VOLUME 24 JANUARY, 1936 NUMBER 1

J W Ashmore

PROCEEDINGS

of

The Institute of Radio Engineers

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Institute of Radio Engineers
Forthcoming Meetings

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January 23, 1936

DETROIT SECTION
January 17, 1936

EMPORIUM SECTION
January 15, 1936

LOS ANGELES SECTION
January 21, 1936

NEW YORK MEETING
January 8, 1936
February 5, 1936

PHILADELPHIA SECTION
January 2, 1936
February 6, 1936

WASHINGTON SECTION
January 13, 1936

PROCEEDINGS OF

The Institute of Radio Engineers

VOLUME 24 | January, 1936 | NUMBER 1

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The Institute of Radio Engineers

GENERAL INFORMATION

INSTITUTE. The Institute of Radio Engineers was formed in 1912 through the amalgamation of the Society of Wireless Telegraph Engineers and the Wireless Institute. Its headquarters were established in New York City and the membership has grown from less than fifty members at the start to several thousand.

AIMS AND OBJECTS. The Institute functions solely to advance the theory and practice of radio and allied branches of engineering and of the related arts and sciences, their application to human needs, and the maintenance of a high professional standing among its members. Among the methods of accomplishing this is the publication of papers, discussions, and communications of interest to the membership.

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Proceedings of the Institute of Radio Engineers
Volume 24, Number 1 *January, 1936*

APPLICATIONS FOR MEMBERSHIP

Applications for transfer or election to the various grades of membership have been received from the persons listed below, and have been approved by the Admissions Committee. Members objecting to transfer or election of any of these applicants should communicate with the Secretary on or before January 31, 1936. These applications will be considered by the Board of Directors at its meeting on February 5, 1936.

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California	Oalkand, 646-63rd St.	Combs, E. E., Jr.
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Indiana	Indianapolis, 6433 Shelley Ave.	Slater, I. M.
Kansas	Kansas City, 901 N. 8th St.	Hollister, H.
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New Jersey	Harrison, RCA Radiotron Division, RCA Manufacturing Co.	Perkins, T. B.
	Merchantville, 606 W. Maple Ave	Trafton, D. C.
	Orange, 449 Mountainview Ave.	Jacobus, H. R.
New York	Buffalo, 56 "C" St.	Bennett, P.A.
	Greenport, L. I., 218-4th Ave.	Wood, R. H.
	Kenmore, 297 W. Girard Blvd	Smith, G. G.
	New York, 463 West St.	Glass, M. S.
	New York, Rm. 1211, 180 Varick St.	Mason, W. P.
	Niagara Falls, 1634-10th St.	Rogers, K. E.
	North Tonawanda, 1531 Kingston Ave.	Manning, F. W.
	Tonawanda, 70 Elmwood Park, E.	Christensen, A. B.
	Yonkers, 710 Warburton Ave.	Kidd, G. B.
Ohio	Cleveland, National Broadcasting Co., 1367 E. 6th St.	Caskey, H. B.
	Lakewood, 1641 Waterbury Rd.	Weber, L. J.
	Sandusky, 1910 Barker St.	Brindle, T. A.
Oklahoma	Tulsa, 247 E. 27th St.	Floyd, F. M.
Oregon	Oakridge	Wolf, B. A.
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	Laredo, 809 Zaragoza St.	Curtis, P. E.
	Lufkin, 124 Fred Ave.	Saxon, M.
Wisconsin	Milwaukee, 2045 N. 57th St.	Eckert, F. A., Jr.
Burma	Rangoon, Post Box 200	Dunkley, E. J.
Canada	London, Ont., Highland Rd.	McIntosh, J. A.
	Toronto, Ont., 374 Brunswick Ave.	Knap, H. B.
	Vancouver, B. C., 1955 Wiley St.	Tupper, B. R.
England	Brentwood, Essex, "Greywalls," Shenfield Common	Ewen, M. E.
	London S. W. 5, 49 Eardley Crescent	Bharucha, J. H.
	London S. W. 13, 55 Lowther Rd., Barnes	Jones, I. C. A.
France	Ermont, (S. et O.), 5 rue des Faillettes	Grossin, H.
South India	Trivandrum, College of Science	Chacko, M. V.

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Canada	Cochrane, Ont., P.O. 92	Tremblay, E. C.

For Election to the Student Grade

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	Santa Paula, R.F.D. 2, Box 222	Teague, D. S. Jr.
Georgia	Atlanta, Box 44, Georgia School of Technology	Jones, W. L., Jr.
	Atlanta, 191 Poplar Circle, N. E.	Winfree, R. W.
Massachusetts	Cambridge A, M.I.T. Dormitories	Lewis, F. D.
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INSTITUTE NEWS AND RADIO NOTES

December Meeting of the Board of Directors

A meeting of the Board of Directors was held on December 4 in the Institute office and those present were: Stuart Ballantine, president; Alan Hazeltine, president-elect; Melville Eastham, treasurer; E. H. Armstrong, director-elect; Arthur Batcheller, H. H. Beverage, director-elect; Alfred N. Goldsmith, Virgil M. Graham, R. A. Heising, J. V. L. Hogan, F. A. Kolster, George Lewis, E. L. Nelson, Haraden Pratt, H. M. Turner, A. F. Van Dyck, L. E. Whittemore, William Wilson, and H. P. Westman, secretary.

Thirty-six applications for Associate and four for Student grades of membership were approved.

Because the first Wednesday in January is New Year's Day, the January meeting of the Board of Directors and the New York meeting will be held on January 8.

A report on the Rochester Fall Meeting held on November 18, 19, and 20 gave a total registration of 336 of whom 294 were from out of town. Approval was granted for the holding of the 1936 Rochester Fall Meeting on November 16, 17, and 18.

A petition for the establishment of an Emporium Section was approved.

The Emergency Employment Service reported a total registration of 790. During the month of November, eleven placements were made, four of which were considered to be permanent in nature.

Radio Emissions of Standard Frequency

The National Bureau of Standards provides standard frequency emissions from its station WWV at Beltsville, Md. On each Tuesday and Friday the emissions are continuous unmodulated waves and on each Wednesday they are modulated by an audio frequency, generally 1000 cycles. There are no emissions on legal holidays.

On all schedules three radio carrier frequencies are transmitted as follows: noon to 1 P.M., Eastern Standard Time, 15,000 kilocycles; 1:15 to 2:15 P.M., 10,000 kilocycles; and 2:30 to 3:30 P.M., 5000 kilocycles. The accuracy of these frequencies will at all times be better than a part in five million.

During the first five minutes of each transmission announcements are given of the station call letters, the frequency of transmission, and the frequency of modulation, if any. For the CW emissions, the announcements are in telegraphic code and are repeated at ten-minute intervals. For the modulated emissions, the announcements are given by voice only at the beginning of each carrier frequency transmission, the remainder of the hour being an uninterrupted audio frequency. The CW emissions are from a twenty-kilowatt transmitter and the modulated transmissions are from a one-kilowatt set.

Information on how to utilize these signals is given in a pamphlet obtainable on request from the National Bureau of Standards, Washington, D. C. Reports from those using this service will be welcomed by the Bureau. As the modulated emissions are somewhat experimental it is particularly desired that users report their experiences outlining methods of utilization, information on relative fading, intensity, etc., on the three carrier frequencies and preferences as to the audio frequency to be furnished.

Committee Work

Membership Committee

A meeting of the Membership Committee was held on December 4, 1935, in the Institute office and was attended by I. S. Coggeshall, chairman; F. W. Cunningham, H. C. Humphrey, C. R. Rowe, E. W. Schafer, C. E. Scholz, E. D. Cook, R. L. Snyder, and C. B. DeSoto. The latter three appeared in response to an invitation to the sections to send representatives.

The general subject under consideration was methods of increasing membership activity within the sections and of coördinating it with the work of headquarters Membership Committee during 1936. Several helpful suggestions were received from the delegates and by correspondence, and positive action was taken on matters introduced by the Philadelphia and Chicago Sections.

New York Program Committee

A meeting of the New York Program Committee was held in the Institute office on December 6 and those present were: George Lewis, chairman; H. H. Beverage, R. A. Heising, Alan Hazeltine, A. F. Van Dyck, and H. P. Westman, secretary.

A tentative program was drawn up covering the New York meetings for the first three months of 1936.

STANDARDIZATION

IRE Standards Executive Committee

A meeting of the Executive Committee of the Institute's Standards Committee was held in the Institute office on December 12 and was attended by Haraden Pratt, chairman; H. F. Olson, J. C. Schelleng, B. E. Shackelford, and H. A. Wheeler.

Reports on the activities of the various technical committees were considered and it is probable that three of these committees will turn over their finished work to the Standards Committee for consideration early in 1936

Technical Committee on Electronics—IRE

The Technical Committee on Electronics of the Institute met in the Institute office on December 5 and those present were: R. W. Larsen, E. A. Lederer, George Lewis, G. F. Metcalf, O. W. Pike, P. T. Weeks, R. M. Wise, and H. P. Westman, secretary and acting chairman.

The material covering the testing of vacuum tubes, graphical symbols, and letter symbols, prepared by its five subcommittees, were reviewed by the committee and final action taken. This material is now in final shape for submission to the Standards Committee.

Technical Committee on Receivers—IRE

A Subcommittee on Definitions of the Technical Committee on Receivers of the Institute met in the Institute office on November 26 and those present were: H. A. Wheeler, acting chairman; A. V. Loughren, and H. P. Westman, secretary. The definitions considered at the previous meetings of the committee were reviewed and some changes made in them. The remaining definitions in the 1933 report which were of interest to this committee were reviewed.

Technical Committee on Transmitters and Antennas—IRE

A meeting of the Technical Committee on Transmitters and Antennas of the Institute was held on November 25 in the Institute office and those present were: J. C. Schelleng, chairman; Raymond Asserson, E. B. Ferrell, Raymond Guy, D. G. Little, E. G. Ports, and H. P. Westman, secretary.

The committee reviewed the report on antennas submitted by its subcommittee and now has only definitions and symbols to complete in order that its final report may be prepared.

Institute Meetings

Chicago Section

A meeting of the Chicago Section was held on November 15 in the RCA Institute's auditorium. Alfred Crossley, chairman, presided and 120 members and guests were present. Twelve attended the informal dinner which preceded the meeting.

A paper on "The Broadcast Antenna" was presented by V. G. Andrews of Doolittle and Faulkner. In it, Dr. Andrews outlined the reasons for the new developments in antenna systems used for broadcasting and the results to be achieved in the way of fading reduction, improved efficiency, and increased low angle radiation. He described various methods of top loading for obtaining uniform cross section, and covered the design of ground systems. Modes of oscillation of various antennas were covered as well as means to achieve them. The principles of the design involved in each case were given and existing antenna structures cited as examples.

The second paper, by Ernest Kohler, an engineer for the Ken-Rad Corporation, was on "Metal Radio Tubes." It covered a technical description of the tubes and the mechanics and problems of their construction. Specific details of the processes used in their manufacture were given. Numerous comparisons with the glass type tubes with particular reference to improvements or losses due to the structures found necessary in the new types were given. It was emphasized that the losses in operation are not in general of a consequential nature.

A nominating committee to propose a slate of officers to be voted on at the December meeting was appointed.

Cincinnati Section

A. F. Knoblaugh, chairman, presided at the November 12 meeting of the Cincinnati Section held at the University of Cincinnati. Fifty-six members and guests were present.

A paper on "Problems and Requirements of High Fidelity Speaker Systems" was presented by H. S. Knowles, chief engineer of Jensen Radio Manufacturing Company. He prefaced the paper with a brief discussion of some of the fundamental concepts of auditory phenomena and then proceeded to prove that at the present time at least, it is impossible to provide strictly high fidelity reproduction because the polar characteristic of the program source cannot be reproduced over existing single channel transmission systems. The problems involved in designing reproducers free from amplitude distortion were covered. It was shown that a reproducer when excited at a given frequency will develop a subharmonic when the input level exceeds a certain value tending to create the illusion that the fundamental pitch has dropped an octave. He concluded by showing that a loud speaker is actually a pair of coupled resonant circuits and demonstrated this by curves of speakers having two resonant peaks. Equalizer circuits by means of which these peaks could be removed were described. Messrs. Bruning, Knoblaugh, Rockwell and others participated in the discussion.

The Nominating Committee presented its recommendations for officers for 1936 and these will be voted on at the December meeting.

Connecticut Valley Section

On November 6, the Connecticut Valley Section met in the Hartford Electric Light Auditorium, Hartford, Conn. J. A. Hutcheson, chairman, presided and twenty-three were present.

A paper on "Radio Communication in Railroad Operation" was presented by S. G. Ellis, a radio engineer for the Westinghouse Electric and Manufacturing Company. In it, he described development work on radio equipment for trains and commercial equipment which resulted from the experimental program. The experimental work was done with transmitters of 1.5 and 15 watts power and receivers of both the superregenerative and superheterodyne types. Work was done on trains from sixty to one hundred and twenty-five cars in length under temperature conditions varying from subzero to 140 degrees, in snow, ice, rain, and fog for a period of time totaling 1500 hours of work while traveling 36,000 miles. Charts illustrating relative performance and photographs showing the difficulty of antenna and apparatus placement were shown.

The commercial train communication system developed consists of a 25-watt transmitter employing two 801 tubes modulated by a similar pair driven by a 59, and shock-mounted in a weatherproof steel box. The transmitter operates between 30 and 40 megacycles with a half-wave horizontal antenna made of a three-fourth-inch brass pipe. The receiver is an eight-tube superheterodyne with four watts of audio output, automatic volume control, and five-microvolt sensitivity. All equipment is dynamotor driven from a separate battery bank. The paper was discussed by Messrs. Bond, Lamb and others.

Detroit Section

A meeting of the Detroit Section was held on October 18 in the Detroit News Conference Room and was presided over by A. B. Buchanan, chairman. Forty were present at the meeting and ten at the dinner which preceded it.

"The Use of Radio in Commercial Aviation" was the title of a paper by E. G. Kowalski who is radio control operator of the Detroit City Airport. He covered first radio as an aid to aerial navigation and discussed the beam system and the weather collecting stations operated by the government. He then considered the point-to-point system provided by individual companies to give specific information to pilots and to enable them to obtain instructions and information while flying. The third part covered the system used at air terminals in directing plane traffic. It is furnished by airports to supply incoming and outgoing planes with information regarding wind direction and velocity, visibility, and other air traffic conditions as well as such specific instructions on landing, taxiing, and taking off as are needed. This system is of short range and is used only when the plane is within five minutes of the airport.

The November meeting was held on the 22nd in the Detroit News Conference Room and presided over by Chairman Buchanan. Thirty-four attended the meeting and twelve were at the dinner which preceded it.

"The Use of Public Address Equipment in Safety and Educational Fields and in Traffic Control" was the subject of three papers. The first of these by E. Anderson of the Research Department of the Detroit Board of Education, covered the educational aspects of the subject. He described the equipment used in the school system in Detroit. He discussed the use of educational programs, particularly the travelog series, and showed how the stimulation received by the students could be made use of by the classroom teacher. Methods of teaching through use of radio programs were outlined.

The second speaker was G. Mulholland of the Safety Division, First Precinct, Detroit Police Department. He described the experiments of the Detroit Police Department in safety education and traffic control. The work done by the sound cars in traffic control and at playgrounds for safety educational work was outlined. A description was given of the street loud speaker system used during the Christmas purchasing season for pedestrian control. In this system, speakers were arranged on busy downtown corners and pedestrian traffic controlled by an officer stationed at a window overlooking the intersection. The paper was concluded with a discussion of the short safety announcements given over radio stations and in the larger theaters.

The third speaker, S. Almas, of KLA Laboratories, covered the technical aspects of the equipment used in the sound cars and on the streets by the police department. Difficulties involved in providing sufficient sound energy to overcome street noises were described. He outlined briefly the troubles encountered in the sound-car equipment and finished his paper with a description of some of the later installations of centralized radio equipment in schools.

Emporium Section

The first meeting of the newly established Emporium Section was held at the Sylvania Club on December 12. The temporary chairman, E. F. Carter, appointed a nominating committee which presented a slate of officers who were unanimously elected. These are Raymond R. Hoffman as chairman, Herbert Ehlers as vice chairman, and L. E. West as secretary-treasurer.

A short historical sketch of the Institute and its section activities was given by H. P. Westman, national secretary. Charles Marshall of the Chicago office and George C. Connor of the New York office of the Hygrade Sylvania Corporation described the activities of the field engineer. It was pointed out that not only did they contact and assist in clearing up difficulties encountered by radio receiving set manufacturers but a substantial proportion of their time is devoted to problems connected with the nonradio applications of tubes. The papers were discussed by Messrs. Graham, Kievit, Jones, and West. Sixty-five members and guests were present.

New York Meeting

The December 4 New York meeting was devoted to a "Review of Radio Developments during 1935." This review was divided into five parts and that on "Radio Communication in the Fixed Services" was covered by C. H. Taylor. Because of his inability to be present, the paper was read by H. H. Beverage. The second part on "Radio Communication in the Mobile Services" was presented by C. N. Anderson. "Radio Broadcast Transmission" by C. M. Jansky, Jr., was presented in his stead by S. L. Bailey. R. H. Langley prepared a report on "Radio Broadcast Reception" which was read by J. V. L. Hogan. The "Allied Fields" were treated by O. H. Caldwell.

It is anticipated that these papers will be published in an early forthcoming issue of the Proceedings. The meeting was attended by 350 members and guests.

Pittsburgh Section

In place of a regular meeting at which papers are presented, the November 19 meeting of the Pittsburgh Section was devoted to a tour of inspection of broadcast station WCAE. The attendance was forty-four and J. G. Allen, vice chairman, was in charge. The new five-thousand-watt RCA transmitter together with its control and monitoring equipment brought forcibly to the attention of the membership the modern trends in transmitter design. Various controls and parts of the transmitter were described by R. Bower, operator in charge of the station at the time of the visit.

Seattle Section

A meeting of the Seattle Section was held on October 4 at the University of Washington and presided over by R. C. Fisher, chairman. Sixty were in attendance.

"Doubling the Available Radio Channels" was the subject of a paper by J. R. Woodyard, instructor in the Electrical Engineering Department of the University of Washington. In it, he described the circuits and equipment used for the reception of frequency and amplitude modulation simultaneously applied to a single carrier. He showed that the receiver could respond either to amplitude or frequency modulation by the process of shifting the carrier through ninety degrees by means of a local synchronized oscillator. This oscillator constitutes the only additional equipment required to equip modern superheterodyne receivers for this reception. It was demonstrated by transmitting two programs simultaneously from a single transmitter, one program being transmitted by frequency modulation and the other by amplitude modulation. Either program was received at will merely by operating a switch at the receiver.

The November meeting of the section was held at the University of Washington. Forty-two were present and Chairman Fisher presided.

A paper on "Interpretation of Patents" was presented by Paul Bliven, a patent lawyer. In it he described the historical background of the patent system which lies in those ancient monopolies granted to people bringing goods from the Orient to England. The body of the paper outlined the fundamental requirements for the grant of a patent among which are novelty, invention, and utility. He described also the various subject matter for which patents may be granted. It was pointed out that only the embodiment of an idea is patentable, not the idea itself. A general discussion which was participated in by Messrs. Eastman, Libby, Willson and others followed in which the author answered numerous questions relative to the safeguarding and protection of inventions. The meeting closed with a discussion of the Washington State Engineers Licensing Law.

Toronto Section

The Toronto Section met on October 28 at the University of Toronto. L. M. Price, chairman, presided and fifty-two members were present.

"New Developments in the Reproduction of Speech and Music" was the subject of a paper by R. A. Hackbusch, chief engineer of the Stromberg-Carlson Telephone Manufacturing Company of Canada. He outlined the complete system of transmitting and receiving sound by radio and pointed out the limiting factors which determine the over-all effectiveness of the system. He then analyzed the receiving system in detail and pointed out the faults of loud speakers and methods of overcoming them by changes in design and the use of suitable materials. The limitations imposed by cabinets were then sketched and the acoustical labyrinth developed to overcome these limitations covered. The use in conjunction with high-frequency loud speakers of deflecting vanes to modify the distribution patterns of their sound output was described. A demonstration using both recorded material and radio broadcast programs was given through the use of a high fidelity receiver. A graphic reproduction of the functioning of the fidelity control was also demonstrated with a cathode-ray oscillograph. A general discussion was participated in by Messrs. Bayley, Blatterman, Smith and others.

Washington Section

A meeting of the Washington Section was held on October 14 at the Potomac Electric Power Company auditorium. It was presided over by E. K. Jett, chairman, and attended by seventy-five. Twenty-five were present at the informal dinner which preceded the meeting.

Two speakers treated the subject of "The Enforcement of Radio Laws, Rules, Regulations, and Treaties." These were G. E. Sterling, inspector-in-charge of the Fourth Radio District and C. A. Ellert of the Broadcast Section, Engineering Department of the Federal Communications Commission. Mr. Sterling treated the subject from the angle of the duties of a radio inspector in a federal office and at the monitoring stations of the Federal Communications Commission. Mr. Ellert described the apparatus and the operation of the monitoring stations giving details of methods used to insure accuracy. The papers were discussed by Messr. Burgess, Dorsey, and Gillett.

The November meeting of the section was held on the 11th and was attended by 180. Forty were present at the dinner which preceded the meeting. It was held in the Potomac Electric Power Company auditorium and presided over by Chairman Jett.

"Some New and Practical Applications of the Cold Cathode Tube" by P. T. Farnsworth and R. L. Snyder of Farnsworth Television, Inc., was presented by the authors. Several types of cold cathode tubes were described. They were used as oscillators and amplifiers which depend on the effects of secondary electron emission for multiplying a very small photoelectric current. A reduction in noise level as compared with thermionic tube amplifiers as high as 100 to 200 times was claimed. Amplification of as high as 240 decibels and oscillator efficiencies of 65 to 95 per cent were specified. Tubes may be used to advantage for frequency multiplication as it is possible to adjust the circuit constants to emphasize harmonics greatly. When operated as a photocell and amplifier combined, a single tube operated a loud speaker directly from a sound film. A fifty-watt oscillator was demonstrated as a telephone transmitting tube and several other models of tubes were shown. The paper was discussed by Messrs. Burgess, Dorsey, McIlwraith, Sterling, and Stevens. The nominating committee appointed at the October meeting submitted its report which will be voted on at the next meeting.

Personal Mention

S. P. Acuna formerly with Revista Telegrafica has established a consulting practice and has become technical editor for the publications of the Radio Club Argentino at Buenos Aires, Argentina.

Benjamin Adler of the RCA Manufacturing Company has been transferred from Atlanta, Ga., to Camden, N. J.

Previously with the Federal Communications Commission, J. H. Barron has established a consulting practice with headquarters in Washington, D. C.

Previously a television consultant, Ivan Bloch has joined the staff of the Electric Home and Farm Authority at Washington, D. C.

R. L. Clark of the Federal Communications Commission has been transferred from Detroit, Michigan, to Washington, D. C.

C. B. Jolliffe formerly with the Federal Communications Commission is now engineer-in-charge of the RCA Frequency Bureau at 30 Rockefeller Plaza, New York City.

T. A. M. Craven has relinquished his consulting practice to become chief engineer for the Federal Communications Commission, Washington, D. C.

F. H. Engel of the RCA Manufacturing Company has been transferred from Harrison, N. J., to Camden, N. J.

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TECHNICAL PAPERS

THE BROADCAST ANTENNA*

By

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Summary—*During the past several years, the trend in broadcast transmitting antennas has been towards the vertical radiator. The performance of these newer antennas has been subject to considerable theoretical discussion. This paper, however, emphasizes the actual field results obtained with tower radiators, and presents data on efficiency, base voltage, base loss, practical design considerations, and cost.*

It is concluded that the conventional broadcast antenna supported by two towers, as used in the past, is definitely outmoded by the single vertical radiator, or, in special cases, by a combination of vertical radiators in a directional array. Antennas used in the broadcast band only, 550 to 1500 kilocycles, are considered.

Introduction

THE design of a broadcast antenna is governed largely by commercial considerations. A broadcast station whose income is derived from advertising sales must usually concentrate on its primary service area. This may be defined as the area in which any home having an average receiver, and desiring to listen to the program transmitted, can receive a satisfactory signal, free of noise and fading. Uncertain, irregular reception at long distance has some prestige value but it seldom contributes to the income of the station. In general, the broadcast receiver is now used to bring entertainment into the home, little interest being shown in the fading, noisier signals of distant transmitters.

In broadcasting, the power output of the transmitter is limited, by both economic and regulatory factors. The problem, then, is to concentrate all of the available radio-frequency power on the primary service area. Only a small fraction of this power is now directed at the population it is desired to serve, most of the power being directed toward the ionosphere and reflected back to earth at distant points.

Increasing the signal intensity in a horizontal direction is only part of the problem. In the case of higher powered transmitters, nighttime fading starts many miles before the field intensity becomes too weak for a satisfactory signal. Radiation above the horizon is completely

* Decimal classification: R320. Original manuscript received by the Institute, May 31, 1935. Presented before New York meeting, April 3, 1935; presented before Washington Section, March 11, 1935.

lost during the daylight hours, this power being robbed from the useful signal. At nighttime, however, conditions are changed so that some of the radiation at angles between forty degrees and seventy degrees above the horizon is returned to earth forty to one hundred and fifty miles from the transmitter. This reflected signal is of negative value since its variation in phase and amplitude causes it to interfere with the signal arriving directly from the transmitting antenna. At night the primary service area ends at that distance from the transmitter where the reflected signal approaches the directly transmitted signal in intensity.

Many experienced antenna engineers doubt the probability of redirecting the high angle radiation and concentrating it along the horizon. Such a development in broadcast antennas would increase the commercial value of the radio station far more than a simple increase in transmitted power. We believe the limit has by no means been reached in the improvement of broadcast transmitting antennas.

Antenna System Losses

The output of a radio station is usually fed to the antenna over a transmission line. The radio-frequency power is then dissipated as follows:

1. Radiation
   (a) Along the horizon to receiving sites. Usually the zone between zero and ten degrees above the horizon includes all such sites.
   (b) Towards the ground; partially reflected back at angles above the horizon and partially dissipated in the earth.
   (c) Into space; either lost or reflected back to receiver sites; causes fading; gives long-distance reception.
2. Transmission line loss in form of heat and radiation.
3. Coupling equipment loss.
   (a) Heat losses caused by resistance in units.
   (b) Transfer of radio-frequency energy to nonradiating surfaces, hence heat loss.
4. Resistive loss of ground system.
5. Dielectric loss at base of antenna.
6. Resistive loss of actual antenna.
7. Power picked up by near-by metallic objects (towers, guys, power wires, building framework, etc.) and either dissipated in the form of heat, or reradiated.

There are other losses which absorb smaller percentages of the radio-frequency power. However, of the divisions of power listed above, only the first one, radiation along the horizon, is of value to the broadcaster.

There is little accurate quantitative data available as to the strength of the signal radiated above the horizon by different types of antennas.[1,2] In the case of the vertical radiator there have been extensive mathematical discussions giving the signal distribution in a vertical plane.[3,4,5,1,6] Attempts have been made to fix the exact height of various types of vertical radiators to minimize the sky-wave signal at points which would otherwise remain outside of the primary service area of the station. To make an actual check of this theory in practice requires measurements in space above the antenna or an extensive study of the reflected sky wave at various distances from the transmitter.

In the case of low powered stations operating at the same frequency simultaneously, fading is not usually a consideration of importance because interference from the side bands of one of the other stations on that frequency ordinarily takes place at a distance from the transmitter much less than the distance to the fading wall. Thus, the primary service area of such a station is limited by cross talk. The first consideration is to obtain a maximum signal along the ground, usually in all directions. In the case of high power clear channel stations, there are no other stations utilizing the same frequency. A serviceable signal from such a station extends out into the zone which is affected by fading. At these stations, it is important to consider both the signal along the ground and a reduction in fading. An antenna is required which will be as effective as possible in minimizing fading.

Evolution of the Broadcast Antenna

The early broadcast antenna consisted of a pair of steel or wooden masts supporting an antenna structure usually consisting of a vertical wire, or cage, with or without a horizontal section consisting of a flat top, or cage. This latter type was known as the "T" or "L" type antenna. It is interesting to note that the majority of broadcast stations in operation today employ this older type of antenna. Most of these have a natural wavelength less than one quarter of the operating

[1] Stuart Ballantine, "High quality radio broadcast transmission and reception," Proc. I.R.E., vol. 22, pp. 564–629; May, (1934).

[2] J. A. Stratton and H. A. Chinn, "Radiation characteristics of a vertical half-wave antenna," Proc. I.R.E., vol. 20, pp. 1892–1913; December, (1932).

[3] A. Sommerfeld, "Über die Ausbreitung der Wellen in der drahtlosen Telegraphie," *Ann. der Phys.*, vol. 28, pp. 665–736; (1909).

[4] F. Eppen and A. Gothe, "Über die schwundvermindernde Antenne des Rundfunksenders Breslau," *Elect. Nach. Tech.*, vol. 10, pp. 173–181; March, (1933).

[5] O. Bohm, *Telefunken-Zeit.*, no. 57, pp. 30, 31, (1931).

[6] M. J. O. Strutt, "Strahlung von Antennen unter dem Einfluss der Erdbodeneigenschaften," *Ann. der Phys.*, ser. 5, vol. 1, pp. 721–750, (1929).

wavelength. Later, it was realized that some gain would be made if larger antennas were employed. Therefore, the same type of antenna was used, but the natural wavelength was increased to one-third or three-eighths wave. In 1924 this problem was investigated[7,8] and it was shown that a marked gain would result if higher antennas were used. Seven years later, in 1931, two such antennas[9] were erected at stations WNAC-WAAB, Boston, and WABC, New York, stations

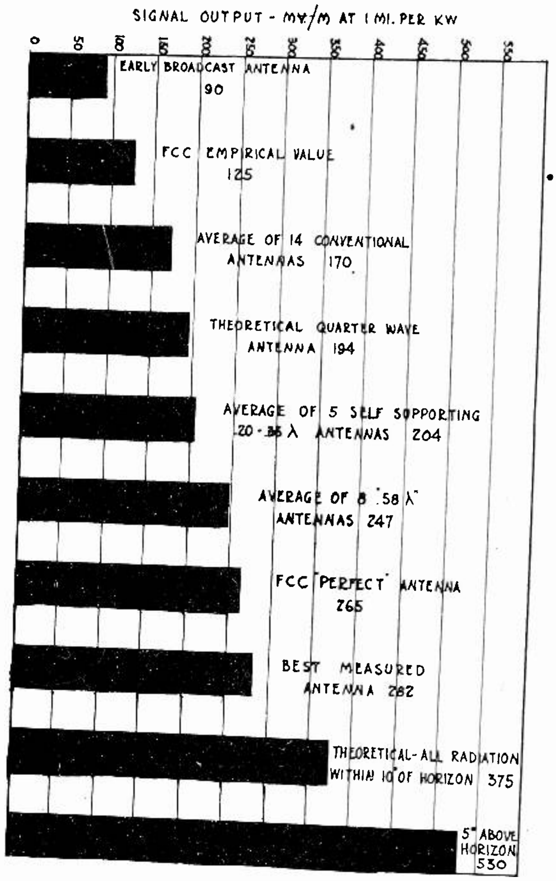

Fig. 1—The evolution of broadcast antennas—signal output.

of the Columbia Broadcasting System. The electrical gains expected from these antennas have been realized in practice. During the past few years a great deal of time has been devoted to a study of the electrical properties of this type of radiator. During the past two years

[7] Stuart Ballantine, "On the optimum transmitting wave length for a vertical antenna over perfect earth," Proc. I.R.E., vol. 12, pp. 833–839; December (1924).

[8] P. P. Eckersley, "Calculation of the service area of broadcast stations," Proc. I.R.E., vol. 18, pp. 1160–1193; July, (1930).

[9] U. S. Patent 1,897,373. Wave antenna. Patent issued to Nicholas Gerten and Ralph L. Jenner, assignors to Blaw Knox Co.

attention has also been given to the self-supporting type which has made an appearance in the field. It appears that the guyed type of vertical mast antenna, or the self-supporting type of mast antenna, is superior to the older conventional antennas, electrically, physically, and economically.

Broadcast Antenna Efficiencies

A study of Fig. 1 indicates the evolution of broadcast antenna efficiencies, and shows, conclusively, that the higher mast type antenna

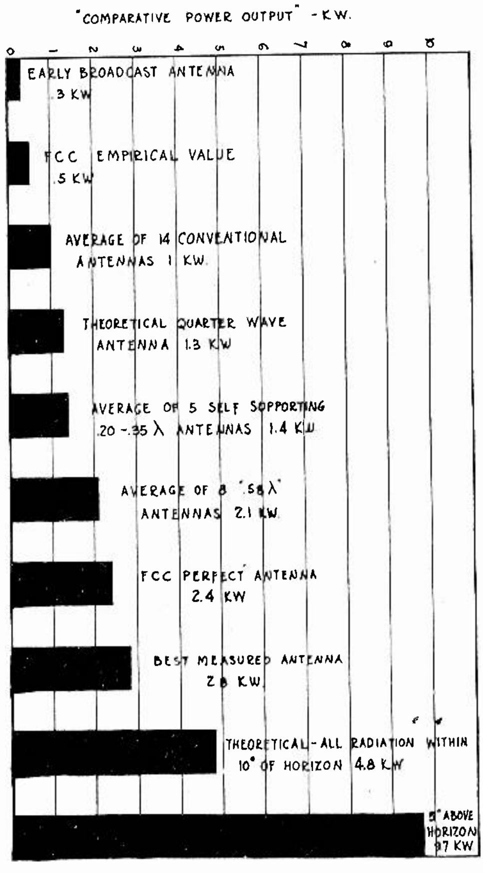

Fig. 2—The evolution of broadcast antennas—comparative power output.

is much better than the older conventional types from an electrical viewpoint. Early broadcast antennas produced an effective field intensity of as little as 100 millivolts per meter at one mile for one-kilowatt antenna input power. The average of fourteen conventional type antennas, recently measured, shows that with one-kilowatt antenna input power the effective field intensity is 169 millivolts per meter at one mile. The average of five self-supporting type mast antennas, from 0.20 to 0.35 wavelength high, shows an average field intensity of

204 millivolts per meter, and measurements made at eight 0.58 wavelength antennas shows the average field intensity to be 247 millivolts per meter at one mile. There is one guyed type vertical mast antenna which has an effective field intensity as high as 280 millivolts per meter. Fig. 2 gives this same information in terms of comparative increase in power. It should be emphasized that these results are based on actual measurements and show the higher antennas to be far more efficient electrically than the older conventional types.

Antenna Terms

Antenna Efficiency

One of the most commonly used terms is "radiation efficiency" or "antenna efficiency." Theoretically, this efficiency should represent the ratio of the radiated power to the input power of the antenna. However, it would be necessary to integrate the power streaming away from the antenna toward every point in space to determine such a figure. As this is not a practical procedure, measurements of field strength are usually made at convenient points on the ground near the antenna. Among American engineers, the field intensity at one mile is the value usually determined, but in the formulation of a figure of merit many different arbitrary standards have been used as a basis of comparison. Thus, 100, 125, 187, 194, and 265 millivolts per meter at one mile for one-kilowatt radiated have, at various times, all been called "100 per cent efficiency."

The Engineering Department of the Federal Communications Commission has arbitrarily defined antenna efficiency as follows:[10] "The antenna efficiency equals 100 per cent if the effective field intensity of the station at one mile, per kilowatt antenna input power, is equal to 265 millivolts per meter."

$$A_{\mathrm{eff}}=\frac{F^2\times 100}{265^2\times P}$$

A_{eff} = antenna efficiency in per cent

F = effective field intensity at one mile, expressed in millivolts per meter[11]

P = antenna input power (in kilowatts).

Actually, whatever the means used to express it, antenna efficiency can only tell the engineer the signal intensity at a given distance, along

[10] Seventh Annual Report of the Federal Radio Commission, 1933, p. 24. Section entitled "Antenna and Radiation Standards."

[11] The measured root-mean-square value of all field intensities at one mile from the antenna in the horizontal plane is termed the "effective field intensity, (F)."

the earth's surface, for a certain antenna input. Efficiency ratings at present are very ambiguous, and since there appears to be no one fundamental value upon which to base efficiency ratings, it is hoped that the method of rating antennas simply in terms of the actual signal

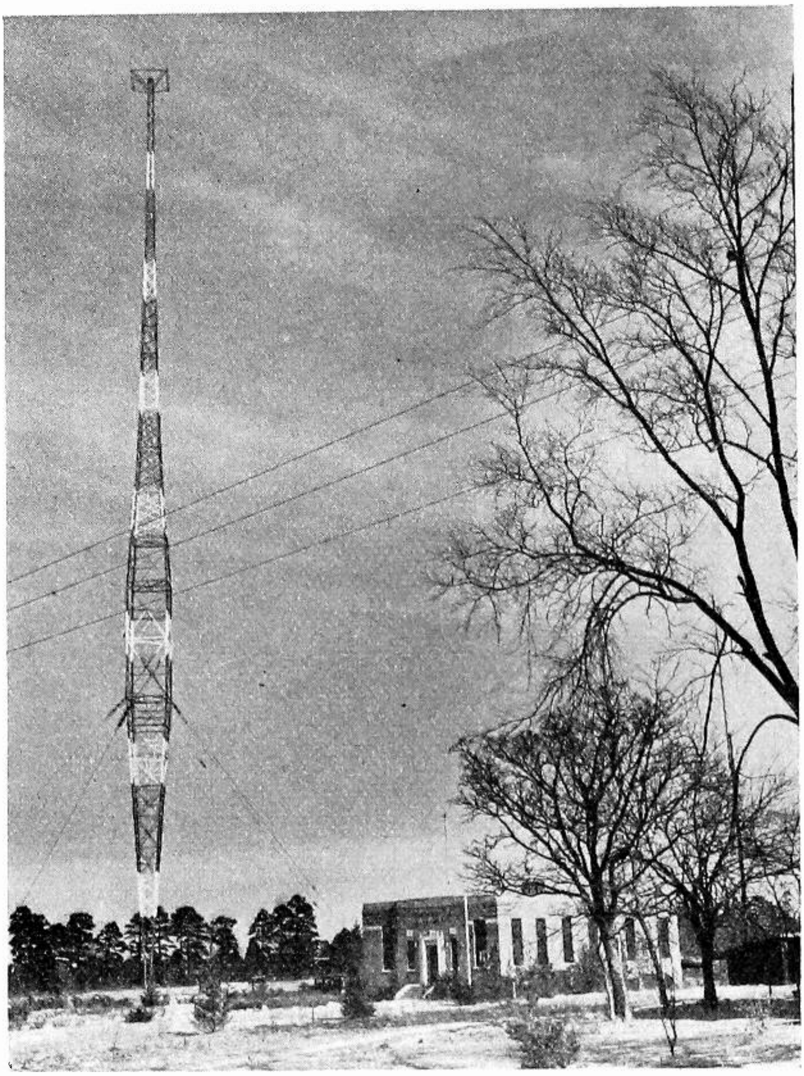

Fig. 3—WBT guyed vertical radiator.

output in millivolts per meter at a given distance for one unit of radiated power will be generally adopted by engineers.

Antenna Length

Early antennas were described in terms of their fundamental wavelength. This was the longest wavelength at which the antenna had zero reactance and the mode of operation of the antenna was given by the ratio of operating wavelength to fundamental wavelength.

Theoretically, a wire in free space will have zero reactance whenever it is any number of quarter wavelengths long; that is, it will have zero reactance when its length is one quarter, one half, three quarters, etc., of a wavelength. Measurements indicate, however, that a vertical wire antenna above earth of finite conductivity has zero reactance at dis-

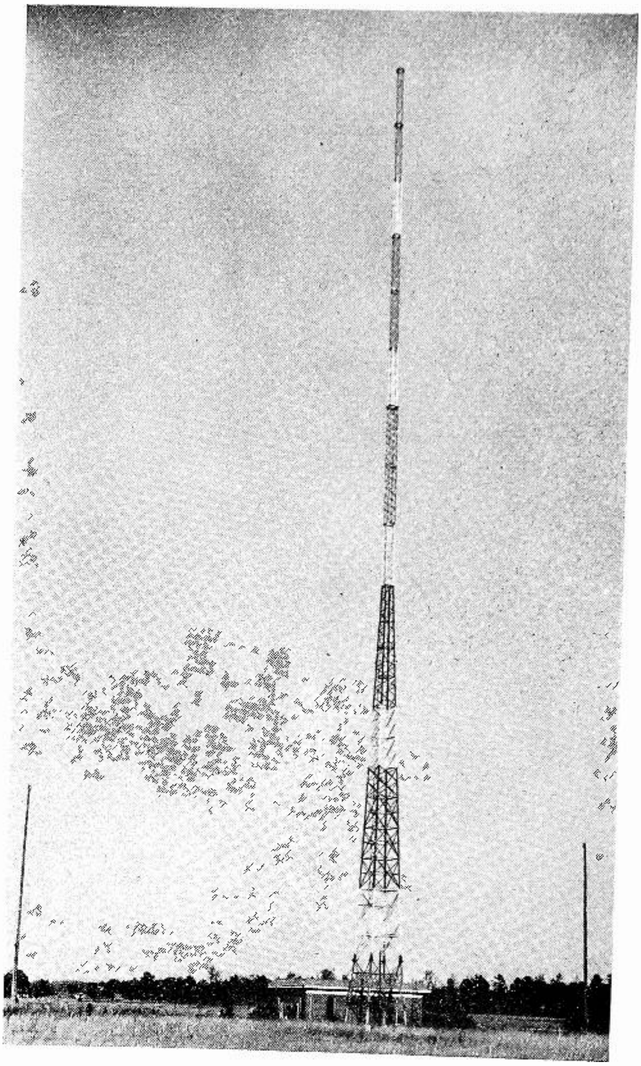

Fig. 4—WDOD self-supporting vertical radiator.

tances about four per cent shorter than the above values; that is, the velocity of propagation along the wire is 0.96 of the free space velocity.

The guyed vertical radiator departs materially from the configuration of a single vertical wire. (See Fig. 3.) This is also true of the self-supporting radiator. (See Fig. 4.) Because of this, the impedance characteristics of such radiators differ from theoretical values obtained with a vertical wire over earth of finite conductivity. Measurements have been made on a number of vertical radiators. The results of such measurements, showing the resistance and reactance characteristics at the base of such antennas, are shown in Figs. 5 and 6.

