VOLUME 17 JUNE, 1929 NUMBER 6

PROCEEDINGS
of
The Institute of Radio Engineers

General Information and Subscription Rates on Page 908

Eastern Great Lakes District Convention

November 18-19, 1929
Rochester, New York

Sponsored by the
Buffalo-Niagara, Cleveland, Rochester and Toronto Sections

All members of the Institute are invited to participate in the very interesting two-day program being arranged

Fifth Annual Convention
Toronto - June, 1930

PROCEEDINGS OF

The Institute of Radio Engineers

Volume 17 June, 1929 Number 6

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The Institute of Radio Engineers

GENERAL INFORMATION

The PROCEEDINGS of the Institute is published monthly and contains papers and discussions thereon submitted for publication or for presentation before meetings of the Institute or its Sections. Payment of the annual dues by a member entitles him to one copy of each number of the PROCEEDINGS issued during the period of his membership.

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Contributors to the PROCEEDINGS are referred to the following page for suggestions as to approved methods of preparing manuscripts for publication in the PROCEEDINGS.

Advertising rates to the PROCEEDINGS will be supplied by the Institute's Advertising Department, Room 802, 33 West 39th Street, New York, N. Y.

Changes of address to affect a particular issue must be received at the Institute office not later than the 15th of the month preceding date of issue. That is, a change in mailing address to be effective with the October issue of the PROCEEDINGS must be received by not later than September 15th. Members of the Institute are requested to advise the Secretary of any change in their business connection or title irrespective of change in their mailing address, for the purpose of keeping the Year Book membership catalog up to date.

All correspondence should be addressed to the Institute of Radio Engineers, 33 West 39th Street, New York, N. Y., U. S. A.

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SUGGESTIONS FOR CONTRIBUTORS TO THE PROCEEDINGS

Preparation of Paper

Form—Manuscripts may be submitted by member and non-member contributors from any country. To be acceptable for publication manuscripts should be in English, in final form for publication, and accompanied by a summary of from 100 to 300 words. Papers should be typed double space with consecutive numbering of pages. Footnote references should be consecutively numbered and should appear at the foot of their respective pages. Each reference should contain author's name, title of article, name of journal, volume, page, month, and year. Generally, the sequence of presentation should be as follows: statement of problem; review of the subject in which the scope, object, and conclusions of previous investigations in the same field are covered; main body describing the apparatus, experiments, theoretical work, and results used in reaching the conclusions and their relation to present theory and practice; bibliography. The above pertains to the usual type of paper. To whatever type a contribution may belong, a close conformity to the spirit of these suggestions is recommended.

Illustrations—Use only jet black ink on white paper or tracing cloth. Cross-section paper used for graphs should not have more than four lines per inch. If finer ruled paper is used, the major division lines should be drawn in with black ink, omitting the finer divisions. In the latter case, only blue-lined paper can be accepted. Photographs must be very distinct, and must be printed on glossy white paper. Blueprinted illustrations of any kind cannot be used. All lettering should be $^3/_{16}$ in. high for an 8 x 10 in. figure. Legends for figures should be tabulated on a separate sheet, not lettered on the illustrations.

Mathematics—Fractions should be indicated by a slanting line. Use standard symbols. Decimals not preceded by whole numbers should be preceded by zero, as 0.016. Equations may be written in ink with subscript numbers, radicals, etc., in the desired proportion.

Abbreviations—Write a.c. and d.c., kc, μf, $\mu\mu$f, emf, mh, μh, henries, abscissas, antennas. Refer to figures as Fig. 1, Figs. 3 and 4, and to equations as (5). Number equations on the right in parentheses.

Summary—The summary should contain a statement of major conclusions reached, since summaries in many cases constitute the only source of information used in compiling scientific reference indexes. Abstracts printed in other journals, especially foreign, in most cases consist of summaries from published papers. The summary should explain as adequately as possible the major conclusions to a non-specialist in the subject. The summary should contain from 100 to 300 words, depending on the length of the paper.

Publication of Paper

Disposition—All manuscripts should be addressed to the Institute of Radio Engineers, 33 West 39th Street, New York City. They will be examined by the Committee on Meetings and Papers and by the Editor. Authors are advised as promptly as possible of the action taken, usually within one month.

Proofs—Galley proof is sent to the author. Only necessary corrections in typography should be made. *No new material is to be added.* Corrected proofs should be returned *promptly* to the Institute of Radio Engineers, 33 West 39th Street, New York City.

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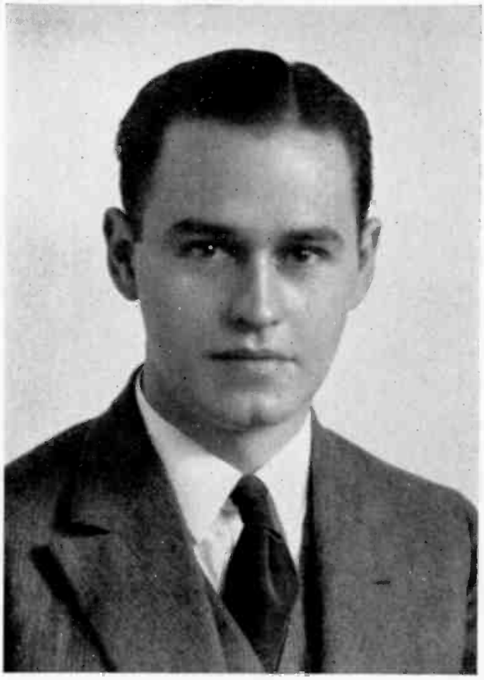

Lewis M. Hull
Member of the Board of Direction, 1929

Lewis M. Hull was born on February 27th, 1899 at Great Bend, Kansas. He graduated from the University of Kansas in 1917 and immediately entered the Radio Section of the Bureau of Standards. He was associated with the Bureau as assistant physicist, and later as associate and consulting physicist, at intervals until 1922. In 1922 he received the degree of Ph.D. in physics from Harvard University.

In 1922 Dr. Hull joined the Radio Frequency Laboratories of Boonton, N. J. as director of research. He is now vice president of that company.

Dr. Hull has served on a number of committees of the Institute, and was appointed Chairman of the Committee on Broadcasting in 1928. He was appointed to membership on the Board of Direction for 1929.

He became a Junior member of the Institute in 1917; Associate in 1919; Member in 1927; and Fellow in 1928. He is a Member of the American Institute of Electrical Engineers and the American Physical Society.

INSTITUTE NEWS AND RADIO NOTES

May Meeting of the Board of Direction

The May meeting of the Board of Direction was held on the first of the month in the office of the Institute in New York City. The following were present: A. Hoyt Taylor, John M. Clayton, Arthur Batcheller, Alfred N. Goldsmith, C. M. Jansky, Jr., R. H. Marriott, and L. E. Whittemore.

The following were transferred or elected to higher grades of membership in the Institute: elected to the Fellow grade: John A. Slee; transferred to the Member grade: James D. Durkee, Edwin B. Dallin, S. S. MacKeown, E. E. Cordrey, Edwin L. Powell, Paul D. Andrews, Leslie F. Curtis, E. G. Shalkhauser, F. L. Schoenwolf, O. W. Pike, John B. Brady; elected to the Member grade: Loy E. Barton, Lloyd A. Briggs, Mortimer Frankel, T. Kono, H. W. Dreyer, Frank N. Calver, E. J. Emery, James K. Nicholson, Alan G. Dryland, J. G. A. Sutherland.

One hundred and sixty-one Associate members and ten Junior members were elected.

The Board appointed C. W. Latimer as alternate to Captain Guy Hill as delegate to the World Engineering Congress to be held in Tokio in October. Captain Hill and Mr. Latimer will present the symposium paper prepared by six members of the Institute.

New York Meetings Suspended During July and August

Carrying out the usual custom there will be no New York meetings during the months of July and August. The Institute office will, however, be open for the transaction of business.

Most of the sections also will suspend their meeting activities during these two months.

Institute Meetings

Atlanta Section

Dr. W. G. Cady presented a paper, "Piezo-electric Crystals as Frequency Standards," before the Atlanta section on April 22nd. The meeting was held in the Physics Laboratory, Georgia Institute of Technology, Atlanta, Georgia, presided over by P. C. Bangs, acting chairman. There were in attendance thirty-four members and guests.

This paper dealt with the crystal itself, classified as to different types and crystal structure, and stressed the point that the symmetry and non-symmetry of the crystal structure was important.

The crystalline structure of Rochelle Salts was explained for the purpose of showing why Rochelle Salts crystals were not used as frequency standards. While such crystals oscillate freely, the structure itself is not adaptable to any great amount of pressure or voltage.

A detailed description was given in regard to the vibration of quartz plates. The important point brought out here was that a vibrating quartz plate takes the shape of a parallelogram when oscillating. This change of shape takes place with each cycle.

Different methods of cutting crystals were explained in detail. Differentiation was made between the Curie type crystal which has a wavelength of about 105 m per mm and the later type thirty-degree crystal which has a wavelength of about 150 m per mm. The later type oscillates more rapidly.

A general discussion followed the presentation of the paper.

Cleveland Section

Dr. M. Luckiesh, director of lighting research, National Lamp Works of General Electric Company, was the speaker at the Cleveland section meeting, which was held April 26th in the Physics Building, Case School of Applied Science, Cleveland, Ohio, and presided over by Bruce W. David, chairman of the section. There were forty-two members and guests in attendance. The paper presented by Dr. Luckiesh was entitled, "Radio Messages of Men, Atoms, and Stars." The speaker stressed the similarity of man's radio messages and the messages sent out in the form of radiated energy by everything in the universe from atoms to stars. The receiving set for these radiated messages is the spectroscope and the message is received through the eye and not the ear. Man's vision limits the number of wavelengths he receives to a relatively narrow band. Perhaps some day man will be able to see life on other planets and even see images from the earth reflected by some other planet but delayed by hours, days, or years, depending upon the distance the reflected light has travelled. A general discussion followed the presentation of this paper.

