Proceedings of the I·R·E

DECEMBER 1943

VOLUME 31 NUMBER 12

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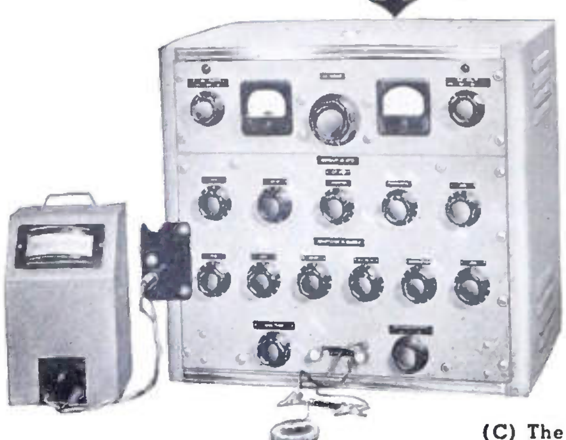
Institute of Radio Engineers

DESIGNS For WAR.... FILTERS

(A) Filter performance is dependent upon three major factors, basic design... Q of coil and capacitor elements... and precision of adjustment. The superiority of UTC products in this field has been effected through many years of research and development on core materials and measuring apparatus. We illustrate below a typical filter formula and some of the UTC apparatus used to determine quantitative and qualitative values:

$$\frac{(LC\pi^2 f_\infty - 1)\left(\frac{1}{Q^2} + 1 - \left(\frac{f_\infty}{f}\right)^2\right)}{\frac{1}{Q^2} + \left(1 - \left(\frac{f_\infty}{f}\right)^2\right)^2} = U_m \text{ (ATTENUATION CONSTANT)}$$



(B) The UTC inductance bridge is capable of four digit accuracy and covers a range from extremely low values to over 100 Hys. The effective resistance and inductance values are direct reading, eliminating the possibility of error in conversion.

(C) The UTC oscillator is direct reading, where the frequency desired is set as in a four digit decade box, and is accurate within 1 cycle at 1,000 cycles. The range is 10 cycles to 100 kc. Accuracy of this type is essential with filters having sharp attenuation characteristics. This instrument is augmented by a UTC harmonic analyzer for the output measuring device.

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Proceedings *of the* I·R·E

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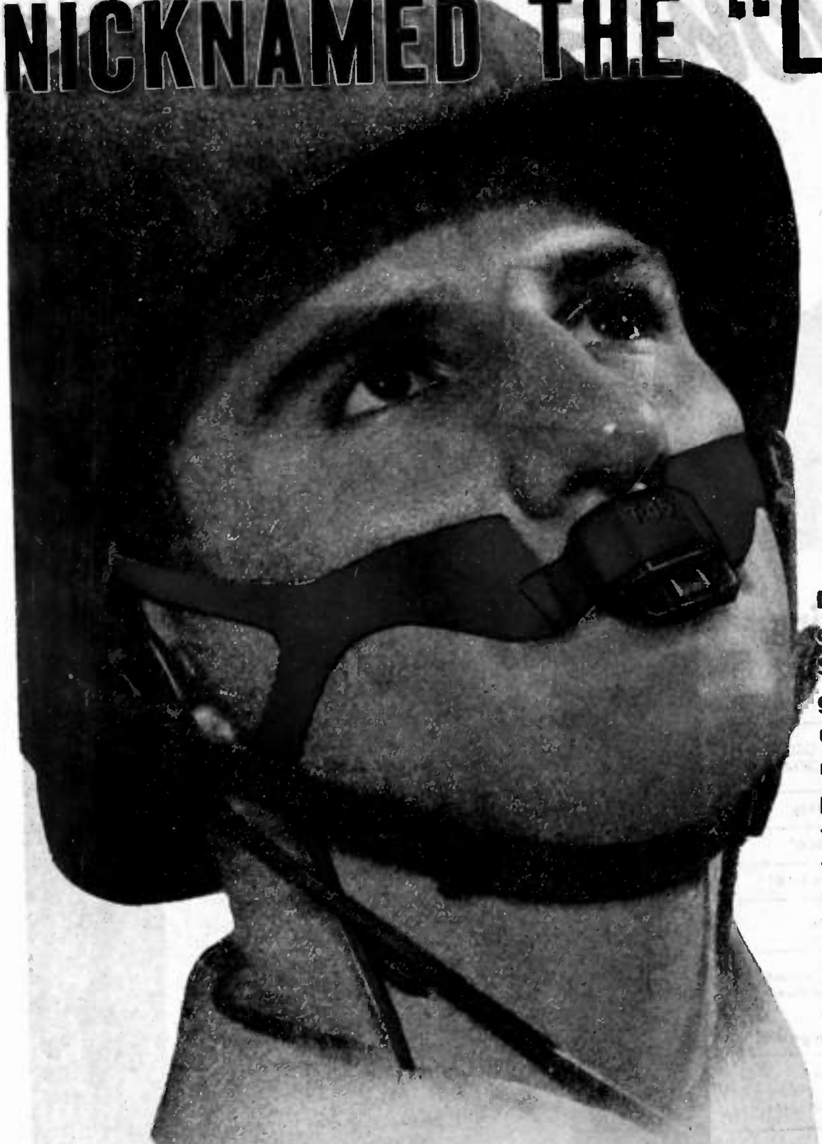

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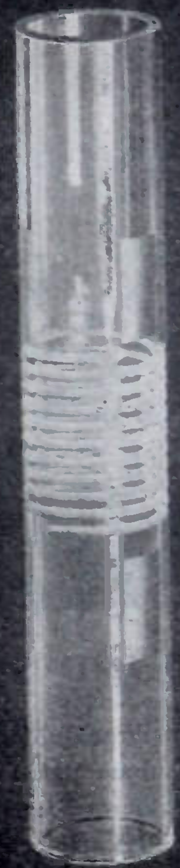

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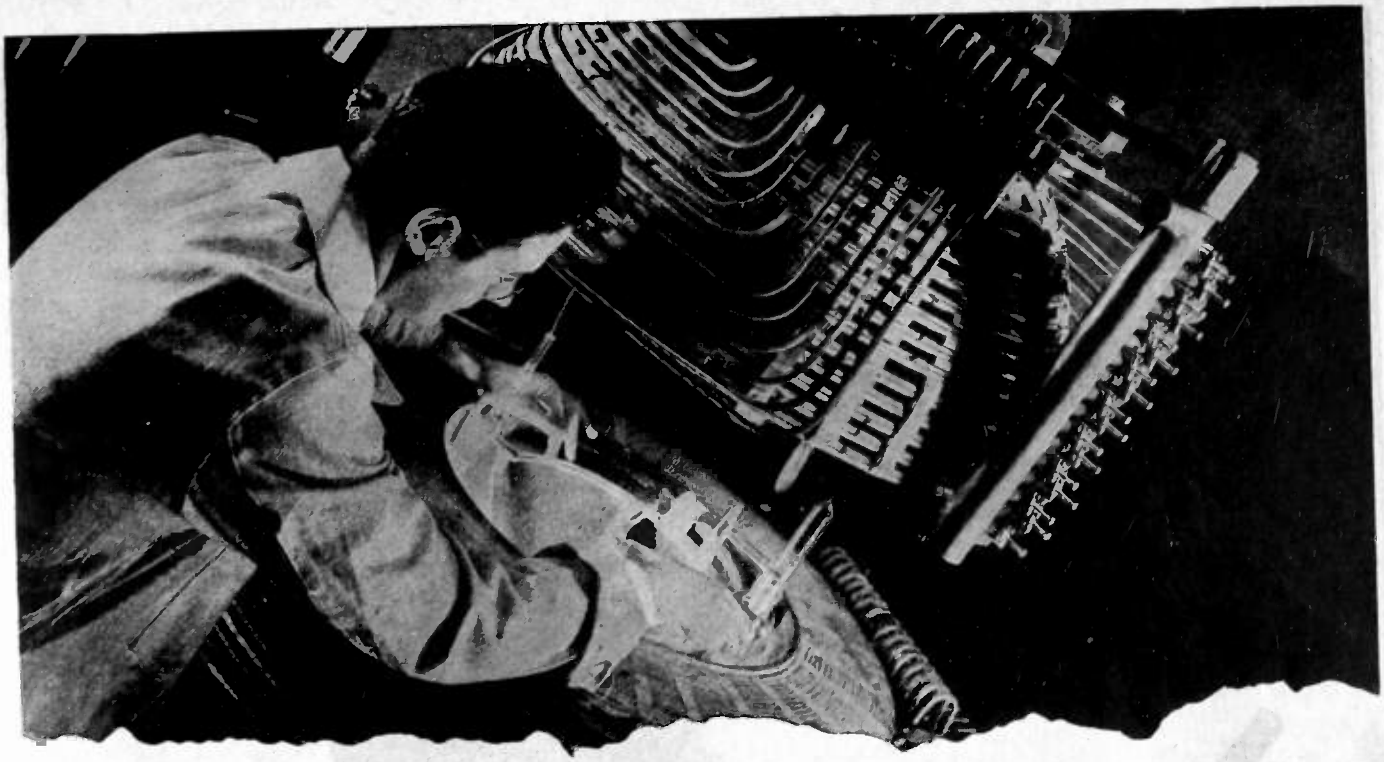

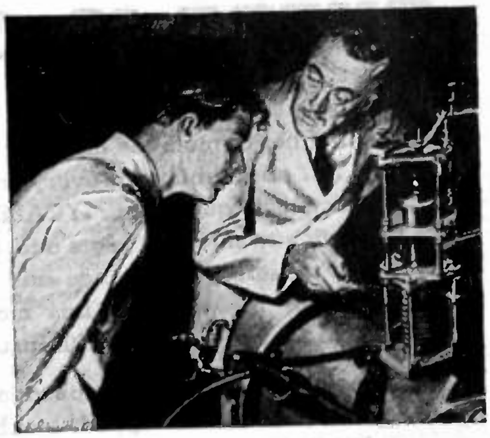

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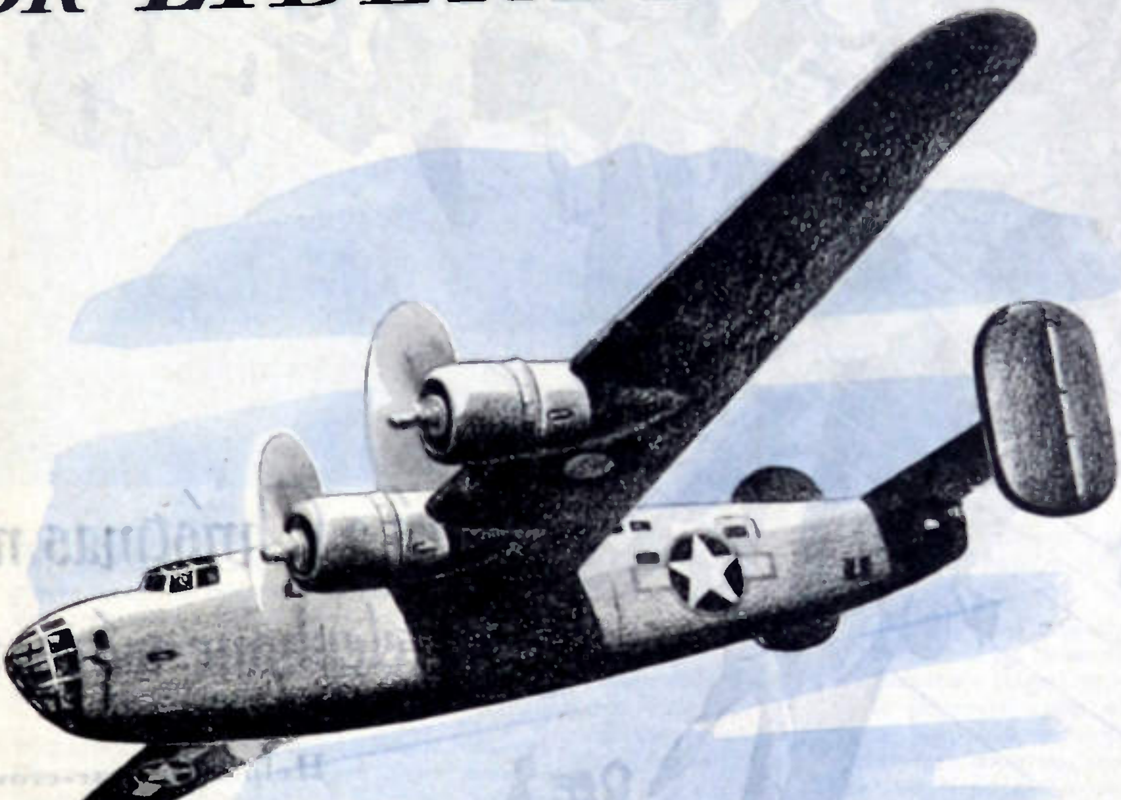

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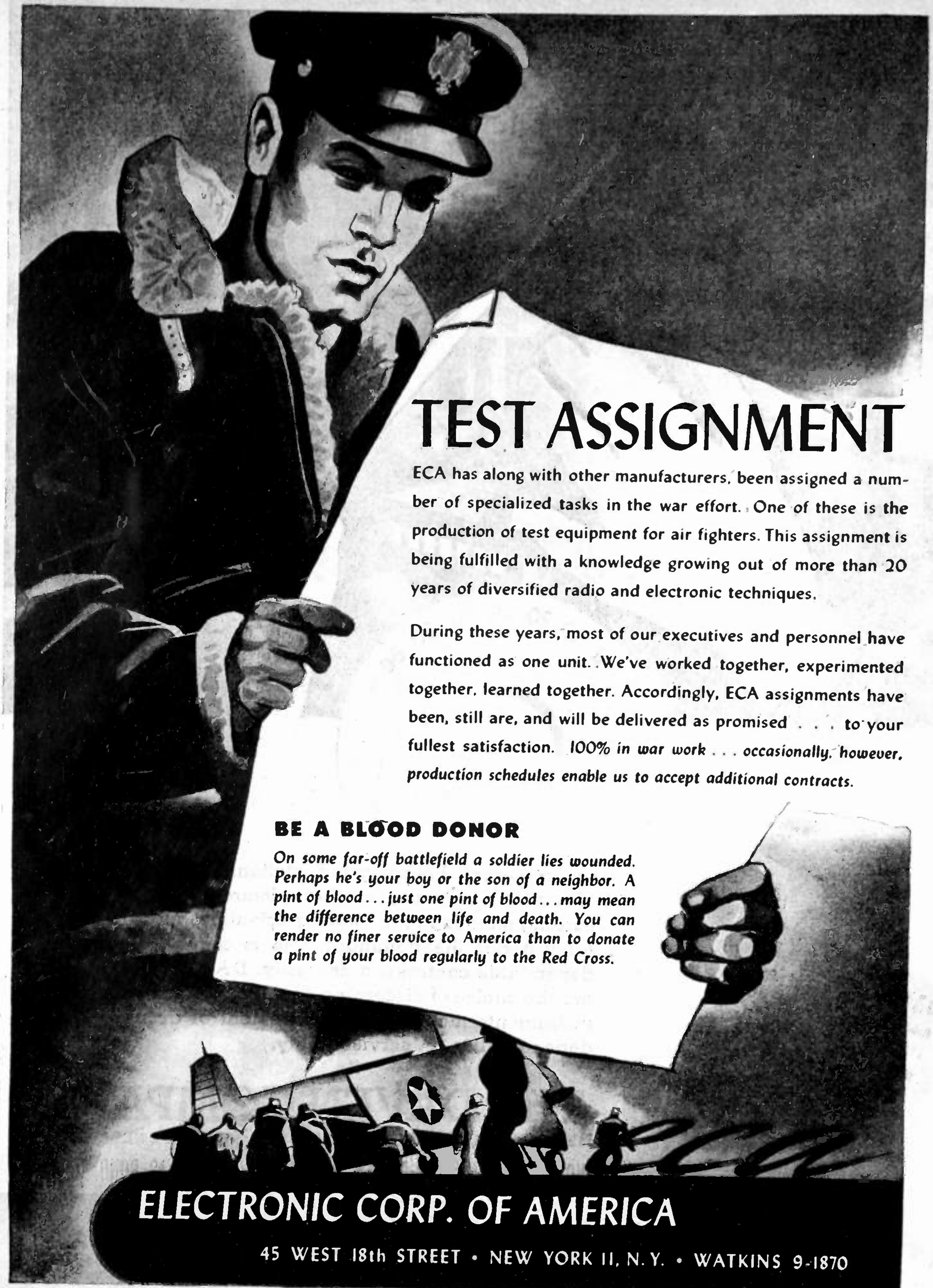
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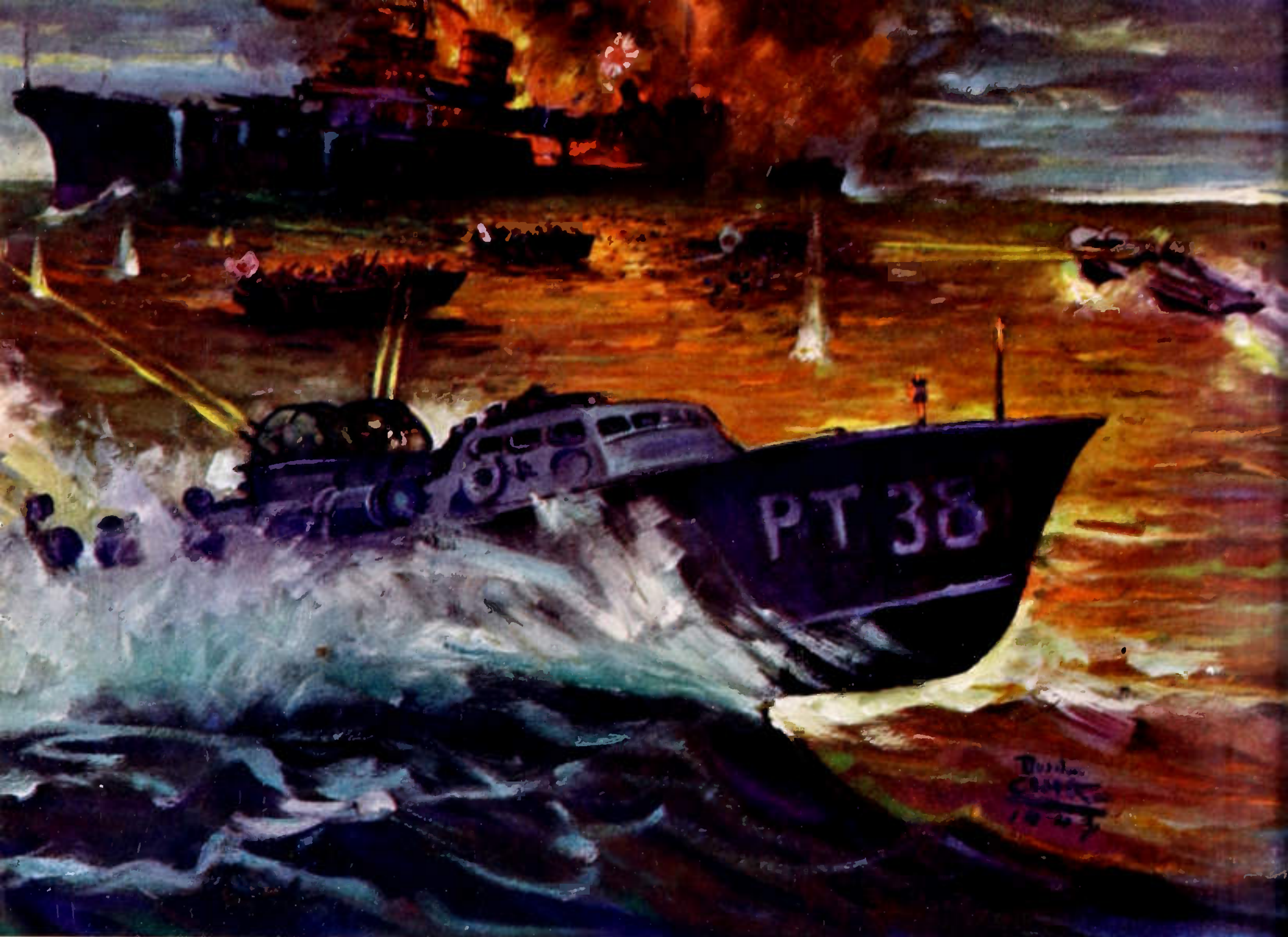

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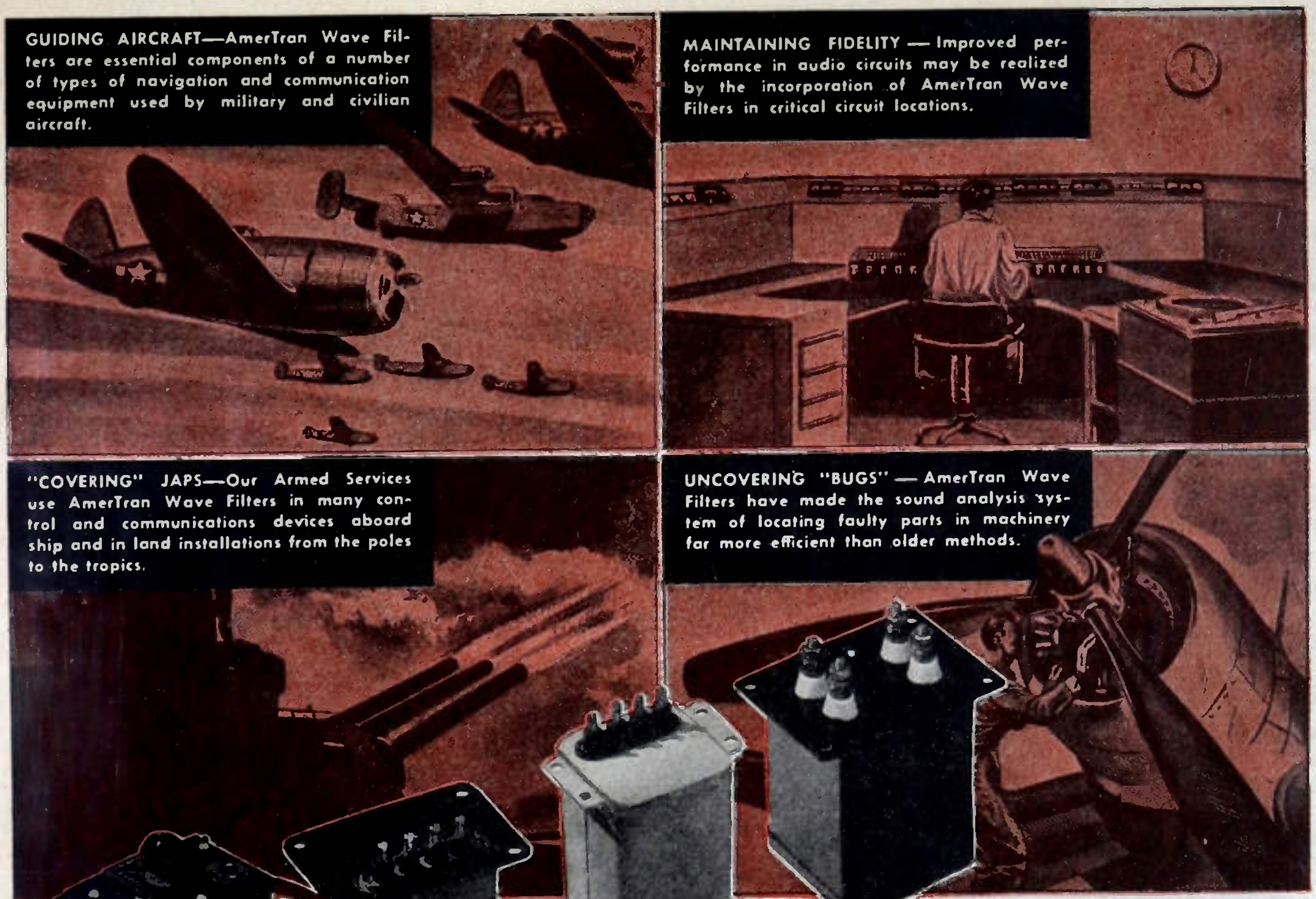
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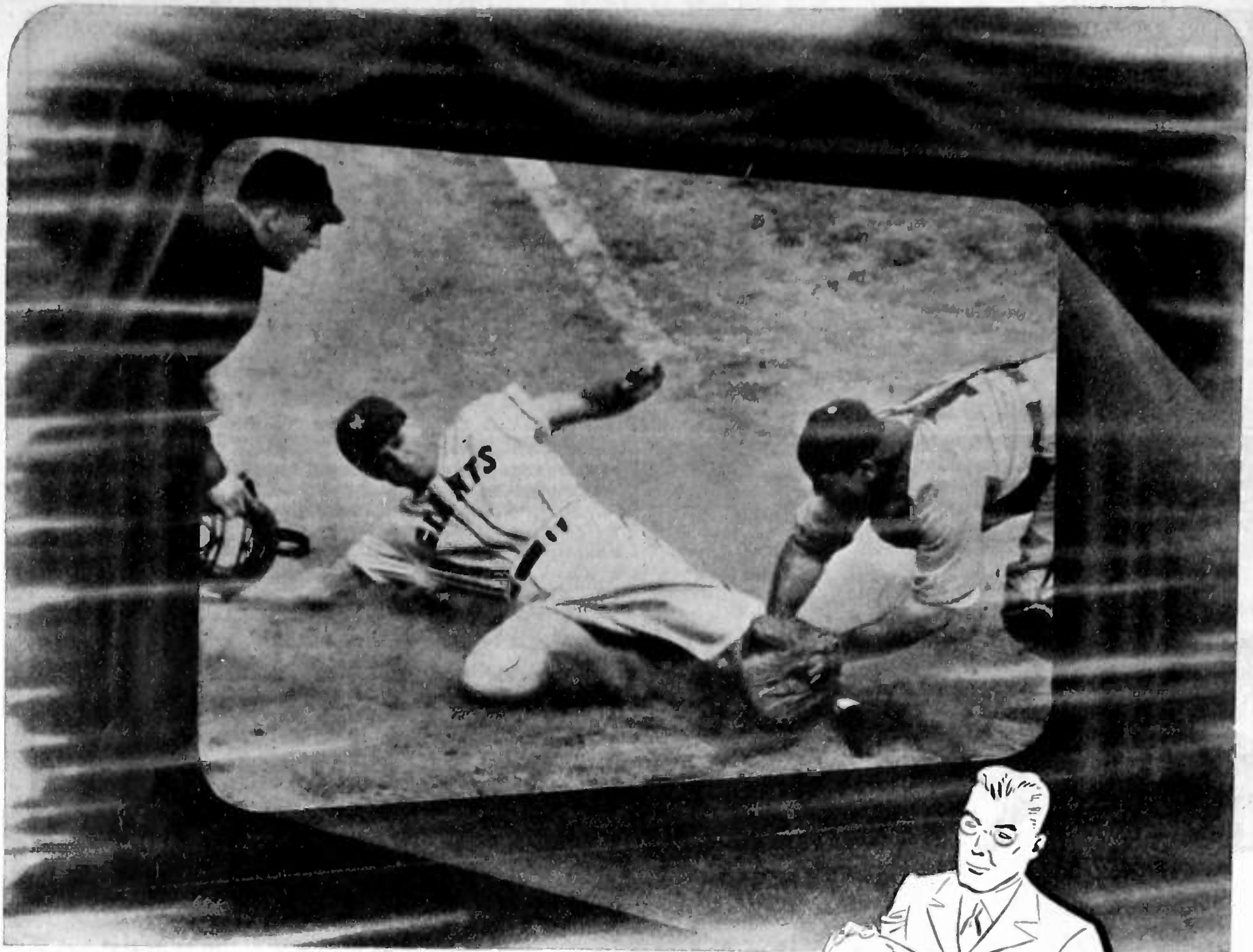

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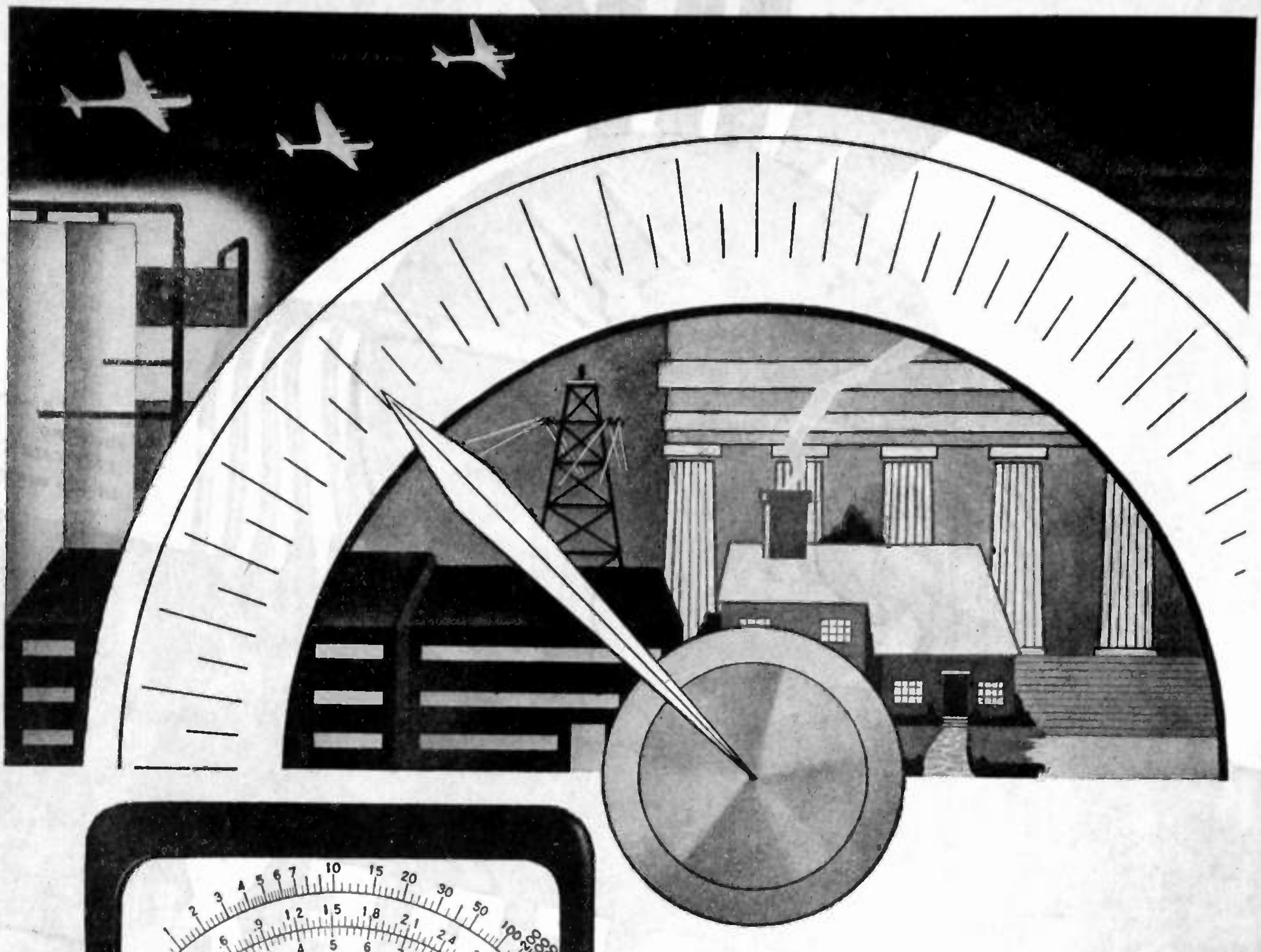

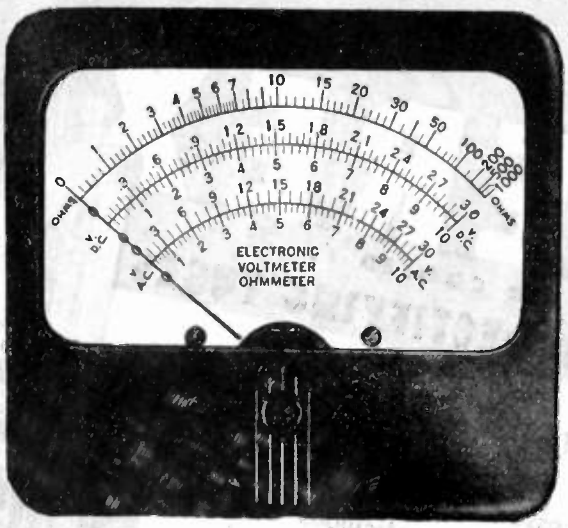

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Commercial and engineering activities are so closely interwoven and so dependent on each other in the radio-and-electronic field that the viewpoints of the business leaders of the industry are of major interest and importance to the engineers. The President of the National Union Radio Corporation, upon invitation, has prepared the following "guest editorial" which, presented in the form in which it was received, will doubtless encourage and stimulate the readers of the PROCEEDINGS to even greater efforts toward that victory which the Allied Nations will undoubtedly achieve.

The Editor

Radio-and-Electronic Engineering Contributions to Victory

S. W. Muldowny

I like the definition of the word engineer which states: "one who carried through an undertaking by skill or astuteness."

Today's undertaking is Victory and the skill and astuteness of American radio engineers is written in the continued advance of our armed forces on land, sea, and in the air. The genius which has made radio such a mighty weapon of offense and defense was certainly latent in our radio engineers before Pearl Harbor, but the inspiration of an aroused nation at war has brought these men to untold heights in their contributions to our war machine. Radio engineering has come of age. Unencumbered by economic pressures which tend to limit the rapidity of technical advances in a normal commercial market, our radio engineers have turned on brain power and won the admiration of our allies and the respect of our enemies. To say that the future of the radio engineer is bright is a conservative statement. If the profession has grown in stature as it has under the duress of emergency conditions, certainly it is the "white hope" toward which we look.

The radio industry recognizes the debt it owes to the skill and astuteness of its engineers. The radio industry looks forward to, and expects the application of this same brain power to peacetime activities. We believe in our engineers. We know that having climbed to unbelievable pinnacles in the Victory undertaking they will rise to even greater heights in dreaming through to practical solutions of commercial needs in a rebuilding of our war-torn world. I can view a thousand times the wonders which I see worked in our laboratories today and still have a feeling of awe at the ingenuity of our engineers from whose brains these miracles have come.

The magic of the word electronic has swept the nation, and some have felt the art may have been oversold. Truthfully, however, we stand but on the threshold of changes in living, in travel, in manufacturing methods, so vast that the prospect is breath taking. It is the electron tube and electronic principles which form the master key to unlock many doorways to a new kind of civilization. The radio-electronic engineer holds this key and thus holds today's power to destroy our enemies, tomorrow's power to bring us the fullest life ever known to mankind.

G. W. Pierce

An eminent member of the Institute is the recipient of an additional honor. Professor Pierce received the Franklin Medal of the Franklin Institute, on April 21, 1943. He was a vice president of The Institute of Radio Engineers in 1915, a member of its Board of Directors from 1915 to 1921, and its president in 1918 and 1919. He was awarded its Medal of Honor in 1929.

The latest recognition of his scientific accomplishments ". . . is awarded annually . . . to those workers in physical science or technology . . . whose efforts . . . have done most to advance a knowledge of physical science or its applications." The medal citation, as read by Dr. Henry Butler Allen, Director of the Franklin Institute at the presentation, stated that the award was "in recognition of his outstanding inventions, his theoretical and experimental contributions in the field of electric communication, and his inspiring influence as a great teacher."

In the presentation speech, Dr. Allen mentioned numerous high lights in Professor Pierce's career. Among these were the following. Professor Pierce has "held the chairs of Rumford professor of physics and Gordon McKay professor of communication engineering at Harvard University." He is "the author of thirty or forty valuable papers in physics and electrical communication, and is the author of two standard books, 'Principles of Wireless Telegraphy' (1910) and 'Electric Oscillations and Electric Waves' (1920)." In studying in the early days of radio communication the action of crystal detectors, he "began a series of systematic investigations to find what other materials possessed the same rectifying properties, and also to discover to what this property is due." He showed that certain then-current theories were inadequate.

He later developed an advanced form of mercury-vapor detector and amplifier, and devised methods for its use "equivalent to the present use of the mercury tube employed to record variable-density sound-on-film." During World War I, he investigated devices for locating submerged submarines, and produced an ingenious and novel form of electrical compensator for use with them.

Turning his attention to piezoelectric oscillators, Professor Pierce "began the study of suitable oscillator circuits for use with quartz crystals. He produced three fundamental types of circuit employing one tube and one set of electrodes on the crystal" which constituted a "considerable simplification" of prior practice. He also "recognized that the phenomenon of magnetostriction could be used to control the frequency of oscillators by mechanical resonance . . ." and "devised a number of oscillator circuits for making use of this property."

In an extemporaneous and graceful speech of acceptance of the medal, Professor Pierce described a novel and interesting series of experiments on an analysis of the "songs" of insects. A parabolic horn reflector was provided with a piezoelectric sound detector at its focus. The supersonic energy radiated by the "singing" insect (e.g., a cricket) was then electrically analyzed as to frequency, and, after heterodyning, converted into audible sound from a loudspeaker and also recorded photographically. The details of the experiments and their results present an unusual combination of electrical skill and entomological aptitude.

In concluding his remarks, Dr. Allen said in part: "Professor Pierce's influence has been impressed widely upon the field of electric communication, for the majority of the men who are engaged upon important work in radio engineering all over the world have studied in his classes. . . . Pierce recognized that the art of communication was neither physics nor engineering, but a combination of the two."

Analysis and Characteristics of Vacuum-Tube Thyratron Phase-Control Circuit*

SAMUEL C. CORONITI†, ASSOCIATE, I.R.E.

Summary—An explanation is presented of a vacuum-tube phase-control circuit of the *RC* type. It is shown that the effective phase angle between the plate- and grid-to-cathode voltages is influenced by the direct rectified current flowing through the control vacuum tube. The effect of the thyratron grid resistance on the stability of operation is discussed. Curves are given to illustrate the degree of control by various types of tubes.

BY SUBSTITUTING a vacuum tube for the resistance in the *RC* component of a phase-control circuit such as Fig. 1, a small grid voltage can be made to control effectively large electrical power. This paper gives an analysis and some operational characteristics of this type of circuit.

In the circuit of Fig. 1, the magnitude of the load current I_L is a function of the phase angle between the plate-cathode and grid-cathode voltage of the thyratron tube.[1–4] Before ionization of the gas within the tube, their magnitudes and phase angles are determined from the alternating-current steady-state conditions of the circuit. In the vacuum-tube—thyratron phase-control circuit, the magnitude of the thyratron grid voltage and its effective phase angle with respect to the plate voltage is determined not only by the alternating-current steady-state conditions, but also by the transient response of the circuit.

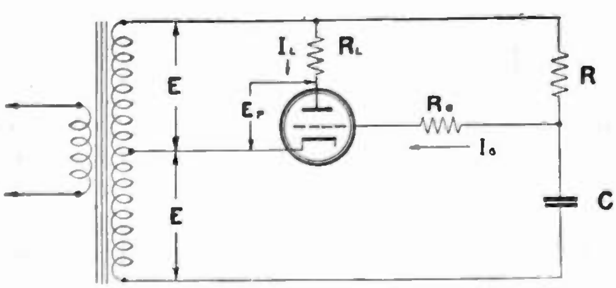

Fig. 1—Basic phase-control circuit.

Fig. 2 is a schematic diagram in which the vacuum tube T_2 replaces the resistance R of Fig. 1. The plate-to-cathode current I_3 at any instant defines its value of resistance, the value of which can be varied from several hundred ohms to several megohms. The operation of this circuit differs in two respects from the circuit of Fig. 1. First, the vacuum-tube resistance is variable for the cycle of operation, and second, current flows through the vacuum tube only during the period of the cycle when the plate is at a positive voltage with respect to its cathode; that is, T_2 behaves as a half-wave rectifier.

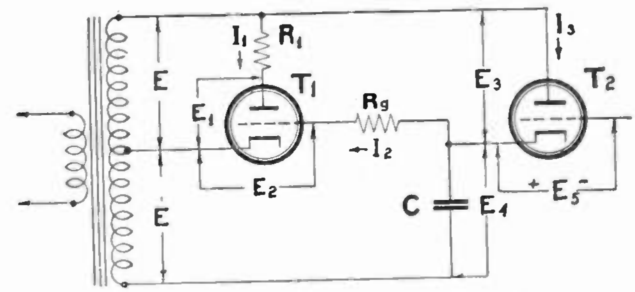

Fig. 2—Vacuum-tube phase-control circuit.

The internal resistance of the T_2 at any time is a function of its grid-bias voltage E_5 and the instantaneous value of the plate-to-cathode voltage E_3. The instantaneous plate voltage E_3 is equal to the secondary voltage $2E$ minus the instantaneous voltage E_4, which is equal to the algebraic sum of the direct and alternating voltages across the capacitor C. The rectification action of tube T_2 raises the direct-current potential of capacitor C to some positive value, the magnitude of which is determined by the internal resistance of the tube, and by the nonconducting period of the thyratron. Since T_2 has a finite resistance during the period that its plate voltage is positive with respect to the cathode, an alternating voltage exists across C. For all practical considerations it can be considered to be constant and to be equal to the static resistance because in the application of the circuit the grid bias E_5 is much greater than recommended values given by e_p-i_p characteristics of the vacuum tubes. For instance, when a 6C5 is used, the value of E_5 is -25 volts, and E_3 is equal to 250 volts.

During the nonconducting period of the thyratron tube T_1 its grid-to-cathode voltage will also be the algebraic sum of the direct and alternating voltages.

Fig. 3 is a graphical representation of the plate and grid voltage of the thyratron tube T_1. For the sake of simplicity, it is assumed that there is no time lag between the application of the switch and the magnitude of the voltage E; that is, when the voltage is applied to the circuit, it has an initial value of zero and progresses toward positive values. In the interval *oa*, the plate and grid voltage of T_1 varies as shown by curves E_1 and $E_{2\,a.c.}$, whereas the direct voltage on the grid due to the rectification action of T_2 rises exponentially as shown by curve $E_{2\,d.c.}$. When the algebraic sum of these

* Decimal classification: R140×R131. Original manuscript received by the Institute, October 19, 1942; revised manuscript received, May 15, 1943. Work done at AgfaAnsco, Binghampton, New York.

† 145 Lake View Avenue, Cambridge, Massachusetts.

[1] W. J. Nottingham, "Characteristics of small grid-controlled hot cathode mercury arcs or thyratrons," *Jour. Frank Inst.*, vol. 211, pp. 271–301; March, 1931.

[2] A. W. Hull, "Hot cathode thyratrons," *Gen. Elec. Rev.*, vol. 37, pp. 390–399; July, 1929.

[3] H. J. Reich, "Theory and Applications of Electron Tubes," McGraw-Hill Book Co., New York, N. Y., 1939, pp. 449–452.

[4] M. M. Morack, "Voltage impulses for thyratron grid control," *Gen. Elec. Rev.*, vol. 37, pp. 288–295; June, 1934.

voltages is equal to or greater than the critical grid voltage E_c of the thyratron, the gas within it ionizes, resulting in a flow of current I_1 (Fig. 2). For instance, at $\omega t=\omega t_1$ the total grid voltage is represented by line $ae=ac-ad=E_c$ where a, c, d, e are points as shown on the volt-ωt diagram of Fig. 3.

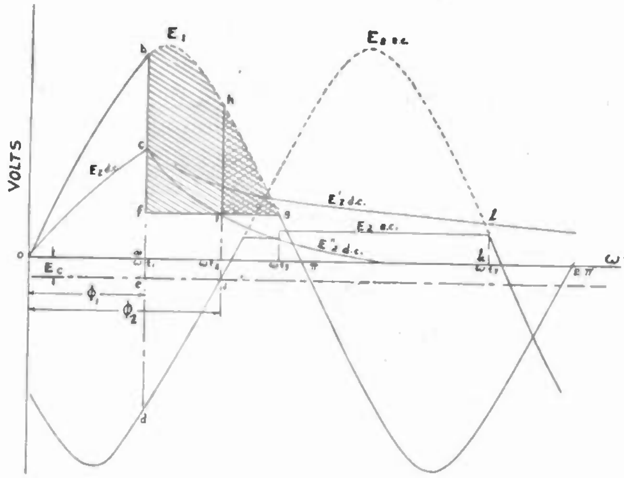

Fig. 3—Graphical operation of circuit of Fig. 2.

The shaded area bfg is proportional to the direct current flowing through the resistance R_1. The effective phase angle between the plate and the grid is denoted by Φ_1. The voltage E_1 across the plate will be reduced to the internal voltage drop of the thyratron. Now without rectification, complete ionization within the thyratron tube occurs when the grid-voltage curve $E_{2\,a.c.}$ intersects the critical-voltage curve E_c at points j or $\omega t=\omega t_2$. In this instant the load current I_1 is proportional to the crosshatched area kig. The phase angle is denoted by Φ_2. Hence, the presence of rectification has the effect of shifting the zero axis of the alternating grid voltage, $E_{2a.c.}$, by some positive value equal to the ordinate alternating current. Stating it differently, the effective phase angle between the grid and plate voltage is decreased from angle Φ_2 to angle Φ_1. The effective phase angle is definitely a function of the static resistance of T_2 for a given value of E_5 and E_3, and of the value of the capacitor C. Increasing or decreasing the negative value of the grid voltage E_5 increases or decreases the phase angle Φ_2, and decreases or increases the direct voltage on the capacitor C. That is, the effective phase angle Φ_1 can be increased or decreased by increasing or decreasing the negative bias value of E_5, or by decreasing or increasing the value of the capacitor C.

The plate current I_1 and the grid current I_2 flow through the thyratron for different periods of the cycle. The plate current is sustained as long as the plate-to-cathode voltage E_1 has a value equal to or greater than the ionization potential of the gas. Hence, it flows during the interval ωt_3-ωt_1. The grid current flows as long as the effective grid voltage E_2 is greater than the value necessary to sustain the ionization of the gas within the grid-cathode region of the tube. The alternating-current component of the grid voltage acquires the shape shown by the solid line $E_{2\,a.c.}$. This voltage is not to be confused with the grid voltage E_2. The sudden jump of $E_{2\,a.c.}$ at ωt_3 is attributed to the decrease of conduction between the grid-cathode space caused by the collapse of ionization between the grid and plate region.

So far the effect of the grid resistance R_g on the behavior of T_1 has been neglected.

During the period oa the condenser C in Fig. 3 accumulates a positive charge. At $\omega t=\omega t_1$, the gas within the thyratron ionizes, thereby providing a leakage path for the charge on the capacitor from the grid to the cathode within the thyratron tube. The accumulated charge on the condenser will leak off through the grid resistance, the rate of leakage depending on the value of R_g and C. The charge leaks off during the interval ωt_5-ωt_1.

Suppose that the values of R_g and C are such that the charge on the condenser decreases as shown by curve $E'_{2\,d.c.}$. At ωt_5 the direct-current potential across the condenser is given by the ordinate kl, and it remains at this value throughout the balance of the period of the cycle of E_1. Since this value is positive, the thyratron will conduct when its plate potential acquires an extremely small positive potential very shortly after $\omega t=2\pi$. Half-wave rectification results and no phase control is accomplished.

In order to have effective phase control, it is necessary to dissipate completely the charge on C during the interval ωt_5-ωt_1. This requirement can be satisfied by reducing the value of R_g or C. It is preferable to decrease R_g. If these values are chosen such that the discharge occurs along the curve $E_2''{}_{d.c.}$, effective phase control is obtained. It can be stated that when the time constant R_gC is greater than period of grid conduction, the thyratron behaves as a half-wave rectifier and the circuit loses its phase-control characteristic, and when the time constant R_gC is less than the period of conduction, the circuit behaves as a phase-control circuit.

