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Standard Broadcast Antenna Systems

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The purpose of this section is to furnish useful information to the engineer, technician, and operator in a broadcasting station. The material is divided into text and handbook types of presentation. An effort has been made to approach each new subject gradually in the text, while in the appendices, design equations and data have been presented in handbook style with the aim of making them most useful to the technician and operating engineer.

First, the single tower is analyzed. It is then used as elements in a two-tower directional-antenna array before going to the more complicated arrays.

Antennas form the dominant theme supported by other items such as coupling networks and monitoring circuits. The topics of adjustments and field measurements are treated in a way thought to be most useful to a man in the field.

The chief purpose of a radio-broadcasting antenna is to radiate the energy supplied by the transmitter efficiently. A simple antenna can do this job quite well. It is usually a vertical tower that radiates the energy equally in all directions along the ground.

A secondary purpose of the antenna system may be to concentrate the amount of radiation in the directions that it is wanted and to restrict the radiation in the directions it is not wanted. This may require a very complicated directional-antenna system if the requirements are great.

The antenna is the last point in the system under the control of the radio-broadcasting station. Radio waves radiated from the transmitting antenna are propagated through space to the receiving antenna. The only control over these propagated waves is in the selection of the antenna site, the polarizations, and the strength of the signals leaving the transmitting antenna. The selection of the antenna site is determined by many considerations, such as ground constants, terrain, distance and direction to populated areas to be served, distance and direction to the areas to be protected, and last but not least the availability of a suitable land area to install the necessary towers and ground system.

For standard broadcast stations, vertical polarization is used because of its superior ground-wave-propagation characteristics and the simplicity of antenna design. The strength of the signal from the transmitting antenna, in any given direction, depends upon the output power of the transmitter and the antenna design. Since the output power is regulated by the Federal Communications Commission for the class of stations involved, the only factors remaining under the engineer's control are the antenna location and design. These factors go hand in hand when designing directional antennas for broadcast purposes.

THE SINGLE-TOWER NONDIRECTIONAL ANTENNA

Current and Voltage Distribution

The vast majority of radio-broadcasting stations have single-tower antennas that are neither top-loaded nor sectionalized. Most of them have an insulator near the ground. Such towers all have a current distribution with a zero value at the top as shown in Fig. 1. The maximum value of current is 90° down from the top on a theoretical antenna, while on all practical antennas it is less than 90° down from the top. This is owing to the fact that the velocity of propagation slows down as the cross section of the tower is increased. For the average uniform-cross-section tower the current maximum is about 84° down from the top.[1]

The general shape of the current distribution on a tower is that of a sine wave given by

$$i_a = I_a \sin (G - y) \qquad [1]$$

Note—Superscript numbers refer to references on page 203.

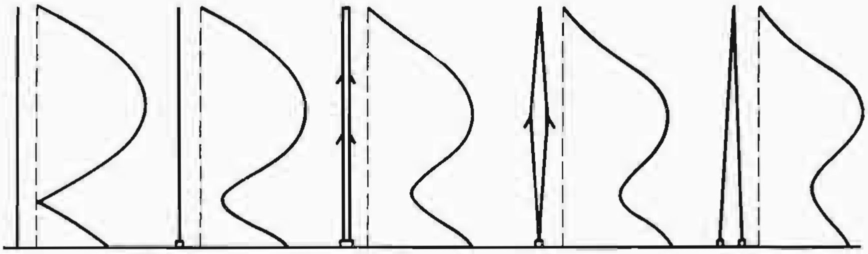


Fig. 1. Practical compared with theoretical current distribution on vertical radiator.

where i_a = current amplitude at distance y above the ground as shown in Fig. 2a, amp
I_a = maximum current amplitude, amp
G = height of antenna, deg
y = height of current element i_a, deg

For most purposes it is entirely satisfactory to consider the current distribution as an exact sine wave through this equation. This is practically true for a vertical wire as shown in Fig. 1b. It is also a good approximation for a uniform-cross-section tower as illustrated in Fig. 1c. For the diamond and tapered types as shown in Fig. 1d and e, the approximation may not be satisfactory.

The general shape of the voltage distribution is very nearly that of a cosine wave as shown in Fig. 2 for the theoretical case and is expressed by the equation

$$e_a = E_a \cos (G - y) \qquad [2]$$

where e_a = voltage amplitude at distance y above the ground as shown in Fig. 2b, volts
E_a = maximum voltage amplitude, volts

and G and y are as defined in Eq. 1. If the tower is not tall enough for the current distribution to have a minimum below the top of the tower, then the maximum value of voltage will be at the top of the tower. It is necessary to visualize the shape of the voltage distribution along the tower because of the need of good insulators at the high-voltage points. If sufficient insulation is not provided in the guy cables or at the tower base, the current may arc-over at these points and disrupt the broadcasting service. If the initial design has poor insulation, then redesign with adequate insulation should be considered.

Some towers are not insulated at the base. Such towers are shunt-fed at some point above the base. This type of tower is less expensive, has no

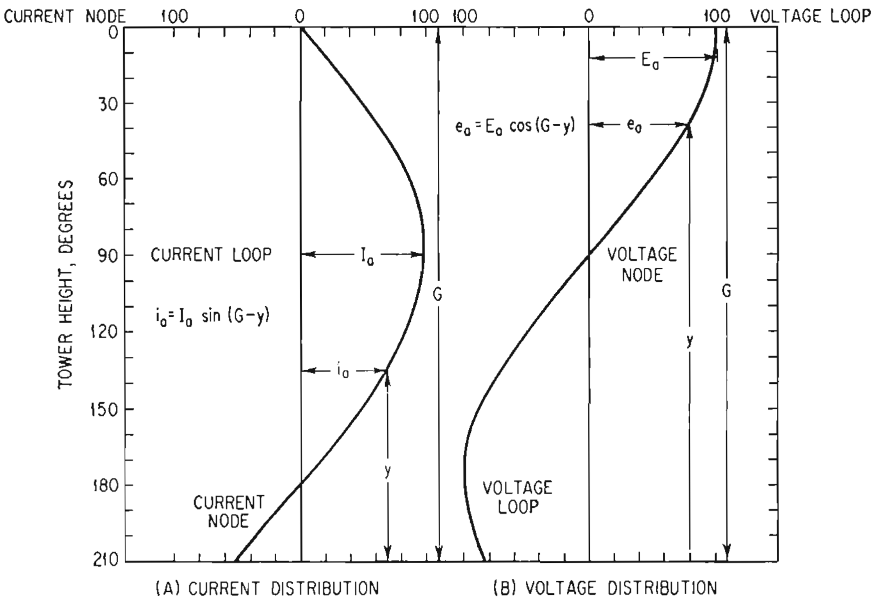


Fig. 2. Theoretical current and voltage distribution on a vertical radiator.

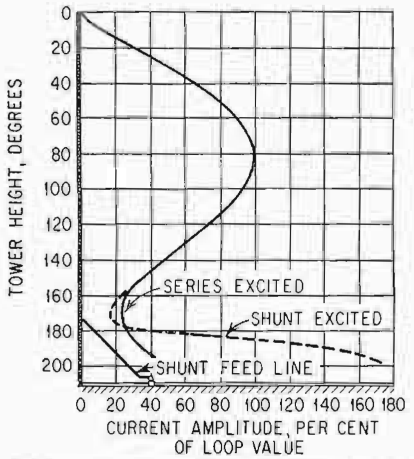


Fig. 3. Experimental current distribution on shunt- and series-fed tower. (Morrison and Smith, *Proc. IRE,* June 1937.)

dangerous base voltage, and is less vulnerable to lightning. The feed line can be used to some extent as the matching network to couple the transmitter or transmission line to the tower. The current distribution above the feed point is essentially the same as for a base-insulated tower.[1] The current below the feed point deviates materially from a sine wave as shown in Fig. 3. This is of little consequence in nondirectional operations. It cannot be tolerated in critical directional-antenna systems because of its effect on deep minima of the radiation pattern.

By proper design the coupling can consist of a slant feed cable with a series capacitor to couple into the transmission line. In general, as the height of the feed point on the tower is raised, the resistance and positive reactance increases. When the horizontal distance to the feed point from the tower base is increased, the resistance and positive reactance decreases. The exact position of the shunt feed line can best be determined by experiment.

The slant cable can consist of two parallel cables properly insulated and having the proper physical dimensions. Such an arrangement can be adjusted to couple directly into a transmission line.

Another method which eliminates the undesirable radiation effects of the slant line is to connect several cables to the tower one-quarter wave ($\lambda/4$) from the base and stretch them down to the ground with insulators at the lower end. These cables form a short-circuited one-quarter-wave ($\lambda/4$) transmission line which is open-circuited at the base, thus making it possible to feed the tower through these cables connected in parallel.

Sectionalized towers have been used for some time and have taken on added importance with the advent of FM and TV broadcasting. A sectionalized tower, in addition to the ordinary base insulator, has one or more insulators in the tower above the base. Very tall towers are often used to support FM and TV antennas in order to achieve the desired height above average terrain for maximum coverage of the FM or TV station. If this is the sole objective, the tower does not have to be sectionalized with insulators.

In some cases a tall tower is sectionalized for the purpose of preventing undesired reradiation when in the vicinity of AM towers, particularly if the other towers are elements of a directional-antenna array. In other cases it is desired to use part or all of the sectionalized tower as active AM radiating elements. This may not be so easy to accomplish as one would expect. Moreover, it requires considerable effort and planning to design the system properly. The Federal Communications Commission will require proof that the antenna system is operating properly, especially if the sectionalized tower is near or part of a directional-antenna array.

Sometimes a tall sectionalized tower is constructed for the purpose of obtaining greater AM broadcast coverage by properly controlling the current distribution on the tower.[2,3] When towers are sectionalized for this purpose, considerable attention is given to the current distributions on the various sections of the tower. This is necessary because the radiation from the current elements on each section must add properly to produce the greatest or optimum field-strength effects. It is well worthwhile to look seriously into these possibilities, even if the tower is to be used for FM and TV operations in addition to AM operation.

Vertical-radiation Characteristic

A nondirectional tower, whether series- or shunt-fed, sectionalized or nonsectionalized, top-loaded or without top loading, has its own vertical-radiation pattern sometimes called its vertical-radiation characteristic. This is simply the amount of signal radiated at all elevation angles above the horizontal plane with respect to the horizontal-plane radiation. Its calculation is usually made using the assumption of sinusoidal current distribution on the radiating portion of the tower.

The current distribution can be controlled by the height and shape of the tower. On a sectionalized tower the magnitude and phase of the current on the lower sections can be controlled with respect to the current on the top section. This permits such a tower to possess a family of vertical-radiation characteristics.[2,3]

The vertical-radiation characteristic of a vertical nonsectionalized, base-insulated tower is given by

$$f(\theta) = \frac{\cos(G \sin\theta) - \cos G}{(1 - \cos G)\cos\theta} \qquad [3]$$

where $f(\theta)$ = vertical radiation characteristic
G = electrical height of antenna, deg
θ = elevation of observation point, deg

The derivation of Eq. 3 is given in Appendix A as a special case for a sectionalized tower when the top section is zero. The curves showing $f(\theta)$ as a function of height were published in several forms in the 4th edition of the "NAB Engineering Handbook."[4] The most useful form is reproduced in Appendix A.

Self-Impedance

A radio tower has a different impedance at every point along its height. Two points are of special interest. One is at the current loop which is the current maximum approximately 90° down from the top of the tower if it is not top-loaded, and the other is at the point where the tower is fed at the base.

Much effort has been made in recent years to find a reliable means of calculating the base impedance of a tower. The average characteristic impedance, usually called Z_0 of the tower appears to play an important role in such calculations.[4]

Assuming a sinusoidal current distribution and the conservation of power between the loop and the base for a simple tower without top loading, the base and loop radiation resistances can be related by the simple equation

$$R_{base} = \frac{R_{loop}}{\sin^2 G} \qquad [4]$$

where R_{base} = base radiation resistance, ohms
R_{loop} = loop radiation resistance, ohms
G = height of tower, deg

This equation for base resistance is quite reliable for antenna heights up to 120°. The loop and base radiation resistance along with the theoretical field strength, assuming a perfect ground, are shown in Appendix A.

It should be remembered that any set of calculations may not and usually will not agree with the actual values determined from measurement after the tower has been constructed. For this reason about the best that one can hope for is to make as intelligent an estimate as possible. The base impedance is affected by stray capacity and inductance effects and may be considerably different from the approximate theory when the tower is of the order of a half wave ($\lambda/2$) high.

The loop impedance of a single tower serves an important role by virtue of the fact that the calculated base impedance usually disagrees with the measured value and also because some towers are fed at or near the loop point. For example, a 90° tower fed at its base is also approximately fed at its loop; thus

$$Z_{base} = Z_{loop} = 36.6 + j21.3 \qquad [5]$$

where Z_{base} = base impedance, ohms
Z_{loop} = loop impedance, ohms
$j = \sqrt{-1}$, making the second term an inductive reactance

This means that the antenna is series resonant, without reactance, when the height G is slightly less than 90°.

Ground System

A single AM tower is not complete without a ground system. To feed power into such a tower it is common practice to couple the output of the transmitter across the base insulator. The tower base forms one terminal, and the ground system forms the other terminal. Simple antenna theory assumes the ground plane to be a perfect conductor which acts like a mirror plane to the radio waves. In practice it is not a perfect conductor and may introduce a series-ground-loss resistance from a fraction of an ohm to several ohms.

A rather common rule of thumb is to use 2 ohms' loss resistance for the copper-wire ground system consisting of 120 radials 90° long. This ground-loss resistance can be decreased by reducing the E loss due to the electric field and the H loss due to the magnetic field.

When the tower is near a half wavelength in height, there is a voltage maximum at the base with a resulting strong electric field that results in high E losses owing to the displacement current passing from the antenna through the earth to the radial ground wires. This loss can be materially reduced by using an expanded copper screen around the antenna base or increasing the number of radial conductors and placing them very near the surface or under a layer of asphalt pavement which has very low loss for the electric displacement current.

The H loss due to the magnetic field extends out a considerable distance. This loss is due to the radial current which divides between the ground conductors and the earth. It can be decreased by increasing the number of conductors and extending their length. This will cause a larger portion of the current to be in the copper radials where the resistance is very low.

A typical ground system may consist of No. 10 copper wire buried 4 to 12 in. deep. Usually this area is made into meadow or grassland that is mowed often enough to prevent tall grass that

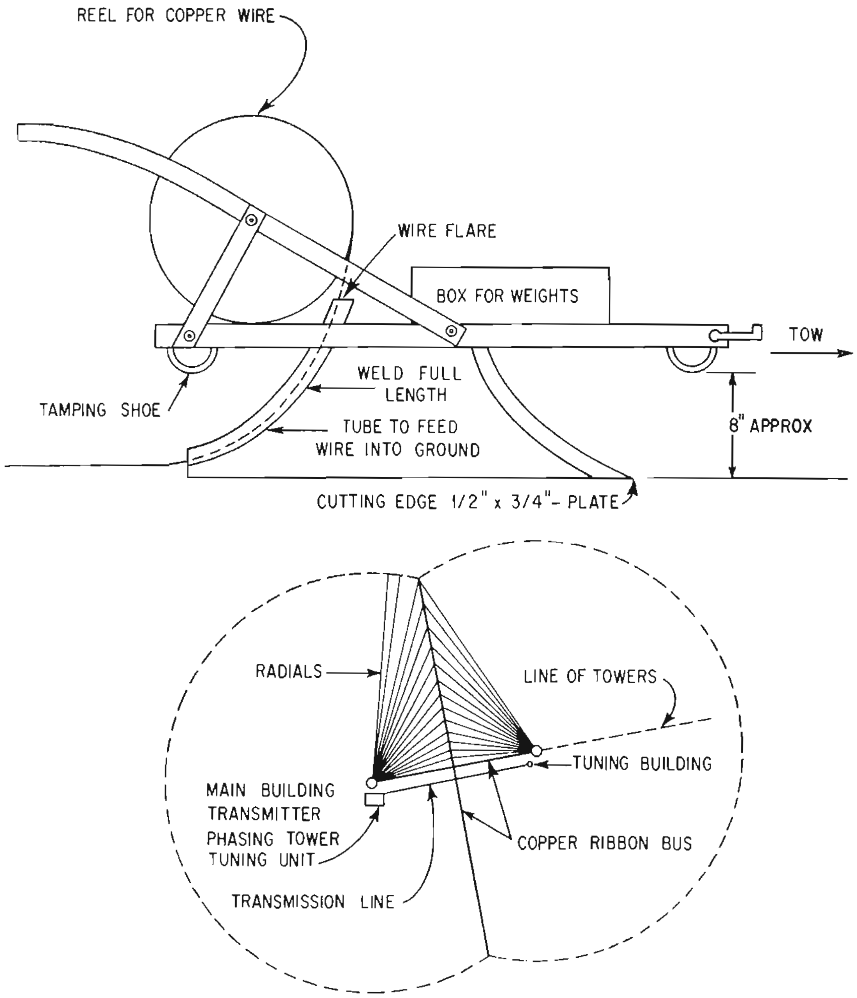


Fig. 4 Design of plow for laying ground wires and typical two-tower ground system.

may cause considerable E loss under certain conditions. If it is necessary to till the soil, the wires must be buried deep enough to avoid mechanical injury.

It is common practice to use a wire plow, as shown in Fig. 4, to lay the ground system. The wire plow consists of a thin vertical steel blade to cut a slit in the ground. At the rear edge of the blade there is a small tube through which the copper wire passes from a wire reel into the ground. The depth of the ground wire can be controlled by the adjustment of the vertical blade with respect to horizontal sled runners or wheels which support the plow mechanism. Soft- or medium-hard-drawn copper wire is easier to handle in the field than hard-drawn copper wire. It can also stand more mechanical stretching before breakage occurs.

The radial wires are usually plowed in, starting from the tower, and driving a tractor pulling the plow toward a guidepost at the edge of the ground system. It is convenient to provide a copper wire or cable ring around the tower base to which each radial ground wire can be mechanically fastened while the radial is being installed. The radial wires must then be soldered or brazed to this ring to provide a good electrical connection. Copper ribbon can then be bonded to the copper ring and run to the ground-system terminal of the antenna. Copper-clad stakes are commonly used to hold the copper ring in place and act as a lightning ground. These stakes are driven down level with the copper ring, and the two are brazed together to form a good electrical connection.

Expanded copper mesh is commonly used inside the copper tie ring or square. Its primary purpose is to terminate the E field. Its secondary job is to carry the radial ground-system current. However, if the amount of copper in the mesh is inadequate, then radial copper straps can be added in this area and bonded to the expanded copper mesh.

Tower Lighting and Painting

The Federal Communications Commission has rules for suitably lighting and painting radio towers so they can be seen from aircraft, thus minimizing their hazard. They are also marked on the aeronautical charts used by aircraft pilots. Part 17, Construction, Lighting and Marking of Antenna Structures, of the FCC Rules and Regulations covers this subject thoroughly.

In brief, it is necessary to provide warning lights at the top and fractional elevation levels of the tower. For tall towers flasher beacons are required at the top and some intermediate levels. For base-insulated towers it is necessary to transfer the a-c power across the base insulator. This is commonly done by RF choke coils, Austin-type power transformers, or the use of quarter-wave ($\lambda/4$) isolating stubs.

Radio towers must be painted with alternate strips of international orange and white paint. The number and width of the strip are covered in the above regulations.

Lightning Protection

Radio towers are vulnerable to lightning; hence it is very important to provide the necessary protection. Lightning rods should be provided at the top of the tower to protect the flasher beacon. Choke coils, large values of resistance, oil-filled insulators, or isolation stubs should be used to drain the static charges across the sectionalizing and base insulators. Ball gaps or horn gaps should be placed across the insulators to carry the high current surges.

The ground system around the tower base should have nonfusible cable or conductor. It is good practice to terminate these cables in copper-clad ground rods not far from the tower base. In some cases the radial ground system itself may be adequate to handle the lightning surges.

An important consideration is the protection of the base current antenna meter and the RF coupling equipment. It is good practice to provide a tower-grounding disconnection switch on the antenna side of the RF coupling equipment for the protection of technical personnel that must maintain it. During operation the antenna terminal of the RF coupling equipment should have a horn-gap path so lightning discharges can be bypassed directly to ground. The lightning paths should be as direct and short as possible to ground.

With regard to the RF antenna meter the best practice is to connect it into the circuit with a double-pole-double-throw (DPDT), make-before-break switch, as shown in Fig. 7a. The meter can then be inserted or removed completely from the circuit during operation. When removed from the circuit, there is very little chance for lightning to injure it because there are no metallic connections to it. The meter is always removed from the circuit except when it is necessary to make a reading. It is sometimes necessary to adjust the length of the shorting loop so it will have the same inductance as the meter loop.

A less expensive and less desirable method is to use a single-pole-double-throw (SPDT), make-before-break switch as shown in Fig. 7b. The least expensive and least desirable method is to use a single-pole (SP) shorting switch as shown in Fig. 7c. The shorting switch shunts most of the current around the RF meter; however, a lightning surge may be sufficient to injure it.

It is difficult to predict what a lightning stroke will do, particularly if it is a direct hit on the tower. The stroke may jump to the transmission-line side of the coupling network; hence it is advisable to provide lightning-protection gaps at the tower end of the transmission line.

Adjustments

A transmission line can usually be coupled into a tower by the use of a series and shunt element. If the tower is in a directional-antenna array, then three tuning components are usually employed so the phase can also be controlled at this point.

Usually it is necessary to design and order the coupling network components before the tower and ground system are installed. In this case sufficient latitude must be allowed for variations of the inductive and capacitive components to match into the tower impedance from the transmitter or transmission line. If the tower and ground system are already installed, antenna input impedance measurements can be made at the operating frequency. It is therefore possible to determine, quite accurately, the value of the components required, but even in this case it is advisable to provide a reasonable tolerance for adjustment in the field.

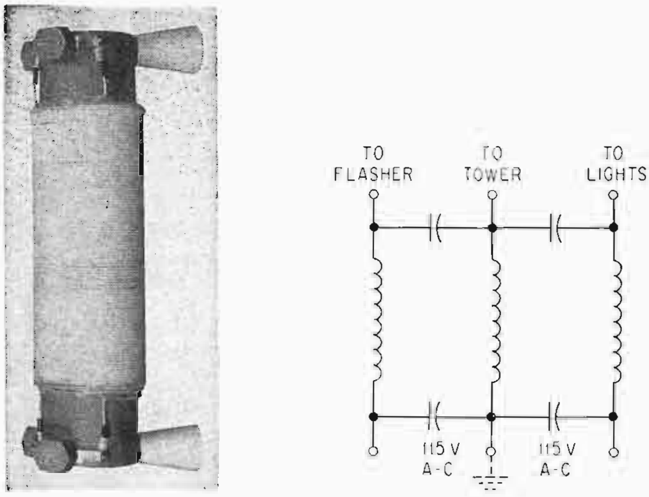
The problem of coupling a transmission line to a sectionalized tower is usually much more involved. The sectionalized tower is, in effect, a collinear vertical directional-antenna system and therefore should be provided with both magnitude and phase control for each of the several elements. For this type of antenna it is usually desired to obtain the optimum vertical-radiation characteristic. If this is done, the results should be checked by field-strength measurements. In this type of antenna it is desirable to provide considerable latitude in variations of the adjustment components.[2,3]

The design information necessary for matching from the transmission line into the antenna is covered in Appendix B.

Inverse Field Strength at 1 Mile

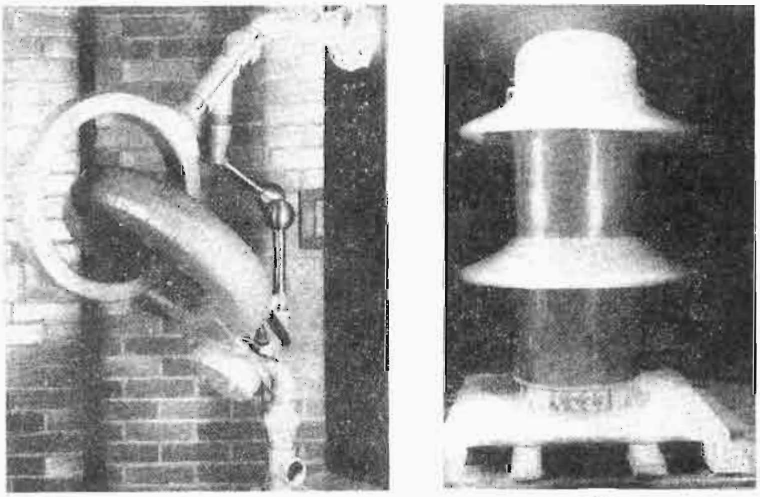
The inverse field strength at 1 mile, sometimes referred to as the unattenuated field strength at 1 mile, is the field strength at 1 mile when the only attenuation is that of distance. It is a theoretical value and is considered primarily for comparison purposes. This concept removes the frequency and ground attenuation effects. Nondirectional- and directional-antenna patterns can be compared on this basis.

The vertical-radiation characteristic, for example, is merely a comparison of the inverse field strength at 1 mile at all elevation angles above the horizontal plane with the inverse field strength at 1 mile in the horizontal plane. When it is necessary to express the radiation from an antenna the value is usually given as the horizontal-plane inverse field strength at 1 mile. It can be expressed either with or without the inherent losses of the antenna system.



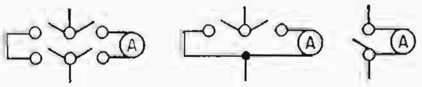
(A) PHOTOGRAPH (B) WIRE DIAGRAM

Fig. 5. Antenna-lighting choke coil.



(a) (b)

Fig. 6. Austin-type transformers: (a) Air-insulated and (b) Oil-insulated.



(A) DPDT MAKE-BEFORE-BREAK SWITCH (B) SPDT MAKE-BEFORE-BREAK SWITCH (C) SP SHORTING SWITCH

Fig. 7. Typical antenna meter switching circuits.

When submitting inverse-field-strength information to the Federal Communications Commission it is common practice to include the antenna-system losses. Nondirectional antennas produce but one such value, since the input power is determined by direct input-power measurements. In other words, the input power is determined by multiplying the measured input resistance by the square of the antenna current measured at the same point. In equation form,

$$P_a = I_a^2 R_a \qquad [6]$$

where P_a = antenna power input, watts
I_a = antenna current, amp
R_a = antenna resistance, ohms

A nondirectional antenna theoretically produces only one value of inverse field strength at 1 mile, which is the same at all horizontal bearings from the antenna. Nondirectional-antenna patterns are graphically described by a circle, the radius of which is the inverse field strength at 1 mile. The pattern of a single-tower radiator is usually considered to be nondirectional. If the feeder is nonsymmetrical, such as in the case of a shunt-fed antenna, or if there are objects which reradiate in the vicinity of the tower, the horizontal pattern will not be circular or nondirectional but will have some directivity.

If the horizontal pattern is directional or nondirectional, its equivalent nondirectional effectiveness can be expressed as the root-mean-square (rms) inverse field strength at 1 mile, or simply its rms value. The rms value is the radius of a circle which has the same area as the pattern formed by all the inverse-field-strength values at 1 mile in all horizontal directions.

A directional-antenna pattern can be quite well described by plotting the inverse-field-strength values at 1 mile at intervals of 10° on polar graph paper. The rms value of the pattern can be obtained by taking the square root of the sum of the squares divided by the number of squared values, thus

$$E_0 = \sqrt{\frac{E_{10}^2 + E_{20}^2 + E_{30}^2 + \cdots + E_{360}^2}{36}} \qquad [7]$$

where E_0 = rms field strength, mv/m
E_{10} = inverse field strength at azimuth angle of 10°, mv/m
E_{20} = inverse field strength at azimuth angle of 20°, mv/m, etc.

The rms value can also be obtained by using a polar planimeter to measure the area and in determining the radius of the circle having the same area. This radius is the rms value of the pattern in the same units used to plot the directional pattern. It is common practice to measure the field strength and plot patterns in millivolts per meter, abbreviated mv/m.

The Federal Communications Commission does not normally require proof of performance measurements on single-tower nondirectional antennas. Therefore measured values of inverse field strength of nondirectional antennas are not in general available to the public at the reference room of the FCC, Washington, D.C. The description of existing towers of nondirectional radio stations is usually not complete except as to height and type. In most directional-antenna proof-of-performance reports it is required to show nondirectional measurements on a single tower in the array either before the other towers are erected or when the other towers are detuned so they will contribute a minimum of reradiation. This information is available at the reference room of the FCC.

Attenuated Field Strength

The attenuated field strength is the amount of signal left after it has been diminished by distance, ground conductivity, ground inductivity, and all other effects encountered by the signal between the antenna and the point of measurement. The field strength is also a function of the operating frequency and unattenuated field strength at 1 mile in the direction from the antenna toward the measuring point.

Only after a tower is constructed is it possible to determine the actual attenuated field strength at any point. Sometimes a test antenna and transmitter are installed at a proposed site to determine more precisely the coverage or interference to be expected. The attenuated field strength is determined by measurement with a field-strength meter properly calibrated and operated.

When the unattenuated rms field strength is needed to prove compliance with minimum requirements, it is necessary to make proof-of-performance measurements on a nondirectional antenna. This consists of a set of attenuated field strength measurements made on each of eight or more radials. The attenuated measurements are then used to determine the unattenuated field strength at 1 mile in the direction of each radial. The manner of taking measurements and the means of analysis are fully described in Part 73 of the FCC Rules and Regulations.

TWO-TOWER DIRECTIONAL ANTENNA

Radiation-Pattern Shape

One purpose of a directional antenna is that of producing a greater radiation in one or more directions than a nondirectional antenna would produce with the same power. Another purpose is to produce a small radiation in one or more directions. The latter consideration is more often required than the former because it is the rule rather than the exception that when a directional antenna is required, its radiation pattern must be so designed that it will not cause interference in any area, thereby depriving that area of one or more existing services. Or if this is not possible, the interference that is created must be limited to population that would feel the impact of losing service least. In other words if interference must be caused to some population, it is more desirable that it be caused to population that already has plenty of service rather than to population that has a meager amount of service or no service at all.

An existing radio station is faced with the same type of interference problem if it desires to change its directional-antenna pattern or if its directional antenna gets out of adjustment. A careful theoretical study of the two-tower directional antenna provides the owner and operator with a very useful tool for maintaining the correct operation.

The two-tower directional antenna, besides being the simplest of directional antennas, is often a basic unit of a three-tower or more array. It may be compared with a single tower in the following respects: The signals produced are but one signal as far as the receiver is concerned. Each tower produces a pattern having an rms value that must be above a specified minimum value according to the FCC rules. The current and voltage distributions on the towers in a directional-antenna array are usually assumed to be sinusoidal and cosinusoidal, respectively, just as they are on a nondirectional antenna. The vertical-radiation characteristic of each tower is defined in exactly the same way. The two-tower operation converges to a single-tower operation if the tower heights are equal, the phasing of the currents in the towers are the same, and the spacing of the two-tower array approaches zero.

This is as far as one can go in comparing the single- and two-tower operations. The two-tower operations are different in the following respects: The spacing of the towers in the two-tower array makes the instantaneous signal of one tower out of phase with the corresponding instantaneous signal for the other tower. This means that the towers can be spaced so that the signal can be made to add or subtract as desired. This, of course, cannot be done with a single tower. Also by means of phasing circuits it is possible to control the instantaneous signals from each of the towers in the two-tower array. Usually the phasing circuit is placed in only one of the tower circuits, since the object of interest is to control the phase of the current in one tower with respect to the phase of the current in the other tower. Furthermore, the magnitude of the current in one tower can be controlled with respect to the magnitude of the current in the other tower. This permits the control of minima depth in the directional antenna pattern. Thus, the spacing, phase, and current ratio controls available in the two-tower array are not available in the single tower, which must have a circular pattern.

To understand better the two-tower directional antenna, consider the following description. Let one tower, say tower 1, be the reference tower and fixed in location. Let the other tower 2 be free to move on a straight line, say due north from tower 1. Let us also by means of the phasing and coupling circuits maintain the same electrical phasing and current magnitude in the two towers. Let us further assume that a person P_n is due north of tower 1 and that another person P_s is due south of tower 1. Now, if tower 2 is gradually moved north, P_n will note that the signal from tower 1 is being received at the same time as before but that the signal from tower 2 is being received sooner. The person P_s will note that the signal from tower 1 is being received at the same time as before but that the signal from tower 2 is being received later. The person P_n might say that the signal from tower 2 leads the signal from tower 1, while the person P_s will say that the signal from tower 2 lags the signal from tower 1.

Actually the persons P_n and P_s can observe only the combination signal from towers 1 and 2. When tower 2 is moved 180° north of tower 1, the person P_n will note an absence of signal. This means that the signal from tower 2 leads the signal from tower 1 by 180°, and since the signals are the same magnitude, they cancel and produce a null effect. The person P_s will also note a null effect which is due to the signal from tower 2 lagging the signal from tower 1 by 180°.