The Cleveland section held a meeting at the Ohio Insulator Co., Barberton, Ohio, on May 3, 1929. D. Schregardus, chairman, presided. Fifty-three members attended the meeting.

A. O. Austin, chief engineer of the Ohio Insulator Co., presented a paper, "High Voltage." A high voltage demonstration was made whereby high voltage testing methods were demonstrated in a million-volt indoor laboratory and a number of fundamental principles of high voltage discharges were discussed. In the outdoor laboratory demonstrations were made of actual power line construction using voltages as high as two or three million, as artificial lightning. The demonstrations were extremely impressive and represented the highest development in designing and testing high voltage equipment. A general discussion followed the presentation of the paper.

Detroit Section

F. A. Firestone, of the Physics Department of the University of Michigan, was the speaker at the January 19th meeting of the Detroit section. E. D. Glatzel, chairman, presided, and forty members and guests attended the meeting.

The paper, "Technique of Sound Measurements," by F. A. Firestone, explained the classical experiments of Prof. D. C. Miller. The recording of the wave form of sounds was explained with reference to the methods used for analysing the records into harmonic components and of synthesizing the components into the original wave form. The ability of the ear to distinguish intensities of tones of the same and different frequencies was discussed. Apparatus used in connection with the detection of undesirable noises in automobiles in motion was shown and demonstrated. This apparatus indicated the intensity of the total noise. It can be adjusted so as to indicate the intensity of a sound of a certain pitch separately from other pitches which may be present.

The annual election of officers was held with the following results: A. B. Buchanan, chairman; Lewis N. Holland, vice-chairman; Walter R. Hoffman, secretary-treasurer.

The Detroit section held a meeting February 15th in the auditorium of the Michigan Bell Telephone Building, Detroit, Michigan, presided over by A. B. Buchanan, chairman. Two hundred and seventy-five members and guests attended.

C. L. Carrington, division superintendent of Installation of Electrical Research Products, Inc., presented a paper, "Sound and Talking Picture Problems and Equipment." The two systems of recording and reproducing the sound of talking motion pictures developed by the company employing him were described, as well as some of the means of eliminating or minimizing bad acoustical conditions encountered in some of the theatres in which the equipment is being installed. A motion picture and a number of the units of equipment comprising the reproducing apparatus were exhibited.

Proposed Lehigh Valley Section

The third organization meeting of the proposed Lehigh Valley section was held April 12th in the Science Bldg., Muhlenberg College, Allentown, Pa. Carlton F. Maylott, temporary chairman, presided. Thirty one members and guests attended the meeting.

Carlton F. Maylott presented a paper, "The Electrical Reproduction of Speech and Music." The history of the talking machine and other recorded sound reproducing mediums and the various steps in their development were treated. A detailed description of the several commercial systems of sound motion pictures was presented and illustrated with blackboard diagrams. By means of an electrically driven turntable and a power amplifier a group of special disk recordings of the Bell Telephone Laboratories were reproduced through an auditorium type of moving-coil speaker, demonstrating the effect on speech and music of successively suppressing various frequencies in both the upper and lower tonal ranges. The committee was fortunate in obtaining for this meeting practically every distinctive type of loudspeaker marketed and this afforded a very interesting demonstration of the evolution of the radio reproducer, from the original crude horn type to the latest and most modern development of the reproducer used in conjunction with sound motion picture presentation.

Los Angeles Section

On April 15th the Los Angeles section held their meeting in the Elite Cafe, Los Angeles, Cal., presided over by T. C. Bowles, vice chairman. There were forty members and guests in attendance.

E. H. Butler presented a paper on "Vacuum Tubes and Their Applications to Radio." The paper covered especially the use of the new 45-type power tube as well as the use of the four-electrode tube as an r-f amplifier and power detector feeding directly into the power tube. The various merits of power tubes and also detectors of various types were discussed. There was quite a discussion regarding the merits of more r-f amplification with only one audio stage or the more common type of two-stage audio amplifier with conventional detector and r-f amplifier.

Philadelphia Section

The Philadelphia section held a meeting on April 25th at the Franklin Institute, Philadelphia, Pa. J. C. Van Horn, chairman, presided. One hundred and sixty members attended the meeting.

C. Francis Jenkins presented a paper, "Latest Phases of Television."

Messrs. Synder, Miller, Arnold, Van Horn, and others participated in the discussion which followed.

Pittsburgh Section

The Pittsburgh section held their regular meeting in the Fort Pitt Hotel, Pittsburgh, Pa. on March 19th. L. A. Terven, vice-chairman of the section, presided. The paper, "Recent Developments in Super Heterodyne Receivers," was presented by G. L. Beers. This paper was published in the March issue of the Proceedings.

Messrs. Allen, Wallace, Terven, Mag, Bricker, Sunnergren, and McKinley participated in the discussion which followed.

The Pittsburgh section held a meeting April 16th in the Fort Pitt Hotel, Pittsburgh, Pa. W. K. Thomas, chairman, presided. There were present twenty-nine members and guests.

Dr. G. A. Scott, of the University of Pittsburgh, presented a paper, "Mechanical Analogies of Electrical Conditions." This paper deals entirely with electro-mechanics. A comparison of the fundamental units of mechanical engineering was made to the fundamental units of electrical engineering through a demonstration of their direct similarity. A derivation of fundamental circuit formulas of electrical engineering was made by a substitution method of well known mechanical formulas.

Messrs. Terven, Hallie, Boardman, McKinley, Smith, Bordini, and Sunnergren participated in the discussion which followed.

The election of officers was held with the following results: L. A. Terven, chairman; A. J. Buzzard, vice-chairman; T. D. Cunningham, secretary; A. Mag, treasurer.

San Francisco Section

The San Francisco section held a joint meeting with the local section of the American Institute of Electrical Engineers, April 10th, at the Engineers' Club, San Francisco, Calif. D. B. Dexter, chairman of the San Francisco section of the American Institute of Electrical Engineers, presided. There were one hundred and sixty members and guests in attendance.

Harry R. Lubcke presented a paper, "Design Equations for Vacuum-Tube Voltmeters." This very interesting paper gave formulas for calculating rapidly the plate external resistor value, the C bias value and plate voltage value for a vacuum-tube voltmeter of any given scale range.

Elmer H. Fisher presented a paper, "Voltage Surges in Audio-Frequency Apparatus." It was shown by means of an oscillograph and an inverted vacuum-tube voltmeter that voltages up to seven hundred volts were developed on the secondary, and up to five hundred in the primary. Mr. Fisher measured the voltages developed in both the secondary and primary audio-frequency transformers when the plate circuit is opened.

F. J. Somers presented a paper, "Characteristics of Electrostatic Loud Speaker." The paper described the construction of a type of electrostatic loud speaker and the method of testing its frequency response in comparison to a standard Jensen Dynamic Cone Speaker. Curves of response of both speakers and a comparison curve of the electrostatic speaker were shown. A model was exhibited.

Washington Section

On April 11th, in the Continental Hotel, Washington, D. C., the Washington section held their regular meeting. F. P. Guthrie, chairman of the section, presided. Eighty-two members and guests attended the meeting.

A paper, "Recent Developments in Radio in Application to Aircraft," by J. H. Dellinger and H. Diamond, was presented by H. Diamond. The paper discussed the radio aids to air navigation being developed and installed on the civil airways of the Department of Commerce. These aids are: communication, course navigation, and fog landing. The paper dealt chiefly with recent progress effected in the double-modulation type directive radio beacon. Methods are given for making this beacon serve four independent courses at variable angles to each other. A new triple-modulation beacon is described whereby twelve beacon courses are obtained making it possible to provide beacon service in every desired direction simultaneously. Special adaptations of the beacon system are described which give promise of facilitating landing in fog.

Messrs. Guthrie, Mirick, Robinson, Worrall, and Dorsey participated in the discussion which followed.

The election of officers was held with the following results: C. B. Jolliffe, chairman; T. McL. Davis, vice-chairman; John B. Brady, secretary-treasurer.

Proceedings of the Institute of Radio Engineers
Volume 17, Number 6 *June, 1929*

GEOGRAPHICAL LOCATION OF MEMBERS ELECTED MAY 1, 1929

Elected to the Fellow Grade

England	London, SW7, 15 Rutland Mews	Slee, John A.

Transferred to the Member Grade

Arkansas	Conway, State Teachers' College	Cordrey, E. E.
California	Pasadena, California Institute of Technology	Mackeown, S. S.
Dist. of Columbia	Bellevue, Naval Research Laboratory	Dallin, Edwin B.
	Washington, Ouray Bldg.	Brady, John B.
	Washington, 6310 5th St. N. W.	Durkee, James D.
	Washington, 3100 Connecticut Ave., Apt. 105	Powell, Edwin L.
Illinois	Chicago, 1917 Warner Ave.	Schoenwolf, F. L.
	Peoria, 147 Cooper Ave.	Shalkhauser, Eric George
Massachusetts	Springfield, 162 Springfield St.	Curtis, Leslie F.
New York	Schenectady, 9 N. Ferry St.	Andrews, Paul D.
	Schenectady, River Road	Pike, O. W.

