VOLUME 24 DECEMBER, 1936 NUMBER 12

PROCEEDINGS
of
The Institute of Radio Engineers

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Institute of Radio Engineers Forthcoming Meetings

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December 15, 1936

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December 18, 1936

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December 2, 1936
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December 14, 1936

PROCEEDINGS OF

The Institute of Radio Engineers

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The Institute of Radio Engineers

GENERAL INFORMATION

INSTITUTE. The Institute of Radio Engineers was formed in 1912 through the amalgamation of the Society of Wireless Telegraph Engineers and the Wireless Institute. Its headquarters were established in New York City and the membership has grown from less than fifty members at the start to several thousand.

AIMS AND OBJECTS. The Institute functions solely to advance the theory and practice of radio and allied branches of engineering and of the related arts and sciences, their application to human needs, and the maintenance of a high professional standing among its members. Among the methods of accomplishing this is the publication of papers, discussions, and communications of interest to the membership.

PROCEEDINGS. The PROCEEDINGS is the official publication of the Institute and in it are published all of the papers, discussions, and communications received from the membership which are accepted for publication by the Board of Editors. Copies are sent without additional charge to all members of the Institute. The subscription price to nonmembers is $10.00 per year, with an additional charge for postage where such is necessary.

RESPONSIBILITY. It is understood that the statements and opinions given in the PROCEEDINGS are views of the individual members to whom they are credited, and are not binding on the membership of the Institute as a whole. Papers submitted to the Institute for publication shall be regarded as no longer confidential.

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GEOGRAPHICAL LOCATION OF MEMBERS ELECTED NOVEMBER 12, 1936

Transferred to the Member Grade

Massachusetts	Longmeadow, 121 S. Park Ave.	Filgate, J. T.
New Jersey	Whippany, c/o Bell Telephone Labs.	Doherty, W. H.
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	New York, Rm. 905, 180 Varick St.	Black, W. L.
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England	Cambridge, 22 Hartington Grove	Carter, C. J.

Elected to the Associate Grade

California	Alhambra, 425 N. Olive Ave.	Swift, F. T., Jr.
	San Diego, Box 202	Ferreira, F.
Colorado	Denver, c/o A. L. Smith, Colorado Geophysical Corp., 610 Midland Savings Bank Bldg.	Hall, H. N.
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New Jersey	Boonton, 81 Dixon Ave.	Cook, L.
	Camden, RCA Manufacturing Co., Inc., 15-2	Van Dyke, W. D.
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	New York, 11 Dominick St.	Baxter, D. C.
	New York, 395 Broome St.	Brunn, R
	Poughkeepsie, Vassar College	Northrup, P. A.
Ohio	Cincinnati, 5024 Oberlin Blvd.	Foster, A. P.
	Cleveland, 1701 W. 25th St.	Plakadis, A.
Pennsylvania	Emporium, c/o Hygrade Sylvania Corp.	Fink, G. A.
	Emporium,	Ostrum, S.
	Emporium, Hotel Warner	Ottemiller, W. H.
	Narberth, 5 Windsor Ave.	Brazee, G.
	Ridgway, P. O. Box 64	Deichman, G. A.
Argentina	Buenos Aires, San Juan 2878-3. p.	Benjamin, M.
Canada	Dartmouth, N. S., Royal Canadian Air Force	Carter, R. D.
	Toronto, Ont., 130 Galley Ave.	Linton, W. R.
England	Bodmin, Cornwall, 8 Beacon Hill	Wills, R. J.
	Chelmsford, Essex, 10 Highfield Rd.	Wells, W. J. M.
	Croydon, Surrey, 38 Temple Rd.	Drewry, E. R.
	Northolt, Middlesex, 14 Kingsmead Dr.	Singh, W. K.
	Romiley, Cheshire, The Oaks	Williams, F. C.
Estonia	Tallinn, Reimanni 9	Work, H. R.
Hungary	Budapest, VI, Podmaniczky ucca 12	Czegledy, G. A.
India	Apollo Bunder, Bombay, c/o I.R.C.C. Co., Ltd., "Radio House"	Gokarn, G. D.
	Santa Cruz, Bombay, Indira Lodge	Gidh, S. G.
Iraq	Basrah, Central Technical Shuba No. 3	Tooni, N. N.
Ireland	Rathfarnham, Co. Dublin, 11 Butterfield Park	Egan, M. C.
Malta	B-Kara, 20 Str. Collegiata	Galea, R. F.
New Zealand	Christchurch, St. Michaels Vicerage, Oxford Ter.	Perry, P. N. M.
South Africa	Durban, 107 Manning Rd.	Blinkhorn, K.
Territory of Hawaii	Honolulu, 1466 S. King St.	Nakamoto, W. H.

Proceedings of the Institute of Radio Engineers
Volume 24, Number 12 *December, 1936*

APPLICATIONS FOR MEMBERSHIP

Applications for transfer or election to the various grades of membership have been received from the persons listed below, and have been approved by the Admissions Committee. Members objecting to transfer or election of any of these applicants should communicate with the Secretary on or before December 31, 1936. These applications will be considered by the Board of Directors at its meeting on January 6, 1937.

For Transfer to the Fellow Grade

California	Stanford University, 659 Salvatierra St.	Terman, F. E.

For Transfer to the Member Grade

Kentucky	South Newport, 206 Main Ave.	Richards, A. P.
New Jersey	Morristown, 45 Franklin St.	Foster, D. E.
New York	Schenectady, 1491 Parkwood Blvd.	Ford, W. A.
Pennsylvania	Emporium, 104 W. 6th St.	Acheson, M. A.
	Emporium, 414 W. 4th St.	Ackman, H. E.
	Emporium, 4 E. 6th St.	Kahl, M. I.
	Emporium, 36 W. 4th St.	Palmer, R. N.
	Emporium, 228 W. Allegheny Ave.	West, L. E.
	Emporium, 520 Vine St.	Wilson, M. D.
	St. Marys, 113 E. Erie Ave.	Bowie, R. M.
	St. Marys, 252 N. St. Marys St.	Hoffman, R. R.

For Election to the Member Grade

Pennsylvania	Philadelphia, 6457 N. Smedley St.	Gillies, J. K.

For Election to the Associate Grade

Alabama	Andalusia, Box 445	Everage, A. E.
California	Inverness	Rezos, G.
	San Francisco, 667-9th Ave.	Hollingsworth, L. M.
	San Pedro, USS Tennessee	Smith, A. J.
Connecticut	Bridgeport, General Electric Co., Bldg. 33E	Chun, M. E.
District of Columbia	Bellevue, Washington, Naval Research Lab.	Fifer, W. H.
	Bellevue, Washington, Radio Materiel School	Rost, E. J.
Illinois	Urbana, 1011 E. Water St.	Phillips, W. E.
Indiana	Fort Wayne, Capehart Corp.	Baxter, J. M.
Iowa	Dubuque, 1595 Atlantic St.	Lutes, R. K.
Kentucky	Louisville, 936 Audubon Park	LaVielle, W. R. R.
	Louisville, 2601 Garland Ave.	Rubin, H. B.
Massachusetts	Springfield, 117 Northampton Ave.	Weeks, G. E.
Michigan	Milford, General Motors Proving Ground	Huber, P.
Missouri	Springfield, 600 E. Page St.	Robinson, C. F.
New Jersey	East Orange, 106 N. Walnut St.	Pritchard, E. M.
	Harrison, RCA Mfg. Co., Inc., RCA Radiotron Div.	Waller, L. C.
	Whippany, Bell Telephone Labs., Box E	Ong, F. C.
	Whippany, Bell Telephone Labs.	Young, L. G.
New York	Brooklyn, 514 Lafayette Ave.	Levi, S.
	New York, 233 Broadway	Beebe, H. G.
	New York, 984 Bronx Park South	Malkin, S.
	New York, Hazeltine Service Corp., 333 W. 52nd St.	Swinyard, W. O.
	New York, 116 Loring Ave.	Taylor, H. S.
	Schenectady, 4 Washington Ave.	Jones, J. L.
	Woodside, L. I., 4054-71st St.	Dostal, F.
Pennsylvania	St. Marys, Hygrade Sylvania Corp.	Price, G. A.
Texas	Dallas, 2211 Commerce St.	Hill, J. O.
	San Antonio, 1719 McKinley Ave.	Ross, C. C.
Washington	Pasco, 25 W. A St.	Lawrence, A. C.
Belgium	Brussels 3, 8 Rue Van Droogen Broeck	Van Hecke, A.
Brazil	Rio de Janeiro, Rua Theophilo Ottoni 74, loja	Damm, G.
Canada	Hamilton, Ont., 35 Catherine St., S.	Pickett, J. H.
	Toronto, Ont., 168 Delaware Ave.	Britney, O. L.
Chile	Santiago, Casilla 3396	Markoff, J. I.
England	Anerley, London S.E. 20, 4 Genoa Rd.	Evans, W. F.
	Leytonstone, London E. 11, 83 Poppleton Rd.	Jones, E.
	Manchester, Lancs., 101 Bellott St.	Olsberg, I.
	Ruislip, Middlesex, 94 Linden Ave.	Cockerell, P.
	Sidcup, Kent, 4 Belton Rd.	Barber, H. J.

Holland	Eindhoven, N.V. Phillips Gloeilampenfabrieken	Van der Mark, J.
India	Bangalore, Indian Inst. of Science, Hebbal P.O.	Patel, I. L.
	Bombay, 775 Parsi Colony, Dadar	Dharap, P. W.
Japan	Kagoshima, Kagoshima Hosokyoku Tempozancho	Sakamoto, R.
Kenya	Nairobi, P.O. Box 1093	Pegrume, S. A.
Latvia	Riga, c/o VEF, Brivibas Gatve 19	Ziemelis, V.
South Africa	Germiston, c/o South African Airways, Rand Airport	Breeze-Carr, A.
Territory of Hawaii	Kahuku, Oahu, c/o R.C.A. Communications, Inc.	Gray, G. P.
Venezuela	Caripito, Standard Oil Co. of Venezuela	Johnson, D. W., Jr.
	Maracaibo, c/o Lago Petroleum Co.	Neidert, J. H.

For Election to the Junior Grade

Illinois	Hinsdale, 113 Mineola St.	Sobotka, H. J.
Indiana	Indianapolis, 605 N. Sherman Dr.	Thompson, M. E.
	Valparaiso, 252 Locust St.	Denton, F. L.
Pennsylvania	Nazareth, R.F.D. 3	Schramm, F. E., Jr.
England	Forest Hill, London S.E.23, 74 Brockley Rise	Davie, O. H.

For Election to the Student Grade

California	Whittier, 1738 Keith Dr.	Grismore, H. M.
Indiana	Portland, 604 W. Arch St.	Spade, J. M., Jr.
Iowa	Ames, 238 Hyland Ave.	Sproul, P. T.
Kansas	Manhattan, 1130 Vattier St.	Caldwell, R. W.
Massachusetts	Boston, 330 Bay State Rd.	Hazeltine, A. V.
	Boston, 82 Gainsboro St.	Turner, R. W.
	Cambridge, 51 Oxford St.	Graef, R. L.
	Cambridge, Box 566, M.I.T. Dormitories	Smith, C. R.
	Cambridge, M.I.T. Dormitories	Wiggin, J. F.
	Cambridge, 8 Dana St.	Wood, D. O.
Nebraska	Holdrege, 915 Sherman St.	Hjelmfelt, R. H.
New Jersey	Allendale, 754 Franklin Turnpike	Phair, R. S.
	Hoboken, 532 River St.	Buchanan, R. L.
New York	Flushing, L. I., 144-23 Barclay Ave.	Duckworth, D. T.
Ohio	Toledo, 2249 Westbrook Dr.	Bissonette, A. J.
	Toledo, 1530 Sabra Rd.	Byram, E. F.
	Toledo, 2053 Dana St.	Hilding, H.
	Toledo, 1516 Lincoln Ave.	Keck, M. W.
	Toledo, 3642 Garrison Rd.	Kinney, B. W.
	Toledo, 526 Valleywood Dr.	Tracy, P. C.
Oklahoma	Norman, 435 College Ave.	Dorsett, L. G.
Pennsylvania	Bethlehem, 229 Warren Sq.	Ford, C. H.

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INSTITUTE NEWS AND RADIO NOTES

November Meeting of the Board of Directors

The regular November meeting of the Board of Directors was held on the 12th in the Institute office. Those present were Alfred N. Goldsmith, acting chairman; Melville Eastham, treasurer; H. H. Beverage, Virgil M. Graham, L. C. F. Horle, C. M. Jansky, Jr., A. F. Murray, E. L. Nelson, Haraden Pratt, H. M. Turner, and H. P. Westman, secretary.

The following were admitted to the grade of Member: C. J. Carter, W. H. Grimditch, J. B. Rock, H. W. Dudley, W. L. Black, and S. Y. White. The following were transferred to Member: A. W. Knight, P. E. A. Griffiths, F. J. Bingley, W. H. Doherty, and J. T. Filgate. Thirty-eight were elected to the Associate grade of membership.

Approval was granted for the holding of a Rochester Fall Meeting in November, 1937. The precise date will be announced later.

The Report of the Tellers Committee on the count of ballots for the election of officers was accepted and the following declared elected:

President, 1937	H. H. Beverage
Vice President, 1937	P. P. Eckersley
Directors, 1937–1939	Ralph Bown
	Alfred N. Goldsmith
	H. M. Turner

Dr. Goldsmith and Secretary Westman were reappointed as representative and alternate, respectively, of the Institute on the Standards Council of the American Standards Association.

Mr. Jansky reported on preparations which are being made for the Institute's participation in the First National Conference on Educational Broadcasting to be held in Washington, D. C., on December 10, 11, and 12, 1936. Arrangements have been made for three papers treating the engineering aspect of the subject.

Twenty-Fifth Anniversary Convention

Preparations for the program of the Institute's Twenty-Fifth Anniversary Convention which will be held in New York City on May 10, 11, and 12, 1937, are getting under way. Authors who are interested in submitting papers for possible presentation at this convention are requested to prepare them in finished form and submit them to the Institute not later than February 1st. In forwarding manuscripts it should be specifically indicated that they are submitted for convention presentation and if they are submitted also for publication in the PROCEEDINGS this should be stated. Manuscripts submitted without a statement as to convention intentions will be assumed to have been forwarded for PROCEEDINGS publication only.

Joint Meeting of the Institute and the American Section, U.R.S.I.

The annual Spring joint meeting of the Institute of Radio Engineers and the American Section of the International Scientific Radio Union will be held in Washington, D. C., the tentative date being April 30, 1937. This meeting is an important feature of the week which attracts to Washington every year an increasingly large number of scientists and scientific societies. Papers on the more fundamental and scientific aspects of radio will be presented. Titles and abstracts of papers offered for this program may be submitted to S. S. Kirby, Technical Secretary, American Section, U.R.S.I., National Bureau of Standards, Washington, D. C., not later than February 15, 1937.

Radio Emissions of Standard Frequency

The National Bureau of Standards provides standard frequency emissions from its station WWV at Beltsville, Md. On each Tuesday and Friday the emissions are continuous unmodulated waves and on each Wednesday they are modulated by an audio frequency, generally 1000 cycles. There are no emissions on legal holidays.

On all schedules three radio carrier frequencies are transmitted as follows: noon to 1 P.M., Eastern Standard Time, 15,000 kilocycles; 1:15 to 2:15 P.M., 10,000 kilocycles; and 2:30 to 3:30 P.M., 5000 kilocycles. The accuracy of these frequencies will at all time be better than one part in five million.

During the first five minutes of each transmission announcements are given of the station call letters, the frequency of transmission, and the frequency of modulation, if any. For the CW emissions, the announcements are in telegraphic code and are repeated at ten-minute intervals. For the modulated emissions, the announcements are given by voice only at the beginning of each carrier frequency transmission, the remainder of the hour being an uninterrupted audio frequency. The CW emissions are from a twenty-kilowatt transmitter and the modulated transmissions are from a one-kilowatt set.

Information on how to utilize these signals is given in a phamphlet obtainable on request from the National Bureau of Standards, Washington, D. C. Reports from those using this service will be welcomed by the Bureau. As the modulated emissions are somewhat experimental it is particularly desired that users report their experiences outlining methods of utilization, information on relative fading intensity, etc., on the three carrier frequencies and preferences as to the audio frequency to be furnished.

Incorrect Addresses

At the end of the technical portion of this issue of the PROCEEDINGS there will be found a list of the names of Institute members whose correct addresses are not known to the Institute. Persons having more recent addresses for these members will confer a favor on them and the Institute by forwarding this information to the Institute.

Index for 1936

An index of all papers published during 1936 is included in this issue. This index includes a list of the papers in the chronological order of their publication, a cross index of authors' names, and a cross index by subjects. It is expected that a complete index for all issues of the PROCEEDINGS published to date will be available in the near future.

Binders

Those who are interested in collecting together the twelve issues of the PROCEEDINGS published during 1936 may find our standard binders of convenience. These binders are of substantial construction and may be used either as temporary transfers or as permanent binders. They are available at $1.50 each and the member's name or PROCEEDINGS volume number will be stamped on it for fifty cents additional.

Committee Work

ADMISSIONS COMMITTEE

A meeting of the Admissions Committee was held in the Institute office on November 12. Those present were C. M. Jansky, Jr., chairman; H. H. Beverage, F. W. Cunningham, R. A. Heising, L. C. F. Horle, F. A. Kolster, E. R. Shute, and H. P. Westman, secretary.

An application for transfer to the grade of Fellow was approved and one for admission to that grade was denied. Of thirteen applications for transfer to Member, eleven were approved and two were tabled. There were four applications for admission to the grade of Member and of these one was approved, two were denied, and one was tabled.

Constitution and Laws Committee

A meeting of the Constitution and Laws Committee was held in the Institute office on November 12 and H. M. Turner, chairman; Austin Bailey, and H. P. Westman, secretary, were present.

The committee completed its examination of the Institute Constitution and is expected to submit its recommendations for modification of that document to the Board of Directors in the near future.

New York Program Committee

The New York Program Committee met in the Institute office on October 15 and those present were Haraden Pratt, chairman; R. R. Beal, G. C. Connor, R. A. Heising, Keith Henney, George Lewis, and H. P. Westman, secretary.

Final arrangements were made for the November 12 meeting and proposals for the program of the December 2 meeting considered.

Tellers Committee

The Tellers Committee, comprised of Haraden Pratt, chairman; Baldwin Guild, A. R. Hodges, and H. P. Westman, secretary, met on October 31 in the Institute office and counted the ballots cast in the election of officers. Its report was presented to the Board of Directors at the November 12 meeting of that body.

Technical Committee on Electronics

A meeting of the Technical Committee on Electronics was held in the Institute office on October 29 and those present were B. J. Thompson, chairman; R. S. Burnap, E. L. Chaffee, F. R. Lack, George Lewis, F. Holborn (representing J. W. Milnor), G. D. O'Neill, O. W. Pike, Dayton Ulrey, P. T. Weeks, Leslie Woods, and H. P. Westman, secretary.

The committee prepared a schedule on which will be based its preparation of a report on developments in its field during 1936 which will be presented at the Annual Meeting of the Institute to be held in January, 1937.

The criticisms of the Standards Committee of the report on definitions, which was considered by it on September 18, were reviewed and replies prepared.

Subcommittee on High-Frequency Tubes—Technical Committee on Electronics

The Subcommittee on High-Frequency Tubes of the Technical Committee on Electronics met in the Institute office on November 9. Those present were B. J. Thompson, chairman; W. D. Hershberger, G. R. Kilgore, F. B. Llewellyn, A. L. Samuel, and H. P. Westman, secretary.

The meeting was devoted to the preparation of material on developments in its field during 1936. This material will be submitted to the Technical Committee on Electronics for inclusion in the report being prepared by that group for presentation at the Annual Meeting of the Institute.

Technical Committee on Transmitters and Antennas

A meeting of the Technical Committee on Transmitters and Antennas was held on November 13 in the Institute office. Those present were J. C. Schelleng, chairman; Raymond Guy, D. G. Little, W. S. Marks (representing P. E. Watson), R. E. Poole, E. G. Ports, D. S. Rau, and H. P. Westman, secretary.

The meeting was devoted to a final consideration of its preliminary reports on definitions and the testing of transmitters and antennas. The material is now to be submitted to the Standards Committee for action by that body.

Institute Meetings

Buffalo-Niagara Section

The Buffalo-Niagara Section devoted its meeting on October 31 to an inspection trip to the new strip and sheet mill of the Bethlehem Steel Company of Lackawanna, N. Y. G. C. Crom, chairman, presided.

In the process of making steel strips, plates about five or six inches thick and six or eight feet long are heated to a bright red color. They are run through large rolls by stages until about a sixteenth of an inch thick, five or six feet wide, and several hundred feet long. These sheets are cleaned, pickled, and cut into strips which are arranged in large piles. Portable annealing ovens are placed over the piles and after annealing the strips are cold-rolled and cut to desired size. Roll motors are as large as 8500 horsepower and conveyor tables are equipped with hundreds of individual small motors driving rollers in pairs.

Chicago Section

A meeting of the Chicago Section was held on October 23rd jointly with the local section of the American Institute of Electrical Engineers and the Western Society of Engineers in the Engineering Building Auditorium.

A paper on "High-Frequency Broad-Band Wire Transmission" was presented by H. A. Affel, toll transmission development director of Bell Telephone Laboratories.

Broad-band wire transmission defines a trend in the art of long-distance communication in which considerably higher frequencies are transmitted than has hitherto been possible. The idea is being applied to open-wire lines, cables, and to the more recently developed coaxial structures. In long circuits, in order to overcome the attenuation at the high frequencies, amplifiers or repeaters are provided at intervals which in some cases are as short as ten miles. These devices have complex regulating arrangements to take care of the effect of temperature and weather variations on the conductor characteristics. The broad-band frequency range which is transmitted can be split up by filters and other devices at the terminals to provide different communication channels as desired, telegraph, telephone, or perhaps television. The whole broad-band technique has been made possible by several recent laboratory developments, such as extremely stable wide frequency range feed-back amplifiers, crystal filters, etc. These different basic developments were described, as well as the essential features of particular systems whose development is under way. These systems include a twelve-channel carrier system for open-wire lines, a twelve-channel carrier system for application to existing cables, and a million-cycle 240-channel coaxial system which is capable also of providing a transmission path for a high definition television system.

The attendance was 400.

Cincinnati Section

A meeting of the Cincinnati Section was held on October 20 at the University of Cincinnati. G. F. Platts, vice chairman, presided and there were eighty members and guests present.

A paper on "Use of the Cathode-Ray Oscillograph in Radio Testing and Service" was presented by Walter Weiss, engineer for the Hickok Electrical Instrument Company.

Because of a large turnout of radio servicemen, Mr. Weiss covered the basic theory of vacuum tubes and cathode-ray tubes in general. The theory of the grid controlled glow discharge type of linear sweep circuit was explained, and the uses of the linear sweep on the oscilloscope were outlined. Frequency modulated oscillators make use of a motor-driven condenser or a frequency control tube similar in circuit to that used in automatic-frequency-control systems. One can apply sixty-cycle voltage to the grid of such a control tube and use a sixty-cycle sweep which will give a double trace pattern or one can apply a saw-tooth wave to the grid of the control tube and with a linear sweep obtain a single trace pattern. The relation between control tube grid voltage and frequency shift must be linear to obtain a pattern which is the true indication of the selectivity curve of the device tested. The paper was discussed by Messrs. Boes, Freeman, Kilgour, and Wells.

Detroit Section

The October meeting of the Detroit Section was held on the 16th in the Conference Room of the *Detroit News*. R. L. Davis, vice chairman, presided and there were fifty-two present. Ten attended the informal dinner which preceded the meeting.

A paper on "Some Concepts of Modern Physics" was presented by G. P. Brewington, professor of physics at Lawrence Institute of Technology. Dr. Brewington pointed out that the properties of a solid body depend on the position of the atoms in the crystal. The atoms are held in position at a point where the forces of attraction are equal to the forces of repulsion. When an object is heated the atoms begin to vibrate about their positions of rest. In setting up this vibration a certain amount of energy is required. As the temperature of a solid is raised, the number and amplitude of these vibrations are increased. This accounts for the energy that must be put into a solid to raise its temperature.

The strength of a solid depends on the interatomic forces. An actual crystal is weaker than these forces would indicate. Several examinations of this weakness indicate that certain surface cracks and intercrystalline material may contribute to this lack of strength.

Some atoms as they collect to form a crystal lose an electron. These electrons form a cloud throughout the crystal. Metals are thought to be of such solids. If an electron is not lost when the atoms pack to form a solid the solid becomes an insulator.

In a crystal the electrons have different properties than in free space in that only certain electron velocities are possible. Electrical and heat conductivity depend on this change or limitation of the properties of electrons in the crystal.

The explanation of magnetism is sought in the spinning electron. Ferromagnetic materials are of the cubic crystal type and possibly this crystal allows a larger number of electrons to line their spins parallel. The axis of easy magnetization depends on the material.

The paper presented at this meeting was broadcast from the ultra-high-frequency transmitter of the *Detroit News*, W8XWJ. Arrangements are being made for the broadcasting of future meetings.

Emporium Section

A meeting of the Emporium Section was held on October 15 at the American Legion Club Rooms with H. A. Ehlers, vice chairman, presiding. There were seventy-three present.

Two papers were presented, the first on "The Cyclotron" was by L. A. DuBridge, professor of physics at the University of Rochester. Dr. DuBridge pointed out that the atom was conceived as being composed of a positively charged nucleus surrounded by a cloud of negative electrons. Within the last decade attempts have been made to study the nucleus alone. Dissociating the nucleus from the electrons requires an enormous force to overcome the binding energies within its "solar" system and the very strong potential barrier immediately surrounding the nucleus. If a correspondingly small particle were subjected to the accelerating force of an enormously high electrical potential, it might be used to break the nucleus away from its electrons and permit its isolated study. The accelerating potential required is of the order of millions of volts, and the "cyclotron" is one answer. The bombarding particles are driven in helical paths from the center by the strong electric field produced by high-frequency oscillations and a very intense magnetic field. Under their influence the particle attains progressively higher velocities until the desired value is reached and the particle discharged at some atom. The effectiveness of the cyclotron is indicated by the large number which have been or are being constructed by research groups. The paper was discussed by Messrs. Bowie, Dehlinger, Fink, and West.

The second paper was given by C. B. Eckel, maintenance engineer for the Hygrade Sylvania Corporation, and was on "The Electrical Equipment of a Modern Vacuum Tube Factory." Mr. Eckel introduced the subject with a brief discussion of the distribution network comprised of the power company's substation and the substation at the point of feed to the plant. Protective systems employed were then discussed in detail.

The Hygrade Sylvania plant employs about 900 motors varying from one-quarter to forty horsepower in size. Descriptions were given of thyratron and other types of accurate timing control devices used in conjunction with some 600 electric welders, automatic machines, and on-off circuits where accurate timing is required. Indicating, control, and recording equipment monitor many of these operations. Approximately 2000 measuring instruments are in use and a meter-instrument department is an essential part of the maintenance program. Tube exhaust, basing, and aging equipment were described and their electrical needs outlined. Test equipment for checking the quality of nearly 200 types of vacuum tubes was covered. Life test racks with their need for complete flexibility as regards connections and voltages applied to each element were described. This department alone consumes an average of 20,000 kilowatt hours of power per month. The paper was discussed by Messrs. Abbot, Dehlinger, and Kievit.

Los Angeles Section

C. R. Dailey, chairman, presided at the November 15 meeting of the Los Angeles Section held at the Los Angeles Junior College. There were sixty present.

