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A MEMBER'S RESPONSIBILITY

In all walks of life it is necessary to be frequently reminded of the principal goal for which we are striving. Amidst the various activities of all engineering institutions we must remember that the main purpose of such professional bodies is to promote the advancement of a branch of science and to afford means for disseminating knowledge.

The terms of reference of the various Standing Committees indicate the scope of the Institution's work in fulfilling those objects; but all these activities, and indeed, the status of the Institution is reflected in the Journal. It is a record of the Institution's progress and more especially a permanent record of papers useful in propagating advancement and knowledge of the science.

There is, therefore, a heavy responsibility in publishing the Journal and the onus largely rests, through the Council, on the Papers Committee. The task of selecting and approving material submitted rests wholly with this Committee, but it is important to note that the Committee does not originate the papers. This responsibility—and it cannot be over-emphasized—rests mainly on the membership.

The Journal enjoys a world wide circulation which is nearly double the actual membership; from its contents the Institution is judged by those who, for various reasons, cannot be in such intimate contact with our activities as the members themselves. It follows, therefore, that the status of the membership itself is judged by the contents of the Journal.

In the last decade especially, the Journal has contained many outstanding contributions to radio science. Some of these have come from non-members, which is an encouraging and welcome indication of the widening scope and circulation of the Journal. It is, therefore, the responsibility of the Committee to ensure that the high standard of papers shall be maintained and an even greater responsibility rests upon the membership to provide the papers. Whilst it may not always be possible for an individual member to contribute, he can play his part by suggestion to the Committee and invitation to authors.

In furtherance of these objects the Council has established six further premiums. The four principal premiums already awarded are well known and the additional premiums will recognize each year the most outstanding papers on specialized branches of radio science. Details will be circulated to the membership and in this way the Council gives further encouragement to the preparation of papers.

Members cannot better serve the Institution than by contributing to radio progress through the pages of the Journal. Scientific work is never complete until it has formed the subject of a paper. On such activity all our future depends; it is the life blood of the practising engineer and requires donation from many, rather than from the few.

H. M.

RADIO FREQUENCY WELDING OF PLASTICS*

by

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A paper read before the North Eastern Section on January 12th, 1949, and the London Section on January 20th, 1949.

SUMMARY

After a brief reference to the welding of thermoplastic materials by the heated tool and hot gas methods, the advantages of the radio frequency process are explained.

It is shown that thermal conduction of the welding electrodes profoundly modifies the power requirements from the associated radio frequency generator when material thickness is small.

The influence of frequency, voltage and variations in physical constants with changing temperature are studied whereby the specification for a suitable radio frequency generator for the work in view may be determined.

The latter part of the paper deals with the problem of designing radio frequency generator circuits to meet pre-selected load requirements. Typical oscillator circuits are discussed and analysed in some detail.

The paper concludes with a review of the present and future trends of radio frequency welding technique.

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1. Introduction

Thermoplastic materials are characterized by the fact that some of their physical properties, for instance, flexibility and ease of deformation, depend to a large extent on temperature. It is common practice to add plasticizers to the basic plastic materials and this addition considerably modifies the properties of the compounds. Such

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plasticized compounds, for instance, polyvinyl chloride containing one or more plasticizers, are used for the manufacture of films and sheets, which are comparatively flexible at room temperature. Generally speaking, the flexibility increases with increasing temperature. If the temperature is raised to a sufficiently high level the material softens considerably, although melting proper does not take place. If the temperature is further increased, decomposition occurs, which results in partial or complete destruction of the original compound.

The majority of thermoplastics exhibit this behaviour when subjected to heat treatment. It should be mentioned, however, that a few thermoplastics have proper and sharply defined melting points; for instance, polythene and nylon have such definite melting points, i.e., change from the solid into the liquid state at a closely defined temperature.

Attempts to unite thermoplastic materials by application of heat date back at least ten years and the earliest method for the " welding " of thermoplastics is described in a patent of the Dow. Chemical Co.[1]

This method consists of using an electrically heated torch and passing a heated stream of inert gas on to the edges of the material to be joined, whilst at the same time a filler rod of identical or similar composition is melted down into the seam. A process like this is, of course, very well suited to materials which flow well in the heated state, but it should be mentioned that success has also been obtained with this method in the United States and Germany for materials which do not flow very readily, for instance, rigid P.V.C.

Another welding method should be mentioned briefly, namely, the direct application of radiant heat to the surfaces to be joined. For the butt welding of thermoplastic pipes, for instance, the following procedure can be adopted :—

An electric heater suitable for transferring heat by radiation is brought into close vicinity of the pipe ends to be heated and, after softening both ends to a predetermined depth, the pipes are butted together so that the molten surfaces interfuse.

This process resembles the resistance butt welding of metals, whilst the previously described hot gas method is comparable with the oxy-acetylene welding of metals.

The application of these two welding processes depends mainly on the thermal properties of the plastics to be welded. A method which is based on the dielectric properties of the material is welding by radio-frequency current. This process makes use of the dielectric losses occurring in a thermoplastic material when the material is subjected to high-frequency alternating fields.

It should be noted that high-frequency welding is, generally speaking, not suitable for materials with low dielectric loss as the rate of heating depends on the loss factor of the plastic. For this reason plastics such as polystyrene and polyethylene cannot be welded by high frequency, at least not with radio frequencies of the order of 50 to 100 Mc/s.

Most other thermoplastic materials have loss factors which are approximately 1,000 times higher and these can be welded easily, particularly in sheet form.

Pressure has to be applied to the sheets during welding in order to obtain real interfusion of the surfaces, i.e., proper welds.

The main advantage of high-frequency welding is to be found in the fact that welding takes place in an extremely short period of time. Welding times are usually a few seconds only and are frequently under 1 second.

The appearance of the finished welded seam is very neat as the surfaces of the welding electrodes, which act as conductors for the high-frequency current, remain relatively cool.

It is possible to weld considerable areas of material in one single operation—a fact which is of great importance for mass production.

2. R.F. Welding of Plastics

2.1. *Thermal Considerations*

High-frequency welding of plastic materials, like any other welding process, requires that the temperature of the material be raised from ambient to softening temperature with a simultaneous application of pressure in order to obtain interflow of the surfaces to be bonded.

The amount of heat required to raise a material of specific heat c from ambient temperature T_1 to the softening temperature T_2 is given by the following relation :—

$$H = c\,(T_2 - T_1).G \ldots\ldots\ldots\ldots\ldots (1)$$

where H = Heat in cals.

c = Specific heat (cal. per gm. per deg. C)

T_1 = Room temperature (deg. C)

T_2 = Softening temperature (deg. C)

G = Weight (grams)

This equation, however, refers to the ideal case only and therefore does not make allowance for any heat losses due to conduction, convection or radiation.

In a recent paper[2] one of the authors has attempted to deal with the problem of heat flow during the welding period in a system comprising a pair of metal electrodes with interposed sheets of plastic material. It is shown that the power required for welding a given quantity of plastic material is greatly in excess of the theoretical requirements worked out from equation (1) by reason of thermal losses in the system. A further interesting fact was established in the course of this investigation, namely that the power density required for satisfactory welding depends on the thickness of the sheets to be welded in a particular manner.[2]

The paper referred to shows that the power density required for the welding of very thin materials is very high, but drops rapidly with increasing sheet thickness to a minimum and increases subsequently at a much less steep rate with further increase in sheet thickness. Fig. 1.

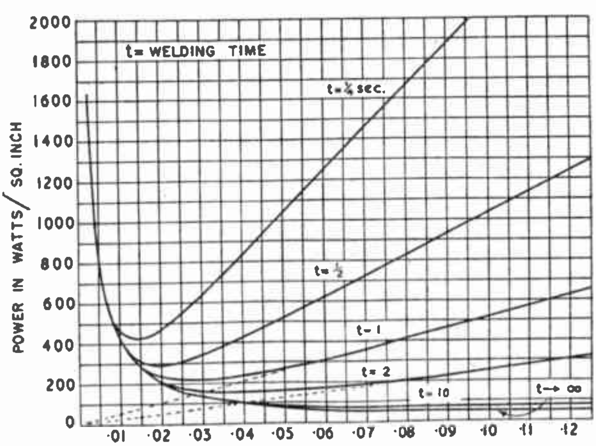

Fig. 1.—Power density required for welding as a function of thickness of material to be welded. Welding time as parameter.

This apparent paradox can be explained in the following way:

In the case of very thin sheets, the heat developed at the interface is transferred rapidly through the sheets into the cold electrodes and most of the electrical energy is used in balancing these heat losses. With increasing sheet thickness, the bad thermal conductivity of the plastic sheets slows up the heat transfer, i.e., less thermal energy is lost through the sheets and the power density for welding can therefore be reduced. For much thicker sheets the actual thermal requirements for welding increase on account of the increased volume of material and the power density has, therefore, to be increased again.

As the welding of relatively thin plastic sheets is of great interest, two methods of overcoming these thermal losses should be mentioned briefly.

The first method consists of providing thermal insulation between the material to be welded and the electrodes. This has the effect of reducing heat flow from the plastic to the electrodes, but presents further electrical problems regarding the supply of high-frequency power through the heat-insulating medium.

The second line of attack is the heating of the metal electrodes themselves. In this case, the temperature gradient between the plastic sheets and the electrodes is reduced by virtue of the increased temperature of the electrodes and heat losses are, therefore, reduced accordingly. The heating of welding electrodes, however, presents a considerable problem, particularly in the case of live electrodes.

2.2. *Electrical Considerations*

2.2.1. *Influence of Frequency*

The following basic formula[4] can be used to evaluate dielectric losses in insulating material of dielectric constant ε and loss angle δ, the voltage gradient being F V/cm at frequency f

$$W = \text{Const}, x\, F^2.\varepsilon.\tan\delta.f. \text{ watts/cm}^3 \ldots (2)$$

It will be seen that the dielectric loss and, therefore, the heating effect is a linear function of the frequency.

For this reason the use of very high frequency appears to be desirable for the welding of plastic materials, since for a given power the voltage gradient required for welding decreases with the square root of the frequency. For technical reasons, however, which will be explained in a later section of the paper, it is not practicable to increase the frequency beyond certain limits which are imposed by the availability of circuit elements. In addition it should be pointed out that the magnitude of the load capacitance in a welding circuit often prevents the use of very high frequencies because of the difficulty of feeding power efficiently to the electrodes.

Loss angle and dielectric constant vary somewhat with frequency, although the variations are not large over a range of, say, 10 to 100 Mc/s.

2.2.2. *Influence of Voltage*

When using a homogeneous electric field in the space between the welding electrodes, the heating effect depends on the square of the voltage gradient, or, assuming constant sheet thickness, on the square of the applied voltage.

The limiting factor in this respect is obviously the breakdown voltage of the material at the particular frequency used; for reasons of safety the actual field strength should be well below the breakdown value for that frequency.

Breakdown strengths of plastic materials as quoted by manufacturers have to be considered carefully as the values given are frequently not applicable for instantaneously applied voltages of steep wave front; it is well known that the conditions for electrical breakdown depend considerably on the rate of application of voltage. Figures quoted for the breakdown strength often refer to specimens of particular thickness and are not necessarily identical with those for much thinner sheets.

