DICTIONARY OF MATHEMATICAL DATA

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DICTIONARY

OF

MATHEMATICAL

DATA

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PRECISE

DEFINITIONS

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PREFACE

This is how it came about : A matriculation student asked for help with a new type of problem—The transformation of formulae—which is more frequently appearing in examination papers “The dog chased the cat across the road,” I said “ Make ‘the cat’ the subject of that sentence” His reply, given in all sincerity, was “The cat chased the dog across the road” finding he had no idea whatsoever of subject and object, active and passive, transitive and intransitive, I pitied his English master How would he translate
A = ʅb into a version of b ? Quite probably b = A ʅ Hence this little dictionary of mathematical terms

This doesn’t sound very logical Neither was one of my boys who didn’t know the Anglo-Saxon word swell but knew quite well the Americanism—*swell*—-and not knowing the mathematical signs < and > yet could cloak his ignorance under a smattering of Italian ! Yes, he knew crescendo and its twin

Do not take the asides too seriously, and you will probably find a few accurate definitions which will be useful in clarifying your ideas about the funny language spoken by mathematicians If you can improve on them, do so—there’s no law against it

WM

March, 1944

D I C T I O N A R Y O F
M A T H E M A T I C A L D A T A

Abacus : The average schoolchild will have met this first on the edge of his high chair or inserted as a decoration in the side of his play-pen Some misguided benefactor will also have given him one as a separate present A rectangular frame across which is stretched a few wires carrying sickly coloured balls This is the “counting frame ” of our very early childhood Not more than one in a million learned to count on it or put it to any use whatever of an educational nature This toy has a long history and is in many forms the counting device throughout Asia to-day

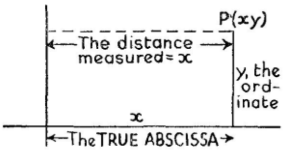
Originally it consisted of mere grooves, cut in a board, and containing pebbles or shells These were the “counters” (This word has survived in our shops) The basic idea was that a counter on one wire was there because a neighbouring wire of lesser order which had been full, had been emptied, and the record kept by the one counter in the next groove This is still the idea we have when we write a number like 728 The 2 means that the “column ” containing the 8 has been full twice already *Our* idea of full is “containing ten”

The modern type dates from the second half of the 17th century In Europe and America it has been superseded by figure manipulation on paper

a, b, c, : Constants, in contrast with x, y, z, variables

Abscissa (PL abscissae) : This adjective dates from the middle of the 17th century and is used now instead of the phrase abscissa-line It means literally “the part cut from the line” It is met with in graph-work and refers always to the axis along which is measured the independent variable (almost always—x) It is the distance we “cut off” this axis, measured from the origin, for any x-value before erecting the ordinate (The end of the ordinate remote from the x-axis is a point on the curve being plotted)

The abscissa of a point on a curve is thus “measured” by drawing a line from the point to the y-axis, parallel to the x-axis and measuring its distance along this line from



Acceleration : This was defined precisely, for mathematical purposes, in the middle of the 17th century Loosely—we mean a “speeding up” even in fits and starts A student’s efforts for a month before an examination is an acceleration, in the loose sense It is a fixed rate of change of velocity For first ideas it is given by

$$f = \frac{v - v_o}{t - t_o},$$

where v and v_o are the final and original velocities at t and t_o, the times at the end and at the beginning of the observations The usual formula is f = v/t, and it is well here and now to convert this to the equally useful form : v = ft

At a later stage the interval of time, $t - t_o$, will have to be made smaller and smaller until it is an “ infinitesimal,” ie, the smallest imaginable quantity of time, then at this limit we shall know the acceleration at the moment under discussion

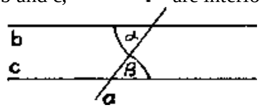
Acute ; The *word means* “ coming to a point,” and in geometry is applied to an angle which is less than a right angle

Addition : Derived from the Latin *ad,* to, and *dare,* to give, the word means "to give to" in the sense of to *cause* to *increase* It is the process by which we find the sum of several quantities of the same kind

Algebra : This major branch of Mathematics has its beginning as the solution of equations with modern symbols in the 17th century, but with much cruder and less convenient apparatus it dates from the 3rd century It deals with the properties and relationships of numbers by using general symbols, and is the medium of work in almost all branches of the science

Algorism : Early arithmetic using numerals instead of counting frames went under this name From the 11th to the 15th century the Algorists sought to supplant the Abacists After four centuries, the Algorists won, at least throughout Europe

Alternate : Derived from Latin *alter,* the other In Geometry this word means " on the other side of"—generally a transversal If a transverses b and c, $\hat{\alpha}$ and $\hat{\beta}$ are interior alternate angles, and $\hat{\alpha} = \hat{\beta}$.

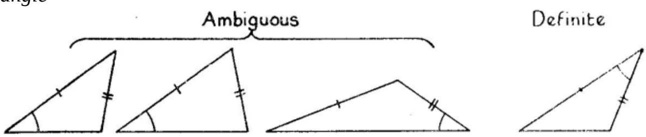


Altitude : In triangles there are three, the distances of the apices from the opposite sides The sides are then called bases—each base being associated with one altitude

In solids the height of the highest point above the base on which it stands is called the altitude

Also, the height of a place above sea-level is called its altitude

Ambiguous Case : A statement that two triangles are alike in every respect, ie, are congruent, will be true if we adopt the standard nomenclature of a triangle, and have aBc in one equal to a'B'c' in the other On the other hand, the statement that two sides and an angle of one are equal to two sides and an angle of the other is the " ambiguous case of congruence," for three pairs of triangles can be drawn which fulfil this condition and ail three are different in shape and size The ambiguity is removed if in the statement we place *included* before *angle*



Amount : A general word meaning quantity ; but, in problems on Interest, having the special meaning of the sum of money which pays for a loan at simple or compound interest, qv Thus, Amount = Principal + Interest

Angle: The Oxford Dictionary describes this rather loosely as "a decree of inclination" It is the difference in direction of two straight lines If the difference is unmeasurably small the lines are said to be parallel and the angle they "contain" is zero A unit angle is *one revolution* and this may be repeated in the "revs" of the mechanic (See Revolution)

Answer : Literally " to swear against," ie, to make a reply to a charge or challenge Hence the answer to a problem must be a correct reply Almost all problems ask for numbers, weights, lengths, heights, gallons, etc, and none asks for an answer Hence, Ans = 4 tons is foolish, while Weight = 4 tons is reasonable

(If one may give away a secret, many problems set in examinations are worked backwards by the problem-master He starts with your answer and works one way, gives you *his* answer as *data* and asks you to get *his data* as your *answer)*

Antilogarithm : See Logarithms If BL = N, B is the base of calculation—for you, 10, L is the logarithm of N And the number N is called the antilogarithm of L

Apex : The highest point of a pyramid or cone The angle point of a triangle when the opposite side is called the base

Appolonius, circa 260-200 BC : Wrote a full treatise on Conic Sections

Appolonius' Theorem : If a median, of length m, is drawn in a triangle, between the sides *a* and *b*, then

$$\begin{aligned} 2\,(c/2)^2 + 2m^2 &= a^2 + b^2 \\ \therefore\ m^2 &= \tfrac{1}{2}\,((a^2 + b^2 - 2(c/2)^2)) \\ \therefore\ m &= \sqrt{\tfrac{1}{2}\,(a^2 + b^2 - c^2/2)} \\ &= \sqrt{\tfrac{1}{4}\,(2a^2 + 2b^2 - c^2)} \\ &= \tfrac{1}{2}\sqrt{2a^2 + 2b^2 - c^2} \end{aligned}$$

(Appolonius lived from approximately 260 BC to approximately 200 BC He studied in Alexandria and taught in Pergamum)

Arc : Part of a curve as distinct from the whole In a circle an arc is called a major arc if greater than a semi-circle and a minor arc if less than a semi-circle curve

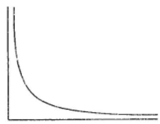
Archimedes, circa 287-212 BC : Wrote brilliant essays on almost all subjects known at his time Laid the foundations of the science of Mechanics of solids and fluids

Area : Surface extent : measured in terms of square inches, or square centimetres for small quantities ; square yards, or square metres for larger quantities ; square miles or square kilometres for large land areas Other units are the *are* in the metric system and the *rood* and *acre* in the English

Arithmetic : This is the whole science and art of counting A start is made with the *cardinal numbers* which are memorised The action of taking them two at a time, three at a time, four at a time, and so on, are memorised in " tables" (What law there is on the Statute Book that forbids the 13 times and other higher tables being learned and places the number 12 as the upper limit, I have been unable to find out 12 x 12—144 is not the summit of mathematical wisdom, even for a child) This is *addition* and its memorisation as *multiplication* There follows substraction and its simplification in *division Involution* and *evolution* are the names of the processes of finding powers and roots So far the subject is abstract, since it deals with number only The completion of the subject is made by applying these numbers and the six processes to measurements of all kinds, through the ideas of *ratio* and *proportion*

Arithmetic Progressions : See Progressions

Asymptote : A line towards which a curve continues to approach, but which it does not touch at any point at a finite distance from the origin Loosely, but rather effectively, described as " tangents at infinity" Any rectangular hyperbola, like that given as the curve for the simplified version of Boyle's Law, pv = k has the two axes of co-ordinates as asymptotes



Average : Imagine the surprise of the teacher who, asked where hens lay their eggs, at being told by one child, "Our hens lay their eggs on an average ! " What *did* Tommy's father mean when he said, "My ten hens lay, on an average, six eggs a day" ? If they laid six on Sunday, four on Monday and 8, 5, 2, 10 and 7 on the remaining days of the week, that would be 42 in all, or the same total as would have been laid had the hens laid six per day This is what is meant by an average A number of quantities is added together and the total is divided by the number of the quantities

There are two different averages of two quantities, the Arithmetic Mean and the Geometric Mean, qv

Avoirdupois : The standard system of weights in use in Great Britian and parts of the Empire for all goods excepting precious metals, stones, medicines, etc The highest unit is the *ion,* the lowest the *dram,* which is composed of 437½ legal *grains* The relationships are :—

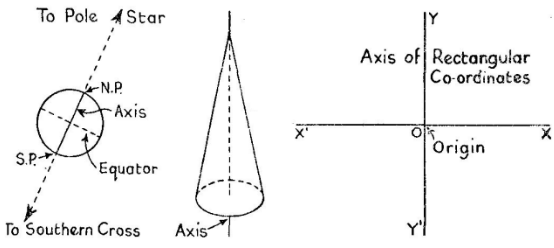
1 ton = 20 cwt = 80 quarters
= 160 stones
= 2,240 pounds (lbs)
= 35,840 ounces (ozs)
= 573,440 drams
= 15,680,000 grams

Axiom : A mathematical proverb "A red sky at night is the shepherd's delight" not only represents the accumulated experience of countless generations of shepherds, it is also a meteorological probability based on facts of observation and their consequences It is "funny ha-ha" to state the opposite if you are young and allowed to be facetious, but if you persist in being sheer pig-headed as you grow older you will be regarded as "funny peculiar" There is a terrible difference

That two quantities (weights, lengths, etc) are equal to the same thing and therefore are equal to one another is a truth almost self-evident to you the first time you meet it It is a mathematical axiom (Axiom is derived from a Greek word, axios, meaning worthy) Philosophers spend large amounts of their time attempting proofs for axioms At our stage we will not quarrel with them until such time that someone can disprove them

Axis : In Roman times wheels did not have axles ; they had axes, and one wheel had one axis That, then, explains this word : axis — axle It is a line, about which a thing or a group of things may be considered to revolve or rotate (These words are used very loosely, but it may be wise to reserve revolve for the case where the axis does not pass through the thing and rotate for the case in which it does That distinction has been observed here)- Examples of axes are (i) the centre line of a cone, (ii) the longest and shortest diameters of an ellipsoid, (iii) any diameter of a sphere Earth's diameter axis ends in the North and South Poles and for purposes of establishing directions in space it is produced in both directions to pass near the Pole Star and the Southern Cross

An axis of symmetry is a line drawn across a plane figure in such a way that it divides the figure exactly into two similar parts The following physical effect may be seen Place the figure on a table-top; stand a mirror vertically along the line of symmetry so as to exclude from view one-half of the figure; note that the reflected portion exactly replaces the hidden portion (Such a line of symmetry in an ellipse, used as an axis of rotation, *is* essential in the development cf the ellipsoid mentioned above) Try this experiment in connexion with two intersecting unequal circles on which a line of centres, common chord, common tangents, and a few radii are drawn Lay the mirror along the line of centres and note all the conclusions that can be drawn



Axes are two lines in a plane or three lines in space, intersecting in an origin Th?y are the lines of reference in graph work

Bacon, Roger, 1214-1294 : Would have shone in any age but glowed like a meteor in the Dark Ages when the inquisition was making the attempt to suppress Science at its birth His main thesis was that no science can flourish on logic but only on careful observation and mathematical calculation He wrote his Opus Major in fifteen months (1267) He wrote authoritatively on Optics, Astronomy and calendar reform He was compelled to spend thirteen years in prison—"for practising magic," the clerical definition at that time of scientific experimenting

Base : The same word as *basis,* the part of an object or diagram on which the object or diagram stands It is a neater word than any used in the dictionary to explain it The only good substitute in our mother-tongue is "ground-work" It has four meanings or shades of meaning in Mathematics :—

(i) The base of a triangle is any one of the three sides chosen for the purpose of making constructions or finding the area—A = ½ (base x vertical height)

(ii) The base of a solid is the plane face on which it stands or is supposed to stand

(iii) The base of the standard triangle of Trigonometrical calculations it the "side against" the angle to contrast it with the "side opposite" (It is understood not to be the hypotenuse, which also lies against the angle under consideration)

(iv) The base of calculations may be our unit of counting or the 10 unit of ordinary logarithms or e, an indeterminate number, 27182818 , used in Naperian Logarithms If B, L, N, chosen in alphabetic order, are arranged thus :—

$$B^L = N,$$

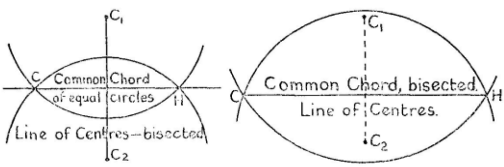
the equation expresses the truth that the *base* must be raised to a power called the *logarithm* in order to give any number under consideration This B is the base of the logarithm of the number Since $2^3 = 8$, 2 is the base, if the logarithm of 8 is 3

Binomial : An expression having only two terms, separated by either a plus sign or a minus sign Examples : $3a + 7b$ and $ax^2 — by^2$

Binomial Theorem: A formula discovered by Sir Isaac Newton by which a binomial quantity may be raised to any power without performing the actual multiplications The work can be performed and understood only after mastering the theory of Permutations and Combinations

Bi-quadratic : An expression involving the fourth power A quartic is the same

Bisect : To cut into two equal parts To do this with ruler measurement, a division by two and a new ruler measurement is never more accurate than the ruler and the operator It is performed best by making the whole line into the Sine of centres of two *equal* circles and drawing the common chord, or mak'ng the whole line into the common chord of two unequal circles and joining their centres (See axis of symmetry)



Brackets : Devices for keeping things together for a time By the conventions of mathematics the things would be separate and the problem under discussion would take on a wholly different meaning Thus, 10x5+ 3 is understood to be 53 the world over But !0 X (5 + 3) is interpreted as 80 The pair of brackets makes the sum of 5 and 3 into a united group of 8 Like book-ends, that serve a similar purpose,, brackets go in *pairs* A bracket has no mathematical meaning—yet some students are careless erTough *to Cmk* the second member of the pair to their ultimate confusion The vinculum (Latin, a bond) is a special bracket, the horizontal straight line which is encountered as follows :—

$$\tfrac{1}{2}(a + b) = \frac{a + b}{2}; \quad \sqrt{}(x - y) = \sqrt{x - y}.$$

Breadth : A special length used principally to distinguish the shorter side of a rectangle from the longer It is just as much a length as the length itself It may be found by dividing the number of units of area in a rectangle by the number of units of length in the length, or b = A/l

Brokerage : A term met with in the Stock Market, and derived from the word Broker (which is the same as the Broacher or the Man who "taps" the man you want to approach), who is the intermediary between the buyer or seller of Stock and the Member of the Stock Exchange, who is entitled to deal in Stocks The brokerage is the commission charged by the broker for his services It varies with the type of stock handled from three-sixteenths to one-half % It would appear from the reading of examination papers that "brokerage ⅛%" is the *only* commission charged (This is a misunderstanding directly traceable to the fact that a teacher or a university lecturer is rarely well-enough *provided* with this world's goods to be able to find out the truth for himself) The commission is charged on the amount of *stock* changing hands, unless it is registered stock, in which case the commission depends on the amount of *money* changing hands

C : Roman symbol for 100 CC = 200 ; CCC = 300 ; CD = 400

Calculus : This, the discovery of Newton and Leibnitz, is one of the highest branches of Mathematics It is based on a full knowledge of Limits and Infinitesimals and deals with (i) rates of change in functions, and (ii) the calculation by the addition of infinitesimals of the lengths of curves, areas of surfaces and volumes of solids which defy elementary treatment Its *name* is derived from the most elementary form of record keeping—with pebbles in a pot Pebbles, to the Romans were *calculi* ; a pebble—*calculus*
(Can you see the connexion ?)