Referring to the impedance characteristics of the guyed mast, Fig. 5, a "half-wave" antenna of this type, is physically only 0.45 of a wavelength high. Following the above method of argument, it could be

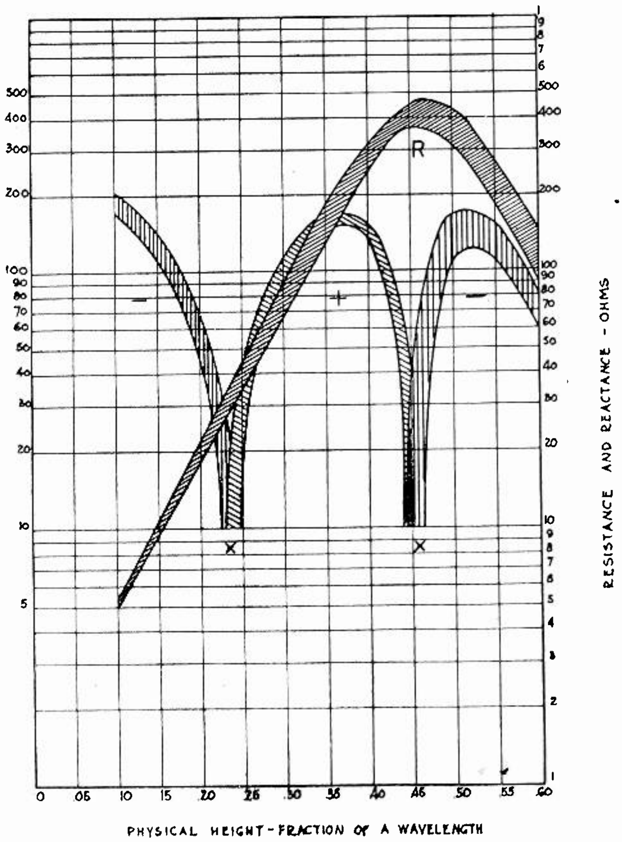

Fig. 5—Antenna characteristics including five existing guyed radiators, resistance R and reactance X.

said that the propagation velocity of this type of antenna is only 90 per cent of the theoretical velocity. Further, if reference is made to Fig. 6 which gives the impedance characteristics of the self-supporting vertical radiator, it could be argued that the velocity of propagation is 66 per cent of the theoretical velocity.

This method of reasoning is based upon the assumption of a sinusoidal distribution of current on the antenna. This assumption is not justified in practice.[12,13,14] It also assumes there is no lumped capaci-

[12] E. A. LaPort, "Increased efficiency from tower antennas—a review," *Electronics*, vol. 7, pp. 238–248; August, (1934).

tance in the antenna itself. When the equivalent circuit of the antenna is considered, it may be seen that this circuit can be tuned to zero reactance at almost any frequency by the choice of a proper shunt condenser. This is exactly what occurs in the case of the self-supporting tower.

If a four to five hundred foot guyed mast and a similar self-supporting tower of a conventional design were measured, it is estimated that

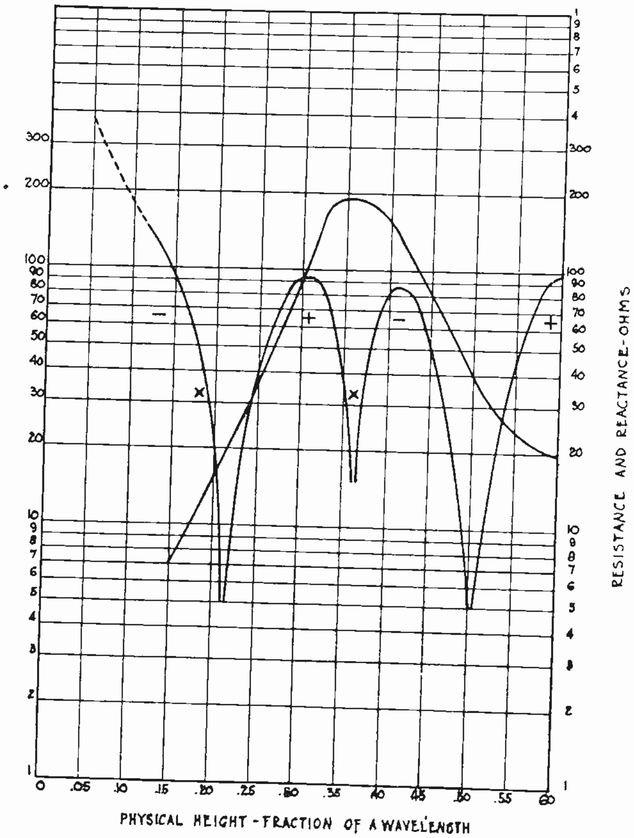

Fig. 6—Average antenna characteristics of three existing self-supporting radiators showing resistance and reactance.

the capacitance to ground of the first thirty or forty feet of the wide base tower would exceed that of the former structure by at least 800 micromicrofarads (including insulators). Taking a guyed mast, 0.47 of a wavelength high, operating at 1080 kilocycles, as an example, Fig. 5 shows its impedance to be 400–j60 ohms. If an 800-micromicrofarad condenser were shunted across the base insulator, the measured im-

[13] Hans Roder, "Discussion on high quality radio broadcasting," Proc. I.R.E., vol. 23, pp. 256–260; March, (1935).

[14] H. E. Gihring and G. H. Brown, "General considerations of tower antennas for broadcast use," Proc. I.R.E., vol. 23, pp. 311–356; April, (1935).

pedance at 1080 kilocycles would be

$$Z = \frac{-j178(400-j60)}{(400-j60)+(-j178)} = 63 - j133 \text{ (ohms)}.$$

The impedance of this mast was thus made similar to that of a wide base self-supporting tower which is given in Fig. 6. Yet no one would argue that the velocity of propagation on the mast itself had changed.

It is our belief in connection with vertical antennas involving structures whose entire length is not of uniform cross section that for the present:

1. The terms "electrical length" and "velocity of propagation" have no significant value.
2. Engineers working with these antennas should standardize on physical height (in fractions of a free space wavelength) as a method of describing the antenna dimensions.

General Considerations

Either the self-supporting or the guyed radiator can be erected at less cost than a two-tower conventional antenna of the same height. It has far greater reliability of operation, has a lower maintenance cost, and the distribution of power is not influenced by supporting structures which tend not only to distort the field pattern, but also to lower the antenna efficiency. The self-supporting antenna has a very practical application in the cases of broadcast transmitters which are located on tall buildings where it is sometimes difficult to erect the older type of antenna system because of physical limitations. However, if an adequate ground is provided, a moderately efficient antenna system can be erected atop a tall building through the use of a vertical mast antenna.

During the past, broadcast engineers have been very hesitant in adopting the vertical mast antenna, particularly the wide base, self-supporting type, because of a fallacy which has existed with regard to high base capacitance being directly related to loss in efficiency.

During the past two years a number of self-supporting towers have been erected, usually with strict limitations on insulator capacitance and tower capacitance to ground. The fear of base capacitance has continued up to the present and measurements of the signal ouput of these antennas operated at higher frequencies led to the conclusion that they would be unsatisfactory for use at heights greater than 0.35 or 0.40, of a wavelength. In 1934 measurements[15] were made which

[15] John Byrne, Ohio State University, paper not published.

showed that the loss in efficiency was due to dielectric losses in the earth near the tower base. With this fact established, it was simply a matter of reducing the radio-frequency voltage gradient in the soil at this point.

Self-supporting vertical antennas have appeared having the first eight or ninety feet constructed of wood, and the radiating section above, insulated from the wood. Various other modifications of the self-supporting structure were developed, including all-steel designs which insert the base insulators ten to thirty feet above ground. Another satisfactory solution has been to construct a well-grounded copper mat or ground screen beneath the tower, thus greatly reducing the high voltage normally impressed across the earth at the tower base. At station WDOD, Chattanooga, Tenn., measurements were made recently which substantiate this point. The station uses a self-supporting tower 0.42 wavelength high, and has installed two sets of insulators, one just above ground, and the other set approximately twenty feet above ground. It has been found, by measurement, that the same antenna efficiency can be obtained using either the higher set of insulators or the lower set of insulators, However, if the lower set of insulators is used, it becomes necessary to install a ground screen to reduce the dielectric losses at the base of the antenna. Such a screen should be erected on or above the earth at the tower base. It should be constructed of noncorrosive metal of good conductivity. The screen must be well connected to the ground system proper. Under normal operating conditions, the ground screen should cover as large an area as is economically feasible. It should, under all circumstances, extend at least ten feet beyond the base of the tower in all directions.

From the present data on broadcast antennas, it appears that the same antenna efficiency can be obtained with nearly any type of vertical mast antenna of a given height, providing the necessary precautions are taken in design and erection, and providing the proper ground system is used.

The Ground System

The proper design of the ground system for use with broadcast antennas has always been important, but the design of a ground system for use with high antennas becomes particularly important, as factors other than ground loss resistance must be taken into account. The radial ground system used with a high antenna should be large in diameter, have a radius equal to at least one-half wavelength, and be buried close to the earth's surface. Fig. 7 shows the empirical relationship between antenna efficiency and ground radius, based on measurements of thirty-five stations. The reasons for a large ground system are to reduce ground resistance loss, dielectric losses, and the absorption of radiation directed towards the ground, to an absolute minimum.

In the design of a ground system, it is necessary that the voltage between the base of the antenna and the ground be accurately estimated in order to reduce the losses previously mentioned. This information is also necessary in order that adequate insulation may be provided. If impedance measurements have been made on antennas of the same general shape as the one contemplated, the voltage may be determined quite accurately in advance. Contrary to popular belief, the highest voltages encountered do not always occur in high resistance

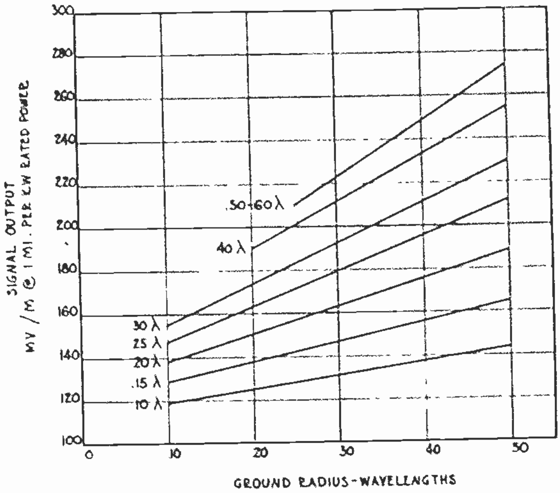

Fig. 7—Empirical relation of signal output in millivolts per meter at one mile for one kilowatt rated power, to ground radius for different height antennas, based on measurements of thirty-five broadcast antennas.

antennas. For instance, a vertical radiator, recently constructed, is 0.47 wavelength high. It has a resistance of 200 ohms at the operating frequency. Its unmodulated fifty-kilowatt carrier voltage, at the base of the antenna, is 4200 volts root-mean-square. There is another fifty-kilowatt station in service, whose antenna resistance is fifteen ohms at the operating frequency. The impedance of this particular antenna is 140 ohms, thus, the base voltage is 8000 volts root-mean-square. Figs. 5 and 6 show the average resistance and impedance characteristics of self-supporting and guyed vertical radiators, and may be used in estimating base losses, insulation requirements, antenna current, antenna loading, and lighting circuit requirements.

Antenna Site

The choice of a station location is definitely related to the antenna design. A site surrounded by soil of the highest possible conductivity is advisable. Actually, the final choice of a station location is governed by considerations of cost and population distribution. Thus, the engineer must often design an efficient radiating system above soil of poor conductivity.

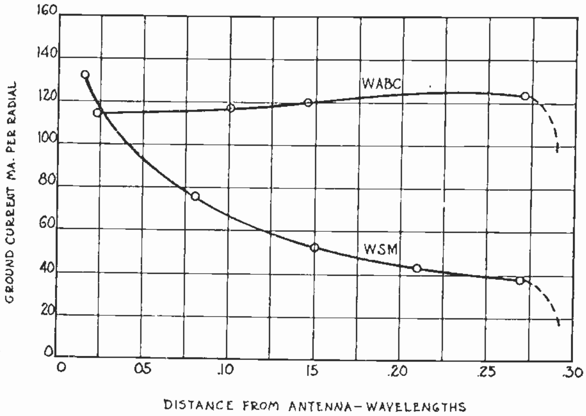

Fig. 8—Current in one conductor of a sixty-radial ground for vertical radiator (50 kilowatts).

If the transmitting antenna is located over poor soil, the ground current will tend to avoid the high resistance earth path and will remain on the lower resistance copper wires of the ground system. This

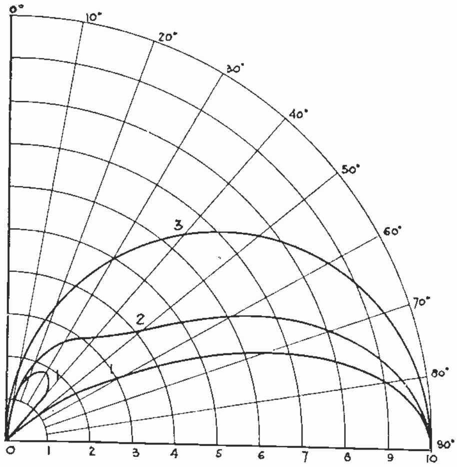

Fig. 9—Vertical polar diagram of a guyed mast antenna.

is shown graphically in Fig. 8, the current on a single ground radial at two fifty-kilowatt transmitters. The antenna and ground systems of the two stations are practically identical, but WSM at Nashville was fortunate in obtaining a station location where the soil conductivity is considerably higher than at any which were available to WABC.

The effect of soil conditions also is evident in the vertical polar pattern of the higher types of antennas. Fig. 9 shows the measured vertical polar diagram for a guyed mast antenna located in a part of Pennsylvania of comparatively low conductivity. Several studies have shown that the current distribution on this antenna structure departs from the assumptions which give the desirable calculated pattern of Fig. 9.[14] Earth of infinite conductity is also assumed in the calculations. Sky-wave and fading measurements indicate that the guyed mast antennas[16] of stations WLW and WSM which are surrounded by high conductivity soil, more nearly approach the theoretical vertical pattern than similar antennas with less favorable ground conditions. It is true that a more satisfactory tower configuration also contributes to this condition.

Directional Antennas

A number of directional antennas have been constructed by broadcast stations during the past two years. Such installations usually employ two or more guyed or self-supporting tower radiators. In some cases, two insulated towers are used to support one or more vertical cables, the towers and vertical cables all being used as a multielement directional radiating system.[17]

The purpose of employing such systems is to fulfill specific interference reduction requirements, consistent with the rendering of maximum public service. In several cases, directional antennas are used to increase the radiated signal in the direction, or directions, of maximum potential coverage.

Directional antenna systems have been installed at stations WJSV, Alexandria, Virginia, and at WKRC, Cincinnati, Ohio, stations of the Columbia Broadcasting System. These antennas are in operation at the present time.

The WJSV antenna system consists of two vertical conductors suspended between two 150-foot steel towers, insulated at their bases. The antennas are three-eighths wave apart ($\theta = 135$ degrees) and the current in the west antenna leads the current in the east antenna by one-eighth wave ($\theta = 45$ degrees). The ten-kilowatt WJSV transmitter is located about four hundred feet from the antenna system and power is transmitted from it to the antenna by a conventional 600-ohm two-conductor open-wire line. Proper phasing is obtained by using transmission lines to each element of such length as to obtain the desired

[16] J. H. DeWitt, Jr., *Jour. Tenn. Acad. Sci.*, vol. 8, p. 95, (1932).
[17] J. F. Morrison, "Controlled radiation for broadcasting," *Bell Lab. Rec.*, vol. 13, pp. 232–237; April, (1935).

phase relations. The field intensity distribution in a horizontal plane is a flattened cardioid, with the minimum signal in an easterly direction. Fig. 10 shows the horizontal polar diagrams of the WJSV conventional antenna and the directional antenna which is now in use. A minimum signal was desired, in this case, at a point one mile east of the station. It can be seen that a reduction in signal intensity of approximately 50:1 has been obtained. In order to determine the

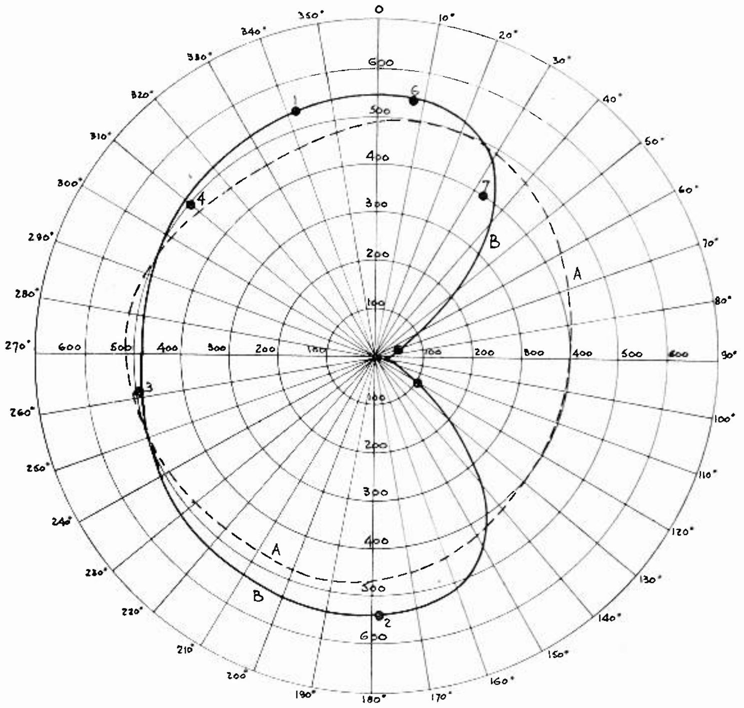

Fig. 10—Signal at one mile—in millivolts per meter for ten kilowatts antenna input power.

stability of the system, an accurate automatic signal intensity recorder was installed one mile east of the WJSV antenna system, which records the signal strength of this station continuously. This equipment has been in operation twenty-four hours a day since July, 1933. The automatic field intensity receiver is specially designed so that its sensitivity is independent of variations in ambient temperature, humidity, and power supply.

The records obtained during the past eighteen months indicate that the stability of the WJSV antenna system is entirely satisfactory. Inasmuch as the cardioid pattern is one of the most difficult to maintain, experience at WJSV has definitely shown that, although desirable,[18] it is, in this case, unnecessary to employ special means for maintaining pattern stability automatically.

The directional antenna system at station WKRC is erected on the roof of the Hotel Alms, Cincinnati, Ohio, and consists of two self-supporting towers 154 feet high and one-eighth wave apart. (Space, $\theta = 45$ degrees.) The current in the north antenna leads the current in the south antenna by 140 degrees ($\theta = 140$ degrees).[19] Fig. 11 is a schematic diagram of this system. In order to obtain the proper phasing, a phase changing circuit is used because of the relatively short transmission lines erected on the roof of a nine-story building. Each line, approximately 110 feet long, has an electrical length of approximately

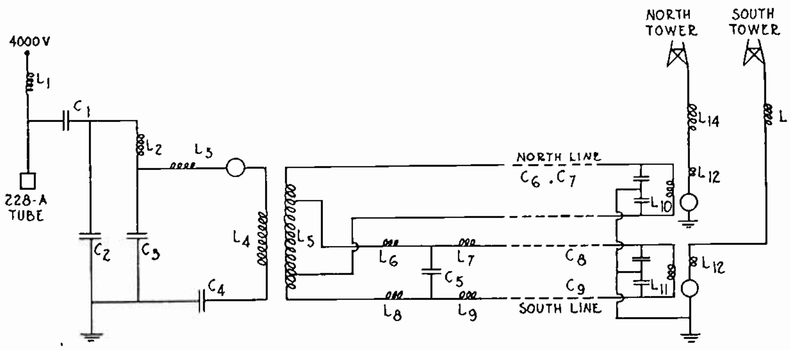

Fig. 11—Schematic diagram of the WKRC directional antenna system.

thirty degrees. The additional eighty degrees is obtained by properly adjusting the artificial line. With this arrangement, the field intensity distribution in a horizontal plane, a pear-shaped pattern, satisfies the requirements of this case.

If now appears that the use of this type of antenna will become more general in the future. Providing a directional antenna is properly designed and installed, it is possible to predict accurately its space pattern in advance.[19,20] However, it is not always possible to predetermine the efficiency of such an antenna system. Directional antennas should find wide application when and if synchronized station operation becomes more general.

[18] F. G. Kear, "Phase synchronization in directive antenna arrays with particular application to the radio range beacon," *Bur. Stand. Jour. Res.*, vol. 11, pp. 123–139; July, (1933).

[19] G. L. Davis and W. H. Orton, "Graphical determination of polar pattern of directional antenna systems." *Bur. Stand. Jour. Res.*, vol. 8, pp. 555–569; RP435 May, (1932).

[20] G. C. Southworth, "Certain factors affecting the gain of directive antennas," PROC. I.R.E., vol. 18, pp. 1502–1536; September, (1930).

Tower Lighting and Painting Requirements

Recommendations governing the marking of obstructions in the vicinity of airports and airways in the United States are set forth in detail in Aeronautics Bulletin No. 16.[21] The general requirements, as they affect radio stations, are summarized below.

Skeleton towers should be painted throughout their height with either alternate bands of chrome yellow or international orange (yellow No. 4 and orange yellow No. 5, respectively, of Color Card No. 3–1) and black, or alternate bands of international orange and white, terminating with either chrome yellow or international orange bands at both top and bottom, depending on color combination used. The width of the chrome yellow or international orange bands should be one seventh the height of the structure for all structures less than 250 feet in height, and from thirty to forty feet for structures over 250 feet in height. The black or white bands should be one half the width of the chrome yellow or international orange bands.

For night marking an aircraft hazard, a red obstruction light consisting of a 100-watt lamp in a red waterproof globe should be mounted at the top of structure.

For radio towers, or towers having a network of wires between the towers, additional fixed red lights consisting of fifty-watt lamps in waterproof globes should be mounted on diagonal corners at the one-third and two-thirds points and so arranged as to be visible from any angle of approach.

Some areas which present a hazard to flying a civil airway may require obstruction marking for night flying by the use of lights of the high intensity fixed projector type. The high intensity fixed projectors should be twenty-four-inch parabolic units using 1000-watt lamps with lamp changers, should be pointed so as to envelop and outline the areas over which flying should be restricted, and should be elevated so that the luminous beams of light will intersect at the height of the obstructions to be cleared. In addition, such hazardous flying areas should be marked with one or more certified landmark beacons as conditions may require to give pilots a long range warning. Such beacons should be similar to the 300-millimeter airways electric code beacons of the double Fresnel lens type with two 500-watt lamps and aviation red color shades, showing not less than six flashes per minute and having a luminous period of not less than 35 per cent. As an alternate system of marking such hazardous flying areas, certified 24-inch rotating landmark beacons equipped with 1000-watt lamps and lamp changers and

[21] Copies may be obtained, without charge, upon request from the Aeronautics Branch, Department of Commerce, Washington, D. C.

with red cover glasses and making six revolutions per minute, may be used.

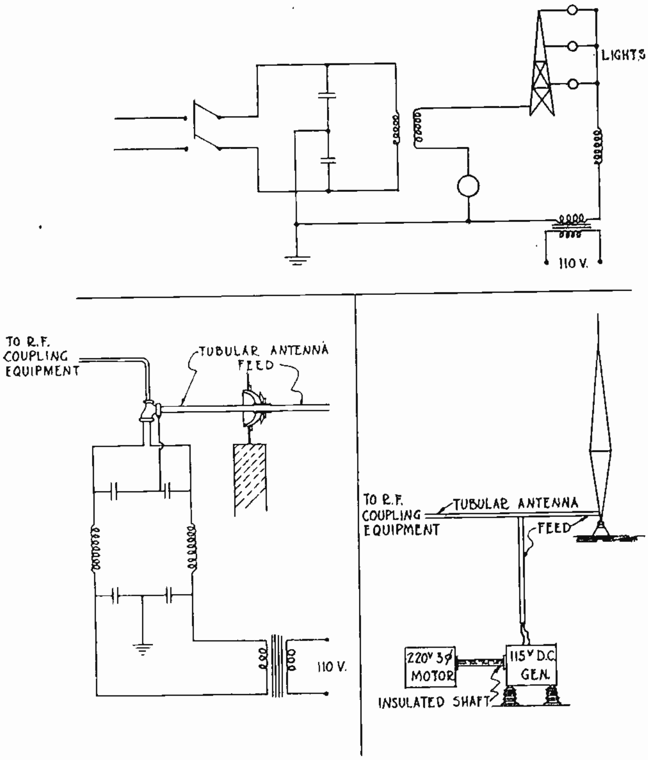

Fig. 12—Three lighting circuits used with vertical radiators.

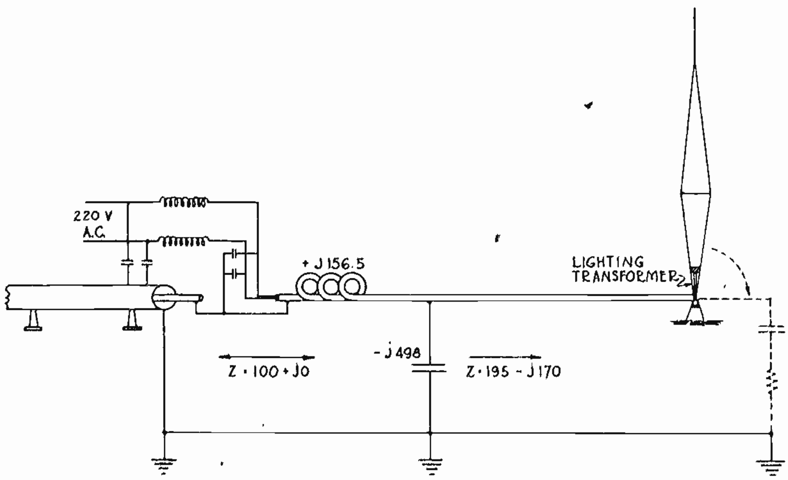

Fig. 13—WLW tower lighting circuit.

All lights marking hazardous flying areas should be exhibited from sunset to sunrise.

At the present time each radio station is being treated as a special case. The regulations outlined above are advisory only. At a new American installation, it is now necessary to submit to the Federal Communications Commission, the plans for lighting and painting of radio towers. The submitted plans may be approved, or they may be returned with additional requirements which must be fulfilled.

Due to the tower obstruction lighting requirements, it is sometimes necessary to furnish as much as four kilowatts of power for lighting the obstruction lamps. In the case of vertical radiators, these lamps are located on a structure at radio-frequency potential, and means must be provided for isolating the lighting circuits on the tower from ground. Figs. 12 and 13 indicate four methods which have been successfully used for this purpose. Use of the insulated generator as shown in Fig. 12 is no longer necessary, since there are commercially available chokes capable of isolating potentials up to 10,000 volts at broadcast frequencies.

Lightning Protection

A tower radiator tends to collect static electrical charges, and is subject to direct lightning hits when an electrical storm takes place in its vicinity. Due regard must be given this point in the design of coupling circuits used to transfer power from the transmitter, or transmission line, to the antenna. It is considered excellent practice to design the coupling network so as to operate the antenna through a very low resistance direct-current path to ground. If this is not feasible, a low resistance choke should be shunted from the antenna leadin to ground. This choke should have a relatively high impedance at the operating frequency. Lighting gaps across insulators are essential, and should have a low impedance path to ground.

Antenna System Costs

Indicative of present costs of antenna systems employing either the guyed type or self-supporting type of tower, are Figs. 14 and 15. The current costs indicated on these graphs are based on average conditions in the field within plus or minus ten per cent. In order to allow a station engineer to estimate more intelligently costs involved in the design of a complete antenna system, the following items are listed which should be included in his estimate: structural steel, insulators, ground system, foundations, erection, obstruction lighting equipment, painting, freight, insurance, and engineering expense.

These items are included in the estimates shown in Figs. 14 and 15. In a particular installation, one or more of the items indicated above may run considerably more than the average. This is particularly true in the cases of foundations, erection, and obstruction lighting.

There has been a lack of coördination between radio engineers and tower manufacturers in the past. As a result of this, insulators having

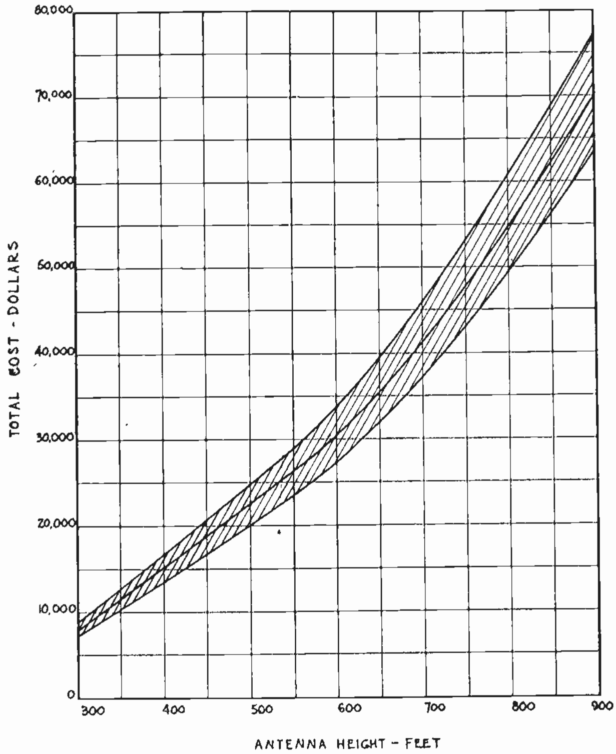

Fig. 14—Estimated cost of guyed vertical radiator complete and ready for operation.

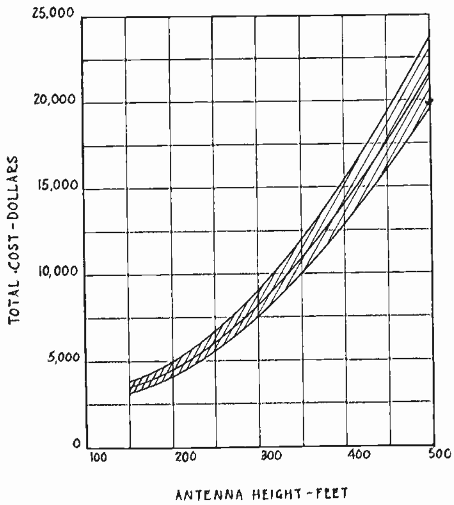

Fig. 15—Estimated cost of self-supporting vertical radiator complete and ready for operation.

an electrical safety factor much greater than necessary have been supplied. As radio engineers and insulator design engineers work closer together, insulator costs, and thus antenna system costs, will be decreased.

Operating Economy

It should be stated here that the most economical method of increasing the coverage of a broadcast station can usually be obtained by improving the antenna system. As an example of antenna economics, consider the following hypothetical case. A one-kilowatt regional station desires to improve its coverage. It applies to the Federal Communications Commission for a power increase to two and one-half kilowatts. If the application is granted, the average station incurs the following expenses:

Litigation	$1,500
New transmitting equipment, complete with tubes and power supply	15,000
Miscellaneous modifications	1,000
Installation of equipment	1,500
	$19,000

The annual operating expenses of the station will be increased by $3000, plus amortization and interest on the above capital expenditure. Even then, the license for the increased power may allow only daytime operation at two and one-half kilowatts.

That is the dollars-and-cents story of the power increase, but what does the millivolt and listener side of the picture show? The average one-kilowatt regional station in the United States now has an effective signal of 125 millivolts per meter at one mile. For 2.5 kilowatts this signal is increased to 198 millivolts, a four-decibel gain in signal intensity. A similar gain in signal can be made by the installation of a 0.25 wavelength self-supporting vertical radiator, whose cost will depend on the operating frequency. The estimated complete installed cost is as follows:

1500 kc	$4,200
1000 kc	6,000
600 kc	14,000

The station would have saved money by purchasing a new antenna rather than higher powered transmitting equipment. A $3,000 a year saving would have been made since no increase in operating costs would be included with the new antenna. The signal increase would have been full time rather than part time.

A Practical Example

A guyed cantilever type vertical radiator, 0.47 wave high, was recently placed in operation at radio station WBT, Charlotte, North Carolina, operating on 1080 kilocycles with a power of fifty kilowatts. This tower replaced a T type conventional antenna suspended between two grounded steel towers two hundred feet high, and five hundred feet apart. The old grid type ground system was replaced by a much larger

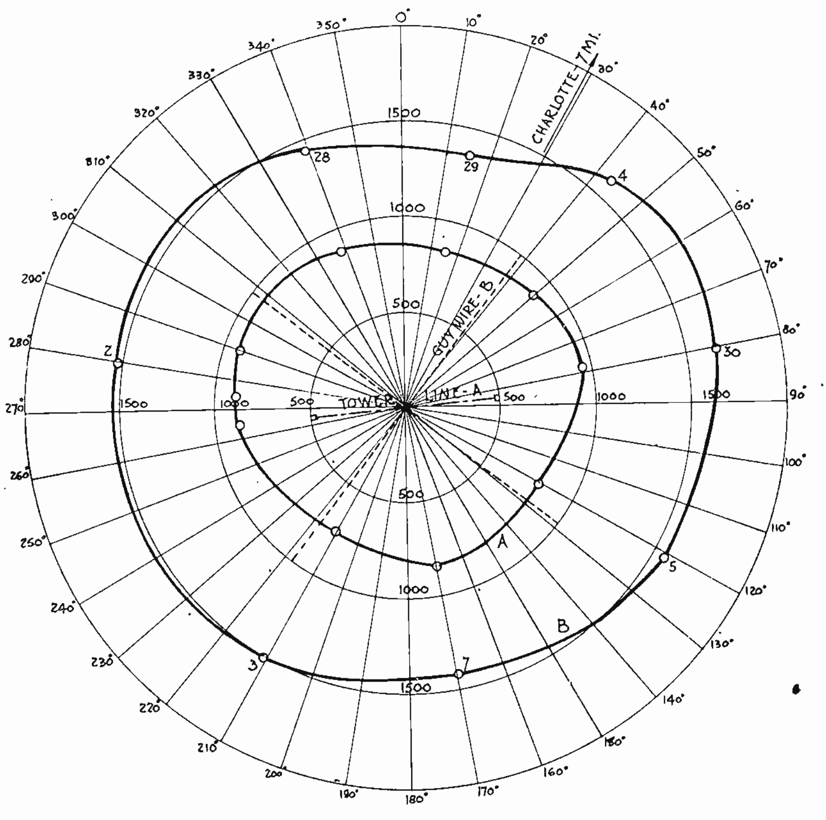

Fig. 16—WBT antenna pattern—millivolts per meter at one mile, power, 50 kilowatts; frequency 1080 kilocycles.

buried ground system. Radials are placed three degrees apart at the base of the mast and extend out 0.35 of a wavelength. A copper mat, 40 feet square, is installed at the base of the tower to reduce to a minimum dielectric losses which take place at this point. Referring to Fig. 16 it can be seen that the use of this new mast antenna has resulted in a signal increase of 77 per cent, equivalent to a power increase from 50 to 150 kilowatts. The fading wall of the station was moved out approximately eight miles, daytime coverage of the station has been increased appreciably, and in addition the secondary service signal intensity has been increased 77 per cent. Reports and field intensity measurements both indicate a marked improvement in the nighttime and daytime service range of this station. This is an excellent example of obtaining maximum transmission efficiency at minimum cost, as to obtain the same increased coverage using the old antenna would have required the installation of a high power amplifier and associated equipment at a cost equal to at least ten times the cost of the new antenna, plus a high additional annual operating cost. Even with such an increase in the transmitter power, the gain in coverage would not have been as great because of the smaller distance to the fading wall.

This example should indicate the economic importance of proper antenna design and the necessity of a careful balance of antenna cost with respect to the complete transmitting plant.

Conclusions

1. Because of their physical configuration, and because tower radiators are built over earth of finite conductivity, current distribution is not sinusoidal. Thus such towers do not perform as would a single vertical wire over perfect earth. At some installations a very close approximation to the ideal is attained.

2. The use of steel structures for merely supporting simple broadcast antennas is outmoded by the vertical radiator.

3. Use of a vertical radiator in place of an older conventional antenna will, in the average case, produce a signal increase equivalent to doubling the power of the transmitter. The exact signal gain will depend upon the efficiency of the existing antenna and the height of the new antenna.

4. The self-supporting radiator may be used effectively up to heights of at least 0.5 wavelength. It can be confidently expected to perform with approximately the same degree of efficiency as the guyed type of antenna.

5. If the self-supporting tower antenna is used, precautions must be taken to prevent excessive dielectric losses in the soil near the tower base. A high base capacitance, of itself, does not contribute to low efficiency.

6. The ground system should be radial in nature, should be buried close to the earth's surface, and should have a maximum radius consistent with economic considerations. In practice, this should mean a radius of at least one-half wavelength and at least one hundred and twenty radials.

7. It is desirable to increase the nonfading area of a transmitter as

much as possible. This is an important factor of design which has not been completely solved. There have been a number of studies which indicate that the optimum height, from the fading viewpoint, ranges between 0.5 and 0.6 of a physical wavelength with the types of radiators so far placed in service. The exact height depends upon the configuration of each individual tower and the soil condition.

8. The vertical radiator is especially well adapted for use in directive antenna systems. Published theoretical methods of pattern calculation agree with field results. However, the efficiency of directive antennas cannot always be predicted and in some cases this determination must be based upon engineering experience.

9. Several modifications of the mast type antenna have been suggested, or have actually made an appearance in the field, including various types of capacitive or inductive top loading, and towers employing insulated sections electrically connected to obtain a control of current distribution. Various advantages are claimed for such antenna modifications.[22] However, there is not sufficient information at this time to justify an accurate opinion of their value. There is a great amount of research work yet to be done and a continued improvement in broadcast antenna efficiency is to be expected.

[22] C. A. Nickle, R. B. Dome, and W. W. Brown, "Control of radiating properties of antennas, Proc. I.R.E., vol. 22, pp. 1362–1373; December, (1934).

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SOME COMMENTS ON BROADCAST ANTENNAS*

By

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Summary—*A constant phase and current antenna is suggested as having more desirable characteristics than sinusoidal distribution.*

It is shown that the maximum nonfading range is a function of the antenna height, frequency, and ground conductivity.

A modified antenna system is suggested which combines a high angle suppressor with a vertical constant phase and current antenna. It is shown that the modified system cancels the high angle lobe of radiation and increases the nonfading range.

THERE is a growing tendency in the broadcast group to accept five-eighths of a wavelength as the maximum desirable height for continuous radiators having sinusoidal current distribution. Ballantine showed[1] that for maximum ground field per watt input the vertical height should be five-eighths of a wavelength. More recently, he has shown that if a maximum nonfading range is desired, the height should be more nearly 0.54 wavelength.

Numerous vertical radiators of the self-radiating type have been constructed and are now in operation. The actual results obtained from the majority of these radiators deviate considerably from the predicted values obtained by calculation based on perfect earth and sinusoidal current distribution. The gap between the actual and predicted results is generally explained by imperfect earth conductivity and nonsinusoidal current distribution. It has been shown that the radiator cross section determines the current distribution to an important degree and if a sinusoidal current distribution is desired the radiator must have a uniform cross section throughout its entire length.

It is probable that a distinct effort will be made to obtain more nearly all of the results predicted by theory for antennas of heights between one fourth and five eighths of a wavelength by proper antenna site selection, ground systems, and close quivalents to uniform cross section of radiator.

It is proposed in this paper to show that:

(1) The ground field strength of a self-radiating vertical radiator is proportional to the square root of the antenna height, if the amplitude and phase of the current is constant throughout the entire length.

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[1] Stuart Ballantine, "On the radiation resistance of a simple vertical antenna over perfect earth," Proc. I.R.E., vol. 12, pp. 823–832; December, (1924).

(2) A constant amplitude of current throughout the radiator is more effective than a sinusoidal distribution.

(3) The optimum antenna height for maximum nonfading range is greater than five eighths of a wavelength and is determined by (a) frequency, and (b) ground conductivity.

An extensive group of antenna experiments were made at Westinghouse Station KDKA, near Saxonburg, Pa., in the spring of 1933. The mechanical support for the various antennas which were tested was a small zeppelin type captive balloon.

During these tests a vertical antenna was developed which approximated a condition of constant phase and constant current amplitude

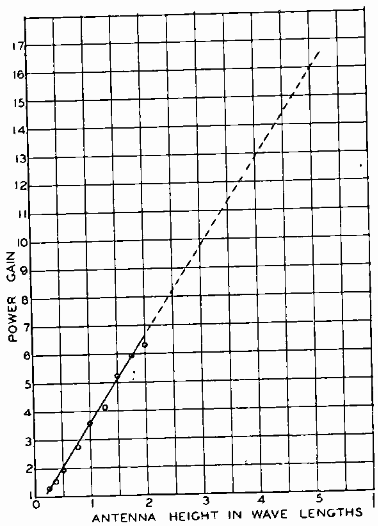

Fig. 1—Power gain vs. antenna height.

throughout its entire length. This current distribution was obtained by rephasing at approximately three-eighths-wave intervals. Rephasing was accomplished by opening the radiating element at the three-eighths-wave intervals and inserting a capacitive reactance equal to the total inductive reactance at these points. Physically these antennas were aluminum wire with sectionalizing insulators at three-eighths-wave intervals and condensers paralleling the insulators.