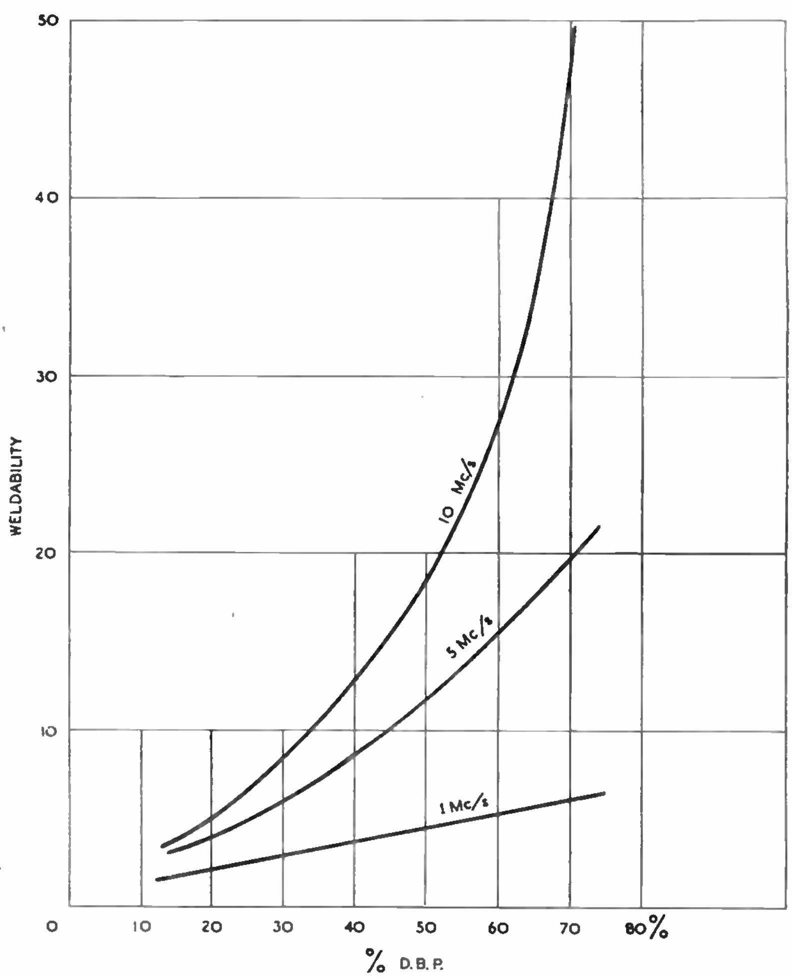

Fig. 2—Influence of plasticizer content on " weldability " ($\varepsilon \times \tan\delta \times 10^3$). Plasticizer: Dibutyl phthalate.

Attention should also be given to the fact that the thickness of the sheets after welding is very much reduced compared with the original thickness so that the same applied voltage results in a much greater field strength at the end of the welding period, if the voltage is maintained throughout the cycle.

2.2.3. *Influence of Variations in Physical Constants of Material with Temperature*

From equation (2) it will have been noted that

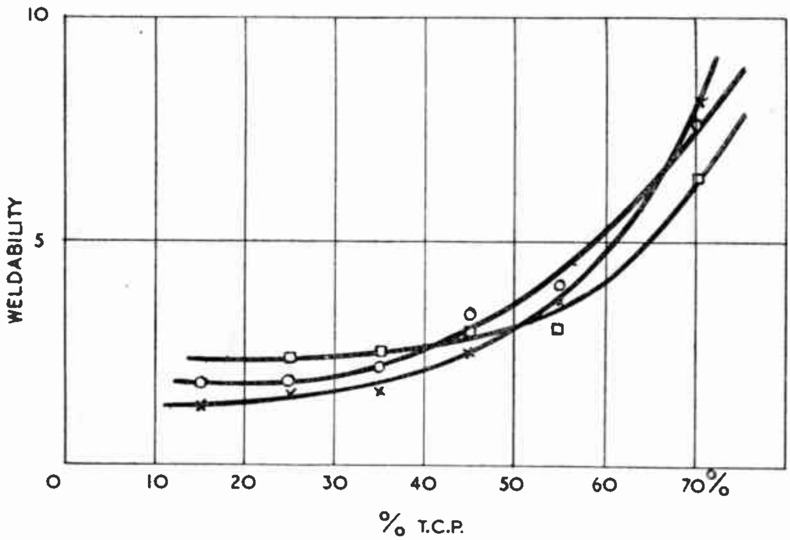

Fig. 3.—Influence of plasticizer content on " weldability " ($\varepsilon \times \tan\delta \times 10^2$). Plasticizer : Tricresylphosphate.
The three curves shown are for frequencies of 1 *Mc/s* [x], 5 *Mc/s* [○] *and* 10 *Mc/s* [□].

heating effect increases in linear proportion to the product of dielectric constant and loss angle. Unfortunately, this product (the loss factor) itself is not strictly constant; the loss angle, at least, varies considerably with temperature. Particularly in the case of plasticized sheet material, the loss angle rises steeply in the range between room temperature and softening temperature. This leads to increasing power absorption in the material during welding.

Furthermore, the increasing temperature causes evaporation of the plasticizer as most of the plasticizers used are somewhat volatile and evaporate at temperatures below the softening point of the plastic.

As has been shown by one of the authors[3] the loss factor of plasticized sheet materials varies considerably with increasing plasticizer content, reference figures 2 and 3. A loss of plasticizer due to evaporation, therefore, is certain to have an influence on the power absorption during the welding period.

2.3. *Mechanical Problems Connected with the Design of R.F. Welding Plant*

The following three types of high-frequency welding machines can be distinguished at present :—

(*a*) Continuous roller welding machine.

(*b*) Jig or bar welders.

(*c*) Press welders.

(*a*) The roller welding machine represents the earliest attempt to use radio-frequency power for the welding of plastics. Basically this type of welder makes use of a pair of counter-rotating rollers which serve as electrodes, and, at the same time, transport the sheet in the direction of the welding seam.

The mechanical design of roller welding machines requires extremely high accuracy in manufacture and it is desirable that the speed of the welding machine should be variable at will by the operator, preferably from zero to full speed.

The latter requirement introduces a considerable complication into the design of the roller welder. Obviously, the amount of power required for welding the material between the roller depends, among other things, on the speed of operation. It is desirable that the power adjustment should be made automatic as it would be impossible for the operator to readjust the power in proportion to the variation of speed at any moment. Various mechanical and electrical devices have been incorporated in roller welding machines for the purpose of inter-relating speed and radio-frequency power supply automatically.

Generally speaking, however, the early expectations regarding this type of welding machine have not been fulfilled, and machines of this type—at least as far as this country is concerned—have made way for other types of welders.

(*b*) The jig or bar welder uses an entirely different principle. Discontinuous welds of finite length are carried out by moving at least one of a pair of electrodes and clamping the material to be welded between the pair of electrodes.

These machines are not self-feeding but are capable of welding some 6 in to 12 in of material in each operation.

The design of such a jig welder is shown in Fig. 4, which represents a typical equipment with 350 watt R.F. output at 30 Mc/s. In this machine the live electrode—which is surrounded by moulded insulating material—is fixed to a stationary electrode holder, whilst the other electrode which is kept at earth potential is clamped to a moving arm, pivoted at the top of the equipment and operated through a pedal-operated lever.

From the mechanical point of view a high degree of accuracy in machining the electrode surfaces is essential. Considering that materials of thickness down to approximately 5 mils have to be welded, the flatness of an electrode of, say, 10-in length has to be within plus or minus 1 to 2 mils in order to guarantee even welding along the whole electrode surface. It is customary to grind these electrodes on a surface grinder in order to obtain accurate alignment between them under all circumstances.

The pressure required for welding depends on the nature of the plastic material, i.e., comparatively low pressures are required for the welding of highly-plasticized P.V.C. sheets for instance, whilst the pressure has to be increased for the welding of the more rigid cellulose acetate and other rigid or semi-rigid materials.

Actual welding pressures range from 20 to 80 lb/sq. in. of electrode surface area. It should be mentioned that this pressure should only be applied to the material until it is sufficiently heated for welding in order to avoid excessive penetration of the electrodes in the weld area. As it is obviously impossible to vary the pressure during the welding period in a mechanically operated machine, means have to be provided to limit the approach between electrodes in such a manner that the operator has a definite control over the final thickness of the weld.

Such mechanically adjusted limiting devices are incorporated in most commercial welding machines ; as a general rule it can be stated that the final gap between the electrodes after completion of the weld should be roughly 50 per cent of the total sheet thickness prior to welding.

(*c*) For the welding of larger articles mechanically operated press welding machines are employed. The basic principle is similar to that of the jig or bar welder, except that press welding machines are usually provided with comparatively large platens which are brought together under pneumatic, hydraulic or mechanical pressure, and between which the actual welding operation takes place.

Fig. 4.—High-frequency jig or bar welder, 350 *watts output, frequency* 28 *Mc/s.*

A machine of this type is shown in Fig. 5. This particular machine has a radio-frequency output of approximately 1·5 kW at 38 Mc/s and a platen area of 18 in × 14 in. The earth-connected bottom electrode is made in the shape of a flat rigid plate whilst the top electrode is fixed to the insulated top platen of the press by means of thumb screws.

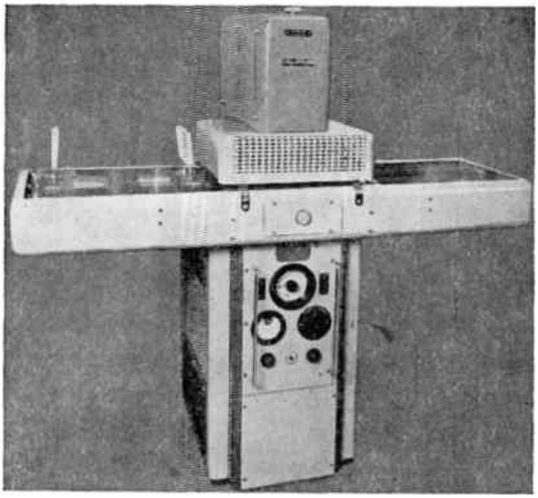

Fig. 5.—Power-operated automatic high-frequency press welder, 1·5 kW output, frequency 38 *Mc/s.*

Special provision has been made for rapid operation by using a two-station sliding tray in the form of a metal sheet sliding over the top of the bottom electrode. This tray is provided with two loading stations consisting of insulating material cut out in the shape of the article to be welded ; in operation one of these stations is under the press for the duration of the welding cycle, whilst the other one is unloaded and reloaded.

The press in this case is toggle operated using an electro-magnet as driving force. The operation of the equipment is completely automatic with the following work cycle :—

After loading the first station with two sheets to be welded, the operator pushes the tray forward until it touches a mechanical stop. This action operates a switch which, in turn, initiates the following cycle :

(i) Mechanical closing of the press and application of pressure.

(ii) Application of the radio-frequency power by means of switching the high tension transformer with simultaneous starting of the automatic timer.

(iii) Completion of the welding cycle and delay period for application of post-welding pressure.

(iv) Mechanical opening of the press.

As soon as the press has opened the operator can move the tray again and start the next cycle which is identical in all respects.

In order to prevent too great a penetration into the plastic material, an adjustment of the fulcrum of the toggle press has been provided which is operated from the micrometer screw at the top of the equipment.

Regarding the mechanical design of press welding machines, it should again be pointed out that extreme accuracy is required for the machining of electrode surfaces, as for the larger weld areas, homogeneous pressure distribution is even more critical than in the case of the straight bar welder. It is obviously much easier to obtain reasonably uniform pressure over a straight line than over an elaborately shaped contour which encloses a large area.

3. R.F. Generators for Welding

3.1. *Typical Oscillator Circuits*

For R.F. welding work, as for other industrial applications of oscillators, many circuit arrangements are possible. The short time cycles generally necessary, as mentioned in earlier sections of the paper, and the variability of the load, both with regard to different work requirements and change of characteristics during the heating cycle, require circuits having particular characteristics.

Either of these circuits must be capable of easy load adjustment or they must be of a type possessing inherently the feature of " constant loading."

A complete realization of the latter ideal is, of course, impossible but a good approximation to it is considered in the following section.

The choice of circuit will not only be governed by the loading requirements, but also by the fact that frequencies lower than 20 Mc/s are not useful in the specialized field of welding.

A circuit typical of the first-mentioned class is illustrated in Fig. 6.

This is essentially a standard form of single-ended oscillator incorporating a fixed-coupling output circuit capable of working into capacitive loads of widely varying impedance and power factor.

Once the grid excitation voltage has been correctly determined by tests, the circuit seldom needs adjustment in this respect. The degree of load coupling needs to be determined experimentally as it is invariably a compromise between the need for close coupling in the interests of circuit efficiency and for loose coupling to avoid the well-known " double tuning bump " effect of Ziehen.

Naturally enough the flexibility of this circuit is not obtained without a disadvantage. This is the somewhat higher loss in the coupling circuits resulting from high primary circuit kVA necessary for frequency stability, i.e., freedom from the Ziehen effect over a useful range of loads.

For output powers of a few hundred watts, the circuit can be designed for frequencies up to about 30 Mc/s. Above that frequency the primary inductance L_1 becomes so small that its construction is physically difficult having regard to the need for maintaining a reasonable coupling coefficient with the secondary (load) coil.