Elected to the Member Grade

Arkansas	Fayetteville, 143 S. Hill St.	Barton, Loy E.
Dist. of Columbia	Washington, c/o Japanese Military Attaché, Portland Hotel	Kono, T.
Illinois	Chicago, 430 S. Green St.	Frankel, Mortimer
New Jersey	Cranford, 31 Spruce St.	Briggs, Loyd A.
New York	Brooklyn, 1244 East 35th St.	Dreyer, Harry W.
England	Hull, c/o British Broadcasting Corp., 26 Bishop Lane	Calver, Frank N.
	London W1, Marconiphone Co., Ltd., 210 Totenham Court Road	Emery, E. J.
	Newcastle-on-Tyne, 5, Selborne Gardens, Jesmond	Nicholson, James K. A.
	Surrey, Windycot, Sugden Road, Long Ditton	Dryland, Alan G.
Straits Settlements	Singapore, c/o United Engineers, Ltd., Battery Road	Sutherland, J. G. A.

Elected to the Associate Grade

Arkansas	Little Rock, 224 Vernon St.	Boone, Virgil
"	Little Rock, 2510 West 16th St.	Bracken, Ben
California	Berkeley, 1645 Capistrano Ave.	Taylor, G. Wesley
	El Cajon, R. No. 1	Gruenwald, Harold C.
	Fullerton, 125 Ellis Place, Apt. 2	Hardison, Arthur A., Jr.
	Hemet, 311 E. Florida Ave.	Sorkness, Hardus
	La Habra, 480 East First St.	Inns, Stephen Henry
	Long Beach, 22 E. 4th St.	Howell, Roger A.
	Los Angeles, 3176 Verdugo Road	Arnold, Harry
	Los Angeles, 3433 La Clede St.	Borden, Charles
	Los Angeles, Western Air Express, 117 W. 9th St.	Hoover, Herbert, Jr.
	Los Angeles, 1619 N. Benton Way	Stolle, R. A.
	Marshall, c/o R. C. A.	Case, Myron D.
	San Francisco, National Broadcasting Co., 111 Sutter St.	Saxton, Alfred H.
	Santa Ana, 300 N. Broadway	Clark, William Moran
	Stanford University, Box 2287	Fisher, E. H.
	Venice, 650 California Ave.	Gleason, Sterling
Connecticut	New Haven, 66 Norton St.	Serig, Howard W.
Dist. of Columbia	Bellevue, Naval Research Laboratory	Bence, Clarence E.
	Bellevue, Naval Research Laboratory	Booth, H. A.
	Bellevue, Naval Research Laboratory	Ferree, John E.
	Bellevue, Naval Research Laboratory	Meyer, R. B.
	Bellevue, Naval Research Laboratory	Moore, Adam H.
	Bellevue, Naval Research Laboratory	Taylor, Samuel
	Bellevue, Naval Research Laboratory	Walker, Guy P.
	Washington, 502 Trenton St.	Curtis, Westley F.
	Washington, 1821 Riggs Place N. W.	Dent, Wm. U.
	Washington, 618 Morris St. N. E.	Doremus, Cornelius W.
	Washington, 508 Rittenhouse St.	Jett, Ewell K.
	Washington, 933 K St. N. W.	Johnson, R. Stanley
	Washington, 2842 Vista St. N. E.	Luke, E. L.
	Washington, 2316–32nd St. S. E.	MacGregor, John J.
	Washington, 226–8th St. S. E.	Noyes, Robert H.
	Washington, 1901 K St. N. W.	Stahl, Barton E.
	Washington, 1738 Mass. Ave. S. E.	Talty, William B.
	Washington, 5125–8th St. N. W.	Ullman, Mervin J.
	Washington, 1608–28th St. S. E.	Wiseman, William W.
Florida	Miami, Pan American Airways, Inc.	Sullinger, Ferris W.
Georgia	Atlanta, 585 Blvd. Place N. E. Apt. 8	Russell, B. A.
Illinois	Champaign, 804 South Prospect Ave.	Duncan, Don C.
	Chicago, 1247 So. Kedvale Ave.	Bank, Maurice

Illinois (cont'd)	Chicago, 3030 N. Halstead St.	Fitzgerald, T. C.
	Chicago, 1145 Bryn Mawr Ave.	Lotter, John G.
	Chicago, 2416 Bryn Mawr Ave.	Moore, James S.
	Chicago, 216 N. Michigan Ave.	Paradise, Maurice E.
	Evanston, 1634 Chicago Ave.	Lund, Russell O.
	Pekin, Box 233	McGarvey, Harold R.
	Rockford, 130 No. Madison St.	Westlund, J. V.
	Urbana, 212 West Vermont St.	Hayden, Gilbert
	Wilmette, 530 Laurel Ave.	Shabino, Clarke L.
Indiana	Marion, c/o Y. M. C. A.	Detrick, Harold M.
	West Lafayette, 330 N. Grant St.	Brown, Wilfred E.
Kansas	Hamlin	Herold, Jos. L.
Kentucky	Louisville, Argonne Hotel	Tillett, Jesse
Maryland	Baltimore, 333 S. Gilmar St.	Ault, Arthur K.
	Baltimore, 2811 Chelsea Terrace	Krebs, William N.
	Brentwood, 4424 Charles St.	Dennary, James E.
	Mount Rainier, P. O. Box 86	Eisenhauer, Harry D.
	Mount Rainier, 3900–31st St.	Russell, Philip T.
Massachusetts	Boston, 7 Goodwin Place	Crawford, John D.
	Brockton, 93 Pleasant St.	Costello, John J.
	Cambridge, 362 Harvard St.	Stark, L. P.
	Holyoke, 49 Cleveland St.	Clemm, Robert Carl, Jr.
	Roxbury, 64 Brunswick St.	Salowitts, Louis W.
	Stoneham, 1 Franklin Road	Brown, Ronald W.
	Worcester, 45 Benefit St.	Gleason, Albert E.
Michigan	Ann Arbor, 715 Church St.	Byram, F. Cameron
	Battle Creek, c/o American Steam Pump Co.	Kinch, Oscar A.
	Detroit, 4217 McClellan Ave.	Carson, Walter E.
	Detroit, 1550 Elmhurst	Green, Walter M.
	Detroit, 1753 West Philadelphia Ave.	Sturman, George G.
	East Lansing, Box 611	Richards, A. R.
	Jackson, Spanks-Withington Co.	Bowmaster, J. M.
	Lakeside, Berrien County	Meier, H. G.
Missouri	Kearney	Thomason, Marvin
	St. Louis, 2551 W. Florissant Ave.	Cassens, Fred Benjamin
	St. Louis, c/o Moloney Elec. Co., 5390 Kingshighway N. W.	Wooley, W. C.
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Volume 17, Number 6 *June, 1929*

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Part II

TECHNICAL PAPERS

Proceedings of the Institute of Radio Engineers
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RECIPROCITY IN ELECTROMAGNETIC, MECHANICAL, ACOUSTICAL, AND INTERCONNECTED SYSTEMS*

By
Stuart Ballantine

(Radio Frequency Laboratories, Inc., Boonton, New Jersey)

Summary—*Recent criticism by Carson of the statement of the reciprocal relations in a radio communication system given by Sommerfeld is supported by a simple example showing the incorrectness of this statement.*

Carson's proof of the extention of Rayleigh's reciprocity theorem to a general electromagnetic system was limited to $\mu=1$ *and to sources consisting of ponderomotive forces on the electricity. A new proof is given under more general conditions,* ϵ, μ, σ *being merely restricted to be scalars and the impressed forces are of the electric type introduced by Heaviside and Abraham. These may be regarded as impressed charges and currents, including ether displacement current. The theorem is finally stated in terms of volume and surface integrals, and thus combines the viewpoints of both Lorentz and Carson.*

The reciprocity relations in a mechanical system are reviewed.

The interconnection of electrical and mechanical systems is next considered and a "transduction coefficient" is defined. This concept is useful in formulating mechanical problems in electric-circuit form. An example of symmetrical transductance (copper coil in steady magnetic field) is given. The subject of units is taken up and it is proposed that in order to bring the mechanical quantities into agreement with the electrical ones when the latter are expressed in "practical" units, the mechanical quantities be expressed in "mechanical-volts," amperes, etc., i.e., in "practical-electric units of the mechanical quantities." A table for converting the principal mechanical quantities from c.g.s. to practical-electrical units is given.

Reciprocity is shown to exist in interconnected electro-mechanical systems with reversible transduction.

The equations of sound propagation in a gas are developed, regarding the velocity and excess pressure as the fundamental quantities. An acoustical reciprocity theorem involving these quantities is then proved.

Interconnections of mechanical and acoustical systems are then discussed and reciprocal relations are shown to exist in the composite system. Such relations are also valid for a system comprising electrical, mechanical and acoustical systems in series connection.

The reciprocity relations in a reversible electrophone are applied in a method of determining the frequency characteristic of an electrophone or microphone. This is called the "method of three electrophones." The overall transmission curves of the devices taken two at a time are measured and from these data the frequency characteristic of any one of them can be calculated. Only one of the electrophones is required to be reversible.

* Dewey decimal classification: R510. Original manuscript received by the Institute, February 26, 1929. Presented before joint meeting of the Institute and the International Union of Scientific Radiotelegraphy, American Section, at Fourth Annual Convention, Washington, D. C., May 15, 1929.

