INTERNATIONAL GEOPHYSICAL YEAR

The third Polar Year will extend from July 1957 to December 1958 and there is much scientific interest in the plans which are now being made by the International Council of Scientific Unions to arrange observations in all latitudes.

The second Polar Year came at a time of minimum sunspot activity, and radio investigation should be helped by the fact that in 1957-58 sunspot activity should be nearer its maximum.

The scope of the geophysical observations to be undertaken will be much wider and more comprehensive than during the two previous Polar Years. Ionospheric physics, meteorology, geomagnetism, and the airglow, are some of the subjects to be specially studied according to the plans now being made by the Comité Spécial de l'Année Géophysique Internationale.

Three meridian chains of stations have been planned. The first of these extends along approximately the meridian of 80° W. from the Arctic, through North America, down the west coast of South America, and into the Antarctic. The second is along the meridian of 10° E., through Western Europe, Tunisia, the Sahara, and along the west coast of Africa to the Antarctic. The third is along the meridian of 140° E., through Soviet territory, Japan, New Guinea and Australia.

These meridian chains are of special importance for the investigation of the general circulation of the atmosphere. The close association between solar activity and terrestrial phenomena, such as geomagnetic disturbances, ionospheric storms and the vagaries of radio propagation, is well established. Little is known, however, about the physical processes involved and in the observations which are to be made in 1957-58

radio methods will form an essential part of the research programme.

Balloons will be widely used to obtain information about the density, pressure and temperature of the atmosphere, of wind velocities and of cosmic ray intensities up to heights of about 20 miles. Aircraft specially fitted with radio equipment will be used for special soundings of the ionosphere. The most important new technical developments to be used in this enterprise will be rockets, and the measuring devices which they carry. Guided missiles capable of reaching heights up to 125 miles, and smaller rockets for launching from balloons or aircraft and capable of reaching heights up to 50 miles, will be equipped with a variety of recording or telemetering instruments. These will enable the atmosphere to be explored to a much greater height.

Some radio observatories will explore the auroral regions by short-wave beams, and the electrical current systems in the atmosphere, which cause some of the observed variations in the Earth's magnetic field, will be investigated.

Intensive observations are desirable during periods of outstanding solar activity, but such times cannot be predicted with any certainty long in advance. A plan has been formulated, therefore, whereby an alert will be transmitted through communication networks whenever unusual solar activity seems likely to occur.

Present indications are that the International Geophysical Year in 1957-58 will be one of the outstanding examples of international co-operation. Much of the means of observation will be of considerable interest to radio and electronic engineers and the results may be of tremendous value, internationally, to communication engineers.

B

NOTICES

Institution Dinner

The date now agreed for the Institution dinner, postponed from the date of the General Election, will be on Wednesday, June 15th, 1955, at the Savoy Hotel, London, W.C.2 (Abraham Lincoln and Manhattan Rooms).

The Immediate Past-President and Mrs. Miller will be the guests of honour and the reception by Rear-Admiral Sir Philip and Lady Clarke will be from 7 p.m. to 7.30 p.m. It is hoped that members who are attending will be accompanied by their ladies. Tickets, which cost £2 5s. each, cover dinner and cocktails, and may be obtained from the General Secretary. Early application is advised.

The Building Appeal

Since the last Annual Report further donations have been received from firms in the industry who are supporting the appeal for £50,000 toward the purchase of a freehold building for the Institution.

The President sent to the membership on March 12th a personal letter appealing to every member to support the Council in this project, and a list of all subscribers will be published in the near future.

Insignia Award in Technology

At a recent meeting of the Council of the City and Guilds of London Institute, the Insignia Award in Technology was conferred upon nine candidates. These included two in the Tele-communications group; Mr. Arthur Horace Watkins (Associate Member) receives the award for his thesis entitled "The Setting-up and Testing of a Wideband Coaxial Telephony Line Link."

This was the second series of such awards based on thesis and interview and brings the total of C.G.I.A.'s in this category to 19. A number of Special Awards have also been conferred on persons of distinction in various industries.

The Insignia Award in Technology was instituted by the City and Guilds of London Institute in November 1952,* and it is stated by the Council that the development of the scheme has been both steady and encouraging. The standard of the award has been maintained at the high level envisaged by the Institute, and of 43 theses so far received from candidates 15 failed to satisfy the assessors and of the remainder nine failed at the interview.

* See *J. Brit.I.R.E.*, 12, p. 550, November 1952.

The Caesium Atomic Frequency Standard

Development of an atomic "clock," the Caesium Atomic Frequency Standard, which is of excep-tional accuracy, has recently been announced by the Director of the Laboratory for Nuclear Science at Massachusetts Institute of Technology. Manu-facture of a commercial model of the clock has been started and will be available by next year.

The primary purpose in developing the apparatus was to build a tool for scientific research, for example, in work on the General Theory of Relativity. Other possible uses for such a precise timing device would include: Navigation—present methods enable a ship to locate its position within only five or ten miles; the caesium device will give a very precise location. Astronomy—very minute changes in the movement of the earth, moon, stars and other bodies can now be measured. Com-munication—synchronization of events in different places can be attained with remarkable accuracy.

Time-keeping in the device is controlled by the oscillation of electrons in the caesium atom, just as a pendulum controls a clock or a balance wheel controls a watch. As an electron revolves around the nucleus of an atom, it "wobbles" very slightly but at a constant rate. This unvarying rate of oscillation is reflected in the frequency of waves that are emitted.

Caesium has a frequency of approximately $9192 \cdot 632$ Mc/s which is the unit of time in the clock. A piece of the metal, about the size of a pinhead and weighing about $0 \cdot 01$ gm, is enclosed in a crucible kept at about $100°C$. A stream of atoms shoot through a hole in the crucible and strike a detector screen, their frequency being measured electronically. The caesium emits atoms at a rate of the order of 10^6 per sec, but the loss is only about 10^{-6} gm per day.

While for scientific work clock hands would not provide sufficiently precise timing, the device can be connected with a conventional but finely calibrated clock face for use in such operations as navigation.

Standard time, with one second equal to 1/86,400th of the mean solar day and generally measured by the frequency of the oscillations of a crystal, is accurate to one part in 10^9. The Atomic Frequency Standard is accurate to one part in 10^{10}; through further development it is expected to obtain accuracy of one part in 10^{12}.

A VERSATILE ELECTRONIC ENGINE INDICATOR*

by

R. K. Vinycomb, B.Sc. (*Associate Member*)†

Read before the Institution in London on February 23rd, 1955.   Chairman: Mr. E. D. Hart, M.A.

SUMMARY

Knowledge of the working pressures within the cylinder of an internal combustion engine is of great importance in engine development work. An outline of the history of devices for obtaining this information is given, followed by the requirements for a satisfactory indicator. The principles of operation, construction and temperature sensitivity of a number of different types of transducers are discussed and examples described. A trolley-mounted cathode-ray engine indicator is described which is arranged in a novel layout for use with several types of transducers. Details are supplied of the pre-amplifiers and pre-circuits employed and special consideration is given to facilities for time or crank-angle sweep, crank-angle marking, beam blanking and photography. The practical operation of the indicator is outlined in order to illustrate how a wide range of measurements can be made.

1. Introduction

1.1. *Historical*

Early in the history of heat engines, it was realized that a knowledge of the cyclic pressures within the working cylinder would provide invaluable data on the performance of the engine. The first mechanism to fulfil this purpose is usually ascribed to James Watt. It appears, however, that the invention of a device which plotted the cylinder pressure on paper is due to John Southern, one of Watt's associates, about 1796,[1] and this has become known as "The Steam Engine Indicator." In principle it comprised a small cylinder connected to the engine cylinder by a passage. The steam pressure was allowed to act on a spring-loaded piston whose displacement therefore was proportional to the applied pressure. A link mechanism connected the piston to a pencil which moved up and down on a paper chart. At the same time the chart was moved at right angles to the direction of pencil travel. A two-dimensional diagram was thus obtained. The necessity of knowing the phase relationship between the movement of the card and the position of the engine piston was clear to Southern, for, in 1796, he wrote to a colleague:

"Great accuracy should be observed at the moment the piston is at the top or bottom, and a person should be stationed opposite the connecting rod to call out the instant it passes the centre of the sun wheel, another person being stationed to observe the indicator pencil at the same moment."

Although it may nowadays be amusing to contemplate the phasing of an indicator by shouts between the operator of the apparatus and an assistant observing the crankshaft, diagrams obtained by this method enabled the working of steam engines to be visualized and much valuable data was collected in this way.

In due course, the advent of higher speeds and, in particular, of the internal combustion engine, necessitated similar cylinder-pressure measurements and, in addition, information regarding the timing and rates of pressure rise during the combustion period was required. All kinds of slow- and medium-speed machines, including spark-ignition and compression-ignition engines and gas compressors have been, and still are being, indicated by the simple mechanical devices of the kind depicted in Fig. 1. In this the drum carrying the paper chart is rotated by means of a cord attached to the engine piston rod, so that a pressure–volume diagram is obtained. By graphical integration of the diagram the "indicated horse-power" of the cylinder can be computed.

The mechanical indicator has the inevitable disadvantage of its inertia, which limits the rate at which it can respond to pressure changes. Even with the latest types, there comes a point

beyond which the accuracy of measurement decreases rapidly as the engine speed increases. The need for instruments capable of indicating faithfully the cylinder pressure fluctuations in high-speed internal combustion engines has led to the development of a number of extremely useful types.[2] One approach to this problem was the design of an indicator in which the engine cylinder pressure is measured by balancing

Fig. 1.— *Part-sectional drawing of modern mechanical diesel engine indicator.* (*With acknowledgments to Messrs. Dobbie, McInnes, Ltd., Glasgow.*)

it against a known steady air pressure in a transducer which detects this point of balance by interrupting an electric circuit.[3] In this case, the recording drum is driven from the engine crankshaft and the indicator card appears as a series of minute holes sparked through black recording paper.

Probably the most versatile indicators use a cathode-ray tube as the display element[1] because of its virtually inertialess response, and the remainder of this paper is confined to this type, which is supplanting all others to an increasing extent.

1.2. *Requirements*

In the early stages of internal combustion engine development, attention was almost wholly centred on measuring the pressure cycle in the cylinder and, in particular, the peak pressures and rates of pressure rise during the combustion period. More recently, it has become evident that a really useful instrument must be able to display, not only cylinder-pressure diagrams, but also many related phenomena. A modern indicator must therefore be capable also of measuring the pressures at different points in the induction and exhaust manifolds and in the fuel-injection system of a diesel engine. Only a further short step is required to the measurement of mechanical events such as the timing and extent of the needle lift in a fuel-injection valve, the vibration of the engine bed and components and the torsional oscillation of the crankshaft. A truly versatile indicator is thus an indispensable tool by means of which the engine designer or fuel technologist can investigate every aspect of an engine's performance.

Although a cathode-ray tube may be used as the display element for all these requirements, a number of different transducers must be employed.

2. Transducers

2.1. *Principles*

There are a number of different types of transducer available for making the same measurement[5] and a number of different measurements to be made. The choice for any particular application must depend on a number of factors, which include the accuracy needed, the temperature, the engine speed, the pressure range, the ease of installation and operation and, not least, the cost. Most known mechano-electric effects have been used in the past as the basis of pressure transducers suitable for electronic engine indication. Before it was possible to design stable high-gain amplifiers, the chief requirement for the transducer was that it should have a high sensitivity, which may
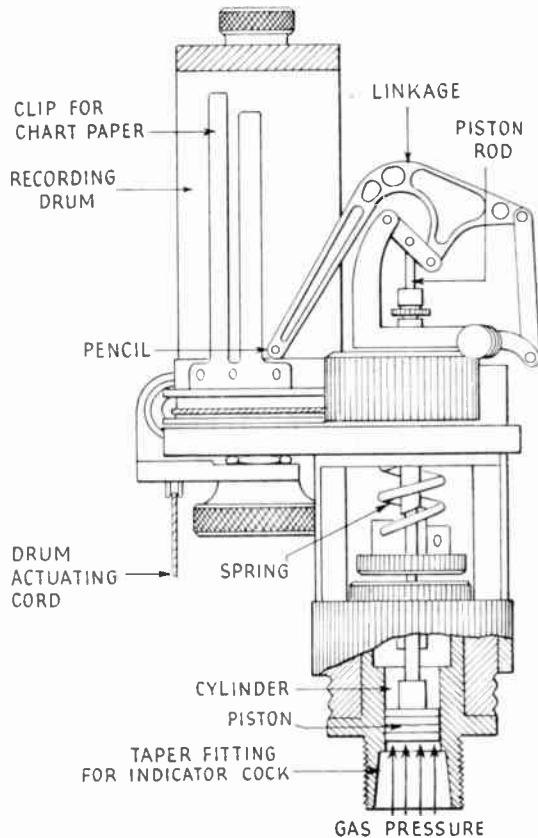
be defined as the output voltage for a change of pressure equal to the maximum for which the particular pick-up was designed. Piezo-electric crystals[6] or a carbon resistive element[7] satisfied this demand, but neither has continued in use because of poor stability under operating conditions. They have been largely superseded by transducers of the electro-magnetic,[8, 9] variable-capacitance[10, 11, 12, 13] and variable-inductance[14] types.

All transducers and their associated equipment must contain a number of basic units, although the function of more than one may be combined in a single component. These are analogous to the parts of the mechanical indicator and may be conveniently termed:

(1) The seal, which prevents the gases from escaping to atmosphere or to other parts of the transducer.

(2) The summing member (e.g., the indicator piston) on which the gas pressure acts to produce a resultant thrust.

(3) The measuring element (e.g., the piston spring) which measures this thrust.

(4) The coupling linkage, which conducts, and possibly amplifies, the output of the measuring element to—

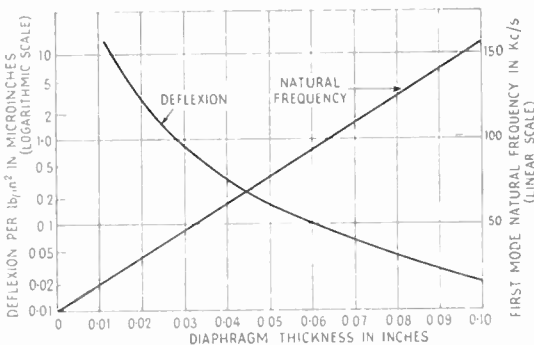(5) The display. This may be graphical, as in the case of the mechanical indicator, or visual.

Fig. 2.—*Theoretical performance of a flat circular diaphragm, ½ in. in diameter, clamped at its periphery.*

It is highly desirable that the relation between applied pressure and resulting output should be linear. This can be met by making the measuring element a linear compliance so that:

$$\delta = \frac{pa}{k},$$

where: δ = deflexion of the measuring element,

      k = stiffness of the measuring element,

      p = intensity of gas pressure,

      a = effective area of summing member.

In a given transducer, therefore, δ is proportional to p.

A further requirement is that faithful response to high rates of pressure rise shall be achieved. This means that the natural frequencies associated with the system must be high. The mechanical natural frequency f_n of the relevant parts of the transducer is given by:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

where m is the effective mass of the moving parts. This must therefore be kept as small as possible while the stiffness is high. A satisfactory transducer thus demands an electrical detecting element which gives as large as possible an output linearly related to the very small mechanical deflexions which are obtained.

2.2. *Practical Systems*

Many transducers employ a metal diaphragm clamped at its periphery and housed within a robust body. The diaphragm thus combines the functions of seal, summing member and measuring element since pressure is applied to one side of it while an electrical sensing element on the other side detects the resulting deflection. Fig. 2 shows the relations between natural frequency, deflexion and thickness of a ½-in diameter clamped steel diaphragm. In a typical case, a diaphragm of 0·050 in thick might be used for a maximum pressure of 500 lb/in.² The maximum deflexion would therefore be 100 microinches and the natural frequency about 80 kc/s.

In electromagnetic transducers, the diaphragm is made part of a magnetic circuit in which there is a permanent magnet and a fixed coil. For small movements of the diaphragm, the flux density in the coil is proportional to the diaphragm deflexion δ.

Thus: $\phi = cp$.

Hence: $e = -\dfrac{d\phi}{dt} = -\dfrac{c\,dp}{dt}$,

where: ϕ = flux,

c = a constant depending on the construction of the transducer,

p = intensity of gas pressure,

e = generated e.m.f.,

t = time.

It is thus apparent that this type of transducer produces an output proportional to the rate of change of pressure with time.[15]

The simplest form of variable–reactance detecting element consists of an insulated electrode mounted in close proximity to the diaphragm so that the electrical capacitance changes in accordance with the deflexion of the diaphragm. The capacitance of a parallel-plate capacitor is given by:

$$C = \frac{aK}{4\pi S},$$

where C = capacitance,

K = permittivity of dielectric,

S = separation of plates.

RETAINING NUT

ELECTRICAL CONNECTION

SPLIT CONICAL RING

MICA BUSHING

FIXED ELECTRODE

DIAPHRAGM

WATER-COOLING JACKET

GAS PASSAGE
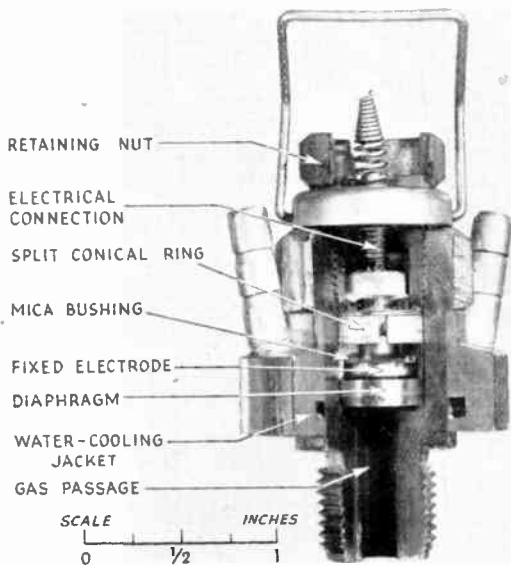
SCALE   INCHES

0   ½   1

Fig. 3.—Cutaway view of a variable-capacitance pressure transducer.

In a simple system in which one plate moved parallel to the axis of the capacitor, the capacitance would thus be inversely proportional to the spacing. However, a clamped diaphragm becomes curved when subjected to pressure.
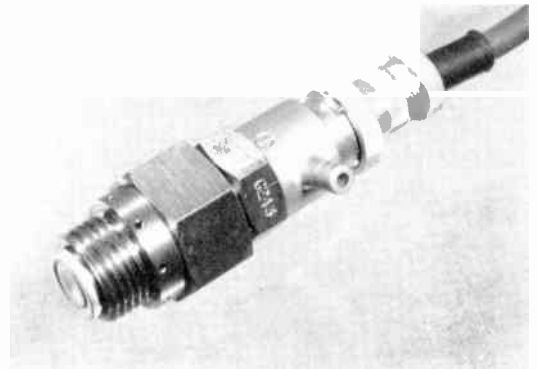

Fig. 4.—Flush-diaphragm pressure transducer employing a variable resistance element. The overall diameter of the diaphragm is approximately ⅝ in.

By curving the fixed electrode also and using a composite dielectric of mica and air,[16] it has been found possible, by empirical means, to produce a transducer in which the pressure–capacitance relationship is linear to within 1 per cent for a capacitance change of 20 pF. This type of transducer is illustrated in Fig. 3.

Another way of detecting the movement of the diaphragm employs a photo-emissive cell.[17] A small shutter fixed to the centre of the diaphragm controls the light falling on the photocell from a miniature electric lamp. The arrangement is such that, within its working limits, the output from the cell is directly proportional to the pressure applied to the diaphragm.

A transducer in which a wire resistance strain gauge is used as the sensing element is shown in Fig. 4. In this case, the diaphragm acts only as the seal, while the restraining force is supplied by a small load cell comprising a hollow tube with one end in contact with the diaphragm and the other fixed in the body of the unit. Two strain gauge windings are bonded to the surface of the tube. In one, the wires run parallel to the axis of the tube while the other consists of a bifilar winding wound circumferentially. The construction of this "strain tube" is shown in

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Fig. 5. The two windings are usually connected in adjacent arms of a simple Wheatstone's bridge. When the strain tube becomes loaded in compression, the axial gauge decreases in resistance, while the circumferential gauge resistance increases due to the effect of Poisson's ratio. These resistance changes thus have an additive effect in causing unbalance of the bridge.

2.3. *Temperature Effects*

The effects of temperature upon transducers may be divided into two classes; those due to temperature alone and those due to temperature gradients. Heat alone causes changes in the physical constants of the materials of the pickup. If inconsistency of the temperature coefficient of expansion is neglected, it can be shown that:[18]

$$\frac{k_{T1}}{k_{T2}} = \left(\frac{1 + r_{T2}}{1 + r_{T1}} \right) \frac{E_{T1}}{E_{T2}},$$

where k_{T1}, k_{T2} = diaphragm stiffness at temperatures T_1 and T_2,

r_{T1}, r_{T2} = Poisson's ratio at the same temperatures,

E_{T1}, E_{T2} = Young's modulus at the same temperatures.

STRAIN GAUGE WINDING   FIXING FLANGE   GAUGE CONNECTIONS

HOLLOW STRAIN TUBE

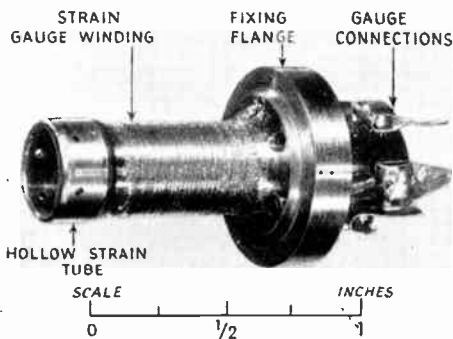SCALE   INCHES
0   1/2   1

Fig. 5.—The sensing element of the transducer shown in Fig. 4.

The output of the transducer is proportional to the stiffness, k, of the measuring element, so when the latter becomes heated it will need application of a lower pressure to reach the designed deflexion. In practice, a temperature rise of 300°F can cause an increase of 25 per cent in sensitivity. Steps are therefore taken to keep the measuring element from being exposed directly to combustion flames by mounting it at the end of a short passage, by water-cooling the transducer or by direct cooling of the measuring element itself.

