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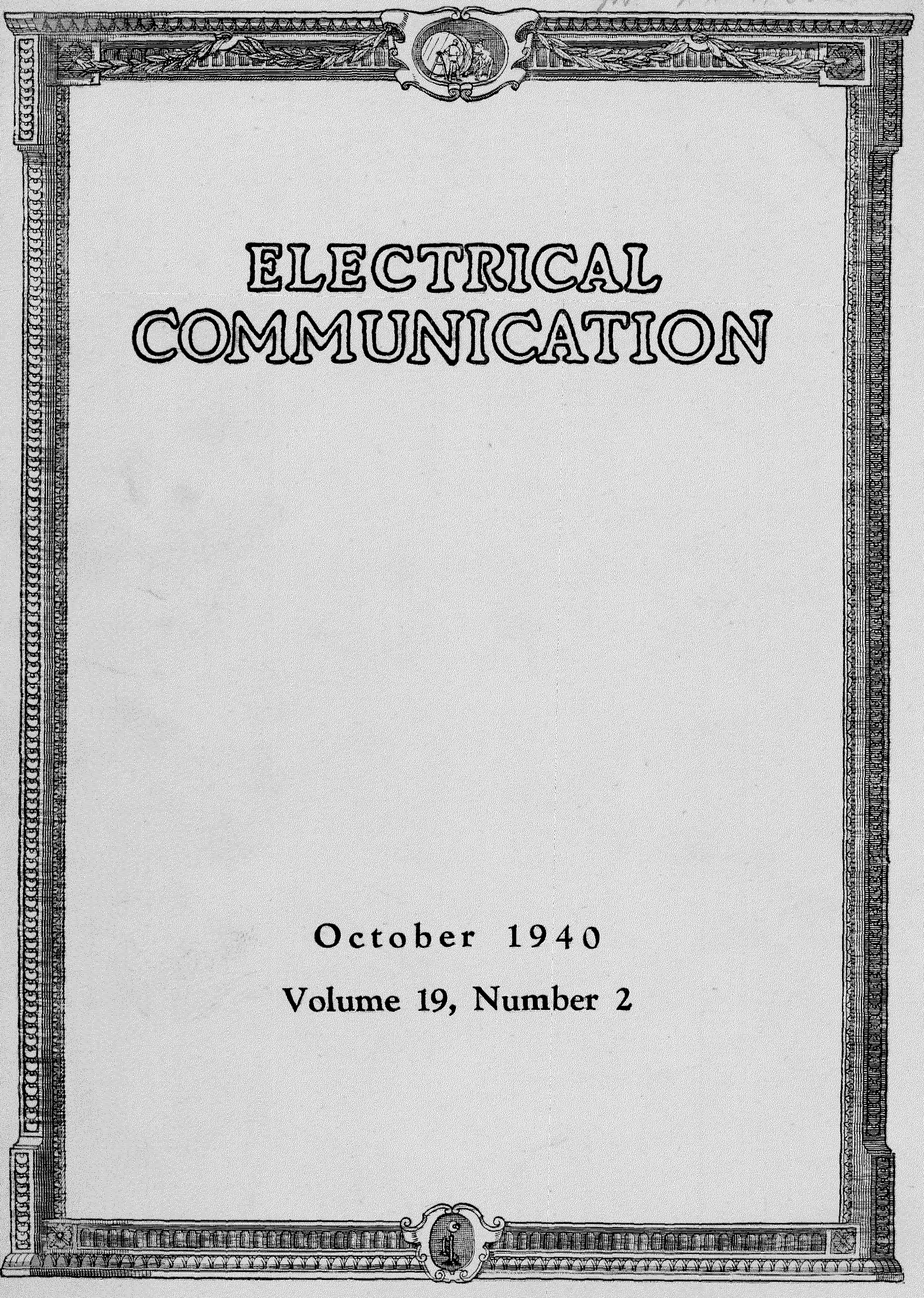

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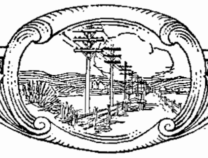

ELECTRICAL COMMUNICATION

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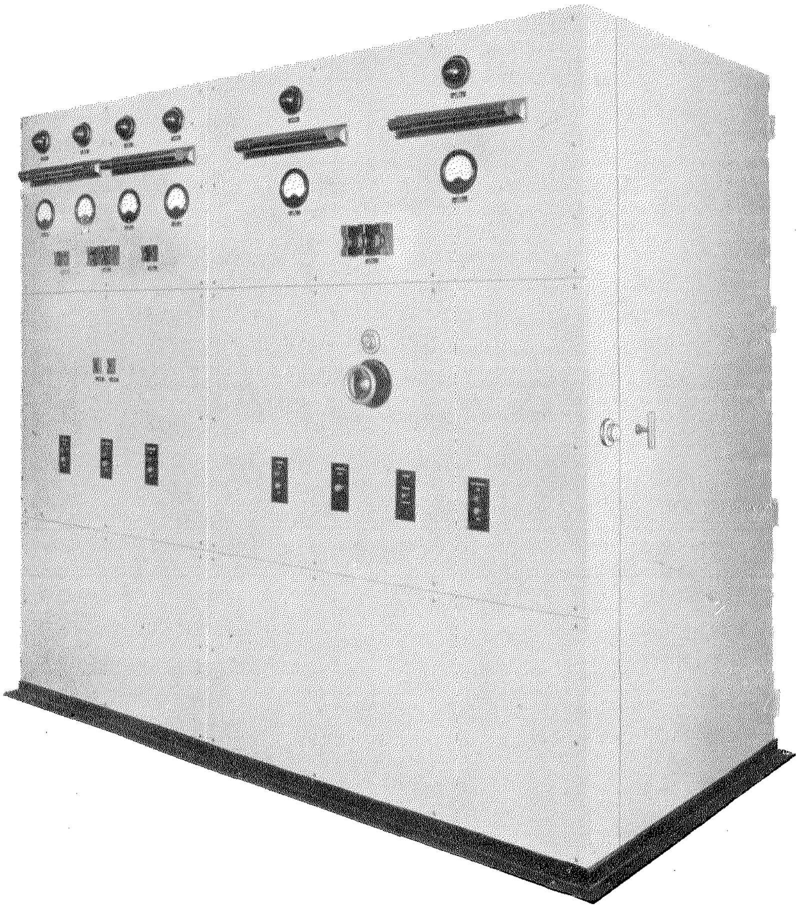

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MAIN AND AUXILIARY HIGH POWER RECTIFIER EQUIPMENT

THIS equipment was designed and constructed by Standard Telephones and Cables Limited, to meet the requirements of a specification prepared by the Radio Branch of the British Post Office Engineering Department for rectifier equipment to supply high tension and grid bias voltages for a high power amplifier.

The rectifier power equipment employs selenium rectifier sets throughout and includes smoothing units and meters. Five separate outputs are obtained as follows :—

H.T.1—300 V, 80 mA for initial amplifier stages. Ripple 0.1%.
H.T.2—Two separate outputs each 1 600 V, 135 mA for intermediate stages. Ripple 1.0%.
H.T.3—12 000 V, 420 mA for final stage. Ripple 1%.
G.B. —450 V, 150 mA for grid bias potentiometers. Ripple 0.1%.

Protective devices are fitted, interlocking the main switches with the two side gates, and H.T.3 is protected by an overload relay. A system of operating switch-buttons ensures that the various supplies can only be switched in a predetermined sequence. The dimensions of the rectifier cubicle are 7 ft. long, 3 ft. 6 in. deep and 6 ft. 6 in. high.

Recent Developments in Esterified Fibrous Insulants

By A. A. NEW, M.Sc., F.I.C.,

Standard Telephones and Cables Limited, London, England

Highly Acetylated Cotopa (Cotopa 60)

IN previous issues of *Electrical Communication* [1, 2, 3] in 1935/6, a short description was given of the acetylation of cotton, using a mild treatment such that considerable combination of acetic acid and cotton takes place without visibly altering the fibrous structure or affecting the mechanical properties. A slight recapitulation may be permitted here in order to show more clearly the relation of recent developments.

Two of the most important insulating materials, cotton and paper, consist mainly of cellulose ; in fact, bleached cotton is practically pure cellulose. Cellulose contains the elements carbon, hydrogen and oxygen only, in nearly the same proportions that they occur in the sugar, glucose ; and, moreover, by suitable treatment, cellulose can be converted almost completely into glucose, indicating a close relationship between the two. Nevertheless it is obvious that there must be some big structural difference underlying the marked contrast between the brittle crystals of glucose which are very soluble in water, and the tough, strong, flexible fibres of cotton completely insoluble in water; the explanation of this lies in the molecular size and aggregation of the two substances.

Cellulose fibres, whatever their origin, whether from cotton, wood, linen, ramie, straw or hemp, etc., consist of long chains built up from glucose molecules, and containing about 100–200 units in a chain. As the glucose units are roughly as broad as they are long, this means that the cellulose molecules or chains are about 100–200 times as long as they are wide. Each link has three hydroxyl groups (–OH) attached to it (being parts of the original glucose molecule), which are rather reactive and which are the cause of many of the chemical reactions in which cellulose takes part.

It has been known for many years that by the action of acetic anhydride on cellulose, combination could take place between these groups and the anhydride to form esters called acetylated cottons. If the reaction is carried out carefully under mild conditions, considerable esterification can be effected while still retaining the outward form and the mechanical properties of the original cotton. In view of the fact that another grade of Cotopa is now being made, this, the original one, is now being designated as Cotopa 30 (because it yields 30 % of combined acetic acid on saponification).

If, however, the reaction is carried on to completion, all the hydroxyl groups in the unit are affected, three times as much esterification takes place, and then the reaction ceases.

If this reaction takes place in the presence of a solvent of cellulose triacetate, such as acetic acid, the cotton swells and finally dissolves to a viscous solution as the esterification approaches completion. The swelling and solution are more rapid and the final solution less viscous the more vigorous the conditions used for the acetylation (i.e., higher temperature and strong catalyst). If this solution is run into water the cellulose triacetate comes out of solution in large flocculent lumps, which on thorough washing and drying yield a white powder which is soluble in chloroform, but which is insoluble in water, alcohol, or acetone, and which is known as "primary" acetate. If, however, the reaction mixture had been only slightly diluted with water and gently heated at about 40°–50° C. for 10–12 hours, a reverse action (saponification) would take place, yielding an incompletely acetylated cellulose, which can be separated by further dilution with water as before. This is called a "secondary" acetate

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[1] For references, see end of article.

and has the commercially valuable property of being soluble in acetone, which makes possible its use in the manufacture of acetate silk, and also in lacquers and dopes.

In the manufacture of Cotopa, a weak catalyst such as zinc chloride is used which does not produce any degradation of the cellulose, but where the soluble triacetate is desired, sulphuric acid catalyst is nearly always used, as it is more powerful ; also a slight degradation of the cellulose is usually desired in order to produce a rather lower viscosity material suitable for use in spinning solutions. For lacquers and dopes a more degraded acetate is generally used, as by this means the most concentrated solutions can be obtained at a suitable viscosity for washing or spraying.

It should be noted that there is no connection between the terms "primary" acetate and "monoacetate," the latter being a term sometimes used previously with reference to celluloses esterified to a degree equivalent to one hydroxyl in three in the cellulose unit being acetylated.

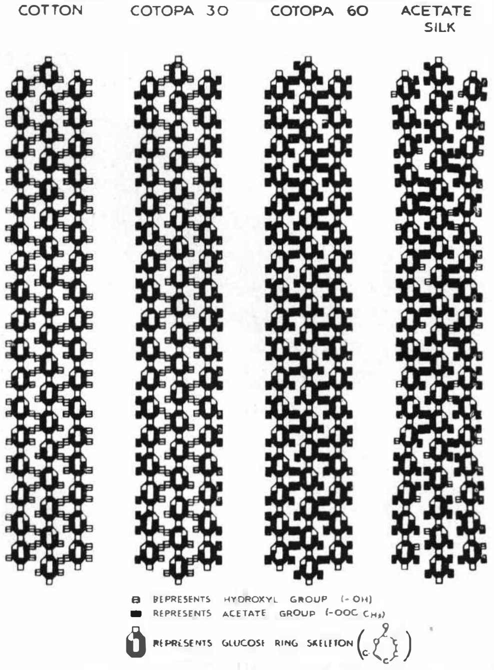
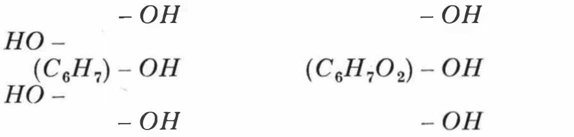


Fig. 1—Pictorial representation of the structure of cellulosic yarns. Each diagram represents one so-called "crystallite" or "micelle." Actually, each diagram should contain of the order of 10 times as many units in length and 20 times in cross section to approach the numbers really involved.

If instead of using a solvent for cellulose triacetate, a substantial proportion of a non-solvent for cellulose triacetate is used in the reaction mixture, the swelling is repressed and the final solution prevented. If this is done using mild reaction conditions and a weak catalyst, such as zinc chloride, a triacetylated cotton can be obtained which has lost but little of the strength and elasticity of the original cotton. It is, however, harder than the original cotton or Cotopa 30, and has slightly different machine-running properties. This new product, which has several new and useful properties, has been designated Cotopa 60 because, on saponification, it yields approximately 60% of combined acetic acid (62.5% being the value for complete triacetylation).

Suitable reaction conditions for producing the types of material described are given in the patent literature.[15]

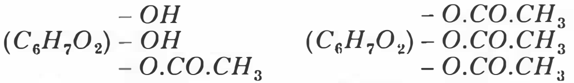
A simplified way of representing the glucose molecule, and also the unit in the cellulose chain, is :—



$$\begin{matrix}HO-\\ \\HO-\end{matrix}(C_6H_7)\begin{matrix}-OH\\-OH\\-OH\end{matrix} \qquad (C_6H_7O_2)\begin{matrix}-OH\\-OH\\-OH\end{matrix}$$

Glucose Cellulose Unit

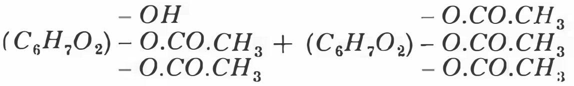
In the same manner Cotopa 30 and Cotopa 60 are represented :—



$$(C_6H_7O_2)\begin{matrix}-OH\\-OH\\-O.CO.CH_3\end{matrix} \qquad (C_6H_7O_2)\begin{matrix}-O.CO.CH_3\\-O.CO.CH_3\\-O.CO.CH_3\end{matrix}$$

Cotopa 30 (Average of units) Cotopa 60 (Cellulose triacetate)

Cellulose Acetate Silk (secondary acetate) by :—



$$(C_6H_7O_2)\begin{matrix}-OH\\-O.CO.CH_3\\-O.CO.CH_3\end{matrix} + (C_6H_7O_2)\begin{matrix}-O.CO.CH_3\\-O.CO.CH_3\\-O.CO.CH_3\end{matrix}$$

Acetate Silk (average effect is the mean of these two units).

A more elaborate two-dimensional idealized representation of these materials is given in Fig. 1. In this illustration only three chains are shown together in each case, but in natural cellulose fibres they are bunched together in so-called "crystallites" or "micelles," of varying size, but averaging about 50 chains in a "micelle." From X-ray measurements it has been discovered that the chains are packed together in

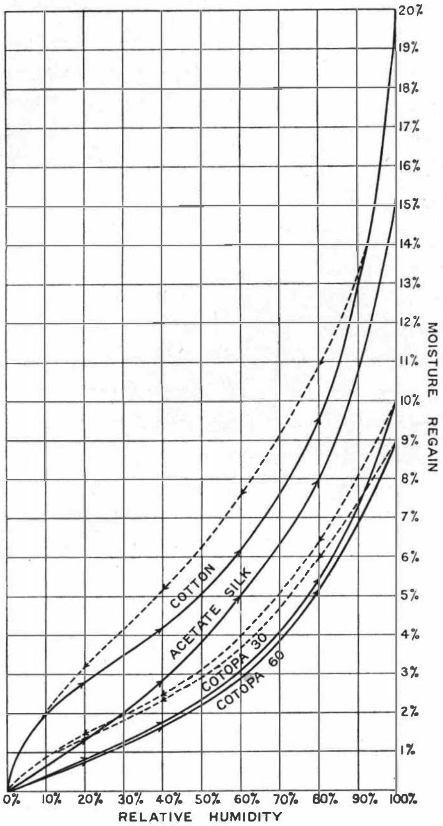


Fig. 2—Moisture regain of acetylated cellulosic yarns.

a regular manner in the micelles with the minimum spacing between them. The micelles themselves are, however, less closely packed together, and it is of numbers of these that the fibrils of cellulosic materials are built up. The fibrils are the minute filamentous units which can just be discerned in the cotton fibre under high magnification, and into which the cellulosic fibres separate on heavy beating. The cellulosic fibres can, therefore, be regarded as having an elongated sponge-like structure with numerous minute fissures between the fibrils and between the micelles constituting them.

Moisture Regain of Cotopa

It has been shown previously that the acetylation of cotton reduces its moisture regain in atmospheres containing water vapour, and a series of curves has been plotted for various degrees of acetylation. These have broadly the same shape as the regain curve for natural cotton, but represent smaller regains at any given relative humidity, and also have a lessened degree of curvature at each end of the curve as compared with untreated cotton. It has been mentioned above that esterification of cotton, using only acetic acid as diluent, produces swelling of the fibre above 30% combined acetic acid content, and it was felt that this was probably enough to account for the fact that the curves for 38.3% and 45.6% combined acetic acid[1] were almost coincident.

Now that the use of inert diluents has enabled the preparation of a full range of acetylated cottons up to triacetate to be effected, it has become possible to investigate the moisture adsorption properties of the whole series. Figure 2 shows the moisture regain loop for cotton acetylated to 0, 30%, and 60% C.A.A.C. The arrows indicate the direction from which moisture equilibrium is being reached in each case, and it is seen that, in common with all other fibrous organic materials having an appreciable regain, the value for acetylated cottons is higher on the desorption curve than on the sorption curve, i.e., the equilibrium values of moisture regain are affected by the recent past history of the sample, and higher values are obtained if the particular humidity in question is reached after being in a saturated atmosphere than is the case if it is reached after first being dried out in an atmosphere of zero humidity. This hysteresis effect is due to some imperfectly elastic feature in the fine structure of the fibres.

The outstanding feature of this set of curves is that, as hinted at in the previous series of curves, hardly any reduction in moisture regain at a given relative humidity takes place above

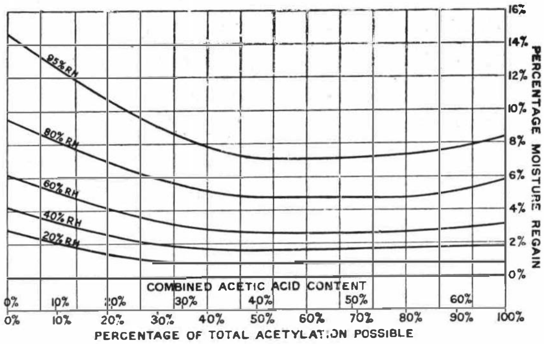


Fig. 3—Acetylated cotton. Relation between moisture regain and degree of acetylation.

40% combined acetic acid content, and, in fact, it apparently rises very slightly again at the highest degrees of acetylation. This effect is shown more clearly in Fig. 3, where the moisture regain is plotted against combined acetic acid content for various humidities.

It is interesting to compare these values with those obtained for cellulose acetate silk and other cellulose derivatives under similar conditions. Ordinary cellulose acetate silk made from secondary acetate (combined acetic acid content 50%–55%) has a moisture regain about 1.8 times that of Cotopa 30, although it is so much more highly acetylated. It is, therefore, clear that factors other than the combined acetic acid content must also affect the moisture regain. This, of course, is not surprising in view of, for instance, the increase in the moisture regain of cotton caused by mercerization, or the considerably higher regains of viscose or cuprammonium silks, or cellulose regenerated from acetic silk, as compared with cotton. All these four materials chemically are cellulose. A consideration of the structure of cellulose leads to the view that these factors can only be the physical space arrangements of the molecules, either the cellulose molecular chains, the attached groups, or combinations of the two.

Beadle and Dahl[4] and also Will[5] found the normal hygroscopic moisture content of a series of nitrates of cellulose to decrease in inverse ratio to the degree of nitration and gave data which, on plotting, yield almost straight line relationships. It should be noted that in the manufacture of cellulose nitrates the cellulose is strongly swelled by sulphuric acid. Sheppard[6] states that the water sorption of cellulose is progressively reduced with degree of acetylation, and quotes data which appear to indicate a similar relationship. In sharp contrast with his values of 9% at saturation for triacetate are the facts that cellobiose octa-acetate and glucose penta-acetate have extremely low moisture adsorptions, being less than 1% at saturation.

It is helpful to realize that the moisture adsorption of textiles is only a portion of a wider phenomenon, and that all vapours are adsorbed to some extent, the amount being a function of their chemical composition, as well as of that of the internal surface of the solid.[7] Cotton adsorbs the largest amounts of the vapours of those liquids that are good solvents of polyhydroxylic compounds, smaller amounts of those which are moderate solvents of the same, and very little of those that are non-solvents. It has also been noted that nitro-cotton adsorbs more than its own weight of good solvents, but only a few per cent. of the vapours of those liquids in which it is insoluble or only slightly soluble.

It is now generally accepted that the adsorption of moisture on the internal surface of solids is due to the formation of monomolecular or multimolecular films at the lower humidities, which is succeeded by capillary condensation at the higher humidities.

With the further information now available, it is possible to attempt an explanation of the phenomena connected with the moisture adsorption of esterified insulants. It is certain from numerous lines of argument that the adsorbed water in cotton is held owing to the hydroxyl groups in the cellulose complex. In order to determine where this water is held, it is helpful to calculate the amounts of water that would be needed to form a monomolecular film on the hydroxyl groups exposed on the surfaces of the cotton hair, the fibrils, the micelles and the molecules, on the basis that most of the attractive force of one hydroxyl group would be satisfied by one water molecule.

These are as follows:—

Hairs	0.05%[8]	
Fibrils	0.42%[8]	
Micelles	8%–11%	(Based on Mark's Micelle of 50–60 molecular chains)
Molecules	33.3%	

The dimensions for the micelle suggested by Mark can be roughly visualized from the idealized cross-section shown in Fig. 4. The lower limit for a monomolecular film is arrived at by considering that there would be alternately one or two hydroxyls available for attraction on the 20 units round the circumference of the cross-section, plus one hydroxyl on each of the 12 units in the second line. The upper limit is obtained by considering 20 units with two hydroxyls free in the outer ring and 12 units with alternately 1 and 2 hydroxyls free in the

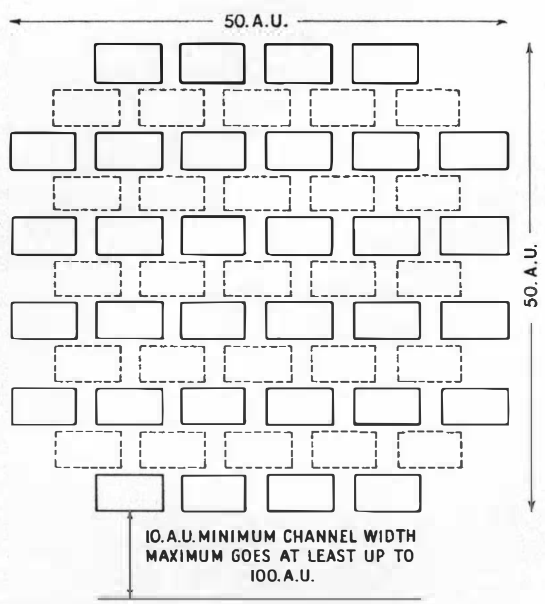


Fig. 4—Idealized cross section of a cellulose micelle (after H. Mark).

second ring. These give :—

Minimum ..	8.2%
Maximum ..	11.3%

The moisture regain of cotton at saturation never exceeds 23%, so that the state is evidently never reached where there is a water molecule to every hydroxyl. The values calculated for a monomolecular film on the cotton hairs and fibrils are much too small to account for the amounts actually found by measurement. The value calculated for a monomolecular film on the surface of micelles of 50 A.U. diameter, however, gives a figure which falls about in the middle of the moisture Regain/Relative Humidity curve corresponding with that portion where the decided curve upward commences.

Consideration of the heat of wetting of dry cellulose and the dielectric constant of cellulose of various moisture contents indicates[9, 10] that the smaller moisture contents are more firmly bound than the greater, and that the greater ones are practically free water.

It is possible, therefore, that water up to about 8% is adsorbed monomolecularly on the internal surfaces of the fibres, but that amounts in excess of this are more loosely held in the form of second, third, etc., layers. One region passes smoothly into the other and there is no sharp line of demarcation, so that, possibly, both mechanisms are operative between 5% and 8% regain. This also probably correlates with the fact that the micelles are not as sharply distinct as bricks in a wall, but are more in the nature of small bundles in which some of the molecular chains pass from one bundle to another along their length. Figure 5, taken from a review by S. H. Clarke,[11] based largely on the work of Frey-Wyssling, shows pictorially the type of arrangement visualized in the case of wood fibres. More recent work by Mark[26, 27] has led to a similar picture consisting of small bundles in the case of regenerated cellulose fibres.

Now considering esterified celluloses, there are the following additional facts to be accounted for :—

1. Cotton can be readily acetylated up to about 30% C.A.A.C. under weak acetylating conditions without alteration of external structure, but above this it is necessary to take special precautions to prevent swelling and solution.

If a calculation is made of the number of acetyl groups needed to cover completely the outside of the micelle represented in Fig. 4, in a similar manner to that employed for the estimation of a monomolecular film of water adsorbed on it, amounts corresponding to a minimum of 23% C.A.A.C. and a maximum of 30% C.A.A.C. are obtained, suggesting that acetylation proceeds easily until the surfaces of all the micelles are acetylated, after which the reaction can only continue by swelling the micelle.

2. On acetylating cotton under mild conditions it is found that the moisture regain steadily decreases with degree of acetylation up to about 30% C.A.A.C., after which it remains nearly constant.

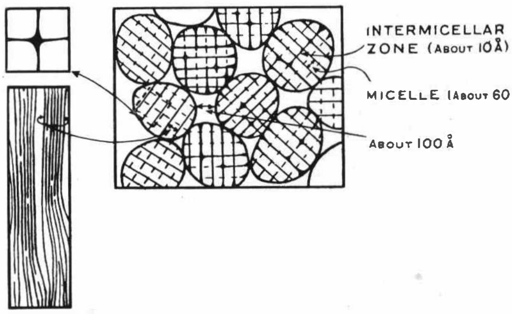


Fig. 5—Microfibrils of a wood fibre, showing approximate relative sizes of micelles and spaces (after Clarke and Frey-Wyssling).

These can be explained as follows :—

(*a*) It is well known that organic acetates are much less soluble in water than polyhydroxy compounds like glycerol or glycols, and similarly there is less attraction between acetate groups on the cellulose complex and water than there is between hydroxyl groups similarly located and water. This is sufficient to account for the fact that there is a reduction in moisture regain on acetylation.

(*b*) That the value is reduced to a lower figure and not to zero is accounted for by there being a definite attraction between acetyl group and water, though smaller than between hydroxyl group and water, as indicated by the adsorption of organic esters and ketones by cotton.[7]

(*c*) The fact that the regain reaches a steady value at 30%–40% C.A.A.C. instead of falling continuously with increasing degree of acetylation can be accounted for by considering the adsorption of moisture to take place on the surfaces of the micelles already described. Once the outside of the micelle is completely acetylated, further acetylation does not change the nature of the surface on which adsorption is taking place.

The high regain of acetate silk and film is accounted for by the fact that (*a*) it has been completely swelled and dissolved and hence has a larger internal surface; (this is confirmed by regenerating the cellulose from same, when it is found to have a considerably higher value than cotton); and (*b*) cellulose acetate composing acetate silk has been partly saponified while in solution, and hence its hydroxyl groups should be evenly distributed, and a proportionate number of hydroxyls are available to increase adsorption on the surfaces of its micelles.

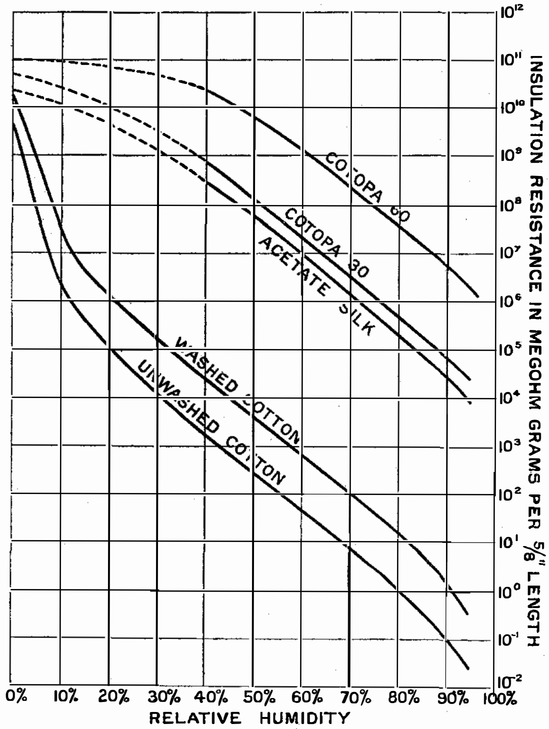


Fig. 6—Relation of direct current insulation resistance of cellulosic yarns to relative humidity.

The very low regains of cellobiose octa-acetate and glucose penta-acetate are readily accounted for on spatial considerations, because, being crystalline substances, their molecules are so closely and regularly packed that they have very little interior surface on which water can be adsorbed.

The above shows broadly what are believed to be the factors governing the moisture adsorption of acetylated celluloses, and hence those which govern most of the electrical properties under humid conditions, but there are many details and related phenomena which it is not possible to go into in a general survey, and which will be dealt with more fully elsewhere.

Direct Current Insulation Resistance of Cotopas

The D.C. Insulation Resistance of Cotopa, like that of other fibrous insulating materials, is governed by three factors :—

1. Chemical and physical structure.
2. Moisture content.
3. Electrolyte content.

The structure involves the degree of acetylation, the degree of swelling, whether before, during, or after acetylation, the evenness of the treatment and the degree of purity of the cellulose.

The factors governing the moisture content have been enumerated in the last section.

The electrolyte content is a variable controlled by the degree of washing given after processing. The insulation resistance curves quoted are for the ordinary degree of electrolyte content normally found, which is low in the case of Cotopa 30 and very low in Cotopa 60.

The D.C. Insulation Resistance of Cotopa 30 is about 500 000 times that of ordinary grey cotton and about 25 000 times that of ordinary washed cotton, when measured at ordinary or high humidities. Cotopa 60 has insulation resistance values about 70 times these figures.

Curves showing the relationship of insulation resistance to humidity have already been published for the range 0–30% Combined Acetic Acid Content. It is now possible to extend these to cover the whole range of acetylation of cotton, as well as an increased humidity range (Fig. 6). It will be seen that the logarithm of insulation resistance/relative humidity relation is roughly a straight line between 85% and 30% Relative Humidity for Cotopa 60, Cotopa 30 and acetate silk, while for cotton the line exhibits slightly more curvature, but is substantially straight between 20% and 80% R.H. below and above which it has a marked curvature. The curves are roughly parallel with each other.