Now, consider person P_e due east and person P_w due west from tower 1 and observing the resulting signal in these directions. They will both note at all times, regardless of the location of tower 2, that both signals arrive at the same time. Hence, the signals are always in phase and add completely. When the towers are spaced 180° and are in phase, it is therefore seen that the pattern has the shape of a figure eight with the lobes east and west and the nulls north and south. This case is illustrated in column 1 of Figs. BC and BI.

One other pattern will be similarly described. Let all the above assumptions hold except that tower 2 is now always phased by means of the electrical phasing circuits to be 90° ahead of tower 1. Now when tower 2 is 90° north of tower 1, the person P_n will note that the signal from tower 2 leads the signal from tower 1 by 180° and therefore complete cancellation occurs, with the result of a null. The person P_s will note that the signal from tower 2 is exactly in phase with the signal from tower 1 and therefore they completely add to form a lobe. The persons P_e and P_w will receive a signal 41 percent greater than the individual signal from tower 1 or 2 because these signals are 90° out of phase. This pattern has the shape of a cardioid with the lobe to the south and the null to the north. This fact can be further explored by referring to column 3, Fig. BC, or column 2, Fig. BH.

The above two patterns just described serve to illustrate the effect of both spacing and phasing. From this it is seen that the pattern shape is affected both by the spacing and phasing. The above discussion pertains only to the pattern shape in the horizontal plane. For the more general case see Appendix B, Two-tower Directional Antennas.

Radiation-Pattern Size

The Federal Communications Commission provides specific amounts of power for the various classes of radio-broadcasting stations. The rules permit the following amounts of power; 100, 250, 500, 1,000, 5,000, 10,000, 25,000, and 50,000 watts. It is therefore necessary to select the value of power to be used and make sure the individual towers will produce enough inverse field strength at 1 mile so the rms of the directional-antenna array will meet minimum radiation requirements of the rules. The pattern used for rms size consideration is the one in the horizontal plane. If the directional antenna is inefficient, the rms pattern may be too small. It is therefore important to be able to determine pattern size.

There are many factors involved in determining the pattern size of a directional-antenna array. The principal ones are phasing, spacing, and height of the towers and the ground-system resistance losses. The pattern size can first be determined assuming no loss in the directional-antenna system. This value is computed from a formula which is based on one tower operating alone, the self-resistance of both towers, the mutual impedance between the two towers, the current ratio, and the relative current phase. The total resistance losses of the directional-antenna system are commonly computed by assigning a series loss resistance to each tower.

The mutual impedance between two towers can be calculated from cumbersome formulas, or it can be found more quickly by graphical means. The mutual impedance between equal-height towers for various spacings is given in Figs. BM and BN. The mutual impedance is referred to the loop, or maximum current position. It is convenient to use the loop values for computation but necessary to use the base mutual-impedance values when tuning up a directional-antenna array.

The procedure for determining pattern size is to use the above factors in the formula, as given in Appendix B, page 210, to calculate the field strength of the reference tower when operating in the directional-antenna array. The field strength from the other tower can then be obtained by applying the field ratio that was used to determine the pattern shape in the horizontal plane. The use of these values of E_1 and E_2 in the horizontal-pattern formula results in the correct-sized pattern.

RMS Field Strength at 1 Mile

It is now possible to calculate the rms field strength of the directional-antenna pattern in the horizontal plane. The appropriate formula in Eq. BN is easy to apply, and the results are accurate. Moreover, these calculations serve as an excellent check on the rms of the plotted pattern which can be measured by using a polar planimeter or Eq. 7.

Monitoring System

Practically all directional antennas have monitoring systems consisting of individual antenna current meters, a common-point input current meter, and an antenna monitor to give the relative phase between the towers. Most directional antennas have antenna meters at each tower base and corresponding remote meters in the transmitter operating room. This makes it possible for the operator on duty to observe the operating conditions continually and make the necessary log entries.

Usually, when the directional antenna is installed, all the antenna meters are calibrated against a meter of known accuracy. Sometimes RF meters are injured in shipment or for some other reason will not give accurate readings. It is good practice to retain an accurate meter so the calibration of all the RF meters can be checked from time to time as needed. The antenna meters at the towers are read daily or weekly, and the remote meters checked for accuracy. Most remote meters are provided with an adjustment so their reading can be made to correspond exactly to the antenna meter.

The calibration of the antenna monitor at the time of the antenna installation is as a rule adequate as long as the monitoring loops and sampling lines stay in good physical and electrical condition. It is important to have an antenna-monitoring system that is more reliable than the directional-antenna system. If the phase readings vary from the licensed value and there is no noticeable change in the antenna current readings, it is advisable to question the antenna monitor before making any readjustments on the directional-antenna system. In such cases the field strength at the monitoring points should be checked. If these readings are normal, the trouble is probably in the antenna-monitor system.

Feeder System

All that is required for a single tower is to match the antenna to the transmission line, and in turn the transmission line must be matched to the transmitter. In most cases the transmitter will match directly into the transmission line, and if the tower is next to the transmitter building, a transmission line is not required. In some cases it is possible to excite the antenna directly from the transmitter output without a transmission line or coupling circuits.

In a directional-antenna feeder system, power-dividing and -phasing networks are required in addition to transmission lines and matching networks. A typical two-tower directional-antenna feeder system is shown in Fig. 8. At least one tower of a two-tower array must have its driving-point impedance transformed to match into a transmission line. The other tower, if not located close to the transmitter, must also be excited through a transmission line.

In a two-tower directional antenna it is necessary to have the required total phase shift from the common point at the transmitter output to each of the towers. The phase shift must be such that the phase of the tower currents meets the design requirements. When a phasing network is employed, it should operate over a favorable control range so the current in tower 2 with respect to the current in tower 1 can be adjusted and maintained at the proper value. In Fig. 8 the current ratio of tower 2 to tower 1 can be adjusted and maintained by the power-dividing network.

It should be pointed out that the phase- and power-division controls are usually not independent because of the mutual-impedance coupling effects between the towers. In other words a change in the phase control may have more effect on the current ratio than on the antenna-monitor reading. The person that must keep the directional-antenna system in adjustment should therefore operate the various controls and obtain a feel of how the system reacts. Before moving any controls, however, the settings should be noted and recorded so it is possible to return to the original operating condition. Here it is assumed that the feeder system is already operating properly. Only persons with the responsibility of operating and maintaining the directional-antenna system need to have this experience.

If the feeder system has been designed and the antenna system is to be tuned up, it is desirable first to determine the feed-point driving-point impedance values. This can be done by following the procedure in Appendix B. It is then necessary to set up the tower coupling networks so they will match into the driving-point impedances determined above. In addition they should have the correct value of phase so the phasing control will be near the center of its range for proper phase of tower 2 with respect to tower 1. The power-dividing network can then be set up to give the approximate division of power, and if there is a matching network between the transmitter and the power divider, it can be set up to give the proper impedance for the transmitter.

A small amount of RF power can now be fed in at the common point, and adjustments made to approximate the required current ratio and phasing between the two towers. If the driving-point impedances at the towers were computed correctly and the tower matching networks were set up properly, it should not be necessary to make any further adjustments at this point other than to add or subtract phase shift.

It should be observed that the driving-point impedances at the towers will not have their correct values until the feeder system is in final adjustment. Therefore, improper meter readings and standing waves on the transmission lines are

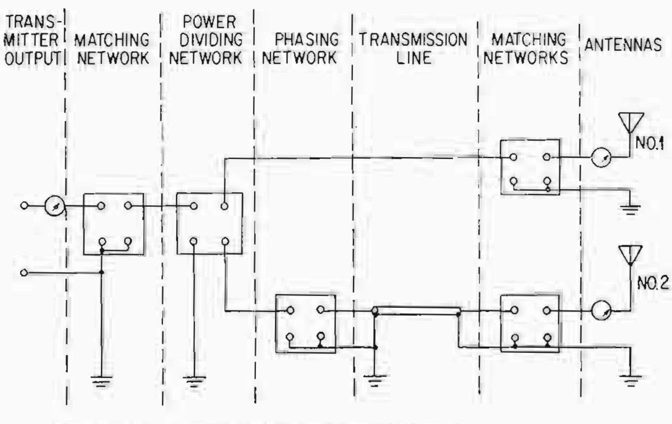


Fig. 8. Block diagram of a directional-antenna feeder system.

to be expected until the final adjustment is approached.

When a directional-antenna array has been correctly adjusted, complete measurement data should be taken before it is turned over to the new user. These data should include not only RF bridge measurements of all components but similar static measurements at several points in the feeder system. These measurements should be made, for example, at the input to each tower impedance-matching network by opening up the transmission line and connecting the RF bridge at this point. Also record open- and short-circuit measurements of all transmission lines.

These measurements can be further supplemented by making dynamic RF voltage readings to ground at a number of points in the feeder system. Also RF current readings should be taken in all network branches.

Then in the event there is a departure from normal performance, the above static and dynamic measurements can be duplicated to assist in the restoration of the feeder system to normal service. This information is particularly helpful if a component becomes defective or is destroyed by lightning.

It is also good practice to make a record of all capacitor and inductor adjustments. It is advisable to mark all coil taps and capacitor settings with lacquer paint. Fingernail polish can be used for this purpose.

Particular attention should be given to make sure that the antenna-monitoring system is in good condition and properly installed. If the sampling lines are of equal length and some of the line must be coiled up, this should be done so that, as nearly as possible, equal lengths are outside the building and will therefore be equally affected by temperature variations. Otherwise, phase variations may occur owing to unequal temperatures of the sampling lines.

The feeder system should be inspected and cleaned regularly. It is advisable to use insect-proof screens over any openings in the housing of feeder system networks. If pressure gauges are used on capacitors or coaxial transmission lines, they should be checked occasionally. These components may be injured if operated without pressure even though the gauge reading appears to be satisfactory.

In the day-to-day operation of a directional-antenna system it should not be necessary to move any controls or make any adjustments. Sometimes temperature and weather conditions will cause slight excursions of the phase and current ratio values. If the tolerance limits are exceeded, then the problem should be analyzed and appropriate corrective measures taken.

DIRECTIONAL ANTENNAS HAVING MORE THAN TWO TOWERS

Comparison with Two-Tower Array

Many directional antennas consist of more than two towers because a two-tower array cannot produce the pattern shape required. It often happens that a two-tower pattern could be used, or it may even be that a nondirectional pattern could be employed if the pattern size were small. Such patterns may be ruled out by the owner because of prestige as to power statements and lack of good service over the urban and rural areas to be covered.

General Case

A general treatment of directional antennas of more than two towers can be limited to an explanation of one tower with respect to a reference point because all other towers are treated in exactly the same manner, since no specific information can be given to distinguish one tower from another. The only facts that need be known about a tower are that it has height, it is sectionalized or not, top-loaded or not, that it has a certain cross-sectional shape and size for each distance above the ground, and that it has a ground system of a specified efficiency. Most of these items have already been discussed.

The important considerations for a general tower in a directional-antenna system are its spacing, phasing, current, and height with respect to the other towers in the array. These four items define its contribution to the radiation characteristics of the array once its individual or single tower characteristics are defined. Hence this general treatment must conclude with an expression that describes the radiation of any tower of a multielement array. This expression is called a vector and is written

$$\dot{E}_k = E_k f_k(\theta) \underline{\left| S_k \cos \phi_k \cos \theta + \Psi_k \right.} \qquad [8]$$

where $\dot{E}_k$ = vector unattenuated inverse field strength at 1 mile for the kth tower while in operation, mv/m

E_k = magnitude of horizontal field strength of kth tower, mv/m

$f_k(\theta)$ = vertical-radiation characteristics of kth tower—always unity y along ground

$\underline{|}$ = vector angle terms are placed in this position. Vector magnitude terms are placed ahead of this angle sign

S_k = spacing of kth tower, deg

ϕ_k = azimuth angle measured clockwise from reference through kth tower, deg

θ = elevation angle from ground or horizontal plane, deg
Ψ_k = electrical phase of current in kth tower, deg

The subscript k was used in this equation to distinguish the radiated field strength of this tower from the other towers in the array. The sum of the vector fields from all the towers gives the total field strength in any direction from the array.

Dropping the subscripts in the above equation for simplicity, the product $Ef(\theta)$ is the magnitude of the vector and $S \cos \varnothing \cos \theta) + \Psi$ is the phase of the vector. The only real value of this general treatment is the meaning it lends to the over-all theory. In fact its understanding is so vital that a clear concept of directional antennas must come from its meaning. As a matter of fact one can treat the whole matter of pattern shape from the vector concept directly without other mathematical complications. If this is done, it is, of course, necessary to know how to add, subtract, multiply, and divide vectors. On the other hand if the mechanism of vectors is not known, one is at a great disadvantage from the start in acquiring a thorough understanding of directional antennas. It is therefore recommended that at least the rudiments of vector analysis be learned by anyone desiring really to understand directional antennas.

Before leaving the general case attention is called to the fact that any directional antenna must be treated as a unit such that the whole operation is considered when any part of the antenna system is changed. For example, it is very important to know how a change in the magnitude or phase of each tower current affects the shape of the pattern, but at the same time it cannot be forgotten that the efficiency is also a factor that must be considered.

The perfect pattern shape and the most efficient operation are seldom attained at the same time. Theoretically this may be possible, but actually the number of towers may be limited or the coverage and protection requirements may have changed after the directional-antenna system was put into operation. Suffice it to say that it is the rule rather than the exception that one or more compromises are necessary before the final operation is attained, and it is best to understand the peculiarities of any particular array so that these compromises can be recognized and dealt with in the most intelligent manner.

Special Cases

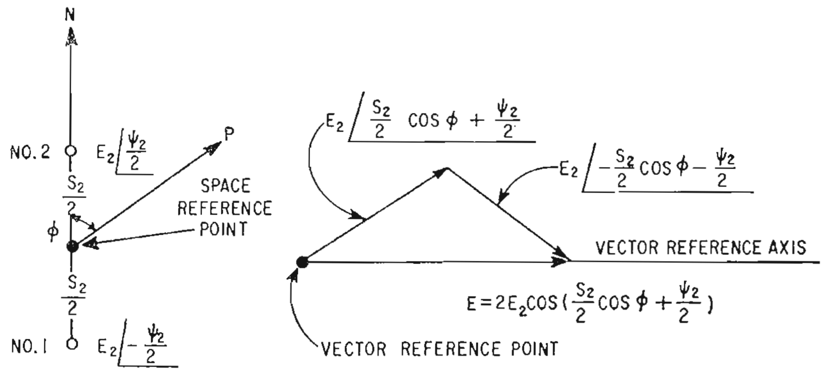
Two-Tower Pattern

The simplest equation for a two-tower pattern is

$$E = 2E_2 \cos\left(\frac{S_2}{2} \cos\phi + \frac{\Psi_2}{2}\right) \qquad [9]$$

where E = inverse field strength at 1 mile, mv/m
E_2 = inverse field strength at 1 mile for each tower acting alone, mv/m
$S_2/2$ = spacing from a reference point midway between the two towers, deg
ϕ = azimuth angle measured clockwise from line of towers, deg
$\Psi_2/2$ = electrical time phase of tower 2 and the negative electrical time phase of tower 1, deg

This equation is for the horizontal plane only, and the terms are especially defined to make the equation simple. The tower heights and current values are assumed to be equal or such that $E_1 = E_2$ along the ground. The spacing S_2 is from tower 1 to tower 2, and the phase Ψ_2 is the phase of the current in tower 2 taken with respect to tower 1 (see Fig. 9).



(A) PLACEMENT PLAN (B) VECTOR DIAGRAM

Fig. 9. Simple two-tower case.

Since a two-tower pattern is symmetrical with respect to the line of towers, it is necessary to compute the values for only one side of the line of towers, that is, Ø from 0 to 180°, and these same values can be used on the other side of the line of towers. For example, cos 10° = cos 350°, and hence the value of E will be the same in these two directions.

The shape of the pattern is controlled by proper selection of spacing S_2 and phasing Ψ_2, while the pattern size is controlled by E_2. For null filling or to determine the radiation above the ground plane, it is necessary to use a more general formula.

Three Towers in Line

A three-tower array will be considered from the simultaneous points of view of theory and practical operation. Three towers are either in line or not in line. If they are in line, as in the case of all two-tower patterns, the three-tower pattern must be symmetrical about the line of towers. Consequently, the pattern shape needs to be computed only on one side of the line of towers.

The three towers may be equally spaced or not equally spaced. If vectors alone are used to analyze the operation, it makes little difference where the towers are located, especially if the current values are not chosen for mathematical simplicity. If simplicity is desired, as it often is, the tower spacings should be made equal. This is covered more thoroughly in Appendix C, Fig. CA.

To relate theory and practice consider an array of three towers in line. The field strength produced by each tower can be represented by a vector at any point in space. The sum of the three vectors at the point is the total field strength produced by the entire array. This sum, or resultant field strength, is itself a vector and can be determined mathematically when all the parameters of the three-tower array are known. The size, or magnitude, of the resultant vector is the all-important part of the resultant field strength as far as the radio receiver is concerned. This is because the receiver does not detect the phase angle; it simply responds to the magnitude of the resultant vector and detects the information on it. The magnitude of the resultant vector at the receiver depends upon the magnitude and phase of the current on each of the three towers, which amounts to a total of six parameters.

The observation that there are six parameters in a three-tower array, which can vary independently of one another, makes it clear that it is necessary to understand which ones are being changed when the array is being adjusted. Otherwise, the possibility of making the wrong adjustment is very great. On the other hand when a desired correction is made by changing the appropriate parameter, the result cannot be wrong. This view is somewhat optimistic because theory does not hold exactly in practice where actual conditions involve the necessity of making compromises. For example, it is not difficult to change a parameter to get a certain field strength at a given point but in so doing the field strength may be adversely affected elsewhere.

An understanding of the directional antenna is quite easily acquired through vector analysis, but in actual field practice it is almost always more efficient to convert the vectors to an ordinary algebraic expression of the field strength at 1 mile. When this is done, the theoretical effects of a change in any one parameter can be quickly determined. The algebraic equations for a three-tower-in-line array are given in Appendix C, Fig. CC.

Three Towers Not in Line

The above treatment of three towers in a straight line should and did receive first consideration because of the number of such directional-antenna arrays in existence. Three towers not in line, sometimes referred to as a dog-leg array, deserves serious consideration because of the number of such arrays that are in existence and because such three-tower patterns naturally fit the predominately unsymmetrical requirements.

There are a number of ways to select the reference point and the reference line in the three-tower array. Only one will be treated here for the sake of clarity and briefness. It is believed to have more significance than other choices because it can be related to the two-tower treatment already covered and to the four-tower paralellogram array.

Further discussion of three towers not in line follows the treatment of the four-tower parallelogram array; hence it is treated as a corollary in Appendix C, Fig. CD.

Four-Tower Parallelogram

Many directional-antenna systems have four towers located at the corners of a parallelogram. The chief reason for the popularity of this type of array is the ease with which the pattern shape can be designed to meet complicated requirements. The design procedure is straightforward, and the computations are relatively easy. Because of these advantages it is surmised that some four-tower arrays have been designed and constructed where three-tower arrays would do the job. This does not necessarily imply that the three-tower array would do a better job. The four-tower array may have better stability in operation and make it possible to modify the pattern without change in tower location, and the efficiency may be better.

When the multiplication form of the four-tower array is used, it can be designed piecemeal by first selecting one two-tower design that will provide the necessary protection in two directions. Then another two-tower array can be selected to protect in two other directions. If the parallelogram design is used, these patterns can be multiplied together and achieve the necessary protection in all four specified directions. The design details are given in Appendix C, Fig. CB.

Arrays of More than Four Towers

It is not believed advisable to go further into the discussion of special cases where more than four towers are involved. If an owner or operator of a multielement array desires to adjust or understand a specific directional-antenna system, it is anticipated that he will take one of two courses: employ a competent consultant to do the work or become sufficiently proficient himself to do the job.

Suffice it to say here that in general the adding of more towers in an array permits protection in more directions or greater protection over wider angles and may in some cases be used to increase the field strength in some directions. The more complicated arrays are usually made up of combinations of two-tower units, and in some cases three-tower units are used. For example, an eight-tower array may be made up of four two-tower units, or a nine-tower array may be made up of three three-tower units.

GENERAL CONDITIONS AND PLANT LAYOUT

The entire plant should be considered from a number of points of view prior to detail study if an existing broadcast station is in need of adjustments or if the broadcast plant is to be built from its beginning.

The location of the transmitter building with respect to the towers is important from the standpoint of access to the transmitter from the road, and if the common point of the directional-antenna system is in the transmitter building, the building should be close to the towers to minimize RF transmission-line losses. If the transmitter building must be located some distance from the towers, it is advisable to feed the RF power over a transmission line to a common point located so that the RF lines from this point to the towers will have a minimum of loss.

The ground-system design for a new directional-antenna array should be laid out such that the copper will be used to best advantage to minimize E and H losses. This does not necessarily mean that a radial system under each tower is the best layout. A study of the H field or ground-current direction at a number of points will help decide how the copper wire should be placed. If the ground system has been installed for some time, it may have deteriorated somewhat. In such cases it is advisable to check for corrosion and mechanical failure of the conductors. The extent of the ground system should be checked against the original construction permit for completeness.

The transmission line, if above the ground, should have a ground strap buried in the ground below it, and this strap should be tied into the ground conductor system. This buried ground strap should be bonded to the ground side of the transmission line at regular intervals.

A general survey of the feeder system in an existing plant will involve not only the location of the matching, phasing, and power-division networks but obtaining information concerning possible inefficiencies and inconveniences in operating arrangements and adjustment controls.

The antenna monitor should be the most reliable indicator of the current of the various towers. If this is not the case, steps should be taken to improve its operation. The location of remote indicating meters should be checked along with their calibration against the antenna ammeters. If any question exists about the current magnitudes, then all the antenna and common-point meters should be calibrated against a meter of known accuracy.

It is good practice to become familiar with the procedure for warming up the transmitter, including starting and stopping operations. This will protect the equipment. Safety precautions, especially when high power is involved, should be obeyed rigorously. It is much better to spend a little more time than to have someone injured or killed.

All tuning controls, including variable capacitors and inductors, should be noted. All taps and settings should be recorded. A complete set of meter and antenna-monitor readings should be recorded before any adjustments are made on an existing system.

The tower lighting, tower insulation, lighting-control circuits, or the phase-monitoring system may require preliminary alterations before tower-impedance or common-point-impedance measurements are made.

The number of transmitters available for regular, auxiliary, or emergency operation should be inspected with regard to switching circuits and studied with regard to the possibility of improvements in convenience and efficiency.

REQUIRED PREADJUSTMENT INFORMATION

Preliminary Computations

Base Driving-Point Impedance

The loop impedance values can be computed or estimated, and from this information the base driving-point impedances of all towers can be estimated as outlined in Appendix B.

Base Driving-Point Currents

Knowing the power and driving-point impedances it is possible to estimate the base driving-point currents. Consideration should be given to the conservation of power principle between the loop and base values as discussed in Appendix B.

Characteristic Impedance of Transmission Lines

In a new or old system it is good practice to measure the characteristic impedance of all transmission lines. This can sometimes be done by measuring the open- and short-circuit impedances and determining the characteristic impedance from the equation

$$Z_0 = \sqrt{Z_{OC} Z_{SC}} \qquad [10]$$

where Z_0 = characteristic impedance, ohms
Z_{OC} = open-circuit impedance, ohms
Z_{SC} = short-circuit impedance, ohms

These values may all contain resistance terms when measured with an RF bridge. When the transmission line is near 90° in length, this method gives very good results. However, when the line is near 180° in length, the open-circuit values will be very large and the short-circuit values will be very small, with the result that Z_0 from the above equation may not be very accurate.

Another method, quite good and acceptable in practice, is to use a decade resistance box at one end of the line and measure the impedance looking in at the other end of the line. Then plot the decade box resistance along the x axis of graph paper and the measured RF bridge input impedance magnitude along the y axis. Where this curve intersects a 45° diagonal line in the first quadrant is located the characteristic impedance magnitude.

This method works quite well for any length of line. Of course, as the line becomes very long, the variation of resistance at the far end will have less and less effect, because if the line is of infinite length, the input impedance will look like the characteristic impedance regardless of the value of load resistance.

This method can be refined in accuracy by first measuring with the RF bridge the values of the decade box resistance. Usually decade resistance boxes have an inductive reactance component that may vary with resistance-value settings. In such cases it may be desirable to parallel the decade resistance box with a small variable capacitor that is adjusted to make the load terminals of the decade box look like a pure resistance. With such refinements it is usually possible to obtain very good results in the field.

Matching Networks

With the above information at hand the transmission line to tower matching networks can be adjusted to transfer the transmission-line-impedance to the driving-point-impedance value. An L section is usually adequate unless the design requires a different amount of phase shift.

The system should be designed for the least amount of phase shift possible consistent with good operating practice because in this way the system efficiency can be maximized.

If it is not convenient to set up dummy driving-point impedances and measure the network from the transmission-line end, then a resistor equal to the characteristic impedance of the transmission line can be connected across the line side of the matching network. The network can then be adjusted until the RF bridge looking in at the tower terminals sees the conjugate value of the driving-point impedance. This means that when the network is connected at this point in a normal fashion, the reactance of the driving-point impedance will be resonated with the reactance of its conjugate value. The resistance of the conjugate impedance is equal to the resistance of the driving-point impedance. Hence the condition for maximum power transfer is achieved.

Probably the only other matching network will be between the power divider and the transmitter. This is the last network to be adjusted. It may be either before or after the common point where the power input to the directional antenna is measured. Usually it is after the common point in order that the common-point input will be a pure resistance at the operating frequency. The loss in this network is then charged against the directional-antenna system, since it is beyond the common point of power measurement.

Phase-Shifting Networks

Usually the phase-shifting networks are designed to operate into the characteristic impedance of the transmission line. Hence, they can be connected directly and maintain an impedance match. It is rather common practice to place the phase-shifting networks in the lines that handle the least amount of RF power, thus minimizing the RF power loss of the feeder system. The transmission line or tower that takes the most power can be run directly to the power-dividing network.

It is usually easier to vary the phase by ganging the rollers of coils rather than using variable capacitors. For this condition a T section would be chosen for a 90° ± 10° phase-retarding network and a π section would be chosen for a 90° ± 10° phase-advancing network as shown in Appendix B, Fig. BZ.

If it is necessary to vary a capacity element, it is sometimes more convenient to place a small fixed capacitor in series with a variable coil so that the series combination will have the correct value at the operating frequency. If the filtering or harmonic properties also have to be considered, this combination may not be satisfactory.

It is important to make sure that the rotating wiping contacts on the phase shifters are in good mechanical condition to wipe smoothly and make good electrical contact continuously. The contacts should feel nearly as cool as the other parts after the equipment has been in operation; otherwise there is too much resistance at the contacts.

Power-Dividing Networks

The power-dividing network usually accepts power from the transmitter output and feeds the proper amounts into transmission lines and phase-shifting networks. For maximum efficiency it is good practice to feed the largest amount of power directly into the transmission line, thus avoiding the power loss in the phase-shifting network.

If power-handling dummy driving-point-impedance loads are used the power divider can be adjusted approximately before connecting to the towers. If this procedure is followed, the approximate value of phase can be inserted if the phase sampling is done at the input to the dummy loads. For L sections in parallel the resistance input to the individual L sections must have the value of resistance needed to absorb the correct ratio of the total power delivered from the common point.

For example, in a two-tower array, the L section resistance inputs can be written

$$R_1 = \frac{V^2}{P_1} \qquad [11]$$

$$R_2 = \frac{V^2}{P_2} \qquad [12]$$

where R_1 = resistance into L section leading to tower 1, ohms
R_2 = resistance into L section leading to tower 2, ohms
V = voltage between common point and ground, volts
P_1 = power to tower 1, watts
P_2 = power to tower 2, watts

The common-point resistance can be written

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

where R is the common-point resistance in ohms.

Component Ratings

The current through and the voltage across each component in the feeder system should be computed to determine the required rating. This information provides a means of choosing the most economically sized component in a new installation. These computations will show up components in an existing system which are underrated and should be changed.

The current rating of capacitors are given by the manufacturer and should not be exceeded during operation for the most adverse temperature and poor adjustment conditions. The voltage ratings are also given by the manufacturer and should be high enough to comply with any surges that may arise, including lightning. The lightning-protection devices should be good enough to allow for reasonable economy in deciding the maximum voltage rating.

The current and voltage rating of coils are related to the voltage gradient between turns of the coil. The distance between conductor centers in adjacent turns should be roughly twice the diameter of the conductor, and the conductor diameter must comply with good engineering standards as to coil losses.

Coarse Adjustments

Meter Calibrations

All meters should be checked for accuracy against meters which are of known accuracy. Calibration charts should be made up for meters that do not register correctly. The current-sampling system on each tower should be checked for accuracy.

Antenna-Monitor System

The antenna monitor should be checked for accuracy. The tower-sampling loops can be placed in parallel and checked for zero phase when the same field is sampled. If tower input currents are sampled, the same current can be sampled by two sampling units to check for accuracy.

Another method is to establish a null off the end of two towers and read the phase which is known quite accurately by theory for this condition. This can be done by setting up a field strength set at some distance and walk the two-tower phase and power-division controls to give a deep null. This also gives an excellent check on unity field-strength ratio. It is helpful to have two-way communication to expedite these measurements.

Common-Point Impedance

After the array is approximately adjusted, it is advisable to measure the common-point input impedance. It may be desired at this time to make input matching network adjustments so a pure resistance load will be presented to the transmitter at the operating frequency.

After final adjustment of the array the common-point input impedance must be measured across a band of frequencies to meet FCC requirements and check on characteristics that may cause objectionable distortion. It is desirable to have a relatively constant value of resistance over the modulation-frequency range.

Low-Power Operation

After the common-point impedance has been adjusted properly, low power can be fed into the system and all meters checked for predicted readings. If errors have been made in the computations, the adjustment will probably be incorrect.

Field-Strength Check

After the array is in reasonable adjustment, it is timely to check the field strength. Usually a nondirectional proof is made by running at least eight radials to determine the rms value and the attenuation in the various directions and establish suitable monitoring points in critical directions.

These measurements will probably indicate what changes are necessary to meet the requirements in the construction permit. Any change in adjustment should be followed by appropriate field-strength measurements properly logged so that further adjustments are always in the correct direction.

As the desired pattern is approached and the feeder system comes into proper adjustment, the full input power can be used.

Fine Adjustments

These adjustments are simply a continuation of the tuning to arrive at the final values. More exact and extensive information is gathered with regard to how the controls affect the field-strength measurements, particularly along radials in the direction of the various minima.

The final adjustment of the array is decided upon after enough information is gathered to show what adjustment of the array gives the desired over-all results. The final adjustment may involve compromises in optimum field strength between the various minima and critical bearings concerned with protection.

During this phase the monitoring points are usually selected. The value of monitoring-point readings should be recorded along with the other meter readings of the system.

When the final pattern adjustments have been made, careful attention is given to establishing the exact power at the common point. The resistance and reactance versus frequency measurements are then run for the common point.

Final Operating Adjustment

With the antenna system operating properly, all meter readings are recorded and maintained while the field-strength measurements are made to prove the shape and size of the horizontal pattern.

The monitoring points have to be photographed in order to provide a reliable means of finding their exact location in the future. The field strength at the monitoring points should be checked at least daily while making the proof of performance field-strength measurements.

The field-strength radial measurements are plotted on FCC logarithmic paper and analyzed to determine the unattenuated field strength at 1 mile. These values are then plotted on polar coordinate paper, and a planimeter can then be used to determine the rms value. This should agree closely with the predicted value.

If only a skeleton or partial proof is required, sufficient directional measurements must be made at points used in the original proof to show by means of ratios between the two measurements at each location that the pattern is basically unchanged. If the average of the ratios for each radial, usually about 5, is between 0.8 and 1.2, it is considered that the pattern is unchanged. If the average deviates outside this range on two or more radials, the array probably needs further adjustment.

MULTIPURPOSE ANTENNA SYSTEMS

Two-Pattern Arrays

Many directional-antenna systems have a different day and night pattern. Usually the nighttime pattern requires deeper minima and in many cases different locations of the towers or more towers in the array.

In the layout and design of such a system the problem should be carefully analyzed and the simplest layout used consistent with good engineering practice. Many times the same matching networks can be used with a day-night transfer relay to change coil turns and capacity values as required for the two conditions of operation.

It is good practice to provide pilot-light circuits to indicate when all RF relays are in their correct positions. If power is applied to a system with the relays not in proper positions, some components may be injured and the transmitter may not be working into a matched load.

Two Transmitters Using Same Towers

There are a number of cases where an antenna system is used by more than one station. If, for example, two radio transmitters use the same towers, it is necessary to add RF filter circuits so energy will not be fed from the output of one transmitter back to the output of the other transmitter and produce cross-modulation products. If this happens, the program of the other radio station will be heard in the background.