AT this meeting there has been presented a criticism by Mr. J. R. Carson of the Sommerfeld-Pfrang reciprocity theorem for radio communication, which had been reviewed by me.[1] Mr. Carson seems entirely justified in emphasizing the superior generality and fundamental character of his generalization of Rayleigh's theorem, and in challenging the scope and validity of the Sommerfeld-Pfrang theorem in the form stated by Sommerfeld. The incorrectness of the Sommerfeld statement in terms of radiated power and received electric force in the case of an antenna which does not act as a dipol may be illustrated by a simple example, privately communicated by Mr. Lester Hochgraf.

Consider two equal antennas A and B some distance apart, oriented as shown in Fig. 1, and excited at such a frequency that there are complete positive and negative current loops. These antennas radiate equal power. Sommerfeld's theorem states that the "received field-strength" (*empfangene Feldstärke*) at B due to the radiation of a certain

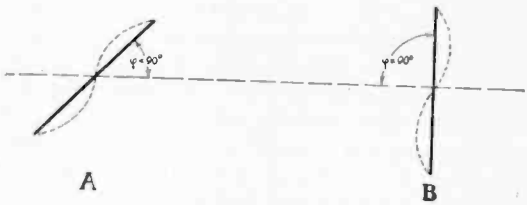

Fig. 1

average power from A will be equal to that at A with radiation of the same power from B. It will be seen, however, that the electric force produced at B by A is finite, whereas that produced at A by B is zero because A is in the equatorial plane of B and in this plane the radiation vanishes (at large distance).[2] This *reductio ad absurdum* seems sufficiently convincing and illustrates in a simple way the purport of Mr. Carson's criticism.

In passing it may be pointed out that the application of the theorem which was made in my review (loc. cit.) to the derivation of the transmitting properties of certain antennas over imperfect earth from their calculated receiving properties is in no way affected by this criticism. The only use to which the theorem was put there was to indicate the reciprocity existing between dipols or current elements, an application for which it is perfectly suitable. The theoretical conclusions of the

[1] Proc. I. R. E., 16, 513; April, 1928.
[2] Cf. Fig. 3, Ballantine: Proc. I. R. E., 12, 838; December, 1924.

review concerning aircraft communication have since been verified experimentally.

2. Carson's Proof of Theorem II. The generalization of Rayleigh's theorem to extended electromagnetic systems given by Carson[3] (Theorem II) is of considerable importance. Carson points out that his proof is restricted to a medium in which $\mu = 1$. We may also note that his impressed forces are ponderomotive forces on the electricity and do not act to produce a displacement current in the ether.

The $\mu = 1$ restriction resulted from Carson's feeling that the current u should be a linear function of E. This does not appear to be necessary, however, and the restriction may be removed by adding to G a term $1/\lambda \cdot \text{curl}\, M$. The justification for this step follows from the fact that the added term, combined with u, vanishes when integrated over all space.

3. Proof of the Reciprocity Relations in an Electromagnetic System. An alternative proof of slightly more general reciprocity relations in an extended electromagnetic system may be constructed along the following lines.

In the first place all the constitutive quantities ϵ, μ, σ are allowed to differ from unity and may vary from point to point. In the second place the impressed forces will be regarded as purely electrical forces acting upon the total current; that is, upon the etheric displacement current as well as upon the current due to the motion of electricity. In other words the impressed forces will not be restricted, as in Carson's discussion, to ponderomotive or mechanical forces on the electricity. Generalized impressed forces of this type were included by Heaviside[4] and by Abraham.[5] A force acting only on the electricity can be replaced by an impressed current; a generalized force of the type now postulated, acting also on the ether, is equivalent to impressed electric and ether displacement currents combined with an impressed charge distribution. Impressed forces of the Heaviside-Abraham type may be objected to as artificial. It is true that in certain simple cases, as for example that of a generator winding revolving in a steady magnetic field, the seat of the impressed emf may be definitely localized, but in many other cases such forces are useful as a cloak for our ignorance of the exact mechanism of generation. In still other cases we may have sufficient knowledge of the generating mechanism but may wish to avoid unnecessary discussion by including only the

[3] J. R. Carson, *Bell System Tech. Jour.*, 3, 393, 1924.
[4] Heaviside, "Electromagnetic Theory," Vol. I, p. 36, (London).
[5] Abraham Foeppl, "Theorie der Electrizitaet," 6th ed., Vol. I, p. 216, (Berlin: 1921).

terminals of the generator in the system. A shielded vacuum-tube generator would constitute an example of this. In such cases the idea of an impressed charge, with its attendant electric and ether displacement currents is perhaps more useful than the ponderomotive F forces acting directly upon the electricity.

Heaviside also introduced impressed magnetic forces and induction. The utility of these seems really questionable and they will be dispensed with here.

The medium is assumed to be linear and isotropic, that is, the constitutive quantities ϵ, μ and σ are scalars independent of the vectors, and the vectors which they connect are always in the same direction and linear vector functions of one another. No attempt will be made to prove that the reciprocity relations fail in the general case where these quantities are second-order tensors. Sommerfeld states, however, that in the case of propagation in a Faraday medium (e.g., ions in the earth's magnetic field) the reciprocal theorem is no longer valid. In this case the ϵ-tensor has the structure:

$$(\epsilon) = \begin{vmatrix} \epsilon_{11} & -i\alpha & 0 \\ i\alpha & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{vmatrix} \quad \epsilon_{11} = \epsilon_{22} \neq \epsilon_{33}$$

which is skew-symmetric. Leaving aside intuitive suspicions, so far as I know it has not been definitely shown by Sommerfeld or anyone else that in this case, or in the case where (ϵ) is symmetric (i.e., $\epsilon_{jk} = \epsilon_{kj}$), or in the still simpler case when all components except ϵ_{11}, ϵ_{22}, ϵ_{33} are zero, the reciprocal relations fail or cannot be properly formulated.

Denoting the impressed electric force by E_0, the field equations for the quasi-stationary state (i.e., $\partial/\partial t = i\omega$) may be written in Gaussian units

$$E_0 = \frac{c}{4\pi\lambda} \operatorname{curl} H - E \tag{1}$$

$$\operatorname{curl} E = -i\omega\mu H/c \tag{2}$$

$$\operatorname{div} \mu H = 0 \tag{3}$$

$$\operatorname{div} \epsilon E = 0 \tag{4}$$

$$\lambda = \sigma + i\omega\epsilon/4\pi. \tag{5}$$

Consider two independent distributions of impressed forces E_0', E_0'', and corresponding total currents C' and C'' ($C = \lambda(E + E_0)$). Multiply (1) by $C = c \operatorname{curl} H/4\pi$ of the other distribution and subtract; then

$$E_0'C'' - E_0''C' = \frac{c}{4\pi}(E'' \text{ curl } H' - E' \text{ curl } H''). \tag{6}$$

Remembering that

$$\begin{aligned} E'' \text{ curl } H' &= \text{div } (H' \times E'') + H' \text{ curl } E'' \\ -E' \text{ curl } H'' &= -\text{div } (H'' \times E') - H'' \text{ curl } E'. \end{aligned} \tag{7}$$

Substituting values of curl E from (2) we have after some cancellation:

$$E_0'C'' - E_0''C' = \frac{c}{4\pi}[\text{div } (E_1' \times H'') - \text{div } (E'' \times H')]. \tag{8}$$

This is a point relation. Upon integrating over a region of space and using the divergence theorem, the result may be formally summarized in the following theorem.

Electromagnetic Reciprocity Theorem

If E_0' and E_0'' are two independent distributions of impressed electric force which act on the total current (convection, conduction, polarization, and displacement current in the ether), and C' and C'' represent the total currents $[=(\sigma + i\omega\epsilon/4\pi)(E+E_0)]$ resulting respectively from the action of E_0' and E_0''; if all quantities vary as $e^{i\omega t}$, and if the properties of the medium ϵ, μ, σ are scalars and independent of the field vectors, then:

$$\iiint [E_0'C'' - E_0''C']dv = \frac{c}{4\pi}\iint [E' \times H'' - E'' \times H']_n ds \tag{9}$$

where the surface integral extends over the boundary of the region of the volume integration.

It may be inferred that the surface integral represents the effect of sources (i.e., E_0C) beyond it, so that if there are no sources outside the

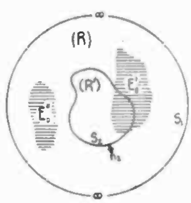

Fig. 2

region of integration the surface integral vanishes. For example, in Fig. 2 the region is shown bounded by a surface S_1 at infinity and a closed surface S_2. It is demonstrated in Note 1 that the S_1-integral vanishes. Hence

$$\iiint_R (\ )dv = -\iint (\ )_{n_2} ds \text{ (outward normal)} \tag{10}$$

but $\iiint_{R+R'} (\ )dv = 0$, hence $\iiint_{R'} (\ )dv = \iint_{S_2} (\ )_{n_2} ds$. This may be expressed in the following corollary:

Corollary I: *If under the conditions of the above theorem, the volume integration extends over all space and $\mu - 1$, $\epsilon - 1$, σ, and $\mathbf{E}_0$ are finitely distributed, then*

$$\iiint_\infty [E_0'C'' - E_0''C']dv = \frac{c}{4\pi}\iint_\infty [E' \times H'' - E'' \times H']_n ds = 0, \tag{11}$$

that is, the surface integral vanishes.

That this is true is almost evident by inspection when it is remembered that at a sufficient distance from a source the field vectors E and H become conjugate, that is, equal in magnitude and normal to each other and to the direction of propagation. To avoid interruption the detailed proof of this proposition is appended as Note 1 at the end of the paper.

The theorem in the form (9) involving both volume and surface integrals is more eloquent than the expressions of either Lorentz or Carson. Lorentz surrounded all sources with a surface integral; Carson used the volume integral over all space, assuming that all external influences could be accounted for by ponderomotive forces on the electricity. Theorem (9) combines both viewpoints. The surface integral is often useful in avoiding inquiries into complicated sources; it is simply necessary to surround the source by a surface and calculate $E \times H$ through it.