Data on this antenna were obtained for various heights up to approximately two wavelengths on a frequency of 980 kilocycles. From these data and resultant calculations Fig. 1 was drawn. This curve, plotted with power gain against antenna height in wavelengths, shows that the power gain is proportional to antenna height. The experimental points check with reasonable accuracy when it is considered that considerable difficulty was experienced in maintaining such long antennas vertical by means of the captive balloon. These data covering from two to five wavelengths were calculated and represent the ratios of the area of a semicircle to the area enclosed under the voltage pattern in the vertical angles for the various antenna heights under consideration. The voltage patterns for the vertical angles for antenna heights from one-eighth to four wavelengths are shown on Figs. 2 to 7 inclusive. The calculations for these patterns are based on a constant phase and current amplitude throughout the entire length of each an-

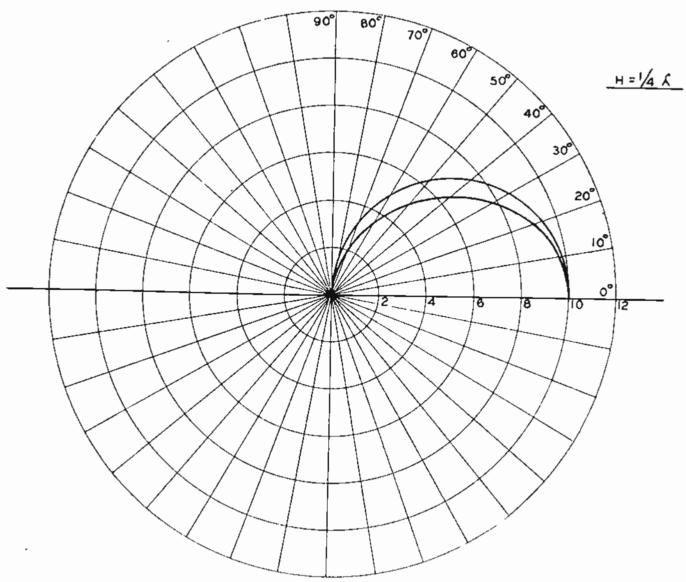

Fig. 2—Vertical radiation pattern for one-quarter-wave antenna.

tenna and for an earth of perfect conductivity. The formula (1) for radiation resistance is

$$R=60[G\ Si(2G) + 1/2 \cos 2G + 1/4G \sin 2\ G-1]$$

where,

R is in ohms
$G=2\pi h/\lambda$
h = height of antenna
λ = is the wavelength
Si = denotes the sine integral

and that the field intensity in millivolts per meter per kilowatt at one mile for any angle θ above the horizon is given by,

$$E = \frac{1180}{\sqrt{R}}\left[\frac{\cos\theta \sin(G \sin\theta)}{\sin\theta}\right]$$

An examination of Figs. 2 to 7 indicates that, (1) at antenna heights from zero to one-half wavelength, only one lobe of radiation is present,

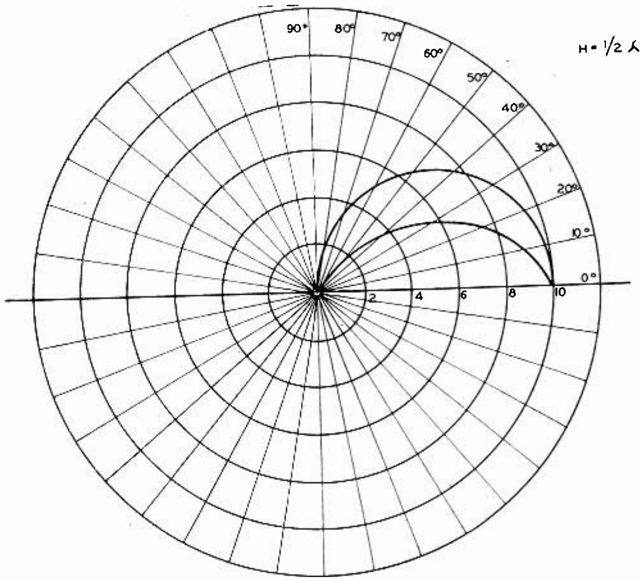

Fig. 3—Vertical radiation pattern for one-half-wave antenna.

the shape of which tends to become flatter as the height approaches one-half wavelength; (2) at antenna heights greater than one-half

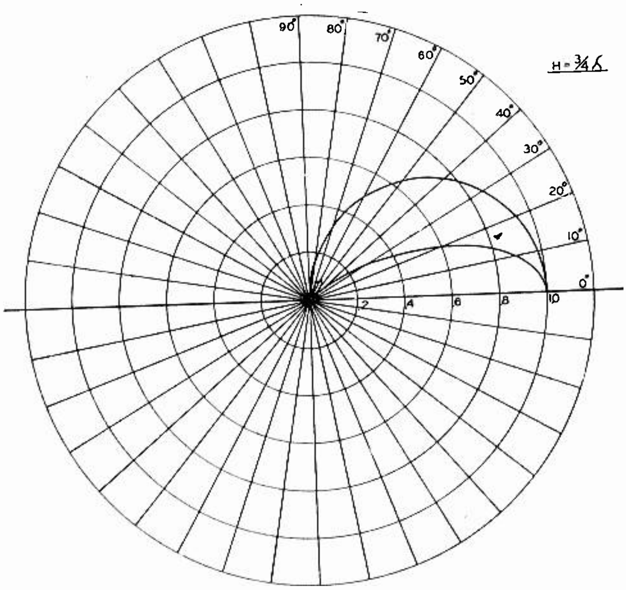

Fig. 4—Vertical radiation pattern for three-quarter-wave antenna.

and less than one wavelength, two lobes of radiation are present, a large low angle and a small high angle. However, as the height is increased from slightly above one-half to slightly below one wavelength the low angle lobe becomes flatter and the high angle one rotates continuously to lower angles and increases somewhat in amplitude; (3)

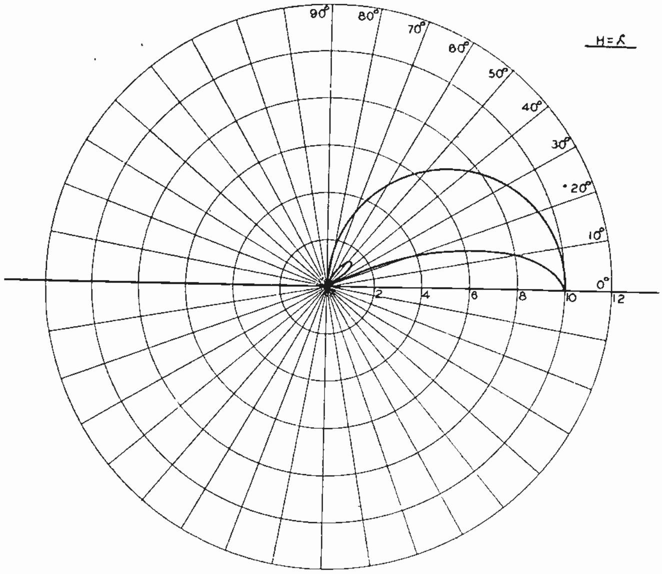

Fig. 5—Vertical radiation pattern for one-wavelength antenna.

with each additional half wavelength of height another high angle lobe of radiation is added with continual thinning of the major low angle lobe.

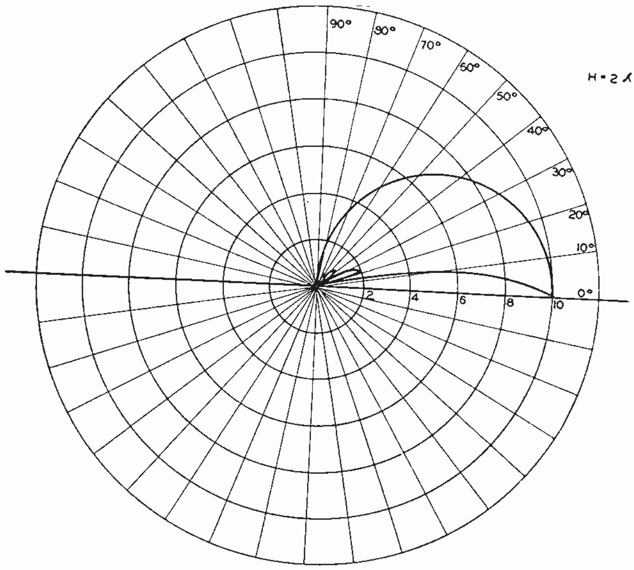

Fig. 6—Vertical radiation pattern for two-wavelength antenna.

A comparison of these curves for antenna heights between one-half and one wavelength shows that the amplitude of the high angle lobe is considerably smaller than for antennas for similar height but having sinusoidal current distribution.

It is noteworthy that high angle radiation from a loss of power standpoint is negligible but from a nonfading range standpoint high angle radiation is of great importance and therefore the limiting factor.

The value of any given antenna is usually based on ground field per watt input and nonfading range.

The ground field may be calculated from (1) and (2) for any antenna height. The nonfading range may arbitrarily be defined as

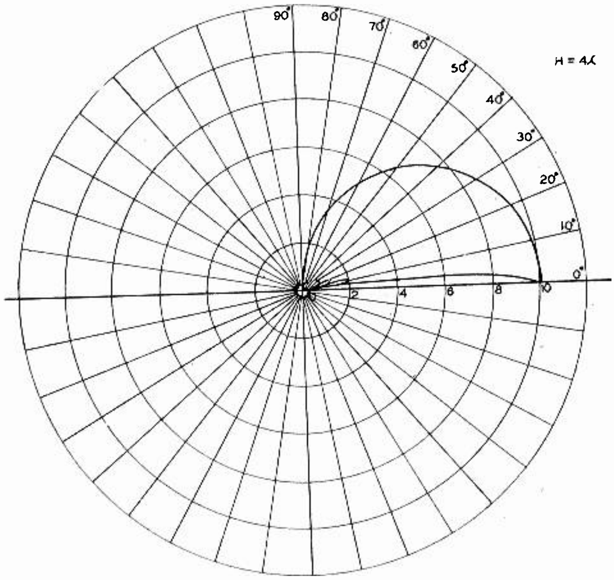

Fig. 7—Vertical radiation pattern for four-wavelength antenna.

the closest point to the transmitter where ground and sky signal have equal intensity. Figs. 8 to 15, inclusive, were calculated and drawn to show the ground and sky signal against distance from the transmitter, for 50 kilowatts radiated at 600-, 1000-, and 1400-kilocycle frequency, and antenna heights from one-eighth to four wavelengths. The ground signal is based on earth conductivities of 1.07×10^{-13} and 3.22×10^{-14} which approximate the range of ground conductivity found in the United States.

The sky-wave signal is computed on the basis of (1) a reflecting layer height of 100 kilometers, (2) a reflection coefficient proportional to the cosine of the angle of incidence, (3) and a vertical receiving antenna.

An examination of Figs. 8 to 15 shows the importance of the various variables chosen, as well as the high degree to which high angle radiations limit the nonfading range of the various antennas shown.

It may be concluded from these curves that there is an optimum antenna height for each frequency and ground conductivity. It does not necessarily follow that the optimum height for one frequency and conductivity is the same for a different frequency and conductivity.

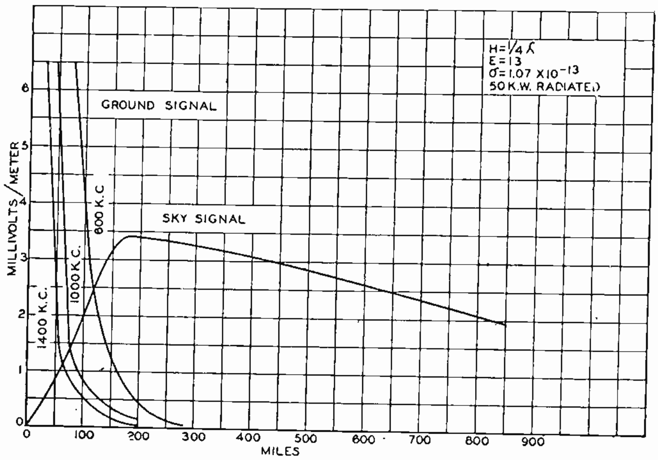

Fig. 8—Ground and sky signal vs. distance for one-quarter-wave antenna.

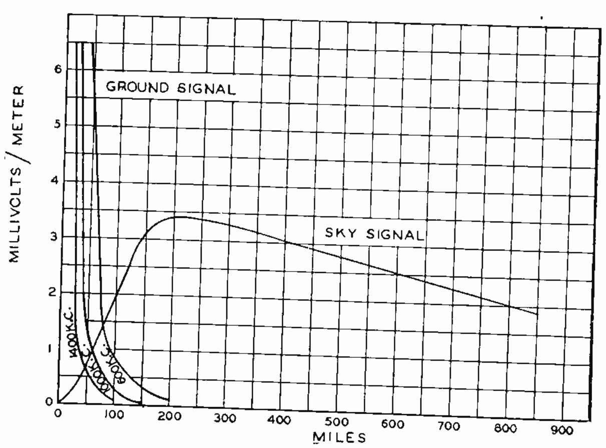

Fig. 9—Ground and sky signal vs. distance for one-quarter-wave antenna, $-h=\frac{1}{4}$, $E=12$, $\sigma=3.22\times10^{-14}$ for 50 kilowatts radiated.

Figs. 16 to 18 inclusive show nonfading range against antenna height for two conductivities and three different frequencies in the broadcast band.

Examination of Fig. 16, which is for a frequency of 600 kilocycles, shows that with an antenna height of three fourths of a wavelength and a conductivity of 1.07×10^{-13}, the first peak in nonfading range is 204 miles. If the antenna height is increased to one wavelength the nonfading range is decreased to 180 miles. A further increase in antenna height up to two wavelengths with the same conductivity produces an increase in the nonfading range to approximately 270 miles. It will be noted that little is to be gained in nonfading range by increasing the antenna height beyond two wavelengths. The curve for 600 kilocycles

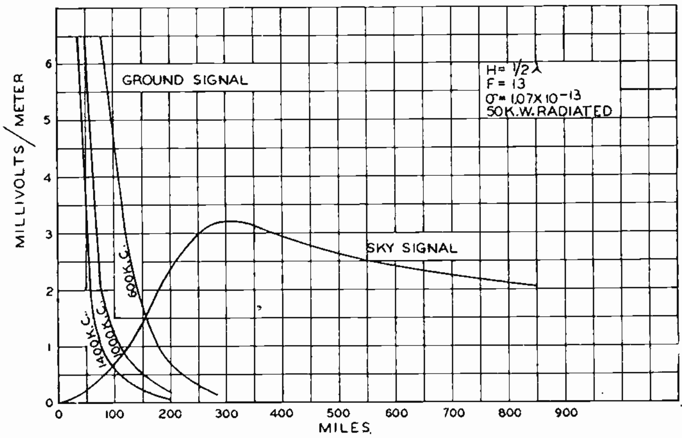

Fig. 10—Ground and sky signal vs. distance for one-half-wave antenna.

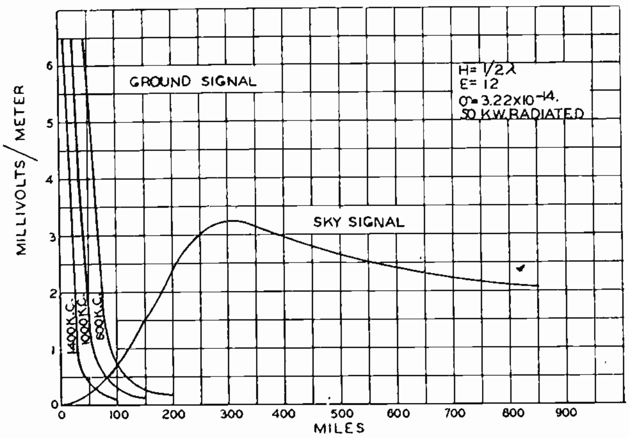

Fig. 11—Ground and sky signal vs. distance for one-half-wave antenna.

and 3.22×10^{-14} conductivity shows the first maximum at three fourths of a wavelength and a distance of 182 miles. Further increase of antenna height causes a sharp decrease in nonfading range to a minimum of 116 miles at one wavelength. A second peak is reached at one and one-half wavelengths and 174 miles, with a second minimum at one and three fourths of a wavelength and 130 miles. However, the fading range from one and three fourths to five wavelengths does not exceed that obtained from the three-fourths wavelength.

Analysis of Fig. 17 for 1000 kilocycles shows a first peak in nonfading range for an antenna height at three-fourths of a wavelength at 182 miles with 1.07×10^{-13} conductivity. With a conductivity of 3.22×10^{-14} the maximum peak is at two wavelengths, only fourteen

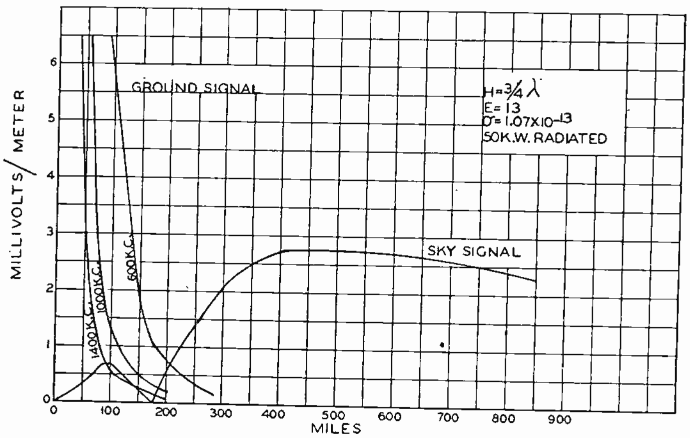

Fig. 12—Ground and sky signal vs. distance for three-quarter-wave antenna.

miles more than the second peak at 1.25 wavelengths, the latter only about thirty-eight miles beyond the half-wave point. Thus, it will be seen that nothing is to be gained in a nonfading range by increasing the

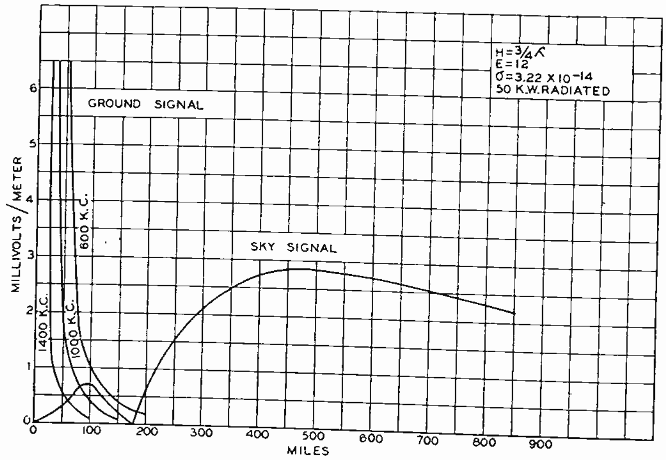

Fig. 13—Ground and sky signal vs. distance for three-quarter-wave antenna.

antenna height from one-half to three-fourths at 1000 kilocycles with a conductivity of 3.22×10^{-14}.

Fig. 18 for 1400 kilocycles shows the first peak in the nonfading range at one half of a wavelength, a second at one and one-fourth and a third at two for a conductivity of 1.07×10^{-13}. Also, a first peak at one-half with a gradual rise from three-fourths to one and three-fourths wavelengths for a conductivity of 3.24×10^{-14}.

As stated at the beginning of this paper, if the antenna height lies between one-half and one wavelength the vertical radiation pattern is

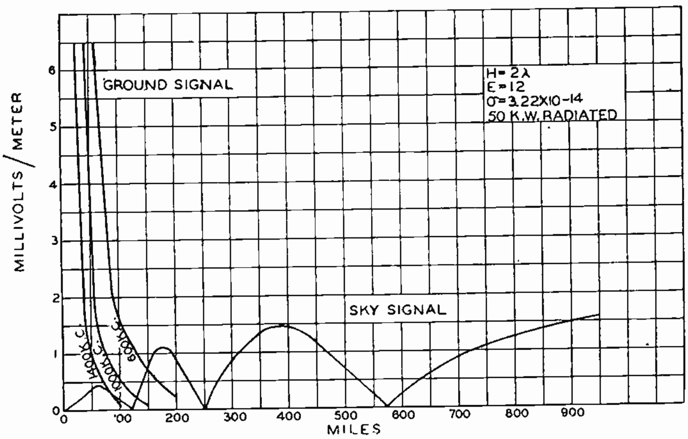

Fig. 14—Ground and sky signal vs. distance for two-wavelength antenna.

composed of one small high angle lobe and a large low angle one. It is obvious that the small height angle lobe has little affect on the overall efficiency but is the factor which limits the nonfading range. Thus, to increase the nonfading range, the solution is to eliminate the high

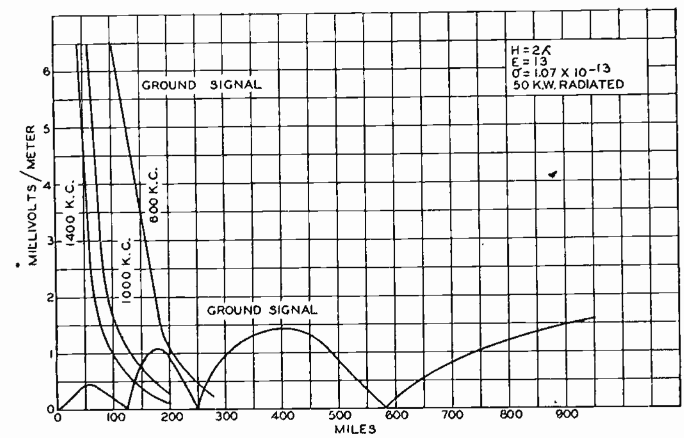

Fig. 15—Ground and sky signal vs. distance for two-wavelength antenna.

angle lobe. Since a small spurious high angle lobe of radiation is inherent in the constant phase and constant current amplitude type of vertical antenna it would seem that its elimination is best obtained by an auxiliary radiator which radiates only a high angle lobe identical to that of the main radiator but in phase opposition. Since the exact shape, amplitude, and angle of this high lobe must be duplicated, the auxiliary antenna must have variables in its construction which will permit adjustments to shape, amplitude and angle of the radiation.

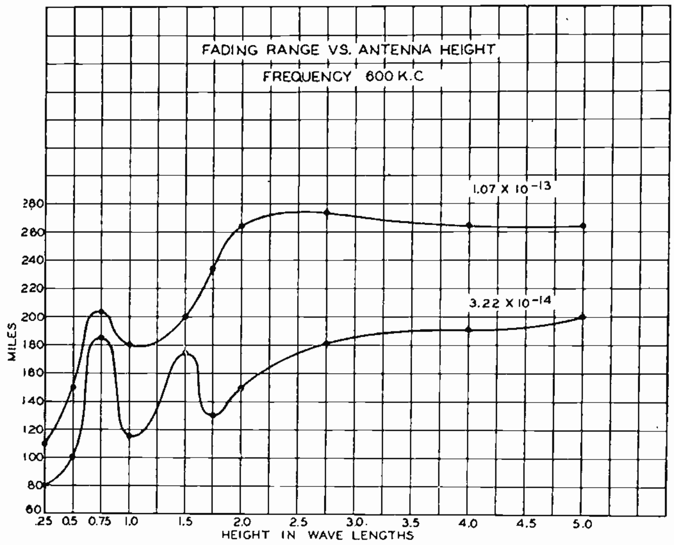

Fig. 16—Fading range vs. antenna height for a transmitting frequency of 600 kilocycles and a ground conductivity of 1.07×10^{-13} and 3.22×10^{-14}.

Frank Conrad, assistant chief engineer of the Westinghouse Electric and Manufacturing Company, developed a high angle radiator for KDKA in 1929 which had all of the above desired characteristics. This

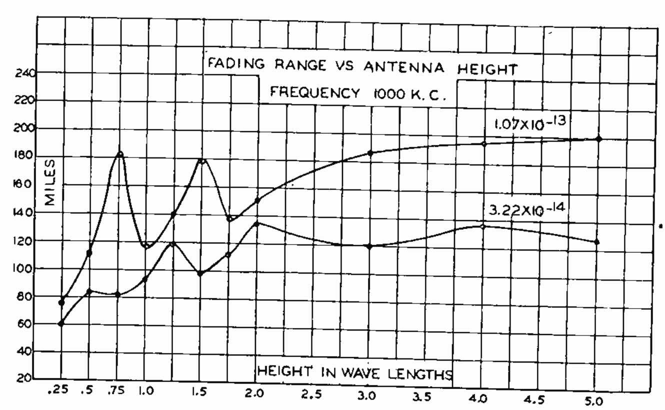

Fig. 17—Fading range vs. antenna height for a transmitted frequency of 1000 kilocycles and a ground conductivity of 1.07×10^{-13} and 3.22×10^{-14}.

antenna consisted of eight low vertical antennas spaced around a circle. By varying the diameter of the circle, the height of the individual radiators, and the phase of the current in them, the desired control of the high angle radiation was obtained.

As the auxiliary radiator takes only a small portion of the total input power to cancel the high angle lobe of the main radiator the over-all efficiency of the combined radiators is only slightly affected.

An examination of the nonfading range for antennas between one-

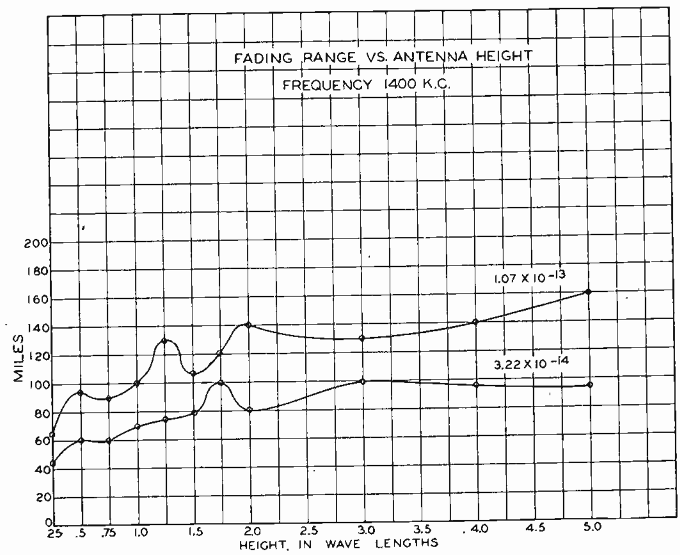

Fig. 18—Fading range vs. antenna height for a transmitted frequency of 1400 kilocycles and a ground conductivity of 1.07×10^{-13} and 3.22×10^{-14}.

half and one wavelength with the high angle lobe of radiation removed reveals that a very considerable increase in coverage will be obtained. It may also be mentioned that the width of the intense fading area will be quite narrow for this type of antenna as the sky-wave signal strength from the main low angle radiation rises rapidly giving a cutoff effect for the ground wave in a relatively few miles.

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A CRITICAL STUDY OF THE CHARACTERISTICS OF BROADCAST ANTENNAS AS AFFECTED BY ANTENNA CURRENT DISTRIBUTION*

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Summary—*The action of broadcast antennas with various current distributions is examined in an endeavor to determine the combinations which are most likely to be useful. In each case, the vertical radiation characteristics, the radiation resistance, and the electric field intensity at the surface of the earth one mile from the antenna are determined. The relative merits with respect to fading suppression are also considered.*

Of the several arrangements examined, some have been very disappointing. In no case where the total antenna height is at all reasonable for broadcast use does the field strength at one mile go to exceptionally high values. For heights of the order of a half wavelength, it is hard to find anything better than a straight vertical wire. Some of the arrangements save a small amount in height, but in no other respect do they improve on the straight wire.

The analysis of the antenna with decreased velocity and antennas spread in the horizontal plane shows that both are definitely ruled out of the picture.

The results show that the Franklin antenna would be very useful if it were not for the great heights required.

I. Introduction

In THE past several years, the antenna systems of broadcast stations have received a great deal of attention. A decade ago, the antenna system generally consisted of a T-shaped antenna suspended between two towers. This type of antenna gave no great control over the vertical radiation pattern. In an attempt to gain this control, a single tower was used as the antenna proper. Antennas of this type were extended to great heights compared to previous practice. Recent studies have shown that there is still room for a great deal of improvement in the design of these structures, since the current distributions on these towers generally differ a great deal from the sinusoidal distribution which had been assumed in the theoretical studies which led to the use of these tall structures. Recently, engineers have shown a great deal of interest in the possibilities of so controlling the current distribution on the antenna that desirable characteristics may be obtained with antennas of rather small height. It is the purpose of this investigation to examine the action of antennas with various current distributions, in an endeavor to determine the combinations which

* Decimal classification: R320. Original manuscript received by the Institute, August 19, 1935.

are most likely to be useful. In each case, we shall calculate the vertical radiation characteristics, the radiation resistance, and the electric field intensity at the surface of the earth one mile from the antenna when one watt of power is fed into the antenna. In the calculation of the electric intensity, the earth will be considered as a perfect conductor, and the antenna system will be considered to be 100 per cent efficient.

II. Method of Determining the Antenna Characteristics

Let us suppose that our antenna is a vertical wire above a perfectly conducting earth. The current is distributed as shown in Fig. 1. We are interested in the electric intensity, F_θ, at a remote point, P, a distance,

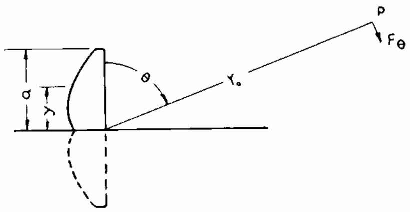

Fig. 1

r_0, from the base of the antenna. The angle between the line drawn from P to the antenna base and the antenna axis is θ. Then we can write

$$F_\theta = +j\frac{60}{r_0} I_0 K f(\theta) \epsilon^{-jkr_0} \tag{1}$$

where I_0 is the current at some reference point along the antenna. This reference point is entirely arbitrary, but is usually chosen either at the base of the antenna or at the point of greatest current. K is called the *form factor* of the antenna and $f(\theta)$ is the *vertical radiation characteristic.* When $f(\theta)$ is plotted on polar coördinate paper, it is sometimes referred to as the *vertical radiation pattern.* The above quantities are defined by the following relations:[1]

$$K = \left[k \sin\theta \int_{y=0}^{y=a} \frac{i_y}{I_0} \cos(ky\cos\theta)dy\right]_{\theta=90^\circ}$$

$$= \left[k\int_{y=0}^{y=a} \frac{i_y}{I_0}\, dy\right] \tag{2}$$

and,

$$f(\theta) = \left[k \sin\theta \int_{y=0}^{y=a} \frac{i_y}{I_0} \cos(ky\cos\theta)dy\right] \Big/ K \tag{3}$$

where,

λ = wavelength
$k = 2\pi/\lambda$
i_y = current in the antenna at a point y units from the ground.

It should be noted that $f(\theta)$ is unity when θ is 90 degrees. In (1), F_θ is given in volts per centimeter when the current is measured in amperes and r_0 is measured in centimeters. If we replace the constant, 60, by 37.25 and measure r_0 in miles, the field strength is then given in millivolts per meter.

The radiation resistance at the reference point and the field intensity at one mile for a given power input are calculated by methods previously described.[1]

It is difficult to determine the relative merits with respect to fading of a number of antenna arrangements by merely inspecting the vertical radiation characteristics. These merits will be weighed in a slightly different fashion. It is assumed that the Heaviside layer height is 100 kilometers (62.5 miles) and that the layer has an efficiency of reflection of 33.3 per cent. We can then compute the magnitude of the reflected ray at any point on the surface of the earth if we know the vertical radiation characteristic of the antenna. We will designate this intensity as F_2. (Fig. 2.) The value of the direct or ground ray, designated

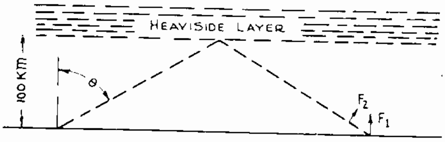

Fig. 2

by F_1, is next computed. We will assume an earth conductivity of 50×10^{-15} electromagnetic units and a wavelength of 300 meters. The ratio F_2/F_1 which is practically zero close to the antenna grows larger as we proceed away from the antenna, until F_1 is negligible compared to F_2. We will define d_1 as the distance along the ground from the antenna to the point where the ratio F_2/F_1 is unity, and call this distance the "100 per cent fading distance." The distance to the point at which the ratio is 0.5 will be designated as $d_{1/2}$, and will be called the "50 per cent fading distance." In most of the cases which we will examine, the values of d_1 and $d_{1/2}$ will be computed. The value of earth conductivity and wavelength used represents average conditions in the broadcast band. While it is admittedly arbitrary to assume a particular Heaviside layer height and reflection coefficient, the procedure is justified in that it does give us a basis for comparing two different antennas.

[1] H. E. Gihring and G. H. Brown, "Tower antennas for broadcast use, Appendix A," Proc. I.R.E., vol. 23, pp. 342–348; April, (1935).

III. Vertical Wire with Sinusoidal Distribution of Current

The characteristics of the vertical wire antenna over a perfect earth have been known for many years. The results will be repeated here since we will use this antenna as a standard of comparison. Let us

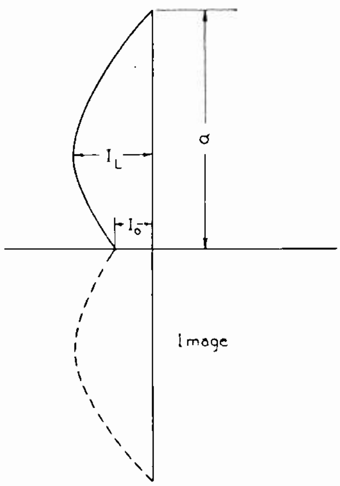

Fig. 3—Vertical wire antenna and image.

suppose that the current is distributed as shown by Fig. 3. Then the current distribution is

$$i_y = \frac{I_0 \cdot \sin (A - ky)}{\sin A} = I_L \sin (A - ky) \tag{4}$$

where I_0 is the current at the base of the antenna, and I_L is the magnitude of the loop current. When a is the height of the antenna and λ is the free space wavelength,

$$A = 2\pi a/\lambda. \tag{4a}$$

For the distribution in question, the vertical radiation characteristic is

$$f(\theta) = \frac{\cos (A \cos \theta) - \cos A}{\sin \theta [1 - \cos (A)]}. \tag{5}$$

This characteristic is plotted in Fig. 4 for a number of values of A. We see that the high angle radiation becomes a minimum when the value of A is about 190 degrees. When the antenna is taller than this, the secondary lobe of radiation becomes prominent. In fact, when the antenna is one wavelength high, there is no radiation along the ground. The form factor, K, referred to the loop of current is

$$K = 1 - \cos(A). \tag{6}$$

The radiation resistance referred to the loop current is

$$\begin{aligned} R_r(\text{ohms}) = 30 \Bigg[& -\frac{\cos(2A)}{2}\{C + \log(4A) - Ci(4A)\} \\ & + \{1 + \cos(2A)\}\{C + \log(2A) - Ci(2A)\} \\ & + \sin(2A)\left\{\frac{Si(4A)}{2} - Si(2A)\right\}\Bigg] \end{aligned} \tag{7}$$

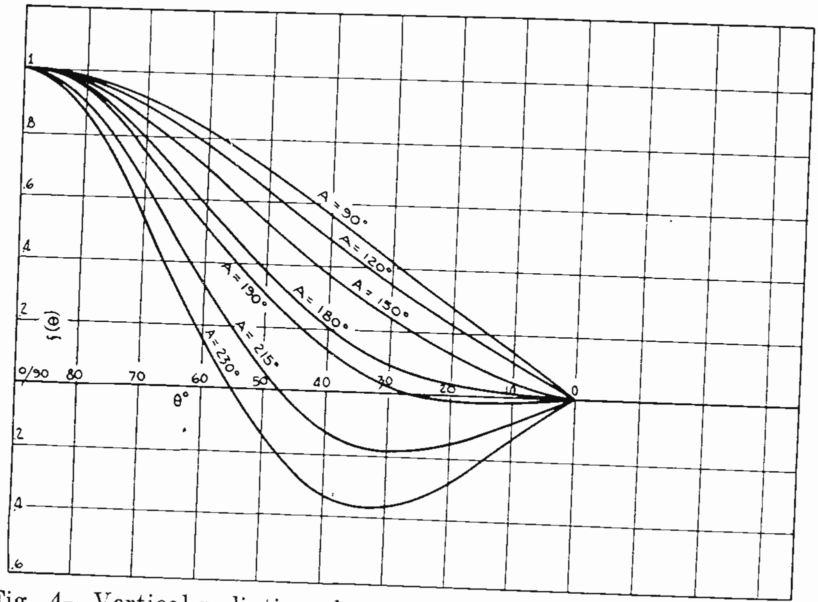

Fig. 4—Vertical radiation characteristics of vertical wire antenna.

where $C = 0.57721$ is Euler's constant, and $Ci(x)$ and $Si(x)$ are respectively the intergral-cosine and the integral-sine as defined on page 19 of the Jahnke-Emde "Funktionentafeln." Fig. 5 shows this radiation resistance plotted as a function of the height, A, in degrees. The field strength at one mile is also shown on this figure. We see that the maximum field strength is obtained when the antenna is 230 degrees tall. However, inspection of Fig. 4 shows that a 230-degree antenna has a rather large high angle lobe of radiation. Examination of the fading distance curve shown on Fig. 5 shows that an antenna whose height is about 190 degrees is the most desirable from the standpoint of fading reduction. For instance, a 190-degree antenna gives a field strength at one mile of 7.8 millivolts per meter for one watt input, while a 230-degree antenna gives 8.7 millivolts per meter, an increase of 11.0 per cent. At the same time, 100 per cent fading occurs at 100 miles if a 190-degree antenna is used, and at 60 miles if a 230-degree antenna is used.

IV. Vertical Wire with Capacity Hat at the Top

Let us now suppose that the vertical wire has a nonradiating capacity area at the top so that the current distribution is as shown in Fig. 6.

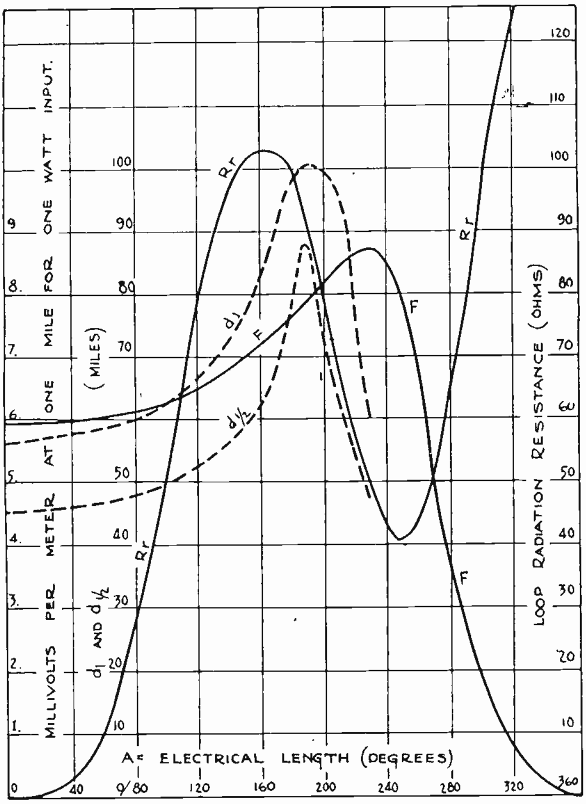

Fig. 5—Characteristics of the simple vertical wire antenna.

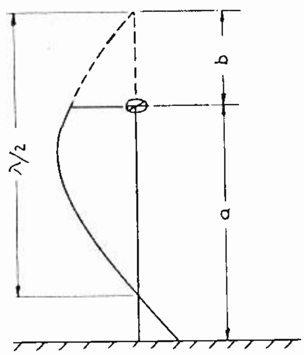

Fig. 6—The vertical wire antenna with a capacity hat at the top.

Then b is the length of the portion of sine wave suppressed by the capacity area. We define the quantities

$$\left.\begin{aligned} B &= 2\pi b/\lambda \text{ radians} = 360b/\lambda \text{ degrees} \\ A &= 2\pi a/\lambda \text{ radians} = 360a/\lambda \text{ degrees} \\ G &= A + B \end{aligned}\right\}. \tag{8}$$

Then $K = \cos(B) - \cos(G)$ (referred to the loop current) (9)

and,

$$f(\theta) = \frac{\cos B \cos(A \cos\theta) - \cos\theta \sin B \sin(A\cos\theta) - \cos G}{\sin\theta[\cos B - \cos G]}. \tag{10}$$

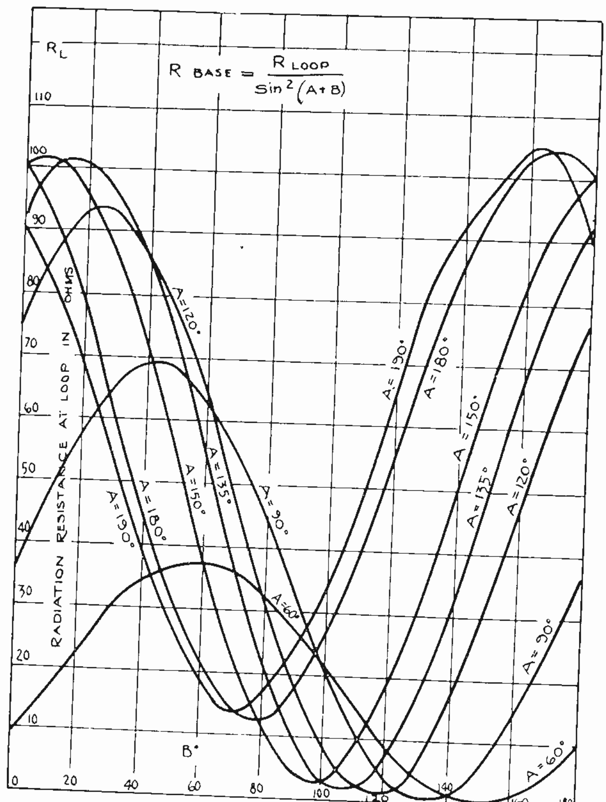

Fig. 7—Loop radiation resistance of the antenna with a hat at the top.

The radiation resistance (referred to the loop current) is

$$\begin{aligned} R_r(\text{ohms}) = 30 \Big[&\sin^2 B \left\{ \frac{\sin(2A)}{2A} - 1 \right\} \\ &- \frac{\cos(2G)}{2} \{C + \log(4A) - Ci(4A)\} \\ &+ \{1 + \cos(2G)\}\{C + \log(2A) - Ci(2A)\} \\ &+ \sin(2G) \left\{ \frac{Si(4A)}{2} - Si(2A) \right\} \Big]. \end{aligned} \tag{11}$$

The radiation resistance as a function of B, with A as a parameter, is shown in Fig. 7.

The field strength at one mile is shown by Fig. 8. We see that in all cases where the antenna is less than 200 degrees tall, the field intensity increases with B for a way, and then drops suddenly to zero, rising again almost as abruptly, The zero point occurs when

$$B = 180 \text{ degrees} - A/2. \qquad (12)$$

When B is zero, the vertical characteristic is given by the curves of Fig. 4. As B is increased the field pattern changes just as if the height of the antenna were increased. For purposes of illustration, the vertical characteristic of an antenna of height 150 degrees is shown by Fig. 9 for a number of values of B. We see that when B is about 47 degrees,

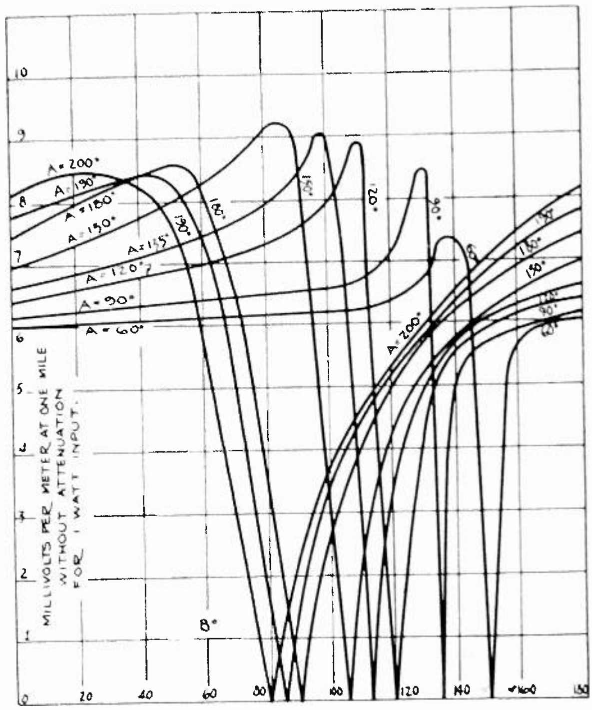

Fig. 8—Field strengths obtainable from the antenna with capacity hat at the top.

the vertical pattern is similar to that obtained with a straight vertical wire 190 degrees tall. For any height, A, we can find the value of B which will give the same vertical pattern as a 190-degree antenna by selecting the value of B which corresponds to a field strength of 7.8 millivolts per meter at one mile on Fig. 8. Since the curve of field intensity peaks, it is important to note that the correct value of B lies on the left side of the peak.