Temperature gradients may be transient ones due to the intermittent application of combustion gases to the unit, or stable ones due to the continuous conduction of heat away from the engine by the transducer. These gradients cause the parts of the transducer to distort due to unequal expansions; momentary distortion is caused by an intermittent gradient and sustained distortion by a steady gradient. The former results in an electrical signal being produced (due to displacement of the summing member and measuring element) which is superimposed upon, and indistinguishable from, the signal due to the application of dynamic engine pressures. The latter effect is of importance in transducers of the "non-generating" type (i.e., those such as the variable–impedance or photoelectric types which have to be supplied with some external form of power) which generally produce an output for a sustained pressure input. This type of pickup is of particular advantage because it enables the steady component of a fluctuating pressure to be measured and, furthermore, the transducer can be calibrated by applying known steady pressures to it. Flat clamped diaphragms are thus particularly liable to distortion if the diaphragm is not kept sufficiently cool. The high sensitivity of transducers employing variable capacitance systems tends to emphasize this trouble, which manifests itself as a slow shifting of the zero-pressure or base line of the diagram on the cathode-ray tube screen. If the zero can be set when the transducer is at working temperature, the inaccuracy is obviated.

In the case of the variable-resistance transducer shown in Fig. 4, severe changes in temperature of the relatively compliant diaphragm affect neither the sensitivity nor the "zero" of the measuring element. Cooling air is used to reduce the temperature of the strain tube. When the two strain-gauge windings are connected in a simple bridge circuit as described previously, the effects of temperature change on the resistance of the windings tend to cancel.

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3. A Complete Equipment

3.1. *Pre-circuits and Pre-amplifiers*

In spite of the large variety of transducers which are available not only for pressure indication, but also for many other measurements on an engine, only two kinds of amplifier are needed to meet nearly all eventualities. The first is a moderately high gain direct-coupled type and the second a modulated radio-frequency carrier type. Appropriate pre-circuits are required in each case between the transducer and the amplifier. Taken together, the pre-circuit and pre-amplifier form the "coupling linkage" analogous to the mechanical levers of the steam engine indicator, and must therefore have a strictly linear input-output characteristic and a flat frequency response from d.c. to the highest frequencies to be encountered.

Two of the most important features to the engineer are the ability of the amplifier systems to respond to sustained inputs, and high stability of the "zero" point when transducers which respond to static pressure are employed. The output from any pickup when a sustained pressure is applied to it then produces a sustained deflexion of the cathode-ray tube beam. Advantages derived from this are that there can be no errors due to lack of response of the amplifier, however slowly the pressure is varying, and the overall system can be calibrated statically by simply applying known steady pressures to the transducer.

When drift takes place, it inevitably introduces inaccuracy into the measurement of absolute pressures if the zero cannot be checked. The amplifiers and power supplies are thus designed to minimize drift. In choosing a suitable direct-coupled pre-amplifier, one having an amplified feedback circuit which compares input and output levels while the amplifier is operating, without affecting the signal,[19] showed outstanding advantages. A two-stage fully-balanced amplifier of this type provided a gain of 100 at a bandwidth of 50 kc/s which is more than ample for engine indicator applications. In the high-frequency carrier system which is employed,[20] the oscillator may be placed close to the transducer and a frequency-modulated signal is conveyed to the amplifier and demodulator by a long cable if necessary. This eliminates nearly all the disadvantages usually associated with carrier systems. The carrier frequency

is approximately 2 Mc/s and a limiter stage in the pre-amplifier ensures immunity from external interference and amplitude changes. The frequency response of the oscillator–amplifier–demodulator system is uniform from zero to about 30 kc/s.

Associated with the drift-corrected direct-coupled pre-amplifier there is a Wheatstone's bridge which enables up to four variable resistance transducers to be connected and balanced and their outputs instantaneously selected for display. A high-tension dry battery is used to energize the bridge. The circuit arrangement is shown in Fig. 6. The four inputs are each intended for pickups or strain gauges having matched pairs of windings. The four
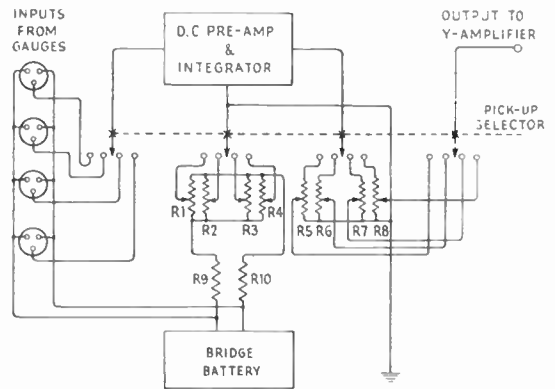
Fig. 6.—Schematic diagram of the pre-circuit used with variable-resistance transducers.

poles of the selector switch are ganged so that at the same time as the pickup is selected, the appropriate ten-turn helical balancing potentiometer (R1, R2, R3 or R4) and attenuator (R5, R6, R7 or R8) are brought into circuit. All four pickups and the ratio arms, R9 and R10, are energized by the bridge battery at all settings of the selector. Attenuation is carried out at a relatively low-impedance point since the output from the pre-amplifier is taken from a cathode-follower stage.

In conjunction with the frequency-modulation amplifier-discriminator there is a panel containing an oscillator and pre-circuits needed for up to three alternative transducers. This is shown in Fig. 7. The output of the discriminator may be passed through differentiating circuits

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if required so that either true pressure or rate-of-change of pressure diagrams can be obtained from variable capacitance transducers. The tuning controls C1, C2, C3 enable the oscillator frequency to be adjusted to 2 Mc/s. The range controls S1, S2 and S3 alter the L/C ratio of the
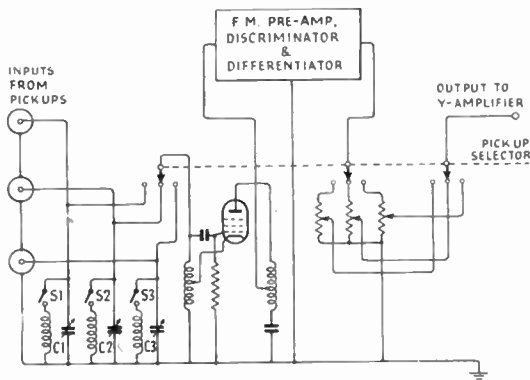

Fig. 7.—*Schematic diagram of the pre-circuit used with variable-reactance transducers.*

tuned circuit so that the required frequency deviation is obtained for incremental capacitances of either 5 pF or 20 pF.

3.2. *Layout and Special Features*

Because of the considerable flexibility required, the equipment comprising the "coupling linkage" and the "display" is built in rack-and-panel form and mounted on a trolley base, as depicted in Fig. 8. Arrangements to meet special requirements can, if necessary, be made by substituting or adding panels. Fig. 9 is a schematic diagram of the apparatus.

Either of the pre–amplifiers may be fed into a balanced, two-stage direct-coupled driver amplifier, having a gain of 800 times, which feeds the Y-plates of the cathode-ray tube. Each stage comprises a cathode-coupled pair of valves. Negative feedback is applied to the first-stage cathodes from each of the final-stage anodes. The cathode-ray tube itself has a 6-in diameter screen and is of the type having a post-deflection accelerator so that maximum brightness of the spot can be obtained when working in high ambient light. It is on a hinged panel which tilts forward for easy viewing.

H.T. and e.h.t. power, valve stabilized in each

case, is supplied to the equipment from built-in packs, operating from a.c. mains. The additional e.h.t. required for the post-deflection accelerator of the cathode-ray tube is obtained from a voltage-doubling rectifier supplied from the secondary of the h.t. transformer. There is also a regulated d.c. supply for the heaters of the valves in the d.c. pre-amplifiers to assist in the elimination of drift and hum.

3.2.1. Crank-angle marking

As nearly all phenomena on an engine are related to the crank revolution, it has been found most useful to mark the diagram with a scale representing crank-angle degrees. This is achieved by fixing a non-magnetic disk to the end of the crankshaft and arranging for its periphery to pass close to a simple electro-magnetic pickup. The disk has steel inserts at accurately-set intervals and these cause the pickup to generate voltage pulses as they pass.
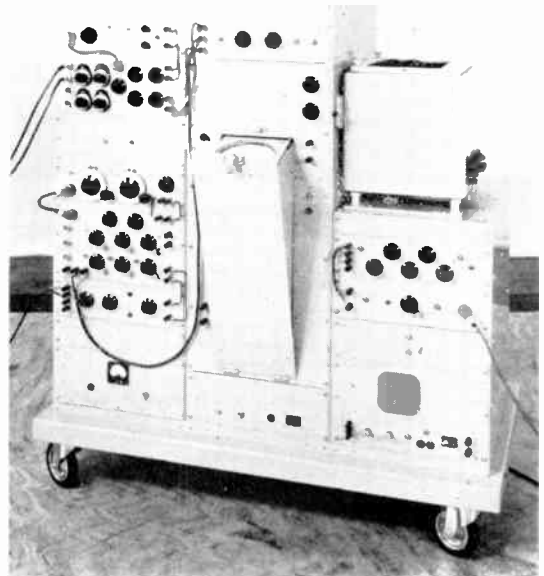

Fig. 8.—*The complete engine indicator equipment.*

These pulses are passed to a shaping amplifier which ultimately superimposes signals on the record. These may conveniently be at 20 deg intervals.

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3.2.2. X-axis sweep

The horizontal axis of the indicator diagram may be provided by a conventional electronic time-base with a push-pull cathode-coupled deflection amplifier in the same panel. The sweep can be synchronized to the engine speed, or a sub-multiple of it, if another simple electro-magnetic pickup is used to generate a pulse from a stud in the engine flywheel, or by some other convenient method. To obviate the possibility of the time-base becoming out of synchronism when the engine speed changes, a pair of contacts, driven by the engine, may be used to discharge the time-base capacitor. This gives positive synchronization but has the dis-advantage that the sweep length varies with engine speed.

The best method of obtaining the horizontal sweep makes use of the photo-cell device shown in Fig. 10. In this case, a helical shutter, driven from the engine, controls the light falling on a photo-emissive cell. The latter is connected to a direct-coupled pre-amplifier whose output is fed to the time-base amplifier. The horizontal position of the spot on the cathode-ray tube screen thus bears a definite relationship to the angular position of the crankshaft under all conditions. In the photograph, a degree-marking disk can also be seen, fitted within the sweep unit case.

3.2.3. Camera

It is often difficult to measure and analyse diagrams presented visually, especially under transient conditions, and so a multi-purpose recording camera has been built into this equipment. This makes possible three kinds of permanent photographic record. "Still" photographs of the diagram on the cathode-ray tube screen may be taken with the film stationary. Alternatively, the horizontal sweep can be stopped and the recording film moved continuously past the camera lens. The "time-base" is thus provided by the movement of the film and this may be increased in speed to spread out the diagram. At the highest speeds, where the maximum temporal resolution is required, a 20 in length of film is wrapped round a drum which is rotated steadily at any desired speed. In this case, the length of record is necessarily
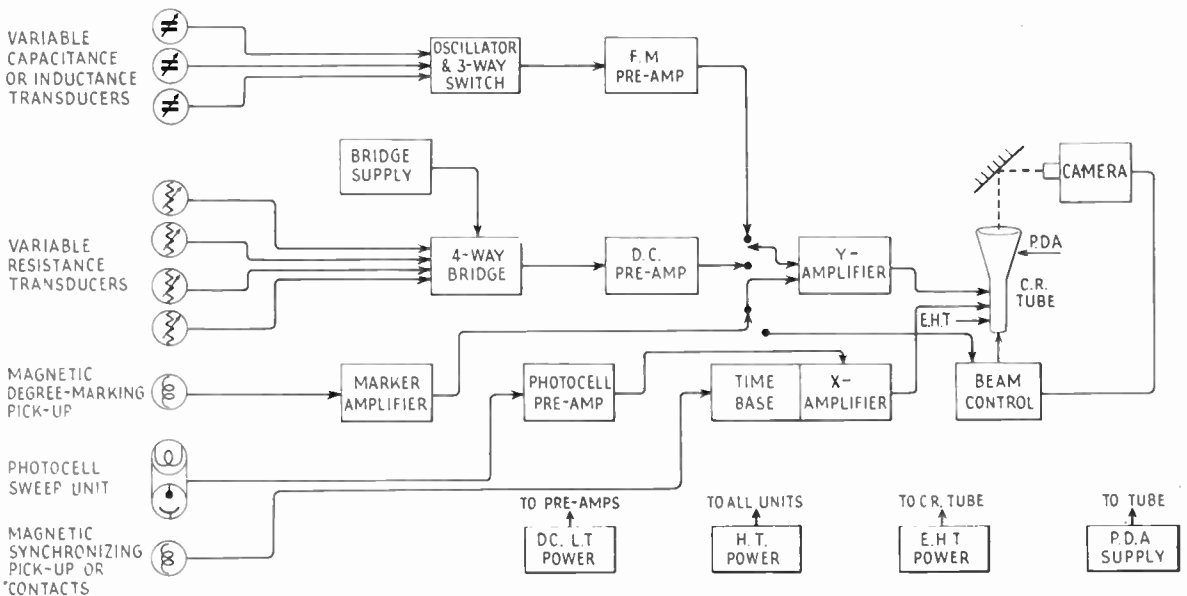

Fig. 9.—Block diagram of the engine indicator equipment.

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limited to the circumference of the drum, but it does enable single diagrams of high resolution to be obtained. Contacts on the camera control the exposure by switching on the cathode-ray tube beam for exactly one revolution of the drum, or for the duration of the film movement
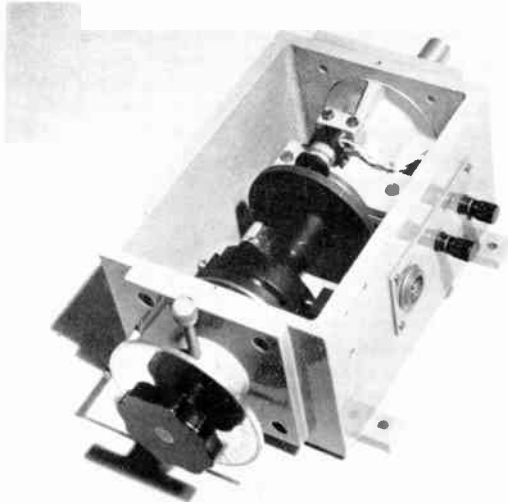

Fig. 10. – *Photo-cell sweep unit for coupling to the engine crankshaft.*

for a "continuous feed" record. The camera motor is normally supplied from a 6-volt automobile-type accumulator.

3.2.4. Beam switching

Since the cathode of the tube is nearly 2-kV negative with respect to the "earthy" anode, it is undesirable to switch the grid circuit directly. Carrier-current beam switches have been very recently discussed[21] and their advantages pointed out when sustained beam-brightening or blanking is required. The particular arrangement used in this indicator, while more complex than some, has the merit that coupling between high- and low-voltage parts of the circuit is by two small capacitors, so that no high-insulation-resistance transformers are required. In addition, no appreciable power is drawn from the oscillator and the system may therefore be used to switch the beams of a number of cathode-ray tube-guns simultaneously. The basic circuit is shown in Fig. 11. One half of the double-triode

V1 oscillates at approximately 3 Mc/s, while the other half is normally cut off by a negative bias obtained from the network R1, R2, R3. Closing the contacts S1 causes this triode to conduct, so cutting off the oscillator by the bias developed across the common cathode resistor R4. The oscillations are fed through capacitors C1 and C2 to a diode rectifier which forms part of the valve V2. The triode portion of this valve is thus held in a non-conducting condition by the rectified signal. No current then passes through the brilliancy control R5, and the tube grid is held negative with respect to its cathode by the potential across the neon valve N1. When oscillation ceases, the triode portion of V2 conducts, and current passes through R5 until the separate diode also conducts and holds the triode cathode and the tube cathode at the same potential. The beams then come on at a brilliancy determined by the setting of R5. When S1 is closed, the beam is on, but may be extinguished momentarily by applying a negative pulse to the MOD terminal. Conversely, the beam is normally off when S1 is open, but may be brought on by the application of a positive pulse to the MOD input.

3.3. *Operation*

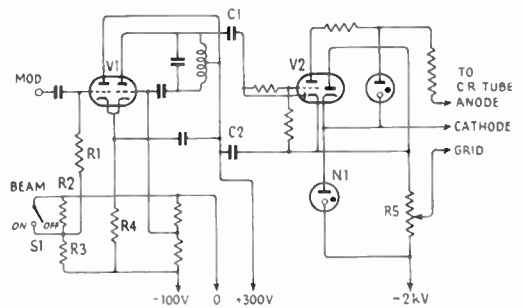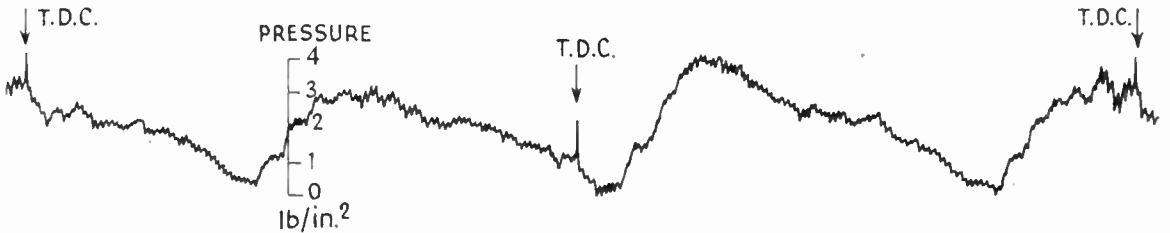Pressure transducers of the variable-capacitance-type are available, having a maxi-

Fig. 11.—*Simplified arrangement of the r.f. beam-switching circuit.*

mum pressure range of a few inches of water-gauge up to about 10,000 lb/in^2. Variable resistance pickups having ranges from a few hundred lb/in^2 up to many tons/in^2 are made.

Other types of pressure, vibration and movement transducers may be used. The pickups
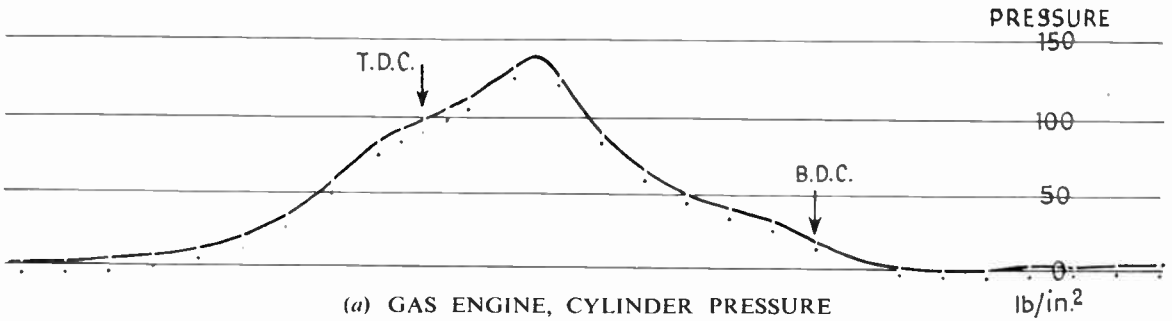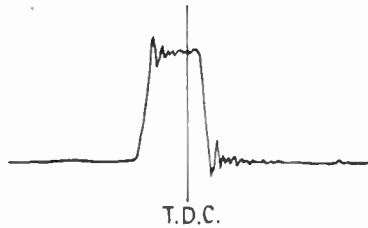
(a) GAS ENGINE, CYLINDER PRESSURE

Obtained from a small gas engine with a variable-resistance transducer. Crankshaft angle marks are superimposed every 20° and at 10° each side of top dead-centre. Calibration lines were superimposed photographically.
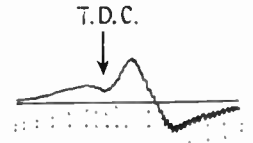

(b) SUPERCHARGED DIESEL ENGINE, EXHAUST MANIFOLD PRESSURE

This record was obtained with a variable-capacitance transducer, a stud on the flywheel being used to indicate the position of top dead-centre. The pressure scale was determined subsequently.
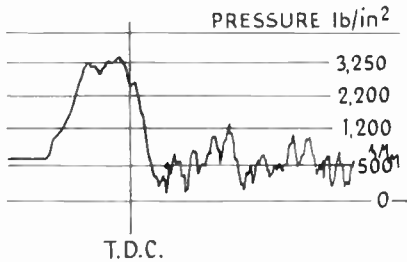
(c) DIESEL ENGINE, FUEL LINE PRESSURE

(d) DIESEL ENGINE, FUEL INJECTOR NEEDLE LIFT

(e) PETROL ENGINE, DIFFERENTIATED CYLINDER PRESSURE

Record from a variable-capacitance transducer fitted to the high-pressure piping between the fuel pump and injector of a diesel engine. Calibration lines superimposed photographically.

A variable-inductance transducer fitted to the fuel injector itself indicates the timing and extent of movement of the injector needle.

This record was obtained from a variable-capacitance transducer. The crank-angle marks are 20° intervals, with additional marks 10° each side of top dead-centre.

Fig. 12.—*Facsimiles of photographic records obtained from the equipment and reproduced at approximately × ⅔.*
(a), (c) and (d) are part of "drum" records; (b) is part of a "continuous feed" record; (e) is a "single shot" photograph.
(Diagrams (b), (c) and (d) by courtesy of Messrs. Davey, Paxman & Co., Ltd., Colchester.)

required for any particular investigation are selected and fitted to the engine, then connected to the appropriate inputs. The selector-switches on the pre-circuit panels are now set to each input in turn so that the bridge balance or tuning controls and the attenuators associated with each transducer may be adjusted. This enables the zero of each signal and its scale on the cathode-ray tube screen to be preset. After connecting up auxiliary devices such as synchronizing and degree-marking pickups, the engine is started and conditions brought to those required. Any of the transducers may now be selected and the resulting diagram observed on the screen. If the electronic time-base is in use, the horizontal sweep may be expanded so that one-quarter of an engine revolution occupies the full diameter of the screen When using the photo-cell sweep unit, the same expansion is possible and, by means of a phasing control on the unit, any part of the diagram can be brought to the centre of the sweep. Degree marks can be superimposed on the diagram, either as sharp upward or downward "spikes" or as beam blanking. Use of the direct-acting horizontal-shift control enables different parts of the diagram to be brought to a convenient part of the screen in order to measure the interval between related events in the cycle in terms of crank angle.

