VOLUME 23 FEBRUARY, 1935 NUMBER 2

PROCEEDINGS

of

The Institute of Radio Engineers

Application Blank for Associate Membership on Page XI

Institute of Radio Engineers Forthcoming Meetings

TENTH ANNUAL CONVENTION
DETROIT, MICHIGAN
July 1, 2, and 3, 1935

JOINT MEETING
American Section, International Scientific Radio Union and Institute of Radio Engineers, Washington D. C.
April 26, 1935

DETROIT SECTION
February 15, 1935

LOS ANGELES SECTION
February 19, 1935

NEW YORK MEETING
February 6, 1935
March 6, 1935

PHILADELPHIA SECTION
February 7, 1935
March 7, 1935

PITTSBURGH SECTION
February 14, 1935

PROCEEDINGS OF

The Institute of Radio Engineers

Volume 23 February, 1935 Number 2

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Technical Papers

The Institute of Radio Engineers

GENERAL INFORMATION

INSTITUTE. The Institute of Radio Engineers was formed in 1912 through the amalgamation of the Society of Wireless Telegraph Engineers and the Wireless Institute. Its headquarters were established in New York City and the membership has grown from less than fifty members at the start to several thousand.

AIMS AND OBJECTS. The Institute functions solely to advance the theory and practice of radio and allied branches of engineering and of the related arts and sciences, their application to human needs, and the maintenance of a high professional standing among its members. Among the methods of accomplishing this is the publication of papers, discussions, and communications of interest to the membership.

PROCEEDINGS. The PROCEEDINGS is the official publication of the Institute and in it are published all of the papers, discussions, and communications received from the membership which are accepted for publication by the Board of Editors. Copies are sent without additional charge to all members of the Institute. The subscription price to nonmembers is $10.00 per year, with an additional charge for postage where such is necessary.

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	Bangalore, Dept. of Elec. Communication Eng., Indian Institute of Science	Mukerji, S. N.
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South Africa	Durban, Natal, African Broadcasting Co.	Harris, A. P.
	Durban, Natal, African Broadcasting Co.	Southorn, B. E.
Switzerland	Solothurn, Hasenmattstrasse 11	Blochlinger, J.

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Elected to the Student Grade

Georgia	Atlanta, 698 Greenwood St., S.W.	Markillie, R. G.
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	Syracuse, 615 S. Beech St.	Hastings, H. D.
Pennsylvania	Philadelphia, 1848 N. Leithgow St.	Gohn, C. T.

Proceedings of the Institute of Radio Engineers
Volume 23, Number 2 *February, 1935*

APPLICATIONS FOR MEMBERSHIP

Applications for transfer or election to the various grades of membership have been received from the persons listed below, and have been approved by the Admissions Committee. Members objecting to transfer or election of any of these applicants should communicate with the Secretary on or before February 28, 1935. These applications will be considered by the Board of Directors at its meeting on March 6, 1935.

For Election to the Associate Grade

California	Los Angeles, 1117 Venice Blvd.	Astorga, L. T.
	Los Angeles, 3120 W. 71st St.	Partridge, A. J.
Connecticut	Waterbury, American Brass Co., Waterbury Brass Goods Branch	Stevens, C. R.
District of Columbia	Washington, 1315 Rittenhouse St. N.W.	Sutherland, J. A.
Illinois	Chicago, 1823 N. Lawndale Ave.	Eltgroth, G. V.
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	Newton, 29 Marlboro St.	Yamins, H. G.
Nebraska	Omaha, 407 N. 40th St.	Meile, J. W.
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	Englewood, 130 Grand Ave.	Fink, D. G.
	Haddonfield, 43 Chestnut St.	Brandt, M. M.
	Merchantville, 3029 Horner Ave.	Gibbs, W. D.
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	Newark, Weston Elec. Instrument Corp.	Morelock, O. J., Jr.
New York	Brooklyn, 4011-7th Ave.	Malmstedt, C. H.
	Buffalo, 1150 Kenmore Ave.	Bratton, E. W.
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	Ithaca, 523 E. Buffalo St.	Wood, J. A., Jr.
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	Philadelphia, Radio Station WCAU	Cohen, A. B.
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Denmark	Copenhagen F., Godthaabsvej 136	Christiansen, E.
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	Ilford, Essex, The Plessey Co.	Bailey, C. E. G.
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Japan	Ishikawaken, Nonoichi Broadcasting Station, Nonoichimachi	Mitui, H.
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Latvia	Riga, Valsts Elektrotechniska Fabrika, Vidzemes sos. 19	Jukna, R. A.
Philippine Islands	Manila, 19 Plaza Ferguson	Grey, J. M.
Sweden	Stockholm, Ala-Vox Radio, Biblioteksgatan 11	Sjoblom, I.

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New Jersey	Berlin, Box 631, Haddon Ave.	Sabati, O.
Pennsylvania	Philadelphia, 4836 N. 9th St.	Horton, J. W.
	Philadelphia, 4048 Baltimore Ave.	Musselman, F. E.
	Philadelphia, 3914 Spruce St.	Slaybaugh, C. W.
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INSTITUTE NEWS AND RADIO NOTES

Annual Meeting of the Board of Directors

The annual meeting of the Board of Directors was held on January 2 at the Institute office and those present were C. M. Jansky, Jr., retiring president; Stuart Ballantine, president elect; Melville Eastham, treasurer; O. H. Caldwell, Alfred N. Goldsmith, R. A. Heising, J. V. L. Hogan, L. M. Hull, E. L. Nelson, Haraden Pratt, H. M. Turner, William Wilson, and H. P. Westman, secretary.

After approving the minutes of the last meeting, the meeting adjourned. Those whose terms of office continued through 1935 then reconvened as the new Board of Directors. Melville Eastham was reappointed treasurer, H. P. Westman as secretary, and Alfred N. Goldsmith as chairman of the Board of Editors for 1935. O. H. Caldwell, Virgil M. Graham, J. V. L. Hogan, George Lewis and A. F. Van Dyck were appointed directors to serve during 1935.

John Scott-Taggert was transferred to the grade of Fellow and C. B. Aiken, V. M. Brooker, T. R. Bunting, A. H. Carson and J. K. Johnson were transferred to Member grade. M. M. Garrison was elected to the grade of Member. Forty-seven applications for Associate membership, two for Junior, and seven for Student grade were approved.

The budget for 1935 was approved and it is estimated that expenditures and receipts will balance.

A joint meeting of the Institute and the American Section of the International Scientific Radio Union is scheduled for April 26, 1935, in Washington, D. C.

The Emergency Employment Service placed four men during December and a total of 695 are now registered.

Joint Meeting of the Institute and the American Section, U.R.S.I.

Arrangements have been made between the Institute of Radio Engineers and the American Section of the International Scientific Radio Union for a joint meeting of the two bodies in Washington, D. C., April 26. This meeting is an important annual feature of the week which attracts to Washington every year an increasingly large number of scientists and scientific societies. The first joint meeting last year was highly successful. There was a large and representative attendance, and a valuable program of papers was given.

For the joint meeting this year, an all-day session for the presentation of papers on the more fundamental aspects of radio problems has been planned. It is certain to be an interesting program. Further details will be given in next month's issue of the PROCEEDINGS.

Committee Work

NEW YORK PROGRAM COMMITTEE

The New York Program Committee met in the Institute office on Monday afternoon, January 7 and those present were A. F. Van Dyck, chairman; Austin Bailey, H. A. Chinn, J. K. Henney, L. C. F. Horle, and H. P. Westman, secretary.

It was necessary to arrange for meetings during February, March, and April and a number of alternative papers were considered satisfactory. The final choice will depend upon obtaining the acceptance of the authors to present them and fitting them into the particular dates open.

STANDARDIZATION

TECHNICAL COMMITTEE ON ELECTRONICS—I.R.E.

A meeting of the Technical Committee on Electronics was held on the morning of January 4 and was attended by B. E. Shackelford, chairman; M. J. Kelly, E. A. Lederer, G. F. Metcalf, O. W. Pike, B. J. Thompson, P. T. Weeks, and H. P. Westman, secretary.

The personnel of the five subcommittees established by this committee was reviewed. The report of the Committee on Electronics of the Sectional Committee on Electrical Definitions of the American Standards Association was recently approved and this together with the 1933 report of the Institute and some material prepared by Dr. Kelly showing wherein these two reports differ will be circulated to members of the committee for future action. Each chairman of a subcommittee reported on the activities of his particular group and obtained such guidance as was necessary for future work.

SUBCOMMITTEE ON ELECTRON BEAM AND MISCELLANEOUS TUBES

TECHNICAL COMMITTEE ON ELECTRONICS—I.R.E.

This subcommittee met on the afternoon of January 3 and those present were G. W. Metcalf, chairman; M. S. Glass, B. J. Thompson and H. P. Westman, secretary.

The scope of the committee was discussed and it was agreed that initial work would be started on cathode ray tubes, magnetrons, and secondary-emission devices. Various members of the committee were asked to prepare notes on these subjects for distribution and action at the next meeting.

SUBCOMMITTEE ON GAS FILLED TUBES

TECHNICAL COMMITTEE ON ELECTRONICS—I.R.E.

O. W. Pike, Chairman; D. V. Edwards, H. E. Mendenhall, P. T. Weeks, and H. P. Westman, secretary, attended a meeting of the above subcommittee operating under the Technical Committee on Electronics of the Institute which was held on the evening of January 3 in the Institute office.

The scope of the committee's activities was briefly outlined and immediate work started on graphical symbols for gas-filled tubes. Those treated covered indications for gas filling in tubes, mercury pool tubes, the indication of internal connections between elements, ignitrons, shield grids, and gas tubes in general. Material is being prepared on ratings of industrial electronic tubes and will be considered at a future meeting of the committee. Definitions were prepared for shield grid, cathode heat shield, and tube heat shield.

Institute Meetings

ATLANTA SECTION

The Atlanta Section met on October 25 at the Atlanta Athletic Club. H. L. Reid, chairman, presided and the attendance was eighteen, half of whom were at the informal dinner which preceded the meeting.

A paper on "Notes on Quarter and Half-Wave Antennas" was presented by N. B. Fowler an engineer of the American Telephone and Telegraph Company. In it he developed formulas for various forms of antenna radiation and presented figures showing the efficiency of quarter and half-wave antennas. The necessity of operating antennas to radiate as effectively as possible was stressed. The paper was discussed by Professor Gerks.

BOSTON SECTION

A meeting of the Boston Section was held at Harvard University on November 22. R. G. Porter, secretary-treasurer, presided and the attendance was 175. Twenty were present at the dinner which preceded the meeting.

A paper on "Ultra-High-Frequency Transmission" was presented by G. W. Pickard of the General Radio Company and A. F. Sise of the Yankee Network.

The past year's work on sixty megacycles was outlined. It covered transmission over long nonoptical paths on land and sea, systematic recording at Seabrook, N. H. from W1XAV at Squantum, Mass. and the results of a boat trip along the New England coast. The records from W1XAV show striking similarities to those obtained at much lower frequencies, while the measurements over water indicate that although the good service radius of an ultra-high-frequency station is approximately twice the optical horizon, signals can be received under normal transmission conditions up to nearly five optical horizons.

In the discussion, C. F. Brooks, Director of Blue Hill Observatory, pointed out the relation of temperature gradients in the lower atmosphere to specific cases of good and poor transmission mentioned by the authors. R. A. Hull, Associate Editor of QST, described his directional work and the relations found between transmission and certain meteorological elements. Additional discussion was presented by W. B. Burgess, and E. B. Dallin.

Buffalo-Niagara Section

The Buffalo-Niagara Section met at the University of Buffalo on November 27. L. E. Hayslett, chairman, presided and fifty-one were in attendance.

Walter Jones, commercial engineer for Hygrade-Sylvania Corporation presented a paper on "Problems Arising in the Manufacture of Present-Day Vacuum Tubes."

The design problems encountered with the addition of each successive element from the two-element vacuum tube to those of seven elements were described. Particular problems in maintaining proper characteristics and various changes in design and receiver requirements were covered. Attention was drawn to the inadequacy of present-day tube testing equipment. Slides illustrated various manufacturing and testing processes. Messrs. Hayes, Nist, Wesselman, and others participated in the discussion.

The paper was preceded by a demonstration of wave motion phenomena with mechanical apparatus by Dr. Hector and short summaries of several papers given at the Rochester Fall Meeting were presented by Eugene Wesselman, and F. M. Schmidt.

The December meeting was held on the 12th at the University of Buffalo. The attendance was 101 and L. E. Hayslett presided.

"Practical Loud Speaker Trends" was the subject of a paper presented by Austin Armer of the Magnavox Company. He gave first a brief resumé of the historical background of loud speaker development. A schematic diagram of a typical loud speaker was shown and the vibrational motion of the various parts described and analyzed in detail. The combination effects and reactions of the motions of these interlinked parts and their various changes in structural material, mass, shape, and size were covered. It was pointed out that "cut and try" methods were still used considerably in loud speaker design. The paper was discussed by Messrs. Crom, Guenther, Hayslett, Waud, and Wesselman, and the meeting was closed with motion pictures showing manufacturing processes employed in the Magnavox Company plant.

Cincinnati Section

A meeting of the Cincinnati Section was held on September 25 at the University of Cincinnati with R. E. Kolo, chairman, presiding. The attendance was forty.

J. R. Nelson of the Raytheon Production Corporation gave a paper on "Output Systems in General Including Direct Coupled Amplifiers." A substantial portion of the paper was devoted to the direct coupled amplifier and its uses. A discussion of the static characteristics of triodes, tetrodes, co-planar grid tubes, and high-mu triodes, with positive bias followed. The paper was discussed by Messrs. Flewelling, Kilgour, and Knoblaugh.

The October meeting of the section was held at the University of Cincinnati on the 23rd. R. E. Kolo presided and eighty-five were present.

A paper on "'Q' Measurements" was presented by J. J. O'Callaghan who covered the subject of radio and intermediate-frequency transformer and coupling unit design. The discussion was based chiefly on the use of equivalent networks to represent conditions found in commercial coils particularly those of the small universally wound type. He covered the varying effects of capacitance and inductance on the tubes which were coupled, the effect of shielding and mechanical construction on the characteristics of the transformer, and touched on the use of iron core coils for intermediate-frequency transformers.

The Nominations Committee was appointed and comprised Messrs. Felix, Barbulesco, and Marshall.

The November meeting was held on the 20th at the University of Cincinnati with R. E. Kolo presiding and fifty-eight were in attendance.

C. E. Haller of the Ken Rad Corporation gave a paper entitled "A Discussion of Cathode Ray Tubes and Equipment." In it he outlined the history of cathode ray tube development. Some of the earlier tubes as well as modern tubes of the simpler types were available for examination. The construction of these tubes and the uses for which they were suitable was explained. Notes were presented on the application of the tube to oscillograph circuits. Power supply and timing circuits were described. The application of cathode ray oscillographs to present measurement problems was discussed.

Cleveland Section

The Case School of Applied Science was the place at which the December 20 meeting of the Cleveland Section was held. F. T. Bowditch, chairman, presided and twenty-six members and guests were present.

"New Tube Requirements of the New Receivers" was the paper presented by W. R. Jones, commercial engineer for Hygrade-Sylvania. He presented first the theory of the action of a triode and in an orderly fashion introduced the screen grid, suppressor grid, and additional elements to complete the multielement tubes now used in broadcast receivers. The functions of these additional electrodes and their effects on tube performance were described.

The election of officers was held and the following elected: chairman, Carl J. Banfer of Sound Systems, Inc.; vice chairman, R. L. Kline of Kladag Radio Laboratories; and secretary-treasurer, J. S. Hill of Broadcast Station WHK.

Connecticut Valley Section

The Connecticut Valley Section met on December 20 at the Hotel Charles in Springfield, Mass. K. S. Van Dyke, chairman, presided and the attendance totaled twenty-nine.

"Intermediate-Frequency Transformer Design" was the subject of a paper by F. H. Scheer of the F. W. Sickles Company. He presented first a classification of the various design considerations. Formulas for computing and methods of measuring the 'Q' of inductances were then presented. Structural considerations in design were explained and the results tabulated. The design of trimming condensers and shield can considerations were outlined. He then discussed coupling in relation to stage gain and selectivity considerations. The resolution of classic inductance theory to practice was accomplished in detail. The material centered largely around wide band or high-fidelity selectors and the design requirements involved. Half of those present participated in the discussion.

In the election of officers, J. A. Hutcheson of the Westinghouse Electric and Manufacturing Company was named chairman; M. E. Bond, and C. B. DeSoto were reëlected vice chairman and secretary-treasurer, respectively.

Detroit Section

A meeting of the Detroit Section at which Samuel Firestone presided was held in the Detroit News Conference Room on December 21 and was attended by fifty-five. Fourteen were at the dinner which preceded the meeting.

R. A. Wolfe, a research physicist in the Department of Engineering Research of the University of Michigan, presented a paper on "Thermionic Emitters." He introduced the subject with a historical development of electron emitters from the original discovery by Edison. After discussing the theory of thermionic activity he described the three types of emitters in general use today which he classified as pure elements, doped emitters such as theoriated tungsten, and oxide coated types. Their advantages and disadvantages and troubles encountered in their production were outlined.

He then introduced a new emitter developed at the University of Michigan which consists of a bariated nickle filament which is made emissive by oxidizing its outer surface. Barium contents as high as one per cent were found possible. Several advantages of these filaments were outlined. The lack of gas given off by them and the small effects of contaminating gasses and mercury vapor on the emission were noted. Between five minutes and one hour are necessary to bring the filament to full activity without preheating or high temperatures. The material can be rolled, drawn, and worked the same as nickle and will stand rough handling. Its emission is as great or greater than from oxide coated types and its work function is the same. The filament does not become inoperative with age. It was felt to be ideally suited for radio tubes.

Officers for 1935 were elected and are as follows: Chairman, A. B. Buchanan of the Detroit Edison Company; vice chairman, H. S. Gould, WMBC; and secretary-treasurer, E. C. Denstaedt of the Detroit Police Department.

New York Meeting

The annual meeting of the Institute was held in the Engineering Societies Building on January 2. Retiring president Jansky opened the meeting and introduced the incoming president, Stuart Ballantine, who then assumed the chair.

A paper by C. B. Jolliffe and A. D. Ring of the Federal Communications Commission on "The Allocation of Broadcast Facilities" was presented by Dr. Jolliffe and is summarized as follows:

"It is estimated that the radio broadcast industry today represents a total investment in transmitting and receiving equipment of approximately a billion dollars. The usefulness of this equipment depends on the maintenance of an orderly and technically sound allocation of the available facilities. The responsibility for this allocation is now vested in the Federal Communications Commission. This paper gives the historical development of the present allocation, the effects thereon of technical developments, of various changes in the radio law and the technical standards used in considering allocation problems. Future developments in broadcast allocation and the possibilities of approaching nearer to an ideal allocation, taking into account engineering and economic principles without artificial restrictions, are discussed."

The second paper of the evening was on the "Case Method Treatment of Certain Broadcast Allocation Problems," by C. M. Jansky, Jr., S. L. Bailey, M. M. Garrison, and G. I. Jones of Jansky and Bailey. It was presented by Mr. Jansky and is summarized as follows:

"This paper presents a discussion of certain fundamental problems involved in the quantitative evaluation of broadcast service and interference on shared channels by a detailed consideration of the steps taken by which the coverage of a certain shared channel station was increased approximately 500 per cent. First, field measurements were made to evaluate the service obtained from the original transmitter site. These led to the selection and removal of the transmitter to a far more favorable location at which a highly efficient radiating system was installed. Permission was then obtained to increase the power of the station from 250 to 1,000 watts after field measurements using an automatic recorder proved that a further large increase in coverage could be obtained with practically no detriment to the service of any other broadcast station."

The attendance was 150.

Philadelphia Section

December 6 was the date of a meeting of the Philadelphia Section at the Engineers Club. It was presided over by Knox McIlwain, vice chairman, and 200 were present. Sixteen attended the dinner.

A paper on "General Considerations of Tower Antennas for Broadcast Use" was presented by H. E. Gihring and G. H. Brown of the RCA Victor Company. In it they summarized the factors influencing the action of towers when used as radiators. It was shown that the results predicted from the simple theory of sinusoidal distribution of current on the tower differ to a major extent from the actual results. A series of measurements using small models of actual antenna structures resulted in data correlating closely with the performance of full size structures and showed that departures from simple theory are due to nonsinusoidal current distribution.

Several types of recently installed antenna towers were shown to be less effective than the simple theory predicted, particularly with regard to reduction of sky wave and fading. Means for correcting current distribution and improving these characteristics were pointed out. The ground system and earth currents were considered from theoretical and experimental viewpoints. A simple method of measuring earth currents was described and it was pointed out that such measurements indicate whether the antenna current is sinusoidal or not. The paper was discussed by Messrs. Kellogg, Murray, and others.

The second paper by W. F. Diehl of the RCA Victor Company was entitled "A Complete Portable Cathode Ray Oscillograph." The instrument weighing only thirty-eight pounds employs a three-inch cathode ray tube with electrostatic deflection in both directions and is completely alternating current operated. Two identical single stage compensated amplifiers linear from twenty cycles to ninety kilocycles are incorporated to increase the sensitivity to one volt per inch on both the horizontal and vertical axes.

Switches are provided to disconnect the amplifiers for high-frequency analysis. Gain controls on each amplifier allow the wave under observation to be spread out in each direction. A linear sweep circuit with a range from twenty cycles to fifteen kilocycles is incorporated. Controls are provided to shift the beam in both horizontal and vertical directions permitting greater equivalent screen size and spot location. The switch is provided so the timing axis may be synchronized with an external source, internal sixty-cycle supply source, or a portion of the voltage of the wave under observation. Synchronizing voltage control, safety switch and fuse are also included.

A new synchronizing impulse generator to be used with the oscillograph for radio and intermediate-frequency circuit analysis was also described. The circuit arrangement results in two waves being on the screen simultaneously when misalignment occurs. Correct alignment is indicated when the two waves coincide perfectly.

Lissajous figures, phase shift, distortion, intermediate and radio-frequency resonance curves were shown and numerous other applications demonstrated with the equipment. Messrs. Shank, Wolfe, and others discussed the paper.

Pittsburgh Section

The Pittsburgh Section met at the Fort Pitt Hotel on December 18. Chairman C. K. Krause presided and the attendance was thirty-four.

W. P. Place, research engineer for Union Switch and Signal Company, presented a paper on "Use of Copper Oxide Rectifiers for Detection and Automatic Volume Control." He stated that no one knew the theory of rectification due to oxide films but pointed out that effective use of these devices could be made in spite of this if their performance characteristics were known.

Many slides were shown of the rectifier and circuits were given for using it as a half and full-wave rectifier, an automatic volume control device, a mixer in a superheterodyne, a second detector, a B battery economiser a volume level control, and for many other unique purposes. A general discussion was held and participated in, among others, by Messrs. Gabler, Krause, Lazich, Noble, Peters, Shreve, Stark, and Wyckoff.

Rochester Section

The Rochester Section held a meeting on December 20 at the Sagamore Hotel at which 118 were present.

N. C. Schmid, sales engineer for the Taylor Instrument Company, presented a paper on "Air Conditioning and its Control as Applied to N.B.C. Broadcast Studios at Radio City." The author described briefly what was meant by air conditioning and discussed in detail the unusually severe requirements of a broadcast studio. Basing many of his explanatory remarks on the large studio at Radio City, he pointed out that a quiet audience was imperative and to accomplish this the audience must be comfortable. Air conditions are controlled accurately by means of automatic controllers actuated by sensitive elements located in the exit air from the room under control. Slides and diagrams showed not only air conditioning apparatus but also the studio arrangements and he described the reasons for the construction used and how these factors influenced the broadcast results.

San Francisco Section

The San Francisco Section met at the Bellevue Hotel on December 19 with Ralph Shermund, vice chairman, presiding. Fourteen attended the informal dinner which preceded the meeting.

A paper on "Broadcast Allocation" was presented by V. F. Greaves, Chief Federal Communications Commission Inspector of the western area. In it he presented a clear picture of the many difficulties encountered in allocating broadcast channels to the best advantage of the country. Many of the difficulties were illustrated by specific examples.

Seattle Section

A meeting of the Seattle Section was held on October 26 at the University of Washington with Howard Mason, chairman, presiding. Forty-six were present.

A paper on "Statewide Radio Communication Systems of the Washington Highway Department" was presented by J. R. Jordon of the State Highway Department. The paper dealt with the advantages of a radio communication system between fixed and mobile stations, the latter for the most part being on snow plows, in controlling traffic in the event of snow slides and similar disturbances and in maintaining close coördination between the activities of the highway, police, forest, and park forces. Plans were outlined for a statewide network of stations consisting of headquarters, key, and regional installations. Mr. John Greig then informally described the technical features of the present highway department equipment.

The November meeting of the Seattle Section was held on the 30th and seventy-five attended. The meeting was at Tumor Institute in Seattle and Howard Mason presided.

A demonstration of X-Ray equipment of the Tumor Institute was presented by J. E. Rose assisted by Doctors Henderson, Loughridge, and Ward. Doctors Rose and Ward are of Tumor Institute and the others are from the Physics Department of the University of Washington.

The audience was conducted in small groups through the room housing the X-ray tube proper. The treatment room where patients are irradiated, the control room, the room where atomic research is conducted, and the room where tubes or "seeds" are filled with radon gas for therapeutic treatment. All then congregated in the lecture room where Dr. Rose discussed unique electrical features of the 800-kilovolt tube and problems involved in its design. Doctors Loughridge and Henderson outlined some research in which they plan to study atomic structures by use of the tube. The paper was closed by Dr. Ward who explained the advantages of hard rays generated by the high-voltage tubes in the treatment of malignant growths and described the technique of such treatment and successes with it.

The December meeting was held on the 28th at the University of Washington. Thirty were present and Howard Mason presided.

A. V. Eastman of the Department of Electrical Engineering of the

University of Washington presented a paper on "Recent Developments and Applications in Mercury-Vapor Tubes." Professor Eastman's paper was devoted to the application of power services of gaseous tubes, particularly the ignitron and thyratron. The function of the grid and baffle in the thyratron and of the ignitor used in the ignitron were described in detail as were the mechanical designs of the tubes, problems of commutation, and circuits for inversion and rectification. The paper was discussed by Messrs. Fisher, Libby, Renfro, Tolmie, Williams, Willson, and others.

The annual election of officers was held and R. C. Fisher was named chairman; E. D. Scott, Puget Sound Power and Light Company, vice chairman; and C. E. Williams, Navy Department, secretary-treasurer.

Washington Section

A meeting of the Washington Section was held on November 12 at the Potomac Electric Power Company Auditorium. T. McL. Davis, chairman, presided and sixty-three attended. Twenty-four were present at the dinner which preceded the meeting.

