ELECTRONIC TECHNOLOGY SERIES

ADVANCED MAGNETISM AND ELECTROMAGNETISM

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ADVANCED MAGNETISM
AND
ELECTROMAGNETISM

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PREFACE

The concepts of magnetism and electromagnetism form such an essential part of the study of electronic theory that the serious student of this field must have a complete understanding of these principles. The considerations relating to magnetic theory touch almost every aspect of electronic development.

This book is the second of a two-volume treatment of the subject and continues the attention given to the major theoretical considerations of magnetism, magnetic circuits and electromagnetism presented in the first volume of the series.*

The mathematical techniques used in this volume remain relatively simple but are sufficiently detailed and numerous to permit the interested student or technician extensive experience in typical computations. Greater weight is given to problem solutions. To ensure further a relatively complete coverage of the subject matter, attention is given to the presentation of sufficient information to outline the broad concepts adequately. Rather than attempting to cover a large body of less important material, the selected major topics are treated thoroughly. Attention is given to the typical practical situations and problems which relate to the subject matter being presented, so as to afford the reader an understanding of the applications of the principles he has learned.

Specific attention is given to the nature of the magnetic field; introduction to magnetism; the magnetic field, definition and description, units for measuring, induction and units; induction lines and magnetic flux; movement of charged particles in magnetic fields; forces on current-carrying wires in magnetic fields; single conductor and rectangular loop; electromagnetism and inductance; magnetic field around a current-carrying conductor; the ampere in the mks

* Schure, A., *Magnetism and Electromagnetism,* New York: John F. Rider Publisher, Inc., 1959.

system; field of a solenoid; Ampère's Law; motional emf; Faraday's Law of induction; Lenz' Law; induced emf in a rotating rectangular loop; self-inductance, circuits combining inductance, resistance and energy in an inductor; ferromagnetic effects; ferro-, para-, and diamagnetic substances; genesis of magnetic effects in ferrous metals; intensity and permeability; characteristics of ferromagnetism; Curie temperature; hysteresis; theories of magnetism, molecular theory and domain theory; magnetic poles and forces; concept of magnetic poles; forces on magnetic poles; torque acting on a strong magnet in a field; forces between magnetic poles; fields associated with magnetic poles; magnetic circuits, reluctance and magnetomotive force; magnetism in technology; eddy currents and eddy-current devices; motors and generators; the cyclotron; the mass spectrograph; the betatron; terrestrial magnetism; and the induction coil.

Mastery of the material presented herein should ensure the student an adequate basis to undertake any work requiring knowledge of magnetic and electromagnetic theory as a prerequisite.

Grateful acknowledgment is made to the staff of New York Institute of Technology for its assistance in the preparation of the manuscript of this book.

New York, N. Y. A.S.
December, 1959

CONTENTS

Chapter 1

THE NATURE OF THE MAGNETIC FIELD

1. Introduction to Magnetism

Our most reliable historical records appear to indicate that magnetic phenomena were recognized and used as aids to navigation as early as the eleventh century by the Chinese. Long before this, the Chinese had learned that an iron rod could be permanently magnetized by stroking it with a fragment of lodestone, a form of naturally magnetic iron ore (magnetite – $FeO{\cdot}Fe_2O_3$). Early applications of magnetism were based upon two essential properties of magnets: the ability of a magnet to exert a force of attraction on small pieces of unmagnetized iron, and the tendency of a long magnet of small axial cross section to line itself up along an approximate north-south line when pivoted at its center. An important facet of the first property is that the forces which exist between magnets (or between magnets and unmagnetized iron) are exerted *over a distance* without the need for contact between the two materials.

Permanent magnets produced by the lodestone-stroking method are quite weak according to modern standards, yet for almost eighteen centuries any advances made in the comprehension of simple magnetic phenomena had to be based upon experiments with magnets made in this fashion. That there was a relationship between current electricity and magnetism was not uncovered until the beginning of the nineteenth century by Hans Christian Oersted, who observed, in 1819, that a freely swinging magnet could be acted upon by a current in a nearby wire. In 1831, Joseph Henry, an American scientist, discovered that an electric current could be induced in a conductor that was near another conductor in which a current was started or stopped; shortly afterward, an extension of this phenomenon was found — a permanent magnet in motion near a conductor could induce a current in the conductor during the time it was in motion. Michael Faraday, working independently in England, encountered the same phenomena as Henry about 11 months later. Since his results were published before Henry's, Faraday is generally accredited as the discoverer.

It is sometimes stated that Oersted's discovery and the findings of Henry and Faraday are exactly reciprocal in nature. This is not quite so. That is:

(Oersted)	A *steady* current produces a *steady* magnetic effect.
(Henry, Faraday)	*Changing* magnetism produces an electromagnetic field (emf). This emf can, of course, cause a current to flow, if the circuit is complete.

The essential points of difference which the reader might well keep in mind are: (1) although magnetism in a state of change (i.e., a magnet in motion, or a current that starts or stops) can give rise to an emf in an open-circuited conductor, an emf applied across an open-circuited conductor cannot produce magnetism, and (2) magnetism may be the outcome of a steady-state electric current, but an electric current cannot be generated by a steady magnetic state.

André M. Ampère proposed, in 1820, that magnetic phenomena might be due to subatomic electric currents associated with electrons in motion. This view now forms a part of modern magnetic theory, which is based upon the belief that all magnetic phenomena are the result of forces that exist between moving electric charges at a subatomic level. Such forces must be distinguished at once from the so-called "Coulomb forces" that arise from the attractions and repulsions of *stationary* electric charges, since magnetic effects are not perceptible until the charges become *dynamic*. Among the tenets of quantum mechanics for which more than adequate experimental verification exists are the proposals that electrons in atoms not only revolve around a central nucleus but also rotate around axes of their own. The latter motion is termed *electron* *spin.* Since the atoms of all elements are characterized by both of these motions, it might be expected that all matter possesses magnetic properties. Modern laboratory techniques have indeed shown this to be true. Although iron, cobalt, and nickel are the only elements that exhibit *strong* magnetic properties, it has been found that small magnetic effects may be obtained with all the elements in the periodic table.

Among the more easily observed characteristics of a permanent magnet is the presence of areas of apparently highly concentrated magnetic force, the *poles.* The interesting nature of the poles is such that the tendency to start with poles as the basis for an analytic study of magnetism is quite strong. The existence of polarity is, however, rather a minor phenomenon in the light of other, more revealing effects. For example, a study of magnetic poles does not provide a very useful base for the study of the nature of a *magnetic field,* whereas a careful description of the underlying properties of a magnetic field can thoroughly illuminate the nature of poles. It is for this reason that we shall start the study of magnetism with an examination of the forces between charges in motion. A description of magnetic poles will follow this in a later chapter.

2. Definition of a Magnetic Field

The concept of *field* has been introduced into physics to describe the action of forces at a distance. A field is a region in space where specific physical effects exist and can be detected. We commonly refer to three important types of fields: gravitational, electrostatic, and magnetic. On the basis of the general definition, a *magnetic field* is a region of space in which magnetic effects can be detected.

When a permanent magnet, such as a compass, is moved around in the vicinity of another strong magnet, the various fixed positions it adopts in different locations differ widely from each other. This suggests that the magnetic field around the strong magnet has a certain *direction* and a specific *strength* at each point. These will be discussed in the following section. The entire surface of the earth lies in a magnetic field in which compasses generally line up in a north-south direction. As a matter of fact, when delicate electrical experimentation must be carried on in a "field-free" space, it is necessary to construct special cubicles shielded with iron or containing artificial field-producing mechanisms which cancel the field of the earth.

The magnetic field of the earth is very weak, however, when compared to artificially generated fields such as one finds around the large cyclotrons in university research centers or around powerful electric generators in central powerhouses.

3. Direction of Magnetic Fields

As pointed out previously, a compass placed in different portions of space permeated by a magnetic field will point in various directions, the specific direction being determined by its location in the field. Hence, a magnetic field must have *direction.* The direction of a magnetic field is defined in terms of certain arbitrary considerations, as explained below.

In analyzing electrostatic phenomena,* it was found convenient to define a *unit charge* in terms of the force exerted by this charge on another identical charge separated from it by a unit distance. To serve as a tool in magnetic analyses, we use a similar definition for a *unit pole.* (A unit pole need not be related in concept to the "poles" of a permanent magnet at all). Thus, a unit pole in magnetism is defined in terms of the force between it and another identical pole located one unit distance away from it. Also, as in electrostatics, we dealt with two kinds of unit charges – plus and minus – similarly, in magnetism, we find two types of unit poles. These are designated as N and S. This symbolism, as the reader is probably aware, arises from the tendency of a compass to align itself in the earth's field so that one end (or pole) points north and the other south.

The direction arbitrarily assigned to a magnetic field is given thus: the direction of a magnetic field is the direction in which an isolated unit N-pole will move if free to do so. Several selected directions are shown in Fig. 1 in accord with this definition. The tangent to the curve at point 1, a particular position of the unit pole as it moves along the path traced by the solid line, is the *direction of the magnetic field* at this point; tangent CD is the direction of the field at point 2, and, similarly, tangent EF is the direction of the magnetic field at point 3. Such a definition provides a relatively concrete picture of what is meant by field direction.

On the basis of Oersted's experiment, it can be deduced that a *unit electric charge* in motion must give rise to an associated mag-

* Schure, A., *Electrostatics,* New York: John F. Rider Publisher, Inc., 1958.

netic field. This, of course, is easily shown by the fact that streams of electrons in free space (as in a cathode-ray tube) can be deflected at will in any direction by orienting the magnetic field through which the stream passes. The force experienced by such a moving charge is due to the reaction between its magnetic field and the field through which it is traveling. Thus, as a second definition, we might state: *a magnetic field exists at a point if a force is exerted on a moving charge at that point.* There is, however, one line along which a charge may move even in a strong magnetic field *without experiencing any force whatever.* This line coincides with the path over which an isolated N-pole would move in the same field (Fig. 1).

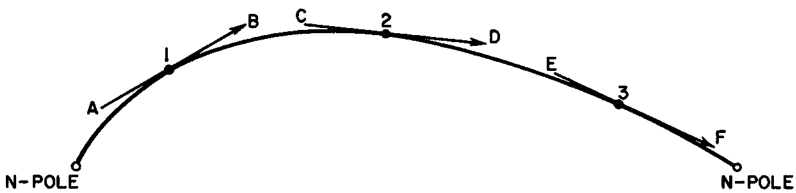

Fig. 1. The direction of a magnetic field as given by the direction of motion of a free unit N-pole. Curve is path of unit N-pole in a magnetic field.

That is, a moving electric charge following the path from 1 to 2 (or from 3 to 2 to 1) in Fig. 1 will not be acted upon by any magnetic forces, regardless of the strength of the magnetic field in which it is immersed. Hence, we are provided with a second definition of line direction by this fact: *the direction of a magnetic field is given by the direction of the motion of an electric charge within the field when the charge direction is such that it experiences zero magnetic force.* Note that this definition does not give the *sense* of the magnetic field direction, only the *line* of action. On the other hand, the definition previously stated (based upon the path of a unit N-pole) gives both the *line* and the *sense* of the field direction. Both definitions are useful, however, as the following sections will demonstrate.

4. Force Acting on a Charge Moving through a Magnetic Field

Suppose we consider a *negative* electric charge moving at right angles to a magnetic field. Working with negative charges right from the start is advisable, since we shall be thinking in terms of electron motion when we discuss magnetic effects of electric currents.

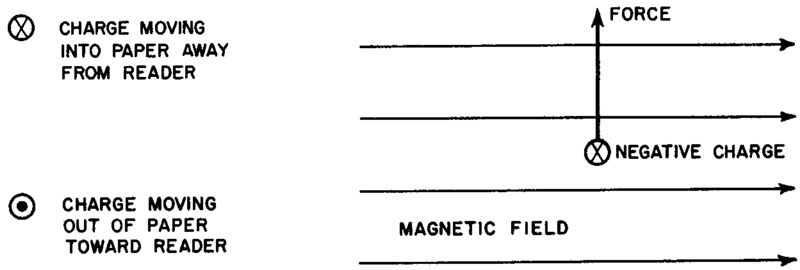

Fig. 2. Force on a negative charge in motion at right angles to a magnetic field.

Identical formulations may be obtained, however, by working with positive charges. In Fig. 2, the magnetic field is shown as having a direction from left to right, and the moving negative charge as progressing into the plane of the paper away from the reader. The accepted convention for charge motion is used here: an electric charge is pictured as an arrow having tail feathers. If the charge moved outward toward the reader, he would see the point of the arrow or a dot inside a small circle. When the charge "arrow" passes through the plane of the paper away from the reader, he sees the tail feathers, or a cross inside the circle.

Such an experiment, performed in a Crooke's tube or a cathode-ray tube, clearly indicates that the force that acts on the moving charge is perpendicular both to the magnetic field and the direction of motion of the charge. In this particular case, the force is upward in the plane of the paper. Thus, there are three directions to consider — in three dimensions. Several different rules-of-thumb have been conceived to serve as mnemonic devices for this situation. The one most convenient for our purposes is illustrated in Fig. 3 and may be described as follows: Using the left hand, orient the thumb, index finger, and middle finger at right angles to each other. Point the *I*ndex finger in the direction of the moving charge (a negative charge in motion constitutes an electron current symbolized by *I*) and the *M*iddle finger in the direction of the *M*agnetic field. The cocked *TH*umb will then point in the direction of the *TH*rust or force acting on the moving negative charge.

The force acting on the charge is perpendicular to both the magnetic field and the direction of motion of the charge *only* if the velocity of the charge is at right angles to the field. This is the condition shown in Figs. 2 and 3. Now suppose that the charge passes through the field at some angle Θ other than 90%. Drawn in perspective, this set of conditions would appear as in Fig. 4 (A). The real direction and speed (velocity) of the negative charge is designated by v, while the component of the velocity at right angles to the plane of the magnetic field is given as v′. The force on the moving charge is at right angles to the magnetic field and v′, but $v' = v \sin \Theta$ since $v' = v \cos \phi$ (i.e., $\sin \Theta = \cos \phi$ because these angles are complementary).

Hence, the force acting on the moving charge is perpendicular to both the field and the component of the velocity at right angles to the field ($v \sin \Theta$). The magnitude of the force may be shown experimentally to be proportional to the right angle velocity component and also to the size of the charge. That is:

$$F = B\, q\, v \sin \Theta \qquad (1)$$

in which q is charge magnitude, $v \sin \Theta$ is the component of the charge velocity perpendicular to the field, F is the force, and B is a proportionality constant; since the force must also be related to the

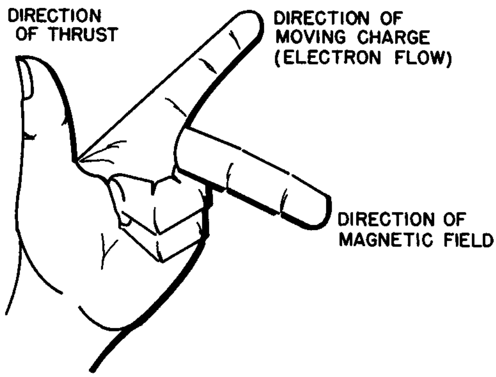

Fig. 3. Left-hand rule to determine the direction of a force acting on a negative charge moving at right angles to a magnetic field.

strength of the magnetic field, then B must implicitly carry a significance connected with field strength. To determine its significance, equation (1) is solved for B:

$$B = \frac{F}{q\, v \sin \Theta} \qquad (2)$$

From equation (2), it is now possible to define the term B, variously called magnetic field strength, flux density, or magnetic induction. Using the mks system of units, when F is given in newtons, q in coulombs, and v in meters per second, then the flux density B is given in *webers per square meter* (w/m^2). One weber per square meter, therefore, is the flux density of a magnetic field in which one coulomb of charge, moving with a component of its velocity at right angles to the magnetic field equal to one meter per second, is

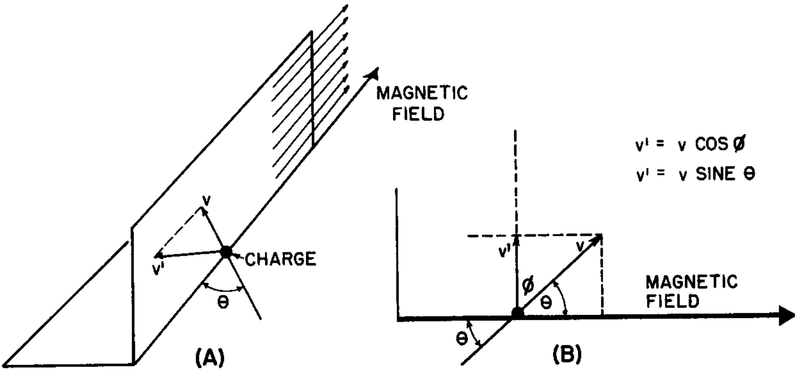

Fig. 4. Resolution of velocity into components.

acted on by a force of one newton. The dimensions of the w/m^2 may be obtained from equation (2). That is:

$$1 \frac{w}{m^2} = \frac{\text{newtons}}{\text{coul} \times \text{meters/sec}}$$

but 1 coul/sec = 1 ampere, so:

$$1 \frac{w}{m^2} = \frac{\text{newtons}}{\text{amp-meters}}$$

The significance of the w/m^2 is, therefore, not apparent from a dimensional conversion. The usefulness of the *weber* as a magnetic unit will, however, be discussed shortly; from this, the physical picture of the value of w/m^2 should become apparent.* The huge electromagnet used in the cyclotron at Massachusetts Institute of Technology generates a flux density of approximately 1.83 w/m^2 or about 18,000 gauss. Certain laboratories report the generation of upwards of 200,000 gauss by means of very heavy instantaneous currents flowing through relatively few turns of wire. The magnetic

* B in the cgs system is given in $maxwells/cm^2$ or *gauss*. The unit conversion from cgs to mks gives 1 w/m^2 = 10^4 gauss.

field of the earth is very weak by comparison, having a flux density of approximately 0.000043 w/m^2 (0.43 gauss) in the vicinity of New York.

Problem 1. A lithium ion bearing a charge equal to that of an electron* is made to enter a magnetic field at right angles with a velocity of 3×10^4 meters per second. The flux density of the field is 10 w/m^2. Find the magnetic force acting on the ion.

Solution. Substituting in equation (1), we have

$$F = 10 \times 1.6 \times 10^{-19} \times 3 \times 10^4 \ (\sin 90°)$$

since $\sin 90° = 1$,

$$= 10 \times 1.6 \times 10^{-15}$$

$$= 4.8 \times 10^{-14} \text{ newton}$$

5. Induction Lines and Magnetic Flux

As in the study of electrostatics, it is convenient to represent a magnetic field in terms of lines of magnetic induction or, more simply, *lines of force*. It must be understood from the start that lines of force are merely *representations* of the conditions that exist in the space around a magnet or a current-carrying conductor and that *they have no real existence.*

Since magnetic lines are representations of conditions and nothing more, we are free to select any number of them that we wish to symbolize a magnetic field of given strength. For example, suppose that we are dealing with a field in which the flux density B is 10 w/m^2. We might represent this field by saying that it contains 100 lines, 1000 lines, or only one line, but the moment the choice is made, the value of the magnetic line has been *unalterably defined.* In short, this procedure defines and describes the magnetic line of force. Returning to the field of flux density 10 w/m^2, it is evident that simple numerical relationships would exist if we should say that such a field contains 10 lines of magnetic force; in this case, a field in which $B = 100$ w/m^2 could be described by stating that it was permeated by 100 lines.

This concept, therefore, permits us to say that the *number of lines of force in a given field passing through one square meter at right angles to the field is numerically equal to B.* Alternatively,

* The charge on an electron in mks units may be taken as 1.6×10^{-19} coulombs.

this definition might be given in this fashion: the number of lines of force passing through a given area perpendicular to the field is the product of the flux density times the area or:

$$\phi = BA \tag{3}$$

in which ϕ is the total number of lines of force or the *total magnetic flux,* B is the flux density, and A is the area through which the lines pass. When B is expressed in w/m^2 and area in m^2, it is seen that ϕ must be given in *webers.* Thus the meaning of B in w/m^2 becomes clear; if the total magnetic flux (ϕ) passing through an area of, say, 100 m^2 is 10 webers, then the *density* (or crowding) of the lines of force is such that B = 0.1 weber for each square meter of perpendicular surface. In concept, this is similar to density in mechanics wherein a substance having a mass of m gm and a volume of V cm^3 has a density D = m/v gm per cm^3.

Problem 2. The number of lines of force passing perpendicularly through a surface of 60 m^2 is 180. What is the total magnetic flux in the surface? What is the flux density?

Solution. The total flux density is numerically equal to the number of magnetic lines of induction and is expressed in webers. That is:

$$\phi = 180 \text{ webers}$$

The flux density is given by equation (3):

$$B = \frac{\phi}{A} = \frac{180}{60} = 3\,\frac{w}{m^2}$$

6. Motion of Charged Particles in Magnetic Field

A primary effect that is of considerable importance in several technological applications of magnetism, i.e., in the operation of the cyclotron and the focusing of television cathode-ray tubes, is the reaction of a moving charge having a given velocity in a magnetic field. It is easier to analyze the behavior of such a particle in two dimensions by representing the magnetic field in terms of lines of force that pass through the paper at right angles; using the same convention as previously described for the motion of a charge, the symbol ⊕ represents a line moving into the paper away from the observer and ⊙ indicates a line moving out of the paper toward the observer.

Consider first an electron projected into a sharply defined magnetic field whose lines emerge from the paper as shown in Fig. 5.

Since the electron moves into the field from the left, its direction is indicated by the arrows labeled *I* (for *I*ndex finger) at right angles to the emerging field lines. With the *M*iddle finger pointing out of the paper (since the *M*agnetic field is out of the paper), the perpendicular *TH*umb shows that the *TH*rust, or force acting on the electron, is upward on the sheet. An unbalanced force must produce acceleration in the direction of the force, hence the direction of the

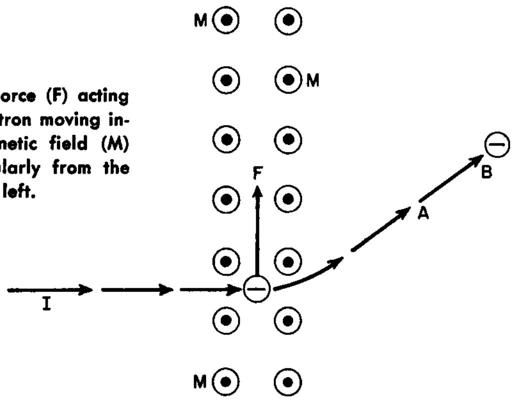

Fig. 5. Force (F) acting on an electron moving into a magnetic field (M) perpendicularly from the left.

electron's velocity must change so that the new velocity has an upward component. If the electron were now to leave the field, its new path would be that indicated by AB in Fig. 5.

