# ELECTRICAL COMMUNICATION

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# ELECTRICAL COMMUNICATION

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## CONTENTS


**Volume 37 — 1961 — Number 1**







**EDITOR, H. P. Westman** — **Subscription: $2.00 per year; 50¢ per copy**

## This Issue In Brief

**International Toll Exchange in Zurich**—As a step to direct dialing by one subscriber to another over long distances in different countries, the Zurich international toll exchange provides for direct distance dialing to the called subscriber by a toll operator who receives instructions from the calling subscriber. This permits skilled operators to handle the problems of international signaling and of different languages when an assistance operator is needed. The many special signals and operating procedures are described.

**Ferroelectric Energy Converters**—The conversion of heat directly into electricity by thermomagnetic converters has received much attention but the analogous ferroelectric process has not been given equivalent study.

If a capacitor using barium-strontium titanate as a dielectric is charged and then heated, its permittivity and capacitance will be reduced and the voltage across it will increase because the charge will remain constant. The charge may then be dissipated and the capacitor cooled to its original temperature. The conversion involves disorienting the molecular dipoles against the aligning force of the electric field. In the analog of mechanically moving a dielectric from a charged capacitor against the force of the field, the mechanical energy is greater when the capacitor is charged, this difference appearing as additional electric energy.

**Characteristics of Waveguides for Long-Distance Transmission**—Open-wire, balanced-pair, coaxial, single-wire, microwave-link, and waveguide methods of transmission are compared as to operating frequency, channel capacity, and repeater spacing.

For long-distance wideband waveguide systems, circular waveguides operating in the $H_{01}$ mode are chosen and for the 4-to-8-millimetre band would be 2 to 3 inches in diameter. Pulse-code modulation is superior to frequency modulation with regard to delay distortion.

Mode conversion due to physical irregularities of the guide causing scattering is important as it also results in reconversion to the original $H_{01}$ mode by later scattering, the reconverted wave being out of phase with the original. Solution to this problem lies in the design of the waveguide itself. The helical and dielectric-coated types are most promising.

The use of pulse-code modulation, regenerative repeaters, and suitable waveguides can produce an advanced system of unequalled transmission capacity.

**Exact Solutions for Ordinary Nonlinear Differential Equations**—Application of the elementary transforms, $y(x) = f(x)\ g(x)$ and $y(x) = 1/[f(x)\ g(x)]$, where $f(x)$ and $g(x)$ are arbitrary functions of $x$, produced two new transforms for a generalized Riccati's nonlinear differential equation of $y(x)$. These new transforms not only linearize a Riccati's equation but also solve it under special conditions where the variable coefficients of the equation are specially related. The transform $y(x) = f(x)/g(x)$ solves a second-order nonlinear differential equation for a nonlinear Langmuir plasma problem. A systematic extension of the Sturm Liouville differential equation yields a second-order nonlinear differential equation of which the exact solution is analytically represented. There is explained a basic concept common to all these approaches for exact solutions of ordinary nonlinear differential equations.

**Class of Solved Riccati's Equations**—Riccati's nonlinear differential equations often appear in the WKBJ method, in aerodynamics, and also in nonuniform-transmission-line problems. In fact, all the second-order ordinary linear homogeneous differential equations with general coefficients can be reduced to generalized Riccati's equations. However, the exact solution for such an equation has not been obtained. This paper presents a new approach, thereby giving exact solutions for a certain class of Riccati's equations in explicit and analytical forms. The class considered in this paper has two arbitrary variable coefficients in specified products.

**World's Telephones—1960**—Almost 134 million telephones were in service throughout the world on 1 January 1960. The increase of 8.8 million telephones during 1959 was the greatest in any single year. During the decade of the 1950's, the telephones in the world practically doubled. Data are given by countries on the number of telephones, per capita utilization, percent automatic, and if privately or governmentally operated.

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## Radio Transmitters

The book, "Radio Transmitters," by L. F. Gray and R. Graham of ITT Federal Laboratories, has just been published. The subject is treated from the practical standpoint in 14 chapters on the following subjects:

Chapter 1—Introduction
Chapter 2—Frequency-Control Techniques
Chapter 3—Radio-Frequency Power Amplifiers
Chapter 4—Power Tubes
Chapter 5—Coupling Circuits
Chapter 6—Amplitude Modulation
Chapter 7—Angle and Pulse Modulation
Chapter 8—Power Supplies
Chapter 9—Control and Protective Circuits
Chapter 10—Cooling
Chapter 11—Radio-Frequency Components
Chapter 12—Transmitter Characteristics
Chapter 13—Transmitter Measurement Techniques
Chapter 14—Hazards Associated with Transmitters

The 14 chapters occupy 450 pages and contain 146 numbered equations, 398 illustrations, and 29 tables. The bibliographies at the end of each chapter contain a total of 586 entries. There is a 19-page appendix of tables and illustrations and a 12-page subject index.

The book measures 6 by 9 inches (152 by 229 millimeters) and is bound in cloth-covered hard covers. It is published by and is available from the McGraw-Hill Book Company, Incorporated, 330 West 42nd Street, New York 36, New York; the price is $12.50.


Figure 1—Courier satellite ground equipment for handling 340 000 words to and from the satellite during its 5-minute exposure to the ground station on each trip around the earth.

## Recent Engineering Developments

**Project Courier Ground Equipment**—The interior of a van housing ground equipment for Project Courier, an active communication satellite system, is shown in Figure 1. This complex of equipment is the ground receiving, transmitting, and controlling apparatus for the system.

The satellite circles the earth in just under two hours and is within sight of its ground stations for but 5 minutes on each pass. Consequently, traffic must be exchanged between ground and satellite with extreme rapidity. Between passes, messages are accumulated from normal 100-word-per-minute teleprinter channels and compacted on magnetic tape. When inactive, the orbiting satellite transmits a low-powered beacon signal for acquisition purposes. The ground station receives this signal when the satellite comes in view and a coded command transmitted to the satellite activates its high-powered transmitter. Simultaneously, then, the satellite transmits its stored traffic to the ground and the ground station sends its accumulated traffic to the satellite at the tremendous speed of 340 000 words in 5 minutes in each direction.

Because the aspect of the satellite constantly changes and because of polarization shifts due to the Faraday effect, the ground antennas are circularly polarized. Fourfold receiving diversity using parametric amplifiers assures noise-free reception of the relatively weak transmissions from the satellite.

The equipment in the van, in addition to the communications and control functions, monitors the condition of the satellite electronic equipment through telemetering channels. In general, the design philosophy was to place as much as possible of the system complexity on the ground to assure a lightweight and reliable satellite.

*ITT Federal Laboratories*

**Doris** means Direct Order Recording and Invoicing System and, exactly as the words indicate, the equipment does the paper work involved in processing and billing an order and also provides an accurate means of inventory control. The installation is now under test in London by Shell-Mex and British Petroleum Limited.

As shown in Figure 2, the operator is surrounded by 7 panels mounting up to 3000 buttons corresponding to the customers served by the company. When a telephoned order is received, the button for that customer is pressed. Facts concerning the product involved, quantity, delivery date, and other pertinent data are then entered into the memory of the system by the operator by punching a series of buttons on the smaller keyboard in front of him.

When the complete order has been taken, another key is pressed and all the information is then punched in tape. If any information is missing, a red light warns the operator that the order is incomplete. The customer's full address, terms of trade, prices, and product description are then extracted from permanent memories and a battery of teleprinters automatically make out the invoice and sales ticket.

At the end of the day the total sales for each

Figure 2—Part of the Doris consol for the Direct Order Recording and Invoicing System utilizing prerecorded data and punched tape for the full processing of orders.

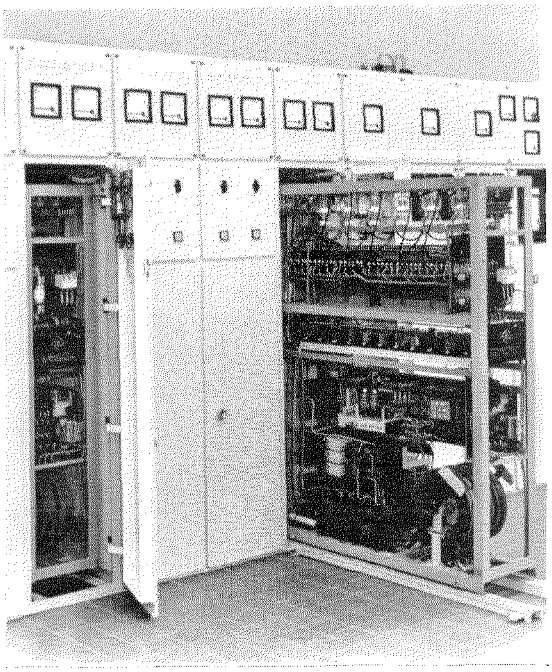


Figure 3—These power supplies are available in capacities up to 75 kilowatts with full-load voltage stabilization and ripple suppression of 1 part in 10 000.

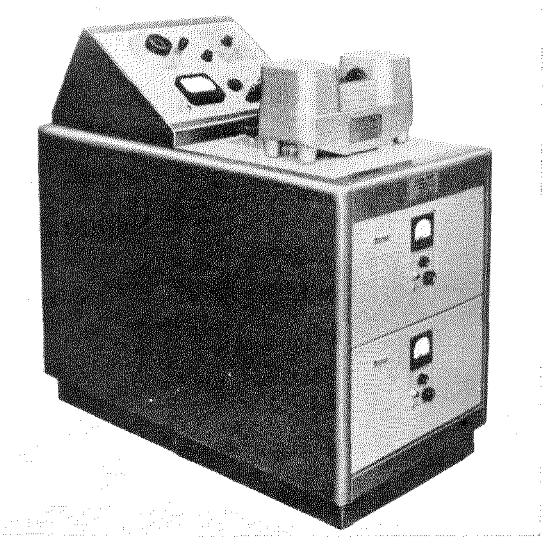


Figure 4—The vibration table standing on the control consol will test a 5-pound load at frequencies up to 50 kilocycles.

product line and other information are printed out for inventory control and sales analysis.

*Creed & Company*

**High-Stability Power Supplies**—Designed to replace storage batteries in laboratories, these power supplies feature full-load voltage stabilities and ripple suppression of 1 part in 10 000. They have found application in nuclear-research laboratories for powering large electromagnets. Available in several capacities up to 75 kilowatts, a 5-unit assembly of 10-kilowatt units is shown in Figure 3.

The output voltage is adjustable between 10 and 100 percent of the rated value. Long-term stabilization is accomplished by motor-driven adjustable transformers and fast stabilization by vacuum-tube regulators. The recovery time is of the order of milliseconds and the long-term drift is not worse that 1 part in 10 000 in 48 hours while undergoing simultaneously $\pm 10$ percent slow and $\pm 3$ percent fast changes in mains voltage and load current and with variations of $\pm 2$ percent in mains frequency.

*Nederlandsche Standard Electric Maatschappij*

**Vibration Exciter**—The model *ST-100* Vibration Exciter for vibration and shock testing, shown in Figure 4, is essentially flat in response from 5 cycles per second up almost to the first resonance at 12 kilocycles per second. The machine is useful to 50 kilocycles per second. The flat low-distortion performance permits testing electronic or other components with an included sweep oscillator, with white noise, and with tape recordings programmed as desired.

The equipment is rated for a load of 5 pounds (2.3 kilograms). When driven with the standard 400-watt audio-frequency amplifier, it delivers a force output of 50 pounds (23 kilograms) or a shock of 5000 *g*.

The exciter unit is shown in the photograph on the console that mounts the sweep oscillator and power amplifier. The vibration table is supported on air bearings. There are 24 jeweled orifices in the bearing surfaces of the base. Air under a few pounds of pressure is driven through the holes and forms a practically friction-free film between the bearing surfaces. The table is also rigidly restrained against motion along any axis except the desired one.

*ITT Industrial Products Division*

**Videx Slow-Scan Television**—Videx transmits pictorial material over voice-channel facilities via either wire lines, carrier systems, or radio links. A view of the camera is given in Figure 5. The receiver resembles an ordinary television receiver except that the picture tube is an Iatron storage tube. The system provides a choice among three resolutions of 400, 275, or 200 lines per inch (15.7, 10.8, or 7.9 lines per millimeter), corresponding respectively to frame-scanning times of 60, 30, or 15 seconds. The viewing tube can provide an extremely bright picture (100 foot-lamberts) and the image persistence is adjustable between 10 seconds and 6 minutes.

*ITT Federal Laboratories*

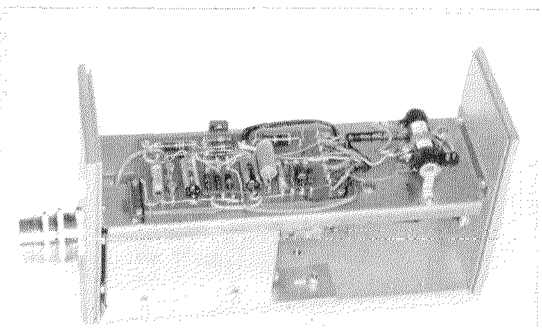


Figure 5—The camera for Videx, a slow-scan television system for wire-line or equivalent transmission links. Resolution from 200 to 400 lines per inch (15.7 to 7.9 lines per millimeter) is provided and image persistence is adjustable from 10 seconds to 6 minutes.

**Navigational Talking Beacon**—A navigational beacon using a vocal recording to give bearings directly has recently been developed for the United States Coast Guard and is shown in Figure 6. Envisioned mainly as an aid to the private small-craft owner, the useful range of the beacon is about 10 miles (16 kilometers).

The microwave beacon consists of a rotating highly directional antenna. The bearing information, a voice recorded on a motion-picture-film sound track, modulates the transmitter in synchronism with the slow rotation of the antenna.

The receiver is about the size of a cigar box and operates on 6 standard flashlight cells. The only controls are an on-off switch and a volume control. In operation, the boatman would hear a series of announcements; near Cape May, New Jersey, he might hear, for instance, "Cape May, 003 – Cape May, 006 – Cape May, 009 –." One announcement would be louder than the

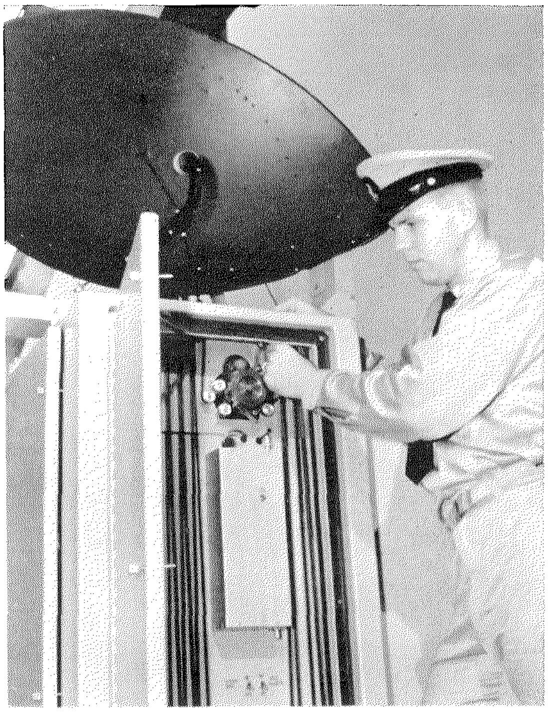


Figure 6—As the paraboloidal antenna slowly rotates, a film strip synchronously modulates the sharp microwave beam to announce the bearing from which the signal would be received at that moment. A simple nondirectional receiver permits the loudest announcement to indicate the direction to the beacon.

others and this gives the boatman the beacon's bearing in relation to his position.

*ITT Federal Laboratories*

**Air Traffic Control**—The three-dimensional display shown in Figure 7 has applications in air-traffic control, missile tracking, submarine warfare, and similar fields where a three-dimensional positional presentation is needed.

The flat screen inside the protective pill-box cover is whirled about a vertical axis faster than the eye can detect. Under control of a computer, radar-type information causes pulsed spots of light from a high-brightness cathode-ray tube to be directed onto the screen at precise instants during rotation. These points of light seem to float in space by reason of persistence of vision. They can be made to move through the volume represented in accordance with the positional information supplied to the computer.

*ITT Federal Laboratories*

Figure 7—As the screen rotates rapidly about its vertical axis, a short-duration pulse of light from a cathode-ray tube illuminates a spot on it which, through synchronous operation, appears to stand still in three-dimensional space.

## Terhune Receives Radio Fall Meeting Plaque

The Radio Fall Meeting Plaque for 1960 was awarded to Harold R. Terhune "for his stellar accomplishments in standardization in the fields of electronic symbology and components."

Mr. Terhune is manager of standards at ITT Federal Laboratories, a division of International Telephone and Telegraph Corporation, in Nutley, New Jersey.

The Radio Fall Meetings, which have been held regularly for over 30 years, are sponsored by the engineering department of the Electronic Industries Association with the cooperation of the professional groups of the Institute of Radio Engineers in the arrangement of the technical programs.

# International Toll Exchange in Zurich


## H. W. HAFFTER

**Standard Telephone et Radio S. A.; Zurich, Switzerland**


Experimental trials were made some years ago on an international basis of semiautomatic telephone operation in which the telephone operator at the originating office dialed directly the number of the wanted subscriber in another country. Originally, only one long-line-access exchange was foreseen in Switzerland for switching the combined terminal and tandem traffic. In the meantime, the international toll lines have been grouped in such a way that several long-line-access exchanges can handle part of the traffic to specific countries abroad. For the international tandem traffic, however, the international toll exchange in Zurich remains the only long-line-access exchange. This situation is illustrated in Figure 1, which shows two of the long-line-access exchanges and the international tandem center at Zurich.

The long-line-access exchange at Zurich has been designed for 4-wire through connection. The rotary finders used as selectors are positioned by phase selection controlled by registers in accordance with the operating principles of the *7E* machine switching system.[1]

When making an international call, the subscriber dials a special-service number that connects him to a toll operator, who receives his instructions. After a certain delay depending on the existing trunking conditions, the subscriber is called back and put through to the called party. In most cases, the subscriber is not aware that two or more telephone operators were involved at home and abroad in setting up the connection.

The constantly increasing telephone traffic between countries requires progressively larger numbers of channels. To this end, multichannel cables of the carrier and coaxial types are being installed. The growing traffic makes more-rapid switching attractive to occupy the channels as fully as possible and to provide service with minimum delay. With the existing methods, such an acceleration in service would require not only additional switching equipment but a larger service staff, which is not only costly but presents in most countries great difficulties in recruiting and training personnel. Such situations and recent technical developments suggest the desirability of introducing new methods of operation.

The simplest solution would be for the subscribers to dial directly the desired international number. This method is already in use over short distances such as between Switzerland and Germany, Austria, and France. It is almost certain to be extended to longer distances but this cannot be done immediately because of technical and other difficulties, one of which is the higher-than-normal metering rates that must be registered automatically on the subscribers' meters. Therefore, part of this expansion program was the introduction of semiautomatic operation in which the toll operator in the

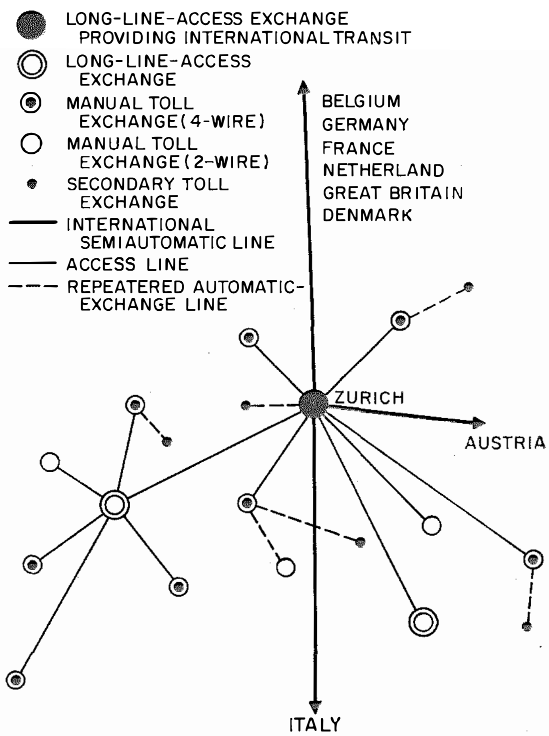



Figure 1—Two long-line-access exchanges and the international tandem center at Zurich.

[1] M. den Hertog and J. Kruithof, "Fundamental Circuits of 7E Rotary Telephone Switching System," *Electrical Communication,* volume 34, pages 56–72; March, 1957.

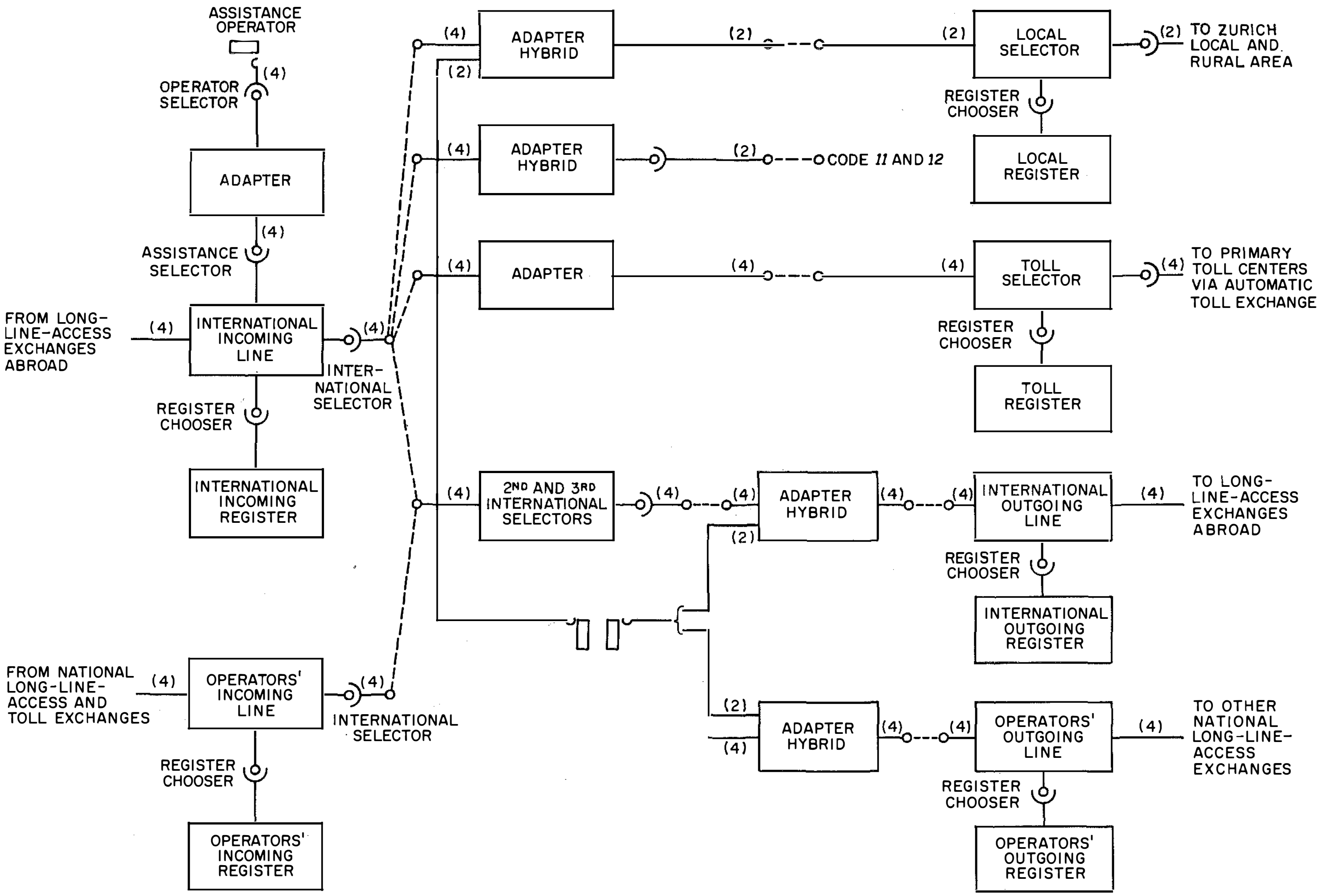



Figure 2—Junction diagram of long-line-access equipment.

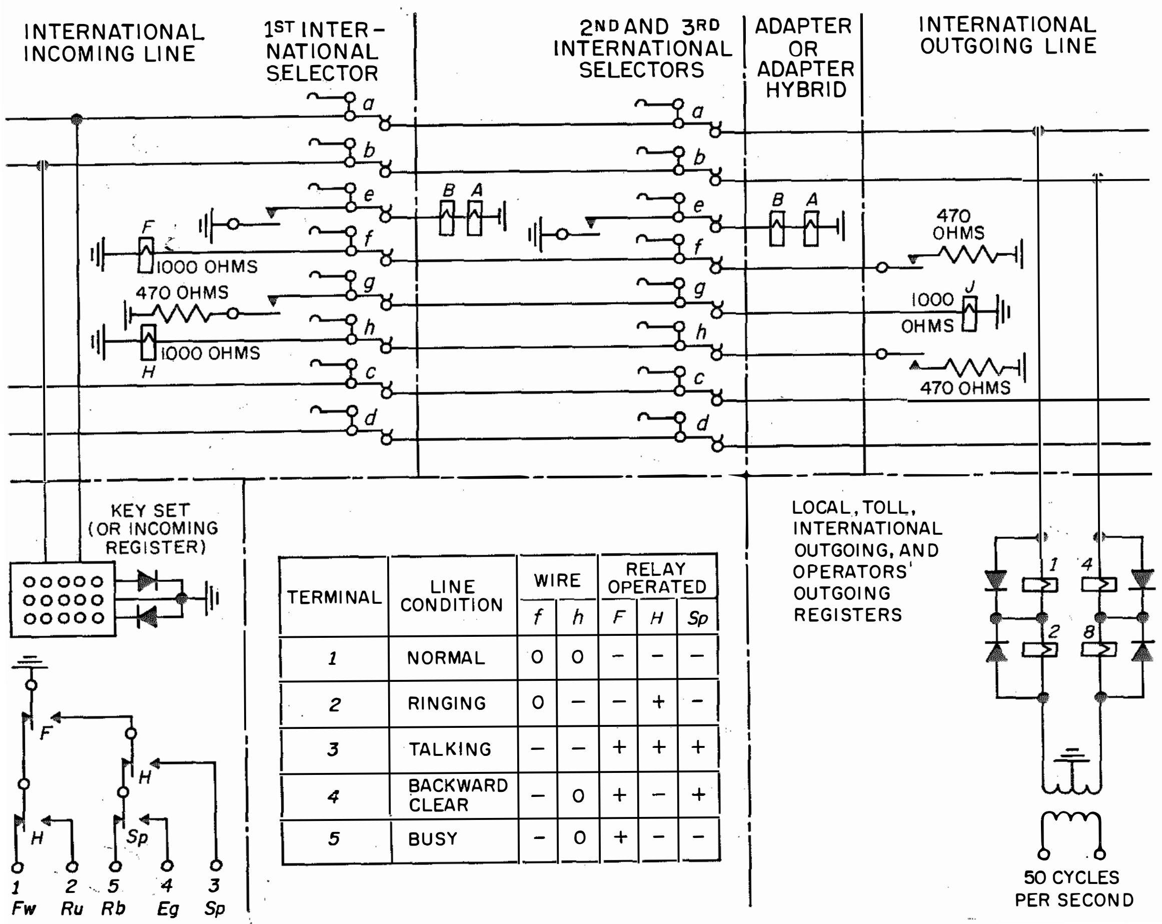
outgoing long-line-access exchange dials directly the number of the called subscriber without the aid of an operator at the incoming exchange.

## 1. Technical Requirements for International Semiautomatic Operation

Many difficulties were encountered in establishing a semiautomatic system for international toll dialing. In each country, various conditions must be met. Language difficulties between the toll operator and the called subscriber may be expected. The number of signals to be transmitted is greater than for a national toll system. Through connections of transit calls toward a third country must also be handled. Furthermore, the ultimate goal whereby a subscriber will be able to dial directly any other subscriber abroad should be kept in mind when designing the semiautomatic system.

There is no doubt that all the requirements can be fulfilled with the switching techniques available but due to the fact that the signaling systems on the international lines are not always the same, a common "language" had to be provided within the long-line-access exchange itself. For this reason, registers are connected to all incoming and outgoing lines as shown in Figure 2. After the phase selections, the incoming register transmits the digits to the outgoing

Figure 3—Signaling within long-line-access exchange.




| TERMINAL | LINE CONDITION | WIRE | | RELAY OPERATED | | |
|---|---|---|---|---|---|---|
| | | f | h | F | H | Sp |
| 1 | NORMAL | 0 | 0 | – | – | – |
| 2 | RINGING | 0 | – | – | + | – |
| 3 | TALKING | – | – | + | + | + |
| 4 | BACKWARD CLEAR | – | 0 | + | – | + |
| 5 | BUSY | – | 0 | + | – | – |

register via two talking wires in a 50-cycle-per-second code. The same happens on outgoing calls made by the operator. The line signals within the long-line-access exchange are sent as direct-current potentials on three wires between incoming and outgoing lines, as shown in Figure 3.

## 2. Signaling Systems

The following signaling systems are used at the long-line-access exchange in Zurich.

### 2.1 The 2-Voice-Frequency System

As recommended by the Comité Consultatif International Télégraphique et Téléphonique, 2 in-band frequencies of 2040 and 2400 cycles per second are used for signaling. The code employs different combinations of the 2 frequencies, transmitted simultaneously or singly. These signals are shown in Table 1.

To minimize operation on false line signals, a delay period is allowed for between the start of a received signal and its recognition. These delay periods must take into consideration the length of the signal itself and are not the same for all signals. For a signal $P$, the delay is between 60 and 100 milliseconds; for $X$ and $Y$, it is 30 to 50 milliseconds; and for $XX$ and $YY$, 160 to 240 milliseconds.

#### *2.1.1 Binary Code*

The binary-code signals are composed of $X$- and $Y$-frequency elements. The signal marking

TABLE 2
BINARY CODE

| Digit or Signal | Binary Number | Code |
|---|---|---|
| 1 | 1 | YYYX |
| 2 | 2 | YYXY |
| 3 | 3 | YYXX |
| 4 | 4 | YXYY |
| 5 | 5 | YXYX |
| 6 | 6 | YXXY |
| 7 | 7 | YXXX |
| 8 | 8 | XYYY |
| 9 | 9 | XYYX |
| 0 | 10 | XYXY |
| Assistance Operator | 11 | XYXX |
| Access to a Particular Operator | 12 | XXYY |
| End of Impulsing | 15 | XXXX |
| Spares | 13 | XXYX |
| | 14 | XXXY |
| | 16 | YYYY |

TABLE 1
2-VOICE-FREQUENCY SIGNALING CODE

| | Designation | Code | Composition of Signals | Direction | Remarks |
|---|---|---|---|---|---|
| 1A | Terminal Seize | PX | | → | = 2040 Cycles Per Second |
| 1B | Transit Seize | PY | | → | = 2400 Cycles Per Second |
| 2A | Terminal, Proceed to Send | X | | ← | X = 100± 20 Milliseconds |
| 2B | Transit, Proceed to Send | Y | | ← | Y = 100± 20 Milliseconds |
| 3 | Digit Signals | | | → | See Table 2 |
| 4 | End of Impulsing | | | → | See Table 2 |
| 5 | Number Received | P | | ← | P= 150±30 Milliseconds |
| 6 | Busy Flash | PX | | ← | |
| | | | | | |
| 7 | Answer | PY | | ← | |
| 8 | Backward Clear | PX | | ← | |
| 9 | Forward Clear | PXX | | → | XX = 350±70 Milliseconds |
| 10 | Release Guard | PYY | | ← | YY = 350±70 Milliseconds |
| 11 | Blocking | PX | | ← | |
| 12 | Forward Transfer | PYY | | → | |

duration is 35 ± 7 milliseconds with a silent spacing interval of the same duration between them. Four such elements form a code signal and using one or the other of the 2 elements, $X$ and $Y$, for each marking position, $2^4 = 16$ combinations are obtained.