Using a RCA 2051 for T_1 and a 6C5 for T_2, the oscillogram traces shown in Fig. 4 were taken for two values

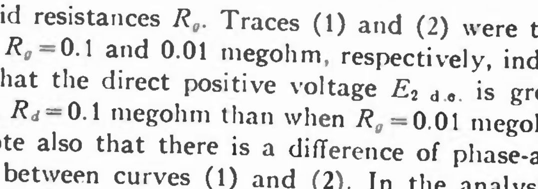

Fig. 4—Trace of oscillograms of the effect of R_g.
--------- Curve 1, $R_g=0.1$ megohm
—·—·- Curve 2, $R_g=0.01$ megohm

of grid resistances R_g. Traces (1) and (2) were taken with $R_g=0.1$ and 0.01 megohm, respectively, indicating that the direct positive voltage $E_{2\,d.c.}$ is greater when $R_d=0.1$ megohm than when $R_g=0.01$ megohm.

Note also that there is a difference of phase-angle shift between curves (1) and (2). In the analysis of Fig. 3, it was assumed that the initial charge on the condenser C was zero. That it is not so is evident from Fig. 4. During the conduction period from grid to cathode of T_1 less charge leaks off for the case of $R_g=0.1$

megohm than for $R_g=0.01$ megohm. Hence, when the cycle of operation repeats, the direct voltage across C will be greater for $R_g=0.1$ megohm than for $R_g=0.01$ megohm.

The behavior of the grid current I_2 is shown in Fig. 5 which is an oscillographic record of the potential across the grid resistance R_g. The high side of the cathode-ray oscilloscope was connected on the condenser side of the grid resistor, and the ground was connected at the grid of the thyratron. The increase in voltage across the resistance R_g from point O to A can be attributed to the flow of very small grid current before total ionization results in the gas. At the instant ionization occurs, the voltage drop across R_g decreases to some negative value, as shown by point C, and then surges back to some positive value, as shown by point D. This can be explained by the fact that at the instant of ionization a large charging current flows from plate to grid to capacitor C charging the capacitor by an amount equal to the difference of direct-current potential across C and the potential of plate T_1 at the instant of ionization. The charging time is approximately equal to R_gC or 1×10^{-5} second. From points D to E, the charge on the capacitor is dissipated, the discharge path being R_g and through the lower half of the secondary winding of the transformer. The time constant for discharge is large because of the large inductance. Point B indicates the collapse of ionization between the plate and the grid. The oscillations shown between points B and F can be caused by the behavior of positive ions within the grid-cathode space[6] of the T_1. The frequency of oscillation was approximately 5000 cycles per second. By increasing the magnitude of R_g, the

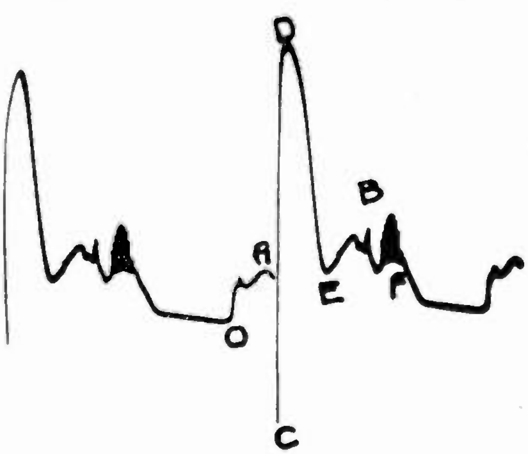

Fig. 5—Oscillogram of grid current I_2.
$T=6C5$ $\quad I_1=430$ milliamperes
$C=0.001$ microfarad $\quad I_2=75$ microamperes
$R_g=10{,}000$ megohms $\quad E_b=18.6$

amplitude and the duration of oscillations were decreased. By increasing the value of capacitor C the amplitude and frequency of oscillation decreased but the duration increased.

Fig. 6 shows experimental curves representing the relationship of load current I_1 and grid voltage E_b for values of C equal to 0.001, 0.01, 0.1, and 1 microfarad.

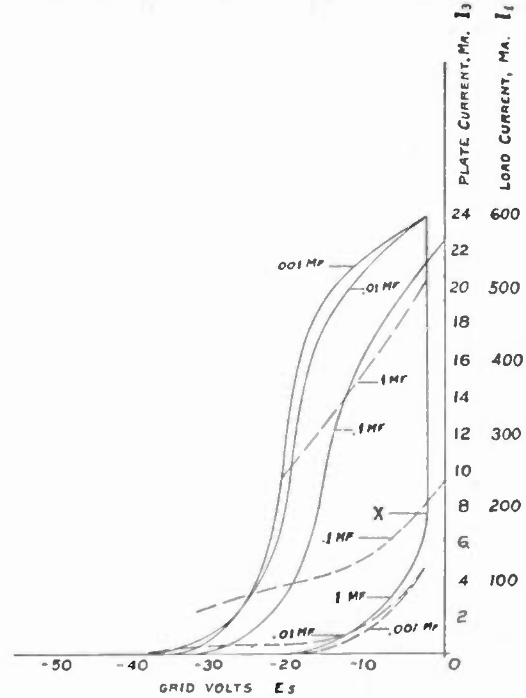

Fig. 6—Effect of capacitance on load current and plate current. 6C5 curves. $R_g=10{,}000$ megohms.
Plate-current curves — — —
Load-current curves ———
Note: The thyratron fired continuously at X.

T_1 and T_2 were a FG57 thyratron and a 6C5. R_1 and R_g were 187 ohms and 10,000 ohms, respectively. For a given value of capacitance C the load current increases with a decrease of negative bias E_b on the grid of T_2. Its internal resistance is decreased, thereby decreasing the phase angle between the grid and plate of T_1. As a result, the T_1 will conduct over a long period of time, since the conduction period of the thyratron is inversely proportional to the phase angle.

For a value of C equal to 1 microfarad, the load current I_1 changed abruptly from 165 to 600 milliamperes (maximum current) when E_b was equal to -2.5 volts. This indicates that the capacitor C was raised to a positive direct-current potential which was unable to decrease during the grid-conduction period to a value lower than the ionization potential of the gas within T_1. The circuit, as a result, behaved as a half-wave rectifier and lost its phase-control characteristic. Upon increasing the value of R_g, the circuit behaves as a half-wave rectifier for larger values of negative grid voltages E_b. For instance, for $R_g=0.1$ megohm, the value of E_b was -28 volts, and for $R_g=1$ megohm, the bias required to affect any measure of control is -282 volts which is much greater than value ordinarily used in practice.

As the value R_g is increased, the value of C, at which T_1 behaves as a half-wave rectifier, decreases. Stating it in another way, the value of the negative voltage E_b must be increased in order to have phase control. For instance, for $R_g=1$ megohm and $C=0.1$ microfarad, no

[6] W. G. Shepherd, "Deionization considerations in a harmonic generator employing a gas-tube switch," Proc. I.R.E., vol. 31, pp. 66–74; February, 1943.

phase control was obtained for a value of E_5 less than -25.4 volts. For $R_g=10$ megohms and $C=0.01$ microfarad, no phase control was obtained for values E_5 less than -25.9 volts.

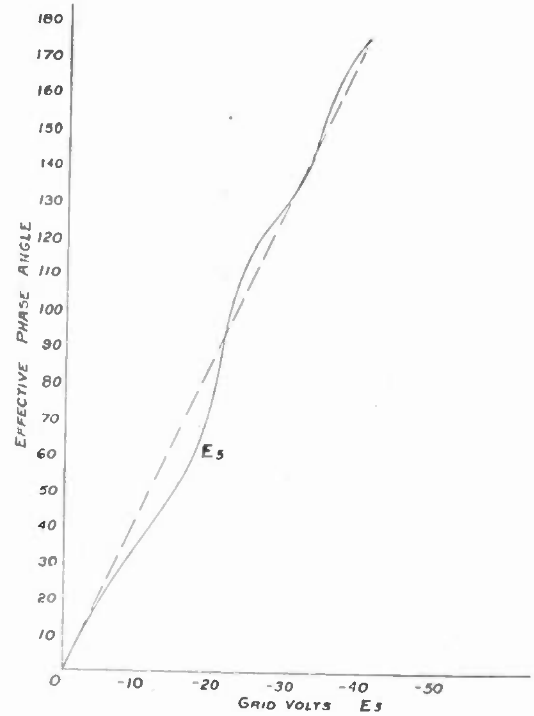

Fig. 7—Phase angle versus E_5.
$\phi=2\tan^{-1}R\omega C$
$C=0.01$ microfarad
$f=60$ cycles

In the basic phase-control circuit, Fig. 1, the thyratron cannot be made to behave as a half-wave rectifier for any combination of R_g and C_1. The curvature of the E_5-I_3 curve is dependent on the value of C because the value of I_3 increases as the value of C increases.[6]

The relationship between the effective phase angle and negative-grid bias E_5 is shown in Fig. 7 as the solid curve. T_1 and T_2 were FG57 and 6C5 respectively, R_g was 10^4 ohms, and $C=0.01$ microfarad. The relationship is approximately linear; especially is it so for phase shifts from 90 to 180 degrees. The author has used this circuit for control purposes, and has found that effective control can be obtained by varying the phase from 90 to 180 degrees.

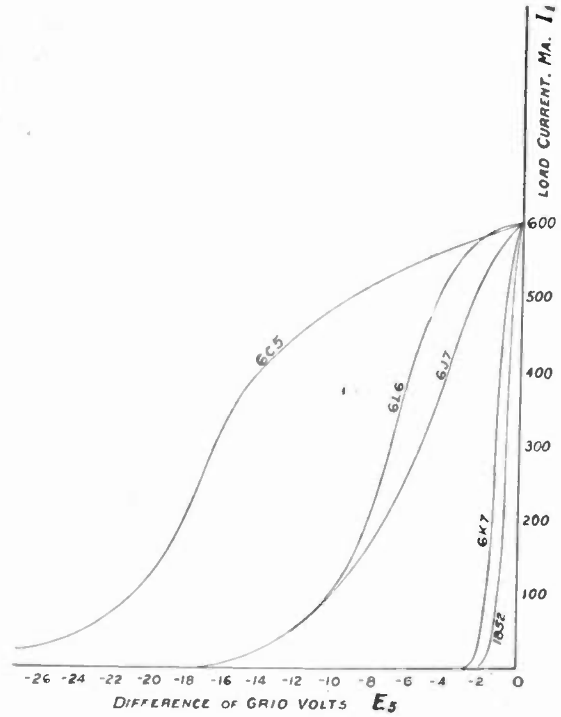

Fig. 8—Effect of E_gI_g characteristics on load current.
$R_g=0.1$ megohm
$C=0.01$ microfarad

It is obvious that the degree of control can be varied by careful selection of vacuum tube T_2. The effect of different tubes is shown graphically in Fig. 8. Note that the abscissa is the "difference of grid voltage." The operating grid bias is determined by the value of capacitor C. For triodes, this voltage is approximately -25 volts, and for pentodes it varies from -67.5 to -90 volts. The screen grids of the pentodes were connected through a proper resistance to the plate.

[6] J. Millman and S. Seely, "Electronics," McGraw-Hill Book Co., New York, N. Y., 1941, p. 457.

Corrections

It has been brought to the attention of the author, Frederick Emmons Terman, that an error appears in his book *Radio Engineers Handbook*, which has been recently published. This error is in equation (117) which appears on page 216 of the *Handbook*. This particular section of the *Handbook* was reprinted in the April, May, and June, 1943, issues of the PROCEEDINGS under the title of "Network Theory, Filters, and Equalizers." The error in question appears on page 234 of the May, 1943, issue of the PROCEEDINGS in equation (38). In place of reading

$$R_1 = R_I\left(\frac{\alpha-1}{\alpha+1}\right)$$

$$R_2 = R_I\left(\frac{2\alpha}{\alpha^2+1}\right)$$

it should be

$$R_1 = R_I\left(\frac{\alpha-1}{\alpha+1}\right)$$

$$R_2 = R_I\left(\frac{2\alpha}{\alpha^2-1}\right).$$

(NOTE: This correction was incorrectly published in the October 1943, issue of the PROCEEDINGS on p. 582.)

The decimal classification for "Heat-Conduction Problems in Presses Used for Gluing of Wood," by George H. Brown, which appeared in the October, 1943, issue of the PROCEEDINGS on pages 537–548, should read "R590×536" instead of R590×R536.

Norman E. Polster has called the attention of the Editor to an error which appeared in the correspondence section of the PROCEEDINGS for November, 1943. The example

$$V_{00'} = \left[\frac{1/r}{1/r+1/R} + \frac{1/R-(1/r+1/R)}{-(1/r+1/R)}\exp\left(-\frac{t}{RC}-\frac{t}{rc}\right)\right]$$

should read

$$V_{00'} = \left[\frac{1/R}{1/r+1/R} + \frac{1/R-(1/r+1/R)}{-(1/r+1/R)}\exp\left(-\frac{t}{RC}-\frac{t}{rc}\right)\right].$$

Theoretical Limitation to Transconductance in Certain Types of Vacuum Tubes*

J. R. PIERCE†, ASSOCIATE, I.R.E.

Summary—The thermal-velocity distribution of thermionically emitted electrons limits the low-frequency transconductance which can be attained in tubes in whose operation space charge is not important. A relation is developed by means of which this dependence may be evaluated for tubes employing electric and magnetic control. This relation is applied to deflection tubes with electric and magnetic control and to stopping-potential tubes. Magnetic control is shown to be inferior to electric control from the point of view of bandwidth and gain.

I. Introduction

IN THE operation of the usual negative-grid tube, the varying fields due to space charge play a determining part in control of the electron current. There are, however, several types of vacuum tube in which the varying fields due to space charge do not have an important effect. Such, for example, are deflection tubes, in which a sharply focused electron beam is deflected by electric or magnetic means past an intercepting edge. This type of tube was known as early[1] as 1912. Another example is the "stopping-potential" or "retarding-field" tube in which electrons are turned back or allowed to proceed through changing the potential of a "stopping-potential" grid placed normal to the electron flow. This sort of action was discussed by Carrara[2] in 1932. Various aspects of such tubes have been discussed in the later literature.

This paper discusses the low-frequency operation of tubes in which the varying effects of space charge are unimportant in the control of electron flow. Its purpose is to express certain limits imposed on the performance of such tubes by the thermal-velocity distribution of electrons leaving the cathode. These are not the only limits to performance, and, indeed, in most actual tubes such factors as electron optical aberration, variation in potential over grids, and contact-potential variations affect the transconductance more than do thermal velocities. Nevertheless, an understanding of the effects of thermal velocities helps in understanding the operation of such tubes and in seeing similarities and differences. Several important points are brought out. For instance, it might be thought that the transconductance of a deflection tube could be increased indefinitely by use of electron-optical means for amplifying the deflection. Such is not the case, and except for aberrations the limiting transconductance of deflection tubes is quite independent of the nature of the fields between the deflection region and the cutoff edge. As another example, it is shown that from the point of view of gain and bandwidth magnetic control is definitely inferior to electric control for any voltages of operation likely to be encountered.

II. Limitations and Terminology

This paper deals with the low-frequency operation, or operation in which the transit times are short compared with the period of the applied signal, of any vacuum tubes in which space charge does not play an important role in the control or sorting regions.

By control region is meant the region in which the velocity of the electron stream is influenced by the control fields. This might be the region of deflection in a deflection tube.

By the region of sorting is meant the region in which electrons are sorted or divided into two streams according to their direction or speed. One of these streams goes to the output electrode of the tube, and the other is usually wasted on some other electrode not connected to the output circuit. The sorting region might be the region between the deflecting plates and the cutoff edge in a deflection tube.

Sometimes the control and sorting regions may be the same, as in what may be called a stopping-potential tube, in which electrons are injected into a retarding field, terminating at the control grid, which acts both as a velocity-controlling and as a sorting space.

One sort of tube with which this memorandum is not concerned may be exemplified by the diode with space-charge-limited current flow. This is a sort of stopping-potential tube in which the electrons leave the thermionic cathode with certain initial velocities. In a diode the sorting region is the region between the thermionic cathode and the potential minimum. There electrons are slowed up. Those having high velocities are able to pass the potential minimum and reach the plate; those with low velocities are returned to the cathode. The control region coincides with this sorting region. However, the fields affecting the velocities of the electrons do not depend solely on the potentials applied to the electrodes, but depend on the current flowing as well. The work to be presented applies as a whole only when space-charge effects are negligible, so that the fields in the control region may be regarded as functions of the control voltage or current only.

In the work presented, many equations are involved, and it is impractical to rewrite all equations in numerical form. Accordingly, the units used will be stated here. Length is measured in centimeters, current in amperes, capacitance in farads, transconductance in mhos. Several constants used are:

* Decimal classification: R139×R262.5. Original manuscript received by the Institute, April 8, 1943; revised manuscript received, May 21, 1943.

† Bell Telephone Laboratories, Inc., New York, N. Y.

[1] Robert H. Goddard, U. S. Patent No. 1,159,209, November 2, 1915. (Application filed, August 1, 1912).

[2] Nello Carrara, "The detection of microwaves," Proc. I.R.E., vol. 20, pp. 1615–1626; October, 1932.

electronic charge	$e=1.59\times10^{-19}$ coulomb
electronic mass	$m=9.00\times10^{-38}$ gram seven
Boltzman's constant	$k=1.37\times10^{-23}$ joule per degree
Permittivity of space	$p_0=8.85\times10^{-14}$ farad per centimeter
Permeability of space	$\mu_0=1.257\times10^{-8}$ weber per ampere turn centimeter

Two special reduced quantities will be used in measuring velocity and potential

$$u = v(m/2kT)^{1/2}$$

$$\Phi = \phi e/kT = 11600\phi/T$$

Here v is velocity in centimeters per second and ϕ is potential in volts. T is the temperature in degrees Kelvin of the thermionic cathode supplying electrons in the vacuum tube under consideration. The units are chosen so that the speed an electron acquires in starting from rest and falling through a potential difference Φ is $u=\sqrt{\Phi}$.

III. General Expression for Transconductance

Consider conditions at a small element of physical space. For this element let us draw a diagram in velocity space u_x, u_y, u_z, shown in Fig. 1. In this diagram S is a surface in velocity space such that all electrons having velocities represented by points to one side of S go to one electrode A and all electrons represented by points lying to the other side of S go to another electrode B.

We can choose arbitrarily the direction in which we consider the current carried by the electrons to flow, as long as all the electrons cross a plane perpendicular to this direction. Let us choose the x direction as the direction of current flow. For electrons leaving a thermionic cathode it has been established[3,4] that anywhere in the electron stream the portion of the current density in the x direction associated with electrons with velocities lying in the range $du_x du_y du_z$ at u_x and $u^2=u_x^2+u_y^2+u_z^2$ is given by

$$dj = (2/\pi)j_0 u_x \epsilon^{-(u^2-\Phi)} du_x du_y du_z. \quad (1)$$

Here Φ is potential measured with respect to the cathode. If we wish to speak of the current crossing an area $dydz$, we may write

$$di = (2/\pi)j_0 u_x \epsilon^{-(u^2-\Phi)} du_x du_y du_z dydz. \quad (2)$$

The total current reaching electrode A may then be obtained by integrating (2) over all regions of velocity space in which electrons are present, in the region on the appropriate side of the surface S, and over the cross section of the beam with respect to y and z.

Suppose that in a time dt, all electrons under consideration suffer a velocity change du. Some current which formerly went to A will now go to B. The amount will be that associated with electrons represented by points in velocity space lying in the elementary volume

$$d\gamma = dsdu \cos\alpha. \quad (3)$$

Here ds is an element of area of the surface S in velocity space, du is the change in velocity and α is the angle between du and the normal to ds. The change in current to A corresponding to this change is then

$$di = (2/\pi)j_0 u_x \epsilon^{-(u^2-\Phi)} dsdu \cos\alpha\, dydz. \quad (4)$$

1. Electric Control

If the change in velocity in the time dt is due to the application of a field dE, the change in velocity will be in the direction of dE and will have a magnitude given by

$$dv = (e/m)dEdt$$

$$du = (m/2kT)^{1/2} \quad dv = (m/2kT)^{1/2}(e/m)dEdt. \quad (5)$$

The time dt is related to the distance of motion in the x direction, dx.

$$dt = (dx/v_x) = (dx/u_x)(m/2kT)^{1/2}. \quad (6)$$

Using (5) and (6) in (4)

$$di = (e/\pi kT)j_0 \epsilon^{-(u^2-\Phi)} dsdE \cos\alpha d\tau. \quad (7)$$

Here $d\tau$ is an element of volume. We shall note that α

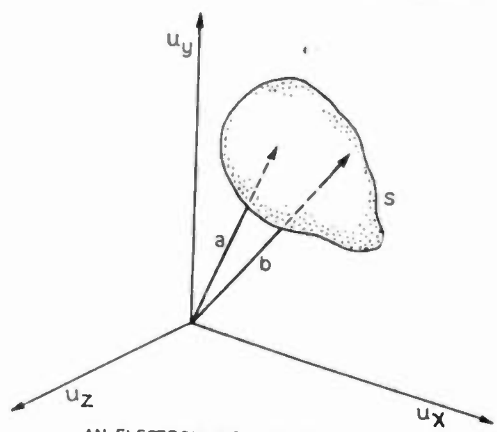

Fig. 1—The sorting surface in velocity space.

is the angle between the field dE and the normal to the surface S.

Relation (7) applies to any sorting type of tube if in the term dE are included all variable fields, both those due directly to changes in control-electrode potential and those due to consequent changes in current and hence in space charge.

In some vacuum tubes the variable fields due to changes in space charge are negligibly small and dE may be taken as the field due to changes in control-electrode potential in the absence of electron flow. If this is so, dE can be related to dV, a voltage increment applied to the control electrode, in terms of the capacitance of the control electrode.

Consider, in the absence of space charge, a tube of force leaving one control electrode, passing through the space which is to be occupied by electron flow, and ending on another electrode. Such a tube of force is shown in Fig. 2. At some point along this tube the cross-sectional area is dA and the electric field is dE. The electric flux is then

$$d\psi = dEdA. \quad (8)$$

This flux is produced by the application of a potential dV between the electrodes on which the tube terminates. Thus the portion of capacitance associated with the part

[3] J. R. Pierce, "Limiting current densities in electron beams," *Jour. Appl. Phys.*, vol. 10, pp. 715–723; October, 1939.
[4] D. B. Langmuir, "Theoretical limitations of cathode-ray tubes," Proc. I.R.E., vol. 25, pp. 977–992; August, 1937.

of an electrode on which the tube terminates is

$$dC = p_0(d\psi/dV). \tag{9}$$

If dw is an elementary length along the tube of force, an elementary volume along the tube is

$$d\tau = dAdw. \tag{10}$$

From (10), (9), and (8)

$$dE = \frac{dCdVdw}{p_0 d\tau}\cdot \tag{11}$$

Putting this value in equation (7)

$$di = \frac{ej_0}{\pi kTp_0}\epsilon^{-(u^2-\Phi)}dsdCdwdV\cos\alpha \tag{12}$$

$$dg_m = \frac{di}{dV} = \frac{ej_0}{\pi kTp_0}\epsilon^{-(u^2-\Phi)}dsdCdw\cos\alpha. \tag{13}$$

The transconductance for the whole control space can be obtained by integrating at each point over the surface S in velocity space, along each tube of force connected with an element of capacitance, and over all elements of capacitance. It must be understood that the only portion of the capacitance of a control element which is effective is that associated with tubes of force crossing the electron flow. Capacitance associated with tubes of force which do not cross the flow will be called stray capacitance.

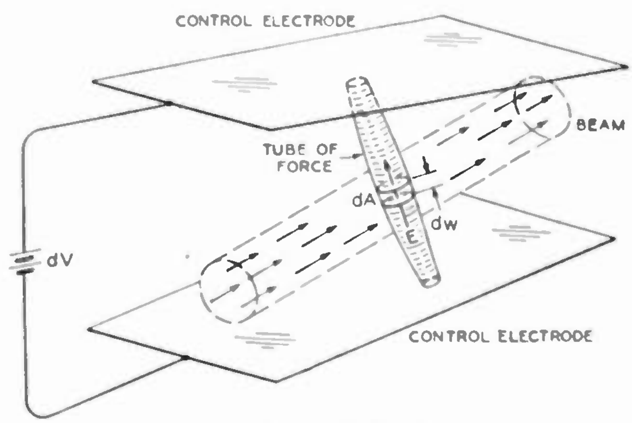

Fig. 2—Control field.

2. Magnetic Control

If a small change in magnetic field dB takes place, an electron in moving a time dt will suffer a change in velocity normal to the change in magnetic field and to the direction of motion, and of magnitude

$$\begin{aligned} dv &= (e/m)vdB\sin\beta dt \\ du &= (m/2kT)^{1/2}dv \\ &= (e/2kT)^{1/2}(e/m)^{1/2}vdB\sin\beta dt. \end{aligned} \tag{14}$$

Here β is the angle between B and u.

Combining (14) with (4) a quantity somewhat analogous to transconductance can be obtained

$$\begin{aligned} dL_m &= di/dB \\ &= j_0(2/\pi)(e/2kT)^{1/2}(e/m)^{1/2}u\epsilon^{-(u^2-\Phi)}dSd\tau\cos\alpha\sin\beta. \end{aligned} \tag{15}$$

There seems little point in reducing this to an expression involving inductance and current rather than B.

As in the case of electric control, the total value of L_m can be obtained by integrating over the surface S in velocity space and over the volume of the control region.

IV. Types of Electron Beams

Tubes for operation at low frequencies with negligible space charge will differ depending on the shape and location of the sorting surface S and on the type of electron beam used. Three types of beams are of interest, and these will now be described.

A parallel or unfocused electron beam is one in which no fields have acted on the beam save along one axis, the direction of motion, i.e., the x direction. Conservation of energy tells us that at a point in space at which the potential is Φ, no electrons are present in the region of velocity space.

$$u_x^2 < \Phi. \tag{16}$$

All values of u_y and u_z are permissible. Thus the electron density is other than zero in a region in velocity space

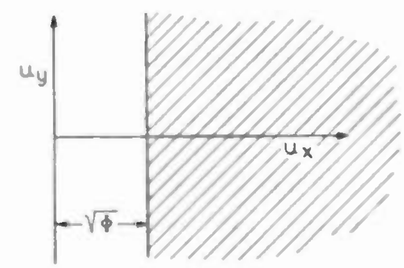

Fig. 3—Parallel beam.

beyond a plane perpendicular to the u_x axis through the point,

$$u_x = \sqrt{\Phi}. \tag{17}$$

This is illustrated in Fig. 3. In using (13) and (15) in the parallel-beam case we must integrate over only those portions of S which lie beyond this plane.

A line-focus beam is one in which fields act in the x direction and one other direction, say, the y direction, but not in the z direction. Conservation of energy tells us concerning the two components of velocity which can be changed by the fields that $u_x^2+u_y^2>\Phi$. One possible state of affairs is that in which electron density is other than zero in a region outside the cylinder $u_r^2=u_x^2+u_y^2=\Phi$ and within a dihedral angle whose sides make an angle θ with the u_xu_z plane. This angle θ will be determined by the angular extent of the aperturing system which is used to eliminate widely diverging electrons from the beam. This line-focus case is illustrated in Fig. 4. Line-focus beams do not necessarily fulfill this condition, but it is a limiting condition for which the current density within a given angular range is as high as possible; its use in conjunction with (13) and (15) will give limiting transconductance for line-focus beams.

In the case of point-focus beams, fields can act on the electrons in all three directions. The energy condition which must be fulfilled is then that

$$u_x^2 + u_y^2 + u_z^2 > \Phi. \tag{19}$$

In this case the condition which gives the highest possible current density is that the density is other than zero in a region outside the sphere

$$u^2 = u_x^2 + u_y^2 + u_z^2 = \Phi \tag{20}$$

and within a cone whose sides make an angle θ with the u_x axis. As in the line-focus case, θ will be determined by the aperturing system. This point-focus case is illustrated in Fig. 5.

When parallel, line-focus, or point-focus beams are referred to henceforward, it will be understood that the electron density is other than zero in regions as described above.

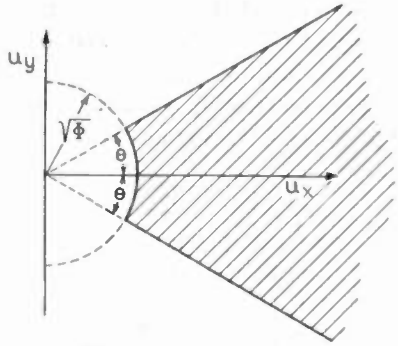

Fig. 4—Line-focus beam.

V. Deflection Tubes

A form of deflection tube is illustrated in Fig. 6. An electron stream from a cathode diverges, is made parallel by an electron lens l_1, passes between deflecting plates d and d', is made convergent by a second lens l_2, impinges on a cutoff edge placed at its focus, whence part of it travels on to the anode. As the voltage between the deflecting plates is changed, the beam is swept past the cutoff edge and the current to the anode is thus made to vary. The deflection may be brought about through magnetic instead of electric means.

A deflection tube may be defined as one in which in the control region the sorting surface S is a plane in velocity space parallel to the direction of electron motion. This will be taken as the $u_x\, u_z$ plane.

1. Electric Deflection

For parallel and line-focus cases, the current density is other than zero from $u_z = \sqrt{\Phi}$ to $u_z = \infty$. We shall assume that the potential is constant throughout the region of deflection. Then, only α will vary in the integration with respect to dw and the integral of $\cos\alpha$ will be the beam width W in a direction normal to S. It will be assumed that W is constant in the region of deflection. Under these conditions, the transconductance is given by

$$g_m = WC\frac{ej_0}{\pi kTp_0}\epsilon^{\Phi}\int_{-\infty}^{+\infty}\int_{\sqrt{\Phi}}^{\infty}\epsilon^{-u_z^2}\epsilon^{-u_x^2}du_z du_x$$

$$= WC\frac{ej_0}{2kTp_0}\epsilon^{\Phi}(1-\operatorname{erf}\sqrt{\Phi}). \tag{21}$$

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_0^x \epsilon^{-u^2}du.$$

g_m is given to better than 12 per cent by the approximate form

$$g_m = \frac{ej_0}{2kTp_0}\,\frac{WC}{\sqrt{\pi\Phi}+1}. \tag{22}$$

The error is about 2 per cent for $\Phi = 5$ and decreases for larger values of Φ.

In the case of a point-focus beam, with the same sorting plane, the electron density is other than zero on regions of the $u_z\, u_y$ plane

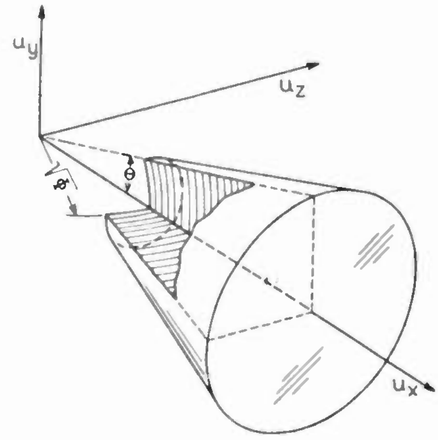

Fig. 5—Point-focus beam.

$$u_z^2 + u_y^2 = u_r^2 > \Phi \tag{23}$$

$$-\theta < \beta < \theta. \tag{24}$$

Here β is the angle with respect to the u_z axis. An appropriate element of area of the plane S is

$$ds = u\,du\,d\beta. \tag{25}$$

Again we shall assume the potential and the beam width constant, and we shall assume θ also constant throughout the region of deflection. As in the previous case, $\cos\alpha$ will be taken care of by measuring W normal to S. Integrating to obtain the transconductance,

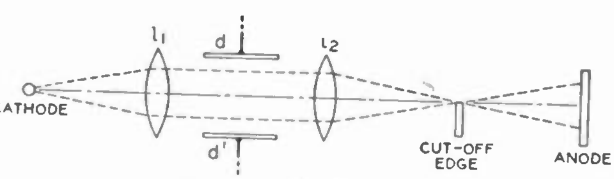

Fig. 6—Deflection tube.

$$g_m = \int_{-\theta}^{+\theta}\int_{\sqrt{\Phi}}^{\infty}\frac{ej_0}{\pi kTp_0}u\epsilon^{-(u^2-\Phi)}du\,d\beta\,dC\,dw$$

$$= \frac{ej_0WC\theta}{\pi kTp_0}. \tag{26}$$

This expression seems independent of Φ. However, for a given current density, θ is a function of Φ and is smaller the larger Φ is.

In the point-focus case it may be desirable to use a beam of circular cross section, so that the beam will be of variable width. In this case the mean width should be used as W. If the beam is converging in the control region there will be a variation of both W and θ, and a mean value of $W\theta$ may be used. Variations in current density over the cross section of the beam also mean variations of θ, and may be taken into account by using a mean value of $W\theta$.

2. Magnetic Deflection

In the case of magnetic-control fields, the only possible sort of tube is the deflection tube, for the magnitude of the velocity cannot be altered by the control field.

Control will be most effective when the magnetic field is perpendicular to the direction of motion, the x direction, and parallel to the sorting surface. In this case, along the sorting surface $v \sin\beta = v_x$, $\cos\alpha = 1$. For parallel and line-focus cases, the current will be other than zero from $u_x = \Phi$ to $u_x = \infty$. It will be assumed that dB and Φ are constant throughout the region of deflection, and that τ is the total volume. Under these conditions

$$L_m = j_0\tau(2/\pi)(e/2kT)^{1/2}(e/m)^{1/2} j_0\tau\epsilon^{\Phi} \cdot \int_{-\infty}^{+\infty}\int_{\sqrt{\Phi}}^{\infty} u_x\epsilon^{-u_x^2}\epsilon^{-u_z^2}du_x du_z$$
$$= (1/\pi)^{1/2}(e/2kT)^{1/2}(e/m)^{1/2} j_0\tau. \quad (27)$$

For the point-focus case

$$L_m = (2/\pi)(e/2kT)^{1/2}(e/m)^{1/2} j_0\tau\epsilon^{\Phi} \cdot \int_{-\theta}^{\theta}\int_{\sqrt{\Phi}}^{\infty} u^2\epsilon^{-u^2}\cos\theta\, d\theta du$$
$$= (2/\pi)(e/2kT)^{1/2}(e/m)^{1/2} j_0\tau \left[\sqrt{\Phi} + (\sqrt{\pi}/2)\epsilon^{\Phi}(1 - \operatorname{erf}\Phi)\right]\sin\theta. \quad (28)$$

For reasonably large values of Φ

$$L_m = (2/\pi)(e/2kT)^{1/2}(e/m)^{1/2} j_0\tau\left[\sqrt{\Phi} + 1/2\sqrt{\Phi}\right]\sin\theta. \quad (29)$$

VI. Stopping-Potential Tubes

A type of stopping-potential tube is shown in Fig. 7. In this tube the electrons are accelerated from the cathode by g_1, and slowed down and controlled between g_1 and g_2. Varying the potential of g_2 determines how many electrons can get past g_2 and reach the plate P.

Fig. 7—Stopping-potential tube.

In this type of tube, the sorting surface S is a plane perpendicular to the direction of electron motion. The sorting electrode, such as g_2 in Fig. 7, is assumed to be at a negative potential $-\Phi_0$ with respect to the cathode. In this case an electron must have an x component of velocity greater than that specified by the potential in order to pass the sorting electrode, and such, in fact, that at $\Phi = -\Phi_0$, the x velocity must be equal to zero. This means that the sorting surface S will be a plane in velocity space perpendicular to the u_x axis at a position $u_x = \sqrt{\Phi + \Phi_0}$. It will be assumed that the applied field acts normal to S, so that $\cos\alpha$ is unity.

For the parallel case, g_m will be

$$g_m = \frac{ej_0WC}{\pi kTp_0}\epsilon^{-\Phi_0}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\epsilon^{-u_y^2}\epsilon^{-u_z^2}du_y du_z$$
$$= \frac{ej_0WC}{kTp_0}\epsilon^{-\Phi_0}. \quad (30)$$

Here W is the width of the beam in the control region perpendicular to S, or in the direction of motion, u_x.

For the line-focus case there is a restriction that the only portion of the surfaces S over which the integration should be carried out is that lying within a dihedral angle 2θ, giving

$$g_m = \frac{ej_0WC}{\pi kTp_0}\epsilon^{-\Phi_0}\int_{-\infty}^{+\infty}\int_{-\sqrt{\Phi+\Phi_0}\tan\theta}^{+\sqrt{\Phi+\Phi_0}\tan\theta}\epsilon^{-u_y^2}\epsilon^{-u_z^2}du_y du_z$$
$$= \frac{ej_0WC}{kTp_0}\epsilon^{-\Phi_0}\operatorname{erf}\left(\sqrt{\Phi+\Phi_0}\tan\theta\right). \quad (31)$$

For the point-focus case, the only portion of the surface S over which the integration should be carried out is that lying within a cone of half-peak angle θ. It seems advisable to use a quantity u_p such that

$$u_p^2 = u_y^2 + u_z^2$$
$$ds = 2\pi u_p du_p.$$

The transconductance will then be given by

$$g_m = \frac{2ej_0WC}{kTp_0}\epsilon^{-\Phi_0}\int_0^{\sqrt{\Phi+\Phi_0}\tan\theta}\epsilon^{-u_p^2}u_p du_p$$
$$= \frac{ej_0WC}{kTp_0}\epsilon\left(1 - \epsilon^{-(\Phi+\Phi_0)\tan^2\theta}\right). \quad (32)$$

It is seen from (29) and (30) that the transconductance is less for point-focus and line-focus beams than for a parallel beam if θ is less than $\pi/2$.

VII. Stopping-Potential Tubes with Positive Stopping-Potential Electrodes

In the case of point-focus and line-focus beams, it is possible to operate the stopping-potential electrode at a positive potential Φ_0 with respect to the cathode. This may occur, for instance, if a beam is focused on a gap between two widely spaced negative control wires, the central region between the wires being positive because of neighboring positive electrodes. In this case, the sorting plane S is at a position in velocity space $x_z = \sqrt{\Phi - \Phi_0}$ from the origin.

In the line-focus case, the only portion of the surface S which should be integrated over is that lying outside of a dihedral angle whose sides cut the surface $u_z^2 + u_y^2 = \Phi$ at $u_y = \sqrt{\Phi - (\Phi - \Phi_0)} = \sqrt{\Phi_0}$ and within the angle θ. The transconductance will then be given by

$$g_m = \frac{2ej_0WC}{\pi kTp_0}\epsilon^{\Phi_0}\int_{-\infty}^{+\infty}\int_{\Phi_0}^{\sqrt{\Phi-\Phi_0}\tan\theta}e^{-u_z^2}\epsilon^{-u_y^2}du_z du_y$$
$$= \frac{ej_0WC}{kTp_0}\epsilon^{\Phi_0}\left(\operatorname{erf}\sqrt{\Phi - \Phi_0}\tan\theta - \operatorname{erf}\sqrt{\Phi_0}\right). \quad (33)$$

For $\theta = \pi/2$, g_m is given to within better than 12 per cent by

$$g_m = \frac{ej_0WC}{kTp_0\sqrt{\pi\Phi_0} + 1}. \quad (34)$$

It may be seen from (33) that in the line-focus case the transconductance is a maximum for $\Phi_0=0$.

In the point-focus case, the integration should be over the part of the plane S for which $\sqrt{\Phi-\Phi_0}\tan\theta > u_p > \sqrt{\Phi_0}$. g_m is thus given by

$$g_m = \frac{2Ej_0WC}{kTp_0}\epsilon^{\Phi_0}\int_{\sqrt{\Phi_0}}^{\sqrt{\Phi-\Phi_0}\tan\theta}\epsilon^{-u_p^2}u_p du_p$$
$$= \frac{ej_0WC}{kTp_0}\left(1-\epsilon^{-(\Phi-\Phi_0)\tan^2\theta}\right). \tag{35}$$

It is seen that in the case of the point-focus beam that when $\theta=\pi/2$, the transconductance is constant for all positive values of Φ.

VIII. A Special Stopping-Potential Tube

Tube designs can be imagined in which the surface S is a cylinder or sphere of radius $\sqrt{\Phi}$ about the origin. This can be the case, for instance, in a tube in which a line-focus or point-focus beam is acted upon in a region of narrow cross section, allowed to diverge, and then made parallel in a sorting region of wide cross section, as illustrated in Fig. 8. Here a beam from a cathode C

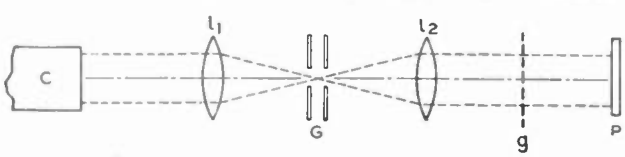

Fig. 8—Special stopping-potential tube.

converges and passes through a control gap G, where its velocity is varied. After diverging and increasing in size it is made parallel by electron lens l and sorted at stopping-potential grid g. Thereafter, the remainder of the beam strikes the output electrode P.

It will be assumed that the lines of force act along the x axis. Thus α will be the angle with respect to the u_x axis.

In the line-focus case, it is convenient to use a quantity $u_r^2=u_x^2+u_y^2$.

Along the surface S, u_r is constant, and $ds=u_r\,d\alpha\,dz=\sqrt{\Phi}\,d\alpha\,dz$.

Accordingly, the transconductance will be

$$g_m = \frac{ej_0WC}{\pi kTp_0}\sqrt{\Phi}\int_{-\infty}^{+\infty}\int_{-\theta}^{+\theta}\epsilon^{-u_z^2}\cos\alpha\,du_z d\alpha$$
$$= \frac{2j_0WC}{\sqrt{\pi}kTp_0}\sqrt{\Phi}\sin\theta. \tag{36}$$

It is apparent that this transconductance may be considerably higher than that for a conventional stopping-potential tube.

In the point-focus case, a quantity u will be used which is constant over the surface S, $u^2=u_x^2+u_y^2+u_z^2=\Phi$.

The element of area will be

$$ds = 2\pi\,u^2\sin\alpha\,d\alpha = 2\pi\,\Phi\sin\alpha\,d\alpha.$$

Accordingly, the transconductance will be

$$g_m = \frac{2ej_0WC}{kTp_0}\Phi\int_0^\theta\sin\alpha\cos\alpha d\alpha$$
$$= \frac{ej_0WC}{kTp_0}\Phi\sin^2\theta. \tag{37}$$

This transconductance also is considerably higher than that of a conventional stopping-potential tube.

IX. Comparison of Electric and Magnetic Control

From the point of view of amplification over a wide band of frequencies, the determining factor is the amount of energy stored in the control field for a given change in current. For instance, if the input capacitance of an electrically controlled tube is doubled, the stored energy for a given input voltage is doubled and the bandwidth over which a given input impedance can be maintained is halved. An important figure of merit of a tube is thus the change in output current squared for unit stored energy in the control field. For an electrically controlled tube this is

$$i^2 = 2g_m^2/C. \tag{38}$$

For a magnetically controlled tube this change is

$$i^2 = 2L_m^2\mu_0/\tau. \tag{39}$$

The ratio of (38) and (39) gives the relative merits of electric and magnetic control from the point of view of bandwidth and gain. This ratio R is

$$R = (g_m/L_m)^2\tau/\mu_0 C. \tag{40}$$

For a line-focus deflection tube and for values of Φ large enough so that approximate forms may be used

$$R = (mc^2/2e\phi). \tag{41}$$

Comparison for the point-focus case leads to the same expression, save for an additional factor $(\theta/\sin\theta)^2$, favorable to electric control.[5]

The quantity $mc^2/2e$ corresponds to a potential of 255,000 volts. The nonrelativistic nature of this work makes it completely inapplicable at that potential. It may be safely concluded, however, that for reasonable potentials electric control will give more gain than magnetic control. Considering that electric control is most effective when applied longitudinally, the case for magnetic control seems almost hopeless from the point of view of bandwidth and gain.

X. Discussion and Numerical Examples

Examining (21), (26), (30), (31), (32), (33), (35), (36), and (37), it may be seen that for a given capacitance, transconductance increases as cathode-current density j_0 is increased and as the width W of the control region is increased. The desirability of high cathode current density is well known, and in the case of negligible space charge quite obvious; twice the current density, twice the output current. The gain through increasing W may be illusory; the larger W, the less the total current must be if space charge is to be avoided. In deflection tubes, an increase in W may be made allowable without other

[5] The comparison can be made more directly as follows: Consider a control field of constant intensity over a unit volume. Imagine an electron moving with a velocity specified by the potential ϕ. The least electric field strength which can produce unit force is $E=(1/e)$. The stored energy is $W_1=p_0/e^2$. The least magnetic field that can produce unit force is $B=(1/e)[2(e/m)\phi]^{-1/2}$. The stored energy is $W_2=(1/\mu_0)(1/e^2)[2(e/m)\phi]^{-1}$. Thus the ratio of stored energies for the same control effect is $R=W_2/W_1=m/2p_0\mu_0 e\phi=mc^2/2e\phi$.

loss if the angular spread of the beam is reduced, thus reducing the total current flow.

When potential enters the various expressions, low potentials appear to be best. This is not conclusive, as higher currents can be used at higher potentials. Thus, for a given tube the current density j_0 increases as the 3/2 power of the voltage, giving a net increase in transconductance for any of the types of tube discussed.

Enlarging the control region perpendicular to W increases both transconductance and capacitance, and results in a gain in their ratio only through reduction of the proportion of useless capacitance associated with edge effects.

It is of interest to substitute plausible figures into the various equations derived. For a deflection tube, from (22), assuming

$$
\begin{aligned}
j_0 &= 0.05 \text{ ampere per square centimeter}\\
T &= 1160 \text{ degrees Kelvin}\\
W &= 0.25 \text{ centimeter}\\
C &= 1\times 10^{-12} \text{ farad}\\
\phi &= 50 \text{ volts } (\Phi = 500)\\
g_m &= 16{,}800\times 10^{-6} \text{ mho}
\end{aligned}
$$

For a stopping-potential tube with a parallel beam, from (30), assuming

$$
\begin{aligned}
j_0 &= 0.05 \text{ ampere per square centimeter}\\
T &= 1160 \text{ degrees Kelvin}\\
W &= 0.25 \text{ centimeter}\\
C &= 0.1\times 10^{-12} \text{ farad}\\
\epsilon^{-\Phi} &= 0.5\\
g_m &= 70{,}500\times 10^{-6} \text{ mho}
\end{aligned}
$$

For the special tube discussed in Section VIII, from (37), assuming

$$
\begin{aligned}
j_0 &= 0.05 \text{ ampere per square centimeter}\\
T &= 1160 \text{ degrees Kelvin}\\
W &= 0.1 \text{ centimeter}\\
C &= 0.1\times 10^{-12} \text{ farad}\\
\phi &= 100 \text{ volts } (\Phi = 1000)\\
\sin\theta &= 0.1\\
g_m &= 563{,}000\times 10^{-6} \text{ mho}
\end{aligned}
$$

Of course, such startling transconductances will not be attained in actual tubes. The transconductances calculated are limiting transconductances. This does not mean that the expressions are inexact, but only that the optimum conditions assumed are not attained in practical tubes. The actual transconductance falls short of the limiting transconductance for the following reasons:

(1) Much of the capacitance in actual tubes is stray capacitance, contributing nothing to the control of the beam.

(2) The assumptions as to the portion of velocity space occupied by electrons, made in Section III, are optimistic.

(3) Because of aberration, misalignment, variation of contact potential over electrodes, and variation in potential over grids, the location and shape of sorting surface S will vary throughout the beam. When this is so the optimum location of the surface may be attained at only a few points in the flow, and at other points the surface S will lie in regions of low or zero current density.

Reason (3) above probably is most important in accounting for the inferior performance of actual tubes. All the defects mentioned above are subject to reduction in some degree.

Neutralization of Screen-Grid Tubes to Improve the Stability of Intermediate-Frequency Amplifiers*

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Summary—**By a simple analysis it is shown that the grid-plate reaction in a screen-grid amplifier stage may be neutralized. This is accomplished by injecting into the screen circuit a small voltage opposite in phase to the plate voltage.**

Practical means for accomplishing this injection are described and a few of the obvious limitations are outlined. Even imperfect neutralization is claimed to be advantageous in improving the operating characteristics of high-gain—narrow-pass-band amplifiers.

For numerous applications it is felt that the increased stability obtainable more than justifies the inclusion of the additional circuit components required.