A paper on "The Conferences of the U.R.S.I. at London and the C.C.I.R. at Lisbon" was presented by J. H. Dellinger, Chief of the Radio Section of the National Bureau of Standards. Dr. Dellinger summarized the problems considered at each of these conferences, the manner in which they were considered and disposed of, and recited new problems assigned for the next session.

The December meeting was held on the 10th in the same auditorium. T. McL. Davis presided; the attendance was seventy-eight; and eighteen were at the dinner held before the meeting.

A. S. Clarke and L. A. Shuttig of the Radio Research Company presented a paper on "Broadcast Transmitter Fidelity." Mr. Clarke discussed in considerable detail the measurements conducted on the type of emissions from broadcast transmitters and reported concerning actual tests on some transmitters. The study was intended to furnish material for improving transmission particularly as regards harmonic content, frequency response, and noise.

Mr. Shuttig then described in detail apparatus developed for making these measurements and demonstrated its operation. A number of those present participated in the discussion of the paper.

At the annual election of officers, E. K. Jett of the Federal Communications Commission engineering staff, was named chairman; C. L. Davis, of the Radio Corporation of America, vice chairman; and W. B. Burgess of the Navy Department, secretary-treasurer.

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TECHNICAL PAPERS

HOT-CATHODE MERCURY RECTIFIER TUBES FOR HIGH POWER BROADCAST TRANSMITTERS*

By

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Summary—*This paper describes the new and large hot-cathode mercury-vapor rectifier tube that is used in the plate power rectifier of the Crosley (WLW) 500-kilowatt broadcast transmitter.*

Design features resulting in improved operation of mercury-vapor tubes are discussed together with the operating characteristics.

An analysis of the advantages of the unit half-wave type of rectifier tube is made.

WITH the solution, several years ago, of certain fundamental problems in gaseous discharges[1,2,3] and improvement in vacuum tube manufacturing and design technique, the development of the hot-cathode mercury-vapor rectifier tube was made possible. Because of the increased efficiency and the reliability of operation of this type of rectifier tube, older forms of high voltage rectifying devices became obsolete for radio transmitter services. Tubes of sufficient capacity were developed[4,5] to supply not only the few kilowatts of power required in small radio transmitters but the several hundred kilowatts required in the larger broadcast transmitters.

This paper describes the design and operating characteristics of a new hot-cathode mercury-vapor rectifier tube that (properly grouped—six tubes) is capable of rectifying over 3000 kilowatts of power at direct voltages suitable for radio broadcast transmitters.

* Decimal classification: R356.3. Original manuscript received by the Institute, June 7, 1934. Presented before Ninth Annual Convention, Philadelphia, Pa., May 28, 1934.

1 K. H. Kingdon, "Neutralization of electric space charge by positive ionization at very low gas pressures," *Phys. Rev.*, vol. 21, April, (1923).

2 H. M. Mott-Smith and Irving Langmuir, "The theory of collectors in gaseous discharges," *Phys. Rev.*, October, (1926).

3 A. W. Hull, "Gas filled thermionic tubes," *Trans. A.I.E.E.*, vol. 47, no. 3; July, (1928).

4 O. W. Pike and H. T. Maser, "A new type of rectifier tube for amateur use," *QST*, vol. 12, no. 2; February, (1929).

5 H. C. Steiner and H. T. Maser, "Hot-cathode mercury-vapor rectifier tubes," Proc. I.R.E., vol. 18, no. 1, pp. 67–87; January, (1930).

Power Supply—500,000-watt Transmitter

The design of a suitable plate power supply for the WLW 500,000-watt transmitter involved a consideration of the available sources for obtaining a high voltage direct-current output of about 1000 kilowatts. Direct-current power outputs of this magnitude could be obtained through parallel operation of rectifier tubes of the size used in the 50,000-watt transmitters; through rectification with a new tube of sufficient capacity to carry the required current directly; or possibly as practiced in Europe, through the use of mercury-arc tank rectifiers.

Parallel operation of tubes whose capacity is smaller than that required to convert directly the desired power involves proper division of the load current, limitation of short-circuit currents through individual tubes to nondestructive values, and the effect of tube life on service. Proper division of the load current may be obtained by anode current-dividing reactors, the use of separate transformer windings for each anode, or separate transformers for each group of tubes. Separate transformers offer perhaps the most effective limitation to short-circuit currents although in most radio rectifiers these do not become excessive. The effect of tube life involves the theoretical consideration that if six tubes, each having an average life of 6000 hours, are used in a rectifier, one tube renewal will be required on the average of every 1000 hours. Tube renewals do not in most cases mean an interruption of service since, in general, adequate warning is given of the approaching end of life and renewals may be made in idle periods. If twelve or twenty-four tubes with the same average life are used to supply the same load, replacements may be required every 500 or 250 hours. Economically too, the cost of the tube complement is greater with parallel operation as with a given design of tube the cost per kilowatt of output decreases with increased tube capacity.

Experience with mercury-arc tank rectifiers for high voltage service is limited in this country although reported in Europe. Certain differences between the multianode tank rectifier and the half-wave sealed-off unit seem outstanding. Economically the initial cost of the tank rectifier (particularly if continuity of service is insured by a spare tank) is considerably higher than that of the equivalent rectifier with half-wave tubes. Obsolescence in a field of rapid technical advancement also requires a high rate of amortization. The charges of interest, amortization, repairs, and service for the tank rectifier must be balanced against the cost of tube replacements. Technically, there are fundamental differences between the two types. In the half-wave unit there is no ionization during the inverse voltage period while in the multianode unit ionization is present during all of this period. The inverse voltage across a half-wave unit for a given output is only half of that between an anode and the cathode of a tank rectifier. In addition, the higher inverse voltage of the tank anode combined with bombardment of the positive ions gives the probability of an increased number of arc-backs.[6] Adaptability of the half-wave unit to the full-wave type of rectifier circuit permits greater utilization of the transformers and the use of standard designs. Water cooling is used in the tank rectifier for vapor pressure control—air cooling with the tube. Efficiency difference, while small in percentage, is in favor of the half-wave unit type of rectifier.

General Characteristics

The outstanding characteristics of hot-cathode gas or mercury-vapor rectifying tubes are those of a low and nearly constant arc-drop while carrying current and of a high impedance during the inverse voltage part of the cycle. Anode current is carried mainly by thermionic emission of electrons from a heated cathode. Gas molecules ionized by the electrons form, in conjunction with the electrons, a "plasma"[7] region of nearly constant potential. This ionized gas effectively neutralizes the space charge existing in high vacuum tubes. In addition, since neutralization of the space charge permits electrons to be drawn through holes and from crevices, heat-shielded and consequently more efficient cathodes may be used.

The use of an "inverted" anode in contrast to one shaped to conform to an equipotential line of the inverse electrostatic field results in a decided improvement in operating characteristics. At the end of the conduction period positive ions in the space diffuse to the walls of the tube or are drawn to the anode. With high inverse voltages these ions obtain sufficient momentum to remove particles from the anode. Continual bombardment causes the walls of the tube to become coated with this sputtered material. With peak inverse potentials below about ten kilovolts this coating gives little trouble, but at higher inverse potentials the changing potential gradients may exceed the critical breakdown value of the gas and cause arc-back. When the anode surrounds the cathode the effects of positive ion bombardment are reduced to a negligible quantity.

The gas pressure for optimum operating conditions governs the temperature limits of operation. Too high a pressure causes breakdown of the gas when the anode potential is negative. Too low a pressure in-

[6] J. Slepian and L. R. Ludwig, "Backfires in mercury arc rectifiers," *Trans. A.I.E.E.*, vol. 51, no. 1; March, (1932).

[7] L. Tonks and Irving Langmuir, "General theory of the plasma of the arc," *Phys. Rev.*, September, (1929).

creases the arc-drop and may result in cathode disintegration through excessive positive ion bombardment. Also, too low a pressure reduces the initial value of current that can be conducted and may cause sputtering of the cathode. In the higher current tubes the optimum vapor pressure range is from about three to ten microns, which corresponds to condensed mercury temperatures of 30 to 50 degrees centigrade. Since the vapor pressure is determined by the temperature of the coldest part of the tube control is easily maintained. Fig. 1 shows the small volume of air required.

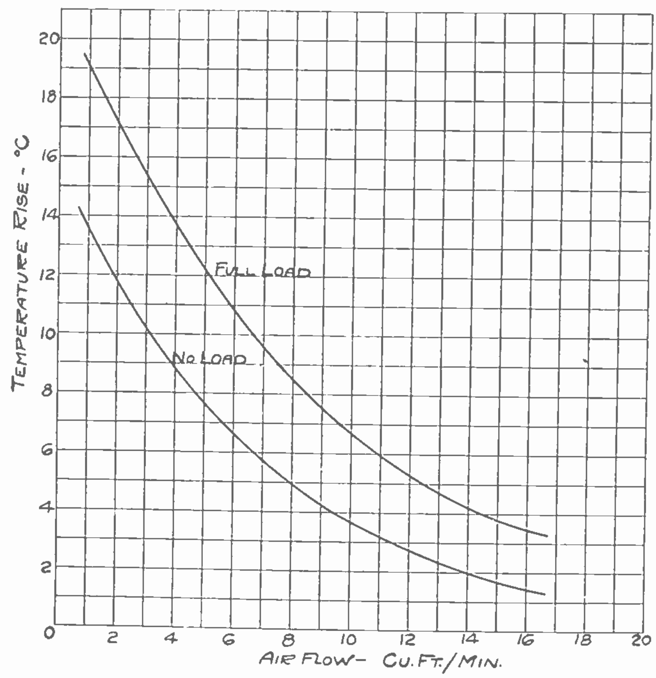

Fig. 1—Curves showing regulation of condensed mercury temperature with forced air cooling.

Radiotron RCA-870

The new rectifier tube, Radiotron RCA-870, is shown in Fig. 2. Physically the tube is approximately ten inches in diameter and twenty-five inches tall. Fig. 3 illustrates the construction of the tube.

The cathode is of the "Wehnelt" or oxide-coated type and consists of a stack of coated disks enclosed by suitable heat shields. A coiled tungsten heater running coaxially with the coated disks supplies heating energy. The efficiency of the shielded cathode is high—the cathode supplying a peak emission of approximately 1.25 amperes per watt in comparison with the usual 0.05 ampere per watt for an open-type ribbon filament. High efficiency, however, is accompanied by an increase in the time required to bring the cathode to operating temperature.

Heating and cooling time temperature curves are shown in Fig. 4. As illustrated, the heating time may be reduced by applying overvoltage to the heater initially and reducing to normal when the operating temperature is reached. Also a spare tube may be operated with the cathode hot without appreciably affecting the life.

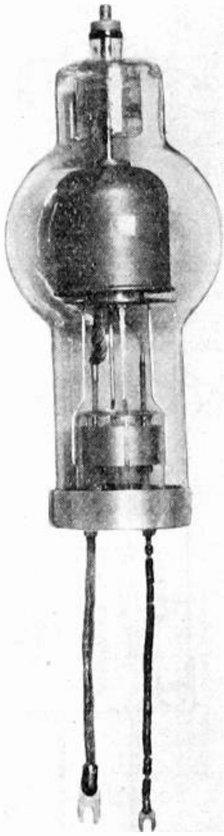

Fig. 2—Hot-cathode mercury-vapor rectifier tube, Radiotron RCA-870.
Ratings:

Cathode heater voltage	5.0 volts
Cathode heater current	65.0 amperes
Peak anode current	450.0 amperes
Average anode current	75.0 amperes
Peak inverse voltage	16,000.0 volts

Tube Ratings

In addition to the usual ratings of maximum peak inverse voltage and maximum peak anode current that represent, respectively, a safe inverse voltage for tube operation and the designed peak cathode emission, it is important in the larger tubes to consider ratings which indicate the heat dissipating ability as a function of load, and the ability of the tube to conduct the high transient currents that occur when the rectifier is short-circuited.

The *maximum average* anode current represents the average load current that the tube is designed to carry continuously. The arc-drop in a mercury-vapor rectifier tube is nearly constant with variation in

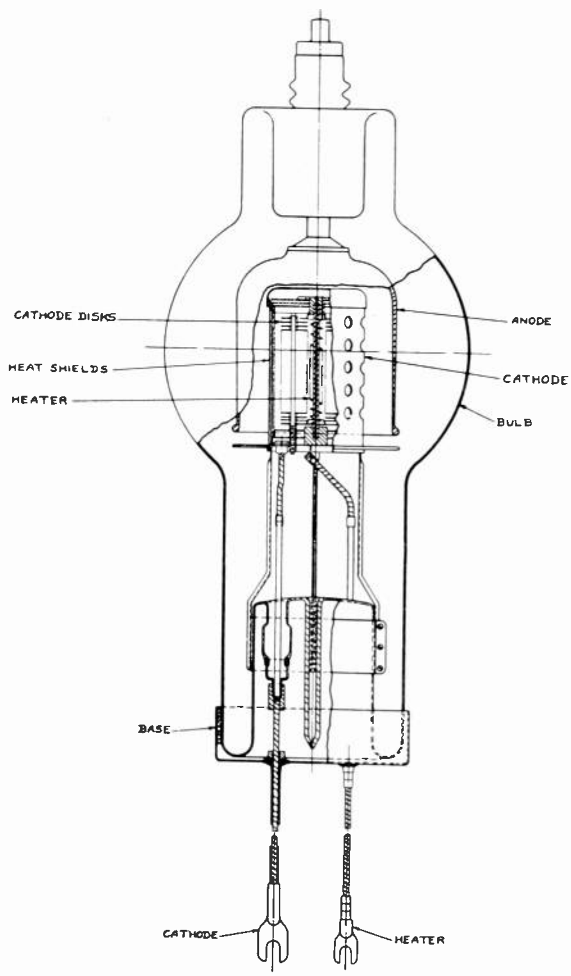

Fig. 3—Longitudinal section through tube showing construction.

load current. Losses in the arc, therefore, vary more nearly with the average value of the load current than with the root-mean-square value. In the case of fluctuating loads or overloads it is necessary to define an integration period over which the load is to be averaged. In common with vacuum tubes which have elements of small mass, and consequently small heat storage capacity, the integration period is much shorter than that encountered in transformers or rotating machinery. Tubes are fundamentally maximum rather than nominal rated devices.

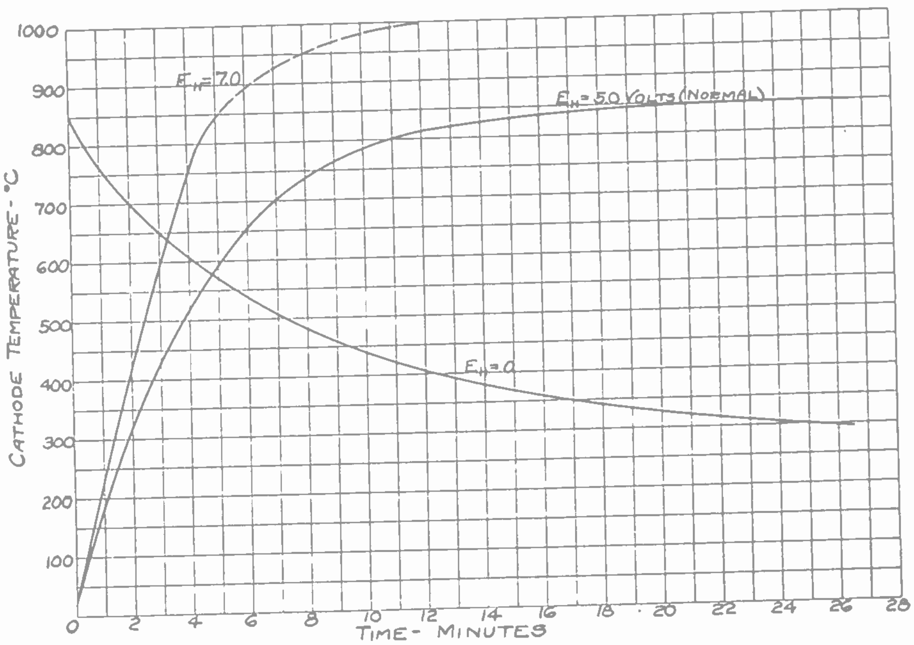

Fig. 4—Curves showing rate of cathode heating and cooling. (Mean cathode color temperatures.)

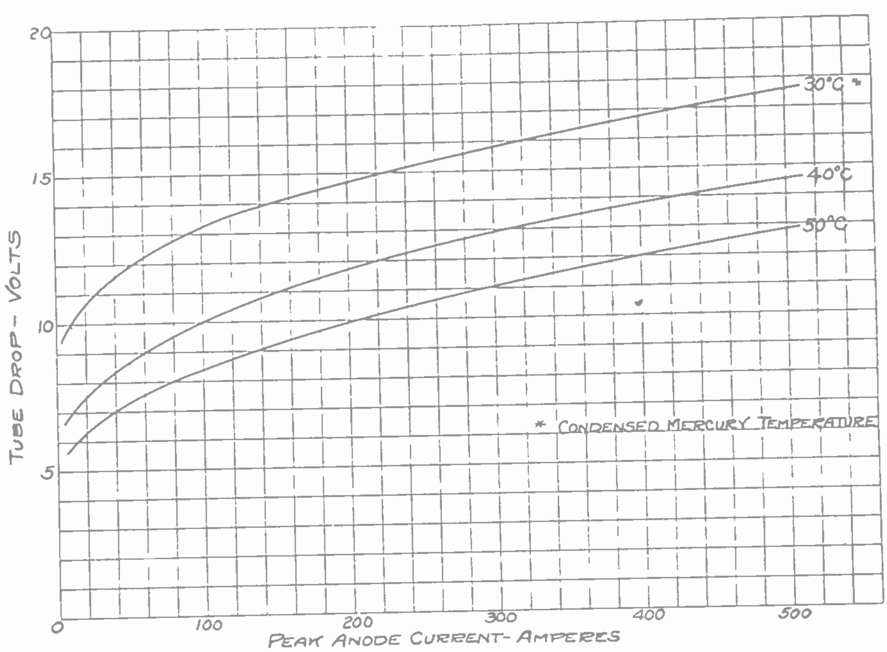

Fig. 5—Curves showing effect of condensed mercury temperature on tube drop voltage.

Under short-circuit conditions whether caused by arcs in the transmitter or arc-backs in the rectifier the current through the tube is limited almost entirely by the leakage reactance of the power transformers and associated voltage regulating equipment. The tubes, because of the arc characteristics of the discharge, offer very little impedance and the cathodes of the tubes are capable of supplying surge

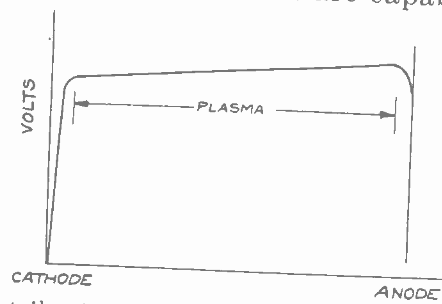

Fig. 6—Voltage distribution between anode and cathode in gas-filled tubes.

emission currents of many times the rated value. In order to indicate the available transient current carrying capacity and the ability of the tube to withstand short-circuit conditions a *maximum surge current*

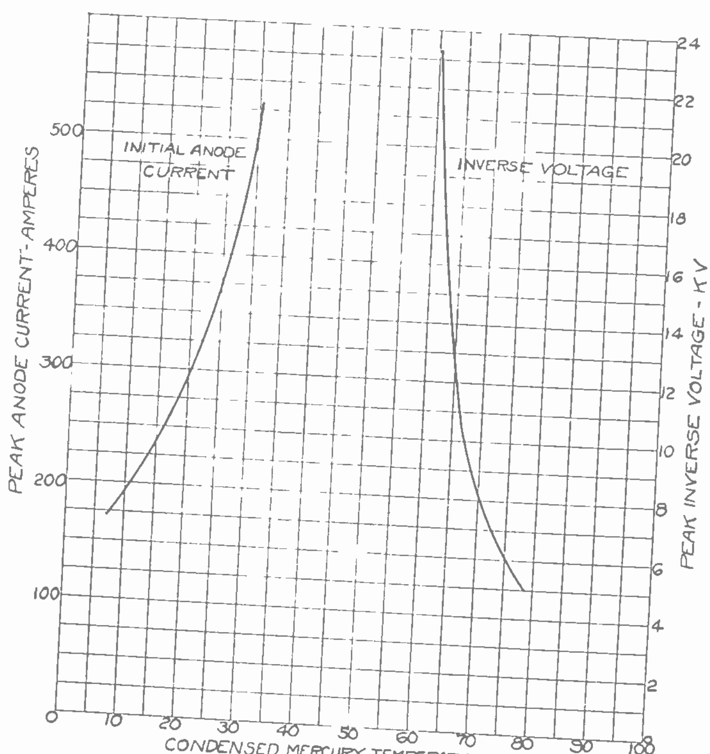

Fig. 7—Curves showing effect of condensed mercury temperature on arc-back voltage and on initial anode current at the start of conduction.

rating sometimes is given. Experience indicates that the impedance of most radio rectifiers in combination with the speed of relaying that is required to protect the transmitting equipment in case of flashover provides ample protection from the rectifier tube standpoint.

Operating Characteristics

The arc-drop voltage as a function of peak anode current for condensed mercury temperature within the recommended operating range is shown in Fig. 5. Fig. 6 illustrates the voltage distribution[8] between the anode and cathode of hot-cathode mercury-vapor tubes of the design shown in Fig. 3.

Fig. 7 illustrates the effect of gas pressure on breakdown of the gas or arc-back as a function of the peak inverse voltage across the tube. The effect of gas pressure upon the initial peak current that can be conducted is also shown. Exceeding these initial peak currents may cause sputtering of the cathode until higher cathode temperatures are reached (automatically) to "balance" the effect of low gas pressure. Interpretation of the arc-back or flash-back curves require a knowledge of the short-circuit impedance of the power transformer. In general, with high impedance transformers "incipient" arc-backs may be commutated without noticeable disturbance in either the tubes or circuit. With low impedance circuits "incipient" arc-backs may develop into disturbances of sufficient magnitude to require clearing of the circuit.

Conclusion

As previously stated, the RCA-870 Radiotron at maximum ratings will convert over 3000 kilowatts at voltages suitable for the plate power supply of radio broadcast transmitters. With this power supply transmitters of over 1000 kilowatts may be built. If, however, the need for greater outputs should arise, the experience and technical information gained in the development of the new rectifier tube indicate that hot-cathode mercury-vapor rectifier tubes of even greater capacity can be built successfully.

In conclusion, acknowledgment is due Messrs. R. B. Ayer and L. E. Record for their able coöperation throughout the development.

[8] E. D. McArthur, "Electronics and electron tubes," *Gen. Elec. Rev.*, March–September, November, December, (1933).

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NOTE ON VACUUM TUBE ELECTRONICS AT ULTRA-HIGH FREQUENCIES*

By

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(Bell Telephone Laboratories, Inc., New York City)

Summary—*The work described in a previous paper*[1] *is extended and revised. An alternative method of solving the fundamental equation is presented. Application to the positive grid oscillator is treated and oscillation criteria in general are discussed.*

Since the previous paper[1] on vacuum tube electronics was published in the Proceedings, further study as well as a series of letters between W. E. Benham in England and the writer have indicated several interesting points in the theoretical side of the development and have brought to light certain flaws in its application. The fundamentals are, however, further supported by this study, and methods of arriving at the results with a somewhat more straightforward attack than that employed originally have been found. It is the purpose of this note to discuss these points with a view toward bringing the theory up to date.

Benham has questioned the validity of extending the boundary conditions as was done in the previous paper on the basis that the resulting formulas gave a value for the alternating-current component of the electric force which became infinite at a virtual cathode. Such a result is naturally not in accord with the physics of vacuum tubes and would seem at first sight to cast doubt upon the validity of the entire development. However, when the fact is taken into account that the expression in question is an approximation obtained by making the assumption that the alternating-current components are small compared with the direct-current components, it is evident that the approximation cannot be expected to be valid at a virtual cathode where the direct-current component of the electric force is zero while the alternating-current components may remain finite. This does not, however, preclude the accuracy of the approximate formula when applied to points where the approximation is valid; namely, anywhere except so close to the virtual cathode that the direct-current component of the force is not large compared with the alternating current.

* Decimal classification: R130. Original manuscript received by the Institute, October 23, 1934.

[1] F. B. Llewellyn, "Vacuum tube electronics at ultra-high frequencies," Proc. I.R.E., vol. 21, pp. 1532–1574; November, (1933).

In order to illustrate this point in a more exact style as well as to exhibit what is thought to be a more straightforward method[2] of arriving at the result, equation (3) in the original paper may be taken as a starting point. This equation expresses the relation between the total current, J, between two parallel planes and the velocity, u, of electrons in the space between them. Thus we have

$$4\pi \frac{e}{m} J = \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x}\right)^2 u \tag{1}$$

but u is a function of x and t so that

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}$$

and (1) may be written

$$4\pi \frac{e}{m} J = \frac{d^2u}{dt^2} = \frac{d^3x}{dt^3}. \tag{2}$$

As explained before, the current J is a function of t only. We take it to consist of two parts, and write

$$4\pi \frac{e}{m} J = K + \phi'''(t) \tag{3}$$

where K is a constant and $\phi'''(t)$ is the third derivative with respect to t of some function of t, say Be^{ipt}. Thus from (2) and (3) we have

$$\frac{d^3x}{dt^3} = K + \phi'''(t). \tag{4}$$

Before the integration of this equation is attempted, it will simplify matters if a change of variable is made by putting

$$t = t_0 + T \tag{5}$$

so that $dt = dT$ and (4) becomes

$$\frac{d^3x}{dT^3} = K + \phi'''(t_0 + T) \tag{6}$$

whence, on successive integrations,

$$\frac{d^2x}{dT^2} = KT + \phi''(t_0 + T) + c_1 \tag{7}$$

$$\frac{dx}{dT} = u = \tfrac{1}{2}KT^2 + \phi'(t_0 + T) + Tc_1 + c_2 \tag{8}$$

$$x = \tfrac{1}{6}KT^3 + \phi(t_0 + T) + \tfrac{1}{2}T^2c_1 + Tc_2 + c_3. \tag{9}$$

[2] A somewhat similar method is employed for analysis of diodes by Johannes Müller, "Elektronenschwingungen in Hochvakuum," *Hochfrequenz. und Elektroakustic*, May, (1933).

We may now put $x=0$ when $T=0$ so that we find from (9) that $c_3=-\phi(t_0)$. This merely determines the origin of our measurement of distance, and imposes no physical conditions. From the physics it can be seen in conjunction with (9) that if there were only the direct-current component present, then ϕ would be zero, so that under conditions of complete space charge we should have zero velocity and acceleration at the origin where $T=0$. This would lead to the result that $c_1=0$, $c_2=0$, $c_3=0$, so that

$$\frac{d^2x}{dT^2} = KT \qquad u = \tfrac{1}{2}KT^2 \qquad x = \tfrac{1}{6}KT^3.$$

The last of these three equations suggests a change of variable which will effect an enormous simplification. We introduce a new variable, τ, in place of T and put in general

$$x = \tfrac{1}{6}K\tau^3. \tag{10}$$

In the absence of alternating current it is evident that τ and T are identical. In the presence of alternating current they will not be the same, so that we write

$$T = \tau + \epsilon \tag{11}$$

and proceed to interpret these quantities.

