (e/m)rA_\theta \quad \ldots\ldots(4)$$

This constant is zero for all electron trajectories in which electrons initially start from the symmetry axis ($r = 0$) and for all electron trajectories through purely electric fields for which the electrons have no initial angular velocity and therefore remain in an axial plane.

It is possible to reduce these equations to two by eliminating the time variable,[5] but unless the approximations can be made which reduce these to the well-known 'paraxial ray equations' it is simpler to integrate eqns. (1), (2) and (3) directly, treating time as the independent variable. The only difficulties which are encountered with this method arise in the determination of skew trajectories which pass close to the axis of a system. Under these circumstances the product $(A_\theta + C/r)C/r^2$ in the right-hand side of eqn. (1) may become extremely large and have very large derivatives with respect to time, causing serious integration errors if a large integration interval is used. The term represents an outward radial acceleration which is large in magnitude for a very short time, and the effect of not taking it into account adequately in calculating skew trajectories is to make these appear to pass too near or even through the axis of a system. This acceleration has no physical significance and is purely a consequence of using a cylindrical co-ordinate system.

The possibility of changing the variables of the integration procedure to eliminate these difficulties was examined, but it would appear that any other set of variables presents equally difficult problems and that only a three-dimensional treatment would be completely satisfactory. However, it was found that by using significantly reduced integration intervals where skew trajectories pass near to a system axis, trajectory calculations could be performed satisfactorily for all trajectories likely to be of practical interest, and the procedure for automatic integration interval control described below provides an adequate safeguard against poor integration of such trajectories.

4.2. *The Integration Method*

Important considerations in choosing a method of integrating equations (1), (2) and (3) on a digital computer were the truncation errors of the integration formulæ, the ease with which integration errors could be assessed, and also controlled (i.e. the ease with which the integration interval could be altered during the course of computation), the stability of the integration formulæ, and computer storage requirements. In view of these considerations, methods such as those of Motz and Klanfer,[3] Liebmann[5] and Goddard[13] were all thought to be much better suited to manual rather than automatic computation, although Liebmann's method has been programmed by Vine.[7] Other methods considered were the Runge-Kutta method,[14] the Fox and Goodwin method,[15] and a rather similar method used by Jennings and Pratt,[6] but it would be difficult to achieve adequate automatic error control with any of these. The methods considered to be most suitable were predictor-corrector methods,[16] whose only disadvantage is that they require a special starting procedure, as opposed to Runge-Kutta methods for example, which are self-starting, but this is not a very serious consideration.

The main problem in choosing particular predictor and corrector formulæ is to decide on the most suitable order of these, i.e. to decide how many terms the formulæ should include. Once this has been decided, the best possible values of the coefficients can be determined from the conditions for maximum accuracy and stability.[16] Important considerations in choosing the order of the formulæ are that as this increases the truncation errors decrease, but the amount of computation required at each step increases, the number of integration steps which must be performed by the starting procedure increases (see below), and changing the integration interval during a calculation becomes more difficult (see below). The question is further complicated by the fact that since truncation errors are also a function of the size of integration interval, the use of high order formulæ may permit the use of quite large integration intervals and thus reduce the work involved in calculating a complete trajectory, in spite of the increased work per step.

This last consideration is particularly important in the calculation of trajectories through magnetic lenses and of paraxial trajectories generally, in which the principal sources of error in the integrations are certainly truncation errors, and this is the reason for the general tendency in works published on the subject to advocate the use of quite high order formulæ. However, in calculating non-paraxial trajectories through electric fields, the maximum permissible size of the integration interval is often in practice not a function of the truncation errors of the predictor-corrector formulæ, but of the errors caused by imperfect detection and allowance for rapid variations in the forces acting on the electrons. These may be due either to large spatial variations of the electric fields or to rapid variations of the type mentioned in the previous section. This point can be simply demonstrated by calculating a few meridional trajectories through a simple two-cylinder lens, for which the accuracy of calculations is almost entirely a function of the integration interval used at the centre of the lens, where the radial electric field experienced by an electron has extremely large derivatives with respect to time. Some trajectories were calculated for the two-cylinder lens studied by Motz and Klanfer,[3] using low-order and high-order formulæ, and the results using the different formulæ all agreed extremely closely even for such large integration intervals that all the results were inaccurate.

The general conclusion is that for accurate and efficient trajectory calculations the order of integration formulæ should be as high, but only as high, as is required to prevent truncation errors from ever becoming significant for such interval sizes as field variations will be likely to permit. After a number of experiments, the following predictor and corrector formulæ were chosen for the integration of eqns. (1) and (2):

$$y_{n+1,1} = y_n + y_{n-2} - y_{n-3} + h^2(5f_n + 2f_{n-1} + 5f_{n-2})/4 \qquad \ldots\ldots(5)$$

$$y_{n+1,2} = 2y_n - y_{n-1} + h^2(f_{n+1,1} + 10f_n + f_{n-1})/12 \qquad \ldots\ldots(6)$$

where y_n and f_n denote the value of the dependent variable and its second derivative at the nth integration step, and h is the integration interval. This predictor-corrector method is due to Milne.[17] The predictor formula has been used alone by Goddard,[13] and the combined use of the two formulæ has been discussed by Jennings and Pratt.[6] The method used for the integration of eqn. (3) is a simplified version of a predictor-modifier-corrector method discussed by Ralston,[16] the predictor and corrector formulæ being the following:

$$y_{n+1,1} = y_{n-3} + 4h(2g_n - g_{n-1} + 2g_{n-2})/3 \qquad \ldots\ldots(7)$$

$$y_{n+1,2} = [9y_n - y_{n-2} + 3h(g_{n+1,1} + 2g_n - g_{n-1})]/8 \qquad \ldots\ldots(8)$$

where in this case g_n denotes the first derivative of the dependent variable.

The calculation of values of f_n and g_n, which are obtained by evaluating the right-hand sides of eqns. (1), (2) and (3), involves the calculation of the radial and axial derivatives of the electric and magnetic vector potentials at a number of points along the trajectory which in general are not coincident with mesh points at which the potentials have been evaluated. To perform the necessary two-dimensional interpolations and differentiations as accurately as possible, the interpolation and differentiation formulæ must make use of the potential values at a large number of mesh points, but if very high order differences are used in such formulæ the errors due to imperfect solution of the finite difference analogues are magnified in the calculated derivatives. In order to determine the best possible order of interpolation and differentiation formulæ, numerical differences of successively high orders were calculated in the radial and axial directions for some typical calculated electric and magnetic fields. With electric fields it was generally found that third-order differences varied smoothly but that fourth-order differences were small but irregular. With the magnetic fields considered, which were obtained from analytical expressions for the field along the axis, fourth-order differences were negligibly small. On these grounds, a four-by-four point Lagrangian formula[12] was considered to be the most suitable for general use. In the particular case where a mesh is locally square the corresponding two-dimensional Everett formula[12] is used because it involves less computation. Values of the magnetic vector potential, which are also required, can be calculated sufficiently accurately by using linear two-dimensional interpolation.

4.3. *The Integration Starting Procedure*

It can be seen from eqns. (5) to (8) that in order to obtain the co-ordinates of an electron at the $(n+1)$th time step it is necessary to have values of its co-ordinates and of f_n and g_n at four previous equally spaced instants of time. Thus the results for the first three steps of an integration must be obtained by a method other than the predictor-corrector methods, and the Runge-Kutta method is used for this purpose. To start the calculation of a trajectory, the computer is supplied with the initial co-ordinates of an electron (r, z, θ) and its initial velocities $(\mathrm{d}r/\mathrm{d}t, \mathrm{d}z/\mathrm{d}t, \mathrm{d}\theta/\mathrm{d}t)$. The constant C is then evaluated using eqn. (4), and then the initial data on the electron are used in solving the following five simultaneous equations by the Runge-Kutta method:

$$\mathrm{d}s/\mathrm{d}t = (e/m)\,\partial V/\partial r - (e/m)^2(A_\theta + C/r)(\partial A_\theta/\partial r - C/r^2) \qquad \ldots\ldots(9)$$

$$\mathrm{d}r/\mathrm{d}t = s \qquad \ldots\ldots(10)$$

$$\mathrm{d}y/\mathrm{d}t = (e/m)\,\partial V/\partial z - (e/m)^2(A_\theta + C/r)\,\partial A_\theta/\partial z \qquad \ldots\ldots(11)$$

$$\mathrm{d}z/\mathrm{d}t = y \qquad \ldots\ldots(12)$$

$$\mathrm{d}\theta/\mathrm{d}t = (e/m)(A_\theta/r + C/r^2) \qquad \ldots\ldots(13)$$

The evaluated right-hand sides of eqns. (9), (11) and (13), and the solutions of eqns. (10), (12) and (13) provide the necessary data for substitution into the predictor-corrector formulæ.

This integration is first performed for three steps using an integration interval supplied with the initial data. The interval is then halved and the integration repeated from the beginning for six steps. If the final results of these two integrations are in close agreement, the original interval is used in subsequent integration by the predictor-corrector methods. Otherwise the interval is halved and the Runge-Kutta integration is repeated until, by comparison of results, a suitable interval is found. The interval which is first used with the predictor-corrector formulæ is not necessarily used throughout the integration, but is subject to an automatic interval changing procedure, which is described in the next section.

4.4. *Automatic Interval Changing*

Since in general the electric and magnetic fields vary more rapidly at some parts of a trajectory than others, the integration interval required to produce a given precision also varies, and therefore it is desirable to be able to assess the errors and modify the integration interval during the course of an integration. A major advantage of predictor-corrector methods is that it is possible to check the precision of calculations at each step by comparing the predicted and corrected results. If the integration errors are primarily truncation errors, then the assessment of the errors in each integration step is quite simple. The truncation errors in the predictor and corrector formulæ of eqns. (5) and (6) are respectively:

$$e_p \simeq -17h^6 y_p^{(6)}/240$$

$$e_c \simeq +h^6 y_c^{(6)}/240$$

where $y^{(6)}$ is the sixth derivative of the dependent variable for some value of time in the interval of integration, and so the error in the corrected results is approximately given by the relation:

$$e_c \simeq (e_c - e_p)/18,$$

i.e. the error in a corrected value is approximately one-eighteenth of the difference between the predicted and corrected values. If the integration errors are largely due to poor detection of rapid variations in the forces acting on electrons, then the assessment of the errors is much more difficult, but it has been found experimentally that the difference between predicted and corrected results always gives an indication of the order, at least, of integration errors.

In performing the integrations the arithmetical differences between the predicted and corrected results for the radial and axial co-ordinates are calculated at each step, and if either of these differences exceed a certain prescribed tolerance all the new values are rejected, the integration interval is halved, and the integration is resumed from the results at the previous step. If for both these variables certain multiples of the differences are smaller than the tolerance for a number of consecutive steps, the integration interval is doubled. If neither of these conditions is satisfied, the interval is not changed. Because halving the interval doubles the number of integration steps required to cover any portion of a trajectory, and vice versa, the tolerance is halved or doubled with the interval, in order to keep the total error over a trajectory within satisfactory bounds. Since halving the interval reduces the errors by a factor which is much larger than 2, this process is quite stable.

The process of doubling the interval merely involves a slightly complex selection of previously calculated results. Halving the interval involves calculating two sets of intermediate results before resuming the integration. The intermediate values of the variables are obtained from the following interpolation formulæ:

$$y_{n-\frac{1}{2}} = (+3y_{n-3} - 20y_{n-2} + 90y_{n-1} + 60y_n - 5y_{n+1})/128$$

$$y_{n-1\frac{1}{2}} = (-5y_{n-3} + 60y_{n-2} + 90y_{n-1} - 20y_n + 3y_{n+1})/128$$

and the intermediate values of f_n and g_n are obtained from the formula:

$$f_{n-\frac{1}{2}} = (-f_{n-2} + 9f_{n-1} + 9f_n - f_{n+1})/16$$

The use of the rejected values y_{n+1} and the corresponding f_{n+1} is justified by the relative smallness of their coefficients, making the effect of their errors on the interpolated results small, while their presence improves the interpolation.

This automatic interval changing procedure has proved to be reasonably satisfactory. The interval doubling procedure is very successful, and the interval halving satisfactorily limits the errors over an integration, but unless very small tolerances are used, errors in the interpolated intermediate results can precipitate several unnecessary reductions of the interval. This tends to reduce the efficiency of the procedure, and improved methods of interval halving are being studied.

5. Applications and Accuracy

The ideal test for the programs described would be on fields for which analytical solutions of the trajectories are known. Unfortunately these solutions are either so simple or so artificial that they cannot give an adequate test of the behaviour of the programs when tracing through fields of practical interest. The programs have therefore been judged rather by the internal consistencies of results produced on typical electrode configurations. Two of these, which are characteristic of distinct classes of imaging device, have been intensively studied; the results indicate that the trajectories can be relied upon to be correct to within a few microns.