This method seems important, since we are able to simulate a 190-degree antenna with much less height. In many cases, we shall desire a value of B so large that it cannot be obtained by a simple capacity area at the top. The desired result can, however, be obtained by inserting an inductance between the top of the vertical wire and the capacity area. Such a scheme was first disclosed by van der Pol[2] many years ago. To obtain the results predicted by theory, it is necessary that inductance so inserted have a very low resistance, since the current at the coil point may be many times the current at the base. To be specific, let us assume that the coil has one ohm of resistance. Let the capacity area and the inductance of the coil be such that the antenna is adjusted to give a vertical field pattern similar to that of a 190-degree antenna. Let us assume that 1000 watts of power is fed into the an-

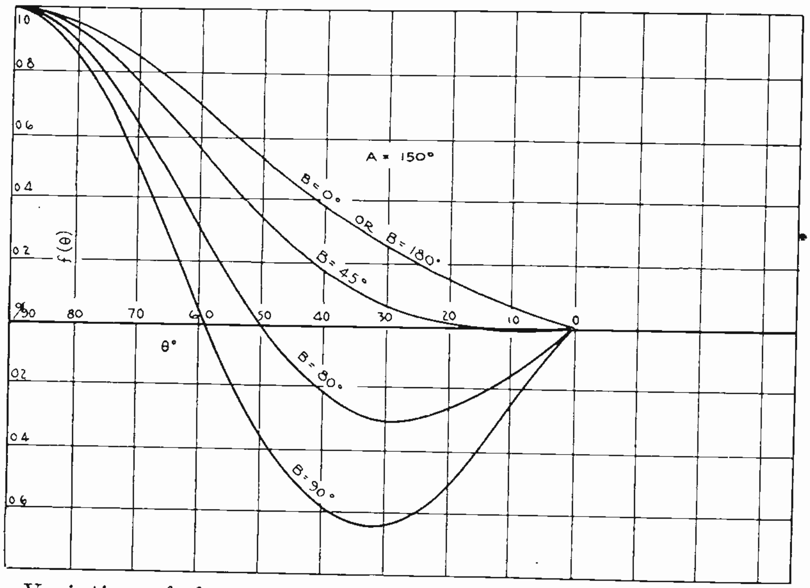

Fig. 9—Variation of the vertical radiation characteristics with top loading.

tenna. Then the conditions for several heights are shown in the following tabulation.

A Degrees	*B* Degrees	Radiation Res. (at loop)	Radiation Res. (at coil)	Total Res. (at coil)	Watts Radiated	Watts Lost in Coil	*F*
150	45	63.5	127.0	128.0	992.0	8.0	246.0
135	77	26.2	27.3	28.3	965.0	35.0	243.0
120	97	11.0	11.2	12.2	918.0	82.0	236.0
90	125	1.5	2.23	3.23	690.0	310.0	205.0

In the above, we have assumed a constant coil resistance. Actually, this resistance will probably be much greater than one ohm for the lower heights. Practically, it is seen that it might be possible to use this scheme for values of *A* as low as 135 degrees, although a height of 150 degrees would be much more desirable. When the antenna is this tall, it is not necessary to insert the coil at the top. An interesting practical example of an antenna of this type is that at Breslau, Germany. Breslau is a sixty-kilowatt station. The antenna is a straight

[2] Balth. van der Pol, *Jahr. der draht. Telegraphie und Telephonie*, Band 13, Heft 3, pp. 217–238, (1918).

vertical wire supported by a wooden tower. The suppression of current at the top is attained by a ring, ten meters in diameter placed at the top of the tower. This ring suppresses about forty meters or one-eighth of a wavelength.[3] The antenna height is 455 feet or 138.5 meters, while the wavelength is 325 meters. The current node is nineteen meters from the ground. The maximum current comes at 100 meters from the ground. No energy is radiated at sixty to sixty-five degrees to the horizon. The horizontal radiation is increased about twenty-five per cent while the radius free from fading is increased about forty per cent. We see that this antenna is 150 degrees tall with forty-five degrees of loading at the top. The intensity along the horizon in the ideal case is twenty-seven per cent greater than that due to a short antenna. This agrees well with the above observed value of twenty-five per cent.

When a steel tower is used for the antenna, it will be necessary to use a larger outrigger or larger coil to attain the same amount of suppression of current at the top of the antenna. This will be especially true if the tower is of nearly uniform cross section from bottom to top. If this last-named condition is not fulfilled, the current distribution will no longer be sinusoidal, and it would then become a rather formidable task to adjust the coil to the correct value to simulate a 190-degree antenna. A method has been devised which will enable one to determine the correct coil setting from measurements made on the ground close to the antenna.

V. Sectionalized Antenna

As stated in the previous section, it may become difficult to place a big enough outrigger on the tower to attain the effect wanted and still have the tower nearly uniform in cross section. One way out of this situation is the so-called "sectionalized antenna." The coil is placed some distance from the top of the antenna, and no outrigger is used. Thus the section above the coil radiates. The current distribution is as shown in Fig. 10. The total antenna height is d, while a is the distance from the earth to the coil. The length of sine wave which would be above the coil point if the coil and top section of antenna were replaced by a straight wire is b. Then we define

$$\begin{aligned} D &= 2\pi d/\lambda \text{ radians} = 360d/\lambda \text{ degrees} \\ A &= 2\pi a/\lambda \text{ radians} = 360a/\lambda \text{ degrees} \\ B &= 2\pi b/\lambda \text{ radians} = 360b/\lambda \text{ degrees.} \end{aligned} \tag{13}$$

[3] *Telefunken Zeit.*, August, (1933).

The vertical radiation characteristic, referred to the loop current, is

$$f(\theta) = \Big[\cos B \cos (A \cos \theta) - \cos \theta \sin B \sin (A \cos \theta) - \cos (A + B) + \frac{\sin B}{\sin (D - A)}\{\cos (D \cos \theta) - \cos (D - A) \cos (A \cos \theta) + \cos \theta \sin (D - A) \sin (A \cos \theta)\}\Big]/K \sin \theta. \tag{14}$$

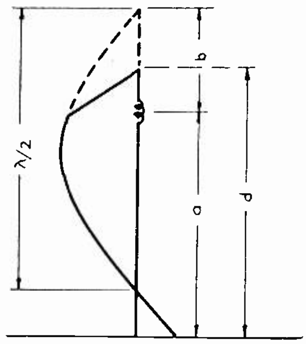

Fig. 10—The sectionalized antenna.

The form factor referred to the same point is

$$K = \left[\cos B - \cos (A + B) + \frac{\sin B}{\sin (D - A)}\{1 - \cos (D - A)\}\right]. \tag{15}$$

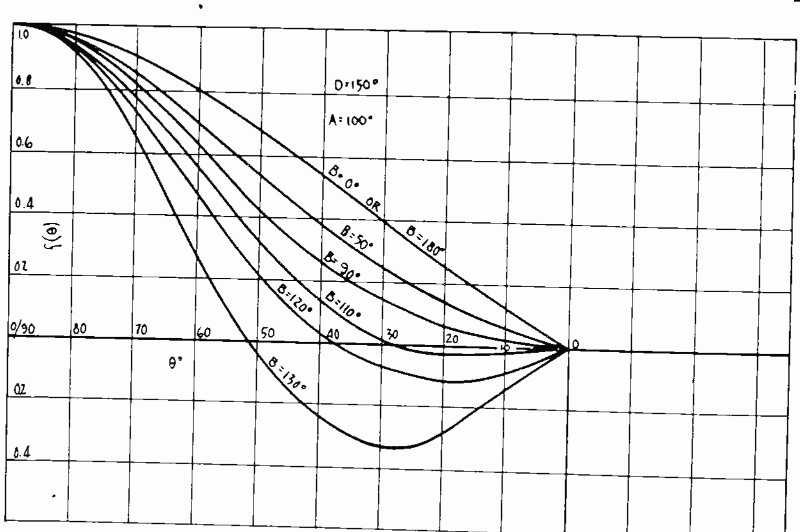

Fig. 11.—Vertical radiation characteristics of a sectionalized antenna.

Let us consider a particular case. We shall choose a total height of $D=150$ degrees, with the coil placed at $A=100$ degrees. Then Fig. 11 shows the vertical radiation characteristics for a number of values of B. This antenna thus has the same characteristic variation as the antenna with the loaded top. The field intensity at one mile and the loop radiation resistance is given in Fig. 12. The resistance referred to the coil point is found from this resistance curve by

$$R_{\text{cp}} = R_{\text{loop}}/\sin^2 B. \tag{16}$$

Likewise,

$$R_{\text{base}} = R_{\text{loop}}/\sin^2 (A + B). \tag{17}$$

Thus, in the above case, when B is 110 degrees (for best fading suppression) the resistance at the coil point is 23.6 ohms and the base resistance is 84.0 ohms.

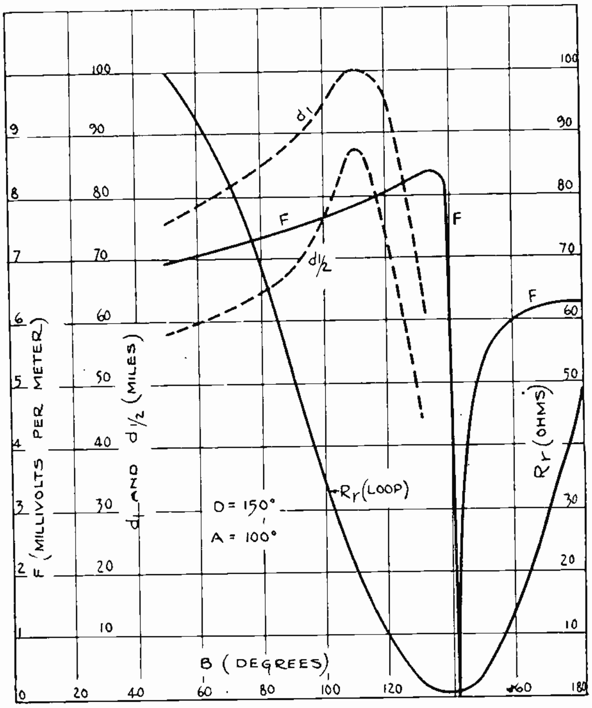

Fig. 12—The dependence of the characteristics of a sectionalized antenna upon the amount of sectionalizing coil.

It is of interest to examine the constants for other coil positions. Let us suppose that for each coil position the coil is readjusted so that the antenna simulates an ordinary 190-degree antenna. The variation of the quantities in question when D is 150 degrees is shown by Fig. 13. Fig. 14 shows a similar set of quantities when D is 120 degrees. From considerations of this sort, we find that the 150-degree height is quite good, while it is very unlikely that we could obtain good efficiencies with a height of 120 degrees.

VI. Antenna with Constant Current

The next case to come to our attention is the antenna with constant current. By this, we mean a vertical antenna so loaded that the current at any point along the antenna has the same magnitude and phase position as the current at any other point. Then if the total height of the antenna is a, we can define

$$A = 2\pi a/\lambda \text{ radians} = 360a/\lambda \text{ degrees}. \tag{18}$$

The form factor is

$$K = A = 2\pi a/\lambda \tag{19}$$

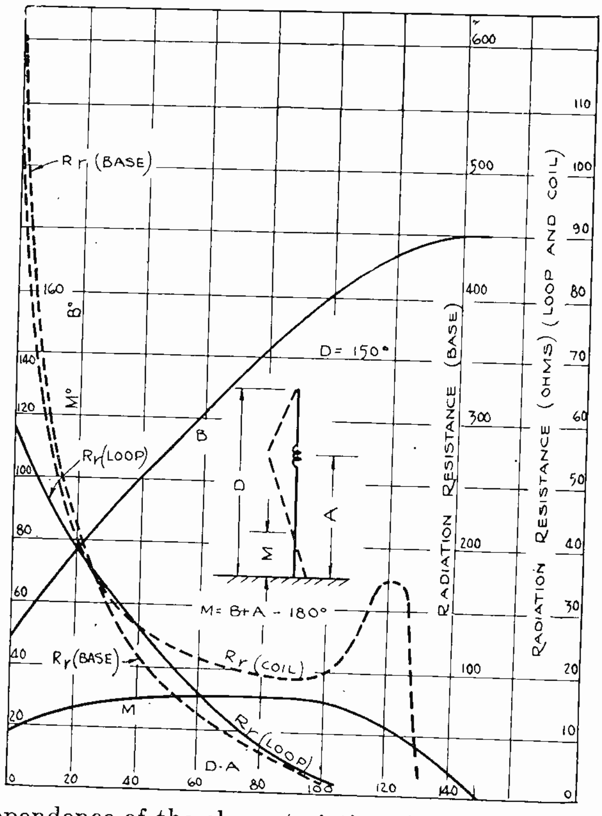

Fig. 13—The dependence of the characteristics of a sectionalized antenna upon sectionalizing coil position. (In each position, the coil is adjusted so that the vertical radiation characteristic is the same as that of a simple 190-degree antenna.)

and the vertical characteristic is

$$f(\theta) = \frac{\tan\theta \sin(A\cos\theta)}{A}. \tag{20}$$

The family of vertical radiation characteristics is shown in Fig. 15. The field strength at one mile and the radiation resistance is shown by Fig. 16. It is seen that this antenna must be practically 190 degrees tall to simulate a 190-degree antenna with sinusoidal distribution. Thus this scheme does not appear to be very promising, for it does not seem reasonable to go to all the trouble involved in getting a constant current along the antenna when the same vertical pattern can be obtained by merely using a straight vertical wire antenna of the same height.

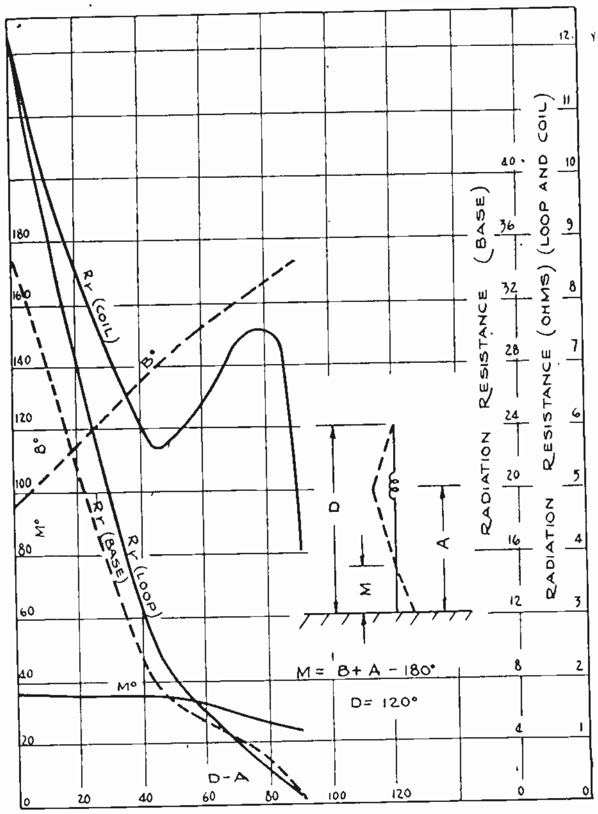

Fig. 14—Similar to Fig. 13, but for a different total height.

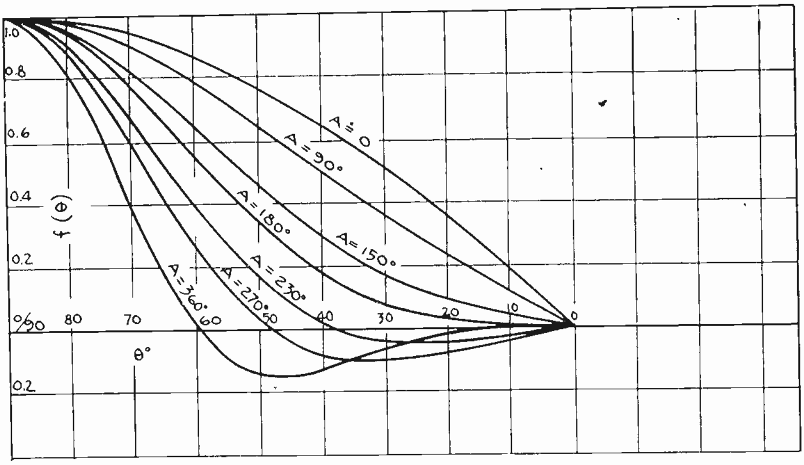

Fig. 15—Vertical radiation characteristics of the antenna with constant current.

VII. Antenna Element Elevated from the Earth

Another possibility of antenna design which suggests itself is to raise an antenna element above the surface of the earth. For convenience, let us assume that the element in question is a half-wave antenna, as shown in Fig. 17. Let h be the distance from the ground to

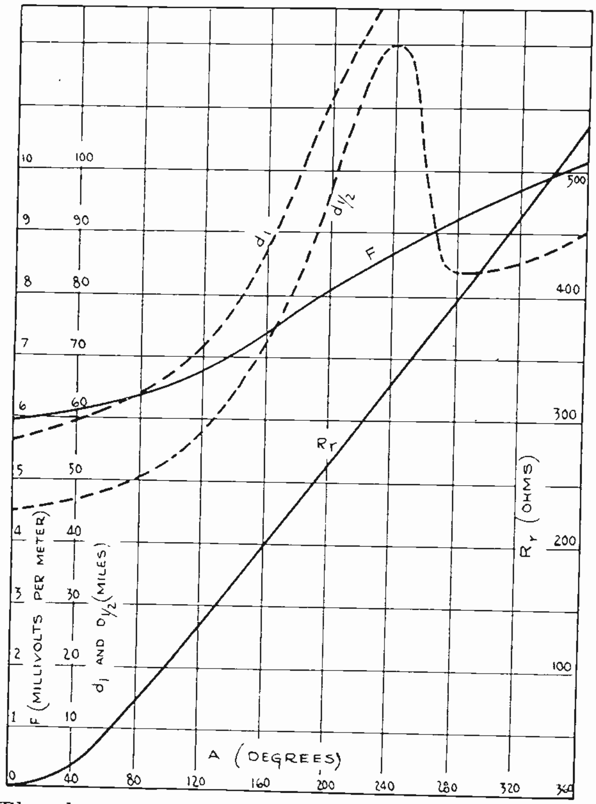

Fig. 16—The characteristics of the antenna with constant current.

the mid-point of the antenna. Then the distance from the ground to the lower end of the antenna is $d = h - \lambda/4$. The form factor of the

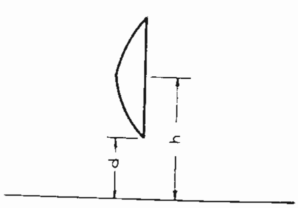

Fig. 17

system is $K = 2$, and the vertical radiation characteristic is

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)\cos(H\cos\theta)}{\sin\theta} \tag{21}$$

where $H = 2\pi h/\lambda$ radians $= 360h/\lambda$ degrees. Since we are considering a half-wave element, the lower end is at the earth's surface when H is

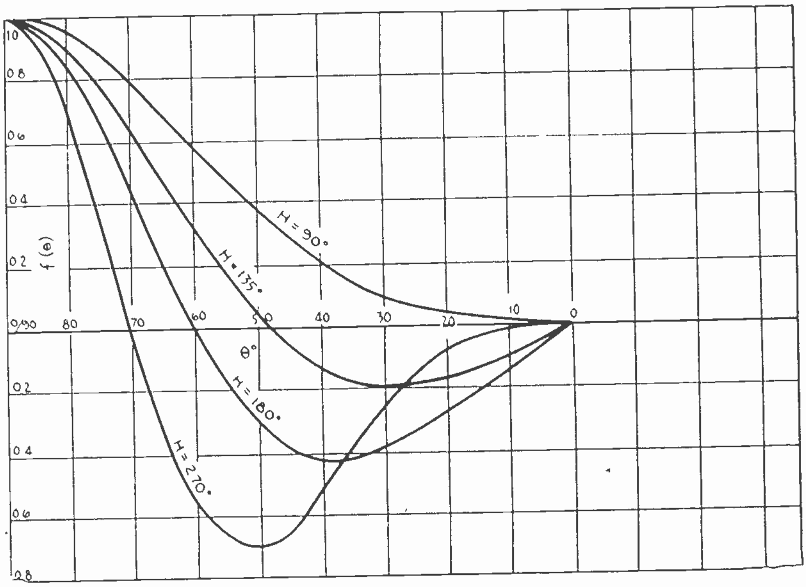

Fig. 18—Vertical radiation characteristics of a half-wave element elevated above the earth.

90 degrees. Fig. 18 shows the vertical radiation characteristic for a number of values of H. The field intensity at one mile and the radiation resistance is shown by Fig. 19. From the fading distance curve on this

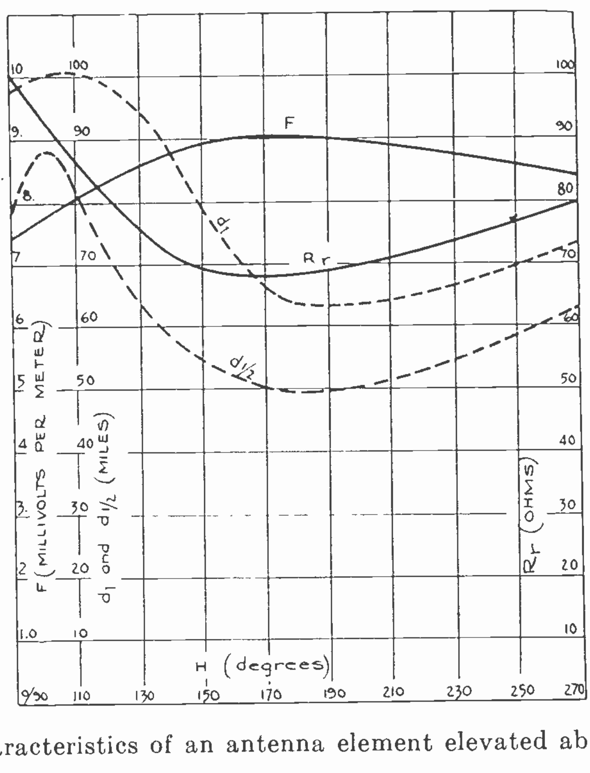

Fig. 19—Characteristics of an antenna element elevated above the earth.

figure, we see that this antenna simulates a 190-degree antenna when H is 102 degrees. This means that the top of the antenna is 192 degrees from the ground.

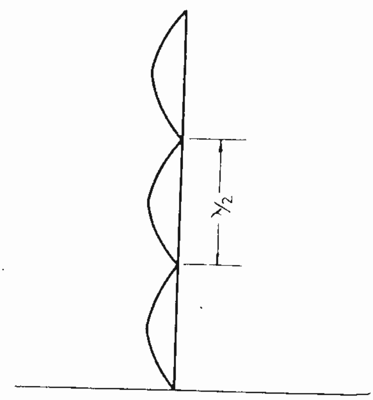

Fig. 20—The Franklin antenna.

Since the vertical characteristic of the half-wave element in space, $\cos\left[(\pi/2)\cos\theta\right]/\sin\theta$, is essentially equivalent to $\sin\theta$, we can take the above vertical characteristics and the field intensity at one mile as indicative of that obtained if the element were shorter than one-half wavelength, where H is still the distance to the center of the element from the surface of the earth.

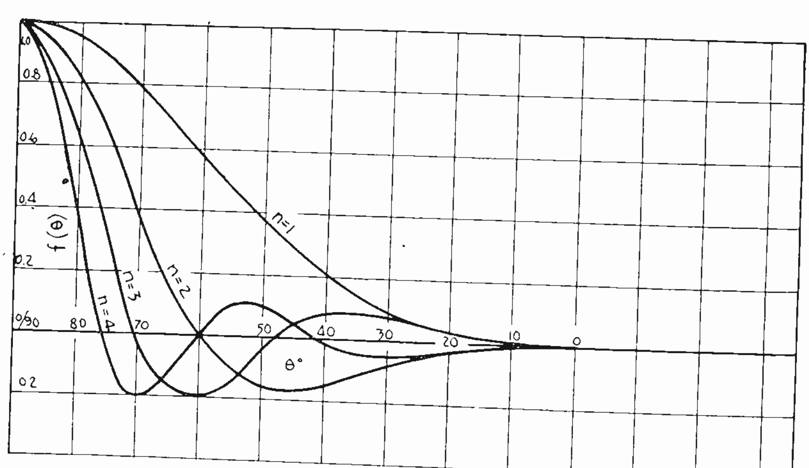

Fig. 21—Vertical radiation characteristics of the Franklin antenna.

VIII. The Franklin Type Antenna

The Franklin antenna, which has been rather useful in short-wave work, consists of a number of half-wave antennas placed end to end on a vertical line, and so fed that the currents in each element are equal and in phase. (See Fig. 20.) Let n be the number of half-wave elements above the earth. Then the form factor is

$$K = 2n. \tag{22}$$

The vertical radiation characteristics for several values of n are shown in Fig. 21.

The field intensity at one mile and the total antenna resistance is given by Fig. 22. We see that very high field strengths are obtained when many sections are used, provided that the losses in the phasing

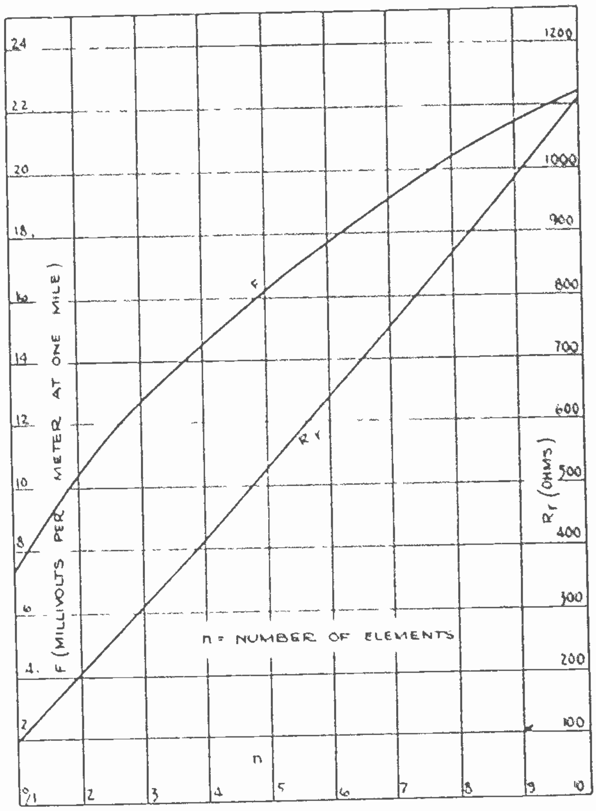

Fig. 22—Field intensity and radiation resistance of the Franklin antenna.

equipment do not become large. It is also extremely unlikely that antennas whose height is more than one wavelength could be built economically.

IX. Antenna with Decreased Velocity

Another arrangement which has been proposed by various engineers from time to time is the antenna with decreased velocity. The idea is to decrease the velocity of propagation of the current waves on the antenna wires so that current nodes are separated by less than one half of the free space wavelength. For instance, if the wire were so loaded that the velocity were but one half the velocity of light in free space, we would have a half sine wave distribution occurring in one fourth of a free space wavelength. Several methods of loading might be utilized. One arrangement might be to arrange the antenna wire in a spiral. The velocity of propagation might also be decreased by surrounding a vertical wire with a mass of dielectric. We shall not interest ourselves in the methods of accomplishing this condition, but shall note the consequences of such an arrangement.

Let us now examine Fig. 23. This shows an antenna of height a loaded at the top with a capacity area. The distance, b, is the actual

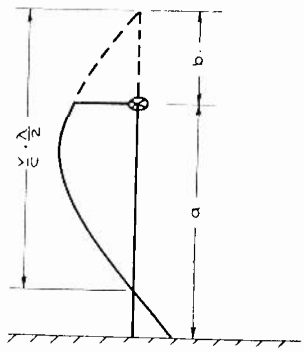

Fig. 23

length of sine wave suppressed by the loading at the top. We shall now define the following quantities:

a = actual height of antenna
b = actual length of sine wave suppressed by loading at the top
c = velocity of propagation of light (or radio waves) in free space ($=3\times10^{10}$ centimeters)
f = frequency of the transmitter
$\lambda = c/f$ = free-space wavelength
v = velocity of current wave on the wire
$A_0 = 2\pi a/\lambda$ (radians) $= 360a/\lambda$ (degrees)
$B_0 = 2\pi b/\lambda$ (radians) $= 360b/\lambda$ (degrees)
$A = A_0/(v/c)$
$B = B_0/(v/c)$
$G = A + B$

Then the form factor (referred to the loop current) is

$$K = \frac{v}{c}[\cos B - \cos G] \tag{23}$$

and the vertical radiation characteristic

$$f(\theta) = \frac{v}{c}\sin\theta\left[\cos(B)\cos(A_0\cos\theta) - \frac{v}{c}\cos\theta\sin B\sin(A_0\cos\theta)\right. \tag{24}$$

$$\left. - \cos G\right] \Big/ K\left[1 - \left(\frac{v}{c}\right)^2\cos^2\theta\right].$$

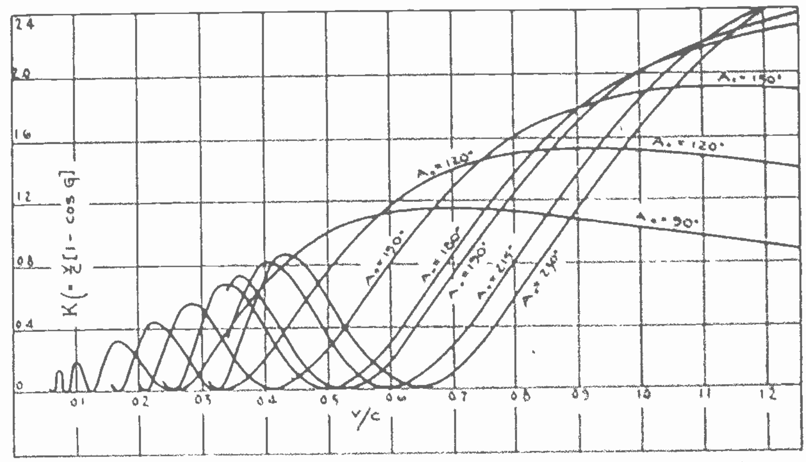

Fig. 24—The form factor of the antenna with decreased velocity.

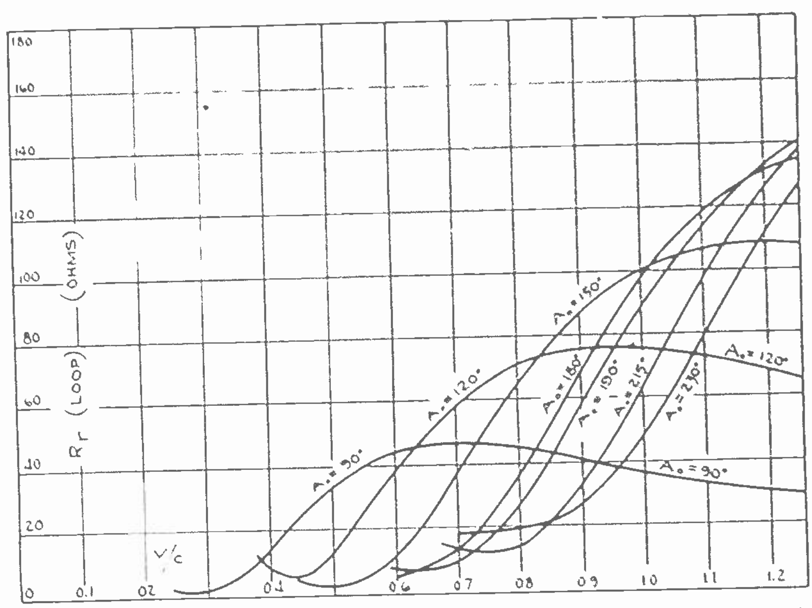

Fig. 25—Loop radiation resistance of the antenna with decreased velocity.

Let us first examine the case when no loading is placed at the top. Then $B = B_0 =$ zero degrees. The form factor for this case is shown as a function of v/c for a number of values of A_0 by Fig. 24. Fig. 25 shows the radiation resistance for the same cases. This radiation resistance is referred to the loop current. To find the radiation resistance at the base, we would use the following relation:

$$R_{\text{base}} = R_{\text{loop}}/\sin^2(G).$$

The radiation resistance at the base for a number of cases is shown by Fig. 26.

The field strength at one mile for one watt input is given by Fig. 27. We that see for a fixed antenna height, the field strength at one mile increases as the velocity is decreased until the field strength reaches about 8.5 millivolts per meter. The field strength then drops rapidly to zero

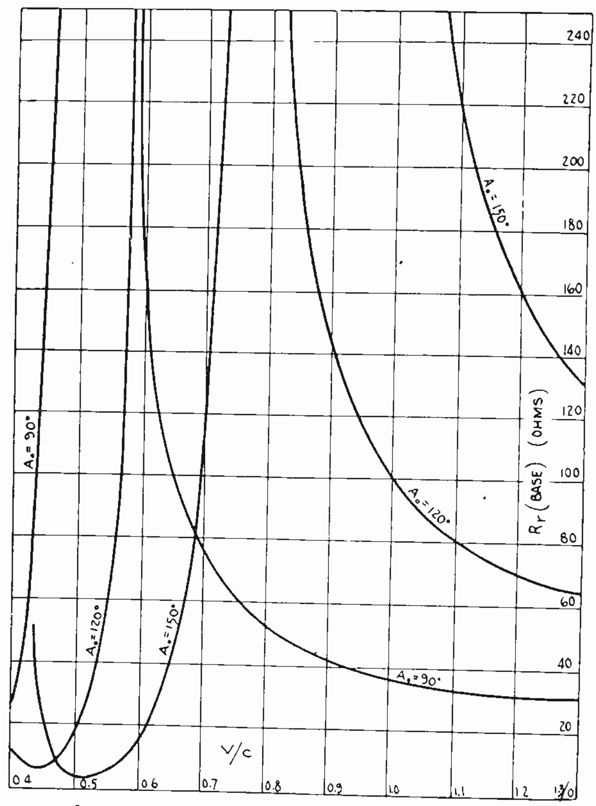

Fig. 26—Base radiation resistance of the decreased velocity antenna.

with further decreases in velocity. It is important to note that the maximum values of field strength occur at the point where the base radiation resistance is extremely small.

The vertical radiation characteristics will now be examined. Let $A_0 = 150$ degrees. Fig. 28 shows the vertical radiation characteristic for a number of values of the ratio, v/c. It is seen that a decrease in velocity for a fixed height changes the vertical characteristics in the same manner as an increase in height of a straight vertical wire. This is true for any value of A_0 which we choose. This fact can be made use of to show the characteristics of the decreased velocity antenna. We shall define a quantity, A_s, in the following fashion: Suppose we have under consideration a specific antenna of height A_0 degrees and a velocity ratio, v/c. Then A_s is the height in degrees of a straight vertical wire antenna

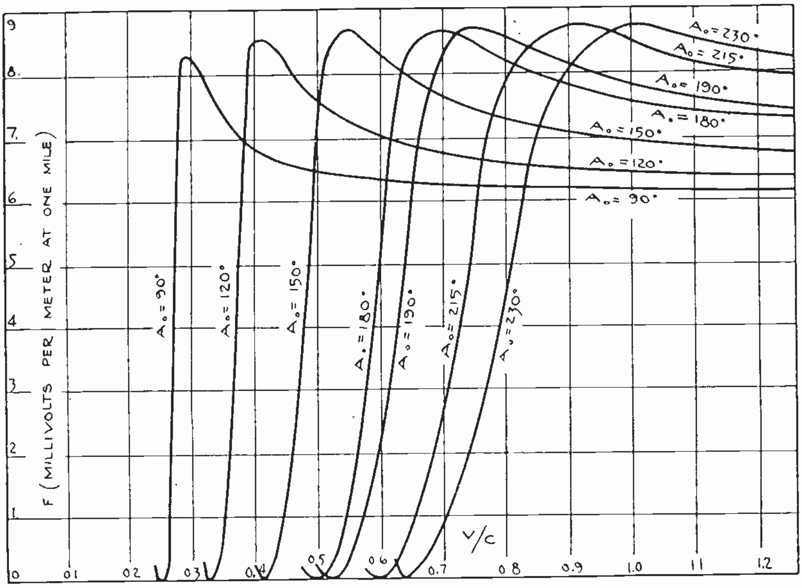

Fig. 27—Field strengths obtainable from the decreased velocity antenna.

which has the same vertical characteristic and field strength at one mile as the decreased velocity antenna under consideration. This quan-

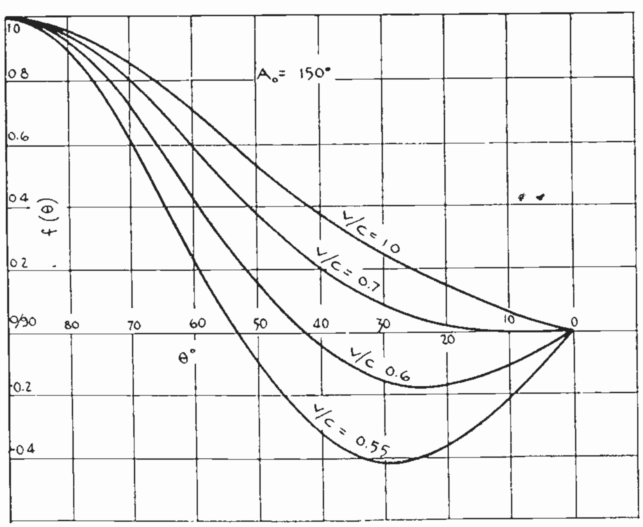

Fig. 28—Vertical radiation characteristics of a decreased velocity antenna.

tity, A_s, is shown as a function of v/c for a number of values of A_0 in Fig. 29. The broken line on this diagram indicates the values of v/c where the current node occurs at the base of the antenna.

The above relations can be used to examine another interesting case. It has been shown previously that it was desirable to build steel tower antennas so that the cross section is substantially constant over the entire length rather than to have the tower much smaller at

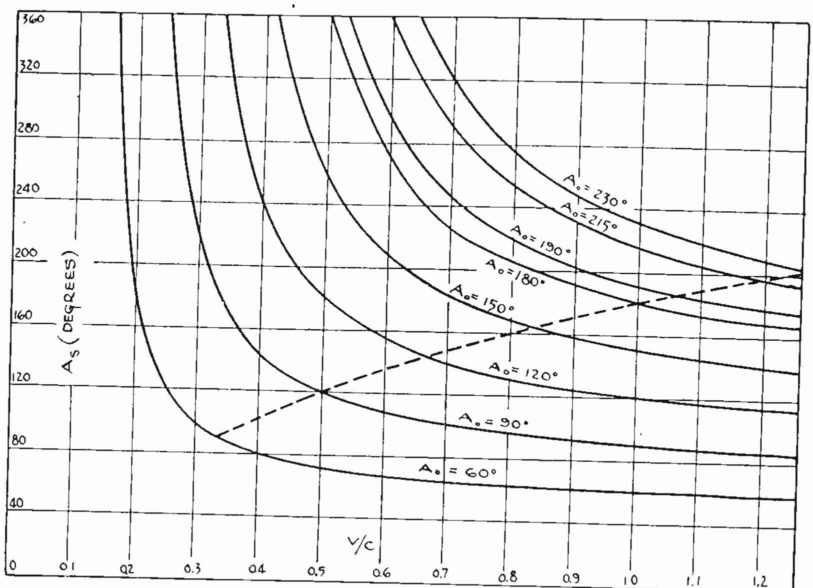

Fig. 29—Curves of similitude of the decreased velocity antenna. A_0 is the actual height of the decreased velocity antenna. A_s is the height of a vertical wire antenna which has the same vertical radiation characteristic and yields the same field intensity at one mile.

the top than at the base. It has been suggested that it is desirable to go still further by making the tower flare out at the top, thus moving the current maximum up the tower. We shall examine an extreme case for purposes of illustration.

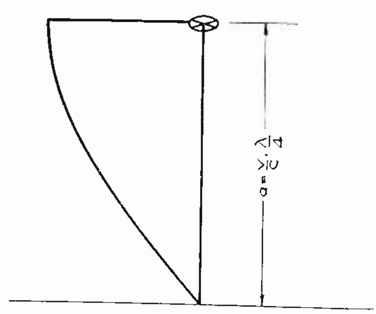

Fig. 30

Suppose that for any particular height antenna, the velocity is so altered and the loading at the top so altered that the current loop occurs at the top and the current node at the bottom of the antenna. This condition implies a velocity greater than the velocity of light when the antenna is greater than one quarter of a free-space wavelength in height. This current distribution is shown by Fig. 30. Fig. 31 shows the

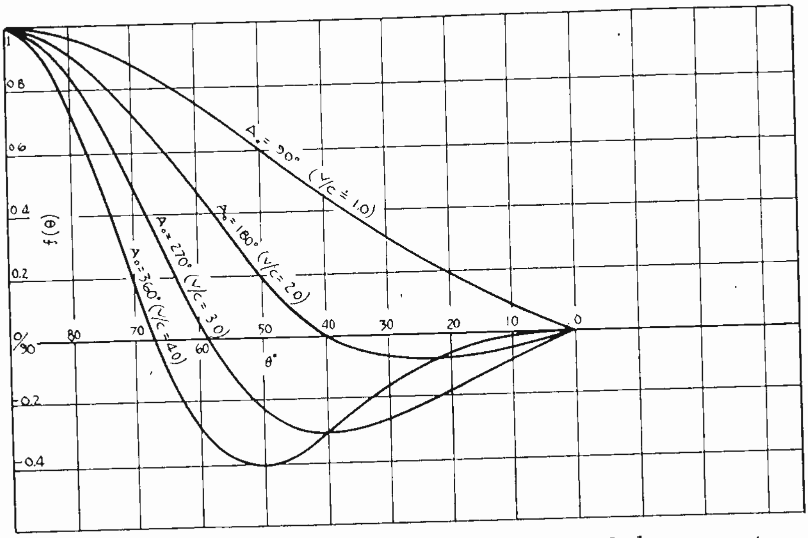

Fig. 31—Vertical radiation characteristics of the current distribution shown in Fig. 30.

vertical radiation characteristic for a number of antenna heights. The corresponding field intensity at one mile and the radiation resistance referred to the current at the top of the antenna is given in Fig. 32.