In the case of cotton, this relationship has been embodied in the following equations by Walker.[12, 13]

1. log. $I.R. = -A\ (\%\ M.C.) + B$
   Range 0 – 3% Moisture Content.
2. log. $I.R. = -A\ (\log.\ \%\ M.C.) + B$
   Range 3–10% Moisture Content.
3. log. $I.R. = -A\ (\%\ R.H.) + B$
   Range 10% Saturation

where the values of A in equation 2 (which covers the most important part of the range) are about 9–11, depending on whether absorption or desorption conditions are being examined and on the method of measurement.

On plotting the log. *I.R.*/log. Moisture Content relationship, all the acetylated textiles exhibit a straight line relation, like those previously examined, but with a slightly smaller slope than cotton. The curves for acetylated yarns are represented to a considerable extent by type 3 equation, and as the log. Moisture content/relative humidity relation is also a straight line over a wide range, it follows that they can also be represented by the type 2 equation. Measurements up to 30% R.H. are up to the present uncertain, owing to the high values involved, so that it is not possible to say whether there is a region corresponding to type 1 equation. Actual values recorded are indicated by dotted lines, but it is not at present possible to say whether these represent the pure material or extremely high resistance leakage paths caused by minute traces of impurity. It has been noted that with the highest degrees of acetylation, and particularly at low humidities, the values tend to be more irregular, presumably owing to the fact that the I.R. values are so high that they are in the range where the slightest trace of impurity or unevenness makes an appreciable difference, and it is, of course, not possible to clean the surfaces of a yarn as meticulously as, for example, a piece of quartz, without disturbing other factors.

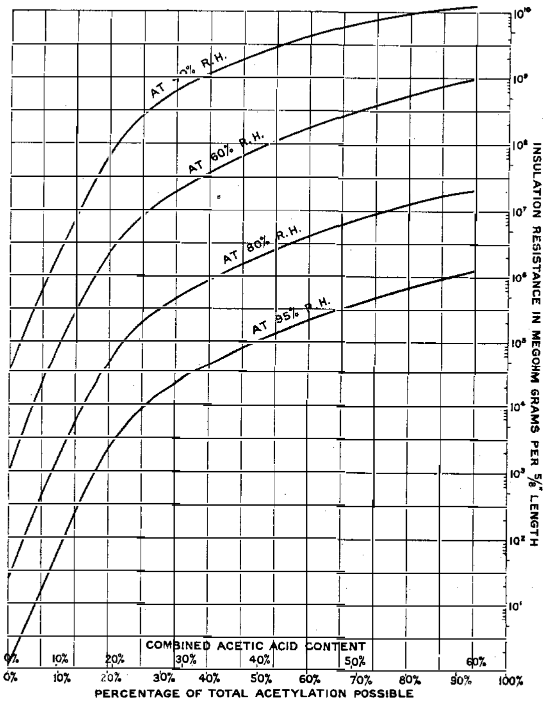


Fig. 7—Relation of direct current insulation resistance of acetylated cotton to degree of acetylation.

The relation of insulation resistance to degree of acetylation is shown in Fig. 7, from which it will be seen that there is a marked increase up to 30%–40% Combined Acetic Acid Content, but that above this there is a marked falling off in the rate of increase. When it is taken into account that the higher acetylated materials are better washed and have a lower electrolyte content, it is clear that the comparative rate of increase is lessened still more. The reduced electrolyte content is not sufficient, however, to account for all the increase in insulation resistance above 30%–40% C.A.A.C.

The above insulation resistance measurements have been made directly on longitudinal samples up to values of 10^8 megohm grams. Higher values have been obtained by altering the form of sample, whereby a larger quantity of textile is measured. It has been established, by repeated experiments of several observers, that the relationships hold equally well for longitudinal tests on yarns or transverse measurements obtained on twisted pairs and yarns wound on tubes, flat plates, etc., provided that the electric stress per unit thickness of textile is not too greatly increased. (There are, of course, probably small changes in the numerical values of the ratios of different materials with moderate changes in stress.)

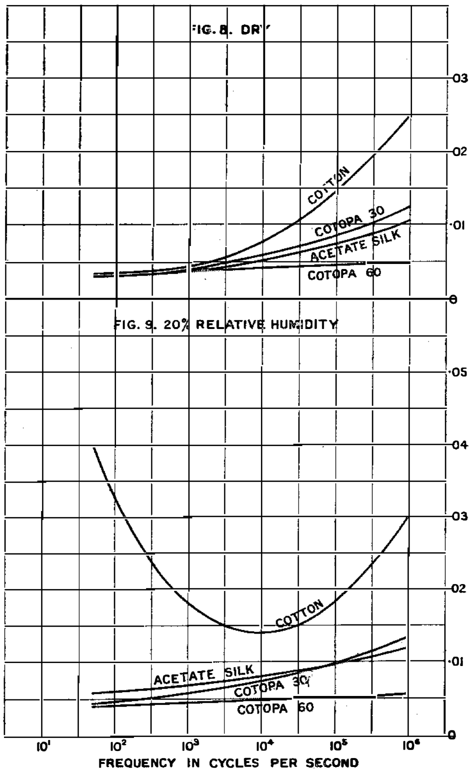
An interesting example of this has arisen in a method of testing employed by a continental manufacturer, where the test sample is formed by twisting one or two threads of yarn very tightly to a diameter of 0.25 mm, until they are on the point of breaking, and then pressing them between metal plates so that the yarn is squashed to a thickness of about 0.2 mm. With the test voltage of 150 volts, this gives a stress of 750 volts/mm, which is much higher than the values actually applied to textile insulated conductors in practice, and considerably above the stress of about 31 volts/mm used in most of the present measurements. Such conditions give insulation resistance values which increase with time, owing to polarization (sometimes several times increase in ten minutes, depending on nature of textile and humidity), but apart from this are quite satisfactory for making measurements on cotton. They are, however, less satisfactory for measuring insulation resistance values on very good materials, such as Cotopa, on account of the combination of minute separation of electrodes and the high stress, leading to a proportion of irregular readings, due to minute patches of contamination or cotton hairs, etc. Such high resistance leaks are of no account in making measurements on ordinary textiles, but are liable to be troublesome when dealing with materials of very high insulation resistance. In practice, of course, they are never noticed, because textiles are invariably employed with at least two and probably four or six laps between conductors.

It is on this account preferable to use several lappings when employing increased stress in a transverse method, whether in the form of twisted pairs or lapped on flat or cylindrical surfaces. The table below gives some measurements on 50-yard lengths of twisted pairs of identical construction, made up with conductors covered with two laps of each of the textiles under discussion.

In making measurements of D.C. insulation resistance, the longitudinal method, such as has been used mainly in obtaining the data on which the curves illustrating this work are based, has the advantage that its results can be calculated to a unit, which is independent of arbitrary considerations, such as pressure and form of sample, with the one exception of the length of the test sample, which, for practical convenience, was originally fixed at 16 mm ($\frac{5}{8}''$). In the present work the same unit, namely, megohm grams per 16 mm ($\frac{5}{8}''$) length, has been retained for the sake of ease of comparison with previous work. In future, however, these measurements will be expressed in megohm grams per centimetre, at a stress of 1 000 volts per cm.

Whenever it has been desired to obtain greater sensitivity of measurement mainly for measuring the very high resistance obtaining at low humidities, recourse has been had to some form of transverse measurement, and this, of course, brings in an extra variable, the pressure applied to the sample. It has been shown by

		D.C. Insulation Resistance of 50-yd. lengths of twisted pairs at 65% Relative Humidity Test voltage (and approx. max. stress)	
	100 volts (200 volts/mm)	200 volts (400 volts/mm)	500 volts (1 000 volts/mm)
Washed Cotton ..	12.6 megohms	11.6 megohms	10.2 megohms
Acetate Silk	5 140 "	5 270 "	5 140 "
Cotopa 30	60 000 "	58 000 "	54 000 "
Cotopa 60	2 500 000 "	2 200 000 "	2 200 000 "



Figs. 8 *and* 9—*A.C. power factor of acetylated cellulosic yarns.*

Denham, Hutton and Lonsdale[14] that, as would be expected, the insulation resistance falls considerably with increase of pressure. Values have been decreased to 1/20th by increasing the pressure from 2 to 170 lbs. This makes it difficult to express the results in the form of any absolute unit. More will be said about this feature in the next section.

Reverting to factors influencing the nature of the material, it has been found that, as is the case with acetate silk, the presence of even minute amounts of combined sulphuric, phosphoric and similar acids greatly reduces the insulation resistance value of the yarn, owing probably to the fact that the inorganic esters of cellulose hydrolyse in the presence of moisture, forming a source of high conductivity electrolyte in the textile. The last traces of this can only be washed out with difficulty; hence it is preferable to avoid its formation at any stage of manufacture.[15]

Alternating Current Characteristics of Cotopa 30 and Cotopa 60

The alternating current characteristics of acetylated cottons[16] are rather more complex than those with direct current, and it is only when a considerable number of measurements are made under carefully controlled conditions that the underlying relationships become apparent.

Figures 8–13 show the Power Factors and approximate relative Permittivities of Cotton, Cotopa 30, Cotopa 60 and Acetate Silk respectively. As mentioned in the last section, obviously the pressure applied, and the degree of packing of the material or, in other words, the degree of dilution of solid dielectric with air, between the electrodes, must affect the values obtained, and hence the measurements have been made at a degree of packing which corresponds roughly to the effective average textile lapping or braiding of a pair of twisted insulated wires.

With looser lappings or braidings, lower values are obtained, while on the other hand,

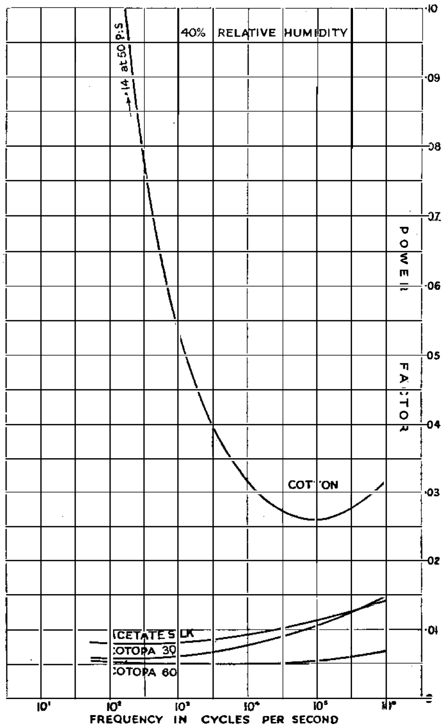


Fig. 10—*A.C. power factor of acetylated cellulosic yarns.*

with tapes and fabrics, especially if heavily calendered, the power factor values are two to three times as great with corresponding increases in permittivity.

It is simplest to consider the dry values first, and it is seen that with cotton there is a continuous and marked rise in power factor with frequency, accompanied by a steady fall in permittivity. With Cotopa 30 the power factor is considerably lower, and while there is still an increase of power factor with frequency, this is much less than with cotton. The permittivity values of Cotopa 30 and 60 are not very different and are much lower than that of cotton. Finally Cotopa 60 has a still lower power factor in which the variation over the frequency range $5 \times 10^1 - 5 \times 10^6$ p:s is reduced to a very small amount.

If the power factor and permittivity are plotted against a degree of acetylation, there is a fall in power factor with increasing degree of acetylation at all the frequencies considered, accompanied by a smaller decrease in permittivity. As acetylation converts hydroxyl groups into acetyl groups and the former are more polar than the latter, it is reasonable to suppose that both power factor and permittivity at any degree of acetylation are complex functions made up from the individual power factors and permittivities of cellulose nucleus, acetyl groups and hydroxyl groups respectively, which in turn are due mainly to the electrostatic forces tending to move the polar groups relative to the rest of the molecule and the resistances to this movement. It is clear that at the higher frequencies the hydroxyl group is the factor having the greatest effect.

It is possible now to see the reason for the curious family of curves obtained at various humidities shown in Figs. 9–12.

As moisture is absorbed by the textile internal surfaces, two more factors are superimposed on those already indicated, and their effects are shown in at least two ways. The first factor is the absorbed water itself and the second the ions which are immediately formed in the water from the traces of salts contained in the textile. The traces of salts were, of course, present when the measurements were made in the dry condition but only contributed a negligible proportion of the effects observed when they were un-ionized. The ionized salts make the water film conducting to extents that have already been described at length in the last section, so that there is now effectively a conducting path in parallel with the dielectric which, as the humidity increases, tends to swamp the other effects, and in fact, at the highest humidities and low frequencies the power factor values approach those obtained by calculation from the D.C. resistance at the same humidity.

The second way in which the effect of the addition is shown is in the form of direct dielectric loss due to the polar nature of the water itself, particularly at the higher frequencies where the absorbed water molecules tend to act like additional hydroxyl groups on the internal surfaces. It is also possible that yet a third type of effect is present, namely that, due to the heterogeneous nature of the dielectrics both as regards dielectric constant and conductivity, charges may be formed alternately at internal surfaces in the manner described by Maxwell and elaborated by Wagner, and add to the power factor. It is, however, proposed to deal with the theoretical aspect of the subject at greater length elsewhere.

Considering now the relative properties of the textiles in question, it is seen that whereas a considerable reduction in dielectric losses at high frequencies and in the dry condition is obtained by the acetylation of cotton, a comparatively small improvement is obtained at low frequencies. When ordinary atmospheric humidities are concerned, however, there is a marked improvement on acetylation at all frequencies, which becomes more and more pronounced the lower the frequency, and may be regarded as ultimately terminating in the D.C. conduction curves at the particular humidity concerned.

On this account it is evident that where high frequencies of the order of 10^5–10^7 are concerned, Cotopa 60 is the most suitable textile insulant to employ,[16] whereas at telephonic frequencies or where D.C. is concerned, Cotopa 30 represents a sufficiently high degree of acetylation.

It should be noted that, whereas under "dry" conditions as obtained in laboratory tests, there is a relatively small improvement in power factor at the lower frequencies by acetylation, in practice, "dry" conditions very frequently represent anything from 0–20% relative humidity. This, and the fact that, on drying, there is only about half the amount of moisture to remove from Cotopa 30 or 60 as compared with cotton, greatly increases the value of Cotopas at low and medium frequencies under "dry" conditions.

All the above comparisons are on a practical basis. If compared on a strictly equal weight per unit volume basis, the power factor and permittivity values for cotton would be higher and those for acetate silk nearer the curve for Cotopa 60.

Purity of Cotopas

It is a common custom with many textiles to add lubricants at some stage in the manufacture either to facilitate the spinning of the yarn or to soften it when made. This is usually done in the case of the harsher or more springy fibres. Another well-known temporary means to the same end for spinning purposes in the case of cotton is to raise the humidity of the atmosphere whereby the cotton fibres take up more moisture and become softer and more pliable.

In the case of Cotopa 30, it is of the same degree of pliability and has the same running properties as ordinary cotton and hence no question is likely to arise of any need to lubricate it. Cotopa 60, however, is slightly harsher than Cotopa 30, although the difference is hardly noticeable on ordinary visual inspection, and it is necessary to consider the question.

Where Cotopa 60 is being used on account of its very low radio frequency losses, it is imperative that no oxygen containing oils such as vegetable or animal oils should be used for softening it, and in fact it is preferable for most cases that no lubricant at all should be used. The vegetable and animal oils are objectionable because they consist of higher fatty alcohols and esters whose power factor at radio frequencies in particular is much higher than that of Cotopa 60, and they are very susceptible to oxidation with corresponding increase in power factor.

For a limited range of purposes, it is possible to use small quantities of a pure mineral oil as a lubricant, but for radio frequency insulation in the dry condition, where a drying process of twenty to fifty hours at 100°–120° C. in the presence of air is employed, it is essential that there should be no lubricant of any sort.

That mineral oils are subject to oxidation when heated in air, particularly in the presence of certain metals, is a well-known phenomenon, especially in connection with power cable insulation, but it is not widely known how rapidly this takes place when mineral oils are spread in a very thin layer, as for instance on textiles where they present a very large surface for reaction, or to what an extent the power factor of the oil, particularly at radio frequencies, is increased by such oxidation.

Figure 14 shows how the power factor at 1 megacycle of a sample of originally pure colourless mineral oil rapidly increases on heating at 120° C. in the presence of copper and with air being slowly bubbled through. Under the conditions employed, in 24 hours over thirty times increase was found to take place. If a sample of Cotopa 60 containing 5% of such an oil were submitted to a drying process in which the oil oxidized at the same rate as the above (and under most practical conditions it would probably be even quicker), it is easy to calculate that on a simple additive basis there would be an increase of 10%–20% in power factor at 1 megacycle after 48 hours' drying, which, of course, would have an appreciable effect on the electrical characteristics of the apparatus in which it was incorporated. Had no oxidation taken place, of course, its effect would have been undetectable.

On this account lubricants and oils of any type are completely excluded in the manufacture of Cotopa, both the 30 and 60 quality, and in the case of the 60 quality, when it is known to be for high frequency insulation, it is invariably washed with a pure hydrocarbon solvent in order to remove the last traces of any oily matters naturally present, and render it absolutely oil free.

Electrolyte Content of Cotopas

It has already been mentioned (on page 76) that one of the factors governing the D.C. insulation resistance of moisture absorbent insulants is the electrolyte content, and it will easily be seen that this factor also enters into the conductivity fraction of the power factor which has such marked effects at high humidities and low frequencies. The effect of removal of electrolyte content by washing from cottons and silks is already well known and only calls for passing reference here.

In view of the fact that all Cotopas after acetylating contain acetic acid and a little catalyst, together with traces of the original natural salts of the cotton from which they were made, they are washed very thoroughly and repeatedly in a natural water of low electrolyte content to an electrolyte content which approximates to that of the best washed cottons normally used for insulating purposes. Expressed quantitatively, the degree of washing is such that the specific electrical conductivity of a standard water extract prepared as described previously (5 grams of textile shaken with 100 cm^3 of distilled water for an hour at 25° C.) is lower than 50 micromhos.

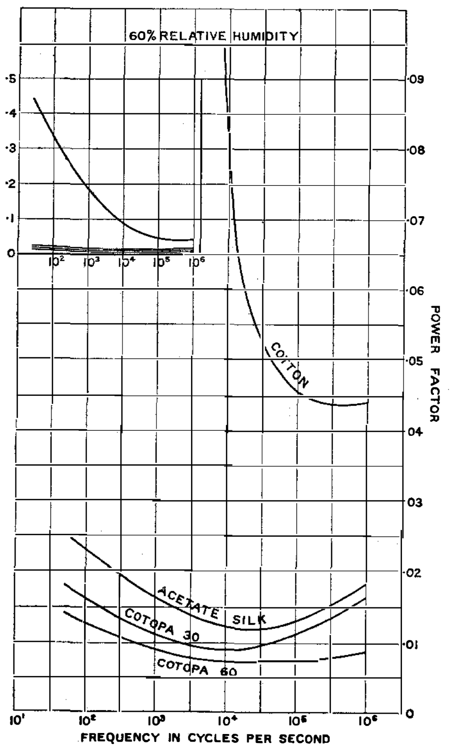


Fig. 11—*A.C. power factor of acetylated cellulosic yarns.*

Cotopa 60, in addition, is specially washed several times with distilled water in which the electrolyte content is negligible, which yields a textile having a lower electrolyte content than any other available at present and having a value of 15–20 micromhos by the method referred to above.

This feature should be taken into account in making any theoretical comparisons regarding relations of structure and D.C. insulation resistance or power factor of the two materials under humid conditions. Its effect is such that if the foregoing curves were recalculated to one standard electrolyte content there would be less difference than that shown between Cotopa 30 and Cotopa 60 in D.C. insulation resistance (and power factor at low frequencies and high humidities). With progressively decreasing relative humidity, the effect decreases and in the dry condition is not detectable. Whether the curves as shown for the two materials were raised or lowered, would depend, of course, on whether values were standardized on the normal electrolyte content of Cotopa 30 or of Cotopa 60.

References have been previously made[2] to "superwashing" ordinary Cotopa 30 in the manner above described, but as it was found possible to reduce the electrolyte content from about 100 micromhos to just below 50 by minor improvements in the ordinary method of washing, it was not felt necessary to reduce it further, and the additional washing process is therefore now only applied to Cotopa 60, for which the name of "Supercotopa" was at one time used. This name has, however, been discontinued, as it seemed to imply merely an extension of the D.C. insulation resistance properties for which Cotopa 30 is valuable, whereas it is desired that Cotopa 60 should be considered as a separate material with distinctive properties for special uses.

It will, of course, be realized that extreme thoroughness of washing can be applied to either type if desired and values as low as 8–10 micromhos can be obtained even on the large scale, but considerably greater effort is required to obtain each successive reduction in electrolyte content below 50 micromhos, so that the limit is set mainly by economic and practical considerations.

Moisture Regain and A.C. Characteristics of Insuwools and Insusilks

The Insuwools and Insusilks are chemical derivatives of wool and silk having lower moisture absorptive properties than the parent materials, and hence improved D.C. insulation resistance under humid conditions while still retaining their well-known resistance to burning. The composition and D.C. insulation resistance characteristics of these materials have already been described,[3] but the following account gives some additional information about their moisture absorption and alternating current characteristics.

From the curves in Figs. 15 and 16 it will be seen that in all cases the treatments produce a reduction in moisture regain at all humidities, but that that produced by the tannic acid reaction is more evident at the high humidities, while acetylation produces a more general effect. It would appear that the effect of the tannic acid treatment is more to reduce the tendency of the protein fibres towards increased absorption above 50–60% R.H., while acetylation is a general factor having the effect of reducing the absorptive ability of the internal surfaces roughly in proportion at any given humidity.

Where the two processes are used, e.g., Insuwool B, and Insusilk C, the effects appear to be superimposed, the tannic acid treatment producing the more marked reduction at the higher part of the humidity range, while the subsequent acetylation causes a roughly proportionate reduction throughout.

These effects are fairly generally reflected in the D.C. Insulation Resistance curves previously given, any minor divergences from the basic principle that the D.C.I.R. is controlled solely by the moisture content (at constant electrolyte content) being probably due to the fact that just as there is a wide range of acetylation of cotton, so there is a similar wide range of possible tannic acid content and a possible range of acetylation, although this latter is not so extensive.

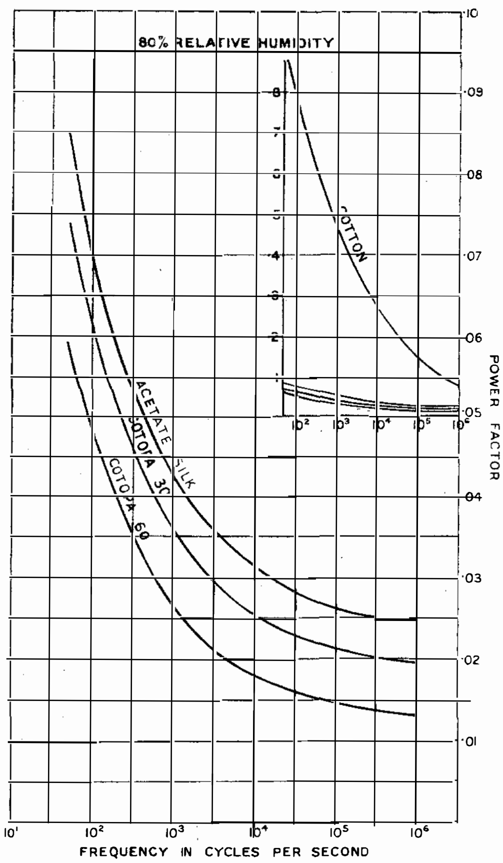


Fig. 12—*A.C. power factor of acetylated cellulosic yarns*

The A.C. characteristics of a number of representative samples of Insuwool and Insusilk yarns have been measured in the same manner as that employed for acetylated cottons and at approximately the same degree of packing and pressure, so that the results are reasonably comparable for practical purposes (Figs. 17 and 18).

As in the case of the acetylated cottons, it is best to consider the curves of these protein and protein derivative materials in the dry condition first, as these are the simplest because of the minimum number of factors operative. In the dry condition, all the Insuwools and Insusilks, like their parent materials if reasonably pure, have fairly flat power factor/frequency characteristics over the range 1–1 000 kc which in all cases and all frequencies are lower than that of cotton. Compared with Cotopa 30, there is a crossover between 10 kc and 100 kc in most

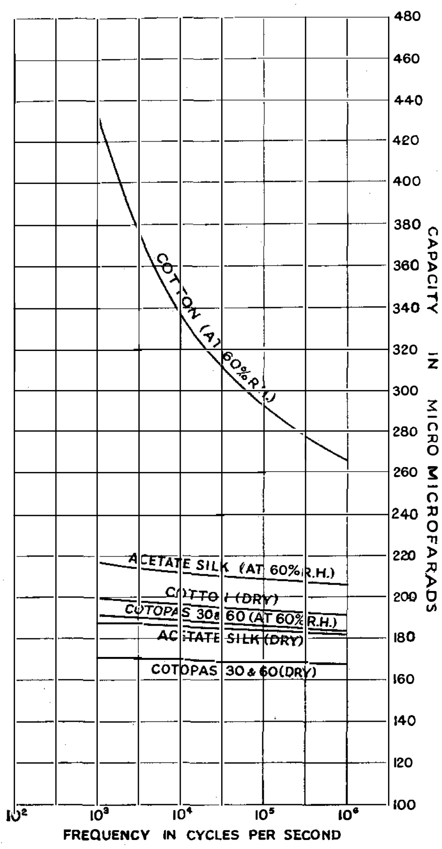


Fig. 13—*Capacities of equal lengths of similar twinned wires insulated to the same radial thickness with various cellulosic yarns.*

cases, and below this Cotopa 30 has the lowest values, while above it the Insuwools and Insusilks have the lower values. Compared with Cotopa 60, the flat type of curve is rather similar, but at no point do the values fall as low as those of Cotopa 60. However, as Insuwool A is cheaper than Cotopa 60, it could reasonably be used to replace it or acetate silk in many places where colour does not matter and improved fire resistance is an advantage.

It will thus be seen that the A.C. characteristics in the dry state are substantially those of the parent materials with a relatively slight improvement due to the processing.

Turning now to the curves of power factor versus frequency under humid conditions, it will be seen that, again, there is an apparent super-position of the effects of the conduction of the absorbed moisture paths and their associated electrolyte (which is most marked at the very low frequencies and falls off with rising frequency) on the dry values together with a dielectric loss effect of the absorbed water itself starting between 100 and 1 000 kc and increasing with frequency. Again, as in the case of the acetylated cottons, the effect of absorbed water is at a minimum at about 100 kc, suggesting that this is a general phenomenon of absorbed water regardless of the surface on which it is absorbed. Both dry and under humid conditions there is the usual fall in permittivity, with increasing frequency and its associated effects. Again, an accurate expression of the exact values of permittivity at any given frequency is difficult owing to the fibrous nature of the material, but it would appear to be of the same order as that of Cotopa 30 and Cotopa 60.

Summarizing briefly, it can be said that the most useful electrical features of the Insuwools and Insusilks are :—

1. High fire resistance characteristic of protein fibres.
2. Greatly improved D.C. Insulation Resistance at high humidities.
3. Greatly improved A.C. Power Factor under humid conditions.
4. Low A.C. Power Factor under dry conditions. This is such that the relatively cheap Insuwool A could be used economically in many positions where a power factor nearly as low as that of Cotopa 60, and at radio frequencies considerably lower than Cotopa 30 or Acetate Silk, is needed and colour is unimportant.

Heat Resistance of Cotopas

For many reasons the heat resistance of insulating materials is of considerable importance; for example, in the drying of cables, cords or coils, the higher the temperature that can be used, the shorter the time for drying, and hence greater throughput of a given drying plant. When as in the case of Cotopas there is only half the amount of moisture to remove as compared with cotton, the effect of its ability to withstand higher temperatures than cotton is considerably magnified. The drying temperature used in any given case is invariably the highest the material will stand without any deterioration in strength or electrical characteristics in the time necessary for removal of moisture. Another case in which the heat resistance of textiles is of advantage is when wires have to be soldered together and the heating up of the wire has a tendency to cause scorching and deterioration of the textile.

It is a matter of common knowledge that cotton scorches in a few seconds if a hot iron is left on it, or that a piece of cotton weakens if left indefinitely in an oven at 100° C., yet relatively little detailed work appears to have been done on this subject. Where studies have been made, they have been mainly concerned with the effects of small amounts of impurities on the rate of deterioration of a given textile or with collecting and analyzing the products of decomposition. It is well established that very small amounts of highly dissociated acids or bases greatly reduce the temperature which cellulosic materials will withstand, and the tendering of cotton on heating, due to traces of mineral acids left in it from dyeing and finishing processes, is well known, as is also the production of this effect by indirect means when cotton dyed with sulphide dye-stuffs is tendered on heating, due to part of the sulphur in or associated with the dye-stuffs being oxidized to sulphuric acid. It is also well known in the acetate silk industry that where (as is generally the case) sulphuric acid has been used as an acetylation catalyst, extreme precautions are taken to remove the last traces of this, because only a small fraction of a percentage will produce a great reduction in the temperature to which the silk can be heated without scorching. No quantitative comparison has apparently been published of the rates of deterioration on heating of textiles which have been meticulously freed from traces of such impurities, such as is the case with textiles used for most electrical insulating purposes.

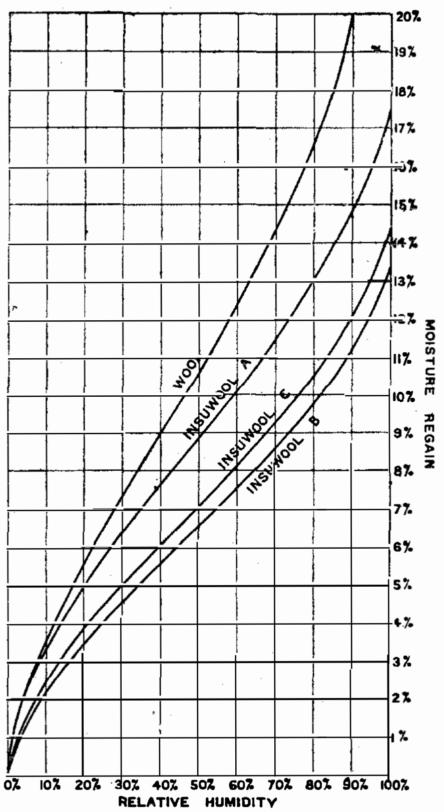


Fig. 15—*Moisture regain of insuwools.*

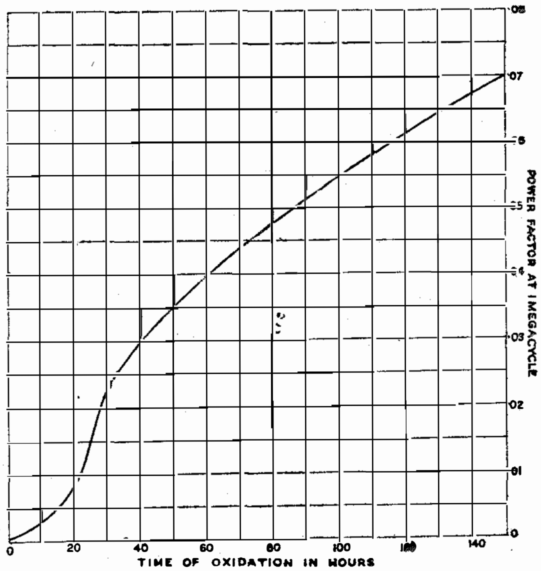


Fig. 14—*Effect of oxidation on power factor at* 1 *megacycle of a refined mineral oil.*

Figures 19–21 show the rate of deterioration in tensile strength of typical yarns of a high degree of purity when heated with free access of air at various temperatures. In practical cases it sometimes occurs that the access of air is limited, in which case the deterioration might be slower, but, of course, the same consideration would be applicable to all kinds, and hence the comparison would remain the same.