Table 2 shows the binary code for this signaling system. The first 10 combinations are used for digital information and the remaining 6 combinations are reserved for other purposes.

The receipt of a complete binary-code signal is acknowledged by a signal returned from the incoming long-line-access or transit exchange. The duration of this signal is 35 ± 7 milliseconds with an $X$ for terminal and a $Y$ for transit acknowledgment.

2.2 Impulse System

The impulse signaling system uses only one frequency, either 2280 or 3000 cycles per second. The digits are sent as ordinary alternating-current pulses. Table 3 shows the line and register signals for this system.

2.3 Multifrequency System

A multifrequency code signaling system is used only for the transmission of register signals over the national access lines. See Table 4. On these lines, the normal line signals of the national toll signaling system remain in force.

## 3. Choice of Signals

3.1 Seizure

The seizure signal is transmitted in the forward direction on placing a call to initiate circuit operation at the incoming end of an international circuit.

For the 2-voice-frequency system, 2 different types of switching signals are available as follows.

(A) A terminal-seizing signal is used at the incoming exchange to assign equipment within the incoming country exclusively for the establishment of the connection to the called party.

(B) A transit-seizing signal is used for establishing the necessary circuit condition at the

TABLE 3
LINE AND REGISTER IMPULSE SIGNALS

| | Designation | Direction | Composition of Signals, Time in Milliseconds |
|---|---|---|---|
| 1 | Seizure | → | 100±20 |
| 2 | Proceed to Send | ← | 100±20 |
| 3 | Digit Signals<br>First Pulse<br>Further Pulses<br>Interval Between Pulses<br>Interval Between Proceed-to-Send and First-Pulse<br>Interdigital Time | → | <br><br>50±10<br>50±10<br><br><br>350/450±50 |
| 4 | End of Impulsing | → | --- |
| 5 | Electrical Busy Signal | ← | --- |
| | Interval Between 5 and 8 | | --- |
| 6 | Digits Received | ← | 100±20 |
| | Interval Between Last Digit and 6 | | ≧150 |
| 7 | End of Selection, Line Free | ← | --- |
| 8 | End of Selection, Line Busy | ← | --- |
| | Interval Between Last Digit and 7/8 | | --- |
| 9 | Answer | ← | 100±20 |
| | Interval Between Answer and 6/7 | | ≧500 |
| 10 | Backward Clear | ← | Normally 70±10 with Interval of 165±15 |
| 11 | Forward Clear | → | ≧500 |
| 12 | Release Guard | ← | --- |
| 13 | Blocking | ← | Continuous or 175±25 with Interval of 450±170 |
| 14 | Forward Transfer | → | --- |

TABLE 4
2-OUT-OF-6 CODE FOR TRANSMITTING NUMBERS BETWEEN REGISTERS

| Digit | A (1) | B (2) | C (4) | D (8) | E (0) | F (5) |
|---|---|---|---|---|---|---|
| 1 | + | | | | + | |
| 2 | | + | | | + | |
| 3 | + | + | | | | |
| 4 | | | + | | + | |
| 5 | + | | + | | | |
| 6 | | + | + | | | |
| 7 | | | + | + | | |
| 8 | | | | + | + | |
| 9 | + | | | + | | |
| 10 | | + | | + | | |
| 11 | + | | | | | + |
| 12 | | + | | | | + |
| 13 | | | + | | | + |
| 14 | | | | + | | + |
| 15 | | | | | + | + |

incoming exchange for switching the call to another international center depending on the routing information received.

3.2 Proceed to Send

The proceed-to-send signal is transmitted from the incoming end of an international circuit. It indicates that the necessary circuit conditions have been made for receiving numerical information. For the 2-voice-frequency signaling system, proceed-to-send signals are provided for both terminal and transit connections.

(A) The terminal proceed-to-send signal indicates that the *numerical* information can be sent for routing the call within the national network of the incoming country.

(B) The transit proceed-to-send signal requires only the transit information for routing the call toward the wanted international exchange in another country, that is, the 2 digits of the international prefix.

3.3 Impulsing

Impulsing signals are transmitted in the forward direction and carry the selective information to complete the call to the wanted subscriber.

For the 2-voice-frequency signaling system, these signals employ a 4-element binary code consisting of $X$ and $Y$ frequencies. For the storage of the binary code at the incoming end, only the $X$ elements result in a definite storage operation. (See also Table 2.)

The impulse system sends the digits in the decimal combination and they are therefore stored as such at the incoming end.

The multifrequency signaling system uses a 2-out-of-6 code for transmission of the numerals from the outgoing operators' register to the distant register as shown in Table 4.

3.4 End of Pulsing

The signal indicating that all the necessary information has been sent is transmitted from the outgoing long-line-access exchange. This signal does not exist in the impulse system.

3.5 Number Received

The number-received signal is transmitted from the incoming international exchange to indicate to the outgoing end that all digits for setting up the connection to the called party have been received. In the impulse system, this signal may also be substituted for by the end-of-selection signal.

3.6 Busy

The busy signal is transmitted in the backward direction for indicating busy conditions during the setting up of a call. The reception of such a signal can be used at the outgoing exchange to provide in an appropriate manner a visual indication to the outgoing operator. If the transmission of such a signal is not possible, as for instance in Switzerland, the outgoing operator will be able to listen to busy tone transmitted from the incoming exchange. In this case, the outgoing operator must note carefully the particular routing of the call to interpret correctly the various acoustic signals from other countries.

With the impulse system, a distinction is being made in some directions by using a separate end-of-selection signal for line-free and line-busy conditions at the incoming exchange. (See Table 3).

3.7 Answer

A signal is transmitted in the backward direction to indicate that the called subscriber has answered. The signal extinguishes the supervisory lamp at the outgoing operator's position. The same signal is used for the reanswer following a backward-clear signal.

3.8 Backward Clear

This signal is transmitted in the backward direction when the called subscriber clears. The supervisory lamp at the operator's position in the outgoing exchange will light.

3.9 Forward Clear

A signal is transmitted in the forward direction on the termination of a call when the operator at the outgoing exchange withdraws the plug from the jack. This signal is recognized by the outgoing, incoming, and transit equipments as the final forward signal of the connection, at the cessation of which all switching apparatus involved will release.

3.10 Release Guard

The release-guard signal is a backward signal transmitted in response to the forward-clear and indicates that the latter has been fully effective in releasing the switching equipment at the incoming end.

3.11 Blocking

A blocking signal is transmitted in the backward direction to indicate the busy condition of the international line at the outgoing end.

3.12 Forward Transfer

The forward-transfer signal is transmitted in the forward direction with the 2-voice-frequency system by the outgoing operator seeking assistance of an operator at the incoming long-line exchange. It has been provided for routing a call to an operator's position at the long-line-access exchange of another country. This assistance operator will be called to intervene at the request of an operator at the outgoing exchange in case of language difficulties, to play the part of interpreter between the controlling operator and the called subscriber, and may, if necessary, be called to identify special acoustic signals or verbal announcements.

3.13 Code *11*

All connections that cannot be obtained automatically are set up manually at the incoming end by the incoming operator, who is summoned by sending the special code *11*. It has been so termed because the 11th code element in the binary series of numbers has been allocated for this purpose. It is used only with the 2-voice-frequency system. (See Table 2.)

3.14 Code *12*

The code used for this signal is the 12th number of the binary code series. Its purpose is to route the call to a particular operator or position at the incoming long-line-access exchange of another country. Code *12* is followed by the other digits that may be necessary to indicate the operator or the position required. It is used only with the 2-voice-frequency system. (See Table 2.)

3.15 Language Code

In the case of the 2-voice-frequency signaling system, the sending of the numerical information representing the national number is preceded by a supplementary digit signal, which is the first to be received by the incoming national register. This signal indicates, in the case of a call extended to the assistance operators, codes *11* and *12*, the language to be spoken by these operators. With the present arrangement, the number of possible languages will not exceed four.

## 4. Equipment at Zurich Exchange

The equipment is shown in Figures 4–11.

The toll-line selectors and registers of the long-line-access exchange in Zurich are situated on two floors of the toll office building. On a third floor is concentrated all the equipment for the 2-voice-frequency system. Table 5 shows the number of lines installed for 1960, anticipated for 1965, and the ultimate capacity of the room.

TABLE 5
2-Voice-Frequency International Lines in Zurich

| International Lines | 1960 | 1965 | Ultimate Capacity |
|---|---|---|---|
| Incoming | 108 | 162 | 270 |
| Outgoing | 108 | 162 | 270 |

The equipment for the impulse system is located on the fourth floor of the same building and includes, in addition to the incoming and outgoing register circuits, the 2nd and 3rd

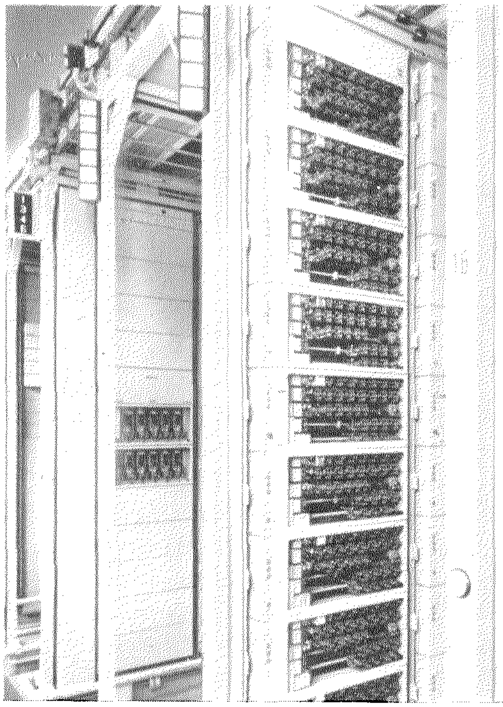
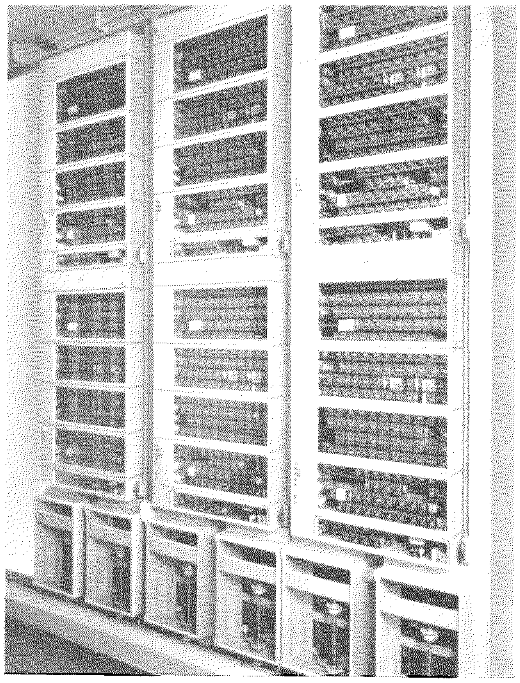


Figure 4—Incoming line and international selector circuit of Figure 2. The selector bay is situated in the same row to the right.



international selectors. Table 6 gives the number of lines.

TABLE 6
IMPULSE-SIGNALING LINES IN ZURICH

| International Lines | 1960 | 1965 | Ultimate Capacity |
|---|---|---|---|
| Incoming | 140 | 200 | 280 |
| Outgoing | 180 | 200 | 280 |

### 4.1 TESTING

Routine test facilities are provided to permit systematic testing of all lines, incoming and outgoing, one after another and in all directions or, if desired, only in a certain direction. Means are also available to test an unterminated line in one direction.

All lines are disconnected from the international circuits during the tests so that the routine testing applies to relay functions and signals generated within the long-line-access exchange.

In accordance with recommendations of the Comité Consultatif International Télégraphique et Téléphonique, robot circuits that act as automatic subscribers are provided. These robots have the same access facilities to the outgoing directions as do the normal circuits but they establish connections to incoming robot circuits in the long-line-access exchanges of other countries.

Signals of specified frequencies and power output are then transmitted and, if within the prescribed limits, the incoming robot returns the same test signals. If not, a clear-back signal is returned instead. The long-line-access exchanges are equipped with both incoming and outgoing robot circuits.

Figure 5—Incoming register for international connections set up via the national access network by an operator at a manual toll office. The translators are shown at the bottom of the bays.

## 5. Method of Operation

### 5.1 Incoming Calls

*5.1.1 Seizure*

The signal receiver transmits the seizing signal from the outgoing end as a direct-current signal to the incoming-line circuit, which will be engaged, and an incoming register will be connected to the line. As soon as the register is ready, it sends the proceed-to-send signal in the backward direction.

*5.1.2 Language Code*

The incoming signaling wires are through-connected to the register, where the code signals can be received. The international prefix is not transmitted on a terminal call. The first numeral received is the language code, that is, *1* for English, *2* for French, *3* for German, and *4* for Italian. After the reception of such a figure, an acknowledgement signal *X* is sent backward in the case of the 2-voice-frequency system.

*5.1.3 Reception of Subscriber's Number*

After the language code, the outgoing exchange sends the national prefix and then the 5- or 6-digit number of the wanted line. The last signal received is the end-of-sending signal, *15* of the binary-code series. The total of 9 or 10 digits is stored in the international incoming register on multicore relays. The incoming line then seizes a trunk circuit to the national toll exchange. A local register is engaged to take control of the selection of the called number. The incoming register transmits the numerical information in coded form to the local register (see Figure 2).

*5.1.4 End of Selection*

As soon as the last figure has been transmitted by the register, it sends the end-of-selection signal backward.

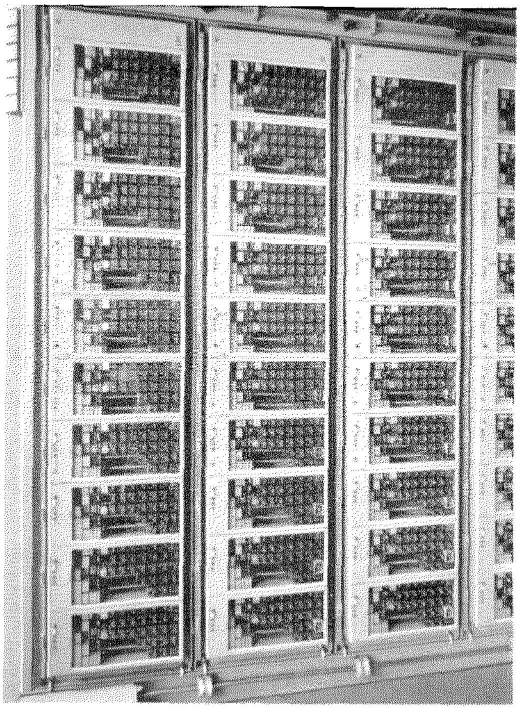


Figure 6—Incoming line circuit and international selector from national toll center. Two relay bays and one selector bay are shown.

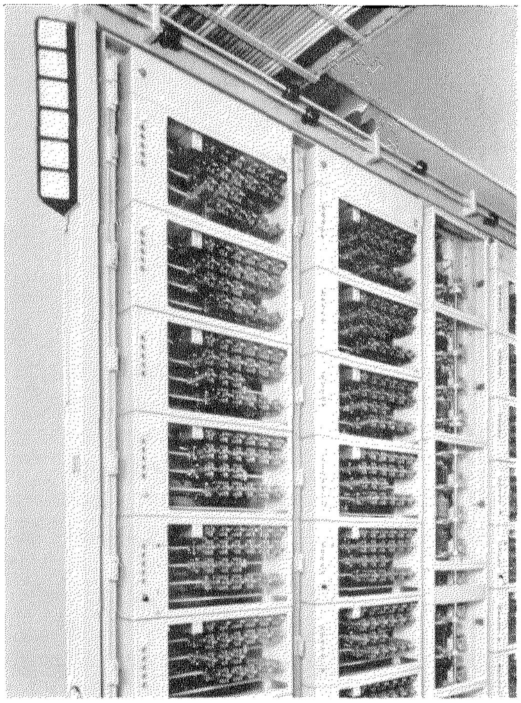


Figure 7—Outgoing international line circuits, impulsing system. The first two bays at the left are in service. Alarm box is shown on first bay at top left.

*5.1.5 Through-Connection*

Immediately after the completion of the national selections, the through-connection is made. The controlling operator at the outgoing exchange can then listen to the ringing or busy tone of the country of destination.

*5.1.6 Answering Signal*

The answer signal received from the national network is transformed by the international incoming line circuit and sent to the outgoing end. With the 2-voice-frequency system, a $PY$ signal is sent backward.

*5.1.7 Backward Clear*

With the 2-voice-frequency system, the supervisory signal from the national system is transformed at the line circuit into the signal $PX$. If the called party replies, the answering signal $PY$ is retransmitted and the connection restored to the talking condition. The signal from the national toll network need not be transformed with the impulse system, both signals being identical.

*5.1.8 Release*

The connection to the national side is broken down on receipt of the forward-clear signal. The line then transmits the release-guard signal to the outgoing end.

5.2 Transit Calls

At present, transit calls are handled only with 2-voice-frequency signaling.

*5.2.1 Seizure*

The seizure is similar to a terminal connection, but the reception of a transit-seize signal $Y$ initiates in the international incoming register a proceed-to-send signal $Y$ sent backward. This confirms to the outgoing exchange that the call is controlled as a transit call by the incoming (transit) register and that the international prefix alone is to be transmitted in response to the proceed-to-send signal $Y$. After receipt of the prefix, a proceed-to-send signal $X$ is sent backward after a timed delay of 100 milliseconds. The outgoing register at the originating exchange then sends the subscriber's number, which is stored in the incoming register for retransmission when the terminal exchange has been reached.

*5.2.2 Selection of Direction*

The direction is selected by marking according to the received international prefix in the arc of the international selector under the control of the incoming register. All the digits stored are retransmitted to the terminal exchange.

*5.2.3 Seizure Signal*

The seizure signal is sent to the next exchange

Figure 8—Adapter circuit for 2-wire/4-wire or 4-wire/4-wire connections, with hybrid coils.

as soon as a free outgoing line is found via the international selector. A proceed-to-send signal from the next exchange is an $X$ signal if the connection terminates there. It is a $Y$ signal if the connection is to be transmitted once more, that is, in the case of a double transit. In that case, the originating exchange would have to send the international prefix twice.

#### *5.2.4 Release*

The clear-forward signal opens the speech path and releases the engaged circuits. The release-guard signal is returned to the originating office.

### 5.3 Outgoing Calls

A subscriber intending to make a call abroad dials the special-service number for the recording operator. When his request has been received and acknowledged by the recording operator, the subscriber hangs up.

The recording operator writes out a ticket and sends it to a controlling operator via a pneumatic ticket-distributing system. The latter operator sets up a connection to the local calling subscriber as indicated by the ticket via the adapter-hybrid circuit and the local selector, shown in Figure 2, to the local or rural area as

Figure 9—Outgoing international register, 3 per bay and impulsing system.

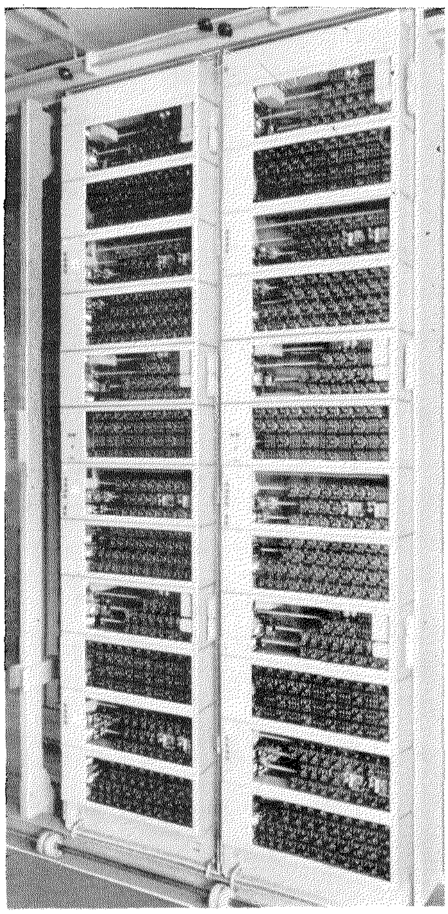
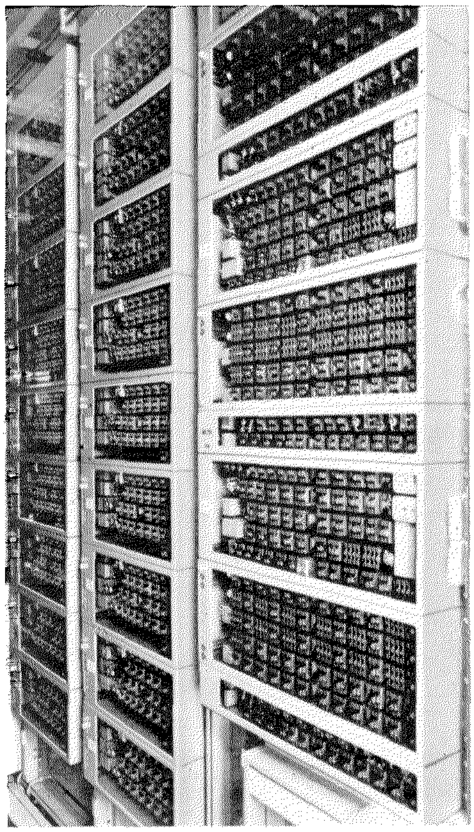


Figure 10—Outgoing international register, 2-voice-frequency system; 3 per bay. The translators are mounted on the same bay. The outgoing lines for 2-voice-frequency signaling are shown on the 2nd and 3rd bays from the right.



required. However, ringing of the calling subscriber is not immediately initiated.

The controlling operator then establishes a connection to the called party abroad via the international outgoing line with the help of an international outgoing register.

When the called party is on the line, the operator rings the calling subscriber, completes the through connection, and supervises the call. The total talking time is marked on the ticket.

No selection is required for outgoing calls at Zurich. The outgoing toll lines of each group or direction are multipled and are accessible at the toll position individually. The outgoing operator inserts the plug in the terminal jack. If all the outgoing circuits are engaged, an alternative route is chosen. The insertion of the plug in the toll-line jack automatically seizes an international outgoing toll line, which engages an international outgoing register that is capable of storing a maximum of 16 digits, including the end-of-pulsing signal.

#### 5.3.1 *Transit*

In the case of a transit call, a transit proceed-to-send signal is returned from the next exchange and eventually from the penultimate exchange. After receipt of the terminal proceed-to-send signal, the rest of the digital information is sent forward; that is, the language code, national prefix, and subscriber's number.

#### 5.3.2 *Call-Back*

The calling subscriber is called back by the outgoing operator via a selective trunk circuit of the national system. A 2-wire connection is made at the toll position. The termination is located at the outgoing line circuit and the through connection to the national network is made.

#### 5.3.3 *Supervision*

The line circuit provides for the same supervisory signals that are used in the national toll system. The supervisory lamp flashes during the transmission of the coded signals. At the end of transmission, it lights steadily until receipt of the answer signal. The ringing or busy signal is also provided for by the supervisory lamp.

### 5.4 Assistance Facilities at Zurich

The intervention of an assistance operator is called for by the transmission of the signal *PYY*, which is received at the international incoming line at Zurich as the forward-transfer signal. It connects an adapter circuit via an assistance selector and selects via an operator selector an assistance operator for the incoming direction, who can answer in the desired language. This operation can be repeated as many times as required by sending the forward-transfer signal *PYY*. The through connection to the position is made via 4 wires.

### 5.5 Calling Operators at Zurich

By sending the following digital information, an incoming operator can be reached from the outgoing exchange.

(A) International code of the country (*66*).
(B) Language digit.
(C) Operator's code, *11* of the binary series.
(D) End-of-sending signal.

On receipt of this information, the line circuit establishes a connection to the position of the incoming operator of the incoming direction. The through connection is made via 2 wires, the termination is provided for at the adapter-hybrid circuit.

To reach a suspended-call operator, similar procedures are followed, with the exception that code *12* and, as the case may be, the required operator's position number must be sent. The call is switched through in a similar manner as code *11*.

## 6. National-Access Network

To avoid the intervention of a second operator on international toll calls initiated by subscribers connected to manual toll or special-service offices (see Figure 1), the Swiss Administration has created a separate network reserved exclusively for handling such calls.

The use of the existing national toll-line network could produce an economy of lines, but it seemed simpler to fulfil the transmission requirements by means of a separate network. This offers some additional advantages, one of which is the possible adaptation of the signaling system to fit the international conditions.

Another advantage is that operators at small outgoing exchanges can handle all calls for international destinations themselves and thus take some of the heavy load off the very-busy operators at the long-line-access exchanges.

The main advantage, however, of a separate national access network lies in the fact that the same quality of service can be offered to all subscribers.

The operator intending to set up an international toll call can reach the long-line-access exchange by sending the appropriate special prefix through which an incoming line and register are seized. The operator now has full access to the available equipment, replacing, so to speak, the second operator mentioned in the introduction to this section. The gain of traffic volume by this measure is of the order of 6 traffic units.

The telephone operators using the access network can also make a choice for routing calls to their destination. As an example, consider the case of an operator at Saint Moritz, wishing to call Stuttgart, via Zurich. If all the lines from Zurich to Stuttgart are busy, then the operator can route the call via Zurich–Basle to Stuttgart. The choice of routes, however, is fixed in advance by the operating branch and classified as 1st, 2nd, and 3rd choices.

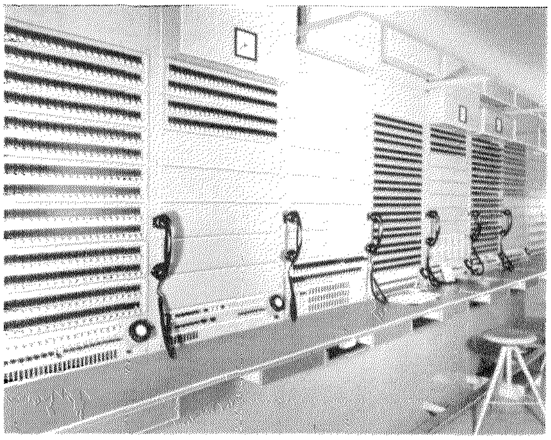


Figure 11—Testing and monitoring desks, with U-links, multiple service lines, tie lines to local exchanges, and telephone circuits.

## 7. Results

By statistical observations made periodically, it is possible to get a picture of the quality of service obtained by the rapid-service method used on semiautomatic toll lines.

It appears that a very-great number of connections are set up at the combined recording and outgoing positions, the average being between 70 and 90 percent of the effective calls. The remainder, 30 to 10 percent of these calls, cannot be put through due to the called party being busy or giving no answer. A small number of calls within the latter figures is of course due to busy toll lines.

The time elapsing from the moment a calling subscriber has recorded the call until the conversation starts is a very-important factor. The statistical observations prove that 70 to 80 percent of the international semiautomatic connections are set up in less than 1 minute and 90 percent of the connections are completed within 2 minutes. The results can be considered as very satisfactory.

# Ferroelectric Energy Converters*

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Thermomagnetic heat-to-electricity converters have been investigated[1,2] extensively. Their ferroelectric analogue, however, has apparently not been studied and this paper fills this gap by reporting on ferroelectric energy converters.

The ferroelectric energy converter is based on the effect of temperature on the dielectric constant of ferroelectric and related materials. This thermodielectric effect permits energy conversion in any dielectric system where the dielectric constant varies with temperature. Only those systems are prospective, however, that have high dielectric constant and dielectric strength, resulting in a relatively high density of electric energy. This paper deals with conversion in ferroelectric materials specifically, but most of the equations are applicable to thermodielectric conversion in general.

Heat can be converted by ferroelectric materials in which the dielectric constant decreases with temperature. This occurs above the Curie point. The small-signal alternating-current permittivity has a well-known peak at the Curie temperature. In heat-to-electric-energy conversion, the direct-current or static dielectric constant $k$ is derived from the capacitance $C$ by

$$C = Q/V = k\epsilon_0 A/d$$

where

$Q$ = electric charge $\int i\, dt$
$V$ = charging voltage
$\epsilon_0$ = dielectric constant of free space
$= 8.86 \times 10^{-14}$ farad/centimeter
$A$ = dielectric area
$d$ = dielectric thickness.

The direct-current dielectric constant $k$ of a commercial barium–strontium titanate is shown in Figure 1 for three different electric fields. Since $k$ varies with both temperature and field strength, the term "dielectric constant" is somewhat a misnomer and the term "permittivity" will be used instead. The measurements of Figure 1 confirm the expected dependence of permittivity on temperature and field strength. It is noteworthy that the small-field direct-current permittivity is practically identical with the alternating-current permittivity up to the Curie point, which, for this particular composition, is approximately 35 degrees centigrade. The alternating-current permittivity decreases again as temperature drops below the Curie point; the direct-current permittivity, however, rises as the temperature drops below the Curie point. This different behavior of alternating- and direct-current permittivities below the Curie point indicates that there is a slow domain wall movement or domain switching process involved in the direct-current polarization that cannot follow fast alternating-current cycling.

Figure 2 shows the effect of field strength on direct-current permittivity at room temperature by taking a room-temperature section through Figure 1. The $k$ values of this curve represent starting values for ferroelectric conversion.

The circuit that has been used for measuring permittivities and converter voltages is shown in Figure 3. The electric charge $Q = \int i\, dt$ was measured with a fluxmeter substituted for a ballistic galvanometer. Its shunt resistor $R_s$ is in series with the capacitor charging and discharging circuit. A three-position switch permits charging from a battery, conversion, and discharging. The voltage was measured with an electrostatic voltmeter. $C$ is the ferroelectric capacitor connected to a series charging resistor $R_c$.

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*Abridgement of American Institute of Electrical Engineers Conference Paper 60-351 presented at its Winter General Meeting in New York, New York, on 4 February 1960.

[1] L. Brillouin and H. P. Iskenderian, "Thermomagnetic Generator," *Electrical Communication,* volume 25, number 3, pages 300–311; September 1948.

[2] J. F. Elliot, "Thermomagnetic Generator," *Journal Applied Physics,* volume 30, number 11, pages 1774–1777; November, 1959.

The conversion experiment consists of the following two steps:

(A) Charging the ferroelectric capacitor $C$ at room temperature $T_1$ and opening the circuit by putting the switch in the middle position.

The charge absorbed by the capacitor is

$$Q = V_1C_1$$

and the energy stored is

$$W_1 = QV_1/2.$$

(B) Heating the capacitor to a temperature $T_2$ causes a voltage increase because permittivity and capacitance decrease as shown in Figure 1, whereas $Q$ remains constant.