One of the authors[4] pointed out in a previous paper an interesting property of circuits of the series capacitance type whereby with proper choice of component values, an oscillator could be designed to give approximately constant output over a range of load capacitances of fixed power factor.

About the same time, W. Schreuer[5] independently discovered this effect and utilized the principle by a neat application of the well-known " π " network. Figure 7 illustrates the general circuit arrangement.

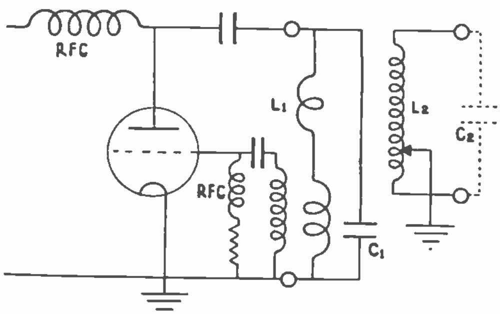

Fig. 6.—Circuit diagram of a typical oscillator with R.F. transformer coupling to the load.

The simplicity of the circuit arising from the absence of any tapped or coupled coils is evident. Apart from the unusual output terminal arrangement, the circuit is essentially of the Colpitts type, the grid-excitation being derived from the potential-divider action of the valve grid-filament and anode-filament capacitances. Once the proper grid-driving voltage has been realized, no further adjustment is necessary unless the circuit is modified in respect of capacitances *Co* and *Cp* to operate over a different range of loads (see section 3, 2.2.).

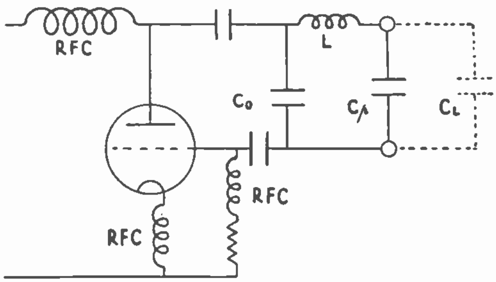

Fig. 7.—Circuit diagram of an oscillator employing a "π" network with direct connection of the load.

The form of circuit just described is useful for frequencies up to about 60 Mc/s provided output capacitances are restricted to some 60 pF. A natural extension of the principle leads to the omission of L from the circuit, resulting in a transmission line which is of length never exceeding λ/8 and is usually considerably shorter. In this way, the frequency may be increased to 80 Mc/s and upwards, or capacitive loads up to 200 pF may be used at somewhat lower frequencies.

3.2. *Investigation of Output Circuit Networks*

Because of the relatively high frequencies required for welding operations compared with general dielectric heating, the development of generators necessitates a good deal of experimental circuit work before prototypes can be built satisfactorily.

Much of this work arises from two main causes. There is first the difficulty of estimating or measuring accurately the exact load impedances into which the generator will have to work. Secondly, there is the ever present difficulty in all high (radio) frequency circuits of making allowances for the unknown factors of stray inductances, capacitances and couplings.

Nevertheless, it is the authors' experience that much time can be saved by intelligent approximation in circuit design so that work begins with components likely to be of the right order of magnitude instead of being arbitrarily chosen.

Whatever type of oscillator is to be used, it is of fundamental importance to ensure that the valve works into its designed load resistance, that the circuit losses are not too high and that the overall circuit Q is reasonable. Accordingly, the circuits discussed in the preceding section are analysed with these objects in view.

3.2.1. *The Coupled Circuit*

When the secondary capacitance is small or the load power factor is high so that the primary kVA exceeds the secondary kVA, the Ziehen effect is absent as the circuits are singly periodic. The design can then be based on resonance in the secondary circuit and the effective resistance of the whole assemblage can be determined[4] by:

$$R_L = \frac{\tan \delta_L}{k^2} \sqrt{\left(\frac{L_1}{C_1}\right)} \quad \ldots\ldots\ldots(3)$$

in units of ohms, farads, henries, $\tan \delta_L$ being the load power factor and k the coefficient of coupling.

It is true that in practice such circuits are not operated at the secondary tune point but rather on the side of the resonance curve approaching the peak. The actual transferred resistance is therefore not as large as that given by the equation and some uncancelled reactance remains to reduce the current ratio i_2/i_1 from its possible maximum.

Nevertheless, by choosing L_1, C_1 and k, such that it is possible to obtain an effective load resistance somewhat lower than that actually required by the valve, it is usually possible fully to load the valve well in advance of the secondary tune point.

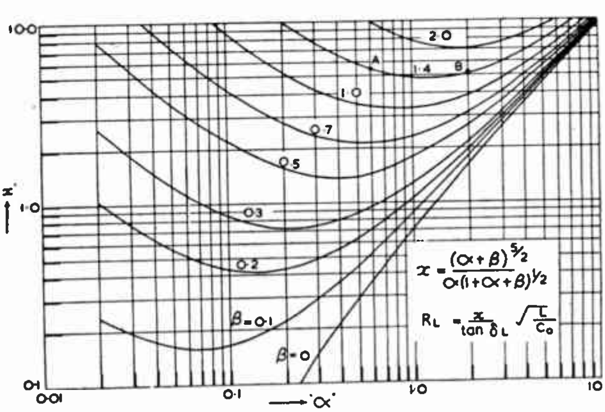

Fig. 8.—Graph of the constant x as a function of α (= C_L/Co) and β (= Cp/Co) used in determining valve loading (equations (4) *and* (10)).

3.2.2. *The "π" Circuit*

As will be evident by inspection, this circuit is of the tapped load type, the oscillatory circuit comprising capacitances Co, Cp and the inductance L.

The function of Cp is twofold. First, it restrains the voltage rise on load C_L when this capacitance is very low and, secondly, it enables a particular point to be chosen on the loading curve for the mean operating condition as will now be shown.

An analysis of this circuit (in appendix 7.1.1) shows that the effective load resistance is given by:

$$R_L = \frac{x}{\tan \delta_L}\left(\frac{L}{Co}\right)^{\frac{1}{2}} . \ 1{,}000 \text{ ohms} \quad \ldots\ldots(4)$$

where x is a function of the ratios C_L/Co and Cp/Co, L is expressed in μH and Co in pF and $\tan \delta_L$ is the load power factor. Writing α for C_L/Co and β for Cp/Co

$$x = \frac{(\alpha + \beta)^{5/2}}{\alpha(1 + \alpha + \beta)^{1/2}} \quad \ldots\ldots\ldots\ldots(5)$$

The variation of x with α and β is shown in Fig. 8.

Once the value of x has been found from equation (4), appropriate values of α and β are selected, according to the load requirements from Fig. 8.

In welding applications, it is generally desirable to arrange that the starting point of a heating cycle is at some place such as A on Fig. 8 since, during the cycle the applied pressure reduces the thickness and the power requirement tends to increase for reasons already given in section 2.1.

On the other hand with certain materials increasing temperature leads to an increase of power factor so that the generator would tend to be overloaded. In these circumstances, it would be preferable to make the starting point at B on Fig. 8 or at some intermediate point between A and B, determined by experiments.

So far the question of frequency has not been mentioned. It is, of course, implicitly settled when Co, C_L, Cp and L are chosen but naturally varies as C_L is changed. The most convenient

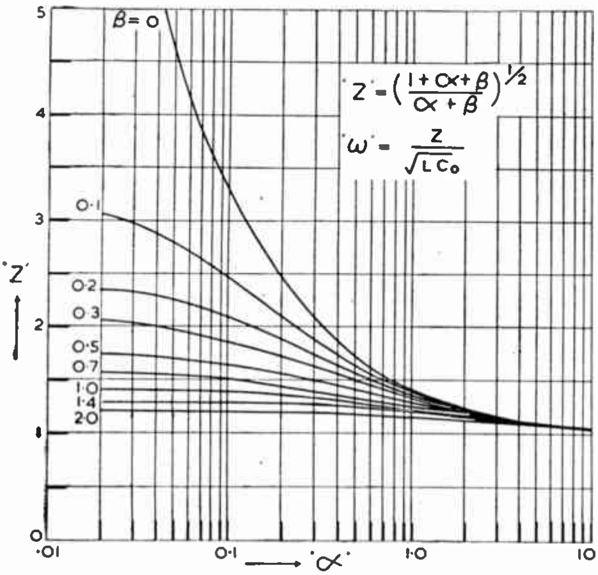

Fig. 9.—Values of the constant z in terms of α *and* β *used in equation* (19) *for determination of operating frequency for the "* π *" oscillator.*

way to observe the changes in frequency is to consider a basic frequency determined by *Co* and L alone. The parallel capacitances C_L and *Cp* in series with *Co* lead to a higher operating frequency which can then be determined in terms of α and β (see appendix 7.1.2). These relations are illustrated in Fig. 9.

Other points of interest are the circuit efficiency, the overall circuit Q and the voltage developed across the work capacitance C_L.

For a given load power factor, it is clear that the efficiency will be controlled by the losses in the inductance and the capacitance ratios α and β.

Denoting the reactance/resistance ratio of the inductance by Q_1 and that of the load ($Q_L = 1/\tan \delta_L$), modified by the capacitances *Co* and *Cp*, by Q_L' the efficiency η is easily shown in appendix 7.1.3. to be :

$$\eta = \frac{1}{1 + Q_L'/Q_1} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

As will be noted from the derivation of this relation the ratio Q_L'/Q_L is purely a function of the capacitance ratios α and β and hence the variation of efficiency under different loading conditions can most simply be shown by graphs, such as Fig. 10, drawn for individual values of β. While the graph shown is for $\beta = 0{\cdot}5$, it is typical of the families of curves and brings out clearly the importance of making Q_1 high unless the load power factor itself ($1/Q_L$) is also high. Except for extremely low values of $\frac{Q_1}{Q_L}$, the influence of α is not great unless α itself becomes smaller than 0·9. In this connection it is of interest to note that the peaks of the efficiency curve are higher and sharper for values of β lower than 0·5 and occur at a lower value of α (e.g., for $\beta = 0{\cdot}1$, maxima of η occur at $\alpha = 0{\cdot}4$). Conversely the efficiency peaks are somewhat lower and much flatter for higher values of β (e.g., for $\beta = 2$, maxima of η occur at $\alpha = 2{\cdot}5$).

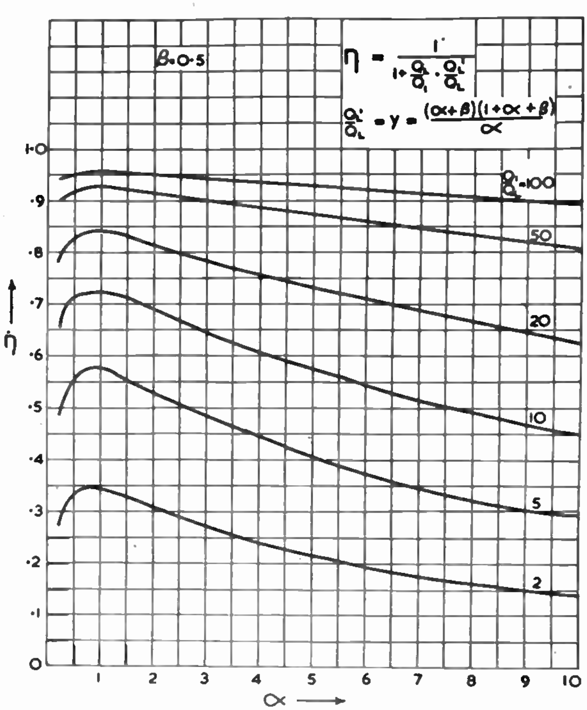

Fig. 10.—Circuit efficiency of the " π *" oscillator as a function of capacitance ratio* C_L/Co*, for a constant value of* Cp/Co*, showing dependence on the reactance/resistance ratio of the main inductance.*

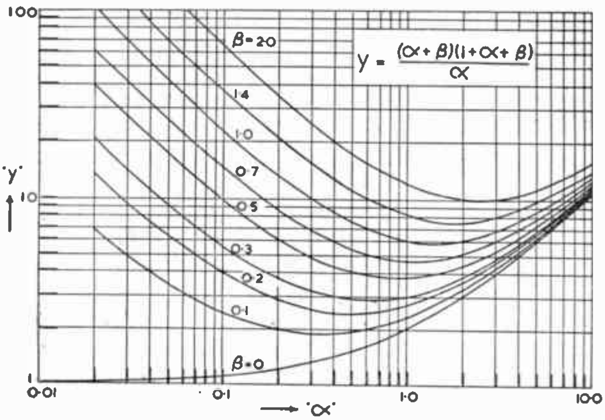

Fig. 11.—Values of the constant y in terms of α and β used in equation (7) for the determination of overall circuit Q for the "π" oscillator.