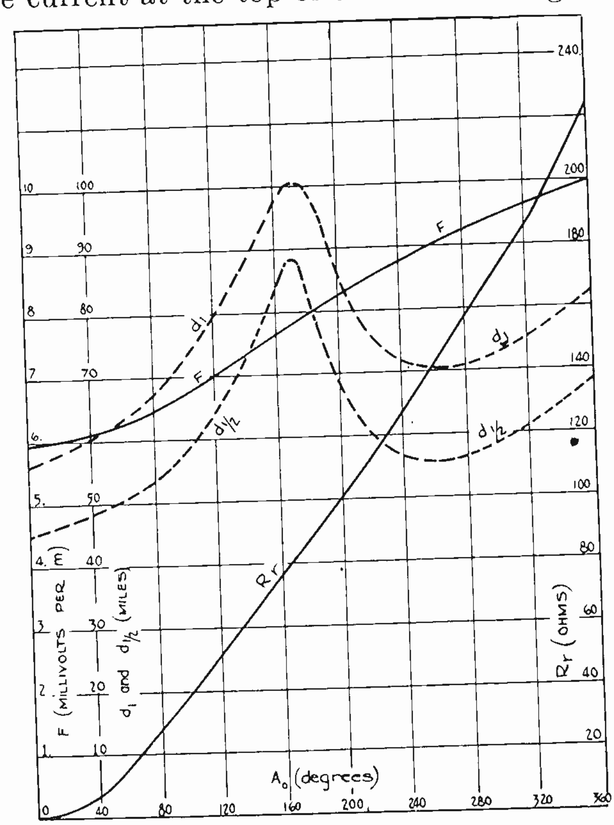

Fig. 32—Field intensity, radiation resistance, and distance to the fading zone of the current distribution shown in Fig. 30.

The distance to the fading zone is also shown. It is seen that if we wish to simulate a vertical wire 190-degree antenna, this type of antenna must be 170 degrees high. This is not a very great saving in height when it is considered that it would be both expensive and difficult to attain the current distribution in question.

It should be further pointed out that all radiation resistances were computed with reference to the vertical component of antenna current. When the velocity of propagation is decreased by spiraling the antenna wire, the radiation resistance referred to the actual current in the wire is the above calculated values multiplied by $(v/c)^2$. This substantially reduces the resistance in all cases where v/c is less than unity.

X. Franklin Type Antennas with Decreased Velocity

We have previously shown that the Franklin antenna composed of half-wave sections gave substantial increases in field strength, but that at the same time the heights became excessive. It has been suggested that the velocity on each element be reduced to one half the velocity

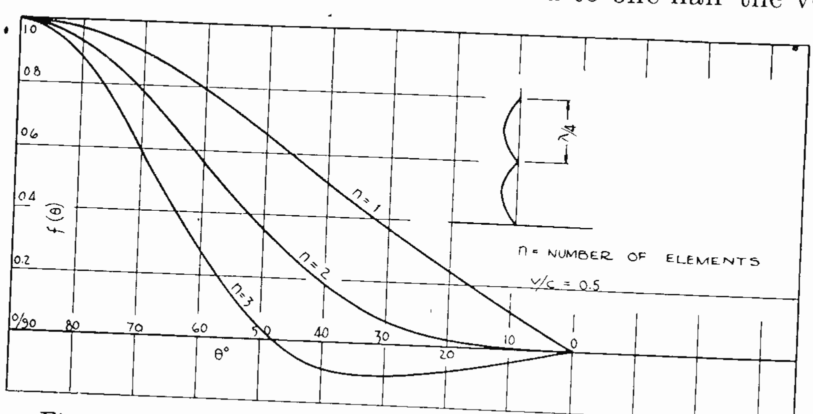

Fig. 33—Vertical radiation characteristics of the Franklin antenna with decreased velocity.

of light so that one half a sine wave of current will occur in a quarter wave of space. This arrangement is supposed to yield large gains in field strength. A few computations have been made concerning this case. Each element is 90 space degrees in length. The ratio, v/c, is 0.5 while n is the number of elements. The vertical radiation characteristics are shown for one, two, and three elements by Fig. 33. We see that the single element gives a vertical characteristic similar to that obtained from a vertical wire, 120 degrees tall. The two-element arrangement yields a vertical characteristic similar to that obtained from an ordinary half-wave antenna, while the three-element antenna is similar to a conventional 215-degree antenna. The following table shows the actual results:

n	mv/m at 1 Mile	R_1 (ohms)	R_2 (ohms)	Total Height (measured in free-space wavelengths)	Height of Equivalent Vertical Wire (measured in free-space wavelengths)
1	6.30	35.0	8.75	0.25	0.333
2	7.67	94.0	23.5	0.5	0.514
3	8.58	169.8	42.45	0.75	0.61

In the above table, R_1 is the total radiation resistance of the system referred to the vertical component of current. If the decreased velocity is obtained by spiraling the conductor, $R_2 = R_1/4$ is the resistance with reference to the actual current in the wire.

XI. Antennas Extended in the Horizontal Plane

We have seen that the process of modifying the current distribution along a vertical axis has not shown any current distribution to be outstanding, when the antenna height is less than a wavelength. The next obvious procedure is to extend the antennas over the horizontal plane. Since we are interested in broadcast antennas which radiate uni-

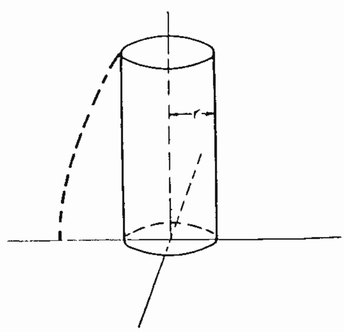

Fig. 34

formly in all directions along the horizontal plane, the antenna elements must be circularly symmetrical. In other words, each element of the antenna must itself be a solid ring of antennas. Of course, this ring can be simulated to a certain extent by placing several vertical antennas on the circumference of a circle. The ideal case, however, is that in which each element is a cylinder. We shall examine this case in detail, since the results of this case will be applicable to the case of several antennas on a circumference. This type of antenna was first discussed by Bohm.[4]

Let us suppose our antenna to be a cylinder, of radius, r. The cylinder is one-quarter wavelength tall and is placed over a perfect earth. (Fig. 34.) It is assumed that the current on this cylinder is sinusoidally distributed. The total current at the base of this cylinder is I_1. Then the form factor is

$$K = J_0(2\pi r/\lambda) \tag{25}$$

[4] O. Bohm, *Zeit. für Hochfrequenz. u. Elect.*, p. 137, October, (1933).

where J_0 is Bessel's function of the first kind and zero order. The vertical radiation characteristic is

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} J_0(2\pi r \sin\theta/\lambda)/K. \tag{26}$$

Fig. 35 shows a plot of $J_0(2\pi r/\lambda)$ as a function of r/λ.

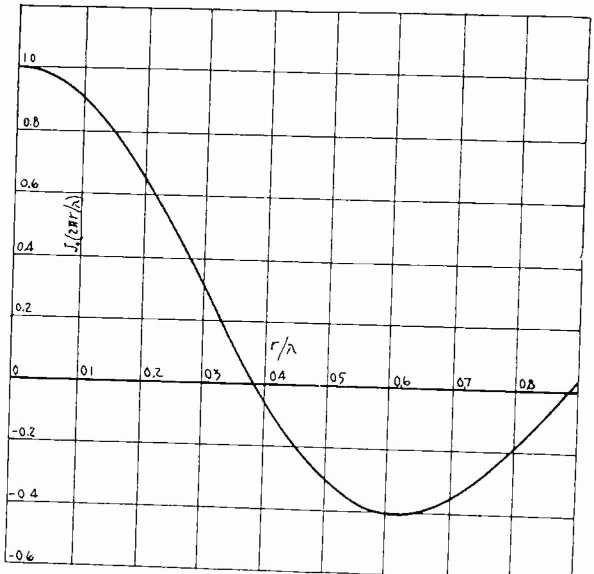

Fig. 35—Form factor of the cylindrical current sheet of Fig. 34.

Fig. 36 shows the vertical radiation characteristics for a number of cylinder radii.

The radiation resistance and the field intensity at one mile for one watt input is shown on Fig. 37. We see that the field strength at the horizon goes to zero whenever $2\pi r/\lambda$ is a root of the Bessel function. The first root occurs when r equals 0.382 wavelength.

While the above results show a rather interesting characteristic, it is evident that a ring of antennas used alone is of little use. We shall next examine the case where another quarter-wave antenna is placed at the center of the ring. The current at the base of this antenna is I_0. The currents in the inner antenna and the outer ring may be related to each other in any phase or ratio. Suppose that this relation is

$$I_1 = I_0(a + jb). \tag{27}$$

Then the form factor (referred to the center antenna) is

$$K = \sqrt{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2} \tag{28}$$

where r is the radius of the ring of antennas.

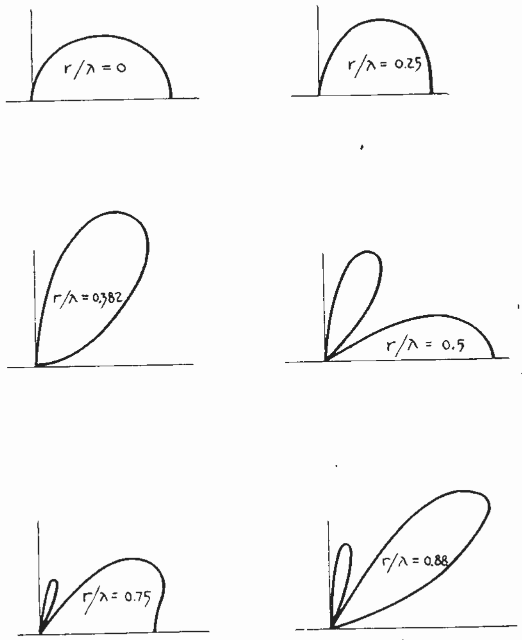

Fig. 36—Vertical radiation patterns of cylindrical current sheets.

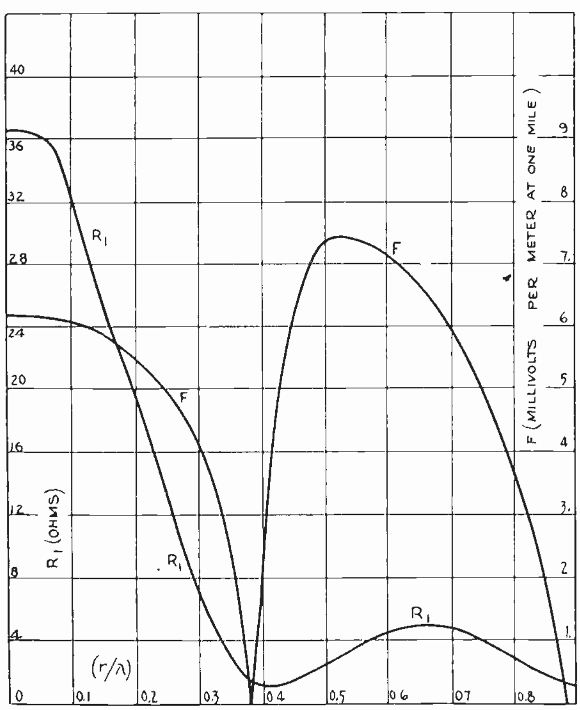

Fig. 37—Field strength and radiation resistance of cylindrical current sheet.

The vertical radiation characteristic is

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{K\sin\theta}\sqrt{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\sin\theta\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\sin\theta\right)\right]^2}. \tag{29}$$

It is obvious from the above expressions that the quantity b must be

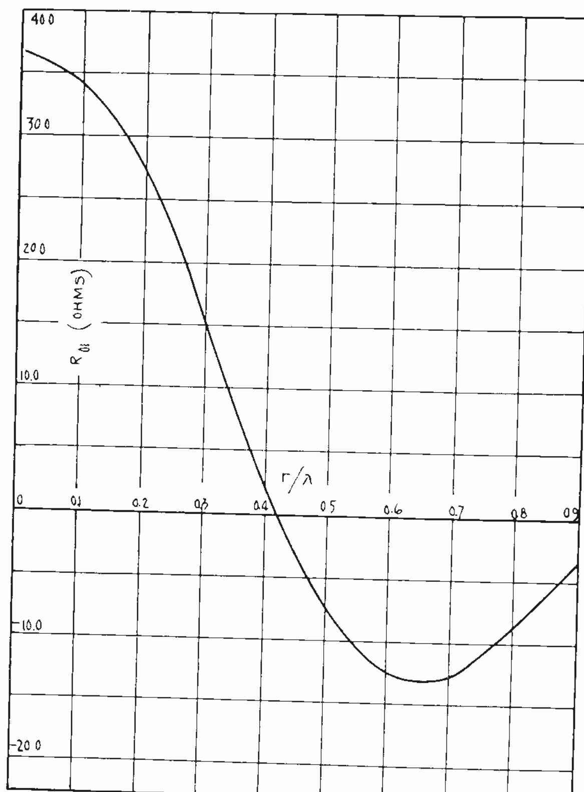

Fig. 38—Mutual resistance between cylinder and concentric wire.

zero to secure sky-wave suppression. It also seems evident that this condition must also be true to secure the maximum field strength along the horizon. That this is true will be shown by the following analysis.

The resistance of the ring of antennas has already been shown. This resistance will be called R_1. The resistance of the center antenna is $R_0 = 36.6$ ohms. The mutual resistance between the center antenna and the ring of antennas, R_{01}, is given as a function of r/λ by Fig. 38. The power input into the antenna is then

$$P = [a^2R_1 + 2aR_{01} + b^2R_1 + R_0]I_0^2 \text{ (watts)}. \tag{30}$$

Then the field strength at one mile for one watt input is

$$F = \frac{37.25\sqrt{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2}}{\sqrt{[a^2R_1 + 2aR_{01} + b^2R_1 + R_0]}} \text{ (mv/m)} \tag{31}$$

or,

$$F^2 = \frac{37.25^2\left\{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2\right\}}{a^2R_1 + 2aR_{01} + b^2R_1 + R_0}. \tag{32}$$

To find the value of b which gives the maximum field strength along the ground, we differentiate the last expression with respect to b and set equal to zero. This yields

$$\frac{dF^2}{db} = \frac{37.25^2\left\{2J_0^2\left(\frac{2\pi r}{\lambda}\right)[a^2R_1 + 2aR_{01} + R_0] - 2R_1\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2\right\}b}{[a^2R_1 + 2aR_{01} + b^2R_1 + R_0]^2} \tag{33}$$

$$= 0$$

or,

$$b = 0. \tag{34}$$

Thus we see that the current in the inner conductor must be either in phase or in phase opposition with the current in the outer ring.

To find the best value of a, we again differentiate, this time with respect to a, and set equal to zero. This yields the value of a, which will give the greatest field strength along the ground as

$$a_m = \frac{R_{01} - R_0J_0\left(\frac{2\pi r}{\lambda}\right)}{R_{01}J_0\left(\frac{2\pi r}{\lambda}\right) - R_1} \tag{35}$$

and the value of a, which gives zero field strength along the ground, is

$$a_0 = -\frac{1}{J_0\left(\frac{2\pi r}{\lambda}\right)}. \tag{36}$$

The values of a_m were substituted back into (31) and the values of maximum field strengths computed. It was found that the largest field strength obtainable for ring diameters less than two wavelengths was about 9.0 millivolts per meter for one watt input into the system, or a gain over a single quarter-wave antenna of forty-six per cent.

Fig. 39 shows the field strength at one mile for one watt input as a function of a, for a number of values of r/λ. Fig. 40 shows the varia-

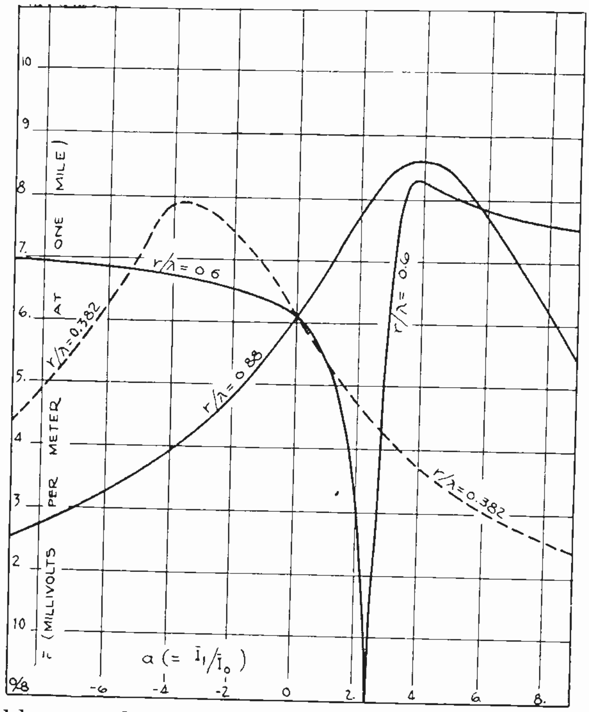

Fig. 39—Field strengths obtained from a cylinder and a concentric wire.

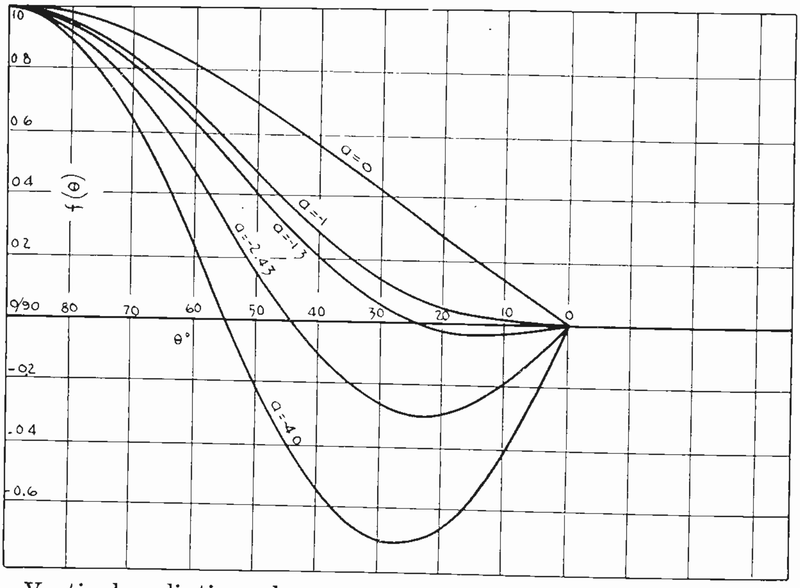

Fig. 40—Vertical radiation characteristics of a cylinder and concentric wire when the cylinder has a radius of 0.382 wavelength.

tion of the vertical characteristic with a when the radius of the outside ring is $r = 0.382$ wavelength. We see that in the neighborhood of

$a=-4.0$, the secondary sky lobe is excessive. This is the point at which the field strength along the ground is greatest. It is invariably true for this type of antenna that the greatest ground signal strength is accompanied by a large secondary lobe.

XII. Conclusion

We have now examined a number of antenna arrangements and current distributions. Some of these arrangements have been very disappointing. In no case where the total antenna height is at all reasonable for broadcast use does the field strength at one mile go to exceptionally high values. For heights of the order of a half wavelength, it is hard to find anything better than a straight vertical wire. Some of the arrangements save a small amount in height, but in no other respect do they improve on the straight wire.

In the case of a straight vertical wire, Fig. 5 shows that an antenna length of 190 degrees (0.528 wavelength) is the most desirable when the power of the station is sufficient so that the night service radius is limited by fading rather than by ground signal deficiency. While it is seen from Fig. 5 that a 230-degree antenna gives ten per cent more signal at one mile, the daytime range is only increased about five per cent. It is apparent that the fading reduction accomplished by the 190-degree antenna far offsets the slight decrease in daytime service range suffered by not extending the antenna height to 230 degrees.

The antenna with the capacity hat at the top is an arrangement which enables a saving in height of antenna. By varying the loading at the top of the antenna, the vertical characteristic of a straight wire antenna of greater physical height can be obtained. This arrangement can be used on antennas of height 135 degrees or better to simulate the pattern of a 190-degree antenna. For antenna heights lower than 135 degrees, the antenna radiation resistance goes to rather small values as the loading at the top is made sufficient to suppress the sky wave. When the antenna itself is a steel tower, it will probably be difficult to keep the tower of nearly constant cross section and still build a large enough capacity hat to be effective with a reasonably small coil.

The sectionalized tower was developed to overcome the difficulties encountered by the tuned hat. Sectionalizing down the tower enables one to build the tower of uniform cross section, so that the results can be predicted in advance. The outstanding advantage of this arrangement is evident when the power is high. In general, the voltage occurring across the sectionalizing insulators is considerably less than occurs across the insulators of the tuned hat on a tower of the same height.

Examination of the curves pertaining to the antenna with constant current shows that it requires a total height of 190 degrees to give results equivalent to those obtained from a 190-degree vertical wire antenna. A further increase in height moves the fading wall still further. Since the distance to the fading wall is already 100 miles, this further increase would be important only in very special cases.

The gain in signal strength obtained by raising a half-wave element from the earth is very slight. It is seen that the best suppression of fading occurs when the top of the half-wave element is 190 degrees off the ground.

The Franklin antenna seems the most promising, in spite of the extreme heights required. It is not unreasonable to speculate on the possibilities of this antenna as a result of improved tower design. Such an arrangement would probably require a wooden tower. A three-section Franklin antenna would be 985 feet high if the operating frequency were 1500 kilocycles. Such an arrangement would give a field intensity at one mile twice as large as that obtained from a simple quarter-wave antenna. This would be the equivalent of quadrupling the power into the lower antenna.

It is of interest to note that the two-section Franklin antenna of total height one wavelength has the same vertical characteristic and field intensity at one mile as obtained from the antenna with constant current of the same height.

The results indicate that the antenna with decreased velocity has no advantage over simpler types. This antenna in general requires a wooden supporting tower. When a wooden tower is used, a straight vertical wire with a capacity hat would be the best arrangement. It has been shown that the Franklin type antenna needs no decreased velocity. In fact, for any given antenna height, it seems that equivalent results can be obtained with a straight vertical wire, either loaded at the top with a capacity hat or sectionalized some distance from the top.

The ring of antennas, while an interesting subject of analysis, is obviously too elaborate an arrangement for the small advantages it offers. In any event, this ring and central antenna can be replaced by a single vertical wire, 150 degrees tall, with the proper amount of loading at the top.

In the above discussion, it was found that most arrangements gave field strengths at one mile of less than 9.0 millivolts per meter. This was due to the fact that the vertical characteristic was not concentrated sufficiently. A simple calculation shows the necessity of concentrating the radiation close to the horizon. Let us suppose that the vertical radiation characteristic is constant within the angle ϕ measured from the earth's surface and is zero at all higher angles, as shown by the sketch in Fig. 41. Then Fig. 41 shows the field strength at one mile for one watt of power into the system as a function of the angle ϕ. It is seen that the effective concentration must be within seventeen degrees to obtain 9.0 millivolts per meter at one mile. To obtain twice

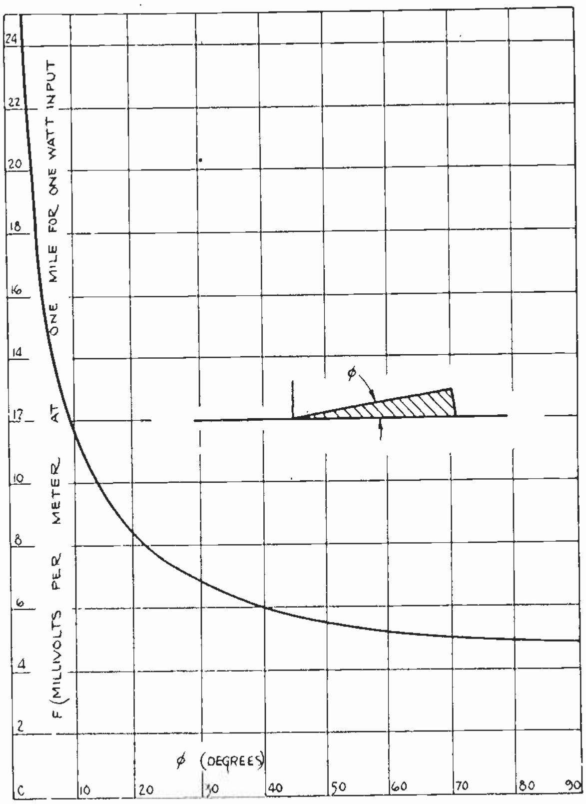

Fig. 41

the signal obtained from a simple quarter-wave antenna, the concentration must be within nine degrees of the earth.

No attention has been given to the relative merits of the various arrangements in reducing ground losses, since these merits will only be present when the ground system is very small. When adequate ground systems are provided, the earth losses can always be reduced to negligible quantities.

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INPUT RESISTANCE OF VACUUM TUBES AS ULTRA-HIGH-FREQUENCY AMPLIFIERS*

By

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Summary—*Vacuum tubes which when operated as voltage amplifiers at low frequency require no measurable grid input power have been found to take very serious amounts of power at ultra-high frequencies. The grid input conductance is shown to be very accurately represented for electrodes of any shape by the expression*

$$g_g = K s_m f^2 \tau^2$$

where g_g is the input conductance, s_m the grid-plate transconductance, f the frequency, and τ the electron transit time. K is a parameter which is a function of the geometry of the tube and the voltage distribution. A physical picture of the effect, a simple theoretical derivation, and experimental proof with conventional tubes are given.

The magnitude of g_g is such that it is the principal limitation for amplifiers at frequencies of the order of 100 megacycles, and it seriously affects the amplification at frequencies as low as fifteen megacycles. The input resistance of a typical commercial tube, the RCA-57, is approximately 20,000 ohms at thirty megacycles. Other commercial tubes, being of the same general construction and size, have input resistances of the same order. The use of very small tubes, such as the RCA-954, with correspondingly short transit times is shown to be a practical means of increasing the amplification obtainable with conventional circuits.

Input capacity variation with frequency is found to be negligible with the RCA-57 even up to eighty megacycles and higher. However, the grid-cathode capacity is a function of the applied voltage; the ratio of this capacity under operating conditions to that with the tube cold having a value of four thirds for its minimum. The change in input capacity from cold to hot is of the order of one micromicrofarad for the RCA-57. No change in grid-screen capacity is indicated.

The plate resistance of screen-grid tubes is found to vary with frequency but with the RCA-57 at eighty megacycles it is over twenty times the grid resistance and thus constitutes a negligible amount of the total loss in the circuit.

Introduction

Vacuum tubes, when used as amplifiers of small voltages at low frequencies, have the unique property, compared with other amplifying devices, of absorbing negligible power from the source. At higher frequencies, circuit reaction due to feedback in the tube may cause an effective input power loss, but such losses may

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be avoided by elimination of the grid-to-plate capacity admittance, either by the use of screen-grid tubes or by various neutralization schemes. The tubes themselves respond substantially instantaneously to all signals up to those commonly called ultra-high frequency. The absence of any apparent inertial effect is due to the fact that for each electron the transfer of charge from cathode to anode takes place at such high velocity that the time of transit is altogether negligible with respect to the period of the alternating signal voltage. The possibility of a change in tube characteristics when the transit time becomes an appreciable fraction of the period has long been recognized. It would be surprising, indeed, if there were no effect when the transit time equals or exceeds a complete period, since several groups of electrons corresponding to various instantaneous values of the voltage must then be simultaneously located between the tube elements. This has often been suggested as a limitation for oscillators at high frequencies. A little thought shows that the effect must be a continuous function of the ratio of the electron transit time to the period of the signal voltage.

The most important effect of the transit time on tube characteristics proves to be a serious inphase or power component of the charging current to the control grid. This may be expressed as a conductance which will be shown in this paper to vary as the square of the frequency and thus rapidly to become of enormous importance as the frequency is increased. The grid-cathode *capacity* varies scarcely at all with frequency in practical cases.

This paper presents a physical explanation of the effect, an experimental verification of the theoretically derived formulas, and measurements of the magnitude of the effect in various commercial tubes.

In order to visualize the relative magnitudes of the quantities involved, one only need consider that the electron transit time in an ordinary receiving tube is of the order of 10^{-9} seconds, which is the period of a thirty-centimeter wave. This is then ten per cent of the period of a three-meter wave and is an appreciable fraction even for much longer waves. Expressed as an angle in radians, the ratio of the electron transit time to the period of the signal is known as the *electron transit angle*, and it is through this transit angle that the transit time enters as a tube parameter.

In 1931 B. J. Thompson, of this laboratory, predicted from elementary considerations in connection with the study of the small "acorn" or "shoe-button" tubes, that the grid input conductance of a triode should become appreciable at frequencies for which the electron transit times from the cathode to the grid or plate were not a negligible fraction of the period of the high-frequency grid voltage. About the same time W. A. Harris, also of this laboratory, derived a theoretical equation for the admittance of a diode, which he extended to apply to a negative-grid triode. This expression is identically equivalent to that given by Llewellyn[1] in his equations (32) and (33) but is applicable only to tubes whose transit time from grid to plate is a negligible fraction of a period of the alternating grid voltage.

In August, 1933, H. O. Peterson of RCA Communications, Inc., observed a very serious damping of a tuned circuit caused by the grid admittance of an RCA-57 pentode, although the tube was operating with sufficient negative bias to prevent the flow of any electron current to the grid. The plate and screen grids were by-passed to ground so that regenerative effects were not present. This effect was called to the attention of B. J. Thompson who recalled his previous work and showed that a loading effect of the observed order of magnitude should exist.

Experimental measurements in this laboratory and obvious dimensional requirements immediately demonstrated that the input conductance of a negatively biased vacuum tube was of the form

$$g_g = K s_m f^2 \tau^2 \tag{1}$$

where,

g_g = input conductance
s_m = grid-plate transconductance
f = frequency
τ = time of electron transit to an arbitrary point in the system
K = a parameter depending upon several factors which could not be definitely determined at that time, but which seemed to depend upon the ratio of τ_1 to τ_2 in some manner; τ_1 being the electron transit time between cathode and control grid and τ_2 that between control grid and screen.[2]

[1] F. B. Llewellyn, "Vacuum tube electronics," Proc. I.R.E., vol. 21, p. 1546; November, (1933).

[2] Llewellyn[1] refers to some experiments performed by J. G. Chaffee, later published in Proc. I.R.E., vol. 22, p. 1018, August, (1934), which indicated a power loss in the grid circuit of a vacuum tube voltmeter. Chaffee's discussion and curves indicate approximately a linear variation of conductance with frequency. Llewellyn in his article suggested as an explanation the possibility of a resistance loss between the cathode and the region of potential minimum between the cathode and grid. Conversations with Dr. Llewellyn have disclosed that he agrees with the correctness of the square-law relation in connection with the class A amplifier. Mr. Chaffee reports that he has submitted a note for publication covering further experiments.

II. Theoretical Considerations

Thompson derived, from general considerations, an equation of the above form in the following manner:

First, consider the process of conduction in a parallel plane diode (Fig. 1) under direct-current operating conditions such that a steady current, I, is flowing in the circuit. Under these conditions the current, I, at the plate is equal to the product of the charge density, ρ, and the electron velocity, v, at that point, and it is therefore customary to state that the plate current is due to the rate of arrival of the elec-

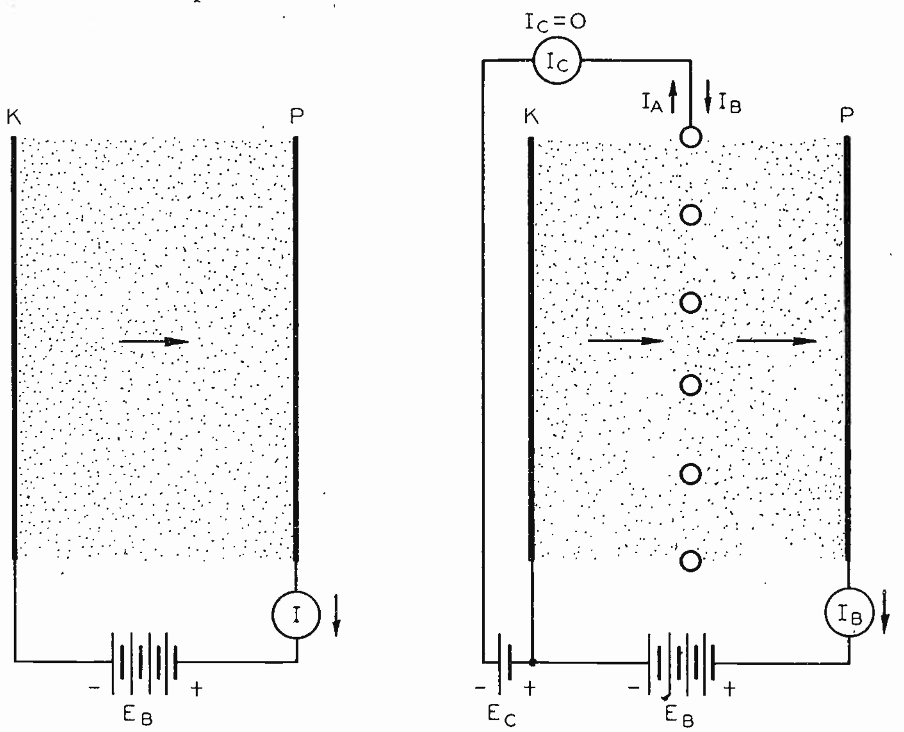


Fig. 1—Distribution of convection current and total current in a diode with direct voltage alone.
Fig. 2—Distribution of convection current and total current in a triode with direct voltage alone.

trons at the plate. It may also be argued, however, with greater rigor, that each electron in the space between cathode and anode induces a charge on the anode determined by its relative proximity thereto and that, since this proximity changes with the electron motion, the charge due to each electron changes with time, and, consequently, there is a current flowing in the plate due to the motion of *each* electron in the space between cathode and plate. Under static conditions this approach yields identical results to those of the first, but it has the advantage of being applicable to high-frequency phenomena, while the first is not. It is to be emphasized, then, that the current flowing in an electrode is due to the inductive action of all charges approaching or receding from it and not due to the instantaneous arrival of charges at the electrode.

Next, consider a triode with negatively biased grid, Fig. 2, under direct-current conditions. From the previous argument it follows that there is a current I_A flowing in the direction indicated by the arrow due to the motion of the electrons between cathode and grid approaching the grid, and another current I_B in the opposite direction due to the motion of the electrons between grid and plate receding from the grid. Since no electrons reach the negative grid, under static conditions the two currents I_A and I_B are equal, and the net current to the grid is zero. In any case, however, where a difference between I_A and I_B arises, either due to arrival of electrons at or to emission of electrons from the grid, or due to high-frequency variations in the convection current, ρv, the grid current will not be zero.

Consider next a triode with negatively biased grid and a small alternating potential superimposed on the direct grid potential, during the portion of the cycle when the potential is increasing in a positive direction, Fig. 3. Since the convection current, represented by the density of the dots, is a function of the instantaneous potential of the grid, the convection current is increasing with time. Further, since the convection current originates at the cathode and propagates at finite velocity across the space, it is greatest near the cathode and least near the plate, as indicated. Thus there is an excess of I_A over I_B and there is a net current flowing into the grid. During the portion of the cycle when the grid potential is decreasing, Fig. 4, exactly opposite conditions obtain and there is an excess of I_B over I_A, with a net current flowing out of the grid.

The relative magnitude of this net grid current may readily be determined. At any instant the difference between the convection currents on the two sides of the grid, measured at two arbitrary points, is proportional to the product of the rate of change of grid potential with time and the transconductance, for this measures the time rate at which the convection current is modulated. This difference is also proportional to the transit time of the electrons between the two arbitrary points, for this is a measure of the space modulation of the convection current. Further, the rate of change of grid voltage is proportional to the product of the root-mean-square magnitude and the frequency of the alternating grid voltage. Thus,

$$I_g = K_1 E_g s_m f T$$

or,

$$Y_g = K_1 s_m f T \qquad (2)$$

when I_g is the alternating grid current, K_1 an undetermined parameter, E_g the alternating grid potential, s_m the transconductance, f the frequency of the alternating grid potential, τ the time of the electron transit, and Y_g the grid admittance. From the foregoing argument it is clear that the admittance is in the nature of a capacitive susceptance.

Consider the moment when the grid potential reaches its maximum positive value. The condition indicated in Fig. 3 still holds, for at the previous instant the potential was increasing, and hence the convec-

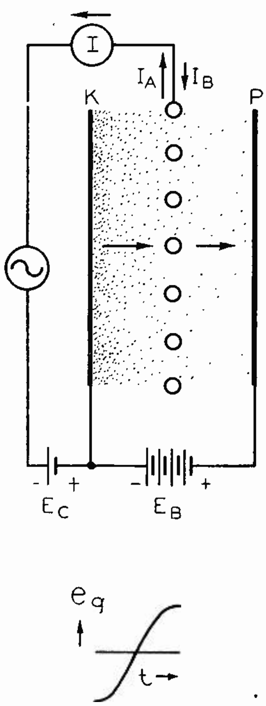


Fig. 3

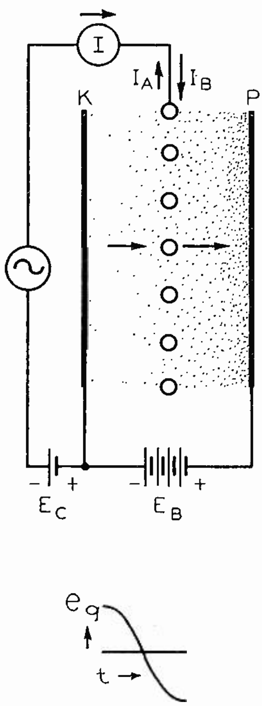


Fig. 4

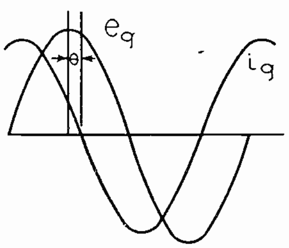


Fig. 5

Fig. 3—Distribution of convection current and total current in a triode with small alternating voltage superimposed on the grid bias, during the portion of the period when the potential is increasing.

Fig. 4—Same as Fig. 3, except with the potential decreasing.

Fig. 5—Alternating grid voltage and current vs. time.

tion current is greatest at the cathode and least at the plate. It is not until some time later that I_A and I_B become equal, this time being determined almost entirely by the time of transit, τ. Thus there is a phase displacement between potential maximum and zero instantaneous grid current, which may be represented by the angle θ, Fig. 5. This shift in phase represents a conductive component, g_g in the grid admittance which may be defined as

$$g_g = Y_g \sin \theta. \qquad (3)$$

For small angles,

$$\theta \sim \sin \theta$$

so (3) may be written as

$$g_g \approx Y_g \theta \tag{4}$$

for small transit angles.

Since θ is proportional to the product of τ and f,

$$g_g = K s_m f^2 \tau^2,$$

which is equation (1).

In using this equation it is to be noted that since τ is not defined as the transit time to any particular point, the parameter K may be a function of τ_2/τ_1, among other things, and is not capable of evaluation by obvious methods. It does, however, permit us to predict several useful relations, among these that if all of the linear dimensions of the tube are increased in a direct ratio, M, the input conductance will increase as M^2. Also, if all of the voltages are changed in a direct ratio, N,—so that to whatever point it refers, τ will be changed by $N^{-1/2}$ and s_m by $N^{1/2}$,—the input conductance will be changed by $N^{-1/2}$. Many limitations due to approximation in the above formula are apparent, among them being the setting of $\sin \theta = \theta$ and the ignoring in the expressions for I and θ of terms involving higher powers of $f\tau$ than the first.

The apparent accuracy of the above formula led the author to seek a rigorous theoretical verification of it and a discussion with D. O. North, of this laboratory, led to the derivation given by him in the accompanying paper.[3]

His derivation, which is in the form of a trigonometric equation, ((21) in his paper) can be reduced, as he shows, to the following form:

$$g_g = \frac{s_m}{180}\left\{9\theta_1^2 + 44\theta_1\theta_2 + 45\theta_2^2 - \frac{2\theta_2}{1+\dfrac{v_p}{v_g}}(17\theta_1 + 35\theta_2) + 20\frac{\theta_2^2}{\left(1+\dfrac{v_p}{v_g}\right)^2}\right\} + \cdots \tag{5}$$

and since $\theta_1 = \omega\tau_1$, $\theta_2 = \omega\tau_2$ where $\omega = 2\pi f$

[3] D. O. North, "Analysis of the effects of space charge on grid impedance," PROC. I.R.E., this issue, pp. 108–136.

$$g_o = \frac{s_m \cdot 4\pi^2 f^2 \tau_1^2}{180} \left\{ 9 + 44\frac{\tau_2}{\tau_1} + 45\left(\frac{\tau_2}{\tau_1}\right)^2 \right.$$

$$\left. - 2\frac{\tau_2}{\tau_1} \frac{\left(17 + 35\frac{\tau_2}{\tau_2}\right)}{1 + \frac{v_p}{v_g}} + \frac{20\left(\frac{\tau_2}{\tau_1}\right)^2}{\left(1 + \frac{v_p}{v_g}\right)^2} \right\} + \cdots$$

so the K in (1) is confirmed as being a function of τ_2/τ_1 and is shown also to be a function of the ratio of electron direct-current velocities v_g and v_p at the grid and plate, respectively.

The trigonometric equation for which (5) is a series approximation was derived on the following assumptions:

(1) The electrodes are parallel planes.
(2) The initial velocity of the emitted electrons is zero and the emission is ample; so the three-halves-power equation holds in the cathode-grid region.
(3) The grid is an equipotential plane surface.
(4) The amplification factor of the grid is high, so that electrons on one side do not appreciably influence the field on the other.
(5) The alternating voltage at the plate is zero.
(6) The alternating voltage at the grid is very small with respect to the effective static potential there.
(7) The space-charge density in the grid-anode space is so slight that the potential distribution between grid and plate is substantially linear.

Of these assumptions, (5) is easily realized by careful by-passing of the plate. (2), (3), (4), (6), and (7) can be approximately realized by obvious expedients and limitations, although these limitations may interfere with the application of the formula to certain practical cases. (1) is difficult to realize in practice, since plane cathodes, to be of practical dimensions, require guard rings which complicate the experimental technique. (1) may be approximated in a cylindrical structure if the cathode, grid, and plate are of such proportions that the lines of force are approximately parallel. With tubes of other than plane structures it is logical to use the true transit time values calculated from the particular structure if possible. In fact, the equation as it stands may be nearly correct for any structure if the proper transit-time values are employed. (3) presents considerable difficulty in the vicinity of cutoff, and likewise (2) to some extent. (6) proves experimentally to be relatively unimportant, since the impedance changes by no readily measurable amount with any value of signal within reason. (6) is similar in this respect to the corresponding limitation imposed in low-frequency amplifier theory by neglecting the curvature of the tube characteristics.