A paper on "Factors Limiting Low-Frequency Response in Dynamic Loud-Speakers" was presented by H. S. Knowles of the Jensen Manufacturing Company. He presented an electrical analogy of the fundamental mechanical design features illustrating the effects of mass, stiffness, and the fluid resistance offered by air. Low-frequency response may be increased by the use of large cones or dual speakers. Distortion at low frequencies was shown in some cases to be so severe that the fundamental frequency comprised only about twenty per cent of the generated sound. Various types of baffles were described such as the pipes used by RCA Victor, Stromberg-Carlson labyrinth, and a development of the Jensen Manufacturing Company. In the latter, the wave from the back of the cone is used as an aiding coefficient giving a three-decibel increase in output and improved low note reproduction resulting from decreased resonance peak, the introduction of another at about forty cycles, and a reduction in harmonic distortion.

The October meeting of the Los Angeles Section was also held in the Los Angeles Junior College with C. R. Dailey presiding. There were seventy present and eighteen attended the dinner which preceded the meeting.

The first paper was on "High-Frequency Reception," by Ralph Gordon of the Radio Frequency Laboratories. In it hediscussed the problems encountered in building ultra-high-frequency receivers emphasizing the necessity for broad intermediate-frequency amplifiers. A description was given of a receiver using a 5000-kilocycle intermediate-frequency amplifier. The theory of noise silencing circuits was described also.

The second paper was on "Observations of Variation in Strength of High-Frequency Signals," by C. Perrine of the Radio-Television Supply Company. In it there was described a series of observations on signal strength in the twenty-meter amateur band with particular reference to signals from Europe. These observations show a distinct twenty-seven-day cycle with predictable variation.

The third paper on "Antennas," by J. N. A. Hawkins of *Radio* magazine, was devoted chiefly to the description of directional antennas for high-frequency transmission and reception.

Philadelphia Section

On October 1st a meeting of the Philadelphia Section was held at the Engineers Club with Irving Wolff, chairman, presiding. Eighteen attended the dinner which preceded the meeting.

A paper on "Intense Sound as a Tool in Research and in Practice" was presented by L. A. Chambers of the School of Medicine of the University of Pennsylvania. In it, Dr. Chambers described equipment for applying intense vibrations at frequencies between one and ten kilocycles to liquid systems. Vibrations of such amplitude as to produce cavitation in the flowing liquids, destroy bacteria and other organisms suspended in the fluid. No adequate explanation of the killing action has been offered but a motion picture through a column of water during vibration indicated that when small water fleas came into the sound field their bodies exploded.

Sonic cavitation also causes emulsification in liquid mixtures and has been used in the preparation of numerous suspensions and dispersions ranging from metallic sols to dairy products. The ability of sonic cavitation to accelerate certain types of chemical reactions and to reduce others has found practicable application in the artificial maturing of distilled spirits and the blending of wines. Aging of whiskey and rum which normally required two to four years can be accomplished by intense vibration in about seven hours. The preferential action of sonic cavitation on certain proteins in a mixture such as that found in bacterial cells has made possible the use of this form of energy in extracting antigenic substances from pathogenic bacteria. These antigens promise to be widely useful in medicine.

The deep-seated nature of the reactions induced in liquid mixtures by cavitation is indicated by the fact that visible light is produced when polar liquids such as water, alcohol, and glycerol are vibrated. While the intensity of the light is related directly to the dipole moment of the molecules involved there is as yet no explanation of this phenomenon. The possibilities of the useful application of intense vibration in a wide variety of fields was stressed rather than the progress made so far in its exploitation.

In the general discussion of the paper, Dr. Hayes of the Naval Research Laboratory spoke of some of the work that he is doing.

San Francisco Section

A meeting of the San Francisco Section was held on October 7 in the Telephone Building. There were twenty-five present and V. J. Freiermuth, vice chairman, presided.

The meeting was devoted to a discussion of two papers which already have appeared in the Proceedings. The first of these, "A Method of Reducing Disturbances in Radio Signaling by a System of Frequency Modulation," by E. H. Armstrong, was reviewed by D. A. Murray of Heintz and Kaufman.

A second paper on "Frequency Modulation Propagation Characteristics," by M. G. Crosby, was reviewed by M. D. Case of RCA Communications.

Another meeting of the section was held on October 21 at the Bellevue Hotel with R. D. Kirkland, chairman, presiding. There were thirty present and twenty of these attended the informal dinner which preceded the meeting.

A paper on "Radio Patents—Seventeen Years After" was presented by H. E. Metcalf of Lippincott and Metcalf. He presented first a brief history of radio patents and the bearing of patents and court action on the radio industry. It was pointed out that although some important patents had expired improvements in the art had rendered the expired patents practically worthless.

Further discussion was held on the possibility of holding a Pacific Coast meeting in Spokane, Washington.

Seattle Section

"Broadcast Transmitters" was the subject of a paper by J. E. Young, transmitter engineer of the RCA Manufacturing Company, given before the October 16 meeting of the Seattle Section which was held in the Engineers Club. E. D. Scott, chairman, presided and there were seventy present. Sixty-five attended the dinner which preceded the meeting.

Discussion was given to the five-kilowatt RCA transmitters installed in the new KOMO-KJR transmitter plant in Seattle. Two transmitters are employed and operate into a single antenna. A four-wire balanced transmission line about 500 feet long couples each transmitter to the antenna. These lines are connected to the antenna through acceptor-rejector type filters. Although the use of a single antenna for two transmitters is not new, its application in the broadcast band where the frequency difference between the two carriers is only fifty kilocycles has not been made before.

After the discussion, the transmitter plant was visited and equipment described inspected.

Washington Section

The October meeting of the Washington Section was held on the 12th in the Potomac Electric Power Company Auditorium. The speaker was J. J. Lamb, technical editor of *QST*, who gave a paper on methods of reducing noise disturbances in communication type instruments. It dealt with the advantages of various types of crystal filter circuits and a method of noise suppression by automatic volume control systems. An interesting demonstration of equipment concluded the paper which was discussed by a number of the ninety members present. The meeting was preceded by an informal dinner at which there was an attendance of twenty-four.

The November meeting of the Section was held on the 9th in the same auditorium. C. T. Solt of the communications division of the U. S. Coast Guard Headquarters gave a paper on "The Development and Application of U. S. Coast Guard Aircraft Radio Equipment." Descriptions of the various equipment used and its installation in many types of airplanes were given. The problems met in designing a loop type antenna for direction finder use aboard the plane where the application of basic aerodynamical principles must be observed were outlined. The advantages of employing a cathode-ray type of course guidance indicator were emphasized. The attendance at the meeting was eight-six and sixty were present at the dinner which preceded it.

Personal Mention

L. C. Verman, previously with the London Shellac Research Bureau, is now with the Industrial Intelligence and Research Bureau, Indian Stores Department, Government Test House, Alipore, Calcutta, India.

Formerly with Lear Developments, Inc., W. L. Webb has joined the radio engineering staff of Radio Research Company, Washington, D. C.

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TECHNICAL PAPERS

COMBINATION HORN AND DIRECT RADIATOR LOUD-SPEAKER*

By
H. F. Olson and R. A. Hackley
(RCA Manufacturing Company, Inc., Camden, New Jersey)

Summary—*A loud-speaker is described consisting of a long horn coupled to one side of a small dynamically driven cone for the reproduction of low frequencies and an acoustic filter for changing the output from the horn to the open side of the cone for the reproduction of the mid- and high-frequency range. A theoretical analysis shows the action of the system. Experimental data substantiate the theory.*

Introduction

THE efficient transformation of electrical variations into the corresponding acoustical vibrations over a wide frequency range is, in general, restricted by practical limitations. The two extreme ends of the audio-frequency range are the most difficult to reproduce with high efficiency. Low efficiency at the high frequencies is primarily the result of the inherent mass reactance of the vibrating system. Inefficiency at the low frequencies is primarily due to small radiation resistance. It is quite well known that a mass controlled diaphragm, driven by a constant force and mounted in an infinite baffle radiates the same energy for all frequencies below the ultimate impedance. However, when such a system is located in a small baffle, three to five feet square, or in a cabinet of the equivalent dimensions, considerable attenuation occurs at the lower frequencies due to a loss in acoustic resistance incurred by circulation from front to back. The response of a practical system mounted in a cabinet is further modified by various resonances of the enclosure, stiffness of the suspension, etc.

There are a number of methods available for obtaining reasonably good efficiency at the lower frequencies when the system is mounted in a small baffle or cabinet. A large radiation resistance may be obtained by using a large diaphragm or cone. A tortuous path or labyrinth coupled to the back of the cone provides another method[1,2] for increasing the radiation resistance by introducing a long path between the front and back of the cone. A horn may be used to increase the

* Decimal classification: R365.2. Original manuscript received by the Institute, September 9, 1936.

[1] J. S. High, U. S. Patent 1,794,957. Filed 1927, Westinghouse.

[2] Benjamin Olney, *Jour. Acous. Soc. Amer.*, vol. 8, no. 2, p. 106, (1936).

radiation resistance presented to a diaphragm. The use of a horn makes it possible to obtain a large ratio of radiation resistance to reactance in the vibrating system at the lower frequencies.

Among the methods referred to above it may be shown that the horn is particularly suitable for use in a wide range loud-speaker system. Smooth response and wide directional characteristics at the high frequencies may be obtained by employing a cone of small diameter. Of course, a small diaphragm operating as a direct radiator is not suitable for low-frequency reproduction because of the limited power output and the large ratio of reactance to radiation resistance as well as a large ratio of reflected electrical impedance to the impedance of the vibrating system. By coupling a horn of a suitable impedance to a small diameter cone good efficiency may be obtained at the low frequencies. The effective radiation resistance of a small cone and horn is equivalent to a large cone. It does not have the undesirable features of a large cone, such as equal radiation from both sides, resonance phenomena when mounted in a cabinet, and a heavy vibrating system.

From the above discussion, it follows that a wide range loud speaker may be built consisting of a long horn[1] coupled to one side of a small dynamically driven diaphragm or cone for the reproduction of low frequencies and an acoustic filter for changing the output from the horn to the open side of the diaphragm for the reproduction of the mid- and high-frequency range.

It is the purpose of this paper to describe a combination of a horn and a direct radiator loud-speaker of such a size as to be suitable for use in radio receivers of the console type, or for other installations in which the available space is relatively small.

Theory

The addition of a horn to a cone loud-speaker provides a means of improving the low-frequency efficiency by increasing the effective radiation resistance. A relatively long horn is required to reproduce efficiently at the low frequencies. Consequently, the horn must be folded to incorporate a suitable system into a cabinet of the conventional size. No measurable loss due to folding occurs if the dimensions at any bend are a fraction of a wave length. For small cabinets of the radio receiver and monitoring type there is no advantage in using a horn for the reproduction of the higher frequencies because the small direct radiator is sufficiently efficient in the mid- and high-frequency ranges. Furthermore, the intensity level of reproduction in small rooms is considerably less than the intensity level of the original sound, and as a consequence some accentuation of low-frequency response is required. Coupling a horn to one side of a direct radiator loud-speaker provides a system of good low-frequency efficiency and makes it possible to use a small light vibrating system for the efficient reproduction and distribution of the mid- and high-frequency ranges from the open side.

A combination of a horn and direct radiator loud-speaker is shown in Fig. 1. The mechanism consists of a six-inch corrugated cone driven by an aluminum voice coil. The back of the cone is coupled to an acoustic capacitance which, in turn, is coupled to the throat of the horn. The equivalent straight axis horn coupled to the cone is shown in Fig. 1D. The equivalent electrical circuit of the vibrating system is shown in Fig. 1C.

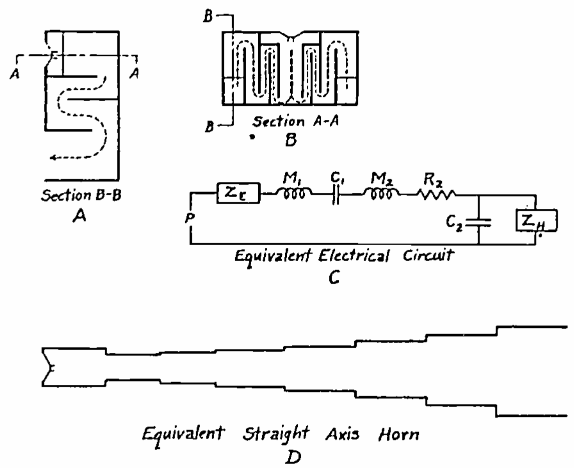

Fig. 1—Combination horn and direct radiator loud speaker.
A. Vertical cross-sectional view.
B. Horizontal cross-sectional view of the top portion.
C. Equivalent electrical circuit of the acoustical system.
D. Equivalent straight axis horn.

The performance of the system may be predicted from an analysis of the equivalent electrical circuit. At low frequencies the impedance of the capacitance C_2 is large compared to the impedance Z_H. Furthermore, the radiation resistance R_H of the horn is larger than the radiation resistance R_2 of the front of the cone. Therefore, the energy is dissipated in R_H and radiated from the horn. In the mid-frequency range the impedance of the capacitance C_2 and the horn throat impedance Z_H are practically the same. Furthermore, the resistance R_2 is comparable to the combination of Z_H and C_2. In this region radiation occurs from both the horn and the direct radiator. At high frequencies the impedance of the capacitance C_2 is small compared to Z_H and the dissipation in the horn is negligible compared to the direct radiation resistance presented to a diaphragm. The use of a horn makes it possible to obtain a large ratio of radiation resistance to reactance in the vibrating system at the lower frequencies.

Among the methods referred to above it may be shown that the horn is particularly suitable for use in a wide range loud-speaker system. Smooth response and wide directional characteristics at the high frequencies may be obtained by employing a cone of small diameter. Of course, a small diaphragm operating as a direct radiator is not suitable for low-frequency reproduction because of the limited power output and the large ratio of reactance to radiation resistance as well as a large ratio of reflected electrical impedance to the impedance of the vibrating system. By coupling a horn of a suitable impedance to a small diameter cone good efficiency may be obtained at the low frequencies. The effective radiation resistance of a small cone and horn is equivalent to a large cone. It does not have the undesirable features of a large cone, such as equal radiation from both sides, resonance phenomena when mounted in a cabinet, and a heavy vibrating system.

From the above discussion, it follows that a wide range loud speaker may be built consisting of a long horn[1] coupled to one side of a small dynamically driven diaphragm or cone for the reproduction of low frequencies and an acoustic filter for changing the output from the horn to the open side of the diaphragm for the reproduction of the mid- and high-frequency range.

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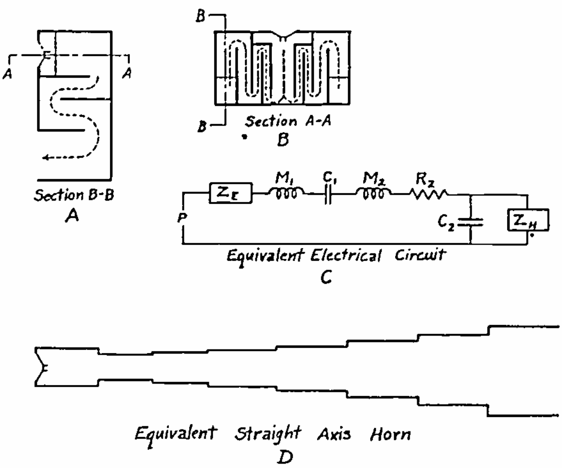

Fig. 1—Combination horn and direct radiator loud speaker.
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B. Horizontal cross-sectional view of the top portion.
C. Equivalent electrical circuit of the acoustical system.
D. Equivalent straight axis horn.

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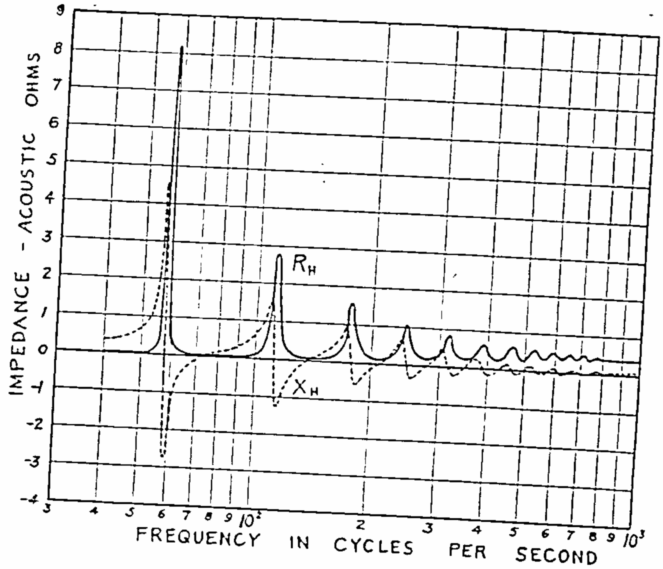

Fig. 2—The acoustic impedance characteristic Z_H at the throat of a horn having a cut off due to flare of 34 cycles, a length of 92 inches, and a mouth area of 300 square inches. R_H = resistive component. X_H = reactive component.

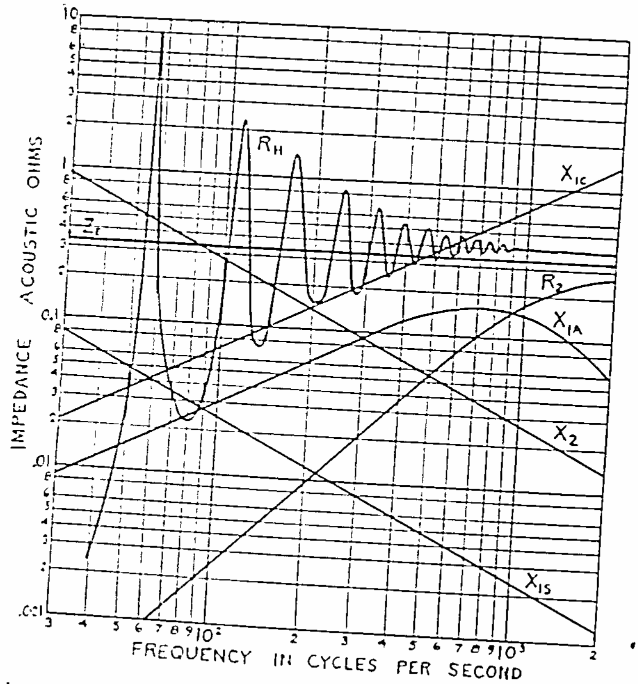

Fig. 3—Acoustic impedance characteristics of the system depicted in Fig. 1.
R_H = acoustic resistance at the throat of the horn.
R_2 = acoustic resistance due to the air load upon the front of the cone.
X_{1C} = acoustic reactance due to the mass of the cone.
$X_{1A} = \omega M_2$ = acoustic reactance due to the mass of the cone and voice coil.
X_2 = acoustic reactance of the capacitance due to the air load upon the front of the cone.
X_{1S} = acoustic reactance of the capacitance due to the volume behind the cone.
Z_E = acoustic reactance of the cone suspension system.
Z_E = acoustic impedance due to the electrical circuit.
Note: X_2 and X_{1S} are negative.

radiation from the cone. The above description gives a physical picture of the action of the direct radiator and horn combination loud-speaker.

The horn used in this loud-speaker, Fig. 1, has a cutoff, due to flare of 34 cycles. The length of the horn is 92 inches. The mouth area is 300 square inches. The acoustic impedance Z_H at the throat computed from the conventional formulas[3] is shown in Fig. 2.

Expressions for the resistive and reactive components of the air load upon a piston in an infinite baffle have been derived by Rayleigh.[4] The acoustic resistance R_2 and reactance X_{11} characteristics for the front side of a six-inch cone computed from these formulas are shown in Fig. 3.

The acoustic reactance of the air chamber behind the cone is given by

$$X_2 = -\frac{1}{\omega C_2}$$

where,

$\omega = 2\pi f$

f = frequency, cycles per second,

$C_2 = \frac{V}{\rho c^2}$, acoustic capacitance,

V = volume of the chamber, cubic centimeters,

ρ = density of air, grams per cubic centimeter,

c = velocity of sound, centimeters per second.

The reactance characteristic of the air chamber is shown in Fig. 3.

The acoustic reactance of the cone and voice coil is given by

$$X_{1C} = \frac{\omega m}{A^2} = \omega M_1$$

where,

m = mass of the cone and voice coil, grams,

A = area of the cone, square centimeters,

M = inertance of the cone and voice coil.

The acoustic reactance of the suspension system is given by

$$X_{1S} = -\frac{S}{\omega A^2} = -\frac{1}{\omega C_1}$$

[3] Olson and Massa, "Applied Acoustics," p. 188, P. Blakiston's Son and Co., Philadelphia, Pa.

[4] Rayleigh, "Theory of Sound," Vol. II, paragraphs 278 and 302, Macmillan, New York, N. Y.

where,

S = stiffness of the suspension system, dynes per centimeter,

C_1 = acoustic capacitance of the suspension system.

The acoustic reactance characteristics of the mass and suspension stiffness of the cone are shown in Fig. 3.

The acoustic impedance due to the electrical impedance of the vacuum tube and voice coil reflected into the acoustic system is

$$Z_E = \frac{(Bl)^2}{ZA^2} \times 10^{-9}$$

where,

B = flux density in the air gap, gauss,

l = length of the conductor, centimeters,

Z = electrical impedance of the vacuum tube and voice coil, ohms.

The acoustic impedance characteristic Z_E of the vacuum tube and voice coil is shown in Fig. 3.

Expressions for the impedance characteristics of the important components of the vibrating system have been stated in the preceding discussion. The characteristics for a certain set of constants are shown in Figs. 2 and 3. By means of the equivalent circuit and the impedance characteristics the performance of the loud speaker may be computed.

In the overlap region where the output changes from the horn to the direct radiator it is important that the phase of the output from the front of the cone and the horn be the same. The choice of constants for the correct phase may be determined from a theoretical analysis.

The power output characteristic of the loud-speaker computed from the equivalent electrical circuit and the characteristics of Figs. 2 and 3 is shown in Fig. 4. It is interesting to note that in spite of the large variations in the horn impedance at the lower frequencies the output is quite uniform.

In the range above 600 cycles the action is the same as that of a conventional direct radiator loud-speaker. The radiation resistance attains its ultimate value at 2500 cycles and remains constant above this frequency. In the case of a mass controlled system the velocity is inversely proportional to the frequency and consequently the output falls off with frequency above the frequency of ultimate resistance. By employing a suitably corrugated cone the effective mass may be reduced and constant output maintained in the region in which the radiation resistance is a constant. With an aluminum voice coil and a six-inch corrugated cone reasonably uniform response may be maintained to 7000 cycles. If it is desired to extend the range a double voice coil[5,6,7] may be used to maintain constant output to 12,000 cycles and above.

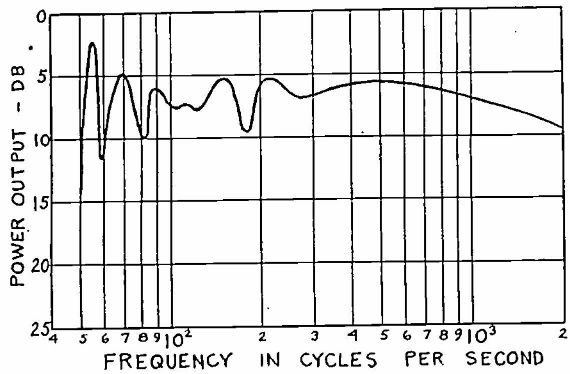

Fig. 4—Power output characteristic of the loud-speaker of Fig. 1 computed from the equivalent electrical circuit and the characteristics of Figs. 2 and 3.

Description of Combination Horn and Direct Radiator

The speaker mechanism used in the combination horn and direct radiator shown in Fig. 1 consists of a six-inch corrugated paper cone with an aluminum voice coil. The cone has a leather outside suspen-

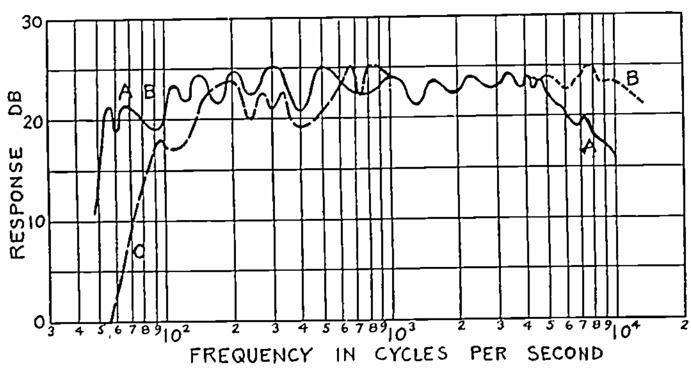

Fig. 5—Experimentally determined response characteristics.
A. Combination horn and direct radiator loud-speaker with single aluminum voice coil mechanism.
B. Same as A with double voice coil mechanism.
C. Single aluminum voice coil mechanism in a large flat baffle.

sion, and a solid izarine center suspension. The air-gap flux density is 12,000 gauss with ten watts field dissipation. A frequency response curve taken on the axis of the speaker mounted in a flat baffle is shown in Fig. 5, curve *C*.

[5] H. F. Olson, Proc. I. R. E., vol. 22, pp. 33–46; January, (1934).
[6] A. Ringel, U. S. Patent 2,007,746.
[7] H. F. Olson, U. S. Patent 2,007,748.

The folded horn used in this combination is equivalent to a straight axis exponential horn having a mouth area of 300 square inches, a throat area of 16 square inches, and a length of 92 inches. The throat area was chosen so that the surge impedance at the throat of the horn matched the combined acoustical and electrical impedance of the vibrating system and its associated electrical circuit. The length and mouth area of the horn were then determined so as to give as low a cutoff frequency as was practical with the space available.

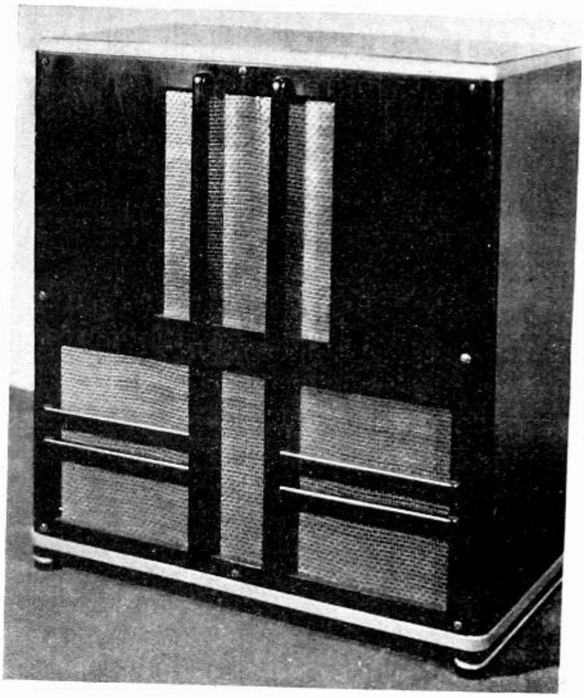

Fig. 6—Commercial design of the combination horn and direct radiator loud-speaker.

Fig. 1B shows a horizontal cross section of the top portion of the horn. The sound comes off the back of the cone and goes into a cavity between the back of the cone and the throat of the horn. This cavity serves the purpose of providing space for the speaker field, and, in addition, acts as an acoustic capacitance which helps to limit the high-frequency response of the horn. Each of the first four sections of the horn is divided into two equal parts. These eight half sections are arranged symmetrically on both sides of the cavity. Thus, the sound, in passing from the cavity to the fifth section of the horn, follows two parallel paths. The fifth, sixth, and seventh sections of the horn occupy the space directly below the top portion of the horn described above, as is shown in Fig. 1A. The sound paths through the various sections of the horn are shown by the broken lines on Figs. 1A and 1B.

The over-all dimensions of this combination speaker are: 27½ inches wide, 15 inches deep, and 30½ inches high.

A photograph of a commercial design[8] of this loud-speaker is shown in Fig. 6.