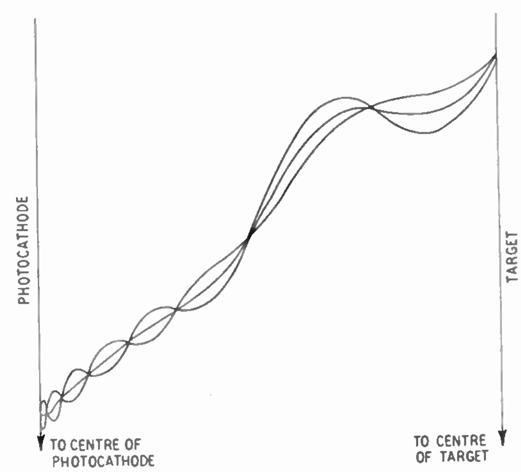

Fig. 3. Off-axis trajectories in image section of image orthicon.

A brief description of these tubes will suffice to distinguish the principles of their operation. The first, shown diagrammatically in section in Fig. 1, is the image section of an image orthicon. In this tube electrons are accelerated from the photo-cathode to the target through a potential of about 460 V. The potential on the intermediate electrode can be chosen to minimize the distorting effects of the electric field. Clearly the resultant field is by no means uniform, nevertheless there is negligible electrostatic focusing, and electrons emitted from a point on the cathode with transverse velocities would spread into a large circle of confusion before arriving at the target. They are prevented from doing so by a longitudinal magnetic field which confines the electron trajectories to helical loops. Typically four focus loops are developed, as illustrated for an off-axis image point in Fig. 3, where the transverse scale is magnified for the sake of clarity. The magnetic field is divergent, being nearly twice as strong at the cathode as at the target. Since the trajectories tend to follow the magnetic lines of force there is a magnification of approximately 4 : 3.

The second tube, an image converter, which relies entirely on an electrostatic field for focusing, is illustrated in Fig. 2. The single accelerating electrode, at target potential of 16 500 V, comprises a cone having an aperture at its smaller end facing the curved photo-cathode. Electrons diverging from a point on the photo-cathode encounter convex equipotentials. These produce a convergent effect on the trajectories so that they pass through the cone aperture in directions intersecting on the target.

Both these tubes fall into the class of cathode lenses, the aberrations of which are discussed by Zworykin *et al.*,[11] who show that the transverse displacement of an electron in the image plane depends on the circumstances of its emission according to the expression:

$$\Delta r = A(\Delta\phi_z \Delta\phi_r)^{\frac{1}{2}}/\phi_0'$$

where $\Delta\phi_z$ and $\Delta\phi_r$ are electron energies corresponding to the components of the emission velocity in the axial and radial directions, ϕ_0 is the extracting field at the photo-cathode and A is a constant characteristic of the imaging system, generally close to $-2M$ (M is the magnification of the system).

This represents the chromatic aberration in the plane of focus of electrons of zero initial axial velocity. For other planes, Beurle and Wreathall[18] use the corresponding expression:

$$\Delta r = Mu_t(u_z - u_1)/(e/m)E_{PC} \qquad \ldots\ldots(14)$$

where u_t and u_z are the transverse and axial velocities of emission, E_{PC} is the extracting field at the photo-cathode, and u_1 is a constant governed by the choice of focal plane in which the aberrations are measured.

It can be argued that the above expression is generally valid,[18] provided that the electron is accelerated rapidly to a velocity very much greater than its velocity of emission, and is not subsequently decelerated to a low velocity. Since these conditions hold for the tubes under consideration, the goodness of fit of the calculated aberrations to eqn. (14) gives an indication of the magnitude of the errors of calculation. A graphical plot of $\Delta r/u_t$ against u_z is most convenient, since inspection of eqn. (14) shows that these quantities should be linearly related with a slope equal to $M/(e/m)E_{PC}$.

Two such plots are shown in Fig. 4 for the axial aberrations of two different configurations for the image section of the image orthicon. These have quite different electrode geometries and potential ratios, but happen to have similar extracting fields. The full lines drawn close to the plotted points are drawn with slopes calculated from the magnifications and extracting fields. The negative slope of the graphs arises from the fact that the trajectories cross the axis at a focus so that the transverse aberration changes sign in successive loops, and in this case the aberration is being measured at the fourth focus node. Estimates of error are complicated by the fact that on this plot the displacement due to a fixed error in Δr is inversely proportional to the initial transverse velocity. This

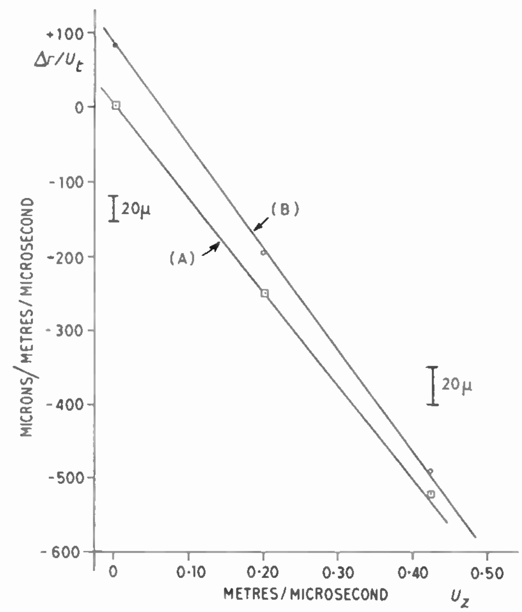

Fig. 4. Axial aberration in image section of image orthicon.

is indicated by the vertical lines showing the spread due to errors of 20 microns at the end points. The set of points for tube A were obtained using a fixed time-step of 10^{-10} seconds throughout the ray-tracings. These involved a total of 144 integration steps for each trajectory. For tube B the time-step was automatically controlled and the number of steps was reduced to only 42. There do not appear to be any errors greater than a few microns for either tube.

A larger set of trajectories have been calculated for the image converter tube. The results are shown in Fig. 5, where the vertical lines represent the effect of errors of one micron on the plotted points. Here again it is clear that a random error not exceeding 2 microns will be sufficient to account for the departures from a straight line locus. Alternatively, there is some indication of a more systematic departure in

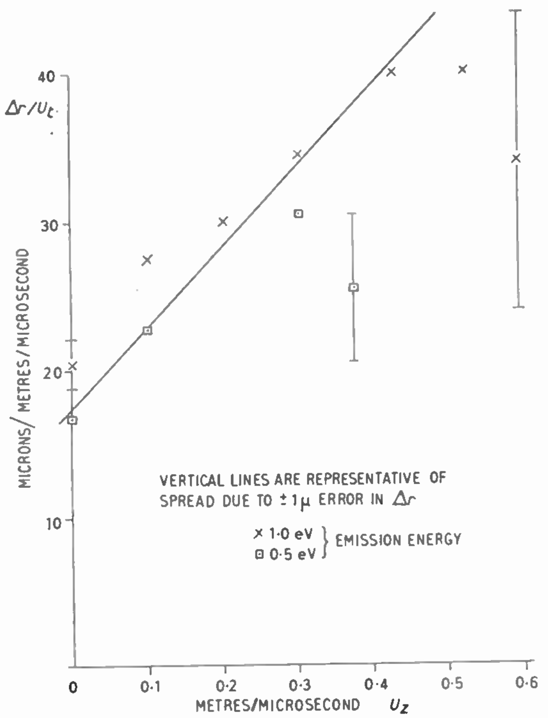

Fig. 5. Axial aberration in infra-red image converter.

Arbitrary locus drawn of slope $\dfrac{M}{(e/m)E_{PC}} = 54{\cdot}5$

the results, in that the lower the radial velocity, the further do the points tend to fall below a linear locus. This could readily be accounted for by an accumulated error of rather less than 3 microns common to all the computed trajectories.

Since velocities are not calculated after the first few steps of the trajectories, an energy check at the end of the calculations can be used to detect errors. Such a check on 20 trajectories plotted through the image orthicons showed all velocities to be correct to just under one part in 10 000, while the root mean square error was less than half of this amount. For the image converter, the largest velocity error in 31 trajectories amounted to less than 5 parts in 10 000, which was again over twice the root-mean-square error.

Having thus established that trajectories can be traced which are reliable to a few microns through fields of practical complexity, it is possible to use the programs for the analysis and correction of electron-optical devices. For example, in a standard design of the image section of the image orthicon similar to that illustrated in Fig. 1, ray-tracing revealed barrel distortion amounting to 0·9% and 'S' distortion amounting to 1·5%. These errors were attributable to the electric field, and, as reported elsewhere,[19] attention to the distorting components of this field led to a reduction of these geometrical errors to only 0·1%, as checked by ray-tracing.

An extremely large amount of aberration was apparent when the infra-red image converter was explored off-axis. The results of these calculations are illustrated in Fig. 6, where the focal surfaces are superimposed on a half-section of the tube. Three principal trajectories are shown from different heights on the photo-cathode, and on these are plotted tangential and sagittal foci. The former were determined by the intersections of trajectories lying in the meridian plane, having initial velocities perpendicular to a principal trajectory corresponding to an energy of one electron volt, and the latter by the points where electrons emitted in a direction perpendicular to the meridian plane, recross that plane. That the tube gives acceptable images, in spite of the gross amounts of field curvature and astigmatism from which it suffers, is attributable to the low emission energies of electrons ejected by infra-red radiation and to the high final voltage which confines the out-of-focus electron trajectories to narrow cones. The corner

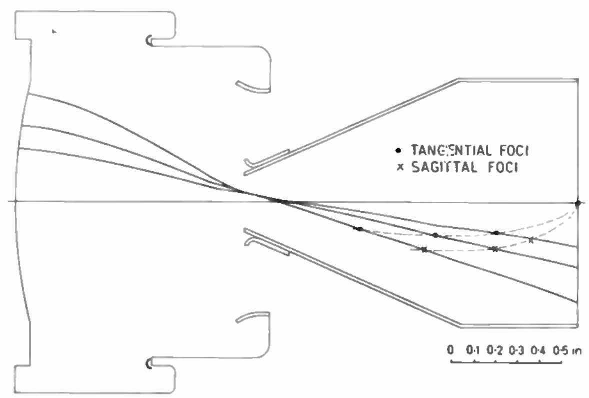

Fig. 6. Infra-red image converter: principal rays and astigmatism.

resolution of about five lines per millimetre obtained in practical tubes can be explained on the assumption that the most probable energy of emission is only about $\frac{1}{16}$eV. Photo-cathodes with sensitivity to shorter wavelengths give rise to electrons having higher initial energies, so that it is clear that improved designs must be developed if tubes of this simple type are to be exploited as light amplifiers for use in the visible part of the spectrum.

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A Novel Ferrite Quarter-wave Plate

By

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Summary: Coupled-wave theory is applied to make an adjustment to the length of an elliptical Faraday rotator to obtain a ferrite quarter-wave plate with non-reciprocal properties. A ferrite rotator can be interposed between two such quarter-wave plates and matrix algebra used to obtain a new reciprocal ferrite phase shifter. A pulsed 180 deg non-reciprocal phase shifter is also described.

1. Introduction

It is the purpose of this paper to apply coupled wave theory to an elliptical Faraday rotator. With appropriate boundary conditions on the ratio of the difference in phase velocities between the coupled waves and the transfer effect due to the magnetized ferrite rod, we can adjust the length of the rotator to obtain circular polarization. Such a rotator consists of an elliptical waveguide in which is placed a ferrite rod magnetized in the direction of propagation as shown in Fig. 1. The difference in phase velocities between the coupled waves is determined by the eccentricity of the waveguide. Distributed coupling between the two coupled waves exists as a result of the tensor permeability of the magnetized ferrite medium. When the d.c. field is in the same direction as that of the r.f. propagation we have one hand of circular polarization. When the d.c. magnetic field is reversed we obtain the other hand of circular polarization. Thus we have a non-reciprocal ferrite element. The transfer characteristic of the ferrite quarter-wave plate is put in compact form with the help of matrix algebra. The matrix algebra of a simple Faraday rotator is also developed. This allows us to describe a new reciprocal ferrite phase shifter which consists of two ferrite quarter-wave plates between which is interposed a conventional Faraday rotator. A non-reciprocal 180 deg phase shifter and an amplitude modulator are also described.

2. Coupled Waveguide Theory Applied to Ferrite Medium

Coupled waveguide theory as applied to a longitudinally biased ferrite medium has previously been discussed.[1, 2] The form of the transfer effect of the off-diagonal component of the tensor permeability has been commented upon by Fox *et al.*,[1] but is not mandatory for what follows. Lack of the exact form of the transfer effect has not inhibited the development of practical Faraday rotators.