THE CONVENTIONAL intermediate-frequency amplifier employing screen-grid tubes is capable of very high gain and may be designed to provide either a broad pass band or a very sharp, highly selective narrow pass band. In most cases it is desirable to have the gain adjustable by varying the grid bias either manually or by means of automatic gain control, more commonly referred to as automatic volume control.

For low-gain intermediate-frequency stages of the wide-pass-band type the reaction effect between the plate and grid circuits can usually be neglected with impunity.

When the stage gain is high, and especially if the intermediate-frequency amplifier is designed for a very narrow pass band, the plate-grid reaction (commonly referred to as the Miller effect) can no longer be neglected safely. To do so usually results in regeneration, instability, or even oscillation. In addition if the gain is variable, there is a pronounced frequency shift as the gain is varied.

The grid input impedance of a vacuum tube with a

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load in the plate circuit is different from the grid input impedance with zero plate load. This is known as the Miller effect. If the load in the plate circuit is a pure resistance the effect is to add a pure capacitive susceptance from grid to cathode. If the plate load is a pure reactance, the effect is to add a pure conductance from grid to cathode. This conductance will be positive if the plate load is capacitive and it will be negative if the plate load is inductive. It is often more convenient to express the susceptance and conductance as equivalent shunt capacitance and resistance.

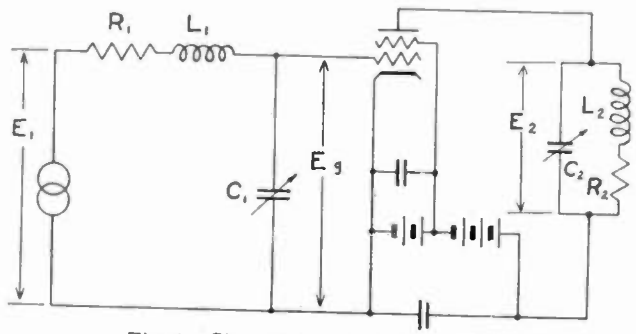

Fig. 1—Circuit of screen-grid tube for demonstrating the Miller effect.

The complete derivation of the input impedance of a vacuum tube with a load in the plate circuit, in the case of a triode, has been given by Terman,[1] who states:

$$\text{input resistance} = R_g = \frac{-1/\omega C_{gp}}{A \sin\theta} \quad (1)$$

$$\text{input capacitance} = C_g = C_{gf} + C_{gp}(1 + A\cos\theta). \quad (2)$$

In the above

C_{gf} = grid-cathode tube capacitance

C_{gp} = grid-plate tube capacitance

$A = E_p/E_g$ = effective grid-plate voltage gain

θ = angle by which voltage across load impedance leads equivalent voltage acting in plate circuit (θ positive for inductive load impedance).

For a screen-grid tube with screen and cathode circuits well by-passed (1) above holds as written; (2) will have one term added as follows:

$$C_g = C_{gf} + C_{gs} + C_{gp}(1 + A\cos\theta) \quad (3)$$

where C_{gs} is the grid-screen tube capacitance.

In Fig. 1 is shown a single-stage amplifier, with the direct-current grid bias omitted for simplicity. E_1 represents the voltage introduced into the tuned input circuit L_1C_1 (with $Q_1 = \omega L_1/R_1$). If we neglect the Miller effect, maximum gain occurs when both the input and output circuits are resonated to the frequency of the voltage E_1.

If we resonate the input circuit by adjusting C_1 with C_2 short-circuited and then remove the short circuit on C_2 the Miller effect can be demonstrated easily. When C_2 is adjusted for exact resonance for the plate circuit, the impedance $L_2C_2R_2$ looks like a pure resistance. Instead of a grid-tuning capacitance of $C_1 + C_{gf} + C_{gs} + C_{gp}$ which occurred when C_2 was short-circuited, we now have a grid capacitance of $C_1 + C_{gf} + C_{gs} + C_{gp} \cdot (1 + A)$. This indicates that the grid circuit will be tuned to a frequency below resonance. If C_1 is now readjusted for resonance by decreasing C_1 by an amount equal to AC_{gp} the over-all gain will increase.

If we now detune the plate circuit so that it becomes inductive (but very close to resonance so that A remains high) a negative conductance will appear across C_1 making the apparent Q of the input circuit rise. If A is very high and the normal Q of the input circuit is high also, it is quite likely that oscillation will take place.

Let us now refer to Fig. 2 where the disposition of the various voltages and capacitances of interest in a screen-grid tube are shown. The plate voltage E_p can be expressed as

$$E_p = -A(\cos\theta + j\sin\theta)E_g. \quad (4)$$

We will next introduce into the screen circuit a voltage that is a fraction of E_p and opposite in phase; this yields

$$E_s = kA(\cos\theta + j\sin\theta)E_g. \quad (5)$$

Equations (1) and (3) can be converted to equivalent susceptances and conductances. If we do this and combine the expressions we find that the total grid admittance is

$$Y_g = j\omega\{C_{gf} + C_{gs} + C_{gp}[1 + A(\cos\theta + j\sin\theta)]\}. \quad (6)$$

If we assign the value zero to A in (6), we obtain

$$Y_g' = j\omega(C_{gf} + C_{gs} + C_{gp}). \quad (7)$$

This is the grid admittance corresponding to perfect neutralization. Let us now inject the value of screen voltage called for in (5) above, and set up the equivalent grid admittance; this gives

$$Y_g'' = j\omega\{C_{gf} + C_{gs}[1 - kA(\cos\theta + j\sin\theta)] + C_{gp}[1 + A(\cos\theta + j\sin\theta)]\}. \quad (8)$$

Equating (7) and (8) we get

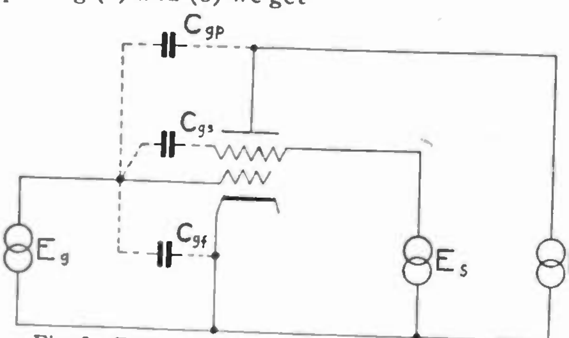

Fig. 2—Equivalent simplified circuit of vacuum tube with capacitive coupling between electrodes.

$$C_{gs}[1 - kA(\cos\theta + j\sin\theta)] - C_{gs} = -C_{gp}[1 + A(\cos\theta + j\sin\theta)] + C_{gp}. \quad (9)$$

Solving for k we obtain

$$k = C_{gp}/C_{gs}. \quad (10)$$

This value of k is seen to be independent of the amplification A and the phase angle θ. Figs. 3, 4, and 5 illustrate practical methods of injecting the required voltage into the screen circuit.

Practical Applications and Limitations

In Fig. 3, $L_1 \cdots L_2$ represents a conventional intermediate-frequency transformer. A third winding of

[1] F. E. Terman, "Radio Engineering," second edition, McGraw-Hill Book Co., New York, N. Y., 1937; pp. 231–233.

very few turns as closely coupled to L_1 as possible provides very close to an impedanceless generator in series with the screen circuit. By connecting this winding with proper polarity with respect to L_1 nearly perfect neutralization is easily obtained experimentally. (Since the transformer is not an ideal transformer, the phase shift between primary and secondary will not be exactly 180 degrees but will depart therefrom by approximately $1/Q$ radians.)

In an experimental 2-stage amplifier operating at 455 kilocycles, set up under the author's direction, a single turn wound over the top of a universal wound coil used as L_1 reduced the frequency shift of the total amplifier for a given gain change from a value of 2.3 kilocycles to a value less than 200 cycles. Before L_3 was added to the intermediate-frequency transformers, stage alignment was very difficult because of a pronounced tendency to oscillate during alignment. This effect was practically absent with L_3 in use.

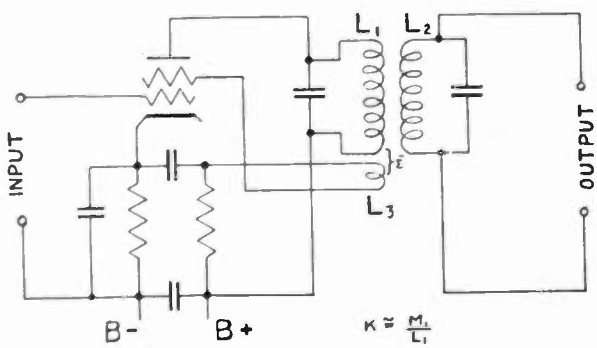

Fig. 3—Inductive method of neutralization of feedback.

Since the grid-screen and grid-plate capacitances of production vacuum tubes are subject to considerable variation, the best neutralization cannot be obtained by a fixed value of coupling between L_1 and L_3. However a major improvement in stability is easily secured with normal production limits on the position of, and number of turns in, L_3. The reduction so obtained will leave a residual reaction that will in general be between 1/5 and 1/25 of the reaction obtained without neutralization, without resorting to the use of preselected vacuum tubes. The advantage of much less frequency shift with automatic frequency control or manual gain control, plus greater ease of production alignment, is very noticeable when several narrow-pass-band stages are cascaded.

For laboratory equipment or specialized applications it would, of course, be possible to arrange the circuit so that the amount of voltage injected into the screen circircuit could be varied for minimum reaction.

For high-frequency applications, where the inductance of L_3 might be detrimental, the circuits shown in Figs. 4 or 5 can be employed. In Fig. 4, and capacitor C_4 should preferably be larger than C_3. For perfect neutralization the ratio of C_2/C_3 should be made equal to $k(=C_b/C_a)$. In addition, the values of R_2 and R_3 should be several times the capacitive reactance of C_3. If the plate and screen circuits are operated at identical direct-current potentials, the simplified circuit of Fig. 5 applies, R_2 should be several times the reactance of C_3 of course, and C_2/C_3 should again equal k.

If one of the elements used to produce the feedback ratio is variable, such as C_2 in Figs. 4 or 5, perfect neutralization is possible at only one frequency. If the circuit is tuned by varying the inductance however, with C_2 and C_3 fixed, a variable frequency neutralized amplifier may be built.

There are, undoubtedly, many who have studied similar projects and papers have been printed describing their work. If proper credit has not been given, it is because the author is not sure to whom the credit is due. Since preparing this paper, it has been brought to the writer's attention that descriptions of the practical methods shown have been covered in the patent literature.

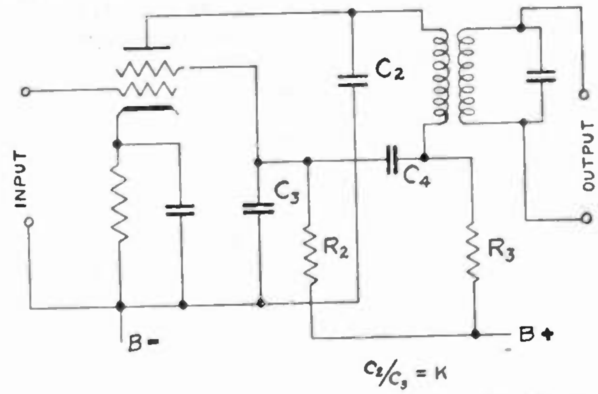

Fig. 4—Capacitive method of neutralization of feedback.

Hazeltine[2] published an historic paper which opened up the entire field of neutralization, at a time when screen-grid tubes were not available. The original screen-grid tube, by virtue of its low grid-plate capacitance, was intended to eliminate the need for neutralization. Subsequent improvement in tubes and circuits has brought about such high possible gain levels that even the small grid-plate capacitances again provide insufficient reaction to make the application of Hazeltines principles advantageous.

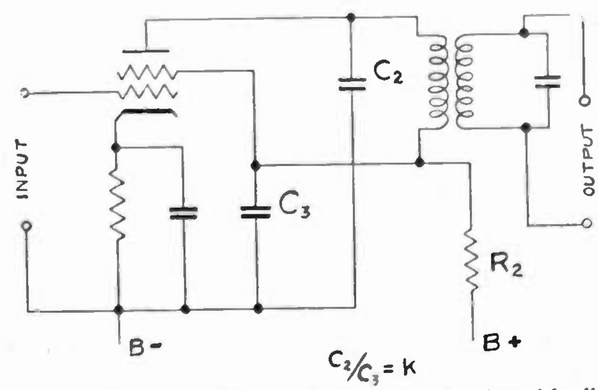

Fig. 5—Simplified capacitive method of neutralization of feedback with screen and plate voltage. Supply at same potential.

[2] L. A. Hazeltine, "Tuned radio frequency amplification with neutralization of capacity coupling," *Proc. Radio Club Amer.*, vol. 2, March, 1923; and *QST*, vol. 7, pp. 7–12; April, 1923.

The Principle of Reciprocity in Antenna Theory*

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Summary—The reciprocity theorem is applied to transmitting and receiving antenna systems in order to establish several important relationships. Formulas are deduced which establish a relation between the receiving current and power of any given antenna and the field intensity of the arriving waves, all the parameters entering into the formulas being the parameters of the same antenna when it is used as a transmitting antenna.

It is shown that, in the case of strong interference, (1) the highest possible directivity is of importance both in the transmitting and in the receiving antennas and (2) the efficiency and the coefficient of exploitation of the receiving antenna are of no importance.

In the case of low interference, it was found that, (1) the directivities of both the receiving and transmitting antennas are of equal importance and (2) the efficiency and coefficient of exploitation of the receiving antenna are just as important as the efficiency of the transmitting antenna.

Introduction

The principle of reciprocity was formulated and established some time ago.[1–5] As we shall demonstrate, this principle proves to be a powerful tool of investigation, establishing a far-reaching analogy in the performance of receiving and transmitting antennas.

The reciprocity principle may have two lines of application to antenna theory. First, it establishes a simple relation between the properties of transmitting and receiving antennas, which permits a deduction of receiving-antennas properties from those of the transmitting antennas, and vice versa. Second, it establishes an interchangeability in the performance of receiving and transmitting antennas. This means that all the differences in the construction of transmitting and receiving antennas, if they are legitimate, may be based either upon stipulations in the statement of the principle, or upon special conditions, and requirements in antenna performance which do not fit into that statement. Thus in antenna design the reciprocity principle yields a criterion for the evaluation and justification of specific comparative peculiarities in the make-up of receiving and transmitting antennas.

In what follows we shall formulate the principle of reciprocity in its relation to the systems which radiate and absorb electromagnetic energy, omitting the justification of the principle itself which is contained in the works of Sommerfeld and Sveshvikova. After that, we shall deduce the corollaries to the principle of reciprocity, along both of the already mentioned lines of application.

Since the theory of transmitting antennas has been more thoroughly developed and better investigated than the theory of receiving antennas, we shall conduct our discussion in the sense of securing the properties and justifying the peculiarities of receiving antenna construction, assuming the properties of transmitting antennas as given and known.

Statement of the Reciprocity Theorem

Let us assume two antennas I and II, which may consist of any combination of conductors, semiconductors, and insulators and may also contain any desired combinations of lumped impedances. Let us mark, on the wires of each of the antennas one arbitrary point, point M_1 on antenna I and point M_2 on antenna II. We can now perform mentally two experiments:

1. Let us apply at the point M_1 of the antenna I an external concentrated electromotive force e_1 of frequency f and at the point M_2 of antenna II an ammeter which will indicate the current I_2 induced by antenna I.

2. Let us apply at the point M_2 an external concentrated electromotive force e_2 of the same frequency f, and at point M_1 of antenna I, an ammeter which will show the current I_1 induced by antenna II.

The electromotive forces and currents in question are shown in Table I where the direction of energy flow is indicated by arrows.

TABLE I

	Antenna I		Antenna II
First experiment	e_1	$\longrightarrow$	I_2
Second experiment	I_1	$\longleftarrow$	e_2

Let us emphasize that points M_1 and M_2 are arbitrary but are the same for both experiments.

It is assumed that the internal impedances of the generators of the electromotive forces e_1 and e_2 as well as the impedances of the ammeters measuring the currents I_1 and I_2 are equal to zero. It is easy to see that this assumption in no way restricts the generality of the cases since when impedances are present they can be considered as a component part of the antenna.

The principle of reciprocity may be formulated as follows: Current I_2 induced in antenna II from antenna I, excited by the electromotive force e_1, is to this electromotive force e_1, as current I_1 induced in antenna I from antenna II excited by the electromotive force e_2 is to that electromotive force e_2, i.e.,

$$I_2/e_1 = I_1/e_2. \tag{1}$$

It is understood that the arrangement of the antennas I and II and the structure of the medium about both antennas are absolutely arbitrary in all respects but one. Namely, the electromagnetic laws which govern

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[1] John R. Carson, "A generalization of the reciprocity theorem," *Bell Sys. Tech. Jour.*, vol. 3, pp. 393–399; July, 1924.

[2] John R. Carson, "Reciprocal theorems in radio communication," Proc. I.R.E., vol. 17, pp. 952–957; June, 1929.

[3] A. Sommerfeld, "Das Reziprozitäts-theorem in der drahtlosen Telegraphie," *Jahr. der drahtlosen Teleg. und Teleph.*, vol. 26, S. 93, 1925.

[4] M. P. Sveshnikova, "The reciprocity theorem in electrodynamics and radiotelegraphy," *Jour. Russian Physico-Chemical Assoc.*, p. 453, 1927.

[5] Stuart Ballantine, "Reciprocity in electromagnetic, mechanical, acoustical, and interconnected systems," Proc. I.R.E., vol. 17, pp. 929–952; June, 1929.

the phenomena must be linear in the sense of the independence of these laws from the amplitude of oscillations. In other words the following equalities must hold true:

$$\left.\begin{aligned} I_2 &= A \cdot e_1 \\ I_1 &= B \cdot e_2 \end{aligned}\right\} \tag{2}$$

where A and B are constants independent of e_1 and e_2. Comparing (1) and (2) we see that the principle of reciprocity is expressed by the equality $A=B$. This last equality holds true for passive linear four-terminal networks of which the system between the terminals of the two antennas is an example.

Fundamental Formulas

Let us assume that the distance between antennas I and II is quite large by comparison with the transmitted wavelengths and the dimensions of the antennas. As seen from the application of the reciprocity theorem to the propagation of electromagnetic waves, the path of a ray going from antenna I to antenna II and that of a ray going from antenna II to antenna I coincide in outline. Furthermore, the angles of rotation of the polarization plane are the same for both rays.[6] Therefore, the length of the path for both rays is the same and the angle between the polarization planes of the sent and of the received rays are identical for both antennas. The proof of these well-known propositions is beyond the scope of this article.

Let us consider the coefficient of directivity D of antenna I in the direction of the ray going toward antenna II, for the case when antenna I is transmitting.

As we know from the theory of transmitting antennas[7] $D_1=(30c^2r^2E_2{}^2/I_{01}{}^2R_{01})$.

Here E_2 = the amplitude of the field intensity about antenna II,

R_{01} = the radiation resistance of antenna I when transmitting referred to the current I_{01} flowing at the point of application of the electromotive force e_1, expressed in ohms.

Further, $I_{01}=e_1/Z_{01}$, where Z_{01} is the complex impedance of the antenna I when transmitting, referred to the electromotive force e_1.

From the above relations we obtain, eliminating I_{01}, $e_1=crE_2Z_{01}\sqrt{30/D_1R_{01}}$.

Similarly for the second antenna we can get $e_2=crE_1Z_{02}\sqrt{30/D_2R_{02}}$.

Substituting the expressions for the electromotive forces in (1) and simplifying we get

$$(I_1Z_{01}/E_1\sqrt{D_1R_{01}})=(I_2Z_{02}/E_2\sqrt{D_2R_{02}}).$$

[6] We neglect, of course, the case when the rays pass through an ionized medium in presence of an external magnetic field, because in such a case the relation (2) and hence the principle of reciprocity are inapplicable.

[7] P. S. Carter, C. W. Hansell, and N. E. Lindenblad, "Development of directive transmitting antennas for R.C.A. Communications, Inc.," Proc. I.R.E., vol. 19, pp. 1773–1843; October, 1931.

Since all of the factors on the left side refer to antenna I and all the factors on the right refer to antenna II we can maintain, in view of the arbitrary antenna construction, that the expression $(IZ_0/E\sqrt{DR_0})$ in which all the factors refer to one antenna, gives a value which does not depend either upon the construction of that antenna or upon the properties of the surrounding medium. In order to determine this value we may consider any antenna. For simplicity's sake let us consider the ideal dipole of Hertz.

Thus let us assume that antenna II is a Hertz dipole situated in air. Let its length be dl and the angle which it forms with the direction of the arriving ray be θ. We then have $I_2=(E_2dl \sin\theta \cos\chi/R_{02})$, where χ is the angle between the planes of polarization of the rays received and sent out in the same direction.

Further, $D_2=(3/2)\sin^2\theta$; $Z_{02}=R_{02}=20m^2(dl)^2$, where $m=2\pi/\lambda$. We therefore get $(I_2Z_{02}/E_2\sqrt{D_2R_{02}})=(\lambda/\pi\sqrt{120})\cos\chi$. Considering the relation (2) we have $(I_1Z_{01}/E_1\sqrt{D_1R_{01}})=(\lambda/\pi\sqrt{120})\cos\chi$. From this, discarding the subscript I we get for any antenna

$$I = (E\lambda\cos\chi/\pi)\left(\sqrt{\frac{DR_0}{120}}\right)\cdot 1/Z_0. \tag{3}$$

This important formula establishes a relation between the receiving current of the antenna at a certain point M and the field intensity of the arriving waves, it being noted that all the parameters entering into the formula (D_0, R_0, Z_0) are the parameters of the antenna when the latter transmits with the electromotive force applied at the same point M. Therefore this formula enables us to build up a theory of receiving antennas upon relations known in the theory of transmitting antennas.

Let us examine for purposes of illustration two particular cases. Let a half-wave unloaded radiator without losses serve as a receiving antenna. Let us place it perpendicularly to the direction of the incoming waves in the plane of the electric field vector $(\chi=0)$. Let us find the current appearing at its mid-point. To do that let us examine a transmitting radiator of the same length excited at the middle. For the latter we have $Z_0=R_0=73.2\Omega$; $D=(120/R_0)\{(\cos[\pi/2\cos\theta])/\sin\theta\}^2$. Here θ is the angle between the axis of the radiator and the direction of the ray which it sends out. Since this direction must coincide with the direction of the arriving ray we must assume $\theta=\pi/2$. Thus in agreement with (3) we get $I=E\lambda/\pi\cdot 1/R_0$, which agrees with the results of direct computing.

For a 1-wavelength radiator we have, according to Ballantine, $E=(2I_{max}/cr)\cdot(1+\cos[\pi\cos\theta]/\sin\theta)$. Also when $\theta=\pi/2$, $D=480/R_{loop}$. Further, $Z_0=R_0=W^2/R_{loop}$. Equation (3) gives $I=E\lambda/\pi\cdot 2/W$, which also agrees with the results of direct computing.

From (3) we get the following direct corollaries:

(1) The transmitting and receiving directivity characteristics of any antenna agree, if the received current is measured at the same point at which the electromotive

force is applied when the antenna is used for transmission.

(2) Any receiving antenna may be considered as a generator with an electromotive force, $e_a=(E\lambda \cos\chi/\pi)\sqrt{DR_0/120}$, and with an internal impedance Z_0.

(3) The resonance curves and therefore the frequency bands for the rays which travel in a definite direction are the same for both the transmitting and receiving antennas if it is assumed that the angle of rotation of the polarization plane depends little upon frequency and if the resonance curves are described by the functions $I/E\lambda=f(\omega)\cdot$constant, for the receiving antenna and $E/e=f(\omega)\cdot$constant for the transmitting antenna.

Both curves are described by the function of the form $f(\omega)=\sqrt{DR_0}/Z_0$.

Let us deduce a few more basic properties of the receiving antennas of any type using (3). In a general case of a loaded receiving antenna the impedance Z_0 consists of the following components:

$$Z_0=R_a+R'+j(X_a+X').$$

Here: R_a = resistive component of the impedance equivalent to the antenna (in relation to the electromotive force)

R' = resistance of the useful load

X_a = reactive component of the impedance equivalent to the antenna

X' = reactance of the load

From the law of conservation of energy it follows that $R_a=R_0+R''$, where R_0 is the radiation resistance of the antenna when transmitting, referred to the current at point M; R'' is the resistance of antenna losses, also transmitting, and referred to the current at point M. Therefore, $Z_0=R_0+R''+R'+j(X_a+X')$.

The power emitted in useful loading will be

$$P'=(1/2)I^2R'=(E^2\lambda^2/\pi^2)(DR_0/240)\cdot(R'/Z_0^2)\cos^2\chi.$$

Let us assume to begin with that there are no losses, i.e., $R''=0$. Then the maximum of useful power, as can be easily shown, will be obtained at $X_a+X'=0$; $R'=R_0$.

Considering these conditions we obtain

$$P_{max}=\frac{E^2\lambda^2}{\pi^2}\frac{D}{960}\cos^2\chi. \tag{4}$$

Thus the maximum power which the receiving antenna can abstract from the waves arriving from a given direction is determined exclusively by the coefficient of directive action of the antenna calculated for that direction with the antenna transmitting.

A still more lucid formula is obtained if the maximum area F_{max} from which the antenna can absorb energy is calculated. Considering the fact that the power passing through the area F_{max}, which is perpendicular to Poynting's vector S, is equal to $P_{max}=F_{max}\cdot S_{eff}=(c/8\pi)E^2\cdot F_{max}$ or, in practical units, $P_{max}=(E^2/30)\cdot(F_{max}/8\pi)$. Equating the powers and taking $\chi=0$, we get

$$F_{max}/\lambda^2=D/4\pi. \tag{5}$$

Thus a maximum area of wave absorption by the antenna, expressed in fractions of wavelength, is equal to the coefficient of the antenna directivity divided by 4π.

This latter result shows that, for instance, the maximum power which can be abstracted from the wave by loops and dipoles which are small in comparison with the wavelength, is absolutely independent of their dimensions, and is equal to $P_{max}=(E^2\lambda^2/960\pi^2)\cdot(3/2)\sin^2\theta=(E^2\lambda^2/640\pi^2)\sin^2\theta$.

Here θ is the angle between the direction of the incoming waves and the axis of the dipole or the loop. When θ is equal to $\pi/2$ we get $P_{max}=(E^2\lambda^2/640\pi^2)$; $(F_{max}/\lambda^2)=(3/8\pi)$.

From (5) it follows that the use of antennas of sharp directivity is essential, when large power is to be abstracted from the waves.

The calculation of the power obtained by complex antennas (such as multidipole arrays, rhombic, V and W types, etc.) is not difficult inasmuch as the coefficients of their directivity are well known from the theory of transmitting antennas. It must be also noted that the results obtained give a lucid physical explanation of the fact that the coefficients of directivity of screen antennas are approximately directly proportional to the area of the screens.

Let us now assume the existence of losses in the antenna. Maximum of useful power emitted in loading will now take place at

$$X_a+X'=0;\qquad R'=R_0+R'' \tag{6}$$

and will be equal to $P'_{max}=(E^2\lambda^2/\pi^2)(D/960)\cdot(R_0/R_0+R'')=P_{max}(R_0/R_0+R'')$.

But the last factor is nothing but the efficiency η_A of the antenna when it transmits. Therefore, $P'_{max}=P_{max}\eta_A$.

This relation gives, for the case of the receiving antenna, an interpretation of the efficiency of a corresponding transmitting antenna. It is equal to the ratio of the maximum useful power which can be taken from the antenna to the greatest power which can be taken from an antenna having no losses.

Let us now consider for a general case, when the conditions of (6) are not met, the factor $\xi_A=P'/P'_{max}$, i.e., the relation of useful power taken from the antenna to the greatest useful power which the antenna can give out when the load impedance is correctly chosen. We can call this factor a coefficient of exploitation of the receiving antenna.

We have

$$\xi_A=\frac{4(R_0+R'')R'}{Z_0^2}=\frac{4(R_0+R'')R'}{(R_0+R''+R')^2+(X_a+X')^2}$$

$$P'=\frac{E^2\lambda^2}{\pi^2}\frac{D}{960}\eta_A\xi_A\cos^2\chi.$$

The coefficient of exploitation obtains its maximum significance in the fulfillment of (6) when $\xi_A=1$.

Distinguishing Characteristics of Receiving and Transmitting Antennas in the Light of Reciprocity Theorem

The preceding discussion shows that the convertability of antennas from receiving into transmitting ones and vice versa follows from the reciprocity theorem. More than that, it follows that the better an antenna radiates energy the better it picks it up.

The relation of the power in a correctly selected useful loading of the receiving antenna II to the power supplied to the transmitting antenna I, i.e., the relation which determines the efficiency of radio transmission, can be represented by $(P_{\text{usef. rec}}/P_{\text{inp. tr}}) = D_1\eta_{A1}D_2\eta_{A2}\cdot A$. Here A is a constant, independent of antenna properties, and determined by the distance between them and by the condition of propagation of electromagnetic waves.

The properties of the transmitting antenna and the properties of the receiving antenna enter into the expression for the ratio of powers quite symmetrically, so that the interchange of the position of the antennas does not alter that ratio.

It follows that if the reciprocity principle would embrace fully the conditions taking place in radio communication, then the construction of the receiving and transmitting antennas should have been absolutely identical. In actual practice, however, we have to consider a series of conditions which are neglected by the reciprocity principle and which do justify the differences in the construction of receiving and transmitting antennas.

Let us determine these conditions. Let us consider the two most substantial groups of factors which we are facing in designing antennas, namely: (1) the nonunidirectivity of radio communications and (2) the nonlinearities which occur, or may occur in antenna work.

Nonunidirectivity of Radio Communications

Only one source and one receiver of electromagnetic waves appears in the formulation of the reciprocity principle. In actual practice, however, one must handle greater numbers of these. It is clear, therefore, that the considerations controlling the design of antennas as regards to their multiplicity, do not fit into the ramifications of the reciprocity principle.

We must note here, first of all, the case of broadcasting which involves the existence of a small number of transmitting antennas and of a great number of receiving antennas. It is obviously expedient in this case to construct relatively cheap receiving antennas and expensive transmitting ones (of high efficiency and with directive characteristics, at any rate, in the vertical plane).

An entirely different picture occurs in point-to-point radio communications when an equal number of receiving and transmitting antennas is used. Here the expenses for the improvement of both the receiving and transmitting antennas are equally justified. But even here we must cope with the nonunidirectivity of radio communications which makes itself especially manifest in the existence of both natural and artificial sources of interference. Let us continue from this point of view. We have to consider two cases: (1) The case of strong interference (with long, medium, and fairly short waves) when the receiver's amplifying capacity cannot be fully employed and the required field intensity of the signal is determined by the interference level, and (2) the case of weak external interference (for very short and ultra-short waves) when the amplification is carried up to the limit determined by the fluctuating noises of the first tube.

1. *The Case of Reduced Amplification.*

The field intensity of the signals is given by the condition $P_{\text{sig}} \geqq A\cdot P_{\text{interf}}$ where P_{sig} and P_{interf} are the power-input components in the useful loading of the antenna due to the signals and to the interference respectively while A is a constant.

We have $P_{\text{sig}} = (\lambda^2/960\pi^2)E^2_{\text{sig}}D_2\xi_2\eta_2$, where D_2, ξ_2, and η_2 are the coefficients of directivity, of exploitation, and the efficiency of the receiving antenna.

Let us assume that the interferences reach the receiving antenna with equal probability from all directions with equal intensity and in random phase relations. Then, $P_{\text{interf}} = (\pi/4)(\lambda^2/960\pi^2)D_{2\,\text{mean}}\xi_2\eta_2\sum E^2_{\text{interf}}$ where $D_{2\,\text{mean}}$ is the mean coefficient of antenna directivity, $D_{\text{mean}} = (\int D\cdot d\omega/\int d\omega)$, $d\omega$ being an elementary spherical angle and the integration being performed over a sphere with the antenna in question at the center. The multiplier $\pi/4$ takes account of the mean probability of the power which is obtained from the addition of oscillations with random (equally probable) phases.

Let us evaluate D_{mean}, taking into consideration that $D = (cr^2E^2/I_0^2R_0) = (cr^2E^2/2P)$; $\int d\omega = 4\pi$, we get $D_{\text{mean}} = (cr^2/8\pi P)\int E^2 d\omega = (cr^2/8\pi P)\int (E^2/r^2)df = (1/P)\int (c/8\pi)\cdot E^2 df$, where df is an element of the surface of the sphere with a radius r.

Since the latter integral is nothing but the power of radiation P we get $D_{\text{mean}} = 1$. Therefore,

$$P_{\text{interf}} = (\pi/4)(\lambda^2/960\pi^2)\eta_2\xi_2\sum E^2_{\text{interf}}.$$

Substituting the expressions for the power due to the signals and to the interferences into the fundamental inequality we get $E^2_{\text{sig}} \geqq (\pi/4)(A/D_2)\sum E^2_{\text{interf}}$.

Since the power of the transmitting radio stations P_{tr} is determined by the expression $P_{\text{tr}} = C\cdot E^2_{\text{sig}}/\eta_1 D_1$ where η_1 and D_1 are the efficiency and coefficient of directivity of the transmitting antenna, and C is a constant, we get as a final result

$$P_{\text{tr}} \geqq (\pi/4)(A/C)(\sum E^2_{\text{interf}}/\eta_1 D_1 D_2).$$

In this manner we obtain the following results: (1) The highest possible directivity is of equal importance both in the transmitting and in the receiving antennas; (2) the efficiency of the transmitting antenna is of considerable importance, while the efficiency and the coefficient of exploitation of the receiving antenna have no significance.

There is no point, therefore, in trying to obtain a high efficiency of the receiving antenna and to adjust precisely its load resistance. The practical usage in the construction of long-wave receiving antennas confirms these results (antennas poorly grounded, "aperiodic" antennas of low coefficient of exploitation, etc.).

Let us also note that with an uneven distribution of interferences in all directions the latter formula must be replaced by $P_{tr} \geqq (\pi/4)(A/C)(D_{interf}\sum E^2_{interf}/\eta_1 D_1 D_2)$, where D_{interf} is a mean coefficient of antenna directivity in relation to the unevenly distributed interferences. It is defined by $D_{interf} = (\sum[E^2_{interf} \cdot D]/\sum E^2_{interf})$.

If this coefficient is less than unity, which occurs during maximum interference from the sides and from the rear, then the application of a receiving antenna with the greatest possible directivity acquires double significance: the useful signal is intensified and simultaneously the interferences are weakened. In this case, therefore, it is rational to make the receiving antenna even more directional than the transmitting one. If D_{interf} is greater than unity, which occurs when the main interferences are coming from direction toward the transmitter, then the directivity of the receiving antenna is of less significance than the directivity of the transmitting one. In a limiting case, when all the interferences proceed in the same direction with the useful signals, the directivity of the receiving antenna loses all significance.

Avoiding a more detailed study of the question, let us note that when the distribution of interferences is undetermined beforehand the first case is more probable, and that this probability increases with the sharpness of antenna directivity and with the nonuniformity of interference distribution in different directions. It is to be believed, therefore, that there are even more grounds for securing a sharp directivity of the receiving antennas than that of the transmitting ones. Such a belief is supported by the fact that the directions of the interferences are sometimes, if only in a general way, known beforehand, and hence special steps can be taken to provide for "dead zones." All this refers to the directivities in the horizontal as well as in a vertical plane.

2. *The Case of Maximum Amplification.*

When the interference level is so insignificant that the useful sensitivity of the receivers is limited by interior noises, the condition for good reception will be $P_{sig} \geqq P_{min}$, where P_{min} is a certain given power.

Considering the fact that $P_{sig} = (E^2\lambda^2/960\pi^2)D_2\xi_2\eta_2$ and on the other hand $E^2 = (P_{tr}D_1\eta_1/C)$, we get $P_{tr} \geqq (C \cdot 960\pi^2/\lambda^2)(P_{min}/D_1\eta_1 D_2\eta_2\xi_2)$.

When the load resistance of the receiving antenna is correctly adjusted, $P_{tr} \geqq (C960\pi^2/\lambda^2)(P_{min}/D_1\eta_1 D_2\eta_2)$.

Thus, in this case, we obtain the following results:

(1) The directivities of both the receiving and transmitting antennas are of equal importance.

(2) The efficiency and coefficient of exploitation of the receiving antenna are just as important as the efficiency of the transmitting antenna.

In the foregoing discussion we neglected the plane of polarization of the waves which reach the receiving antenna. Where it is permissible to think that the plane of polarization of the greater part of interferences is more or less constant in time and is different from the plane of polarization of the signals, we should then have an additional way of selection from interferences.

Nonlinearities

Let us now consider the second group of conditions which determine the differences in the construction of receiving and transmitting antennas. These are the nonlinearities which do, or may take place in the performance of antennas.

Before all else this group includes phenomena of nonlinear characters which occur at high antenna voltages, such as the breakdown of insulators and discharges into the air (corona in long waves, and standing arcs or torches in short waves). Considerations concerning the prevention of these phenomena, while playing an important part in powerful transmitting-antenna design, are of no significance in the case of receiving antennas. Thus the difference in the insulation of receiving and transmitting antennas is justified. Furthermore, in the case of the receiving antennas there is no need of increasing the capacitance and radiation resistance for lowering of voltage. In the case of reduced amplification the radiation resistance has its principal significance in the efficiency and the conditions of adjustment of load resistance. In the cases of maximum amplification even these considerations vanish.[8]

It is natural, therefore, to use in the case of long waves, antennas of one wire. On the other hand for purposes of short-wave reception, systems of low radiation resistance (such as directors, passive reflectors, etc.) may be more widely used.

Also let us note that the multiple use of antennas for purposes of reception is simpler than for purposes of transmission since in the former case the increase of antenna voltage does not have to be considered.

The second group of nonlinearities is manifest in those cases where the work of antennas depends upon vacuum tubes. As a typical instance of these, we may mention the well-known method of spaced antennas, employed in short-wave schemes, designed to counteract fading (diversity system). Inasmuch as this method gives good results only when the signals arriving from different antennas are combined after detection, it is dependent upon the nonlinear characteristics of the tubes and, therefore, cannot be adopted to transmitting antennas.

The arrangement of the coupling circuits between the tubes and antennas (tank circuits of the output stages of transmitters and the input circuits of receivers) will be different too since these are impedance transformers with the magnitude of the required impedance

[8] It must be noted, however, that the criteria for passing the entire frequency band through antennas apply also to the receiving antenna.

transformation being determined by the characteristics of the tubes. Further differences are determined by nonlinear phenomena due to overheating and to breakdown of insulation in couplings.

In feeding through transmission lines with a traveling wave, two transformers are required, one between the antenna and the line and the other between the line and the tubes. The first transformer (if nonlinear phenomena connected with high power are neglected) must be the same for both transmission and reception because, in the first case, consideration of feeder losses requires matching the equivalent antenna resistance and the characteristic impedance of the line, while in the latter case it is necessary to match the loading and equivalent antenna resistance.

Conclusions

In summarizing we may say that:

(1) The principle of reciprocity enables us to obtain all of the characteristics of receiving antennas from the known characteristics of the transmitting ones, avoiding involved direct computing, and in a very simple manner.

(2) The principle of reciprocity gives a convenient criterion for evaluating and explaining the peculiarities of receiving-antenna construction as compared with that of transmitting antennas, and vice versa.

Antenna Arrays Around Cylinders*

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Summary—Arrays of dipoles around spires or other vertical supports are useful for the broadcasting of ultra-high-frequency waves. To the author's knowledge no sound method of computing the radiation patterns which takes into account the effect of the support has previously been presented. The three arrangements of dipoles around a vertical cylinder which are of interest are (1) an array of vertical dipoles, (2) an array of horizontal dipoles whose axes lie on the circumference of a circle, and (3) an array of horizontal dipoles whose axes point radially outward. There are several phase relationships for the currents which are of practical interest for each of these arrays.

The necessary number of dipoles in various types of arrays to obtain a horizontal radiation pattern approaching a circle within any specified tolerance are shown by curves. The interference phenomena caused by a plane wave passing a vertical cylinder are discussed and shown graphically. Several radiation patterns for one dipole near a cylinder are discussed. A detailed study of a 4-element horizontal dipole array surrounding a cylinder whose diameter is 1.27 wavelengths, or whose periphery approximates that of the Chrysler Building spire at the assigned television frequency, has been made. Both horizontal and vertical patterns for three different phase relationships of the dipole currents have been calculated.

Formulas for the radiation patterns for arrays of all three types having various numbers of elements and fed in several different ways are tabulated.

The method of obtaining a rigorous solution of Maxwell's equations for a dipole near a long cylinder is outlined. By making use of the reciprocity theorem infinite integrals in the terms of the Fourier-Bessel series are avoided. When the expression for the field from one dipole and cylinder has been obtained it is a simple matter to develop the expression for an array of any number of elements.

When the diameter of the support is large in terms of wavelength arrays of two or more tiers are necessary to avoid waste of energy in high-angle radiation. A substantially circular horizontal pattern can always be obtained.

Introduction

For television and sound broadcasting with ultra-short waves it is often desirable to use arrays of radiators surrounding vertical projections from tall buildings such as the Empire State Building mooring mast and the Chrysler Building spire. Usually such structures are good conductors at high frequencies, being constructed with stainless steel or other metallic surfaces. One would naturally expect the currents which flow in such conducting structures to have a substantial effect upon the radiation patterns of antenna systems which they support. This paper shows how such characteristics may be calculated and discusses in detail some arrangements of importance in practice. If we assume a supporting structure to be a long perfectly conducting cylinder a rigorous mathematical solution for the radiation fields is possible. In practice where the vertical structures may not be of cylindrical form the radiation patterns should differ little from those computed for a cylindrical form if the periphery of the theoretical cylinder is made equal to the periphery of the actual structure. For broadcast purposes we are usually interested in obtaining a substantially circular radiation pattern in the horizontal plane and as little waste of energy as possible at high angles above the horizon.

Unfortunately, the mathematical theory necessary to develop the expressions giving the radiation characteristics is not readily understandable by one unfamiliar with the vector calculus, the partial differential equations of wave motion, and the properties of Bessel functions. An attempt will be made to give a nonmathematical explanation of the physical principles involved but first we shall discuss the characteristics of some arrangements of practical importance.

General Considerations—Circular Arrays

In the design of a circular antenna array we usually wish to use the minimum number of units which will produce a satisfactory horizontal radiation pattern. The term "circular array" as used here is intended to include all arrays consisting of any number of units evenly spaced at equal distances from a fixed point. The array may consist of two units at diametrically opposite points, three units at the corner of an equilateral

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triangle, four units at the corners of a square, etc.

Usually, but not always, the number of units required to obtain a substantially circular pattern in the horizontal plane is at least as great when the array surrounds a conducting cylinder as when the same array is free from the cylinder. It is, therefore, well to give some

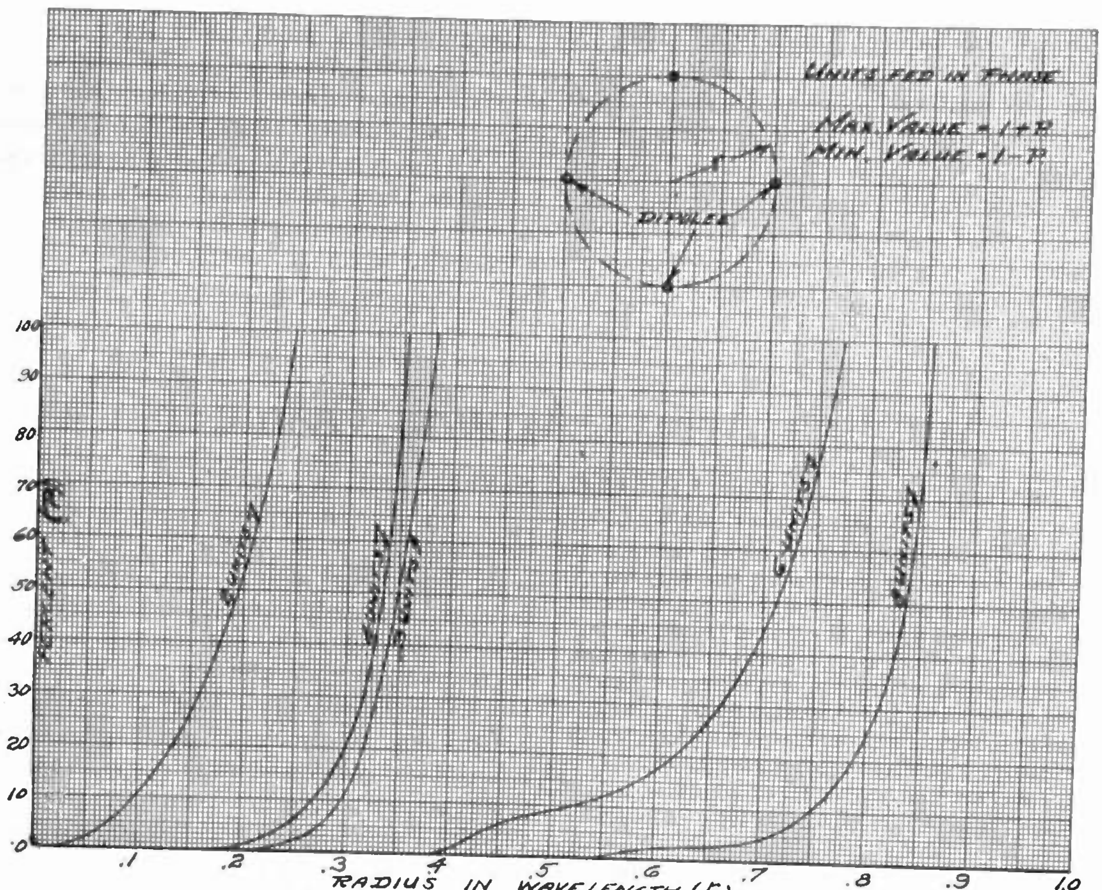

Fig. 1—Vertical diople arrays. Variation in field strength with horizontal angle. (In phase.)

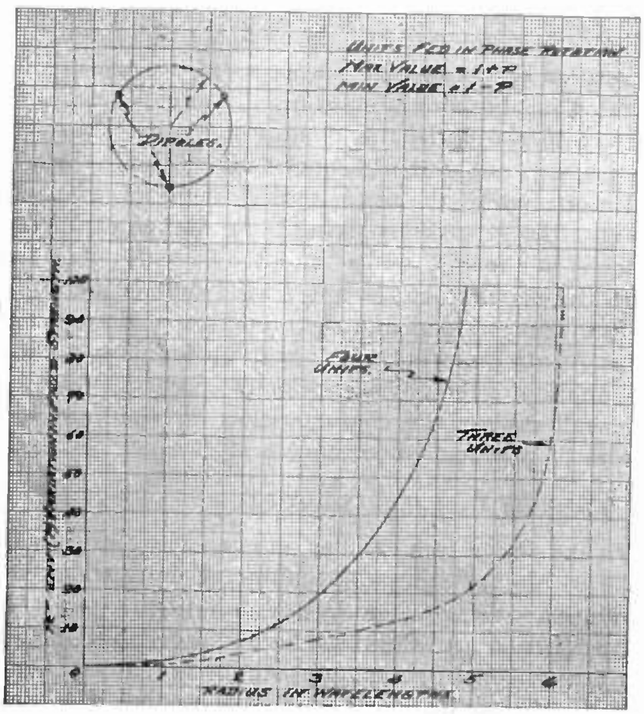

Fig. 2—Vertical dipole arrays. Variation in field strength with horizontal angle. (Phase rotation.)