In the feeder-system design for two transmitters using the same towers it is usually possible to design the networks such that the T, τ, or L sections perform the necessary filtering action in addition to the required impedance transformation and phase-shift functions.

The metering circuits must also be filtered so they will not respond to the undesired frequency. If a meter has two RF currents of different frequencies, it will give an rms response providing it is not frequency-sensitive.

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APPENDIX A
SINGLE-TOWER ANTENNAS

Theoretical Vertical-Radiation Characteristics

Formulas

Sectionalized Top-loaded Tower. The current distribution on the bottom section of the tower as shown in Fig. A is given by

$$i_a = I_a \sin (G - y) \qquad [A\text{-}1]$$

The current distribution on the top section is given by

$$i_c = I_c \sin (H - y) \qquad [A\text{-}2]$$

At the insulator height $y = A$, $I_c = i_a$, $G - A = B$, and

$$I_c \sin (H - A) = I_a \sin B \qquad [A\text{-}3]$$

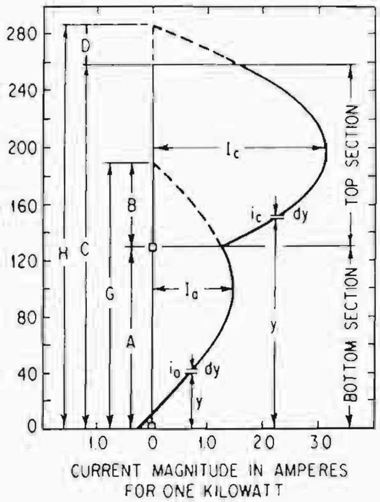


Fig. AA. Theoretical current distribution on a top-loaded sectionalized tower.

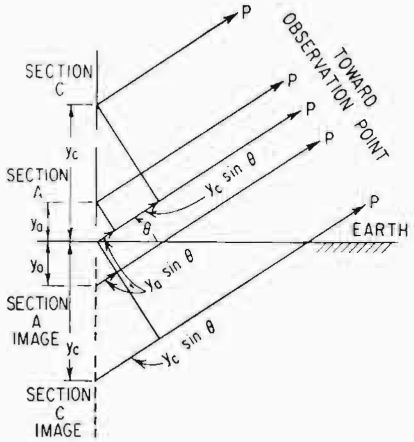


Fig. AB. Geometry to determine field from sectionalized tower with images.

or $$I_c = \frac{\sin B}{\sin (H - A)} I_a \qquad \text{[A-4]}$$

The inverse field or unattenuated field strength toward at 1 mile the observation point P at an elevation angle θ produced by a vertical element dy at the base of the antenna would be $dy \cos \theta$. The field from any other similar element of the antenna or its image would have a phase different from zero as depicted in Fig. B. The addition of these vector fields from an element and its image on sections A and C is shown in Fig. ACa and b, respectively. It is noted that the sine components cancel. The total field at the point P is, therefore,

$$E_\theta = K2I_a \cos\theta \left[\int_0^A i_a \cos (y \sin \theta)\, dy + \int_A^C i_c \cos (y \sin \theta)\, dy \right] \qquad \text{[A-5]}$$

where K is a constant such that E_θ will be in the units desired. K cancels out in $f(\theta)$. Substituting from Eqs. A-1, A-2 and A-4, performing the indicated integration, and dividing the result by itself when $\theta = 0$ give

$$f(\theta) = \frac{\cos B \cos (A \sin \theta) - \cos G + \dfrac{\sin B \cos (H - C) \cos (C \sin \theta)}{\sin (H - A)} - \dfrac{\sin B \sin \theta \sin (H - C) \sin (C \sin \theta)}{\sin (H - A)} - \dfrac{\sin B \cos (H - A) \cos (A \sin \theta)}{\sin (H - A)}}{\cos \theta \left\{ \cos B - \cos G + [\sin B / \sin (H - A)] (\cos \overline{H - C} - \cos \overline{H - A}) \right\}} \qquad \text{[A-6]}$$

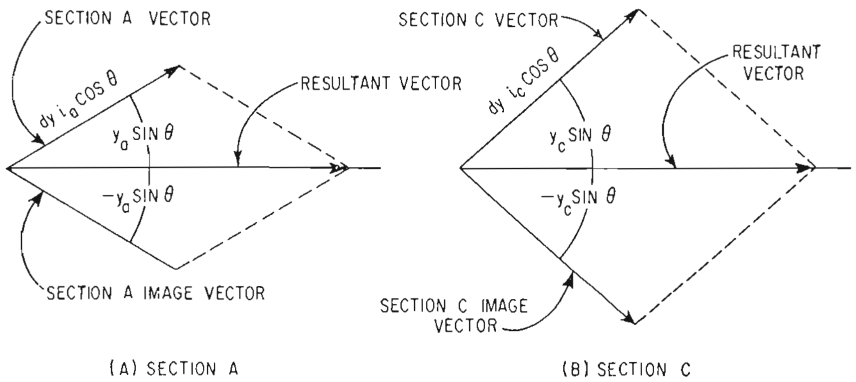


Fig. AC. Vector diagrams of field strength at point *P*.

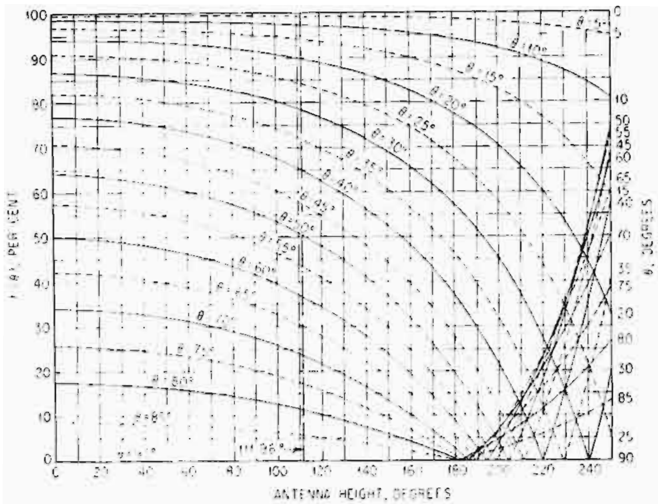


Fig. AD. Vertical-radiation charateristic as a function of electrical tower height for various values of elevation angle.

This is the vertical-radiation-characteristic equation for a two-section sectionalized tower. The same procedure can be applied if more than two sections are involved.

Top-loaded Tower. Referring to Fig. A-1 it is noted that the sectionalized antenna can be reduced to a nonsectionalized top-loaded antenna by making the top section of zero length but at the same time arranging for top loading such that B is unchanged. This can be done by letting $C = A$ and $H = C$ in Eq. (A-6), which then reduces to

$$f(\theta) = \frac{\cos B \cos (A \sin \theta) - \cos G - \sin B \sin \theta \sin (A \sin \theta)}{\cos \theta (\cos B - \cos G)} \qquad [A\text{-}7]$$

This is the vertical-radiation characteristic for a top-loaded tower of height A and top-loaded to a height of $G = A + B$.

Ordinary Vertical Tower. The ordinary tower without top loading can be obtained from Eq. (A-6) by letting $C = A$, $H = C$, and $B = 0$ or in Eq. (A-7) by letting $A = G$ and $B = 0$ to obtain

$$f(\theta) = \frac{\cos (G \sin \theta) - \cos G}{\cos \theta (1 - \cos G)} \qquad [A\text{-}8]$$

This is the same as Eq. 3 in the text. Table A gives values of $f(\theta)$ for a useful range of tower heights, and Fig. D gives this information in graphical form.

Theoretical Self-Impedance and Radiation

It is useful to know the theoretical loop and base resistance of a vertical radiator. This information is presented graphically in Fig. AE along with the theoretical inverse field strength at 1 mile.

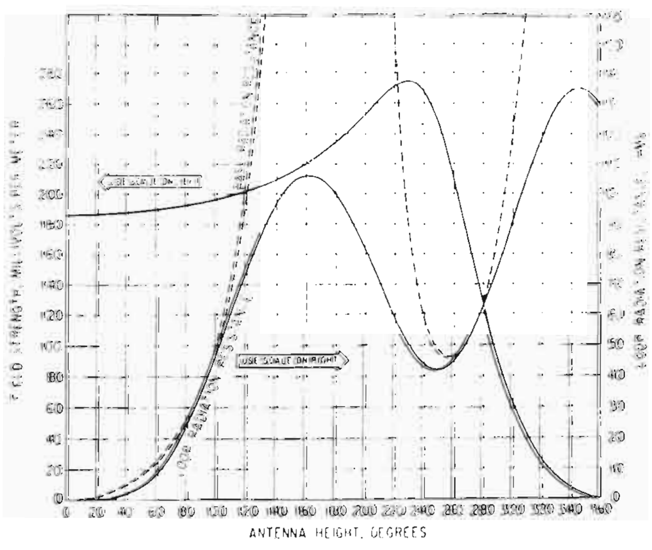
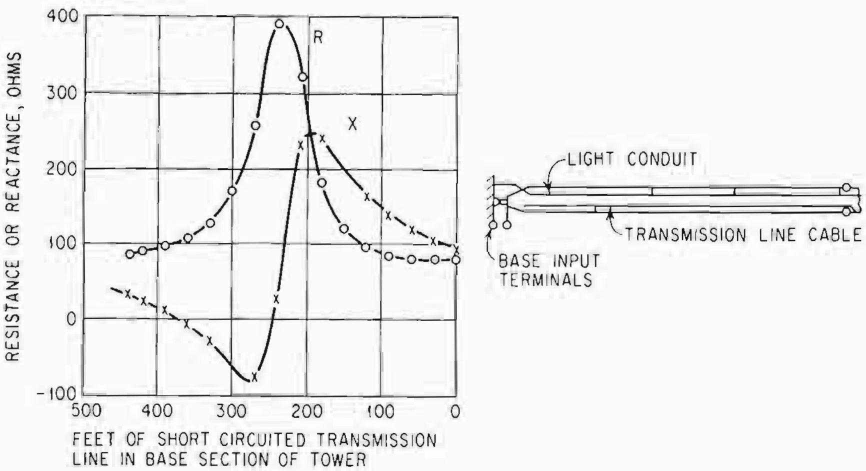
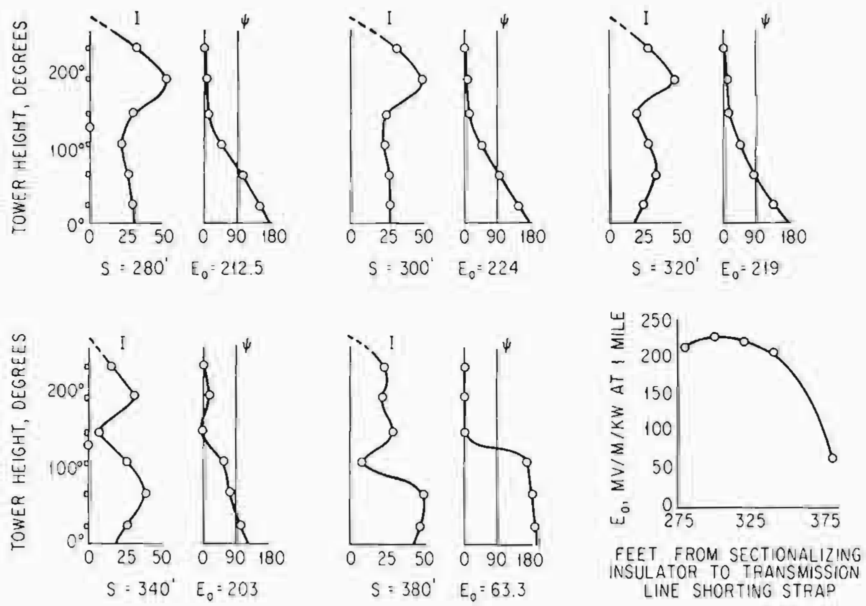


Fig. AE. Inverse field strength at 1 mile for 1 kw, loop and base radiation resistance as a function of tower height over a perfectly conducting earth.

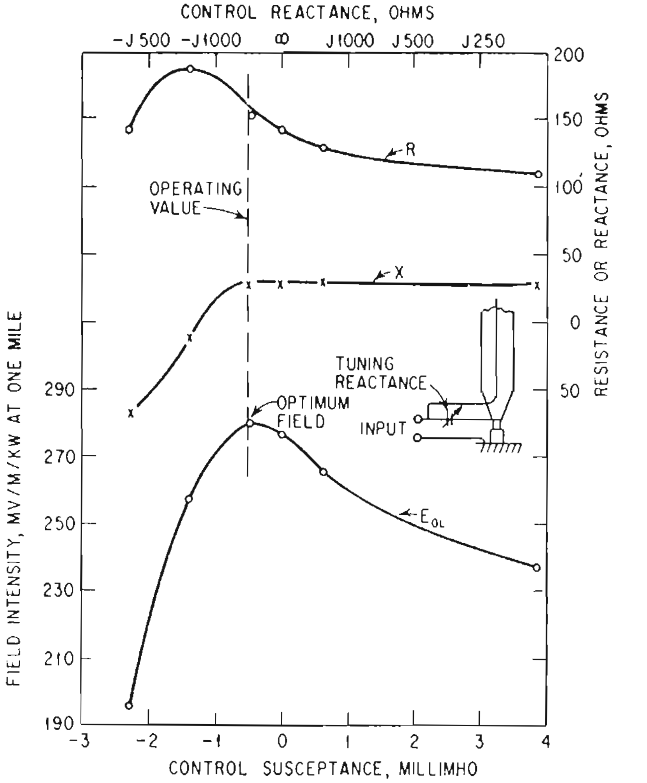


(A) - BASE IMPEDANCE OF 258° TOWER AS A FUNCTION OF LOADING REACTANCE ACROSS SECTIONALIZING INSULATOR

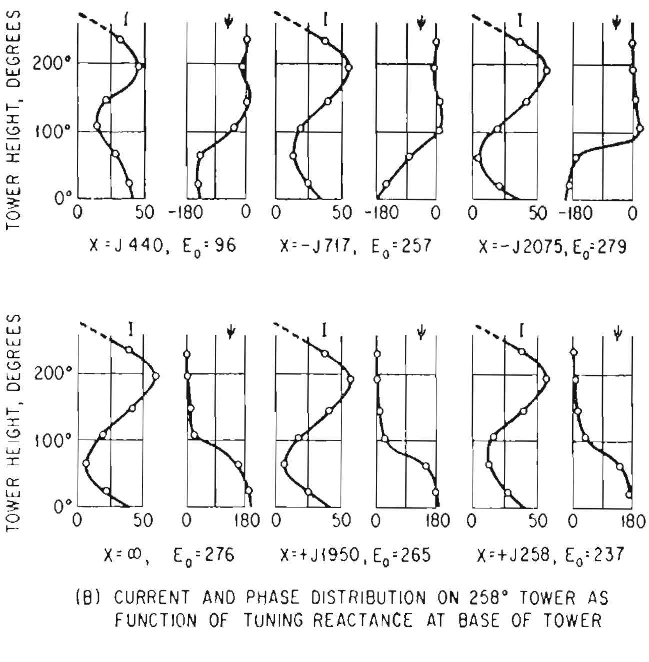


(B) - CURRENT AND PHASE DISTRIBUTION ON 258° TOWER AS FUNCTION OF LOADING REACTANCE ACROSS SECTIONALIZING INSULATOR IN TERMS OF SHORTED TRANSMISSION LINE LENGTHS

Fig. AF. Performance of sectionalized tower with a top section excited through reactance element from bottom section.



(A) INPUT IMPEDANCE AND FIELD STRENGTH OF TOP LOADED SECTIONALIZED 258° TOWER AS FUNCTION OF TUNING REACTANCE



(B) CURRENT AND PHASE DISTRIBUTION ON 258° TOWER AS FUNCTION OF TUNING REACTANCE AT BASE OF TOWER

Fig. AG. Performance of sectionalized tower with top and bottom sections excited independently.

Table A-1. Vertical-radiation Characteristic $f(\theta)$

θ°	Tower height, G° 0	5	10	15	20	25	30	35	40
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9962	0.9962	0.9962	0.9961	0.9961	0.9961	0.9960	0.9960	0.9959
10	0.9848	0.9848	0.9847	0.9846	0.9845	0.9843	0.9841	0.9839	0.9836
15	0.9659	0.9659	0.9657	0.9655	0.9653	0.9649	0.9644	0.9639	0.9632
20	0.9397	0.9394	0.9393	0.9390	0.9386	0.9379	0.9372	0.9362	0.9351
25	0.9063	0.9062	0.9058	0.9054	0.9047	0.9037	0.9026	0.9012	0.8996
30	0.8660	0.8658	0.8654	0.8648	0.8638	0.8626	0.8610	0.8592	0.8571
35	0.8192	0.8188	0.8186	0.8176	0.8164	0.8148	0.8129	0.8106	0.8080
40	0.7660	0.7658	0.7653	0.7642	0.7628	0.7610	0.7587	0.7561	0.7530
45	0.7071	0.7069	0.7062	0.7051	0.7035	0.7014	0.6989	0.6960	0.6925
50	0.6428	0.6423	0.6418	0.6406	0.6390	0.6368	0.6341	0.6309	0.6272
55	0.5736	0.5732	0.5726	0.5714	0.5697	0.5674	0.5647	0.5615	0.5577
60	0.5000	0.4947	0.4990	0.4979	0.4961	0.4940	0.4914	0.4882	0.4846
65	0.4226	0.4222	0.4217	0.4203	0.4191	0.4171	0.4143	0.4117	0.4084
70	0.3420	0.3412	0.3412	0.3404	0.3390	0.3372	0.3351	0.3325	0.3297
75	0.2588	0.2579	0.2584	0.2575	0.2564	0.2550	0.2533	0.2513	0.2490
80	0.1736	0.1695	0.1732	0.1727	0.1720	0.1710	0.1697	0.1684	0.1668
85	0.0871	0.0844	0.0869	0.0869	0.0864	0.0858	0.0852	0.0844	0.0836

θ°	45	50	55	60	65	70	75	80	85
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9958	0.9957	0.9956	0.9955	0.9953	0.9952	0.9950	0.9948	0.9946
10	0.9832	0.9830	0.9824	0.9819	0.9815	0.9809	0.9804	0.9795	0.9788
15	0.9625	0.9617	0.9607	0.9597	0.9585	0.9573	0.9559	0.9544	0.9527
20	0.9339	0.9325	0.9309	0.9289	0.9272	0.9251	0.9227	0.9200	0.9173
25	0.8978	0.8957	0.8934	0.8908	0.8879	0.8848	0.8813	0.8776	0.8735
30	0.8546	0.8519	0.8487	0.8453	0.8416	0.8375	0.8328	0.8278	0.8224
35	0.8050	0.8015	0.7977	0.7934	0.7887	0.7836	0.7779	0.7718	0.7651
40	0.7485	0.7449	0.7410	0.7358	0.7305	0.7244	0.7180	0.7109	0.7103
45	0.6886	0.6769	0.6791	0.6735	0.6675	0.6608	0.6536	0.6457	0.6372
50	0.6230	0.6186	0.6130	0.6073	0.6009	0.5936	0.5862	0.5777	0.5686
55	0.5535	0.5486	0.5427	0.5373	0.5308	0.5236	0.5159	0.5075	0.4984
60	0.4804	0.4759	0.4705	0.4648	0.4587	0.4518	0.4441	0.4361	0.4271
65	0.4042	0.4002	0.3954	0.3898	0.3843	0.3779	0.3710	0.3630	0.3556
70	0.3263	0.3216	0.3190	0.3141	0.3089	0.3031	0.2970	0.2906	0.2842
75	0.2463	0.2433	0.2400	0.2363	0.2323	0.2279	0.2232	0.2181	0.2127
80	0.1649	0.1629	0.1622	0.1576	0.1557	0.1515	0.1492	0.1457	0.1408
85	0.0826	0.0816	0.0804	0.0791	0.0777	0.0761	0.0739	0.0726	0.0707

θ°	90	95	100	105	110	115	120	125	130
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9944	0.9942	0.9939	0.9937	0.9934	0.9931	0.9927	0.9923	0.9919
10	0.9781	0.9770	0.9760	0.9749	0.9738	0.9725	0.9712	0.9697	0.9681
15	0.9509	0.9489	0.9468	0.9445	0.9420	0.9393	0.9363	0.9337	0.9297
20	0.9143	0.9110	0.9074	0.9035	0.8993	0.8947	0.8898	0.8845	0.8788
25	0.8691	0.8642	0.8590	0.8534	0.8473	0.8407	0.8336	0.8259	0.8175
30	0.8165	0.8102	0.8033	0.7959	0.7878	0.7791	0.7698	0.7597	0.7489
35	0.7579	0.7501	0.7417	0.7320	0.7228	0.7122	0.7000	0.6886	0.6754
40	0.6946	0.6855	0.6759	0.6656	0.6541	0.6420	0.6288	0.6157	0.5999
45	0.6279	0.6180	0.6073	0.5958	0.5834	0.5702	0.5560	0.5408	0.5245
50	0.5591	0.5487	0.5373	0.5253	0.5124	0.4987	0.4838	0.4689	0.4511
55	0.4886	0.4781	0.4669	0.4548	0.4419	0.4281	0.4134	0.3977	0.3809
60	0.4178	0.4078	0.3969	0.3854	0.3730	0.3600	0.3460	0.3310	0.3151
65	0.3470	0.3378	0.3279	0.3174	0.3061	0.2942	0.2813	0.2680	0.2536
70	0.2766	0.2687	0.2598	0.2509	0.2413	0.2311	0.2203	0.2091	0.1969
75	0.2067	0.2007	0.1937	0.1866	0.1790	0.1709	0.1623	0.1533	0.1437
80	0.1377	0.1331	0.1281	0.1237	0.1180	0.1130	0.1064	0.1005	0.0941
85	0.0686	0.0664	0.0640	0.0614	0.0588	0.0559	0.0529	0.0497	0.0464

Table A-1. Vertical-radiation Characteristic $f(\theta)$ (*Continued*)

θ°	Tower height, G°							
	135	140	145	150	155	160	165	170
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9915	0.9910	0.9905	0.9899	0.9893	0.9886	0.9878	0.9870
10	0.9663	0.9645	0.9624	0.9602	0.9577	0.9551	0.9522	0.9491
15	0.9259	0.9219	0.9175	0.9127	0.9075	0.9019	0.8956	0.8889
20	0.8725	0.8657	0.8584	0.8504	0.8418	0.8324	0.8222	0.8110
25	0.8085	0.7988	0.7883	0.7769	0.7645	0.7511	0.7366	0.7207
30	0.7372	0.7245	0.7108	0.6961	0.6801	0.6628	0.6440	0.6237
35	0.6612	0.6460	0.6293	0.6118	0.5926	0.5720	0.5496	0.5254
40	0.5837	0.5664	0.5477	0.5276	0.5060	0.4828	0.4577	0.4305
45	0.5070	0.4882	0.4681	0.4466	0.4235	0.3979	0.3719	0.3432
50	0.4330	0.4137	0.3932	0.3710	0.3473	0.3219	0.2948	0.2657
55	0.3631	0.3440	0.3237	0.3020	0.2786	0.2542	0.2278	0.1996
60	0.2981	0.2802	0.2611	0.2407	0.2190	0.1960	0.1713	0.1451
65	0.2382	0.2222	0.2051	0.1867	0.1675	0.1469	0.1256	0.1019
70	0.1838	0.1716	0.1555	0.1399	0.1237	0.1065	0.0881	0.0687
75	0.1335	0.1227	0.1114	0.0994	0.0866	0.0732	0.0590	0.0439
80	0.0868	0.0796	0.0719	0.0634	0.0547	0.0458	0.0359	0.0256
85	0.0428	0.0390	0.0351	0.0309	0.0265	0.0218	0.0169	0.0117
θ°	175	180	185	190	195	200	205	210
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9861	0.9851	0.9840	0.9828	0.9815	0.9801	0.9784	0.9766
10	0.9455	0.9418	0.9375	0.9330	0.9278	0.9222	0.9159	0.9089
15	0.8815	0.8733	0.8645	0.8547	0.8438	0.8319	0.8186	0.8038
20	0.7988	0.7855	0.7708	0.7548	0.7370	0.7175	0.6958	0.6718
25	0.7034	0.6845	0.6638	0.6412	0.6168	0.5888	0.5585	0.5250
30	0.6015	0.5774	0.5510	0.5222	0.4907	0.4561	0.4179	0.3757
35	0.4991	0.4706	0.4395	0.4057	0.3687	0.3283	0.2839	0.2350
40	0.4013	0.3696	0.3353	0.2979	0.2573	0.2129	0.1645	0.1112
45	0.3122	0.2788	0.2427	0.2036	0.1612	0.1152	0.0650	0.0103
50	0.2344	0.2008	0.1646	0.1256	0.0834	0.0378	−0.0118	−0.0657
55	0.1658	0.1370	0.1022	0.0649	0.0247	−0.0186	−0.0655	−0.1161
60	0.1171	0.0873	0.0553	0.0211	−0.0155	−0.0550	−0.0973	−0.1431
65	0.0772	0.0509	0.0228	−0.0071	−0.0391	−0.0733	−0.1100	−0.1494
70	0.0481	0.0261	0.0029	−0.0220	−0.0483	−0.0765	−0.1065	−0.1388
75	0.0280	0.0111	−0.0069	−0.0259	−0.0461	−0.0676	−0.0905	−0.1150
80	0.0148	0.0033	−0.0089	−0.0218	−0.0354	−0.0499	−0.0633	−0.0818
85	0.0062	0.0004	−0.0057	−0.0122	−0.0191	−0.0264	−0.0341	−0.0424
θ°	215	220	225	230	235	240	245	250
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9746	0.9723	0.9697	0.9668	0.9635	0.9597	0.9551	0.9504
10	0.9011	0.8925	0.8826	0.8580	0.8586	0.8442	0.8275	0.8084
15	0.7873	0.7689	0.7481	0.7247	0.6981	0.6679	0.6333	0.5935
20	0.6450	0.6151	0.5815	0.5438	0.5010	0.4525	0.3970	0.3334
25	0.4877	0.4462	0.3997	0.3475	0.2887	0.2220	0.1462	0.0593
30	0.3291	0.2772	0.2188	0.1548	0.0821	0.0000	−0.0931	−0.1992
35	0.1810	0.1214	0.0551	−0.0188	−0.1016	−0.1946	−0.2997	−0.4191
40	0.0529	−0.0117	−0.0814	−0.1620	−0.2501	−0.3489	−0.4599	−0.5853
45	−0.0498	−0.1156	−0.1881	−0.2683	−0.3572	−0.4562	−0.5671	−0.6917
50	−0.1243	−0.1885	−0.2588	−0.3363	−0.4216	−0.5163	−0.6216	−0.7394
55	−0.1711	−0.2310	−0.2962	−0.3677	−0.4462	−0.5327	−0.6285	−0.7351
60	−0.1924	−0.2461	−0.3040	−0.3674	−0.4364	−0.5125	−0.5958	−0.6882
65	−0.1918	−0.2375	−0.2880	−0.3406	−0.3989	−0.4625	−0.5322	−0.6089
70	−0.1733	−0.2107	−0.2503	−0.2935	−0.3402	−0.3909	−0.4463	−0.5069
75	−0.1411	−0.1691	−0.1992	−0.2315	−0.2664	−0.3042	−0.3452	−0.3899
80	−0.0992	−0.1182	−0.1380	−0.1600	−0.1826	−0.2079	−0.2346	−0.2639
85	−0.0511	−0.0605	−0.0705	−0.0812	−0.0927	−0.1051	−0.1185	−0.1330

Sectionalized Tower Measurements

Two cases of interest are presented in Figs. AF and AG. In Fig. AF, it will be noted that the maximum field strength is 224 mv/m for 1-kw input when the tower is driven at the base and an inductive reactance in the form of a short-circuited transmission line is connected across the sectionalizing insulator. The current and phase distribution are of particular interest for this condition. A standing wave of varying amplitude exists on the top section with very little phase shift, while on the lower section a traveling wave characterized by a constant amplitude and progressive phase shift is evident.

When both the upper and lower tower sections are driven as shown in Fig. AG, the maximum inverse field strength for 1-kw input is 279 mv/m. In this case there is a very rapid phase shift of 180° and the current drops to a very low value. At the bottom of the tower there is a build-up of current that is approximately 180° out of phase with the current on the top section. It is this combination that reduces high angle radiation and is responsible for the strong ground-wave field strength.

APPENDIX B
TWO-TOWER DIRECTIONAL ANTENNAS

Pattern Formulas

General Equation

The inverse field strength from a two-tower directional antenna as shown in Fig. BA is given by

$$E = E_1 f_1(\theta) \sqrt{2F} \sqrt{\frac{1+F^2}{2F} + \cos(S\cos\phi\cos\theta + \Psi)} \qquad \text{[B-1]}$$

where E = inverse field strength at 1 mile, mv/m
E_1 = inverse field strength at 1 mile from tower 1 when operating in array, mv/m
$f_1(\theta)$ = vertical-radiation characteristic of tower 1

$$F = \frac{E_2 f_2(\theta)}{E_1 f_1(\theta)} \qquad \text{[B-2]}$$

= ratio of field strength from tower 2 to tower 1

where E_2 = inverse field strength at 1 mile from tower 2 when operating in array, mv/m
$f_2(\theta)$ = vertical-radiation characteristic of tower 2
S = spacing between tower 2 and tower 1, deg
ϕ = azimuth angle from line of towers, deg
θ = elevation angle, deg
Ψ = electrical phase angle of current in tower 2 with respect to tower 1, deg

The above terms as shown in Fig. BB are written without subscripts where possible for a simple two-tower array. When more than two towers are involved, the term F in Eq. B-2 is written F_{21} to designate the ratio between tower 2 and tower 1. Similarly, the spacing would be marked S_{21}, which is the distance from tower 2 to tower 1 that is located at the space reference point. The electrical phase in general is written Ψ_{21} to designate the phase of the current in tower 2 with respect to tower 1.

Minimum-Depth Term

The minimum-depth term by definition is

$$\frac{1+F^2}{2F} \qquad \text{[B-3]}$$

Because of its significance in Eq. B-1, Table A has been prepared. This table lists the values of F, $1/F$, F^2, and the minimum-depth term. The minimum-depth term is unchanged if $1/F$ is used in the place of F.

An inspection of the radicand in Eq. B-1 shows that E in the minima can be made larger than zero providing the minimum-depth term is not completely canceled by the cosine term. The exact size of E in the minima can be fixed by choice of the value of the minimum-depth term. Conversely, if the equation for E is in the form of Eq. B-1 and if F is also given, then Table B-1 can be used to check the accuracy of the formula quickly. If F is not given, it can be found from the table.

General Value of Field Ratio

The general form of F is given in Eq. B-2. The values of $f(\theta)$ for various tower heights are given in Appendix A, Fig. AD and Table A. Hence, in the design of a two-tower directional antenna, it is convenient first to select the value of the minimum-depth term. The corresponding value of F can be found in Appendix B, Table A, and by Eq. B-2 the ratio of E_2 over E_1 can be determined. This procedure can be reversed to check a given equation.

For pattern computations, it is convenient to fix θ and vary ϕ; thus the information can be used to meet the requirement in Paragraph 3.150 of Part 3, FCC Rules. The data required are field strength E as a function of azimuth angle ϕ for 5° intervals of elevation angle θ from 0 to 60°.

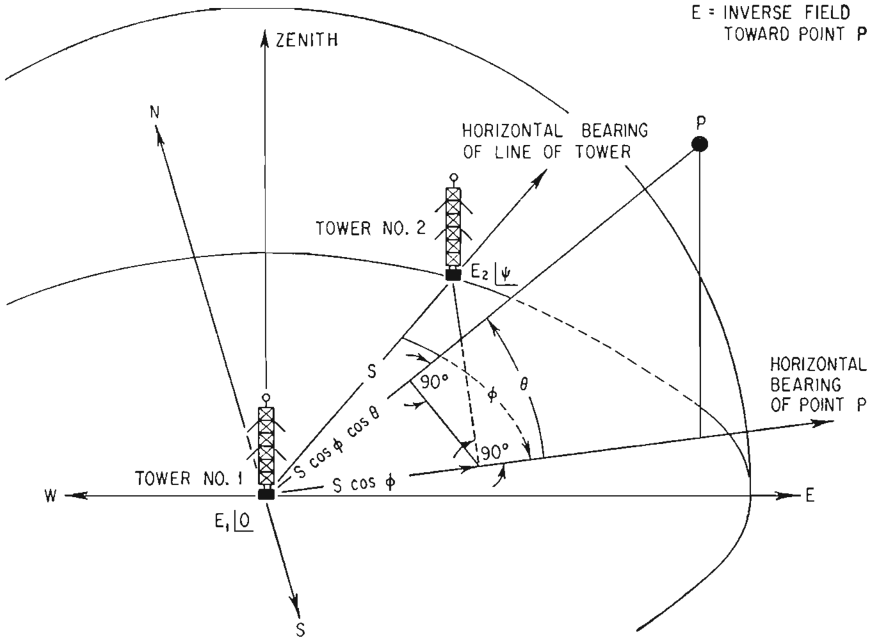


Fig. BA. Space view of two-tower directional antenna.