The results show that acetylation of cotton to 30% Combined Acetic Acid Content (Cotopa 30) produces a considerable improvement in the resistance to heating. This apparently falls off very slightly on further acetylation (Cotopa 60). It can be seen, however, that Cotopa 30 can be dried at 150° C. or even 160° C. without any noticeable deterioration being observed practically. Similarly, it will withstand the heat of soldering irons and heated wires better than acetate silk and far better than cotton. Acetate silk particularly tends to melt as it decomposes, and the same is true of the highest degrees of acetylation in the fibrous form (Cotopa 60), though at a higher temperature than acetate silk. The effect is confined in the case of the latter to roughly the last 10% of acetylation and has been proposed as a rough guide to the degree of acetylation.

Similar measurements have been made on other textiles not quoted in detail here. However, it can be said that natural silk is about the least resistant, being worse than cotton, and that wool and insuwools are a little better than cotton. The considerable improvement in heat resistance on acetylation is probably due to the greater resistance to oxidation of acetyl groups as compared with hydroxyl groups and their alteration of the spatial arrangements in the cellulose complex.

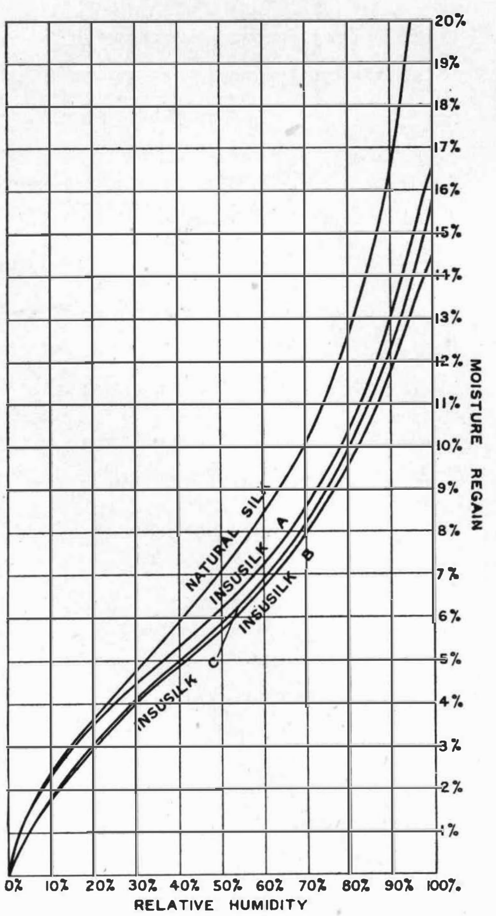


Fig. 16—Moisture regain of insusilks.

Acetylated Paper

As paper consists mainly of cellulose, it would be expected that it would acetylate in a manner analogous to that occurring in the case of yarn. Broadly speaking, this is true, and paper of many kinds can be acetylated[17] in the same baths and in about the same time as cotton, but there are various features of the acetylation of paper which distinguish it sharply from that of cotton, and make it a more complicated problem.[18, 19] The main ones of these are:—

1.—Paper may frequently contain about 10% of ligno-cellulose, a material whose chemical composition is not yet completely elucidated but which may be built up from somewhat similar units to cellulose. The most definite things known about its structure are that it contains methoxyl $(-0.CH_3)$ groups, and that it reacts in some way with acetylating and benzoylating agents, suggesting the presence of hydroxyl groups. It affects the acetylating process by dissolving into the liquor to some extent and fouling it, and secondly by constituting a second system of material interlocked with the cellulose which may affect its electrical properties out of all proportion to its amount.

2.—Being a sheet material, the small changes in mechanical dimensions that take place at different stages of the acetylating process are of far more consequence with paper than with cotton yarn.

3.—As paper is a considerably cheaper raw material than cotton, the costs of acetylation represent a bigger proportion of the cost of the original material and hence economic considerations are even more vital than in the case of acetylated cotton.

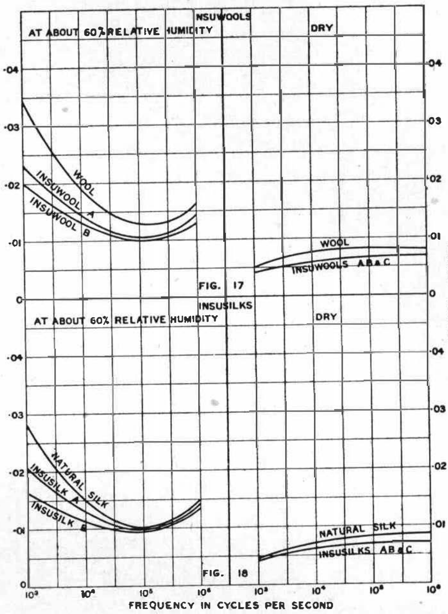
4.—Acetylation of cellulose is an exothermic reaction, and allowance must be made for the heat evolved. This is more important when dealing with paper, because of the need to keep down the ratio of liquor to paper for economic reasons.

The acetylation process can be represented graphically in several ways. With somewhat different scales, the curve already given for cotton represents the course of the reaction for single sheets or loose fibres immersed in a weak acetylating mixture. With stronger conditions, the process gathers speed all the way, until approaching total acetylation, when the scarcity of hydroxyl groups to be acetylated causes it to slow down. With still weaker conditions, there appears to be an asymptotic approach to a certain degree of acetylation which depends on the conditions. As with cotton, solution of cellulose triacetate in the bath becomes marked as the degree of acetylation exceeds 25%–35%, but this can be repressed by the use of inert diluents. If the proportion of anhydride to cellulose is limited, this is another factor tending to cause asymptotic approach to some fixed degree of acetylation.

Properties

The properties that make acetylated paper valuable as a commercial material may be classed as follows :—

1 *a.* Electrical properties under humid conditions.
  b. Electrical properties under dry conditions.
2. Greater stability under various conditions.



Figs. 17 *and* 18.—*A.C. power factors of insuwools and insusilks.*

1 *a.* The moisture regain of papers generally is higher than that of cotton under the same conditions, reflecting the fact that the paper fibres have a larger internal surface. By acetylation this moisture regain is considerably decreased by amounts depending on the degree of acetylation.

As in the case of cotton, this results in a considerable increase in D.C. insulation resistance under atmospheric conditions. This is, however, not so large as is the case with cotton, probably owing to the presence of lignin. Nevertheless, substantial improvements of the order of 100 times or more are obtained with acetylation to a degree of 25%–30% C.A.A.C.

Other electrical properties which depend on moisture content are also improved, particularly the A.C. conductivity. Power factor and permittivity are all considerably reduced, the effect being more pronounced the lower the frequency.

1 *b.* Under dry conditions, the parallel with cotton still holds closely. The D.C. insulation resistance is only slightly better than the starting material, but the A.C. conductance, power factor and permittivity are all improved, the improvement being small at power frequencies but increasing with frequency until at radio frequencies the power factor for 25%–30% C.A.A.C. is reduced by about 40%, and at the highest degree of acetylation (62.5% C.A.A.C.) reduced to about one-third that of the original paper.

A valuable feature is that the power factor does not increase so rapidly with temperature as is the case with untreated paper.

From the above short *résumé* of properties it will be seen that acetylated paper has possibilities of useful application to many electrical problems in the construction of power and telephone cable, condensers, joints, panels, bushings, and electrical apparatus generally.

2. Stability. In four respects at least, other than those concerned with electrical properties, acetylated paper has markedly improved properties when compared with ordinary paper.

(*a*) Properties having application to electrical manufacture. Firstly, its variation in dimensions with change in humidity is much less than that of paper, suggesting a use for offset printing, where accurate register is required, but also indicating that cables or other apparatus insulated with acetylated paper will slacken up less on drying.

Secondly, acetylated paper, like acetylated cotton, has a greater resistance to heat and oxidation and can be heated to higher temperatures than ordinary paper. It is, therefore, possible to use increased temperatures to speed up drying processes of cable or condensers, which will be made still more rapid by the fact that there is considerably less moisture to drive off.

(*b*) Properties having little application to electrical work. It is well known that paper loses its strength when wetted, owing to the fibres being enabled to slip over each other. This effect is greatly reduced with acetylated paper, and hence it has a much higher wet strength[18] amounting to as much as twelve times that of the paper from which it was made in the case of ordinary wrapping paper. This makes it suitable for many wrapping or display purposes where water is likely to be encountered.

Acetylated paper is also more resistant to bacterial action and decomposition than is the case with untreated paper, and hence can stand much longer exposures in wet conditions before it decays.

The above indicates generally the nature and possibilities of acetylated paper, which, together with results obtained with different types of acetylated paper prepared from various types of original material, will be given in detail in the next part of this paper.

Test Methods

Degree of Acetylation.—As the materials described in the foregoing sections include ones having a considerably greater combined acetic acid content than those covered by the method previously described, and as saponification of the higher acetylated celluloses is more difficult than that of the lower ones, the following method has been devised to cover any type of acetylated cellulose, such as acetylated cotton, Cotopa 30, Cotopa 60, acetylated paper, acetate rayon, etc., regardless of the magnitude of the combined acetic acid content.

About 0.5 gram of the material should be reduced to a state of fine subdivision, e.g., yarn should be cut up into pieces not longer than 1 cm: cord should be treated similarly, and then separated into its component threads: acetylated paper should be cut up into pieces not longer than 1 cm × 0.5 cm, etc.

This should then be dried in an oven at 110° C. for 30 minutes, and immediately transferred to a tared weighing bottle, which should be allowed to cool in a desiccator and weighed.

The material should then be transferred to a 250 ml conical flask of alkali resistant glass, fitted with a ground-in stopper with short air condenser tube 8″ long and $\frac{3}{16}$″ internal diameter; 10 ml of ethyl alcohol should then be added and the flask heated on a water bath until the alcohol boils; 10 ml of normal sodium hydroxide solution should then be added, and the flask shaken to ensure that the sample is completely covered by the liquid, before replacing on the water bath.

The flask should remain on the boiling water bath for 15 minutes, with careful shaking every few minutes, and at the end of this period about 100 ml of carbon-dioxide-free distilled water should be run in through the inverted air condenser to wash in any drops of alkali which may have splashed up. The solution should then be back titrated with normal sulphuric acid, using 2 drops of 0.2% phenolphthalein solution as indicator (or pH titration meter if available); when the apparent endpoint is reached, the solution should be boiled gently for 1 minute and then, if necessary, more acid added drop by drop to reach the true endpoint.

The Combined Acetic Acid Content is calculated as follows:

$$\frac{\text{No. of ml. Normal Alkali used in Saponification} \times 0.06 \times 100}{\text{Weight of dry sample}}$$

which expresses it as a percentage of the dry weight of the material.

It has been found that, owing to a number of effects of which the attack of the alkali on the cellulose itself is probably the most important, a small amount of alkali is used up under the above conditions, even by pure cellulose itself. With unbleached cellulose and cellulose regenerated from acetylated unbleached cellulose, an apparent value of 0.8% C.A.A.C. is usually obtained, while with bleached cellulose and cellulose regenerated from acetylated bleached cellulose one of about 0.35% C.A.A.C. is generally found. In expressing the results of an analysis by the above method, therefore, such amounts should be deducted from the apparent figures, depending on the degree of bleaching of the material. When this has been done, the results are believed to be accurate to ± 0.3% C.A.A.C., although the difficulty of choosing an absolute standard for this type of analysis is fully apparent.

As several methods of expressing degree of acetylation are in use, the relation of the more common ones, namely, combined acetic acid content, acetyl content and percentage of total possible acetylation are given in Fig. 22, together with the gain in weight of cellulose on acetylation. In the case of the last named, an allowance must be made if any solution takes place in the bath.

Measurement of Alternating Current Characteristics

Owing to the wide range of frequencies covered, the following series of test sets was employed :—

Frequency Range	
50 p:s	50-cycle Schering Bridge.[20]
50 p:s–3 kc.	Conductance and Capacity Bridge.[21]
1 kc–1 mc.	General Radio Radio Frequency Bridge, Type 516.C.[22]
50 p:s.–10 mc.	Radio Frequency Schering Bridge.[23]
10 kc–50 mc.	N.P.L. Resonance Apparatus.[24]

As, however, they have been fully described elsewhere, it is not proposed to deal with them in detail here.

A few words may be said, however, about the type of test condensers used. Broadly speaking, four types have been used, as follows :—

1.—Yarns have usually been wound on 2″ dia. polished brass tubes, and covered by a lapping of copper tape tightly applied, to provide the second electrode. A ½″ clear margin at least is left between the edge of the brass tube and the end turns of textile, and also between the latter and the extreme turns of copper tape at each end. The width of the smaller electrode is of the order of 100 times the thickness of the dielectric, so that errors due to fringing effects are small compared with other sources of error, and can be allowed for. From the thickness of dielectric layer and its weight the degree of packing or bulk density can be determined, but on this the degree of accuracy of measurement is not very high. With different materials and a normal lapping it usually varies between 0.4 gm/cm^3 and 0.8 gm/cm.^{3}

2.—An alternative to method 1 has been the use of optically flat electrodes, 6″ × 6″, somewhat on the lines of electrodes used by Denham, Hutton and Lonsdale.[14] By this means, measurements have been made possible under an accurately determined pressure, but the calculation of bulk density, which is an essential factor, still depends on accurate measurement of the small distance separating the plates. If all textiles were of the same degree of compressibility, of course, control of the pressure would automatically control the bulk density, but unfortunately textiles vary widely in this respect. A type of electrode in which pressure can be varied continuously and both pressure and electrode separation be measured accurately is now being constructed to overcome this difficulty and increase the accuracy of measurement.

3.—A few measurements have been made for comparative purposes by lapping copper wires, making twisted pairs and carrying out the measurements on these, but obviously, fringing errors must be fairly large here.

4.—Many of the measurements on fabrics and on papers have been made by building up small condensers with tinfoil leaves to the desired capacity, and pressing together in a standard condenser clamp. As these measurements are made on materials which are less compressible, they are more accurate, but, of course, always represent higher bulk densities than the yarn tests. Values usually range between 0.5 gm/cm^3 and 1.0 gm/cm^3 for fabrics, and from 0.6 gm/cm^3 to 1.2 gm/cm^3 for papers. For solid films, of course, the values go correspondingly higher. Owing to the small thickness of dielectric compared with its other dimensions, fringing errors are sufficiently small to be ignored.

5.—A considerable number of tests have been made with mercury electrodes, but as the simpler method described in (4) checked up satisfactorily with this one, and as the mercury electrodes are more difficult to manipulate and considerably more costly, hence precluding large numbers of tests being made simultaneously, they were only used for a proportion of the tests.

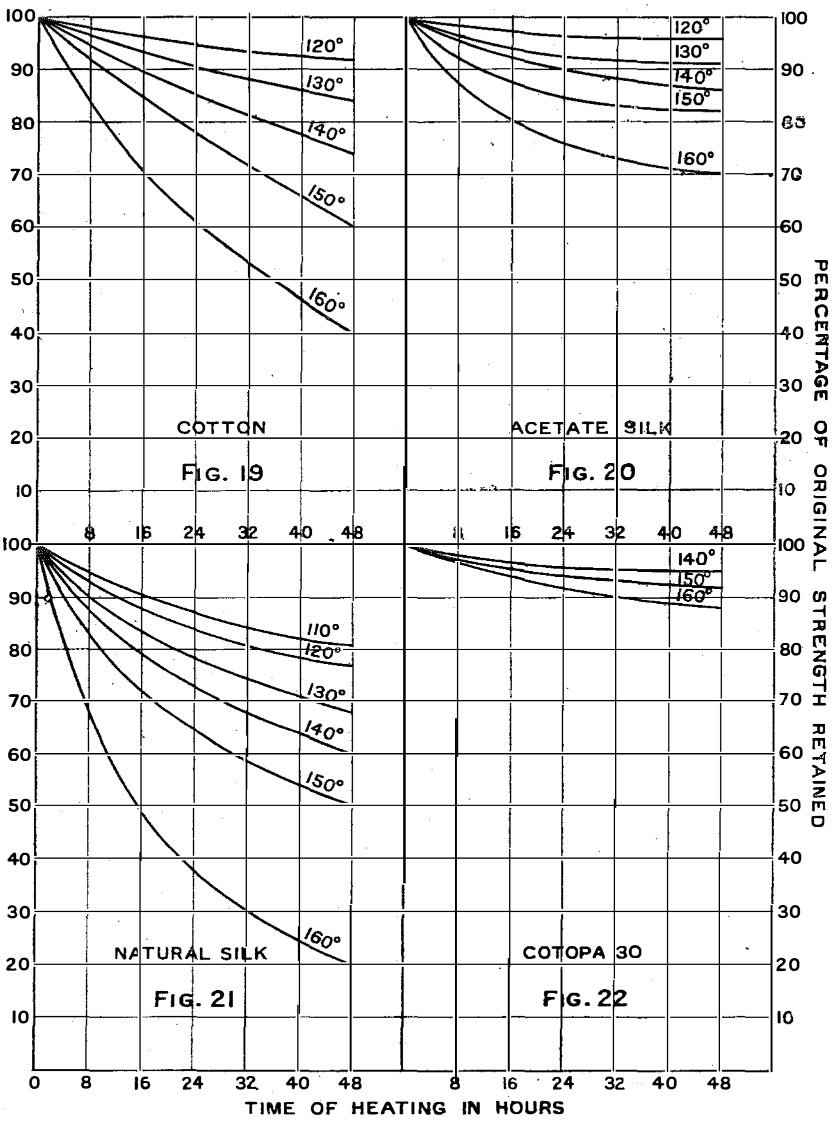
Where measurements have been made at stated humidities, these have been controlled by sulphuric acid and distilled water solutions, as previously described. The type of electrode used makes the attainment of equilibrium very slow, and on this account either the D.C. insulation resistance or power factor at 50 cycles is measured periodically until no further change takes place, as a criterion that equilibrium has been reached. A further check, of course, can be made by weighing before and after humidifying, when the amount of regain will indicate how closely equilibrium has been reached.

Electrical Aspects of Some Other New Textiles

Since the publication of the last paper in this series, several other new textiles have been placed on the market, and a brief description is given below of the five most important, in order that a comparison may be made with the esterified products described in detail in this paper.

Cellulose Triacetate Silk.—This is a rayon similar in appearance to ordinary acetate (di-tri or secondary acetate silk) or natural mulberry silk. It is stated to be made straight from the triacetate without the use of solvents, and it is claimed that its individual filaments are as fine as those of natural silk, and hence that it can be used to replace the latter in the insulation of fine wires. It has been found, however, that its tensile strength is about the same as ordinary acetate silk, and hence is considerably lower than that of mulberry silk of the same denier, which lessens the possibility of its use in this application.

It has a very high D.C. insulation resistance, and this and its A.C. characteristics are very similar to those of Cotopa 60 at the same degree of packing. Actually, being a rayon, it will pack a little closer in any practical case, and hence would give higher power factors and permittivities than the latter. Its price is about 3–5 times that of Cotopas; hence there is unlikely to be any competition between them. It is of German origin.



Figs. 19–22—*Resistance to heat deterioration of insulating yarns.*

Nylon.—Nylon (Neophil) is also a rayon, and has the distinction of being the first 100% synthetic fibre. It is of American origin (but production has now started in Great Britain) and is remarkable for its fine counts and high tensile strength, about 2 or 3 times that of cotton. Its D.C. insulation resistance under humid conditions is about equal to that of acetate silk, and much the same is true of its A.C characteristics. It would appear to have an assured future in the hosiery trade, but does not appear likely, as at present produced, to displace any of the established textiles in the electrical industry.

Vinyon.—This is another all-synthetic rayon of American origin. Its most outstanding property is that it is non-inflammable, but it also has a very low softening temperature, completely losing its strength at 80° C.

Its D.C. insulation resistance is about half that of ordinary acetate yarn under atmospheric conditions. Its A.C. properties are difficult to compare with other textiles, because it is unusual amongst textiles in having a power factor frequency characteristic which falls as the frequency increases (under dry as well as wet conditions). This effect is characteristic of the synthetic resin from which it is made. At 50 cycles, its power factor is about 10 times that of Cotopa 30, whereas at 1 mc it is about half-way between the values for Cotopa 30 and Cotopa 60. It is stated to have complete resistance to a wide range of chemicals and, hence, to be very suitable for making filtering cloth for many purposes.

Glass Yarns.—Glass fibres are really in a class by themselves, because they constitute the only practical inorganic yarn used for insulating purposes. Asbestos, of course, is an inorganic fibrous material, but its spinning and tensile strength properties by itself are very poor. The property in which glass fibres surpass all others is in superior heat resistance. The best organic yarn for heat resistance, Cotopa 30, will stand 24 hours at 160° C., with hardly any deterioration, but glass fibres can be heated up to 600° C. without more than a slight reduction in tensile strength.

The electrical properties of fibres made from soda glass are somewhat similar to those of unpurified cotton, but considerably better characteristics are possessed by fibres made from glasses more suitably chosen.

Synthetic yarns made from proteins.—A number of these have appeared in the last few years and have interesting textile and dyeing properties, but none appear to have any electrical characteristics of outstanding value.

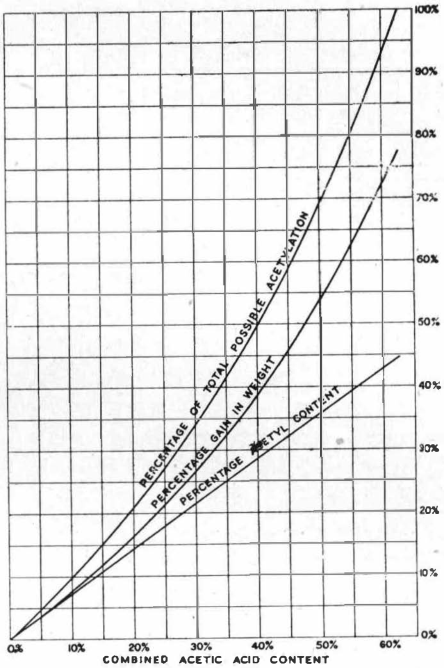


Fig. 23—Relation of combined acetic acid content to : (1) acetyl content ; (2) gain in weight ; (3) percentage of total possible (tri) acetylation.

Summary and General Comparison of Esterified Insulation with Other Fibrous Insulating Materials

In the foregoing pages it has been shown that Cotopa 30 is a textile insulating material having all the mechanical properties of cotton, including those of easy running on insulating machines, coupled with enormously greater D.C. insulation resistance, marked improvement in A.C. properties under most conditions, and considerably greater heat resistance than cotton. These features make it particularly suitable as insulation for many types of instrument and magnet wires, as well as switchboard wire, bank wire, switchplate wire, also for telephone and instrument cords, whether for ordinary or tropical use, and for various types of cable, such as switchboard cable, leading-in cable, and repeater station cable, which normally have to work partly or wholly in contact with the air and are subject to the effects of fluctuations in atmospheric humidity.

Its nearest rival is the best type of ordinary acetate silk, which has about one-half to one-

Insulation	A.C. leakance of 6-foot samples of twinned conductors in micromhos at 800 p : s. 15° C. and 70% Relative Humidity
Double Cotopa lapped and lacquered	0.013
Double Cotopa lapped	0.017
Enamelled, Triple Acetate Silk lapped and Silk braided	0.017
Single Acetate Silk and Wool lapped	0.026
Double Tussah Silk and Single Cotton lapped	0.053
Enamelled, Double Silk and Single Cotton lapped	0.109
Double Acetate Silk and Double Cotton lapped	0.317
Double Cotton lapped	1.37

third the insulation resistance of Cotopa under any given humidity conditions, coupled with more difficult running properties and a lower decomposition temperature.

The table shown above gives an idea of the comparison of various types of insulation under practical conditions.

Crestol.—Crestol is a very lustrous form of Cotopa 30, which combines a pleasing appearance with the valuable electrical properties of Cotopa 30 described above.

If particularly low high frequency loss and phase angle are required, coupled with mechanical properties similar to those of cotton, Cotopa 60 is preferable,[16] especially if the dielectric can be dried and kept in a sheath or container. On this account it has been used in considerable quantities as the dielectric in various forms of coaxial cable for the transmission of radio frequencies. Its use in the London–Birmingham coaxial cable has been described by Angwin and Mack,[25] by whom it was shown that, owing to the low cost of Cotopa, such cable, although of somewhat larger dimensions than a cable insulated with more perfect dielectrics, compares favourably with such cables. Cotopa 60 is also suitable for cords or instrument wire for high frequency work, whether working at atmospheric humidity or dry. It will, of course, provide a considerably higher D.C. insulation resistance than any other textile available at present at anything like the same cost, but in most cases where purely D.C. considerations govern the choice, it is usually found that the values obtained with Cotopa 30 are sufficiently high for most practical purposes.

Where low inflammability, coupled with high insulation resistance and good A.C. characteristics, whether dry or at atmospheric humidity, are required, the *Insuwools* and *Insusilks* provide improved insulation for instrument wires, cords and cables, such as ribbon cable, etc.

The above materials are made in yarn, fabric and tape form, and Cotopa 30 and 60 are also available as a "bias cut" tape, which is a type without selvedge, cut on the bias and hence less readily split or torn, and which possesses the useful property of more closely fitting non-cylindrical-shaped objects (e.g., various types of coils, joints and other types of apparatus) than the tapes of normal construction. Where a paper type insulant having better D.C. and A.C. characteristics than ordinary types of paper is required, *Acetylated paper* is valuable for many purposes in cables, instruments, compound dielectrics and also for various non-electrical purposes. It also has the advantage that it can be dried more rapidly than ordinary paper, thus enabling a greater throughput of cable from a given drying plant.

Acknowledgments

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Harmonic Current Generation in Polyphase Rectifier Circuits*

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IT is not very generally realized that the occurrence of current harmonics on the A.C. side of rectifier equipment is indissolubly linked with, and is in fact determined by, the generation of voltage harmonics on the D.C. side, a feature of polyphase rectification which was discussed in an earlier article.[1] The necessary connection between the two groups of harmonics can be demonstrated by considering the energy transfer process implicit in rectifier operation, as may be seen from Kübler's analysis of the active and reactive power oscillations at harmonic frequencies occurring on the A.C. side of a polyphase rectifier system as the result of power production on the D.C. side from a rectilinear output current and a non-rectilinear terminal voltage.[2] The phenomenon of harmonic current generation in polyphase rectifier circuits is perhaps more easily to be understood, however, by analysing the wave-form of the appropriate circuit currents.

As before, the customary assumption of an infinitely inductive rectifier load will be made, so that the rectifier system delivers to the load a direct current of constant magnitude. In the ideal case illustrated by Fig. 1 the D.C. load is carried by each rectifier phase in turn, the load period being $1/p$th of the phase-voltage cycle. Successive phases thus carry blocks of direct current having a rectangular wave-form, the current-conducting period per phase being $2\pi/p$ electrical radians, and the instantaneous value of the current being constant and equal to I_d. The mean value of the phase-current is thus—

$$I_m = \frac{1}{2\pi}\int_{-\pi/p}^{+\pi} I_d.\, d\theta = \frac{I_d}{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

while its r.m.s. value is, similarly—

$$I = \sqrt{\left[\frac{1}{2\pi}\int_{-\pi/p}^{+\pi/p} I_d^2.\, d\theta\right]} = \frac{I_d}{\sqrt{p}} \quad \ldots\ldots\ldots\ldots(2)$$

As usual, the rectangular current wave of Fig. 1 can be represented by the Fourier series $A_0 + \Sigma A_n \cos n\,\theta + \Sigma B_n \sin n\,\theta$. The constant term A_0 here represents the mean value of the current wave I_m. The Fourier coefficients of the nth harmonic are then given by—

$$A_n = \frac{1}{\pi}\int_{-\pi/p}^{+\pi/p} I_d \cos n\theta .\, d\theta = \frac{2I_d}{n\pi}\sin\frac{n\pi}{p}$$

and

$$B_n = \frac{1}{\pi}\int_{-\pi/p}^{+\pi/p} I_d \sin n\theta . d\theta = 0.$$

The amplitude of the nth phase-current harmonic is $\sqrt{(A_n^2 + B_n^2)}$ so that its r.m.s. value is consequently—

$$I_n = \frac{\sqrt{2}}{n\pi}\sin\frac{n\pi}{p}\,.\, I_d = \frac{\sqrt{2p}}{n\pi}\sin\frac{n\pi}{p}\,.\, I \quad \ldots(3)$$

It will be observed that $I_n = 0$ for all values $n = mp$, where m is any integer. This is the converse of the condition obtaining in the case of the output voltage where, it will be recollected, the occurrence of any individual harmonic can only be associated with a value of n which is a multiple of the number of rectifier phases. Thus

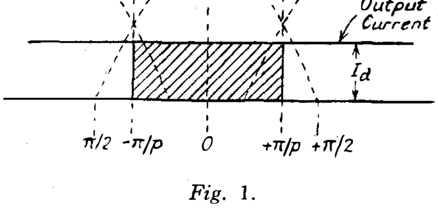

Fig. 1.

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* Reprinted from *The Electrician*, January 19th, 1940.

[1] *Electrical Communication*, Vol. XVIII, April, 1940, p. 271.

[2] *Elektrotechnik und Maschinenbau*, 1937, Vol. 55, p. 457.

TABLE I

OUTPUT-VOLTAGE AND INPUT-CURRENT HARMONICS (%)

Number of Rectifier Phases.	$p = 2$		$p = 3$		$p = 4$		$p = 6$		$p = 12$	
% R.M.S. Harmonic. / Order of the Harmonic, n.	V_n/V_d	I_n/I	V_n/V_d	I_n/I	V_n/V_d	I_n/I	V_n/V_d	I_n/I	V_n/V_d	I_n/I
1	—	63.66	—	67.51	—	63.66	—	55.13	—	40.37
2	47.13	—	—	33.76	—	45.01	—	47.75	—	39.01
3	—	21.22	17.71	—	—	21.22	—	36.75	—	36.75
4	9.44	—	—	16.88	9.44	—	—	23.88	—	33.76
5	—	12.73	—	13.50	—	12.73	—	11.03	—	30.13
6	4.05	—	4.05	—	—	15.00	4.05	—	—	26.01
7	—	9.09	—	9.50	—	9.09	—	7.88	—	22.95
8	2.25	—	—	8.44	2.25	—	—	11.94	—	16.88
9	—	7.07	1.77	—	—	7.07	—	12.25	—	12.25
10	1.43	—	—	6.75	—	9.00	—	9.55	—	7.80
11	—	5.79	—	6.14	—	5.79	—	5.01	—	3.67
12	0.99	—	0.99	—	0.99	—	0.99	—	0.99	—
13	—	4.90	—	5.19	—	4.90	—	4.24	—	3.10
14	0.73	—	—	4.74	—	6.43	—	6.82	—	5.57
15	—	4.24	0.63	—	—	4.24	—	7.35	—	7.35
16	0.56	—	—	4.50	0.56	—	—	5.97	—	8.44
17	—	3.74	—	3.97	—	3.74	—	3.25	—	8.86
18	0.44	—	0.44	—	—	5.00	0.44	—	—	8.67
19	—	3.35	—	3.56	—	3.35	—	2.91	—	7.93
20	0.36	—	—	3.38	0.36	—	—	4.78	—	6.75
21	—	3.03	0.32	—	—	3.03	—	5.25	—	5.25
22	0.29	—	—	3.07	—	4.92	—	4.34	—	3.55
23	—	2.76	—	2.83	—	2.76	—	2.40	—	1.75
24	0.25	—	0.25	—	0.25	—	0.25	—	0.25	—
25	—	2.45	—	2.70	—	2.45	—	2.21	—	1.16

any particular harmonic present in the output voltage is absent from the input current, and vice versa. The existence of this fundamental harmonic relation is apparent from an inspection of Table I in which the r.m.s. values of the harmonic currents given by (3), and expressed as percentages of the r.m.s. input current I, are tabulated alongside those of the harmonic voltages given by the corresponding D.C. relation—

$$V_n = \frac{\sqrt{2}}{n^2 - 1} \cdot V_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

established in the previous article.[1]

(a) Retarded Commutation (Grid-control).