Performance Characteristics

An over-all measured response frequency characteristic of the combination horn and direct radiator loud-speaker is shown in Fig. 5. The measured response is the sound pressure at a distance of five feet on the axis of the loud speaker. The measured response does not take account of the directional characteristics. Above 100 cycles the directional characteristics do not vary appreciably with frequency. Below 100 cycles the radiation pattern is somewhat broader. The theoretically predicted power output, Fig. 4, indicated uniform response above 50 cycles. The slight deviation between Fig. 4 and Fig. 5 is due to a broadening of the directional pattern below 100 cycles which causes a corresponding reduction in the measured sound pressure in this range. The uniform measured response, Fig. 5, and the close agreement with Fig. 4, substantiates the analyses of the action and performance of the combination horn and direct radiator loud-speaker.

A comparison of the response of the combination loud-speaker with the response of the same speaker mounted in a large flat baffle, Fig. 5*C*, shows the increase in response due to the horn. Fig. 3 shows that the radiation resistance of the horn, R_H, below 200 cycles, is fifteen or more times that of the front of the cone, R_2, and is comparable with the acoustic impedance of the electrical circuit, Z_E. Since R_2 is the radiation resistance of a six-inch cone in an infinite baffle, this explains the increase in response due to the horn. Furthermore, it shows that the low-frequency response is limited in a small cone, mounted in an infinite baffle, due to the large value of Z_E in comparison with R_2. By employing a large cone it is possible to obviate this difficulty. To equal the response of the horn, with all other constants the same, would require a cone diameter of from 18 to 24 inches. However, a large cone is undesirable for the reproduction of high frequencies due to the inefficiency incurred by its large mass reactance. The very narrow beam of high-frequency radiation which results from a large vibrating area is another disadvantage of a large cone.

The radiation resistance R_2 does not increase but remains a constant above 2500 cycles. Therefore, to maintain constant output above 2500 cycles it is necessary to reduce the effective mass of the vibrating system. This is accomplished by suitable corrugation of the cone.

[8] The commercial design of the loud-speaker shown in Fig. 5 was carried out by J. Vassos and J. D. Seabert.

The corrugations also serve to maintain uniform directional characteristics due to progressive phase shift between sections. Referring to Fig. 5 it will be seen that uniform response is maintained to 7000 cycles with a single aluminum coil. The loss in response above 7000 is due to the mass reactance of the coil. For certain applications it is desirable to maintain uniform response above 7000 cycles. By employing a double coil[5] driving system the effective mass is reduced, and good response may be maintained to 12,000 cycles as shown in Fig. 5.

Conclusion

The combination horn and direct radiator loud-speaker incorporates the following features: a light vibrating system for the efficient production of high-frequency radiation; a small diameter cone for wide angle distribution of the high-frequency radiation; a horn coupled to the vibrating system for the efficient production of low-frequency radiation.

The speaker is suitable for applications requiring a relatively small acoustic power output and a wide audio-frequency range, and where the amount of space available is definitely limited. Such speakers are desirable for wide range radio receivers, broadcast monitoring, sound motion picture recording monitoring, centralized radio, etc.

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THERMOCOUPLE AMMETERS FOR ULTRA-HIGH FREQUENCIES*

By
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Summary—*Thermocouple ammeters are shown to have errors at frequencies from 10 to 100 megacycles depending almost entirely on the skin effect in the heated member. The use of a tubular heater of suitable dimensions is shown to reduce the error to smaller acceptable values.*

AMMETERS of the thermocouple type have been in use for many years for the measurement of high-frequency currents, and have generally been considered as the most satisfactory instruments available for such measurements. Hot-wire expansion type instruments have very nearly ceased to be used because of the difficulty of compensating out errors due to ambient temperature

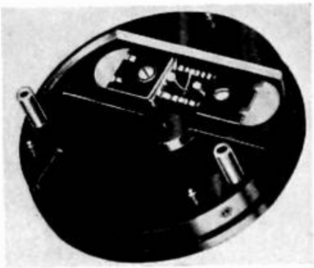

Fig. 1—Thermocouple converter in ammeter base.

variations. These variations, as well as errors which might exist due to heating of the terminals, can be entirely compensated for in the thermocouple type.

Fig. 1 shows the general arrangement of a typical thermocouple converter, with compensating strips. These strips, to which are attached the cold ends of the couple, are insulated from the terminals with thin mica. For satisfactory temperature compensation they must be thermally equivalent to the heater wire or strip.[1,2]

The entire theory of the thermocouple ammeter rests on the production of heat, that is, the heater wire or strip having a resistance R is heated by the current I passing through it, the total heat produced

* Decimal classification: R242.12. Original manuscript received by the Institute, September 16, 1936. Presented before Rochester Fall Meeting, November 16, 1936.

[1] U. S. Patents 1,407,147 and 1,456,591.

[2] W. N. Goodwin, Jr., "The compensated thermocouple ammeter," *Elec. Eng.* vol. 55, pp. 23–33; January, (1936).

being I^2R. This heat raises the temperature of the strip an amount depending on the thermal drop from the center of the heated wire to the heavy terminals, since most of the heat is dissipated by conduction to them.

However, at high frequencies the value of R will increase due to skin effect, whereby the total heat produced is increased. This, in turn, still further raises the temperature of the heater wire, and since the thermocouple and direct-current instrument connected to it merely indicate temperature, the instrument reads high.

Assuming a given instrument deflection under conditions of low-frequency current and the same deflection at a lesser value of high-frequency current, the fact that the same deflection is had means that the same temperature was attained in the heater wire in both cases, or that

$$I^2R = I_{HF}^2R_{HF}$$

since the other factors of heat dissipation and conduction are not affected by the frequency of the heating current. Solving for I_{HF}, we have

$$I_{HF} = I\sqrt{\frac{R}{R_{HF}}}.$$

Assuming the instrument to have been correctly calibrated on low frequency, then the true high-frequency current is equal to the indication multiplied by the square root of the ratio of low-frequency to high-frequency resistance of the heater.[3]

By using high resistance platinum alloys radio-frequency instruments were originally designed to have negligible skin effect errors up to about three megacycles, the highest frequencies in commercial use at that time. Recently however, the use of ultra-high frequencies of the order of 100 megacycles, has made it necessary to redesign the heating conductors in which the skin effect errors for these frequencies become serious for currents greater than about one ampere. As an example a standard five-ampere instrument has a heater of eleven-mil diameter platinum alloy wire which has a resistance at this frequency of 2.57 times the direct-current or low-frequency resistance, as obtained from standard tabulations for high-frequency resistance of round wires.[4] The square root of the reciprocal of this value is 0.624 which is the theoretical correction factor for an instrument of this range at this frequency.

An experimental check against this theory is difficult since no standard ammeter exists which has a negligible error at ultra-high frequencies. It is necessary to use some sort of a comparator between

[3] Discussion of reference No. 2, John H. Miller, *Elec. Eng.* vol. 55, pp. 407–408; April, (1936).

[4] Smithsonian Physical Tables, Eighth Edition, Tables 432–534.

low-and high-frequency currents having a higher order of accuracy than the standard thermocouple, and this appears to be answered through the use of straight filament tungsten lamps, using a photocell arrangement as a measure of the heating current.

This method of measuring high-frequency current has been suggested before,[5] and investigators here and abroad have made limited use of the method. Work was undertaken some time ago in the Weston laboratories to obtain experimental data as to the high-frequency errors

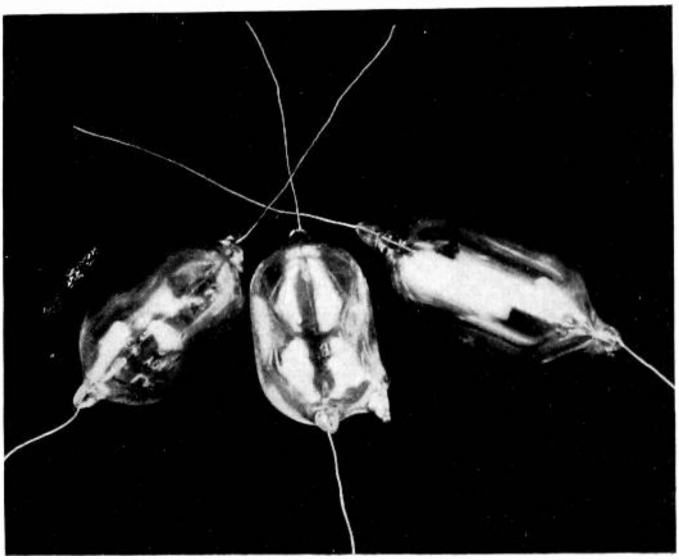

Fig. 2—Special straight filament calibrating lamps.

in thermal ammeters and particularly to check as to whether or not the skin effect is the entire or at least the major source of error.

The reason for the use of a tungsten lamp as a high-frequency standard is simply that a great number of watts may be dissipated in a tungsten filament per unit length, hence a relatively high current can be carried in a filament having a relatively small skin effect. For example, the five-ampere instrument having a heater with an R_{HF}/R ratio of 2.57 may be checked with a tungsten filament lamp having a similar ratio of 1.065.

Fig. 2 shows the special type of lamp used, having a straight filament about two centimeters long.

The actual procedure for making a check is shown in Fig. 3. A conventional oscillator using a pair of type 800 tubes in push-pull was used. The frequency was checked by finding nodes on a pair of Lecher wires coupled loosely to the oscillator, using a thermal ammeter for checking the current maxima.

[5] *QST*, vol. 16, p. 38; June, (1932). Note on use of lamp and photocell for measuring antenna current.

Two lamps were used, one in each lead to the instrument, although the same results were had later with a single lamp. A wooden box enclosed the instruments, lamps, and photocell. The light was picked up by a standard photronic cell several inches away, and indicated on a 0 to 50-microampere meter. Shielding was used on the photocell circuit on the theory that some radio-frequency energy might be picked up and rectified by the photocell, but when the shielding was omitted and the lamps short-circuited, there was no reading on the microammeter so the shielding was omitted in the interest of simplicity.

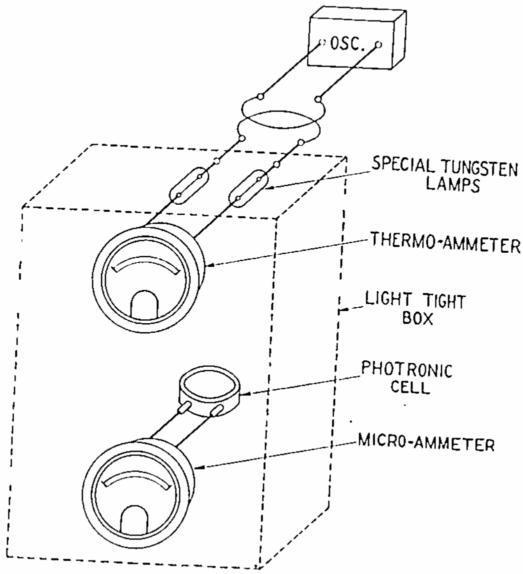

Fig. 3—Schematic layout of calibrating method.

Using lamps with a normal rating corresponding to about half scale on the thermal ammeter being checked, a reading, I_1 was first made on high-frequency current. The illumination from the lamps, picked up by the photocell gave a reading M.

The pickup loop was then removed and sixty-cycle energy introduced with a value giving the same reading M on the microammeter, or the same illumination. The reading on the thermal instrument was recorded as I_2.

Direct current was then introduced to give the same illumination, and the drop across each lamp was taken. From this the direct-current resistance of each lamp was obtained at its operating point, and, measuring the filament length, the resistance in ohms per unit length was obtained. From this value, and the frequency, the ratio R_{RF}/R for the filament may be obtained from the Smithsonian Tables,[4] and recorded as F.

[4] Smithsonian Physical Tables, *loc. cit.*, Tables 532–534, pp. 449–451.

The net correction factor for the instrument then becomes,

$$\frac{I_2}{I_1}\sqrt{\frac{1}{F}}.$$

The actual corrections obtained in this manner were entirely consistent, but the errors shown were larger by a few per cent than indicated by the computed skin effect factor above, particularly at the higher frequencies. It is believed that circuit contours are largely responsible, since the theory is predicated on a straight wire, whereas a loop always exists because of the terminals coming out of the back of the instrument.

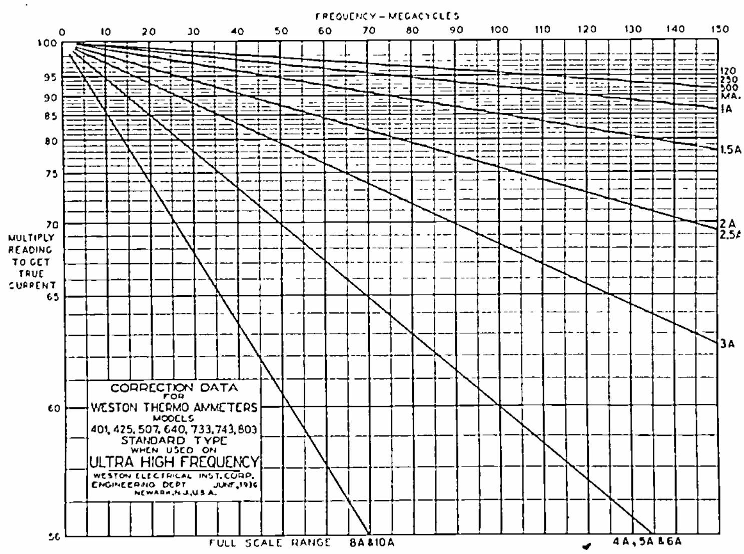

Fig. 4—Chart giving correction factors for standard line of thermoammeters using solid heaters.

Tests were run on a large number of stock Model 425 thermoammeters; the results were then averaged and compared with the theoretical correction factors. In all cases good agreement was had, with the experimental errors always slightly larger than the calculated values. Fig. 4 shows a summary of these correction factors plotted on a special co-ordinate chart to give straight-line results.

As a matter of interest, a number of five-ampere instruments of different makes were tested at 80 megacycles with the results given below.

Make	Reading for Three Amperes Radio-Frequency 80 Megacycles	Correction Factor
A	4.37	0.686
B	4.53	0.662
C	4.63	0.648
D	4.86	0.617
E	5.13	0.585

The solution of the problem of reasonable accuracy at high frequencies is evidently the obtaining of a heater wire or strip which will have a smaller skin effect and still carry the current; or, as an alternative, some form of compensation. Compensation with networks introduces difficulties due to the very low resistance of the heater; in a five-ampere instrument it is only about 0.04 ohm. Further, compensation is usually obtainable only at one frequency, and large errors exist at other frequencies.

A very short heater of small cross section may be used, and such arrangements have been successful but are difficult to manufacture. For example, the eleven-mil diameter heater wire referred to previously, which is about 1/4 inch long, can be replaced by a three-mil wire only 1/32 inch long. Welding a small thermocouple to the center of a heater as short as this is difficult, but has been done satisfactorily. The center of this short heater attains the same temperature as the larger one, and the overload capacity is the same. Because the wire is smaller in diameter the correction factor changes to 0.85 at 80 megacycles, a very considerable improvement.

The use of a tubular heater was also considered, and fine platinum alloy tubes were finally obtained with a one-mil wall. A five-ampere instrument constructed with a tube 28 mils in diameter with a one-mil wall has a correction factor of 0.95 at 80 megacycles, and this probably approaches a satisfactory solution, since the theoretical correction is 0.98, and circuit contours appear to account for the remaining errors.

It is interesting to note that if a wall thickness of one mil is maintained, and the tube diameter increased for larger currents, such a heater design will maintain this small theoretical correction with no apparent upper limit. Circuit contours and the crowding of the current to one side of the tube will undoubtedly determine the true correction for any given installation.

To reduce these errors and to eliminate overheating of the instrument base and case, the instrument must be kept out of radio-frequency fields or be well shielded.

Acknowledgment

The assistance and counsel of Mr. W. N. Goodwin, Jr., inventor of the compensated thermocouple ammeter, is gratefully acknowledged. Thanks are also due to Mr. H. Bercovitz and Mr. A. R. Thiel for their assistance in performing the calculations and experiments.

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APPLICATION OF CONVENTIONAL VACUUM TUBES IN UNCONVENTIONAL CIRCUITS*

By
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Summary—*This paper describes some out-of-the-ordinary, yet simple and interesting, vacuum tube applications. It illustrates how conventional vacuum tubes can be used in circuits which impose unusual requirements for grid current, noise, and life. Analysis of circuit requirements and knowledge of tube performance characteristics frequently make possible adjustment of operating conditions to give the desired results. A number of circuits to indicate the practicability of this procedure are described, and are used to illustrate simple and logical methods of analyzing circuit operation.*

1. A two-stage photo amplifier relay circuit operating directly on the alternating-current line and using a voltage divider, one resistor, and three condensers as circuit parts.

2. A sensitive photo amplifier circuit using a pentode as the load resistor for a phototube and a standard tube as a reliable and sensitive electrometer tube feeding a low priced indicating instrument.

3. A modification of (2) to provide variable-range, variable-sensitivity characteristics.

4. A simple vacuum tube circuit in which standard unselected tubes can be used to multiply currents on the order of 10^{-12} amperes by a definite factor (fixed by the circuit elements and not by the tubes) to such values that they can be easily read on an insensitive milliammeter.

5. A simple capacitance operated relay working on the alternating-current line, using metal tubes and only a few inexpensive circuit parts.

I. Introduction

VACUUM tubes are becoming more and more widely used as means of doing things that have, heretofore, been done with difficulty, inconvenience, or not at all.

If we desire our vacuum tubes to operate with the same degree of reliability and length of life that we expect of other mechanical and electrical links in our modern machines, we must use the same factors of safety in the operation of the tubes as in the use of the other mechanical and electrical parts. If we fail to do this and let an electronic device be placed in service after only initial tests to show that it will work, we can expect trouble after the device has been in service for a period of time. The results of this procedure have led many people to be skeptical of the reliability of electronic devices.

When we do consider the requirements of circuits and tube char-

* Decimal classification: 621.375.1. Original manuscript received by the Institute, July 6, 1936. Presented before Eleventh Annual Convention, Cleveland, Ohio, May 13, 1936.

acteristics and use the same factor of safety in the design of these circuits that we would use in the design of other mechanical or electrical equipment, we will have little to fear from the unreliability or the short life of electronic apparatus.

Simplicity of an electronic device is another important factor in determining its success or failure for a particular application. It seems needless to say that it is uneconomical to replace existing reliable apparatus with complicated new electronic equipment; yet too often we find ourselves so involved in the electronic problems that we lose sight of simpler and more direct methods of accomplishing our purpose.

It is believed that the following circuits will prove of interest to the designer of electronic devices.

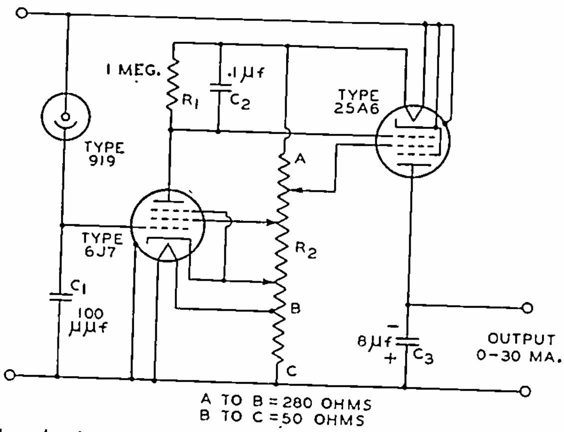
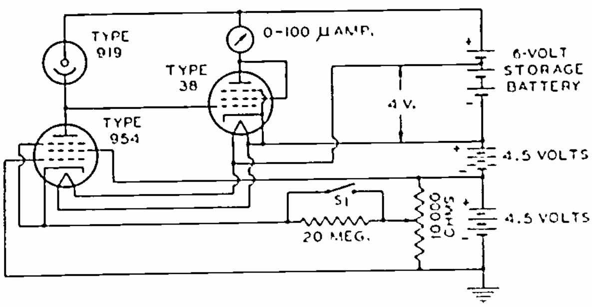


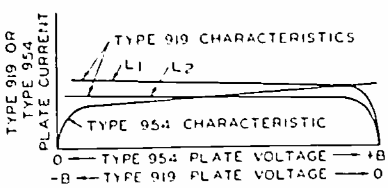
Fig. 1—A simple two-stage alternating-current operated photo amplifier relay circuit.

II. A Simple Two-Stage Alternating-Current Operated Photo Amplifier Relay Circuit

Fig. 1 shows a simple two-stage photo amplifier relay circuit operating directly on the alternating-current line. The simplicity of the circuit is illustrated by the fact that the complete list of circuit parts includes only one voltage divider resistor, one plate load resistor, and three condensers. The circuit shown in Fig. 1 consists of a high impedance phototube feeding through a voltage amplifier or buffer stage into a power output stage. The filament voltage of the buffer stage has been reduced to reduce the temperature of, and, hence, the electron emission from the grid of the buffer tube. The plate current of the buffer stage is kept at a minimum in order to reduce the electron bombardment of the gas molecules within the tube and hence the gas current to the grid. The bias to the grid of the buffer stage is obtained by means of the rectifying action of the grid itself. This method of obtaining the grid bias keeps the effective bias and plate current of the tube constant, regardless of large fluctuations in contact potential between the grid and the cathode. The impedance of the condenser C_1 acts as a load impedance for the phototube. Condenser C_1 is charged up to a definite negative potential on one half of the alternating-current cycle and is allowed to discharge through the phototube on the other half of the cycle. The amount that is discharged by the phototube determines the working potential on the grid of the buffer stage. The size of C_1 can be set to any desired value to control the desired sensitivity range of the relay.

As the 6J7 conducts on only one half of the alternating-current cycle, it acts as a rectifier and so a negative direct-current potential





Figs. 2 and 3—A sensitive light intensity indicator.

is built up on its plate. This negative direct-current potential is suitable for use as bias and signal for the output stage. The plate of the 6J7 is returned through its load to a point on the voltage divider (one side of the heater of the 25A6) in order to obtain a zero working bias on the grid of the 25A6 output tube without the necessity of having the 6J7 cut off. This allows the 6J7 to be operated about the center of its characteristic. As the output of the 6J7 buffer stage has relatively low impedance, it is suitable for driving the grid of the 25A6 power output stage, which in turn is capable of handling relatively large amounts of power to operate a relay.

This circuit finds its principal use in applications where relatively small amounts of light are available and where the light variations last not less than one tenth of a second.

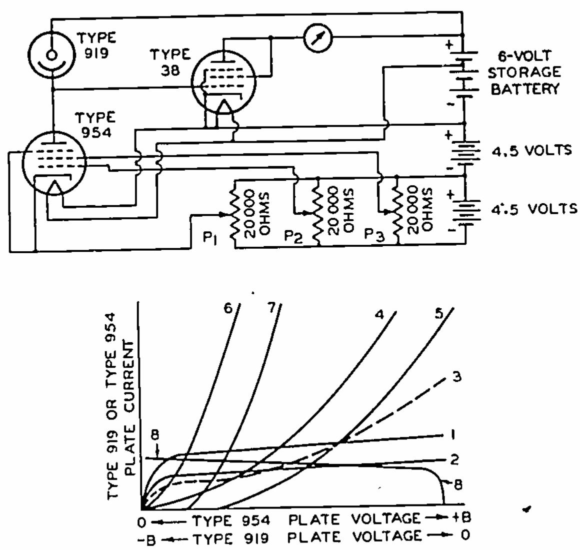
III. A Sensitive Light Intensity Indicator

Fig. 2 shows a sensitive photo amplifier circuit that can be used for accurately matching the intensities of amounts of light. With this circuit, it is easily possible to indicate light differences or changes which may amount to small parts of one per cent. In this circuit arrangement, the high impedance 954 pentode acts as a load impedance for the 919 high impedance vacuum type phototube. It can be seen by reference to Fig. 3 that the potential of the common connection between the 919 and the 954 is determined by the intersection of the 954 and 919 characteristics. It is also evident that a small change of light on the phototube will result in an output of several volts. This output voltage is applied to the grid of a 38 output tube, the plate current of which is indicated on a 200-microampere meter. Because the phototube with the 954 load has an extremely high output impedance, it is necessary to operate the 38 so that its grid input impedance is extremely high. To reduce the grid emission to a minimum, the voltage to the heaters of the 38 and the 954 is reduced to four volts. The possibility of emission from the heaters to the grid is eliminated by operating the heaters at a potential positive with respect to the plate of the 954 and the grid of the 38. Gas current to the grid of the 38 is kept at a minimum by keeping the potentials within the 38 low so as to minimize the ionization of any gas that may be in the tube. Because all of the high impedance external connections are made to electrodes brought out from the tops of the tubes, external leakages are reduced to a minimum. External leakage can be greatly reduced by carefully cleaning the tubes and coating them with a nonhygroscopic wax. This can be done by dipping the tubes in hot ceresin wax and holding them under the surface of the wax until the greater part of the moisture on the glass is boiled off. Care should be taken not to scorch the wax. Ceresin wax has long been used by research men for reducing the effects of moisture leakages in high impedance direct-current circuits. Dr. Rentschler of the Westinghouse Lamp Company very kindly made available to the writer information on the use of ceresin wax.

This circuit finds application where it is desirable to indicate very small percentage variations of an amount of light. For instance, it can be used to indicate the absorption of light by a fluid and, consequently, to indicate or control the concentration of certain chemicals in suspension or solution. The use of monochromatic light can be used to advantage when it is desired to isolate a particular constituent. This circuit also finds application in color matchers and in indicating small changes of small amounts of light. For instance, when measuring small changes of small amounts of light, it has been demonstrated that a change of light intensity on the order of two millionths of a lumen is sufficient to swing the output meter over its full scale.

IV. A Variable Range, Variable Sensitivity, Light Variation Indicator

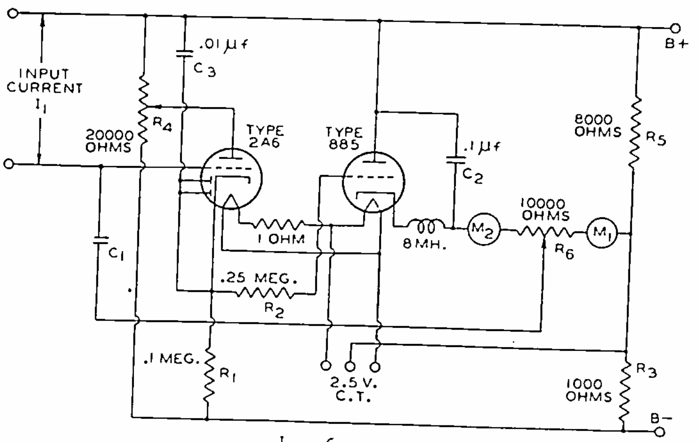
It is sometimes desirable to make the sensitivity of the light intensity meter indicator less for small percentage changes of light. The sensitivity can be reduced to any desired degree by varying the plate characteristics of the 954 between those of a pentode and those of a triode. This variation is produced in the arrangement shown in Fig. 4 by properly adjusting P_2 and P_3 to control the relative potentials on the control grid and the screen grid. When the No. 2 grid of the 954 is positive with respect to the cathode, the 954 has a high impedance pentode characteristic. Changing the No. 1 grid bias changes



Figs. 4 and 5—A variable range, variable sensitivity, light variation indicator.

the height of the characteristic as shown in Fig. 5 by curves 1 and 2. As the potential of the No. 2 grid is made more negative, the characteristic of the 954 changes to that shown by curve 3. With zero bias on the No. 1 and the No. 2 grids, the characteristic curve is as shown by curve 4. When a negative bias is placed on the No. 2 grid, the shape of the characteristic is unchanged but it is shifted along the voltage axis as shown by curve 5. If the No. 1 grid is biased positively, the slope of the characteristic is increased. Curve 6 shows the effect of positive bias on the No. 1 grid (bias on the No. 2 grid for this curve is zero). Placing a negative bias on the No. 2 grid shifts this characteristic along the axis as shown by curve 7.