A much more important result is obtained when we consider the electron as remaining in a uniform field throughout its motion. Since the force acting on the particle is directed at right angles to the velocity direction *at any instant,* the electron is subject to a *constant accelerating force* perpendicular to its motion. Such a force must produce circular motion, because only in a circle is the radius always perpendicular to the tangent to the curve at any point. This is illustrated in Fig. 6.

Referring to Fig. 6, at point 1 the electron has an instantaneous velocity v which is tangent to the curve at that point. It is subject to a force *F perpendicular to v* and directed toward point 0. As it moves from 1 to 2, it experiences central acceleration, since F is always perpendicular to the particular tangent representing v at any instant; the same is true from 2 to 3 and then back again to 1. Thus, a charged particle moving with constant speed at right angles to a magnetic field must describe a circle whose plane is also perpendicular to the field. The force F is then a *centripetal force* which can be described by either of two equations. Considered mechanically only, centripetal force is given by:

$$F = \frac{mv^2}{R} \qquad (4)$$

in which m is the mass of the moving particle, v is the linear or tangential velocity taken to have a constant magnitude (although

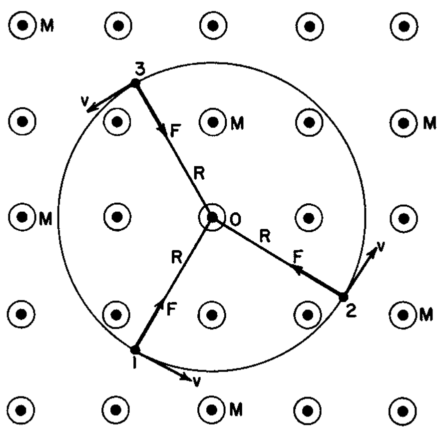

Fig. 6. Circular motion is produced when a constant unbalanced force is exerted perpendicular to the tangent at the particle on the curve.

its direction changes from point to point), and R is the radius of the circle of rotation.

The second equation is taken from equation (1) by taking sin Θ equal to unity, since the electron is moving at right angles to the field (see Problem 1 and Solution), and solving for F. That is:

$$F = B\,q\,v \qquad (5)$$

Since F represents the same force in both cases, we may set the right-hand side of equation (4) equal to the right-hand side of equation (5) to obtain:

$$B\,q\,v = \frac{mv^2}{R} \qquad (6)$$

from which the radius of rotation may be deduced as:

$$R = \frac{mv}{Bq}^{*} \qquad (7)$$

* The reader should check this equation dimensionally as an exercise.

Problem 3. An electron moving with constant speed at right angles to a magnetic field of flux density B = 20 w/m^2 describes a circle having a radius R. If the electron's linear velocity is 5×10^7 m, find the radius of rotation. (Mass of an electron = 9×10^{-31} kgm; charge on an electron = 1.6×10^{-19} coul.)

Solution. Substituting in equation (7):

$$R = \frac{9 \times 10^{-31} \times 5 \times 10^7}{20 \times 1.6 \times 10^{-19}}$$

$$= 1.4 \times 10^{-5} \text{ meters (or 0.0014 cm)}$$

7. Force on Current-Carrying Wire in a Magnetic Field

An electric current consists of electrons in motion from a point of negative to a point of positive (relatively speaking) potential. Having derived the equation that describes the motion of an electron in a magnetic field, it follows that sufficient information is now available to enable us to obtain a relation that defines the force acting on a current-carrying wire. Many electrical devices are based upon the force that exists under these conditions: electrical motors, moving coil voltmeters, ammeters, and wattmeters are a few examples of such devices.

The equation that describes the magnitude of an electric current (i) is:

$$i = \frac{n\,q}{t} \tag{8}$$

in which i is the current, n is the number of charges each of magnitude q, and t is the time during which the charges flow. [The more familiar form of this equation is I = Q/t. In this form, Q is the total charge or number of charges times the unit charge, and I is the total current. Equation (8) is the more fundamental statement.] If this current flows through a length of wire (L) and the moving charges have a velocity v, then the time of flow may be given as:

$$t = \frac{L}{v} \tag{9}$$

since distance = velocity × time. The factor L/v may then be substituted for the time (t) in equation (8) to yield:

$$i = \frac{n\,q\,v}{L} \tag{10}$$

Suppose now that the wire is placed in a magnetic field so that the current charges move past the lines of force at right angles as illustrated in Fig. 7. Once again it is more convenient to represent the moving charges in the plane of the paper and the magnetic field perpendicular to it. Under the conditions shown in Fig. 7, the length of wire L experiences a total force F (upward); the total force may be considered as the resultant of all the individual forces

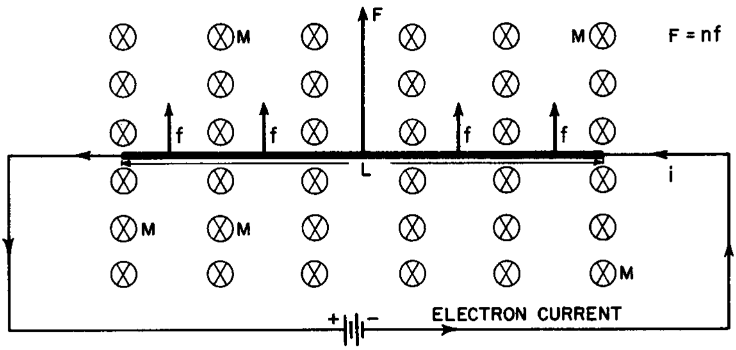

Fig. 7. Force acting on a current-carrying wire at right angles to a magnetic field.

(f) acting on the particles in the same direction. Since the force on a particle (right angle provision where $\sin \Theta = 1$) is B q v, then the total force F is:

$$F = n\ B\ q\ v \qquad (11)$$

Dividing both sides of the equation through by L, we have as the force per unit length:

$$\frac{F}{L} = \frac{n\ B\ q\ v}{L} \qquad (12)$$

As derived in equation (10), however, (n q v) /L is the current in the wire. Thus:

$$\frac{F}{L} = B\ i \qquad (13)$$

so that the total force acting on the wire is:

$$F = B\ i\ L \qquad (14)$$

As previously remarked, this is the force acting on a wire of length L in a field of flux density B when a current of intensity i flows through the wire, *provided that the wire is oriented at right angles to the magnetic field.* The identity of the product B i L and the product B q v, however, enables us to use equation (1) directly and write, for the force acting on a wire oriented at an angle Θ to the field:

$$F = B\ i\ L \sin \Theta \tag{15}$$

It should be noted from the analysis of units given in Section 4 that, since B may be measured in newtons (amp-meter), current must be stated in amperes and wire length in meters if the force is to be measured in newtons. That is:

$$F\ \text{(newtons)} = B \left(\frac{\text{newtons}}{\text{amp-meters}}\right) \times i\ \text{(amp)} \times L\ \text{(meters)}$$

Problem 4. A copper wire 0.5 m long is immersed at right angles in a magnetic field having a flux density of 1000 w/m². If the length of wire has a mass of 100 gm, how much current must be passed through it so that the gravitational force acting on it will be exactly neutralized? (Assume that current direction and field direction are such that the wire will tend to rise when current flows.)

Solution. We want the upward force on the wire for right-angle conditions and will, therefore, use equation (14). Solving for i, we have

$$i = \frac{F}{B\,L}$$

The weight of the wire must first be converted to newtons. Thus:

$$\begin{aligned} w\ \text{(newtons)} &= m\ \text{(gm)} \times 10^{-3} \times 9.8 \\ &= 9.8 \times 10^{-3} \end{aligned}$$

Thus, an upward magnetic force of 9.8×10^{-3} newtons is required to neutralize gravity. Therefore, the current

$$\begin{aligned} i &= \frac{9.8 \times 10^{-3}}{10^{-3} \times 0.5} \\ &= 19.6 \text{ amperes} \end{aligned}$$

8. Analysis of Forces on Current-Carrying Loop

Electric motors and measuring instruments make use of the forces that exist on complete circuits (closed loops) in magnetic fields. Let us review briefly the force conditions that apply to a *couple* before analyzing the forces on current-carrying loops.

A mechanical couple is produced when two equal but oppositely directed forces act on the same rigid body in a non-colinear manner.

(That is, their lines of application are not colinear.) Forces applied by the hands to a steering wheel of an automobile, or the action of the earth's magnetic field on a balanced compass needle constitute practical couples. An ideal couple cannot produce translatory motion but only rotation (Fig. 8). The *moment of the couple,* or

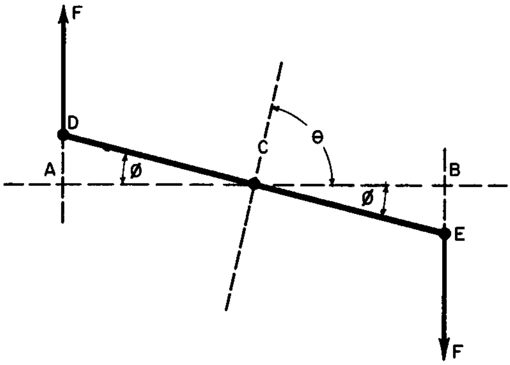

Fig. 8. The action of a mechanical couple.

the *torque* produced, is given as the product of either one of the two identical forces by the perpendicular distances between their lines of application. In Fig. 8, the moment is:

$$\text{torque} = F \times AB \tag{16}$$

and in terms of the distance between the points of application of the two forces (DE):

$$\text{torque} = F \cos \phi \ DE \tag{17}$$

It is more convenient to describe the torque in terms of the angle between the horizontal (AB) and the normal to the rigid body. Since this angle Θ is complementary to ϕ, then:

$$\text{torque} = F \sin \Theta \ DE \tag{18}$$

Equation (18) is applicable to the conditions which control the torque on a rectangular loop of wire carrying a current in a magnetic field.

Referring first to Fig. 9 (B), it is seen that a loop of wire, in which a current flows in the direction of the arrows, is immersed in magnetic lines that are entering the plane of the paper. The loop can rotate on a horizontal axis at right angles to the magnetic field. Imagine now that you are viewing the system end on, looking at it from the right side of the paper with your line of vision skimming over the paper surface. From this point of view, the lines of force have a direction from left to right [M in Fig. 9 (A)], the electron current I emerges from the paper at the upper wire cross section and returns to the paper at the lower cross section, and the forces exerted on the shaded sides of the loop have the directions given by the heavy arrows.

Both these sides fulfill the conditions called for in the derivation of equation (14); furthermore, the two equal forces applied to the sides of the loop constitute a couple. From equation (14), the magnitude of each force is F = B i L, so that the torque acting on the whole loop is given by the expression:

$$\text{torque} = F \sin \Theta \, d \qquad (19)$$

in which Θ is the angle between the magnetic field and the plane of the loop, and d is the diameter of the loop in the direction shown in Fig. 9. [Identities should be established for DE in Fig. 8 and d in

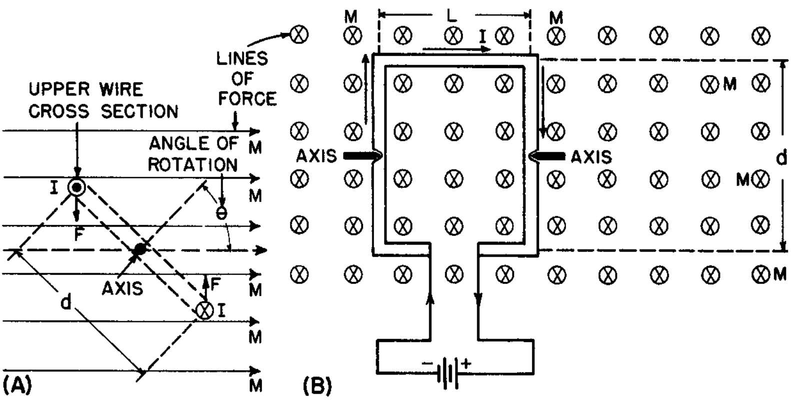

Fig. 9. Torque exerted on a rectangular loop of wire carrying a current in a magnetic field.

Fig. 9 (A) on the one hand, and for the Θ's in those figures on the other.] From equations (14) and (19) we can obtain:

$$\text{torque} = B \; i \; L \; d \sin \Theta \qquad (20)$$

in which L is the length of the conducting wire at right angles to the field. (The reader should again establish the identity of L in Fig. 9 and L in Fig. 7.)

It becomes evident at once that equation (20) can be simplified by noting that the product Ld is equal to the *area* (A) of the loop, hence:

$$\text{torque} = B\ i\ A\ \sin\Theta \qquad (21)$$

in which B is the flux density (w/m^2), i is the current (amp), A is the area of the loop (m^2), and Θ is the angle between a normal to the plane of the loop and the magnetic field; for these units, torque will be expressed in newton-meters (n-m).

The armature of an electric motor or the moving coil of a galvanometer is made up of many loops in series. It can be shown by means of the integral calculus that the torque exerted on a coil of *any shape* having n concentric turns (closely wound) is given by the product of n and the right-hand side of equation (21) or:

$$\text{torque} = n\ B\ i\ A\ \sin\Theta \qquad (22)$$

Equation (22) applies equally well to a closely wound solenoid; in this case, Θ is the angle between the major axis of the solenoid and the direction of the magnetic lines of force.

Problem 5. At what orientation of the major axis of a solenoid will the coil attain equilibrium in the magnetic field?

Solution: From a purely mathematical standpoint, the torque acting on a solenoid becomes zero when Θ takes on a value of *either* zero degrees or 180 degrees, since, for both of these angles of orientation with the field, the sine of Θ is zero. Physically, however, the equilibrium attained by reducing Θ to zero [Fig. 9 (A)] is a rather precarious one, since the slightest angular displacement would then produce a torque that would rotate the solenoid through 180°. This is very much like saying that a solid cone resting on its apex is in equilibrium when its axis is vertical or pointing toward the center of the earth. Anyone who has balanced a cone this way knows that the equilibrium is very unstable.

Thus, the orientation of the solenoid with respect to the field should be such that the forces constituting the couple be directed *away* from the center of rotation rather than toward it, if stable equilibrium is to be realized. In a real system, of course, this could occur with $\Theta = 0 = 180°$, depending upon the reference line from which Θ is measured and upon the directions of both the field and the electron current flow.

Problem 6. The plane of a rectangular loop of conductor 4 cm × 8 cm is oriented at right angles to a magnetic field having a flux density of 0.1 w/m^2. If a current of 5 amperes flows through the wire, (a) what torque acts on the loop? (b) what is the maximum torque that can be realized with the same total length of wire by changing the shape of the loop, if necessary, under the same conditions of current and flux density?

Solution. (a) The torque is obtained by substituting in equation (21) as follows:

$$\text{torque} = 10^{-1} \times 5 \times 32 \times 10^{-4}$$
$$= 1.6 \times 10^{-8} \text{ n-m}$$

(b) When B, i, and Θ are all constant as in this example, then the torque is directly proportional to the area of the loop; hence, maximum torque is obtained when area is maximum. For a given perimeter, a square has the maximum area of any polygon; the area of a square of a given perimeter is also greater than a circle having a circumference equal to this perimeter. Hence, converting the rectangular coil given in the example to a square, the area becomes 36 cm^2 (each side is now 6 cm in length) or 36×10^{-4} m, so that:

$$\text{torque} = 10^{-1} \times 5 \times 36 \times 10^{-4}$$
$$= 1.8 \times 10^{-8} \text{ n-m}$$

9. Review Questions

1. Explain in detail why the Oersted effect and Faraday's process of electromagnetic induction are approximately, but not exactly, reciprocal in nature.
2. Define a magnetic field. What arbitrary method has been chosen to describe the direction and sense of a magnetic field?
3. We have used a negative charge in motion to derive certain rules and laws of moving charges in magnetic fields. In the older textbooks (first published before it was ascertained that an electric current is composed of electrons rather than positive charges in motion), a unit *positive* charge is employed to derive similar rules and laws. Describe a "hand rule" which will give the same answers for field direction, charge direction, and thrust as our left-hand rule provides for moving electrons, using a positive charge as your unit charge.
4. Show that the force acting on a charge moving through a magnetic field parallel to the lines of force is zero. What force acts on a unit charge moving at 45° to the lines of force with a velocity of 1 m, if the flux density to the field is 1 w/m^2?
5. Prove that a newton of magnetic force could be expressed in terms of weber/amperes per meter.
6. What is the total magnetic flux passing through a plane that measures 2 m $\times$ 8 m, if the flux density is 0.03 w/m^2?
7. Show in a rigorous manner that the path of an electron with uniform speed moving at right angles to a magnetic field is a circle if the field is uniform in strength. Show that a charged particle that enters a magnetic field at some angle other than 90° will move in a helical path, provided that it maintains uniform speed and that the magnetic field is uniform throughout the path of the particle.
8. An electron is to be introduced at a velocity of 2×10^5 m into a uniform

magnetic field at right angles to the field. It is desired that the radius of rotation of the electron be 1.0 cm. Find the magnitude of the flux density required to accomplish this.

9. Does the angle Θ in equation (15) have the same meaning as the angle Θ in equation (21)? Explain your answer.
10. Prove with the help of the equations developed in this chapter that the maximum torque experienced by a motor armature is obtained when the axis perpendicular to the plane of the armature coils is also perpendicular to the direction of the lines of force.

Chapter 2

ELECTROMAGNETISM AND INDUCTANCE

10. Magnetic Field around a Current-Carrying Conductor

In the discussions about the forces acting upon and the motion of charges in magnetic fields presented in Chapter 1, experimental evidence was cited to prove that charges in motion were acted upon by forces resulting from interplay between the charge and the magnetic field. The work of Hans Christian Oersted clearly demonstrates that moving charges give rise to magnetic fields of their own. Combining the implications of these two phenomena leads to the logical guess that the mutual effect of these fields upon each other is responsible for the forces described in Chapter 1. We shall accept this hypothesis for the moment, and, using it as a jumping-off point, we shall investigate the relationships between the motions of charged particles and the magnetic fields they produce.

A single wire conductor, mounted vertically and passing through a plane surface such as a sheet of poster board, may be used to trace the magnetic field around the wire when a current passes through it. Looking down at the top of the conductor, so that it is seen in cross section, one can sprinkle iron filings on the poster board and then pass a surge of heavy current through the conductor. The filings will align themselves in concentric circles around the wire as shown in Fig. 10. This establishes the direction but not the sense of the field. If a small compass is then moved around the plane near the wire while the current is flowing, the direction in which the north-seeking end of the magnet points is an indication of the sense of the field in accordance with the accepted definition. In addition, it is found that the compass needle as a whole comes to rest in each position tangent to the filing circle at that point. In Fig. 10 (B), the electron current in the conductor cross section is shown emerging from the plane of the paper; for this condition, it is found that the circular magnetic field has a clockwise direction; with the electron current entering the paper, the field is counterclockwise [Fig. 10 (C)]. A simple mnemonic device for remembering the relationship between electron current and field direction, often called the *left-hand rule for wires*, may be stated thus: if the thumb of the left hand is outstretched at right angles to the other fingers and then

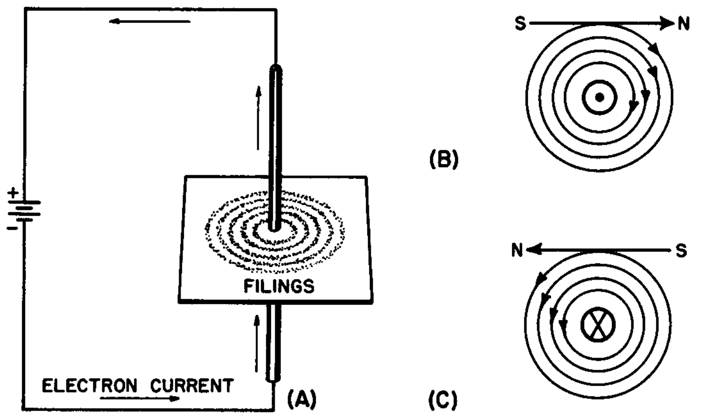

Fig. 10. Magnetic field around a current-carrying wire.

pointed in the direction of the electron current, the fingers will then encircle the wire in the direction of the lines of force.

11. Ampère's Formula (The Biot Law)

The qualitative results of Oersted's experiment led to further investigation of magnetic fields engendered by electric currents by three French physicists of the 18th century: André Ampère, Jean B. Biot, and Felix Savart. Although we shall not directly derive the law first proposed by Biot and later published by Ampère, the factors that must enter the equation are quite easily seen from relatively simple considerations. Reference should be made to Fig. 11, in which a current is shown passing through a vertical wire and generating a magnetic field in the plane ABCD, which is perpendicular to the wire. The Ampère-Biot-Savart argument is based on the fact that current in every tiny element of the wire makes its contribution to the magnetic field. Consider one small section of wire Δs located

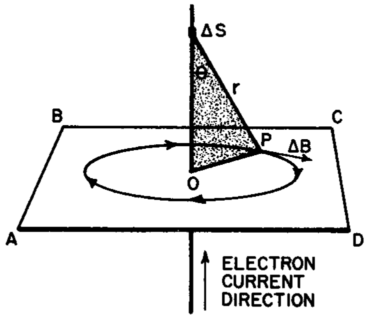

Fig. 11. Illustrating the approach to the problem of computing magnetic flux density due to a current in a wire.

at a distance r from the point P which lies on the circular magnetic line in the plane ABCD. At point P, tangent to the magnetic line and lying in the plane ABCD, there is a certain flux density ΔB due to the current element Δs. The plane containing Δs, P, and O is then drawn in. From the geometry of the figure, it is evident that this plane is perpendicular to ABCD; also the flux density vector ΔB lies in ABCD and is perpendicular to the triangular plane formed by Δs, P, and O.

The magnitude of the flux density ΔB is a function of several other variables: the length of the element Δs and the current i flowing through it must certainly have a hand in establishing the size of ΔB; the distance r between the current element and the plane ABCD must also enter into the calculations; and finally, the angle Θ must be a determining factor. A rigorous mathematical treatment of the problem, supported by experimental evidence, shows that ΔB is directly proportional to the product of the length of the wire element Δs, the current i, and the sine of Θ, and is inversely proportional to the square of the distance r. That is:

$$\Delta B \propto \frac{i\ \Delta s \sin \Theta}{r^2} \qquad (23)$$

Stated in the form of an equation, this becomes:

$$\Delta B = K \frac{i\ \Delta s \sin \Theta}{r^2} \qquad (24)$$

in which K is a constant of proportionality having a numerical value determined by the units chosen for the other quantities.