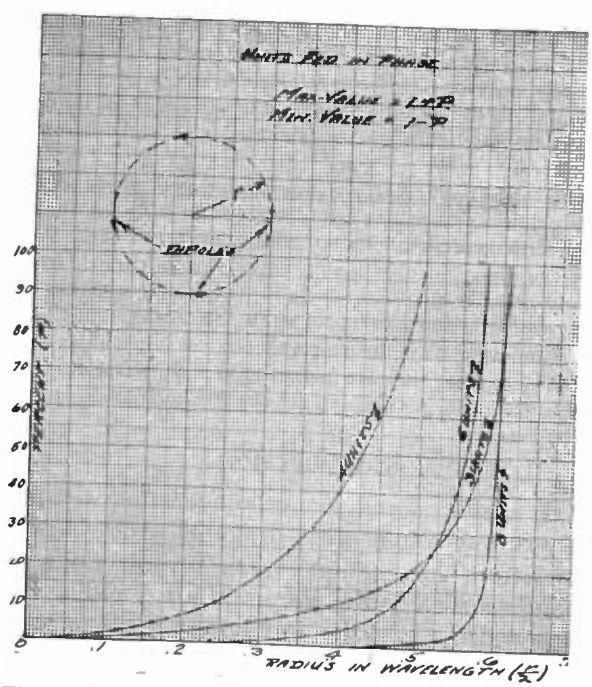

Fig. 3—Horizontal dipole arrays. Variation in field strength with horizontal angle.

consideration to circular arrays by themselves before proceeding to a study of the effects of a cylinder.

Consider first arrays of vertical dipoles fed in phase. A perfect circular pattern will, of course, be obtained by the use of a single unit. If two units are used the spacing must be quite small to prevent a large variation in field with direction. As the diameter of the circle is increased the number of units must be increased to prevent a large variation in radiation with direction. The curves of Fig. 1 have been plotted to show the effects of diameter and number of units on the horizontal pattern.

field with the radius for 3- and 4-unit arrays of this type.

For horizontally polarized radiation we may wish to use an array of horizontal dipoles, where the axes of these dipoles lie on the circumference of a circle. We assume these units to be fed in phase, i.e., that the instantaneous directions of the currents are all in the same sense around the circumference. Fig. 3 shows the manner in which the pattern varies with the radius and the number of units. It will be noticed that the 3-unit array is very much superior to the 4.

Another arrangement of interest for the radiation of

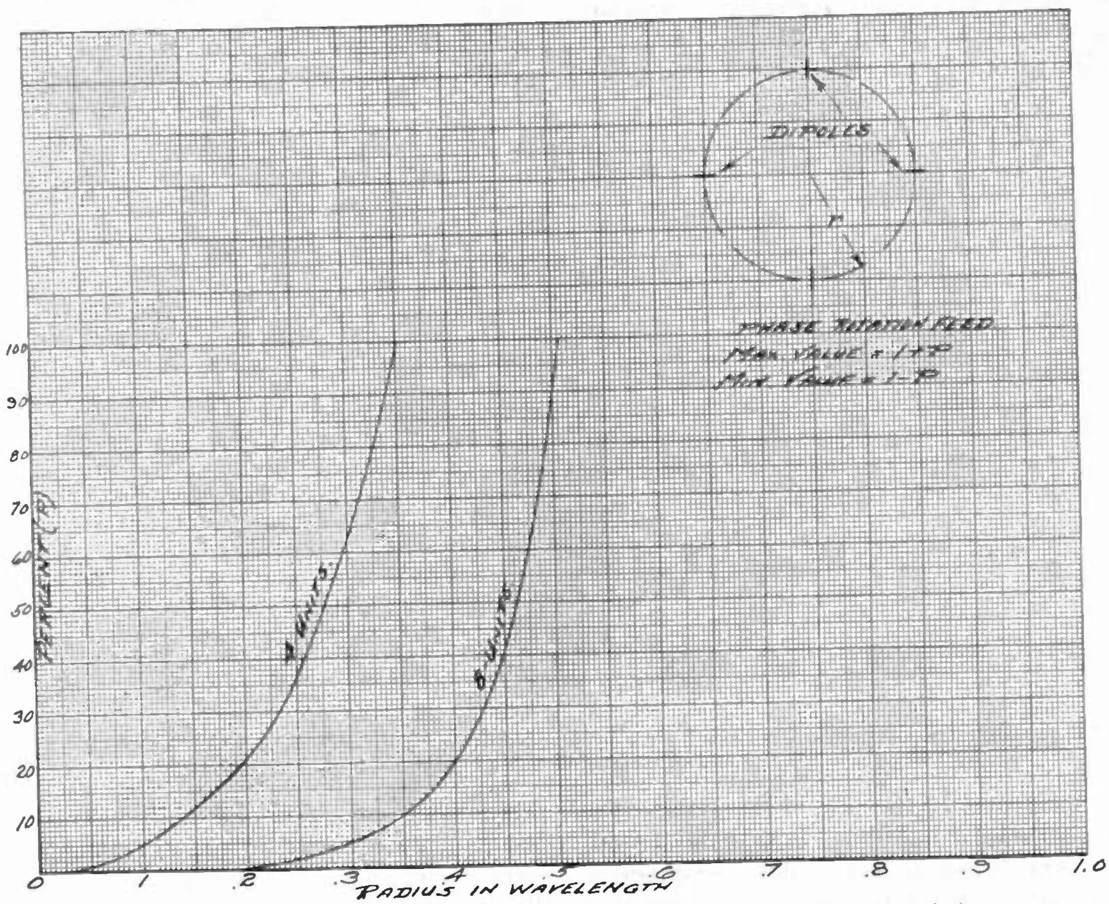

Fig. 4—Radial dipole arrays. Variation in field strength with horizontal angle.

The per cent variation in field strength with direction is plotted as a function of the radius of the circle in wavelengths for arrays of 2, 3, 4, 6, and 8 units. For any particular tolerance we may immediately find the minimum number of units which can be used for a given radius. For example, if we specify the variation as less than 10 per cent we find that at least 3 units must be used for a radius of one-quarter wavelength. For a half-wavelength radius 6 units are necessary. It is of interest to note that a 3-unit array is superior to one of 4 units.

Other arrays of vertical dipoles of interest are those of 3, 4, and 6 units fed in phase rotation, that is, in which the current in each dipole leads that in the preceding dipole by 360 degrees/n where n is the total number, and each unit is considered in order following around the array. Fig. 2 shows the per cent variation in

horizontally polarized waves is an array of horizontal dipoles in which the dipole axes are directed radially. The pairs of units are fed in a phase relation corresponding to their position on the circle. In a 4-unit array the pairs are in quarter phase relation while in an 8-unit array adjacent pairs have a 45-degree phase relationship. Curves similar to the preceding are shown in Fig. 4 for this array.

So far we have failed to mention the effect of the radius and number of dipoles in circular arrays upon the radiation patterns in vertical planes. Although we may obtain a nearly circular horizontal pattern from arrays of considerable radius by using a sufficient number of units, the radiation at high angles to the horizon increases greatly with increase in diameter. To avoid this waste of energy it becomes necessary to use two or more tiers.

Diffraction by a Tall Cylinder

Before proceeding with a discussion of the effects of a cylinder upon near-by dipole antennas it may be well to try to get a picture of the phenomena of diffraction by a tall cylinder. Let us assume that we have a vertically polarized electromagnetic wave coming from a distant source and that in its path is placed a long vertical cylinder. The electric field of this wave sets up currents in the surface of the cylinder which are responsible for a secondary electromagnetic wave diverging outwardly from the cylinder. In the case of reflection from a plane surface we ordinarily picture a main and reflected wave, the reflected wave traveling in the opposite direction to the direct wave for the condition of perpendicular incidence. In the case of the cylinder the effect is somewhat similar but differs in that the secondary waves travel in all directions including the direction directly in back of the obstruction. The curves in Fig. 5 show a picture of the phenomenon for a cylinder having a diameter of 0.32 wavelength. To the side of the cylinder directly toward the source the combination of primary and secondary fields results in a standing wave in a manner quite similar to that for reflection from a plane conductor. At all points on the surface of the conductor the electric-field intensity must of course be zero. At distances greater than about a wavelength from the cylinder the maxima and minima of the standing wave are $\frac{1}{4}$ wavelength apart as would be expected but in the region close to the cylinder the law governing the total field is rather complex due to the fact that the secondary wave is a diverging, cylindrical wave. In the direction directly behind the conducting obstruction the intensity of the electric field builds up in a smooth curve and gradually approaches the strength of the primary field as the distance increases. There is no standing-wave phenomenon in this direction.

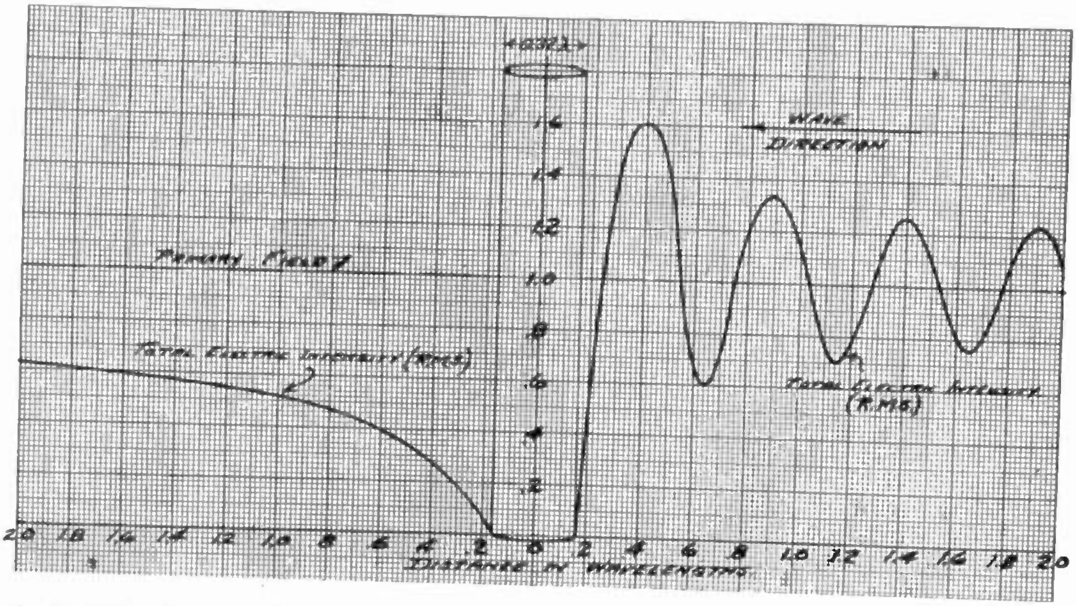

Fig. 5—Diffraction of vertical polarized wave by long vertical cylinder. (Front and back.) Total electric field directly in front of and behind cylinder. See Fig. 6 for intensity at right angles.

This result would be expected from physical reasoning since both primary and secondary waves are traveling in the same direction and no rapid change of phase between these two components can take place. The total field intensity for directions at right angles to the direction of the primary wave is shown in Fig. 6. Here we have an interference phenomena similar to that for the direction directly in front of the cylinder but differing in that the distance between maxima and minima is now approximately a half wavelength rather than a quarter wavelength as in the preceding case. This is in accordance with physical intuition since in this direction the phase between the primary and secondary fields depends only upon the phase of the secondary field so that positions of opposition and addition occur only half as fast as they occurred in the direction in front of the cylinder. This phenomenon is quite similar to the condition when surface waves on the water encounter an obstruction such as a lighthouse on a circular foundation or a government "can" buoy. A close scrutiny of the wave phenomenon near a can buoy in a light breeze when the waves are short will be found very enlightening. The diameter of these buoys is too small compared to a wavelength of water waves, present under other than light breezes, to produce the interference phenomena of interest here.

The current distribution on the surface of the cylinder is shown in Fig. 7. The broken curve shows the phase angle with reference to the phase of the electric force of the primary field at the axis of the cylinder.

Vertical Dipole and Cylinder

In order to show the effect of a conducting cylinder upon the horizontal radiation pattern of a vertical dipole for one range of conditions, a fixed distance of 0.24 wavelength from the axis of the cylinder to the dipole

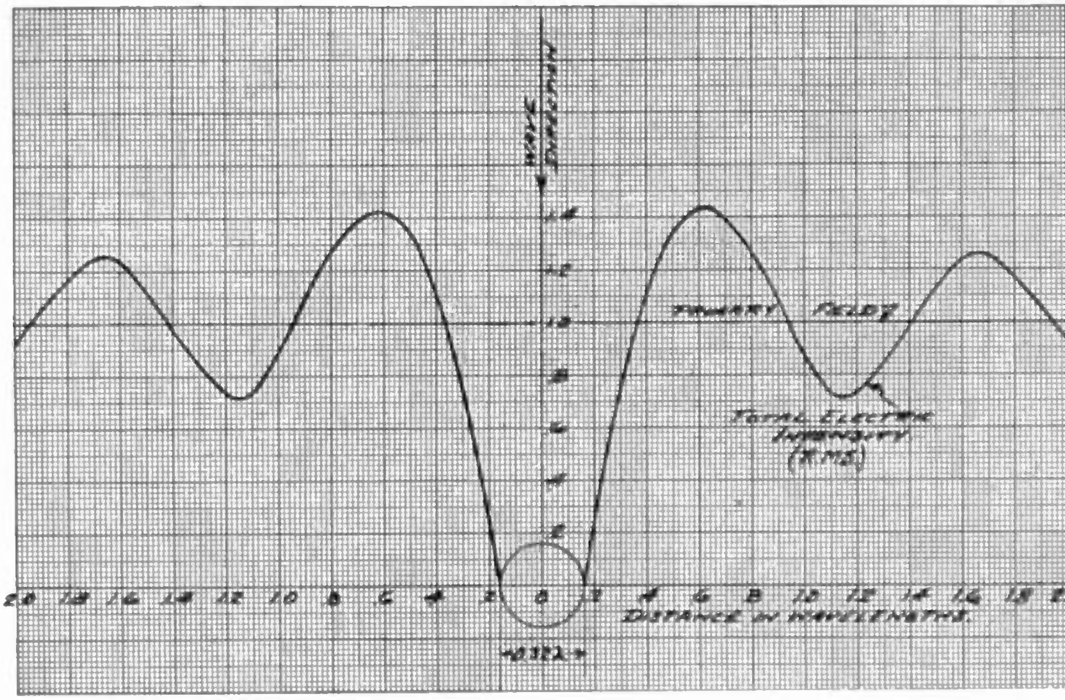

Fig. 6—Diffraction of vertical polarized wave by long vertical cylinder. (At side.) Total electric field in direction at right angles to primary wave direction. See Fig. 5 for intensity in other directions.

has been assumed and the radius of the cylinder allowed to increase from a very small value until the cylinder becomes of such a size as to be almost in contact with the radiator. Fig. 8 shows the pattern when the radius of the cylinder is 0.0016 wavelength, which is equivalent to about 0.4 inch at a frequency of 50 megacycles. It will be noted that the radiation diagram does not differ greatly from a circle. In Fig. 9 the radius of the cylinder has been increased to about 0.03 wavelength. It is seen that the distortion from a circular pattern has considerably increased over that shown in the first-mentioned drawing. However, there is no definite relationship between the geometrical shadow as indicated and the shape of the pattern. In Fig. 10 the radius of the cylinder is 0.08 wavelength. Although there is no indication in the pattern of a demarcation between lighted and shadow regions the radiation is very much less at the middle of the shadow region than in the opposite

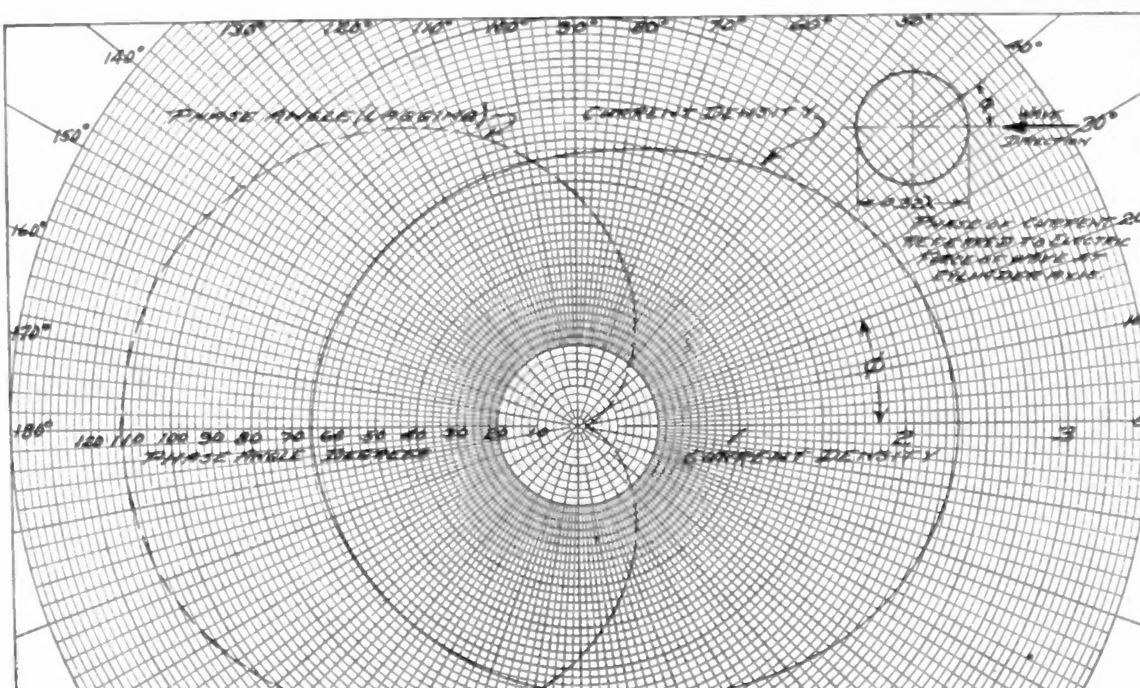

Fig. 7—Current distribution on cylinder for plane wave vertically polarized.

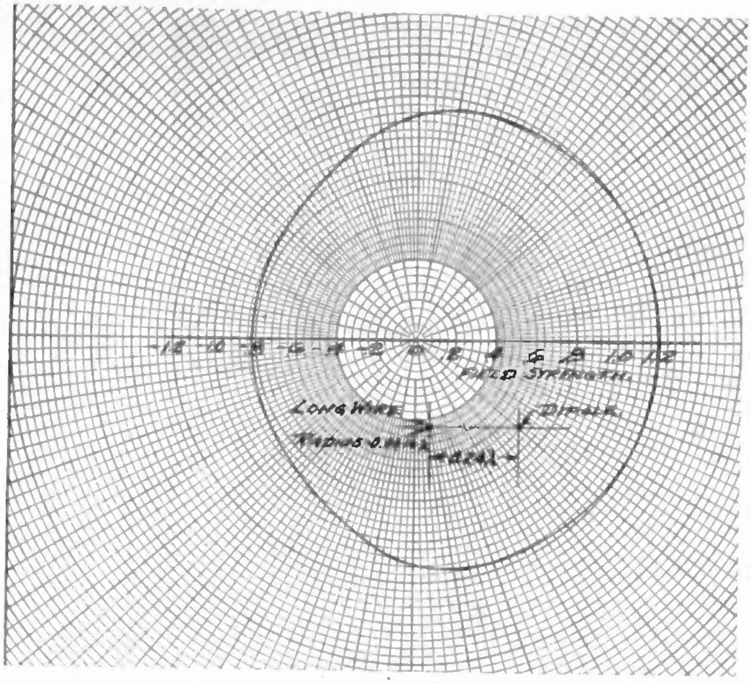

Fig. 8—Vertical dipole and long wire.

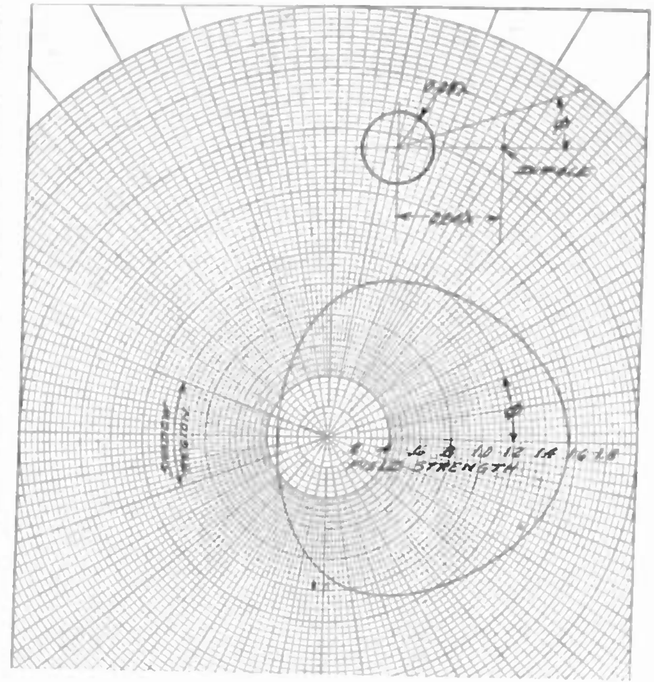

Fig. 10—Vertical dipole and cylinder. (B)

direction. In Fig. 11 the radiation pattern is shown for a cylinder radius of 0.16 wavelength. This pattern is quite unidirectional and the depression in the diagram tends to approximate coincidence with the shadow area. In Fig. 12 the cylinder radius is increased to nearly 0.24 wavelength so that it nearly touches the dipole. The shadow area for this condition covers an angle of 180 degrees and the radiation pattern tends to follow the outline of the shadow area.

It should be emphasized that the general characteristics brought out by the preceding radiation patterns are not necessarily typical. For larger cylinders there are pseudo resonance effects so that under some conditions more radiation may take place in the direction corresponding to the center of the geometric shadow area than in the opposite direction. Also it should not be assumed that the pattern for an array of dipoles can be obtained by direct addition of the diagrams shown. These diagrams only show the absolute magnitude of the field intensity as a function of direction and give no

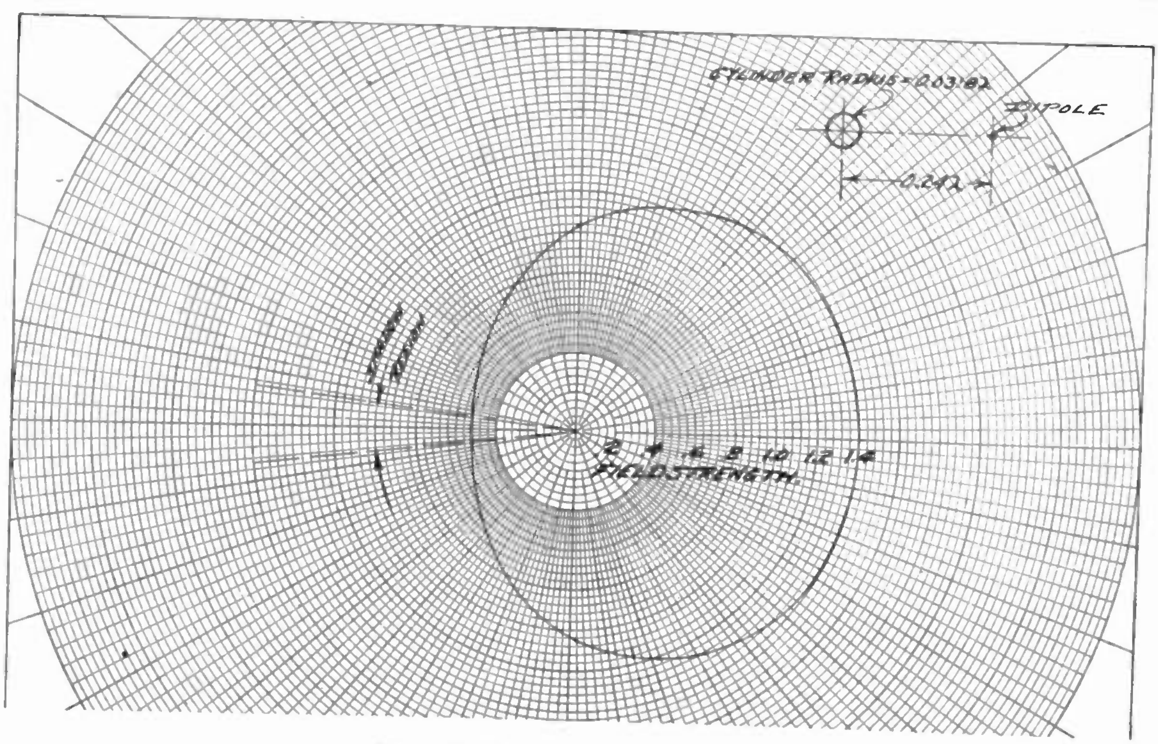

Fig. 9—Vertical dipole and cylinder. (A)

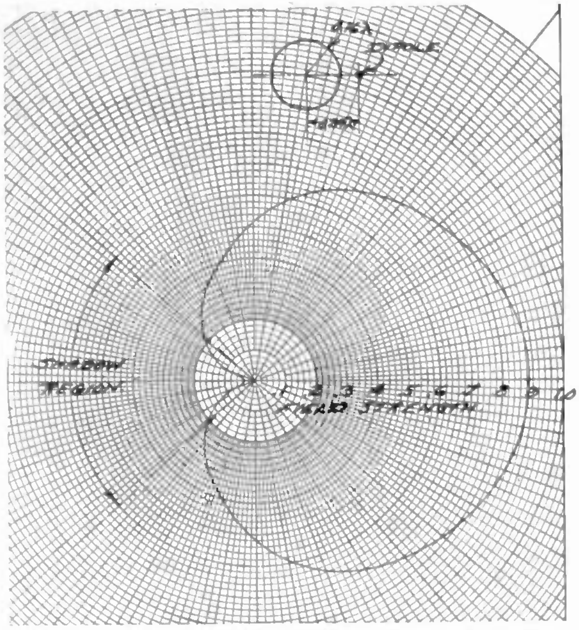

Fig. 11—Vertical dipole and cylinder. (C)

indication of the variation in the phase of the waves with rotation about the center of the cylinder. The phase relations vary in a rather complex manner with direction.

Fig. 13 shows a pattern for a vertical dipole and a cylinder when the radius of the cylinder is 0.383 wavelength and the distance of the dipole from the cylinder axis is 0.878 wavelength. It will be noted that this pattern is quite different from those patterns for a smaller cylinder which we just considered. The radius of this cylinder was purposely chosen of a value such as to result in a zero constant term in the Fourier series expression for the field as a function of direction angle.

Arrays of Vertical Dipoles Around a Cylinder

If to the arrangement of Fig. 13 we add a second dipole in a diametrically opposite position to the original

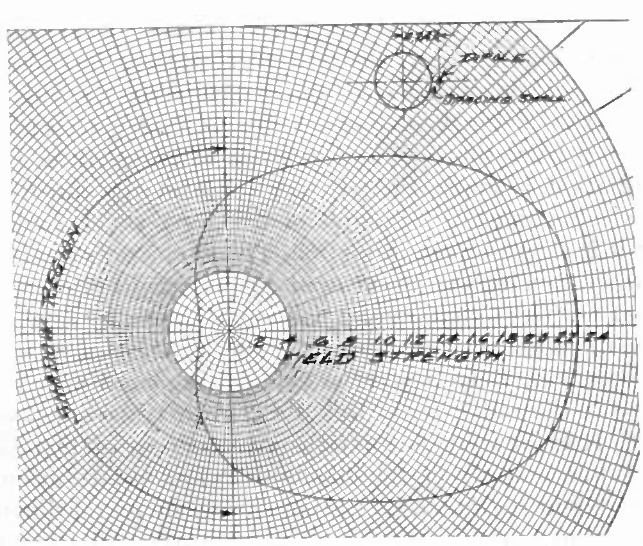

Fig. 12—Vertical dipole and cylinder. (D)

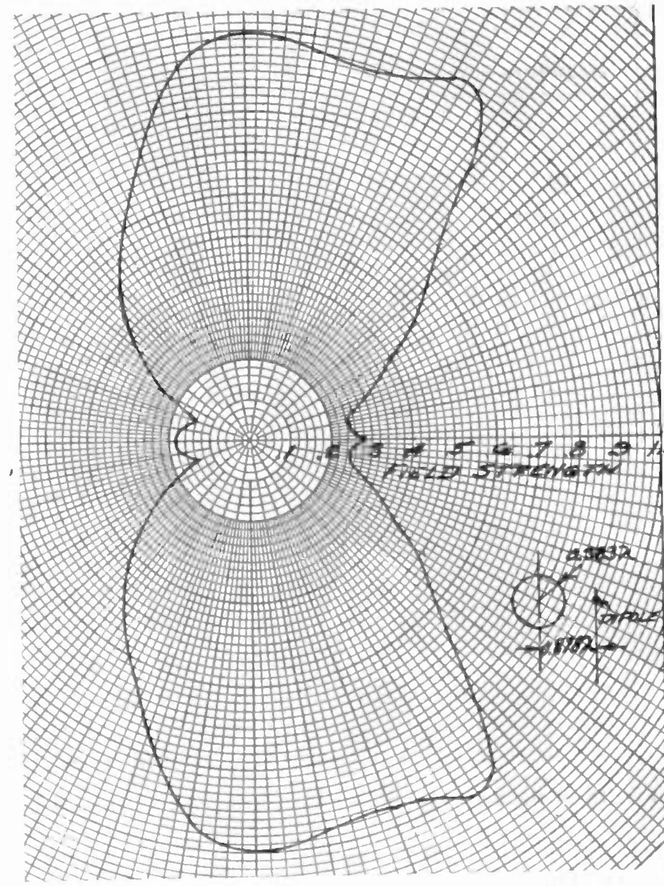

Fig. 13—Vertical dipole and cylinder. Horizontal Pattern.

dipole and feed the two units in phase with each other we obtain the horizontal radiation pattern of Fig. 14. The addition of two more radiators so as to form a square external to the cylinder results in the pattern of

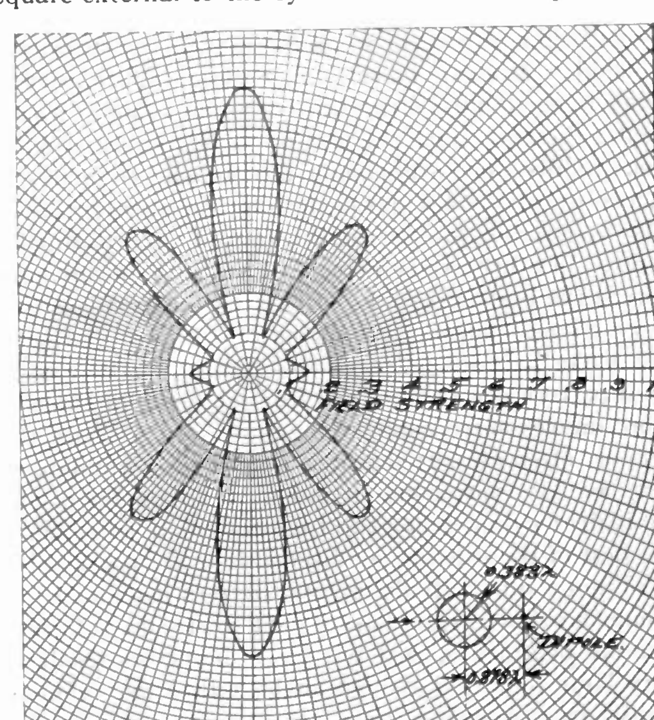

Fig. 14—Two vertical dipoles and cylinder. Dipoles fed in phase. Horizontal pattern.

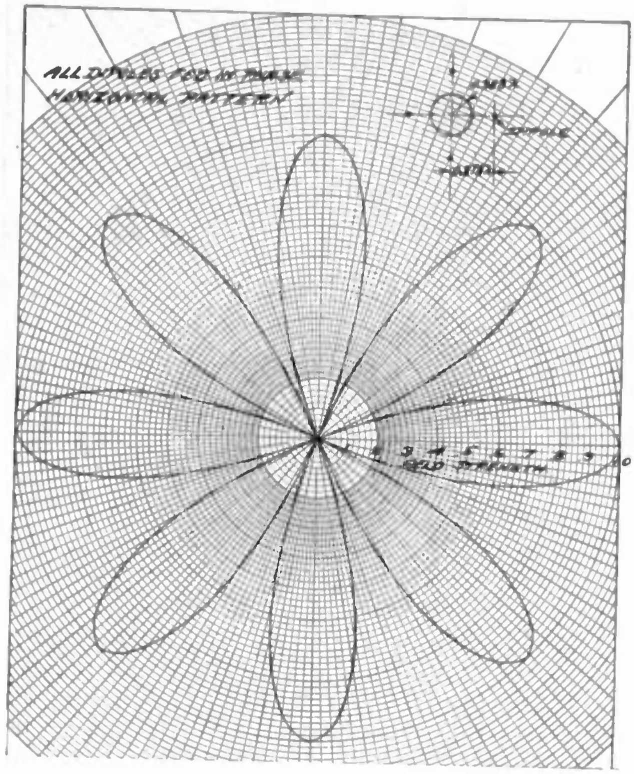

Fig. 15—Four vertical dipoles and cylinder. All dipoles fed in phase. Horizontal pattern.

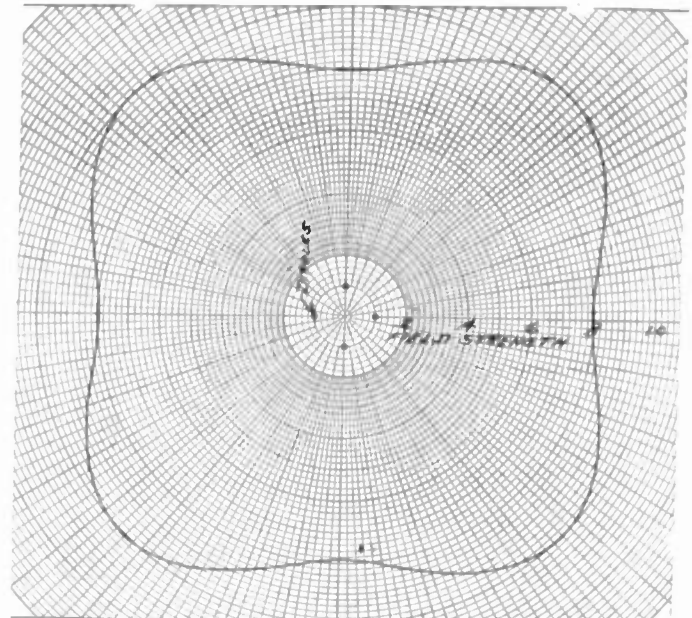

Fig. 16—Field-strength pattern. Four vertical dipoles. Phase rotation.

Fig. 15. There are four ears and four zeros in this pattern and it is almost the same as the radiation characteristic for the same four dipoles without the conducting cylinder.

Fig. 15 is a good illustration of the fact that usually, although not always, if, from the broadcast viewpoint, the pattern for a particular array of dipoles alone is poor it will still be poor when the same array surrounds a cylinder. As a general rule, when designing an antenna array to surround a structure, the units being at a particular distance from the axis of the structure, a sufficient number of units should first be chosen to result in a substantially circular pattern for the array by itself. The best radius for a circular array of vertical dipoles is zero. In other words a single dipole without any support is the best unit that can be used for broadcasting. If we have a choice of the diameter of the support and can make it a small fraction of a wavelength a very good pattern can be obtained from an array of three or more dipoles surrounding the support. In practice the support often already exists as a spire on a building so that we have no choice of either the diameter of the equivalent cylinder nor the frequency and must make the best of the existing conditions. It is always possible to obtain a substantially circular pattern from an array around a cylinder regardless of its diameter if a sufficient number of units is used but large diameters result in very large radiation at high angles unless two or more arrays are arranged in tiers.

When the diameter of the support is not great a useful arrangement for broadcasting vertically polarized waves is an array of 4 dipoles fed in phase rotation. The current induced in the cylinder is negligible when its radius is a small fraction of a wavelength with such an arrangement and if the dipole centers are at a radius of less than $\frac{1}{3}$ wavelength the pattern is circular within ± 10 per cent. Fig. 16 shows the pattern when the dipoles are at a radius of one-quarter wavelength.

Experience shows that in most locations the signal-to-noise ratio is better when horizontal polarization is used in transmission than when vertical polarization is used. For this reason we shall confine our attention primarily to antennas radiating horizontally polarized waves.

Horizontal Dipole and Vertical Cylinder

When a horizontal dipole is placed near a conducting cylinder currents flow around the cylinder and their effect upon the radiation field is quite different than when they flow up and down as in the case of a vertical dipole. Fig. 17 shows the horizontal pattern for a horizontal dipole placed 0.24 wavelength from the axis of a vertical cylinder having a radius of 0.16 wavelength. This pattern shows two phenomena of importance. The radiation is approximately the same in the direction corresponding to the center of the geometrical shadow as it is in the reverse direction. This fact shows how far astray conclusions based upon geometrical optics may lead us. The radiation of a horizontal dipole by itself in the direction corresponding to a continuation of its axis is zero but, when a vertical cylinder is placed near the dipole, the radiation in this direction may be substantial. It is far from zero for the arrangement of Fig. 17.

While a study of the effects of cylinders of various sizes upon the radiation pattern for a horizontal dipole placed at various distances might be interesting much labor is involved in making the necessary computations.

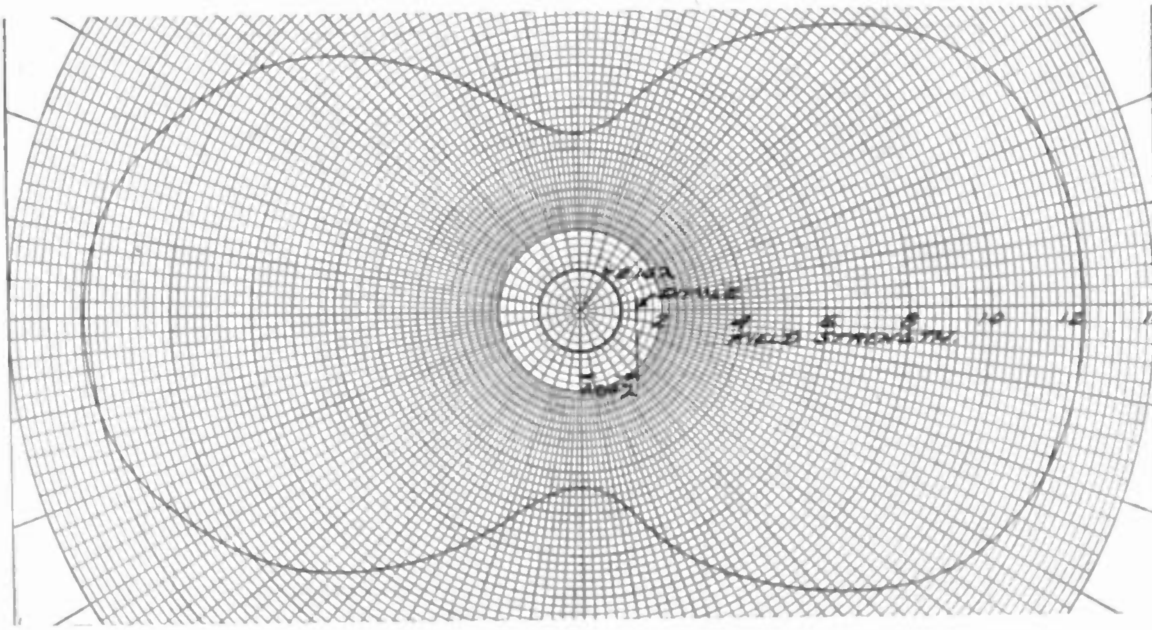

Fig. 17—Horizontal dipole and vertical cylinder. Horizontal pattern.

Since our primary interest here is in the characteristics of the arrays we shall pass on.

Horizontal Dipoles around Cylinders—Dipole Axis Circumferential

A detailed study of arrays of four elements surrounding a cylinder having a radius of 0.637 wavelength or a periphery of 4 wavelengths has been made. It is understood that the spire of the Chrysler Building has approximately this periphery at the frequency of the television transmitter located there. The openings in the spire of this building limited the number of radiators which could be used to four. There are three methods of feeding such an array which are of interest:

1. All the elements may be fed in phase.
2. Diametrically opposite elements may be fed in phase while the pairs of such elements are fed in quarter-phase relation.
3. Diametrically opposite units of a pair may be fed in phase opposition while the pairs are fed in quarter-phase relation.

Before proceeding it is important to have clearly in mind proper definitions of "in phase" and "in phase opposition." For our purposes here we shall define "in phase" as that condition when the instantaneous currents in the various dipoles of the circular array have the same direction when viewed looking toward the axis of the cylinder from a position outside of the cylinder. In other words, if the currents in the four units are in phase and their instantaneous directions are represented by arrows all arrows will point in the same direction if we walk around the cylinder and view them facing the cylinder. This viewpoint is opposite to that ordinarily taken when considering one pair of horizontal dipoles but any other definition in connection with a circular array would cause confusion. In accordance with this definition the third method of feeding mentioned above is equivalent to a phase-rotation feed.

(a) *In-Phase Feed*

Fig. 18 shows the horizontal pattern when the above-stated arrangement of four dipoles and cylinder are fed in phase. The field distribution is far from circular. The dashed curve in the drawing shows the horizontal radiation pattern for the same array without the cylinder. It will be noted that the maxima of one pattern correspond to the mimima of the other.

The horizontal pattern is only one part of the story. We also need to know the distribution in vertical planes.

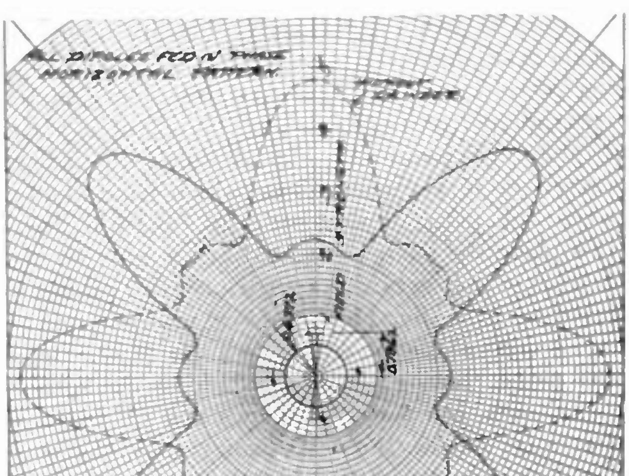

Fig. 18—Four horizontal dipoles and cylinder. (I).

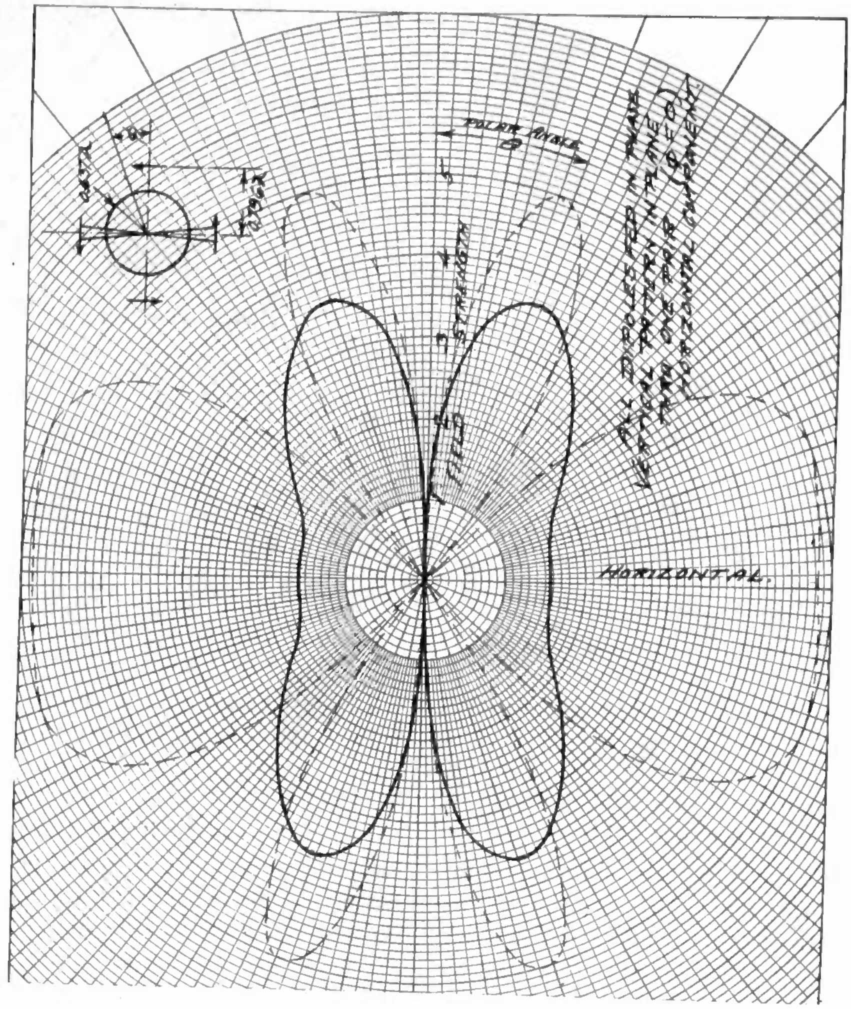

Fig. 19—Four horizontal dipoles and cylinder. (A).

Fig. 19 shows the pattern in a vertical plane containing the centers of two diametrically opposite dipoles. At an angle of 20 degrees to the vertical the field strength is over twice that in the horizontal directions. Thus the energy radiated at this high angle is about five times that radiated horizontally. Fig. 20 shows the radiation characteristic in a vertical plane which passes halfway between the pairs of diametrically opposite dipoles. In this plane the energy radiated at an angle of about 35 degrees to the vertical is approximately $2\frac{1}{2}$ times that radiated horizontally. Fig. 21 shows the radiation pattern in a vertical plane making an angle of 22.5 degrees to the plane passing through a pair of diametrically opposite units. For waste of energy at high angles this is the worst of the three vertical planes, the energy radiated at an angle of 25 degrees to the vertical being more than 7 times that radiated horizontally. If a second tier of units is placed at a height of 0.575 wavelength above the first set no radiation can take place at an angle of 30 degrees to the vertical and the patterns in vertical planes will all become fairly satisfactory. However, the horizontal pattern is, of course, unchanged and is far from ideal for broadcasting.

(b) Two Dipoles of Pair in Phase—Pairs in 90-degree Phase Relation

Fig. 22 shows the horizontal pattern for four dipoles around a cylinder when the feeding system is connected so as to result in this condition. This pattern brings out the result of a peculiar phenomenon which needs explanation. First it should be emphasized that this is *not* the condition of phase rotation feed. At first sight it might seem that the pattern should repeat itself in each quadrant of the circle whereas the pattern shown repeats itself only in each semicircle. It will be noted that the pattern shown in dashed lines for the four units without the cylinder is cyclic for each quadrant. When a dipole is placed near a cylinder, radiation takes place in the direction of the dipole axis even though the radiation in this direction from the dipole alone is zero. The field in this direction is due to currents set up in the surface of the cylinder. When two diametrically opposite dipoles are fed in phase this is also true. The currents responsible for this radiation in the direction of the axes of the dipole pair are not in phase with the dipole currents which cause them. Let us call one pair of opposite dipoles A and the other B, and assume the current in A

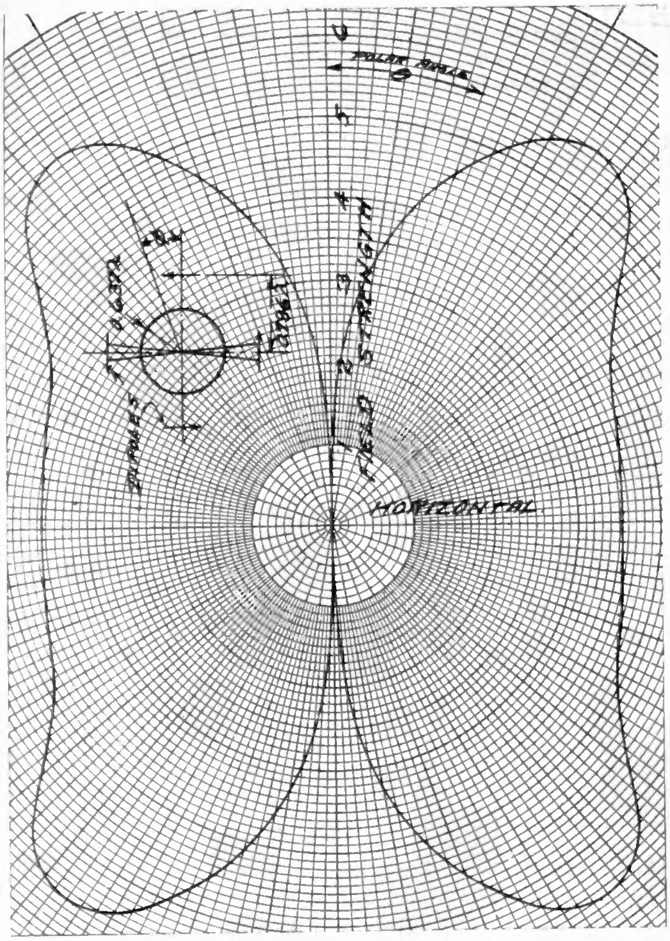

Fig. 20—Four horizontal dipoles and cylinder. (B) All dipoles fed in phase. Vertical pattern. In phase halfway between pairs ($\phi=45$ degrees). Horizontal component.