It is of interest to examine the overall circuit Q factor since with all self oscillators it is desirable to keep this factor in the range 12 to 20, if possible; lower values may lead to oscillator instability and higher values to disproportionate circuit loss. In appendix 7.3.4 it is shown that the same circuit parameters that influence the efficiency govern the overall Q factor. The expression derived is:

$$Q = \frac{y\,Q_L}{1 + y\,Q_L/Q_1} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

where y is a function of α and β only and is expressed by

$$y = \frac{(\alpha+\beta)\,(1+\alpha+\beta)}{\alpha} \quad \ldots\ldots\ldots(8)$$

This latter relation is given in graphical form by Fig. 11.

The load voltage is clearly independent of the frequency, where reasonable matching has been performed, being settled only by the capacitance ratios.

Inspection of Fig. 13*b* shows that:

$$\frac{V_L}{V_o} = \frac{1}{\alpha+\beta} \text{ (approximately)} \ldots\ldots(9)$$

where V_L is the load voltage and V_o the alternating anode/grid voltage.

A particularly important feature of the "π" circuit is to be found in the relatively constant voltage gradient in the material during the heating cycle. This means that a higher level of power may be used than is possible with circuits depending on the degree of resonance in a secondary coupled circuit. This is a matter of great importance when materials such as cellulose acetate have to be welded as it is essential to work close to the limiting voltage gradient for breakdown; it is under these conditions that relative constancy of load voltage is so desirable.

3.2.3. *The "Transmission Line" Circuit*

When the operating frequency of the "π" circuit is to be raised considerably, the inevitable minimum of valve and stray capacitances make it necessary for the series inductance to be reduced to a very low value, often only a fraction of 1 μH.

At this stage, it becomes necessary to replace lumped inductance by the distributed inductance of a (nearly) short-circuited line less than λ/4 in length.

The exact analysis of such a circuit is, however, somewhat difficult on account of the awkward expression for frequency in terms of the characteristic impedance of the line and its length.

By using an approximation in determining the frequency as given in appendix 7.2.1, it has been possible to derive an expression for the effective resistance of a short line end-loaded with a finite impedance. This analysis, given in appendix 7.2.2, also requires other small approximations to be made before a reasonably simple solution for design purposes is reached, but it does indicate the principal features of the transmission line circuit.

The expression for the load effectively presented to the valve is found to be closely similar to that previously derived for the circuit except that L is now expressed differently. The same constant x can be used and the curves of Fig. 8 are also applicable to the transmission line circuit.

Denoting v = velocity of E/M propagation
l = length of line (cm)
Z_o = characteristic impedance of line (ohms)
C_o = valve output capacitance (pF)
$\tan \delta_L$ = power factor of load capacitance, C_L.

the valve loading is given by :—

$$R_L = \frac{x}{\tan \delta_L} \cdot \left(\frac{Zol}{Cov}\right)^{\frac{1}{2}} \cdot 10^6 \text{ (ohms)} \quad \ldots .(10)$$

Again, the frequency is implicit in this relation and so for design purposes it is evaluated only after selection of capacitances Co and Cp, having regard to the probable range of load capacitances, in conjunction with trial values of l and Zo.

Indeed, unless the material to be welded has a pronounced maximum power factor at a particular frequency (and this is not general), the actual frequency is only important in so far as it is high enough to restrain the load voltage gradient to a value suitable for that material.

With a given line input voltage, itself fixed by the requirements of the oscillator valve, the load voltage is, of course, controlled by the line characteristics and its termination.

The output voltage across the load capacitance can therefore be determined from ordinary transmission line equations as shown appendix 7.2.3, and is found to depend on :

$$Vo = V_L [\cos \varphi - (Zo/X) \sin \varphi] \quad \ldots .(11)$$

Where $\varphi = \tan^{-1} (2\pi l/\lambda)$, $X = 1/\omega (C_L + Cp)$, and Vo and V_L are the line input and output voltages respectively.

Similarly, the load current can be found in terms of the line input current which is itself known from the input voltage and the reactance of Co at the determined frequency. Since this frequency is closely related to the resonance frequency of $Lo.Co$ for self oscillation, the input current to the line must necessarily be the image of the current through Co.

3.3. *Oscillator Performance*

Circuits have been designed according to the principles just given and tests on the prototype oscillators have proved that predetermination of the loading conditions is well worth while.

The very nature of plastic welding, however, precludes accurate measurements of true power output and this has to be estimated, generally in two ways. One is the obvious way of observing what the complete apparatus will accomplish with a given load in a specified time. The alternative method is to ensure that the work test piece fully loads the oscillator to its rated input power and to deduct the estimated and/or measured losses in the valve and circuits from this power. With the "π" and "line" circuits the principal loss is the anode dissipation and this can often be checked by the aid of an optical pyrometer.

In the authors' experience, the predetermined output is not reached on the first test ; only small modifications are usually necessary to redress the lost power.

With the line circuit, in particular, it has been found that the loss of output is accompanied by an operating frequency considerably lower than planned. This effect is easily cured by reducing the value of the "added part of Co" until the correct frequency is reached when rated output will generally be secured.

The effect just discussed can be shown to be the result of ignoring the transmission line formed by internal anode and grid connections from the electrodes to the terminals. This results in a capacitance across the external line input terminals that is greater than the sum of the anode-grid capacitance and the deliberately added capacitance. The operating frequency is thus reduced.

4. Present Trend of Technique Reviewed

The radio-frequency welding of plastics is a comparatively new branch of engineering, and it is, therefore, difficult to predict the trend of development for the future.

The various types of welding machines which have been described in the earlier part of this paper will no doubt be improved considerably from the point of view of both mechanical design and generator performance. While it is expected that there will be no major changes in the design of welding equipment, it is certain that there will be improvements to existing designs which will make their operation more foolproof and overcome some of the difficulties encountered at present when using materials of varying physical properties. It is almost certain that this welding process will be applied to mass production of articles on an ever-increasing scale, and that automatic welding machines will largely supersede the manually operated types.

The advent of new thermoplastic materials will no doubt have an influence on the design of welding equipment, and particularly on the desirable generator characteristics. It has already been found, for instance, that the welding of cellulose acetate film is facilitated by certain modifications in generator and feeding circuit design.

A type of welding machine which combines the features of the roller welder and those of the bar welder has recently been developed.[6, 7, 8, 9] This machine uses feed rollers for intermittent feeding of the plastic sheets to be welded, and a reciprocating bar as a welding electrode.

Whether this type of welding machine will finally prove to be a competitor to the static types of welders is difficult to forecast.

In conclusion, it should be stated that R.F. welding of plastics has proved to be a very valuable method of manufacture in many instances, in spite of initial difficulties inevitable with a technique which has been known only for a comparatively short time.

5. Acknowledgments

The authors desire to thank the directors of Redifon, Ltd., for permission to publish this paper based on development work carried out in their industrial electronics laboratory during the past two years.

They also gratefully acknowledge the helpful discussions with W. Schreuer, of the laboratory, who was primarily responsible for the development of the "π" and "line" oscillators.

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10. M. R. Gavin, "Triode Oscillators for Very Short Wavelengths," *Wireless Engineer*, 1939, Vol. 16, p. 287.
11. F. E. Terman, Radio Engineers' Handbook, 1943, McGraw Hill Publishing Co.

* *Corrigenda to "Dielectric Heating by the R.F. Method."*

Page 133 : *Between equations* (9) *and* (10) *read "expressing* tan δ_1 *as* ωC_1R_1 *and* tan δ_L *as* $\omega C_L R_2$."

Page 135 : *The constant in equations* (15) *and* (16) *is* 14.2, *not* 2.24.

Page 144 : *The footnote in column* 2 *should end "... in quadrature with* V_1."

7. APPENDICES

7.1. *Analysis of the "π" Circuit*

7.1.1. *Effective Resistance presented to the Valve*

Taking into account all interelectrode capacitances of the oscillator valve, the circuit of Fig. 7 can be redrawn as in Fig. 12*a* which reduces to Fig. 12*b*. Resistance R_1 is added to allow for losses in the inductance when circuit efficiency is investigated later on.

The circuit is now simplified by replacing the load with its equivalent resistance and capacitance as in Fig. 13*a* and the vector diagram† of Fig. 13*b* is drawn for the resonance condition $\omega^2 L C_{eq} = 1$, since the difference in the result for R_L compared with that calculated for the

† Note that for the sake of clarity the reactance voltages have purposely been made short compared with the resistance voltage drops.

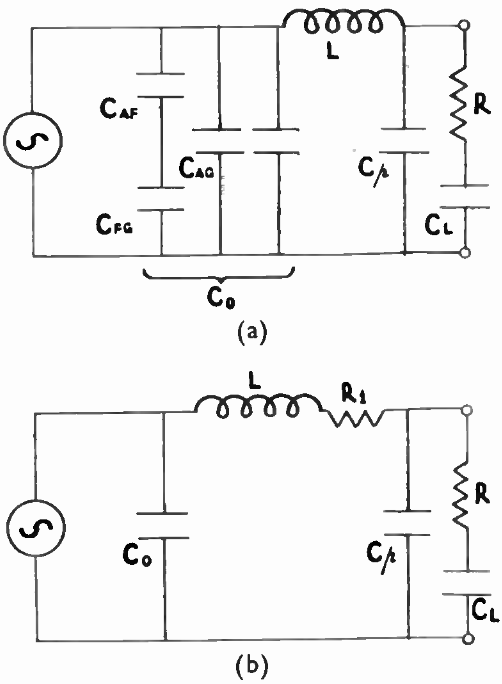


Fig. 12.—Diagram of equivalent circuits for the "π" oscillator.

actual operating frequency (at which OV will come into phase with i_1) is negligible for practical circuits. Also R_1 will be neglected in the calculation of R_L as it should be small compared with R_O in the interest of efficiency: the main problem is generally to make R_L low enough for fully loading the valve.

Two relations can be obtained immediately by inspection of the vector diagram:

$$\frac{i_2}{i_1} = \frac{1}{\omega C_o R_o} \quad \ldots\ldots\ldots\ldots (12)$$

$$\text{and } R_L i_1 = \frac{i_2}{\omega C_o} \quad \ldots\ldots\ldots\ldots (13)$$

Substituting equation (12) into equation (13) gives

$$R_L = \frac{1}{\omega^2 C_o^2 R_o} \quad \ldots\ldots\ldots\ldots (14)$$

Expressing the equivalent resistance[4] R_o in terms of the original circuit components, the load power-factor and the frequency, two further relations are obtained:

$$R_o = \frac{\tan \delta_L}{\omega C_L}\left(\frac{C_L}{C_L + C_p}\right)^2$$

$$= \frac{C_L \tan \delta_L}{\omega (C_L + C_p)^2} \quad \ldots\ldots\ldots\ldots (15)$$

$$\omega^2 \left\{\frac{C_o (C_L + C_p)}{C_o + C_p + C_L}\right\} L = 1 \quad \ldots\ldots (16)$$

When equations (15) and (16) are substituted in equation (14), a short manipulation leads to:

$$R_L = \frac{L^{1/2} (C_L + C_p)^{5/2}}{\tan \delta_L \,.\, C_o^{3/2} \,.\, C_L (C_o + C_p + C_L)^{1/2}} \quad \ldots\ldots\ldots\ldots (17)$$

The insertion of α for C_L/C_o and β for C_p/C_o leads directly to equations (4) and (5) in the paper.