Since

$$Q = V_1C_1 = V_2C_2,$$
$$V_1/V_2 = C_2/C_1.$$

The increased voltage $V_2$ is a measure of the increase in electric energy, since

$$W_2 = QV_2/2,$$

or

$$W_2/W_1 = V_2/V_1.$$

The energy increase due to conversion is

$$\Delta W = W_2 - W_1 = Q(V_2 - V_1)/2$$
$$= \tfrac{1}{2}Q\Delta V/2.$$

The energy $W_2$ can be dissipated in a load. The capacitor can then be cooled to its starting temperature $T_1$, thus completing a temperature cycle.

The conversion mechanism involves disorienting the molecular dipoles against the aligning force of the electric field. An analog is the mechanical removal of a dielectric from between two capacitor plates against the force of a field. The amount of mechanical energy required is larger when the capacitor is charged, the difference in work appearing as additional electric energy in the capacitor. In the analogous case of a thermodielectric converter, it will take more thermal energy to raise the charged capacitor to $T_2$ than it

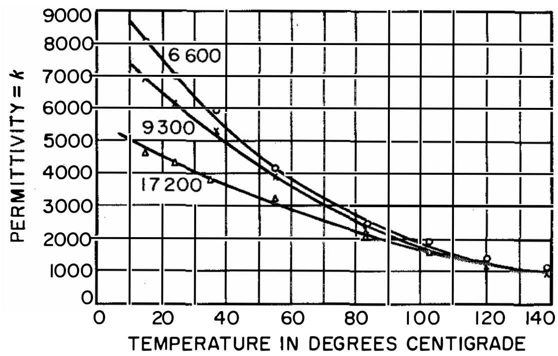


Figure 1—Direct-current permittivity of barium–strontium titantate versus temperature. The curves are labeled with the electric field stress in volts per centimeter.

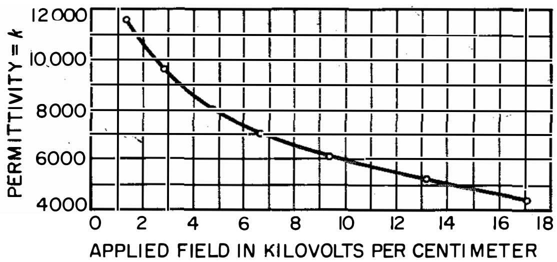


Figure 2—Direct-current permittivity of barium–strontium titanate as a function of electric field at room temperature.

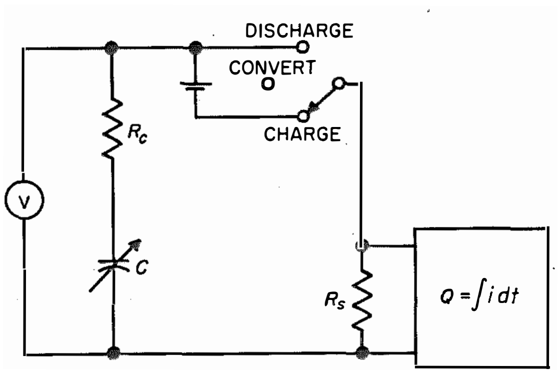


Figure 3
Circuit for measurement of converter performance.

requires to heat it in the uncharged state. This means that the specific heat of the ferroelectric is field-dependent.

Thermodielectric conversion can also be described by a charge–voltage diagram such as Figure 4. Assuming the capacitance constant and field-independent, charge and discharge curves are idealized as straight lines. The slope of these lines is proportional to the capacitance at a given temperature. The first step is charging at temperature $T_1$. The electric energy

$$W_1 = QV_1/2$$

appears as a triangle in this graph. Then, heat is applied to the capacitor while the charge $Q$ remains constant. This conversion phase raises the capacitor voltage to $V_2$. Finally, the total energy $W_1 + \Delta W$ can be taken from the capacitor. The converted energy $\Delta W$ appears as a triangle enclosed by the charge, heat, and discharge curves. The cycle is completed by cooling the capacitor to the starting temperature $T_1$.

Actual voltage measurements using the circuit of Figure 3 are plotted in Figure 5. These were taken with a commercial 0.5-millimeter-thick barium–strontium titanate. In the upper curve, the capacitor was charged to 330 volts and heated. The voltage rises to about 1600 volts as expected from permittivity measurements. At this point the combined effect of temperature and electric field causes noticeable leakage and the charge loss becomes particularly evident in the second part of this curve for decreasing temperature. There is a noticeable voltage loss over the same temperature range.

The lower curve in Figure 5 does not indicate any appreciable leakage because of the lower voltage range. This curve is particularly noteworthy because it starts from zero voltage. This was made possible by previously polarizing the dielectric material (a well-established process for preparing piezoelectric ferroelectrics). Polarizing results in a bound charge that becomes free on heating and is consequently raised to a higher potential level by thermodielectric conversion. In this experiment, the voltages with decreasing temperature are slightly higher than the voltages with increasing temperature. At room temperature there is a net charge left that was frozen in by polarizing and is very useful to provide a starting charge for a practical energy converter, thus avoiding batteries or other means of charging.

For complete elimination of batteries or other charging means in a continuously operating converter, it will be necessary to compensate for charge leakage through the dielectric. This

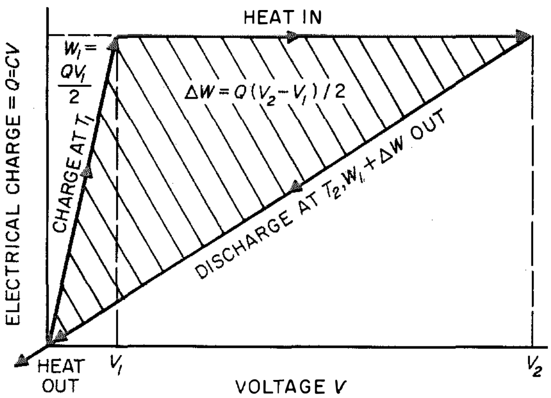


Figure 4
Basic operating cycle of ferroelectric converter.

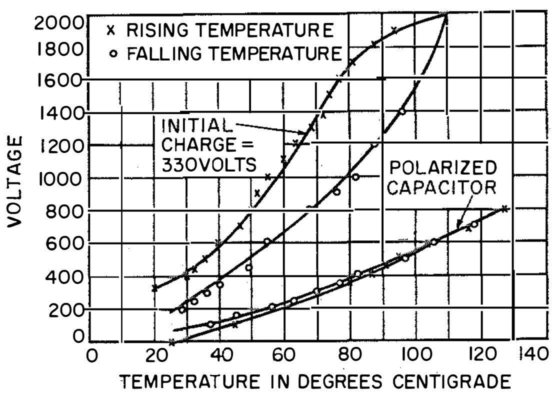


Figure 5
Voltage of ferroelectric energy converter.

can be done by using some of the output energy to restore lost charge. An experiment has been made that shows this to be feasible. An isolating transformer operating from the alternating-current output of the generator and a small silicon rectifier were used to obtain the high direct voltage needed to sustain a constant charge.

The use of a polarized ferroelectric tends to obscure the difference between the thermodielectric and the pyroelectric phenomenon characterized by a current per unit area

$$i = \mathrm{d}p/\mathrm{d}T$$

where $p$ is the polarization of the material.[3] Apparently, this effect does not convert thermal energy. It can be used, however, to store an electric charge in a sort of heat-activated battery. In contrast, the thermodielectric effect is characterized by a differential $\mathrm{d}k/\mathrm{d}T$ that is not restricted to pyroelectric materials. In fact, it is not even restricted to solids.

In actual energy converters, a multiplicity of converter capacitors can be heated and cooled alternately and supply a common load through diodes. The fact that temperature cycling is required leads to circuits generating alternating current; a simple example is shown in Figure 6, where two converter capacitors, $C_1$ and $C_2$, operate in push-pull. Each capacitor has to be charged initially, which can be accomplished by the above polarizing method. Heating one capacitor while cooling the other will force a charge $\Delta Q$ through the load. Upon reversing the heating and cooling, the same charge will be pumped back. A transformer and rectifier can supply the charge loss due to leakage. The output frequency of a single capacitor pair is limited by the rate of temperature cycling; it can be increased by using multiple pairs supplying a common load.

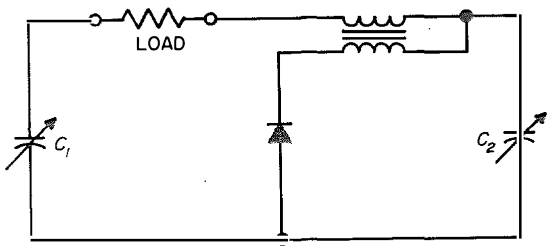


Figure 6—Alternating-current ferroelectric generator.

Temperature cycling can be obtained rather conveniently in a spinning satellite body covered with ferroelectric converters. Solar energy would be absorbed by those converter elements facing the sun while the others would radiate heat into space. The surface absorption and emission coefficients as well as the speed of rotation would determine the operating temperature range, which in turn determines the choice of material. Pure barium titanate has a Curie point of approximately 120 degrees centigrade. A large variety of ferroelectric substances are known[4] that range in Curie points from approximately zero to nearly 900 degrees Kelvin. Also, antiferroelectric and related high-$k$ materials appear suitable for energy conversion.

It is quite apparent that thermodielectric energy converters are capable of generating high output voltages limited only by the dielectric strength of the converters, provided that the starting voltage is high enough. It appears feasible to start with a relatively low voltage and to build it up in a cascade circuit. Such a proposed circuit, shown in Figure 7, consists of 3 converter capacitors $C_1 > C_2 > C_3$. Initially, these capacitors are charged by polarization or from an outside source. Heating of $C_1$ will raise its potential and will drive its charge into $C_2$ and $C_3$, thus raising their voltage. Next, $C_2$ is heated and its charge is driven into $C_3$, further raising its voltage. Lastly, $C_3$ is heated, driving its charge through

[3] A. G. Chynoweth, "Dynamic Method for Measuring the Pyroelectric Effect with Special Reference to Barium Titanate," *Journal Applied Physics*, volume 27, number 11, pages 78–84; January, 1956.
[4] C. Pulvari, "Ferroelectric Materials Survey with Particular Interest in Their Possible Use at High Temperatures," Wright Air Development Center Technical Note *56-467*, Armed Services Technical Information Agency Document *110489*.

a high-impedance load that should be voltage dependent to prevent discharge prior to the voltage increase. In discharging $C_3$, the charge returns to $C_1$, which has been cooled in the meantime. Thus, through repeated sequential heating of the capacitors, the same charge can be circulated in the cascaded circuit. If $n$ capacitors are cascaded in this fashion, the output voltage $V_n$ would approach

$$V_n = V_1(V_2/V_1)^n$$

where $V_1$ is the starting voltage and $V_2/V_1$ is the voltage gain per stage.

The efficiency of ferroelectric energy converters can be derived by using the approach[1] of Brillouin and Iskenderian for the thermomagnetic converter. A much-simpler estimate can be made by relating the electric output energy $\Delta W$ to total heat input according to

$$\eta = \Delta W/(\Delta H + \Delta W).$$

In this approach, the total input heat in the denominator is $\Delta H$, that required to heat an uncharged capacitor, plus $\Delta W$, that portion of the heat input converted into electric energy. The above equation can be written:

$$\eta = \frac{Q\Delta V/2}{cAd\Delta T + Q\Delta V/2}$$

where $c$ is the specific heat of the ferroelectric in joules per cubic centimeter. Furthermore, since $Q = pA$ and $V = dE$, where $E$ is the electric field,

$$\eta = \frac{p\Delta E}{2c\Delta T + p\Delta E}.$$

This indicates that for good efficiency, $p\Delta E$ should be large in comparison with the thermal properties $2c\Delta T$ of the ferroelectric.

The output of a converter per unit of weight is of prime importance in satellite and related applications. The output power $P$ can be

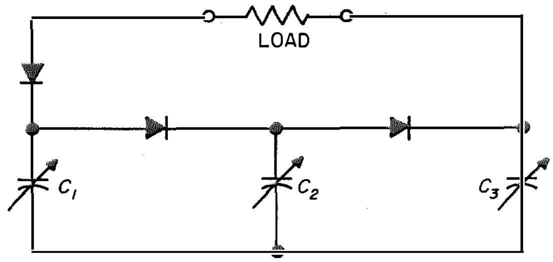


Figure 7—High-voltage ferroelectric generator circuit.

derived from the energy gain $\Delta W$ as

$$P = \Delta Wf$$

where $f$ is the number of temperature or discharge cycles per second. It must be divided by the weight $F$ of a converter:

$$P/F = \Delta Wf/Ads.$$

High cycling rates are desirable for high output. The maximum cycling rate is determined by heat source, heat sink, and heat transfer mechanism. Ultimately, thermal shock will limit the cycling rate.

Summarizing the present results, ferroelectric energy converters offer unique characteristics such as high alternating and direct voltages. This contrasts to existing heat-to-electricity converters such as thermoelectric and thermionic types. The conversion efficiency will be substantially lower than these latter converters unless material characteristics can be improved. The output per unit weight appears high where a relatively high rate of temperature cycling can be achieved. The cost of the converter is likely to be lower than other types as long as ceramic ferroelectrics can be used.

The author is indebted to W. L. Harries for his study of the thermodynamic aspects of this converter and for many helpful discussions. Also, the interest and support of F. A. Muller and P. E. Lighty are deeply appreciated.

# Characteristics of Waveguides for Long-Distance Transmission*

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Discussion of a waveguide communication system is given and the system is compared with existing communication media. The principal properties of waveguide—the medium of communication—are discussed in some detail. The particular significance of helical and coated waveguides is pointed out and the design formulas included. The phenomenon of mode conversion–reconversion, which is peculiar to a waveguide communication system, is discussed in general and the basic theory as applicable to design is also discussed. Design features of components such as bends, transducers, tapers, et cetera, are analyzed. The effect of waveguide discontinuities is analyzed in some detail and various aspects of signal distortion are also considered.

## 1. Introduction

Man, in his constant efforts towards progress, finds it necessary to establish better, quicker, and more-numerous communication systems. It is even said that the complexity of communication systems grows in proportion to the development of civilization. As the conventional means of communication become gradually utilized to capacity and the traffic continues to increase, one, naturally, turns towards more-exotic means of communication. Waveguide is a possible winner[1–3] and in this connection special acknowledgment should be made of the very-important pioneering and continuing work of the Bell Telephone Laboratories in this field. Although the final assessment can only be made against the background of economy, a detailed performance assessment must be the first step. For a number of reasons, as we shall see subsequently, such a waveguide is essentially a circular tube about 1 to 3 inches (25 to 76 millimetres) in diameter and the waves involved are in the millimetric band. Such a pipe—the medium of communications—would be laid in the ground or at the bottom of oceans for hundreds and thousands of miles.

To bring out any advantages that a waveguide communication system may have over existing systems, the principal features are shown tabulated in Table 1. From this it is evident that the main advantage of a waveguide communication system is its large communication capacity. Provided that very-short millimetric waves are used, then, in principle, several hundreds of television channels, or the equivalent in speech channels, can be accommodated in a single pipe. The waveguide is also characterized by small attenuation, and in contrast to microwave links it is a screened system. Whether such enormous communication capacities will ever be required is another matter; but, if present statistics are anything to go by, then these capacities will be required in less than 20-years time, if not for telephone communication then for purposes such as data transmission, color and cinema television, as well as for purposes not yet conceived.

A waveguide communication system is essentially simple (Figure 1). The video channels are grouped together (say, by frequency division) and passed to a coder where, for reasons to be discussed, it is placed in pulse-code modulation. The coded signal then activates an on–off modulator operating on one of the many carriers in the whole spectrum carried by the waveguide. The carriers are combined through a suitable channel insertion filter and passed through a transducer to the waveguide. At convenient intervals along the waveguide, say, every 10 miles (16 kilometres) regenerative repeaters are inserted to amplify and restore the quality of the signal.

For effective operation, the terminal equipment (modulation/demodulation and coding/decoding) is designed to handle nanosecond

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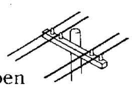
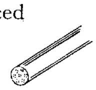
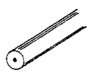
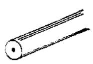
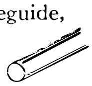
[1] Superior figures indicate the literature references at the end of this paper.

pulses. This in itself is a complication but it is a necessary one to achieve pulse-code modulation of the wide video band and derive the advantage of regeneration. Regeneration is a necessary requirement because of the large signal distortion, even with waveguides of moderate length (say, 10 miles (16 kilometres)). It has been established beyond any doubt that for long-distance communication purposes only modulation systems capable of

TABLE 1
COMPARISON OF COMMUNICATION MEDIA
One Superchannel = 4 Megacycles per Second

| Type | | Operating Frequency in Megacycles per Second | Channel Capacity | Repeater Spacing in Miles (Kilometres) or Attenuation |
|---|---|---|---|---|
|  Open Wire | One Pair | 0·036 to 0·084 (Go) 0·092 to 0·140 (Return) | 12 + 3 + (1) = 15 + (1) | 60 to 70 (97 to 113) (0·4 Decibel/Mile = 0·25 Decibel/Kilometre) |
| | Maximum, 16 Pairs | As Above | 240 + (16) | |
| Balanced pair  | One Pair | 0·012 to 0·252 | 60 | 12 to 30 (20 to 48) (4 Decibels/Mile = 2·5 Decibels/Kilometre) |
| | Maximum, 24 Pairs | As Above | 1440 | |
| Coaxial, Great Britain |  | Up to 4 | 960 (One Superchannel) | 6 (10) |
| | | Up to 12 | 3 Superchannels | 3 (5) |
| Coaxial, United States |  | To about 3 | 720 | 8 (13) |
| | | L3 System | 1 Television Channel + 600 Speech Channels | 4 (6·4) |
| Single-Wire Transmission Line | | About 100 to 1000 | Probably a Few Television Channels | About 2 to 10 (3·2 to 16) |
| Microwave Links  | | Below 500 | Maximum of 60 | 40 to 50 (64 to 80) |
| | | 500 to 1000 | Up to 120 | 40 to 50 (64 to 80) |
| | | 2000 | 240 × 6 or 1 Television × 6 | 30 to 40 (48 to 64) |
| | | 4000 | 600 × 6 or 1 Television × 6 | 25 to 30 (40 to 48) |
| | | 6000 to 8000 | 600 × 6 or 1 Television × 6 Maximum 2 Television × 6 | 25 to 30 (40 to 48) |
| | | 11 000 | Less than 600 | Less than 20 (32) |
| Long-Haul Waveguide, $H_{01}$ Mode  | | 30 000 to About 100 000 | 1000 Superchannels = Several Hundreds of Television Channels = Several Hundred Thousands of Speech Channels | 20 to 40 (32 to 64) (Attenuation 2 to 4 Decibels/Mile = 1·2 to 2·5 Decibels/Kilometre) |

regeneration can be used, and of these pulse-code modulation has definite technological advantages.

This paper is not an attempt at a balanced summary of the subject matter, but is a presentation of the opinions of the authors.

## 2. Summary of Properties of Waveguides

There are four principal factors influencing the performance of a waveguide: (A) Its geometry both in shape and size (in terms of the wavelength), (B) its mode of operation, (C) the value of the surface impedance of its walls, and (D) the nature and magnitude of the tolerances on the cross section.

For long-distance communication, waveguides of circular cross section operating in the $H_{01}$ mode have been chosen. The choice is chiefly a matter of technological preferences based on a number of considerations. Such waveguides have relatively low dispersion and low attenuation, can be constructed to discriminate against unwanted modes, and the tolerances on the cross section are less exacting than for other waveguides and modes.

The exact waveguide properties and construction is a matter of compromise between a number of conflicting factors such as cost of terminal equipment and repeaters, permissible signal distortion, service reliability, et cetera. And, since these factors can be traded for each other, it is impossible to give a specification for a waveguide under any given conditions except in the light of accumulated experimental experience. But, to fix orders of magnitude, a waveguide operating in the 4- to 8-millimetre band would be about 2 to 3 inches (51 to 76 millimetres) in diameter.

There are two principal sources of signal distortion with waveguides: (A) delay distortion,[6] and (B) mode conversion–reconversion phenomena.[1,4] Delay distortion arises (as in most communication media) because of the nonlinearity of the phase characteristic $\beta(\omega)$. This leads to the undesirable effect that the high-frequency components of the signal travel faster than the low-frequency components, a phenomenon known as dispersion. A corresponding distortion due to the variation in the attenuation characteristic $\alpha(\omega)$ is, in waveguides, negligible.[6]

The actual bandwidth limitation due to delay distortion depends to a large extent on the modulation method and is particularly troublesome with frequency modulation. Pulse-code modulation, on the other hand, is rather immune, but even here 10-nanosecond modulation pulses would be unrecognizable after a journey down a typical waveguide, say, 30 miles (48 kilometres) long; regeneration is, therefore, called for. In addition, a certain amount of equalization of the $\beta(\omega)$ characteristic may be necessary.

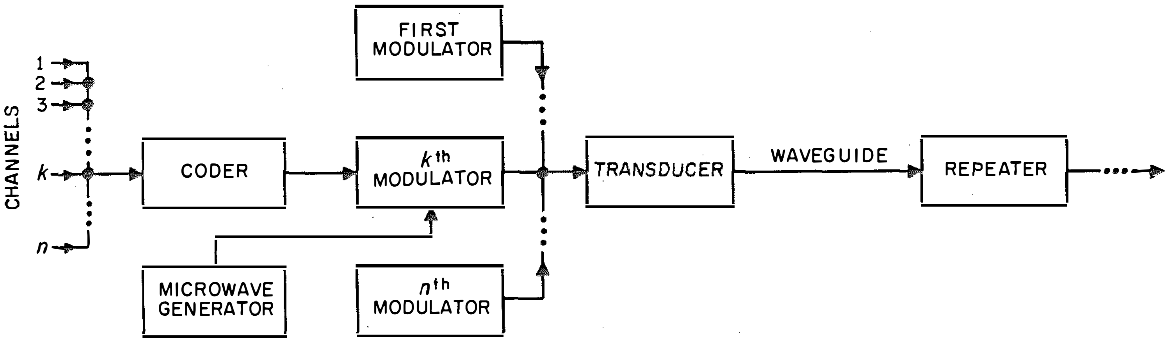



Figure 1—A possible communication system using waveguide.

Mode conversion–reconversion is a new phenomenon, peculiar to waveguide transmission systems. It arises because a typical waveguide suitable for long-distance transmission can support a large number of modes in addition to the desired one ($H_{01}$), and because of irregularities in the waveguide (small dents, changes in diameter and cross-sectional shape, small bends and kinks, et cetera). In consequence, at each minute irregularity, the preferred transmission mode ($H_{01}$) becomes scattered and partially transformed into parasitic modes. The process is known as mode conversion and leads to a gradual diffusion of energy into the parasitic modes and, therefore, increased attenuation. With some exceptions, this process of mode conversion is actually not particularly harmful. It cannot, however, exist on its own, but, by reciprocity, it is accompanied by the complementary process of mode reconversion. In this process, the energy now partially carried in the unwanted modes becomes reconverted (by being scattered by subsequent irregularities) back into the $H_{01}$ mode. But, since the group velocities of the parasitic modes differ from that of the $H_{01}$ mode, the reconversion is not in phase with the signal carried in the $H_{01}$ mode. Further, since the degree of distortion is phase dependent and the phase of the reconverted signal is related to the distance between the conversion and reconversion points in terms of the wavelength, the whole process of mode conversion–reconversion is frequency sensitive. The overall frequency–attenuation curve is, therefore, very irregular, leading to considerable signal distortion. Because of the nature of mode conversion–reconversion phenomena, it is inconceivable that suitable terminal equipment could be constructed to counteract its effects. Mode conversion–reconversion effects are minimized by suitable waveguide design.

There is one particular aspect of mode conversion–reconversion that calls for special attention: this arises through bending of the waveguide. The circular waveguide carrying the $H_{01}$ mode can support simultaneously the $E_{11}$ mode, having identical velocity and termed a degenerate mode. In a straight waveguide, this is of no direct consequence, but if the waveguide is bent in an arc of a circle, the two modes become coupled with subsequent exchange of energy between them. This leads to large attenuation and signal distortion. Once again, the effect is counteracted by judicious waveguide design.

It is possible to design a whole series of components[1,5] such as specially tailored bends and corners, tapers, mode transducers, et cetera, but the design is not as straightforward as for components for conventional single-mode rectangular waveguide. The waveguide can support a large number of modes and, therefore, any changes of the waveguide geometry must be made with great care, bearing in mind mode conversion–reconversion phenomena. This requires a detailed study of wave transmission in multimode waveguides and the coupling effects due to irregularities; a field relatively unexploited in its analytical and experimental aspects.

Despite all the planning difficulties, a properly designed waveguide will give satisfactory service and will handle successfully a bandwidth well in excess of 20 gigacycles per second, which is equivalent to an information bandwidth of about 1000 megacycles per second, a capacity not offered by any other existing communication medium.

## 3. Delay Distortion in Uniform Waveguides

As is well known with waveguides, both $\alpha(\omega)$ and $\beta(\omega)$ are functions of frequency. It can be shown,[6] however, that the effect of $\alpha(\omega)$ on signal distortion is negligible in comparison with $\beta(\omega)$. Here, therefore, only the effect of the $\beta(\omega)$ characteristic will be examined.

The phase propagation coefficient $\beta(\omega)$ is given by

$$\beta = (\omega^2 - \omega_c^2)^{1/2}/c \qquad (1)$$

where $c$ is the velocity of light and $\omega_c$ is the cutoff frequency of the waveguide, which is a geometrical parameter. These relations are illustrated in Figure 2.

The phase $v_{ph}$ and group velocities $v_g$ are defined by

$$\frac{v_{ph}}{c} = \frac{\omega_o}{(\omega_o^2 - \omega_c^2)^{1/2}} = \frac{c}{v_g}. \tag{2}$$

These quantities are frequency dependent.

If the signal is composed of several frequencies, then, naturally, the signal will be distorted because the high-frequency components will travel faster then the low-frequency ones. The differential phase delay between the extreme components of the modulating envelope is a measure of signal distortion: it is the delay-distortion term. For a waveguide operated at a frequency substantially above cutoff the bandwidth can be expressed[6]

$$f = (19/F)(f_o/l)^{1/2}(\delta\phi)^{1/2} \tag{3}$$

where

$f$ = bandwidth in megacycles per second
$F$ = (cutoff frequency)/(carrier frequency)
$l$ = length of waveguide in kilometres
$f_o$ = carrier frequency in gigacycles per second
$\delta\phi$ = permissible phase delay between the carrier and the sideband in radians.

It has been assumed that $f$ is small in proportion to carrier frequency.

For example, for a typical rectangular waveguide, $F = 0{\cdot}6$, and therefore at the carrier frequency of 10 gigacycles per second and a length of 18·6 miles (30 kilometres), we get $f = 9{\cdot}4$ (1 radian)$^{1/2}$ megacycles per second.

The permissible amount of delay distortion $\delta\phi$ is a function of the modulation method. Clearly, with amplitude modulation if $\delta\phi = \pi/2$, then inversion of sidebands takes place: a condition of severe distortion. For amplitude-, frequency-, and many other modulation methods, the maximum possible value of $\delta\phi$ is only a small fraction of 1 radian.

For pulse-code modulation, however, $\delta\phi$ can take quite-substantial values; once again, therefore, pulse-code modulation is superior to the more-common modulation methods.

The modulation methods for wideband application must be analyzed individually. For example, with pulse transmission systems, the energy contained in a pulse of finite duration gradually becomes spread out on the time scale. For a given waveguide length, the degree of pulse distortion is a function of pulse duration. But up to a certain minimum pulse duration, the pulse is quite recognizable and most of the energy is contained in the time interval equal to the duration of the transmitted pulse. The pulse distortion for pulses of shorter duration becomes quite severe and increases as pulse duration decreases[6]; the energy also becomes diffused over a longer and longer time interval. The actual pulse width for which this happens is related to the geometry of the waveguide cross section and waveguide lengths (Figure 3).

Typical waveguides for long-distance transmission have $F = 0{\cdot}15$, then at a frequency of 35 gigacycles per second, pulses as short as 10 nanoseconds can be transmitted without significant distortion over a distance of some

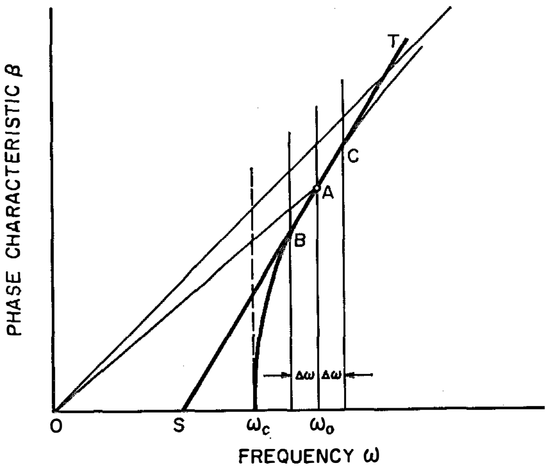



Figure 2—$\beta(\omega)$ characteristic of a perfect waveguide.

18·6 miles (30 kilometres). Such performance is sufficient for many applications, but if it were required to transmit even shorter pulses, some equalization of the waveguide characteristic would be necessary. Such equalization is best carried out at the terminal equipment.

## 4. Properties of Uniform Waveguides and Their Uses

### 4.1 Simple Waveguides

The reasons for choosing the circular waveguide operated in the circular electric mode $H_{01}$ for long-distance transmission have been explained in section 2. Such waveguides can support, beside the desired $H_{01}$ mode, a multitude of other modes all of which are undesirable. The total number of modes that can possibly propagate in a waveguide with diameter $D$ is known as the modulus of overmoding, $M$. This is approximately given by

$$M = 2{\cdot}55(D/\lambda_0)^2. \quad (4)$$

The larger the modulus of overmoding, the larger the number of possible modes and, consequently, the greater the engineering difficulties in preventing mode conversion–reconversion phenomena. Yet, a large ratio $D/\lambda_0$ is necessary to have low attenuation and dispersion. Of necessity, therefore, the proportions of a waveguide are a compromise. Typically, $D/\lambda_0$ is of the order of 10 or more and the modulus of overmoding is, therefore, at least about 200 to 400.

Each mode propagates with a characteristic group velocity given by

$$v_g/c = c/v_{ph} = (1 - F^2)^{1/2}. \quad (5)$$

The commonest waveguide is a plain metallic pipe where the wall of the waveguide presents a surface resistance $R_s$ given[7] by

$$R_s = \frac{(\pi f \mu/\sigma)^{1/2}}{(\mu_o/\epsilon_o)^{1/2}} = R_m \quad (6)$$

where $\sigma$ is the conductivity of the metal, $\mu$ its permittivity, and $(\mu_o/\epsilon_o)^{1/2} = 377$ ohms = the free-space impedance. This surface resistance gives rise to attenuation which for $E_{mn}$ modes is given[7] by

$$\alpha_{mn}^{(e)} = \frac{8{\cdot}686}{D/2} \times \frac{R_s}{(1 - F^2)^{1/2}} \quad (7)$$

and for $H_{mn}$ modes by

$$\alpha_{mn}^{(h)} = \frac{8{\cdot}686}{D/2} \times \frac{F^2}{(1 - F^2)^{1/2}} \times \left[1 + \frac{1}{F^2}\frac{(m/\chi_{mn}^{(h)})^2}{1 - (m/\chi_{mn}^{(h)})^2}\right] R_s. \quad (8)$$

These attenuations are in decibels per meter if $D$ = diameter of the waveguide in meters.