In the case of a grid-controlled rectifier system in which the instant of commutation is artificially delayed by the so-called "ignition angle" α, the individual rectifier phase currents are no longer in phase with their corresponding phase voltages. The axis of current conduction per phase then no longer coincides with the point of maximum phase voltage. In other words, referring to Fig. 1, the phase voltage normally effective between the limits $\theta = -(\pi/p)$ and $\theta = +(\pi/p)$ generates a phase current flowing not during this same interval, but during the corresponding interval from $\theta = (\alpha - \pi/p)$ to $\theta = (\alpha + \pi/p)$. These two intervals are of the same duration, but the latter lags the former by the angle α. It is evident that under these conditions of forced commutation neither the magnitude nor the wave-form of the individual rectifier phase-currents is affected. So that the mean and r.m.s. values given by (1) and (2) remain valid. The Fourier coefficients of the nth harmonic, however, in this case become—

$$A_n = \frac{1}{\pi} \int_{\alpha - \pi/p}^{\alpha + \pi/p} I_d \cos n\theta . d\theta = \frac{2I_d}{n\pi} \sin \frac{n\pi}{p} . \cos n\alpha$$

and

$$B_n = \frac{1}{\pi} \int_{\alpha - \pi/p}^{\alpha + \pi/p} I_d \sin n\theta .\ d\theta = \frac{2I_d}{n\pi} \sin \frac{n\pi}{p} . \sin n\alpha .$$

The amplitude of the nth phase-current harmonic is $\sqrt{(A_n^2 + B_n^2)}$ and its angular displacement is $\varphi_n = \tan^{-1} (B_n/A_n)$. Its r.m.s. value is consequently—

$$I_n = \frac{\sqrt{2}}{n\pi} \sin \frac{n\pi}{p} . I_d = \frac{\sqrt{2p}}{n\pi} \sin \frac{n\pi}{p} . I \quad ..(5)$$

while its phase angle is $\varphi_n = n\alpha$. A comparison of (5) with (3) shows that *the magnitude of the phase current harmonics is unaffected by the retardation of the instant of phase commutation*, only their relative phase position is altered thereby ; but each harmonic is displaced by the angle $n\alpha$ from its normal position. As the actual phase displacement is thus proportional to the order of the harmonic, the relative displacement with respect to the phase-voltage wave is the same for all harmonics. In other words, the entire (rectangular) current wave is displaced by the angle α without any change in wave-form taking place.

The above statement is only valid if the rectifier phase currents remain constant in magnitude throughout the successive periods of current conduction, whatever the value of the rectifier load or the ignition angle; that is, if the load inductance is infinite. The input-current wave-form will alter, however, if the rectifier load is predominantly resistive. In the case of a pure resistance load, where the input-current wave is a replica of the output-voltage wave during the same interval, the alteration in wave-form is at a maximum since the bodily displacement of the input-current wave through the phase angle α alters the part of the phase-voltage wave which is effective in generating the phase-current wave.

(b) Finite Commutation (Overlapping).

So far, consideration has only been given to the ideal case of polyphase rectification in which the commutation of the load current I_d from one rectifier phase to the next is assumed to take place instantaneously. Consider now the actual case in which, due to the inevitable presence of circuit reactance, commutation does not take place instantaneously but occupies a finite time represented in Fig. 2 by the angle of overlap u. During this commutation interval the successive rectifier phase currents overlap in such a way that their instantaneous sum remains equal to the load current I_d.

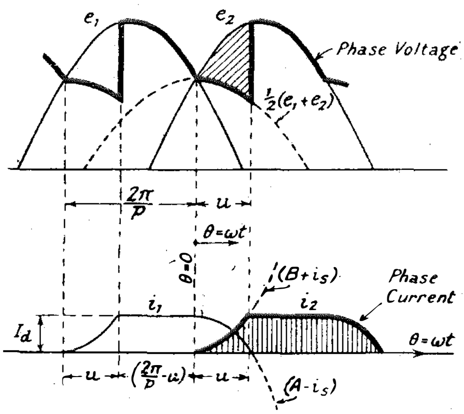

Fig. 2.

Referring to Fig. 2, it can be shown[3] that the phase-current wave consists of the three parts—

$$i=\left(\frac{1-\cos\theta}{1-\cos u}\right)I_d=w(\theta).\ I_d \quad \text{for } o<\theta<u,$$

$$i=I_d \qquad \text{for } u<\theta<\frac{2\pi}{p},$$

$$i=\left[\frac{\cos(\theta-2\pi/p)-\cos u}{1-\cos u}\right]I_d$$

$$=[1-w\,(\theta-2\pi/p)]\,I_d$$

$$\text{for}\ \frac{2\pi}{p}<\theta<\left(\frac{2\pi}{p}+u\right),$$

where $\theta=0$ is reckoned from the point of intersection of succeeding phase-voltage waves. The Fourier coefficients thus become—

$$A_n=\frac{I_d}{\pi}\left[\int_0^u w(\theta)\cos n\theta.\ d\theta+\int_u^{\pi/p}\cos n\theta.\ d\theta\right.$$

$$\left.+\int_{2\pi/p}^{2\pi/p\,+\,u}\left\{1-w\left(\theta-\frac{2\pi}{p}\right)\right\}\cos n\theta.\ d\theta\right]$$

$$=\frac{I_d}{\pi}\left[\int_0^u\left\{\cos n\theta-\cos n\left(\theta+\frac{2\pi}{p}\right)\right\}w(\theta)d\theta\right.$$

$$\left.+\int_u^{2\pi/p\,+\,u}\cos n\theta.\ d\theta\right]$$

and, similarly—

$$B_n=\frac{I_d}{\pi}\left[\int_0^u\left\{\sin n\theta-\sin n\left(\theta+\frac{2\pi}{p}\right)\right\}w(\theta)d\theta\right.$$

$$\left.+\int_p^{2\pi/\mathrm{p}\,+\,u}\sin n\theta.\ d\theta\right]$$

where $n=2, 3\ \ldots$ etc. The integration gives

$$A_n=\frac{2I_d\sin\dfrac{n\pi}{p}}{n\pi(n^2-1)(1-\cos u)}$$

$$\left[X\sin\frac{n\pi}{p}+Y\cos\frac{n\pi}{p}\right]$$

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[3] *Cf.* the author's *Mercury-arc Current Convertors*, pp. 26 *et seq.* (Sir Isaac Pitman & Sons, 1940, 2nd Edn.).

and

$$B_n = \frac{2I_d \sin\frac{n\pi}{p}}{n\pi(n^2-1)(1-\cos u)} \left[Y \sin\frac{n\pi}{p} - X \cos\frac{n\pi}{p}\right]$$

where X and Y are functions of n and u having the values—

$X = n \cos nu \sin u - \sin nu \cos u,$

$Y = \cos nu \cos u + n \sin nu \sin u - 1.$

The amplitude of the nth phase-current harmonic is given by $\sqrt{(A_n^2 + B_n^2)}$. The corresponding r.m.s. value is thus—

$$I_n = \frac{\sqrt{2}I_d \sin\frac{n\pi}{p}}{n\pi(n^2-1)(1-\cos u)} \cdot \sqrt{[X^2 + Y^2]}$$

$$= I_{n_0} \cdot \frac{\sqrt{[(\cos nu - \cos u)^2 + (\sin nu - n \sin u)^2]}}{(n^2-1)(1-\cos u)} \quad \ldots\ldots 6)$$

where I_{n_0} is the ideal value of I_n, when $u = 0$, as given by (3). Equation (6) approximates to the form—

$$I_n = I_{n_0} \cdot \frac{\sqrt{\left[\frac{1}{2}(1 - \cos nu) + \frac{nu}{4}(nu - 2 \sin nu)\right]}}{(n^2-1)(1-\cos u)} \quad \ldots\ldots(6a)$$

The phase angle of the nth harmonic is then—

$$\varphi_n = \tan^{-1}\left(\frac{B_n}{A_n}\right) = \left(\frac{n\pi}{p}\right) - \tan^{-1}\left(\frac{X}{Y}\right) \quad \ldots\ldots(7)$$

It is interesting to note that while the values of the sine and cosine components B_n and A_n, expressed as ratios of their ideal values when $u = 0$, are functions of p as well as of n and u, the ratio of their resultant I_n to its ideal value I_{n_0} is independent of the number of rectifier phases, and depends only on the order of the harmonic and the angle of overlap of the input currents.

In the case of the fundamental component, for which $n = 1$, one finds—

$$A_1 = \frac{I_d \sin\frac{\pi}{p}}{2\pi(1-\cos u)} \left[(2u - \sin 2u) \sin\frac{\pi}{p} + (1 - \cos 2u) \cos\frac{\pi}{p}\right]$$

and

$$B_1 = \frac{I_d \sin\frac{\pi}{p}}{2\pi(1-\cos u)} \left[(1 - \cos 2u) \sin\frac{\pi}{p} - (2u - \sin 2u) \cos\frac{\pi}{p}\right].$$

Hence the r.m.s. value of the fundamental component of the phase-current wave is—

$$I_1 = \frac{\sqrt{2}I_d \sin\frac{\pi}{p}}{4\pi(1-\cos u)} \cdot \sqrt{[(1 - \cos 2u)^2 + (2u - \sin 2u)^2]}$$

$$= I_{1_0} \cdot \frac{\sqrt{(u^2 - 2u \sin u \cos u + \sin{}^2 u)}}{2(1-\cos u)} \quad \ldots\ldots(8)$$

and its phase angle with reference to the zero axis $\theta = 0$ is—

$$\varphi_1 = \tan^{-1}\left(\frac{B_1}{A_1}\right) = \left(\frac{\pi}{p}\right) - \tan^{-1}\left(\frac{2u - \sin 2u}{1 - \cos 2u}\right) \quad (9)$$

As $\theta = \pi/p$ is the axis of symmetry of the phase-voltage wave, it is seen that the fundamental component of the phase current lags the phase voltage by the so-called "displacement angle" φ defined by—

$$\tan \varphi = \frac{2u - \sin 2u}{1 - \cos 2u} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

which approximates very closely to $\varphi = \frac{2}{3}u$, as shown in Table II. The effect of circuit reactance is thus to cause a phase displacement φ of the power-carrying component of the input current with respect to the rectifier phase voltage whose amount may be taken as being two-thirds of the angle of overlap u.

TABLE II

u	0°	15°	30°	45°	60°	75°	90°
ϕ	0°	9° 59′	19° 55′	29° 43′	39° 19′	48° 37′	57° 31′
$\frac{2}{3}u$	0°	10°	20°	30°	40°	50°	60°

(c) *Finite and Retarded Commutation (Overlapping with grid-control).*

In the general case, where both a forced and a natural delay take place in the phase-commutation process, represented by the ignition angle α and the succeeding angle of overlap u respectively (Fig. 3), the expressions for the three parts of the phase-current wave become—

$$i = \left[\frac{\cos\alpha - \cos\theta}{\cos\alpha - \cos(\alpha + u)}\right] I_d$$

$$= w(\theta, \alpha).\, I_d \qquad \text{for } \alpha < \theta < (\alpha + u),$$

$$i = I_d \qquad \text{for } (\alpha + u) < \theta < \left(\frac{2\pi}{p} + \alpha\right)$$

$$i = \left[\frac{\cos(\theta - 2\pi/p) - \cos(\alpha + u)}{\cos\alpha - \cos(\alpha + u)}\right] I_d$$

$$= [1 - w\{(\theta - 2\pi/p), \alpha\}] I_d$$

$$\text{for } \left(\frac{2\pi}{p} + \alpha\right) < \theta < \left(\frac{2\pi}{p} + \alpha + u\right).$$

The Fourier coefficients for the nth harmonic thus become—

$$A_n = \frac{2I_d \sin\frac{n\pi}{p}}{n\pi(n^2 - 1)(\cos\alpha - \cos\overline{\alpha + u})} \left[X \sin\frac{n\pi}{p} + Y \cos\frac{n\pi}{p}\right]$$

and

$$B_n = \frac{2I_d \sin\frac{n\pi}{p}}{n\pi(n^2 - 1)(\cos\alpha - \cos\overline{\alpha + u})} \left[Y \sin\frac{n\pi}{p} - X \cos\frac{n\pi}{p}\right]$$

where X and Y are functions of n, u and α having the values—

$$X = n \cos nu \sin(\alpha+u) - \sin nu \cos(\alpha+u) - n \sin\alpha$$

$$Y = \cos nu \cos(\alpha+u) + n \sin nu \sin(\alpha+u) - \cos\alpha.$$

The amplitude of the nth phase-current harmonic is given by $\sqrt{(A_n^2 + B_n^2)}$ so that its r.m.s. value is—

$$I_n = \frac{\sqrt{2} I_d \sin\frac{n\pi}{p}}{n\pi(n^2 - 1)(\cos\alpha - \cos\overline{\alpha + u})} \cdot \sqrt{[X^2 + Y^2]}$$

$$= I_{n0} \cdot \frac{\sqrt{[(\cos nu \cos\alpha - n \sin nu \sin\alpha - \cos\overline{\alpha+u})^2 + (\sin nu \cos\alpha + n \cos nu \sin\alpha - n \sin\overline{\alpha+u})^2]}}{(n^2 - 1)(\cos\alpha - \cos\overline{\alpha + u})} \quad \ldots\ldots(11)$$

where I_{n0} is the ideal value of I_n, when both $\alpha = 0$ and $u = 0$, as given by (3). Equation (11) approximates to the form—

$$I_n = I_{n0} \cdot \frac{\sqrt{[\frac{1}{2}(1 - \cos nu)(\cos^2\alpha + n^2 \sin^2\alpha)]}}{(n^2 - 1)(\cos\alpha - \cos\overline{\alpha + u})} \quad \ldots\ldots\ldots\ldots\ldots\ldots(11a)$$

The phase angle of the nth harmonic is given by equation (7), as before.

In the case of the fundamental component, for which $n = 1$, one finds—

$$A_1 = \frac{I_d \sin\frac{\pi}{p}}{2\pi(\cos\alpha - \cos\overline{\alpha+u})} \left[V \sin\frac{\pi}{p} + W \cos\frac{\pi}{p}\right]$$

and

$$B_1 = \frac{I_d \sin\frac{\pi}{p}}{2\pi(\cos\alpha - \cos\overline{\alpha+u})} \left[W \sin\frac{\pi}{p} - V \cos\frac{\pi}{p}\right]$$

where $V = 2u + \sin 2\alpha - \sin 2(\alpha + u)$
and $W = \cos 2\alpha - \cos 2(\alpha + u)$.

Hence the r.m.s. value of the fundamental component of the phase-current wave is—

$$I_1 = \frac{\sqrt{2} I_d \sin\frac{\pi}{p}}{4\pi(\cos\alpha - \cos\overline{\alpha + u})} \cdot \sqrt{[V^2 + W^2]}$$

$$= I_{10} \cdot \frac{\sqrt{[u^2 + u(\sin 2\alpha - \sin\overline{2\alpha + 2u}) + \sin^2 u]}}{2(\cos\alpha - \cos\overline{\alpha + u})} \quad (12)$$

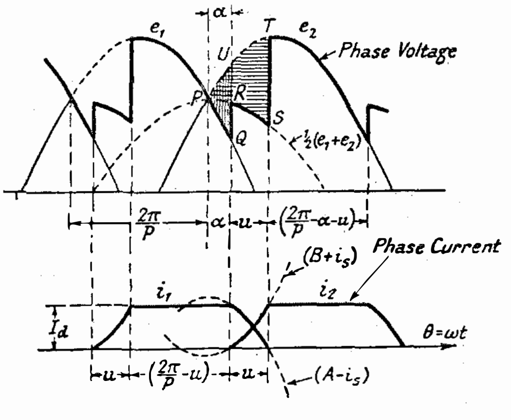

Fig. 3.

In the general case, therefore, the angle φ by which the fundamental component of the phase-current wave lags behind the phase-voltage wave is given by—

$$\tan\varphi = \frac{V}{W} = \frac{2u + \sin 2\alpha - \sin 2\,(\alpha + u)}{\cos 2\alpha - \cos 2\,(\alpha + u)} \quad \ldots .(13)$$

which gives $\varphi = (\alpha + \frac{2}{3}u)$ as a close approximation.

Similar relations can be established for the harmonics present in the current drawn by a polyphase rectifier system from the A.C. supply, i.e., the primary current of the rectifier transformer.[4] It will be found that there exists a unique relation between these current harmonics and the voltage harmonics on the D.C. side, a relation which the author has termed the "harmonic law" of polyphase rectification.[5]

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[4] *Vide* the author's paper cited below; also R. D. Evans and H. N. Muller: *Harmonics in the A.C. Circuits of Grid-Controlled Rectifiers and Invertors,* Technical Paper No. 38–39 presented at the A.I.E.E. Winter Convention, New York, January, 1939.

[5] *Cf.* the author's paper entitled *Harmonic Power Consumption of Polyphase Rectifier Systems* published in the *I.E.E. Journal*, 1940, Vol. 86, p. 568; see also p. 118 of this journal.

Dr. F. B. Jewett and Dr. O. E. Buckley

DR. F. B. JEWETT, former President of the Bell Telephone Laboratories, has recently been appointed a member of the National Defence Research Committee of the U.S.A. Government.

This Committee is empowered to correlate and support scientific research on the mechanisms and devices of Warfare, to aid and supplement the experiments and research activities of the Army and Navy, to conduct research for the creation and improvement of instrumentalities, methods and materials of War.

The Chairman of this Committee is Dr. V. Bush, President of the Carnegie Institution. The Representative of the Navy is Rear-Admiral H. G. Bowen, and of the Army, Brigadier-General G. V. Strong. The other members with the initial division of the work are: Armour and Ordnance—Dr. R. C. Tolman of the California Institute of Technology, Chairman; Bombs, Fuels, Gases and Chemical problems—Dr. J. B. Conant, President of Harvard University, Chairman; Communication and Transport—Dr. F. B. Jewett, Chairman; Detection, Controls and Instruments—Dr. K. T. Compton, President of Massachusetts Institute of Technology, Chairman; Patents and Inventions—Mr. C. P. Coe, U.S. Commissioner of Patents, Chairman.

Latest news is that Dr. Jewett has resigned the Presidency of the Bell Telephone Laboratories and has become Chairman of the Board, which will enable him to give more time to the Government as President of the National Academy of Science.

Dr. O. E. Buckley, well known to many of our readers, has succeeded Dr. Jewett as President of the Bell Telephone Laboratories.

Design Factors Influencing the Economical Size and Spacing of Multi-channel Telephone Repeaters

By K. G. HODGSON, B.A., A.M.I.E.E., and A. H. ROCHE, B.Sc., A.C.G.I., A.M.I.E.E.,

Standard Telephones and Cables Limited, London

SUMMARY

Past practice in the design of telephone circuits has in two respects been founded largely on an empirica basis. On the one hand, such circuits have been designed as complete entities and without reference to the way in which they might be linked up to form international circuits. On the other hand, the prevailing tendency has been to consider that an increase in the number of channels calls for a corresponding increase in the power-handling capacity of the amplifiers. It is the object of the present paper to put forward a logical basis on which any system may be designed in terms of the longest circuit of which it might form a link, and to determine in the case of any such system the most economical size and spacing of the repeater stations. The paper demonstrates that it may be economical in multi-channel systems[1] *to space repeaters more closely together and to use amplifiers of lower power than might have been expected from an approach to the subject based on considerations which have prevailed in the past.*

IN planning circuits for a particular route, the repeater spacing must be determined in relation to noise requirements. As it is nowadays no longer possible to regard any circuit as an individual entity, but only as a link in a much larger circuit chain, it is necessary that the longest chain likely to occur should determine the design of the circuits forming its several links. At the same time, the overall circuit requirements with regard to noise should, of course, be in accordance with the C.C.I.F. recommendations.[2]

As noise considerations are thus the determining factor in circuit design, it is necessary at the outset to discuss the possible sources of noise, or interference, in a multi-channel system. The principal sources are :—

(*a*) *Terminal Equipment.*—In a well designed system the terminal noise is chiefly due to band filter "spill-over" and is unintelligible.

(*b*) *Repeater Equipment.*—Repeater noise consists of unintelligible cross talk due to non-linearity in the amplifiers and noise due to shot effect in valves.

(*c*) *Line Circuit.*—The noise originating in the line circuit is due to pick-up from external sources or to the E.M.F. of thermal agitation (commonly known as thermal noise).

Since the noise on the complete circuit is the sum of all the above, it is desirable to allocate limits to each in order to ensure that the overall requirements will be met. It is obvious that the limits to be met by each part of the system must be more severe than the overall limits. Further, the level of noise which can be permitted in, for instance, terminal equipment will depend (*a*) on the number of terminal equipments which on the average are included in a circuit of the agreed maximum length, and (*b*) on the relative cost of improvements in this respect in terminal, and line and repeater equipment.

The noise which occurs in line and repeater equipment may conveniently be considered as a

(1) Noise due to intermodulation and dependent upon the presence and level of signals in the repeaters. The magnitude of such noise appearing at a point of zero relative level varies directly as the relative level at the repeater outputs if, as is usually the case, products of the second order are the controlling factor. (This is subject to limitation due to the amplifier overload point.)

(2) Thermal noise, valve noise or any externa noise which may be picked up. This does not depend upon the presence of signals. The magnitude of such noise measured at a point of zero relative level varies directly as the minimum relative level which occurs on the circuit.

Since noise of both the above types has a random distribution, the total noise appearing in the circuit is represented by the power sum of the two. The relative magnitudes of the two types of noise in any given system will depend upon the relative level at the repeater outputs

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[1] For references, see end of article.

an increase of x db. in this relative level will cause an increase of x db. in the intermodulation noise and a decrease of x db. in thermal noise. Thus, the total noise power with varying relative level at repeater outputs may be expressed by an equation of the form

$$p = \frac{p_1}{s} + sp_2 ,$$

where p is the total noise power at a point of zero relative level; p_1 and p_2 are, respectively, the noise powers at a point of zero relative level due to intermodulation, and to random noise when the relative level at the repeater outputs has a particular value; s is a factor representing the power ratio corresponding to a change in the relative level at repeater outputs.

It is readily seen that, with s as variable, p has its minimum value when

$$\frac{p_1}{s} = sp_2$$

and consequently, in a given system, the optimum condition for noise is reached when random noise equals intermodulation noise.

The limitations introduced by intermodulation and random noise may therefore, at any rate in the case of a single line section, be considered quite separately, each source being allowed to contribute up to half the maximum permissible noise power. At a later stage it will be shown that design work on a single section basis may be extended to cover the practical multi-section case.

The proportioning of the line equipment on this basis is the subject of the ensuing sections, and may be summarized as follows:—

(1) Minimum receiving signal level is first determined from considerations of noise which does not depend upon the presence of signals.
(2) Maximum transmitting signal level is determined from consideration of intermodulation due to amplifier non-linearity.
(3) For optimum conditions interference due to noise and to intermodulation must be of equal magnitude.
(4) The maximum permissible attenuation of a system consisting of a single line section is given by (1) and (2).
(5) It is shown that as the number of line sections is increased the repeater gain must be reduced, and that if the gain reduction is effected by increase of feedback the permissible transmitting level will remain substantially unaltered.

It should be noted that if the system is extended to n sections by the method suggested in (5) above, all design work may be done in terms of a single section length, and, conversely, any system of n sections may be reduced to its equivalent single section for purposes of comparison.

Curves have been drawn showing:

(*a*) Total circuit range in db. against number of sections for various values of single section attenuation, and actual section attenuation.

(*b*) Equivalent single-section attenuation against number of sections for various values of total attenuation and actual section attenuation.

Since α, the equivalent single section attenuation, depends upon

(1) Repeater output power,
(2) Number of channels,
(3) Noise requirements, and
(4) Circuit equivalent,

the effect on the layout of a complete circuit of a change in any of these factors can readily be determined from the curves.

Although this paper is chiefly concerned with the design of systems, such as coaxial cable systems, involving a large number of channels, the same general principles may be applied, subject to modifications in the computation of mean signal power and peak voltages in the amplifiers, to the design of systems giving as few as 12 channels.[3]

The technical considerations which are outlined provide a starting point for a consideration of the economic advantages of different sizes and designs of cables and repeaters. Thus, in the design of a system to meet given traffic requirements (i.e., number of channels, noise, band width and circuit equivalent), a satisfactory line circuit may be realized in a variety of ways, since there are three variables, viz.,

(1) Cable attenuation,
(2) Repeater output power,
(3) Repeater spacing (line section attenuation),

and in any particular case values assigned to any two of these variables will determine the third.

DETERMINATION OF MINIMUM RECEIVING LEVEL

In a coaxial cable system the protection against external interference, at any rate for frequencies above 50–60 kc, is such that the major sources of interference in the absence of signals may be assumed to be thermal noise and "shot effect" in the repeater valves.

In multi-conductor cable systems the same conditions generally apply, but open-wire systems are considerably more liable to interference from sources such as nearby power lines, radio stations, etc., which frequently exceeds that due to thermal noise, especially in particular channels.

On cable systems noise is in general evenly distributed over the frequency spectrum, and if the signal level at the sending end of the cable is the same for all channels the greatest interference occurs in the channel of highest frequency.

A similar tendency, of course, exists in open-wire systems, but noise due to external sources is often the predominating factor and may be greatest in any channel.

Having determined for a given type of system the level of noise to be expected in the worst channel, we can decide the minimum level to which signals on the line may be allowed to fall.

In order to avoid any possible confusion between "nominal circuit level" and "signal power" at any point in the system, the following terms will be used:

The "*sending toll test board*" refers to the two-wire circuit at the sending end, and represents either the two-wire input to the terminating set or, where pad switching is employed, the subscriber side of the switching pad. The nominal circuit level at this point is defined as "Reference Level" or zero "Relative Level."

The "*receiving toll test board*" refers to the corresponding point at the receiving end of the circuit. The "Relative Level" at this point is "-*q*" where "*q*" is the circuit equivalent.

The "*Relative Level*" at any point in the circuit is defined by the gain or loss between the sending toll test board and that point.

The R.M.S. power of a signal wave or of interference is specified in terms of db. relative to 1 mW.

Suppose that in a system consisting of one line section, the relative level is allowed to fall to $-S_1$ db., then the receiving gain will be S_1-q, where q is the circuit equivalent in db. If S_0 is the noise power in the absence of signals, integrated over one channel band, expressed in db. below 1 mW, at the point of minimum relative level, then the noise power at the receiving toll test board is S_0-S_1+q db. below 1 mW.

If the maximum permissible noise power in one channel band at the receiving toll test board due to interference which occurs in the absence of signals is z db. below 1 mW, then

$$S_1 \underset{<}{=} S_0+q-z \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The C.C.I.F. requirement for total noise at the receiving toll test board is expressed as 2 mV, Psophometric E.M.F.[2] If the noise is uniformly distributed over the frequency spectrum it can be shown by integration of the psophometric curve that this is equivalent to a noise power of approximately 54.8 db. below 1 mW, for a band of 3.1 kc and, therefore, in a system designed to meet this requirement, z must be equal to 54.8 db. plus an allowance for noise due to other causes.

If the frequency distribution of the noise is random, the total noise in a system of "n" sections is given by the power sum of the noise due to individual sections and, therefore, the minimum relative level must be increased by $10 \log_{10} n$ db. Equation 1 thus becomes:

$$S_n = S_0+q-z-10 \log_{10} n \ldots\ldots\ldots (2)$$

DETERMINATION OF MAXIMUM TRANSMITTING LEVEL

In any system in which a number of channels is passed through the same amplifier, intermodulation products resulting from the non-linear characteristics of the amplifier cause interference between channels. These intermodulation products may be simple harmonics of a single frequency signal or may be complex products of any order resulting from the interaction of two or more signal frequencies.

The distribution of these intermodulation products, and their amplitude in relation to the degree of non-linearity of the amplifier characteristic, are discussed in detail on a statistical basis in a paper by Jacobsen,[3] in which the following conclusions are reached.

(1) In a wide band system the distribution of the intermodulation products is uniform over the frequency range, at any rate to a first approximation.

(2) The amplitudes of the intermodulation products are related to the mean total signal power at the output of the amplifier.

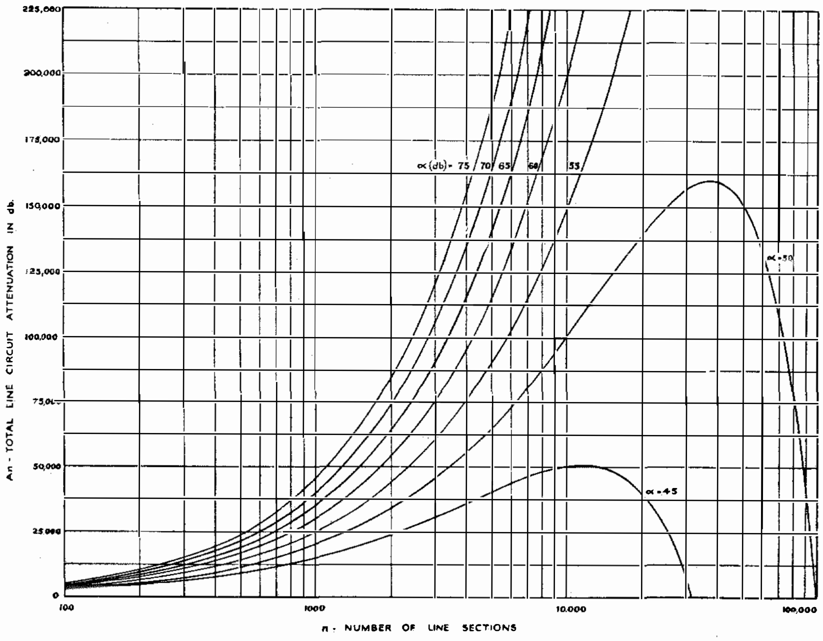

Fig. 1.

(3) The amplitudes of the intermodulation products of a given order bear a certain relation to the amplitude of the harmonic of the same order generated by a sine wave of equal power, but this relation varies both with the number of channels and the order of the intermodulation products.

For a given amplifier there is a certain output power at which the harmonics have increased to such levels that the intermodulation products are too great for the particular circuit requirements. An adjustment may be made by applying negative feedback round the amplifier which, as is well known, causes a decrease in the level of all harmonics in direct proportion to the reduction in gain.[4]

The above statement is subject to certain limiting conditions, which must now be considered briefly. If the input-output characteristic of an amplifier with a large amount of negative feedback is plotted for a fundamental sine wave frequency, it will be found to be represented by a straight line until a certain value of output is reached, when the slope will change abruptly.

The point at which this abrupt change of slope takes place, expressed in db. above 1 mW, is termed the "overload point," and is the point at which a theoretical amplifier with infinite feedback would start giving distortion.