Horizontal-Plane Equation

In the ground plane Eq. B-1 reduces to

$$E = E_1 \sqrt{2F} \sqrt{\frac{1 + F^2}{2F} + \cos(S \cos \phi + \Psi)} \quad \text{[B-4]}$$

where F is the ratio of E_2 over E_1 for equal or unequal height towers. Thus Eq. B-4 can be used for $\theta = 0$ and Eq. B-1 can be used at elevation angles up to 60° as required by the above rules.

If the ratio of E_2 over E_1 is given but E has not been expressed in the form of Eq. B-1 or B-4, it is convenient to use Table A to obtain the values to be used in the above equations.

The minimum-depth term usually appears more than once in directional-antenna-pattern equations for arrays having more than two towers. Three towers in line and parallelogram arrays can use two or more of the multiplication radicands given in Eq. B-1.

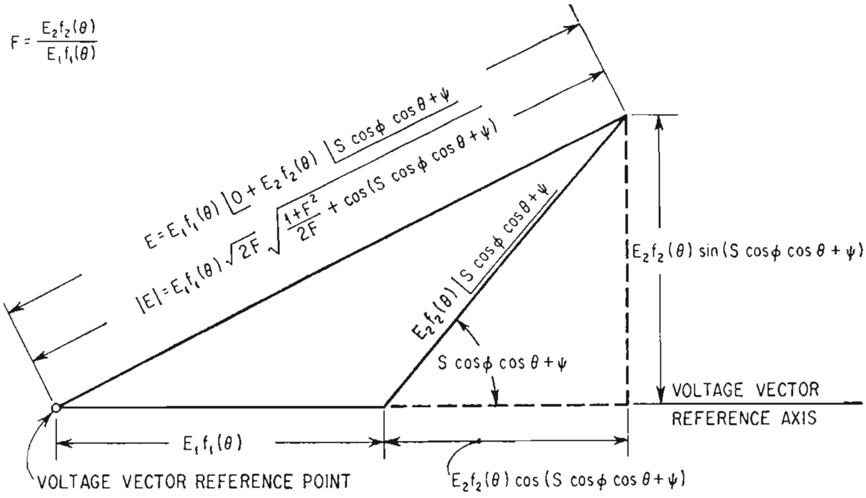


Fig. BB. Voltage vector diagram for two-tower directional antenna.

Systematization of Two-Tower Patterns

The systematization has been divided into relatively large steps of 45° for phasing and spacing in Figs. BC to BF. The spacing extends to four wavelengths or 1,440°. The more useful range of spacing up to one wavelength is given in small steps of 15° for both spacing and phasing in Figs. BG to BL.

Pattern Size

Field Strength of Reference Tower

In order to determine the pattern size, it is rather common practice to compute the value of field strength that the reference tower will radiate when operating in the directional-antenna array. This value can then be used in Eq. B-1 to determine the pattern shape at the correct size. The value of E_1 can be computed from the following equation, sometimes called the loop impedance formula, thus

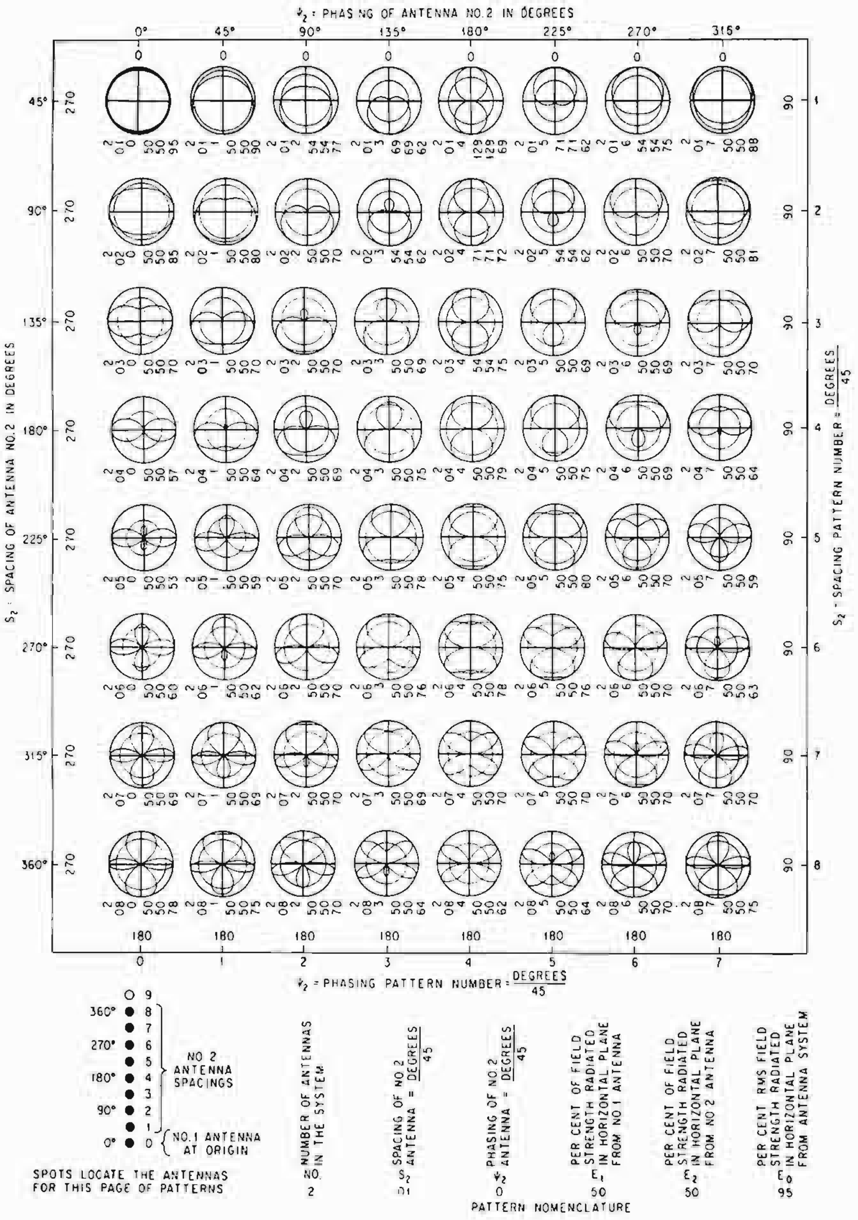


Fig. BC. Systematization of two-tower patterns in steps of 45 to 1,440°.

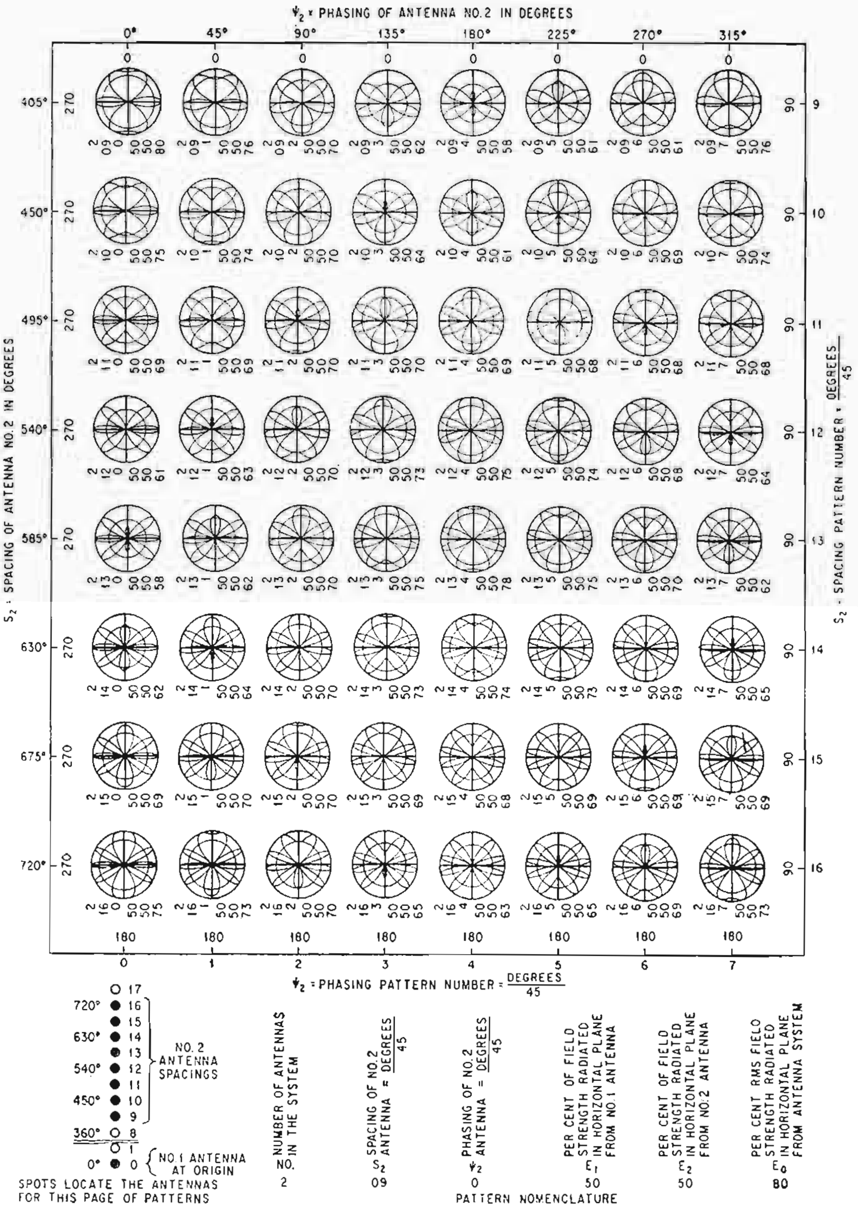


Fig. BD. Systematization of two-tower patterns in steps of 45 to 1,440°.

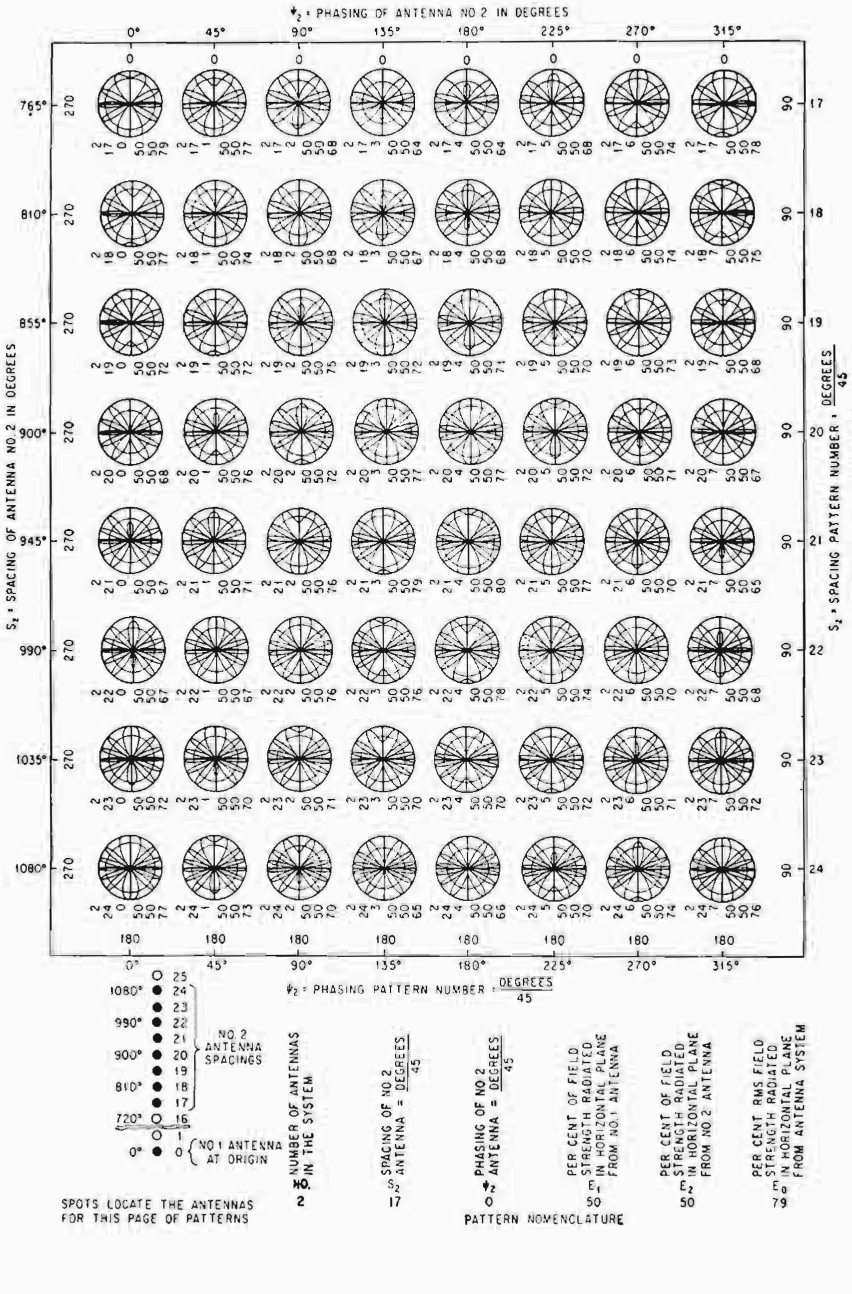


Fig. BE. Systematization of two-tower patterns in steps of 45 to 1,440°.

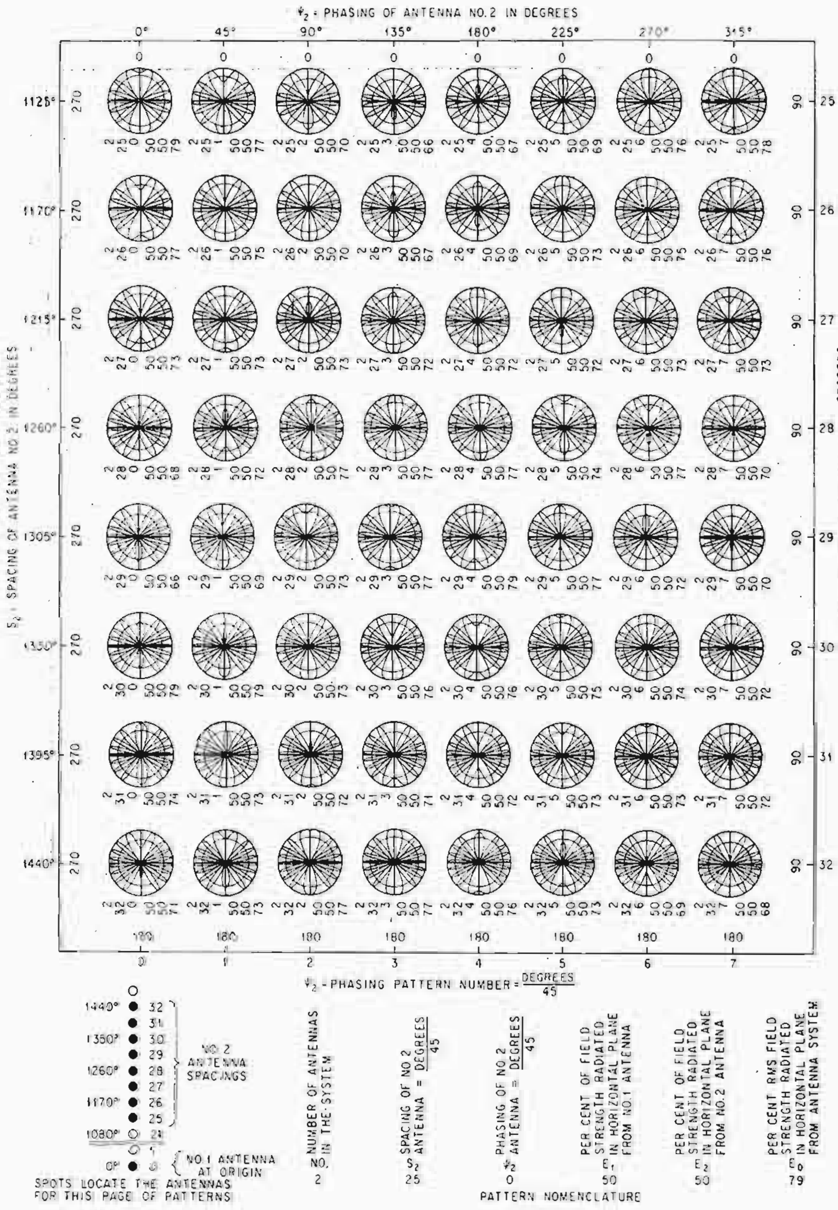


Fig. BF. Systematization of two-tower patterns in steps of 45 to 1,440°.

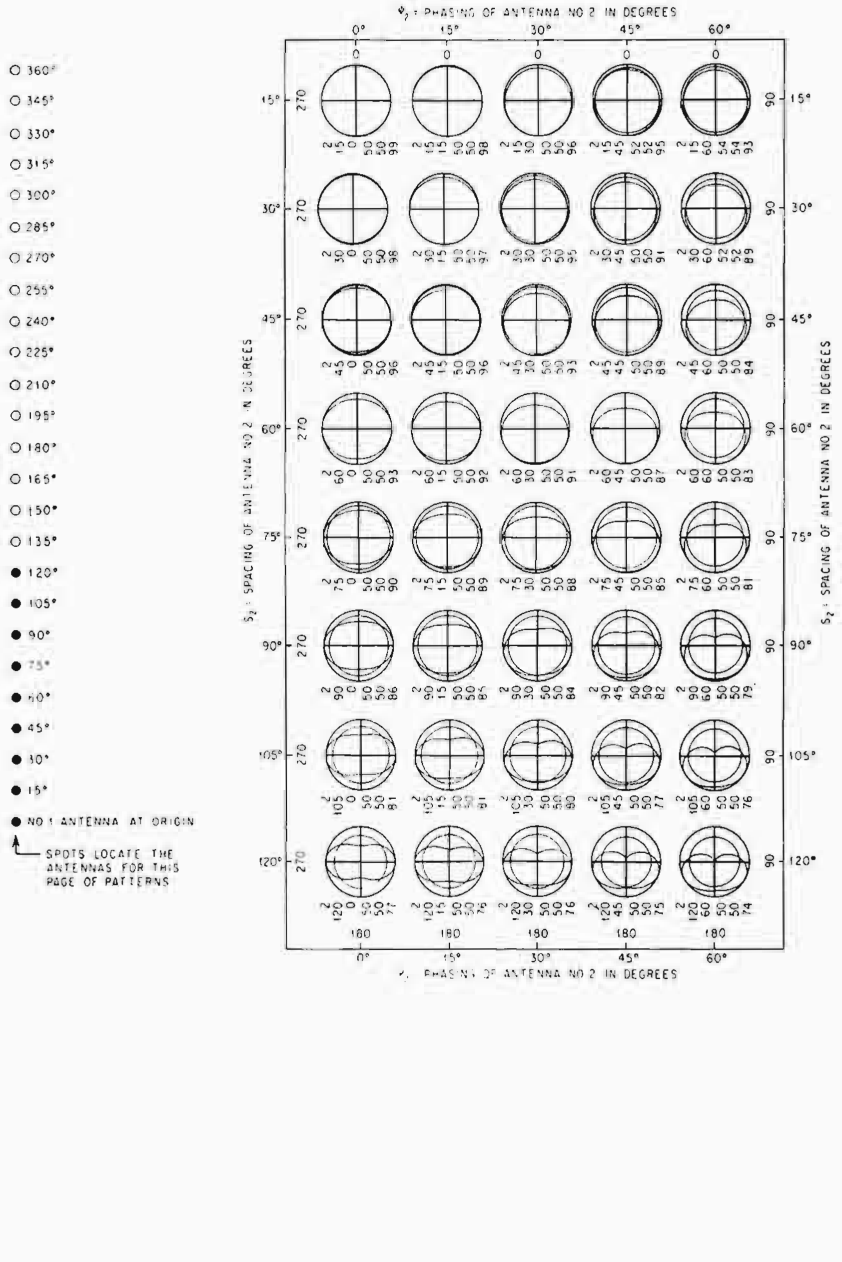


Fig. BG. Systematization of two-tower patterns in steps of 15 to 360°.

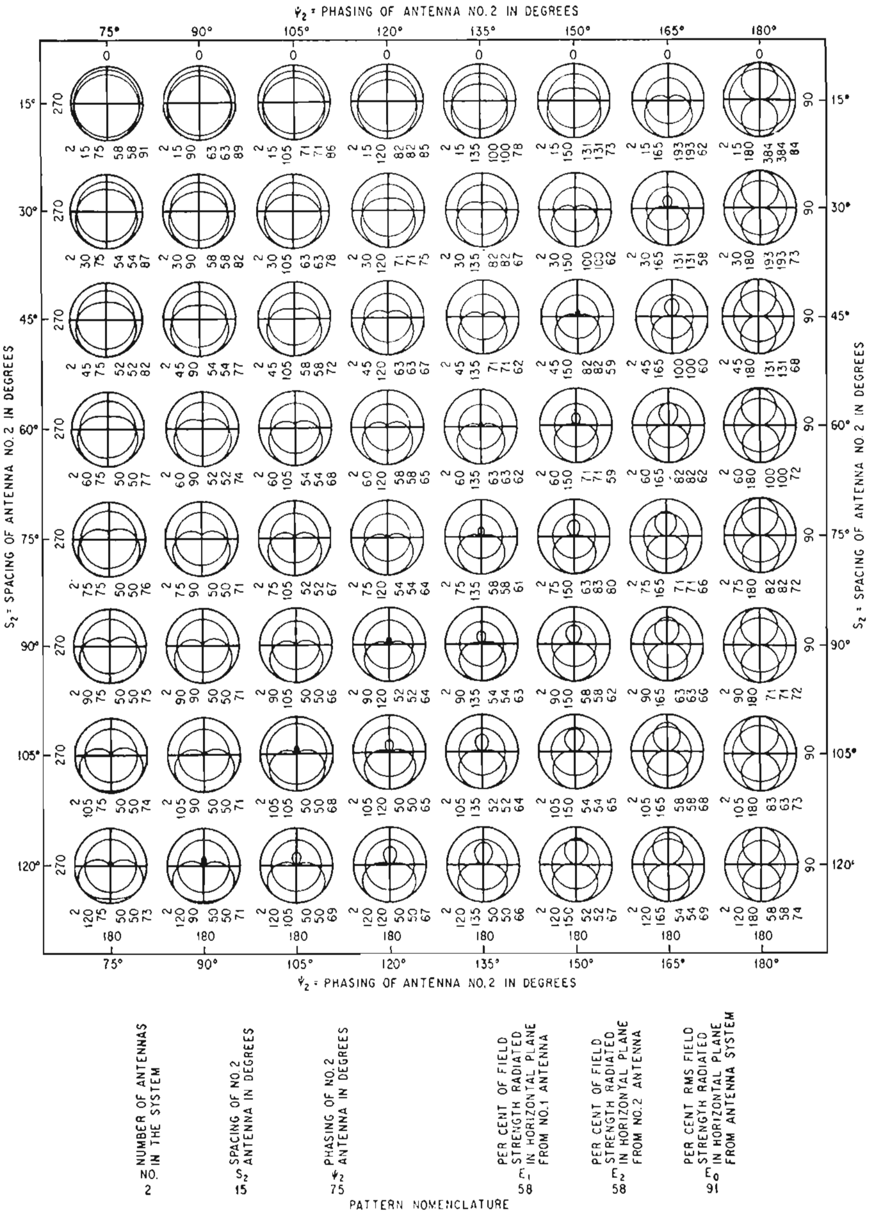


Fig. BH. Systematization of two-tower patterns in steps of 15 to 360°.

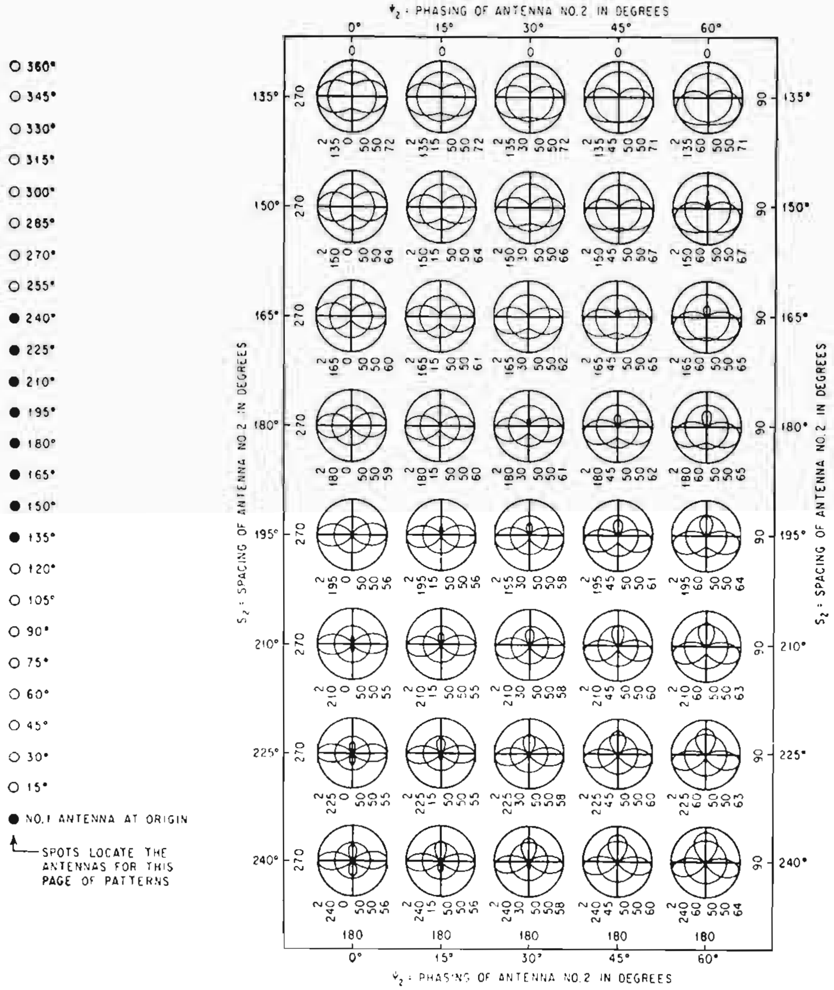


Fig. B1. Systematization of two-tower patterns in steps of 15 to 360°.

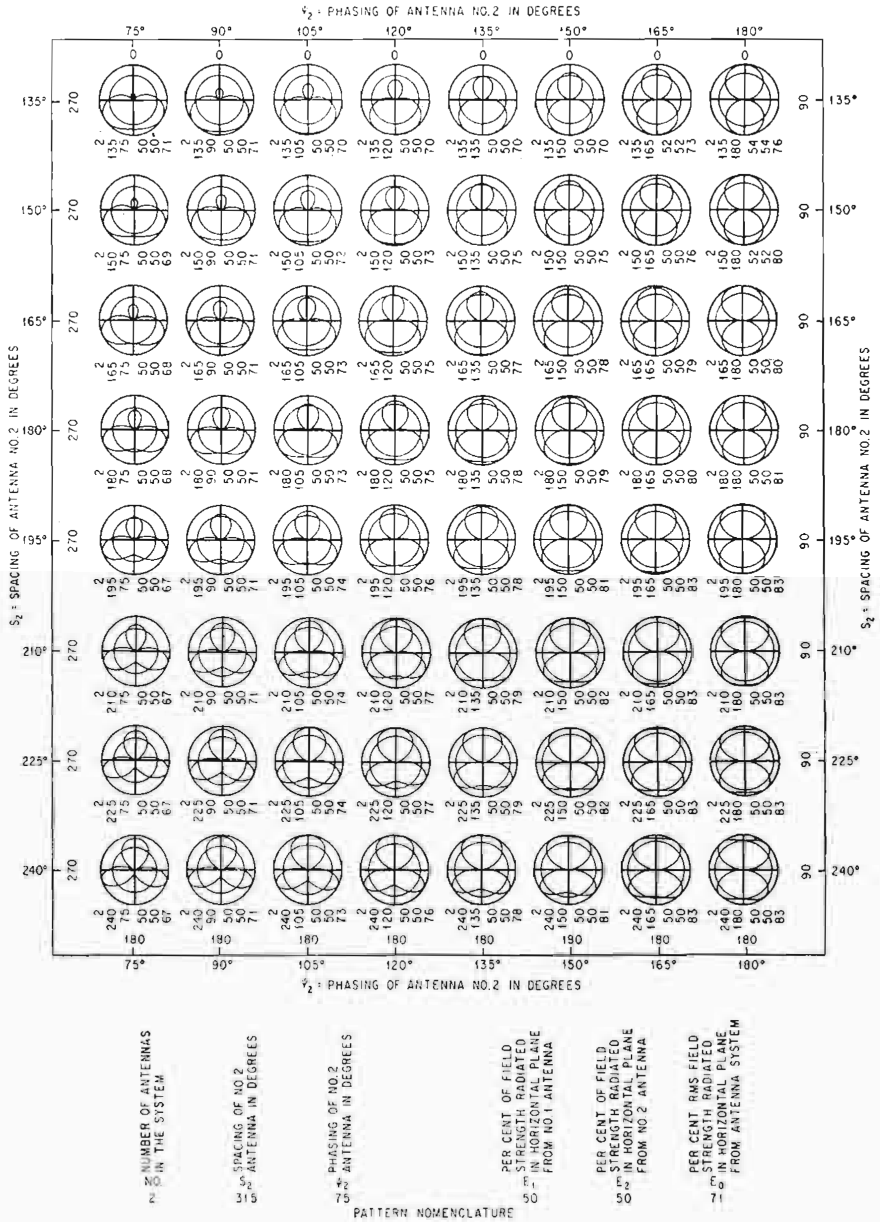


Fig. BJ. Systematization of two-tower patterns in steps of 15 to 360°.

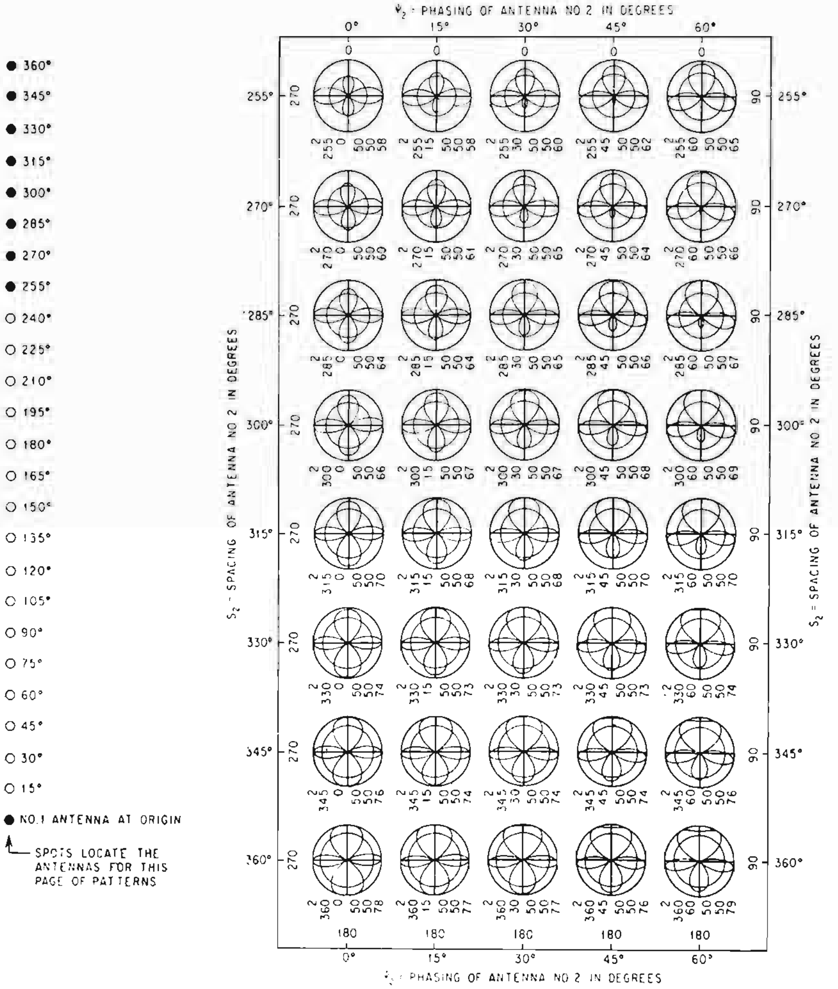


Fig. BK. Systematization of two-tower patterns in steps of 15 to 360°.