7.1.2. *Resonance Frequency*

The frequency is determined, as soon as C_{eq} is known, in the usual way. This equivalent capacity is clearly:

$$C_{eq} = \frac{C_o (C_p + C_L)}{C_o + C_p + C_L} = \frac{C_o (\alpha + \beta)}{1 + \alpha + \beta} \quad (18)$$

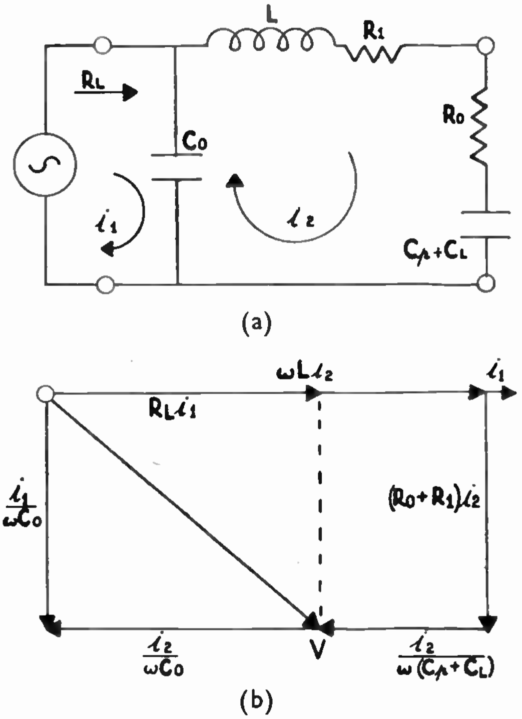


Fig. 13.—Simplified equivalent circuit of the "π" oscillator and vector diagram of related voltages and currents (note—the reactance vectors have purposely been shown short compared with the resistance vectors to secure greater clarity).

When the load terminals are short-circuited the frequency is obtained from $\omega_0^2 L\, Co = 1$, and since Co will in general be less than the equivalent load capacitance $(Cp + C_L)$ and thereby be the principal factor in frequency determination, it is convenient to write :

$$\omega = \omega_0 . z = \frac{z}{(L\,.\,Co)^{\frac{1}{2}}} \qquad \text{..........(19)}$$

Where $z = \left(\dfrac{1 + \alpha + \beta}{\alpha + \beta}\right)^{\frac{1}{2}}$

as will be clear from inspection of equation (18).

7.1.3. *Circuit Efficiency*

Figure 13*a* shows that the main oscillatory current, i_2, flows through R_1 and Ro in series. The circuit efficiency is hence given by :

$$\eta = \frac{Ro}{Ro + R_1} \qquad \text{..................(20)}$$

On putting Q_L for $1/\tan \delta_L$ and Q_1 for $\omega L/R_1$,

$$R_1 = \frac{\omega L}{Q_1} \text{ and } Ro = \frac{C_L}{\omega (C_L + Cp)^2 Q_L}$$

giving :

$$\frac{R_1}{Ro} = \frac{\omega^2 L\,(C_L + Cp)^2 Q_L}{C_L\, Q_1} \qquad \text{.........(21)}$$

but $\omega^2 L = \dfrac{1}{C_{eq}} = \dfrac{Co + Cp + C_L}{Co\,(Cp + C_L)}$ ......(22)

Substitution of equation (22) into equation (21) leads to :

$$\frac{R_1}{Ro} = \frac{Q_L\,(C_L + Cp)\,(Co + C_L + Cp)}{Q_1\, Co\, C_L}$$

$$= \frac{Q_L}{Q_1} \cdot \frac{(\alpha + \beta)\,(1 + \alpha + \beta)}{\alpha} \qquad (23)$$

If $f(\alpha, \beta)$ in equation (23) be termed y :

$$\eta = \frac{1}{1 + yQ_L/Q_1} \qquad \text{................(24)}$$

This is the relation plotted in Fig. 10.

7.1.4. *Overall Circuit Quality*

Designating the reactance/resistance ratio of the whole oscillatory circuit as Q and making substitutions for R_1 and Ro as in the previous section :

$$Q = \frac{\omega L}{\dfrac{\omega L}{Q_1} + \dfrac{C_L}{\omega\,(Cp + C_L)^2 Q_L}} \qquad \text{.....(25)}$$

On multiplying through by ω and substituting for $\omega^2 L$ from equation (22), Q is given directly in terms of Q_1, Q_L and capacitances only :

$$Q = \frac{Q_1\, Q_L\,(Co + Cp + C_L)}{Q_L\,(Co + Cp + C_L) + Q_1\, Co\, C_L\,(Cp + C_L)} \qquad \text{..(26)}$$

This is evidently of the form,

$$Q = \frac{Q_1\, Q_L'}{Q_1 + Q_L'}, \text{ where}$$

$$\frac{Q_L'}{Q_L} = \frac{(Cp + C_L)\,(Co + Cp + C_L)}{Co\, C_L}$$

$$= \frac{(\alpha + \beta)\,(1 + \alpha + \beta)}{\alpha} \qquad \text{........(27)}$$

This involves the same $f(\alpha, \beta)$ as in equations (23) and (24) and leads directly to :

$$Q = \frac{y\,.\,Q_1\, Q_L}{Q_1 + yQ_L} = \frac{y\,.\,Q_L}{1 + yQ_L/Q_1} \qquad \text{....(28)}$$

Where the efficiency has already been determined from equation (24), for a known value of the ratio Q_L/Q_1, equation (28) may be simplified to :

$$Q = \eta\,.\,y\,.\,Q_L \qquad \text{..................(29)}$$

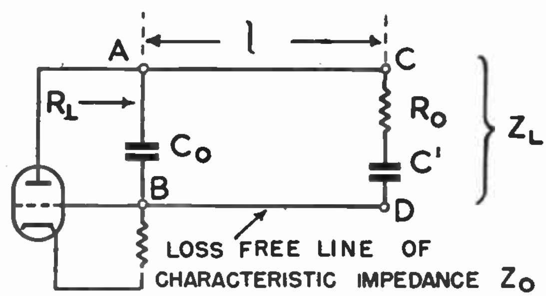


Fig. 14.—Simplified diagram of the "transmission line" oscillator used for determination of valve loading.

7.2. *Analysis of the "Line" Circuit*

7.2.1. *Determination of Resonance Frequency*

Taking the essential transmission line circuit to be as shown in Fig. 14 and ignoring losses in the line and the power factor of the load connected to terminals CD, as these have only second order effects on the resonance frequency, the input reactance of the line is :

$$X_i = \frac{Zo\,(Z_L + j\,Zo \tan \varphi)}{Zo + j\, Z_L \tan \varphi} \qquad \text{........(30)}$$

where X_i = input reactance of line to right of terminals AB

Z_L = impedance (actually reactance) of load C'

$C' = Cp + C_L$

φ = electrical length of line = $2\pi l/\lambda$

l = line length.

Replacing Z_L by $1/j\omega C'$ and rearranging, gives:

$$X_i = \frac{jZo\,(\omega C'Zo \tan\varphi - 1)}{\omega C'Zo + \tan\varphi} \quad \ldots\ldots(31)$$

For oscillation, the total circuit reactance must be zero, hence:

$$\frac{jZo\,(\omega C'Zo \tan\varphi - 1)}{\omega C'Zo + \tan\varphi} + \frac{1}{j\omega Co} = 0$$

which simplifies to:

$$\tan\varphi = \frac{\omega(C' + Co)\,Zo}{\omega^2 C'CoZo^2 - 1} \quad \ldots\ldots\ldots\ldots(32)$$

Putting C_{eq} for $C'Co/(C' + Co)$, equation (32) can be rewritten:

$$\tan\varphi = \frac{1}{\omega C_{eq}Zo - 1/\omega(C' + Co)Zo} \quad \ldots(33)$$

or, writing X_{Ceq} for $1/\omega C_{eq}$ and $X_{(C'+Co)}$ for $1/\omega(C' + Co)$, as:

$$\tan\varphi\left(\frac{Zo}{X_{Ceq}} - \frac{X_{(C'+Co)}}{Zo}\right) = 1$$

Provided that Zo^2 is much greater than the product of the two reactances indicated,* then equation (33) can be further simplified to:

$$\tan\varphi = \frac{1}{\omega C_{eq}\,Zo} \quad \ldots\ldots\ldots\ldots\ldots\ldots(33a)$$

Equation (33*a*) can be solved graphically[10] but a reasonably accurate answer can be obtained algebraically by noting that tan φ approaches φ when l/λ is small, as it generally is in practical arrangements. With this further approximation:

$$\frac{2\pi l}{\lambda} = \frac{1}{\omega C_{eq}Zo}$$

or

$$\frac{2\pi lf}{v} = \frac{1}{2\pi f C_{eq} Zo}$$

* Generally, this inequality can be arranged without sacrificing other desirable circuit properties. If this condition is not fulfilled, the error in calculating the frequency from equation (33*a*) may be considerable, and the analysis in the next section will be inapplicable.

where v = velocity of E/M propagation.

$$\text{Hence } f^2 = \frac{v\,(\text{cm/sec})}{4\pi^2 C_{eq\,(pF)}\,Zo\,l_{(cm)}}\,(\text{Mc/s})^2 = \frac{7{\cdot}61 \times 10^8}{C_{eq}.Zo.l} \quad \ldots\ldots\ldots\ldots(34)$$

Also, under the same conditions

$$\omega^2 = \frac{v}{C_{eq}.Zo.l.} \quad \ldots\ldots\ldots\ldots\ldots(35)$$

7.2.2. *Valve Load at Terminals AB*

The problem resolves itself into finding the equivalent resistance at the input end of the line. The reactive component of the input impedance is not required as it has already been used in the previous section, to determine the frequency.

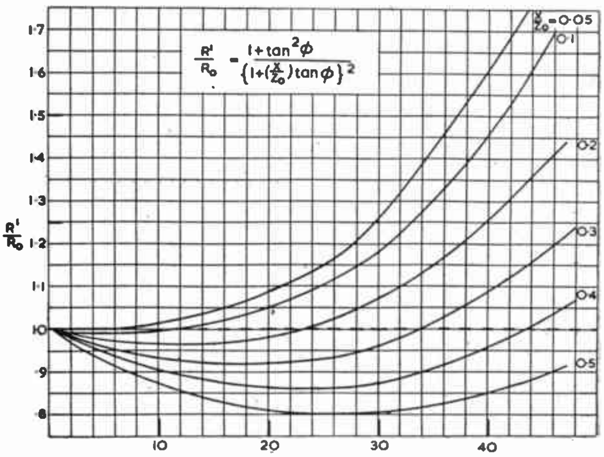
Then,[11] if X is the reactance of Cp and C_L in parallel:

$$Zi = Zo\left\{\frac{(Ro - jX) + jZo\tan\varphi}{Zo + j(Ro - jX)\tan\varphi}\right\}$$

On rationalizing this expression and extracting the "real" part, denoted R′:

$$R' = \frac{Ro\,Zo^2(1 + \tan^2\varphi)}{(Zo + X\tan\varphi)^2 + Ro^2\tan^2\varphi} \quad \ldots(36)$$

As the second term in the denominator is negligible compared with the first item, it is ignored. Dividing both numerator and denominator by Zo^2 leads to:



$\varphi° = 2\pi l/\lambda$

Fig. 15.—The effect of transmission-line length (electrical) on the equivalent series resistance of an unmatched impedance loaded line. The parameters are ratios of negative reactance of terminal load to characteristic impedance of the line.

$$R' = \frac{Ro(1+\tan^2\varphi)}{\left\{1+\left(\frac{X}{Zo}\right)\tan\varphi\right\}^2} \quad \text{.........(37)}$$

This relation is graphed in Fig. 15 which clearly shows that for a short line (e.g., 20 electrical degrees or less) terminated in a capacitance whose reactance is small compared with the characteristic impedance of the line the ratio R'/Ro does not vary seriously from unity. As these conditions do obtain in practice, the analysis is completed under the assumption that $R' = Ro$.

The effective resistance into which the valve looks is then that of a rejector circuit comprising Co in parallel with Ro and an effective inductance Lo which, however, does not need to be explicitly determined at this stage. The load resistance is thus :

$$R_L = \frac{\omega^2 Lo^2}{Ro} = \frac{1}{\omega^2 Co^2 Ro} \quad \text{.........(38)}$$

Taking equation (15) and inserting α and β to obtain Ro in terms of Co, load power-factor and frequency, there results :

$$Ro = \frac{\alpha \tan\delta_L}{\omega Co(\alpha+\beta)^2} \quad \text{..............(39)}$$

It is now only necessary to substitute ω from equation (35) into equation (39), and the expression for Ro, thus modified, into equation (38) to obtain :

$$R_L = \frac{(\alpha+\beta)^{5/2} Zo^{1/2} l^{1/2}}{\alpha(1+\alpha+\beta)^{1/2} Co^{1/2} v^{1/2}} \cdot \frac{1}{\tan\delta_L} \quad \text{................(40)}$$

$$= \frac{x}{\tan\delta_L}\left(\frac{Zo.l}{Co.v}\right)^{1/2} \quad \text{..........(41)}$$

In equation (41) the factor x is the same as that determined for the "π" circuit ; the equation also shows that the effective inductance Lo is $Zo\,l/v$. Since $v = \omega\lambda/2\pi$ the reactance of Lo is $jZo.2\pi l/\lambda$ and this expression is the same as that for a short-circuited line when short enough[11] to allow the approximation $\varphi = \tan\varphi$ to be made without undue error.