τ is the time it would take an electron to move from the origin to a point $x=\frac{1}{6}K\tau^3$ if there were no alternating component to the current. T is the time which is actually required by the electron to reach the same point, and differs from τ by an amount ϵ. When the alternating component of the current is small compared to the direct current, then the difference between τ and T will be small, so that an analysis based upon the premise that $\epsilon \ll \tau$ will be an acceptible approximation for small values of alternating current.

On this basis we can write

$$\phi(t_0 + T) = \phi(t_0 + \tau + \epsilon) = \phi(t_0 + \tau) + \epsilon\phi'(t_0 + \tau) + \tfrac{1}{2}\epsilon^2\phi''(t_0 + \tau) + \cdots$$

so that, with the value of c_3 already found, (9) becomes

$$\tfrac{1}{6}K\tau^3 = \tfrac{1}{6}K(\tau + \epsilon)^3 + \phi(t_0 + \tau) + \epsilon\phi'(t_0 + \tau) + \tfrac{1}{2}\epsilon^2\phi''(t_0 + \tau) + \cdots + \tfrac{1}{2}(\tau + \epsilon)^2c_1 + (\tau + \epsilon)c_2 - \phi(t_0).$$

In solving this under the assumption that ϵ is small compared with τ it will be a permissible approximation to neglect all powers of ϵ higher than the first. The result is

$$\epsilon = -\frac{\phi(t_0 + \tau) + \frac{1}{2}\tau^2 c_1 + \tau c_2 - \phi(t_0)}{\frac{1}{2}K\tau^2 + \phi'(t_0 + \tau) + \tau c_1 + c_2}. \tag{12}$$

It was pointed out above that the c's are all zero in the absence of alternating current so that it may be inferred that they are of the same order of magnitude as the alternating-current components. This means that the term $\frac{1}{2}K\tau^2$ in the denominator of (12) is by far the largest term, and would seem to indicate that the others may be neglected with the same degree of approximation involved in the neglect of the higher powers of ϵ. Before this step is taken, however, it must be noted that near the origin where τ is very small, the term $\frac{1}{2}K\tau^2$ approaches zero while the terms involving ϕ remain different from zero. Thus, any solution where the denominator of (12) is replaced by $\frac{1}{2}K\tau^2$ cannot be employed for values of τ so small that the remaining terms become appreciable.

Keeping this in mind we write

$$\epsilon = -\frac{\phi(t_0 + \tau) + \frac{1}{2}\tau^2 c_1 + \tau c_2 - \phi(t_0)}{\frac{1}{2}K\tau^2} \tag{13}$$

except when $\tau \to 0$.

With the aid of (11) and (13) the value of u can be found from (8) and gives, to the same approximation,

$$u = \frac{1}{2}K\tau^2 - \frac{2}{\tau}\phi(t_0 + \tau) + \frac{2}{\tau}\phi(t_0) + \phi'(t_0 + \tau) - c_2. \tag{14}$$

In this expression the constant c_2 is still undetermined and may be found if the value of u at a certain value of τ is given. Thus, the condition, when $\tau = \tau_1$

$$\begin{aligned} u &= \tfrac{1}{2}K\tau_1^2 + f(t) \\ &= \tfrac{1}{2}K\tau_1^2 + f(t_0 + \tau_1) + \cdots \end{aligned} \tag{15}$$

can be used to find c_2, giving

$$c_2 = -\frac{2}{\tau_1}\phi(t_0 + \tau_1) + \frac{2}{\tau_1}\phi(t_0) + \phi'(t_0 + \tau_1) - f(t_0 + \tau_1) \tag{16}$$

so that

$$\begin{aligned} u = \tfrac{1}{2}K\tau^2 &- \frac{2}{\tau}\phi(t_0 + \tau) + \frac{2}{\tau}\phi(t_0) + \phi'(t_0 + \tau) \\ &+ \frac{2}{\tau_1}\phi(t_0 + \tau_1) - \frac{2}{\tau_1}\phi(t_0) - \phi'(t_0 + \tau_1) + f(t_0 + \tau_1). \end{aligned} \tag{17}$$

Here we have u expressed in terms of t_0 whereas it is more useful to express it in terms of t. This can be done by using the relation,

$$t_0 = t - \tau - \epsilon$$

neglecting, as before, terms of the order $|\epsilon^2|$. The result is

$$u = \tfrac{1}{2}K\tau^2 - \frac{2}{\tau}\phi(t) + \frac{2}{\tau}\phi(t - \tau) + \phi'(t) + \frac{2}{\tau_1}\phi(t - \tau + \tau_1)$$
$$- \frac{2}{\tau_1}\phi(t - \tau) - \phi'(t - \tau + \tau_1) + f(t - \tau + \tau_1). \tag{18}$$

This equation is in agreement with (18) in the original electronics paper. In order to reduce it to the same form it is only necessary to put

and,
$$\left.\begin{aligned} \phi'''(t) &= Be^{ipt} \\ f(t) &= (M + iN)e^{ipt} \end{aligned}\right\}. \tag{19}$$

It is thought that the above analysis, dealing with ordinary differential equations instead of the partials employed in the previous paper, illustrates better the extent of the approximations and their significance, at the same time eliminating the cumbersome arbitrary functions which are now replaced by constants of integration. In particular, the reason why the solution cannot be applied directly to points very close to the origin is explained by (12) and (13).

In the original paper, (18) was used to find the potential difference between the origin and a point corresponding to τ_1. It would seem at first sight that the difficulty at the origin should cause the result to be in doubt. The fact that this is not the case may be shown, however.

To do this we return to (7) which gives the acceleration of the electrons, and note that

$$\frac{e}{m}E = \frac{d^2x}{dt^2} = K(\tau + \epsilon) + \phi''(t_0 + \tau) + c_1$$

whence from (13)

$$\frac{e}{m}E = K\tau + \phi''(t_0 + \tau) - \frac{2}{\tau^2}\phi(t_0 + \tau) + \frac{2}{\tau^2}\phi(t_0) - \frac{2}{\tau}c_2$$

or, with (16)

$$\frac{e}{m}E = K\tau + \phi''(t_0 + \tau) - \frac{2}{\tau^2}\phi(t_0 + \tau) + \frac{2}{\tau^2}\phi(t_0) + \frac{4}{\tau\tau_1}\phi(t_0 + \tau_1)$$
$$- \frac{4}{\tau\tau_1}\phi(t_0) - \frac{2}{\tau}\phi'(t_0 + \tau_1) + \frac{2}{\tau}f(t_0 + \tau_1).$$

When t_0 is expressed in terms of t this becomes

$$\frac{e}{m}E = K\tau + \phi''(t) - \frac{2}{\tau^2}\phi(t) + \frac{2}{\tau^2}\phi(t-\tau) + \frac{4}{\tau\tau_1}\phi(t-\tau+\tau_1)$$
$$- \frac{4}{\tau\tau_1}\phi(t-\tau) - \frac{2}{\tau}\phi'(t-\tau+\tau_1) + \frac{2}{\tau}f(t-\tau+\tau_1). \quad (20)$$

The force E is related to the potential V through the equation

$$V_b - V_a = -\int_a^b E dx = -\frac{1}{2}\int_{\tau_a}^{\tau_b} K\tau^2 E d\tau \qquad (21)$$

where the integration is carried out by the use of (20), regarding t as a constant.

If the lower limit τ_a in (21) approaches zero, where the approximation used in finding (20) leads to erroneous results, it is still true from (20) that τ^2E approaches zero as τ becomes small. This means that the potential varies very little in the neighborhood of the origin, and is a true result in the presence of space charge. It follows, then, that as τ^2E does not affect the answer for small values of τ_1 the equation used for E in that region is inconsequential, and the approximate form (20) will serve just as well as an exact one. Farther away from the origin where the form of E does influence the result, the approximation is sufficiently good for most practical purposes, and can be made as exact as we like by taking the alternating component of the current to be small compared with the direct current.

The difficulty near the origin is therefore of more academic than practical importance although naturally it is imperative that its significance be investigated thoroughly so that proper reliance may be placed upon the final results.

The next point in the previous paper which requires further attention is the application of the fundamental equations to the performance of an oscillator tube operating in the Barkhausen manner with its grid at a high positive potential while its plate is operated at a potential just far enough positive to prevent electrons from turning back toward the grid. It is also postulated that the grid-plate spacing is just sufficient to allow complete space charge in the grid-plate region with the formation of a virtual cathode close to the plate. Under these conditions the equations were applied to find the relation between current and potential difference. It has been found by the writer, however, that in this application an error in algebraic signs has occurred which affects (34) and (35) and alters Figs. 11 and 15. Figs. 12 and 13 being intended primarily to illustrate a theoretical principle, will still illustrate this point as they stand.

In regard to the algebraic signs, it will be remembered that the fundamental equations, (24-a) in the original paper, or (21) in this note, were written for the condition that the origin was taken at the virtual cathode and that the electrons moved away from the origin. In the grid-plate region the origin will be at the virtual cathode near the plate, but the electrons will be moving toward the origin. Fig. 1 illustrates the comparative relations in the two regions. Near the

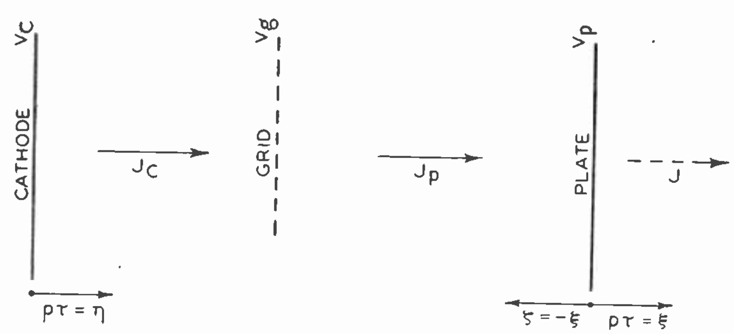


Fig. 1—Nomenclature for positive grid triode.

cathode, the assumed alternating current, J_c, is in the same direction as the electron motions so that $p\tau = \eta$ where η is the transit angle and is taken as positive. Near the plate conditions are different. If the plate were perforated so that electrons could pass through it, then in the space to the right of the plate the current and velocity would both have the same direction just as they do near the cathode, and the transit angle $\xi = p\tau$ would be a positive number.

To the left of the plate, however, with the same origin, values of $p\tau$ would be negative, so that if the transit angle is to be regarded as a positive number ζ, then we must put $\zeta = -\xi$. When this has been done, the integration of (21) between the limits $\xi_a = p\tau_a = 0$ and $\xi_b = p\tau_b = \xi_1 = -\zeta$ gives the following in place of (34) in the previous paper:

$$V_g - V_p = -\left(\frac{2m}{e}\,\frac{\alpha^3}{9p^2}\right)(M + iN)\left[(1 - \cos\zeta) - i(\zeta - \sin\zeta)\right] + J_p Z_p. \qquad (22)$$

It will be noted that the difference between this and the original equation lies in the algebraic signs in the first term on the right, the remaining symbols having the same meaning as in the previous paper.

In order to investigate oscillation conditions, it will be convenient to extend the treatment given in the previous paper in such a way that a better physical picture may be obtained. To this end, we introduce the following symbols:

$$\begin{aligned}
A &= 1 - \cos \zeta \\
B &= \zeta - \sin \zeta \\
D &= \tfrac{1}{6}\zeta^3 - \frac{4}{\zeta}(1 - \cos \zeta) + 2 \sin \zeta \\
F &= \frac{1}{\eta}(\eta - \sin \eta) \\
H &= \frac{1}{\eta}(1 - \cos \eta) \\
I &= 1 + \cos \eta - \frac{2}{\eta} \sin \eta \\
K &= \frac{2}{\eta} - \sin \eta - \frac{2}{\eta} \cos \eta \\
P &= 2 \cos \eta + \eta \sin \eta - 2 \\
Q &= \eta + \tfrac{1}{6}\eta^3 - 2 \sin \eta + \eta \cos \eta.
\end{aligned} \tag{23}$$

Using these symbols, we can write:

$$Z_p = -i \frac{12r_p}{\zeta^4} D \tag{24}$$

$$\frac{2m}{e}\frac{\alpha^3}{9p^2}(M + iN) = \frac{4\pi J_0}{p^2}(M + iN)$$

$$= -\frac{12r_p}{\zeta^4} J_c (I + iK) \text{ for complete space charge} \tag{25}$$

$$= -\frac{12r_p}{\zeta^4} J_c (F + iH) \text{ for no space charge} \tag{26}$$

$$V_c - V_g = J_c Z_c = -\frac{12r_c}{\eta^4} J_c (P + iQ) \text{ for complete space charge} \tag{27}$$

$$= -\frac{iJ_c}{pC} \text{ for no space charge} \tag{28}$$

where C is the cathode-grid capacitance.

In these expressions r is the magnitude of the low-frequency tube impedance and the subscripts c and p refer to cathode and plate regions, respectively.

In the triode now under consideration there is a certain approximation in taking V_g to be the potential of the grid wires, whereas it is more nearly the potential of a point midway between two of the grid wires. For the negative grid triode it would not be permissible to assume that these two potentials are the same, because the electrons are constrained to flow through only a part of the space between the grid wires. On the other hand, the positive grid tube does not force this condition on the electrons, and their density varies only slightly at different points in the grid plane. Consequently the approximation of calling V_g the potential of the grid wires in the positive grid tube may be expected to yield reasonably valid results. In its application, the reservation should always be made that V_g is properly the potential of the space between grid wires, and that there may be a certain impedance (which is small with positive grid tubes) between that point and the wires themselves.

Keeping this in mind, (22) may be written in the form

$$V_g - V_p = \mu(V_c - V_g) + J_p Z_p \tag{29}$$

where, for complete space charge, and since $r_p/r_c = \zeta^2/\eta^2$, we have

$$\mu = -\frac{\eta^2}{\zeta^2}\,\frac{(I + iK)(A - iB)}{(P + iQ)}. \tag{30}$$

For no space charge near the cathode the value of μ is

$$\mu = \frac{12 r_p i p C}{\zeta^4}(F + iH)(A - iB). \tag{31}$$

Equation (29) taken in conjunction with (30) and (31) shows that in many respects the operation of the positive grid triode is similar to that of the negative grid tube, while in other respects conditions are quite different. The equivalent circuit of the positive grid triode is shown in Fig. 2 and illustrates the similarities as well as the differences.

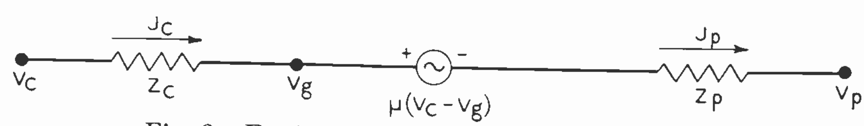


Fig. 2—Equivalent circuit for positive grid triode.

Foremost among the differences is the location of the grid V_g in series with the cathode-plate circuit. This condition comes about solely because of the assumption that the grid potential is the same as that of the space between grid wires, and has nothing to do with the mathematics dealing with conditions within the electron stream. It is not improbable that an equivalent circuit applicable to all conditions could be obtained by supposing the grid wires to be coupled to the electron stream in the space between wires through a suitable impedance. In the case of the negative grid tube this impedance would be a pure capacity, and at low frequencies its reactance would be high enough to mask any other impedance in the cathode-grid path, while at high frequencies its impedance has decreased more rapidly than that of the electron stream between cathode and the grid plane, leaving the resistive component of the electron stream an important factor in determining the cathode-grid impedance. Such a convention would be in general accord with experimental data and deserves further investigation at a future date.

With the positive grid tube and at the very high frequencies at which these tubes operate this impedance between grid wires and electron stream is quite certainly low enough to be neglected as a first approximation, especially when applied in the manner illustrated in Fig. 3 which will also be used for illustration of oscillation conditions, for here the impedance Z_0 may be considered as including the impedance between grid wires and the electron stream.

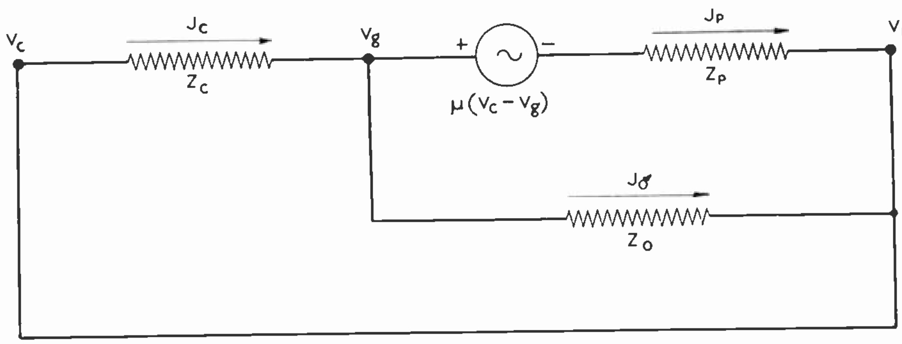


Fig. 3—Oscillator circuit with positive grid triode.

In the circuit of Fig. 3 an impedance Z_0 is connected between grid and plate terminals of the tube while the cathode and plate potentials are taken to be the same for alternating as well as for direct current so that a short may be placed between them. The circuit equations may be written in the determinant form

$$\begin{array}{cc} J_c & J_p \\ \hline \end{array}$$

$$\begin{vmatrix} z_c + Z_0 & -Z_0 \\ \mu z_c - Z_0 & Z_p + Z_0 \end{vmatrix} = 0$$

whence the required value of Z_0 is given by

$$\frac{1}{Z_0} + \frac{1}{z_c} + \frac{\mu}{Z_p} = 0. \tag{32}$$

In this equation the factor p is contained in the various terms. It will be remembered that the solution of the fundamental equation (1) was obtained by writing the alternating current in the form of the exponential e^{ipt}. The various steps in the development would have followed in exactly the same manner if we had written e^{mt}, say, where m is complex in general. Thus (32) will hold if p is replaced by $(-im)$ and the current now appears as a transient which will either build up or die down depending on whether the real part of m is positive or negative. Thus, writing

$$m = \alpha + i\beta$$

it is clear that p is merely the value taken by β when α is zero.

Consider the function of m given by (32):

$$f(m) = \frac{1}{Z_0} + \frac{1}{z_c} + \frac{\mu}{Z_p}.$$

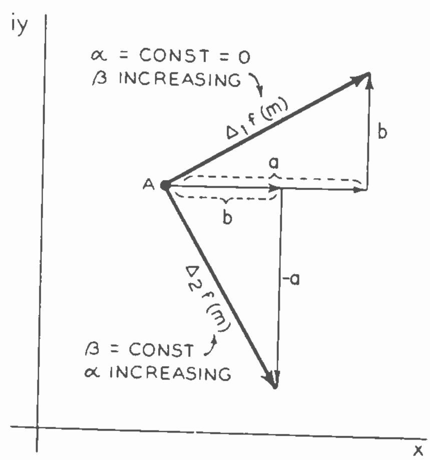


Fig. 4—Properties of complex variables.

Suppose that we take α to be zero and plot the real part of this function against the imaginary part for various values of $\beta = p$. We are used to considering such a plot as representing the effect of a frequency change. Actually it is a special case of a much more general function.

In Fig. 4 $\Delta_1 f(m)$ represents a short segment of such a plot obtained by holding α equal to zero and increasing β. Now suppose that α were to be made just a little different from zero and the curve were replotted for the same range of values of β. What would be the relation between the new curve thus obtained and the original $\Delta_1 f(m)$ with which we started?

The answer may be found in one of the fundamental properties of complex variables which is demonstrated in any text on the subject.[3] This property may be stated as follows:

If,

$$f(m) = x + iy \tag{33}$$

is a monogenic function of $m = \alpha + i\beta$, then

$$\frac{\partial x}{\partial \alpha} = \frac{\partial y}{\partial \beta} \quad \text{and} \quad \frac{\partial x}{\partial \beta} = -\frac{\partial y}{\partial \alpha}. \tag{34}$$

Thus, if α is held constant while β is varied, we have from (33)

$$\Delta_1 f(m) = \left(\frac{\partial x}{\partial \beta} + i\frac{\partial y}{\partial \beta}\right)\Delta\beta \tag{35}$$

and this gives the short segment of the curve shown in Fig. 4 and marked $\Delta_1 f(m)$.

Suppose now that we return to the point marked A in the figure and find what happens to the function if β is held fixed but α is varied. Under this condition, from (33)

$$\Delta_2 f(m) = \left(\frac{\partial x}{\partial \alpha} + i\frac{\partial y}{\partial \alpha}\right)\Delta\alpha$$

but, from (34) this may be written

$$\Delta_2 f(m) = \left(\frac{\partial y}{\partial \beta} - i\frac{\partial x}{\partial \beta}\right)\Delta\alpha. \tag{36}$$

Comparison of (35) and (36) now shows two things. The first is that $\Delta_1 f$ and $\Delta_2 f$ are perpendicular to each other, and the second is that when $\Delta\beta$ and $\Delta\alpha$ are both positive, then $\Delta_2 f$ considered as a vector points to the right with respect to $\Delta_1 f$ also considered as a vector. This means that points just to the right of the curve correspond to positive values of α while those just to the left correspond to negative values of α.

The problem now is to show how this apparent digression has a very important bearing on the prediction of oscillation properties from (32). In Fig. 5 the curved segment represents a short portion of the

[3] For instance, "Functions of a Complex Variable," by T. S. Fiske.

curve of $f(m)=1/Z_0+1/z_c+\mu/Z_p$ plotted for zero values of α and increasing values of β as shown by the arrow. Equation (32) demands that $f(m)$ shall be zero. This means that a curve representing $f(m)$ will pass through the origin for some value of $m=\alpha+i\beta$. The particular curve drawn in the figure shows that when $m=0+i\beta$ the curve passes very near the origin. What change should be made in α in order that the curve shall be shifted to the right a sufficient amount to pass through the origin?

We have found above that points lying just to the right of the curve for $\alpha=0$ represent positive values of α while points lying just to the left of the curve represent negative values. In Fig. 5 the origin is a point lying just to the right of the locus of $\alpha=0$ and hence a positive

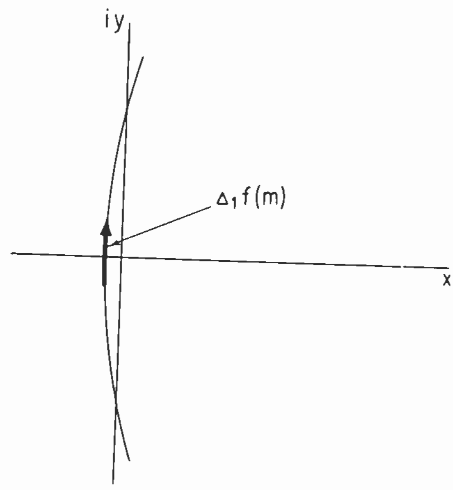


Fig. 5—Segment of typical curve representing locus of zero decrement.

value of α will satisfy (32). This means that transient currents will build up in amplitude so that oscillations will start when the curve $f(0+i\beta)$ is located as in Fig. 5. If the origin lay just to the left of the curve, then (32) would be satisfied by a negative value of α and transients would die out.

These conclusions apply only so long as the origin lies near enough to the curve so that no singular or branch points are contained in the region containing the origin and the neighboring segment of the curve. The only way in which we can assure ourselves that this condition is fulfilled is to find a value of the external impedance Z_0 such that the curve for $\alpha=0$ passes exactly through the origin. Then, when this is done the curve may be shifted slightly to right or left by placing a noninductive resistance in parallel with Z_0 so that the value of resistance which will just allow oscillations to start is thus determined.

In the actual application of this procedure to the solution of (32) a simplification occurs when it is realized that the impedance Z_0 is under our control to the extent inherent in the properties of passive circuit elements. Hence, since the remaining terms are determined by the properties of the vacuum tube, it will be convenient to plot the graph of the portion of the equation

$$F(m) = \frac{1}{z_c} + \frac{\mu}{Z_p} \tag{37}$$

in the ordinary manner, regarding m as equal to ip. When this is done it will be relatively easy to determine what properties of Z_0 are necessary in order to cause the curve to pass either just to the right or just to the left of the origin and thus to find whether oscillations can be produced with possible values of Z_0.

The values of the real and imaginary components of z_c have been computed and are given Fig. 1 of the previous paper for the condition of no space charge. Likewise Z_p is given in Fig. 10 of the same paper. This leaves only the computation of μ to be performed. Writing (37) in terms of the symbols in (23), we have for complete space charge

$$\frac{1}{z_c}+\frac{\mu}{Z_p}=\frac{\eta^2\zeta^2}{12r_p(P+iQ)}\left[-1+\frac{1}{D}(KA-IB)-i\frac{1}{D}(IA+KB)\right] \tag{38}$$

and for no space charge

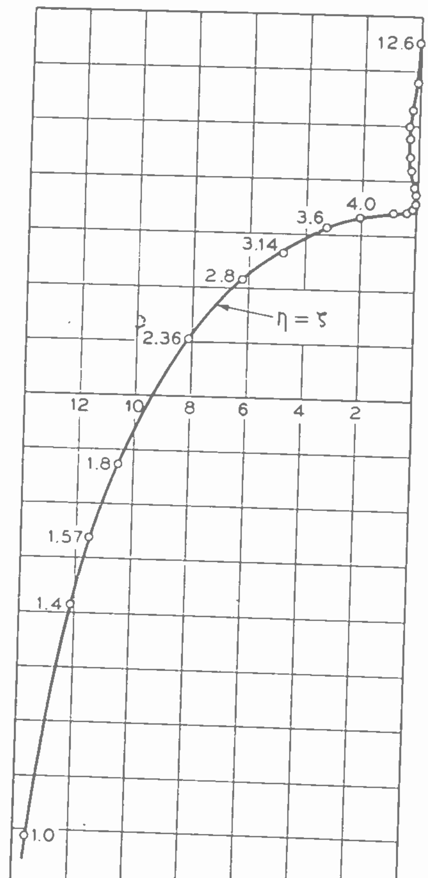
$$\frac{1}{z_c} + \frac{\mu}{Z_p} = pC\left[-\frac{FA+HB}{D} + i\left(1 - \frac{HA-FB}{D}\right)\right]. \tag{39}$$

The latter of these is the more easy to visualize. Here all of the symbols are intrinsically positive at all frequencies, whence it follows that the real part of $F(ip)$ is always negative. Thus it is clear that positive values of the resistive component of Z_0 will be required to shift the curve toward the origin. This, at least, is satisfactory since a negative resistance requirement for Z_0 would have been awkward to obtain practically, and moreover, experiment has shown that negative external resistances are not required for positive grid oscillators.