III. Experimental Procedure

In carrying out the experimental work reported in this paper a direct substitution method was chosen as suitable for reasonably ac-

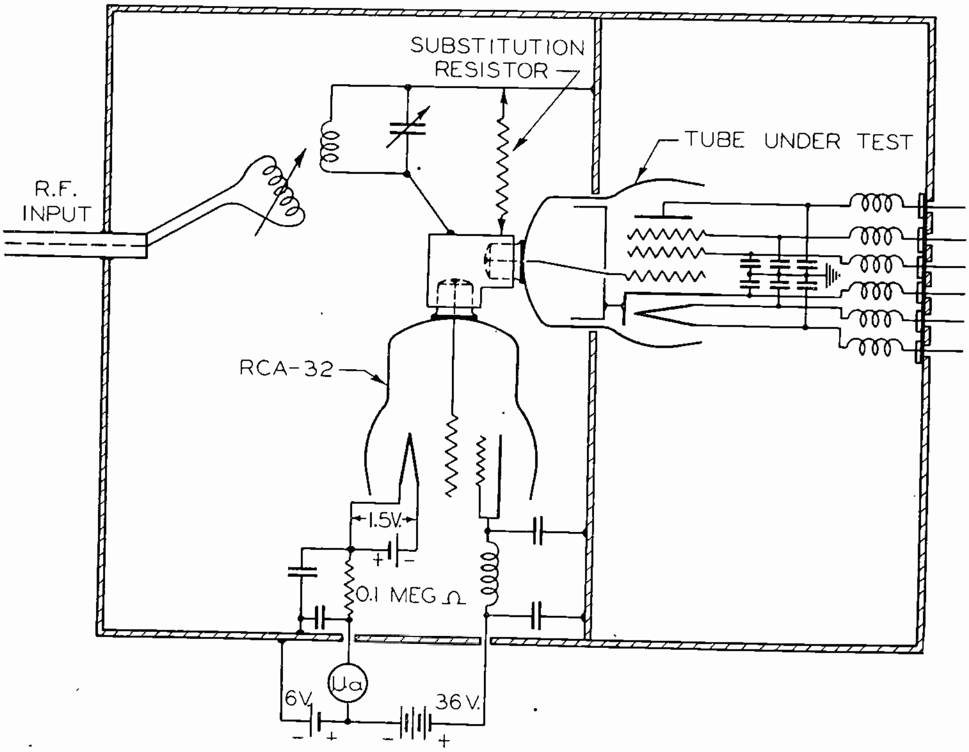


Fig. 6—Schematic diagram of test circuit for investigating grid conductance and capacity.

curate measurements of input conductance with little chance for serious error. The apparatus was placed in a copper box having two compartments. The tube under test was mounted inside one of these, and the tuned circuit with its variable inductive coupling coil and a tube voltmeter were placed in the other. Of the tube under test only the grid cap extended into this second compartment. A special socket with built-in by-pass condensers was used for the tube under test, and chokes or resistors with additional by-pass condensers were used to prevent coupling between the input circuit and the leads from the tube under test or the tube voltmeter. Fig. 6 shows a schematic diagram of the circuit including the tube voltmeter, which is of a type devised by A. W. Hull of the General Electric Research Laboratory. This voltmeter may be considered practically equivalent to the slide-back peak type in that it requires only a few microamperes of cathode current for operation. The use of the small fixed grid bias shown was suggested by L. S. Nergaard of this laboratory. This reduces the cathode current to a very low value even when a cathode resistor of relatively low value is used and thus affords additional sensitivity. The impedance of the tuned circuit, including the tube voltmeter and the cold capacity of the tube under test, was about 75,000 ohms at thirty megacycles. The desirable feature of a high impedance tuned circuit is that maximum sensitivity is thereby obtained. Different coils were used for higher frequencies and their poorer L/C ratio gave much lower impedance, but good sensitivity was still obtained since the tube impedance was correspondingly lower at these high frequencies.

The substitution resistors employed were "grid leaks" of the metallized-glass-rod type. Direct tests have shown that these are entirely reliable up to about seventeen megacycles for values of resistance less than one-fourth megohm. No direct tests have been made at higher frequencies* but comparative tests between the one-watt and one-fourth-watt

* *Note added in proof:* Recent measurements on the effective resistance of these metallized resistors indicate a considerable decrease in their resistance as the frequency is increased. These measurements were obtained by A. V. Haeff of this laboratory. Two editorials in *The Wireless Engineer*, vol. 12, pp. 291–295; June, (1935), and pp. 413–414; August, (1935), discuss this effect and ascribe it to the fact that the resistors behave as short transmission lines having distributed resistance and capacity. The results obtained by Haeff can be explained only partly by the simple theory given in the above editorials, but any correct theory must include this obvious condition as one of the principal causes of resistance variation.

Extrapolations from measurements made at about 120 megacycles indicate that the resistance values of the resistors at thirty megacycles should be about as follows:

R_{dc}	R_{ac}
250,000	190,000
200,000	170,000
150,000	130,000
100,000	95,000

Below 100,000 ohms the correction at thirty megacycles is in general less than the experimental error. The curves numbered 6 and 7 in Fig. 8 should be corrected accordingly. Although the nature of the effect is such that it would change the slope of these two curves of Fig. 8 somewhat, the magnitude of this change is small and it is sufficiently accurate to redraw these curves parallel to their orginal position. The corrected curves should be drawn with the exact slope of —2, the error being in the original plotting of the points. This error is less than that due to experiment in this case.

The curves of Figs. 10 and 11 should be corrected in accordance with the above, and also some of the values of Table I. This error is seen to have no effect on the validity of the theory presented in this paper and is merely mentioned at this time for accuracy and as a precaution to be taken in some high-frequency measurements employing such resistors at somewhat higher frequencies.

sizes having the same direct-current resistance but of different physical dimensions show no difference, and as it hardly seems probable that both sizes should have the *same* frequency error it may be safely assumed that the error is not of importance. The conducting film on these resistors is so thin and of such high specific resistance that skin effects should not cause any appreciable error. This film is said to be continuous for the lower values of resistance, up to a few hundred thousand ohms at least, so no loss due to capacity coupling to isolated patches of coating not in metallic contact with the main body of the resistor should be present. These resistors may be combined in *parallel* but not in *series*, since their own shunt capacity and stray capacities cause part of the current to flow around the resistors instead of through them. The effect of this capacity may be eliminated by retuning with the parallel combination but not with the series. Experimental checks substantiate this argument. Capacity coupling to resistors not in the circuit must be avoided, preferably by removal of the resistors from the shielding compartment except when they are actually connected.

This digression upon the behavior of resistors is made because of the interest in the subject expressed by many people with whom the author has discussed it, and because of the lack of such data in high-frequency measurement literature.

The actual method of measurement is as follows: A resistor of a value thought to be of the order of the input resistance of the tube under test is connected across the tuned circuit while the tube under test is in place, but either cold or biased considerably beyond cutoff, and preferably with no plate or screen voltage applied. The circuit is then tuned for maximum deflection of the tube voltmeter and the coupling from the oscillator is varied until a voltage somewhat less then one volt is indicated. The exact value of voltage need not be known as the method is strictly one of substitution, but it should be the smallest value which will give a good deflection on the voltmeter. The resistor is then removed from the shield compartment and all voltages are then applied to the tube under test. The circuit is retuned to resonance while the bias of the tube under test is varied, for a particular value of screen voltage, until the same voltmeter deflection is obtained as previously existed with the resistor in place. This is repeated for several values of screen voltage, and a curve is plotted. The entire process is then repeated for other values of resistance. Fig. 7 is a sample curve sheet taken on an RCA-57 tube. This method is independent of errors in voltmeter calibration, of voltmeter resistance, and of variation of resistance with signal amplitude, since all readings are taken with the same deflection on the meter. Variation of resistance with signal amplitude is found to be practically absent, but the method is independent of it even if any such variation should exist in a particular case.

Variation of tube input capacity with frequency is not of easily measurable magnitude with transit angles of the order of those obtained with commercial tubes at ordinary frequencies. There is, however, an important variation of tube capacity with applied direct voltages, almost independent of frequency, which can be easily mea-

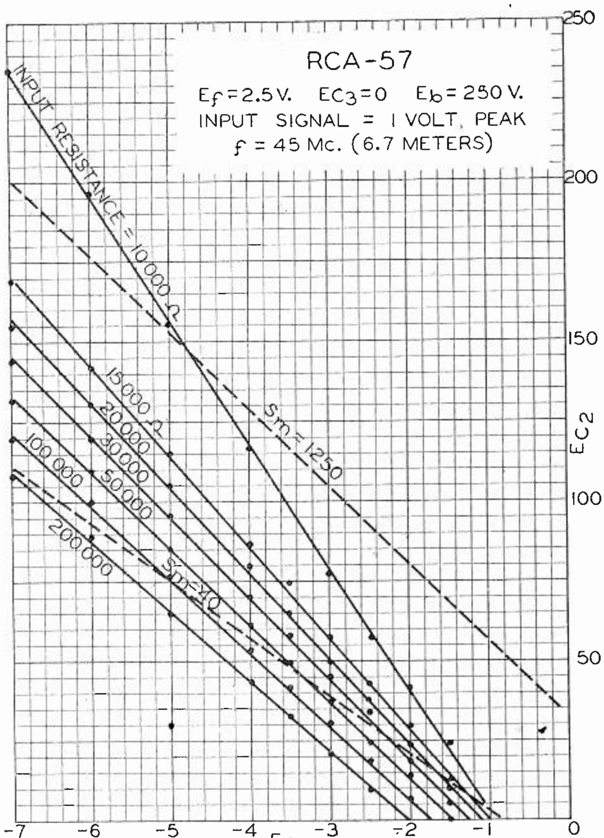


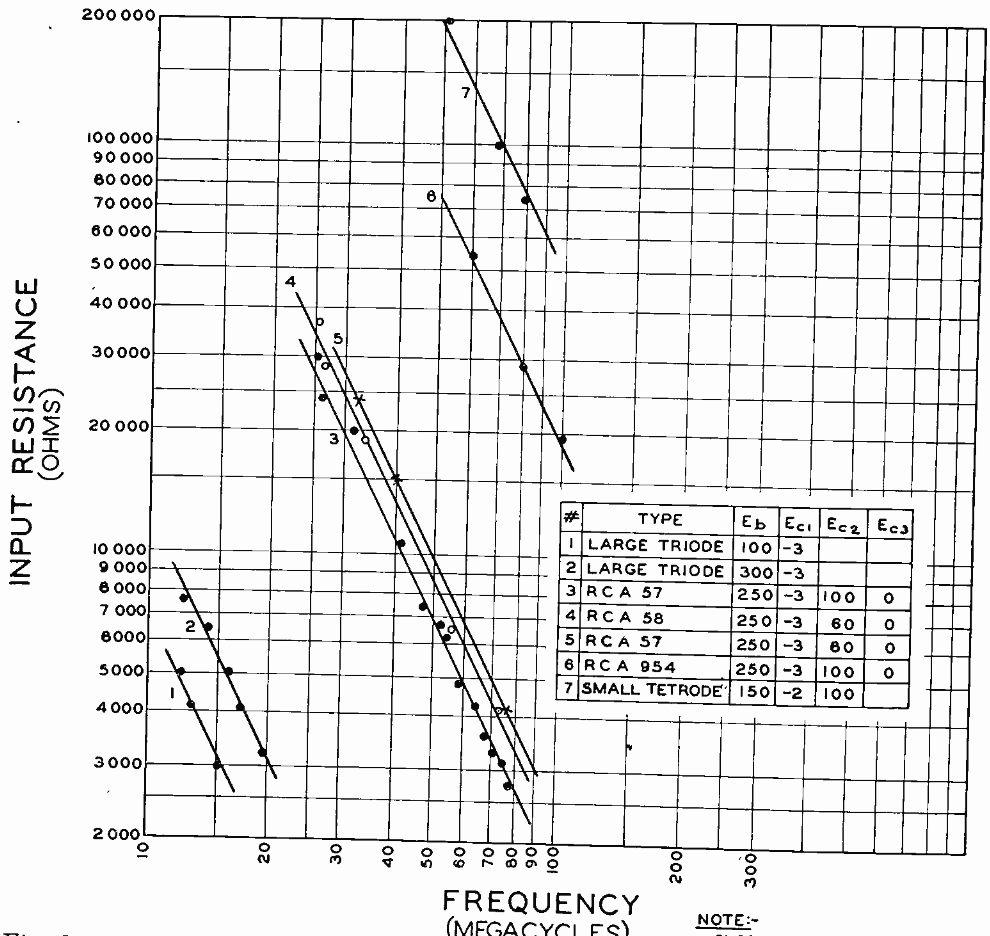
Fig. 7—Screen-grid voltage vs. control-grid voltage for constant input conductance and for constant transconductance (or plate current).

sured. The expression derived by North[3] shows the form of this variation and also shows that the variation of capacity with frequency should be small for small transit angles. This change in capacity with voltage makes it necessary to retune the circuit for each change in applied voltage.

Plate resistance of tetrodes or pentodes may be measured in exactly the same way as input resistance, and in the case of the RCA-57 a rough measurement has shown it to be of the order of 50,000 ohms at eighty megacycles and to be very high at thirty megacycles. It should, theoretically, contain a term which varies inversely as the square of the frequency. The grid resistance at these frequencies is so low in each case that the corresponding value of plate resistance is of no great importance in determining the gain to be obtained with the tube.

IV. Experimental Results

Fig. 8 shows experimentally determined curves of input resistance as a function of frequency for several types of tubes under different



#	TYPE	E_b	E_{c1}	E_{c2}	E_{c3}
1	LARGE TRIODE	100	-3		
2	LARGE TRIODE	300	-3		
3	RCA 57	250	-3	100	0
4	RCA 58	250	-3	60	0
5	RCA 57	250	-3	80	0
6	RCA 954	250	-3	100	0
7	SMALL TETRODE[4]	150	-2	100	

Fig. 8—Input resistance vs. frequency for several typical tube types. The large triode (R-104 of Table I) and small tetrode[4] are special laboratory tubes.

operating conditions. It is seen that (1) holds to a high degree of precision.

Due to the difficulties encountered in obtaining tubes of exactly similar characteristics even of supposedly identical dimensions, it is practically impossible to construct experimental tubes of different sizes

[4] B. J. Thompson and G. M. Rose, Jr., "Vacuum tubes of small dimensions for use at extremely high frequencies," Proc. I.R.E., vol. 21, pp. 1707–1721; December, (1933).

to demonstrate directly the variation of g_g with s_m and τ to the same precision as with frequency. However, in the case of the small tetrode and the RCA-57, having transconductances of 700 and 1200, respectively, and spacings of the order of one to four or five, the ratio of the input conductances, which according to the theory should be of the order of one to twenty or forty, is seen to be equal to one to twenty-eight, which is in good agreement with the equation.

Table I represents experimental data obtained on a number of specially constructed tubes, the dimensions of which are given, compared with the values of input resistance calculated from (5). The experimental check of (5) is entirely satisfactory except in the case of the R-104 triode. This may possibly be due to the fact that in all of the tubes except R-104 the cathode diameter is comparable with that of the grid, and hence the parallel plane formula is more applicable to them. The experimental value for the RCA-57 is seen to lie between the calculated values obtained by using the major and minor diameter of the control grid. It should be remembered that, since the input conductance varies as the square of the transit angle, a few electrons with a large transit angle can absorb as much power as a relatively large number with a smaller angle. Therefore the large major diameter of the RCA-57 control grid may be responsible for a considerable part of the input loading even though relatively few of the electrons take paths in the general direction of this diameter. Electrons escaping from the end of the anode may also have long transit times. This effect may be large with tube R-104 since the cathode extends almost to the open ends of the anode.

TABLE I

Tube serial No.	R-192	R-191	R-190	R-189	*	R-104	RCA-57
Cathode diam. (in.)	0.054	0.054	0.054	0.054	0.054	0.082	0.054
Grid diam. (in.)	0.125	0.125	0.125	0.125	0.125	1.50	0.092 × 170
Wire size (in.)	0.0045	0.0045	0.0045	0.0045		0.020	0.0033
Turns per inch	16.5	22	28	45		4.0	42.5
Plate diam. (in.)	1.000	0.500	0.300	0.175	0.125	3.00	(screen) 0.265
E_b	150	150	150	150		300	E_{c2} 100
E_c	−5	−5	−7	−5.5		−3	−3
μ	12	12.65	11.9	13.3		10.9	20**
I_b	3.85	4.25	4.5	4.4		11.5	2.5
s_m	645	925	975	1000	1000	720	1480**
V_g	7.5	6.40	6.50	6.35	6.35	22.5	2.57
f	30 mc	30 mc	30 mc	30 mc	30 mc	15 mc	30 mc
θ_1	0.264	0.298	0.293	0.294	0.294	0.98	Max. 0.73 Min. 0.24 r-m-s mean 0.54
θ_2	0.375	0.179	0.086	0.026	0	0.244	Min. 0.06 Max. 0.13 r-m-s mean 0.10
Input resistance (calc. by eq. (5)) (ohms)	32,000	49,600	90,000	164,000	230,000	13,000	Max. 50,000 Min. 11,600 by mean θ_1 and θ_2 23,200
Input resistance (observed) (ohms)	23,000	45,000	75,000	140,000		5,500	19,000

* Limiting case with $\theta_2 = 0$.
** As triode.

Another source of possible failure of (5) to check with experimental measurements, on some tubes, is the loading effect of positive ions which may be emitted from coated cathodes in considerable numbers. There is evidence that these ions from some cathodes may cause the space current to be more than twice that calculated from the three-halves-power equation. Ions are known at times to make many trips around a cathode of small diameter, relative to the anode, and in such

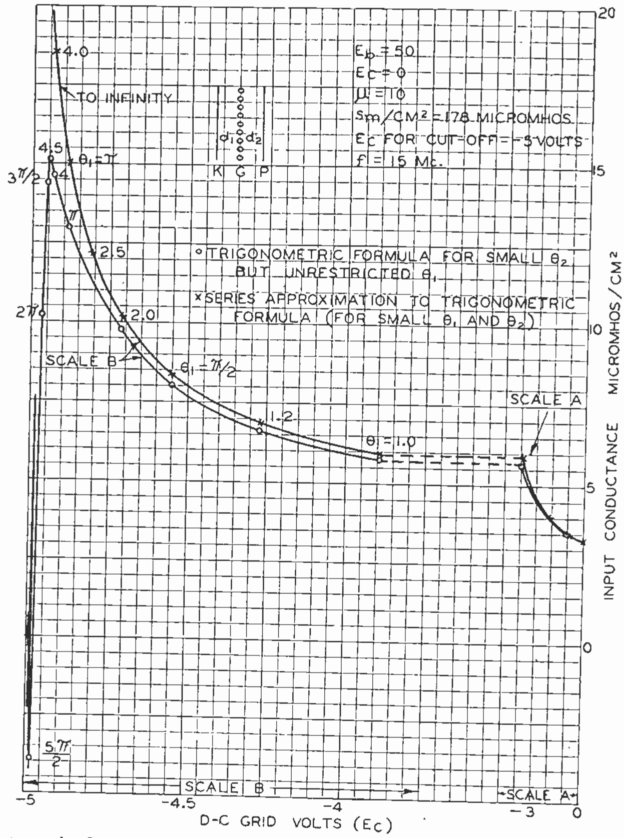


Fig. 9—Electronic loading equation. Comparison of series approximation (eq. (5)) with exact trigonometric equation.

cases their effective transit angles could reach very large proportions. The effect may also contribute to the low measured input conductance value of tube R-104.

Equation (5) is subject to a further limitation that θ_1 and θ_2 be small in comparison with unity, but the transcendental equation from which it is derived is not so limited. A check of (5) shows that θ_1 may become almost as great as π without seriously affecting its validity. The same is true of θ_2. Thus (5) may be considered adequately precise for all practical transit angles except for tubes operating near cutoff, for which the complete expression must be used. This is illustrated in

Fig. 9 which is calculated for an imaginary ideal triode with perfect cutoff. The variable nature of the cutoff in actual tubes smooths out the oscillations indicated in Fig. 9 so that they may be ignored except at very high frequencies for which θ_1 and θ_2 are large even far from cutoff.

Figs. 10 and 11 show the measured values of input resistance as a function of grid bias and direct plate voltage for two special laboratory

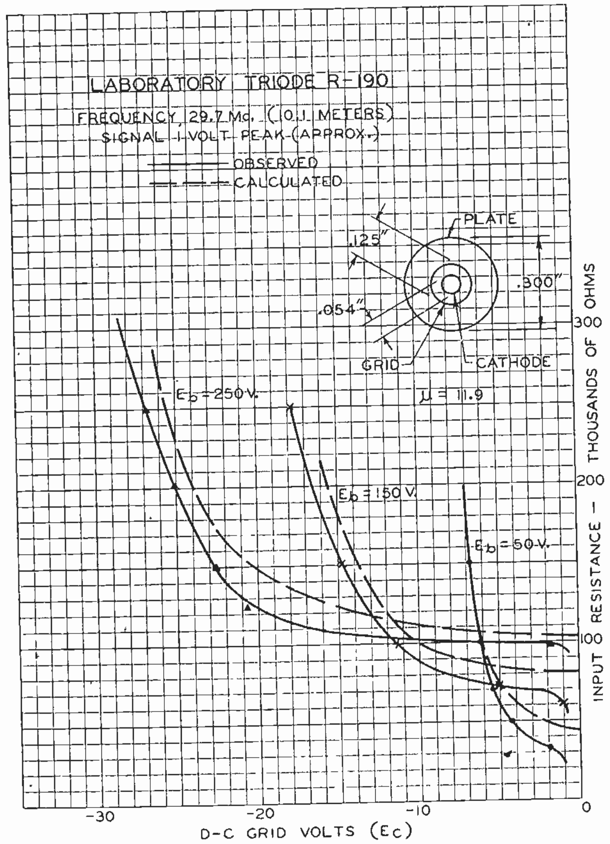


Fig. 10—Electronic grid circuit loading vs. grid bias with various plate voltages. Grid-plate spacing corresponding to usual practice.

tubes whose dimensions are shown in the insets. The calculated values were obtained from (5).

The opposite trend of the curves of Fig. 9 with respect to those of Figs. 10 and 11 is due to the departure of the transconductance of the actual tubes from the theoretical variation with the square root of the effective grid voltage, which is assumed in Fig. 9. This departure is considerable and is due largely to grid side-rod cutoff effects on the electron stream and to irregularities in the grid. In the actual electron stream, however, the theoretical distribution of potential is still closely approximated, and since both input conductance and transconductance vary together with the area of the electron beam, the values of input conductance calculated from the measured transconductance may be expected to be substantially correct. Some tubes show a pronounced minimum of input resistance vs. grid voltage. The tube of Fig. 10 suggests a minimum of resistance for $E_b = 250$ volts and $E_c = 5$, but the curve is too flat to show this with certainty.

The measurements adequately demonstrate that the input conductance results from the electron transit times in both the cathode-to-grid and grid-to-plate spaces, and that for parallel plane or conven

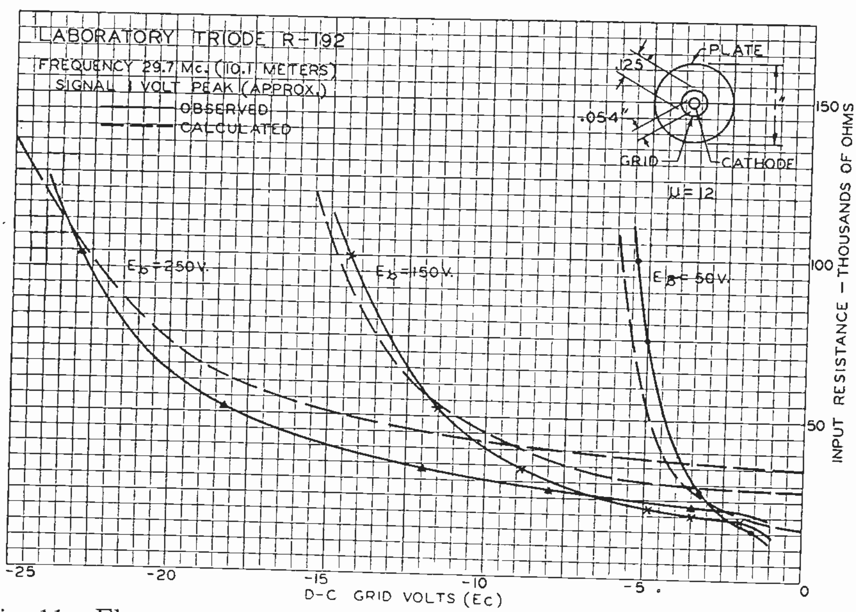


Fig. 11—Electronic grid circuit loading vs. grid bias with various plate voltages. Abnormally large grid-plate spacing.

tional cathode type tubes the magnitude of the effect may be calculated with sufficient accuracy for design purposes by (5).

V. Effects of Electronic Loading on Amplification of Ultra-High-Frequency Signals

The large value of grid conductance, relative to the transconductance, at frequencies of the order of thirty megacycles and higher, proves to be by far the most important cause of the low amplifications observed experimentally at such frequencies. It is readily apparent that a tube cannot amplify at all when the internal input loss is equal to the power output. With commercial tubes at frequencies of the order of 100 megacycles, the electronic loading loss alone is equal to the output power, since the grid input conductance is then equal to the transconductance. Circuit losses, if large, would tend to lower this frequency limit considerably, but they prove to be of practically negligible magnitude in comparison with the electronic loading in many cases. Tuned interstage coupling circuits with an over-all impedance of the order of 75,000 ohms at thirty megacycles have been made with ordinary helical coils. This value was measured in a circuit with all tubes and shields in place but with the tubes cold. With the tubes hot and biased below cutoff, the impedance was substantially the same. The usual composition tube bases and sockets were not used. By the use of a smaller coil in the same circuit, an impedance of over 6000 ohms could be obtained at eighty megacycles, the difference being largely due to the reduced L/C ratio. (A standard but slightly altered variable condenser having a range of one to twenty micromicrofarads was used and the tube capacities totaled about ten micromicrofarads.) Transmission-line interstage tuners can be made to have very high over-all impedance even at frequencies higher than 300 megacycles. The transconductance of screen-grid tubes is only slightly reduced at high frequencies and the plate resistance has been found to remain relatively high with respect to the grid resistance. Rough measurements showed the plate resistance of a standard '57 tube, excluding dielectric loss in the base, to be over 50,000 ohms at eighty megacycles, at which point the grid resistance was about 2700 ohms. The phase angle of the transconductance can have no effect on the total gain obtainable with well-shielded screen-grid tubes used as amplifiers.

In spite of the good values of circuit impedance which can be obtained, the maximum possible gain of the '57 tube is only about fifteen at thirty megacycles and it is reduced to unity somewhere near 100 megacycles, since at this frequency the input conductance becomes equal to the transconductance.

A suggested figure of merit for high-frequency amplifier tubes is the frequency at which the input conductance becomes equal to the transconductance. This, in addition to the static characteristics, would allow the calculation of the performance of the tube at all frequencies and would show the upper frequency limit of the tube for amplifier use.

With tubes of the dimensions of the RCA-954, the input resistance at a given frequency is some twenty times better than for standard tubes. This permits signals of a frequency four or five times higher to be amplified with the same gain. Gain of unity or better has actually been obtained at about 430 megacycles using RCA-954 pentodes.

The construction of intermediate-frequency amplifiers operating at thirty megacycles and having a gain of fifty or sixty per stage should be possible with these tubes although neutralization of the grid-plate capacity might be necessary with the high load impedance which this would necessitate.

Tubes used as superheterodyne converters should permit operation at a much higher frequency than the same tubes used as amplifiers.

Increasing the voltage applied to conventional types of tubes would improve conditions considerably but cannot be carried very far because of the excessive dissipation resulting.

The use of very small tubes, such as the RCA-954, provides an entirely satisfactory method of obtaining amplification at frequencies up to 300 megacycles, and by an extension of this principle it seems possible to extend the limit to perhaps 1000 megacycles with conventional circuits and reasonable voltages.

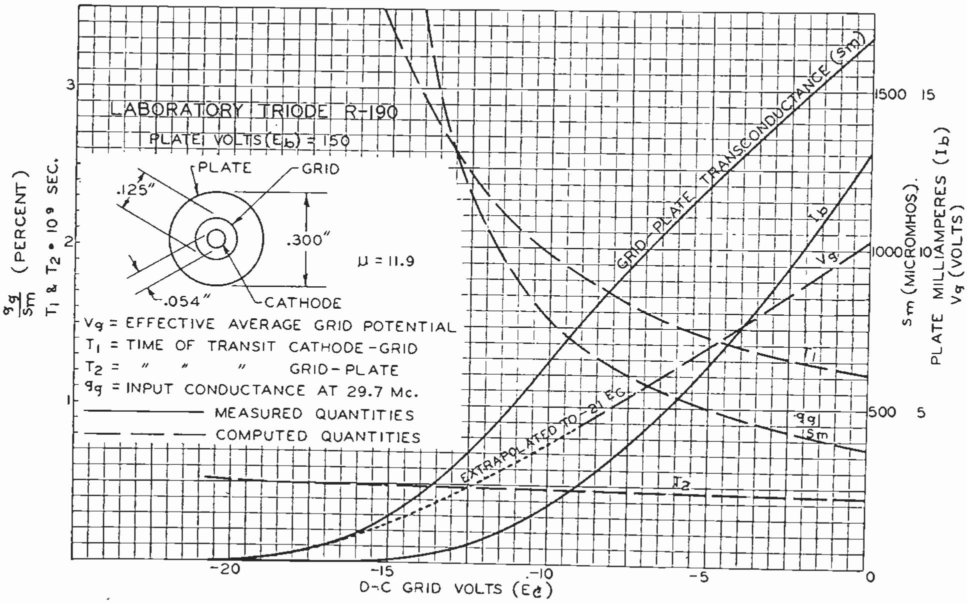


Fig. 12—Transit time and related quantities as obtained from measured values of I_b, s_m and the dimensions of the tube vs. grid-bias voltage.

Appendix I

Calculation of Electron Transit Times in Vacuum Tube Structures

The application of (5) requires the calculation of the electron transit times τ_1 and τ_2, which are the times of transit of an electron from cathode to grid and from grid to plate, respectively. The transit times are functions of the geometry of the tube and the effective voltages at the elements.

Calculation of Effective Voltages

In triodes, the effective voltage of the plate is simply the applied direct voltage, and in screen-grid tubes the pitch of the screen is usually so fine that the screen may be considered to be practically equivalent to a solid sheet of metal at the applied screen potential.

The effective control-grid potential, V_g, may be calculated entirely from the physical dimensions and applied potentials of the tube, but irregularities in the structure of practical tubes make the following method much more reliable. The ratios b/a, or $\log r_P/r_G$ in the expressions below, are the only terms which cannot be determined electrically, and since they are terms of relatively small magnitude they need not be measured accurately, a good estimate usually being sufficient.

Considering the tube as an equivalent diode with the potential V_g at its anode, located at the plane of the grid, we have—

For parallel-plane tubes[5]

$$I_b = A V_g^{3/2} = A\left(\frac{\dfrac{E_b}{\mu}+E_c+e}{1+\dfrac{1}{\mu}+\dfrac{4}{3\mu}\dfrac{b}{a}}\right)^{3/2}.$$

For cylindrical tubes[5]

$$I_b = A V_g^{3/2} = A\left(\frac{\dfrac{E_b}{\mu}+E_c+e}{1+\dfrac{1}{\mu}+\dfrac{2}{3\mu}\log\dfrac{r_P}{r_G}}\right)^{3/2}.$$

A = proportionality constant
E_b = plate voltage
E_c = grid-bias voltage
e = contact potential
μ = amplification factor
a = cathode-to-grid spacing (cm)
b = grid-to-plate spacing (cm)
r_G = radius of grid (cm)
r_P = radius of plate (cm)
r_k = radius of cathode (cm)

In either case

$$s_m = \frac{dI_b}{dE_c} = \frac{3}{2} A V_g^{1/2} \frac{dV_g}{dE_c}$$

$$V_g = \frac{3I_b}{2s_m}\frac{dV_g}{dE_c};$$

where I_b, μ, and s_m are determined experimentally with some arbitrary setting for E_b and E_c. It may be seen that the two factors, A and e, which are not always easily determined, automatically drop out of the expression for V_g.

The value of dV_g/dE_c, which is

$$\frac{1}{1+\dfrac{1}{\mu}+\dfrac{4}{3\mu}\dfrac{b}{a}} \quad \text{or} \quad \frac{1}{1+\dfrac{1}{\mu}+\dfrac{2}{3\mu}\log\dfrac{r_P}{r_G}}.$$

[5] These formulas have been published by B. D. H. Tellegan, *BZN-Physica*, 5e, p. 301, (1925).

for plane and cylindrical structures, respectively, is nearly equal to unity if μ is very large. However, it should not be neglected as a factor in most cases.

These expressions are for the space-charge-limited condition. In the absence of space charge, the factors $(4/3)(b/a)$ and $(2/3)\log(r_P/r_G)$ in the expression for dV_g/dE_c should be replaced by b/a and $\log(r_P/r_G)/\log(r_G/r_K)$, respectively.

The value of V_g, so obtained, is satisfactory for values of grid bias near zero and down to about one third of the cutoff value. Extrapolation to transconductance cutoff may then be employed, Fig. 12.

Calculation of Transit Times

For parallel plane structures (space-charge-limited)

$$\tau_1 = \int_0^a \frac{dx}{v} \quad \text{where } v = \sqrt{\frac{2Ve}{m}}$$

V = voltage at point x between cathode and grid.

v = electron velocity at x.

a = cathode-grid spacing (cm).

$$I = \frac{1}{9\pi}\sqrt{\frac{2e}{m}}\,\frac{V^{3/2}}{x^2} = \frac{1}{9\pi}\sqrt{\frac{2e}{m}}\,\frac{V_g^{3/2}}{a^2};$$ since the current density is the same at all points in a parallel plane structure.

$$\therefore \quad V = V_g\left(\frac{x}{a}\right)^{4/3}; \text{ so } \tau_1 = \frac{a^{2/3}}{v_g}\int_0^a \frac{dx}{x^{2/3}}$$

$$= \frac{a^{2/3}}{\sqrt{\frac{2eV_g}{m}}}\int_0^a \frac{dx}{x^{2/3}} = \frac{3a}{\sqrt{\frac{2eV_g}{m}}} = \frac{3a}{5.95\cdot 10^7 V_g^{1/2}}.$$

This is exactly 1.5 times the time required with the electrons uniformly accelerated, as is the case in the absence of space charge.

For concentric cylinders (space-charge-limited)

$$\tau_1 = \int_{r_0}^{r_1} \frac{dr}{v} \qquad v = \sqrt{\frac{2Ve}{m}}.$$

The plate current for concentric cylinders[6] is

[6] The functions β and $d\beta/d\gamma$ have been defined and partly tabulated by Langmuir and Blodgett, *Phys. Rev.*, vol. 22, p. 347–356, (1923).

$$I = \frac{2}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}}{r\beta^2} = \frac{2}{9}\sqrt{\frac{2e}{m}}\frac{V_g{}^{3/2}}{r_1\beta_1{}^2} \qquad \begin{aligned}\beta &= f\left(\frac{r}{r_0}\right)\\ \beta_1 &= f\left(\frac{r_1}{r_0}\right)\end{aligned}$$

$$\therefore \quad V = V_g\left(\frac{r\beta^2}{r_1\beta_1{}^2}\right)^{2/3}; \text{ so } \tau_1 = \frac{(r_1\beta_1{}^2)^{1/3}}{\sqrt{\frac{2eV_g}{m}}}\int_{r_0}^{r_1}(r\beta^2)^{-1/3}dr.$$

This may be written

$$\tau_1 = \frac{K(r_1 - r_0)}{5.95\cdot 10^7 V_g{}^{1/2}}; \text{ in which } K = \frac{3}{2}\frac{\frac{d(\beta^2\epsilon^\gamma)}{d\gamma}}{\epsilon^\gamma - 1} \quad \text{where } \gamma = \log r_1/r_0.$$

The value of K ranges from 1.5 to 3 for internal cathodes, depending upon the value of r_1/r_0, and from 3 to infinity for external cathodes, Fig. 13.

For concentric spheres (space-charge-limited)—

An analogous value of K can be shown to range from 1.0 to 3 for concentric spheres with internal cathodes, and from three to infinity for spheres with external cathodes. The plate current for concentric spheres[7] is

$$I = \frac{4}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}}{\alpha^2} = \frac{4}{9}\sqrt{\frac{2e}{m}}\frac{E_b{}^{3/2}}{\alpha_1{}^2} \qquad \begin{aligned}\alpha &= f\left(\frac{r}{r_0}\right)\\ \alpha_1 &= f\left(\frac{r_1}{r_0}\right).\end{aligned}$$

And the equation for the transit time in spherical structures is

$$\tau_1 = \frac{3}{2}\frac{(r_1 - r_0)}{v_g}\frac{\epsilon^\gamma}{\epsilon^\gamma - 1}\frac{d(\alpha^2)}{d\gamma} \qquad \gamma = \log\frac{r}{r_0}.$$

Calculation of τ_2 (with negligible space charge):

Since the potential distribution between grid and plate is linear, for parallel planes

$$\tau_2 = \frac{b}{\frac{1}{2}(v_G + v_P)} = \frac{2b}{5.95\cdot 10^7(V_g{}^{1/2} + E_b{}^{1/2})} \qquad b = \text{grid-plate spacing (cm).}$$

[7] The function, α, has been calculated by I. Langmuir and K. G. Blodgett. *Phys. Rev.*, vol. 24, p. 49, (1924).

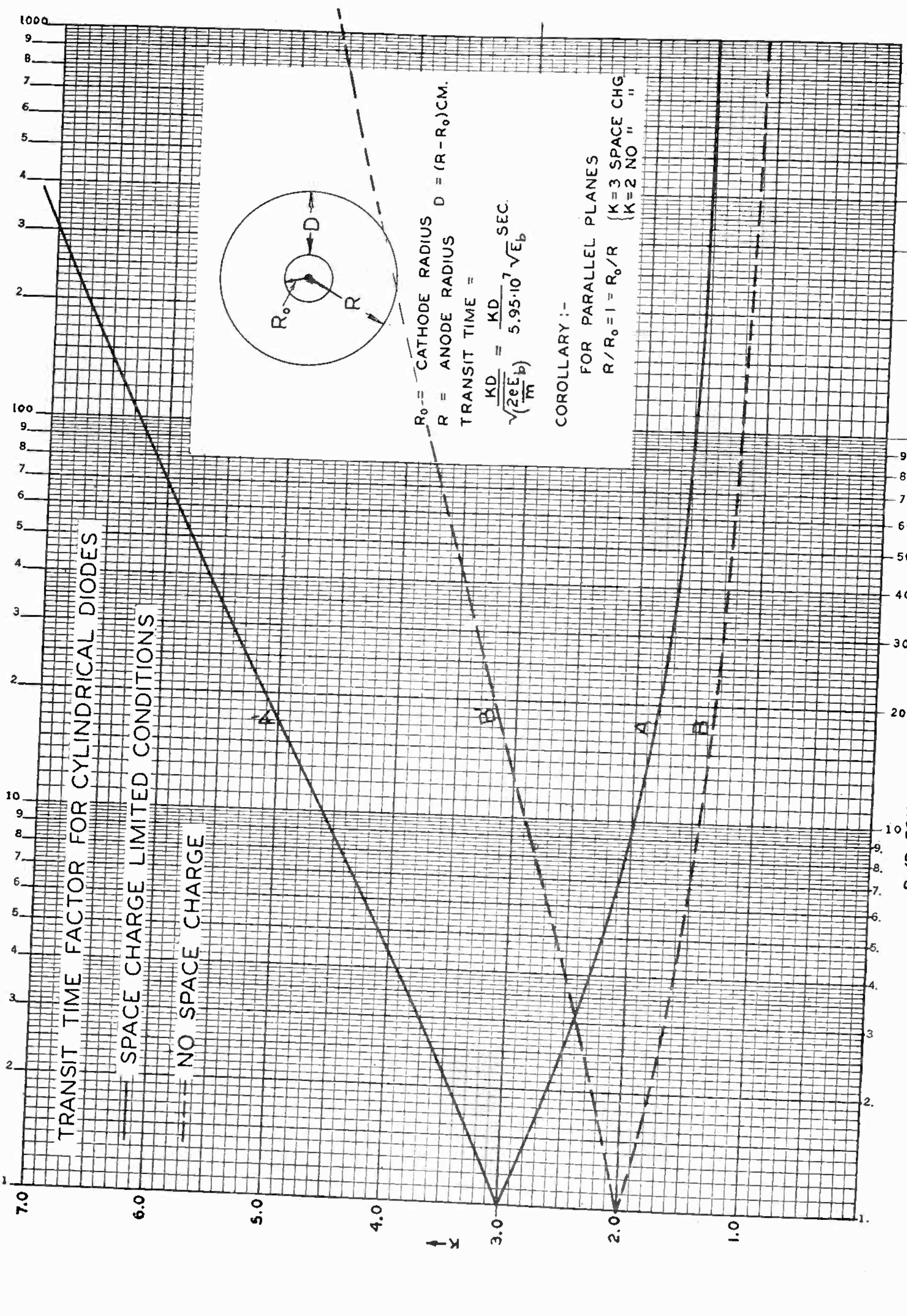


Fig. 13—Transit-time factor for cylindrical diodes.

For cylinders with negligible space charge[8]

$$\tau_2 = \frac{2r_2}{5.95 \cdot 10^7 \sqrt{E'}} \frac{1}{\epsilon^{E_b/E'}} \int_{\sqrt{V_g/E'}}^{\sqrt{E_b/E'}} \epsilon^{x^2} dx$$

where,

$$E' = \frac{(E_b - V_g)}{\log r_P/r_G}.$$

For small ratios of r_P/r_G, say less than 2, the value of τ_2 may be found approximately by assuming the plate and grid to be parallel planes, so

$$\tau_2 = \frac{2(r_P - r_G)}{5.95 \cdot 10^7 (V_g^{1/2} + E_b^{1/2})}.$$

[8] $\int_0^x \epsilon^{x^2} dx$ is tabulated in Jahnke and Emde "Tables of Functions with Formulas and Curves," Second (revised) Edition; B. G. Teubner, Leipzig and Berlin, p. 106, (1933).

DISCUSSION

J. G. Chaffee:[1] The data presented by Thompson and Ferris at the Rochester Fall Meeting on November 13, 1934, indicate that the active grid loss in vacuum tubes increases as the square of the frequency. In my paper[2] in which measurements on this type of grid loss were first reported it was shown that the loss in a vacuum tube increased almost directly with the frequency over the region embraced by the measurements (7–18 megacycles). Thompson and Ferris limited their discussion to vacuum tubes operated as class A amplifiers, while the discussion in my paper related to vacuum tube voltmeters. That it is this difference in operating conditions which accounts for our apparent disagreement is shown by the following set of data.

The active grid loss in a certain vacuum tube was measured by the method outlined in my paper when biased first an an amplifier and then as a detector or voltmeter. The results are shown in Fig. 1. The static characteristics of this tube are shown in Fig. 2, in which the operating points at which measurements were made are marked to correspond with the designations in Fig. 1. Curve A, taken with the tube operated as an amplifier, shows by its slope that for this condition the active grid loss increases as the square of the frequency in agreement with the findings of Thompson and Ferris. Curve B was secured with the same grid bias but with reduced plate voltage to obtain an operating point suitable for plate curvature detection. In curve C this same condition was obtained with a higher grid bias and with plate voltage corresponding to curve A.

[1] Bell Telephone Laboratories, New York City.
[2] "The determination of dielectric properties at very high frequencies," Proc. I.R.E., vol. 22, pp. 1009—1020; August, (1934).