Calendar : From earliest times man has endeavoured to measure Time, of the steady passing of which he has an inner consciousness From observations of sun, stars and moon ; through steadily burning candles, sand glasses and water clocks ; up to pendulum, hair-spring-controlled chronometers and electric impulses ; he has gradually perfected his instruments and more closely kept track of this most elusive of "weights and measures" His greatest difficulty has been to make days measure months and years They are as unrelated as were the length of Henry l's arm to the "length of 20 men's feet end to end on Sunday morning after church" The best that could be done with this *odd* pair was to let one be 5½ of the other !

A month is not quite 29¼ days A year has been known from the days of ancient Egypt to be about 365 days long Julius Caesar enlisted the help of the Alexandrians in fixing it at 365¼ days He also abandoned the moon as a fixer of civil time despite its great apparent advantages The Church clings to this anachronism for fixing the date of Easter Julius Caesar fixed the first day of the year so that the vernal equinox should fall on March 25th He named the fifth month after himself—July (A later monarch, not to be out-done, chose the sixth month as *his* monument, and changed it to August)

But the year is not quite 365¼- days long It is 10 minutes 48 seconds short of this, and in 400 years this deficiency amounts to three whole days The Julian calendar went merrily along for sixteen centuries, by which time the Spring Equinox was falling on March 11th Pope Gregory and his advisers undertook to correct matters They eliminated three leap years from the 400 year cycle, so that 1600 was a leap year and 2000 will be, but 1700, 1800 and 1900 were not This revised Gregorian Calendar is now in universal use, and it will take 3300 years for the time reckoning to accumulate an error of one *day* We have thus measured the year to within 26 seconds

(for those wanting to read the full story of Time Measurement, consult book of that name by L Bolton, MA, Geo Bell & Sons, Ltd)

Cancelling : Why should 6/16 be "cancelled" into 3/8 ? Cancel means to blot out, to suppress, to strike out Surely "simplify" is a better verb ; three-eighths is simpler than six-sixteenths Or must we forever tolerate the slovenly striking out process that spoils most work on fractions ? 3⅞/5¾ simplifies to 31/46 by multiplying the numerator and divisor by 8 What student, or, for that matter, what teacher ever cancels by multiplying ? Truly 6 cancels —6 in an addition problem and 10 cancels 10 in a subtraction problem, for in both cases nothing is left Some attempt must be made to understand terms used and to avoid the confusion made by words like cancel and reduce

Cap : The name given to a segment of a sphere which is less than a hemisphere
If its circular base has diameter, *d*, and height, *h*, then its curved area and volume are:-

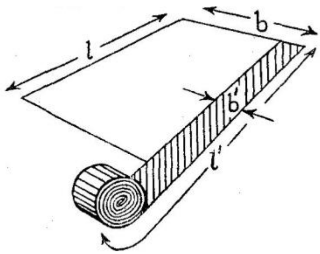
$$A = \pi (d^2 + 4h^2)/4 \; ; \; V = \pi h (3d^2 + 8h^2)/12.$$

Cardinal : The first time we meet this word is probably in our History, when we are told of Cardinal Wolsey We then note his alleged features advertising woollens Someone then tells us that 1, 2, 3, 4, 5, are the cardinal numbers and life becomes very confused If, after that, you start Latin and find that *cardinalis* means a hinge, you will probably give up in despair ever trying to understand this world Yet this hinge idea Is at the bottom of the Cardinal Wolsey idea—the Cardinal is a pivot about which the Roman Catholic Church works ; and Cardinal Numbers are at the very centre of all mathematical ideas and around them all mathematics has been built

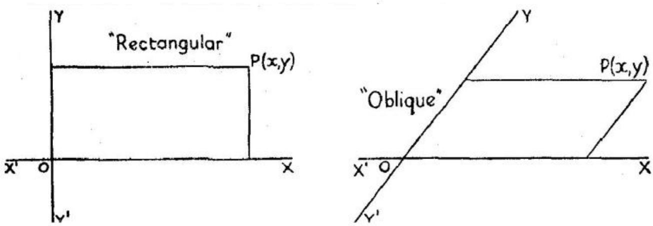
As important as a hinge to a door or a Cardinal-Prince to a Church were man's fingers (digits) to early counting schemes So 1, 2, 3, 4, 5, etc,, are the Cardinal Numbers (Cf Ordinal)

Carpet and Wallpaper Problems : These elementary problems present no difficulty once we realise that the total area of carpet used equals the total area of floor covered Each has a rectangular shape but the dimensions are not a bit alike The long thin rectangle of carpet is always seen as a cylinderical roll

But $lb = l'b'$ hence the length of carpet is lb/b'



Cartesian Co-ordinates : Rene Descartes' name is immortalised in this phrase and in the Cartesian Diver, which amuses students taking elementary physics A mathematician of the 17th century, he explored and established new methods of analysing (and thus solving) problems in mathematics His discoveries are largely associated with coordinates—the measurement along axes which "fix" the position of a point in a graph With three unknowns, *x, y, z, and three* axes meeting in a point, the position of any point in space may be related to the three axes (See Axis)



Rectangular and oblique co-ordinates of space cannot be represented on paper other than pictorially and should be constructed with wires

Catenary : The curve of a hanging chain with equation in cartesian co-ordinates:-

$$y = \frac{e^x + e^{-x}}{2}$$

Centre : In a circle, the point equidistant from all points on the circumference In an ellipse—the point of intersection of the axes In a sphere, the centre is equidistant from all points on the surface

Centroid : The “centre of gravity” The point in a plane sheet or a solid body at which might be concentrated the whole weight of the sheet or body without alteration in the equilibrium of the sheet or body Thus we do not have to think of the weight of every separate molecule in a sphere—but of the *total* weight acting vertically (ie, in the direction of a plumb-line) through the centre of the sphere

If the plane sheet has two lines of symmetry, the centroid is at their intersection If there is one line of symmetry only the centroid lies on this and its position is found fairly easily If the sheet has no line of symmetry, the position of the centroid may not easily be found, and its discovery may be a matter that involves the use of the Calculus

Characteristic : That portion of a Logarithm which is not given in the tables and which must be known beforehand Most text-books give two rales for finding the characteristic by inspection; one only is necessary and it is much easier than either usually given Take the first figure in a number and its name Then the rule is :—

All unit numbers	0
10’s and 10ths numbers	1 and —1
100’s and 100ths numbers	2 and *—2*
1,000’s and 1,000ths numbers	3 and —3

and so on

Chord : A straight line joining two points on a curve If produced both ways the line is called a secant Thus a chord may be thought of as the portion of a secant *inside* the curve

Cipher : From the Arabic word *cafara*—to be empty Still used for Zero Our word decipher—to make clear the meaning, generally of a mystery, etc, comes from this source The emptier a thing is, the more likely are some people to be sure it contains something—not *revealed*—so they persevere with their deciphering

Circle : A plane figure, bounded by a circumference, every point of which is at a fixed distance (the radius) from a fixed point (the centre) The word is used loosely for the circumference in the phrase—“draw a circle” It is derived from a diminutive form of the word *circus,* a ring, *circulus,* a small ring

Circumcircle : The circle which passes through the three points at the vertices of a triangle or through the vertices of a regular polygon or the vertices of a cyclic quadrilateral

Circumference : The full arc of a circle Its length is not measurable in terms of the diameter $C = \pi d$ Almost exactly $113c = 355d$ (This relationship is true as far as the sixth place of decimals—the error is therefore not more than one in ten millions)

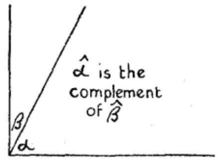
Coefficient: A number which *multiplies* some quantity, known or unknown Thus 3a has 3 as a coefficient In the equation : Expansion = k (length), k is called the coefficient of expansion of length and is a constant for each material Thus a length of copper wire expands 0000017 of its length for 1°C rise in temperature

In the expression $ax+ by = c$, the unknowns x and y are said to have *literal* coefficients

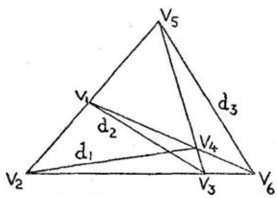
Combinations : If we are given a number of individual things, say n, the number of ways in which a smaller number, say c, can be chosen from the larger group n is written nCc and in this no notice is taken of the arrangement of the group once it is chosen This type of grouping is called a combination to distinguish it from a different grouping called a permutation, qv

Compass, Compasses : The plural word is the more correct An instrument for drawing a circle, having a point which may be fixed, and a point that is moveable at a fixed distance and capable of tracing the circumferential line

Complementary : Two angles are so described if their sum is a right angle Note the spelling—no i The word *with* i, complimentary, is the Italian derivative of the Latin word with e It has a similar meaning but is reserved for occasions of courtesy Both words mean fulfilment, the first of a right angle, the second of a person's capacity for flattery



Complete Quadrilateral : This has six vertices and three diagonals as shown



Complex Numbers : Numbers with composition a + ib, ie, a + b $\sqrt{-1}$ where a and b have real values

Compound Interest : The type of interest given or received when it is understood by the borrower and the lender that interest added periodically will not be withdrawn, but will be considered as part of the capital sum, the Principal The formula— a matter more often than not for the use of logarithms—is
$A=P\ (1\ r/100)^n$ By this we find first the compound amount; the Compound Interest is found as an afterthought, and is often forgotten In the equation given for the Amount, P is the Principal, r is the bank rate per cent, per payment and n the number of payments made

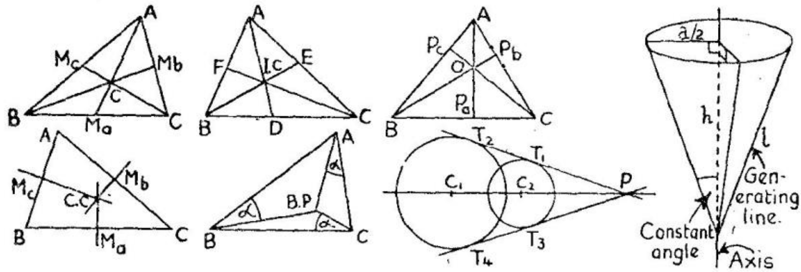
Compound Practice : The method by means of which the *cost* of a quantity of some commodity expressed in units and sub-units (Ex : 6 tons, 5 cwts, 2 qrs, 17 lbs) is found when we know the price of the commodity, ie, the amount to be paid for a unit quantity, a sum involving £, s and d In all but very simple cases it is best effected by decimalising both quantities, finding their product, and de-decimalising the resulting sum of money

Compound Proportion : When a quantity is dependent for its value on several factors, each of which is likely to vary at the same time as any other, the quantity is in compound proportion with the variants The best example is provided by Simple Interest, where the interest received depends on (a) the amount of money lent, (b) the time for which it is lent, and (c) the current rate of interest

Concentric : Describing two or more circles drawn with a common centre

Concurrency : The property among straight lines of having a point in common This is of considerable importance in triangles The medians, the bisectors of the angles, the perpendiculars at the middle points of the sides, and th*e* perpendiculars are concurrent in the centroid, the in-centre, the circumcentre and the orthocentre respectively Brochard's two points in a triangle are points of concurrency, but the law of their construction is somewhat beyond the scope of elementary work

In the case of intersecting circles the line of centres and a pair of common tange are concurrent



Cone : A solid generated by a straight line moving in space with one point in the line fixed The special cone met with in Geometry, etc, is caused by this movement under slightly simplified conditions

Imagine a circular disc and a straight length of wire passing through the centre at right angles to the plane of the disc If any point is chosen on this wire and from it a string is drawn tightly to the edge of the disc, then the string will generate a "right circular cone" if it is caused to pass through successive points on the rim of the disc Now substitute circle for circular disc; axis for wire; straight line for string; and circumference for rim of disc—and you have the idea of the geometrical cone The generating line maintains a constant semi-vertical angle with the axis of the cone

The cone forms the geometrical basis of all Conic Sections, qv

The area of the curved surface may be found from *(a)* the height and the diameter of the base, or (*b*) the length of the generating line and the diameter of the base The volume is found in terms of the diameter of the base and the height The formulae are—

$\triangle\,(a) = \frac{1}{4}\,\pi\, d\,\sqrt{d^2 + 4h^2} \qquad \triangle\,(b) = 4\,\pi\, l\,\sqrt{l^2 - h^2} \qquad V = \pi\, d^2h/12$

Congruency : The property in triangles that two or more shall be "alike in every respect" This is true in two independent ways only -

(1) Two sides and the included angle of one must equal two sides and the included angle of the other

(2) One side and the angles at its extremities in one must equal one side and the angles at its extremities in the other

The first case is referred to in the paragraph on the Ambiguous Case

If a triangle has to be turned over to fill these conditions the pair is regarded as being congruent

A third condition of congruency, derived from the two above with the aid of a well- known property of isosceles trianges, is :

(3) If three sides of one triangle equal three of another, the triangles are congruent

Conic Section : A plane section through a cone The direction and the position of the plane of the section will determine the nature of the Section obtained It may be (1) a point, (2) a straight line, (3) a pair of straight lines, (4) a circle, (5) an ellipse, (6) a parabola, (7) a hyperbola, qv The nature of every one of these is included in the co-ordinate equation :—

$$ax^2 + 2hxy + by^2 \text{ -f- } 2gx + 2fy + c = 0$$

Constant : A quantity which does not vary for a specified time in a specified place Examples : a, h, b, g, f, c, in the preceding paragraph ; the coefficients of expansion , the constant of Gravity, qv

Constant of Gravity : The acceleration with which objects in the neighbourhood of the earth's surface fall earthwards *in vacuuo* A compact, heavy body is very little affected by air resistance Its value is always represented by *g* and is approximately 3216 feet per second per second (322 ft per sec per sec is usually taken in examples)

Continued Fractions : Fractions of the type

$$1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots$$

which means that the denominators are more and more involved as we move to the right If we take first one, then two, then three of the plus signs and ignore the rest we get this series of results : 1½, 1.4, 1.41667, 1.41379, 1.41428, 1.41420, followed by three results 1.41421 if we do not pursue the results beyond six places of decimals and give them correctly to five in each case If the first, third, fifth, etc, terms are taken in series we have 1.5, 1.41667, 1.41428, descending in value, and 1.4, 1.41379, 1.41420, in ascending value The fourth term and fifth terms of these odd and even choices are 1.41421, By taking as many places as we like we may make the final term as near in value to the square root of 2 as we please—1.414213

Another useful continued fraction gives the value of one-quarter of π This is

$$\frac{1}{1}+\frac{1^2}{2}+\frac{3^2}{2}+\frac{5^2}{2}+\frac{7^2}{2}+\ldots\ldots$$

Taken this far only, and multiplying by 4 gives a whole value for II of 2.97600, and taken one step further the value obtained is 3.28572 Perseverance will make the results fluctuate and come to rest eventually at 3.14159

Continuity—The state of being free from disjointed breaks as, for example, the curves of the ellipse and the catenary The continuity of the tangent curve in trigonometry is not quite so evident The subject has wide applications in the study of the Calculus

Contour : A line marking the emergence of a horizontal plane through any irregular surface such as, specially, a hillside

Contracted Methods : Devices for completing multiplication and division of decimals by decimals without first finding figures of less value than the data of the problem warrant our finding (Not the method most children follow of doing vitally important work on scraps of paper or in the margin and giving answers of doubtful origin)

If w have to multiply together 36.814 and 3.1415926 as an exercise in pure arithmetic we are entitled to obtain a final right-hand figure of 4 which will have a value of four ten thousand millionths if, however, the 36.814 is a measurement which represents the best we can do to ascertain the diameter of a pipe which might just as well be 36.813 or 36.815, had we used a different measuring instrument, then the circumference, however we find it, will not be more accurate than the measured diameter We shall thus have found *eight* unnecessary decimal fractions—all those beyond the thousandths The total work necessary is shown :- 36.8140

3.1415926

110.4420
3.6814
1.4726
0.0368
0.0184
0.0033
0.0001

115.6546
= 115.655

Compare this with the usual approximation, using three-and-one-seventh for π, and the full result, the one for accuracy and the other for unnecessary work :—

Three times.. .
One-seventh ....
Three-and-one-seventh times

The correct result is 115.6545899764

Division is performed similarly—to the end of eliminating unnecessary figures Instead of making each partial remainder ten times as large numerically and ten times as small in name, we make the divisor ten times as small numerically and the partial quotient ten times as small as well : 90a/10a = 9a/a = 9 Example

217.25
32.815)7149.22
566.22
238.07
8.37
1.81
0.17

Convention : Literally, a coming together. A meeting, and the agreements reached by the delegates.

Converse : The opposite approach. Ex. : It is possible to show that the base angles of an isosceles triangle are equal. It is possible to show that the converse theorem is true—that a triangle is isosceles if it has two equal angles—by the process *reductio ad absurdum,* i.e., by making the former demonstrably true proposition false unless the second is true.

Co-ordinates : Cartesian, using abscissae (x-units) and ordinates (y-units), These were the discovery of Descartes and Fermat in the early 17th Century. Polar, using a pole and a radius vector, were the discovery of Fontana who worked in the 18th Century. (See Cartesian and Polar co-ordinates).