The required form of Z_0 may be derived by reference to Fig. 6 which shows the real part of (39) plotted against the imaginary part for two ratios of transit angle. It will be seen in both cases that an anti-resonant circuit will be satisfactory in shifting the curves toward the origin. The impedance of a moderately good tunable circuit may be written

$$\frac{1}{Z_0} = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right). \tag{40}$$

The plot of this function would be a straight line parallel to the imaginary axis located at a distance $1/R$ to the right of it. Thus at the resonant frequency where the curve crosses the axis of reals, the addition of Z_0 to the graph of Fig. 6 would move the segment of the curve between $\zeta = 1.8$ and $\zeta = 2.36$ to the right by an amount proportional to $1/R$. Also, the greater the antiresonant impedance R, the less



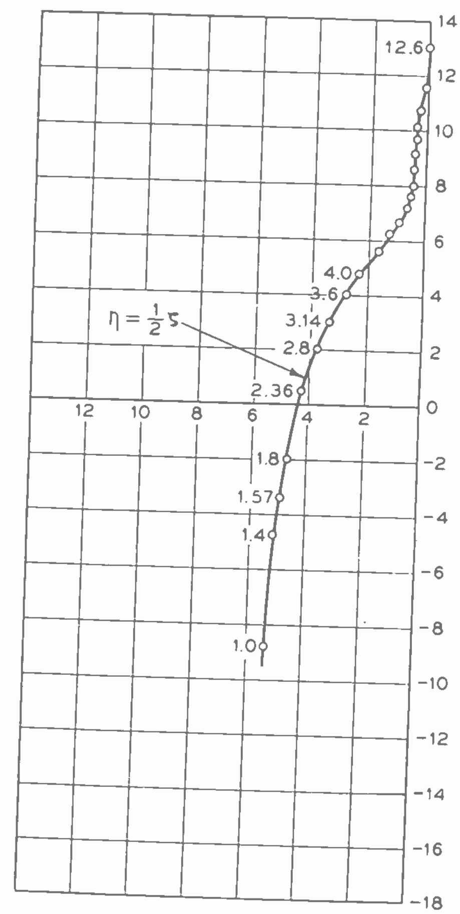


Fig. 6—Zero decrement locus for positive grid triode when $\eta = \zeta$.
Fig. 7—Zero decrement locus for positive grid triode when $\eta = \frac{1}{2}\zeta$.

would the curves of Fig. 6 be shifted. It follows then, that for oscillations

$$R > \frac{C_p}{C_c} \frac{r_p}{2.2} \qquad \text{when } \eta = \tfrac{1}{2}\zeta \qquad (41)$$

and,

$$R > \frac{C_p}{C_c} \frac{r_p}{4.65} \qquad \text{when } \eta = \zeta \qquad (42)$$

where C_p and C_c are the plate-grid and cathode-grid capacities, respectively, measured when the vacuum tube is cold, that is, when no electrons are in the interelectrode spaces.

As explained in the earlier paper, the condition that $\eta = \zeta$ is a limiting condition that cannot be attained in actual practice because the grid captures a large number of electrons in their passage through its mesh. The curve for $\eta = \frac{1}{2}\zeta$ corresponds to a condition that can be duplicated experimentally, and shows the possibility of oscillations when the transit angle between grid and plate is about two radians. This indicates a lower frequency than the erroneous curves of the original paper, and corresponds better with experimental conditions, although it must be remembered that these equations apply directly to parallel plane structures whereas the cylindrical form is more often used in practice. This difference will affect the conditions between cathode and grid to a much greater extent than conditions between grid and plate, as the relative radii of grid and plate cylinders are usually more nearly equal than those of cathode and grid. The equations will be affected mostly through the μ-factor in (29) or the velocity $(M+iN)$ in (22). They apply only when all electronics hit the plate.

The same procedure as that just outlined could be carried through in connection with (38) instead of (39). However, the possibility of obtaining new information from carrying out the lengthy calculations does not at the present appear to warrant the time which would be required.

Finally we come to discuss the analysis of conditions when electrons are turned back at a virtual cathode and return again toward the grid. Benham called the writer's attention to the fact that (40) in the original paper does not follow from (39). To see why this is, we have to remember that u_a refers to an electron traveling in one direction while u_b refers to another electron traveling in the opposite direction. Hence the relation

$$\frac{du_a}{dt} = \frac{du_b}{dt}$$

holds for a single value of t only; namely, the instant when the two electrons are passing each other. Hence, the equation is not a general one and cannot be integrated blindly as was done in the original paper.

This means that analysis of the negative plate Barkhausen oscillator is still lacking, and that (49), (52), and (53) together with Figs. 14 and 16 of the original paper are not correct. It is greatly to be regretted that these errors have occurred, particularly as they tend to weaken the presentation of the fundamental method of approach. Revision of the faulty equations is now in progress and it is hoped that the correct solution for the negative plate tube can be presented at an early date.

Proceedings of the Institute of Radio Engineers
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A METHOD OF MEASURING NOISE LEVELS ON SHORT-WAVE RADIOTELEGRAPH CIRCUITS*

By

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Summary—*This method is essentially a means for continuously measuring the percentage of elapsed time during which the noise level exceeds a certain predetermined voltage level. The predetermined voltage level may be made any level at which data are desired. The device consists of a biased tube circuit such that when input signal exceeds a certain level, the output tube passes a normal value of plate current through a ballistic meter of slow period. If the input continuously exceeds the threshold level, the ballistic meter reads "100 per cent." The data obtained are particularly useful in connection with the engineering of short-wave radiotelegraph circuits, since these are usually operated through vacuum tube relay devices at the receiving end, set to register normal output whenever the signal "marks" at any level above a controllable threshold value.*

THE general function of noise data is to enable a prediction to be made of the signal strength required to afford a given quality of service, or conversely, to enable prediction of the quality of service possible with a given signal level available. In nearly all cases of telegraphic communication, intelligence is transmitted by on-off keying of the transmitter carrier. The carrier is broken to form dots and dashes of full radiation separated by spaces of no radiation. In the following discussion, "carrier-on" will be spoken of as "mark" and "carrier-off" will be spoken of as "space." The durations of these elements of mark and space will be inversely proportional to the rate at which intelligence is transmitted.

Radio-frequency noise energy passing through the receiving system may cause false "mark" to be registered where there should be "space" or may in some cases oppose the signal to cause "space" to appear where there should be "mark." If these false elements are present, the speed of transmission may be slowed down to a rate at which the lengths of the shortest signaling elements are two or three times as long as the false elements. If the false elements are relatively long but occur infrequently, it will be preferable to transmit at a higher rate and send each word or message twice in succession. It accordingly follows that noise data indicating the lengths and frequency of occurrence of false elements will be useful in predicting the effects of noise on circuit quality.

* Decimal classification: R270. Original manuscript received by the Institute, May 15, 1934; revised manuscript received by the Institute, July 9, 1934.

A device for indicating per cent "mark" together with a means for measuring durations of "marks" will therefore yield useful data. In Fig. 1 is shown a circuit which will indicate per cent "mark." With

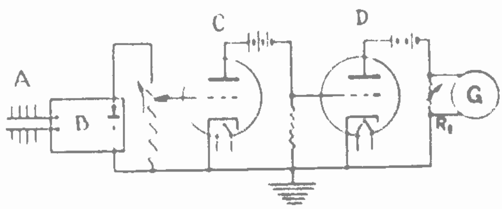

Fig. 1—Schematic diagram circuit for measuring per cent mark.

no signal (space element) the plate of tube C draws current through a resistor so as to place a large negative bias on the grid of tube D. For this condition, tube D draws zero plate current. Any value of rectified signal (mark element) above a certain value will bias the grid of tube C negative beyond cut-off. For this condition, tube C draws no plate current and the grid bias of tube D is such as to allow plate current to flow. This flow of plate current actuates ballistic galvanometer G. The period of galvanometer G may be made quite long so as to make the reading a fairly steady indication of average percentage "mark." The calibration of the system will be determined by the amount of plate current drawn by tube D for "mark" and by the value of the resistor R_1.

By some additions to this circuit, we can also provide means for measuring durations of the "mark" elements. The way this may be accomplished is shown in Fig. 2. This circuit functions the same as

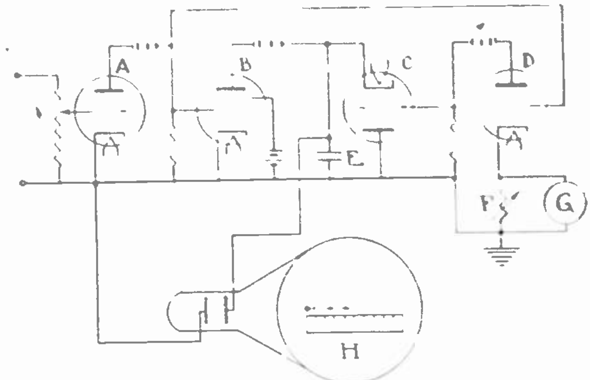

Fig. 2—Circuit for measuring per cent mark and duration of impulses.

the circuit of Fig. 1 to cause tubes B and D to draw plate current whenever rectified signal above a certain level flows through the input resistor. Plate current through tube D causes galvanometer G to read "per cent mark." Plate current is at the same time drawn by tube B.

This may be a screen-grid tube having the characteristic of drawing constant plate current over a wide range of plate voltage. Hence, the voltage drop across capacitor E will increase at a constant rate during

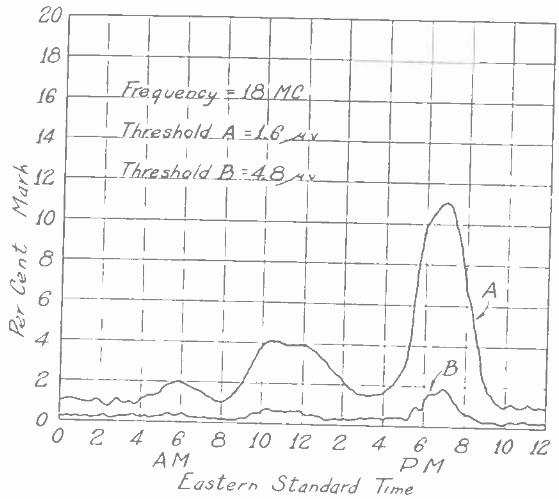

Fig. 3—Diurnal variation of per cent mark due to noise.

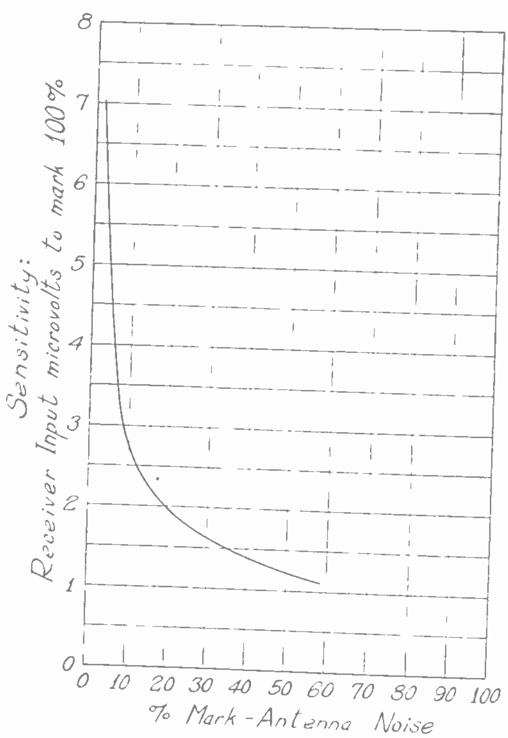

Fig. 4—Per cent mark versus receiver sensitivity.

the length of time that tube B draws plate current. Consequently, the voltage drop across the capacitor reaches a value proportional to the duration of the "mark" element. The peak values reached for the successive mark elements are currently indicated on the cathode ray tube H. At the end of each mark element, the charge on the capacitor is discharged through tube C which is connected so as to become conductive during all "space" elements. The scale on the cathode ray tube may be calibrated to read milliseconds, the sensitivity being governed by the capacitance of the condenser and by the operating constants of tube B.

These circuits may be used with automatic recording instruments. Such combinations may be applied to a variety of research problems. Fig. 3 indicates average results obtained in some test runs made with the circuit of Fig. 1 and a recording galvanometer. The antenna used was situated at Riverhead, New York, and was sharply directive for reception from the northeast.

By taking "per cent mark" readings for various values of receiver sensitivity, interesting information as to the nature of noise voltage may be obtained. Fig. 4 indicates the results obtained in a typical set of such readings. It will be noted that only two per cent of the time did the noise voltage exceed 7 microvolts at the receiver input. The noise voltage was found to exceed 1.5 microvolts 35 per cent of the time, and 1.2 microvolts 55 per cent of the time. This curve indicates that signal voltage greater than 8 microvolts would be required to obtain a radiotelegraph channel free from noise under this set of conditions.

Selected References

(1) Karl G. Jansky, "Directional studies of atmospherics at high frequencies," Proc. I.R.E., vol. 20, pp. 1920–1932; December, (1932).

(2) R. K. Potter, "High-frequency atmospheric noise" Proc. I.R.E., vol. 19, pp. 1731–1765; October, (1931).

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THE IONIZING EFFECTS OF METEORS*

By
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Summary—*It is shown that a meteor of average velocity has enough energy to cause ionization of atmospheric gases by impact. Recent experimental work by Frische and others on collisions of ions is interpreted as supporting the hypothesis that meteoric collisions do result in ionization. The afterglow of nitrogen is considered as a possible example of the process by which a meteor train remains glowing for a period of minutes and the coincidence of the region in which such trains are generally observed and of the E region of the upper atmosphere is pointed out. The spectra of bright meteors, while not showing atmospheric lines, are shown not to be inconsistent with the above hypothesis.*

The behavior of the transatlantic short-wave radio telephone circuits of the American Telephone and Telegraph Company, during 1930, 1931, and 1932, is examined for possible meteoric effects. It is concluded that, in general, a rather large shower is necessary to affect them appreciably. This was to be expected since these circuits are normally under a continuous bombardment by random meteors. It seems possible that a certain degree of the variability (rapid fading, etc.) of received signals over such paths is due to this bombardment.

Results of radio pulse studies of the upper atmosphere, particularly by Schafer and Goodall, which are strongly suggestive of meteoric ionization, especially at times of special meteoric activity, are (1) sudden increases in ionizaton in the E region lasting for a period of minutes or less, and (2) increases of longer duration with maxima coincident in time with those of observed meteoric activity. Such tests made during the Leonid shower of November, 1932, were successful in correlating sudden increases in ionization in the E region with the visual observations of a number of bright meteors passing overhead. For the brightest meteor observed, the ionization increased to a value in excess of summer noon conditions.

It is pointed out that meteoric showers might take place in the F region which would be unobservable by ordinary visual means.

Taking into account the energy spent by the meteor in ionization, a mass for the brightest meteor, for which correlative data was obtained is roughly calculated to be 0.3 gram. Its estimated brightness was −1 magnitude.

The recombination coefficient at the height of the E region is calculated from the rate of decrease of ionization after the passage of a meteor, to be less than 0.2×10^{-8} *cubic centimeters per second.*

Introduction

The first suggestion that meteors affect radio waves was published by Nagaoka[1] in 1929 in the *Proceedings* of the Imperial Academy, Tokyo. The possibility that the passage of meteors through the upper atmosphere might effect radio transmission was con-

* Decimal classification: R113.5 Original manuscript received by the Institute, October 5, 1934.

[1] *Proc. Imp. Acad.* (Tokyo), vol. 5, p. 233, (1929).

sidered by the writer in 1930, who was then unaware of Nagaoka's suggestion, and a preliminary review of meteoric and available radio data was published,[2] which indicated that the passage of a meteor of sufficient energy might produce enough ionization through the upper atmosphere to affect radio transmission. According to Nagaoka's hypothesis the passage of meteors should reduce the electron content of the region through which they pass by a clean-up effect, while according to the hypothesis of the present author an increase in ionization should result from the dissipation of the energy of the meteor.

The present investigation has proceeded along two general lines: the search for evidence of ionization by meteors from meteoric and physical data and the search for evidence of such ionization from radio data. The results of these are presented in Parts II and III. In Part I a very brief résumé of the physical nature of the properties of the upper atmosphere important in radio transmission is given. Part IV is concerned with further discussion of various phases of the problem.

Part I. The Ionosphere

The general distribution of ionization[3] in the atmosphere is as shown in Fig. 1. At the latitude of Deal, New Jersey (40° 15′), it has been found that during a severe magnetic storm, the regions are greatly

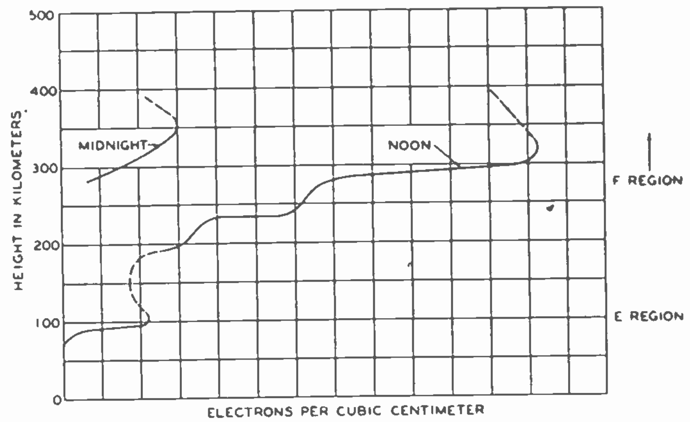

Fig. 1—The value of the maximum in the E region is of the order of 10^6 electrons per cubic centimeter and in the F region from 10^6 to 10^7.

disturbed and it is not possible to differentiate clearly between them. Besides this the ionization may be increased. The well-known correlation of the appearance of the aurora at such times and the coincidence of its position with that of the ionosphere indicate that the light of the

[2] *Phys. Rev.*, vol. 37, p. 1668, (1931).

[3] For the fine structure of the ionosphere see Schafer and Goodall, *Nature*, vol. 131, no. 3318, p. 804; June 3 (1933).

aurora is due to this abnormally large ionization of the ionosphere. In view of this it seems probable that the faint permanent aurora which is known to be present at any place and on any night is also the glow of the ionized gases of the ionosphere. Slipher's[4] recent work on the light of the night sky, in which he found great numbers of radiations throughout the whole spectrum, confirms this idea. Until measures of the heights at which this latter type of aurora occurs are obtained, however, such a coincidence must remain speculative.

Part II. Evidence of the Ionizing Effects of Meteors From Meteoric Data

According to the old theory which is found in text books, a meteor becomes heated by friction with the gases of the atmosphere until its surface liquefies and vâporizes, thus surrounding it with a luminous gas mantle. If small, it will be entirely consumed in this manner. The only effect on the atmospheric gases postulated by this theory is that of heating. At the time that this theory was first proposed ionization as such was unknown. One of the purposes of this paper is to show that when meteoric data are reconsidered in the light of our present knowledge, strong evidence is found that meteors act as ionizing agents in the upper atmosphere and in particular in those regions which are of especial importance to radio transmission.

This evidence may be had from two sources: (1) theoretical considerations based on undisputed data concerning meteors, and (2) observational data, the character of which indicates ionization phenomena.

Theoretical

Consider a meteor entering the atmosphere with a velocity of 40 kilometers per second. It will suffer collisions with atmospheric molecules, the energy of which will depend on the mass of the molecule as well as on this velocity. It may readily be shown that the energy of such an impact is more than sufficient (1) to dissociate or ionize the molecule, (2) to ionize the constituent atoms, or (3) to release a molecule from the body of the meteor. For instance the energy of impact of a molecule of nitrogen would be 230 volts, of oxygen 262, of helium 33, and of hydrogen 16. The ionization potentials of these atoms are: nitrogen 14.48, oxygen 13.56, helium 24.47 and hydrogen 13.54. The second and third ionization potentials are 34.94 and 54.88 for oxygen and 29.50 and 47.4 for nitrogen.[5] Ordinary lattice and dissociation potentials are even less than the ionization potentials.

[4] Paper presented at meeting of Am. Phil. Soc., April, (1933).
[5] C. E. Moore, "Multiplet Table," Princeton, (1933).

It remains to be shown that molecular and atomic collisions at such velocities actually do result in ionization. Unfortunately, we do not have any experimental data bearing directly on this problem. In an investigation of positive ion impacts in various gases, Frische[6] has roughly determined the effective energy at which ionization sets in for several gases. For nitrogen this energy value was of the order of 170 volts which is less than that given above. Thus we may interpret these data as indicating that meteoric impacts with nitrogen and also with oxygen molecules (since their masses are not greatly different) would result in ionization of these gases.

Lindemann and Dobson,[7] Sparrow,[8] and Maris[9] have formulated theories attempting to describe in greater detail the physical nature of meteor phenomena. These have departed from the older "textbook" theory in the following ways: According to both the theory of Lindemann and Dobson, and that of Sparrow, the meteor begins to shine when the temperature of its surface becomes high enough so that copious evaporation takes place. The former assume that such heating can only be due to a gas cap formed in front of the meteor; that heating of the meteor is inappreciable before this cap forms. Sparrow assumes that direct impact of the air molecules on the meteor itself will cause sufficient heating and that the assumption of a gas cap to explain such heating is unnecessary. According to Maris' theory, the direct impacts of air molecules on the surface of the meteor cause miniature explosions liberating a number of molecules of the meteor at each impact. The subsequent collisions of these liberated molecules, with air atoms or molecules, result in atomic or molecular excitation which gives the light of the meteor trail.

A good deal of controversy has arisen over some of the assumptions and conclusions of these theories, none of which is of particular concern in this discussion. The one point on which they all agree is that the impacts at the meteor's surface will be inelastic and that ionization will occur. Maris[10] states . . . "the energy of complete inelastic impact of an air-molecule with a meteoric mass is generally over 150 volts, which is equivalent to the ionization potential of very soft X-ray radiation; we would expect then a large percent of the radiation of the meteor trail to be in the ultra-violet or even soft X-ray region as suggested by Lindemann and Dobson."[11] If this assumption is

[6] *Phys. Rev.*, vol. 43, p. 160, (1933).
[7] *Proc. Roy. Soc.*, vol. 100, series A, p. 411, (1922).
[8] Ap. J., vol. 63, p. 90, (1926).
[9] *Terr. Mag.*, vol. 34, no. 4, p. 309, (1929).
[10] *Loc. cit.*, p. 315.
[11] *Loc. cit.*, p. 418.

justified, such radiation would provide an additional means of ionization since quanta in this region are exceedingly effective for the ionization of the air. Maris[12] further concludes, "thus less than 0.02 of the energy of the meteor will be expended directly against air resistance and the remaining 0.98 must be spent as suggested by Lindemann and Dobson[13] in ionization and excitation-impacts beyond the main mass of the meteor which lead to radiation of light."

Observational

Meteor Trains

It is well known that occasionally a meteor will leave a glowing train which remains in the sky for a period ranging from a few seconds to more than an hour. Considering, for the present, only those trains which are observed at night, the following facts have been established: They are self-luminous; they range from ten to thirty miles in length and are often several kilometers or more in diameter; they expand rapidly and, as mentioned above, may persist for a good many minutes. Obviously they cannot be explained on any basis of high temperature; it would be impossible for such an amount of material in any state to remain incandescent at the conditions encountered in the upper atmosphere for such long periods.

The mechanism of the process involved must be one in which energy is stored for an appreciable length of time in the gas and then emitted as light. The only such mechanism that is known, which is not inconsistent with the facts, is the storage of this energy in the atoms or molecules by excitation or ionization. When they return to the normal neutral state, light is emitted. An experimental fact lends particular support to this assumption. It is the production in the laboratory of a similar phenomenon (afterglow in air) by electrical discharge.

C. C. Trowbridge[14] has shown that meteor trains are similar to the afterglow in color, type of spectrum, rate of diffusion, and nature of decay; also that they are both produced at about the same gas pressures. He was able to obtain a duration of the afterglow of only 19 minutes. Knipp[15] with nitrogen alone has recently extended this to 187 minutes.

According to Ruark and Urey[16] the nitrogen afterglow spectrum is due to the neutral molecule. They conclude that the active nitrogen is probably composed of both unexcited and metastable atoms, mole-

[12] *Loc. cit.*, p. 316.
[13] *Loc. cit.*, pp. 418 and 423.
[14] Ap. J., vol. 26, p. 114, (1907).
[15] *Phys. Rev.*, vol. 39, p. 181, (1932).
[16] "Atoms, Molecules, and Quanta," p. 512, (1930).

cules in a metastable state, and a large amount of ordinary nitrogen. None of these are ions; i.e., none has actually lost an electron, but such states differ from the ionic state only in the degree of excitation. The afterglow represents merely a by-product of the intense ionization of the primary discharge, without which it has never been produced. Reasoning along these lines, if the train of a meteor is identical with the afterglow observed in the laboratory, its presence may be taken as an indication of an intense ionization which fostered it.

It seems possible that the train may be due partially to the afterglow and partially, at least in its first stages, to the emission from ions which have been slow in recombining.

One of the most important results of Trowbridge's investigation was the discovery that the trains were produced only in a particular region or layer of the upper atmosphere. The mean height of this layer was found to be 87 kilometers. None were observed to extend above 103 kilometers nor below 70 kilometers. Moreover the longest train observed was the nearest to being horizontal; i.e., the exciting meteor had traveled the greatest distance inside this layer. At the time of this investigation the ionosphere as such was unknown (though the suggestion of a conducting upper region had been made). In the light of our present knowledge, however, it is at once apparent that this is the *E region or lower layer of the ionosphere.* This coincidence is, in itself, good evidence that meteor trains are ionization phenomena.

Little is definitely known about the spectra of meteor trains. Trowbridge reviewed all of the meteor spectra data available up to the time of his death in 1918. His results and conclusions were published posthumously.[17] From visual spectroscopic observations he was able to show that the spectra of meteor trains were not unlike those of the afterglow in various gases. These visual observations are of course rather unreliable, the comparisons being made from memory.

It is probable that the spectrum of the light given off during the passage of a meteor may be quite different from that of the glowing train which remains. Thus the light of the meteor itself might be due mainly to the bright mantle of vapor closely surrounding it, which distills off the meteor nucleus, while the train is of larger dimensions, less bright, and due mainly to the ionization of the atmosphere in the neighborhood of the trail. In support of this hypothesis are a number of observations of trains changing color. Unfortunately the pure spectrum of a train taken independently of that of the meteor has never been photographed.

Trains seen in daylight are apparently due to a different cause.

[17] *Nat. Acad. Sci.*, vol. 10, p. 24, (1924).

Trowbridge[18] shows that the heights at which they usually appear (from 40 to 65 kilometers) are below the region in which the night trains are seen. They have the appearance of smoke so that there is good reason to believe that they are due to the reflection of sunlight by dust resulting from the disintegration of the meteor.

Meteor Spectra

The spectra of very bright meteors have been obtained in a number of cases by means of objective prisms. These all consist of bright lines and show little or no continuous emission—usually none at all. The line spectra may differ greatly from one to another.