7.2.3. *The Load Voltage*

Since the choice of oscillator valve settles the line input voltage, this being the r.m.s. anode-grid alternating voltage which is normally of value $V_{DC}/\sqrt{2}$ if V_{DC} is the H.T.D.C. supply voltage, it is only necessary to know how the voltage varies along a short line under different terminating conditions ; the input voltage (Vo) divided by the ratio of line input and output voltages will then determine the load voltage (V_L).

From transmission line theory[11] these voltages are related by :

$$Vo = V_L\cos\varphi + j\,I_L\,Zo\sin\varphi \quad \text{.......(42)}$$

where I_L is the load current and φ is $2\pi l/\lambda$, as before. Noting that I_L is V_L/Z and writing $Z = Ro - jX$, where $X = 1/\omega(Cp + C_L)$, equation (42) becomes :

$$\frac{Vo}{V_L} = \cos\varphi + \frac{j\,Zo}{Ro - jX}\sin\varphi \quad \text{......(43)}$$

Ignoring Ro as being small compared with X :

$$\frac{Vo}{V_L} = \cos\varphi - (Zo/X)\sin\varphi \quad \text{.......(44)}$$

GRADUATESHIP EXAMINATION, MAY 1949

PASS LIST

A total of 224 candidates sat the May, 1949 Graduateship Examination. This list contains the results of all the successful candidates.

ELIGIBLE FOR TRANSFER OR ELECTION TO GRADUATESHIP OR HIGHER GRADE OF MEMBERSHIP.

The following candidates have passed the entire examination or having previously passed or been exempt from part of the examination have now passed the remaining subject(s).

1999	ACHARYA, Khadri Samachar S. (S)	Trichinopoly
1979	ALI, Ahmad (S)	Dacca, Pakistan
2093	ASQUITH, Stanley Francis William (S)	Hockley, Essex
2072	ATKINSON, James Gibson (S)	Glasgow
2238	BARKER, John Kenneth (S)	Cheltenham
2140	BARNES, George Raymond (S)	Blackburn
2194	BATES, James William (S)	Coalville, Leics
2164	BEECROFT, William Douglas (S)	Bradford
2114	CAVANAGH, Edward (S)	Gateshead, Durham
2101	CLIFTON, Frank Ronald (S)	London, W.4
2139	CONLON, James (S)	London, S.W.12
2133	CROSDALE, Frank (S)	Manchester
2161	ELDRIDGE, Dennis Arthur George (S)	Enfield, Middx
2218	EVE, Godfrey Arnold (S)	London, N.8
2141	FARRAR, George Henry (S)	Bradford
2090	FELLOWS, Horace (S)	Wolverhampton
2082	FLAVELL, John Aubrey	Ipswich
2129	FRANKS, Percy Ronald (S)	Luton, Beds
2044	GARDE, Vinayak C. (S)	Bombay
2239	GIFKINS, Geoffrey Charles (S)	Hertford
2123	HALSALL, James Richard (S)	Liverpool
2216	HICKS, Charles Ernest (S)	London, E.13
1987	HOLYWELL, Keith Harold (S)	Melbourne, Australia
2048	HUBBARD, John Valentine (S)	Godalming, Surrey
2062	JARMAN, Eric (S)	Warrington, Lancs
2170	JUSTICE, James William Henry (S)	London, N.5
1982	KAMESWARARAO, Gummuluru Satya (S)	Trichinopoly
2160	KEMP, Alan (S)	Darwen, Lancs
2060	KINALLY, Dennis Raymond (S)	Morden, Surrey
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RADIO SERVICING CERTIFICATE EXAMINATION—MAY 1949

Of the 153 candidates who sat the examination which took place in May this year, 89 candidates satisfied the examiners in both the written and practical parts, 24 candidates passed the written examination but were referred in the practical test and seven candidates completed the examination, having been referred in the practical test in the May 1948 examination.

PASS LIST

The following candidates satisfied the examiners in the entire examination :

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The following candidates who were referred in the Practical Examination in May 1948 now qualify for the certificate :

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MARR, John Halliday	Glasgow
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SHINGLETON, Ronald John	Birmingham
WITT, Harold William	Buckfast, Devon
WRIGHT, Hyman	Harrow, Middx

The following candidates satisfactorily passed the written papers but were referred in the Practical Examination :

BAKER, Brian Thomas	Bromley, Kent
BATES, Milton	Winchester
BERRIMAN, Michael	Hull
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IMPEDANCE TRANSFORMATION IN FOLDED DIPOLES*

by

Rudolf Guertler, Dr.Tech.†

SUMMARY

It is pointed out that the impedance of a folded dipole relative to that of a simple dipole can be adjusted by employing conductors of different diameters for the separate elements of the folded dipole. Increased impedance ratios can be obtained by the use of additional elements.

It is shown that the impedance ratio can be obtained from the current ratio and suitable expressions are derived. Practical examples are given.

1.0. The Folded Dipole as Impedance Transformer

Any folded dipole [1, 2] has a higher impedance at the input terminals than a simple dipole at the same place in any antenna or antenna array. This property of impedance transformation explains the increasing use of folded dipoles especially at very high frequencies.

The simplest folded dipole comprises two conductors of equal diameters Fig. 1 and gives a step-up impedance transformation of 4 : 1. By employing elements of different diameters as in Fig. 2, any desired step-up transformation ratio can be achieved.

If a high transformation ratio is desired it is practicable to use more than two elements with parallel axes although they need not be in the same plane. A practically important application of three elements is shown in Fig. 3 ; the axes of the three conductors are in the same plane, the outer elements being identical with each other but generally different from the fed middle element from which they have equal separation.

Assuming that the radiation from a folded dipole does not differ much from that of a simple dipole at the same place[3], it is possible to compute the transformation ratio and consequently the impedance at the feeding point if the ratio of the currents in the elements of the folded dipole is known.

In Fig. 2 the current (r.m.s.) at the feeding point is designated by J_1 and the current in the centre of the auxiliary element by J_2. It is assumed for simplicity that the dimensions of the dipole "match" the frequency so that the input impedance is real.

* Reprinted from Proc.I.R.E. Australia, April, 1949.
† Standard Telephones and Cables Pty., Ltd., Sydney.
U.D.C. 621.396.671.011.2.

In any array in which the fed element is a simple dipole, let the input resistance at the feeding point be R_0. When the simple dipole is replaced by a folded dipole, let the new input resistance be R_1.

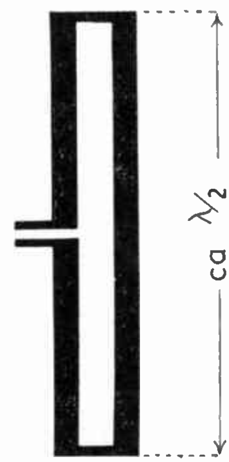

Fig. 1.—Folded dipole of two elements of equal diameter.

Then with the above-mentioned assumption of equal radiation, we have the following relation :

$$J_1^2 R_1 = (J_1 + J_2)^2 R_0 \qquad (1)$$

The folded dipole therefore gives the resistance transformation ratio u, where u is given by the following expression :—

$$u = R_1/R_0 = [(J_2/J_1) + 1]^2 = (n + 1)^2 \qquad (2)$$

in which n is the current ratio given by

$$n = J_2/J_1 \qquad (3)$$

We can state the resistance transformation if we know the current ratio. The computation of the current ratio is the object of the following sections.

2.0. Comparison of a Folded Dipole with a Simple Dipole of Equal Configuration

Consider the dipole of Fig. 4, which is physically like the folded dipole of Fig. 2, except that the auxiliary element is broken and fed in parallel with the first element. The electrical difference is mainly this, that in Fig. 2 out of phase " line " currents are superimposed on the in phase " antenna " currents i_1, i_2. Since the out-of-phase " line " currents are negligible compared to the " antenna " currents at the centre points and at the feeding point of the dipole elements[3,4] we do not need to consider them. Consequently we shall calculate the current partition in a simple unfolded dipole, comprising two or more conductors in parallel as in Fig. 4. The result will be an approximation suitable for engineering design of folded dipoles.

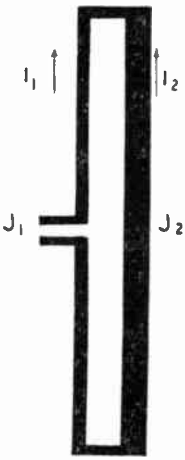

Fig. 2.—Folded dipole of two elements of unequal diameter.

3.0. The Field Equations in Four-Dimensional Form as Basis for Investigation of Folded Dipoles

To describe the electro-magnetic relations in the antenna of Fig. 4, we start with the field equations. To enjoy the advantages of more concise expression we shall use them in the four-dimensional representation.[5,6,7] In addition we shall choose the potential form which is accentuated by the problem—computation of current and charge distribution.

The four-potential is designated by Φ, the four-current by **P**, the six-vector of the electromagnetic field by **F**. Div and Curl are differential

Fig. 3.—Folded dipole of three elements.

operations which may be represented by the four-dimensional differential operator,

$$\Diamond = \mathbf{k}_1 \frac{\delta}{\delta x_1} + \mathbf{k}_2 \frac{\delta}{\delta x_2} + \mathbf{k}_3 \frac{\delta}{\delta x_3} + \mathbf{k}_4 \frac{\delta}{\delta x_4},$$

where

$$x_1 = x,\ x_2 = y,\ x_3 = z,\ x_4 = jct$$

and $\mathbf{k}_1$ are the unit vectors of the four-dimensional space.

$$\square = \Diamond \cdot \Diamond = \frac{\delta^2}{\delta x_1{}^2} = \frac{\delta^2}{\delta x_2{}^2} + \frac{\delta^2}{\delta x_3{}^2} + \frac{\delta^2}{\delta x_4{}^2}$$

$$= \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2} - \frac{1}{c^2} \cdot \frac{\delta^2}{\delta t^2}$$

is the four-dimensional form of the Laplace operator

$$\nabla^2 = \Delta.$$

The field equations are now

$$\mathbf{F} = \mathrm{Curl}\, \Phi \quad \ldots\ldots\ldots\ldots (4)$$

$$\mathrm{Div}\, \Phi = 0 \quad \ldots\ldots\ldots\ldots (5)$$

$$\square \Phi = \mathbf{P} \quad \ldots\ldots\ldots\ldots (6)$$

$$\mathrm{Div}\, \mathbf{P} = 0 \quad \ldots\ldots\ldots\ldots (7)$$

Since we are interested in the current distribution i_1, i_2 on the two conductors of the dipole Fig. 4, we do not require the knowledge of field strengths so we can neglect Eq. (4) in the further considerations. It is obvious that on both conductors all points of the cross-sections placed in plane p orthogonal to the axes of the conductors have equal potential at any moment. We may also assume that the currents are flowing only in the surface of conductors, an assumption permissible for practical purposes. The currents therefore have only components parallel to the axes. Thus the four-current of an element is

$$\mathbf{P} = (1/c)i\,\mathbf{k}_3 + j\rho\mathbf{k}_4, \quad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

if the z-axis is parallel to the axes of the conductors. In the formula " i " designates the current and " ρ " the charge for unit length of a conductor. Also the four-potential has in this case two components only,

$$\Phi = A_z\,\mathbf{k}_3 + j\,\varphi\,\mathbf{k}_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

where " A_z " denotes the vector potential and " φ " the scalar potential.