Equation (6) leads to another form. Since curl $H = 4\pi C/c$, C being as before the total current;

$$E_0'C'' - E_0''C' = E_0''C' - E'C'', \tag{12}$$

and

$$\iiint_\infty [E''C' - E'C'']dv = 0. \tag{13}$$

This does not involve the impressed forces E_0 and is a general relation.

Considering for a moment the case contemplated in Carson's theorem, in which the impressed forces are of the nature of ponderomotive forces on the electricity and do not act on the displacement current (which is equivalent to saying that there are no impressed charges), equation (1) would be written

$$E_0 = \frac{c}{4\pi} \text{curl } H - \left(\lambda' + \frac{i\omega}{4\pi} \right) E \tag{14}$$

where $\lambda' = \sigma + i\omega(\epsilon - 1)/c$. We should then have Carson's result (Theorem II)

$$\iiint_\infty [E_0' u'' - E_0'' u'] dv = 0. \tag{15}$$

Here u represents the current due to the motion of electricity. When we have to deal with impressed currents u_0 due to the motion of electricity (1) may be written

$$\text{curl } H = 4\pi\lambda E/c + 4\pi u_0, \tag{16}$$

and the following reciprocal relations obtain

$$\iiint_\infty [E'' u_0' - E' u_0''] dv = 0 \tag{17}$$

note that at the field vector E replaces E_0 in this expression. An example of such an impressed current would be furnished by electrons acted upon by mechanical force.

If in (11) we substitute $C = \lambda(E + E_0)$ then:

$$\iiint_\infty [E_0' C'' - E_0'' C'] dv = \iiint_\infty \lambda [E_0' E'' - E_0'' E'] dv, \tag{18}$$

which furnishes another useful relation involving only the E's.

4. Example of the Application of the Electromagnetic Theorem. Calculation of the Effective Height of a Receiving Antenna. The physical significance of the reciprocity relations expressed by equation (11) may be brought out by means of a practical example. The following application to the calculation of the effective height of a receiving antenna is due to R. M. Wilmotte[6] and is simple and instructive.

The impressed voltage acting upon an element dx of the antenna (Fig. 3) is the component along the wire of the electric vector in the wave. Identify this with E_0 in (11) and call it $E(x)$ where x is the distance along the antenna wire. Our problem is to find the value of a localized emf E, which when inserted in some branch of the electric

[6] R. M. Wilmotte, *Jour.* I. E. E., 66, 966, 1928. Communication from the National Physical Laboratory, Teddington, England.

circuit connected to the antenna terminals AB will produce in this branch the same current which is produced by the action of the distributed force from the wave. The reciprocity theorem states that in any element dx the current $I(x)$ produced by E will be equal to the

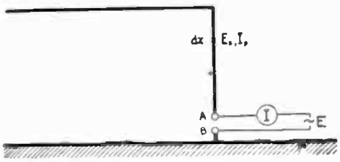

Fig. 3

current at the point of application of E due to an equal voltage $E(x)$ impressed at dx. Stated mathematically, for each element dx, $EI = E(x)I(x)$. This is true for each element of the antenna and is therefore true for the whole structure. Applying it to the whole antenna

$$E_0 I = \int_0^l E(x) I(x) \cdot dx. \tag{19}$$

We define the "effective height" as usual as $h = E/E'$; where E' is the electric force in the wave. If $E(x)$ is uniformly distributed along the antenna, $E(x)/E' = 1$. The possibility of this calculation rests upon knowing $I(x)$. Wilmotte,[7] in an important piece of experimental work, has verified in the case of an antenna of simple geometry, the approximate correctness of the old assumptions (Abraham, Pierce, etc.) of a sinusoidal distribution of current along the antenna.

In the case of a vertical antenna with E at the base, $I(x) = I \sin 2\pi x/\lambda / \sin 2\pi l/\lambda$, and

$$h = \frac{1}{\sin 2\pi l/\lambda} \int_0^l \sin \frac{2\pi x}{\lambda} \cdot dx = l \frac{4n_0}{\pi n} \frac{\sin^2 \pi n/4n_0}{\sin \pi n/2}, \tag{20}$$

where n_0 is the fundamental frequency and n the operating frequency. At the fundamental $n = n_0$ and $h = 2l/\pi$.

An interesting case is that of the short rod antenna proposed by F. H. Drake[8] for reception on airplanes in connection with a highly sensitive receiver. When the airplane is in horizontal flight the horizontal fusilage members contribute nothing, so that since $n = n_0$ the effective height is equal to half the actual height.

[7] R. M. Wilmotte, *Jour.* I. E. E., 66, 617, 1928.
[8] F. H. Drake, *Contributions from the Radio Frequency Laboratories* No. 8, October 1928; Proc. I. R. E., 17, 306; February, 1929; *Aero Digest*, October, 1928.

5. Mechanical Systems. By a mechanical system is meant a system composed of moving parts having mass, compliance, and resistance (dissipation), these parts being usually amenable to direct physical observation. Elastic solids, fluids and gases are really also mechanical systems, but for convenience will be treated as acoustical. The reciprocal relations in a mechanical or dynamical system which can be described by means of linear equations have been discussed by the late Lord Rayleigh.[9] Reciprocity exists between the displacements and forces, although it is often convenient, especially when employing electric-circuit analogies of the mechanical system, to replace the displacements by the velocities.

If a set of impressed forces F_1', F_2', $F_3' \cdots$ produce displacements x_1', x_2', $x_3' \cdots$, and another independent set of forces F_1'', F_2'', $F_3'' \cdots$ produce displacements x_1'', x_2'', $x_3'' \cdots$, then

$$F_1'x_1''+F_2'x_2''+\cdots=F_1''x_1'+F_2''x_2'+\cdots, \tag{21}$$

or generally

$$\sum_n (F_n'x_n''-F_n''x_n')=0. \tag{22}$$

If we are dealing with velocities the x's in (22) are simply replaced by v's.

The circumstances of validity in the mechanical system are the same as those for an electrical system as discussed in Sect. 3, viz. quasi-stationary state with quantities varying as $e^{i\omega t}$, linearity in the relations between forces and displacements, and complete reversibility. Reversibility should not be confused with friction. Failure of reciprocity would follow if elastic hysteresis or an equivalent phenomenon were present. Friction will also introduce non-linearity when the velocity is small. All these practical limitations must be taken into consideration in making applications of the theoretical theorems.

6. Electro-Mechanical Systems. Since there are reciprocal relations between impressed electric forces and currents in an electric system, and between impressed forces and velocities in a mechanical system, they can be shown to hold in an interconnected electromechanical system, provided that the *interconnections* are linear and reversible.

In considering such composite systems I have found the use of an *interaction*, or *transduction coefficient* convenient. This may be defined as follows: let F be the mechanical force acting upon an element of a mechanical system as the result of a current I flowing in an element of

[9] Rayleigh, "Theory of Sound," Art. 108, (London: 1877).

an electrical system. Then the *electro-mechanical transduction* T_{em} between these elements is defined as

$$F_0 = T_{em} I .$$

Similarly if E_0 represents the electromotive force due to an element of a mechanical system moving with velocity v, then the *mechano-electrical transduction* T_{me} is defined by

$$E_0 = T_{me} v .$$

If the transformation is reversible $T_{em} = T_{me}$; and if the interaction is linear T is independent of I or v. The dimensions of T are $[M^{1/2}L^{-3/2}]$. The "mutual energy" is equal to 1/2 TIv.

This concept is of considerable utility. Recent years have seen an increasing use of electrical analogues of mechanical systems. By this means we can bring to bear upon mechanical problems a great deal of knowledge concerning electrical networks which has resulted from the economic necessity for studies of electrical problems. In such formulations the role of the transduction coefficient is something like that of a mutual inductance between the real electrical circuit and the electrical analogue of the mechanical system.

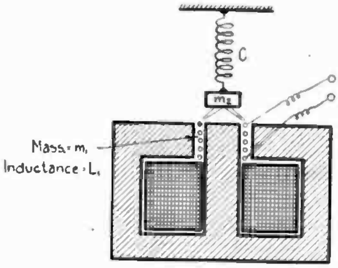

Fig. 4

As a simple example of this point of view consider the electromechanical system shown in Fig. 4. The coil is situated in a uniform magnetic field, H_0. We shall neglect eddy currents. The ponderomotive force in each element of volume is, generally,

$$F_0 = \frac{1}{c}[I \times B] - \frac{1}{8\pi} H^2 \text{ grad } \mu . \tag{23}$$

In the above case if the coil is of copper $B = H$, $\mu = 1$, so that

$$F_0 = \frac{1}{c}[I \times H_0] \tag{24}$$

on each element of the coil. Hence $T_{em}=H_0/c$. In the mechano-electrical process the induced emf due to a motion of an element of the wire with velocity v is

$$E_0=\frac{1}{c}[v\times H_0] \tag{25}$$

so that $T_{me}=H_0/c$. The processes are thus reversible and the transductance is symmetrical, $T_{me}=T_{em}$.