Upon rewriting Miller's[3] equation for coupled waveguides with unequal phase velocities for a system of magnetic waves coupled through the off-diagonal component of the tensor permeability as shown in Fig. 1, we have:

$$\frac{\mathrm{d}E_y^{\pm}}{\mathrm{d}z} = -\mathrm{j}\beta_y E_y^{\pm} \pm K_{xy} E_x^{\pm} \qquad \text{......(1)}$$

$$\frac{\mathrm{d}E_x^{\pm}}{\mathrm{d}z} = \mp K_{yx} E_y^{\pm} - \mathrm{j}\beta_x E_x^{\pm} \qquad \text{......(2)}$$

in which β_y, β_x are the perturbed phase velocities in the y, x directions;

E_y^+, E_x^+ are the complex electric wave amplitudes in the y, x direction when applied d.c. magnetic field is in the same direction as that of r.f. propagation;

E_y^-, E_x^- are the complex electric wave amplitudes when the applied d.c. field is in the direction opposite to that of r.f. propagation;

$+K_{xy}$, $-K_{yx}$ are real quantities and represent the transfer effect of the off-diagonal component of the tensor permeability for the applied d.c. magnetic field in the same direction as that of r.f. propagation; and

$-K_{xy}$, $+K_{yx}$ are real quantities and represent the transfer effect of the off-diagonal component of the tensor permeability for the applied field in the direction opposite to that of r.f. propagation.

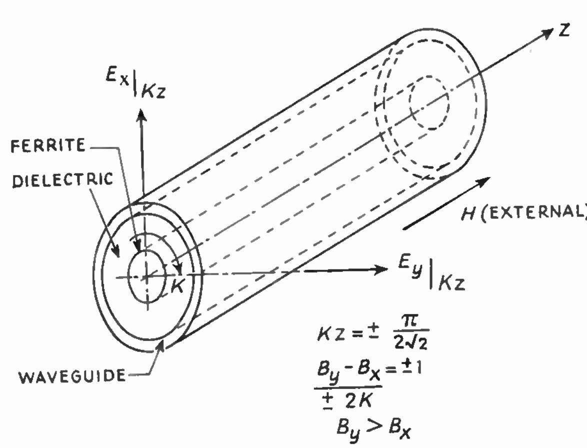

Fig. 1. Elliptical Faraday rotator.

† Formerly Raytheon Company, Waltham, Mass., U.S.A., now with Sylvania Electric Products Inc.

If we let $|K_{xy}| = |K_{yx}| = |K|$, eqns. (1) and (2) reduce to:

$$E^{\pm}_{y|Kz} =$$

$$\left\{\cos\left(\left[\frac{(\beta_y-\beta_x)^2}{4K^2}+1\right]^{\frac{1}{2}}(\pm Kz)\right)\right\}\exp -\mathrm{j}\frac{(\beta_y+\beta_x)}{2}z - \left\{\frac{-\mathrm{j}\dfrac{(\beta_y-\beta_x)}{\pm 2K}}{\left[\dfrac{(\beta_y-\beta_x)^2}{4K^2}+1\right]^{\frac{1}{2}}}\,.\,\sin\left(\left[\frac{(\beta_y-\beta_x)^2}{4K^2}+1\right]^{\frac{1}{2}}(\pm Kz)\right)\right\}\times$$

$$\times \exp -\mathrm{j}\frac{(\beta_y+\beta_x)}{2}z \quad \ldots\ldots(3)$$

$$E^{\pm}_{x|Kz} = \left\{\frac{-1}{\left[\dfrac{(\beta_y-\beta_x)^2}{4K^2}+1\right]^{\frac{1}{2}}}\,.\,\sin\left(\left[\frac{(\beta_y-\beta_x)^2}{4K^2}+1\right]^{\frac{1}{2}}(\pm Kz)\right)\right\}\exp -\mathrm{j}\frac{(\beta_y+\beta_x)}{2}z \quad \ldots\ldots(4)$$

Equations (3) and (4) satisfy the boundary conditions

$$E^{\pm}_{y|0} = 1, \qquad E^{\pm}_{x|0} = 0, \qquad \beta_y > \beta_x,$$

$$\left.\frac{\mathrm{d}E^{\pm}_y}{\mathrm{d}z}\right|_0 = -\mathrm{j}\beta_y, \qquad \left.\frac{\mathrm{d}E^{\pm}_x}{\mathrm{d}z}\right|_0 = \mp K$$

3. Simple Faraday Rotation

Upon writing $\dfrac{(\beta_y-\beta_x)}{\pm 2K} = 0$ in eqns. (3) and (4), we have for $\beta_y = \beta_x = \beta$,

$$E^{\pm}_{y|Kz} = \cos(\pm Kz)\,.\,\exp -\mathrm{j}\beta z = \cos Kz\,.\,\exp -\mathrm{j}\beta z \quad \ldots\ldots(5)$$

$$E^{\pm}_{x|Kz} = -\sin(\pm Kz)\,.\,\exp -\mathrm{j}\beta z = \mp\sin Kz\,.\,\exp -\mathrm{j}\beta z \quad \ldots\ldots(6)$$

Combining $E^{\pm}_{y|Kz}$ and $E^{\pm}_{x|Kz}$, we obtain a linearly polarized wave rotating with quantity Kz. Reversing the direction of r.f. propagation or the direction of the d.c. magnetic field reverses the rotation. This represents simple Faraday rotation in a ferrite medium.

When the boundary conditions are changed from $E^{\pm}_{y|0} = 1$, $E^{\pm}_{x|0} = 0$, $\beta_y > \beta_x$ to $E^{\pm}_{y|0} = 0$, $E^{\pm}_{x|0} = 1$, $\beta_y > \beta_x$, eqns. (5) and (6) become:

$$E^{\pm}_{y|Kz} = \pm\sin Kz\,.\,\exp -\mathrm{j}\beta z \quad \ldots\ldots(7)$$

$$E^{\pm}_{x|Kz} = \cos Kz\,.\,\exp -\mathrm{j}\beta z \quad \ldots\ldots(8)$$

Equations (5) to (8) imply the general matrix equation.

$$\exp -\mathrm{j}\beta z\begin{bmatrix}\cos Kz\ , & \pm\sin Kz\\ \mp\sin Kz\ , & \cos Kz\end{bmatrix}\,.\,\begin{bmatrix}E^{\pm}_{y|0}\\ E^{\pm}_{x|0}\end{bmatrix} = \begin{bmatrix}E^{\pm}_{y|Kz}\\ E^{\pm}_{x|Kz}\end{bmatrix} \quad \ldots\ldots(9)$$

4. Ferrite Quarter-wave Plate

Equations (3) and (4) satisfy the condition

$$|E^{\pm}_{x|Kz}| = |E^{\pm}_{y|Kz}|$$

for the two boundary conditions,

$$\frac{(\beta_y-\beta_x)}{\pm 2K} = \pm 1 \quad \ldots\ldots(10)$$

and

$$\left[\frac{(\beta_y-\beta_x)^2}{4K^2}+1\right]^{\frac{1}{2}}(\pm Kz) = \frac{\pm\pi}{2} \quad \ldots\ldots(11)$$

Introducing eqn. (10) into eqn. (11) gives

$$Kz = \frac{\pi}{2\sqrt{2}} \quad \ldots\ldots(12)$$

Upon satisfying the above, eqns. (3) and (4) reduce to

$$E^{\pm}_{y|\pi/2(\sqrt{2})} = \frac{-\mathrm{j}}{\sqrt{2}}\,.\,\exp -\mathrm{j}\frac{(\beta_y+\beta_x)z}{2} \quad \ldots\ldots(13)$$

$$E^{\pm}_{x|\pi/(2\sqrt{2})} = \frac{\mp 1}{\sqrt{2}}\,.\,\exp -\mathrm{j}\frac{(\beta_y+\beta_x)z}{2} \quad \ldots\ldots(14)$$

If we take $E^{+}_{x|\pi/(2\sqrt{2})}$ and $E^{+}_{y|\pi/(2\sqrt{2})}$, we obtain one hand of circular polarization. Similarly, $E^{-}_{x|\pi/(2\sqrt{2})}$ and $E^{-}_{y|\pi/(2\sqrt{2})}$ yield the other hand of circular polarization, so that we have a non-reciprocal device.

When the initial boundary conditions are changed from $E^{\pm}_{y|0} = 1$, $E^{\pm}_{x|0} = 0$, $\beta_y > \beta_x$ to $E^{\pm}_{y|0} = 0$, $E^{\pm}_{x|0} = 1$, $\beta_y > \beta_x$, eqns. (13) and (14) become

$$E^{\pm}_{y|\pi/(2\sqrt{2})} = \frac{\pm 1}{\sqrt{2}}\,.\,\exp -\mathrm{j}\frac{(\beta_y+\beta_x)z}{2} \quad \ldots\ldots(15)$$

$$E^{\pm}_{x|\pi/(2\sqrt{2})} = \frac{+\mathrm{j}}{\sqrt{2}}\,.\,\exp -\mathrm{j}\frac{(\beta_y+\beta_x)z}{2} \quad \ldots\ldots(16)$$

This yields another set of circularly polarized waves, differing in phase by 180 deg from the first set.

Equations (13) to (16) imply the general matrix equation

$$\exp -\frac{(\beta_y+\beta_x)z}{2}\cdot\begin{bmatrix}\frac{-j}{\sqrt{2}}, & \frac{\pm 1}{\sqrt{2}}\\ \frac{\mp 1}{\sqrt{2}}, & \frac{+j}{\sqrt{2}}\end{bmatrix}\cdot\begin{bmatrix}E^{\pm}_{y|0}\\ E^{\pm}_{x|0}\end{bmatrix}=\begin{bmatrix}E^{\pm}_{y|\pi/(2\sqrt{2})}\\ E^{\pm}_{x|\pi/(2\sqrt{2})}\end{bmatrix} \quad \ldots\ldots(17)$$

If the major axis instead of the minor one of the elliptical waveguide is made to coincide with the y-axis, eqn. (17) becomes

$$\exp -\frac{(\beta_y+\beta_x)z}{2}\cdot\begin{bmatrix}\frac{+j}{\sqrt{2}}, & \frac{\pm 1}{\sqrt{2}}\\ \frac{\mp 1}{\sqrt{2}}, & \frac{-j}{\sqrt{2}}\end{bmatrix}\cdot\begin{bmatrix}E^{\pm}_{y|0}\\ E^{\pm}_{x|0}\end{bmatrix}=\begin{bmatrix}E^{\pm}_{y|\pi/(2\sqrt{2})}\\ E^{\pm}_{x|\pi/(2\sqrt{2})}\end{bmatrix} \quad \ldots\ldots(18)$$

This last equation will be needed later on when we butt two such quarter-wave plates with their major axis crossed.

5. Switched Reciprocal Phase Shifter

By interposing a length of ferrite for which $\beta_y = \beta_x = \beta$ between two identical ferrite quarter-wave plates, a reciprocal ferrite phase shifter is obtained. This can be shown by evaluating the following matrix product:

$$\exp -j\{(\beta_y+\beta_x)z+(\beta)z_1\}\cdot\begin{bmatrix}\frac{-j}{\sqrt{2}}, & \frac{\pm 1}{\sqrt{2}}\\ \frac{\mp 1}{\sqrt{2}}, & \frac{+j}{\sqrt{2}}\end{bmatrix}\cdot\begin{bmatrix}+\cos Kz, & \pm\sin Kz\\ \mp\sin Kz, & +\cos Kz\end{bmatrix}\cdot\begin{bmatrix}\frac{-j}{\sqrt{2}}, & \frac{\pm 1}{\sqrt{2}}\\ \frac{\mp 1}{\sqrt{2}}, & \frac{+j}{\sqrt{2}}\end{bmatrix}\cdot\begin{bmatrix}E^{\pm}_{y|0}\\ E^{\pm}_{x|0}\end{bmatrix}=\begin{bmatrix}E^{\pm}_{y}\\ E^{\pm}_{x}\end{bmatrix}$$

This reduces to

$$E^{\pm}_y = 1\,.\exp -j\{(\beta_y+\beta_x)z+(\beta-K)z_1+\pi\}$$

and $E^{\pm}_x = 0$, for $E^{\pm}_{y|0} = 1$, $E^{\pm}_{x|0} = 0$.

Any amount of reciprocal phase shift can be obtained by adjusting the length z_1 of the centre section.