It is advisable to digress here for a moment to discuss current practice in handling the proportionality constant. Several possible courses are open to us: (1) K might be arbitrarily set equal to unity and the unit of current in equation (24) defined in terms of flux density and length; (2) current, length, and flux density might be independently defined in units already selected in other branches of physics, in which case K would have to be determined experimentally; (3) or in accordance with modern trends, the value of K might be selected so that the useful and practical laws derived from the basic statement (and, incidentally, utilized to a much greater extent than the basic statement) may be stated in the simplest possible terms. This last method is the one we shall use in the developments that follow.

To derive the dimensions of K in the mks system, let us solve equation (24) for K. Thus:

$$K = \frac{\Delta B \ r^2}{i \ \Delta s \ \sin \Theta} \tag{25}$$

Substituting mks units for the quantities on the right side of the equation, we have:

$$K = \frac{\text{webers}}{\text{m}^2} \times \text{m}^2 \times \frac{1}{\text{amp}} \times \frac{1}{\text{m}} \tag{26}$$

$$= \frac{\text{webers}}{\text{amp-meters}}$$

To establish a magnitude for K that is commensurate with quantities met in practice, it is set equal to 10^{-7} webers/amp-meter.

$$K = 10^{-7} \text{ webers/amp-meter} \tag{27}$$

If this value for K is used in the derivations of practical laws for finding flux density in loops, solenoids, toroids, etc., it is found that the term 4π constantly makes an appearance in the final form of the relationship. Since it appears most often in the numerator, it is convenient to define a new proportionality constant containing 4π in the denominator so that cancellation can occur. That is, if we define the new constant – generally symbolized μ_o – thus:

$$K = \frac{\mu_o}{4\pi} \tag{28}$$

then, when $\mu_o/4\pi$ is substituted for K in equation (24), all succeeding equations containing 4π in the numerator can be written in much simplified form.

The numerical value of μ_o is easily obtained from equations (27) and (28):

$$\mu_o = 4\pi \times 10^{-7} \text{ webers/amp-meter} \tag{29}$$

$$= 1.257 \times 10^{-6} \text{ webers/amp-meter}$$

And, finally, expressing the Biot law in terms of the new constant μ_o, we can write:

$$\Delta B = \frac{\mu_o \; i \; \Delta s \sin \Theta}{4\pi r^2} \tag{30}$$

Before we turn to specific examples of the use of the Biot law in deriving practical equations, it is worthwhile examining the implications of the general form in equation (30). Since $\sin \Theta = 0$, when Θ is zero — that is, when P becomes coincident with the axis of the current element (Fig. 11) — then the flux density on the axis must be zero. Similarly, if P is allowed to move vertically at a constant distance r from the wire, the maximum value of flux density through P will be obtained when this point lies in a plane perpendicular to the axis of Δs. In such a plane, $\Theta = 90°$, and $\sin \Theta$ attains its maximum value of 1. Thus, the flux density is greatest in a plane at right angles to the current element axis.

12. Special Cases — Applications of Biot's Law

The expression for Biot's Law described in equation (30) applies to the flux density produced by a very small current element in a given small section of wire. To extend the concept so that it will encompass systems of practical nature encountered in real physical problems, the contribution of every current element in the system must be added vectorially to that of every other element. A summation of this kind is best handled by the use of the integral calculus and so is beyond the scope of this work; on the other hand, certain relatively simple cases can be examined for the purpose of gaining insight into the general method whereby the Biot law is applied.

Flux density in a current loop. If a current flows through a perfectly circular loop of wire, the flux density produced at the geometric center can be determined from the general form of the Biot law without too much difficulty. The conditions are illustrated in Fig. 12. Since the point under consideration is the center of the circle, then r is the same for all current elements, regardless of their position in the loop; similarly, the same current exists in each element, so that i is a constant for this system. This leaves only $\Delta s \sin \Theta$ for summation. Further simplification is achieved by noting that the angle Θ must be 90° for every current element, since all the elements in the circumference of a circle coincide with the tangent at that point; with $\Theta = 90°$, sine Θ is unity, leaving for summation

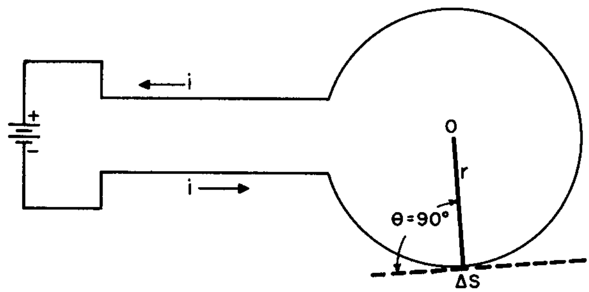

Fig. 12. Conditions for determining the flux density at the center of a circular wire loop carrying a current.

only Δs. For a circle, the summation of all the elements of the circumference must be the same as the circumference itself, hence the summation of $\Delta s = 2\pi r$.

Substituting the terms obtained above in equation (30) yields:

$$B = \frac{\mu_o \, i \, 2\pi r}{4\pi r^2} \tag{31}$$

which reduces to:

$$B = \frac{\mu_o \, i}{2 \, r} \quad \text{(for a single loop)} \tag{32}$$

(Note the cancellation of π's which results from the use of μ_o rather than K.)

If the single loop is replaced by a coil of wire consisting of n turns, and if all the turns have approximately the same radius with the length of the coil made very small compared to the radius, then each turn contributes the same flux to the center point O so that equation (32) may be modified to:

$$B = \frac{\mu_o \, i \, n}{2 \, r} \text{ (for a coil of n turns)} \tag{33}$$

Problem 7. Calculate the flux density at the center of a closely wound short coil having 100 turns, wound from a length of wire measuring 50 meters. The current flowing in the coil is 5 amperes.

Solution. The circumference of this circle must be 0.5 m, since 50 m are used to make up 100 turns. The radius r is then:

$$r = \frac{0.5}{2\pi} = \frac{0.5}{6.28} = 0.0795 \text{ m}$$

Therefore:

$$B = \frac{1.26 \times 10^{-6} \times 5 \times 10^{2}}{2 \times 7.95 \times 10^{-2}}$$

$$= 0.00397 \text{ w/m}^2$$

Flux density in a solenoid. A solenoid differs from the coil described in the previous paragraph in that its axial length L is generally substantially greater than the radius of each turn. By means of a series of steps similar to those used for determining the flux density of a flat coil [equation (33)], it can be shown that:

$$B = \frac{\mu_o \, n \, i}{L} \tag{34}$$

for a solenoid of n turns, axial length L, carrying a current of i amperes. The flux density of the field thus described refers to any point near the axis of a long solenoid and well within its end boundaries. These qualifications arise from the fact that the field inside a solenoid is not absolutely uniform, growing slightly weaker at greater distances from the axis and at close approaches to the ends. It is interesting to note that the radius of the solenoidal coil form does not appear in the equation, hence, within the limits set by the original specifications (length substantially greater than radius), the radius of the coil form is unimportant.

Flux density around a long straight wire. The Biot law, applied to a straight current-carrying conductor, provides a very simple expression for flux density, provided that the wire is very long compared to the distance between it and the point P at which the flux density is measured (Fig. 13). If the perpendicular distance between the wire and the point of interest is s, then the flux density is given by the relation:

$$B = \frac{\mu_o \, i}{2 \, \pi \, s} \tag{35}$$

This equation, known as the Biot-Savart law because it was empirically derived by these men before the mathematical analysis had been performed, is very useful for determining the force that exists between two wires carrying currents. The force, in turn, serves as the basis for defining the mks *ampere,* as is shown in the next section.

13. The mks Ampere

Before the widespread adoption of the mks system of units, it was necessary to carry three different systems of units along in many electrical calculations. Relations in electrostatic work were expressed in esu's (electrostatic units), equations for magnetic relationships were given in emu's, and electrical engineering used the so-called practical units involving the ampere, volt, and ohm. A great unification has resulted since the mks system became accepted; the use of the mechanical units in the mks system (meter, kilogram, second)

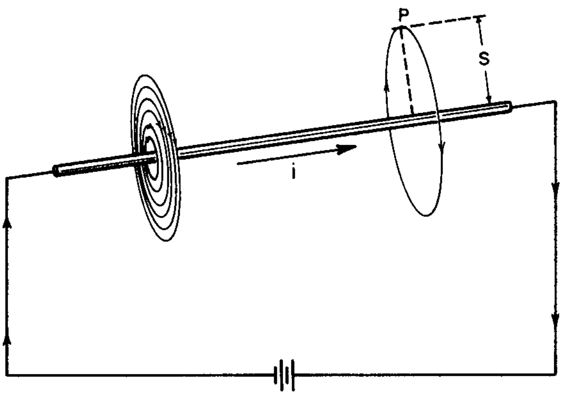

Fig. 13. Parameters for determining the flux density around a long, straight wire carrying a current.

now makes it possible to express all equations in electrostatics and magnetism in a single set of units. The *coulomb,* as we have used it throughout this series, is defined as the quantity of charge which passes a given point in each second when one ampere of current flows. The quantity of charge in the electrostatic system, however, is defined in terms of Coulomb's Law, has an entirely different value from the mks coulomb, and is now called the *statcoulomb.**

* 1 statcoulomb = 2.08×10^9 electrons; 1 mks coulomb = 6×10^{18} electrons.

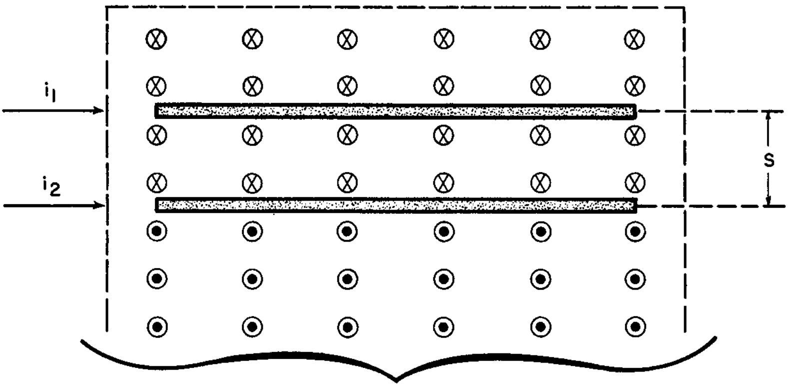

Fig. 14. Determination of the force between two parallel straight conductors of infinite length.

Since the ampere is a *fundamental* unit in the mks system (the coulomb is a *derived* unit), it is necessary to define it in terms of the basic mks mechanical units. This is what we propose to do in this section.

Consider a very long wire carrying a current i_1 parallel to a second very long wire carrying a current i_2 in the same direction, separated from each other by a relatively short distance *s*, as shown in Fig. 14. If the wires are very long (ideally, infinite in length), then equation (35) gives the flux density around each of the conductors in terms of i_1 and i_2, respectively.

Let us first visualize the situation in terms of the force produced *on* i_1 by the magnetic field *due to* i_2. The field around the wire carrying i_2 consists of concentric rings in planes perpendicular to the straight wire (Fig. 13). Thus, the concentric rings viewed on edge, as in Fig. 14, would present the appearance of "dots" and "x's" as shown. The sense of the field is obtained, of course, from the left-hand rule for wires, explained in Section 10. This situation is quite analogous to that pictured in Fig. 7 and discussed in Section 7. Applying the "three-finger rule" (Fig. 3), it is at once evident that the upper wire must experience a downward thrust and that the magnitude of this force as given in equation (14) is:

$$F = B\ i_1\ L$$

where B is the flux density, i_1 is the current, and L is the length of the wire.

For a wire of infinite length, this expression has no real meaning, since it would imply the existence of an infinite force. To circumvent this difficulty, we divide the equation through by L, thus obtaining a statement of the *force per unit length* or:

$$\text{Force on upper wire per unit of length} = \frac{F}{L} = B\ i_1 \qquad (36)$$

The flux density produced by the current in the lower wire (B) is given by equation (35). Substituting i_2 for i in this equation and combining with equation (36), we have:

$$\frac{F}{L} = \frac{\mu_o\ i_1\ i_2}{2\pi s} \qquad (37)$$

Now let us suppose that we had two infinitely long wires separated from each other in free space by exactly *1 meter*, each wire carrying *unit current.* This unit of current — that is, the *ampere* — may now be defined in terms of the force acting between the wires. Thus, *the ampere is defined as that current which, when flowing in each of two infinitely long parallel conductors separated by 1 meter in free space, gives rise to a force of 2 × 10⁻⁷ newtons per meter length on each conductor.*

The correspondence of this definition with the logic formerly used in Section 11 for obtaining the value of μ_o [see equation (29)] may easily be tested by substitution in equation (37).

$$2 \times 10^{-7}\ \frac{n}{m} = \frac{\mu_o \times 1\ \text{amp} \times 1\ \text{amp}}{6.28 \times 1\ \text{meter}} \qquad (38)$$

from which

$$\mu_o = 1.257 \times 10^{-6}\ \text{webers/amp-meter}$$

Problem 8. Two parallel wires, each 100 meters in length, are separated by 2 cm. Each wire carries a current of 50 amperes oppositely directed. (a) Find the total force between the two wires, and (b) determine whether the force is attraction or repulsion.

Solution. (a) Since the length of the wires is so much greater than their separation, equation (37) may be taken as very closely applicable. Substituting:

$$\frac{F}{L} = \frac{1.26 \times 10^{-6} \times 50 \times 50}{6.28 \times 2 \times 10^{-2}}$$

$$= 2.5 \times 10^{-2}\ \text{newtons/meter}$$

so $F = 2.5$ newtons for the full 100 meter length

(b) Refer to Fig. 14. Assume i_1 now directed to the left rather than the right. This will cause the *I*ndex finger to point to the left; with the *M*iddle finger pointing down, the *TH*rust is upward. Hence, the wire carrying i_1 will be repelled by the wire carrying i_2.

If the wire carrying i_2 (Fig. 14) is considered the current-carrying conductor immersed in the field produced by i_1, it is evident that when the currents flow in the same direction, the wires will attract *each other* (thrust on upper wire down, thrust on lower wire up), and when the currents flow in opposite directions, the wires will repel *each other*. The same conclusions may have been arrived at by applying Newton's Third Law of Reaction.

14. Field Forms — Descriptive Characteristics of Magnetic Lines

It is often valuable to pictorialize the gross forms of magnetic fields around current-carrying conductors of various shapes and structures. The types in which we are especially interested are: (1) a long straight wire or bar, (2) a single circular turn or group of turns forming a coil whose diameter is much greater than its length, (3) a solenoid, and (4) a toroid.

Long, straight wire. This case has already been discussed in detail. See Fig. 13 and accompanying text. The field form is described as a series of concentric circles in planes perpendicular to the conductor.

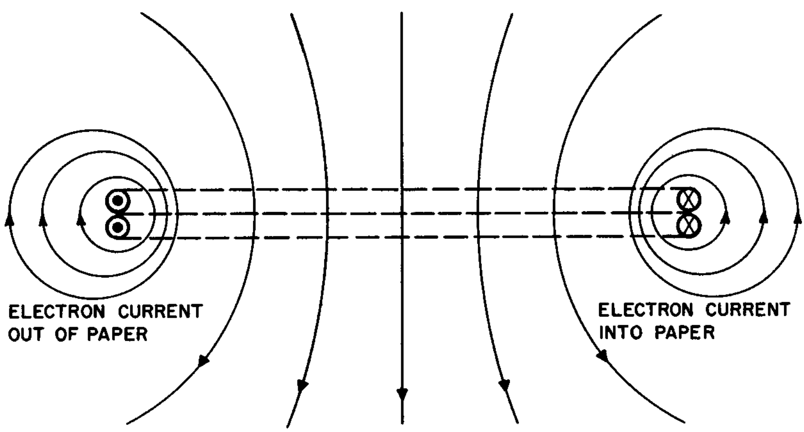

Fig. 15. Field due to current flowing through a single circular turn or coil of which the diameter is much larger than the length.

Single circular turn or coil in which diameter >> length. Viewed in cross section, the gross field of this structure appears as in Fig. 15. Magnetic lines through the center of the coil are intensified in strength as a result of the addition of lines from above and below directed in the same sense (from left to right in this case). Equation (33) gives the flux density of such a coil.

Solenoid. When the coil described in the section above is modified so that its axial dimension becomes large compared to its diameter, the coil is then a solenoid [see equation (34)]. Each wire in the long winding contributes to the central magnetic field, causing the flux density through the core – directed along the axis – to become substantially greater than it would be for a single turn. Although a definite field exists on the outside of the coil, the core field is considerably stronger and of much greater importance in electrical technology. It is the core field which makes electromagnets practical in all forms of alarm and control equipment, motors, generators, and a multitude of other devices. The solenoidal field is schematically diagrammed in Fig. 16. When the left-hand rule for wires is applied to each individual turn, it is seen that the circular lines of force surrounding each turn have the same sense in the core, resulting, therefore, in total flux density equivalent to the sum of the individual flux densities surrounding the turns. A common mnemonic rule for determining the direction of the core field in solenoids, called the *left-hand rule for coils,* is stated as follows: if the fingers of the left hand encircle the coil in the direction of the

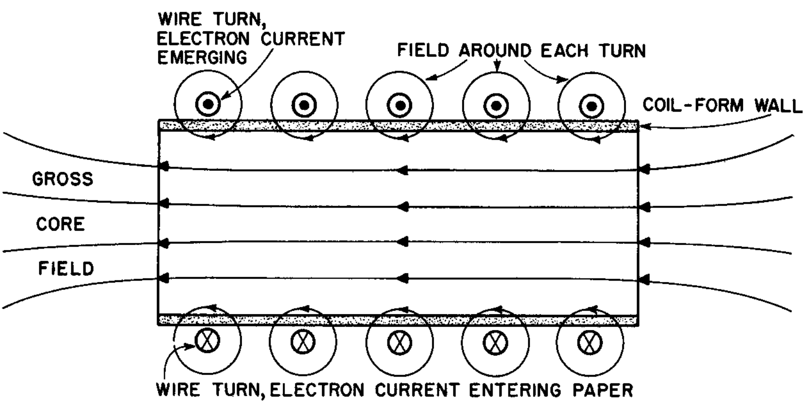

Fig. 16. The field of a solenoid. Note that the individual field contributions of each turn add up to produce a core field of high flux density.

electron current, the thumb held at right angles to the fingers will then point in the direction of the magnetic field. Note that this rule can be applied to any coil, even one having a single turn, but, since it probably finds its widest use in connection with solenoids, it is sometimes referred to as the "solenoid rule."

Toroid. A toroid is a coil wound on a doughnut-shaped form (the torus of the mathematician). It may be considered as the limiting condition attained by a solenoid as its ends are bent around smooth-

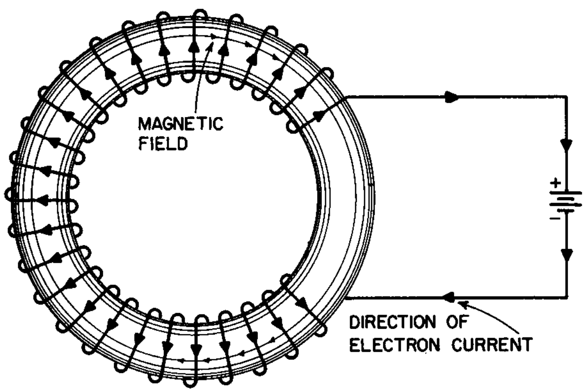

Fig. 17. A toroid is a solenoid deformed into a continuous ring.

ly toward each other to form a continuous ring. Figure 17 shows the general appearance of a toroidal winding.

An important property of a current-carrying toroid is that the magnetic field is almost entirely contained within the structure as the core field. Unlike a solenoid, the toroidal field does not have "ends" or points where lines of force emerge or enter the core. Thus, any attempts to discuss the "polarity" of a toroid field is fruitless. This is just one more demonstration of the shortcomings of the notion of magnetic poles as a tool for analyzing magnetic effects. [The magnitude of a toroidal field may be determined from equation (34).]

15. Review Questions

1. State the two left-hand rules used in electromagnetism. Show by a diagram that the rule for coils can be deduced from the rule for wires. What would you have to alter to apply these rules to circuits described in terms of conventional (plus to minus) current flow?

2. Explain why the proportionality constant K in the Biot law, as given in equation (24), is changed to a new constant (μ_o) in the final statement [equation (30)]
3. Prove by means of the Biot law that the flux density surrounding a current-carrying straight rod is greatest in a plane perpendicular to the rod.
4. Prove that the numerical value of μ_o is 1.26 when K = 10^{-7} webers/amp-meter.
5. A coil having a diameter of 5 cm and an axial length of 0.5 cm is wound with seven turns of wire. What is the flux density through the center of the coil, when the current flowing in the coil is 3.5 amperes?
6. Find the flux density along the axis of a solenoid that fills a cardboard core 0.36 cm in diameter and 12 cm in length. The solenoid contains 14 turns per cm, and the current intensity is 500 ma.
7. Determine the force of attraction between two straight current-carrying conductors for the following conditions: wire #1 carries a current of 3 amperes in a given direction; wire #2 is parallel to wire #1 and carries a current of 8 amperes in the same direction; each wire is 1 km in length, and the separation between them is 1 cm. (Express the answer in newtons/meter.)
8. Draw the field forms for a single circular turn of wire, a solenoid, and a toroid. Assume constant currents flowing through each; select any current direction.
9. Define the mks ampere. Is this the same ampere that is used in solving Ohm's Law problems in which E is in volts and R in ohms?
10. Determine the flux density at a point 8 cm from a very long wire in free space (or air), when the current flowing through the wire is 10 amperes. Describe the direction of the flux at this point.

Chapter 3

INDUCTION AND INDUCTANCE

16. Induced Electromotive Force

The discovery by Faraday and Henry in 1831 (see Chapter 1), that a conductor moving relative to a magnetic field would develop an emf across its ends, was the event that ushered in the present age of electrical and electronics technology. Prior to the discovery of induction, the only significant sources of emf were the modifications of Alexander Volta's chemical generator — the simple cell.