Co-secant : The reciprocal of the sine ratio, q.v., and equal in value to the secant of the complementary angle.

Co-sine : The reciprocal of the secant ratio, q.v., and equal in value to the sine of the complementary angle.

Cosine Formula : The trigonometrical expression of the extension of Pythagoras' theorem.
In the nomenclature of the standard triangle it is :— $a^2 = b^2 + c^2 — 2bc \cos A$.
or $b^2 = c2 + a2 — 2ca \cos B$,
or $c^2 = a^2 + b2 — 2ab \cos C$.
By transforming the equations we are able to find the angles in terms of the sides. For example :—

$$\text{Cos } A = (b^2 + c^2 — a^2)/2bc, \text{ etc.}$$

Cost Price : The price at which a commodity or object is first acquired. Note that the retailer's Cost Price is the manufacturer's Selling Price, unless there are other intermediaries acquiring a little income, between the manufacturer and the consumer.

Co-tangent : The reciprocal of the tangent ratio and equal to the tangent of the complementary angle. If B is the right angle in the triangle aBcAbC, then Cot A = 1 /tanA and Cot A = Tan C.

Counting : A vast subject which ranges from fingers (digits) through pebbles (calculi) and the abacus to digits written on paper but still counted on our fingers. The Calculus is the highest flight of counting—that and nothing more. All of man's efforts to make an orderly survey of, and to find the total of his possessions first, to his final estimation of the number of molecules comprising the universe is—counting.

Coversine, etc. : Coversine Θ = 1- sine Θ ; Havercosine Θ = ½ (coversine 0) ; Haversine Θ = ½ (Versine Θ) ; Versine Θ = 1 — cosine Θ. Abbreviated respectively to covers Θ, havercos0, haversin Θ and vers Θ. Occasionally used by some writers but not of much importance in everyday life, except to navigators.

Cross-multiplication : When two ratios are equal, they give four numbers in proportion, and are often met with thus:-

$$\frac{a}{b} = \frac{c}{d}$$

" Cross multiply " is a loose, unexplained, almost slangy way of saying "Multiply each ratio (fraction) by the least common denominator," in this case bd. The full result of doing this is :—

$$\frac{bda}{b} = \frac{cbd}{d},$$
$$\frac{b}{b}.da = cb.\frac{d}{d},$$
$$da = cb.$$

It is tantamount to agreeing that, when a : b = c : d, then " the product of the means equals the product of the extremes." It is high time such mumbo-jumbo (q.v.) was allowed to die-a natural death.

Cube : The second of the five regular solids. It has six equal square faces, twelve edges and eight corners. A cube of unit edge (inch, cm., ft., metre, mile, Km.) is a unit of volume.

In indices, the cube is the third power, one higher than the square. It is represented by x^3, called x-cubed, and means x.x.x.

Cubic Expression and Equation : One having no power higher than the third, which it contains. Example :—

$$7x^3 — 2x + 4 = 0.$$

Curve : Any line which obeys some law of construction and thus may be represented algebraically by an equation. This equation will involve two unknowns, x and y, if the curve lies in a plane, and three, x, y and z, if it does not lie in a plane.

Cyclic Quadrilateral : (Greek : kuklos—a circle). A figure of four straight sides, joining consecutive points on a circle. Its chief property is that pairs of opposite angles are supplementary.

Cycloid : The curve traced out by a point on a radius of a circle, while the circle roils, without slipping, along a fixed tangent-line. Usually the point is considered to be on the circumference. The curve determines the shape of cogwheels in a rack and pinion.

Cylinder : The surface and the solid within the surface generated by a line moving through space and maintaining a fixed direction in space.

Usually, a point on the generator describes a definite curve. If this is a circle the cylinder is of the jam-jar variety. If this is cut off between two circular ends, it is called a right circular cylinder. The area of the curved surface is the product of perimeter of base and length, i.e., π dl, where d and l measure the units of length in the diameter and the length. We have then :—

$$\begin{aligned} \text{Curved area} &= \pi dl \\ \text{Plane area} &= 2\pi (d/2)^2, \\ &= \tfrac{1}{2}\pi d^2 \\ \text{Volume} &= \pi(d/2)^2 l, \\ &= \tfrac{1}{4}\pi d^2 l. \end{aligned}$$

D : 500 in Roman numerals.

Day : There are three distinct "days," of which the one we measure with household clocks and B.B.C. "pips" is the commonest. Its subdivisions are

1 day = 24 hours = 1,440 minutes = 86,400 seconds.

This day is the average length of the 375¼ days that measure the year (q.v.). The time interval between any two times on two successive days on which the sun crosses the meridian is not constant and our mid-day given on a clock differs by as much as 16¼ minutes behind to 14½ minutes in front of the true mid-day shown by a sundial at different times during the year.

Decimal System : (Latin : decem—ten). All our counting has been on the decimal system since Greek and Roman times. The system depends on our giving a new name to the larger group whenever we have ten of a simpler group. We proceed from units to millions, etc., through tens, hundreds, thousands, tens of thousands and hundreds of thousands. The system now includes the method of changing our vulgar fractions into series of tenths, hundredths, thousandths, etc.—a process that sometimes terminates and sometimes does not. A decimal point always separates the units from the tenths— and should not, under any circumstances, be "moved." This is a bit of mathematical mumbo-jumbo, q.v.

Deduction : A process of making a statement clear by reasoning logically from a previous statement.

Definition : A clear statement of meaning ; full but without redundant ideas. (Something between the scanty ideas found in a dictionary and the full treatment of an idea found in a text-book or encyclopaedia. The paragraphs of this book are attempts at defining words and ideas.

Degree : The stage at which your tutors' educations last received university notice. This is not to be confused in any way with their present mental attainments, as in nine cases out of ten it "crowns" a career ending in the early twenties. With some it is the end; with others the beginning of their mental life.

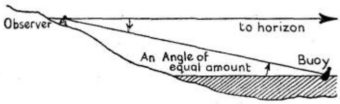
One three-hundred and sixtieth part of a revolution.

The number of times a factor has been multiplied together to give the quantity under discussion. Thus x^3 is said to be of the third degree ; $ab^2c^3d^4$ is of the tenth degree.

Denominator : (Latin : nomen—a name). See last definition. A degree is one three-hundred and sixtieth of a revolution. Shortly, we write : 1° = 1/360 revolution. But note that, in writing the 1 and the 1 above the vinculum, we are talking of things, parts, ideas and whatnot. In the case of the 360, when written *fully,* it is not a number, but a name as shown by the the ending.

Much care is needed in the first year of dealing with fractions, that this subtle distinction, a number over a name, is thoroughly understood. You will then avoid the pitfall of "multiplying the numerator and the denominator by six" or any other number. You will take the trouble, mentally, of separating the whatever-it-is into six times as many parts and take six times as many of the smaller parts in order to keep the same portion, or fraction of the thing you had originally.

Depression : Generally the angle of depression ; the angle through which the line to the horizon (horizontal) must be lowered in order to sight some distant point.



A mental state preceding examinations, if the conscience is not clear.

Descartes, 1596-1650 : One of France's most brilliant mathematicians. Founder of analytical geometry based on plane and spatial co-ordinates. (The man to blame for graph-paper and graphs).

Detached Co-efficients : Much of the tedium in multiplication and division in algebra may be eliminated if the expressions are written in ascending or descending order of powers of one of the variables and any term missing from the series represented by zero. The co-efficients need then only be written, the operations performed, and the variables added in their higher or lower powers at the end. Example :

$(4x^2 + 7x^3 — 3) \times (4 — 3x+ 7X^2 - x^4 + x^5) = (7 + 4 + 0 - 3) (1 - 1 + 0 + 7 -3 + 4)$ arranged in descending order of x powers. These should now be placed as for a straight-forward multiplication problem and 24 partial products obtained before combining them into, 9 different kinds, the highest being x^3 times x^5, i.e., x^8,'the result is :—

$$7x^{3}\,{}^{-}3x^{7} - 4x^{6} + 46x^{5} + 10x^{4} + 16x^{3-}5x^{2+}9x - 12.$$

Diagonal : A line joining " corner points " of a plane figure by crossing the figure. It must pass, as the Greek *dia* says, *through the angles.*

Diagonal Scale : See Scales.

Diagram : The drawing made to illustrate a statement, a definition, or a theorem proof.

Diameter : The measure through—and in relation to—a circle, the line, and its length, passing from side to side through the centre. The longest chord in a circle.

Difference : The remainder of a problem in subtraction. What is the difference between having £5 of your own and being £5 in debt ? £10 ! A person £5 in debt has less money by £5 than a debt-free person "who hasn't a penny to bless himself with." His state may be looked upon as minus £5. — £5, using the sign in this second accepted way. Hence :—

£5 — (— £5) = £10,

where the first minus means subtract or find the difference, and the second means the opposite of plus, in the sense of relative value. This is a man-made standard of wealth, as fictitious as sea-level for determining altitudes, or the temperature of thawing ice for determining warmth or cold.

Digit: - (Latin : a finger). An integer less than 10. (A little used measure of length, ¾- in., a finger breadth).

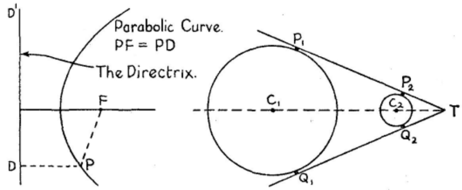
Dimensions : Measurable magnitudes of length, breadth, height, thickness, depth, area, volume, etc. The number of factors in a given product, thus, a^3b^2c, meaning aaabbc, is of sixth dimension.

The phrase "three dimensions" means length, breadth and thickness.

Direction : We should have sense of direction but this has been lost by most civilised individuals. In a room the direction is fixed generally by the length, breadth and height of the room. Outside, the directions are North-South, East-West, up and down, as shown by the plumb-line.

In the larger universe, one fixed line is taken as the extension of the earth's axis.

Directrix : The straight line which, with the foci, determines the conic sections as algebraical curves. A conic section may be defined as the locus of a point which moves around a point and towards, or away from, a line in such a way that the distance from the point maintains a constant ratio with the distance from the line.



Direct Tangents : The two common tangents to two circles which meet the line of centres at a point not between the centres.

Discount : An allowance of cash made by the seller when payment is made at the time of sale. A rebate.

The difference between the face value of a bill, due for payment at some specified future date, and its Present Worth.

Division : Separation into factors of which one is the divisor and the other ,the quotient.

Dodecahedron : The fourth regular solid ; having twelve equal faces, each a regular pentagon. It has 20 corners, 12 faces and 30 edges.. See *Regular Solid.*

e or € : 27182818285 . . . . The result of adding together as many terms as is convenient of the series:-

$$1 + 1 + \tfrac{1}{2} + \frac{1}{3.2} + \frac{1}{4.3.2} + \frac{1}{5.4.3.2.} + \ldots\ldots$$

It is as indeterminate as π. it is the basis of calculation of Naperran Logarithms, which preceded Common Logarithms.

Eccentricity : The ratio of distance from focus to distance from directrix. It is represented by e, not to be confused with e, above. If $e > 1$, the locus of the point will be a hyperbola ; if $e = 1$, a parabola ; if $e < 1$, an ellipse. *See Conic Sections, etc.*

E-circle : One of three circles lying outside a triangle and touching one side and two sides produced. Its centre lies on the point of concurrency of the bisector of one interior and two exterior angles of the triangle.

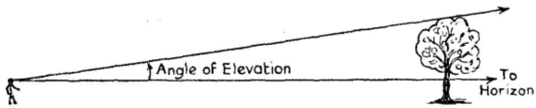
Eena, mena, mina, mo : Schoolchild's vulgarisation of Anglo-Saxon counting, one, two, three and some more, or, etc.

Einstein (1879) : The famous author of the Theory of Relativity. Part I, published in 1905, the “special theory” dealt with problems in optics and thermo-dynamics. Part II, published ten years later, the “general theory,” is the new theory of gravitation. The theory was tried out during the eclipse of 1919 and its verification established its author’s fame. In 1933 he accepted the life appointment of Head of the Mathematical School in the institute of Advanced Studies, Princeton, U.S.A.

Elements : The great compilation of geometrical fact, theory and proof that is associated with the name of Euclid.

Elevation : Horizontal view of an object. Side elevation and front elevation usually of a building.

The angle of elevation is the angle that a line to the horizon (horizontal) must pass through in order to sight some distant point.



A mental state associated with the success rewarding work well done. (See Depression).

Eleven: The first "teen,” if we were consistent in using decimal *system of* counting. “One-leave” is the Anglo-Saxon for “ten is left if one is taken away.” (Cf. Twelve).

Ellipse : A conic section made by a plane. The angle between the axis of the cone and the plane must be greater than the angle between the axis and any generating line of the cone.

The locus of a point which moves in such a way that its distance from the focus is a constant (less than unity) times its distance from the directrix.

The locus of a point that moves in such a way that the sum of its distances from two foci is constant and equal to the major axis.

Enunciation : A precise statement.

Epicycloid : A cycloid drawn *upon* (i.e., on the outside of) a circle instead of upon a straight line. One of the curves which determine the shape of cog-wheels.

Equal : Having the same number of units of length, weight, money, etc. Numbers or expressions that have no difference.

(In schoolboy language in the mathematics room, the word is a substitute for a hundred verbs. Sixteen men = £3 12s. is a common form of “ sixteen men *earn* £3 12s.” ; 14 lbs. of tea = 55 shillings is given for “14 lbs. of tea *cost* 55s.”. And so on. This silly habit is fraught with danger and should be overcome).

Equation : An expression stating the equality of two quantities not obviously the same. We treat as an axiom the statement $2 \times 2 = 4$, but $ax^3 + bx + c = 0$ is a quadratic trinomial equation needing verification and capable of solution, giving two values of x if a, b and c are known.

Equi-angular : Having equal angles.

Equi-tateral : Having equal sides. Specially in reference to a triangle—having three equal sides—the simplest regular plane figure.

Equivalent : Having equal worth. (See Reduction).

Euclid, circa 330-275 B.C. : A most successful Alexandrian teacher. His fame rests almost entirely on the Elements (of geometry), mainly a compilation of the works of earlier geometricians, Pythagoras, Hippocrates, Eudoxus, chiefly. It was the standard work for nearly 2,000 years—surely a record no other text-book has achieved. Euclid wrote authoritively also on optics, astronomy and music

Evaluate, Evaluation : A learned way of saying "find the value of" and " finding the value of."

Evolution : The unfolding of a curve like a spiral. The extraction of equal factors (roots) of a quantity.

Exponent: That which " points out." The number that tells the power. In x^3, $a^{-½}$, p^q, $(x — y)^{n-1}$etc., 3, - ½, qn-1 are the exponents.

Exponential Functions : Є = 2.7182818285 . . . . ; $Є^{-1}$ = 0.367879 . . . . ; $Є^2$ = 7.389057 . . . . ; $Є^{-2}$ = 0.135335 . . . . ; $\sqrt{Є} = Є^{½}$ = 1.648721 ....

Corresponding to the circular functions in Trigonometry, sine, secant, tangent, is a series of hyperbolic functions based on these values of Є. They are hyperbolic sine Θ , written sinh Θ, cosh Θ and tanh Θ, etc., and the values are :— sinh Θ = $(e\Theta - e^{-\Theta})/2$,

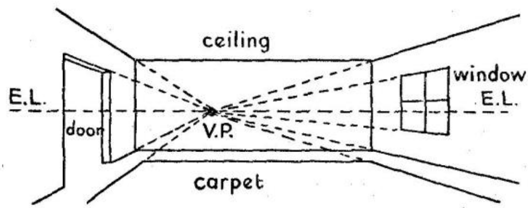
tanh Θ = $(e\Theta - e^{-\Theta})/(e\Theta + e^{-}\Theta)$,

sech Θ = $2/(e^{\Theta} + e^{-\Theta})$.

Points having ordinates which are the average of the ordinates of $y = e^x$ and $y = e^{x}$ for the same abscissa lie on the curve called the catenary, q.v.

Extension of Pythagoras* Theorem : The square on any side of a triangle is equal in area to the sum of the areas of the squares on the other sides, increased or decreased (according to whether the angle opposite the side chosen is obtuse or acute) by the amount of area in a rectangle formed by one of these sides and the projection on it of the other. (See Cosine Formula).

Eye-level : A theoretical horizontal line at eye-level corresponding in practice with the horizon. The level on which is found the point to which sets of other than vertical parallel lines converge in a perspective drawing.



Factorials : The multiple, continued product n (n —1) (n —2) . . . . 3.2.1, is called factorial n and is written n! or $\underline{n|}$. The first six factorials are very useful and should be memorised—1, 2, 6, 24, 120, 720. 10! is 3628800. 20! is 243290 x 10^{13}. These are an essential feature in the theory of Combinations and Permutations, q.v.

Field Book : The book used "on the job" by surveyors. In it the entries consist of the familiar central line of distances from some starting point, with offsets at right angles and occasional triangulation checks.

