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Power Dividers for Directional Antenna Systems

Fred Damm
Collins Radio Company
Dallas, Texas

GENERAL

A directional antenna system is a complex network of radiators, branching, and coupling circuits that are integrated into a single closely coupled system. Radio frequency energy is fed to the individual radiating elements of the array or system in the proper proportions and phase angle relationship to produce the desired radiation pattern. The control of the amount of power flowing to each radiator and its phase angle relationship to that in the other radiators is the function of a system of networks for dividing the power, shifting its phase, and matching the various impedances encountered in the array. This feeder system should have a degree of flexibility and a range of adjustment that permits this function to be accomplished at the initial tune up and at any time that the performance of the system is affected by external changes.

TYPICAL FEEDER SYSTEM

A block diagram of a typical feeder system is shown in Fig. 1. The power divider is a branching circuit that divides the total transmitter power between the individual radiators. The proportions in which this power is divided is determined by parameters of the array. Power division can be accomplished by a variety of different circuit configurations and generally takes a form that is determined by the personal preference of the designer or the engineer who is responsible for the initial tune up.

The power divider is often preceded by a matching network to give a more precise and wider range of adjustment of the input impedance of the feeder system and to provide a degree of isolation to the input impedance from short term variations occurring in the array.

The phase control of phase shifting networks are generally lagging T networks. They usually have unit impedance transformation, and have a characteristic impedance equal to that of the transmission lines that they feed. It is expedient to use networks which shift the phase by 90° since greater excursions of phase around this value can be obtained without affecting the characteristic impedance seriously. However, the actual phase shift used is dictated by the overall phase requirements of the entire system and it is not always possible to use a shift of 90° in all of the networks. It is wise, however, to manipulate the phases to affect a shift of as near 90° in as many networks as is possible.

The primary purpose of the antenna matching networks is to transform the complex operating impedance of the antennas to the characteristic impedance of the transmission lines that feed them. This impedance is a function of the antenna's self-impedance, the mutual impedances between the antenna, and the phase and magnitude of the field radiated from the antenna.[1,2] In some arrays, the impedance of one or two antennas may have a negative resistive component. This requires that the feed system provide for feeding power from the antenna back to the power divider. The phase shift in these networks is a part of the overall phase problem and must be considered along with that of the phase shfting networks and the transmission lines.

Design of networks that have specific transformation and phase properties has been discussed by several authors.[2]

POWER DIVIDERS

Any power divider is a form of one of the basic circuits shown in Fig. 2, or a combination of several of them. No matter which of these basic circuits is used, the input impedance is a complex impedance that may be transformed to any value of input resistance that is desired. The load

[1]Superscript numbers in text refer to references at the end of the chapter.

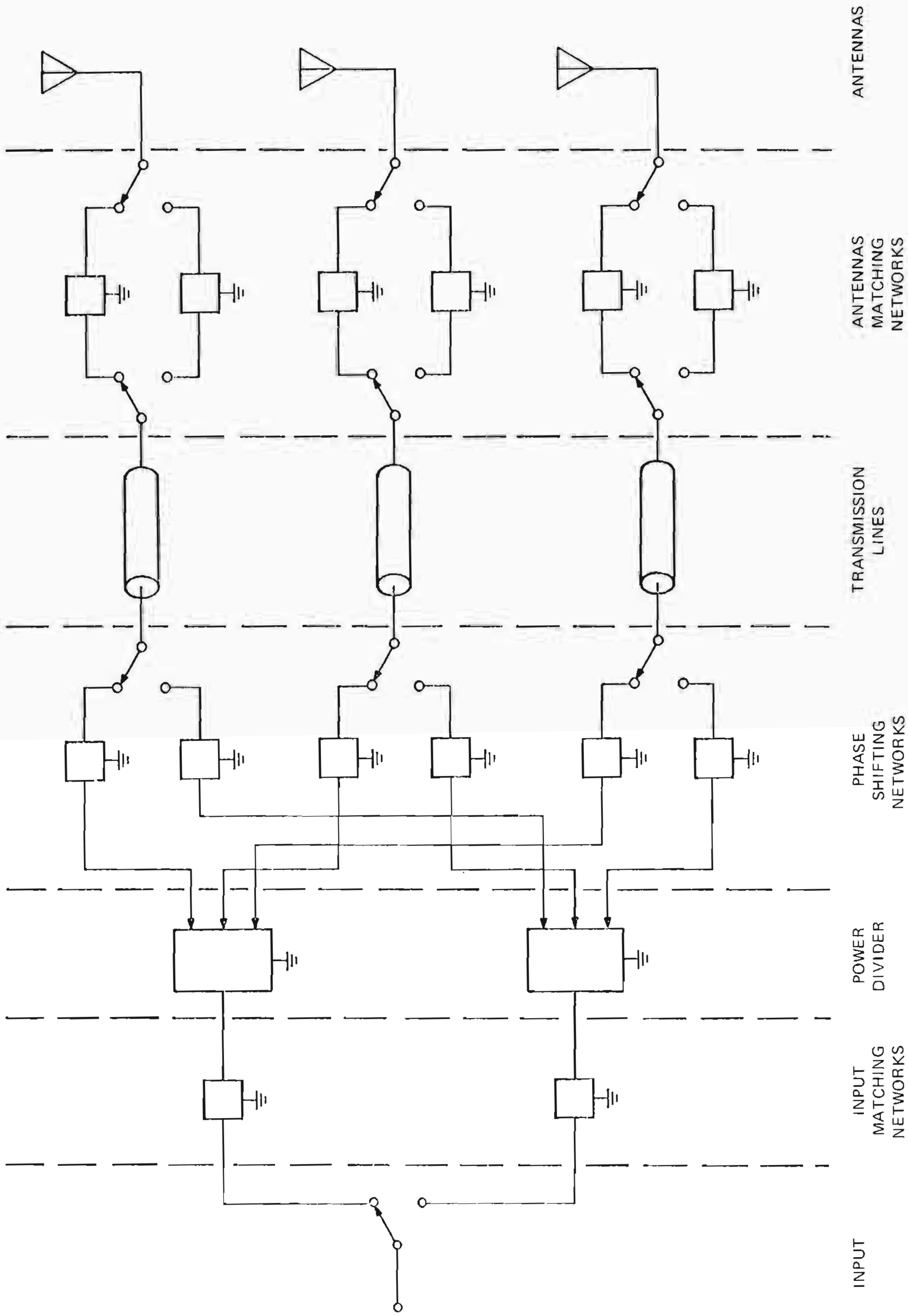

Fig. 1. Typical directional antenna feeder system.