For $E^{\pm}_{y|0} = 0$, $E^{\pm}_{x|0} = 1$, we have $E^{\pm}_y = 0$ and

$$E^{\pm}_x = 1\,.\exp -j\{(\beta_y+\beta_x)z+(\beta+K)z_1+\pi\}$$

By independently modulating the rotator section of the phase shifters, matrix multiplication indicates that it is possible to modulate the phase shifter reciprocally through $\exp -(j2Kz)$. In this instance, the modulating envelope need not necessarily be pulsed.

6. Switched Non-reciprocal 180-degree Phase Shifter

Butting two quarter-wave plates in such a manner that their major axes are crossed produces a 90 deg rotator in the magnetized state. This can be shown by expanding the matrix product formed by eqns. (17) and (18) with $E^{\pm}_{y|0} = 1$, and $E^{\pm}_{x|0} = 0$. Upon doing this,

$$E^{\pm}_y = 0,$$

and

$$E^{\pm}_x = 1\,.\exp -j\left\{(\beta_y+\beta_x)z\frac{\pm\pi}{2}\right\}$$

For this arrangement, E_x is shifted through 180 deg when the applied field is reversed. This feature can be used to advantage in a pulsed 180 deg phase shifter since this reversal in phase is independent of path length and ought, therefore, to be broadband.

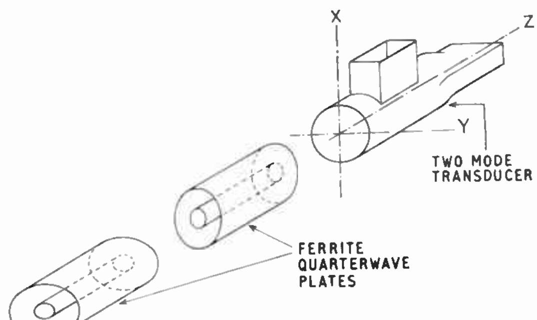

Fig. 2. Two-mode transducer and 90 deg rotator.

The two linearly polarized modes in the elliptical waveguide section can be coupled to separate rectangular waveguides with the help of a two-mode transducer.[4] Such a two-mode transducer is shown in Fig. 2. Energy polarized along the x-direction in

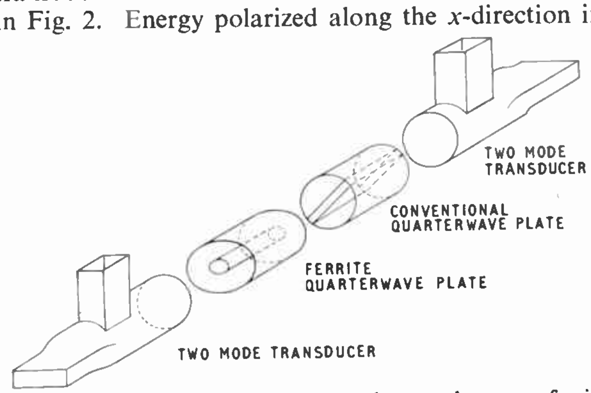

Fig. 3. Circulator with two two-mode transducers, a ferrite quarter-wave plate and reciprocal quarter-wave plate.

the elliptical waveguide is transmitted to the main rectangular waveguide through a suitable transition. Energy polarized along the y-direction would ordinarily divide in the round section between both rectangular waveguides, however, the main waveguide is cut-off for the y-polarized wave; by adjusting the plane of this waveguide with reference to the side one the energy can be made emergent at the side waveguide.

When used in conjunction with a two-mode transducer, the 90 deg rotator described above becomes an electrical variable attenuator. This arrangement is shown in Fig. 2. By replacing one of the ferrite quarter-wave plates by a reciprocal one and by using two two-mode transducers we have a circulator. This is shown in Fig. 3.

7. Practical Considerations

If we write K in terms of z in eqn. (12) and let $z = 2\lambda_{gy}$ for example, we have $K = \pi/4\sqrt{2}\lambda_{gy}$ if we now substitute for K in terms of λ_{gy} in eqn. (10) we obtain $\beta_y/\beta_x = 1{\cdot}18$ so that a phase difference of 18% is required between β_x and β_y, where λ_{gy} is the guide wavelength associated with polarization in the y direction.

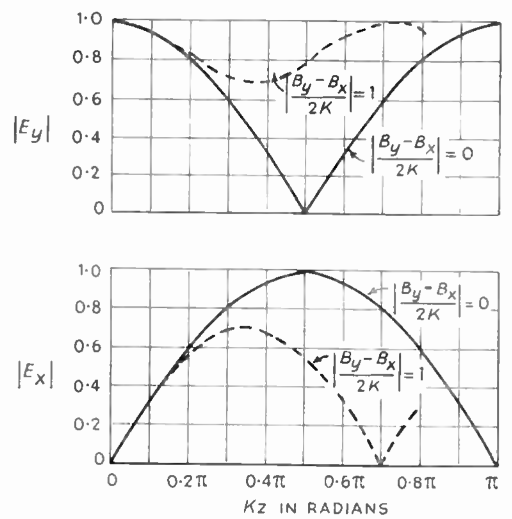

Fig. 4. Graphs of wave amplitudes vs. Kz.

In Fig. 4 the wave amplitudes $|E_y|$ and $|E_x|$ are plotted against Kz for

$$\left|\frac{\beta_y - \beta_x}{2K}\right| = 0 \quad \text{and} \quad \left|\frac{\beta_y - \beta_x}{2K}\right| = 1.$$

Experimentally we note that for $\left|\frac{\beta_y - \beta_x}{2K}\right| = 0$, the quantity $Kz = \pi/(2\sqrt{2})$ is equivalent to a $62\frac{1}{2}$ deg rotator so that by making such a rotator, we satisfy one of our boundary conditions. One way the other boundary condition can be satisfied is by making flats on the dielectric, say, until $\beta_y/\beta_x = 1{\cdot}18$.

Finally, we note that the dispersion of $|E_x|$ and $|E_y|$ is zero at the plane where the wave is circularly polarized; hence, the ferrite quarter-wave plate is inherently more broadband than an elementary 45 deg rotator.

8. Conclusion

Coupled wave theory has been applied to an elliptical ferrite rotator, and has resulted in a ferrite quarter-wave plate which has non-reciprocal properties. This should afford simple switching of an antenna from the left hand of circular polarization to the right hand of circular polarization. By switching a transmitter between two appropriate quarter-wave plates it is possible to switch circularly polarized waves through 180 deg over broad frequency bands. Finally, with such quarter-wave elements, a new pulsed non-reciprocal 180 deg phase shifter is obtained, and reciprocal phase shift structures are described.

9. Acknowledgments

The author expresses his gratitude to Mr. John Q. Owen for many helpful discussions, to Dr. C. Bowness for suggesting the co-ordinate system, and to the Raytheon Company, Special Microwave Devices Operation, for permission to publish this paper.

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Some Bandwidth Compression Systems for Speech Transmission

By

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Summary: The nature of speech is discussed, and the channel capacity required for transmitting speech signals by conventional means is considered. A review is given of methods which have been suggested by various workers for processing the speech signals so that they may be transmitted over channels of lower capacity.

A description is given of a simple 'formant tracker' for speech analysis, using a system of successive binary comparisons. Tests which have been carried out with the apparatus are described, including its use with a simple speech synthesizer to demonstrate the principle of analysis-synthesis telephony.

Part 1. TYPICAL SYSTEMS

1. Introduction

The frequency range occupied by the sounds of speech extends approximately from 60 c/s to 10000 c/s. The transmission of electrical signals which are an exact analogue of the speech signals therefore requires a bandwidth of 10 kc/s.

Since the overall channel capacity of any transmission system is finite, the number of simultaneous conversations which can be carried on using a particular cable or radio link is limited. An increase in the number of such conversations can be obtained by reducing the channel capacity required by each.

Channel capacity is usually measured in bits per second. The bit (contraction of 'binary digit') is the amount of information associated with the selection of one out of two alternatives of equal probability. It may therefore be used as a measure of the smallest discrete change of signal level which is detectable in the presence of noise.

The theoretical maximum capacity C of a transmission channel of bandwidth W c/s for a signal/noise power ratio P/N has been shown by Shannon[1] to be

$$C = W \log_2\left(1+\frac{P}{N}\right) \text{bits/second} \qquad \ldots\ldots(1)$$

For a given signal/noise ratio the theoretical channel capacity and bandwidth are thus directly related; and methods of reducing the required capacity are often referred to as systems for bandwidth economy or compression. The channel capacity required by a signal having a bandwidth of 10 000 c/s and a signal/noise ratio of 30 dB (i.e. 1000 : 1) is

$$C = 10\,000 \log_2(1+1000) \simeq 100\,000 \text{ bits/second} \qquad \ldots\ldots(2)$$

In practice it is found that a considerable reduction in channel capacity is possible, without serious loss of intelligibility.

2. The Nature of Speech

Speech is produced by the excitation of the vocal cavities of the throat, nose and mouth with acoustic stimuli. The excitation may be provided by the larynx in the form of a recurrent series of impulsive shocks due to air passing through the vibrating vocal chords ('larynx excitation', producing 'voiced' sounds) or by turbulent air flow through restricted passages such as between the teeth, between the tongue and palate, and so on ('fricative excitation', producing 'unvoiced' sounds). For some sounds both types of excitation may be present.

Owing to resonance effects in the vocal cavities, some of the overtones produced by the excitation process are reinforced relative to the remainder. The frequency regions in which reinforcement takes place are referred to as vocal resonances, formant frequencies, or more simply as formants. A vowel sound may have four or five such formant frequencies associated with it. The first formant (i.e. the formant of lowest frequency) usually has the greatest amount of energy associated with it. The other formants are numbered second, third, fourth, etc., in ascending order of frequency, corresponding in general to descending order of amplitude.

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Figure 1 shows the selective effect or filter characteristic of the vocal tract. For whispered (unvoiced) vowel sounds the fricative excitation sets up a random noise spectrum which may be considered to be uniform over the audio range. When this is modified by the filter characteristic, the resultant noise output has a spectral envelope with a shape similar to that of Fig. 1, in which three resonance peaks (at the first three formant frequencies) may be observed. Hissing sounds, such as 'ssss' and 'sh', are characterized by noise occupying a wide band of frequencies at the higher end of the speech spectrum.

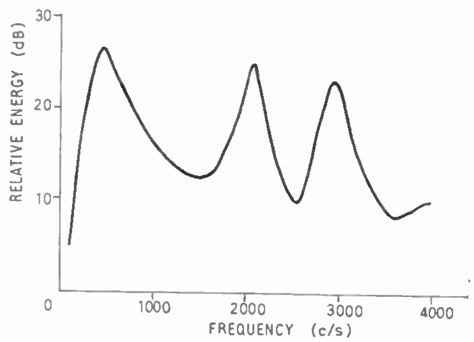

Fig. 1. Typical filter characteristic of vocal tract.

With voiced sounds, the vocal folds of the larynx open and close with a repetition frequency which lies between 60 and 240 c/s for the average male voice and is higher for women and children. The shape of the larynx pulse (i.e. volume velocity of air plotted against time) is subject to considerable variation but those shown in Fig. 2(*a*) may be taken as typical. The frequency spectrum of the vibrations is rich in harmonics, the amplitude of which falls off in a manner similar to that indicated in Fig. 2(*b*) which has been drawn for a fundamental vibration frequency of 100 c/s and hence has harmonics at 100-c/s intervals. When this spectrum is modified by the filtering action of the vocal tract, the result is as shown in Fig. 3. The three formant peaks may still be observed, though the envelope is not continuous. The frequencies of these peaks do not necessarily coincide with pitch harmonics.

Full discussion of the above process has been given by Fant[2] and by Cherry.[3]

3. Compression Schemes

These may be divided broadly into two categories. In the first the electrical signal transmitted remains an analogue of the speech sounds, even though it may be a very imperfect or distorted analogue. The methods described in Sections 3.1 to 3.5 are of this type. The

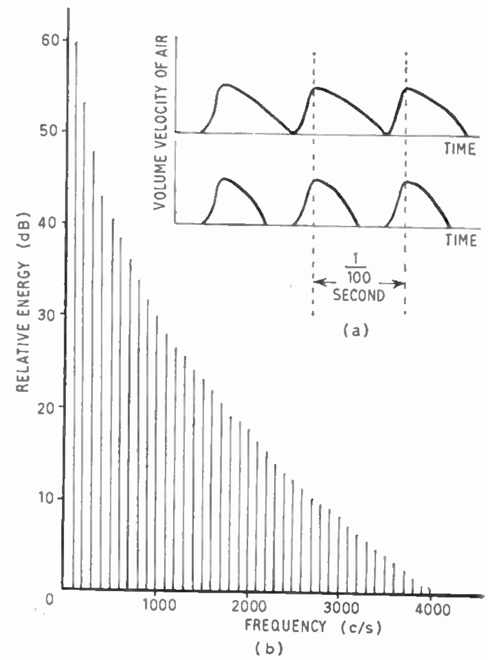

Fig. 2. Typical larynx pulses and harmonic spectrum.

second category uses analysis-synthesis techniques. Narrow-bandwidth code signals are derived from the speech by means of an 'analyser', and transmitted over the channel to the receiver. Here the code signals are arranged to control an artificial talking device, or 'speech synthesizer'. Methods described in Section 3.6 are of this type. Hybrid systems are dealt with in Section 3.7.