From this analysis it can be seen that the 954 phototube load can be adjusted to give practically any desired positive impedance load at any desired current and at any desired voltage across the tube. This means that the full scale reading of the output meter can be made to cover a fraction of a per cent light variation, a 100 per cent light variation, or any desired amount of variation between these two extremes. A photo amplifier such as this finds application as a densitometer for use in connection with the analysis of the photographically recorded spectra and for use in connection with a suitable monochrometer or light filter as a means of measuring the absorption lines or the concentration of certain chemicals in solution. These are but two of a large number of possible applications for this type of circuit.



$$\frac{I_1}{I_2} = \frac{C_1}{C_2}(F_1)(F_2)$$

Fig. 6—A vacuum tube current multiplier circuit for small currents from a high impedance course.

V. A Vacuum Tube Current Multiplier Circuit for Small Direct Currents from a High Impedance Source

Fig. 6 shows a vacuum tube current multiplier circuit that will multiply a small direct current from a high impedance source by a definite factor which is practically unaffected by the tube characteristics or by supply voltage variations. Essentially the operation of the circuit is as follows: The signal current I_1 is used to discharge condenser C_1; after the charge on C_1 has been reduced to a certain value just below the grid current point of the buffer stage, the potential on the grid of the buffer stage will be such as to make the current through the buffer stage just sufficient to cause a certain voltage drop across the cathode load resistor. This drop will decrease the bias on the 885 gas triode sufficiently to cause the tube to "break down." When this occurs, condenser C_2 is discharged and then charged in the opposite direction by the action of the inductance in series with the cathode of the 885 gas triode. While condenser C_2 is being discharged through the 885, C_3 holds the cathode of the buffer stage at an essentially constant potential and C_1 is charged to a potential equal to the potential change across C_2, modified by a factor F_1 determined by the position of the slider on R_6. This statement is true when the plate potential of the buffer stage is adjusted so that the 885 breaks down at the instant that the control grid starts to draw grid current. The adjustment of the circuit can be made by varying the slider on R_1 until the 885 ceases to relax and then backing off on the control slightly. The output meter M_1 reads I_2+I_1, and the output meter M_2 reads I_2-I_1, but because I_1 is usually extremely small compared to I_2, both I_1+I_2 and I_2-I_1 can generally be considered equal to I_2. Thus, the equation

$$\frac{I_1}{I_2} = \frac{C_1}{C_2} F_1$$

can be written without appreciable error as

$$\frac{I_1}{I_2 + I_1} = \frac{C_1}{C_2} F_1$$

or,

$$\frac{I_1}{I_2 - I_1} = \frac{C_1}{C_2} F_1.$$

These equations hold true only when the actual discharge time for the 885 is extremely small with respect to the time taken to build up the charge on condenser C_2 through resistor R_6. This is usually the case. Where a high degree of accuracy is required, a correction factor F_2 to account for the time required for discharge should be applied to the reading of the output meter. This correction factor, which is slightly less than unity, will apply for all readings of the instrument.

It should be noted here that the grid-to-cathode capacitance as well as the grid-to-plate capacitance of the buffer stage, or any capacities in the input circuit, are not to be considered a part of C_1 for the computation of the current ratios in the circuit. When the 885 discharges C_2, C_1 is charged as explained above to a voltage equal to the voltage change across C_2 times F_1. After this the charge is allowed to distribute itself between C_1 and C_x without changing its actual value. C_x is the sum of all the capacitances from grid to ground; this sum includes the buffer stage capacitances of the grid to plate and the grid to cathode, as well as the capacitances in the signal current source. Since this is true, a definite current will remove this charge in a definite time regardless of the size of C_x.

The most accurate form of the expression for current ratios in this circuit is

$$\frac{I_1}{I_2} = \frac{C_1}{C_2}(F_1)(F_2).$$

The accuracy of this formula is probably as great as the accuracy of the reading of the output meter, when I_2 is equal to the meter reading M_1 minus I_1, or I_2 is equal to $(M_1+M_2)/2$, when F_1 is a correction factor due to the setting of the slider on potentiometer R_6, and F_2 is a correction factor due to the time of conductance per cycle of the 885. The measured current I_1 includes leakages in the setup and the grid current of the buffer stage.

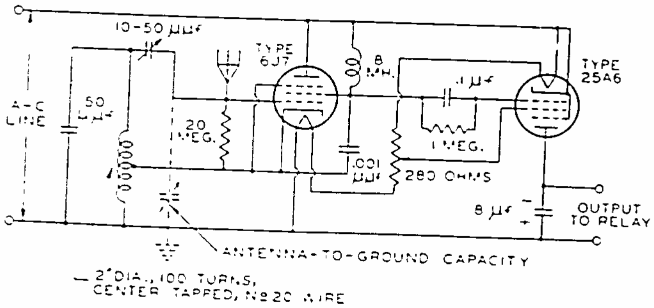


Fig. 7—A metal tube capacitance operated relay operating directly from the alternating-current line.

It is interesting to note that practically all of the time C_1 is being discharged by means of I_1 the plate current of the buffer stage is zero. This means that there can be no gas current to the grid during this time. The heater voltage of the buffer stage is lowered to reduce the grid emission. The heater is returned to a potential positive with respect to the grid of the buffer stage to avoid the possibility of emission from the heaters to the grid. This type of circuit is capable of measuring currents on the order of 10^{-12} amperes. It finds application in the measurement of small phototube currents, of leakage currents, or of any other small currents in high impedance circuits. If test condensers are connected in place of C_1, I_2 will be inversely proportional to the leakage of the condenser expressed directly in units of resistance per unit of capacity.

VI. A Simple Capacitance Operated Relay Working Directly on the Alternating-Current Line

Fig. 7 shows a new and simple form of capacitance operated relay that can be made up cheaply from standard radio parts.

In this circuit operating on the alternating-current line, the sensitive element consists of a pentode oscillator, the feedback of which is determined by the difference in ratio between the inductance of the two parts of the oscillator coil and the ratio between C_1 and the antenna-to-ground capacitance. Thus, the intensity of oscillation of the oscillator varies rapidly with a change of C_1 or a change in the antenna-to-ground capacitance C_2. Because the cathode of the oscillator is at a radio-frequency potential, and because the control grid of the output tube is by-passed for high frequencies through suitable by-pass condensers to the cathode, a negative direct voltage equal to the peak radio-frequency voltage on the cathode of the 6J7 is built up across the grid leak and condenser due to the rectifying action of the grid of the 25A6 output tube. This voltage appears on the grid of the 25A6 output tube. The 6J7 oscillator oscillates at high frequency on one half of the alternating-current cycle and builds up the above-mentioned negative charge on the grid of the output tube. During this time, the output tube has negative plate voltage and so is nonconducting. On the other half of the alternating-current cycle, the 6J7 oscillator has negative plate voltage and so ceases oscillating. The negative charge built up on the grid of the output tube does not have time to leak off during this interval and, hence, is effective in controlling the plate current of the output tube during the positive plate voltage interval.

The sensitivity of this circuit to small changes in capacitance can be increased by increasing the resistance of the choke coil feeding the screen of the 6J7. When this resistance is more than about 15,000 ohms, the circuit will become unstable; i.e., the relay will not operate and release on the same value of capacitance. This increase in sensitivity is caused by the fact that as the intensity of oscillation of the 6J7 is increased, its plate current and its screen current decrease. This causes the screen voltage and, hence, the mutual conductance of the 6J7 to increase; the increase, in turn, helps to increase the intensity of oscillation.

This type of circuit finds application in connection with door-openers, counters, etc., and has even been used as a foul-line indicator for bowling alleys.

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THE BAND-PASS—LOW-PASS ANALOGY*

By
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Summary—*The analogy between low-pass and band-pass circuits is shown and a simple method of deriving a band-pass filter from a low-pass filter is given. Simple circuits illustrating the analogy are shown with their performance curves.*

A. Introduction

THE theory of modulation and the theory of band-pass filters have been well covered in the literature. It is believed however that the following analogy results in greater ease of understanding and greater simplicity in calculation in certain instances.

The similarity between the wave form of a signal which is to be used for modulation, and the envelope of the resulting modulated wave is at once apparent. An equally accurate similarity exists between the circuits of low-pass filters suitable for passing the former, and band-pass circuits suitable for passing the latter. While this similarity is no doubt well understood in certain quarters, it has not previously been clearly pointed out in the literature.

Proposition No. 1

A circuit series resonant to a frequency f_0 has a positive reactance at higher frequencies and a negative reactance at lower frequencies. If f_1 and f_2 are frequencies at which the reactance values are equal in magnitude and opposite in sign, then the inductance alone has this same magnitude for its reactance at a frequency $f_1 - f_2$. That is, if

$$X = \omega_1 L - \frac{1}{\omega_1 C} = -\left(\omega_2 L - \frac{1}{\omega_2 C}\right)$$

then $X = (\omega_1 - \omega_2)L$. The frequency of resonance is the geometric mean of f_1 and f_2.

Proof: The reactance of the series resonant circuit is

$$X = \omega_1 L - \frac{1}{\omega_1 C} = -\left(\omega_2 L - \frac{1}{\omega_2 C}\right)$$

$$= \omega_0 L\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = -\omega_0 L\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right).$$

* Decimal classification: R386. Original manuscript received by the Institute, June 30, 1936; revised manuscript received by the Institute, August 17, 1936.

Dividing out $\omega_0 L$ and collecting like powers of ω_0 gives

$$\frac{\omega_1 + \omega_2}{\omega_0} = \frac{\omega_0(\omega_1 + \omega_2)}{\omega_1\omega_2}$$

$$\omega_0 = \sqrt{\omega_1\omega_2}$$

$$f_0 = \sqrt{f_1 f_2}.$$

This proves the second portion of the proposition. It follows that

$$\omega_1 = \frac{\omega_0^2}{\omega_2}$$

$$X = \omega_0 L\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_0^2/\omega_2}\right)$$

$$= (\omega_1 - \omega_2)L.$$

PROPOSITION NO. 2

A circuit parallel resonant to a frequency f_0 has a negative reactance at higher frequencies and a positive reactance at lower frequencies. If f_1 and f_2 are frequencies at which the reactance values are equal in magnitude and opposite in sign, then the capacitance alone has this same magnitude for its reactance at a frequency $f_1 - f_2$, (providing the losses of the circuit are negligible at f_1 and f_2).

The proof of this second proposition is easily demonstrated by a method similar to that used for the first.

PROPOSITION NO. 3

If in a given low-pass filter a condenser is added in series with each inductance, of the proper value to tune it to a frequency f_0; and if an inductance is added in parallel with every (previously present) condenser of the proper value to tune it to parallel resonance at frequency f_0; then the band width of the new band-pass filter is exactly the same as the band width of the previous low-pass filter at every attenuation ratio. In the band-pass filter the frequency f_0 is the geometric mean of any two frequencies of equal attenuation.

The truth of this proposition is readily established as follows: If f_1 and f_2 are any two frequencies whose geometric mean is f_0, then each resonant circuit of the band-pass filter has the same absolute value of impedence at f_1 that it has at f_2, and the impedance of the corresponding element of the low-pass filter has the same value at frequency $(f_1 - f_2)$ as shown in propositions 1 and 2. It follows that the attenuation of the low-pass filter at the frequency $(f_1 - f_2)$ is the same as the attenuation of the band-pass filter at frequencies f_1 and f_2. The difference in the sign of the reactance of each resonant circuit at f_1 and at f_2 does not make the attenuation different at the two frequencies because the reactive impedances are at right angles to the impedance of the resistive meshes regardless of sign.

In Fig. 1, several simple circuits are presented as illustrations of the usefulness of the proposition.

At the left are shown three of the simplest possible low-pass filters. In the center are the three band-pass filters resulting from applying proposition No. 3 to these circuits. The performance of the circuits is given on the right.

In applying this analogy there is a two-to-one factor which is apt

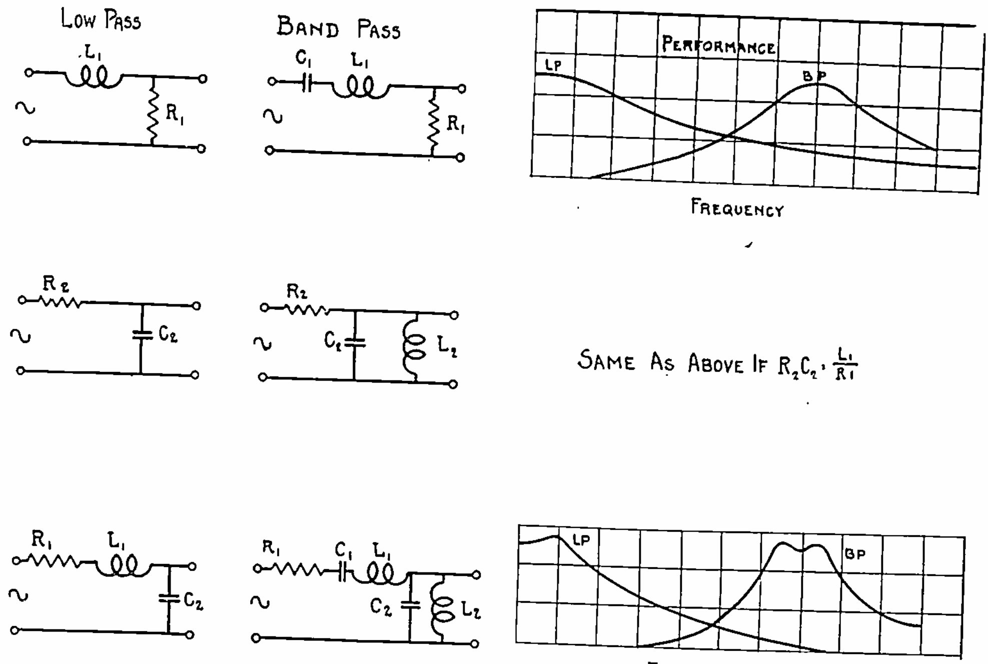

Fig. 1—Circuits illustrating proposition No. 3.

to be a source of confusion. The bad-pass filter passes the same frequency band as the corresponding low-pass filter, but since it is necessary to pass both side bands, the modulation is limited to a frequency band one half as great.

It is realized that a strict application of the analogy will often result in a set of values which are impracticable. Usually this defect can be remedied by multiplying all impedances by a suitable integer or fraction so as to obtain more reasonable values.

The analogy works in the reverse direction only in case the given band-pass filter consists of a network in which every mesh is resonant to the same frequency.

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CALCULATION OF THE SELF-INDUCTANCE OF PLANE POLYGONAL CIRCUITS*

By
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THE present article on the calculation of self-inductances of flat polygonal circuits is based upon the papers of V. I. Bashenoff[1] and presents a further development. Three methods have been offered by V. I. Bashenoff for the calculation of the self-inductances of circuits in the form of plane irregular polygons of round wire without re-entrant angles. According to the first method, the circuit under examination is replaced by a right triangle having the same perimeter and area. This substitution is possible only in the case of $l/\sqrt{s} \geqq 4.8284$, l being the length of the circuit and s its area. Moreover, F. W. Grover has indicated that in the case of $4 \leqq l/\sqrt{s} \leqq 4.8284$, the circuit under examination may be replaced by a rectangle having the same perimeter and area.

The second method is more general. It is based on the fact that the self-inductance of a polygonal circuit may be represented by the sum of the self-inductances of the separate sides and of their mutual inductances; namely,

$$L = \Sigma L_i + \Sigma M_{ik}.$$

Using the classical formula of Neumann,

$$M = \int\int \frac{dl_1 dl_2}{R} \cos \epsilon,$$

V. I. Bashenoff has reduced the latter expression to

$$L = 2l\left(1 + \frac{\nabla}{100}\right)\left(\log_\epsilon \frac{2l}{r} - a_k + \mu\delta\right) \tag{1}$$

when l is the total length of the wire, r the radius of the section of the wire, ∇ a correction for the omission of the terms of the order r/l, $\mu\delta$ a

* Decimal classification: R231. Original manuscript received by the Institute, May 15, 1936.

[1] V. I. Bashenoff, "Abbreviated method for calculating the inductance of irregular plane polygons of round wire," Proc. I.R.E., vol. 15, pp. 1013–1039; July, (1927). V. I. Bashenoff, Supplementary note to "Abbreviated method for calculating the inductance of irregular plane polygons of round wire," Proc. I.R.E., vol. 16, pp. 1553–1558; November, (1928). Westnik theor. i exper. elektrot., (1930).

term allowing for the internal flux of the wire, a_k a coefficient which is the same for similar circuits and depends to a first approximation only on the ratio $l/\sqrt{s}$. The value of a_k, according to V. I. Bashenoff, should be determined from a curve $a_k = F(l/\sqrt{s})$, which he obtained from the calculation of a_k for several circuits and extrapolated for values of $l/\sqrt{s} > 4.7$.

Finally, the third method presents the result of the analysis of the expression a_k in the case of a rectangle. With the help of the well-known formula of Sumec for the self-inductance of a rectangle, V. I. Bashenoff demonstrated that in the case of a rectangle the term $\log_\epsilon l^2/s$ is predominant in the expression of a_k. Assuming this to be

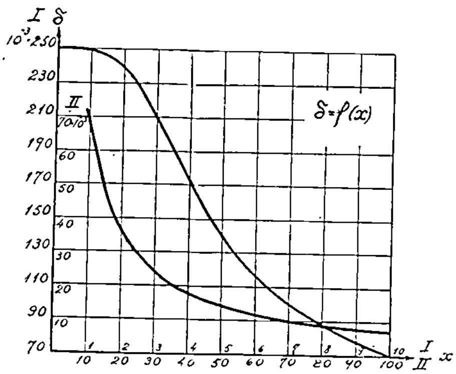

Fig. 1

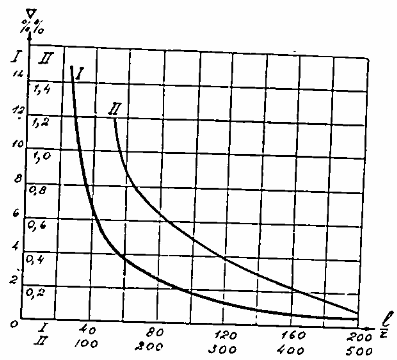

Fig. 2

true also in the case of circuits which are not rectangles, V. I. Bashenoff presents (1) in the following form:

$$L = 2l\left(1 + \frac{\nabla}{100}\right)\left(\log_\epsilon \frac{2s}{rl} - \phi + \mu\delta\right) \tag{2}$$

where $\phi = a_k - \log_\epsilon l^2/s$ varies from $\phi = -0.080$ in the case of a circle to $\phi = 0.30$ in the case of an isosceles triangle, for which $l/\sqrt{s} \cong 13$. The values of ϕ, as well as the values of a_k, are the same for similar circuits. To calculate the correction $\mu\delta$ it is necessary to determine the value of δ from the curve in Fig. 1 as a function of $x = 0.281\ r\sqrt{\mu f/\rho}$, where f is the frequency in cycles, ρ the specific resistance of material of the wire in microhms × centimeters, μ its permeability in units of the C.G.S.M. system. The correction ∇ for the omission of the terms of the order r/l may be considered to be independent of the shape of the circuit. Fig. 2 gives the values of this correction in the case of a square.

The purpose of the present article is to prove the possibility of the application of both the last-mentioned methods to calculate the inductance of circuits with re-entrant angles. The experimental determinations of the self-inductances of a series of circuits with re-entrant angles agree well with Bashenoff's curve $a_k = F(l/\sqrt{s})$ in its calculated part; but the extrapolated portion of this curve does not give satisfactory results either for circuits with re-entrant angles or without. Considering (1) and (2) we shall simplify them as follows:

$$L = 2l\left[\log_e \frac{2l}{r} - a_l\right] \tag{1'}$$

$$L = 2l\left[\log_e \frac{2s}{rl} - \phi\right] \tag{2'}$$

as in practice the corrections $\mu\delta$ and [illegible] may be considered with sufficient precision as independent of the shape of the circuit.

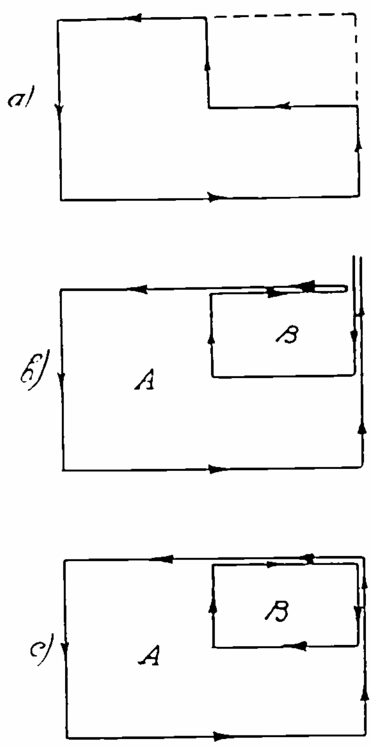

Fig. 3

Consider circuits with re-entrant angles and take the case of a rectangle one quarter of which has been removed as in Fig. 3a. From the point of view of its electromagnetic properties, a circuit of this kind is equivalent to two rectangular circuits A and B with currents of equal intensity flowing in opposite directions, (Figs. 3b and 3c), the two sides of the rectangles coinciding.

The self-inductance of this circuit L_{A-B} then will be

$$L_{A-B} = L_A + L_B - 2M_{AB},$$

where M_{AB} is the mutual inductance of rectangles A and B which have one right angle in common between them. The expression for the self-inductance of a rectangular circuit being known, the problem of calculating L_{A-B} is reduced to the determination of M_{AB}. In case the area of the rectangle B is equal to a quarter of that of rectangle A, the circuit B, provided there is no current in this circuit, is traversed by one fourth of the flux of circuit A and, consequently,

$$M_{AB} = 1/4 L_A.$$

Hence,

$$L_{A-B} = L_A + L_B - 2M_{AB} = L_A + L_B - 1/2 L_A = 1/2 L_A + L_B.$$

The latter expression may be reduced to the form of (2) and (2′), for

$$1/2 L_A = l_A\left[\log_\epsilon \frac{2s_A}{rl_A} - \phi_A\right]; \quad L_B = 2l_B\left[\log_\epsilon \frac{2s_B}{rl_B} - \phi_B\right],$$

and as

$$l_A = 2l_B = l_{A-B}; \quad s_A = 4s_B = 4/3\, s_{A-B}; \quad \phi_A = \phi_B = \phi',$$

by omitting the index $A-B$ we obtain

$$L = l\left[\log_\epsilon \frac{2s}{rl} + \log_\epsilon \frac{4}{3} - \phi'\right] + l\left[\log_\epsilon \frac{2s}{rl} + \log_\epsilon \frac{2}{3} - \phi'\right]$$

$$= 2l\left[\log_\epsilon \frac{2s}{rl} + \frac{1}{2}\log_\epsilon \frac{8}{9} - \phi'\right] = 2l\left[\log_\epsilon \frac{2s}{rl} - \phi\right],$$

where $\phi = \phi' - 1/2 \log_\epsilon 8/9 = \phi' + 0.059$. We have thus reduced the expression for L_{A-B} to the form of (2′).

By means of similar transformations, expressions for the self-inductances of plane polygonal circuits with re-entrant angles having the form of (2′) may be obtained, provided that the shape of the removed part be known and, besides, that this removed part be traversed by a calculable portion of the flux of the fundamental polygon. For all regular polygons the portion of the flux passing through the removed circuit may be easily determined, provided this circuit be bounded by radii drawn from the center of gravity of the fundamental polygon towards its vertexes or between its apothems; or between a radius and an apothem (Fig. 4). The value of the coefficient ϕ for such a polygon with re-entrant angles may be expressed in terms of coefficients ϕ_1 and ϕ_n for the fundamental polygon and for the part removed from it.

Suppose that the area s_n of the removed part of a regular polygon is $1/n$ of its total area s_1. Let us designate the perimeters of the fundamental polygon and its removed part by l_1 and l_n, and let us use k to designate the ratio of the sum of the length of the above-mentioned radii or apothems, (Fig. 4), to the $1/n$th part of the perimeter of the fundamental polygon. In this case we shall have:

$$L = L_1 + L_n - 2M_{1n} = L_1 + L_n - \frac{2}{n} L_1 = \frac{n-2}{n} L_1 + L_n$$

where,

$$L_1 = 2l_1 \left[\log_e \frac{2s_1}{rl_1} - \phi_1 \right] \quad \text{and} \quad L_n = 2l_n \left[\log_e \frac{2s_n}{rl_n} - \phi_n \right].$$

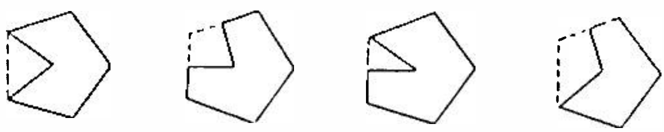

Fig. 4

Let us replace in these expressions, l_1, l_n, s_1, and s_n, the values in terms of perimeter l and area s of the polygon with re-entrant angle, using the following relations which exist:

$$l = l_1 - \frac{1}{n} l_1 + k \frac{l_1}{n} = l_1 \frac{n-1+k}{n}.$$

$$l_n = \frac{l_1}{n} + k \frac{l_1}{n} = l_1 \frac{1+k}{n} = l \frac{1+k}{n-1+k}$$

$$s_1 = \frac{n}{n-1} s$$

$$s_n = \frac{1}{n-1} s.$$

We shall now obtain for L

$$L = 2l \left[\log_e \frac{2s}{rl} + \log_e \frac{n-1+k}{n-1} - \frac{1+k}{n-1+k} \log_e (1+k) - \phi_1 \frac{n-2}{n-1+k} - \phi_n \frac{1+k}{n-1+k} \right].$$

Thus, we reduce the expression for L to (2′) and obtain for the coefficient ϕ the expression

$$\phi = \frac{1+k}{n-1+k} \left[\log_e (1+k) + \phi_n \right] + \frac{n-2}{n-1+k} \phi_1 - \log_e \frac{n-1+k}{n-1}$$

The values of ϕ and $a_k = \phi + \log_e l^2/s$, calculated by means of this formula for a series of different circuits with re-entrant angles, are given

TABLE I

$\frac{l}{\sqrt{s}}$	a_k	ϕ	Shape of circuit	$\frac{l}{\sqrt{s}}$	a_k	ϕ	Shape of circuit	$\frac{l}{\sqrt{s}}$	a_k	ϕ	Shape of circuit
3.54	2.45	−0.080		4.77	3.40	0.274		8.26	4.24	0.018	
3.64	2.56	−0.0235		4.80	3.29	0.152		8.95	4.67	0.285	
3.72	2.64	0.0035		4.83	3.33	0.182		8.99	4.66	0.271	
3.81	2.71	0.0355		5.00	3.27	0.051		9.39	4.49	0.014	
4.00	2.85	0.081		5.09	3.50	0.267		9.54	4.59	0.077	
4.08	2.87	0.076		5.10	3.49	0.232		9.80	4.86	0.290	
4.15	2.95	0.102		5.42	3.63	0.253		10.20	4.92	0.276	
4.30	3.04	0.120		5.60	3.73	0.287		10.59	5.01	0.293	
4.37	3.01	0.067		5.67	3.70	0.225		10.84	4.84	0.073	
4.40	3.09	0.130		5.77	3.62	0.110		11.22	4.86	0 010	
4.56	3.17	0.139		5.84	3.75	0.223		11.30	5.15	0.296	
4.56	3.20	0.164		5.87	3.81	0.273		12.00	5.25	0.282	
4.58	3.20	0.156		6.67	3.82	0.028		12.00	5.27	0.298	
4.62	3.20	0.140		6.94	4.14	0.260		12.95	5.42	0.299	
4.69	3.21	0.170		7.01	4.15	0.252		13.07	5.21	0 069	
4.71	3.21	0.135		7.70	4.17	0.087		14.42	5.34	0.006	
4.76	3.33	0.208		8.01	4.41	0.276		16.66	5.69	0.065	

in Table I together with the values of ϕ and a_k for circuits without re-entrant angles.