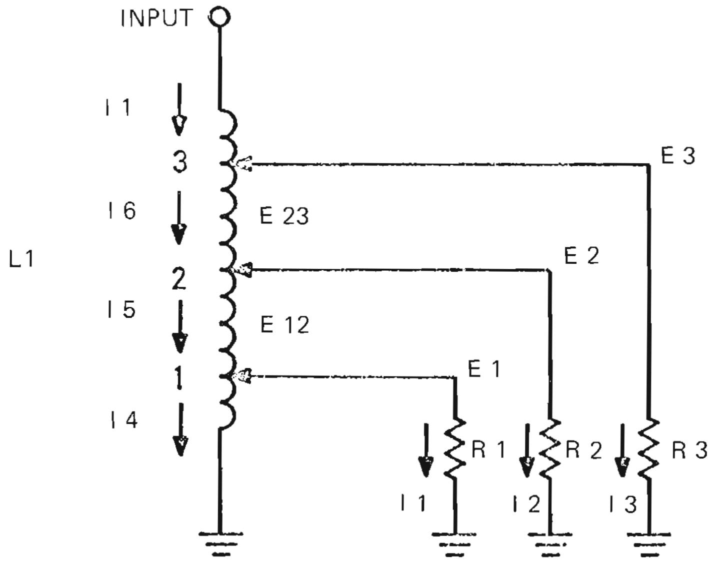

FIG. 2A. SERIES POWER DIVIDER

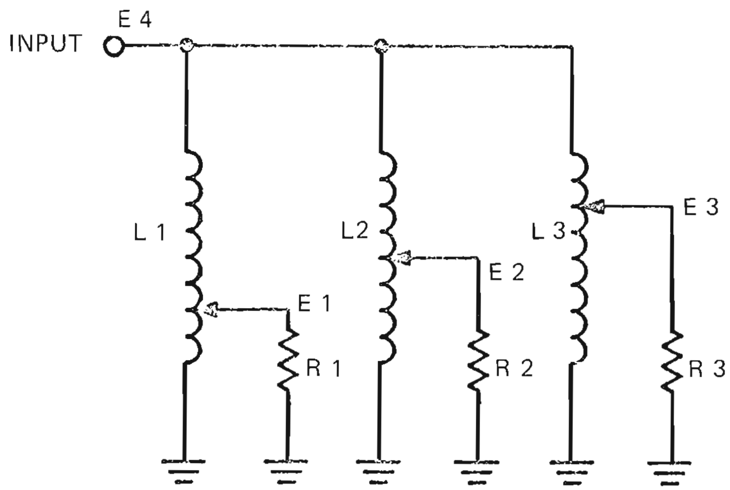

FIG. 2B. SHUNT POWER DIVIDER

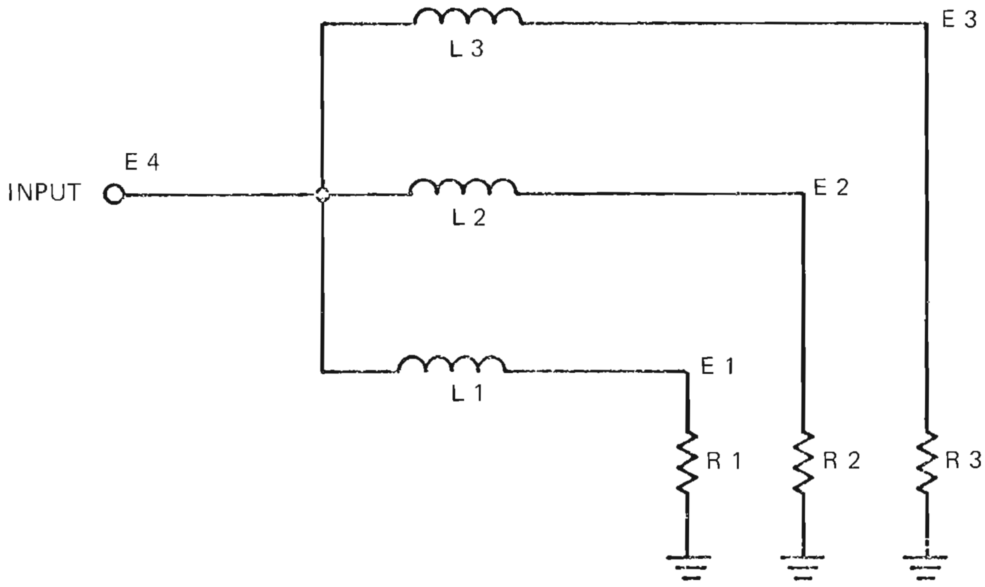

FIG. 2C. HYBRID (SHUNT SERIES) POWER DIVIDER

Fig. 2.

resistances R_1, R_2, and R_3 are the input impedances of the phase shifting networks that feed each of the lines to the antenna. Assuming that the lines are properly terminated and the phase shifting networks have a unity transformation ratio, the resistance of the loads is equal to the characteristic impedance of the lines. The voltage developed across each load is equal to the square root of the product of the power fed to the antenna and the impedance of the lines. The phase relationship between these voltages depends upon the transformation that occurs within the power divider.

The magnitude of the phase difference between these voltages is greater where the voltage difference is greater and less where the voltage difference is less, being zero when the voltages are equal. An exception to this rule occurs when a tower with a negative resistance exists and power is fed into the power divider from this tower. Phase difference may be reduced by increasing the circulating current in the divider, but, higher losses occur and a compromise between phase difference and efficiency must be reached.

DESIGN OF POWER DIVIDERS

The design of power dividers of the type shown in Fig. 2a by algebraic methods is tedious and time consuming. However, it can be accomplished easily and quickly by graphical means.[1] The accuracy of this method is as good as design by the use of a slide rule and it gives a good visual representation of the currents and voltages involved and their relationship to each other. Design errors and poor assumptions can be recognized and corrections can be quickly made.