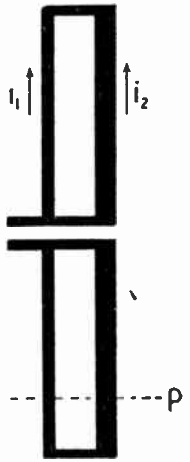

Fig. 4.—Simple dipole of same physical elements as folded dipole of Fig. 2.

The currents will be distributed on both elements in such a way that the above-mentioned condition of equal potentials on the conductor cross-sections in any plane p, Fig. 4, will be fulfilled.

Before starting with the calculation of the retarded potentials,[7, 8] we consider a further simplification.

As a consequence of the equation of continuity (7) which establishes the time-space conservation of electrical charge, it follows that the ratio of charge densities equals to ratio of currents ; i.e., from

$$\mathbf{P}_2 : \mathbf{P}_1 = n$$

follows

$$\rho_2 : \rho_1 = i_2 : i_1 = n \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Therefore we can confine ourselves to the calculation of the ratio of charges. The same fact is expressed by Eq. (5) which states that the four-potential is " divergence-free."[9] Thus the vector potential of the conductor elements in a plane p is in proportion to the scalar potentials. Consequently we may confine ourselves to the calculation of the scalar potential.

We assume further a sinusoidal distribution of currents and charges along the dipole. In addition it will be obvious from the derivation below that the current ratio is not critically dependent on the current and charge distribution along the antenna. Now we substitute the actual charge distribution on each cylindrical conductor by a line charge in parallel with the axis of the cylinder, and which produces as far as possible the same potential distribution.

On the basis of these assumptions and simplifications we are able to compute relatively simply with sufficient approximation for practical purposes the potential determining the current distribution. The calculation is given in the Appendix.

4.0. Calculation of Current Ratio and Impedance Transformation in the Two-Element Folded Dipole

In the Appendix the normalized scalar potential (i.e., for $\rho_{max} = 4\pi\epsilon_0$ in the element 1) is derived for a point in the proximity of a conductor in the plane p if the plane is placed through an end of the dipole.

The formula reads

$$\varphi = \log_e \frac{\lambda^{n+1}}{\delta\,\delta'^{n}} - \frac{n+1}{2}\,C_{in}\,2\pi \ldots(11)$$

where δ and δ' are the distances of the reference point and the axes of the conductor 1 and 2.

The " end effect " has not been considered in this formula. Nevertheless it produces the same rule for designing as formulae derived for planes p which are not placed through an end of the dipole.

We employ the formula for computation of the current ratio " n " if the diameters and the separation of the dipole elements are fixed. As stated above, the potential is the same for all elements of the cross-sections of *both* conductors in a plane p. We consider the potential at the points X_1 and X_2 of Fig. 5, and that in a plane p through the end of the dipole at which Eq. (11)

holds under the idealizations as assumed in section 3. The distance of X_1 from O_2 and of X_2 from O_1 is approximately $O_1 O_2 = s$. For the reference point X_1, we must therefore in Eq. (11) put $\delta = a_1$, $\delta' \simeq s$, and we obtain the potential

$$\varphi(X_1) = \log_e \frac{\lambda^{n+1}}{a_1 s^n} - \frac{n+1}{2} C_{in} 2\pi \ldots (12)$$

Correspondingly we obtain the potential for X_2 at the same instant if we put $\delta \simeq s$ and $\delta' = a_2$:—

$$\varphi(X_2) = \log_e \frac{\lambda^{n+1}}{s a_2^n} - \frac{n+1}{2} C_{in} 2\pi \ldots (13)$$

Both values (12) and (13) of the potential must be equal, hence we obtain immediately,

$$a_1 s^n = a_2^n s \ldots\ldots (14)$$

The current ratio in question for both conductors is thus approximately

$$n = \log \frac{s}{a_1} \Big/ \log \frac{s}{a_2} \ldots\ldots (15)$$

The impedance transformation follows now from Eq. (2),

$$u = R_1/R_0 = (n+1)^2$$
$$= \left(\log \frac{s^2}{a_1 a_2} \Big/ \log \frac{s}{a_2}\right)^2 \ldots\ldots (16)$$

These formulae are suitable for design of folded dipoles if the transformation ratio is fixed, and for computation of the transformation ratio of a given dipole.

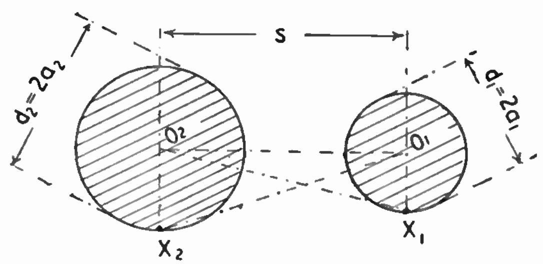

Fig. 5.—Cross section through the dipole of Fig. 4 in the plane P.

Sometimes the following formula obtained from Eq. (15) is more convenient :—

$$n - 1 = \log \frac{a_2}{a_1} \Big/ \log \frac{s}{a_2} \ldots\ldots (17)$$

It is immaterial whether we use natural or decade logarithms.

The formulae give a good approximation if,

(a) $a_2/a_1 \geqq 1$ and $s/a_2 \geqq 2{\cdot}5$

or

(b) $a_2/a_1 < 1$ and $s/a_1 \geqq 2{\cdot}5$.

5.0. Calculation of Current Ratio and Impedance Transformation of a Folded Dipole Comprising More Than Two Elements

As for the two-element dipole we obtain formulae for design of multi-element folded dipoles applying Eq. (28).

We shall consider only one such type which is of practical interest, namely the symmetrical three-element dipole of Fig. 3, in which the axes of the three elements are in a common plane. If a_1 denotes the radius of the inner element which is fed, and a_2 denotes the radius of any of the outer equal elements, s denotes the separation of the inner element from any outer element, m denotes the current ratio for one outer element to the fed inner element, we obtain approximately

$$m = \log \frac{s}{a_1} \Big/ \log \frac{s}{2a_2} \ldots\ldots (18)$$

or, more convenient for some problems,

$$m - 1 = \log \frac{2a_2}{a_1} \Big/ \log \frac{s}{2a_2} \ldots\ldots (19)$$

The impedance transformation ratio is obviously given by

$$u = (2m+1)^2$$
$$= \left(\log \frac{s^3}{2a_1^2 a_2} \Big/ \log \frac{s}{2a_2}\right)^2 \ldots\ldots (20)$$

Especially it is clear from (18) and (19) that currents in the conductors of a three-element dipole of Fig. 3 are equal, i.e., that $m = 1$ only if $a_1 = 2a_2$, that is to say if the inner element has twice the thickness of an outer element. For this case the current ratio is practically (i.e., approximately) independent of the separation.

Another specially interesting case is $m = 2$, i.e., a transformation ratio $u = 25$; according to Eq. (19) it is achieved if $s/2a_2 = 2a_2/a_1$, i.e., if the diameter of an outer element is the geometric mean value of the radius of the inner element and the spacing.

The simple approximative formulae (18) to (20) become inaccurate if the separation is too small. The currents and the charges in the outer elements are shifted considerably to the outer parts of the outer elements so that the substituting linear charges ought to be placed some distance outside the axes of the outer elements.

For practical purposes the formulae (18) to (20) may be used for a_2/a_1 between 0·5 and 5 if $s/2a_2 > 2{\cdot}5$, and for $a_2/a_1 < 0{\cdot}5$ if $s/a_1 > 2{\cdot}5$.

6.0. Measurements on Two-Element and Three-Element Folded Dipoles

J. O'Shannassy, B.Sc., and E. J. Wilkinson performed various measurements on folded dipoles at 150 Mc/s. They published a part of the results in " Amateur Radio," Melbourne, January 1948, p. 7. Because their measurements are very interesting, the following series are quoted from the article mentioned.

Two-Element Folded Dipole

1. Series of measurements (constant spacing) :

Dimensions			Transformation Ratio u	
$2a_1$ in.	$2a_2$ in.	s in.	Measured	Calculated from n of Eq. (15)
⅜	⅜	1½	3·96	4·0
¼	⅜	1½	4·6	4·8
¼	½	1½	6·08	5·7
0·19	½	1½	8·3	6·5

2. Series of measurements (constant diameters):

Dimensions			Transformation Ratio u	
$2a_1$ in.	$2a_2$ in.	s in.	Measured	Calculated from n of Eq. (15)
¼	½	½	8·89	9·0
		1	6·19	6·25
		1½	5·67	5·7
		2	5·48	5·44
		2½	5·25	5·3

Later on Mr. O'Shannassy made experiments on three-element folded dipoles which are not yet concluded because it proved very difficult to measure high S.W.R. values correctly. The writer quotes from a letter of Mr. O'Shannassy therefore only two tentative measurements on symmetrical three-element dipoles (Fig. 3) :

Dimensions			Transformation Ratio u	
$2a_1$ in.	$2a_2$ in.	s in.	Measured	Calculated from m of Eq. (18)
¼	¼	1½	11·0	14
¼	¼	1	12·5	16

7.0 Recapitulation

The folded dipole of two or more elements is being used as an impedance transformer, e.g., to match an aerial to a line of higher characteristic impedance.

The impedance transformation ratio can be calculated if the current ratio is known. To find this the two-element folded dipole Fig. 2 is compared to a simple dipole of equal physical construction Fig. 4, i.e., of two elements in parallel. It is obvious that the charge and current distributions in both types of dipoles are essentially much the same. The charge distribution in the simple two-element dipole can be approximately calculated under the assumption that the (retarded) potentials on co-planar cross-sections of both elements are equal.

Measurements seem to prove the practical applicability of the approximate formulae.

8.0. Acknowledgments

The author wishes to acknowledge his indebtedness to Mr. K. W. Magee (director of Austronic Engineering Laboratories, Melbourne) for the lively discussions which stimulated the investigations[12] which provided the basis for this Paper, and to express his appreciation to Messrs. Standard Telephones & Cables Pty., Ltd., for assistance in preparing the Paper for publication.

He also wishes to thank Messrs. J. O'Shannassy and E. J. Wilkinson of P.M.G.'s Department, Melbourne, for their measurements of which those on three-element dipoles were undertaken at the personal request of the writer.

9.0. Appendix—Calculation of Retarded Potential in the Proximity of a Folded Dipole

The first step is to draw according to Fig. 6 a system of co-ordinate axes through a folded dipole as in Fig. 2, or a single dipole of the same physical construction as in Fig. 4. The z-axis coincides with the straight line charge equivalent to the fed element 1. The potential at any point X is the sum of the potential of conductor 1 and the potential of conductor 2.

First we shall calculate the potential produced by the charges on conductor 1 only. The charge per unit length of conductor 1 will be designated by ρ, thus $\rho\, dz$ will be the charge of a conductor element of length dz. The distance of the charge

$\rho\, dz$ from the reference point X may be called r, the distance of point X from the z-axis may be δ, and ζ denotes the height of X over the xy-plane.

The retarded potential[6, 7, 8] is given by

$$\psi = \frac{1}{4\pi\varepsilon_0}\int_{z=0}^{\lambda/2} \frac{1}{r}[\rho]\, dz \ldots\ldots\ldots\ldots\ldots (21)$$

where (ρ) denotes the retarded charge.

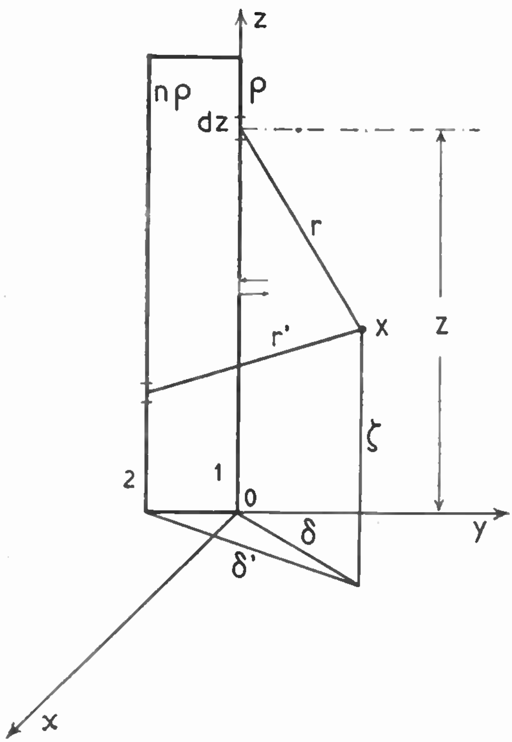

Fig. 6.—Folded dipole showing co-ordinate system for calculation of the retarded potential.