The subject of units deserves some attention. If E_0 is expressed in volts, I in amperes, F in dynes, and v in cm per sec., then

$$\begin{aligned} F_0 &= 10^{-1}IH_0 \text{ dynes amp} \\ E_0 &= 10^{-8}vH_0 \text{ volts, cm/sec} \end{aligned} \tag{26}$$

therefore $T_{em}=10^{-1}$; $T_{me}=10^{-8}$. T_{me} can be adjusted to have the same numerical value as T_{em} when E and I are expressed in practical units by expressing F and v in appropriate units. We shall call these units in this particular case mechanical volts and mechanical amperes, respectively, or alternatively, practical electric units of mechanical force and velocity. A collection of the principal electro-mechanical analogues and of conversion factors from c.g.s. mechanical units to "practical electric" mechanical units is given in Table I.

TABLE I

Table for conversion of mechanical quantities expressed in c.g.s. units to units based upon the practical electrical system as may be required in the formulation of electrical analogues of mechanical systems.

Mechanical Quantity	Symbol	Definition	Electrical Analogue	c.g.s. Unit	Value in Practical Electrical units
displacement	x	x	charge	1 cm	10^{-8} mech. coulombs
force	F	F	voltage	1 dyne	10 mech. volts
velocity	v	dx/dt	current	1 cm/sec	10^{-8} mech. amps.
energy	W	Fx	energy	1 erg	10^{-7} joules.
power	P	Fv	power	1 erg/sec	10^{-7} watts
resistance	R	F/v	resistance	1 dyne/cm/sec	10^{9} mech. ohms.
mass	m	$F/i\omega v$	inductance	1 gram	10^{9} mech. henrys.
compliance	C	x/F	capacitance	1 cm/dyne	10^{-9} mech. farads
stiffness	s	F/x		1 dyne/cm	

According to this point of view the sytem in Fig. 4 may be represented as shown in Fig. 5. The transduction is represented by T, and symbolically as a mutual inductance. In this "mutual inductance" representation only the inductance directly concerned with the interaction is included in the primary coil and only the masses of the mechanical elements associated with the transfer are included in the secondary "coil." The so-called *motional impedance* of the system is easily seen to be the impedance between the electrical terminals AB, and the *damped impedance* that between AB with the secondary open.

This representation in familiar electric form enables us to see at a glance what reactions the mechanical system will have upon the electrical system, and conversely the effect of the electrical circuit upon the motion of the mechanical system when an impressed mechanical force is acting. The latter effect is usually neglected by writers on this subject but obviously becomes of importance when the transfer is high. The conditions for maximum transfer between the systems can also be immediately formulated.

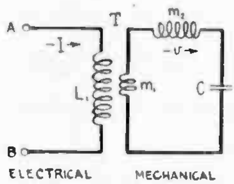

Fig. 5

The demonstration of the reciprocal relations in the interconnected system now proceeds along the usual lines. Let a set of impressed electric forces E_1', $E_2' \cdots E_n'$ act in the electrical system, and a set of mechanical forces F_1', F_2', $\cdots$, F_n' act in the mechanical system; let the corresponding currents in the electrical system be I_1', I_2', $\cdots$, I_n' and the velocities in the mechanical system be v_1', v_2', $\cdots$; let also F'', E'', I'', v'' represent a second set of such forces, currents and velocities. All quantities vary as $e^{i\omega t}$.

$$
\begin{aligned}
E_1 &= I_1 z_1 + I_2 m_{12} + I_3 m_{13} + \cdots v_1 T_{11} + v_2 T_{12} + \cdots \\
E_2 &= I_1 m_{12} + I_2 z_2 + I_3 m_{23} + \cdots v_1 T_{21} + v_2 T_{22} + \cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
F_1 &= I_1 T_{11}' + I_2 T_{12}' + I_3 T_{13}' + \cdots v_1 Z_1 + v_2 M_{12} + \cdots \\
F_2 &= I_1 T_{21}' + I_2 T_{22}' + I_3 T_{23}' + \cdots v_1 M_{21} + v_2 Z_2 + \cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
\qquad (27)
$$

Here T_{jk} represents the electric force in mesh j due to velocity v_k of mechanical mesh k, and T_{jk}' represents the mechanical force in mechanical mesh j due to the electric current in mesh k; z and m represent electrical self and mutual impedances, and Z and M represent corresponding mechanical impedances.

Set up another system of equations for the other set of forces E'' and F''. Multiply line 1 of (27) by I_1'', line 2 by I_2'' and so on, and line 1 of the second set by I_1', etc., and subtract. Then if the connections are perfectly reversible, that is $M_{jk}=M_{kj}$; $m_{jk}=m_{kj}$, and $T_{jk}=T'_{jk}$, we obtain the following general reciprocal relations

$$\sum_n E_n'I_n''+F_n'v_n''=\sum_n E_n''I_n'+F_n''v_n'. \tag{28}$$

Considering the simple case in which all the forces except two are zero

$$E'I''=F''G', \tag{29}$$

which is an expression of simple reciprocity. In words this states that: if in an interconnected electro-mechanical system a unit electric force impressed in an electrical mesh produces a certain velocity in a mechanical mesh, then an equal mechanical force acting in the mechanical mesh will produce a current in the electrical mesh which is numerically the same as the velocity previously produced in the mechanical mesh.

Equation (28) is correct as it stands if F is expressed in dynes, v in cm per sec., and E and I are expressed in electromagnetic c.g.s. units, or electrostatic c.g.s. units. In order to be correct when E and I are expressed in volts and amperes, F must be expressed in *mechanical volts* and v in *mechanical amperes* as above. If E and I are in volts and amperes, and F and v are in dynes and cm per sec., then a factor of 10^7 will occur, that is

$$E'I''=10^7F''v'. \tag{30}$$

In the more general expression (28) this factor will also precede Fv whenever they occur.

A word may be said as to the linearity and reversibility in practical interconnections between electrical and mechanical systems. In the example of Fig. 4 reversibility may be safely assumed, but in general when the system contains ferro-magnetic materials, hysteresis and the non-linearity of the $B-H$ relation demand some caution. When the amplitudes are small the higher order effects can generally be ignored. Eddy currents not depending upon magnetic behavior will not disturb the reversibility or linearity.

7. Acoustical Systems. Turning now to the propagation of sound of small amplitude in air, it will be convenient for reference purposes to preface the discussion of the reciprocal relations by a brief statement of the principal equations. These will be given in modern vector form and the excess pressure p and velocity v will be regarded as the fundamental quantities, analogous to E and H of the electromagnetic field and F and v of the mechanical system.

If we ignore viscosity and have a ponderomotive force $\mathbf{F}_0$ per unit mass acting upon the body of the gas, the motion may be described by Euler's equation

$$F_0 = \frac{\partial v}{\sigma t} + \frac{1}{2} \text{ grad } v^2 - v \times \text{ curl } v + \frac{1}{\rho} \text{ grad } P. \tag{31}$$

In addition we have the equation of continuity

$$\frac{\partial \rho}{\partial t} + \rho \text{ div } v + v \text{ grad } \rho = 0. \tag{32}$$

If the motion is irrotational, curl $v=0$, and if the amplitudes are small we may neglect the terms grad $(v \cdot v)$ in (31) and $v \cdot$grad ρ in (32); so that

$$F_0 = \partial v/\partial t + \text{grad } P/\rho; \tag{33}$$

$$\partial \rho/\partial t + \rho \text{ div } v = 0. \tag{34}$$

A further relation between ρ and P is furnished by considering the adiabatic expansion and compression of the gas

$$P/P_0 = (\rho/\rho_0)^\gamma, \tag{35}$$

where for air $\gamma = 1.4$, and P_0 and ρ_0 represent the steady values of these quantities. Now eliminate P from (31) and (32) by means of (35) and let the excess pressure $p = P - P_0$; then

$$F_0 = \partial v/\partial t + \text{grad } p/\rho_0, \tag{36}$$

$$\text{div } v = -\frac{1}{\gamma P_0} \frac{\partial p}{\partial t}. \tag{37}$$

These equations and the boundary conditions are sufficient to determine p and v. Take the divergence of (36)

$$\text{div } F_0 = \frac{\partial}{\partial t} \text{ div } v + \frac{1}{\rho_0} \nabla^2 p = -\frac{1}{\gamma P_0} \frac{\partial^2 p}{\partial t^2} + \frac{1}{\rho_0} \nabla^2 p_0, \tag{38}$$

from (37) so that p satisfies the wave-equation

$$\nabla^2 p - \frac{\rho_0}{\gamma P_0} \frac{\partial^2 p}{\partial t^2} = \rho_0 \text{ div } F_0. \tag{37}$$

Similarly, taking the gradient of (37)

$$\text{grad div } v = \nabla^2 v - \frac{1}{\gamma P_0} \frac{\partial}{\partial t} \text{ grad } p = \frac{\rho_0}{\gamma P_0} \left(\frac{\partial^2 v}{\partial t^2} - \frac{\partial F_0}{\partial^{\text{·}}} \right), \tag{40}$$

so that v also satisfies the wave-equation

$$\nabla^2 v - \frac{\rho_0}{\gamma P_0} \frac{\partial^2 v}{\partial t^2} = -\frac{\rho_0}{\gamma P_0} \frac{\partial F_0}{\partial t}. \tag{41}$$

The energy relations involving these quantities are interesting. Multiply (36) by $v\rho_0$ to get the rate at which work is done by the impressed force $\boldsymbol{F}_0$ per unit volume:

$$\rho_0 F_0 v = \rho_0 v \partial v/\partial t + v \text{ grad p}. \tag{42}$$

But $v \cdot \text{grad } p = \text{div } (v \cdot p) - p \text{ div } v$. Substituting the value of div v from (37) and integrating over a region of space

$$\underset{(1)}{\iiint \rho_0 F_0 v dv} = \frac{\partial}{\partial t} \iiint \left[\underset{(2)}{\frac{1}{2}\rho_0 v^2} + \underset{(3)}{\frac{1}{2}\frac{1}{\gamma P_0}p^2} \right] dv$$

$$+ \underset{(4)}{\iint p \cdot v_n ds}. \tag{43}$$

This states that the rate at which work is done by the impressed forces (1) is equal to the rate of increase of the kinetic energy (2) plus the rate of increase of the energy of compression (3) plus the rate at which energy is escaping across the boundary (4). The flow of acoustic power is represented by the vector Pv, which is therefore analogous to the Poynting-vector $E \times H$ of the electromagnetic theory.