The fundamental qualitative aspects of induction are best illustrated by familiar simple experiments in which a magnet is moved in or out of a solenoid or in which a coil of wire is rotated in a magnetic field. If the coil circuit is closed, a current flows as a result of the induced emf. Relative motion between the conductor and field is necessary; the largest emf is induced when the field is made as strong as possible, the conductor is moved as fast as possible, and the relative direction of motion of the conductor is perpendicular to the lines of force.

From these qualitative observations, it is then possible to state that an emf is induced in a conductor when the magnetic flux linked with the conductor is changed. We shall be concerned with the computation of the magnitude of this induced emf in the next section.

17. Faraday's Law

There are several different paths that may be taken to arrive at a quantitative expression of Faraday's Law for induced emf magnitude. One of the simplest of these involves a consideration of the work and energy relationships in the arrangement shown in Fig. 18.

A straight conductor AB rides on a pair of conductors GE and HF *without friction.* As AB moves, it cuts through a magnetic field of uniform density, emerging from the page as shown by the dots, and, since the rider moves in the plane of the paper, the angle of cutting is 90°. A current flows in the loop formed by AEFB, hence an emf must be induced across AB. We can now derive the magnitude of this emf.

Assume that AB has moved horizontally to the left over the distance Δs to a new position CD and that the motion was uniform. As a result of this motion, a current flowed in the rider due to induction. But a current-carrying wire in a magnetic field is acted

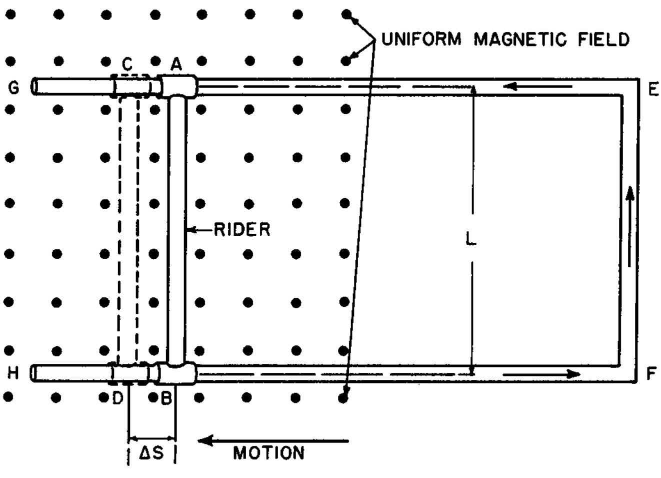

Fig. 18. Arrangement used to investigate quantitatively the factors that govern the magnitude of induced emf.

upon by a force given by equation (15) — $F = B\ i\ L \sin \Theta$ — so that another force, equal and opposite to F as given by equation (15), must have been exerted on AB in order to produce uniform motion.*

The actual value of this force is, of course, B i L, since Θ is 90°, and $\sin \Theta = 1$ established by original statement that the rider moves

* This follows from Newton's Second Law of Motion, since there can be no unbalanced forces acting if the resultant motion is uniform. An unbalanced force produces acceleration.

perpendicularly to the field. The action of force F over the distance Δs results in work being done on the rider, hence we may say:

$$\Delta W = F \, \Delta s \qquad (39)$$

in which ΔW is the work done. Confining our units to the mks system, F would be expressed in newtons, Δs in meters, and ΔW in newton-meters or *joules*. This immediately gives:

$$\Delta W = \Delta s \; B \; i \; L \qquad (40)$$

As the rider moves from AB to CD, it sweeps out an area represented by the space inside ABCD. This area is the product of the distance moved by the length (L) of the rider or $A = \Delta sL$. Substituting in equation (40), we have:

$$\Delta W = B \; i \; A \qquad (41)$$

in which A is the area in meters^2.

It is necessary at this point to refer back to equation (3) in which total flux was defined in terms of flux density and area as $\phi = BA$, where ϕ is total magnetic flux, A is the area through which the flux lines pass, and B is the flux density. Replacing the product $B \times A$ in equation (41) with ϕ enables us to write:

$$\Delta W = i \; \Delta\phi \qquad (42)$$

where $\Delta\phi$ is the total magnetic flux through which the rider passes as it moves from AB to CD.

Suppose, now, that the time required to move the rider from AB to CD is given by Δt. Then the work done in this process is also expressible in terms of current (i) and emf (e) as:

$$\Delta W = e \; i \; \Delta t \qquad (43)$$

from which:

$$e = \frac{\Delta W}{i \; \Delta t} \qquad (44)$$

but since $\Delta W = i \; \Delta\phi$ [equation (42)], then:

$$e = -\frac{\Delta\phi}{\Delta t} \qquad (45)$$

Note the negative sign in equation (45). This is merely a way of denoting that the induced current flowing in loop AEFB gives rise to a magnetic field which *opposes* the initial field, hence the induced emf is considered negative when $\Delta\phi/\Delta t$ is positive and vice versa. (The matter of opposition of initial and induced fields is considered in the next section.)

Equation (45) is a mathematical statement of Faraday's Law. Verbally, it may be read as: the emf induced by moving a conductor relative to a magnetic field is equal to the rate of change of the magnetic flux through which the conductor cuts. If magnetic flux is expressed in webers and time is stated in seconds, then *e is given in volts.**

Thus, when the flux linked with a conductor changes at the rate of 1 weber per second, we would expect to find an induced emf of 1 volt appear across the ends of the conductor.

The significance of Faraday's Law is considerably more profound than it first appears to be. Not only does it apply to a conductor moving through a field, but it also applies to circuits where no mechanical motion is present at all. That is, as long as flux linkages are changed, the induced emf will have a magnitude as predicted by Faraday's Law. Thus, induction coils, inductances, transformers, and generators of all types are based upon this relationship. Furthermore, equation (45) may be extended to encompass coils of many turns, shapes, and sizes by appropriate modifications. For a coil of small axial length as compared with diameter, each turn has induced in it an emf equal to that induced in every other turn if the motion and the field are maintained uniform. Since the turns are in series-aiding, the emf induced across the ends of the coil is merely the sum of the individual emf's or:

$$e = n \frac{\Delta\phi}{\Delta t} \tag{46}$$

where n is the number of turns.

Problem 9. A straight conductor placed near another conductor carrying a current is linked by a magnetic flux of 6×10^{-4} weber. When the current in the second conductor is cut off, the magnetic flux drops to zero in 0.0001 second. What is the average emf induced in the first conductor?

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* This is a good example of the usefulness of the mks system. Faraday's Law, expressed in mks units, does not contain an unwieldy constant, yet provides an answer in a practical unit like the volt. The reader should verify for himself that equation (45) will come out in volts if ϕ is in webers and t in seconds. Hint: refer to the material following equation (2) in Section 4; also, remember that a volt = 1 joule/coulomb and that a joule = 1 newton-meter.

Solution. If the flux falls off as a linear function of time, then the induced emf is the same at every instant. Since this problem does not specify this linear relationship, then application of Faraday's Law will provide the *average* magnitude of the induced emf during the entire fall-off interval. Substitution in equation (46):

$$e = \frac{6 \times 10^{-4} \text{ weber}}{10^{-4} \text{ sec}}$$

$$= 6 \text{ volts}$$

It should be observed that a field that drops off very quickly will induce a larger emf than one which decreases slowly with time, since Δt appears in the denominator. This immediately explains why a fast decay time in the lagging edge of a square wave is capable of producing such high voltages in television flyback transformers. The relationship between fast flux changes and induced voltages is very important in understanding the operation of many industrial devices.

Faraday's Law may also be expressed in another form that is often quite useful. By substituting equation (40) into equation (43), we can obtain:

$$\Delta s \; B \; i \; L = e \; i \; \Delta t \qquad (47)$$

This equation may be solved for e to yield:

$$e = \frac{\Delta s \; B \; i \; L}{\Delta t \; i} = \frac{\Delta s \; B \; L}{\Delta t} \qquad (48)$$

However, the change of distance with respect to time is *velocity*, hence:

$$e = v \; B \; L \qquad (49)$$

where v is velocity in meters per second. This equation gives the magnitude of the voltage induced in a conductor of length L meters, moving with a velocity of v meters per second at *right angles* to a magnetic field of flux density B webers/meter^2. As a last step, it can be seen that if the velocity vector of the moving conductor makes an angle Θ with the direction of the field, then only the perpendicular component with respect to the field causes induction. In this case, the perpendicular component is $v' = v \sin \Theta$, and equation (49) becomes:

$$e = B \; L \; v \sin \Theta \qquad (50)$$

Problem 10. A single-turn rectangular loop pivoted between the pole pieces of a permanent magnet having a uniform field is rotated at such speed that the linear velocity of the field-cutting crossbars is 20 cm/sec. If the length of each of the two crossbars is 5 cm and the flux density of the field is 5×10^{-3} w/m², (a) what voltage is induced across the terminals of the loop when it cuts the field perpendicularly? (b) when it cuts at 45°? (c) when the crossbars are moving parallel to the field?

Solution. (a) The two crossbars are in series-aiding, hence, the emf appearing across the loop terminals is twice the emf induced in either crossbar. Thus:

$$e = 5 \times 10^{-3} \times 5 \times 10^{-2} \times 2.0 \times 10^{-1} \times \sin 90°$$

$$= 50 \times 10^{-6} \text{ volt} = 5.0 \times 10^{-5} \text{ volt}$$

$$\text{total } e = 10 \times 10^{-5} \text{ volt}$$

(b) When $\Theta = 45°$, $\sin \Theta = 0.707$, so that:

$$e = 10.0 \times 10^{-5} \times 0.707 = 7.07 \times 10^{-5} \text{ volt}$$

(c) When $\Theta = 0°$, $\sin \Theta = 0$ and

$$e = 0 \text{ volt}$$

18. Lenz' Law

Consider Fig. 18 once again. When the system is motionless, no forces or currents exist since nothing has been done to bring them into being. If the system is frictionless and completely removed from the magnetic field, a small *momentary* force applied to the rider in a leftward direction would cause it to move off to the left with uniform velocity until it ran off the ends of the loop (Newton's First Law of Motion, law of inertia). Now if the loop and rider are immersed in a uniform magnetic field and the rider is given a momentary push to the left, it will come to rest almost instantly. This *must* occur for the following reason: as the rider begins to cut through the field, an emf is induced across its ends, resulting in a current flowing through loop AEFB. Thus, the kinetic energy imparted to it by the momentary force is converted almost instantly into electrical energy in the loop, thus reducing the kinetic energy to zero. (Actually the kinetic energy approaches zero asymptotically, but this point is unimportant here.) From a practical standpoint, therefore, *a retarding force must be acting on the wire to bring it to rest.* This force arises because the rider is now a current-carrying conductor in a magnetic field; such a conductor is acted upon by a force at right angles to both the field and the current direction. To keep the rider moving with uniform velocity, then, it is necessary to exert a constant force toward the left just equal to the retarding force under these specific conditions.

These considerations show that the *direction of the current* induced in the closed loop must be such as to create a magnetic field around the rider *which opposes the motion of the system.* The small arrows around the loop in Fig. 18 show the direction this current must have to generate the required retarding force. Using the left-

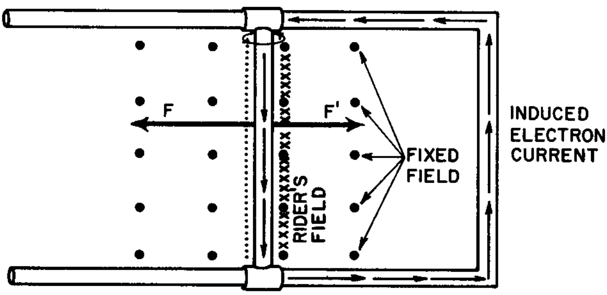


Fig. 19. Interplay of magnetic fields and resulting forces in the induction of a current.

hand rule for wires, it can be seen that this electron current direction produces a magnetic field emerging from the paper at the left of the rider and going into the paper at the right. The high flux density resulting from the addition of the fixed field and the rider's field at the left of the rider generates a force perpendicular to the rider and directed toward the right. Referring to Fig. 19, the large black dots represent the fixed magnetic field in which the rider-loop system is immersed; the smaller dots and crosses show the direction of the circular lines of force around the rider as it is pushed to the left by applied force F, and F′ represents the retarding force generated by the high flux density to the left of the moving rider.

This action is duplicated exactly in any conducting system that is forced to move in a magnetic field; it appears in identical form when moving magnetic lines due to a varying current are made to cut through nearby conductors (as in a transformer). The concept of opposition is generally stated as follows: whenever an emf is induced in a conductor due to the relative motion of the conductor across a magnetic field, the direction of the emf is such as to produce a current in a closed circuit whose resultant magnetic field opposes the relative motion of the system. This statement is known as Lenz' Law and finds constant application in all phases of electrodynamics.

Several examples of the method of application of Lenz' Law to practical situations are given in Fig. 20.

(A) A straight conductor (small circle at center) is swept downward through a magnetic field directed toward the right. Since opposition must occur, the flux density must build up under the wire

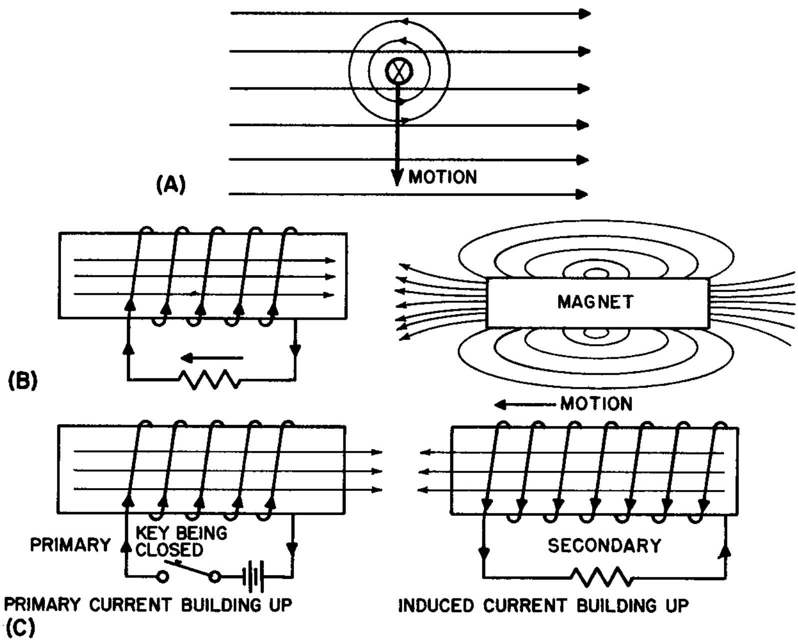


Fig. 20. Use of Lenz' Law in determining direction of induced current in circuits involving conductors and coils.

as it moves downward in order to generate an upward retarding force. This requires that the induced current enter the paper to produce circular lines in a counterclockwise direction as shown. In this way, the conductor's field reinforces the fixed field below the wire.

(B) A permanent magnet with field direction shown is plunged into a hollow solenoid at the left. The induced current in the solenoid must then be in that direction which will give rise to a new field to the right so that field opposition may occur. The left-hand rule for coils is then applied to predict the current direction shown.

(C) Upon closing the primary circuit at the left, the electron current builds up in the direction illustrated; as the current grows, its accompanying field also grows outward, moving toward the right. The current in the secondary must then flow in such a direction as to generate a new magnetic field to the left in order to oppose the growing primary field. Current direction in the secondary is obtained by applying the left-hand rule for coils.

19. The Generator Principle

In a sense, the generator principle was introduced in a basic form in Problem 10. Modern dynamos are generally modifications of a rotating rectangular loop in a magnetic field. Instead of a single turn, the rotating armature consists of many turns; the armature coil is terminated in either slip rings (a-c generators) or commutator segments (d-c generators). These practical considerations will be discussed further in a later chapter. For the present, we are concerned with deriving two important expressions around which all generator designs are concentrated. Reference should now be made to Fig. 21. Loop ABCD [Fig. 21 (A)] has length and height dimensions a and b, respectively; it is placed in a field at right angles to the paper (entrant field) and can rotate about axis PQ. In Fig.

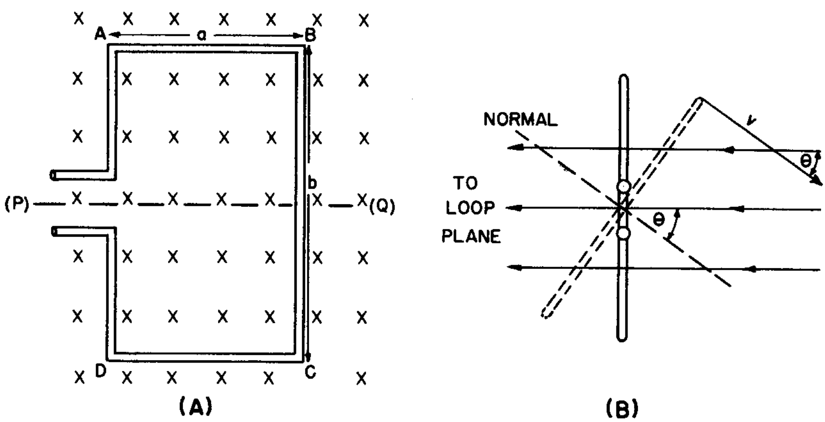


Fig. 21. (A) Rectangular loop in a magnetic field. PQ is the axis of rotation. (B) End view of loop with rotational axis turned so that end P is toward reader.

21 (B), the loop has been turned so that end P of the axis is toward the reader. A normal drawn through the center of the plane of the loop makes an angle Θ with the lines of force of the fixed field. Θ is thus selected so that the crossbars AB and DC will be cutting through the field perpendicularly when Θ is 90° and 270°. The vertical legs AD and BC are therefore parallel to the field *at all times,* so that the induced emf across these members is always zero. Now, we refer to equation (50), $e = B\ L\ v \sin \Theta$ (where $L = a$ in this case). In this form, the equation is not quite as useful as it might be, because it expresses the induced emf for only one of the crossbars in terms of the linear instantaneous velocity (v) of the crossbar. What we are looking for is an expression that will be applicable to a rotating coil of n turns moving with an angular velocity ω.

This is not difficult to obtain. The linear instantaneous velocity of a particle in uniform circular rotation is defined:

$$v = \omega\ r \tag{51}$$

in which ω is the angular velocity in radians per second and r is the radius of rotation. In the case of the loop, the radius of rotation is half of side b, so that we may write:

$$v = \omega \frac{b}{2} \tag{52}$$

Substituting equation (52) into equation (50) gives:

$$e = \frac{1}{2}\ B\ a\ b\ \omega \sin \Theta \tag{53}$$

which is the emf induced in *each* of the two crossbars. As these are in series-aiding, the total induced emf is twice that given in equation (53). In addition, the product $a \times b$ is the *area* (A) of the coil. So, for a single-turn loop, the instantaneous emf is

$$e = B\ A\ \omega \sin \Theta \tag{54}$$

For a coil of n turns having the same orientation of axis, the instantaneous induced emf is, as before, given by the expression:

$$e = n\ B\ A\ \omega \sin \Theta \tag{55}$$

Maximum emf is induced when the crossbars AB and CD are moving perpendicularly to the magnetic field, that is, when $\sin \Theta = 1$. Hence,

$$E = n\ B\ A\ \omega \tag{56}$$

where E = maximum induced emf. Also:

$$e = E \sin \Theta \tag{57}$$

Equation (57) is one of the most significant expressions in electrodynamics. It states that the instantaneous induced emf in any coil rotating about an axis perpendicular to a uniform magnetic field is always the product of the maximum emf and the sine of the angle at that particular moment.

In linear notation, distance = velocity × time. Similarly, in angular notation, the angle through which a rotating system turns (Θ) is distance of motion. By analogy, $\Theta = \omega\ t$, which is the same as saying that angular distance = angular velocity × time. Since there are $2\ \pi$ radians in one rotation, a particle which rotates once per second must have an angular velocity $\omega = 2\ \pi$ radians per second; or if it rotates f times per second, then $\omega = 2\ \pi\ f$ radians per second. The number of rotations per second is *frequency*, a very important term in electrodynamics. Finally, the equation $\Theta = \omega\ t$ may be rewritten as $\Theta = 2\ \pi\ f\ t$ and substituted in equation (57) to give a very useful expression:

$$e = E \sin 2\ \pi\ f\ t \tag{58}$$

The instantaneous emf developed by a rotating coil is thus seen to be a function of time. When e is plotted against t, the familiar sine curve is generated as shown in Fig. 22.

20. Self-Inductance

Our discussion of induced emf's has thus far been confined to considerations involving no fewer than two circuit elements (that is, emf's induced by a conductor moving through a magnetic field, a magnet moving into a coil, etc.). It has been shown that an induced emf appears in any circuit when the magnetic flux linking the circuit grows or decays, i.e., changes in any way. Thus, an increasing or decreasing current flowing in a conductor produces a changing magnetic field which, in turn, varies the flux linkage in the conductor itself; hence, a varying current can induce an emf within the conductor that carries it. This phenomenon is known as self-inductance.

An emf resulting from self-inductance is still bound by the restrictions of Lenz' Law: its polarity must be such that the current it pro-

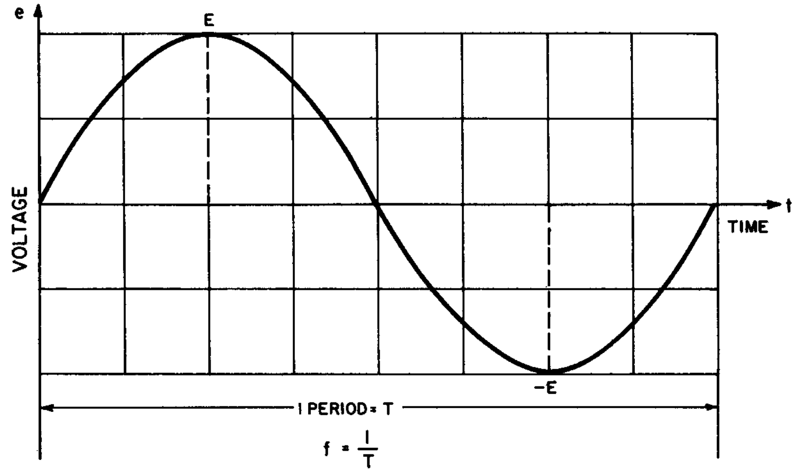


Fig. 22. The graph of equation (58) generates a sine curve in which the amplitude is E, the frequency is f, and the period is 1/f.

duces must oppose the motion in the system. This is a point to bear in mind. In the simple solenoid circuit of Fig. 23, each time the key is depressed or released, flux lines build up or decay around each turn of the coil. These link with adjacent turns so that an emf must develop which opposes the *change* of current. When the key is closed, the circuit current grows; hence, the induced emf has a direction such that it attempts to prevent the growth of the current. Conversely, as the circuit is broken, the original current attempts to drop off in magnitude and the induced emf tends to maintain the flow.