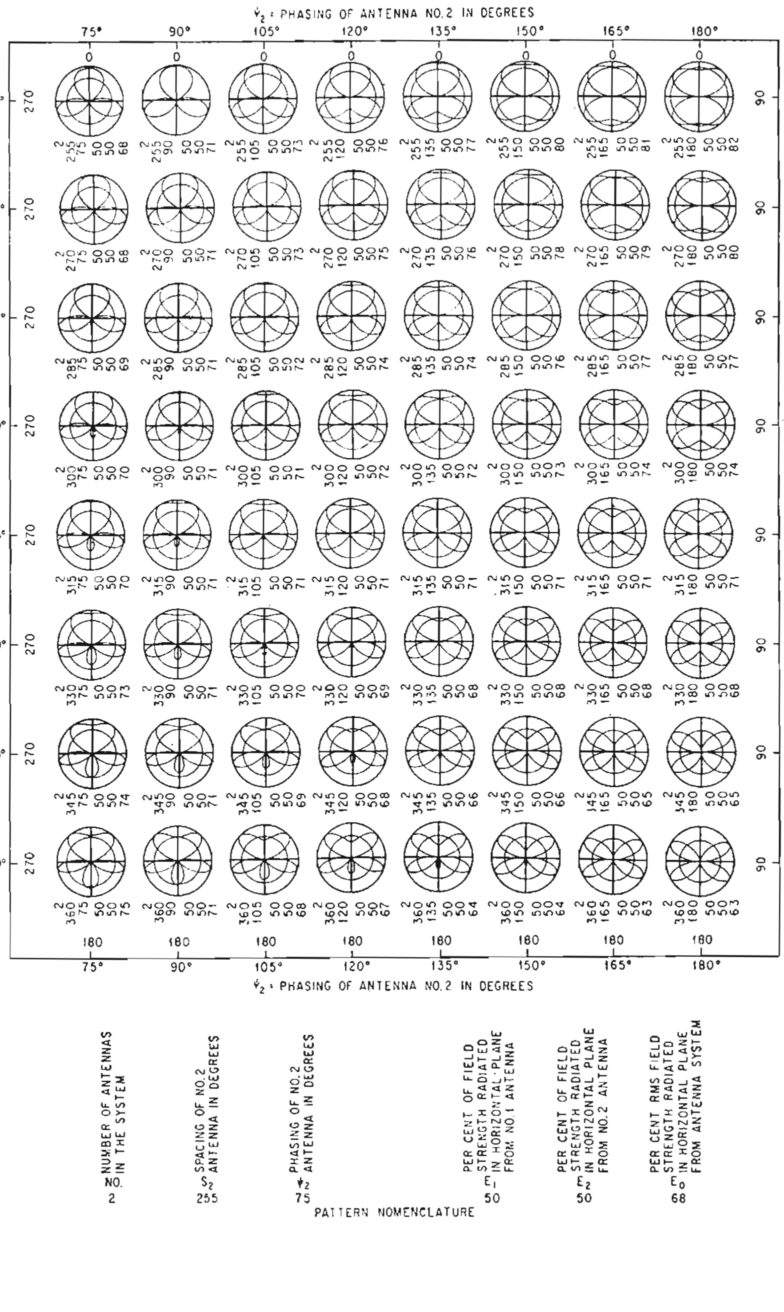


Fig. BL. Systematization of two-tower patterns in steps of 15 to 360°.

$$E_1 = E_{1s}\sqrt{\frac{R_{11}}{R_1 + M^2 R_2}}$$

$$= E_{1s}\sqrt{\frac{R_{11}}{(R_{11} + R_{L1} + R_{C1}) + M^2(R_{22} + R_{L2} + R_{C2})}} \quad \text{[B-5]}$$

where E_1 = inverse field strength at 1 mile for tower 1 while operating in the array, mv/m

E_{1s} = inverse field strength at 1 mile for tower 1 operating alone as a standard reference antenna, mv/m

R_{11} = loop self-resistance of tower 1, ohms

R_{L1} = loss resistance assumed at loop of tower 1, ohms

R_{C1} = coupled resistance at loop of tower 1 from other towers while array is in operation, ohms

R_{22} = loop self-resistance of tower 2, ohms

R_{L2} = loss resistance assumed at loop of tower 2, ohms

R_{C2} = coupled resistance at loop of tower 2 from other towers while array is in operation, ohms

M = ratio of current at loop of tower 2 divided by current at loop of tower 1

The values of E_{1s}, R_{11}, and R_{22} can be obtained from Fig. A-5 for simple towers that are not sectionalized or top-loaded. For other types of towers these values must be assumed or calculated. The values of R_{L1} and R_{L2} are usually assumed to be 2 ohms each.

Coupled-Resistance Formula

The values of coupled resistance from the other towers are given by

$$R_{C1} = MZ\cos(\Psi + \gamma) \quad \text{[B-6]}$$

$$R_{C2} = \frac{Z}{M}\cos(-\Psi + \gamma) \quad \text{[B-7]}$$

where Z = magnitude of loop impedance between the two towers, ohms

M = magnitude of current ratio of loop current in tower 2 divided by tower 1

γ = angle of loop mutual impedance, deg

Ψ = electrical phase angle of current in tower 2 with respect to tower 1, deg

These equations can be used with reasonable accuracy for simple nonloaded towers.

The above equations are written without the use of magnitude signs and subscripts for the sake of simplicity. The exact vector expressions are

$$Z_{21} = |Z_{21}|\,\underline{|\gamma_{21}} \quad \text{[B-8]}$$

and

$$M_{21} = \frac{I_2}{I_1} = |M_{21}|\,\underline{|\Psi_{21}} \quad \text{[B-9]}$$

where the currents are vector values having magnitude and phase angle. The current values in this equation are determined when the directional antenna is designed.

Mutual-Impedance Curves

The value of mutual impedance for most tower heights and spacing is given in Fig. BM. The loop mutual impedance between quarter-wave towers is shown in Fig. B-BN. These values can be used in the above equations for coupled resistance. For towers of unequal height the mutual impedance can be computed or reference can be made to curves already computed.[4]

Top-Loaded and Sectionalized Towers

The preceding equations can be applied to top-loaded and sectionalized towers but with considerable complication. The field strength E_{1s} and the self-resistance R_{11} are in general not available. They can be found in the literature for a few special cases.[3] The mutual impedance between sectionalized towers likewise is not readily available. It must be calculated for any special case if it is to be determined accurately. It is easier to determine the mutual effects by graphical solutions to the necessary accuracy.

Horizontal-Plane RMS Field Strength

The rms field strength for a two-tower array can be determined from

$$E_0 = E_1\sqrt{1 + F^2 + 2F\cos\Psi J_0(S)} \quad \text{[B-10]}$$

where E_0 = rms in verse field strength at 1 mile, mv/m

E_1 = inverse field strength at 1 mile for reference tower 1 while operating in array, mv/m

F = ratio of magnitude of field strength from tower 2 divided by tower 1

Ψ = electrical phase of field from tower 2 with respect to tower 1, deg

$J_0(S)$ = Bessel function of first order for tower spacing S

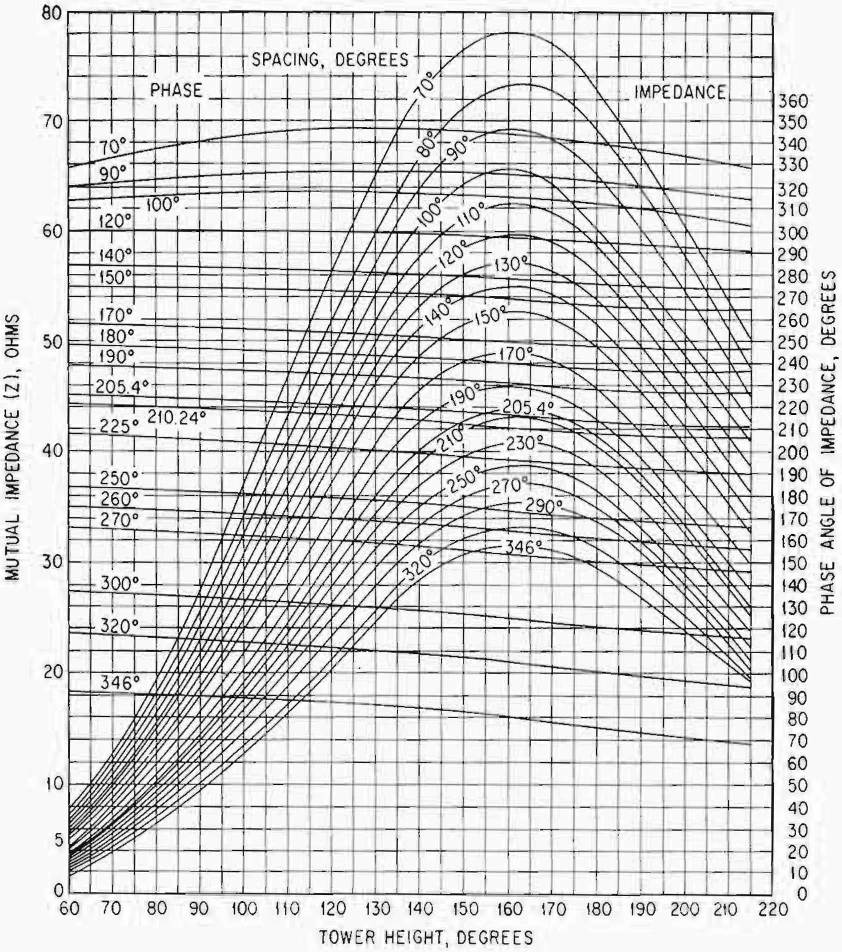


Fig. BM. Loop mutual impedance and phase angle between two towers of equal height.

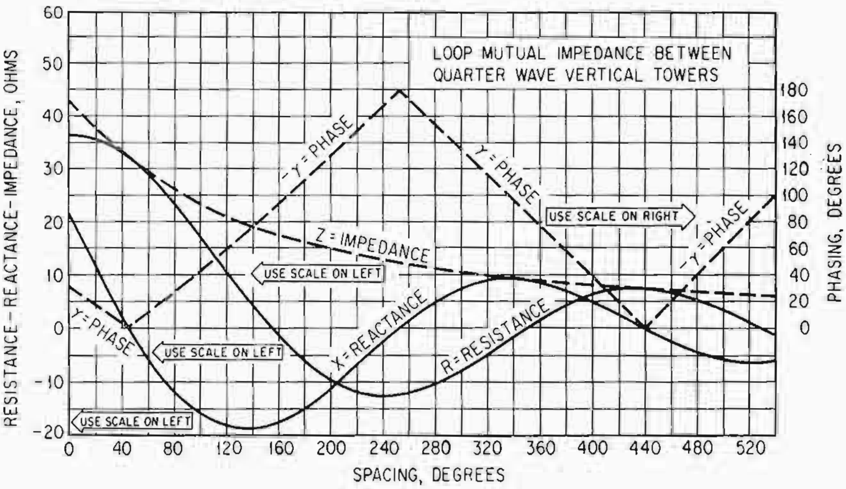


Fig. BN. Loop mutual impedance between quarter-wave vertical towers.

Usually the teams F and M are identical. However, with unequal-height towers and top-loading or sectionalized towers these ratios may have different values. Table B can be used to obtain the desired Bessel-function values. Interpolation can be used if necessary.

Horizontal RMS Field-Strength and Power Gain

The field-strength gain of any directional-antenna array can be written

$$\sqrt{g_0} = \frac{E_0}{E_{1s}} \qquad \text{[B-11]}$$

where $\sqrt{g_0}$ = field-strength gain by definition in the horizontal plane
E_0 = rms inverse field strength at 1 mile, mv/m
E_{1s} = inverse field strength at 1 mile for tower 1 operating alone as a standard reference antenna, mv/m

The power gain for a two-tower array is given by

$$g_0 = \frac{1 + F^2 + 2F\cos\Psi J_0(S)}{1 + F^2 + 2F\cos\Psi(R_{12}/R_{11})} \qquad \text{[B-12]}$$

where g_0 = directivity or power gain
F = ratio of magnitude of field strength from tower 2 divided by tower 1
Ψ = electrical phase of field from tower 2 with respect to tower 1, deg
$J_0(S)$ = Bessel function of first kind and zero order for tower spacing S
R_{12} = mutual loop resistance between towers, ohms
R_{11} = loop-radiation self-resistance of tower 1, ohms

It is of interest to know whether a particular antenna system is a gainer or a loser as compared with a standard reference antenna. This can be determined by the following:

$$E_0 = E_{1s}\sqrt{g_0} \qquad \text{[B-13]}$$

Now, if 90° towers are used and the field ratio F = 1, Eq. B-12 substituted in Eq. B-13 gives

$$E_0 = 195\sqrt{\frac{1 + \cos\Psi J_0(S)}{1 + \cos\Psi(R_{12}/36.6)}} \qquad \text{[B-14]}$$

The solution of this equation is shown in Fig. B-15 for various values of tower current phasing and tower spacing. It gives the theoretical field without loss for 1-kw operation.

Power to Provide System Losses

Because of losses in the transmission lines and matching, phasing, and power-division networks, plus other losses in the system such as resistance losses in the tower and ground system and dielectric losses in the insulators, an overfeeding of power is allowed by FCC at the common point.

The calculation of the amount of overfeeding of power at the common point is made as follows:

For stations with directional antennas authorized to radiate 5 kw of power or less, the measured

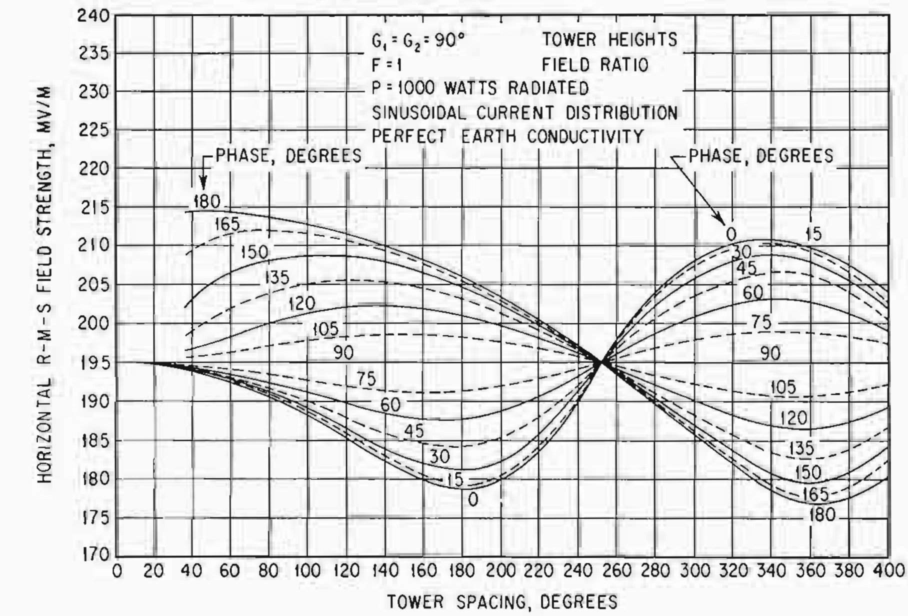


Fig. BO. Horizontal rms field strength of two-tower directional antenna.

common-point resistance is assumed to be 92.5 percent of its measured value, and for all other directional antennas, it is assumed to be 95 percent. This arbitrary reduction in resistance amounts to increasing the current at the common point by $1/\sqrt{0.925} = 1.0389$, or approximately a 4 percent increase for values of transmitter power up to and including 5 kw. For transmitters with power above 5 kw the antenna current can be increased $1/\sqrt{0.95} = 1.0252$, or approximately a 2.5 percent increase.

Another way of saying this is that for a 1-kw station 1,081 watts can be fed in at the common point, while for a 50-kw station 52,105 watts can be fed in at the common point. For the 1-kw station there is 81 watts available for feeder-system loss, and in the 50-kw station there is 2,105 watts available for feeder-system loss.

Feeder System

Loop and Base, Impedance and Current

The first approximation of the base resistance of an ordinary base-insulated tower that is not top-loaded or sectionalized is given by

$$R_b = \frac{R_a}{\sin^2 G} \qquad \text{[B-15]}$$

where R_b = base resistance of tower, ohms
R_a = loop resistance of tower, ohms
G = electrical height of tower, deg

The first approximation of the base current for the above tower is

$$I_b = I_a \sin G \qquad \text{[B-16]}$$

where I_b = base current of tower, amp
I_a = loop current of tower, amp

These equations are consistent with the theory that the power at the loop is equal to the power at the base; that is,

$$I_b{}^2 R_b = I_a{}^2 R_a \qquad \text{[B-17]}$$

These equations are most accurate for single towers operating alone where the cross section is small and uniform so the current distribution will be approximately sinusoidal.

A second approximation of the base impedance depends upon the tower acting like a transmission line from the loop to the base and is given by

$$Z_b = Z_0 \frac{Z_a \cos(G - 90) + jZ_0 \sin(G - 90)}{Z_0 \cos(G - 90) + jZ_a \sin(G - 90)} \qquad \text{[B-18]}$$

where Z_b = base impedance of tower, ohms
Z_a = loop impedance of tower, ohms
Z_0 = average characteristic impedance of tower, ohms
G = electrical height of tower, deg

A second approximation for the base current also depends upon the tower acting like a transmission line and is written

$$I_b = |I_a| \left[\sin G + j\frac{R_a}{2Z_0}(1 - \cos G)\right] \qquad \text{[B-19]}$$

where I_b = complex value of tower base current, amp
$|I_a|$ = magnitude of tower loop current, amp
R_a = loop resistance of tower, ohms

Z_0 and G are defined following Eq. B-18. The first term in this equation is a sinusoidal term corresponding to Eq. B-16, which corresponds to the antenna current that causes the radiation. The second term is the feed current which supplies the radiated power from the base to the loop. It should be noted that this equation does not give the phase of the base current with respect to the loop current. This change in phase between the loop and base current is expressed by the more general equation

$$I_y = |I_a| \left[\sin(G - y) + j\frac{R_a}{2Z_0}(\cos y - \cos G)\right] \qquad \text{[B-20]}$$

where I_y is the current at any height y on the tower in amperes. The other terms are defined in Eq. B-19. Equation B-20 reduces to Eq. B-19 when $y = 0$. It gives the phase as well as the magnitude of the current at any point on the tower; hence at the current loop,

$$I_a = |I_a| \left[1 + j\frac{R_a}{2Z_0}(\cos y - \cos G)\right] \qquad \text{[B-21]}$$

The difference in the phase of the current I_a in Eq. B-21 and the current I_b in Eq. B-19 is the additional phase shift that must be provided for in the feeder system.

Conservation of power between the loop and base may not exist in Eqs. B-18 and B-19 as it does in Eqs. B-15 and B-16. However, Eqs. B-18 and B-19 usually give a better answer for the base impedance values when the towers are operating in a directional-antenna system.

These equations can be used in the feeder-system design. However, it is advisable to provide adequate range so proper adjustments can be made when the system is put into operation.

Networks for Matching Impedances

General. A directional-antenna system usually requires impedance-matching networks at several points such as from the transmission lines to the towers and perhaps from the transmitter to the common point of the antenna feeder system. These networks are usually made up of lumped constants in the form of L, T, or π sections.

If an impedance load is not a pure resistance, it can be made to look like a pure resistance by adding a reactance element in series or parallel that will make the load either series- or parallel-resonant. If this is done, the treatment of the impedance-matching networks can be simplified. Therefore in this section networks to match between pure resistance values of R_1 and R_2 will be treated.

L Sections. An L section is the simplest way to match between two resistors R_1 and R_2. It is made up of an inductor for Z_2 and a capacitor for Z_3 with a small, fixed phase lag depending upon the ratio $r = R_1/R_2$. Or it can be constructed with a capacitor for Z_2 and an inductor for Z_3 with a small phase advance as shown in Figs. P and Q.

The phase shift in an L section is fixed by the ratio $r = R_1/R_2$. This is because there are only two reactance elements. The size of the shunt element across R_1 controls the size of R_2, while the series element is used to resonate the circuit so only resistance appears at the R_2 terminals.

The design equations for an L section matching between resistors R_1 and R_2 are

$$Z_2 = \pm jR_2\sqrt{r-1} = \pm j\frac{R_1}{a} \qquad \text{[B-22]}$$

$$Z_3 = \mp j\frac{R_1}{\sqrt{r-1}} = \mp j\frac{R_1}{b} \qquad \text{[B-23]}$$

$$\cos\beta = \frac{1}{\sqrt{r}} \qquad \text{[B-24]}$$

where Z_1 = reactance of series arm, ohms
Z_2 = reactance of shunt arm, ohms
R_1 = larger terminating resistance, ohms
R_2 = smaller terminating resistance, ohms
r = R_1/R_2 ratio
β = phase shift, deg

The $\pm j$ in Eq. B-22 and $\mp j$ in Eq. B-23 means simply that if $+j$ or an inductor is used for Z_1, then $-j$ or a capacitor must be used for Z_3.

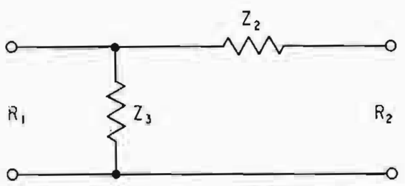


Fig. BP. General L-section impedance-matching network.

T and π Sections. T and π sections made up of reactance arms are widely used in the directional-antenna feeder systems. With the three reactance elements it is possible to control the amount of phase shift in addition to the input and output resistance values.

The efficiency of such a network is implicitly determined by the ratio $r = R_1/R_2$ and the phase shift β. There is no choice between T and π sections, whether advancing or retarding the phase, as far as efficiency is concerned. The loss increases with the ratio r and tends to increase for very small or very large phase shifts. For very high transformation ratios r of, say, 10 or more, it is advisable to use two or more sections in tandem. This will increase the stability and reduce the loss.

The design equations for a T or π section are:

$$a = \frac{r\sin\beta}{\sqrt{r} - \cos\beta} \qquad \text{[B-25]}$$

$$b = \sqrt{r}\sin\beta \qquad \text{[B-26]}$$

$$c = \frac{\sqrt{r}\sin\beta}{1 - \sqrt{r}\cos\beta} \qquad \text{[B-27]}$$

where a = design factor as shown in Figs. R and S
b = design factor as shown in Figs. R and S
c = design factor as shown in Figs. R and S
r = R_1/R_2 ratio, greater than 1
β = phase shift, deg

The graphs of a, b, and c are shown in Figs. T through X where r is assumed to be equal or greater than unity. The terminals at R_1 can be placed at either the load or generator end of the section; hence the above equations can be applied to any case.

It is interesting to note that these design curves also apply for the L section where $a = r/\sqrt{r-1}$, $b = \sqrt{r-1}$, and $c = \infty$.

Transmission Lines

General. Most directional antennas require one or more RF transmission lines. They are usually operated nonresonant, which means that the load is made equal to the characteristic impedance of the transmission line. In this case the wave travels from the transmitter end to the load end and is completely absorbed by the load; therefore, there are no standing waves on the line. For this condition of operation the power loss and standing waves on the line are a minimum.

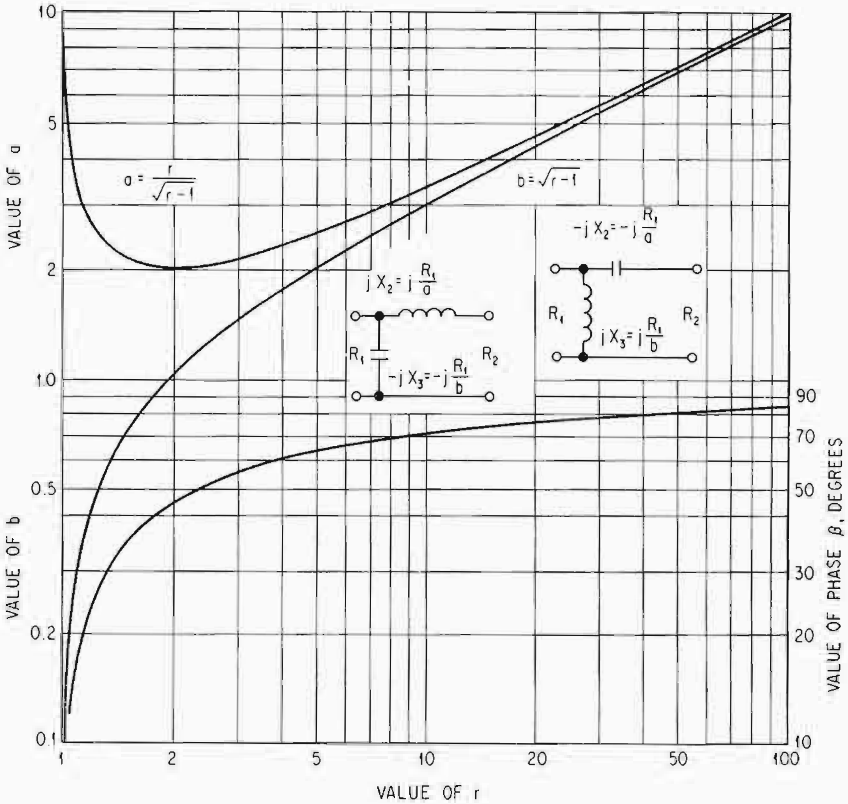


Fig. BQ. L-section design chart.

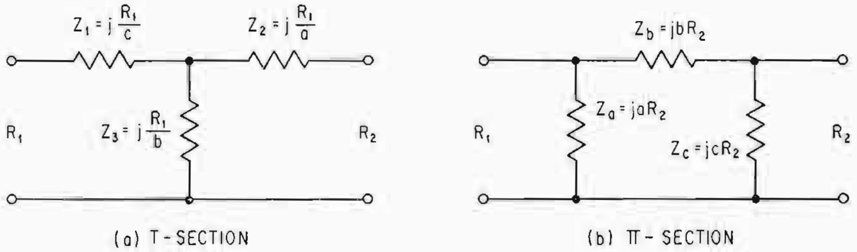


Fig. BR. General T- and π section impedance-matching network.

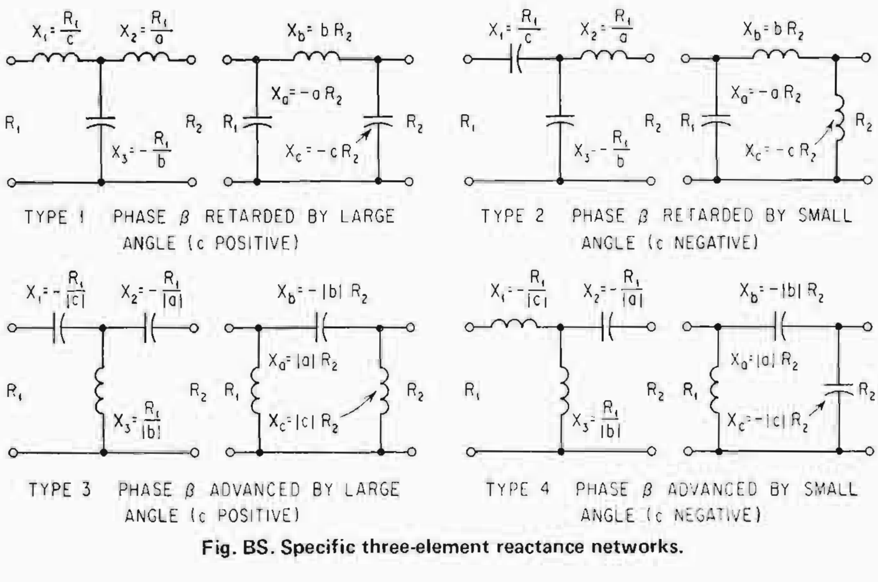


Fig. BS. Specific three-element reactance networks.

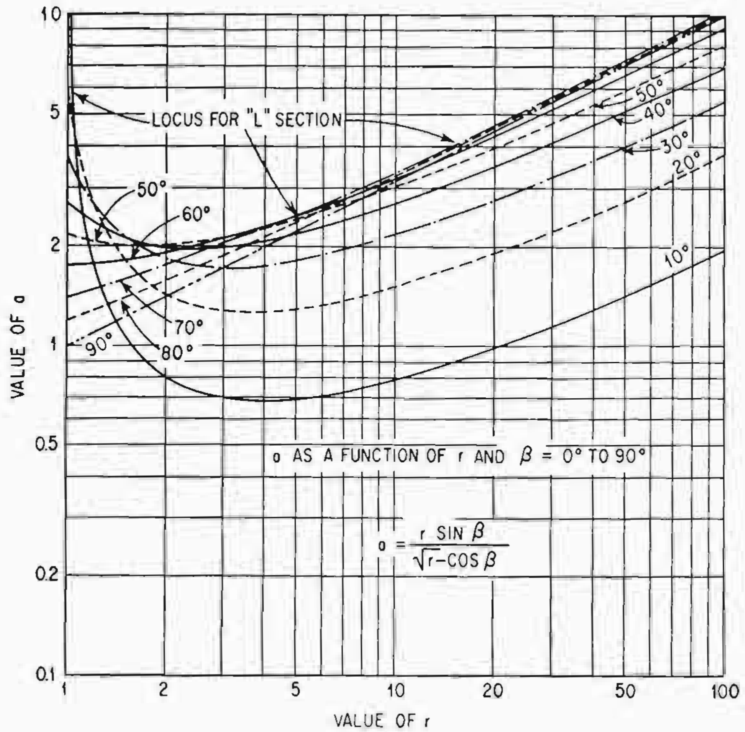


Fig. BT. Design chart for *a* as function of *r* and $\beta = 0$ to 90°.

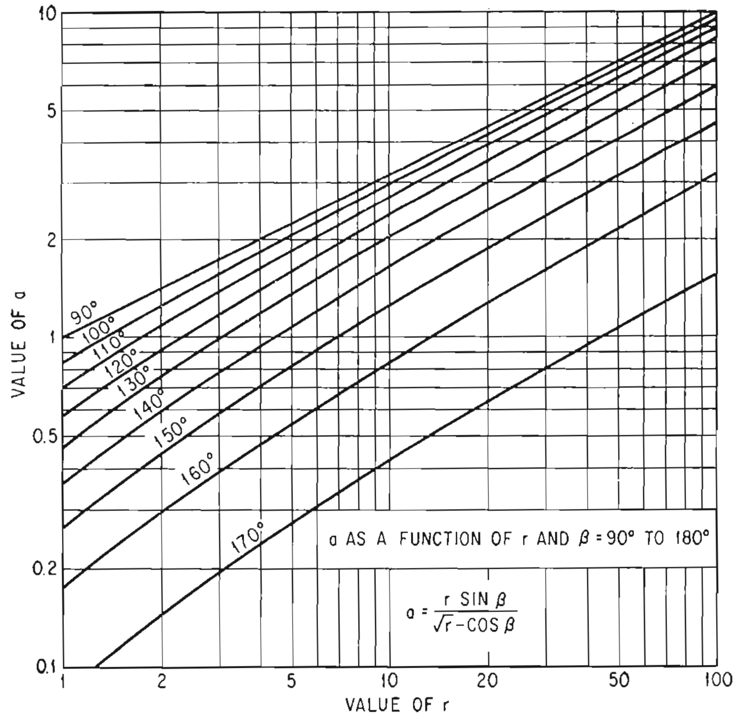


Fig. BU. Design chart for *a* as function of *r* and $\beta = 90$ to 180°.

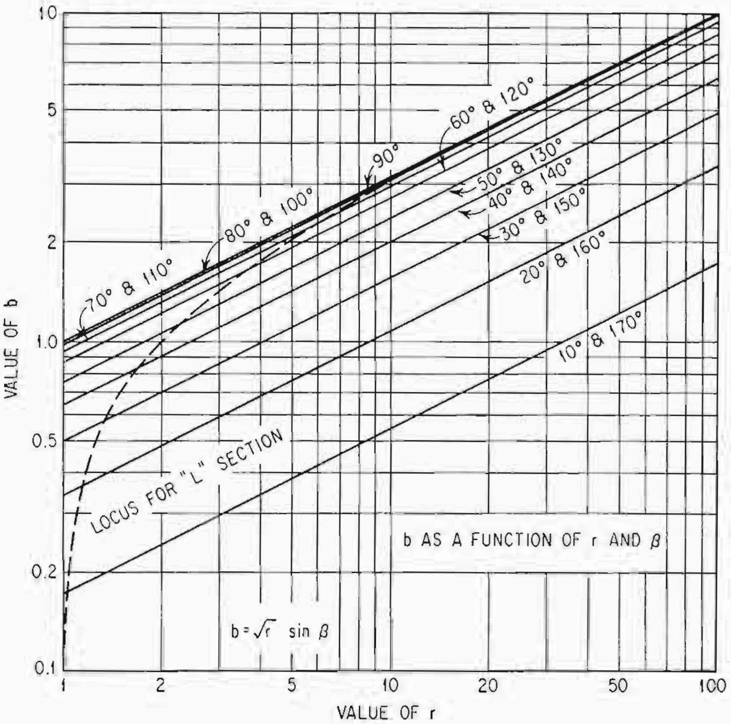


Fig. BV. Design chart for b as a function of r and β.