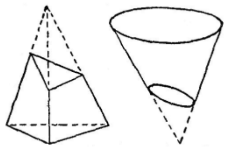
Figures and Fingers : Both these words are derived from the Latin word *digit.* Unfortunately, we have four fingers and a thumb on both hands, and from this fortuitous circumstance we have our basis TEN for counting. Few numbers could have been worse. Twelve, with its factors, 1, 2, 3, 4, 6, 12, would have been much more useful. We use Arabic figures, 0, 1, *2,* 3, 4, 5, 6, 7, 8, 9. These superceded the combined Roman and Etruscan letter-symbols—I, V, X, L, C, D, M. The V (5), L (50), D (500) were due to the Etruscans. Combinations by these symbols were almost impossible for more than simple addition and subtraction. Compare the facility of handling CLXXXVIII and 188 for the idea one hundred and eighty eight. !

Focus (pi. Foci) : Latin for hearth ; hence, in mechanics and mathematics, the point from which rays of light, energy, force, radii-vectores; etc., emerge. Particularly, in an ellipse, those two points on the major axis (i.e., the longest chord) which are distant half the major axis from the ends of the minor axis (i.e., the shortest chord). ..

Formula : An expression of a rule in symbols.

Fraction : (Latin : fractus—broken). A part of a whole thing. Vulgar fraction—a common fraction having two parts, a numerator and a denominator (q.v.) and said to be proper when the numerator is less than the number of parts into which the whole thing has been divided and improper if it is greater. An improper fraction may be exchanged for an equivalent mixed number, i.e., a mixture of whole numbers and proper fraction. Thus, 19/7 is 2 and 5/7 and is written 25/7. A decimal fraction follows the laws of decimal notation associated with our ordinary counting into the region of quantities smaller than wholes. A decimal fraction is in fact a series of vulgar fractions added together. Every *denominator* in *this* series is a power of TEN. It is omitted, because the position of the numerator in the series tells the denominator that should be associated with it. Thus, 0.30718 means 3/10 + 7/1000 + 1/10000 + 8/100000.

Frustum : (Latin, a fragment). When applied to pyramids or cones the word means the difference between a *complete pyramid or cone,* and the small portion removed from the region of the apex by means of a plane section cut.



Function : The power of one quantity to determine a change in another to correspond in some way with a change in itself. Thus, if y = 3x2 + 2x - 5, we may say that y is a function of x, for, when some change is made in the value of x, the values of $3x^2$ and 2x change and accordingly the value of y is changed. If x is zero, y is -5 ; but if x is - 5, y is 80. We express this function idea of the dependability of y on x by the symbol $y = f(x)$.

g : The number of feet per second added per second to the velocity of a body falling freely to earth. It differs from place to place over the earth's surface, as, for examples : London, 32.19 ; Paris, 32.18 ; New York, 32.16. Taken as 32.2 in general problems.

Galileo, 1564-1642 : Laid the foundations of the modern science of Dynamics. He ended his medical studies after watching the swinging lamp in Pisa University. He studied mathematics intently and began his independent researches. Became Professor of Mathematics in Pisa in 1589. Demonstrated a fundamental truth from the top of the Leaning Tower, He gave many very accurate definitions of hazily understood ideas. Prepared the way for Newton. A great astronomer.

G.C.M. : Standing for greatest common measure. A phrase now generally superseded by "highest common factor." This is a pity in a way for, in a practical sense, we do find the greatest common measure of two or more weights, capacities, sums of money, etc. Are not our tables still called weights and measures?

Gematria : The most pernicious branch of Numerology, and a species of witchcraft. The word is a corruption of geometry. It dabbled in the exchange of numbers for words and especially the names of persons and places. Once a person was saddled with a number, to which the letters of his name added, elementary arithmetic could be performed and he could be shown to be equal to this that, or the other according to the wish of the Mathematician performing the trick. (How many well-known men from the Popes to Luther and the ex-Kaiser have not been the 666 of the Book of Revelations !). Remember that figures do not lie—but liars can "figure." It has revived to-day along with astrology in the care of the editors of our allegedly enlightened national newspapers.

Generating Line : A line, the movement of which, according to a simple understandable law, generates a plane or curved surface. Examples : a radius generates a plane circle. A line, fixed at one point and moving in such a way that another point in the line describes a closed curve, generates a cone. A line that moves in such a way that a point in it describes a closed curve and simultaneously a second point describes an equal dosed curve lying in the same direction in a parallel plane, will generate a cylinderical surface. The arc of a semi-circle, rotating about its diameter, will generate the surface of a sphere.

Geometric Progression : See Progressions.

Geometry : A major branch of Mathematics, the origin of which is lost in antiquity. Originally it was the science of earth-measurement as the Greek word suggests. Egyptian priests more than 5,000 years ago brought this art of geometrical surveying to the pitch of perfection. The temples and pyramids are a tribute to their work. The work of early geometricians resulted in the mapping of the heavens and opened the way for the great era of exploration.

Now it is the science of the magnitudes of length, area and volume of lines, surfaces and solids and their inter-relationships and properties. It penetrates the structure of atoms and attempts an explanation of the structure of the universe.

"Gozintas" : Thus was written for me the phrase "Goes-intos." A small school in Bedfordshire always learned a number like 63 in four ways : 7x9 = 9x7 = 63, and 63/9 = 7, and 63/7 = 9. They managed this and reached an alarming standard in Mental Arithmetic by chanting in addition to their "twice times" their "two gozintas," and so on. The colossal ignorance of schoolchildren, and of most adults for that matter, of numbers up to 100 is one of their greatest handicaps.

Gradient: Slope. A loosely used term, but generally meaning the *tangent* of the angle of slope. It is necessary to check that the *sine* is not meant by the questioner.

Grade : The French (decimal) system of sub-division of an angle. There are 100 grades in a right angle and therefore 9 degrees = 10 grades. The grade is sub-divided into 100 minutes and each minute into 100 seconds. The full table is thus 1 revolution = 4 right angles,

= 400 grades,

= 40,000 minutes,

= 4,000,000 seconds.

Gramme : The mass of 1 cubic centimetre of distilled water weighed in vocuuo at a temperature of 4°C. Its weight is the Metric unit.

Graph : The line drawn through points plotted with reference to two axes, to represent the relationship that exists between two quantities. Thus the relationship existing between x and y in the equation $y = mx + n$ is represented by a straight line for all values of m and n. The graph of $x^2/a^2 + y^2/b^2 = 1$ is an ellipse with major axis 2a and minor axis 2b.

Gunter's Chain : The surveyor's chain of 22 yards, divided into 100 links of 7.92 inches each.

Harmonic Progression : See Progressions.

Hemisphere : Half a sphere formed by bisecting it by means of a plane passing through the centre.

Hexagon : A regular plane figure with six equal angles, and by implication six equal sides.

Hero's Formula : The area of a triangle may be found if we know the lengths of the sides (and therefore the semi-perimeter) from the formula :—

$$\triangle = \sqrt{s\,(s - a)\,(s - b)\,(s - c)}.$$

This venture in the continued product of four quantities of length—a genuine fourth dimension (see Dimension)—was due to Hero, who achieved fame in Alexandria in the first century B.C. (He achieved among other things a working model of a steam engine !).

Highest Common Factor : Exactly what the words imply. These should be *treated* as factors and not set out by the continued division process of obsolete text-books. Consider 189, 495 and 630. If the laws of divisibility by 2, 3, 5 and 11 are known, it is easy to write :—

$$189 = 3^3.\ 7,$$
$$495 = 3^2.\ 5.\ 11,$$
$$\text{and } 630 = 2.\ 3^2.\ 5.\ 7.$$

The only complete column is that containing 3, and this occurs twice. The H.C.F. is therefore 9.

Horizon : The circle at ground level, limiting one's vision. ("Visibility good" assumed). If one's eye is h feet above the plane then the distance to the horizon is $1.23 \sqrt{h}$ miles, very nearly.

Horizontal : The line joining the eye to any point on the horizon. Differs from a spirit level line by such an insignificant amount as to be the same for all practical purposes. Any line parallel to the one to the horizon. A horizontal plane is one on which two horizontal lines intersect.

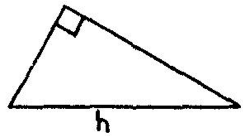
Hyperbola : A conic section made by a plane making a smaller angle with the axis of the cone than that made by the generating line.

The locus of a point which moves in such a way that its distance from a focus is a constant greater than unity times the distance from the directrix.

The locus of a point that moves in such a way that the difference of its distances from two foci is constant.

Hypocycloid : A cycloid under (i.e., inside) a circle. Compare with Cycloid and Epicycloid. The curve is used in designing the shapes of cog-wheels.

Hypotenuse : The longest side of a right-angled triangle. The side *under* the right angle.



I : 1 in Roman numerals. II = 2 ; III = 3 ; IV = 4 ; VI = 6 ; VII = 7 ; VIII = 8 ; IX = 9 ; XI = 11, etc.

Icosahedron : The fifth regular solid, with 20 equal equilateral triangle faces, 12 corners and 30 edges.

Identity : An equation which is true for *all* values of the unknown. Example :- $x^3-1 = (x-1)(x^2 + x + 1)$

Imaginary : Not existing in the world of measurable quantities, $\sqrt{-1}$, for example.

In-circle : The circle inscribed in a triangle.

Income Tax : A tax levied directly on Income by the Government through the Commissioners of Income Tax. it is a tax of so many shillings and pence "in the £," and was once 6d. It was thought that this would ruin the country. We have learned different. It is now 10/- in the £. It is subject to several rebates. Gross income becomes nett income after the payment of Income Tax. To make the necessary conversion, multiply by the fraction (£1—amount of tax in the £)/£ 1. Thus, with a tax of 4s. 6d. in the £, the fraction 15s. 6d./£l or 31/40 will make the necessary conversion.

Index, Indices : (Latin, the forefinger). The exponent of a power, indicating the number of factors in the product. Thus, in x^3, the 3 is the index. Indices is the plural.

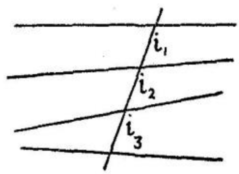
Indirect Tangents : Same as Transverse Tangents, q.v.

Infinity : A place or idea about which most schoolboys and all theologians totally untrained scientifically, presume to know a great deal, but which is stiff a matter of some doubt and difficulty to the mathematician. It is represented by the symbol ∞. At this stage it is difficult to give a cast-iron definition of it. If we think of it as the number of times we can take 0 from any number but 0, we shall not be far out, i.e., $n/0 = \infty$. To say that it is the largest number you can think of is just ridiculous, because anyone can add another one.

Integer : Latin for whole. A whole number as distinct from a mixed number or a fraction.

Interest: See Compound Interest and Simple interest.

Intercept: Literally, the part taken between. Intercepts are the portions of a line cut off by a number of transversals.

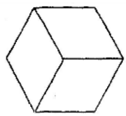


Inverse Proportion : The equality of two ratios, of whi.ch one is the reciprocal of that expected in the normal way. Thus, if 6 men earn £10, then 10 men should earn 10/6 of £10, or £100/6 or £16 2/3. But, if 6 men have enough food for 10 days, the same amount of food would last 10 men 6/10 times as long, or 6 days. This unexpected topsy-turvydom occurs in most problems which involve consumption.

Involution : The process of finding Powers.

Irrational : A quantity which cannot be expressed as a ratio between two integral numbers. It can be known only approximately—but to any degree of accuracy required The square root of any prime number greater than 1 is irrational. So are Є and π.

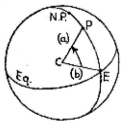
Isometric : Of equal measure. A method of making projections, in which the plan of a cube appears as a regular hexagon and all faces look alike, and all edges of equal length.



Isosceles : Greek for equal legs. Used in connection with triangles and trapeziums, implies that these figures have two equal sides. In the case of the trapezium, the sides are *not* the parallel ones.

L : 50 in Roman numerals. XL = 40 ; CX = 60 ; LXX = 70 ; LXXX = 80.

Latitude : For any point on the earth, the angle between (a) the line joining the point to the centre of the earth, and (*b*) the line joining the point on the equator due north or south of it, to the centre of the earth. Such a latitude is said to be N. or S. according to which hemisphere it lies in. The North Pole is 90°N., all points on the Equator are 0°, and the South Pole is 90°S.



Lb. or lb.: Originally crossed in the same manner as the £ sign for money, is a contraction for librum or libra, pound or pounds and is reserved for use in weights.

Least Common Denominator : The L.C.M. of several denominators, which must be found before fracti6ns may be added or subtracted.

Least Common Multiple : Exactly what the words imply. These should be developed from factors, when they are simple to understand. In example given for H.C.F., q.v., the L.C.M. is $2.3^3. 5. 7. 11 = 6930$, the continued product of the highest number of factors in every column.

Leibnitz, 1646-1716 : A brilliant youth, who, by the age of twelve had taught himself to read Latin with ease and was then studying Greek. Before he was twenty he had mastered all the available textbooks on mathematics, philosophy, theology and law. His claim to fame as the co-discoverer of the Calculus is now recognised.

Length : The first dimension, in comparison with area, the second, and volume, the third. A specified length, such as a mile or a centimetre may be a well-known unit as these are, or an odd quantity like 3 ml., 5 fur., 2 poles, 4 yds. To comprehend this fully we need to know how many of some standard unit of length it is necessary to take to measure the distance from A to B along some prescribed track 3m. 5f. 2p. 4y. long. In this case it might be 6,395 yds. or, not quite so comprehendable, 3.63352 . . . miles. The English unit is the statute mile for large distances and the yard, 1 /1760 of a mile, for short distances. The metric unit is the metre, q.v.

Level : The idea involved in the spirit-level. A straight line, the ends of which are equidistant from the centre of the earth. A plane, on which every line drawn, fulfils this condition.

Limit : A word that has a vague meaning in the phrase : "He's the limit." This implies that no additional indulgence (in vice or virtue, buffoonery or what you will) would produce any sensible alteration in the state (of vice, etc.) already attained. This is, when defined adequately, what limit means in mathematics. For example, if to 1 we add to ½, to 1½ to ¼, to 1¾ ⅛ and proceed thus to obtain as much of the infinite series

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \ldots$$

as we please, we know that time and patience will not let us complete the total—yet we know that the total is 2. It can never be reached by adding a finite number of terms but is known as the limit to which the total grows as the number of terms is increased indefinitely. We may say in like phraseology that the limiting value of tangent x as x approaches 90° is oo. (A more accurate definition lies in the domain of higher mathematics).

Try to understand that most good mathematical work at this stage is limited largely by the dexterity of your fingers when handling instruments and in later life most of your work will be limited in its usefulness by your skill in handling the appropriate tools and machines.

A good exercise in limits is to apply to the Compound Interest formula (q.v.) an ever-increasing number of payments of ever-decreasing amount to keep the total annual rate constant and to discover thus the law of natural growth. Another example : The quotient $(X^n — 1)/(X — 1)$ becomes more and more equal to n as X is made more and more equal to 1. The proof involves use of the Binomial Theorem.

Line : The path traced by a point moving in any manner. A line is said to be straight if no shorter line can be drawn between its extremities. (Note that the straight line is not the shortest *distance* between two points, a confusing statement made frequently).

Literal Equation : An equation of a more general nature than usual, having coefficients of no specified values. Thus the literal quadratic expression $ax^2 + bx + c$ will be zero if $x = (-b + \sqrt{b^2 — 4ac})/2a$, or if

$x = (-b + \sqrt{(b^2 — 4ac)})/2a$, and this covers all cases, for all values of a, b and c.

Litre : The metric quantity measure corresponding in use to our pint. It measures the amount of liquids or materials measured as liquids which will fill a volume of 1 cub. dm. or 1,000 ccs. It is the equivalent of 1.76 pints nearly.

Locus, Loci : The complete path traced by a point constrained to move in a given way. It may be an almost useless path derived from plotting points from an involved equation, or it may be some extremely useful curve like one of the conic sections or one of the cycloids, q.v.

Logarithm : If $B^L = N$, then L is the-logarithm of N to the base B. Thus, since $2^3 = 8$, 3 is the logarithm of 8 to the base 2. The usual base is 10 (Briggs' logarithms) and then logarithms may be defined as the powers of ten which give the numbers being considered. Very few of these are simple numbers. They are known very accurately— and can be computed to sixteen places of decimals, but the four-figure tables in common use will give an accuracy of one in a thousand. Seven-figure tables, with their one-in-a-hundred-thousand standard of accuracy, are the stand-by of the engineers whose feats include the Mont Cenis tunnel and the Sydney Harbour bridge.

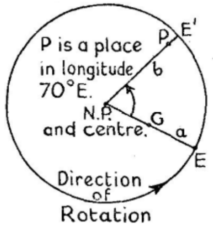
The logarithm is in two parts, the Characteristic and the Mantissa, q.v.

The logarithms of all trigonometrical ratios are tabulated in the same way as those of numbers, and save considerable time ; as, for example :— $(640 \tan 16° \cos 34°)/\pi$

$$=10^{2.8062 + 1.4575 + 1.9186 - 0.4971}$$

$$= 10^{1.6852} = 48.42$$

Longitude : For any point on the earth, the angle between *(a)* a fine joining a point in the Gulf of Guinea, on the Equator and south of Greenwich to the centre of the earth, and *(b)* a line joining the point on the Equator, north or south of the point under consideration, to the centre of the earth. Longitude is east or west, according as the place ies to the left or right of Greenwich, when the earth is viewed from above the North Pole.



Loss : The result of selling for a sum of money less than the cost price. The difference between these buying and selling prices.