If ρ_{max} represents the maximum of charge density ρ with respect to time and space, and if we assume sinusoidal distribution of charge along the dipole, the retarded charge density along conductor 1 is

$$[\rho] = \rho_{max} \cos 2\pi \frac{z}{\lambda} \cos \omega \left(t - \frac{r}{c}\right)$$

or by suitable choice of zero time more simply

$$[\bar{\rho}] = 4\pi\varepsilon_0 \cos 2\pi \frac{z}{\lambda} \cos 2\pi \frac{r}{\lambda} \quad \ldots (22)$$

where

$$[\rho] = 4\pi\varepsilon_0 [\rho]/\rho_{max}$$

denotes the "normalized" retarded charge which shall be used from now on. By substituting the expression (22) in Eq. (21) we obtain the formula for the "normalized" scalar potential due to conductor 1,

$$\psi = \int_{z=0}^{\lambda/2} \frac{1}{r} \cos 2\pi \frac{r}{\lambda} \cos 2\pi \frac{z}{\lambda} dz \ldots (23)$$

For convenience all quantities may be considered as measured in electrical angular degrees, i.e., we write now r instead of $2\pi r/\lambda$; z instead of $2\pi z/\lambda$; ζ instead of $2\pi\zeta/\lambda$; and δ instead of $2\pi\delta/\lambda$. The limits have also to be taken in angular scale, so that Eq. (23) becomes

$$\psi = \int_{z=0}^{\pi} \frac{1}{r} \cos r \cos z\, dz \quad \ldots\ldots\ldots\ldots (24)$$

Applying the co-ordinate transformation $z - \zeta = u$ we obtain for (24) the expression

$$\psi = \int_{u_1=-\zeta}^{u_2=\pi-\zeta} r^{-1} \cos r \cos (u + \zeta)\, du$$

$$= \int_{u_1}^{u_2} (u^2+\delta^2)^{-\frac{1}{2}} \cos (u^2+\delta^2)^{\frac{1}{2}} \cos (u+\zeta)\, du.$$

This we transform by means of the addition theorem of trigonometric functions in

$$\psi = \tfrac{1}{2} \cos \zeta \left[\int_{u_1}^{u_2} \frac{1}{r} \cos (r + u)\, du + \int_{u_1}^{u_2} \frac{1}{r} \cos (r - u)\, du\right]$$

$$- \tfrac{1}{2} \sin \zeta \left[\int_{u_1}^{u_2} \frac{1}{r} \sin (r + u)\, du - \int_{u_1}^{u_2} \frac{1}{r} \sin (r - u)\, du\right]$$

Introducing new variables for the arguments $(r + u)$ or $(r - u)$, we reduce the individual integrals to the cosine integral C_i or sine integral S_i. Substituting the limits we obtain the potential in numerically calculable form,

$$\psi = \tfrac{1}{2}\cos\zeta[-C_i(r_1+u_1)+C_i(r_1-u_1)+C_i(r_2+u_2) -C_i(r_2-u_2)]+\tfrac{1}{2}\sin\zeta[S_i(r_1+u_1)+S_i(r_1-u_1) -S_i(r_2+u_2)-S_i(r_2-u_2)] \quad \ldots\ldots\ldots\ldots(25)$$

putting for simplicity

$$r_1 = \sqrt{u_1^2+\delta^2},\ r_2 = \sqrt{u_2^2+\delta^2}.$$

Using the well-known definition[10, 11]

$$C_i(x) = \gamma + \log_e x - C_{in}(x),$$

where γ denotes the Euler constant, we can transform Eq. (25) in a more suitable form for our purpose :—

$$\psi = \tfrac{1}{2}\cos\zeta\,[2\log_e\frac{r_2+u_2}{r_1+u_1} + C_{in}(r_1+u_1) - C_{in}(r_1-u_1) - C_{in}(r_2+u_2) + C_{in}(r_2-u_2)] + \tfrac{1}{2}\sin\zeta[S_i(r_1+u_1)+S_i(r_1-u_1)-S_i(r_2+u_2) - S_i(r_2-u_2)] \quad \ldots\ldots\ldots\ldots(26)$$

To simplify as much as possible the following considerations we choose a reference point X in the xy-plane, i.e., we put $\zeta = 0$. Thus $u_1 = 0$, $u_2 = \pi$, $r_1 = \delta$, $r_2 = \sqrt{x^2+\delta^2}$. Hence the potential of a point in the xy-plane is

$$\psi = \log_e\frac{\pi+\sqrt{\pi^2+\delta^2}}{\delta} - \tfrac{1}{2}C_{in}(\pi+\sqrt{\pi^2+\delta^2}) + \tfrac{1}{2}C_{in}(-\pi+\sqrt{\pi^2+\delta^2}) \quad \ldots\ldots\ldots\ldots(27)$$

For a reference point in the proximity of the dipole is $\delta^2 \ll \pi^2$, thus with a good approximation

$$\psi \cong \log_e\frac{2\pi}{\delta} - \tfrac{1}{2}C_{in}2\pi \quad \ldots\ldots\ldots(28)$$

Since the charge of the second conductor (see Fig. 6) is n - times larger according to Eq. (10) the potential produced by the second element at point X is

$$\psi \cong n(\log_e\frac{2\pi}{\delta'} - \tfrac{1}{2}C_{in}2\pi),$$

hence the total potential

$$\varphi = \psi + \psi' = \log_e\frac{(2\pi)^{n+1}}{\delta\delta'^n} - \frac{n+1}{2}C_{in}2\pi.$$

By returning from the angular scale to length scale we have to write again $2\pi\delta/\lambda$ and $2\pi\delta'/\lambda$ instead of δ and δ', so we obtain finally for the potential in a plane through an end of the dipole of Fig. 4 the formula

$$\varphi = \log_e\frac{\lambda^{n+1}}{\delta\,\delta'^n} - \frac{n+1}{2}C_{in}2\pi \quad \ldots(11)$$

which also holds for the folded dipole of Fig. 2, at least with good enough approximation for practical purposes.

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TRANSFERS AND ELECTIONS TO MEMBERSHIP

Subsequent to the publication of the list of elections to membership which appeared in the July issue f the Journal, a meeting of the Membership Committee was held on July 14th, 1949. Twelve proposals for irect election to Graduate or higher grade of membership were considered, and seventeen proposals for ·ansfer to Graduate or higher grade of membership.

The following list of elections was approved by the General Council : ten for direct election to Graduate r higher grade of membership, and fifteen for transfer to Graduate or higher grade of membership.

Direct Election to Full Member

OWSHER, Edward Albert Henry — Edgware, Middlesex

Direct Election to Associate Member

[ANNURKAR, Lakshaman Gopal — Poona, India
.ICKMAN, Samuel Reginald, Major — Chesterfield

Direct Election to Associate

:ARMENT, Paul Maxwell — Felixstowe
;AYFER, Herbert Wiggins — London, S.W.12
[ARRIS, Raymond John — Basingstoke
:NEESHAW, Robert William — Swindon
HARRATT, Herbert Henry — Leeds, 6, Yorks.

Direct Election to Graduate

EARSEY, Richard Stanley — Oswestry, Shropshire
OCHE, John — London, W.5

·ansfer from Associate Member to Full Member

RUCE, William Norman — Edinburgh, 9

Transfer from Associate to Associate Member

MAIL, James Elliot — Edinburgh, 9
AYLOR, Harold — Birmingham, 15
HOMPSON, Robert Frederick — Natal, S. Africa

Transfer from Graduate to Associate Member

MORTON, George Basil Stanley — Bletchley, Bucks
SRIVASTAVA, Shambhu Saran, M.Sc. — Allahabad, India

Transfer from Student to Associate Member

PORRITT, Brian Longshaw Day — Johannesburg, S. Africa

Transfer from Student to Associate

BLACK, Walter Stuart — Thetford, Norfolk
KENNEDY, Cyril Richard — Smethwick, Staffs.
McGOVERN, Michael Christopher — Balbriggan
MENZEL, Raoul — Tel Aviv, Israel
PRATT, Charles Albert Allen — Croydon
QURESHY, Abdul Majid — Karachi, Pakistan
SPROSON, James, M.B.E. — Manchester, 14

Transfer from Student to Graduate

SINHA, Kailash Nath — Meerut City, India

STUDENTSHIP REGISTRATIONS

In addition to the list of Studentship Registrations published in the August issue of the Journal, the ıllowing fourteen studentship proposals were dealt with at the meeting of the Membership Committee held ı September 7th, 1949, and these have now been approved by Council.

HAWDHARY, Basant Singh, B.Sc. — London, W.4
ENT, Arthur Henry James — Whitley Bay
OEL, Radhey Shyam — Delhi
EIGHTMAN, Anthony Norman — Clacton-on-Sea
ANKSHEAR, Peter Maurice — Hawke's Bay, New Zealand
[cCRUM, Sidney David — Glasgow, S.W.2
MEDBURY, Robert Colin — Perth, Australia
PEGRUME, Peter Hillyard — Twickenham
REANEY, Donald — Liverpool
ROBINSON, John — Hull
STEVENSON, Roy Neville — Ilford, Essex
TUCKER, John Drew — London, S.W.18
UNDERHILL, Walter Thomas — Chelmsford Essex
WINTER, Simon Chapman — London, N.W.4

NOTICES

Birmingham Pilot Transmitter

The B.B.C. has been transmitting a television test picture from a low-power transmitter situated at Frankley Beeches, Birmingham, between the hours of 10 a.m. to 1 p.m., and 3 to 6 p.m. These transmissions started on August 16th, and are for the benefit of the radio industry in the Midlands to enable dealers to install and adjust television receivers before the service opens from the Sutton Coldfield television transmitter. It is intended to continue the test transmissions for the remainder of the four weeks, and then move the transmitter to Wolverhampton and Coventry.

It has been found, however, that the test transmissions are causing more interference than was expected to the measurements work in progress at Sutton Coldfield, and in order not to impede the completion of this work it has been decided to close the morning test transmissions and extend the afternoon ones from 3 to 8 p.m. as from Monday, August 29th.

Radio Components Exhibition, 1950

The seventh annual Exhibition of British Components, valves and test gear for the radio, television, electronic and telecommunication industries, will be held in the Great Hall, Grosvenor House, Park Lane, London, W.1, from Monday, April 17th, to Wednesday, April 19th, 1950.

Admission is by invitation of the organizers, the Radio and Electronic Component Manufacturers' Federation (22 Surrey Street, Strand, London, W.C.2).

Electronics Courses

The South East London Technical College, Lewisham Way, London, S.E.4, is to run a number of courses of lectures during the next evening-class session. These are to include such subjects as Communication Networks, Radio Frequency and Electronic Measurements and Industrial Electronics.

Application for admission should be made to the Head of the Electrical Engineering Department.

Radio Reconstruction in Hungary

The Hungarian News Service states that all transmission stations in Hungary which were damaged during the war have been rebuilt. During 1948 there were 8,756 hours of broadcasting, of which 87 hours were devoted to advertising the products of nationalized industries.

It is also stated that half a million new wireless sets will be manufactured during the present Five Year Plan.

D.S.I.R. Exhibits

The Department of Scientific and Industrial Research are exhibiting two interesting equipments at Radiolympia.

These are a Storm Locator and an automatic Ionospheric Recorder for predicting the best short wave communication frequencies. The Storm Locator will be demonstrated during an actual location period, but the Ionospheric Recorder will not be demonstrated owing to the interference which would be caused to other exhibits.

The two equipments have been developed by the Department's Radio Research Organisation.

J. W. Ridgeway

The Council is pleased to note the election of J. W. Ridgeway (Member) as Chairman of the British Radio Valve Manufacturers' Association for the current year.

Mr. Ridgeway remains as Chairman of the Radio Industry Council. He is also a director of Ediswan Ltd. in addition to being manager of the firm's radio division.

Scientific Films

The Scientific Film Association of Great Britain, 4 Great Russell Street, London, W.C.1, will appreciate receiving information regarding privately produced films of engineering or scientific interest.

The third International Scientific Film Congress will be held in Brussels from September 30th to October 5th, 1949. Details of the Convention can be obtained on application to the Association.