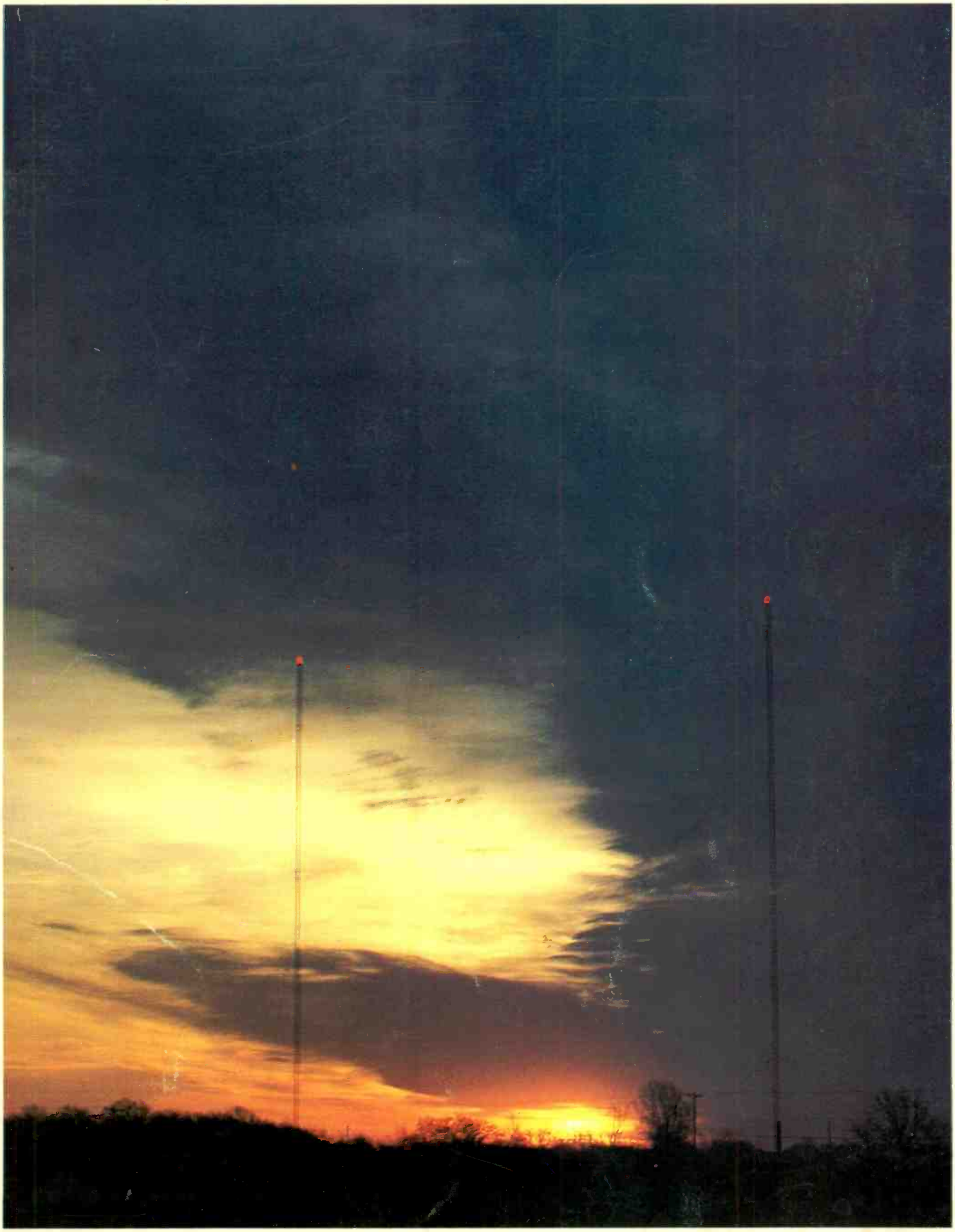
Directional Antenna Handbook



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Directional Antenna Handbook

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Some chapters have appeared in 1977 and 1978 issues of **Broadcast Engineering**.

Preface

It is my hope that this book will fill a special need in our industry that I have observed over the past 25 years. In some ways it may be easier to explain to the reader what the book is not, rather than what it is.

This book is not written as a college text. It is not written as a complete treatise of all mathematical formulas ever designed by engineers to calculate directional formulas. Nor is the book an effort to impress the reader with all my knowledge of the subject.

This book is intended to equip the station engineer with the basic knowledge required to design his own pattern. The book also will provide the reader a basic math review as well as some of the shortcuts employed by engineers. The book begins with simple, basic antennas and goes from there to progressively more sophisticated systems.

It is hoped that this book will serve as a reference work to those in the field and become a standard of review, covering all normal types of directionals as well as normal methods of computation.

The reader will note there is greater emphasis on two-element and three-element designs. The majority of directional antennas in use today are one of these types. It is believed that each reader will find a pattern like his own to study, or one very similar. Please keep in mind that for comparative convenience most patterns in this book are designed to beam northward. The reader needs to merely rotate the page of the book to see his own pattern along its major axis.

I must express my grateful thanks to Pat for help in typing these notes, formulas and tables.

The Author

1

Introduction to the design of directional antennas

Basic design considerations

Any single vertical tower will radiate an equal amount of signal in all directions. For all practical purposes assume this to be a perfect circular pattern. With the addition of a second vertical tower, inserted into the field radiated by the first tower, the pattern of this first tower will no longer be circular. In other words, the non-directional pattern has been destroyed! Depending upon the electrical height, physical spacing, and current induced in this second antenna, the original circular pattern may be offset, elongated, or even approximate a figure-eight pattern shape.

Sometimes a station ends up with an unwanted directional pattern, when it had hoped for a circular one. This occurs when a non-directional antenna is erected too close to a reflecting object, such as a water tank, a hi-line tower, a metal smoke stack, or a structure capable of re-radiating a broadcast signal. When this second object is "unwanted," a station has no practical control over its effects, nor can it use the object to achieve any particular desired pattern. Obviously, the best type (and from FCC's viewpoint, the only kind they will license) is one where each element of the directional system is *controlled.*

The earliest known directional antenna licensed by the FCC was in about 1935 to stations WSUN, St. Petersburg, Florida; and WTMJ, Milwaukee, Wisconsin. Prior to this time all AM stations were considered non-directional, or at least no deliberate efforts had been made to generate anything other than a circular pattern. Some early stations, particularly those employing "flat top" antennas, noted a directional effect to their coverage. In some cases this was due to the metal supporting towers. However, in the case of WSUN and WTMJ, each station erected two towers and purposely restricted its nighttime pattern in the direction of the other.

These early efforts were successful, and the next few years saw a rapid increase in the use of DAs, both nighttime and daytime. According to recent FCC station lists, there are at least 1,502 stations utilizing DAs in the United States today.

Design factors

Three factors can determine the radiation pattern of any basic daytime pattern. These are electrical spacing between the towers, the amplitude of the tower currents in each element; and the phase angle in degrees between the currents in each element. Once an antenna system has been constructed, the electrical spacing is, of course, fixed. Thus the only variables a station has at that point are currents, and phase angles. By manipulating these parameters, the design engineer can achieve almost any pattern, in addition to the one he desires.

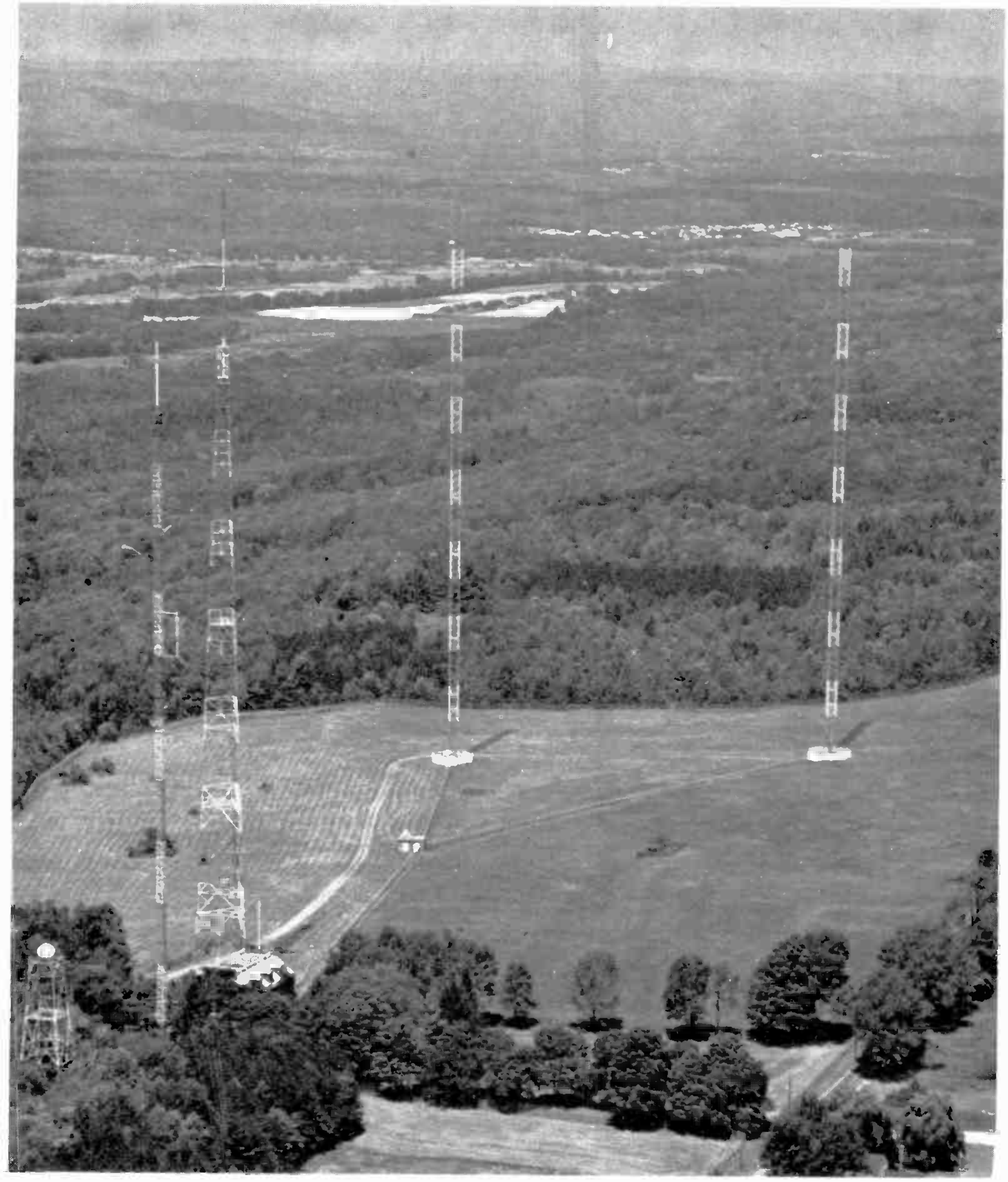
For the purpose of design, it is assumed that each tower, plus its associated ground system, acts as an individual antenna system and radiates its own signal equally in all directions. This is an important fact to remember. Each and every antenna in any directional antenna system is assumed to radiate a perfect circle.

How is it possible to achieve any specific pattern, if all a station ever gets from any tower is a perfectly radiated circle? The trick really comes in knowing how to combine these circles, both as to relative size and as to time-phase relationships. In examples where equal height of towers are used, the relative size of these circles is directly proportionate to the ratio between the individual base currents. Hence, if a station has twice the base current in one tower than in the other, the radius of the individual circles will be on a ratio of two to one.

Special considerations

The ground system of each tower should be equal, from a design standpoint. If they are not, this could result in some non-circular radiation from any given tower. For example, if one tower has fewer ground radials in a given direction, than other towers in the system, the radiation from that tower in that direction may be impaired due to increased ground losses on that side of the tower.

Another consideration is the antenna ground system that has individual ground wires of one tower overlapping those from an adjacent tower. If the



The background two-tower is WTIC-AM. Other antennas and towers are for WWUH-FM, WEDH-TV, WTIC-FM, and WFSB-TV.

tower spacing is such that the radials overlap, each ground wire must be terminated where it meets another and the ends bonded together. The reason for bonding is to eliminate any high resistance joints that can cause power losses in the ground return path. Bonding also eliminates the possibility of corroded joints which could cause cross modulation or harmonic signals.

Two-tower DAs

Figure 1 shows the simplest and most common type of directional antenna: the two-tower pattern. As a beginning point, let's say these two towers are of equal height, and each has a full and complete ground system. For the first "basic" approach, assume each tower has an equal amount of current flowing in it, and that each tower receives its RF energy at the same identical instant. That is zero time phase difference.

Figure 2 shows what will happen under these conditions as only the physical spacing, or distance, between the respective towers is changed. The pattern to the upper left is for one-quarter wavelength spacing. The next is for three-eighths, the next for one-half; and the pattern at the bottom is for one full wavelength spacing. By varying only the spacing, and holding currents and phase angles constant, the station ends up with several widely varying patterns.

Now let's use the approach of holding the spacing fixed, at one-quarter wavelength, holding the individual tower currents equal, but this time varying the phase angle relationship. Figure 3 shows how this parameter can produce widely varying shapes. The last parameter is shown in Figure 4, as it is varied. In this last case the spacing was held constant at one-quarter wavelength; the phase angle was held constant at 135°; and the current ratio was then varied. The effects of equal currents in both towers, and where the currents are 2:3 and 2:1 are shown.

In these examples, each of the three basic design parameters are varied separately to show how each can, in and of itself, create directional patterns. In proceeding to the design of a particular pattern shape, the design engineer may choose to use one, two, or all three of these basic factors in combination. Chapter 2 will go into detail on how this is accomplished. But first let's complete a basic understanding of the fundamentals of directional antenna patterns.

Other factors

The first and most obvious of these other design factors is, of course, the amount of power to be fed to this directional antenna pattern. While this isn't one of the "big-three" factors, it is an important consideration in any pattern design. In some cases, as a design engineer proceeds with a particular allocation study, where the use of a directional antenna is necessary, he finds that higher power can be used. In other cases he may discover that it will take one or two additional towers, to permit the power he is seeking. In this type of situation the design engineer is faced with an economic decision. More later on this point.

The second of these other factors is one of tower orientation. Obviously, a station can point, or aim, a given pattern in any direction it chooses. In other words, it can use the same basic pattern and point it north, or south, or east, or any other direction it may want.

The term "Tower Line" is often used in connection with pattern orientation. The Tower Line is the bearing or direction the row of towers points to. Some types of patterns have more than one tower line, i.e., a parallelogram or dog-leg configuration.

A third factor to consider is electrical tower height. While it is customary to construct most directionals with elements one-quarter wavelength in height, many may use either taller or shorter electrical height towers. The taller a tower is, the more efficient a radiator it will be, up to a height of five-eighths of a wavelength.

Figure 6, from Section 73.18 of the Rules, shows this relationship. For example,(a tower of 0.25 wavelength has for 1 kW of power an efficiency of 196 MV/M. But for this same power, a 0.50 wavelength tower has 237 MV/M. This is an increase of 41 MV/M over a quarter wavelength tower, yet the power is the same. A 0.625 wavelength tower's efficiency becomes 274 MV/M.)This is almost a 50 percent increase in radiated signal, which in power ratio, corresponds to more than a doubling of the transmitter power. If a station desired the maximum efficiency from any given pattern, at any given power, it would use a 5/8 wavelength tower. However, economics may rule out this choice.

In the design of nighttime antenna patterns, note the signal energy radiated at elevation angles above the horizon. This is shown in the reproduction of FCC Figure 5. It shows that for different electrical tower heights, different vertical patterns exist. Keep in mind that these are the vertical patterns of single towers, not directional antennas! If several single towers are combined, as would be the case in a directional antenna, the vertical pattern of the combination might be quite different. This vertical effect is referred to as the "Vertical Form Factor" of a tower.

It should suffice to point out that by using different heights for the elements in a nighttime directional pattern, you can place vertical angle nulls at the elevation angle most suited to providing skywave protection to any given co-channel station.

Important definitions

A common expression is "Radiation Resistance." This is the value computed at the point of current measurement. In most cases this is at the input of the tower. This can sometimes be referred to as "Base

Figure 1

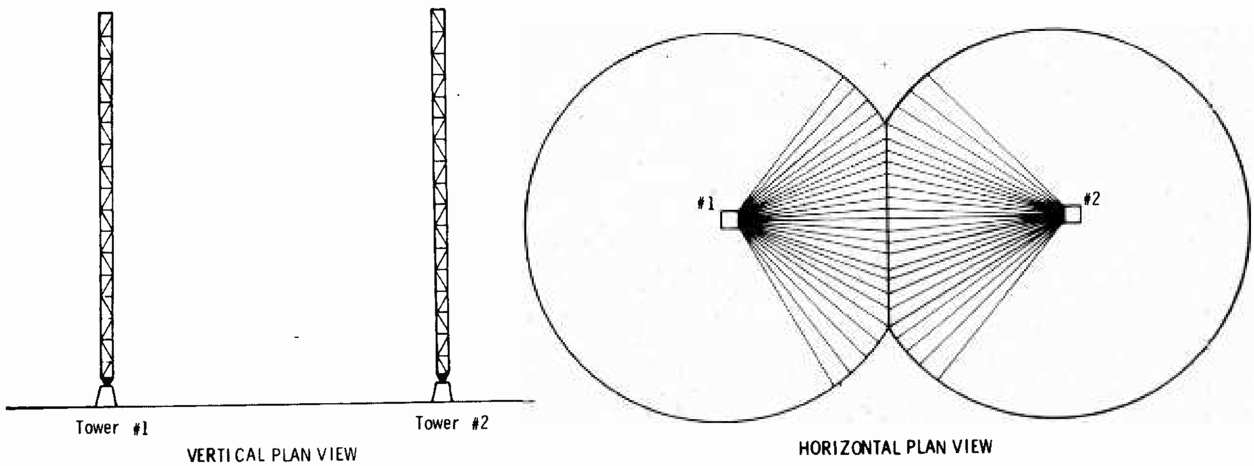
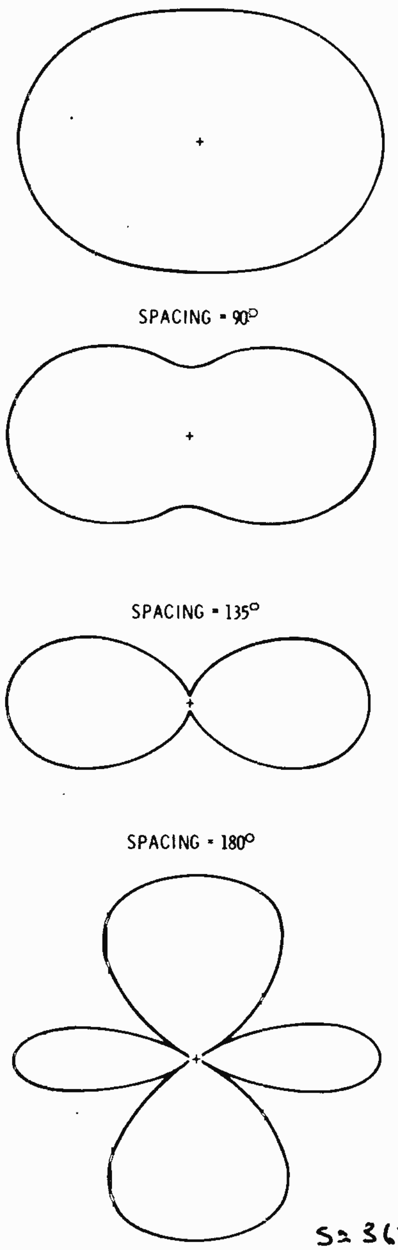


TABLE 1
VERTICAL RADIATION CHARACTERISTICS

TOWER HEIGHT =		90°	180°	225°
VERTICAL ANGLE =	0°	1.0000	1.0000	1.0000
	10°	.9781	.9418	.9011
	20°	.9143	.7855	.6450
	30°	.8165	.5774	.3291
	40°	.6946	.3696	.0529
	50°	.5591	.2008	-.1248
	60°	.4178	.0873	-.1924
	70°	.2766	.0261	-.1733
	80°	.1377	.0033	-.0993
	90°	.0000	.0000	.0000



S≈360

Figure 2 E1 = E2 and Phase Angle = 0

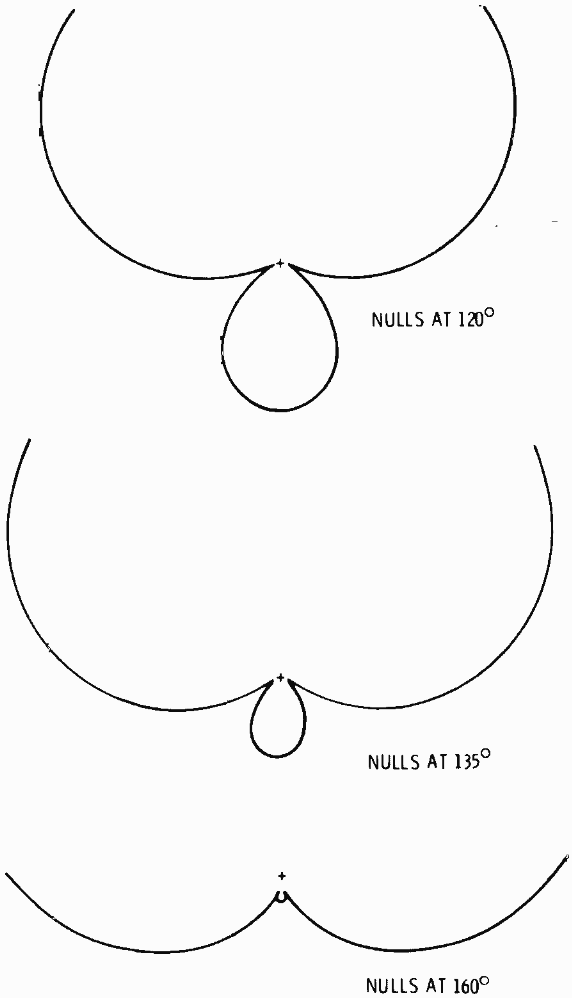


Figure 3

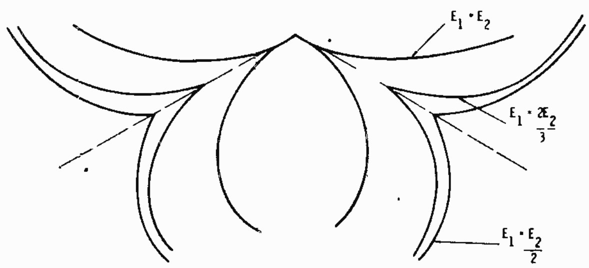
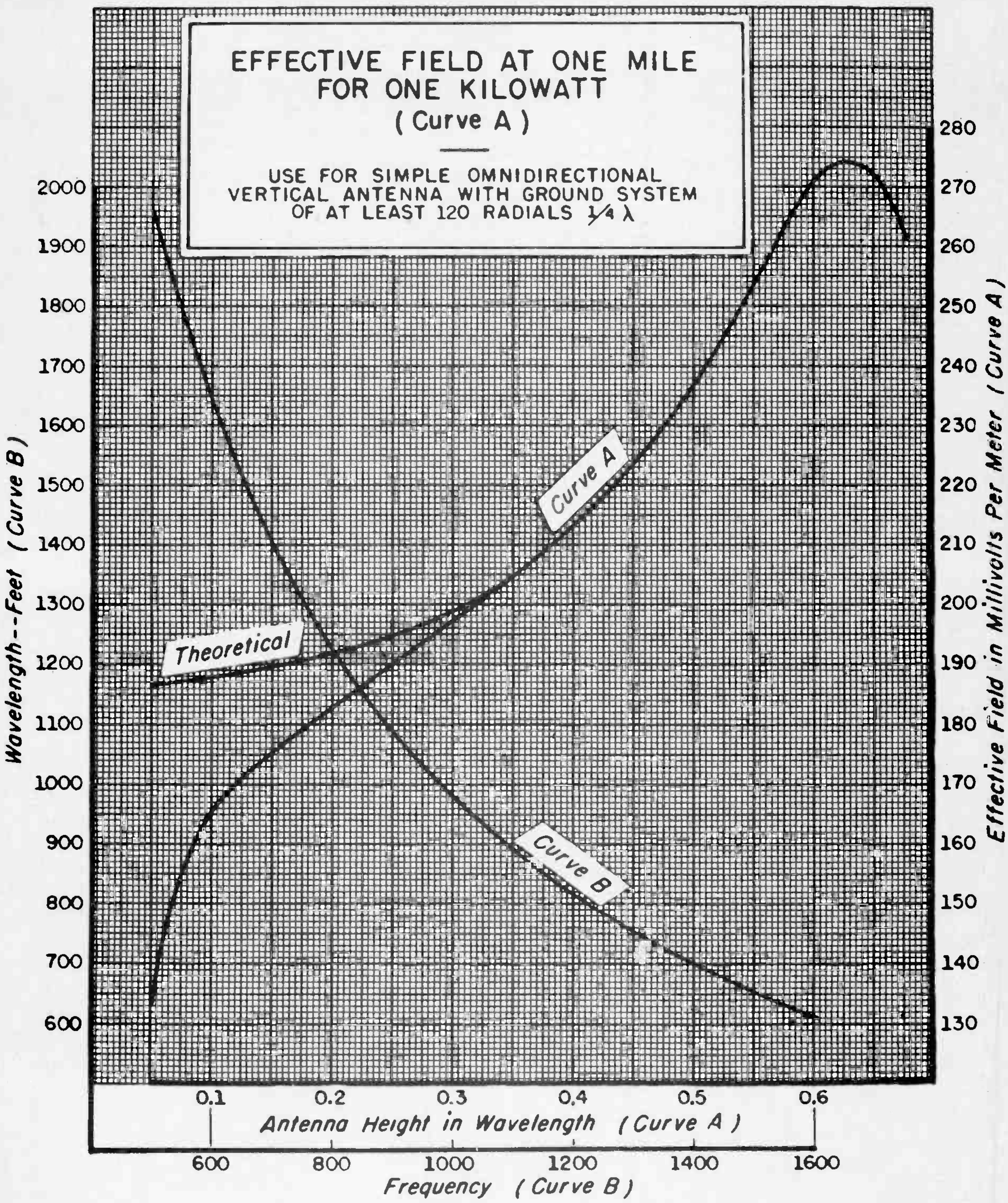


Figure 4



EFFECTIVE FIELD AT ONE MILE
FOR ONE KILOWATT
(Curve A)
USE FOR SIMPLE OMNIDIRECTIONAL
VERTICAL ANTENNA WITH GROUND SYSTEM
OF AT LEAST 120 RADIALS 1/4 λ
Theoretical
Curve A
Curve B
Wavelength--Feet (Curve B)
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
Effective Field in Millivolts Per Meter (Curve A)
0.1
0.2
0.3
0.4
0.5
0.6
Antenna Height in Wavelength (Curve A)
600
800
1000
1200
1400
1600
Frequency (Curve B)

Figure 5

Radiation Resistance," also commonly used is the expression "Loop Radiation Resistance." This is defined as the resistance at the point of maximum current in the tower. Theoretically, this occurs at a distance one-quarter wavelength below the top of the tower. To compute the power in a tower, multiply the square of the RF current in amps at this radiation resistance point, times the resistance in ohms.

Another term is "Radiation Measurement Standards." In the comparison of antennas, the signal is referred to in millivolts per meter (MV/M) at a distance of one mile. In fact, this standard of comparing signals at one mile distance is used so commonly that most engineers don't even say "at one mile"—it is just automatically understood. Thus Radiation Measurement means the signal strength in MV/M at one mile. The measurement of radiation efficiency expressed in millivolts per meter per amp will be referred to. For every tower there is some finite value of radiated signal at one mile in MV/M that will exist when one amp of current flows in that tower at its Radiation Resistance Point. For example, a common reference is the one-quarter wave tower with 1 kW of power. This is generally recognized as a standard. A quarter wave tower will radiate, at 1 kW of power, a signal of 196 MV/M at one mile. Also, knowing that such a tower has a theoretical radiation resistance of 36.6 ohms, the following formula can be applied:

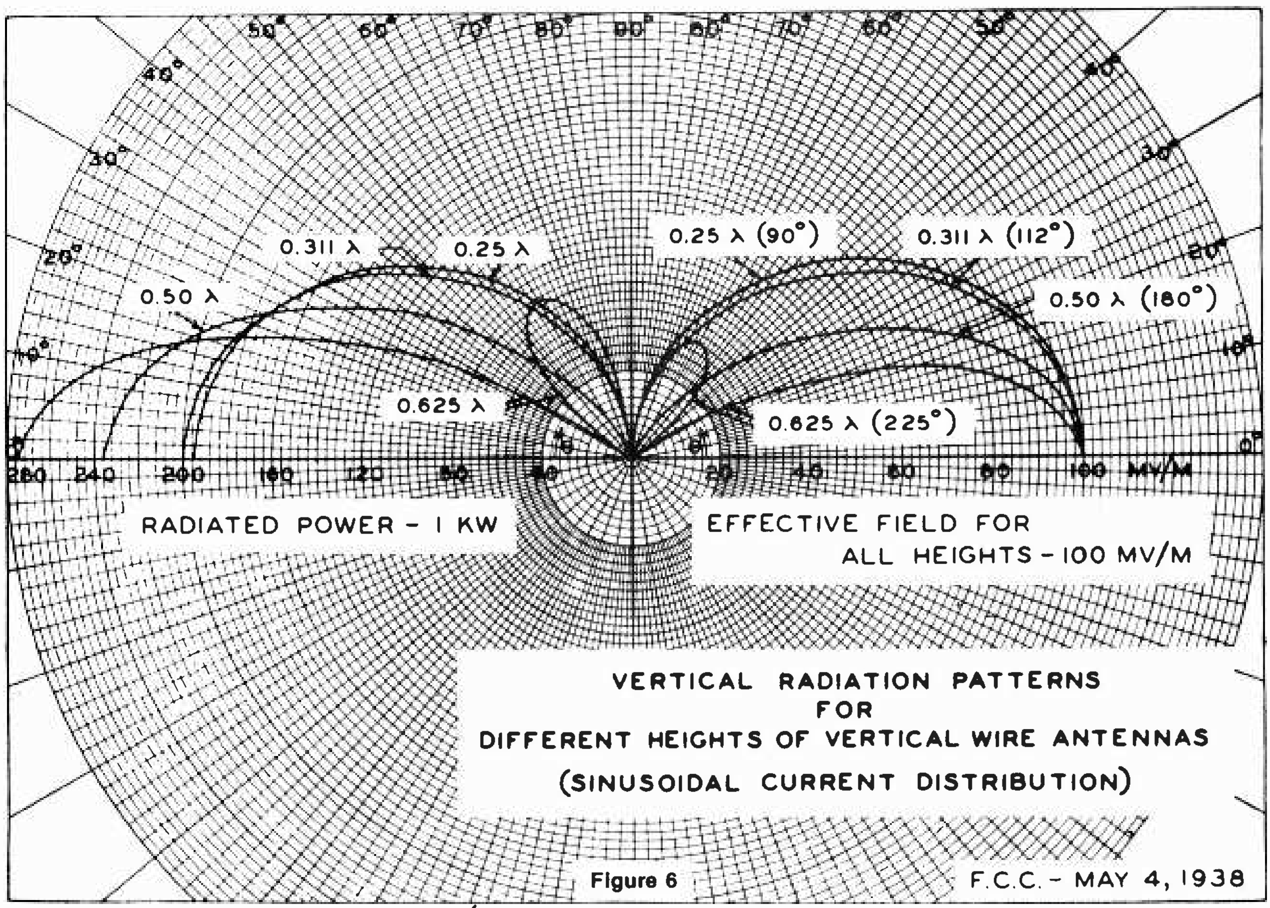
$$E/ = \frac{196\ MV/M}{\sqrt{\dfrac{1000\ watts}{36.6}}} = 37.4\ MV/M/Amp.$$

This is simple Ohm's Law, where

$$I = \sqrt{\frac{Power}{Resistance}} \qquad \sqrt{\frac{100}{2}} = \sqrt{50}$$

In this case each amp of current flowing in this tower will generate a signal of 37.4 MV/M. With a base current of 2.0 amps, the signal would be 74.8 MV/M. Thus the field intensity radiated at one mile is "directly proportional" to the amount of current flowing in the antenna. This is an important consideration to keep in mind.

You often hear about "Antenna Resonance." In practice, the design engineer defines this as the condition which occurs when the Radiation Resistance is pure resistance. This resonance occurs between 0.22 and 0.23 wavelength. For towers of lower heights, the reactance is capacitive. For towers taller than resonance, the reactance is inductive.



VERTICAL RADIATION PATTERNS FOR DIFFERENT HEIGHTS OF VERTICAL WIRE ANTENNAS (SINUSOIDAL CURRENT DISTRIBUTION)

Figure 6 F.C.C. - MAY 4, 1938

$1000 \div 36.6 = 27.32 \quad \sqrt{5.22} \quad 196 \div 5.22 = 37.497$ mv/A

2

Basic concepts of directional antennas

Need for directionals

At this point it might be interesting to look at some of the reasons or necessities for installing directional antennas. By far the most common reason is the need for a new station to protect a station already on a channel. Designing a DA to obviously interfere with a fellow broadcaster is not considered "playing the game." In other words, all new stations must afford protection to existing stations.

A second common reason for the use of directional antennas is to protect other stations when seeking power increases. These would be cases where a straight non-directional power increase would overlap some other station.

A third reason, not as common as those above, would be to prevent useless radiation over areas where no people live. A classic example would be a non-directional station located on the sea coast.

Basic pattern concepts

Figure 1 shows the typical pattern with its ground system. This is identical to that in Chapter 1 and is redrawn to show the comparison of this to our mathematical approach. Hoping we could do it all without math? Sorry. I'll try to keep it simple and explain every term and step, giving examples by using actual stations' patterns.

Figure 2 represents the vector approach to this same two-tower directional pattern. Picture the view looking down from an airplane at 1000 feet over the top of the array. The tower signs represent the top view of the towers. Now take an engineer and place him on the ground at a distance of a mile away, at some angle (Ø) off the tower line. This engineer uses his field intensity meter to measure the "total signal" so that he can tell the amount of radiation at his particular bearing. If the engineer walked a circle around the pattern, taking readings of signal intensity at every 5° point, his readings could be plotted and the complete shape of the directional pattern seen.

In design, with the use of simple mathematics, the signal the engineer would measure could be predicted had he stood one mile away in any given direction.

It was said the engineer would measure the "total signal." Each tower radiates its own signal. In the example, two individual signals will arrive at the point where the engineer is standing. His field meter is not able to differentiate between the separate individual signals. He reads the combined signals. Hence the reading on his F.I. meter represents the "total" signal. With three towers, the total signal would be the vector sum of these signals, and so forth.

Oh-oh, what is this vectorial sum business? This is just a mathematical way of saying the two signals add in a relationship defined by the design of the pattern.

Point "P" in Figure 2 represents the spot where the engineer is standing with his field meter. Towers one and two are marked by appropriate numbers. The letter "S" represents the electrical spacing between the two towers, normally expressed in degrees. These two towers lie along the same tower-line, referred to as TL.

The signal from tower one to our engineer observer follows the arrow from the tower, which in relation to the tower-line is an angle of Ø. The same angle is assumed to exist from tower two. The signal from tower two should not be the same angle since this can occur only if the two vectors are parallel. However, in theoretical design assume the path to each tower from the observer is parallel, even though in reality it is not.

This is a difficult concept to accept. The design engineer assumes them to be parallel lines when he is standing at a distance greater than ten times the tower spacing. The reason for this is that mathematically the sine and the tangent of angles less than 6° are equal.

Now that the terms are defined, let's see what happens to the two signals. Tower one is the reference tower. So its signal is arbitrarily taken as the basis for comparing the signal relationships of all the other towers in any directional antenna pattern.

If the engineer is standing at point "P," then the sum total signal he observes is the resulting addition of the two individual tower vectors. If the two towers have equal currents, then the only thing which can affect the vector sum at point "P" is the phase

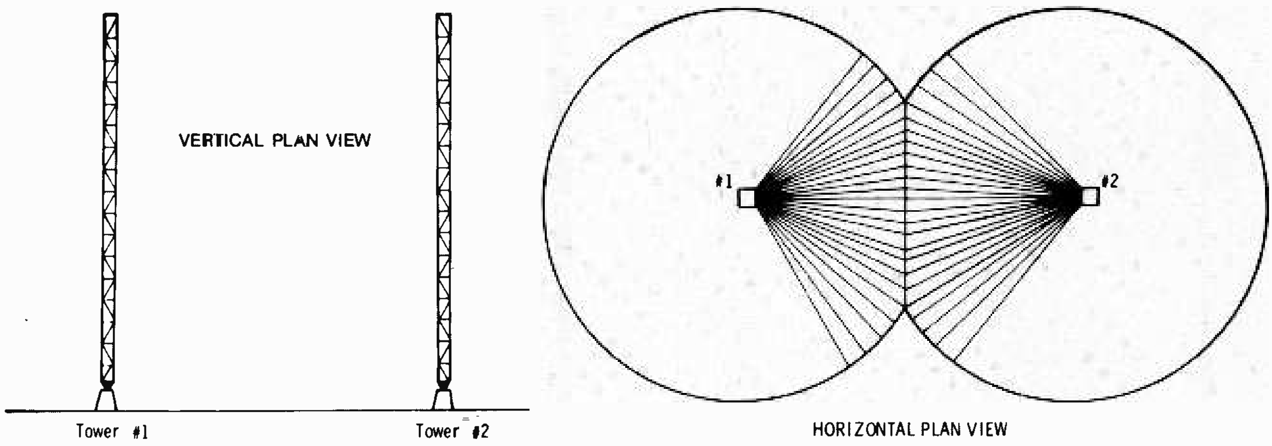


Figure 1

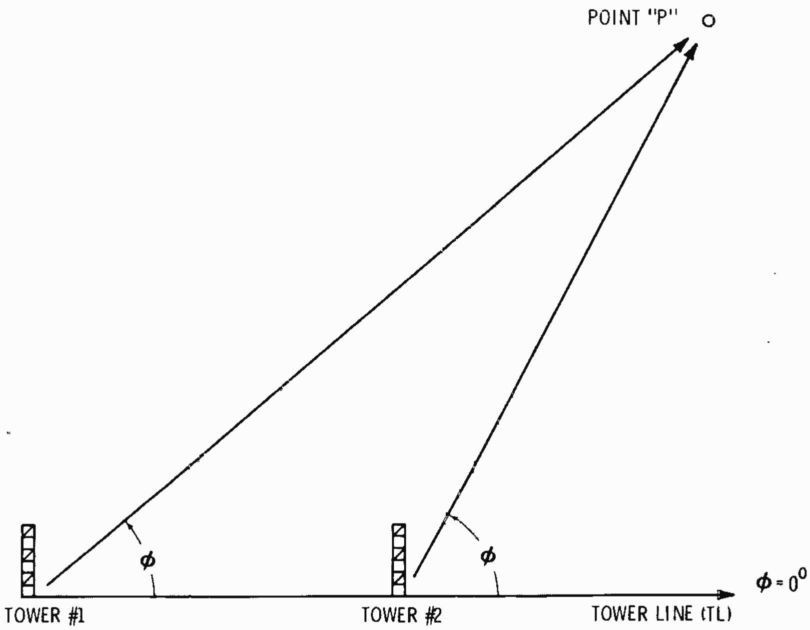


Figure 2

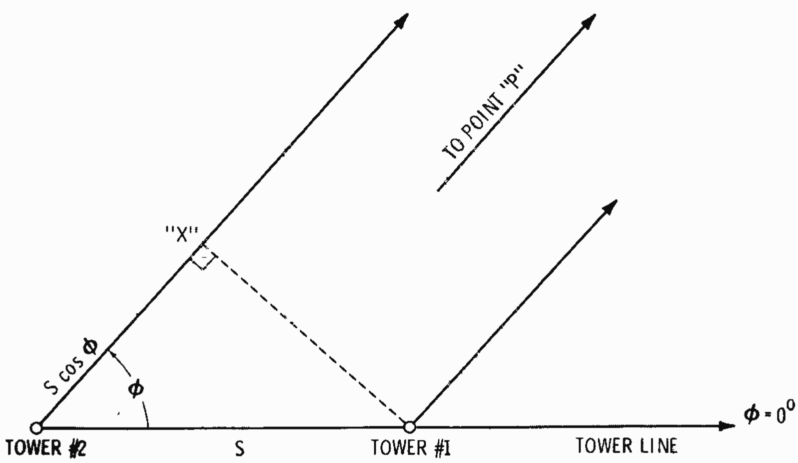


Figure 3

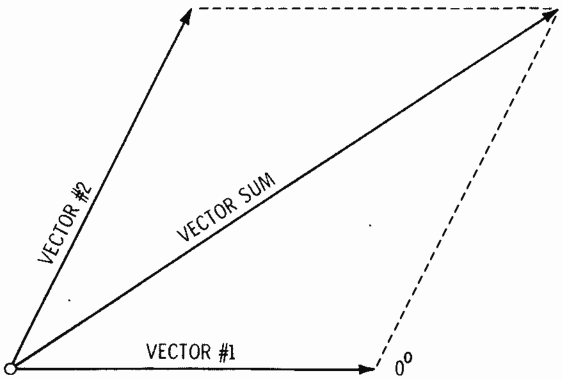


Figure 4

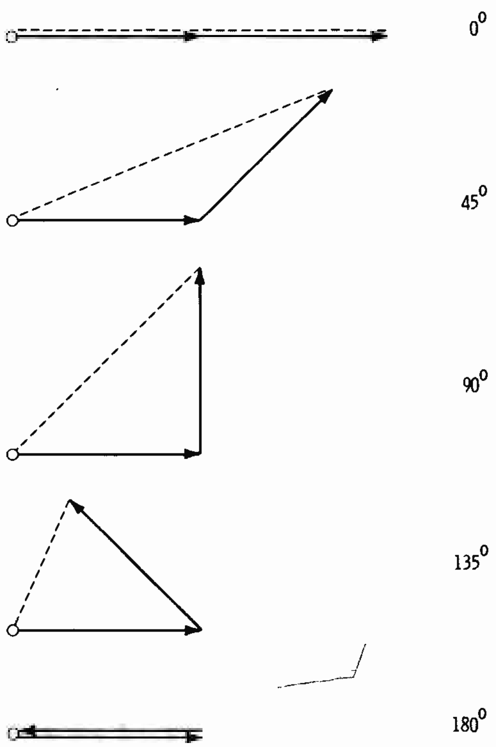


Figure 5

relationship between the two signals. Two factors control this relationship. One factor is the phase angle between the currents induced into each tower. This is the angle you might measure on a station antenna monitor.

The other factor is the "space phase," due to the difference in the length of the paths from each tower to point "P." For example, if point "P" were at right angles to the tower-line (TL) the path lengths would be equal. If this same point "P" were along the tower-line, the "space phase" would be equal to electrical spacing (in degrees) between the two towers. The "space phase" will vary between zero and S. The rate at which it varies can be expressed by this simple formula of Equation 1.

$$\text{Space Phase} = S \cos \emptyset$$

The variation is the cosine of the angle (Ø) off the tower-line. Since Ø equals zero along the TL, and the cosine equals 1.00, "space phase" equals "S." Also, when Ø=90°, the cosine of the "space phase" equals 0, or the condition of equal path lengths. Thus our formula works.

This "space phase" relationship is shown in Figure 3. Point "X" is at the corner of a right triangle formed by the tower spacing and "S cos Ø."

Obviously the distance from tower two is greater than the distance from tower one by the length between point "X" and tower two. As shown, this length can be calculated by Equation 1. Predict the space phase for any given bearing to point "P" by multiplying the term S by the cosine of the bearing angle, as measured from the tower-line.

Computations for directional patterns having two towers need be calculated over just one-half of the circle. This is because any two-tower pattern is symmetrical about its tower-line. For example, the "total signal" at 110° off the TL was to be the same as that at 250°; i.e., each side of the TL is the same as its corresponding angle. In other words, the right-hand side mirrors the left-hand side.

Let's put the terms together and see the result. There is the combination of two signals or two vectors at point "P." The reference tower is always assumed to be a vector of 1.0 units in length lying at an angle of 0°. This is vector one in Figure 6. The vector from tower two is the same length as tower one, because equal currents were assumed. Tower two's angle of relationship to that of tower one's vector is affected by the two phases discussed above. These two factors are added as follows:

$$\beta = \Psi + (S \cos \emptyset)$$

The first term is the Greek letter "phsi" and stands for the phase angle between individual currents, or the value occurring on a station's antenna monitor. This is a constant value for any given pattern. The last part of Equation 2 is the space phase from Equation 1. The Greek letter (β) "beta" represents the combined angle, at any given bearing, and expresses the angle between tower one and tower two vectors at point "P."

For example, when $\beta=0°$, our resultant total signal is the sum of the two vectors, I_1, I_2 = 2 I units. If, however, $\beta=180°$, the vectorial sum will be 0 units. Thus the angle of β really controls the vectorial sum, which controls the pattern shape. If $\Psi=90°$ and $S=90°$ is assumed, the following vector sums shown in Figure 5, at 0°, 45°, 90°, 135° and 180° would be found off the tower-line.

In the early days of designing directionals, many engineers simply plotted the vector sums at 10° intervals and thereby determined their overall pattern shape. It is usually a bit more professional to compute these vector sums at 5° intervals, except in cases where a smaller angular change is important. In Table 1, the tabulation used to compute these vector sums is shown. The operation shown at the top of each column in Table 1 is performed, and each column in turn is modified, or processed by the next adjacent one. Column "J" represents the vectorial sums expressed in units. These are plotted on Figure 6.

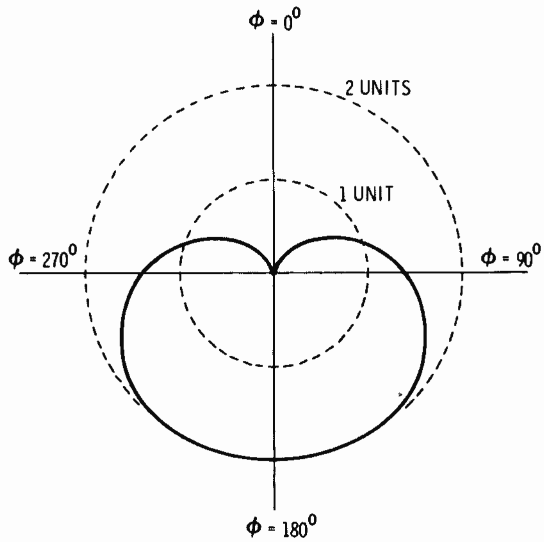


Figure 6

Computing power

In order to relate the unit vector pattern to some specific size, the power of the designed station as well as the RMS efficiency of the pattern must be known. For any given pattern shape there will be an RMS efficiency for 1 kW, for 500 watts, or even for 50 kW. The relationship bewteen these RMSs varies according to good old "Ohm's Law." Using the RMS at 1 kW as a standard, hence by simple Ohm's Law the RMS at any other power is the square root of that power divided by 1 kW, times the RMS at 1 kW. This is shown in Equation 3.

$$RMS_{(X\ Power)} = RMS_{(1\ kW)} \times \sqrt{\frac{X\ kW}{1\ kW}}$$

How does one know what the RMS efficiency is for a given pattern? This is calculated by a formula known as the Bessel Function Method. For now let's pass over this step and see how to apply the RMS efficiency to the unit vector pattern computed in Table 1.

In the example, assume the RMS efficiency was calculated to be 196 MV/M for 1 kW and it is planned to operate at 1 kW. By definition, the RMS of any pattern represents the radius (in MV/M) of a circle which will contain the same total area as the area which is encompassed by our pattern. Thus the area of the pattern is πR^2.

* In order to convert the pattern shape to this area, a conversion factor is needed. This is commonly referred to as "K." K is the constant by which each vector sum is multiplied to arrive at the final pattern expressed in MV/M. This is determined from Equation 4.

$$K = \frac{\text{RMS efficiency}}{\sqrt{\dfrac{\text{RMS unit vectors}}{\text{No. of bearings calculated}}}}$$

For the RMS efficiency in the example I assumed 196 MV/M. The number of bearings calculated means the number computed, normally 36. The RMS of the unit vectors is found by taking each of the individual vector sums, shown on Table 1, column "J" for each of the 36 bearings; squaring that value; adding up the total, dividing by 36 and then taking the square root. In fact, RMS stands for the root-of-the-mean-of-the-squares.

Table 2 shows how the unit vectors of Table 1, column "J" have been taken, squared, added; and, how I determined "K" by Equation 4 and applied that factor to arrive at the final pattern. This is plotted on Figure 7.

Nulls and lobes

If the currents in the two towers are not precisely equal, their individual vectors will not perfectly cancel at some bearing and cannot produce a pattern null having zero MV/M. In design, the engineer will often use unequal currents in order to "fill in" a given null, thus giving a more stable pattern, and one that is easier to adjust, since I have yet to find anybody who can tune to a null of zero MV/M.

TABLE 1
CALCULATING UNIT VECTORS

$$E = f(\Theta)\,[F_1\underline{/0^\circ} + F_2\underline{/\Psi + S\cos\phi\cos\Theta}]$$

$$\text{SUBSTITUTING} = 1.0[1.0\underline{/0} + 1.0\underline{/90^\circ + 90^\circ\cos\phi\cdot 1.0}]$$

A

B

C

D→J

A	B	C	D	E	F	G	H	I	J*
ϕ	90 cos A	90 + B	cos C	1 + D	E^2	sin C	G^2	F + H	$\sqrt{I}$
0°	90.0	180.0	-1.000	0.000	0.000000	0.000	0.00000	0.00000	0.000
10°	88.6	178.6	-.999	.001	.000001	.024	.00059	.00059	.024
20°	84.5	174.5	-.995	.005	.000025	.096	.00920	.00920	.096
30°	77.9	167.9	-.978	.022	.000480	.209	.04390	.04440	.211
40°	68.9	158.9	-.933	.067	.004500	.359	.12900	.13400	.366
50°	57.8	147.8	-.846	.154	.023700	.533	.28400	.30800	.555
60°	45.0	135.0	-.707	.293	.085800	.707	.50000	.58500	.765
70°	30.8	120.8	-.512	.488	.238000	.858	.73700	.97600	.988
80°	15.6	105.6	-.269	.731	.534000	.963	.92800	1.46200	1.209
90°	0.0	90.0	0.000	1.000	1.000000	1.000	1.00000	2.00000	1.414
100°	-15.6	74.4	.269	1.269	1.610000	.963	.92800	2.53800	1.593
110°	-30.8	59.2	.512	1.512	2.286000	.858	.73700	3.02300	1.738
120°	-45.0	45.0	.707	1.707	2.914000	.707	.50000	3.41400	1.848
130°	-57.8	32.2	.846	1.846	3.408000	.533	.28400	3.69200	1.921
140°	-68.9	21.1	.933	1.933	3.736000	.359	.12900	3.86500	1.966
150°	-77.9	12.1	.978	1.978	3.912000	.209	.04390	3.95600	1.989
160°	-84.5	5.5	.995	1.995	3.980000	.096	.00920	3.98900	1.997
170°	-88.6	1.4	.999	1.999	3.998000	.024	.00059	3.99800	1.999
180°	-90.0	0.0	1.000	2.000	4.000000	0.000	0.00000	4.00000	2.000

*REPRESENTS LENGTH OF UNIT VECTORS

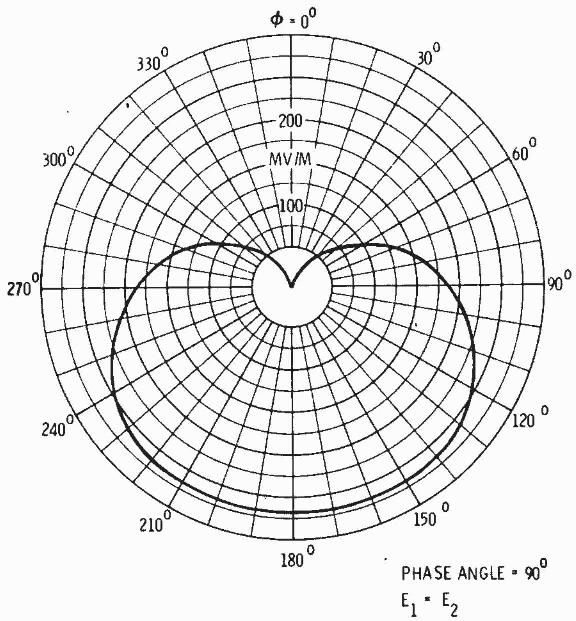


Figure 7

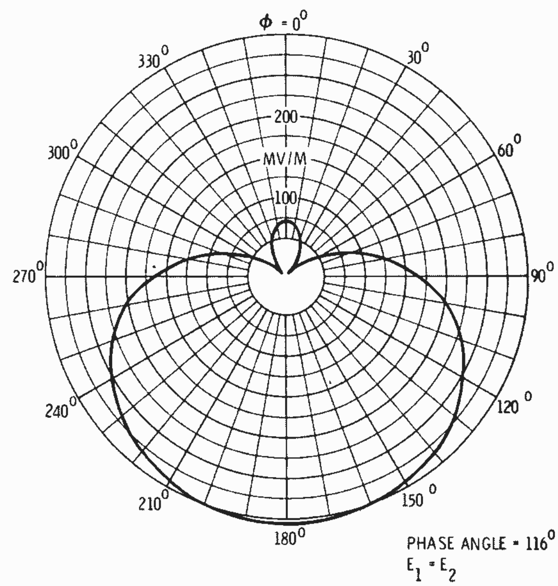


Figure 8

TABLE 2
CONVERTING UNIT VECTORS TO F.I.

$$E_T = K\left[E_{unit}\right] \text{ MV/M}$$

A	B*	C	D
∅	E_{unit}	B^2	K•B
0°	0.000	0.00000	0.0 MV/M
10°	.024	.00059	3.3
20°	.096	.00920	13.1
30°	.211	.04440	29.1
40°	.366	.13400	50.6
50°	.555	.30800	76.7
60°	.765	.58500	106.0
70°	.988	.97600	139.9
80°	1.209	1.46200	167.5
90°	1.414	2.00000	196.0
100°	1.593	2.53800	220.8
110°	1.738	3.02300	241.0
120°	1.848	3.41400	256.1
130°	1.921	3.69200	266.3
140°	1.966	3.86500	272.5
150°	1.989	3.95600	275.7
160°	1.997	3.98900	276.9
170°	1.999	3.99800	277.1
180°	2.000	4.00000	277.2
		38.056	

*COLUMN J
FROM TABLE 1

38.056 (19 BEARINGS)
x 2
76.112 (38 BEARINGS)
\- 0.0 (-FIRST BEARING)
\- 4.0 (-LAST BEARING)
72.112 (36 BEARING)

$$K = \frac{196 \text{ MV/M/KW}}{\sqrt{\frac{72.112}{36}}}$$

K = 138.6 MV/M/KW

138.98526

The example used a phase angle (Ψ) which gave just one pattern null. If instead of Ψ=90°, Ψ=116° was used, the resulting pattern of Figure 8 would be obtained. This example produced two nulls, with a minor lobe in between. Thus it is the phase angle Ψ which is important in producing the number of nulls, as well as their placement angle along the tower-line. The current ratio, as noted, will affect the depth of the nulls and will also affect the amplitude of any minor lobes. As the nulls fill in, the lobes will grow.

Different bearings

Note that math angles are not the same as bearing angles. Mathematical convention established that math angles are measured counter-clockwise from the X-axis (east). Navigational convention has established that bearings be noted clockwise, beginning at true north. This would correspond to a math angle of +90°. Not only does each system have a different reference point, but the angles are measured in opposite directions. Conversion from one system to the other is therefore obvious.

3

Design of two-tower systems

This chapter deals with the most common type of directional: the two-tower pattern. The concepts developed in Chapter 2 will be expanded, with examples of the more common pattern shapes and at least one example of each of the standard methods of calculation.

Two-tower addition formula

There are two basic ways to calculate the two-tower radiation pattern. These are referred to as the addition method and the multiplication method. Equation 1 is the addition form:

$$E = Kf(\Theta) \cdot (E_1 + E_2 \underline{\angle \Psi + S \cos \Theta \cos \emptyset)}$$

As explained in Chapter 2, these terms represent the tower vectors (E_1 and E_2), the phase angle relationship between the two towers (Ψ), and physical spacing in degrees (S). This formula can be rewritten as shown in Equation 2, by separating the sine and cosine terms:

$$E = Kf(\Theta) \cdot [(E_1 + E_2 \cos(\Psi + S \cos \Theta \cos \emptyset))^2 + (E_2 \text{ sine } (\Psi + S \cos \Theta \cos 0))^2]^{1/2}$$

Using this formula, you can compute the results shown in Table 1. In this example the horizontal plate pattern has been calculated, thus F(Θ) and cos Θ) can be assumed to be 1.0 and 0° respectively. The constant "K" was computed, as shown in Equation 4 of Chapter 2. In this example is a simple cardioid pattern.