Two methods of making measurements in the Y-direction are possible. In either case, it is assumed that the transducer has been previously calibrated statically so that its sensitivity is known in terms of the arbitrary divisions of the calibrating dials. The continuously variable sensitivity controls for each transducer may be set so that some convenient relationship exists between the divisions on the transparent graticule in front of the tube screen and pressure applied to the pickup. In this case, the height of the diagram on the screen may be readily converted to pressure. Alternatively, the pressure between any two points on the diagram may be read off by bringing these two points successively to the same line on the graticule by means of the calibrating dials. The change in reading of the dial may then be converted to pressure.

When a photographic record is required, the cathode-ray tube mounting is swung on its hinges into the main panel, the tube beam switched off and the camera shutter opened. Pressing the appropriate trip button on the camera then brings on the beam for the exposure required for a "still" shot, a "continuous-feed" record or a "drum" diagram, as the case may be. 70-mm wide unperforated film or paper is used in the camera which can be loaded or unloaded in daylight. Calibration lines may be superimposed on the record by switching off the signal and re-exposing the film with the cathode-ray tube spot shifted vertically known amounts by means of the calibrating dials. Typical indicator cards are shown in Fig. 12.

4. Conclusion

Some of the principles employed in this indicator are not new and details of several generally known circuits have therefore not been included in this paper. The equipment is, however, a serious attempt to provide electronic equipment which embodies the latest practice with regard to engine-indicating techniques and, in addition, to provide facilities for making many kinds of related measurements which have only received brief mention.

The novel layout of the equipment has been adopted so as to make it as flexible as possible while keeping to logical grouping of controls and ease of operation. An example of the way in which this form of construction proves itself is in a version of the equipment in which a double-gun cathode-ray tube is used with two entirely separate amplifier channels. This enables two related phenomena to be displayed or photographed simultaneously.

5. Acknowledgments

6. References

1. H. W. Dickinson and R. Jenkins, "James Watt and the Steam Engine," p. 228. (The Clarendon Press, Oxford, 1927.)

2. F. J. Woodcock, "Some electrical methods of measuring mechanical quantities," *Proc. Instn Elect. Engrs*, **97**, Part I, p. 136, 1950.

3. H. Wood, "The R.A.E. electrical indicator for high-speed internal combustion engines and gauge for maximum pressures," *Proc. Instn Mech. Engrs*, **104**, pp. 137-146, 1923.

4. A. Towle, "The cathode-ray oscillograph and its application to engine indicators," *Proc. Instn Auto. Engrs*, **32**, p. 754, 1937.

5. R. K. Vinycomb, "Electronic engine indicators, with special reference to some recent American designs." *Proc. Instn Mech. Engrs*, **164**, p. 195, 1951.

6. "The piezo-electric indicator," *The Engineer*, **167**, p. 152, 1949.

7. E. M. Dodds, "Recent developments in engine indicators," *Proc. Instn Auto. Engrs*, **32**, p. 179, 1937.

8. E. S. L. Beale and R. Stansfield, "The Standard-Sunbury engine indicator," *The Engineer*, **160**, p. 167, 1935.

9. E. S. L. Beale and R. Stansfield, "High-speed engine indicators," *The Engineer*, **163**, p. 240, 1937.

10. C. H. W. Brookes-Smith and J. A. Colls, "The measurement of pressure by electro-static systems," *J. Sci. Instrum.*, **16**, p. 361, 1939.

11. C. E. Grinstead, R. N. Frawley, F. W. Chapman and H. F. Schultz, "An improved indicator for measuring static and dynamic pressures," *S.A.E. Journal*, **52**, p. 534, 1944.

12. A. H. B. Walker, "An improved electronic engine indicator," *Engineering*, **162**, p. 361, 1946.

13. S. E. Goodall and R. B. Smith, "Instantaneous fluid-pressure recording equipment," *Engineering*, **148**, p. 127, 1939.

14. F. D. Smith, E. H. Lakey and H. Morgan, "The Admiralty cathode-ray oscillograph engine indicator," *Proc. Instn Mech. Engrs*, **143**, p. 39, 1940.

15. E. S. L. Beale and R. Stansfield, "The Standard-Sunbury engine indicator, Mk 6," *The Engineer*, **192**, p. 215, 1951.

16. M. O. W. Wolfe, "The measurement of fluctuating fluid pressure," *Aircraft Engineering*, **21**, p. 368, 1949.

17. J. G. G. Hempson, "Engineers' problems in the measurement of stress and pressure," *Engineering*, **166**, p. 425, 1948.

18. C. S. Draper and Y. T. Li, "A new high-performance indicator of the strain-gauge type," *J. Inst. Aeronautical Sci.*, **16**, p. 593, 1949.

19. J. A. Colls, British Patent No. 670,801.

20. R. K. Vinycomb, "Electronic aids to vibration measurement," *J.Brit.I.R.E.*, **14**, p. 115, 1954.

21. M. J. Goddard, "The development and design of direct-coupled cathode-ray oscilloscopes for industry and research," *J.Brit.I.R.E.*, **15**, p. 179, 1955.

RADIO COMPONENT SHOW

This year's exhibition of components, valves and test gear, organized by the Radio and Electronic Components Manufacturers' Federation, and open from April 19th to 21st, was the twelfth of the series. The expansion in scope of this last exhibition compared with the previous ten post-war shows was well demonstrated in the exhibits which were described as the "raw materials" of electronic development.

The number of exhibitors continues to increase, there being no fewer than 141 stands this year—11 more than last year. The Federation has stated that it may be necessary to seek larger accommodation, since the capacity of the Great Hall at Grosvenor House is obviously limited. At the close of the exhibition, Mr. C. M. Benham, B.Sc., A.C.G.I. (Member), chairman of the R.E.C.M.F., said that more than 20,000 persons had visited it, . including engineers from 23 countries abroad.

The ever-increasing export figures achieved by the Federation's members were referred to by its president, Sir Robert Renwick (Member), when he formally opened the show, and he pointed out that direct exports had risen by 30 per cent. during the year and in the first quarter of 1955 had attained the monthly value of £1 million.

One of the more interesting features of this year's exhibition was the application of printed circuit technique to actual equipments. Previously, with one or two exceptions, only specimens of the circuit printer's art have been seen, but many manufacturers are now in a position to supply either the basic circuits or special components for use with the printed circuit, and, in some instances, complete prefabricated units; it seems likely that it will only be a short time before this technique is extended to domestic radio and television sets.

In order to take advantage of the reduced operator-time involved in wiring up radio chassis, it is desirable to make the attachment of components as straightforward as possible and consequently the developments by two leading manufacturers, Multicore Solders and Enthoven Solders, of special materials, including cleaning agents, solders and fluxes, are particularly interesting.

Examples of components intended for use with printed circuits were miniature potentiometers, shown by A.B. Metal Products and Morganite, capacitors by Dubilier, transformers by Parmeko, and strip-type plugs and valve-holders by McMurdo.

The forthcoming extension of television broadcasting to Band III was catered for by new types of aerials, both the adaptor type for use with existing Band I aerials by the addition of stubs, and special high-gain independent aerials for the shorter wavelength to be used in fringe areas. The higher frequency will naturally make necessary coaxial feeders of much lower transmission loss, and Aerialite and British Insulated Callender's Cables were among the manufacturers showing suitable examples. Both the Plessey Company and Sydney S. Bird featured multi-channel turret television tuners on their stands, enabling the selection of up to 12 channels in the two bands and, in addition, N.S.F. showed a similar unit covering 13 channels; the usual method of selection is by the switching of incremental inductances.

There is, of course, considerable interest currently within the radio industry in the B.B.C.'s introduction of frequency-modulated sound broadcasting at very high frequencies, and components designed for sets receiving these broadcasts included a turret tuner by Sydney Bird and a special ganged variable capacitor by Wingrove and Rogers; the increased a.f. response now possible is being borne in mind by Reproducers and Amplifiers, who can supply alternative diaphragms for loudspeakers. The majority of valve manufacturers exhibiting at the show included in their ranges triodes suitable for operation as grounded grid amplifiers in both the v.h.f. bands as well as new output pentodes.

Other advances in valves and tubes extended several manufacturers' ranges of valves for special duty, while the trend in cathode-ray tubes is towards the 21-in diagonal screen. Wide-angle scanning, up to 90 deg, is a feature of the new tubes introduced by Mullard and Edison Swan.

There was evidence of the consolidation of production types of transistors by all the leading manufacturers, and Mullard stated that by next year the majority of British hearing aids for export and private purchase will use only transistors.

The Services' research establishments sponsored a combined stand as usual and here again some of the future trends of the radio component industry could be seen. Glass dielectric capacitors have always been an attractive objective by virtue of the cheapness of glass as compared with mica, and S.R.D.E. showed that in some features it was much superior to the older pattern; the temperature coefficient of $+120$ parts in 10^6 per deg C should be improved by further development.

PHYSICAL SOCIETY EXHIBITION

This year's Physical Society Exhibition was the first to be held in the Royal Horticultural Society's New Hall, London, a change made necessary by the increasing pressure on space which had been experienced at Imperial College where the previous 38 exhibitions have been held. During the period from April 25th to 28th, over 18,000 persons visited the exhibition, and the easier accessibility of the Hall, as well as the advantage of having all the stands together, seemed to be generally appreciated.

The increasing use of transistors in all fields of communications and allied engineering was to be seen in the exhibits of several manufacturers. The General Electric Company's Research Laboratories showed a number of arrangements for carrying out tests on semi-conductors generally; Standard Telephones & Cables demonstrated some of the more striking applications of this device in a two-wire telephony repeater, and in a "wire-less" telephone in which germanium diodes and triodes provided full facilities for picking up telephone conversations.

As in the case of the new technique of printed circuits, transistor techniques call for special components, and a particularly interesting display was given by Fortiphone. This included transformers of 0·20 in and 0·25 in thickness, miniature volume controls and switches, and a two-way cable with an outside diameter of 0·024 in including insulation and an outer layer of nylon. This same firm also showed a miniature magnetic amplifier using transductors of about 7 cm³ in volume, and a sub-miniature torque motor. Transistor h.t. generators are a further potentially very useful application and production types were shown having, for instance, an output of 30 V at 100 µA for 1·5 V input, the efficiency being about 65 per cent.

A number of firms showed electronic computers of various types. These included the Ferranti packaged computer No. 1 ("Pegasus"), which is a medium-size general-purpose digital computer suitable for a wide variety of calculations in industry and research. It incorporates an immediate access store of 43 single-word registers consisting of nickel delay lines and a larger magnetic drum store of 4,096 words capacity. Elliott Brothers showed a small general-purpose analogue computer intended to be particularly suitable for the simulation of air-

craft control system calculations, and based on standard units similar to those employed in the very large computer, TRIDAC, built for the Royal Aircraft Establishment. Saunders-Roe introduced the "Miniputer," which is a very compact "table top" analogue machine for solving linear simultaneous differential equations, having time as the independent variable.

An increasing number of manufacturers are now building strain-gauge equipment for a variety of applications and there were several examples of multichannel bridges and display units. Boulton Paul Aircraft showed a six-channel recording oscillograph incorporating a 2-kc/s oscillator for energizing transducers and strain-gauge bridges,* while Elliott Bros. demonstrated a three-channel automatic balancing bridge.† Accelerometers and pressure transducers were shown by several firms including J. Langham Thompson.‡

Many of the Services' research establishments, as well as the establishments of the Department of Scientific and Industrial Research, were represented, and of particular interest was a spectrum analyser for the range 2,400 to 4,500 Mc/s with panoramic frequency display developed by the Admiralty Signals and Radar Establishment. The major interest in this equipment was that it uses a backward wave voltage-tuned oscillator called a Carcinatron, a voltage variation from about 150 to 1,200 V tuning the valve over this range: alternatively this sweep can be provided by a sawtooth voltage and a maximum frequency coverage on the oscilloscope of 1,000 Mc/s can be obtained. Difficulties which might result from the confusion of image and signal frequencies at the screen are obviated by the use of a low intermediate frequency so that the two signals appear almost superimposed.

Items shown by the National Physical Laboratory included a magnetic digital storage drum of improved design, and a multiple-position electrically-steered beam for ultrasonic flaw detection.§

This review was compiled immediately after the exhibition and a further report on other items of interest will be given in the June "Journal."

* L. W. Blick, *J. Brit.I.R.E.*, 14, p. 603, December 1954.
† D. L. Johnston, *J. Brit.I.R.E.*, 14, p. 613, December 1954.
‡ J. L. Thompson, *J. Brit.I.R.E.*, 14, p. 583, December 1954.
§ G. Bradfield, *J. Brit.I.R.E.*, 14, p. 303, July 1954.

THE USE OF CORRELATION TECHNIQUES IN THE STUDY OF SERVOMECHANISMS*

by

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Chairman: Gp. Capt. G. Thripp

SUMMARY

Methods of analysis and synthesis of linear servomechanisms in the past have largely been based upon the frequency-response approach. Analogue computer techniques have also been widely used, particularly for transient-input tests. Random inputs which statistically resemble the expected inputs may be more valuable than sine-wave or simple transient inputs for the testing of many servomechanisms. With such input functions the use of correlation functions and of devices for measuring auto- and cross-correlation becomes important. Random input functions and correlator measurement used together with analogue computer techniques are shown to be most valuable in the study of non-linear servomechanisms and other non-linear devices.

1. Introduction

The linear networks used in communication systems have long been studied by calculating or measuring their transfer functions, $H(j\omega)$, for sinusoidal inputs. In early work the amplitude function, $|H(j\omega)|$, was emphasized more than the phase, arg $H(j\omega)$, because audio signals were of greatest interest, and the ear is insensitive to phase distortion. During the 1930's the development of feedback amplifiers, carrier telephony and the beginnings of television led to an increased interest in the phase shift and corresponding time delay of networks. During World War II, radar and servomechanism developments intensified this interest and the specification or measurement of both gain and phase response became routine among engineers.

Because of the growing use of the Laplace transform operational calculus during the 1940's and because the new networks being used were often tested for their response to transients, some arguments arose as to the relative desirability of sinusoidal and transient testing. However, it was soon generally realized that the mathematical

expression for sinusoidal response, $H(j\omega)$, also gave the operational expression $H(s)$ for impulse response, the inverse transform of which in turn would give the response $c(t)$ to any input $r(t)$ through the use of the convolution integral,

$$c(t) = \int_{-\infty}^{\infty} h(\tau)\, r\,(t - \tau)\, d\tau \quad \ldots\ldots\ldots(1)$$

In the decade since the end of World War II, non-linear systems have assumed growing importance, and a number of analytical methods have been proposed for dealing with such systems. At the same time there has been an ever-increasing interest in noise and information theory. The essential resemblance between noise and information-bearing signals has been recognized, and the design of optimum linear networks for the separation of noise and signals has been studied. Also the concepts of auto- and cross-correlation have been borrowed from the statistician and put to work.

However, there has been no development of general methods for the analysis, and certainly not for the synthesis, of the great class of all networks, of which linear networks are only a small sub-class. Certain methods, such as phase-plane analysis, have been developed for the study of limited classes of non-linear networks, while the "describing-function" method of Kochenberger[1] and others has proved useful for systems with single, non-essential non-linearities. Significantly, however, Kochenberger's work has been extended by Booton[2,3]

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to the study of non-linear systems with random inputs.

Advances in analytical methods must be accompanied by corresponding advances in methods of measurement. Thus sinusoidal generators were generally available for amplitude response tests in the 30's and phase-shift or time-delay tests began to be made towards the end of this period. In the next decade pulse generators and wide-band oscilloscopes became available, and were used for transient testing of linear networks. Some attempts have been made to use sinusoidal and transient test methods on non-linear networks, but have been generally unsuccessful because of the amplitude sensitivity of such networks. Random input test signals have been used with linear systems, particularly for noise figure measurements, and, because such inputs may be made to resemble signals statistically, they have been proposed for more general use.[4] Correlators have also been developed[5,6,7,8,9] and have been used to study the characteristics of signals as well as the response of networks.

A simple example will illustrate the difficulties encountered if an attempt is made to extend methods of linear analysis to a system containing a non-linear element. If two linear resonant rotating shafts, each with inertia, damping and a spring connected to the frame of reference, are coupled together by a linear spring, then the motion of each for any torque function applied to one shaft may be solved by the methods of operational calculus. Now if the coupling has some backlash which is small compared to the total motion of either shaft, this backlash may be ignored and the same methods used to give a useful approximate solution. For larger amounts of backlash better results may be obtained, for sinusoidal inputs only, by the "describing-function" method of Kochenberger, which takes into account the effect of the non-linearity on the fundamental frequency only. This method is approximate because it assumes that higher harmonics are filtered out, which is not always the case; also, it cannot readily be used for other than sinusoidal inputs.

If the amount of backlash is relatively large it may be necessary to make an exact transient solution, taking advantage of the fact that this simple system is piecewise linear. Such a step-by-step solution is extremely tedious, however.

If we consider that in this system, as in most communication and servo systems, our real interest lies in the *resemblances* between the input and output wave-forms, we are led to consider:

(a) The use of random input signals which statistically resemble the expected input signals.

(b) The measurement of the cross-correlation between input and output, as a measure of the resemblance between these quantities.

In its simplest form the cross-correlation is merely the integrated product of the input and output time functions. This will be a maximum if they closely resemble one another, and will be reduced by anything (such as increasing backlash) which causes the input to less closely resemble the input.

This paper will attempt to show that, by means of a combination of statistical inputs and correlation measurements, the quantitative analysis and comparison of non-linear systems may be profitably undertaken. Synthesis, which must in any case follow analysis, will not be discussed in any detail.

2. Correlation Functions

The auto-correlation function[10,11] of a continuous stationary process $f(t)$ is given by

$$\phi_{ff}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t)f(t + \tau)\, dt \quad \ldots \ldots (2)$$

The cross-correlation function of this continuous process $f(t)$ and another, $g(t)$, is given by

$$\phi_{fg}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t)\, g(t + \tau)\, dt \quad \ldots \ldots (3)$$

These correlation functions are actually measures of the mean relationship existing among the products of all pairs of points of the time series considered, where these points are separated in time by some delay τ. It can be shown that the auto-correlation function $\phi_{ff}(\tau)$ has a maximum at $\tau = 0$ and is asymptotic, in the absence of periodic components, to the mean square of the time function. Consequently, if the mean is zero, the auto-correlation function tends to zero as the delay τ approaches infinity.

Cross-correlation is a measure of the coherence between two random functions. For two random

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functions which are independently generated, cross-correlation produces a constant which is the product of the individual mean values of the functions. In this case the functions are said to be incoherent; and, in the particular and common case where one or both has a zero mean, the cross-correlation function is zero everywhere.

It is often desirable to express correlation functions in normalized form. Thus, in the cross-correlation case,

$$_n\phi_{fg}(\tau) = \lim_{T \to \infty} \frac{\dfrac{1}{T}\displaystyle\int_0^T f(t)\,g(t+\tau)\,\mathrm{d}t}{\left[\dfrac{1}{T}\displaystyle\int_0^T f^2\mathrm{d}t \;\cdot\; \dfrac{1}{T}\displaystyle\int_0^T g^2\mathrm{d}t\right]^{\frac{1}{2}}} \quad \dots\dots(4)$$

$$= \frac{\phi_{fg}(\tau)}{[\phi_{ff}(0).\phi_{gg}(0)]^{\frac{1}{2}}}.$$

The auto-correlation $\phi_{ff}(\tau)$ of a stationary time series and its power spectrum $\Phi_{ff}(\tau)$ are Fourier transforms of one another (Weiner-Kintchine theorem),

$$\Phi_{ff}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{ff}(\tau)e^{-j\omega\tau}\mathrm{d}\tau, \quad \dots\dots\dots(5)$$

$$\phi_{ff}(\tau) = \int_{-\infty}^{\infty} \Phi_{ff}(\omega)\,e^{j\omega\tau}\mathrm{d}\omega. \dots\dots\dots\dots(6)$$

Similar relations may be obtained for cross-correlation functions.[3]

3. Relation between Cross-correlation and Impulse Response in Linear Networks

In a linear system the input $r(t)$ and output $c(t)$ are related by the convolution integral (1) which may be rewritten as

$$c(t) = \int_{-\infty}^{\infty} h(\sigma)\,r(t-\sigma)\,\mathrm{d}\sigma, \quad \dots\dots\dots(7)$$

where $h(t)$ is the time response of the network to a unit impulse, $\delta(t)$.

The cross-correlation between input and output, for a stationary random input, is

$$\phi_{rc}(\tau) = \lim_{T \to \infty} \frac{1}{T}\int_0^T r(t)\,c(t+\tau)\,\mathrm{d}t. \quad \dots\dots(8)$$

From (7) and (8),

$$\phi_{rc}(\tau) = \lim_{T \to \infty}\frac{1}{T}\int_0^T r(t)\int_{-\infty}^{\infty} h(\sigma)r(t+\tau-\sigma)\,\mathrm{d}\sigma\mathrm{d}t. \quad (9)$$

Inverting the order of integration,

$$\phi_{rc}(\tau) = \int_{-\infty}^{\infty} h(\sigma)\,\mathrm{d}\sigma \lim_{T \to \infty}\frac{1}{T}\int_0^T r(t)r(t+\tau-\sigma)\,\mathrm{d}t$$

$$= \int_{-\infty}^{\infty} h(\sigma)\,\phi_{rr}(\tau-\sigma)\,\mathrm{d}\sigma$$

$$= \int_{-\infty}^{\infty} h(\sigma)\,\phi_{rr}(\sigma-\tau)\,\mathrm{d}\sigma, \dots\dots\dots(10)$$

since ϕ_{rr} is an even function.