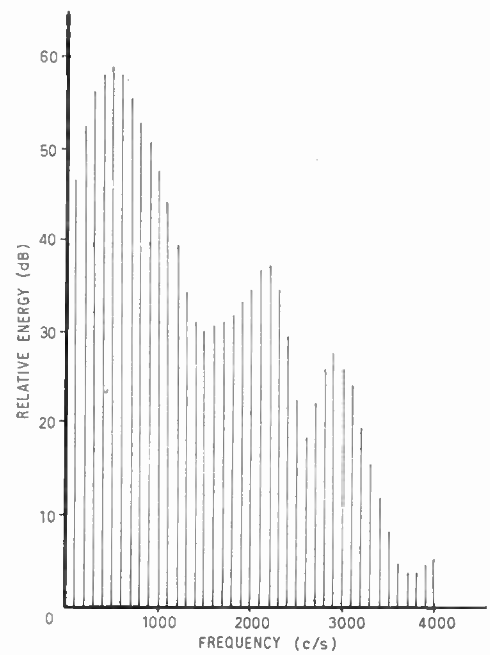

Fig. 3. Typical energy spectrum of voiced vowel-sound.

3.1. *Band-pass Filtering*

If a speech signal is confined by means of a filter to the range 150 c/s to 4500 c/s the intelligibility and the quality (or 'naturalness') are found to be quite satisfactory. A 9-kc/s separation between broadcasting stations is common and this permits a speech bandwidth of 4·5 kc/s to be used, with double-sideband operation. Intelligibility remains adequate, though quality suffers, if the frequency band is reduced to 300–3400 c/s, as in the normal telephone system. The single-sideband channel separation is 4 kc/s, to allow for the fact that the filters used do not have a perfectly sharp cut-off. Low-grade circuits are sometimes operated in which two separate 500–2000 c/s signals occupy one standard 4-kc/s channel. In such cases the intelligibility for isolated words is only about 60%; but for connected speech, with suitable repetitions where required, it can be adequate. The quality, however, is not really satisfactory for commercial use, and recognition of individual speakers is difficult. It has been found by Kryter[4] that intelligibility with such a system can be improved by dividing the available 1500 c/s into three separate 500-c/s bands, having their centres spaced within the range 500 c/s to 3000 c/s.

3.2. *Frequency Division*[5, 6]

Several schemes have been devised in which the frequencies present in the speech signal are divided by a factor of say two or three before transmission, and multiplied back to their correct values at the receiver. An elementary way of achieving this is to record the speech, and to replay it at a slower speed for transmission over a narrow-bandwidth channel. Upon reception the slowed-down speech is re-recorded, and then played back at a faster rate to give normal speech. Although a simple arrangement of this kind enables a narrower bandwidth to be used, the actual saving of channel capacity is nil, since a greater time is needed to pass the signal. Hence more complex processing is needed. For example, since the speech signal usually includes repetitive waveforms, it may be sampled at suitable intervals and the portions of the signal between the samples may be rejected. The frequencies contained in the samples may then be divided down, and transmitted in the same time as the original signal would have taken. At the receiving end the frequencies in the samples are restored to normal, and the reduction in time which then occurs is compensated for by an appropriate number of repetitions of each sample. By use of 'Doppler' techniques it is possible to arrange for the sampling and frequency division to be carried out as a continuous process at the transmitter, and for the frequency expansion and repetition of samples to be carried out continuously at the receiver.

3.3. *Time-assignment Speech Interpolation*[7]

It is sometimes possible to obtain better utilization of communication channels without actually reducing the bandwidth of the individual signals. In time-assignment speech interpolation, for instance, the pauses in speech (which on average take up more than 50% of the time of a conversation) are filled by interleaving utterances due to different speakers. Thus any talker may be disconnected from his channel when he stops speaking for any reason (such as to listen to a reply). The channel is switched automatically to another speaker, and when the original talker recommences to speak his conversation will probably be carried by quite another channel. Hence the total number of conversations carried at peak periods can be doubled, with no loss of intelligibility or quality. The complex electronic switching apparatus is, however, very expensive, and the method is only economically suitable for special circuits such as trans-oceanic cables.

3.4. *Interrupted Speech*[8]

Another way of improving the utilization of a channel, though with some lowering of intelligibility, would be to interrupt the speech continuously at a frequency of say 40 c/s. With equal 'on' and 'off' periods it would be possible to time-multiplex two conversations on to a single channel. Experiments have shown that with this 50% duty cycle the intelligibility is slightly better than would be obtained by halving the bandwidth of the two signals and frequency-multiplexing them on to the channel.

3.5. *Speech Clipping*[9]

Under conditions of low signal/noise ratio, some improvement in intelligibility can be obtained by speech clipping. Speech has a dynamic intensity range of about 30 dB, and an amplifier which is adjusted so as not to overload on loud passages may not give sufficient amplification to the weaker sounds. By using a system of amplitude compression the intensity of weaker sounds can be raised and the intelligibility improved. The compression process may be carried to the limit, giving an infinitely clipped rectangular wave of constant amplitude. This retains only the information contained in the zero-crossings of the original speech wave, yet the intelligibility is not much less than that of normal speech. Unfortunately, the quality is harsh and unpleasant.

The increase in effective signal power obtained by speech clipping may approach 12 dB, and from eqn. (1) a corresponding improvement in the capacity of the channel concerned is possible.

3.6. *Analysis–Synthesis Methods: The Vocoder*[10, 11]

One way of improving channel capacity is to transmit, instead of the normal speech currents, only sufficient coded information to permit the 'remaking' of the speech by suitable apparatus at the receiving end. The coded information can be passed in the form of signals requiring a channel capacity less than that of the speech currents, so that the line or other transmission system is able to accommodate more channels within its pass range of frequencies. This is the basis of speech analysis–synthesis telephony systems, such as fixed-channel vocoders and parametric (formant-tracking) devices.

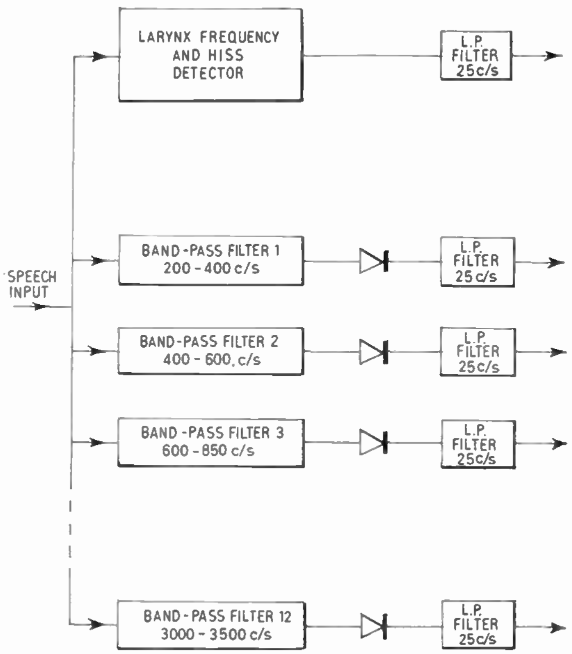

Fig. 4. Channel vocoder analyser.

The 'fixed-channel' or 'frequency-band' vocoder (voice CODER) was first demonstrated in America by Dudley, and has undergone extensive development. In this system an analysis of the power spectrum of speech is made by means of a number of band-pass filters, which divide the audio-frequency spectrum into say, twelve adjacent bands (see Fig. 4). The energy in each band is measured, and twelve narrow-bandwidth signals are obtained, which vary as the energy varies. An additional signal gives the pitch of the larynx signal, and is also used to specify whether the speech is 'voiced' or 'unvoiced'. At the receiving end this pitch signal is used to switch into circuit either a relaxation oscillator which acts as a 'buzz' source for voiced sounds, or a noise generator which acts as a 'hiss' source for unvoiced sounds (Fig. 5). The pitch signal also controls the fundamental frequency of the buzz source.

The line spectrum of the buzz source or the continuous spectrum of the hiss source is filtered into twelve frequency bands, corresponding to those of the band-pass filters of the analyser. The magnitude of the output in each band is controlled by the appropriate line signal, using a system of modulators. The outputs of these modulators are then combined to produce artificial speech similar to the original. An overall bandwidth compression of 10 : 1 in the transmitted signals is possible.

3.6.1. Pattern-recognition vocoders[12]

Continuous speech may be segmented into about forty different fundamental acoustic elements called 'phonemes'. A device which could recognize these would be able to produce a series of signals which would specify the phonemic content of the speech. In the phonetic-pattern recognition vocoder the characteristic phonetic patterns, corresponding to the steady-state spectra of a limited number of speech sounds, are 'recognized' by the analyser, and narrow-bandwidth control signals corresponding to these patterns are transmitted to a modified vocoder synthesizer for the production of artificial speech. Many investigations have been made into requirements and possibilities of recognition systems for words, digits, and speech segments. So far it has not been possible to develop a completely satisfactory analysis–synthesis system using segmentation techniques, although in theory they offer the greatest possibilities in channel economy. According to Fant,[13] if a certain amount of redundancy is retained

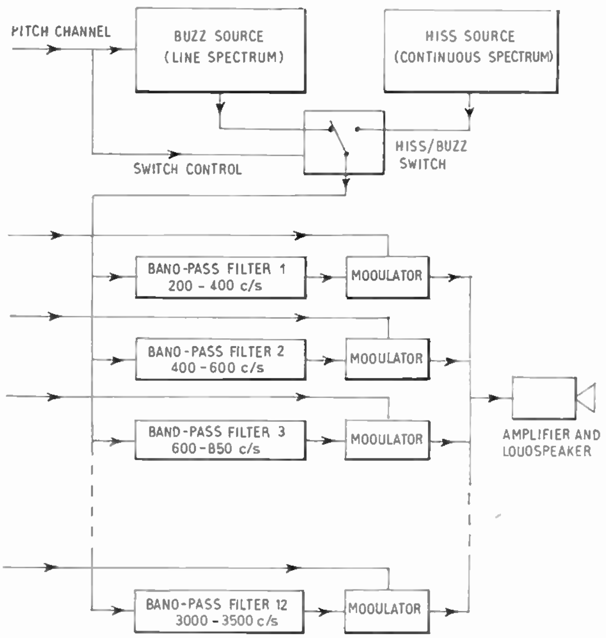

Fig. 5. Channel vocoder synthesizer.

as an insurance against errors, phonetic-pattern matching devices should be usable with transmission links having an information handling capacity of only 150–300 bits/second, compared with 2000–4000 bits/second for conventional frequency-band vocoders, and 500–1000 bits/second for resonance vocoders such as the parametric type mentioned below. The corresponding figure for a conventional telephone system with a bandwidth of 3400 c/s and a signal/noise ratio of 30 dB is 34 000 bits/second. Digital computers, with their extensive storage facilities and their ability to perform high-speed switching, offer considerable promise in the development of pattern-recognition vocoders. The high cost of the computers would probably militate against their extensive use for commercial systems, but they are already proving a powerful research tool for 'analysis by synthesis' investigations, and for simulating new and complex types of vocoder.[14]

3.6.2. Formant vocoders

A speech synthesizer or 'parametric artificial talking device' was built by Lawrence[15] at the Signals Research and Development Establishment (S.R.D.E.) at Christchurch in the early nineteen-fifties. It was used to demonstrate that intelligible speech could be synthesized from a small number of slowly varying 'parameter' signals, specifying the type of excitation and the properties of the three most prominent resonances (or formants) in the speech spectrum, and requiring only very narrow-bandwidth channels for their transmission. A typical modern version of the Lawrence synthesizer is used for speech research in the Department of Phonetics of Edinburgh University, under Abercrombie.[16] It makes use of eight synthetically-produced voltage/time signals. Three of these, designated F_1, F_2 and F_3, are made to control variable-frequency oscillators, to reproduce the frequencies of the first three formants in the speech sample to be synthesized. Another voltage/time signal specifies the amplitude of the fricative (hiss) excitation, while two others specify the amplitude and the frequency (F_0) of the larynx excitation.