In the examples shown, vectors with closed arrowheads represent currents and those with open arrowheads represent voltages. If rectangular coordinate paper is used, the printed divisions on the paper can be used for the magnitude of the vectors and a divider and straight edge used to transfer them to the vectors. If plain paper is used, an electrical engineers scale is used for decimal scale divisions and different scales may be used for voltages and currents.

Fig. 3 is a vector diagram of the voltages and currents found in the power divider of Fig. 2a. From antenna impedance and power division calculations,[1,2] the amount of power flowing to each of the loads is known and knowing the resistance of the loads, the magnitudes of the currents and voltages can be calculated. Since the loads are assumed to be resistive, the currents and voltages are in phase for a given load.

The current and voltage I_1 and E_1 for the load with the least power are laid out as shown in the diagram. The current in the bottom end of the inductor, I_4, is determined by the voltage, E_1, and

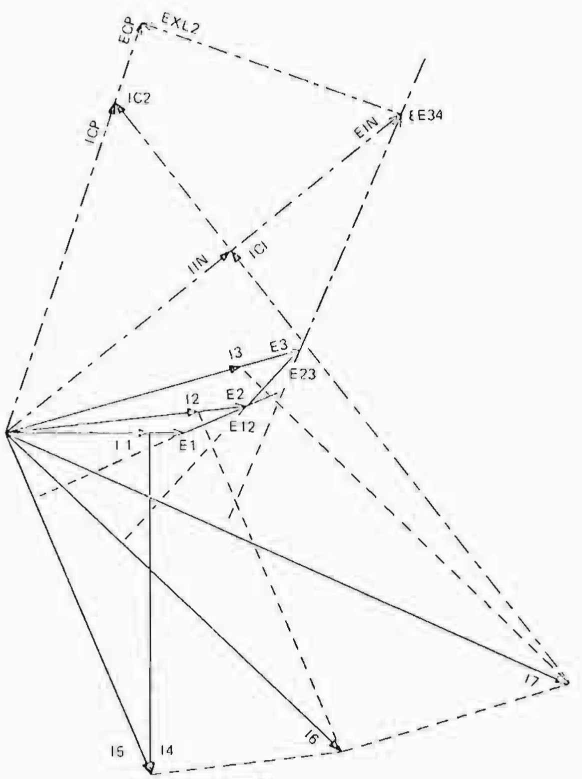

Fig. 3. (Scales I-50 E-20).

the reactance of that portion of the inductor. Since the value of this reactance has not been determined, a value of the magnitude of I_4 can be assigned to it. The relative phase of the currents in all the loads is dependent on the magnitude of this current so some care should be used when assigning a value to it. The magnitude of this current should be two to four times the value of I_1 and if I_1 is relatively small, I_4 should be chosen to be at least as large or slightly larger than I_3, the current in the load receiving the most power.

The vector sum of I_1 and I_4 is I_5, the current flowing in the inductor above the number 1 tap. The voltage developed across the portion of the coil between taps 1 and 2 leads I_5 by 90°. The voltage E_{12} is added vectorially to E_1 at right angles to I_5 to a distance from the source equal to E_2. E_2 and I_2 are then laid out on the diagram and I_2 and I_5 added vectorially to give I_6. The voltage E_{23} across taps 2 and 3 leads I_6 by 90° and is laid out to extend E_2 in a direction which is at right angles to I_6 to a distance from the source equal to E_3. This procedure is continued for as many loads as are required, in this case three, until the current into the top end of the coil and the total voltage across R_3 are determined. In our diagram, these are I_7 and E_3.

The resistive component of the impedance seen at the input (at tap 3) is then

$$R = \frac{P}{I_7^2} \text{ where } P \text{ is the total transmitter power.}$$

The magnitude of the impedance at the input is

$$Z = \frac{E_3}{I_7^2}.$$

The reactive component of this impedance is

$$X_L = +j Z \sin \quad \cos^{-1} \frac{R}{Z} .$$

The Q of the power divider is then

$$Q = \frac{X_L}{R}$$

and may be extended to any greater value desired by increasing the amount of reactance above the upper tap. The Q of the circuit would also be greater if I_4 were initially given a higher value.

The circuit may be resonated to produce the desired input resistance. The input resistance is

$$R_{in} = (Q^2 + 1) R.$$

Since R has been determined, Q may be extended to result in the desired value for R_{in}. The value of Q then is

$$Q = \sqrt{\frac{R_{in}}{R} - 1}$$

The resonating capacitor has a reactance of

$$-j x_C = -j \frac{R_{in}}{Q}$$

R_{in} may be adjusted to the value desired for a common point resistance or may be transformed to that value by means of an L or T network.[3,4]

Problem: Design a power divider for a 3-tower directional array where the power of 10 kilowatts is divided and fed to the antennas in the following proportions, 1,800 watts, 3,200 watts, 5,000 watts by means of transmission lines of 50-ohm impedance. See Fig. 4.

The voltages at the loads are

$$E_1 = \sqrt{PR} = \sqrt{1800 \times 50} = 300 \text{ volts}$$

$$E_2 = \sqrt{PR} = \sqrt{3200 \times 50} = 400 \text{ volts}$$

$$E_3 = \sqrt{PR} = \sqrt{5000 \times 50} = 500 \text{ volts}$$

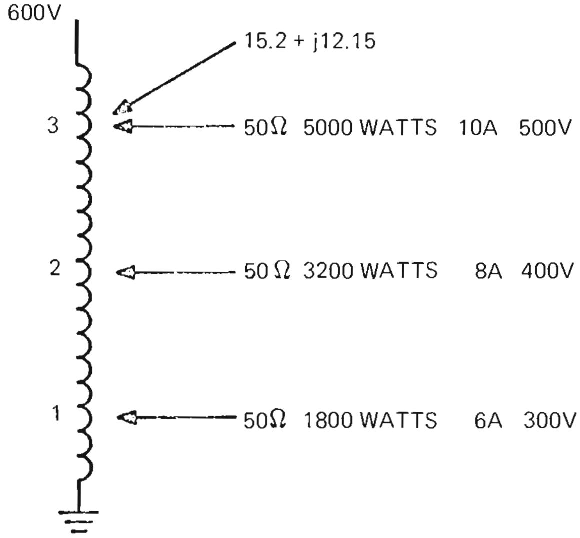

Fig. 4.