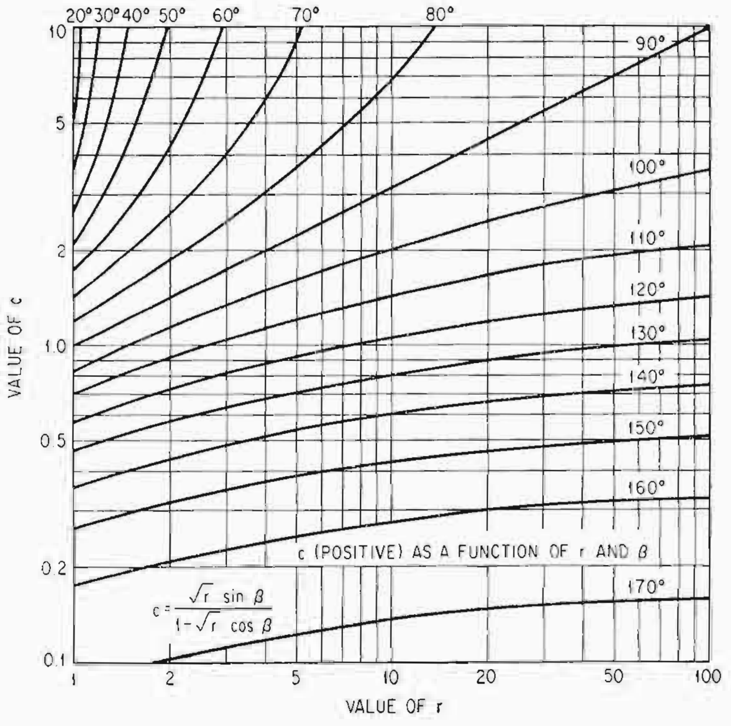


Fig. BW. Design chart for c (positive) as a function of r and β.

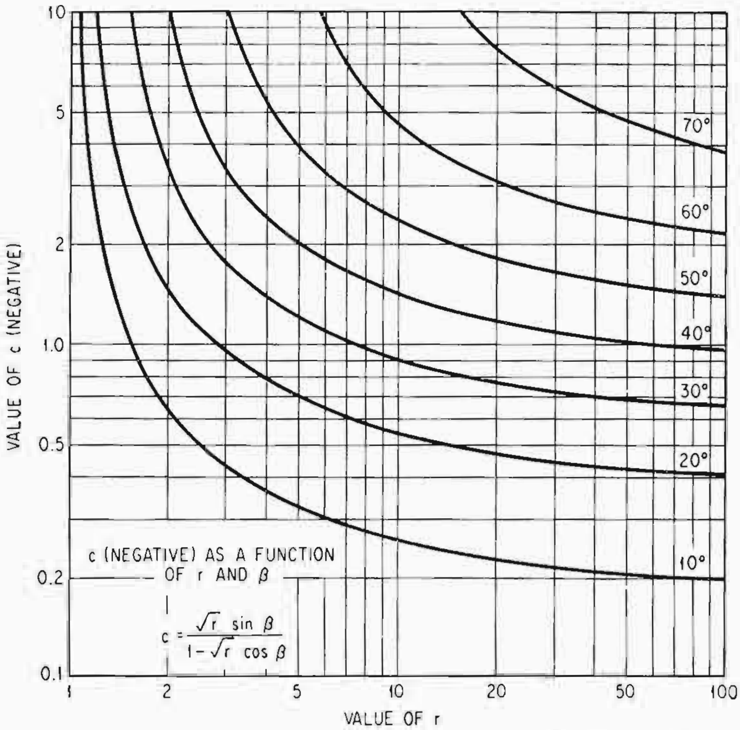


Fig. BX. Design chart for *c* (negative) as function of *r* and β.

The general equations for the voltage and current at the sending and receiving end of a transmission line are

$$E_s = E_r \cosh \sqrt{ZY}\,l + I_r Z_0 \sinh \sqrt{ZY}\,l \qquad \text{[B-28]}$$

$$I_s = I_r \cosh \sqrt{ZY}\,l + \frac{E_r}{Z_0} \sinh \sqrt{ZY}\,l \qquad \text{[B-29]}$$

where E_s = sending end voltage, volts
E_r = receiving end voltage, volts
$Z = R + j\omega L$ = series impedance per unit length, ohms
$\gamma = G + j\omega C$ = shunt admittance per unit length, ohms
l = length of line in same units as Z and γ
$Z_0 = \sqrt{Z/Y}$ = characteristic impedance, ohms
$\gamma = \sqrt{ZY}$ = propagation constant, or hyperbolic angle per unit length, radians

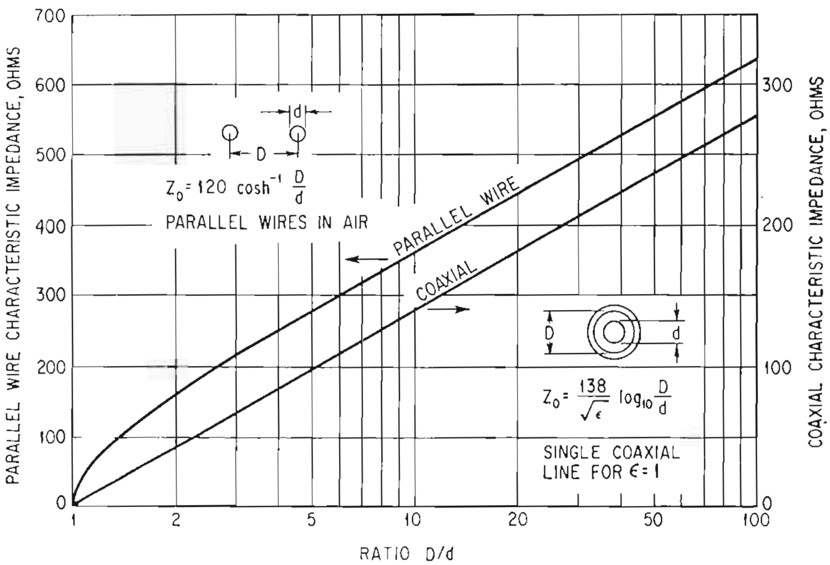


Fig. BY. Characteristic impedance of coaxial and two-wire transmission lines.

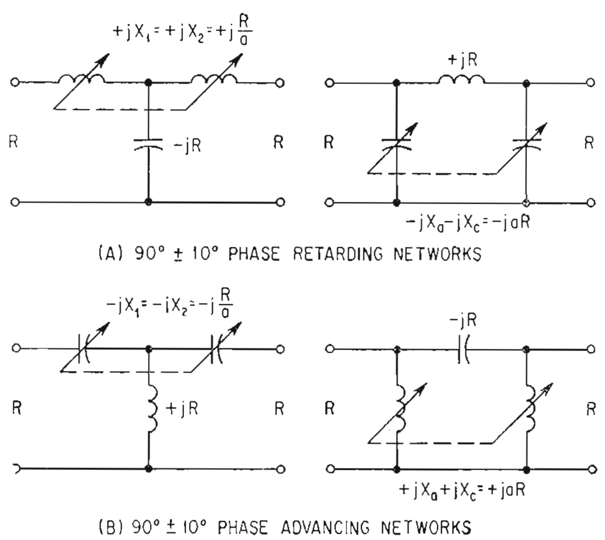


Fig. BZ. Phase-shifting networks.

Characteristic impedance. The characteristic impedance at radio frequency can be taken as a pure resistance,

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \text{[B-30]}$$

where L = inductance per unit length, henrys
C = capacitance per unit length, farads

For a single coaxial transmission line this equation can be written

$$Z_0 = \frac{138}{\sqrt{\epsilon}} \log_{10} \frac{D}{d} \qquad \text{[B-31]}$$

where ϵ = dielectric constant
D = inside diameter of outer conductor
d = outside diameter of inner conductor in the same units as D

For a parallel two-wire transmission line the characteristic impedance is

$$Z_0 = 120 \cosh^{-1} \frac{D}{d} \qquad \text{[B-32]}$$

where D = spacing between conductor centers
d = diameter of conductors in same units as D

The characteristic impedance of coaxial and two-wire transmission lines is shown in Fig. Y.

Propagation constant. The other factor of interest is the propagation constant given by

$$\sqrt{ZY} = \gamma = \alpha + j\beta \qquad \text{[B-33]}$$

where γ = propagation constant, radians per unit length
α = attenuation constant, nepers per unit length
β = phase constant, radians per unit length

Usually the attenuation in the line can be neglected in the design of directional antennas, particularly if the line is designed for low loss and is operated nonresonant.

The phase shift in an air-dielectric line will be only a few percent less than would occur in free space. For a coaxial line with ceramic beads the velocity factor is 0.85, while with solid polyethylene insulation the velocity factor is approximately 0.66. For two-wire lines insulated with ceramic spacers at intervals of a few feet the velocity factor is about 0.975. These factors must be taken into consideration when determining the phase shift in transmission lines of a directional-antenna system.

Phase-Shifting Networks

In addition to the necessary phase shift in the networks and transmission lines of a feeder system, it is usually desirable to have a phase-shift network the control on which shifts the phase with little or no impedance transformation. This is readily accomplished in a 90° T or π section that has unity impedance transformation, $r = 1$, between input and output terminals.

In such a 90° T or π section the reactance arms all have the same magnitude and are equal to $R = R_1 = R_2$. The shunt arm in the T section or the series arm in the π section is held constant while the other two arms are varied in unison to shift the phase (see Figs. BZ and BA).

The value of the reactance in the series arms of a T section is

$$Z_1 = Z_2 = \pm jR \frac{1 - \cos\beta}{\sin\beta} = \pm j\frac{R}{a} \qquad \text{[B-34]}$$

where $Z_1 = Z_2$ = series-arm reactance, ohms
$R = R_1 = R_2$ = terminating resistances, ohms
β = phase shift, deg

and the shunt arm of the T section is

$$Z_3 = \mp jR \frac{1}{\sin\beta} = \mp j\frac{R}{b} \qquad \text{[B-35]}$$

where Z_3 is the shunt-arm reactance in ohms and the other values are given above.

The value of the resistance in the shunt arms of a π section is

$$Z_a = Z_c = \pm jR \frac{\sin\beta}{1 - \cos\beta} = \pm jaR \qquad \text{[B-36]}$$

where $Z_a = Z_c$ = shunt-arm resistance, ohms
$R = R_1 = R_2$ = terminating resistances, ohms
β = phase shift, deg

and the series arm of the π section is

$$Z_b = \mp jR\sin\beta = \mp jbR \qquad \text{[B-37]}$$

where Z_b is the series-arm reactance in ohms and the other values are defined above.

The phase shift for the approximation that $b = 1$ gives good answers for $\pm 10°$ and can be written

$$\beta = \cos^{-1}\left(1 - \frac{R}{X_1}\right) = \cos^{-1}\left(1 - \frac{R}{X_a}\right) \qquad \text{[B-38]}$$

where X_1 and X_a are the reactance values as shown in Fig. Ba and the other values are defined above for the T- and π-section phase-shifting network.

Power-Dividing Networks

The driving-point impedance at the base of each tower in a directional-antenna system must be fed the correct amount of power to make the array operate properly. The usual practice is to provide a power-dividing network near the common-point input to the feeder system as shown in Fig. 8.

Typical power-dividing networks are shown in Fig. Ba. Terminals 1 and 2 are the output terminals to towers 1 and 2, respectively. The transmitter is connected directly or through a matching section to the terminals marked IN. A common practice is to start with the phase at the antenna loop current and compute the phase shift back through the matching networks, transmission lines, and phase shifters to the output of the power divider. In case the power division is nearly equal and the feed lines need to be out of phase, the push-pull circuit in Fig. Ba-a may be suitable. Where the feeder lines are in phase the series- or parallel-resonant circuits of Fig. Ba-b or c are applicable. In some cases the feed lines may be in quadrature phase, so the circuit of Fig. Ba-d can be used. If the power input to the feeder lines is known, then L sections can be designed to give the proper division as shown in Fig. Ba-e.

Small and Large Values of Variable Reactance

The design of a feeder system should be such that adjustments can be made with ease in the field. Some ideas are given here that may be of help in new designs or modifications of existing designs to make them easier to adjust.

If a very low value of capacitive reactance is required, it can be obtained easily by placing an inductor in series with a capacitor as shown in Fig. Ca-a. This arrangement makes it easy to obtain equivalent capacity values up to infinity when the circuit becomes series-resonant. Thus, it is possible to obtain values of inductive or capacitive reactance near zero values. This arrangement is often used in the shunt or series arms of a T or π section, so the correct value can be easily obtained. It is usually more satisfactory than providing a variable capacitor. In this arrangement care must be taken not to exceed the current rating of the capacitor. This discussion neglects the resistance component, which is usually very small.

Sometimes it is necessary to obtain an inductive reactance larger than the reactance of available inductors. In such cases it is possible to parallel the coil with a very small capacitor. If the capacitor tap on the coil is moved as shown in Fig. Bc-b, the desired value of inductive reactance can be achieved.

Adjustments

Theoretical Mesh Circuit Equation

In order to understand the operation of a directional-antenna system, it is desirable to understand how the circuit performs theoretically. With this understanding, it is easier to make the necessary adjustments. For a two-tower array the input terminals of the two towers can be considered to be a mesh circuit; hence the following simultaneous equations apply:

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \qquad \text{[B-39]}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} \qquad \text{[B-40]}$$

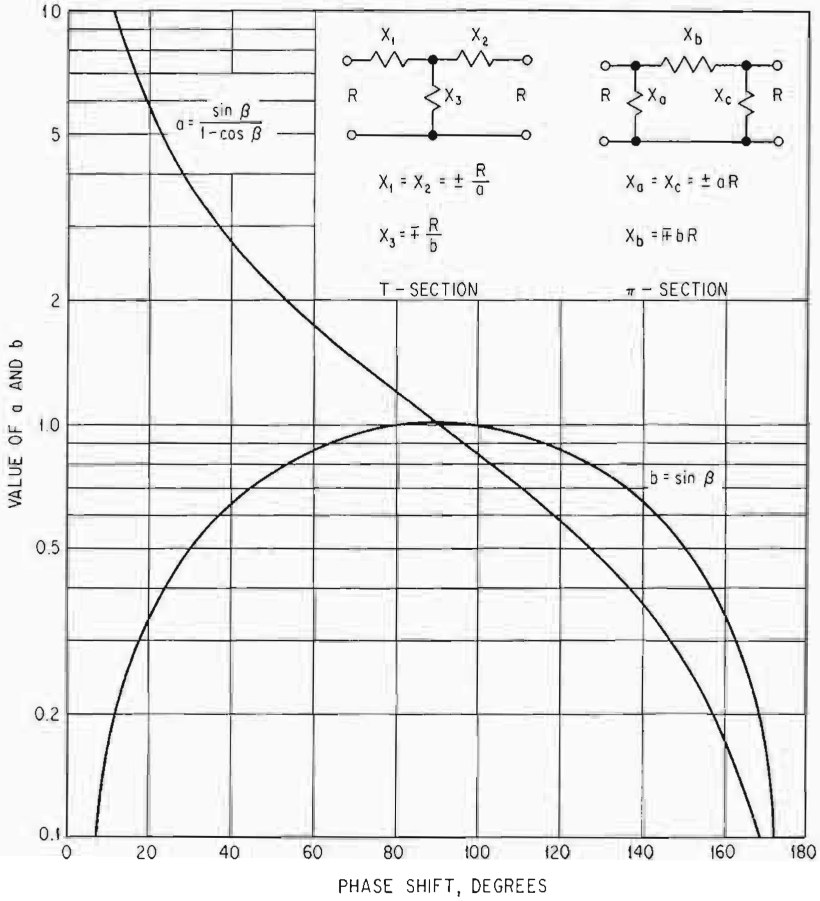


Fig. Ba. Phase-shifting-network curves.

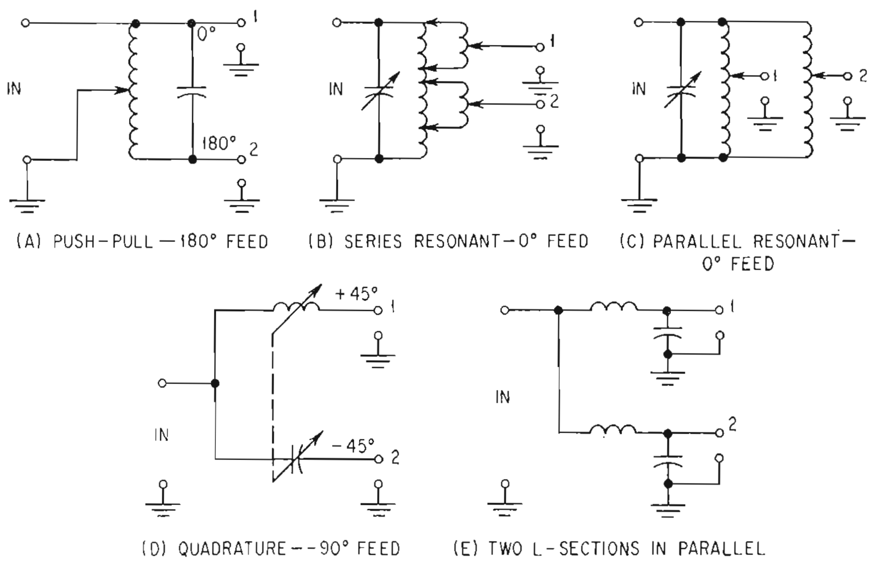


Fig. Bb. Typical power-dividing networks.

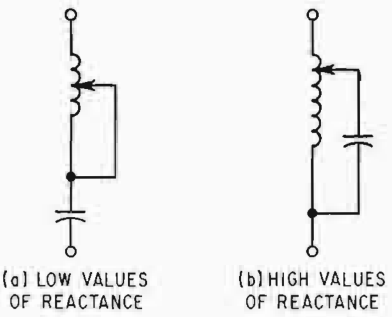


Fig. Bc. Methods of varying reactance.

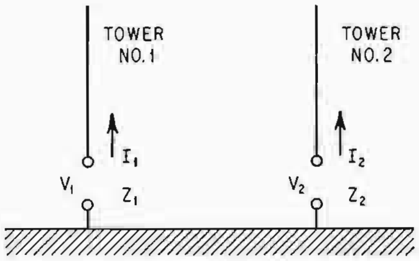


Fig. Bd. Two-tower input terminals.

where V_1 and V_2 = vector effective voltage at input terminals of towers 1 and 2 respectively, volts

I_1 and I_2 = vector effective current at input terminals of towers 1 and 2 respectively, amp

Z_{11} and Z_{22} = self-impedance of towers 1 and 2 respectively, ohms

$Z_{12} = Z_{21}$ = mutual impedance between towers 1 and 2, ohms

All the terms in the above mesh equations are complex quantities.

From this set of simultaneous equations it is possible to define the driving point impedance of each tower, thus

$$Z_1 = \frac{V_1}{I_1} = Z_{11} + \frac{I_2}{I_1} Z_{12} \qquad \text{[B-41]}$$

$$Z_2 = \frac{V_2}{I_2} = \frac{I_1}{I_2} Z_{12} + Z_{22} \qquad \text{[B-42]}$$

where Z_1 = driving-point impedance of tower 1 while array is in operation, ohms

Z_2 = driving-point impedance of tower 2 while array is in operation, ohms

The resistance component R_1 of Z_1 and R_2 of Z_2 is pure radiation resistance if there is no loss in the system. The directional-antenna system can be designed using this theoretical basis, and then from a knowledge of the system losses the driving-point impedances can be estimated with fair accuracy. (See Fig. Bd.)

Measured Base Self-Impedance

If the towers are approximately 90° high or less and the spacing is not very close, then the self-impedance can be measured by leaving the terminals of tower 2 open while measuring the impedance of tower 1 with an RF bridge at the operating frequency. This can be seen by inspecting Eq. B-41, where $I_2 = 0$; hence only Z_{11} will be measured for this condition.

Similarly, Z_{22} can be measured by leaving the terminals of tower 1 open while measuring at the terminals of tower 2.

Measured Base Mutual Impedance

This can be done by inserting a variable reactance in series with the terminals of tower 2 when the terminals of tower 1 are open and adjusting it so that only a pure resistance R_{22} remains. This can be done with an RF bridge. Tower 2 is now tuned to resonance at the operating frequency.

The next step is to drive tower 1 with a suitable voltage at the operating frequency and note the currents in towers 1 and 2 when tower 2 is tuned to resonance.

From these measurements the magnitude of the mutual impedance to a first approximation is

$$|Z_{12}| = -\frac{|I_2|}{|I_1|} R_{22} \qquad \text{[B-43]}$$

The angle of the mutual impedance can best be approximated by using theoretical information. If the loop mutual-impedance phases are used, they must be delayed by the effective electrical distance of the loop above the tower base. If base mutual-impedance phases are used, they also must be delayed by the same effective electrical distance because they do not provide for time delay of the current to reach the loop position.

If the phase-monitoring system has been installed and is calibrated to read properly, it can be used to measure the phase of I_2 with respect to I_1 and thus provide the necessary phase angle of Z_{12}. This is a good way to check the theoretical values. It may be even a better check on the antenna-monitor calibration.

The theoretical value of the magnitude of Z_{12} written $|Z_{12}|$ can be used in lieu of the above measured value. If the towers are near 180° in height, it is advisable to measure the mutual impedance. Also, if the spacing is less than 90°, the mutual-impedance values should be measured. In other words if the mutual impedance is large, it should be measured for best results.

Estimated Base Driving-Point Impedance

The driving-point impedance Z_1 can now be estimated by using the above values of Z_{11} and Z_{12} along with the current ratio:

$$M_{21} = \frac{I_2}{I_1} \qquad [B\text{-}44]$$

as specified in the directional-antenna design. When these values are substituted in Eq. B-41, the driving-point impedance for tower 1 while the array is in operation results.

Similarly it is possible to obtain the driving-point impedance for tower 2.

Estimated Base Driving-Point Current

Assuming no loss in the tower, insulators, or ground system, the authorized power input must be

$$P = |I_1|^2 R_1 + |I_2|^2 R_2 \qquad [B\text{-}45]$$

Since the ratio of current is known, we can write

$$|I_1| = \frac{P}{R_1 + |M_{21}|^2 R_2} \qquad [B\text{-}46]$$

and then,

$$|I_2| = |I_1|\,|M_{21}| \qquad [B\text{-}47]$$

The above authorized power does not include the power allowed by FCC for feeder-system losses.

Feeder-System Adjustment

A good way to set up the feeder system is to make up dummy driving-point impedance loads with the aid of an RF bridge. If small components are available, only RF bridge measurements can be used. If larger resistors are used which will not change value when heated up with power, then the whole feeder system can be set up and adjusted for the correct power division using dummy driving-point impedance loads.

It is possible to adjust each transmission line to the tower input network using the dummy load, since the tower input impedance will not have the correct value until the whole directional-antenna system is operating properly. After these matching networks are adjusted with the RF bridge, it should not be necessary to make any further adjustments at this point. The networks at the towers can then be connected, and the completion of the feeder-system adjustments can usually be made at the common point where the power division and phasing controls are located.

Tower with Negative Resistance

In some directional-antenna systems a tower will have a negative resistance at the driving point. This means that power must be removed from the tower terminals. While the initial adjustments are made, this power can be dissipated, but for the final operation it is usually desirable to feed this power back into the system in order to maintain high efficiency.

In Fig. Be-a the antenna system is properly adjusted to give the required pattern shape but the power from tower 2 is being dissipated into a driving-point impedance with a negative resistance component. If the driving-point impedance of the negative tower is matched into a transmission line, this power can be fed back to the common-point input point as shown in Fig. Be-b. It is necessary to adjust the phase and magnitude of the feedback voltage properly so it will equal the magnitude and phase of the voltage where the feedback system is connected at the common point. Then the two circuits can be connected in parallel. The feedback system is used to control the phase and magnitude of the feedback voltage. When the feedback voltage is connected across the common-point input terminals, the input resistance will increase because the negative resistance in parallel with a positive resistance will have a resistance value larger than the positive resistance value alone.

In the design of the feeder system it is usually possible simply to tap on to the power-dividing network in the usual manner with the circuit to the tower having a negative resistance. In this case the phase shifts must be figured in the reverse direction owing to the reverse direction of power flow.

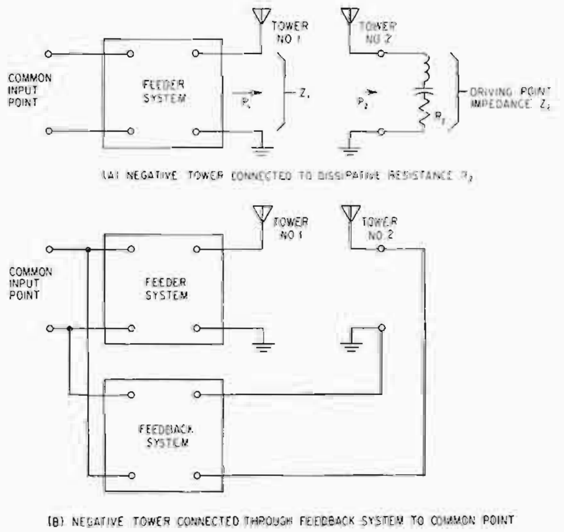


Fig. Be. Addition of power from tower with negative resistance.

TABLE A
Minimum-Depth Term[a]

F	$\frac{1}{F}$	F^2	$\frac{1+F^2}{2F}$	F	$\frac{1}{F}$	F^2	$\frac{1+F^2}{2F}$
0.995	1.0050	0.99002	1.00001	0.745	1.3422	0.55502	1.0436
0.990	1.0101	0.98010	1.00005	0.740	1.3513	0.54760	1.0457
0.985	1.0152	0.97022	1.00011	0.735	1.3605	0.54022	1.0478
0.980	1.0204	0.96040	1.00020	0.730	1.3698	0.53290	1.0499
0.975	1.0256	0.95062	1.00032	0.725	1.3793	0.52562	1.0522
0.970	1.0309	0.94090	1.00046	0.720	1.3888	0.51840	1.0544
0.965	1.0363	0.93122	1.00063	0.715	1.3986	0.51122	1.0568
0.960	1.0417	0.92160	1.00083	0.710	1.4084	0.50410	1.0592
0.955	1.0471	0.91202	1.00106	0.705	1.4184	0.49702	1.0617
0.950	1.0526	0.90250	1.00132	0.700	1.4285	0.49000	1.0643
0.945	1.0582	0.89302	1.0016	0.695	1.4388	0.48302	1.0669
0.940	1.0638	0.88360	1.0019	0.690	1.4492	0.47610	1.0696
0.935	1.0695	0.87422	1.0023	0.685	1.4598	0.46922	1.0724
0.930	1.0752	0.86490	1.0026	0.680	1.4705	0.46240	1.0753
0.925	1.0810	0.85562	1.0030	0.675	1.4814	0.45562	1.0782
0.920	1.0869	0.84640	1.0035	0.670	1.4925	0.44890	1.0812
0.915	1.0929	0.83722	1.0039	0.665	1.5037	0.44222	1.0844
0.910	1.0989	0.82810	1.0045	0.660	1.5151	0.43560	1.0875
0.905	1.1049	0.81902	1.0050	0.655	1.5267	0.42902	1.0909
0.900	1.1111	0.81000	1.0056	0.650	1.5384	0.42250	1.0942
0.895	1.1173	0.80102	1.0062	0.645	1.5503	0.41602	1.0977
0.890	1.1236	0.79210	1.0068	0.640	1.5625	0.40960	1.1012
0.885	1.1299	0.78322	1.0075	0.635	1.5748	0.40322	1.1049
0.880	1.1363	0.77440	1.0082	0.630	1.5873	0.39690	1.1086
0.875	1.1428	0.76562	1.0089	0.625	1.6000	0.39062	1.1125
0.870	1.1494	0.75690	1.0097	0.620	1.6129	0.38440	1.1164
0.865	1.1560	0.74822	1.0105	0.615	1.6260	0.37822	1.1205
0.860	1.1627	0.73960	1.0114	0.610	1.6393	0.37210	1.1246
0.855	1.1695	0.73102	1.0123	0.605	1.6528	0.36602	1.1289
0.850	1.1764	0.72250	1.0132	0.600	1.6666	0.36000	1.1333
0.845	1.1834	0.71402	1.0142	0.595	1.6806	0.35402	1.1378
0.840	1.1904	0.70560	1.0152	0.590	1.6949	0.34810	1.1425
0.835	1.1976	0.69722	1.0163	0.585	1.7094	0.34222	1.1472
0.830	1.2048	0.68890	1.0174	0.580	1.7241	0.33640	1.1521
0.825	1.2121	0.68062	1.0186	0.575	1.7391	0.33062	1.1571
0.820	1.2195	0.67240	1.0197	0.570	1.7543	0.32490	1.1621
0.815	1.2269	0.66422	1.0210	0.565	1.7699	0.31922	1.1675
0.810	1.2345	0.65610	1.0223	0.560	1.7857	0.31360	1.1728
0.805	1.2422	0.64802	1.0236	0.555	1.8018	0.30802	1.1784
0.800	1.2500	0.64000	1.0250	0.550	1.8181	0.30250	1.1841
0.795	1.2578	0.63202	1.0264	0.545	1.8348	0.29702	1.1899
0.790	1.2658	0.62410	1.0279	0.540	1.8518	0.29160	1.1959
0.785	1.2738	0.61622	1.0294	0.535	1.8691	0.28622	1.2021
0.780	1.2820	0.60840	1.0310	0.530	1.8867	0.28090	1.2084
0.775	1.2903	0.60062	1.0327	0.525	1.9047	0.27562	1.2149
0.770	1.2987	0.59290	1.0343	0.520	1.9230	0.27040	1.2215
0.765	1.3071	0.58522	1.0361	0.515	1.9417	0.26522	1.2284
0.760	1.3157	0.57760	1.0379	0.510	1.9607	0.26010	1.2354
0.755	1.3245	0.57002	1.0397	0.505	1.9802	0.25502	1.2426
0.750	1.3333	0.56250	1.0417	0.500	2.0000	0.25000	1.2500

* $(1 + F^2)/2F$ where either F or $1/F$ is the ratio of the inverse field strengths.