If the coil in Fig. 23 had a different diameter, number of turns, turn spacing, core material, or overall shape, the flux linkages would be different in number and extent. Many structural factors contribute to the number of flux lines that link any circuit. We are not concerned, however, with specific dimensions or characters at this point but are interested in general relationships. For example, irrespective of the structure or form of the circuit, the flux density B at any point in the circuit is directly proportional to the current responsible for its existence. This follows directly from equation (24) for the Biot law, which, in this form, does not relate to any specific case. This proportionality, stated in an equation, is:

$$B = k\,i \qquad (59)$$

and since the total flux (ϕ) at any point is proportional to B, we may write:

$$\phi = k' \, i \qquad (60)$$

In these statements, k and k′ are proportionality factors which remain constant for any given circuit geometry. For a coil of n turns, we rewrite the mathematical statement of Faraday's law [equation (46)]:

$$e = -n \frac{\Delta\phi}{\Delta t}$$

Note the inclusion of the minus sign. This is self-induction in which the direction of the induced emf *opposes* or is opposite in direction to the flux change that produces it. But if $\phi = k' \, i$, then:

$$\Delta\phi = k' \, \Delta i \qquad (61)$$

so that the equation (46) becomes:

$$e = -nk' \frac{\Delta i}{\Delta t} \qquad (62)$$

since both n and k′ are constants *for any given system*, then the product is a constant symbolized by a single letter L. Hence, we have:

$$e = -L \frac{\Delta i}{\Delta t} \qquad (63)$$

if i is in amperes and t in seconds, e is in volts, and L has the dimensions of volt-seconds per ampere. One volt-sec/amp is called

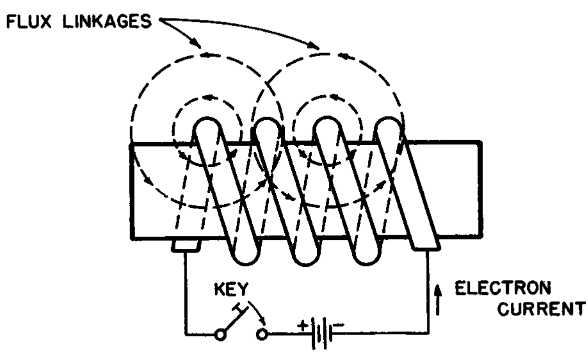


Fig. 23. Flux linkages in a solenoid. When the key is opened or closed, the linkages vary in accordance, inducing an emf that opposes the *change* in current.

one *henry*, and the quantity L is termed inductance. The *henry* is therefore defined as that amount of inductance which will develop an induced emf of one volt across its terminals, when the current through it is changing at the rate of one ampere per second.

Problem 11. A filter choke has an inductance of 10 henries and carries a current of 500 ma. When the switch is opened, the current drops to zero in 0.01 second. What is the magnitude of the induced emf?

Solution. Using equation (63):

$$e = 10 \times \frac{0.5}{0.01}$$

$$= 500 \text{ volts}$$

Note the extremely high voltage developed in this case. Although the figures used are admittedly somewhat unusual in common practice, they are quite possible. Hence, even when an unknown circuit is *opened* by means of an exposed switch, too much caution cannot be exercised.

21. Energy in an Inductor

A circuit element in a form which provides relatively large amounts of inductance is called an *inductor* (even a straight length of wire has some inductance, but it is not an inductor because its

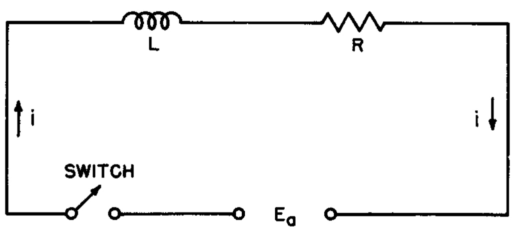


Fig. 24. All inductors contain resistance which may be shown as a series element in the circuit.

form has not been intentionally adjusted to provide a larger inductance). Flat coils and solenoids may be called inductors, regardless of their core material, although, as we shall see, the magnitude of an inductance is profoundly affected by the type of core used.

It is physically impossible to wind a resistance-free inductor, since all conducting material has some specific resistance. Hence any circuit containing an inductor must also contain a certain minimum resistance; this resistance, regardless of its site, may be shown as a series element, as in Fig. 24.

When the switch in Fig. 24 is open, no current flows in the circuit. Upon closure, the rate of growth of the current during the first instant of time is very large. This gives rise to a large induced emf in the inductor which opposes the applied emf from the source (E_a). At this instant, the *back emf* or *counter emf* of self-induction is, by equation (63), $e = L\ (\Delta i/\Delta t)$, so that the net emf causing current to flow is:

$$e_n = e_a - L \frac{\Delta i}{\Delta t} \tag{64}$$

Since the second term in the equation is large, the net emf (e_n) is small.

As the current rises, its rate of change must diminish, because it begins to approach its steady-state value as determined by R. At the same time, due to the decreased rate of change of i, the back emf also decreases, making the net emf grow larger. Steady-state current is reached when $I = E_a/R$. In theory, the steady state cannot be reached in finite time since any change of current, no matter how small, causes a back emf which prevents the net emf from ever attaining the value of E_a. Practically, however, steady-state current is assumed to flow after a time 5 (L/R) has passed. (The significance of the ratio L/R will be discussed very shortly.)

As a result of the *time-delay* introduced by L in an L/R circuit, the growth of the current through an inductor in series with a resistor follows an exponential curve obtained by plotting the equation:

$$i = \frac{E}{R} \left[1 - \epsilon^{(-Rt/L)} \right] \tag{65}$$

where ϵ is the base of Naperian logarithms.

When the switch is opened (after steady-state current has been reached), the current again requires a finite time to drop to zero, this time following the relation:

$$i = \frac{E}{R}\ \epsilon^{(-Rt/L)} \tag{66}$$

Figure 25 illustrates the so-called "Universal Time Constant" curves. It should be observed from equation (66) that if t is made equal to L/R, then $i = I/\epsilon$ or about 37% of the maximum value of I. This means that one *time constant period* (that is, a period of time equal to L/R) after the switch is opened, the current will fall to about 37% of its maximum value; after two TC intervals, the current is down to approximately 13% of maximum; and after five TC intervals, it is down to a point well below 0.1% of maximum. Thus, the significance of the ratio L/R is clear: during one period of time equal to L/R for any given inductor and resistance, the current will drop to 37% of maximum when the switch is opened; similarly, when the switch is closed, the current will rise to within 37%

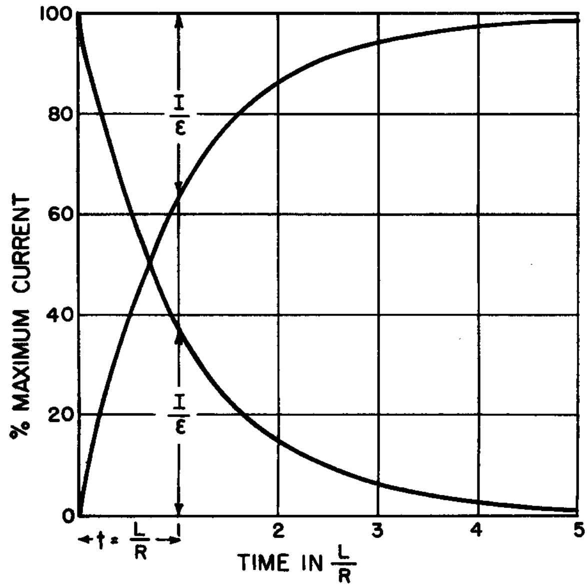


Fig. 25. Universal Time Constant curves for the growth and decay of current in a series circuit containing an inductor and resistance.

of maximum during one L/R period. (The reader is urged to verify the implication that appears throughout this section, that is, that the ratio L/R has the dimensions of time.)

The discussion just completed is very important for a thorough understanding of the behavior of inductors. It also helps to clarify the derivation of the equation that expresses the *energy stored in an inductor.*

In the circuit of Fig. 24, while the current is rising toward its maximum value (E_a/R), there is a counter emf that changes from the full value of the applied voltage to zero. As the time constant curves indicate, this variation is not linear, since at any instant the counter emf is given by L $(\Delta i/\Delta t)$ and the current by i. The power input to the inductor is, therefore, variable and may be expressed by the equation:

$$p = i\ L\ \left(\frac{\Delta i}{\Delta t}\right) \tag{67}$$

in which p is the instantaneous magnitude of the power input. With the attainment of steady-state current, current variations cease and p becomes zero. All this means is that the power supplied to the inductor to build up the magnetic field is now stored as potential energy in the field itself. Thus, when the field collapses upon the removal of the source emf, the energy must be returned to the system by induction. If equation (67) is multiplied through by Δt, it takes the form:

$$p\ \Delta t = i\ L\ \Delta i \tag{68}$$

Since power × time = energy (or work), equation (68) gives the instantaneous work that must be done on the inductor L for a given current i and a given change in current Δi. To find the total work necessary to make the current rise from zero, through the intermediate values, to its maximum value I, recourse must be had to the integral calculus. This operation yields a final equation for the energy stored in the magnetic field of an inductor as follows:

$$W = \frac{1}{2}\ L\ I^2 \tag{69}$$

in which L is the inductance in henries, I is the current in amperes, and W is the energy in joules. (If the reader tires of our constant urging to perform dimensional checks on these equations, let him remember that a really clear grasp of the principles is much easier to realize when the dimensions are fitted into one another properly *by the reader* as an exercise.)

Problem 12. A circuit consists of an inductor and resistance in series. The inductance is 2 henries and the resistance is 2 ohms. If, after a time interval of 1 second after switch closure, the current is 1.48 amperes, how much energy is stored in the inductor when the current reaches steady-state maximum?

Solution. The time constant of this system is:

$$TC = \frac{L}{R} = \frac{2}{2} = 1 \text{ sec}$$

After an interval of 1 second (or one TC period), the current is 37% *below* maximum or 63% of its maximum value. Hence, for a current of 1.48 amperes after one TC period, the maximum value will go to 1.48/0.63 or 2.35 amperes. Hence, I = 2.35 amp. The energy stored in the inductor is found from equation (69):

$$W = \frac{1}{2} \times 2 \times (2.35)^2$$
$$= 5.5 \text{ joules}$$

22. Review Questions

1. Give the general mathematical statement for Faraday's Law which provides an expression for determining instantaneous value of induced emf in terms of magnetic flux rate of change. Explain the significance of the negative sign.
2. A straight conductor 10 cm long is moved at a velocity of 50 cm/sec through a magnetic field of flux density equal to 8×10^{-3} webers/m^2. The long axis of the conductor cuts through the magnetic field at an angle of 30°. If the resistance of the conductor is 0.001 ohm, find the current that flows in it while it is moving at uniform speed through the field.
3. Explain, in terms other than those used in this book, just what the significance of Lenz' Law is. In what way does this law make use of energy conservation?
4. In Fig. 20 (C), the conditions shown exist when the switch has just been closed. Redraw this diagram and show the magnetic field direction in the secondary winding just after the primary switch is opened. Explain your conclusions.
5. The frequency of a sinusoidal voltage wave is 500 cycles per second. At what time during the first cycle (consider the cycle to start at t = 0) does the instantaneous voltage reach its maximum negative value? Make use of equation (58) in this problem.
6. A sinusoidal current flows through an inductance. At what part (s) of each cycle is the back emf greatest? least? Give your answer in radians.
7. Explain the meaning of time constant interval.
8. What is the time constant of a 400-millihenry choke coil whose resistance is 65 ohms?
9. Explain the statement ". . . the current flowing in a coil having a resistance of 10 ohms requires an infinite time to grow to exactly 1.0 ampere under an applied voltage of 10 volts."
10. It is desired that a certain coil store up 8.6 joules of potential energy while a steady current of 4.0 amperes flows through it. What is the smallest inductance the coil can have?

Chapter 4

FERROMAGNETIC EFFECTS

23. Magnetic Behavior of Matter

It is generally believed that all substances have magnetic properties, although some exhibit magnetic effects to a much more marked degree than others. A piece of iron or steel, when placed at the margins of a magnetic field, will attempt to move into the strongest part of the field; often, the force that prompts such movement is a large one. This action gives rise to the term *ferromagnetic,* or the property of tending to move into the strongest portion of a magnetic field. In addition to iron and iron alloys, *cobalt* and *nickel* display the property of ferromagnetism.

Paramagnetic substances are also acted upon by forces toward the stronger field strengths, but differ from ferromagnetic materials in that the effect is much less pronounced. Gadolinum and liquid oxygen are best classified in this way. It is interesting to note at this point that ferromagnetic materials become paramagnetic in behavior at high temperatures. This effect will be discussed under theories of magnetism.

The last classification, that of *diamagnetic* substances, includes elements such as copper, silver, and bismuth. Diamagnetic materials are impelled *away from* the regions of high flux density in magnetic fields. In no instance, however, has a diamagnetic substance been discovered which displays quantitative effects that approach the ferromagnetic materials in degree. From a practical point of view, ferromagnetic elements and alloys are of the greatest importance and have been investigated theoretically and experimentally to a greater extent than the other groups.

24. The Nature of Magnetization

It is common knowledge that the magnetism of a solenoid (or any type of coil) can be tremendously strengthened by inserting an iron core inside the windings while a current flows. That is, the presence of a ferromagnetic material causes a large increase in flux density (B) over and above the flux density that would exist as a result of the flow of the same current in the same coil in a vacuum.

Mathematical and experimental evidence supports the view that this increase in flux density in a ferromagnetic substance is due to the magnetic effects of the electrons that go into the structure of the very atoms of the ferrous material. An electron in motion is a moving charge and hence constitutes an *electronic current* within the atom. As might be anticipated, such electronic currents produce magnetic fields of their own which may or may not supplement the field produced by the magnetizing current. (In diamagnetic elements, for example, the electronic currents appear to be such as to weaken the field of the magnetizing current. That is, a solenoid with an air core exhibits a larger flux density than the same solenoid wound on bismuth or silver.) In addition to the revolution of the atomic electrons around the nucleus, they also spin on their own axes much like the planets of the solar system. In the iron atom, however, a situation may be shown to exist which is *unique* among the elements. Four of the electrons in this atom have a direction of spin that is not compensated for by other electrons with opposite spin direction. As a result, the iron atom contains unbalanced electronic currents that give rise to tiny magnetic fields in themselves, completely independently of external currents or fields. Even a small volume of iron contains many millions of atoms, however, so that the resultant magnetic field for this volume is zero, due to the random orientation of its atoms. A magnetic field applied from the outside can, on the other hand, reorient the atoms so that all the electron spins are in the same direction, making the net flux density very much larger than it was initially.

The term *magnetic dipole* is now applied to any atom in which there is an unbalanced electronic current, as in iron or other ferromagnetic materials. When a group of magnetic dipoles are located so that they come under the influence of a magnetic field, they tend to turn so that the plane of each current is at right angles to the external field and the direction of charge motion is such as to produce flux lines in the same direction as those of the external field.

This explains the excess flux density observed when ferromagnetic materials are placed in a magnetic field. It might also be mentioned that this theory fits in very well with the behavior of ferromagnetic substances in a magnetic field while they are heated to increasingly higher temperatures: thermal agitation, by its very nature, reacts against any aligning process; this makes it more difficult to bring the dipoles around to axial coincidence, hence, the observed ferromagnetic effects become much less intense for a given field when the temperature of the core material is high.

25. The Intensity of the Magnetic Field

The reader is reminded that, in Section 5, the magnetic field was first introduced as consisting of *lines of magnetic induction* or "more simply, lines of force." Now that we have reached the study of ferromagnetism, it is convenient to distinguish between these phrases more rigorously. A line of magnetic induction will retain the meaning it has had up until now: a descriptive term applied to a constituent of a magnetic field produced by a current flowing through a conductor, the conductor being in air or a vacuum. We shall reserve the phrase *line of force*, however, for the component of a magnetic field in a paramagnetic, ferromagnetic, or diamagnetic substance.

For instance, the flux density produced by a coil in a vacuum is given as B webers/meter^2. This field then consists of B lines of magnetic induction, as described in Section 5. An iron core is now inserted in the coil; this results in a substantial increase in the strength of the magnetic field. The intensity of the field now present in the core is certainly a function of *both* the original flux density produced by the coil current and some factor that depends upon the nature of the core insofar as its electronic currents are concerned. Expressing this in the form of a simple equation, we have:

$$B = \mu H \tag{70}$$

in which B is flux density in w/m^2, H is *field intensity* (units to be derived later), and μ is some constant that depends upon the nature of the core. The common name for the constant is *permeability*. Putting this equation into words, we might say that, if a given current-carrying coil in a vacuum had produced B lines of magnetic induction, then, when the coil is given a core, the field intensity it produces (H) is B/μ *lines of force,* where μ is the permeability of the core. If the permeability of a vacuum is now made μ_o [the same μ_o as appeared in equation (28) for the first time; that is 1.257×10^{-6} weber per ampere-meter], then H becomes a new magnetic vector defined by a modification of equation (70) thus:

$$H = \frac{B}{\mu_o} \tag{71}$$

From equation (71), *unit field intensity* is that field intensity which produces a flux density 1.257×10^{-6} weber per meter^2 in a vacuum.

It is much more difficult to define H inside *matter* having a permeability different from μ_o. Using a relatively simple case, however, it may be shown that the field intensity inside a toroid* (Fig. 17) depends only upon the number of turns, the current, and the length of the winding and not upon the magnetic medium. From equation (34), the flux density in a toroid (same as solenoid) is:

$$B = \frac{\mu_o\, n\, i}{L}$$

where B is flux density in w/m^2, μ_o is the permeability of a vacuum, n is the number of turns of wire, i is the current, and L the total length of the winding. However, equation (71) tells us that $\mu_o H$ may be substituted for B. If we do this, the μ_o factors on each side of the equation cancel, leaving:

$$H = \frac{n\, i}{L} \tag{72}$$

This provides us with a unit for field intensity; for if we substitute mks quantities in equation (72), we obtain:

$$H = \frac{\text{number of turns} \times \text{amperes}}{\text{meters}} \tag{73}$$

so that the units for H are *ampere-turns per meter.* Thus, a field intensity of one ampere-turn per meter in a vacuum produces a flux density of 1.257×10^{-6} w/m^2.

The *relative permeability* of any substance (unfortunately, this is often contracted to just *permeability* by engineering textbooks) is

* A toroid rather than a straight solenoid is used here since the field of a toroid is confined to the material of its core, making a rigorous treatment substantially easier.

defined as the ratio of the permeability of the substance to the permeability of a vacuum or:

$$\mu_r = \frac{\mu}{\mu_o} \tag{74}$$

in which μ_r is the relative permeability. Using the same system of measure in both numerator and denominator yields a dimensionless number for μ_r. The relative permeability of a vacuum is, of course, unity. Relative permeabilities of diamagnetic substances are less than unity, and those of paramagnetic and ferromagnetic substances are greater than unity. The μ_r of special alloys of iron, for instance, may run to figures in excess of tens of thousands. Relative permeability is not a constant for all conditions, even for the same substance. It is affected by the degree of magnetization of the material and depends, to a certain extent, upon what magnetic processes the material has undergone in the past. The *maximum* relative permeabilities of a few common substances are:

Pure commercial annealed iron	6000 to 8000
Silicon steel for transformers	10,000
Pure cobalt	170
Permalloy (iron 21.5%, nickel 78.5%)	83,000

Problem 13. A ring solenoid (or toroid) of iron has a mean circumference of 20 cm and a cross-sectional area of 0.5 cm^2. It contains a uniform winding of 200 turns of wire. When the current in the winding is 0.05 ampere, the total flux in the toroid is 3×10^{-6} weber. Find B, H, μ, and μ_r for the iron core.

Solution. To find B, use equation (3):

$$\phi = B\,A$$

so

$$B = \frac{\phi}{A} = \frac{3 \times 10^{-6} \text{ weber}}{0.5 \times 10^{-4} \text{ meter}^2}$$

$$= 6.0 \times 10^{-2} \text{ w/m}^2$$

To find H, use equation (72):

$$H = \frac{n\,i}{L}$$

$$= \frac{200 \times 0.05}{0.20}$$

$$= 50 \text{ amp-turns/m}$$

To find μ, use equation (70):

$$\mu = \frac{B}{H} = \frac{6.0 \times 10^{-2} \text{ w/m}^2}{50 \text{ amp-turns/m}}$$

$$= 1.2 \times 10^{-8} \text{ w/amp-m}$$

To find μ_r, use equation (74):

$$\mu_r = \frac{\mu}{\mu_o}$$

$$= \frac{1.2 \times 10^{-8}}{12.57 \times 10^{-7}}$$

$$= 0.0945 \times 10^4 = 945 \text{ (dimensionless)}$$

26. Behavior of Ferromagnetic Substances

If the interior of a solenoid hollow core is filled with a bundle of soft iron wires and a current is passed through the coil, the magnetic strength of the solenoid is increased greatly as compared with the air-core condition. It is customary to consider that the magnetic intensity H inside the solenoid is the cause of a numerically greater flux density B where $B = \mu H$. Even when the core material is not changed, μ is *not* a true constant. That is, the flux density is not a linear function of the magnetic intensity. To make matters worse, the permeability of a given material appears to depend in large measure upon what magnetic events have already occurred within the core material prior to the measurements under discussion. This is evident when we consider that a permanent magnet (previously nothing but a bar of steel) retains a substantial magnetic field even after the magnetizing current has ceased flowing.

Thus, the relationship between H and B is quite complex. In practice, the manner in which one varies with the other is displayed in either tabular or graphical form. A typical magnetization curve is given in Fig. 26.

The way in which flux density varies with field intensity is shown by the solid-line curve; the nonlinearity (and hence the inconstancy of μ) is quite evident, of course. The permeability of the core may be found at any point by solving the ratio of B/H at that point. By doing this for several points, it is then possible to draw in the curve that demonstrates the relationship between μ and H (broken-line curve).

Several important conclusions may be drawn directly from the curves of Fig. 26. (a) When the field intensity is small, the value of B is determined almost entirely by the electronic currents within the iron atoms, because, in this region, very tiny increases in ampere-turns per meter result in large increases in flux density.