In the case where $r(t)$ is "white" noise, ϕ_{rr} is a unit impulse, so that (10) becomes

$$\phi_{rc}(\tau) = \int_{-\infty}^{\infty} h(\sigma)\,\delta(\sigma-\tau)\,\mathrm{d}\sigma$$

$$= h(\tau) \dots\dots\dots\dots\dots\dots(11)$$

Thus the cross-correlation of the input and output of a linear network, for the case where the input is white noise, is equal to the impulse response of the network. Since the impulse response gives the response to any other input according to (1) or (7), the measurement of input-output cross-correlation with white noise input would be quite valuable if it were an easier measurement to perform.

The more general case given by (10) is also valuable, for a random input $r(t)$ may be limited to some spectrum corresponding to that occupied by expected input signal and the value of ϕ_{rr} may be determined by using (6).

For an ideal servo one would expect $\phi_{rc}(\tau)$ to approach the form of an impulse at $\tau = 0$. In practical cases the peak response $\phi_{rc}(max)$ will be less than unity (after normalization) and this peak will occur at some time τ_m after $\tau = 0$. The values of $\phi_{rc}(max)$ and τ_m will give a measure of the "goodness" of the linear servo for the given input spectrum, while the whole response ϕ_{rc} will give a still more complete picture of affairs. A weighted average value of ϕ_{rc} may be used if signals with different spectra may be expected at different times.

This use of ϕ_{rc} will be clearer if we relate it to the mean square error. This error is

$$\overline{E^2} = \lim_{T \to \infty} \frac{1}{T} \int_0^T (r - c)^2 \, dt, \qquad \ldots \ldots \ldots (12)$$

and has been used as a criterion in the design of predictors[12,14] and servomechanisms.[3] It can also be written

$$\overline{E^2} = \lim_{T \to \infty} \frac{1}{T} \int_0^T (r^2 + c^2 - 2rc) \, dt \quad \ldots (13)$$

$$= \phi_{rr}(0) + \phi_{cc}(0) - 2\phi_{rc}(0).$$

If r and c are normalized, then the normalized mean square error is

$$\overline{E_n{}^2} = 1 + 1 - 2 \frac{\phi_{rc}(0)}{[\phi_{rr}(0) \cdot \phi_{cc}(0)]^{\frac{1}{2}}}$$

$$= 2 \{1 - {}_n\phi_{rc}(0)\} \ldots \ldots \ldots \ldots \ldots (14)$$

Thus, as would be expected, the normalized error is $\sqrt{2}$ if input and output are completely incoherent.

In a more general case we may be interested in the delayed response $c(t - \tau_1)$ to an input $r(t)$, in which case

$$\overline{E_n{}^2} = 2 \{1 - {}_n\phi_{rc}(\tau_1)\} \quad \ldots \ldots \ldots \ldots \ldots (15)$$

Note that the error depends only upon the departure from unity of the normalized cross-correlation at $\tau = 0$, or $\tau = \tau_1$. *Thus it appears that the full cross-correlation curve is more meaningful than the r.m.s. error.*

Furthermore, we wish to propose the extension of the use of the cross-correlation function to non-linear servos. Such a suggestion requires that we consider the practical means for measurement of this function.

4. Measurement of Correlation Functions

As indicated by the basic equations (2) and (3) for auto- and cross-correlation, the operations which must be performed in a correlator are:

  (i) Delay of a function of time.

 (ii) Multiplication of a function by the same function delayed or, in the case of cross-correlation, by another function delayed.

(iii) Integration or averaging, over a long period of time.

Some of the early digital and analogue correlators,[5,6] though capable of good precision,

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were somewhat slow. Some newer analogue devices are more rapid,[7,8] and in the Bell-Rideout correlator[9] a speed was attained which was shown to be close to the maximum theoretically possible. This maximum speed, in a correlator of given band-width, depends upon the integration time T, which cannot of course approach infinity, but must be limited to some period which reduces error to the value of about 1 or 2 per cent., usually permissible in an analogue device. Since operational amplifier integrators give increasing error as T increases, there is actually an optimum value of T for this correlator.

The Bell-Rideout correlator was built using Philbrick computer units[15] and, because of its band-width (15 kc/s), is well adapted to the study of electrical networks and filters set up on high-speed or repetitive analogue computers such as the Philbrick computer.

5. Correlation Studies, using Scatter-diagram Techniques with a High-speed Computer

Although the correlator[9] referred to above is satisfactory for the computer study of non-linear systems with random inputs, a scatter-diagram technique[18] has proved simpler for rapidly obtaining approximate results, and all studies to be reported on these have been made by means of this scheme. Early work on this method was done by Middleton[16] and Sugar.[17] They have shown that the scatter diagrams of two random electrical variables may be oscilloscopically displayed, and that a simple relationship exists between the resultant pattern and the correlation coefficient, if a Gaussian amplitude distribution is assumed for the noise. In general, if a randomly varying voltage $x(t)$ is applied to one pair of plates of a cathode-ray tube, and another such voltage $y(t)$ to the other plates, the resulting trajectory will form a random two-dimensional pattern. This pattern, if integrated by photography or by use of a long-persistence screen, will show some variations in density just as does an ordinary scatter graph. The light distribution will be governed by some joint probability density, $W(x,y,\tau)$, where τ represents a possible delay (or advance) of one random variable, $x(t)$, with respect to the other, $y(t)$.

Middleton and Sugar have shown that for a pure Gaussian noise distribution the pattern is elliptical, with the intensity being greatest at the centre, and dropping off toward the edge. They

have also shown that in this case the correlation function is given by

$$\phi_{xy}(\tau) = \frac{1 - (a/b)^2}{1 + (a/b)^2}, \quad \ldots\ldots\ldots\ldots(16)$$

if the r.m.s. voltage applied to each set of oscilloscope plates is equal, and a and b (functions of τ) are the co-ordinates of the ellipse formed by following some fixed density level in the pattern.
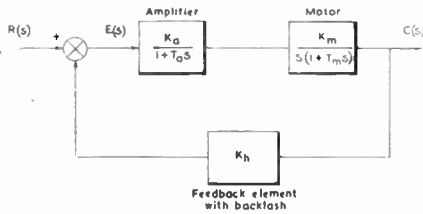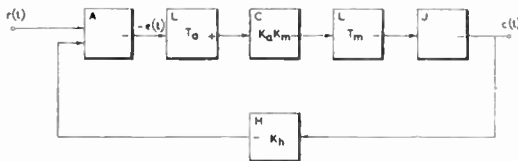

Fig. 1.—*Block diagram of a non-linear servo.*

Fig. 2.—*Computer set-up for the servo of Fig. 1. Here A is an adder, C a scale-changer, and J an integrator. The L-units give the simple lags, and H is a hysteresis or backlash unit.*

By making use of a somewhat different interpretation of the quantities obtained from the scatter diagram, Burford[18] has recently shown that qualitative information concerning the correlation coefficient may be found from the scatter diagram, no matter what statistical distribution is assumed. Instead of considering the joint probability distribution $W(x, y, \tau)$ to be a surface in x, y, $W(x, y, \tau)$ space, he considers it to be a mass density over the x, y plane. He has shown that no matter what distribution is assumed the correlation function is described by

$$\phi = \frac{1 - k}{1 + k}, \quad \ldots\ldots\ldots\ldots(17)$$

where the quantity k is defined as the ratio of the extreme values of inertia of the distribution, which occur at some angle θ and at right angles to θ, that is,

$$k = \frac{E(d^2)|_{\theta_1}}{E(d^2)|_{\theta_1 + \pi/2}} \quad \ldots\ldots\ldots\ldots\ldots(18)$$

Thus, in the special case of a Gaussian distribution the results of Burford are identical to the results of Middleton and Sugar. However, it has now been shown that for *any* distribution the correlation coefficient may be found from the ratio of the extreme values of inertia. Therefore a means is available for determining correlation coefficients for non-Gaussian distributions such as those resulting from non-linear networks. However, the scatter diagrams are at present only suited to giving a quick semi-quantitative picture of a correlation function, since no device has as yet been constructed to determine quickly the ratio of the extreme values of inertia of these diagrams.

6. Some Experimental Results

Consider the servo shown in block diagram form in Fig. 1. This is a type 1 position control servo with backlash in the feedback path. This servo may be set up on an analogue computer shown in Fig. 2, with a hysteresis unit used to introduce the backlash.
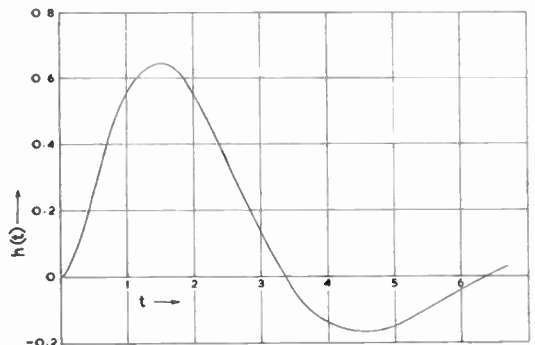

Fig. 3.—*Calculated response of the linear servo of Fig. 2 $(K_h = 0)$ to a unit impulse.*

If the hysteresis unit is set at zero $(K_h = 0)$, the servo is linear, and if we assume $T_a = T_m = 0.5$, $K_a K_m = 1$, this servo has a loop gain

$$G(s)H(s) = \frac{1}{s(1 + 0.5s)(1 + 0.5s)}, \quad \ldots(19)$$

and $\quad \dfrac{C(s)}{R(s)} = \dfrac{4}{s^3 + 4s^2 + 4s + 4} \quad \ldots\ldots\ldots(20)$

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For a unit impulse, $R(s) = 1$, and the inverse Laplace transform of (20) then gives

$$c(t) = h(t) = 0.48e^{-3.129t}$$
$$+ 1.33e^{-0.1355t} \sin(1.041t - 21.15°)$$
$$\dots\dots\dots\dots(21)$$

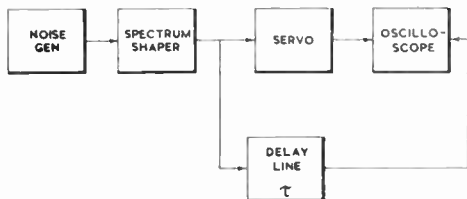This response is plotted in Fig. 3.

Fig. 4.—*Set-up for determining the scatter diagrams for the computer representation of a servo.*

The scatter diagrams discussed in the preceding section may easily be plotted for the computer set-up of Fig. 2 by the use of a cathode-ray oscilloscope. Because of the time scale used here, (1 p sec = 1/2400th sec was the computer unit of time), photographs gave "integrated" pictures of the scatter diagrams with an exposure of only a few seconds. The parameter τ in the cross-correlation formula (3) was introduced by using a high-quality delay line[9] in the set-up of Fig. 4. The results are as shown in the photographs of Fig. 6. In this linear case the scatter diagrams are all ellipses and the cross-correlation may easily be determined from their major axes, to a fair degree of accuracy. The result is that of the uppermost curve of Fig. 5. Note that this is very nearly equal to the impulse response curve replotted from Fig. 3, as it should be from (11). If backlash is introduced the scatter diagrams are no longer elliptical, as may be seen from the results from $K_h = 15$ in Fig. 7. The cross-correlation can still be determined from these diagrams by the methods outlined in Section 5, but only with much more difficulty than in the linear case. The various curves obtained from the scatter diagrams for non-zero values of K_h are also shown in Fig. 5, but are rather approximate. However, they show that the effect of increasing backlash, K_h, or decreasing input level, is to cause the peak cross-correlation to decrease, and to move out to larger values of τ.

It is not mathematically possible, in the non-linear case, to speak of any simple relationship

between the impulse response and these cross-correlation curves. The kind of linearization used by Kochenberger and Booton may be used, however, the equivalent linear servo for any given noise level-backlash relation being determined by changing servo gain, and by changing or adding to the lags in the system, so that a linear cross-correlation curve is obtained which agrees with the non-linear cross-correlation curve to a close approximation. Here some method of mechanizing the scatter-diagram technique would be desirable, or some of the faster correlators discussed above might be used.

The use of "white" noise (of a band-width which is at least considerably wider than the overall servo band-width), although meaningful in the linear case, and useful in the slightly non-linear case, is somewhat less useful in extremely (or essentially) non-linear systems. If the random noise input is band-limited to correspond to the spectral content of the signals which may be applied to the system, the cross-correlation
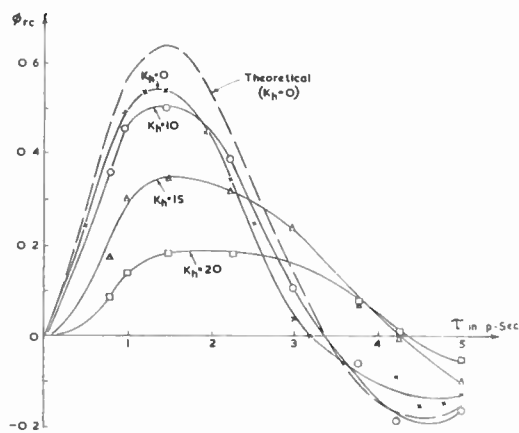

Fig. 5.—*Input—output cross-correlation curves for several values of K_h obtained from Figs. 6 and 7 and other data for $K_h = 10$ and 20.*

will be generally more meaningful, particularly if the system under consideration is a servo. Cross-correlation at $\tau = 0$ will no longer be zero (indicating a maximum value of error there) as it is in the case of "white" noise input. The designer's problem is to make $\phi_{rc}(max)$ as

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near unity as possible, and to make it occur as small a value of τ as possible. Since these quantities vary with input noise power level, we may choose to plot $\phi_{rc}(max)$ and τ_m against this power level, or, inversely, against backlash K_h with a fixed input level.

The data of Fig. 5 have been used to plot such a pair of curves, as shown in Fig. 8. From these curves it is possible to determine interpolated values of peak cross-correlation and time of occurrence, for any input noise level. This example is for the case of white noise input— a band-limited noise would give larger values of $\phi(max)$. The r.m.s. error may also be determined from these curves by use of the simple relationships shown in (14) and (15).
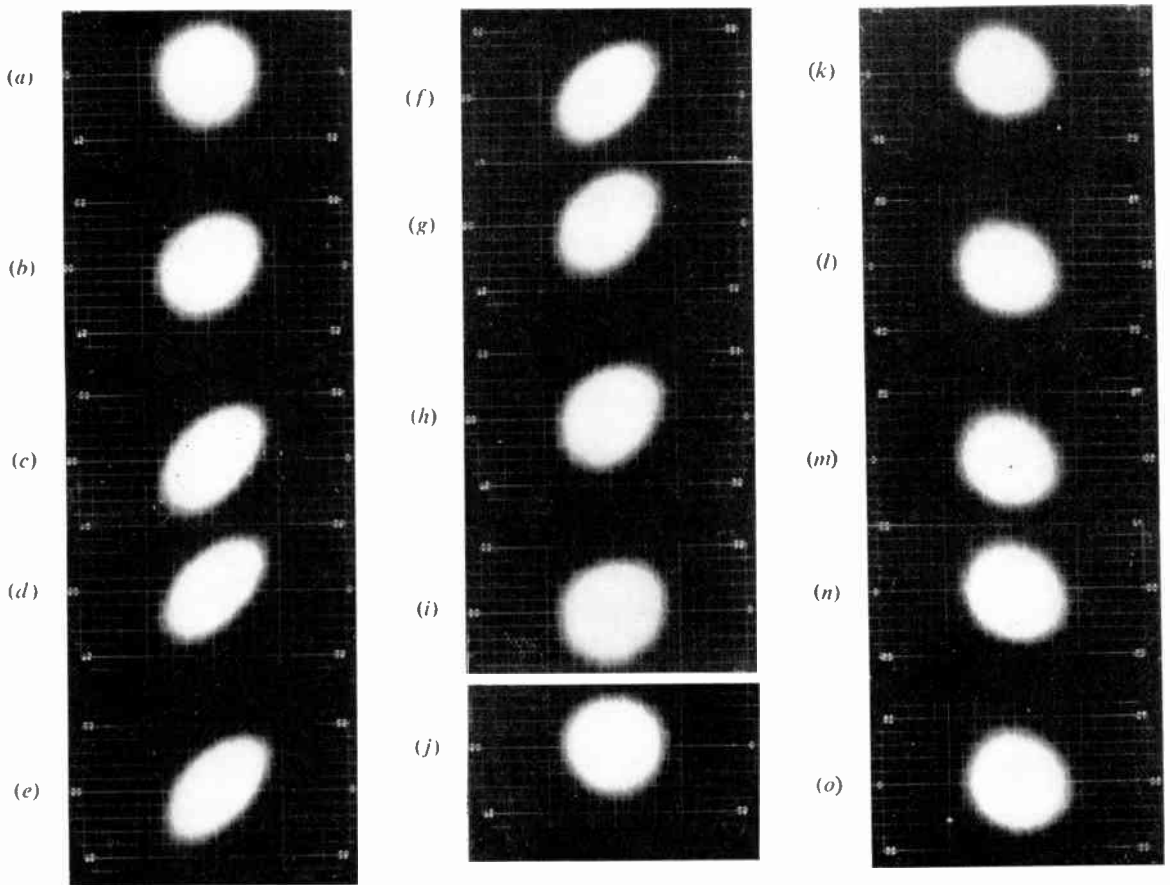

Fig. 6.—Input–output cross-correlation scatter diagrams for white noise input to the linear servo of Fig. 2 ($K_h = 0$).

(a) $\tau = 0$

(b) $\tau = 0.50$ p sec

(c) $\tau = 1.00$ p sec

(d) $\tau = 1.25$ p sec

(e) $\tau = 1.50$ p sec

(f) $\tau = 2.00$ p sec

(g) $\tau = 2.25$ p sec

(h) $\tau = 2.50$ p sec

(i) $\tau = 3.00$ p sec

(j) $\tau = 3.25$ p sec

(k) $\tau = 3.75$ p sec

(l) $\tau = 4.25$ p sec

(m) $\tau = 4.50$ p sec

(n) $\tau = 4.75$ p sec

(o) $\tau = 5.00$ p sec

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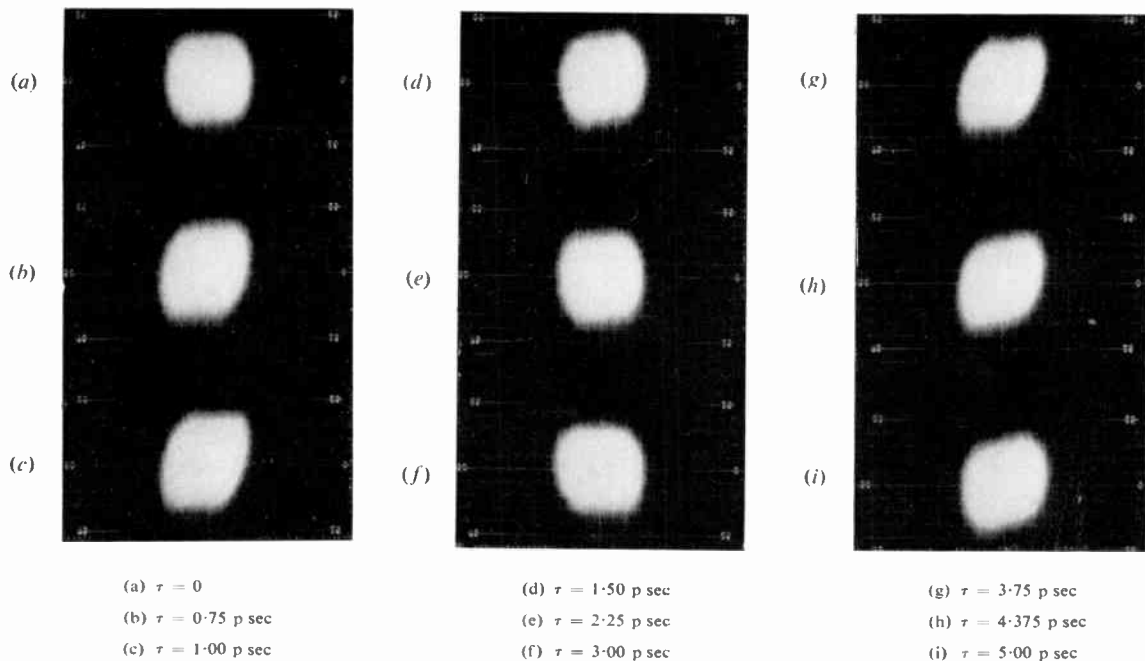| (a) $\tau = 0$ | (d) $\tau = 1 \cdot 50$ p sec | (g) $\tau = 3 \cdot 75$ p sec |
| (b) $\tau = 0 \cdot 75$ p sec | (e) $\tau = 2 \cdot 25$ p sec | (h) $\tau = 4 \cdot 375$ p sec |
| (c) $\tau = 1 \cdot 00$ p sec | (f) $\tau = 3 \cdot 00$ p sec | (i) $\tau = 5 \cdot 00$ p sec |

Fig. 7.—*Input–output cross-correlation scatter diagrams for white noise input to the non-linear servo of Fig. 2 ($K_h = 15$).*

7. Conclusion

A new method for testing and analysing both linear and non-linear systems has been described. This method is particularly well-suited to use in analogue computer studies of servomechanisms. Depending, as it does, upon the use of statistical inputs, and the use of correlator measuring devices, it is much more general than the sinusoidal and step-function testing methods of the past, which are only useful for the testing of linear systems, and a few restricted groups of non-linear systems. In this method a family of cross-correlation curves, or a single pair of curves obtained from such a family, replaces the gain and phase-versus-frequency curves which are used to describe linear systems. The effects of noise, whether originating within or without the system, may also be determined by this method.

8. References

1. R. J. Kochenberger, "A frequency response method for analysing and synthesizing contactor servomechanisms," *Trans. Amer. Inst. Elect. Engrs*, **69**, Part I, pp. 270-283, 1950.
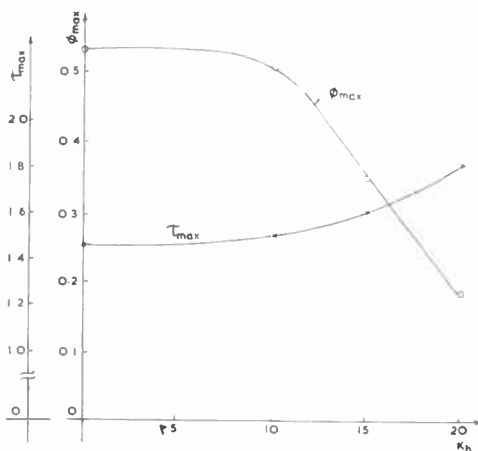

Fig. 8.—*Peak cross-correlation, $\phi_{rc}(max)$, and time of its occurrence, τ_m, obtained from Fig. 5 and plotted against K_h.*

2. R. C. Booton, Jr., "An optimization theory for time-varying linear systems with non-stationary statistical inputs," *Proc. Inst. Radio Engrs*, **40,** pp. 977-981, August 1952.