The other two parameter signals respectively specify the centre frequency of the 'hiss' signal, and the amplitude of a 'hiss through formants' signal, which gives the effect of aspiration in words like 'high'. Since changes in the configuration of the vocal cavities, and in the positions of the tongue, teeth, etc., are the result of muscular movements, the rate at which they can take place is limited. Hence the corresponding changes in the formant frequencies and in the excitation can also only take place slowly.

The stages in the synthesis of a phrase of speech are:

(i) The drawing of eight voltage/time graphs on a 'parameter sheet' to represent eight signals.

(ii) The production of eight equivalent electrical signals from these graphs.

(iii) The changing of the voltage/time variations F_1, F_2 and F_3 into frequency/time variations corresponding to the slow variations of formant frequencies 1, 2 and 3. Concurrently with this, the use of the remaining signals for the production and treatment of suitable 'pitch' and 'hiss' sounds.

(iv) The combination of the signals to actuate a loudspeaker or telephone receiver, and hence produce sound waves corresponding to intelligible speech.

The stages in a complete analysis system would be:

(i) The division of the band of frequencies covered by the speech into three narrower bands, covering the ranges of the three main formants.

(ii) The analysis of each band, continuously or at appropriate time intervals, to find the frequency at which the most energy is instantaneously concentrated—i.e. the frequency of the formant concerned.

(iii) The changing of the frequency/time variations of (ii) into voltage/time variations corresponding to the F_1 to F_3 signals.

(iv) Simultaneously with (ii) the extraction of the other five signals giving coded information as to the type, intensity, and frequency of the excitation function.

(v) The passing of the eight parameter signals over a single-line pair or other link by carrier-telephony methods.

(vi) Use of the eight parameter signals to operate a synthesizer of the Lawrence type, as if they were the eight voltages obtained by scanning the parameter sheet.

The eight parameters mentioned are not the only ones which could be used for an analysis-synthesis system. Work on such systems by Stead[17] and his co-workers at S.R.D.E. has shown that good results are obtained using the four signals for the frequencies F_0, F_1, F_2 and F_3, together with three signals to specify the energy levels in the first formant band, the combined second and third formant bands, and the high-frequency band respectively, and one signal to indicate whether the speech is voiced or unvoiced. At the Royal Institute of Technology at Stockholm the use of eleven parameters has enabled artificial speech of very natural quality to be synthesized. On the other hand, the original version of the Lawrence synthesizer used only four signals to control the parameters, and a 'simple speech synthesizer', developed at Bristol

University by Billings, Woollons and Gill[18] enables certain words and phrases to be synthesized intelligibly, though in whispered or laryngitic form, using only two variable parameters (the frequencies of the first and second formants). A modified version of this simple synthesizer has been used by the author, to enable tests to be made on the simple formant tracker described in Part 2 of the present paper. These tests, which are described in Section 8.3, show that some words and phrases can undergo running analysis and be re-synthesized in an intelligible form, using the two formant-frequency parameters only.

3.7. *Combination of Methods: Semivocoders*

Combinations of the methods and techniques mentioned above have been used with success. Channel vocoders have been designed which use a base-band, that is they transmit the low-frequency part of the spectrum without any coding. This avoids the difficult process of determining accurately the pitch of the larynx excitation. The ear is able to utilize the pitch harmonics present in the base-band to give the effect of the fundamental larynx pitch, even if the latter has been severely attenuated, as it usually is on the public network. The degree of bandwidth compression obtained with these 'semivocoders' is between 2 : 1 and 5 : 1, as compared with the 10 : 1 of conventional vocoders, but the intelligibility and quality are improved. The base-band technique is sometimes also used with formant vocoders.

A system which uses a frequency division technique at the formant frequencies is the Vobanc (voice BANd coder) described by Bogert,[19] which gives a 2 : 1 reduction in bandwidth. The speech band is divided into three parts by means of filters. The three ranges are 200 to 1000 c/s, 1000–2000 c/s and 2000–3200 c/s. Each band usually contains one of the three main vowel formants. Frequency dividers halve the frequency of the strongest harmonic in each of these three bands, and the divided signals are passed through filters having pass bands half as wide as the original bands. Therefore the signals sent over the line or radio link have only half the bandwidth of the original speech sounds. The information transmitted is mainly that contained in the strongly resonant regions of the voice sounds. Consonant articulation figures of 80–90% have been reported, but the quality is not very satisfactory.

A somewhat similar system is the Codimex (COmpression DIvision Multiplication EXpansion) of Daguet,[20] which requires a total bandwidth of 750 c/s. The 300–3400 c/s band is subdivided into three bands, 300–700 c/s, 700–2000 c/s and 2000–3400 c/s, selected to accommodate each of the three principal formants. The amplitude and frequency information is separated from each of the three bands and is compressed by a factor of eight before being transmitted by a frequency-multiplex method. Articulation tests giving a 72·3% score on nonsense syllables and 98·3% on sentences have been reported.

4. Comparison Between Various Systems

An evaluation of some bandwidth compression systems has been carried out by Stevens, Hecker, and Kryter.[21] The apparatus used included (i) a 16-channel conventional vocoder, (ii) a 7-parameter formant vocoder, (iii) a semivocoder with a base-band of 250–750 c/s and 13 conventional channels for the higher frequencies, (iv) a spectrum-sampling system using three filters with pass bands spaced over the speech-frequency range and having a total bandwidth of 900 c/s and (v) a reference system consisting of a low-pass filter with a cut-off frequency of 1500 c/s and a sloping characteristic of 18 dB/octave. Tests were carried out to find the intelligibility of phonetically-balanced words and of nonsense syllables involving typical consonant and vowel features. Voice quality and talker recognition were also compared.

Each of the systems gave better results in some tests than others. Any attempt at a comparative assessment would need to take into consideration the varying degrees of compression involved, and the order of importance to be attached to the various measures of performance. On balance, the reference system and the semivocoder appear to have given the best overall performance in the tests, and the formant vocoder the worst, while the channel vocoder and spectrum-sampling system gave intermediate results. On the other hand, although the relative bandwidth requirements are not stated, it is possible that the formant vocoder was about eight or nine times more economical in channel capacity than the reference, four or five times more so than the semivocoder or the spectrum sampling system, and twice as economical as the conventional vocoder. The authors have been careful to point out that the systems tested were developed by various organizations, with different applications in mind and under various constraints as to physical size, component reliability, bandwidth, and intelligibility. Furthermore, some were production models and others experimental prototypes. Nevertheless, the results represent a worthwhile attempt to evaluate the good and bad points of selected systems.

Part 2. A SIMPLE FORMANT TRACKER

5. Outline of Requirements

The frequencies and relative amplitudes of the formant components of a particular speech sound may vary considerably from speaker to speaker, and to a lesser extent with successive utterances of the same speech by the same person. Furthermore, the frequency ranges within which the three principal formants lie are not separate and distinct, but overlap. Peterson and Barney[22] have shown, from tests on vowels with a number of male and female adult speakers, that the first formant frequency may lie between 210 and 1040 c/s, the second between 570 and 3100 c/s and the third between 1400 and 3900 c/s. The ranges and the overlap are even greater when children are included. For the sake of simplicity, however, the assumption may be made that for most of the time the formants lie within frequency bands which do not overlap. A speech signal occupying the frequency range 200 c/s to 3400 c/s may thus be divided into three arbitrary bands, covering say 200 to 1000 c/s, 1000 to 2600 c/s and 2600 to 3400 c/s respectively. It is then assumed that within each of these three bands there is at any instant one particular frequency (the formant frequency) at which the energy concentration is greatest.

A simple formant tracker has been designed to identify the first two formant frequencies, and to provide two output voltages, of which the instantaneous amplitudes are respectively proportional to the two formant frequencies. These voltages are equivalent to the F_1 and F_2 voltages required for application to a Lawrence-type synthesizer. Similar apparatus could be used for the tracking of the third formant, thus providing the F_3 voltage.

6. Possible Methods for Formant Tracking

A variety of methods of identifying the formant frequencies has been suggested by a number of authors.[11,23,24,25] The identification may take place at the original speech frequencies, or the frequency band occupied by the speech may first be translated to a higher-frequency part of the spectrum. Since comparison usually involves the use of band-pass filters, and since these can be made smaller, more uniform in performance and less expensive at frequencies above the audio range, frequency translation is often adopted. It is also necessary to decide whether to treat the speech-frequency band as a whole, in order to find the energy maxima wherever they occur, or whether say three separate bands, as described in Section 5, should be sampled individually.

Comparison between all filter outputs within a formant band can take place simultaneously, or the outputs may be sampled in rapid succession by means of a rotating switch or its electronic equivalent. A third possibility, suggested by Billings,[11] is to make binary comparisons, i.e. to compare filter outputs in pairs, in each case passing only the output of greater magnitude. The number of outputs is thus halved, and can be progressively reduced and identified. This is the method adopted for the formant tracker to be described. The method has the merit of simplicity, and also offers the possibility of presenting the output voltage of the tracker in amplitude—quantized form, though the apparatus is at present arranged to give a continuously varying output.

Figure 6 indicates some of the alternatives which were considered when deciding the initial design of the apparatus. Each horizontal line shows a number of possibilities, those adopted at present appear on the right of the diagram. Several of the others are, however, also being explored. In particular, methods of using transistors for switching and the control of switching are being investigated.

7. General Description of the Apparatus

The speech signal from the microphone and preamplifier has a frequency range of about 100 to 8000 c/s. As shown in Fig. 7 it is translated to the range 10 100 to 18 000 c/s by means of a ring modulator, and fed to two band-pass filters which have pass bands of 10 200 to 11 000 c/s and 11 000 to 12 600 c/s

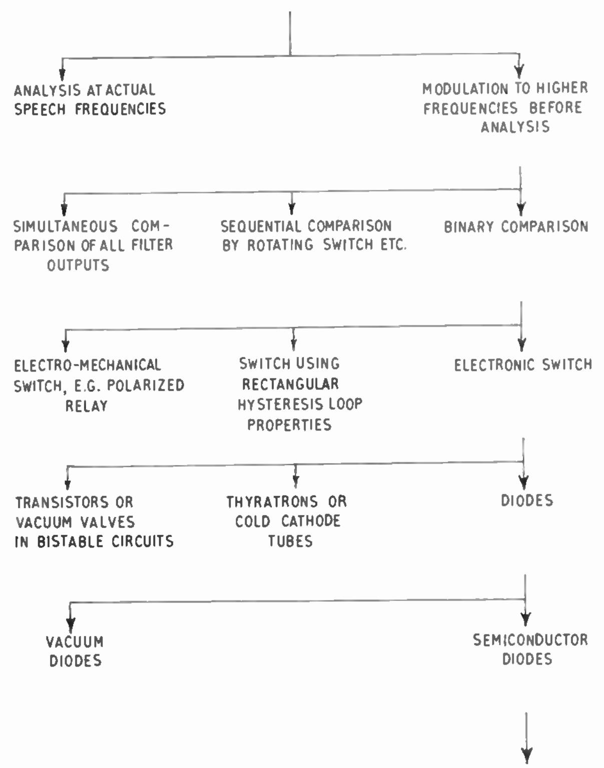

Fig. 6. Alternatives considered when designing the simple formant tracker.

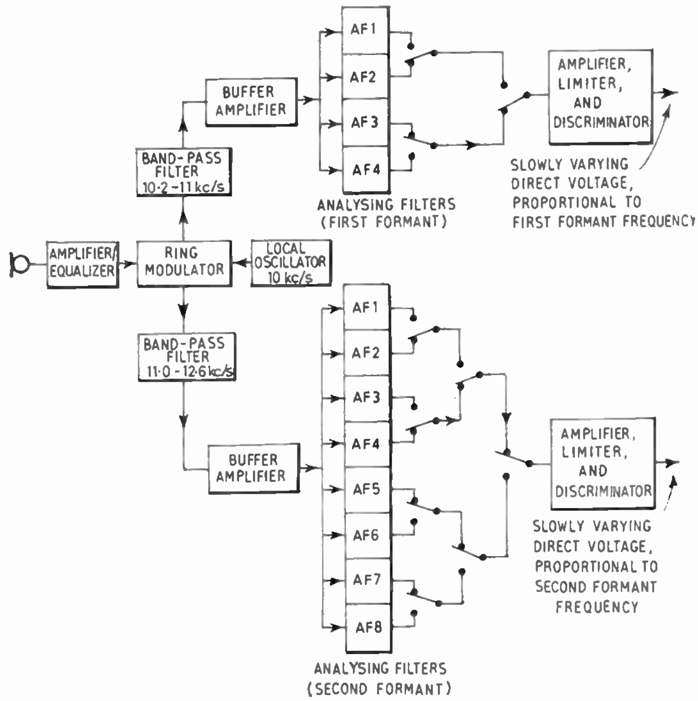

Fig. 7. Block diagram of simple formant tracker.

respectively. These correspond to the first and second formant bands (200 to 1000 c/s and 1000 to 2600 c/s respectively). In a complete commercial system it would also be necessary to use a third band-pass filter covering 12 600 to 13 400 c/s for the third formant component (2600 to 3400 c/s), a fourth covering 13 400 to 18 000 c/s for the detection of hiss components above 3400 c/s and a fifth covering 10 100 to 10 250 c/s for the fundamental larynx frequency (taken as 100 to 250 c/s). In the interest of simplicity these have been omitted in the present apparatus. A programme of work for including them is, however, being instituted, using similar circuits and techniques to those used for the first two formants.