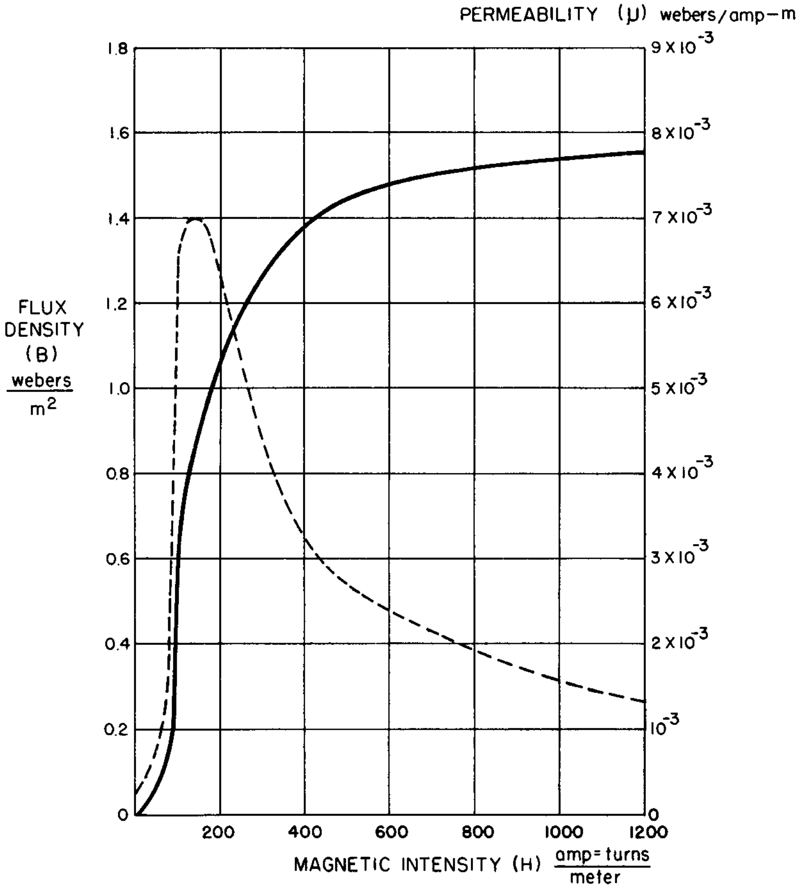


Fig. 26. Magnetization curve for a typical specimen of soft annealed iron of high purity. The broken-line curve is the graphical relation between permeability and magnetic field intensity.

(b) At about 700 ampere-turns per meter, the curve begins to flatten out, requiring large changes in H to cause virtually linear variations in B. This shows that the core is now *saturated* (from 700 ampere-turns per meter up) and that any further increase in B must be produced by increasing the magnetizing current from the external source. (c) The permeability of the core reaches its peak value long before saturation is approached. Permeability drops sharply as the core goes into the condition of saturation.

27. The Curie-Weiss Law

As a ferromagnetic material undergoes a rise in temperature, its magnetic properties begin to diminish in extent. The change with rising temperature is gradual, but at a certain temperature, known as the *Curie point,* there is a transition from ferromagnetic properties to paramagnetic properties. In 1895, Pierre Curie proposed that the intensity of the magnetic field produced under fixed conditions of current and core material above this point varied inversely as the absolute temperature of the system. Later, in 1907, Pierre Weiss showed that this was not strictly true and that the intensity was a function of the *excess* absolute temperature above this point. This statement is now known as the Curie-Weiss Law. For soft iron, the Curie point is in the order of 800°C (1073° K). Although the Curie-Weiss Law is useful only in highly specialized applications, it has been of considerable importance in revealing the relationships between thermal molecular energy and magnetism; in other words, it has been helpful in establishing a firmer foundation for modern magnetic theory.

28. Hysteresis

Suppose that we had started with an unmagnetized piece of annealed iron in obtaining the curves in Fig. 26. The magnetizing current responsible for the gradual increase of H (and consequently the increase of B) would, therefore, have begun at zero and would have been increased until the flux density had become approximately 1.52 w/m^2. If the magnetizing current is now decreased at exactly the same rate as it was increased, it might be anticipated that the flux density B would diminish to zero at the same instant that the current dropped to zero. This does not occur. Starting with maximum flux density, we shall want to trace the variation of the magnetic field as the magnetizing current is changed. The most common way to do this is to plot the changes of B *vs* H for a given material. When this is done, the curve of Fig. 27 is obtained; for various ferromagnetic materials, the general form of this curve remains the same, but the area enclosed within the loop differs widely.

In Fig. 27, we assume a starting field intensity H of 1000 amp-turns/meter and a flux density of 1.5 w/m^2 (point a). The current is then gradually reduced to zero, bringing H to zero as well; but at

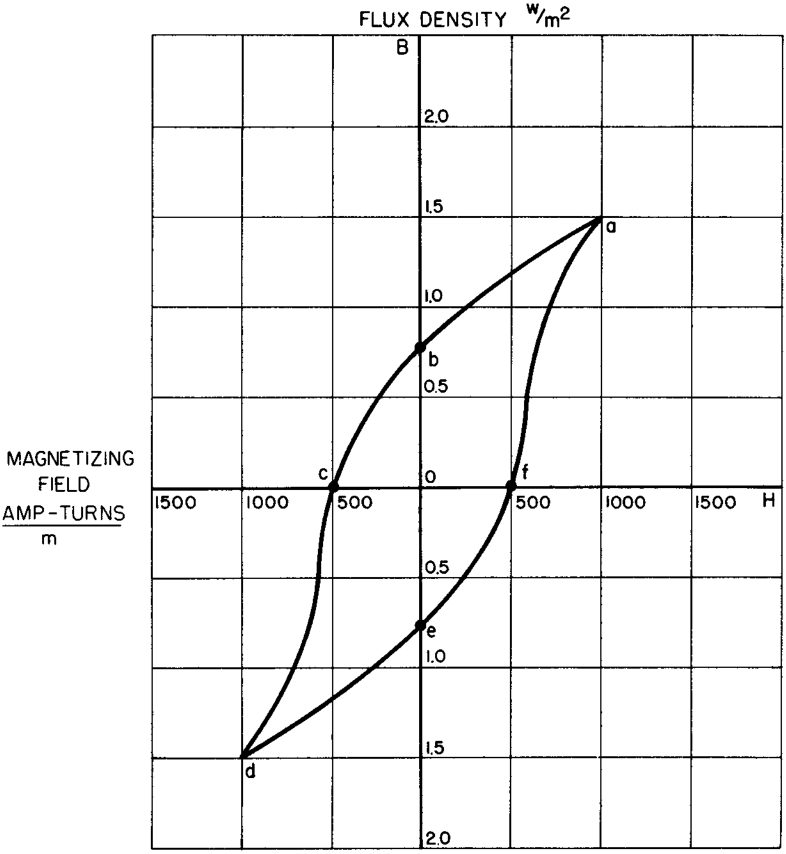


Fig. 27. Plot of B *vs* H for a specific sample of ferromagnetic material. Different materials yield the same general curve form but different areas inside the loop.

this point, the ferromagnetic core retains a flux density (point b) of approximately 0.56 w/m². Now the current is reversed and slowly increased in the new direction. A point is reached (c) where the flux density decreases to zero, requiring an H value of 500 amp-turns/meter to accomplish this. Further increases in current (in the second direction) bring the flux density to point d, where B is equal in intensity but opposite in direction as compared with its initial value. Retracing along path d-e-f-a, it is seen that, as the current is reduced to zero once again, a flux density of 0.56 w/m² is again retained, decreasing to zero only when the magnetizing current has produced a field intensity of 500 amp-turns/meter (point f). As H is further increased, B returns to its initial value at point a.

Several new terms, extensively used in electromagnetic technology, may be obtained directly from this diagram:

Hysteresis is the name given to the *phenomenon* wherein the flux density *lags* behind the magnetizing field. The closed curve a-b-c-d-e-f is called the *hysteresis loop.*

Retentivity (or remanence) represents the flux density retained by the specimen when the value of H has dropped to zero. Retentivity is measured by the length of ordinate o-b (or o-e).

Coercive force (*or coercivity*) is a measure of the magnetic field intensity, in a reverse direction, required to reduce the flux density to zero after magnetization (abscissa o-c or o-f).

Work area refers to the area of the closed hysteresis loop. This area may be shown to be proportional to the energy that must be expended to reverse the direction of the magnetic field in a given sample of ferromagnetic material. In a-c circuits particularly, where rapid periodic reversals must occur constantly, the work done on the core appears in the form of heat. Thus, core materials such as high-grade silicon steels or special alloys are selected because the areas inside their hysteresis loop are relatively small. Such alloys, therefore, have low retentivity and small coercive forces. At the other extreme, powerful permanent magnets such as those found in P-M speakers (e.g., various alnico formulas) must display both high retentivity and high coercive forces.

The progress that has been made in this direction in recent years is illustrated by these figures:

	Retentivity	*Coercivity*
Carbon steel formerly used for permanent magnets	0.95 w/m^2	$3.6 \times 10^3 \ \frac{\text{amp-turns}}{\text{meter}}$
Alnico 5, the modern material for permanent magnets	1.25 w/m^2	$44.2 \times 10^3 \ \frac{\text{amp-turns}}{\text{meter}}$

Problem 14. What is the permeability of the specimen of iron used as a core in the solenoid described by Fig. 27 when the magnetizing field is zero? when it is 500 amp-turns/meter?

Solution. Permeability is defined by equation (70). When the magnetizing field is zero, then:

$$\mu = \frac{B}{H} = \frac{0.56}{0} = \text{indeterminate}$$

That is, permeability as a concept *cannot be applied to permanent magnets,* since it is meaningless in this connection.

When the magnetizing field is 500 amp-turns/meter, then

$$\mu = \frac{0}{500} = 0$$

For this particular core material, permeability is zero when H = coercivity of the substance.

29. Molecular Theory of Magnetism

Because a permanently magnetized needle can be broken repeatedly into smaller and smaller fragments, each of which still displays the same magnetic characteristics of its parent, Ewing proposed that the same effect might continue to occur until one reached the molecules of the iron. In short, Ewing's model of a permanent magnet consists of a bar of steel in which all the molecules are aligned in the same direction, each molecule being in itself a tiny permanent magnet. Ewing also supposed that each of these natural molecular magnets has the same strength, even in an unmagnetized bar, but that their axial orientations are random, in this case, producing a statistical cancellation of effects.

To magnetize a bar of ferromagnetic material, it is necessary to apply forces which align the little natural magnets. This may be accomplished by flux lines produced by an electric current or by means of other magnets used to stroke the unmagnetized material so that its molecules line up all pointing the same way. Ewing considered experimental evidence in formulating this theory; one of the most important confirmations (apparently) of the molecular theory of magnetism relates to the initial difficulty of causing permanent magnetism to appear in certain materials and the corresponding retentivity of these same materials. Soft iron and permalloy are easily magnetized while cobalt steel and modern alnico alloys are very difficult to magnetize. One may visualize little "friction" between the turning molecules in the first instance and much "friction" in the second. On this basis, a material difficult to magnetize should possess substantially more retentivity than one easy to magnetize. This is borne out experimentally, of course, and was considered more or less conclusive evidence that the molecular model was at least a good start in explaining magnetism.

This elementary account of magnetism does not explain any quantitative relations, nor does it attempt to explain why the atoms and molecules of ferromagnetic substances are themselves magnets. The discovery of the electron and electron spin have made possible reasonable arguments in which these effects, combined with quantum mechanics and wave mechanics, provide an extremely credible explanation of observed magnetic properties. Although we cannot undertake to review all of the phases of modern magnetic theory in a book of this scope, we shall present some of the fundamental concepts involved in the modern domain theory of magnetism.

30. The Domain Theory of Magnetism

In 1907, Pierre Weiss published an important new approach to the explanation of ferromagnetism. His reasoning may be summarized as follows: (1) the flux density due to electronic (interatomic) currents is negligible in diamagnetic and paramagnetic substances as compared with the density produced by the applied field; (2) in ferromagnetic materials, the flux density (electronic currents) may be very much larger than that caused by the external field; (3) thus, in ferromagnetic substances, there must be present some active influence other than the applied field, which tends to line up the molecular "magnets." Weiss suggested that molecular interactions of such nature as to produce the observed effects must occur.

Essentially, the *domain theory* of magnetism is patterned after Weiss' original analysis. Bolstered by much experimental and theoretical investigation in recent years, the domain theory provides a fairly complete picture of ferromagnetic effects. Although the quantitative ramifications of the theory are complex, certain aspects of it can be rather simply stated.

Rather than consider each individual molecule as a magnetic entity that acts independently of all the other molecules, the modern theory states that ferromagnetic substances contain small separate areas called domains, in which all the magnetic molecules are aligned in one direction, even in the absence of an external magnetic field. This spontaneous interaction results in a large number of domains that are haphazardly oriented with respect to one another, so that the net magnetic field is zero. This situation is pictured in Fig. 28 (A). In an actual specimen of ferromagnetic material, of course, the domains would be of various sizes and shapes, but the resultant, with a sufficiently large number of domains, would be statistically zero.

F. H. Bitter has shown by means of some rather ingenious experimental techniques that a magnetic powder dusted on a ferromagnetic sample tends to collect along the domain boundaries. Microscopic inspection is required to locate these lines of concentration. This same experimental method reveals that an unmagnetized (i.e., zero *net* magnetization) specimen becomes magnetized in two ways.

(a) *Boundary displacement* [Fig. 28 (B)]. In a weak applied field, the specimen tends to show its own ferromagnetic field, since

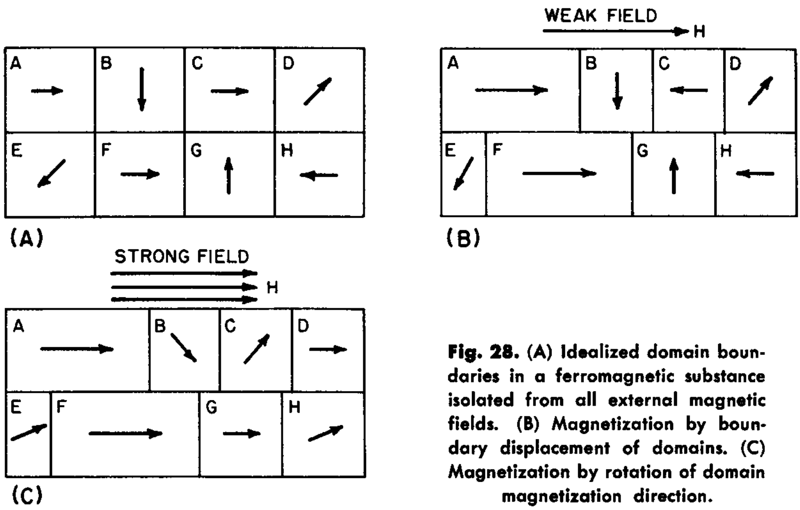


Fig. 28. (A) Idealized domain boundaries in a ferromagnetic substance isolated from all external magnetic fields. (B) Magnetization by boundary displacement of domains. (C) Magnetization by rotation of domain magnetization direction.

favorably oriented domains tend to grow at the expense of domains the natural direction of which opposes the applied field direction. Thus, in Fig. 28 (B), domains A and F have displaced their boundaries under the action of the weak field to raise the net magnetization to some value greater than zero.

(b) *Rotation of domain direction* [Fig. 28 (C)]. As the intensity of the external field is increased, the direction of molecular alignment in even the unfavorably oriented domains begins to change, causing a further increase in the net flux density. As pictured in Fig. 28 (C), the specimen has approached magnetic saturation, with all the domains rotated close to parallelism with the strong external field.

Individual domain volumes show considerable variation. Bitter reports domains in some instances smaller than 10^{-6} cm^3 and, in other cases, somewhat larger than 10^{-2} cm^3. Even these tiny volumes contain molecules that number around 10^{20}.

The fact that domain growth and rotation are both *discontinuous* rather than smooth effects gives rise to what is commonly called the Barkhausen Effect. When a ferromagnetic specimen is made the core of a coil and then placed in a gradually increasing external field, the coil, serving as a transducer for a high-gain amplifier, will produce clicks in a loudspeaker. Thus, the domains *snap* into new positions and change boundaries in jumps. This helps explain why a hysteresis loop forms in normal practice; the domains do not snap back as readily when the field is reduced or removed. The ones that remain in alignment after the field is gone are those which give us permanent magnets.

31. Review Questions

1. Distinguish between the terms: ferromagnetic, diamagnetic, and paramagnetic. Give examples of substances that fit each classification.
2. Explain what is meant by an "electronic current" in an atom. Why is the iron atom unique with respect to its electronic currents?
3. Define magnetic permeability. In what way is permeability related to B and H?
4. Distinguish carefully between flux density and magnetic field intensity.
5. Using equations presented in this chapter, show that the units for measuring field intensity in the mks system are ampere-turns per meter.
6. What is meant by relative permeability?
7. Exactly what is hysteresis? Draw a representative hysteresis curve for a ferromagnetic substance, labeling each part clearly. Define coercive force and retentivity in terms of the hysteresis curve.
8. Explain in terms of the hysteresis curve why heat is generated in the iron core of a transformer used for a-c circuits.
9. In what ways does Ewing's molecular theory of magnetism fail to account for experimental phenomena that are now well known?
10. Describe the domain theory of magnetism and show why this theory is to be preferred over Ewing's theory.

Chapter 5

MAGNETIC POLES AND FORCES

32. The Concept of Magnetic Poles

When a bar of a ferromagnetic material such as steel is placed in a strong magnetic field and then removed, the bar will be found to be permanently magnetized. That is, it will attract other bits of iron, nickel, or cobalt with a force that depends upon the nature of the material and the strength of the original field. When iron filings or small compasses are distributed around the bar, the magnetic field now present as a result of the bar alone may be traced. This field appears as shown in Fig. 29. The flux density near the ends of the bar is appreciably greater than near its middle; thus, the magnetic field set up in the space around the bar magnet appears to be caused by some unique properties of the ends of the magnet. Before the relationship between the electrical aspects of atomic structure and magnetism were recognized, it was thought that the magnetism of a bar or horseshoe magnet was due to "magnetic charges" that resided in the ends of the bar. For this reason, these ends were called the *poles* of the magnet. Although little credence is given the notion of magnetic charges today, the pole concept is still often convenient in certain approaches to magnetism.

A freely suspended bar magnet at many points on the earth's surface tends to align itself in a north-south direction. The end that points in a general northerly direction is referred to as the north-seeking pole, the north pole, or the N-pole; similarly, the opposite end is the south-seeking pole, the south or S-pole of the magnet.

Experiment shows that like poles repel each other, while unlike poles attract each other. Let us return for a moment to Section 3, in which the arbitrarily assigned direction of a magnetic field was discussed. It will be recalled that the direction of a magnetic field is the direction in which an isolated unit N-pole will move when placed under the influence of the field. Since like poles repel each

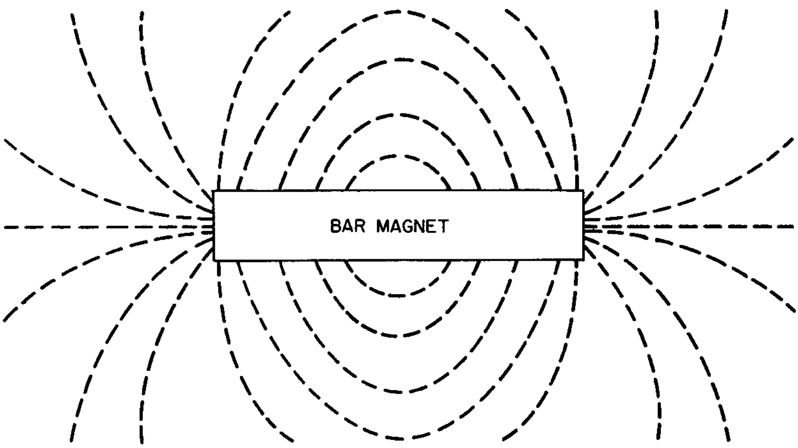


Fig. 29. Magnetic field around a bar of permanently magnetized steel.

other, it is evident that we must then assume that lines of force *emerge from the N-pole* of a bar magnet and *re-enter at the S-pole.* In this way, the direction assigned to the field in our initial discussion is maintained consistent (Fig. 1).

33. Inverse-Square Law for Magnetic Pole Forces

Consider a long, needle-like bar of steel that has been permanently magnetized. Two reasonable assumptions concerning its poles may be made at once: (a) the poles are so far apart that each may be considered as being completely isolated from the other; and (b) the pole to be considered is located at a point since the bar is so small in diameter compared to its length. The moment we make these assumptions, we may use a relation established in Section 7. According to equation (14), the force acting on a length of wire L carrying a current i in a field of flux density B (see Fig. 7) is:

$$F = B\ i\ L$$

Suppose now that we consider the magnetic field near the *point pole* defined by the two assumptions just listed. Lines of force must *emerge* from an N-pole; furthermore, these lines must be radial in all directions like pins in a spherical pin cushion. Suppose further that we place a wire loop carrying a current i around the point pole and calculate the force acting on the loop due to those radial lines of force which intersect it at right angles (Fig. 30). From equation (14), the force (ΔF), acting on a small element of the wire (ΔL), is:

$$\Delta F = B\ i\ \Delta L \qquad (75)$$

This force acts directly outward at right angles to the paper. (The reader should verify this, using the three-finger left-hand rule.)

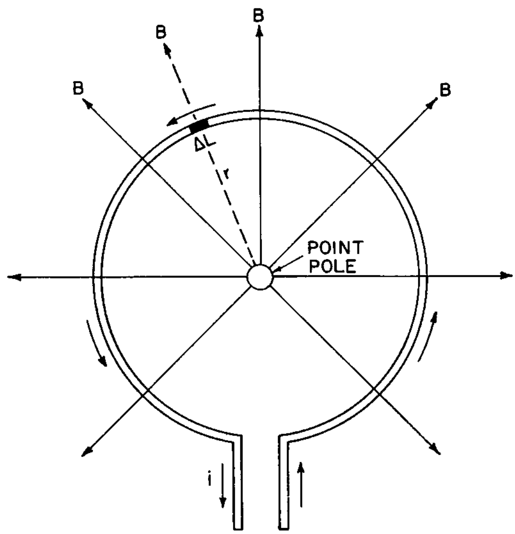


Fig. 30. Circular loop of wire carrying current i placed with point pole at its geometric center.

Since this is the direction of all the elemental forces acting on all the elemental lengths, then the total force is merely the sum of the elemental forces or:

$$F = B\ i\ (2\pi r) \qquad (76)$$

in which $2\pi r$ is the circumference of the loop or the sum of all the elemental lengths.