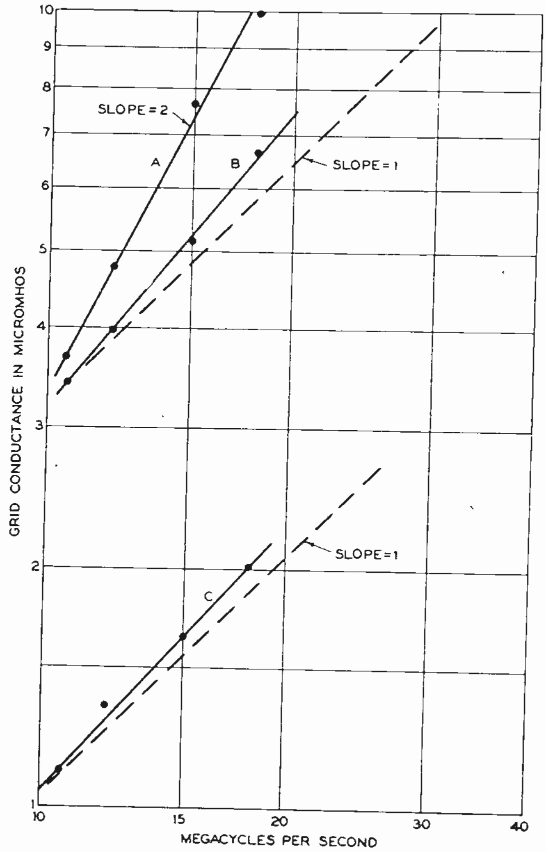


Fig. 1—Active grid conductance vs. Frequency
Curve A: $E_p = 180v$ $E_c = -9v$ $I_p = 4.5$ ma.
Curve B: $E_p = 67v$ $E_c = -9$ $Iv_p = 112\mu$a.
Curve C: $E_p = 180v$ $E_c = -31v$ $I_p = 112\mu$a.

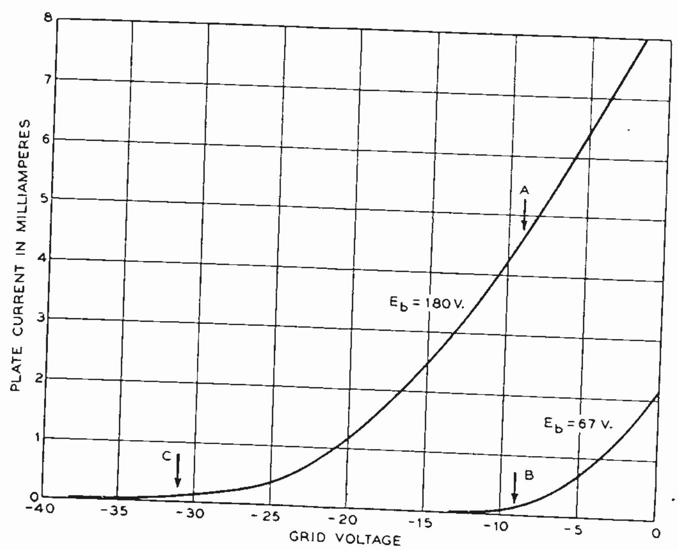


Fig. 2—Static characteristics of tube used in obtaining data shown in Fig. 1.

Both of these latter curves have a slope of approximately 1.1, so that in these cases the grid loss is roughly a linear function of frequency.

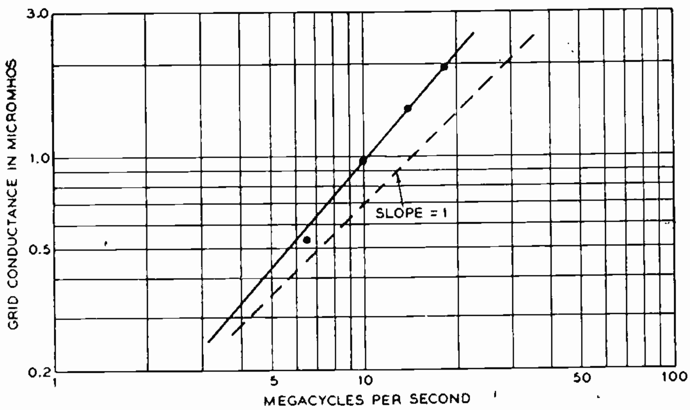


Fig. 3—Data given in original paper replotted to conform with Fig. 1.

For comparison the data given in my original paper, plotted in similar fashion, are shown in Fig. 3, which is seen to agree closely with curve C.

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ANALYSIS OF THE EFFECTS OF SPACE CHARGE ON GRID IMPEDANCE*

By

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Summary—*Previous theory of transit-time phenomena in high vacuum diodes is extended and augmented to provide an explanation of the high-frequency behavior of high-μ amplifiers with parallel plane electrodes. For mathematical reasons the analysis is restricted to triodes with plate at alternating-current ground and to tetrodes with screen grid at alternating-current ground. Expressions for internal input loading and capacity are derived, showing the dependence upon frequency, voltages, and tube dimensions, and it is shown how the theory in its present form can be made quantitatively applicable to many commercial tubes of cylindrical design.*

In agreement with both elementary theory and observation, the theory shows that at the threshold of the effect the input loading varies as the square of the frequency. For the RCA-57 there is calculated an internal input resistance of 21 megohms at one megacycle, dropping to 2100 ohms at 100 megacycles. These figures are in excellent agreement with actual measurement, and illustrate the tremendous importance of transit times in the design of tubes for ultra-high frequencies. It is likely that internal input power losses of this character, together with closely allied losses in transconductance, are primarily responsible for high-frequency failure of both amplifiers and oscillators.

"Hot" input capacity exceeds the "cold" value. The magnitude and dependence upon tube parameters is given; the increase is primarily due to space charge but also depends upon τ_2/τ_1, the ratio of the electron transit time between control grid and plate to the transit time between cathode and control grid. In agreement with observation, the theory indicates very slight frequency dependence.

There is included a brief account of temperature-limited diodes, illustrating their possibilities as a source of high-frequency negative resistance.

Introduction

UNTIL recently it was not only convenient but effective for the engineer to picture his vacuum tube as an "inertialess" device. For moderately high-frequency applications it is still correct and useful to look upon the electron carriers as entities moving instantaneously from one electrode to another, for the time of transit is usually of the order of 10^{-9} seconds. But when the frequency is pushed above five to ten megacycles, the old concept crumbles and its utility fades away. For if the electron has a transit time comparable to the period of the oscillation it is used to promote, the space charge, and therefore the potential distribution, is radically altered, and the elec-

* Decimal classification: R130. Original manuscript received by the Institute, June 26, 1935.

tron is subjected to high-frequency forces whose phases vary in a marked fashion even while the electron is moving from one electrode to another closely adjacent.

Benham[1,2] provided the first analysis of the high-frequency problem. His second paper particularly presents an excellent analysis of the electronics of a space-charge-limited parallel plane diode. Müller[3] followed this with a treatment producing the same net results, but from a somewhat different mathematical attack, which appears to the present author to be at once more concise, easier to grasp, and less cumbersome in manipulation. Llewellyn,[4,5] in this country, has probed the problem further, in his first paper pursuing the method of Benham, and in his second showing the equivalence of Müller's solution. In these publications extensions of the theory have been made largely to explain the operation of positive grid tubes; e.g., Barkhausen oscillators. In this paper, as well, a theory for high-μ, negative grid triodes and multielectrode tubes will be based upon the parallel plane diode solution which will be found in the Appendix.

In order to examine the electronic behavior of tubes possessing grid electrodes, certain simplifying assumptions will have to be made. First, we treat only the parallel plane array, for as yet no manageable solution of the cylindrical diode has been found. (Benham illustrates the difficulties encountered in his 1931 paper.) Second, we assume that space-charge density in regions remote from the cathode plays no significant part in the determination of the potential at any point. That this assumption is invalid for a treatment of the cathode-grid space is well known; e.g., note the enormous difference between the direct-current solutions for temperature-limited and space-charge-limited diodes. Still it is justifiable to make the assumption in a discussion of any of the other spaces in the tube provided electrons which pass the control grid at a potential of at least a volt or two never later enter regions of lower potential. In usual practice, e.g., in tetrode amplifiers, the electrons are continually accelerated as they progress towards the plate, the space-charge density falling off inversely to the velocity. Third, we confine our discussion to high-μ grids so that, in

[1] W. E. Benham, "Theory of the internal action of thermionic systems at moderately high frequencies—Part I", *Phil. Mag.*, vol. 5, p. 641; March, (1928).

[2] W. E. Benham, "Theory of the internal action of thermionic systems at moderately high frequencies—Part II," *Phil. Mag.*, vol. 11, p. 457; February, (1931).

[3] Johannes Müller, "Electron oscillations in high vacua," *Hochfrequenz. und Elektroakustik*, vol. 41, p. 156; May, (1933).

[4] F. B. Llewellyn, "Vacuum tube electronics at ultra-high frequencies," Proc. I.R.E., vol. 21, p. 1532; November, (1933).

[5] F. B. Llewellyn, "Note on vacuum tube electronics at ultra-high frequencies," Proc. I.R.E., vol. 23, p. 112; February, (1935).

speaking of the image of an electron in its environment of conductors, we can with confidence assert that an electron situated between any pair of electrodes is imaged in only these two. Fourth, it will be assumed that the trajectories are straight lines and that all electrons possess the same transit times. At first these last two assumptions may be thought a very serious limitation upon the validity of the ensuing analysis, the third because no matter how high a μ the grids may possess there is always a certain amount of electron-field penetration, the fourth because of the action of the grid wires upon electron paths. In the absence of clinching theoretical justification, and a statement of

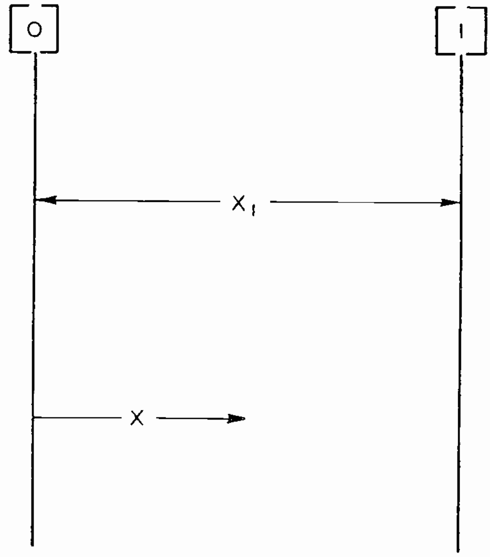

Fig. 1

percentage errors, sufficient *a posteriori* vindication exists in the excellent agreement between the conclusions of this theory and observation.

Part I—Fundamental Theory

No attempt will be made in this paper to discuss triodes with external plate impedance; the author hopes to discuss this problem at a later date. Here we shall confine ourselves to the relatively simpler problem of triodes with grounded plate, and tetrodes with grounded screen grid. In Fig. 1, the plane [0] represents a source of electrons which move to the right to impinge upon or to penetrate the plane [1]. With so general a statement at the start, we shall later find it a simple matter to convert our findings to a variety of uses. For example, we can take [0] to be the cathode and [1] to be the plate of a temperature-limited diode. Usually, however, we shall consider [0] the grid and [1] the plate or screen grid. In any case we shall assume either

[0] or [1] to be at alternating-current ground potential so that although alternating currents may flow in either electrode the alternating-current field existing between the two will be represented by an alternating voltage on only one electrode.

Let us then place a voltage $V_0(1+N\cos\omega t)$ on the plane [0] and a voltage V_1 on [1], thus putting [1] at alternating-current ground potential. For convenience we write

$$k = (V_1/V_0)^{1/2}. \tag{1}$$

We shall suppose, further, that the charge density ρ and the velocity v are given at the plane [0] as functions of the time t, thus

$$\left.\begin{aligned} \rho(t) &= \rho_0 + \rho_a \cos\omega t + \rho_b \sin\omega t \\ v(t) &= v_0 + v_a \cos\omega t + v_b \sin\omega t \end{aligned}\right\}. \tag{2}$$

The quantities ρ_a/ρ_0, v_a/v_0, ρ_b/ρ_0, v_b/v_0, N are all to be assumed small so that squares and cross products can be discarded. As a result the conduction current ψ will be

$$\left.\begin{aligned} &\psi = \rho v = \psi_0 + \psi_a \cos\omega t + \psi_b \sin\omega t \\ \text{wherein,}\quad &\psi_0 = i_0 = \rho_0 v_0,\ \psi_a = \rho_a v_0 + \rho_0 v_a,\ \psi_b = \rho_b v_0 + \rho_0 v_b \end{aligned}\right\}. \tag{3}$$

The dynamics of a single particle of mass m and charge e is then completely represented by

$$\frac{d^2x}{dt^2} = \frac{eV_0}{mx_1}[(1-k^2) + N\cos\omega t] \tag{4}$$

and its integrals. Suppose the charge emerges from [0] at the instant t_0. Successive integrations give the velocity at time t,

$$\frac{dx}{dt} = \frac{eV_0}{mx_1}\left\{(1-k^2)(t-t_0) + \frac{N}{\omega}(\sin\omega t - \sin\omega t_0)\right\} + v(t_0) \tag{5}$$

and the position at time t,

$$\begin{aligned} x = \frac{eV_0}{mx_1}\Bigg\{&(1-k^2)\frac{(t-t_0)^2}{2} \\ &- \frac{N}{\omega^2}[(\cos\omega t - \cos\omega t_0) + \omega(t-t_0)\sin\omega t_0]\Bigg\} + (t-t_0)v(t_0). \end{aligned} \tag{6}$$

The current flowing from [0] to [1] at time t as a result of the motion of this particle is shown directly from image theory to be $(e/x_1)(dx/dt)$. Hence the current per unit area of [0] at time t due to charges which emerged from [0] between the instants t_0 and t_0+dt_0 is

$$di = \frac{\psi(t_0)}{x_1} \frac{dx}{dt} dt_0. \tag{7}$$

The total current per unit area at time t will be the integral of this expression over t_0 from $t_0=t-\tau$ to $t_0=t$ where τ is itself a function of t and represents the time of transit of that electron which at the instant t is just reaching [1]. That is,

$$i = \int_{t-\tau}^{t} \frac{\psi(t_0)}{x_1} \cdot \frac{dx}{dt} \cdot dt_0$$

where τ is defined by (6) in which one sets $x=x_1$, $t-t_0=\tau$. The integral gives

$$\begin{aligned}
\frac{i}{i_0} = {} & \frac{eV_0}{mx_1^2} \cdot \frac{1-k^2}{2} \cdot \tau^2 + \frac{v_0\tau}{x_1} \\
& + \cos \omega t \left\{ \begin{aligned} & \frac{eV_0}{mx_1^2} \tau^2 \left[N \frac{1-\cos\theta}{\theta^2} + \frac{\psi_a}{\psi_0}(1-k^2) \frac{\cos\theta + \theta\sin\theta - 1}{\theta^2} \right. \\
& \left. - \frac{\psi_b}{\psi_0}(1-k^2) \frac{\sin\theta - \theta\cos\theta}{\theta^2} \right] + \frac{v_0\tau}{x_1} \left[\frac{\psi_a}{\psi_0} \frac{\sin\theta}{\theta} \right. \\
& \left. - \frac{\psi_b}{\psi_0} \frac{1-\cos\theta}{\theta} + \frac{v_a}{v_0} \frac{\sin\theta}{\theta} - \frac{v_b}{v_0} \frac{1-\cos\theta}{\theta} \right] \end{aligned} \right\} \\
& + \sin \omega t \left\{ \begin{aligned} & \frac{eV_0}{mx_1^2} \tau^2 \left[N \frac{\theta - \sin\theta}{\theta^2} + \frac{\psi_a}{\psi_0}(1-k^2) \frac{\sin\theta - \theta\cos\theta}{\theta^2} \right. \\
& \left. + \frac{\psi_b}{\psi_0}(1-k^2) \frac{\cos\theta + \theta\sin\theta - 1}{\theta^2} \right] - \frac{v_0\tau}{x_1} \left[\frac{\psi_a}{\psi_0} \frac{1-\cos\theta}{\theta} \right. \\
& \left. + \frac{\psi_b}{\psi_0} \frac{\sin\theta}{\theta} + \frac{v_a}{v_0} \frac{1-\cos\theta}{\theta} + \frac{v_b}{v_0} \frac{\sin\theta}{\theta} \right] \end{aligned} \right\}
\end{aligned} \tag{8}$$

in which, as is customary, we have defined the transit angle

$$\theta = \omega\tau. \tag{9}$$

Equation (6) becomes

$$\begin{aligned}
1 = {} & \frac{eV_0}{mx_1^2} \cdot \frac{1-k^2}{2} \cdot \tau^2 + \frac{v_0\tau}{x_1} \\
& + \cos \omega t \left\{ \frac{eV_0}{mx_1^2} \tau^2 N \frac{\cos\theta + \theta\sin\theta - 1}{\theta^2} + \frac{v_0\tau}{x_1} \left[\frac{v_a}{v_0} \cos\theta - \frac{v_b}{v_0} \sin\theta \right] \right\} \\
& + \sin \omega t \left\{ \frac{eV_0}{mx_1^2} \tau^2 N \frac{\sin\theta - \theta\cos\theta}{\theta^2} + \frac{v_0\tau}{x_1} \left[\frac{v_a}{v_0} \sin\theta + \frac{v_b}{v_0} \cos\theta \right] \right\}.
\end{aligned} \tag{10}$$

This equation makes apparent the truth of the statement above, that τ is itself a function of t. And since this is the case we must be careful not to neglect the dependence in evaluating τ for use in (8). The question, however, can be met handily by simply subtracting (10) from (8), thus ridding (8) of direct-current terms involving τ. When this has been done we again assume all modulation coefficients to be small and evaluate τ from (10) from which the cosine and sine terms are discarded. That is to say, we evaluate τ from the direct-current equation

$$1 = \frac{eV_0}{mx_1^2}\,\frac{1-k^2}{2}\,\tau^2 + \frac{v_0\tau}{x_1}. \tag{11}$$

If all charges in motion originate at a zero-potential plane with zero emission velocities, then

$$-eV_0 = \tfrac{1}{2}mv_0^2. \tag{12}$$

(*Note:* The use of (12) requires all direct current potentials to be negative for positive ions, or, conversely, positive when we speak of electrons in which event the implicit sign of e is negative.) Consequently,

$$\frac{v_0\tau}{x_1} = \frac{2}{k+1}; \quad -\frac{eV_0}{mx_1^2}\tau^2 = \frac{2}{(k+1)^2}. \tag{13}$$

And for evaluation of θ one has, for electrons

$$\tau(\text{sec}) = \frac{3.365 \times 10^{-8}}{k+1}\,\frac{x_1(\text{cm})}{\sqrt{V_0(\text{volts})}}. \tag{14}$$

Subtracting (8) from (10) and using (13) we have

$$\begin{aligned}\frac{i}{i_0} = 1 + \cos\omega t \Bigg\{ & \frac{\psi_a}{\psi_0}\cdot 2\,\frac{\cos\theta+\theta\sin\theta-1}{\theta^2} - \frac{\psi_b}{\psi_0}\cdot 2\,\frac{\sin\theta-\theta\cos\theta}{\theta^2} \\ & + \frac{2}{k+1}\Bigg[\frac{\psi_a}{\psi_0}\cdot\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2} \\ & + \frac{\psi_b}{\psi_0}\cdot\frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^2} + \frac{v_a}{v_0}\cdot\frac{\sin\theta-\theta\cos\theta}{\theta} \\ & + \frac{v_b}{v_0}\cdot\frac{\cos\theta+\theta\sin\theta-1}{\theta}\Bigg] \\ & - \frac{2N}{(k+1)^2}\cdot\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2}\Bigg\} \end{aligned} \tag{15}$$

$$+\sin\omega t\left\{\frac{\psi_a}{\psi_o}\cdot 2\,\frac{\sin\theta-\theta\cos\theta}{\theta^2}+\frac{\psi_b}{\psi_0}\cdot 2\,\frac{\cos\theta+\theta\sin\theta-1}{\theta^2}\right.$$
$$-\frac{2}{k+1}\left[\frac{\psi_a}{\psi_0}\cdot\frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^2}\right.$$
$$-\frac{\psi_b}{\psi_0}\cdot\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2}+\frac{v_a}{v_0}\cdot\frac{\cos\theta+\theta\sin\theta-1}{\theta}$$
$$\left.\left.-\frac{v_b}{v_0}\cdot\frac{\sin\theta-\theta\cos\theta}{\theta}\right]+\frac{2N}{(k+1)^2}\cdot\frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^2}\right\}.$$

This expression can be analyzed briefly as follows: The N terms arise from the interaction of the modulation voltage with the unmodulated part of ψ; i.e., with i_0. The other terms are a mixture of three effects rather difficult to separate at this stage, namely, the modulation in ψ at [0], the moduation in v at [0], and the variation in total charge content between [0] and [1] due to the variations in ψ, v, and the modulation voltage. It must be remembered that the *electrode* capacity ("cold" capacity) of amount $1/4\pi x_1$ has *not* been included in this expression, so that any evaluation of the out-of-phase or sine component of (15) will not be complete without the addition of this cold capacity current. Equation (15) purports to give the currents arising from the motion of "free" charges alone.

Part II—Temperature-Limited Diode

For this case we take [0] to be a source emitting charges of zero velocity in insufficient numbers to alter seriously the linear potential function. Thus $\psi_a=\psi_b=v_a=v_b=0$. The current from [1] to [0] will then be the negative of (15):

$$i = -i_0+\frac{2N}{(k+1)^2}\,i_0\left[\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2}\cos\omega t\right.$$
$$\left.-\frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^2}\sin\omega t\right].$$

Now, to insure zero emission velocities, we must set [0] at zero potential and transfer the modulation to the plate so that [1] has a potential $V_1(1+N\cos\omega t)$, then,

$$\frac{N}{(k+1)^2}\rightarrow -N\cdot$$

and,

$$i = -i_0 - (NV_1)2\frac{i_0}{V_1}\left[\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2}\cos\omega t - \frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^2}\sin\omega t\right].$$

Remembering that in the case of electron carriers i_0 must be negative, or in the case of positives V_1 must be negative, we have in either case an alternating-current conductance of amount

$$\left.\begin{aligned} G &= \frac{|i_0|}{|V_1|}\cdot 2\cdot\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^2} = \frac{|i_0|}{|V_1|}\frac{\theta^2}{6}\left[1-\frac{\theta^2}{15}+\cdots\right. \\ &\text{and an increment to the cold capacity of amount} \\ \Delta C &= \frac{|i_0|}{|V_1|}\tau\cdot 2\cdot\frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^3} = \frac{|i_0|}{|V_1|}\frac{\tau}{3}\left[1-\frac{3\theta^2}{20}+\cdots\right. \end{aligned}\right\}. \quad (16)$$

One will observe that here we have a curious reversal of the situation for space-charge-limited diodes, for which the inset of high-frequency effects is manifested by a *lowering* of the conductance and a *decrease* in the cold capacity.[1,2] The physical differences are not easy to analyze and depict. But a clearer view is had by comparing these results with those obtained by Benner[6] for charges shot at a uniform rate and with constant velocity between a pair of deflection plates. Benner's theory found a positive loading, as we have found, but a *decrease* in the capacity. This decrease was explained, probably correctly, as due to the inertial or inductive-lag effects in the electronic alternating-current velocities. In our case the same effects are present but are dominated by the variation in total charge content arising from the fact that the transit time is a function of t, which is not true in Benner's problem. In fact, our formula differs substantially from Benner's only by reason of our use of the *complete* equation (10) instead of only the direct-current portion; i.e., aside from the trivial difference that in our case charges move with a direct-current acceleration whereas Benner's direct-current charge velocities are constant. It seems evident then that Benner's conclusions are *not* pertinent to structures in which the alternating- and direct-current fields are aligned. And to the author it appears fruitless for Benham to contrast his analytical findings for the change in capacity of a space-charge-limited diode with Benner's formula. Nor does it seem appropriate for Sil[7] to compare any of his experimental measure-

[1,2] *Loc. cit.* Part II. See discussion of diode in the Appendix to this paper.

[6] S. Benner, "Alteration of the dielectric constant of a highly rarefied gas by electrons," *Ann. der Phys.*, s. V. b. III, p. 993, (1929).

[7] B. C. Sil, "On the variation of the interelectrode capacity of a triode at high frequencies," *Phil. Mag.*, vol. 16, p. 1114, (1933).

ments of the plate-grid capacity of a positive-grid triode with Benner's expressions.

It will be noted that (16) provides an opportunity for producing negative resistance oscillations; the conductance has negative maxima at approximately alternate roots of $\theta = 3 \tan \theta$. The largest negative conductance obtainable is $G \approx -(1/5)|i_0|/|V_1|$, when $\theta \approx 7.45$ radians. In view of the large θ required this method would have practical difficulties unless the frequency of operation were very high so that the spacing could be kept reasonably small. Then one could be sure of obtaining a reasonably large i_0 with a small enough V_1 to give a good negative conductance without danger of running into space-charge limitation. This last difficulty could certainly be avoided by the effective but wasteful use of a positive grid of fine mesh near the cathode.

The author wishes here to take exception to Benham's statement[2] that "The fact which emerges when space charge is taken into account is that whenever there is a transit of electrons between a space-charge limited cathode and an anode there is a negative resistance within the system for some value of pT. The negative resistance property may thus be regarded as inherent in the space charge itself." The first sentence is certainly true, but if the second was meant to convey the impression that space-charge limitation is *essential* to a negative resistance diode, (16) seems to disprove the contention. Hence, contrary to Benham's views, this author feels that a suitable theory of Barkhausen-Kurz oscillations can be built upon a basis of complete temperature limitation, and hopes to present such an analysis shortly.

Part III—Screen-Grid Amplifier

Here [0] will be considered the control grid, biased negatively; [1] will then be the screen grid with a high positive bias as usual, and at alternating-current ground potential. Alternatively [1] may be considered the plate of a triode, but since our theory is limited it will still have to be placed at alternating-current ground so that this does not represent a practical amplifier although it is precisely the arrangement described by W. R. Ferris[8] in the accompanying paper setting forth the experimental end of the grid-loading problem.

V_0 will then represent, not the grid-bias voltage, but the effective potential in the grid plane as determined from the well-known μ formulas, or experimentally by direct observation of static characteristics. The modulation voltage $NV_0 \cos \omega t$ is not quite the impressed grid

[2] *Loc. cit.*, Part II, p. 497.
[8] W. R. Ferris, "Input resistance of vacuum tubes as ultra-high-frequency amplifiers," Proc. I.R.E., this issue, pp. 82–105.

voltage. In a triode the voltage effective at the grid plane is

$$V_0 = \sigma\left(E_c + \frac{E_b}{\mu} + \epsilon\right)$$

where ϵ is a contact potential, and where, to good approximation,

$$\sigma = \left[1 + \frac{1}{\mu}\left(1 + \frac{4}{3}\frac{x_1}{x_0}\right)\right]^{-1} \tag{17}$$

x_0 and x_1 being the grid-cathode and grid-anode spacings respectively.

(For a cylindrical structure $\sigma = \left[1 + \frac{1}{\mu}\left(1 + \frac{2}{3}\log\frac{x_0 + x_1}{x_0}\right)\right]^{-1}$ provided the ratio of grid-to-cathode diameter exceeds 10.) Hence the actual alternating voltage applied to the grid is $NV_0/\sigma \cos \omega t$. The "static" or low-frequency transconductance is consequently

$$s_m = \frac{3}{2}\sigma\frac{|i_0|}{|V_0|}. \tag{18}$$

In the case of a tetrode, inasmuch as the plate is usually well shielded from the rest of the tube by the screen grid, it will be feasible to use these relationships also, forgetting the plate entirely and treating the screen grid as though it were the anode itself.

We shall calculate the total current from grid to ground. In the Appendix, (10) and (11) furnish us with the current from ground to grid in the cathode-grid space to which we shall hereafter refer as the first space (θ_1), while (15) above gives us the current from grid to ground in the grid—screen-grid space which we shall call the second space (θ_2). The sum of these currents, taken in the correct manner with due regard for signs, gives us the total current flowing from control grid to ground (with the exception of the "cold capacity" current in the second space). One must remember that in all of these equations i_0 is implicitly negative if electrons are the carriers. Since the grid is biased negatively and can draw no electrons to it, the direct current will be found to vanish when the total grid current is obtained. Defining C_1 and C_2 as the "cold" capacity per unit area of the first and second spaces respectively, we obtain the following expression for the total grid-to-ground current, *including all capacity currents:*

$$
\begin{aligned}
\frac{i}{i_0} = NF\Bigg\{&\left(\frac{\bar{i}_a}{i_0}\right) - 2\cdot\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right) + 2\cdot\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)\\
&-\frac{2}{k+1}\Bigg[\frac{2(1-\cos\theta_2)-\theta_2\sin\theta_2}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)\\
&+\frac{2\sin\theta_2-\theta_2(1+\cos\theta_2)}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)+\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2}\cdot\left(\frac{\bar{v}_a}{v_0}\right)\\
&+\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2}\cdot\left(\frac{\bar{v}_b}{v_0}\right)\Bigg]\\
&+\frac{2}{(k+1)^2}\cdot\frac{2(1-\cos\theta_2)-\theta_2\sin\theta_2}{\theta_2^2}\cdot\frac{1}{F}\Bigg\}\cos\omega t \qquad (19)\\
-NF\Bigg\{&-\left(\frac{\bar{i}_b}{i_0}\right)+2\cdot\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)+2\cdot\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)\\
&-\frac{2}{k+1}\Bigg[\frac{2\sin\theta_2-\theta_2(1+\cos\theta_2)}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)\\
&-\frac{2(1-\cos\theta_2)-\theta_2\sin\theta_2}{\theta_2^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)+\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2}\cdot\left(\frac{\bar{v}_a}{\nu_0}\right)\\
&-\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2}\cdot\left(\frac{\bar{v}_b}{v_0}\right)\Bigg]\\
&+\frac{2}{(k+1)^2}\cdot\frac{2\sin\theta_2-\theta_2(1+\cos\theta_2)}{\theta_2^2}\cdot\frac{1}{F}+\frac{V_0}{i_0}\cdot\frac{\omega C_2}{F}\Bigg\}\sin\omega t.
\end{aligned}
$$

All of the quantities $(\bar{i}_a/i_0)$, $(\bar{\psi}_a/\psi_0)$, $\cdots$, F are functions of θ_1 only and are defined by the set (11) in the Appendix.

To analyze, we shall choose a network consisting of a conductance g_g per unit area paralleled by a susceptance which is most conveniently pictured as a capacity C per unit area. For such a network the electrical behavior is represented by,

$$\frac{i}{|i_0|} = \frac{NV_0}{\sigma|i_0|}\{g_g\cos\omega t - \omega C\sin\omega t\}. \qquad (20)$$

Comparison of (19) and (20) yields expressions for the equivalent g_g and C between grid and ground.

Input Loading

Using (18),

$$\frac{g_g}{s_m}\cdot\frac{3}{2F}=\left(\frac{\bar{i}_a}{i_0}\right)-2\cdot\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)+2\cdot\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)$$
$$-\frac{2}{k+1}\left[\frac{2(1-\cos\theta_2)-\theta_2\sin\theta_2}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)\right.$$
$$+\frac{2\sin\theta_2-\theta_2(1+\cos\theta_2)}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)+\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2}\cdot\left(\frac{\bar{v}_a}{v_0}\right)\tag{21}$$
$$\left.+\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2}\cdot\left(\frac{\bar{v}_b}{v_0}\right)\right]$$
$$+\frac{2}{(k+1)^2}\cdot\frac{2(1-\cos\theta_2)-\theta_2\sin\theta_2}{\theta_2{}^2}\cdot\frac{1}{F}.$$

Here then we have the complete expression for grid loading in a space-charge-limited screen-grid amplifier (or in a triode with plate at alternating-current ground). As expected it turns out to be proportional to s_m. (Incidentally, for tetrodes, the s_m used here must not be confused with the listed s_m for a given tube; in these expressions it means the *total* s_m and will be larger than the listed value by the ratio of the sum of the screen and plate currents to the plate current.) The loading is otherwise a function of three parameters, θ_1, θ_2, and k. The space required to portray adequately the behavior against variations in these three parameters (what one needs is a four-dimensional space!) is too great to be feasible in this paper. There are, however, some interesting characteristics to be observed out of hand.

First of all, since θ_2 can generally be kept small compared with θ_1, we give the reduction of (21) from which terms of order $\theta_2{}^4$ and higher have been discarded:

$$\frac{g_g}{s_m}\cdot\frac{3}{2F}=\theta_1\left(\frac{E_b}{E_0}\right)+\frac{\theta_2{}^2}{4}\left(\frac{\bar{\psi}_a}{\psi_0}\right)+\frac{2}{3}\,\theta_2\left(1-\frac{\theta_2{}^2}{10}\right)\left(\frac{\bar{\psi}_b}{\psi_0}\right)$$
$$-\frac{2}{k+1}\left[\frac{\theta_2{}^2}{12}\left(\frac{\bar{\psi}_a}{\psi_0}\right)+\frac{\theta_2}{6}\left(1-\frac{3\theta_2{}^2}{20}\right)\left(\frac{\bar{\psi}_b}{\psi_0}\right)+\frac{\theta_2{}^2}{3}\left(\frac{\bar{v}_a}{v_0}\right)\right.\tag{22}$$
$$\left.+\frac{\theta_2}{2}\left(1-\frac{\theta_2{}^2}{4}\right)\left(\frac{\bar{v}_b}{v_0}\right)\right]+\frac{1}{(k+1)^2}\cdot\frac{\theta_2{}^2}{6F}.$$

The first term is a function of θ_1 alone, comes out of the third relation in set (12) of the Appendix, and represents the inphase component of the displacement current at the grid in the first space.

Perhaps the most interesting form for the loading is the approximation for θ_1 and θ_2 both small:

$$\frac{g_g}{s_m} = \frac{1}{180}\left[(9\theta_1^2 + 44\theta_1\theta_2 + 45\theta_2^2) - \frac{2}{k+1}\theta_2(17\theta_1 + 35\theta_2) + 20\frac{\theta_2^2}{(k+1)^2}\right]. \tag{23}$$

We note that the loading is an even function of the frequency, in agreement with simple theoretical deductions which show that the Maclaurin series expansion of any resistance develops in only *even* powers of ω. Naturally (23) vanishes for vanishing ω; the low-frequency input impedance of a high vacuum tube is purely capacitive. If θ_2 is set equal to zero there remains $g_g/s_m = \theta_1^2/20$, a value derivable from the work of Benham, Müller, or Llewellyn on diodes; so that this is another check on the mathematical accuracy of our derivation. Since k is of the order $\sqrt{\mu}$, if the tube has a sufficiently high μ the k terms are negligible and only the first three terms are significant, the first representing first space loading and the other two arising from second space behavior of the current stream. It is now evident that θ_2 may contribute predominantly. For if $\theta_1 = \theta_2$ and the k terms are inconsequential, the second space loads the grid 10 times as much as does the first space. The contributions are equal when $\theta_1 \approx 5.75\ \theta_2$, a practical ratio.

The detrimental results of input loading are discussed at length in the accompanying paper by W. R. Ferris.[8] It will suffice to note here that the ideal amplifier is one which requires no input operating energy. At low frequencies the ideal is approached by the use of external input loads with resonant impedances as high as possible. Our results indicate that no matter how painstaking our efforts to build up a power-saving external circuit, at sufficiently high frequencies the efforts are futile because of the *internal* load shunt arising *within the tube itself.*

A brief calculation will illustrate both this contention and the manner in which g_g can be calculated from (23), listed tube characterisitics, and tube dimensions. It should be borne in mind that although this analysis postulated parallel plane structures, it may be expected to portray as well the behavior of cylindrical structures provided the ratios of element diameters are not too large. The conclusions are probably valid enough for cylindrical tubes with equipotential cathodes, but cannot be expected to serve for tubes with fine wire filaments. The RCA-57 is a suitable structure for example. There is listed an s_m of 1185 micromhos for 100 volts on the screen and plate, a grid bias of -3 volts, a plate current of two milliamperes, and a screen current of 0.5 milliampere. With a screen μ of about 20, $\sigma \approx 0.945$. The appropriate s_m will be $1185(2+0.5)/2 = 1480$ micromhos, and from (18) the effec-

[8] *Loc. cit.* Exact equations for transit times are developed for plane, cylindrical, and spherical structures.

tive potential in the grid plane is $V_0 \approx 2.4$ volts. Hence $k=(100/2.4)^{1/2}=6.45$. A rough estimate of the transit times can be had from (14) above and from (13) in the Appendix. This requires a knowledge of the spacings (x_0 and x_1). A better estimate on the basis of a cylindrical structure[8] gives approximately $\tau_1=5.4\times\tau_2=2.9\times 10^{-9}$ seconds. For small transit angles (23) will serve; it shows

$$g_g(\text{micromhos}) \approx 0.048 \times (\text{frequency in megacycles})^2.$$

At around 100 megacycles this represents a loading of 480 micromhos, or 2100 ohms, which practically destroys the usefulness of the tube for small signal power. At one megacycle the loading has dropped to a resistance of 21 megohms which is so far in excess of the impedance developable in the external grid circuit that for this frequency the tube may be said, practically speaking, to consume no input power. The observations of Ferris[8] on the RCA-57 may be seen to check rather nicely with the above theoretical evaluation of loading at the threshold or inset of the effect. At thirty megacycles, he records an input resistance of 19,000 ohms, while the theoretical expression above yields for this frequency a value, 23,200 ohms. For this tube and frequencies above 100 megacycles, one must resort to (22) or (21) for an estimate of g_g, for when $\theta \gg 1$, (23) is invalid.

Input Capacity

Returning to a comparison of (19) and (20) for the equivalent capacity C between grid and ground, we use the approximate relationship

$$C_1 = \frac{3}{4}\sigma\frac{|i_0|}{|V_0|}\tau_1 \tag{24}$$

where C_1 is the "cold" capacity per unit area in the first space, and find

$$\begin{aligned}
\frac{C}{C_1}\cdot\frac{3}{4}\cdot\frac{\theta_1}{F} = &-\left(\frac{\bar{i}_b}{i_0}\right)+2\cdot\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)\\
&+2\cdot\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)-\frac{2}{k+1}\Bigg[\frac{2\sin\theta_2-\theta_2(1+\cos\theta_2)}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_a}{\psi_0}\right)\\
&-\frac{2(1-\cos\theta_2)-\theta_2\sin\theta_2}{\theta_2{}^2}\cdot\left(\frac{\bar{\psi}_b}{\psi_0}\right)\\
&+\frac{\cos\theta_2+\theta_2\sin\theta_2-1}{\theta_2}\cdot\left(\frac{\bar{v}_a}{v_0}\right)-\frac{\sin\theta_2-\theta_2\cos\theta_2}{\theta_2}\cdot\left(\frac{\bar{v}_b}{v_0}\right)\Bigg]\\
&+\frac{2}{(k+1)^2}\cdot\frac{2\sin\theta_2-\theta_1(1+\cos\theta_2)}{\theta_2{}^2}\cdot\frac{1}{F}+\frac{C_2}{C_1}\cdot\frac{3}{4}\cdot\frac{\theta_1}{F}\cdot
\end{aligned} \tag{25}$$

The expression for C_1 is derived by comparison with the capacity of a diode. From the set (8) in the Appendix, by evaluation of the potential gradient at the anode, we find the "hot" capacity of a diode $C_H=(4/3)C_C=(|i_0|/|V_0|)\tau_1$ where C_C is the "cold" capacity. Hence $C_C=\frac{3}{4}(|i_0|/|V_0|)\tau_1$; but if we replace the plate of the diode by the grid of a triode or tetrode, the capacity is lowered in the ratio $C_1/C_C=\sigma$. Unfortunately σ itself has "hot" and "cold" values; the 4/3 in (17) approaches 1 as the space charge is made to vanish. However, if $\mu\gg 1$, or if $x_1/x_0\ll 1$, this difference is slight and therefore (17) may as well used to define σ under all conditions of space charge.

For small θ_2 and arbitrary θ_1:

$$\begin{aligned}\frac{C}{C_1}\cdot\frac{3}{4F}=&\left(\frac{\bar{E}_a}{E_0}\right)+\frac{2}{3}\left(\frac{\theta_2}{\theta_2}\right)\left(1-\frac{\theta_2^2}{10}\right)\left(\frac{\bar{\psi}_a}{\psi_0}\right)\\&-\frac{1}{4}\left(\frac{\theta_2}{\theta_1}\right)\theta_2\left(1-\frac{\theta_2^2}{18}\right)\left(\frac{\bar{\psi}_b}{\psi_0}\right)\\&-\frac{2}{k+1}\left(\frac{\theta_2}{\theta_1}\right)\left[\frac{1}{6}\left(1-\frac{3\theta_2^2}{20}\right)\left(\frac{\bar{\psi}_a}{\psi_0}\right)-\frac{\theta_2}{12}\left(1-\frac{\theta_2^2}{15}\right)\left(\frac{\bar{\psi}_b}{\psi_0}\right)\right.\\&\left.+\frac{1}{2}\left(1-\frac{\theta_2^2}{4}\right)\left(\frac{\bar{v}_a}{v_0}\right)-\frac{\theta_2}{3}\left(1-\frac{\theta_2^2}{10}\right)\left(\frac{\bar{v}_b}{v_0}\right)\right]\\&+\frac{1}{3}\cdot\frac{1}{(k+1)^2}\cdot\left(\frac{\theta_2}{\theta_1}\right)\left(1-\frac{3\theta_2^2}{20}\right)\frac{1}{F}+\frac{C_2}{C_1}\cdot\frac{3}{4F}.\end{aligned}\tag{26}$$

For θ_1 and θ_2 both small:

$$\begin{aligned}C=\frac{4C_1}{3}\Bigg\{&\left[1+\left(\frac{\theta_2}{\theta_1}\right)\left(1-\frac{1}{k+1}+\frac{1}{3(k+1)^2}\right)\right]\\&-\frac{\theta_1^2}{1200}\left[13+88\left(\frac{\theta_2}{\theta_1}\right)+165\left(\frac{\theta_2}{\theta_1}\right)^2+120\left(\frac{\theta_2}{\theta_1}\right)^3\right]\\&+\frac{1}{k+1}\cdot\frac{\theta_1^2}{600}\cdot\left(\frac{\theta_2}{\theta_1}\right)\left[29+95\left(\frac{\theta_2}{\theta_1}\right)+120\left(\frac{\theta_2}{\theta_1}\right)^2\right]\\&-\frac{1}{(k+1)^2}\cdot\frac{\theta_1^2}{20}\cdot\left(\frac{\theta_2}{\theta_1}\right)^3\Bigg\}+C_2.\end{aligned}\tag{27}$$

It will again be noted that this equation is consistent with electrical theory, for since (θ_2/θ_1) is independent of frequency, (27) contains only *even* powers of ω.

There is a marked difference between the input capacity of diodes and control-grid tubes. In the brief discussion of a diode in the Appendix the actually measurable low-frequency "hot" capacity is found to be $(3/5)C_1$, whereas the apparent theoretical (but unmeasurable) "hot" capacity is $(4/3)C_1$ when calculated from the potential gradient at the anode. This operating drop in capacity is explained by Benham as due to the inertial lag in electronic velocities.