The curve $a_k = F(l/\sqrt{s})$ plotted according to this table shows that in the most unfavorable case the values of a_k diverge by no more than 4.3 per cent of the average values given by this curve (Fig. 5).

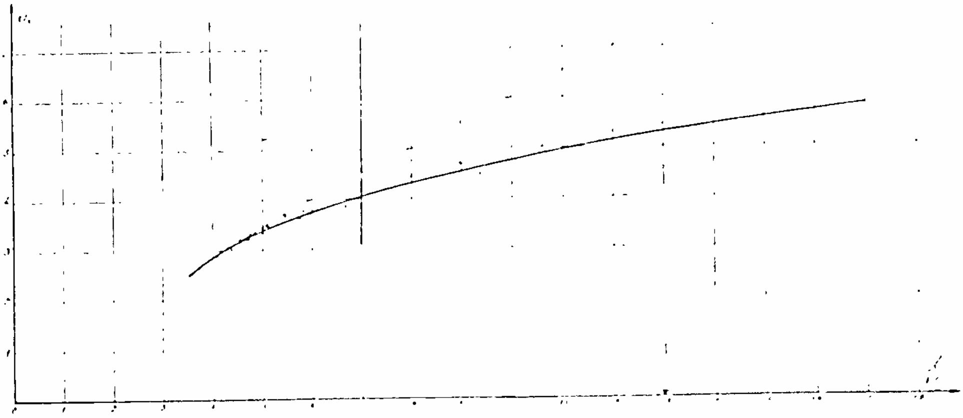

Fig. 5

——————— curve $a_k = F(l/\sqrt{s})$

— — — — extrapolated part of Bashenoff's curve

Moreover a comparison of this curve with the extrapolated part of Bashenoff's curve shows that the extrapolation is not precise. This accounts for the divergence of experimental results from values calculated with the help of the extrapolated part of the curve.

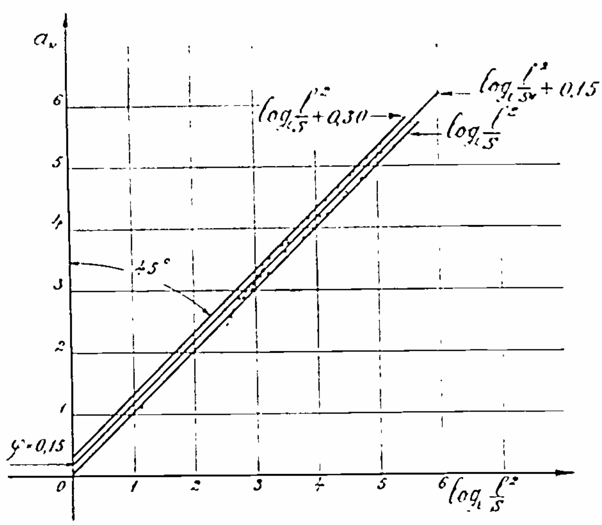

Fig. 6

Fig. 6, which contains the values of a_k as a function of $\log_e l^2/s$, shows that all these values, with the exception of the first two points, lie in the region between the limiting lines $a_k = \log_e l^2/s$ and $a_k = \log_e l^2/s + 0.30$. In other words, (2) and (2′) of V. I. Bashenoff in which a_k is supposed to equal $\log_\epsilon l^2/s+\phi$, (ϕ lying within the limits from 0 to 0.30), may be applied to circuits with re-entrant angles as well as to those without them. In plotting for these circuits the values of ϕ as a function of $l/\sqrt{s}$, we obtain a series of dispersed points. Evidently, the coefficient ϕ may not be considered as a function of $l/\sqrt{s}$, but is determined by other characteristics of the circuit also.

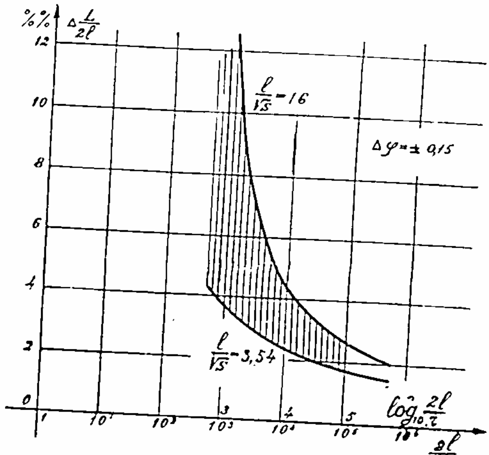

Fig. 7

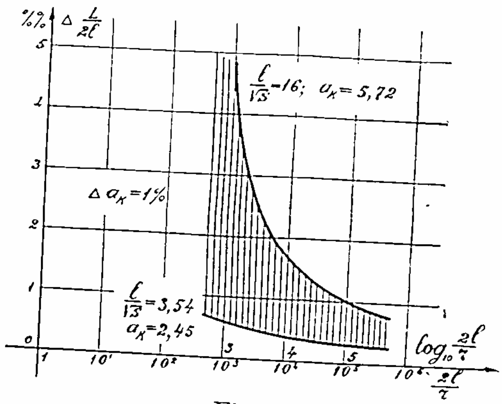

Fig. 8

Admitting for ϕ as does Bashenoff the average value of 0.15, we obtain

$$L = 2l\left(1+\frac{\nabla}{100}\right)\left(\log_\epsilon \frac{2s}{rl} - 0.15 + \mu\delta\right) \tag{3}$$

and it is possible to apply this formula both to circuits with re-entrant angles and to those without them.

The curves of Figs. 7 and 8 give an idea of the error possible in using (3) and (1). The examination of these curves proves that (3) and (1) give a precision of the same order.

Since the correction $\mu\delta$ for the internal flux is, for high frequencies, equal to zero, and for direct current to 0.25 (in the case of $\mu=1$), we obtain

$$L = 2l\left(1+\frac{\nabla}{100}\right)\left(\log_\epsilon \frac{2s}{rl} - 0.15\right) = 2l\left(1+\frac{\nabla}{100}\right)\log_\epsilon \frac{1.72s}{rl}$$

$$\cong 2l\left(1+\frac{\nabla}{100}\right)\log_\epsilon \frac{\sqrt{3}s}{rl}$$

for high frequencies, and

$$L = 2l\left(1+\frac{\nabla}{100}\right)\left(\log_\epsilon \frac{2s}{rl} + 0.10\right) = 2l\left(1+\frac{\nabla}{100}\right)\log_\epsilon \frac{2.21s}{rl}$$

$$\cong 2l\left(1+\frac{\nabla}{100}\right)\log_\epsilon \frac{\sqrt{5}s}{rl}$$

for direct current and $\mu=1$.

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CONCERNING NEW METHODS OF CALCULATING RADIATION RESISTANCE, EITHER WITH OR WITHOUT GROUND*

By
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(Stanford University, California)

Summary—*New general methods of computing radiation resistance, either with or without ground, are described and illustrated by example. The formulas are general, exact, and practical, and allow the calculations to be made for any form of antenna over a plane earth of arbitrary characteristics. As the mathematical basis of these methods has been explained elsewhere the present paper gives no proofs or derivations of formulas but rather illustrates the method of use of the theory by computing radiation resistances for a number of typical antennas. In connection with these computations various timesaving graphical methods are explained and tables of functions and expansions useful in the calculations are given.*

A. Preliminary Discussion of Formulas

In two previous papers [1,2] methods were explained and formulas given whereby it is possible to compute, with a minimum of labor, the total power radiated from any given arbitrary current distributions over a plane earth of arbitrary characteristics. In the present paper we explain in detail the use of these formulas by actually carrying through the calculations for a number of practical cases. As the methods used represent an entirely new attack on the problem, we shall first sketch the results qualitatively to orient the reader and then proceed to clarify the details by work on specific examples.

First of all, the physical principles needed were understood by Maxwell and embodied in his equations. In principle, then, all antenna problems, with or without ground, were solved by Maxwell. But considerable technique of a purely mathematical nature is needed to find even the simplest solutions of Maxwell's equations that are useful for

* Decimal classification: R120×R144. Original manuscript received by the Institute, August 10, 1936. Presented before I.R.E.-U.R.S.I. joint meeting, Washington, D. C., May, 1, 1936.

[1] W. W. Hansen, *Phys. Rev.*, vol. 47, p. 139; January, (1935).

[2] W. W. Hansen and J. G. Beckerley, *Physics*, vol. 7, p. 220; June, (1936). We deeply regret that several errors found their way into this paper. They are as follows: In equation (6) and also in (7) the b on the left should have a minus sign for a superscript. On the right of the first equation in (7) the first term in the square brackets should be multiplied by $e^{-imz'}$ and the second term by $e^{+imz'}$; the right of the second equation in (7) should be multiplied by $e^{+imz'}$. Moreover the sign of A in (7) and (8) and of E and B in (10) should be reversed. Finally in (9) the $\bar{A}^{+}_{1,-n}$ should be replaced by $\bar{A}^{+}_{1,n}$ etc., and similar changes made in (12). None of these changes would affect the results of calculations except for phase shifts. For the understanding of the method however it is absolutely necessary that equation (7) be corrected as specified above.

present purposes. This technique was built up along a single line of attack by a series of investigators, all of whom considered the simplest possible problem; namely, that of finding the field due to an infinitesimal dipole. First there was Hertz who found the field of a dipole in free space. Later Sommerfeld and a long sequence of workers after him worked on the problem of a dipole above a plane earth of arbitrary characteristics. Now any current distribution can be made up of dipoles and so one can say again that the problem is solved for any antenna. But the actual addition of dipole fields may be cumbersome and is likely to become tedious if, in addition, one must perform the two integrations needed to get the total power radiated. Clearly there is room for more mathematical technique to assist in the solution, and it is the object of the present work to supply this. To do this a new point of view is taken in which[3] the current distribution in the antenna is considered as made up of dipole, quadrupole, octopole, etc., all at the origin instead of distributed dipoles. The parts (dipole, quadrupole, . . .) used to make up this pseudo current distribution are so chosen that the energy radiated from any part is independent of the presence or absence of any other part so that we can add the various radiated energies without the necessity of taking interference effects into account. For this reason calculations of the total radiation are much simplified.

To see how this is possible we may consider the matter from the point of view of the fields. We express the fields as a sum of various vector functions A_s', thus writing $E=-1/c\sum_s a_s A_s'$ with the a_s constants depending on the current distribution and the A_s' vector functions of the spherical co-ordinates, r, θ, ϕ. These functions A_s' are especially devised so that (a) they are solutions of a vector wave equation, and (b) if we take the product of two different ones of the set, say A_s' and $A_{s'}'$, and integrate over the surface of a sphere, then

$$\int A_s' \cdot A_{s'}' \sin\theta \, d\theta \, d\phi = 0 \qquad s \neq s'. \tag{1}$$

Now in computing the total radiation we shall want the energy in a thin spherical shell which involves $\int E^2 \sin\theta \, d\theta \, d\phi$. If we use the above assumed expression for E and compute this integral, then, due to the property (b) of the A_s', all cross-product terms drop out and we are left with a simple expression involving only $\sum a_s^2$. This is similar to the situation in a Fourier analysis where, if $f(x)$ is known to be given by $f(x)=\sum_n a_n \sin nx$ then $\int f^2 \, dx = \pi \sum_n a_n^2$.

[3] The qualitative explanation here supposes the use of spherical co-ordinates. The same principles are used in cylindrical co-ordinates.

The advantage of this method is that in finding the energy radiated we do not need to compute the fields, nor do we need to perform the integrations over angle (i.e., over the surface of a sphere) of the Poynting vector. We therefore expect a very considerable reduction of the labor required provided that (a) the coefficients a_s are easily found, and (b) the series $\sum_s a_s^2$ converges well. Moreover the same attack can be used in cylindrical co-ordinates, whereupon it is very little more troublesome to take into account the effect of the ground.

Now in actual fact the coefficients are easily found, and in general we need only a few of them. To show that this is so, and to explain quantitatively the workings of the present methods, we will write down the formulas used and apply them to specific examples.

In spherical co-ordinates the results are expressed in terms of the following notation. Let,

$$\left.\begin{aligned} &\xi_{l,n}(r,\theta,\phi) \\ &\quad = \left[\frac{(2l+1)(l-|n|)!}{8l(l+1)(l+|n|)!}\right]^{1/2}\frac{1}{(kr)^{1/2}}J_{l+1/2}(kr)P_l^{|n|}(\cos\theta)e^{in\phi} \\ &\mathbf{A}_{2,l,n}(r,\theta,\phi) = k\nabla\times(\mathbf{r}\xi_{l,n}) = -\frac{1}{k}\nabla\times\mathbf{A}_{3,l,n} \\ &\mathbf{A}_{3,l,n}(r,\theta,\phi) = -\frac{1}{k}\nabla\times\mathbf{A}_{2,l,n} \end{aligned}\right\} \quad (2)$$

where $\xi_{l,n}$ is a solution of the scalar wave equation, k the wave number ω/c, and r, θ, ϕ spherical polar co-ordinates. All vectors are in bold face.[4] We also use functions $\mathbf{A}_s'$ which are generated in the same way as the $\mathbf{A}_s$ except that the Bessel function in $\xi_{l,n}$ is replaced by a Hankel function of the first kind. Then, if $\mathbf{i}$ is the current in the antenna we define

$$a_s = \int \overline{\mathbf{A}}_s \cdot \mathbf{i}\, d\tau \quad (3)$$

and then find[5] the field $\mathbf{E}$

$$\mathbf{E} = -\frac{1}{c}\sum_s a_s\mathbf{A}_s' \quad (4)$$

[4] In the course of the mathematical development of the theory a third function denoted by $\mathbf{A}_{1,l,n}$ appears. But this function never appears in the expressions for the fields or the energy radiated, and we therefore do not define or discuss further this function. The functions $\mathbf{A}_{2,l,n}$ and $\mathbf{A}_{3,l,n}$ are essentially those used for different purposes by Mie and others. See G. Mie, *Ann. der Phys.*, vol. 25, p. 377, (1908); H. Bateman, "Electrical and Optical Wave Motion"; M. Born, "Optik" (p. 274 *et seq.*); G. Wolfsohn, *Handbuch der Physik*, vol. 20, p. 307 *et seq.*

[5] The letter s denotes the complete set of numbers needed to specify an $\mathbf{A}$. Summation over all s means summation over both kinds of $\mathbf{A}$ with l running from zero to ∞ and n from $-\infty$ to $+\infty$. Heaviside-Lorentz units are used. The expression for $\mathbf{E}$ is valid outside a sphere which surrounds all currents.

and the total energy radiated per unit time

$$\int S \cdot d\sigma = \frac{1}{2c} \sum_s | a_s |^2. \tag{5}$$

We may note that formally the results are what would be obtained if we expanded the current distribution i in a series of orthogonal functions A_s and then said that the sum of the squares of the expansion coefficients is a measure of the radiation. These formulas are in general useful both for finding the field at various angles and for finding the energy radiated. They are not applicable when a ground of finite conductivity is present.

For purposes of computation we need explicit expressions for the A_s. By differentiation we find that[6]

$$\left.\begin{aligned} A_{2,l,n} &= kh \frac{1}{(kr)^{1/2}} J_{l+1/2}\, e^{in\phi}\left[k_\theta \frac{in}{\sin\theta} P_l^{|n|} - k_\phi \frac{d}{d\theta} P_l^{|n|}\right] \\ A_{3,l,n} &= khe^{in\phi}\left\{k_r\left[-\frac{l(l+1)}{(kr)^{3/2}} J_{l+1/2} P_l^{|n|}\right]\right. \\ &\left. + \frac{1}{(kr)^{1/2}}\left[J_{l+3/2} - \frac{(l+1)}{kr} J_{l+1/2}\right]\left[k_\theta \frac{d}{d\theta} P_l^{|n|} + k_\phi \frac{in}{\sin\theta} P_l^{|n|}\right]\right\} \end{aligned}\right\} \tag{6}$$

where,

$$h = \left[\frac{(2l+1)(l-|n|)!}{8l(l+1)(l+|n|)!}\right]^{1/2}.$$

In each case the argument of the Bessel function is kr and of the P function (associated Legendre polynomial) $\cos\theta$.

Numerical values may easily be obtained. Tables of half-order Bessel functions are common and the P functions are simple combinations of $\sin\theta$ and $\cos\theta$. For small values of kr the power series expressions are useful. To save others the work of making an equivalent tabulation we have included Table I which gives explicit formulas in terms of elementary functions for all the factors needed to construct the A_s up to $A_{2,4,4}$ and $A_{3,4,4}$. Table I is divided into subtables numbered 1, 2, 3, $\cdots$ 7, so that if we denote $\langle 5\rangle$ to mean "the reading from subtable 5," etc., the components become simply

$$\left.\begin{aligned} A_{2,l,n} &= k\langle 1\rangle\langle 2\rangle e^{in\phi}[k_\theta i\langle 6\rangle - k_\phi\langle 7\rangle] \\ A_{3,l,n} &= k\langle 1\rangle e^{in\phi}[k_r\langle 3\rangle\langle 5\rangle + k_\theta\langle 4\rangle\langle 7\rangle + k_\phi i\langle 4\rangle\langle 6\rangle]. \end{aligned}\right\} \tag{7}$$

Also included in Table I are subtables 2a, 3a, 4a, which give power series expressions, good for small values of kr, to be used in place of subtables 2, 3, and 4 when convenient. We have also made graphs of the various functions of r which occur. These appear in Fig. 1.

[6] The symbols k_r, k_θ, k_ϕ represent unit vectors in the various directions.

TABLE I

TABLES FOR THE CALCULATION OF THE COMPONENTS OF THE SPHERICAL A_s

[See Equations (6) and (7)]

Subtable 1. Values of $h=\left[\frac{(2l+1)\ (l-|n|)!}{8l(l+1)\ (l+|n|)!}\right]^{1/2}$

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
$l=1$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{8}\sqrt{6}$			
$l=2$	$\frac{1}{12}\sqrt{15}$	$\frac{1}{24}\sqrt{10}$	$\frac{1}{48}\sqrt{5}$		
$l=3$	$\frac{1}{24}\sqrt{42}$	$\frac{1}{48}\sqrt{14}$	$\frac{1}{240}\sqrt{35}$	$\frac{1}{1440}\sqrt{210}$	
$l=4$	$\frac{3}{40}\sqrt{10}$	$\frac{3}{80}\sqrt{2}$	$\frac{1}{80}$	$\frac{1}{1120}\sqrt{14}$	$\frac{1}{2240}\sqrt{7}$

Subtable 2. Values of $\frac{1}{(kr)^{1/2}}J_{l+1/2}(kr)$

$l=1$	$\sqrt{\frac{2}{\pi}}\ \frac{1}{kr}\left(\frac{\sin kr}{kr}-\cos kr\right)$
$l=2$	$\sqrt{\frac{2}{\pi}}\ \frac{1}{kr}\left[\left(\frac{3}{k^2r^2}-1\right)\sin kr-\frac{3}{kr}\cos kr\right]$
$l=3$	$\sqrt{\frac{2}{\pi}}\ \frac{1}{kr}\left[\left(\frac{15}{k^3r^3}-\frac{6}{kr}\right)\sin kr-\left(\frac{15}{k^2r^2}-1\right)\cos kr\right]$
$l=4$	$\sqrt{\frac{2}{\pi}}\ \frac{1}{kr}\left[\left(\frac{105}{k^4r^4}-\frac{45}{k^2r^2}+1\right)\sin kr-\left(\frac{105}{k^3r^3}-\frac{10}{kr}\right)\cos kr\right]$

TABLE I (*Continued*)

Subtable 3. Values of $-\frac{l(l+1)}{(kr)^{3/2}} J_{l+1/2}(kr)$

$l = 1$	$-2\sqrt{\frac{2}{\pi}}\frac{1}{k^2r^2}\left[\frac{\sin kr}{kr} - \cos kr\right]$
$l = 2$	$-6\sqrt{\frac{2}{\pi}}\frac{1}{k^2r^2}\left[\left(\frac{3}{k^2r^2} - 1\right)\sin kr - \frac{3}{kr}\cos kr\right]$
$l = 3$	$-12\sqrt{\frac{2}{\pi}}\frac{1}{k^2r^2}\left[\left(\frac{15}{k^3r^3} - \frac{6}{kr}\right)\sin kr - \left(\frac{15}{k^2r^2} - 1\right)\cos kr\right]$
$l = 4$	$-20\sqrt{\frac{2}{\pi}}\frac{1}{k^2r^2}\left[\left(\frac{105}{k^4r^4} - \frac{45}{k^2r^2} + 1\right)\sin kr - \left(\frac{105}{k^3r^3} - \frac{10}{kr}\right)\cos kr\right]$

Subtable 4. Values of $\frac{1}{(kr)^{1/2}}\left[J_{l+3/2}(kr) - \frac{(l+1)}{kr}J_{l+1/2}(kr)\right]$

$l = 1$	$\sqrt{\frac{2}{\pi}}\frac{1}{kr}\left[\left(\frac{1}{k^2r^2} - 1\right)\sin kr - \frac{1}{kr}\cos kr\right]$
$l = 2$	$\sqrt{\frac{2}{\pi}}\frac{1}{kr}\left[\left(\frac{6}{k^3r^3} - \frac{3}{kr}\right)\sin kr - \left(\frac{6}{k^2r^2} - 1\right)\cos kr\right]$
$l = 3$	$\sqrt{\frac{2}{\pi}}\frac{1}{kr}\left[\left(\frac{45}{k^4r^4} - \frac{21}{k^2r^2} + 1\right)\sin kr - \left(\frac{45}{k^3r^3} - \frac{6}{kr}\right)\cos kr\right]$
$l = 4$	$\sqrt{\frac{2}{\pi}}\frac{1}{kr}\left[\left(\frac{420}{k^5r^5} - \frac{195}{k^3r^3} + \frac{10}{kr}\right)\sin kr - \left(\frac{420}{k^4r^4} - \frac{55}{k^2r^2} + 1\right)\cos kr\right]$

TABLE I (*Continued*)

Subtable 5. Values of $P_l^{|n|}(\cos\ \theta)$

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
$l=1$	$\cos\theta$	$\sin\theta$			
$l=2$	$\frac{1}{2}(3\cos^2\theta-1)$	$3\sin\theta\cos\theta$	$3\sin^2\theta$		
$l=3$	$\frac{1}{2}(5\cos^3\theta-3\cos\theta)$	$\frac{3}{2}\sin\theta(5\cos^2\theta-1)$	$15\sin^2\theta\cos\theta$	$15\sin^3\theta$	
$l=4$	$\frac{1}{8}(35\cos^4\theta-30\cos^2\theta+3)$	$\frac{5}{2}\sin\theta(7\cos^3\theta-3\cos\theta)$	$\frac{15}{2}\sin^2\theta(7\cos^2\theta-1)$	$105\sin^3\theta\cos\theta$	$105\sin^4\theta$

Subtable 6. Values of $\frac{n}{\sin\theta}P_l^{|n|}(\cos\theta)$

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
$l=1$	0	1			
$l=2$	0	$3\cos\theta$	$6\sin\theta$		
$l=3$	0	$\frac{3}{2}(5\cos^2\theta-1)$	$30\sin\theta\cos\theta$	$45\sin^2\theta$	
$l=4$	0	$\frac{5}{2}(7\cos^3\theta-3\cos\theta)$	$15\sin\theta(7\cos^2\theta-1)$	$315\sin^2\theta\cos\theta$	$420\sin^3\theta$

TABLE I (*Continued*)

Subtable 7. Values of $\frac{d}{d\theta} P_l^{|n|}(\cos\theta)$

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
$l=1$	$-\sin\theta$	$\cos\theta$			
$l=2$	$-3\cos\theta\sin\theta$	$3(2\cos^2\theta-1)$	$6\sin\theta\cos\theta$		
$l=3$	$-\frac{3}{2}\sin\theta\,(5\cos^2\theta-1)$	$\frac{3}{2}(15\cos^3\theta - 11\cos\theta)$	$15\sin\theta\,(3\cos^2\theta-1)$	$-45(\cos^3\theta - \cos\theta)$	
$l=4$	$-\frac{5}{2}\sin\theta\,(7\cos^3\theta - 3\cos\theta)$	$\frac{5}{2}(28\cos^4\theta - 27\cos^2\theta + 3)$	$30\sin\theta\,(7\cos^3\theta - 4\cos\theta)$	$-105(4\cos^4\theta - 5\cos^2\theta+1)$	$-420\sin\theta\,(\cos^3\theta-\cos\theta)$

Subtable 2a. As $kr\to 0$, values of $\frac{1}{(kr)^{1/2}} J_{l+1/2}(kr)$

$l=1$	$\frac{1}{3}\sqrt{\frac{2}{\pi}}\,kr\left[1-\frac{k^2r^2}{10}+\frac{k^4r^4}{280}-\cdots\right]$
$l=2$	$\frac{1}{15}\sqrt{\frac{2}{\pi}}\,k^2r^2\left[1-\frac{k^2r^2}{14}+\frac{k^4r^4}{504}-\cdots\right]$
$l=3$	$\frac{1}{105}\sqrt{\frac{2}{\pi}}\,k^3r^3\left[1-\frac{k^2r^2}{18}+\frac{k^4r^4}{792}-\cdots\right]$
$l=4$	$\frac{1}{945}\sqrt{\frac{2}{\pi}}\,k^4r^4\left[1-\frac{k^2r^2}{22}+\frac{k^4r^4}{1144}-\cdots\right]$

TABLE I (*Continued*)

Subtable 3a. As $kr \rightarrow 0$, values of $-\frac{l(l+1)}{(kr)^{3/2}} J_{l+1/2}(kr)$

$l = 1$	$-\frac{2}{3}\sqrt{\frac{2}{\pi}}\left[1 - \frac{k^2r^2}{10} + \frac{k^4r^4}{280} - \cdots\right]$
$l = 2$	$-\frac{2}{5}\sqrt{\frac{2}{\pi}}\, kr\left[1 - \frac{k^2r^2}{14} + \frac{k^4r^4}{504} - \cdots\right]$
$l = 3$	$-\frac{4}{35}\sqrt{\frac{2}{\pi}}\, k^2r^2\left[1 - \frac{k^2r^2}{18} + \frac{k^4r^4}{792} - \cdots\right]$
$l = 4$	$-\frac{4}{189}\sqrt{\frac{2}{\pi}}\, k^3r^3\left[1 - \frac{k^2r^2}{22} + \frac{k^4r^4}{1144} - \cdots\right]$

Subtable 4a. As $kr \rightarrow 0$, values of $\frac{1}{(kr)^{1/2}}\left[J_{l+3/2}(kr) - \frac{(l+1)}{kr} J_{l+1/2}(kr)\right]$

$l = 1$	$-\frac{2}{3}\sqrt{\frac{2}{\pi}}\left[1 - \frac{k^2r^2}{5} + \frac{3k^4r^4}{280} - \cdots\right]$
$l = 2$	$-\frac{1}{5}\sqrt{\frac{2}{\pi}}\, kr\left[1 - \frac{5k^2r^2}{42} + \frac{k^4r^4}{216} - \cdots\right]$
$l = 3$	$-\frac{4}{105}\sqrt{\frac{2}{\pi}}\, k^2r^2\left[1 - \frac{k^2r^2}{12} + \frac{k^4r^4}{396} - \cdots\right]$
$l = 4$	$-\frac{1}{189}\sqrt{\frac{2}{\pi}}\, k^3r^3\left[1 - \frac{7k^2r^2}{110} + \frac{9k^4r^4}{5720} - \cdots\right]$

For use in deciding in any given case which a_s can be expected to be zero by reason of symmetry qualitative diagrams that make plain the general properties of the first few A_s are very useful. We have reproduced a few such diagrams in Fig. 2.