The attenuation of the $H_{01}$ mode (the wanted mode) is[7]

$$\alpha_0 = \frac{8{\cdot}686}{D/2} \times \frac{F^2}{(1 - F^2)^{1/2}} R_m \quad (9)$$

Thus for a copper waveguide at the free-space wavelength of 8·7 millimetres, it is 1·2 decibels/mile (0·75 decibel/kilometre).

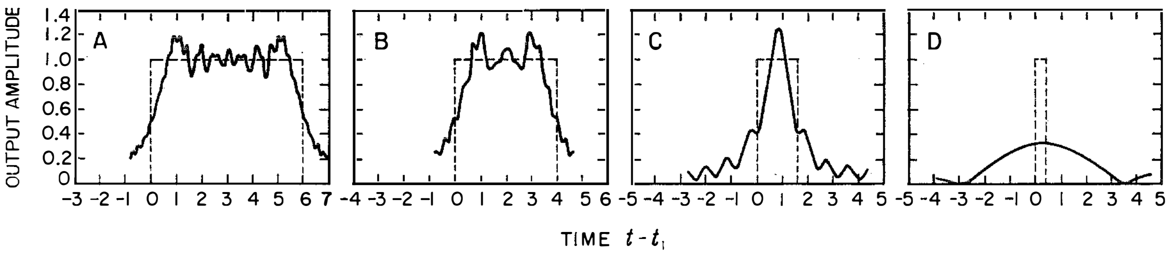



Figure 3—Pulse distortion in waveguides exhibiting increasing dispersion in the order $A$ to $D$.

Most of the unwanted modes have attenuation and group velocities differing substantially from the $H_{01}$ mode. Thus attenuations of 10 to 50 times that of the $H_{01}$ mode are common and group velocities of most modes differ from the $H_{01}$ mode by as much as 10 per cent or more. A few of the modes, however, have properties similar to those of the $H_{01}$ mode. For example, the $E_{11}$ mode has group velocity almost equal to that of the $H_{01}$ mode and a few of the modes, notably the $H_{02}$, $H_{03}$, $H_{12}$, $H_{13}$, $H_{22}$, and $H_{23}$, have attenuations only 3 to 10 times as much as the $H_{01}$ mode. These modes, therefore, tend to be particularly troublesome and the only way to deal with them satisfactorily is through judicious use of special waveguides.

### 4.2 Special Waveguides

Waveguides of special construction are necessary, as explained in the earlier sections, principally to combat or at least to minimize the mode conversion–reconversion effects. For such applications, disk waveguides were first to be suggested and various elliptic, corrugated, and more-complex structures were tried for negotiating bends. It is now clear after many years of research that only helical and dielectric-coated waveguides are likely to be of any extensive use. Such waveguides behave as if their surface impedance were anisotropic.

The theory of propagation in anisotropic waveguides can be quite involved, but since waveguides for long-distance transmission have, in general, small surface impedance, an approximate treatment[8,9] is adequate, and this will be given here.

The propagation coefficient in such waveguides is

$$\gamma = \gamma_o + \delta\gamma = j\beta_o + (\alpha + j\delta\beta) = \alpha + j\beta \quad (10)$$

where the quantity $\delta\gamma (= \alpha + j\delta\beta)$ is composed of the attenuation coefficient $\alpha$ and a quantity $\delta\beta$ modifying the phase-change coefficient $\beta_o$ of the waveguide with zero surface impedance. The coefficient $\alpha$ is proportional to surface resistance $R_s$ and $\delta\beta$ to surface reactance $X_s$, these being respectively the real and imaginary parts of surface impedance $Z_s$. With anisotropic waveguides the quantity $Z_s$ may be described in terms of the matrix of its components; most simply in terms of its two principal components $Z_\eta$ and $Z_\xi$ which are surface impedance components along the two principal axes. The sum of the projection of these components along the $\phi$ and $z$ directions define the circumferential $Z_\phi$ and axial $Z_z$ components of the surface impedance; it is these components that are used in the subsequent equations. $Z_\phi$ can be regarded as the surface impedance presented to the currents flowing in the waveguide wall in the circumferential direction and $Z_z$ a corresponding quantity for currents flowing in the axial direction.

In such waveguides the attenuation of $E$-waves is given[9] by

$$\alpha^{(e)} = 2R_z/D\,(1 - F^2)^{1/2} \quad (11)$$

nepers and for $H$-waves by

$$\alpha_{mn}^{(h)} = \frac{2}{D}\frac{F^2}{(1 - F^2)^{1/2}} \times \left[R_\phi + R_z \frac{1 - F^2}{F^4}\left(\frac{m}{\pi}\frac{\lambda_o}{D}\right)\right] \times \left[1 - \left(\frac{m}{\pi}\frac{\lambda_o}{D}\right)^2\right]^{-1} \quad (12)$$

nepers, where the meaning of symbols is as before.

In the above equations if $X_z$, $X_\phi$ are substituted for $R_z$ and $R_\phi$ respectively, then $\delta\beta$ will be obtained.

### 4.3 Dielectric-Coated Waveguides

A metallic surface exhibits a surface impedance given[8] by

$$Z_m = (1 + j)\,R_m \quad (13)$$

where $R_m$ is given by (6). The surface of a copper tube ($\sigma = 5{\cdot}8 \times 10^7$ mho/meter), therefore, presents at $\lambda_o = 8{\cdot}7$ millimetres a surface impedance of $(1 + j) \times 1{\cdot}28 \times 10^{-4}$, relative to free space (about 0·048 ohm).

If, now, the metallic tube is coated with a thin layer of dielectric, then its surface impedance will be enhanced as given in Table 2,

TABLE 2

SURFACE IMPEDANCE OF DIELECTRIC-COATED METAL SURFACE*

| Surface Impedance | Real Part | Imaginary Part |
|---|---|---|
| $Z_z = R_z + jX_z$ | $R_z = tk_o(\tan\delta/\epsilon_r)$ | $X_z = tk_o(1 - 1/\epsilon_r)$ |
| $Z_\phi = R_\phi + jX_\phi$ | $R_\phi = \frac{1}{3}(tk_o)^3(\epsilon_r \tan\delta)$ | $X_\phi = \frac{1}{3}(tk_o)^3(\epsilon_r - 1)$ |

* These values must be augmented by $Z_m = R_m + jX_m$ ($R_m = X_m = (\pi f\mu/\sigma)^{1/2}(\epsilon_o/\mu_o)^{1/2}$), the surface impedance of the metal.

where $t$ = thickness of dielectric layer, $k_o = 2\pi/\lambda_o$, $\epsilon_r$ = relative permittivity, and $\delta$ = loss angle of dielectric. Clearly, by choosing suitable materials and the thickness of the dielectric layer, it is possible to obtain within wide limits any required surface impedance components and, consequently, the attenuation and phase velocity of the $E$ and $H$ modes can be varied relative to the $H_{0n}$ modes. By way of illustration, at the free-space wavelength of 8·7 millimetres, for $\epsilon_r = 2$, $\tan\delta = 0{\cdot}15$, and $t = 0{\cdot}046$ millimetres, the surface impedance components become $Z_z = 2{\cdot}5 \times 10^{-3} + j\,1{\cdot}7 \times 10^{-2}$ and $Z_\phi = 3{\cdot}7 \times 10^{-6} + j\,3{\cdot}3 \times 10^{-4}$ over and above the value of $Z_m$. If we substitute these values in (11) and (12), we find that for a waveguide of 7-centimetre diameter, there is a 20-fold increase in attenuation of $E$ and $H$ modes (other than $H_{0n}$) relative to the $H_{01}$ at the expense of only a 3-per-cent increase in attenuation of the $H_{01}$ mode. The surface reactance $X_z$ is, at the same time, increased by a factor of 140, leading to a substantial change in phase velocity for most modes.

The increase in relative attenuation is necessary to cope with mode conversion–reconversion, due to irregularities in the waveguide, while the increase in the surface reactance $X_z$ is necessary for bends.

### 4.4 Ring or Disk Structures

Any periodic array of coaxial disks or rings of different surface impedances falls into this class. A simple approximate treatment is possible if the pitch of such a structure is small in comparison with the wavelength, as follows:

Let $Z_1$ and $Z_2$ be the surface impedance of the individual elements, thickness $t_1$ and $t_2$ respectively, then the anisotropic components of effective surface impedance are given by

$$\left.\begin{aligned} Z_z &= \frac{Z_1 t_1 + Z_2 t_2}{t_1 + t_2} \\ Z_\phi &= \frac{Z_1 Z_2 (t_1 + t_2)}{Z_2 t_1 + Z_1 t_2}. \end{aligned}\right\} \quad (14)$$

Since $|Z_2| > |Z_1|$ and $t_2 \ll t_1$ in most applications, these expressions can be further simplified to

$$\left.\begin{aligned} Z_z &= Z_2 t_1/(t_1 + t_2) \\ Z_\phi &= Z_1(t_1 + t_2)/t_1. \end{aligned}\right\} \quad (15)$$

Thus given $Z_1$, $Z_2$, $t_1$, and $t_2$, the surface impedance components $Z_z$ and $Z_\phi$ can be calculated. Attenuation and phase propagation coefficients can be obtained from (11) and (12).

The quantities $Z_1$ and $Z_2$ are determined from the knowledge of the actual physical structure. This can be a lengthy calculation, but since in practical applications only an estimate of the value of the surface impedance is needed, it is permissible to make a number of rather-drastic approximations. For example, for a corrugated waveguide (Figure 4), $Z_1$ is the copper intrinsic impedance and $Z_2$ is the input impedance to a parallel-plane transmission line, length $l$, and separation of plates $t_2$. Thus

$$\left.\begin{aligned} Z_z &= R_m[1 + l/(t_1 + t_2)] \\ &\qquad + j2\pi \frac{l}{\lambda_0} t_2/(t_1 + t_2) \\ Z_\phi &= Z_m(t_1 + t_2)/t_1 \end{aligned}\right\} \quad (16)$$

where $Z_m$ is the surface impedance of the metal; see (13).

It will be observed that $Z_\phi$ has increased a little, in proportion of $(t_1 + t_2)/t_1$, but that $Z_z$ is highly reactive. Such waveguides are particularly suitable for negotiating bends.

Clearly, other structures can be calculated in an analogous manner.

### 4.5 Helical Waveguides

It will be observed from (15) that $Z_z$ and $Z_\phi$ can be adjusted within very-wide limits to any desirable values provided that the proportions of the microstructure of the waveguide surface are suitably chosen. This feature makes the disk waveguides, and their variants, extremely attractive. But, unfortunately, such waveguides are difficult if not impossible to manufacture in length. It is for this reason that helical waveguides are preferred.

It can be shown that if the pitch of the helical waveguide is very small, as with waveguides wound with a fine wire, and the surface impedance is not too large (say less than $10^{-1}$, that is, some 40 ohms), then the helical waveguide behaves as a corresponding ring structure.

A particularly important structure is shown in Figure 5; this shows a helix of insulated copper wire set in a suitable resin and backed by a resistive layer of surface impedance $R$, the whole structure enclosed in a suitable protective jacket. If the wire diameter is $t_1$ and the gap between the wires $t_2$, then provided that $t_2$ is not too small, the structure behaves approximately as a corresponding disk structure (Figure 6) with $l = t_1$.

Using the procedure outlined before, (15) can be used where $Z_1 = Z_m$ = copper intrinsic impedance and $Z_2$ = input impedance of a transmission line length $l$ terminated in a resistive impedance $R$. Clearly, therefore,

$$\left.\begin{aligned} Z_z &= R\frac{t_2}{t_1+t_2} + j2\pi\frac{l}{\lambda_0}\frac{t_2}{t_1+t_2} \\ Z_\phi &= Z_m\frac{t_1+t_2}{t_1}. \end{aligned}\right\} \quad (17)$$

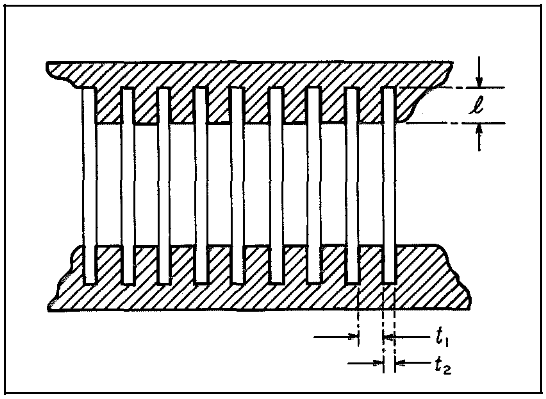



Figure 4—Corrugated circular waveguide.

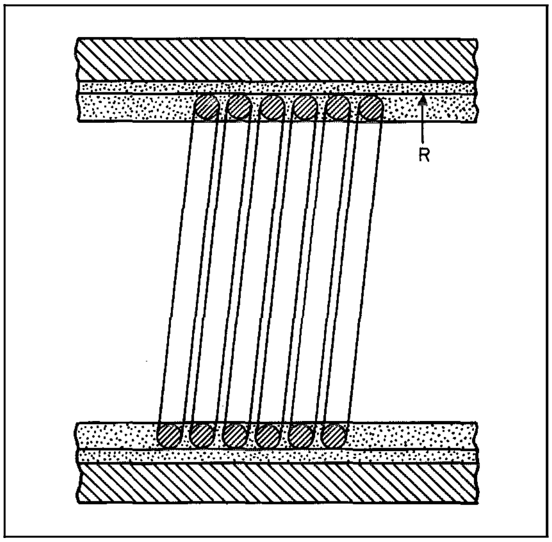



Figure 5—Helical waveguide.

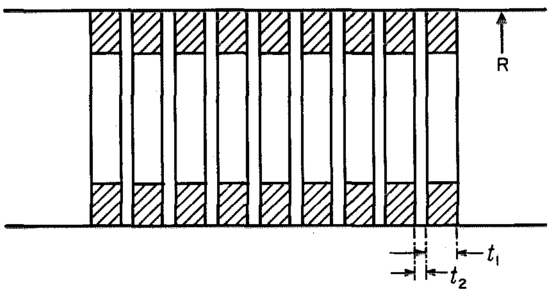



Figure 6—Ring structure with resistive layer.

As a numerical example, consider a helical waveguide with following constants: $t_1 = l = \lambda_0/10$, $t_2 = t_1/2$, $R_m = 10^{-4}$ (approximately copper impedance), $R = 10^{-2}$ (approximately 4 ohms) then, neglecting the lay angle of the helix, at a free-space wavelength of 8·7 millimetres the surface impedance components become

$$Z_z = 0{\cdot}003 + j0{\cdot}021$$
$$Z_\phi = 1{\cdot}5(1 + j) \times 10^{-4}.$$

The propagation coefficients follow from (7) and (8). Thus for a waveguide 7 centimetres in diameter there is a 50-per-cent increase in the attenuation of the $H_{01}$ mode, and a 30-fold increase in the attenuation of most $E$-modes. The attenuation of the lower-order $H$-modes has also been increased by a factor of 30, while the surface reactance is increased 200 times.

## 5. Basic Theory of Nonuniform Waveguides

Theory of nonuniform waveguides is fundamental to thorough understanding of transmission aspects as well as design of many waveguide components peculiar to long-distance transmission by waveguide. Thus, commonly, waves are conveyed from an oscillator *via* a single-mode rectangular waveguide, through a transducer to a circular waveguide, then through a taper to the main waveguide run.

We shall see that a mode transducer, a bent waveguide, as well as a taper are particular embodiments of nonuniform transmission lines. The main waveguide run is also a nonuniform transmission line in the sense that it can carry a large number of modes and all minute irregularities act as coupling elements between the lines; each line representing one particular mode.

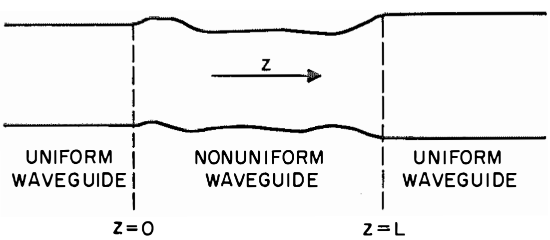



Figure 7—Schematic of a nonuniform waveguide.

In general, we represent a nonuniform waveguide by its equivalent circuit: $n$ coupled transmission lines, where $n$ is the number of possible modes of propagation (see Figure 7).

For convenience in analysis, nonuniform waveguides can be subdivided into (A) straight waveguides, and (B) bent waveguides.

### 5.1 Straight Waveguides

The first general solution to transmission in nonuniform waveguides was obtained[10] by Stevenson in terms of a series of transverse wave functions. Through such an approach it is possible to obtain[11] equivalent generalized telegraphist's equations of the waveguide. Adopting Reiter's approach,[12,13] it can be shown[17] that for the forward and backward traveling waves the following equations hold

$$\left.\begin{aligned} \frac{dA_i^+}{dz} &= -\,j\beta_i A_i^+ - \frac{1}{2}\frac{d(\ln K_i)}{dz}A_i^- \\ &\quad + \sum_p \left(S_{ip}^+ A_p^+ + S_{ip}^- A_p^-\right) \\ \frac{dA_i^-}{dz} &= -\frac{1}{2}\frac{d(\ln K_i)}{dz}A_i^+ + j\beta_i A_i^- \\ &\quad + \sum_p \left(S_{ip}^- A_p^+ + S_{ip}^+ A_p^-\right) \end{aligned}\right\} \qquad (18)$$

where

$z$ = coordinate along waveguide axis
$A_i^\pm$ = amplitude coefficients of forward ( + ) and backward ( − ) traveling waves on $i$th line (corresponding to $i$th mode)
$S_{ip}^\pm$ = corresponding coupling coefficients between the waves on lines $i$ and $p$ (a parameter related to geometry of waveguide)
$K_i$ = wave impedance of $i$th line
$\beta_i$ = propagation coefficient.

The above are exact equations for a set of lossless nonuniform transmission lines where $S$, $K$, and $\beta$ are, in general, functions of $z$. These equations are an exact representation of transmission in a nonuniform waveguide. In principle, therefore, given the functional dependence of the waveguide geometry on the coordinate $z$, the propagation problem can be solved, but the computational difficulties are in most cases formidable. With practical waveguides, however, it is possible to proceed with a number of simplifying assumptions leading in many cases to rather simple solutions.[14–17]

In the first place, we assume that the waveguide geometry changes slowly and consequently $S$, $K$, and $\beta$ are slowly varying functions of $z$. The coupling between the lines per unit length will therefore be small. If the input at one end of the waveguide is a pure mode, amplitude $A_o$, then the boundary conditions are simply

$$\left.\begin{array}{ll} A_m^+(0) = A_o & A_m^-(L) = 0 \\ A_i^+(0) = 0 & A_i^-(L) = 0 \quad (i \neq m). \end{array}\right\} \quad (19)$$

With these assumptions only the coupling between line $m$ and the remaining lines need be considered; the cross-coupling between the lines can be neglected and so can the reflected waves. Then a simplified solution[17] is

$$A_m^+(z) = A_o \exp\left(-j\int_0^z \beta_m dz\right). \quad (20)$$

The amplitude of the remaining waves are

$$\left.\begin{array}{l} A_m^-(0) = -\exp\left(-j\int_0^L \beta_m dz\right)\int_0^L \left(S_{mm}^- - \dfrac{1}{2}\dfrac{d(\ln K_m)}{dz}\right)\exp\left(-j2\int_0^Z \beta_m dz\right)dz \\ \begin{array}{c} A_i^+(L) \\ A_i^-(0) \end{array} = \pm\exp\left(-j\int_0^L \beta_i dz\right)\int_0^L S_{im}^{\pm}\exp\left(-j\int_0^z (\beta_m \mp \beta_i)dz\right)dz. \end{array}\right\} \quad (21)$$

In the derivation of these expressions it has been assumed that none of the modes are at cutoff. When dealing with components such as tapers, for example, some modes are of necessity at the cutoff at some cross-section along the taper, and it would appear that (20) and (21) are not a sound basis for design. Nevertheless, it appears[18] that the use of (20) and (21) is usually justified.

It is to be observed that usually it is only the lower-order modes that need be at all considered (typically $H_{02}$, $H_{03}$, $H_{12}$, $H_{13}$, $H_{22}$, $H_{23}$; compare section 4.1.).

The $\beta$ coefficients of these modes differ but very little from that of the $H_{01}$ and consequently [from (21)] only the coefficients of the forward traveling waves $A^+$ need be considered; the coefficients $A^-$ are very-much smaller and can, therefore, be neglected.

An interesting proposition can be proven using (21). In general, a pure mode at the input to a waveguide will produce at the output a combination of modes. For a good transducer the output consists of an almost pure mode of the desired kind. The purity of this mode can now be made as high as desired merely by increasing the length of the transducer: the longer and more gradual the transducer the purer is the mode at the output.

### 5.2 Propagation in Bent Waveguides

The general coupled-transmission-line equations for a bent waveguide of arbitrary cross-section were given[19] by Shimizu from which the relations for a circular waveguide follow. The particular case of a bent circular waveguide was analyzed earlier[20] by Jouguet, who described the field in a curved waveguide by perturbation calculus and obtained the field in terms of an expansion in powers of $1/R$, $R$ being radius of curvature of waveguide.

It can be shown that if a curved section of waveguide is interposed between two straight waveguides (Figure 8) and a pure $H_{01}$ mode is incident at one end of the curved waveguide, then the wave appearing at the other end is composed of the $H_{01}$, $E_{11}$, and a series of $H_{1n}$ modes,[19–21] but the exact proportion of each of the modes depends on the geometry of the bend. The equivalent circuit of the bend is therefore $n + 2$ coupled transmission lines: one for the $H_{01}$, one for the $E_{11}$, and $n$ lines one for each of the $H_{1n}$ modes. The coefficients of the coupling matrix are constants related to the geometry of the bend. But it can be shown, further, that if the radius of curvature is large, the series of $H_{1n}$ modes may be neglected, since these are of $1/R^2$ or higher order, and therefore only the $H_{01}$ and $E_{11}$ modes need be considered. The equivalent circuit is then, simply, two coupled transmission lines.

Jouguet's analysis can be further extended[2] to waveguides of arbitrary surface impedance. It transpires that, provided the radius of bend is not too small, the field inside a curved waveguide can be represented by

$$\Psi = A(\mathcal{H}_{01} + \tau \mathcal{E}_{11})e^{-\gamma l} \tag{22}$$

where $\mathcal{H}_{01}$ = field of $H_{01}$ mode, and $\mathcal{E}_{11}$ = field of $E_{11}$ mode. That is, the field is a linear combination of the $H_{01}$ and $E_{11}$ modes in a definite proportion. The proportionality factor can take two possible values given by

$$\tau^2 - 2p\tau - 2 = 0 \tag{23}$$

where

$$p = \frac{h_0\beta_0}{k_0^2}\frac{R}{s}\, jZ_s. \tag{24}$$

If the surface impedance and the radius of curvature are small, $p$ is small and $\tau$ is approximately $(2)^{1/2}$; therefore, the expression for the field[2] can be put into the form

$$\Psi = \mathcal{H}_{01}\cos\delta\beta_R l + j(2)^{1/2}\mathcal{E}_{11}\sin\delta\beta_R l \exp(-\gamma_o l) \tag{25}$$

where

$$\left.\begin{aligned} \delta\beta_R &= k_0/h_0R(2)^{1/2} \\ &\text{and} \\ \gamma_1 &= j(\beta_0 \pm \delta\beta_R) + (\alpha_E + \alpha_H)/2 \end{aligned}\right\} \tag{26}$$

with $\beta_0$ the phase propagation coefficient of the straight waveguide and $\alpha_E$ and $\alpha_H$ the attenuation coefficients of the $E_{11}$ and $H_{01}$ modes in the straight waveguide.

Two important conclusions must be made: (A) The energy is carried alternately in the $H_{01}$ and $E_{11}$ modes, and (B) the mean attenuation of the complete normal mode (25) is equal to the mean of those for the $E_{11}$ and $H_{01}$ modes.

Evidently, the waveguide is unsuitable for the transmission of the $H_{01}$ mode around bends. However, the examination of the above equations shows that a different situation arises if $p$ is a large real number. Large real values of $p$ are obtained for highly reactive surfaces ($X_s$ a large number), and in such waveguides the $H_{01}$ and $E_{11}$ modes propagate almost independently of each other, but the attenuation of the $H_{01}$ mode is slightly increased[2]:

$$\alpha_1 = \alpha_0 + \alpha_R \tag{27}$$

where $\alpha_0$ is the attenuation in the straight waveguide and is given [compare (12)] by

$$\alpha_0 = \frac{2}{D}\frac{F^2}{(1 - F^2)^{1/2}} R_\phi. \tag{28}$$

The additional attenuation due to the bend is given by

$$\alpha_R = \frac{D}{4R^2}\frac{R_s/|Z_s|^2}{F^2(1 - F^2)^{1/2}}. \tag{29}$$

Clearly, it is possible by choosing a sufficiently large surface reactance to bring this increase in attenuation below any predetermined level. Such values of surface reactance can conveniently be obtained either by coating the waveguide surface with a layer of dielectric or by corrugating the waveguide surface in a suitable manner.

The ratio $\alpha_R/\alpha_0 = \nu_R(R_z/R_\phi)$ defines a quality factor of the bent waveguide. With the help of Figure 9 it is possible to determine the required value of surface reactance for a given value of radius of curvature and quality factor $\nu_R$. Clearly, helical waveguides of appropriate proportions (section 4) can be designed for the purpose of negotiating bends.

With suitably designed waveguides, bends of even a few hundred feet in radius can be negotiated quite successfully. A further improvement can be achieved by increasing gradually the radius of bend by means of a suitable transition curve[22] (a "tapered transition"). This is yet another embodiment of a set of non-uniform coupled transmission lines.

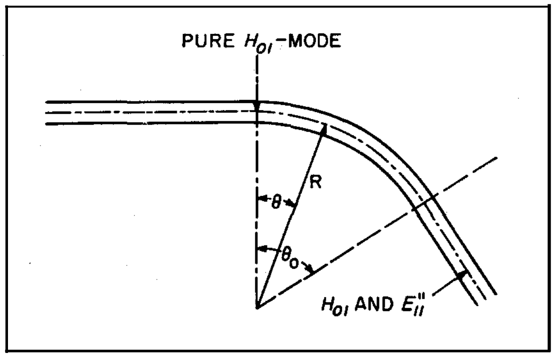



Figure 8—Curved circular waveguide.

## 6. Transducers

A transducer is a waveguide configuration which achieves a required change of mode pattern and/or pattern size.

### 6.1 Tapers

A taper is the simplest illustration: it achieves a change in waveguide diameter and, if properly designed, leaves the mode pattern unaltered yet changed in size.

The rigorous theory[23,24] of tapers can be very involved indeed but for our applications a number of approximations are permissible, resulting in an adequate and simple design procedure.

Consider two coaxial waveguides joined by a conical taper and suppose that the larger waveguide (radius $a$) is operated substantially above cutoff (overmoded waveguide) and the smaller waveguide (radius $b$) carries a pure $H_{01}$ mode. The taper will, evidently, give rise to a series of unwanted modes besides the $H_{01}$ mode, but because of the symmetry of the structure only the $H_{0n}$ modes will be present. If, now, the diameter of the smaller waveguide is sufficiently small to permit only the $H_{01}$ mode to propagate, the amplitudes of the $H_{0n}$ modes in the larger waveguide will

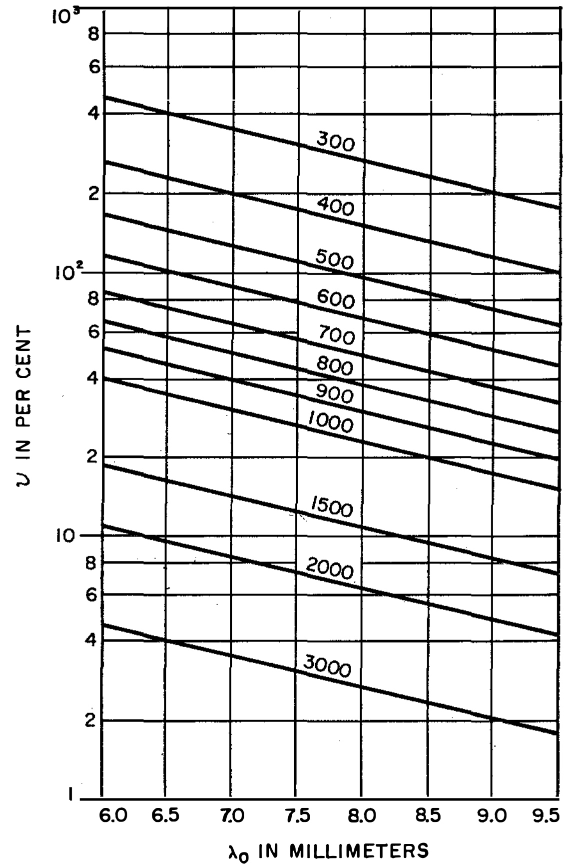



Figure 9—Quality factor $\nu$ of a bent waveguide of 7-centimeter diameter as a function of wavelength. $|Z_s|R$ in ohm-meters is the parameter indicated by the figures on the curves.

be a simple function of parameter $\sigma$ given by

$$\sigma = \pi \frac{a}{\lambda} \frac{a-b}{L} \tag{30}$$

where $L$ is the length of the taper and $\lambda$ the free-space wavelength.

These relations are obtained from simple analytical considerations[25] by expanding the fields in the circular waveguides in cylindrical harmonics and the field in the conical region in spherical harmonics and equating the fields in the cross-sections radius $a$ and $b$ respectively.

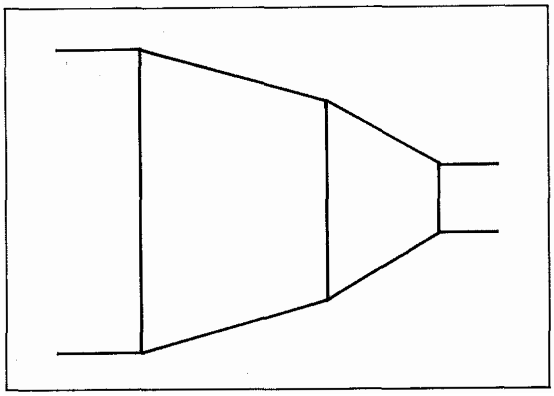


Figure 10—Representation of a two-section conical taper.

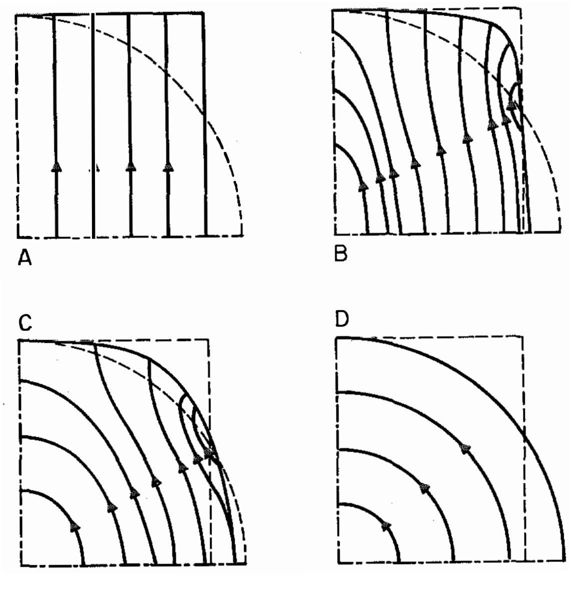



Figure 11—Gradual mode transformation from the $H_{01}$ circular to $H_{02}$ rectangular (only one quadrant shown).