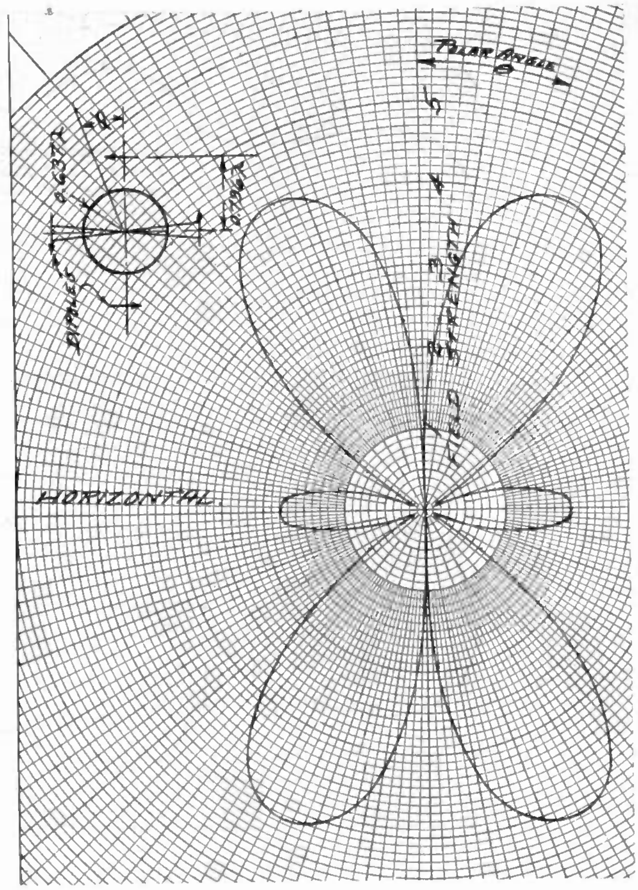

Fig. 21—Four horizontal dipoles and cylinder. (C). All dipoles fed in phase. Vertical pattern in plane at 22.5 degrees to plane of pair. ($\phi=22.5$ degrees.) Horizontal component.

of zero phase and the current in pair B of $+90$ degrees phase. The current in the cylinder responsible for radiation in the direction of the axes of A, as already stated will have a phase angle which we may designate as ψ. The radiation due to pair B in this direction is proportional to the current in B but the current leads that in A by 90 degrees. For this direction the effective current may be written as $I_1=F\angle\psi+j1$. In the direction of the axis of B we then have $I_2=jF\angle\psi+1$. The magnitudes of these two effective currents are not generally equal since $|I_1|^2=1+F^2+2F\sin\psi$ and $|I_2|^2=1+F^2-2F\sin\psi$. Thus the radiation in these two perpendicular directions generally is not equal. In attempting to give a simple explanation for this peculiarity in the pattern we have neglected the current in the cylinder due to pair B which is effective in producing radiation in the direction of the axes of pair A and passed over some other matters rather loosely.

It will be noted that, under the assumed feeding relations, the pattern for the dipoles with the cylinder is much better than for the four dipoles alone. Although the radiation diagram departs considerably from a circle it should be fairly satisfactory for broadcast purposes. In order for such a system to be efficient the radiation at high angles must be reasonably low. That such is not the case will become evident upon inspection of Figs. 23, 24, and 25.

Fig. 23 shows the pattern in a vertical plane passing through the pair of dipoles fed in leading quarter phase. The energy radiated at angles in the vicinity of 20 degrees to the vertical is more than five times that radiated horizontally. In the vertical plane passing halfway between the dipole pairs the energy radiated in a direction 35 degrees to the vertical is nearly three times that radiated horizontally, the pattern for this vertical plane being shown in Fig. 25. The radiation in the vertical plane passing through the pair in lagging phase is a maximum in the vicinity of 60 degrees to the vertical and the energy is nearly twice that radiated horizontally (see Fig. 24). It is thus evident that two tiers are necessary in order to obtain an efficient antenna of this type. If two tiers are spaced vertically by a distance of 0.55 wavelength so as to cancel radiation at and near a vertical angle of 25 degrees an efficient system is obtained.

(c) *Four Horizontal Dipoles Fed in Quarter Phase Rotation*

The horizontal pattern for this condition is shown in Fig. 26. Here we have a pattern which repeats itself in each quadrant but is not symmetrical for right- and left-hand rotation. With this type of feed an observer looking in from outside the cylinder would have no way

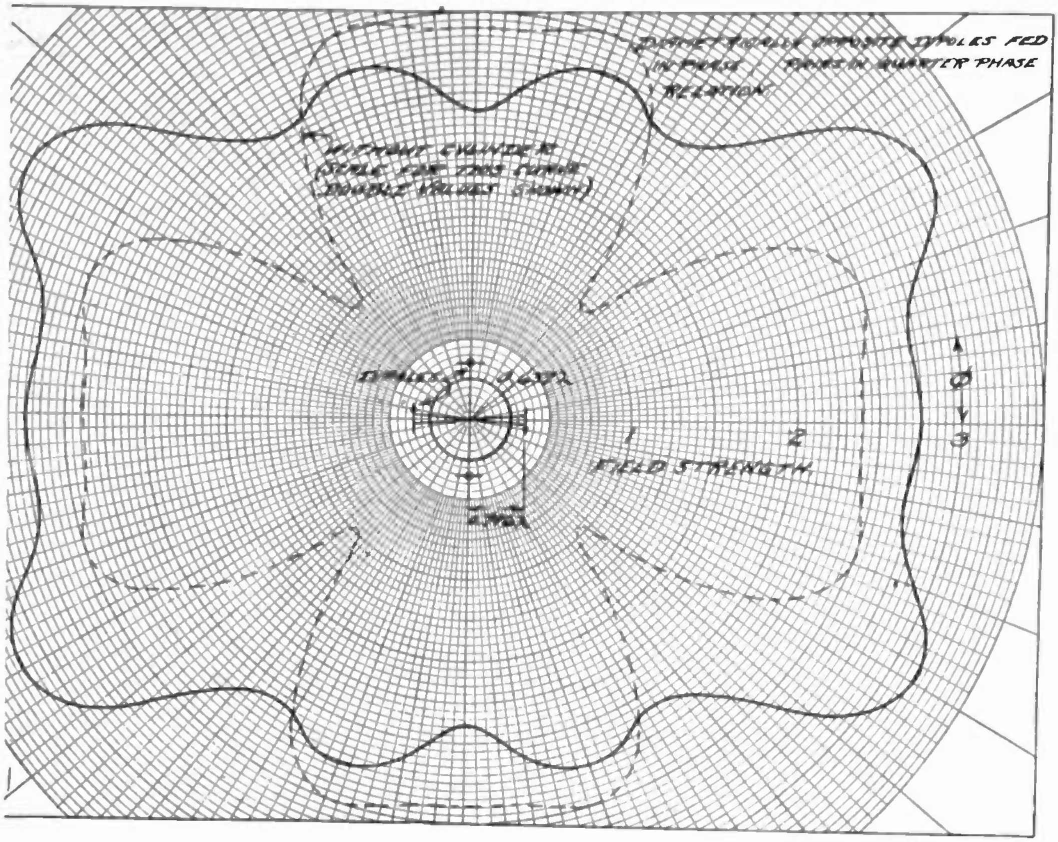

Fig. 22—Four horizontal dipoles and cylinder. (D). Horizontal pattern.

of distinguishing one quadrant from another so that the radiation pattern would necessarily be cyclic for each quadrant. Since such an observer would theoretically be able to distinguish between right- and left-hand phase rotation it would not be unreasonable to expect the pattern to be dissimilar for right- and left-hand rotation. The pattern for the same four dipoles without the cylinder which is shown in dashed lines is symmetrical for right- and left-hand rotation. The horizontal pattern for this method of feed is, from a broadcast viewpoint, considerably inferior to that discussed under (*b*) and we have not taken the trouble to calculate the corresponding vertical patterns for this condition.

Horizontal Dipoles around Cylinder—Dipole Axes Radial

For broadcasting the only useful arrangements of this type are arrays fed in phase rotation. A substantially circular pattern may be obtained with 4 dipoles when the radius of the array is less than 0.14 wavelength. If the inner conductors of concentric transmission lines are allowed to project a short distance outwardly from the cylinder as in Fig. 27 the system is in effect equivalent to an array of dipoles in close proximity to the cylinder.

The horizontal pattern for a 4-phase array like that of the figure when the diameter of the cylinder is 0.48 wavelength is shown in Fig. 28. It will be noted that there is considerable variation in the field strength with direction. For this diameter a 6-unit, 6-phase array would be satisfactory.

General Theory

The majority of engineers probably do not realize that rigorous solutions of the fundamental electromagnetic laws are only possible under a very limited number of highly idealized conditions. Most of the so-called laws of electrical engineering are only approximations which become increasingly more inaccurate as the frequencies involved become higher and higher or, more specifically, as the sizes of the elements concerned become greater in terms of the wavelengths involved. At the other extreme we have the laws of geometrical optics which are reasonably accurate only when the dimensions concerned are of the order of hundreds of wavelengths or greater. The conditions in many of the problems connected with ultra-high-frequency radio practice lie within the region between the two extremes mentioned and neither electrical engineering methods nor the laws of geometrical optics can be relied upon to predict results. The only safe procedure then is either to try to find a solution of Maxwell's equations under idealized conditions which approximate actual conditions or to forget all theory and rely entirely upon experimental measurements.

If all the currents in a system of conductors are known the fields throughout all space may, theoretically at least, be immediately determined. For many antenna problems we have a fairly reliable knowledge of the currents and make use of this principle. For example we assume the current in a thin-wire half-wave dipole to be of sine-wave form and upon this assumption determine

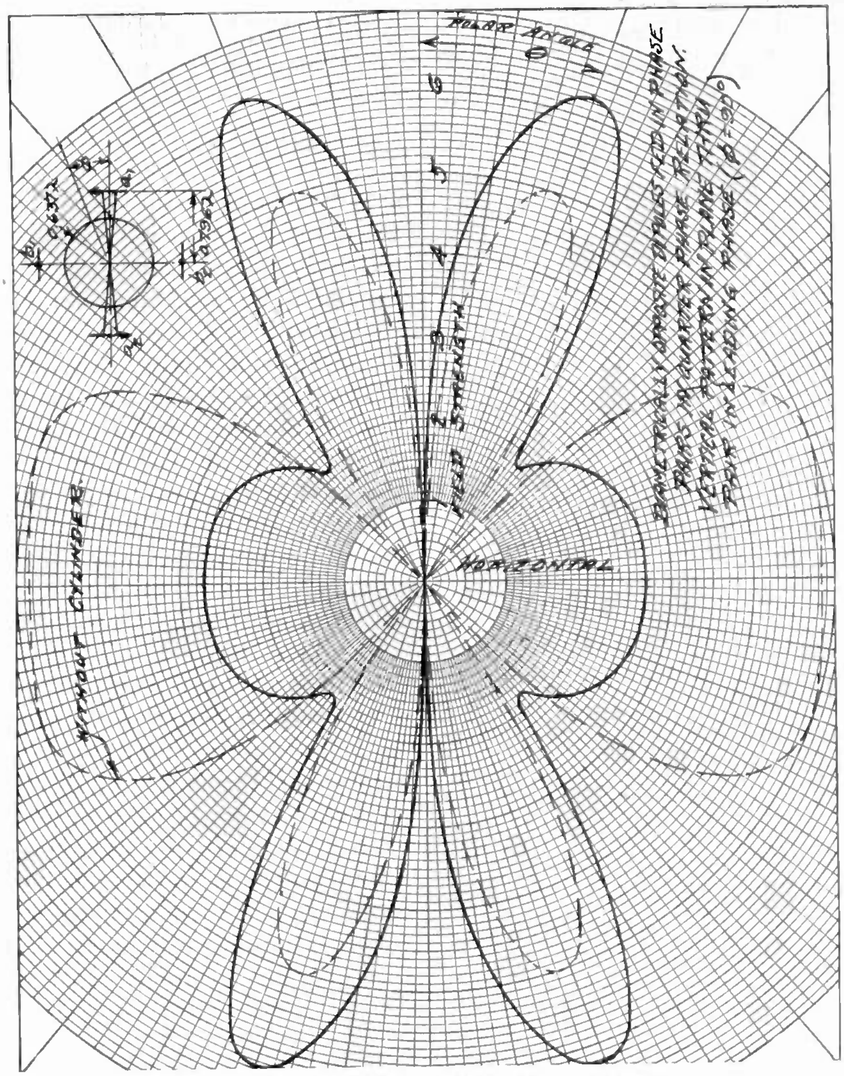

Fig. 23—Four horizontal dipoles and cylinder. (E).

the fields. Both rigorous theory and measurements indicate that such an assumption is sufficiently accurate for most purposes. When a dipole is located near a cylinder we are in complete ignorance of the currents flowing on the cylinder surface so that this method is useless.

All electromagnetic fields in space must be waves consistent with Maxwell's laws. If we assume all bodies to be perfectly conducting, a legitimate assumption for high frequencies since the radiated energy is usually many times the energy converted into heat, the tangential component of electric force at the surfaces of the bodies must be zero since a perfect conductor can support no voltage. This knowledge alone must serve as the foundation for the solution of this type of problem.

Consider a horizontal dipole located near a very tall conducting cylinder. For a given current in the dipole we may determine the field everywhere in space due to the dipole alone and will henceforth call this the primary field. This primary field is the driving force for currents around the conducting cylinder. These currents in the cylinder set up a secondary electric field in a more or less similar manner to the counter electromotive force in an electric motor. This secondary electric force must

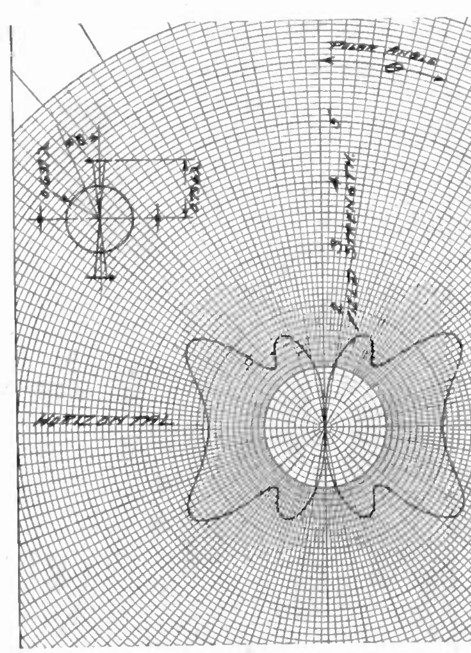

Fig. 24—Four horizontal dipoles and cylinder. (F). Diametrically opposite dipoles fed in phase. Pairs in quarter-phase relation. Vertical pattern in plane through lagging pair. (ϕ=0.)

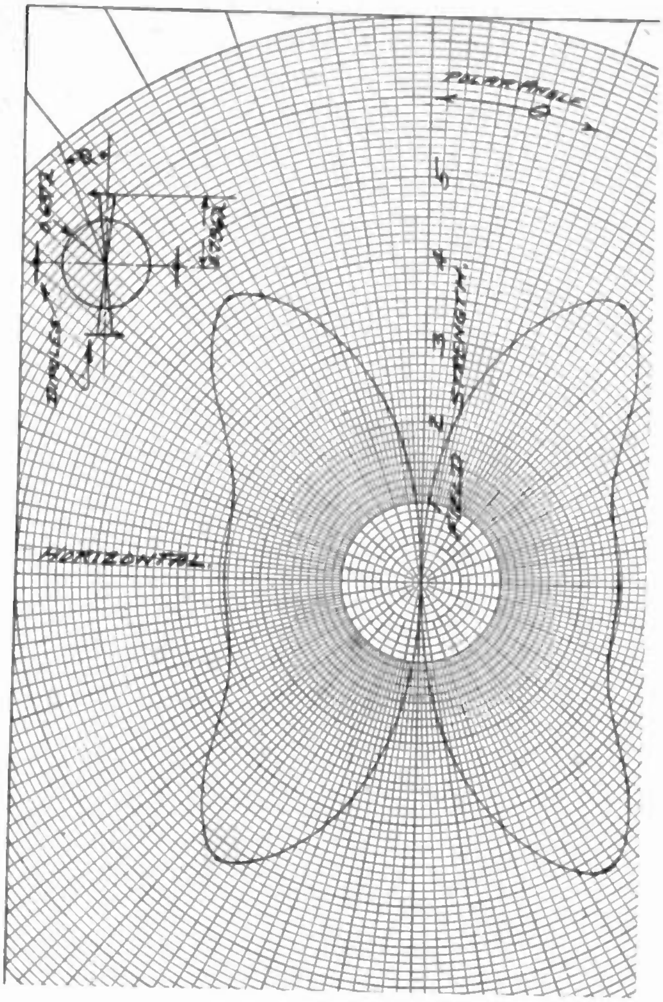

Fig. 25—Four horizontal dipoles and cylinder. (G). Diametrically opposite dipoles fed in phase. Pairs in quarter-phase relation. Vertical pattern in plane halfway between pairs. ($\phi = 45$ degrees.)

exactly counterbalance the primary electric force everywhere on the surface of the cylinder and result in zero total electric force tangential to the cylinder. In accordance with Maxwell's laws this secondary field must be a constituent of a secondary system of electromagnetic waves diverging outwardly from the cylindrical surface. The electric and magnetic forces of this system of waves must follow the laws of cylindrical symmetry; i.e., the waves are cylindrical. The secondary electromagnetic field may be expressed as the sum of a number of waves, the intensity of the field of each one of which must undergo a cyclic variation expressed in terms of the sine and/or cosine of an integral multiple of the horizontal direction angle, and at the same time must vary with radial distance in a manner consistent with the fundamental electromagnetic laws.

In order to determine the amplitude of each of the secondary waves it is first necessary to break up the primary field from the dipole into a number of cylindrical waves having the axis of the cylinder as their axis of symmetry. When the mathematical expression for the primary field is so expanded the result is an infinite series wherein each term contains an infinite integral. In other words we have an infinite sum of an infinite sum of waves of infinitesimal amplitude. To avoid the

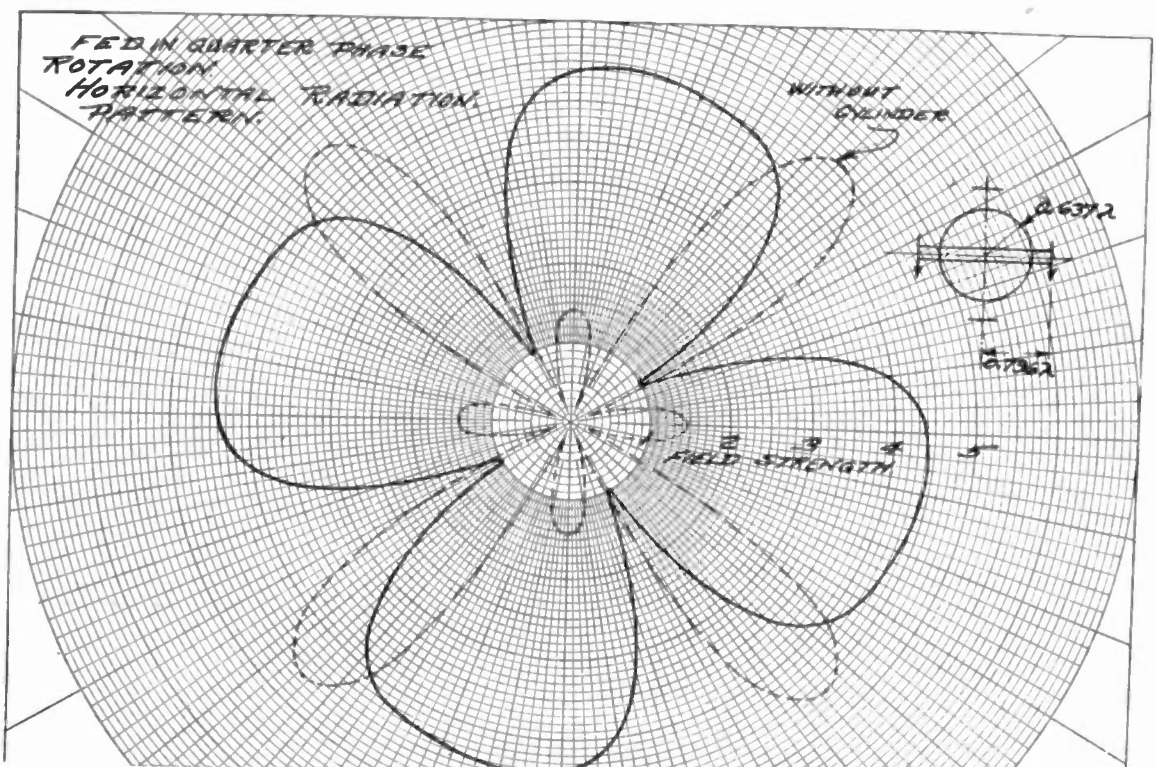

Fig. 26—Four horizontal dipoles and cylinder. (H).

difficulties inherent in working with infinite integrals we may make use of the reciprocity law in the following way: Imagine a dipole (1) located at a great distance from the cylinder and a second dipole (2) located near the cylinder. The wave from (1) is essentially plane upon arrival at the cylinder. In accordance with the reciprocity law if a current of 1 ampere in (1) results in a certain voltage at the terminals of (2) we may interchange the position of current and voltage without altering the result. The plane traveling wave from (1) may be expressed as an infinite sum of standing cylindrical waves. This series is devoid of infinite integrals. Now, by equating the sum of the primary and secondary tangential electric forces of each wave of the infinite series of waves to zero at the surface of the cylinder we obtain the amplitudes of each of the secondary electromagnetic waves. Then by applying the reciprocity principle we obtain an expression which gives us the total radiation field from the dipole and cylinder. If we locate dipole (2) in various positions around the cylinder and apply the reciprocity principle we may immediately obtain the directive pattern for a dipole located at any specified position near the cylinder. Having obtained the expression for the field due to one dipole and the cylinder we may add the fields due to any number of other dipoles including the effect of the cylinder, these additional dipoles being fed in any desired phase relation and located at any specified position and thus obtain the over-all radiation characteristic.

In making use of the reciprocity theorem to avoid infinite integrals in the series we lose a knowledge of currents on the cylinder surface. However, if we assume a high stack of closely spaced dipoles, rather than a single unit, the expression for the primary field of the near-by source when expanded into a Fourier series is

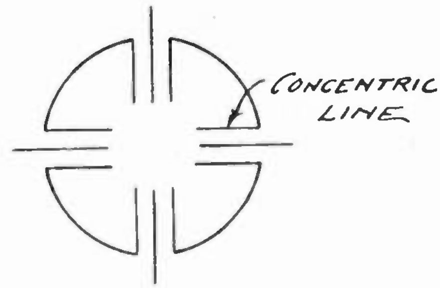

Fig. 27—Radial radiators and cylinder.

devoid of infinite integrals. The cylinder currents may then be obtained without difficulty.

As described the processes would appear to be simple straightforward procedures. Unfortunately, the mathematical technique involved is considerable. The mathematical development shown should be in sufficient detail to be clear to one having some familiarity with the vector calculus, partial differential equations and the theory of Bessel functions. A bibliography of the publications referred to by the author is given. There are, no doubt, many other references but only those available to the author are given.

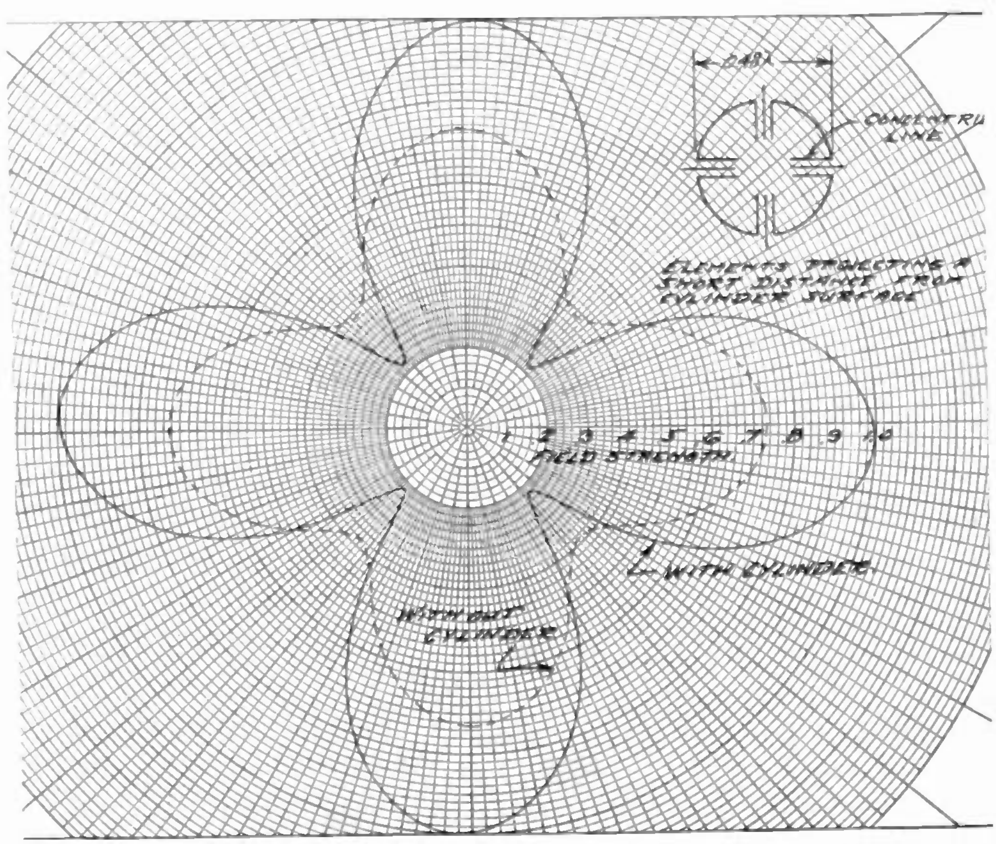

Fig. 28—Four radial horizontal dipoles and cylinder units fed in phase rotation.

Formulas for Radiation Patterns—Arrays Around Cylinders

The following table of formulas covers most arrays likely to be used in practice.

Symbols

a = radius of cylinder
b = radius of radiator circle
$k = 2\pi/\lambda$
$j = \sqrt{-1}$
$J_n(\ )$ = nth order Bessel function of first kind
$U_n(\ )$ = nth order Hankel function of second kind
$J_n'(\ )$, $U_n'(\ )$ = derivatives with respect to their arguments
ϵ = Newman's number = 1 for $n=0$ and 2 for $n=2, 3, 4$, etc.
θ = polar angle to vertical axis
ϕ = horizontal angle (longitude)
E_ϕ = electric field in direction ϕ (horizontal)
E_θ = electric field in direction θ (vector in vertical plane)
V_n, W_n, B_n, and D_n are expressions involving nth order Bessel functions as given with the single-unit formulas for each type of array

Horizontal Dipoles—Axes Circumferential

One Unit

$$E_\phi = -\left[jW_0 + 2j\sum_{n=0}^{\infty}(j)^n W_n \cos n\phi\right] \quad (1)$$

where

$$W_n = J_n'(kb \sin\theta) - \{J_n'(ka\sin\theta)/U_n'(ka\sin\theta)\} U_n'(kb\sin\theta)$$

$$E_\theta = (-j2\cos\theta/kb\sin\theta)\sum_{n=1}^{\infty}(j)^n\cdot n\cdot V_n \sin n\phi. \quad (2)$$

where

$$V_n = [J_n(kb\sin\theta) - \{J_n(ka\sin\theta)/U_n(ka\sin\theta)\} U_n(kb\sin\theta)]$$

Two Units—Diametrically Opposite, In Phase

$$E_\phi = -2j\{W_0 - 2W_2\cos 2\phi + 2W_4\cos 4\phi - \cdots\} \quad (3)$$
$$E_\theta = (j4/kb\sin\theta)\{2V_2\sin 2\phi - 4V_4\sin 4\phi + 6V_6\sin 6\phi - \cdots\}. \quad (4)$$

Two Units—Phase Opposition

$$E_\phi = 4\{W_1\cos\phi - W_3\cos 3\phi + W_5\cos 5\phi - \cdots\} \quad (5)$$
$$E_\theta = (4\cos\theta/kb\sin\theta)\{V_1\sin\phi - 3V_3\sin 3\phi + 5V_5\sin 5\phi - \cdots\}. \quad (6)$$

Three Units—In Phase

$$E_\phi = -j3\{W_0 - j2W_3\cos 3\phi - 2W_6\cos 6\phi + \cdots + (j)^{3n}\cdot 2W_{3n}\cos 3n\phi + \cdots\} \quad (7)$$
$$E_\theta = (-6\cos\theta/kb\sin\theta)\{3W_3\sin 3\phi - j6V_6\sin 6\phi - 9V_9\sin 9\phi + \cdots + 3n(j)^{3n}V_{3n}\sin 3n\phi + \cdots\} \quad (8)$$

Three Units—Phase Rotation (3-Phase)

$$E_\phi = 3\exp(j\phi)\{W_1 + jW_2\exp(-j3\phi) - jW_4\exp(j3\phi) + W_5\exp(-j6\phi) - W_7\exp(j6\phi) + \cdots\} \quad (9)$$
$$E_\theta = -j(3\cos\theta/kb\sin\theta)\exp(j\phi)\{V_1 - j2V_2\exp(-j3\phi) + j4V_4\exp(j3\phi) - 5V_5\exp(-j6\phi) - V_7\exp(j6\phi) + \cdots\} \quad (10)$$

Four Units—In Phase

$$E_\phi = -j4\{W_0 + 2W_4\cos 4\phi + 2W_8\cos 8\phi + \cdots\} \quad (11)$$
$$E_\theta = (-j8\cos\theta/kb\sin\theta)\{4V_4\sin 4\phi + 8V_8\sin 8\phi + 12V_{12}\sin 12\phi + \cdots\}. \quad (12)$$

Four Units—Phase Rotation (4-Phase)

$$E_\phi = 4\exp(j\phi)\{W_1 - W_3\exp(-j4\phi) + W_5\exp(j4\phi) - W_7\exp(-j8\phi) + W_9\exp(j8\phi) - \cdots\} \quad (13)$$
$$E_\theta = (-j4\cos\theta/kb\sin\theta)\exp(j\phi)\{V_1 + 3V_3\exp(-j4\phi) + 5V_5\exp(j4\phi) + 7V_7\exp(-j8\phi) + 9V_9\exp(j8\phi) + \cdots\} \quad (14)$$

Four Units—Units of Pair in Phase, Pair in Quarter Phase

$$E_\phi = -j2\sqrt{2}\exp(j\pi/4)\{W_2\cos 2\phi + j2W_4\cos 4\phi + 2W_6\cos 6\phi + \cdots\} \quad (15)$$
$$E_\theta = (4\sqrt{2}/kb\sin\theta)\exp(j\pi/4)\cos\theta\{2V_2\sin 2\phi - j4V_4\sin 4\phi + 6V_6\sin 6\phi - \cdots\}. \quad (16)$$

Six Units—In Phase

$$E_\phi = -6j\{W_0 + 2W_6\cos 6\phi + 2W_{12}\cos 12\phi + \cdots\} \quad (17)$$
$$E_\theta = (-j12\cos\theta/kb\sin\theta)\{6V_6\sin 6\phi + 12V_{12}\sin 12\phi + \cdots\}. \quad (18)$$

Six Units—Phase Rotation (6-Phase)

$$E_\phi = 6\exp(j\phi)\{W_1 + W_5\exp(-j6\phi) + W_7\exp(j6\phi) + W_{11}\exp(-j12\phi) + W_{13}\exp(j12\phi) + \cdots\} \quad (19)$$
$$E_\theta = (-j6\cos\theta/kb\sin\theta)\exp(j\theta)\{V_1 - 5V_5\exp(-j6\phi) + 7V_7\exp(j6\phi) - 11V_{11}\exp(-j12\phi) + 13V_{13}\exp(j12\phi) + \cdots\}. \quad (20)$$

Eight Units—In Phase

$$E_\phi = -j8\{W_0 + 2W_8\cos 8\phi + 2W_{16}\cos 16\phi + \cdots\} \quad (21)$$
$$E_\theta = (-j8\cos\theta/kb\sin\theta)\{8V_8\sin 8\phi + 16V_{16}\sin 16\phi + \cdots\}. \quad (22)$$

Eight Units—Phase Rotation (8-Phase)

$$E_\phi = 8\exp(j\phi)\{W_1 - W_7\exp(-j8\phi) + W_9\exp(j8\phi) - W_{15}\exp(-j16\phi) + W_{17}\exp(j16\phi) + \cdots\} \quad (23)$$
$$E_\theta = (-8j\cos\theta/kb\sin\theta)\exp(j\phi)\{V_1 + 7V_7\exp(-j8\phi) + 9V_9\exp(j8\phi) + 15V_{15}\exp(-j16\phi) + 17V_{17}\exp(j16\phi) + \cdots\}. \quad (24)$$

Horizontal Dipoles—Axes Radial

Single Unit

$$E_\phi = (-j2/kb\sin\theta)\sum_{n=1}^{\infty}(j)^n\cdot n\cdot B_n\sin n\phi \quad (25)$$

where

$$B_n = [J_n(kb\sin\theta) - \{J_n'(ka\sin\theta)/U_n'(ka\sin\theta)\} U_n(kb\sin\theta)]$$

$$E_\theta = j\cos\theta\left[D_0 + 2\sum_{n=1}^{\infty}(j)^n D_n\cos n\phi\right] \quad (26)$$

where

$$D_n = [J_n'(kb\sin\theta) - \{J_n(ka\sin\theta)/U_n(ka\sin\theta)\} U_n'(kb\sin\theta)].$$

Two Units—In Phase

$$E_\phi = (j4/kb\sin\theta)\{2B_2\sin 2\phi - 4B_4\sin 4\phi + 6B_6\sin 6\phi - \cdots\} \quad (27)$$

$$E_\theta=j2\cos\theta\{D_0+2D_2\cos 2\phi+2D_4\cos 4\phi+\cdots\}. \quad (28)$$

Two Units—Phase Opposition

$$E_\phi=(4/kb\sin\theta)\{B_1\sin\phi-3B_3\sin 3\phi+5B_5\sin 5\phi-\cdots\} \quad (29)$$

$$E_\theta=-4\cos\theta\{D_1\cos\phi-D_3\cos 3\phi+D_5\cos 5\phi-\cdots\} \quad (30)$$

Three Units—In Phase

$$E_\phi=(6/kb\sin\theta)\{3B_3\sin 3\phi+j6B_6\sin 6\phi+9B_9\sin 9\phi+12B_{12}\sin 12\phi+\cdots\} \quad (31)$$

$$E_\theta=j3\cos\theta\{D_0-j2D_3\cos 3\phi-2D_6\cos 6\phi+j2D_9\cos 9\phi+\cdots\}. \quad (32)$$

Three Units—Phase Rotation (3-Phase)

$$E_\phi=(-j3/kb\sin\theta)\exp(j\phi)\{B_1-j2B_2\exp(-j3\phi)-j4B_4\exp(j3\phi)-5B_5\exp(-j6\phi)-7B_7\exp(j6\phi)-j\cdots\} \quad (33)$$

$$E_\theta=-3\cos\theta\exp j\phi\{D_1+jD_2\exp(-j3\phi)-jD_4\exp(j3\phi)+D5\exp(-j6\phi)D_7\exp(j6\phi)+j\cdots\}. \quad (34)$$

Four Units—In Phase

$$E_\phi=(-j8/kb\sin\theta)\{4B_4\sin 4\phi+8B_8\sin 8\phi+\cdots\} \quad (35)$$

$$E_\theta=j4\cos\theta\{D_0+2D_4\cos 4\phi+2D_8\cos 8\phi+\cdots\}. \quad (36)$$

Four Units—Units of Pair in Phase, Pairs in Quarter Phase

$$E_\phi=(4\sqrt{2}\,\epsilon^{+j(\pi/4)}/kb\sin\theta)\{2B_2\sin 2\phi-j4B_4\sin 4\phi+6B_6\sin 6\phi-j8B_8\sin 8\phi+\cdots\} \quad (37)$$

$$E_\theta=j2\sqrt{2}\,\epsilon^{j(\pi/4)}\cos\theta\{D_0+j2D_2\cos 2\phi+2D_4\cos 4\phi+j\cdots\}. \quad (38)$$

Six Units—In Phase

$$E_\phi=(-j12/kb\sin\theta)\{6B_6\sin 6\phi+12B_{12}\sin 12\phi+\cdots\} \quad (39)$$

$$E_\theta=j6\cos\theta\{D_0+2D_6\cos 6\phi+2D_{12}\cos 12\phi+\cdots\}. \quad (40)$$

Six Units—Phase Rotation (6-Phase)

$$E_\phi=(-j6/kb\sin\theta)\exp(j\phi)\{B_1-5B_5\exp(-j6\phi)-7B_7\exp(j7\phi)+\cdots\} \quad (41)$$

$$E_\theta=-6\cos\theta\exp(j\phi)\{D_1+D_5\exp(-j6\phi)+D_7\exp(j7\phi)+\cdots\} \quad (42)$$

Eight Units—In Phase

$$E_\phi=(-j16/kb\sin\theta)\{8B_8\sin 8\phi+16B_{16}\sin 16\phi+\cdots\} \quad (43)$$

$$E_\theta=j8\cos\theta\{D_0+2D_8\cos 8\phi+2D_{16}\cos 16\phi+\cdots\} \quad (44)$$

Eight Units—Phase Rotation (8-Phase)

$$E_\phi=(-j8/kb\sin\theta)\exp(j\phi)\{B_1+7B_7\exp(-j7\phi)+9B_9\exp(-j7\phi)+15B_{15}\exp(j16\phi)+\cdots\} \quad (45)$$

$$E_\theta=8\cos\theta\exp(j\phi)\{D_1+D_7\exp(-j7\phi)+D_9\exp(-j7\phi)+D_{15}\exp(-j16\phi)+\cdots\}. \quad (46)$$

Vertical Dipoles

One Unit

$$E_\theta=\sin\theta\left[V_0+2\sum_{n=1}^{\infty}(j)^nV_n\cos n\phi\right] \quad (47)$$

where

$$V_n=[J_n(kb\sin\phi)-\{J_n(ka\sin\theta)/U_n(ka\sin\theta)\}U_n(kb\sin\theta)].$$

Two Units—In Phase

$$E_\theta=2\sin\theta[V_0+2V_2\cos 2\phi+2V_4\cos 4\phi+\cdots]. \quad (48)$$

Two Units—Phase Opposition

$$E_\theta=j4\sin\theta[V_1\cos\phi-V_3\cos 3\phi+V_5\cos 5\phi-\cdots]. \quad (49)$$

Three Units—In Phase

$$E_\theta=3\sin\theta[V_0+2V_2\cos 2\phi+2V_4\cos 4\phi+\cdots]. \quad (50)$$

Three Units—Phase Rotation

$$E_\theta=3\sin\theta\exp(j\phi)[V_1+jV_2\exp(-j3\phi)-jV_4\exp(i3\phi)+V_5\exp(-j6\phi)-V_7\exp(j7\phi)+\cdots] \quad (51)$$

Four Units—In Phase

$$E_\theta=4\sin\theta\{V_0+2V_4\cos 4\phi+2V_8\cos 8\phi+\cdots\}. \quad (52)$$

Four Units—Phase Rotation (4-Phase)

$$E_\theta=j4\sin\theta\exp(j\phi)\{V_1-V_3\exp(-j4\phi)+V_5\exp(j4\phi)-V_7\exp(-j8\phi)+V_9\exp(j8\phi)-\cdots\}. \quad (53)$$

Four Units—Units of Pair in Phase, Pairs in Quarter Phase

$$E_\theta=2\sqrt{2}\exp j(\pi/4)\sin\theta\{V_2\cos 2\phi+j2V_4\cos 4\phi+2V_6\cos 6\phi+j2V_8\cos 8\phi+\cdots\}. \quad (54)$$

Six Units—In Phase

$$E_\theta=6\sin\theta\{V_0+2V_6\cos 6\phi+2V_{12}\cos 12\phi+\cdots\}. \quad (55)$$

Six Units—Phase Rotation (6-Phase)

$$E_\theta=j6\sin\theta\exp(j\phi)\{V_1+V_5\exp(-j6\phi)+V_7\exp(j6\phi)+V_{11}\exp(-j12\phi)+V_{13}\exp(j12\phi)+\cdots\}. \quad (56)$$

Eight Units—In Phase

$$E_\theta=8\sin\theta\{W_0+2W\cos 8\phi+2W_{16}\cos 16\phi+\cdots\}. \quad (57)$$

Eight Units—Phase Rotation (8-Phase)

$$E_\theta=j8\sin\theta\exp(j\phi)\{V_1-V_7\exp(-j8\phi)+V_9\exp(j8\phi)-V_{15}\exp(-j16\phi)+V_{17}\exp(j16\phi)+\cdots\}. \quad (58)$$

Multitier Arrays of All Types

For an array of n tiers fed in phase multiply the field from one tier by $\{\sin(n/2kS\cos\theta)/\sin(kS/2\cos\theta)\}$, where S is the spacing and θ the angle to the vertical.

Mathematical Theory

Fig. 29 shows the system of co-ordinates which will be used and is self-explanatory. For convenience a polar angle θ, properly belonging to a spherical co-ordinate system, is used together with the usual cylindrical co-ordinates. In order to carry out the method of solution which has been outlined we must first start with the fundamental differential equations of electromagnetic theory. If, in accordance with standard engineering practice, we let the time function be represented by the real part of $\exp(j\omega t)$ the fundamental laws[1] for free space become

$$c\,\text{curl}\,\overline{H}=(\delta\overline{E}/\delta t)\ \text{or}\ \overline{E}=(-j/k)\,\text{curl}\,\overline{H} \quad (59)$$

$$c\,\text{curl}\,\overline{E}=(-\delta\overline{H}/\delta t)\ \text{or}\ \overline{H}=(j/k)\,\text{curl}\,\overline{E} \quad (60)$$

$$\text{div}\,\overline{E}=\text{div}\,\overline{H}=0 \quad (61)$$

where $\overline{E}$ = electric vector
$\overline{H}$ = magnetic vector

[1] Heaviside-Lorentz units are used for simplicity.

c = velocity of light $=3\times10^{10}$ centimeters per second
ω = angular frequency
$j=\sqrt{-1}$
and $k=\omega/c=2\pi/\lambda$, λ being the wavelength in centimeters.

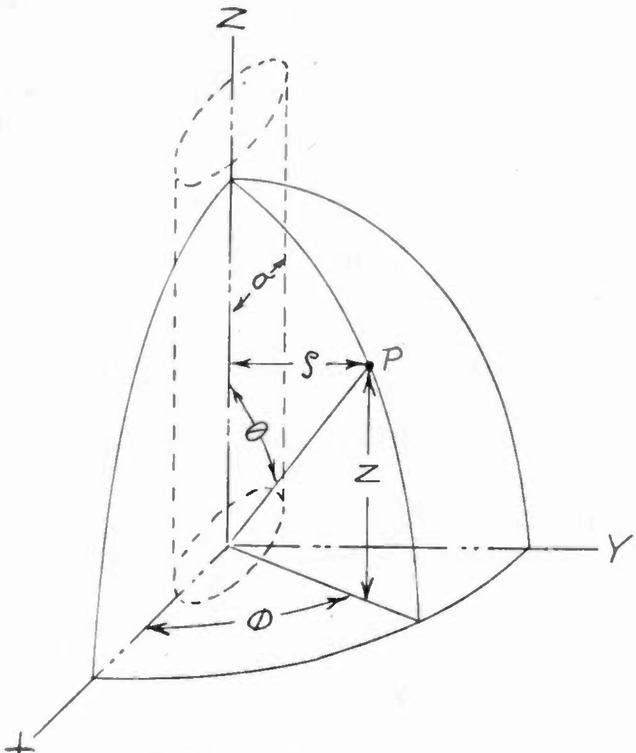

Fig. 29—Co-ordinate system used.

By differentiating and combining the above relations we obtain the vector-wave equations

$$\nabla^2\overline{E} + k^2\overline{E} = 0 \qquad (62)$$

$$\nabla^2\overline{H} + k^2\overline{H} = 0. \qquad (63)$$

Let us first consider the scalar-wave equation

$$\nabla^2\psi + k^2\psi = 0 \qquad (64)$$

which, written in cylindrical co-ordinates, becomes

$$\frac{\delta^2\psi}{\delta\rho^2} + \frac{1}{\rho}\frac{\delta\psi}{\delta\rho} + \frac{1}{\rho^2}\frac{\delta^2\psi}{\delta\phi^2} + \frac{\delta^2\psi}{\delta z^2} + k^2\psi = 0. \qquad (65)$$

We shall not take the time to consider the mathematical details involved in the solution of this partial differential equation.[2–5] A general solution which represents diverging waves is

$$\psi = \sum_{n=0}^{\infty} U_n(k\rho\sin\theta)\left[A_n\cos(kz\cos\theta) + B_n\sin(kz\sin\theta)\right] \cdot \left[C_n\cos n\phi + D_n\sin\phi\right] \qquad (66)$$

where U_n is the Hankel function of the second kind of order n, which is usually written[6] as $H_n^{(2)}$ but which has been designated here as U_n to avoid confusion with the magnetic vector $\overline{H}$.

A_n, B_n, C_n, D_n are arbitrary constants.

"n" in connection with the problem at hand, must be an integral number or zero.

If we replace the Hankel function in the above relation by a Bessel function of either the first or second kind we have a solution of the wave equation representing standing cylindrical waves.

Now suppose we multiply the scalar-wave function ψ by a constant vector in the z direction (vertical). We then have a solution of the vector-wave equation. This vector function, however, cannot in general represent either the electric or magnetic force of a possible electromagnetic wave since it violates the continuity law, i.e., the divergence is not zero. Nevertheless it is very useful. If we let the electric force (E) equal the curl of this function we have an electromagnetic wave consistent with the fundamental laws. This wave has no vertical component of electric force. Another electromagnetic wave may be obtained from the same vector function by letting the magnetic force H be equal to curl of this function. This wave has no vertical component of magnetic force. The most general type of electromagnetic field may be considered as a combination of these two types of waves.

Since the amplitude of the vector function is entirely arbitrary let us for convenience designate the function in accordance with its use. When the electric field is equal to its curl we shall call the function the "electric vector potential" whereas when the magnetic field is obtained from it by curling we will use the term "magnetic vector potential." It should be remembered that both types of potentials are the same function excepting for differences in the arbitrary constants. Calling the first type $\overline{F}$ and the second $\overline{A}$ we may write the following relations giving the electric and magnetic forces in the most general type of electromagnetic field as follows:

$$\overline{E} = (-J/k)\,\text{curl}\,(\text{curl}\,\overline{A}) - \text{curl}\,\overline{F}$$
$$= (-j/k)(\text{grad div}\,\overline{A} + k^2\overline{A}) - \text{curl}\,\overline{F} \qquad (67)$$

$$\overline{H} = \text{curl}\,\overline{A} - (j/k)\,\text{curl}\,(\text{curl}\,\overline{F})$$
$$= \text{curl}\,\overline{A} - (j/k)(\text{grad div}\,\overline{F} + k^2\overline{F}). \qquad (68)$$

A. Horizontal Dipole and Cylinder—Dipole Axis Circumferential

At a great distance from the vertical cylinder and in the xz plane in a polar direction θ let us assume a horizontal dipole, i.e., a dipole with its axis perpendicular to the xz plane. The electric field from such a dipole may be represented in the vicinity of the cylinder by

$$\overline{E}_y = \exp\left[j(kx\sin\theta + kz\cos\theta)\right]$$
$$= \exp\left[j(k\rho\sin\theta\cos\phi + kz\cos\theta)\right] \qquad (69)$$

if we leave out the amplitude factor and use the origin of co-ordinates as our reference point for phase.