When using (4) to find the directive pattern of an antenna approximate expressions for the A_s' valid at large distances are needed. Using asymptotic expansions for the Hankel functions these are easily found to be

$$\left.\begin{aligned} A'_{2,l,n} &= h\sqrt{\frac{2}{\pi}}\,\frac{e^{ikr}}{r}(-i)^{l+1}e^{in\phi}\left[k_\theta\frac{in}{\sin\theta}P_l{}^{|n|} - k_\phi\frac{d}{d\theta}P_l{}^{|n|}\right] \\ A'_{3,l,n} &= h\sqrt{\frac{2}{\pi}}\,\frac{e^{ikr}}{r}(-i)^{l+2}e^{in\phi}\left[k_\theta\frac{d}{d\theta}P_l{}^{|n|} + k_\phi\frac{in}{\sin\theta}P_l{}^{|n|}\right] \end{aligned}\right\} \qquad (8)$$

or

$$\left.\begin{aligned} A'_{2,l,n} &= \langle 1\rangle\sqrt{\frac{2}{\pi}}\,\frac{e^{ikr}}{r}(-i)^{l+1}e^{in\phi}[k_\theta i\langle 6\rangle - k_\theta\langle 7\rangle] \\ A'_{3,l,n} &= \langle 1\rangle\sqrt{\frac{2}{\pi}}\,\frac{e^{ikr}}{r}(-i)^{l+2}e^{in\phi}[k_\theta\langle 7\rangle + k_\theta i\langle 6\rangle] \end{aligned}\right\}.$$

If we use cylindrical co-ordinates the results are as follows:[7]

$$\xi_n(\rho, \phi, z:\theta') = e^{i(kz\cos\theta'+n\phi)}J_n(k\rho\sin\theta') \qquad (9)$$

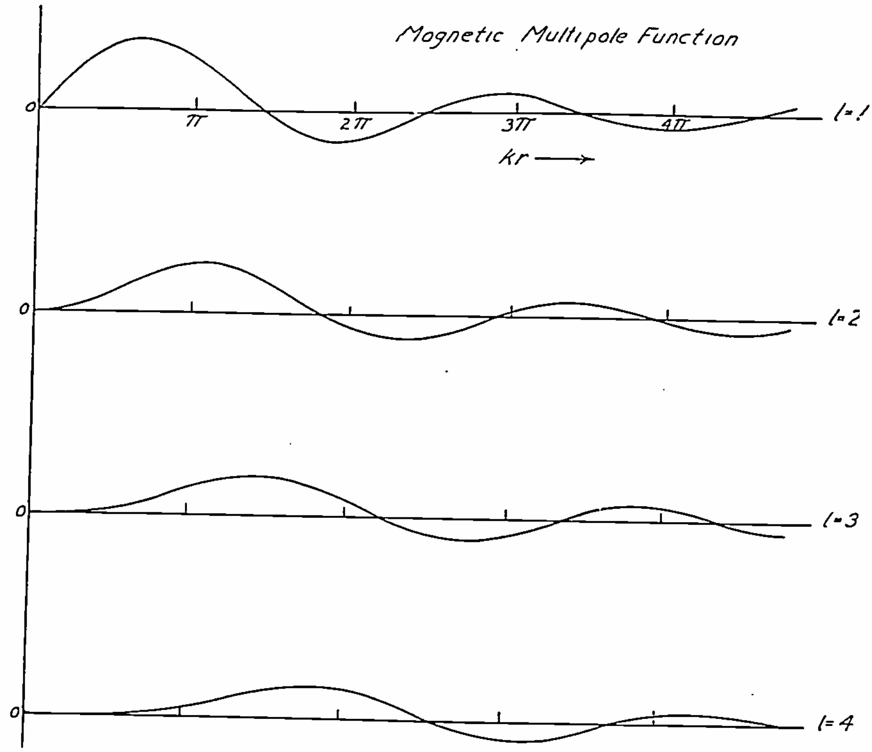
is a solution of the scalar-wave equation.[8] The vector functions are[9]

$$\left.\begin{aligned} A_{2,n}(\rho, \phi, z:\theta') &= \nabla\times k_z\xi_n = \frac{1}{k}\nabla\times A_{3,n} \\ A_{3,n}(\rho, \phi, z:\theta') &= \frac{1}{k}\nabla\times A_{2,n}. \end{aligned}\right\}. \qquad (10)$$

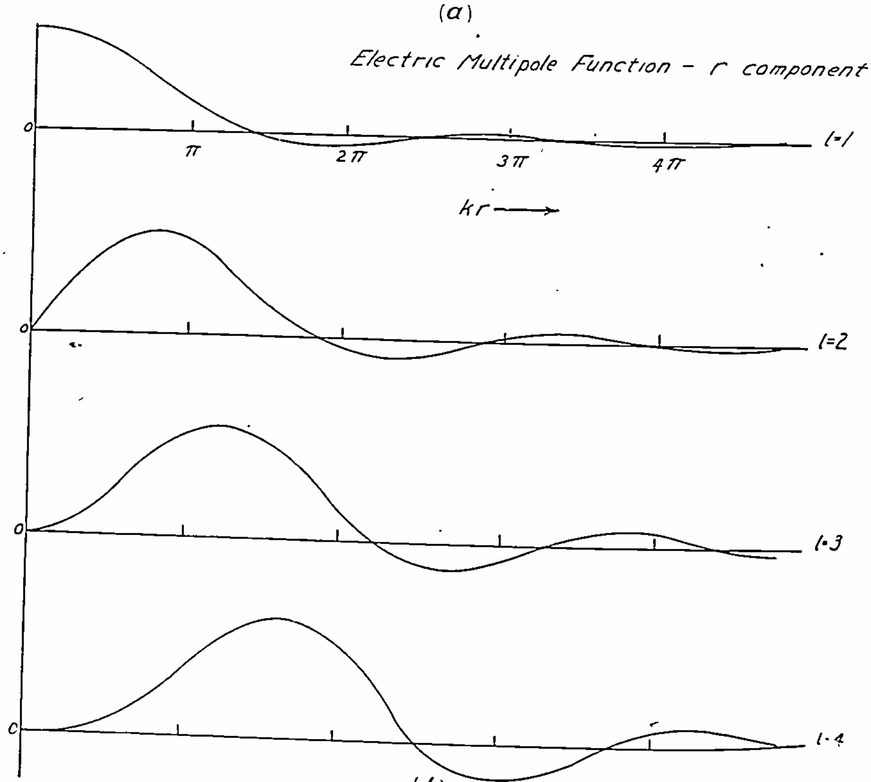
[7] Those familiar with Sommerfeld's theory will find it instructive to compare the present theory with that one. To do this one uses a current distribution i corresponding to an elementary dipole. For example a vertical dipole Δz long at $z=0$ gives $f_{2,n}=0$, $f_{3,0}=i_0\Delta z/2\pi k$, all other $f_{3,n}=0$. Putting in $f_{3,0}=$constant in the various equations it will be found that many of them can be compared directly with those of Sommerfeld.

[8] ρ, ϕ, z are the standard circular cylindrical co-ordinates, with the z direction perpendicular to the ground. The subscript n has practically the same significance in both spherical and cylindrical A. It will be noted that the notation in cylindrical co-ordinates has been changed from that which is given in the reference of footnote (2), the most important change being the introduction of θ' which corresponds to l and m of (2) in the following way: $l=k\sin\theta'$ and $m=k\cos\theta'$. One of the obvious advantages of θ' is that it eliminates the necessity of "plus" and "minus" functions (i.e., A^+, A^-, f^+, f^- etc.)—*loc. cit.* Another advantage is that θ' is connected with the directional effects of antennas, as will be shown in a future paper.

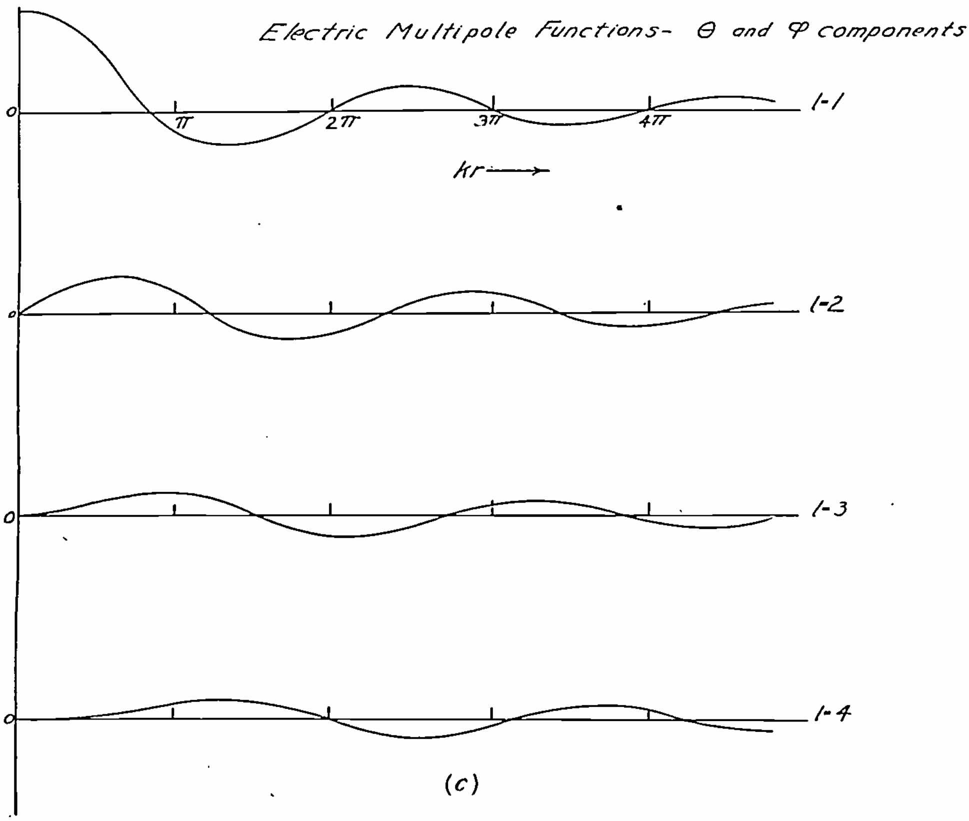
[9] These functions are exactly those suited to represent the fields in the "wave guides" of Southworth and Barrow. See *Bell Lab. Rec.*, vol. 14, p. 283; May (1936).


Magnetic Multipole Function
0
π
2π
3π
4π
kr →
l=1
l=2
l=3
l=4

(a)

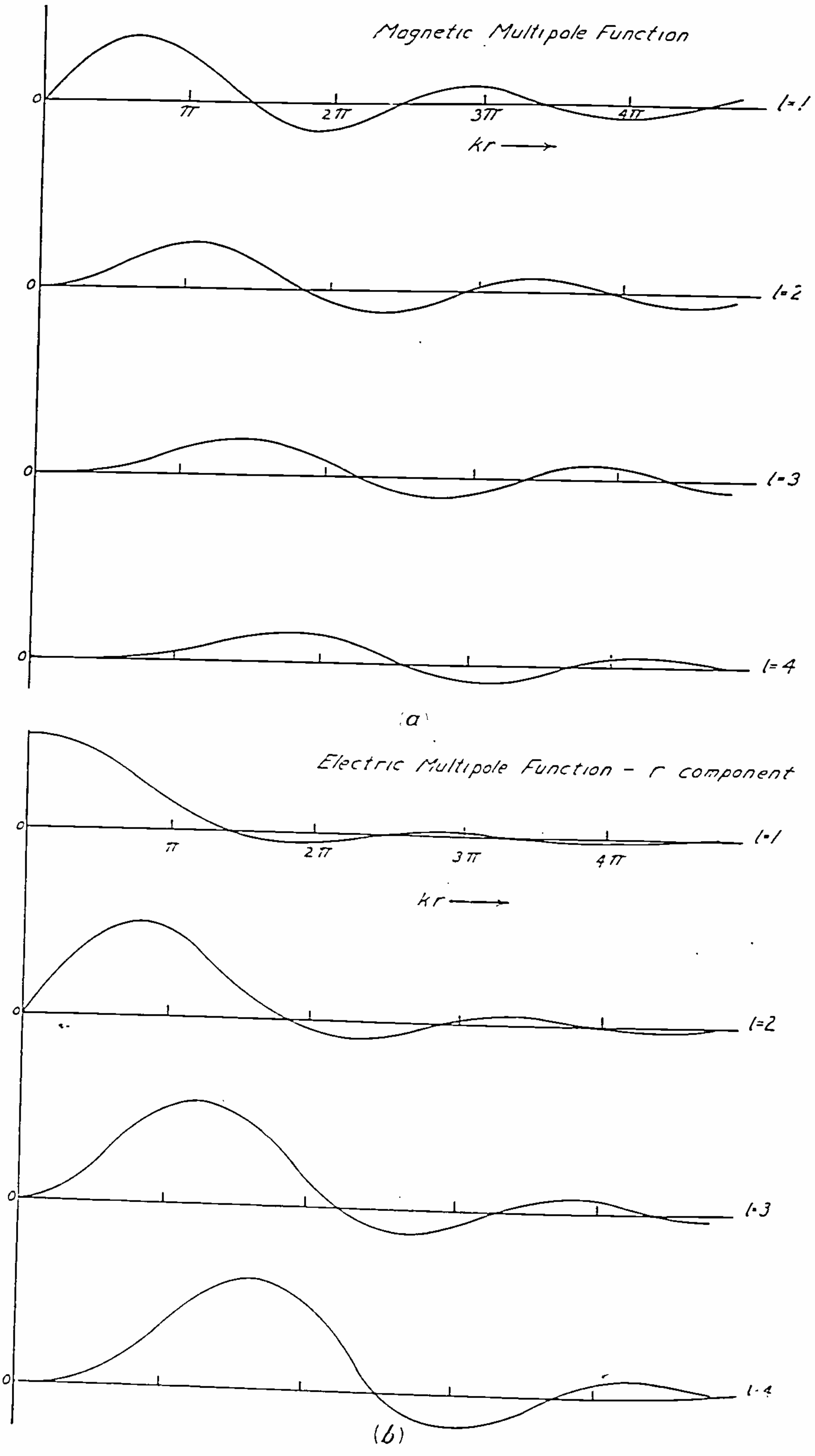

Electric Multipole Function – r component
0
π
2π
3π
4π
kr →
l=1
l=2
l=3
l=4

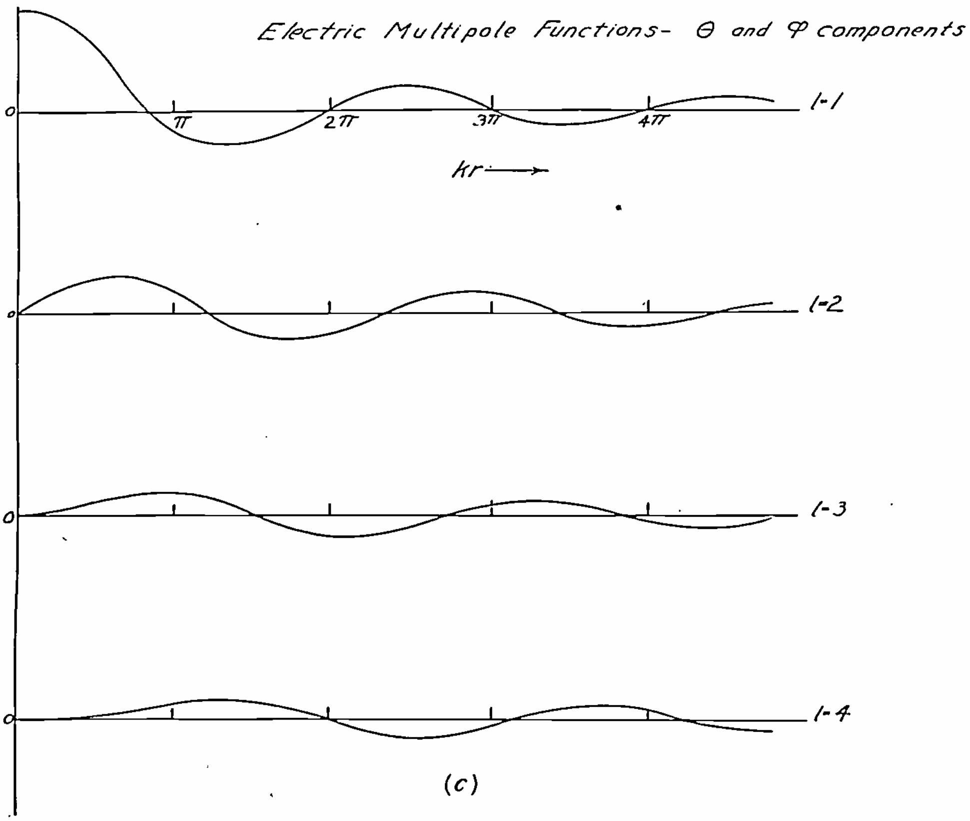
(b)



(c)

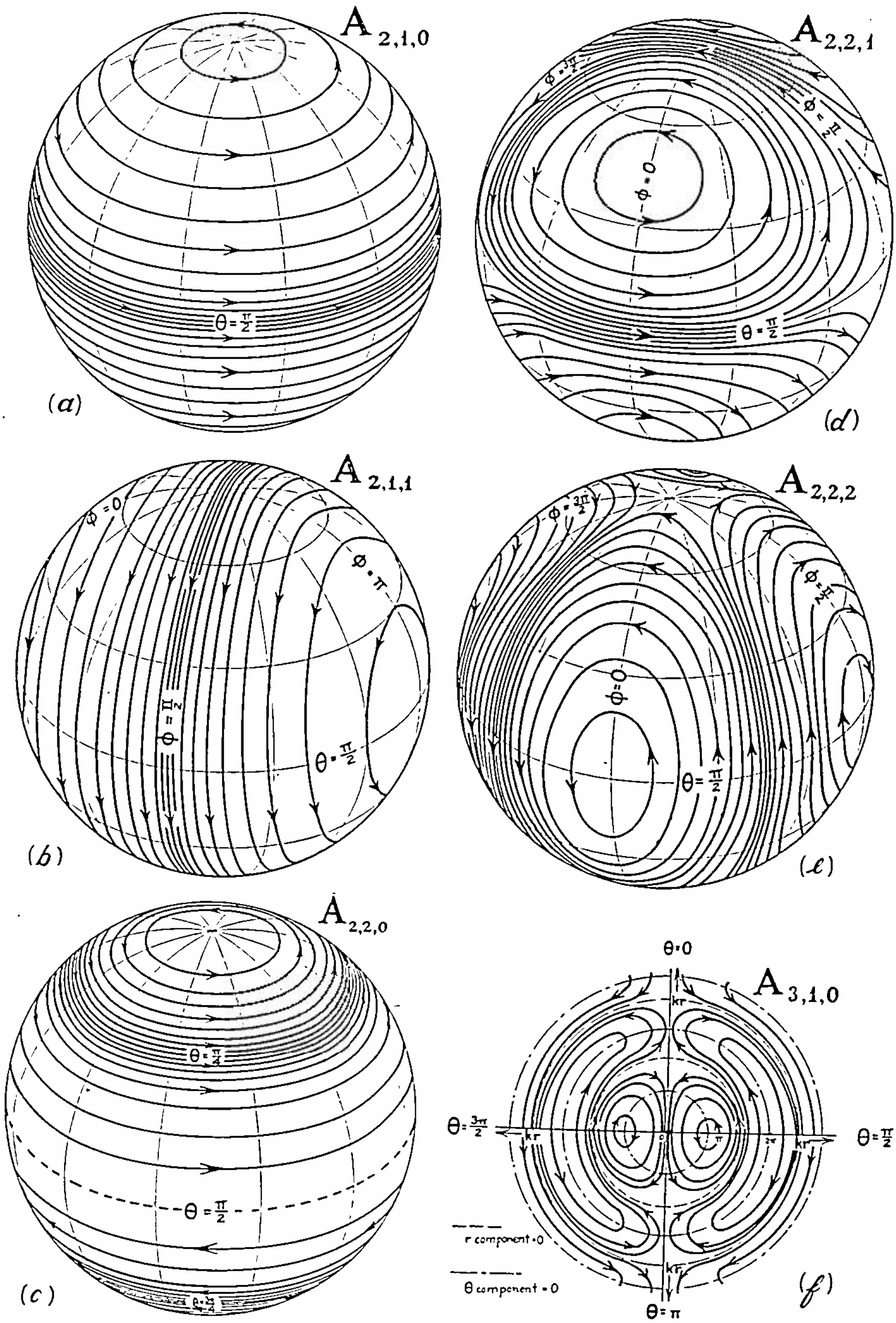
Fig. 1—Each of the components of the vector functions A_s can be factored into a function of r, a function of θ, and a function of ϕ. In this figure we have plotted the various functions of r for l values from 1 to 4 for all the components of the functions $A_{2,l,n}$, $A_{3,l,n}$. The first group of four graphs (a) labelled "magnetic multipole functions" gives the dependence on r of both the θ and ϕ components of $A_{2,l,n}$. There is no r component. The next four graphs (b) illustrate the r dependence of the radial component of $A_{3,l,n}$. This group is labelled "Electric multipole functions—r component." The last four (c) are similar graphs for the θ and ϕ components of $A_{3,l,n}$. Unfortunately these last two sets of graphs (b) and (c), are plotted upside down, as may be seen from values for r factor of $A_{3,l,n}$ given in Table I.


Magnetic Multipole Function
0
π
2π
3π
4π
kr
l=1
l=2
l=3
l=4
(a)
Electric Multipole Function – r component
l=1
l=2
l=3
l=4
(b)

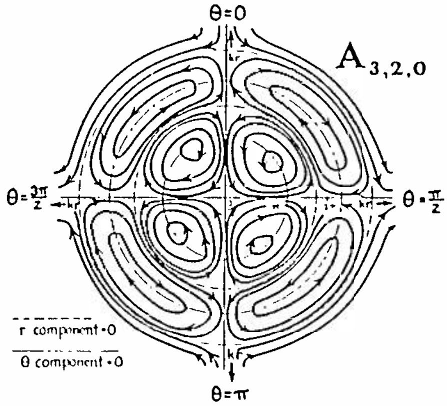


(c)

Fig. 1—Each of the components of the vector functions A_s can be factored into a function of r, a function of θ, and a function of ϕ. In this figure we have plotted the various functions of r for l values from 1 to 4 for all the components of the functions $A_{2,l,n}$, $A_{3,l,n}$. The first group of four graphs (a) labelled "magnetic multipole functions" gives the dependence on r of both the θ and ϕ components of $A_{2,l,n}$. There is no r component. The next four graphs (b) illustrate the r dependence of the radial component of $A_{3,l,n}$. This group is labelled "Electric multipole functions—r component." The last four (c) are similar graphs for the θ and ϕ components of $A_{3,l,n}$. Unfortunately these last two sets of graphs (b) and (c), are plotted upside down, as may be seen from values for r factor of $A_{3,l,n}$ given in Table I.



A2,1,0
θ = π/2
(a)
A2,2,1
φ = 3π/2
φ = π/2
φ = 0
θ = π/2
(d)
A2,1,1
φ = 0
φ = π
φ = π/2
θ = π/2
(b)
A2,2,2
φ = 3π/2
φ = π/2
φ = 0
θ = π/2
(e)
A2,2,0
θ = π/4
θ = π/2
(c)
A3,1,0
θ = 0
kr
θ = 3π/2
θ = π/2
r component = 0
θ component = 0
θ = π
(f)



(*g*)

Fig. 2—This group of illustrations is intended to give a qualitative picture of the first few of the functions A_s. We have drawn lines to give direction of the vectors A; and since $\nabla \cdot A = 0$, it has been possible for us to arrange the lines so that their density is a measure of the magnitude of A. Strictly speaking we have considered only the real part of A_s. Picture (a) shows magnitude and direction of the vector function $A_{2,1,0}$ on the surface of a sphere centered on the origin. This might be for example the electric field around a "magnetic dipole" or small loop antenna. There are three possible independent orientations of such an antenna so we are not surprised when (b) which represents $A_{2,1,1}$ turns out to be essentially the same except for orientation. Figure (c) is a representation of $A_{2,2,0}$. An electric field of this type might be radiated by two opposing magnetic dipoles. Figures (d) and (e) represent $A_{2,2,1}$ and $A_{2,2,2}$ which are essentially the same. The functions $A_{3,l,n}$ are much harder to depict as they have in general three components. At large distances the r component vanishes by comparison with the others and pictures like those of the $A_{2,l,n}$ could be drawn. We have not done this, however, as the resulting lines, for given l and n values are exactly at right angles to the lines for the $A_{2,l,n}$ with the same l and n and it is thought unnecessary to draw pictures having so simple a relation to those already shown. We have given up trying to represent functions with all three components and have therefore restricted ourselves to pictures of $A_{3,1,0}$ and $A_{3,2,0}$ which have no ϕ component. Figures (f) and (g) show lines of these functions in a plane ϕ=constant; i.e., a plane containing the polar axis. Figure (f) will be recognized as the field due to an electric dipole. It may be mentioned that the $A_{3,1,1}$ is the same function differently oriented. Figure (g) is the field associated with an electric quadrupole composed of two dipoles end to end with currents 180 degrees out of phase.

Then we define "oscillator strengths" f_s which correspond to the coefficients a_s of the spherical case

$$\left.\begin{aligned} f_{2,n}(\theta') &= \frac{1}{2\pi k^2 \sin^2 \theta'} \int i \cdot \bar{A}_{2,n} d\tau \\ f_{3,n}(\theta') &= \frac{1}{2\pi k^2 \sin^2 \theta'} \int i \cdot \bar{A}_{3,n} d\tau \end{aligned}\right\}. \tag{11}$$

We also need to take in the characteristics of the ground. For this purpose we introduce reflection coefficients

$$\left.\begin{aligned} \alpha_2 &= \frac{\mu \cos \theta' - \sqrt{\epsilon'\mu - \sin^2 \theta'}}{\mu \cos \theta' + \sqrt{\epsilon'\mu - \sin^2 \theta'}} \\ \alpha_3 &= \frac{\epsilon' \cos \theta' - \sqrt{\epsilon'\mu - \sin^2 \theta'}}{\epsilon' \cos \theta' + \sqrt{\epsilon'\mu - \sin^2 \theta'}} \end{aligned}\right\} \tag{12}$$

with,

$$\epsilon' = \epsilon + \frac{i\sigma}{kc} \tag{13}$$

where ϵ, μ, σ are the dielectric constant, permeability, and conductivity, respectively, of the ground. We may note that the α are independent of the form of the antenna.

With this notation the field is found to be[10]

$$\begin{aligned} E = -\frac{k^2}{2c} \sum_{n=-\infty}^{+\infty} \int_0^{i\infty} \Big\{ \Big[f_{2,n}(\theta') + \alpha_2(\theta') f_{2,n}(\pi - \theta') \Big] A_{2,n} \\ + \Big[f_{3,n}(\theta') + \alpha_3(\theta') f_{3,n}(\pi - \theta') \Big] A_{3,n} \Big\} \sin \theta' \, d\theta' \end{aligned} \tag{14}$$

where the integration path is from 0 to $\pi/2$ to $\pi/2 + i\infty$. This holds for elevations above ground greater than that of the highest current. A similar expression can be written for elevations above the ground and below the lowest current. The total energy radiated per second is given by

$$\begin{aligned} \int S \cdot d\sigma = \frac{\pi k^4}{4c} \sum_{n=-\infty}^{+\infty} \int_0^{\pi/2} \Big[\,| f_{2,n}(\theta') |^2 + 2R\Big(\bar{f}_{2,n}(\theta')\alpha_2(\theta') f_{2,n}(\pi - \theta')\Big) \\ + | f_{2,n}(\pi - \theta') |^2 + | f_{3,n}(\theta') |^2 \\ + 2R\Big(\bar{f}_{3,n}(\theta')\alpha_2(\theta') f_{3,n}(\pi - \theta')\Big) \\ + | f_{3,n}(\pi - \theta') |^2 \Big] \sin^3 \theta' \, d\theta'. \end{aligned} \tag{15}$$

[10] It will be noted that this equation for E differs from that in equation (10) of reference (2) by a minus sign. See note on reference (2).