It can be shown that the $H_{02}$ mode is largest in amplitude among the modes produced by the taper. Considering, therefore, the conversion into the $H_{02}$ mode only we get for the taper length

$$L \approx \frac{0 \cdot 57}{p^{1/2}} \frac{a}{\lambda} (a-b) \tag{31}$$

where $p$ is power conversion coefficient, which is the ratio of the permitted power in the $H_{02}$ mode to that in the $H_{01}$ mode.

The length of such a taper may be excessive for some applications, but a further improvement is possible by using a two-section conical taper (Figure 10). For such a taper[26] the performance of the taper can be considerably improved by choosing correctly the proportions of the two cones. Optimum performance is possible at only one frequency, but by correct design it is possible to keep the sum of the energy carried in the $H_{02}$ and $H_{03}$ modes below any required level over a substantial bandwidth.[26]

More-elaborate design procedures have been devised by several investigators,[18,24,27] but it appears[28] from results on antenna theory that provided the taper contour is "smooth and gradual" and that the derivatives at the taper ends are zero, there will be very-little mode conversion even over a wide bandwidth.

### 6.2 Mode Transducers

There is no general design procedure for mode transducers with the exception of a few isolated instances. It is possible in a few simple cases to predict from the changes in the waveguide geometry the type of mode produced and with a little engineering intuition to design sometimes a reasonably satisfactory transducer. In most cases, however, even the intuitive approach is of no avail. Recently a satisfactory design procedure has been proposed[29] and confirmed experimentally. This is explained below.

Consider a mode transducer connecting two uniform waveguides $A$ and $B$. According to an established theorem, if the change from waveguide $A$ to waveguide $B$ is effected very gradually, then a pure mode in waveguide $A$ will produce a substantially pure mode in waveguide $B$: the purity of the mode in waveguide $B$ will be the higher the longer the transducer.

To determine the modes present at the ends of the transducer, it is necessary to choose several cross-sections along its length and to determine the eigenfunctions in each. If the number of cross-sections chosen is sufficient, then from the similarity between the successive field patterns of field distribution it is possible in detail to examine the passage of any given mode along the whole length of the transducer.

To design a mode transducer, the desired modes in the uniform waveguides are given and it is then required to determine the surface of the transducer for reasonable mode purity. We then choose a cross-sectional wave function $\Psi$ (as a continuous function of $z$) such that it becomes the desired mode at each end of the transducer (at $z = 0$ and $z = L$, where $L$ is the length of the transducer). This wave function determines the surface of the transducer; provided that $\Psi$ is a slowly varying function, the modes involved will be substantially pure. The design procedure is essentially numerical or graphical.

As an illustration consider the $H_{01}$ circular to $H_{01}$ rectangular mode transducer. For convenience and better illustration we propose to design the transducer in two parts: (A) $H_{01}$ circular to $H_{02}$ rectangular and $(B)$ $H_{02}$ rectangular to $H_{01}$ rectangular.

For the first transducer a convenient choice for the wave function is

$$\Psi_m = g_1(z) J_o\left(\frac{\pi}{w}(x^2 + y^2)^{1/2}\right) + g_2(z)\cos\frac{\pi}{w}x \quad (32)$$

where, to satisfy the requirements at each end of the transducer, the conditions are

$$\begin{aligned} g_1(0) &= 1 \quad g_1(L_1) = 0 \\ g_2(0) &= 0 \quad g_2(L_1) = 1 \end{aligned} \quad (33)$$

with $L_1$ = length of transducer and $x$, $y$ = Cartesian coordinates in transverse plane.

The diameter $d$ of the circular waveguide and the width $2w$ of the rectangular waveguide are adjusted so that their cutoff numbers are equal $(d/w = 2{\cdot}44)$.

Evidently the function $\Psi_m$ reduces at the ends of the waveguide to the eigenfunctions of the circular and rectangular waveguides respectively. The surface of the transducer for any intermediate values of $z$ is most easily determined graphically as follows. For any required value of $z$, curves of $\Psi_m =$ *constant* are plotted; these represent the lines of electric field. But a permissible boundary must be normal to such lines and, therefore, the cross-sectional contour can be mapped. Figure 11 shows four contours of such a transducer, and since we know that the transducer must be gradual, in most cases four contours are sufficient to determine the surface of the transducer.

Since the wave functions of the $H_{02}$ and $H_{01}$ modes have the same mathematical equation, the obvious choice for the wave function of the second transducer is[30]

$$\Psi_m = \cos(\pi/w)x.$$

The curves $\Psi_m =$ *constant* are simply straight lines and, therefore, the transducer has the shape shown in Figures 12 and 13; the dimension $\delta$ changes slowly from $\delta = 0$ to $\delta = h$.

A further taper is necessary to connect the two transducers since $h$ is not equal to the diameter of the circular waveguide. Thus the complete $H_{01}$-circular-to-$H_{02}$-rectangular transducer consists of the above three parts.

## 7. Slightly Irregular Waveguides

The general problem of propagation in slightly nonuniform waveguides was investigated[31] by Katzenelenbaum and in a number of specific cases[32,33] by Morgan and Iiguchi.

In the case of a waveguide carrying an $H_{01}$ mode at a frequency substantially above cutoff, a quasi-optical approach[34] can be adopted leading to simple derivation of relevant equations. The problem can be stated with reference to Figure 14. The cross-section of waveguide $A$ is circular and that of $B$ differs a little but in an arbitrary manner. By assuming that the field in the aperture is substantially the field as produced by the $H_{01}$ mode in waveguide $A$ according to geometrical optics, simple integral relations can be derived for the intensity of the modes produced in waveguide $B$. There are only $H$-modes produced and the amplitude of the $H_j$ mode is given[34] by

$$A_j = \int_{S_0} \nabla_t \Psi_j \, \nabla_t \Psi_{01} \, dS \qquad (35)$$

where

$S_o$ = aperture
$\nabla_t$ = gradient operator in transverse plane
$\Psi_j$ = wave function of $H_j$ mode in waveguide $B$
$\Psi_{01}$ = wave function of $H_{01}$ mode in waveguide $A$.

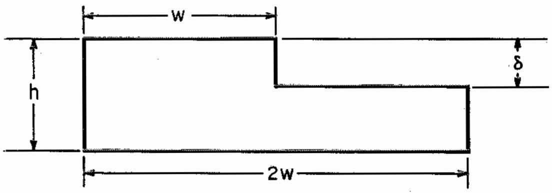



Figure 12
Cross-section of $H_{01}$ to $H_{02}$ rectangular transducer.

Only $H_{1n}$ modes will be generated in the case of a junction formed by two offset circular waveguides of identical diameters if the exciting mode is a pure $H_{01}$ mode. Only $H_{0n}$ modes are produced if there is a small change in waveguide diameter and the waveguides are coaxial. Similar deductions can be made for other discontinuities without actually solving (35).

For the two coaxial waveguides of slightly different diameter (Figure 14), (35) leads to particularly simple relations. In this case the amplitude $A_{0n}$ of the $H_{0n}$ mode produced by the incident $H_{01}$ mode is given by

$$\frac{A_{0n}}{A_{01}} = \frac{2\chi_1\chi_n}{\chi_1^2 - \chi_n^2}\left(\frac{r_2 - r_1}{r_1}\right) \qquad (36)$$

where $\chi_i$ is the $i$th root of $J_1(x) = 0$.

Consider now two such steps in tandem. Clearly the contributions from the conversions must be combined taking the correct phases into account. It can then be shown[35] that if the inserted length of the waveguide is one-half beat wavelength (of $H_{01}$ and $H_{02}$) long and its diameter the mean of the two extreme waveguides, then the mode conversion is minimized. The same procedure can be applied to other $H_{0n}$ modes and a suitable

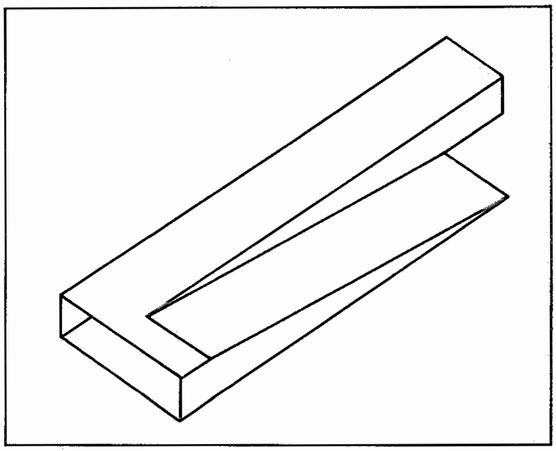


Figure 13—Perspective view of the complete $H_{01}$ to $H_{02}$ rectangular transducer of Figure 12.

transition between two waveguides of different diameters designed.

## 8. Mode Conversion–Reconversion

As mentioned in section 2, mode conversion–reconversion is a phenomenon existing in any over-moded waveguide, such as would be used for long-distance transmission. The level of mode conversion or reconversion separately is a function of waveguide tolerances, but the combined effect of mode conversion–reconversion, which determines the signal distortion, is also a function of waveguide design or, more precisely, it is related to the surface impedance of the waveguide.

For technological reasons one cannot reduce the mode conversion process beyond a certain limit, related to the waveguide tolerances. For given tolerances, however, the mode conversion–reconversion process can be reduced, in principle, below any predetermined level by choosing suitable surface impedance. (The conversion to $H_{0n}$ modes has been neglected, since this is not usually troublesome.) The remaining problem is then a technological one, of realizing such surface impedance. The actual design is a compromise between what is desirable from the mode conversion–reconversion point of view and what is practicable or at all possible to achieve using existing methods.

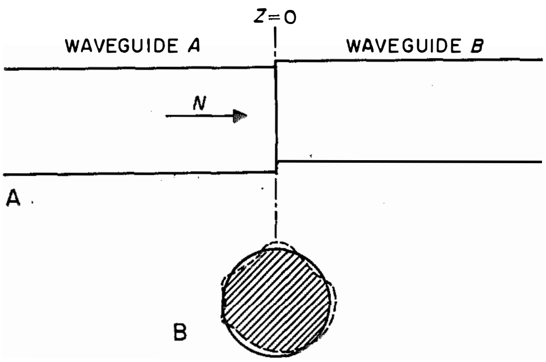



Figure 14—Junction of two waveguides of slightly different cross-section.

Briefly, for good performance on bends, waveguides must have high surface reactance in the axial direction ($X_z$ components) while the circumferential component ($X_\phi$) remains small. For good discrimination against unwanted modes produced by typical small irregularities, large attenuation for unwanted modes and small attenuation for the $H_{01}$ mode is called for: that is large $R_z$ and small $R_\phi$.

Due to waveguide irregularities a large number of modes become coupled together. If the coupling coefficients representing the aggregate effect of waveguide irregularities contain a periodic component, considerable signal distortion takes place[4]: in effect, the $\beta(\omega)$ characteristic becomes broken up by regularly spaced absorption bands. But if the irregularities are spaced in a random manner no such extreme signal distortion will take place; but on the other hand, the $\alpha$ and $\beta$ characteristics become irregular in a random manner with a root-mean-square value proportional to $(c/\Delta\beta)^2$ ($c$ is the value of the coupling coefficient $c$, and $\Delta\beta$ the difference between the phase propagation coefficients of the $H_{01}$ mode and the parasitic mode) leading to crosstalk between the components of the coded signal. In general, the signal distortion due to mode conversion–reconversion is a function of the quantity $(c/\Delta\beta)^2/\Delta\alpha$ where $\Delta\alpha$ is the difference between the attenuations of the parasitic mode and the $H_{01}$ mode. For this reason, the more the group velocities of the parasitic modes differ from that of the $H_{01}$ mode and the greater the attenuation of the parasitic modes, the smaller the signal distortion due to mode conversion–reconversion. As discussed in section 4, special waveguides can bring about such changes and hence their significance.

## 9. Conclusions

Waveguides for long-distance communications already form a widely specialized subject, too wide to be discussed adequately in an article of this size. But we have discussed the main constituents of a waveguide communication system pointing out the necessity of regenerative modulation method, such as pulse-code modulation.

Special waveguides have been discussed in some detail because, in the light of present knowledge, such waveguides—either of the helical type or dielectric coated—are the only types likely to succeed.

Mode conversion phenomena are intimately connected with overmoded waveguides, of the type used for long-distance transmission. Deep understanding of mode conversion processes is essential for successful designs and accordingly a theory has been outlined. The design of such components as tapers, mode transducers, and bends rests on mode conversion theory.

Various sources of signal distortion have been analyzed in some detail, but it has been stressed that if pulse-code modulation is used and regenerative repeaters, as well as special waveguides, a most-advanced system of unequalled capacity will result.

## 10. Acknowledgments

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32. S. P. Morgan, "Mode Conversion Losses in Transmission of Circular Electric Waves Through Slightly Non-Circular Guides," *Journal of Applied Physics*, volume 21, pages 329–338; April, 1950.

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## Use of Statistical Moments for Specifying Noise in Long-Distance Telephone Circuits

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Equation (4), page 199, of this paper, which appeared in volume 36, number 3 of *Electrical Communication,* should read as follows:

$$10 \log B = y^2/r \times 2\cdot 17 \text{ decibels,} \qquad (4)$$

# Exact Solutions for Ordinary Nonlinear Differential Equations

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Application of the elementary transforms, $y(x) = f(x)\,g(x)$ and $y(x) = 1/[f(x)g(x)]$, where $f(x)$ and $g(x)$ are arbitrary functions of $x$, produced two new transforms for a generalized Riccati's nonlinear differential equation of $y(x)$. These new transforms not only linearize a Riccati's equation but also solve it under special conditions where the variable coefficients of the equation are specially related. The transform $y(x) = f(x)/g(x)$ solves a second-order nonlinear differential equation for a nonlinear Langmuir plasma problem. A systematic extension of the Sturm–Liouville differential equation yields a second-order nonlinear differential equation of which the exact solution is analytically represented. There is explained a basic concept common to all these approaches for exact solutions of ordinary nonlinear differential equations.

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Increasing interest in nonlinear phenomena has encouraged mathematicians to solve nonlinear differential equations exactly. The recent advances in analog and digital computers are expanding the use of nonlinear differential equations through numerical analysis. The tremendous aids rendered by these computers to evaluate any expression numerically opened a new era in which almost all of the problems previously unsolved by analytical means can be approximately solved. While the use of computers has been helpful in reducing the time required for routine computation, it must be recognized that the detailed knowledge obtained through the analytical solution is lost. As cited by many authors,[1] the hazard of overemphasizing the usefulness of computers may result in the neglect of analytical solutions as is the case for nonlinear differential equations.

Compared with the vast amount of theorems and lemmas for linear differential equations, little has been done for the nonlinear differential equation[2,3] despite the fact that there are many nonlinear systems. Approximate solutions for these equations can be obtained by solving "accompanying" linear differential equations.[4] These approximate solutions are useful, but the exact solutions are much more valuable. With computers numerical solutions for any nonlinear differential equation can be obtained, but these answers contain an unknown amount of accumulated truncation error and round-off error inherent to calculus of finite differences. Error analysis for analog computers requires a quality control analysis of the components used in computers. In the two-point boundary-value problems, the accumulated error traverses between the two end points. The initial-value problem suffers from unmanageable propagation of errors, as the number of steps increases in using the method of finite calculus. Laplace's equation, together with Cauchy's conditions, exhibits an exponential growth rate for the error.[5] If the exact solutions of nonlinear differential equations are evaluated numerically, the errors accumulated during the computation are always known.

The purpose of this paper is to provide an elementary approach for exact solutions of a

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[1] A. H. W. Beck, "Space-Charge Waves and Slow Electromagnetic Waves," Pergamon Press, New York, New York; 1958: pages 1–2.

[2] E. F. Beckenbach, editor, "Modern Mathematics for the Engineer," McGraw-Hill Book Company, New York, New York; 1956: S. Lefschetz, Chapter 1, "Linear and Nonlinear Oscillations," page 13.

[3] E. F. Beckenbach, editor, "Modern Mathematics for the Engineer," McGraw-Hill Book Company, New York, New York; 1956: C. B. Morrey, Chapter 16, "Nonlinear Method," page 389.

[4] R. Kalaba, "On Nonlinear Differential Equations, the Maximum Operation, and Monotone Convergence," *Journal of Mathematics and Mechanics*, volume 8, pages 519–574; July, 1959.

[5] I. Sugai, "Numerical Solution of Laplace's Equation, Given Cauchy Conditions," *IBM Journal of Research and Development*, volume 3, pages 187–188; April, 1959.

few nonlinear differential equations. The three differential equations considered include Riccati's equation, a second-order nonlinear differential equation for a plasma problem, and a second-order nonlinear differential equation obtained from the extension of the Sturm–Liouville differential equation.

## 1. The Transform $y = fg$

The generalized Riccati's nonlinear differential equation, which often appears in the WKBJ* method, in solutions for nonuniform transmission lines†, and in aerodynamic problems,[6] is considered first. It is represented by

$$y' + P(x)\,y + Q(x)y^2 = R(x) \qquad (1)$$

where the unknown function is $y(x)$ and the independent variable is $x$. Primes designate derivatives with respect to $x$ throughout this paper. Variable coefficients $P(x)$, $Q(x)$, and $R(x)$ are arbitrary functions of $x$. Two transforms that linearize (1) have been given before,[7] such as the "conventional" transform,

$$y = z'/Qz \qquad (2)$$

and a "new" transform,

$$y = Rz/z'. \qquad (3)$$

These two transforms linearize (1) as (4) and (5), respectively.

$$z'' + (P - Q'/Q)z' - QRz = 0 \qquad (4)$$

and

$$z'' - (P + R'/R)z' - QRz = 0 \qquad (5)$$

where $z(x)$ is an arbitrary function.

Another transform that linearizes (1) is found by the systematic application of the transform

$$y = f(x)g(x) \qquad (6)$$

where $f(x)$ and $g(x)$ are arbitrary functions of $x$.

When (6) is substituted into (1), the result is

$$f'g + fg' + Pfg + Qf^2g^2 = R. \qquad (7)$$

Equating the first or the second term, respectively, of the left-hand side to $R$ gives (3) and (5) identically. Alternatively, setting the sum of the first and fourth terms of the left-hand side to zero, there results

$$f' + Qf^2g = 0. \qquad (8)$$

The remainder is arranged as

$$f = R/(g' + Pg). \qquad (9)$$

Combining of (8) and (9) yields

$$g'' - (R'/R - P)g' - R[Q - (P/R)']g = 0. \qquad (10)$$

This linear differential equation is derived from a generalized Riccati's nonlinear differential equation, (1), by a new transform:

$$y = Rg/(g' + Pg). \qquad (11)$$

In essence, (2) and (3) achieve the purpose of linearization and, in general, (4) and (5) are not solved exactly. Whereas (11) not only accomplishes the task of linearization but also solves the original Riccati's equation if a special condition among parameters $P$, $Q$, and $R$ is

$$(P/R)' = Q. \qquad (12)$$

It is interesting to note that the "conventional" transform (2), for a generalized Riccati's equation[8] is not obtained from (6) by all methods. The other possible combination is to make the sum of the second and

* Wentzel, Kramers, Brillouin, and Jeffreys.

† Added in proof: I. Sugai, "A New Exact Method of Nonuniform Transmission Lines," *Proceedings of the IRE*, volume 49, pages 627–628; March, 1961. I. Sugai, "D'Alembert's Method for Nonuniform Transmission Lines," *Proceedings of the IRE*, volume 49, pages 823–824; April, 1961.

[6] M. Bateman, "Partial Differential Equations of Mathematical Physics," Dover Publications, New York, New York; 1944: pages 491–496.

[7] I. Sugai, "Riccati's Nonlinear Differential Equation," *American Mathematical Monthly*, volume 67, pages 134–139; February, 1960.

[8] G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge University Press, London, England; second edition, 1944: pages 1–2 and 85–94.

the fourth terms of the left hand of (7) equal zero. This yields the same results as in (9), (10), and (11).

## 2. The Transform $y(x) = 1/[f(x)\,g(x)]$

The counterpart of the transform discussed in the previous section is:

$$y(x) = 1/[f(x)g(x)]. \qquad (13)$$

This transform reduces (1) to

$$-f'g - fg' + Pfg + Q = Rf^2g^2. \qquad (14)$$

The familiar technique to divide (14) in two parts is repeated again. The sum of the first and the fourth terms is equated to zero. That is to say,

$$f'g = Q \qquad (15)$$

and the remainders of (14) are written as

$$-g' + Pg = Rfg^2. \qquad (16)$$

The combination of (13), (15), and (16) has perfect agreement with the previous results of (2) and (4). The other combination, the second and fourth terms of (14), also produces the identical results.

Something different is obtained if other combinations are used. For instance, let the first term of the left-hand side of (14) equal the right-hand side of (14), or,

$$-f' = Rf^2g. \qquad (17)$$

Then the remaining terms of (14) are arranged to give

$$f = Q/(g' - Pg). \qquad (18)$$

When (18) is substituted in (17), a linearized second-order differential equation is derived,

$$g'' - (P + Q'/Q)g' - Q[(P/Q)' + R]g = 0. \qquad (19)$$

Therefore, a new transform,

$$y(x) = (g' - Pg)/Qg \qquad (20)$$

linearizes (1) and this new transform solves (1) exactly if the special following condition among variable coefficients is met:

$$(P/Q)' = -R. \qquad (21)$$

The identical results of (19), (20), and (21) are obtained by choosing the other combination: by making the sum of the second term of the left-hand side and the right-hand side of (14) zero. It is noteworthy that the application of (13) to (1) does not give the transform (3), while in the previous section, (6) did not yield (2). Here the existence of a sort of duality is detected. The "new" transform of the published result[7] and the "conventional" transform[8] also exhibit dualism.

The special relationships among variable coefficients $P(x)$, $Q(x)$, and $R(x)$ are given by (12) and (21). These relationships were not realized by using the previous transforms (2) and (3). Equations (12) and (21) were obtained as byproducts rather than as main goals. A similarity is observed in the theory of a linear second-order differential equation. The use of the adjoint equation of a second-order differential equation examines the characteristics of self-adjoint equations and the orthogonality of solutions. The byproduct of this process is that when

$$y'' + P(x)y' + Q(x)y = 0 \qquad (22)$$

this equation is solved exactly[9] if

$$P' = Q. \qquad (23)$$

To utilize the byproducts (12) and (21) effectively, an elementary manipulation is worthwhile. It is clear that if (22) is directly made into Riccati's equations by

$$z = y/y' \qquad (24)$$

and

$$z = y'/y \qquad (25)$$

the equations are, respectively,

$$z' - Pz - Qz^2 = 1 \qquad (26)$$

[9] P. M. Morse and H. Feshbach, "Methods of Theorectical Physics," McGraw-Hill Book Company, New York, New York; 1953: volume 1, pages 526–529.

and

$$z' + Pz + z^2 = -Q. \quad (27)$$

These two equations, (26) and (27), have different coefficients for $z^2$ and $z^0$. It is possible to derive a Riccati's equation from (22) so that the variable coefficients for $z^2$ and $z^0$ are identical. The technique is to use well-developed results[10] for the solution of a nonuniform transmission line in the reverse order. Since $P(x)$ and $Q(x)$ in (22) are independent, it is permissible to introduce a second-order linear differential equation for $w(x)$, an arbitrary function of $x$, such that

$$w'' - (P + Q'/Q)w' + Qw = 0. \quad (28)$$

Then define $z(x)$ in a different way from (24) and (25), letting

$$z(x) = [y(x) - w(x)K(x)]/[y(x) + w(x)K(x)] \quad (29)$$

where

$$K(x) = \{[-\exp(-2\int P dx)]/Q\}^{1/2}. \quad (30)$$

The required Riccati's equation for $z(x)$ is

$$z' + P_1(x)z + Q_1(x)z^2 = Q_1(x) \quad (31)$$

where

$$P_1(x) = -2[-Q(x)]^{1/2} \quad (32)$$

and

$$Q_1(x) = -K'(x)/2K(x). \quad (33)$$

A totally different approach, in which the primary purpose is to obtain a simple relationship among $P(x)$, $Q(x)$, and $R(x)$ for an assumed exact solution of a differential equation, is treated in section 5.

## 3. The Transform $y = f/g$

There are no all-purpose formulas that solve all ordinary nonlinear differential equations. Therefore, in this section an elementary philosophy is given for exact solutions of a few such equations.

The basic concept is: for ordinary nonlinear differential equations of $y(x)$, first use transforms $y = fg$, $y = 1/(fg)$, or $y = f/g$. The original equation in terms of $y(x)$ is next partitioned into two differential equations; one for $f(x)$, for instance, the other for $g(x)$. The equation for $f(x)$ should be a solvable type like Bernoulli's nonlinear differential equation, or a first-order linear differential equation, or any other solvable type. For this equation, $g(x)$ and its derivatives may be included in variable coefficients. The other differential equation for $g(x)$ must not contain $f(x)$ and its derivatives. A more stringent condition is that $g(x)$ must be solved exactly analytically. Relating the variable coefficients to each other in this equation for $g(x)$ sometimes provides an exact solution for $g(x)$. Once $g(x)$ has been solved exactly, it is then substituted back into variable coefficients of a solvable equation for $f(x)$.

An illustrative example for this statement is taken from a recent publication[11] in which, with a few assumptions, M. V. Konyukov solved exactly a second-order ordinary nonlinear differential equation that arises from nonlinear Langmuir electron oscillations in a plasma. He solved

$$nn'' - 2(n')^2 + 4\pi e^2 n^2 (n - n_0)/m = 0 \quad (34)$$

where $e$ and $m$ are the charge and mass of an electron and $n$ and $n_0$ are the electron and positive ion densities. The independent variable in (34) is time, which is represented by $x$ in this paper. The unknown function is $n(x)$ and all the rest of the quantities are constant. Konyukov's exact solution for (34) is

$$n = n_0/[1 - b \sin(\omega_0 x + \phi)] \quad (35)$$

[10] L. R. Walker and N. Wax, "Nonuniform Transmission Lines and Reflection Coefficients," *Journal of Applied Physics*, volume 17, pages 1043–1045; December, 1946.

[11] M. V. Konyukov, "Nonlinear Langmuir Electron Oscillations in a Plasma," *Journal of Experimental and Theoretical Physics of the Academy of Sciences of the USSR* (English translation), volume 37 (10), pages 570–571; March, 1960.

where

$$\omega_0 = (4\pi e^2 n_0/m)^{1/2}. \quad (36)$$

$\omega_0$ is the plasma frequency due to small perturbations and two arbitrary constants, $b$ and $\phi$, conform with specified initial conditions on $n(x)$.

This exact solution is re-examined from the viewpoint of the transform $y = f/g$. A slightly generalized version of (34) is

$$yy'' + P(x)y'^2 + Q(x)\,y^3 + R(x)\,y^2 = 0. \quad (37)$$

As mentioned before, $P(x)$, $Q(x)$, and $R(x)$ are arbitrary functions of $x$. The elementary transform

$$y = f(x)/g(x) \quad (38)$$

is applied to (37), giving

$$ff''g^2 - f^2gg'' + Pf'^2g^2 - 2(P+1)ff'gg' + (P+2)f^2g'^2 + Qf^3g + Rg^2f^2 = 0. \quad (39)$$

Wherever $P$ appears, the square of the $f$ and $g$ product is missing. This suggests the investigation of the conditions (A): $P = 0$, (B): $P = -1$, and (C): $P = -2$. It turns out that (A) offers nothing useful, (B) has a nontrivial solution if $Q$ and $R$ are both constants. In this case, the sum of the second and sixth terms of (39) are equated to zero:

$$g'' = Qf. \quad (40)$$

This eliminates $f$ and its derivatives from the remainders of (39), and

$$g(x) = c\exp(-Rx)^{1/2} + d\exp[-(-Rx)^{1/2}] \quad (41)$$

where $c$ and $d$ are arbitrary constants.

Using (41), $f(x)$ is readily solved via (40) and the final solution for $y(x)$ is given by evaluating (38). (C) offers an interesting result that happens to be the case of nonlinear Langmuir electron oscillations. The sum of the second, fourth, and sixth terms of (39) is set to zero:

$$fg'' - 2f'g' - Qf^2 = 0. \quad (42)$$

The remaining terms of (39) are grouped:

$$ff'' - 2f'^2 + Rf^2 = 0. \quad (43)$$

This meets the requirements of the statement given at the beginning of this section. Equation (42) is a solvable differential equation for $g'(x)$. Equation (43) does not contain $g(x)$ and its derivative. Now, let

$$h(x) = f(x)/f'(x) \quad (44)$$

then (43) reduces to a Riccati's equation

$$h' - Rh^2 = -1. \quad (45)$$

For a very few special cases, (45) can be solved exactly for $h(x)$ as shown[12] by several authors. The problem is, however, that the exact solution for $h(x)$ must always be obtainable, which can only be done by adjusting the variable coefficients. There is only one variable coefficient, $R(x)$, and in the light of published[7] results, $h(x)$ is always solved exactly if $R$ is constant. Therefore, by substituting the dependent variables $h(x)$, $f(x)$, and $g(x)$ in a sequential order, $y(x)$ is solved exactly. In Konyukov's case, $R$ and $Q$ are both constant, whereas the above result indicates that $Q$ can be a function of $x$. In a word, by re-examining the paper of Konyukov, it is found that

$$yy'' - 2(y')^2 + Q(x)y^3 + ay^2 = 0 \quad (46)$$

is always solved exactly, where $a$ is any constant.

At first glance, the above illustrative example, (46), does not give an impression that it is solvable exactly. A systematic application of an elementary transform, such as $y = f/g$, provides a logical path to the exact solution. The uses of the elementary transforms, $y = fg$, $y = 1/fg$, $y = f/g$, or any other simple ones, may solve many other ordinary nonlinear differential equations.

[12] H. Betz, P. B. Burcham, and G. M. Ewing, "Differential Equations with Applications," Harper and Brother Publishers, New York, New York; 1954: pages 40–45.

## 4. Extension of Sturm–Liouville Equation

In this section a slightly different approach is taken; a well-known linear differential equation is made nonlinear. In some cases this is very useful. For instance, there is no complete exact solution for the Sturm–Liouville differential equation, which has so many applications in boundary-value and eigenvalue problems. It is a linear differential equation, yet its general solution is unknown. Only a handful of specialized cases have been worked out[13,14] for mathematical physics. Specialized cases are identified with such familiar names as the functions of Mathieu, Legendre, Bessel, and Hermite to list a few. The Sturm–Liouville differential equation under consideration is represented by

$$[y'P(x)]' = -Q(x,\lambda)y \tag{47}$$

where

$$Q(x,\lambda) = \lambda R(x) + S(x) \tag{48}$$

is often used and $\lambda$ is an eigenvalue. The case in which $P$ depends on $\lambda$ is mentioned[15] by others. In (48), $R(x)$ and $S(x)$ are called weighting and potential functions, respectively. The generalization proposed in this section makes (47) nonlinear:

$$[(y')^k P]' = -Qy^k \tag{49}$$

where $k$ in any real number. The transformation

$$P(y')^k = zy^k \tag{50}$$

reduces (49) to

$$z' + kP^{-(1/k)}z^{(1+k)/k} = -Q. \tag{51}$$

From (51) it is evident that the special case $k = 1$ (the Sturm–Liouville case) provides a Riccati's equation and, except in a very few special cases[16] and under special conditions such as (12) and (21), $z(x)$ is not solved exactly. However, an interesting fact is that the $k = -1$ case gives an exact solution for $z(x)$. By similar sequential substitutions, the exact solution of

$$(P/y')' = -Q/y \tag{52}$$

is found to be

$$y = a_1 \exp q(x) \tag{53}$$

where

$$q'(x) = P(x)/\{\int[P(x) - Q(x)]dx + a_2\} \tag{54}$$

and $a_1$ and $a_2$ are two arbitrary constants of integrations.[17]

There are many other ways to derive the steps from (52) to (54); two of them are

$$z = y/y' \tag{55}$$

and

$$z = y'/y. \tag{56}$$

These give, respectively,

$$Pz' + P'z = P - Q \tag{57}$$

and

$$Pz' - P'z + (P - Q)z^2 = 0. \tag{58}$$

Equation (58) is a well-known Bernoulli's nonlinear differential equation. It is quite conceivable that other elementary transforms may produce interesting and useful results from (51) for other values of $k$. The example considered in this section used the Sturm–Liouville equation of unknown general solution. The application of the basic concept presented in this section can be used with any other differential equation.