Reciprocal relations in acoustics have been stated by Helmholtz.[10] We shall, however, derive the relations anew and in somewhat greater generality in terms of the quantities F, p, and v which are here regarded as fundamental, rather than in terms of the "sources" and velocity-potential of the historical treatment.

Consider two independent sets of impressed forces F_0', F_0'', pressures p', p'' and velocities v', v''. Multiply (36) by the v of the other set and subtract

$$F_0'v'' - F_0''v' = \frac{1}{\rho_0}(v'' \text{ grad } p' - v' \text{ grad } p''), \tag{44}$$

in the quasi-stationary state (all quantities varying as $e^{i\omega t}$). But

$$v \text{ grad } p = \text{div } (vp) - p \text{ div } v;$$

hence, substituting the value of div v from (37),

$$\rho_0(F_0'v'' - F_0''v') = \text{div } (v''p' - v'p''). \tag{45}$$

This is a point relation. The result of integration over a region of space with the employment of the divergence theorem may be expressed in the following theorem.

[10] Lord Rayleigh, "Theory of Sound"; Vol. 2, p. 132, (London: 1878).

ACOUSTICAL RECIPROCITY THEOREM

If in an acoustical system comprising a medium of uniform density ρ_0 propagating irrotational vibrations of small amplitude, a distribution of impressed ponderomotive force F_0' per unit mass per unit volume varying as $e^{i\omega t}$ produces a velocity v' and pressure p', and a second distribution of impressed force F'' produces velocity v'' and pressure p'', then

$$\rho_0 \iiint (F_0'v'' - F''v')dv = \iint (v''p' - v'p'')_n ds, \qquad (46)$$

where the surface integral is to be taken over the boundaries of the region of volume integration.

Corollary I: *If the volume integral extends over all space and the distributions are finite:*

$$\iiint_\infty (F_0'v'' - F_0''v')dv = 0. \qquad (47)$$

The surface integral, as in the electrical theorem, represents the effect of impressed forces or other sources outside the region of volume integration, or those introduced at the boundaries. If there are no such sources the surface integral vanishes. This proposition is demonstrated in Note 2 at the end of the paper. Either the volume or the surface integral may be used, the surface integral being then taken over surfaces enclosing all sources.

The irrotational waves considered above are of a simple "compressional" type.

Body forces of the type F_0 postulated above are conceivable but are seldom encountered in practical acoustical problems. What we are generally concerned with are boundary forces arising from the motion of a mechanical boundary or discontinuity. The transfer of energy to the acoustical system may also be achieved by a thermodynamic process, as in the thermophone. An example of the F_0 type of body force would be the direct electrical excitation of an ionized gas; an example of the boundary type of force would be provided by a moving diaphragm, or piston. In this case the reciprocal relations are exhibited by the surface integral, that is

$$\iint (p'v'' - p''v')_n ds = 0. \qquad (48)$$

Either p or v may be regarded as "impressed." In the case of the radiation of sound from a vibrating piston, for example, either the impressed mechanical force on the piston or its velocity may be specified.

The above demonstration relates to sound propagation in a gas and should be supplemented by a similar demonstration for elastic solid and liquid media since the gas system may have such boundaries. The equations and the proof for irrotational vibrations in these media are the same as those given if the compressibility of the gas $(1/\gamma P_0)$ is replaced by the reciprocal of the bulk modulus $1/k$ for the solid or liquid, or $1/(\lambda+2\mu)$ for the solid.

It may be remarked in passing that a special type of force is required to set up pure rotational waves (transverse or shear waves) in an isotropic elastic solid, and in general this type of vibration will not be excited in the solid by an incident irrotational wave from the gas.

8. Mechano-Acoustical Systems. Let us now consider the reciprocal relations in interconnected mechanical and acoustical systems. A mechanically driven piston is an example of such interconnection. This sort of transduction differs from that between an electrical circuit and a mechanical system in that the mechanical and acoustic systems are really both mechanical. It is, in fact, more nearly analogous to the interconnection of an electrical network and an electrical space-

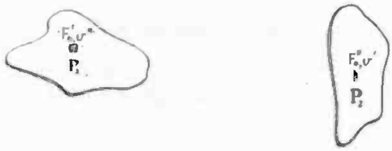

Fig. 6

radiation or receiving system, both of these processes being electrical. The forces in the acoustic system are mechanical forces, and the velocities are physical velocities, and both are measured in the same system of units which is used for the corresponding mechanical quantities. An extended argument is therefore not necessary to establish reciprocity in the connected system, and it is merely necessary to identify p and v of (46) with the force and velocity of the mechanical system. The acoustic pressure is the force per unit area of the mechanical system, and the velocities must be the same.

As an example of reciprocity in such a system consider two diaphragms A and B (Fig. 6). If we impress on a unit area at P_1 a force F_0' the velocity v' produced at a point P_2 of the other diaphragm will be equal to the velocity v'' which would be produced at P_1 if an equal force F'' were impressed on unit area at P_2. An important use of this theorem is the deduction of the receiving properties of a sound transducer from its transmitting properties, and vice versa.

9. Interconnected Electrical, Mechanical and Acoustical Systems. Reciprocal relations have been shown to hold in an electro-mechanical system and in a mechano-acoustical system, therefore they hold in a composite system involving all three systems in interconnection in

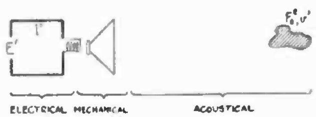

Fig. 7

this order. (Reversible transduction directly between electric and acoustical systems is rare and a mechanical system will usually intervene between them.) The reciprocity in such a composite electric-mechanic-acoustic system as shown in Fig. 7 may be expressed as follows, F_0 being an acoustical body-force

$$\iiint_{\infty} \rho_0 F_0{}'' v' w = E' I''. \tag{49}$$

If we deal with *pressure* in the sound field, p'' will replace $\rho_0 F_0{}''$. In words: if a generator of emf E' (in em or es c.g.s. units) operates in an electrophone circuit and produces at a point P in the sound field a velocity v', then a numerically equal force in dynes exerted at P on the gas material per unit volume, or communicated as a pressure, will

Fig. 8

produce a current I'' in the electrical mesh equal to the previously produced velocity v' at P.

The transduction must of course be reversible. An electromagnetic or electrostatic device (e.g., condenser microphone) would probably be sufficiently reversible, but a thermophone would certainly violate the requirements because of the irreversibility of the Joulean effect.

As a further extension of the composite system we may set up two reversible electrophones, as shown in Fig. 8, and state that in the terminating electrical circuits

$$E'I'' = E''I'. \tag{50}$$

The reciprocal relations are then the same as if the system were entirely electrical.

10. Example of the Application of the Extended Reciprocity Relations. Frequency Calibration of an Electrophone. The above extended reciprocity relations between two reversible electrophones has an interesting and useful application in a method for determining the frequency response characteristics of an emitting electrophone or a receiving microphone. This may be called the "method of three electrophones" and is analogous to the method of measuring the resistance of a ground by the method of three grounds, or of the effective height of an antenna by using two additional antennas. The frequency characteristic of the electrophone usually desired is that in a certain direction, say normal to the sound-emitting area. (Characteristics in other directions may obviously be obtained in the same way.)

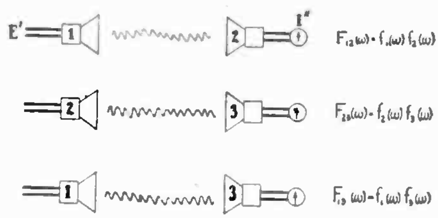

Fig. 9

The electrophones are used two at a time (Fig. 9) and are situated so far apart that the wave-front is substantially plane and the interaction between them is negligible. The purpose of this is to obtain data which will be fundamentally related to plane waves; investigations based on divergent waves may be carried out by proper adjustments of separation. One unit is excited by a generator E' of variable frequency and the current (or equivalent) in the other is measured. Let the frequency characteristics of the individual electrophones with respect to plane waves be denoted by $f_1(\omega)$, $f_2(\omega)$ and $f_3(\omega)$, respectively. Then for transmission from No. 1 to No. 2, for example, the measurement will give $F_{12}(\omega) = f_1(\omega)f_2(\omega)$. The transmission characteristic of No. 2 to No. 3 (F_{23}) is next obtained, and then that from No. 1 to No. 3 (F_{13}). This experimental information suffices to determine f_1, f_2, or f_3. For

$$\begin{aligned} F_{12}(\omega) &= f_1(\omega)f_2(\omega) \\ F_{23}(\omega) &= f_2(\omega)f_3(\omega) \\ F_{13}(\omega) &= f_1(\omega)f_3(\omega), \end{aligned} \tag{51}$$

which is a set of simultaneous functional equations to be solved for the f's. The solutions are

$$\begin{aligned} f_1(\omega) &= (F_{12}F_{13}/F_{23})^{1/2} \\ f_2(\omega) &= (F_{12}F_{23}/F_{13})^{1/2} \\ f_3(\omega) &= (F_{13}F_{23}/F_{12})^{1/2}. \end{aligned} \qquad (52)$$

If absolute values for the f's are desired care must be taken to measure properly the F's for each pair. For most purposes, however, all that is required is a curve of the relative variations of $f(\omega)$ with frequency; in this case absolute values of the F's are of no importance.