The output of each filter is passed through a buffer amplifier with a low-impedance output, suitable for feeding a parallel bank of series-tuned circuits which act as band-pass filters. In the first-formant unit there are four such simple analysing filters, shown as AF1 to AF4, each having a half-power bandwidth of 200 c/s, the centre frequencies being 10 300, 10 500, 10 700 and 10 900 c/s. In the second formant unit there are eight filters (AF1 to AF8) also with pass bands of 200 c/s, and having equally spaced centre frequencies of 11 100, 11 300 c/s, etc., up to 12 500 c/s. (It is possible that improved results would be obtainable with a greater number of filters, each having a narrower pass-band, and it is intended to experiment in due course with eight 100-c/s filters for the first formant and sixteen for the second.)

The amplitude of the output voltages from each analysing filter depends on the energy distribution within the formant band concerned. Considering the second formant band, the amplitudes of the outputs of the analysing filters AF1 to AF8 are compared in pairs, AF1 against AF2, AF3 against AF4, and so on. By means of crystal-diode switching units, shown as simple switches in Fig. 7, it is arranged that only one (the greater) of the outputs from each pair is passed on, so that the original eight outputs are reduced to four. These four outputs are again compared in pairs, and the greater one of each pair is passed on, reducing the outputs to two. A further comparison gives a single output, at the instantaneous frequency of the second formant. In Fig. 7, for instance, the frequency is one coming within the range of filter AF4.

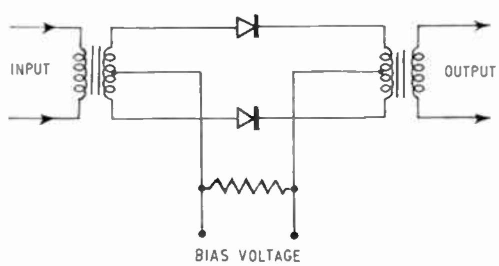

Fig. 8. Elementary diode switch.

The action in the case of the first formant band is similar, but since there are only four 200-c/s filters the number of binary comparisons required is less than for the second formant.

The single output for each band is fed into an amplifier, limiter and frequency-modulation discriminator, from which is obtained the final output voltage. This has an amplitude proportional to the instantaneous value of the frequency of the formant concerned.

Since the formant frequencies cannot change very rapidly the amplitude of each output voltage can only change slowly, and only a narrow-bandwidth channel is needed for its transmission.

7.1. *Diode Switches*

The basic switching unit used throughout consists of a pair of transformers, coupled through two CV448 crystal diodes (Fig. 8). When a direct-voltage bias is applied to the centre-tappings of the transformers, the passage or otherwise of a comparatively small alternating-voltage signal depends on whether the crystals are biased in the forward or the reverse direction.

By combining two of these units with a common output transformer (shown in Fig. 9) it is possible to select one of two signals, since the bias across one diode switch is opposite in polarity to that across the other. The biasing (or switching) voltage for each pair is obtained by means of a circuit of the type shown in Fig. 10.

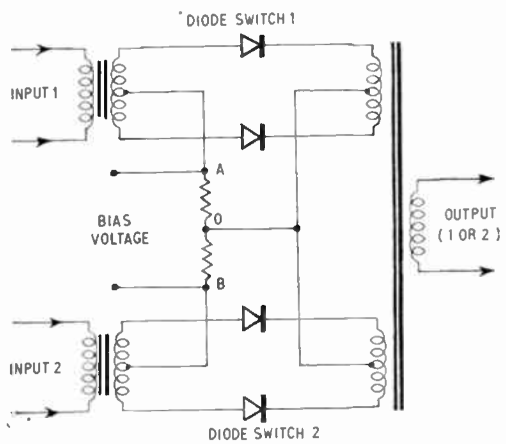

Fig. 9. Combination of two diode switches.

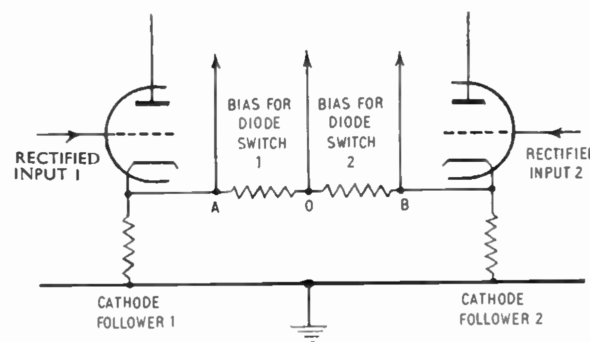

Fig. 10. Method of obtaining bias voltages.

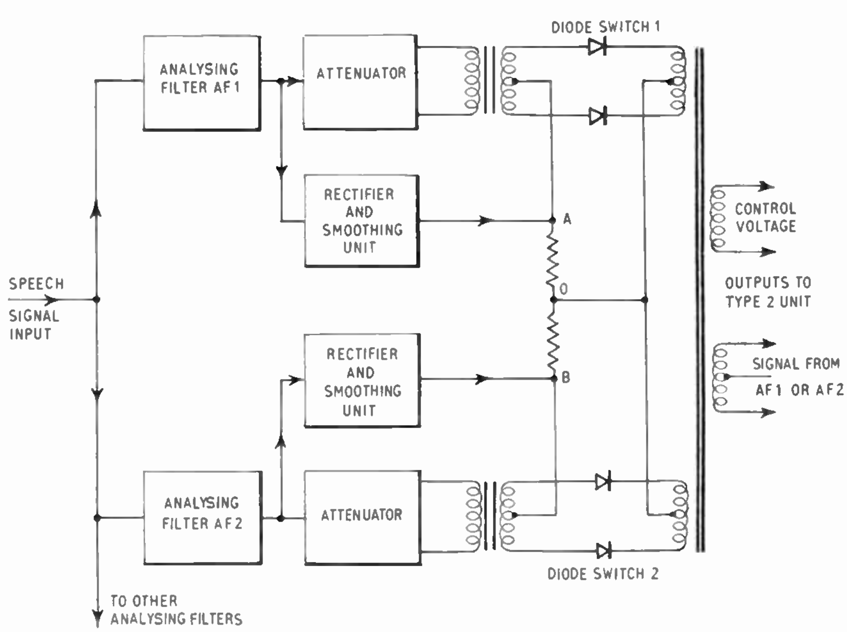

Fig. 11. Type 1 switching unit, for initial comparisons.

A portion of the signal from each of the two analysing filters concerned is rectified and smoothed, and the resulting slowly-varying direct voltages are fed to two cathode followers. The difference in voltage between the two cathodes at any instant provides the biasing voltage required.

7.2. *Type 1 Switching Units for Initial Comparisons*

Two main types of unit are used as building blocks. Type 1 is used for the first set of comparisons, on the signals arriving direct from the filters (Fig. 11); type 2 is used for subsequent comparisons.

The input filters, AF1, AF2, etc., are simple series-tuned circuits, with their 200-c/s pass-bands arranged to overlap at the -3 dB points. In an ideal system more complex filters would be used, with 'square-topped' response curves overlapping at a fraction of a decibel below the maxima.

Consider two adjacent filters in, say, the second formant unit, such as AF1 and AF2. The respective amplitudes of the alternating output voltages from these filters depend on the frequency of maximum energy concentration in the second-formant range in the applied signal. Unless this formant frequency happens to coincide temporarily with the cross-over frequency of the response curves of the two filters, the amplitudes of the output voltages are not equal. By means of fixed attenuators, a fraction (about 1/2000) of each of these alternating voltages is passed on as a main signal, through cathode-follower valves and transformers.

At the same time, the full amplitudes of the alternating voltages from the filters are rectified and smoothed and applied to another pair of cathode followers. Slowly-varying direct voltages of up to 120 V in amplitude are obtained at the cathodes. These direct voltages are applied to points A and B (Fig. 11) and because they differ in amplitude they provide bias voltages of opposing polarities across A0 and B0, causing the diode switches to block the signal path from one filter, and free the path from the other. Only the greater of the two signals from AF1 and AF2 is passed on, in attenuated form, to the appropriate primary winding of the output transformer of the type 1 unit.

Simultaneously, the outputs from the other pairs of filters are passed on, by means of other type 1 switching units, to three other output transformers.

In order to ensure correct switching in this and subsequent stages, it is necessary to set the maximum values of the various voltages before use, using a signal generator in place of the microphone. With a sinusoidal input of 2 V r.m.s. applied to the analysing filter, and link 0 open, the direct voltages between A and earth and between B and earth are each set to 100 V at the frequencies of maximum response of the filters with which they are associated. The correspond-

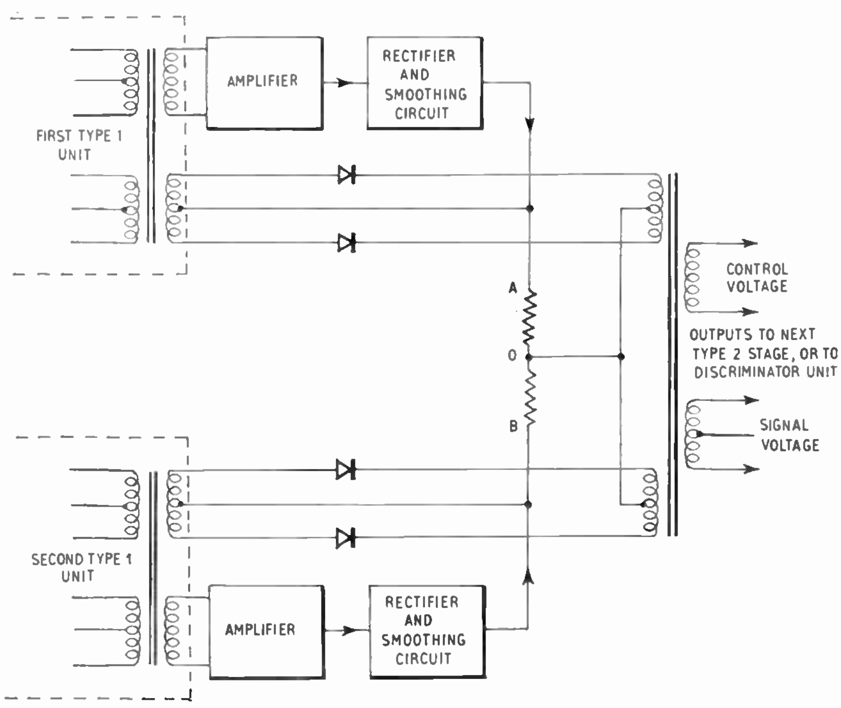

Fig. 12. Type 2 switching unit, for subsequent comparisons.

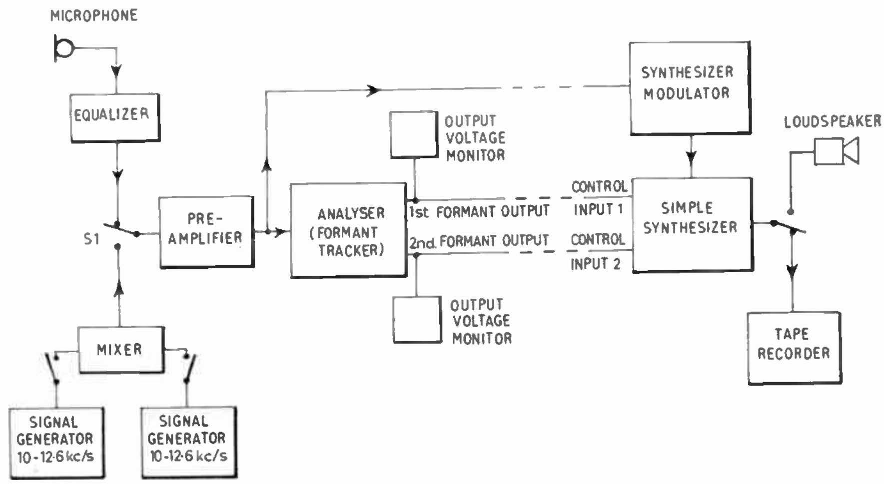

Fig. 13. Arrangement of apparatus for testing.

ing voltages in the signal paths are each set to 30 mV r.m.s. at the peak frequencies of the filters.