3. R. C. Booton, Jr., "Non-linear Servo-mechanisms with Random Inputs," Dynamic Analysis and Control Laboratory Report No. 70 (Massachusetts Institute of Technology, August 20th, 1953).

4. C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, **27,** pp. 279 and 623, July and October 1948.

5. H. E. Singleton, "A Digital Electronic Correlator," Research Laboratory of Electronics Report No. 152 (Massachusetts Institute of Technology).

6. M. J. Levin and J. B. Reintjes, "A five-channel electronic analog correlator," *Proc. Nat. Electronics Conf., Chicago*, **8,** p. 647, 1952.

7. J. N. Holmes and J. M. C. Dukes, "A speech-waveform correlator with magnetic-tape delay and electronic multiplication," *Proc. Instn Elect. Engrs*, **101,** Part III, pp. 225-237, July 1954.

8. W. R. Bennett, "The correlatograph," *Bell Syst. Tech. J.*, **32,** pp. 1173-85, September 1953.

9. H. Bell and V. C. Rideout, "A high-speed correlator," *Trans. Inst. Radio Engrs (Electronic Computer Group)*, **EC-3,** No. 2, p.30, June 1954.

10. G. I. Taylor, "Diffusion by continuous movements," *London Math. Soc. Proc.*, Ser. 2, **20,** pp. 196-212, 1920.

11. A. Kolmogoroff, "Interpolation und Extrapolation von stationären züfalligen Folgen," *Bull. Acad. Sci. U.R.S.S.*, Ser. Math., **5,** pp. 3-14, 1941.

12. N. Weiner, "The Extrapolation, Interpolation and Smoothing of Stationary Time Series" (John Wiley & Sons, Inc., New York, 1949).

13. N. Weiner, "Generalized harmonic analysis," *Acta Mathematica*, **55,** pp. 117-258, 1930.

14. Robert Cohen, "Some Analytical and Practical Aspects of Weiner's Theory of Prediction," Research Laboratory of Electronics Report No. 69 (Massachusetts Institute of Technology, June 2nd, 1948).

15. Geo. A. Philbrick Researches, Inc., "Catalog and Manual" (Boston, U.S.A., 1951).

16. C. D. Middleton and W. Gottschalk, "Microwave Noise Study," Raytheon Mfg. Co., Quarterly Report No. 1, February 1st, 1953, to May 1st, 1953.

17. G. R. Sugar, "Estimation of correlation coefficients from scatter diagrams," *J. Appl. Phys.*, **25,** pp. 354-357, March 1954.

18. T. M. Burford, "Qualitative evaluation of correlation coefficients from scatter diagrams," *J. Appl. Phys.*, **26,** pp. 56-57, January 1955.

19. H. Bode and C. E. Shannon, "A simplified derivation of linear least square smoothing and prediction theory," *Proc. Inst. Radio Engrs*, **38,** pp. 417-425, April 1950.

20. R. R. Bennett, "Analog computing applied to noise studies," *Proc. Inst. Radio Engrs*, **41,** pp. 1509-1513, October 1953.

21. J. H. Laning, Jr., and R. H. Battin, "An Application of Analogue Computers to Problems of Statistical Analysis," Cyclone Symposium II, Part 2, pp. 79-88 (Reeves Instr. Corp. N.Y., 1952).

22. W. P. Caywood, R. C. Lyman and W. M. Kaufman, "Generalized servomechanism evaluation" (Abstract only), *Trans. Inst. Radio Engrs (Information Theory Group)*, No. PGIT-3, p. 150, March 1954.

23. J. L. Lawson and G. E. Uhlenbeck, "Threshold Signals" (McGraw-Hill Book Co., New York, 1950).

24. Y. W. Lee, T. P. Cheatham and J. B. Wiesner, "Application of correlation analysis to the detection of periodic signals in noise," *Proc. Inst. Radio Engrs*, **38,** pp. 1165-1171, October 1950.

25. D. C. Sather, "Experimental Determination of Random Noise Correlation Coefficients by Use of Scatter Diagrams obtained with an Analog Computer," M.S. Thesis, University of Wisconsin, 1954.

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APPLICANTS FOR MEMBERSHIP

New proposals were considered by the Membership Committee at a meeting held on May 5th, 1955, as follows: 19 proposals for direct election to Graduateship or higher grade of membership and 32 proposals for transfer to Graduateship or higher grade of membership. In addition, 49 applications for Studentship registration were considered. This list also contains the names of four applicants who have subsequently agreed to accept lower grades than those for which they originally applied.

The following are the names of those who have been properly proposed and appear qualified. In accordance with a resolution of Council and in the absence of any objections being lodged, these elections will be confirmed 14 days from the date of the circulation of this list. Any objections received will be submitted to the next meeting of the Council with whom the final decision rests.

Transfer from Associate Member to Member

GOODYEAR, Sydney. *Corbridge.*
HILL, Wg.-Cdr. Ieuan Edward, R.A.F. *B.A.O.R.* 19.

Direct Election to Associate Member

ATKINSON, Eric James. *London, N.17.*
JOHNSON, Frederick, B.Sc., M.Sc., Ph.D. *Bolton.*
PARAMESWARAN, Flg.-Off. Nurani Sivaramakrishnan, B.Sc., M.A., I.A.F., *Tambaram, Madras.*

Transfer from Associate to Associate Member

BROWN, Lieut.-Com. Kenneth Stanley, R.N. *Arundel.*

Transfer from Graduate to Associate Member

BOTTOMLEY, Frederick Walter. *Liverpool.*
CAMERON, Peter McDougall. *London, S.W.19.*
PETERS, John Manning, B.Sc.(Eng.). *Haslemere.*
RAO, Capt. B. Siva Nagendra, B.Sc., Indian Signals. *Mhow.*
THOMAS, Flt.-Lt. Newton Vedanayagam, B.Sc. I.A.F. *Bangalore.*
TRAIL, Gordon James. *London, W.C.1.*

Transfer from Student to Associate Member

COPELAND, Keith. *Holland-on-Sea.*

Direct Election to Associate

BHALLA, Narindar Singh, M.Sc. *Ghatsila, Bihar.*
BOOTH, Charles Hector. *Greenford.*
CHRISTIE, Norman McGregor. *Auckland.*

CLARKE, Frederick John. *St. Albans.*
DAVIS, Jack Haskell. *Baltimore.**
DUNELL, Wilfred Maurice. *Loughton.*
HENNINGS, Harry. *Lagos.*
METCALFE, George Russell Francis. *London, S.W.16.*
PORTER, Francis James Garvie. *Manchester.*
STEVENSON, Kenneth. *Blantyre, Nyasaland.*

Transfer from Student to Associate

BELL, Ronald James. *Trinidad.*
BULLEY, Frank Noel. *Abridge.*
CHAN, York Chye. *Singapore.*
COATES, George Frederick Rudolph. *West Wickham.*
EMERY, Trevor Munslow. *London, N.W.9.*
NEYLAN, James Patrick. *Warrington.*
SUN, Tai Lum. *Singapore.*
WILLIAMSON, Robert. *Didcot.*

Direct Election to Graduate

HICKSON, Sub-Lieut. John, R.N. *Lee-on-Solent.*
LARKIN, Richard Walter Edward. *Seaford.*
VADGAMA, Gulab Maganlal. *Nairobi.**

Transfer from Student to Graduate

ABHYANKAR, Moreshwar Kashinath. *Poona.*
BATES, Leslie John. *Dorking, Surrey.*
JAWA, Bal Raj, B.A. *New Delhi.*
ROW, Edward Francis. *Chessington.*
SEABROOK, Gordon Dennis. *Romford.*
ZEIHER, Graham. *Abingdon.*

STUDENTSHIP REGISTRATIONS

ALEXANDER, Richard. *New Delhi.*
ANTIA, Jal Merwanji, B.Sc.(Hons.). *London, S.W.6.*
ARORA, Jagdish Chander. *Delhi.*

BACON, Philip Morgan. *Wembley.*
BAGCHI, Proshanto Kanti. *Bangalore.*
BALAKRISHNAN NAIR, K. R. *Delhi.*
BECKLEY, Herbert Reginald, B.Sc. *London, N.W.4.*
BHACKA, Sam Jamshedji. *Bangalore.*

CURRAN, Andrew Brewster. *Edinburgh.*

D'SILVA, Tarcisius Joseph, B.Sc. *Madras.*
DIAS, Cyril Francis. *Delhi.*
DUGGAL, Jagdish Mittar. *Srinagar.*

FONG TSE MO, Chypre Fong. *Hong Kong.*

GILBERT, John Lodge. *Chislehurst.*
GLENDENNING, Major James Alfred, R.E.M.E. *Rawalpindi.*
GRANTHAM, Colin. *Rossendale.*

HOMBURGER, E. Raphael. *Haifa.*

IRVING, Thomas Clark. *Manchester.*

JAGGI, Inder Singh. *Ambala.*
JAIN, Lt. Ajit Pershad, M.Sc., Indian E.M.E. *Delhi.*
JOGLEKAR, Madhav Vishnu. *Dhond.*

KHAN, Abdul Jamal. *Karachi.*

LAKSHMANA RAO, Motamarry. *Anakapalle.*
LOW PING HUNG. *Kuala Trengganu, Malaya.*
LUND, Hugh Forsyth. *Pietermaritzburg.*

MOORE, Ernest Henry. *Chessington.*

MUKHERJEE, Probodh Kumar. *Lonavla, Bombay.*

NARASIMHAN, Sirangam Ramaswamy. *Melapalayam, South India.*

PANDEHIS, Kyriakos Charles. *London, W.12.*
PANDEY, Girish Chandra. *Delhi.*
PANIKER, K. Viswanatha. *Bombay.*
PANTHAKY, Jal-Khurshed. *London, W.2.*
PURICH, Andrew. *Montreal.*

RAM NATH TANDON, B.Sc. *New Delhi.*
RATCLIFFE, Norman. *Lower Darwen, Lancashire.*
RUDD, Dennis. *Winscombe.*

SETHI, Uttam Chand. *New Delhi.*
SHIVA SWAMY, Chamarajanagar Rangaswamy, B.Sc. *Bangalore.*
SIVASANKARA PILLAI, A. *Mavelikara, Cochin State.*
SOBTI, Mahinder Nath, B.A. *Delhi.*
SOPER, Richard Maxwell. *Hemel Hempstead.*
SPACKMAN, Charles Bradwell. *London, S.W.14.*
STRANGE, Norman Edward. *London, E.4.*
STROYNOWSKI, Slawomir Stanislaw. *London, N.4.*
SUDARSHAN LAL UPPAL. *New Delhi.*
SURIJ BHAN. *Jullunder City, Punjab.*
SWAMINATHAN, Kashi Ramamurthi, B.Sc. *Madras.*

TYAGI, Birbal Singh, B.A. *Bangalore.*

WALTERS, Henry George. *Bath.*

* Reinstatements.

MEASUREMENT OF THE RESOLVING TIME OF SCALING CIRCUITS*

by

P. K. Patwardhan, M.Sc., (Graduate)†

SUMMARY

An account of the double-pulse method of measurement of the resolving times of some common types of scaling circuits is introduced by an elementary statistical analysis of the random processes met in radiation counting. Other possible methods of measurement are also mentioned. The influence of circuit parameters on resolving time is pointed out, and the scope for further improvement is indicated.

1. Introduction

In nuclear physics, scale-of-n circuits are used for scaling down the high input rate of pulses, which are randomly distributed in time, in order to bring the rate within the range of a conventional mechanical register. Scaling circuits, whether binary or decade, consist essentially of scale-of-two units so arranged as to give an over-all scale-of-n circuit.

2. The Resolving Time of Scaling Circuits

The basic scale-of-two circuit has two distinct and reversible states of equilibrium. The trigger pulse generates transients which transfer the circuit from one state to the other. The cyclic operation consists of three parts: (1) equilibrium, (2) transient, and (3) reciprocal equilibrium. The usefulness of a scaling unit is judged by two factors: the stability of the electronic unit and its resolving time. The stability is defined as the probability of obtaining proper scaling action during a given interval, which must be independent of normal variations in power supply voltage, valve characteristics, component values or in the triggering waveform. The resolving time is the interval between the start of the initial trigger pulse and the arrival in the reciprocal stable state, when the circuit is ready for re-triggering. The inherent "insensitive" or "dead" time is the contributory effect of the associated equipment (amplifier, discriminator, scaling unit, counting rate meter) as well as the

detecting device (Geiger-Müller counter or ionization chamber). To minimize the resultant dead time, each should have a fast response. To apply accurate counting loss correction, the first-mentioned unit should have the longest dead time.

As well as reducing the high input rate, a scale-of-n circuit regularizes the randomly distributed input pulses, the spacing of pulses in the output having only a relatively small variation. The higher the scaling factor, the smaller is this variation.

3. Application of the Theory of Probability[1]

Two problems are of interest, to find the fraction of the input pulses lost on account of the dead time (counting loss), and to find how the regularizing action of a scaler reduces counting losses of a mechanical register.

In the first problem, let n be the "output" counting rate when τ is the insensitive period after each count, and let N be the "input" rate. It is assumed that a second input pulse during the insensitive period $t \leqslant \tau$ will neither produce an output pulse nor extend the insensitive time. It can be shown that n and N are related by $(N - n) = nN\tau$, this expression being the counting loss per second. Then follows

$$N = \frac{n}{1 - n\tau} \text{ and } n = \frac{N}{1 + n\tau} \ldots\ldots\ldots(1)$$

The fraction lost is therefore proportional to the output rate, the maximum output rate as $N \to \infty$ being τ^{-1}. For $\tau = 10^{-4}$ sec, an output rate of 100 per sec gives 10 per cent. counting loss, and the rate must be not more than 10 per sec to give only 1 per cent. counting loss.

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Let it now be assumed that a new input pulse arriving during the insensitive period will not give an output count, but will extend the insensitive time by an amount τ after its arrival. This modifies the conditions underlying equation (1). If the input rate is high, always spaced less than τ, the system will clearly remain permanently blocked. It will give an output only when the spacing between two successive pulses is $\geqslant \tau$. Thus,

$n = N \times$ (probability of no input pulse during time τ after a given pulse).

Since the mean number u of input pulses expected during the interval τ is $N\tau$, the probability that no count be received in the interval τ is, according to the Appendix:

$$P_0(N\tau) = e^{-N\tau} \text{ and } n = Ne^{-N\tau} \ldots\ldots\ldots(2)$$

For $(N\tau)^2 \ll 1$, equations (1) and (2) both give approximately

$$n \simeq N(1 - N\tau) \simeq N(1 - n\tau) \ \ldots\ldots\ldots(3)$$

For $N\tau > 1$, the output blocks. The maximum of equation (2) occurs for $N\tau = 1$, which gives $n = (\tau e)^{-1}$.

Equations (1) and (2) are extreme cases of single recovery processes; what actually happens is something intermediate, depending upon the spacing between the input pulses. The usual way of obtaining counting losses is to assume that either equation (1) or (2) applies, and to restrict measurements to the region where they are given closely by equation (3). This requires that $(N\tau)^2 \ll 1$.

Application to the regularizing action[1] of a scaling unit is as follows. Let the scaling factor be m and let n be the output rate for a mean random input rate of mn per sec. Let the resolving time of the whole equipment be negligible, so that counting losses are due entirely to the mechanical register. The problem then is to find the probability $q_m(x)\,dx$ that the actual timing spacing between output pulses lies between x and $x + dx$, where $x = nt$ is the timing in units of average spacing between output pulses. Also the probability $Q_m(x)$ that the next output pulse will have arrived by $t = x/n$ is required to be found.

During the time $t = x/n$ there are N input pulses, where $N = 0, 1, 2, \ldots$ The probability u that there is exactly a given number N is given by the Poisson distribution formula (see Appendix), and is the mean number of counts

expected during the interval $t = x/n$, such that $u = mnt = mx$. The next output pulse will have been obtained if $N \geqslant m$. The probability $Q_m(x)$ that this will have happened is just the total probability that $N \geqslant m$. This is given by the Poisson distribution as:

$$Q_m(x) = \sum_{N=m}^{N=\infty} P_x(mx)$$

$$= \left[1 - \sum_{0}^{m-1} P_x(mx) \right] dx \ \ldots\ldots\ldots(4)$$
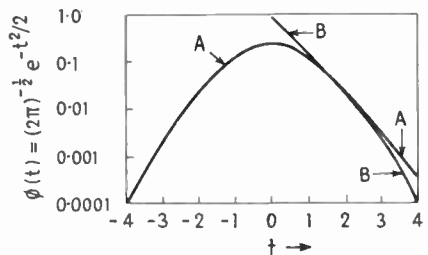
in which $P_x(mx) = (mx)^N e^{-mx}/N!$

Fig. 1.—Probability curves.[1]

Curve A shows the Gaussian or the "error" function $\phi(t) = (2\pi)^{-\frac{1}{2}} \exp(-t^2/2)$. For a large count, this gives the probability that the count will deviate from its mean expected value by t to $t + dt \times$ the standard deviation of the count.

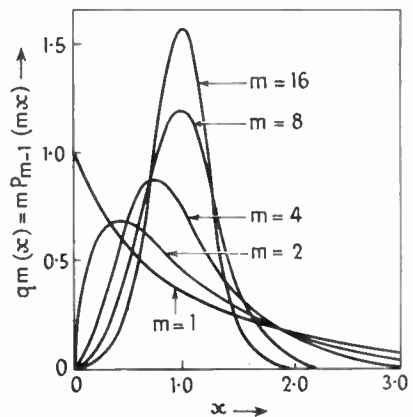Curve B shows the probability that the deviation exceeds $t \times$ the standard deviation.

Fig. 2.—Regularizing curves.[1]

The probability that the next output pulse will arrive between x and $x + dx$ is $q_m(x)dx$ which is the increase in $Q_m(x)$ during dx.

So:
$$q_m(x) = \frac{d}{dx} Q_m(x)$$

$$= \sum_{m}^{\infty} \left[mP_{x-1}(mx) - mP_x(mx) \right]$$

and
$$q_m(x)dx = mP_{m-1}(mx)dx$$

$$= \frac{m(mx)^{m-1}e^{-mx}}{(m-1)!} \cdot dx \quad \dots \dots (5)$$

It is easily shown that this distribution is normalized and has a mean spacing mx such that $x = 1$.

The standard deviation of x is σ, and of mx is $m^{\frac{1}{2}}$, so that for x

$$\sigma = m^{-\frac{1}{2}} \dots \dots \dots \dots \dots \dots \dots (6)$$

where σ is the standard deviation of the output pulse spacing from the mean value. Thus for a scale-of-64 circuit, σ is one-eighth of the value of the mean spacing.

In Fig. 1, curve A shows the Gaussian error function $\phi(t) = (2\pi)^{-\frac{1}{2}} \exp(-t^2/2)$. For a large count, this gives the probability that the count will deviate from its mean expected value by t to $t + dt$ times the standard deviation of the count. Curve B shows the probability that the deviation exceeds t times the standard deviation. When the scaling factor m is not too small, the Gaussian distribution holds good, and Fig. 1 may be used. Half the value from curve B is used to calculate counting losses, because only deviations in which the spacing is less than the mean give counting losses.

In Fig. 2, $mP_{m-1}(mx)$ is plotted as a function of x for $m = 1, 2, 4, 8$ and 16. These curves show directly the shape of the regularizing curves for the scales of 1 (no scaling), 2, 4, 8 and 16. They are normalized so that the probability of the next count arriving during $x \leqslant x_1$ is the area under the curve for $x \leqslant x_1$. Thus less than 1 per cent. of the counts will have arrived by $x = \frac{1}{2}$ for a scale of 16, and about 10 per cent. for $x = 8$. From the curves of Fig. 2 it can be seen that even small scaling factors can be of great help, especially when the use of large scaling factors is considered too expensive.

4. Methods and Apparatus for the Measurement of Resolving Time[2,3]

The resolving time of a Higginbotham type of scaler (Fig. 3) is determined entirely by the time constant of the anode to grid-coupling components. However, this requires that the capacitances C1, C2 be large compared with the stray capacitances, which is generally so.
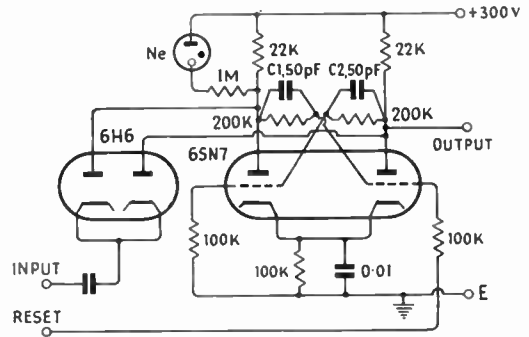

Fig. 3.—*Higginbotham scaling circuit.*[3]

If random pulses are passed through a switching circuit that is alternately conducting and non-conducting, the switching period can be adjusted so that one pulse only is gated, that is, occurs during a conducting period. Measurement of this period gives the resolving time of the circuit. It should be noted that the resolving time is the same for regular or random pulses, as the scaling circuit can only regulate the average rate of random pulses to give an acceptable loss.

When the resolving time of the circuit is not accurately known one employs counting rates which are sufficiently low to keep down the losses. If, however, the resolving time is known to an accuracy of, say, 10 per cent., then the counting losses up to 10 per cent. can be tolerated, if an accuracy of 1 per cent. is required. In any system where the rate of pulse production is high (e.g. an atomic pile), there is a greater probability that one pulse will be followed by another in a given short time following the first pulse. In such a case the counting system should have a fast response, i.e. a short resolving time. The counting circuit will have a single resolving

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time if it comprises a long chain of identical scaling units or if the resolving time of each element in the chain is just sufficient for it to pass the pulses that occur when the input pulse rate is a maximum $1/T_1$, where T_1 is the resolving time of the first element of the chain.