Two-tower multiplication formula

Here's how to develop the same pattern with the multiplication method, using the formula in Equation 3:

$$E = Kf(\Theta) \sqrt{\frac{1+M^2}{2M} + \cos(\Psi + S \cos \emptyset \cos \Theta)}$$

In this formula, all terms are the same as in the addition formula except for term "M." This represents the ratio of tower two's vector divided by tower one's vector:

$$\frac{E_2 f_2(\Theta)}{E_1 f_1(\Theta)}$$

Table 2 contains the data used to compute the pattern. Note that the final column in Table 1 is identical with the last column in Table 2, except it is 141% larger. Figure 1 represents the polar graph of this pattern.

At this point you may say that's all great, but where did Equation 3 come from? Also, why are there no sine terms? The reason for this last point is that in this formula the equation has been written around the mid-point between the two towers. In such a step, the sine terms from each tower will have opposite polarity, hence will cancel each other at each and every bearing calculated.

I will outline the method used to develop Equation 3. Let the expression (Ψ+S cos Ø cos Θ) be represented by the term "X." We can then let E_1 = 1.0 and rewrite it in Equation 4 as:

$$E = Kf(\Theta)\sqrt{(1.0+E_2 \cos \text{"X"})^2 + (E_2 \text{ sine "X"})^2}$$

Multiplying out we get:

$$E = Kf(\Theta)\sqrt{\frac{1.0+2E_2 \cos \text{"X"}+E_2^2}{\cos^2\text{"X"}+E_2^2 \text{ sine}^2\text{"X"}}}$$

Since $\text{sine}^2+\cos^2 = 1.0$ (from simple trig) we can substitute:

$$E = Kf(\Theta)\sqrt{1.0+2E_2 \cos \text{"X"}+E_2^2}$$

By substituting for X and dividing out the $2E_2$ term we get:

TABLE 1
CALCULATING UNIT VECTORS

$$E = f(\Theta)\,(F_1 \angle 0^\circ + F_2 \angle \Psi + S\cos\emptyset\cos\Theta)$$

$$\text{SUBSTITUTING} = 1.0\,(1.0\angle 0 + 1.0\,\angle 90^\circ + 90^\circ \cos\emptyset\; 1.0)$$

A

B

C

D→J

A	B	C	D	E	F	G	H	I	J*
Ø	90 cos A	90 + B	cos C	1 + D	E^2	Sin C	G^2	F + H	$\sqrt{I}$
0°	90.0	180.0	-1.000	0.000	.000000	0.000	0.00000	0.00000	0.000
10°	88.6	178.6	-.999	.001	.000001	.024	.00059	.00059	.024
20°	84.5	174.5	-.995	.005	.000025	.096	.00920	.00920	.096
30°	77.9	167.9	-.978	.022	.000480	.209	.04390	.04440	.211
40°	68.9	158.9	-.933	.067	.004500	.359	.12900	.13400	.366
50°	57.8	147.8	-.846	.154	.023700	.533	.28400	.30800	.555
60°	45.0	135.0	-.0.707	.293	.085800	.707	.50000	.58500	.765
70°	30.8	120.8	-.512	.488	.236000	.858	.73700	.97600	.988
80°	15.6	105.6	-.269	.731	.534000	.963	.92800	1.46200	1.209
90°	0.0	90.0	0.000	1.000	1.000000	1.000	1.00000	2.00000	1.414
100°	-15.6	74.4	.269	1.269	1.610000	.963	.92800	2.53800	1.593
110°	-30.8	59.2	.512	1.512	2.286000	.858	.73700	3.02300	1.738
120°	-45.0	45.0	.707	1.707	2.914000	.707	.50000	3.41400	1.848
130°	-57.8	32.2	.846	1.846	3.408000	.533	.28400	3.69200	1.921
140°	-68.9	21.1	.933	1.933	3.736000	.359	.12900	3.86500	1.966
150°	-77.9	12.1	.978	1.978	3.912000	.209	.04390	3.95600	2.989
160°	-84.5	5.5	.995	1.995	3.98000	.096	.00920	3.98900	1.997
170°	-88.6	1.4	.999	1.999	3.998000	.024	.00059	3.99800	1.999
180°	-90.0	0.0	1.000	2.000	4.000000	0.000	0.00000	4.00000	2.000

*REPRESENTS LENGTH OF UNIT VECTORS

TABLE 2
MULTIPLICATION FORMULA

$$E_{UNIT} = \left(\frac{1 + M^2}{2M} + \cos(\Psi + S\cos\emptyset)\right)^{1/2}$$

WHERE S = 90°, Ψ = 90°, and M = 1.0

A	B	C	D	E	F
Ø	90 cos A	90 + B	cos C	1 + D	$E^{1/2}$
0	90.0	180.0	-1.0000	.0000	.000 MV/M
10	88.6	178.6	-.9997	.0003	.017
20	84.5	174.5	-.9950	.0046	.068
30	77.9	167.9	-.9780	.0220	.148
40	68.9	158.9	-.9330	.0670	.259
50	57.8	147.8	-.8460	.1540	.392
60	45.0	135.0	-.7070	.2930	.541
70	30.8	120.8	-.5120	.4880	.698
80	15.6	105.6	-.2690	.7310	.855
90	0.0	90.0	.0000	1.0000	1.000
100	-15.6	74.4	.2690	1.2690	1.126
110	-30.8	59.2	.5120	1.5120	1.229
120	-45.0	45.0	.7070	1.7070	1.306
130	-57.8	32.2	.8460	1.8460	1.359
140	-68.9	21.1	.9330	1.9330	1.390
150	-77.9	12.1	.9780	1.9780	1.406
160	-84.5	5.5	.9950	1.9950	1.412
170	-88.6	1.4	.9990	1.9990	1.413
180	-90.0	0.0	1.0000	2.0000	1.414

$$E = Kf(\Theta)\,\sqrt{2E_2}\sqrt{\frac{1.0+E_2^2}{2E_2} + \cos(\Psi + S\cos\emptyset\cos\Theta)}$$

The $2E_2^2$ term outside the radical is a constant (for any given design), so it can be included in the K term. Thus Equation 4A can be written as:

$$E = Kf(\Theta)\sqrt{\frac{1+E_2^2}{2E_2} + \cos(\Psi+S\cos\emptyset\cos\Theta)}$$

Equation 3 is the most practical, the least time consuming, and results in the least chance for error. This is without a doubt the most widely used method among engineers. Equation 3 is identical to Equation 4A, except it is more common to substitute M for E_2.

I have tabulated in Table 3 the full and complete calculation via the multiplication method of a cardiod type two-tower pattern. The pattern of Table 3 is graphically displayed in Figure 4.

Rarely used method

There is another method rarely used in calculating two-tower patterns, shown in Equation 5 mostly for its historic value, and not as a common or generally accepted method of developing patterns. This is referred to as the "half-angle formula." It can only be used when the fields of each tower are equal. For this the reference point is assumed to be half-way between the two towers.

$$E = Kf(\Theta)\left[\cos\left(\frac{\Psi}{2}+\frac{S}{2}\cos\emptyset\cos\Theta\right)\right]$$

This formula is derived from Equation 3 by using the old trig fact that,

$$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$$

All formulas really represent different trig relationships.

As an example of this method, Table 4 lists the math used to compute a pattern like that used by WKAM, as well as many others. A graphical plot is shown in Figure 5.

Two-towers by computers

Most consultants now resort to the aid of a computer in calculating directional patterns. It will be helpful to understand how the computer calculates a basic two-tower, or multi-tower, pattern.[1] In essence this is done by the addition method, similar to Equation 1. One tower is written as the reference tower ($E_1 \angle 0°$). Then each of the other towers is "added" to the reference tower, one at a time, regardless of the number of other towers. The computer program developed by Don Markley and the author was written to accommodate up to 12 towers. In Equation 6, each of the other towers is added in by:

$$\text{Reference tower} + E_n \underline{\angle \Psi_n + S_n \cos\Theta \cos(\emptyset - \delta)}$$

1. Jones, R.A. and Markley, D.L.: Antennas By Computer, *Broadcast Engineering*, March 1967.

The only new term is the Greek letter δ. This is to account for the fact that not all the towers may lie in a straight line. For each tower, other than the reference tower, this represents the angle between true north and the reference tower. Figure 2 shows how this angle is determined. For every other tower, beyond this reading, this δ would be a different value. The one exception would occur when all towers are on a straight line. In such a case δ would be a constant angle.

Phase angle determination

At this point it would be helpful to show how to calculate the correct value of phase angle (Ψ) to produce a null at any desired bearing. Once the angle of the tower line of a given two-tower pattern is established, and the spacing between the towers is set, the next step is to compute the phase angle. Figure 3 shows the relationship between the tower line, the spacing and the phase angle needed to produce a null at any desired angle. Keep in mind that when a null occurs in a two-tower pattern it means that the vectors from each tower are 180° out of phase and result in a cancelling of the total signal at that angle.

Knowing this fact, we can write the formula to be used as:

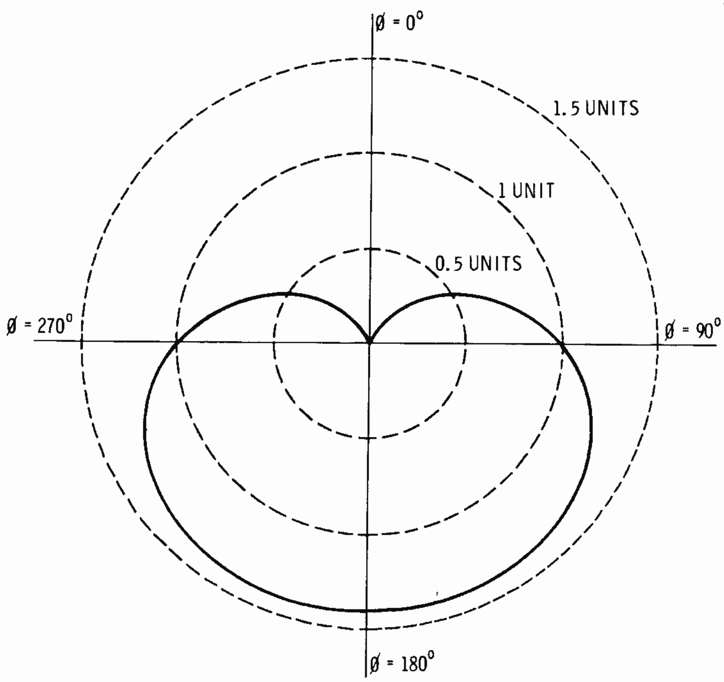


Figure 1

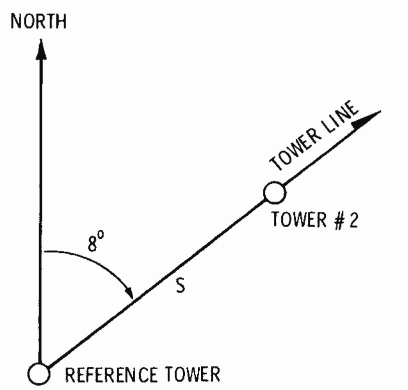


CHART SHOWING HOW ANGLE 8 IS DETERMINED

Figure 2

$$\pm 180^\circ = \Psi + S \cos \beta \text{ (Equation 7) or,}$$

$$\Psi = \pm 1.0 = 180^\circ \pm S \cos \beta \text{ (Equation 8)}$$

It is recognized that the cosine of 180° is always -1.0. This Ψ can be either a negative or a positive value. The angle β represents the azimuth angle from the line of the towers to the desired null bearing. With a little experience you will easily learn whether this phase angle (Ψ) is negative or a positive. Generally speaking, if the null angle is between zero and ninety degrees it is positive. A negative sign is used when the null falls between 90° and 180°.

Null fill

In summary, there are two basic ways to compute two-tower patterns. These are called the addition form and the multiplication form. In each case there is one term which is constant for each individual bearing. This is the term (Ψ+S cos Ø cos Θ). In fact, once you have calculated this term, the only other variables are the individual fields radiated by each of the two towers.

If we then say in the two-tower pattern that the null bearings are set, we can vary the "depth" of this null by varying the field ratios. When the individual fields are equal, the nulls will be "pulled in" to a theoretical zero signal. As the ratio between E_1 and E_2 goes up, this null fills in more and more. By the time this ratio gets down to 100/1 you will have, for all practical purposes, a non-directional antenna.

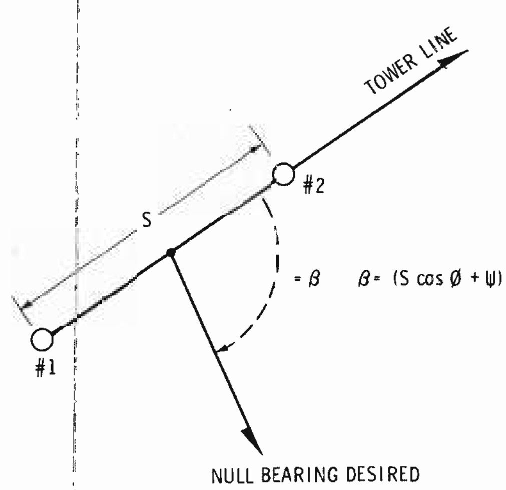


Figure 3

TABLE 3
CARDIOID PATTERN

Formula:

$$E = Kf(\Theta)\left[\frac{1+M^2}{2M} + \cos(\Psi + S\cos Ø \cos\Theta)\right]^{1/2}$$

Assumptions: f(Θ) = 1.0, M = 1.0, Ψ = -90°, S = 90°, Θ = 0°

A	B	C	D	E	F	G
Ø	90 cos A	B-90	cos C	1+D	√E	K•F
0	90 A	0.0°	1.000	2.000	1.414	254.9 MV/M
10	88.6	-1.4	.999	1.999	1.414	254.8
20	84.6	-5.4	.996	1.996	1.412	254.6
30	77.9	-12.1	.978	1.978	1.406	253.5
40	68.9	-21.1	.933	1.933	1.390	250.6
50	57.9	-32.1	.847	1.847	1.359	245.0
60	45.0	-45.0	.707	1.707	1.306	235.5
70	30.8	-59.2	.512	1.512	1.229	221.6
80	15.6	-74.4	.269	1.269	1.126	203.0
90	0.0	-90.0	0.0	1.000	1.000	180.3
100	-15.6	-105.6	-.269	.731	.855	154.1
110	-30.8	-120.8	-.572	.428	.654	117.9
120	-45.0	-135.0	-.707	.293	.541	97.5
130	-57.9	-147.9	-.847	.153	.391	70.5
140	-68.9	-158.9	-.933	.067	.259	46.7
150	-77.9	-167.9	-.978	.022	.148	26.7
160	-84.6	-174.6	-.996	.004	.063	11.4
170	-68.6	-178.6	-.999	.003	.017	3.0
180	-90.0	-180.0	-1.000	.000	.000	0.0
				18.942		
				x 2		
				37.884		
				- 2.000		
				35.884		

$$K = \frac{180}{\sqrt{\frac{35.884}{36.0}}} = 180.3$$

TABLE 4
THREE LEAF CLOVER

Equation: $E = Kf(\Theta)\left[\cos\left(\frac{\Psi}{2} + \frac{S}{2}\cos\Theta\cos Ø\right)\right]$

Assumptions: Ψ = 23.8°, S = 203.8°, f(Θ) = 1.0, Θ = 0°

A	B	C	D	E	F
Ø	$\frac{S}{2}$ cos A	$\frac{\Psi}{2}$+B	cos C	D^2	K•D
0	101.9	113.8	.4035	.1628	127.8
10	100.3	112.2	.3778	.1427	119.6
20	95.7	107.6	.3023	.0914	95.7
30	88.2	100.1	.1754	.0307	55.5
40	78.0	89.9	0.0000	0.0000	0.0
50	65.5	77.4	.2181	.0476	69.0
60	50.9	62.8	.4570	.2089	144.7
70	34.8	46.7	.6858	.4703	217.2
80	17.7	29.6	.8695	.7560	275.4
90	0.0	11.9	.9785	.9575	309.9
100	-17.7	-5.8	.9949	.9898	• 315.1
110	-34.8	-22.9	.9212	.8486	291.7
120	-50.9	-39.0	.7771	.6039	246.1
130	-65.5	-53.6	.5934	.3521	187.9
140	-78.0	-66.1	.4051	.1641	128.3
150	-88.2	-76.3	.2368	.0561	74.9
160	-95.7	-83.8	.1079	.0117	34.2
170	-100.3	-88.4	.0279	.0007	8.8
180	-101.9	-78.1	-.9000	0.0000	0.0
				5.8949	
				x 2	
				11.7898	
				- .1628	
				11.6270	

$$K = \frac{180}{\sqrt{\frac{11.627}{36}}} = 316.7 \text{ MV/M}$$

Well-formed nulls

At this point it should be pointed out that the nulls we have been talking about are those that you will find at a great distance over conductive flat earth. In other words, as you walk in closer and closer into a null, it will not hold. This is true for short distances, generally those less than 10 times the greatest element spacing. Near the directional array, predicted nulls cannot be deep, and may not seem like nulls at all. This is due to what I call the "parallax effect."

In some arrays the "inductive field" also will destroy the null in close to the towers. This generally occurs within five times the tower height. To use common FCC language, the null is considered not to be "well-formed."

Several factors actually affect the true signal you would observe on your field intensity meter as you walk closer and closer to the array. These are the angular displacement from parallel of each of the towers' signals. As we have noted in previous comments, the designer always assumed that all signals from all towers arrive parallel, and when you are standing at an observation point that is more than 10 times the greatest element spacing, they are. But here, close to the towers, they are not.

If you were to walk directly into the middle of a two-tower pattern, you would find that instead of being parallel, the two signals would be arriving from exactly opposite directions (180°).

A second factor is the difference in path lengths. Previously, we had talked about the fact that the difference in path lengths was calculated by S cos Ø. This can no longer be true because with non-parallel signals the angle Ø to the observor is not the same.

The third factor you must apply as a correction factor is to account for the fact that the loop antenna on the observer's field intensity meter will discriminate.

At this point you might question how or why the field meter's loop antenna will not read all signals equally, from all towers. This is because the nature of a loop antenna is to peak in the plane of the loop and reject along the axis of the loop. It is normally assumed that this discrimination varies as a function of the cosine of the angle between the plane of the loop and the angle of the respective incoming signals.

As a close to this consideration of two-tower patterns, the most common pattern shapes will be shown. These are the super cardioid with null fill, and the figure eight pattern. Table 6 is the calculation for the super cardioid by use of the multiplication method. Table 5 is the mathematical solution for a figure eight, again based upon the multiplication method. Figures 6 and 7, respectively, show the graphical plot of each of these designs.

It is physically impossible to give an example of every single kind of two-tower in use. It is therefore hoped that the reader will either see his type of pattern, or a representative of it, in the examples used.

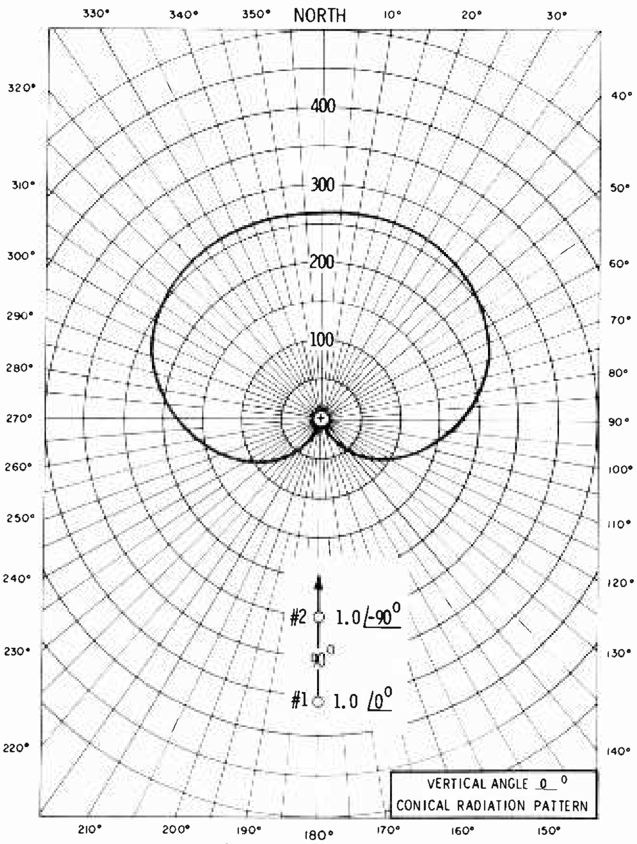


Figure 4 TABLE 3

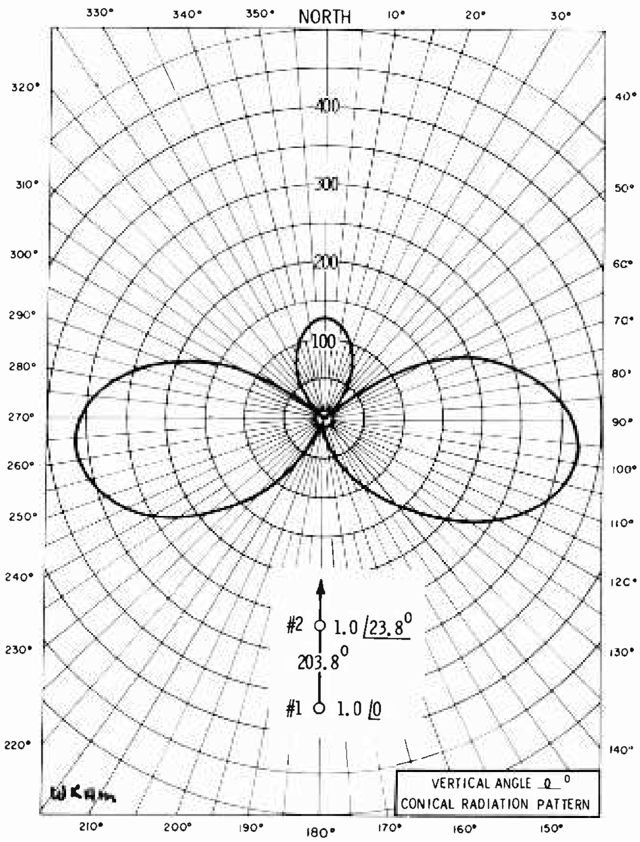


Figure 5 TABLE 4

TABLE 5
FIGURE EIGHT DESIGN

Formula:

$$E = Kf(\Theta)\left[\frac{1 + M^2}{2M} + \cos(\Psi + S\cos\phi\cos\Theta)\right]^{1/2}$$

Assumptions: $f(\Theta) = 1.0$, $M = 1.0$, $\Psi = 0°$,

$S = 180°$, $\Theta = 1.0$

A	B	C	D	E	F	G	
Ø	S cos A	Ψ+B	cos C	1+D	√E	K•F	
0	180.0	180.0	-1.0000	0.0000	0.0000	0.0	MV/M
10	177.2	177.2	-.9988	.0012	.0346	7.5	
20	169.2	169.2	-.9823	.0177	.1330	28.7	
30	155.8	155.8	-.9121	.0879	.2960	63.8	
40	137.8	137.8	-.7408	.2592	.5090	109.8	
50	115.8	115.8	-.4352	.5648	.7510	161.9	
60	90.0	90.0	0.0000	1.0000	1.0000	215.7	
70	61.4	61.4	.4787	1.4787	1.2160	262.3	
80	31.2	31.2	.8554	1.8554	1.3620	293.8	
90	0.0	0.0	1.0000	2.0000	1.4140	• 304.9	
100	-31.2	-31.2	.8554	1.8554	1.3620	293.8	
110	-61.4	-61.4	.4787	1.4787	1.2160	262.3	
120	-90.0	-90.0	0.0000	1.0000	1.0000	215.7	
130	-115.8	-115.8	-.4352	.5648	.7510	161.9	
140	-137.8	-137.8	-.7408	.2592	.5090	109.8	
150	-155.8	-155.8	-.9121	.0879	.2960	63.8	
160	-169.2	-169.2	-.9823	.0177	.1330	28.7	
170	-177.2	-177.2	-.9988	.0012	.0346	7.5	
180	-180.0	-180.0	-1.0000	0.0000	0.0000	0.0	
				12.5298			
				25.0596			

$$K = \frac{180\text{ MV/M}}{\sqrt{\frac{25.0596}{36}}} = 215.7$$

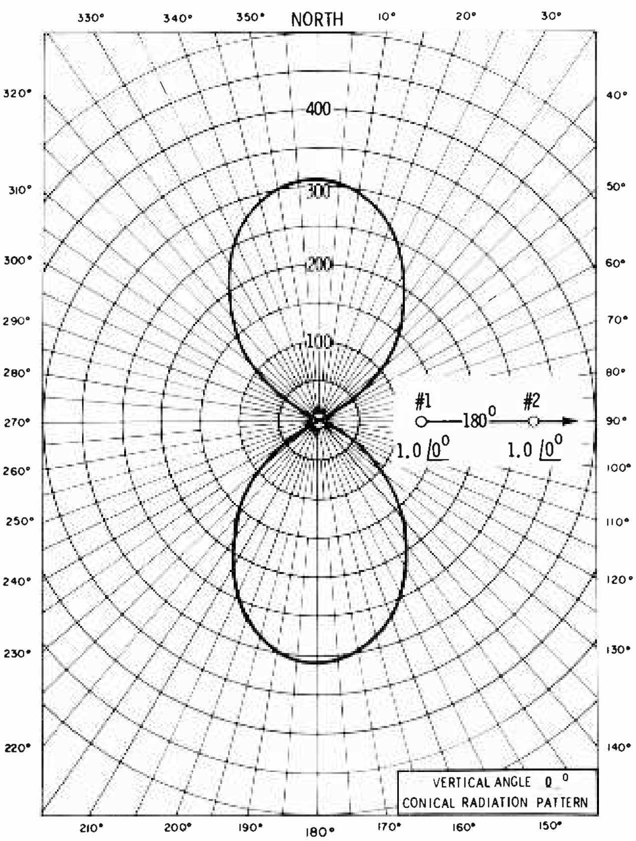


Figure 6 TABLE 5

TABLE 6
SUPER-CARDIOD-FILLED NULL

Formula:

$$E = Kf(\Theta)\left[\frac{1 + M^2}{2M} + \cos(\Psi + S\cos\phi\cos\Theta)\right]^{1/2}$$

Assumptions: $f(\Theta) = 1.0$, $M = 0.5$, $\Psi = -116.4$,

$S = 90$, $\Theta = 0°$

A	B	C	D	E	F	G	
Ø	S cos A	B - Ψ	cos C	D+1.250	√E	K•F	
0	90.0	-26.4	.8957	2.145	1.465	258.6	MV/M
10	88.6	-27.8	.8846	2.134	1.461	257.9	
20	84.6	-31.8	.8499	2.099	1.449	255.7	
30	77.9	-38.5	.7826	2.033	1.426	251.7	
40	68.9	-47.5	.6756	1.925	1.387	244.8	
50	57.9	-58.5	.5225	1.772	1.331	234.9	
60	45.0	-71.4	.3189	1.568	1.253	221.1	
70	30.8	-85.6	.0767	1.326	1.152	203.3	
80	15.6	-100.8	-.1874	1.063	1.031	181.9	
90	0.0	-116.4	-.4446	.805	.897	158.3	
100	-15.6	-132.0	-.6691	.581	.762	134.5	
110	-30.8	-147.2	-.8406	.409	.639	112.8	
120	-45.0	-161.4	-.9477	.302	.549	96.9	
130	-57.9	-174.3	-.9950	.255	.505	89.1	
140	-68.9	-185.3	-.9957	.254	.504	88.9	
150	-77.9	-194.3	-.9690	.281	.530	93.5	
160	-84.6	-201.0	-.9336	.316	.563	99.4	
170	-88.6	-205.0	-.9063	.344	.586	103.4	
180	-90.0	-206.4	-.8957	.354	.595	105.0	
				19.966			
				x 2			
				39.932			
				- 2.145			
				- .354			
				37.433			

$$K = \frac{180}{\sqrt{\frac{37.433}{36}}} = 176.5$$

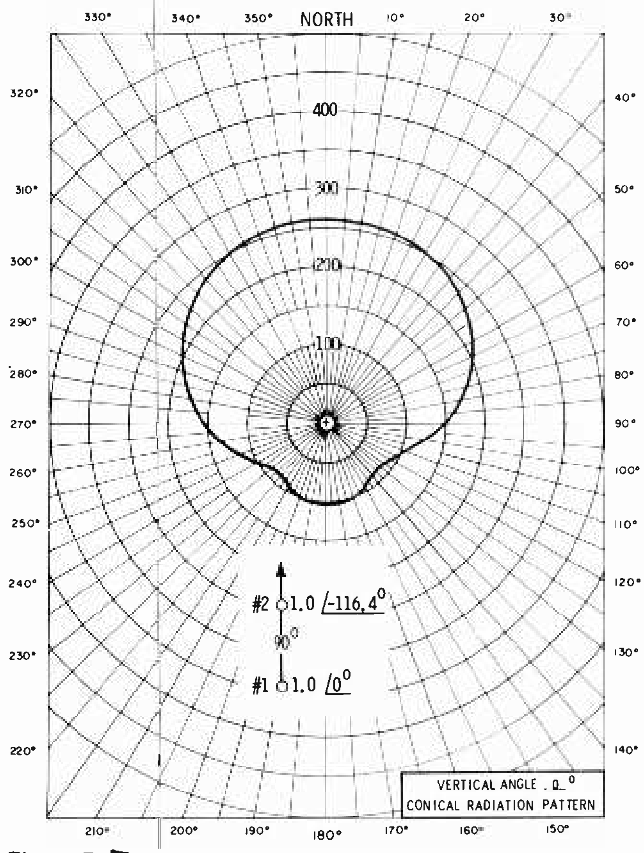


Figure 7 TABLE 6

4

Design of three-tower systems

This chapter will consider both the basic three-tower pattern shapes as well as the basic methods of computation, with useful examples.

Three-tower arrays are usually required when a two-tower design will not provide enough nulls, or where the protection required must be over a wider arc than can be achieved with a single null. There are two common methods of computing typical three-tower patterns. These are, like the two-tower patterns, the addition method and the multiplication method.

Addition formula

Like the two-tower formulas, the addition form for three towers consists of adding the respective tower vectors and employing basic trigonometry steps. In fact, what really has been done is that a third tower has been added into the two-tower pattern at the mid-point or reference spot. This can be represented as follows (Equation 1):

$$E = Kf(\Theta)(E_1 + E_2 \angle \underline{\Psi + S\cos\emptyset\cos\Theta}) + E_3 \angle \underline{\Psi - S\cos\emptyset\cos\Theta})$$

In this formula, E_1 is assumed to be the center tower of a three-tower in-line array.

The most common approach says that each end tower is equally spaced from the center (reference) tower, (this is more by custom than necessity); and that each has the same phase angle, but opposite signs. Typical three-tower patterns will produce at least two nulls on each side of the tower line. If these two nulls are to be of equal depth then the magnitude of each end tower (E_2 and E_3) must be equal. When this occurs a special case results that can be computed by the following formula (Equation 2):

$$E = Kf(\Theta)[F_1 + 2F_2 \angle \underline{S\cos\emptyset\cos\Theta + \Psi}]$$

There are similarities between this formula and the one in Chapter 3 for the "half angle formula." Except that here spacing is S instead of S/2, and the phasing is Ψ instead of $\Psi/2$. One special advantage to this formula is that the nulls can be filled equally by changing the phase of the center (reference) tower. The greater the number of degrees introduced, the greater will be the amount of null fill. There are no sine terms because they cancel, being equal and of opposite signs.

The example of Table 1 shows the step-by-step calculation of a typical three-tower pattern by this method. Figure 3 is a graphic representation of the vectors of the three towers in Table 1 and Figure 2. At Bearings A and B the reader can see that all three vectors add up to zero, hence these are the two null bearings. The size of the minor lobe is represented by the amount of overlap of the three vectors at C.

By using this vector plot, it is easy to see that the angle or arc between the two nulls can be changed by increasing or decreasing the magnitude of the end vectors. It must be recognized that the size of the minor lobe will be affected by the arc between these nulls. The wider this arc, the larger the size of this lobe. The nulls can be moved forward or backward along the tower line by rotating the beginning point of the end vectors.

Seldom used method

In addition to the general method outlined above, there is a way of considering a three-tower design as being the sum of a non-directional tower added to a two-tower pattern. Figure 1A shows how this relationship is accomplished. As with the four-tower addition methods, signs of (+) and (-) are assigned to each lobe. This method works only for a case of three towers being on the same plane (straight line).

Table 3 is a compilation of the mathematical solution to this method. A polar plot is shown on Figure 9.

Three-tower multiplication formulas

This is a very interesting way of designing a three-tower pattern. It consists of literally taking two separate two-tower patterns and multiplying them so that the result is an equivalent three-tower pattern.[1] In using this method one must make certain assumptions.

One assumption is that the spacing between the towers of each individual two-tower pattern is equal to that between adjacent towers on the final three-tower array. Another assumption is that the

1. Jones, R.A.: Two-Tower Tests. *Broadcast Engineering*, February 1968.

TABLE 1
THREE-TOWER SPECIAL CASE

$$E = Kf(\Theta)\ (F_1 + 2F_2 \cos \underline{/S \cos \emptyset \cos \Theta + \Psi)}$$

Where
$F_1 = 1.7$, $F_2 = 1.0$, $S = 90°$, $\Theta = 0°$, and
$\Psi = 167.6°$

$$[1.7 + 2.0 \cos \underline{/90 \cos \emptyset \cdot 1 + 167.6°}]$$

A	B	C	D	E	F	G
Ø	90 cos A	B+167.6	2.0•cos C	1.7+D	E^2	E•K
0	90.0	257.6	-.429	1.270	1.613	149.3 MV/M
10	88.6	256.2	-.477	1.222	1.495	143.6
20	84.5	252.1	-.615	1.085	1.178	127.5
30	77.9	245.5	-.829	.870	.758	102.2
40	68.9	236.5	-1.104	.596	.355	70.0
50	57.8	225.4	-1.404	.295	.087	34.6
60	45.0	212.6	-1.684	.015	.....	1.7
70	30.8	198.4	-1.897	-.198	.039	23.3
80	15.6	183.2	-1.997	-.297	.088	34.9
90	0.0	167.6	-1.953	-.253	.064	29.7
100	-15.6	152.0	-1.765	-.065	.004	7.6
110	-30.8	136.8	-1.458	.242	.058	28.4
120	-45.0	122.6	-1.077	.622	.367	73.0
130	-57.8	109.8	-.677	1.022	1.045	120.0
140	-68.9	98.7	-.302	1.397	1.953	164.1
150	-77.9	89.7	.010	1.710	2.926	200.9
160	-84.5	83.1	.240	1.940	3.765	227.9
170	-88.6	79.0	.382	2.081	4.333	244.5
180	-90.0	76.6	.463	2.163	4.680	254.1

$$K = \frac{129}{\sqrt{\frac{43.36}{36}}} = 117.5$$

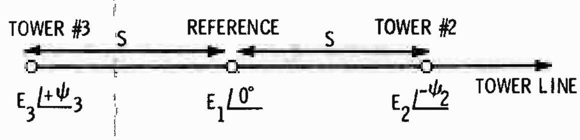


Figure 1

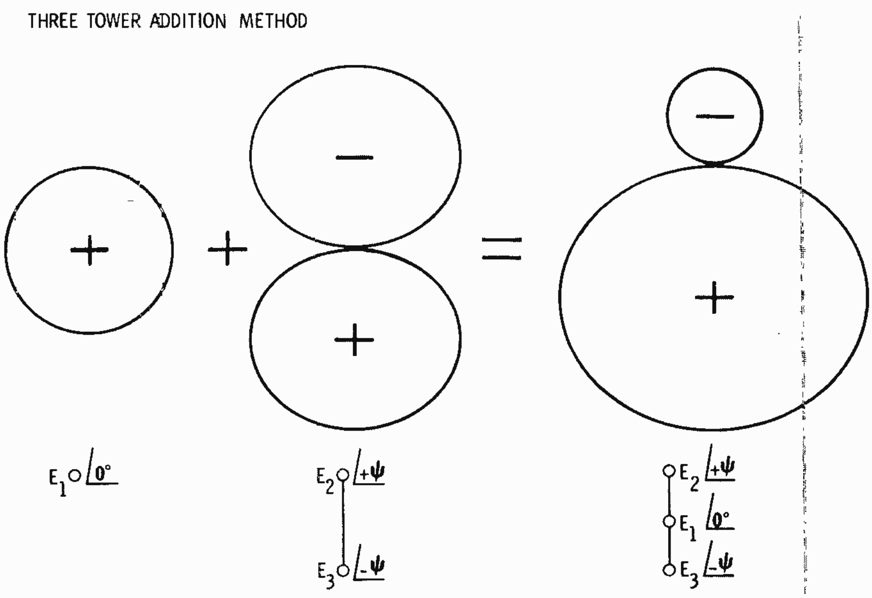


Figure 1A

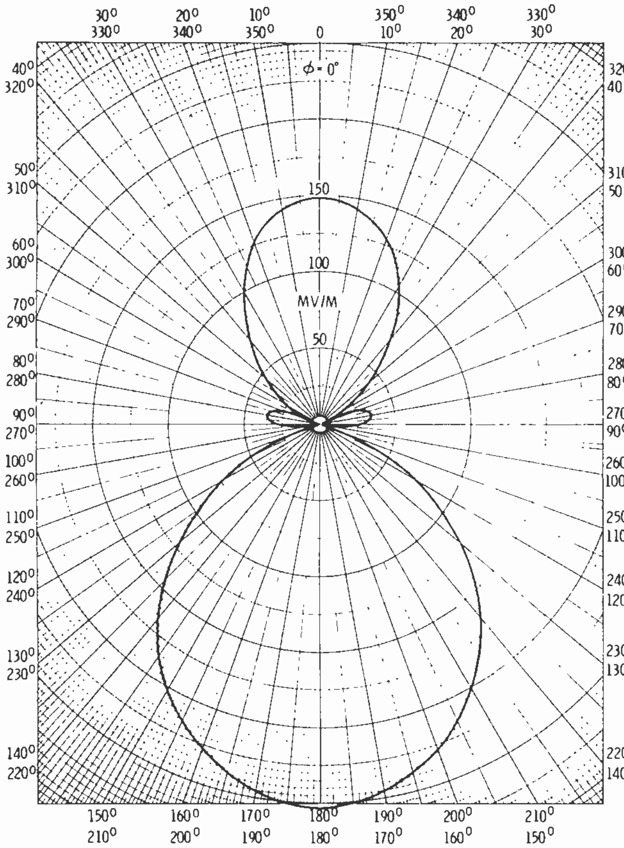


Figure 2

reference tower line for each two-tower pattern is the same as the reference for the final pattern. Also, all towers are assumed to be of equal height. If Equation 3 of Chapter 3 is used to represent a typical two-tower pattern, and you multiply the radical terms together, then (Equation 3):

$$E = K^1 f(\Theta)\sqrt{\frac{1+M_1^2}{2M_1} + \cos(\Psi_1 + S\cos\Theta\cos\emptyset)}\ x$$
$$K^1 f(\Theta)\sqrt{\left[\frac{1+M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\Theta\cos\emptyset)\right]}$$

Actually there was an extra $Kf(\Theta)$ term from the second pattern, but since $f(\Theta)$ was the same for each, one $f(\Theta)$ will suffice. Also, the K term from the second equation combines into a new K^1 term.

Figure 4 shows how two, two-tower patterns can be used to produce a three-tower array. The next step is to show how to move from the basic design values of each individual pattern to arrive at the final three-tower value. In Figure 4 the design values are below each of the two-tower patterns. The end tower is assumed to be the reference tower, hence it is taken as having a value of $1.0\angle 0°$.

The center tower is calculated as follows (Equation 4):

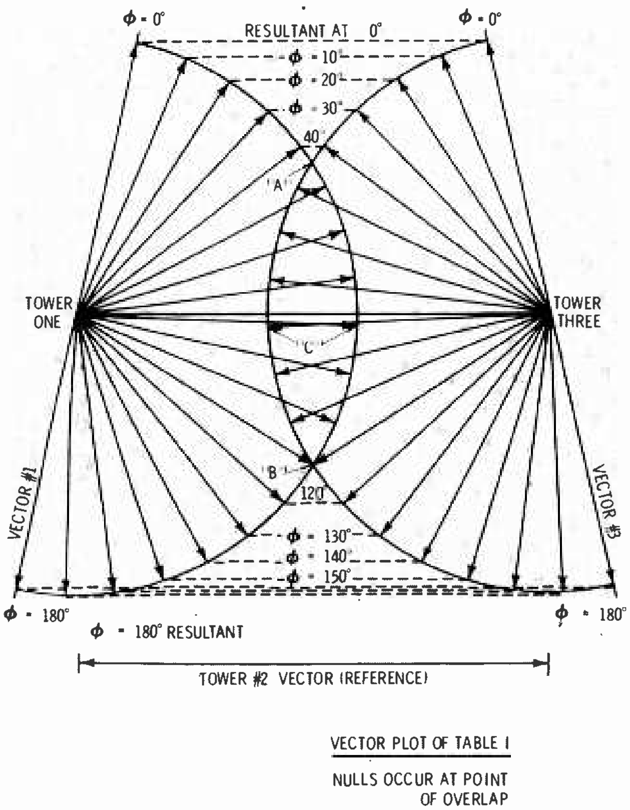


Figure 3

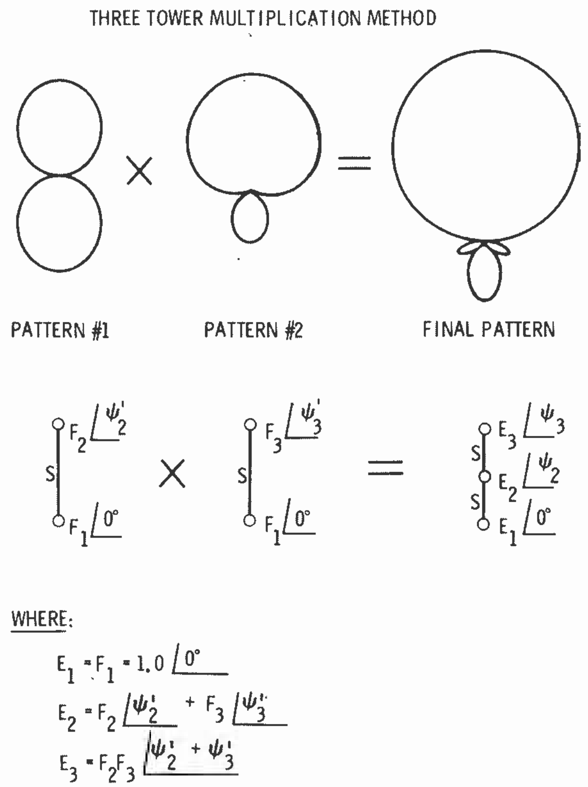


Figure 4

TABLE 2
THREE-TOWER MULTIPLICATION

$$E = Kf(\Theta)\left[\left(\frac{1+1^2}{2} + \cos(180 + 90\cos\emptyset)\right) \times \left(\frac{1+1^2}{2} + \cos(-116.4 + 90\cos\emptyset)\right)\right]^{1/2}$$

A	B	C	D	E	F	G	H	I	J	K
Ø	90 cos A	180+B	cos C	1+D	B-116.4	cos F	1+G	E•H	$I^{1/2}$	$J \cdot K^1$
0	90.0	270.0	.000	1.000	-26.4	.896	1.896	1.896	1.377	399.3 MV/M
10	88.6	268.5	-.024	.976	-27.8	.885	1.885	1.839	1.356	393.2
20	84.5	264.5	-.096	.904	-31.9	.849	1.849	1.671	1.293	375.0
30	77.9	257.9	-.209	.791	-38.5	.783	1.783	1.410	1.188	344.5
40	68.9	248.9	-.359	.641	-49.5	.649	1.649	1.057	1.028	298.1
50	57.8	237.8	-.533	.467	-58.6	.521	1.521	.710	.843	244.5
60	45.0	225.0	-.707	.293	-71.4	.319	1.319	.386	.621	180.0
70	30.8	210.8	-.859	.141	-85.6	.077	1.077	.152	.389	112.8
80	15.6	195.6	-.963	.037	-100.8	-.187	.813	.030	.173	50.2
90	0.0	0.0	-1.000	.000	-116.4	-.445	.555	.000	.000	0.0
100	-15.6	164.4	-.963	.037	-132.0	-.669	.331	.012	.111	32.2
110	-30.8	149.2	-.859	.141	-147.2	-.641	.159	.022	.149	43.2
120	-45.0	135.0	-.707	.293	-161.4	-.948	.052	.015	.123	35.7
130	-57.8	122.2	-.533	.467	-174.2	-.995	.005	.002	.048	13.9
140	-68.9	111.1	-.359	.641	-185.3	-.996	.004	.002	.048	13.9
150	-77.9	102.1	-.209	.791	-194.3	-.969	.031	.024	.156	45.2
160	-84.5	95.5	-.096	.904	-200.9	-.934	.066	.059	.244	70.8
170	-88.6	91.4	-.024	.976	-205.0	-.906	.094	.092	.303	87.9
180	-90.0	90.0	.000	1.000	-206.4	-.896	.104	.104	.322	93.4

K = 289.8

TABLE 3
THREE-TOWER ADDITION BY SPECIAL CASE

Equation: $E = Kf(\Theta)\left[ND + \left(1 + \cos(\Psi + S\cos\Theta\cos\emptyset)\right)\right]^{1/2}$

Assumptions: $ND = 1.198$, $\Psi = 180°$, $f(\Theta) = 1.0$, $\Theta = 0°$

A	B	C	D	E	F	G	H	I
Ø	S cos D	Ψ+B	cos C	1+D	$\sqrt{E^*}$	F+1.198**	G^2	K•G
0	180.0	0.0	1.0000	2.0000	1.4140	2.612	6.822	283.9 MV/M
10	177.2	-2.8	.9988	1.9988	1.4130	2.611	6.817	283.8
20	169.2	-10.8	.9823	1.9823	1.4080	2.606	6.791	283.3
30	155.8	-24.2	.9121	1.9121	1.3830	2.581	6.661	280.6
40	137.8	-42.2	.7408	1.7408	1.3190	2.517	6.335	273.6
50	115.8	-64.2	.4352	1.4352	1.1980	2.396	5.741	260.4
60	90.0	-90.0	0.0000	1.0000	1.0000	2.198	4.831	238.9
70	61.4	-118.6	-.4787	.5213	.7220	1.920	3.636	208.7
80	31.2	-148.8	-.8554	.1446	.3800	1.578	2.490	171.5
90	0.0	-180.0	-1.0000	0.0000	0.0000	1.198	1.435	130.2
100	-31.2	-211.2	-.8554	.1446	.3800	.868	.753	94.3
110	-61.4	-241.4	-.4787	.5213	-.7220	.476	.226	51.7
120	-90.0	-270.0	.0000	1.0000	-1.0000	.198	.039	21.5
130	-115.8	-295.8	.4352	1.4352	-1.1980	.000	.000	0.0
140	-137.8	-317.8	.7408	1.7408	-1.3190	-.121	.015	13.1
150	-155.8	-335.8	.9121	1.9121	-1.3830	-.185	.034	20.1
160	-169.2	-349.2	.9823	1.9823	-1.4080	-.210	.044	22.8
170	-177.2	.357.2	.9988	1.9988	-1.4130	-.215	.046	23.4
160	-180.0	-360.0	1.0000	2.0000	-1.4140	-.216	.047	23.5

*Second lobe assigned (-) sign

**N-D magnitude

$$K = \frac{180}{\sqrt{\frac{98.756}{36}}} = 108.7$$

52.812
x 2
105.625
- 0.047
- 6.822
98.756

FIG 9

$$\text{No. } 2 = F_1 \angle \Psi_1 + F_2 \angle \Psi_2$$

and for the other end tower we use this formula (Equation 5):

$$\text{No. } 3 = F_1 \times F_2 \angle \Psi_1 + \Psi_2$$

The terms are those shown in Figure 4. These two-tower design values are combined by the usual vector mathematics to achieve the final values. For an example, a two-tower pattern with nulls at 90°

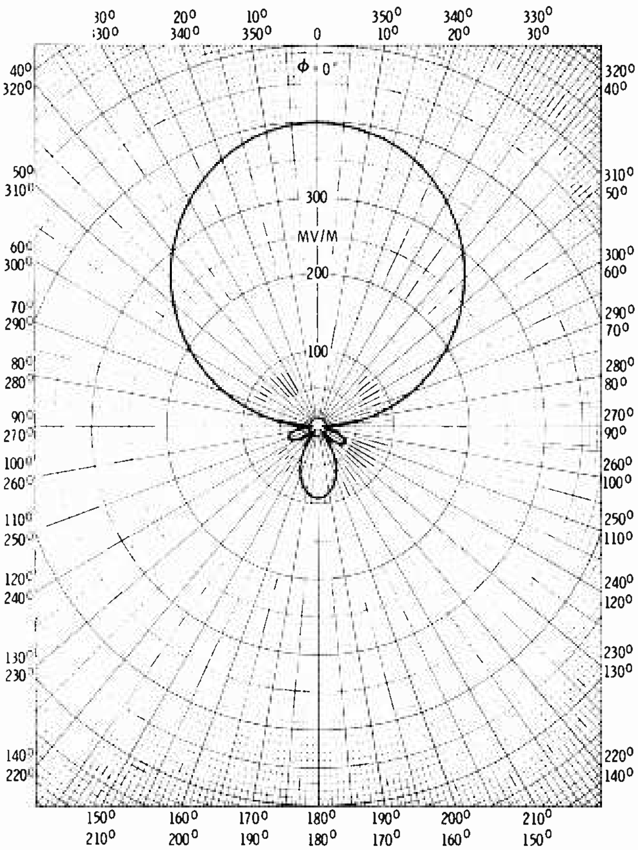


Figure 5

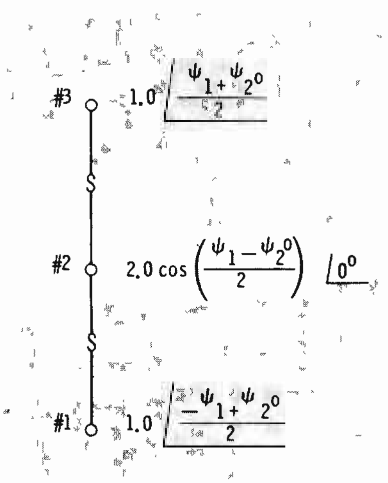


Figure 6

and a second with nulls at 135° off the tower line has been chosen. The calculation is shown in Table 2, and results in the pattern plotted in Figure 5.

One of the special advantages of using this multiplication method is that each null can be placed separately, and the depth of each null is independent of the other nulls. In the foregoing three-tower addition formula, there is no such design flexibility. For this reason almost all design engineers use the multiplication method.

A special case of the three-tower multiplication formula occurs when the magnitudes of towers F_1 and F_2 are equal to the reference tower. In such a case Equation 3 can be rewritten as follows (Equation 6):

$$E = Kf(\Theta)\sqrt{[1+\cos(\Psi_1 + S\cos\Theta\cos\emptyset)]} \times Kf(\Theta)\sqrt{[1+\cos(\Psi_2 + S\cos\Theta\cos\emptyset)]}$$

For this condition zero nulls will be produced in the final three-tower array. It will be customary to change the reference tower of the final pattern from the end to the center tower. The phase angle of the center tower is then changed to $\angle 0°$ by subtracting out a $\Psi_1+\Psi_2$ from each term in Equations 4 and 5. The end results are shown in Figure 6.

This special case can be refined even one more step, if the center tower of the new pattern is made equal to unity. Equation 6 can then be rewritten (Equation 7):

$$E = Kf(\Theta)\left[\frac{1}{2E} + \cos(\Psi + S\cos\Theta\cos\emptyset)\right]$$

In this formula, E represents the field ratio of each of the end towers, as compared to unity for the center tower. All the other terms are as previously explained.

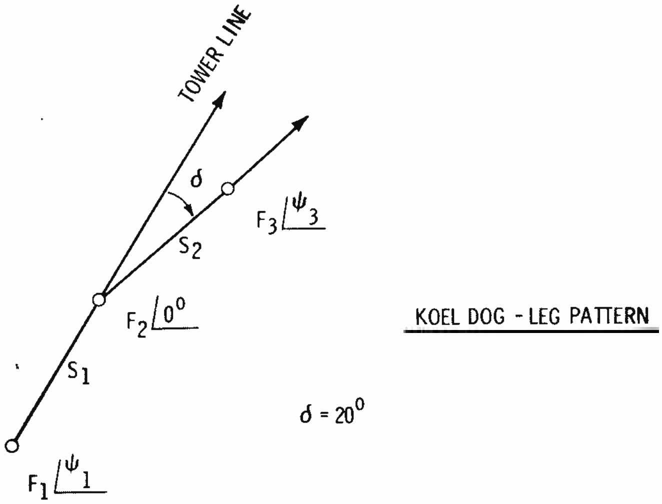
Null filling

Producing other than zero depth nulls in a typical three-tower multiplication array can be done in one of two ways. First, consider the case where you want both nulls filled an equal amount. The easiest way to do this is to shift the reference tower angle away from 0° by a few degrees. The greater the shift introduced, the greater will be the null fill.

If, however, you want to fill only one null or to fill each pair of nulls by a different amount, then the value of F_1 or F_2 in Figure 4 must be made slightly different than unity. As pointed out above, unity in E_1 and E_2 will produce zero depth nulls. As the relative magnitude of F_1 or F_2 decreases, the greater will be the "filling in" of any given null.

Three-tower dog-leg arrays

There is one other three-tower pattern form besides the usual "in-line" arrays. These, as the name "dog-leg" implies, are cases where the three towers are not in a straight line. There are no shortcut formulas for this type of array. You will have to use



FORMULA FOR ANY DOG - LEG ARRAY

$$E_t = K\left[F_2 + F_3\angle\psi_1 - S_1\cos\phi + F_3\angle\psi_3 + S_2\cos(\phi-\delta)\right]$$

SUBSTITUTING FOR THE KOEL DESIGN:

$$E_t = K\left[1.0 + 0.66\angle 146 - 80\cos\phi + 0.60\angle -140 + 90\cos(\phi-20)\right]$$

Figure 7

Equation 1, the general addition case of a three-tower array. The most common need for a dog-leg type pattern is that used to produce a lopsided shape to a basic three-tower pattern. The degrees of lopsidedness of course depends upon the spacing of this third tower, the amount of energy it radiates, and its phase angle. The only simple way to design a pattern like this is by the cut and try method, although vector algebra is a useful tool. Figure 7 contains the data used to calculate the dog-leg design at KOEL. Figure 8 is a graphical plot of same.

A second, more dramatic, example of a dog-leg array is that calculated in Table 6 and graphically displayed in Figure 12. In this array, it can be seen that the towers are at right angles plus greatly different in spacing.

In review, Table 4 is a calculation of a basic three-tower pattern using the addition method. Its results are plotted in the polar graph of Figure 10. A typical figure eight used to achieve high gain in the major lobe, coupled with maximum suppression is demonstrated in Figure 11, and calculated in Table 5.

The basic three-tower pattern and basic step-by-step calculations have been shown. And, if your particular pattern was not one of those used, there was a representative one. Keep in mind that except when using one of the addition methods, the final base current ratios and final base phase angles are not the same as those used in the mathematical computations.