F is the force exerted *on the loop of wire by the point N-pole.* From Newton's Third Law (action and reaction), F must also represent the magnitude of the force exerted *by the loop on the N-pole.*

Now let us turn to the flux density produced by the current i flowing in the loop of radius r. (The symbol B was used for the flux density due to the point pole, hence, we shall use B′ to represent the flux density due to i.)

From equation (32), we have:

$$B' = \frac{\mu_o\ i}{2r} \tag{77}$$

so that B′ is the flux density at the center of the loop in Fig. 30 due to the current i. At this point we are interested in the development of another equation which tells us the force exerted *by the loop on the N-pole.* [We already have one, that is, equation (76)]. To do this, it is first necessary to define the unit of *pole strength. Unit pole strength in the mks system is defined as that magnetic pole strength which, when placed in a field of one weber per m^2 flux density, will experience a force of one newton.* Thus, pole strength of any pole is given by this definition as:

$$m = \frac{F}{B} \tag{78}$$

in which F is the force in newtons acting on the pole and B is the flux density in w/m^2 in which the pole is immersed. In Section 4, it was shown that one w/m^2 is the same as one newton/amp-meter. From equation (78) and this fact, it is seen that the unit of pole strength is the *ampere-meter.*

Equation (78) may be written:

$$F = B'\ m \tag{79}$$

in which B′ is the flux density produced by the circular loop of wire, m is the strength of the point pole of our previous discussion, and F is the force acting on the pole caused by the current in the loop. Substituting equation (77) in equation (79):

$$F = \frac{\mu_o\ i\ m}{2r} \tag{80}$$

This is the second equation for force acting on a point pole in the center of a loop of wire carrying a current i. Thus, equations (80) and (76) may be equated as follows:

$$B\ i\ (2\pi r) = \frac{\mu_o\ i\ m}{2r} \tag{81}$$

Solving for B, we obtain:

$$B = \frac{\mu_o\ m}{4\pi r^2} \tag{82}$$

Note that the current factors cancel out. Hence, this relation gives the flux density produced by a point pole of strength m at a distance r from the pole.

If another point pole having a strength m′ is now placed a distance r from the first point pole, the force acting on this pole (since it will be immersed in a field of flux density B) is given by equation (79) and may be written:

$$F = B\ m' \tag{83}$$

And, finally, substituting the value of B as given in equation (82) into equation (83), we have:

$$F = \frac{\mu_o\ m\ m'}{4\pi r^2} \tag{84}$$

The implications of this equation are extremely important. Since μ_o and 4π are constant quantities, the fraction $\mu_o/4\pi$ may be replaced by K and equation (84) written in this form:

$$F = K\ \frac{m\ m'}{r^2} \tag{85}$$

The similarity between this expression and Coulomb's equation for the force between electrostatic charges is strikingly evident. Carried one step farther, the law of universal gravitation also is given in identical form (see chart below). From this it is clear that common physical field phenomena all follow the same inverse-square law; this leads to the natural speculation that all field phenomena must be interrelated in nature. The three field equations are:

electrostatic $F = K'\ \frac{q\ q'}{r^2}$ (q, q′ are electric charges)

magnetic $F = K\ \frac{m\ m'}{r^2}$ (m, m′ are pole strengths)

gravitational $F = G\ \frac{M\ M'}{r^2}$ (M, M′ are masses)

Problem 15. Two equal magnetic poles repel each other with a force of 0.04 newton when separated from each other by 2.0 cm. Find the pole strength of each pole.

Solution. From equation (84):

$$F = \frac{\mu_o \, m \, m'}{4\pi r^2}$$

since m = m', we may write the above as:

$$m \text{ (or } m') = \sqrt{\frac{4\pi r^2 F}{\mu_o}}$$

so that:

$$m = \sqrt{\frac{12.57 \times 4 \times 10^{-4} \times 0.04}{12.57 \times 10^{-7}}}$$

$$= \sqrt{1.6}$$

$$= 1.27 \text{ amp-meters}$$

34. Magnetic Circuits

Although we are completely aware that lines of magnetic flux due to a steady current through a coil do not move, we may still refer to the "path" of the flux lines as a *magnetic circuit.* This phrase arises because it may be shown that there is a strong similarity between an electric circuit to which Ohm's Law applies and this magnetic circuit for which an analogous expression may be found. This is best approached by considering the magnetic conditions in a closed toroid containing n turns, carrying a current i, having a core of permeability μ, and a circumference L. From equation (34), which is applicable to a toroid as well as a solenoid:

$$B = \frac{\mu \, n \, i}{L}$$

Also, from equation (3):

$$\phi = BA$$

Substituting (34) in (3), we have:

$$\phi = \frac{\mu \, n \, i \, A}{L} \qquad (86)$$

If the factors μ and A are moved into the denominator of a fraction, we may write:

$$\phi = \frac{n \, i}{\frac{L}{\mu A}} \qquad (87)$$

The reason for rearranging terms as in equation (87) becomes understandable when we analyze what each of the terms signifies. The agency that produces the magnetic flux is the product $n \times i$, since the field intensity is directly proportional *only* to this product [refer to equation (72)]. Hence $n \times i$ is analogous to emf in electric circuits, because it is emf that behaves as the agency for the production of current flow. For this reason, the product $n \times i$ is referred to as the *magnetomotive force.*

The term in the denominator of equation (87) expresses the difficulty with which lines of force can form a "flow" pattern in the medium. The longer the core, the greater the resistance to formation of closed magnetic loops; if either the permeability or the area of the core is increased, the flux lines find it *less* difficult to form. In general, the ratio $L/\mu A$ gives the *reluctance* of the medium to the appearance of high density flux patterns. The ratio is therefore called the *reluctance* of the magnetic circuit and is quite analogous to the resistance of an electric circuit.

Thus, the presence of mmf (*magnetomotive force*) in the numerator and $\mathcal{R}$ (*reluctance*) in the denominator provides an equation that is comparable in sense and application with Ohm's Law for electricity. That is, if total flux (ϕ) is considered the analog of current, mmf the analog of emf, and reluctance the analog of resistance, then we may set up the following comparison:

Electricity	*Magnetism*
$I = \frac{E}{R}$	$\phi = \frac{\text{mmf}}{\mathcal{R}}$ (88)
Resistance of a wire having a length L, a cross-sectional area A, and a conductance σ, is given by:	Reluctance of a flux path of length L, permeability μ, and cross-sectional area A, is given by:
$R = \frac{L}{\sigma A}$	$\mathcal{R} = \frac{L}{\mu A}$

The parallelism between conductance and permeability is obvious; it is not a perfect parallelism, however, because conductance is not dependent upon the current in the conductor (assuming that the heat produced by the current is negligible), whereas permeability is not a constant and is very much a function of the flux density. In addition to this, the current in an electric wire is confined to the metal while magnetic flux tends to leak outside of the core material to a very significant extent. Nevertheless, the analogous behavior of electric and magnetic circuits is very helpful in the design of transformers, motors, and generators.

Magnetomotive force, being a product of number of turns and current strength, has the unit *ampere-turns* in the mks system. The unit for reluctance has not yet been assigned a special name; its composite name is, therefore, ampere-turns per weber. This is obtained by solving equation (88) for reluctance (ϕ is measured in webers in the mks system, so that $\mathcal{R} = \text{mmf}/\phi$ or ampere-turns/weber).

Problem 16. A coil of wire is wound on a rectangular iron frame whose permeability is 0.002 weber/amp-m. The effective perimeter of the core is 30 cm, and its cross-sectional area is 5 cm². The coil contains 200 turns and the current flowing through it is 2.0 amperes. Calculate the flux (ϕ) in the core in webers.

Solution. Using equation (88), we first find the mmf.

$$\text{mmf} = n\ i = 200 \times 2.0 = 400 \text{ ampere-turns}$$

Next, the reluctance is determined:

$$\mathcal{R} = \frac{L}{\mu A} = \frac{0.3 \text{ meter}}{0.002 \text{ w/amp-m} \times 0.005 \text{ m}^2}$$

$$= 30{,}000 \text{ amp-turns/weber}$$

Hence, the flux is:

$$\phi = \frac{\text{mmf}}{\mathcal{R}} = \frac{400}{30{,}000} = 0.013 \text{ weber}$$

35. Review Questions

1. Explain why the concept of magnetic poles is no longer accepted as a fundamental aspect of magnetism.
2. Is the north magnetic pole of the earth a magnetic pole having N or S characteristics? Explain.
3. Using the pole convention, why do we say that lines of force "emerge from the N-pole and re-enter at the S-pole" of a magnet? Is this consistent with field direction arbitrarily selected in Chapter 1?
4. Define the mks unit of pole strength.
5. Show that the unit of pole strength in the mks system is the ampere-meter.
6. Two equal magnetic poles of 0.75 = amp-meter strength are separated from each other by 20 mm. Assuming they are of unlike polarity, calculate in newtons the force that acts between them. Would you arrive at a different

numerical answer if the poles were of the same type? How would your answer differ?

7. Define magnetomotive force.
8. Show that mmf is measured in ampere-turns.
9. Define reluctance.
10. Show that reluctance is measured in ampere-turns/weber.

Chapter 6

MAGNETISM IN TECHNOLOGY

36. Scope of Applications of Magnetism

The technological applications of magnetic and electromagnetic effects are not only tremendously varied, but many of the individual applications are, in themselves, suitable subjects for lengthy textbooks and treatises. Although a book of this type cannot pretend to cover magnetic technology in all its diversified phases, some remarks on this subject – particularly where it deals with the use of the concepts thus far developed – are certainly called for.

We shall therefore confine our discussions to those applications that have received light or cursory treatment in books dealing with electrical and electronic technology. In particular, we shall be interested in aspects of magnetism that are capable of adequate treatment in this chapter, such as eddy currents, the cyclotron, the mass spectrograph, the betatron, terrestrial magnetism, and the induction coil. For the reasons given, we shall omit the transformer, electric motors, generators, electrical measuring instruments, and other topics of like nature which may be found in almost any textbook.

37. Eddy Currents

A few years prior to Faraday's description of the principles of electromagnetic induction, Arago accidentally made a discovery that was not at all understood at the time. A horizontal copper disc about four inches in diameter was spun manually around a vertical axis. Suspended above the disc and mounted on a vertical axis was a permanent bar magnet. As the copper disc rotated, the bar magnet was dragged around with it in the same direction. Somehow, a material with no detectable magnetic properties – copper, for instance – was able to act upon a bar magnet over a distance. After Faraday's discovery of induced emf's, Arago's disc could be explained. The rotating copper disc, moving through the magnetic field of the bar magnet, served as a conductor in which electric currents could be induced.

In our discussions thus far, we have limited ourselves to currents induced in wire-like conductors. The effects of such well-defined

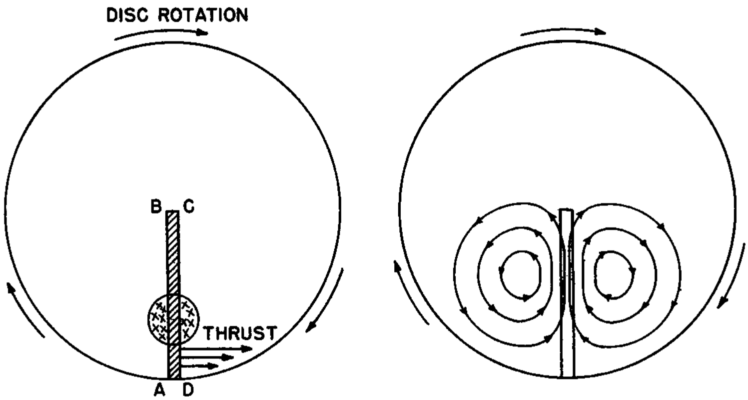


Fig. 31. A copper disc rotated in a magnetic field develops eddy currents which oppose the motion of the disc.

current paths have been thoroughly covered. On the other hand, currents such as those in Arago's disc flow through relatively large masses of metal, and the paths they take are extremely difficult to determine exactly. These loops and whirls of current, like those tiny localized flurries in a swiftly moving river, are called *eddy currents.*

To appreciate the nature of eddy currents and their effect upon electrical machinery, consider a disc rotating in a sharply defined magnetic field at right angles to it (Fig. 31). The cross-sectional area of the field is small compared to the area of the disc.

A small rectangular section of the copper disc (the rectangle BCDA) is passing from right to left in the upper diagram through the magnetic field shown inside the small circle. According to Lenz' Law, the current induced in and around BCDA must have a direction such as to oppose the motion of the disc. Hence, the thrust acting on the current in BCDA must act from left to right as shown in the figure. Knowing the direction of *TH*rust (*TH*umb) and *M*agnetic field (*M*iddle finger), the three-finger left-hand rule tells us that the electron current induced in the vicinity of BCDA must flow from the rim of the disc toward the center. Since it is assumed that all of the disc material is equally electrically conductive, such currents must flow in closed loops (not necessarily circular), as illustrated in the lower drawing of Fig. 31. As the disc continues to move through the field, the eddy currents shift position and direction so as to oppose the direction of rotation continuously and steadily.

Arago's disc is thus simply explained: the bar magnet induces eddy currents that oppose the rotation of the copper disc; by Newton's Third Law, the force on the disc reacts by producing a torque on the magnet, causing it to follow the moving copper wheel.

Eddy currents in this form have been used in many applications. Certain types of automobile speedometers are refined forms of Arago's disc. A small permanent magnet is linked to the wheels of the car through cables. As it spins, it drags with it a copper disc to which the speedometer needle is secured. The torque acting on the disc is proportional to the angular velocity of the bar magnet. A return spring on the disc (stretch proportional to stress applied) opposes the torque on the disc, hence the angle through which the needle turns is proportional to the speed of the wheels.

Experimental eddy-current braking systems have been used successfully on several railroads in this country. Copper discs coupled to the train wheels rotate between the poles of a powerful electromagnet. When the magnet is unenergized, there is no retarding force on the discs. Upon operation of the motorman's braking lever, however, a heavy current flows through the coils of the electromagnet. Now, the disc, spinning through a strong field, is opposed by the side thrusts described in the previous paragraphs, and a braking force is applied to the train. Anyone who has had the experience of riding in an eddy-current braked train can appreciate the smoothness and silent action of this system.*

Eddy currents flowing in the iron core of a transformer can produce serious heat losses. (Hysteresis losses and eddy-current losses taken together constitute the so-called "iron losses" of iron-core mag-

* Eddy-current brakes were used for many years on the all-aluminum cars of the Fulton Street elevated BMT line, plying between Queens and Brooklyn in New York City.

netic devices used on a-c.) Referring to Fig. 32, a transformer using a solid iron core [Fig. 32 (A)] develops eddy currents in a slice of its body [Fig. 32 (B)] as illustrated. (There is no air gap in the core; the section has been removed only to illustrate the paths of the eddy currents. These eddy currents are present in the section while it is in place, and the core is one solid piece.) Since this section has a large area, its resistance is quite small, permitting large

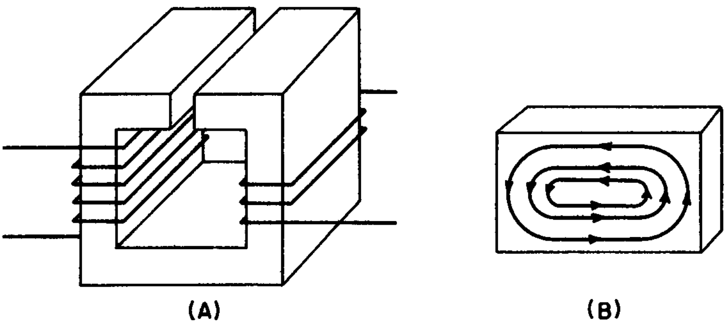


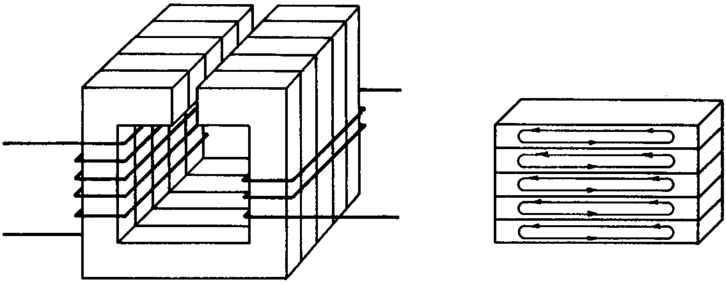
Fig. 32. (A) A transformer with a solid core. A section has been removed for explanation purposes only. (B) Eddy currents in the removed section.

currents to develop as a result of a given induced emf. The I^2R loss may then be a substantial percentage of the total power consumed by the transformer.

An appreciable reduction in eddy-current loss is realized by forming the core of thin laminations, insulated from each other by varnish, shellac, or the natural oxides that form on the surface of the iron. Such a transformer is shown in Fig. 33. In this instance, whatever eddy currents there are must flow in confined areas of high resistance. Thus, the current magnitudes are very substantially diminished with a consequent reduction in the I^2R loss.

38. The Cyclotron

The cyclotron has seldom been out of the news since its invention in 1931 by E. O. Lawrence and M. S. Livingstone at Berkeley, California. The first of the "big" atom smashers, the cyclotron in operation is based to an enormous degree upon the behavior of charged particles in magnetic fields. The cyclotron is to be discussed here not only because it is an important modern technological application of electromagnetism but also because it is an excellent example of an instrument which "grew out of an equation." The feasibility of the cyclotron as a practical device was determined mathematically from several of the equations we have already discussed. As a matter



(A) (B)

Fig. 33. (A) A transformer with a laminated iron core. (B) Eddy-current paths in a section of this core must flow in very much "thinner" conductors having a higher resistance than the solid core.

of fact, were it not for these equations, Drs. Lawrence and Livingstone would never have attempted its construction.

First, let us briefly review the basic principles and construction of the cyclotron (see Fig. 34). A pair of hollow metal chambers, called "dees" because they are shaped like the letter "D," are separated from each other by a small distance with their straight edges parallel. Near the center of the "pill-box" thus formed is placed a source of ions, generally the nuclei of heavy hydrogen. The dees are located in a highly evacuated region between the poles of a huge electromagnet where the flux density is large and quite uniform. The field produced by the electromagnet is arranged perpendicular to the large faces of the dees (or perpendicular to the plane of the paper in Fig. 34). The dees, electrically insulated from each other, are also connected across a source of high-frequency voltage, so that the charge on each dee reverses at a predetermined, fixed rate with respect to the other dee.

At the start of the operation, a deuteron is emitted by the source S. (A *deuteron* is the positively charged nucleus of heavy hydrogen.) Assume that D_2 becomes positive at this instant with respect to D_1, the difference in potential being about 50,000 volts. The deuteron will thus be accelerated into D_1, but, as soon as it enters this chamber, the electric field can no longer affect its motion due to the shielding effect of the enclosing metallic chamber. In this region, however, the moving positive particle is deflected into a *circular path* according to the principles presented in Section 6, Chapter 1, and in Figs. 5 and 6. Taking the radius of this circular orbit as r_1, the deuteron moves in a tight semicircle within D_1 until it is ready to emerge into the gap between the dees. If the polarity of the two dees remained the same, the deuteron would be halted at this point.

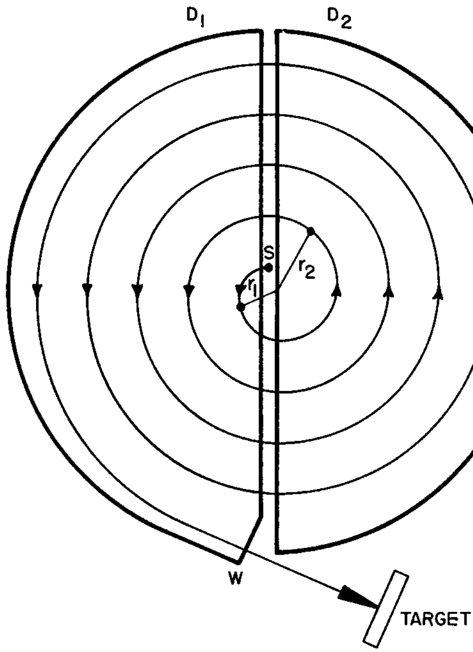


Fig. 34. Schematic construction features of the Lawrence cyclotron.

When this point is reached, however, the potential between dees has been reversed, and the deuteron is further accelerated, moving now into D_2. Since its linear velocity has increased, the radius of the new semicircular path must increase to r_2. This process is repeated over and over again (many more circuits than are shown in the illustration) until eventually the deuteron has a kinetic energy as great as it would have if it had been accelerated through a potential difference of 18 to 22 million volts in a Van de Graaff or linear accelerator.* At the height of their energy content, the deuterons emerge from the "window" at W and are used for disintegrating other atoms in the target space.

To appreciate the importance of the equations developed for the motion of particles in magnetic fields, consider this implication of the cyclotron's design. The oscillating electric field used to provide successive accelerations to the deuterons each time they cross the gap is supplied by a fixed-frequency oscillator. That is, the frequency does not gradually increase or decrease to accommodate possible changes in particle speed; rather, the frequency is constant and is determined by a single initial setting. This means that the *angular velocity* of the deuteron *must* remain the same *regardless of the radius of the path.* Is this possible when the path radius gradually increases to form a spiral? Let us see what the relevant equations have to say about this matter.

The radius of the orbit taken by a charged particle moving in a path at right angles to the magnetic field was shown to be, by equation (7):

$$R = \frac{mv}{Bq}$$

where R is the path radius, m is the mass of the particle, v is the linear velocity of the particle, B is the flux density, and q the particle charge.

The angular velocity in radians per second of any object moving in a circle is given by the relation:

$$\omega = \frac{v}{R} \tag{89}$$

with v in meters per second and R in meters.

If the value of R as given in equation (7) is substituted for R in equation (89), we have:

$$\omega = \frac{v}{\frac{m\,v}{B\,q}} = \frac{B\,q}{m} \tag{90}$$

* Refer to *Electrostatics* by A. Schure, John F. Rider Publisher Inc., for a complete description of the Van de Graaff generator.