Since no vertical component of electric force exists the electric field of this wave can be derived from a vector function $\overline{F}$ in accordance with the relation,

$$\overline{E}_y = -\,\text{curl}_y\,\overline{F}_z = \overline{E}_\phi\cos\phi + \overline{E}_\rho\sin\phi. \qquad (70)$$

[2] W. E. Byerly, "Fourier Series and Spherical Harmonics," Ginn and Co., New York, N. Y., 1893.
[3] F. S. Woods, "Advanced Calculus," Ginn and Co., New York, N. Y., 1926.
[4] H. Bateman, "Electrical and Optical Wave Motion," Cambridge University Press, London, England, 1915.
[5] E. T. Whitaker and G. N. Watson, "Modern Analysis," Cambridge University Press, London, England, 1927.
[6] E. Jahnke and F. Emde, "Tables of Functions," B. G. Teubner, Leipzig, Germany, 1933 and 1938.

Hence $\quad \delta\overline{F}/\delta x = \exp[j(kx\sin\theta + kz\cos\theta)]. \quad (71)$

A solution of this partial difference equation for $\overline{F}$ which gives the proper values of all the field components is

$$\overline{F} = \overline{Z}_1(-j/k\sin\theta)\exp[j(kx\sin\theta + kz\cos\theta)]$$
$$= (-j\overline{Z}_1/k\sin\theta)\exp[j(k\rho\sin\theta\cos\phi + kz\cos\theta)]. \quad (72)$$

By well-known expansions in Bessel functions we have the relation

$$\exp[j(k\rho\sin\theta\cos\phi)] = \sum_{n=0}^{\infty}\epsilon_n(j)^n J_n(k\rho\sin\theta)\cos n\phi \quad (73)$$

where ϵ_n = Newman's number = 1 when $n=0$ and 2 for all other values of n.

We then obtain for the primary vector function

$$\overline{F}_{\text{pri}} = \overline{Z}_1(-j/k\sin\theta)\exp(jk\cos\theta) \cdot \sum_{n=0}^{\infty}\epsilon_n(j)^n J_n(k\rho\sin\theta)\cos n\phi. \quad (74)$$

This expression represents the sum of a number of standing cylindrical waves. The secondary-wave function must be of the same form excepting that it must represent diverging waves. Substituting the Hankel function in place of the Bessel function and putting in the unknown constants (a_n) we obtain for the secondary field:

$$\overline{F}_{\text{sec}} = \overline{Z}_1(-j/k\sin\theta)\exp(jkz\cos\theta) \cdot \sum_{n=0}^{\infty}\epsilon_n(j)^n a_n U_n(k\rho\sin\theta)\cos n\phi. \quad (75)$$

Adding the two expressions we have for the total wave function:

$$\overline{F}_{\text{tot}} = \overline{Z}_1(-j/k\sin\theta)\exp(jkz\cos\theta) \cdot \sum_{n=0}^{\infty}\epsilon_n(j)^n[J_n(k\rho\sin\theta) + a_n U_n(k\rho\sin\theta)]\cos n\phi. \quad (76)$$

To determine the amplitudes (a_n) of the secondary waves we make use of the boundary condition that the total electric force tangential to the surface of the cylinder must be zero, i.e., when $\rho = a$, $\overline{E}_\phi$ must be zero.

Then $\quad \text{curl}_\phi \overline{F}_{\text{tot}} = 0$ when $\rho = a \quad (77)$

from which

$$a_n = \{-J_n'(ka\sin\theta)/U_r'(ka\sin\theta)\} \quad (78)$$

where the primes indicate derivatives with respect to the argument. The electric force $\overline{E}_\phi$ acting on the horizontal dipole with its axis circumferential and located at the position $(b, \phi, 0)$ becomes

$$\overline{E}_{\phi\text{tot}} = -\text{curl}_\phi \overline{F}_{\text{tot}} = -j\sum_{n=0}^{\infty}\epsilon_n(j)^n[J_n'(kb\sin\theta)$$
$$-\{J_n'(ka\sin\theta)/U_n'(ka\sin\theta)\}U_n'(kb\sin\theta)]\cos n\phi. \quad (79)$$

By the reciprocity law the horizontal electric force at a distant dipole due to a horizontal dipole located at the above-stated position near the cylinder is also given by the above expression. In (79) the first term in the brackets represents the primary field and the second term the secondary field. When the distance of the dipole from the axis of the cylinder is considerably greater than the radius of the cylinder the Bessel-Fourier series expression for the primary field converges much more slowly than the series for the secondary field and under such conditions it may save labor to compute the primary field without the series expansion, separately compute the secondary field from the series, and then combine the two to obtain the total field.

In directions other than in the horizontal plane and one vertical plane a horizontal dipole radiates waves having a vertical component of electric force so that in addition to the field pattern for the horizontal component of electric field we must consider radiation patterns for the other component of electric force. In order to do this let us assume a distant electric dipole whose axis lies in the xz plane and is perpendicular to the ray for the origin. In the resulting electromagnetic wave the magnetic vector is entirely horizontal and is given by the expression

$$\overline{H}_y = \exp[j(kx\sin\theta + kz\cos\theta)]. \quad (80)$$

For reasons similar to those already explained the magnetic vector can be derived from a magnetic-vector potential in accordance with the relation

$$\overline{H}_y = \text{curl}_y \overline{A}_z = \overline{H}_\phi\cos\phi + \overline{H}\rho\sin\phi. \quad (81)$$

Then

$$(-\delta\overline{A}/\delta x) = \exp[j(kx\sin\theta + kz\cos\theta)]. \quad (82)$$

The proper solution of this equation for our purposes is

$$\overline{A} = \overline{Z}_1(j/k\sin\theta)\exp[j(kx\sin\theta + kz\cos\theta)]$$
$$= \overline{Z}_1(j/k\sin\theta)\exp[j(k\rho\sin\theta\cos\phi + kz\cos\theta)]. \quad (83)$$

Expanding in the same manner as previously and assuming secondary waves with amplitudes b_n we obtain

$$\overline{A}_{\text{ntot}} = \overline{Z}_1(j/k\sin\theta)\exp(jkz\cos\theta) \cdot \sum_{n=0}^{\infty}\epsilon_n(j)^n[J_n(k\rho\sin\theta) + b_n U_n(k\rho\sin\theta)]\cos n\phi. \quad (84)$$

Since

$$\overline{E} = (-j/k)(\text{grad div }\overline{A} + k^2\overline{A}) \quad (85)$$
$$E_z = (-j/k)\{(\delta^2\overline{A}/\delta z^2) + k^2 A\} = -jk\sin^2\theta\overline{A} \quad (86)$$

and

$$E_\phi = \frac{-j}{k}\left(\frac{1}{\rho}\frac{\delta^2 A}{\delta\phi\delta z}\right) = (-j)\exp(jkz\cos\theta)\frac{\cos\theta}{\rho\sin\theta} \cdot \sum_{n=0}^{\infty}\epsilon_n(j)\cdot n[J_n(k\rho\sin\theta) + b_n U_n(k\rho\sin\theta)]\sin n\phi. \quad (87)$$

Both tangential components E_ϕ and E_z of electric force must be zero when $\rho = a$, resulting in

$$b_n = -\{J_n(ka\sin\theta)/U_n(ka\sin\theta)\}. \quad (88)$$

The electric field along a horizontal dipole located at the position $(b, \phi, 0)$ is then

$$E_\phi = -j2\frac{\cos\theta}{kb\sin\theta}\sum_{n=1}^{\infty}(j)^n\cdot n[J_n(kb\sin\theta)$$
$$-\{J_n(ka\sin\theta)/U_n(ka\sin\theta)\}U_n(ka\sin\theta)]\sin n\phi. \quad (89)$$

From the recurrence formula for Bessel functions we may put (89) in the following form (90) which may be more convenient than (89) for some conditions:

$$E_\phi = -j\cos\theta\sum_{n=1}^{\infty}(j)^n[J_{n-1}(kb\sin\theta) + J_{n+1}(kb\sin\theta)$$
$$+\{J_n(ka\sin\theta)/U_n(ka\sin\theta)\}(U_{n-1}(ka\sin\theta)$$
$$+U_{n+1}(kb\sin\theta)]\sin n\phi. \quad (90)$$

Again, by the reciprocity principle, the electric field in the direction of the assumed dipole due to the dipole located in the specified position near the cylinder is also given by the last relation. To summarize, we have for the case of a horizontal dipole located near a vertical cylinder:

Horizontal electric force

$$E_\phi = j\sum_{n=0}^{\infty} \epsilon_n (j)^n [J_n'(kb \sin\theta) - \{J_n'(ka \sin\theta)/U_n'(ka\sin\theta)\} U_n'(kb\sin\theta)] \cos n\phi. \quad (91)$$

Vertically polarized electric force[7]

$$E_\theta = -j2 \frac{\cos\theta}{kb\sin\theta} \sum_{n=1}^{\infty} (j)^n \cdot n[J_n(kb\sin\theta) - \{J_n(ka\sin\theta/U_n(ka\sin\theta)\} U_n(kb\sin\theta)] \sin n\phi. \quad (92)$$

B. Horizontal Dipole with Axis Radial

For this condition we may proceed in the same manner as that explained under A. The only difference is that we are now interested in the radial component of total electric force near the cylinder rather than in the circumferential. The radial component $\overline{E}_\rho$, due to the horizontally polarized wave represented by the vector function $\overline{F}_Z$, is then

$$\overline{E}_\rho = -\text{curl}_\rho \overline{F}_z = -(1/\rho)(\delta \overline{F}/\delta\phi). \quad (93)$$

At the position of the near-by dipole ($\rho = b$, $\phi = \phi$, $z = 0$) we obtain

$$E_\rho = -j \frac{2}{kb\sin\theta} \sum_{n=1}^{\infty} (j)^n \cdot (n)[J_n(kb\sin\theta) - \{J_n'(ka\sin\theta)/U_n'(ka\sin\theta)\} U_n(kb\sin\theta)] \sin n\phi. \quad (94)$$

Again, using the reciprocity principle this expression gives us the horizontal electric-field intensity of the horizontally polarized wave at a great distance from the dipole when the dipole is located at the position ($\rho = b$, $\phi = \phi$, $z = 0$). The radial component of electric force for regions near the cylinder due to the wave having a purely horizontal $\overline{H}$ vector derived from the vector function $\overline{A}$ is given by

$$\overline{E}_\rho = (-j/k) \text{ grad}_\rho \text{ div } \overline{A}_z = (-j/k)(\delta^2/\delta\rho\delta z)\overline{A}_z = (-j/k)(\delta/\delta\rho)(jk\cos\theta \overline{A}_z) = \cos\theta(\delta A_z/\delta\rho) \quad (95)$$

and

$$\overline{E}_\rho = j\cos\theta \sum_{n=0}^{\infty} \epsilon_n (j)^n [J_n' kb\sin\theta) - \{J_n(ka\sin\theta)/U_n(ka\sin\theta)\} U_n'(kb\sin\theta)] \cos n\phi \quad (96)$$

when $\rho = b$, and $z = 0$.

By reciprocity this expression also gives the electric vector of the wave from the cylinder and near-by dipole polarized in vertical planes.

Thus for the field from a horizontal dipole with axis pointed outwardly together with a vertical cylinder we have

(*a*) *Horizontally polarized wave component*

$$E_\phi = \frac{-j2}{kb\sin\theta} \sum_{n=1}^{\infty} (j)^n \cdot n[J_n(kb\sin\theta) - \{J_n'(ka\sin\theta)/U_n'(ka\sin\theta)\} U_n(kb\sin\theta)] \sin n\phi. \quad (97)$$

(*b*) *Vertically polarized wave component*

$$E_\theta = j\cos\theta \sum_{n=0}^{\infty} \epsilon_n (j)^n [J_n' kb\sin\theta - \{J_n(ka\sin\theta)/U_n(ka\sin\theta)\} U_n'(kb\sin\theta)] \cos n\phi. \quad (98)$$

C. Vertical Dipole and Cylinder

This condition is the simplest to analyze. As described under A we assume a dipole lying in a vertical plane at a great distance, thus resulting in a wave in which the magnetic field is purely horizontal. In this case we do not need the second assumption of a distant horizontal dipole since the wave from the latter can produce no electric force parallel to the direction of the axis of the cylinder and its near-by dipole. The vertical component of the electric force in the vicinity of the cylinder is then determined from (84), (86), and (38). By reciprocity this expression gives the total field E_θ along a distant dipole perpendicular to the ray. The result is

$$E_\theta = \sin\theta \sum_{0}^{\infty} \epsilon_n (j)^n [J_n(kb\sin\theta) - \{J_n(ka\sin\theta)/U_n(ka\sin\theta)\} U_n(kb\sin\theta)] \cos n\phi. \quad (99)$$

D. Dipoles Very Close to Cylinder

When the distance between the surface of the cylinder and the dipole is only a small fraction of a wavelength the two terms in the brackets representing the primary and secondary fields are nearly equal so that the formulas become nearly useless in the forms given. However, useful forms may be obtained by letting $b = a + \Delta a$, expanding the Bessel functions by Taylor's theorem, and neglecting higher-order terms. The process is straightforward and needs no discussion.

E. Dipole Arrays and Cylinders

All of the expressions for the field components which have been developed for the three orientations of a dipole near a cylinder may be expressed in the general form

$$E = \sum_{n=0}^{\infty} S_n(a, b, \theta, n) \begin{Bmatrix} \cos n\phi \\ \sin n\phi \end{Bmatrix} \quad (100)$$

where $S_n(a, b, \theta, n)$ is a function of a, b, θ, and n. When a second unit is placed in a position $\phi = \phi_0$ and fed so that its current leads that in the first by a phase angle ψ the field from the second unit becomes

$$E = \exp(j\psi) \sum_{n=0}^{\infty} S_n(a, b, \theta, n) \begin{Bmatrix} \cos n(\phi - \phi_0) \\ \sin n(\phi - \phi_0) \end{Bmatrix}. \quad (101)$$

If we have an array of m dipoles equally spaced by an angle $2\pi/m$, and the current in the Pth dipole has a phase angle ψ_p the total field for the array is then

$$E = \sum_{p=1}^{m} \sum_{n=0}^{\infty} \exp(j\psi_p) S_n(a, b, \theta, n) \cdot \begin{Bmatrix} \cos[n\{\phi - 2\pi(P/m)\}] \\ \sin[n\{\phi - 2\pi(P/m)\}] \end{Bmatrix}. \quad (102)$$

[7] To conserve words we shall define a vertically polarized wave as one whose *magnetic* force is entirely horizontal and "vertically polarized," electric force as the force belonging to a vertically polarized wave. Thus a vertically polarized electric force as defined here is not necessarily vertical.

For most of the arrays of practical interest this formula becomes considerably simpler upon expansion due to the cancellation of some of the terms. We shall not take the space to show the details of development of the expressions for the various arrays.

If an array consists of n tiers spaced apart by a distance S and fed with equal currents in phase and E_1 is the field from one tier the total field E_n becomes

$$E_n = E_1\{1 + \exp(j\,kS\cos\theta) + \exp(j\,2kS\cos\theta) + \cdots + \exp[j(n-1)kS\cos\theta]\} \tag{103}$$

$$= E_1 \frac{1-\exp(jnkS\cos\theta)}{1-\exp(jnkS\cos\theta)}$$

$$= E_1 \exp\left(j\frac{n-1}{2}kS\cos\theta\right)\frac{\sin\{(n/2)kS\cos\theta\}}{\sin(kS/2\cos\theta)} \tag{104}$$

and

$$|E_n| = E_1 \frac{\sin\{n(kS/2)\cos\theta\}}{\sin(kS/2)\cos\theta} \tag{105}$$

the limit for $\theta = 90$ degrees being $E_n = nE_1$.

For a two-tier array this reduces to

$$E_z = 2E_1\cos\{(kS/2)\cos\theta\}. \tag{106}$$

F. Line Sources or Very High Stacks of Dipoles

It is of advantage in certain cases to know the behavior of waves from line rather than point sources. The viewpoint is somewhat similar to that taken in the treatment of ordinary radio-frequency transmission lines where the laws of propagation are strictly true only when the line section under consideration is part of a line of infinite length. There are many cases in practice where such an approximation is sufficiently

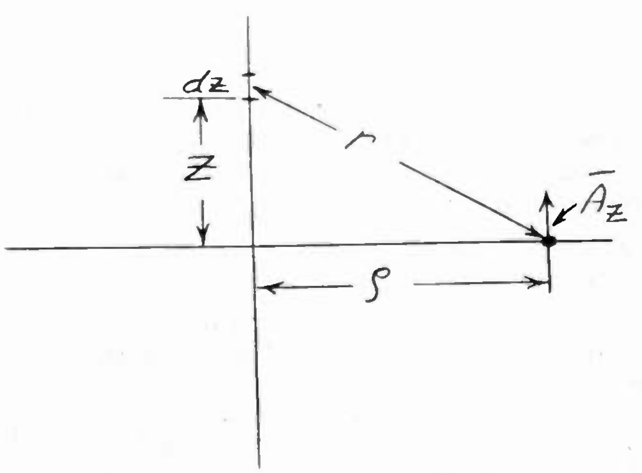

Fig. 30—Vector potential from line source.

accurate. By a line source for waves is meant a line of point sources all oscillating in phase. The elements of the line may be vertical electric doublets, horizontal electric doublets, or vertical magnetic doublets, the latter being equivalent to a stack of small horizontal loop antennas. Let us consider first a line source of vertical electric dipoles. Referring to Fig. 30 the current moment of an element dz is Idz and the retarded vector potential

$$d\bar{A}_z = (\bar{Z}_1 I/4\pi C)\{\exp(-jkr)/r\}dz \tag{107}$$

where

$$r^2 = \rho^2 + z^2.$$

The total vector potential $\bar{A}$ at P is then

$$\bar{A} = \frac{\bar{Z}_1 I}{4\pi C}\int_{-\infty}^{+\infty}\exp\left(\frac{-jkr}{r}\right)dz. \tag{108}$$

Let us change the variable in the integral by letting $r = \rho\cosh\alpha$, then, since

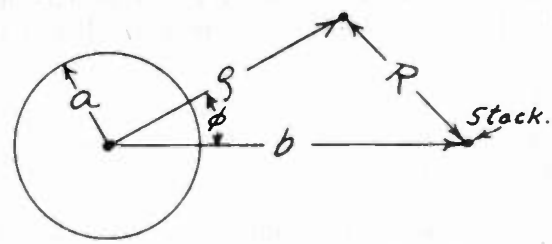

Fig. 31 Co-ordinates for cylinder and dipole stack.

$$\cosh^2\alpha - \sinh^2\alpha = 1,\quad z = \rho\sqrt{\cos^2\alpha - 1} = \rho\sinh\alpha,$$

and

$$dz = \rho\cosh\alpha\, d\alpha = r d\alpha$$

and the vector becomes

$$\bar{A} = \frac{\bar{Z}_1 I}{4\pi C}\int_{-\infty}^{+\infty}\exp(-jk\rho\cosh\alpha)d\alpha. \tag{109}$$

This is a standard form, for the Hankel function[6] and therefore

$$\bar{A} = \bar{Z}_1 I\{(-j)/4C\}U_0(k\rho). \tag{110}$$

The electric and magnetic forces may then be obtained by

$$\bar{H} = \text{curl}\,\bar{A},\quad \bar{E} = (-j/k)(\text{grad div}\,\bar{A} + k^2 A)$$
$$= -jkA \text{ since div}\,\bar{A} \equiv 0. \tag{111}$$

Hence the diverging wave from a vertical line source of vertical electric dipoles may be represented by $U_0(k\rho)$, disregarding constants.

Now consider a vertical stack of horizontal dipoles, the axes of the dipoles lying in the direction of the y axis. The vector potential then becomes

$$\bar{A} = \bar{A}_y = \bar{y}_1 U_0(k\rho) = U_0(k\rho)(\rho_1\sin\phi + \bar{\phi}_1\cos\phi). \tag{112}$$

If the axes of a stack of horizontal dipoles are radial and lie along the x axis

$$\bar{A} = A_x = \bar{x}_1 U_0(k\rho) = U_0 k\rho(\rho_1\cos\phi - \phi_1\sin\phi). \tag{113}$$

For a vertical string of magnetic dipoles the electric vector potential is given by

$$\bar{F} = \bar{F}_z = \bar{Z}_1 U_0(k\rho). \tag{114}$$

G. Cylinder and Stack of Vertical Dipoles

If a high stack of dipoles rather than a single dipole is assumed near a cylinder it is not necessary to make use of the reciprocity theorem to avoid infinite integrals in the Fourier series for the waves. The advantage of using the direct method of approach is that we gain an exact knowledge of the fields in the near vicinity of the

[6] Loc. cit., p. 218.

cylinder and the currents flowing along its surface as well as the radiation fields at great distances. In Fig. 31 assume a stack of vertical dipoles located at $\rho=b$, $\phi=0$. The vector potential $\overline{A}\epsilon$ at any position ρ, ϕ may be represented by

$$\overline{A}_z = \overline{Z}_1 U_0(kR), \tag{115}$$

where $R=\sqrt{\rho^2+b^2-2\rho b\cos\phi}$. In order to satisfy boundary conditions we need to express the primary wave $\overline{A}$ as a sum of cylindrical waves referred to axis of the cylinder. From the addition theorem for Bessel functions

$$U_0(kR) = \sum_{n=0}^{\infty} \epsilon_n U_n(kb) J_n(k\rho) \cos n\phi \text{ when } \rho < b \tag{116}$$

$$U_0(kR) = \sum_{n=0}^{\infty} \epsilon_n U_n(k\rho) J_n(kb) \cos n\phi \text{ when } \rho > b. \tag{117}$$

(It may be of interest to note that (116), and (117) prove the reciprocity theorem for this particular condition).

For the secondary waves we have a similar series with $U_n(K\rho)$ in place of $J_n(K\rho)$ to represent diverging waves. As under (E) we determine the unknown amplitudes from the law that the total electric force E_z along the surface of the cylinder must be zero and thus obtain for the total wave vector

$$A_z = \sum_{n=0}^{\infty} \epsilon_n [U_n(kb) J_n(k\rho) - U_n(kb)\{J_n(k\rho)/U_n(k\rho)\} U_n(k\rho)] \cos n\phi \quad \text{when } \rho < b \tag{118}$$

and

$$A_z = \sum_{n=0}^{\infty} \epsilon_n [J_n(kb) - \{J_n(ka)/U_n(ka)\} U_n(kb)] U_n(k\rho) \cos n\phi \quad \text{when } \rho > b \tag{119}$$

and

$$E_2 = (-j/k)(\text{grad div } A_z + k^2 A_z) = -jkA_z. \tag{120}$$

For great distances (ρ) the asymptotic value of $U_n(k\rho)$ is

$$U_n(k\rho) \sim \sqrt{\frac{2}{\pi}} \frac{\exp\{-(jk\rho+\pi/4)\}}{\sqrt{k\rho}} \exp\left(jn\frac{\pi}{2}\right)$$

$$= \sqrt{\frac{2}{\pi}} j^n \frac{\exp\{-jk\rho+\pi/4\}}{\sqrt{k\rho}}. \tag{121}$$

Thus for fixed large distances we obtain the same expression as that given under (E) for $\theta=90$ degrees, i.e., the horizontal plane.

H. Stacks of Horizontal Dipoles Near Cylinder—Dipole Axes Circumferential

For this condition we proceed as before to start with but our vector potential is now

$$\overline{A}_v = \overline{A}_R + \overline{A}_\phi \tag{122}$$

$$A_R = U_0(kR) \sin\phi \tag{123}$$

and

$$A_\phi = U_0(kR)\cos\phi \tag{124}$$

from which

$$\overline{H} = \overline{H}_z = \text{curl } A_v = -kU_1(kR)\cos\phi \tag{125}$$

$$E_R = (-1/R)(\delta\overline{H}_z/\delta\phi) = k^2\{U_1(kR)/k\rho\}\sin\phi \tag{126}$$

$$E_\phi = (-\delta H_z/\delta R) = k^2 U_1'(kR)\cos\phi. \tag{127}$$

Since the electric force has no vertical component the fields may be derived from a vector function $\overline{F}_z$. We then have

$$\overline{E} = -\text{curl } \overline{F}_z \tag{128}$$

and $$\overline{H} = (-j/k)(\text{grad div } F_z - k^2 F_z) = -jk\overline{F}_z. \tag{129}$$

Then

$$H_z = -kU_1(kR)\cos\phi = -jk\overline{F}_z \tag{130}$$

and

$$F_z = -jU_1(kR)\cos\phi = (-j/k)U_0'(kR)\cos\phi. \tag{131}$$

But

$$U_1(kR)\cos\phi = \sum_{m=-\infty}^{m=+\infty} U_{1+m}(kb) J_m(k\rho)\cos m\phi, \rho<b \tag{132}$$

$$= U_1 J_0 + (U_2 J_1 - U_0 J_1)\cos\phi + (U_3 J_2 - U_1 J_2)\cos 2\phi + \cdots$$

$$= -\sum_0^\infty \epsilon_n U_n'(kb) J_n(k\rho)\cos n\phi, \rho < b. \tag{133}$$

Hence

$$F = j\sum_0^\infty \epsilon_n U_n'(kb) J_n(k\rho)\cos n\phi, \ \rho < b. \tag{134}$$

Assume secondary

$$F_z = F_{z\text{sec}} = a_n U_n'(kb) U_n(k\rho)\cos n\phi. \tag{135}$$

Then the total electric force

$$E_{\phi(\text{tot})} = \text{curl } F_z = (\delta F_z/\delta\rho). \tag{136}$$

Hence when $\rho=a$

$$(\delta F_z/\delta\rho) = \text{zero and } a_n = \{J_n'(ka)/H_n'(ka)\} \tag{137}$$

and

$$E_\phi = jk\sum_0^\infty \epsilon_n [U_n'(kb) J_n'(k\rho) - \{J_n'(ka)/U_n'(ka)\} U_n'(kb) U_n'(k\rho)]\cos n\phi \quad \text{when } \rho < b \tag{138}$$

and

$$E_\phi = jk\sum_0^\infty \epsilon_n [J_n'(kb) - \{J_n'(ka)/U_n'(ka)\} U_n'(kb)] U_n'(k\rho)\cos n\phi \quad \text{when } \rho > b. \tag{139}$$

This becomes the same as the expression developed by the reciprocity theorem when we substitute the asymptotic expression for $U_n'(k\rho)$.

J. Stack of Horizontal Dipoles Near Cylinder—Dipole Axes Radial

The procedure for this condition is almost identical with that under (H) except that we start with a vector potential

$$\overline{A}_z = U_0(kR)\{\overline{\rho}_1\cos\phi - \overline{\phi}_1\sin\phi\}. \tag{140}$$

The result is

$$E_\phi = -jk\sum_0^\infty \epsilon_n [U_n'(kb) J_n'(k\rho) - \{J_n'(ka)/U_n'(ka)\} U_n'(kb) U_n'(k\rho)]\sin n\phi, \ \rho < b \tag{141}$$

$$E_\phi = -jk\sum_0^\infty \epsilon_n [J_n'(kb) - \{J_n'(ka)/U_n'(ka)\} U_n'(kb)] U_n'(k\rho)\sin n\phi, \ \rho > b \tag{142}$$

$$E_r = -k\sum_0^\infty \epsilon_n [U_n'(kb) J_n(k\rho) - \{J_n'(ka)/U_n'(ka)\} U_n'(kb) U_n(k\rho)]\sin n\phi, \ \rho < b. \tag{143}$$

K. Currents in Cylinder

Having developed the expressions giving the fields in the immediate vicinity of the cylinder from stacks of dipoles located near the cylinder we may immediately determine the current distribution in the surface of the cylinder from the law that the current density in the surface is equal to c times the magnetic field adjacent to the surface. For a stack of vertical electric dipoles we thus obtain for the current density on the cylinder

$$I = cH_\phi = c \operatorname{curl} \phi A_z = -c\sum_0^\infty \epsilon_n [U_n(kb)J_n'(ka) - U_n(kb)\{J_n(ka)/U_n(ka)\}U_n'(ka)] \cos n\phi. \quad (144)$$

For the stack of horizontal dipoles with circumferential axes the current flowing around the cylinder is

$$I = cH_z = -j\omega F_z = \omega \sum_0^\infty \epsilon_n [U_n'(kb)J_n(ka) - U_n'(kb)\{J_n'(ka)/U_n'(ka)\}U_n(ka)] \cos n\phi. \quad (145)$$

When the dipole axes of the stack are radial the current density is

$$I = -\omega \sum_0^\infty \epsilon_n [U_n'(kb)J_n(ka) - \{J_n'(ka)/U_n'(ka)\}U_n'(kb)U_n(ka)] \sin n\phi. \quad (146)$$

The distribution of current in the cylindrical surface for a dipole stack should be approximately the same as that for a single dipole at least in the plane containing the dipole.

L. Horizontal Dipole and Dielectric Cylinder

By a process similar to that used for a perfectly conducting cylinder we may develop expressions for the radiation pattern from a dipole or arrays of dipoles near a dielectric cylinder. When such a cylinder is assumed we must consider three wave components, the primary wave, the external or reflected secondary waves, and the internal or refracted secondary waves. In the boundary between air and the dielectric cylinder the following conditions must hold. The tangential components of both total electric and total magnetic force must be continuous in crossing the boundary. Also the total radial components of magnetic induction and electric displacement must be continuous. We shall not take up the space required to show the procedure. It results in the following expression for the magnetic field H_z of the secondary wave

$$H_{z\,sec} = \sum_{n=0}^\infty \epsilon_n J^n a_n U_n(k\rho) \cos n\phi \quad (147)$$

where

$$a_n = -\frac{J_n(ka)J_n'(k_2a) - \eta J_n'(ka)J_n(k_2a)}{U_n(ka)J_n'(k_2a) - \eta U_n'(ka)J_n(k_2a)}, \quad (148)$$

in which if e_2 is the dielectric constant of the cylinder

$$\eta = \sqrt{e_2} = \text{refraction coefficient} \quad (149)$$

$$k_2 = \eta k = \sqrt{\epsilon_2}k = \sqrt{e_2}\,x(2\pi/\lambda) = \text{propagation constant in cylinder.} \quad (150)$$

The cylinder need not be a pure dielectric. If its conductivity is σ (in Heaviside-Lorentz units) we may use a complex dielectric constant $\epsilon_2 = \epsilon_0 - j\sigma/\omega$. However, the arguments of the Bessel functions then become complex and the values of the functions are difficult to evaluate[8] except when the ratio of conductivity to frequency is either quite large or quite small compared to ϵ_0. Similar formulas may be developed in the same way for a vertical dipole in dielectric cylinder but we shall not write them.

Additional References

(1) N. W. McLachlan, "Bessel Functions for Engineers," Oxford University Press, London, England, 1934.

(2) W. W. Hansen and J. G. Beckerley, "Concerning new methods of calculating radiation resistance," Proc. I.R.E., vol. 24, pp. 1594–1622; December, 1936.

(3) W. W. Hansen and L. M. Hollingsworth, "Design of flat-shooting antenna arrays," Proc. I.R.E., vol. 27, pp. 137–144; February, 1939.

(4)* J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Publishing Co., New York 18, N. Y., 1941.

* This excellent treatise was published after writing this report. Chapter VI, "Cylindrical Waves," gives a detailed treatment of cylindrical-wave functions.

[8] Works Project Administration tables are in the process of computation. Certain of these tables are obtainable from the National Bureau of Standards, Washington, D. C.; others are to be published by the Columbia University Press, New York, N. Y.

The Radiation Field of a Symmetrical Center-Driven Antenna of Finite Cross Section*

CHARLES W. HARRISON, JR.†, ASSOCIATE I.R.E. AND RONOLD KING†, ASSOCIATE I.R.E.

Summary—A simple method is advanced for calculating the distant field of a symmetrical center-driven antenna of known effective cross section. This is accomplished by introducing two fictitious antennas carrying currents of simple analytical form and of such *amplitude* and *phase* as to approximate closely the actual distribution of current. In the case of thick antennas, this departs significantly from the sinusoidal distribution ordinarily assumed. It is demonstrated that for antennas of moderate thickness the *shape* of the vertical field pattern differs only very slightly from that for an indefinitely thin antenna carrying a simple sinusoidal current. The magnitude of the field of the actual antenna differs by only a few per cent from that of the indefinitely thin antenna if the maximum value of the square root of the sum of the squares of the quadrature components of the current in the former is made equal to the maximum sinusoidal current in the latter. The phase of the field for the actual antenna differs from that of the indefinitely thin antenna. The cross section of a cylindrical antenna in which the magnitude of the current approximates the measured distribution of current along on an actual antenna is defined to be the effective cross section of the actual antenna.

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Introduction

In a recent paper the writers have discussed the distribution of current in a symmetrical center-driven antenna of nonvanishing radius.[1] One of the conclusions reached was that the distribution of current in an antenna of appreciable thickness differs considerably from a simple sine curve. Accordingly, it is to be expected that the distant field of an actual antenna will differ from that computed for an indefinitely thin antenna of the same length carrying a sinusoidally distributed current. This is also suggested by the work of L. V. King.[2]

Rather intricate equations (originally due to Hallén)[3] were derived[1] for calculating the correct distribution of current for cylindrical antennas. It is possible, by an exceedingly tedious procedure, to determine by direct integration the distant field using this correct formula for the current. A simpler method, which is certainly adequate for all practical purposes, is to approximate each of the quadrature components of current in the antenna by a fictitious antenna carrying a current of simple analytical form which is a satisfactory equivalent of the actual distribution. Then by properly combining the fields calculated for each of these fictitious antennas in the distant zone, the radiation field of the actual antenna may be ascertained to a good approximation at least for cylindrical antennas of known radius.

A correct and also practical method for calculating the electric field of an antenna with a nonsinusoidally distributed current—and strictly this means all antennas which are not extremely thin—is to determine the field due to each quadrature component of the current, and then combine these into a resultant by taking the square root of the sum of the squares. Only if the phase angle of the current can be shown to be constant *along the entire length of the antenna* (as it can approximately, for example, for cylindrical antennas with half-lengths near a quarter wavelength) is it correct to assume that the phase is independent of the distance along the axis of the antenna. In such cases it is legitimate to use the square root of the sum of the squares of the quadrature components of the current as the current-distribution function.

The experimental determination of the distribution of current along an antenna is difficult because it is usually not possible to determine separately the quadrature components. A small wavemeter moved along parallel and quite close to the antenna, as described, for example, by Gihring and Brown,[4] measures the magnitude of the quadrature components to a satisfactory approximation, but does not permit determining the phase angle of the currents as was pointed out in general terms by L. V. King.[5] Since in most cases this angle varies with distance along the antenna, it is obviously quite incorrect to assume that if the magnitude of the current is known, one may simply ignore the phase angle or assume that it reverses if the magnitude becomes small and then large. This is, for example, what was done by Gihring and Brown in calculating both the electric field and the radiation resistance of antennas with nonsinusoidally distributed currents. These investigators also recognized and pointed out the limitations of this procedure. Actually the magnitude of the current along any antenna of finite thickness can never reach zero except at the end of the antenna, and neither the phase nor the direction of the current can be determined using a wavemeter which reads current on a thermal milliammeter.

The present approach to the problem of determining accurately the distant field of symmetrical center-driven antennas (or of vertical base-driven antennas over a highly conducting plane) of unknown effective cross section is, first, to plot the observed distribution of current as measured, for example, using a wavemeter; and second, by a judicious application of the formulas for the current distribution in moderately thick cylindrical antennas, to obtain curves for the square root of the sum of the squares of the quadrature components of the current in antennas of several different radii. Then by a curve-fitting process, an estimate of the effective antenna cross section is made in terms of an approximate equivalent circular cross section (whether the actual cross section be triangular, square, etc.). In this way one has achieved the desired end, namely, that of obtaining satisfactory approximations for the quadrature components of the current in the actual antenna.

The analysis for the symmetrical center-driven antenna may be applied rigorously to an antenna over a perfectly conducting earth, and the fields obtained in this paper apply exactly to this case. In practice, when the earth is not perfectly conducting, radial currents flowing along the surface of the earth also have quadrature components which affect the distant field. The present analysis includes the effect on the field pattern of quadrature currents in the antenna, but does not take into account the effect of such currents in an earth of finite conductivity. The effect of these latter on the distant field of short antennas, or of sufficiently thin antennas for which a sinusoidal distribution of current is a satisfactory approximation, has been determined by Strutt.[6] In the case of a thick

[1] Ronold King and Charles W. Harrison, Jr. "The distribution of current along a symmetrical center-driven antenna," Proc. I.R.E., vol. 31, pp. 548–567; October, 1943.

[2] L. V. King, "On the radiation field of a perfectly conducting base insulated cylindrical antenna over a perfectly conducting plane earth, and the calculation of radiation resistance and reactance," *Phil Trans. Royal Soc.* (London), vol. 236, pp. 394–422; November 2, 1937.

[3] E. Hallén, "Theoretical investigations into the transmitting and receiving qualities of antennas," *Nava Acta Upsalienses*, ser. 4, vol. 11, no. 4, pp. 3–44; 1938.

[4] H. E. Gihring and G. H. Brown, "General considerations of tower antennas for broadcast use," Proc. I.R.E., vol. 23, pp. 311–356; April, 1935.

[5] See page 382 of footnote reference 2.

[6] M. J. O. Strutt, "Strahlung von Antennen unter dem Einflusz der Erdbodeneigenschaften," *Annal. der Phys.*, vol. 1, pp. 721–772; 1929.

antenna above an imperfect earth both effects must be superimposed.

The Distant Field Due to Simple Distributions of Current

The distant electric field of a symmetrical center-driven antenna may be calculated conveniently by use of the formula

$$E_\theta^r = \frac{j\omega \Pi}{4\pi} \frac{e^{-j\beta R_0}}{R_0} \int I_{z'} e^{j\beta z' \cos\theta} \sin\theta dz'. \quad (1)$$

Here the integration is to be carried out over the length of the antenna. The following notation is used.

- E_θ^r is the electric field in the distant or radiation zone of the antenna in volts per meter
- ω is 2π multiplied by the frequency
- Π is the magnetic constant of space given numerically by $\Pi = 4\pi \times 10^{-7}$ henry per meter
- β is the propagation constant in radians per meter, and equals $2\pi/\lambda$ where λ is the wavelength
- R_0 is the distance from the point of field calculation in the distant zone to a convenient reference origin 0 at the center of a center-driven antenna or at the base of a vertical base-driven antenna over a perfectly conducting half-space
- $I_{z'}$ is the complex current amplitude in amperes flowing in the element dz'
- z' is the distance from the reference origin at the center of the antenna to the element dz'
- R_0, θ, and ϕ form a set of spherical co-ordinates with origin at 0. ϕ does not appear in (1) because rotational symmetry obtains

Assuming a current distribution of the form

$$I_{z'} = I_m \sin \beta(h - |z'|) \quad (2)$$

and using (1), one obtains

$$E_\theta^r = j60 I_m \frac{e^{-j\beta R_0}}{R_0} \left[\frac{\cos(H\cos\theta) - \cos H}{\sin\theta}\right]. \quad (3)$$

This is the well-known relation for the distant field of an antenna of half-length h, $(H = \beta h)$, referred to the maximum current.

If the current is of the form

$$I_{z'} = I_0 \left[\frac{\cos\beta z - \cos H}{1 - \cos H}\right], \quad (4)$$

one obtains

$$E_\theta^r = j60 I_0 \frac{e^{-j\beta R_0}}{R_0} \left[\frac{1}{(1-\cos H)\sin\theta}\left(\sin H \cos(H\cos\theta) - \frac{\cos H \sin(H\cos\theta)}{\cos\theta}\right)\right]. \quad (5)$$

The equation for a triangular current distribution is

$$I_{z'} = \frac{I_0(h - z)}{h}. \quad (6)$$

Using (6) in conjunction with (1) gives

$$E_\theta^r = j60 I_0 \frac{e^{-j\beta R_0}}{R_0} \frac{\sin\theta}{H\cos^2\theta}[1 - \cos(H\cos\theta)]. \quad (7)$$

If one assumes a uniform current distribution, i.e., one that is independent of z',

$$I_{z'} = I_0 \quad (8)$$

then

$$E_\theta^r = j60 I_0 \frac{e^{-j\beta R_0}}{R_0} H \sin\theta. \quad (9)$$

The Distant Field of a Moderately Thick Antenna

An inspection of the curves for the current distribution in antennas of nonvanishing cross-section[7] reveals the very interesting fact that in all cases the quadrature components of current may be roughly approximated by the distributions given by (2) and (4) as shown in equation (70) of reference 1. This may be written in the following equivalent form.

$$I_z = I_z'' + jI_z' = I_0''\left[\frac{\cos\beta z - \cos H}{1 - \cos H}\right] + jI_m' \sin(H - \beta|z|). \quad (10)$$

The expression for the distant field due to this distribution is obtained using (2) and (4) in (1). It is

$$E_\theta^r \approx j60 I'_m \frac{e^{-j\beta R_0}}{R_0}\left[\frac{\cos(H\cos\theta) - \cos H}{\sin\theta} + \frac{jC}{(1-\cos H)\sin\theta}\left(\sin H\cos(H\cos\theta) - \frac{\cos H \sin(H\cos\theta)}{\cos\theta}\right)\right]. \quad (11)$$

It has been convenient to introduce the amplitude ratio for the quadrature components in the form $C = I_0''/I_m'$. I_0'' is the component of current in phase with the driving voltage at the center of the antenna; I_m' is the maximum value of the sinusoidally distributed component of current in quadrature with the driving voltage.

For the special case, $H = \pi$ or $h = \lambda/2$ (11) becomes

$$E_\theta^r \approx j60 I_m \frac{e^{-j\beta R_0}}{R_0}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta} + \frac{jC}{2\sin\theta}\frac{\sin(\pi\cos\theta)}{\cos\theta}\right]. \quad (12)$$

In this case an alternative representation, using the distribution,

$$I_z = I_0''[1 - z/h] + jI_m' \sin(H - \beta|z|) \quad (13)$$

gives an equally satisfactory approximation. With (2) and (6) used in (1) one obtains

$$E_\theta^r \approx j60 I_m \frac{e^{-j\beta R_0}}{R_0}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta} + \frac{jC\sin\theta}{\pi\cos^2\theta}[1 - \cos(\pi\cos\theta)]\right]. \quad (14)$$

Curves for the distant field have been computed using (11) for several antennas of different length but

[7] Figs. 19 and 20 of footnote reference 1.

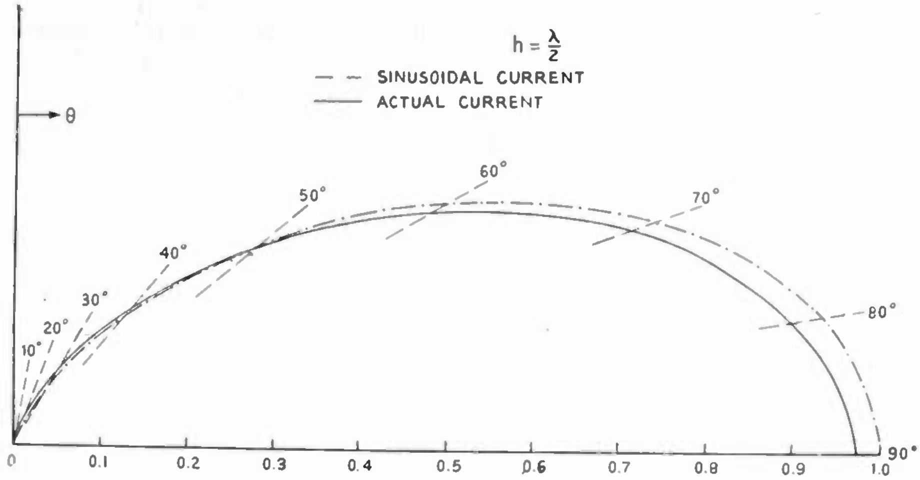

Fig. 1—Vertical field patterns of an isolated center-driven antenna of radius a and half-length $h=\lambda/2$ or of a base-driven antenna of length $h=\lambda/2$ erected vertically over a perfectly conducting plane. The curve marked "sinusoidal current" is for an indefinitely thin antenna for which $a=0$; the curve marked "actual current" is computed for an antenna for which $h/a=75$.

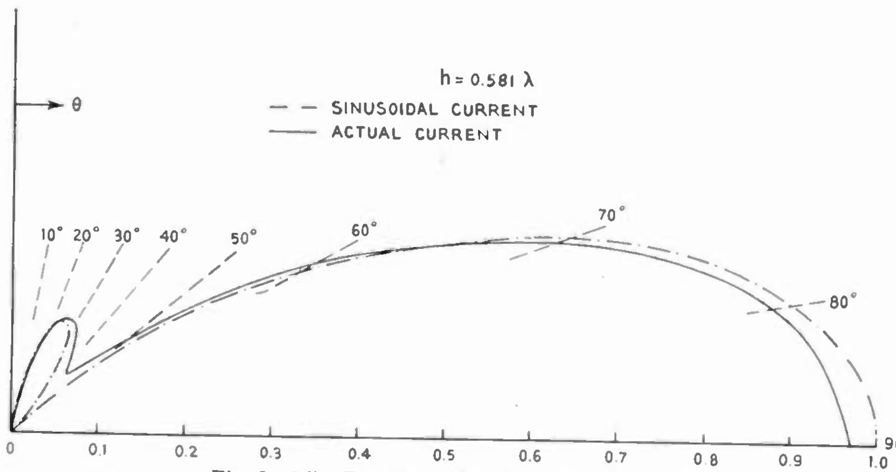

Fig. 2—Like Fig. 1 but with $h=0.581\lambda$, $h/a=45$.

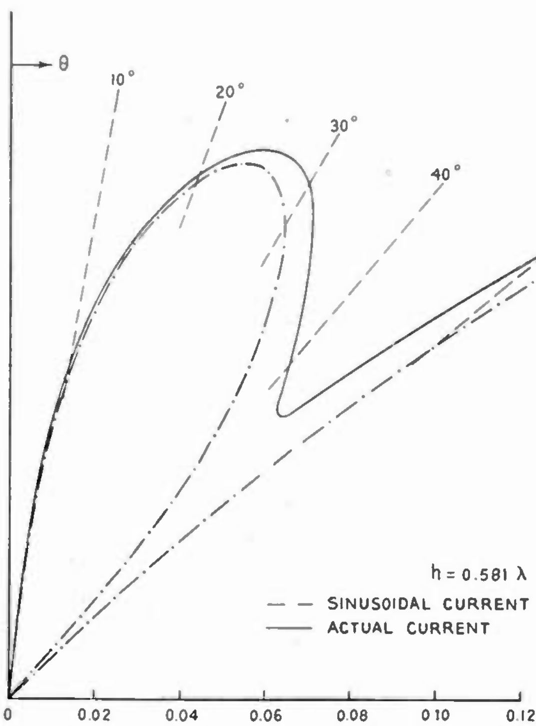

Fig. 3—An enlarged section of Fig. 2.