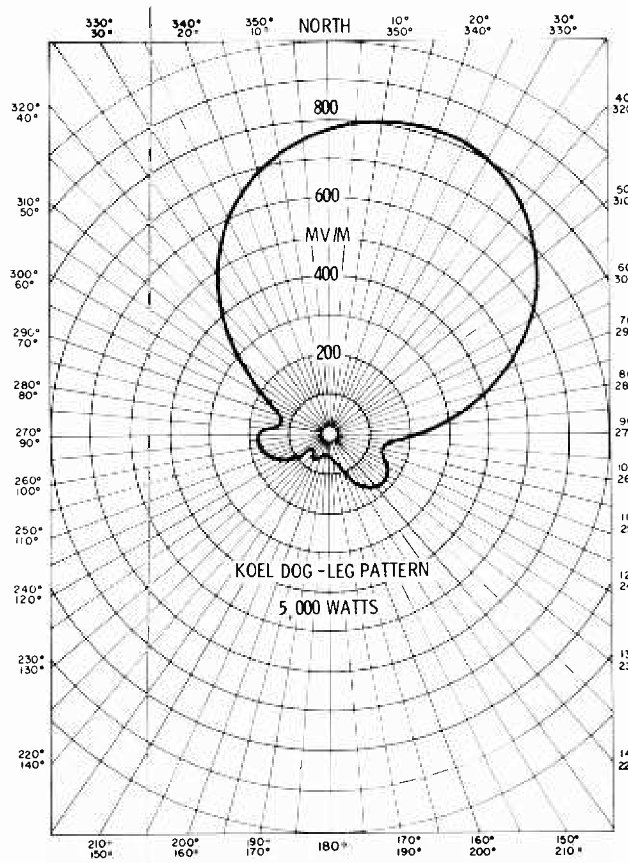


Figure 8

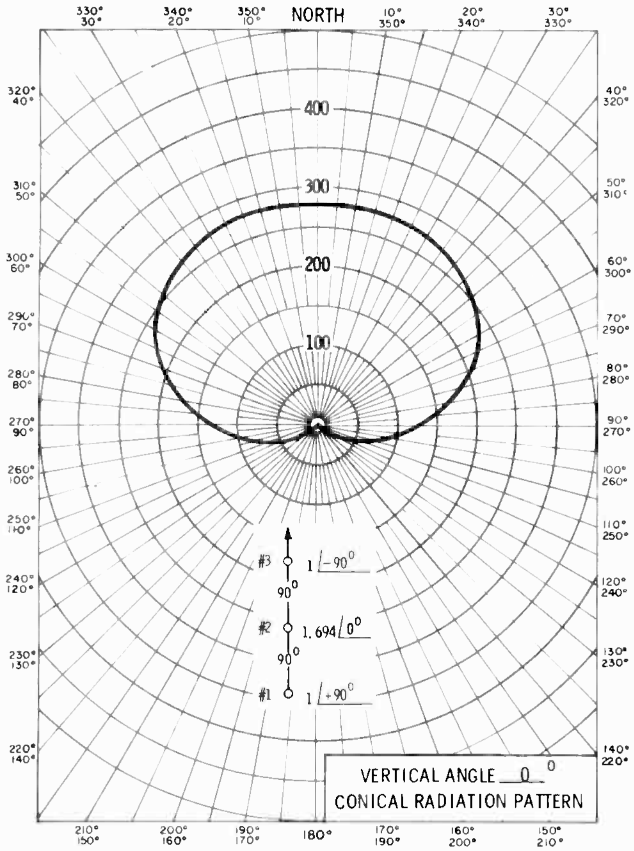


Figure 9

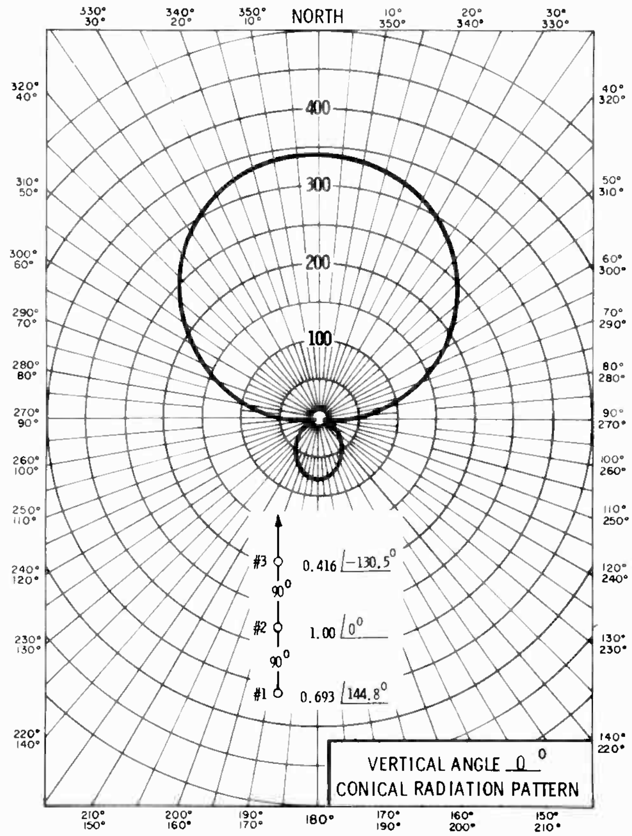


Figure 10

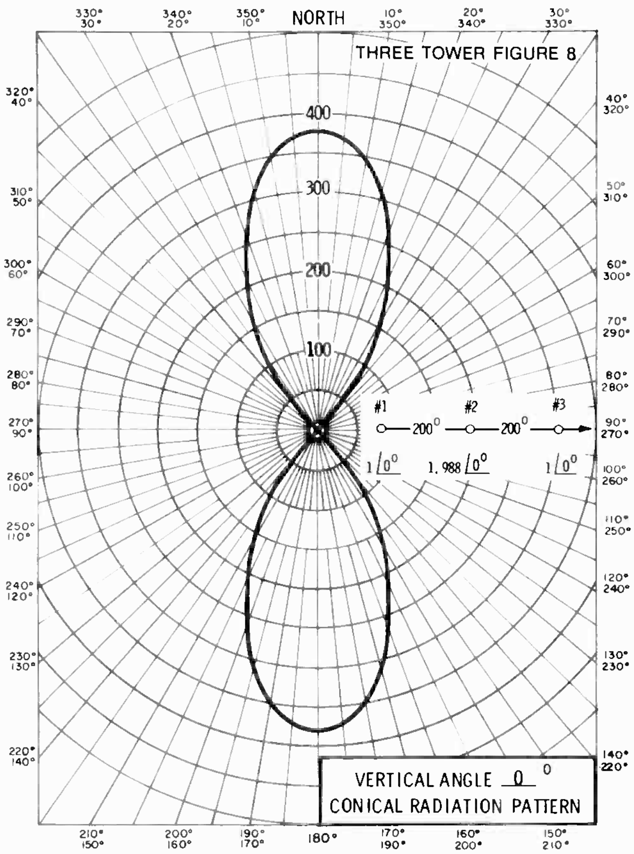


Figure 11

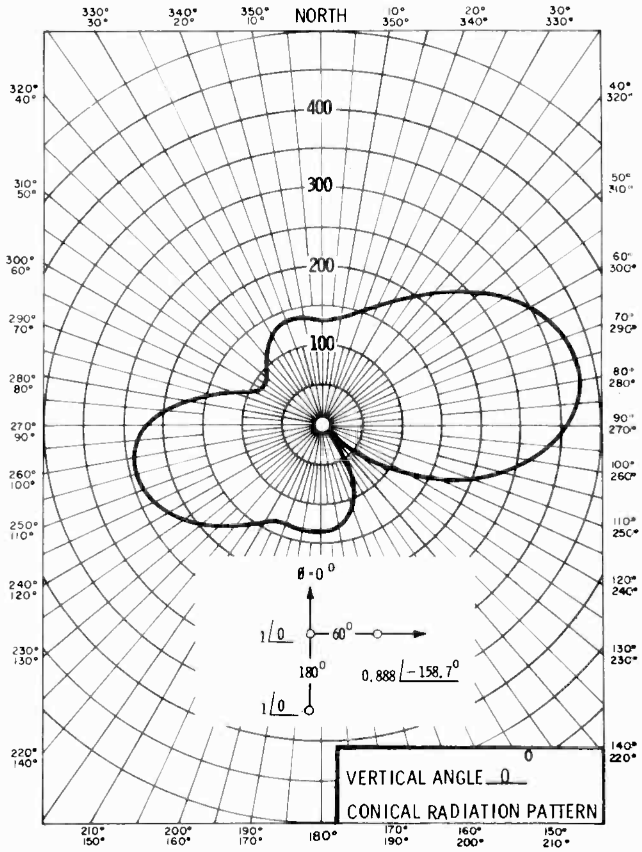


Figure 12 TABLE 6

TABLE 4
THREE-TOWER BY ADDITION

Formula: $E = Kf(\Theta)\,[E_1 \angle 0 + E_2 \angle \underline{\Psi_2 + S\cos\emptyset\cos\Theta} + E_3 \angle \underline{\Psi_3 + S\cos\emptyset\cos\Theta}]$

Assumptions: $E_1 = 1.000$, $E_2 = 0.416$, $E_3 = 0.693$, $\Psi_2 = -130.5°$, $\Psi_3 = +144.8°$, $f(\Theta) = 1.0$, $\Theta = 0°$

A	B	C	D	E	F	G	H	I	J	L	M	N	O	P
∅	S cos A	Ψ_2+B	.416 cos C	.416 Sin C	Ψ_3-B	.693cosF	.693 Sin F	1+D+G	I^2	E+H	L^2	J+M	$\sqrt{N}$	K•O
0	90.0	-40.5	.316	-.270	54.8	.399	.566	1.715	2.943	.296	.088	3.031	1.741	341.3 MV/M
10	88.9	-41.6	.311	-.276	55.9	.388	.574	1.699	2.888	.298	.089	2.977	1.725	338.1
20	84.6	-45.9	.289	-.297	60.2	.344	.601	1.633	2.668	.304	.093	2.761	1.662	325.7
30	77.9	-52.6	.253	-.330	66.9	.271	.637	1.525	2.325	.307	.095	2.420	1.556	304.9
40	68.9	-61.6	.198	-.366	75.9	.169	.693	1.367	1.868	.306	.094	1.962	1.400	274.4
50	57.9	-72.6	.124	-.397	86.9	.037	.692	1.161	1.349	.295	.087	1.436	1.198	234.8
60	45.0	-85.5	.033	-.415	99.8	-.118	.683	.915	.837	.268	.072	.909	.954	186.8
70	30.8	-99.7	-.070	-.410	114.0	-.282	.633	.648	.420	.223	.049	.469	.685	134.2
80	15.6	-114.8	-.175	-.378	129.2	-.438	.537	.387	.149	.159	.025	.174	.417	81.7
90	0.0	-130.5	-.270	-.316	144.8	-.566	.399	.164	.027	.083	.006	.033	.181	35.5
100	-15.6	-146.1	-.345	-.232	160.4	-.653	.232	.002	.000	.000	.000	.000	.000	0.0
110	-30.8	-161.3	.394	-.133	175.6	-.691	.053	.085	.007	-.080	.006	.013	.114	22.3
120	-45.0	-175.5	-.415	-.033	189.8	-.683	-.118	.098	.009	-.151	.023	.032	.179	35.1
130	-57.9	-188.4	-.412	.061	202.7	-.639	-.267	.051	.003	-.206	.043	.046	.214	41.9
140	-68.9	-199.4	.393	.138	213.7	-.576	-.385	.030	.001	-.246	.061	.062	.249	48.8
150	-77.9	-208.4	-.366	.198	222.7	-.509	-.469	.125	.016	-.272	.074	.090	.300	58.8
160	-84.6	-215.1	-.340	.239	229.4	-.451	-.526	.209	.044	-.287	.082	.126	.355	69.6
170	-88.9	-219.4	-.321	.264	233.7	-.410	-.558	.268	.072	-.295	.087	.159	.399	78.2
180	-90.0	-220.5	-.316	.270	234.8	-.399	-.566	.285	.081	-.296	.088	.169	.411	80.5
												16.776		
											x	2		
												33.552		
												-3.031		
												- .169		
												30.352		

$$K = \frac{180}{\sqrt{\frac{30.352}{36}}} = 196.0$$

TABLE 5
THREE-TOWER FIGURE EIGHT

$$\text{Formula: } E = Kf(\Theta)\left[\left(\frac{1 + M_1^2}{2M_1} + \cos(\Psi_1 + S\cos\emptyset\cos\Theta)\right) \text{x} \left(\frac{1 + M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\emptyset\cos\Theta)\right)\right]^{1/2}$$

Assumptions: $M_1 = M_2 = 1.0$, $\Psi_1 = +7°$, $\Psi_2 = -7°$, $f(\Theta) = 1.0$, $\Theta = 0°$, $S = 200°$

A	B	C	D	E	F	G	H	I
∅	S cos A	B+7	1+cos C	B-7	1+cos E	D•F	$\sqrt{G}$	K•H
0	200.0	207.0	.109	193.0	.026	.002	.053	10.1 MV/M
10	196.9	203.9	.086	189.9	.015	.001	.036	6.9
20	187.9	194.9	.034	180.9	.000	.000	.000	0.0
30	173.3	180.3	.000	166.3	.028	.000	.000	0.0
40	153.2	160.2	.059	146.2	.169	.010	.099	18.9
50	128.5	135.5	.287	121.5	.478	.137	.370	70.8
60	100.0	107.0	.708	93.0	.948	.671	.819	156.8
70	68.4	75.4	1.252	61.4	1.478	1.850	1.360	260.4
80	34.7	41.7	1.747	27.7	1.885	3.293	1.815	347.6
90	0.0	7.0	1.993	-7.0	1.993	3.972	1.993	381.6
100	-34.7	-27.7	1.885	-41.7	1.747	3.293	1.815	347.6
110	-68.4	-61.4	1.478	-75.4	1.252	1.850	1.360	260.4
120	-100.0	-93.0	.948	-107.0	.708	.671	.819	156.8
130	-128.5	-121.5	.478	-135.5	.287	.137	.370	70.8
140	-153.2	-148.2	.169	-160.2	.059	.010	.099	18.9
150	-173.3	-166.3	.028	-180.3	.000	.000	.000	0.0
160	-187.9	-180.9	.000	-194.9	.034	.000	.000	0.0
170	-196.9	-189.9	.015	-203.9	.086	0.001	0.036	6.9
180	-200.0	-193.0	.026	-207.0	.109	.002	.053	10.1
						15.900		
						x 2		
						31.800		
						- .002		
						- .002		
						31.796		

$$K = \frac{180}{\sqrt{\frac{31.796}{36}}} = 191.5$$

TABLE 6
THREE-TOWER DOG LEG

Equation: $E = Kf(\Theta)\,[1\angle 0 + 1\angle \overline{-S_2 \cos \phi \cos \Theta} + .888\angle \overline{\Psi_3 + S_3 \cos \Theta \cos (\phi - 90)}]$

Assumptions: $E_1 = 1.0$, $E_2 = 1.0$, $E_3 = .888$, $\Psi_2 = 0°$, $\Psi_3 = -158.7$, $S_2 = 180°$, $S_3 = 60°$

A	B	C	D	E	F	G	H	I	J	L	M	N	O	P	Q
ϕ	180 cos A	cos B	Sine B	A-90	60 cos E	F-158.7	.888 cos G	.888 Sine G	1+C+H	J^2	D+I	M^2	L+M	$\sqrt{O}$	KxP
0	-180.0	-1.000	0.000	-90	0.0	-158.7	-.827	-.322	.827	.684	.322	.103	.787	.887	134.2 MV/M
10	-177.3	-.999	-.047	-80	10.4	-148.3	-.756	-.467	.755	.570	.514	.264	.834	.913	138.0
20	-169.1	-.982	-.189	-70	20.5	-136.2	-.662	-.592	.644	.414	.781	.609	1.023	1.011	152.9
30	-155.9	-.913	-.408	-60	30.0	-128.7	-.555	-.693	.468	.219	1.101	1.212	1.431	1.196	180.9
40	-137.9	-.742	-.670	-50	38.6	-120.1	-.445	-.768	.187	.035	1.438	2.068	2.103	1.450	219.3
50	-115.7	-.433	-.901	-40	45.9	-112.8	-.344	-.819	.223	.049	1.720	2.958	3.007	1.734	262.3
60	-90.0	0.000	-1.000	-30	51.9	-107.8	-.271	-.845	.729	.531	1.845	3.404	3.935	1.983	300.0
70	-61.6	.475	-.879	-20	56.4	102.3	-.189	-.868	1.286	1.653	1.747	3.052	4.705	2.169	328.1
80	-31.2	.855	-.518	-10	59.0	-99.7	-.149	-.875	1.706	2.910	1.393	1.940	4.850	2.202	333.2
90	0.0	1.000	0.000	0	60.0	-98.7	-.134	-.878	1.866	3.482	.878	.771	4.253	2.062	312.0
100	31.2	.855	.518	10	59.0	-99.7	-.149	-.875	1.706	2.910	.357	.127	3.037	1.743	263.7
110	61.6	.475	.879	20	56.4	-102.3	-.189	-.868	1.286	1.653	.011	.000	1.653	1.285	194.5
120	90.0	.000	1.000	30	51.9	-107.8	-.271	-.845	.729	.531	.155	.024	.555	.745	112.7
130	115.7	-.433	.901	40	45.9	-112.8	-.344	-.819	.223	.049	.082	.007	.056	.236	35.8
140	137.9	-.742	.670	50	38.6	-120.1	-.445	-.768	.187	.035	.098	.009	.044	.209	31.7
150	155.9	-.913	.408	60	30.0	-128.7	-.555	-.693	.468	.219	.285	.081	.300	.547	82.6
160	169.1	-.982	.189	70	20.5	-138.2	-.862	-.592	.644	.414	.403	.162	.576	.759	114.8
170	177.3	-.999	.047	80	10.4	-148.3	-.756	-.467	.755	.570	.420	.176	.746	.864	130.6
180	180.0	-1.000	0.0	90	0.0	-158.7	-.827	-.322	.827	.684	.322	.104	.788	.887	134.3
190	177.3	-.999	.047	100	-10.4	-169.1	-.872	-.168	.873	.762	.121	.014	.776	.880	133.3
200	169.1	-.982	.189	110	-20.5	-179.2	-.888	-.012	.870	.757	.177	.031	.788	.886	134.3
210	155.9	-.913	.408	120	-30.0	-188.7	-.877	.134	.790	.624	.542	.294	.918	.958	144.9
220	137.9	-.742	.670	130	-38.6	-197.3	-.848	.264	.590	.348	.934	.872	1.220	1.104	167.1
230	115.7	-.433	.901	140	-45.9	-204.6	-.807	.369	.240	.058	1.270	1.613	1.670	1.293	195.5
240	90.0	0.000	1.000	150	-51.9	-210.6	-.764	.452	.236	.055	1.452	2.108	2.163	1.471	222.5
250	61.6	.475	.879	160	-56.4	-215.1	-.726	.510	.749	.561	1.389	1.929	2.490	1.578	238.7
260	31.2	.855	.518	170	-59.0	-217.7	-.703	.543	1.152	1.327	1.061	1.126	2.456	1.567	237.0
270	0.0	1.000	0.000	180	-60.0	-218.7	-.693	.555	1.307	1.708	.693	.480	2.188	1.479	223.8
280	-31.2	.855	-.518	-170	-59.0	-217.7	-.703	.543	1.152	1.327	.025	.0006	1.333	1.155	174.7
290	-61.6	.475	-.879	-160	-56.4	-215.1	-.726	.510	.749	.561	.369	.136	.697	.834	126.2
300	-90.0	0.0	-1.000	-150	-51.9	-210.6	-.764	.452	.236	.055	.548	.303	.355	.896	90.1
310	-115.7	-.433	-.901	-140	-45.9	-204.6	-.807	.369	.240	.058	.532	.283	.341	.583	88.4
320	-137.9	-.742	-.670	-130	-38.6	-197.3	-.848	.264	.590	.348	.406	.164	.512	.716	108.3
330	-155.9	-.913	-.408	-120	-30.0	-188.7	-.877	.134	.790	.624	.274	.075	.699	.836	126.5
340	-169.1	-.982	-.189	-110	-20.5	-179.2	-.888	-.012	.870	.757	.201	.040	.797	.893	135.1
350	-177.3	-.999	-.047	-100	-10.4	-169.1	-.872	-.168	.873	.762	.215	.046	.808	.899	136.0
												55.914			

$$K = \frac{188.6}{\sqrt{\frac{55.914}{36}}} = 151.3 \text{ MV/M}$$

5

Design of four-tower systems

As is the case with three-tower patterns, there are two basic methods of computing four-tower designs: the addition method and the multiplication method. As you might suspect, there are special cases here, too. One new factor which comes into play with any four-tower pattern is tower placement. You can have all four towers in a straight line (the so-called in-line array), or in the form of a parallelogram.

The general equation (1) for a four-tower pattern is the same as that of a three-tower pattern, plus one more term. This is shown below (Equation 1):

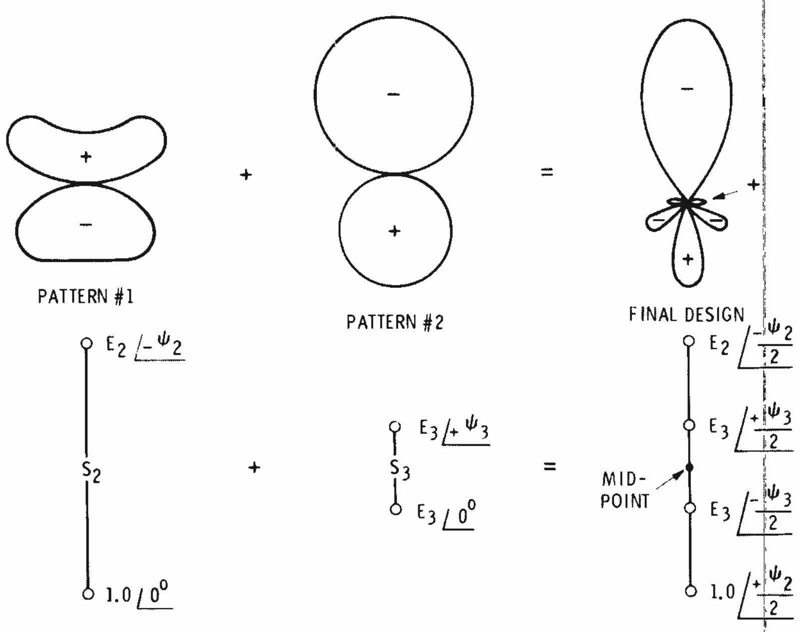
$$\begin{aligned} E = Kf(\Theta)\,[1.0 \angle 0 &+ E_2 \angle \Psi_2 + S_2 \cos\Theta \cos(\emptyset - \delta) \\ &+ E_3 \angle \Psi_3 + S_3 \cos\Theta \cos(\emptyset - \delta) \\ &+ E_4 \angle \Psi_4 + S_4 \cos\Theta \cos(\emptyset - \delta)] \end{aligned}$$

As with the use of computers in Chapter 3, I have written in the Greek letter to represent the shift from the reference bearing on each tower. If all towers are in a straight line, then $\delta = 0$.

In designing four-tower patterns one usually looks at the end result as being the product of two or three two-tower patterns or the sum of two two-tower patterns. Because of this I'll first show an example of an addition method for an in-line array, then a multiplication method for a parallelogram pattern. Keep in mind both methods apply to each type of tower configuration.

Four-tower addition formula

In using this method you first must calculate the pattern of each of the individual two-tower patterns to be added. The negative and positive signs are added appropriately to each lobe. These ± signs must be carefully observed when the two patterns are added. For an example I used the WTAQ nighttime



FOUR TOWER ADDITION FORMULA (TOWERS IN LINE)

Figure 1

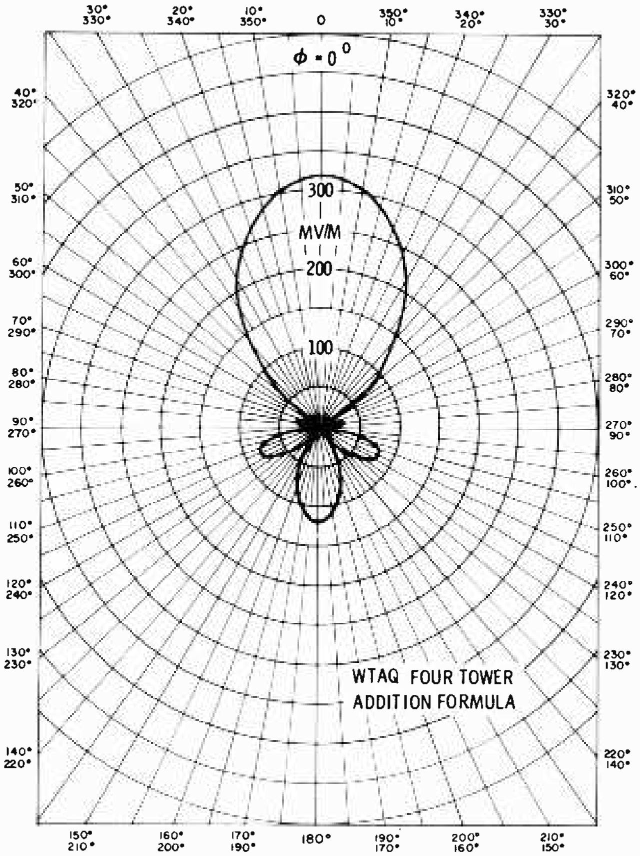


Figure 2

pattern, consisting of a two-tower 90° spaced cardioid and a 270° spaced four-leaf-clover pattern. Figure 1 shows how each pattern looks, as well as the end result. The calculated data is in Table 1.

The lobes to the north on each are (+), so they add. To the sides and the back these lobes are of opposite signs, hence they cancel. Keep in mind that you can vary not only the individual pattern nulls of each two-tower pattern, but also the relative amplitude of each tower, hence the null depth. In the final four-tower design the nulls are determined not by the location of the nulls in the two-tower patterns, but by the bearings at which the (+) and (-) lobes are of equal magnitude, for only at those points will cancellation occur. Although this is an awkward method, it is still in common use.

Four-tower multiplication formula—parallelogram

This method is similar to the addition form, except you multiply the two-tower patterns. For this example I've chosen two two-tower patterns that will combine in a parallelogram shape. The close-spaced pattern is a familiar 90° super cardioid. For the wide-spaced array I've chosen a 200° spaced figure eight, with

TABLE 1
FOUR-TOWER ADDITION FORM

$$E = Kf\Theta \left[E_2 \underline{/ \frac{\Psi_2}{2} + \frac{S_2}{2} \cos \emptyset \cos \Theta} + E_3 \underline{/ \frac{\Psi_3}{2} + \frac{S_3}{2} \cos \emptyset \cos \Theta} \right]^*$$

We Substitute: $E_2 = 1.0,\ \frac{\Psi_2}{2} = -76,\ \frac{S_2}{2} = 135,\ E_3 = 2.1,\ \frac{\Psi_3}{2} = 97,\ \frac{S_3}{2} = 45$

$$E_T = K\ [1.0 \cos (-76 + 135 \cos \emptyset) + 2.1 \cos (97 + 45 \cos \emptyset)]$$

A	B	C	D	E	F	G	H	I	J	
Ø	135 cos Ø	B-76	cos C	45 cos A	97+E	2.1 cos F	D+G	H^2	H•K	
0	135.0	59.0	.515	45.0	142.0	-1.655	1.140	1.2990	321.0	MV/M
10	132.9	56.9	.546	44.3	141.3	-1.639	1.093	1.1950	306.0	
20	126.8	50.8	.632	42.3	139.3	-1.592	.960	.9220	270.0	
30	116.9	40.9	.756	38.9	135.9	-1.508	.752	.5660	214.0	
40	103.4	27.4	.888	34.5	131.5	-1.392	.504	.2540	141.7	
50	86.8	10.8	.982	28.9	125.9	-1.231	.248	.0620	69.6	
60	67.5	-8.5	.989	22.5	119.5	-1.034	.045	.0020	6.2	
70	46.2	-29.8	.868	15.4	112.4	-.800	-.068	.0050	19.2	
80	23.4	-52.6	.607	7.8	104.8	-.536	-.071	0.0050	19.5	
90	0.0	-76.0	.242	0.0	97.0	-.256	.014	.0002	4.0	
100	-23.4	-99.4	-.163	-7.8	89.2	.029	-.134	.0180	37.5	
110	-46.2	-122.2	-.533	-15.4	81.6	.307	-.226	.0510	64.0	
120	-67.5	-143.5	-.804	-22.5	74.5	.561	-.243	.0590	68.8	
130	-86.8	-162.8	-.955	-28.9	68.1	.783	-.172	.0290	48.2	
140	-103.4	-179.4	-1.000	-34.5	62.5	.969	-.031	.0010	8.5	
150	-116.9	-192.9	-.975	-38.9	58.1	1.109	.134	.0180	38.6	
160	-126.8	-202.8	-.921	-42.3	54.7	1.213	.292	.0850	81.5	
170	-132.9	-208.9	-.875	-44.3	52.7	1.273	.398	.1580	111.8	
180	-135.0	-211.0	-.857	-45.0	52.0	1.293	.435	.4950	122.0	

*Half angle formula added to a second half-angle formula.

K = 281.5, RMS = 136

small side lobes. These are shown in Figure 3, and the data used to calculate them in Table 2. The question now is, how does one go from the design values of the respective two-tower patterns to the final four-tower values?

If you assume one corner of the parallelogram as the reference tower ($1.000 \angle 0°$), then the nearest tower has the same phase and field ratio as that of the close-spaced (90°) pattern. Likewise, the closest wide-spaced tower has the same values as that of the

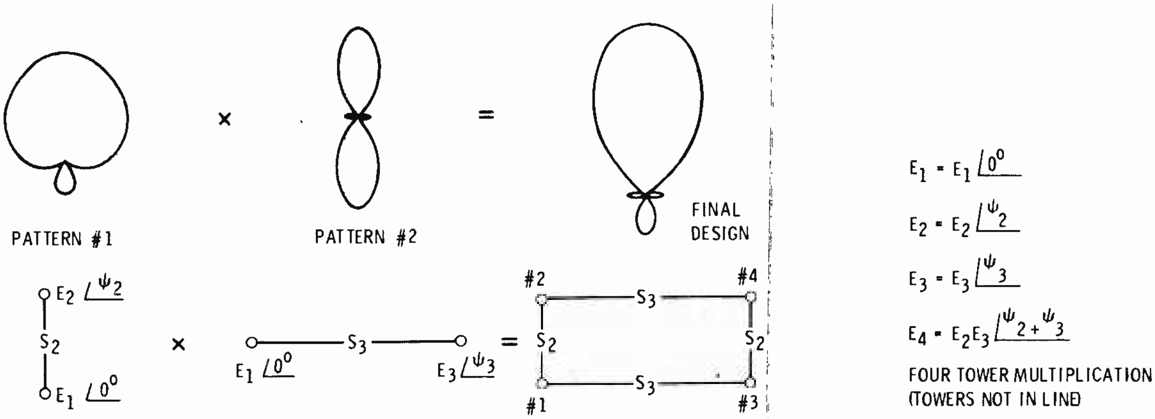


Figure 3

TABLE 2
FOUR-TOWER MULTIPLICATION METHOD, PARALLELOGRAM

$$E_T = Kf(\Theta)\left[\left(E_1 + E_2 \angle \Psi_2 + S_2 \cos \emptyset \cos \Theta\right) \cdot \left(E_1 + E_3 \angle \Psi_3 + S_3 \cos \Theta \cos (\emptyset - \delta)\right)\right]$$

Where

$E_1 = E_2 = E_3 = 1.0$, $\Psi_2 = -116.4$, $S_2 = 90°$, $\Psi_3 = 0°$, $S_3 = 200$, $\delta = -90$

Substituting

$$E_1 = K\left[\left(1 + \cos(-116.4 + 90 \cos \emptyset)\right) \cdot \left(1 + \cos\left(0° + 200 \cos (\emptyset - 90)\right)\right)\right]^{1/2}$$

A	B	C	D	E	F	G	H	I	J	K	L
Ø	90 cos A	B-116.4	cos C	1+D	A-90	200 cos F	cos G	1+H	E•I	$J^{1/2}$	$K \cdot K^1$
0	90.0	-26.4	.896	1.896	-90	0.0	1.000	2.000	3.7920	1.947	497.0 MV/M
10	88.6	-27.8	.885	1.885	-100	-34.7	.822	1.822	3.4340	1.853	472.0
20	84.5	-31.9	.849	1.849	-110	-68.4	.368	1.368	2.5290	1.590	405.0
30	77.9	-38.5	.783	1.783	-120	-100.0	-.173	.827	1.4740	1.214	309.0
40	68.9	-49.5	.849	1.649	-130	-128.5	-.622	.378	.6230	.789	201.0
50	57.8	-58.6	.521	1.521	-140	-153.2	-.892	.108	.1640	.405	103.0
60	45.0	-71.4	.319	1.319	-150	-173.2	-.993	.007	.0090	.096	24.4
70	30.8	-85.6	.077	1.077	-160	-187.9	-.990	.010	.0110	.104	26.5
80	15.6	-100.8	-.187	.813	-170	-196.9	-.957	.043	.0350	.187	47.7
90	0.0	-116.4	-.445	.555	-180	-200.0	-.939	.061	.0340	.184	46.9
100	-15.6	-132.0	-.669	.331	-190	-196.9	-.957	.043	.0140	.119	30.3
110	-30.8	-147.2	-.841	.159	-200	-187.9	-.990	.010	.0016	.039	9.9
120	-45.0	-161.4	-.948	.052	-210	-173.2	-.993	.007	.....	.019	4.8
130	-57.8	-174.2	-.995	.005	-220	-153.2	-.892	.108	.....	.023	5.9
140	-68.9	-185.3	-.996	.004	-230	-128.5	-.622	.378	.0015	.039	9.9
150	-77.9	-194.3	-.969	.031	-240	-100.0	-.173	.827	.0260	.160	40.8
160	-84.5	-200.9	-.934	.066	-250	-68.4	.368	1.368	.0900	.300	76.5
170	-88.6	-205.0	-.906	.094	-260	-34.7	.822	1.822	.1710	.414	105.6
180	-90.0	-206.4	-.896	.104	-270	0.0	1.000	2.000	.2080	.456	116.3

$K^1 = 255$,
RMS = 196

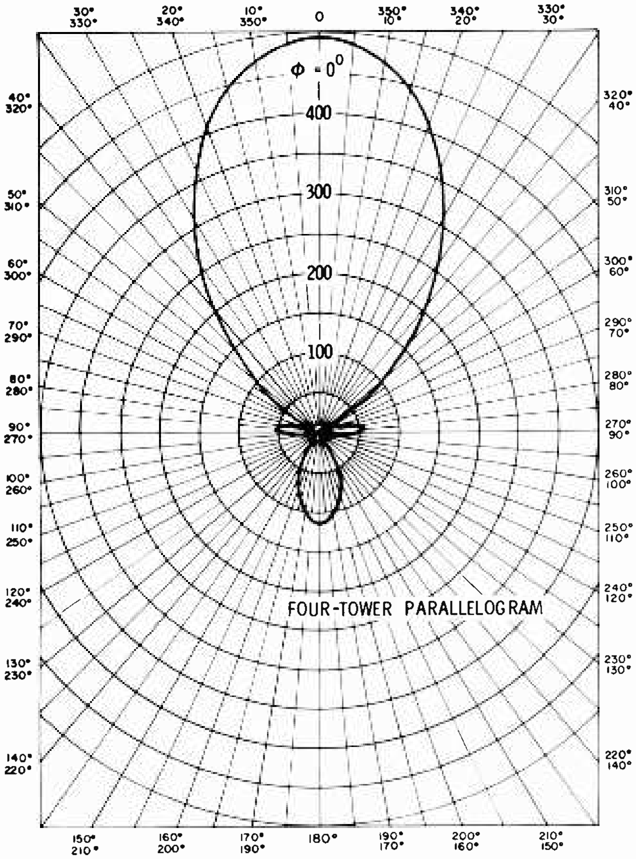
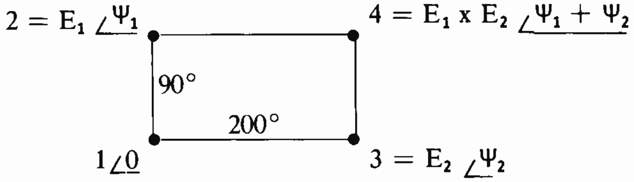


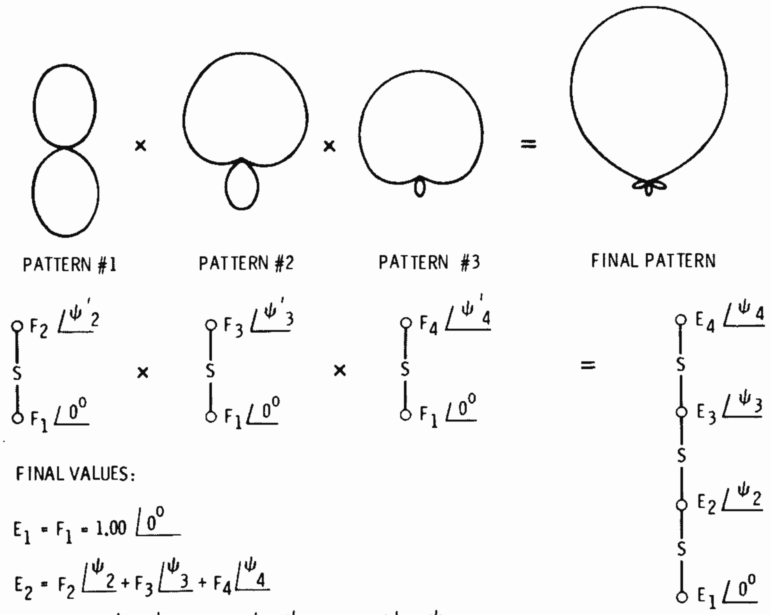
Figure 4

200°-spaced pattern. The opposite corner tower has the multiplication of the two-corner tower. Thus we have (Equation 2):



Another version of the four-tower multiplication method is that of the in-line array, where one more term is added to Equation 1 of the three-tower formula in Chapter 4. This can be written as (Equation 3):

$$E = Kf(\Theta)\sqrt{\left[\frac{1+M_1^2}{2M_1} + \cos(\Psi_1 + S\cos\Theta\cos\emptyset)\right]} \times$$
$$Kf(\Theta)\sqrt{\left[\frac{1+M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\Theta\cos\emptyset)\right]} \times$$
$$Kf(\Theta)\sqrt{\left[\frac{1+M_3^2}{2M_3} + \cos(\Psi_3 + S\cos\Theta\cos\emptyset)\right]}$$



FINAL VALUES:

$E_1 = F_1 = 1.00 \angle 0^0$

$E_2 = F_2 \angle \psi_2 + F_3 \angle \psi_3 + F_4 \angle \psi_4$

$E_3 = F_2F_3 \angle \psi_2 + \psi_3 + F_2F_4 \angle \psi_2 + \psi_4 + F_3F_4 \angle \psi_3 + \psi_4$

$E_4 = F_2F_3F_4 \angle \psi_2 + \psi_3 + \psi_4.$

Figure 5

Four-tower multiplication in-line

Equation 3, above, shows the formula to be used. For an example I've taken Figure 4 of Chapter 4 and added one more two-tower design value, having nulls 155° True. This will produce three pairs of nulls in the final pattern (shown in Figure 3). The end result is that both sides as well as the rear arc of the pattern are well suppressed. All useful energy is directed into the major lobe.

Figure 5, at the bottom, sets forth the method by which the design engineer determines the final operating current ratios and phase angles of each individual tower. In this method the letter E_n represents the "design" values of each of the pairs that go to make up the final four-tower pattern. F_1 is assumed to be equal to 1.00 with an angle of $\angle 0°$.

As a final thought I'll show how separate two-tower designs can be added together at their mid-point. WTAC's pattern represents one designed by this approach. Figure 7 shows the two individual shapes with assigned values of (+) and (-) for the respective lobes. Keep in mind that wherever lobes have equal signs the respective magnitudes add, and where opposite, they subtract. At any bearing where the

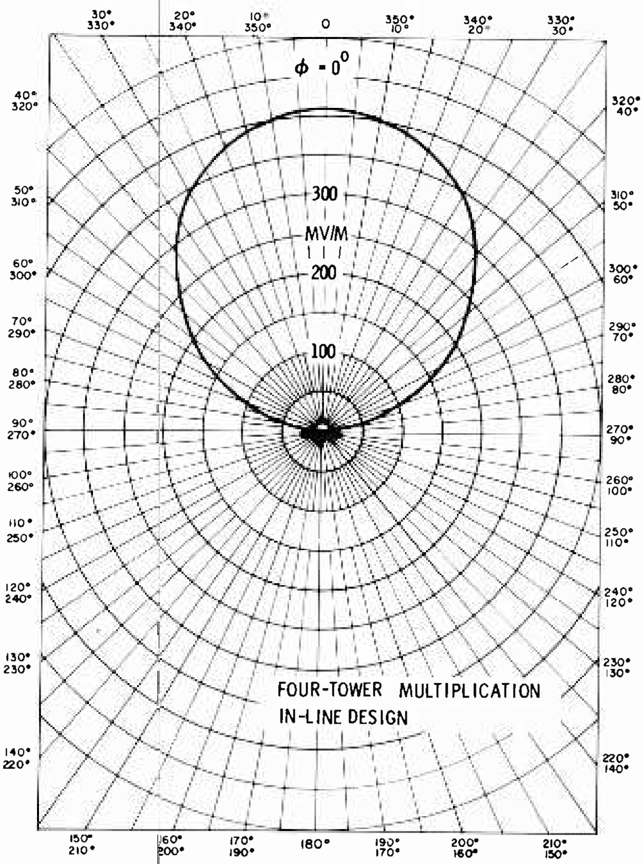


Figure 6

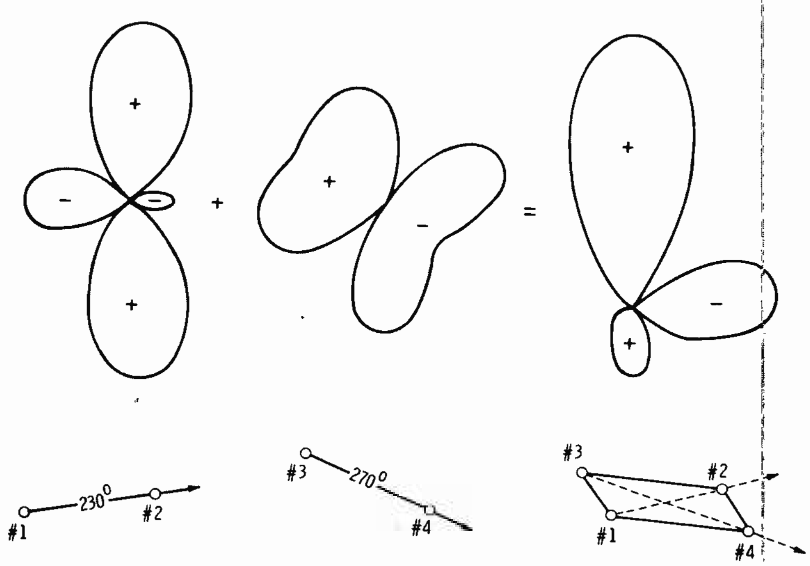


Figure 7

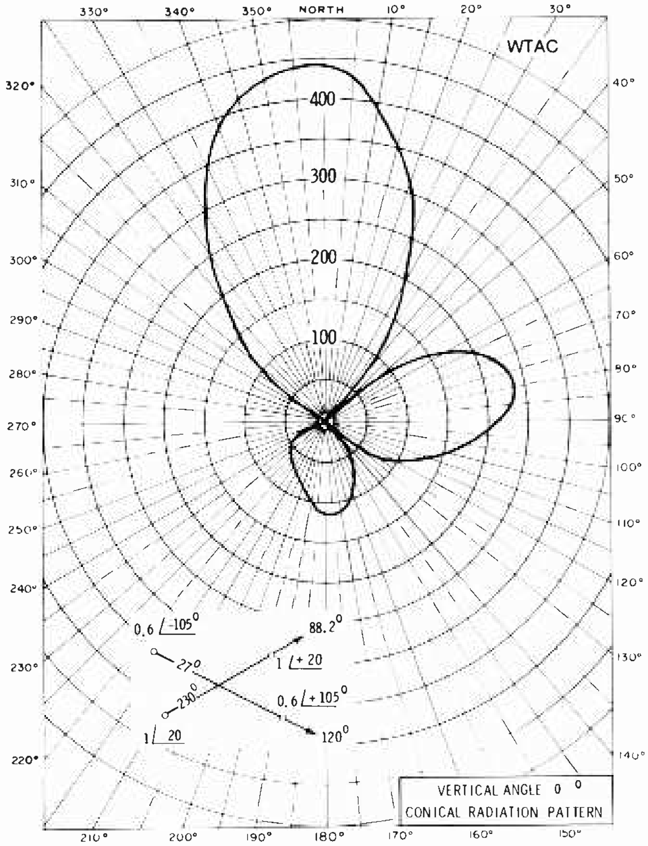


Figure 8

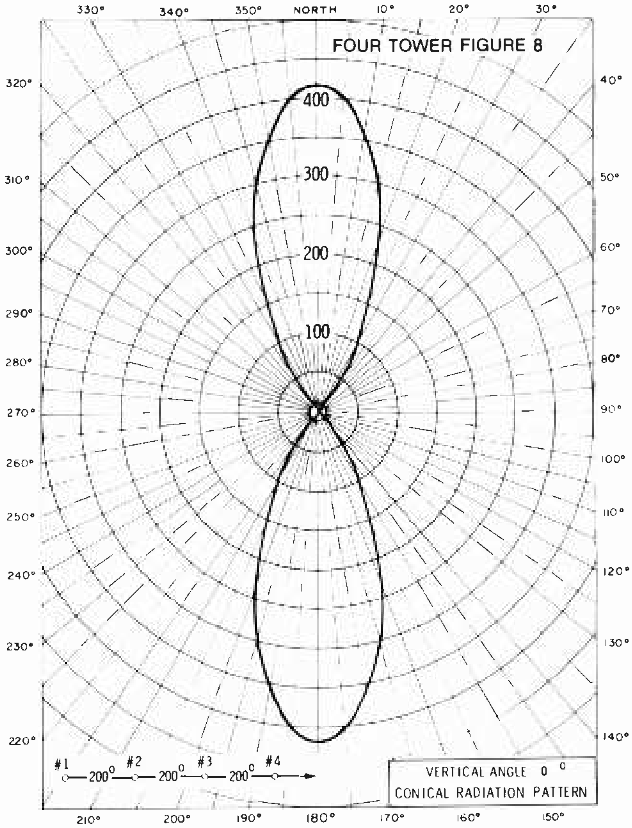


Figure 9

TABLE 3
FOUR-TOWER MULTIPLICATION METHOD, IN-LINE

$$E = Kf(\Theta)\left[\left(\frac{1+M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\emptyset\cos\Theta)\right)x\left(\frac{1+M_3^2}{2M_3} + \cos(\Psi_3 + S\cos\emptyset\cos\Theta)\right)x\left(\frac{1+M_4^2}{2M_4} + \cos(\Psi_4 + S\cos\emptyset\cos\Theta)\right)\right]^{1/2}$$

A	B	C	D	E	F	G	H	I	J	K	L	M	N
Ø	90 cos A	B-116.4	cos C	1 + D	180 + B	cos F	1 + G	B-98.4	cos I	1 + J	E•H•K	$L^{1/2}$	$M \cdot K^1$
0	90.0	-26.4	.896	1.896	270.0	.000	1.000	-8.4	.989	1.989	3.7710	1.9420	408.0 MV/M
10	88.6	-27.8	.885	1.885	268.5	-.024	.976	-9.8	.985	1.985	3.6510	1.9110	401.0
20	84.5	-31.9	.849	1.849	264.5	-.096	.904	-13.9	.970	1.970	3.2930	1.8140	381.0
30	77.9	-38.5	.783	1.783	257.9	-.209	.791	-20.5	.937	1.937	2.7320	1.6530	347.0
40	68.9	-49.5	.649	1.649	248.9	-.357	.641	-29.5	.870	1.870	1.9770	1.4060	295.0
50	57.8	-58.6	.521	1.521	237.8	-.533	.467	-40.6	.759	1.759	1.2490	1.1170	235.0
60	45.0	-71.4	.319	1.319	225.0	-.707	.293	-53.4	.596	1.596	.6160	.7850	165.0
70	30.8	-85.6	.077	1.077	210.8	-.859	.141	-67.6	.381	1.381	.2090	.4580	96.2
80	15.6	-100.8	-.187	.813	195.6	-.963	.037	-82.8	.125	1.125	.0340	.1840	38.6
90	0.0	-116.4	-.445	.555	0.0	-1.000	0.0	-98.4	-.146	.854	0.0000	0.0000	0.0
100	-15.6	-132.0	-.669	.331	164.4	-.963	.037	-114.0	-.407	.593	.0070	.0850	17.8
110	-30.8	-147.2	-.841	.159	149.2	-.859	.141	-129.2	-.632	.368	.0080	.0910	19.1
120	-45.0	-161.4	-.948	.052	135.0	-.707	.293	-143.4	-.803	.197	.0030	.0550	11.5
130	-57.8	-174.2	-.995	.005	122.2	-.533	.467	-156.2	-.915	.085	.0002	.0140	3.0
140	-68.9	-185.3	-.996	.004	111.1	-.359	.641	-167.3	-.975	.024	.....	0.0078	0.0
150	-77.9	-194.3	-.969	.031	102.1	-.209	.791	-176.3	-.998	.002	.....	0.0070	0.0
160	-84.5	-200.9	-.934	.066	95.5	-.096	.904	-182.9	-.998	.002	.0001	.0110	2.3
170	-88.6	-205.0	-.906	.094	91.4	-.024	.976	-187.0	-.993	.007	.0006	.0250	5.3
180	-90.0	-206.4	-.896	.104	90.0	0.0	1.000	-188.4	-.989	.011	.0011	.0330	6.9

K^1 = 210, RMS = 196 MV/M

magnitudes are of opposite sign and equal in magnitude, the final pattern will contain a null. For example, at 47° and 122° are nulls in the final pattern. Figure 8 is a polar plot of the final pattern, with the calculations shown in Table 4.

As a comparison of the degree of major lobe gain I have taken the basic figure eight pattern of Chapter 3 (2-towers) and Chapter 4 (3-towers) and added a fourth tower to produce the design shown in Figure 9.

Table 5 is the step-by-step computation used to calculate this design. Forward gain is achieved by widening the arc of the pattern minima. Or, in other words, by narrowing the beam. The top of the major lobe has been increased from 304.9 to 381.6 MV/M to 420.3 MV/M. From a two-tower to a four-tower you have gained:

$$\frac{420.3 \text{ MV/M}}{304.9 \text{ MV/M}}$$

Or a ratio of 1.378. This relates to an equivalent power increase of 1.900 times.

If it is assumed that the two-tower pattern had 1 kW of power, this would require increasing the transmitter power into the two-tower to 1.90 kW in order to equal the 1 kW signal you would get off the tip of the four-tower pattern. Thus doubling the number of tower almost doubles the effective power.

TABLE 4
FOUR-TOWER PARALLELOGRAM BY ADDITION

$$E = Kf(\Theta)\left[\cos\left(\frac{\Psi_1}{2} + \frac{S_1}{2}\cos\Theta\cos\emptyset\right) + M\cos\left(\frac{\Psi_2}{2} + \frac{S_2}{2}\cos\Theta\cos\emptyset\right)\right]$$

Where Ψ_1 = -20°, S_1 = 230°, M = 0.7, Ψ_2 = 105°, S_2 = 270°

A	B	C	D	E	F	G	H	I	J	L
Ø	115 cos A	B-10	cos C	A-31.8	135 cos E	F+105	.6 cos G	D+H	I^2	K•J
0	115.00	105.00	-.259	-31.8	114.7	219.7	-.461	-.720	.518	214.4 MV/M
10	113.25	103.30	-.230	-21.8	125.3	230.3	-.383	-.613	.376	182.6
20	108.00	98.00	-.139	-11.8	132.2	237.2	-.326	-.465	.216	138.5
30	99.60	89.60	.007	-1.8	134.9	239.9	-.301	-.294	.086	87.6
40	88.10	78.10	.206	8.2	133.6	238.6	-.313	-.107	.011	31.9
50	73.90	63.90	.440	18.2	128.2	233.2	-.359	.081	.007	24.1
60	57.50	47.50	.676	28.2	118.9	223.9	-.432	.244	.060	72.7
70	39.30	29.20	.873	38.2	106.1	211.1	-.513	.360	.130	107.2
80	19.90	9.90	.985	48.2	89.9	194.9	-.579	.406	.165	120.9
90	0.00	-10.00	.985	58.2	71.1	176.1	-.598	.387	.472	115.2
100	19.90	-29.90	.866	68.2	50.1	155.1	-.544	.322	.104	95.9
110	-39.20	-49.30	.652	78.2	27.6	132.6	-.406	.246	.061	73.2
120	-57.50	-67.50	.383	88.2	4.2	109.2	-.198	.185	.034	53.1
130	-73.90	-83.90	.106	98.2	-19.3	85.7	.045	.151	.023	44.9
140	-88.10	-98.10	-.141	108.2	-42.1	62.8	.274	.133	.018	39.6
150	-99.60	-109.60	-.335	118.2	-63.8	41.2	.452	.117	.014	34.8
160	-108.00	-118.00	-.470	128.2	-83.5	21.5	.558	.088	.008	26.2
170	-113.25	-123.25	-.548	138.2	-100.6	4.4	.598	.050	.003	14.9
180	-115.00	-125.00	-.574	148.2	-114.7	-9.7	.591	.017	.000	5.1
190	-113.25	-123.25	-.548	158.2	-125.3	-20.3	.562	.014	.000	4.2
200	-108.00	-118.00	-.470	188.2	-132.1	-27.1	.534	.064	.004	19.0
210	-99.60	-109.60	-.335	178.2	-134.9	-29.9	.520	.165	.034	55.1
220	-88.10	-98.10	-.141	-171.8	-133.6	-28.6	.526	.385	.148	114.6
230	-73.90	-83.90	.106	161.8	-128.2	-23.2	.551	.657	.432	195.6
240	-57.50	-67.50	.383	151.8	-118.9	-13.9	.583	.966	.933	287.7
250	-39.20	-49.30	.652	141.8	-106.1	-1.1	.600	1.252	1.568	372.7
260	-19.90	-29.90	.866	131.8	-89.9	15.1	.579	1.445	2.088	430.3
270	0.00	-10.00	.985	121.8	-71.1	33.9	.498	1.483	2.199	441.6
280	19.90	-9.90	.985	111.8	-50.1	54.9	.345	1.320	1.742	393.0
290	39.20	29.20	.873	101.8	-27.6	77.4	.131	1.004	1.008	309.7
300	57.50	47.50	.676	91.8	-4.2	100.8	-.112	.564	.318	167.9
310	73.90	63.90	.440	81.8	19.3	124.3	-.342	.098	.010	29.2
320	88.10	78.10	.206	71.8	42.1	147.1	-.504	-.298	.089	88.7
330	99.60	89.60	.007	61.8	63.8	168.8	-.589	-.582	.339	173.3
340	108.00	98.00	-.139	51.8	83.5	188.5	-.593	-.732	.536	217.9
350	113.25	103.25	-.230	41.8	100.6	205.6	-.541	-.771	.594	229.6
									14.897	

$$K = \frac{191.5 \text{ MV/M}}{\sqrt{\frac{14.897}{36}}} = 297.8$$

TABLE 5
FOUR-TOWER FIGURE EIGHT

Formula:

$$E = Kf(\Theta)\left[\left(\frac{1 + M_1^2}{2M_1} + \cos(\Psi_1 + S\cos\emptyset\cos\Theta)\right) x\right.$$
$$\left(\frac{1 + M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\Theta\cos\emptyset)\right) x$$
$$\left.\left(\frac{1 + M_3^2}{2M_3} + \cos(\Psi_3 + S\cos\emptyset\cos\Theta)\right)\right]^{1/2}$$

Assumptions:

$M_1 = M_2 = M_3 = 1.0$, $\Psi_1 = +7°$, $\Psi_2 = -7°$, $\Psi_3 = 0°$,
$S = 200°$, $f(\Theta) = 1.0$, $\Theta = 0°$

A	B	C	D	E	F	G	H	I	J
∅	200 cos A	B+7	1 + cos C	B-7	1 + cos E	1 + cos B	D•F•G	H	K•I
0	200.0	207.0	0.109	193.0	0.026	.060	0.000	0.000	0.0 MV/M
10	196.9	203.9	0.086	189.9	0.015	.043	0.000	0.000	0.0
20	187.9	194.9	0.034	180.9	0.001	.009	0.000	0.000	0.0
30	173.3	180.3	0.000	166.3	0.028	.007	0.000	0.000	0.0
40	153.2	160.2	0.059	146.2	0.169	.107	.001	.033	4.9
50	128.5	135.5	0.287	121.5	0.478	.377	.052	.227	33.8
60	100.0	107.0	0.708	93.0	0.948	.826	.554	.744	110.9
70	68.4	75.4	1.252	61.4	1.478	1.368	2.530	1.591	237.2
80	34.7	41.7	1.747	27.7	1.885	1.822	6.000	2.449	365.1
90	0.0	7.0	1.993	-7.0	1.993	2.000	7.944	2.818	420.3
100	-34.7	-27.7	1.885	-41.7	1.747	1.822	6.000	2.449	365.1
110	-68.4	-61.4	1.478	-75.4	1.252	1.368	2.530	1.591	237.2
120	-100.0	-93.0	0.948	-107.0	.708	.826	.554	.744	110.9
130	-128.5	-121.5	0.478	-135.5	.287	.377	.052	.227	33.8
140	-153.2	-146.2	.169	-160.2	.059	.107	.001	.033	4.9
150	-173.3	-166.3	.028	-180.3	0.000	.007	0.000	0.000	0.0
160	-187.9	-180.9	0.001	-194.9	0.034	.009	0.000	0.000	0.0
170	-196.9	-189.9	.015	-203.9	0.086	.043	0.000	0.000	0.0
180	-200.0	-193.0	.026	-207.0	.109	.060	0.000	0.000	0.0
							26.216		
							x 2		
							52.432		

$$K = \frac{180}{\sqrt{\frac{52.432}{36}}} = 149.1$$

6

Design of five-tower systems

This chapter deals with five-tower directional systems. Although it may seem odd, six-tower systems are much more common than five-tower systems. The reason for this is the ability to achieve a greater variety of patterns with six than with five, plus there is some simplicity in the design mathematics.