[13] H. Magenau and G. M. Murphy, "The Mathematics of Physics and Chemistry," D. Van Nostrand Company, New York, New York; 1943: pages 253–255.

[14] E. T. Whittaker and G. N. Watson, "A Course of Modern Analysis," third edition, Cambridge University Press, London, England; 1920: pages 203–206.

[15] F. B. Hildebrand, "Advanced Calculus for Engineers," Prentice-Hall Incorporated, New York, New York; 1948: pages 201–202.

[16] B. Kamke, "Differentialgleichungen Lörungrmethoden und Lösungen," Chelsea Publishing Company, New York, New York; 1958: pages 295–299.

[17] I. Sugai, "The Solutions of Nonuniform Transmission Line Problems," *Proceedings of the IRE*, volume 48, pages 1489–1490; August, 1960.

## 5. Related Variable Coefficients

The historical originality of this method has not been ascertained. It employs related variable coefficients to solve differential equations exactly. Piaggio showed[18] the use of this idea in his book. For two arbitrary functions of $x$, $P(x)$, and $Q(x)$, a general solution of a second-order linear differential equation for $y(x)$ is not available in

$$y'' + P(x)\,y' + Q(x)y = 0. \qquad (22)$$

If the solution of the type

$$y = a \sin (nx + b) \qquad (59)$$

is assumed, where $a$ and $b$ are arbitrary constants, the compatibility between (22) and (59) demands that $P$ and $Q$ be related by

$$n^2 = nP(x) \cot (nx + b) + Q(x). \qquad (60)$$

An extension of (59) uses

$$y = a \exp[-u(x)] \sin[nx + w(x) + b]. \qquad (61)$$

Equation (61) represents amplitude and phase variations with respect to $x$ and this expression is employed to obtain the exact solutions of nonuniform-transmission-line problems.[17]

A similar useful procedure[19] has been attributed to Madelung. This basic concept (if variable coefficients are somehow related, exact solutions are always possible) expands the usual techniques[20] employed in the WKBJ method.

Let

$$y = a \exp s(x) \qquad (62)$$

where $a$ is an arbitrary constant and

$$s(x) = \int [z(x) - P(x)/2] dx. \qquad (63)$$

Substitution of (62) and (63) into (22) yields a Riccati's equation for an arbitrary function which, in turn, is not always solvable. To avoid fruitless cycling of substitutions, the WKBJ method assumes an approximation requiring that $P(x)$ vary slowly with respect to $x$ over the interval of interest. If $s(x)$ is defined differently from (63), what would be the result? Let

$$s(x) = \int t(x,P,Q) dx. \qquad (64)$$

Consequently, the Riccati's equation for $t$ is

$$t' + \alpha(x,P,Q)t + \beta(x,P,Q)t^2 = \gamma(x,P,Q). \qquad (65)$$

Here again the special conditions given by (12) and (21) become useful. Another approach is to adjust the expression for $t$ and relate $P$ and $Q$ so that (65) is reduced to either Bernoulli's nonlinear equation ($\gamma = 0$) or a first-order linear equation ($\beta = 0$). Two simple examples are

$$t = (z - Q)/(z + P) \qquad (66)$$

for which

$$1 + P + Q = 0 \qquad (67)$$

or

$$(P/Q)' + 1 = 0 \qquad (68)$$

is required; and for

$$t = (z - P)/(z + P) \qquad (69)$$

the condition (67) or

$$1 + Q = P \qquad (70)$$

is required.

When (67), (68), and (70) are compared with (60), more loosely related states are realized in (67), (68), and (70) than in (60). A similar method is now applied directly to the Riccati's equation (1). Let

$$y(x) = a \tan nx \qquad (71)$$

[18] H. T. H. Piaggio, "An Elementary Treatise on Differential Equations and their Applications," G. Bell and Sons, Limited, London, England; 1920: page 199.

[19] F. B. Hildebrand, "Introduction to Numerical Analysis," McGraw-Hill Book Company, New York, New York; 1956: pages 255–256.

[20] F. B. Hildebrand, "Advanced Calculus for Engineers," Prentice-Hall, Incorporated, New York, New York; 1948: pages 196–197.

where $a$ is an arbitrary constant. Then (71) reduces (1) to

$$(n + Qa)\tan^2 nx + P\tan nx - (R/a - n) = 0 \quad (72)$$

which is further rearranged to

$$2(n + Qa)\tan nx = -P \pm [P^2 + 4(n + Qa)(R/a - n)]^{1/2}. \quad (73)$$

If $n$ and $a$ are real, (73) not only relates $P$, $Q$, and $R$, but it also strongly restricts (73) to real numbers. This gives one additional condition that

$$P^2 + 4(n + Qa)(R/a - n) \geqslant 0. \quad (74)$$

When $n$ and $a$ are imaginary, (73) and (74) must be properly satisfied with a hyperbolic tangent function so that $y(x)$ in (71) is real.

The common characteristics of these approaches are that the types of solutions are assumed beforehand. Elaborate extension of (71) and types of solutions other than (71) will not be pursued further here.

## 6. Numerically Solved Nonlinear Differential Equations

Nonlinear differential equations arising from physical problems are numerous. When analytical solutions are not obtainable, problems are solved on computers. Many such equations have been solved on computers but only a few solutions have been published and widely distributed. Some of those published results are listed below, indicating that their exact solutions are still not available. Unless specified, the first few lower-case letters are arbitrary constants.

$$y'' - yy' = 0 \quad (75)$$ [21]

$$y''' - yy'' = 0 \quad (76)$$ [21]

$$y^{3/2}y'' + 2x(4 + y') + 2y^{1/2}(4 - y') - 8y = 0. \quad (77)$$ [22]

Equation (77) arises in the theory of turbulence.

$$y'' = x(a + y^{1/2}x^{-1/2})^3 \quad (78)$$ [23]

$$xy'' = y^{3/2}. \quad (79)$$ [24]

Equations (78) and (79) appear in quantum mechanics.

$$y'' + ay' + \exp(-x) + \exp[b - c/(y + d)] = 0 \quad (80)$$ [25]

$$y' = axy^{-1/3} \quad (81)$$ [26]

$$y'' + ay' + by^{1/2}\exp(-c/y) = 0 \quad (82)$$ [27]

$$y''(x + b) = ay(y + x + b). \quad (83)$$ [28]

Equation (83) is used in connection with the diffusion of flames.

[21] V. M. Falkner, "A Method of Numerical Solution of Differential Equations," *Philosophical Magazine*, volume 21, pages 624–640; 1936.

[22] S. Chandrasekhar, "On Heisenberg's Elementary Theory of Turbulence," *Proceedings of Royal Society of London*, volume 200A, pages 20–23; 1949.

[23] N. Metropolis and J. R. Reitz, "Solutions of the Fermi-Thomas-Dirac Equation," *Journal of Chemical Physics*, volume 19, pages 555–573; 1951.

[24] N. H. March, "Thomas-Fermi Field for Molecules with Tetrahedral and Octahedral Symmetry," *Proceedings of Cambridge Philosophical Society*, volume 48, pages 665–682; 1952.

[25] G. Blanch, "On Numerical Solution of Equations Involving Differential Operators with Constant Coefficients," *Mathematical Tables and Other Aids to Computation*, volume 6, pages 219–223; 1952.

[26] T. E. Sterne, "The Accuracy of Numerical Solutions of Ordinary Differential Equations," *Mathematical Tables and Other Aids to Computation*, volume 7, pages 159–164; 1953. Analytical approximate solutions of this equation are discussed in I. Sugai, "Approximate Solutions for a First Order Nonlinear Differential Equation," *American Mathematical Monthly*, now in press.

[27] J. E. Maxfield, "A Useful Technique in Programming for Analog Computer," *Mathematical Tables and Other Aids to Computation*, volume 8, pages 233–234; 1954.

[28] B. H. Worsley, "Solutions of a Nonlinear Differential Equation Arising in the Theory of Diffusion of Flames," *Mathematical Tables and Other Aids to Computation*, volume 9, pages 112–116; 1955.

$$(y'')^2 = yy' \qquad (84)^{29}$$

$$y'' + y' + y^2 = 0. \qquad (85)^{30}$$

Equation (85) is called a "jury" problem.

$$y'' - (1 - x^2)y' + y = \cos x. \qquad (86)^{30}$$

Equation (86) is a variation of van der Pol's equation.

## 7. Conclusions

The study presented shows that two useful transforms, (11) and (20), are derived by systematic application of elementary transforms $y = fg$ and $y = 1/fg$. The transform $y = f/g$ generalizes the second-order nonlinear differential equation arising from nonlinear Langmuir electron oscillations. A slight generalization of the Sturm–Liouville linear differential equation resulted in the exact solution for a second-order nonlinear differential equation.

The techniques presented are so universal that engineers and scientists should be encouraged to utilize the above-mentioned elementary transforms to obtain exact analytical solutions of ordinary nonlinear differential equations before the problems are programmed for numerical solution.

[29] A Solution of the Equation $(y'')^2 = yy'$," Publication *A.R.L./T.1/Maths 2.7, 16 p.* of the Admiralty Research Laboratory; Teddington, Middlesex, England.

[30] F. J. Warner, "On the Solution of "Jury" Problems with Many Degrees of Freedom," *Mathematical Tables and Other Aids to Computation,* volume 11, pages 268–271; 1957.

# A Class of Solved Riccati's Equations

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Riccati's nonlinear differential equations often appear in the WKBJ* method, in aerodynamics, and also[1,2] in nonuniform-transmission-line problems. In fact, all the second-order ordinary linear homogeneous differential equations with general coefficients can be reduced to generalized Riccati's equations. However, the exact solution for such an equation has not been obtained. This paper presents a new approach, thereby giving exact solutions for a certain class of Riccati's equations in explicit and analytical forms. The class considered in this paper has two arbitrary variable coefficients in specified products.

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For three arbitrary variable coefficients, $P(x)$, $Q(x)$, and $R(x)$, a generalized Riccati's nonlinear differential equation for $y(x)$ is

$$y'(x) + P(x)y(x) + Q(x)y^2(x) = R(x) \quad (1)$$

where the prime denotes the derivative with respect to $x$.

The conventional transform[3] and new transform[4] linearize (1), yet the resultant differential equations are of the second order with variable coefficients and are not solved exactly for all cases. The extended work[5] of the previous paper[4] uncovered two slightly more complicated transforms for (1):

$$y = Rz(x)/[z'(x) + Pz(x)] \quad (2)$$

and

$$y = [z'(x) - Pz(x)]/[Qz(x)] \quad (3)$$

where $z(x)$ is any arbitrary function of $x$.

These two transforms respectively linearize (1) as

$$z'' + (P - R'/R)z' + R[(P/R)' - Q]z = 0 \quad (4)$$

and

$$z'' - (P + Q'/Q)z' - Q[(P/Q)' + R]z = 0. \quad (5)$$

The exact solution of (1) is obtainable if the variable coefficients are so related that

$$(P/R)' = Q \quad (6)$$

or

$$(P/Q)' = -R. \quad (7)$$

The basic idea presented in this paper is the extension of transforms (2) and (3). That is, to find means to reduce (1) to a first-order linear differential equation or into any solvable nonlinear differential equation, by suitable transforms in the change of variables. This idea was explained in another paper[6] in conjunction with the solution for nonuniform-transmission-line problems. We consider here the reduction of (1) to a first-order linear ordinary differential equation.

At first, the utility of specialized transforms is demonstrated to solve a Riccati's equation. For instance in

$$y' + Py^2 = Q' + Q^2P \quad (8)$$

two transforms described in the previous paper[4] are

$$y = g'(x)/[Pg(x)] \quad (9)$$

and

$$y = (Q' + Q^2P)f(x)/f'(x) \quad (10)$$

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* Wentzel, Kramers, Brillouin, and Jeffreys.

[1] I. Sugai, "The Solutions of Nonuniform Transmission Line Problems," *Proceedings of the IRE*, volume 48, pages 1489–1490; August, 1960.

[2] O. Buneman, "A Small Amplitude Theory for Magnetrons," *Journal of Electronics and Control*, volume 3, pages 1–62; July, 1957: page 31.

[3] G. N. Watson, "A Treatise on the Theory of Bessel Functions," second edition, Cambridge University Press, London, England; 1944: pages 1–2 and 85–94.

[4] I. Sugai, "Riccati's Nonlinear Differential Equations," *American Mathematical Monthly*, volume 67, pages 134–139; February, 1960.

[5] I. Sugai, "Exact Solutions for Ordinary Nonlinear Differential Equations," *Electrical Communication*, volume 37, number 1, pages 47–55; 1961.

[6] I. Sugai, "A New Exact Method for Nonuniform Transmission Lines," *Proceedings of the IRE*, volume 49, pages 627–628; March, 1961.

where

$$y = f(x)/g(x) \quad (11)$$

was used: $f(x)$ and $g(x)$ are arbitrary functions of $x$. Equations (9) and (10), respectively, reduce (8) to

$$g'' - (\log_e P)'g' - P(Q' + Q^2P)g = 0 \quad (12)$$

and

$$f'' - [\log_e (Q' + Q^2P)]'f' - P(Q' + Q^2P)f = 0. \quad (13)$$

These two equations, (12) and (13), are in general not exactly solvable. The fruitless cycles of substitutions for variables among (8), (9), and (12) give no concrete solution for (8). When approximate solutions may be useful, various approximate methods are used. A famous one is the WKBJ method in quantum physics. In the field of electrical engineering, many efforts to obtain approximate solutions of (1) have been made for the problems of nonuniform transmission lines, in which the reflection coefficient forms a generalized Riccati's nonlinear differential equation. Numerous published papers for nonuniform-transmission-line problems are collected[7,8] in bibliographies. Suppose a different type of transform is used for (8): What would be the result? A useful transform† for (8) is

$$y = (PQz + z')/(Pz). \quad (14)$$

Then a first-order linear differential equation for $z'(x)$ is obtained by substituting (14) into (8),

$$Pz'' + (2P^2Q - P')z' = 0. \quad (15)$$

Therefore, $y(x)$ is solved exactly for (8) as

$$y = Q + \frac{\exp(-\int PQdx)}{c + \int P \exp(-\int PQdx)dx} \quad (16)$$

where $c$ is any arbitrary constant of integration. This illustration convincingly demonstrates that (14) is much more useful than (11) to solve (8) exactly. In (16) there is no restriction on the relationships between variable coefficients $P$ and $Q$. It is somewhat interesting to note the symmetry of (8), which is also a Riccati's equation for $Q(x)$. Yet this is merely an accidental case because the equations in the list below do not all present this symmetry.

Next, it is worthwhile to consider systematically the reason why the general transform, such as (11), does not reduce any Riccati's equation to a first-order linear differential equation. Thus, (1) is reduced to

$$w'(x) + Qw^2(x) = S(x) \quad (17)$$

where $w(x)$ is an arbitrary function of $x$, and $S(x)$ is

$$S(x) = R + (P/2Q)' + P^2/4Q. \quad (18)$$

And, further, $y(x)$ is related to $w(x)$ via

$$y(x) = w(x) - P/2Q. \quad (19)$$

Without loss of generality, (17) can be used in place of (1) in the following treatment to simplify the algebra involved. This treatment concerns the method of specifying $f(x)$ in terms of $g(x)$ in

$$w(x) = f(x)/g(x). \quad (20)$$

Substitute (20) into (17), then

$$f'g + fg' + Qf^2 = Sg^2. \quad (21)$$

Now, let

$$f(x) = V(x)g(x) + T(x)g'(x) \quad (22)$$

where $V(x)$ and $T(x)$ are to be specified by $Q(x)$ and $S(x)$. Combination of (21) and (22) yields

[7] H. Kaufman, "Bibliography for Nonuniform Transmission Lines," *IRE Transactions on Antennas and Propagation*, volume AP-5, pages 218–220; October, 1955.

[8] I. Sugai, "Supplementary Bibliography for Nonuniform Transmission Lines," (Submitted to *IRE Transactions on Antennas and Propagation*).

† Added in proof: This transform was shown in a recent note. I. Sugai, "D'Alembert's Method for Nonuniform Transmission Lines," *Proceedings of the IRE*, volume 49, pages 823–824; April, 1961.

$$Tgg'' + (T' + 2QVT)gg' + (V' + QV^2 - S)g^2 + T(QT - 1)g'^2 = 0. \quad (23)$$

Equation (23) is always solved exactly if

$$T = 1/Q \quad (24)$$

and

$$V' + QV^2 = S. \quad (25)$$

However, (25) is only (17). This proves the point that unless a product or a division between $Q$ and $S$ appears in special form as in (8), (17) is not solved exactly by using (22). Of course, expressions other than (22) may provide solvable nonlinear differential equations instead of (23).

The class of solvable Riccati's equations considered in this paper is of a form similar to (8). It is desirable to eliminate the second term of the right-hand side in (8) because this term restricts (8) to a small specialized class of (17). Nevertheless, the past efforts to solve Riccati's equations had been restricted to specified forms of variable coefficients $P(x)$, $Q(x)$, and $R(x)$; for example, $x^2$, $\sin x$, and $\exp x$, for these coefficients as in existing books.[9,10] A numerical solution of a Riccati's equation by a modern high-speed digital computer is reported[11] in a recent paper. In contrast, little has been done for extensive work on such equations with arbitrary variable coefficients. Necessitated by physical studies and engineering problems, great interest for exact solutions for nonlinear differential equations has grown considerably in the last decade. This trend has been accelerated by wider availability of modern high-speed digital computers. Exact solutions of these equations are valuable for unequivocal checks on numerical evaluations by computers, while the converse is not true. Approximate numerical solutions, no matter to how many digits the program may be carried, do not, in general, check exact solutions, simply because the errors committed always propagate and accumulate. Even with linear differential equations, there is a problem from the unmanageable exponential growth of errors with the increase in the steps used for the calculus of finite difference.[12] Much less is known about complicated nonlinear differential equations. In the light of these circumstances, it may be of service to enumerate a certain class of solved Riccati's equations together with useful transforms that reduce Riccati's equations directly to first-order linear differential equations.

The purpose of this paper is to present the basic idea for solving a certain class of Riccati's equations: the intermediate steps dealing with transforms and solutions of these equations are avoided. There are fourteen transforms listed below in pairs, exhibiting the dualism between two transforms as described[4] in the former paper. However, it is appropriate to cite the first case of the transforms listed below to explain the intermediate steps omitted. Take Transform 1,

$$y = z/(Pz + Qz'). \quad (26)$$

Substitute this into a generalized Riccati's equation,

$$y' + \alpha(x)y + \beta(x)y^2 = \gamma(x) \quad (27)$$

where $\alpha$, $\beta$, and $\gamma$ are undetermined variable coefficients and only $\alpha$ can become zero, typical of Riccati's class of equations.

[9] D. Kamke, "Differentialgleichungen Lösungsmethoden und Lösungen," third edition, Chelsea Publishing Company, New York, New York; 1958: pages 290–299.

[10] H. Betz, P. B. Burcham, and G. M. Ewing, "Differential Equations with Applications," Harper and Brother Publishers, New York, New York; 1954: pages 40–45.

[11] H. H. Howe, "Note on the Solution of Riccati's Differential Equation," *Journal of Research of the National Bureau of Standards*, volume 64B, pages 95–98; April–June, 1960.

[12] I. Sugai, "Numerical Solution of Laplaces' Equation, Given Cauchy Conditions," *IBM Journal of Research and Development*, volume 3, pages 187–188; April, 1959.

Substitute (26) into (27):

$$Qz'^2 - P'z^2 - (Q'z' + Qz'')z + (Pz + Qz')\alpha z + \beta z^2 = (P^2z^2 + Q^2z'^2 + 2PQzz')\gamma. \quad (28)$$

There are four places where $z^2$ appears; on the other hand, there are two places where $z'^2$ appears. The key idea is to eliminate terms of $z^2$ and $z'^2$ in (28), thus degenerating (28) into a first-order linear differential equation. Because $z'^2$ appears in two places, the unique choice for $\gamma(x)$ is made first,

$$\gamma(x) = 1/Q(x). \quad (29)$$

The coefficients for $z^2$ are collected to specify $\alpha(x)$ and $\beta(x)$,

$$-P' + \alpha P + \beta - P^2/Q = 0. \quad (30)$$

Note that there are three ways of specifying while retaining (27) in Riccati's class of equations. If

$$\alpha = 0 \quad (31)$$

then

$$\beta = P' + P^2Q. \quad (32)$$

If

$$\alpha = P/Q \quad (33)$$

then

$$\beta = P'. \quad (34)$$

If

$$\alpha = (\log_e P)' \quad (35)$$

then

$$\beta = P^2/Q. \quad (36)$$

Judging from these developments, it is quite obvious that a transform of type

$$y = z'/(Pz + Qz') \quad (37)$$

does not solve a Riccati's equation. If (37) is substituted in (27), there is only one place where $z^2$ appears. This forces $P$ to zero to eliminate the term of $z^2$. If $P$ is zero in (37), $y$ is not related to another variable $z$. Following the above steps, the transforms listed below are obtained and may be applied for exact solutions for a certain class of Riccati's nonlinear differential equations. Only fourteen transforms in pairs are listed below but it is possible to find many more transforms in pairs.

Transform *1*:

$$y = z/(Pz + Qz') \quad (38)$$

solves:

$$y' + (p' + P^2/Q)y^2 = 1/Q \quad (39)$$

$$y' + Py/Q + P'y^2 = 1/Q \quad (40)$$

$$y' + (\log_e P)'y + P^2y^2/Q = 1/Q. \quad (41)$$

Transform *2*:

$$y = (Pz + Qz')/z \quad (42)$$

solves:

$$y' + y^2/Q = P' + P^2/Q \quad (43)$$

$$y' - Py/Q + y^2/Q = P' \quad (44)$$

$$y' - (\log_e P)'y + y^2/Q = P^2/Q. \quad (45)$$

Transform *3*:

$$y = z/(z + PQz') \quad (46)$$

solves:

$$y' - [\beta(x) - 1/PQ]y + \beta(x)y^2 = 1/PQ \quad (47)$$

where $\beta(x)$ is any arbitrary function of $x$.

Transform *4*:

$$y = (z + PQz')/z \quad (48)$$

solves:

$$y' + [\gamma(x) - 1/PQ]y + y^2/PQ = \gamma(x) \quad (49)$$

where $\gamma(x)$ is any arbitrary function of $x$.

Transform *5*:

$$y = PQz/(Pz + Qz') \quad (50)$$

solves:

$$y' + (P - Q')y^2/Q^2 = P \quad (51)$$

$$y' + Py/Q + (Q^{-1})'y^2 = P \quad (52)$$

$$y' - (\log_e Q)'y + Py^2/Q^2 = P. \quad (53)$$

Transform *6*:

$$y = (Pz + Qz')/PQz \qquad (54)$$

solves:

$$y' + Py^2 = (P - Q')/Q^2 \qquad (55)$$

$$y' - Py/Q + Py^2 = (Q^{-1})' \qquad (56)$$

$$y' + (\log_e Q)'y + Py^2 = P/Q^2. \qquad (57)$$

Transform *7*:

$$y = z/(PQz + z') \qquad (58)$$

solves:

$$y' + [(PQ)^2 + (PQ)']y^2 = 1 \qquad (59)$$

$$y' + PQy + (PQ)'y^2 = 1 \qquad (60)$$

$$y' + [\log_e PQ]'y + (PQ)^2y^2 = 1. \qquad (61)$$

Transform *8*:

$$y = (PQz + z')/z \qquad (62)$$

solves:

$$y' + y^2 = (PQ)^2 + (PQ)' \qquad (63)$$

$$y' - PQy + y^2 = (PQ)' \qquad (64)$$

$$y' - [\log_e PQ]'y + y^2 = (PQ)^2. \qquad (65)$$

Transform *9*:

$$y = Pz/(Pz + Qz') \qquad (66)$$

solves:

$$y' - [\beta(x) - P/Q]y + \beta(x)y^2 = P/Q \qquad (67)$$

where $\beta(x)$ is any arbitrary function of $x$.

Transform *10*:

$$y = (Pz + Qz')/Pz \qquad (68)$$

solves:

$$y' + [\gamma(x) - P/Q]y + Py^2/Q = \gamma(x) \qquad (69)$$

where $\gamma(x)$ is any arbitrary function of $x$.

Transform *11*:

$$y = Qz/(Pz + Qz') \qquad (70)$$

solves:

$$y' + \{[\log_e (P/Q)]' + P/Q - Q\beta(x)/P\}y + \beta(x)y^2 = 1 \qquad (71)$$

where $\beta(x)$ is any arbitrary function of $x$.

Transform *12*:

$$y = (Pz + Qz')/Qz \qquad (72)$$

solves:

$$y' - \{P/Q + [\log_e (P/Q)]' - Q\gamma(x)/P\}y + y^2 = \gamma(x) \qquad (73)$$

where $\gamma(x)$ is any arbitrary function of $x$.

Transform *13*:

$$y = PQz/(z + z') \qquad (74)$$

solves:

$$y' + [1/PQ + (1/PQ)']y^2 = PQ \qquad (75)$$

$$y' + y + (1/PQ)'y^2 = PQ \qquad (76)$$

$$y' - (\log_e PQ)'y + y^2/PQ = PQ. \qquad (77)$$

Transform *14*:

$$y = (z + z')/PQz \qquad (78)$$

solves:

$$y' + PQy^2 = [1/PQ + (1/PQ)'] \qquad (79)$$

$$y' - y + PQy^2 = (1/PQ)' \qquad (80)$$

$$y' + (\log_e PQ)'y + PQy^2 = 1/PQ. \qquad (81)$$

# World's Telephones—1960 *

Telephones of the world underwent a tremendous gain of 8 800 000 in the year 1959—not only the largest gain for any year of record, but exceeding by a million that in any previous year. At the beginning of 1960, the world's telephones had risen to 133.6 million. The United States accounted for nearly half of all the telephones added and continues to account for more than half the world total in service. In the tables below are included comparative data for continental areas back to 1950. These data indicate the highly important growth in telephones that is taking place not only in North America, but in the other continental areas. In round numbers, the count of telephones practically doubled during the decade from 1950 to 1960 and even trebled for Asia.

Following their admission to the Union, statistics for Alaska and Hawaii in this issue of the report are included in the total for the United States, although data for Hawaii also appear under Oceania. There are also some new entries for Africa.

* Abridgement from the 1960 issue of a booklet, "The World's Telephones," published yearly by the chief statistician's office of the American Telephone and Telegraph Company, New York, New York.