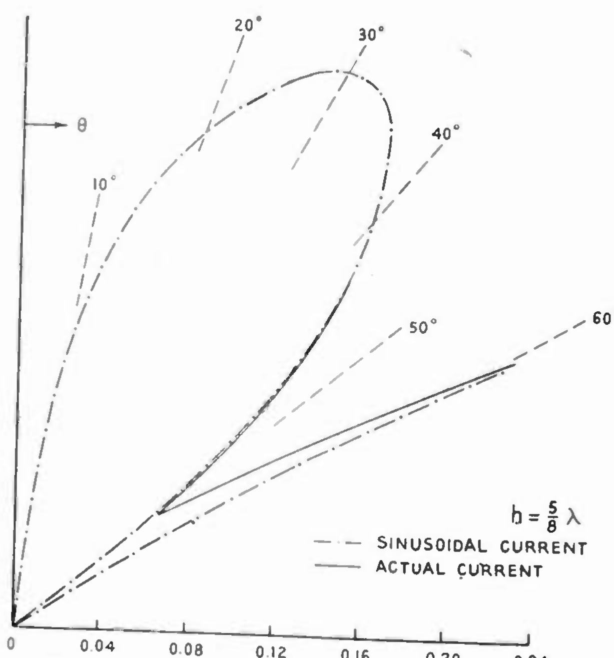

Fig. 4—A section like Fig. 3 but for $h=5/8\lambda$; $h/a=75$.

all of the same thickness ($\Omega=2\ ln\ (2h/a)=10$), with the exception of that for which $h=0.581\lambda$. For this case $\Omega=9$. In all cases the maximum value along the antenna of $\sqrt{(I_z')^2+(I_z'')^2}$ was set equal to I_{max} in a simple sinusoidal distribution.

It is clear from the curves shown in Figs. 1 to 6 that the field patterns of the thick antenna differ only slightly from those of the infinitely thin antenna. They are somewhat rounded off where the fields of the simple sine-wave antenna pass through zero. However, this effect is quite small for antennas even as thick as implied by $\Omega=10$. It is less for antennas of smaller effective cross section. In magnitude the fields calculated for the actual and the sinusoidal distributions differ by only a few per cent. In phase they are, of course, entirely different.

Figs. 2 and 3 show the distant field computed from (11) for an antenna with $h=0.581\lambda$ or $H=3.64$ using

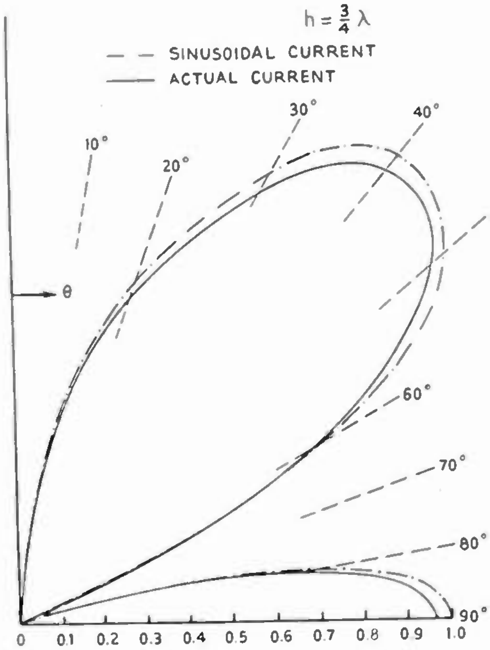

Fig. 5—Like Fig. 1 but with $h=3\lambda/4$.

the distribution of current computed for this length and for $\Omega=9$ by L. V. King.[2] The data are given in his Fig. 4 on page 393. The ratio factor C was determined in this case using his figure. A comparison of Figs. 2 and 3 with the distant field computed by L. V. King[2] (using of course this same distribution of current), as given in his Figs. 6 and 7 on pages 398 and 399, shows appreciable difference. The very considerable increase in the minor lobe with increasing thickness as obtained by L. V. King is not verified. Since both components

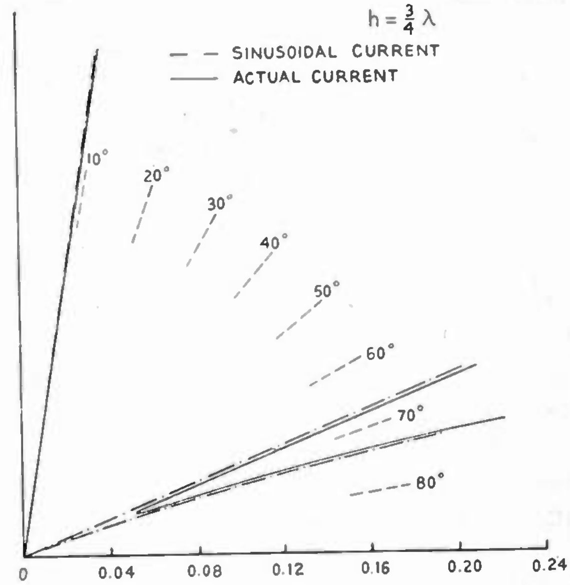

Fig. 6—An enlarged section of Fig. 5.

of the current are very well represented by (10) with a correctly adjusted factor C, it is difficult to see how the present simple and straightforward procedure (once the distribution of current is given) can be much in error. Consequently, the writers do not believe that minor lobes in the field pattern of moderately thick antennas are significantly different from those of extremely thin antennas of the same length except where the latter have sharp zeros.

Conclusion

The results of the present analysis indicate that for purposes of computing *the shape* of the pattern of the distant field even for relatively thick antennas a simple sinusoidal distribution is entirely adequate if it is understood that sharp zeros are rounded off. For more accurate determinations of the distant field including both its magnitude and phase the method described above may be used with only a small increase in complexity.

Corrections

The following typographical changes should be made in "The Distribution of Current Along a Symmetrical Center-Driven Antenna," by Ronold King and Charles W. Harrison, Jr., Proc. I.R.E., vol. 31, pp. 548–567; October, 1943.

page 549, column 2, line 16 from bottom: change . to , and T to t
line 8 from bottom: change interval to internal
page 550, column 1, equation (12b) change subscript r to p on A
page 551, column 1, line after equation (24): change (15) to (22)
line 2 after equation (24): change (13) to (15)
page 551, column 2, equation (36): change $-$ to $+$ before $1/\Omega$
page 552, column 2, line 14 after equation (50): change length to half-length
page 565, column 2, equation (83): change $-$ to $+$ before $\}$
page 566, column 1, equation (84): change $-$ to $+$ before $\}$
equation (86): move first brace to between $-$ and F_0.
equation (91): change $-$ to $+$ after F_0.

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December 21	January 5	January 6	January 3	January 3

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Institute News and Radio Notes

Board of Directors

At the regular meeting of the Board of Directors, which took place on October 6, 1943, the following were present: L. P. Wheeler, president; S. L. Bailey, E. F. Carter, W. L. Everitt, H. T. Friis, Alfred N. Goldsmith, editor; G. E. Gustafson, O. B. Hanson, R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; G. T. Royden (guest), F. E. Terman, B. J. Thompson, H. M. Turner, H. A. Wheeler, W. C. White, and W. B. Cowilich, assistant secretary.

The actions of the Executive Committee taken at its meeting on September 7, 1943, were ratified.

President Wheeler reported that the Executive Committee had approved 114 applications for Associate grade and 90 for Student grade.

The recommended changes to be made as the result of the adopted Constitutional amendments, which were given in the joint report from Chairmen G. T. Royden and B. J. Thompson of the Admissions and Membership Committees, respectively, were discussed at length. The actions taken included the decision to mail the proposed Bylaws amendments to the Board members, as prescribed by the Constitution; to transfer nearly all those of present Member status to the new Senior Member grade, and all Juniors to the Associate grade; to use dark blue as the emblem color for the Senior Member grade and light blue as that for the new Member grade; to incorporate the recommended changes in the general membership-application blank and to make appropriate changes in other printed material of the Institute; and, to have the Membership Committee take steps to notify present Associates concerning transfer to the new Member grade.

Secretary Pratt recorted on the recent meetings of the Radio Technical Planning Board which he attended as the Institute representative, and stated that Dr. W. R. G. Baker had been elected RTPB Chairman. A vote was taken on two proposed changes in the "RTPB—Organization and Procedure."

The Institute Committee on the RTPB consisting of Chairman Pratt, also the representative; B. J. Thompson, also the alternate; H. M. Turner, and Editor Goldsmith, was unanimously reappointed.

Treasurer Heising, as chairman of the Investment Committee, gave a report on matters pertaining to the Institute investments, which were discussed at length. The actions taken included the approval of arrangements for amending the Institute charter, and of the following bank resolution:

"Resolved: That any two of the following: viz., the President, the Secretary, the Treasurer and the Editor, be and they hereby are expressly authorized at all times to make and execute, in name of The Institute of Radio Engineers, Inc., any and all necessary powers of attorney, acts of assignment and/or instruments of transfer, for the sale, assignment and transfer of any and all bonds standing in the name of The Institute of Radio Engineers, Inc."

President Wheeler stated that, as a result of the joint report submitted by F. B. Llewellyn and H. A. Wheeler, the Executive Committee had taken steps toward increasing the distribution of technical information and the activities of the technical committees.

The subject of professional representation of radio engineers was introduced and provision was made for appointing a committee to study the subject.

Editor Goldsmith reviewed several editorial matters, including the papers available for the PROCEEDINGS, the preparation of a special presentation for the membership and the industry on certain postwar matters, and the status of the paper-supply situation for 1943 and 1944.

H. P. Westman was appointed alternate representative on the ASA Sectional Committee on Standards for Drawings and Drafting Room Practices, and Martin Matheson and Frank L. Egner to the Papers Procurement Committee.

It was announced by President Wheeler that Dr. B. E. Shackelford had accepted the chairmanship of the arrangements committee on the Winter Technical Meeting, to take place in New York City during January, 1944.

Consideration was given to a letter from Professor E. H. Armstrong.

President Wheeler called attention to recent correspondence from The Royal Electrical and Mechanical Engineers, a new British society which was recently formed.

The Institute Representatives in Colleges, listed on page 589 of the October, 1943, issue of the PROCEEDINGS, were appointed for the term ending June, 1944, with the changes indicated below:

University of Illinois: A. J. Ebel
Oregon State College: A. L. Albert

Executive Committee

The Executive Committee meeting, held on October 5, 1943, was attended by L. P. Wheeler, president; Alfred N. Goldsmith, editor; R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; H. A. Wheeler, and W. B. Cowilich, assistant secretary.

Approval was granted to 114 applications for admission to Associate grade and 90 to Student grade.

Assistant Secretary Cowilich reported on several office matters including the overtime work of the staff during September, and stated that the winter working schedule, calling for longer hours, became effective in October.

The need for larger office quarters was given further consideration.

Matters pertaining to the Buenos Aires and New York Sections were discussed.

The recommendation was made to the Board of Directors that Martin Matheson and Frank L. Egner be appointed to the Papers Procurement Committee.

Editor Goldsmith explained that the restricted paper allotment, granted by the War Production Department, necessitate sharply limiting forthcoming issues of the PROCEEDINGS with respect to number of pages and copies printed and that, despite the severe paper economies put into practice, the situation must be regarded as difficult. It was also indicated that steps are being taken to prepare an appeal to the WPB relative to the paper needed for printing the PROCEEDINGS in 1944.

Among the other editorial matters, reported by Editor Goldsmith, it was noted that the quantity and quality of papers on hand are of satisfactory order, and that a special article on certain postwar matters might soon be made available to the membership and the industry.

Announcement was made of the availability of reprint copies of the "Standards on Electronics, 1938," in the new format. It was decided to reprint immediately a limited number of copies of the "Standards on Electroacoustics, 1938," also to be in the new format.

The appointment of H. P. Westman, as alternate Institute representative on the ASA Sectional Committee on Standards for Drawings and Drafting Room Practices, was recommended to the Board of Directors.

President Wheeler called attention to the acceptance of Dr. B. E. Shackelford for the chairmanship of the committee on arrangements for the Institute's Winter Technical Meeting, which will be held in New York City during January, 1944.

The report on expanding the technical-committee and related activities, prepared jointly by F. B. Llewellyn and H. A. Wheeler was discussed and followed by certain actions.

Matters pertaining to Institute investments were given consideration.

A recent letter from the American Television Society, on postwar problems of television, was read and discussed.

The recommended changes in Bylaws, membership-application blank, emblem colors, and related matters, necessitated by the adopted Constitutional amendments and contained in the joint report of Chairmen G. T. Royden and B. J. Thompson of the Admissions and Membership Committees, respectively, were considered and referred to the Board of Directors.

President Wheeler called attention to the arrival of petitions for the formation of Institute Sections at Dayton, Ohio, and San Diego, California.

It was decided to refer the requests for permission to form student branches at Northwestern Technological Institute and Drexel Institute of Technology, to the Institute Representatives at those colleges.

The appointment of A. J. Ebel, as Institute Representative at the University of Illinois, was recommended to the Board of Directors.

Contributors

Philip S. Carter

Philip S. Carter (A'29–M'38) was born in 1896 at Glastonbury, Connecticut. He received the A.B. degree in mechanical engineering from Stanford University in 1918 and later during that year he was with the Signal Corps of the U. S. Army. During 1919–1920, Mr. Carter was with the General Electric Company; since 1920 he has been with R.C.A. Communications, Inc., and RCA Laboratories. In 1940 he received the Modern Pioneer Award from the National Association of Manufacturers. He is a Member of the American Mathematical Society and Sigma Xi.

Samuel S. Coroniti

Samuel S. Coroniti (A '31) was born in Keiser, Pennsylvania, in 1908. He received the B.S. degree in engineering physics from Lehigh University in 1931 and the M.S. degree in physics from the University of Michigan in 1933. He was in the X-ray division of the Westinghouse Lamp Company in 1934; with the General Electric Company and RCA Manufacturing Company during 1935–1937; and with Agfa Ansco from 1938 to 1942. Since 1942, Mr. Coroniti has been at the Radiation Laboratory of the Massachusetts Institute of Technology.

C. A. Hultberg

C. A. Hultberg (M'34) was born in Chicago, Illinois, on March 4, 1904. He received the B.E.E. degree from the University of Detroit in 1928. During 1924 and 1925, while attending the university, he was in charge of radio interference-elimination work for the City of Detroit, Department of Street Railways. He was a design engineer with Universal Wireless Communications Company during 1929 and 1930 and the following three years were spent with the Stewart Warner Alemite Corporation as a radio development engineer. From August, 1933, to October, 1934, he was engaged by Hygrade Sylvania Corporation on research and circuit analysis. During 1934–1935, he acted as chief engineer of L' Tatro Products Corporation. In 1936, he joined the engineering department of Crosley Radio Corporation, as a receiver development engineer, for a period of two years. In 1938, he was engaged by Dominion Electrohome Industries, Ltd., of Canada as chief engineer. Early in 1942 Mr. Hultberg was appointed engineering consultant and director of design of Dominion Electrohome Industries Ltd., and of the Radio Production Alliance, with joint laboratories in Kitchener, Ontario, Canada. He is also registered with the Association of Professional Engineers of the Province of Ontario.

For a biographical sketch of C. W. Harrison, Jr., and Ronold King, see the Proceedings for October, 1943.

Mikhail S. Neiman

Mikhail S. Neiman (M'42) is a native of Sevastopol, U.S.S.R. He was graduated from the Politechnical Institute of Leningrad in 1928 and received the D.Sc. degree from the Electrotechnical Institute of Leningrad in 1939. From 1928 to 1935 he was a research engineer in the Leningrad Radio Research Institute of Glavradioprom (Central Radio Industry) and in charge of its laboratory from 1935 to 1941. Dr. Neiman became a professor at the Electrotechnical Institute of Leningrad in 1939. Since 1941 he has been in charge of the radio division of the communication department of the Government Purchasing Commission of the U.S.S.R. in the United States.

J. R. Pierce

J. R. Pierce (S'35-A'38) was born at Des Moines, Iowa, on March 27, 1910. He received the B.S. degree in 1933 and the Ph.D. degree in 1936 from the California Institute of Technology. In 1936 Dr. Pierce became a member of the Technical Staff of the Bell Telephone Laboratories, where he is engaged in electronics research.

Proceedings of the I·R·E

Published Monthly by
The Institute of Radio Engineers, Inc.

VOLUME 31—1943

The Institute of Radio Engineers, Inc.
330 West 42nd Street
New York 18, N.Y.

GENERAL INFORMATION

The Institute

The Institute of Radio Engineers serves those interested in radio and allied electrical-communication fields through the presentation of publication and technical material.

Membership has grown from a few dozen in 1912 to more than eleven thousand. There are several grades of membership, depending on the qualifications of the applicant, with dues ranging from $3.00 per year for Students to $10.00 per year for Senior Members and Fellows.

PROCEEDINGS, Standards Reports, and any other material published in a given year are sent to members without further payment for that year.

The PROCEEDINGS

The PROCEEDINGS has been published without interruption from 1913 when the first issue appeared. Over 2200 technical contributions have been included in its pages and portray a currently written history of developments in both theory and practice. The contents of every paper published in the PROCEEDINGS are the responsibility of the author and are not binding on the Institute or its members. Text material appearing in the PROCEEDINGS may be reprinted or abstracted in other publications on the express condition that specific reference shall be made to its original appearance in the PROCEEDINGS. Illustrations of any variety may not be reproduced, however, without specific permission from the Institute.

The first issue of the PROCEEDINGS was published in 1913. Volumes 1, 2, and 3 comprise four issues each. Volume 4 through volume 14 contain six numbers each and each succeeding volume is made up of twelve issues.

In 1939, the name of the PROCEEDINGS of the Institute of Radio Engineers was changed to the PROCEEDINGS OF THE I.R.E. and the size of the magazine was enlarged from six by nine inches to eight and one-half by eleven inches.

Subscriptions

Annual subscription rates for the United States of America, its possessions, and Canada, $10.00; to college and public libraries when ordering direct, $5.00. Other countries, $1.00 additional.

Back Copies

The Institute endeavors to keep on hand a supply of back copies of the PROCEEDINGS for sale for the convenience of those who do not have complete files. However, some issues are out of print and cannot be provided.

All back issues of the PROCEEDINGS of the I.R.E., which are available, are priced at $1.00 per copy. Price includes postage in the United States and Canada. Postage to other countries is ten cents per copy.

A discount of 25 per cent will be allowed to members of the Institute in good standing; accredited public and college libraries will be granted a discount of 50 per cent.

1913–1915 Volumes 1–3 Quarterly

1913 Vol. 1 January (a reprint)

1916–1926 Volumes 4–14 Bimonthly

1917	Vol. 5	April, June, August, December
1918	Vol. 6	April, June, August, December
1919	Vol. 7	December
1920	Vol. 8	April, June, August, October, December
1921	Vol. 9	February, April, June, August, October, December
1922	Vol. 10	All 6 issues
1923	Vol. 11	February, April, June
1924	Vol. 12	August, October
1925	Vol. 13	April, June, August, October, December
1926	Vol. 14	All 6 issues

1927–1938 Volumes 15–26 Monthly

1927	Vol. 15	April, June, July, October, December
1928	Vol. 16	February, March, May to September, inc.
1929	Vol. 17	April, May, November
1930	Vol. 18	January, May, June, August, October
1931	Vol. 19	January to July, inc., September to December, inc.
1932	Vol. 20	January, March to July, inc., October to December, inc.
1933	Vol. 21	All 12 issues
1934	Vol. 22	January to October, inc., December
1935	Vol. 23	January to August, inc., October to December, inc.
1936	Vol. 24	January, March, June
1937	Vol. 25	April, June to October, inc., December
1938	Vol. 26	February, May to October, inc.

1939–1942 Volumes 27–30 Monthly

New Format—Large Size

1939	Vol. 27	March to June, inc., December
1940	Vol. 28	February to April, inc., June to August, inc.
1941	Vol. 29	January to June, inc., August to December, inc.
1942	Vol. 30	January to May, inc., October to December, inc.
1943	Vol. 31	February to December, inc.

CONTENTS OF VOLUME 31—1943

Volume 31, Number 6, June, 1943

Volume 31, Number 7, July, 1943

Volume 31, Number 8, August, 1943

Volume 31, Number 9, September, 1943

Volume 31, Number 10, October, 1943

Volume 31, Number 11, November, 1943

Volume 31, Number 12, December, 1943

INDEX TO AUTHORS

Numbers refer to the chronological list. Light-face type indicates papers, **bold-face** type indicates discussions, and *italics* refer to books and book reviews.

INDEX TO SUBJECTS

This listing includes technical, sociological, economic, and general papers as well as books and book reviews.

NONTECHNICAL INDEX

Awards

Biographical Notes

Committee Personnel

Constitution and Bylaws

Constitution for Sections

Conventions and Meetings

Editorials

Election of Officers

Group Photographs

Miscellaneous

Photographs

Report of the Secretary—1942

Representatives in Colleges

Representatives on Other Bodies—1943

Resolutions

Current I. R. E. Standards

In addition to the material published in the PROCEEDINGS, Standards on various subjects have been printed. These are available at the prices listed below.

Price

STANDARDS ON ELECTROACOUSTICS, 1938
Definitions of Terms, Letter and Graphical Symbols, Methods of Testing Loudspeakers. (vi+37 pages, 6×9 inches.) $0.50

STANDARDS ON ELECTRONICS, 1938
Definitions of Terms, Letter and Graphical Symbols, Methods of Testing Vacuum Tubes. (viii+59 pages, 6×9 inches.) .... Out of Print

STANDARDS ON ELECTRONICS: DEFINITIONS OF TERMS, SYMBOLS, 1938
A Reprint (1943) of the like-named section of "Standards on Electronics, 1938." (viii+8 pages, 8 1/2×11 inches.) $0.20

STANDARDS ON ELECTRONICS: METHODS OF TESTING VACUUM TUBES, 1938
A Reprint (1943) of the like-named section of "Standards on Electronics, 1938." (viii+18 pages, 8 1/2×11 inches.) $0.50

STANDARDS ON TRANSMITTERS AND ANTENNAS, 1938
Definitions of Terms, Graphical Symbols, Methods of Testing Transmitters, Methods of Testing Antennas. (vi+42 pages, 6×9 inches.) .... Out of Print

STANDARDS ON TRANSMITTERS AND ANTENNAS: DEFINITIONS OF TERMS, 1938
A Reprint (1942) of the like-named section of "Standards on Transmitters and Antennas, 1938." (vi+8 pages, 8 1/2×11 inches.) $0.20

STANDARDS ON TRANSMITTERS AND ANTENNAS: METHODS OF TESTING, 1938
A Reprint (1942) of the like-named section of "Standards on Transmitters and Antennas, 1938." (vi+10 pages, 8 1/2×11 inches.) $0.50

STANDARDS ON RADIO RECEIVERS, 1938
Definitions of Terms, Graphical Symbols, Methods of Testing Broadcast Radio Receovers. (vi+58 pages, 6×9 inches.) .... Out of Print

Price

STANDARDS ON RADIO RECEIVERS: DEFINITIONS OF TERMS, 1938
A Reprint (1942) of the like-named section of "Standards on Radio Receivers, 1938." (vi+6 pages, 8 1/2 ×11 inches.) $0.20

STANDARDS ON RADIO RECEIVERS: METHODS OF TESTING BROADCAST RADIO RECEIVERS, 1938
A Reprint (1942) of the like-named section of "Standards on Radio Receivers, 1938." (vi+20 pages, 8 1/2 ×11 inches.) $0.50

STANDARDS ON RADIO WAVE PROPAGATION: DEFINITIONS OF TERMS, 1942
(vi+8 pages, 8 1/2×11 inches.) $0.20

STANDARDS ON RADIO WAVE PROPAGATION: MEASURING METHODS, 1942
Methods of Measuring Radio Field Intensity, Methods of Measuring Power Radiated from an Antenna, Methods of Measuring Noise Field Intensity. (vi+16 pages, 8 1/2×11 inches.) $0.50

STANDARDS ON FACSIMILE: DEFINITIONS OF TERMS, 1942
(vi+6 pages, 8 1/2×11 inches.) $0.20

NORMAS SOBRE RECEPTORS DE RADIO, 1938*
A Spanish-language translation of "Standards on Radio Receivers, 1938," by the Buenos Aires Section of the Institute of Radio Engineers. (vii+64 pages, 6×9 inches.) .... Two Argentine Pesos (Postpaid)

* *Not carried in stock at I. R. E. Headquarters in New York. Obtainable only from Señor Domingo Arbó, Editor of Revista Telegrafica, Peru, 165, Buenos Aires, Argentina.*

ASA Standards

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AMERICAN STANDARD: STANDARD VACUUM-TUBE BASE AND SOCKET DIMENSIONS
(ASA C16.2–1939.) (8 pages, 7 3/4×10 5/8 inches.) .. $0.20

AMERICAN STANDARD: MANUFACTURING STANDARDS APPLYING TO BROADCAST RECEIVERS
(ASA C16.3–1939.) (16 pages, 3/4×10 5/8 inches.) .. $0.20

AMERICAN STANDARD: LOUDSPEAKER TESTING
(ASA C16.4–1942.) (12 pages, 7 3/4×10 5/8 inches.). $0.25

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Radio Technical Planning Board Members at New York, October 13

First Row—Paul V. Galvin, president RMA; Howard Frazier, representing NAB; Dr. W. R. G. Baker, chairman RTPB; Haraden Pratt, representing IRE; Bond Geddes, treasurer RTPB; K. B. Warner, representing ARRL. Second Row—E. J. Content, representing FM Broadcasters, Inc.; Fred D. Williams, alternate RMA; A. S. Wells, representing RMA; B. J. Thompson, alternate IRE; H. W. Holt, representing Independent Broadcasters, Inc.; Dr. G. T. Harness, representing AIEE; Frank J. Martin, observer for NEMA.

John A. Hutcheson

It was announced recently by Dr. L. W. Chubb (M'21–F'40), director of the Westinghouse Research Laboratories, that John A. Hutcheson has been appointed associate director of the laboratories.

Mr. Hutcheson (A'28–M'30), for the last three years has been manager of engineering at the Baltimore radio division of the Westinghouse Electric and Manufacturing Company. In that post he directed a large staff of engineers developing new types of radio equipment for the armed forces and designing secret electronic devices for military use. His immediate assignment at the laboratories will be to direct wartime microwave research but his scope of responsibility will include all phases of research engineering, Dr. Chubb said.

John A. Hutcheson

Soon after he received his Bachelor of Science degree in electrical engineering at North Dakota in 1926, Mr. Hutcheson joined Westinghouse as a graduate student and was assigned to radio engineering work. He has been in the radio engineering department of the Company ever since, first at East Pittsburgh, Pennsylvania, then at East Springfield, Massachusetts, and finally at Baltimore. He was named engineering manager at Baltimore in 1940.

In the early thirties, he helped design the world's most powerful broadcast transmitter at radio station WLW in Cincinnati, Ohio, and was in charge of Westinghouse television development from 1938 until the war temporarily postponed that work. He also directed engineering for the Westinghouse X-ray division at Baltimore during the last year.

In 1939 he was appointed to the National Television Standards Committee by the Federal Communications Division to study television broadcasting and receiving and to prepare recommendations for standards throughout the industry. He is serving the National Defense Research Committee as a member of the division which is doing advanced electronics work under direction of

(Continued on page 36A)

IRC TYPE MP RESISTORS

Keeping America's newest broadcasting F M and television transmitters operating with a minimum of interruption or distortion is a challenging job. There's no room for chance with thousands of dollars of air-time and talent services at stake. To make certain that transmitters and control instruments will function perfectly under their full power loads—often running to 50 kilowatts—daily tune-up tests at off-time periods have become standard practice. But to throw this unbridled wattage out over the regular antenna could conceivably cause air-signal havoc . . . squeals . . . crashes . . . shot noises.

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I.R.E. People

(Continued from page 34A)

Dr. Karl T. Compton, president of the Massachusetts Institute of Technology. He is also a member of a special advisory committee appointed by Secretary of War Stimson for research and development on war equipment.

Earlier this year he received an honorary doctor's degree from the University of North Dakota in recognition of his achievements in radio and electronics engineering.

C. A. Priest

C. A. Priest (A'24–M'38) has been appointed manager of the transmitter division of the General Electric electronics department, Dr. W. R. G. Baker (A'19–F'26), vice president in charge of the department, has announced. In this capacity, Mr. Priest will assume the responsibility for the operations of the Syracuse, New York, plant of the division, and will have his headquarters in that city. Mr. Priest was engineer of the radio transmitter engineering Division at Schenectady, New York, before his new appointment.

C. A. Priest

Mr. Priest was born in Solon, Maine, and was graduated from the University of Maine in 1922 with the degree of B.S. in electrical engineering. Three years later, he received his E.E. degree. He entered the testing department of the General Electric Company in Schenectady in June, 1922, and in August of the same year was assigned to the transmitter section of the radio department where he has been since, with the exception of the period from August, 1927, to May, 1928, when he was sent to Japan as an employee of the International General Electric Company. While in Japan, he was engaged as a sales engineer on radio apparatus. In May, 1928, Mr. Priest returned to the United States and to the General Electric radio department, where he became particularly active in the high-power work of the transmitter section.

Mr. Priest was named designing engineer of radio transmitters in January of 1930, and in July, 1938, became engineer in charge of the radio transmitter engineering division.

(Continued on page 38A)

I.R.E. People

(Continued from page 36A)

R. H. Siemens

R. H. Siemens (A'41) has been appointed chief engineer of RCA Victor Argentina, wholly-owned RCA subsidiary company in Buenos Aires, it was announced on October 28, 1943, by J. D. Cook, managing director of RCA Victor's International Division. He succeeds Paul Bennett, who has returned to Camden headquarters.

Siemens joined RCA in 1933, as a vacuum-tube application engineer at the Company's Harrison, New Jersey, engineering laboratories, and four years later he was assigned to field work in Chicago. In 1938, he was transferred to Camden as engineer in charge of the design and development of a complete line of small radio receivers.

R. H. Siemens

When the manufacture of these sets was centralized at the Company's new Bloomington, Indiana, plant, Siemens also went there as development engineer. While there he also gained further experience in the design, development, and production engineering of radio transmitters, radio receivers, and specialized equipment for the United States armed forces. Later he was assigned to RCA's Government Equipment Section at Camden. Here he handled research contracts with all divisions of the United States armed forces and with the Office of Scientific Research and Development.

His present assignment is not his first experience in South America. In 1939, he designed and installed two 400-watt short wave transmitting stations in Bogota, Colombia. At the time he also developed all-wave radio receivers that found wide acceptance in Colombia, and short-wave radio link circuits for intercommunication of a long-distance radio system in Bogota.

Prior to joining RCA, Mr. Siemens was chief engineer of the ICA Export Corporation where he produced long and short-wave receivers for the South American and European markets. He was also a development engineer for the Fada Company, and owner of Radio Construction Laboratories, in New York, which specialized in the development of custom-built receivers and battery eliminators. Mr. Siemens holds Bachelor of Science and Electrical Engineering degrees from Cooper Union University, in New York City.

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SECTION MEETINGS

Buffalo-Niagara

"Present-Day Direct Recording on Wire and Disk" by Richard Blinzler, Buffalo Broadcasting Corporation; October 20, 1943.

Cleveland

"Radio Installations in the Amazon River Valley," by R. A. Fox, WGAR Broadcasting Company; September 23, 1943.

Cincinnati

"Recent Improvements in Phonograph Reproduction," by J. D. Reid, Crosley Corporation; October 12, 1943.

Connecticut Valley

"A New Electron Microscope," by Igor Bensin, General Electric Company; September 13, 1943.

"The Theory and Application of Industrial Heating," by J. P. Jordan, General Electric Company; October 15, 1943.

Dallas-Fort Worth

Sound Pictures, "Television," from General Electric Company; "The Jap Zero," and "Mission Accomplished," from the Army Air Corps; October 15, 1943.

Detroit

"Electronics at Work," by Carl Madsen, Westinghouse Electric and Manufacturing Company; September 17, 1943.

Emporium

"You Can Teach an Old Dog New Tricks," by Robert Manhardt, Sylvania Electric Products, Inc.; October 9, 1943.

Election of Officers, October 9, 1943.

Indianapolis

"Personnel Planning for the Post War," by F. H. Kirkpatrick, Radio Corporation of America; October 15, 1943.

"Engineering News Reviews" (illustrated), by H. I. Mertz, Vice Chairman; October 15, 1943.

Kansas City

"Methods Used to Effect Mass Production of Quartz Crystals," by M. E. Hall, Jr., Universal Television Company, Inc.; October 19, 1943.

"Use of Supersonic Frequencies in Materiel Inspection," by Boley Andrews, The Vendo Company; October 19, 1943.

Los Angeles

"Design and Production of High-Q Audio Reactors," by C. A. Campbell, Attic-Lansing Corporation; September 21, 1943.

New York

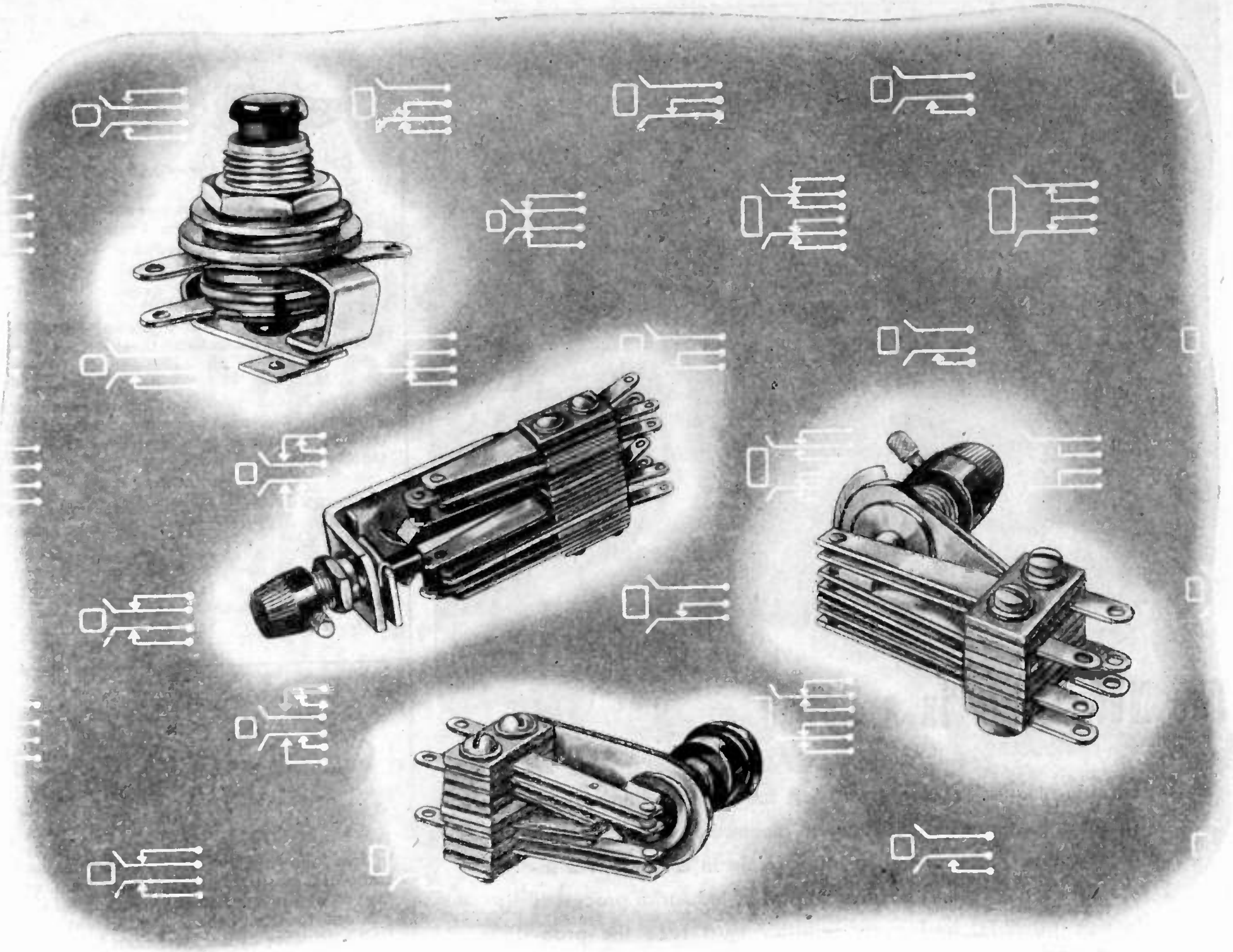
"Application of Electronics to Medical Science," by D. H. More, College of Physicians and Surgeons of Columbia University; October 6, 1943.

"Need for an Instrument to Measure pH in Localized Areas of the Mouth," by Bernerd Thomas School of Dental and Oral Surgery of Columbia University; October 6, 1943.

"An Amplifier For Low-Level Photoelectric Currents," by J. L. Nickerson, Department of Physiology of Columbia University; October 6, 1943.

"Electronic Apparatus For Recording Electric Potentials in Nerves and Muscles" (with demonstrations), W. M. Rogers, Department of Anatomy of Columbia University; October 6, 1943.

(Continued on page 42A)



SECTION MEETINGS

(Continued from page 40A)

"Electronic Energy Analyses in Psycho-Physiological Research," by John Lynn and Charles Scheer, Department of Neurology of Columbia University; October 6, 1943.

Philadelphia

"RCA Laboratories at Princeton," by E. W. Engstrom, RCA Laboratories; October 7, 1943.

Pittsburgh

"The Production of the Radio Program; Adventures in Research," by Phillips Thomas, Westinghouse Electric and Manufacturing Company; October 4, 1943.

Portland

"Design of Antenna Tuning Networks," by V. J. Andrew, Victor J. Andrew Company; August 25, 1943.

St. Louis

"Communications in Civilian Defense," by O. S. McDaniel, United States Army; October 1, 1943.

MEMBERSHIP

The following indicated admissions and transfers of membership have been approved by the Admissions Committee. Objections to any of these should reach the Institute office by not later than December 31, 1943.

Transfer to Senior Member

Atkins, C. E., 1100 Oak Ave., Evanston, Ill.
Brauer, H. H., 76 East Boulevard, Rochester, N. Y.
James, V. N., 2841 Dyer St., University Park, Dallas, Texas
Martin, H. B., Radiomarine Corporation of America, 75 Varick St., New York, N. Y.
Persons, C. B., 1559—19 St., North, Arlington, Va.
Samuel, A. L., Bell Telephone Laboratories, 463 West St., New York, N. Y.
Sharpe, M. O., 135 North Park Dr., Arlington, Va.
Town, G. R., 148 Colebourne Rd., Rochester, N. Y.

Admission to Senior Member

Attwood, S.S., Room 277, W. Engineering Bldg., Ann Arbor, Mich.
Riblet, H. B., 93–18 Lamont Ave., Elmhurst, L. I., N. Y.

Transfer to Member

Alverson, J. G., 3438 Niolopua Dr., Honolulu, T. H.
Ashton, J. O., Hurl Towers Apts., Greenwich, Conn.
Buckinghan, E., 3006 Gough St., San Francisco, Calif.
Harrison, C. W., Jr., Cruft Laboratory, Harvard University, Cambridge, Mass.
Hidy, J. H., 365 Stewart Ave., A-24, Garden City, L. I., N. Y.

The following admissions and transfers were approved by the Board of Directors on November 3, 1943.

Admission to Senior Member

Barker, P. L., Grafton Hotel, Connecticut Ave., Washington, D. C.

(Continued on page 44A)

MEMBERSHIP

(Continued from page 42A)

Levy, S. J., 600 Fourth Ave., Bradley Beach, N. J.
McCreary, H. J., 320 Lewis Ave., Lombard, Ill.
Moore, J. R., Box 12, Dutch Neck, N. J.
Ryder, R. M., Bell Telephone Laboratories, Murray Hill, N. J.

Transfer to Senior Member

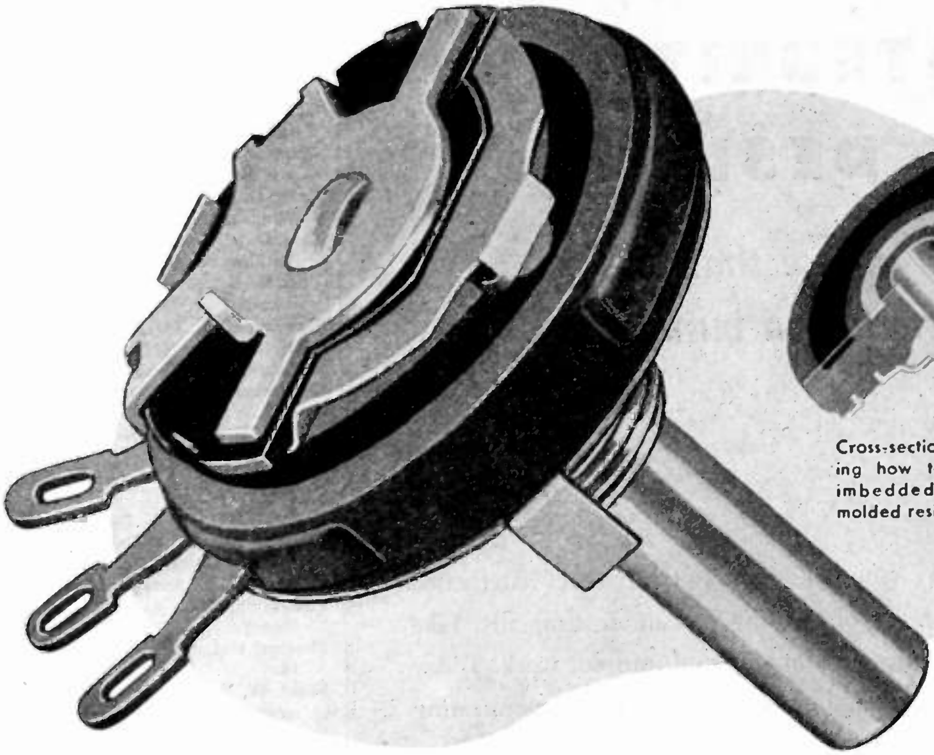
Breazeale, W. M., Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.
Bronwell, A. B., Northwestern University, Evanston, Ill.
Chipp, R. D., 4805—14 St., N.W., Washington D. C.
Epstein, D. W., RCA Laboratories, Princeton, N. J.
Gibson, W. T., Lopen House, Seavington, Somerset, England
Hector, L. G., 57 State St., Newark, N. J.
Hunt, A. B., Box 369, Montreal, Que., Canada
Jenks, D. W., Electronic Tube Engineering Division, General Electric Company, Schenectady, N. Y.
King, R. W., Cruft Laboratory, Cambridge, Mass.
Lidbury, F. A., Box 346, Niagara Falls, N. Y.
Nicoll, F. H., RCA Laboratories, Princeton, N. J.
Paddon, J. W., Canadian Department of Munitions and Supply, Washington, D. C.
Pollack, D., Groton Long Point, Conn.
Shanck, R. B., 134 Manor Rd., Douglaston, L. I., N. Y.
Silver, M., 140 E. 28 St., New York, N. Y.

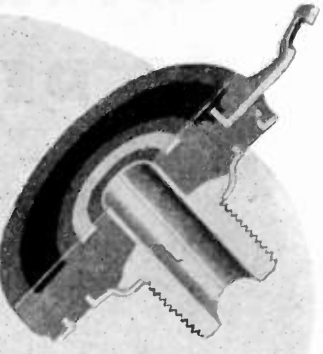
The following admissions to Associate were approved by the Board of Directors on November 3, 1943.

Adams, T. N., 953 Victory Dr., S.W., Atlanta, Ga.
Arnold, N. E., 109 Whitman Ave., Collingswood, N. J.
Arp, H. E., CAA Signals Training Center, R.F.D. 2, Box 10, Ft. Worth, Texas
Auxter, C. N., Fifth Student Sq., Bks., Fort Myers, Fla.
Bailey, A. D., Department of Electrical Engineering, University of Illinois, Urbana, Ill.
Barr, E. W., 1519 Olympian Way, S.W., Atlanta, Ga.
Barron, J. P., A.P.O. 825, c/o Postmaster, New Orleans, La.
Battie, R. G., 730 Leavenworth St., San Francisco, Calif. (transfer)
Bauler, J. W., 5028 Greene St., Philadelphia, Pa.
Bergan, K. M., 706 W. Second St., Northfield, Minn.
Berman, I., 485 W. 187 St., New York, N. Y.
Blinoff, W., Jr., 5820 N. Kenmore Ave., Chicago, Ill.
Blue, N. E., 2912 N. 12 St., Philadelphia, Pa.
Boland, C. E., 1169 Boulevard, N.E., Atlanta, Ga.
Booth, W. S., 169 Darrington St., S.W., Washington, 20, D. C. (transfer)
Boudon, A. P., 12½ E. Fourth St., Emporium, Pa.
Bouman, L. F., c/o Royal Netherlands Indies Airways, 521 Fifth Ave., New York, N. Y.
Brittain, V. M., 1204 N.W. 20 Ave., Portland, Ore.
Brown, E. E., 518 Russell St., Covington, Ky.
Brown, J., 410 Union Ave., Irvington, 11, N. J.
Buckley, E. F., 150 Bedford Rd., Toronto, 5, Ont., Canada (transfer)
Buitenkant, N., 1154 First Ave., New York, N. Y.
Burnett, K. H., 42 Oakland St., West Springfield, Mass.
Bush, B. E., Jr., North 1534 Cedar St., Spokane, Wash.
Butterfield, W. S., A.P.O. 958, c/o Postmaster, San Francisco, Calif.
Cadwallader, R. W., Jr., 1815 N. 50, Seattle, 3, Wash.
Campbell, E. W., 308 N. Cheney St., East Point, Ga.

(Continued on page 46A)

Here's THE ONLY VARIABLE RESISTOR with a SOLID MOLDED ELEMENT

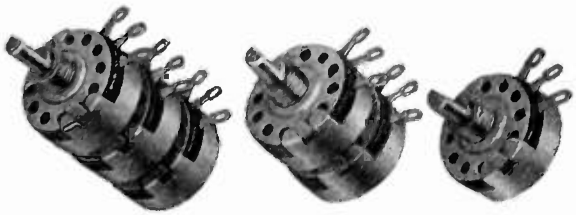




Cross-section view showing how terminals are imbedded in solid, molded resistor element.

The resistor element in the Allen-Bradley Type J Bradleyometer has substantial thickness (approx. 1/32 in. thick) and is molded as a single unit with the insulation, terminals, face plate, and threaded bushing. It is not a film, spray, or paint type resistor. Reliability and compactness are assured by this simple construction which eliminates all rivets, welded or soldered connections, and conducting paints. During manufacture, resistor material may be varied throughout its length to provide practically any resistance-rotation curve. Bradleyometers meet Army and Navy 200-hour salt spray tests.

Bradleyometers are the only continuously adjustable composition type resistors (only one inch in diameter) having a rating of two watts with a good safety factor. Available in resistance values as low as 50 ohms. They can be supplied for rheostat or potentiometer uses, with or without a switch. Write for specifications today. Allen-Bradley Company, 114 W. Greenfield Ave., Milwaukee 4, Wis.



Type J Bradleyometers may be used seporately or in dual or triple construction to fit any particular control need.

FIXED RESISTORS

Type GB Insulated 1-Watt Fixed Resistor

Type EB Insulated ½-Watt Fixed Resistor

Actual size sectional views of Brodleyunits showing the molded homogeneous resistor material, insulation, and imbedded lead wires. Overall lengths: Type GB—3 7/16 in.; Type EB—3 3/8 in.