£. s. d. : Money signs. Singular : Librum, solidus, denarius. Plural : Libra, solidi, denarii.

M : Roman numeral for 1,000. The highest denomination used. MM = 2.000. If larger numbers were needed a bar was placed over a number and this made its value one-thousand-fold greater.

Magic Squares : While affording to-day a mathematical diversion for the end of term these squares were, five centuries ago, part of the paraphenalia of magic as then practiced. They have no practical value whatsoever. The columns, rows and diagonals have numbers with like totals. Example:-

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

The same number must not be used twice. A greater degree of skill was needed to make a magic cube, and, I suppose, it cost more as a charm and it was alleged to have a correspondingly greater potency.

Magnitude : Any quantity that may be measured—in length, area, volume, weight, time, force, etc.

Mantissa : Latin for makeweight. The fractional part of a logarithm, i.e., the part that is always positive and is the amount added to the characteristic below, to make the exact logarithm. It is necessary to know the mantissas of the range 10 to 100 for these are repeated in the range from 1 to 10 and in the range from 100 to 1,000.

Mathematicians : A teacher of mathematics is not necessarily a mathematician. More the pity—for most people's loathing for "maths." is a direct result of having been taught this most fascinating subject by someone doing bits of this and bits of that to fill out a time-table. A mathematician also can be a very indifferent teacher. Euclid and Pythagoras were probably both teachers of merit and mathematicians of the first water. A mathematician's fame may rest on one brilliant contribution to the science or on the abundance of work of a high order which clarifies what has gone before and paves the way for the work of those who follow. The contributions of a dozen or so of the greatest mathematicians of all times are mentioned among these definitions. There are tens of thousands of others of equal or lesser fame. The work and often hazardous lives of many of them can be followed in W. W. R. Ball's " Short History of Mathematics."

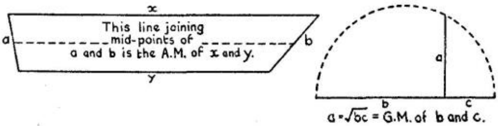
Mathematics : This word is almost alike in English, Latin and Greek. It arose originally from a Greek word meaning "to learn." Mathematics and learnings have always been closely associated. The basic and most fundamental branch of SCIENCE, it covers every activity of man as the *calculating animal*. It is generally thought of in two great divisions—pure mathematics and applied.

Pure mathematics falls into two parts, Analysis and Geometry. Analysis covers the subjects Arithmetic, Algebra, Trigonometry, Calculus, and all has grown steadily around the idea of natural counting and that 1 + 1=2. Geometry takes space for its province and the simpler plane geometry of the first few books of Euclid is only one aspect of the larger subject.

Applied mathematics is what its name suggests. Without the most abstruse mathematics little headway would have been made in the sciences of Dynamics, Hydrostatics ; the physics of Heat, Light, Sound, Magnetism and Electricity ; Astronomy. In a humbler but no less dispensible way it is handmaiden to History, Geography, Geology ; commerce. Statistical and actuarial science is ninety-five per cent, numerical calculation.

Maximum : Biggest. A maximum point is a point on a curve, on both sides of which the points on the curve have shorter ordinates. Cf. Minimum.

Mean : Either the arithmetic or the geometric average of two quantities. The A.M. is one-half of the sum ; the G.M. is the square root of the product of the two quantities.



Mechanics : The branch of Mathematics dealing with movement, and the forces acting on related bodies. This subject provides the student of pure mathematics with scores of fictitious, idealised problems.

Median : A line in a triangle joining a corner point to the middle point of the opposite side. There are three in every triangle.

Mensuration : Measureation, in fact, especially of lengths, areas and volumes and formulae for these for all regular lines, surfaces and solids. The calculation of these quantities for irregular lines, areas and volumes can be effected only by means of the Calculus.

Metre : The length discovered in the effort to find a scientific measurement. A committee of scientists set up at the time of the French Revolution divided a quadrant of the great circle through the Poles and Paris, decimally, into ten million parts. One part was called a METRE, and was a length of approximately 39.37 inches. It is now no longer what it was as the length of the quadrant was not measured exactly. It is now— like the yard—the distance between fine lines marked on a bar of gold under fixed climatic conditions.

Metric System : The French and now the scientific system of measurements based on the decimal system allied to the metre, gram, litre and seconds units adopted by a committee set up by the revolutionaries in France. It is referred to sometimes as the c.g.s. (centimetre-gram-second) system.

Minute : This is the same word as the adjective minute. It was probably as indefinite in quantity as *a minute portion* when first used. It is now precisely one-sixtieth of (a)an hour of time, (b) a degree of revolution.

Minimum : Smallest. A minimum point is a point on a curve, on both sides of which, the points on the curve have longer ordinates. Cf. Maximum.

Money : English units of money ; the Pound sterling (£), the Shilling (£1/20), the Penny (£1/240), and the Farthing.(£1/960). Multiples and fractions of these make the convenience of Exchange going under the general title of Money. There are twelve coins in all—in 1944—three having been withdrawn from circulation, and one is almost obsolete. There must be a score of other money systems in use in the world. The ones with which it is necessary to be acquainted are ;—

American, U.S.A., Dollar ($) and Cent (1&/100),
French, Franc (F) and Centime (1F/100).
German, Mark (M) and Pfennig (1M/12).

The numbers of dollars, francs and marks exchangeable for £1 are so variable that it is useless giving them for any one time, in the hope they would be applicable at another.

Month : From the Anglo-Saxon Monath from mona, the moon. The average time interval from one new moon to another—about 29¼days. There was a new moon in 1939 on November 11th at 7.54 a.m. The previous one was on October 12th at 8.30 p.m. The time interval was 29 days 11 hrs. 24 mins. Between the two new moons in February and March the time interval was 29 days 17 hrs. 21 mins. The June-July interval was 29 days 7 hrs. 26 mins.

Multiplication : The act of "folding many times," giving a product for the answer. It is really repeated addition, and when memorised in the " --------times table" chants, is a tremendous economiser of time. To know that seven-nines-are-sixty-three means that, in future,

$9 + 9 + 9 + 9 + 9 + 9$ does not have to be "worked out" with coloured beads, counters, and what-not. We know it is 63 always.

Mumbo-jumbo : The incantations of savage tribes to their gods. There is a great deal left in mathematics as taught by very select private "Public" schools, the primary and secondary schools, the colleges and universities. It is a bad mixture of (1) badly digested mathematical fact, (2) the failure to express laws in readable English, (3) the readiness of most folk to do anything which " delivers the goods," however untidy, uneconomic, or stupid the process may be. Examples, which you should dear up for yourselves, are :—

"Take it over to the other side and change the sign."

"Multiply (or *divide)* the numerator and the denominator by n."

"Cross multiply."

"Move the decimal point."

"Turn upside down and multiply."

"Bar one " (in logarithms).

"Reduce 1 ton " to, say, 20 cwt.

"30 knots per hour."

"Two minuses make a plus."

Napier, 1550-1617 : Main business in life—political and religious controversy! Mathematics and science were his favourite diversions. The inventor of Logarithms.

Naperian Logarithms : The *original* logarithms computed by Napier to the base e. This base, 2.7102816285 ....is of tremendous value and ranks with π as the most important of constants. These logarithms have little value in decimal calculations, and "common" logarithms were computed from them by Briggs, a close friend of Napier. These are obtained by multiplying the Naperian logs by the constant 0.43429 .... Note the use of "common" ; it might just as well have been "*vulgar*" in a different age.

Newton, 1642-1727: The greatest English name in Mathematics. "The extraordinary abilities of N. enabled him within a few years to perfect the more elementary . . . processes and to distinctly advance every branch of mathematical science then studied as well as to create some new subjects." (W. W. R. Ball). Name is associated with discoveries in optics, gravitational laws, series. Co-discoverer of the methods of the Calculus with Leibnitz. Wrote major work : Principia.

Negative : The opposite of Positive, q.v.

Nine Points Circle : The circle which passes through

(1) three feet of the perpendiculars of a triangle,

(2) three middle points of the sides of a triangle,

(3) three points on the perpendiculars mid-way between the orthocentre of the triangle and the vertices.

Normal : The line drawn across a curve at right angles to the tangent, the curve, the tangent and the normal having one common point. In a circle the radii are normals. In an ellipse the normals bisect the angles between the focal distances lines. (See third definition of Ellipse). The joint lines in timber and masonry arches are, in good work, normal lines to the curves.

North : The primary idea of direction on the face of the earth. It was developed in the hemisphere early civilization occupied. Billions of tons of matter spin slowly in space, about the earth's axis. (Slow and fast are relative to one another ; points on the equator move at 1,000 miles per hour, points near the poles make their journey round the poles each day). This mass obeys the same laws as those which control a top spinning under the lashings of a whip. It spins in three dimensions and has three distinct movements, of which the predominant one is the rotation. In the immensity of space the stars do not rise and set when seen from the ends of this axis about which the earth rotates. It is these ends of the axis which are called the Poles. The one nearest to us in the British Isles is called the North Pole. A direction is said to be North if, by following it unswervingly, we arrive at the North Pole. It can be found by finding (a) the North Star, which is directly over the North Pole (at night-time), or (b) the direction of the sun at mid-day, and reversing this.

Nothing, Nought: See Zero.

Number : One of the oldest senses man possesses, and distinct from the mental process of counting. One does not count in deciding that three articles have been substituted for two. Number now means that abstract total which is different from all others, for example, 13.

Numerals : A number is expressed in writing by a word or a symbol. The Greeks and Hebrews used the first letters of their alphabets for units, the next letters for tens and the next for hundreds. (Not having enough letters to write 999, the Greeks reintroduced some letters that had dropped out of use). The Romans economised very much, using only I, V, X, L, C, D and M for 1, 5, 10, 50, 100, 500 and 1,000 respectively. The symbols for 5, 50 and 500 were probably adopted from the Etruscians whose culture in that age was securely separated from that of Rome by the Appenines. The folk of those times could not interpret the paradoxes of Zeno because they could do nothing with a series of fractions like

$$I + \frac{I}{X} + \frac{I}{C} + \frac{I}{M} + \cdot\cdot\cdot\cdot\cdot$$

In the first place their symbols had run out. No, the Greeks and Romans made nothing of fractions.

Credit for introducing into Europe probably from India the symbols we now use must go to the Arabs who pushed their culture through North Africa and into Spain in the Middle Ages. There was no word for million in the Anglo-Saxon. War budgets of money and casualties, ideas on inter-stellar space and the delicate intricacies of electrons were pleasures in store for a much later age—ours. There was a Greek, Anaxagoras (circa 500-428 B.C.), who "ventured to assert that the sun was larger than the peninsular portion of Greece," just a few hundred miles across ! No wonder his age was satisfied with 999. The intrinsic value of the Arab notation is the use of positional figures, the adaption in written symbols of the values of the numbers on the wires of the abacus. Finally, the introduction of the symbol for zero, 0, to register the emptiness of a particular wire removed the last obstacle to the use of symbols and writing as against the use of the abacus. "The dawn of nothing," to quote a brilliant Persian astronomer-algebraist-calendar-reformer-poet, marked the deciding victory of the algorists over the abacists.

The decimal notation and all that has meant in calculation would never have arrived in a world where letter numerals prevailed.

Numerator : (Latin—numerus, a number). Anyone or anything which numbers or counts. Especially the number of parts taken, of a whole thing, and is written above the vinculum of a fraction. *All* the figures following the decimal point in a decimal fraction are numerators of fractions with power-of-ten denominators understood.

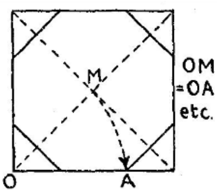
Numerology : A species of witchcraft that attemps to explain character, and destiny, by finding the number-value of names, etc. Now featured in a weekly column of several of our allegedly great " Dailies."

Oblique : Slanting, sloping. Neither perpendicular nor horizontal.

Obtuse : Blunt; greater than a right angle and smaller than a straight angle.

Octahedron : The third regular solid having eight equal equilateral triangular faces, twelve edges and six vertices.

Octagon : An eight-sided plane figure. If it is regular the sides will be equal in length and each angle will be 135°, i.e., 1½ right angles. It is very easily drawn from a square, by marking along each side from each corner, in both directions, a distance equal to half the diagonal.

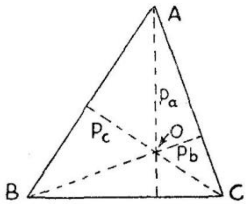


Ordinal : The *order* numbers of counting, as first, second, third, etc., whereby quantities are arranged in order and each assigned a place. The number 16 on a house means 16th house, and is an ordinal and not a cardinal.

Ordinate : A line drawn from a point on a curve parallel to one of the axes. Used In the plural, co-ordinates means both ordinate and abscissa of a point. More precisely the ordinate is the line (and its length) erected at the end of the abscissa line to measure the second variable. (See Cartesian and Co-ordinates).

Origin : From Latin verb—to rise ; hence, the beginning. A fixed point from which measurement is made and especially the point at the intersection of Cartesian axes. Also the pole-point of polar co-ordinates.

Ortho-centre *:* The point of intersection *in* a triangle *of* the three ortho-lines, i.e., the "perpendiculars" of the triangle.



Orthogonal Projection : Projection of an object on to a plan-plane or elevation- plane by means of lines drawn perpendicular to the planes of the plan or elevation. This method of projection gives the picture outline of the object "as seen from infinity" above, or from the front, back, or either side.

Parabola : The conic section formed by cutting the cone by a plane parallel to a generating line.

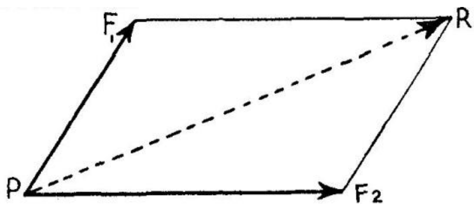
The locus of a point moving at equal distances from a focus and a directrix.

It is the path of flight in vacuuo of a projectile, and has the interesting and highly useful property that rays of light from infinity parallel to the axis converge at the focus, and inversely, rays of light leaving the focus emerge parallel to the axis. This is the basic idea of the searchlight beam.

Parallel : In original Greek—*para allelos*—contracted into parallelos. Para means by *the side of* ; allelos means *one another*. It is well worth studying this translation, for in it is embodied the useful idea of parallelism. It is understood that the distance apart remains the same throughout the section under discussion. In practice, we cannot be expected to know what happens to our parallel lines a million miles away from where we are contemplating them. In modern geometrical language, parallel lines are straight lines which intersect only "at infinity," i.e., at no measurable distance.

Parallelogram : A quadrilateral bounded by two pairs of parallel lines.

Parallelogram of Forces : If two forces act on a body in different directions but at one point, then the parallelogram, completed as in the sketch, gives a means of finding the combined effect of the two forces. It acts along, and is proportional to the length of the diagonal shown and is called the resultant.



Parallelepiped : Parallel and epi and pedos, where *epi* means upon and *pedos* the ground in the Greek language. The solid consisting of parallelogram sides, on a parallelogram base and with a parallelogram top, i.e., it is enclosed by six parallelogramic faces.

Parity, contracted to usual form Par : Equality, used in connexion with stocks and shares, when the buying or selling price is the original price—as shown on the share certificates, i.e., £100 share costs once more £100, or when £7,699 of stock realises £7,699 cash on sale.

Pedal Triangle : The triangle formed by joining the feet of the perpendiculars in another triangle.

Pentagon : A figure with five angles and sides. If regular, each angle is 108°.

Percentage : The second decimal fraction, *hundreths, may* be referred to as "per cent", short for the Latin, per *centum* (symbol %), "for one hundred." Thus, 17/100 may be thought of as 17%. 3.692 is 369.2%. A percentage is therefore a proportion per cent.

Perfect Numbers : If 2^n — 1 is a prime number, then 2^{n-1} (2n — 1) is called a perfect number. Euclid was interested in these numbers. Each perfect number is equal to the sum of all its factors with the exception of itself. Thus 28 = 14 + 7 + 4 + 2 + 1, and 6 = 3 + 2 + 1. The next smallest perfect number is 496, and it is a profitable exercise to show its perfection. A waste of human endeavour has discovered that the ninth perfect number is one of 37 known digits ! Redolent of the times when the magic properties of 3 and 13 were more important than their primeness.

Perimeter : Greek for "the measure around." Used for ellipses, quadrilaterals, etc., in the same way as circumference is used for circles.

Permutations : The number of *arrangements* of p things, taken from n things, taking into account the number of internal arrangements of the p things is written : ${}^np_p$. This type of grouping is known as a permutation. (Cf. Combinations). It is a more thorough arranging. *Per* means thorough and *mutare* to change. Contrasted values are:-

$${}^{10}P_4 = 10.9.8.7 = 5040$$

$${}^{10}C_4 = (10.9.8.7/1.2.3.4) =270$$

and in the general case,

$${}^{10}Pp = n\,(n-1)\,(n-2)\,(n-3)\ldots\ldots(n-p+1),$$

$${}^nC_c = \frac{n\,(n-1)\,(n-2)\,(n-3)\ldots\ldots(n-c+1)}{1.2.3.4\ldots\ldots\ldots c}$$

" **Pi " or** π : The ratio of the lengths of the circumference and the diameters of a circle. An indeterminate quantity known decimally to hundreds of places (though only the first few have any practical application). In Hebrew times the ratio was thought to be 3. Very roughly it is 3 1/7. 3.14 is good and 3.1416 is as accurate as is needed for the most exact engineering. It is calculated from series to be 3.1-415926536

The vulgar fraction 355/113 agrees with this value as far as the sixth decimal figure.