Five towers

There are really only two five-tower configurations that can be employed. The most common is the in-line array, shown in Figure 1. The only other design is that of a box or rectangle wherein the fifth tower is at the mid-point of the other four. This is shown in Figure 2, and is really a "special case."

The in-line array can either be a two-tower system multiplied by a three-tower system, or four-tower system multiplied by themselves. A third possible way to look at this is to consider it as a four-tower parallelogram (in-line) with a fifth tower at the mid-point. These different basic concepts are shown in Figure 3.

The most common approach is that of 3B. The following formula can be used (Equation 1):

$$E = Kf(\Theta)\Big[\Big(\frac{1+E_2^2}{2E_2} + \cos(\Psi_2 + S\cos\emptyset\cos\Theta)\Big) \times \Big(\frac{1+E_3^2}{2E_3} + \cos(\Psi_3 + S\cos\emptyset\cos\Theta)\Big) \times \Big(\frac{1+E_4^2}{2E_4} + \cos(\Psi_4 + S\cos\emptyset\cos\Theta)\Big) \times \Big(\frac{1+E_5^2}{2E_5} + \cos(\Psi_5 + S\cos\emptyset\cos\Theta)\Big)\Big]^{1/2}$$

The similarity between this formula and that of Equation 3 in Chapter 4 reveals the addition of a fourth and fifth term to the basic three-tower multiplication formula, as well as a fifth term to the basic four-tower multiplication formula (Equation 3 of Chapter 5).

Keep in mind that the designer has the same two major advantages here that he had with the three-tower and the four-tower designs. These are the facts that the bearings of each pair of two-tower nulls can be individually controlled, and that the depth of each two-tower null can be filled as much or

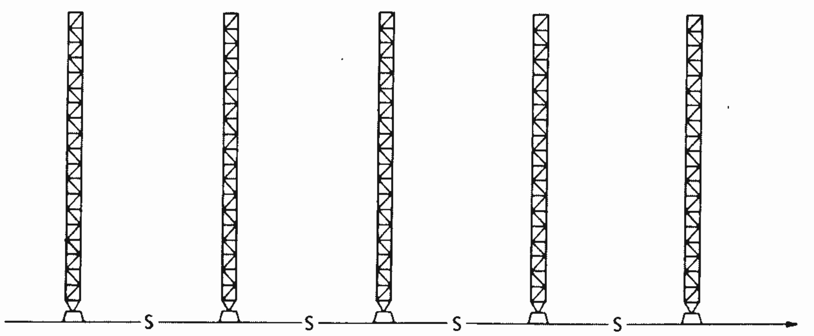


DIAGRAM OF ONE OF TWO BASIC FIVE–TOWER CONFIGURATIONS

Figure 1

as little as desired. The example is that of WIMS nighttime. The value selected for each pair is shown on Figure 4. Table 1 is a step-by-step calculation of this pattern. Keep in mind that as with any in-line array, the designer only need compute halfway around the pattern, since all in-line designs are symmetrical about their tower lines. Figure 5 is the polar graph of WIMS.

To arrive at the final base current ratios and phases, it is necessary to convert the design values of each two-tower pair to the final tower values. Again the WIMS design is used as the example. It is suggested that you follow the equation shown in Figure 4 to convert individual pairs into final values.

Table 2 is a calculation of a five-tower in-line figure eight array. This type of pattern will achieve the maximum amount of gain off the two ends of the propellor blades. It also achieves very good suppression over a wide arc between the two major lobes.

Figure 6 is a graphical plot of the five-tower plot of Table 2.

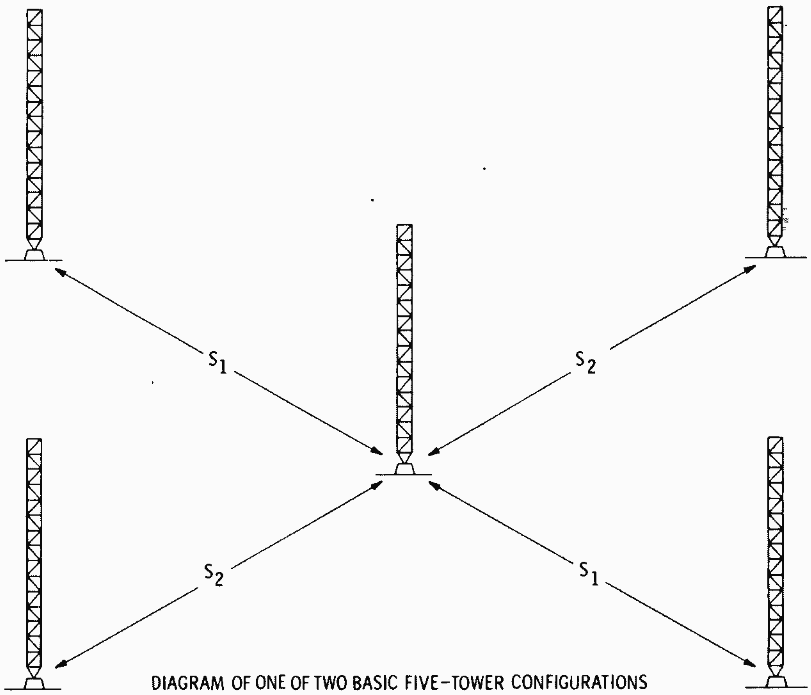


Figure 2

FIVE - TOWER CONFIGURATIONS

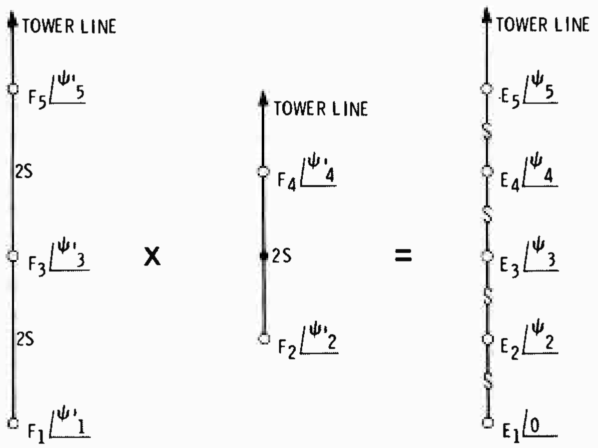


Figure 3A

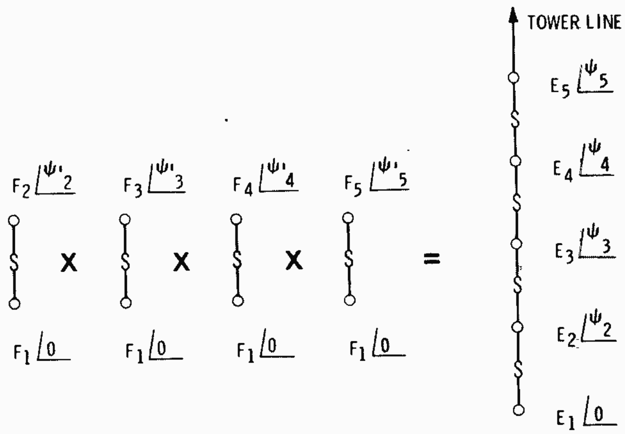


Figure 3B

TABLE 1
FIVE-TOWER MULTIPLICATION IN-LINE DESIGN

$$E = K\,[E_1 + E_2\,\underline{/\Psi_2 + S\cos\emptyset} + E_3\,\underline{/\Psi_3 + 2S\cos\emptyset} + E_4\,\underline{/\Psi_4 + 3S\cos\emptyset} + E_5\,\underline{/\Psi_5 + 4S\cos\emptyset}$$

Where: $E_1 = E_2 = E_3 = E_4 = 1.0$; $S = 90°$, $\Psi_2 = 165°$, $\Psi_3 = 170°$, $\Psi_4 = 132.5°$, $\Psi_5 = -90°$, $E_5 = 0.582$

Substituting:**

$$E = K\left[(1 + S\cos\emptyset + \Psi_2)(1 + S\cos\emptyset + \Psi_3)(1 + S\cos\emptyset + \Psi_4) + \left(\frac{1 + 0.582^2}{2(0.582)} + S\cos\Psi_5\right)\right]^{1/2}$$

A	B	C	D	E	F	G	H	I	J	K	L	M
Ø	90 cos A	165 + B	1 + cos C	170° + B	1 + cos E	132.5-B	1 + cos G	B-90	1.15 + cos I	D•F•H•J	$K^{1/2}$	$L•K^1$
0	90.0	255.0	.7411	260.0	.8264	-42.5	1.7373	0.0	2.1500	2.2876	1.5120	963.3 MV/M
10	88.6	253.6	.7076	258.6	.8023	-42.9	1.7205	-1.4	2.1490	2.1000	1.4490	923.1
20	84.5	251.6	.6497	254.5	.7328	-48.0	1.6691	-5.4	2.1450	1.7040	1.3050	831.4
30	77.9	242.9	.5444	247.9	.6238	-54.6	1.5793	-12.1	2.1280	1.1413	1.0680	680.4
40	68.9	233.9	.4108	238.9	.4835	-63.6	1.4446	-21.1	2.0830	.5976	.7730	492.5
50	57.8	222.8	.2662	227.8	.3283	-74.7	1.2638	-32.2	1.9960	.2203	.4690	298.8
60	45.0	210.0	.1339	215.0	.1808	-87.5	1.0436	-45.0	1.8570	.0469	.2165	137.9
70	30.8	195.4	.0340	200.0	.0603	-101.7	.7972	-59.6	1.7360	.0028	.0533	33.9
80	15.6	180.5	.0000	185.6	.0048	-116.9	.5476	-74.4	1.4190	.0000	.0000	0.0
90	0.0	165.0	.0340	170.0	.0152	-132.5	.3244	-90.0	1.1500	.0002	.0139	8.8
100	-15.6	149.4	.1392	154.4	.0982	-148.1	.1510	-105.6	.8811	.0018	.0426	27.1
110	-30.8	134.6	.3015	140.0	.2339	-183.3	.0422	-120.8	.6439	.0019	.0437	27.8
120	-45.0	120.0	.5083	125.0	.4264	-177.5	.0009	-135.0	.4430	.0001	.0095	6.0
130	-57.8	107.2	.7042	112.2	.6221	-190.3	.0161	-147.8	.3038	.0021	.0463	29.5
140	-68.9	96.1	.8937	101.1	.8075	-201.4	.0689	-158.9	.2171	.0108	.1039	66.2
150	-77.9	87.1	1.0500	92.1	.9834	-210.0	.1375	-167.9	.1722	.0239	.1547	98.5
160	-84.5	80.4	1.1650	85.5	1.0785	-217.0	.2014	-174.6	.1544	.0390	.1976	125.9
170	-88.6	76.4	1.2350	81.4	1.1495	-221.1	.2464	-178.6	.1503	.0526	.2293	146.1
180	-90.0	75.0	1.2590	80.0	1.1736	-225.5	.2627	-180.0	.1500	.0582	.2413	153.7

**Converted to multiplication formulas.

$K^1 = 626.9$, RMS = 400 MV/M

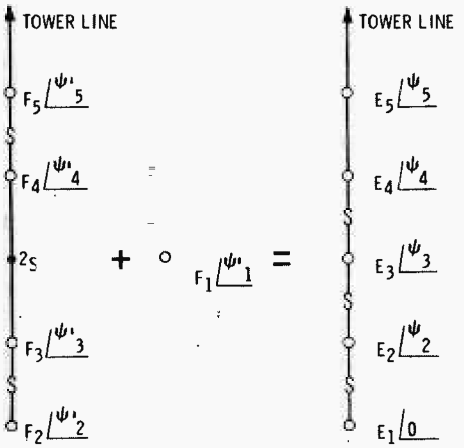


Figure 3C

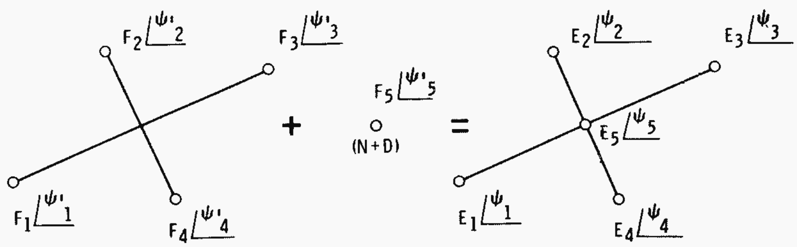
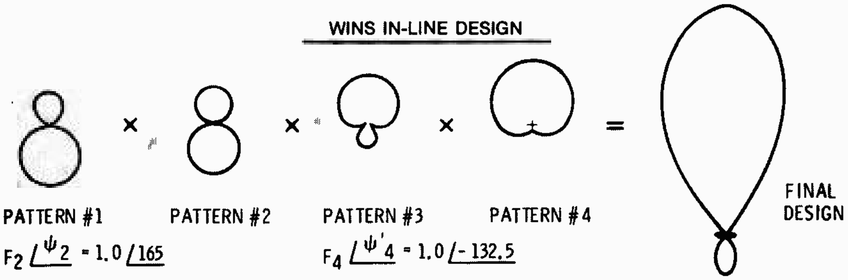
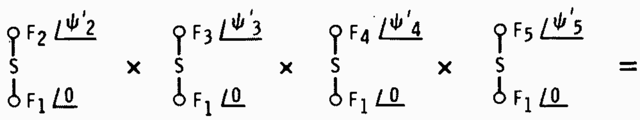


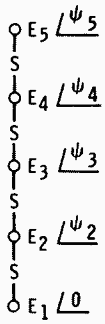
Figure 3D

WINS IN-LINE DESIGN



$F_2 \angle \psi 2 = 1.0 \angle 165$ $\quad F_3 \angle \psi' 3 = 1.0 \angle 170$ $\quad F_4 \angle \psi' 4 = 1.0 \angle -132.5$ $\quad F_5 \angle \psi' 5 = 0.582 \angle -90$





DETERMINATION OF BASE VALUES

$S = 90^0$

$E_1 \angle 0 = F_1 \angle 0$

$E_2 \angle \psi 2 = F_2 \angle \psi' 2 + F_3 \angle \psi' 3 + F_4 \angle \psi' 4 + F_5 \angle \psi' 5$

$= 2.7720 \angle -161.35^0$

$E_3 \angle \psi 3 = F_2F_3 \angle \psi' 2 + \psi' 3 + F_2F_4 \angle \psi' 2 + \psi' 4 + F_2F_5 \angle \psi' 2 + \psi' 5 +$

$F_3F_4 \angle \psi' 3 + \psi' 4 + F_3F_5 \angle \psi' 3 + \psi' 5 + F_4F_5 \angle \psi' 4 + \psi' 5$

$= 3.2660 \angle 43.6^0$

$E_4 \angle \psi 4 = F_2F_3F_4 \angle \psi' 2 + \psi' 3 + \psi' 4 + F_2F_3F_5 \angle \psi' 2 + \psi' 3 + \psi' 5 +$

$F_3F_4F_5 \angle \psi' 3 + \psi' 4 + \psi' 5 + F_2F_4F_5 \angle \psi' 2 + \psi' 4 + \psi' 5$

$= 1.9294 \angle -105.10^0$

$E_5 \angle \psi 5 = F_2F_3F_4F_5 \angle \psi' 2 + \psi' 3 + \psi' 4 + \psi' 5$

$= 0.582 \angle 112.50^0$

Figure 4

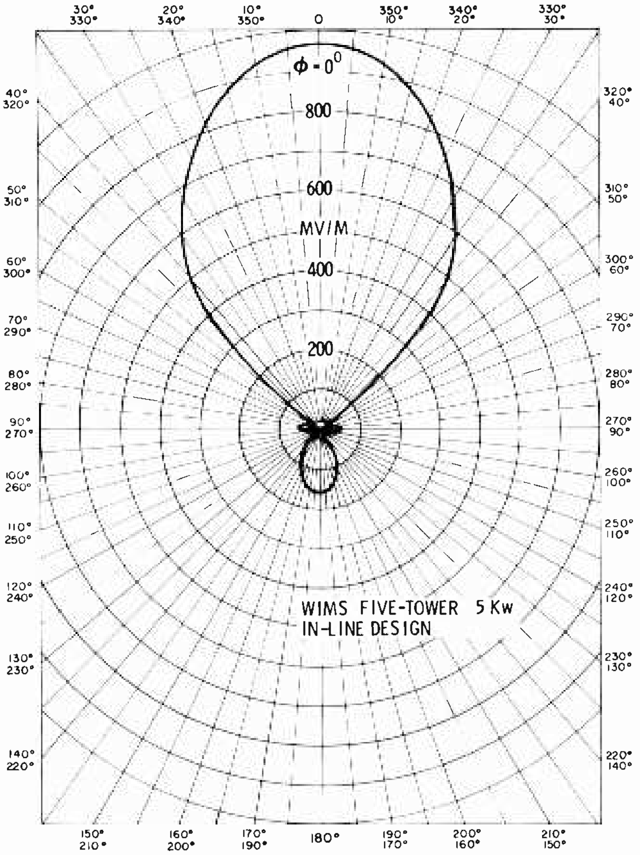


Figure 5

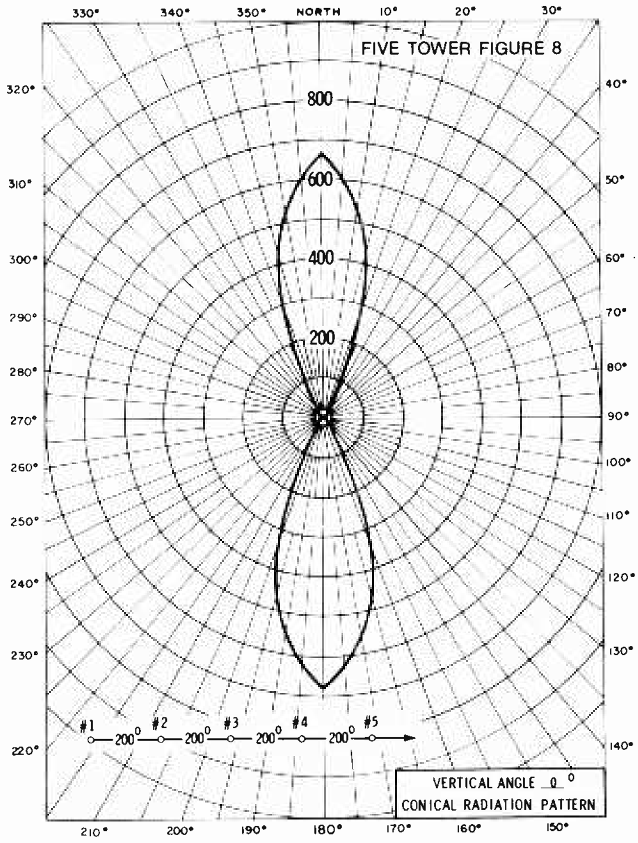


Figure 6

TABLE 2
FIVE-TOWER FIGURE EIGHT

Equation: $$E = Kf(\Theta)\left[\left(\frac{1+M_1^2}{2M_1} + \cos(\Psi_1 + S\cos\Theta\cos\emptyset)\right) \times \left(\frac{1+M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\Theta\cos\emptyset)\right) \times \left(\frac{1+M_3^2}{2M_3} + \cos(\Psi_3 + S\cos\Theta\cos\emptyset)\right) \times \left(\frac{1+M_4^2}{2M_4} + \cos(\Psi_4 + \cos\Theta\cos\emptyset)\right)\right]^{1/2}$$

Assumptions: $M_1 = M_2 = M_3 = M_4$, $\Psi_1 = +7°$, $\Psi_2 = -7°$, $\Psi_3 = +38°$, $\Psi_4 = -38°$,
$S = 200°$, $f(\Theta) = 1.0$, $\Theta = 0°$

A	B	C	D	E	F	G	H	I	J	L	M	N
Ø	200 cos A	B+7	1+cos C	B-7	1+cos E	B+38	1+cos G	B-38	1+cos I	D•E•H•J	√L	K•N
0	200.0	207.0	0.109	193.0	.026	238.0	.470	162.0	.048	0.000	0.000	0.0 MV/M
10	196.9	203.9	0.086	189.9	.015	234.9	.425	158.9	.067	0.000	0.000	0.0
20	187.9	194.9	0.034	160.9	.001	225.1	.304	149.9	.135	0.000	0.000	0.0
30	173.3	180.3	0.000	166.3	.028	211.3	.146	135.3	.289	0.000	0.000	0.0
40	153.2	160.2	0.059	146.2	.169	191.2	.019	115.2	.574	0.000	.010	1.8
50	128.5	135.5	0.287	121.5	.478	166.5	.028	90.5	.991	0.004	.061	11.1
60	100.0	107.0	0.708	93.0	.948	138.0	.257	62.0	1.469	0.253	.503	91.5
70	68.4	75.4	1.252	61.4	1.478	106.4	.718	30.4	1.863	2.475	1.573	286.3
80	34.7	41.7	1.747	27.7	1.885	72.7	1.297	-3.3	1.998	8.533	2.921	531.6
90	0.0	7.0	1.993	-7.0	1.993	38.0	1.788	-38.0	1.788	12.698	3.563	648.4
100	-34.7	-27.7	1.885	-41.7	1.747	3.3	1.998	-72.7	1.297	8.533	2.921	531.6
110	-68.4	-61.4	1.478	-75.4	1.252	-30.4	1.863	-106.4	.718	2.475	1.573	286.3
120	-100.0	-93.0	0.948	-107.0	.708	-62.0	1.469	-138.0	.257	0.253	0.503	91.5
130	-128.5	-121.5	0.478	-135.5	.287	-90.5	.991	-166.5	.028	0.004	0.061	11.1
140	-153.2	-146.2	0.169	-160.2	.059	-115.2	.574	-191.2	.019	0.000	0.010	1.8
150	-173.3	-166.3	0.028	-180.3	0.000	-135.3	.289	-211.3	.146	0.000	0.000	0.0
160	-187.9	-180.9	0.001	-194.9	.034	-149.9	.135	-225.1	.304	0.000	0.000	0.0
170	-196.9	-169.9	.015	-203.9	.086	-158.9	.067	-234.9	.425	0.000	0.000	0.0
180	-200.0	-193.0	.026	-207.0	.109	-162.0	.048	-238.0	.470	0.000	0.000	0.0
										35.228		

$$K = \frac{180}{\sqrt{\frac{335.228}{36}}} = 182.0$$

7

Design of six-tower systems

As with five-tower arrays, there are two basic configurations: the in-line or the parallelogram shapes (shown in Figure 1). There are several ways to think of the basic combinations used. The in-line patterns can be looked upon as individually multiplied together or as a three-tower combined with a second three-tower. They can even be looked at as a four-tower combined with a two-tower, as shown in Figures 2 and 3.

The parallelogram shape is probably more common in the six-tower design than the in-line. It is almost always looked upon as a direct multiplication of a three-tower by a two-tower, as shown in Figure 3. The parallelogram has two advantages over the in-line. First, it requires less real estate. Secondly, it can produce a much narrower major lobe. Optional arrays include the three-tower in-line closed-spaced multiplied by the two-tower wide-spaced, or the three-tower wide-spaced multiplied by the two-tower close-spaced.

Six-tower in-line

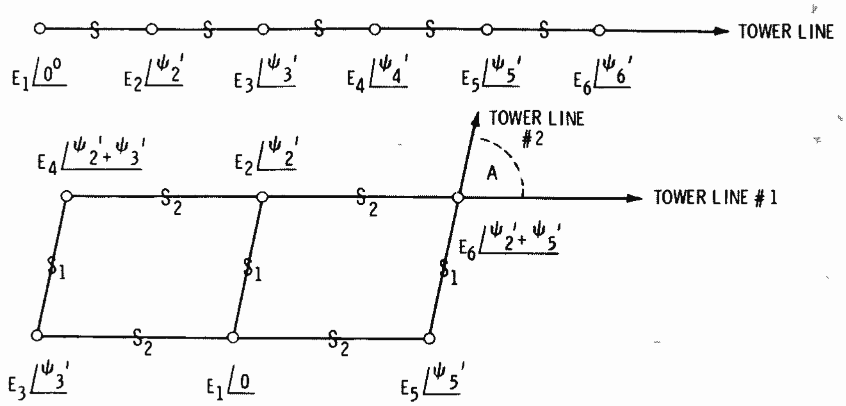
The general equation (1) for the in-line can be written as follows:

$$E = Kf\,(\Theta)\Big[\Big(\frac{1+F_2}{2F_2} + \cos\,(\Psi_2 + S\cos\emptyset\cos\Theta)\Big)x$$
$$\Big(\frac{1+F_3}{2F_3} + \cos\,(\Psi_3 + S\cos\emptyset\cos\Theta)\Big)x$$
$$\Big(\frac{1+F_4}{2F_4} + \cos\,(\Psi_4 + S\cos\emptyset\cos\Theta)\Big)x$$
$$\Big(\frac{1+F_5}{2F_5} + \cos\,(\Psi_5 + S\cos\emptyset\cos\Theta)\Big)x$$
$$\Big(\frac{1+F_6}{2F_6} + \cos\,(\Psi_6 + S\cos\Theta\cos\emptyset)\Big)\Big]^{1/2}$$

The terms are the same as previously explained. The advantages are the same as those of Equation 1 in Chapter 6, namely: each pair can be so designed as to effect its own angle and depth of null.

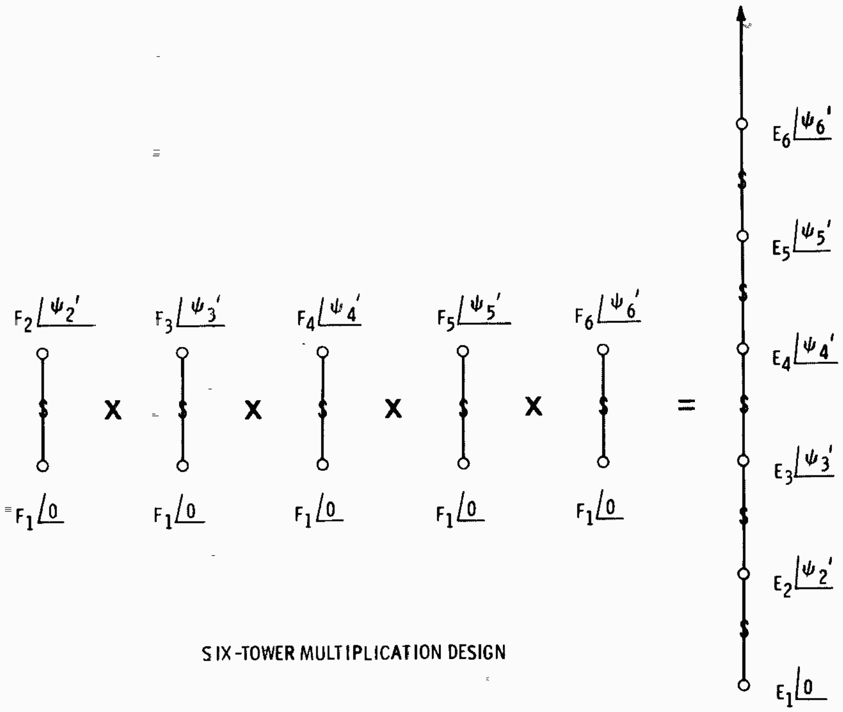
One danger with six-tower in-line is that if the spacing is 90° or less between adjacent towers, the overall radiation efficiency may be very low. (Chapter 10 will discuss the reason for this in depth.)

As with the five-tower array, you need to convert from the individual pairs to the final operating values. This is shown in Table 1. The example shows



TWO BASIC TYPES OF SIX-TOWER CONFIFURATIONS

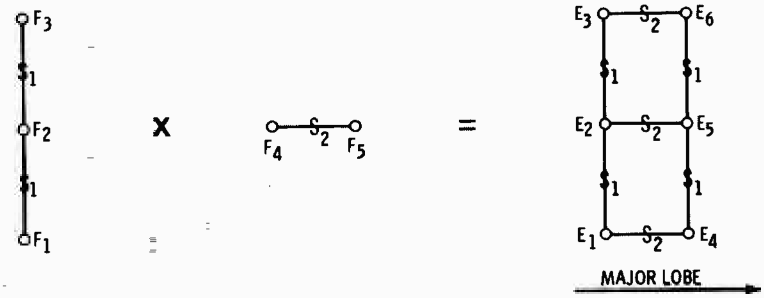
Figure 1



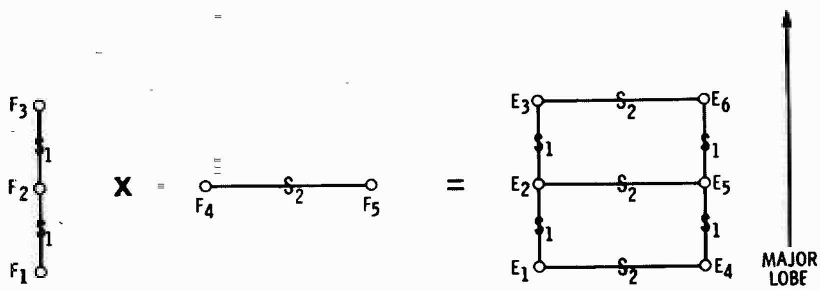
SIX-TOWER MULTIPLICATION DESIGN

Figure 2

SIX-TOWER PARALLELOGRAMS



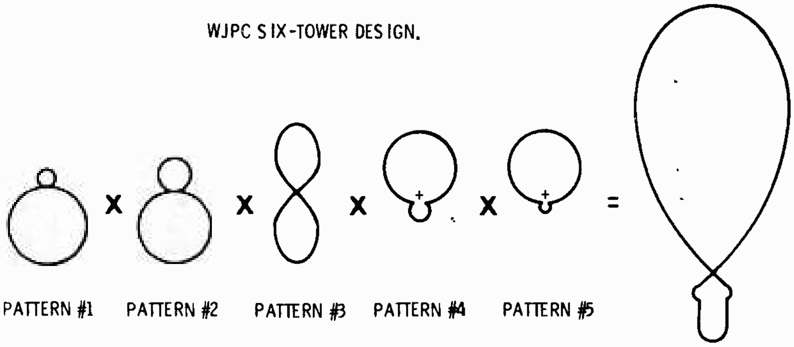
WIDE SPACED THREE X CLOSE SPACED TWO

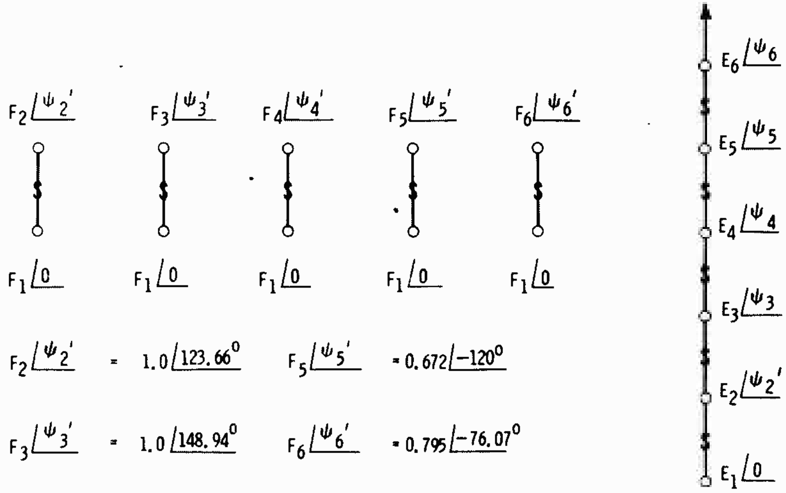


CLOSE SPACED THREE X WIDE SPACED TWO

Figure 3

WJPC SIX-TOWER DESIGN.





$F_2\angle\psi_2' = 1.0\angle 123.66^\circ$ $\qquad F_5\angle\psi_5' = 0.672\angle -120^\circ$

$F_3\angle\psi_3' = 1.0\angle 148.94^\circ$ $\qquad F_6\angle\psi_6' = 0.795\angle -76.07^\circ$

$F_4\angle\psi_4' = 1.0\angle 180^\circ$ $\qquad S = 120^\circ$

Figure 4A

DETERMINATION OF BASE OPERATING VALUES

$E_1\angle 0 = F_1\angle 0 = 1.0\angle 0^\circ$

$E_2\angle\psi_2' = F_2\angle\psi_2' + F_3\angle\psi_3' + F_4\angle\psi_4' + F_5\angle\psi_5' + F_6\angle\psi_6'$

$= 2.5553\angle 180.11^\circ$

$E_3\angle\psi_3' = F_2F_3\angle\psi_2'+\psi_3' + F_2F_4\angle\psi_2'+\psi_4' + F_2F_5\angle\psi_2'+\psi_5' +$
$F_2F_6\angle\psi_2'+\psi_6' + F_3F_4\angle\psi_3'+\psi_4' + F_3F_5\angle\psi_3'+\psi_5' +$
$F_3F_6\angle\psi_3'+\psi_6' + F_4F_5\angle\psi_4'+\psi_5' + F_4F_6\angle\psi_4'+\psi_6' +$
$F_5F_6\angle\psi_5'+\psi_6'$

$= 3.235\angle 15.5^\circ$

$E_4\angle\psi_4' = F_2F_3F_4\angle\psi_2'+\psi_3'+\psi_4' + F_2F_3F_5\angle\psi_2'+\psi_3'+\psi_5' +$
$F_2F_3F_6\angle\psi_2'+\psi_3'+\psi_6' + F_3F_4F_5\angle\psi_3'+\psi_4'+\psi_5' +$
$F_3F_4F_6\angle\psi_3'+\psi_4'+\psi_6' + F_4F_5F_6\angle\psi_4'+\psi_5'+\psi_6' +$
$F_2F_4F_5\angle\psi_2'+\psi_4'+\psi_5' + F_2F_4F_6\angle\psi_2'+\psi_4'+\psi_6' +$
$F_3F_5F_6\angle\psi_3'+\psi_5'+\psi_6' + F_2F_5F_6\angle\psi_2'+\psi_5'+\psi_6'$

$E_5\angle\psi_5' = F_2F_3F_4F_5\angle\psi_2'+\psi_3'+\psi_4'+\psi_5' +$
$F_2F_3F_4F_6\angle\psi_2'+\psi_3'+\psi_4'+\psi_6' +$
$F_2F_3F_5F_6\angle\psi_2'+\psi_3'+\psi_5'+\psi_6' +$
$F_2F_4F_5F_6\angle\psi_2'+\psi_4'+\psi_5'+\psi_6' +$
$F_3F_4F_5F_6\angle\psi_3'+\psi_4'+\psi_5'+\psi_6'$

$= 1.6451\angle 54.37^\circ$

$E_6\angle\psi_6' = F_2F_3F_4F_5F_6\angle\psi_2'+\psi_3'+\psi_4'+\psi_5'+\psi_6'$

$= 0.5342\angle -103.47^\circ$

Figure 4B

the WJPC six-tower in-line design. The pairs used to produce the pattern of Figure 4 are shown in Figure 5.

Six-tower parallelograms

The six-tower parallelogram can be computed, based upon the following design formula (Equation 2):

$$E = Kf(\Theta)\left[\left(\frac{1+F_2}{2F_2} + \cos(\Psi_2 + S_1 \cos \emptyset \cos \Theta)\right) \times \left(\frac{1+F_3}{2F_3} + \cos(\Psi_3 + S_1 \cos \emptyset \cos \Theta)\right) \times \left(\frac{1+F_4}{2F_4} + \cos(\Psi_3 + S_2 \cos(\emptyset - A) \cos \Theta)\right)\right]^{1/2}$$

Most of these terms are self-evident. The usual tower spacing term "S" has been replaced with an S_1 and an S_2. The logic for this should be obvious, but in case it is not, S_1 represents the spacing between towers of the two-tower pattern.

One new term is introduced; this is (∅-A). Because the six towers do not lie along the same axis (or

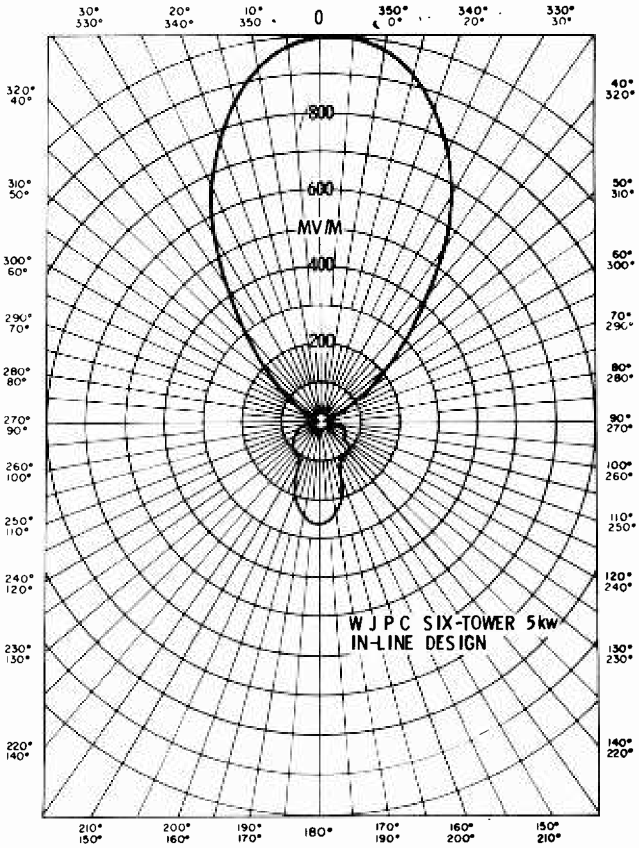
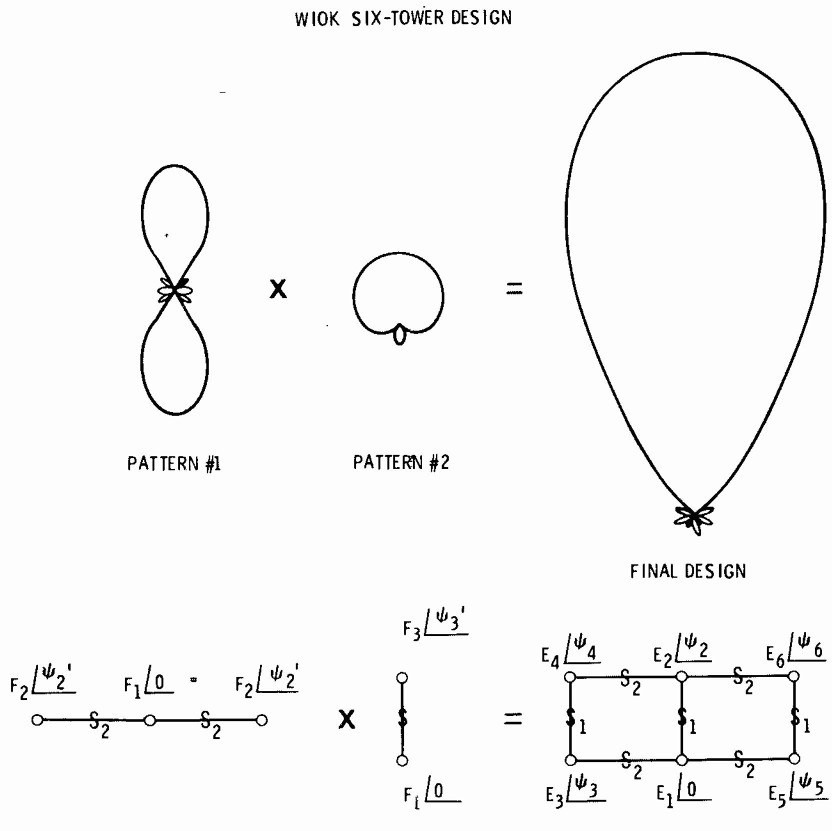


Figure 5



DETERMINATION OF BASE VALUES

$E_1\angle 0 = F_1\angle 0 = 1.82\angle 0^\circ$

$E_2\angle\psi_2 = F_1F_3\angle\psi_3' = 1.82\angle -92.4^\circ$

$E_3\angle\psi_3 = F_2\angle\psi_2' = 1.00\angle 0^\circ$

$E_4\angle\psi_4 = F_2F_3\angle\psi_2'+\psi_3' = 1.00\angle -92.4^\circ$

$E_5\angle\psi_5 = F_2\angle\psi_2' = 1.00\angle 0^\circ$

$E_6\angle\psi_6 = F_2F_3\angle\psi_2'+\psi_3' = 1.00\angle -92.4^\circ$

$S_1 = 90^\circ$

$S_2 = 217^\circ$

Figure 6

TABLE 1
SIX-TOWER IN-LINE DESIGN

$$E = K\Big[\Big(\frac{1+F_2^2}{2F_2} + \cos(\Psi_2 + S\cos\emptyset)\Big) \times \Big(\frac{1+F_3^2}{2F_3} + \cos(\Psi_3 + S\cos\emptyset)\Big) \times \Big(\frac{1+F_4^2}{2F_4} + \cos(\Psi_4 + S\cos\emptyset)\Big) \times \Big(\frac{1+F_5^2}{2F_5} + \cos(\Psi_5 + 5\cos\emptyset)\Big) \times \Big(\frac{1+F_6^2}{2F_6} + \cos(\Psi_6 + S\cos\emptyset)\Big)\Big]^{1/2}$$

A	B	C	D	E	F	G	H	I	J	K
∅	120 cos A	B+123.66°	1+cos C	B+148.94	1+cos E	B+180	1+cos G	B-120	1.080+cos I	B-76.07
0	120.00°	243.66°	.56160	268.94°	.98250	300.00	1.50000	0.00	2.0800°	43.93°
10	118.18	241.84	.53330	267.18	.95080	298.18	1.47220	-1.82	2.0795	42.11
20	112.76	236.42	.45190	261.76	.85670	292.76	1.38690	-7.24	2.0720	36.69
30	103.92	227.58	.32980	252.92	.70630	283.92	1.24060	-16.08	2.0409	27.85
40	91.92	215.58	.19020	240.92	.51400	271.92	1.03350	-28.08	1.9623	15.85
50	77.13	200.79	.06724	226.13	.30700	257.13	.77730	-42.87	1.8129	1.06
60	60.00	183.66	.00244	209.00	.12540	240.00	.50000	-60.00	1.5800	-16.07
70	41.04	164.70	.03390	190.04	.01532	221.04	.24570	-78.96	1.2715	-35.03
80	20.84	144.50	.18250	169.84	.01570	200.84	.06543	-99.16	.9208	-56.77
90	0.00	123.66	.44090	149.00	.14280	180.00	.00000	-120.00	.5800	-76.07
100	-20.84	102.82	.77230	128.16	.38210	169.16	.06543	-140.84	.3046	-96.91
110	-41.04	82.62	1.12260	107.96	.69160	138.96	.24570	-161.04	.1343	-117.11
120	-60.00	63.66	1.43840	89.00	1.01750	120.00	.50000	-180.00	.0800	-136.07
130	-77.13	46.53	1.68360	71.87	1.31120	102.87	.77730	-197.13	.1244	-153.20
140	-91.92	31.74	1.84730	58.08	1.52870	88.08	1.03350	-211.92	.2312	-167.99
150	-103.92	19.74	1.93920	46.08	1.69360	76.08	1.24060	-223.92	.3597	-179.99
160	-112.76	10.90	1.98080	37.24	1.79610	67.24	1.38690	-232.76	.4748	-188.83
170	-118.18	5.48	1.99480	30.82	1.85880	61.82	1.47220	-238.18	.5527	-194.25
180	-120.00	3.66	1.99750	29.00	1.87460	60.00	1.50000	-240.00	.5800	-196.07

TABLE 2
SIX-TOWER MULTIPLICATION DESIGN

$$E_T = K\,[I_1 + \cos(\Psi_1 + S_1\cos\emptyset)]^* \cdot [\cos\frac{\Psi_2}{2} + \frac{S_2}{2}\cos(\emptyset\text{-}\delta)]^{**}$$

Where:

$\Psi_1 = 0°$, $S_1 = 217°$, $\Psi_2 = 92.4°$, $S_2 = 90°$, $\delta = 90°$, $I_1 = 0.91$,

$$E_T = K\,[0.91 + \cos(217\cos\emptyset)] \cdot [\cos 46.2° + 45\cos(\emptyset\text{-}90)]$$

A	B	C	D	E	F	G	H	I	J	K	L	M
∅	cos ∅	217xB	cos C	0.91+D	∅ - 90	cos F	45.G	H+46.2°	cos I	ExJ	K^2	K•301
90	0.0000	0.0	1.000	1.910	0	1.000	45.0	91.2	0.02094	0.04000	0.001600	12.10 MV/M
100	-0.1735	-37.6	.792	1.702	10	0.9850	44.3	90.5	0.00873	0.01488	0.000220	4.20
110	-0.3420	-74.2	.272	1.182	20	0.9400	42.3	88.5	0.02618	0.03095	0.000960	9.30
120	-0.5000	-108.5	-.314	.596	30	0.8660	39.0	85.2	0.08368	0.04980	0.002500	19.90
130	-0.6430	-139.6	-.762	.148	40	0.7660	34.5	80.7	0.16160	0.02390	0.000570	7.20
140	-0.7660	-166.2	-.971	-.061	50	0.6430	29.0	75.2	0.25500	0.01555	0.000240	4.70
150	-0.8660	-188.0	-.990	-.080	60	0.5000	22.5	68.7	0.36300	0.02900	0.000850	8.70
160	-0.9400	-204.0	-.913	-.003	70	0.3420	15.4	61.6	0.47500	0.00142	0.000002	0.43
170	-0.9850	-214.0	-.829	.081	80	0.1735	7.8	54.0	0.58800	0.04770	0.002280	14.30
180	-1.0000	-217.0	-.799	.111	90	.0000	0.0	46.2	0.69200	0.07690	0.005910	23.20
190	-0.9850	-214.0	-.829	.081	100	-0.1735	-7.8	38.4	0.78300	0.06340	0.004020	19.10
200	-0.9400	-204.0	-.913	-.003	110	-0.3420	-15.4	30.8	0.85900	0.00258	0.000007	0.78
210	-0.8660	-188.0	-.990	-.080	120	-0.5000	-22.5	23.7	0.91500	0.07320	0.005370	22.10
220	-0.7660	-166.2	-.971	-.061	130	-0.6430	-29.0	17.2	0.95500	0.05830	0.003410	17.60
230	-0.6430	-139.6	-.762	.148	140	-0.7660	-34.5	11.7	-0.97900	0.14500	0.021100	43.60
240	-0.5000	-108.5	-.314	.596	150	-0.8660	-39.0	7.2	0.99210	0.59130	0.349600	177.90
250	-0.3420	-74.2	.272	1.182	160	-0.9400	-42.3	3.9	0.99770	1.18100	1.400000	355.00
260	-0.1735	-37.6	.792	1.702	170	-0.9850	-44.3	1.9	0.99950	1.70200	2.910000	512.00
270	0.0000	0.0	1.000	1.910	180	-1.0000	-45.0	1.2	0.99980	1.91000	3.660000	574.00

*Special case of a 3-tower with equal current ratios in end towers.
**Special case of a 2-tower (half-angle formula).

K = 301, RMS = 179

$$K = \frac{179 \text{ MV/M}}{\sqrt{\dfrac{12.7441}{36}}} = 301 \text{ MV/M}$$

L	M	N	O
1.026 + cos K	D•F•H•J•L	$\sqrt{M}$	N•K[1]
1.74500	3.004300	1.73330	1002.50 MV/M
1.76700	2.743000	1.65620	957.90
1.82700	2.032300	1.4255	824.50
1.90900	1.125900	1.06110	613.70
1.98700	0.393900	.62760	362.80
2.02600	.058930	.24270	140.40
1.98700	.000480	.02190	12.67
1.94500	.000315	.01770	10.27
1.69700	.000292	.01710	9.89
1.26800	.000000	.00000	0.00
.89520	.005260	.07256	41.97
.55970	.014335	.11970	69.25
.29490	.017260	.13140	76.00
.12230	.026090	.16150	93.40
.04810	.032400	.18000	104.10
.02600	.038100	.19520	112.90
.03767	.088250	.29700	171.80
.05647	.170400	.41280	238.00
.06474	.213700	.46230	267.00

K[1] = 578, RMS = 400

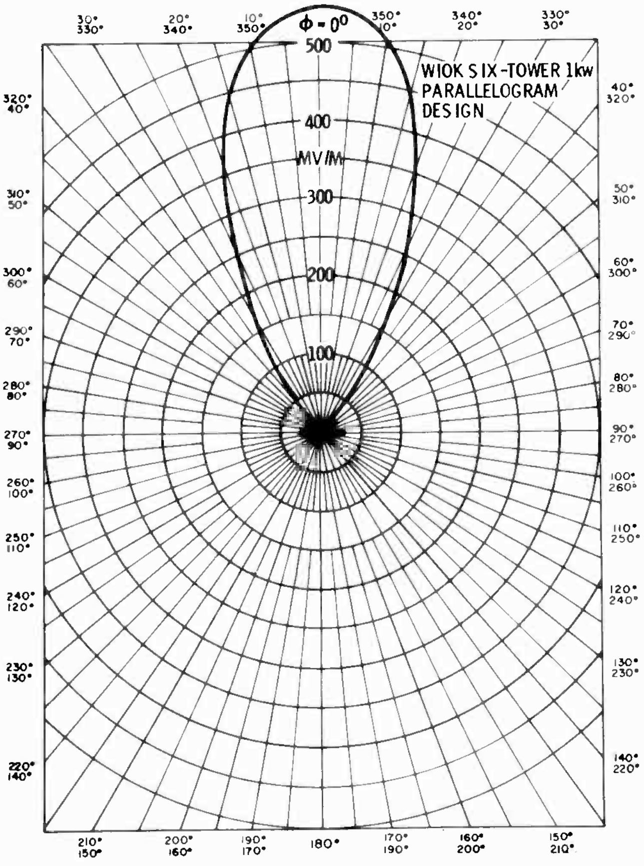
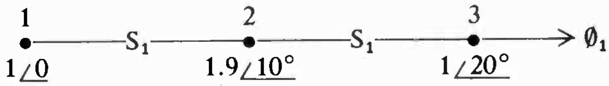


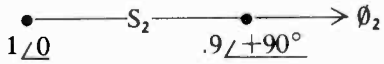
Figure 7

tower-line), but rather along two axis, let this term stand for the angular displacement between the two axis (or tower-lines).

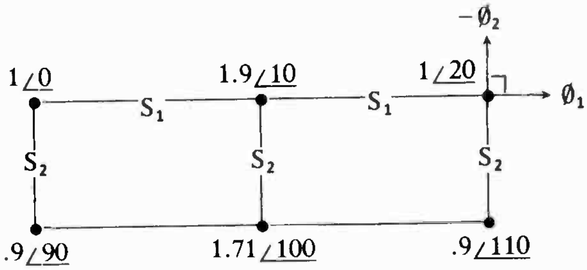
Table 2 is a calculation using this method. In arriving at the final base operating impedances, one merely determines the "three-tower" parameters first. The second row of towers then has its phase added to that of each front-row towers. The current ratio is the product of the amplitude of the second tower in the two-tower times each of the towers in the other row. The math is shown in the following example. Let the three-tower be represented by:



And let the two-tower be shown as:



The product would then look like this:



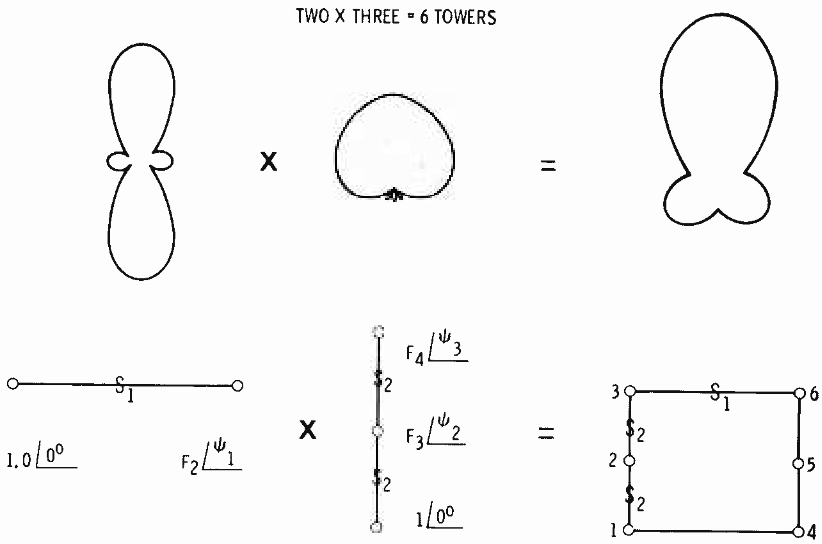
Equation 2 can be modified because the end towers of the three-tower wide-spaced are of equal currents. This is the "little-used method" described in Chapter 4.

Therefore, the six-tower parallelogram formula can be rewritten as follows (Equation 3):

$$E = Kf(\Theta)\left[\left(\frac{F_1}{2} + \cos(S_2 \cos \emptyset \cos \Theta)\right) x \left(\frac{1+F_3^2}{2F_3} + \cos(\Psi + S_1 \cos \Theta \cos(\emptyset - A))\right)\right]^{1/2}$$

Figure 6 shows the breakdown of how the two patterns are multiplied together to achieve the final six-tower design. Also shown are how the design engineer would determine the final base values by combining the separate two patterns' values. Figure 7 is a polar graph of the final design.

The second type of six-tower parallelogram is that formed by the two times three. Figure 8 is a display of this, with Figure 9 being the polar graph. The equation and the step-by-step math is tabulated in Table 3.