In an investigation of all available photographically recorded meteor spectra, P. M. Millman[19] identified lines of iron, calcium, and ionized calcium, also magnesium and ionized magnesium, manganese, aluminum, and chromium. In none of the spectra was he able to identify lines due surely to the atmosphere. But as H. N. Russell[20] has pointed out the ultimate lines of the atmospheric gases are so far in the ultra-violet that this result is not surprising. Furthermore, if, as suggested above, the light of the small incandescent mantle of vapor is much more intense than the light of the larger train we should not expect the lines of the latter to show. Thus the fact that meteor spectra do not include lines due to the atmospheric gases is not necessarily inconsistent with the idea that these gases are ionized by the meteor. These spectra do prove that the elements of which the meteor is composed are highly excited and, in some cases at least, are highly ionized when they leave the main mass of the meteor.

Meteoric Glow

At times of great meteoric showers a number of observers have noted a curious glow in the sky. It has often been reported as being prominent in the region of the meteoric radiant. Challis,[21] however, records it on Nov. 13, 1866, as being "throughout the heavens." According to his observations and those of his assistants it was of the same appearance as the auroral light but was never accompanied by streamers. The earth's magnetic field as recorded at Greenwich was unusually quiet during the night.

Suppose that each individual meteor on its passage through the upper atmosphere left a train of ionized gas. For the fainter meteors these trains would have been invisible individually but taken as a

[18] *Pop. Sci. Monthly*, vol. 79, p. 191, (1911).
[19] *Annals Harvard Col. Obs.*, vol. 82, p. 6, (1932).
[20] Discussion at Meeting of A.A.S., Dec., 1932.
[21] Challis, M.N.R.A.S., 27, 75, 1867.

whole, the combined ionization might well have been great enough to produce the effect observed. This explanation was first suggested by Trowbridge. From his reasoning one would infer that the glow was restricted to the lower layer. This may not necessarily be true, for if a meteor can produce ionization in the lower layer it must, a fortiori, produce it higher up, before it has penetrated so deeply. Thus the glow is possibly due to excitation and ionization, present throughout the whole ionosphere.

PART III. EVIDENCE OF THE IONIZING EFFECTS OF METEORS FROM RADIO DATA

Let us calculate the order of magnitude of the ionic concentration which may be produced by a meteor, for comparison with the ionic concentration generally assumed in order to explain short-wave radio phenomena. If the mass of the meteor is taken as one gram, the velocity as 40 kilometers per second the path length as 200 kilometers, and the range of ionization around the path as one-half kilometer,[22] the concentration of ionization on the assumption that a large proportion of the energy of the meteor goes into ionization, would be of the order of 10^6 per cubic centimeter. The maximum ionic density deduced from radio data is also of the order of 10^6 electrons per cubic centimeter. Since there are something like 10^{13} molecules in each cubic centimeter at a height of 100 kilometers, only a negligibly small fraction need be ionized for maximum radio effects.

CORRELATIONS AT TIMES OF METEORIC MAXIMA

The data of the New York to London radiotelephone short-wave circuits for 1930, 1931, and 1932 were examined with particular regard to those times of the year when activity from a known meteoric radiant was to be expected. Since a magnetic disturbance even of moderate character is apt to affect these circuits adversely it was necessary to exclude all those days during which such a disturbance was in progress. No correlation was found[23] except possibly in the case of the Leonids which, being near the maximum in their 33-year cycle, were more than usually active.

[22] Meteor trains, several kilometers in diameter, have been seen a number of times.

[23] The disturbance of August 11 and 12, 1930, noted by E. Quack, *Elek. Nach. Tech.*, vol. 8, p. 46, (1931), and suggested by him as being possibly due to a meteoric cause was also observed in the above-mentioned data. However, on August 11 and 12, 1932, in spite of moderate magnetic activity on the 12th, the circuits were generally good throughout the day and night although the meteoric activity was not greatly different from that of the same period of 1930 according to visual observations.

It may be concluded that in general, for these circuits, a meteoric shower of good proportions is necessary for a major effect. This is not surprising considering that normally the number of random meteors passing through or near some portion of the path is great enough to provide continuous bombardment. Furthermore, the increase in the number of meteors at the times in question is generally only a few fold. Minor effects can only be determined by data taken over a good many years.[24]

Variability of Signals

If we assume with Shapley[25] that 10^9 meteors strike the earth each twenty-four hours, and that the radius of the influence of each extends to one-half kilometer from its path, we can roughly calculate the average rate at which the path of the transatlantic radio waves will be affected, i.e., will pass through the ionization or ionic turbulence produced by a meteor. Taking two thirds of the actual length of 5600 kilometers as being in the ionosphere this rate turns out to be 300 per hour or five per minute. On the assumption that each meteor introduces a certain amount of ionization, it is evident that under normal conditions some ionic turbulence is to be expected.

The amplitude of the received signals over this path varies continually with a normal rate of fading, according to Potter,[26] of from five to one hundred per minute. Often this rate increases to between 500 and 10,000 per minute. Now it is known that the received signal is generally made up of a number of components which have traveled over different paths. The rôle of the meteor in producing this fading is, therefore, not at all likely to be a direct one; the fading will generally result from variations of the interference pattern at the receiving antenna. These variations can only be produced by *changes* in the transmission medium; they are thus an indication of a degree of turbulence which is inconsistent with a strict interpretation of the classical idea, according to which the upper atmosphere exhibits isothermal equilibrium. We know from visual observations that meteors do introduce turbulence into the upper atmosphere.[27] It, therefore, seems possible that a certain degree of variability of the received signal is due to the influence of random meteors. There is no other cause of such turbulence for which we have any direct data.

[24] G. W. Pickard has made a study of five years reception data from WBBM and Nauen, Germany, with particular reference to meteor showers and hourly rates of observed meteors; Proc. I.R.E., vol. 19, p. 1166; July, (1931). Although his data seem rather meager for this kind of a correlation study, he believes that his results indicate a relation.

[25] Harvard Circular No. 317, (1928).

[26] Proc. I.R.E., vol. 18, p. 638; April, (1930).

[27] L. J. Wilson, *Publ. Leander McCormick Obs.* 2, Part 4.

Radio Pulse Studies

A preliminary perusal of radio pulse data taken by Schafer and Goodall and by others indicated that this line of investigation was a promising one. It was found that none of the earlier pulse measurements, so far as could be learned, were made at times of special meteoric activity. A series of tests were therefore planned for such times. The first was made on the nights of July 27–28 and 28–29, 1931. Subsequent tests were made during the Perseid and Leonid showers of both 1931 and 1932. Unfortunately, weather conditions were poor throughout most of the tests. On three of the nights, however, the sky was clear at least part of the time.

The transmitter and receiver[28] were operated by Messrs. Schafer and Goodall, respectively, while the meteor observations were made by the writer.

It is now well known that the upper atmospheric ionization often increases during the nighttime hours.[29] If the cause of such increases is meteoric in origin, one would expect that the effect would be particularly marked during meteor showers. This was actually found to be the case in the lower layer for all of the meteor showers during which tests were made. Moreover the greatest effects were observed during the Leonid showers of 1931[30] and 1932.

Evidence of Individual Effects

On several occasions during some of their earlier work, Schafer and Goodall observed rather sudden increases in the ionization in the E region at night which lasted for a comparatively short interval. Such a case is shown as the dotted dip in the curve of Fig. 2 at 7:40 P.M. This was an unusual occurrence and might be interpreted to mean that a meteor had passed directly overhead or nearly so at that time, leaving sufficient ionization to reflect the wave from this region. Then as recombination set in the ionization decreased until it was no longer sufficient to reflect the waves of the frequency used and the waves penetrated to the F region once more.

Throughout all of the meteor showers during which observations were made intermittent reflections of this type were observed. They were from virtual heights ranging from 100 to 200 kilometers and appeared and disappeared suddenly, lasting in general only for a few seconds. *They are apparently a definite characteristic of such observations made during meteor showers* and are not nearly so prevalent at other

[28] For description of method and apparatus see PROC. I.R.E., vol. 20, p. 1131; July, (1932).

[29] The after midnight dip of Fig. 2 is an example of this.

[30] PROC. I.R.E., vol. 20 p. 1941; December, 1932.

times. It seems reasonable to assume that when they are found at other times, they are due to random meteors such as may be observed on any night.

During the Leonid shower of 1931 this type of reflection was prominent throughout all three nights on which observations were made but was most frequent on the night of November 16–17, the night of maximum for the shower as determined visually.

The suddenness with which this type of reflection disappears may be explained by the character of the mechanism of recombination. The decrease of ionization is at first very rapid, gradually tapering off. Now the reflection of any particular frequency exhibits a threshold effect;

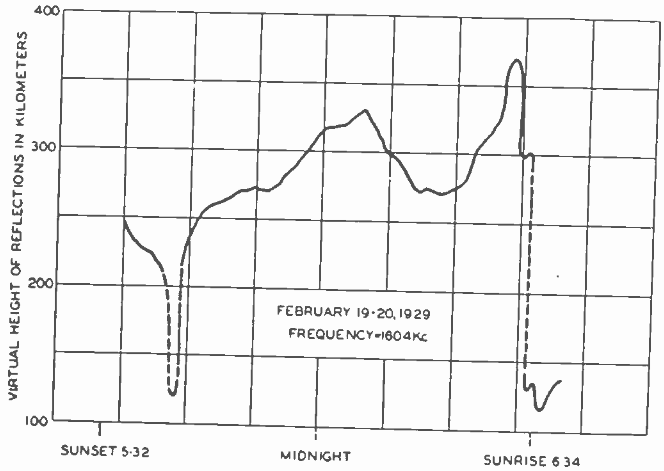

Fig. 2.

above a certain value of ionic density, reflection occurs and below it reflection does not occur. For an increase in ionization which raises it to a magnitude not greatly in excess of the threshold value, the drop back to this level will be correspondingly rapid. For a large increase, however, the ionic density should remain above the threshold value a good deal longer. We shall see further on that this is apparently so.

The most direct evidence of an ionizing effect due to a meteor would be, of course, the correlation of a sudden increase in ionization with the visual observation of the passage of the meteor overhead. We were able to obtain this correlation once on July 29, 1931, and a number of times on the nights of November 14–15 and 15–16, 1932.

Observations on July 28–29, 1931

This was the time of maximum for the activity from the δ Aquariid radiant. A few clouds appeared from time to time and an almost full moon further hindered the watch for meteors.

The transmitter and receiver were located one kilometer apart and the observer watching for meteors was situated on a small rise of ground between the two buildings. The three locations were connected with a system of microphones and loud speakers giving very efficient and continuous communication between all three points. Observations were started at 9:45 P.M., E.S.T. and were continued until daybreak at about 4:00 A.M. The watch for meteors was concentrated directly overhead.

The frequency used was 2398 kilocycles. Reflections from the F region were received throughout the night with the exception cited below. Reflections from the E region began to appear in the interval between 10:40 and 11:15 and continued throughout the night.

The first meteor to pass near the zenith was observed at 1:05. It was probably an Aquariid of about the second magnitude and passed, according to estimate, within five degrees of the zenith. Coincident with its appearance the reflections from the E region increased in strength and a multiple reflection from this region appeared. Meteors were then observed as follows:

1:12 A.M. 1^m, $Z = 10°$, probably an Aquariid.
1:18 A.M. 3^m, $Z = 0°$, red and faint.
1:21 A.M. 1^m, $Z = 5°$, possibly a Perseid.
1:25 A.M. 2^m, $Z = 5°$, possibly a Perseid.

Z is the zenith distance.

From 1:05 until 1:55 reflections from the E region were strong or very strong and a multiple reflection was present. The virtual height of the reflections during this period gradually decreased from 100 to 85 kilometers. These conditions indicate a strong increase in ionization. At 1:32 the ionic density rose so high that the waves were not able to penetrate the layer and the reflections from the F region disappeared. This blanketing action continued until 1:51 when the reflections from the F region began to appear again. The multiple reflection disappeared at 1:55 and from then on reflections from both regions were received in varying strength until dawn.

At 2:00 A.M. a third magnitude meteor passed close to the zenith but no marked effect was observed. A number of other meteors were recorded but these did not pass near the zenith and no marked effects attributable to them were observed.

Sudden increases in ionization were not recorded for each of the meteors noted above but this does not mean that such increases did not occur. After the appearance of the meteor at 1:05 the ionization was so far above the threshold value for the frequency used that it became rather insensitive to such increases. In such cases a frequency nearer the critical value would show the effects to a much greater degree. By the time of the tests in 1932 the apparatus had been improved, facilitating a quick change of tuning, so that the frequency could be kept more nearly at or near the critical value.

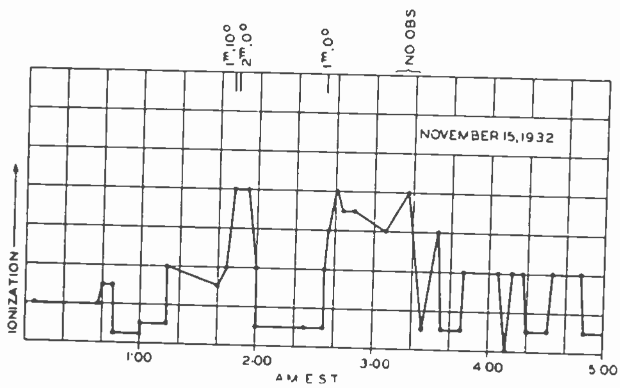

Fig. 3.

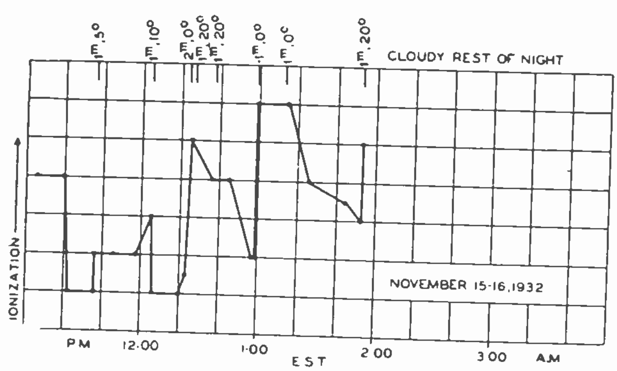

Fig. 4.

Observations on November 14–15 and 15–16, 1932

Weather conditions were generally favorable throughout the first night and for the first part of the second night. However the moon was nearly full so that the fainter meteors could not be seen.

The transmitter and receiver were located in adjoining rooms in the building and the meteor observer was stationed about twenty feet to the south of the building. Independent records were kept.

In order to determine the ionic density, the radio frequency of the pulses must be changed. This necessitates a retuning of both transmitter and receiver and for a wide range of frequencies a number of coils must be used which have to be changed in both transmitter and receiver. Since as near a continuous watch as possible was desired, the radio apparatus was set on an optimum frequency and left there, changing only at regular periods according to a prearranged program or when specific occasion demanded it. The ordinates of Figs. 3 and 4 are therefore only roughly proportional to the ionic density. They give, however, a fairly reliable indication of the relative variations of the ionization.

A total of thirty-one meteors was observed on the first night and nineteen on the second. Only those which passed within twenty degrees of the zenith are recorded on the figures. They are indicated at the proper times by the short lines at the top of each figure. The magnitude and the zenith distance for each is recorded. They are also listed below.

Time	Magnitude	Zenith Distance	Leonid?	Train
		November 14–15		
1:47	1	10°	yes	3 sec.
1:49	2	0	no	—
2:34	1	0	yes	1 sec.
		November 15–16		
11:35	1	5	yes	1 sec.
12:04	1	10	yes	5 sec.
12:24	2	0	yes	—
12:26	1	20	yes	—
12:36	1	20	yes	—
12:58½	−1	0	yes	3 sec.
1:12	1	0	yes	1 sec.
1:53	1	20	yes	—

As shown by the figures at the time of each of these meteors the ionization increased or tended to remain at a high value. The curve of ionization is not a continuous one so that the actual variation of ionization for the passage of each meteor could not be determined. Neglecting those meteors which passed twenty degrees or more from the zenith, for each of the remaining, the coincidence of increase of ionization with the observation of the meteor was within the experimental error (one-half minute) and in one case (mentioned below) in which the disposition of apparatus and observers was particularly favorable, this coincidence was as exact as could be determined. Furthermore *for all of the major increases in ionization a meteor was observed to pass nearly overhead.* There were a few moderate increases on the first night, November 15, for which a meteor near the zenith was not recorded. These may have been due to shooting stars of low visibility but relatively high ionizing power; the full moon prevented observation of the fainter meteors.

When the bright meteor was sighted at 12:58½ A.M. the meteor observer and the operator at the receiver called out simultaneously. As near as could be told, the ionization suddenly increased at exactly the same time the meteor was sighted.

The apparatus was set on the lowest frequency and at the above time strong lower layer reflections appeared. The frequency range was immediately run through to find the critical frequency. Eight minutes later the frequency run had been completed; the equipment was tuned to 8500 kilocycles and no reflections occurred though a minute or so earlier 7600 kilocycles had shown reflections. 7600 was then tried again and reflections came in strongly, indicating that the ionization was of the order of 10^6 electrons per cubic centimeter. *This value is greater than that usually observed at noon on a summer day*, i.e., when the sun's effect is greatest.

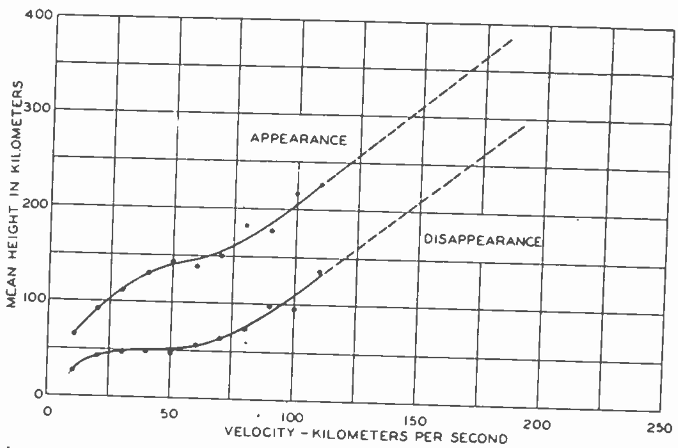

Fig. 5—Dependence of the observed heights of meteors on their velocities.

Part IV. Additional Deductions and Discussion

Meteoric Activity in the Upper Layer

The general relation that meteors of greater velocity appear and disappear higher in the atmosphere than those of lesser velocity is well established. The two curves of Fig. 5, showing this relation, were plotted from data published by Kopff.[31] They are for mean heights. There does not seem to be any reason why these curves should not be extrapolated to greater altitudes. On this basis, at heights of 250 to 300 kilometers (the F region of the ionosphere) only those shooting stars with velocities ranging from about 150 to 170 kilometers per second would be seen. The hazy appearance of telescopic meteors believed to be of high altitude is in keeping with the greater rarity of the atmosphere in this region. Because of this hazy appearance and their greater speed it seems likely that such meteors if of average size would be much harder to observe.

Shooting stars of these hyperbolic speeds have been observed tele-

[31] Handbuch der Astrophysik 4, 482, 1929.

scopically by the Harvard-Cornell expedition to Arizona.[32] The results of this expedition should shed much needed light on this subject. Preliminary reports have indicated that the number of such meteors is much greater than was expected.

It is not unlikely, therefore, that the F region may experience meteoric showers at random times which could only be observed visually by telescopic means but which would be very effective in disturbing radio transmission through this region. It is generally accepted that the F region is the more important one in short-wave transmission over long distances, hence such showers furnish a possible cause of the unexplained disturbances, mentioned in Part III, which do not coincide with magnetic disturbances.

The Size of Meteors

The calculation of the size of a meteor is usually based on the assumption that the major part of its energy goes into radiation. If some of the energy is stored up for an appreciable amount of time in atmospheric ionization and dissociation the usual computation of the size of a meteor will need revision. It would be possible by employing a plurality of radio pulse equipments to ascertain the extent as well as the density of the resultant ionization and from these data alone to arrive at a minimum size and mass for the meteor.

On the assumption that the refractive area effective in returning the waves back to earth was of the order of several square wavelengths, the minimum mass of several of the meteors for which data were available was calculated. This calculation was made as follows: Since the longest wavelength used was about 180 meters,[33] it was assumed that a cylinder of average radius equal to one-half kilometer and length depending on the observed length of trail was filled with gas, ionized to the extent that there were n electrons per cubic centimeter. (The value of n was estimated from the radio data.) Now knowing the amount of energy necessary to ionize one atom or molecule, a value for the total energy, spent in ionization along the path, could be calculated and set equal to the kinetic energy of the meteor. Knowing also the velocity of the meteor (if from a known radiant) a minimum mass could be computed. For the brightest one observed, on November 16, of magnitude equal to -1, this mass was 0.3 gram which is of the same order as those generally calculated on the old assumptions. Since the ionic den-

[32] S. L. Boothroyd, paper presented at meeting A.A.A.S., June 22, 1932.

[33] Since reflections were received on this wavelength immediately after the passage of the brightest meteor observed, the large area over which the ionization was apparently spread, in this case at least, could not have been caused by diffusion in the ordinary sense.

sity was not determined until eight minutes after the passage of the meteor and the assumptions were very rough, little significance can be placed on this estimate; the actual mass may have been much larger.

Recombination Coefficient

On the assumption that the sudden increase in ionization observed at 12:58½ A.M., November 16, 1932, was caused only by the meteor observed at that time, it becomes possible to set an upper limit to the recombination coefficient in the region through which the meteor passed; i.e., the neighborhood of the lower layer of the ionosphere. The ionic density was calculated (from the radio data) to be of the order of 10^6 electrons per cubic centimeter eight minutes after the passage of the meteor. From the formula for the ionic density n after time t,

$$n = \frac{n_0}{1 + \alpha n_0 t}$$

the ionic density n_0 at $t = 0$ is

$$n_0 = \frac{n}{1 - \alpha n t}.$$

Putting $n = 10^6$ per cubic centimeter, and $t = 500$ seconds, n_0 will be infinite when the recombination coefficient α equals 0.2×10^{-8} cubic centimeter per second. Therefore α must be smaller than this value. This agrees with the work of Lenz[34] according to which a value of the order of 10^{-10} at 0.01 millimeter pressure would be expected. An improvement in the mechanical details of the pulse equipments now in use will enable a rapid and continuous determination of the ionic density. This should enable an exact determination of α by this method.

Dust Effects

Though the several investigators, mentioned above, have concluded that the passage of a meteor will result in some degree of ionization, no one heretofore seems to have considered the possible effects of such ionization on radio transmission through the upper atmosphere. Hantaro Nagaoka,[1] recognizing the similarity of the mean height at which meteors are observed and the height of the lower layer of the ionosphere, proposed the theory that meteors will affect radio transmission by *reducing* the ionization already present. This action, he thought, takes place by an electron clean-up effect due to the dust resulting from the disintegration of the meteor.

[34] Zeits, f. Physik, 76, 660, 1932.

In his first paper he assumed a size for the dust particles of from 0.1 to 0.01 micron, but in a later paper[35] he estimates them to be of molecular dimensions. As stated above it is likely that only a very small fraction of the molecules at the heights of the ionosphere are ionized, so that it is not apparent why molecules left by the meteor should be more effective in attaching electrons to themselves than those already present. On the other hand, the evidence given above strongly supports the hypothesis that the passage of meteors through the upper atmosphere increases the ionization.

Acknowledgment

The writer wishes to express his thanks to Mr. J. C. Schelleng, under whose general supervision the experimental work was carried on, for the benefit of his advice and criticism, to Mr. J. P. Schafer and Mr. W. M. Goodall for their whole-hearted cooperation and for access to their data, and to Prof. H. N. Russell and Prof. J. Q. Stewart, who have contributed much through their many suggestions.

[35] Sci. Pap. Inst. Phys. Chem. Res. Tokyo, 15, 297, 183, 1931.

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ON THE IMPEDANCE OF A VERTICAL HALF-WAVE ANTENNA ABOVE AN EARTH OF FINITE CONDUCTIVITY*

By

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Summary—*The complex impedance of a vertical half-wave antenna located any distance above an earth of given conductivity and dielectric constant is calculated by the "induced electromotive force" method. Based on the assumption* $k_0 \ll |k|$ *(where* k_0 *and* k *are the wave numbers for the atmosphere and earth, respectively), the results are applicable down to about 10 meters for any earth except a very dry soil. The calculation is based on the Sommerfeld-von Hoerschlemann expression for the field of a dipole above a half space of arbitrary electrical character. After splitting the total impedance into three parts,* $Z = Z_1 + Z_2 + Z_3$, *the component* Z_1 *is shown to be the self-impedance of the antenna,* Z_2 *the mutual impedance between the antenna and its perfect image, and* Z_3 *an impedance component due to the finite conductivity of the earth.* Z_3 *is found to be proportional to two factors, one of which depends on the conductivity and dielectric coefficient of the earth and the wavelength and the other of which depends only on the ratio* h/λ, *where* h = *antenna height and* λ = *wavelength.* Z_3 *is put in a form suitable for the computation of any given case and curves are shown for four typical examples. For* $\lambda > 10$ *meters and all except very dry soil, the effect of the finite conductivity is quite small and the assumption, often made, of a perfectly reflecting earth thus is justified for a large number of cases. The impedance is, except for very short waves or exceedingly dry soil, substantially that obtained for a perfectly conducting earth. A principle of similitude is stated, in which two antennas over the same kind of earth and having equal values of* h/λ *have identical impedances.*

Introduction

A knowledge of the effective impedance of a given antenna as measured at its terminal connections is of fundamental importance in the design and operation of the antenna, its transmission line, and the associated power supply. The reduction of this class of problems to a more or less conventional type of electric circuit theory by Pistolkors,[1] Bechmann,[2] Carter[3] and other investigators[4–11] represents a decided step forward in the design of a suitable antenna and transmission line for both transmitter and receiver. While a part of the effective impedance, the so-called "radiation resistance," has been

* Decimal classification: R120. Original manuscript received by the Institute, November 10, 1933; revised manuscript received by the Institute, May 25, 1934.

[1] Numbers refer to Bibliography.

known for the commonly used antenna configurations for many years, and forms the subject of a large number of papers, it is only within the last four or five years that a satisfactory method of calculating the imaginary part of this impedance has been available. As a result of these recently developed methods, the complex impedance of almost any simple or composite antenna construction can now be calculated with relative ease, *provided* that the earth is either neglected entirely or assumed to be a perfectly reflecting plane of infinite extent.