Ptolemy used the value 3° 8''30" which stood for $3 + 8/60 + 30/(60)^2$, which reduces to 3.1416 an error of less than one in forty thousand.

Plan : The orthogonal projection of an object on a horizontal plane.

Plane : A surface on which any three points lying in one direction lie in a straight line.

Playfair's Axiom : Through a point one line only can be drawn parallel to another line.

Point: -A "that." Consult any dictionary or geometry text-book : *A point is that which ....* An idea originating from the Latin word which means to prick. Hence it is best understood as a practical thing. Theoretically a point merely indicates a position—and lacks all other properties, including size.

The decimal point is a stop, written above the usual position on the line, to mark the change from units to tenths as the eye moves to the right. Example : 3.1416.

Polar Co-ordinates : The system of location of a point, by means of (a) an initial line containing an origin called a pole, (b) a line joining the pole to the point, (c) the angle this line makes with the initial one. The co-ordinates written rΘ are called the radius vector and the vectoral angle.

Polygon : A plane figure with many straight sides. When the figure is regular the names are in order of number of angles, equilateral triangle, square, regular pentagon, hexagon, etc., using Greek numerals throughout. If *n* be the number of sides, and if Θ be the size of an interior angle, n Θ+ 360° = n x 180°, or Θ = 180° (n — 2)/n

Positive : Whenever we can look either of two ways along a road ; whenever we have a choice of having money "in hand " or of going into debt ; when we think of temperature as being "hotter than this " or "colder than that " ; we are compelled to fix some standard from which we make our measurement, and then we are compelled to define one viewpoint in such a way that there can be no doubt that we are talking of it and not the other. One is called positive (from Latin *posit*—I place). The other is then called negative (q.v.). The following positive ideas are now fixed :—

(a) Possession, in contrast with debt,
(b) Altitude above sea-level,
(c) Anticlockwise rotation,
(d) Force of compression, in contrast with force of tension in linkages,
(e) Temperatures, centigrade, higher than that of freezing point of water.

In other subjects, it is possible to fix positive and negative temporarily for the purpose of the problem.

One unfortunate thing in mathematics is that the Latin word for more, i.e., *plus (sign* +). now serves two purposes : it means " add on," and it has been used to indicate the choice of positive. This is the source of all the trouble with signs. The same remarks apply to *less,* i.e., minus (sign —). It is necessary therefore to decide whether + (or —) stands for a verb or an adjective.

Postulate : It is taken for granted among all reasonable people that a straight line can be drawn between any two given points. It is a demand we make upon ourselves that this must be so. *Postulate* is merely the Latin word for (a thing) *demanded.*

Power ; Name given to the force applied to the one end of a pulley rope or one end of a lever, in order to displace the weight at the other.

The exponent, which tells how many equal factors have been multiplied together, to give a final product. The 6 in x^6 means that x.x.x.x.x.x is the product, and it is said to be of the "sixth power."

Practice : The arithmetical process, by which the cost of a number of things is found. If the price of one is known ("Simple Practice"), or the arithmetical process by which the cost of a quantity of material is found, if the price of some unit quantity is known ("Compound Practice").

Also, according to an old proverb, that which *makes perfect.*

Present Worth : The value to-day of a bill due to be paid at some time in the future. It is found as the reverse problem of Compound Interest, q.v.

P.W. = (Value of Bill)/(1 + r/100)n.

Prime : A whole number not divisible without remainder by any other whole number except 1. We may say that it is a whole number that occurs at the right-hand side of the first line of its own "times table," and not at the right-hand side anywhere else. Thus, 1 X 13 = 13. This is the sole appearance of 13 as a product, and on the left are its only factors.

Prism : Have you ever seen a tree-trunk being sawn into logs, or steel rails being sawn into lengths ? If you have, you have a good idea of a prism, for this word meant "sawn-off" to the Greeks. The geometrical definition means two equal parallel ends (the saw-cuts) Joined by straight lines forming parallelograms. In the case of the right prism, the side-faces are rectangular.

Problem : A difficulty that can be unravelled by applying our rules of thought to the facts known.

Product: The result of multiplying together two or more numbers.

Profit and Loss : The difference between the buying price and the selling price of an article (as it passes through one person's hands) is a Profit if the S.P. is greater than the C.P. and a Loss if less.

Progressions : A series of numbers, in which the steps *from* any one to the next obey one or other of three rules of construction :—

(i) In an Arithmetic Progression the steps follow a law of equal increment, i.e., each step is of equal amount ascending or descending. Thus, numerically, 3, 7, 11, 15 , and 6½, 3⅜, ¼, - 2⅞ . . . . are arithmetic progressions. The symbolic form is a. a + d, a + 2d, a + 3d, etc., where a always stands for the first term, and d stands for the common difference. The sum of the first terms from 1 to n is (n/2) (2a + n — 1.d).

(ii) In a Geometric Progression the *steps* follow a law of equal ratio, i.e., each term is a fixed number of times, or a fixed fraction of, the term immediately preceding it.
Thus, numerically, 3, 9, 27, 81 . . . . , 5, 1 2/3, 5/9, 5/27 ....½ , — 1, 2, — 4...........
are geometric progressions. The symbolic form is a, ar, ar^a . . .. where a is the first term, and r the common ratio of change. The sum of the first n terms is then :—

$$a\left(\frac{1 - r^n}{1 - r}\right), \text{ or } a\left(\frac{r^n - 1}{r - 1}\right).$$

(iii) A series is in Harmonic Progression if the reciprocal of every term forms a series in arithmetic progression.

Projection : A throwing forward ; methods of drawing plans and elevations by approved technique. Types are isometric and orthogonal, q.v.

Proof: A satisfactory explanation of a problem, leading to its solution. A proof can be no more satisfactory than (a) the postulates (axioms), from which we start, and (b) the laws of thought we obey. Many of Euclid's axioms are now doubted and the laws of logic we inherit from the Greeks are not the cast-iron affairs they were half a century ago. So the immortal query : What is TRUTH ? is as difficult to answer as it always has been. Perhaps there *is* no answer.

Proportion : This may be Simple, Compound or Inverse, q.v.

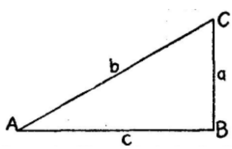
Proposition : A sentence in which something is affirmed or denied. In mathematics it consists of the theorem and the demonstration of its truth.

Protractor : A circular disc on which is marked radii at degree (half degree and quarter degree on larger ones) intervals for the setting out of angles. It was originally the Egyptian clock and calendar. The 360 degrees probably arose from the computed length of the year as 360 days in those remote times.

Pyramid : The Pyramids are found in Egypt, and when we think of a pyramid we think of a solid of the same shape, i.e., having a square base and an apex above it joined to each corner of the square base. The sides are thus triangular. The word now covers all solids with bases bounded by straight lines and having an apex. Tetrahedron is the special name for a triangular pyramid. The exact formula for computing the volume of a pyramid was known to the Egyptian pyramid builders : First find the area of the base, then find the volume of the prism on the same base and as high as the pyramid, then divide by 3.

Pythagoras, circa 569-500 B.C.: Systematised all the mathematics of his day and in his school laid the foundation of rigorous proof on which geometry is now firmly established. He probably first demonstrated the theorem immortalised by Euclid in Elements Bk. 1 47-48, and based on the Egyptian rope-stretchers' triangle. He studied Progressions.

Pythagoras' Theorem : This, the best-known proposition of Geometry, states that the square on the hypotenuse of a right-angled triangle has an area equal to the sum of the areas of the squares on the other two sides, i.e., $b^2 = a^2 + c^2$, in the general triangle if B is 90°



It has a long and interesting history, beginning in the antiquity of the Nile valley and delta. The rope-stretchers knew the fact that a rope of twelve length units, marked off in sections of 5, 4 and 3, would, if held taut, between these points fold round into a perfect right-angled triangle as shown. The extreme accuracy of the priest-built pyramids and temples can be explained thus. This problem intrigued Pythagoras in the sixth century B.C., and he found several other possible positions for the knots in the rope. He discovered the relationship $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$ and probably proved it.

The proof 1 47 in Euclid's "Elements," dealing with the above, is probably one of the few proofs Euclid could claim as his own. (Euclid lived, it is believed, between 330 and 275 B.C.. and was probably educated in Athens).
The Trigonometrical version of this very important idea is $\text{Sin}^2\Theta + \text{Cos}^2\Theta = 1$.

Quadrant: Portion of a circle enclosed by one-quarter of the circumference and two radii at right angles.

Quadratic Expression and Equation : An expression Involving the second and no higher power of the unknown quantity is called a quadratic expression because it is obtained by multiplying together two linear expressions. As *linear* implies the idea of "line," so *quadratic* involves the idea of "surface," and quadratic expressions invariably accompany problems on area.

A quadratic equation is a statement of the equality in area of certain squares and rectangles. We can add and subtract quantities only if these are alike in name and kind ; hence, if we write a quadratic equation, $ax^2 + bx + c = 0$, we mean that *a squares* of side x, at present unknown, together with *b rectangles*, of side x, as length and *unit* width, and c *unit squares* make no area at all. Thus a, *b, c or* x will necessarily be negative, or *a, b, c and* x are zero, which reduces the equation to nonsense.

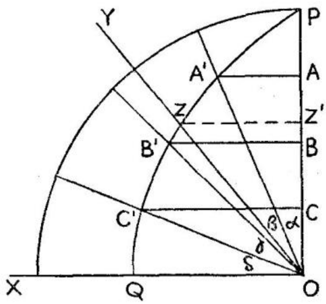
The solution gives a choice of *two* values for x :—

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It is very useful to know from these solutions that if x_1 is one value of x and x_2 the other, then

$$X_1 + x_2 = -b/a,$$
$$X_1X_a = c/a.$$

Quadratrix : A non-Euclidean bit of geometry, the invention of Hippias (circa 420 B.C.), by means of which an angle can be trisected or divided into any two parts.



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A, B, C, are points on the radius of a quadrant and α. β, γ, δ, ... are the corresponding sub-divisions of the right angle. (In the sketch these angles are quarters of a right angle). Through A, B. C, . . ., perpendiculars are drawn to the radius to meet the other radii in A', B' C'. . . respectively. The smooth curve PA'B'C' ... Q is the curve called the quadratrix.

The method of its construction is also the key to its use. Let XOY be the angle to be trisected. Place QO and XO as shown with OY crossing the quadratrix in Z. Find Z' on initial radius. Trisect OZ' (not shown). Draw horizontals through these points to the quadratrix and join points of intersection to 0, These joins trisect the angle.

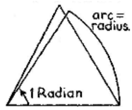
Quadrilateral : Four-sided. A quadrilateral is a figure with four straight line sides.

Quotient : The "answer" to a division "sum."

Radian : The angle unit of higher mathematics and of little use to mathematical tyros. The angle subtended at the centre of a circle by an arc equal in length to the radius. It is independent of the size of the circle. The radius may be marked round the circle six times and there is a portion left. Since the circumference is π x the diameter, the semi-circumference is π' x the radius. Hence, semi-circumference/

radius = π, and π radians fill up a straight angle. Thus, π radians = 180°.
From this we *calculate* :—

$$\begin{aligned} 1 \text{ radian} &= 180°/3.1415926\ldots \\ &= 57.2957795\ldots \text{ degrees} \\ &= 57.3 \text{ degrees approximately.} \\ &= 57° \, 17' \, 44.8062\ldots'' \end{aligned}$$



Radicle : The root sign —√ .

Radius : The distance from the centre of a point on the circumference of a circle.

Radius Vector : One of the two co-ordinates in Polar co-ordinates. Its length and sense are measured from some pole on a plane or in space. (See Polar Co-ordinates).

Rates : The sums of money that burghers have to pay to the Local Authority for the services it renders them in the form of street-lighting, refuse disposal, parks maintenance, police, education, etc. The payment is made on the assessed value of the property. The following proposition is true :—

$$\begin{aligned} \frac{A}{R} &= \frac{\text{Assessment of one property}}{\text{Rateable Value of the Town}} \\ &= \frac{\text{Rates paid for one property}}{\text{Estimated Expenditure of Town}} \\ &= \frac{r}{E} \end{aligned}$$

The calculation of the rote for the year is determined by the fraction :—

$$\frac{\text{Estimated Expenditure of Town}}{\text{Rateable Value of the Town}}$$

which must be equal to :—

$$\frac{\text{Rates to be paid for one property}}{\text{Rateable Value of the property}}$$

This always works out to so many shillings and pence in the £. Thus, if A/R = r/E, as above, A.E = r.R, and E/R = r/A.

Ratio : The relationship of any one quantity, called the antecedent, to another similar quantity, called the consequent. Examples :—A/R, r/E, E/R, r/A, of last paragraph, and the Trigonometrical Ratios, q.v.

Rational : A quantity is so described when it is capable of being written as a ratio the antecedent and consequent of which are whole numbers.

Reciprocal : The reciprocal of a fraction is that fraction formed by exchanging the numerical values of the numerator and denominator in the original fraction. The reciprocal of x/y is y/x ; the reciprocal of 1/a is a ; the reciprocal of p is 1/p.

Rectangle : A parallelogram with right angles.

Recurring Decimal : A decimal fraction which repeats a figure like 0.333 . . . . (called "0.3 recurring " and written 0.3, the value of 1/3), or which repeats a set of figures like 0.142857, the value of 1/7.

Reduction : A lady who indulges in a "reducing course" and thereby "reduces" her weight *from* 11 stones to 154 lbs. would not be unduly elated with success. Yet that is what our arithmetic books call Reduction. We reduce £1 to 20 shillings ! We reduce 36 ins. to a yard !

Obviously, either we are *not* reducing and Reduction is not the *correct* name for it, or we *are* reducing and Reduction means staying put ! (This sentence needs a little careful thought). The truth is that no Reduction problem in the elementary text-books ever reduced anything. We just find equivalents. A sign for equals cannot lie between two quantities if one has been reduced.

Here, then, is what is meant by the subject of Reduction but not by the word itself ; the equating of one quantity to another having a larger number of smaller parts or a smaller number of larger parts in its composition.

Examples : £3 ≡ 24 halfcrowns ; 16s. £ 192d. ; 96 lbs. ≡ 6 st. 12 lbs. The triple bar sign is used for congruency between triangles and may be used also for equivalencies between other kinds of quantities.

Reflex Angle : An angle greater than 180° and less than 360°.

Regular Solid : There are only six regular solids :—

1.The regular tetrahedron.	4. The regular dodecahedron.
2.The cube,	5. The regular icosahedron.
3.The regular octahedron.	6. The sphere.

If P be the number of points or corners of a regular solid, F the number of faces, and E the number of edges, then for the five rectilinear solids

$$P+F=E+2$$

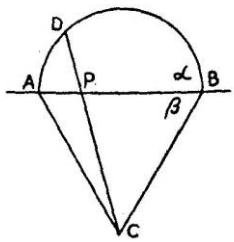
This formula was discovered by L. Euler (1707-1783).

Remainder Theorem : This provides us with a simple means of calculating the remainder when a function of some unknown quantity is divided by a linear expression of the same unknown. The remainder is of identical form with the dividend except — *(or* the unknown is written the opposite of the known term in the divisor. Example 1 :— $(3x^2 + 2x + 4) \div (x + 5) = 3x - 13 + 69/(x + 5)$. This 69 remainder is $3(-5)^2 + 2(-5) + 4$. Example 2 :—$(ax^2 + bx + c) \div (x - p) = ax + (b + pa) + (ap^2 + pb + c)/(x - p)$.

The particular value of this theorem is that it enables us to discover linear factors of expressions of higher powers by establishing the fact that there is no remainder when they are divided by the linear expression.

Renaldinus : The name of this 16th century monk is associated with one discovery— the method of drawing regular polygons in circles.

Let α be a semi-circle and β an equilateral triangle, having line AB as diameter and one side respectively. Let the diameter be divided into any number of equal parts, n. and let P be the second point of sub-division from one end. so that AP/AB = 2/n. If the line CP produced intersects circumference of semi-circle in D. and AD is joined, then AD is the length of the side of a polygon of n sides that can be inscribed in the full circle. The vertices so found very closely approximate those of the true polygon constructed by Euclidean methods, if possible.



Revolution : 360° The largest angle that can be *drawn* is one that falls short of a revolution by some exceedingly small amount. It is then the largest possible reflex angle, and a revolution is the limit to its size.

The completion of a circuit, round some centre or focus.

Rhombus : A quadrilateral with all sides of the same length.

Rhombohedron : The parallelepiped with all faces rhombuses.

Rider : A problem bearing on the main proposition and generally considered at the same time. Thus, from the proposition : *All angles in* the some segment ore equal to *half the angle at the centre,* follows the rider *The angle in a semi-circle is a right angle.*

Right Angle : 90°. Quarter of a revolution.

Rise : The difference between two levels. The perpendicular in the standard right-angled triangle.

Rope Stretchers : See Pythagoras' Theorem.