4.1. Cascade Counting

A variable-frequency oscillator is used as the pulse source. Two or three identical scale-of-two circuits are connected in series, to bring the high input rate within the range of the mechanical register. The oscillator frequency is increased until the oscillation period is smaller than the resolving time of the first stage. The system will become blocked at this stage as two successive pulses will not give separate counts. The resolving time is given by $1/f$ where f is the frequency at which the circuit begins to lose count. In the above procedure an oscilloscope is preferable to a mechanical register.

4.2. Oscilloscopic Test

This is a modification of the above method. Consider a ring-of-ten decade scaling circuit. If the output at the tenth stage is seen on the oscilloscope, it will appear as a square wave. Over the top of the square wave will be seen small pulses which will be equal to $(n - 1)$ where n is the scaling factor, nine in this case. The input oscillator frequency is increased until some of the small pulses are seen to be missing, when this happens the circuit is not resolving. The resolving time is given again by $1/f$.

4.3. Double-pulse Method

The double-pulse method will be found to be simpler and more accurate than the methods so far described. It works on the following principle. Two trigger pulses having fast rise times and suitable amplitudes are fed to the scaling circuit under test. The time delay between the two pulses can be varied. The scaling circuit output is normally a rectangular waveform and is displayed on an oscilloscope having a suitable sweep speed. The delay between the pulses is made shorter until, at one stage, the circuit fails to resolve, the rectangular waveform disappears, and the pulses are seen separately. The interval between the pulses, then displayed, will be the minimum separation allowable if the circuit is to resolve. This will, therefore, give the resolving time.

4.4. Experimental Apparatus

The apparatus used for this experiment consists essentially of a multiple-trigger generator and the Du Mont Synchroscope 256D.

The multiple-trigger generator[3] supplies two pulses of variable amplitude and time delay. In use, the delay of one pulse is fixed and the other varied. The two pulses have a rise time of 0·2 μsec and length 1·3 μsec, and their maximum amplitude is 175 V. The two outputs can be placed in parallel, if required.

Time markers at 10 and 50 μsec are available on the synchroscope, whose maximum writing speed is 1 in. per μsec.

5. Measurement of Resolving Times of Scaling Circuits

The positive trigger from the synchroscope is used for synchronizing the multiple-trigger generator. Both pulses from the generator are fed to the input of the scaling circuit and its output observed on the synchroscope.

5.1. 6SN7—Scale-of-two[3]

The conventional 6SN7 scaler (see Fig. 3) was tested first. The time interval between the pulses when the circuit just failed to trigger, was noted, and the resolving time was found to be 7·5 μsec.

$$\text{Counting error} = \frac{\text{Number lost}}{\text{Number counted}}$$

$$= \frac{Nn\tau}{n} = N\tau$$

For this 6SN7 circuit, assuming a 1 per cent. error, the maximum counting rate is 1,333 per sec for random pulses. It can, however, deal with continuous repetition rates up to 13,333 per sec.

5.2. B. Ramchandra's Scale-of-two[5]

This particular scale-of-two circuit does not require such a large trigger voltage, which is injected at the grid of a triode valve forming the common cathode impedance. The circuit was altered by the replacement of the original valves (6SN7 and 9002) by 6J6's, and the resolving time was found to be 8 μsec. The circuit diagram of the modified circuit is shown in Fig. 4.
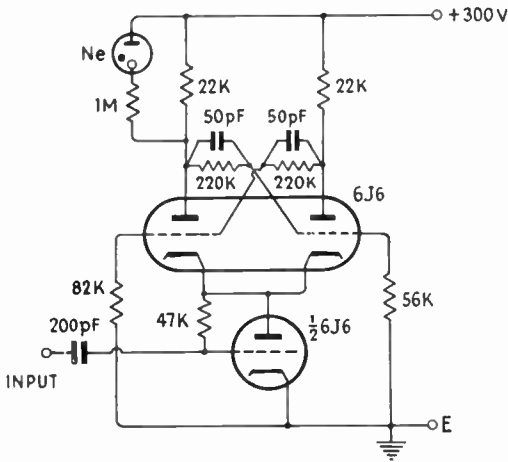
Fig. 4.—*B. Ramchandra's type modified scaling circuit.*[5]

5.3. *6J6—Scaling Circuit*[6]

This is the usual locking-trigger type of scaling circuit (Fig. 5). Replacing of 1N34 germanium crystal diodes by 6AL5 diodes did not affect the performance appreciably, though the stability was better in the latter case. This circuit was thought to have a resolving time of less than 1 μsec, which could not be measured with the pulses available from the multiple-trigger generator.

Although the rise time of the pulses was short the decay time was long and when the pulses were separated by a time interval of 1·3 μsec or less, the rising part of one overlapped with the
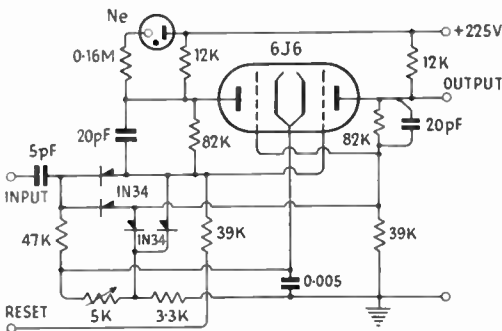

Fig. 5.—*6J6 scaling circuit.*[6]

falling part of the other. It was, therefore, not possible to make measurements below 1·3 μsec without first sharpening the pulses.

5.3.1. *Pulse-sharpening devices*

(*a*) One method of obtaining pulses having widths of the order of 0·1 μsec or so, would be to use blocking oscillators with suitable pulse transformers. For this purpose the Westinghouse 132-DW or 132-DW2 pulse transformers[3] would have been quite suitable, but were not available. Air-cored r.f. pulse transformers using 20 turns of 22 S.W.G., having a 1 : 1 ratio were tried, but the coupling was found to be too small and the efficiency poor.
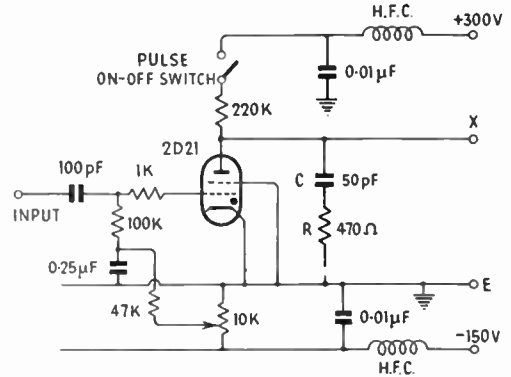

Fig. 6.—*Pulse-sharpening circuit.*[7]

(*b*) A second approach was to use a delay line with a 1-μsec delay. A single-trigger pulse from the trigger generator was injected at one end of the delay line and the first two reflections were used as trigger pulses for the scaling circuit. In this way the pulse width could be reduced to 0·7 μsec. Using this method, the 6J6 scaling circuit was found to have a resolving time of 1·3 μsec. The accuracy of this method was limited by multiple reflections and the pulse width of 0·7 μsec.

(*c*) In the circuit shown in Fig. 6,[7] the pulses are produced by the discharge of a capacitor C through R and the thyratron, the output being taken from P. The thyratron is 2D21 valve, having an ionization time between 0·05 and 1 μsec, and de-ionization time of 130 μsec. This xenon-filled tetrode is capable of
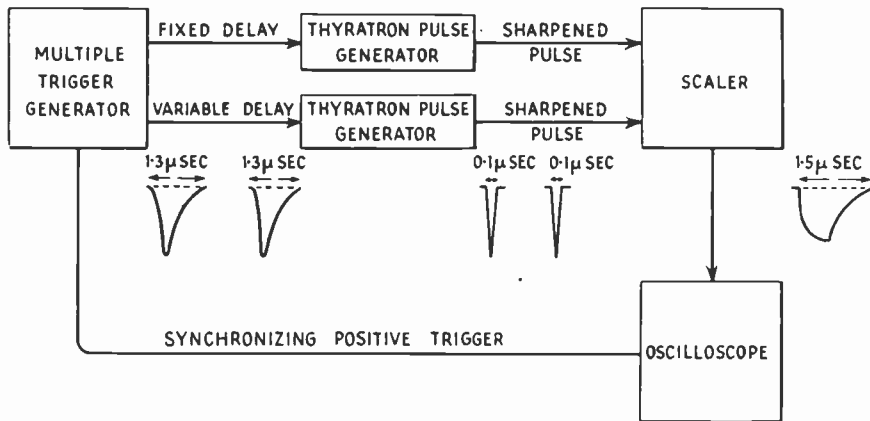
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Fig. 9.—Block diagram of testing equipment.

passing 500 mA peak current. The ionization time falls with increasing triggering voltage and is approximately 0·3 μsec for a pulse of 60 V amplitude. The de-ionization time can be reduced by holding the grid highly negative between triggering pulses, a holding potential of approximately 95 V was found suitable.

As the shape of the output pulse and its amplitude are dependent on the inductance of the leads forming the output network and on the stray capacitance between point X and earth, special care has to be taken in wiring, making it as compact as possible.

Two such units were made and used to sharpen the multiple-trigger generator pulses. The output pulses obtained had an amplitude of 100 V and the R.C time constant (Fig. 6) was adjusted to give a pulse width of 0·1 μsec. The rise time of the sharpened pulses was very high, while the time delay between the two pulses could be made as short as 0·2 μsec, without mutual interference. The block diagram of the complete testing arrangement is given in Fig. 9.

When these triggers were fed to the 6J6-scaling circuit it showed the resolving time to be 1·3 μsec in conformity with the previous result.

5.4. Improved 6J6-Circuit

The 6J6 circuit which gave a resolving time of 1·3 μsec was improved as follows. It was observed that the discharge portion of the output rectangular pulse was quite sharp,

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though the charging portion showed a trailing edge. Germanium crystal diodes 1N34 were used from the grids to a tap on the cathode resistor to discharge the 20-pF cross-coupling capacitors, through the low-input impedance of the opposite valve.

Furthermore, it was estimated, from the analysis[2] of transient conditions during triggering, that the current required for charging the stray capacitance plus the coupling capacitance and the input capacitance of the scope (approximately 100 pF) must be large to shorten the charging time. This meant the use of smaller anode loads, which would necessitate higher gain valves, if the change-over action is to remain fast. This was achieved to a certain extent by reducing the anode loads from 12 kΩ to 3 kΩ. The improved circuit gave a resolving time of 0·25 μsec.

5.5. 6AK5—Scaling Circuit

It was felt at this stage that if higher gain valves like the 6AK5 were used, one could utilize higher currents for charging up the stray capacitances, as smaller anode loads of 1,000 Ω or less could be used without slowing down the change-over action. Such a circuit was tried, using 1 kΩ anode loads for the 6AK5 valves while the remainder of the circuit (Fig. 7) was unaltered. The resolving time was the same as in the last case, except that the charging portion of the output pulse was much sharper. This condition is essential, particularly at higher

repetition rates. For example, in scintillation counter work or in electronic computers, very fast pulses and repetition rates of 4 Mc/s are quite common. A circuit having a resolving time of 0·25 μsec, which produces a sharply defined rectangular output waveform, can easily handle such rates.

5.6. *Another type of 6AK5—Scaling Circuit*[8]

Another circuit using 6AK5 valves for work at 5 Mc/s (Fig. 8) repetition rate was tried, and is described below. The essential feature of this circuit, as in the previous cases, which allows its operation at such high speeds is the use of 1N34 germanium diodes between the grids and a tapping point on the cathode load. This permits use of large coupling capacitors which can be discharged quickly after each triggering to a voltage, from which the action can again be initiated.

The higher gain and higher transconductance of the 6AK5 valve permits the use of plate loads as low as 670 Ω with heavy charging currents. The resolving time was 0·25 μsec, as in the last case, although the waveform was more sharply defined.
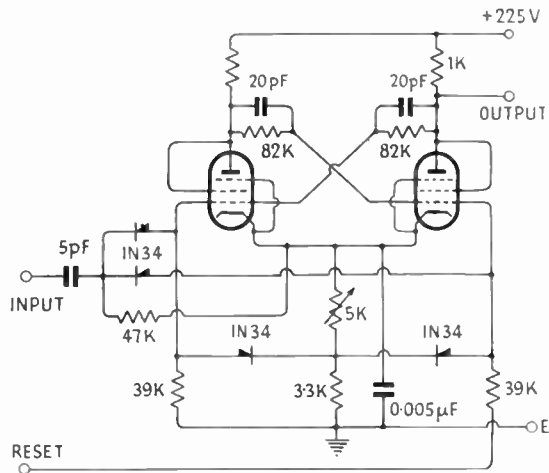

Fig. 7.—6AK5 scaling circuit.

It was, however, not certain whether the resolving time of the circuit was better than this. The reason being that the trigger pulses themselves were 0·1 μsec wide and their separation could not be less than 0·2 μsec to avoid overlapping.

6. Results

The results obtained with the various scaling circuits are given in Table 1.

One thing becomes very obvious from these results, that although for continuously recurring pulses we do not need the resolving time to be very small, yet for random pulses it must be so, otherwise the counting loss is high. The theory behind this from a statistical point of view has already been discussed in earlier sections.
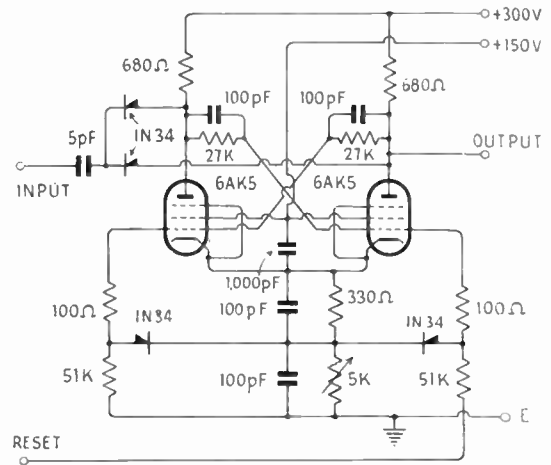

Fig. 8.—Scaling circuit using 6AK5 valves.[8]

The effects of changes in the circuit parameters on the resolving time and other considerations for its improvement can be summarized as follows:—The resolving time depends ultimately on the equilibrium potentials and the values of the circuit components. The smallest interval time τ in which the circuit resolves two successive pulses with similar characteristics is called the resolving time for pulse pairs. It depends principally upon the switching and transfer characteristics. The resolving power for periodically recurring pulses is determined by the recovery time; because, in this case, the recovery to equilibrium values has to be almost complete. The resolving time for random as well as for uniformly spaced pulses is the same.

The initiation of the regenerative transfer cycle has a certain time delay which depends on the steepness of the rectangular switching

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waveform, on the equilibrium values of the grid potentials, on the gain of the stages, on $\Delta E_a/\Delta E_g$, on the intrinsic time constants of the unit (the fastest anode transient change) and in particular on such components as the plate loads and the grid-anode coupling capacitors. To decrease this time delay, tubes should have high transconductances, high $\Delta E_a/\Delta E_g$, the smallest possible circuit capacitances (except the anode-grid capacitors which would have values several times greater than the tube capacitances) and low ΔE_g (grid voltage excursions). The problem can be reduced to a great extent to the choice of the valves.

In the design of scaling circuits with very short resolving times, all the above points must be considered; in addition, the transient response can be improved by series inductance compensation, while attention must be paid to the characteristics of the triggering waveforms. Stabilized and well-filtered power supplies are also necessary.

The important waveforms have been photographically recorded. Fig. 10 shows 1·3-μsec wide pulses directly from the multiple-trigger generator. The delay between them is variable. Fig. 11 shows the same pulses after being sharpened by the pulse-sharpening circuit. The pulse width is about 0·1 μsec, the rise times are still smaller. So the time delay between them can be made as small as 0·2 μsec without overlapping. They are here seen separated by

about 1 μsec. The minimum resolving time they can measure is 0·25 μsec. Fig. 12 shows the rectangular output waveform of a scaler when it is just resolving. The total width of the waveform is about 1·5 μsec. The resolving time is about 1·25 μsec. This is the waveform at the output of the unimproved 6J6 scaler using sharpened pulses.

7. Conclusion

The double-pulse method of measuring the resolving time of scaling circuits has distinct advantages over the other methods mentioned in section 4.

Its advantages are:

(1) Being an oscilloscopic method it is quicker to use, and permanent records of the waveforms can be made.

(2) Due to the time markers measurements are accurate.[2]

(3) Does not require much apparatus.

The following suggestions on future development are made:

(1) The measurements depend upon the correct shape and sharpness of the triggering pulses. When scalers with resolving times of the order of 0·01 μsec or less are to be tested one would require pulse widths of the order of 0·001 μsec or less, so that special circuits must be designed. Moreover, the screening problem then becomes acute.

Table 1

| No.	Type of scaler	Sharpness of output waveform	Resolving time measured	Maximum continuous repetition rate	Maximum number of random counts for 1 per cent. counting loss
			(μsec)	(per sec)	(per sec)
1	6SN7—Higginbotham ..	medium	7·5	133,333	1,333
2	B. Ramchandra's scaler	poor	8·0	125,000	1,250
3	6J6—lock-in type ..	medium	1·3	769,230	7,692
4	6J6—improved	good	0·25	4,000,000	40,000
5	6AK5—in 6J6 circuit ..	good	0·25	4,000,000	40,000
6	6AK5—other type ..	very good	0·25 or less	4,000,000	40,000
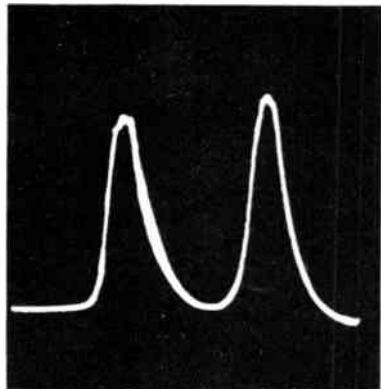
Fig. 10.—*This figure shows the waveforms of the multiple-trigger pulses.[3] One is a variable-delay trigger and the other the fixed-delay trigger. The delay between them can be varied. The pulses have almost the same amplitude and are* 1·3 *μsec wide.*

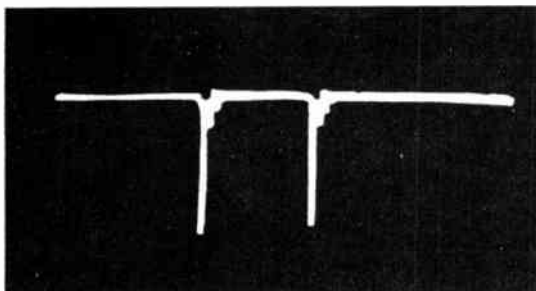Fig. 11.—*These are the sharpened pulses, coming from the thyratron pulse generator.[7] They have widths of the order of* 0·1 *μsec. The rise times are extremely small. The minimum time delay for which they can work is about* 0·2 *μsec.*
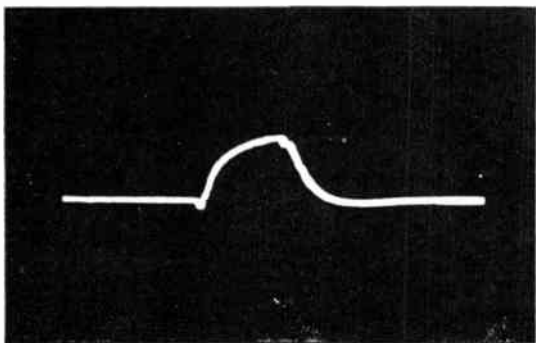

Fig. 12.—*This shows the rectangular output waveform of the scaler when it is triggering properly but is barely resolving two closely spaced pulses. The resolving time corresponding to this position is about* 1·25 *μsec.*

(2) For observing such sharp pulse waveforms one must have an oscilloscope with very fast writing speeds.

However, for the usual type of laboratory scaling circuits, the method described is quite efficient and adequate.

8. Acknowledgments

9. Bibliography

1. L. J. Rainwater and C. S. Wu, "Applications of probability theory to nuclear particle detection," *Nucleonics*, **1**, p. 60, October 1947.

2. W. L. Buys, "Analysis of scale units," *Nucleonics*, **3**, p. 49, November 1948.

3. W. C. Elmore and M. L. Sands, "Electronics Experimental Techniques," pp. 111-114; pp. 206-209; pp. 356-359. (McGraw-Hill, New York, 1949).

4. C. W. Johnstone, "A High-Speed Scaler Design," Declassified document. AECU-364 (LADC-669).

5. B. Ramchandra Rao, "A new type of scale-of-two unit," *J. Scientific and Industrial Research*, **9B**, p. 185, 1950.

6. Warren H. Bliss, "Recorder and timer for short intervals," *Electronics*, **19**, p. 126, November 1947.

7. C. S. Fowler, "Narrow pulse generator," *Wireless Engineer*, **27**, p. 265, October/November 1950.

8. H. E. Singleton, "A digital electronic computer," *Proc. Inst. Radio Engrs*, **38**, p. 1422, December 1950.

10. Appendix

In our particular problem let A be the number of atoms present in a sample before the start of the counting period. Each atom has the probability p, say, of disintegrating and producing a count which is detected during the counting interval; p will be very small, as it is the product of separate probabilities that a given atom will disintegrate multiplied by the probability that the given disintegration will be detected in the given time interval, and both of these factors

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are very small. We want to find out the probability P_n that exactly "n" counts will be obtained during the counting period.

This problem, which is known as the Bernoulli's problem, is very similar to the throws of a single die such that n sixes are obtained in A successive throws. The solution is,

$$P = (1/6)^n (5/6)^{A-n} \frac{A!}{n!(A-n)!} \quad \cdots \cdots (7)$$

Here p replaces $1/6$ and $(1-p)$ replaces $5/6$ so that,

$$P_n = \frac{p^n (1-p)^{A-n} A!}{n!(A-n)!} \quad \cdots \cdots \cdots \cdots (8)$$

If u indicates $n_{av.}$ or the average number of counts obtained, we get some interesting results.

$$\sum_0^A P_n \equiv [p + (1-p)]^A \equiv 1 \cdots \cdots \cdots \cdots (9)$$

The term P_n is the $(n+1)$th term in the above-mentioned binomial expansion. The mean value,

$$u \equiv n_{av.} \equiv \sum_0^A n P_n = \sum_1^A n P_n$$

$$= pA \sum_0^M \frac{p^m (1-p)^{M-m}}{(M-m)!\,m!} M! = pA$$

where $M = A - 1$ and $m = n - 1$, and using equation (9).