The current in the loads are

$$I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{1800}{50}} = 6 \text{ amperes}$$

$$I_2 = \sqrt{\frac{P}{R}} = \sqrt{\frac{3200}{50}} = 8 \text{ amperes}$$

$$I_3 = \sqrt{\frac{P}{R}} = \sqrt{\frac{5000}{50}} = 10 \text{ amperes}$$

Referring to Fig. 3, the scale used for current is 50 divisions per inch and voltage scales are 20 per inch. Draw E_1, 300 volts, and I_1, 6 amperes, and add I_4, 14 amperes lagging I_1 by 90°. The value of I_4 was chosen arbitrarily. The vector sum of I_1 and I_4 is labeled I_5 and has a value of 15.2 amperes.

E_{12} is drawn from the end of E_1 and is perpendicular to I_5. E_2 is drawn to coincide with E_{12} at a point where E_2 has a value of 400 volts. A pair of dividers may be used for this purpose. I_2 is drawn along E_2 to a length of 8 amperes. I_2 and I_5 are added vectorially resulting in a value of 19.1 amperes for I_6. The procedure is continued until I_7 id determined and a value of 25.7 amperes is measured.

The input resistance then is

$$R = \frac{P}{I_7^2} = \frac{10000}{(25.7)^2} = 15.2 \text{ ohms}$$

The magnitude of the impedance is

$$Z = \frac{E_3}{I_7} = \frac{500}{25.7} = 19.45 \text{ ohms}$$

The reactive component of the impedance is

$$X_L = +j\,Z \sin(\cos^{-1}\frac{R}{Z}) = +j\,19.45 \sin(\cos^{-1}\frac{15.2}{19.45})$$

$$= j\,12.15 \text{ ohms.}$$

The common point impedance may then be adjusted to 50 ohms or a value larger than this and then transformed to 50 ohms by an L or T network.

For example, adjust the input impedance to 70 ohms and then transform to 50 ohms using an L network as shown in Fig. 5.

The Q of the circuit is

$$Q = \sqrt{\frac{R_{in}}{R} - 1} = \sqrt{\frac{70}{15.2} - 1} = 1.9$$

The resonating capacitor has a reactance of

$$X_{C1} = -j\frac{R_{in}}{Q} = -j\frac{70}{1.9} = -j\,36.8$$

To resonate the circuit, the inductive reactance of the divider must be adjusted to

$$+j\,X_L = +j\,RQ = +j\,(15.2)(1.9) = +j\,28.9$$

Since at tap 3 we have a reactance of $+j$ 12.15, the value of reactance to be added is 28.9 – 12.15 = 16.75.

The reactance of the inductor to be added above tap 3 is $+j$ 16.75 ohms.

The total reactance of L_1 may be determined by adding up the reactance of all of its parts. These are determined by dividing the voltage across each part by the current through it and adding them together, thus

$$j\frac{E_1}{I_4} = j\frac{300}{15} = j\,21.4$$

$$j\frac{E_{12}}{I_5} = j\frac{105}{15.2} = j\,6.92$$

$$j\frac{E_{23}}{I_6} = j\frac{125}{19.1} = j\,6.55$$

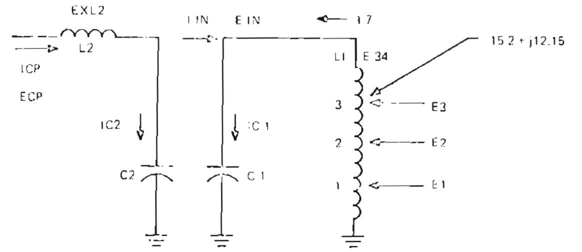

Fig. 5.

Total reactance is

$$j\,X_L = j\,21.4 + j\,6.92 + j\,6.55 + j\,16.75 = 51.62.$$

The components of the L matching network are designed as follows:

$$X_{L2} = +j\,R(\text{common point})\,Q_2$$

$$X_{C2} = -j\frac{R_{in}}{Q_2}$$

Q_2 is the Q of the L network and is

$$Q_2 = \sqrt{\frac{R_{cp}}{R_{in}} - 1} = \sqrt{\frac{70}{50} - 1} = .633$$

Then the reactance of L_2 is

$$X_{L2} = +j\,(50)\,(.633) = +j\,31.6$$

$$X_{C2} = \frac{-j\,(70)}{.633} = -j\,110.5.$$

The reactance of C_1 and C_2 may be combined into one value

$$X_c = \frac{1}{\frac{1}{-jX_{C1}} + \frac{1}{-jX_{C2}}} = \frac{1}{\frac{1}{-j36.8} + \frac{1}{-j110.5}} = -j27.6$$

The vector diagram for the power divider Fig. 3 has been extended to show the graphical design of the rest of the circuit as we have just designed it.

Currents and voltages at the divider input and the common point input for the total power are

$$I_{cp} = \sqrt{\frac{P_t}{R_{cp}}} = \sqrt{\frac{10000}{50}} = 14.14 \text{ amperes}$$

$$E_{cp} = \sqrt{P_T\,R_{cp}} = \sqrt{(10000)\,(50)} = 707 \text{ volts}$$

$$I_{in} = \sqrt{\frac{P_T}{R_{in}}} = \sqrt{\frac{10000}{70}} = 11.95 \text{ amperes}$$

$$E_{in} = \sqrt{P_T R_{in}} = \sqrt{(10{,}000)\,(70)} = 837 \text{ volts}$$

The current I_7 in the divider and the current in C_1 add vectorially to give us the current into the resonated divider I_{in} and the current in C_1, I_{C1}, is leading the voltage across it by 90°. Since the current I_{in} and E_{in} are in phase, resonant condition, they are drawn as shown in Fig. 3. E_{in} is 837 volts and when drawn as shown will coincide with E_{34} which is the voltage from tap 3 to the top of the inductor and is drawn perpendicular to I_7, the current in the top end of the inductor.

I_{C2} is added to I_{C1} for the total capacitor current. The common point current I_{CP} is 14.14 amperes and is drawn to add vectorially with I_{C2}, determining the length of I_{C2}. The voltage at the common point E_{CP} is drawn in phase with I_{CP} and has a magnitude of 707 volts. Since the common point current flows through $L2$ the voltage E_{XL2} developed across $L2$ will lead the current by 90°. The vector sum of E_{in} and E_{XL2} is E_{CP}.