Root : A number which, when multiplied by itself the necessary number of times, produces a given number. The same *word,* as that used for the members below ground, which join up to produce the trunk above.

$\sqrt{3} \times \sqrt{3} = 3$. $1.732\ldots \times 1.732 \ldots = 3$. $\therefore$ $\sqrt{3} = 1.732$

The square root is written without a number, to show it *is* the square root, as above. (1.732 ... is the "side" of a square of "size" 3). Higher types of root are indicated by a number in front of the radicle:-

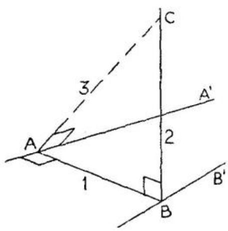
$$3\sqrt{7} \times 3\sqrt{7} \times 3\sqrt{7} = 7 = 1.913 \times 1.193 \times 1.193$$

$$\therefore 3\sqrt{7} = 1.193$$

Roots of an Equation : The value(s) of the unknown(s) in an equation which make the total value of all the terms zero. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then it is known that $\{-b \sqrt{(b^2 -4ac)}\} /2a$ are the values of α and β and the roots are related as follows :— $\alpha + \beta = -b/a$; $\alpha\beta = c/a$.

Rotation : Turning on an axis as contrasted with passing round an orbit. Is positive if counterclockwise when observed from above. The hands of a clock therefore rotate in a negative direction. (Why ? See the entry under Longitude).

Rule of Three Perpendiculars : One of the fundamental ideas of solid geometry and trigonometry of three dimensions. Let AA' be any straight line. Let AB be any line at right angles to AA' at A. Let BC be any line at right angles to AB and the *plane* BAA' at 8. Let C be any point in BC. Then a fine joining C to A is at right angles *to* the original line AA'. The line BB' is added to give the impression of two kerbstone lines in a well-made street and BC the line of an upright telegraph pole. The proof of this is a simple exercise in the congruency of three sets of triangles.



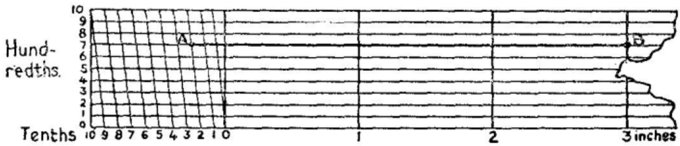
Scaler : See Vector.

Scale : The ratio of the size of a drawing to the size of the object drawn. Linear dimensions are used for the ratio, so that an object drawn to a scale of 1/12 will be 12 times as long, broad and deep as the drawing ; 144 times as big in area, and 1,728 times as big in volume and weight, as a scale model made of the same material.

A map scale of 320 miles to the inch is sometimes given R.F. = 1:20,000,000. This R.F., for representative fraction, is a very approximate ratio ; in truth it is 1:20,275,200. Work this out for yourself.

A scale is also the name for a ruler, used by draughtsmen.

A *diagonal scale* is a device by means of which second sub-divisions of the first subdivisions of a ruler may be shown accurately. If a ruler shows inches and tenths, it is not easy to pick off by means of dividers a length of, say, 3.27 inches. The 0.07 portion would be guessed as being “a little more than half-way between 3.20 and 3.30.” By using the seventh transversal of a triangle whose side had been divided into ten equal parts, we get seven-tenths of the base, and if the base is 0.1 inch long, the seventh transversal will be 0. 07 exactly. The diagram explains itself. The line AB is 3.27 inches long.



Scalene : Crooked. Having three unequal sides. This definition might apply to a right-angled triangle, but such a triangle would not be referred to as scalene. Scalene is generally meant to describe all triangles other than isosceles, equilateral, and right- angled.

Science : This word derives from Latin *scientia*, knowledge. It is a little more than that. It covers every branch of knowledge and mental activity like chemistry, geology, physics, zoology and in fact all the sciences which, for their proper understanding, find mathematical calculation necessary.

Secant : A straight line which cuts across a curve in one or more points. If the straight line and curve have only two common points the part of the segment between the points is a chord of the curve.

A trigonometrical ratio : Hypotenuse over side against, or slope over level, q.v.

Second : A secondary minute portion of time or revolution—one-sixtieth of one of the first minute portions ; therefore, 1/3600 of an hour or degree.

Section : The plane area exposed in a solid object, when it is cut across by a plane surface.

Sector : The portion of a circle included between two radii and an arc joining their ends.

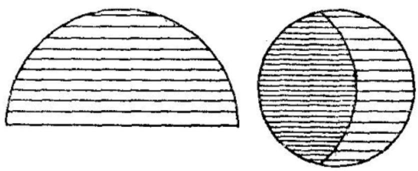
The portion in an ellipse included between two lines joining points on the ellipse to a focus, and the portion of curve joining their ends.

Segment : The portion of any area bounded by an arc and the chord joining its ends.

Selling Price : The price for which a salesman parts with goods. Note that one person's S.P. is some other person’s Buying Price.

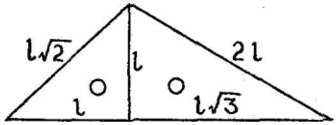
Semi-circle : Half a circle, usually bounded by half the circumference and a diameter. Until you have hours of leisure time after studying mathematics for many more years, do not attempt to find how much rope a donkey may be allowed if tethered to a point on the circumference of a circular field so that he is able to graze over half the grass.

That semi-circle is a real problem.



Series : Any sequence of numbers, such that any term is obtained from its predecessor by prescribed rules.

Set-squares : Instruments giving two lines at right angles for establishing, say, a corner to a square. The hypotenuse of these wooden or celluloid right-angled triangles is always one or other of two patterns giving (a) half an equilateral triangle (angles 30°, 60°, 90°), (b) an isosceles triangle (angles 45°, 45°, 90°). The relative lengths of the sides cannot be too well known as they are the basis of much elementary work in trigonometry. If the two squares have a short side of equal length. l, then the "45" has a second side of length, l and the "60 " a second side of length, 21, as shown. The remaining sides are then l.√2 in the "45" and l.√3in the "60." The trigonometrical ratios for 30°, 45° and 60°, eighteen in all, can be calculated from these facts.



Shares : One type of holding in the Capital of a Company. (See Stocks).

Sieve of Aratosthenes : A device advanced by a contemporary of Archimedes and Apollonius, who was, among other things, Librarian at Alexandria. Suppose all the numbers from 1 upwards are written down in order. Starting with 4, cancel every number that is even, i.e., every alternate number ; similarly cancel every third number from 3 onwards ; leaving 5 cancel every number not already cancelled if it ends in 5 or 0. It is not necessary to deal with any first number greater than the square root of the highest number in the original list :—

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

This work will eliminate all but the prime numbers less than 100. W. W. R. Ball has stated that 300 hours of work would be needed by this method to find all the prime numbers less than a million.

Signs : Marks of approved design which indicate definite operations : plus minus -, the multiplication and division signs, x, ÷ ; the radicle, √, etc.

Significant Figures : A number of significant figures is always mentioned, for example 4 significant figures. This means that, whatever the result obtained from the data of the problem, the fifth figure in the series found, it may be the 6 in 128.9657 and all of lesser value need not be quoted. (Their accuracy is doubtful—not because they have not been found accurately, but because the data is only approximate). The answer expected would be 128.9. This is not the same thing as "giving the answer correct to four figures," as in that case it would be 129.0. When the number begins with zeros after the decimal point, these are not counted as significant figures.

Similar Triangles, etc. : Geometrical figures are similar if their corresponding angles are equal and all corresponding sides are in the same proportion.

Similarly : This word introduces a step in a proof, and implies that much in the way of repetition has been omitted. It must be used sparingly. It is better to say something unimportant in a proof twice than to omit something important.

Simple Interest: The arrangement made between borrower and lender, whereby the tribute paid for the loan is paid at intervals. If the units are £100, 1 year, and 1%, then the Simple Interest will be £1. This £1 varies, in simple direct proportion with the principal, the time and the rate, i.e., in compound direct proportion with all three. Thus, for a principal of £P, borrowed over n years at a rate of r% p.a., the Simple Interest is £1 X £P/£100 x n yr /1 yr. x r% /1%, or as it is usually written after simplification.

S.I. = £Prn/100.

Simple Practice : The method by means of which we find the cost of a number of things when we know the price of one.

Simple Proportion : The equality of two ratios gives four quantities in Simple Proportion : as $^{a}/b = {}^{e}/d$. If any three of these quantities are known the fourth can be found thus :— 5

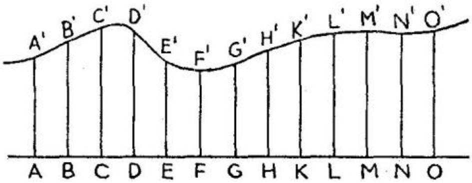
$a = b.^{c}/d \qquad c = d.^{a}/b$

$b = a.^{d}/c \qquad d = c.^{b}/a$

Simpson's Rule : This formula, published in 1743, by Thomas Simpson (1710-61), gives an approximate value for an area or a volume when the data is limited.

Let ABC . . . . O be some straight line, AA', OO' perpendiculars at its extremities which intersect, *once only*, a curve A'B'C' ... O'. The whole area must be divided into an even number of equal sections. Then the area AA'O'O is given by :—

$$\frac{1}{3} AB \; [AA' + OO' + 4\,(BB' + DD' + FF' + \ldots) + 2\,(CC' + EE' + GG' + \ldots)]$$



If the distance between any two ordinates is h, the sum of the first and last ordinates A, the sum of the even-numbered ordinates B, and the sum of the odd-numbered ordinates C, then the formula as met with usually is :—

Area = h/i . (A + 4B + 2C).

If for the lengths AA', BB', CC' . . . . OO', we have plane cross-sections of some solid having areas α, β, γ.......... .... and the distance between the.sections is h, the formula is :— Volume = $h/_3$. (A 4- 4B + 2C), where A is the sum of the end faces, B the sum of the areas of section occupying even positions, and C the sum of the areas occupying odd positions.

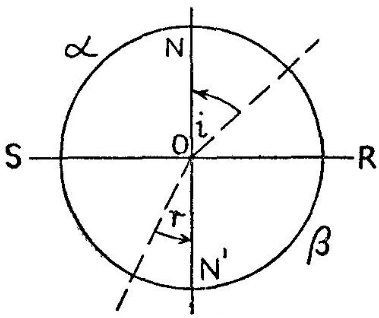
The formula may be applied to rectilinear solids, by making only one cross-section midway between the two ends.

Simultaneous : " Belonging to both together," literally.

Sine Ratio : The ratio of side opposite (perpendicular, rise) to the hypotenuse (slope).

Sine Formula : if triangle AcBaCb is circumscribed by a circle of diameter d, then a/SinA = b/SinB = c/SinC = d.

Sine Rule in Refraction : O is a point in a surface common to two media α and β . It is intersected by a vertical circle with centre at O. The broken line shows a ray of light passing from medium α to medium β . NON' is a normal to the surface. Angle i is called the angle of incidence and angle r the angle of refraction. Sine i / sin r is a constant for any two specified media and is called the refractive index between the two media.



Slide-rule : This is neither a slide nor a rule, nor yet a ruler. It is a device for mechanically adding together or separating into any two parts the mantissae of logarithms. (A dozen “extras" are given with more expensive types of rule, adapted for special purposes). The numbers marked on the sliding portions are the “anti-logarithms,” and the points at which they are placed are measured in terms of the logarithm mantissae. Very simply : 2x3 = 6 appears on the “ slide” because 0.3010300 + 0.4771213 = 0.7781513 and the points marked 2, 3 and 6 are placed at 0.301 . . . k in., 0.477 ... k in. and 0.7781 ... k in. as accurately as possible from the end of the rule, the constant k determining the length of the ruler and in consequence its value from the point of view of accurate use.

Slope : The direction which, with level and rise, gives a practical right-angled triangle.

The slope forms the hypotenuse.

Solid : A word used loosely to indicate anything, liquid, solid or gas, which occupies space.

Solid Geometry : What the name implies—geometry of three dimensions, an advance on the geometry of plane surfaces in both difficulty and practical usefulness. The solid geometry of regular solids with straight line edges is not beyond the understanding of a senior student, but the solid geometry of most curved surfaces involves a considerable knowledge of the Calculus.

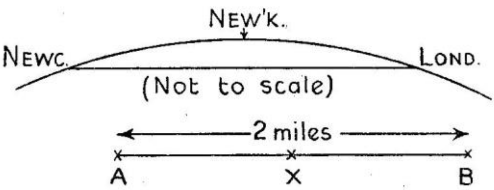
Solidus : The sloping dividing line between numerator and denominator in a simple fraction, thus π = 355/113. Needs to be used very sparingly and carefully to distinguish it from figure 1.

Solution : The whole process of analysis of the data of a problem, and synthesis of the facts which produces the answer. Sometimes we use the word loosely for the answer only.

Space : The whole or any portion of the continuous extension in three mutually perpendicular directions in which we and the universe exist. (Take this bit at a time— for safety).

Sphere : The sixth and last regular solid. It is formed by the rotation of a circular plan about one of its diameters. It has area πd^2 and volume $\pi d^3/6$ in terms of its diameter.

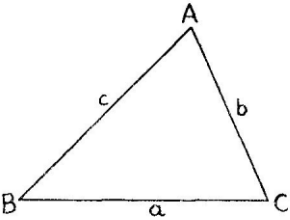
Spherical Trigonometry : The trigonometry of triangles, the sides of which are arcs of great circles on a spherical surface. An essential study for intending map-makers, navigators. Surveyors do not need the refinements introduced on plane trigonometry unless the territory they cover is very considerable in extent. Newark, almost at sea- level on the river Trent, is somewhere near midway between London and Newcastle, both at sea-level. A chord of the great circle of the earth through London and Newcastle passes about 2 miles below Newark, which lies on the arc. If two corks A and B are two miles apart on a straight length of canal or in a line on the sea coast, then a cork at X, midway between them, stands about 8 inches above the straight line AB.



Square : The quadrilateral with the largest number of special properties four equal sides, four right angles, equal diagonals, four lines of symmetry.

Square Inch, etc.—A unit of area equal to the area of a square of edge one inch, etc.

Standard Triangle : The nomenclature of the triangle is shown. Capital letters for points, in anti-clockwise rotation and small letters for sides ; a opposite A, etc. For preliminary work with the standard right-angled triangle it is best to have the right angle at B, or C.



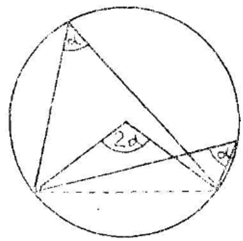
Straight Angle : Half a revolution or two right angles ; 180°.

Straight Line : That line which passes directly between its extremities in such a way that its length is a minimum.

In Cartesian co-ordinates, its equation is $y = mx + n$, where m is the tangent of the angle of slope of the line and n the length of the intercept between the origin and the point where the line crosses the y-axis. An alternative form is $ax + by + c = 0$, where a, b and c are constants convertible into "m" and "n" constants by simple transformations. (Sec Transposition of Terms).

Style : It is n by-word among comedians that doctors (men of medicine) cannot or will not write legibly. In the long experience of the writer only two mathematical scribblers have been encountered. One was a boy fellow student, one a student he helped part of the way towards his finals. Both are successful. But how they conveyed to the examiners their prowess is quite beyond understanding. In the exactest of all human activity style, neatness, lay-out, order, call it what you will, is nine-tenths of the battle. Fifty points on style could be mentioned but two will have to suffice : (i) the fulcrum of a balance is un-moved whatever the transaction, and the = sign in a problem should be kept in the most rigid possible vertical column : (ii) the decimal point separates units from tenths—it, too, should be as far as possible in a vertical column and not allowed to wander at will.

Subtend : "To extend under," or to be opposite, in the sense that an angle at the ccntre, or at the circumference is subtended by an arc of the circle.



Subtraction : The action of taking away.

Sum : See Total. An amount of money. The whole process of an arithmetical soliution.

Supplementary : Describes the angle which, added to another, makes a straight angle ; or two angles which total 180°. Note : S for supplementary and straight. Must not be confused with Complementary, q.v.

Surd: (Same root as absurd). Examples: $\sqrt{3}$, $\sqrt{7}$, unending but not recurring decimals. It is not possible to write a surd as the ratio of two whole numbers.

Surface : Exterior part of a solid. Measured in terms of length *and* breadth, a surface has *area* - a unit of the second dimension.

Symbol : A letter standing for a quantity or an object. A for apple, m for mouse. The usual “unknown quantity” is x but this is not very helpful at times. “Let m be the number of men engaged and b the number of boys” is much better than the usual x and y and the continuous doubt as to which stands for which throughout the problem.

Symmetry : The property of being shaped in such a way that-one-half may be folded over to fit exactly on the other. Thus, a square is symmetrical about its middle lines and its diagonals, a circle about every diameter.

Tables : Lists of numbers, references or other items arranged systematically to show their interdependence. Especially the multiplication tables; tables of money, length and weight; logarithms; trigonometrical ratios.

Tangent : (From Latin tangere, to touch). The position taken up by a secant (q.v.), when the intercepted chord is no longer a measurable quantity. The two points of intersection are so close together as to be indistinguishable but *there are* two *always*. The line at right angles to a radius at its end remote from the centre of the circle.