FINAL VALUES:

TOWER 1 = $1.00\angle 0$

$2 = F_3\angle\psi_2$

$3 = F_4\angle\psi_3$

$4 = F_2\angle\psi_1$

$5 = F_2 \cdot F_3\angle\psi_2 + \psi_1$

$6 = F_2 \cdot F_4\angle\psi_3 + \psi_1$

Figure 8

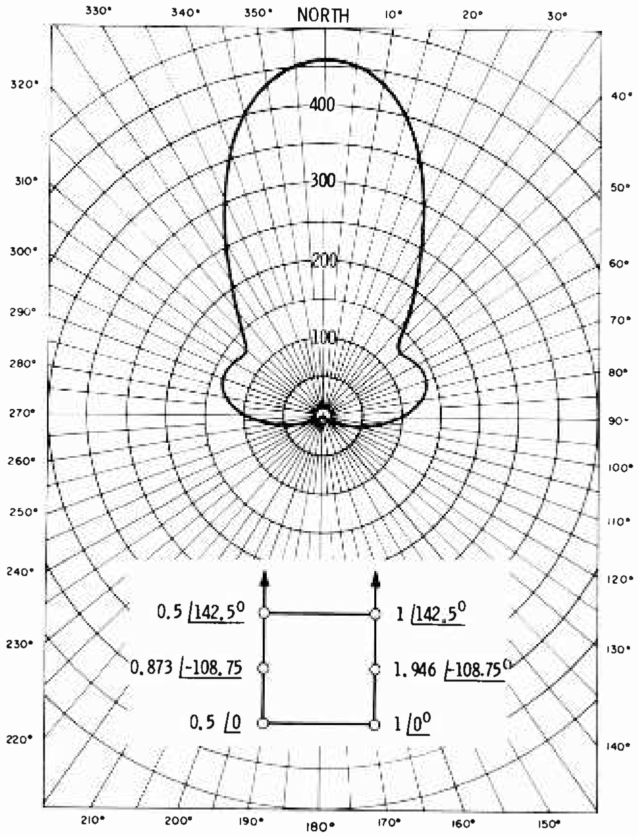


Figure 9

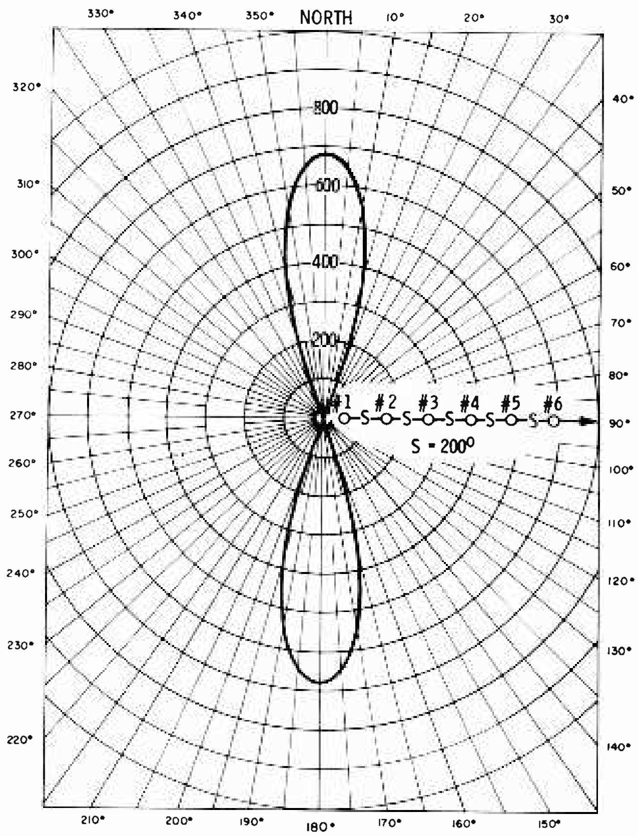


Figure 10

The final six-tower to look into is like Figure 4, an in-line array. However, in this case the adjacent tower spacing has been increased to the point where the maximum signal is broadside to the line of towers, not along the row as was the case in Figures 4 and 5. Figure 10 is a polar plot, which can be constructively compared with the figure eight pattern of a two-tower, a three-tower, a four-tower, etc. The maximum lobe signal has increased from 304.9 MV/M for the two-tower to 675.4 MV/M with the six-tower and can be ratioed to find the equivalent increase in power. This works out to 4.9 kW. By tripling the number of towers, power has been increased almost five times!

TABLE 3
SIX TOWER TWO BY THREE

Equation:
$$E = Kf(\Theta)\Big[\Big(\frac{1 + M_1^2}{2M_1} + \cos(\Psi_1 + S_1 \cos\Theta \cos(\emptyset - \delta))\Big) \times$$
$$\Big(\frac{1 + M_2^2}{2M_2} + \cos(\Psi_2 + S_2 \cos\Theta \cos\emptyset)\Big) \times$$
$$\Big(\frac{1 + M_3^2}{2M_3} + \cos(\Psi_3 + S_2 \cos\Theta \cos\emptyset)\Big)\Big]^{1/2}$$

Assumptions: $M_1 = 0.5$, $M_2 = M_3 = 1.0$, $\Psi_1 = 0°$, $\Psi_2 = -95.4°$, $\Psi_3 = 122.1°$, $S_1 = 240°$, $S_2 = 90°$, $\delta = -90°$, $f(\Theta) = 1.0$, $\Theta = 0°$

A	B	C	D	E	F	G	H	I	J	L	M
Ø	A-90	240 cos B	1.25 + cos C	90 cos A	E-95.4	1 + cos F	E-122.1	1 + cos H	D•G•I	√J	K•L
0	-90	0.0	2.225	90.0	-5.4	1.995	-32.1	1.847	8.1980	2.863	454.9 MV/M
10	-80	41.6	1.998	88.6	-6.8	1.993	-33.5	1.833	7.3060	2.703	429.5
20	-70	82.1	1.388	84.6	-10.8	1.986	-37.5	1.793	4.9320	2.721	352.9
30	-60	120.0	.750	77.9	-17.5	1.954	-44.2	1.717	2.5160	1.586	252.0
40	-50	154.2	.349	68.9	-26.5	1.895	-53.2	1.599	1.0570	1.028	163.3
50	-40	183.8	.252	57.8	-37.6	1.792	-64.3	1.434	.6470	.805	127.9
60	-30	207.8	.366	45.0	-50.4	1.637	-77.1	1.223	.7330	.856	136.0
70	-20	225.5	.549	30.8	-64.6	1.429	-91.3	.977	.7660	.875	139.0
80	-10	236.3	.695	15.6	-79.8	1.177	-106.5	.716	.5860	.765	121.6
90	0	240.0	.750	0.0	-95.4	.906	-122.1	.469	.3190	.565	89.8
100	10	236.3	.695	-15.6	-111.0	.642	-137.7	.260	.1160	.341	54.2
110	20	225.5	.549	-30.8	-126.2	.409	-152.9	.109	.0240	.156	24.8
120	30	207.8	.366	-45.0	-140.4	.229	-167.1	.025	.0020	.045	7.2
130	40	183.8	.252	-57.8	-153.2	.107	-179.9	.000	.0000	.000	0.0
140	50	154.2	.349	-68.9	-164.3	.027	-190.4	.016	.0001	.012	1.9
150	60	120.0	.750	-77.9	-173.3	.007	-200.0	.060	.0003	.017	2.7
160	70	82.1	1.388	-84.6	-180.0	.000	-206.7	.107	.0000	.000	0.0
170	80	41.6	1.998	-88.6	-184.0	.002	-210.7	.140	.0005	.024	3.8
180	90	0.0	2.225	-90.0	-185.4	.004	-212.1	.153	.0010	.037	5.9
									27.203		
									x 2		
									54.406		
									-8.198		
									- .001		
									46.207		

$$K = \frac{180}{\sqrt{\frac{46.207}{36}}} = 158.9$$

TABLE 4
SIX-TOWER FIGURE EIGHT

$$\text{Equation: } E = Kf(\Theta)\Big[\Big(\frac{1 + M_1^2}{2M_1} + \cos(\Psi_1 + S\cos\Theta\cos\emptyset)\Big) \times$$
$$\Big(\frac{1 + M_2^2}{2M_2} + \cos(\Psi_2 + S\cos\Theta\cos\emptyset)\Big) \times$$
$$\Big(\frac{1 + M_3^2}{2M_3} + \cos(\Psi_3 + S\cos\Theta\cos\emptyset)\Big) \times$$
$$\Big(\frac{1 + M_4^2}{2M_4} + \cos(\Psi_4 + S\cos\Theta\cos\emptyset)\Big) \times$$
$$\Big(\frac{1 + M_5^2}{2M_5} + \cos(\Psi_5 + S\cos\Theta\cos\emptyset)\Big)\Big]^{1/2}$$

Assumptions: $M_1 = M_2 = M_3 = M_4 = M_5 = 1.0$, $\Psi_1 = +7°$, $\Psi_2 = -7°$, $\Psi_3 = 0°$, $\Psi_4 = 38°$, $\Psi_5 = -38°$, $S = 200°$, $f(\Theta) = 1.00$, $\Theta = 0°$

A	B	C	D	E	F	G	H	I	J	L	M	N	O
Ø	200 cos A	B + 7	1 + cos C	B-7	1 + cos E	1 + cos B	B + 38°	1 + cos H	B-38°	1 + cos J	D•F •H•I•L	√M	K•N
0	200.0	207.0	0.109	193.0	0.026	0.060	238.0	.470	162.0	.048	.0000	.000	0.0 MV/M
10	196.9	203.9	0.086	189.9	0.015	0.043	234.9	.425	158.9	.067	.0000	.000	0.0
20	187.9	194.9	0.034	180.9	0.001	0.009	225.9	.304	149.9	.135	.0000	.000	0.0
30	173.3	180.3	0.000	166.3	0.028	0.007	211.3	.146	135.3	.289	.0000	.000	0.0
40	153.2	160.2	0.059	146.2	0.169	0.107	191.2	.019	115.2	.574	.0000	.003	0.4
50	128.5	135.5	0.287	121.5	0.478	0.377	166.5	.028	90.5	.991	.0014	.038	5.1
80	100.0	107.0	0.708	93.0	0.948	0.826	138.0	.257	62.0	1.469	.2090	.457	62.2
70	68.4	75.4	1.252	61.4	1.478	1.368	106.4	.718	30.4	1.863	3.3860	1.840	250.6
80	34.7	41.7	1.747	27.7	1.885	1.822	72.7	1.297	-3.3	1.998	15.5480	3.943	537.0
90	0.0	7.0	1.993	-7.0	1.993	2.000	38.0	1.788	-38.0	1.788	24.5940	4.959	675.4
100	-34.7	-27.7	1.885	-41.7	1.747	1.822	3.3	1.998	-72.7	1.251	15.5480	3.943	537.0
110	-68.4	-61.4	1.478	-75.4	1.252	1.368	-30.4	1.863	-106.4	.718	3.3860	1.840	250.6
120	-100.0	-93.0	0.948	-107.0	.708	.826	-62.0	1.469	-138.0	.257	.2090	.457	62.2
130	-128.5	-121.5	0.478	-135.5	.287	.377	-90.5	.991	-166.5	.028	.0014	.038	5.1
140	-153.2	-146.2	0.169	-160.2	.059	.107	-115.2	.574	-191.2	.019	0.0000	.003	0.4
150	-173.3	-166.3	0.028	-180.3	.000	.007	-135.3	.289	-211.3	.146	.0000	.000	0.0
160	-187.9	-180.9	0.001	-194.9	0.034	.009	-149.9	.135	-225.9	.304	.0000	.000	0.0
170	-196.9	-189.9	0.015	-203.9	0.086	.043	-158.9	.067	-234.9	.425	.0000	.000	0.0
180	-200.0	-193.0	0.026	-207.0	0.109	.060	-162.0	.048	-238.0	.470	.0000	.000	0.0
											62.880		

$$K = \frac{180}{\sqrt{\frac{62.880}{36}}} = 136.2$$

8

Design of eight-, nine-, and twelve-tower systems

This chapter focuses on the largest directional antennas currently approved and in use by U.S. stations. These are not common, but are practical.

Eight-tower arrays

Presently, none of these exist in the United States. There had been one for many years, which eventually added a third row to change from eight to twelve towers. This was necessary in order to increase power.

There can be only one basic configuration of eight towers: the basic parallelogram. It can be looked at either as two rows of four or four rows of two (see Figure 1). The general equation (1) is:

$$E = Kf(\Theta)\Big[\Big(\frac{1+F_2^2}{2F_2} + \cos(\Psi_2 + S_1 \cos\emptyset \cos\Theta)\Big) \times$$
$$\Big(\frac{1+F_3^3}{2F_3} + \cos(\Psi_1 + S_1 \cos\emptyset \cos\Theta)\Big) \times$$
$$\Big(\frac{1+F_4}{2F_4} + \cos(\Psi_4 + S_1 \cos\emptyset \cos\Theta)\Big) \times$$
$$\Big(\frac{1+F_5}{2F_5} + \cos(\Psi_5 + \cos\Theta \cos(\emptyset\text{-}A)\Big)\Big]^{1/2}$$

The first three parts of Equation 1 represent the three pairs of patterns used to develop a typical four-tower in-line multiplication array. The last part of the equation is that used to "multiply" the four-tower by the two-tower. The term (Ø-A) represents the offset between the individual tower lines. This is the angle in degrees to be measured between the tower-line bearing of the four-tower in-line and the tower-line bearing of the two-tower in-line.

For an example, assume a hypothetical design. In this case, designed for maximum suppression along the sides and the rear of the pattern.

Figure 2 shows how this basic eight-tower array can be formed. For this example (Figure 3, 3A), a four-tower wide-spaced multiplied by a two-tower close-spaced has been used. Note that the tower-lines are not at right angles. That is to say, they are not displaced by 90°. This was selected to show several facts. First, the final pattern is not symmetrical even though each of the individual patterns were symmetrical. Secondly, with this type of a design the engineer must calculate entirely around the pattern, not just halfway (as was the case with most of the previous examples). Finally, how to treat this deplacement angle (Ø-A).

Table 1 is a complete calculation of this pattern. As noted, it shows 36 lines of calculations, not just 18. Note that the equation at the top of Table 1 is not the same as Equation 1, above. What is going on here? The answer is nothing. This demonstrates the flexibility possible in calculating patterns.

In Equation 1, it was said the first three terms or pairs represent the four-tower design, while the last term is that of a two-tower. In Equation 1, the offset is shown in the two-tower pattern, although it could be in either, but not in both. In Table 1, the offset (Ø-A) was chosen to be in the four-tower pattern. The first term of Table 1 represents the two-tower pattern, and therefore is similar to the last term of Equation 1. But it may appear that the four-tower part of Equation 1 is not the same as the last of Table 1's equation. Actually they are. One method of computing a four-tower (multiplication method) was substituted for a second method (addition method). The final answer is still the product of a four-tower times a two-tower. The basic four-tower equation used here is similar to that of WTAQ shown in Chapter 5 of this manual.

In arriving at the final operating values one first must convert the individual two-tower pairs of the four-tower in-line into their base current ratios. Then the values of the back row can be determined from a multiplication of each front row field ratio by the magnitude of the second tower of the two-tower. The phase angles of each of the rear towers is a simple addition of the phase angle of the second tower of the two-tower phase, to the phase angle of each of the corresponding towers in the front row. This is self-evident, and of course is basically identical with that of the six-tower multiplication form. Figure 3A shows how this is accomplished. The phase angles of the front row reflect the phase angle of the two-tower. The difference in phase angle between adjacent towers of the wide-spaced reflect the differences in the four-tower phases. In the examples these are all zero degrees.

Nine-tower arrays

As with eight towers there can be only one basic shape: three times three, as shown in Figure 4. While it is technically feasible to have nine towers all in a straight line, it probably never will be proposed.

Surprisingly, one of the first nine-tower arrays ever

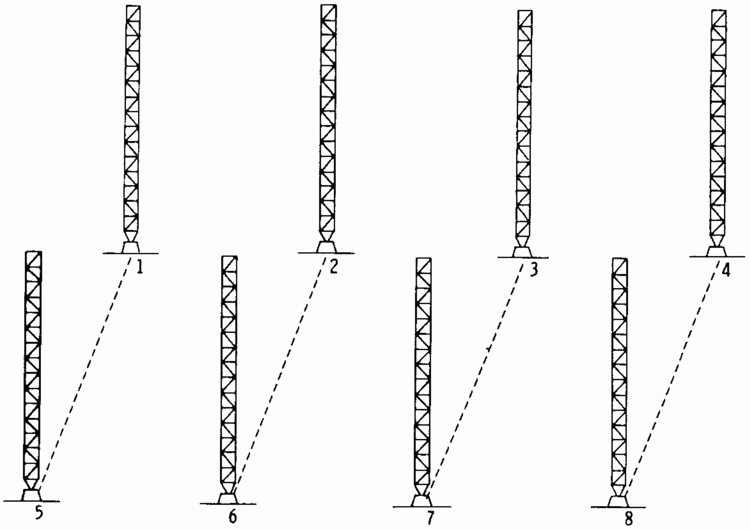
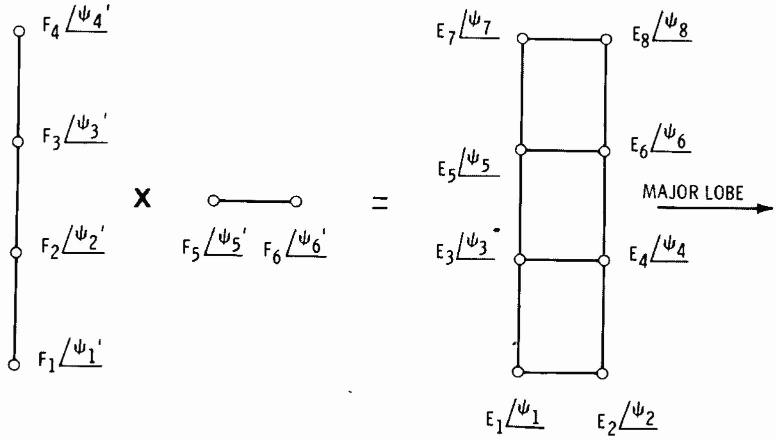


DIAGRAM OF BASIC EIGHT-TOWER CONFIGURATIONS

Figure 1



ANALYSIS OF HOW AN EIGHT-TOWER CAN BE PRODUCED AS THE PRODUCT OF A FOUR-TOWER TIMES A TWO-TOWER

Figure 2

constructed was that of WDGY, more than 25 years ago. The following general formula (Equation 2) can be used to calculate the shape of a nine-tower array:

$$E = Kf(\Theta)\left[\left(\frac{1+F_2}{2F_2} + \cos(\Psi_2 + S_1 \cos\Theta \cos\emptyset)\right) x \left(\frac{1+F_3}{2F_3} + \cos(\Psi_3 + S_1 \cos\Theta \cos\emptyset)\right) x \left(\frac{1+F_4}{2F_4} + \cos(\Psi_4 + S_2 \cos(\emptyset\text{-}A) \cos\Theta)\right) x \left(\frac{1+F_5}{2F_5} + \cos(\Psi_5 + S_2 \cos(\emptyset\text{-}A \cos\Theta))\right)\right]^{1/2}$$

The first two parts of the above equation are those of a three-tower design; the last two parts represent the other three-tower design. Although this is not the only formula available, it represents the one which gives the greatest flexibility in the design of null bearings and null depth.

Figure 5 shows how a three-tower is multiplied by a

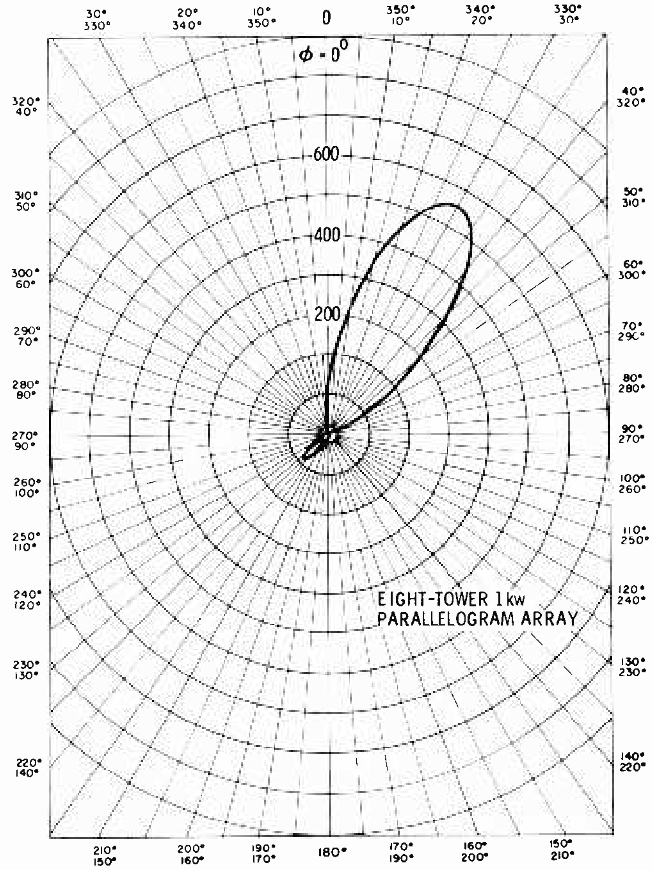
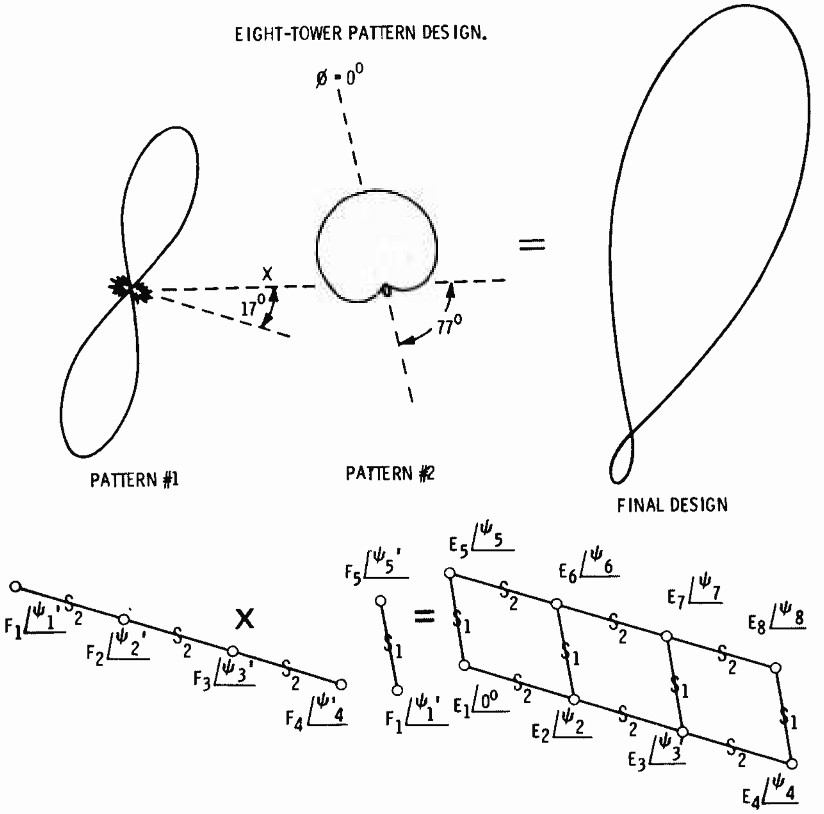


Figure 3



DETERMINATION OF BASE OPERATING VALUES

$E_1\angle 0$	=	$F_1\angle 0$	=	$1.00\angle 0^0$
$E_2\angle\psi_2$	=	$F_2\angle\psi_2'$	=	$2.76\angle 0^0$
$E_3\angle\psi_3$	=	$F_3\angle\psi_3'$	=	$2.76\angle 0^0$
$E_4\angle\psi_4$	=	$F_4\angle\psi_4'$	=	$1.00\angle 0^0$
$E_5\angle\psi_5$	=	$F_5F_1\angle\psi_5'+\psi_1'$	=	$1.00\angle -90^0$
$E_6\angle\psi_6$	=	$F_5F_2\angle\psi_5'+\psi_2'$	=	$2.76\angle -90^0$
$E_7\angle\psi_7$	=	$F_5F_3\angle\psi_5'+\psi_3'$	=	$2.76\angle -90^0$
$E_8\angle\psi_8$	=	$F_5F_4\angle\psi_5'+\psi_4'$	=	$1.00\angle -90^0$

$S_1 = 95^0$

$S_2 = 215^0$

$A = -120^0$

Figure 3A

three-tower. Table 2 is a calculation of a nine-tower pattern. As with Table 1, a slightly different formula than that of Equation 2 has been used. The equation on Table 2 is a three-tower written around the mid-point multiplied by a three-tower written around its mid-point with the offset. This is a special case of the basic three-tower when the end towers have equal magnitudes and are of equal and opposite phase angle.

The last step is designing the final operating values of each tower base current and phase angle. This is best accomplished by determining the base values of each of the two three-tower patterns. The operating values for each should be written around the center tower, as in the example.

To arrive at the final values, one merely multiplies one pattern by the other. Or, tower five is the product of the two reference towers, so obviously it retains the value $1.00\angle 0°$. Towers four and six really have the end-tower values of the first three-tower

TABLE 1
EIGHT-TOWER DESIGN

$$E_T = Kf(\Theta)\left[\frac{1+E_5^2}{2E_5} + \cos(\Psi_5 + S_{15}\cos\phi\cos\Theta)\right]^{1/2} \times \left[\left(E_1\cos\left(\frac{S_{23}}{2}\cos(\phi\text{-}120)\cos\Theta\right)\right) + \left(E_2\cos\frac{S_{14}}{2}\cos(\phi\text{-}120)\cos\Theta\right)\right]$$

Where:

$E_5 = 1.0$, $E_1 = 1.0$, $E_2 = 2.76$, $\Psi_5 = -90°$, $S_1 = 95°$, $S_{14} = 645°$, $S_{23} = 215°$

A	B	C	D	E	F	G	H	I	J	K	L	M	N
ø	95 cos A	B-90	cos C	1+D	√E	A-120	322.5 cos G	cos H	107.5 cos G	2.76 cos J	I+K	F•L	M•K[1]
0	95.0	5.0	.9962	1.9962	1.4128	-120	-161.7	-.9494	-53.7	1.6339	.6845	.96710	118.4 MV/M
10	93.5	3.5	.9981	1.9981	1.4135	-110	-110.3	-.3469	-36.8	2.2100	1.8631	2.63300	322.5
20	89.3	-0.7	.9999	1.9999	1.4140	-100	-55.9	.5606	-18.6	2.6160	3.1766	4.49100	550.0
30	82.3	-7.7	.9909	1.9909	1.4109	-90	0.0	1.0000	0.0	2.7600	3.7600	5.30500	650.0
40	72.8	-17.2	.9553	1.9553	1.3983	-80	55.9	.5606	18.6	2.6160	3.1766	4.44180	535.7
50	61.1	-28.9	.8755	1.8755	1.3695	-70	110.3	-.3469	36.8	2.2100	1.8631	2.55150	312.5
60	47.5	-42.5	.7373	1.7373	1.3180	-60	161.7	-.9494	53.7	1.6339	.6845	.90220	110.5
70	30.8	-59.2	.5120	1.5120	1.2296	-50	207.3	-.8886	69.1	.9846	.0960	.11600	14.4
60	16.5	-73.5	.2840	1.2840	1.1331	-40	247.0	-.3907	82.3	.3698	-.0209	-.02370	2.9
90	0.0	-90.0	.0000	1.0000	1.0000	-30	279.3	.1616	93.1	-.1492	.0124	.01240	1.5
100	-16.5	-106.5	-.2840	.7160	.8462	-20	303.0	.5446	101.0	-.5266	.0180	.0152	1.9
110	-30.8	-120.8	-.5120	.4880	.6985	-10	317.6	.7384	105.8	-.7515	-.0131	-.00920	1.1
120	-47.5	-137.5	-.7373	.2627	.5125	0	322.5	.7933	107.5	-.8299	-.0366	-.01870	2.3
130	-61.1	-151.1	-.8755	.1245	.3528	10	**	**	**	**	.0131	.00460	0.6
140	-72.8	-162.8	-.9553	.0447	.2114	20	.....	.....	.....	.....	.0180	.00380	0.5
150	-82.3	-172.3	-.9909	.0091	.0954	30	.....	.....	.....	.....	.0124	.00120	0.1
160	-89.3	-179.3	-.9999	.0000	.0000	40	.....	.....	.....	.....	.0203	.00000	0.0
170	-93.5	-183.5	-.9981	.0019	.0436	50	.....	.....	.....	.....	.0960	.00018	0.01
180	-95.0	-185.0	-.9962	.0038	.0616	60	.....	.....	.....	.....	.6845	.00260	0.3
190	*	*	*	*	.0436	70	.....	.....	.....	.....	1.8631	.08120	10.9
200	.....	.....	.....	.....	.0000	80	.....	.....	.....	.....	3.1766	.00000	0.0
210	.....	.....	.....	.....	.0954	90	.....	.....	.....	.....	3.7600	.35870	43.9
220	.....	.....	.....	.....	.2114	100	.....	.....	.....	.....	3.1766	.67150	80.9
230	.....	.....	.....	.....	.3528	110	.....	.....	.....	.....	1.8631	.65730	80.5
240	.....	.....	.....	.....	.5125	120	.....	.....	.....	.....	.6845	.35080	42.9
250	.....	.....	.....	.....	.6985	130	.....	.....	.....	.....	.0960	.06700	8.2
260	.....	.....	.....	.....	.8462	140	.....	.....	.....	.....	-.0209	-.01770	2.2
270	.....	.....	.....	.....	1.0000	150	.....	.....	.....	.....	.0124	.01240	1.5
280	.....	.....	.....	.....	1.1331	160	.....	.....	.....	.....	.0180	.02040	2.5
290	.....	.....	.....	.....	1.2296	170	.....	.....	.....	.....	-.0131	-.01610	1.9
300	.....	.....	.....	.....	1.3180	-180	-322.5	.7933	-107.5	-.8299	-.0366	-.04820	5.9
310	.....	.....	.....	.....	1.3695	-170	-317.6	.7384	-105.8	-.7515	-.0131	-.01790	2.2
320	.....	.....	.....	.....	1.3983	-160	-303.0	.5446	-101.0	-.5266	.0180	.02520	3.1
330	.....	.....	.....	.....	1.4109	-150	-279.3	.1616	-93.1	-.1492	.0124	.01750	2.1
340	.....	.....	.....	.....	1.4140	-140	-247.0	-.3907	-82.3	.3698	-.0209	.03010	3.7
350	.....	.....	.....	.....	1.4135	-130	-207.3	-.8886	-69.1	.9846	.0960	.13570	16.6

*Duplicates 0° to +180°
**Duplicates 0° to -180°

$K^1 = 122.51$ RMS = 196.0

pattern, while towers two and eight have the end-tower values of the other three-tower design. This is because multiplying either by the other's reference tower, is the same as multiplying a number by $1.0\angle 0^\circ$, which produces the same number as the original number. The phase angles are merely added from each basic pattern as shown in Figure 6.

This leaves the four corner-tower values to be determined. Tower one is really the product of tower two times four. Tower three is then the product of tower two times six. Likewise, the other two corner towers can be seen to be the products of tower four times tower eight and tower six times tower eight. For a graphic picture of this, see Figure 6A.

Twelve-tower arrays

There are no more than two arrays with this many towers now existing or proposed, which lends some idea as to their rarity. Unlike nine-towers which can

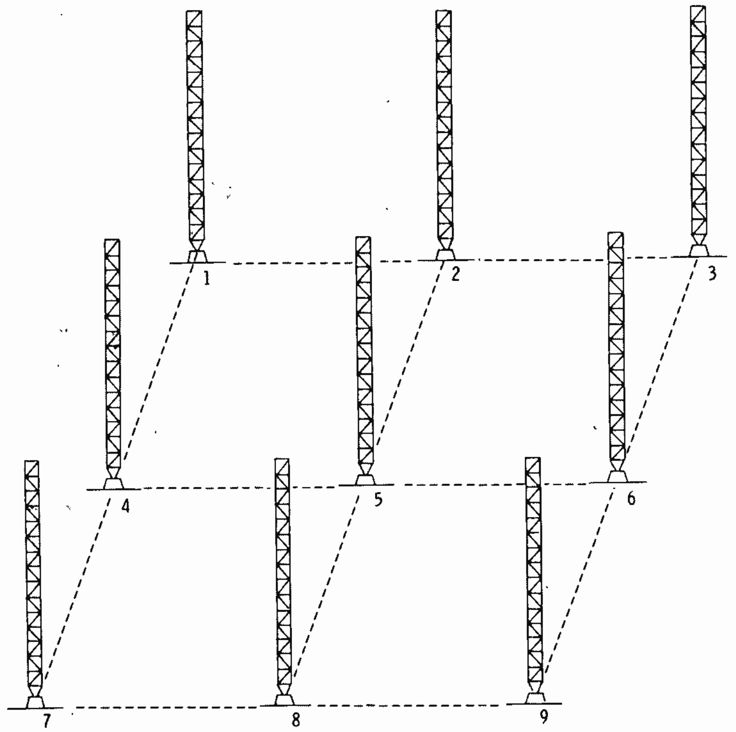
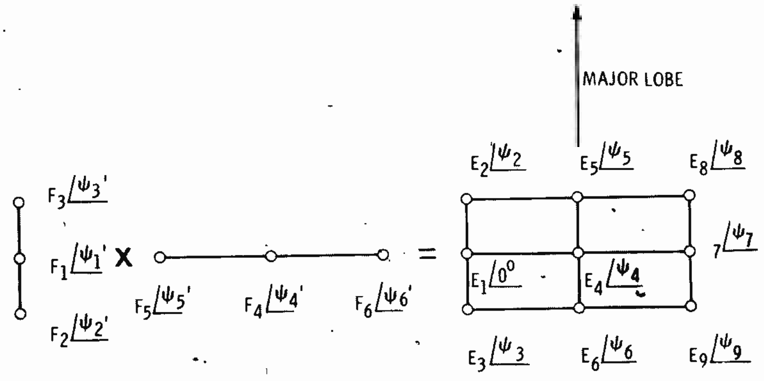


DIAGRAM OF BASIC NINE-TOWER CONFIGURATION

Figure 4



ANALYSIS OF HOW A NINE-TOWER CAN BE CONSTRUCTED AS THE PRODUCT OF A THREE TOWER TIMES A THREE-TOWER.

Figure 5

have only one basic "floor-plan," twelve can be looked upon as three rows of four towers or four rows of three towers.

The following example has assumed a three-tower pattern multiplied by a four-tower. As with the foregoing, a hypothetical design which is assumed to produce maximum suppression along the sides and the rear of this pattern has been chosen. The only application possible for a pattern like this would be on a clear channel, where wide-angle protection is required to be provided to the skywave service area of a class I-A or I-B station.

For the twelve-tower design, the multiplication form is the basic formula (Equation 3):

TABLE 2A
NINE-TOWER DESIGN

$$E = Kf(\Theta)\left[\left(\frac{F_1}{3} + \cos(\Psi_3^1 + S_1 \cos\Theta \cos\emptyset)\right) \times \left(\frac{F_2}{3} + \cos(0° + S_2 \cos(\emptyset\text{-}90)\cos\Theta)\right)\right]^{1/2}$$

Where

$F_1 = 1.00$, $\Psi_3^1 = -94°$, $S_1 = 90$, $F_2 = 1.016$, $S_2 = 200°$

$$E = Kf\Theta\left[1.0 + 2(.5)\underline{\angle(90 \cos \emptyset\text{-}94°)}\right] \times \left[1.0 + 2(.508)\underline{\angle 200 \cos(\emptyset\text{-}90) + 0°}\right]$$

Using the special case where end towers are equal

So

$$E_1 = Kf\Theta\left[F_1 + 2F_2 \underline{\angle S_2 \cos\emptyset \cos\Theta + \Psi_3}\right] \times \left[F_1 + 2F_3 \underline{\angle S_3 \cos(\emptyset\text{-}90°) + \Psi_3}\right]$$

Work each out, square, multiply, then take square root.

$$E_1 = K_1 f\left[1.0 + 1.0 \cos \underline{\angle 90 \cos(\emptyset)\ \text{-}94°}\right] \cdot \left[1.0 + 1.016 \cos \underline{\angle 200 \cos(\emptyset\text{-}90°)}\right]$$

TABLE 2B
NINE-TOWER DESIGN

FIGURE 6

$$E_T = Kf(\Theta)\left[E_1 + 2E_2 \cos(S_2 \cos\emptyset \cos\Theta + \Psi_2)\right] \times \left[E_1 + 2E_3 \cos(S_3 \cos 0 \cos \emptyset\text{-}A) + \Psi_3\right]$$

Where:

$E_1 = 1.0$, $E_2 = 0.5$, $S_2 = 90°$, $\Psi_2 = -94°$, $E_3 = 0.508$, $S_3 = 200$, $\Psi_3 = 0$, $A = 90$

A	B	C	D	E	F	G	H	I	J	K	L
Ø	90 cos A	B-94°	cos C	D+1	A-90	200 cos F	E cos G	1.016•H	1+I	E•J	K•K¹
0	90.0	-4.0	.9975	1.9975	-90	0.0	1.0000	1.0160	2.0160	4.02700	4179.6 MV/M
10	88.6	-5.4	.9956	1.9956	-80	34.7	.8220	.8351	1.8351	3.66200	3800.8
20	84.6	-9.4	.9866	1.9866	-70	68.4	.3681	.3739	1.3739	2.72930	2832.7
30	77.9	-16.1	.9608	1.9608	-60	99.9	-.1719	-.1746	.8254	1.61840	1679.7
40	68.9	-25.1	.9066	1.9066	-50	128.5	-.6225	-.6325	.3675	.70060	727.1
50	57.8	-36.2	.8069	1.8069	-40	153.2	-.8926	-.9069	.0131	.02370	24.6
60	45.0	-49.0	.6560	1.6560	-30	173.2	-.9929	-1.0088	-.0088	.01430	14.8
70	30.8	-63.2	.4508	1.4508	-20	187.9	-.9905	-1.0063	-.0063	.00910	9.4
80	15.6	-78.4	.2010	1.2010	-10	196.9	-.9565	-.9718	.0282	.03380	35.1
90	0.0	-94.0	-.0697	.9303	0	200.0	-.9397	-.9547	.0453	.04210	43.8
100	-15.6	-109.6	-.3354	.6646	10	*	*	*	.0282	.01870	19.4
110	-30.8	-124.8	-.5707	.4293	20	.....	.....	.....	-.0063	.00270	2.8
120	-45.0	-139.0	-.7547	.2453	30	.....	.....	.....	-.0088	.00215	2.2
130	-57.8	-161.8	-.9499	.0501	40	.....	.....	.....	-.0131	.00066	0.7
140	-68.9	-162.9	-.9558	.0442	50	.....	.....	.....	.3675	.01580	16.4
150	-77.9	-171.9	-.9900	.0100	60	.....	.....	.....	.8254	.00820	8.5
160	-84.6	-178.6	-.9999	.0000	70	.....	.....	.....	1.3739	.00000	0.0
170	-68.6	-182.6	-.9989	.0011	80	.....	.....	.....	1.8351	.00200	2.1
180	-90.0	-184.0	-.9976	.0024	90	.....	.....	.....	2.0160	.00480	5.0

*Second half duplicates first half because 3-tower widespaced is symmetrical about the tower line.

$K^1 = 1037.9$, RMS = 1385 MV/M

$$E = Kf(\Theta)\Bigg[\left(\frac{F_1^2+F_2^2}{2F_2} + \cos(\Psi_2 + S_1 \cos\Theta \cos\emptyset)\right) x$$
$$\left(\frac{F_1^2+F_3^2}{2F_3} + \cos(\Psi_3 + S_1 \cos\Theta \cos\emptyset)\right) x$$
$$\left(\frac{F_1^2+D_4^2}{2F_4} + \cos(\Psi_4 + S_2 \cos(\emptyset\text{-}A) \cos\Theta)\right) x$$
$$\left(F_1^2+F_5^2 + \cos(\Psi_5 + S_2 \cos(\emptyset\text{-}A) \cos\Theta)\right) x$$
$$\left(F_1^2+F_6^2 + \cos(\Psi_6 + S_2 \cos(\emptyset\text{-}A) \cos\Theta)\right)\Bigg]^{1/2}$$

The first two terms represent a typical three-tower design, whereas the last three are a typical four-tower in-line design. As with the six-, eight-, and nine-tower designs, the (Ø-A) term represents the angle between the respective tower lines. The most common value for these multiple element arrays is ±90°. When one uses A = 90°, the final pattern will be symmetrical about the tower-line of the three-tower pattern.

Figure 9A shows how a three-tower is multiplied by a four-tower. As with the example of an eight-tower, two patterns that are offset by 120° have been used. This will produce a final pattern that is not symmetrical, even though each of the basic patterns were symmetrical.

Table 3 is a complete set of calculations. Because of the unsymmetrical offset, it is necessary to calculate 36 lines instead of 18 lines. Note that the equation at the top of Table 3 is not completely the same as Equation 3 above. Here too, the flexibility of

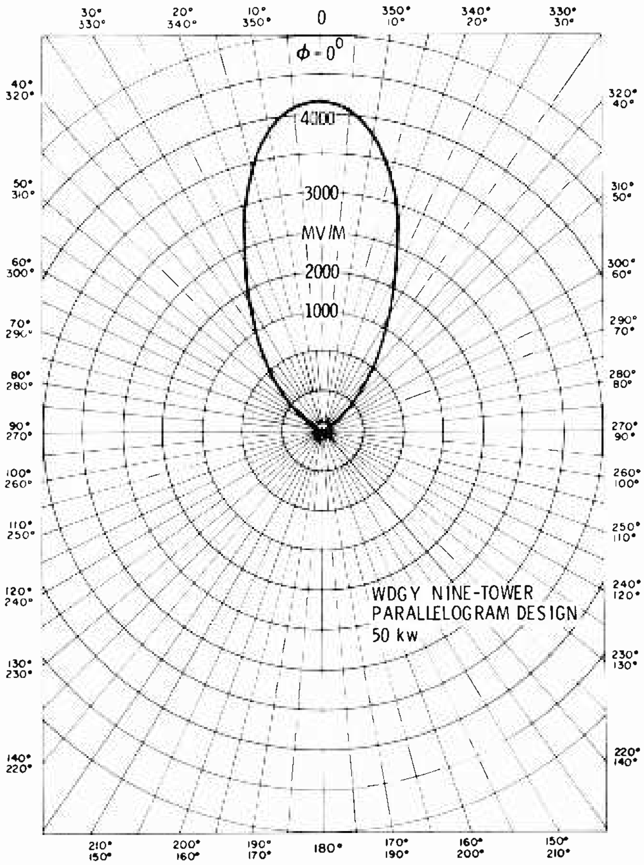
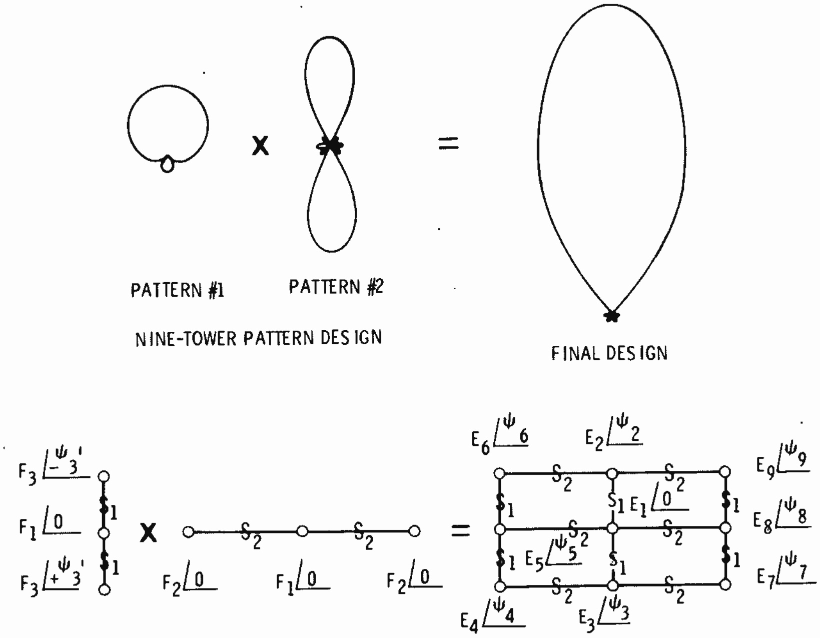


Figure 6 TABLE 2B



DETERMINATION OF BASE OPERATING VALUE:

$E_1\angle 0$ = $F_1\angle 0$ = $1.00\angle 0$

$E_2\angle\psi_2$ = $F_3\angle -\psi_3'$ = $0.500\angle -94^0$

$E_3\angle\psi_3$ = $F_3\angle +\psi_3'$ = $0.500\angle +94^0$

$E_4\angle\psi_4$ = $F_2F_3\angle 0+\psi_3'$ = $0.254\angle +94^0$

$E_5\angle\psi_5$ = $F_2F_1\angle 0+0$ = $0.508\angle 0^0$

$E_6\angle\psi_6$ = $F_2F_3\angle -\psi_3^0+0$ = $0.254\angle -94^0$

$E_7\angle\psi_7$ = $F_2F_3\angle 0+\psi_3'$ = $0.254\angle +94^0$

$E_8\angle\psi_8$ = $F_2F_1\angle 0+0$ = $0.508\angle 0^0$

$E_9\angle\psi_9$ = $F_2F_3\angle 0-\psi_3'$ = $0.254\angle -94^0$

$S_1 = 90^0$

$S_2 = 200^0$

$A = 90^0$

Figure 6A

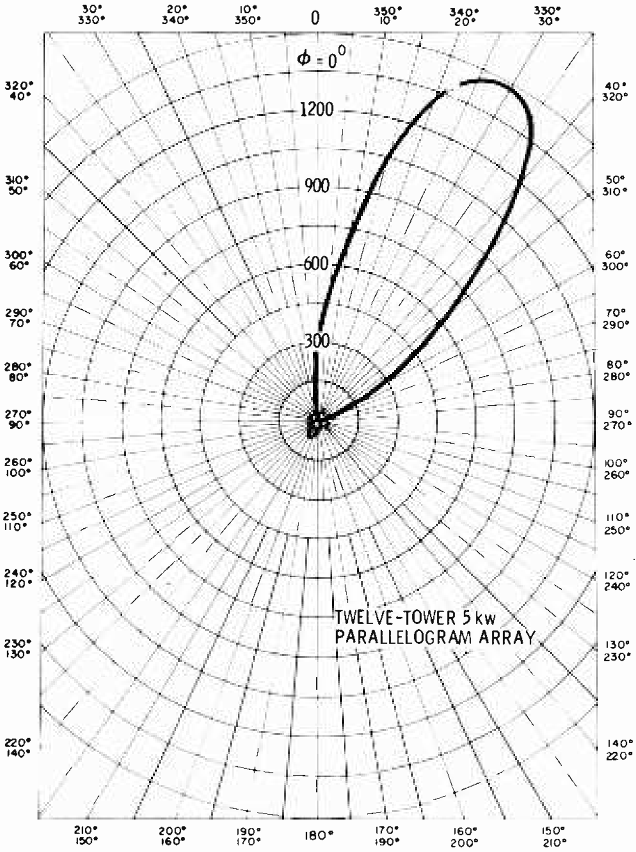
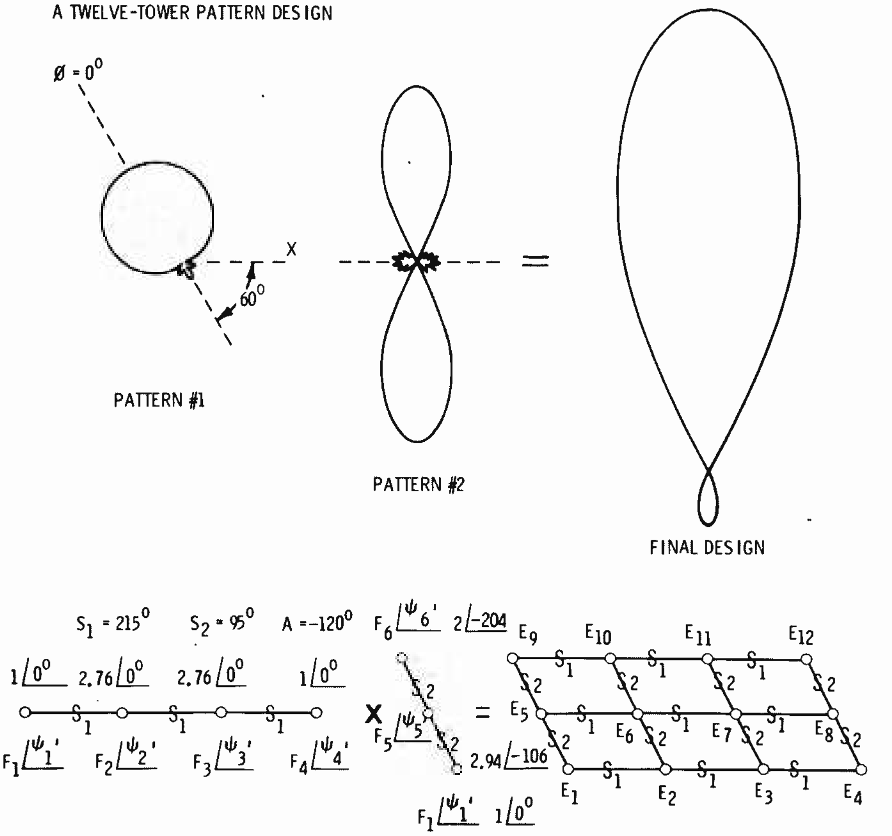


Figure 9



DETERMINATION OF BASE OPERATING VALUES.

$E_1 \angle \psi_1 = F_1 \angle \psi_1' = 1.00 \angle 0^0$

$E_2 \angle \psi_2 = F_1F_2 \angle \psi_1' + \psi_2' = 2.76 \angle 0^0$

$E_3 \angle \psi_3 = F_1F_3 \angle \psi_1' + \psi_3' = 2.76 \angle 0^0$

$E_4 \angle \psi_4 = F_1F_4 \angle \psi_1' + \psi_4' = 1.00 \angle 0^0$

$E_5 \angle \psi_5 = F_1F_5 \angle \psi_1' + \psi_5' = 2.98 \angle -106^0$

$E_6 \angle \psi_6 = F_2F_5 \angle \psi_2' + \psi_5' = 8.22 \angle -106^0$

$E_7 \angle \psi_7 = F_3F_5 \angle \psi_3' + \psi'_5 = 8.22 \angle -10$

$E_8 \angle \psi_8 = F_4F_5 \angle \psi_4' + \psi_5' = 2.98 \angle -10$

$E_9 \angle \psi_9 = F_1F_6 \angle \psi_1' + \psi_6' = 2.00 \angle -20$

$E_{10} \angle \psi_{10} = F_2F_6 \angle \psi_2' + \psi_6' = 5.52 \angle -20$

$E_{11} \angle \psi_{11} = F_3F_6 \angle \psi_3' + \psi_6' = 5.52 \angle -20$

$E_{12} \angle \psi_{12} = F_4F_6 \angle \psi_4' + \psi_6' = 2.00 \angle -20$

Figure 9A

9

Design of nighttime directional antennas

This chapter will examine the special considerations required of nighttime arrays and top-loaded towers, as well as the special problem that falls on the designer when the towers are unequal in height or when the shapes are not uniform.

Skywave propagation

You probably realize, or are at least aware, that during hours of darkness a given radio tower will broadcast a signal both along the ground as well as via the ionosphere to distant areas. The reason one does not hear (at least most of the time) distant signals during the daytime is not due to the fact that towers fail to radiate signals skyward during daylight hours; but, that such emissions are absorbed in the lower ionosphere and fail to return to the surface of the earth. Figure 1 shows nighttime skywaves. FCC Figure 6 predicts the value of Θ and Φ.

Unfortunately, the FCC Rules assume that skywaves are turned on and off at sunset or sunrise. There is, of course, a transitional period each day. Fortunately for the directional antenna designer, the FCC Rules assume that once skywave conditions occur, they are constant. While this is not true, it nevertheless is a realistic approach to nighttime designs. If conditions were different from hour to hour, you could conceivably have one pattern needed for 8:00 p.m., a second one for 10:00 p.m. and even a third one for midnight. This just would not be practical. The FCC has, in my opinion, employed a most practical application on these physical principles.

Vertical characteristics

Each vertical radio tower has certain properties insofar as its generation of signals at various vertical angles above the horizon. This often is referred to as the Vertical Radiation Characteristic. FCC Figure 5 represents such a typical pattern for non-directional towers, with heights of 45° - 225°. Thus, one of the two tools that allow the design to restrict radiation at vertical angles is the electrical tower height.

Taking a purely theoretical example where a distant station whose vertical angle lies at 36° above the horizon needs protection, you could use a tower of 225°. This would give a perfect null, regardless of the power or signal along the ground.

The second tool is the ability to place nulls and major lobes at various vertical angles by controlling the currents, phases, and tower spacings of the various elements of the array. It can generally be stated that whenever you design a null on the ground, you also will have that null follow vertically along some calculable path. I don't mean that a null on the ground will go straight up vertically from that point.

Vertical nulls

An example that will show the principle of vertical nulls is a two-tower design with a single pair of nulls at 45 degrees off the tower-line. Figure 3 shows a polar plot of this simple pattern. Figure 4 is a composite path of this pair of nulls, or rather the route these nulls will take as one looks around the end of the pattern. The degree marks along the horizontal scale represent degrees from north on the ground. The degree marks on the vertical scale represent elevations above the ground. These two nulls come closer and closer together as they increase their elevation above the ground, until finally at 45° they merge.

The next question is how to employ this information. Let's assume you must protect a co-channel station (WXXX) by skywave whose vertical angle from you is 25° vertically and lies at a horizontal bearing of 215° True. Is the null at this bearing? Well, it is on the ground, but if you trace a line straight up from 215° True, to an elevation of 25° vertical, there is no null! The null at 25° vertical elevation actually lies 3° clockwise from 215°. The solution to the problem would be to rotate the tower line 3° counter-clockwise, so that the null on the ground now lies at 222° True and the 25° vertical null lies directly overhead of 215° True. Figure 5 shows how this would look on the ground as well as vertically. (Note the difference between Figure 6 and Figure 3.)

Design protections to more than one co-channel nighttime station could require "spreading" or foreshortening the number of degrees along the surface between the two nulls in the example. Figure 7 shows how with two nulls 35° off the tower-line, you can still meet the protection to WXXX at 215° True and vertically at 25°, plus protection to WYYY at 180° True and 34° vertically.

A formula now can be written to calculate the phase angle, if the horizontal bearing at which a vertical null is required, is known. Or, in other words, if $\emptyset$ and Θ are known, and the spacing between any two towers is known, Ψ can be found as follows (Equation 1):

$$\pm 180 = \Psi + S \cos \emptyset \cos \Theta$$

This is the formula for a two-tower design. As an example, assume the vertical null has to be at 30°, and the bearing is on the tower-line, and that S = 90°. Solving for Ψ, it is equal to -102.0°. To find the horizontal bearing at which the null occurs you can calculate as follows (Equation 2):

$$\cos \emptyset = \frac{\pm 180 + \Psi}{S}$$

In this formula the cosine of $\emptyset$ would be looked up in a cosine table and it would represent the number of degrees off the tower-line the null falls. By knowing the $\emptyset$ bearing of the vertical null, it is a simple matter of subtraction to arrive at the horizontal shift of the null.

Vertical cosine effects

As shown in Figure 2, whatever the tower spacing difference between the two towers, they decrease by the cosine of the vertical angle, from a reference of zero degrees at the horizon. The equations shown in the earlier segments of this series all had a cosine Θ term in them. For example, Equation 3 of Chapter 3 was as follows:

$$E = Kf(\Theta)\sqrt{\frac{1+M^2}{2M} + \cos(\Psi + S \cos \emptyset \cos \Theta)}$$

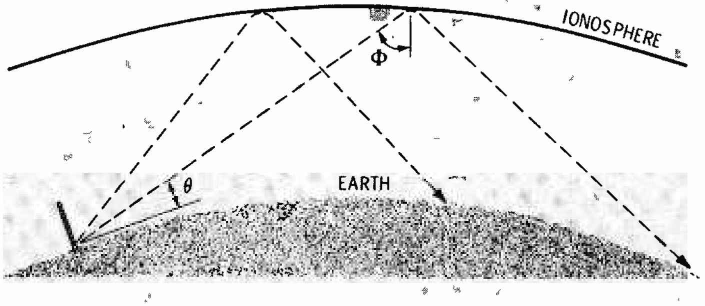
When patterns are designed purely in the horizontal plane, the cosine of 0° is 1,000, hence this multiplier has no effect. Proceeding vertically, this cosine term multiplier becomes smaller and smaller until at 90° it cancels completely the influence of S cos $\emptyset$.

This cosine term when multiplied times (S cos $\emptyset$) is the actual cause of rotation of the horizontal null as it proceeds upward in Figures 4, 5, and 7.

Vertical attenuation

Besides the influence of the cosine of the vertical angle Θ, the vertical attenuation also must be taken into account. Or as commonly written, $f(\Theta)$. Sometimes it is referred to as the vertical form factor.

The total energy radiated by a single tower, in a directional array, is distributed in the vertical plane. This takes the form of a half-donut shape, commonly called a hemi-toroid. As the tower height increases, the amount of energy both along the surface as well as at specific vertical angles will change. FCC Figure 5 shows some typical heights. The FCC Rules are basically concerned only with the total energy directed along the ground. In fact, different classes of



PATH OF RADIO SIGNALS REFLECTED BACK TO EARTH, SHOWING THE HIGHER THE ANGLE θ THE SHORTER THE DISTANCE.

Figure 1

stations have different minimum RMS limits on the horizontal plane RMS.

The energy at any given vertical angle varies because the current flowing upwards in the tower is not distributed uniformly. Standing waves exist in every tower. The radiation which occurs from any point along the antenna is proportioned to the current flowing at that point. The magnitude of each of these point currents, as well as its phase relationship, is due to its height above ground on the tower. The mathematical summation of these infinitesimal currents will determine the total radiated energy over the half hemisphere.