TELEPHONES IN CONTINENTAL AREAS—JANUARY 1, 1960

| Area | Total | | | Privately Operated | | Automatic | |
|---|---|---|---|---|---|---|---|
| | Number in 1960 | Percent of World | Per 100 Population | Number 1960 | Percent of Total | Number 1960 | Percent of Total |
| North America | 76 036 400 | 56.9 | 38.7 | 75 201 000 | 98.9 | 71 120 800 | 93.5 |
| Middle America | 1 008 000 | 0.7 | 1.5 | 909 200 | 90.2 | 836 100 | 82.9 |
| South America | 3 145 900 | 2.4 | 2.3 | 1 526 300 | 48.5 | 2 704 000 | 86.0 |
| Europe | 40 340 900 | 30.2 | 7.0 | 6 779 000 | 16.8 | 33 613 100 | 83.3 |
| Africa | 1 904 500 | 1.4 | 0.8 | 32 100 | 1.7 | 1 386 200 | 72.8 |
| Asia | 8 110 000 | 6.1 | 0.5 | 4 956 100 | 61.1 | 5 393 600 | 66.5 |
| Oceania | 3 054 300 | 2.3 | 18.7 | 223 500 | 7.3 | 2 353 400 | 77.1 |
| World | 133 600 000 | 100.0 | 4.6 | 89 627 200 | 67.1 | 117 407 200 | 87.9 |

YEARLY TOTAL NUMBER OF TELEPHONES IN SERVICE

| Area | 1959 | 1958 | 1957 | 1956 | 1955 | 1950 |
|---|---|---|---|---|---|---|
| North America | 71 799 300 | 68 484 000 | 64 720 700 | 60 422 900 | 56 691 700 | 43 423 700 |
| Middle America | 910 800 | 835 900 | 772 800 | 733 100 | 700 300 | 523 900 |
| South America | 2 999 600 | 2 845 000 | 2 695 300 | 2 568 300 | 2 422 900 | 1 657 000 |
| Europe | 37 598 100 | 35 218 700 | 32 510 000 | 29 990 000 | 27 787 000 | 20 299 000 |
| Africa | 1 768 600 | 1 663 200 | 1 546 100 | 1 411 200 | 1 247 400 | 805 600 |
| Asia | 6 855 700 | 6 062 500 | 5 229 500 | 4 708 800 | 4 261 200 | 2 468 400 |
| Oceania | 2 867 900 | 2 690 700 | 2 525 600 | 2 365 700 | 2 189 500 | 1 522 400 |
| World | 124 800 000 | 117 800 000 | 110 000 000 | 102 200 000 | 95 300 000 | 70 700 000 |

## Telephones by Countries as of January 1, 1960

| Country or Area | Number of Telephones | Per 100 Population | Percent Automatic | Telephones by Type of Operation | |
|---|---|---|---|---|---|
| | | | | Private | Government |
| NORTH AMERICA | | | | | |
| Canada | 5 439 023 | 30.85 | 84.5 | 4 604 019 | 835 004 |
| Greenland | 0 | — | — | — | — |
| St. Pierre and Miquelon | 397 | 7.94 | 0.0 | 0 | 397 |
| United States (1) | 70 597 000 | 39.52 | 94.2 | 70 597 000 | 0 |
| MIDDLE AMERICA | | | | | |
| Bahamas | 9 814 | 6.82 | 98.5 | 0 | 9 814 |
| Bermuda | 11 500 | 25.56 | 100.0 | 11 500 | 0 |
| British Honduras | 925 | 1.01 | 0.0 | 0 | 925 |
| Canal Zone (2) (3) | 7 803 | 26.91 | 100.0 | 0 | 7 803 |
| Costa Rica | 15 360 | 1.34 | 78.2 | 14 703 | 657 |
| Cuba | 191 414 | 2.84 | 92.2 | 191 414 | 0 |
| Dominican Republic | 19 019 | 0.65 | 89.2 | 18 694 | 325 |
| El Salvador | 15 930 | 0.62 | 66.1 | 0 | 15 930 |
| Guadeloupe and Dependencies | 2 939 | 1.10 | 0.0 | 0 | 2 939 |
| Guatemala | 22 000 | 0.59 | 85.0 | 0 | 22 000 |
| Haiti | 4 400 | 0.13 | 86.0 | 0 | 4 400 |
| Honduras | 5 862 | 0.31 | 83.6 | 0 | 5 862 |
| Martinique | 5 709 | 2.09 | 71.5 | 0 | 5 709 |
| Mexico | 491 800 | 1.46 | 79.6 | 490 960 | 840 |
| Netherlands Antilles | 12 116 | 6.15 | 99.8 | 3 635 | 8 481 |
| Nicaragua | 8 000 | 0.55 | 70.0 | 0 | 8 000 |
| Panama | 25 754 | 2.48 | 85.0 | 25 124 | 630 |
| Puerto Rico | 81 724 | 3.46 | 73.8 | 75 734 | 5 990 |
| Virgin Islands (United Kingdom) | 1 | 0.01 | 0.0 | 0 | 1 |
| Virgin Islands (United States) | 3 440 | 11.47 | 0.0 | 3 440 | 0 |
| West Indies Federation: | | | | | |
| Antigua | 796 | 1.35 | 0.0 | 0 | 796 |
| Barbados | 8 811 | 3.66 | 100.0 | 8 811 | 0 |
| Cayman Islands | 52 | 0.58 | 0.0 | 0 | 52 |
| Dominica | 541 | 0.81 | 0.0 | 0 | 541 |
| Grenada | 1 380 | 1.47 | 100.0 | 1 380 | 0 |
| Jamaica | 33 049 | 1.95 | 98.3 | 33 049 | 0 |
| Montserrat | 121 | 0.86 | 0.0 | 0 | 121 |
| St. Kitts | 325 | 0.53 | 0.0 | 0 | 325 |
| St. Lucia | 650 | 0.68 | 70.2 | 0 | 650 |
| St. Vincent | 426 | 0.51 | 0.0 | 0 | 426 |
| Trinidad and Tobago | 30 751 | 3.72 | 88.2 | 30 751 | 0 |
| Turks and Caicos Islands | 45 | 0.64 | 0.0 | 0 | 45 |
| SOUTH AMERICA | | | | | |
| Argentina | 1 244 133 | 5.99 | 86.7 | 93 513 | 1 150 620 |
| Bolivia | 21 010 | 0.61 | 92.4 | 21 010 | 0 |
| Brazil | 964 013 | 1.48 | 83.0 | 927 381 | 36 632 |
| British Guiana | 5 430 | 0.97 | 15.5 | 0 | 5 430 |
| Chile | 183 519 | 2.43 | 74.4 | 183 019 | 500 |
| Colombia | 265 628 | 1.90 | 98.0 | 17 822 | 247 806 |
| Ecuador (4) | 27 000 | 0.64 | 95.0 | 500 | 26 500 |
| Falkland Islands and Dependencies | 417 | 18.95 | 0.0 | 0 | 417 |
| French Guiana | 937 | 3.02 | 0.0 | 0 | 937 |
| Paraguay | 9 883 | 0.57 | 91.3 | 0 | 9 883 |
| Peru | 102 385 | 0.96 | 85.2 | 102 385 | 0 |
| Surinam | 5 183 | 1.79 | 97.0 | 0 | 5 183 |
| Uruguay | 137 048 | 4.99 | 78.0 | 1 350 | 135 698 |
| Venezuela (4) | 180 000 | 2.70 | 97.0 | 180 000 | 0 |
| EUROPE | | | | | |
| Albania | 5 500 | 0.35 | 50.0 | 0 | 5 500 |
| Andorra | 200 | 2.86 | 0.0 | 0 | 200 |
| Austria | 653 413 | 9.25 | 93.6 | 0 | 653 413 |
| Belgium | 1 084 594 | 11.88 | 86.2 | 0 | 1 084 594 |
| Bulgaria (5) | 54 347 | 0.77 | 39.4 | 0 | 54 347 |
| Guernsey and Dependencies | 12 033 | 27.35 | 28.1 | 0 | 12 033 |
| Jersey | 16 622 | 28.17 | 62.1 | 0 | 16 622 |
| Total | 28 655 | 27.82 | 47.8 | 0 | 28 655 |
| Czechoslovakia | 936 099 | 6.88 | 80.4 | 0 | 936 099 |
| Denmark | 1 019 582 | 22.17 | 55.4 | 902 359 | 117 223 |
| Finland | 571 483 | 12.89 | 78.5 | 423 717 | 147 766 |
| France | 4 084 843 | 9.06 | 77.1 | 0 | 4 084 843 |
| Germany, Democratic Republic | 1 237 796 | 7.16 | 94.9 | 0 | 1 237 796 |
| Germany, Federal Republic | 5 516 226 | 9.98 | 99.0 | 0 | 5 516 226 |
| Gibraltar (2) | 2 559 | 9.84 | 100.0 | 0 | 2 559 |

(1) Includes Alaska; excludes Hawaii, which is listed under Oceania.
(2) Excludes telephone systems of the military forces.
(3) Data are for the fiscal year ending June 30, 1959.
(4) Estimated.
(5) January 1, 1948, the most recent date for which statistics are available.
(6) Data are for the fiscal year ending March 31, 1960.
(7) January 1, 1947, the most recent date for which statistics are available.
(8) This telephone system has been under government operation since 1949.

TELEPHONES BY COUNTRIES AS OF JANUARY 1, 1960—*Continued*

| Country or Area | | Number of Telephones | Per 100 Population | Percent Automatic | Telephones by Type of Operation | |
|---|---|---|---|---|---|---|
| | | | | | Private | Government |
| Greece | | 190 504 | 2.30 | 92.9 | 0 | 190 504 |
| Hungary | | 459 000 | 4.62 | 78.0 | 0 | 459 000 |
| Iceland | | 37 274 | 21.42 | 67.8 | 0 | 37 274 |
| Ireland | | 145 881 | 5.13 | 73.4 | 0 | 145 881 |
| Italy | | 3 517 908 | 7.15 | 96.5 | 3 517 908 | 0 |
| Liechtenstein | | 3 844 | 24.03 | 100.0 | 0 | 3 844 |
| Luxemburg | | 46 689 | 14.32 | 94.6 | 0 | 46 689 |
| Malta and Gozo | (6) | 14 842 | 4.53 | 62.5 | 0 | 14 842 |
| Monaco | | 7 252 | 34.53 | 100.0 | 0 | 7 252 |
| Netherlands | | 1 500 693 | 13.15 | 99.2 | 0 | 1 500 693 |
| Norway | (3) | 693 743 | 19.50 | 69.3 | 60 696 | 633 047 |
| Poland | | 812 358 | 2.76 | 77.4 | 0 | 812 358 |
| Portugal | | 364 858 | 4.01 | 72.8 | 252 532 | 112 326 |
| Rumania | (7) | 127 153 | 0.77 | 75.8 | 126 131 (8) | 1 022 |
| San Marino | | 400 | 2.67 | 100.0 | 0 | 400 |
| Spain | | 1 641 395 | 5.47 | 79.0 | 1 621 220 | 20 175 |
| Sweden | | 2 637 336 | 35.30 | 86.4 | 0 | 2 637 336 |
| Switzerland | | 1 562 360 | 29.65 | 100.0 | 0 | 1 562 360 |
| Turkey | (4) | 260 000 | 0.97 | 85.0 | 0 | 260 000 |
| Union Soviet Socialist Republics | | 4 022 633 | 1.91 | 49.5 | 0 | 4 022 633 |
| United Kingdom | (6) | 7 848 000 | 15.03 | 81.7 | 0 | 7 848 000 |
| Yugoslavia | | 236 292 | 1.27 | 75.1 | 0 | 236 292 |
| AFRICA | | | | | | |
| Algeria | | 178 327 | 1.70 | 82.9 | 0 | 178 327 |
| Angola | | 7 865 | 0.17 | 98.0 | 0 | 7 865 |
| Ascension Island | | 62 | 14.29 | 77.4 | 62 | 0 |
| Basutoland | | 1 000 | 0.15 | 10.0 | 0 | 1 000 |
| Bechuanaland | | 487 | 0.14 | 0.0 | 0 | 487 |
| Cameroon | | 6 659 | 0.21 | 59.0 | 0 | 6 659 |
| Cameroons, North | | 267 | 0.03 | 0.0 | 0 | 267 |
| Cameroons, South | | 200 | 0.03 | 0.0 | 0 | 200 |
| Cape Verde Islands | | 120 | 0.06 | 0.0 | 0 | 120 |
| Central African Republic | (4) | 1 000 | 0.08 | 0.0 | 0 | 1 000 |
| Chad | (4) | 2 000 | 0.08 | 0.0 | 0 | 2 000 |
| Comoro Islands | | 0 | — | — | — | — |
| Congo Republic (Formerly Belgian) | | 28 013 | 0.20 | 85.4 | 0 | 28 013 |
| Congo Republic | (4) | 4 600 | 0.56 | 50.0 | 0 | 4 600 |
| Dahomey | | 1 871 | 0.11 | 70.5 | 0 | 1 871 |
| Ethiopia | | 10 964 | 0.05 | 81.9 | 0 | 10 964 |
| Gabon | (4) | 1 000 | 0.23 | 0.0 | 0 | 1 000 |
| Gambia | | 650 | 0.22 | 98.6 | 0 | 650 |
| Ghana | | 24 917 | 0.51 | 49.7 | 0 | 24 917 |
| Guinea | (4) | 3 000 | 0.11 | 72.0 | 1 860 | 1 140 |
| Ifni | | 130 | 0.24 | 0.0 | 0 | 130 |
| Ivory Coast | | 6 286 | 0.20 | 84.7 | 0 | 6 286 |
| Kenya | | 37 159 | 0.57 | 80.8 | 0 | 37 159 |
| Liberia | (4) | 2 100 | 0.15 | 100.0 | 600 | 1 500 |
| Libya | (4) | 9 400 | 0.80 | 60.0 | 0 | 9 400 |
| Malagasy | | 12 995 | 0.24 | 46.2 | 0 | 12 995 |
| Mali, Republic of | | 2 723 | 0.06 | 64.6 | 0 | 2 723 |
| Mauritania | | 255 | 0.03 | 0.0 | 0 | 255 |
| Mauritius and Dependencies | | 8 673 | 1.38 | 8.4 | 0 | 8 673 |
| Morocco | | 129 899 | 1.22 | 88.6 | 19 758 | 110 141 |
| Mozambique | | 10 606 | 0.17 | 84.7 | 0 | 10 606 |
| Niger | | 1 321 | 0.05 | 0.0 | 0 | 1 321 |
| Nigeria | | 37 246 | 0.11 | 45.2 | 0 | 37 246 |
| Portuguese Guinea | | 369 | 0.06 | 0.0 | 0 | 369 |
| Reunion | | 5 743 | 1.76 | 0.0 | 0 | 5 743 |
| Rhodesia and Nyasaland: | | | | | | |
| Northern Rhodesia | | 21 882 | 0.92 | 93.1 | 1 639 | 20 243 |
| Nyasaland | | 5 152 | 0.18 | 89.0 | 0 | 5 152 |
| Southern Rhodesia | | 78 541 | 2.59 | 87.3 | 0 | 78 541 |
| Total | | 105 575 | 1.28 | 88.6 | 1 639 | 103 936 |
| Ruanda-Urundi | | 1 814 | 0.04 | 91.6 | 0 | 1 814 |
| Saint Helena | | 129 | 2.58 | 0.0 | 0 | 129 |
| São Tomé and Principe | | 448 | 0.69 | 84.4 | 0 | 448 |
| Senegal | | 16 619 | 0.64 | 72.7 | 0 | 16 619 |
| Seychelles and Dependencies | | 193 | 0.45 | 100.0 | 193 | 0 |
| Sierra Leone | | 4 062 | 0.17 | 78.8 | 41 | 4 021 |
| Somali | | 1 996 | 0.10 | 0.0 | 0 | 1 996 |
| Somaliland, French | | 865 | 1.24 | 100.0 | 0 | 865 |
| South West Africa | | 14 829 | 2.64 | 41.1 | 0 | 14 829 |
| Spanish Guinea | | 997 | 0.46 | 86.2 | 997 | 0 |
| Spanish North Africa | | 7 209 | 4.65 | 100.0 | 7 209 | 0 |
| Spanish Sahara | | 52 | 0.27 | 0.0 | 0 | 52 |
| Sudan | | 21 391 | 0.18 | 73.3 | 0 | 21 391 |
| Swaziland | | 1 543 | 0.55 | 50.2 | 0 | 1 543 |
| Tanganyika | | 13 711 | 0.15 | 72.7 | 0 | 13 711 |
| Togo | | 1 633 | 0.14 | 65.7 | 0 | 1 633 |
| Tunisia | | 24 762 | 0.63 | 52.1 | 0 | 24 762 |
| Uganda | | 13 217 | 0.20 | 76.9 | 0 | 13 217 |
| Union of South Africa | (6) | 918 217 | 6.17 | 68.8 | 0 | 918 217 |
| United Arab Republic: Egypt | | 219 469 | 0.86 | 81.2 | 0 | 219 469 |
| Upper Volta | | 1 242 | 0.03 | 0.0 | 0 | 1 242 |
| Zanzibar and Pemba | | 1 556 | 0.51 | 71.1 | 0 | 1 556 |

TELEPHONES BY COUNTRIES AS OF JANUARY 1, 1960—*Continued*

| Country or Area | Number of Telephones | Per 100 Population | Percent Automatic | Telephones by Type of Operation | |
|---|---|---|---|---|---|
| | | | | Private | Government |
| **ASIA** | | | | | |
| Aden Colony | 4 937 | 3.25 | 100.0 | 0 | 4 937 |
| Aden Protectorate | 0 | — | — | — | — |
| Afghanistan (4) | 7 300 | 0.06 | 30.0 | 0 | 7 300 |
| Bahrain | 3 239 | 2.23 | 100.0 | 3 239 | 0 |
| Bhutan | 0 | — | — | — | — |
| Brunei | 677 | 0.83 | 96.6 | 0 | 677 |
| Burma | 11 978 | 0.06 | 71.0 | 0 | 11 978 |
| Cambodia | 3 043 | 0.06 | 0.0 | 0 | 3 043 |
| Ceylon | 35 550 | 0.37 | 96.0 | 0 | 35 550 |
| China, Mainland (5) | 244 028 | 0.05 | 72.9 | 94 945 (8) | 149 083 |
| China, Taiwan | 67 116 | 0.64 | 60.2 | 0 | 67 116 |
| Cyprus | 16 600 | 2.95 | 96.0 | 0 | 16 600 |
| Hong Kong | 95 920 | 3.20 | 100.0 | 95 920 | 0 |
| India (6) | 424 221 | 0.10 | 56.8 | 3 374 | 420 847 |
| Indonesia | 115 006 | 0.13 | 11.5 | 0 | 115 006 |
| Iran (4) | 92 600 | 0.45 | 62.0 | 0 | 92 600 |
| Iraq (6) | 46 471 | 0.66 | 76.0 | 0 | 46 471 |
| Israel | 102 273 | 4.90 | 95.9 | 0 | 102 273 |
| Japan (6) | 4 864 858 | 5.21 | 69.6 | 4 864 858 | 0 |
| Jordan (4) | 20 500 | 1.25 | 69.0 | 0 | 20 500 |
| Korea, Republic of | 92 530 | 0.40 | 48.7 | 0 | 92 530 |
| Kuwait | 2 500 | 1.13 | 80.0 | 0 | 2 500 |
| Laos | 701 | 0.04 | 52.8 | 0 | 701 |
| Lebanon | 45 239 | 2.64 | 93.1 | 0 | 45 239 |
| Macao | 2 036 | 0.94 | 100.0 | 0 | 2 036 |
| Malaya | 70 238 | 1.03 | 70.2 | 0 | 70 238 |
| Maldive Islands | 0 | — | — | — | — |
| Muscat and Oman | 201 | 0.04 | 100.0 | 201 | 0 |
| Nepal | 0 | — | — | — | — |
| Netherlands New Guinea | 1 852 | 0.26 | 13.2 | 0 | 1 852 |
| North Borneo | 2 797 | 0.66 | 97.4 | 0 | 2 797 |
| Pakistan | 69 579 | 0.08 | 67.0 | 0 | 69 579 |
| Philippine Republic | 99 973 | 0.40 | 72.0 | 87 272 | 12 701 |
| Portuguese India | 548 | 0.08 | 0.0 | 0 | 548 |
| Portuguese Timor | 494 | 0.10 | 0.0 | 0 | 494 |
| Qatar | 1 612 | 3.66 | 100.0 | 1 612 | 0 |
| Ryukyu Islands (2) | 7 168 | 0.83 | 35.2 | 0 | 7 168 |
| Sarawak | 2 983 | 0.44 | 72.8 | 0 | 2 983 |
| Saudi Arabia (3) | 22 865 | 0.35 | 37.5 | 0 | 22 865 |
| Sikkim | 0 | — | — | — | — |
| Singapore | 57 523 | 3.57 | 100.0 | 0 | 57 523 |
| Thailand | 38 247 | 0.17 | 83.7 | 32 027 | 6 220 |
| Trucial Oman | 0 | — | — | — | — |
| United Arab Republic: Syria | 48 900 | 1.05 | 84.0 | 0 | 48 900 |
| Viet-Nam, Republic of | 14 519 | 0.10 | 80.9 | 0 | 14 519 |
| Yemen | 0 | — | — | — | — |
| **OCEANIA** | | | | | |
| Australia | 2 122 251 | 20.88 | 76.5 | 0 | 2 122 251 |
| British Soloman Islands | 342 | 0.32 | 0.0 | 0 | 342 |
| Caroline Islands | 250 | 0.58 | 0.0 | 0 | 250 |
| Christmas Island | 50 | 1.67 | 100.0 | 50 | 0 |
| Cocos (Keeling) Islands | 61 | 6.10 | 100.0 | 0 | 61 |
| Cook Islands | 273 | 1.52 | 0.0 | 0 | 273 |
| Fiji Islands | 6 624 | 1.71 | 58.4 | 0 | 6 624 |
| Gilbert and Ellice Islands | 0 | — | — | — | — |
| Guam | 13 263 | 18.42 | 100.0 | 0 | 13 263 |
| Mariana Islands (less Guam) | 350 | 4.38 | 71.4 | 0 | 350 |
| Marshall Islands | 651 | 4.65 | 100.0 | 0 | 651 |
| Nauru | 0 | — | — | — | — |
| New Caledonia and Dependencies | 3 069 | 4.32 | 72.4 | 0 | 3 069 |
| New Hebrides Condominium | 310 | 0.53 | 79.0 | 0 | 310 |
| New Zealand (6) | 686 021 | 28.95 | 71.3 | 0 | 686 021 |
| Niue Island | 79 | 1.58 | 0.0 | 0 | 79 |
| Norfolk Island | 50 | 5.00 | 0.0 | 0 | 50 |
| Papua and New Guinea | 5 262 | 0.28 | 79.9 | 0 | 5 262 |
| Pitcairn Island | 0 | — | — | — | — |
| Polynesia, French | 1 136 | 1.40 | 0.0 | 0 | 1 136 |
| Samoa, American | 370 | 1.85 | 100.0 | 0 | 370 |
| Samoa, Western | 777 | 0.74 | 0.0 | 0 | 777 |
| Tokelau Islands | 0 | — | — | — | — |
| Tonga (Friendly) Islands | 620 | 0.98 | 0.0 | 0 | 620 |
| United States: Hawaii | 223 464 | 33.76 | 100.0 | 223 464 | 0 |

## TELEPHONE CONVERSATIONS DURING 1959

Data were not available for all countries

| Country or Area | Thousands of Conversations | | | Average Conversation Per Person |
|---|---|---|---|---|
| | Local | Long Distance | Total | |
| Algeria | 108 300 | 19 800 | 128 100 | 12.4 |
| Argentina | 3 678 200 | 44 400 | 3 722 600 | 180.6 |
| Australia | 1 400 000 | 130 000 | 1 530 000 | 152.1 |
| Belgium | 563 800 | 112 600 | 676 400 | 74.3 |
| Bermuda | 13 500 | 27 | 13 527 | 307.4 |
| Brazil | 5 217 500 | 78 400 | 5 295 900 | 82.5 |
| Canada | 9 044 800 | 205 400 | 9 250 200 | 530.3 |
| Ceylon | 84 400 | 5 700 | 90 100 | 9.4 |
| Channel Islands | 15 000 | 600 | 15 600 | 151.5 |
| Chile | 431 300 | 22 100 | 453 400 | 60.7 |
| Colombia | 803 500 | 4 900 | 808 400 | 58.5 |
| Costa Rica | 49 900 | 1 000 | 50 900 | 45.2 |
| Cuba | 568 400 | 7 500 | 575 900 | 86.0 |
| Cyprus | 14 600 | 1 800 | 16 400 | 29.4 |
| Czechoslovakia | 698 500 | 92 700 | 791 200 | 58.3 |
| Denmark | 1 080 900 | 230 100 | 1 311 000 | 288.3 |
| Dominican Republic | 67 100 | 400 | 67 500 | 23.3 |
| United Arab Republic: Egypt | 563 700 | 15 000 | 578 700 | 22.8 |
| Ethiopia | 25 300 | 900 | 26 200 | 1.2 |
| Germany, Democratic Republic | 776 700 | 156 100 | 932 800 | 54.0 |
| Germany, Federal Republic | 3 205 800 | 928 200 | 4 134 000 | 75.2 |
| Ghana | 18 600 | 3 500 | 22 100 | 4.5 |
| Greece | 450 600 | 9 700 | 460 300 | 55.7 |
| Iceland | 85 500 | 2 000 | 87 500 | 508.7 |
| Ireland | 117 100 | 16 500 | 133 600 | 46.9 |
| Israel | 217 900 | 5 500 | 223 400 | 108.4 |
| Italy | 5 008 000 | 413 900 (1) | 5 421 900 | 110.5 |
| Jamaica, West Indies | 120 000 | 1 100 | 121 100 | 72.5 |
| Japan (2) | 12 000 000 | 842 700 | 12 842 700 | 138.5 |
| Lebanon | 66 700 | 10 300 | 77 000 | 45.3 |
| Malaya | 196 000 | 17 100 | 213 100 | 31.8 |
| Mexico | 973 700 | 16 000 | 989 700 | 29.7 |
| Mozambique | 14 600 | 500 | 15 100 | 2.4 |
| Netherlands | 978 600 | 404 800 | 1 383 400 | 121.9 |
| Nigeria | 27 000 | 2 800 | 29 800 | 0.9 |
| Norway (3) | 515 700 | 49 800 | 565 500 | 159.7 |
| Philippine Republic | 621 600 | 1 200 | 622 800 | 25.2 |
| Portugal | 388 600 | 45 600 | 434 200 | 48.0 |
| Puerto Rico | 154 400 | 3 500 | 157 900 | 67.3 |
| Singapore | 150 900 | 1 100 | 152 000 | 96.2 |
| South West Africa | 14 300 | 2 200 | 16 500 | 29.8 |
| Spain | 2 432 400 | 113 800 | 2 546 200 | 85.2 |
| Sweden (4) | 2 408 100 | 110 200 | 2 518 300 | 337.1 |
| Switzerland | 610 800 | 523 800 (1) | 1 134 600 | 216.7 |
| Thailand | 65 061 | 32 | 65 093 | 3.0 |
| Trinidad and Tobago, West Indies | 94 700 | 8 300 | 103 000 | 126.1 |
| Tunisia | 28 800 | 7 200 | 36 000 | 9.2 |
| United Kingdom (2) | 3 842 000 | 386 000 | 4 228 000 | 81.3 |
| United States | 84 971 000 | 3 206 000 | 88 177 000 | 496.1 |
| Uruguay | 380 400 | 7 400 | 387 800 | 142.1 |
| Viet-Nam, Republic of | 18 500 | 200 | 18 700 | 1.4 |
| Yugoslavia | 371 800 | 29 600 | 401 400 | 21.8 |

(1) Three-minute time-units. The number of completed conversations is not available.
(2) Conversations during the year ending March 31, 1960.
(3) Conversations during the year ending June 30, 1959.
(4) Conversations during the year ending June 30, 1960.

# United States Patents Issued to International Telephone and Telegraph System; February–October 1960

Between February 1 and October 31, 1960, the United States Patent Office issued 233 patents to the International System. The names of the inventors, company affiliations, subjects, and patent numbers are listed below.

H. H. Abelew, Mackay Radio and Telegraph Company, Radio Receiver with Means to Detect Signals Below Noise Level, 2 956 151.

R. T. Adams, ITT Laboratories, Gyroscope, 2 947 178.

P. R. Adams, B. Alexander, and R. I. Colin, ITT Laboratories, Aerial Navigation Indicator, 2 938 204.

P. R. Adams, G. B. Speen, and C. C. Miller, Jr., ITT Laboratories, Gyroscopes, Gyroscope Stabilized Systems, and Gas Bearings for Same, 2 940 318.

H. Adelaar, Bell Telephone Manufacturing Company (Antwerp), Automatic Starting Circuit for Electrical Pulse Distributors, 2 928 939.

D. F. Albanese, ITT Laboratories, Time-Position Modulator Using Nonlinear Saturable Element, 2 928 054.

M. Arditi, ITT Laboratories, Atomic Frequency Standard, 2 951 992.

M. Arditi, ITT Laboratories, Directional Couplings, 2 951 218.

M. Arditi, ITT Laboratories, Gas Cell for Frequency-Selective System, 2 955 262.

J. Augustin, and H. Wlodarczak, Lorenz Werke (Stuttgart), One-Way Link System Between Tape Back-Spacing Means and Lever-Operated Code Cancelling Punch, 2 937 702.

M. C. E. Batalille, Laboratoire Central de Télécommunications (Paris), Impulse Counting and Storing Circuit, 2 928 988.

A. H. W. Beck, Standard Telecommunication Laboratories Limited (London), Electron Velocity-Modulation Tubes, 2 939 991.

J. I. Bellamy and T. L. Bowers, Kellogg Switchboard and Supply Company, Switching System and Translator Therefor, 2 928 903.

W. Berthold, J. Rottgardt, and G. Heller, Lorenz Werke (Stuttgart), Storage Tube, 2 950 409.

J. Bernutz, Mix & Genest Werke, (Stuttgart), Multicontact Arrangement for Multiswitches, 2 936 340.

J. F. Bigelow, Capehart–Farnsworth Company, Electric Filter, 2 927 205.

N. A. Blake, Federal Electric Corporation, Distortion-Correcting System, 2 939 910.

R. A. Boomer and P. E. Alexander, Federal Electric Corporation, Pilot-Light Assembly, 2 932 820.

F. H. Bray and R. G. Knight, Standard Telephones and Cables Limited (London), Automatic Telecommunication Exchange Equipment, 2 954 438.

T. G. Brown, Jr., ITT Laboratories, Delay Device, 2 951 906.

C. Buff, Mackay Radio and Telegraph Company, Narrow-to-Wide-Band Converter, 2 932 730.

H. Burr, Standard Telephones and Cables Limited (London), Electric Cables, 2 947 652.

H. G. Busignies, P. R. Adams, and M. Rogoff, ITT Laboratories, Electronic Spectroanalyzer Systems, 2 927 501.

L. R. Cain, Kellogg Switchboard and Supply Company, Telephone System with Digit-Translating Trunk Repeaters, 2 926 218.

E. F. Carr, Federal Electric Corporation, Modulation-Synchronizing Pulse Generator, 2 952 811.

E. F. Carr, Federal Electric Corporation, Monostable Multivibrator, 2 952 784.

A. M. Casabona, J. S. Engel, and C. Lucanera, ITT Laboratories, Omnirange Beacon Antennas, 2 939 141.

K. W. Cattermole, Standard Telephones and Cables Limited (London), Electric Pulse Communication Systems, 2 927 162.

K. W. Cattermole and R. B. Herman, Standard Telephones and Cables Limited (London), Pulse-Producing Device, 2 956 176.

G. Chamberlain and M. P. Nashman, ITT Laboratories, Waveguide Coupling, 2 928 059.

P. A. Childs, S. F. Gold, and W. A. G. Walsh, Standard Telephones and Cables Limited (London), Equipment Unit Mounting Arrangements, 2 950 156.

P. A. Childs, S. F. Gold, and W. A. G. Walsh, Standard Telephones and Cables Limited (London), Multiple-Panel Mounting Arrangement, 2 928 555.

J. L. Culbertson, Kellogg Switchboard and Supply Company, Toll-Discriminating Outgoing Repeater System, 2 934 605.

J. L. Culbertson, Kellogg Switchboard and Supply Company, Voltage Multiplier, 2 956 183.

P. P. Danesi, Royal Electric Corporation, Cord Connector, Design Patent 188 419.

D. W. Davis, ITT Laboratories, Image Reproducing Device, 2 925 525.

D. W. Davis, ITT Laboratories, Storage-Tube Construction, 2 932 764.

K. L. DeBrosse and W. J. Williams, ITT Laboratories, Display Centering Device, 2 931 940.

E. deFaymoreau and M. Mandel, ITT Laboratories, Azimuth Receiver, 2 924 822.

E. M. Deloraine and H. G. Busignies, ITT Laboratories, Radio Receiving and Transmitting System, 2 943 318.

E. M. Deloraine and H. G. Busignies, International Telephone and Radio Corporation, Radio Repeating System, 2 931 031.

E. M. Deloraine, H. G. Busignies, and P. G. Chevigny, International Telephone and Radio Manufacturing Corporation and Federal Telephone and Radio Company, Protective Circuit for Radio Repeater, 2 950 400.

A. R. Denz and E. B. DuBois, Kellogg Switchboard and Supply Company, Adjustable Equalizer, 2 926 314.

G. A. Deschamps, ITT Laboratories, Rotation-Sensing Device, 2 954 700.

M. Dishal and M. Rogoff, ITT Laboratories, Aerial-Navigation Beacon System, 2 924 820.

S. H. M. Dodington, ITT Laboratories, Antenna Decoupling Arrangement, 2 947 987.

S. H. M. Dodington, ITT Laboratories, Delay Device, 2 947 954.

S. H. M. Dodington and R. L. Johnson, ITT Laboratories, Comparison Circuits, 2 924 821.

D. L. A. Driver, Standard Telephones and Cables Limited (London), Article-Holding Devices Controlled by Shock, 2 939 668.

R. F. Durst and A. E. Machala, ITT Components Division, Pressure-Resistant Insulators for Selenium Rectifiers, 2 934 683.

H. Eggerding, ITT Laboratories, Construction of Electrical Devices, 2 955 260.

H. F. Engelmann and M. Arditi, ITT Laboratories, Adjustable Attenuator, 2 924 793.

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P. T. Farnsworth, Federal Electric Corporation, Cathode-Ray Tube, 2 941 100.