Thus,

$$u = n_{av.} = pA \quad \cdots \cdots \cdots \cdots \cdots (10)$$

When p is very small and A is very large and $n \ll A$, equation (8) becomes simplified by substitutions,

$$A(A-1)(A-2)\ldots(A-\overline{n+1}) \simeq A^n,$$

$$(1-p)^{A-n} \simeq e^{-p(A-n)} \simeq e^{-u}, \text{ and}$$

$$p^n A^n = u^n \text{ giving,}$$

$$P_n \simeq \frac{u^n e^{-u}}{n!} \quad \cdots \cdots \cdots \cdots \cdots \cdots (11)$$

This is the Poisson distribution.

In the derivation of the above equation, it is assumed that "p" is the same for all the atoms, but it can be shown that the above distribution is much more general. The above equation can be alternatively derived by using the law of large numbers.

Inauguration of V.H.F.-F.M. Service in Great Britain

The first of the B.B.C.'s very-high-frequency transmitting stations at Wrotham, Kent, was brought into full service on May 2nd, 1955. It transmits the Home Service, Light Programme and Third Programme in parallel with the existing long- and medium-wave stations and is intended to serve a potential audience of some 13 millions in Greater London and the south-east of England. Freedom from interference caused by other stations and electrical apparatus, an extension of the coverage of the programmes to certain areas at present poorly served, and improved quality because of increased bandwidth in the receiver, are some of the advantages expected.

Members will know that the B.B.C. carried out long-term trials with an experimental installation at Wrotham to compare the relative advantages of amplitude and frequency modulation. Details of the transmitters were published in the *Journal* four years ago.[1] Special comparator receivers used in connection with the subjective assessment of the two systems have been described in a paper by Beaumont[2] and subsequently problems of v.h.f. broadcasting have been discussed in papers by Brinkley[3] and Adorian.[4] In the latter paper, presented during his tenure of office as President, Mr. Adorian disclosed details of the field strength contours of the experimental transmitters. On April 13th last, an Institution meeting was held in London under the chairmanship of Mr. Adorian to discuss the new service, and an account of the discussion will be published in the June issue of the *Journal*.

On the basis of the experience gained from the prolonged tests from Wrotham, the B.B.C.'s Engineering Division put forward a scheme for providing a frequency-modulation v.h.f. service which would cover some 98 per cent. of the population.[5,6] This scheme was divided into two parts:

Stage I, which will cover the main centres of population as quickly as possible and bring a v.h.f. service to about 83 per cent. of the population, comprises five high- and five medium-power stations; it was approved by H.M. Postmaster-General in July 1954, and work is in hand. A map giving the estimated coverage has been published by the B.B.C. Stage II, which

will require about 16 further high-, medium- and low-power stations, has not yet been approved; it is hoped that about nine more stations will be completed by the end of 1957, and that v.h.f. coverage of 98 per cent. of the population will be obtained within four years.

The timetable for the opening of the Stage I stations allows for the Pontop Pike medium-power transmitters to open during May, followed by the Northern Ireland and North-East Scotland stations later in 1955. The remaining six will follow during 1956.

Wavelengths for the three programmes at the first ten stations will be grouped in the frequency ranges 88·1–90·1 Mc/s, 90·3–92·3 Mc/s and 92·5–94·5 Mc/s for the Light, Third and Home programmes respectively, the Wrotham frequencies being 89·1, 91·3 and 93·5 Mc/s; channel spacings are 200 kc/s.

At Wrotham, the original transmitters were designed so that they could be adapted to work as either f.m. or a.m. transmitters depending upon the outcome of the tests. Since the cessation of the experimental transmissions in March 1955, work has been in progress at Wrotham to install additional transmitters and carry out modifications to those already on site together with the associated aerial, drive, programme input and other auxiliary equipment.

The two existing transmitters will normally carry the Light and Third programmes respectively, two new 4·5 kW transmitters will act as spares. Two further new transmitters, each of 10-kW output, will operate together and carry the Home Service. The layout and design of the equipment for the Home Service transmitters has been so arranged that they will, ultimately, be automatic in operation. Certain ancillary equipment common to all three services will also be automatic.

The output and combining arrangements of these six transmitters are unusual and are of considerable interest. In the case of the first two pairs, the failure of the normal transmitter is covered by the reserve; manual switching of the outputs and application of power to the reserve being necessary. The outputs of whichever transmitters of these two pairs are operating are fed to a combining circuit, at the output of which the Light and Third programmes appear

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as a combined signal. This output is then split into two halves, each half being fed to one section of a further combining circuit.

The output of each 10-kW Home Service transmitter is arranged to feed the other section of this combining circuit. Thus, at the outputs of the second combining circuit the signal appears as two half-power combinations of Home, Light and Third signals, and under normal conditions these two half-power combined signals are taken over separate feeders to the two halves of the aerial system. By this means the effect of faults developing in the transmitters is reduced to a minimum and the failure of any one transmitter or of one-half of the aerial would be almost undetected except by listeners on the fringe of the service area. It is intended that the other high-power stations will also use pairs of transmitters for each programme. In addition, a very comprehensive emergency switching installation is provided to guard against the failure of the various combining circuits and the aerial system.

Duplicate FMQ drives are installed for each service and these are fed simultaneously with the programme. Automatic changeover arrangements ensure that either drive may be selected, with the other acting as spare. The "FMQ" system of frequency modulation was developed by Marconi's Wireless Telegraph Co., Ltd., and comprises a quartz crystal oscillator connected through a quarter-wave network to a balanced modulator, the susceptance of which varies with the modulating signal, and in turn varies the frequency generated by the crystal oscillator. The crystal is specifically cut so that it does not produce spurious harmonics within the operating range. The chief advantage claimed for this system of frequency modulation is that the circuits are much simpler than those of other systems, and therefore easier to maintain.

The output of the crystal oscillator is passed through three stages of frequency doubling and one tripling stage to produce the required carrier frequency. In the case of the two original transmitters, there then follow five stages of amplification. The first two are conventional push-pull stages while the remaining three are single-ended earthed-grid stages with coaxial-line tuning elements. The output stage consists of two BR128 valves in parallel, giving an output of 25 kW. Supplies at 6 kV and 3 kV for the

valve anodes are obtained from hot-cathode mercury-vapour rectifiers in the power conversion plant which is installed behind the transmitter.

The 4·5 kW stand-by transmitters are generally similar in electrical design to the earlier ones; the first two stages are push-pull and these feed two single-ended earthed-grid stages provided with coaxial-line tuning elements, one of which forms the output stage.

The new 10-kW transmitters for the Home Service also follow closely on the design of the earlier transmitters, having two push-pull stages followed by three single-ended earthed-grid stages and these three stages are again provided with coaxial-line tuning elements. The final stage has two BR.191B valves operating in parallel.

The filaments of all the valves are a.c. heated and all six transmitters use air-cooled valves; the cooling system of the 25-kW transmitters can be re-circulated within the building for heating in cold weather.

Since the transmissions are automatically monitored at Broadcasting House no master control position is installed at Wrotham. If a programme fault occurs, a signal will be given on the alarm and on the indication panel in the control room. Provision is included for the unattended operation of the Home Service transmitter in the early morning.

References

1. "B.B.C. v.h.f. transmitting station." *J.Brit.I.R.E.*, **11**, p. 72, February 1951.
2. F. H. Beaumont, "The design of a comparator a.m./f.m. broadcast receiver." *J.Brit.I.R.E.*, **13**, p. 131, March 1953.
3. J. R. Brinkley, "Very high-frequency sound broadcasting—the case for amplitude modulation." *J.Brit.I.R.E.*, **11**, p. 585, December 1951.
4. P. Adorian, "The future of broadcasting." *J.Brit.I.R.E.*, **13**, p. 81, February 1953.
5. "The B.B.C. scheme for v.h.f. broadcasting." *B.B.C. Quarterly*, **1**, p. 62, 1951.
6. P. A. T. Bevan, "Some aspects of v.h.f. sound broadcasting and f.m. broadcast stations (with reference to the performance of the B.B.C.'s Wrotham v.h.f. station)." *Electronic Engineering*, **27**, pp. 96-101, 147-153, March and April 1955.

HEAVY-DUTY, HIGH-STABILITY D.C. POWER SUPPLIES*

by

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SUMMARY

Series-valve stabilizers are considered as a simple step-by-step development of the cathode follower. By shunting a resistor or barretter across the series valve, substantially constant loads of several amperes can be stabilized to $\pm 0 \cdot 01$ per cent. For operation from a.c. mains, a heavy-duty d.c. power unit comprises a medium-power series valve stabilizer complementing an unstabilized rectifier and filter circuit. This latter supplies up to 95 per cent. of the load current, preferably through barretters. If the barretters precede the rectifier, a variable-ratio current transformer can be used to adjust their effective regulating current. Practical details of heater supply stabilization, protective delay circuits, and rectifier and filter economics are discussed. Full circuits are given of two typical units providing outputs of 250 and 500 watts with efficiencies of 35 and 65 per cent. respectively. The latter includes a device providing audible warning when supply-voltage variations exceed the range stabilized automatically.

1. Introduction

Much has been written already on the subject of d.c. voltage stabilizers and on the series valve stabilizer[1] in particular. This circuit is now available commercially in a variety of forms and so the majority of light-current engineers seldom have need to concern themselves with the finer points of stabilizer design.[2, 3] On the other hand, literature on the subject has tended to become increasingly specialized. Indeed, some of the mathematical treatments are so thorough as to be almost discouraging. Consequently, there is little inducement for engineers to familiarize themselves with the principles of stabilizer design and so to keep fully in view what can and cannot be readily achieved.

It is hoped that this paper will prove useful to the engineer who is occasionally called upon to design stabilized d.c. power supplies. Practical details are given of the means by which conventional circuits can be adapted to provide power outputs of the order of 1 kW and it will be shown that even heavy-duty units of this kind may be considered as a simple step-by-step development of the cathode-follower circuit. Approximate expressions are developed to indicate the performance of each circuit, but the mathematics has been simplified intentionally by neglecting the effects of certain refinements.

This means that in many cases the performance can be made better than indicated here. In particular, the output impedance can usually be made zero or even negative should this be required.

2. Simple Degenerative Stabilizers

A cathode follower can be used as a simple voltage stabilizer as shown in Fig. 1. A reference potential, E_{ref}, must be applied to the grid, but as no current is drawn from this source, a miniature h.t. battery of the hearing-aid type can be used and its working life is substantially the same as its shelf life. The output voltage is equal to the sum of E_{ref} and the bias voltage at which V1 operates. It can be seen on inspection that the output impedance is given by the well-known expression, $Z_o = 1/g_m$. The circuit operates from zero current to the maximum permissible cathode current for the valve. The supply voltage must not exceed $(E_{a(max)} + E_{ref})$, where $E_{a(max)}$ is the maximum anode-cathode voltage permissible for the valve. The minimum supply voltage must usually exceed the output voltage by at least 100V if a useful current is to be supplied. Hence the total acceptable range of supply voltage is given by

$$E_{i(max)} - E_{i(min)} = (E_{a(max)} + E_{ref}) -$$
$$- (E_{ref} + 100)$$
$$= E_{a(max)} - 100$$

A better performance can be obtained by using a second valve, as in Fig. 2, to increase the effective mutual conductance of the cathode

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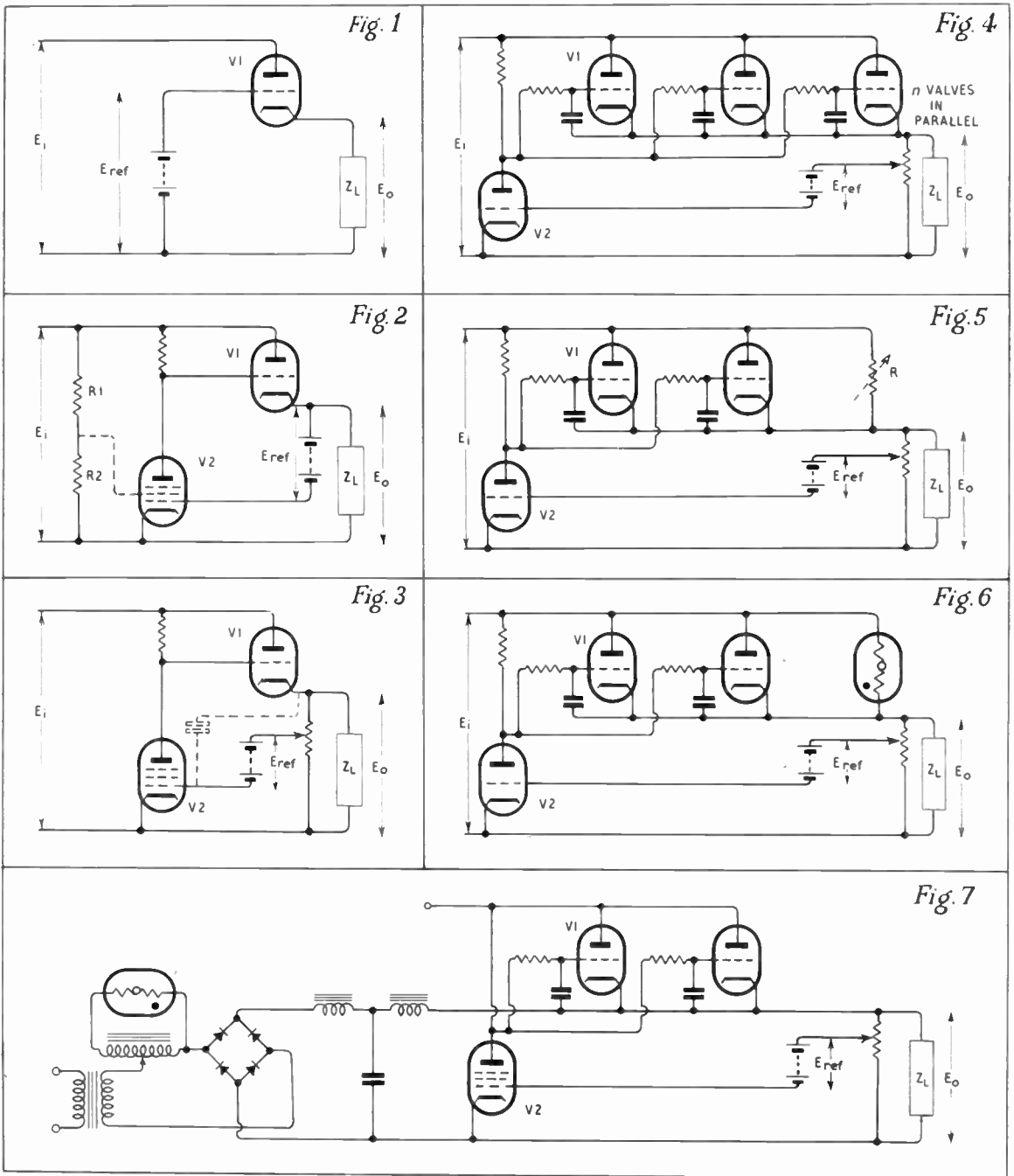
Figs. 1–7.—*Development of the series-valve stabilizer from the basic cathode-follower circuit.*
(For description see Table 1).

Table I

APPROXIMATE EXPRESSIONS RELATING TO CIRCUITS OF FIGS. 1-7

Circuit	Output Impedance z_o	Output Current Range	Supply Voltage Range
Fig. 1	$\dfrac{1}{g_m}$	0 to $I_{a(max)}$	$(E_{a(max)} - 100 \text{ V})$
Fig. 2	$\dfrac{1}{A.g_m}$	0 to $I_{a(max)}$	$(E_{a(max)} - 100 \text{ V})$
Fig. 3	$\dfrac{E_o}{E_{ref}} \cdot \dfrac{1}{A \cdot g_m}$	0 to $I_{a(max)}$	$(E_{a(max)} - 100 \text{ V})$
Fig. 4	$\dfrac{E_o}{E_{ref}} \cdot \dfrac{1}{A \cdot n \cdot g_m}$	0 to $n \cdot I_{a(max)}$	$(E_{a(max)} - 100 \text{ V})$
Fig. 5	$\dfrac{E_o}{E_o/R + E_{ref} \cdot A \cdot n \cdot g_m}$	$\dfrac{E_i - E_o}{R}$ to $\left(\dfrac{E_i - E_o}{R} + n \cdot I_{a(max)}\right)$	$n.I_{a(max)}.R$
Fig. 6	$\dfrac{E_o}{E_o/R_{hot} + E_{ref} \cdot A \cdot n \cdot g_m}$	$\dfrac{E_i - E_o}{R_{hot}}$ to $\left(\dfrac{E_i - E_o}{R_{hot}} + n \cdot I_{a(max)}\right)$	$nI_{a(max)}.R_{hot}.S$ or E plateau, whichever is less
Fig. 7	$\dfrac{E_o}{E_o/t^2 \cdot R_{hot} + E_{ref} \cdot A \cdot n \cdot g_m}$	$\dfrac{E_i - E_o}{t^2 R_{hot}}$ to $\left(\dfrac{E_i - E_o}{t^2 R_{hot}} + n \cdot I_{a(max)}\right)$	$n.I_{a(max)}.t^2 R_{hot}.S$ or $t.E$ plateau, whichever is less

(For key to symbols, see text)

follower. The improvement is directly proportional to the stage gain, A, of V2. Thus, the output impedance is reduced to $1/A.g_m$ whilst the current and voltage ranges remain unaffected. The output voltage is now equal to E_{ref} less the bias voltage of the amplifier valve, V2. This bias is usually about 1 or 2 V and, if the value of A is large, it will change only by a few millivolts if the supply voltage, E_i, varies. Changes in the heater supply voltage of V2 effectively change the bias on this valve, however, and this change in bias is added to, or subtracted from, the output voltage. Where the best performance is required, therefore, it is important to stabilize the heater supply to V_2. This may be effected conveniently by means of a barretter[4] or a constant-voltage transformer. Alternatively, the heater may be supplied through a series resistor from the stabilized output. If the load current is constant, the heater may often be supplied more efficiently by connecting it in series[5] with the load.

Stabilization of heater supply to V1 is of less importance, as the effect of the equivalent bias shift in this valve is reduced at the output terminals in proportion to the amplification, A, of the degenerative feedback through V2.

If the screen of V2 is connected to a potential divider across the supply, as suggested in Fig. 2, a proportion of the supply variation is applied to the screen of V2 and produces an excursion of the grid potential of V1 in the opposite sense. By a judicious proportioning of R1 and R2, perfect stabilization can be achieved for a given set of conditions and extending over a range limited by the curvature of the valve characteristics. If the source impedance of the supply is sufficiently high, this technique can even be used to provide a stabilized supply with zero or negative output impedance.

By the addition of screen grid compensation, all the circuits described hereafter can be made to show an improvement in performance.

It will be appreciated that the degenerative nature of the basic circuit, and of the others to be discussed, makes the output impedance tend towards zero whether or not compensation is applied to the screen grid. The approximate expressions quoted for the output impedance remain useful, therefore, as an indication of the

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order of magnitude to be expected, even though under some conditions the value may become negative or zero.

3. Variable Output-voltage Stabilizers

In many cases, a continuously variable adjustment of the output voltage is required. Since it is undesirable to draw current from the reference voltage battery, as would be done if a potentiometer were connected across it, the reference voltage cannot be changed continuously. A potentiometer may, however, be connected across the stabilized supply, as in Fig. 3, and the circuit will then operate to maintain substantial equality between E_{ref} and the proportion of E_o selected by the potentiometer. Consequently, movement of the slider of the potentiometer towards the negative output rail results in an increase in the output voltage, E_o.

Tapping down the potentiometer in this way clearly reduces the effective internal gain of the feedback loop in the proportion E_o/E_{ref}. Consequently the output impedance increases to $E_o/(E_{ref}.A.g_m)$.

If the output voltage is to be varied over a wide range, a constant value of E_{ref}, equal to the minimum required value of E_o, leads to a large ratio E_o/E_{ref} at the maximum output voltage. In consequence the output impedance rises and the stabilizing action deteriorates. This difficulty may be overcome by switching in additional reference batteries to increase the value of E_{ref} when higher outputs are required. In this way, E_o/E_{ref} need never exceed 2.

Hum ripple in the output can be reduced substantially by the addition of a feedback capacitor as suggested in Fig. 3. With this arrangement, the a.c. impedance of the output can at all times be kept as low as that of the circuit shown in Fig. 2. Where an output is required at 400–500V, it is usually uneconomic to increase the reference voltage beyond 90–120V. The addition of the feedback capacitor then offers an improvement of 4 to 5 : 1 in the hum level which is readily attained in practice. It may, however, be necessary to decouple the grid of V1 to avoid parasitic oscillations.

The minimum output voltage which can be stabilized by this circuit is equal to the sum of the "bottoming voltage" of V2 and the grid base of V1. In general, this approximates to about 50V.

When the load current is to exceed the value conveniently handled by a single series valve, V1, other valves may be connected in parallel with it, as indicated in Fig. 4. All the power valves should have matched characteristics and individual decoupling of the grids becomes essential to prevent oscillation. Provided these precautions are taken, however, the parallel arrangement of n valves performs as a single valve of proportionately increased current capacity and mutual conductance. Consequently, the output impedance is reduced in proportion to n, and becomes $E_o/(E_{ref}.A.n.g_m)$ and the maximum output current is increased to $n.I_{a(max)}$.

4. Heavy-current Stabilizers

Load currents amounting to several amperes cannot be supplied conveniently by the type of circuit indicated in Fig. 4 since a dozen or more power valves would be required in parallel. Quite apart from the expense of this arrangement, a practical limitation may be encountered in the difficulty of maintaining stability.