Any tower will radiate the same total amount of power as any other tower, but the radiated fields will be distributed differently. The most efficient tower is one which will produce the greatest groundwave signal, since that is where the listeners generally reside. In the design of a given directional pattern the engineer can take advantage of these differences in individual efficiency both horizontally and vertically.

Conical planes

When speaking about the vertical elevation plane for a specific vertical angle, you really don't mean a "plane," but rather a pattern as described on a conical surface. For example, the only plane pattern is that on the ground. At an elevation of +5° the ratio of base to altitude on the conical surface is large. As the vertical angle increases, this ratio of base to altitude on the cone becomes unity at V = 45° and proceeds down to a very small ratio at V = 85°.

This next comment may seem ludicrous after the above paragraph. But, the FCC requires that these conical surface patterns be submitted on plain paper! These may be either polarily or rectangularly displayed. Figure 9 shows a comparison of such a plot for WKAM at an elevation of 30° vertically.

The equation (4) to be used to calculate this WKAM 30° conical surface is as follows:

$$E_{30°} = Kf\,(\Theta)\left[\frac{1+M^2}{2M} + \cos\,(\Psi + S\cos\emptyset\cos\Theta)\right]^{1/2}$$

Table 1 contains the calculations. As with earlier pattern calculations, it is only necessary to compute halfway around the pattern, since the pattern is symmetrical about the tower-line. Probably the only new figure here is the f(Θ) value. The factor "K" as well as the other values have been previously explained. You may either compute the value of f(Θ) for a given vertical elevation angle or, more conveniently, it can be pulled out of a table of vertical form factors.[1]. It should be pointed out that this f(Θ) is dependent upon both the electrical height of the tower as well as the vertical elevation angle.

For those wishing to compute them, the following formula can be used (Equation 5):

1. NAB Engineering Handbook, pp 2-16.

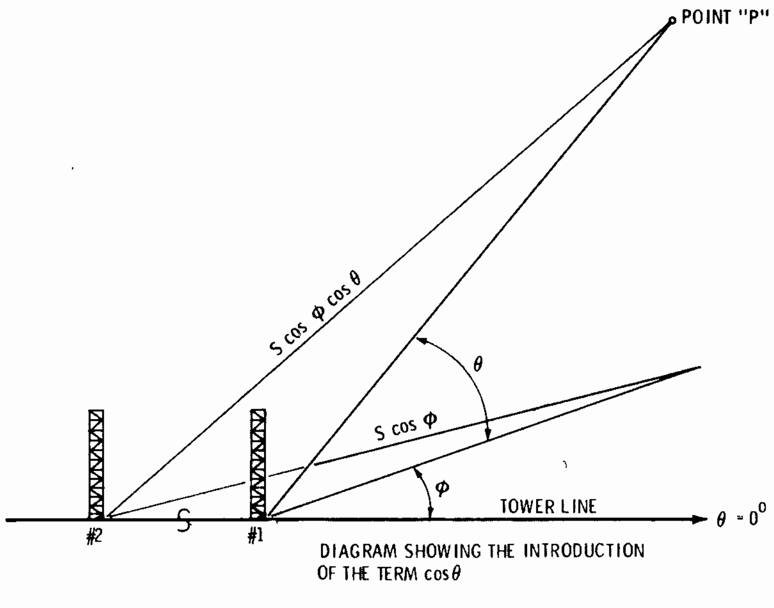


Figure 2

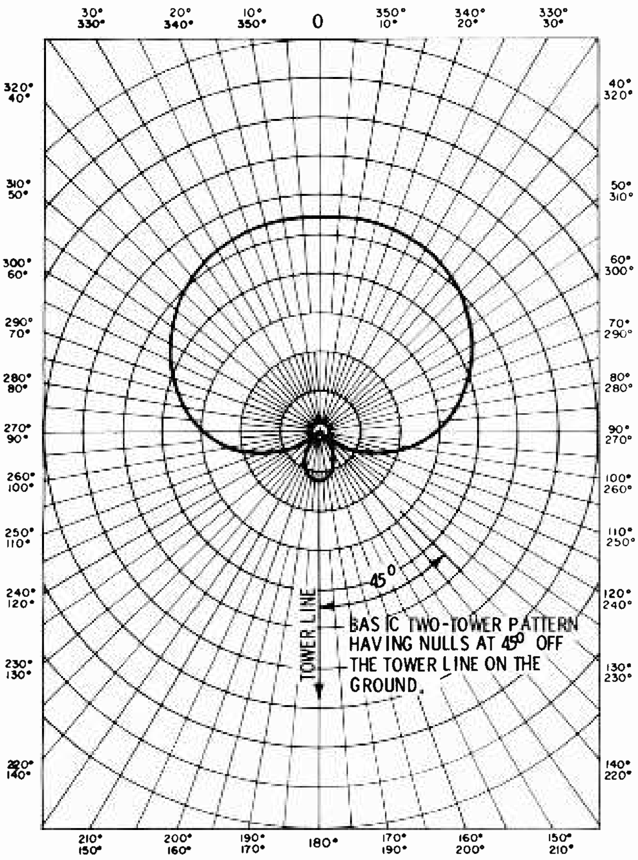


Figure 3

$$F(\Theta) = \frac{\cos(G \sin \Theta) - \cos G}{\cos \Theta (1 - \cos G)}$$

In this formula Θ represents the vertical angle above the ground. The letter G is a symbol to represent the electrical height of the towers. Normally all towers in a DA-N are of equal height. If they are not, it will require an individual calcualtion for each tower at each elevation.

If you care to confirm the value for f(Θ) used in Table 1, the electrical tower height at WKAM is 110°. Both towers are equal, which is why the f(Θ) term is shown outside the radical sign (square root).

Vertical planes are interesting in that the calculation gives a three-dimensional view of a given horizontal plane pattern. They are also necessary to prove the absence of co-channel interference at night.

Top-loaded towers

Some stations use top-loading on their towers which is often accomplished by tying the upper guy wires to the top of the tower and then adding a horizontal skirt wire around the lower end of these top-loading segments. This is shown in Figure 10.

In the ground plane the effect of top-loading is not considered other than it may raise the RMS efficiency of a given tower. The maximum that can be expected would be +10%. But when talking about designing nighttime patterns where top-loaded towers are used, the effect must be considered. This manifests itself as a change in the f(Θ) factor. The following equation (6) is used:

$$f(\Theta) = \frac{\cos B \cos (A \cos \Theta) - \cos \Theta \sin B \sin (A\text{-}\cos \Theta) - \cos G}{\cos \Theta (\cos B - \cos G)}$$

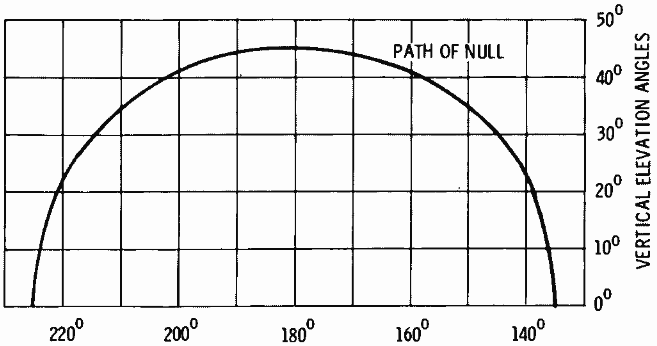


Figure 4

Where: A = the electrical height of the vertical length of the tower that is top-loaded.
B = the electrical length of the portion of the sine wave effectively added by the top-loading.
G = A + B

In the previous equations this new value of f(Θ) would be substituted for the normal values. This new f(Θ) would be a constant for any tower at a given frequency, and vertical angle (Θ). But if you change the frequency, you would obviously change the value of electrical lengths A and B, and thus f(Θ) would change.

Unequal tower heights

Up to this point I have written about designing antenna systems where the radiations from the individual elements were directly proportional to the currents in the towers or as it is often referred to, the field ratio. This condition will occur when each tower within a given system is of equal height and cross-section to each of the others. If not, then a new factor needs to be entered into our design considerations.

The new factor can be looked upon as a ratio of the respective tower efficiencies. For example, a quarter-wave tower operating by itself (energized by 1000 watts of power) will generate a signal at one mile proportional to its amperes of base current:

$$E_{MV/M/Amp} = \frac{196MV/M}{\sqrt{\frac{1000}{36}}} = 37.2 \text{ MV/M/Amp}$$

In other words, for each and every ampere of current that will ever flow in that tower we will measure 37.2 MV/M at a distance of one mile. Thus 2.0 amperes would produce 74.4 MV/M, etc.

But let's look at what would happen if our tower were a half-wave tower. Then the MV/M/Amp would be:

$$E_{MV/M/Amp} = \frac{237.0}{\sqrt{\frac{1000}{870}}} = 221.0 \text{ MV/M/Amp}$$

We now generate for each ampere of current flowing in this second tower a signal at one mile of 221.0 MV/M.

Let's look into the condition that will exist in a simple cardioid pattern consisting of two towers, whereby one is a quarter-wave and the other is a half-wave. And let's assume you wish to achieve a zero null. From the earlier chapters it is apparent that to create a zero null you must have equal radiated fields from each of the two towers. Or the MV/M at one mile must be equal in magnitude, and of course at the null bearing they must arrive 180° displaced in phase. The actual currents that must flow to produce these equal fields are not equal, but are inversely proportional to the MV/M/Amp calculated for the individual tower heights. The tower with the lowest efficiency (i.e. MV/M/Amp) must have the higher total amperes to create the null.

In the example it's assumed each tower contributes a total field intensity, at one mile, of 98.0 MV/M. For the quarter-wave this requires 2.63 amps of current. And for the half-wave it requires 0.44 amps. The base current ratio would be 5.98 or almost six times. Yet the field ratio would be 1.0:1.0 - Unity.

Most design engineers today prefer to specify equal height for all towers since it allows base currents and base current ratios to be equal to designed field ratios. It is obviously easier to see what is going on within a directional pattern when this condition

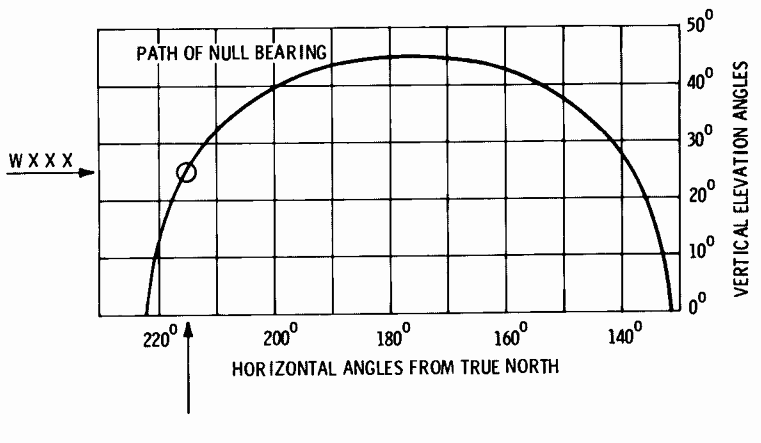


Figure 5

occurs, rather than having to interpret the additional factor of unequal MV/M/Amp.

A second area to keep in mind while designing directional systems is the difference in cross-sectional areas, or in the extreme condition, towers of varying cross-sectional widths. Probably the self-supporting or free-standing tower would be the classic example of this condition. In the first case where there are equal heights, but different cross-sectional weights (even though each tower is uniform) there will be a minor difference in the MV/M/Amp due to the variations in electrical properties of each tower.

In the case of self-supporting towers there is an additional effect due to the variation in width along the tower. This is the problem that exists due to the fact that the current flowing in the tower is not sinusoidally distributed. In the early chapters I stated that one of the prime assumptions necessary to the theory of design is sinusoidal current distribution.

The ideal way to know the difference factor to apply is to operate each tower as a non-directional tower, then apply a correction factor based upon the MV/M/Amp ratio.

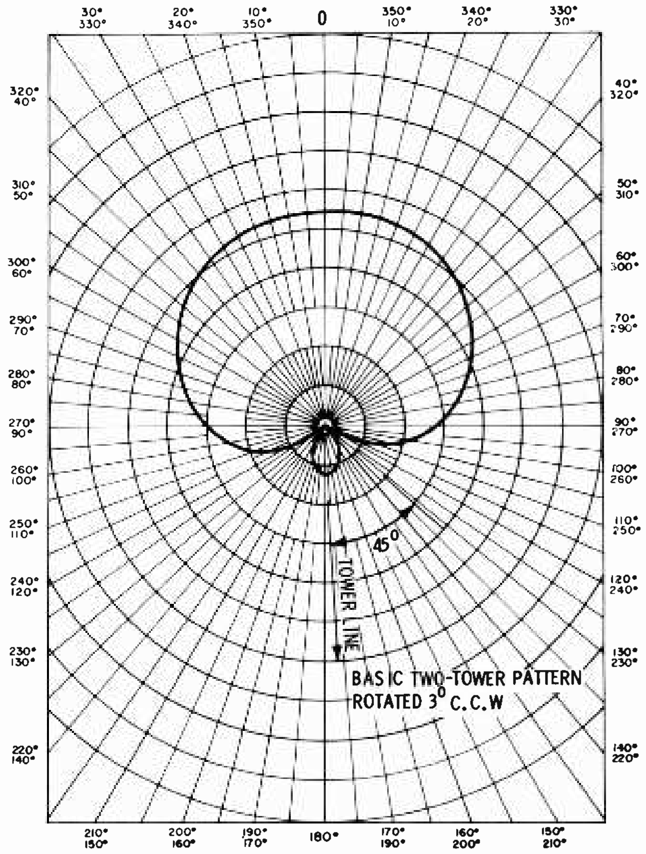


Figure 6

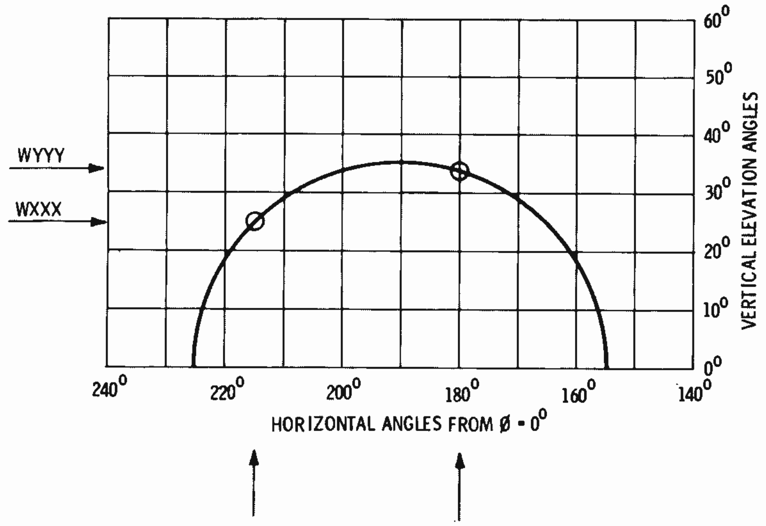


Figure 7

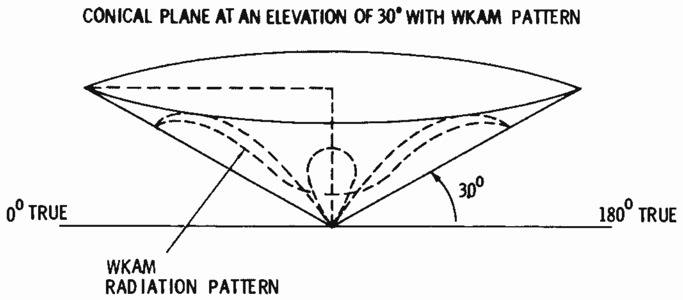


Figure 8

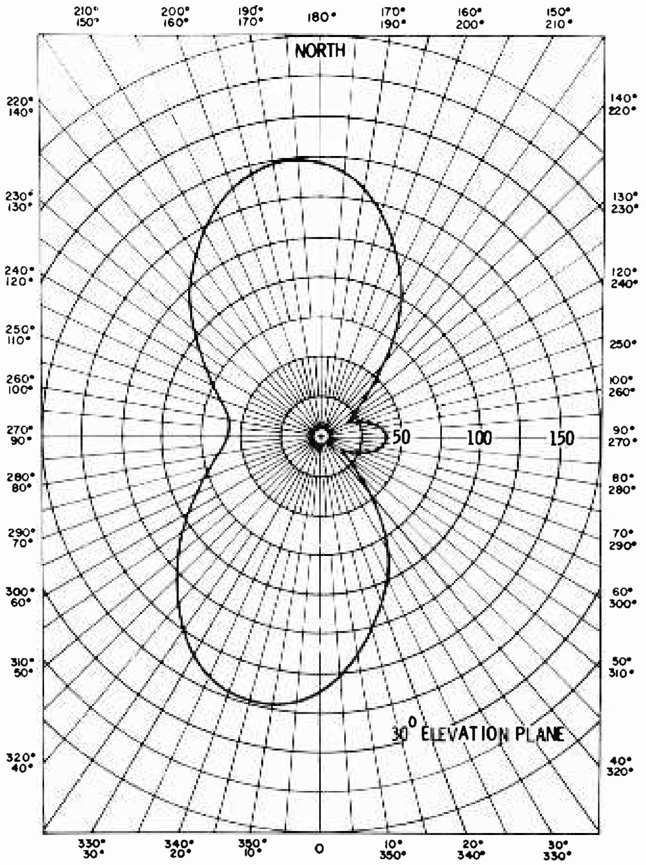


Figure 9

TABLE 1
VERTICAL ELEVATION DATA

Bearing	5°	10°	15°	20°	25°	30°
	MV/M	MV/M	MV/M	MV/M	MV/M	MV/M
93°T	95.0	91.0	82.0	71.0	58.0	42.0
83 103	91.0	87.0	77.0	67.0	52.5	38.4
73 113	76.0	72.0	63.0	54.0	41.1	29.1
63 123	50.5	47.2	41.2	33.9	25.9	19.7
53 133	25.4	24.3	23.5	22.9	25.9	31.0
43 143	45.4	45.8	48.8	52.0	55.0	58.0
33 153	93.5	94.0	93.0	93.0	92.0	91.0
23 163	142.0	141.0	138.0	134.0	129.0	123.0
13 173	183.0	181.0	175.0	169.0	160.0	150.0
3 183	210.0	207.0	200.0	191.0	180.0	168.0
353 193	217.0	214.0	207.0	198.0	186.0	174.0
343 203	206.0	204.0	190.0	190.0	160.0	169.0
353 213	180.0	178.0	174.0	169.0	162.0	153.0
323 223	145.0	143.0	143.0	140.0	137.0	133.0
313 233	108.0	109.0	109.0	108.0	109.0	110.0
303 243	75.5	77.0	79.0	82.0	85.0	88.0
293 253	50.4	52.0	57.5	60.0	67.0	72.0
283 263	36.4	38.2	42.5	48.2	55.0	62.0
273	32.8	34.4	38.5	43.8	51.0	58.0

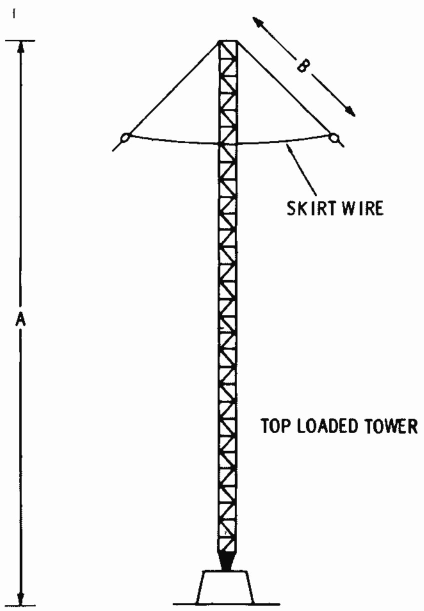


Figure 10

10

Calculating the RMS efficiency

This chapter deals with the subject of pattern efficiency or RMS, as it's normally referred to. It will be demonstrated why some patterns "work" and why others do not. Sometimes a pattern can be designed whose shape looks great, fits all the requirements, but when constructed it doesn't work.

In the early days of designing directionals, the design engineer would generally guess at the RMS of his pattern. Past experience was usually the basis for such guestimates. Also, in the early days the FCC was less concerned about RMS efficiency. The only limits were that the minimum efficiency for power and class of station were met (Section 73.189 of the Rules) or that the point of interference to another station was not exceeded. This was both reasonable and proper by the FCC since there were fewer broadcast stations in those days.

In time though, several stations were granted which could not achieve the first of the above criteria. I won't cite any of these since it is not my intent to embarrass other engineers or stations, but I will cite one case with which I had first-hand knowledge. It won't embarrass the station involved, because they have since dropped their directional and now operate non-directionally. In about 1964, station WRSW was granted a three-tower daytime 1000 watt pattern. This is shown in Figure 1. While I did not design the original pattern nor was I involved in the theoretical aspects, I was retained by WRSW to construct and to "tune" the pattern. After seven weeks of struggling with tuning moves, overheating of parts, impossibly low drivepoint impedances, I finally realized that for 1000 watts input power, the most we could radiate was about 300 watts! At this point it was obvious that the design engineer had goofed on his RMS prediction.

The solution was to reexamine the basic pattern to see what changes could be made in current ratios and phase angles to hold essentially the same pattern shape, but to enhance the RMS. Figure 2 is the result. Note the similarity in patterns between one and two. But look at the phase angle and current ratios. Pattern two achieved a measured RMS of 203 MV/M. In this case WRSW was lucky that a similar pattern could be found. Other stations have not been so fortunate.

Major lobe vectors

One of the quick and cheap methods of looking at the probable efficiency of any given pattern is to study the vector relationship of the individual towers as they combine to produce the major lobe. Figure 3 is a plot of these vectors for each of the WRSW patterns. The dashed line represents the resultant signal in the major lobe. The reference tower was selected as having a value of 1.0 units. Even without determining the individual tower signals in MV/M, the three vectors in pattern two add up to a much greater resultant.

Let's look at another concept exemplified by this case. To achieve the same resultant signal (R) from both patterns, it is obvious that the individual signals (in MV/M) from each of the three towers would have to be much greater in pattern two. In order to achieve stronger fields for each tower, one would have to generate higher antenna currents. As tower currents go up, so do the losses per tower. Ohm's Law states that this will occur in direct relationship to the square of the current. In pattern two, if it took twice the base currents to achieve the true pattern sizes, losses would be quadrupled! And with higher losses it would require higher base currents, which would increase the losses in an almost never-ending cycle. The point should be obvious—maximize the vectors in the major lobe for greatest efficiency! This brings up the concept of pattern gain.

Pattern power gain

The vectors of Figure 3 reveal that in some patterns the combination of vectors is more efficient. If these combinations result in more total signal along the ground, overall, than would be achieved by a single tower (ND) operating with the same power level, then a pattern is said to have gain (g).

This measurement of pattern gain is thus the gain along the surface, and not the gain at some vertical angle. It would follow, from the law of conservation of energy, that in order to achieve "gain" you must conserve somewhere at vertical angles. One way to predict if gain will be achieved in the horizontal plane, is to look at the rate of attenuation of the

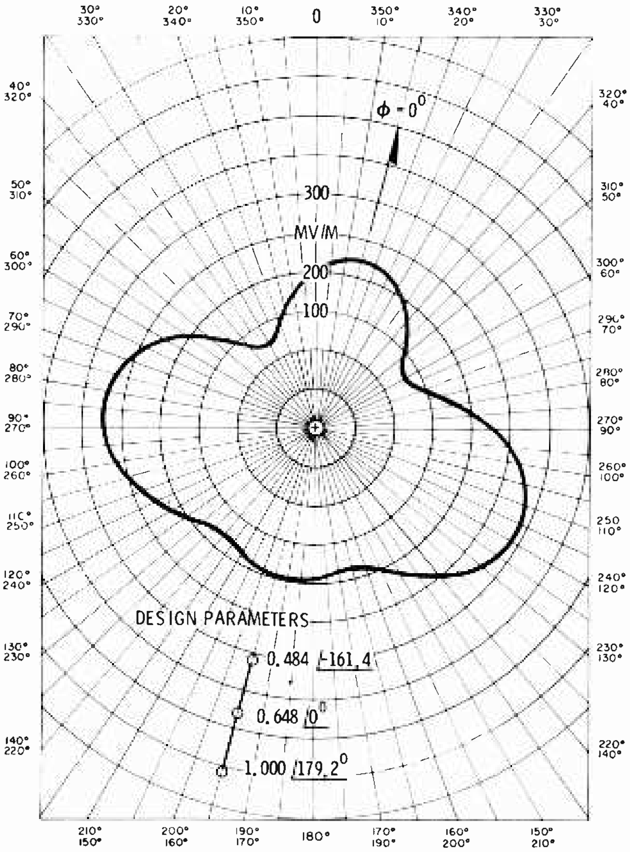


Figure 1

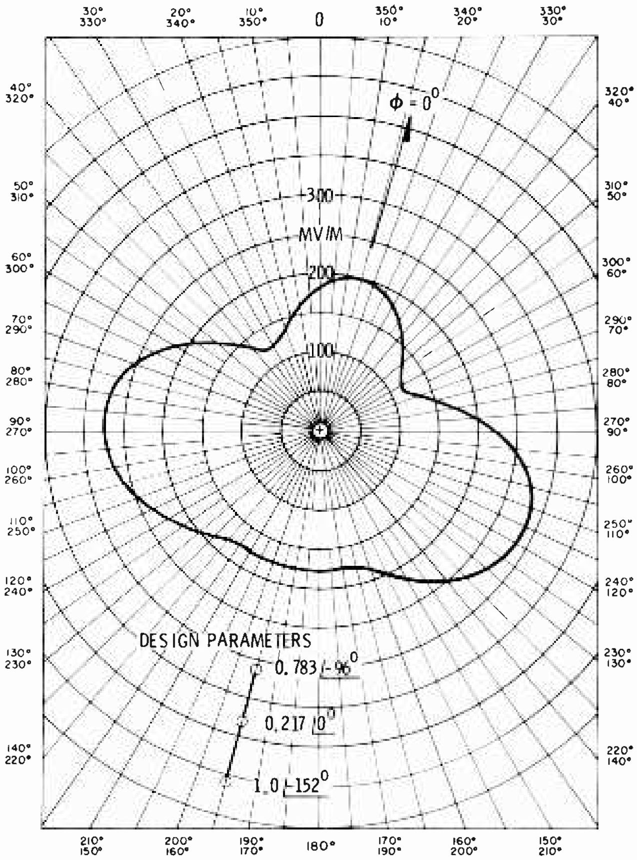


Figure 2

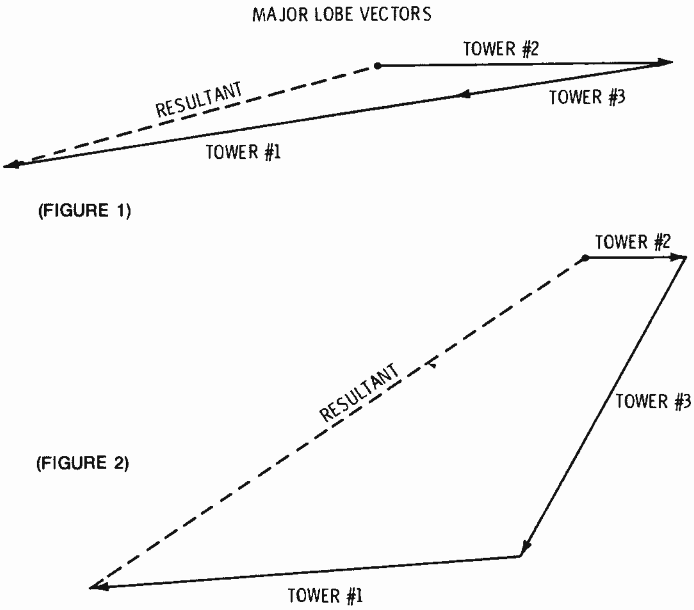


Figure 3

vertical field intensities. For example, if there are large overhanging lobes, it is a good bet the signal along the ground will be less than the equivalent of a single (ND) tower.

To calculate the gain of any antenna pattern, first compute or measure the directional RMS. This is then divided by the field intensity at one mile from a single reference tower having the same height with the same power. Gain is expressed as follows (Equation 1):

$$g = \left(\frac{E_0}{E_1}\right)^2$$

The figure E_0 is used to represent the RMS of the DA, and E_1 is the horizontal field intensity at one mile of the Number 1 or reference tower. To use this equation, E_0 and E_1 should be the same power.

Why be concerned with power gain at this point? In addition to the above reasons, the best reason is because the coverage of a radio station can be increased if for the same number of watts, the horizontal RMS can be increased. Maximum coverage per watt is still the ultimate goal.

Efficiency of directional antenna

This is one way of expressing antenna gain, demonstrated by the following formula (Equation 2):

$$E_0 = E_1 \sqrt{g}$$

It is necessary to use the square root of g in order to determine the RMS voltage of the final design. No losses are assumed in Equation 2.

Since Equation 2 was written upon the basis of no-loss in either the reference antenna (E_1) or the directional array (E_0), g truly represents the inherent property of the DA to be a gainer or a loser of horizontal RMS. If the system has losses, and every one ever built has, Equation 2 must be modified downward to account for these losses. This can be written as follows (Equation 3):

$$E_{0L} = E_1 \sqrt{g} \cdot \sqrt{\gamma}$$

In this equation, E_{0L} is the RMS of the directional with losses. The new term γ can be computed from the following expression (Equation 4):

$$\gamma = \frac{P_R}{P_R + P_L}$$

The power efficiency (γ) is proportional to the power radiated divided by the sum of the power radiated plus the power lost. The term γ must always be less than unity.

TABLE 1
BESSEL FUNCTION VALUES

TOWER SPACING =	90°	135°	180°	225°	270°	360°
ELEVATION ANGLE (0)						
0°	0.470	0.030	-.306	-.400	-.265	.222
10°	0.484	0.047	-.290	-.400	-.285	.200
20°	0.524	0.105	-.240	-.400	-.336	.126
30°	0.584	0.200	-1.500	-.365	-.390	-.025
40°	0.666	0.338	-.010	-.264	-.399	-.236
50°	0.760	0.500	0.210	-.062	-.270	-.397
60°	0.850	0.680	0.470	.242	-.072	-.304
70°	0.928	0.845	0.733	.597	.450	.140
80°	0.980	0.960	0.928	.888	.840	.724
90°	1.000	1.000	1.000	1.000	1.000	1.000

Power flow integration

This is one of the three generally accepted methods used by design engineers to calculate the RMS of a given directional pattern. Don't let that word "integration" throw you. I don't intend to go through any calculus steps. It suffices to say that in order to determine the total amount of power that will be generated from a given directional pattern, measure or compute the total energy over the semi-hemisphere of the array. Calculus integration is one mathematical way to sum up this energy. Fortunately for the design engineer, this has been reduced to a single term called a Bessel Function (J_0). The Bessel Function will vary with the tower spacing between the elements, and will vary with the vertical elevation angle. Table 1 contains a partial list of some common J_0 values. Complete tables or graphs of Bessel Functions are available in math handbooks or in the *NAB Handbook.*

Table 2 shows how to begin with the basic formula for a two-tower pattern and expand it through to find the E_S value of the total signal. Line one represents the basic formula of a two-tower pattern. This is then changed from Polar Coordinates to Rectangular Coordinates, squared, and simplified by $\text{Cos}^2+\text{Sin}^2=1.0$. The fourth line represents the general equation for the horizontal plane. Line five is modified to show the same general equation modified by introducing Vertical Angles (Θ). The standard integration over the hemisphere in line six is very difficult. However, in order to avoid this integration, use the summation of the individual calculations of the RMS, taken at 10 degree vertical intervals. Note the similarity of this approach to the summation technique used in Table 4. To obtain an idea of what a three-tower pattern would look like at a vertical angle of Θ, use the following equation; assuming all towers are equal in height (Equation 5):

$$E_\Theta = f(\Theta)\,[E_1^2 + E_2^2 + E_3^2 + 2E_1 E_2 \cos \Psi_{12} J_0 (S_{12} \cos \Theta) + 2E_1 E_3 \cos \Psi_{13} J_0 (S_{13} \cos \Theta) + 2E_2 E_3 \cos \Psi_{23} J_0 (S_{23} \cos \Theta)]^{1/2}$$

For a directional antenna with more elements, the number of terms will increase. It will be obvious that the general equation consists of the following terms under the radical: first, the square of all the individual antenna field intensities, and second, the terms written as twice the product of the field intensity of each pair of towers multiplied by the cosine of the difference in phase times the Bessel Function for the spacing and elevation angle of that pair of towers. The power radiated then, in terms of E_Θ^2, can be written as (Equation 6):

$$P = \frac{1}{R_C} \int_0^{2\pi} \int_0^{\pi/2} E^2 d^2 \cos \Theta \, d \Theta \, d \emptyset$$

For the standard hemispherical field intensity produced by the directional antenna system, of any general pattern, write (Equation 7):

$$E_S = \sqrt{\int_0^{\pi/2} E_\Theta^2 \cos \Theta \, d \Theta}$$

This is the exact formula for determining the size of the directional antenna pattern. However, the integration for even the general case is quite tedious. Therefore a practical and useful solution is to determine the value of E_Θ at a number of elevation angles and to replace the integral in Equation 7, with a summation. It is common to do this at 10 degrees of elevation intervals. The approximate equation (using the trapezoidal rule from mathematics) will be written as (Equation 8):

$$E_S = \sqrt{\frac{\pi}{18}\left[\frac{E_0^2}{2} + \sum_1^8 E_{10}^2 \cos (10°)\right]}$$

Where E_S = the standard hemispherical FI produced by a directional antenna, where E_0 = the RMS in the horizontal plane, and where E_{10} is the RMS of each of the specified elevation planes. Table 3 shows a summation calculated for a basic two-tower pattern. For each vertical plane the RMS decreases. The last column is a tabulation of the RMS field intensity for each 10° elevation angle plane.

There are two steps to Table 3. The first consists of columns A through F, which allow the designer to calculate the RMS gain in relationship to tower one, or the tower taken as unity. These individual ratios are then squared, multiplied by the cosine of their respective vertical elevation angle, and summed. The last step is to arrive at a computation for the radiated field intensity from tower one. To do this, since it is known that the power of 1 kW over a hemisphere will produce 152.1 MV/M, divide this standard by the summation and arrive at a value for E_1. To calculate the RMS in the horizontal plane, multiply E_1 x the gain in the horizontal plane. In the example this is 145.0 MV/M x 1.355 = 196.48 MV/M.

Resistance method of pattern size

The above method could be used to find the base and loop mutual resistance of each element in a given antenna system. It does not yield, however, the drive point impedances, but it is a very practical and useful method for determining the RMS of a pattern. The resistance method is in essence an extention plus simplification of the Power Flow Integration Method.

TABLE 2
TWO-TOWER POWER-FLOW DERIVATION

$E_T = E_1 \angle 0 + E_2 \angle S_2 \cos \emptyset + \Psi_2$

Changing to rectangular coordinates and squaring.

$E_T^2 = E_1^2 + E_2^2 \cos^2 (S_2 \cos \emptyset + \Psi_2) +$
$E_2^2 \sin^2 (S_2 \cos \emptyset + \Psi_2) +$
$2 E_1 E_2 \cos (S_2 \cos \emptyset + \Psi)$

Since $\cos^2 + \sin^2 = 1.0$, we can simplify as follows:

$E_T = E_1^2 = E_2^2 + 2E_1 E_2 \cos (S_2 \cos \emptyset + \Psi_2)$

Integration over the horizontal field:

$$E_T^2 = \int_0^{2\pi} E^2 d\emptyset =$$

$2 [E_1^2 + E_2^2 + 2E_1 + E_2 \cos \Psi_{12} J_0 (S_{12})]$

Integration for any angle

$$E_{T\Theta}^2 = \int_0^{2\pi} E_{d\emptyset}^2 = E_1^2 f_1 (\Theta)^2 + E_2^2 F2 (\Theta)^2 + 2E_1 f_1 (\Theta) E_2 f_2 (\Theta) \cdot \cos \Psi_{12} J_0 (S_{12} \cos \Theta)$$

The standard Hemispherical field intensity will be:

$$E_S = \sqrt{\int_0^{\pi/2} E_{T\Theta}^2 \cos \Theta \, d \Theta}$$

(While this is the exact formula for calculating the size of any two-tower directional pattern, the general case is

$$E_S = \sqrt{\frac{\pi}{18}\left[\frac{E_T^2}{2} + \sum_1^8 E_{10th}^2 \cos (10_n)\right]}$$

Begin by comparing the relationship between field intensity and antenna current. This can be expressed as MV/M/Amp. The value of field intensity is dependent upon the distance the observer is from the antenna, the electrical height, and the elevation angle (Θ). The General Equation is written (Equation 9):

$$E = \frac{R_C I \, [\cos (G \sin \Theta) - \cos G]}{2\pi d \qquad \cos \Theta}$$

If the terms are not familiar by now, E is the field intensity in V/M, R_C is the resistance of free space (376.71 ohms) and I is the distance in meters. If at this point the letter "h" is substituted for the expression:

$$h = \frac{\cos (G \sin \Theta) - \cos G}{\sqrt{\cos \Theta}}$$

Then Equation 9 can be rewritten as follows (Equation 10):

$$E = \frac{R_C I}{2\pi d} \frac{h}{\sqrt{\cos \Theta}}$$

The self and mutual resistance terms can be found by the following method (Equation 11).

$$R_{12} = \frac{R_C}{2} \int_0^2 h_1 h_2 J_0 (S_{12} \cos \Theta) \, d \Theta$$

$$= \frac{R_C}{36} [(1 - \cos G_1) (1 - \cos G_2) J_0 (S_{12})]$$

$$+ \sum_1^8 (h_1)_{\Theta_{10}} (h_2)_{\Theta_{10}} J_0 (S_{12} \cos \Theta_{10})$$

Since this integration is quite laborious, it is more simple to graphically determine the value of h_1 and h_2. This can be read from the graph of Figure 6. Also the (J_0) Bessel Function can be read from Table I or standard Bessel Function Graphs. The convenience of this method lies in its ability to compute mutual resistance between towers of unequal heights.

Table 4 shows how to calculate the mutual resistance between two towers: one of 90° and one of 150° spaced 200° apart. This would yield a value of 21.44 ohms. In order to convert this loop value to its base equivalent it is only necessary to divide it by (Sin G_1 Sin G_2). In the example then:

$$\frac{21.44 \text{ ohms}}{(1.0)\,(0.5)} = 42.88 \text{ ohms}$$

To find the power radiated from the two-tower, use the following formula (Equation 12):

$$P_r = I_1^2 R_{11} + I_2^2 R_{22} + 2 I_1 I_2 \cos \Psi_{12} R_{12}$$

If the pattern's power is known, we can find the size of the hemispherical RMS by the following (Equation 13):

$$E_S = \sqrt{\frac{P_r R_c}{2 d^2}}$$

But since the horizontal RMS (E_0) is of interest, the following equation can be substituted for a two-tower pattern (Equation 14):

$$E_0 = \sqrt{E_1^2 + E_2^2 + 2 E_1 E_2 \cos \Psi_{12} J_0 (S_{12})}$$

If the towers in this two-tower array are of equal height, then the current ratios can be substituted for the field ratios.

If Equation 14 is solved for E_1, then recognize that the current in tower one is proportional to the square root of the power radiated by the first tower divided by the operating resistance of the first tower. This can be written as an expression having numerators and denominators that are very similar, the only difference being that the denominator contains the resistance ratio, while the numerator contains the Bessel Function (Equation 15):

$$E_0 = E_{1S} \sqrt{\frac{E_1^2 + E_2^2 + 2 E_1 E_2 \cos \Psi_{12} J_0 (S_{12} \cos \Theta)}{E_1^2 + E_2^2 + 2 E_1 E_2 \cos \Psi_{12} \frac{R_{12}}{R_{11}}}}$$

In the equation, E_0 is the horizontal RMS expressed in MV/M. E_{1S} is the horizontal field intensity of antenna 1, acting as a reference antenna of P_R watts. R_{12} is the mutual loop resistance between towers 1 and 2, while R11 is the self loop radiation resistance. These latter two quantities are each expressed in ohms.

Driving point impedance

This last method is most useful since it allows the designer to determine both the drive point impedance of each tower operating in the directional antenna but also allows the introduction of losses. This method is demonstrated with a three-tower pattern.

Figure 5 shows the design values of field ratio and phase angles, as well as the tower spacing. Since the towers are of equal heights, current ratios can be substituted for field ratios. The self resistance as well as the mutual impedance can be looked up in standard graphs or texts. In order to solve for each tower we would use the basic equation (Equation 16):

$$Z_{01} = R_{11} + \frac{I_2}{I_1} M_{12} + \frac{I_3}{I_1} M_{13}$$

The term Z_{01} is the operating impedance of tower one. Likewise I will solve for the Z_{02} and Z_{03}. The three by four squares in Figure 5 show how to compute these values.

The next step in the solution is to take the three operating resistances (1.56, 2.74, and 5.42 ohms) and compute the actual current of tower one. This is done by taking the current ratio, squaring it, multiplying each by its respective base resistance, then adding

TABLE 3
RMS CALCULATION BY SUMMATION

$$E_\Theta\,[E_1^2 f(\Theta_1) + E_2^2 f(\Theta_2) + 2\,E_1 f(\Theta_1)\,E_2 f(\Theta_2) \cos \Psi_{12}\, J_0 (S_{12} \cos \Theta)]^{1/2}$$

If the height of $E_1 = E_2$, then $f(\Theta_1) = f(\Theta_2)$ and this term can be removed from the radical,

let: $E_1 = 1.0$, $E_2 = 1.0$, $\Psi_{12} = -110°$, $S = 90°$, and the power be 1000 watts

A	B	C	D	E	F	G	H	J
θ	f(θ)	$J_0 S_{12}$ = 90°	2 cos Ψ_{12}	C•D+2.0	B/√E	F^2	cos B	G•H
0	1.000	0.470	-0.173	1.836	1.355	.....	.....	.....
10	.978	0.485	-0.173	1.831	1.323	1.750	0.985	1.724
20	.914	0.525	-0.173	1.817	1.232	1.518	0.939	1.425
30	.816	0.584	-0.173	1.800	1.095	1.199	0.866	1.038
40	.695	0.666	-0.173	1.768	.924	.854	0.766	.654
50	.559	0.760	-0.173	1.736	.736	.542	0.643	.348
60	.418	0.850	-0.173	1.704	.545	.297	0.500	.148
70	.277	0.928	-0.173	1.678	.359	.129	0.342	.044
80	.138	0.980	-0.173	1.659	.177	.031	0.173	.005
							Sum =	5.386

$$E_S = \sqrt{\frac{\pi}{18}\left(\frac{[E_0^2]}{2} + \sum_1^8 E^2_{\Theta_{10}} \cos \Theta_{10}\right)}$$

$$E_S = \sqrt{\frac{\pi}{18}\left(\frac{1.355^2}{2}\right) + (5.386)}$$

$E_S = 1.049$ Since 1 kW hemisphere = 152.1 MV/M

$$E_S = \frac{152.1 \text{ MV/M}}{1.049} = 145.0 \text{ MV/M}$$

RMS in the horizontal plane =

145.0 MV/M x 1.355 = <u>196.48 MV/M</u>

TABLE 4
CALCULATION OF MUTUAL RESISTANCE

$$R_{12} = \frac{R_c}{2\pi} \int_0^{\pi/2} h_1\, h_2\, J_0 (S_{12} \cos \Theta)\, d_\Theta,$$ can be approximated by the following

$$= \frac{R_c}{36}\left[\frac{(1\text{-}\cos G_1)(1\text{-}\cos G_2)(J_0)}{2} + \sum_1^8 h_{1(10)}\, h_{2(10)}\, J_0 (S_{12} \cos \Theta)\right]$$

If $G_1 = 90°$, $G_2 = 150°$, and $S_{12} = 200°$

$\sum_1^8 =$

θ	$h_1(90°)$	$h_2(150°)$	$J_0(S_{12} \cos \theta)$	Sum
10	0.97	1.77	-0.37	-.635
20	0.89	1.54	-0.34	-.466
30	0.76	1.20	-0.27	-.246
40	0.61	0.88	-0.13	-.069
50	0.45	0.55	0.09	.022
60	0.29	0.32	0.37	.034
70	0.16	0.15	0.88	.016
80	0.06	0.05	0.91	.003
				-1.341

$$R_{12} = \frac{376.7}{36}\left[\frac{(1.0)(1.865)(-0.38)}{2} -1.341\right]$$

$$= 10.46\,[-0.709\ -1.341]$$

<u>R_{12} = 21.44 ohms (Loop)</u>

To convert to base mutual, divide by

$$R_{12(Base)} = \frac{R_{12}}{(\sin G_1)(\sin G_2)} = \frac{21.44}{(1.0)(.50)} = 42.88 \text{ ohms}$$

INTEGRATION OF POWER FLOW OUT OF HEMISPHERE

$$E_S = \int_0^{2\pi}\int_0^{\pi/2} E^2 \, d_\theta \, d\phi$$

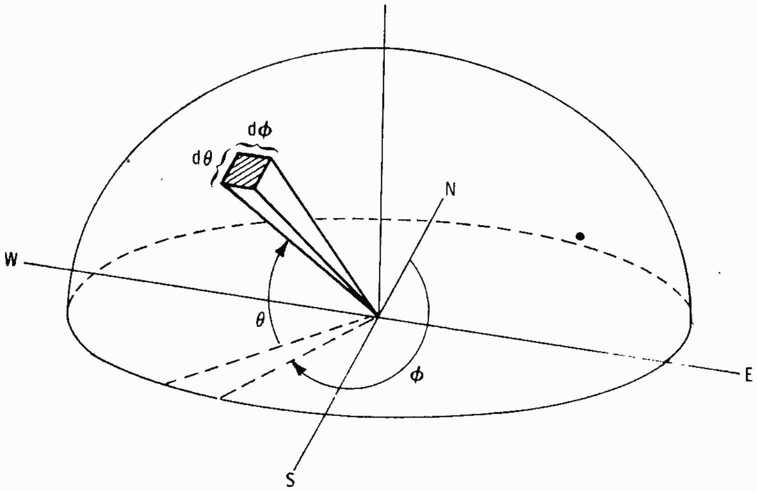
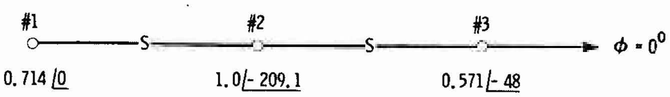


Figure 4.



GIVEN: S = 70°, G = 78°

SELF RESISTANCE (R_{11}) = 24.5 OHMS

MUTUAL IMPEDANCE (70°) = 19.2/- 22

MUTUAL IMPEDANCE (140°) = 13.2/- 77

	TOWER #1	TOWER #2	TOWER #3	ANSWER
Z_{01} =		(1/- 209.1)(19.2/- 22) / 1.0/0 26.9 /- 231.1 - 16.88 + J20.9	(.571/- 48)(13.2/- 77) / .(714/0 10.56/- 125 - 6.06 - J8.65	24.5 + J0 - 16.88 + J20.9 - 6.06 - J8.65 1.56 + J11.25
Z_{02} =	(.714/0)(19.2/- 22) / 1/- 209.1 13.71 /187.1 - 13.6 - J1.69		(.571/- 48)(19.2/- 22) / 1/- 209.1 10.97 /139.1 - 8.16 + J7.17	24.5 + J0 -13.60 - J1.69 -8.16 + J7.17 2.74 + J5.48
Z_{03} =	(.714/0)(13.2/- 22) / 0.571/- 48 16.5 /- 29 14.41 - J8.0	(1.0/-209.1)(19.2/-22) / 0.571/- 48 33.6/- 183.1 - 33.5 + J1.82		24.5 + J0 14.42 - J8.0 - 33.4 + J11.82 5.42 - J6.18

OPERATING RESISTANCES ARE:

R_1 = 1.56 OHMS R_2 = 2.74 OHMS R_3 = 5.42 OHMS

Figure 5

and equating it to the proposed power (1000 watts) (Equation 17):

$$1000 \text{ Watts} = \sum_{1}^{3} I^2 R$$

Substituting in the example, plus adding 1.0 ohm loss in series with each tower gives:

$$1000 = I_2^2 \, [(.714)^2 \, (1.56 + 1.0) + (1.0)^2 \cdot (2.74 + 1.0) + (.571)^2 \, (5.42 + 1.0)]$$

Solving for I_2 results in 11.84 amps, with loss. Proceed to calculate the actual field intensity radiated by the tower two (reference). At this point it is necessary to calculate the MV/M/Amp of a tower of this electrical height.

Study FCC Figure 8 to determine the Unattenuated Field Intensity for 1 kW and this tower height. Using the self-resistance (R_{11}) value of 24.5 ohms (Equation 18):

$$\text{Field Int./Amp} = \frac{192.8 \text{ MV/M}}{\sqrt{\dfrac{1000 \text{ Watts}}{24.5 \text{ Ohms}}}} = 30.2 \text{ MV/M/Amp.}$$

This means that tower two, with a base current of 11.84 amps, will radiate a field intensity of 357.6 MV/M. Tower one would be 255.3 MV/M and tower three would be 204.2 MV/M.

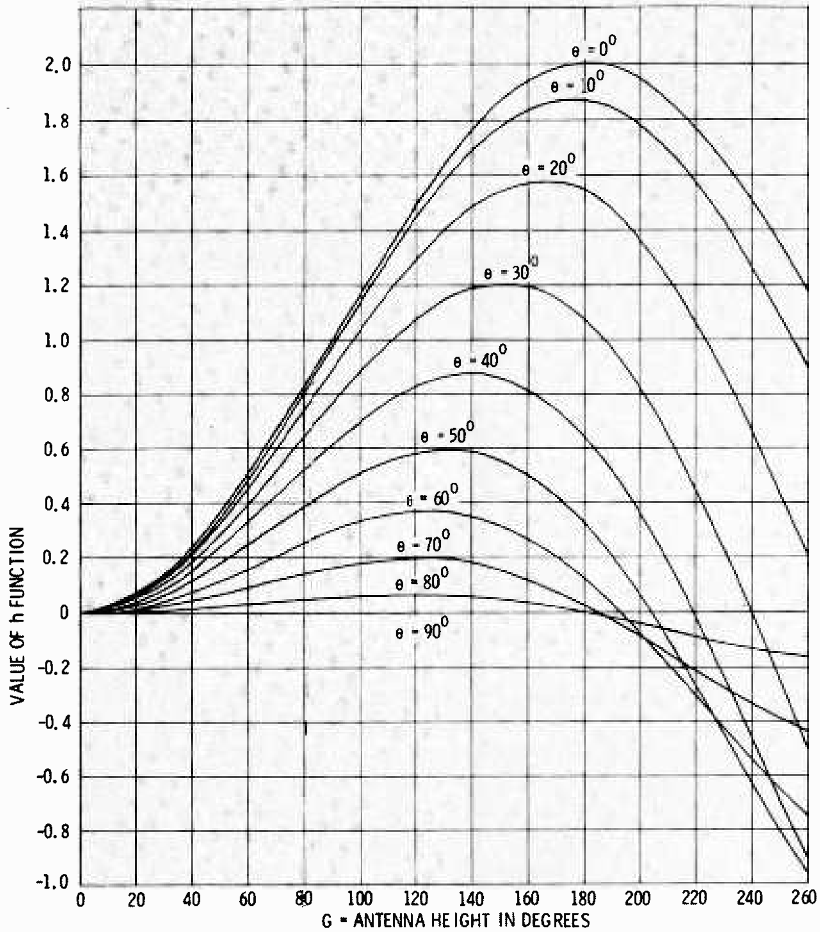
Equation 14 determines the gain times the field intensity of tower two (our reference tower) to determine the RMS (Equation 19):

$$E_0^2 = E_1^2 + E_2^2 + E_3^2 + 2\, E_1 E_2 \cos \Psi_{12} J_0 (S_{12}) + 2\, E_1 E_3 \cos \Psi_{13} J_0 (S_{13}) + 2 E_2 E_3 \cos \Psi_{23} J_0 (S_{23})$$

$$E_0 = 357.6 \sqrt{0.510 + 1.00 + .326 + 357.6 \sqrt{1.428} \cos (-209.1)\,(0.656) + .815 \cos (-48^\circ)\,(-.038) + 1.142 \cos (102.9)\,(0.656)}$$

$$E_0 = 192.2 \text{ MV/M}$$

It is important to keep terms straight, and not to mix measured values with theoretical values. After all, the real purpose here is to predict the theoretical RMS of any given pattern. In actual practice the measured RMS may be greater or lesser than theory predicts.

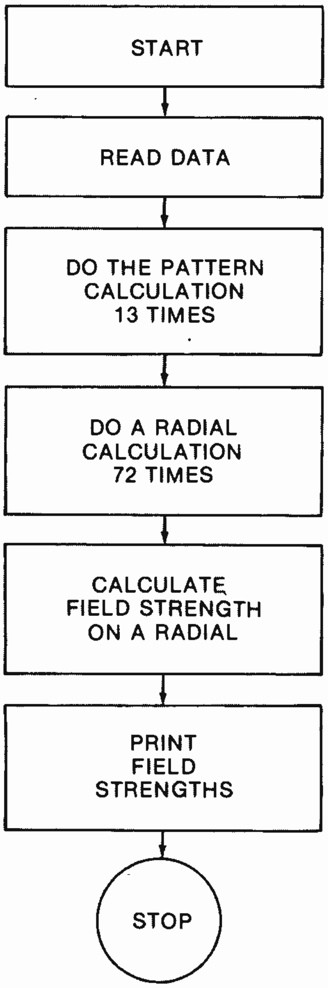


VALUE OF h FUNCTION PLOTTED FOR VARIOUS VALUES OF ELEVATION ANGLE θ AS A FUNCTION OF ANTENNA HEIGHT G

Figure 6

11

The FCC standard method, plus use of computers and hand calculators



Flow Chart shows instructions given computer to solve the problem.

Figure 1

This chapter will discuss the use or "help" that computers and pocket calculators can be to the design engineer. The FCC's standard method of calculating directional antennas for submission with FCC applications will then be considered.

Using digital computers

Besides relieving the design engineer of much of the relatively non-productive work, any computer has two significant advantages: great speed and a high degree of accuracy. Accessibility of computers is easy with almost all large companies owning or sharing them, as well as all major universities and even many high schools.

By using one of the new IBM models, the simple two-tower daytime pattern calculated by longhand in Chapter 2 can be computed and printed out in no more than two minutes. (Or about the time it takes to unpack your pocket calculator, sharpen a couple of pencils, pick up a pad of paper, and get ready to go to work.)

If you are afraid the computer may replace the chief engineer some day, there is little cause for concern because all computers have one fault—they are stupid. They must be told in a very specific way, everything they are to perform. This seems incongruous when you think of their speed and accuracy, but it is true. In fact, a computer even has to be told when to stop.

Computer programs

The instructions you give to a computer in order for it to work a specific problem is called a "program." The program must list precisely and in correct sequence each step to be performed. Most computer programmers rarely have any knowledge of the electronic and mechanical principles involved in computer operations. This is because one does not need to know how a computer functions if you can speak its language.

The instructions contained in a given computer program must be in a "language" which can be understood by the writer and the computer. Computer languages, although similar to the basic spoken word, have some rather unique names, such

TYPICAL PRINTOUT FROM IBM COMPUTER

FOR A VERTICAL ANGLE OF 0. THE FOLLOWING RESULTS ARE OBTAINED

PHI	ETH MV	EST MV	PHI	ETH MV	EST MV	PHI	ETH MV	EST MV	PHI	ETH MV	EST MV
5	460.76	483.86	95	55.94	59.29	185	93.79	98.82	275	3.92	9.12
10	461.32	484.46	100	20.33	22.85	190	94.12	99.16	280	34.08	36.69
15	459.99	483.06	105	12.41	15.37	195	93.36	98.37	285	71.17	75.17
20	456.69	479.59	110	38.82	41.56	200	91.68	96.60	290	110.93	116.76
25	451.22	473.85	115	60.19	63.72	205	89.47	94.29	295	152.07	159.89
30	443.31	465.55	120	76.08	80.29	210	87.31	92.04	300	193.32	203.15
35	432.62	454.32	125	86.66	91.36	215	85.88	90.54	305	233.47	245.28
40	418.77	439.78	130	92.50	97.47	220	85.72	90.37	310	271.48	285.17
45	401.40	421.55	135	94.45	99.50	225	87.03	91.74	315	306.53	321.96
50	380.22	399.31	140	93.59	98.61	230	89.48	94.30	320	338.02	355.01
55	355.05	372.89	145	91.17	96.07	235	92.25	97.20	325	365.60	383.96
60	325.88	342.27	150	88.40	93.18	240	94.19	99.24	330	389.16	408.70
65	292.91	307.67	155	86.34	91.02	245	94.07	99.11	335	408.79	429.31
70	256.59	269.55	160	85.61	90.26	250	90.69	95.57	340	424.71	446.02
75	217.61	228.64	165	86.33	91.01	255	83.04	87.57	345	437.26	459.19
80	176.89	185.91	170	88.13	92.89	260	70.38	74.34	350	446.79	469.20
85	135.54	142.55	175	90.38	95.25	265	52.30	55.51	355	453.69	476.44
90	94.79	99.96	180	92.44	97.40	270	28.78	31.30	360	458.26	481.24

FOR A VERTICAL ANGLE OF 5. THE FOLLOWING RESULTS ARE OBTAINED.