P. T. Farnsworth, Capehart–Farnsworth Corporation, Color Television Receiver, 2 951 113.

H. Feissel, Laboratoire Central de Télécommunications (Paris), Pulse-Code-Modulation Demodulator, Reissue Patent 24 790.

F. R. Fegan, ITT Components Division, Rectifier Stack and Spacing Contact Washer Therefor, 2 934 684.

L. G. Fischer, ITT Laboratories, Voltage Ratio Indicators, 2 928 091.

H. Fliegner, Mix & Genest Werke (Stuttgart), System for Separating and Conveying Flat Articles, 2 952 457.

S. G. Fong, ITT Laboratories, Radiation-Sensitive Device, 2 951 944.

O. C. From, ITT Laboratories, Magnetic-Field Detecting Device, 2 930 974.

K. N. Fromm, Federal Electric Corporation, Frequency-Determining System, 2 926 304.

K. N. Fromm and N. A. Blake, ITT Laboratories, Flat-Copy Scanner, 2 941 033.

T. Fujii, Nippon Electric Company, Limited (Tokyo), Reflex Klystrons, 2 950 415.

S. Gallee, Lorenz Werke (Stuttgart), Switch-Controlled Counting System, 2 927 206.

R. F. Gates and P. E. Kendall, Federal Electric Corporation, Wave Analyzer, 2 927 272.

R. H. Geiger, ITT Laboratories, Traveling-Wave Electron-Discharge Device, 2 924 739.

J. H. Gesell, ITT Laboratories, Coaxial Cable, 2 934 586.

J. C. Gibson and T. L. Bowers, Kellogg Switchboard and Supply Company, Nonblocking Link Access, 2 945 211.

F. P. Gohorel, Compagnie Générale de Constructions Téléphoniques (Paris), Automatic Telephone Systems, 2 932 695.

J. Grambow, Schaub Werk (Pforzheim), Broadcasting Receiver, 2 924 662.

H. Grayson, R. A. G. Dunkley, and T. H Walker, Standard Telecommunication Laboratories Limited (London), Magnetic Switch-Amplifiers, 2 947 946.

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G. Grimsen and J. Augustin, Lorenz Werke (Stuttgart), Tabulator Mechanism for Teleprinters, 2 956 120.

J. C. Groce, J. K. Bates, and D. G. Hunt, ITT Laboratories, Signal Translation System, 2 930 987.

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L. B. Haigh and J. I. Bellamy, Kellogg Switchboard and Supply Company, Terminal-Per-Station Television Party-Line Switching System, 2 929 881.

J. Handley, Creed & Company Limited (Croydon), Printing Telegraph Apparatus, 2 944 111.

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W. Hatton and G. F. McCarthy, ITT Laboratories, Coordinate Switch, 2 942 069.

H. Havstad, ITT Laboratories, Amplitude-Sensitive Circuit, 2 928 002.

W. H. Heiser, ITT Laboratories, Multiar Circuit, 2 934 707.

J. A. Henderson, D. L. Johnson, and E. W. Koenig, Federal Electric Corporation, System for Separating and Extracting Related Recorded Information, 2 941 083.

J. F. Heney and P. E. Dorney, ITT Laboratories, Gas Discharge Tube, 2 945 149.

A. J. Henquet and J. J. Perrot, Compagnie Générale de Constructions Téléphoniques (Paris), Concentrators of Telephone Lines or the Like, 2 944 115.

R. J. Heppe, ITT Laboratories, Synchronizing System, 2 947 981.

W. G. Hill, Standard Telephones and Cables Limited (London), Coaxial Cable Wound on a Drum and Process of Winding, 2 939 907.

R. C. P. Hinton and L. C. Deschuytere, ITT Laboratories, Party-Line Detector System, 2 938 956.

R. C. P. Hinton, B. Dzula, and A. L. M. Fettweis, ITT Laboratories, Tape-to-Card Converter, 2 945 221.

R. C. P. Hinton, R. B. Page, and J. Larkin, ITT Laboratories, Error-Detection Systems, 2 950 464.

J. D. Holland and R. A. Buck, Standard Telephones and Cables Limited (London), Radio Telegraph Systems, 2 938 077.

O. Holstein, Lorenz Werke (Stuttgart), Clutch Apparatus, 2 924 317.

O. A. H. Holstein, Lorenz Werke (Stuttgart), Function Controling Apparatus for Printing, 2 939 911.

J. B. Horodyski, ITT Laboratories, High-Voltage Pulse Generator, 2 947 884.

H. L. Horwitz and M. E. Homan, Federal Telephone and Radio Company, Party-Line Automatic Telephone System, 2 924 663.

R. W. Hughes, ITT Laboratories, Data-Processing Transposition System, 2 955 280.

R. W. Hughes, ITT Laboratories, Hybrid Circuits, 2 947 952.

G. D. Hulst, ITT Laboratories, Analog-to-Digital Translator, 2 943 311.

G. D. Hulst, ITT Laboratories, Readout System, 2 934 653.

R. W. Hunter, ITT Laboratories, Information Storage Tube, 2 927 239.

H. F. Jarger, ITT Laboratories, Electromechanical Filter Arrangement, 2 928 057.

K. Jekelius, Lorenz Werke (Stuttgart), Amplitude Limiter, Particularly for Television Signals, 2 923 768.

N. C. Joehlin, ITT Laboratories, Pulse-Code Translator, 2 943 310.

C. E. Jones, Jr., ITT Laboratories, Narrow-Band Video Communication System, 2 955 159.

A. G. Kandoian, ITT Laboratories, Aircraft Antenna System, 2 934 761.

M. A. Karpeles, ITT Laboratories, Course-Guidance System, 2 943 321.

A. J. Katz, ITT Laboratories, Radio Altimeter Systems, 2 928 085.

A. Kleinle and R. F. Weiss, Mix & Genest Werke (Stuttgart), Selenium Rectifier, 2 956 218.

J. I. Kovalsky, ITT Laboratories, Cooling System for Electronic Apparatus, 2 933 903.

I. A. Krause, ITT Laboratories, Single-Sideband Communication System, 2 938 114.

P. W. Kruse, ITT Laboratories, Radiation Amplifier, 2 928 006.

A. P. Kundrotas, Kellogg Switchboard and Supply Company, Fast-Switching Transistor Telegraph Repeater, 2 937 236.

J. J. B. Lair, ITT Laboratories, Detector Circuit, 2 939 003.

G. Laube, Lorenz Werke (Stuttgart), Space-Function Release Arrangement for Printing Telegraph System, 2 956 122.

J. S. LeGrand and R. H. Myers, ITT Laboratories, Sampling and Correcting System, 2 952 016.

H. S. Leman, Standard Telephones and Cables Limited (London), Telephone Subscribers' Instruments, 2 950 351.

J. A. Leno and A. M. Searle, Standard Telephones and Cables Limited (London), Printed Electrical Circuits, 2 943 966.

H. L. Levin, ITT Laboratories, Electron Gun, 2 938 134.

H. L. Levin, ITT Laboratories, Electron-Gun Support, 2 943 227.

H. L. Levin, ITT Laboratories, Traveling-Wave Electron-Discharge Device, 2 928 019.

L. Lewin, Standard Telephones and Cables Limited (London), Waveguide Coupling Arrangement, 2 952 820.

A. Lieb, Lorenz Werke (Stuttgart), Voltage-Indicator Tubes, 2 927 237.

P. E. Lighty, ITT Laboratories, Semiconductor Devices, 2 928 030.

B. B. Mahler and S. D. Hammond, ITT Laboratories, Position Selector System, 2 951 194.

M. Mandel, ITT Laboratories, Aerial-Navigation System, 2 938 205.

T. J. Marchese, ITT Laboratories, Constant-Impedance Attenuator, 2 951 999.

T. J. Marchese, ITT Laboratories, Traveling-Wave Electron-Discharge Device, 2 927 832.

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F. P. Mason, Creed & Company Limited (Croydon), Circuits for Providing a Variable Bias Potential, 2 954 428.

M. J. Maynard, ITT Laboratories, Path-Recording System, 2 927 833.

G. F. McCarthy and A. N. Gulnick, ITT Laboratories, Coordinate Switching Unit, 2 942 068.

E. M. S. McWhirter, International Standard Trading Corporation, Photographic Control Apparatus, 2 947 217.

O. S. Meixell, ITT Laboratories, Radio-Direction-Finding System, 2 928 089.

L. C. Merrill, Federal Electric Corporation, Glow Lamp, 2 952 793.

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F. A. Muller, ITT Laboratories, Transistor, 2 939 056.

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Y. Nannichi, Nippon Electric Company, Limited (Tokyo), Transistor, 2 940 023.

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B. J. Norris, Federal Electric Corporation, Screen-Grid Amplifier, 2 926 309.

F. H. Numrich, ITT Laboratories, Signal Detection Circuit, 2 934 642.

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A. D. Odell and H. F. Hartley, Standard Telecommunication Laboratories Limited (London), Electrical Circuit Using Two-Electrode Devices, 2 944 164.

R. K. Orthuber and G. Papp, ITT Laboratories, Electron-Multiplier Mosaics, 2 931 914.

R. K. Orthuber, L. R. Ullery, and C. V. Stanley, ITT Laboratories, Radiation-Amplifying Device, 2 936 379.

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E. G. Parker, ITT Laboratories, Omnidirectional Beacon Antenna, 2 928 087.

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S. B. Pickles and J. S. Engel, ITT Laboratories, Omnirange Beacon Antenna, 2 938 208.

R. L. Plouffe, Jr. and R. E. Hufnagel, ITT Laboratories, Instantaneous Amplitude Compressor and Expanders for Pulse-Time-Modulation Transmission Systems, 2 951 905.

L. Pollack and H. Goldman, ITT Laboratories, Velocity-Modulation Electron-Discharge Device, 2 934 672.

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A. Preisman, International Telephone and Radio Manufacturing Corporation and Federal Telephone and Radio Company, Radio Jamming Systems, 2 953 677.

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G. C. Randolph and G. R. Polem, ITT Laboratories, Voltage-Ratio Device, 2 934 274.

N. Raver, ITT Laboratories, Bistable Circuit, 2 955 214.

R. C. Renick and H. J. Mills, ITT Laboratories, Signal Decoder, 2 928 045.

L. J. Rennenkampt, ITT Laboratories, Automatic Frequency Search and Track System, 2 951 150.

A. G. Richardson, ITT Laboratories, Positional Orientation Evaluator, 2 930 129.

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O. Shapiro and R. E. Troell, ITT Laboratories, Wide-Band Automatic-Frequency-Control Systems, 2 924 783.

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K. J. Staller, Federal Telephone and Radio Company, Electrical Lead Conductor, 2 927 953.

A. T. Starr, K. W. Cattermole, and J. Price, Standard Telephones and Cables Limited (London), Pulse Coding Arrangement for Electric Communication Systems, 2 954 550.

K. Steinbuch, Mix & Genest Werke (Stuttgart), Method and Apparatus for Determining the Position of a Printed or Written Item with Respect to an Identification Mark Applied Thereon, 2 947 972.

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H. F. Sterling, Standard Telecommunication Laboratories Limited (London), Manufacture of Semiconductor Material, 2 955 966.

I. R. Studebaker and W. W. Greutman, Federal Electric Corporation, Cathode-Ray Tube System, 2 951 180.

H. H. Synder, ITT Laboratories, Traveling-Wave Electron-Discharge Device, 2 938 138.

H. Stobbe, A. Glas, and H. Heitmann, Mix & Genest Werke (Stuttgart), Line Identification Arrangements, Particularly for Telephone Systems, 2 945 916.

M. A. Strivens, Standard Telecommunication Laboratories Limited (London), Formation of Ceramic Mouldings, 2 939 199.

R. D. Sutherland and W. A. Catchpole, Standard Telephones and Cables Limited (London), Semiconductor Devices, 2 939 205.

H. E. Thomas and S. A. DeMars, ITT Laboratories, Automatic Damping Means in Deflection Circuits, 2 927 998.

M. F. Toohig, ITT Laboratories, Method of Increasing the Thickness of Fine-Mesh Metallic Screens, 2 940 873.

M. F. Toohig, ITT Laboratories, Method of Making Storage Electrode for Charge-Storage Tube, 2 947 651.

F. P. Turvey, ITT Laboratories, Gated-Trigger Predetermined Binary Counter, 2 947 944.

E. H. Ullrich and G. F. Klepp, Standard Telephones and Cables Limited (London), Electric Discharge Devices, 2 928 976.

R. Urtel, Lorenz Werke (Stuttgart), Method and Apparatus of Controlling Direct-Current Components in Television Signals, 2 936 335.

M. C. Vosburgh, E. Baum, and S. A. Demars, ITT Laboratories, Electronic Marker Generator, 2 938 949.

R. R. Waer, ITT Laboratories, Automatic Phasing System for Multichannel Pulse-Code-Modulation Systems, 2 927 965.

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A. W. Wallens and B. W. Evershed, Creed & Company Limited (Croydon), Tape Storage Apparatus, 2 956 270.

H. Walter, Lorenz Werke (Stuttgart), Electromagnetic Relay Spring Assembly, 2 956 140.

H. Walter, Lorenz Werke (Stuttgart), Relays with Guided Contact Spring, 2 939 932.

M. Wagner, Suddeutsche Apparatefabrik Gesellschaft m.b.H. (Nuremberg), Electrolytic Capacitor, 2 923 866.

B. A. Wallace and E. Van Wie, Kellogg Switchboard and Supply Company, Armature Retaining Assembly for an Electromagnetic Relay, 2 929 001.

D. A. Weir, Standard Telecommunication Laboratories Limited (London), Testing Arrangements, 2 936 443.

D. A. Weir and J. Rice, Standard Telecommunication Laboratories Limited (London), Binary Adder–Subtracter, 2 926 851.

R. C. Wertman, ITT Laboratories, Traveling-Wave Electron-Discharge Device, 2 934 674.

A. D. White, ITT Laboratories, Gas-Tube Microwave Detector, 2 928 000.

W. J. Williams and P. E. Kendall, Federal Electric Corporation, Black-Body Radiation Source, 2 952 762.

R. H. Williamson, Kellogg Switchboard and Supply Company, Decade Counting Device, 2 936 955.

J. G. Willis, Standard Telephones and Cables Limited (London), Carrier-Wave Modulators and Demodulators, 2 943 271.

E. P. G. Wright and C. Parker, Standard Telecommunication Laboratories Limited (London), Mounting Arrangements for Components of Electrical Circuits, 2 942 332.

E. P. G. Wright and J. Rice, Standard Telecommunication Laboratories Limited (London), Computing Systems, 2 927 306.

E. P. G. Wright and J. Rice, Standard Telecommunication Laboratories Limited (London), Information Storage Equipment, 2 932 009.

E. P. G. Wright and J. Rice, Standard Telecommunication Laboratories Limited (London), Timing Equipment, 2 927 305.

E. P. G. Wright, J. Rice, and J. D. Reynolds, Standard Telecommunication Laboratories Limited (London), Electrical Storage of Information, 2 932 688.

E. P. G. Wright, D. A. Weir, and J. Rice, Standard Telecommunication Laboratories Limited (London), Electric Signaling Systems, 2 952 732.

E. P. G. Wright, D. A. Weir, and J. Rice, Standard Telecommunication Laboratories Limited (London), Teleprinter Exchange System Incorporating Storage Devices, 2 952 731.

E. P. G. Wright, D. A. Weir, and J. Rice, Standard Telephones and Cables Limited (London), Information-Handling Equipment, 2 945 625.

S. Yamato and T. Sudo, Nippon Electric Company, Limited (Tokyo), Communication System with Interlocking Calling System, 2 932 729.

N. H. Young, Jr., ITT Laboratories, Direction-Finding System with Phase Comparing and Indicating System, 2 928 088.

N. H. Young, Jr., ITT Laboratories, Electric Amplifier Control, 2 928 051.

## Pulse-Code-Modulation Demodulator

Reissue 24 790

H. Feissel

A pulse-code demodulator in which the individual pulses of the signal are weighted in amplitude in accordance with the pulse position and separately delayed to occur simultaneously at a given position, and are then combined to produce a composite pulse of an amplitude dependent on the code composition. This arrangement is believed to be used in many pulse-code demodulators.

## Transistor

2 939 056

F. A. Muller

The patent discloses a *pnpn* transistor in which connections are made to three of the four layers, the unconnected layer between the base and the collector being known as a "hook" because it acts to trap electrons or holes. This imparts a desirable snap-action or toggle effect to the transistor, which is now generally called a controlled rectifier. The patent covers such a device in which the emitter has been made by alloying, which is the most-common method for manufacturing silicon controlled rectifiers at present. Controlled rectifiers are expected to be of great value in the electric power control field.

## Method and Apparatus for Determining the Position of a Printed or Written Item with Respect to an Identification Mark Applied Thereto

2 947 972

K. Steinbuch

A circuit for determining the position of a stamp on envelopes that have varying degrees of contrast and may also have writing. By photoscanning, a wave is obtained having narrow peak pulses for the writing and broad pulse of lower peak amplitude in the stamp area. The wave passes through two parallel paths. One path has a low-pass filter, reducing the narrow peaks and producing a pulse of higher amplitude than these peaks at the stamp location. The other path has a high-pass filter, a rectifier to pass only the negative components, and then a low-pass filter. Thus, the second path produces a negative wave corresponding largely to the components of the original wave representing the writing. The waves from the two paths are combined algebraically, leaving a broad pulse corresponding to the stamp.

## Electrical Storage of Intelligence

2 932 688

E. P. G. Wright, J. Rice, and J. D. Reynolds

An arrangement designed for multiplex telegraphy in which the messages are individually stored in the form of coded electric signals on a plurality of storage tracks. Each message has an identification signal and is also given a code indicating its priority. An arrangement is then provided to remove the messages from the storage record in succession by priority by a series of control pulses. This is part of Strad.

## Course Guidance System

2 943 321

M. A. Karpeles

An airborne equipment for use with beacons such as tacan, which will enable the craft to maintain a course when beacon signals are lost. A calculator determines the direction of heading and velocity from the received beacon signals and corrects for deviations from course. Upon loss of signals, the stored information is used to calculate the proper heading by reverse operation.

## Contributors to This Issue

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H. W. Haffter was born in Berne, Switzerland, on 16 August 1900. After studying electrical engineering, he spent a short time in England.

In 1925, he joined the circuit laboratory of Bell Telephone Manufacturing Company in Antwerp and was soon transferred to the circuit design department. After 4 years in Switzerland, he returned to Antwerp in 1938. In 1940, he was transferred to Standard Telephone & Radio in Zurich, where he is head of the circuit department.

Mr. Haffter is a member of the Institution of Swiss Engineers and Architects. He has lectured on automatic telephony at the Evening College of Technology in Zurich.

S. R. HOH

S. R. Hoh was born on 24 February 1915 in Germany. He received the degree of Dipl.-Ing. from the Technische Hochschule Danzig in physics in 1941.

From 1941 to 1945, he was engaged in research and development in the supersonic missile field. After the war, he came to the United States to work at Wright Field for the United States Air Force on instrumentation and materials.

In 1958, Mr. Hoh joined ITT Federal Laboratories, where he has been active in the development of energy converter systems for the generation of electric power and of ferroelectric materials and devices.

A. E. KARBOWIAK

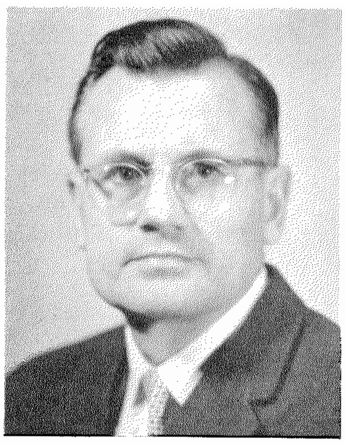
A. E. Karbowiak was born on 1 March 1923 in Warsaw, Poland. He received the B.Sc. and the Ph.D. degrees in electrical engineering from London University.

From 1951 until 1954, he was at University College, London, carrying out research on surface waves. In 1954, he joined Standard Tele-

H. W. HAFFTER



S. R. HOH



A. E. KARBOWIAK

communications Laboratories, where he was responsible for research on various aspects of long-distance communication by waveguide. Since 1958, he has been head of the microwave laboratory.

He is an Associate Member of the Institution of Electrical Engineers, England.

### LASZLO SOLYMAR

L. Solymar was born in Budapest, Hungary, on 24 January 1930. He received the Diploma of Electrical Engineering from the Technical University of Budapest in 1952. He served as a faculty assistant from 1952 to 1953. A higher Hungarian degree was conferred on him in 1956.

From 1953 to 1956, he was a research engineer at the Research Institute of Telecommunication in Budapest. He then joined the research staff of Standard Telecommunication Laboratories, where he has worked on the generation and transmission of microwaves.

### IWAO SUGAI

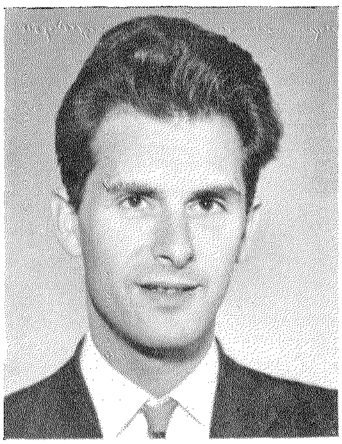
Iwao Sugai was born in Tokyo, Japan, in 1928. He received a B.S. degree in engineering from the University of California at Los Angeles in 1955 and an M.S. degree from California Institute of Technology in 1956.

During 1958 and 1959, he was an associate engineer at the IBM Research Center.

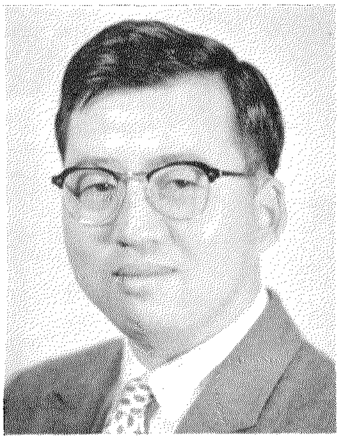
In November 1959, he joined ITT Federal Laboratories as a development engineer in the electron-tube laboratory. His interests are in the application of digital computers and nonlinear differential equations to the nonlinear-field theories of electron tubes. He is the author of two mathematical papers in this issue.

Mr. Sugai is a member of the Institute of Radio Engineers and of its Professional Group on Electron Devices.

LASZLO SOLYMAR



IWAO SUGAI



# International Telephone and Telegraph Corporation

## Principal Divisions and Subsidiaries

### North America

#### COMMERCIAL GROUP

Components Division, Clifton, N. J.
  Kuthe Laboratories, Inc., Newark, N. J.
Industrial Products Division, San Fernando, Calif.
International Standard Electric Corporation, New York
IT&T Electronics Service Company of Canada Ltd., Town of Mount Royal, P.Q.
ITT Distributor Products Division, Lodi, N. J.
ITT Intelex Service Division, New York
  Airmatic Systems Corporation, Rochelle Park, N. J.
  Intelex Systems Incorporated, New York
ITT Kellogg, Chicago
Kellogg Credit Corporation, New York
Royal Electric Company (Quebec) Limited, Pointe Claire, P.Q.
Royal Electric Corporation, Pawtucket, R. I.
  Electric Cords & Supply Corporation, Los Angeles
Standard Telephones & Cables Mfg. Co. (Canada) Ltd., Montreal

#### DEFENSE GROUP

Federal Electric Corporation, Paramus, N. J.
International Electric Corporation, Paramus, N. J.
ITT Communication Systems, Inc., Paramus, N. J.
ITT Federal Laboratories, Nutley, N. J.

### Europe, Middle East, Africa

#### AUSTRIA

Standard Telephon und Telegraphen Aktiengesellschaft, Czeija, Nissl & Co., Vienna

#### BELGIUM

Bell Telephone Manufacturing Company, Antwerp
ITT Europe, Inc., Brussels

#### DENMARK

Standard Electric Aktieselskab, Copenhagen

#### FINLAND

Oy Suomen Standard Electric AB, Helsinki

#### FRANCE

Compagnie Générale de Constructions Téléphoniques, Paris
  Les Téléimprimeurs, Paris
International Standard Engineering, Inc., Paris
Laboratoire Central de Télécommunications, Paris
Le Matériel Téléphonique, Paris

#### GERMANY

Standard Elektrik Lorenz Aktiengesellschaft, Stuttgart
  Divisions:
    Bauelemente Werk S.A.F., Nuremberg
    Informatikwerk, Stuttgart
    Kabelwerk, Stuttgart
    Lorenz Werke, Stuttgart
    Mix & Genest Werke, Stuttgart
    Schaub Werk, Pforzheim

#### IRAN

Standard Electric Iran A.G., Tehran

#### ITALY

Fabbrica Apparecchiature per Comunicazioni Elettriche Standard S.p.A., Milan

#### NETHERLANDS

Nederlandsche Standard Electric Maatschappij N.V., The Hague

#### NORWAY

Standard Telefon og Kabelfabrik A/S, Oslo

#### PORTUGAL

Standard Eléctrica, S.A.R.L., Lisbon

#### SPAIN

Compañía Radio Aérea Marítima Española, S.A., Madrid
Standard Eléctrica, S.A., Madrid

#### SWEDEN

Standard Radio & Telefon AB, Stockholm

#### SWITZERLAND

Standard Téléphone et Radio S.A., Zurich

#### TURKEY

Standard Elektrik ve Telekomünikasyon Limited Şirketi, Ankara

#### UNION OF SOUTH AFRICA

Standard Telephones and Cables (South Africa) (Proprietary) Limited, Boksburg East, Transvaal

#### UNITED KINGDOM

Creed & Company Limited, Croydon
International Marine Radio Company Limited, Croydon
Standard Telephones and Cables Limited, London
  Kolster-Brandes Limited, Sidcup
  Standard Telecommunication Laboratories Limited, London

### Latin America

**Manufacturing and sales**

#### ARGENTINA

Capehart Argentina S.A.I.C. (50% owned), Buenos Aires
Compañía Standard Electric Argentina, S.A.I.C., Buenos Aires

#### BRAZIL

Standard Eléctrica, S.A., Rio de Janeiro

#### CHILE

Compañía Standard Electric, S.A.C., Santiago

#### CUBA

Equipos Telefónicos Standard de Cuba, Havana*

#### MEXICO

Industria de Telecomunicación, S.A. de C.V. (50% owned), Mexico City
Standard Eléctrica de México, S.A., Mexico City

#### VENEZUELA

Standard Telecommunications C.A., Caracas

**Telecommunication operations**

**ARGENTINA**

Compañía Internacional de Radio, S.A., Buenos Aires

**BOLIVIA**

Compañía Internacional de Radio Boliviana, La Paz

**BRAZIL**

Companhia Rádio Internacional do Brasil, Rio de Janeiro
Companhia Telefônica Nacional, Curitiba and Pôrto Alegre

**CHILE**

Compañía de Teléfonos, de Chile, Santiago
Compañía Internacional de Radio, S.A., Santiago

**CUBA**

Cuban American Telephone and Telegraph Company (50% owned), Havana
Cuban Telephone Company, Havana*
Radio Corporation of Cuba, Havana

**PERU**

Compañía Peruana de Teléfonos Limitada, Lima

**PUERTO RICO**

Puerto Rico Telephone Company, San Juan
Radio Corporation of Puerto Rico, San Juan

**VIRGIN ISLANDS**

Virgin Islands Telephone Corporation, Charlotte Amalie

## Far East and Pacific

**AUSTRALIA**

Standard Telephones and Cables Pty. Limited, Sydney

**HONG KONG**

International Standard Electric Corporation, Kowloon (branch office and warehouse)

**PHILIPPINES**

ITT Philippines, Incorporated, Manila

## Worldwide Cable and Radio Telegraph Operations

American Cable & Radio Corporation, New York
All America Cables and Radio, Inc., New York
Commercial Cable Company, The, New York
Globe Wireless Ltd., New York
Globe Wireless Ltd. Philippines, Manila
Mackay Radio and Telegraph Company, New York
Sociedad Anónima Radio Argentina, Buenos Aires

## Associate licensees for manufacturing and sales

**AUSTRALIA**

Austral Standard Cables Pty. Limited, Melbourne

**FRANCE**

Lignes Télégraphiques et Téléphoniques, Paris

**ITALY**

Società Italiana Reti Telefoniche Interurbane, Milan

**JAPAN**

Nippon Electric Company, Limited, Tokyo
Sumitomo Electric Industries, Limited, Osaka

**SPAIN**

Marconi Española, S.A., Madrid

* Cuban properties seized under expropriation decrees of August 1960.

# THE WORLD OF ITT

## North America:

19 000 employees
34 locations (factories, major laboratories, service units)
4 000 000 square feet of floor space

## Europe, Middle East, Africa:

93 000 employees
87 locations (factories, major laboratories, marine radio companies)
11 500 000 square feet of floor space

## Latin America (excluding Cuba):

17 000 employees
15 locations (factories; telephone, radiotelephone, and radiotelegraph operating companies)
1 000 000 square feet of floor space

## Far East and Pacific:

3 000 employees
5 locations (factories and major warehouses)
650 000 square feet of floor space

## Total:

132 000 employees, including headquarters and service personnel in the United States, but not including American Cable & Radio Corporation personnel at home or abroad
141 locations
17 150 000 square feet
Sales representatives in most countries

# Principal ITT System Products

## Telecommunication Equipment and Systems

Automatic telephone and telegraph central office switching systems
Private telephone and telegraph exchanges—PABX, PBX
Carrier systems: telephone, telegraph, power-line
Long-distance dialing and automatic message-recording equipment
Switchboards: manual, central office, toll
Telephones: desk, wall, coin-operated
Automatic answering and recording equipment
Intercommunication, paging, and public-address systems
Microphones and loud speakers
Microwave radio systems: line-of-sight, over-the-horizon
Parametric amplifiers
Data-transmission systems
Teleprinters and facsimile equipment

## Military/Space Equipment and Systems

Aircraft weapon systems
Missile fuzing, launching, guidance, tracking, recording, and control systems
Electronic countermeasures
Power systems: ground-support, aircraft, spacecraft, missile
Radar
Simulators: missile, aircraft, radar
Ground and environmental test equipment
Programmers
Infrared detection and guidance equipment
Global and space communication
Nuclear instrumentation
Antisubmarine warfare systems
System management: worldwide, local

## Industrial/Commercial Equipment and Systems

Distance-measuring and bearing systems: Tacan, DMET, Vortac, Loran
Instrument Landing Systems (ILS)
Air-traffic control systems
Direction finders
Ground and airborne communication
Data-link systems
Inverters: static, high-power
Power-supply systems
Altimeters
Flight systems (autopilot)
Information-processing and document-handling systems
Electronic computers
Analog-digital converters
Mail-handling systems
Pnuematic tube systems
Broadcast transmitters: AM, FM, TV
Point-to-point radio communication
Mobile communication: air, ground, marine, portable
Closed-circuit television: industrial, aircraft, and nuclear radiation
Instruments: test, measuring
Oscilloscopes: large-screen, bar-graph
Magnetic amplifiers and systems
Alarm and signaling systems
Telemetering

## Consumer Products

Television and radio receivers
High-fidelity phonographs and equipment
Refrigerators, freezers
Air conditioners
Hearing aids
Incandescent lamps
Home intercommunication equipment
Electrical housewares

## Cable and Wire Products

Multiconductor telephone cable
Telephone wire: bridle, distribution, drop
Switchboard and terminating cable
Telephone cords
Submarine telephone and telegraph cable and systems
Coaxial cable
Aircraft cable
Power cable
Domestic cord sets
Fuses and wiring devices
Wire, general-purpose

## Components and Materials

Semiconductors: selenium, germanium, silicon
Power rectifiers, metallic
Transistors
Diodes: tunnel, zener
Capacitors: wet, dry, ceramic
Ferrites
Tubes: power, transmitting, traveling-wave, rectifier, receiving
Picture tubes
Relays and switches: telephone, industrial
Magnetic counters
Resistors
Varistors
Fluorescent starters
Transformers
Quartz crystals
Crystal filters
Printed circuits
Hermetic seals
Magnetic cores

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