The situation is quite different, however, when the earth (idealized as the half space beneath the antenna) is considered to possess a finite conductivity. For such a problem almost no progress has been made. Although the radiation *resistance* for this condition forms the subject of a group of several papers, no previous calculation of the *complex impedance* of an antenna above a finitely conducting earth has, to the author's knowledge, been published. In fact, nearly every question involving electromagnetic wave propagation about a finitely conducting medium has presented considerable difficulties and most such problems are still only partially solved. One of the earliest and perhaps the most fundamental attack on the problem of radio transmission above an earth of arbitrary electrical character was made by Sommerfeld[12,13] in 1909 and his expressions form the basis of the present work, although the problems considered are themselves different.[1–23]

The radiation resistance and the real and imaginary parts of the antenna terminal impedance are three significant quantities in any antenna design. The radiation resistance is a fictitious property attributed to the antenna which, when multiplied by the current squared, yields the value of the radiant energy of the system;* it is determined solely by the radiation component of the total field (assuming the field resolved into the usual radiation, induction, and static components). The antenna impedance is an actual, measurable quantity and expresses the relation between applied voltage and resultant current, both as to magnitude and phase, at the antenna terminals. The real part of this impedance is determined by the total power expended as radiation, heat, etc. There is experimental evidence that the heat loss in a practical antenna is negligible compared to the radiation loss; consequently, the real part of the terminal impedance is quite approximately equal to the radiation resistance and thus in turn depends only on the radiation field. In this paper, the power dissipated in heat within the radiator will be assumed negligible. The reactive part of the terminal impedance is of equal importance with the real part for the proper

* As generally used in antenna theory, the term "radiation resistance" does not include the effect of RI^2 losses in the antenna itself.

matching of antenna and line. Essential for its determination are the induction and static components of the field, while the radiation component contributes nothing to its value. This situation is quite similar to the reaction of a ground plane on the current distribution in a vertical antenna, where, similarly, the radiation field plays no part in the determination of the reaction on the antenna.

At present, there are two principle methods in use for calculating the power radiated by an antenna. The first is sometimes called the "Poynting vector method," and consists in determining the Poynting vector for every point of a surface inclosing the radiator and then integrating the normal component of energy flow over this surface. Usually, this process can only be carried through by taking a surface at a great distance from the radiator; as a result, it gives only the real part of the impedance, or the radiation resistance, and, as used in antenna theory, does not give a value of resistance including the effect of RI^2 losses in the conductors of the radiator. The important work of Strutt[16] for vertical and horizontal radiators above a plane having arbitrary electrical constants has this inherent limitation and thus gives no information as to the reactive component of the impedance. The second method has been called the "induced electromotive force method," as it is based on a calculation of the total effective back electromotive force appearing at the radiator terminals by the elements of the radiator itself; either the Hertzian vector or the scalar and vector potentials may be used and the calculation can be completed for most of the antenna configurations of practical interest. The great power of the induced electromotive force method lies in the fact that it gives the real *and* the imaginary components of the impedance and thus furnishes the basis for an antenna circuit theory, which, as just discussed, the Poynting vector method was unable to do.

In the present paper the induced electromotive force method will be used in connection with the expressions of Sommerfeld and von Hoerschlemann to derive an expression for the complex impedance of a half-wave vertical radiator located at any height above a finitely conducting ground. The electric constants of the earth and air are here contained in the wave number:*

$$k^2 = \frac{\epsilon\mu\omega^2 + i\mu\sigma\omega}{c^2}$$

where,

* The rational (i.e., 4π-free) mixed (i.e., electric quantities measured in electrostatic units, magnetic quantities in electromagnetic units) system of units will be used throughout this paper.

ϵ = dielectric constant (rational e.s.u.)

σ = conductivity (rational e.s.u.)

μ = permeability = unity

c = velocity of light ($3 \cdot 10^{10}$ cm per sec)

$\omega = 2\pi f$, f = frequency (cycles per sec)

λ = wavelength in vacuum (cm)

$i = \sqrt{-1}$

For the atmosphere the wave number has the value $k_0 = \omega/c = 2\pi/\lambda$.

The following analysis will be based on the underlying assumption

$$k_0 \ll |k|, \tag{1}$$

where k_0 refers to the atmosphere and k to the earth. While this assumption imposes a certain limitation on the range of application of the results, it is thought that a majority of the cases of practical interest are within the scope of the investigation. The inequality (1) may be written $N \gg 1$, where $N = |k|/k_0$. The meaning of the $\gg$ depends upon the accuracy desired; for most practical purposes, including those of the present paper, it will be sufficient to require that N be larger than or about equal to 10. Zenneck,[14] Fleming,[21] quite recently Feldman,[24] and others have given values for the electrical constants of different kinds of earth and water at short wavelengths; typical approximate values might be selected as follows:

	ϵ	σ
sea water	80	5×10^{11}
fresh water	80	6×10^{8}
wet soil	30	2×10^{9}
dry soil	7	1×10^{8}

The effect of the assumed inequality between wave numbers of atmosphere and earth may be considered as fixing a minimum wavelength below which the derived formulas will not be valid. It is important to examine the values taken on by N for several wavelengths and the above types of earth. The expression for N is

$$N = \left| \sqrt{\left[\epsilon^2 + \left(\frac{\lambda\sigma}{2\pi c}\right)^2\right]^{1/2}} \right|$$

and it is easily found

sea water	$\lambda = 1$ meter,	$N = 16.6$
	$\lambda = 0.1$ meter,	$N = 9.2$

wet soil	$\lambda =$ 100 meters,	$N = 10.5$
	$\lambda =$ 10 meters,	$N = 5.7$
	$\lambda =$ 1 meter,	$N = 5.5$
fresh water	$\lambda =$ 100 meters,	$N = 9.3$
	$\lambda =$ 50 meters,	$N = 9.0$
	$\lambda =$ 10 meters,	$N = 8.9$
dry soil	$\lambda =$ 1000 meters,	$N = 7.3$
	$\lambda =$ 100 meters,	$N = 3.0$
	$\lambda =$ 10 meters,	$N = 2.6.$

Thus, it is seen that the case of an antenna over sea water is covered quite well for wavelengths down into the decimeter region, while the cases of wet soil and fresh water are fairly accurately represented down to about 10-meter wavelengths; these regions include almost the entire wavelength range of contemporary importance. Only the case of a very dry soil is not well represented, although the results are still useful as a first approximation.

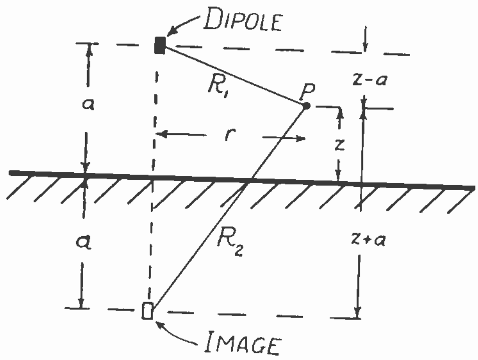

Fig. 1—Dipole above half space.

The Nature of the Field Above a Finitely Conducting Plane

According to von Hoerschlemann,[13] the field in the air produced by a vertical dipole located at a height a above the finitely conducting half space may be derived from the following Hertzian vector:

$$\Pi_0 = \frac{e^{ik_0R_1}}{R_1} - \frac{e^{ik_0R_2}}{R_2} + \int_0^\infty \frac{2k^2}{N} J_0(lr)e^{-\sqrt{l^2-k^2_0}(z+a)} l\,dl \tag{2}$$

$$N = k^2\sqrt{l^2 - k_0^2} + k_0^2\sqrt{l^2 - k^2}.$$

The significance of r, R_1, R_2, z, a may be immediately seen from Fig. 1, which presents the postulated geometric arrangement; J_0 is the zero order, first kind Bessell function and l is the variable of integration.

The electric field intensity, with which we are alone concerned in the following work, is then given by

$$E = k_0^2\Pi_0 + \frac{\partial^2\Pi_0}{\partial z^2}\cdot \tag{3}$$

It has been shown by Sommerfeld[12] that under the assumed condition $k_0 \ll |k|$ the second term in N may be treated as a correction term, since the principle contribution to the integral then comes from the pole at $l^2 = k_0^2$, and l^2 neglected in comparison with k^2. With the abbreviation $a = ik_0^2/k$ (a complex number) it follows that

$$\frac{k^2}{N} \cong \frac{1}{\sqrt{l^2 - k_0^2} - \frac{ik_0^2}{k}} = \frac{1}{\sqrt{l^2 - k_0^2} - a}\cdot \tag{4}$$

The negative sign before a is taken because $\sqrt{l^2 - k^2}$ is, even for $l = 0$, to be taken with a positive real part, and k is to be the root of k^2 with positive imaginary part.

Following Sommerfeld for the moment and considering only the third term of (2), which has now become

$$\Pi_{03} = 2\int_0^\infty \frac{1}{\sqrt{l^2 - k_0^2} - a} J_0(lr) e^{-\sqrt{l^2-k_0^2}(z+a)} l\,dl \tag{5}$$

a simplification may be made as follows: the product $\Pi_{03} \cdot e^{az}$ is formed and differentiated with respect to z. After multiplication with e^{-az}, the result may be written

$$e^{-az}\frac{\partial}{\partial z}(\Pi_{03} \cdot e^{az}) = -2\int_0^\infty J_0(lr) e^{-\sqrt{l^2-k_0^2}(z+a)} l\,dl. \tag{6}$$

Making use of the following relation

$$\frac{\partial}{\partial z}\left(\frac{e^{ik_0R_2}}{R_2}\right) = -\int_0^\infty J_0(lr) e^{-\sqrt{l^2-k_0^2}(z+)a} l\,dl \tag{7}$$

between the symmetrical solutions of the wave equation in cylindrical and spherical coördinates, (5) may be written at once as

$$\frac{\partial}{\partial z}(\Pi_{03} \cdot e^{az}) = 2e^{az} \cdot \frac{\partial}{\partial z}\left(\frac{e^{ik_0R_2}}{R_2}\right). \tag{8}$$

Now, if $F(x)$ is a function whose derivative is $f(x)$, the definite integral $\int_\Omega^x f(y)dy = F(x) - F(\Omega)$ exists, where Ω is an arbitrary constant and may conveniently be chosen so that $F(\Omega) = 0$. Performing this operation on (8) gives

$$\Pi_{03} \cdot e^{\alpha z} = 2 \int_{\Omega}^{z} e^{\alpha\zeta} \cdot \frac{\partial}{\partial \zeta} \left(\frac{e^{ik_0 R_\zeta}}{R_\zeta} \right) d\zeta \tag{9}$$

$$R_\zeta = \sqrt{r^2 + (\zeta + a)^2}, \quad R_2 = \sqrt{r^2 + (z + a)^2}.$$

The value of Ω required to make $\Pi_{03}(\Omega) \cdot e^{\alpha\Omega}$ vanish may be taken as $-\infty$, which value also insures the convergence of the inetgral; $\Omega = i\infty$ is another equally usable value. Partially integrating (9) gives

$$\Pi_{03} = 2\frac{e^{ik_0 R_2}}{R_2} - 2\alpha e^{-\alpha z} \int_{-\infty}^{z} \frac{e^{ik_0 R_\zeta + \alpha\zeta}}{R_\zeta} d\zeta \tag{10}$$

whereupon we are ready to consider again Π_0 in its entirety.

Substituting (10) into (2) gives the expression:

$$\Pi_0 = \frac{e^{ik_0 R_1}}{R_1} + \frac{e^{ik_0 R_2}}{R_2} 2\alpha e^{-\alpha z} \int_{-\infty}^{z} \frac{e^{ik_0 R_\zeta + \alpha\zeta}}{R_\zeta} d\zeta \tag{11}$$

which completely determines the field under the assumed condition $k_0 \ll |k|$. The three terms in (11) give rise to three separate components of the field, which will be denoted by the subscripts 1, 2, and 3, so that (11) may be abbreviated as

$$\Pi_0 = \Pi_1 + \Pi_2 + \Pi_3.$$

The first part Π_1 is immediately recognizable as a spherical wave originating at the dipole source and constitutes the direct or primary radiation. The second part Π_2 may likewise be recognized as a spherical wave, but it has the geometric image dipole as its source instead of the dipole itself; its amplitude and phase are identical with the primary wave. The third part Π_3 is a wave of more complicated type but it may also be associated with the image dipole source, as the vertical distance occurs as $(z+a)$, and thus appears to be emitted from the image. The second and third parts together constitute the secondary radiation. Obviously, the second part Π_2 is nothing more than a perfect image, such as would exist for an earth of infinite conductivity. Considered in this respect, the third part Π_3 may be thought of as a correction term which takes into account the finite conductivity of the earth. Our main concern here is with the impedance due to this component of the total field, as the other two components may be taken from previous calculations by Pistolkors, Bechmann, Carter and others. The problem of calculating the impedance of a vertical radiator over a finitely conducting earth has thus been resolved into the calculation of the three additive impedances due respectively to (a) the radiator itself, (b) a second radiator identical with the first and located in the image position, and (*c*) the effect of a more complicated type of radiation due to the finite conductivity of the earth.

While the preferable procedure would be to carry through the complete analysis for an arbitrary value of r and let r approach zero at the end, the resulting mathematical difficulties make it advisable to put r equal to zero at the start. However, a particularly careful study of the latter process must be made, as the third term in (11),

$$\Pi_3 = -2\alpha e^{-\alpha z} \int_{-\infty}^{z} \frac{e^{ik_0 R_\zeta}}{R_\zeta} d\zeta \tag{11a}$$

where,

$$R_\zeta = \sqrt{r^2 + (\zeta + a)^2}$$

has two singular points at $\zeta = -a \pm ir$ respectively. With $r = 0$, the above expression becomes

$$\Pi_3 = -2\alpha e^{-\alpha z} \int_{-\infty}^{z} \frac{e^{ik_0(\zeta + a)}}{\zeta + a} d\zeta \tag{11b}$$

which indicates a path of integration passing through the singular point at $\zeta = -a$. Expression (11b) may be used in place of (11a) provided certain conditions are satisfied; these conditions may be determined* from a consideration of the path of integration over the Riemann surface of the integrand together with the physical condition that Π_3 must vanish as $h \to \infty$.

The Impedance of a Half-Wave Radiator

Expression (11) determines the field of an infinitesimal dipole, while the radiator under consideration has a length equal to one half of the wavelength of the radiation. A theoretical determination of the current distribution for such a radiator above a ground plane has never been made, and consequently a particular form of distribution must be assumed. Wilmotte[25] has given very strong experimental evidence that the distribution is sinusoidal, at least within the accuracy of practical measurement. Based on this and other evidence, a sinusoidal current distribution will be taken; remembering that $k_0 = 2\pi/\lambda$, the current is seen to be defined by

$$I = I_0 \sin k_0 z \tag{12}$$

* The discussion of this situation has been omitted for the sake of economy in printing. Details of the integration of Z_3, to follow, have also been deleted from the final draft of this paper for the same reason. The author will be glad to supply this material to anyone especially interested therein.

where I_0 is the current maximum measured at A, Fig. 2. Since the Hertzian vector is proportional to the current amplitude, (11) is to be multiplied by (12). The tangential component of the electric field intensity on the wire axis may then be evaluated as the sum of three components $E_z = E_1 + E_2 + E_3$, corresponding to the three components of (11).

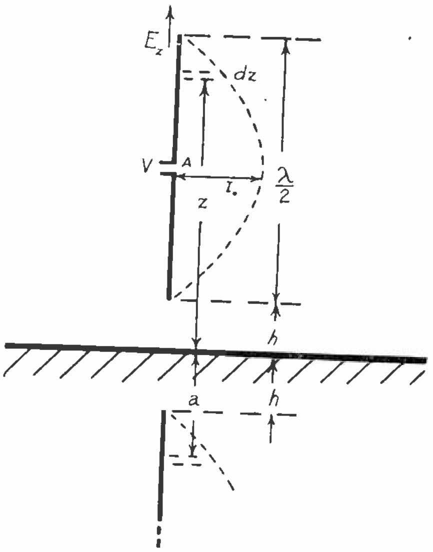

Fig. 2—Vertical half-wave antenna above half space.

The impedance of the antenna at the center terminals A may now be determined by an application of the generalized reciprocity theorem;[26,27,28] the form given by Carson is best suited to the nature of the problem at hand. Considering the case depicted in Fig. 2, it may be reasoned by the reciprocity theorem that if the current I_0 at the point A induces an electromotive force $E_z \cdot dz$ in the wire element dz, then an identical current I_0, if sent through the element dz, must induce the same voltage at A, both as regards magnitude and phase. But by the hypothesis (12), the actual current in dz is $I_0 \sin k_0 z$. If the electromotive force induced at A by the current in dz is denoted by dV, then the above argument may be expressed analytically as

$$\frac{E_z \cdot dz}{I_0} = \frac{dV}{I_0 \sin k_0 z}.$$

The total electromotive force induced at A is the integral of the contributions from each element, so

$$V = \int_h^{h+\lambda/2} E_z \sin k_0 z \, dz$$

V is actually a back electromotive force, and consequently the applied voltage necessary to maintain the assumed current distribution is $-V$. The ratio $Z=-V/I_0$ therefore defines the impedance of the radiator as measured at A. Since E_z is directly proportional to I_0, the impedance expression is correctly written as

$$Z = \int_h^{h+\lambda/2} E_1 \sin k_0 z dz + \int_h^{h+\lambda/2} E_2 \sin k_0 z dz + \int_h^{h+\lambda/2} E_3 \sin k_0 z dz \tag{13}$$
$$= Z_1 + Z_2 + Z_3.$$

In (13) the component of the total impedance defined by Z_1 is obviously the self-impedance of the antenna and has been calculated by a number of workers. The value of this quantity may therefore be taken from previous publications;[3] it is,

$$Z_1 = 73.3 + i42.6 \text{ ohms}. \tag{14}$$

The impedance Z_2 may be identified as the mutual impedance of two exactly similar half-wave colinear radiators. That Z_2 is really the mutual impedance, and not some other function, follows from the fact that the current in the image is exactly equal to the current in the antenna. This may be easily seen by writing Kirchoff's voltage equation for the antenna; in the notation of conventional circuit theory this gives

$$V = I_1 Z_{11} + I_2 Z_{12} + V_3.$$

But $I_1 = I_2$ identically, so

$$V = I_1(Z_{11} + Z_{12}) + V_3$$

and finally

$$Z = -V/I_1 = Z_{11} + Z_{12} - V_3/I_1.$$

Thus, $Z_2 = Z_{12}$ and the second contribution of the total impedance may be found from previously published formulas for the mutual impedance of two identical colinear radiators. The expression for Z_2 as given by Carter[3] is

$$\begin{aligned} Z_2 = & -15 \cos 2k_0h \Big[-2Ci[4k_0h] + Ci[2k_0(2h-l)] + Ci[2k_0(2h+l)] \\ & - \ln\left(\frac{4h^2 - l^2}{h^2}\right)\Big] + 15 \sin 2k_0h \Big[2Si[4k_0h] - Si[2k_0(2h-l)] \\ & - Si[2k_0(2h+l)]\Big] - i15 \cos 2k_0h \Big[2Si[4k_0h] - Si[2k_0(2h-l)] \end{aligned} \tag{15}$$

$$- Si[2k_0(2h + l)]\Big] + i15 \sin 2k_0h \Big[2Ci[4k_0h] - Ci[2k_0(2h - l)]$$

$$- Ci[2k_0(2h + l)] - \ln\left(\frac{4h^2 - l^2}{h^2}\right)\Big] \text{ ohms}$$

where,

$$Ci = \text{cosine integral*}$$
$$Si = \text{sine integral}$$
$$l = \lambda/2$$

and the curve of Fig. 21, page 1022 of his article shows this part of the impedance in a very clear way. Since $k_0 = 2\pi/\lambda$, Z_2 is a function of h/λ alone and the impedance of a vertical half-wave antenna above a perfectly conducting surface (equal to Z_1+Z_2) does not depend on either the wavelength or height alone, but only on their ratio. This fact will be used later to state a more general principle of similitude.

The Effect of a Finite Earth Conductivity on the Impedance

Expressions have now been found for the first two components Z_1 and Z_2 of the impedance. A completion of our knowledge of this function requires the evaluation of Z_3, which will now be undertaken. Z_3 is zero for infinite conductivity and becomes larger as the conductivity decreases, due to the exponential factor $e^{-\alpha z}$ in (11). The last integral of (13) must be evaluated to determine the exact way in which Z_3 changes with conductivity and antenna height, which in turn will require an evaluation of Π_3. It is thought convenient to summarize here the remaining calculation in the following three steps:

$$\left.\begin{array}{lll}
\text{(given)} & \Pi_3(z, a) = -2\alpha e^{-\alpha z} \displaystyle\int_{-\infty}^{z} \frac{e^{ik_0(\zeta + a) + \alpha\zeta}}{\zeta + a} d\zeta \\
\text{(I)} & \Pi_3(z) = \displaystyle\int_{h}^{h+\lambda/2} \Pi_3(z, a) \sin k_0(a - h) da \\
\text{(II)} & E_3(z) = k_0^2\Pi_3(z) + \dfrac{\partial^2}{\partial z^2}\Pi_3(z) \\
\text{(III)} & Z_3 = -\displaystyle\int_{h}^{h+\lambda/2} E_3(z) \sin k_0(z - h) dz
\end{array}\right\} \quad (16)$$

The actual details of carrying through the above indicated calculations are quite lengthy and contribute little to an understanding of

* For definition and tables see Jahncke-Emde, "Funktionen Tafeln mit Kurven."

the problem. For this reason, together with a desire for economy, the intermediate steps are omitted† here and a passage to the end result made. After completing the calculations in question, Z_3 becomes

$$Z_3 = -600\sqrt{4\pi}\cdot\alpha\cdot H \tag{17}$$

where,

$$H = -\left[Q(k_0[2h+\lambda]) + Q(k_0 2h) + 2Q\left(k_0\left[2h+\frac{\lambda}{2}\right]\right) - i4\pi\right]$$
$$+\frac{3}{4}e^{-ik_0 2h}\left[Q(2k_0[2h+\lambda]) + Q(2k_0 2h) - 2Q\left(2k_0\left[2h+\frac{\lambda}{2}\right]\right)\right]$$
$$+\frac{1}{2}ik_0 e^{-ik_0 2h}\left[(2h+\lambda)Q(2k_0[2h+\lambda])\right.$$
$$\left.+\,2hQ(2k_0 2h) - 2\left(2h+\frac{\lambda}{2}\right)Q\left(2k_0\left[2h+\frac{\lambda}{2}\right]\right)\right]$$

$$Q(x) = Ci(x) + iSi(x) + i\frac{\pi}{2}$$

which is the desired result. Z_3 is seen to be formed from the product of two functions, α and H. α contains the characteristics of the earth and is independent of the height of the antenna above earth. H, however, is a function of h/λ alone and does not depend on the material constants of the earth. Both α and H are complex quantities.

The function H has been computed* for the range of h/λ of principle interest and is shown in Fig. 3. The absolute magnitude decreases with increasing h/λ in an exponential type curve. The phase angle follows a straight line except for $h/\lambda < 1/16$, where it is slightly curved. Due to the rapid decrease of H with increasing height h, it is to be expected that Z_3 is only of consequence when the antenna is located a fraction of a wavelength above the earth. Expression (17) and the curve for H allow the component of the impedance due to the finitely conducting properties of the earth to be computed for any arbitrary type of earth (within the limitation $k_0 \ll |k|$) with ease, as the remaining computation consists of a simple multiplication. The factor $\alpha = ik_0/k$ depends on the conductivity and dielectric constant of the

† As mentioned before, these details will be gladly supplied to anyone especially interested.

* Computation involves taking the difference of numbers of almost equal magnitudes. Many significant figures must be carried along to obtain reasonable accuracy in the results. Interpolation from available tables (Jahncke-Emde) of sine and cosine integral functions does not give sufficiently accurate values; consequently, these functions were computed from series expansions for the desired values of the argument.

earth and on the wavelength of the radiation. Curves expressing α as a function of λ for the four previously selected sets of values of σ and ϵ are reproduced in Fig. 4. Because α covered such an enormous range

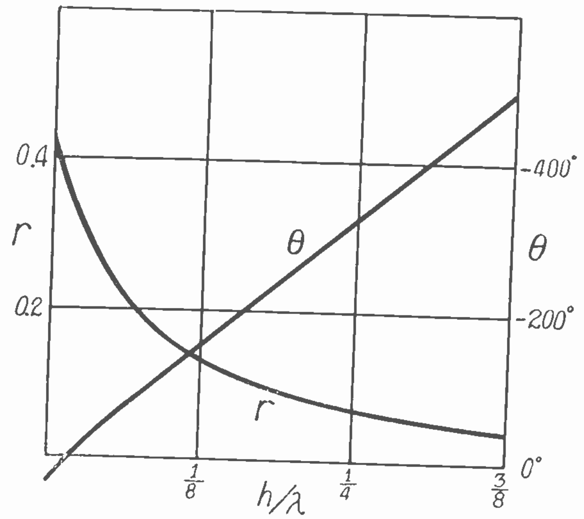

Fig. 3—Curves for absolute magnitude and phase angle of $H = re^{i\theta}$.

of values, the curves have been plotted to a logarithmic scale which is otherwise without significance. These curves predict, by their rapid decrease with increasing λ, that Z_3 will be of no importance for large

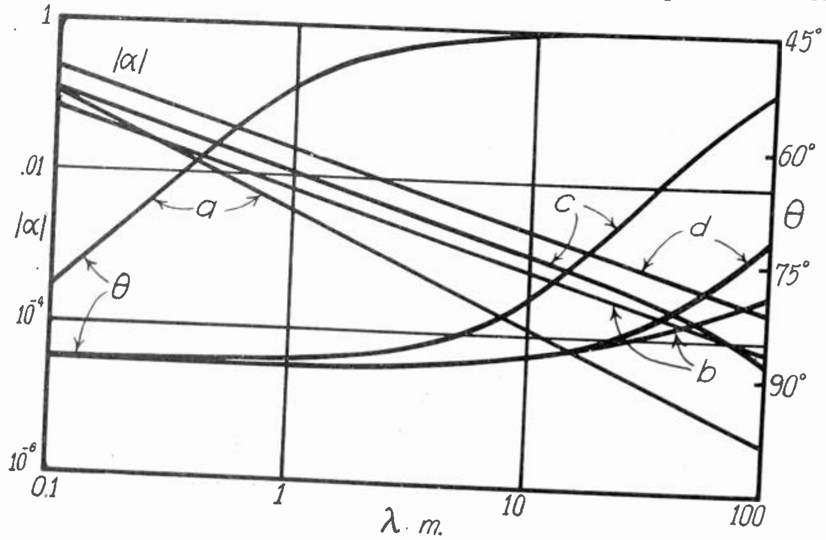

Fig. 4—Curves of $\alpha = |\alpha| e^{i\theta}$ for
a—sea water
b—fresh water
c—wet soil
d—dry soil

wavelengths; however, for sufficiently short wavelengths Z_3 may be of large magnitude and so may have to be taken into consideration.

The following values for α are obtained for the special case of $\lambda = 15$ meters.