Here R signifies that the real part of the following expression is to be taken. The part of the energy which is eventually dissipated in the ground and the part that is radiated into space may be found separately, if desired. The formulas are: the energy into the ground,

$$\frac{\pi k^4}{4c}\sum_n \int_0^{\pi/2}\Big[\,|f_{2,n}(\pi-\theta')|^2 - |\alpha_2(\theta')f_{2,n}(\pi-\theta')|^2 + |f_{3,n}(\pi-\theta')|^2 - |\alpha_3(\theta')f_{3,n}(\pi-\theta')|^2\Big]\sin^3\theta'\,d\theta' \qquad (16)$$

the energy radiated upward,

$$\frac{\pi k^4}{4c}\sum_n \int_0^{\pi/2}\Big[\,|f_{2,n}(\theta')|^2 + |\alpha_2(\theta')f_{2,n}(\pi-\theta')|^2 + 2R\Big(\bar{f}_{2,n}(\theta')\alpha_2(\theta')f_{2,n}(\pi-\theta')\Big) + |f_{3,n}(\theta')|^2 + |\alpha_3(\theta')f_{3,n}(\pi-\theta')|^2 + 2R\Big(\bar{f}_{3,n}(\theta')\alpha_3(\theta')f_{3,n}(\pi-\theta')\Big)\Big]\sin^3\theta'\,d\theta'. \qquad (17)$$

At least in the present form (14) is not useful for purposes of finding the field as it would involve the numerical integration of a rapidly oscillating function.[11] However, formula (15) for computing the total energy radiated per second is easily computed. If a ground is present this formula is the only one available. If the ground is neglected either (5) or (15) may be used to find the power radiated. In general the spherical co-ordinate formula is to be preferred, as one has only to sum over discrete indexes l and n, whereas in (15) one of these summations is replaced by an integration. In many cases, however, the relation between the geometry of the antenna and the co-ordinates is such that the f_s are much easier to compute than the a_s and this may more than overbalance the labor of a numerical integration. Examples of this are broadside and end-fire arrays, special cases of which are computed presently.

Again, in cylindrical co-ordinates, for actual evaluation of the f_s we need explicit expressions for the A_s. These are

[11] Since the completion of the present paper one of the authors has succeeded in getting a simple approximation to equation (17) which is valid at large distances. Thus we are able to find the directional pattern. In fact if r, θ, ϕ are spherical polar co-ordinates with the polar axis coincident with the present z axis then at large distances we find

$$E = -\frac{k^3}{2c}\frac{e^{ikr}}{kr}\sum e^{in\phi}\{k_\theta i^{-(n+1)}[f_{3,n}(\theta')+\alpha_3(\theta')f_{3,n}(\pi-\theta')]+k_\phi i^{-n}[f_{2,n}(\theta')+\alpha_2(\theta')f_{2,n}(\pi-\theta')]\}.$$

It is hoped that the method of derivation and the details of the behavior at the surface of the ground will be published in *Physics*.

$$\left.\begin{aligned} A_{2,n} &= e^{i(kz\cos\theta'+n\phi)}\left[k_\rho \frac{in}{\rho} J_n(k\rho \sin\theta') - k_\phi \frac{d}{d\rho} J_n(k\rho\sin\theta')\right] \\ A_{3,n} &= e^{i(kz\cos\theta'+n\phi)}\left[k_\rho i \cos\theta' \frac{d}{d\rho} J_n(k\rho \sin\theta') \right. \\ &\qquad \left. - k_\phi \frac{n\cos\theta'}{\rho} J_n(k\rho\sin\theta') + k_z k \sin^2\theta' J_n(k\rho\sin\theta')\right]. \end{aligned}\right\} \quad (18)$$

In Fig. 3 qualitative drawings of a few of these vector functions are drawn. It is considered that the functions J_n, etc., are too well known

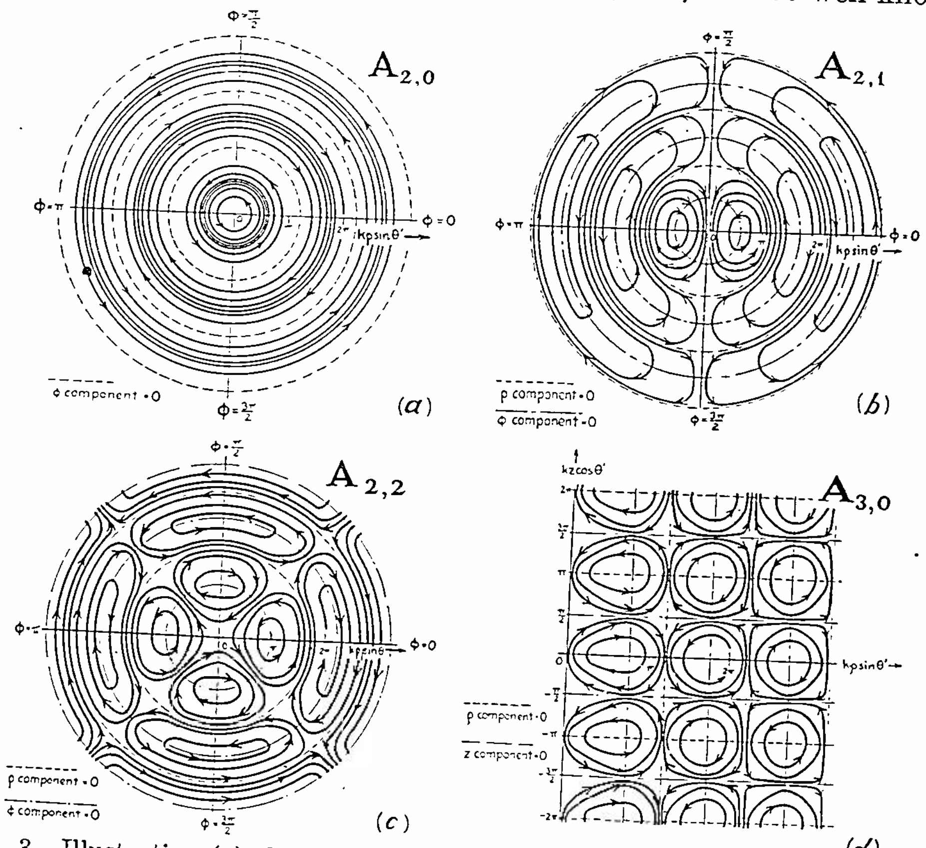


Fig. 3—Illustration (a) shows lines of the function $A_{2,0}$ in a plane z = constant. This is related to the electric field around an infinitely long solenoid excited with alternating current. Figure (b) shows the function $A_{2,1}$ which is related to the field around two long solenoids with currents in opposite directions. Figure (c) is a representation of $A_{2,2}$. The functions $A_{3,n}$ present the same difficulty as the $A_{3,l,n}$: they have in general three components. We have therefore confined ourselves to $A_{3,0}$ which has two components only. A picture of this in a plane ϕ = constant is given in (d). This might be the electric field caused by an infinitely long wire with a current distribution cos $(k \cos\theta')z$.

to be worth plotting and that their power series are easily found. We have therefore omitted a table equivalent to Table I.

B. Examples in Spherical Co-ordinates (Without Ground)

Perhaps the best way to begin working with the present formulas is to consider the use of the qualitative pictures of Fig. 1 in deciding which coefficients a_s vanish. We first remind ourselves of what we are going to do by repeating in words the instructions of formula (3) for finding the a_s. We are directed to multiply the value of the current at each point in the antenna by the value at that point of a particular vector function A_s and to add the results (integrate) over the antenna. Moreover we must notice that both the current and the functions A_s are vectors and the notation $i \cdot A_s$ means that we are to multiply the magnitudes together and then multiply by the cosine of the angle between i and A_s; or, to put it another way, we take the component of i in the direction of A_s and multiply by $|A_s|$ (or vice versa). Thus if the current is everywhere at right angles to a particular A_s the associated a_s will be zero. For example, if we have a very short antenna of length $\Delta z << \lambda$ (a dipole) with the center of its length at the origin and pointed in the direction $\theta = 0$, we see by looking at Fig. 2 that the coefficient $a_{3,1,0}$ will be finite. On the other hand $a_{3,1,1}$, $a_{3,1,-1}$ will be zero because the associated A_s (not drawn in Fig. 2) are at right angles to $A_{3,1,0}$ at the origin. All the other A_s are zero at $r=0$ (see Fig. 1) so that the accompanying a_s vanish.

It is common for large groups of a_s to be zero because of symmetry. Thus if we have a straight antenna pointed along the polar axis and with a current distribution symmetrical with respect to the origin, the coefficient $a_{3,1,0}$ will be important. On the other hand $a_{3,2,0}$ will vanish, because, as one finds from Table I, if the r component of $A_{3,2,0}$ is positive when $\theta = 0$ then it will also be positive when $\theta = \pi$, so that i and $A_{3,2,0}$ will be in the same direction above the origin and in opposite directions below the origin. Integrating over the whole antenna will give zero.

We may now work out the case of a dipole more quantitatively. Looking at Table I we find that at $r=0$ the r component of $A_{3,1,0}$ is $-k(1/6\pi)^{1/2} \cos\theta$ so that multiplying by the current which is a constant[12] i_0 and integrating over the length of the dipole Δz we find $a_{3,1,0} = -i_0 \Delta z k(1/6\pi)^{1/2}$. As explained above, all other a_s vanish. The summation of (5) therefore reduces to a single term, and we find for the total energy radiated[13] per second: $1/2c|a_{3,1,0}|^2 = (i_0 \Delta z k)^2/12\pi c$.

It is plain that placing the dipole at the origin, as above, makes

[12] In this case as well as in all following cases we omit the time dependent factor $e^{-i\omega t}$ which does not enter into any of the integrations or other operations.

[13] Formulas (5) and (15) give the power radiated in ergs per second when the current is in Heaviside-Lorentz units. To obtain the radiation resistance in practical ohms, let i_0 be unity and multiply the resulting answer by $(240\pi c)$.

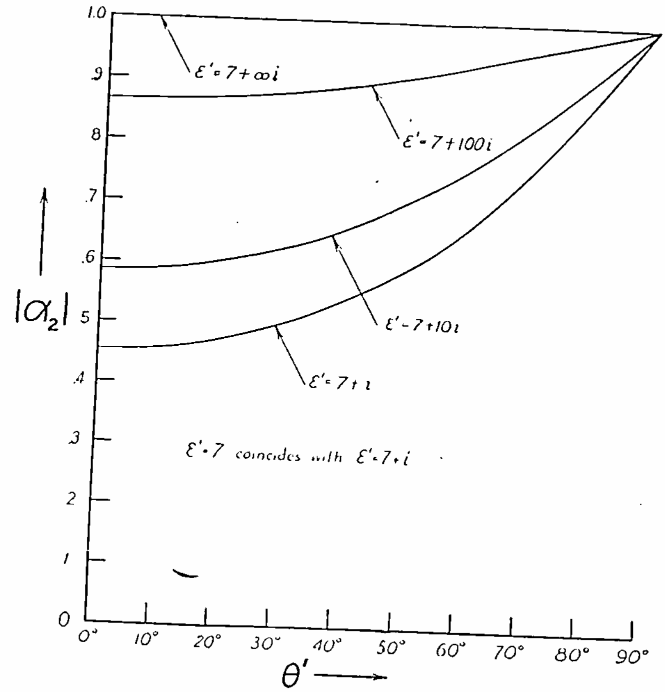


Fig. 4—Absolute value of the complex reflection coefficient α_2 plotted against θ' for various earth conditions as specified by ϵ'.

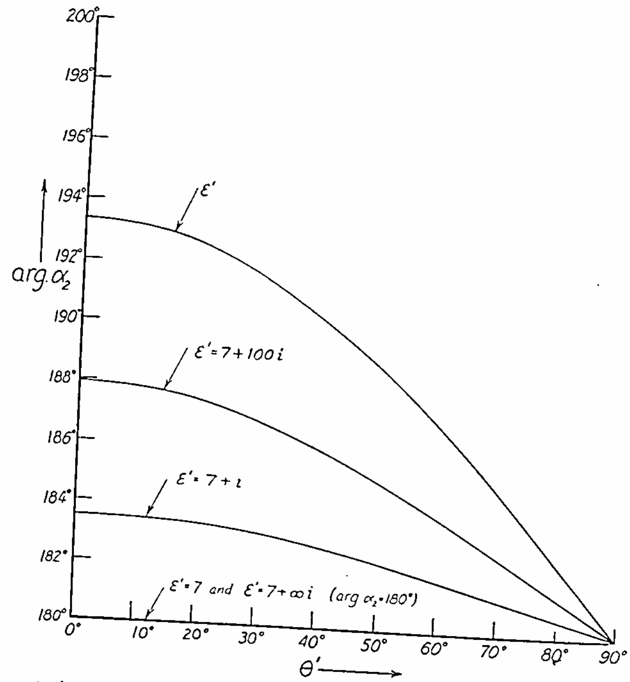


Fig. 5—Argument (phase angle) of the complex coefficient α_2 plotted against θ' for various earth conditions as specified by ϵ'.

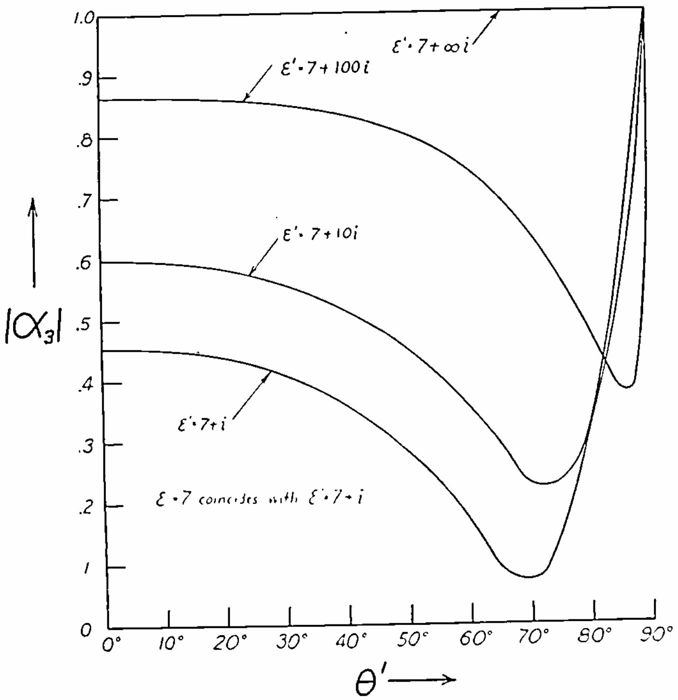


Fig. 6—Absolute value of the complex reflection coefficient α_3 plotted against θ' for various earth conditions as specified by ϵ'.

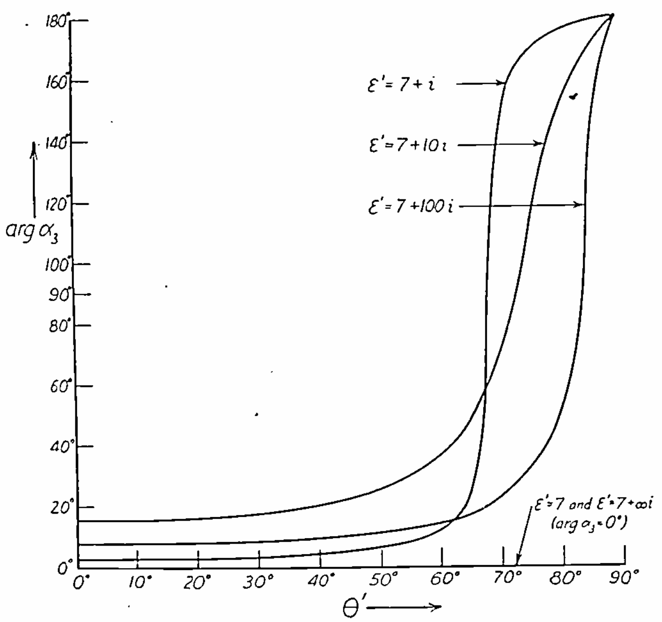


Fig. 7—Argument (phase angle) of the complex coefficient α_3 plotted against θ' for various earth conditions as specified by ϵ'.

$\epsilon'=7+i$, $\epsilon'=7+10i$, $\epsilon'=7+100i$, and $\epsilon'=7+\infty i$, ϵ' being defined by (13). These correspond to $\mu=1$, $\epsilon=7$, $\sigma=10^8$ sec^{-1} with $\lambda=0$, 18.8, 188, 1880, and ∞ meters respectively. The α_2 that appear later and the α_3 that are used here[14] are plotted in Figs. 4, 5, 6, and 7. Once the α are known a single numerical integration gives the power radiated. Another integration using (16) will determine the power which is dissipated in the ground. The reflection coefficients α do not vary rapidly so that the work of numerical integration is quite simple. The results are given in Table III. The f_s are fortunately not rapidly oscillating functions, so that, even in the case of an end-fire array consisting of nine half-wave antennas spaced one-quarter wave length, the function $f_{2,1}$, for example, has the comparatively simple form shown in the graph of Fig. 8. (The factor $i\pi k^2 \sin\theta'$ was introduced for convenience of plotting.)

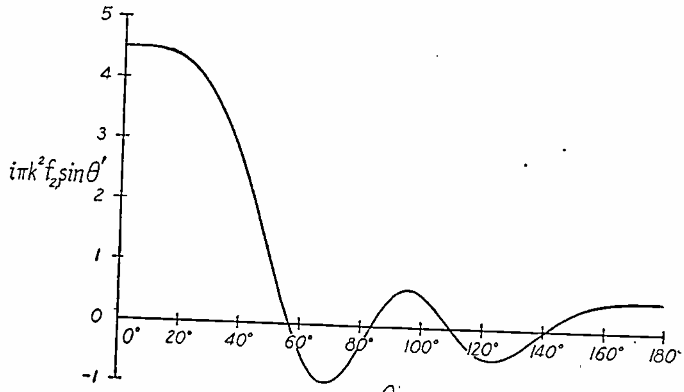


Fig. 8—The function $i\pi k^2 \sin\theta' f_{2,1}(\theta')$ plotted against θ'.

We next consider a vertical quarter-wave antenna with the lower end grounded. The calculation is exactly the same as for the dipole treated above, except that $A_{3,0}$ is no longer constant over the antenna so that we must perform an integration to find $f_{3,0}$. We multiply the current strength $i_0 \cos kz$ by the value of the z component of $\overline{A}_{3,0}$ on the axis ($\rho=0$, $\phi=0$), i.e., $k \sin^2\theta' e^{-ikz\cos\theta'}$ and integrate over the length of the antenna. This integral is elementary, and the value of $f_{3,0}$ is at once expressible in terms of sines and cosines. The resulting values of $f_{3,0}$ are multiplied by α, etc., as directed by (15) and integrated numerically to find the energy radiated. The results are given in Table III. We may

[14] The method used for calculating the α is explained in Appendix I. For values of the constants ϵ and σ, see the following articles: W. L. Barrow, Proc. I.R.E., vol. 23, p. 153; March, (1935); J. Zenneck, *Ann. der Phys.*, vol. 23, p. 846; September, (1907); J. A. Fleming, "Principles of Wireless Telegraphy," Third Edition, p. 800; and C. B. Feldman, Proc. I.R.E., vol. 21, pp. 764–802; June, (1933). The values of ϵ and σ chosen in this paper hardly represent all those which will be found in practice, but they do represent the greatest deviation from the characteristics of perfect reflection which can normally be found.

note that when the wave length is great the earth acts as a perfect reflector. For $\lambda=\infty$ or $\epsilon'=7+\infty i$ then the radiation resistance should be just half that of a half-wave antenna in free space as found above.

TABLE III

Type of Antenna	Ground Characteristics as Specified by Value of ϵ'	Formula Used	Remarks	Power Radiated in Ergs/Sec with Peak Current at Loop of One H.L.U. Unit of Current	Power Lost in Ground	Radiation Resistance Measured in Ohms at Current Loop
$\lambda/2$	1	(5)	Only two terms of series needed	$0.304/\pi c$		73
Infinitesimal dipole of length $\Delta z \ll \lambda$ placed just above ground with axis vertical	1 7 $7+i$ $7+10i$ $7+100i$ $7+\infty i$	(15) and (16)	Only one f needed here and its value is read directly from (11) and (18) without integration	$0.083(k\Delta z)^2/\pi c$ $0.076(k\Delta z)^2/\pi c$ $0.076(k\Delta z)^2/\pi c$ $0.085(k\Delta z)^2/\pi c$ $0.123(k\Delta z)^2/\pi c$ $0.166(k\Delta z)^2/\pi c$	$0.041(k\Delta z)^2/\pi c$ $0.035(k\Delta z)^2/\pi c$ $0.035(k\Delta z)^2/\pi c$ $0.033(k\Delta z)^2/\pi c$ $0.022(k\Delta z)^2/\pi c$ 0	$19.9(k\Delta z)^2$ $18.3(k\Delta z)^2$ $18.3(k\Delta z)^2$ $20.4(k\Delta z)^2$ $29.5(k\Delta z)^2$ $39.8(k\Delta z)^2$
$\lambda/4$ vertical, bottom end grounded	1 7 $7+i$ $7+10i$ $7+100i$ $7+\infty i$	(15)	One term of series (15) used, all other terms vanishing	$0.081/\pi c$ $0.070/\pi c$ $0.068/\pi c$ $0.070/\pi c$ $0.099/\pi c$ $0.152/\pi c$		19.4 16.8 16.3 16.8 23.8 36.5
$\lambda/2$ vertical, without ground	1	(15)	Work practically identical with that for $\lambda/4$	$0.302/\pi c$		72.5
$\lambda/2$ horizontal, $\lambda/2$ above ground	1 7 $7+i$ $7+10i$ $7+100i$ $7+\infty i$	(15)	Two terms of series (15) used	$0.303/\pi c$ $0.290/\pi c$ $0.288/\pi c$ $0.278/\pi c$ $0.279/\pi c$ $0.293/\pi c$		72.7 69.7 69.2 66.7 67.0 70.3
Broadside array of five $\lambda/2$, spacing of $\lambda/2$, excited in phase	1	(15)	The z axis is oriented to pass through centers of $\lambda/2$ elements. Two terms of series (15) used	$1.17/\pi c$		281
Same array with elements excited out of phase to act as an end-fire antenna	1	(15)	Work involves only slight changes from that in next above	$2.07/\pi c$		497
Broadside of nine $\lambda/2$, spaced $\lambda/4$, excited in phase	1	(15)	Same as above	$4.02/\pi c$		961
Same array excited as end-fire antenna	1	(15)	Same as above	$3.64/\pi c$		875

The slightly imperfect agreement is an indication of errors in numerical integration where no effort has been made to keep the accuracy better than about one per cent.

As a further check of this type we have calculated the radiation resistance of a vertical half-wave antenna without ground using the

Now the maximum value of p that is used is 1 and $|\epsilon'|$ is always greater than about 7 if there is any ground at all.[14] We can therefore expand $\sqrt{\epsilon'-p^2}$ and neglect all except the first two terms without making any appreciable error. Thus we get

$$\alpha_2 \cong \frac{\sqrt{1-p^2} - \sqrt{\epsilon'} + (p^2/2\sqrt{\epsilon'})}{\sqrt{1-p^2} + \sqrt{\epsilon'} - (p^2/2\sqrt{\epsilon'})}. \tag{20}$$

From this point on we use graphical methods as illustrated in Fig. 9 to get the numerator and denominator of α_2 which are then divided by

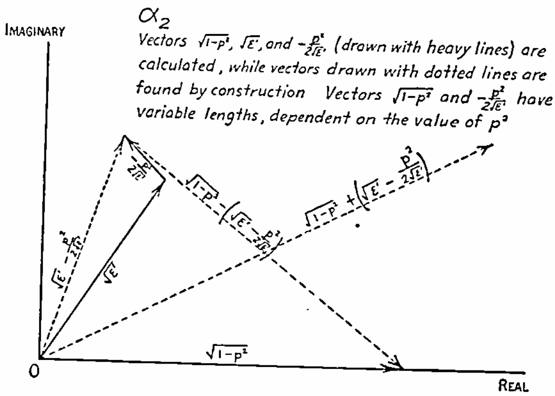


Fig. 9—Graphical computation of α_2.

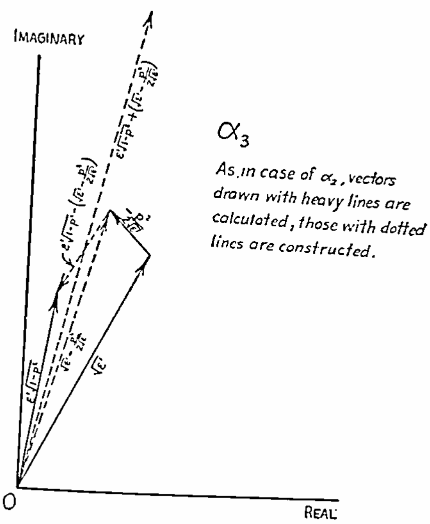


Fig. 10—Graphical computation of α_3.

dividing moduli and substracting arguments in the normal manner.

The same methods are used for α_3 which, with the same notation, can be written

$$\alpha_3 = \frac{\epsilon'\sqrt{1-p^2} - \sqrt{\epsilon'-p^2}}{\epsilon'\sqrt{1-p^2} + \sqrt{\epsilon'+p^2}} \tag{21}$$

and, on expanding and neglecting terms,

$$\alpha_3 \cong \frac{\epsilon'\sqrt{1-p^2} - \sqrt{\epsilon'} + (p^2/2\sqrt{\epsilon'})}{\epsilon'\sqrt{1-p^2} + \sqrt{\epsilon'} - (p^2/2\sqrt{\epsilon'})}. \qquad (22)$$

Computation is then done graphically as illustrated in Fig. 10.

Appendix II

The integral

$$\int_0^{n\pi} \frac{\sin x}{x^{3/2}} J_{l+1/2}(x)dx$$

which occurs in the computation of the a_s for tilted wire and other antennas may be evaluated simply as follows:

$$\int_0^{n\pi} \frac{1}{x}\left(\frac{\sin x}{x^{1/2}}\right) J_{l+1/2}(x)dx = \sqrt{\frac{\pi}{2}} \int_0^{n\pi} \frac{1}{x} J_{1/2}(x) \cdot J_{l+1/2}(x)dx$$

$$= (-1)^n \frac{n\sqrt{\pi}}{l(l+1)} J_{l+1/2}(n\pi).$$

The integration is by means of formula 5, p. 166, Jahnke and Emde "Funktionentafeln," (1928). The value of $J_{l+1/2}(n\pi)$ may be looked up in tables or computed quickly from the asymptotic series, which in this case is exact. This integral with the sine replaced by a cosine is handled by the same methods.

BOOK REVIEWS

Principles of Radio Engineering, by R. S. Glasgow, McGraw-Hill Book Co., New York and London. 1936. 520 pp. 344 illustrations. Price $4.00.

This book should be a very acceptable textbook for an undergraduate course in the radio engineering field. While fundamentals are emphasized, a good balance is maintained with applications and matters of practical convenience. Following the first two chapters which review alternating-current theory and the application of complex quantities, and then treat series and parallel resonant circuits, there is a well-placed chapter on the properties of coils and condensers. Chapters on coupled and oscillatory circuits complete the first quarter of the book. The next two quarters deal with vacuum tubes, the treatment of audio-frequency amplifiers and modulation being somewhat emphasized. The book closes with a relatively long chapter on antennas and wave propagation. (90 pages—There are 14 chapters in the book.)

The author points out that the treatment of iron-core inductances, coupled circuits, graphical methods of determining amplifier performance, push-pull circuits, antennas, and radio-frequency transmission lines receive more attention than is sometimes the case in books of this type. A large assortment of problems for solution is given at the end of each chapter. The make-up of the book and the arrangement of the many figures supplement the author's general plan and very clear treatment to make a promising textbook.