Distortion may, therefore, arise in a practical amplifier due to two causes :

(*a*) The input may be sufficient to cause the "overload point" to be exceeded, or
(*b*) If the "overload point" is never exceeded, distortion can occur owing to the curvature of the valve characteristic.

It is of importance to note that, while feedback may be employed to reduce the distortion resulting from (*b*) it will have no effect on that resulting from cause (*a*).

In order that expressions may be derived for the distortion produced by the curvature of the valve characteristic (cause (*b*)) it will be assumed for the present that any distortion due to the "overload point" being exceeded is sufficiently small to be negligible. Further consideration will, however, be given to this point in a later section.

For the purpose of deriving an expression for the maximum relative transmitting level (or transmitting gain) we shall summarize the conclusions reached in the detailed analysis given elsewhere.[3]

If a given amplifier is worked at a certain total output power p (expressed in db. above 1 mW), at which the values of all the sine wave harmonics are known, the total power of the intermodulation products produced by a complex wave due to a given number of channels may be derived by adding weighting constants to the figures for the sine wave harmonics of the various orders, and summing the result in terms of power. If the frequency band is wide and covers r channels, the total power of the intermodulation products may be considered to be divided equally among them, so that the disturbance produced in each channel will be 10 $\log_{10}r$ db. below the total distortion output of the amplifier.

The value of p at which the total power of intermodulation products is such that the interference requirements are just met, represents the maximum permissible transmitting power, and is a function of the number of channels, the quality of subscribers' lines and subsets (i.e., the average speech power at a point of zero level), the amplifier characteristics, the circuit equivalent and, of course, the distortion requirements. From these considerations is determined the maximum transmitting gain β at which the desired interference requirements can be met.

It follows that the maximum relative level at the output of the sending terminal (and at intermediate repeaters) is also given by β.

From the foregoing it is clear that, for a system consisting of a single line section, the maximum permissible attenuation is given by the difference between the maximum permissible relative transmitting level as determined by the amplifier characteristic, and the minimum relative level to which the signals may be permitted to fall on account of thermal noise. If α db. represents the maximum single section attenuation, then

$$\begin{aligned}\alpha &= \beta + S_1 \\ &= \beta + S_0 + q - z \text{..................} (3)\end{aligned}$$

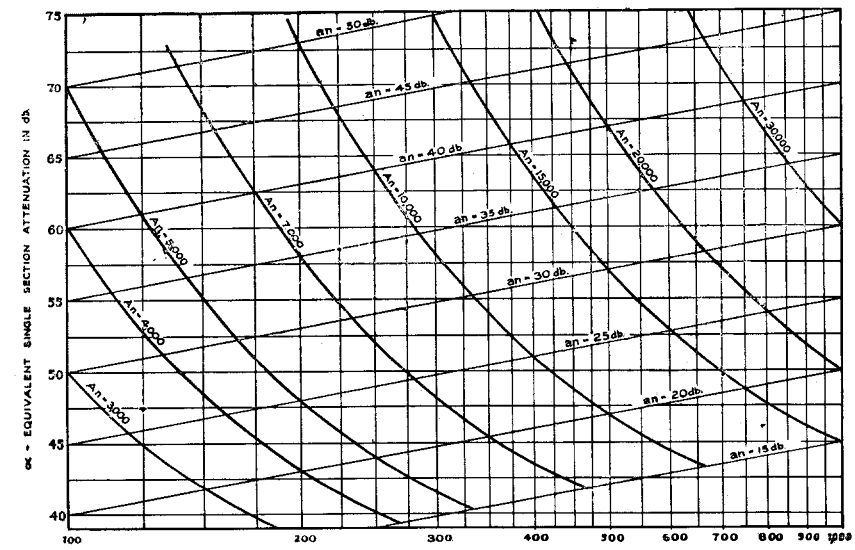

Fig. 2.

For the case of a system of n sections, we have seen that the noise considerations require that the minimum relative level on the circuit must be increased by 10 $\log_{10}n$ db. from that required for a single section. From equation (2) the permissible section attenuation must, therefore, be decreased by 10 $\log_{10}n$ db. and the required receiving (or repeater) gain will also be decreased correspondingly. If this reduction in gain is brought about by means of increased negative feedback, it is clear that the distortion products in the repeater will also be improved by 10 $\log_{10}n$ db. The improvement obtained in the distortion products is, therefore, exactly equal to the increase in distortion produced by the increase in the number of amplifiers, assuming that the distortion due to successive repeaters is additive on a power basis. As a result of the above considerations, it will be seen that no decrease in transmitting level is necessary as the number of sections increases.

This result, which is of considerable practical importance, assumes that it is always possible to decrease the gain of the amplifier to the required value by means of an increase in feedback. While this is substantially true over a wide range it should be mentioned that, in practice, a point is reached where a further increase of feedback is rendered extremely dangerous owing to the difficulty of meeting Nyquist's stability rule for the $\mu\beta$ path. After this point is reached, it would be necessary to reduce the gain by other means, and the desired improvement in distortion products would not be obtained. Under these conditions some decrease in relative transmitting level would be necessary. In the present paper, however, it is assumed that such a point is never reached.

Up to this point we have defined an amplifier as being capable of handling a certain power, without reference to its characteristics as regards non-linearity and gain, which are also relevant to this discussion. The necessary requirements for non-linearity are considered elsewhere,[3] but it is convenient at this point to mention a relation which is not evident at first sight between the gain of an amplifier and the value of a single section attenuation which can be used in practice.

Suppose that for a particular application we have an amplifier which is just capable of giving a transmitting gain of β with sufficient margin as regards overload point, and whose feedback has been adjusted so that intermodulation is just low enough. The gain of the amplifier under these conditions is g_1. If now α, as determined from equation (3), is greater than g_1, it would seem that the equivalent single section attenuation should be limited to g_1. An improvement can, however, be effected by a reduction in both feedback and output level.

The reduction in feedback increases the harmonic distortion by an equal amount for all harmonics, while a reduction in output level reduces the harmonics by a quantity which is greater the higher the order of the harmonic. We shall, therefore, be pessimistic if we consider that the second harmonic is the controlling factor.

Suppose that the feedback is reduced by y db. The gain will then be increased by approximately y db. and all harmonics increased by y db. In order to maintain the second harmonic at the same relative level, β must be reduced by y db. If the new value for the single section attenuation is $\alpha0$, and if this is equal to the new value of the amplifier gain, we have :

$$\alpha0 = g_1 + y$$

and from equation (3) :

$$\alpha0 = \beta - y + S_0 + q - z$$

Eliminating y we have :

$$\alpha0 = \tfrac{1}{2}\ (S + g_1 + S_0 + q - z) \quad \ldots\ldots(4)$$

This modified formula enables the best use to be made of a given amplifier, but departs from the optimum conditions in that the margin between the total signal output power and the overload point of the amplifier is unnecessarily large. No such adjustment can be made if the gain of the amplifier is greater than the value of α given by equation (3), since any increase in relative transmitting level would reduce the overload margin below the required value.

From the foregoing paragraphs we have seen that the permissible attenuation for a single section system depends upon the maximum relative level derived from amplifier considerations, and the minimum relative level derived from noise considerations and that, for a multi-section system, the permissible section attenuation must be decreased. Expressing these results mathematically, if a_n is the section attenuation for a system of n sections, we have :

$$a_n = \alpha - 10\ \log_{10} n \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

and

$$A_n = n\ (\alpha - 10\ \log_{10} n) \quad \ldots\ldots\ldots\ldots(6)$$

where A_n is the total attenuation of the whole system.

It is of interest to note that, whatever the value of α, the repeater section attenuation a_n for the maximum value of A_n is always equal to 4.34 db. or 0.5 neper. In order to show this, it is convenient to express the attenuations in nepers rather than db. ; thus in this notation

$$a_n = \alpha - \tfrac{1}{2}\ \log_e n$$

$$A_n = n\alpha - \frac{n}{2} \log_e n \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\frac{dA_n}{dn} = \alpha - \tfrac{1}{2}\ \log_e n - \tfrac{1}{2}$$

which for a maximum value of A_n gives

$$\log_e n = 2\alpha - 1$$

and hence

$$a_n = \alpha - \tfrac{1}{2}(2\alpha - 1) = \tfrac{1}{2}\ \text{neper} \ldots\ldots(8)$$
$$= 4.34\ \text{db.}$$

While the above is of interest, it is not of great practical importance in view of the fact that the number of repeaters increases very rapidly compared with the total attenuation as the maximum condition is approached ; therefore the arrangement would usually be uneconomical.

Up to this point we have not considered distortion due to peaks in the total signal wave which exceed the overload point of the amplifier, either as a basic limitation or as an interference which is cumulative with increases in the

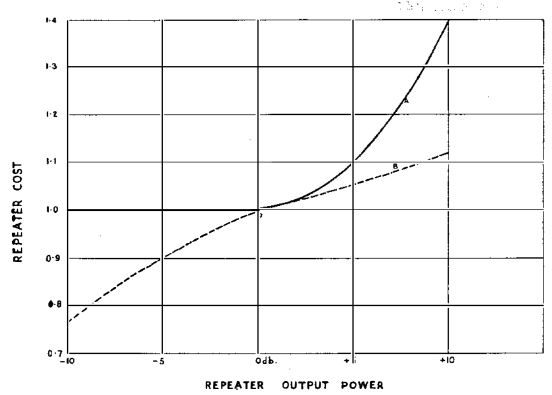

Fig. 3*a*.

number of sections. The study of this problem[3] indicates that in the vicinity of the overload point, interference varies very steeply with changes in the total signal level—decreasing by about 8 db. for 1 db. decrease in signal level. On this basis, even if the overload point is a limitation, 250 repeater sections in tandem would demand a transmitting level only 3 db. lower than that required for a single section.

In practice it is seldom possible to design a repeater which exactly fits given circuit requirements as regards overload point, harmonics and gain, and there is usually some additional margin which may be allocated either to the overload point or harmonic distortion. Since a given margin has a greater effect on overload distortion, this appears to be the most favourable method of using such margin. Consequently the assumption that limitations due to the overload point may be neglected in system design is believed to be justified.

CURVES

The curves shown in Figs. 1 and 2 have been drawn with respect to four parameters:

α—the equivalent single section attenuation.
n—number of line sections.
A_n—total attenuation of circuit.
a_n—repeater section attenuation.

Figure 1, in which A_n is plotted against n for different values of α, shows that A_n reaches a maximum value for a finite number of sections, and that this maximum increases very rapidly with increasing values of α.

In Fig. 2, α is plotted against n for different values of A_n and for different values of a_n, and this method of plotting results in a very convenient chart for the ready comparison of differently constituted types of circuit.

EFFECT ON CIRCUIT COST

Having evaluated the technical factors controlling the planning of multi-channel systems, it becomes possible to study the question from the economic standpoint. Analysis of the problem as a whole is a matter of considerable complexity and is beyond the scope of this paper, but it is proposed briefly to examine the economic consequences of variations in the output power capacity of repeater amplifiers, particularly as applied to coaxial cable systems.

In assessing the annual cost of operating a single repeater it is necessary to take into account the initial cost of buildings, amplifiers and power equipment, and the running cost comprising maintenance, valve replacements and power. Some of these items, in particular buildings and maintenance, may vary rather widely with different operating practices. It is assumed, however, that a large proportion of repeater stations will be of the unattended type, housed in comparatively simple structures, and will be operated on A.C. power supplied either locally or over the cable. Under these conditions curve *A* of Fig. 3a represents the estimated variation in annual cost with repeater output power for one repeater station. As the power is reduced, the cost decreases less and less rapidly until a point is reached (arbitrarily designated zero db.) below which it remains constant, or may even increase slightly owing

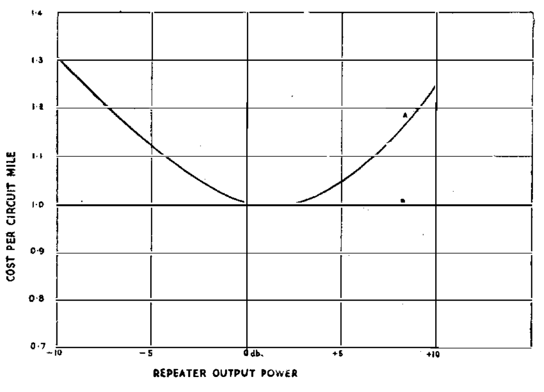

Fig. 3*b*.

to the increased cost of making small valves of very low consumption.

Based on this curve and the technical requirements derived in the paper, curve A of Fig. 3b represents the variation with repeater power of the annual cost per circuit mile of repeater stations for a given cable and frequency range.

It is found that for a given repeater station cost curve, the curve giving cost per circuit mile retains substantially the same shape over a wide range of values of total circuit attenuation (A_n). Thus, curve A of Fig. 3b may be considered as representative of the variation of equipment cost with repeater power over a wide range of cable sizes and frequency ranges.

The striking feature of these curves is that the point of minimum cost per circuit mile lies at or near the point below which it is not possible to reduce the cost of a single repeater. Curve B of Fig. 3a has been computed as a check on the validity of this general conclusion. This curve represents the manner in which the cost of a single repeater station would have to vary with repeater power in order that the equipment cost per circuit mile should be independent of repeater power (curve B of Fig. 3b). This curve shows that an increase in power of ten times above our arbitrary zero level point could be justified only if it involved an increase in repeater station cost of less than 12%. It is, therefore, evident that quite large errors in the assumptions upon which curve A of Fig. 3a is based would not seriously modify the conclusions which have been drawn.

The use of repeaters of low power on coaxial cables presents a further advantage in that the cable itself can more readily be used for the transmission of power to unattended repeaters. This consideration is of especial importance in cases where coaxial circuits are being added to existing cable routes already equipped with repeater stations, since it is often desirable to feed as many as four or five unattended stations in tandem from an existing attended station.

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The Application of Styrene to H.T. Cable Systems

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PART IV

Since Part III of the present series was published in July, 1938, much research has been undertaken to extend and simplify styrenation technique in the light of experience gained under service conditions. A review of various installations and details of recent developments are given. New methods of making styrene joints, plugs and terminations have fulfilled the long-felt aim of so simplifying and reducing costs that general use can be made of barrier action in cables. Bonded styrenated paper is also introduced as a new product which is likely to have widespread commercial application.

IN previous parts of this article a sketch has been made of how the peculiar properties of styrene are controlled and adapted for use in cable systems. It will be recalled that styrene can be produced in its monomeric form, in which condition it appears as a thin clear liquid of about the consistency of water, boiling at 144° C. In this state it is a good solvent of oil, etc., rather like benzene. By the application of heat, however, monostyrene can be changed into polystyrene by a process known as polymerisation, by which is meant merely a re-grouping of the styrene molecules. Polystyrene is a hard clear glass-like substance having an extremely low power factor—about 0.0002 at all normal frequencies—combined with very high dielectric strength. It is, moreover, quite insoluble in oil although it is soluble in several aromatic or chlorinated hydrocarbons, the most important of the former group being its own monomer, i.e., monostyrene. Such properties are made use of in the technique of styrenation.

The chief application of styrene to cable systems so far has been the provision of plugs or barriers, either at cable ends or joints, or even at intermediate points in a cable run. These barriers are for the purpose of restricting the flow of oil or compound in the cable, limiting hydrostatic pressures due to gradients, avoiding mutual contamination between the cable compound and the compound in glands at termination, and generally securing the hydraulic isolation of cable lengths.

Other applications take advantage of the excellent electrical and mechanical characteristics of polystyrene, and these will be dealt with in due course.

Matters which have already been discussed in the preceding parts of this paper include:—

(1) The stabilization of monomeric styrene so that it can be stored indefinitely in this condition and transported, even through the tropics, without unwanted thickening due to premature polymerisation.

(2) The de-stabilization and subsequent control of the rate of polymerisation.

(3) The incorporation of plasticisers to modify the physical properties of the polystyrene, in particular to make it tougher.

(4) The impregnation of paper with styrene and its application.

Field Experience

Styrene joints made by the "hot process" have been in continuous service on a 66-kV cable system since 1934 and have given no trouble. Many joints, besides barrier terminations and plugs on 33-kV systems have also given complete satisfaction. As is inevitable, however, in evolving a new technique, there has been a small number of failures and, as attention is always focused on breakdowns, especially when a new process is involved, it is necessary clearly to distinguish between essential and merely accidental features.

Out of over 200 commercial installations there have been 3 breakdowns. The first occurred on an early hot process joint on a newly-installed 33-kV H.S.L. type cable, and it

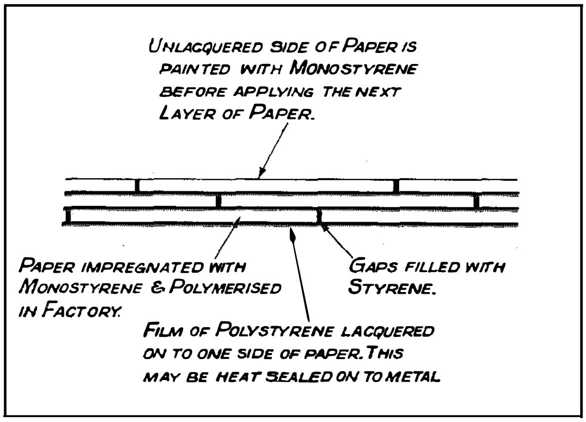

Fig. 1—Diagrammatic section of build-up of styrenated paper as applied in joint construction.

was found that the fault was due to the ineffective bonding of the earth screen over the joint build-up. The styrene, on polymerisation, had formed an insulating layer under the bonding wire, and breakdown was initiated by sparking occurring between this floating screen and earth. Revised instructions were issued in the jointing specification and this trouble has been eliminated.

The second fault was on a joint for an 11-kV, 3-core belted type cable. The method adopted was to fill the joint with partially polymerised monostyrene, and thereafter to complete polymerisation by heating. It has since been found that while satisfactory joints can be so made, there are circumstances in which gaseous bubbles are generated in styrene during polymerisation; this was the probable cause of failure. A new method of solid filling, to be described, has now been adopted, so that this trouble should not recur.

A third fault on a 33-kV joint in a 3-core, H-type cable was due to ground subsidence in the vicinity of the joint, and no blame can be attributed to its construction.

Besides these three electrical faults, there have been a few cases of failure to establish complete barrier action in the case of plugs made in cables of large cross-section by displacing the original cable oil with styrene. Methods of greater reliability are described in this paper, so that on account of field experience gained this problem is now solved.

Thus, although styrenation is a completely new procedure in the technique of cable-jointing, there has been a comparatively small number of faults.

Recent Developments

Styrenated Paper.—The first type of styrene joint was made by the so-called "hot process," implying that the paper tapes were impregnated with monostyrene and only polymerised after they had been lapped on to the joint by maintaining the latter at a temperature of 120° C. for about three days. This heating period was effectively reduced to eight hours by the use of pre-polymerised styrenated paper tapes, painted with monostyrene as they were lapped. The latest type of styrenated paper, however, has succeeded in eliminating the heating period altogether. This is achieved by coating the original paper with films of polystyrene lacquer, and so increasing its polystyrene content. When such paper is lapped as already described, the build-up is sufficiently firm to provide a positive barrier without any subsequent heating period. If coated on one side only, the paper is wrapped on with the coated side inwards, and

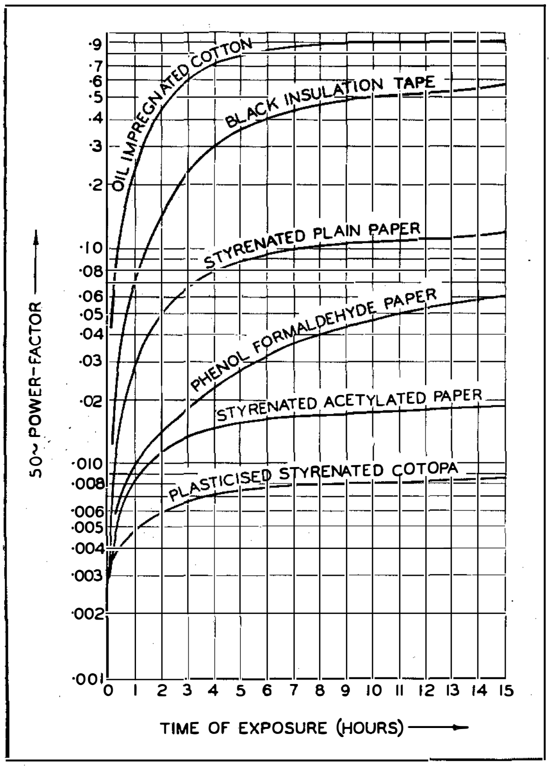

Fig. 2—The effect on various tapes of exposure to an atmosphere of relative humidity 80% at 20° C. as indicated by the 50-cycle power-factor.

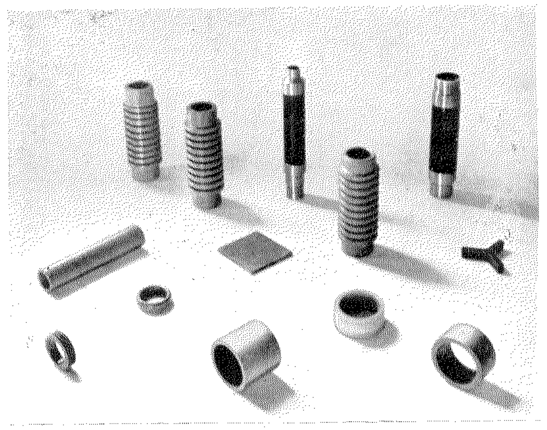

Fig. 3—Various articles made from bonded styrenated paper. The two plain bushes in the background have been fitted with copper end caps ready for attaching to a cable end.

then the outside is painted with monostyrene before the next layer is applied. When its polystyrene film comes in contact with the previous layer of paper moistened with monostyrene, adhesion takes place almost immediately, there being just time to adjust and tighten the tape, which causes styrene to ooze into and fill the gaps between papers. The first paper to be applied can be made to adhere firmly to a metal surface, such as a copper ferrule, by "heat sealing," i.e., by pressing it firmly into contact with the metal surface which has been heated. Figure 1 shows diagrammatically the resulting build-up.

Not only does coated styrenated paper eliminate the subsequent heating period, but the electrical characteristics remain excellent. It may thus be used for the very highest voltages.

The paper employed is a particular grade which satisfies supertension power cable specifications, and which will readily "take" the monostyrene during vacuum impregnation. Following impregnation and polymerisation in roll form the paper is unwound at high temperature, and after coating with polystyrene lacquer, cut into strips and rewound as jointing rolls. As the paper is inevitably exposed during these operations, it is vacuum-dried before packing in sealed tins, and it is of interest to note that the paper may be almost as easily re-dried as if the styrene were not present. While there is some exposure during application, the time involved is usually too short to allow of any substantial moisture absorption. This is made evident by referring to Fig. 2. The effects of moisture regain may be mitigated by the use of acetylated paper where circumstances warrant.

It might be thought that pure polystyrene film could be used instead of styrenated paper, but experiments have shown that the additional "backbone" provided by the paper, together with a lowering of the thermal resistivity from about 1 000 to 570 thermal ohm-cm is a definite advantage. Moreover, the power-factor of built-up insulation is not just a simple function of the power-factor of the component materials, and the electrical characteristics of joints made with pure polystyrene tape were found to be no better than those made with styrenated paper, while the mechanical characteristics of the latter were far superior.

Besides its known application to joints, the potentialities of styrenated paper have been explored in relation to other fields of use, and it has become increasingly evident that, in the bonded form, it fulfils a long-felt want on account of its low power-factor and ionization values, and its resistance to "tracking." Thus,

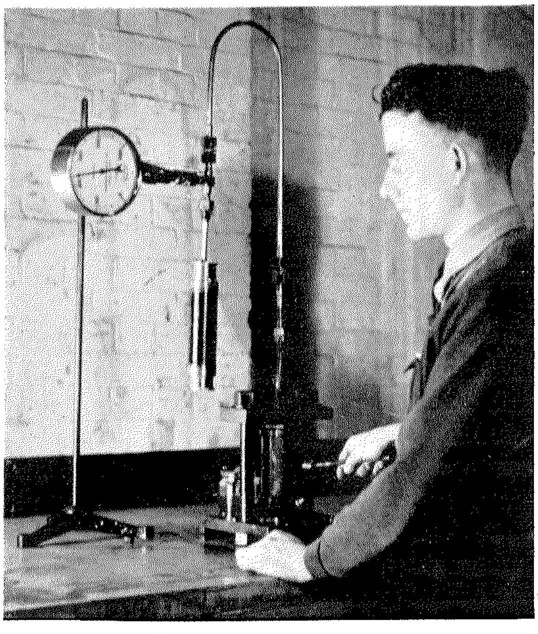

Fig. 4—Hydraulic pressure test on a styrenated paper bushing fitted with copper end caps. The bushing illustrated must withstand a pressure of 200 lb./sq. inch without leaking.

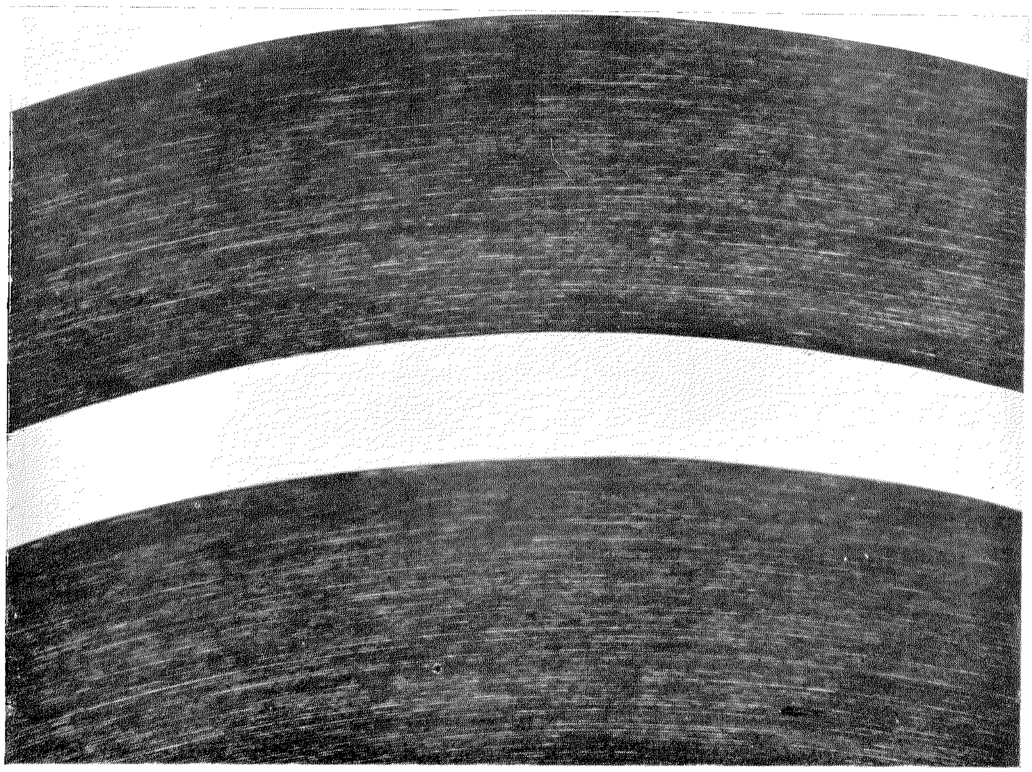

Fig. 5—Wafer sections of bushings indicating degree of compactness of winding. The larger bushing is of phenol formaldehyde and the smaller one of bonded styrenated paper. A more revealing comparison is provided by the ionization curves of Fig. 6.

coated styrenated paper, which may also be acetylated, is now used with much success in the construction of bonded paper bushings and sleeves. These may be either plain or of the condenser type with foil inserts. Blocks of styrenated paper may likewise be made by uniting sheets, and Fig. 3 illustrates various articles so constructed. It should be noted that the bonded paper can be readily machined, and a process has also been devised for so firmly uniting copper end caps on to bushings that they will withstand considerable hydraulic pressure without leaking. Figure 4 shows such a bushing being tested.

It is inevitable that bonded styrenated paper should be compared with the well-known synthetic resin bonded paper of the phenol formaldehyde type. There are two main grades of this latter material, viz.: Grade I, the principal characteristics of which are relatively low water absorption, high resistivity, low dielectric loss at radio frequencies, and good machining properties; and Grade II, which has high electric strength at high temperature, and is generally employed on oil-immersed plant and on low voltage apparatus in air. Grade I is covered by British Standard Specification (B.S.S.) No. 547–1934, and Grade II by B.S.S. No. 316–1929.

Styrenated paper boards and tubes, both plain and acetylated, have been tested to these specifications and the following generalizations can be made:

Plain bonded styrenated paper satisfies the requirements of the Grade II specification.

Acetylated bonded styrenated paper satisfies the requirements of both the Grade I and Grade II specifications.

Both types have an electric strength of over 500 volts/mil (20 kV/mm).

As styrene is a thermo plastic, it has a lower softening point than phenol formaldehyde, which is a thermo setting material. Thus, while measurements obtained by the V.D.E. procedure[1] give a value in excess of 200° C. Martens for laminated phenol formaldehyde, the yield point of styrenated paper is 60–70° C. Martens.

Measurements made by the tentative A.S.T.M. method for arc resistance[2] indicate that styrenated paper is considerably more resistant than phenol formaldehyde paper, and this has been confirmed by direct comparisons under service conditions, as in Fig. 7.

Wafer sections of each type of bush reveal that similar compactness may be obtained (Fig. 5), but, as styrenated paper is made by vacuum impregnation of the paper with monostyrene before polymerisation, a much more intimate penetration of the paper fibres is possible than with the lacquering process usually followed in the case of phenol formaldehyde. This fact is reflected in the measurement of ionization as in Fig. 6.

From Fig. 8 it will be observed that the power factor at high frequencies of styrenated paper is considerably lower than that of phenol formaldehyde, and these curves also illustrate the gain to be expected in using acetylated paper.

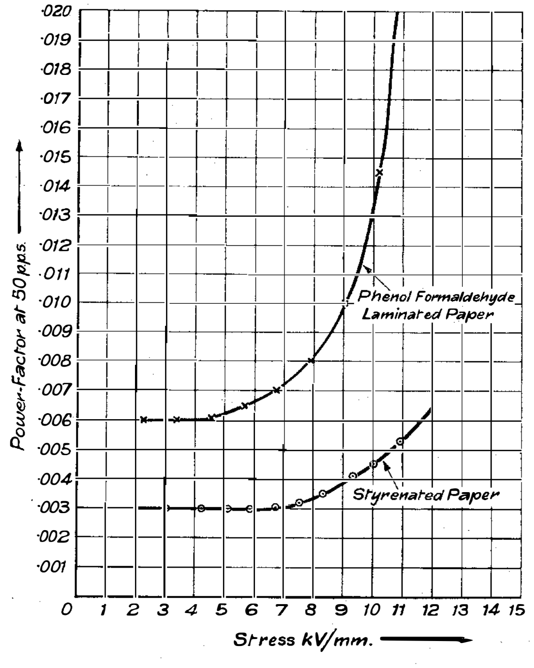

Fig. 6—Comparative ionization curves of phenol formaldehyde and bonded styrenated paper bushings. Both samples vacuum-dried.