Earth	α (rational e.s.u.)	
	Abs. Magnitude	Phase Angle
a. Sea water	0.000 046 7	45°
b. Fresh water	0.001 840	90°
c. Wet soil	0.000 727	76°
d. Dry soil	0.003 830	87°

Fig. 5 shows curves for the absoute magnitude in ohms and the phase angle in degrees of Z_3 for these four types of earth.* The small

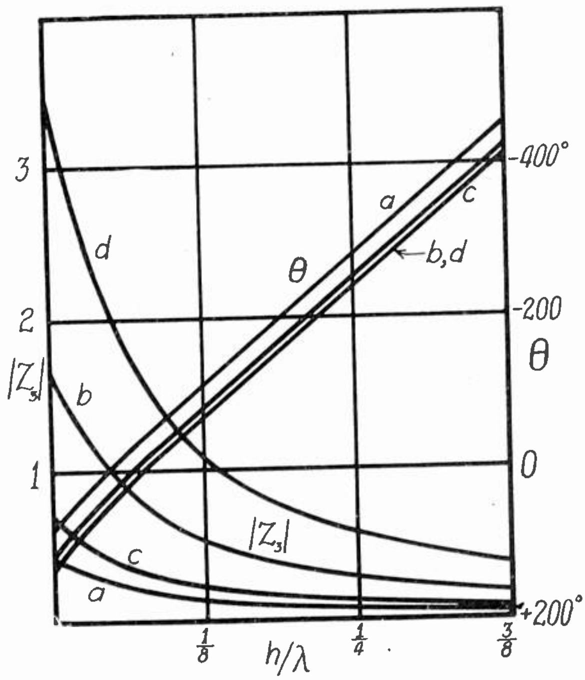

Fig. 5—Curves of absolute magnitude and phase angle for component of antenna impedance due to the finite conductivity of the earth: $Z_3 = |Z_3| e^{i\theta}$. For $\lambda = 15$ meters and

a—sea water
b—fresh water
c—wet soil
d—dry soil

magnitude of Z_3 compared to $Z_1 = 73.3 + i\ 42.6$ is at once striking. It is clear that for most earth conditions the total antenna impedance at a wavelength of 15 meters is substantially unaffected by the finite conductivity (as opposed to infinite conductivity, the condition often assumed). The assumption of a perfectly reflecting earth in prob-

* The case of dry soil does not quite satisfy the condition $k_0 \ll |k|$ and so represents only a first approximation to the correct value.

lems dealing with antenna impedance, current distribution, and generally where the reaction of the ground plane on the radiator is the subject of investigation thus seems to be well justified on theoretical grounds for a large range of wavelengths and many sorts of earth. It is not to be concluded from this that the above assumption is justified for problems dealing with the propagation of waves along or from the surface of the earth.

As the wavelength is made continuously shorter, the magnitude of

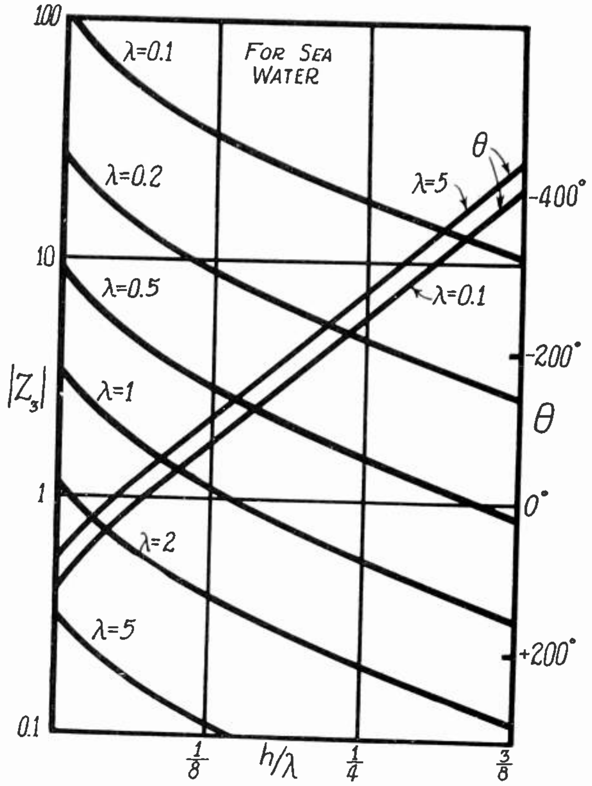

Fig. 6—Curves of $Z_3 = |Z_3| e^{i\theta}$ for sea water and several different wavelengths.

Z_3 increases, but the present analysis ceases to be valid for too short waves. For the case of sea water, however, the validity extends into the decimeter wavelength region and so this type of earth has been used to illustrate the change of Z_3 with wavelength. Fig. 6 shows a family of curves of Z_3 vs. h/λ for different values of λ (a semilogarithmic scale is used as a matter of convenience). At a wavelength of 0.1 meter, the value of Z_3 is comparable to the self-impedance of the antenna and larger than the perfect conductivity impedance component Z_2. It is of nonnegligible magnitude below 1.0 meter. A similar situation exists for other kinds of earth, but generally Z_3 will become important for much longer wavelengths than was the case in the illustration given. Approximately, when the particular earth under consideration has an α of the order of magnitude 10^{-3}, the impedance will be materially different from that obtained under the assumption of a perfectly reflecting earth. Expression (17) gives an approximation to the true value of the impedance Z_3 under these conditions; an exact complete solution is not known, although Strutt[16] has given expressions for the real part of the impedance. His results show a marked difference in the radiation resistance over that obtained by assuming the earth to have an infinite conductivity, in agreement with the findings here. Again using the case of sea water as an illustration, the real part

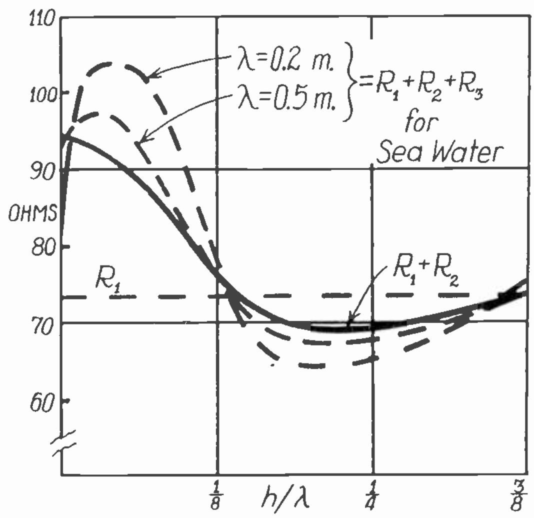

Fig. 7—Curves of effective antenna resistance in free space (R_1); above perfectly conducting earth (R_1+R_2); and above sea water ($R_1+R_2+R_3$) for two wavelengths as indicated.

of Z_3 (i.e., the effective antenna resistance) for the wavelengths of 0.2 and 0.5 meters is shown in Fig. 7. For purposes of comparison R and R_1+R_2 (R_2 taken from Carter's values) are shown separately. Even for the ultra-short waves represented in Fig. 7, it may be seen that the principal deviation from R_1 is caused by R_2. As previously mentioned, the total deviation will be many times larger for dry soil and will then be of importance for considerably longer wavelengths.

Similitude in Antenna Theory

The total effective impedance of an antenna operated under conditions where the earth may be considered perfectly conducting is given by $Z=Z_1+Z_2$. Since h and λ enter into the appropriate expressions only in the ratio h/λ, the impedances of two half-wave vertical antennas having identical values of h/λ are the same. A principle of similitude then exists which may be expressed for two antennas a and b by

$$Z_a = Z_b \quad \text{if} \quad h_a/\lambda_a = h_b/\lambda_b. \tag{18}$$

On the basis of (18), a method of models might be employed to study the performance of a proposed antenna at a given wavelength by carrying out experiments with a more convenient structure and a wavelength for which h/λ has the same value as that of the proposed antenna and wavelength. When the finite conductivity of the earth must be taken into consideration, (18) no longer holds exactly because α depends upon λ alone and not upon the ratio h/λ (see Fig. 6). In a large number of instances, usually when $k_0 \ll |k|$, Z_3 is very small, which is the case discussed above. It may be that α_a is so close in value to α_b that the above-stated condition for similitude is sufficient. When the magnitude of α is appreciably different for the two wavelengths under consideration, the conditions for similitude given in (18) are insufficient; they are then thought too involved to be of practical significance. In order that two antennas above different types of earth have identical impedances, (18) must be satisfied and in addition α_a must equal α_b. Thus, it is seen that similitude considerations in antenna design necessarily involve the electrical properties of the earth above which the antennas are constructed.

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(No pretense of completeness is assumed)

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THE PHASE AND MAGNITUDE OF EARTH CURRENTS NEAR RADIO TRANSMITTING ANTENNAS*

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Summary—*This paper deals with various electrical magnitudes involved in the process of radiation from an ordinary antenna. The calculations presented are based on the simplified assumptions of a highly conducting earth and a sinusoidal distribution of antenna current. Also, they neglect any components of the near-by field that may be associated with the flat top. The paper is divided into four main parts and an appendix. In the first part, the relative magnitudes and phases of earth currents associated with radiation from antennas of four representative proportions are calculated. The results are shown in Figs. 3 and 4. In the second part, both the displacement current density and the electric intensity are studied quantitatively. The results are shown in Fig. 5. The third part consists of an experimental verification of the calculated magnetic flux near an antenna. The agreement is evident from Fig. 6. In the last section, the theory is applied to a simple half-wave antenna with a view to localizing the earth losses. These are found to be greatest at a distance from the base of the antenna of 0.35λ. This is indicated by Fig. 7. An appendix points out the magnitude of error in neglecting components of the near-by field associated with the flat top.*

I. THE EARTH CURRENTS

IN THE operation of the usual transmitting antenna, the conduction current in the antenna diminishes as we proceed upward along the antenna. This is explained by displacement currents which are assumed to flow from the antenna, through space to the conducting plane below. This conducting plane completes the circuit by forming the return path to the base of the antenna. If this plane is not a perfect conductor, some power must be expended in returning the current to the base of the antenna. It is the purpose of this discussion to determine the phase and magnitude of these earth currents, and thus estimate the amount of energy lost in the conducting plane. The earth currents are derived under the assumption that the plane below the antenna is a perfect conductor. Such an assumption of course implies no losses in the earth. Despite this implication, a first approximation to the earth losses for an earth of finite conductivity may be obtained if we further assume that the phase and magnitude of the earth currents do not change appreciably when the conductivity of the earth becomes finite, but remains large.

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Since it can be shown that the contribution to the field made by the horizontal portion of a T antenna is small, we shall interest ourselves only with the field due to the vertical portion. With this reservation, we see that the field and therefore the distribution of earth currents will be symmetrical about the antenna.[1]

Across any cylindrical surface of radius, x, with axis coincident with the antenna, there will be a flow of current, $\bar{I}_z$, (Fig. 1). By Kirchhoff's current law, the current, $\bar{I}_0$, at the base of the antenna is

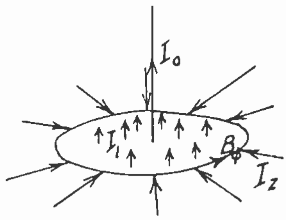

Fig. 1

made up of this zone current and the total displacement current, $-\bar{I}_1$, flowing downward into the area enclosed by the circle of radius x.

$$\bar{I}_0 = (-\bar{I}_1) + \bar{I}_z. \tag{1}$$

We now make use of the fact that the line integral of the tangential component of magnetic intensity around any closed path is equal to the total current threading the path. This is arrived at mathematically by the relation[2]

$$\text{curl } \overline{B} = \mu\left(\bar{\imath} + p\frac{dF}{dt}\right) = \mu(\bar{\imath} + j\omega pF). \tag{2}$$

By applying Stokes's theorem, we see

$$2\pi x\overline{B}_\phi = \mu(\bar{I}_0 + \bar{I}_1) = \mu\bar{I}_z \tag{3}$$

or,

[1] To take account of the contribution of the horizontal portion would complicate the problem far beyond the importance of this contribution. The earth currents due to the flat top not only are out of phase in time with the contribution made by the vertical portion of the antenna but also have a different direction so that the situation is very complicated. This statement does not mean that the effect of the flat top has been neglected entirely, for its effect upon the current distribution of the antenna itself is very important. It should be noted that the derived expressions are rigorous when the length of the flat top approaches zero.

[2] The rationalized practical system of units is used throughout the discussion. In this system, magnetic intensity, H, is expressed in ampere-turns per centimeter. Magnetic flux density, B, is expressed in webers per square centimeter ($=10^8$ maxwells per square centimeter). Permeability, $\mu = B/H = 4\pi\times10^{-9}$ for free space. Electric intensity, F, is expressed in volts per centimeter. Displacement, D, is expressed in coulombs per square centimeter. Permitivity, $p = D/F = 8.85\times10^{-14}$ for free space.

$$\bar{I}_z = \frac{2\pi x \overline{B}_\phi}{\mu} \tag{4}$$

where B_ϕ is the magnetic flux density in space at the surface of the earth, a distance x from the base of the antenna.

Making use of the vector expressions for the field due to an elementary length of current,[3] we see that the B vector at the surface due to a small element taken a distance, y, along the antenna and due to the image of this element is

$$dB_\phi = \frac{\mu \bar{I}_0}{2\pi \sin G'} \sin (G' - ky)\left[\frac{\epsilon^{-jkr}}{r^2} + jk\frac{\epsilon^{-jkr}}{r}\right]\frac{x}{r}\, dy \tag{5}$$

where $r^2 = x^2 + y^2$ and[4] $G' = (2\pi/\lambda)(a+b')$. The current at the point y (Fig. 2) is $\bar{I}_0 \sin (G' - ky)/\sin G'$ where $k = 2\pi/\lambda = \omega/c$.

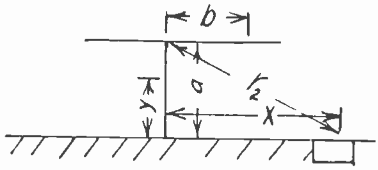

Fig. 2

Then the contribution to the zone current, from (4), is

$$d\bar{I}_z = \frac{\bar{I}_0 \sin (G' - ky)}{\sin G'}\left[\frac{\epsilon^{-jkr}}{r^2} + j\frac{k\epsilon^{-jkr}}{r}\right]\frac{x^2}{r}\, dy. \tag{6}$$

The total zone current is obtained by integration over the vertical portion of the antenna.

$$\bar{I}_z = \frac{\bar{I}_0 x^2}{\sin G'}\int_{y=0}^{y=a} \frac{\sin (G' - ky)}{r}\left[\frac{\epsilon^{-jkr}}{r^2} + j\frac{k\epsilon^{-jkr}}{r}\right] dy. \tag{7}$$

The expression may be integrated by means of a transformation. Noting that x is a parameter, we see that (7) becomes

$$\bar{I}_z = \frac{-\bar{I}_0 x}{\sin G'}\frac{\partial}{\partial x}\int_{y=0}^{y=a} \sin (G' - ky)\, \frac{\epsilon^{-jkr}}{r}\, dy. \tag{8}$$

Then,

[3] Abraham and Becker, "The Classical Theory of Electricity and Magnetism," Blackie and Son, Limited. Translated by John Dougall, 1932; equation (44a), p. 225, and Section 12, p. 227.

[4] The equivalent length, b', is obtained from the actual length, b, by the equation, $\cot (kb') = \frac{1}{2} \cot (kb)$. R. M. Wilmotte, "General formulae for the radiation distribution of antenna systems," *Jour. I.E.E.* (London), vol. 68, p. 1174, (1930).

$$\bar{I}_z = \frac{-\bar{I}_0 x}{2j \sin G'} \frac{\partial}{\partial x}\left[\epsilon^{jG'} \int_{y=0}^{y=a} \frac{\epsilon^{-jk(r+y)}}{r} dy - \epsilon^{-jG'} \int_{y=0}^{y=a} \frac{\epsilon^{-jk(r-y)}}{r} dy\right]. \tag{9}$$

By making the substitution, $v=r+y$, in the first integral and $w=r-y$ in the second integral, (9) is readily integrated. When the indicated differentiation is performed and the terms combined, we see that

$$\bar{I}_z = \frac{\bar{I}_0}{\sin G'}\left[j \cos B'(\cos kr_2 - j \sin kr_2) - j \cos G'(\cos kx - j \sin kx) + \frac{a}{r_2} \sin B'(\cos kr_2 - j \sin kr_2)\right] \tag{10}$$

where,

$$B' = 2\pi b'/\lambda,$$

and,

$$r_2^2 = a^2 + x^2.$$

It is interesting to note the very simple form taken by the zone current when the antenna is a quarter wavelength. Then $B'=0$ degrees and $G'=90$ degrees and

$$\bar{I}_z = j\bar{I}_0[\cos kr_2 - j \sin kr_2]. \tag{11}$$

Examination of (10) shows that, as x becomes large, $\bar{I}_z$ reduces to a rotating vector of constant magnitude. For x large, $r_2 \doteq x$, and

$$\bar{I}_z = \frac{j\bar{I}_0}{\sin G'}(\cos B' - \cos G')(\cos kx - j \sin kx). \tag{12}$$

Fig. 3 shows the total zone current and the components of this current in their proper phase relationship, for four different antennas. Each horizontal row of vectors depicts the mode of variation of zone current with distance from the antenna for a particular antenna. In each case, the vector at the extreme left represents the current at the base of the antenna. Antenna No. 4 is an exception to this case. Here the vector at the left represents the current at the center of the antenna.

The components of zone current shown in Fig. 3 represent the contributions made by the so-called radiation and induction terms of the magnetic flux density. The radiation component increases with distance and approaches a limit, while the induction term decreases with an increase of distance.

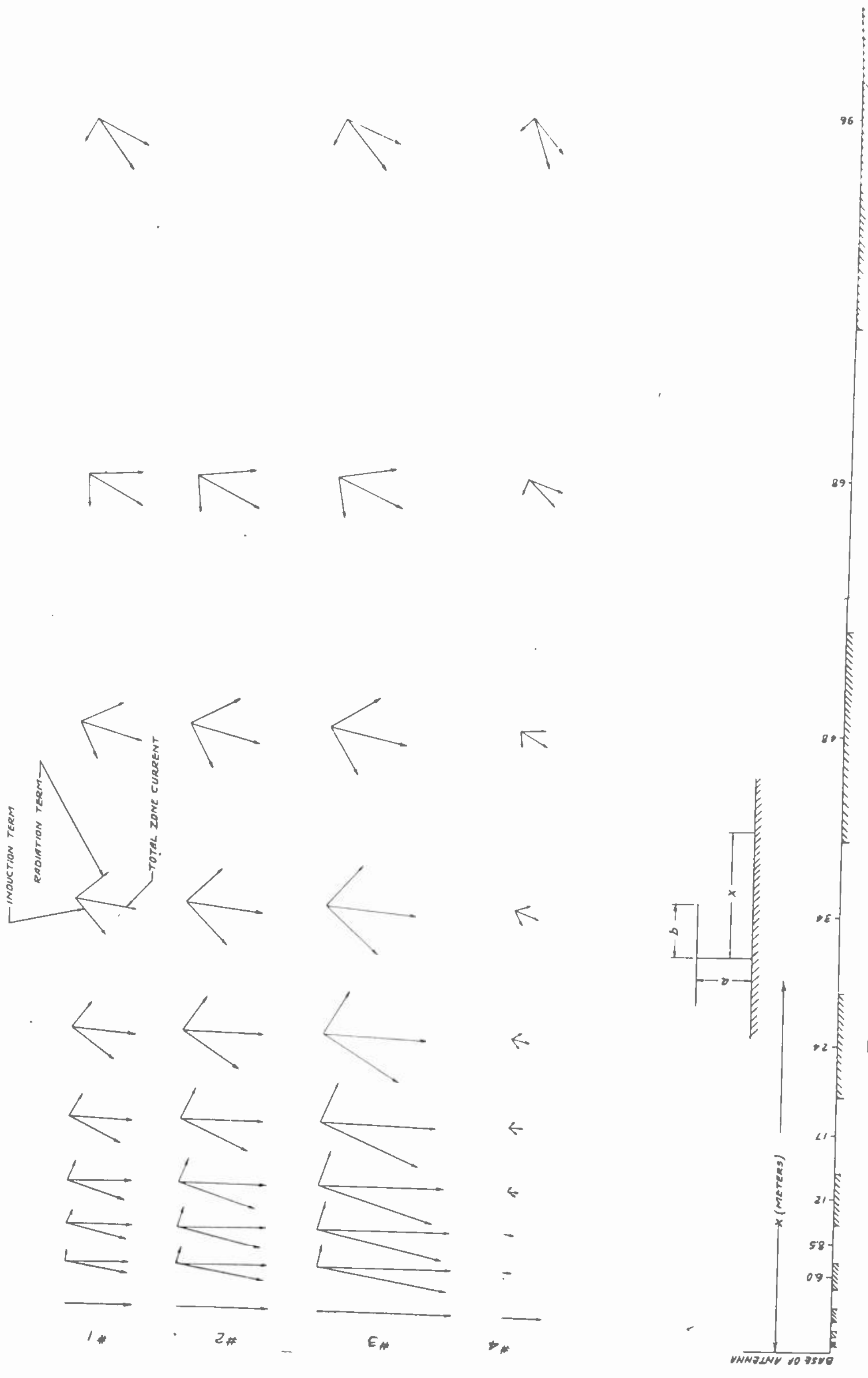

Fig. 3—Phase and magnitude of earth currents near radio antennas.

TABLE I

Distance from Antenna Base (m)	Antenna No. 1			Antenna No. 2			Antenna No. 3			Antenna No. 4		
	Induction Term	Radiation Term	Total Zone Current	Induction Term	Radiation Term	Total Zone Current	Induction Term	Radiation Term	Total Zone Current	Induction Term	Radiation Term	Total Zone Current
0	1.0	0	1.0	1.445	0	1.445	2.17	0	2.17	0	0	0 Loop =0.604
6.0	0.96	0.1725	1.0	1.335	0.235	1.366	2.04	0.344	2.07	—	—	0.0801
8.5	0.945	0.232	1.0	1.309	0.312	1.354	1.965	0.446	2.022	—	—	0.121
12	0.915	0.309	1.0	1.25	0.407	1.328	1.832	0.557	1.922	0.1395	0.0678	0.1602
17	0.88	0.403	1.0	1.17	0.518	1.29	1.545	0.671	1.779	0.167	0.1086	0.222
24	0.824	0.508	1.0	1.06	0.627	1.242	1.392	0.762	1.588	0.204	0.1725	0.293
34	0.748	0.624	1.0	0.93	0.737	1.191	1.113	0.832	1.392	0.238	0.262	0.384
48	0.653	0.737	1.0	0.767	0.827	1.132	0.847	0.87	1.215	0.273	0.39	0.505
68	0.536	0.830	1.0	0.601	0.893	1.078	0.626	0.897	1.112	0.287	0.552	0.617
96	0.424	0.880	1.0	0.452	0.927	1.025	0.448	0.91	1.015	0.301	0.728	0.801
∞	0	1.0	1.0	0	0.982	0.982	0	0.9225	0.9225	0	1.208	1.208

TABLE II

Zone Radius (meters)	Ratio of Zone Resistance to Radiation Resistance			
	No. 1	No. 2	No. 3	No. 4
Antenna Height (a)	75	44	21	150
$2b'$	0	34	108	0
Radiation Resistance	36.6	17.52	7.7335	100 (Loop)
6.0	0.0298	0.0558	0.125	0.0000705
8.5	0.0298	0.0550	0.1225	0.000161
12	0.0298	0.0528	0.1110	0.000314
17	0.0298	0.0498	0.095	0.00054
24	0.0298	0.0462	0.0755	0.000945
34	0.0298	0.0425	0.05825	0.001617
48	0.0298	0.0384	0.0443	0.00279
68	0.0298	0.0348	0.0372	0.00417
96	0.0298	0.0314	0.0310	0.00704
Σ of resistance of zones from 6 to 96 m inclusive	9.816	7.1285	5.4265	1.767
	Wave length =300 m Resistivity =10^4 ohm-cm^3		Skin thickness =503.3 cm	

If we assume that the phase and magnitude of earth currents depicted above do not change appreciably when the conductivity of the earth becomes finite, but remains large, we are enabled to form a picture of the magnitude of the power losses in the earth.[5] It is further postulated that all the zone current travels in a layer of earth of thickness, $s = 1/\sqrt{\pi\mu\gamma_e f}$, the skin thickness of the earth. In this expression, f is the frequency of the driving voltage measured in cycles per second, μ is the permeability of the material, and γ_e is the conductivity of the earth measured in mhos per centimeter cube of material. (On the diagrams, this is indicated as "mho-cm,[3]" a convenient shorthand notation.) Then the resistance of a narrow zone of radii, x_2 and x_1, referred to the current at the base of the antenna is

$$R_z = (I_z/I_0)^2 \frac{1}{2\pi\gamma_2 s} \log_e \frac{x_2}{x_1}. \tag{13}$$

In Fig. 3, the zones are all taken of such a width that $x_2/x_1 = \sqrt{2}$. Thus, for a quarter-wave antenna, where $|I_z| = |I_0|$, the resistance of these zones remains constant.

The values shown in Table II were obtained by the above procedure. The values given in this table enable one to estimate the magnitude of the losses in the vicinity of the antenna, and to form opinions as to the advisability of using radial ground systems of great length and a great number of wires.

The diagrams of Fig. 3 were obtained on the basis of a constant radiated power of 36.6 watts. Fig. 4 shows the variation of root-mean-square zone current with distance from the antenna, when the radiated power is 1000 watts.

II. The Displacement Current Densities and the Electric Intensities

From (10) it is very easy to obtain the displacement current density and the vertical electric intensity at the surface. $\bar{I}_z$ is the current flowing toward the antenna at a distance, x. At a distance, $x+\Delta x$, the current flowing inward is $\bar{I}_z'$. Then $\bar{I}_z' - \bar{I}_z = \bar{I}_d$, the displacement current flowing upward from the area between the two circles of radii, x and $x+\Delta x$. If Δx becomes small, the displacement current becomes

$$\bar{I}_d = \frac{d\bar{I}_z}{dx} \Delta x. \tag{14}$$

[5] The above assumptions are only valid when the earth has a high conductivity. This conductivity should be great enough so that the conduction earth currents predominate greatly over the displacement currents in the earth.