A-B Quality

ALLEN-BRADLEY

FIXED & VARIABLE RADIO RESISTORS

QUALITY

MEMBERSHIP

(Continued from page 44A)

Carley, W. S., Georgetown, Ky. (transfer)
Carlstrom, T. H., 321 E. Alleghany Ave., Emporium, Pa.
Chadbourne, H. L., 2745—29 St., N.W., Washington, D. C.
Clarke, A. C., "Ballifants," Bishops Lydeard, Taunton, Somerset, England
Cohn, G. I., 265 Woodland Rd., Lake Forest, Ill. (transfer)
Cole, D. R., Comm. Depr., Gen. Sec., Antiaircraft Artillery School, Camp Davis, N. C.
Cox, E. B., 30 Girton Ave., London, N.W., 9, England
Downey, R. F., 132 Jay St., Apt. 8, Schenectady, N. Y.
Dresch, R., 4044 Harding Ave., Cheviot, Ohio
Drougalis, J. V., 101-21—107 St., Richmond Hill, L. I., N. Y.
Elliott, A. W., 157 Maplewood Ave., Toronto, Ont., Canada
Ewald, E. E., RCA Manufacturing Co., Lancaster, Pa.
Fasick, R. C., 1739 W. Devon Ave., Chicago, Ill.
Felton, W. W., 603 Hillborn Ave., Swarthmore, Pa.
Ferl, R. H., 1240 Edmond Dr., Rosemead, Calif.
Fichtl, R. R., 69 Sinniger Pl., Maywood, N. J.
Finkelstein, R. L., A. P. O., 640, c/o Postmaster, New York, N. Y.
Fletcher, K. L., 3333 N. Marshfield Ave., Chicago, Ill.
Floyd, W. B., Jr., Electrical Engineering Department, Stanford University, Calif.
Formby, J. K., 39 Victoria St., Burwood, N. S. W., Australia
Franklin, P. J., 3855 Rodman St., N.W., Washington, 16, D. C.
Freeman, R. W., 117 Nott Ter., Schenectady, N. Y.
Fritschi, W. W., Bell Telephone Laboratories, 463 West St., New York, N. Y.
Fromer, J., 114 E. Third St., New York, 3, N. Y.
Furedy, F., 1400 Lakeshore Dr., Chicago, Ill.
Garity, N. O., 58 W. 72 St., Chicago, Ill.
George, D. E., 88 Blaine Ave., Buffalo, 8, N. Y. (transfer)
Gieffers, H. W., 4045 N. LaPorte Ave., Chicago, 41, Ill. (transfer)
Gilmore, A. C., Jr., 209 Post Ave., Lyndhurst, N. J.
Glade, H. E., 1537 S. 12 St., Milwaukee, 4, Wis.
Graim, R. J., 970 National Press Bldg., Washington, D. C.
Greene, M. B., 227 North Aphir, Apt. B., Stockton, Calif.
Gruenberg, E. L., 3014 Clarendon Rd., Brooklyn, N. Y.
Hall, T. S., 514 Pilchuck Path, Everett, Wash.
Harrell, L. B., 311 W. Cleveland Ave., East Point, Ga.
Hayden, L. B., 2130 Como Ave., St. Paul, 8, Minn.
Hearsum, D. R., 1785 Massachusetts Ave., Washington, D. C.
Herbert, C., U. S. Coast Guard, 414 Insurance Bldg., Seattle, Wash. (transfer)
Hicken, J., 54 Elm St., Belmont, 78, Mass.
Hinshaw, R. M., c/o W. E. Hinshaw, Box 475, Mt. Home, Idaho
Holland, H. A., 5038 Waveland Ave., Chicago, 41, Ill.
Holtz, R. F., 6159 Cedar Ave., Merchantville, N. J.
Hudgins, W. D., Radio Material Officer, Navy 117, c/o Fleet Post Office, New York, N. Y.
Hughes, W. R., 10849 Parr Ave., Sunland, Calif.
Jacob, C. W., Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.
Jeffries, C. D., Radio Research Laboratory, Harvard University, Cambridge, Mass.
Jones, F. F., Hq. Fourth Airways Communications Region, 581 Peachtree St., Atlanta, Ga.
Jordan, W. H., Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.
Kahal, R., 330 E. 71 St., New York, 21, N. Y.

MEMBERSHIP

Klemm, G. H., 189 Belleville Rd., New Bedford, Mass. (transfer)
Kelly, J. M., 645 N. Jackson St., Arlington, Va.
Keller, C. A. R., 4 W. Sixth St., Emporium, Pa.
Kelsey, L. E., 941 N. Linwood Ave., Indianapolis, 1, Ind.
Kessel, R. L., 151 S. Wisconsin Ave., Villa Park, Ill.
King, B. A., Jr., 10 Alden Pl., Bronxville, N. Y.
Kipp, L., 8268—168 St., Jamaica, L. I., N. Y.
Lonrad, W. L., 1133 Chislett St., Pittsburgh, 6, Pa.
Kopp, R., 736 Rock Creek Church Rd., N. W., Washington, 10, D. C.
Koury, E. J., 45 W. 75 St., New York, N. Y.
Kunz, K. S., 128 Oxford St., Cambridge, 40, Mass.
Lindberg, A. F., 7714 N. Haskins Ave., Chicago, 26, Ill.
Lindsey, J. G., 5732 N. Winthrop Ave., Chicago, 40, Ill.
Lumsden, R. B., 59 Station Road, Mimico, Ont., Canada
McKowen, P. B., Hadnot Point, New River, N. C.
Meader, H., 3430 South Hill St., Los Angeles, Calif. (transfer)
Meadows, F. D., RCA Victor Division, 501 North LaSalle, Indianapolis, Ind.
Melius, F. G., 1482 Iglehart Ave., St. Paul, 4, Minn.
Middleton, R. G., 8 Rossie St., Mystic, Conn.
Moffatt, V. J., 6938 South Laflin St., Chicago, 36, Ill.
Mollenauer, J. H., R.D. 1, 84, Pa.
Mozley, R. F., 351 Front St., Hempstead, L. I., N. Y.
Murphy, C. H. S., Naval Research Laboratory, Anacostia Station, D. C. (transfer)
Nidzyn, J. J., Merrick Rd., Speonk, L. I., N. Y.
Norris, C. A., 628 Huron St., Toronto, Ont., Canada
Odell, N. H., 229 Cobbs Hill Dr., Rochester, N. Y.
Pardell, M. K., 1031 Pierson Rd., Flint, Mich.
Penners, B. A., 757 E. 13, Idaho Falls, Idaho
Pine, C. C., 3015 Morgan St., Apt. 57, Seattle, 8, Wash.
Pinsky, J. D., 3536 W. 13 Pl., Chicago, Ill.
Poulson, W. A., Rm. 657, Bell Telephone Laboratories, 463 West St., New York, N. Y.
Redman, W. T., 35 S. Howard St., Bellevue, Pittsburgh, 2, Pa.
Revercomb, H. E., 7069 Sheryl Dr., Norfolk, 5, Va. (transfer)
Rice, S. M., 1902—23 St., S.E., Apt. 93B, Washington, D. C. (transfer)
Rothschild, R. F., 54 Willow Ave., Hempstead, L. I., N. Y. (transfer)
Rowlands, T. E., 186 James St., Ottawa, Ont., Canada
Rubin, A., 115 Amboy St., Brooklyn, N. Y.
Schmidt, P. S., Research Laboratory, Stromberg-Carlson Co., Rochester, N. Y.
Schwartz, H. E., Jr., 620 Main St., New Rochelle, N. Y. (transfer)
Scullin, C. H., 111 Dodd St., East Orange, N. J.
Scully, J. F., Officers Club, Camp Murphy, Fla.
Selleck, R. S., 6730 Fourth Ave., Los Angeles, 43, Calif. (transfer)
Shallcross, G. P., 4713 Pensacola Ave., Chicago, Ill.
Smith, B., Jr., 111½ Miller St., Maryville, Tenn.
Sonnemann, H., 91 Ft. Washington Ave., New York, 32, N. Y.
Speight, N. H., 145 Wellington St., West, Toronto, Ont., Canada
Stann, L., 3936 W. 56 Pl., Chicago, Ill.
Starek, R. A., 107 W. Fourth St., Emporium, Pa.
Steen, W. J., Applied Physics Laboratories, Johns Hopkins University, Silver Spring, Md.
Stenger, F. J., 6619 Pine Ave., Bell, Calif.
Stiepel, G. W., 22-17—119 St., College Point, L. I., N. Y.
Strong, J. A., 3128 Dove St., San Diego, Calif.
Sylvester, T. D., 2414 Telegraph Ave., Berkeley, 4, Calif.
Tomaschuk, L., 6815 Avenue des Erables, Montreal, Que., Canada

(Continued on page 48A)

MEMBERSHIP

(Continued from page 47A)

Thompson, R. B., Hq. Field Engineering Force, Western Electric Co., 195 Broadway, New York, N. Y.
Thompson, R. R., 48 Schley Blvd., Bremerton, Wash.
Toomey, J. W., 2925 O St., S.E., Washington, 20, D. C.
Veeck, H., 2009 Fairlawn Ave., S.E., Washington, 20, D. C.
Venable, D., 1511 Bolton St., Baltimore, 17, Md.
Wathen, R. L., Sperry Gyroscope Co., Garden City, L. I., N. Y.
Weinstein, G., 705 East Thomas, Seattle, Wash.
Wheeler, H. S., "Kildare," Manby Rd., Gt. Malvern, Worcestershire, England
Whittemore, H. B., 160 High St., Middletown, Conn.
Wilkins, O. L., 3805 W St., S.E., Washington, 20, D. C.
Williams, C. K., 680 Lee St., S.W., Atlanta, Ga.
Wilson, J. H., Civil Aeronautics Administration, Box 23, Ogden, Utah
Wolff, J., 27 Howard Ave., Brooklyn, N. Y.
Woodward, P. J., 6956 Calumet Ave., Chicago, Ill.
Young, M. L., 64 E. College, Oberlin, Ohio

Incorrect Addresses

Listed below are the names and last-known addresses of the members of the Institute whose correct addresses are unknown. It will be appreciated if anyone having information concerning the present addresses of the persons listed will communicate with the Secretary of the Institute.

Alley, C. L., 2508 N. Spaulding, Chicago, Ill.
Altman, F. J., Box 2917, Tampa, Fla.
Armstrong, E. A. Armstrong, 473 Embarcadero, Palo Alto, Calif.
Armstrong, H. W., 4431 N. Rockwell St., Chicago, Ill.
Baker, N. A., 276 Church St., Newton, Mass.
Baltimore, D. M., 222 Babcock St., Brookline, Mass.
Barkley, F., Box 21, Medfield, Mass.
Bartelink, E. H., R.F.D. 3, West Wilton, Ballston Spa, N. Y.
Barth, E. G., 71st Coast Artillery, Fort Story, Va.
Baxter, Charles, 298 Banbury Rd., Oxford, England
Beckman, J. A., 2309 Chickasaw St., Cincinnati, Ohio
Black, Jr., A. O., 1714 Massachusetts Ave., N.W., Washington 6, D. C.
Bloom, Abraham, Rumson Area "C.E.S.L.," Rumson, N. J.
Bode, R. H., 1200 Louisiana, Lawrence, Kan.
Bramhall, E. H., 22 Bay State Rd., Boston, Mass.
Brelsford, Howard, 4550 Linden Ave., Norwood, Ohio
Brown, J. N., 148 Elm St., Franklin, Pa.
Buchter, T. W., Fort Monmouth New Jersey.
Burr, R. P., 214 Foulke Hall, Princeton, N. J.
Calister, R. A., 19 S. Wolcott St., Salt Lake City, Utah
Campbell, R. E., 410 W. 110 St., New York, N. Y.
Capps, J. H., 1409 Monroe St., Corvallis, Ore.
Cheeks, J. A., 2300 Fairdale Ave., Cleveland, Ohio.
Christenson, C. K., 334 Lincoln Ave., Palo Alto, Calif.
Cockrell, H. E., 126 W. Hoover St., Ann Arbor, Mich.
Cole, Hubert, 2401 Ohio Ave., Cincinnati, Ohio

(Continued on page 54A)

POSITIONS OPEN

The following positions of interest to I.R.E. members have been reported as open. Apply in writing, addrssing reply to company mentioned or to Box No. ....

The Institute reserves the right to refuse any announcement without giving a reason for the refusal.

PROCEEDINGS of the I.R.E.

330 West 42nd Street, New York 18, N.Y.

RADIO ENGINEER

Experienced in the manufacture and testing of ultra-high-frequency apparatus; must be capable of taking complete charge of war projects. Splendid opportunity. War workers at highest skill need not apply.

Inquiries will be kept confidential. Please state age, experience and salary expected. Write Box 288.

ELECTRICAL AND RADIO DESIGN ENGINEERS

Familiar with analysis and design of complex circuits similar to those used in radio transmitter equipment. Should have five years full-time commercial or research experience. Must have B.S. in E.E., or equivalent; thorough grounding in engineering electronics and familiarity with high-voltage rectifier systems. Apply in writing, to Personnel Office, Radiation Laboratory, University of California, Berkeley, California.

RADIO TECHNICIAN

In Brooklyn war plant. Must be able to use test equipment, to set up and use laboratory test instruments and supervise production testing of radio parts and electronic equipment. Will consider men with amateur radio experience. State age, education, experience. Availability certificate required. Write to Box 308.

ELECTRONIC TUBE DESIGN ENGINEER

Experience in actual design and manufacture of large high-vacuum tubes, or electrical and mechanical design, as well as in process, test, and application techniques. Essential workers need release statement. Write to Box 309, giving complete details including salary expected.

ELECTRONIC DESIGN ENGINEERS

One of the largest manufacturers of radio equipment, located in Eastern Massachusetts, has openings for several engineers. Work involves design and development of electronic apparatus having a wide field of application both now and after the war.

A Master's degree, or a B.S. degree in Electrical Engineering with two years' experience in electronic work, would be desirable but not absolutely necessary, depending upon the individual. Those now employed in an essential activity must be able to obtain release. Applicants should submit their qualifications and salary expected to Box 307.

RADIO ENGINEERS AND TECHNICIANS

A progressive company with a sound background in radio and electronics needs, at once, several men with training and experience in any phase of the radio industry. The work open is vital to the war effort but offers a promising post-war future for the right men. College degree or equivalent experience necessary. Men now engaged at highest skill on war production should not apply. Write Box 294.

INSTRUCTORS IN ELECTRICAL ENGINEERING

Instructors in Electrical Engineering needed to teach civilians and A.S.T.P. trainees in Electrical Engineering, in prominent Eastern university. Work will be mostly in communications, some power, and in the Electrical Engineering laboratories. Basic salary $2400-3200. Opportunity for advancement. Box 310.

(Continued on page 51A)

POSITIONS OPEN

(Continued from page 50A)

ELECTRICAL OR CHEMICAL ENGINEER

. . . thoroughly versed in the theory of liquid and solid dielectrics for the position of chief engineer. To direct the research, development and general laboratory on capacitors and capacitor applications. This is an unusual opportunity for a capable engineer interested in his present and postwar future. Write to Industrial Condenser Corp., 1725 W. North Ave., Chicago, Ill.

RADIO ENGINEERS

Transcontinental & Western Air, Inc. has openings at Kansas City for three Radio Engineers in the Communications Department. Applicants should have completed an electrical or radio engineering course, or should have had one to two years of practical experience. These openings are permanent.

For additional details and application forms, write to Personnel Department, Transcontinental & Western Air, Inc., Kansas City, Missouri.

PATENT ATTORNEYS

Patent attorneys, who are electronic physicists and electrical or radio engineering graduates who have maintained contact with the field of high-frequency electronics, radio manufacture, carrier-current telephony, and light-current circuit design and computing, can make a substantial contribution in research or development jobs with one of the National Defense Research Committee laboratories located in the East. The project is secret but is one of the most urgent of all research jobs now under way for the Government.

An electrical engineering background in light currents is essential, and amateur radio experience, inventive ability and ingenuity in the design and layout of radio equipment would be of considerable help.

Facilities for specialized refresher training and orientation in the particular field may be available. Anyone who possesses these qualifications and is interested in a vital wartime development job for the duration may get further details on request. All inquiries will be held confidential. Address Box 299.

ELECTRONIC ENGINEER

Electronic engineer with M.A., Ph.D., or the equivalent in physics, for research and design in geophysics. Experience in filter design and sound recording is desirable. Write to Independent Exploration Company, 901 Esperson Building, Houston, Texas.

ENGINEER

AAA-1 eastern manufacturer, over 75 years operation and leader in growing industry, has immediate permanent position for chemical, electrical, electronic or chemical-metallurgical engineer to organize and increase efficiency of production activities. Send full details of experience to Box 301.

RADIO OR ELECTRICAL ENGINEER

Capacitor manufacturer located in New Bedford, Mass., wants an electrical or radio engineer—man or woman—for equipment- and circuit-development work.

Permanent postwar future for right person. This firm has excellent laboratory facilities and is a leader in its field.

Applicant should be college graduate with a degree—or equivalent experience—in radio engineering or electrical engineering.

Interview in Boston, New Bedford or New York can be arranged. Traveling expenses paid to place of interview.

Write fully, giving age, education, experience, etc. Address reply to Box 302.

RADIO ENGINEERS

Well-established international corporation, 100% in war work with definite postwar possibilities, needs several radio engineers who are familiar with the construction or use of automatic-receiving equipment. Also two transmitter engineers familiar with 40 k.w. equipment.

Applicants should have college degree or approximately ten years experience in radio. Openings in Chicago and New York. Salaries from

(Continued on page 52A)

POSITIONS OPEN

(Continued from page 51A)

$100.00 a week dependent upon experience and ability.

In reply please give complete details of experience, age, education, present and former employers, present earnings and your telephone number. Enclose recent photo if available. Address reply to Box 303.

RADIO ENGINEER

EDUCATION: Minimum of two years college in Electrical Engineering.

EXPERIENCE: Minimum of two years in radio test or engineering, or five years in electrical control work (power station or telephone central-office wiring, etc.).

Must be of a type qualified to interpret and clarify with inspectors and responsible executives electrical specifications, problems of manufacture, test and inspection. Address Box 290.

RADIO AND ELECTRONIC ENGINEERS

First, we are seeking the services of one or two trained engineers who have had ample experience in electronic engineering. The men selected will not only be concerned with present war production, but should eventually develop key positions in postwar operation.

Second, we are also looking for a few young engineers who have had good schooling and background to be trained for specialized work with us.

This is an excellent opportunity for men who qualify to connect with a progressive, highly regarded manufacturer of transmitting tubes. Many special benefits will be enjoyed in your association with this company.

Write at once giving complete details of past experience. Interviews will be promptly arranged. Persons in war work or essential activity not considered without statement of availability. Chief Engineer, United Electronics Company, 42 Spring Street, Newark, New Jersey.

PHYSICIST OR ELECTRICAL ENGINEER

Leading manufacturer of industrial radio-frequency equipment desires the services of a physicist or electrical engineer to direct developmental and applications laboratory. This field is expanding rapidly and offers excellent opportunities for advancement. Position of a permanent nature. Present activities devoted entirely to the war effort. Address replies to Box 306.

SOUND AND PROJECTION ENGINEERS

Openings exist for sound and projection engineers. Several years experience in the installation and maintenance of 35 mm motion-picture equipment of all types required. Must be draft exempt or over draft age and free to travel anywhere in the United States. Basic starting salary $3200. U. S. Army Motion Picture Service, Engineering and Maintenance Division, 3327-A Locust Street, St. Louis, Missouri.

RADIO ENGINEERS

Permanent radio-engineering position in Southern California for men with creative and design aptitude, especially with UHF circuits. Starting salary and advancement depends upon the engineer's experience and ability.

Applications are solicited from persons that are not using their highest skills in war work.

Write complete qualifying educational training and experience to Chief Radio Engineer, Bendix Aviation, Ltd., in care of The Shaw Company, 816 W. 5th Street, Los Angeles 13, California.

The foregoing positions of interest to I.R.E. members have been reported as open. Apply in writing, addressing reply to company mentioned or to Box No. . . . .

Incorrect Adresses

(*Continued from page 48A*)

Crabb, R. C., 1229 Mt. Vernon Ave., Dayton, Ohio
Crain, C. M., 510½ W. 18 St., Austin, Tex.
Cramer, K. H., 740—16 Ave., N., Seattle, Wash.
Cunningham, J. C., Rm. 12, Yatoka Hall, University of New Mexico, Albuquerque, N. M.
Dacewicz, L. N., Box 305, College Station, Durham, N. C.
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Fish, William, 621 Leverette, Fayetteville, Ark.
Fitting, F. N., 1583 Massachusetts Ave., Cambridge, Mass.
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Gardner, F. H., 5831 Philadelphia Dr., Dayton, Ohio
Gardner, R. S., 45 E. High St., Ballston Spa, N.Y.
Garrett, E. T., 808 Union Ave., Belleville, Ill.
Gates, H. W., 506 Wisconsin Ave., Oak Park, Ill.
Geisert, W. O., 2512 E. Union St., Seattle, Wash.
Gerstein, Michael, A.P.O. 958, c/o Postmaster, San Francisco, Calif.
Gervais, W. A., 16 Mellen St., Cambridge, Mass.
Gibson, Robert, University Station, Gainesville, Fla.
Glan, George, 471 Grand Ave., Dayton, Ohio
Goodstine, Herman, 148 Bissell St., Manchester, Conn.
Guest, J. W., 1336 Glenwood Rd., Glendale, Calif.
Haacke, E. M., 276 University Ave., Kingston, Ont. Canada
Hagenbuch, W. H., Graduate House, Massachusetts Institute of Technology, Cambridge, Mass.
Halsall, Michael, 123 Waverly Pl., New York, N. Y
Hamrey, S. D., A.E.T.C., Austin Hall, Harvard Univ., Cambridge, Mass.
Harvey, H. F., 333 W. 88 St, New York, N. Y.
Hawley, P. F., 7339 S. Coles Ave., Chicago, Ill.
Henrich, W. H., 144 Brattle St., Cambridge, Mass.
Hodgers, R. W., 339 W. Berry St., Fort Wayne, Ind.
Hoeper, H. B., 153 Arden Park, Detroit, Mich.
Hoffman, R. W., A.P.O. 617, Douglas Aircraft Corp. c/o Postmaster, New York City
Hofheimer, R. W., Harvard Univ., Cambridge, Mass.
Hollyer, Jr., R. N., 15906 Blackstone, Detroit, Mich.
Homewood, Charles, Casoc Bahrein Island, Persian Gulf.
Hykal, F. A., 14150 Young St., Detroit, Mich.
Inns, S. H., 1241 Agate St., San Diego 9, Calif.
Instrall, R. C., Catterick Camp, Yorkshire, England.
Jackson, J. K., 144 Brattle St., Cambridge, Mass.
Jellinek, Ernest, 13 State St., Schenectady, N. Y.
Jennett, Jr., N. E., Y.M.C.A., Rm. 422, Hunt Ave. Boston, Mass.
Johnson, J. L., Forest Ave., Eden Ter., Catonsville, Md.
Johnston, R. B., Cruft Laboratory, Harvard University, Cambridge, Mass.
Jungerman, J. A., Bowles Hall, Berkeley, Calif.
Kannanstine, F. M., 1922 W. Gray Ave., Houston, Texas
Kantenberger, W. J., 740 S Hampton Rd., Dallas, Texas
Kaye, Philip 1025 Linwood Ave., Apt. A, St. Paul, Minn.
Kelar, Joseph, 52 S. Walnut St., East Orange, N. J.
Kercher, T. C., Atlantic Fleet Schools, Radio Nob, Norfolk, Va.
Kleeb, G. F., 145 Grant Ave., Etna, Pa.
Knox, D. N., c/o Douglas Aircraft Co., A.P.O. 617, Postmaster, N. Y.
Koch, R. F., 436—43 St., West Palm Beach, Fla.
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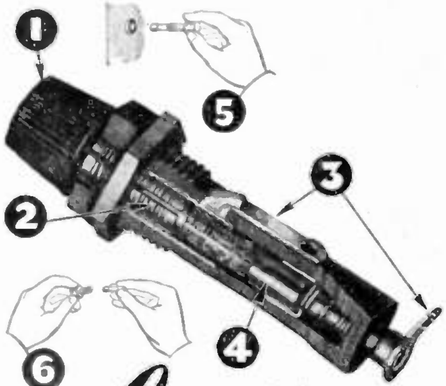

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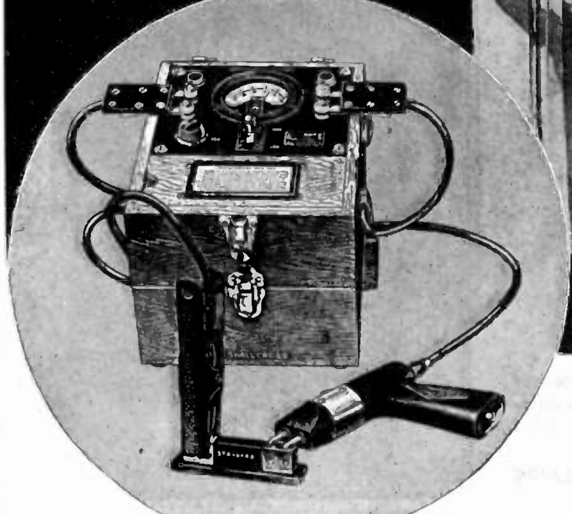

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Lee, W. C., 14 Shrewsbury St., Christchurch, New Zealand
Lewis, H. G., 16 Langdon St., Madison, Wis.
Lines, S. R., 1309 Easton Ave., South Bellingham, Wash.
Little, W. A., 605 Mt. Prospect Ave., Newark, N. J.
Lundwall, G. F., Second Communications Sqdn., Patterson Field, Ohio
Lyman, C. H., 308 S. Sixth St., Bozeman, Mont.
Mack, R. A., Leverett F-41, Cambridge, Mass.
Marriner, A. W., Headquarters, Army Air Forces, Washington, D. C.
McCord, H. W., 1928 Geddes Ave., Ann Arbor, Mich.
McKee, D. I., 38 N. Woodward Ave., Dayton, Ohio
McLeod, K. A., 9727 Sutherland Rd., Silver Spring, Md.
McReynolds, Z. A., Box 99, Palestine, Texas
Mesch, F. A., 913½ South Jenkins, Norman, Okla.
Michaelson, M. W., 707—28 St., Sioux City 18, Iowa
Muller, F.A., 740 Riverside Dr., Apt. 5G, New York N. Y.
Nelson, A. L., Riverview Add, Rt. 10, North Kansas City, Mo.
Newson, J. V., H.Q. 209 Group, Royal Air Force, Middle East Forces
Ogden, B. W. 26 M. Dod Hall, Princeton, N. J.
Pennycook, W. D., Box 1828, Stanford, Calif.
Perrino, Salvatore, Shell Pipe Line Corp., Rm., 1209, Shell Bldg., Houston, Texas
Predmore, E. E., 1746 Cambridge St., Cambridge, Mass.
Procter, E. N., 2276 Shattuck Ave., Berkeley, Calif.
Pumphrey, F. H., 4621—23 St. N., Arlington, Va.
Putzrath, F. L., 15 W. Harrison, Iowa City, Iowa
Radcliffe, J. C., Eagle River, Wisconsin
Regen, B. R., 1224 Cottage Pl., N.W., Canton, Ohio
Reich, A. L., 927 Beacon Ave., Los Angeles, Calif.
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Ringwalt, D. L., 3003 Seventh, S.E., Washington, D. C.
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Rissler, H. D., WHO, Central Broadcasting Co., Hughesville, Wis.
Rocke, Arthur, 100 Varick St., New York, N. Y.
Ross, J. M., 406 Stewart Ave., Ithaca, N. Y.
Ryan, E. L., 180 N. Broadway, Apt. 9, Lexington, Ky.
Schauf, E. L., North English, Iowa
Sherken, J. I., 1829 E. 14 St., Brooklyn, N. Y.
Sheve, H. C., 1023 Flower Ave., Takoma Park, Md.
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Silberstein, Richard, 34 Bexhill Dr., Kensington, Md.
Slater, F. R., R.C.A. Communications, Bolinas, Calif.
Steiner, J. R., 1151 Alba St., Mobile, Ala.
Stevens, A. M., 404 Riverside Dr., Apt. 11B, New York, N. Y.
Stites, F. H., 209 Greene Ave., Brooklyn, N. Y.
Stokes, E. G. C., c/o Forres School, Swanage, Dorset, England
Story, H. O., Almo, Ky.
Tandberg, W. E., 1730 La Loma Ave., Berkeley, Calif.
Taylor, A. H., Box 90, Rt. 2, Anacostia Station, D. C.
Thomas, H. E., 102 Elm Ave., Haddonfield, N. J.
Thompson, Kenneth, 19 Garden St., Cambridge, Mass.
Thornberry, H. C., Y.M.C.A., 316 Huntington Ave., Boston, Mass.
Tristani, Jr., J. E., N. T. Sch., Harvard Univ., Cambridge, Mass.
Tuckerman, L. P., Federal Telegraph Co., 200 Mt. Pleasant Ave., Newark, N. J.
Vandermause, O. J., 1457 Fairmont St., N. W., Washington 9, D. C.

(Continued on page 58A)

Incorrect Addresses

(Continued from page 56A)

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Wakefield, E. H., 5646 Kenwood Ave., Chicago, Ill.
Waychus, F. J., A P O 4002, c/o Postmaster, New York, N. Y.
Wiggin, J. F., 27 Priscilla La., Schenectady, N. Y.
Williams, L. B., 18 Prescott St., Cambridge, Mass.
Willner, M. J., Box 2355, Georgia School of Technology, Atlanta, Ga.
Wolf, S. B., 225 W. Lutz St., West Lafayette, Ind.
Wood, W. L., Fort Bragg, N. C.
Woodruff, C. W., Georgia School of Technology, Atlanta, Ga.

Television Relay Network

Nation-wide television networks made possible by automatic radio relay stations and other new developments, which eventually are expected to lead to finding the key to international television, are a postwar prospect, according to Ralph R. Beal, assistant to the vice president in charge of RCA Laboratories.

A radically new form of "lighthouse" radio relay station developed by the Radio Corporation of America engineers will make relaying of television programs a relatively simple matter, according to Mr. Beal. He envisages that these unattended relay stations located 20 to 50 miles apart will not only link television stations into national networks but will open up a new era in international communications, through development of trunk lines over such vast areas as Russia and China.

"It is to be expected," he stated, "that television stations will first go on the air in such broadcasting centers as New York, Chicago, and Los Angeles. But there is every indication that alert broadcasters will keep pace with them in such localities as Boston, Philadelphia, Washington, Pittsburgh, Cleveland, Detroit, St. Louis, Kansas City, Omaha, Denver, and San Francisco. It seems logical to assume that the first television network linked by radio relay stations will be formed along the Atlantic Seaboard.

"But television will not be limited to the larger cities. The radio map will be dotted with stations in cities like Schenectady, Utica, Syracuse, Minneapolis, Erie, Buffalo, Louisville, and many others. By the use of radio relays, these two will become outlets for the television network which before many years pass after the war, will weave from the east across the Mississippi and the midwest plains to meet a Pacific Coast link striking eastward across the Rockies. A relay station atop Pike's Peak might well be the key station to complete a transcontinental television chain."

Pointing out that radio relay stations may also bring about a vast change in world-wide communications, Mr. Beal explained that, "the routes of these radio relays will extend to any part of the world. They can go through the jungles, from island to island, across mountains and the polar wastes. Neither tropical heat, nor arctic snow,

(Continued on page 60A)

Television Relay Network

(Continued from page 58A)

neither fog nor hurricane will 'cut' the global lines. They can be built to be practical, efficient, and foolproof.

"Just think what television trunk lines will mean to China," he continued. "I have been there and I feel that I know how welcome the new art of radio relaying will be to the millions of Chinese, for it will bring them communications, entertainment, and education on a scale they have never known. What a gigantic task it would be to wire all China and its great open spaces for sound. But how much easier it will be to do the job by 'wireless,' to dot the countryside with relatively inexpensive radio relay transmitters that will given to China a trunk-line system of communication for television, radio, telephone, and telegraph. Even the Himalayas will be no barrier to such radio relaying. Their high mountain peaks will speed the process, for relay stations at such altitudes can reach far beyond the horizons of the valley. China will then have a new Burma Road—a road of television."

It was explained that radio relaying will be a comparatively simple process. The relay transmitters will operate on microwaves with the energy concentrated almost in a beeline. Practically all the power is made to serve a useful purpose; it is not scattered as in broadcasting. Therefore, relatively small amounts of power will operate the relay transmitters. The apparatus is neither cumbersome nor complicated. It is simple and compact. It could not be otherwise and still perform in the domain of tiny wavelengths which bring radio men so close to the frontiers of light, he said.

"We know, of course," continued Mr. Beal, "that ultra-short waves and centimeter waves travel in a straight line and leave the earth on a tangent at the horizon. The area of the earth's surface touched by such waves, is much like that touched by a stick held against a basketball. Obviously, if we use high towers or antennas on lofty buildings or mountain peaks, we capture and retransmit the waves at higher levels, and therefore their effective range is lengthened. With the use of radio relay stations, the average range is about 30 miles, depending upon the terrain and various other factors. It is interesting to recall that an airplane over Washington, D. C., carrying a television receiver intercepted the pictures from the NBC aerial on the dome of the Empire State Building 200 miles away. But for such long-distance reception of the ultra-short waves, the plane had to go up 20,000 feet."

The radio relay system is to be no one-way ethereal street as Mr. Beal charts it. Multiple channels make it all the more promising in efficiency, flexibility, and service. The relay towers will handle numerous circuits, for example, down and back from New York to Washington. Furthermore, the circuits can be multiplied to any reasonable extent, not only to carry one television program but several simultaneously, as well as frequency-modulation sound broadcasts, telegraphic traffic, and facsimile. In fact, relay circuits should be among the busiest in the air.

The main relay system, envisaged by Mr.

(Continued on page 62A)

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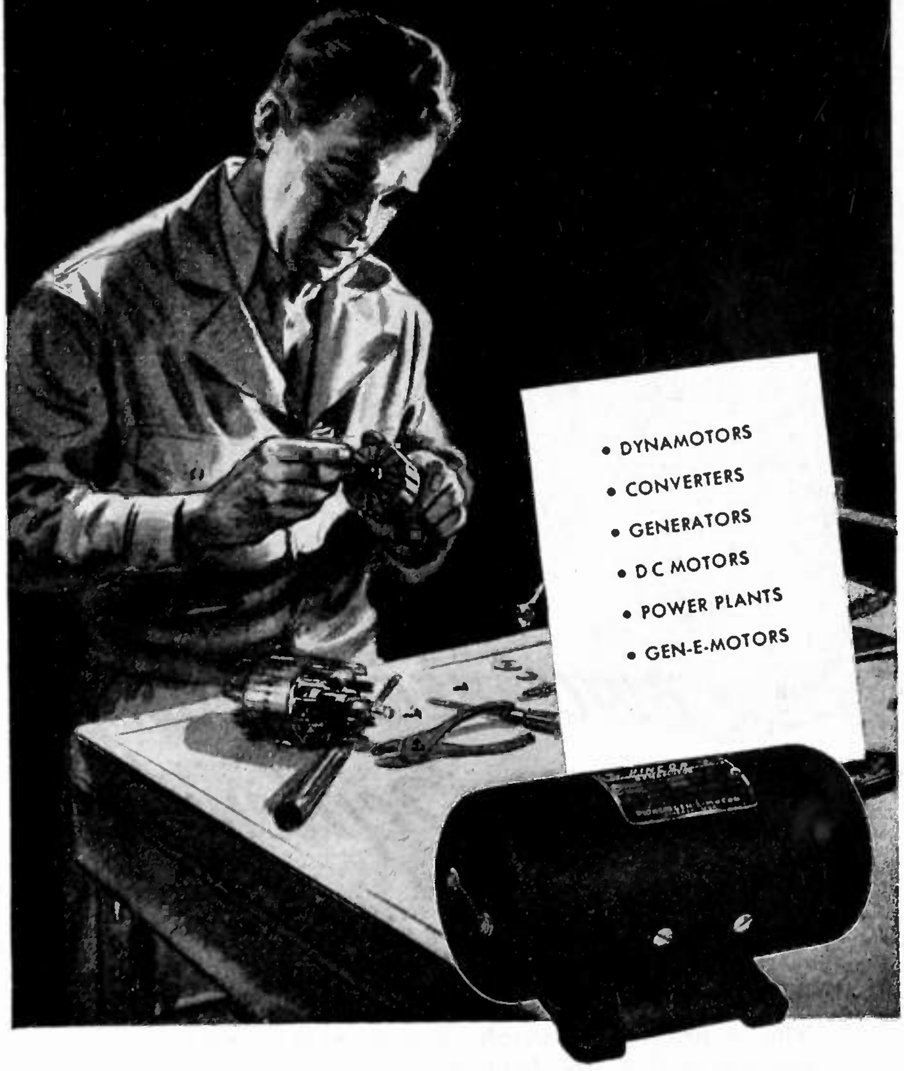

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Television Relay Network

(Continued from page 60A)

Beal, will be like a great intercity spine, becoming interstate and eventually transcontinental. The ribs will spread to television stations. To illustrate its possibilities, he describes it as it is likely to function between New York and Washington: While the NBC television program is being broadcast from the aerial on top of the Empire State Building, a different program will be originating in Washington. Both programs will be fed simultaneously into the relay system leading from Manhattan Island through Philadelphia to the nation's capital. One Philadelphia station can elect to broadcast the program from New York, while another taps the relay channel carrying the program staged in Washington.

Also, if a New York station, aside from the NBC transmitter, desires to broadcast the Washington program it can do so by tapping the relay channel. In this way, the relay system becomes a trunk line that can be tapped at will by the television stations, thereby affording greater freedom of program selection and operation. The relay enhances variety in programming, because there may be four or five relay channels simultaneously carrying different programs, which can be selected by the main television stations.

"Of course it will be understood," concluded Mr. Beal, "that I have spoken of these technical developments from the standpoint of the engineer. I realize, as do others, that it will take money to establish such a radio relay system as I have described. Indeed, it will take more than money. It will require a sympathetic and helpful attitude on the part of governmental agencies concerned with licensing and regulation and the daring spirit of the American industrial pioneers who have led the way in so many new developments."

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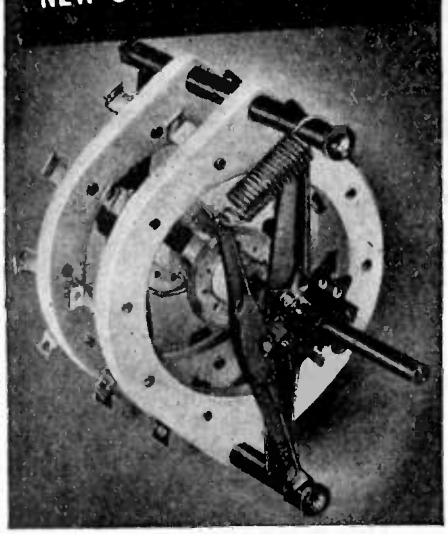

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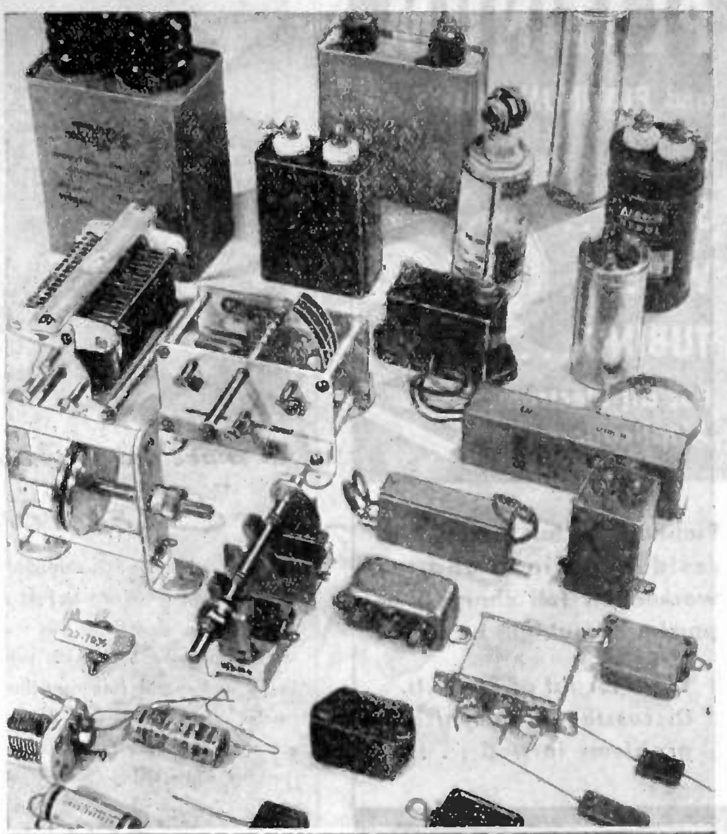

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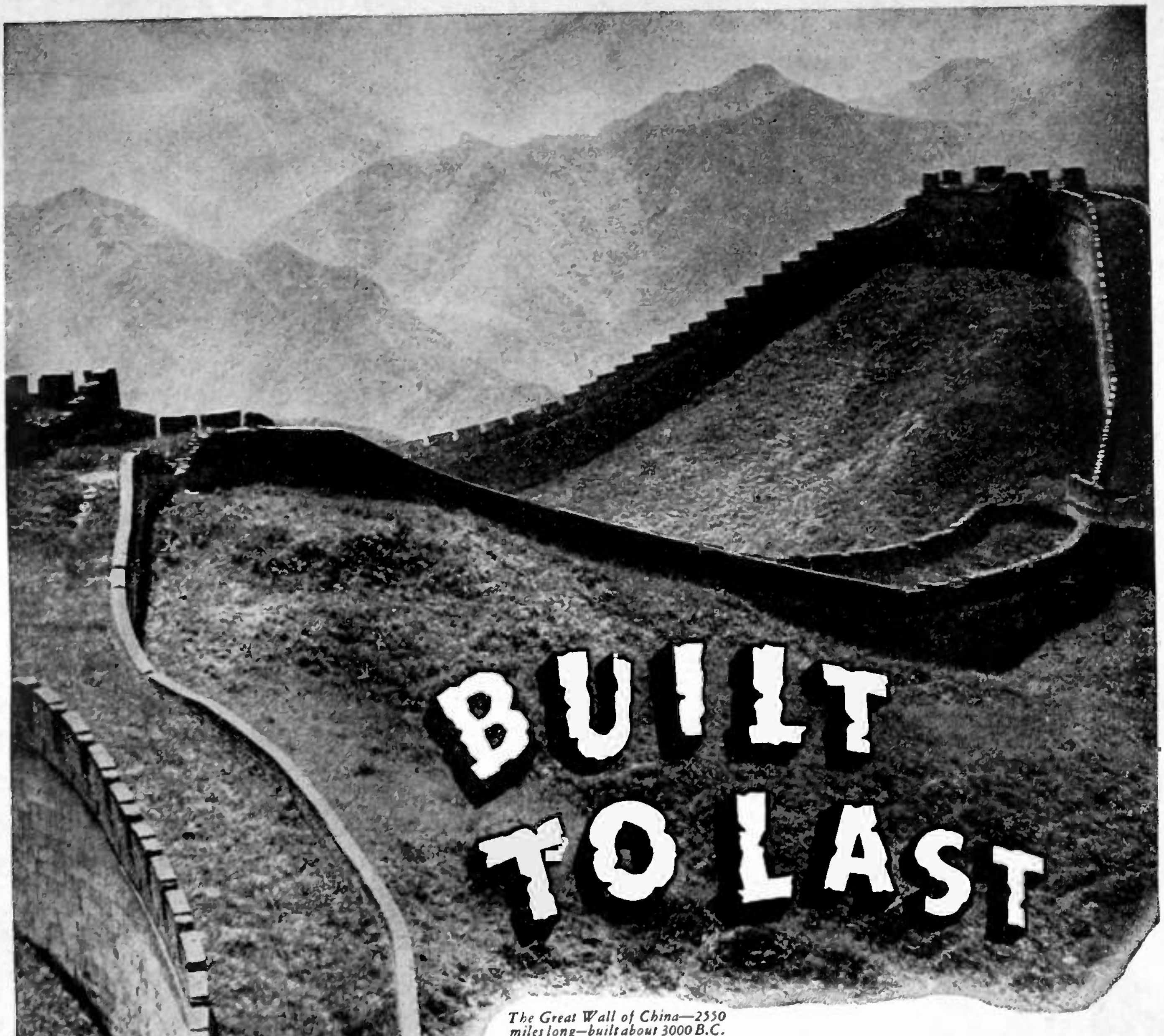

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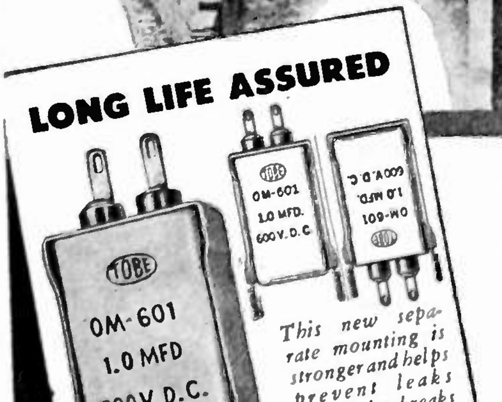

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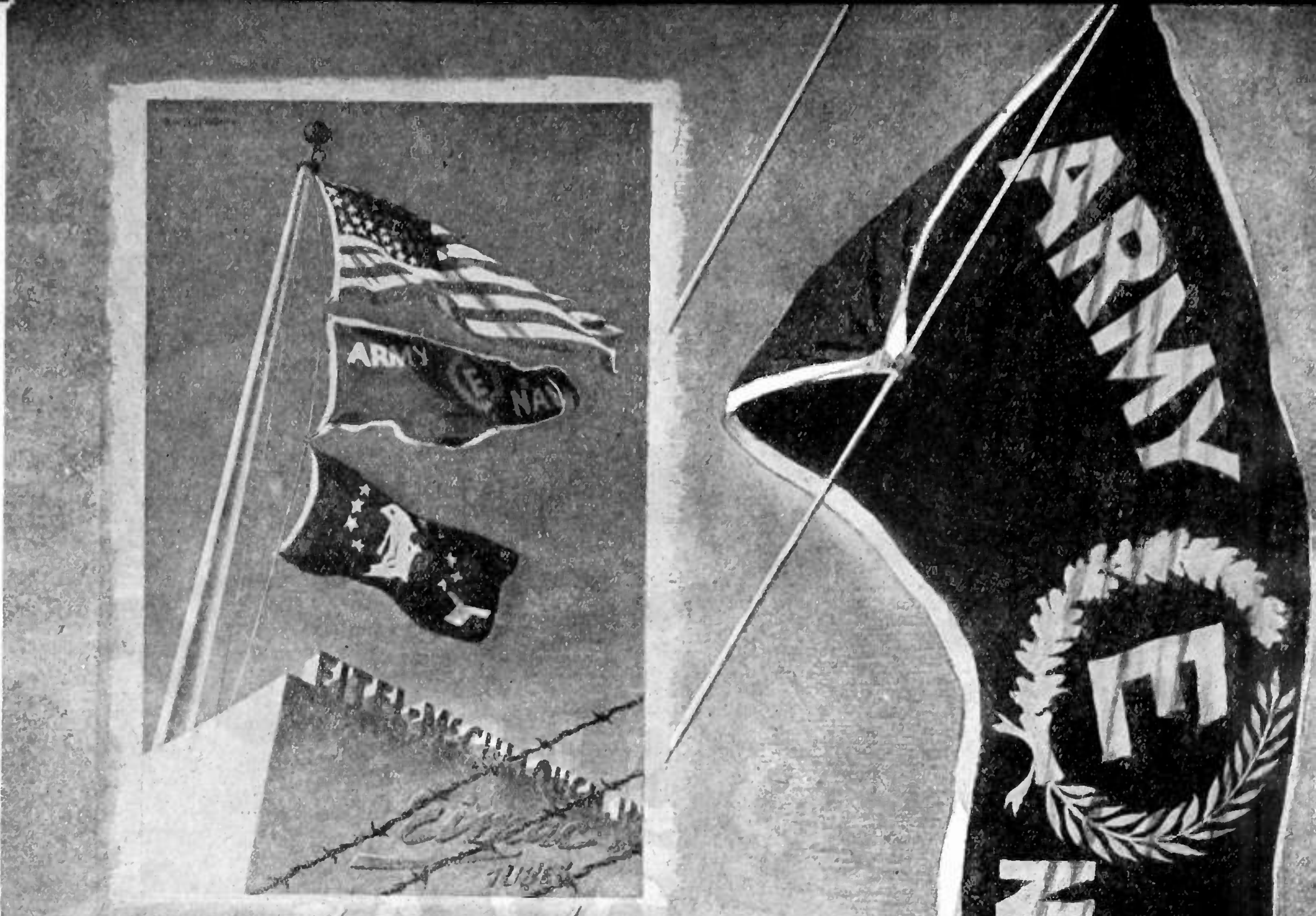

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