The reviewer regrets that he is not able to find clearly recognizable in the text the new graphical methods for determining the distortion in amplifiers which the author mentions in the preface. Footnote references are in general well selected, many of the references being to papers of historical importance, and a fair proportion to publications of the past five years. These references, however, do not seem to emphasize the recent developments quite to the extent which the preface suggests.

It is not inexcusable to limit very seriously the discussion of loud-speakers as the author has done, yet in view of the extended treatment given to audio amplifiers a better balance might have been attained. The treatment of the piezoelectric crystal is limited to the quartz-crystal oscillator, and consists mainly of a few practical notes. A typographical error in one of the footnotes to this section unfortunately credits Morrison instead of Marrison with the work on the precision standard of frequency. The reviewer has, however, been unable to find other errors of this sort, and the book seems to be quite free from misprint.

The most serious defect of the book in the eyes of the reviewer is the almost complete neglect of short-wave work. In view of present tendencies in the art and current development programs of the large commercial organizations it would seem that the young graduate looking for a job would fare better in his search if his intuitive approach to radio engineering problems were colored by an early familiarity with the behavior of ultra-high frequencies.

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Review of Principles of Electric and Magnetic Measurements, by P. Vigoureux and C. E. Webb. Published by Prentice-Hall, Inc., 70 Fifth Ave., New York City. 386 pages + 6 pp. index. Price $5.00.

This is a contribution which carries the prestige of the British National Physical Laboratory. In the treatment of the various types of Wheatstone bridges and related methods, the authors have succeeded in bringing out important fundamental principles of value to students and practicing engineers. The emphasis is on direct- and low-frequency alternating-current measurements. Only about thirty pages are devoted to the important subject of high-frequency measurements which is a disappointment to radio engineers. About the same space is given to a general discussion of thermionic tubes and associated circuits. The last third of the book is on magnetism and various magnetic measurements.

In the expression at the bottom of page 8 the coefficient of M should be 2 according to the usual definition of mutual inductance. Fig. 2, page 31, is upside down.

†H. M. Turner

Radio Operating Questions and Answers, by A. R. Nilson and J. L. Hornung. Published by McGraw-Hill Book Company, Price $2.50. 427 pages.

This text is intended for students preparing for U. S. Government radio operator's license examinations. The scope of the material covers the requirements of all classes of licenses from the amateur to the highest radiotelephone and radiotelegraph grade.

An understanding of radio operating and theory on the part of the reader is assumed. As the title implies, the text is in the form of questions and answers and is intended to indicate the general methods to be followed when taking an examination.

The characteristics, circuit details, operation, and appearance of commercial telegraph transmitters of the spark, arc, and tube types are covered. Broadcast, aeronautical, police radio and amateur apparatus and operation are likewise discussed in detail. Photographs of the equipment, together with sketches and diagrams, assist the student in his understanding of the subject.

The material covered by this edition (the sixth) of this book includes all that formerly presented and, in addition, further information pertaining to transmitting and receiving equipment, general radio theory, and radio laws. The new material is, for the most part, presented in the form of an addendum and is not placed in proper relation to the subject matter of former issues. This results in some confusion in the use of the book.

Although this is a newly revised book, modern equipment and practices are not covered adequately in several respects. In some instances, there is a question as to the scientific accuracy of the material as it has been presented. Nevertheless, this book is probably one of the best of its kind available and it fills a definite need in the education field.

*H. A. Chinn

Physik und Technik der Ultrakurzen Wellen: Erster Band, Erzeugung Ultrakurzwelliger Schwingungen (Physics and Engineering Development of Ultra Short Waves: Vol. 1, Generation of Ultra-Short-Wave Oscillations),

† Yale University, New Haven, Connecticut.
* Columbia Broadcasting System, New York City.

by H. E. Hollmann. Julius Springer, Berlin, Germany. 315 pages, 381 illustrations. Price—RM 36. In German.

When a new branch of science or engineering is started, a period of a number of years ensues during which those who are interested in learning something about it must go to original references in numerous periodicals in order to obtain the knowledge they desire. Under such conditions a text which attempts to collect and co-ordinate this information is most welcome.

This book by Hollmann is the first of a set of two volumes which is intended to cover the short-wave field below ten meters with emphasis on wave lengths below one meter. Since a large proportion of this work is of very recent origin it is not possible at present to select the results which will endure from developments of lesser importance, so that the reader will find the inclusion of descriptions of numerous types of apparatus satisfying. A bibliography with 295 references is included at the end of the book for those who are interested in original articles.

On the theoretical side, particularly as regards the more rigorous mathematical development, the book is much less complete than it is for descriptive material. The theoretical work of Benham, Llewellyn, Müller and others is gone over with extreme brevity.

Chapter headings are as follows: The generation of quasi-optical waves by means of sparks; The generation of ultra-short waves using feedback; The "stopping" field method; The magnetron; The generation of ultra-short waves by electron beams. In the introduction a short review is given of the use of infra red and heat rays for communication. In the second volume, of which the reviewer has seen only a proposed table of contents, the author intends to treat such subjects as reception and detection, radiation and transmission, applications, and measurement methods for ultra-short waves.

These books by Hollmann are recommended for those who are interested in learning something about laboratory or commercial developments in generating frequencies above thirty megacycles or as reference texts in this field.

†Irving Wolff

† RCA Manufacturing Company, Inc., RCA Victor Division, Camden, New Jersey.

Proceedings of the Institute of Radio Engineers
Volume 24, Number 12 *December, 1936*

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page are not available from the Institute but may be obtained without charge by addressing the publishers.

"Micarta in the Radio Industry" and "Where Can You Use Micarta" are the names of two booklets issued recently by the Westinghouse Electric and Manufacturing Company of East Pittsburgh, Pa.

Allen B. Du Mont Laboratories, Inc., Upper Montclair, N. J., has issued a leaflet describing new developments in cathode-ray tubes, oscillographs, and accessory apparatus.

General Manufacturing Company of 8066 S. Chicago Ave., Chicago, Ill., has issued several leaflets giving circuit and design data on some multiwave receivers using their products.

"A Practical Radio and Communication Engineering Course for Home Study" is the title of a booklet issued by the Smith Practical Radio Institute of 7502 Renwood Dr., Parma, Ohio.

The Magnavox Company of Fort Wayne, Ind., has issued a leaflet giving standard specifications of their loud-speakers.

The "Bullet" electrodynamic microphone is described in a leaflet issued by the Transducer Corporation, 30 Rockefeller Plaza, New York, N. Y.

Catalog No. 500, describing "True-Fidelity" transformers, has been issued by the Thordarson Electric Manufacturing Company, 500 W. Huron St., Chicago, Ill.

The AVA-8 crystal attachment unit for controlling the frequency of receivers is described in a leaflet issued by the Aviation Radio Section of the RCA Manufacturing Company, Inc., Camden, N. J.

Hygrade Sylvania Corporation of Emporium, Pa., has issued Engineering News Letters No. 31 on the "Modulation Capabilities of Infinite Impedance Detectors" and No. 32 on "Some Design Precautions for Audio Power Output Stages."

CONTRIBUTORS TO THIS ISSUE

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Kalantaroff, P.: Born, 1892. Graduated from electromechanical faculty, Leningrad Industrial Institute, and attached to department of theoretical electrotechnics, 1920; department of theory of alternating currents, 1930 to date. Received degree of doctor of technical science, 1935. Nonmember, Institute of Radio Engineers.

Landon, V. D.: See Proceedings for November, 1936.

Miller, John H.: Born June 6, 1893, at Oak Park, Illinois. Received B. S. degree in electrical engineering, University of Illinois, 1915. Westinghouse Electric and Manufacturing Company, 1915–1917; radio development section, Signal Corps, U. S. Army, 1915–1917; vice president in charge of engineering, Jewell Electrical Instrument Company, 1919–1931; assistant chief engineer, Weston Electrical Instrument Corporation, 1931 to date. Member, American Institute of Electrical Engineers and Radio Club of America. Associate member, Institute of Radio Engineers, 1919; Member, 1925.

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PROCEEDINGS

OF

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VOLUME 24

1936

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Number 3; March, 1936

Part I

Part II

Technical Papers

Number 4; April, 1936

Part I

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Number 5; May, 1936

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NUMBER 6; JUNE, 1936

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NUMBER 7; JULY, 1936

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Technical Papers

NUMBER 8; AUGUST, 1936

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Number 9; September, 1936

Number 10; October, 1936

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Number 12; December, 1936

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AUTHOR INDEX

Numbers refer to the chronological list. Bold-face type indicates papers, light-face type indicates discussion, and italics refer to books and book reviews.

SUBJECT INDEX

INCORRECT ADDRESSES

Listed below are the names and the last-known addresses of one hundred and twenty-eight members of the Institute whose correct addresses are unknown. It will be appreciated if anyone having information concerning the present addresses of any of the persons listed will communicate with the Secretary of the Institute.

Allston, William F.	c/o Tropical Radio Telephone Company, Box 488, Hialeah, Fla.
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Blessing, G. W.	Gregory W. Blessing and Company, 106 Jarvis St., Toronto, Ont., Canada
Booth, Albert E.	11 Buckingham Mount, Headingley, Leeds 6, Yorkshire, England.
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Brewster, Grant	c/o T. A. Long, Box 50–51 Cristobal, Canal Zone.
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Coblentz, Orhan R.	1420-39th Ave., North, Seattle, Wash.
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Cooke, Charles G.	2953 Colerain Ave., Cincinnati, Ohio.
Cumming, L. Gordon	Shore Acres, Cape Elizabeth, Maine.
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Davis, Harold	536½ N. San Vincente Blvd., West Hollywood, Calif.
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Isserstedt, Siegfried G.	1590 Bathurst St., Toronto, Ont., Canada.
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Jackson, Wilbur M.	Box 125, Faculty Exchange, College Station, Tex.
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Jefferson, Sidney	52 Cambridge St., London, W.2, England.
Jensen, Jens O.	534 Anastasia Ave., Coral Gables, Fla.
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Likel, Harry C.	207 Park Pl., Brooklyn, N. Y.
Loukota, Douglas H.	1967½ N. Wilcox Ave., Los Angeles, Calif.
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Scofield, Robert W.	40-22 Lawrence St., Flushing, L. I., N. Y.
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Toth, E.	233 Littleton Ave., Newark, N. J.
Treitel, Leo	1 Seaman Ave., New York, N. Y.
Tumleson, James E.	4 South Portland Ave., Brooklyn, N. Y.
Valentine, Russell D.	6848 Ingram St., Forest Hills, N. Y.
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Weagant, Roy A.	101 Forest Rd., Douglas Manor, L. I., N. Y.
Weeden, William N.	1 Alden Pl., Bronxville, N. Y.
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Wiebach, William T.	207 Avenue C., Rochester, N. Y.
Williams, Aaron F.	RCA Victor Co., Inc., Bldg. 6, Camden, N. J.

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The Institute of Radio Engineers

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To the Board of Directors

Gentlemen:

I hereby make application for Associate membership in the Institute of Radio Engineers on the basis of my training and professional experience given herewith, and refer to the members named below who are personally familiar with my work.

I certify that the statements made in the record of my training and professional experience are correct, and agree if elected, that I will be governed by the constitution of the Institute as long as I continue a member. Furthermore I agree to promote the objects of the Institute so far as shall be in my power, and if my membership shall be discontinued will return my membership badge.

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Sec. 2: * * * Applicants shall give references to members of the Institute as follows: * * * for the grade of Associate, to three Fellows, Members, or Associates; * * * Each application for admission * * * shall embody a full record of the general technical education of the applicant and of his professional career.

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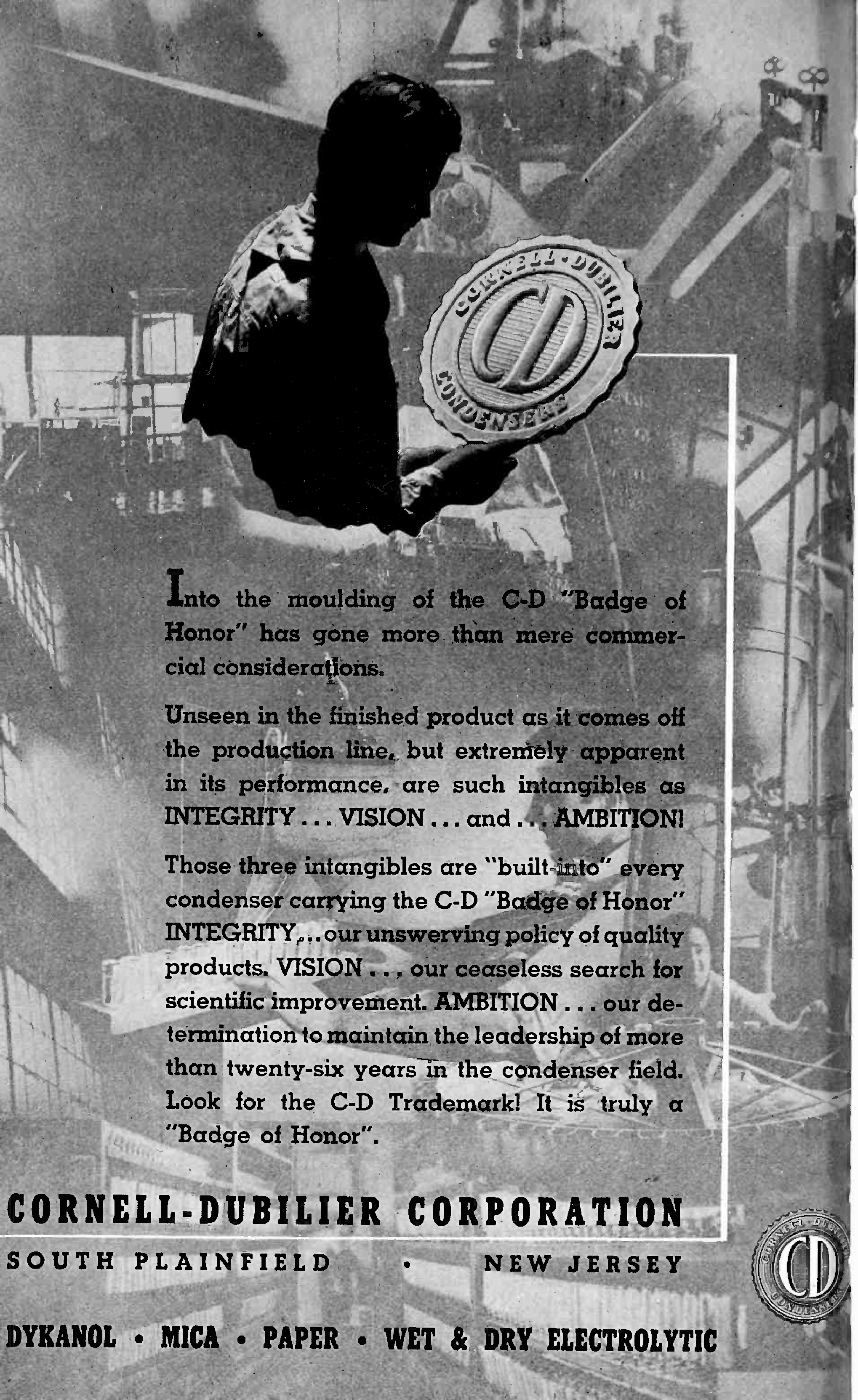
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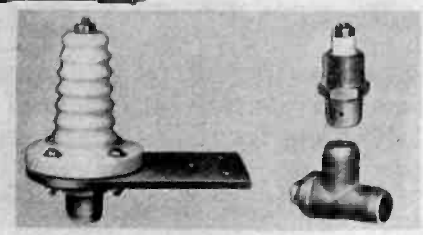

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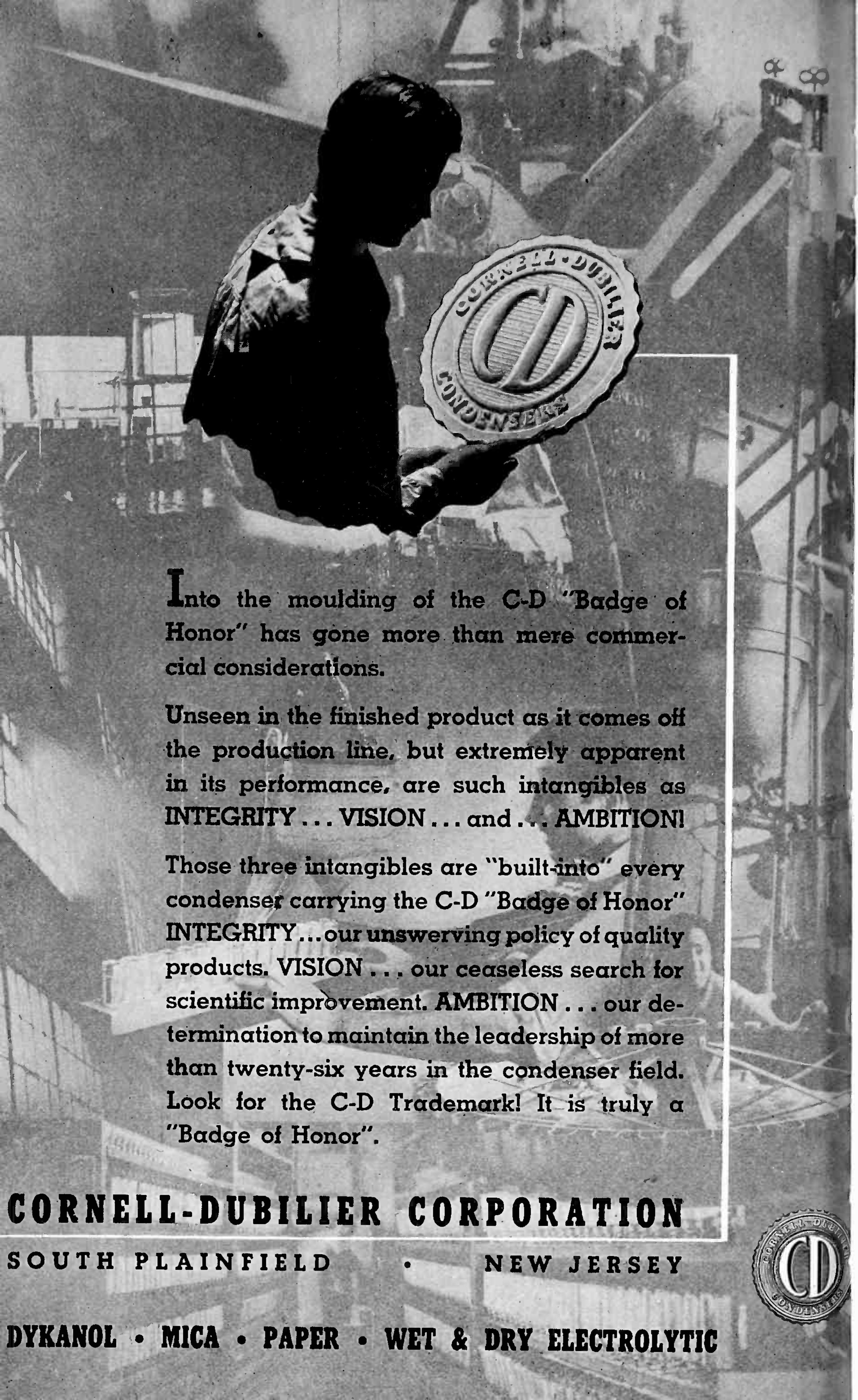
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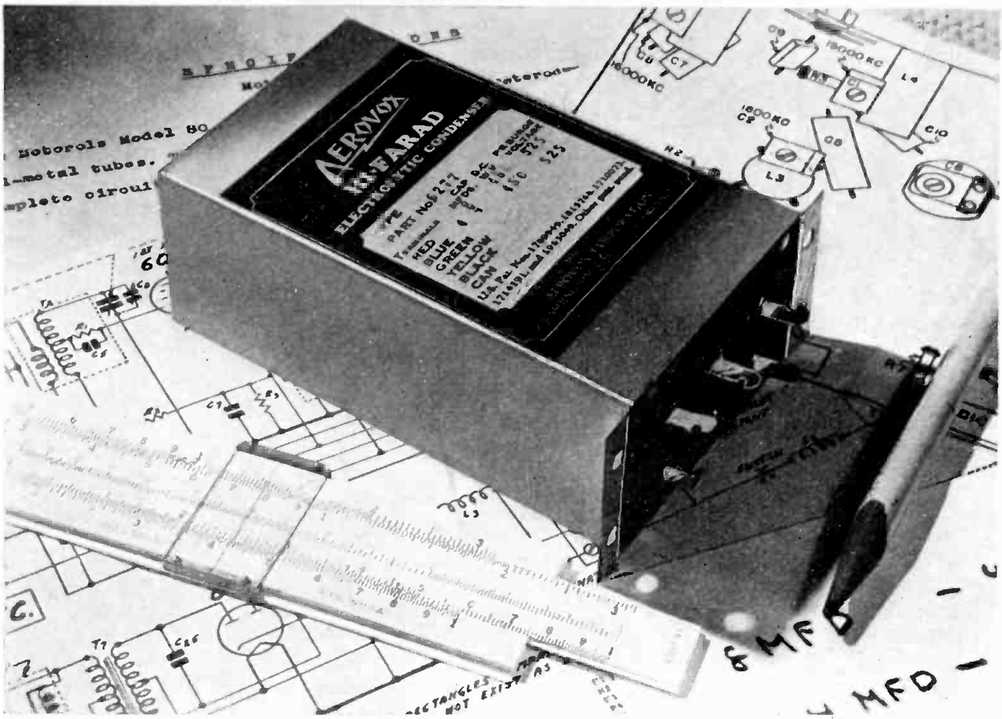

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VOLUME 23 (1935) of the Proceedings is now available in bound form to members of the Institute. It may be obtained in Buckram or Morocco leather for $9.50 and $12.00 respectively. Foreign postage is $1.00 additional per volume.

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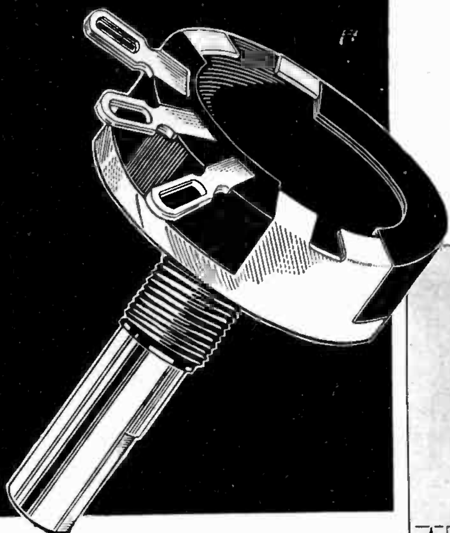

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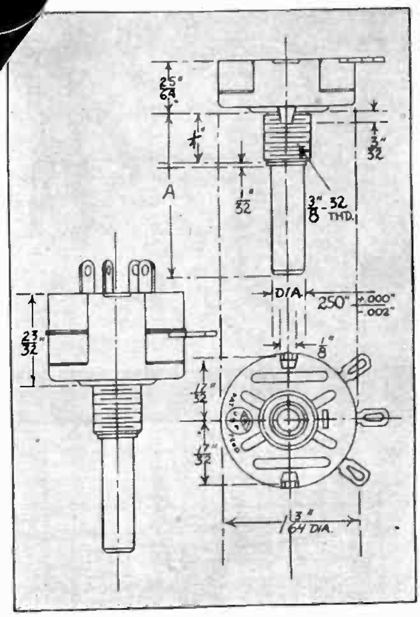

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EMPLOYMENT PAGE

RADIO ENGINEERS

In the rapidly growing field of radio transmission and reception, a large engineering and manufacturing company in England is augmenting its staff. The following positions require to be filled :—

1. RADIO TRANSMITTING EXPERT, familiar with present-day practice and design. The applicant must have laboratory experience together with proved ability to give practical manufacturing expression to his work.

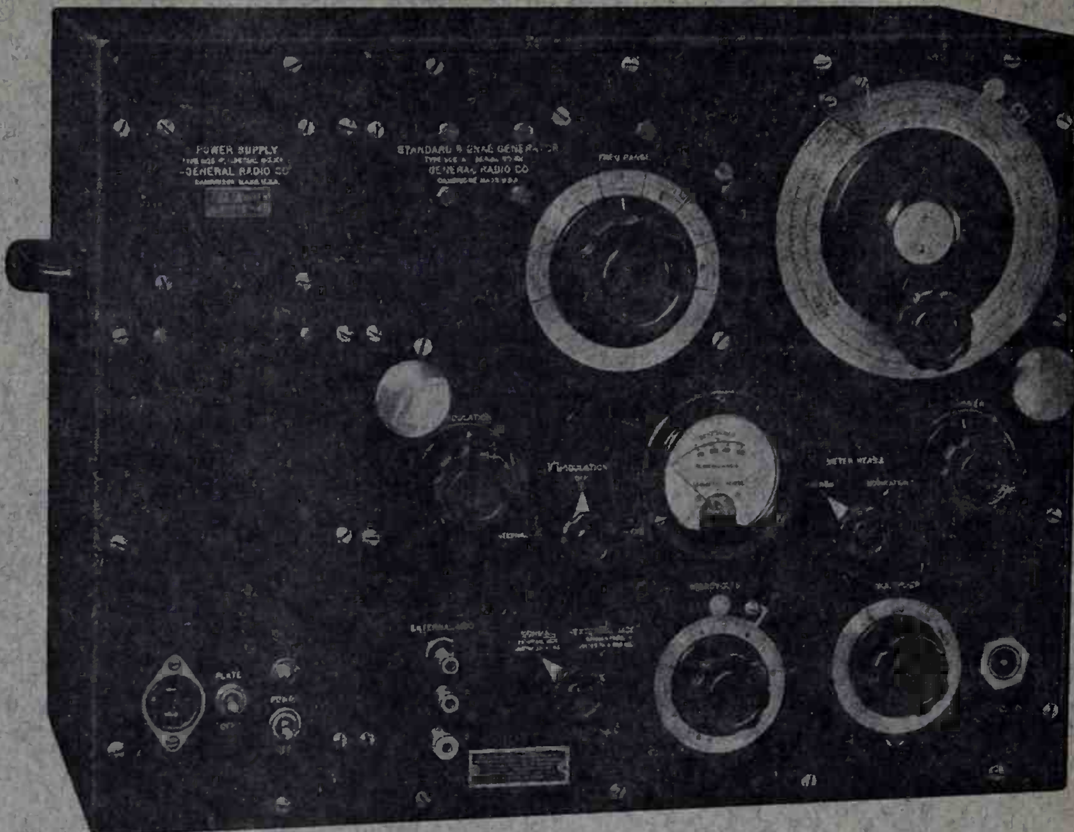
2. RADIO RECEIVER ENGINEERS, with up-to-date knowledge of radio application. First-class theoretical and practical experience. A knowledge of modern ultra-short-wave practice and cathode-ray technique would be advantageous. Write stating age, past experience in detail, and salary required. Box No. 164, care of The Institute of Radio Engineers.

TUBE MANUFACTURER WANTS ENGINEER thoroughly experienced in design, development and manufacturing of radio receiving and transmitting tubes. Good knowledge of present type tubes and actual factory experience essential. Desirable opportunity for the right man. Location, foreign country. Reply Box No. 168, care of The Institute of Radio Engineers stating experience and salary expected.

LARGE RADIO MANUFACTURER in Great Lakes district has good opening for capable radio engineer in receiver development. Man with experience in design of battery operated receivers preferred, Box No. 169, care of The Institute of Radio Engineers.

WANTED: Advertising solicitor for radio publication. Man must have sales ability, write good letter, and have general technical knowledge of radio. Give experience record and salary expected. Box No. 167, care of The Institute of Radio Engineers.

WANTED—Development Engineer on electroacoustic and electromechanical devices by established manufacturer. Should have good communications engineering background and experience in applied acoustics. Write fully including experience and minimu acceptab e starting salary. Box No. 172, care of the Institute of Radio Engineers.



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