7.3. *Type 2 (Second- or later-stage) Switching Unit*

The input to this type of unit is taken from the output transformers of two type 1 units (Fig. 12). The two signals are passed straight into diode switches via the centre-tapped secondary windings, but only the greater of the two signals is allowed to pass on to the output transformer of the type 2 unit. The direct voltages for biasing the diode switches are derived from alternating voltages obtained from additional secondary windings on the input transformers. With a step-up ratio of 1 : 4 these alternating voltages have maximum r.m.s. values of about 120 mV. They are amplified by two RC-coupled stages before being rectified, smoothed, and applied via cathode followers to points A and B for operating the diode switches.

In an eight-filter tracker, such as is used for the second formant in the present simple apparatus, the signals from two type 2 units are fed into one further similar unit, which selects the larger of the two signals and passes it on to the discriminator unit. There are thus four type 1 switching units, followed by two type 2 units, which in turn feed the final type 2 unit.

In a four-filter panel such as is used in the present apparatus for the first formant, only one type 2 switching unit is necessary, following two type 1 units.

If sixteen filters were used instead of eight for the second formant panel, eight type 1 units would be needed, followed in succession by four type 2 units, two type 2 units and one type 2 unit.

7.4. *Discriminator Unit*

This is a conventional f.m. detector of the Round-Travis double-tuned-circuit type, using two crystal diodes. The incoming signal is amplified and limited by two resistance-capacitance coupled triodes, and a pentode with low anode and screen-grid potentials. The output from the discriminator is a slowly varying direct voltage, whose instantaneous amplitude is proportional to the frequency of greatest energy concentration in the original speech.

8. Performance of the Formant Tracker

A variety of tests has been carried out with the apparatus. A typical arrangement is shown in Fig. 13. With switch S1 in the lower position, the formant tracker was fed with two audio-frequency sinusoidal voltages, representing respectively a 'wanted' signal (corresponding to a pitch harmonic at a formant peak frequency) and a smaller 'unwanted' signal. With S1 in the upper position the tracker was used with an input derived from 'live' speech.

8.1. *Tests Using Signal Generators*

By adjusting the amplitudes and frequencies of the two sinusoidal voltages it was possible to ascertain the extent to which the formant tracker output voltage was affected by the presence of an unwanted signal. It was found convenient to interpret the consequential errors in the output voltage in terms of equivalent errors in the apparent frequency of the true formant.

It was found that if there was to be complete freedom from error over the greater part of each formant band the amplitude of the unwanted signal had to be kept some 6 to 12 dB below that of the wanted signal. The actual value depended on the frequencies of the signals relative to the peak frequencies of the analysing filters. In general, the most unfavourable cases occurred when the frequency of the wanted signal corresponded to a crossover frequency between two filters (at which the response of each filter was 3 dB below its peak value) while at the same time the unwanted signal was at the frequency of a filter peak. At frequencies other than these it was found in general that the amplitude of the unwanted signal could be allowed to approach to within about 4 dB (and in some favourable cases to within 1 dB) of the amplitude of the wanted signal, before the apparent error in the frequency of the first formant reached ± 75 c/s or that of the second formant reached ± 100 c/s.

8.2. *Tests Using a Microphone and C.R.O.*

The output voltages corresponding to the first and second formants of typical vowels and diphthongs, spoken by various male English speakers were displayed on a double-beam direct-coupled cathode-ray oscillograph. By careful choice of scales it was possible to estimate the apparent formant frequencies of the vowels. As was expected, there was some spread in the results for different speakers, but the formant patterns appear consistent with those published by other authors.[22, 26, 27] Qualitative results are obtainable when a sound spectrograph[27] is available, and the tracker output voltages for samples of speech by individual speakers can be compared with the corresponding spectrograms.[28]

Two interesting deviations from the accepted pattern of formant frequencies were noted in the tests with male speakers. These occurred in the second formants of [ɔ] (as in 'hawed') and [u] (as in 'who'd'), for which Peterson and Barney[22] quote second-formant frequencies of 840 c/s and 870 c/s respectively. These frequencies are well below the arbitrarily chosen lower limit of 1000 c/s for which the second-formant circuits of the tracker were designed. As a result, the tracker output voltage took up its mean or no-signal position at the centre of the second-formant band in each case, thus giving apparent errors of over 1000 c/s.

If the tracker had been intended for use only by male speakers it would have been permissible to arrange the circuits so that the crossover frequency between first and second formants was 800 c/s instead of 1000 c/s. The tracker would not, however, have then been suitable for some of the higher-frequency first formants of the female voice. The overlap between the frequency ranges of the formants constitutes a major difficulty in the design of a practical system, which would have to be suitable for use by men, women, and children. In advanced types of system[25] the band limits are sometimes made variable, and are controlled automatically by the speech signals themselves.

8.3. *Tests with Simple Speech Synthesizer*

Another series of tests with speech input signals was carried out by feeding the two slowly-varying output voltages of the formant tracker to a 'simple speech synthesizer',[18] thus forming a rudimentary speech analysis-synthesis system. A number of vowels, diphthongs, and short sentences which the simple synthesizer was known to be able to produce were spoken into the microphone, and several of them were reproduced in whispered form with fair intelligibility.

9. Future Developments

The simple apparatus which has been described is useful for demonstrating the basic concept of speech analysis and synthesis, but the facilities it provides are, of course, limited. A number of improvements are contemplated, in order to form a more complete analysis-synthesis system.

Modifications to the analyser will include controlled equalization of the input speech waveform and automatic control of gain, control of the formant bands to obviate overlap, improved binary-comparison circuits, and provision of a pitch extractor, a third-formant tracker, and a fricative-excitation detector. Experiments will be carried out using switching circuits similar to those already in use, but with the bias connections reversed, to enable the minima in the speech frequency spectrum to be located (Fig. 1).

Appropriate modifications to the synthesizer will also be carried out.

10. Acknowledgments

The work described in Part II of the paper was carried out in the Department of Electrical Engineering of Bristol University. The author wishes to thank Professor G. H. Rawcliffe for the facilities provided, and Professor A. R. Billings (now of the University of Western Australia) for his guidance. He also wishes to thank Mr. A. J. Eales for providing facilities for the work to be continued in the Department of Electrical Engineering of the Bristol College of Science and Technology.

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DISCUSSION

Under the chairmanship of Mr. A. I. Forbes Simpson, M.B.E.

Mr. J. Powell: Telephone channels are generally allocated a 4-kc/s band of frequencies, while nominal 3-kc/s bands are used on trans-oceanic links and there have been experiments with still narrower-band channels of a nominal 2-kc/s bandwidth.

The transmission in the 4-kc/s band is substantially from 300 c/s to 3400 c/s whereas in the two latter cases the lower limit of the frequency range is reduced to 200 c/s. I understand that subjective tests have been carried out which indicate quite clearly that it is possible to improve the quality of received speech if the receiving instrument accepts the frequencies between 200 c/s and 300 c/s, and so I was particularly interested in the recordings played by Mr. Williams. However, I could detect hardly any change when the lowest frequency transmitted was altered from 200 c/s to 300 c/s and I would like Mr. Williams to enlarge on his description of the filters used in the experiments; in particular I would like to know the slope of the attenuation/frequency characteristic beyond the half-power points.

Mr. L. J. Doubell: Having separated the speech into discrete bands, and passed these bands through filters, it is surely important that these filters should all introduce the same transit time and phase shift, so that the separate speech bands have the same time and phase relationship at the outputs of the filters, as they had at the inputs.

Mr. D. Wilkinson: An inherent disadvantage of the vocoder is that the cost per channel is high. This cost might be acceptable in systems where the terminal equipment represents a small fraction of the total cost, e.g. coaxial cable systems, but in multichannel radio systems it would be cheaper to provide the extra bandwidth. I think it would be fair to say that the most obvious use of vocoders is on military circuits, and these are carried predominantly by radio. I would have thought that, in many cases, it would be better to increase the element of redundancy in speech (so increasing the bandwidth) in order to achieve a measure of protection from jamming and interference.

Is there any indication as to what signal/noise ratio would be required in the narrow-band coded channel for a given subjective signal/noise ratio at the synthesizer output? If a 100-times bandwidth reduction is accompanied by a 10-dB increase in signal/noise requirements then, from a radio standpoint, there would be little advantage in using a vocoder.

To what extent is the individuality of a caller's speech likely to be affected by a practical vocoder? Can this individuality be retained solely at the expense of bandwidth, or is it dependent solely on the degree of sophistication of the vocoder?

Mr. D. Bayley: Since it seems that a good deal of the individual characteristics of a speaker are contained in the fundamental frequency and overtones of the larynx tone does Mr. Williams envisage the possibility of replacing the uniform spectrum noise source by some type of pitch-controlled periodic waveform generator? Can Mr. Williams also say what maximum compression ratio he hopes to be able to achieve?

Mr. R. F. Bye: I have always understood that producing speech is more complex than reducing a speech waveform to a number of varying voltages. The speech produced artificially by PAT was much clearer than by the compressed speech signals. Can the voltages produced by the 'formant tracker' be used to operate PAT, and by passing these voltages over a line to PAT, could this save bandwidth?

AUTHOR'S REPLY

I agree with Mr. Powell that the quality of received speech is improved if the receiving instrument accepts the frequencies between 200 c/s and 300 c/s. This was only marginally apparent from my demonstration of speech with and without these frequencies, because the filter used does not have a very sharp cut-off. It is a commercially available variable electronic filter, with a slope of 24 dB/octave in the attenuation/frequency characteristic at frequencies considerably removed from the half-power points. There is, however, some rounding of the characteristic at these -3 dB points, so that when the filter was set for a nominal low-frequency cut-off of 300 c/s there was still an appreciable 200-c/s speech component present. It will be recalled that when the low-frequency half-power cut-off was adjusted to 400 c/s or 500 c/s the degradation of quality was much more marked, and this was because of the much greater attenuation which then occurred in frequencies between 200 c/s and 300 c/s. We must, of course, distinguish between quality and intelligibility. The low frequencies are necessary for good quality, but they add comparatively little to the intelligibility.

The filters mentioned by Mr. Powell were used for demonstrating the effect of filtering on ordinary speech. Those referred to by Mr. Doubell on the other hand are used with the formant tracker. I would agree that it is desirable that the filters used for the tracker should all introduce the same transit time and phase shift. It does not appear to be essential, however, because the various rectifying and smoothing processes result in slowly-varying direct voltages which do not seem to be seriously affected by any phase shifts and time delays occurring in the present filters.

Mr. Wilkinson's comment on the economics of vocoder systems draws attention to this very important factor in deciding whether a vocoder system is appropriate to any given set of conditions. The provision of extra bandwidth is not always possible, though the time may yet come when the use of millimetre and optical wavelengths will provide bandwidth in abundance.

I agree that an element of redundancy is desirable as a safeguard against jamming and interference. The degree of redundancy required will, however, depend on the system used and the conditions under which it operates. On the question of signal/noise ratio, it has been estimated by Flanagan[29] that, on the average, signal/noise ratios of 33, 24, and 20 dB and bandwidths of 7·1, 6·7, and 5·3 c/s are sufficient for the transmission of signals specifying the frequencies of the first, second, and third formants respectively. For the transmission of speech by conventional methods, a signal/noise ratio of 30 dB is normally considered satisfactory.

Turning to the effect of a practical vocoder on the individuality of a caller's speech, it is probable that no vocoder yet developed is perfect in this respect. Identification of speakers may certainly be improved by increasing bandwidth (as in hybrid vocoders, for instance) but it is also true that some improvement may be sought by increasing the complexity of the vocoder. The circuits for dealing with the larynx tone are particularly important in this respect, as suggested by Mr. Bayley, and I am proposing to carry out experiments with a controllable waveform generator. The degree of compression for which I am aiming is about 15 to 1.

I heartily agree with Mr. Bye that the speech produced artificially by PAT, the Lawrence synthesizer, sets a very high standard of naturalness and intelligibility. It is almost inevitable however that live speech submitted to an analysing process before being re-synthesized by PAT would be somewhat less perfect, because of the additional degradation which would occur in even the best of analysers. The result might, however, still be better than that obtained with a narrow-bandwidth system of conventional telephony. There would be no difficulty in principle in using the voltages produced by the simple formant tracker to operate PAT, but some alteration to the output circuits would probably be necessary.

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