The magnitude of these vector currents and voltages may be scaled and their ratio determined to give the reactance of $L2$ and C ($C1 + C2$)

$$X_{L2} = +j\frac{E_{XL2}}{I_{cp}} = +j\frac{445}{14.14} = +j\,31.6$$

$$X_c = -j\frac{E_{in}}{I_c} = -j\frac{837}{30.3} = -j\,27.6$$

These values agree with those determined earlier.

Problem: Design a power divider for a three-tower directional array where the power of 10 kilowatts is divided and fed to the antennas in the following proportion: 7800 watts, 3200 watts, -1000 watts. Transmission line impedance is 50 ohms. The solution of this problem will be covered only far enough to illustrate the procedure necessary to accommodate the negative power flow.

The load currents and voltages are

$$E_1 = \sqrt{(1000)(50)} = 224 \text{ volts}$$

$$E_2 = \sqrt{(3200)(50)} = 400 \text{ volts}$$

$$E_3 = \sqrt{(7800)(50)} = 625 \text{ volts}$$

$$I_1 = \sqrt{\frac{1000}{50}} = 4.47 \text{ amperes}$$

$$I_2 = \sqrt{\frac{3200}{50}} = 8 \text{ amperes}$$

$$I_3 = \sqrt{\frac{7800}{50}} = 12.5 \text{ amperes.}$$

Since P_1 is negative, either the current or voltage must be negative. A negative current results in a better phase angle relationship between output powers so the design will consider only a negative current for I.

The circuit diagram is shown in Fig. 6 and the vector diagram in Fig. 7. Symbols used in Fig. 2 are used in this example and the procedure is the same as used in the first problem except that now I_1 is negative so the vector is drawn in a direction opposite that of the voltage E_1. The outputs are more nearly in phase in this example. This is due to

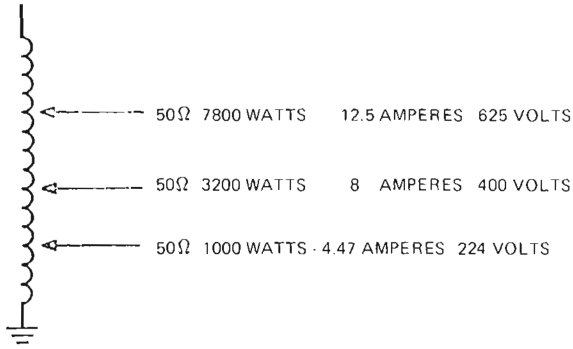

Fig. 6.

the effect of the negative current flow, I_1. I_7 in this example is 21.7 amperes and the input resistance is

$$R = \frac{P}{I_7^2} = \frac{10000}{(21.7)^2} = 21.3 \text{ ohms.}$$

The magnitude of the impedance at tap 3 is

$$Z = \frac{E_3}{I_7} = \frac{625}{21.7} = 28.8 \text{ ohms.}$$

The reactive component of the impedance is

$$X_L = j\,Z \sin \cos^{-1}\frac{R}{Z} = +j\,28.8 \sin \cos^{-1}\frac{21.3}{28.8}$$

$$= +j\,19.4 \text{ ohms.}$$

The divider impedance of $21.3 + j\,19.4$ may then be transformed to the desired common point impedance, as was done in the first example.

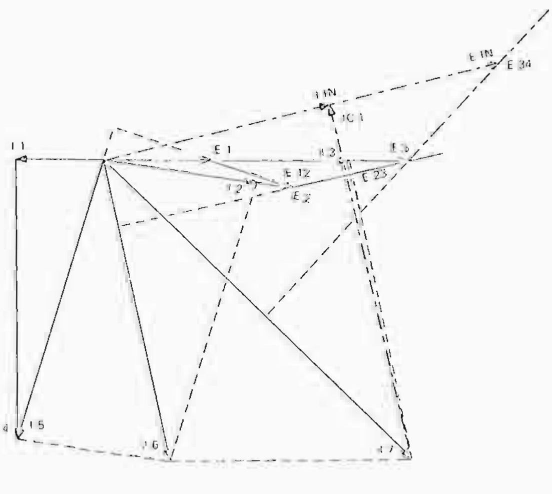

Fig. 7.

SHUNT POWER DIVIDER

The shunt power divider, as shown in Fig. 2b consists of a group of variable inductors in parallel between the common input point and ground. The loads are connected from the variable taps to ground and each is adjustable from maximum voltage to zero voltage. The impedance at the input of each of these branches depends upon the position of the tap and is a pure reactance when the tap is at the minimum position and an impedance consisting of the reactance of the total inductor in shunt with the load resistance at the maximum position. Both the total reactance of the inductor and the resistance of the load are fixed quantities for any given application. The series input impedance of each branch is determined by finding the series impedance of the load resistance and the portion of the inductor in shunt with it and adding to this series impedance the reactance of the remainder of the coil. The solution of this problem is complicated by the fact that all of these branches must be connected to a common voltage source and develop a voltage at its output that is proportional to a specified power output.

A series-parallel conversion chart is a valuable tool for the solution of the problem. This chart is illustrated in Fig. 8 and consists of a family of resistance circles on the x axis and a family of reactance circles on the y axis superimposed on rectangular grid lines. The solution of the series impedance of a reactance and resistance in parallel is accomplished by selecting the parallel resistance circle and following it to the point where it coincides with the parallel reactance circle. The vertical distance of this point from the x axis represents the series reactance and the horizontal distance from the y axis represents the series resistance. There remains, however, the reactance of the remainder of the inductor above the tap which must be added to the series reactance of the impedance for the shunt position. The locus of all of the impedance points, as the tap is moved from the top to the bottom of the inductor, is a curve beginning at the junction of the load resistance circle and the circle representing the total reactance of the inductor, terminating on the y axis at the end of this same reactance circle. A family of these curves for values to total coil reactances of from $+j50$ to $+j150$ ohms and a load resistance of 50 ohms have been drawn on the conversion chart.

The procedure for the design of a shunt power divider will be demonstrated by an example.