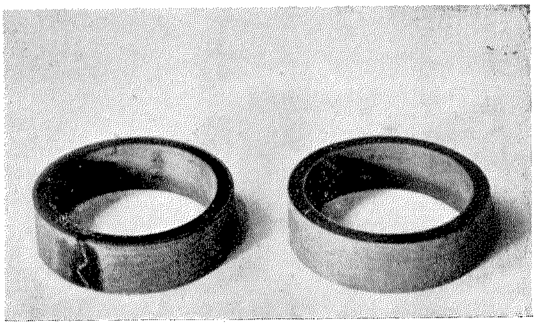

Fig. 7—Styrenated paper has good arc-resistance. The bushing on the left is of the usual phenol formaldehyde type, that on the right is of bonded styrenated paper. Both have been subjected to the same electric discharge test.

Styrene-rubber Cotopa Tape.—Another useful accessory which prevents mutual contamination between the joint and cable is cotton tape impregnated with a mixture of rubber and polystyrene. This is a form of inert tape which, on being moistened with monostyrene, becomes adhesive, and is useful for binding over the end of the built-up insulation to prevent either the egress of oil from, or the ingress of monostyrene into, the cable. To avoid the usual trouble due to moisture pick-up during handling, the cotton is acetylated into the well-known form of cotopa. Careful note should be made of this question of moisture absorption and what it involves. The most revealing test is the 50-cycle power-factor measured with progressive exposure, and reference should again be made to Fig. 2 which gives characteristic data, both for this and other materials. There are, of course, many other applications for this superior form of adhesive tape besides that mentioned.

Solid Filling with Plasticised Styrene.—A recent development relates to the homogeneous filling of confined spaces with plasticised polystyrene. Such filling cannot always be achieved by filling with monostyrene and thereafter polymerising, on account of the tendency of a mass of monostyrene to generate gaseous bubbles on polymerising at elevated temperature, and while this question is still under review,

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[1] For description of this test see "Plastics for Electrical Purposes," by A. R. Dunton, *Electrical Times*, March 31st, 1938, Vol. 93, p. 471.

[2] Method originally outlined by Race and Warner, *General Electric Review*, Vol. xxxviii, February, 1935, p. 97.

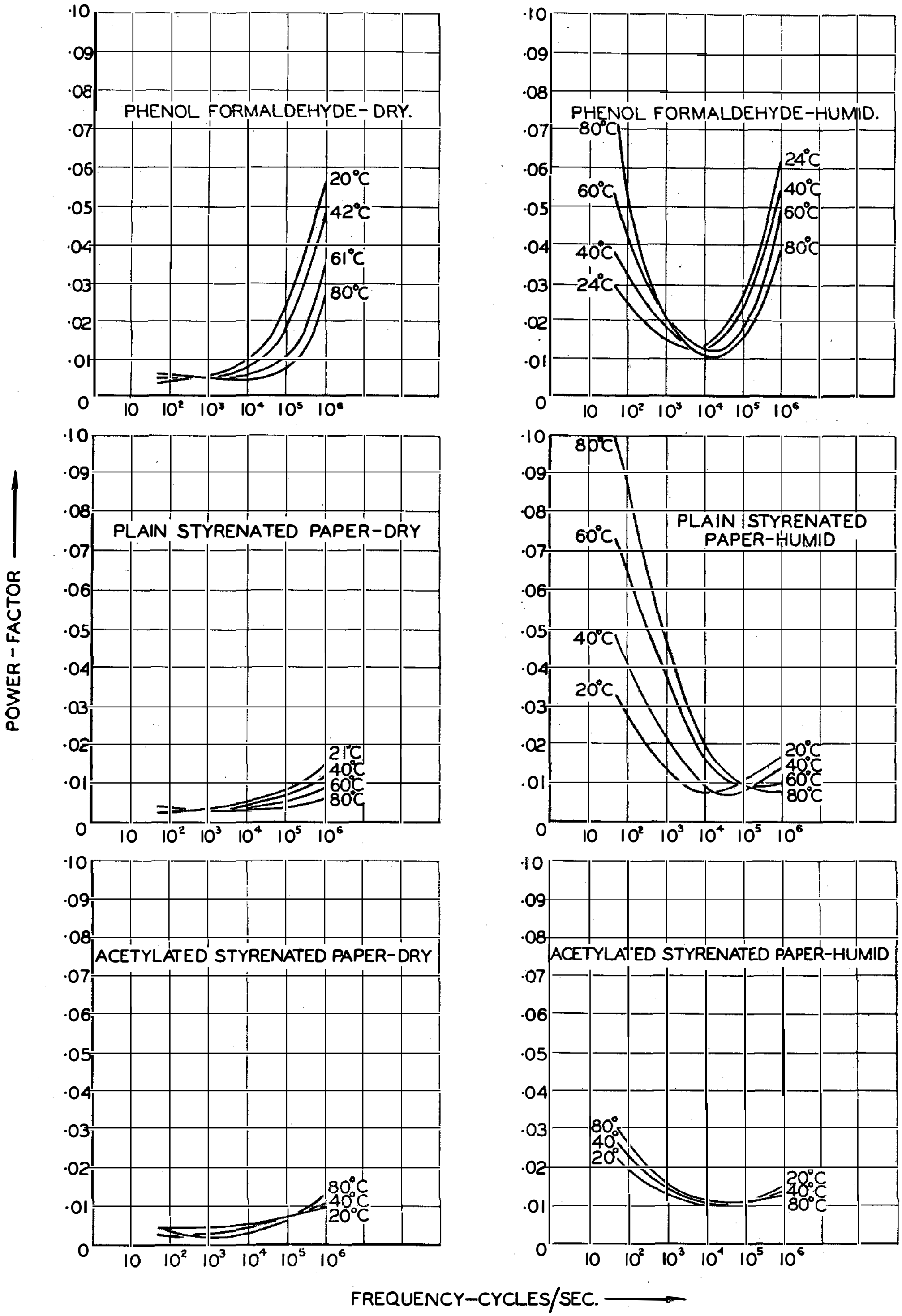

Fig. 8—*The power-factor frequency temperature characteristics of phenol formaldehyde and styrenated paper. "Humid" means* 8 *days' conditioning in atmosphere of relative humidity* 80% *at* 20° *C. Specimens were in the form of bonded tubes of diameter* 1.5 *inch and radial thickness* 0.050 *inch.*

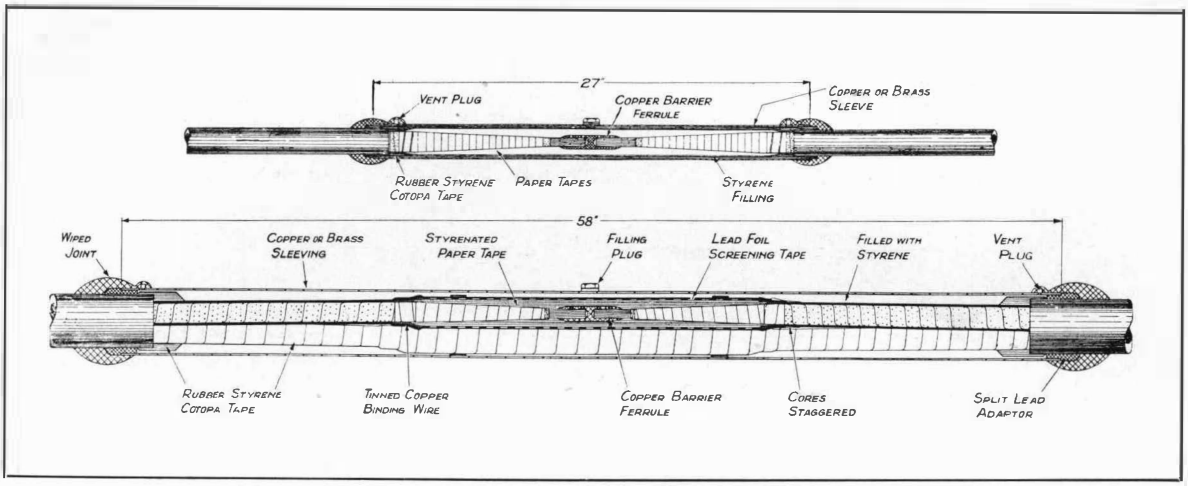

Fig. 9—Single-core and three-core styrene barrier joints for 33-kV cables.

a practical solution has been found which involves first filling the space with monostyrene and then displacing this with plasticised polystyrene by forcing it in under pressure. It is found, as a rule, that most of the monostyrene can be displaced ; what little remains is absorbed by the polystyrene, and will polymerise in course of time. Two examples of spaces which have to be filled in this way are the zone between the insulation and the metal sleeve in a joint, and the space between the bushing and the cable insulation in a termination. In the case of the lower voltages, lapping may be dispensed with altogether, the plastic styrene serving both as an insulating medium and a solid barrier. The plasticiser chosen is one which does not involve undue softening at temperatures up to 100° C., and the resulting homogeneous filling is mechanically adequate. The material is, moreover, a very good insulant, is unaffected by oil, and will adhere both to metal and oil-impregnated paper.

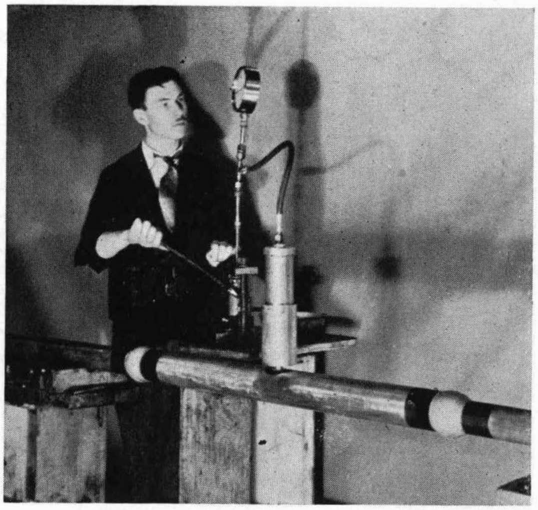

Fig. 10—Filling 3-core 33-kV styrene joint with plasticised styrene which occupies the spaces between cores and completes the barrier action.

Specific Applications

Joints.—The styrene joint, as first proposed, enabled a barrier joint of very modest dimensions to be constructed, and the only objection to its use was the necessity for heating it for a period to complete the polymerisation. With the advent of coated styrenated paper, styrene rubber cotopa tape and solid filling technique, the essential details of the joint have not been altered, but the heating period has been eliminated. The electrical characteristics comprising breakdown strength, power-factor and ionization remain so good that the joints may be used for the highest voltages. In the past, the field of use for styrene barrier joints was restricted to cases where a barrier joint was absolutely essential, and in those cases the styrene joint was a welcome advance from the orthodox type of barrier joint, both with regard to dimensions and costs. It is generally admitted, however, that it is desirable to make every joint a barrier,

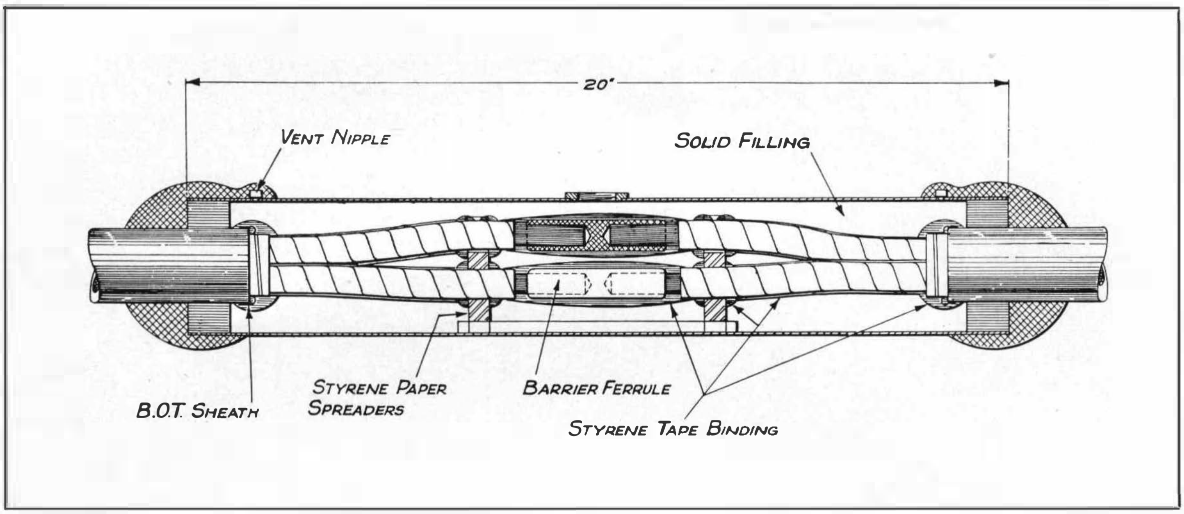

Fig. 11—Joint for 11-kV belted type 3-core cable made by spacing the cores and solid fillings with plasticised styrene.

and since the cost of the latest styrene joint is little more than that of ordinary joints, this aim can now be economically secured.

The small dimensions of typical styrene barrier joints are evident from Fig. 9. In the case of the 3-core joint, solid filling has made possible the inclusion of the three cores in a common sleeve. Figure 10 shows such a joint being filled.

Belted type cables are invariably of the low to medium tension type, and rarely exceed 11 kV. In such cases the usual paper lapping method of insulating each core may be wholly or partly dispensed with, and a construction as indicated in Fig. 11 adopted. In this case the solid filling material provides a mechanical barrier and also serves as the main insulation.

Tests have been carried out to answer the following questions :—

(1) *Is the joint a complete barrier to cable oil at the pressures which might be anticipated in practice ?*

Joints have been consistently tested at pressures up to 100 lb./sq. inch, for 48 hours at 20° C. followed by 40 lb./sq. inch at 80° C. without leaking. After long periods of service under life test conditions they have been re-tested with like results.

(2) *Are values of power-factor and thermal resistivity sufficiently low to avoid the possibility of breakdown due to thermal instability on supertension cables ?*

Typical power-factor voltage temperature curves for a 33-kV joint are given in Fig. 12. The thermal resistivity of styrenated paper is about 570 thermal ohms-cm, and analysis has shown that the question of thermal instability hardly arises, even on 132-kV joints.

(3) *What is the breakdown value of the joint in relation to the cable ?*

It has been found that in the case of a new solid-type cable and joint subjected to short-

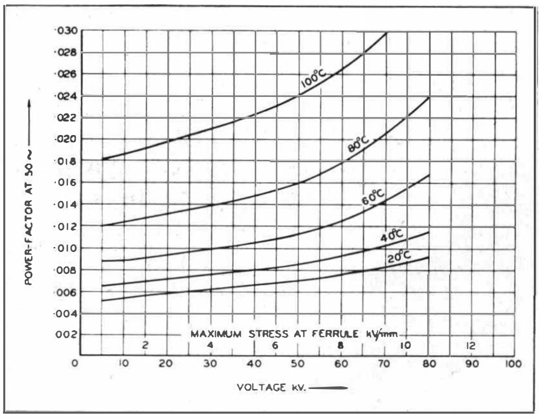

Fig. 12—Typical power-factor-voltage-temperature curves for a 33-kV styrene joint.

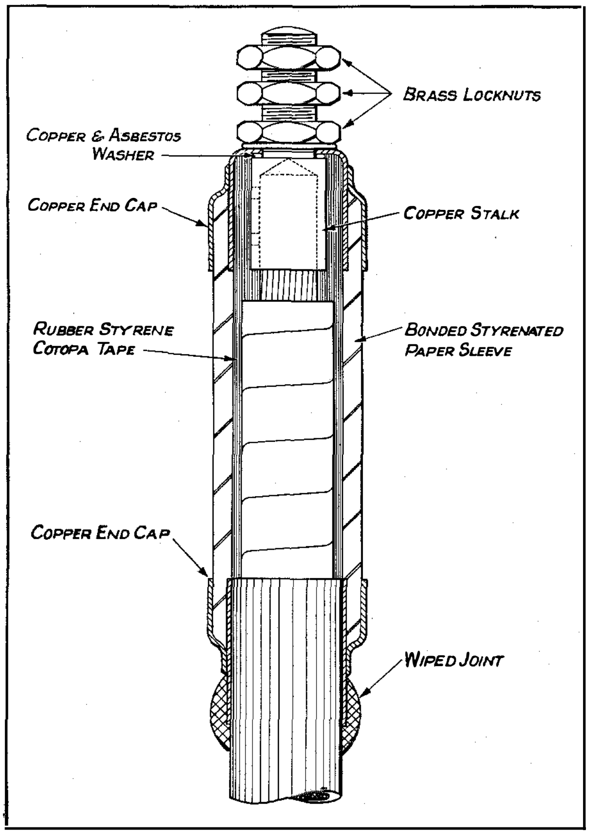

Fig. 13—Barrier termination for a 1 sq. inch 11-kV cable.

time breakdown tests, the cable may be slightly stronger than the joint, although there is still a large factor of safety. After a period of service, however, the position is reversed. The cable deteriorates but the joint actually appears to improve.[3]

(4) *What is the effect of heavy current loading cycles on the joint characteristics?*

The joint remains a barrier and maintains its electrical strength at all practical conductor temperatures. The power-factor improves with time. This is probably due to the auto-polymerisation of the residual monostyrene used for painting the tapes during jointing.

Plugs.—Initially plugs or barriers in cables were made by displacing the oil in the cable with monostyrene and then polymerising. The preferred method is now simply to sever the cable and insert a joint; this, besides obviating the need for heating, permits the use of a barrier ferrule, so that any possible leakage along the strand is eliminated, a factor of importance in cables of large cross-section. The design of plugs thus becomes identical with that of joints. In certain cases the conductor can be left intact and the interstices between the strands filled with solder.

Terminations.—As in the case of plugs, barrier terminations were first made by dis-

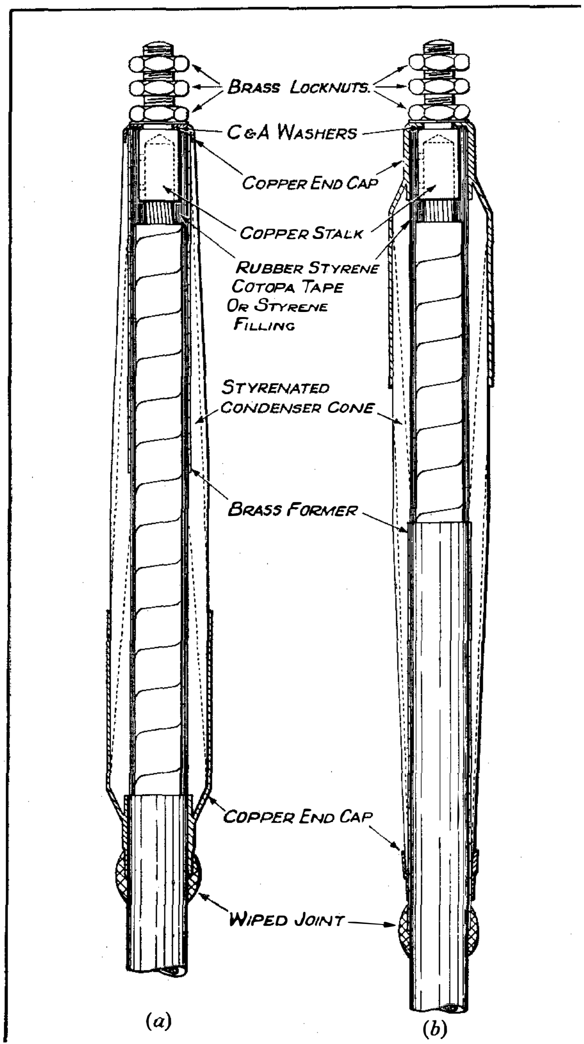

Fig. 14—The condenser cone made of bonded styrenated paper and mounted either normally as in (b) or inverted as in (a) serves both as a barrier termination and as a means of electrically reinforcing the end.

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[3] T. R. Scott and R. C. Mildner—"Long Period Ageing Tests on Solid-Type Cables," *J.I.E.E.*, Vol. 85, No. 511.

placing the oil at the cable end with monostyrene and then polymerising by heating to 120° C. for about three days. A second alternative was to taper the cable papers and lap-on styrenated paper as in joint construction; but an easier method is to employ a hot rolled bushing fitted with copper end caps, as in Fig. 13. The copper stalk is soldered to the cable strand and styrene-rubber cotopa tape wrapped over the end. After painting the outside of the latter with styrene lacquer the styrenated paper bushing is slipped on and the lower cap plumbed to the lead. An oil-tight seal between the copper stalk and the end cap of the bushing is effected by means of a washer. The bushing is made a tight fit over the tape, and the effect of the styrene lacquer is to cause the latter to swell and completely fill the space between the cable and the bushing. As an alternative to using cotopa tape, the end may be solid-filled with plasticised styrene, but either method allows the end to be quickly assembled on site. The 11-kV end illustrated can be completed within two hours.

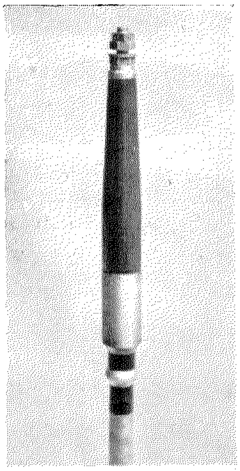

Fig. 15—*Inverted condenser cone barrier termination on a cable end.*

Condenser Bushings and Condenser Cones.—The insertion of metallic foils in a styrenated paper bushing during winding is a simple matter, so that condenser bushings of any of the recognized types may be formed. In particular, the well-known Condenser Cone[4] can be made to combine the functions of a stress grading device and a barrier termination by forming it with styrenated paper and fitting copper end caps. The cone is mounted over the cable end either in the normal or inverted position as indicated in Fig. 14. A photograph of such an inverted cone is given in Fig. 15. The time taken to fit a cone on to an end is no longer than that required for a simple bushing, that is, about 2 hours.

Acknowledgment

The authors wish to acknowledge the help afforded them by their laboratory staff, in particular Mr. E. C. Lee who has given generous help in present developments.

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[4] J. K. Webb, "The Condenser Cone," *Electrical Communication*, October, 1933.

J. K. Webb, "Condenser Cones for Cable Testing," ibid. April, 1937.

Harmonic Power Consumption of Polyphase Rectifier Systems*

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SUMMARY

The aims of the present paper are (1) *to establish the general harmonic law of polyphase rectification,* (2) *to evaluate the distortion factor of the primary currents in a universal form, and* (3) *to derive a genera expression for the correction factor which takes into account the effect of finite commutation on the shape of the primary current wave. In considering these aims it is to be remembered that—*

(1) *The existence of such an harmonic law has been known for some time, although its basis has hitherto remained obscure.*

(2) *The distortion factor has so far been evaluated only in specific cases and without any attempt at generalization.*

(3) *The possibility of being able to derive a general expression for the primary-current correction factor has hitherto not been suspected.*

CONTENTS

1. INTRODUCTION

THE most important aspect of polyphase rectifying apparatus, considered as an A.C. load, is its harmonic loading of the supply system : for the fact that the rectification process is bound up with the consumption of wattless, non-reactive power—harmonic power, in short—is fundamental to a proper understanding of rectification phenomena. This fact, one that is characteristic not merely of rectifier circuits but of all A.C. circuits containing impedances which are functions of time,[1] leads at once to a unique relation between the input and output sides of any rectifying apparatus comprising a supply transformer and a polyphase rectifier system, viz., the relation between the occurrence of particular harmonics in the D.C. terminal voltage on the one hand, and in the transformer primary currents on the other hand. It should be emphasized here that the phenomenon of harmonic generation is inherent in the rectification process and that its origination in no way depends upon the type and design of the rectifier or its associated transformer.

2. LIST OF SYMBOLS

p = number of rectifier phases.

n = order of a current or voltage harmonic.

m = any integer.

u = angle of overlap.

v_d = instantaneous value of rectifier output voltage.

V_d = mean value of rectifier output voltage.

V_n = r.m.s. value of the nth output voltage harmonic.

e = instantaneous value of primary phase voltage.

E = r.m.s. value of primary phase voltage.

i_d = instantaneous value of rectifier output current.

I_d = mean value of rectifier output current.

i = instantaneous value of primary current.

I = r.m.s. value of primary current.

i_n = instantaneous value of the nth primary current harmonic.

I_n = r.m.s. value of the nth primary current harmonic.

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* Reprinted from *Journal I.E.E.*, June, 1940.

[1] For numbered references, see end of this article.

I_1 = r.m.s. value of the fundamental component of the primary current.

I' = r.m.s. value of the total harmonic content of the primary current

$$= \sqrt{(I^2 - I_1^2)} = \sqrt{\left[\frac{1}{2\pi}\int_0^{2\pi}\Sigma i_n^2 \,.\, d\theta\right]}$$

φ = phase angle between i_1 and e.

I_p = active component of $I_1 = I_1 \cos\varphi$.

I_r = reactive component of $I_1 = I_1 \sin\varphi$.

P_d = mean value of rectifier power output $= V_d I_d$.

P = active power input to transformer primary $= 3EI_p$.

P_R = reactive power input to transformer primary $= 3EI_r$.

P_H = harmonic power input to transformer primary $= 3EI'$.

P_A = apparent power input to transformer primary $= 3EI$.

μ = distortion factor of the primary current $= I_1/I$.

$\cos\varphi$ = displacement factor of the primary current $= I_p/I_1$.

λ = power factor of the primary current $= I_p/I$.

α_n = voltage harmonic ratio $= V_n/V_d$.

β_n = current harmonic ratio $= I_n/I_1$.

3. *THE HARMONIC LAW OF POLYPHASE RECTIFICATION*

In the ideal case of a rectifier installation devoid of transformer or other circuit reactance, as well as of all resistance, the instantaneous D.C. power output of the rectifier system must of necessity be equal to the instantaneous A.C. power input to the primary winding of the rectifier transformer. Assuming a symmetrical three-phase supply system, so that the primary phase voltages e_R, e_Y and e_B all have the same r.m.s. value E, and denoting the corresponding primary currents by i_R, i_Y and i_B, we then obtain the fundamental power relation

$$v_d i_d = e_R i_R + e_Y i_Y + e_B i_B \quad \ldots\ldots\ldots(1)$$

where v_d and i_d are the instantaneous output voltage and current. It is usual to assume that the rectifier load is infinitely inductive, so that the several rectifier phase currents have a rectangular wave form which is symmetrical about the maximum phase-voltage ordinate. Under these circumstances the primary current wave can contain only sine terms, each harmonic being of the form $i_n = \sqrt{2}I_n \sin n\theta$ where I_n is the r.m.s. value and n the order of the harmonic; whilst the output voltage wave can contain only cosine harmonics, of the form $v_n = \sqrt{2}V_n \cos n\theta$ between the limits $\theta = -(\pi/p)$ and $\theta = +(\pi/p)$, corresponding to the current-conducting period of each rectifier phase $(2\pi/p)$. We thus obtain the following further relations:—

$$\begin{aligned}
v_d &= V_d + \sqrt{2}\Sigma V_n \cos n\theta.\\
e_R &= \sqrt{2}E \sin\theta.\\
e_Y &= \sqrt{2}E \sin(\theta + 2\pi/3).\\
e_B &= \sqrt{2}E \sin(\theta + 4\pi/3).\\
i_d &= I_d.\\
i_R &= \sqrt{2}\Sigma I_n \sin n\theta.\\
i_Y &= \sqrt{2}\Sigma I_n \sin n(\theta + 2\pi/3).\\
i_B &= \sqrt{2}\Sigma I_n \sin n(\theta + 4\pi/3).
\end{aligned}$$

On substituting these several values in equation (1) one finally obtains, after some rearrangement of terms and trigonometrical reduction, the following general power equation:—

$$\begin{aligned}
&V_d I_d + \sqrt{2}I_d \Sigma V_n \cos n\theta\\
&\quad = 2E\Sigma I_n\left[\left(1 - \cos n\pi \cos\frac{n\pi}{3}\right)\sin n\theta \sin\theta\right.\\
&\qquad \left. + \sqrt{3}\left(\cos n\pi \sin\frac{n\pi}{3}\right)\cos n\theta \cos\theta\right]. \quad (2)
\end{aligned}$$

Now in a symmetrical three-phase system the instantaneous sum of the currents is always zero, so that $i_R + i_Y + i_B = 0$. But in such a system the instantaneous sum of the harmonics i_n of order $n = 3, 6, 9$, etc., is not zero, and hence these triplen-current harmonics must themselves all be zero. Furthermore, the only symmetrical p-phase rectifier systems which can be supplied from a three-phase A.C. system are those having values of p which are multiples of 3. If $n = mp$, where m is any integer, then, since p is triplen, n must be triplen also. Also, as is well known in rectifier theory, the several voltage harmonics v_n are zero for all values of n other than $n = mp$.[2] Thus only certain triplen harmonics can occur in the output voltage; whereas it is precisely these triplen harmonics which are absent from the primary current. This fundamental harmonic relationship is a unique characteristic of polyphase rectification. To obtain a quantitative expression of this relationship it is necessary to evaluate equation (2). The two bracketed expressions on the right-hand side of this equation become zero for all values of n that are multiples of 3. For all non-triplen values of n, on the other hand, the first expression has the value 3/2, whilst the second expression has the value $\pm \frac{1}{2}\sqrt{3}$ depending on whether $n = (3m \pm 1)$, where m is any integer. So that, on developing the sine and cosine products in the usual manner by putting

$$\sin n\theta \sin \theta = \tfrac{1}{2}[\cos (n-1)\theta - \cos (n+1)\theta]$$

$$\text{and } \cos n\theta \cos \theta = \tfrac{1}{2}[\cos (n-1)\theta + \cos (n+1)\theta]$$

equation (2) finally takes the basic form

$$\begin{aligned} V_d I_d + \sqrt{2} V_3 I_d \cos 3\theta + \sqrt{2} V_6 I_d \cos 6\theta + \ldots \\ = 3EI_1 + 3E(I_2 - I_4) \cos 3\theta \\ + 3E(I_5 - I_7) \cos 6\theta + \ldots\ldots (3) \end{aligned}$$

Inspection of this equation at once leads to two subsidiary relations of fundamental importance to rectifier theory, namely, the active power relation

$$V_d I_d = 3EI_1 \quad \ldots\ldots\ldots\ldots\ldots (4)$$

and the harmonic power relation

$$\sqrt{2} V_n I_d = 3E(I_{n-1} - I_{n+1}) \quad \ldots\ldots (5)$$

Equation (4) indicates that the D.C. power output, $P_d = V_d I_d$, is equal to the true A.C. power input, $P = 3EI_1$, i.e., the power carried by the fundamental component of the primary current—a relation which is evident from energy considerations alone. Equation (5) expresses the fundamental "harmonic law" of polyphase rectification, which may be stated as follows :—

> *Every harmonic occurring in the D.C. terminal voltage of a polyphase rectifier system is inevitably associated with two harmonics, of the next lower and next higher order, in the alternating current drawn by the rectifier transformer from the three-phase A.C. supply.*

In other words, the nth voltage harmonic on the D.C. side is always associated with the $(n-1)$th and the $(n+1)$th primary-current harmonics.