TABLE A
Minimum-Depth Term (*Continued*)

F	$\frac{1}{F}$	F^2	$\frac{1+F^2}{2F}$	F	$\frac{1}{F}$	F^2	$\frac{1+F^2}{2F}$
0.495	2.0202	0.24502	1.2576	0.345	2.8985	0.11902	1.6218
0.490	2.0408	0.24010	1.2654	0.340	2.9411	0.11560	1.6406
0.485	2.0618	0.23522	1.2734	0.335	2.9850	0.11222	1.6600
0.480	2.0833	0.23040	1.2817	0.330	3.0303	0.10890	1.6802
0.475	2.1052	0.22562	1.2901	0.325	3.0769	0.10562	1.7010
0.470	2.1276	0.22090	1.2988	0.320	3.1250	0.10240	1.7225
0.465	2.1505	0.21622	1.3078	0.315	3.1746	0.09922	1.7448
0.460	2.1739	0.21160	1.3170	0.310	3.2258	0.09610	1.7679
0.455	2.1978	0.20702	1.3264	0.305	3.2786	0.09302	1.7918
0.450	2.2222	0.20250	1.3361	0.300	3.3333	0.09000	1.8167
0.445	2.2471	0.19802	1.3461	0.295	3.3898	0.08702	1.8424
0.440	2.2727	0.19360	1.3564	0.290	3.4482	0.08410	1.8691
0.435	2.2988	0.18922	1.3669	0.285	3.5087	0.08122	1.8969
0.430	2.3255	0.18490	1.3778	0.280	3.5714	0.07840	1.9257
0.425	2.3529	0.18062	1.3890	0.275	3.6363	0.07562	1.9557
0.420	2.3809	0.17640	1.4005	0.270	3.7037	0.07290	1.9869
0.415	2.4096	0.17222	1.4123	0.265	3.7735	0.07022	2.0193
0.410	2.4390	0.16810	1.4245	0.260	3.8461	0.06760	2.0531
0.405	2.4691	0.16402	1.4371	0.255	3.9215	0.06502	2.0883
0.400	2.5000	0.16000	1.4500	0.250	4.0000	0.06250	2.1250
0.395	2.5316	0.15602	1.4633	0.245	4.0816	0.06002	2.1633
0.390	2.5641	0.15210	1.4770	0.240	4.1666	0.05760	2.2033
0.385	2.5974	0.14822	1.4912	0.235	4.2553	0.05522	2.2451
0.380	2.6315	0.14440	1.5058	0.230	4.3478	0.05290	2.2889
0.375	2.6666	0.14062	1.5208	0.225	4.4444	0.05062	2.3347
0.370	2.7027	0.13690	1.5364	0.220	4.5454	0.04840	2.3827
0.365	2.7397	0.13322	1.5524	0.215	4.6511	0.04622	2.4331
0.360	2.7777	0.12960	1.5689	0.210	4.7619	0.04410	2.4860
0.355	2.8169	0.12602	1.5859	0.205	4.8780	0.04202	2.5415
0.350	2.8571	0.12250	1.6035	0.200	5.0000	0.04000	2.6000

TABLE B
Bessel Function, J_0 ($S \cos \theta$)

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
4	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
5	0.998	0.998	0.998	0.999	0.999	0.999	1.000	1.000	1.000
6	0.997	0.997	0.998	0.998	0.998	0.999	0.999	1.000	1.000
7	0.996	0.996	0.997	0.997	0.998	0.998	0.999	1.000	1.000
8	0.995	0.995	0.996	0.996	0.997	0.998	0.999	0.999	1.000
9	0.994	0.994	0.995	0.995	0.996	0.997	0.998	0.999	1.000
10	0.992	0.993	0.993	0.994	0.996	0.997	0.998	0.999	1.000
11	0.991	0.991	0.992	0.993	0.995	0.996	0.998	0.999	1.000
12	0.989	0.989	0.990	0.992	0.994	0.996	0.997	0.999	1.000
13	0.987	0.988	0.989	0.990	0.992	0.995	0.997	0.998	1.000
14	0.985	0.986	0.987	0.989	0.991	0.994	0.996	0.998	1.000
15	0.983	0.983	0.985	0.987	0.990	0.993	0.996	0.998	0.999
16	0.981	0.981	0.983	0.985	0.985	0.992	0.995	0.998	0.999
17	0.978	0.979	0.981	0.984	0.987	0.991	0.994	0.997	0.999
18	0.976	0.976	0.978	0.982	0.985	0.990	0.994	0.997	0.999
19	0.973	0.974	0.976	0.980	0.986	0.989	0.993	0.997	0.999
20	0.970	0.971	0.973	0.977	0.982	0.987	0.992	0.996	0.999
21	0.967	0.968	0.971	0.975	0.980	0.986	0.992	0.996	0.999
22	0.964	0.965	0.968	0.973	0.973	0.985	0.991	0.996	0.999
23	0.960	0.961	0.965	0.970	0.977	0.983	0.970	0.975	0.999
24	0.957	0.958	0.962	0.967	0.974	0.982	0.985	0.995	0.999
25	0.953	0.954	0.958	0.965	0.972	0.980	0.988	0.994	0.999
26	0.949	0.951	0.955	0.962	0.970	0.979	0.987	0.994	0.998
27	0.945	0.947	0.953	0.959	0.968	0.977	0.986	0.994	0.998
28	0.941	0.943	0.948	0.956	0.965	0.976	0.985	0.993	0.998
29	0.937	0.939	0.944	0.953	0.963	0.974	0.984	0.993	0.998
30	0.933	0.935	0.940	0.949	0.960	0.972	0.983	0.992	0.998
31	0.928	0.930	0.936	0.946	0.958	0.970	0.982	0.992	0.998
32	0.924	0.926	0.932	0.942	0.955	0.968	0.981	0.991	0.998
33	0.919	0.921	0.928	0.939	0.952	0.966	0.979	0.990	0.998
34	0.914	0.916	0.924	0.935	0.945	0.964	0.978	0.990	0.997
35	0.905	0.912	0.919	0.931	0.941	0.962	0.977	0.985	0.997
36	0.904	0.907	0.915	0.927	0.943	0.960	0.976	0.989	0.997
37	0.899	0.901	0.910	0.923	0.940	0.957	0.974	0.986	0.997
38	0.893	0.896	0.905	0.919	0.937	0.955	0.973	0.987	0.997
39	0.888	0.891	0.900	0.915	0.933	0.953	0.971	0.987	0.997
40	0.882	0.885	0.895	0.911	0.930	0.950	0.970	0.986	0.996
41	0.876	0.880	0.890	0.906	0.926	0.948	0.968	0.985	0.996
42	0.870	0.874	0.885	0.902	0.923	0.945	0.967	0.984	0.996
43	0.864	0.868	0.880	0.897	0.919	0.943	0.965	0.984	0.996
44	0.858	0.862	0.874	0.892	0.915	0.940	0.963	0.983	0.996
45	0.852	0.856	0.868	0.888	0.913	0.937	0.962	0.982	0.995
46	0.845	0.850	0.862	0.883	0.908	0.935	0.960	0.981	0.995
47	0.839	0.843	0.857	0.878	0.904	0.932	0.958	0.980	0.995
48	0.832	0.837	0.851	0.873	0.900	0.929	0.957	0.980	0.995
49	0.825	0.830	0.845	0.868	0.896	0.926	0.955	0.979	0.994
50	0.818	0.823	0.838	0.862	0.891	0.923	0.953	0.978	0.994

TABLE B
Bessel Function, J_0 (S cos θ) (*Continued*)

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
51	0.812	0.817	0.832	0.857	0.887	0.920	0.951	0.977	0.994
52	0.804	0.810	0.826	0.851	0.883	0.917	0.949	0.976	0.994
53	0.797	0.803	0.820	0.846	0.878	0.913	0.947	0.975	0.994
54	0.790	0.796	0.813	0.840	0.874	0.910	0.945	0.974	0.993
55	0.783	0.789	0.807	0.835	0.869	0.907	0.943	0.973	0.993
56	0.775	0.782	0.800	0.829	0.865	0.904	0.941	0.972	0.993
57	0.767	0.774	0.793	0.823	0.860	0.900	0.939	0.971	0.993
58	0.760	0.767	0.786	0.817	0.855	0.897	0.937	0.970	0.992
59	0.752	0.759	0.779	0.811	0.850	0.893	0.935	0.969	0.992
60	0.744	0.751	0.772	0.805	0.845	0.890	0.933	0.968	0.992
61	0.736	0.743	0.765	0.799	0.840	0.886	0.930	0.967	0.991
62	0.728	0.735	0.758	0.792	0.835	0.883	0.928	0.966	0.991
63	0.720	0.728	0.750	0.786	0.830	0.879	0.926	0.965	0.991
64	0.712	0.720	0.743	0.780	0.825	0.875	0.923	0.964	0.991
65	0.703	0.712	0.735	0.773	0.820	0.871	0.921	0.963	0.990
66	0.695	0.703	0.728	0.766	0.815	0.867	0.919	0.962	0.990
67	0.686	0.695	0.720	0.760	0.810	0.863	0.916	0.960	0.990
68	0.678	0.686	0.712	0.753	0.804	0.860	0.914	0.959	0.989
69	0.669	0.678	0.704	0.746	0.798	0.856	0.911	0.958	0.989
70	0.660	0.669	0.696	0.739	0.792	0.851	0.909	0.957	0.989
71	0.652	0.661	0.688	0.732	0.787	0.847	0.906	0.956	0.988
72	0.642	0.652	0.680	0.725	0.781	0.843	0.904	0.954	0.988
73	0.634	0.643	0.672	0.718	0.776	0.839	0.901	0.953	0.988
74	0.625	0.635	0.664	0.711	0.770	0.835	0.898	0.952	0.987
75	0.615	0.626	0.656	0.703	0.764	0.830	0.896	0.950	0.987
76	0.606	0.617	0.648	0.697	0.758	0.826	0.893	0.949	0.987
77	0.597	0.607	0.639	0.689	0.752	0.822	0.890	0.948	0.986
78	0.588	0.599	0.631	0.682	0.746	0.818	0.887	0.947	0.986
79	0.578	0.589	0.622	0.674	0.740	0.813	0.884	0.945	0.986
80	0.569	0.580	0.613	0.667	0.734	0.808	0.882	0.944	0.985
81	0.559	0.571	0.605	0.659	0.728	0.804	0.879	0.943	0.985
82	0.550	0.561	0.596	0.652	0.722	0.799	0.876	0.941	0.985
83	0.540	0.552	0.587	0.644	0.715	0.794	0.873	0.939	0.984
84	0.531	0.543	0.579	0.636	0.709	0.790	0.870	0.938	0.984
85	0.521	0.535	0.570	0.628	0.702	0.785	0.867	0.937	0.983
86	0.511	0.524	0.561	0.620	0.696	0.780	0.864	0.935	0.983
87	0.502	0.515	0.552	0.612	0.689	0.775	0.861	0.934	0.983
88	0.492	0.505	0.543	0.604	0.683	0.770	0.858	0.933	0.982
89	0.482	0.495	0.534	0.596	0.678	0.766	0.854	0.931	0.982
90	0.472	0.486	0.525	0.588	0.670	0.761	0.851	0.929	0.981
91	0.462	0.476	0.516	0.580	0.663	0.756	0.848	0.928	0.981
92	0.452	0.466	0.506	0.572	0.656	0.751	0.845	0.926	0.981
93	0.442	0.456	0.497	0.564	0.649	0.746	0.842	0.924	0.980
94	0.432	0.447	0.488	0.556	0.643	0.741	0.839	0.923	0.980
95	0.422	0.437	0.479	0.547	0.636	0.735	0.835	0.921	0.979
96	0.412	0.427	0.470	0.539	0.629	0.730	0.832	0.920	0.979
97	0.402	0.417	0.461	0.531	0.622	0.725	0.829	0.918	0.978
98	0.392	0.407	0.451	0.522	0.615	0.720	0.825	0.916	0.978
99	0.382	0.397	0.442	0.514	0.608	0.714	0.822	0.915	0.977
100	0.372	0.387	0.432	0.506	0.601	0.709	0.818	0.913	0.977

TABLE B
Bessel Function, J_0 (S cos θ) (*Continued*)

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
101	0.361	0.377	0.423	0.497	0.594	0.703	0.815	0.911	0.977
102	0.352	0.367	0.414	0.489	0.587	0.698	0.812	0.909	0.976
103	0.341	0.357	0.404	0.480	0.579	0.693	0.808	0.908	0.976
104	0.331	0.347	0.395	0.471	0.572	0.687	0.804	0.906	0.975
105	0.321	0.337	0.385	0.463	0.565	0.682	0.801	0.904	0.975
106	0.311	0.327	0.375	0.454	0.558	0.676	0.797	0.902	0.974
107	0.301	0.317	0.366	0.446	0.551	0.671	0.793	0.900	0.974
108	0.290	0.307	0.356	0.437	0.543	0.665	0.790	0.899	0.973
109	0.281	0.297	0.347	0.428	0.536	0.660	0.786	0.897	0.973
110	0.270	0.287	0.337	0.419	0.528	0.654	0.783	0.895	0.972
111	0.260	0.277	0.328	0.411	0.521	0.648	0.779	0.893	0.972
112	0.250	0.267	0.318	0.402	0.513	0.642	0.775	0.891	0.971
113	0.240	0.257	0.309	0.393	0.506	0.636	0.771	0.890	0.971
114	0.230	0.247	0.299	0.385	0.498	0.630	0.767	0.888	0.970
115	0.220	0.237	0.290	0.376	0.491	0.625	0.763	0.886	0.970
116	0.210	0.227	0.280	0.367	0.484	0.619	0.760	0.884	0.969
117	0.200	0.217	0.271	0.359	0.476	0.613	0.756	0.882	0.969
118	0.190	0.208	0.261	0.350	0.468	0.607	0.752	0.880	0.968
119	0.180	0.198	0.252	0.341	0.461	0.601	0.748	0.878	0.968
120	0.170	0.188	0.242	0.332	0.453	0.596	0.744	0.876	0.967
121	0.160	0.178	0.233	0.323	0.445	0.590	0.740	0.874	0.967
122	0.150	0.168	0.223	0.314	0.438	0.584	0.736	0.872	0.966
123	0.140	0.158	0.214	0.306	0.430	0.578	0.732	0.870	0.965
124	0.130	0.148	0.204	0.297	0.422	0.572	0.728	0.868	0.965
125	0.120	0.139	0.195	0.288	0.415	0.565	0.724	0.866	0.964
126	0.111	0.129	0.185	0.279	0.407	0.559	0.720	0.864	0.964
127	0.101	0.120	0.176	0.270	0.400	0.553	0.716	0.861	0.963
128	0.092	0.110	0.167	0.261	0.392	0.547	0.712	0.859	0.963
129	0.082	0.101	0.158	0.253	0.384	0.541	0.707	0.857	0.962
130	0.072	0.091	0.148	0.244	0.376	0.535	0.703	0.855	0.961
131	0.063	0.082	0.139	0.235	0.368	0.529	0.699	0.853	0.961
132	0.053	0.072	0.129	0.227	0.360	0.522	0.695	0.851	0.960
133	0.044	0.063	0.120	0.218	0.353	0.516	0.690	0.848	0.960
134	0.035	0.053	0.111	0.209	0.345	0.510	0.686	0.846	0.958
135	0.026	0.044	0.102	0.201	0.337	0.503	0.682	0.844	0.958
136	0.017	0.035	0.093	0.192	0.329	0.497	0.678	0.842	0.958
137	0.007	0.026	0.094	0.183	0.321	0.491	0.673	0.840	0.957
138	−0.002	0.017	0.075	0.175	0.314	0.485	0.669	0.837	0.957
139	−0.011	0.008	0.066	0.166	0.308	0.478	0.665	0.835	0.956
140	−0.020	−0.001	0.058	0.158	0.299	0.472	0.660	0.833	0.955
141	−0.029	−0.010	0.049	0.149	0.291	0.466	0.656	0.830	0.955
142	−0.037	−0.019	0.040	0.141	0.283	0.459	0.652	0.828	0.954
143	−0.046	−0.028	0.031	0.132	0.275	0.453	0.647	0.826	0.954
144	−0.055	−0.036	0.022	0.124	0.267	0.446	0.642	0.823	0.953
145	−0.064	−0.045	0.013	0.115	0.259	0.440	0.638	0.821	0.952
146	−0.072	−0.053	0.005	0.107	0.252	0.434	0.634	0.819	0.952
147	−0.080	−0.062	−0.003	0.099	0.244	0.427	0.629	0.817	0.951
148	−0.088	−0.070	−0.012	0.090	0.236	0.420	0.625	0.814	0.950
149	−0.097	−0.078	−0.020	0.082	0.229	0.414	0.620	0.812	0.950
150	−0.105	−0.087	−0.029	0.073	0.221	0.408	0.615	0.809	0.949

TABLE B
Bessel Function, J_0 (S cos θ) (*Continued*)

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
151	−0.113	−0.095	−0.037	0.065	0.213	0.401	0.611	0.807	0.948
152	−0.121	−0.103	−0.045	0.057	0.205	0.395	0.606	0.805	0.947
153	−0.129	−0.111	−0.053	0.049	0.198	0.388	0.602	0.802	0.947
154	−0.137	−0.119	−0.062	0.040	0.190	0.382	0.597	0.800	0.946
155	−0.145	−0.126	−0.070	0.032	0.183	0.375	0.592	0.797	0.945
156	−0.153	−0.134	−0.078	0.024	0.175	0.368	0.587	0.795	0.945
157	−0.160	−0.142	−0.085	0.017	0.167	0.362	0.583	0.792	0.944
158	−0.167	−0.149	−0.093	0.009	0.160	0.356	0.578	0.790	0.943
159	−0.175	−0.157	−0.101	0.001	0.152	0.349	0.573	0.787	0.943
160	−0.182	−0.164	−0.108	−0.007	0.145	0.343	0.569	0.785	0.942
161	−0.189	−0.172	−0.116	−0.015	0.137	0.336	0.564	0.782	0.941
162	−0.196	−0.179	−0.123	−0.023	0.130	0.330	0.559	0.780	0.940
163	−0.203	−0.186	−0.131	−0.030	0.122	0.323	0.555	0.777	0.940
164	−0.210	−0.193	−0.138	−0.037	0.114	0.317	0.550	0.774	0.939
165	−0.217	−0.200	−0.146	−0.045	0.107	0.310	0.545	0.772	0.938
166	−0.223	−0.207	−0.153	−0.053	0.100	0.303	0.541	0.769	0.937
167	−0.230	−0.213	−0.159	−0.060	0.093	0.297	0.536	0.767	0.937
168	−0.236	−0.220	−0.167	−0.068	0.085	0.290	0.531	0.764	0.936
169	−0.242	−0.226	−0.173	−0.075	0.078	0.284	0.526	0.761	0.935
170	−0.249	−0.232	−0.180	−0.082	0.070	0.277	0.521	0.759	0.935
171	−0.255	−0.239	−0.187	−0.089	0.063	0.271	0.516	0.756	0.934
172	−0.261	−0.245	−0.194	−0.097	0.056	0.264	0.511	0.753	0.933
173	−0.266	−0.251	−0.200	−0.104	0.049	0.258	0.506	0.751	0.932
174	−0.272	−0.257	−0.207	−0.111	0.042	0.251	0.502	0.748	0.932
175	−0.278	−0.263	−0.213	−0.118	0.035	0.245	0.497	0.746	0.931
176	−0.283	−0.269	−0.220	−0.125	0.027	0.238	0.492	0.743	0.930
177	−0.289	−0.274	−0.226	−0.131	0.020	0.232	0.487	0.740	0.929
178	−0.294	−0.279	−0.232	−0.138	0.013	0.226	0.482	0.737	0.928
179	−0.299	−0.285	−0.238	−0.145	0.006	0.219	0.477	0.735	0.927
180	−0.304	−0.290	−0.244	−0.151	−0.001	0.212	0.472	0.732	0.927
181	−0.309	−0.295	−0.249	−0.158	−0.008	0.206	0.467	0.729	0.926
182	−0.314	−0.300	−0.255	−0.164	−0.015	0.200	0.462	0.726	0.925
183	−0.319	−0.305	−0.261	−0.171	−0.022	0.193	0.457	0.723	0.924
184	−0.323	−0.310	−0.266	−0.177	−0.028	0.187	0.452	0.721	0.923
185	−0.328	−0.315	−0.272	−0.183	−0.035	0.180	0.447	0.718	0.923
186	−0.332	−0.319	−0.277	−0.190	−0.041	0.174	0.442	0.715	0.922
187	−0.336	−0.324	−0.282	−0.196	−0.048	0.167	0.437	0.712	0.921
188	−0.340	−0.328	−0.287	−0.202	−0.055	0.161	0.432	0.709	0.920
189	−0.344	−0.332	−0.292	−0.208	−0.061	0.155	0.427	0.706	0.919
190	−0.348	−0.336	−0.297	−0.214	−0.068	0.148	0.422	0.704	0.918
191	−0.351	−0.340	−0.302	−0.219	−0.074	0.142	0.417	0.701	0.918
192	−0.355	−0.344	−0.307	−0.225	−0.081	0.136	0.412	0.698	0.917
193	−0.358	−0.348	−0.311	−0.231	−0.087	0.129	0.407	0.695	0.916
194	−0.362	−0.352	−0.316	−0.236	−0.094	0.123	0.402	0.692	0.915
195	−0.365	−0.355	−0.320	−0.242	−0.100	0.117	0.397	0.689	0.914
196	−0.368	−0.359	−0.324	−0.247	−0.106	0.110	0.392	0.686	0.913
197	−0.371	−0.362	−0.328	−0.252	−0.113	0.104	0.387	0.683	0.913
198	−0.374	−0.365	−0.332	−0.257	−0.119	0.098	0.382	0.680	0.912
199	−0.376	−0.368	−0.336	−0.262	−0.125	0.092	0.377	0.677	0.911
200	−0.379	−0.371	−0.340	−0.267	−0.131	0.086	0.372	0.674	0.910

TABLE B
Bessel Function, J_0 (S cos θ) (*Continued*)

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
201	−0.381	−0.374	−0.344	−0.272	−0.137	0.079	0.367	0.671	0.909
202	−0.383	−0.376	−0.347	−0.277	−0.143	0.073	0.361	0.668	0.908
203	−0.386	−0.379	−0.351	−0.282	−0.149	0.067	0.356	0.665	0.907
204	−0.388	−0.381	−0.354	−0.287	−0.154	0.061	0.351	0.662	0.906
205	−0.390	−0.383	−0.357	−0.292	−0.160	0.055	0.346	0.659	0.905
206	−0.391	−0.385	−0.360	−0.296	−0.166	0.049	0.341	0.656	0.904
207	−0.393	−0.387	−0.363	−0.301	−0.172	0.043	0.336	0.653	0.904
208	−0.394	−0.389	−0.366	−0.305	−0.177	0.037	0.331	0.650	0.903
209	−0.396	−0.391	−0.369	−0.309	−0.183	0.031	0.326	0.647	0.902
210	−0.397	−0.393	−0.372	−0.313	−0.188	0.025	0.321	0.644	0.901
211	−0.398	−0.394	−0.374	−0.317	−0.193	0.019	0.316	0.641	0.900
212	−0.399	−0.396	−0.377	−0.321	−0.199	0.013	0.311	0.638	0.899
213	−0.400	−0.397	−0.379	−0.325	−0.204	0.007	0.306	0.635	0.898
214	−0.401	−0.398	−0.382	−0.329	−0.209	0.001	0.301	0.632	0.897
215	−0.401	−0.399	−0.384	−0.333	−0.214	−0.004	0.296	0.629	0.896
216	−0.402	−0.400	−0.386	−0.336	−0.220	−0.010	0.291	0.626	0.895
217	−0.402	−0.401	−0.388	−0.340	−0.225	−0.016	0.285	0.623	0.894
218	−0.403	−0.401	−0.389	−0.343	−0.230	−0.022	0.280	0.620	0.893
219	−0.403	−0.402	−0.391	−0.346	−0.235	−0.027	0.275	0.616	0.892
220	−0.403	−0.402	−0.393	−0.350	−0.239	−0.032	0.270	0.613	0.892
221	−0.403	−0.403	−0.394	−0.353	−0.244	−0.038	0.265	0.610	0.891
222	−0.402	−0.403	−0.395	−0.356	−0.249	−0.044	0.260	0.607	0.890
223	−0.402	−0.403	−0.397	−0.359	−0.254	−0.049	0.255	0.604	0.889
224	−0.402	−0.403	−0.398	−0.362	−0.258	−0.055	0.250	0.601	0.888
225	−0.401	−0.403	−0.399	−0.364	−0.263	−0.061	0.245	0.597	0.887
226	−0.400	−0.402	−0.400	−0.367	−0.267	−0.066	0.240	0.594	0.886
227	−0.399	−0.402	−0.400	−0.370	−0.272	−0.072	0.235	0.591	0.885
228	−0.398	−0.401	−0.401	−0.372	−0.276	−0.077	0.230	0.588	0.884
229	−0.397	−0.401	−0.402	−0.374	−0.280	−0.083	0.225	0.585	0.883
230	−0.396	−0.400	−0.402	−0.377	−0.284	−0.088	0.220	0.581	0.882
231	−0.395	−0.399	−0.402	−0.379	−0.288	−0.093	0.215	0.578	0.881
232	−0.393	−0.393	−0.403	−0.381	−0.292	−0.099	0.210	0.575	0.880
233	−0.392	−0.397	−0.403	−0.383	−0.296	−0.104	0.205	0.572	0.879
234	−0.390	−0.396	−0.403	−0.385	−0.300	−0.109	0.200	0.568	0.878
235	−0.389	−0.395	−0.403	−0.387	−0.304	−0.114	0.195	0.565	0.877
236	−0.387	−0.393	−0.402	−0.388	−0.308	−0.119	0.190	0.562	0.876
237	−0.385	−0.391	−0.402	−0.390	−0.312	−0.124	0.185	0.559	0.875
238	−0.383	−0.389	−0.402	−0.391	−0.315	−0.129	0.180	0.555	0.874
239	−0.380	−0.388	−0.401	−0.393	−0.319	−0.134	0.175	0.552	0.873
240	−0.378	−0.386	−0.401	−0.394	−0.322	−0.139	0.170	0.549	0.871
241	−0.376	−0.384	−0.400	−0.395	−0.326	−0.144	0.165	0.546	0.870
242	−0.373	−0.382	−0.399	−0.397	−0.329	−0.149	0.160	0.543	0.869
243	−0.371	−0.380	−0.398	−0.398	−0.333	−0.154	0.155	0.540	0.868
244	−0.368	−0.377	−0.397	−0.399	−0.336	−0.159	0.150	0.536	0.867
245	−0.365	−0.375	−0.396	−0.399	−0.339	−0.164	0.145	0.533	0.866
246	−0.363	−0.372	−0.395	−0.400	−0.342	−0.168	0.140	0.530	0.865
247	−0.359	−0.370	−0.393	−0.401	−0.345	−0.173	0.135	0.526	0.864
248	−0.356	−0.367	−0.392	−0.401	−0.348	−0.178	0.130	0.523	0.863
249	−0.353	−0.364	−0.390	−0.402	−0.351	−0.183	0.125	0.520	0.862
250	−0.350	−0.361	−0.389	−0.402	−0.353	−0.187	0.120	0.516	0.861

TABLE B
Bessel Function, J_0 ($S \cos \theta$) (*Continued*)

$S°$	Elevation angle, $\theta°$ 0°	10°	20°	30°	40°	50°	60°	70°	80°
251	−0.346	−0.358	−0.387	−0.402	−0.356	−0.192	0.116	0.513	0.860
252	−0.343	−0.355	−0.385	−0.403	−0.359	−0.196	0.111	0.510	0.859
253	−0.339	−0.352	−0.383	−0.403	−0.361	−0.201	0.106	0.506	0.858
254	−0.335	−0.349	−0.381	−0.403	−0.364	−0.205	0.101	0.503	0.857
255	−0.332	−0.346	−0.379	−0.403	−0.366	−0.209	0.097	0.500	0.856
256	−0.328	−0.342	−0.377	−0.402	−0.368	−0.214	0.092	0.496	0.855
257	−0.324	−0.339	−0.374	−0.402	−0.371	−0.218	0.087	0.493	0.854
258	−0.320	−0.335	−0.372	−0.402	−0.373	−0.222	0.082	0.489	0.852
259	−0.316	−0.331	−0.369	−0.401	−0.375	−0.226	0.077	0.486	0.851
260	−0.312	−0.327	−0.367	−0.401	−0.377	−0.231	0.072	0.483	0.850
261	−0.307	−0.324	−0.364	−0.400	−0.379	−0.235	0.067	0.479	0.849
262	−0.303	−0.320	−0.361	−0.400	−0.380	−0.239	0.063	0.476	0.848
263	−0.299	−0.316	−0.358	−0.399	−0.382	−0.243	0.058	0.472	0.847
264	−0.294	−0.312	−0.356	−0.398	−0.384	−0.247	0.053	0.469	0.846
265	−0.290	−0.307	−0.353	−0.397	−0.386	−0.251	0.049	0.466	0.844
266	−0.285	−0.303	−0.350	−0.396	−0.387	−0.255	0.044	0.462	0.843
267	−0.280	−0.299	−0.346	−0.395	−0.389	−0.259	0.039	0.459	0.842
268	−0.276	−0.294	−0.343	−0.393	−0.390	−0.262	0.035	0.455	0.841
269	−0.271	−0.290	−0.340	−0.392	−0.391	−0.266	0.030	0.452	0.840
270	−0.266	−0.285	−0.336	−0.391	−0.393	−0.270	0.026	0.449	0.839
271	−0.261	−0.281	−0.333	−0.389	−0.394	−0.274	0.021	0.446	0.838
272	−0.256	−0.276	−0.330	−0.387	−0.395	−0.277	0.017	0.442	0.837
273	−0.251	−0.271	−0.326	−0.385	−0.396	−0.281	0.012	0.439	0.835
274	−0.246	−0.267	−0.322	−0.384	−0.397	−0.284	0.007	0.435	0.834
275	−0.241	−0.262	−0.318	−0.382	−0.398	−0.288	0.003	0.432	0.833
276	−0.235	−0.257	−0.314	−0.380	−0.399	−0.291	−0.002	0.429	0.832
277	−0.230	−0.252	−0.310	−0.378	−0.399	−0.294	−0.006	0.425	0.831
278	−0.225	−0.247	−0.306	−0.376	−0.400	−0.298	−0.011	0.422	0.830
279	−0.219	−0.242	−0.302	−0.374	−0.401	−0.301	−0.015	0.418	0.829
280	−0.214	−0.236	−0.298	−0.372	−0.401	−0.304	−0.020	0.415	0.827
281	−0.208	−0.231	−0.294	−0.370	−0.402	−0.307	−0.024	0.411	0.826
282	−0.203	−0.226	−0.289	−0.367	−0.402	−0.311	−0.028	0.408	0.825
283	−0.197	−0.221	−0.285	−0.365	−0.402	−0.314	−0.033	0.404	0.824
284	−0.192	−0.215	−0.281	−0.362	−0.402	−0.317	−0.037	0.401	0.823
285	−0.186	−0.210	−0.276	−0.360	−0.403	−0.320	−0.042	0.397	0.822
286	−0.181	−0.205	−0.272	−0.357	−0.403	−0.323	−0.046	0.394	0.820
287	−0.175	−0.199	−0.267	−0.354	−0.403	−0.326	−0.050	0.390	0.819
288	−0.169	−0.194	−0.263	−0.351	−0.403	−0.328	−0.055	0.387	0.818
289	−0.163	−0.188	−0.258	−0.349	−0.403	−0.331	−0.059	0.384	0.817
290	−0.157	−0.182	−0.253	−0.346	−0.402	−0.334	−0.063	0.380	0.816
291	−0.151	−0.177	−0.248	−0.343	−0.402	−0.337	−0.068	0.377	0.814
292	−0.146	−0.171	−0.243	−0.340	−0.402	−0.339	−0.072	0.373	0.813
293	−0.140	−0.166	−0.238	−0.336	−0.401	−0.342	−0.076	0.370	0.812
294	−0.134	−0.160	−0.233	−0.333	−0.401	−0.344	−0.080	0.366	0.810
295	−0.128	−0.154	−0.228	−0.330	−0.400	−0.346	−0.084	0.363	0.809
296	−0.122	−0.148	−0.223	−0.326	−0.400	−0.349	−0.089	0.359	0.808
297	−0.116	−0.143	−0.218	−0.323	−0.399	−0.351	−0.093	0.356	0.807
298	−0.110	−0.137	−0.213	−0.320	−0.398	−0.354	−0.097	0.352	0.805
299	−0.104	−0.131	−0.208	−0.316	−0.397	−0.356	−0.101	0.349	0.804
300	−0.098	−0.125	−0.203	−0.313	−0.396	−0.358	−0.105	0.345	0.803

TABLE B
Bessel Function, J_0 (S cos θ) (*Continued*)