Problem: Design a shunt power divider, Fig. 9, for a three-tower directional array where the total power of 10 kilowatts is divided and fed to the antennas in the following proportions, 1,800 watts, 3,200 watts and 5,000 watts by means of transmission lines of 50-ohm impedance. The total reactance of each of the variable inductors is as assumed to be $+j100$ ohms. We then use the curve connecting the end points of the 100 ohm reactance circle where it coincides with the y axis at one end and the 500 ohm resistance circle at the other end. This curve is the locus of all values of branch input impedance as a 50-ohm load connection is moved from the top to the bottom of the 100-ohm inductor. A reference must now be established for one of the branches to which the other branches are related.

When the load of one of the branches is set at the top of the inductor, the branch parallel resistance is 50 ohms and the branch parallel reactance is $+j100$ ohms. The series resistance is 40 ohms and the series reactance is $+j20$ ohms. This position is shown on Fig. 8. It is desirable however to move the operating point down the coil to allow for adjustment latitude. Adjustment to Point 1, Fig. 8, results in a parallel resistance, Rp, of 70 ohms and a parallel reactance, X_p, of 75 ohms.

If the antenna receiving the most power (5,000 watts) is connected to this tap, the input voltage required is

$$E = \sqrt{PR_p} = \sqrt{5000 \times 70} = 591.6 \text{ volts.}$$

This voltage then is the input voltage for all of the branches.

The parallel resistance for the other branches may then be determined by

$$Rp = \frac{E^2}{p}$$

For the second branch, the parallel resistance is

$$Rp = \frac{(591.6)^2}{3200} = 109.4 \text{ ohms}$$

and for the third branch, the parallel resistance is

$$Rp = \frac{(591.6)^2}{1800} = 194.4 \text{ ohms.}$$

The points on our locus curve corresponding to these parallel resistance values are shown as Point 2 and Point 3. The parallel reactances for these points are read from the chart and are $+j73$ and $+j80$, respectively.

We then have three parallel resistance values and three parallel reactance values which when in shunt result in the input impedance.

The resistance values are again 70, 109.4, and 194.4 ohms. Combined in parallel, they are

$$Rp \text{ in} = \frac{1}{\dfrac{1}{Rp_1} + \dfrac{1}{Rp_2} + \dfrac{1}{Rp_3}}$$

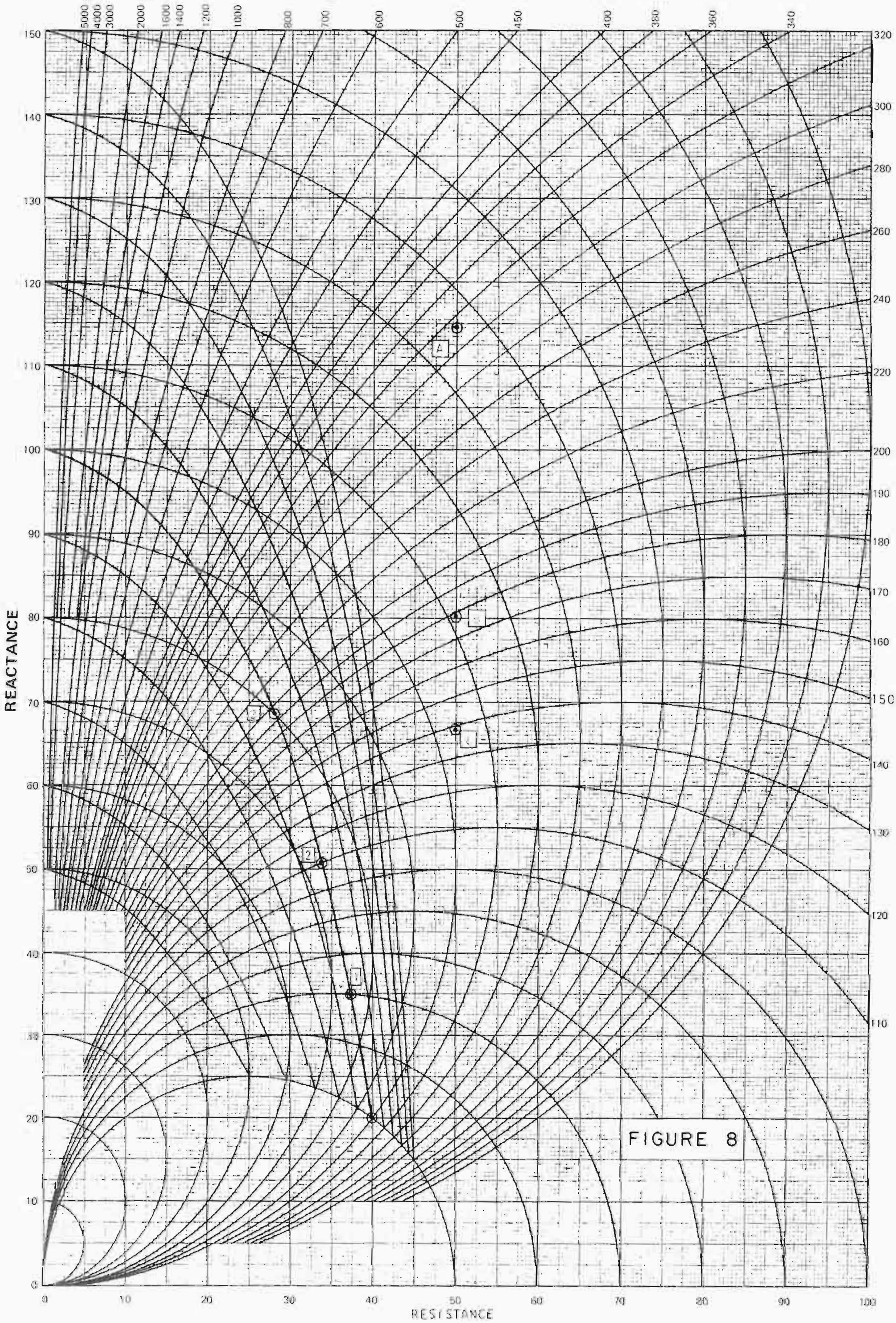

Fig. 8. A series-parallel conversion chart. The X axis is a family of resistance circles and Y axis is a family of reactance circles.