On putting $\alpha_n = V_n/V_d$, the voltage harmonic ratio, and $\beta_n = I_n/I_1$, the current harmonic ratio,

TABLE I

THE HARMONIC FREQUENCY SPECTRUM

Number of rectifier phases		$p = 3$		$p = 6$		$p = 9$		$p = 12$	
Percentage harmonic		α_n	β_n	α_n	β_n	α_n	β_n	α_n	β_n
Order of the harmonic, n	1*	—	100.0	—	100.0	—	100.0	—	100.0
	2	—	50.0	—	—	—	—	—	—
	3	17.71	—	—	—	—	—	—	—
	4	—	25.0	—	—	—	—	—	—
	5	—	20.0	—	20.0	—	—	—	—
	6	4.04	—	4.04	—	—	—	—	—
	7	—	14.3	—	14.3	—	—	—	—
	8	—	12.5	—	—	—	12.5	—	—
	9	1.77	—	—	—	1.77	—	—	—
	10	—	10.0	—	—	—	10.0	—	—
	11	—	9.1	—	9.1	—	—	—	9.1
	12	0.99	—	0.99	—	—	—	0.99	—
	13	—	7.7	—	7.7	—	—	—	7.7
	14	—	7.1	—	—	—	—	—	—
	15	0.63	—	—	—	—	—	—	—
	16	—	6.2	—	—	—	—	—	—
	17	—	5.9	—	5.9	—	5.9	—	—
	18	0.44	—	0.44	—	0.44	—	—	—
	19	—	5.3	—	5.3	—	5.3	—	—
	20	—	5.0	—	—	—	—	—	—
	21	0.32	—	—	—	—	—	—	—
	22	—	4.5	—	—	—	—	—	—
	23	—	4.3	—	4.3	—	—	—	4.3
	24	0.25	—	0.25	—	—	—	0.25	—
	25	—	4.0	—	4.0	—	—	—	4.0

* Fundamental component.

division of equation (5) by equation (4) gives

$$\sqrt{2}\alpha_n = \beta_{n-1} - \beta_{n+1} \quad \ldots\ldots\ldots\ldots(6)$$

which is the quantitative expression, in its simplest form, of the harmonic law implicit in (5). Furthermore, as is well known in rectifier theory,[2] the voltage harmonics are defined by $V_n = \sqrt{2}V_d/(n^2-1)$, so that $\alpha = \sqrt{2}/(n^2-1)$. Hence equation (6) becomes

$$\beta_{n-1} - \beta_{n+1} = \frac{2}{n^2-1} = \frac{1}{n-1} - \frac{1}{n+1}$$

a relation which is only satisfied by the condition that $\beta_n = 1/n$, i.e., that

$$I_n = \frac{1}{n} \cdot I_1 \quad \ldots\ldots\ldots\ldots\ldots(7)$$

The harmonic law of polyphase rectification thus leads to the important conclusion that the r.m.s. values of the primary-current harmonics are inversely proportional to their harmonic frequency and are independent of the number of rectifier phases. This relation stands in marked contrast to the corresponding relation for the rectifier phase currents. In that case, as has been shown previously,[3]

$$I_n = \frac{\sqrt{2}}{n\pi}\sin\frac{n\pi}{p}\cdot I_d$$

so that

$$I_n = \frac{1}{n}\cdot I_1\left[\frac{\sin(n\pi/p)}{\sin(\pi/p)}\right] \quad \ldots\ldots\ldots(8)$$

In interpreting equation (6) it is to be remembered that $n = mp$, where p is the number of rectifier phases and m is any integer, and that in practice p can only have values which are multiples of 3. The harmonic law of polyphase rectification thus leads to the numerical harmonic relationships of Table I, in which α_n and β_n are expressed as percentages.

The importance of the conclusion expressed by (7) is that it at once enables one to evaluate the distortion factor of the primary current, that is to say, the ratio of the r.m.s. fundamental component I_1 to the r.m.s. resultant $I = \sqrt{(I_1^2 + I_2^2 + I_3^2 + \ldots.)}$. If the r.m.s. value of all the harmonics be denoted by I', then, since $n = (mp \pm 1)$ on the primary side of the rectifier transformer, we have

$$I'^2 = \Sigma I_n^2 = I_1^2\sum\left(\frac{1}{n^2}\right) = I_1^2\sum_{m=1}^{m=\infty}\left(\frac{1}{mp\pm 1}\right)^2$$

$$= I_1^2\sum_{m=1}^{m=\infty}\left[\left(\frac{1}{mp+1}\right)^2 + \left(\frac{1}{mp-1}\right)^2\right]$$

$$= I_1^2\sum_{m=1}^{m=\infty}\left[\frac{2(m^2p^2+1)}{(m^2p^2-1)^2}\right]$$

Making use of the infinite series

$$\pi^2\operatorname{cosec}^2(\pi\theta) = \frac{1}{\theta^2} + 2\sum_{m=1}^{m=\infty}\left[\frac{m^2+\theta^2}{(m^2-\theta^2)^2}\right]$$

one thus obtains

$$I'^2 = I_1^2\left(\frac{\pi^2}{p^2}\operatorname{cosec}^2\frac{\pi}{p} - 1\right)$$

so that

$$I^2 = I_1^2 + I'^2 = \frac{\pi^2}{p^2}\operatorname{cosec}^2\frac{\pi}{p}\cdot I_1^2.$$

The distortion factor is then simply

$$\mu = \frac{I_1}{I} = \frac{p}{\pi}\sin\frac{\pi}{p} \quad \ldots\ldots\ldots\ldots\ldots(9)$$

a result which is well known in rectifier theory.

4. HARMONIC GENERATION ON THE PRIMARY SIDE OF RECTIFIER TRANSFORMERS

So far the r.m.s. value I of the primary current has been evaluated indirectly by summation of the fundamental and harmonic components. Its direct evaluation from a consideration of the primary-current wave form is of importance, however, in that it demonstrates the correctness of the analysis leading up to the basic power relation expressed by equation (3), from which the harmonic law expressed by equation (6) automatically follows. The condition that the number of phases in a symmetrical polyphase rectifier system fed from a three-phase A.C. supply must be a multiple of 3 leads, in the first place, to symmetry of the primary current wave form about the maximum phase-voltage ordinate. In the second place, a little consideration will show that it means also that each primary phase must carry the equivalent of two or more rectifier phase currents (i.e., transformer secondary currents). As the result, the primary winding of the rectifier transformer carries a symmetrical alternating current having a stepped wave form, as indicated in Fig. 1 which shows the positive half-wave only. The wave form of diagram (*a*) occurs in the case of those transformer connections which result in phase coincidence between the primary and secondary phase voltages. The alternative wave form of diagram (*b*) occurs in the case of those connections giving rise to a phase difference of π/p between these voltages. However, both wave forms contain the same harmonics, and have the same r.m.s. value and the same distortion factor. They are distinguished only by specific differences in phase between the fundamental component and the several harmonics.[4]

In either case, and as will appear subsequently, the height of the middle step H_1 is equal to the peak value of the equivalent sine wave, i.e., the sinusoid having the same r.m.s. value I as the stepped wave. The width of the several steps is $2\pi/p$, the current-conducting period per rectifier phase, and they are symmetrically disposed with regard to the maximum phase-voltage ordinate. If $\theta = \omega t$ be reckoned from this axis, then the middle step H_1 extends from $-\pi/p$ to $+\pi/p$; and the subsequent pairs of steps H_k extend from $\theta = -(2k-1)\pi/p$ to $\theta = -(2k-3)\pi/p$ and from $\theta = +(2k-3)\pi/p$ to $\theta = +(2k-1)\pi/p$. In the case of wave form (*a*) the last pair of steps H_l extend from $\theta = -\pi/2$ to $\theta = -(\pi/2 - 2\pi/p)$ and from $\theta = +(\pi/2 - 2\pi/p)$ to $\theta = +\pi/2$; so that $(2l-1)\pi/p = \pi/2$, or $l = \frac{1}{4}(p+2)$. In the case of wave form (*b*) the corresponding limits of H_l are $\theta = -(\pi/2 - \pi/p)$ and $\theta = -(\pi/2 - 3\pi/p)$, and $\theta = +(\pi/2 - 3\pi/p)$ and $\theta = +(\pi/2 - \pi/p)$; so that $(2l-1)\pi/p = (\pi/2 - \pi/p)$, or $l = \frac{1}{4}p$. In general, therefore,

$$l = \tfrac{1}{4}(p + 1 \pm 1) \quad \ldots\ldots\ldots\ldots (10)$$

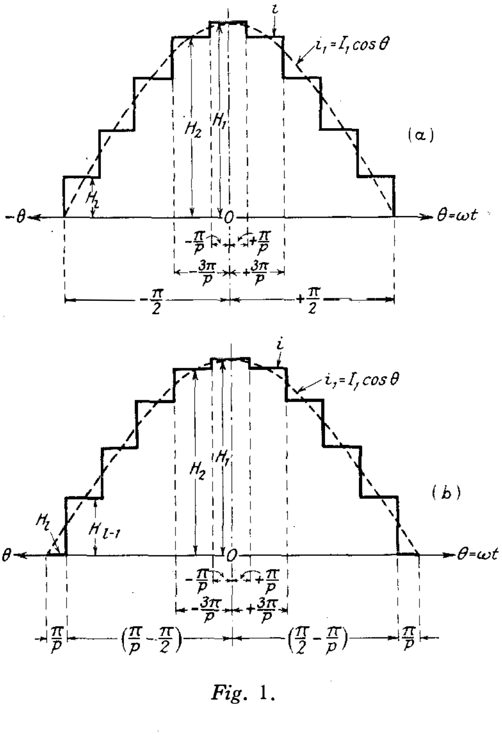

Fig. 1.

Furthermore, the height of each symmetrically placed pair of steps is clearly given by $H_k = \cos(k-1)2\pi/p$, where $k = 1, 2, 3, \ldots\ldots l$.

The primary-current wave can be represented by the Fourier series $i = \Sigma A_n \cos n\theta$, in which the coefficient of the nth harmonic is given by

$$A_n = \frac{2}{\pi}\left[\int_{-\pi/p}^{+\pi/p} H_1 \cos n\theta\, d\theta + 2\int_{\pi/p}^{3\pi/p} H_2 \cos n\theta\, d\theta + \ldots\right.$$

$$\left. + 2\int_{(2k-3)\pi/p}^{(2k-1)\pi/p} H_k \cos n\,\theta + \ldots + 2\int_{(2l-3)\pi/p}^{(2l-1)\pi/p} H_l \cos n\theta\, d\theta\right].$$

Bearing in mind that $n = (mp \pm 1)$, where m is any integer, and substituting the value of H_k given above, one finds

$$A_n = \frac{2H_1}{\pi}\left[\int_{-\pi/p}^{+\pi/p} \cos(mp \pm 1)\theta . d\theta + 2\sum_{k=2}^{k=l} \cos(k-1)\right.$$

$$\left. \frac{2\pi}{p}\int_{-\pi/p}^{+\pi/p} \cos(mp \pm 1)\left\{\theta + (k-1)\frac{2\pi}{p}\right\} d\theta\right]$$

$$= \frac{2H_1}{\pi}\left[\frac{2\sin(mp \pm 1)\frac{\pi}{p}}{(mp \pm 1)} + \frac{4\sin(mp \pm 1)\frac{\pi}{p}}{(mp \pm 1)}.\right.$$

$$\left. \sum_{k=2}^{k=l} \cos(k-1)\frac{2\pi}{p} \cos(k-1)(mp \pm 1)\frac{2\pi}{p}\right]$$

$$= \pm\frac{4H_1}{n\pi}\sin\frac{\pi}{p}.\left[1 + 2\sum_{k=2}^{k=l}\cos^2(k-1)\frac{2\pi}{p}\right]$$

$$= \pm\frac{4H_1}{n\pi}\sin\frac{\pi}{p}\cdot\left[\frac{p}{4}\right].$$

This final result is obtained by summation of the (cosine)^2 series and substitution of the value of l given by (10). The r.m.s. value of the nth harmonic is thus

$$I_n = \frac{1}{\sqrt{2}}A_n = \frac{p}{n\pi\sqrt{2}}\sin\frac{\pi}{p}\cdot H_1 \ldots\ldots (11)$$

On putting $n = 1$, one finds for the r.m.s. fundamental component

$$I_1 = \frac{p}{\pi\sqrt{2}}\sin\frac{\pi}{p}\cdot H_1 \ldots\ldots\ldots\ldots (12)$$

Substituting H_1 from (12) in (11) then gives $I_n = I_1/n$, which is the universal harmonic relation of equation (7). The validity of the foregoing analysis, based on the stepped wave forms of Fig. 1, is thereby established.

The r.m.s. value of the primary current is given by

$$I^2 = \frac{1}{\pi}\left(H_1^2\cdot\frac{2\pi}{p} + 2H_2^2\cdot\frac{2\pi}{p} + \ldots 2H_k^2\cdot\frac{2\pi}{p} + \ldots + 2H_l\cdot\frac{2\pi}{p}\right)$$

$$= \frac{2H_1^2}{p}\cdot\left[1 + 2\sum_{k=2}^{k=l}\cos^2(k-1)\frac{2\pi}{p}\right]$$

$$= \frac{2H_1^2}{p}\cdot\left[\frac{p}{4}\right]$$

$$= \frac{H_1^2}{2}$$

and is thus

$$I = \frac{1}{\sqrt{2}}H_1 = \frac{\pi}{p}\operatorname{cosec}\frac{\pi}{p}\,.\,I_1 \ldots\ldots (13)$$

The height of the middle step H_1 is, therefore, the peak value of a sine wave whose r.m.s. value I is that of the actual stepped wave. Furthermore, equation (13) at once gives for the distortion factor of the primary current

$$\mu = \frac{I_1}{I} = \frac{p}{\pi}\sin\frac{\pi}{p} \ldots\ldots\ldots\ldots (14)$$

which is the same result as (9), as is to be expected.

5. *THE EFFECTS OF TRANSFORMER REACTANCE*

In the case of an actual rectifier system the effect of transformer reactance is to introduce a finite interval of phase commutation—represented by the so-called angle of overlap u—which results in a modification of the ideal stepped wave form shown in Fig. 1. In consequence of this modification the individual harmonics as well as the fundamental component of the primary current are altered both in magnitude and phase, resulting in a change in the r.m.s. value of the resultant current together with a displacement of the fundamental component from its position of phase coincidence with the primary phase voltage. Both changes are a function of the overlap angle u, the former producing a change in the harmonic power consumption and the latter giving rise to a consumption of reactive power. That is to say, in addition to the wattless power expended in harmonic generation, a further proportion of the total or apparent power drawn from the A.C. supply is utilized in overcoming electromagnetic inertia, represented mainly by the inevitable inductance of the transformer windings.

The concept of reactive power, of course, arose originally in connection with sinusoidally alternating currents and voltages, where the magnitude of the reactive power is given by the r.m.s. product of the voltage and the component of the current in phase quadrature with that voltage. In the general case where the current is not sinusoidal, but its several components—fundamental and harmonics—are regarded as being such, and the generating voltage remains sinusoidal, the reactive power is given by the r.m.s. product of the voltage and the quadrature component of the fundamental sinusoidal current. Thus if I_1 denote, as before, the r.m.s. value of the fundamental component of the primary current and if φ be its phase angle with respect to the primary phase voltage, then the power or in-phase component of the fundamental is $I_p = I_1 \cos\varphi$, whilst the reactive or quadrature component is $I_r = I_1 \sin\varphi$. Hence the active power consumption of the rectifier system is $P = 3EI_p = 3EI_1 \cos\varphi$, whilst the corresponding reactive power consumption is then $P_R = 3EI_r = 3EI_1 \sin\varphi$. The harmonic power consumption remains, of course, $P_H = 3EI' = 3E\sqrt{(I^2 - I_1^2)} = P_A\sqrt{(1-\mu^2)}$, where $P_A = 3EI$ is the total or apparent power consumption. The power factor of the primary current is thus $\lambda = P/P_A = \mu\cos\varphi$; the induction factor is $P_R/P_A = \mu\sin\varphi$; whilst the harmonic factor is $P_H/P_A = \sqrt{(1-\mu^2)}$.

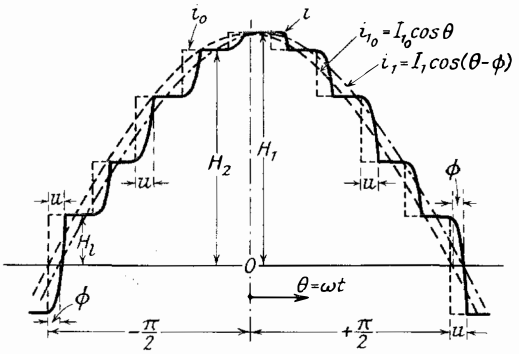

Fig. 2.

The effect of transformer reactance, then, is to modify the primary current from the ideal wave form shown in Fig. 1 to that of Fig. 2.* Here the individual steps no longer have vertical sides but are flanked by the overlapping sections of the individual rectifier phase-current waves. A little consideration will show that the rising sections are represented by the function $(H_k - H_{k+1}) \,.\, w\{\theta + (2k-1)\pi/p\}$, whilst the falling sections are similarly represented by the function $(H_k - H_{k+1}) \,.\, [1 - w\{\theta - (2k-1)\pi/p\}]$, where $w(\theta) = (1 - \cos\theta)/(1 - \cos u)$ is the well-known function determining the growth of the rectifier phase currents during the intervals of overlap.[5] The primary-current wave is in this case represented by the Fourier series $i = \Sigma A_n \cos n\theta + \Sigma B_n \sin n\theta$, the coefficients A_n and B_n as usual being defined by

$$A_n = \frac{2}{\pi}\int_0^{\pi} i \cos n\theta \,.\, d\theta$$

$$B_n = \frac{2}{\pi}\int_0^{\pi} i \sin n\theta \,.\, d\theta \,.$$

By analyzing the wave form of Fig. 2 along the lines established for Fig. 1, a general expression for the primary current i can be obtained in terms of symmetrically disposed pairs of steps. Integration of the products $i\cos\theta$ and $i\sin\theta$ between the limits $\theta = -(2l-1)\pi/p$ and $+(2l-1)\pi/p$, where l is given by equation (10), then gives for the first-order Fourier coefficients

$$A_1 = H_1\frac{p}{\pi}\sin\frac{\pi}{p} \,.\, \left[\frac{1-\cos 2u}{4(1-\cos u)}\right]$$

and

$$B_1 = H_1\frac{p}{\pi}\sin\frac{\pi}{p} \,.\, \left[\frac{2u - \sin 2u}{4(1-\cos u)}\right] .$$

Hence the active and reactive components of the primary current become

$$I_p = I_1\cos\varphi = \frac{1}{\sqrt{2}}A_1 = I_{1_0}\left[\frac{1-\cos 2u}{4(1-\cos u)}\right] \qquad \ldots\ldots(15a)$$

and

$$I_r = I_1\sin\varphi = \frac{1}{\sqrt{2}}B_1 = I_{1_0}\left[\frac{2u-\sin 2u}{4(1-\cos u)}\right] \qquad \ldots\ldots(15b)$$

where I_{1_0} is the r.m.s. value of the fundamental component in the ideal case when $u = 0$, as given by (12). The phase angle φ by which the fundamental primary current lags behind its corresponding phase voltage is thus given by

$$\tan\varphi = \frac{2u - \sin 2u}{1 - \cos 2u} \qquad \ldots\ldots\ldots\ldots(16)$$

which is the same relation as that obtained in the case of the rectifier phase currents.[3] That this must be so is evident from magnetic energy considerations alone, the reactive power loading on the secondary side of the rectifier transformer being transmitted without change to the primary side. The displacement factor of the primary current is thus

$$\cos\varphi = \frac{1}{\sqrt{(1 - 2u\cot u + u^2\operatorname{cosec}^2 u)}} \qquad \ldots(17)$$

As is well known in rectifier theory, (16) or (17) gives as a close approximation $\varphi = \frac{2}{3}u$.

The change in harmonic content brought about by the change in wave form due to rectifier phase-current overlap is also reflected in the r.m.s. value I. Integration of i^2 between the limits defined above gives for the r.m.s. value of the primary current

$$I^2 = \frac{2}{p}\left(H_1^2 + 2\sum_{k=2}^{k=l} H_k^2\right) - 2\sum_{k=1}^{k=l}(H_k - H_{k+1})^2 \,.\, \frac{1}{\pi}\int_0^u [w(\theta) - w^2(\theta)]d\theta$$

with $H_{k+1} = 0$ when $k = l$ (Fig. 2). The first term of the above expression is

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* Only the wave form of diagram (*a*) has been repeated here. The subsequent analysis, however, applies equally to the alternative wave form of diagram (*b*).

$$\frac{2H_1^2}{p}\left[1+2\sum_{k=2}^{k=l}\left(\frac{H_k}{H_1}\right)^2\right]$$
$$=\frac{2H_1^2}{p}\left[1+2\sum_{k=2}^{k=l}\cos^2(k-1)\frac{2\pi}{p}\right]$$
$$=\frac{2H_1^2}{p}\left[\frac{p}{4}\right]$$
$$=\frac{H_1^2}{2}$$

and corresponds to I^2 in the ideal case where $u=0$, as may be seen by reference to the derivation of equation (13). The integral in the second term is the current overlap function $\psi(u)$, well known in rectifier theory. The above expression for I^2 thus leads to

TABLE II

p	2	3	4	6	12
κ	4	4.5	4	3	1.61

$$I=\frac{H_1}{\sqrt{2}}\cdot\sqrt{[1-\kappa\,.\,\psi(u)]}$$
$$=I_0\sqrt{[1-\kappa\,.\,\psi(u)]}\,\ldots\ldots\ldots\ldots(18)$$

where $I_0=H_1/\sqrt{2}$ is the value of I in the ideal case where $u=0$, and the factor κ is a function of p alone, viz., :—

$$\kappa=4\sum_{k=l}^{k=l}\left(\frac{H_k-H_{k+1}}{H_1}\right)^2,$$

with $H_{k+1}=0$ when $k=l$,

$$=8\sin^2\frac{\pi}{p}\sum_{k=1}^{k=(l-1)}\left[1-\cos(2k-1)\frac{2\pi}{p}\right]$$
$$+4\cos^2(l-1)\frac{2\pi}{p}$$
$$=8\sin^2\frac{\pi}{p}\,.\left[\frac{p}{4}\right]$$
$$=p\left(1-\cos\frac{2\pi}{p}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

The effect of transformer reactance is therefore to reduce the r.m.s. primary current by the factor

$$\sqrt{[1-p(1-\cos 2\pi/p)\,.\,\psi(u)]}$$

as compared with the well-known reduction factor $\sqrt{[1-p\,.\,\psi(u)]}$ in the case of the secondary (or rectifier phase) current. Table II gives values of the factor κ for several values of p.*

Equations (15a) and (15b) give for the r.m.s. value of the fundamental component

$$I_1=\sqrt{(I_p^2+I_r^2)}$$
$$=I_{1_0}\,.\,\frac{\sqrt{[u^2-2u\sin u\cos u+\sin^2 u]}}{2(1-\cos u)}\ \ldots(20)$$

which is the same relation between the actual and ideal values of the fundamental component as that obtaining in the case of the rectifier phase currents.[3] It is to be noted, however, that in the case of the primary currents not only is the reduction factor of the fundamental component I_1 independent of p, but those of its sine and cosine components (viz., I_r and I_p)

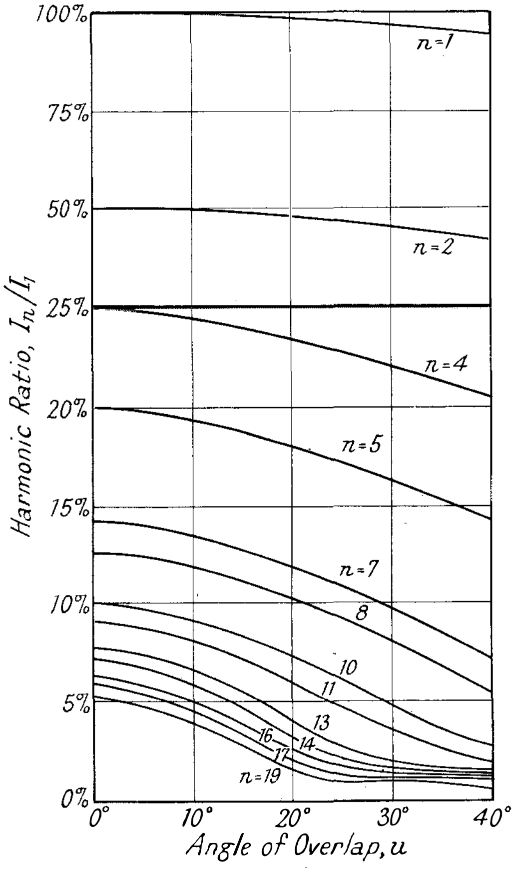

Fig. 3.

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* O. K. Marti and W. Winograd in their book *Mercury-Arc Power Rectifiers* (McGraw-Hill, 1930) have analyzed the primary-current wave form in the specific case of 12-phase rectification, arriving at the numerical value $\kappa=1.61$. No attempt has been made so far, however, to obtain a general expression for κ which can be universally applied, viz., equation (19) above.

are also independent of p and are functions of u alone, which is not the case for the rectifier phase currents. It can be shown that this applies equally to the several primary-current harmonics. Integration of the products $i \cos n\theta \, . \, d\theta$ and $i \sin n\theta \, . \, d\theta$ between the same limits as before gives for the nth-order Fourier coefficients

$$A_n = H_1 \cdot \frac{p}{n\pi} \sin\frac{\pi}{p} \left[\frac{\cos nu \cos u + n \sin nu \sin u - 1}{(n^2-1)(1-\cos u)}\right] \quad (21a)$$

and

$$B_n = H_1 \cdot \frac{p}{n\pi} \sin\frac{\pi}{p} \left[\frac{n \cos nu \sin u - \sin nu \cos u}{(n^2-1)(1-\cos u)}\right] \quad \ldots\ldots(21b)$$

from which the r.m.s. value of the nth harmonic is found to be

$$I_n = \frac{I_{1_0}}{n} \cdot \frac{\sqrt{[(\cos nu - \cos u)^2 + (\sin nu - n \sin u)^2]}}{(n^2-1)(1-\cos u)} \quad \ldots\ldots\ldots(22)$$

where, as before, I_{1_0} is the r.m.s. value of the fundamental in the ideal case where $u = 0$. On comparing (22) with (20) it will be seen that, although the fundamental harmonic law still holds in the case where transformer reactance is taken into account—that is to say $n = (mp \pm 1)$, where m is any integer—the corollary relation expressed by (7) only applies in the ideal case. Due to the change in primary-current wave form brought about by the overlapping of the rectifier phase currents during the commutation process, the r.m.s. values of the primary-current harmonics no longer remain inversely proportional to the harmonic frequency, but are further reduced by the factor

$$\frac{2}{n^2-1} \cdot \sqrt{\left[\frac{(\cos nu - \cos u)^2 + (\sin nu - n \sin u)^2}{u^2 - 2u \sin u \cos u + \sin^2 u}\right]}.$$

This is clearly shown by Fig. 3, which indicates the variation in r.m.s. value of the several primary-current harmonics given in Table I as a function of the angle of overlap u.

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Statistical Control of the Quality of Telephone Service*

By W. F. NEWLAND and E. E. NEAL

YOU are no doubt familiar with the application of statistical methods of controlling the quality of production in industries manufacturing physical products. Such control is largely applied to properties such as dimensions of piece parts or stampings, to breaking strain or hardness, or to chemical properties. The material used may be subjected to tests during the various processes of manufacture, from the raw state to the partly finished or finished article. In the telephone manufacturing industry, which concerns itself with large-scale production of piece parts, statistical methods of control of quality have been widely applied. In the case of a telephone "service," however, some distinct differences appear. The quality of telephone " service " is a much less tangible thing than the physical property of material. A telephone service can be good, bad, fairly good, rather bad, and so on. It will readily be appreciated, therefore, that in the field of " service," as distinct from the field of " production," the application of statistical methods will call for a technique peculiar to its own special needs. This paper endeavours to explain the technique used in the Post Office in controlling the quality of the service it supplies to its telephone subscribers.

In devising a scheme for controlling the quality of service, three basic considerations emerge :—

(*a*) A method of measuring quality in arithmetical terms must be evolved.

(*b*) So great is the number of calls handled that it will be impossible to test quality except by sampling.

(*c*) From the sample data so obtained, some method of appraisement must be evolved which will not ignore the fact that the data represent merely a sample, and will consequently be subject to sampling fluctuations.

THE METHOD OF SAMPLING

The Telephone Administration has for many years carried out systematic tests of the quality of service it gives to subscribers by testing a small proportion of public telephone calls during their entire progress—that is, from the time the caller lifts his receiver, until the distant subscriber answers or the connection is severed at the termination of the call.

Briefly, this test is carried out by means of special equipment located at a central point. One special junction is provided between this point and each London exchange. At the local exchange between six and fifteen subscribers, the number being dependent upon the subscribers' rate of calling, are joined to a local selective equipment for two-day periods. This equipment ensures that the first party calling, when the common junction is free, shall be connected thereto, and extended to the central point. Immediately the subscriber lifts his receiver, this condition is signalled forward to the centre. Special equipment receives the dial impulses and actuates a visual display of the numerical digits dialled. The electrical conditions relating to all subsequent operations are also signalled forward, including the operation of the call-counting meter associated with the subscriber's line if the call is successfully established.

The data required to express quality of service

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must include not only definite knowledge of whether or not a particular condition occurs in its correct sequence in the setting up of a call, but also the moment of time at which each operation occurs, counting from the time the subscriber lifts his receiver. In the case of manual service these time-values are of major importance. They refer, for instance, to such operations as the time taken for the " home " operator to answer the subscriber's calling signal, the distant operator to answer the signal at her exchange, the speed of attention to the caller's recall signal—that is, the movement of his switch-hook—and how long the distant subscriber permits ringing to be sent out over his line before he answers. The need for data on these conditions will be readily apparent so far as manual service is concerned, and similar data are equally useful in the case of automatic service. For instance, the times elapsing between the subscriber lifting his receiver and beginning to dial, completing dialling, the answer of the distant party, and the termination of the call by both parties are all, individually, matters which directly affect the revenue-earning capacity of the equipment brought into use. They determine also the quantity of plant which is required for the system and therefore the capital outlay.

SUMMARIZING THE SAMPLE DATA

In setting up the various types of call in common demand—that is, local calls, calls to other exchanges, toll calls and trunk calls—some 75 different operating conditions may occur. Our practice is to record the time elapsing from the commencement of a call until the application of such of the 75 conditions as occur and are signalled forward. The recorded particulars show, therefore, whether a particular condition occurred, and if so when it occurred. Some of these conditions require for their appreciation close knowledge of telephone technical practices, but many will readily be understood by all users of the telephone. Some of these are shown in the headings of Tables I and II. These two tables are typical summaries of the data taken from the rough recording sheets. The data themselves are, however, hypothetical, and have been inserted for explanatory purposes. We have chosen a control period of one month. This choice is based purely on practical considerations. On the one hand, a very short control period would, for instance, reflect temporary variations in the staffing of the exchanges, both operating and maintenance, which would have been rectified in the ordinary course before the statistical data for each of the many London exchanges had been analyzed. On the other hand, any attempt to eliminate these effects by spreading the tests over a lengthy period would delay investigation and action when such action was clearly called for. Moreover, from the point of view of office organization and machinery, the work of summarizing and circulating duplicated copies of the summaries can most conveniently be arranged on the basis of one month. These considerations will no doubt be in mind when later in the paper the size of our sample is dealt with.

The data in Table I relate to automatic working and in Table II to manual working. Entries are made on the sheets in respect of each exchange. The average condition is shown at the foot of the table. You will no doubt recognize in the headings some of the troubles you have yourselves experienced.

It will be seen from these summaries that quality of service is susceptible to measurement, at least in comparative arithmetical terms, and as all telephone systems will in the main be subject to each of the conditions mentioned, the system of measurement will have universal application. Plant and operating performance can be checked against earlier figures of the same exchange, against those of other exchanges, and against the average " All London " figures.