S°	Elevation angle, θ°								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
301	−0.092	−0.119	−0.198	−0.309	−0.395	−0.360	−0.109	0.342	0.802
302	−0.086	−0.113	−0.192	−0.305	−0.394	−0.362	−0.113	0.338	0.801
303	−0.080	−0.107	−0.187	−0.301	−0.393	−0.364	−0.117	0.335	0.799
304	−0.076	−0.101	−0.182	−0.298	−0.392	−0.366	−0.121	0.332	0.798
305	−0.068	−0.095	−0.176	−0.294	−0.391	−0.368	−0.125	0.328	0.797
306	−0.062	−0.090	−0.171	−0.290	−0.390	−0.370	−0.129	0.325	0.796
307	−0.056	−0.084	−0.165	−0.286	−0.388	−0.372	−0.133	0.321	0.794
308	−0.050	−0.078	−0.160	−0.282	−0.387	−0.374	−0.137	0.318	0.793
309	−0.044	−0.072	−0.155	−0.278	−0.385	−0.376	−0.141	0.314	0.792
310	−0.038	−0.066	−0.149	−0.273	−0.384	−0.377	−0.145	0.311	0.791
311	−0.032	−0.060	−0.144	−0.269	−0.382	−0.379	−0.149	0.307	0.789
312	−0.026	−0.054	−0.138	−0.265	−0.380	−0.380	−0.152	0.304	0.788
313	−0.020	−0.048	−0.133	−0.261	−0.379	−0.382	−0.156	0.300	0.787
314	−0.014	−0.042	−0.127	−0.256	−0.377	−0.383	−0.160	0.297	0.786
315	−0.008	−0.036	−0.122	−0.252	−0.375	−0.385	−0.163	0.293	0.784
316	−0.002	−0.030	−0.116	−0.248	−0.373	−0.386	−0.167	0.290	0.783
317	0.004	−0.024	−0.110	−0.243	−0.371	−0.387	−0.171	0.286	0.781
318	0.010	−0.018	−0.104	−0.239	−0.369	−0.389	−0.175	0.283	0.780
319	0.016	−0.013	−0.099	−0.234	−0.367	−0.390	−0.178	0.279	0.779
320	0.022	−0.007	−0.093	−0.229	−0.365	−0.391	−0.182	0.276	0.777
321	0.028	−0.001	−0.088	−0.225	−0.363	−0.392	−0.185	0.273	0.776
322	0.034	0.005	−0.082	−0.220	−0.360	−0.393	−0.189	0.269	0.775
323	0.039	0.011	−0.076	−0.215	−0.358	−0.394	−0.192	0.266	0.773
324	0.045	0.017	−0.071	−0.211	−0.356	−0.395	−0.196	0.262	0.772
325	0.051	0.023	−0.065	−0.206	−0.353	−0.396	−0.200	0.259	0.771
326	0.056	0.028	−0.059	−0.201	−0.351	−0.397	−0.203	0.255	0.770
327	0.062	0.034	−0.053	−0.196	−0.348	−0.397	−0.206	0.252	0.768
328	0.068	0.040	−0.048	−0.191	−0.345	−0.398	−0.210	0.248	0.767
329	0.073	0.046	−0.042	−0.186	−0.343	−0.399	−0.213	0.245	0.766
330	0.079	0.051	−0.037	−0.182	−0.340	−0.399	−0.217	0.241	0.764
331	0.085	0.057	−0.031	−0.177	−0.337	−0.400	−0.220	0.238	0.763
332	0.090	0.062	−0.025	−0.172	−0.334	−0.400	−0.223	0.234	0.762
333	0.095	0.068	−0.020	−0.167	−0.331	−0.401	−0.227	0.231	0.760
334	0.101	0.073	−0.014	−0.162	−0.328	−0.401	−0.230	0.227	0.759
335	0.106	0.079	−0.009	−0.157	−0.325	−0.402	−0.233	0.224	0.758
336	0.111	0.084	−0.003	−0.152	−0.322	−0.402	−0.236	0.221	0.756
337	0.116	0.090	0.003	−0.147	−0.319	−0.402	−0.239	0.218	0.755
338	0.122	0.095	0.008	−0.141	−0.316	−0.402	−0.242	0.214	0.754
339	0.127	0.100	0.014	−0.136	−0.313	−0.403	−0.245	0.211	0.752
340	0.132	0.106	0.020	−0.131	−0.310	−0.403	−0.249	0.207	0.750
341	0.137	0.111	0.025	−0.126	−0.307	−0.403	−0.252	0.204	0.749
342	0.142	0.116	0.031	−0.121	−0.303	−0.403	−0.255	0.200	0.748
343	0.147	0.121	0.036	−0.116	−0.300	−0.403	−0.258	0.197	0.746
344	0.151	0.126	0.042	−0.111	−0.297	−0.403	−0.261	0.193	0.745
345	0.156	0.131	0.047	−0.105	−0.293	−0.402	−0.263	0.190	0.744
346	0.161	0.136	0.052	−0.100	−0.290	−0.402	−0.266	0.187	0.742
347	0.166	0.141	0.058	−0.095	−0.286	−0.402	−0.269	0.183	0.741
348	0.170	0.146	0.063	−0.090	−0.282	−0.402	−0.272	0.180	0.740
349	0.175	0.151	0.068	−0.084	−0.279	−0.401	−0.275	0.176	0.738
350	0.170	0.155	0.073	−0.079	−0.275	−0.401	−0.278	0.173	0.737

TABLE B
Bessel Function, J_0 ($S \cos\theta$) (*Concluded*)

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
351	0.184	0.160	0.079	−0.074	−0.271	−0.400	−0.281	0.169	0.735
352	0.188	0.164	0.084	−0.069	−0.268	−0.400	−0.283	0.166	0.734
353	0.192	0.169	0.089	−0.064	−0.264	−0.399	−0.286	0.163	0.732
354	0.197	0.173	0.094	−0.058	−0.260	−0.399	−0.289	0.159	0.731
355	0.201	0.178	0.099	−0.053	−0.257	−0.398	−0.291	0.156	0.730
356	0.205	0.182	0.104	−0.048	−0.253	−0.397	−0.294	0.152	0.729
357	0.209	0.187	0.109	−0.043	−0.249	−0.397	−0.297	0.149	0.727
358	0.213	0.191	0.114	−0.038	−0.245	−0.396	−0.299	0.146	0.726
359	0.216	0.194	0.119	−0.033	−0.241	−0.395	−0.302	0.142	0.724
360	0.220	0.199	0.124	−0.027	−0.237	−0.394	−0.304	0.139	0.723
361	0.224	0.203	0.128	−0.022	−0.233	−0.393	−0.307	0.136	0.722
362	0.228	0.207	0.133	−0.017	−0.228	−0.392	−0.309	0.132	0.720
363	0.231	0.211	0.138	−0.012	−0.224	−0.391	−0.312	0.129	0.719
364	0.234	0.215	0.143	−0.006	−0.220	−0.390	−0.314	0.125	0.717
365	0.238	0.218	0.147	−0.001	−0.216	−0.389	−0.316	0.122	0.716
366	0.241	0.222	0.152	0.004	−0.212	−0.388	−0.319	0.119	0.714
367	0.244	0.226	0.156	0.009	−0.208	−0.387	−0.321	0.116	0.713
368	0.247	0.229	0.161	0.014	−0.204	−0.386	−0.323	0.113	0.712
369	0.250	0.233	0.165	0.019	−0.199	−0.384	−0.325	0.109	0.710
370	0.253	0.236	0.170	0.024	−0.195	−0.383	−0.327	0.106	0.708
371	0.256	0.239	0.174	0.029	−0.191	−0.381	−0.330	0.103	0.707
372	0.259	0.242	0.178	0.034	−0.186	−0.380	−0.332	0.099	0.705
373	0.262	0.245	0.182	0.039	−0.182	−0.378	−0.334	0.096	0.704
374	0.264	0.248	0.186	0.044	−0.178	−0.377	−0.336	0.093	0.702
375	0.267	0.251	0.190	0.049	−0.173	−0.375	−0.338	0.089	0.701
376	0.269	0.254	0.194	0.054	−0.169	−0.374	−0.340	0.086	0.700
377	0.271	0.257	0.198	0.059	−0.164	−0.372	−0.342	0.083	0.698
378	0.274	0.260	0.202	0.064	−0.160	−0.370	−0.344	0.079	0.697
379	0.276	0.262	0.206	0.069	−0.156	−0.369	−0.346	0.076	0.695
380	0.278	0.265	0.210	0.074	−0.151	−0.367	−0.348	0.073	0.694
381	0.280	0.267	0.213	0.079	−0.147	−0.365	−0.349	0.070	0.692
382	0.282	0.270	0.217	0.084	−0.142	−0.363	−0.351	0.066	0.691
383	0.283	0.272	0.220	0.088	−0.138	−0.361	−0.353	0.063	0.689
384	0.285	0.274	0.224	0.093	−0.133	−0.359	−0.355	0.060	0.688
385	0.287	0.276	0.227	0.097	−0.128	−0.357	−0.357	0.057	0.686
386	0.288	0.278	0.230	0.102	−0.124	−0.355	−0.358	0.053	0.685
387	0.290	0.280	0.233	0.107	−0.119	−0.353	−0.360	0.050	0.684
388	0.291	0.282	0.237	0.111	−0.115	−0.351	−0.362	0.047	0.682
389	0.292	0.284	0.240	0.116	−0.110	−0.349	−0.363	0.044	0.681
390	0.293	0.285	0.243	0.120	−0.106	−0.347	−0.365	0.040	0.679
391	0.295	0.287	0.246	0.125	−0.101	−0.345	−0.366	0.037	0.678
392	0.296	0.288	0.249	0.129	−0.097	−0.342	−0.368	0.034	0.676
393	0.296	0.290	0.251	0.134	−0.092	−0.340	−0.369	0.031	0.675
394	0.297	0.291	0.254	0.138	−0.087	−0.338	−0.371	0.028	0.673
395	0.298	0.292	0.257	0.142	−0.082	−0.335	−0.372	0.025	0.671
396	0.298	0.294	0.259	0.146	−0.078	−0.333	−0.374	0.021	0.670
397	0.299	0.295	0.262	0.151	−0.073	−0.331	−0.375	0.018	0.668
398	0.299	0.296	0.264	0.155	−0.069	−0.328	−0.376	0.015	0.667
399	0.300	0.297	0.267	0.159	−0.064	−0.326	−0.378	0.012	0.665
400	0.300	0.297	0.269	0.163	−0.060	−0.323	−0.379	0.009	0.664

TABLE B
Bessel Function, J_0 $(S \cos \theta)$ *(Concluded)*

$S°$	Elevation angle, $\theta°$								
	0°	10°	20°	30°	40°	50°	60°	70°	80°
401	0.300	0.298	0.271	0.167	−0.055	−0.321	−0.380	0.006	0.662
402	0.300	0.298	0.273	0.171	−0.050	−0.318	−0.381	0.003	0.661
403	0.300	0.299	0.275	0.175	−0.046	−0.315	−0.382	0.000(−)	0.659
404	0.300	0.299	0.277	0.179	−0.041	−0.313	−0.383	−0.003	0.658
405	0.300	0.300	0.279	0.183	−0.036	−0.310	−0.385	−0.006	0.656
406	0.299	0.300	0.281	0.186	−0.032	−0.307	−0.386	−0.009	0.655
407	0.299	0.300	0.283	0.190	−0.027	−0.304	−0.387	−0.012	0.653
408	0.298	0.300	0.284	0.194	−0.023	−0.302	−0.388	−0.016	0.652
409	0.298	0.300	0.286	0.198	−0.018	−0.299	−0.389	−0.019	0.650
410	0.297	0.300	0.287	0.201	−0.013	−0.296	−0.390	−0.022	0.648
411	0.296	0.300	0.289	0.204	−0.009	−0.293	−0.390	−0.025	0.647
412	0.296	0.299	0.290	0.208	−0.004	−0.290	−0.391	−0.028	0.645
413	0.295	0.299	0.291	0.211	0.000(+)	−0.287	−0.392	−0.031	0.644
414	0.294	0.298	0.293	0.215	0.005	−0.284	−0.393	−0.034	0.642
415	0.292	0.298	0.294	0.218	0.009	−0.281	−0.394	−0.037	0.641
416	0.291	0.297	0.295	0.221	0.014	−0.278	−0.394	−0.040	0.639
417	0.290	0.297	0.295	0.224	0.018	−0.275	−0.395	−0.043	0.638
418	0.289	0.296	0.296	0.227	0.023	−0.272	−0.396	−0.046	0.636
419	0.287	0.295	0.297	0.230	0.027	−0.269	−0.396	−0.049	0.635
420	0.286	0.294	0.298	0.233	0.032	−0.266	−0.397	−0.052	0.633

APPENDIX C DIRECTIONAL ANTENNAS HAVING MORE THAN TWO TOWERS

A Three-Tower-in-Line-Array Pattern Shape

Three towers in line spaced an equal distance between adjacent towers and with the end towers having equal inverse fields and phasings that are equal but opposite in sign are shown in Fig. CA-1. The resultant inverse field strength at 1 mile in the direction of any point P in the horizontal plane is shown in the vector diagram of Fig. CA-b. The resultant vector E always lies along the vector reference axis. This simplifies the pattern computations.

The resultant inverse field strength E at 1 mile in the direction of point P in the horizontal plane is given by

$$E = E_1\underline{|0} + E_2\underline{|\beta_2} + E_3\underline{|\beta_3} \qquad \text{[C-1]}$$

where E = resultant inverse field strength at 1 mile, mv/m
E_1 = inverse field strength from tower 1 at 1 mile while array is in operation, mv/m
E_2 = inverse field strength from towers 2 and 3 at 1 mile while array is in operation, mv/m

$$\beta_2 = -\beta_3 = S_2 \cos\phi + \Psi_2 \qquad \text{[C-2]}$$

where S_2 = spacing from tower 1 to 2 and 3, deg
ϕ = azimuth angle to observation point P, deg
Ψ_2 = phase of tower 2 with respect to tower 1, deg

The magnitude of E in the above equation can be written

$$E = E_1(1 + 2F_2 \cos\beta_2) \qquad \text{[C-3]}$$

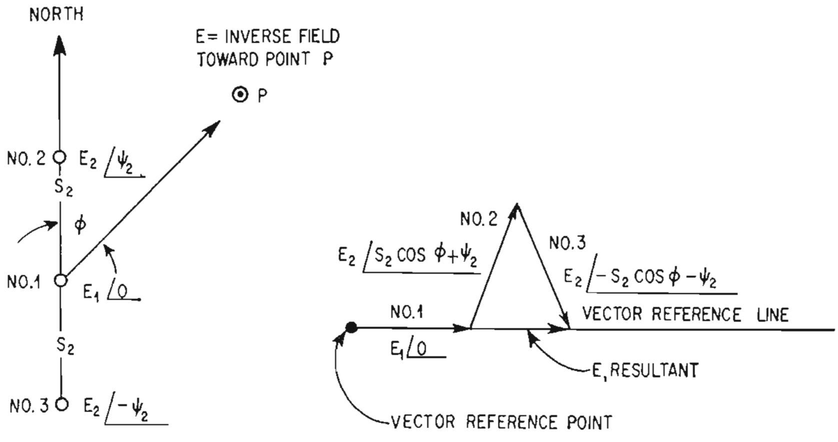
where $F_2 = E_2/E_1$. This equation is in convenient form for computing the horizontal pattern. It can also be used to determine the effect of any change in the design parameters. For example, for a given spacing $S_2 = 90°$ and direction $\phi = 60°$, the equation can be written

$$E = E_1[1 + 2F_2 \cos(45 + \Psi_2)] \qquad \text{[C-4]}$$

If $F_2 = 0.5$, then the equation becomes

$$E = E_1[1 + \cos(45 + \Psi_2)] \qquad \text{[C-5]}$$

Now when Ψ_2 is selected properly, E can be made to have any value from 0 to $2E_1$. If E is to be zero at 60 and 300°, since the pattern is symmetrical,



(A) PLACEMENT PLAN (B) VECTOR DIAGRAM

Fig. CA. Three-tower-in-line array.

then $\cos(45 + \Psi_2) = -1$ or $\Psi_2 = 135°$. For this condition the horizontal-pattern equation is written:

$$E = E_1 [1 + \cos(90 \cos \phi + 135)] \qquad \text{[C-6]}$$

When ϕ is varied from 0 to 180°, the complete pattern can be determined because it is symmetrical around the line of towers.

In general a three-tower-in-line array will produce a pattern with four nulls. Actually the pattern of Eq. C-6 has four nulls, but two occur at $\phi = 60°$, and two occur at $\phi = 300°$. This results in wider angles of low field strength at these two bearings than would be the case with single nulls.

The multiplication form for three towers in line is a more useful form and in the horizontal plane is written

$$E = 2E_1 \sqrt{F_2 F_3} \left[\left(\frac{1 + F_2^{\,2}}{2F_2} + \cos \beta_2 \right) \left(\frac{1 + F_3^{\,2}}{2F_3} + \cos \beta_3 \right) \right]^{1/2} \qquad \text{[C-7]}$$

where

$F_2 = E_2/E_1$

$F_3 = E_2/E_1$

$\beta_2 = S_2 \cos \phi + \Psi_2$

$\beta_3 = -S_2 \cos \phi - \Psi_3$

and the other values are defined above. Each parenthesis term in Eq. C-7 gives the pattern shape for two towers. The resulting pattern is produced by multiplying these terms together and taking the square root. With this design the direction of the nulls can be controlled for each pair of towers, the center tower in this case acting like two towers. If F_2 and F_3 are other than unity, a minimum rather than a null will result. Thus it is possible to fill in one pair of nulls independently of the other set.

Attention is directed to the minimum-depth terms

$$\frac{1 + F_2^{\,2}}{2F_2} \quad \text{and} \quad \frac{1 + F_3^{\,2}}{2F_3}$$

which can be determined from Appendix B, Table A. The actual values of F_2 and F_3 or their reciprocal values can be used to obtain the same result. If the actual reciprocal values are used to make adjustments in the field, this fact should be made clear in the report to FCC and there should be no objection on their part.

Four-Tower Parallelogram-Array Pattern Shape

General Case

The plan configuration of a four-tower parallelogram array is shown in Fig. B along with the related vector diagram. The general equation for the pattern can be written

$$\text{[C-8]}$$

$$E = E_1 f_1(\theta)[1 + F_2^{\,2} + F_3^{\,2} + F_4^{\,2} + 2F_2 \cos \beta_2 + 2F_3 \cos \beta_3 + 2F_4 \cos \beta_4 + 2F_2 F_4 \cos(\beta_2 - \beta_4) + 2F_2 F_3 \cos(\beta_2 - \beta_3) + 2F_3 F_4 \cos(\beta_3 - \beta_4)]^{1/2}$$

where E = inverse field strength at 1 mile in direction of point P, mv/m

$\beta_2 = S_2 \cos(\phi_2 - \phi) \cos \theta + \Psi_2$

$\beta_3 = S_3 \cos(\phi_3 - \phi) \cos \theta + \Psi_2$

$\beta_4 = S_4 \cos(\phi_4 - \phi) \cos \theta + \Psi_2$

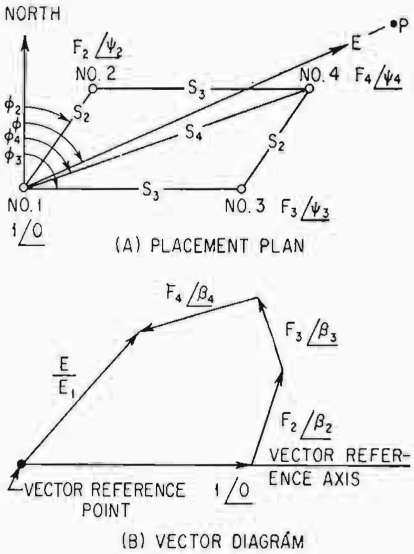


Fig. CB. General parallelogram array.

$$F_2 = \frac{E_2 f_2(\theta)}{E_1 f_1(\theta)}$$

$$F_3 = \frac{E_3 f_3(\theta)}{E_1 f_1(\theta)}$$

$$F_4 = \frac{E_4 f_4(\theta)}{E_1 f_1(\theta)}$$

E_1, E_2, E_3, and E_4 = inverse field strength at 1 mile, produced by towers 1 to 4, respectively when, the array is in operation, mv/m

$f_1(\theta)$, $f_2(\theta)$, $f_3(\theta)$, and $f_4(\theta)$ = vertical radiation characteristic of towers 1 to 4, respectively

This equation is valid for towers of unequal height but is of a form seldom used. if the restriction $\beta_4 = \beta_2 + \beta_3$ and $F_4 = F_2F_3$ is applied, Eq. C-8 can be written

$$E = 2E_1 f_1(\theta)\sqrt{F_2F_3}\left[\left(\frac{1+F_2^2}{2F_2} + \cos\beta_2\right)\left(\frac{1+F_3^2}{2F_3} + \cos\beta_3\right)\right]^{1/2} \qquad \text{[C-9]}$$

where the terms are defined above.

It is noted that tower 4 must have the following field ratio:

$$F_4 = F_2F_3 = \frac{E_2 f_2(\theta)\, E_3 f_3(\theta)}{E_1^2 f_1^2(\theta)} \qquad \text{[C-10]}$$

If $E_1 = E_2 = E_3$, then the vertical-radiation characteristics of tower 4 must be

$$f_4(\theta) = \frac{f_2(\theta)\, f_3(\theta)}{f_1^2(\theta)} \qquad \text{[C-11]}$$

Since all $f(\theta)$ values are unity along the ground, this condition cannot be detected in the horizontal plane. However, if the parallelogram array is operated at night when high angle radiation is involved, it is necessary to compute the radiation at various elevation angles and the above equation must be considered.

The greatest practical value of Eq. C-9 is that it gives information concerning the exact location of the four minima, two for each of the two-tower patterns unless spacings greater than 180° are used, and in such cases there may be four minima per two-tower pattern. This information is useful in the design and adjustment of the parallelogram array. It comes from the condition when $\beta = 180°$.

Then the minimum-depth terms give useful information concerning the value of field strength to be expected in these directions. They are helpful in shaping the pattern to meet the design requirements.

The expression inside the brackets completely defines the pattern shape. The factor $2E_1 f_1(\theta)\sqrt{F_2F_3}$ outside the bracket defines the pattern size, especially the term E_1, which will be evaluated in a later section of this appendix.

Special Three-Tower-in-Line Case

If tower 2 is placed on top of tower 3, a three-tower-in-line array will result as shown in Fig. C-2. This arrangement is redrawn in Fig. C-3.

Special Three Towers Not in Line

If in Eq. C-8 the condition that $F_4 = 0$ is imposed, the equation reduces to

$$E = E_1 f_1(\theta)\left[2F_2\left(\frac{1+F_2^2}{2F_2}+\cos\beta_2\right) + 2F_3\left(\frac{1+F_3^2}{2F_3}+\cos\beta_3\right) - 1 + 2F_2F_3\cos(\beta_2-\beta_3)\right]^{1/2} \qquad \text{[C-12]}$$

This equation is essentially an addition formula for a three-tower-not-in-line array as shown in Fig. CD plus the variable term $2F_2F_3 \cos(\beta_2 - \beta_2) - 1$. The -1 merely translates the sum of the other three terms to the left one unit. Hence it remains only to find how the pattern changes with the addition of the two patterns plus the term $2F_2F_3 \cos(\beta_2 - \beta_3)$. The effect of change in parameter values can be noted in the computations. The information can be very useful in making adjustments on this type of array.

Pattern-Size Determination

Four-Tower Array

The equation to determine the field strength from the reference tower 1 is

$$E_1 = E_{1S}\sqrt{\frac{R_{11}}{R_1 + M_{21}^2R_2 + M_{31}^2R_3 + M_{41}^2R_4}} \qquad \text{[C-13]}$$

where E_1 = inverse field strength at 1 mile in horizontal plane from tower 1 while operating in the array, mv/m

E_{1S} = inverse field strength at 1 mile in horizontal plane from tower 1 operating alone, mv/m

R_1, R_2, R_3, and R_4 = driving-point resistance values at input terminals of towers 1 to 4 respectively, ohms

M_{21}, M_{31}, and M_{41} = loop-current ratios of towers 2, 3 and 4 to tower 1, respectively

Only magnitude values are used in the above equation. The driving-point resistances, as in Eq. B-5, are made up of self-, mutual-, and loss-resistance terms.

They can be written

$$R_1 = R_{11} + R_{L1} + R_{C1} \qquad \text{[C-14]}$$

$$R_2 = R_{22} + R_{L2} + R_{C2} \qquad \text{[C-15]}$$

$$R_3 = R_{33} + R_{L3} + R_{C3} \qquad \text{[C-16]}$$

$$R_4 = R_{44} + R_{L4} + R_{C4} \qquad \text{[C-17]}$$

where R_{11}, R_{22}, R_{33}, and R_{44} = self-loop radiation resistance of towers 1 to 4, respectively, ohms

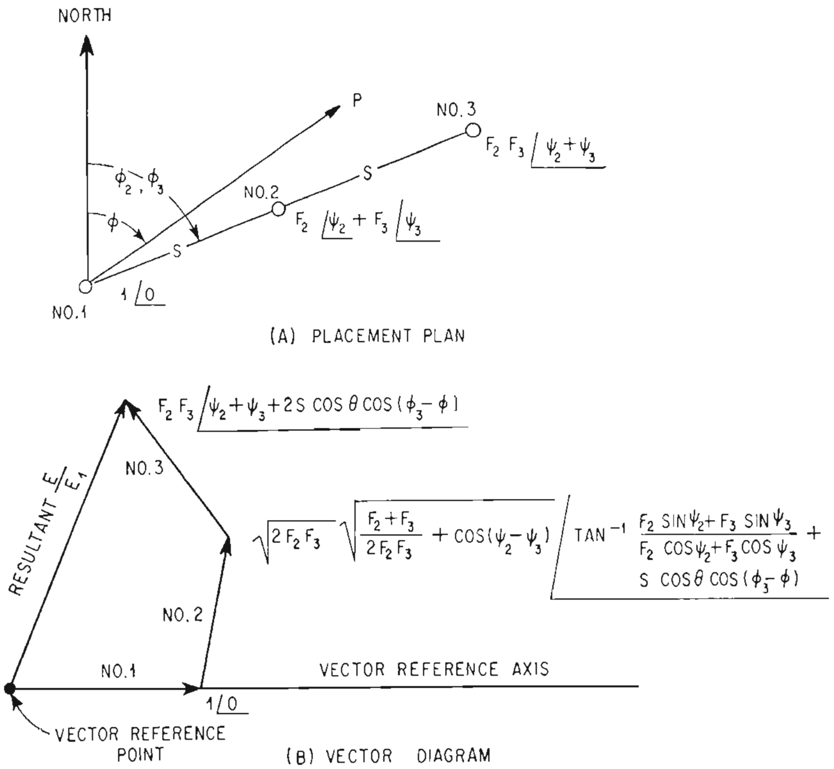


Fig. CC. Special three-tower-in-line array.

R_{L1}, R_{L2}, R_{L3}, and R_{L4} = loop-loss resistance of towers 1 to 4, respectively, ohms

R_{C1}, R_{C2}, R_{C3}, and R_{C4} = loop-coupled resistance of towers 1 to 4, respectively, ohms

In Eq. C-13 the value of E_{1s}, R_{11}, R_{22}, R_{33}, and R_{44} can be obtained from Appendix A, Fig. E, R_{L1}, R_{L2}, R_{L3}, and R_{L4} are usually assumed to be 2 ohms for towers 90° high or taller. The values of coupled resistance can be obtained from the mutual-impedance terms in mesh circuit equations, which can be written

$$R_{C1} = M_{21}Z_{12} \cos(\Psi_{12} + \gamma_{12}) + M_{31}Z_{13} \cos(\Psi_{13} + \gamma_{13}) + M_{41}Z_{14} \cos(\Psi_{14} + \gamma_{14}) \quad [C\text{-}18]$$

$$R_{C2} = M_{12}Z_{21} \cos(\Psi_{21} + \gamma_{21}) + M_{33}Z_{23} \cos(\Psi_{23} + \gamma_{23}) + M_{42}Z_{24} \cos(\Psi_{24} + \gamma_{24}) \quad [C\text{-}19]$$

$$R_{C3} = M_{13}Z_{31} \cos(\Psi_{31} + \gamma_{31}) + M_{23}Z_{32} \cos(\Psi_{32} + \gamma_{32}) + M_{43}Z_{34} \cos(\Psi_{34} + \gamma_{34}) \quad [C\text{-}20]$$

$$R_{C1} = M_{14}Z_{41} \cos(\Psi_{41} + \gamma_{41}) + M_{24}Z_{42} \cos(\Psi_{42} + \gamma_{42}) + M_{34}Z_{43} \cos(\Psi_{43} + \gamma_{43}) \quad [C\text{-}21]$$

where Z_{pq} = Z_{qp} = magnitude of loop mutual impedance between tower p and tower q, ohms

γ_{pq} = γ_{qp} = angle of loop mutual impedance between tower p and tower q, deg

Ψ_{pq} = $-\Psi_{qp}$ = electrical phasing of tower q with respect to tower p, deg

The loop coupled reactance X_{c1}, X_{c2}, X_{c3}, and X_{c4} of towers 1 to 4, respectively can be calculated from Eqs. C-18 to C-21 by replacing cos by sin throughout. The coupled reactance values are not needed for the determination of the size of E_1. However, they are useful when it comes to determining the driving-point impedances and setting up the matching networks at the towers.

The mutual-impedance terms required above for ordinary equal-height towers are shown in Fig. B-14. The mutual impedance between top-loaded towers may be assumed equal to the mutual impedance between the same towers not top-loaded in most cases.

The above equations can also be applied to top-loaded sectionalized towers but with major problems. The radiation E_{1s} and self-resistance values are not readily available in general. Also, the mutual impedances must be calculated except for special cases which are available.

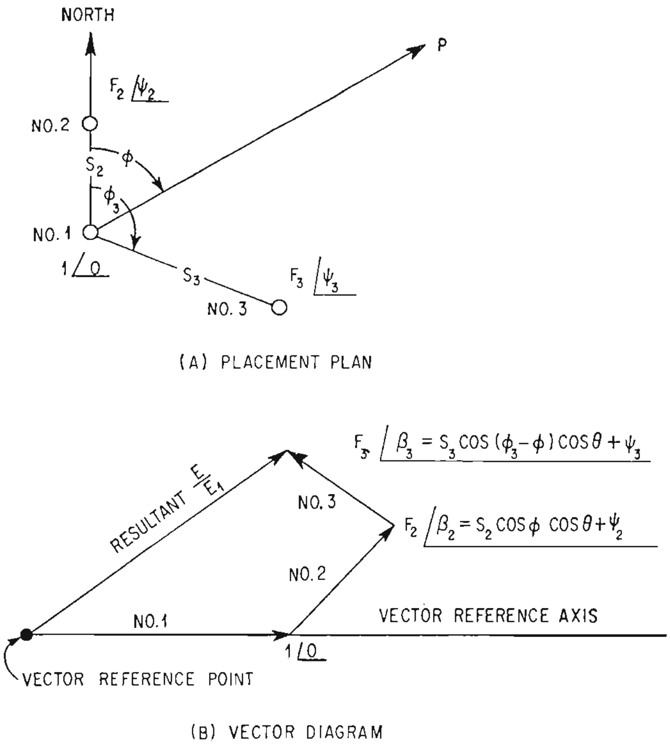


Fig. CD. Special three-tower-not-in-line case.

When E_1 is determined by Eq. C-13, the pattern can be plotted to its exact size.

Three-Tower Array

The equation to determine the field strength for a three-tower array can be written

$$E_1 = E_{1s}\sqrt{\frac{R_{11}}{R_1 + M_{21}{}^2 R_2 + M_{31}{}^2 R_3}} \qquad \text{[C-22]}$$

where the values are defined following Eq. C-13. The terms involving tower 4 in Eqs. C-18, C-19, and C-20 will vanish.

Horizontal-Plane RMS Field Strength of Four-Tower Array

The equation for the rms field strength in the horizontal plane of a four-tower array is

$$\begin{aligned} E_0 = E_1 [1 &+ F_2{}^2 + F_3{}^2 + F_4{}^2 \\ &+2F_2 \cos \Psi_{12} J_0(S_{12}) \\ &+2F_3 \cos \Psi_{13} J_0(S_{13}) \\ &+2F_4 \cos \Psi_{14} J_0(S_{14}) \\ &+2F_2F_3 \cos \Psi_{23} J_0(S_{23}) \\ &+2F_2F_4 \cos \Psi_{24} J_0(S_{24}) \\ &+2F_3F_4 \cos \Psi_{34} J_0(S_{34})]^{1/2} \end{aligned} \qquad \text{[C-23]}$$

where

- E_0 = rms inverse field strength in horizontal plane, mv/m
- E_1 = inverse field strength at 1 mile produced by tower 1 while operating in array, mv/m,
- F_2, F_3, and F_4 = inverse field-strength ratios of towers 2, 3 and 4 to tower 1, respectively.
- Ψ_{pq} = electrical phasing of tower q with respect to tower p as designated by the subscript numbers, deg
- S_{pq} = spacing of tower q to tower p as designated by the subscript numbers, deg
- $J_0(S_{pq})$ = Bessel function of first kind and zero order of the spacing as designated by the subscripts and given in Appendix B, Table B.

A good check on the arithmetic is to measure the pattern area with a planimeter.

For a four-tower parallelogram array Eq. C-23 reduces to

$$\begin{aligned} E_0 = E_1 [1 &+ F_2{}^2 + F_3{}^2 + F_4{}^2 \\ &+2(F_2 + F_3F_4) \cos \Psi_{12} J_0(S_{12}) \\ &+2F_4 \cos \Psi_{14} J_0(S_{14}) \\ &+2F_2F_3 \cos \Psi_{23} J_0(S_{23}) \\ &+2(F_3 + F_2F_4) \cos \Psi_{13} J_0(S_{13})]^{1/2} \end{aligned} \qquad \text{[C-24]}$$

where the terms are defined following Eq. C-23.

Horizontal-Plane RMS Field Strength of Three-Tower Array

The equation for the rms field strength in the horizontal plane of a three-tower array is

$$\begin{aligned} E_0 = E_1 [1 &+ F_2{}^2 + F_3{}^2 \\ &+2F_2 \cos \Psi_{12} J_0(S_{12}) \\ &+2F_3 \cos \Psi_{13} J_0(S_{13}) \\ &+2F_2F_3 \cos \Psi_{23} J_0(S_{23})]^{1/2} \end{aligned} \qquad \text{[C-25]}$$

where the terms are defined following Eq. C-23.

For the special three-tower-in-line array of Fig. C-1 the above equation reduces to

$$\begin{aligned} E_0 = E_1 [1 &+ 2F_2{}^2 \\ &+4F_2 \cos \Psi_{12} J_0(S_{12}) \\ &+2F_2{}^2 \cos (2\Psi_{12}) J_0(2S_{12})]^{1/2} \end{aligned} \qquad \text{[C-26]}$$

where the terms are defined following Eq. C-23.