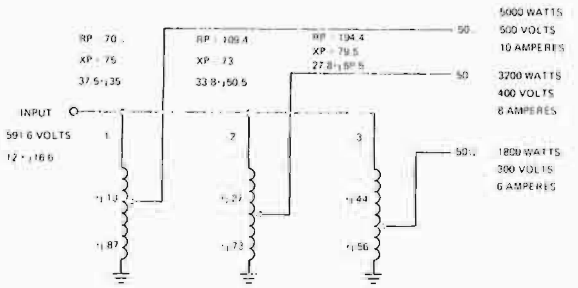

Fig. 9.

$$= \frac{1}{\frac{1}{70} + \frac{1}{109.4} + \frac{1}{194.4}} = 35 \text{ ohms.}$$

The reactance values are again $+j75$, 73, and 80 ohms. Combined in parallel they are

$$Xp \text{ in } = \frac{1}{\frac{1}{+j75} + \frac{1}{+j73} + \frac{1}{+j80}} = j25.3.$$

The total power is

$$P = \frac{E^2}{R} = \frac{(591.6)^2}{35} = 10{,}000 \text{ watts.}$$

The parallel resistance of 35 ohms and reactance of $+j25.3$ are then converted to series resistance and reactance

$$R_S = \frac{Rp\ Xp^2}{Rp^2 + Xp^2} = \frac{(35)\ (25.3)^2}{(35)^2 + (25.3)^2} = 12 \text{ ohms}$$

$$X_S = \frac{Rp^2\ Xp}{Rp^2 + Xp^2} = \frac{(35)^2\ (25.3)}{(35)^2 + (25.3)^2} = +j16.6 \text{ ohms.}$$

The series impedance is $12 + j16.6$ ohms.

If all of the taps are moved up on the inductors, the series resistance is raised. If larger inductors are used, the series resistance is also raised. This series impedance may then be transformed to a common point impedance by adding series inductance and resonating the circuit with a shunt capacitor as was done in the examples for a series power divider. The input impedance may be transformed to any higher value desired and then transformed to the common point by means of an L or T network, as was done in an earlier example.

The phase angle of the currents in the load referred to the input voltage may now be determined. This phase angle is the difference between the input impedance phase angle and the phase of the current in the input with respect to the current in the output. The current in the load and the current in the shunt portion of the inductor have a 90° phase difference and their vector sum is the input current. Then the phase angle between the input current and load current is

$$\beta = \tan^{-1} \frac{I_S}{I_L}.$$

where I_S is the inductor shunt current and I_L is the load current. The input impedance phase angle which also is the phase relationship between input voltage and current is

$$\phi = \tan^{-1} \frac{X\text{in}}{R\text{in}}.$$

The phase difference between these is the phase relationship between input voltage, which is our reference, and the output current.

$$\theta = -\tan^{-1} \frac{X\text{in}}{R\text{in}} + \tan^{-1} \frac{Ix}{I_L}.$$

The input impedances for the branches can be found in Fig. 8 by reading the x and y axis dimensions for the points labeled 1, 2, and 3. They are

Branch 1 (5000 watts) $37.5 + j35$

Branch 2 (3200 watts) $33.8 + j50.5$

Branch 3 (1800 watts) $27.8 + j68.5$

The currents in the loads are determined by the relation

$$I_L = \sqrt{\frac{P}{R}}$$

and are for

$$\text{Branch 1, } I_{L1} = \sqrt{\frac{5000}{50}} = 10 \text{ amperes}$$

$$\text{Branch 2, } I_{L2} = \sqrt{\frac{3200}{50}} = 8 \text{ amperes}$$

$$\text{Branch 3, } I_{L3} = \sqrt{\frac{1800}{50}} = 6 \text{ amperes.}$$

The currents in the shunt portion of the inductors are determined by the relation

$$I_S = \frac{E}{X} \frac{\sqrt{PR}}{X}$$

where X is the reactance of the shunt portion.

The X can be determined from Fig. 8 by dropping our impedance points vertically to the 50-ohm parallel resistance circle and from this point following the parallel reactance circle to the y axis where the shunt reactance is read. This may be checked by measuring the distance from the impedance point to the 50-ohm circle and subtracting this value from 100.

The shunt reactance for Point 1 is found to be 87 ohms. This distance between Point 1 and the 50-ohm circle is measured and found to be 13 and confirms the value 87.

The shunt reactance for Point 2 is likewise found to be 73, and the distance from the point to the 50-ohm circle is measured and found to be 27 confirming the value 73.

In the same manner, the reactance for Point 3 is found to be 56. The distance here is 44, which confirms the value 56.

The shunt currents are then found to be

$$I_{S_1} = \frac{\sqrt{PR}}{X} = \frac{\sqrt{5000 \times 50}}{87} = 5.75 \text{ amperes}$$

$$I_{S_2} = \frac{\sqrt{PR}}{X} = \frac{\sqrt{3200 \times 50}}{73} = 5.48 \text{ amperes}$$

$$I_{S_3} = \frac{\sqrt{PR}}{X} = \frac{\sqrt{1800 \times 50}}{56} = 5.36 \text{ amperes.}$$

The phase angles of the load currents then related to the input voltage are

$$\theta_1 = -\tan^{-1}\frac{35}{37.5} + \tan^{-1}\frac{5.75}{10} = -13^\circ$$

$$\theta_2 = -\tan^{-1}\frac{50.5}{33.8} + \tan^{-1}\frac{5.48}{8} = -21.8^\circ$$

$$\theta_3 = -\tan^{-1}\frac{68.5}{27.8} + \tan^{-1}\frac{5.36}{6} = -26.2^\circ.$$

The phase difference between outputs using θ_1 as a reference are

$$\theta_1 = 0^\circ$$

$$\theta_2 = -8.8^\circ$$

$$\theta_3 = -13.2^\circ.$$

HYBRID (SHUNT-SERIES) POWER DIVIDER

The hybrid power divider, as shown in Fig. 2c, consists of a group of L networks each of which transforms the load (transmission line) impedance to a value of resistance that is determined by the power flowing into its load. Since the input impedance of each of these networks is greater than the output impedance, the shunt legs of the L networks are in parallel at their inputs and can all be combined as one reactance value equal to the reactance of all of the shunt legs in parallel. This reactance is generally capacitative and is not shown in Fig. 2c. The input impedance of each of the branches is determined by first selecting the desired input parallel resistance and determining the voltage developed across this resistance by the total power input. The input parallel resistance of each branch is then determined by the following relationship:

$$R = \frac{E^2}{P}$$

where E is the voltage across the input and P the power fed to each branch load.