PHI	ETH MV	EST MV	PHI	ETH MV	EST MV	PHI	ETH MV	EST MV	PHI	ETH MV	EST MV
5	452.96	475.68	95	54.99	58.30	185	91.87	96.79	275	3.87	8.99
10	453.53	476.27	100	20.05	22.53	190	92.18	97.12	280	33.54	36.12
15	452.21	474.89	105	12.10	15.03	195	91.46	96.37	285	69.93	73.87
20	448.93	471.44	110	38.04	40.74	200	89.87	94.71	290	108.94	114.67
25	443.51	465.76	115	59.07	62.54	205	87.80	92.54	295	149.30	156.97
30	435.68	457.53	120	74.73	78.87	210	85.81	90.46	300	189.77	199.42
35	425.11	446.43	125	85.20	89.82	215	84.53	89.12	305	229.17	240.77
40	411.42	432.07	130	91.03	95.91	220	84.49	89.08	310	266.50	279.94
45	394.29	414.08	135	93.03	98.01	225	85.85	90.50	215	300.93	316.06
50	373.41	392.17	140	92.27	97.21	230	88.28	93.04	320	331.88	348.57
55	348.64	366.15	145	89.94	94.77	235	90.98	95.86	325	359.02	377.05
60	319.95	336.04	150	87.22	91.94	240	92.83	97.80	330	382.23	401.42
65	287.55	302.03	155	85.15	89.76	245	92.62	97.59	335	401.58	421.73
70	251.88	264.59	160	84.34	88.92	250	89.21	94.01	340	417.29	438.23
75	213.61	224.43	185	84.92	84.92	255	81.61	86.06	345	429.69	451.24
80	173.85	182.51	170	86.56	91.24	260	69.10	73.00	350	439.12	461.15
85	133.08	139.96	175	88.66	93.43	265	51.30	54.46	355	445.95	468.32
90	93.11	98.09	180	90.59	95.46	270	28.18	30.65	360	450.49	473.08

FOR A VERTICAL ANGLE OF 10. THE FOLLOWING RESULTS ARE OBTAINED

PHI	ETH MV	EST MV	PHI	ETH MV	EST MV	PHI	ETH MV	EST MV	PHI	ETH MV	EST MV
5	430.27	451.85	95	52.25	55.39	185	86.43	91.07	275	3.74	8.61
10	430.82	452.43	100	19.24	21.61	190	86.69	91.35	280	31.99	34.45
15	429.53	451.08	105	11.21	14.05	195	86.08	90.71	285	66.36	70.09
20	426.34	447.72	110	35.79	38.35	200	84.76	89.33	290	103.17	108.60
25	421.07	442.19	115	55.80	59.09	205	83.07	87.56	295	141.27	148.53
30	413.47	434.21	120	70.80	74.74	210	81.52	85.94	300	179.48	188.61
35	403.25	423.48	125	80.95	85.34	215	80.67	85.05	305	216.73	227.69
40	390.07	409.85	130	86.72	91.38	220	80.94	85.33	310	252.05	264.76
45	373.63	392.39	135	88.88	93.64	225	82.43	86.89	315	284.69	299.02
50	353.66	371.43	140	88.39	93.12	230	84.79	89.36	320	314.09	329.88
55	330.03	346.62	145	86.32	90.96	235	87.26	91.95	325	339.93	357.01
60	302.74	317.97	150	83.78	88.30	240	88.83	93.59	330	362.08	380.27
65	272.00	285.70	155	81.70	86.12	245	88.40	93.13	335	380.62	399.72
70	238.21	250.23	160	80.68	85.06	250	84.90	89.47	340	395.72	415.56
75	202.01	212.25	185	80.90	85.29	255	77.45	81.68	345	407.68	428.13
80	164.26	172.64	170	82.09	86.54	260	65.40	69.10	350	416.81	437.71
85	125.95	132.47	175	83.76	88.28	265	48.39	51.39	355	423.44	444.68
90	88.23	92.96	180	85.36	89.95	270	26.43	28.79	360	427.85	449.31

Figure 2

as Fortran, Cobol and Algol. These are abbreviations for much longer names such as FOR-mula-TRANSla-tion (FORTRAN), COmmon Business Oriented Language (COBOL), or ALGOrithmic Language (ALGOL). The language used depends upon the computer and its manufacturer. In addition, each manufacturer needs to develop a program that translates the source language (FORTRAN) into an object language which actually controls the circuitry and micro-devices within the computer and its related equipment.

In order to write a program, it first becomes necessary to understand the problem which you wish the computer to solve, from the computer's viewpoint. Then select the basic computer language which best suits that type of problem. FORTRAN is generally used for engineering and other mathematical applications.

To program a computer, it is first necessary for the reader to "learn" the language to be used. Learning a computer language is much like learning any language; there are rules of grammar and of spelling

CALCULATOR PROGRAM BY STEPS

001	*LBLA	21 11
002	RCL5	36 05
003	COS	42
004	ENT↑	-21
005	RCL0	36 00
006	x	-35
007	ENT↑	-21
008	RCL1	36 01
009	+	-55
010	STOD	35 14
011	RCLD	36 14
012	COS	42
013	ENT↑	-21
014	RCL2	36 02
015	+	-55
016	x^2	53
017	STOE	35 15
018	RCLD	36 14
019	SIN	41
020	x^2	53
021	RCLE	36 15
022	+	-55
023	ST+6	36-55 06
024	RCL4	36 04
025	ST+5	35-55 05
026	RCL5	36 05
027	3	03
028	6	06

Figure 2A

TYPICAL CALCULATOR PRINTOUT

Bearings		Theoretical		Standard		Parameters	
0.00	***	406.52	***	427.08	***	MV/M 125.00	0
5.00	***	445.30	***	467.78	***	-117.50	1
10.00	***	480.58	***	504.80	***	0.55	2
15.00	***	511.70	***	537.47	***	-83.50	3
20.00	***	538.32	***	565.41	***	5.00	4
25.00	***	560.33	***	588.51	***	0.00	5
30.00	***	577.84	***	606.89	***	0.00	6
35.00	***	591.15	***	620.87	***	444.00	7
40.00	***	600.71	***	630.90	***	395.41	8
45.00	***	607.05	***	637.56	***	0.00	9
50.00	***	610.78	***	641.47	***	0.00	A
55.00	***	612.52	***	643.30	***	1.00	B
60.00	***	612.87	***	643.67	***	0.00	C
65.00	***	612.38	***	643.16	***	0.00	D
70.00	***	611.54	***	642.27	***	0.00	E
75.00	***	610.71	***	641.40		13.42	I
80.00	***	610.18	***	640.84	***		
85.00	***	610.08	***	640.74	***		
90.00	***	610.45	***	641.13	***		
95.00	***	611.19	***	641.90	***		
100.00	***	612.07	***	642.82	***		
105.00	***	612.75	***	643.54	***		
110.00	***	612.79	***	643.55	***		
115.00	***	611.68	***	642.42	***		
120.00	***	608.82	***	639.41	***		
125.00	***	603.59	***	633.93	***		
130.00	***	595.39	***	625.32	***		
135.00	***	583.64	***	612.99	***		
140.00	***	567.86	***	596.42	***		
145.00	***	547.67	***	575.23	***		
150.00	***	522.90	***	549.23	***		
155.00	***	493.55	***	518.42	***		
160.00	***	459.88	***	483.08	***		
165.00	***	422.40	***	443.74	***		
170.00	***	381.94	***	401.29	***		
175.00	***	339.68	***	356.54	***		
180.00	***	297.21	***	312.39	***		
185.00	***	256.71	***	269.91	***		
190.00	***	221.10	***	232.58	***		
195.00	***	194.15	***	204.34	***		
200.00	***	175.72	***	189.23	***		
205.00	***	175.69	***	189.20	***		
210.00	***	192.20	***	202.30	***		
215.00	***	212.85	***	223.94	***		
220.00	***	237.36	***	249.62	***		
225.00	***	262.62	***	276.11	***		
230.00	***	286.68	***	301.34	***		
235.00	***	308.39	***	324.11	***		
240.00	***	327.07	***	343.71	***		
245.00	***	342.37	***	359.77	***		
250.00	***	354.11	***	372.08	***		
255.00	***	362.20	***	380.57	***		
260.00	***	366.61	***	385.20	***		
265.00	***	367.35	***	385.97	***		
270.00	***	364.40	***	382.88	***		
275.00	***	357.78	***	375.94	***		
280.00	***	347.50	***	365.15	***		
285.00	***	333.61	***	350.58	***		

Figure 2B

and much memorizing to be done. The more you know of the language, the greater will be the speed and facility with which you can "converse" with the computer.

Today, there are so many computer programs already written and available, that it is rarely necessary to "write" a new one.

The next step in designing the pattern by computer is to feed the data-processing cards (punched cards) into a data-processing reader. In some computers this input is via perforated tape or keyboard input. The output from the computer can be in one of many forms: punched cards, perforated tape, magnetic tape, typewriter print out, or line printer output.

Pattern calculations

It is most common to write and/or use a program that has as its final output the radiation pattern in V/M or MV/M, with computations at 5° horizontal intervals and 5° vertical intervals. These vertical computations would be limited to the zone between 0° and 60°, the limits required by the FCC in support of nighttime directional applications.[1] The program must be capable of handling any number of towers in any physical or electrical configuration, as well as individual tower heights and for any frequency.

The basic equation the computer is programmed to solve is the general equation (1) covering any directional antenna system.[2]

$$E = \sum_{K=1}^{K=n} E_k \, f_k(\Theta) \, \angle B_k$$

These terms can generally be recognized by the reader as commonly used in the hand calculation of two-, three-, and four-tower designs. The only new expression here is Carl Smith's use of the term B_k (Equation 2):

1. *Antenna Patterns by Computer;* B/E March 1967.

2. *Theory and Design of Directional Antennas,* Carl Smith.

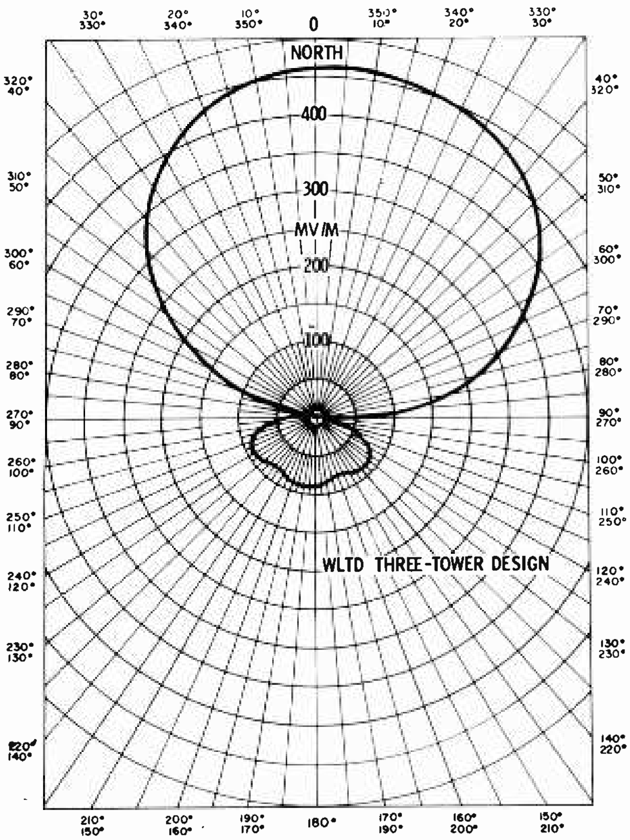


Figure 3

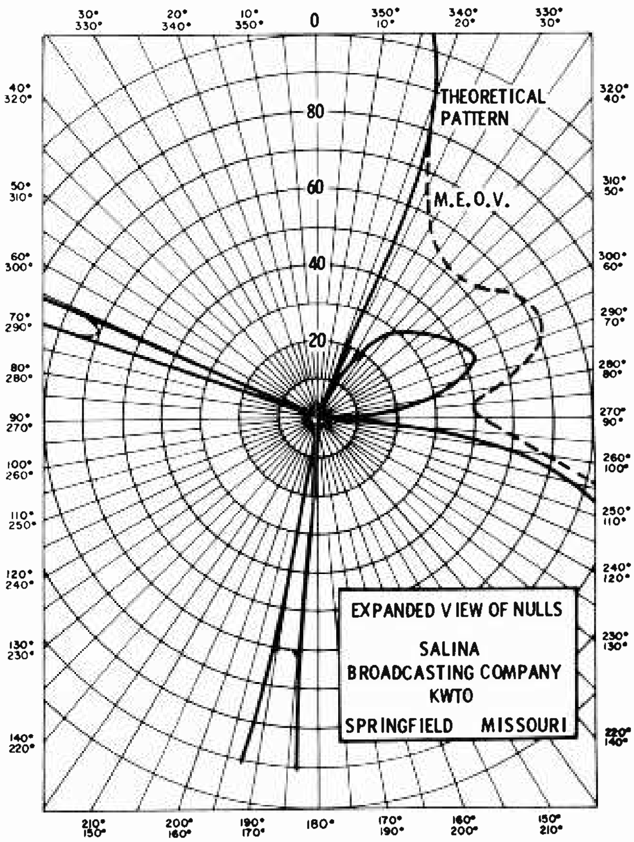


Figure 4

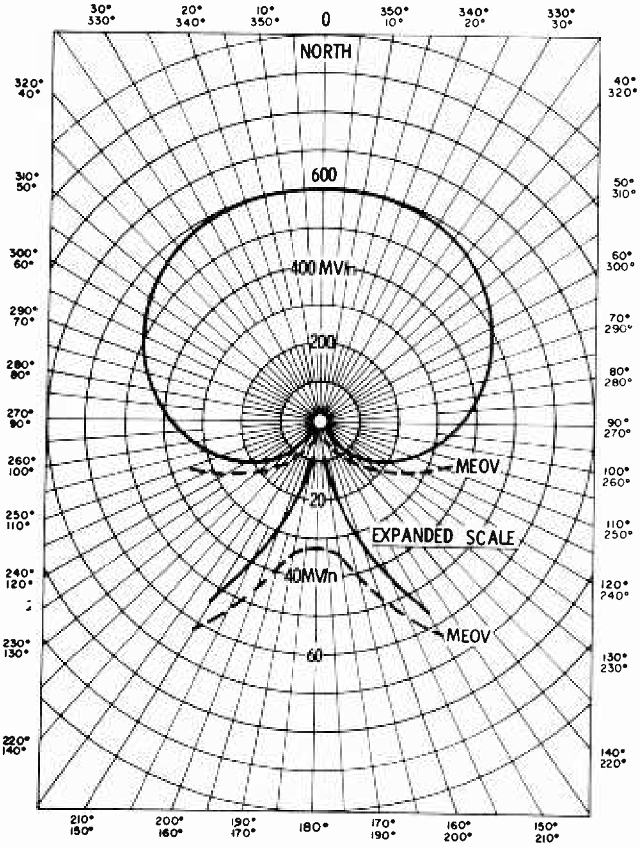


Figure 5

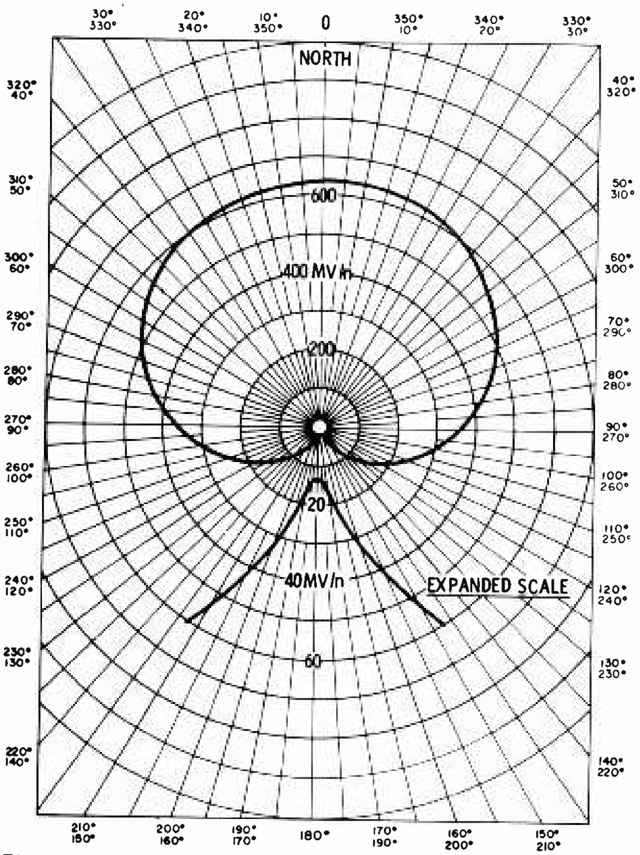


Figure 6

$$B_k = S_k \cos \Theta \cos (\emptyset_k - \emptyset) + \Psi_k$$

The terms used in these two equations are defined as follows:

- E = the total effective field intensity vector at unit distance (D) for the antenna array with respect to the voltage-vector reference axis.
- K = the Kth tower in the directional antenna system.
- n = the total number of towers in the antenna array to be calculated; i.e. n = two, n = three, etc.
- E_k = the magnitude of the individual field at the unit distance, produced by the K_{th} tower acting alone.
- $f_k(\Theta)$ = the vertical radiation characteristics of the K_{th} tower.
- Θ = the vertical elevation angle of the observer at point "P" measured above the horizon in degrees.
- S_k = electrical spacing of the K_{th} tower in the horizontal plane from the reference point.
- $\emptyset_k$ = true horizontal azimuth orientation of the K_{th} tower with respect to the reference point.
- $\emptyset$ = true horizontal azimuth angle of the direction to the observation point "P."
- Ψ_k = time phasing portion due to the electrical phase angle of the voltage (or current) in the K_{th} tower with respect to the reference axis.

This formula is really the addition form discussed earlier, which is not the speediest way to calculate a pattern. But while shortcuts may be applied to some patterns, only a general equation can apply to all systems of any number and configuration. Also keep in mind that while the computer may have to work a little longer to do everything the hard way with this generalized pattern, the difference in time normally will be in the order of a few seconds.

The punched cards fed into the computer contain the system parameters, field ratios for each tower, the phase angle for each tower, the electrical spacing between each tower and the reference tower, the electrical height of each tower, and the shift from reference of each tower's individual bearing or tower-line. It is also necessary to tell the computer the power level and in some cases even the RMS or efficiency factor.

If the computer printout agrees with your slide-rule design to eight significant places, you can feel confident in the mathematics. The probability of any error in the computer is negligible.

Flow chart

A "flow chart" helps show how a computer attacks an antenna design problem. An analogy to a flow chart might be a highway road map; they both show the route to follow in traveling from point A to point B.

The first step taken by the computer is to read the punch cards. The second step is to calculate the pattern 13 times (once for each vertical pattern from 0° to 60°). This is called an execution statement, or in some computers, a "do-loop." What this really does is tell the computer to execute a series of operations a specified number of times (in this case 13 times).

The second execution statement instructs the computer to calculate the signal strength for each 5° of azimuth in each elevation plane. This step actually tells the computer how to compute the required performances. This may seem backward, but in writing a computer program, it is necessary to think like a computer. And recall I said all computers are dumb!

Figure 1 shows that this second execution statement requires that the calculation be made 72 times. This should be obvious since 360° divided by 5° increments corresponds to 72 bearings. When all calculations have been made by the computer, the program next tells the machine to give the answer in a readable form. This is normally referred to as the print statement. In addition to the answers, the computer must be told how to print the information, i.e., whether to print the data in rows, in columns, or any other desired configuration. Usually for aid in plotting polar patterns, it is helpful to ask the computer to print the data in tabulation form. It is customary to have the data grouped by vertical planes. Figure 2 shows a portion of the ground plane data computed for the nighttime array of Figure 3.

The last step in the computer program is to instruct the computer to "stop."

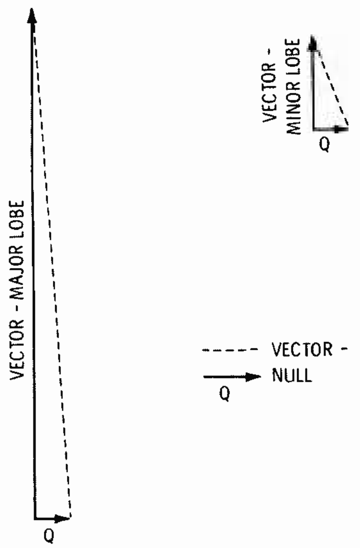
This brief look into the world of computers in the design of directional antennas reveals they are a useful tool for speed, for accuracy, and for final verification. But the computer has a long way to go before it can replace the intuitive experience of a design engineer.

Pocket calculators

Today one seldom sees a slide rule used or even a Chinese abacus. The electronic pocket calculator has replaced these devices. The pocket calculator is fast, can compute to six or eight decimal places, and can do trig functions without resorting to tables. A pocket calculator can be used "manually" to perform each and every step that an IBM computer does. Or one

TABLE 1
BASIC THEORETICAL PATTERN

A	B	C	D	E	F	G	
Bearing	70 cos A	B-110°	cos C	1+D	$E^{1/2}$	F x 447	
0	70.0	-40.0	0.766	1.766	1.330	595.0	MV/M
10-350	68.9	-41.1	0.754	1.754	1.326	593.0	
20-340	65.8	-44.2	0.717	1.717	1.310	585.0	
30-330	60.6	-49.4	0.650	1.650	1.266	574.0	
40-320	53.6	-56.4	0.553	1.553	1.246	556.0	
50-310	45.0	-65.0	0.423	1.423	1.193	533.0	
60-300	35.0	-75.0	0.259	1.259	1.124	501.0	
70-290	23.9	-86.1	0.068	1.068	1.034	462.0	
80-280	12.2	-97.8	-.136	0.864	0.931	416.0	
90-270	0.0	-110.0	-.342	0.658	0.812	362.0	
100-260	-12.2	-122.2	-.538	0.467	0.684	305.0	
110-250	-23.9	-133.9	-.689	0.311	0.558	250.0	
120-240	-35.0	-145.0	-.819	0.181	0.426	190.0	
139-230	-45.0	-155.0	-.906	0.094	0.307	137.0	
140-220	-53.6	.163.6	-.960	0.040	0.200	89.0	
150-210	-60.6	-170.6	-.985	0.015	0.123	55.0	
160-200	-65.8	-175.8	-.995	0.005	0.071	31.7	
170-190	-68.9	-178.9	-.999	0.001	0.031	13.8	
180	-70.0	-180.0	-1.000	0.000	0.000	0.0	



VECTORS SHOWING HOW Q FACTOR EFFECTS MAJOR LOBE, MINOR LOBES, OR NULLS. DASHED LINE REPRESENTS RESULTANT.

Figure 7

can use the the programmable calculator like the HP-97. With the built-in memories these units possess, they can perform several of the manual steps a pocket calculator would do.

In fact, with the HP-97 I have written a program about its memories such that it can compute up to a six-tower directional pattern. The accuracy is identical with an IBM, but of course its speed is much reduced. For example, to compute a single pattern on IBM will take four or five minutes to "think" about the problem, then print out the solution in 10-15 seconds. Whereas the HP-97 type will require 20-25 minutes to think about a problem, and then 15-18 minutes to print out the solution. Figure 2A shows a portion of the program written for an HP-97, and Figure 2B is a portion of the print-out one would obtain.

Standard patterns

There is one final area in which the design engineer must become familiar and must be willing to take into account. This revolves around the fact that no directional antenna constructed ever tuned up to perfectly fit its theoretical design. But furthermore, even if it could be so initially adjusted, there is no way it could forever be held in such a state.

Historically, the FCC has conceived different approaches to this problem of pattern shape variations. Initially it was their concern of the magnitude of such variations along bearings toward co-channel, or if daytime, also toward adjacent channel stations. In the early days it was the responsibility of the design engineer to determine how much above the designed values of radiation these upper limits were to be proposed. In other words, it was a judgment predicated upon experience, maximum allowable radiation before interference would result, and the levels to which he felt the pattern could be contained.

Later, the concept of MEOV (Maximum Expected Operating Values) was used. As the name implies, this was the value of radiation beyond which deviations of the initial tune-up plus the day-to-day operation would not exceed. Figure 4 is an example of such a use of MEOV. Most of the DAs currently licensed by the FCC are of this type. Only the newer directionals granted since 1971 must conform to the new FCC standard pattern concept.

Essentially the new standard pattern incorporates the old MEOV concept, plus some new factors. One of these is the use of a safety factor of 1.05 times the theoretical pattern in all directions. Under the old MEOV concept it was customary to plot MEOVs only over nulls and minor lobes, plus along bearings toward other stations to whom interference could result.

Another way to look at this standard pattern is to see it as the last step to be taken after you have designed your theoretical pattern, prior to its submission to the FCC.

Comparing methods

Figure 4 was a 5 kW pattern designed by the old method with MEOV limits. The formula for this is as follows (Equation 3):

$$E = K\left[\frac{1 + M^2}{2M} + \cos(\Psi + S\cos\emptyset)\right]^{1/2}$$

Figure 5 is a plot of this pattern. Figure 6 is a plot of the same basic 5 kW two-tower pattern, this time with the addition of the two extra factors.

The first step in determining the final standard pattern is to calculate the old theoretical pattern by Equation 3, as shown in Table 1. Then this design is multiplied by the following (Equation 4):

$$E_{STD} = 1.05\,[E^2_{theo} + Q^2]^{1/2}$$

In this equation, E_{STD} represents the final design. It is expressed in MV/M. E_{theo} represents the "basic" theoretical pattern, or the old theoretical pattern. The term Q is a factor applied in quadrative to each and every bearing. Quadrative, for those who may not recall their high school math, is when a second number is added at right angles. Figure 7 shows how the vector Q would be added to the major lobe, a null and a minor lobe. Obviously Q will not have a significant effect except in areas or bearings of the old theoretical pattern where the radiation is zero or very weak.

Computing Q

The factor Q is stated by the FCC to be an orthogonal component. In this respect it always will be a constant for any given pattern. It is calculated in one of two ways. Whichever yields the larger constant is the one to use. One of the methods is to figure the value of Q by multiplying 6.0 times the square root of the power in kilowatts. This is written as (Equation 5):

$$Q = 6.0\,(P_{kw})^{1/2}$$

If the power in the pattern were 1 kW, this would calculate out to 6.0. If 5 kW it is 13.4, if 10 kW it is 19.0 and so forth. The FCC Rules say that for powers of less than 1.0 kW, 6.0 is the minimum factor that can be used for Q.

The second method of computing Q is by multiplying 0.025 times the RSS of the unit fields. The RSS term is not to be confused with the RMS of a pattern as explained in the previous chapter. Each individual tower of any pattern will radiate its own signal. Each tower, regardless of how many towers there are in a pattern, if it could be measured alone, would produce at one mile an unattenuated signal of E_n MV/M.

In the foregoing two-tower example you would expect two signals. Had three towers been used, you would have three signals, and so forth. In computing RSS the individual signals are taken from each tower and squared. This is followed by adding each of the individual squares, then taking the square root of the total.

To demonstrate how to do this, take the individual fields for the two-tower in the above 5 kW example. Each of the towers would have a signal of 316 MV/M. These when squared are 100,000 and 100,000. The square root of 200,000 is 447 MV/M. This would be the RSS, for this 5 kW pattern.

Having arrived at the RSS, you would then multiply the 447 MV/M times 0.025 which computes to 11.2 MV/M. At this point the design engineer would stop and compare which value of Q yields the greater factor; six times the square root of 5 kW is 13.4 MV/M, whereas 2.5% of the RSS is 11.2 MV/M. This may seem like a lot of extra work, but remember that for any given pattern Q is always a constant and needs to be calculated just once. In the case of nighttime patterns the factor Q must be applied against the vertical form factor.

The Q factor is applied in quadrative to the old theoretical pattern. This is done by taking the original field intensity at any given bearing, squaring it, then adding the square of Q, and taking the square root of the total. Table 2 shows how the design engineer would take the values from Figure 5 to achieve the final data plotted in Figure 6. Table 2 shows the addition of both the linear and orthogonal components. The end result of Column E is multiplied by 1.05 and yields the final Standard Radiation Pattern. Figure 2 shows the print-out of both the theoretical and the Standard Patterns.

TABLE 2
STANDARD RADIATION PATTERN

A	B	C	D	E	F
Bearing	Theo. Pat.	B^2	$C+Q^2$	$D^{1/2}$	1.05 x E
0	595.0	354,000	354,180	595.0	624.0 MV/M
10-350	593.0	352,000	352,180	593.0	622.0
20-340	585.0	342,000	342,180	585.0	614.0
30-330	574.0	333,000	333,180	574.0	602.0
40-320	556.0	309,000	309,180	556.0	584.0
50-310	533.0	285,000	285,180	533.0	560.0
60-300	501.0	251,000	251,180	501.0	525.0
70-290	462.0	213,500	213,680	462.0	485.0
80-280	416.0	173,000	173,180	416.0	436.0
90-270	362.0	131,100	131,280	362.0	380.0
100-260	305.0	93,000	93,180	305.0	320.0
110-250	250.0	62,500	62,680	250.0	262.0
120-240	190.0	36,100	36,280	191.0	201.0
130-230	137.0	18,770	18,950	138.0	145.0
140-220	89.0	7,920	8,100	90.0	94.5
150-210	55.0	3,030	3,210	56.6	59.4
160-200	31.7	1,005	1,185	34.4	36.2
170-190	13.8	190	370	19.3	20.3
180	0.0	0	180	13.4	14.1

12

Review of basic math

This is a brief review of the fundamentals of vector math as well as trigonometry. I'm certain most of you knew or grasped these concepts back in your school days.

Trigonometry

In simple words, trigonometry is the use of triangles to solve math problems. How these triangles are constructed and how the various angles, sides, altitudes, bases, etc., combine in orderly fashion is the science of trig. But to be able to understand the use of vectors, you first must understand trig. As has already become apparent, the design of directional antennas is based on vectors.

The angle between any two adjacent sides of a triangle can be measured in units called degrees or units called radians. Figure 1 shows how a circle is divided up into 360° total. In nautical science each degree is subdivided into seconds. There are 60 seconds in each minute and 60 minutes in each degree. In computing directional designs, most engineers resort to the use of decimal divisions of a degree simply because it makes calculating easier. For example, 6° 5′ 23″ can be written as 6.0897.

Angles between two lines can also be measured in radians. An angle whose arc is exactly equal to the length of either side is one radian. If that sounds confusing, take the radius of a circle and divide it into the circumference of the same circle and you will find it divides 2π times. The angle along the circumference covered by one radius is an angle of one radian. One radian equals 57.2958°.

Consider definitions of some other terms, such as complementary angles. These are any two angles whose sum equals 90°. This can be written mathematically as follows (Equation 1):

$$A = (90° - B)$$
and
$$B = (90° - A)$$

A second common term is supplementary angles. These are any two angles whose sum is 180°. This is written mathematically as follows (Equation 2):

$$A = (180° - B)$$
and
$$B = (180° - A)$$

Trigonometric functions

In talking about triangles it is commonly said that there are just six possible functions, or if expressed in relation to the sides of a right triangle, as ratios. These are the sine, the cosine, the tangent, the cotangent, the secant, and the cosecant. Table 1 demonstrates these basic relations. One way of remembering them is that three of the functions are reciprocals of the other three. The cotangent is the reciprocal of the tangent; the secant is the reciprocal of the cosine; and the cosecant is the reciprocal of the sine.

These ratios of one side to another side do more than define these six separate functions. They actually produce them! Let's look at a couple of examples.

If the angle at "A" in Figure 3 is 45°, each of the smaller sides is of the same length. From the definitions in Table 1, it can be seen that the sine of

$$45° = \frac{1.0}{\sqrt{2}} = 0.707.$$

Likewise the cosine of

$$45° = \frac{1.0}{\sqrt{2}} = 0.707.$$

$$\text{The tangent of } 45° = \frac{1.0}{1.0} = 1.000.$$

In the case of "A" being 60°,

$$\text{the sine} = \frac{\sqrt{3}}{2.0} = 0.866.$$

$$\text{The cosine} = \frac{1.0}{2.0} = 0.500.$$

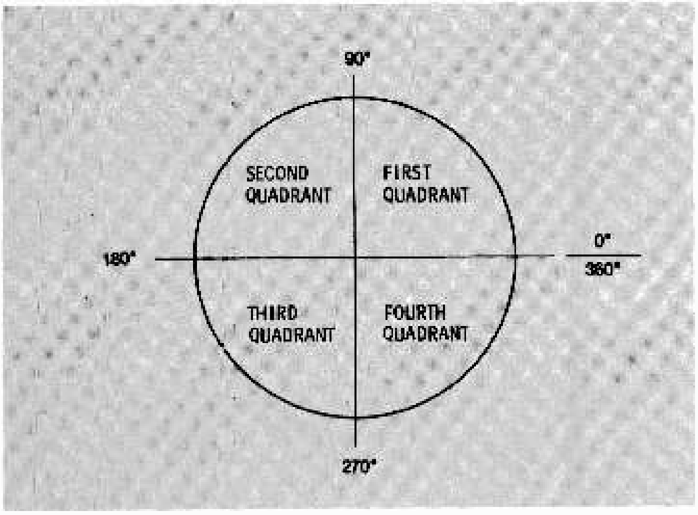


Figure 1

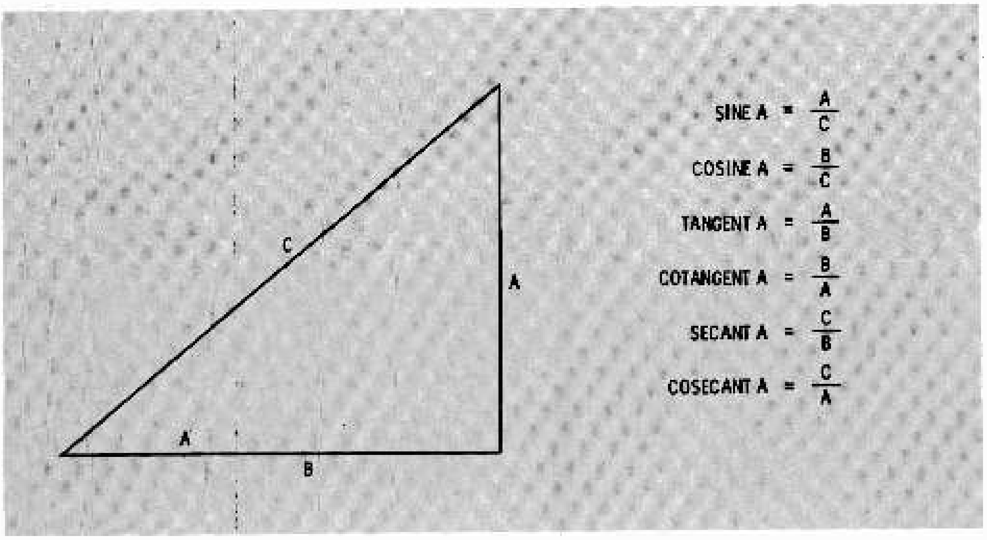


Table 1

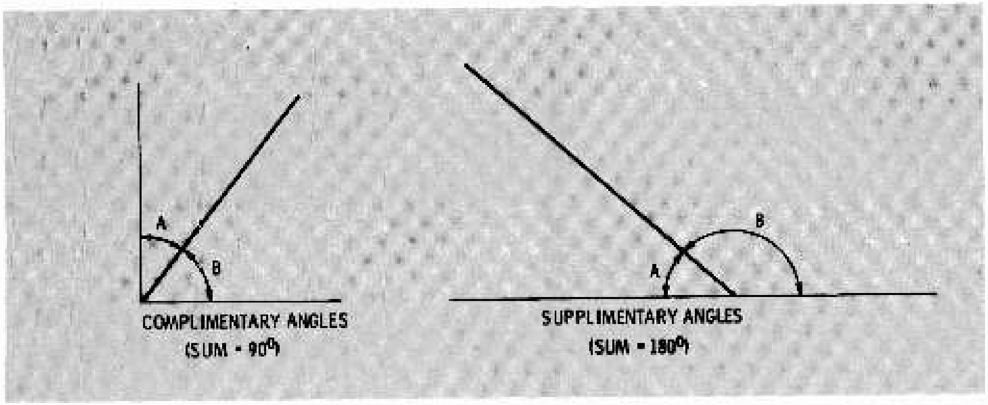


Figure 2

$$\text{And the tangent} = \frac{\sqrt{3}}{1} = 1.732.$$

If you had an angle where you did not know the number of degrees, but knew it was part of a right-angle triangle, all you would need to do is measure with your ruler the length of each of two sides, calculate their ratio, then look up in a trig table to find the angle corresponding to that ratio. Thus the numbers you find in trig tables are not mythical values, but represent actual ratios of selected sides in any and all right triangles, having that given angle.

Something funny happens, of course, should the angle of "A" in Figure 4 begin to get closer and closer to either zero degrees or 90 degrees. The angle A, as it approaches zero, causes its opposite side (a) to become smaller and smaller, and at the same time causes side (b) to approach the length of side (c), until they actually become equal when "A" is zero. Conversely, as angle "A" approaches 90°, side (a)

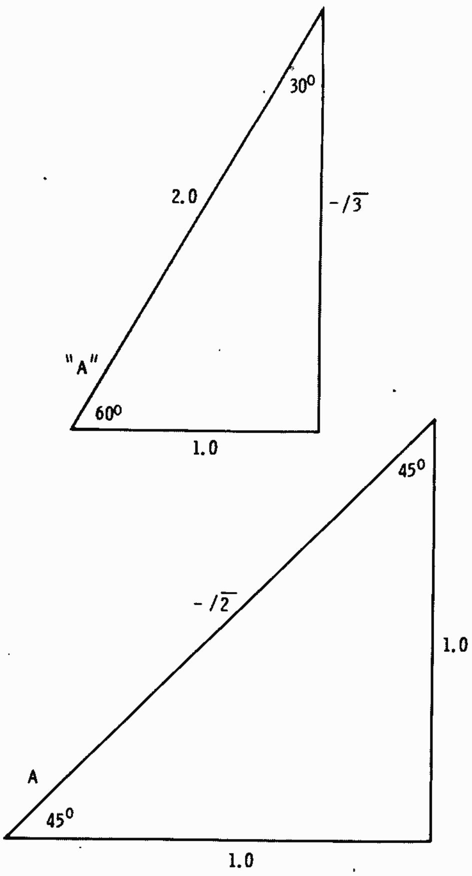


Figure 3

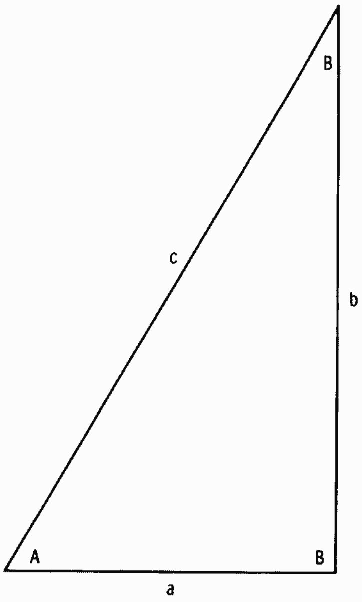


Figure 4

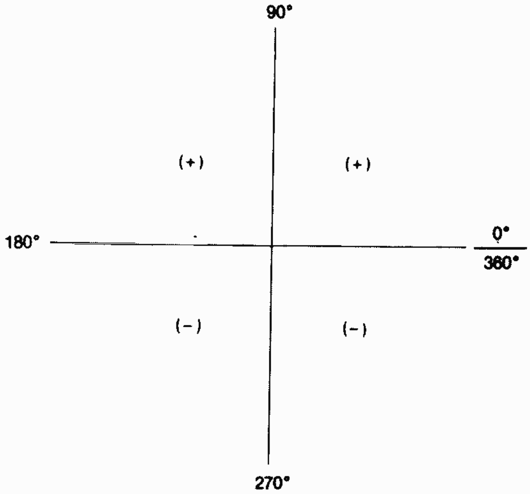


Figure 5

becomes larger and larger until its length equals that of side (c) when angle "A" reaches 90°. In this event side (b) has disappeared and becomes zero.

To summarize: for any angle there will be definite values of these six functions. Conversely, when any of the six functions is known, the angle is "defined." Tables have been prepared in much detail down to tenths and even hundredths of a degree. These are helpful, and necessary, unless you own one of the new pocket calculators.

Right triangles

I have discussed how any angle can be described as a ratio of any of the two sides of its triangle. But, this can be taken one step farther by saying that if we know the angle, and one of the sides of a given triangle, the other sides can be determined.

In our example above, I said:

$$\text{Sine A} = \frac{a}{c}$$

From this you can also say that (Equation 3):

$$C\,(\text{Sine A}) = \left(\frac{a}{c}\right) C \quad \text{or,}$$

$$c = \frac{a}{\text{Sine A}}$$

Similar equations can be written for cosine functions, tangents, cotangents, cosecants and secants.

Angles greater than 90°

In angles greater than 90 degrees, the values of (a) and (b) become negative on occasion, in accordance with the rules of Cartesian coordinates. Figure 5 shows how a circle of 360° can be divided into four quadrants, by two perpendicular intersecting lines. As pointed out in Chapter 1, zero degrees is assumed to lie to the right, at three o'clock. In the next figure I've described which of the sides are positive and which are negative for each quadrant. Keep in mind that each quadrant is 90° wide. Thus the first quadrant is 0° to +90°, the second quadrant is +90° to +180°, the third quadrant is +180° to +270° (or -90° to -180°), and the fourth quadrant is +270° to +360° (or 0° to -90°).

By the use of Figure 6 you can tell the sine of any function in any quadrant. For example, to find the sine of the angle A in the third quadrant, we recall from above that the sine is a/c. Note that in this third quadrant, *a* has a negative sign in front of it. The answer would be -a/c. Likewise, to find the sign of the tangent in the second quadrant, recall that tangent = a/b. This yields an answer of a/-b. And you could say the tangent of any angle in the second quadrant will carry a minus sign. Where there is a double negative sign, such as the tangent in the third quadrant, the answer is a positive sign.

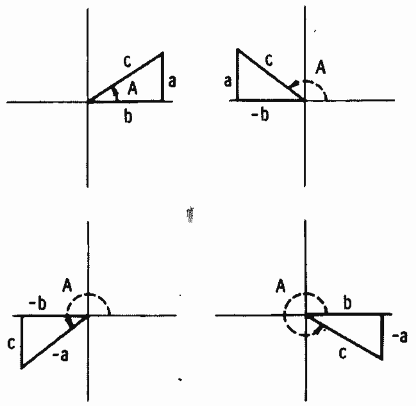


Figure 6

Graphs of trig functions

If you plot the sine function from 0-360° and then beyond, you produce what is called a sine wave. This is usually expressed as y = sine x, where x is in radians or degrees. Figure 7A shows this wave over two cycles (0-720°).

Likewise you can plot a cosine wave from 0-360° and beyond. This for comparison purposes is shown in Figure 7B. The magnitude of either the sine or the cosine wave varies between +1.0 and -1.0, and each of these waves repeats itself. Note that in Figure 7, the cosine wave is similar to the sine wave, except it is displaced by 90° or $\pi/2$ radians.

Tangent curves

The tangent curve (shown on Figure 8) is not like the sine or cosine, since while it repeats itself each cycle, it is not continuous. The tangent function begins at zero and rises toward positive infinity at 90°. It then changes signs to a negative infinity approach down to zero magnitude at 180° and then repeats itself over the second half of the circle. The tangent can have the value of anything from $+\infty$ to $-\infty$.

The co-tangent curve is the inverse of that of the tangent. At zero degrees its value is $+\infty$ and at 90° its value has decreased to zero.

The other two trigonometric functions are the secant and the cosecant. These, like the cotangent, are of less importance. They are respectively the inverse of the cosine and the inverse of the sine. Therefore they will vary in magnitude from +1 to -1.

Vectors

Here is where to use all that trigonometric knowledge and relationships. A scaler quantity has only length or magnitude, but a vector has both magnitude and direction. The direction of any vector has to be measured from some reference point, hence the need for angles or radians of displacement.

Looking at a simple difference between a scaler and a vector, if you say the wind is blowing 30 mph this is a scaler. If, however, you say the wind is 30 mph from the east, you have a vector.

Vectors are usually represented by arrows, where the length of the arrow is its magnitude and the direction the arrow is pointing is its direction. Vectors can be added graphically or trigonometrically. There is no limit to the number of vectors that can be added.

This is useful in the design of DAs, where it is common to let the signal radiated by each tower be a vector. The length of the respective vectors will be proportional to the respective radiated fields from each tower. At any given bearing each of these vectors will have some angular displacement. If you know, or if you have computed these angles, you can then add all the vectors together and arrive at the correct (total) radiation at that bearing. Carrying this one step further, if you repeat this vector addition each 10° around a circle you will be able to plot the final directional pattern. In the early days many engineers used this method to design their patterns.

In order to be able to algebraically add, subtract, multiply, or divide vectors, first have a logical notation system. Two notation systems are commonly used: the Cartesian and the Polar Coordinate Systems. The Polar is more often used in the design of DAs. The Cartesian method is usually used in the calculation of Tee or Ell networks, or in odd complex numbers like R+jX.

In the Cartesian system each vector is divided into its components of X and Y. As an example, let the vector Z on Figure 9 be the sum of vectors X and Y. By the use of (+) and (-) signs any vector can be

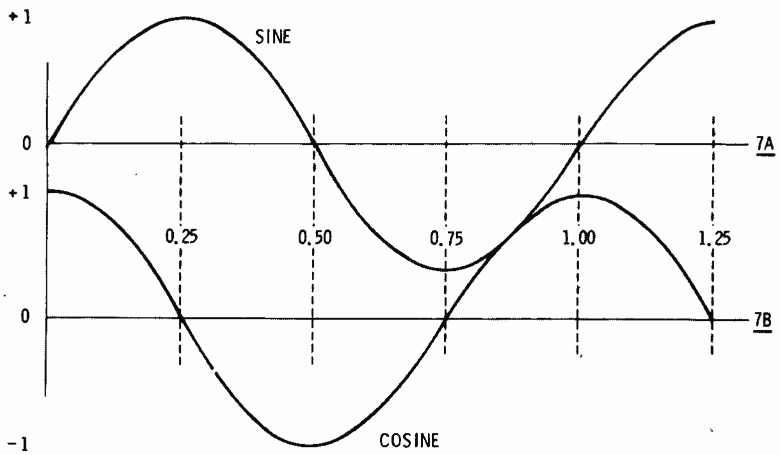


Figure 7

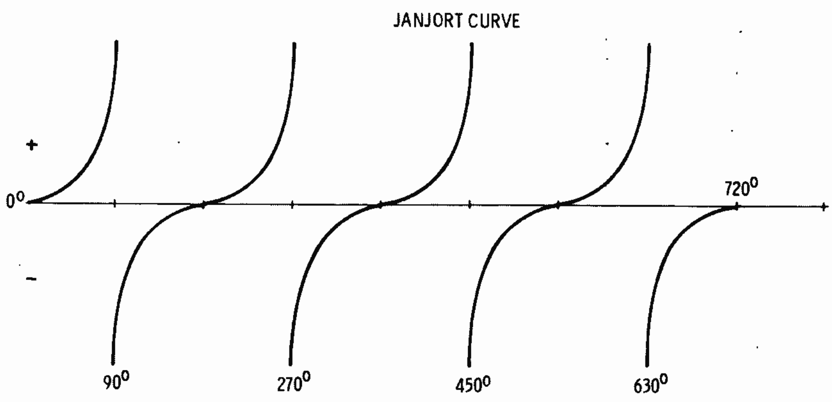


Figure 8

described in terms of X and Y unit vectors. In the example the vector Z will be:

$$Z = 3 + j4$$

You may ask at this point where did the term (j) come from? This letter is used to denote the Y component. Keep in mind that along the X axis it is positive to the right from zero, and negative to the left. Along the Y axis the sign is positive above the axis and negative below.

The coordinates 3 + j4 describe the location of the vector Z in our example, but they do not describe the scaler value. This can be found easily by use of the Pythagorian theorum. Or in simple language, the sum of X+Y equals the square root of the sums of the squares (Equation 4):

$$Z = \sqrt{3^2 + 4^2} = 5 .$$

Addition of vectors

This is easily accomplished by adding the horizontal components of each vector together, then adding the vertical components and applying the Pythagorian theorem. The best analysis of this is a comparison between addition of the sine and the cosine components of any vector. The cosine terms are usually expressed in a horizontal plane like the X axis, while the sine terms are expressed vertically like the Y axis. In fact the (+) and (-) signs are the same. Figure 10 shows the similarity. That is, the cosine to the right of the vertical axis is (+), just as the X is (+). Conversely, the sine is (+) above the horizontal axis just as is the Y term.

Adding the vectors results in the following (Equation 5):

$$A + B = (x_1 + x_2) + j\,(y_1 + y_2) \quad \text{or,}$$
$$A + B = (\cos_1 + \cos_2) + J\,(\text{Sin}_1 + \text{Sin})_2$$

Subtraction of vectors

Subtraction is the opposite of addition. Here one merely separates each vector into its horizontal and its vertical components. Subtraction is thus accomplished by subtracting the horizontal components and subtracting the vertical components (Equation 6):

$$A - B = (x_1 - x_2 + j\,(y_1 - y_2)$$

Multiplication of vectors

This is a little more tricky than simple addition or subtraction. The operation of multiplication of two vectors is actually a simple algebraic operation. It must be pointed out that the imaginary number (j) when multiplied by itself equals a(-1). This is commonly written as (Equation 7):

$$j^2 = -1$$

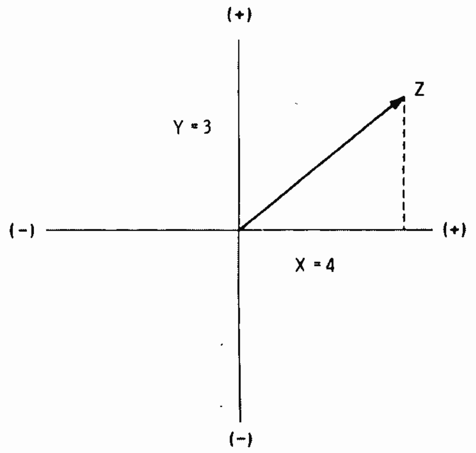


Figure 9

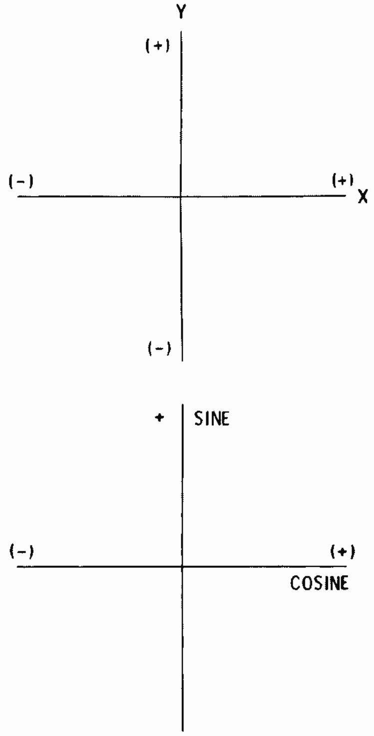


Figure 10

The multiplication of the two vectors A and B gives the following results (Equation 8):

$$A \cdot B = (x_1 + j\,y_1) \cdot (x_2 + j\,y_2)$$
$$= x_1\,x_2 + j\,x_1\,y_2 + j\,x_2\,y_1 + j^2 y_1\,y_2$$
$$= x_1\,x_2 - y_1\,y_2 + j\,(x_1\,y_2 + x_2\,y_1)$$

The actual use of multiplication of vectors is rare in the design of directional antennas. And I have shown it here primarily for reference. The same is true of division of vectors.

Division of vectors

Again the basic operation is one of algebraic division. The imaginary term (j) is taken as a -1 (Equation 9):

$$\frac{A}{B} = \frac{x_1 + j\, y_1}{x_2 + j\, y_2}$$

$$= \frac{(x_1 + j\, y_2)\,(x_2 - j\, y_2)}{(x_2 + j\, y_2)\,(x_2 - j\, y_2)}$$

$$= \frac{x_1\, x_2 + y_1\, y_2 + j\,(x_2\, y_1 - x_1\, y_2)}{x_2^2 + y_2^2}$$

Polar coordinates

So far I have discussed vectors when they are expressed in Cartesian coordinates. The more frequent use in designing directionals is to express vectors on their polar coordinates; this is written as a magnitude plus a vectorial angle with an arbitrary reference axis. In Figure 13 the magnitude of vector A is shown as 30 MV/M at an angle of 60° True. This would be written as (Equation 10):

$$30\ \text{MV/M}\ \angle 60T$$

Any polar coordinate can be converted to its Cartesian equivalent and vice versa. For example, a vector (Equation 11):

$$A = x + j\, y = \sqrt{x^2 + y^2}\ \angle \tan^{-1} = \frac{x}{y}$$

In this conversion the $\tan^{-1}$ means the angle of the tangent, or arc tangent. This can be written because

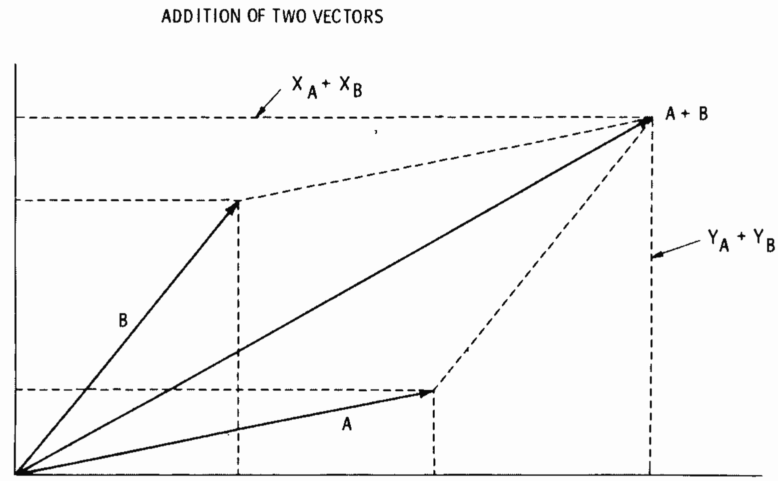


Figure 11

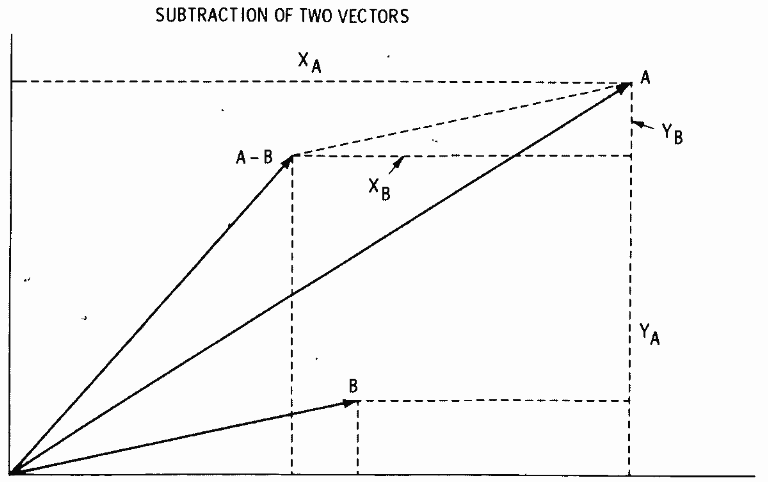


Figure 12

the separate component vectors x and y *are always* assumed to be the lesser sides of a right triangle.

To convert from Polar coordinates to Cartesian coordinates, divide the vector into separate products of its sine and cosine terms. Using the example in Table 1 (Equation 12):

$$A \angle B^\circ = A \cos B^\circ + j\, A \sin B^\circ$$
$$30\ \text{MV/M}\ \angle 60^\circ = 30 \cos 60 + j\, 30 \sin 60$$
$$= 15 + j\, 26$$

One thing to keep in mind when dealing with vectors is that you can only add or subtract like kinds of vectors. That is, you can add voltage vectors to voltage vectors and current to current, but not voltage to current. This I trust is self-evident.