A trigonometrical ratio, p/b, “rise”/ “level”.

Taxes : The moneys raised by the governments of the day, by adding a sum to the price of a commodity. Thus, in buying a gallon of petrol, for example, the two shillings paid goes in about equal parts to the government, *direct,* and to the petrol combine to cover their costs and profits. They range over almost every commodity purchased. The *direct* tax, in contrast to these *indirect taxes*, is called the Income Tax, q.v., and a tax levied on fortunes left is called the Death Duties.

Teens : The numbers from 13 to 19. Eleven and twelve do not fall into this classification because they are based on the dozen principle of counting.

Tetrahedron : The simplest solid. (No portion of space may be bounded by less than four plane surfaces). It has 4faces, 6 edges and 4 corners, if the faces are equilateral triangles it is one of the regular solids, q.v.

Thales, circa 640-550 B.C. : One of the Seven Sages of Greece. Founder of the earliest mathematical school, A geometrician and astronomer.

Theorem : Greek for “behold,” and is the first word used in bringing to the notice of students some truth needing proof. One British people would translate it, “ Look-you."

Time : The continuous sequence of events. It is axiomatic that equal amounts of a continuous activity, like a swing of a pendulum or a flow in a river or the spin of a flywheel, take place in equal portions of time.

Total : The result of adding together a number of quantities of the same name. The *sum*.

Transposition of Terms : Usually performed, without understanding by the first formula in the entry : Mumbo-jumbo.

The re-arrangement of the terms in an equation, by which an item appearing on one side is caused to appear on the other (after disappearing from its original position), without alteration in the truth of the equation. It is performed *only* in one or other of the following four ways, all of which depend upon axiomatic rules of reasoning :—

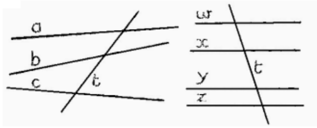
(i) Equal quantities may be added to both sides m an attempt to eliminate a negative quantity from either side of the equation. *

(ii) Equal quantities may be subtracted from both sides to eliminate a positive quantity from one side.

(iii) Each si-de may be multiplied by the same number—thereby eliminating a denominator from one side, and causing this quantity to appear in the numerator on the other side.

(iv) An equal fraction of both sides may be taken, thus causing a numerator on one side to disappear and to re-appear in the opposite denominator.

Transversal : The line which crosses other lines, and especially one which crosses parallel *lines* as shown.



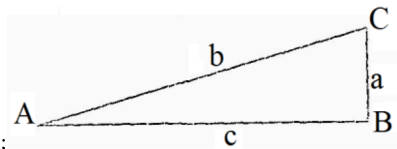
Transverse Tangents ; The two common tangents to two circles which meet the line of centres at a point between the centres. There are no transverse tangents to intersecting circles.

Trapezium *:* A quadrilateral with *one* pair of parallel sides, in an isosceles trapezium the non-parallel sides are equal.

Triangle : The simplest regular area. It is impossible to bound a plane surface by less than three straight lines. Equilateral and isosceles are used to describe triangles in which the three sides and two sides only are equal.

Triangulation : The breaking up of a surface into a collection of triangles for the better study of its area.

Trigonometrical Ratios : These are three *in* number for any angle, and these and their reciprocals make the six sets of ratios listed in books *of* tables. The ratios are :— Sine, Tangent and Secant, with their reciprocals Cosecant, Cotangent and Cosine. If we postulate a standard right-angled triangle as shown in which b is the hypotenuse and B the right the ratios are, for the angle A,

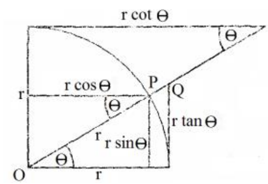


Sine A =a/b ; Tangent A = a/c ; Secant A = b/c ;

a

Cosecant A — b/a ; Cotangent A = c/a ; Cosine A = c/b.

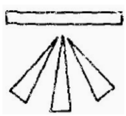
If c and a are the plan and elevation of a radius of a circle, as shown, then six *lengths* proportional to the ratios are easily found as shown—four marked on the diagram and OQ - rSecΘ and OR = rCscΘ



Three ratios of the six are “co” -ratios. This implies that the ratio of one angle is the *co-ratio* of the complementary angle. (Note the two co’s). Examples :—

Sin 60° = Cos 30° ; Tan 82° = Cot 8° ; Sec 41° = Csc 49°,

Trigonometrical Survey : The planning and mapping of a country by means of triangulation and the application to the triangles of trigonometrical formulae for calculating the lengths of the sides, The corner points of the triangles are called trigonometrical stations. The bench-mark (arrowhead and bar), is one of the signs of the survey, the point above the arrowhead and the level bar are known for exact position and height above level.

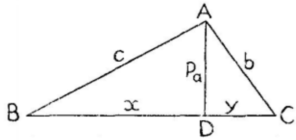


Trigonometry : This word breaks up into parts tri-gon-o-metry. We find in it the following Greek elements :—a part of *treis* meaning three, a part of *gonia*, an angle, the "o" is euphonistic, and a part derived from metres, a measurer or *metria*, a measurement. The subject is thus the complete study of the measurement of triangles, by means of ratios. If a perpendicular is dropped from the vertex where is found the largest angle of the triangle to opposite side, this will always divide the longest side of the triangle at an internal point, it is in terms of the trigonometrical ratios of the angles of the two right angled triangles so formed, and existing between the perpendicular, the parts of the "base'' and the two undivided sides, that the complete measurement is made, in the triangle shown, the perpendicular p_a divides a into two parts, x and y, giving two right- angled triangles ABD, ACD.

$$p = c \text{ Sin } B = b \text{ Sin } C,$$

and $$a = x + y = b \text{ Cos } C + c \text{ Cos } B$$

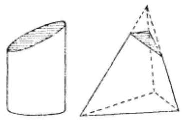
are the equations used to solve triangles. The "Sine Rule" and the "Cosine Rule," q.v., are the remaining methods of attack on problems.



Trisect : To cut into three equal parts. The trisection of an angle by Euclidean means has proved impossible.

Troy : A system of weights measurement, wherein 12 ozs. avoirdupois is called 1 lb. troy. (Maybe from the town Troyes to the S.E. of Paris). Thus, 1 ib. troy is ¾-lb. avoir. Used almost entirely for weighing precious metals and stones.

Truncate : To cut away the top. To replace a solid angle by a plane surface. A truncated cone or pyramid is the difference between the original and the part removed. This may seem obvious, but it is strange how many fail to apply this item of common sense to problems.



"Turn upside down" : Fifth item of Mumbo-jumbo, q.v., lampooned in the story of Father William by a very brilliant mathematician—the Rev. Charles Lutwidge Dodgson, who preferred to be known as Lewis Carroll, writer of Alice in Wonderland.

(*a*) Unity can assume a million garbs : 66/66 ; 2½/2½ ; (x — y)/(x— y) ; in fact any ratio of two equal quantities.

(*b*) Whether we multiply or divide first when we have the chance of doing either and must do both eventually, does not alter the result. This may be proved by a few examples with small numbers.

$$3 \times 6 \div 7 = 18 \div 7 = 2\tfrac{4}{7};$$
$$3 \div 7 \times 6 = 3/7 \times 6 = 18/7 = 2\tfrac{4}{7}.$$

From (*a*) and (*b*) we may demonstrate :—

$$\frac{13}{15} \div \frac{19}{23} = \frac{13/15}{19/23} \times \frac{23/19}{23/19}$$

since we have multiplied by a ratio equal to unity. We may continue :—

$$\frac{13/15}{19/23} \times \frac{23/19}{23/19} = \frac{(13/15) \times (23/19)}{(19/23) \times (23/19)}$$

since it does not matter in what order we perform operations of multiplication and division. The complicated denominator is now unit and need not be written.

$$\frac{13}{15} \div \frac{19}{23} = \frac{13}{15} \times \frac{23}{19}$$

and we have "turned upside down and multiplied." Division by 19/23 is multiplication by 23/19 only when you know how, and why.

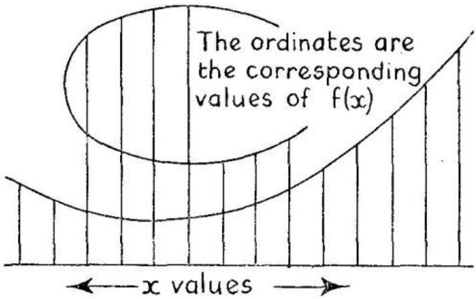
Twelve : The second "teen" by rights. But the Anglo-Saxon counting scheme was based on the dozen concept as well as the decimal, and 12 is the number from which we take 2 to have 10 left. Twa-lif-tien = two leaves ten.

Unit : A single thing. Unit number, the figure next to and on the left of the decimal point. The numbers 1 to 9.

V : 5 in Roman numerals. IV, VI, VII, VIII, XV, XVI, XVII, XVIII, XXV, etc., are 4, 6, 7, 8, 15, 16, 17, 18 and 25 respectively.

Vanishing Points : Two points on eye-level (q.v.) to which the sets of horizontal parallel lines converge in the drawing of a " perspective " drawing.

Variation : The way in which one quantity changes when its value is determined by another and this quantity changes according to some law. Generally used in the phrase " variation of a function." If y = f (x) and x changes, the function changes and "y varies with x." Example :—If $y = ax^2 + bx + c$, and x varies according to the straight line law, i.e., it increases or decreases in equal steps, then the function, represented by y, changes its value according to the parabolic curve ordinates.

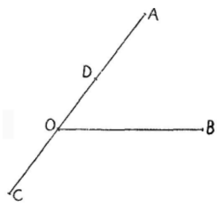


Vector : A quantity not fully defined unless mention is made of three qualities h possesses : magnitude, direction, sense.

(1) A journey from D to A is distinct from a journey from O to A in magnitude.

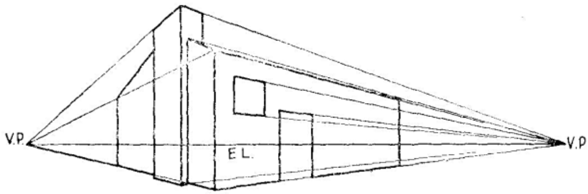
(2) A journey from O to A is distinct from one from O to B in direction, but equal in amount.

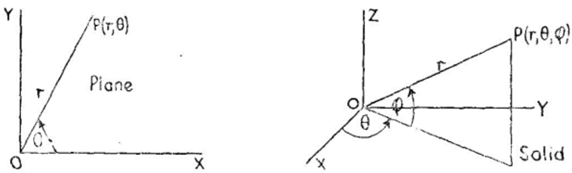
(3) A journey from O to C is distinct from one from D to A, although in the same direction and equal in amount. It is opposite in sense.



Quantities which are vectors are velocities, accelerations, advances, forces, in contrast with this type of quantity is the scalar quantity like weight and time.

Vectorial Angles : One of the two co-ordinates in polar co-ordinates, measured in precise direction from initial positions. In plane geometry, OX, OY are perpendicular to one another and Θ measured anti-clockwise from the OX direction is the vectorial angle, in sol id geometry. OX, OY, OZ are mutually perpendicular and Θ and ∅, *a*s shown, are the vectorial angles. The points P are referred to as P (rΘ) and P(r,Θ,∅) respectively.





Velocity : Rate of change of position ; and s/t is average velocity over a distance s, if t is the time interval occupied during the change of position. The velocity, at any *moment,* is the value of the ratio s/t when t becomes infinitesimally small.

Vertex : Apex and vertex are synonyms.

Vertical : At right angles to the base of a triangle. In the direction of a plumb-line in space. Vertical plane ; one containing a vertical line.

Volume : The measure of the amount of space occupied by a solid, liquid or gas, and involving three dimensions, length, breadth and height.

Vulgar : Used by the common people. To one person knowing fully the meaning of 3.1415926 . . . there will be ninety-nine knowing the meaning of 3 1/7. Vulgar fractions of small denominators are as old an idea as number, and their meaning would be impressed on early man as soon as he felt the need of sharing his possessions among a number of sons.

Week : A perfectly arbitrary sub-division of time having no scientific use but based probably on the superstition value of 7.

Weight : The force exerted by a mass, due to the attraction of every particle in the universe on every other particle, and especially the force exerted by the collection of mass called the earth, on any loose object near its surface.

Work : The quantity obtained by multiplying together the magnitude of a force (in pounds or tons weight) and the distance through which the force is able to displace the body acted upon. The equation of work is W =; F.s.

X : Roman numeral 10. IX, XI, XII, XIII, XX, XXX, XL, etc., are 9, 11, 12, 13, 5.0, 30, 40 respectively.

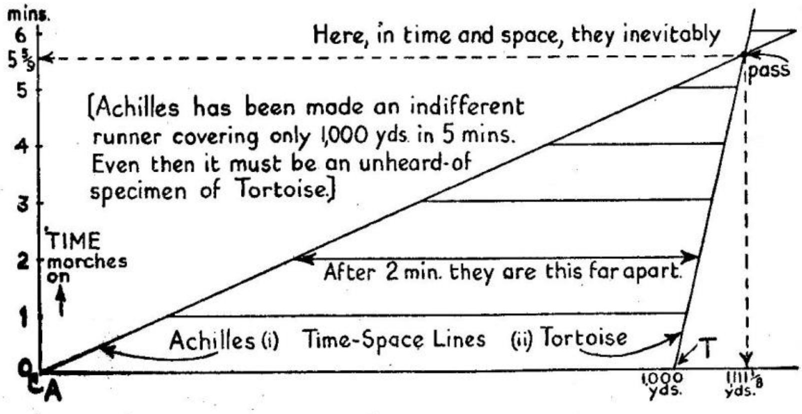
. . . x, y, z : Usually used for variables in contrast to a, b, c . . used for constants.

Year : This longest unit of time (centuries and millenia notwithstanding) is determined by the tilt of the earth's axis at 231° from a vertical to its orbital plane. In consequence of this tilt the northern hemisphere has summer and the southern hemisphere winter at one and the same time. The seasons have always been matters of life and death to man since he emigrated from the always warm sub-tropical home in which he first dwelt. His most ancient monuments were made for the task of measuring this vital period of time. It is a tribute to the careful observation that must have been done in ancient Egypt that the record of one year was known to cover 365 days. It was fixed as 365¼ by Julius Caesar. With an infinitesimal error it is now known to be 365 days 5hrs. 49 mins. 2 secs. Had the Julian year remained it was found to accumulate 3 days' error in 400 years and after sixteen centuries the Spring equinox was falling on March 11th. Pope Gregory's advisors undertook to correct matters. They eliminated February 29th from all years ending in 00 unless the other figures divided by 4. Thus 1600 was a leap year and 2000 will be but 17,00, 1800 and 1900 were not. It will now take 3,300 years for the calendar to accumulate an error of one day.

Zeno : Was born in 495 B.C. He was executed 60 years later "for conspiracy against the State "— probably as a thinker he undermined someone's ignorant obsessions. (People are still executed in many parts of the world for having *dangerous thoughts).* He propounded several paradoxes, of which the one concerning Achilles and the tortoise is the best known. The solution, as we know to-day, involves a knowledge of infinite series insofar as these cover the subject of recurring decimals. Two straight line graphs give the solution immediately.

Suppose Achilles can run ten times as fast as his tortoise to which he gives 1,000 yards start. Achiiles runs this 1,000 yards and, by the time he completes it, the tortoise is 100 yards ahead. Achilles continues running ; by the time he covers this 100 yards the tortoise is ten yards ahead. Achilles runs the 10 yards and the tortoise is still a yard ahead. Achilles covers this and the tortoise is 1/10 yard ahead. Achilles, on this reckoning, can never catch up with the tortoise. In fact he does, and passes him in exactly 111 1/9 yards. Can you solve this paradox that he both cannot and yet does pass the tortoise ?

It is from doubting "fundamental" things that changes are made. Thus, Galileo upset the pundits of his age , Newton brought low his critics, and Einstein has revolutionised aill the laws of Physics.



Zero : One of the first and seeming unsurmountable difficulties of the early abacists was to keep a record of the empty column ; 61 might mean (on a five-wire frame), 61,000, 60,100. 60,010, 6,0001, 6,100, 6,010, 6,001, 610, 601, or 61.

The Hindus called the blank—sunya, but they did not connect it with our ideas of nothing. The clever folk of the ancient world could not think of nothing as a number. It is a moot point still whether nought is a unit number.

The word sunya passed through Arabic becoming *sifr* ("empty") to *zephirum* in Italy by the 13th Century. It culminated eventually in zero. *Cifr* passed into Germany and became *cifra* and thence into England as *cipher,* when it became a symbol of mystery and a secret sign ! ! ! The word *decipher,* to unravel a mystery, is a legacy of those times.

The word for the idea *nothing* is now zero. But thanks to the bad enunciation of the British people and their general laziness, we shall soon have a new name for nothing. Zero for the mathematical *idea* ; nought for the symbol was good. The man in the street would say "ought." The telephone people, the Stock Exchange and the B.B.C. are conspiring very successfully to substitute the fifteenth letter of the alphabet. O dear !

(I am indebted to "Number, the Language of Science," by T. Dantzig (Geo. Allen

Unwin, Ltd.) for the history of the word zero).

Zone of a Sphere : The difference between two caps, the circular faces of which are parallel. (See Cap).

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