Knowing both the input and output resistance, an L network for each branch may be designed or Fig. 8 may be used in the following manner. From the output load resistance value on the x axis, follow a straight line up to each of the input parallel resistances. The input impedances of each of the branches are found at the point where the vertical line crosses each input parallel resistance circle. Both series and parallel value of input resistance and reactance may be read at these points. The parallel reactance of all branches are combined in shunt to give the total parallel reactance of the input. If this is inductive, a capacitor having a reactance equal to this reactance is connected across the input, resulting in a resonant condition and an input resistance equal to that of the originally selected value.

Problem: Design a power divider, Fig. 10, for a three-tower directional antenna array where the total power of 1 kilowatt is divided and fed to the antennas in the following proportions: 200 watts, 350 watts, and 450 watts by means of transmission lines of 50-ohm impedance.

Assume that an input impedance of 62.5 ohms is specified. This may be transformed to any common point impedance desired by means of a T network.

The input voltage for our divider is found by the following relationship.

$$E = \sqrt{PR} + \sqrt{1000 \times 62.5} = 250 \text{ volts}$$

The input parallel resistances for each branch is then

$$R_1 = \frac{E^2}{P_1} = \frac{(250)^2}{200} = 312.5 \text{ ohms}$$

$$R_2 = \frac{E^2}{P_2} = \frac{(250)^2}{350} = 178.6 \text{ ohms}$$

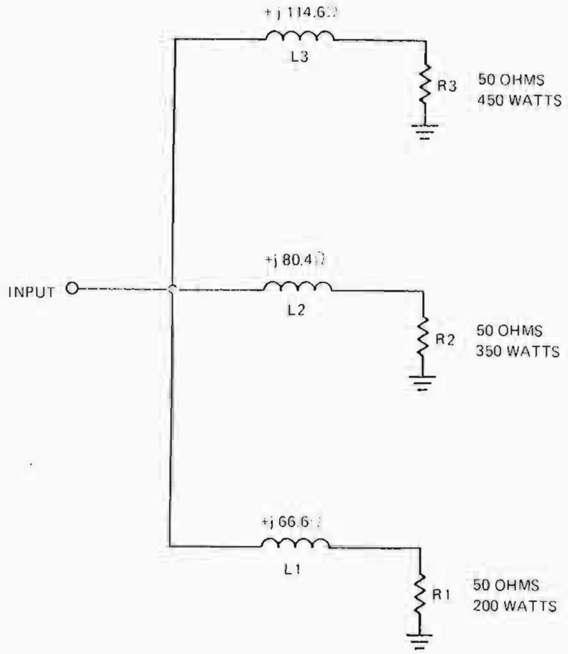

Fig. 10.

$$R_3 = \frac{E^2}{P_3} = \frac{(250)^2}{450} = 138.9 \text{ ohms.}$$

Moving in a vertical line up from the 50-ohm point on the x axis to a Point A on a 312.5-ohm resistance circle, we find a point whose rectangular coordinates are 50-ohm resistance and 114.6-ohm reactance. These are the series components of the input impedance, and the reactance value 114.6 ohms is the value of the Series Inductor $L1$. The parallel components of this impedance are found by following the circles to their indexes. The parallel resistance is the value we previously determined 312.5 ohms and the parallel reactance is found to be 136.2 ohms by following the reactance circle to the y axis. The series and parallel components are then:

$$R_{1s} = 50 \text{ ohms.}$$

$$X_{1s} = +j\,114.6 \text{ ohms}$$

$$R_{1p} = 312.5 \text{ ohms}$$

$$X_{1p} = +j\,136.2 \text{ ohms.}$$

In the same manner, the series and parallel components for the input impedance of Branch 2 are found to be at Point B.

$$R_{2s} = 50 \text{ ohms}$$

$$X_{2s} = +j\,80.4 \text{ ohms}$$

$$X_{2p} = 178.6 \text{ ohms}$$

$$X_{2p} = +j\,111 \text{ ohms.}$$

And for Branch 3, the values are found at Point C.

$$R_{3s} = 50 \text{ ohms}$$

$$X_{3s} = +j\,66.6 \text{ ohms}$$

$$R_{3p} = 138.9 \text{ ohms}$$

$$X_{3p} = +j\,104 \text{ ohms.}$$

The parallel resistances are then combined to give us the input parallel resistance.

$$\frac{1}{\frac{1}{R_{1p}} + \frac{1}{R_{2p}} + \frac{1}{R_{3p}}} = \frac{1}{\frac{1}{312.5} + \frac{1}{178.6} + \frac{1}{138.9}} = 62.5.$$

The parallel reactances are then combined to give us the input parallel reactance.

$$\frac{1}{\frac{1}{X_{1p}} + \frac{1}{X_{2p}} + \frac{1}{X_{3p}}} = \frac{1}{\frac{1}{+j136.2} + \frac{1}{+j111} + \frac{1}{+j104}} = +j38.6.$$

A capacitor of 38.6-ohm reactance is then connected across the input to resonate the circuit and to provide a resistive input impedance of 62.5 ohms. As mentioned before, this impedance may be transformed to any value by means of a T network.

The phase shifts encountered in the branches are given by this relationship

$$\theta = \tan^{-1} \frac{X_s}{R_s}$$

and

$$\theta_1 = \tan^{-1} \frac{114.6}{50} = 66.5°$$

$$\theta_2 = \tan^{-1} \frac{80.4}{50} = 58.1°$$

$$\theta_3 = \tan^{-1} \frac{66.6}{50} = 53.1°$$

An interesting special case of this power divider is called the quadrature power divider. Two outputs are required to have a phase difference of 90° with any power division ratio. Two L networks are used with one being a lagging network with an inductor in the series arm and a capacitor in the shunt arm and the other a leading network with a capacitor in the series arm and an inductor in the shunt arm. Then, if the input parallel resistance has the same value as the loads, the shunt arms will have equal values of reactance of opposite sign and can be omitted and the power divider is a branched network with a capacitor in one series arm and an inductor in the other. The solution of this divider is accomplished in the same manner as the hybrid divider with the only special requirements being that the input parallel resistance is the same as the load resistances, the phase difference is 90° and only two outputs are used.

CONCLUSION

The generally accepted types of power dividers have been described and design examples have been shown. Many combinations of these types are possible and possibly desirable depending upon the particular application involved.

These combinations are left to the ingenuity of the reader.

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