Benham has also pointed out that this effect should not be expected to obtain in a triode because the electrons go through the grid and ultimately give their energy to the plate. Hence, he argues, the triode input capacity should for low frequencies be always $C > (C_1 + C_2)$. This argument is substantiated by Benham's measurements of triode hot capacities, and is in agreement with (27) above.

Benham[2] suggested an empirical formula

$$\frac{C}{C_1 + C_2} = 1 + As_m,$$

this arising from a comparison of curves for measured input capacity with tube characteristics. His measurements, however, went all the way to cutoff, in which event one would certainly expect $C/(C_1+C_2) \to 1$ as a limit. Our formula holds only for the region within which the three-halves power law serves adequately, thus suggesting for this domain a modification of Benner's formula as developed below.

The low-frequency C is given by the first brackets in (27) and is independent of ω:

$$C = \frac{4C_1}{3}\left[1 + \frac{\tau_2}{\tau_1}\left(1 - \frac{1}{k+1} + \frac{1}{3(k+1)^2}\right)\right] + C_2.$$

From (18) and (24), $2C_1 = s_m \tau_1$, so that

$$\frac{C}{C_1 + C_2} = \frac{\left[\frac{4}{3} C_1 + C_2\right] + s_m \frac{2}{3} \tau_2 \left[1 - \frac{1}{k+1} + \frac{1}{3(k+1)^2}\right]}{C_1 + C_2} \tag{28}$$

over the range of voltage for which the three-halves power law holds and $\theta_1 \ll 1$, $\theta_2 \ll 1$. This compares with Benham's formula, although it must be remembered that both τ_2 and k are somewhat sensitive to variations in grid bias and therefore are themselves slowly varying functions of s_m.

The dependence of C upon the frequency is almost nil, as may be seen from a brief example. If one assumes $\theta_2/\theta_1 = 1/5$, $\tau_1 = 10^{-9}$ seconds, $C_2 = (3/4)C_1$, $\sigma = 1$, $k = 6.5$, there results, according to (27), between the C at 100 megacycles and the C for zero frequency a difference of only 1.2 per cent! Observations of Ferris of these laboratories have never shown a measurable shift in C even at the limit of available frequency, eighty megacycles, although the difference between "hot" and "cold" capacity was readily observed.

Significance of Screen-Grid—Plate Transit Angle

Applying this theory to operation of screen-grid amplifiers, we are confronted with effects arising in the third space, between screen grid and plate. What and how large are the effects due to plate voltage swing and to θ_3? Since, with a good screen, variations in plate voltage have little effect upon the behavior of the tube currents on the cathode side of the screen grid, the grid loading is unaltered by the presence of an external plate impedance, regardless of the value of θ_3. Furthermore, if $\theta_3 \ll 1$, the plate current will be closely equal to ψ at the screen grid, as regards both modulus and phase, minus the fraction stopped by the screen-grid wires, plus the cold capacity current between plate and screen grid. To evaluate ψ at the screen grid, one must take into consideration the fact that existence of a velocity modulation at the control grid produces, *per se*, an alteration of both phase and modulus of the convection current at the screen grid. This results from the tendency for particles possessing higher initial velocities to overtake those with lower initial velocities. It is planned to give the details of this portion of the analysis at a later date. Suffice it to say for the present that, subject to the same limitations as were imposed above, the theory indicates that in the instance of sufficiently small θ_1 and θ_2 the modulus of s_m departs only *very slightly* from the "static" value, whereas the phase angle is roughly

$$-\left[\frac{11}{30}\theta_1 + \theta_2\left(1 - \frac{1}{3(k+1)}\right)\right].$$

If θ_3 is not negligibly small, it will manifest itself through a phase shift in the effective s_m accompanied by a change in modulus, which at the outset is a *decrease*. No attempt will be made here to calculate the actual plate current for large θ_3, but the reader will see that this can readily be done by a repetition of the methods used above for the second space. In most practical cases these effects can be divested of practical importance because of the ease with which θ_3 can be diminished through the use of the customary high screen and plate potentials. In the case of suppressor-grid pentodes the problem is obviously more complex.

It appears, consequently, that internal failure of tetrode amplifiers at high frequencies is more likely to be a result of grid loading than of loss in s_m. Failure of oscillators may very well be due partially to improper feed-back phase.

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It is a pleasure to recognize here the coöperation of Mr. W. R. Ferris, of this laboratory, whose experience and whose frequent counsel have been an inestimable aid. I am further indebted to him for performing the laborious task of computing the curves which appear in the Appendix.

APPENDIX

Here we shall derive the solution for space-charge-limited parallel plane diodes. Inasmuch as Llewellyn[5] has developed this solution in a very clear and unified fashion from the point of view of Müller, we shall not start from the fundamentals but from the appropriate place in Llewellyn's paper. The reader is referred to this exposition for the basic equations, the justification of certain critical steps in the treatment, and the development of the theory up to the point at which we break in.

We restate here, so that they may easily be borne in mind, the limitations imposed by our mathematical idealization. It is assumed that the cathode possesses an inexhaustible supply of electrons ready to be emitted with zero velocity, so that the direct-current solution would give the usual three-halves power law for current versus voltage, and the potential has the same sign throughout the space, the potential (V_0) being proportional to the four-thirds power of the distance. In actuality there exists, because of initial velocities, a virtual cathode very close to the actual cathode surface, the potential at this point being slightly depressed below the cathode voltage. Consequently, only those electrons reach the plate whose emission velocities are sufficient to enable them to surmount the potential barrier represented by the virtual cathode. In the absence of a modulating voltage those electrons which do not get through the barrier contribute to the current only a very weak noise spectrum having a maximum at tremendously high frequencies (characterized by the time of transit from the instant of emergence to the instant of return to the cathode). Those which do penetrate the barrier to constitute the space current correspond to the conduction current in the idealized model we are about to discuss. In any case our model is likely to violate fact to a gross extent only if the modulation field can penetrate the barrier. But even in this case, and despite the great density of electrons in the region between cathode and virtual cathode, it may be shown that the electrons which do not cross the barrier contribute in most practical instances an insignificant amount to the high-frequency impedance of the tube. Calculations of this contribution are somewhat lengthy; the author intends to discuss this conclusion in detail in another place. The electrons which do cross the barrier produce effects different from those predicted from our model only provided the time of transit from cathode to virtual cathode is an appreciable fraction of the total transit time. Later in these pages we shall make calculations showing this ratio to be insignificant.

There is one other difference between the model and reality. Whereas the model electrons have zero emission velocities, in actuality there exists a Maxwellian spread at the virtual cathode, so that some electrons reach the plate in shorter transit times. Yet again one may expect the model to serve adequately for it will be recalled that the distribution is exponential according to the square of the normal velocity with a maximum for zero velocity. Consequently, the great majority of electrons in the space-current stream will have transit times deviating a negligible amount from that assigned to the model. This will be especially true if the plate voltage is high in comparison with the voltage corresponding to the average velocity of emission from the virtual cathode.

Perhaps the most important limitation is one imposed upon our model for the sake of mathematical manipulation. The equations are valid only so long as the alternating component of current is so small in comparison with the steady component that squares of the ratio can be neglected.

Proceeding with the derivation we start with (14) in Llewellyn's paper in which we use v (rather than his "u") to denote velocity:

$$v = \frac{1}{2} K\tau^2 - \frac{2}{\tau}\phi(t_0 + \tau) + \frac{2}{\tau}\phi(t_0) + \phi'(t_0 + \tau) - C_2.$$

Here we determine C_2 by requiring that $v=0$ when $\tau=0$; hence

$$C_2 = -\phi'(t_0) \tag{1}$$

so that,

$$v = \frac{1}{2} K\tau^2 - \frac{2}{\tau}\phi(t_0 + \tau) + \frac{2}{\tau}\phi(t_0) + \phi'(t_0 + \tau) + \phi'(t_0). \tag{2}$$

Now let v be transformed by means of the relationship (Llewellyn, equations (5) and (11)):

$$t_0 = t - \tau - \epsilon \tag{3}$$

at the same time neglecting products of ϵ with ϕ or any of its derivatives, i.e., terms of order ϵ^2. There results:

$$v = \frac{1}{2} K\tau^2 - \frac{2}{\tau}\phi(t) + \frac{2}{\tau}\phi(t-\tau) + \phi'(t) + \phi'(t-\tau). \tag{4}$$

This, then, is the correct expression for the velocity at time t of an electron which is found at a point in the space characterized by a transit time τ, the electron having started from rest at the cathode at a time $(t-\tau)$. The relationship between x, the position, and τ, the transit time, is given by Llewellyn, equation (10):

$$x = \tfrac{1}{6}K\tau^3. \tag{5}$$

and the total current

$$\frac{4\pi e}{m} J = K + \phi'''(t) \tag{6}$$

so that K represents the direct component of current and $\phi'''(t)$ the alternating part. Now we shall define

$$\left.\begin{aligned} J &\equiv i_0(1 + M\cos\omega t) \\ K &= \frac{4\pi e}{m} i_0 \\ \phi'''(t) &= 4\pi\frac{e}{m} i_0 M\cos\omega t \end{aligned}\right\} \quad \text{thus requiring} \tag{7}$$

Substitution of (7) into (4) gives

$$\frac{v}{v_0} = 1 + M\left[2\,\frac{2\sin\theta - \theta(1+\cos\theta)}{\theta^3}\cos\omega t + 2\,\frac{2(1-\cos\theta) - \theta\sin\theta}{\theta^3}\sin\omega t\right]$$

where,

$$v_0 = 4\pi\frac{e}{m} i_0 \frac{\tau^2}{2} = \text{the direct-current velocity}$$

and,

$$\theta \equiv \omega\tau = \text{the “transit angle”}.$$

Similarly we take Llewellyn's equation for the field (E),

$$\frac{e}{m}E = K\tau + \phi''(t_0+\tau) - \frac{2}{\tau^2}\phi(t_0+\tau) + \frac{2}{\tau^2}\phi(t_0) - \frac{2}{\tau}C_2$$

which becomes in our case, after use of (1) and (3),

$$\frac{e}{m}E = K\tau + \phi''(t) - \frac{2}{\tau^2}\phi(t) + \frac{2}{\tau^2}\phi(t-\tau) + \frac{2}{\tau}\phi'(t-\tau)$$

or,

$$\frac{E}{E_0}=1+M\left[2\,\frac{\sin\theta-\theta\cos\theta}{\theta^3}\cos\omega t+\frac{\theta(\theta-2\sin\theta)+2(1-\cos\theta)}{\theta^3}\sin\omega t\right]$$

where,

$$E_0=4\pi i_0\tau=\text{the direct-current field}$$

The potential is found from the relation

$$E=-\frac{\partial V}{\partial x}\text{; thus}$$

$$V=-\int_0^x E\,dx=-\frac{1}{2}K\int_0^\tau \tau^2 E\,d\tau,\quad\text{so that}$$

$$\frac{V}{V_0}=1+M\left[8\,\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^4}\cos\omega t\right.$$

$$\left.+4\,\frac{\dfrac{\theta^3}{3}+2\theta(1+\cos\theta)-4\sin\theta}{\theta^4}\sin\omega t\right]$$

where,

$$V_0=-2\,\frac{e}{m}(\pi i_0\tau^2)^2=\text{the direct-current potential.}$$

The charge density ρ comes out of the relation,

$$\rho=\frac{1}{4\pi}\,\frac{\partial E}{\partial x}=\frac{1}{2\pi K\tau^2}\,\frac{\partial E}{\partial \tau}$$

$$=\frac{m/e}{2\pi K\tau^2}\left[K+\frac{4}{\tau^3}\phi(t)-\frac{4}{\tau^3}\phi(t-\tau)-\frac{4}{\tau^2}\phi'(t-\tau)-\frac{2}{\tau}\phi''(t-\tau)\right]$$

or,

$$\frac{\rho}{\rho_0}=1+M\left[2\,\frac{2(\theta\cos\theta-\sin\theta)+\theta^2\sin\theta}{\theta^3}\cos\omega t\right.$$

$$\left.+2\,\frac{2(\theta\sin\theta+\cos\theta-1)-\theta^2\cos\theta}{\theta^3}\sin\omega t\right]$$

where,

$$\rho_0=\frac{1}{2\pi\dfrac{e}{m}\tau^2}=\text{the direct-current charge density.}$$

Finally, we want the value of ψ, $(=\rho v)$, the conduction current. This comes from the relationship

$$J=\psi+\frac{1}{4\pi}\frac{\partial E}{\partial t}.$$

Therefore,

$$\frac{\psi}{\psi_0}=1+M\left[2\frac{\theta\sin\theta-(1-\cos\theta)}{\theta^2}\cos\omega t+2\frac{\sin\theta-\theta\cos\theta}{\theta^2}\sin\omega t\right]$$

where,

$$\psi_0=i_0=\text{the direct current}.$$

This last could, of course, have been obtained by taking the product of our expressions for ρ and v and discarding terms of order M^2. Collecting these results we have:

The direct-current solution

$$\left.\begin{aligned}
x_0 &= \frac{2}{3}\pi\frac{e}{m}i_0\tau^3\\
v_0 &= 2\pi\frac{e}{m}i_0\tau^2\\
E_0 &= 4\pi i_0\tau\\
V_0 &= -2\frac{e}{m}(\pi i_0\tau^2)^2\\
\rho_0 &= 1\Big/\left(2\pi\frac{e}{m}\tau^2\right)\\
\psi_0 &= \rho_0 v_0 = i_0
\end{aligned}\right\} \tag{8}$$

The alternating-current solution

$$\frac{i}{i_0}=1+M\cos\omega t$$

$$\frac{v}{v_0}=1+M\left[2\frac{2\sin\theta-\theta(1+\cos\theta)}{\theta^3}\cos\omega t+2\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^3}\sin\omega t\right]$$

$$\frac{E}{E_0}=1+M\left[2\frac{\sin\theta-\theta\cos\theta}{\theta^3}\cos\omega t+\frac{2(1-\cos\theta)+\theta(\theta-2\sin\theta)}{\theta^3}\sin\omega t\right]$$

$$\frac{V}{V_0}=1+M\left[8\frac{2(1-\cos\theta)-\theta\sin\theta}{\theta^4}\cos\omega t\right.$$
$$\left.+4\frac{-4\sin\theta+2\theta(1+\cos\theta)+\frac{1}{3}\theta^3}{\theta^4}\sin\omega t\right]$$

$$\frac{\rho}{\rho_0}=1+M\left[2\,\frac{2(\theta\cos\theta-\sin\theta)+\theta^2\sin\theta}{\theta^3}\cos\omega t\right.$$
$$\left.+2\,\frac{2(\theta\sin\theta+\cos\theta-1)-\theta^2\cos\theta}{\theta^3}\sin\omega t\right]$$

$$\frac{\psi}{\psi_0}=1+M\left[2\,\frac{\theta\sin\theta-(1-\cos\theta)}{\theta^2}\cos\omega t+2\,\frac{\sin\theta-\theta\cos\theta}{\theta^2}\sin\omega t\right]. \quad (9)$$

One may easily verify the equivalence of the direct-current solution to Child's solution of the space-charge-limited diode, using the first equation of the set (8) to replace the parameter τ by the spacing x_0. It must be remembered that i_0 is the current flowing from cathode *to* plate, so that if electrons are the carriers i_0 and e are both negative; this will be seen to give all of the expressions in (8) the correct sign.

The set (9) expresses the alternating-current behavior of each quantity in terms of its direct-current value and two additional parameters, the current modulation M and the transit angle θ. The coefficients of the $\cos\omega t$ and $\sin\omega t$ terms give, respectively, in each expression, the components in and out of phase with the current. It is preferrable to phase everything against the voltage V, and this can evidently be done by making the transformation

$$t \rightarrow t + \delta$$

where,

$$\tan\omega\delta = b/a$$

and,

$$a = \frac{8}{\theta^4}[2(1-\cos\theta)-\theta\sin\theta]\ ;\ b = \frac{4}{3\theta^4}[\theta^3+6(\theta+\theta\cos\theta-2\sin\theta)].$$

We shall also introduce a new quantity N, the voltage modulation (i.e., the ratio of plate voltage swing to plate bias voltage), and eliminate M through the relationship

$$N = M\sqrt{a^2+b^2}.$$

We shall also define

$$F(\theta) = \frac{4}{9}\,\frac{1}{a^2+b^2}.$$

Under this rotation the set (9) transforms into

$$\left.\begin{aligned}
\frac{V}{V_0} &= 1 + N\cos\omega t\\
\frac{i}{i_0} &= 1 + NF\left[\left(\frac{\bar{i}_a}{i_0}\right)\cos\omega t + \left(\frac{\bar{i}_b}{i_0}\right)\sin\omega t\right]\\
\frac{v}{v_0} &= 1 + NF\left[\left(\frac{\bar{v}_a}{v_0}\right)\cos\omega t + \left(\frac{\bar{v}_b}{v_0}\right)\sin\omega t\right]\\
\frac{E}{E_0} &= 1 + NF\left[\left(\frac{\bar{E}_a}{E_0}\right)\cos\omega t + \left(\frac{\bar{E}_b}{E_0}\right)\sin\omega t\right]\\
\frac{\rho}{\rho_0} &= 1 + NF\left[\left(\frac{\bar{\rho}_a}{\rho_0}\right)\cos\omega t + \left(\frac{\bar{\rho}_b}{\rho_0}\right)\sin\omega t\right]\\
\frac{\psi}{\psi_0} &= 1 + NF\left[\left(\frac{\bar{\psi}_a}{\psi_0}\right)\cos\omega t + \left(\frac{\bar{\psi}_b}{\psi_0}\right)\sin\omega t\right]
\end{aligned}\right\} \qquad (10)$$

in which terms of the type $(\bar{\xi}_a/\xi_0)$, (ξ_b/ξ_0), and F are functions of the transit angle alone, thus,

$$\left.\begin{aligned}
F = \frac{4}{9} \div \left(\left\{\frac{8}{\theta^4}[2-2\cos\theta-\theta\sin\theta]\right\}^2 + \left\{\frac{4}{3\theta^4}[\theta(\theta^2+6)+6\theta\cos\theta-12\sin\theta]\right\}^2\right) &= \left[1+\frac{13}{300}\theta^2-\cdots\right.\\
\left(\frac{\bar{i}_a}{i_0}\right) = \frac{18}{\theta^4}[2-2\cos\theta-\theta\,\text{sins}\,\theta] &= \frac{3}{2}\left[1-\frac{1}{15}\theta^2+\cdots\right.\\
\left(\frac{\bar{i}_b}{i_0}\right) = \frac{3}{\theta^4}[-\theta(\theta^2+6)-6\theta\cos\theta+12\sin\theta] &= -\frac{9}{20}\theta\left[1-\frac{5}{126}\theta^2+\cdots\right.\\
\left(\frac{\bar{v}_a}{v_0}\right) = \frac{6}{\theta^4}[2-2\cos\theta-\theta\sin\theta] &= \frac{1}{2}\left[1-\frac{1}{15}\theta^2+\cdots\right.\\
\left(\frac{\bar{v}_b}{v_0}\right) = \frac{6}{\theta^7}[(\theta^4+12\theta^2+48)+(\theta^4+12\theta^2-48)\cos\theta-2\theta(\theta^2+24)\sin\theta] &= \frac{1}{10}\theta\left[1-\frac{41}{840}\theta^2+\cdots\right.\\
\left(\frac{\bar{E}_a}{E_0}\right) = \frac{3}{\theta^6}[(\theta^4+8\theta^2+24)+4(\theta^2-6)\cos\theta-2\theta(\theta^2+12)\sin\theta] &= \left[1-\frac{13}{240}\theta^2+\cdots\right.\\
\left(\frac{\bar{E}_b}{E_0}\right) = \frac{6}{\theta^7}[12(\theta^2+2)+(\theta^4-24)\cos\theta-4\theta(\theta^2+6)\sin\theta] &= \frac{3}{40}\theta\left[1-\frac{11}{210}\theta^2+\cdots\right.\\
\left(\frac{\bar{\rho}_a}{\rho_0}\right) = \frac{6}{\theta^6}[-8(\theta^2+3)-(\theta^4+4\theta^2-24)\cos\theta+2\theta(\theta^2+12)\sin\theta] &= \left[1-\frac{17}{120}\theta^2+\cdots\right.\\
\left(\frac{\bar{\rho}_b}{\rho_0}\right) = \frac{6}{\theta^7}[-48-2(\theta^4+12\theta^2-24)\cos\theta-\theta(\theta^4+4\theta^2-48)\sin\theta] &= \frac{9}{20}\theta\left[1-\frac{22}{315}\theta^2+\cdots\right.\\
\left(\frac{\bar{\psi}_a}{\psi_0}\right) = \frac{6}{\theta^6}[-6(\theta^2+4)-(\theta^4+6\theta^2-24)\cos\theta+\theta(\theta^2+24)\sin\theta] &= \frac{3}{2}\left[1-\frac{7}{60}\theta^2+\cdots\right.\\
\left(\frac{\bar{\psi}_b}{\psi_0}\right) = \frac{6}{\theta^5}[(\theta^2+12)-(\theta^2+12)\cos\theta-\theta(\theta^2+6)\sin\theta] &= \frac{11}{20}\theta\left[1-\frac{61}{924}\theta^2+\cdots\right.
\end{aligned}\right\} \qquad (11)$$

These equations can be partially verified by means of the following identities which arise from the neglect of all terms containing powers of the current modulation M higher than the first:

$$\left(\frac{\bar{\psi}_a}{\psi_0}\right) = \left(\frac{\bar{\rho}_a}{\rho_0}\right) + \left(\frac{\bar{v}_a}{v_0}\right)$$

$$\left(\frac{\bar{\psi}_b}{\psi_0}\right) = \left(\frac{\bar{\rho}_b}{\bar{\rho}_0}\right) + \left(\frac{\bar{v}_b}{v_0}\right)$$

$$\left(\frac{\bar{i}_a}{i_0}\right) = \left(\frac{\bar{\psi}_a}{\psi_0}\right) + \theta\left(\frac{\bar{E}_b}{E_0}\right) \qquad (12)$$

$$\left(\frac{\bar{i}_b}{i_0}\right) = \left(\frac{\bar{\psi}_b}{\psi_0}\right) - \theta\left(\frac{\bar{E}_a}{\bar{E}_0}\right).$$

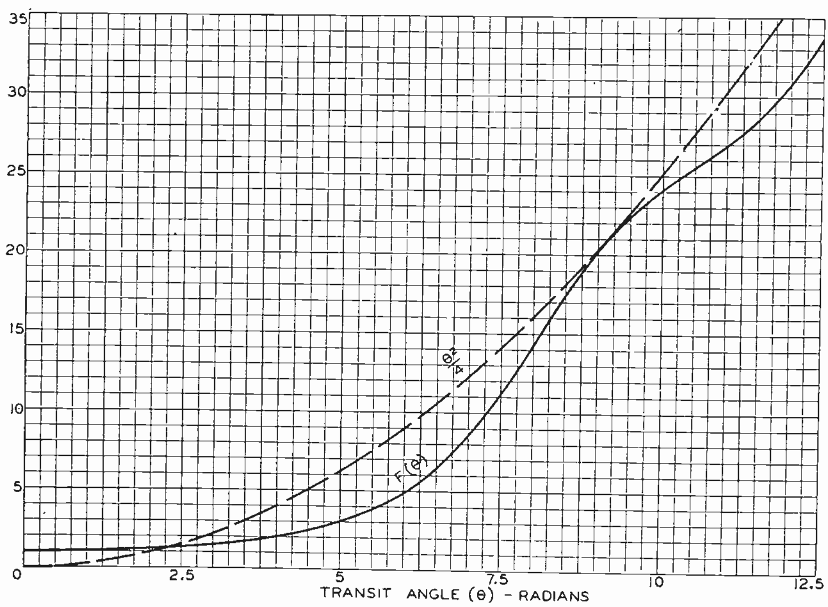

Fig. 2

In Fig. 2 the function F is plotted out to $\theta=4\pi$. For larger θ, the curve sinuates closely about $\theta^2/4$, which is also shown for comparison. All the rest of the set (11) can be found in Fig. 3, the interpretation of which needs this explanation. Any quantity (ξ_a) in the figure is actually $F\times(\bar{\xi}_a/\xi_0)$, i.e., the inphase coefficient for ξ/ξ_0 when the voltage modulation is unity ($N=1$). In order to keep i_b on the sheet, i_b/θ was also plotted. It is to be noted that $i_a\equiv(3/2)(G/G_0)$, where G_0 is the conductance of the diode, G_0 its zero-frequency conductance; $i_b/\theta\equiv -(3/4)(C/C_c)$, where C is the capacity of the diode, C_c its "cold" capac-

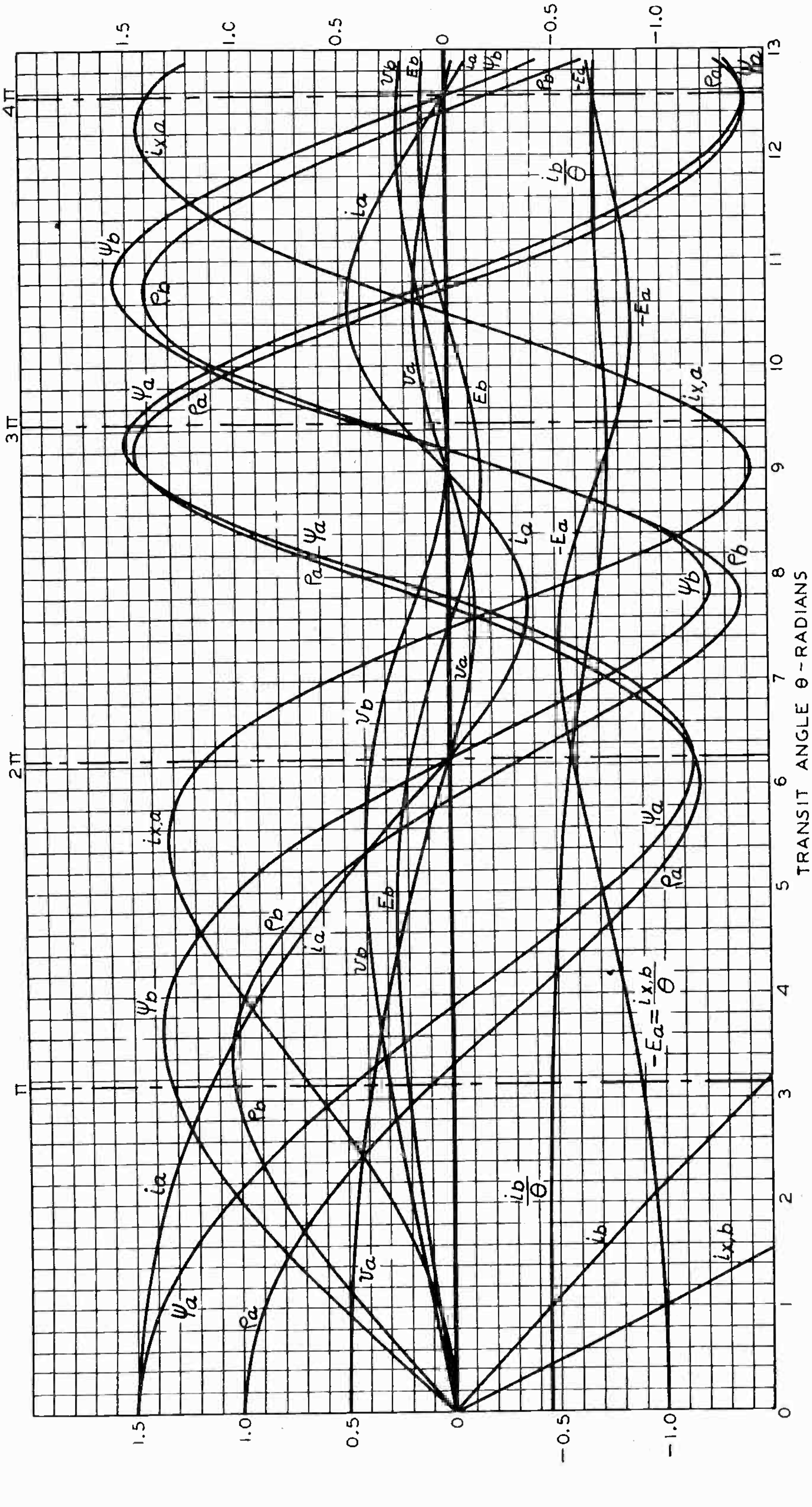
TRANSIT ANGLE θ - RADIANS
π
2π
3π
4π
1.5
1.0
0.5
0
-0.5
-1.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
ψ_a
ψ_b
ρ_a
ρ_b
i_a
i_b
$i_{x,a}$
$i_{x,b}$
v_a
v_b
E_b
$-E_a$
$\frac{i_b}{\theta}$
$-E_a = \frac{i_{x,b}}{\theta}$

Fig. 3

ity. Two additional curves are shown: $i_{x,a}=i_a-\psi_a=\theta E_b$ and $i_{x,b}=i_b-\psi_b=-\theta E_a$. Again it is to be observed that $i_{x,a}\equiv(3/2)(G/s_m)$, $i_{x,b/\theta}=-(3/4)(C/C_1)$ where s_m and C_1 are the transconductance and input "cold" capacity of a hypothetical triode possessing zero grid-plate transit angle and zero grid-plate capacity, and G, C are the input conductance and capacity respectively.[9]

It is not our purpose here to discuss the significance of these equations in respect to diodes *as such*, this having been done at considerable length by Benham, Müller, and Llewellyn. However, it does seem to the author that the equations (11) are of more interest than those given previously. For, inasmuch as we now have all quantities phased with respect to the voltage, it becomes a simple matter to pick out the parallel connected circuit elements which symbolize the electrical behavior of the device. Furthermore it is usually the voltage swing which is known or measurable and not the current.

For example, using the approximate formulas in the set (11)

$$i = i_0 + Ni_0\frac{3}{2}\cos\omega t - Ni_0\frac{9}{20}\theta\sin\omega t.$$

One can see from (8) that the low-frequency $r_p=(2/3)(V_0/i_0)$, while the cold capacity is $C_c=(3/4)\tau(i_0/V_0)$, so that

$$i = i_0 + (NV_0)\left[\frac{1}{r_p}\cos\omega t - \frac{3}{5}\omega C_c\sin\omega t\right].$$

Clearly then the appropriate alternating-current equivalent network valid for vanishingly small transit angles is the customary diode resistance shunted by a capacity which is, however, only three fifths the cold capacity of the tube. As others have already pointed out, this is to be expected, for the mass of each charged particle behaves, for small transit angles, like an inductance so that the drop in measurable capacity is hardly astonishing. That the factor is three fifths is fortuitious and depends upon the geometry we have chosen, namely, parallel planes. Needless to say, it is out of the question to hope to secure a decent check of this factor unless one takes extreme precautions to insure the reality of the various assumptions we have made for our model, particularly the three-halves power law.

In applying these formulas to a particular structure it becomes necessary to evaluate the transit angle θ. Now the transit time τ can be obtained from any of the set (8) but generally it will be most convenient to combine the first two of the set, thus:

[9] Compare (22) and (26) in the main body of the paper.

$$\tau(\text{sec}) = \frac{3x_0}{v_0} = \frac{3x_0}{\sqrt{2\frac{e}{m}V_0}} = 5.048 \times 10^{-8}\frac{x_0(\text{cm})}{\sqrt{V_0(\text{volts})}}. \quad (13)$$

It is of interest to observe that this transit time is three halves that which obtains under temperature-limited conditions.

We return, now that we have a measure of τ, to the question of the virtual cathode in a real tube and the ratio of the transit time between cathode and virtual cathode to that for the virtual-cathode—plate space in a real diode. We shall seek an upper limit to this ratio; that is manifestly obtained from consideration of those electrons which come through the virtual cathode with zero velocity. Although space charge has a much different distribution on the cathode side of the virtual cathode, so that the potential is of a somewhat different form, it will be accurate enough to assume that the transit time is determined in both spaces by an equation having the form of (13). Then

$$\left(\frac{\tau_m}{\tau}\right)^2 \approx \left(\frac{x_m}{x_0}\right)^2 \cdot \frac{V_0}{V_m}$$

where τ is the transit time computed for the model, τ_m is the transit time out to the potential minimum V_m, and x_m is the distance of that point from the cathode. But since $\log_e (I_s/I) = eV_m/kT$, wherein I_s/I is the ratio of emission to plate current, we have

$$\left(\frac{\tau_m}{\tau}\right)^2 \approx 5\left(\frac{1000}{T}\right)\frac{V_0}{\log_{10}\left(\frac{I_s}{I}\right)}\left(\frac{x_m}{x_0}\right)^2.$$

An upper limit to x_m is given by Langmuir and Compton[10]; viz., for

$$I_{s/I} > 7, \quad x_m(\text{cm}) < 0.0016(1000I)^{-1/2}\left(\frac{T}{1000}\right)^{3/4}.$$

Therefore,

$$\frac{\tau_m}{\tau} \approx \frac{0.0016}{x_0}\left(\frac{5V_0}{\log_{10}\left(\frac{I_s}{I}\right)}\right)^{1/2}\left(\frac{T}{1000}\right)^{1/4}(1000I)^{-1/2}. \quad (14)$$

In a typical case we may choose, for an oxide-coated cathode,

[10] I. Langmuir and K. T. Compton, "Electrical discharges in gases—Part II," *Rev. Mod. Phys.*, vol. 3, p. 244; April, (1931).

$$V_0 = 3 \text{ volts}$$

$$\frac{I_s}{I} = 100, \text{ although this is probably too small}$$

$$x_0 = 0.05 \text{ cm}$$

$$T = 1000° \text{ Kelvin}$$

$$I = 0.004 \text{ amp/cm}^2$$

So that (14) gives

$$\frac{\tau_m}{\tau} = 0.04$$

which is small enough that it may be neglected in general,[1] particularly in view of the fact that the space charge, being very dense in the region close to the cathode, must prevent appreciable penetration of the high-frequency field unless the frequency is very high. Thus, in effect, even though this calculation had shown τ_m/τ to be significantly large, one would expect the model to serve well for reasonably low frequencies, for the grid is still unable to "see" the electrons until they move past the virtual-cathode barrier.

[1] *Loc. cit.*, pp. 656–669, invokes the hypothesis of a large τ_m to acount for his observation that the frequency variation of diode rectification is independent of anode voltage. The illustration he uses ($x_0 = 5$ mm, $x_m = 1$ mm) is, he admits, an extreme example. Too, it is not clear how his conclusions can quantitatively substantiate the discrepancy between his theory and experiment. In view of the analysis given above for a practical design, it seems to the present author that the resolution of his difficulty must be found elsewhere.

Proceedings of the Institute of Radio Engineers
Volume 24, Number 1 *January, 1936*

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page may be obtained without charge by addressing the publishers.

Elastic Stop Nut Corporation, P.O. Box 38, Elizabeth, N. J., has issued Catalog A-43 giving design information on the use of elastic stop nuts.

"Voltrol," a manual voltage regulator is described in Bulletin No. 136 issued by the Acme Electric and Manufacturing Company of 144 Hamilton Avenue, Cleveland, Ohio. Their Bulletin No. 137 describes step-down transformers of the 220- to 110-volt variety and their variable voltage adjuster. Another leaflet describes their "Windoette" supply units for neon signs.

Catalog "36" of the Hammarlund Manufacturing Company of 424 West 33rd Street, New York, covers condensers, coil forms, sockets, transformers, chokes, shields and other products for receiving and transmitting. Leaflets 146 and 147 cover trimmer and air padding condensers, respectively.

Molded resistors are described in Leaflet 271B issued by the S. S. White Dental Manufacturing Company, Industrial Division, 10 East 40th Street, New York.

Dynamic speakers and high fidelity speakers with curvilinear diaphragms are described in leaflets issued by the Magnovox Company of Fort Wayne, Ind.

Volume and tone controls, fixed and adjustable resistors, transmitter grid leaks, line pads, attenuators and numerous other resistors are described in the 1936 catalog of Electrad, Inc., of 175 Varick Street, New York City.

The Erie Resistor Corporation of Pennsylvania has issued a booklet "Check Everything" describing the performance of their resistors.

Supplement to Bulletin 464 describes amateur transmitting and receiving type tubes and is available from the RCA Manufacturing Company of Camden, N. J. A piezoelectric calibrator for checking frequencies between 100 and 50,000 kilocycles is described in a leaflet as is a frequency modulator for aligning receivers in conjunction with a cathode-ray oscillograph.

A leaflet issued by the Spaulding Fiber Company of 310 Wheeler Street, Tonawanda, New York, describes the insulating materials manufactured by that organization.

Cornell-Dubilier Corporation of 4377 Bronx Boulevard, New York City has issued Catalog No. 128 covering capacitors for radio purposes and Catalog No. 130 on units for power factor correction.

Ferris Instrument Corporation of Boonton, N. J., has issued booklets covering model 15A master oscillator type standard signal generator and model 17B microvolter covering a range from 100 to 30,000 kilocycles.

All types of glass and metal tubes are covered in a technical data chart issued by the Raytheon Production Corporation of 55 Chapel Street, Newton, Mass.

Bulletin No. 41 of the Triplett Electrical Instrument Company of Buffton, Ohio, covers electrical measuring instruments and testing sets.

Solar Manufacturing Corporation of 599 Broadway, New York City, has issued Catalog No. 7-5 on radio capacitors.

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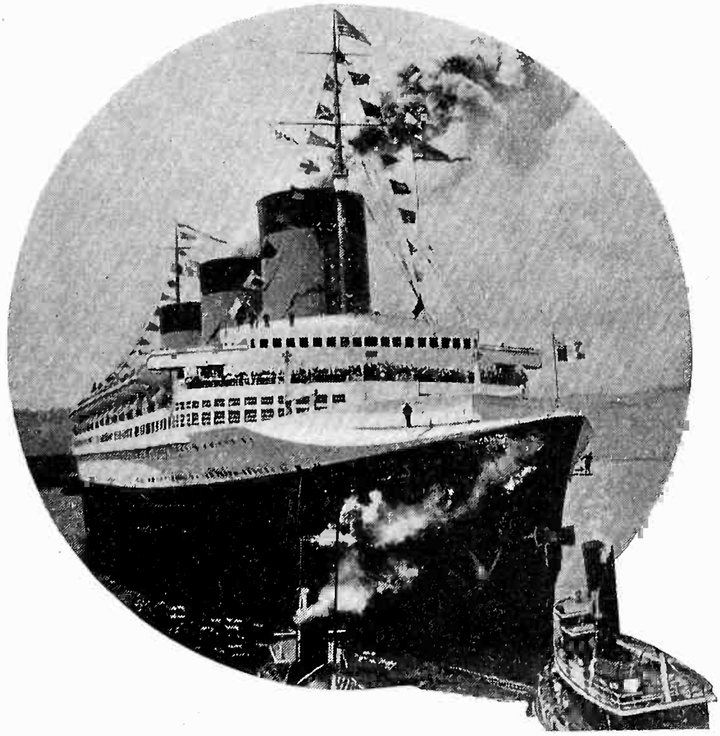

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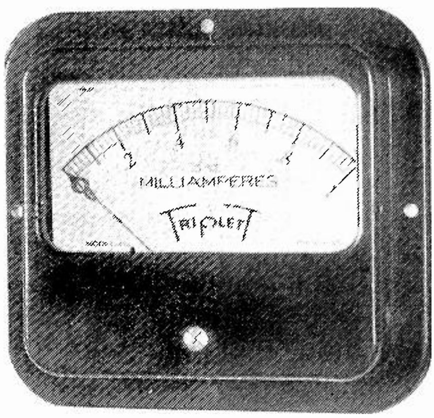

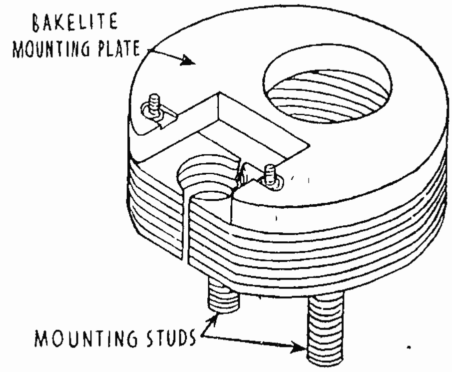

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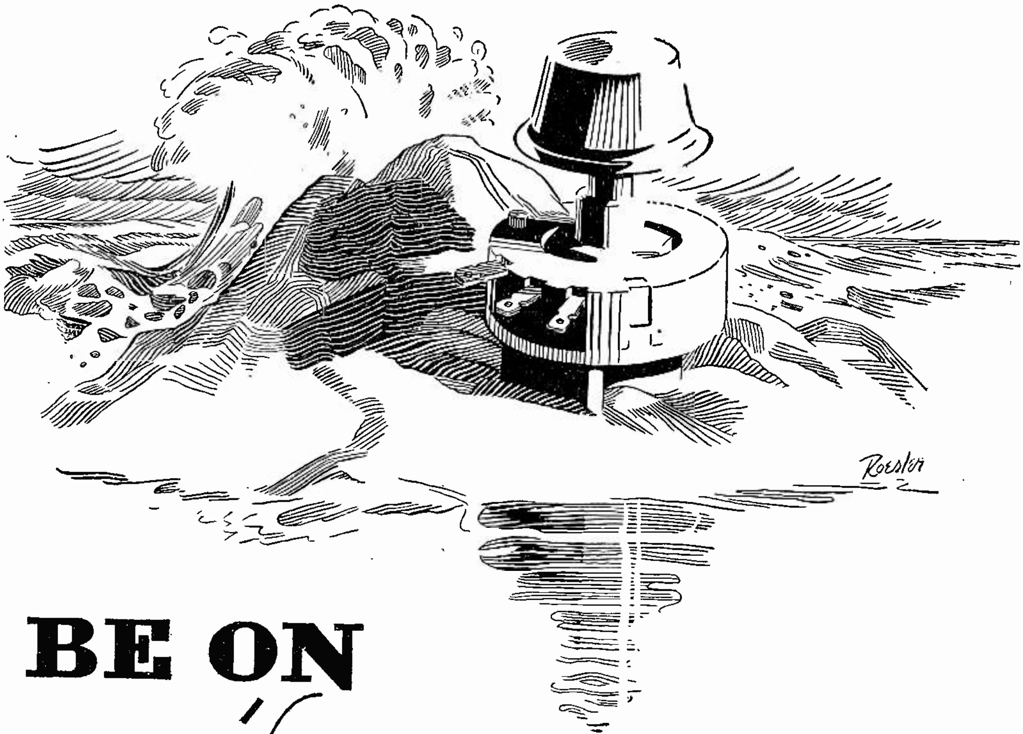

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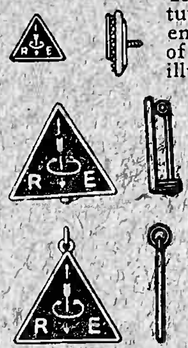

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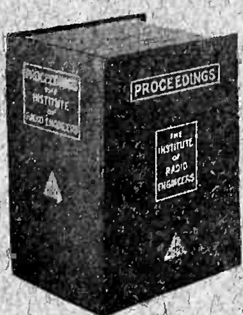

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