If the load is to be substantially constant, the majority of the power valves may, however, be replaced by a resistor,[4] as shown in Fig. 5. The resistance is chosen so that the greater part of the load current is supplied through R and the feedback circuit ensures that the series valves supply the remainder. Any variation of the supply voltage, E_i, produces a change in the current through R which will result in a complementary change in the current passed by the series valves. It follows, therefore, that if the value of E_i changes by more than $n.I_{a(max)}.R$ the stabilizer will run out of control unless the value of R is re-adjusted. In practice, R is adjusted manually to keep the anode current of the power valves at a mid-range value. Such adjustment can, of course, be made without affecting the stabilized output voltage.

The power efficiency of a d.c. stabilizer is given approximately by E_o/E_i. When large-output currents are required, it becomes desirable to raise the efficiency as far as possible. This implies that $E_i \approx E_o$. Such a condition requires a low value of R, however, and consequently a severe restriction of the acceptable range of E_i.

Conversely, a high value of E_i may be used to increase the permissible percentage variation in

E_i, but in this case the efficiency soon becomes very poor. Practical circuits necessitate a compromise, the efficiency falling from 67 per cent. to 25 per cent. as E_i/E_o is increased from 1·5 to 4·0.

With the circuit shown in Fig. 5, the choice of input voltage, E_i, may be influenced by a desire to obtain from the series valves the maximum cathode current permitted by the manufacturers' rating. This demands that the anode to cathode voltage (E_i-E_o) shall be sufficiently high, yet not so high that the anode dissipation is exceeded. This consideration, therefore, determines the efficiency which may be realized and the range of supply voltages over which stabilization can be maintained, even with manual adjustment of R.

Since it is desirable to keep the series valves working in the middle of their available range, it is clear that, with a constant load current, the manual adjustment of R serves to keep constant the current flowing through R. A logical development is to replace R by one or more barretters, as shown in Fig. 6. Slow changes in E_i are thus accommodated automatically as long as the barretter is working on the plateau of its current stabilization characteristic. The range of available output current is not changed by the use of a barretter, but the acceptable range of supply voltages may be extended by a factor S to $n.I_{a(max)}.R_{(hot)}.S$ where S is the stabilization ratio of the barretter and R_{hot} is its mid-range hot resistance. If S is very high, however, i.e. if the plateau in the barretter characteristic has virtually zero slope, then the range of E_i becomes approximately equal to the voltage range over which the plateau extends.

Theoretically, the range of acceptable supply voltages is not improved in respect of abrupt supply variations. As the barretter produces a substantial correction within a fraction of a second, however, it is permissible momentarily to exceed the anode dissipation of the series valves. When barretters are used, therefore, the circuit may be designed so that the series valves operate normally at about 75 per cent. of their rated anode dissipation. The anode to cathode voltage (E_i-E_o), must then be chosen so that the cathode current can rise momentarily to an anode dissipation about 50 per cent. above the rated value.

Although supply variations tend to occur as a progression of small changes, the importance of the transient response must not be under-estimated. When barretters are used, the series valves are concerned solely with the stabilization of transients, since slow changes are accommodated by the barretters themselves. Moreover, the mean current passed by the series valves may represent only 4 per cent. of the load current. In this case, a 2 per cent. variation in (E_i-E_o) will momentarily produce a 50 per cent. variation in the series-valve current. In a stabilizer having an overall efficiency of 50 per cent., this transient will be produced by an abrupt supply variation of 1 per cent. It will be seen, therefore, that the barretter should have as rapid a response as possible so that the series valves may operate under conditions favourable to the stabilization of successive variations.

5. Stabilizers with Extended Supply-voltage Range

By using a number of small barretters, rather than a single large one, a satisfactory speed of response can be obtained. This procedure has also the merit of enabling cheap, mass-produced barretters to be used, spares for which can readily be obtained. Moreover, barretters may be switched in and out of circuit to adapt the stabilizer to different load currents.

Unless the current capacity of the series valves is greatly in excess of the current passed by a single barretter, it is desirable to have some method of adjusting matters so that the series valves work in the middle of their current range. This can be done by shunting the barretters with a resistance variable over a range sufficient to bridge the steps in the total current output of the barretters. This has, however, the disadvantage of off-setting, to some extent, the automatic control provided by the barretters. The same objection applies to the use of a variable resistance connected in series with the barretters. At the expense of a reduction in efficiency, a better result can be obtained by connecting the variable resistance in parallel with the load.

The ideal would, of course, be found in a barretter with a smoothly adjustable regulating current. In an a.c. circuit, this can be simulated by using a continuously variable transformer as a current transformer to change the effective barretter current by a factor $1/t$. If the stabilizer is operated from smoothed, rectified a.c., the barretter and variable current transformer can precede the rectifier, as shown in Fig. 7. In order

to maintain a reasonable plateau in the transformed barretter characteristic, it is generally necessary to restrict to about 3 : 1 the total range of transformed current. Barretters may conveniently be switched into circuit to double the regulated current at each step, however, and these large steps are more than spanned by adjustment of the variable transformer.

When the load current is varying continuously, the barretters may be replaced by a saturable reactor with feedback control.[6] The reactor then provides low-speed current control over a wide range so that the series valves are required only to provide rapid control over a restricted range. Such a system makes it possible to increase still further the power efficiency of the unit.

6. Rectifier and Filter Circuits

In many cases, no d.c. mains supply is available and the stabilizer must operate from a.c. mains. The a.c. supply can, of course, be rectified and smoothed, but special problems arise. Since the stabilizer may be able to handle instantaneous supply variations of no more than 5V, it is clearly essential to reduce to less than 1V the ripple voltage in the unstabilized d.c. supply.

When a 3-phase supply is available, polyphase rectification may be considered. This offers the apparent advantages of reduced ripple amplitude and higher ripple frequency. The resulting economies which may be effected in the filter circuit are, however, offset by the expense of the double-wound 3-phase transformer which is required. Moreover, unbalance in the 3-phase supply, or inequality of the rectifier resistances lead, respectively, to 100 c/s and 50 c/s ripple components. It is found that a filter adequate to cope with these components as well as the principal ripple at, say, 300 c/s, costs almost as much as a filter for a bridge rectifier on a single-phase supply.

Using a single-phase supply, one has the option of employing an auto-transformer or even of dispensing with an input transformer entirely. The reduction in weight may be useful, and in many cases there may be no inconvenience in the fact that the d.c. output cannot be earthed. A relatively small double-wound transformer is sufficient to provide h.t. for the series valves.

The selenium bridge rectifier which supplies the greater part of the d.c. can, of course, be obtained as a single unit. It has been found, however, that mass-produced selenium rectifiers, intended for use in radio receivers, offer an economic alternative when used in appropriate series-parallel arrangements. In the event of an accidental overloading of the rectifier bridge, only two of the component units are destroyed. This is because, in any arm of the bridge, failure of one of the rectifier units protects other units in the same arm by short-circuiting them. A short-circuit in one arm usually produces overloading and failure of the diametrically opposite arm. Here, too, however, the failure of one rectifier protects the others. With two arms short-circuited, the bridge presents a low impedance to the a.c. supply and the supply fuses are blown.
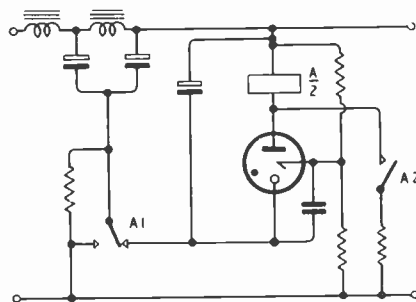

Fig. 8.—*Simple delay circuit using cold-cathode trigger tube. The capacitors of the filter circuit are used as the timing element and are charged slowly before application of the full voltage.*

Replacement costs with the multi-unit rectifier are measured in shillings rather than pounds. Also, because the units are readily obtainable, the apparatus can be returned to service within hours rather than months. Compared with a single bridge rectifier, the array of smaller units may occupy more space. On the other hand, the greater flexibility of mounting often outweighs this drawback.

When the barretters are used in the a.c. supply, as shown in Fig. 7, the voltage ratings of the rectifier and electrolytic filter capacitors may be reduced. It must be remembered, however, that if the load current is interrupted, the full a.c. supply voltage is applied to the rectifier. Some form of automatic cut-out may, therefore, be desirable.

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As the series valves usually heat up more rapidly than the amplifier, there is a danger that the output voltage will exceed the stabilized value until the amplifier becomes operative. When the amplifier valve obtains its heater supply from a low-impedance source, the heating time of the series valves is often extended sufficiently if these heaters are supplied from a substantially constant-current source. Thus, for example, the amplifier can be connected to a constant-voltage transformer which will produce a current surge through the cold heater and bring it rapidly to operating temperature. The series valves, on the other hand, can be connected through a series resistor to a transformer winding providing about twice the required heater voltage. In this way the current surge through the series-valve heaters is limited to about 1·5 times the operating current. In consequence the heating time of the series valves is extended considerably.

When the amplifier valve derives its heater supply from the stabilized output, a separate delay circuit becomes necessary. Fig. 8 shows a simple cold-cathode valve circuit which can be used to provide this delay and at the same time provide gradual charging of the filter capacitor. By using this capacitor as the timing capacitor of the delay circuit, a 30-second delay is readily provided during which the electrolytic capacitor is charged slowly towards its working voltage. If the apparatus has been idle for several months, the delay is extended automatically until the filter capacitor has re-formed sufficiently to reduce the leakage current to a safe value.

7. Reference Voltage

All the circuits discussed above have used one or more hearing-aid-type h.t. batteries as a reference voltage. For some applications it is preferable to use a high-stability neon tube for this purpose. This neon tube usually has a
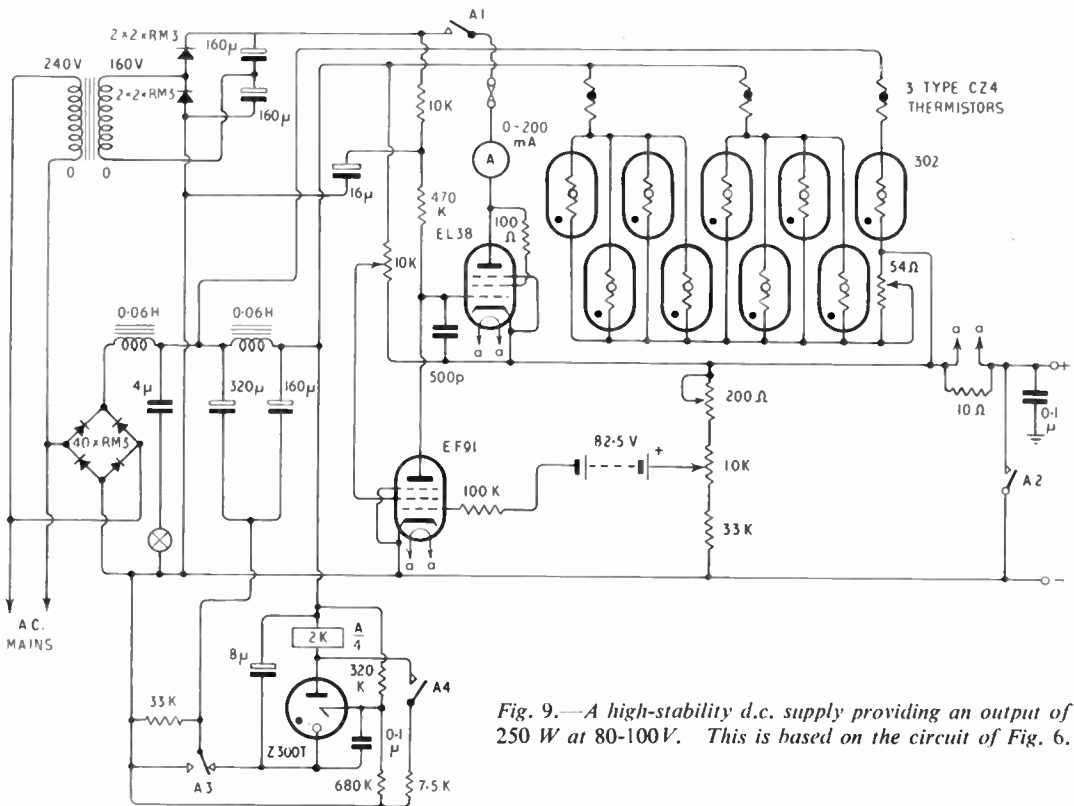

Fig. 9.—A high-stability d.c. supply providing an output of 250 W at 80-100V. This is based on the circuit of Fig. 6.
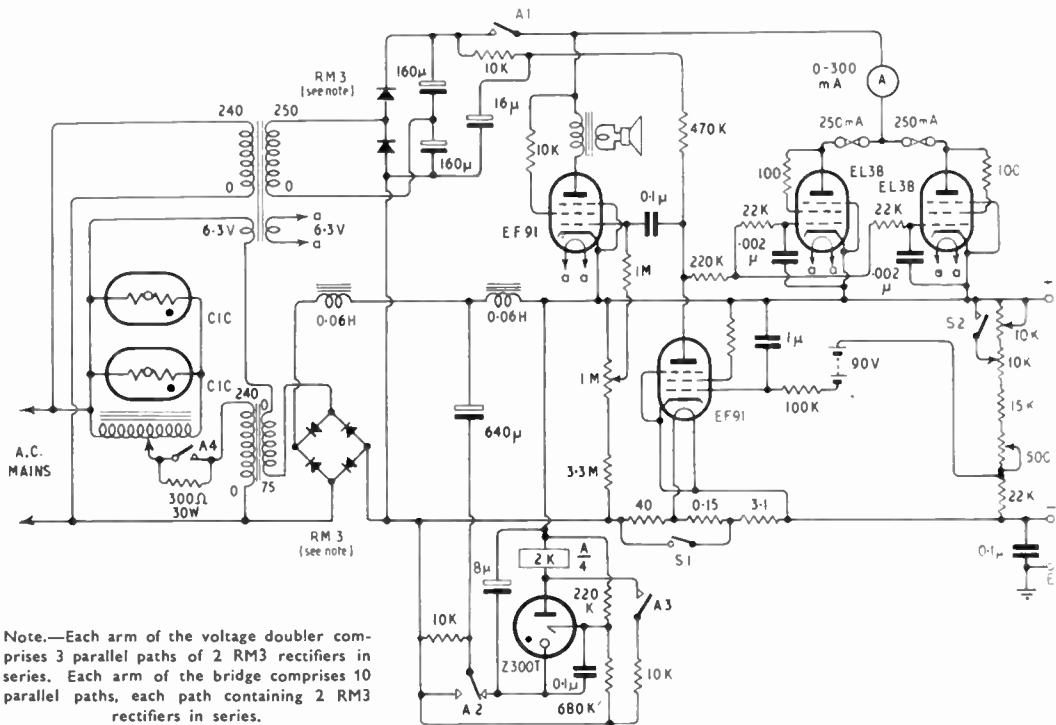
Fig. 10.—A heavy-duty stabilized d.c. supply providing up to 500 W at two alternative voltages between 150 and 230V. By basing the circuit on Fig. 7, the efficiency at maximum output has been raised to 65 per cent.

longer working life than the battery but, unless it is operated at constant current, it remains sensitive to supply voltage variations. Where the best possible short-term stability is required, the dry battery is difficult to excel.[7] It requires replacement at 2-year intervals and has an appreciable temperature coefficient. On the other hand it eliminates the warming-up period needed before a neon stabilizer attains full stability and provided the battery is mounted under the chassis and away from hot components its stability is very good.

8. Typical Circuits

Figures 9 and 10 show circuits of stabilized power units embodying the principles outlined above. The circuit of Fig. 9 was designed to operate a 250-W lamp at voltages ranging from 80 to 100V. With supply voltages ranging from 235 to 245V, the output is held automatically within 10 mV of the pre-set value. Larger supply

variations can be accommodated by adjusting the variable resistor in series with the barretters so that the milliammeter is kept roughly at a mid-scale reading. The output ripple is about 5 mV peak-peak and the power efficiency about 35 per cent.

There are several interesting points about this circuit. One of the pilot lamps is operated by ripple current from the filter circuit. This lamp is extinguished only if the delay circuit brings the filter capacitors into circuit correctly. By supplying one barretter from the junction of the two filter chokes, it has been possible to balance out part of the residual ripple at the filter output. The delay circuit initially short-circuits the output in order to over-run the barretters at first and so bring them rapidly up to operating temperature. To restrict the initial surge through the cold barretters, however, it is necessary to connect thermistors in series with them, as shown.

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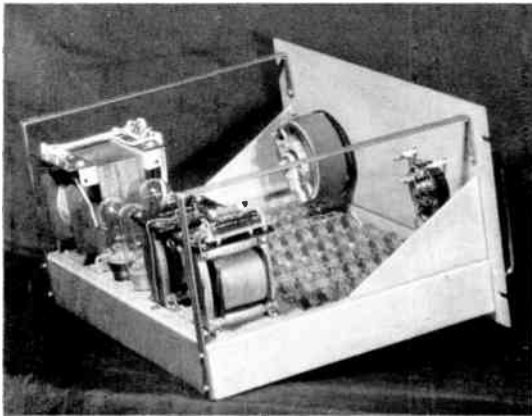
(a)
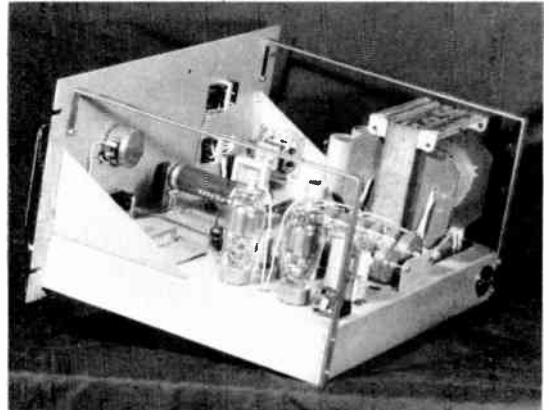
(b)

Fig. 11.—(a) Upper chassis of the 500-W stabilized d.c. supply shown in Fig. 10. The bridge rectifier may be seen clearly, consisting of 80 radio-type selenium rectifiers. Also visible are the barretters, variable-ratio current transformer, booster transformer and filter chokes.

(b) Lower chassis of the 500-W stabilized d.c. supply. With the exception of the delay circuit and the "rasper" loudspeaker (visible on far side of chassis), this chassis differs little from a conventional 300-mA series-valve stabilized power unit.

(c) Complete 500-W stabilized d.c. power unit, indicating simplicity of controls.
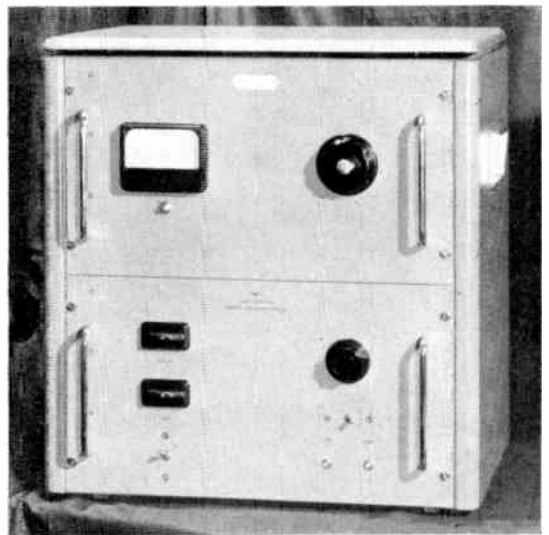
(c)

The circuit shown in Fig. 10 was designed to operate a 500-W lamp at two alternative pre-set voltages in the range 150–230V. In this case the barretters precede the bridge rectifier and, through a variable transformer, regulate the current passing through a booster transformer. This arrangement is largely responsible for the higher efficiency, up to 65 per cent., which is realized with this circuit. Since the regulating action of the barretters is not reduced by series resistors or thermistors, this circuit still controls automatically over a supply voltage range of more than ± 5V.

The alternative output voltages are selected by operation of the ganged switches, S1 and S2. The 40-Ω resistor introduced when S1 is opened ensures that the d.c. output required from the bridge rectifier shall be much the same on high- and low-output settings. This eases considerably the design of the barretter-controlled boost circuit. The 500-Ω variable resistor in the feedback potentiometer allows small adjustments to be made to the preset output voltages to allow for the effects of temperature changes on the reference batteries.

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As the 500-W unit is used in total darkness, a device is included to give audible warning when the anode current of the series valves becomes too high or too low. As soon as the limit of stabilization is reached, the ripple in the output increases abruptly. This ripple appears, amplified, at the anode of the error-voltage amplifier and to this is coupled a second EF91 biased to operate as an anode bend detector. The detector delivers sufficient power to produce a disagreeable noise in a miniature loudspeaker which has consequently been referred to as a "rasper." If necessary, the operator can readjust the variable transformer whilst still in complete darkness. By setting it midway between the two limits at which the audible warning is produced, he can thus maintain the stabilized output when supply variations exceed the range which is stabilized automatically.

9. Conclusion

The series-valve d.c. stabilizer circuit can readily be adapted to regulate outputs of the order of 1 kW with a short-term accuracy of better than ± 0.01 per cent. When the d.c. load is substantially constant, the stabilizer can be designed to operate from unstabilized single-phase a.c. mains with an efficiency of over 60 per cent. With the exception of the chokes and transformers, standard radio-type components may be used throughout.

10. Acknowledgment

11. References

1. F. V. Hunt and R. W. Hickman. "On electronic voltage stabilizers." *Review of Scientific Instruments*, 10, p. 6, January 1939.

2. J. J. Baruch. "Zero-impedance power supply termination." *Electronics*, 24, pp. 240–244, August 1951.

3. R. D. Trigg. "Voltage stabilization with series valve control." *Electronic Engineering*, 25, No. 304, pp. 254–256, June 1953.

4. R. J. Hercock. "A voltage stabilizer for photometer lamps." *Journal of Scientific Instruments*, 25, p. 320, September 1948.

5. C. Morton. "An electronic voltage stabilizer with self-regulated heater supply." *Electronic Engineering*, 24, No. 288, p. 65, February 1952.

6. D. C. Bakeman and J. E. Richardson. "Regulated d.c. supply has high efficiency." *Electronics*, 27, p. 165, October 1954.

7. F. A. Benson. "The design of series-parallel valve voltage stabilizers." *Electronic Engineering*, 24, No. 289, pp. 118–119, March 1952.