Before proceeding with the subsequent treatment of the data, we will outline very briefly the switching system adopted by the Post Office. The automatic switching system comprises in principle a switch of ten positions, operated under the control of the impulses of the subscriber's dial.

As the first digit is dialled—say digit 2 of a four-figure system—the switch steps to position 2 and automatically searches for a free channel from this position to the next switching stage—i.e., the stage controlling all numbers 2 000-2 999, as distinct from all other positions controlling numbers beginning with 0-1 and 3-9.

TABLE I

Monthly Summary of Main Service Criteria—Automatic Exchanges

(These figures are hypothetical and for explanatory purposes only)

Exchange	Number of Observations			Percentage of Calls Completed			Busy Tone. All Calls‡†	Percentage of calls not completed due to :—												Percentage Number Tone at 31 Secs. from Completion of Dialling, C.C.I. Positions	Total Number of Calls to "O" Observed	Average Time until Operator Answered. Assistance Positions	Exchange
								Number Unobtainable			No Reply. Sub. Abandoned After Prescribed Waiting Period				Subscriber Abandoned Prematurely		Wrong Exchange or Number† All Calls	Miscellaneous Causes, All Calls					
											R.T. O.K.	No Tone			R.T. O.K.	No. Tone							
	Automatic	Distant Manual	All Calls	Automatic†	Distant Manual†	All Calls†		Automatic†	Distant Manual†	All Calls†	All Calls†	Automatic†	Distant Manual†	All Calls†	All Calls†	All Calls†		Subscriber	Plant or Operator				
Item Number on Blue Print	2	3	6	26	27	29	31	32	33	34	35	37	38	39	36	42	43	44	45	59	4	10	
Exchange *A*	125	117	244	77.6	71.1	75.9	10.8	1.2	0	0.6	3.6	5.0	3.8	5.0	3.5	2.8	1.2	0.6	0.6	9.1	2	4.7	
,, *B*	124	126	253	78.6	73.8	76.4	9.5	1.6	1.6	1.6	2.7	4.1	5.5	5.7	0.7	2.0	1.1	0	0.4	9.5	3	19.8	
,, *C*	260	205	470	77.5	77.9	77.6	9.4	1.3	1.7	1.4	3.4	3.1	4.7	3.8	1.1	1.5	1.3	0.9	0	7.4	5	4.5	
,, *D*	133	94	228	78.4	77.0	78.2	8.3	1.3	1.1	1.1	2.6	7.8	6.7	7.2	0.9	2.1	0.4	1.3	0	10.8	1	5.4	
....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
Total for Automatic Exchanges	9 298	5 645	15 490	81.9	80.3	80.8	9.1	1.8	1.7	1.7	3.1	4.4	4.6	4.5	0.8	1.1	1.0	0.9	0.4	8.2	547	9.8	

† See pp. 130, 133.

‡ See p. 136.

There will be a number of channels from position 2 to the range 2 000-2 999, and the first disengaged one will be taken, extending the caller to the second stage. This stage also terminates on a 10-position switch. Suppose the next digit dialled be a 3, the channels now chosen will serve the numbers (2)300 to (2)399, the figure in brackets having already been determined. Each stage is similar, until at the final stage the four-digit number has been completely dialled and the wanted line connected. You will see that on any one call the total number of possible combinations of links, from the first switch to the last, will be large. For instance, any link in group 2 of the first switch might be associated with any link in group 3 of the second switch, and with any one of the group next required. It is not possible, therefore, to say beforehand just what links or channels will be brought into use for any given call, unless no calls are passing through the exchange, when the link or channel will be the first in each case. As the free channels are tested in order beginning with the first, early channels will be most often in use, and end channels most rarely. If the call is directed to a distant automatic exchange, the same system of selection applies. These are the conditions subscribers' traffic meets. From the point of view of plant performance under working conditions, our sample calls give us an unbiased picture of working conditions in all sections.

APPRAISEMENT OF THE SAMPLE DATA

Reverting to Table I, it will be seen that, broadly speaking, the conditions classified are of one type—that is, the call either succeeds or fails. It may, for instance, fail because the " busy " condition is met, the " number unobtainable " tone is received, or a wrong number is given. It will be seen, however, from Table II that there is a second broad category relating to speeds—i.e., speed of operator's answer (item 5), or attention to recall signals (flashing supervisory signal, item 14), or " disconnect " connecting cords at the termination of the call (item 19), etc. The category which concerns itself with the two alternative conditions, success or failure, will be considered first.

To illustrate the technique we have applied to the problem, a few more important terms of the data will be dealt with. One term of importance to the subscriber is the proportion of calls not successful because either the exchange plant or the distant subscriber is engaged. The Telephone Administration is interested in both cases, firstly because it has to balance economic plant provision against the desire to ensure a satisfactory service, and secondly because a call reaching a distant subscriber's number which is already engaged utilizes plant without producing revenue.

It is clearly desirable to fix some limit above which investigation will be made to determine the source of " busy tone," and either increase plant provision if that is necessary, or locate subscribers who have too few lines for their telephone traffic. In Table I the percentage of calls lost over the whole of London owing to plant or subscriber being engaged, is shown as 9.1. As this percentage was based on over 9 000 calls, it may be assumed to be very near the real average—so near, in fact, that to two significant figures it no doubt represents the real average.

If the size of a sample of observed calls at any one exchange be 200, the expansion of the binomial $(0.909 + 0.091)^{200}$ will give the expected distribution of the proportion of failures, if the service has been subjected to control for some time, and has reached a stable condition. This binomial distribution will have a mean value of 18.2, and the sum of the first 24 terms of the expansion—i.e., those corresponding to 0.1 . . . . 23 faulty calls—is 0.900478. If, therefore, we set up an upper control limit of 23 calls meeting the busy tone in 200 or 11.5 per cent. and investigate all cases exceeding this percentage, we shall include not more than one case in ten where the high figure is due to sampling fluctuations. The same reasoning holds for the remaining items of this type—namely, calls lost by meeting number unobtainable tone, or no tone, or by obtaining a wrong number, etc. Such items have been marked † on Tables I and II. On a theoretical basis, therefore, statistical methods of controlling the quality of service are readily applicable to certain main criteria of service shown in these Tables.

In its practical application, the system of erecting control limits about a binomial distribution would entail, as a first consideration, the expansion of a large number of binomial ex-

TABLE II

Monthly Summary of Main Service Criteria—Manual Exchanges

(*These figures are hypothetical and for explanatory purposes only*)

Exchange	Total Local and Junction Calls	Average Time until Operator Answered	Percentage of Calls Answered in :		Average Time to Disconnect	Position on Total Operation (Cols. 5 and 19)	Average Time from Comp. of Repetition until First Supervision	Average Time to :		Percentage of :			Percentage of Wrong Exchange or Number Connection†	Percentage of :			Position on Total Irregularities
			10 Secs. or less‡	20 Secs. or less‡				Answer Subs. Flash given Satisfactorily	Supervise after Conn. of Engaged Tone or Guard Flash	Calls Completed†	Line Engaged†	No Reply†		Failed to Repeat Properly	Supervised Inadequately	Total Operators' Irregularities	
Item Number on Form TF75W and 76W	3	5	8		19	—	—	14	15	21	22	24	36	43	45	48	—
Exchange *A*	203	7.5	80.5	94.5	6.6	52	70.3	7.0	10.4	87.7	7.9	3.3	1.2	3.1	5.2	21.9	40
,, *B*	174	6.3	85.5	94.7	6.1	33	50.2	9.1	12.1	88.6	8.2	1.9	1.1	2.7	8.2	23.4	18
,, *C*	150	4.6	95.0	99.4	4.2	7	68.3	6.5	13.5	85.2	12.0	1.8	0.8	2.8	3.3	20.9	15
,, *D*	199	5.8	87.9	94.9	6.1	56	62.1	9.8	11.6	83.9	11.1	7.2	1.7	7.0	12.7	38.7	15
,, *E*	161	6.4	82.7	96.2	6.8	47	66.3	6.1	16.5	87.5	9.2	3.9	2.8	3.7	10.8	34.5	37
,, *F*	109	8.2	78.6	93.7	6.3	55	99.5	4.9	20.0	89.9	6.9	3.9	1.4	3.5	7.2	29.1	33
,, *G*	164	6.9	84.9	96.6	4.9	29	67.0	7.2	10.0	90.2	6.7	2.2	1.2	4.3	5.6	20.6	12
,, *H*	139	3.5	86.2	92.3	6.7	58	67.2	6.3	13.1	85.7	10.2	1.8	1.9	2.8	3.2	19.9	28
,, *I*	185	5.9	87.2	97.9	5.5	24	76.9	9.0	12.9	89.1	9.2	3.7	1.6	4.2	5.7	13.2	14
....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total All London	9720	5.1	86.2	96.3	5.5	—	60.8	9.0	12.2	89.3	9.2	2.7	1.9	3.1	4.2	20.7	—

† See pp. 130, 133.

‡ See p. 136.

pressions with varying values of n, p, and q. The Administration holds the view that too much regard should not be paid to keeping all samples of the same size. Indeed, it specifies that the total number of observed calls should be determined approximately from the expression 150 + 0.2 per cent. of the total originated busy hour calls. This works out in practice to a varying figure of between 200 and 400 calls per month at each exchange. It is natural, therefore, to enquire whether use could be made of the tabled values of the normal curve. In this connection we are not directly concerned with the disparity between the individual tail readings of the values of binomial and normal distributions, but rather with the sum of values beyond a fixed upper limit. Further, because the number defective can only increase in fixed stages of one call, we shall usually obtain an observed percentage defective a little on one side or the other of a precise limit, without being able to attach significance to a small excess. For instance, if we observe 150 calls we may lose either 10 or 11 calls, representing 6.67 or 7.3 per cent. defective, and no intermediate values are possible. If, therefore, the fixed upper limit was 7 per cent., the fact that 7.3 per cent. were defective is not definitely significant. It follows that if we maintain unequally sized samples, control limits based on binomial distributions must in any case tend to some loss of precision.

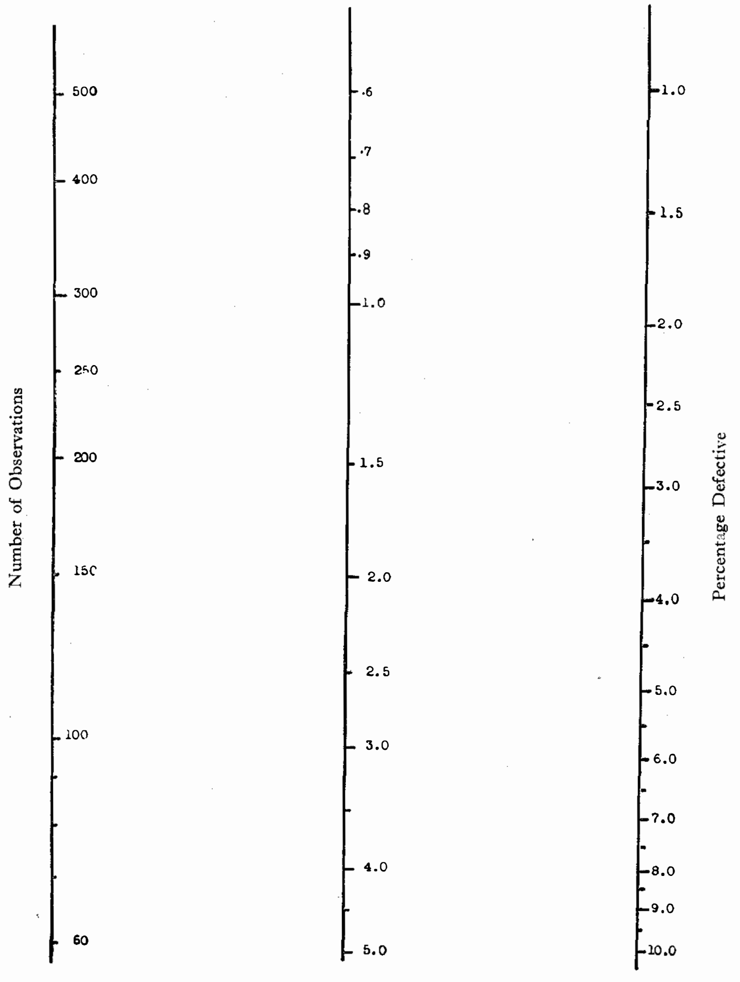

Fig. 1—*ABAC for probable limits of sampling error.*

In order to obtain a measure of the disparity between the tail areas of binomial and normal distributions within the regions met in practice, binomial expansions were worked out and compared with normal distributions of the same mean. The comparison between the two distributions took, in principle, the form of erecting an ordinate which included between itself and the origin, x per cent. of the total area of each distribution curve. The tail areas beyond this ordinate for each pair of distributions of the same means were then compared. This introduced the conception of a fractional number of defective calls, but it had the advantage of permitting the upper limit (x per cent.) to be constant for all sizes of the sample. The differences between the two distributions are not large. On a sample of 150 calls with a mean percentage defective of 2 per cent., 90 per cent. of the area of the distribution curve included all values from 0 to 3.1 per cent. defective for a binomial distribution and from 0 to 3.5 per cent. in the case of the normal distribution. For a percentage defective of 10, comparative figures were 12.9 per cent. and 13.1 per cent.

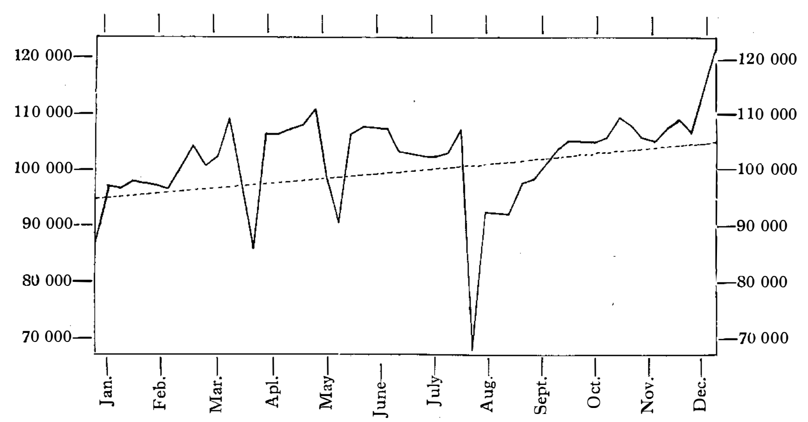

Fig. 2—Total calls per week for each week of the year at a typical exchange.

When " n " is large, and " q " reaches or exceeds 0.1.—i.e., 10 per cent.—there is, of course, very little difference between the binomial and the normal distributions. Within the range of percentage defective met in practice (2-10 per cent.), and with the size of the sample in use, there is very little error introduced by using the tabled values of the normal curve. Furthermore, it becomes easy to fix control limits for varying numbers of calls observed and percentages defective. For example, the " All London " figure of busy tones is shown in Table I as 9.1 per cent. for automatic working, and this average has been based on a large number of calls. It can be taken as the probability of failure of this type. Taking " n " as the number of calls in the sample at the exchange under consideration, the control limit will be $\sqrt{npq}\ 100t/n$ per cent. above the mean of 9.1, " t " being the factor determining what proportion of the total area of the normal distribution curve shall fall outside the limits of the ordinate representing the control limit.

From the equation $f(t) = 100t\sqrt{\dfrac{p(1-p)}{n}}$ an *ABAC* can readily be constructed, from which the height of the control limit above the mean can be read for a large range of values of " n " and " p." Such an *ABAC* is shown in Fig. 1.

The value of " t " at present adopted by the Post Office is 1.28, which, if the distribution of the percentage defective followed the normal curve, and the quality of service was standard, would assure that in the long run the limit would be exceeded by sampling fluctuations only once in ten times.

Pending further experience, an additional allowance or tolerance of one-tenth of the percentage defective is allowed beyond the figure arrived at from the *ABAC*. There is a further reason for giving this tolerance. It is obviously undesirable that a sample giving a percentage defective beyond the control limit by a very small amount—say 0.1 or 0.5 per cent.—should be the subject of special investigation. It would be better to limit such special action to samples which disclosed a service materially below standard. From an examination of the results of past investigation spread over a number of years, it was found that this additional factor would generally exclude cases where such special investigation had proved abortive, but would include all cases where trouble had been located.

Appraisement of the sample of service is quite easily undertaken locally. For example, assume exchange X observed 200 calls, of which 24, or 12 per cent., received the busy tone. The appraiser sets his rule at 9.1 per cent. defective (the All London average) and at 200 calls. Reading the middle scale of Fig. 1 gives 2.6. He adds to 9.1 per cent., his reading together with one-tenth of 9.1 per cent. making a total of 12.6 per cent. defective, which represents the upper permissible limit. His sample contains a lower proportion defective, and his exchange therefore demands no special investigation in respect of this item. All the items marked † in Tables I and II can be so treated.

Referring now to the second type of service criteria, which are concerned with speeds, it has long been an established practice for the monthly summaries of samples of manual exchange service to form the subject of short talks to the operators at the exchanges. Exchanges have been listed in order of merit, and every effort has been made to improve standards of operating by these and similar methods. Comparison between one exchange and another was necessary, although the number of calls in the samples varied between exchanges. The monthly average speeds of the same exchange also varied, owing not only to seasonal traffic effects and the staffing position—e.g., unforeseen heavy sick absences among the staff, etc.—but also to the size of the exchange and methods of distributing subscribers' calling signals and to the type of junction circuit to other exchanges. These conditions influence so considerably the speeds attained that they take command of the situation.

It will perhaps be of interest to consider a few of these conditions in detail. Figure 2 shows the total traffic handled at a typical exchange for each week of the year. The seasonal variations are well marked. April to June is a period of heavy telephone traffic. This is followed by a period of depression from July to September, after which traffic generally rises to the Christmas peak. The pre-bank-holiday traffic peaks and the post-bank-holiday depressions are well marked. During the year additional subscribers have been accepted, so that for the succeeding year the traffic will be correspondingly higher. The dotted line on the figure exhibits this tendency. In shape, the curve for the following year will follow approximately a similar trend; the bank-holiday characteristics and the early summer and late autumn periods always exhibit themselves.

These tendencies, together with a knowledge of the probable number of new subscribers to be expected in the future, and also the subscribers' probable rate of calling, will form the basis of a determination of the probable staff requirements for the coming year. The staff estimates must, of course, be made months ahead, and the staff be recruited and trained at the Post Office Operating Schools and placed in position in time to meet traffic needs. Provision must also be made for staff losses due, for instance, to resignations on account of marriage. The holidays of the staff will need to be planned reasonably early in the year, so that the number away simultaneously may be regulated to enable full advantage to be taken of the seasonal drop in traffic (July to mid-September). During the month of February the greatest incidence of staff sickness is to be expected.

Figure 3 shows the traffic passing through a typical exchange during each half-hour of the day on the busiest day of the week. Because a 24-hour service is given, it is possible so to order

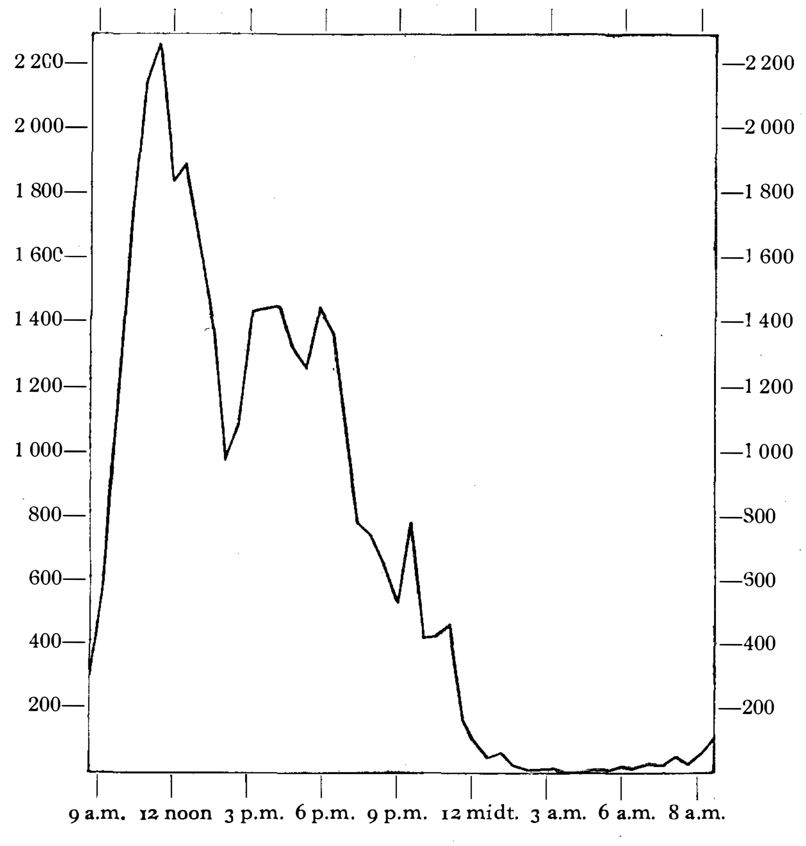

Fig. 3—Calls originated at a typical exchange during each half-hour of the day and night.

TABLE III

Incoming Analysis of Main Service Criteria

(*These figures are hypothetical and for explanatory purposes only*)

Manual Exchanges	Calls Incoming from :		Total Calls Observed	Percentage of Calls :					Percentage of C.C.I. Calls, No Tone at :	
					Not Completed Owing to :					
	Automatic Exchanges	Manual Exchanges		Completed	No Tone, Sub. Waited	Busy Tone	No Reply	Other Causes	20 Secs.	40 Secs.
A	133	201	334	78.5	4.5	13.0	1.2	1.2	3.8	0.8
B	503	326	829	79.3	1.0	7.7	2.2	2.3	16.0	2.2
C	142	94	236	91.3	1.0	4.9	2.8	1.9	7.1	1.9
D	474	258	732	80.9	5.7	6.8	1.9	3.2	10.8	3.5
E	332	255	587	74.9	2.6	11.1	2.3	1.3	14.6	1.6
F	173	189	368	81.9	3.2	9.5	1.8	2.8	9.7	0.6
…………	...	...	...	...	...	...	...	...	...	...
…………	...	...	...	...	...	...	...	...	...	...
…………	...	...	...	...	...	...	...	...	...	...
Total	14 982	12 361	27 343	81.2	2.7	9.1	2.2	2.1	11.3	1.5

Automatic Exchanges	Calls Incoming from :		Total Calls Observed	Percentage of Calls :				
					Not Completed Due to :			
	Automatic Exchanges	Manual Exchanges		Completed	No Tone, Sub. Waited	Busy Tone	No Reply	Other Causes
U	219	199	418	80.2	3.6	10.6	2.8	1.2
V	273	109	382	81.5	1.0	7.9	1.7	2.0
W	923	405	1 328	82.2	3.4	8.7	2.4	1.1
X	228	188	416	77.8	1.3	10.8	2.7	3.0
Y	181	138	319	86.9	1.2	16.0	3.1	2.2
Z	235	188	423	86.8	2.4	17.1	2.8	2.0
…………	...	...	...	...	...	...	...	...
…………	...	......	...	...	...	...	...	...
…………	...	...	...	...	...	...	...	...
Total	26 400	10 924	37 323	85.7	2.2	9.1	2.2	2.0
Grand Total	41 382	23 289	64 666	85.5	2.3	9.1	2.2	2.0

the hours of attendance of the staff that after allowing for meals the actual number of staff employed during each half-hour may be very closely related to the traffic. With these considerations in mind, it will perhaps be agreed that the effectiveness of our efforts to exercise some control over operating speeds will be somewhat influenced by our inability to control the volume of traffic to be handled at any moment. It is to be expected, therefore, that operating speeds will generally reflect periods of traffic pressure and average speeds will be statistically unstable.

It is becoming increasingly recognized that the quality of operating, so far as speeds are concerned, is quite inadequately measured by *average* speeds. The shape of the frequency distribution of these speeds is the real criterion of quality. For instance, it is of little satisfaction to a subscriber complaining of delay in getting attention from the exchange for some 2 minutes on an urgent call finally maturing after his client had left his office, to be told that on an *average*, calls are answered in no more than 10 seconds. What the subscriber desires is an assurance that ***all*** his calls will be answered within a reasonable maximum period, even if the average speed of attention is a little longer than at present.

It seems likely that the frequency distribution of speeds of answer would approximate to the normal, if the quality of operating approached ideal conditions and that poor quality operating of the same average speed would more nearly be represented by an asymmetrical curve with an extended tail in the region of slower speeds.

The problem of operating speeds had been met in the past by adopting two criteria: one was the average speed of answer, and the other the percentage of calls attended to in 10 seconds or less and in 20 seconds or less. It was the Administration's concern that certain figures should be very closely reached. If, judged on this basis, service was continuously poor at one exchange compared with the average All London figures, local effort was directed to improve the position.

It was, however, necessary to recognize that the figures relating to the samples taken from any one exchange were subject to sampling fluctuations, if wasted effort in searching for trouble which did not exist was to be avoided. The question immediately at issue was whether the same simple technique of appraisement already outlined could be applied here. In repeated sampling spread over a number of months, it was found that the percentages relating to calls answered in 10 seconds or less and 20 seconds or less were distributed in a form surprisingly near the normal curve. The same method of statistical control can therefore be applied. A typical example of procedure relating to speeds (marked‡ in Table II) is as follows. The All London figures in Table II are 86.2 per cent. and 96.3 per cent. of calls answered in 10 and 20 seconds, respectively. An exchange has taken 200 test calls. What are the permissible limits? Reading from the *ABAC*, we have

	For 10 secs.	For 20 secs.
Percentage defective ...	13.8	3.7
ABAC reading	3.2	1.7
10% tolerance	1.38	0.37
	18.4%	5.8%
Control limit for calls answered	81.6%	94.2%

If the percentage of calls answered within the respective periods did not fall below 81.6 and 94.2, no special investigation would be called for.

It is, perhaps, interesting to observe that the influences under control determining this particular quality of service—namely, speed—comprise not only operating effort and skill, but also staff provision. Indeed, here we have applied an acid test to the proper co-ordination of staff and traffic determined through the staff duty-rotas arranged during the previous week. In this case, therefore, we are establishing a criterion, not only of operating efficiency, but also of good exchange management as well.

There remain two other major items—namely, average time to disconnect and the percentage of operating irregularities. They are mentioned because they are typical of their class.

Dealing firstly with the time taken by the operator to disconnect, the logical way would, of course, be to treat the case on the same basis as the speeds of answer, but it was decided not to do this for the following reasons:—

(1) The shape of the distinction curves of speeds of clear (i.e., time taken by the operator to disconnect) are much more symmetrical than those of the speeds of answer, owing to our methods of operating, and more control is exhibited.

(2) Because of this nearer approach to the normal, it is considered that the expense and time involved in extracting speeds answered in 10 and 20 seconds are not worth while.

In order to apply the same fundamental procedure as before, it is necessary to specify an appropriate standard deviation, σ. This is calculated at infrequent intervals from the distribution of speeds of clear obtained from one of the monthly samples themselves. The value of σ so obtained is assumed to hold for the ensuing months. The permissible variation from the All London average for this item is $t\sigma/\sqrt{n}$, where t has the same value as before.

The remaining typical item—operating irregularities—presents considerable difficulty. Some of these items are shown in Table II, columns 43, 45 and 48. They appear to exhibit an almost complete absence of statistical control.

This condition is, however, brought about by our method of recording the irregularities. For instance, an incorrect repetition of the wanted number by the operator would give rise to a wrong number. This would be recorded as two operating irregularities—i.e., an incorrect repetition and also a wrong number. If the operator's repetition was corrected by the caller before the call was passed forward and the correct number was consequently obtained, only one operating irregularity would, of course, be recorded. There are a number of other similar conditions. Taking the problem as it stands—that is, without attempting to recast the basis of recording operating irregularities—it is clearly desirable that one method of setting up control limits should be adopted throughout. It has been decided, therefore, that the standard deviation shall be calculated from time to time and used as in the previous case, it being for the moment assumed that the distribution of the proportion of failures would follow more or less a normal distribution. It is anticipated that it may eventually be practicable to make some alteration, probably in recording methods, which will clarify the position.

In connection with the former of the two items—namely, speed of clear and operating irregularities—a second *ABAC* has been prepared. Its *A* and *C* scales refer respectively to the number of calls in the exchange sample and values of σ. The *B* scale shows the value of $t\sigma/\sqrt{n}$ by which the All London average shall be increased to form the control limit appropriate to the particular size of exchange sample under consideration.

From the foregoing review of Post Office procedure, it will be seen that it is easily possible to supply the local controlling staff with copies of the *ABAC*s, together with very brief but adequate instructions which will enable them to work out the upper control limits and appraise the service on one common basis, either as satisfactory or as calling for attention.

SAMPLES DISCLOSING SUB-STANDARD SERVICE

We now come to the position at which quality of service has been appraised. Unless effective steps can be taken to remedy weakness disclosed by the sampling system, the problem of control is not wholly solved. Fortunately, effective steps are readily practicable. They consist of an examination of the rough recording sheets, a scrutiny of the failures, the relative time in the progress of the call at which failure occurred, and its nature. From this information it is possible to deduce the likely origin of the trouble. Suspected equipment can be tested rapidly and automatically by machines designed to set up all the conditions met in practice, and, where a failure occurs, to cease operation, and give a visual indication to the attendant that faulty conditions have been located. If the need arises, test calls may be passed through items of plant suspected of weakness and their operations checked.

BEST SIZE OF SAMPLE

There is a point of interest regarding the *ABAC* of Fig. 1. One can read off the chart the reduction in control limits which could be achieved were the size of the sample increased.

Within the range of percentages defective met in telephone practice the best size of sample falls at about 400 calls. The Administration has now under consideration the desirability of revising the basis upon which the size of sample is determined, and it is anticipated that in the near future this will be between 400 and 500 calls. On the present basis, however, it may be of interest to know that we sample about 1 call in 4 000. With the development of the telephone system and a corresponding increase in the number of subscribers served from each exchange, it is probable that in the future not more than 1 call in 5 000 will be sampled after allowing for an increase in the size of sample.

Before leaving the subject of sampling telephone service, it might be mentioned that except on calls to numbers on the same exchange (and in London these are only a small percentage of the total), the quality of the service is determined by the originating exchange and also the called exchange. It is easily possible, however, to re-summarize the original data under the name of the called exchange. A typical summary is shown in Table III. The figures shown are again hypothetical. The same procedure as already outlined can be applied here.

In conclusion, one or two other matters may be mentioned. Through the use of statistical tools in controlling the quality of service there has grown an appreciation that quality of service in nearly all its phases is susceptible to mathematical analysis. It becomes possible to experiment in new arrangements of feeding traffic to the operating staff, and to determine with some precision the effect of such new arrangements. By this means it may be possible to create conditions which will result in frequency distributions of speeds of operating being of the best shape to meet public needs.

The present basis for the provision of operating staff is, broadly speaking, 200 valued calls per operator per hour. Valuation of the calls is made by timed observation of the various manipulations of the operator. Because calls arrive randomly—that is, the calls originating in unit time follow a Poisson distribution—an additional allowance of about 27 per cent. of the actual operating time is added. This allowance recognizes that on occasion calls will originate in bunches, and it is designed to ensure an average speed of answer to a subscriber's calling signal of 5 seconds.

At first glance there seem to be two problems here : (*a*) the shape of the frequency distribution of speeds, and (*b*) the random order in which calls reach the exchange. Can we evolve some method of distributing calls only to disengaged staff, and so submit the difficulties arising from both conditions to a greater measure of control ?

It is along these lines that many of us are now thinking. There will then remain the human problems, which are, no doubt, the more difficult.