Thus, the angular velocity of the deuteron is completely independent of the linear velocity of the ion and also of the radius of the ion's orbit. This is the condition that must be met if an electric field oscillator of constant frequency is to be capable of providing the necessary acceleration thrust at exactly the right instant — as the ion crosses the gap between dees. As equation (90) shows, the angular velocity of the deuteron is dependent only upon the flux density of the field and the charge-to-mass ratio of the ion. The effectiveness of the cyclotron is therefore determined by the constancy of all three of these factors (B, q, and m). No trouble is experienced with B and q, but constant mass becomes a problem, as particles move with increasingly greater velocities in the attempt to obtain ever more energy. As a particle begins to approach the speed of light, significant increases in mass occur, causing the particle to lose synchronization, or to get "out of phase" with the oscillating field as a result of its changing angular velocity. The synchrotron and synchrocyclotron compensate for this effect, as does the betatron which is to be discussed in a later section.

39. The Mass Spectrograph

The original form of the mass spectrograph is attributed to J. J. Thomson and his work with so-called *positive rays* (moving positively charged ions). Important improvements in the instrument were subsequently made by the English physicist, F. W. Aston, and by A. J. Dempster and K. Bainbridge in the United States, to mention only a few of the scientists who have done significant research with this instrument. In Aston's original research, the mass spectrograph was used to confirm his original supposition that two forms of neon exist with atomic masses almost exactly 20 and 22, respectively. The instrument is designed to separate and record path differences of moving ions having slightly different masses, making it possible to determine isotope masses and other atomic masses with great precision. All forms of the mass spectrograph utilize the effects of charges moving through a magnetic field but differ somewhat in other details. We shall describe the operation of Dempster's mass spectrograph in terms of the equations developed in this book for charges moving through magnetic fields.

Dempster's instrument is shown schematically in Fig. 35. The element to be studied is vaporized by electric heating, and the atoms

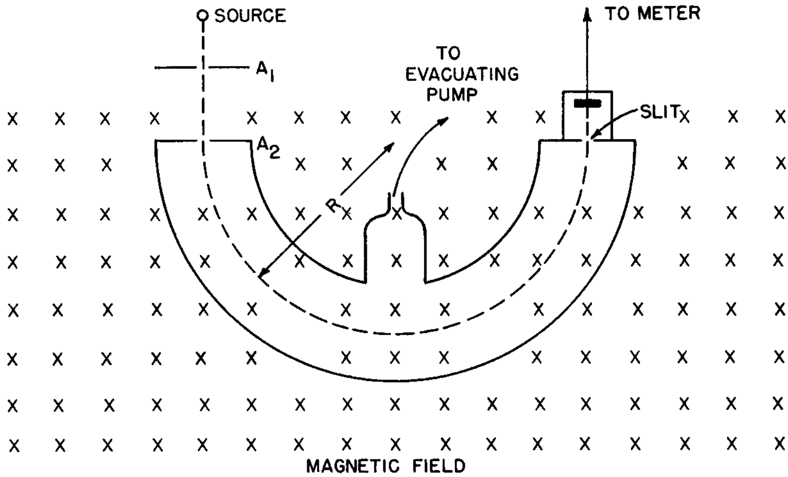


Fig. 35. Schematic diagram of the Dempster Mass Spectograph.

in the vapor are then ionized by bombardment with electrons from a hot filament. The positive ions emerge through the hole in anode A_1. This anode is made positive with respect to A_2, so that the ions are accelerated through the second hole (in A_2), passing into a region permeated by a constant magnetic field at right angles to the plane of the paper. The ions which appear inside the semicircular chamber through the hole in A_2 all have approximately the same energy, having been acted upon by the same electric field between the anodes.

If E is the potential difference between the anodes, a positive ion carrying a charge q will acquire energy given by:

$$\text{K.E.} = E\,q \tag{91}$$

Since this is kinetic energy, it may be represented by the equation involving the mass and velocity of the particle or:

$$\text{K.E.} = \frac{1}{2}\,m\,v^2 \tag{92}$$

Equating:

$$E\,q = \frac{1}{2}\,m\,v^2 \tag{93}$$

And solving for v:

$$v = \sqrt{\frac{2\ E\ q}{m}} \tag{94}$$

Equation (94) gives the velocity of the particle in terms of electric potential, charge, and mass. We shall use this shortly.

As the beam of ions emerges into the magnetic field, it is deflected into the circular path shown by the broken line in the figure. Particles having a given mass will then pass through the slit and on to an electrode connected to an electrometer or other device for measuring ion current.

From equation (7), the radius of the path followed by the ion is:

$$R = \frac{m\ v}{B\ q}$$

This equation when solved for m becomes:

$$m = \frac{B\ q\ r}{v} \tag{95}$$

The value for v obtained in equation (94) may now be substituted into equation (95) to yield:

$$m = \frac{B\ q\ r}{\sqrt{\frac{2\ E\ q}{m}}} \tag{96}$$

Squaring both sides:

$$m^2 = \frac{B^2\ q^2\ R^2\ m}{2\ E\ q} \tag{97}$$

Simplifying:

$$\frac{m}{q} = \frac{B^2\ R^2}{2\ E} \tag{98}$$

This is the form in which the equation is generally used. In the apparatus of Fig. 35, only those ions moving in a path of radius R can pass through the last slit to the meter electrode. The mass-to-charge ratio of these ions (m/q) is obviously determined by the potential E and the flux density of the magnetic field B, as shown in equation (98). Thus, assuming that all ions carry the same charge, only ions of a particular mass can pass through the slit when the electric field and magnetic field are both held constant. The mass could be calculated directly from equation (98), but this is not generally done in practice. The voltage needed to make ions of known mass follow the path of radius R is measured first; then, since the mass-to-charge ratio is known, the product $B^2 R^2$, which is characteristic of a specific instrument, is readily obtained. The instrument is then said to be "standardized."

Following the standardization process, the potential E is set to any desired value, and the ion current is determined by the electrometer. Since $B^2 R^2$ has been previously determined and E can be measured, the m/q ratio of particular ions that succeed in reaching the meter electrode under these conditions can be calculated. The number of these ions is proportional to the ion current as read on the meter. Now, as E is set to different values, ions of various masses (depending upon the adjustment of E) pass to the meter, successively. Thus, the masses and their relative quantities can be calculated from the known settings of E and the meter readings.

40. The Betatron

In Section 38, an important limitation of the cyclotron was mentioned: the loss of synchronization between the varying magnetic field and the moving electron due to the increase of mass that the moving particle experiences as its velocity grows. This limitation becomes serious only when the accelerated particle approaches the speed of light. For example, when the particle velocity is 1/10 the speed of light, the mass increase is only 0.5% of the rest mass; that is, if the rest mass is unity, then the so-called *relativistic mass* at 1/10 the speed of light is 1.005. But, if the particle velocity is 99% of the velocity of light, then the relativistic mass will be more than 7 times greater than the rest mass. Obviously, this will have a pronounced effect upon the angular velocity of the particle in the cyclotron [refer to equation (90)].

It is for this reason that the cyclotron is never used to accelerate electrons. Since particle energy is equal to $\frac{1}{2}$ m v^2, an electron having a mass that is 1840 times less than that of a single proton must be accelerated to substantially higher velocities than the latter in order to realize the same amount of energy. (If m is small, v must be large to keep the product $\frac{1}{2}$ m v^2 constant.) When an electron is given an energy of only one million electron volts, it must be moving at a speed of about 0.9 that of light; its relativistic mass is then about 2.5 times its rest mass, causing its angular velocity to

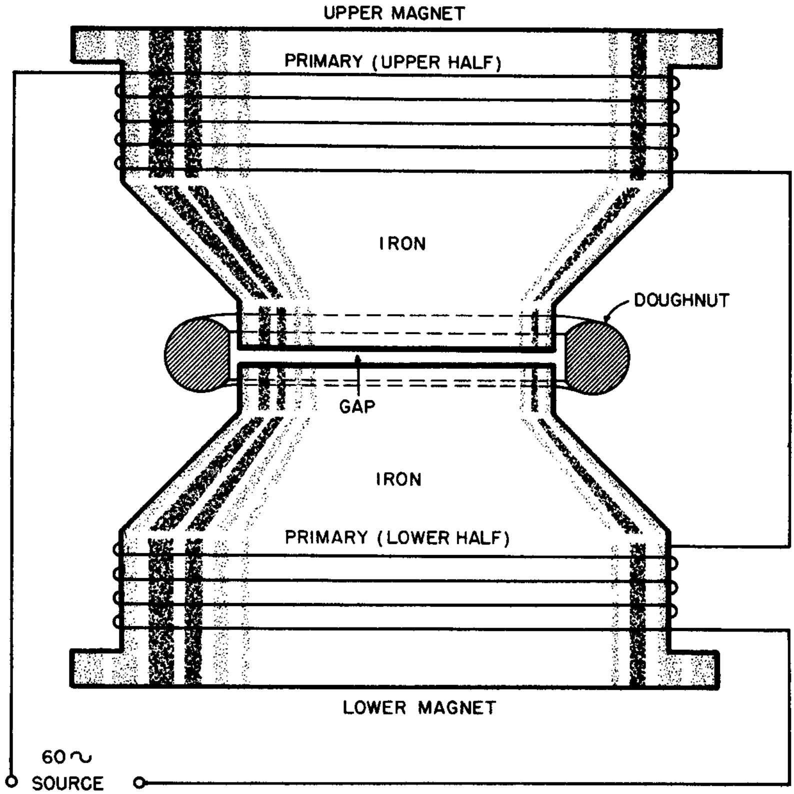


Fig. 36. Schematic representation of the 100-Mev General Electric Betatron.

change more than enough to destroy the cyclotron's synchronization.

The first successful electron accelerator of the betatron type was designed and constructed in the United States in 1940 by Donald W. Kerst at the University of Illinois. The first betatron produced electrons with less than 2.5 Mev (millions of electron volts) ; it was followed later by a 20-Mev machine and finally by a 100-Mev accelerator, both constructed by General Electric Co. Our discussion will be based on the 100-Mev machine which is pictured schematically in Fig. 36.

Betatron action is comparable to that of an ordinary transformer. If a changing current flows in the primary winding, an electromotive force will be developed in the secondary which, in the case of the betatron, is an evacuated toroidal-shaped chamber called the "doughnut." This emf will produce a thrust on any charged particles that happen to be present in the doughnut.

The iron core of the electromagnet is specially shaped to produce not only the required emf for particle thrust, but also a strong magnetic field of uniform intensity in the hole of the doughnut. The acceleration process is begun by injecting electrons into the doughnut from a thermal emitter; these are first given a preliminary thrust by an external electric field (not shown) having a potential differ-

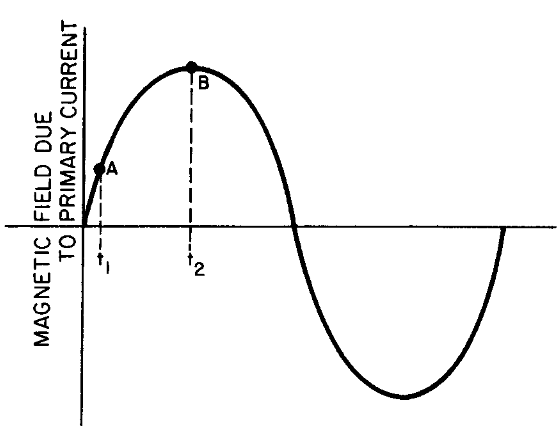


Fig. 37. A single cycle of the current that energizes the primary magnets of the Betatron. Only one-quarter of the cycle is used for accelerating electrons.

ence of about 50,000 volts. Upon entrance into the acceleration chamber, the electrons thus have a velocity close to one half that of light.

The electron must be injected at the proper time relative to the current cycle in the electromagnet, however. The magnet is energized by an alternating pulsed current of quasi-sinusoidal form passed through the primary coils (Fig. 37). The current then produces a magnetic field of the shape shown in Fig. 37. Electron injection occurs at t_1 just when the magnetic field is beginning to build up rapidly. The field at this time has two functions, one to apply a side thrust on the electrons so that they move in a circular path and the other to impart a forward thrust (the field is *increasing* in strength) which increases their velocity.

In the cyclotron, the particles move in a spiral path, since their velocity is *increasing* while the field remains *uniform* and *constant.*

The increasing magnetic field in the betatron keeps the electrons in a *circular* path during the time interval t_2-t_1. Equation (7) shows that this is possible, provided the flux density B grows proportionately to the increase of the product of mass m and velocity v. The waveform of the energizing current must be adjusted to provide just such a growth of flux from A to B (Fig. 37). Thus, the electrons are held to a fairly stable, circular path, with energy being acquired during each lap. At the instant when the field intensity reaches point B, a strong current pulse is sent through an auxiliary coil (not shown in the drawing) which causes the magnetic field to change suddenly. At the same instant, the electrons are thrust out of their stable path to fall upon a target or emerge from the machine to be used for other purposes.

To afford the reader some idea of the dimensions of the 100-Mev betatron and the energies involved, we present the figures below:

Doughnut diameter: 1.08 meters
Magnet weight: 135 tons
Number of laps per electron during time interval t_2-t_1: 250,000 laps
Energy gained during each lap: 400 ev
Total energy (250,000 laps): 100 Mev
Electron velocity (final): 99.99% of light
Relativistic mass: 200 times rest mass.

We should like to pass one more idea along to the reader. If the electrons had not been removed from the doughnut at the time point B in the magnetic field intensity had been attained, the diminishing field from this point on would have applied a decelerating thrust to the electrons. Therefore, a little less than one quarter of a full cycle is used for acceleration purposes, electrons being injected at point A and removed at point B. Since synchronization between energizing current and electron velocity is not necessary, the problem of relativistic mass increase *versus* thrust timing becomes one of controlling only the pattern of magnetic field growth as described previously.

41. Terrestrial Magnetism

Although historical evidence indicates that the Chinese used magnetic compasses for navigation before Western civilization had made any scientific progress, it remained for William Gilbert, an Englishman, first to suggest that the earth behaves as a great magnet. The external lines of magnetic force that surround the earth emerge from the surface over the whole southern magnetic hemisphere and re-enter the surface over the entire northern hemisphere. To attempt to explain this pattern (Fig. 38) in terms of magnetic poles requires the assumption that the earth is a uniformly magnetized

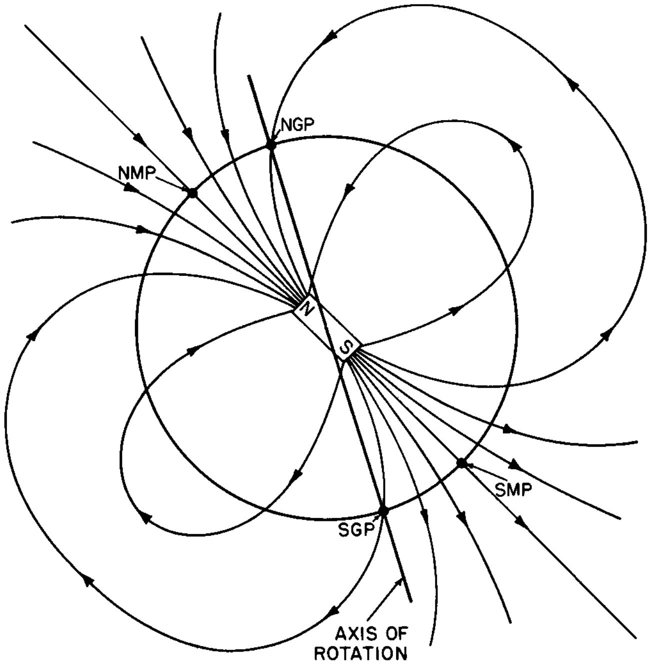


Fig. 38. The magnetic field of the earth can be explained either by assuming that the earth is a nearly uniformly magnetized sphere or by assuming that the core is a short, stubby magnet whose poles are far removed from the surface.

sphere with distributed poles or that the core of the earth is a very short magnet oriented somewhat as shown in Fig. 38. When we refer to the north magnetic pole of the earth, we implicitly suggest two things: (a) there is a place on the surface of the northern hemisphere where the lines of force re-enter the earth vertically as they would do at the S-pole of a bar magnet and (b) this point is *near* the geographic north pole of the earth. At the present time, the approximate location of the magnetic north pole is at latitude

70°N, longitude 96°W. From the point of view of field direction, it must be noted that the magnetic north pole of the earth exhibits S-pole characteristics; similarly, the magnetic south pole is, according to the conventions we have adopted, an N-pole.

The compass points exactly north and south in very few regions on the earth. The angular distance the compass varies from true north is called the *declination* at that point; declination changes in an irregular fashion and must be measured at a given point to be determined precisely. That is, there is no recognized law which permits its calculation. Maps of large areas, such as the United States, are available in which points of equal declination are con-

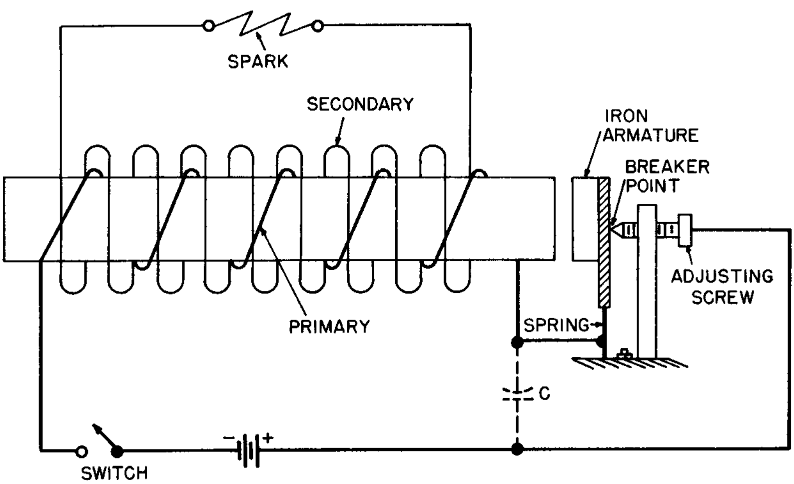


Fig. 39. Standard induction coil with a mechanical interrupter or circuit breaker.

nected by irregular lines called *isogonic lines*. In the United States, at points on a line running through Michigan, Ohio, eastern Kentucky, and Tennessee, the compass points directly toward the true north. All points east of this line have declinations west of north; all localities west of this line have declinations east of north. In northeastern Maine, for example, the declination is about 20 degrees west.

The earth's field is horizontal only at the magnetic equator; at all other points it makes some angle of emergence or entry with the surface. The angle, measured between the horizontal and the line of force, is called the *angle of dip or inclination*. The angle of dip in New York City is approximately 72°; at the magnetic poles the angle is 90°, as mentioned previously.

42. Induction Coils

In general, an induction coil is a device in which a high voltage is induced in a coil by the change in current flowing through a nearby coil, this change in current resulting from the "break" of the

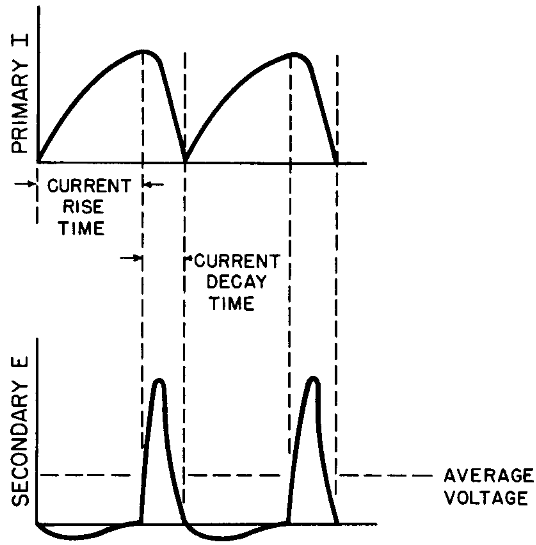


Fig. 40. (A) The primary current of an induction coil resembles a sawtooth wave because the current rise time is greater than the decay time. (B) The secondary voltage consists of a large pulse in one direction and a small amplitude pulse in the other direction.

circuit in the nearby coil. Figure 39 pictures an induction coil with a mechanical breaker system. When Sw is closed, current begins to flow in the primary winding. The rise of current magnitude occurs comparatively slowly because the inductance of the primary winding is quite large, causing the L-R time constant to be correspondingly large. When the magnetic field produced in the core by the primary current reaches a sufficient intensity, the armature is attracted, breaking the circuit at the contact point. Although some sparking occurs as the circuit is opened, the resistance of the air gap being formed by the retreating armature rises very rapidly, causing the primary current to decay to zero in a much shorter time than was required for it to go from zero to maximum on the make. The graph of primary current *vs* time for two cycles of interrupter action is given in Fig. 40 (A).

It is important to note that the secondary voltage [Fig. 40 (B)] induced by the changing primary current is alternating in general nature but that the average voltage is undirectional; that is, the resultant of the two halves of the secondary voltage cycle contains a large d-c component since the voltage induced during the fast decay interval is much greater than that induced during the rise time. For this reason, an induction coil may be used to energize d-c devices such as demonstration cathode-ray tubes and similar high-voltage apparatus.

A capacitor is often connected across the contact points of the interrupter to absorb the energy of the spark that tends to leap the gap as the armature pulls away. The capacitor stores this energy during the break interval and discharges back into the circuit on the make. This retards the corrosion and pitting of the contacts. Upon discharge, in this case, however, the secondary voltage waveform is considerably different. As the capacitor discharges, oscillation is produced in the circuit consisting of the primary inductance and the capacitor. This oscillation is short-lived, being highly damped by the resistance of the primary winding. In addition, it does not affect the production of the high-voltage pulse, since oscillation ceases when the gap resistance becomes high.

43. Review Questions

1. Using the three-finger left-hand rule, prove that the currents in the lower portion of Fig. 31 would take the reverse direction if the disc were rotated counterclockwise.
2. Explain why laminating the core of an inductor or transformer helps to reduce the iron losses.
3. Explain why the path of a charged particle in a cyclotron is a spiral. (A constant magnetic field normally causes a circular orbit.)
4. Prove that the angular velocity of a deuteron in a cyclotron is independent of the linear velocity of the deuteron in the "dees."
5. Discuss the reason (s) why a cyclotron cannot be used successfully to accelerate electrons to very high energies; what other particles besides deuterons would be satisfactory in the cyclotron?

6. What is the function of a mass spectrograph? In A. J. Dempster's design, how is the "spectrum" or isotopic content of an element displayed?
7. Explain carefully why the relativistic mass increase of an electron in a betatron does not affect the performance of the instrument.
8. Write the ratio that must be kept constant if the electron paths in the betatron are to remain circular throughout the growth of the first quarter of the magnetic cycle.
9. Explain why the magnetic field pattern of the earth could not be due to a "magnet" lying along the entire length of the axis of rotation.
10. Discuss the current and voltage waveforms in the primary and secondary coils (respectively) of an induction coil. Explain why an induction coil can be used to activate many types of apparatus that require d-c.

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