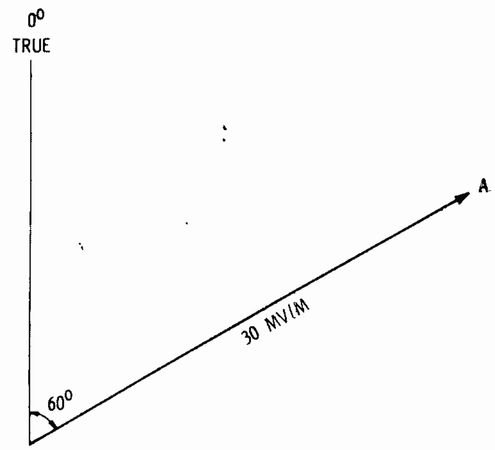


Figure 13

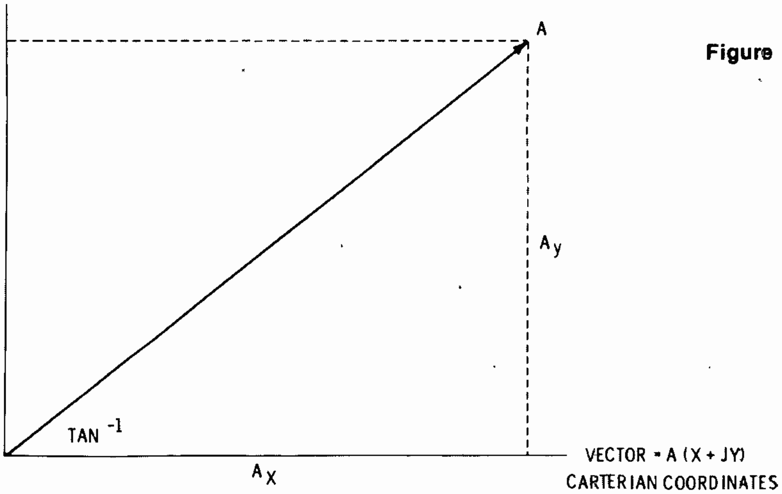


Figure 14A

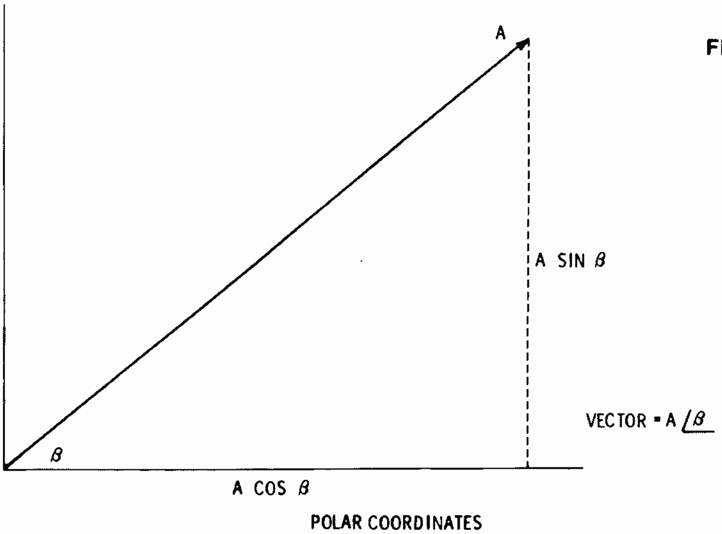


Figure 14B

Appendix

Vertical Form Factor—The following Table 1 contains a summary of the f(θ) term for every tower height between 0° and 250° of electrical heights. The vertical intervals are in 5° increments between the horizon and 85°.

The values not shown on this table can be interpolated between chart values or the theoretical values can be computed from the following equation:

$$f(\theta) = \frac{\cos(G \sin\theta) - \cos G}{(1-\cos(G))\sin\theta}$$

Where: $f(\theta)$ = Vertical radiation characteristics
G = Electrical height of antenna in degrees
θ = Elevation angle in degrees

ANTENNAS, TOWERS, AND WAVE PROPAGATION
TABLE 1. VERTICAL-RADIATION CHARACTERISTIC f(θ)

Tower height, G°

θ°	0	5	10	15	20	25	30	35	40
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9962	0.9962	0.9962	0.9961	0.9961	0.9961	0.9960	0.9960	0.9959
10	0.9848	0.9848	0.9847	0.9846	0.9845	0.9843	0.9841	0.9839	0.9836
15	0.9659	0.9659	0.9657	0.9655	0.9653	0.9649	0.9644	0.9639	0.9632
20	0.9397	0.9394	0.9393	0.9390	0.9386	0.9379	0.9372	0.9362	0.9351
25	0.9063	0.9062	0.9058	0.9054	0.9047	0.9037	0.9026	0.9012	0.8996
30	0.8660	0.8658	0.8654	0.8648	0.8638	0.8626	0.8610	0.8592	0.8571
35	0.8192	0.8188	0.8186	0.8176	0.8164	0.8148	0.8129	0.8106	0.8080
40	0.7660	0.7658	0.7653	0.7642	0.7628	0.7610	0.7587	0.7561	0.7530
45	0.7071	0.7069	0.7062	0.7051	0.7035	0.7014	0.6989	0.6960	0.6925
50	0.6428	0.6423	0.6418	0.6406	0.6390	0.6368	0.6341	0.6309	0.6272
55	0.5736	0.5732	0.5726	0.5714	0.5697	0.5674	0.5647	0.5615	0.5577
60	0.5000	0.4947	0.4990	0.4979	0.4961	0.4940	0.4914	0.4882	0.4846
65	0.4226	0.4222	0.4217	0.4203	0.4191	0.4171	0.4143	0.4117	0.4084
70	0.3420	0.3412	0.3412	0.3404	0.3390	0.3372	0.3351	0.3325	0.3297
75	0.2588	0.2579	0.2584	0.2575	0.2564	0.2550	0.2533	0.2513	0.2490
80	0.1736	0.1695	0.1732	0.1727	0.1720	0.1710	0.1697	0.1684	0.1668
85	0.0871	0.0844	0.0869	0.0869	0.0864	0.0858	0.0852	0.0844	0.0836

θ°	45	50	55	60	65	70	75	80	85
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9958	0.9957	0.9956	0.9955	0.9953	0.9952	0.9950	0.9948	0.9946
10	0.9832	0.9830	0.9824	0.9819	0.9815	0.9809	0.9804	0.9795	0.9788
15	0.9625	0.9617	0.9607	0.9597	0.9585	0.9573	0.9559	0.9544	0.9527
20	0.9339	0.9325	0.9309	0.9289	0.9272	0.9251	0.9227	0.9200	0.9173
25	0.8978	0.8957	0.8934	0.8908	0.8879	0.8848	0.8813	0.8776	0.8735
30	0.8546	0.8519	0.8487	0.8453	0.8416	0.8375	0.8328	0.8278	0.8224
35	0.8050	0.8015	0.7977	0.7934	0.7887	0.7836	0.7779	0.7718	0.7651
40	0.7485	0.7449	0.7410	0.7358	0.7305	0.7244	0.7180	0.7109	0.7103
45	0.6886	0.6769	0.6791	0.6735	0.6675	0.6608	0.6536	0.6457	0.6372
50	0.6230	0.6186	0.6130	0.6073	0.6009	0.5936	0.5862	0.5777	0.5686
55	0.5535	0.5486	0.5427	0.5373	0.5308	0.5236	0.5159	0.5075	0.4984
60	0.4804	0.4759	0.4705	0.4648	0.4587	0.4518	0.4441	0.4361	0.4271
65	0.4042	0.4002	0.3954	0.3898	0.3843	0.3779	0.3710	0.3630	0.3556
70	0.3263	0.3216	0.3190	0.3141	0.3089	0.3031	0.2970	0.2906	0.2842
75	0.2463	0.2433	0.2400	0.2363	0.2323	0.2279	0.2232	0.2181	0.2127
80	0.1649	0.1629	0.1622	0.1576	0.1557	0.1515	0.1492	0.1457	0.1408
85	0.0826	0.0816	0.0804	0.0791	0.0777	0.0761	0.0739	0.0726	0.0707

θ°	90	95	100	105	110	115	120	125	130
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9944	0.9942	0.9939	0.9937	0.9934	0.9931	0.9927	0.9923	0.9919
10	0.9781	0.9770	0.9760	0.9749	0.9738	0.9725	0.9712	0.9697	0.9681
15	0.9509	0.9489	0.9468	0.9445	0.9420	0.9393	0.9363	0.9337	0.9297
20	0.9143	0.9110	0.9074	0.9035	0.8993	0.8947	0.8898	0.8845	0.8788
25	0.8691	0.8642	0.8590	0.8534	0.8473	0.8407	0.8336	0.8259	0.8175
30	0.8165	0.8102	0.8033	0.7959	0.7878	0.7791	0.7698	0.7597	0.7489
35	0.7579	0.7501	0.7417	0.7320	0.7228	0.7122	0.7000	0.6886	0.6754
40	0.6946	0.6855	0.6759	0.6656	0.6541	0.6420	0.6288	0.6157	0.5999
45	0.6279	0.6180	0.6073	0.5958	0.5834	0.5702	0.5560	0.5408	0.5245
50	0.5591	0.5487	0.5373	0.5253	0.5124	0.4987	0.4838	0.4689	0.4511
55	0.4886	0.4781	0.4669	0.4548	0.4419	0.4281	0.4134	0.3977	0.3809
60	0.4178	0.4078	0.3969	0.3854	0.3730	0.3600	0.3460	0.3310	0.3151
65	0.3470	0.3378	0.3279	0.3174	0.3061	0.2942	0.2813	0.2680	0.2536
70	0.2766	0.2687	0.2598	0.2509	0.2413	0.2311	0.2203	0.2091	0.1969
75	0.2067	0.2007	0.1937	0.1866	0.1790	0.1709	0.1623	0.1533	0.1437
80	0.1377	0.1331	0.1281	0.1237	0.1180	0.1130	0.1064	0.1005	0.0941
85	0.0686	0.0664	0.0640	0.0614	0.0588	0.0559	0.0529	0.0497	0.0464

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ANTENNAS, TOWERS, AND WAVE PROPAGATION

TABLE 1. VERTICAL-RADIATION CHARACTERISTIC f(θ) (CONTINUED)

	Tower height, G°							
θ°	135	140	145	150	155	160	165	170
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9915	0.9910	0.9905	0.9899	0.9893	0.9886	0.9878	0.9870
10	0.9663	0.9645	0.9624	0.9602	0.9577	0.9551	0.9522	0.9491
15	0.9259	0.9219	0.9175	0.9127	0.9075	0.9019	0.8956	0.8889
20	0.8725	0.8657	0.8584	0.8504	0.8418	0.8324	0.8222	0.8110
25	0.9085	0.7988	0.7883	0.7769	0.7645	0.7511	0.7366	0.7207
30	0.7372	0.7245	0.7108	0.6961	0.6801	0.6628	0.6440	0.6237
35	0.6612	0.6460	0.6293	0.6118	0.5926	0.5720	0.5496	0.5254
40	0.5837	0.5664	0.5477	0.5276	0.5060	0.4828	0.4577	0.4305
45	0.5070	0.4882	0.4681	0.4466	0.4235	0.3979	0.3719	0.3432
50	0.4330	0.4137	0.3932	0.3710	0.3473	0.3219	0.2948	0.2657
55	0.3631	0.3440	0.3237	0.3020	0.2786	0.2542	0.2278	0.1996
60	0.2981	0.2802	0.2611	0.2407	0.2190	0.1960	0.1713	0.1451
65	0.2362	0.2222	0.2051	0.1867	0.1675	0.1469	0.1256	0.1019
70	0.1838	0.1716	0.1555	0.1399	0.1237	0.1065	0.0881	0.0687
75	0.1335	0.1227	0.1114	0.0994	0.0866	0.0732	0.0590	0.0439
80	0.0868	0.0796	0.0719	0.0634	0.0547	0.0458	0.0359	0.0256
85	0.0428	0.0390	0.0351	0.0309	0.0265	0.0218	0.0169	0.0117
θ°	175	180	185	190	195	200	205	210
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9861	0.9851	0.9840	0.9828	0.9815	0.9801	0.9784	0.9766
10	0.9455	0.9418	0.9375	0.9330	0.9278	0.9222	0.9159	0.9089
15	0.8815	0.8733	0.8645	0.8547	0.8438	0.8319	0.8186	0.8038
20	0.7988	0.7855	0.7708	0.7548	0.7370	0.7175	0.6958	0.6718
25	0.7034	0.6845	0.6638	0.6412	0.6168	0.5888	0.5585	0.5250
30	0.6015	0.5774	0.5510	0.5222	0.4907	0.4561	0.4179	0.3757
35	0.4991	0.4706	0.4395	0.4057	0.3687	0.3283	0.2839	0.2350
40	0.4013	0.3696	0.3353	0.2979	0.2573	0.2129	0.1645	0.1112
45	0.3122	0.2788	0.2427	0.2036	0.1612	0.1152	0.0650	0.0103
50	0.2344	0.2008	0.1646	0.1256	0.0834	0.0378	-0.0118	-0.0657
55	0.1658	0.1370	0.1022	0.0649	0.0247	-0.0186	-0.0655	-0.1161
60	0.1171	0.0873	0.0553	0.0211	-0.0155	-0.0550	-0.0973	-0.1431
65	0.0772	0.0509	0.0228	-0.0071	-0.0391	-0.0733	-0.1100	-0.1494
70	0.0481	0.0261	0.0029	-0.0220	-0.0483	-0.0765	-0.1065	-0.1388
75	0.0260	0.0111	-0.0069	-0.0259	-0.0461	-0.0676	-0.0905	-0.1150
80	0.0148	0.0033	-0.0089	-0.0218	-0.0354	-0.0499	-0.0633	-0.0818
85	0.0062	0.0004	-0.0057	-0.0122	-0.0191	-0.0264	-0.0341	-0.0424
θ°	215	220	225	230	235	240	245	250
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9746	0.9723	0.9697	0.9668	0.9635	0.9597	0.9551	0.9504
10	0.9011	0.8925	0.8826	0.8580	0.8586	0.8442	0.8275	0.8084
15	0.7873	0.7689	0.7481	0.7247	0.6981	0.6679	0.6333	0.5935
20	0.6450	0.6151	0.5815	0.5438	0.5010	0.4525	0.3970	0.3334
25	0.4877	0.4462	0.3997	0.3475	0.2887	0.2220	0.1462	0.0593
30	0.3291	0.2772	0.2188	0.1548	0.0821	0.0000	-0.0931	-0.1992
35	0.1810	0.1214	0.0551	-0.0188	-0.1016	-0.1946	-0.2997	-0.4191
40	0.0529	-0.0117	-0.0814	-0.1620	-0.2501	-0.3489	-0.4599	-0.5853
45	-0.0498	-0.1156	-0.1881	-0.2683	-0.3572	-0.4562	-0.5671	-0.6917
50	-0.1243	-0.1885	-0.2588	-0.3363	-0.4216	-0.5183	-0.6216	-0.7394
55	-0.1711	-0.2310	-0.2962	-0.3677	-0.4462	-0.5327	-0.6285	-0.7351
60	-0.1924	-0.2461	-0.3040	-0.3674	-0.4364	-0.5125	-0.5958	-0.6882
65	-0.1918	-0.2375	-0.2880	-0.3406	-0.3989	-0.4625	-0.5322	-0.6089
70	-0.1733	-0.2107	-0.2503	-0.2935	-0.3402	-0.3909	-0.4463	-0.5069
75	-0.1411	-0.1691	-0.1992	-0.2315	-0.2664	-0.3042	-0.3452	-0.3899
80	-0.0992	-0.1182	-0.1380	-0.1600	-0.1826	-0.2079	-0.2346	-0.2639
85	0.0511	-0.0605	-0.0705	-0.0812	-0.0927	-0.1051	-0.1185	-0.1330

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TABLE 2
BESSEL FUNCTIONS, $J_0(S \cos \theta)$

	ELEVATION ANGLE θ°								
5°	0°	10°	20°	30°	40°	50°	60°	70°	80°
45°	0.852	0.856	0.868	0.888	0.913		0.962	0.982	0.998
50°	0.818	0.823	0.838	0.862	0.891	0.923	0.953	0.978	0.994
55°	0.783	0.789	0.807	0.835	0.869	0.907	0.943	0.973	0.993
60°	0.744	0.751	0.772	0.805	0.845	0.890	0.933	0.968	0.992
65°	0.703	0.712	0.735	0.773	0.820	0.871	0.921	0.963	0.990
70°	0.660	0.669	0.696	0.739	0.792	0.851	0.909	0.957	0.989
75°	0.615	0.626	0.656	0.703	0.764	0.830	0.896	0.950	0.987
80°	0.569	0.580	0.613	0.667	0.734	0.808	0.882	0.944	0.985
85°	0.521	0.535	0.570	0.628	0.702	0.785	0.867	0.937	0.983
90°	0.472	0.486	0.525	0.588	0.670	0.761	0.851	0.929	0.981
95°	0.422	0.437	0.479	0.547	0.636	0.735	0.835	0.921	0.979
100°	0.372	0.387	0.432	0.506	0.601	0.709	0.818	0.913	0.977
105°	0.321	0.337	0.385	0.463	0.565	0.682	0.801	0.904	0.975
110°	0.270	0.287	0.337	0.419	0.528	0.654	0.783	0.895	0.972
115°	0.220	0.237	0.290	0.376	0.491	0.625	0.763	0.886	0.970
120°	0.170	0.188	0.242	0.332	0.453	0.596	0.744	0.876	0.967
125°	0.120	0.139	0.195	0.288	0.415	0.565	0.724	0.866	0.964
130°	0.072	0.091	0.148	0.244	0.376	0.535	0.703	0.855	0.961
135°	0.026	0.044	0.102	0.201	0.337	0.503	0.682	0.844	0.958
140°	-.020	-.001	.058	.158	.299	.472	.660	.833	.955
145°	-.064	-.045	.013	.115	.259	.440	.638	.821	.952
150°	-.105	-.087	-.029	.073	.221	.408	.615	.809	.949
155°	-.145	-.126	-.070	.032	.183	.375	.592	.797	.945
160°	-.182	-.164	-.108	-.007	.145	.343	.569	.785	.942
165°	-.217	-.200	-.146	-.045	.107	.310	.545	.772	.938
170°	-.249	-.232	-.180	-.082	.070	.277	.521	.759	.935
175°	-.278	-.263	-.213	-.118	.035	.245	.497	.746	.931
180°	-.304	-.290	-.244	-.151	-.001	.212	.472	.732	.927
185°	-.328	-.315	-.272	-.183	-.035	.180	.447	.718	.923
190°	-.348	-.336	-.297	-.214	-.068	.148	.422	.704	.918
195°	-.365	-.355	-.320	-.242	-.100	.117	.397	.689	.914
200°	-.379	-.371	-.340	-.267	-.131	.086	.372	.674	.910
205°	-.390	-.383	-.357	-.292	-.160	.055	.346	.659	.905
210•	-.397	-.393	-.372	-.313	-.188	.025	.321	.644	.901
215°	-.407	-.399	.384	-.333	-.214	-.004	.296	.629	.896
220°	-.403	-.402	-.393	-.350	-.239	-.032	.270	.613	.892
225°	-.401	-.403	-.399	-.364	-.263	-.061		.597	.887
230°	-.396	-.400	-.402	-.377	-.284	-.088	.220	.581	.882
235°	-.389	-.395	-.403	-.387	-.304	-.114	.195	.565	.877
240°	-.378	-.386	-.401	-.394	-.322	-.139	.170	.549	.871
245°	-.365	-.375	-.396	-.399	-.339	-.164	.145	.533	.866
250°	-.350	-.361	-.389	-.402	-.353	-.187	.120	.516	.861
255°	-.332	-.346	-.379	-.403	-.366	-.209	.097	.500	.856
260°	-.312	-.327	-.367	-.401	-.377	-.231	.072	.483	.850
265°	-.290	-.307	-.353	-.397	-.386	-.251	.049	.466	.844
270°	-.266	-.285	-.336	-.391	.393	-.270	.026	.449	.839
275°	-.241	-.262	-.318	-.382	-.398	-.288	.003	.432	.833
280°	-.214	-.236	-.298	-.372	-.401	-.304	-.020	.415	.827
285°	-.186	-.210	-.276	-.360	-.403	-.320	-.042	-.397	.822
290°	-.157	-.182	-.253	-.346	-.402	-.334	-.063	.380	.816
295°	-.128	-.154	-.228	-.330	-.400	-.346	-.084	.363	.809
300°	-.098	-.125	-.203	-.313	-.396	-.358	-.105	.345	.803
305°	-.068	-.095	-.176	-.294	-.391	-.368	-.125	.328	.797
310°	-.038	-.066	-.149	-.273	-.384	-.377	-.145	.311	.791
315°	-.008	-.036	-.122	-.252	-.375	-.382	-.163	.293	.784
320°	.022	-.007	-.093	-.229	-.365	-.391	-.182	.276	.777
325°	.051	.023	-.065	-.206	-.353	-.396	-.200	.259	.771
330°	.079	.051	-.037	-.182	-.340	-.399	-.217	.241	.764
335°	.106	.079	-.009	-.157	-.325	-.402	-.233	.224	.752
340°	.132	.106	.020	-.131	-.310	-.403	-.249	.207	.750
345°	.156	.131	.047	-.105	-.293	-.402	-.203	.190	.744
350°	.170	.155	.073	-.079	-.275	-.401	-.278	.173	.737
355°	.201	.178	.099	-.053	-.257	-.398	-.291	.156	.730
360°	.220	.199	.124	-.027	-.237	-.394	-.304	.139	.723

TABLE 3
MAGNITUDE OF MUTUAL LOOP IMPEDANCE BETWEEN TWO EQUAL HEIGHT TOWERS

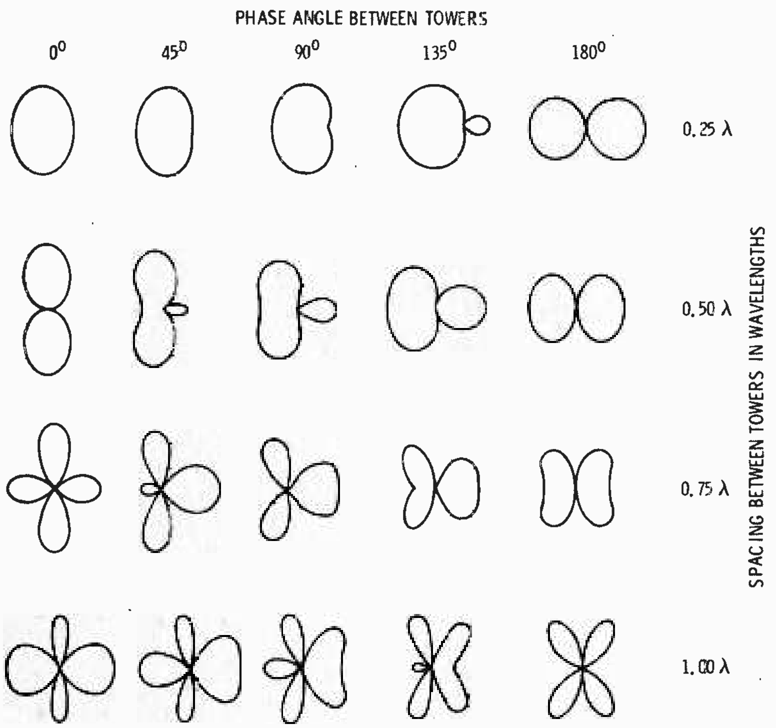
	TOWER HEIGHTS (G) IN DEGREES											
Spacing	60°	70°	80°	90°	100°	110°	120°	140°	160°	180°	200°	220°
60°	8.4	13.7	20.7	29.3	39.1	50.0	59.5	76.5	83.2	78.5	65.0	49.8
80°	7.4	12.2	18.5	26.2	34.8	45.0	52.4	67.0	73.2	69.5	59.3	45.0
100°	6.5	11.0	16.7	23.6	31.2	39.4	47.0	59.8	65.5	62.2	52.5	40.3
120°	5.8	9.9	15.1	21.2	28.2	35.6	42.7	54.4	59.5	56.5	47.5	36.2
140°	5.2	8.9	13.7	19.3	25.7	32.5	39.2	49.9	54.7	52.0	43.4	32.5
160°	4.6	8.0	12.4	17.6	23.7	30.0	36.1	46.2	50.6	48.1	39.9	29.3
180°	4.3	7.3	11.4	16.2	21.7	27.6	33.4	43.0	47.4	45.0	37.0	26.7
200°	4.0	6.7	10.4	15.0	20.1	25.7	27.3	40.2	44.5	42.4	34.6	24.4
220°	3.7	6.1	9.6	13.9	18.7	24.0	25.7	37.8	42.0	40.2	32.7	22.6
240°	3.4	5.8	9.0	12.9	17.5	22.3	24.3	35.7	39.7	38.2	31.1	21.2
260°	3.1	5.4	8.3	12.0	16.4	21.0	23.0	33.8	37.7	36.3	29.7	20.0
280°	2.9	5.0	7.8	11.4	15.5	19.8	21.8	32.0	36.0	34.7	28.4	19.1
300°	2.7	4.7	7.4	10.6	14.6	18.7	20.7	30.3	34.3	33.3	27.4	18.3
320°	2.6	4.4	7.0	10.1	13.7	17.7	19.7	28.9	32.7	32.0	26.5	17.7
340°	2.5	4.2	6.6	9.5	13.0	16.9	18.9	27.6	31.4	30.8	25.6	17.2
360°	2.4	4.0	6.3	9.0	12.4	16.0	18.0	26.3	30.1	29.7	24.7	16.8
380°	2.3	3.8	6.0	8.6	11.8	15.2	17.3	25.2	29.0	28.5	24.0	16.4
400°	2.1	3.6	5.7	8.3	11.2	14.6	16.6	24.2	27.9	27.6	23.4	16.0
420°	2.0	3.4	5.4	7.8	10.8	14.0	15.9	23.2	26.9	26.7	22.7	15.6
440°	1.9	3.3	5.2	7.5	10.4	13.3	15.4	22.4	25.9	25.8	22.0	15.3
460°	1.8	3.2	5.0	7.2	9.9	12.8	14.8	21.4	25.0	25.0	21.5	15.0
480°	1.8	3.0	4.8	7.0	9.5	12.3	14.2	20.7	24.1	24.2	20.9	14.7

TABLE 4
PHASE ANGLE OF MUTUAL LOOP IMPEDANCE BETWEEN TWO EQUAL HEIGHT TOWERS

	TOWER HEIGHTS (G) IN DEGREES											
Spacing	60°	70°	80°	90°	100°	110°	120°	140°	160°	180°	200°	220°
60°	-25	-19	-15	-11	-8	-8	-6	-6	-8	-12	-20	-27
80°	-33	-30	-28	-27	-25	-24	-25	-26	-27	-30	-34	-39
100°	-46	-44	-43	-42	-42	-42	-42	-43	-44	-48	-52	-54
120°	-60	-60	-60	-60	-60	-60	-60	-61	-63	-66	-68	-69
140°	-77	-77	-77	-78	-78	-78	-79	-80	-82	-84	-87	-86
160°	-93	-93	-94	-94	-95	-96	-97	-98	-102	-104	-106	-104
180°	-111	-111	-112	-112	-113	-114	-115	-118	-121	-123	-125	-123
200°	-129	-129	-130	-130	-131	-132	-133	-136	-138	-142	-144	-143
220°	-147	-148	-149	-150	-151	-152	-153	-155	-158	-161	-164	-163
240°	-166	-166	-167	-168	-169	-170	-172	-174	-177	180	176	177
260°	175	174	173	173	172	171	170	167	164	160	157	157
280°	157	156	155	154	153	152	150	148	145	140	137	136
300°	137	137	136	135	134	133	132	128	125	121	117	115
320°	118	117	116	115	114	113	112	109	106	102	98	95
340°	98	98	97	96	95	94	93	90	87	83	78	75
360°	79	79	78	78	77	76	74	72	68	64	59	55
380°	60	60	59	58	57	56	55	52	49	44	39	35
400°	41	41	40	39	38	37	35	33	29	25	20	15
420°	21	21	20	19	18	17	16	14	10	6	0	-5
440°	2	1	0	-1	-2	-3	-4	-6	-10	-13	-19	-25
460°	-18	-18	-19	-20	-21	-22	-23	-25	-29	-33	-39	-45
480°	-38	-38	-39	-40	-41	-42	-43	-45	-48	-52	-58	-64

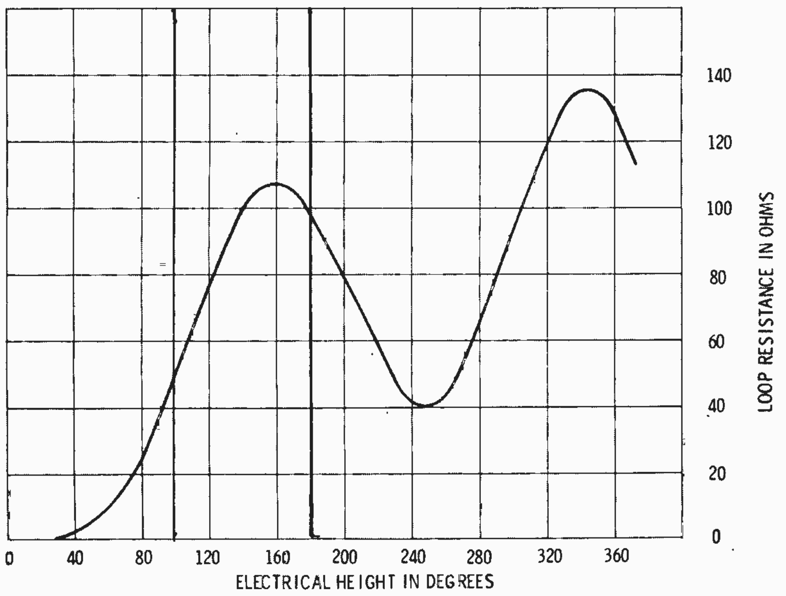
TABLE 5
TRIGONOMETRIC FUNCTIONS

Degrees	Sine	Tangent	Cotangent	Cosine	
0	.0000	.0000	......	1.0000	90
1	.0175	.0175	57.2900	.9998	89
2	.0349	.0349	28.6360	.9994	88
3	.0523	.0524	19.0810	.9986	87
4	.0698	.0699	14.3010	.9976	86
5	.0872	.0875	11.4300	.9962	85
6	.1045	.1051	9.5144	.9945	84
7	.1219	.1228	8.1443	.9925	83
8	.1392	.1405	7.1154	.9903	82
9	.1564	.1584	6.3138	.9877	81
10	.1736	.1763	5.6713	.9848	80
11	.1908	.1944	5.1446	.9816	79
12	.2079	.2126	4.7046	.9781	78
13	.2250	.2309	4.3315	.9744	77
14	.2419	.2493	4.0108	.9703	76
15	.2588	.2679	3.7321	.9659	75
16	.2756	.2867	3.4874	.9613	74
17	.2924	.3057	3.2709	.9563	73
18	.3090	.3249	3.0777	.9511	72
19	.3256	.3443	2.9042	.9455	71
20	.3420	.3640	2.7475	.9397	70
21	.3584	.3839	2.6051	.9336	69
22	.3746	.4040	2.4751	.9272	68
23	.3907	.4245	2.3559	.9205	67
24	.4067	.4452	2.2460	.9135	66
25	.4226	.4663	2.1445	.9063	65
26	.4384	.4877	2.0503	.8988	64
27	.4540	.5095	1.9626	.8910	63
28	.4695	.5317	1.8807	.8829	62
29	.4848	.5543	1.8040	.8746	61
30	.5000	.5774	1.7321	.8660	60
31	.5150	.6009	1.6643	.8572	59
32	.5299	.6249	1.6003	.8480	58
33	.5446	.6494	1.5399	.8387	57
34	.5592	.6745	1.4826	.8290	56
35	.5736	.7002	1.4281	.8192	55
36	.5878	.7265	1.3764	.8090	54
37	.6018	.7536	1.3270	.7986	53
38	.6157	.7813	1.2799	.7880	52
39	.6293	.8098	1.2349	.7771	51
40	.6428	.8391	1.1918	.7660	50
41	.6561	.8693	1.1504	.7547	49
42	.6691	.9004	1.1106	.7431	48
43	.6820	.9325	1.0724	.7314	47
44	.6947	.9657	1.0355	.7193	46
45	.7071	1.0000	1.0000	.7071	45
	Cosine	**Cotangent**	**Tangent**	**Sine**	**Degrees**



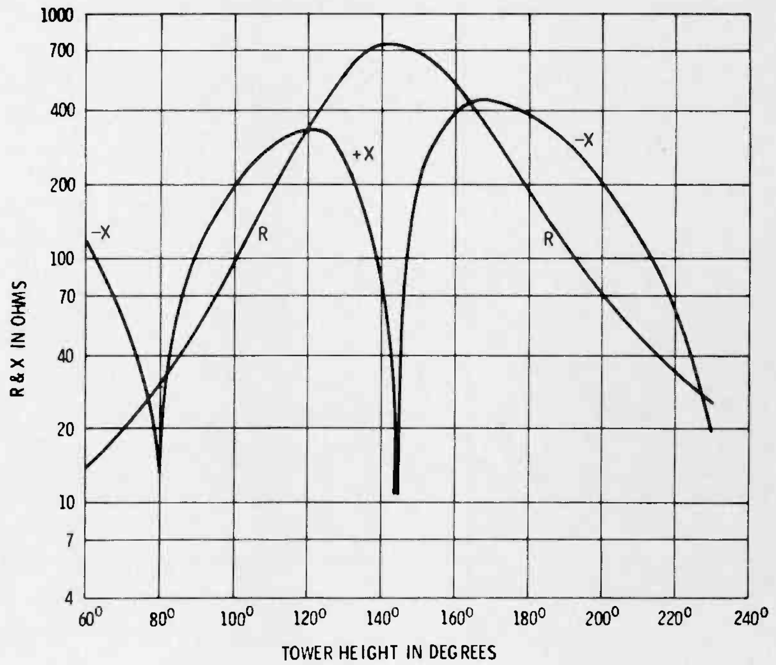
TYPICAL PATTERNS FOR A TWO-TOWER ARRAY FED WITH EQUAL CURRENTS AND EQUAL HEIGHTS, AND WITH VARIOUS SPACINGS AND PHASE ANGLES

Figure 1



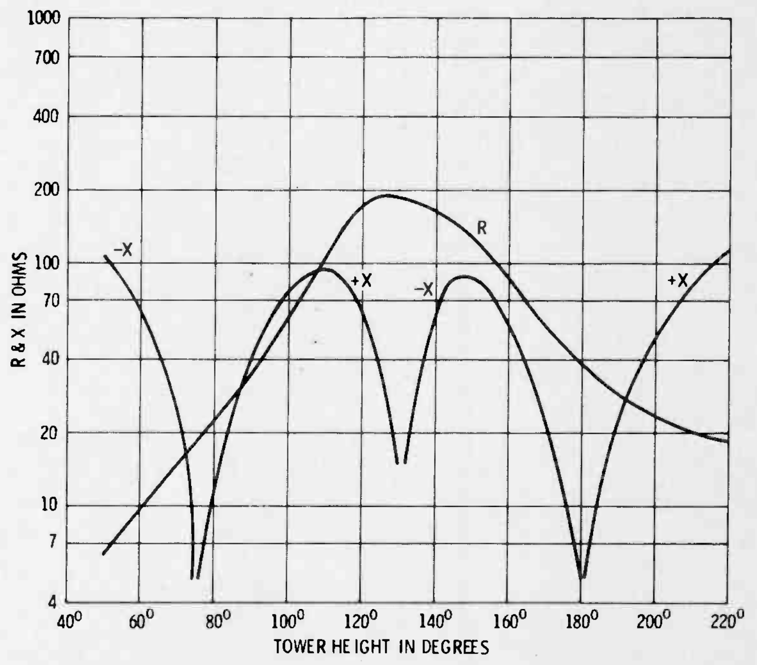
LOOP RESISTANCE FOR TOWERS AS A FUNCTION OF TOWER HEIGHT.

Figure 2



EMPIRICAL RESISTANCE AND REACTANCE CURVES FOR GUYED UNIFORM CROSS SECTIONAL TOWERS OF DIFFERENT HEIGHTS.

Figure 3



EMPIRICAL RESISTANCE AND REACTANCE CURVES FOR SELF SUPPORTING TOWERS OF DIFFERENT HEIGHTS.

Figure 4

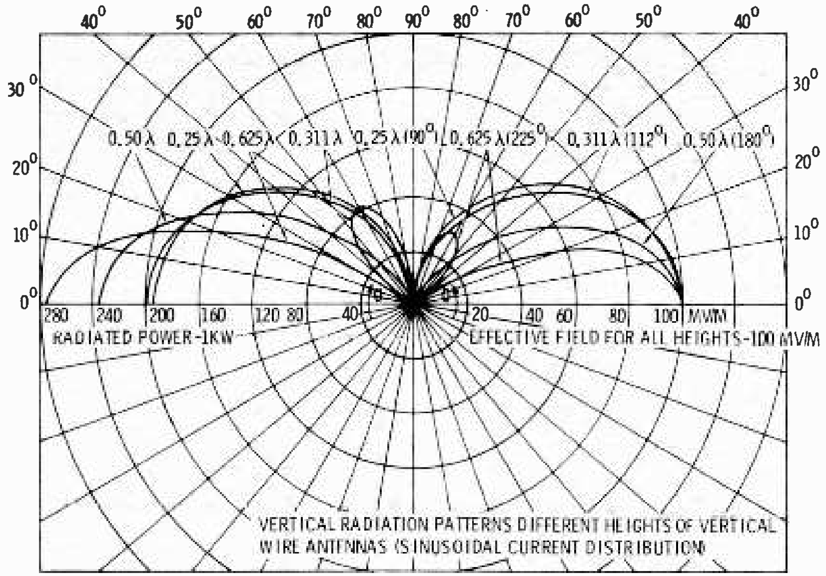


Figure 5

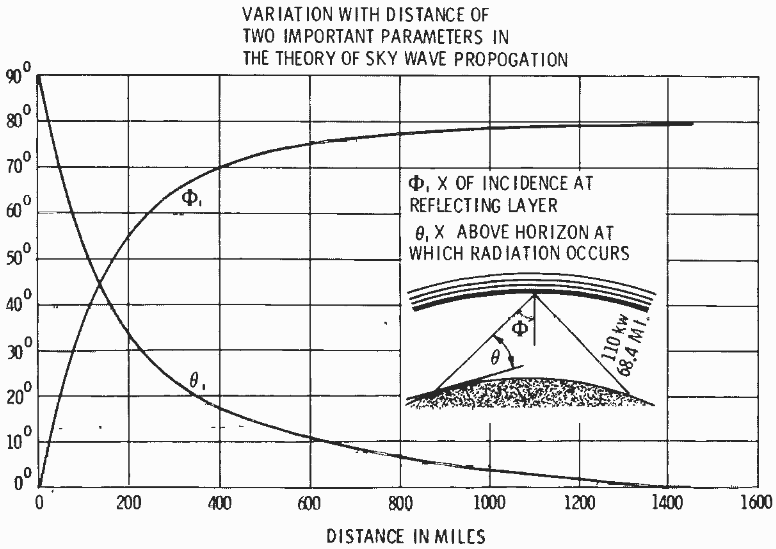


Figure 6

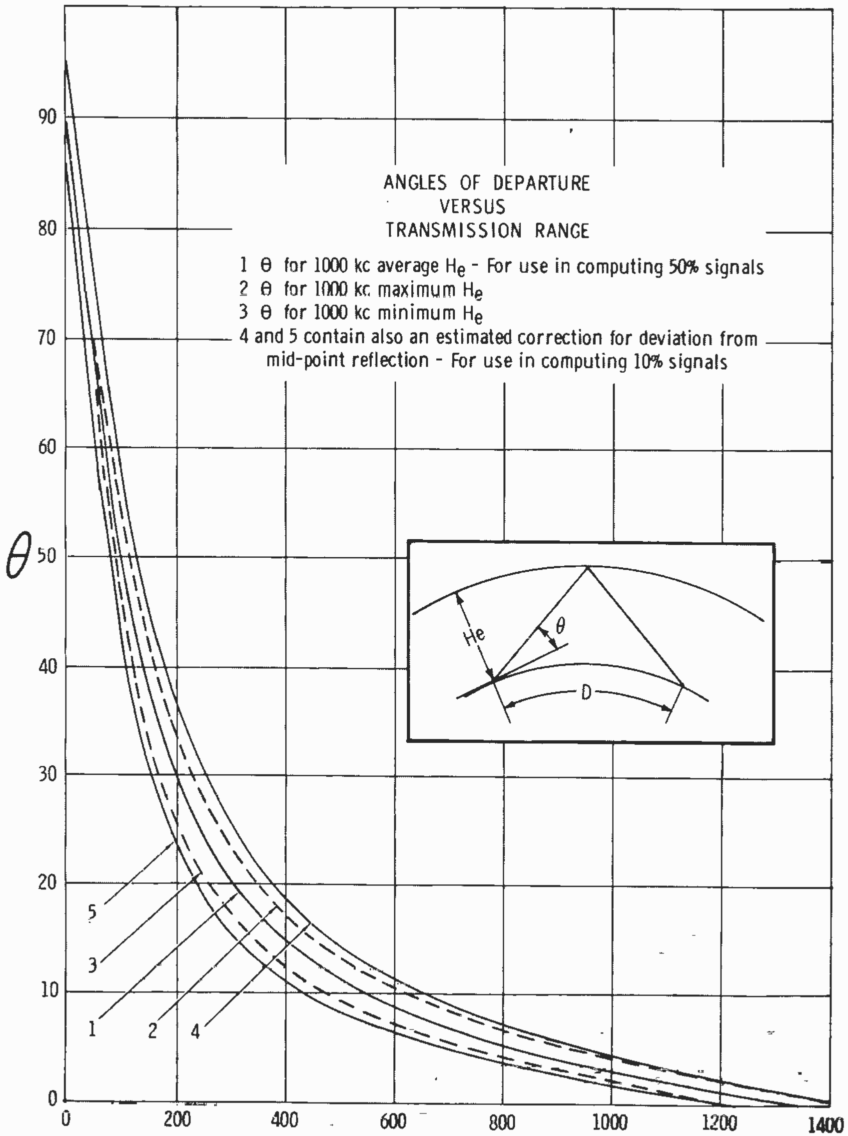
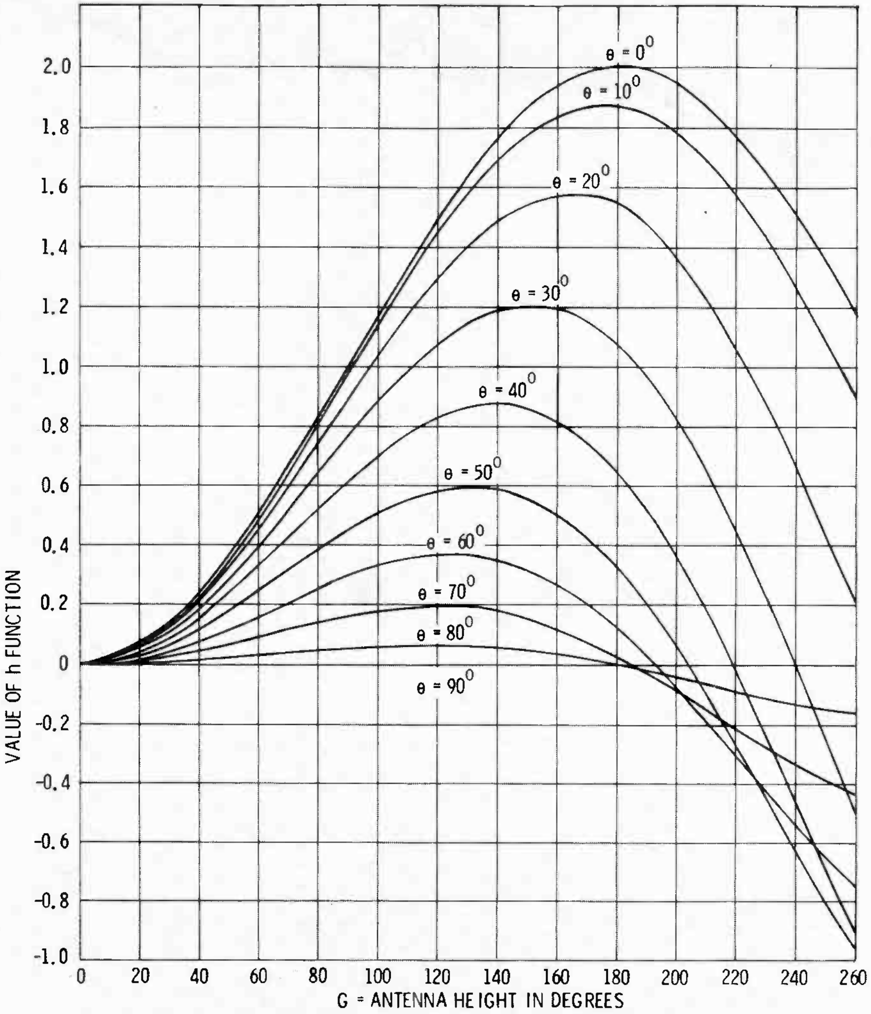


Figure 6A



VALUE OF h FUNCTION PLOTTED FOR VARIOUS VALUES OF ELEVATION ANGLE θ AS A FUNCTION OF ANTENNA HEIGHT G

Figure 7

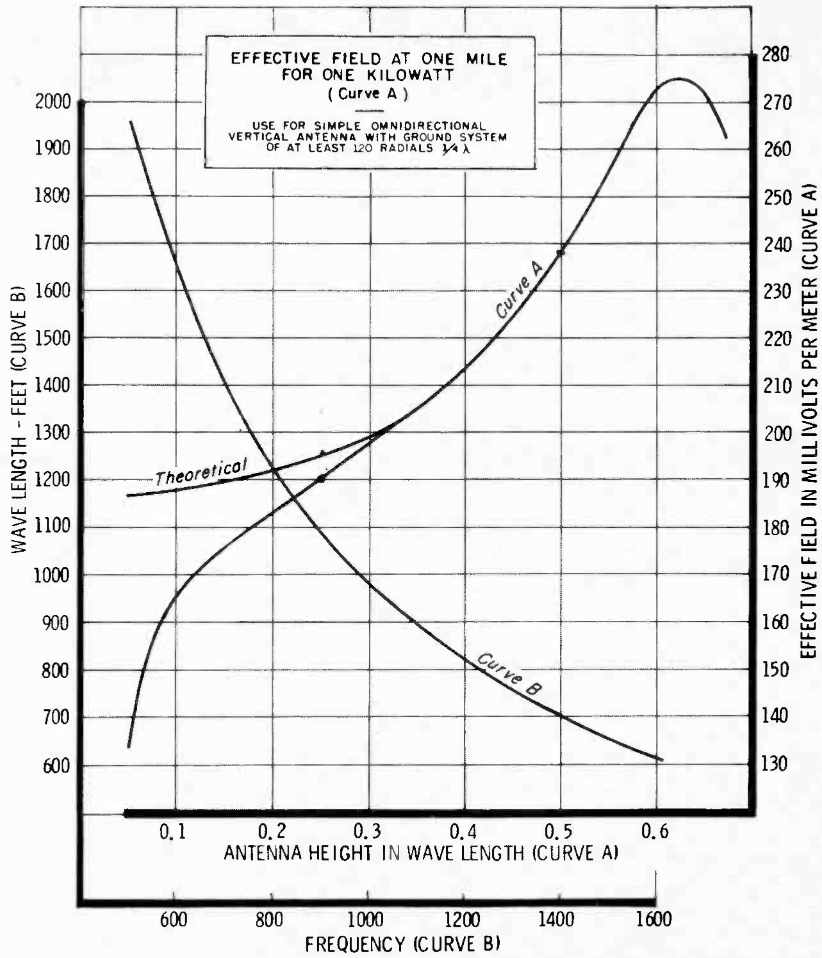


Figure 8

Glossary

ANTENNA ARRAY The number of towers and their physical configuration, i.e.; in-line, dog-leg, parallelogram, etc.

ANTENNA RESONANCE This is a condition which occurs when the base impedance of a given tower has zero reactance and pure resistance.

AXIS OF ARRAY This is the bearing line of the towers, if all towers are in a straight row, or the principal axis of design, if not.

AZIMUTH ANGLE The horizontal bearing from true north of the desired calculation.

CENTER OF ARRAY The point within the antenna array from which all calculations are made in the design.

CURRENT RATIOS This is the ratio of current in the Nth tower divided by the current in the reference tower.

DIRECTIONAL ANTENNA A system employed to control the radiation in given directions.

DRIVE POINT IMPEDANCE This is the actual operating impedance of a given tower within an array and consists of self impedance plus mutual impedance from all other towers in that given array. For each pattern this is a different value.

DIRECTIVITY A measure of the ability to concentrate energy in a preferred direction. Exact directivity is a ratio of maximum lobe power to the average power.

DOG-LEG ARRAY This describes an array of three towers or more where the towers are not in a straight row. The spacing between may or may not be uniform.

ELEMENT OF ARRAY This is one tower of an array of towers.

ELEVATION ANGLE The angle above the horizon at which a calculation is made. Commonly called the vertical angle and represented by the Greek letter θ.

FIELD RATIO The radiated signal of the Nth tower, as compared to the radiated signal of the reference tower.

FLOW CHART A block diagram of the procedure or sequence of steps taken by a computer or calculator to arrive at a final design.

GROUND PLANE This is one other word for horizontal plane and refers to the plane along the surface of the earth.

GROUND SYSTEM This is the physical system of wires, straps and screens that form the earth connection from each tower of the antenna system.

HEIGHT-ELECTRICAL The height of the tower(s) expressed in degrees of a wavelength at the design frequency. This is commonly represented by the letter "G."

HEIGHT-PHYSICAL The height of the tower(s) expressed in feet or in meters.

HORIZONTAL ANGLE The angle at which a horizontal calculation is made, measured in a clockwise direction, with reference to true north as 0°. This is represented by the Greek letter ϕ.

IMPEDANCE-MUTUAL Ratio of voltage to current at the loop or base of a second tower (Nth) as influenced by the first tower (reference). See Mutual Coupling.

IMPEDANCE-SELF Ratio of voltage to current at a given point on a tower, usually the base of the tower. Sometimes called the natural impedance.

IN-LINE ARRAY This is a directional antenna where all of the towers, regardless of the number, are in a straight line or row.

INVERSE DISTANCE This refers to the mathematical relationship of signal attenuation with distance from the antenna. Signal equals the reciprocal of distance and vice versa.

INVERSE DISTANCE FIELD INTENSITY This is the field intensity one would measure at a given distance, if there were no losses due to the passage of the signal over the surface of the earth, i.e. perfect soil conductivity.

LOBE-MAJOR That part of the pattern containing the greatest amount of signal. A pattern can have more than one major lobe. For example a figure eight would have two equal major lobes.

LOBE-MINOR This is a small lobe between two minimas. A given pattern can have any number of minor lobes.

MID-POINT The center reference spot about which all design calculations have been made. This may or may not be the same as the physical center of the array.

MINIMA This is another word for null and refers to any bearing where the radiated signal is reduced to zero, or a small amount. A given pattern can have more than one minima.

MUTUAL COUPLING A term which stands for the effect of current flowing in one tower as induced in the Nth tower. This depends upon current ratios, phase angles, tower spacing, and tower heights.

MUTUAL RESISTANCE See Impedance—Mutual.

NO-LOSS CONDITION This is a theoretical condition whereby the ohmic losses are ignored and all power is assumed to result in useful radiated energy.

NON-DIRECTIONAL PATTERN This is the pattern one would measure from a single tower acting by itself as one element of a directional antenna. Theoretically assumed to radiate uniform signal intensity in all directions.

NULL—See Minima.

NULL-FILL This describes the condition of a null that has not been reduced to zero signal. It will occur when the vectors do not all cancel at a given bearing.

OPERATING IMPEDANCE See drive point impedance.

ORIENTATION OF ARRAY The axis of the array. Commonly called the tower line. Arrays of more than two towers can have more than one tower line.

PARALLELOGRAM ARRAY This is a name given to an array of four, six, eight, nine or twelve towers whose opposite side dimensions form the sides of a parallelogram.

PARAMETERS The design values used by the engineer for computing any directional antenna and can be divided into variable and fixed.

VARIABLE: field ratio, phase angle, RMS or power frequency.

FIXED: Spacing, orientation, electrical height.

PATH LENGTH This is the distance, usually expressed in degrees, of the distance from a tower in an array to the point of the observer. The path length from one point in space to each tower in an array is usually different.

PHASE ANGLE The angular time displacement of current flowing in the Nth tower in relation to the reference tower current, expressed in ±degrees. This is represented by the Greek letter Ψ.

PHASE DELAY The time interval between the arrival of energy at the Nth tower in relation to the reference tower, commonly expressed in ±degrees.

POLAR COORDINATES A mathematical system of notation whereby the location of a point is defined by a magnitude and an angle.

POLAR PATTERN A graph of the unattenuated field intensity in a 360° circular configuration, referenced to a 1-mile radius.

POWER DIVISION This represents the proportion of the total power which is consumed in each tower of the array. The sum of the individual powers from all towers will equal the power input to the antenna system.

POWER GAIN This is a ratio of the equivalent power it would take from a single non-directional tower to achieve the radiated signal at the tip of the major lobe, divided by the power into the directional system.

POWER—RADIATED This is the amount of power consumed by the antenna, and represents the total power minus the system losses.

Q FACTOR This is the orthogonal component used in the FCC Standard Method of directional antenna computation. It is 2.5% of the R.S.S. or 6.0/power in kilowatts, whichever is greater, for a given pattern.

R.M.S. A measurement of the given radiation patterns efficiency, as expressed in MV/M and is equivalent to the radius of a non-directional tower having the same power radiated.

RADIATION EFFICIENCY This is a measure of the ability of a given antenna pattern to radiate the full power applied. This is represented by the Greek letter η.

RADIATION PATTERN See Polar Pattern.

RADIATION RESISTANCE—BASE This is the resistance one would measure at the base of a tower within an array, that represents that portion of the energy radiated into free space. The loss resistance of a given tower is assumed to be in series with the radiation resistance.

RADIATION RESISTANCE—LOOP This is the resistance one would measure at the point one-quarter wavelength below the top of a given tower, at the point of maximum current.

RECTANGULAR COORDINATES A mathematical system of notation whereby the location of a point is defined by its ± distance along the "X" axis and the "Y" axis.

REFERENCE—TOWER The one tower in each antenna array having a current ratio of 100%, and a phase angle of 0°, in relationship to all the other antennas within a given array.

SELF-RESISTANCE This is the resistance part of the self impedance of a single element acting independently, i.e. the natural non-directional resistance.

SHIFT FROM REFERENCE This is the angular displacement ± in degrees from the axis of the array to a given tower. Sometimes the shifts of each tower are referred to the angular displacement from true north.

SPACE PHASE The difference in path length between the signals from two towers as measured at the point of the observer "P." This is commonly computed as S cos Φ.

SPACING This represents the physical separation between any two towers in an antenna system. It can be expressed in electrical degrees or in physical dimensions. It is represented by the letter "S."

SYMMETRICAL PATTERN This refers to a pattern whose right half is a mirror of its left side. This condition will exist when all the antenna towers are in a straight row.

TIME DISPLACEMENT This represents the difference in time that one would measure between the arrival of two signals from two towers. It is expressed as S cos Φ.

TOP LOADING This refers to the physical appurtenances added to the upper portion of a tower to increase its equivalent electrical height.

TOWER LINE See Orientation of Array.

TOWER SPACING The electrical or physical separation between towers, usually referred to one tower as the reference tower.

UNIT VECTORS This is the vector which represents the proportional signal from each tower, expressed not in actual MV/M but in arbitrary units. The reference tower is normally assumed to be 1.0 units.

VECTORS A mathematical unit having both magnitude and directivity.

VERTICAL ANGLE See Elevation Angle.

VERTICAL ATTENUATION See Vertical Form Factor.

VERTICAL ELEVATION PLANE The angle above the horizon at which a calculation is made, commonly expressed by the Greek letter θ.

VERTICAL FORM FACTOR The configuration of radiation at all vertical angles above the horizon to the zenith, expressed as $f(\theta)$.