The displacement current density is

$$\bar{J}_d = \frac{\bar{I}_d}{2\pi x \cdot \Delta x} = \frac{1}{2\pi x}\frac{dI_z}{dx} \tag{15}$$

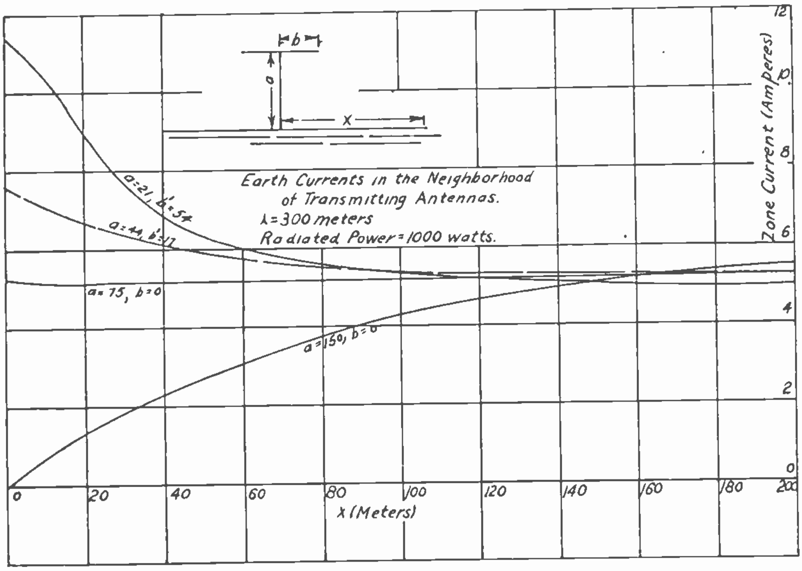

Fig. 4—Variation of zone current with distance from the base of the antenna.

or,

$$\bar{J}_d = \frac{\bar{I}_0}{2\pi \sin G'}\Bigg[\left\{\frac{k}{r_2}\cos kr_2 \cos B' - \frac{k}{x}\cos kx \cos G' - \frac{a}{r_2{}^3}\cos kr_2 \sin B' - \frac{ka}{r_2{}^2}\sin kr_2 \sin B'\right\}$$
$$+ j\left\{\frac{-k}{r_2}\sin kr_2 \cos B' + \frac{k}{x}\sin kx \cos G' + \frac{a}{r_2{}^3}\sin kr_2 \sin B' - \frac{ka}{r_2{}^2}\cos kr_2 \sin B'\right\}\Bigg]. \tag{16}$$

The vertically upward electric intensity is obtained from the fact that

$$\bar{J}_d = jp\omega\bar{F} \tag{17}$$

or,

$$\bar{F} = -j\frac{\bar{J}_d}{p\omega} = -j\frac{\mu c}{k}\bar{J}_d \tag{18}$$

so that,

$$\bar{F} = \frac{\mu c \bar{I}_0}{2\pi \sin G'} \Big[-j \frac{\cos B'}{r_2} (\cos kr_2 - j \sin kr_2)$$

$$+ j \frac{\cos G'}{x} (\cos kx - j \sin kx)$$

$$+ j \frac{a}{kr_2{}^3} \sin B'(\cos kr_2 - j \sin kr_2)$$

$$- \frac{a}{r_2{}^2} \sin B'(\cos kr_2 - j \sin kr_2) \Big]. \tag{19}$$

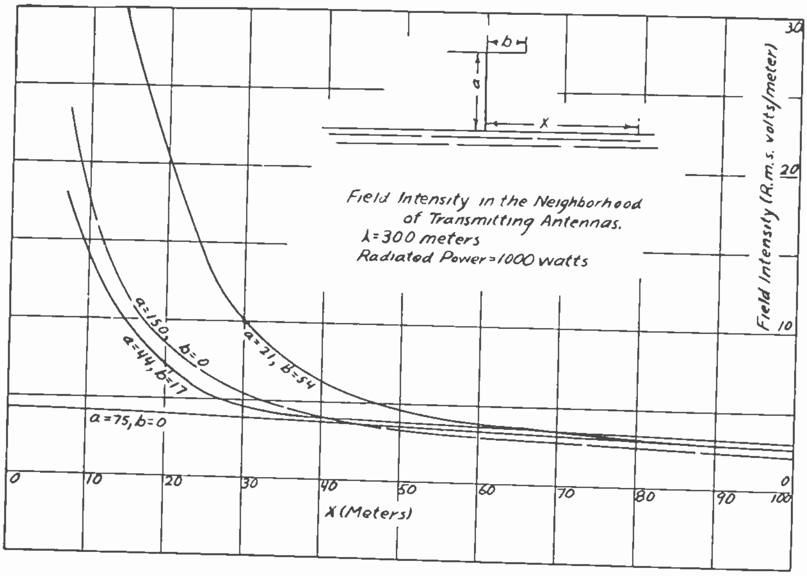

Fig. 5—The vertical electric intensity at the surface of the earth as a function of the distance from the base of the antenna.

For a quarter-wave antenna, (19) reduces to

$$\bar{F} = -j \frac{\mu c}{2\pi r_2} \bar{I}_0(\cos kr_2 - j \sin kr_2). \tag{20}$$

For x large, (19) becomes

$$\bar{F} = -j \frac{\mu c \bar{I}_0}{2\pi x \sin G'} (\cos B' - \cos G')(\cos kx - j \sin kx). \tag{21}$$

Fig. 5 shows the electric intensity in the neighborhood of the four transmitting antennas already referred to. The computations were made for a radiated power of 1000 watts, rather than for the same antenna current. It should be noted that the field of a quarter-wave antenna close to the antenna is much smaller than for the other antennas. This feature would indicate that the currents induced in the structures near the base of the antenna would be much less than for the other types under consideration.

These expressions for displacement current density and electric intensity should prove useful in arriving at estimated values of losses occurring in the vicinity of antenna structures.

III. An Experimental Verification

While it is, in general, difficult to obtain a direct experimental check of the foregoing relations, one such check was made. When the antenna in question has no flat top, (10) becomes

$$\bar{I}_z = \frac{\bar{I}_0}{\sin G'}[\{\sin kr_2 - \sin kx \cos G'\} + j\{\cos kr_2 - \cos kx \cos G'\}] \tag{22}$$

where,

$$G' = \frac{2\pi a}{\lambda}.$$

From (4) and (22),

$$\bar{B}_\phi = \frac{\mu \bar{I}_0}{2\pi x \sin G'}[\{\sin kr_2 - \sin kx \cos G'\} + j\{\cos kr_2 - \cos kx \cos G'\}] \text{ (webers/cm}^2\text{)}. \tag{23}$$

The experimental set-up to check this equation was an entirely vertical antenna driven by a short-wave oscillator. The earth was covered by large sheets of copper screen. A wave meter, tuned to resonance, was placed on the screen, and successive readings of the wave meter current were taken as the wave meter was moved from the base of the antenna. From these readings, the magnetic flux density was computed.

From a given wave meter current, the induced voltage is

$$E_i = I_{wm}R_{wm}. \tag{24}$$

But,

$$E_i = \omega B_\phi NA \tag{25}$$

or,

$$B_\phi = (I_{wm}R_{wm})/(\omega NA) \text{ (webers/cm}^2\text{)} \tag{26}$$

where R_{wm} is the resistance of the wave meter circuit, N is the number of turns in the wave meter coil, and A is the cross-sectional area of this coil. These values are

$$R_{wm} = 10.3 \text{ ohms (measured)}$$
$$N = 5 \text{ turns}$$
$$A = 32.3 \text{ cm}^2$$
$$\omega = 2\pi(42.2)(10^6) = 2.65 \times 10^8.$$

The antenna height was 148.0 centimeters while the wavelength was 710.0 centimeters. The antenna current, I_0, was one ampere.

Fig. 6 shows values of B as obtained from (23) and from the experimental values substituted in (26). The variation of the experimental values from the theoretical curve in the region, $x/\lambda < 0.06$, is no

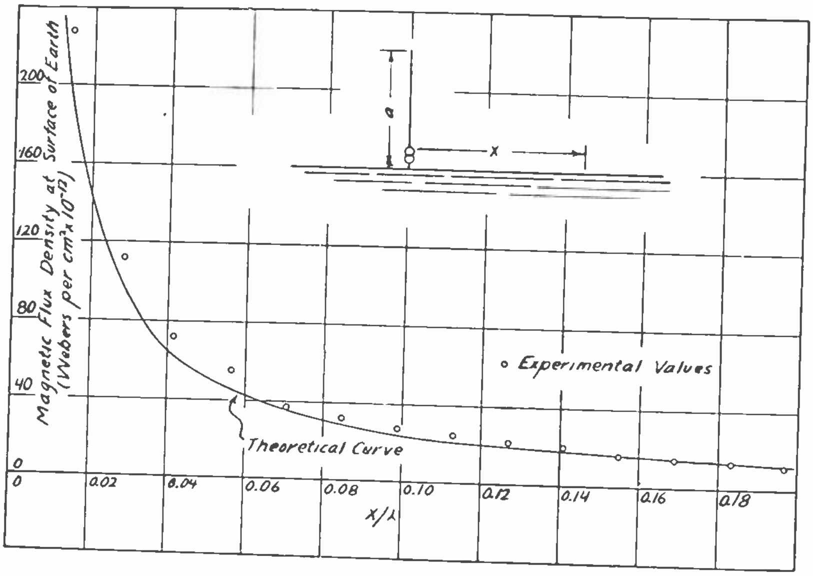

Fig. 6—The magnetic flux density at the surface of the earth in the neighborhood of a vertical antenna. (The solid curve was computed from (23). The circles indicate experimental values.)

doubt caused by the field set up by the circuit of the oscillator itself. In this region in question, small deflections of the wave meter were observed when the antenna was completely removed and the only field was that from the oscillator itself.

IV. A Simple Application of the Theory (The Half-Wave Antenna)

It is interesting to examine the earth losses in the vicinity of a half-wave antenna. If the zone current is considered to flow in a layer of thickness, $s = 1/\sqrt{\pi\mu\gamma_e f}$, the power expended in a zone of radius, x, and length, dx, is

$$dP_z = |I_z|^2 \frac{dx}{2\pi x \gamma_e s}. \tag{27}$$

Then the resistance of this zone (referred to the current at the center of the antenna) is

$$dR_z = \left| I_z/I_0 \right|^2 \frac{dx}{2\pi x \gamma_e s}. \tag{28}$$

From (10), the zone current of a half-wave antenna is

$$\bar{I}_z = j\bar{I}_0\left[\epsilon^{-j2\pi r_2/\lambda} + \epsilon^{-j2\pi x/\lambda}\right] \tag{29}$$

where $r_2^2 = (\lambda/2)^2 + x^2$.

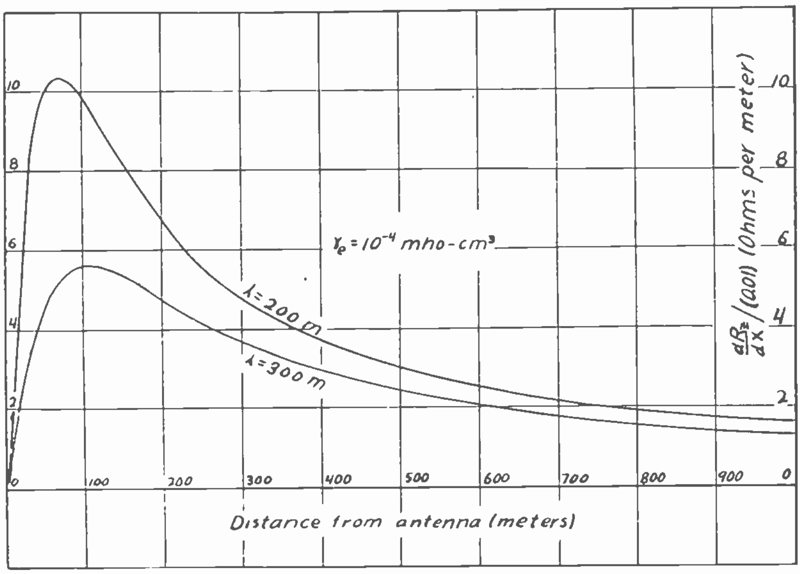

Fig. 7—The distribution of earth losses in the neighborhood of a half-wavelength antenna.

Then,

$$(I_z/I_0)^2 = \left(\cos\frac{2\pi r_2}{\lambda} + \cos\frac{2\pi x}{\lambda}\right)^2 + \left(\sin\frac{2\pi r_2}{\lambda} + \sin\frac{2\pi x}{\lambda}\right)^2$$

$$= 2\left[1 + \cos\frac{2\pi}{\lambda}(r_2 - x)\right]. \tag{30}$$

Substituting (30) in (28),

$$\frac{dR_z}{dx} = \frac{1}{\pi\gamma_e s}\,\frac{\left[1 + \cos\frac{2\pi}{\lambda}(r_2 - x)\right]}{x}. \tag{31}$$

This equation expresses the losses in ohms per unit length. This function is plotted in Fig. 7 for wavelengths of 200 and 300 meters, and an earth conductivity of $\gamma_e = 10^{-4}$ mho/cm³. It is to be noted that the maximum losses occur at a point 0.35λ distance from the base of the antenna. It is particularly desirable that the earth resistivity be low in the neighborhood of this point or that conducting wires be buried there.

It is from an analysis of this type that one is enabled to arrive at definite ideas as to the effect of buried networks and the most effective placing of such a network.

Appendix

The Earth Currents Associated with the Field of the Horizontal Section of the T Antenna

In the main body of the paper, any contribution to the earth currents due to currents in the horizontal portion of a T antenna has been

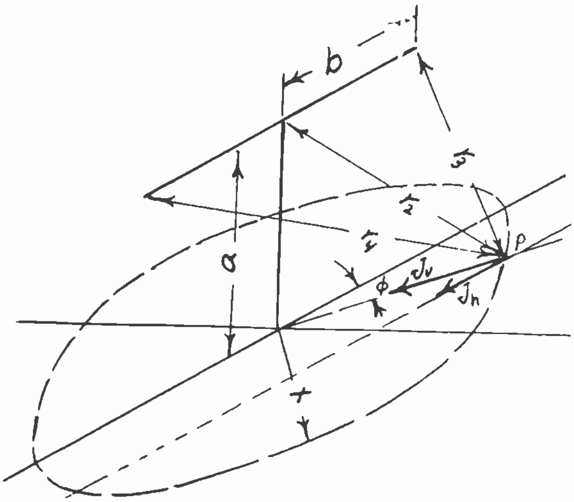

Fig. 8

neglected. We have seen that the earth currents contributed by the vertical section flow radially inward toward the base of the antenna, and that the current density is the same at all points on the periphery of a circle whose center is at the base of the antenna. The current density due to the horizontal portion is parallel to the horizontal wire. (Fig. 8.) Thus we see that the current density due to the horizontal portion ($\bar{J}_h$) flows in a different direction than does the current density due to the vertical portion, ($\bar{J}_v$). Not only is the direction different, but the two components are not in time phase.

The current densities referred to above are surface densities. Thus, at point P on the surface of the earth, a distance, x, from the base of the antenna, the surface density of current due to the vertical portion flows radially inward (Fig. 8) and is given by

$$\bar{J}_v = \bar{I}_z/(2\pi x) \qquad \text{(amperes/cm)} \tag{32}$$

where $\bar{I}_z$ is given by (10).

The component, $\bar{J}_h$, at the same point is

$$\bar{J}_h = \frac{j\bar{I}_0 \quad \sin B'}{4\pi \sin G' \sin B} \cdot \frac{a}{a^2 + x^2}\left[\epsilon^{-jkr_3} - \epsilon^{-jkr_4} - 2j\frac{x}{r_2}\epsilon^{-jkr_2} \sin B \cos\phi\right] \qquad \text{(amperes/cm)} \tag{33}$$

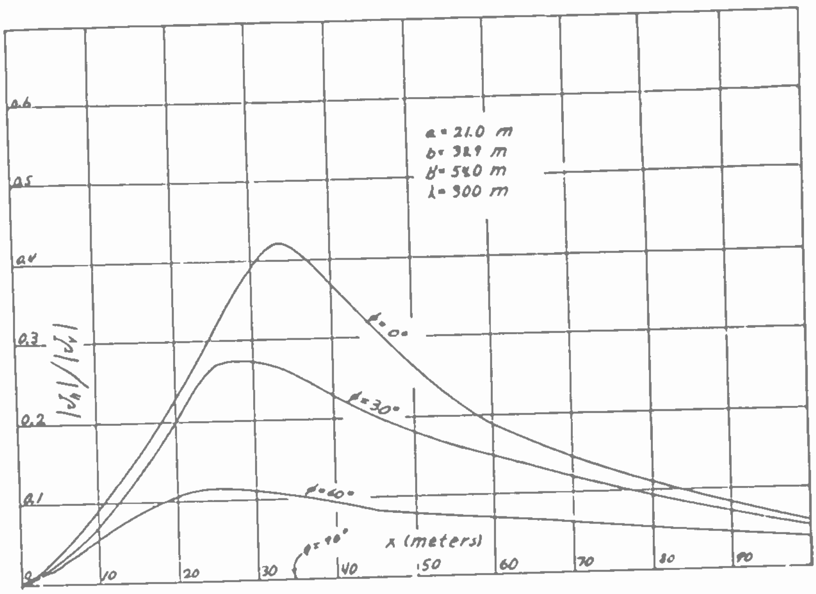

Fig. 9

where,

$r_3 = \sqrt{(b - x\cos\phi)^2 + a^2 + (x \sin\phi)^2}$

$r_4 = \sqrt{(b + x\cos\phi^2) + a^2 + (x\sin\phi)^2}$

$r_2 = \sqrt{a^2 + x^2}$

a = height of antenna

b = half length of horizontal section

x = distance from base of antenna to point P

ϕ = angle between the line joining the base of the antenna to point P and the line on the earth parallel to the horizontal section

$B = 2\pi b/\lambda$

$G' = 2\pi(a + b')/\lambda$

$B' = 2\pi b'/\lambda$

b' = equivalent length defined by footnote 4

λ = wavelength

$k = 2\pi/\lambda$

$\bar{I}_0$ = current at the base of the antenna

Fig. 9 shows the ratio, $|J_h|/|J_v|$, plotted with the distance, x, as a variable and the angle, ϕ, as a parameter, for Antenna No. 3. This is an extreme case, yet we see that only at distances in the neighborhood of 35 meters from the base, directly under the horizontal section is the earth current density, J_h, comparable to the term, J_v. As the flat top shortens, the peak of the curve, $\phi = 0$ degrees, moves closer to the antenna base and decreases in value.

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page may be obtained without charge by addressing the publishers.

Heavy duty type air circuit breakers are described in Catalog No. 5 issued by the Roller-Smith Company, 233 Broadway, New York City.

Aerovox condensers and resistors are described in a catalog issued under that name by the Aerovox Corporation of 70 Washington Street, Brooklyn, N.Y.

Application Note No. 43 on cathode ray curve tracing apparatus for aligning tuned circuits was issued by the RCA Radiotron Company, Harrison, N. J. A technical discussion on the determination of oscillator circuit constants in superheterodyne receivers is known as laboratory series No. UL-8.

The all-wave test oscillator and an all-wave superheterodyne coil kit are described in leaflets issued by J. W. Miller Company, 5917 S. Main Street, Los Angeles, Calif.

A leaflet issued by the Lightning Calculator Company of 8 Henry Street, Bogota, N. J. describes devices for eliminating pencil work in making many commonly needed computations.

Catalog "35" of the Hammarlund Manufacturing Company, 424 West 33rd Street, New York City covers, condensers, coil forms, sockets, transformers, chokes, shields, and other products of that organization.

Their G-12 high fidelity electrodynamic loud speaker is described in a leaflet issued by The Rola Company of Cleveland, Ohio.

A 10 to 6 ampere battery charger for home use is described in a leaflet issued by the Automatic Electrical Devices Company, 324 E. 3rd Street, Cincinnati, Ohio.

A new portable potentiometer is described in Volume 1, No. 1 of "Electrical Measurements" issued by the Sensitive Research Instrument Corporation of 4545 Bronx Boulevard, New York City.

Model 10B microvolter, having a range from 150 to 20,000 kilocycles, and its use in radio-frequency measurements are described in two booklets issued by the Ferris Instrument Corporation of Boonton, N. J. Another booklet describes alternating current operated and battery operated standard signal generators.

The ACR-136 communications receiver covering from 540 kilocycles to 18 megacycles is described in a leaflet issued by the RCA Victor Company of Camden, N. J.

Application Note No. 44 on operating conditions for the 6A6 has been released by the RCA Radiotron Company of Harrison, N. J.

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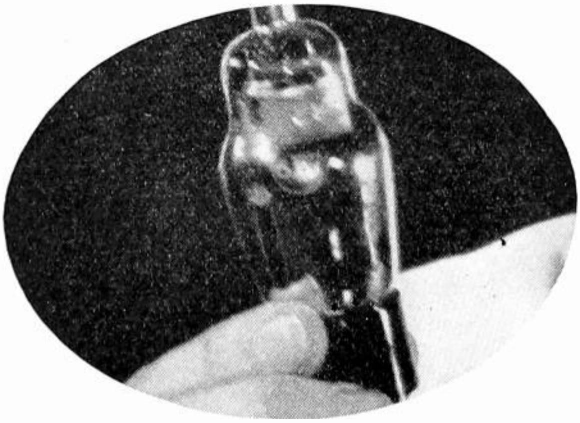
Peterson, Harold O.: Born November 3, 1899, at Blair, Nebraska. Received B.S. degree in electrical engineering, University of Nebraska, 1921. Testman, General Electric Company, 1921–1922. Engaged in development of radio communications equipment, Radio Corporation of America, 1922–1929; in charge, radio communication receiver development laboratory, RCA Communications, Inc., 1929 to date. Associate member, Institute of Radio Engineers, 1922; Member, 1931.

Skellett, A. M.: Born July 14, 1901, at St. Louis, Missouri. Received A.B. degree, 1924; M.S. degree, Washington University, 1927; Ph.D. degree, Princeton University, 1933. Physicist, Westinghouse Electric and Manufacturing Company, summer, 1926; instructor in electrical engineering, 1927–1928; assistant professor of physics, University of Florida, 1928–1929. Chief engineer, station WRUF, 1928–1929; Bell Telephone Laboratories, Inc., 1929 to date. Member, American Physical Society and American Astronomical Society. Member, Institute of Radio Engineers, 1930.

Steiner, H. Carlton: Born October 5, 1902, at Larned, Kansas. Received B.S. degree, University of Kansas, 1926. Student engineer, General Electric Company, 1926–1927; research laboratory, 1927–1930; vacuum tube engineering department, 1930 to date. Associate member, Institute of Radio Engineers, 1929.

THE TUBE THAT CAME FROM A POINT OF VIEW



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● True scientists have always been driven by a passion for truth. As some one phrased it, they were "born into a scientific attitude of mind." And paramount to all their other interests is their unwavering *desire to know.*

It is a proud brotherhood numbering such great figures as Galileo, Newton and Edison as members. And Sylvania is proud that their engineers, in a modest way, have deserved inclusion in this group. For it was their research that led to the development of the 6.3 volt tubes that made auto radio possible. And at various other times they have contributed to the growth of our industry with important research and experimental work.

It is a policy of Sylvania—just as much so as its AaA1 credit rating—that these engineers shall continue to make important contributions.

Set manufacturers are invited, at all times, to write for engineering consultation on circuit and design problems. Our facilities are placed at your disposal without cost or obligation. Hygrade Sylvania Corporation, Emporium, Pa.

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The Institute of Radio Engineers

Incorporated

330 West 42nd Street, New York, N.Y.

APPLICATION FOR ASSOCIATE MEMBERSHIP

(Application forms for other grades of membership are obtainable from the Institute)

To the Board of Directors

Gentlemen:

I hereby make application for Associate membership in the Institute of Radio Engineers on the basis of my training and professional experience given herewith, and refer to the members named below who are personally familiar with my work.

I certify that the statements made in the record of my training and professional experience are correct, and agree if elected, that I will be governed by the constitution of the Institute as long as I continue a member. Furthermore I agree to promote the objects of the Institute so far as shall be in my power, and if my membership shall be discontinued will return my membership badge.

....
(Sign with pen)

....
(Address for mail)

.... (Date)

....
(City and State)

Sponsors:

(Signature of references not required here)

Mr. ....

Address ....

City and State ....

Mr. ....

Address ....

City and State ....

Mr. ....

Address ....

City and State ....

The following extracts from the Constitution govern applications for admission to the Institute in the Associate grade:

ARTICLE II—MEMBERSHIP

Sec. 1: The membership of the Institute shall consist of: * * * (c) Associates, who shall be entitled to all the rights and privileges of the Institute except the right to hold any elective office specified in Article V. * * *

Sec. 4. An Associate shall be not less than twenty-one years of age and shall be a person who is interested in and connected with the study or application of radio science or the radio arts.

ARTICLE III—ADMISSION AND EXPULSIONS

Sec. 2: * * * Applicants shall give references to members of the Institute as follows: * * * for the grade of Associate, to three Fellows, Members, or Associates; * * * Each application for admission * * * shall embody a full record of the general technical education of the applicant and of his professional career.

ARTICLE IV—ENTRANCE FEE AND DUES

Sec. 1: * * * Entrance fee for the Associate grade of membership is $3.00 and annual dues are $6.00.

ENTRANCE FEE SHOULD ACCOMPANY APPLICATION

(Typewriting preferred in filling in this form) No............

RECORD OF TRAINING AND PROFESSIONAL EXPERIENCE

Name ....
(Give full name, last name first)

Present Occupation ....
(Title and name of concern)

Business Address ....

Permanent Home Address ....

Place of Birth.......................... Date of Birth............ Age.....

Education ....

Degree ....
(college) (Date received)

TRAINING AND PROFESSIONAL EXPERIENCE

DATES	

Record may be continued on other sheets of this size if space is insufficient.

Receipt Acknowledged............ Elected............ Deferred............

Grade .... Advised of Election........ . This Record Filed..........

These tests prove it!

Only

ERIE RESISTORS
GIVE BALANCED PERFORMANCE

Here is a complete cross-section of the performance of carbon resistors. It shows the existence of a complete lack of uniformity in the characteristics of many makes.

To obtain these data, standard production Erie units were tested with six leading competitive types for voltage coefficient, temperature coefficient, and humidity, load and noise characteristics.

The results obtained show conclusively that only Erie Resistors can be counted on to give all-round performance in every respect.

Switch to Erie Resistors 100%. You can depend on their balanced trouble-free operation.

The lack of uniformity in the various competitive makes of resistors tested can be traced in the table below by the wide fluctuation in their respective values. The readings have been taken at the maximum point on the horizontal axis.

Note that only Erie Resistors maintain a uniformly good position while the other makes change widely.

COMPETITIVE RESISTOR CHARACTERISTICS

MAKE	Voltage Coefficient % Change, R	Temperature Coefficient % Change R	Load Characteristics % Change, R	Humidity Characteristics % Change, R	Noise Characteristics Microvolts
ERIE	1.80	1.10	2.00	22.00	300
RESISTOR "A"	2.45	4.55	14.60	16.00	490
RESISTOR "B"	3.10	12.85	8.60	40.50	2,700
RESISTOR "C"	3.55	1.20	10.20	61.00	225
RESISTOR "D"	5.30	10.25	7.30	34.50	1,250
RESISTOR "E"	9.10	1.75	7.50	29.50	215
RESISTOR "F"	13.85	1.20	9.10	42.00	2,900

The fourth of a series of advertisements dealing with the balanced performance of Erie Resistors

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