It may be noted that in the special case of two electrophones having identical frequency characteristics it is merely necessary to measure the frequency-transmission F for this pair since $F_{12}(\omega) = f_{12}(\omega)f_{21}(\omega)$ and $f_1 = (F_{12})^{1/2}$. This match will rarely be sufficiently exact in practical apparatus, except perhaps with precision microphones of the air-damped variety which have been accurately made and adjusted as to membrane tension.

It is also worth noticing that this method of calibration is equivalent to a Rayleigh-disk calibration in that it also takes into account the effect of reflection by the microphone.[11]

As will be seen from Fig. 9 this application of reciprocity requires only one of the electrophones (No. 2) to be *reversible*. Also note that one of them, No. 1, is always used as a source, and that the other, No. 3, is always used as a receiver. Thus a thermophone (irreversible) might be used at No. 1 as a source, with a reversible electrophone No. 2 as a comparator, to calibrate a carbon microphone, No. 3 (irreversible). It is needless to say that in order to obtain fundamental data the work should be performed in the open air and at a sufficient height to avoid disturbance due to reflection from the earth.

Note 1

It is required to evaluate the integral

$$\iiint_{\infty} [E_0'C'' - E_0''C']dv. \qquad (1)$$

To do this we shall make use of the electromagnetic potentials A and ϕ, between which and the field vectors the following well-known relations exist

[11] For an account of this effect and of another method for evaluating it, see Stuart Ballantine, *Contributions from the Radio Frequency Laboratories*, No. 9, December 1928; *Phys. Rev.*, **32**, 988, 1928; Proc. I. R. E., **16**, 1639; December, 1928.

$$B = \text{curl } A; \quad E = -\text{grad } \phi - i\omega A/c. \tag{2}$$

also $$\Delta^2 A - \left(\frac{i\omega}{c}\right)^2 A = \text{dal } A = -i\omega E_0/c - 4\pi u/c - \text{curl } H \tag{3}$$

$$\text{dal } \phi = -4\pi\rho \tag{4}$$

$$\text{div } A = -i\omega\phi/c \tag{5}$$

$$M = (\mu - 1)H; \quad B = \mu H \tag{6}$$

u is the current due to the motion of electricity; ρ is the charge (exclusive of impressed charge, $\rho_0 = \text{div } E_0/4\pi$). Equation (6) of Sect. 3 may be written, since curl H = curl B − curl M:

$$E_0'C'' - E_0''C' = \frac{c}{4\pi}(E'' \text{ curl } B' - E' \text{ curl } B'' - E'' \text{ curl } M' + E' \text{ curl } M''). \tag{7}$$

Substituting for E and B their values (2) in terms of the potentials, we have

$$E_0'C'' - E_0''C' =$$

$$\begin{aligned}
&(1) \quad \text{grad } \phi'' \text{ dal } A' - \text{grad } \phi' \text{ dal } A'' \\
&(2) \quad + \frac{i\omega}{c}(A''\Delta^2 A' - A'\Delta^2 A'') \\
&(3) \quad + \text{grad } \phi'' \text{ curl } M' - \text{grad } \phi' \text{ curl } M'' \\
&(4) \quad + \frac{i\omega}{c}(A'' \text{ curl } M' - A' \text{ curl } M'').
\end{aligned} \tag{8}$$

We will consider these terms in order. Substitute in (1) the value of dal A from eq. (3), then

$$(1) \quad = -\text{grad } \phi''\left(\frac{4\pi u'}{c} + \frac{i\omega E_0'}{c}\right) + \text{grad } \phi'\left(\frac{4\pi u'}{c} + \frac{i\omega E_0'}{c}\right) \tag{a}$$

$$-\text{grad } \phi'' \text{ curl } M' + \text{grad } \phi' \text{ curl } M''. \tag{b}$$

Note that term (1b) cancels term (3). Transform (a) by means of the theorem

$$\iiint D \text{ grad } \psi dv = \iiint \text{div } (\psi D) dv - \iiint \psi \text{ div } D \cdot dv$$

and since div $\mathbf{C}=0$

$$(1a)\quad =\frac{-i\omega}{c}\iiint(\varphi''\,div\,\mathrm{E}'-\varphi'\,div\,\mathrm{E}'')dv$$

$$+\frac{i}{c}\iint[\phi'(4\pi\boldsymbol{u}''+i\omega\boldsymbol{E}_0'')-\phi''(4\pi\boldsymbol{u}'+i\omega\boldsymbol{E}_0')]_n ds.$$

If the integration embraces all space the integral over the infinitely distant surface vanishes for a finite distribution of $\boldsymbol{E}_0$ and $\boldsymbol{u}$. As to the volume integral, note that div $\boldsymbol{E}=4\pi\rho$, the value of which is given by (4) above. Making this substitution and applying Green's theorem

$$(1a)\quad =\frac{i\omega}{c}\iiint(\phi''\Delta^2\phi'-\phi'\Delta^2\phi'')dv$$

$$=\frac{i\omega}{c}\iint\left(\phi''\frac{\partial\phi'}{\partial n}-\phi'\frac{\partial\phi''}{\partial n}\right)ds.$$

If the surface is at infinity, $\partial/\partial n=-i\omega/c$ so that this term also vanishes. Terms (1)+(3) are therefore $=0$.

Term (2) vanishes for similar reasons. Applying Green's theorem

$$(2)\quad =\frac{i\omega}{c}\iiint(A''\Delta^2A'-A'\Delta^2A'')dv$$

$$=\iint\left(\frac{\partial A'}{A\partial n}-A'\frac{\partial A'}{\partial n}\right)ds.$$

This can be regarded as a scalar equation for each component of $\boldsymbol{A}$. As before, $\partial/\partial n=-i\omega/c$ for the infinitely distant surface so that the integral over all space is zero.

The remaining term (4) may be transformed by means of the theorem

$$\iiint \boldsymbol{P}\,\mathrm{curl}\,\boldsymbol{Q}dv=-\iint(\boldsymbol{P}\times\boldsymbol{Q})_n ds+\iiint\boldsymbol{Q}\,\mathrm{curl}\,\boldsymbol{P}dv$$

so that

$$(4)\quad =\frac{i\omega}{c}\iiint(\boldsymbol{M}'\,\mathrm{curl}\,\boldsymbol{A}''-\boldsymbol{M}''\,\mathrm{curl}\,\boldsymbol{A}')dv$$

$$-\iint(\boldsymbol{A}''\times\boldsymbol{M}'-\boldsymbol{A}'\times\boldsymbol{M}'')_n ds.$$

If, as before, the distribution of $\mu-1$ is finite the surface integral vanishes. As to the volume integral, since curl $A=B$ and $M=(\mu-1)B/\mu$

$$(4) \qquad = \frac{i\omega}{c}\iiint \frac{\mu-1}{\mu}(B'B''-B''B')dv=0.$$

This completes the demonstration that the integral (1) vanishes when taken over all space.

Note 2

It is required to show that in the acoustical case

$$\rho_0 \iiint (F_0'v''-F_0''v')dv = \iint (v''p'-v'p'')_n ds \qquad (1)$$

generally, and if the integration is over all space the result is equal to zero. Let there be two distributions of impressed force F_0' and F_0'' of the type $e^{i\omega t}$. Then according to equation (49)

$$\Delta^2 v' - k^2 v' = AF_0' \qquad (2)$$

$$\Delta^2 v'' - k^2 v'' = AF_0'' \qquad (3)$$

where $k^2 = -\rho_0\omega^2/\gamma P_0$ and $A = -i\omega\rho_0/\gamma P_0$. Multiply (2) by v'' and (3) by v' and subtract

$$A(F_0'v''-F_0''v') = v''\Delta^2 v' - v'\Delta^2 v''. \qquad (4)$$

Integrate this over a region of space and transform the right-hand side by means of Green's theorem

$$A\iiint (v''\Delta^2 v' - v'\Delta^2 v'')dv = \iint \left(v''\frac{\partial v'}{\partial n} - v'\frac{\partial v''}{\partial n}\right)_n ds. \qquad (5)$$

If we consider an integral over all space and take the surface to be that of a sphere, $\partial/\partial n = -i\omega/c$, and the surface integral vanishes, giving

$$\iiint (F_0'v''-F_0''v')dv=0. \qquad (6)$$

The properties of the medium P_0 and ρ_0 have been assumed to be uniform. The theorem can also be proved for the case where they vary from point to point, but this paper is already long and this work will not be reproduced.

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RECIPROCAL THEOREMS IN RADIO COMMUNICATION*

By
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Summary—*Two reciprocal theorems, the generalized Rayleigh theorem and the Sommerfeld-Pfrang theorem are of great theoretical importance in radio-communication. A careful analysis of these theorems and their mathematical derivations shows that they are quite distinct and their practical fields of application different. In particular it shows that the Sommerfeld-Pfrang theorem labors under restrictions, implicit in its mathematical derivation, which seriously limit its field of practical applicability.*

IN his "Theory of Sound", Rayleigh states and proves the following *reciprocal theorem:*

I. *Let there be two circuits of insulated wire A and B and in their neighborhood any combination of wire circuits or solid conductors in communication with condensers. A periodic electromotive force in the circuit A will give rise to the same current in B as would be excited in A if the electromotive force operated in B.*

This theorem is of the greatest usefulness in electric circuit theory and in communication engineering, and is employed extensively. One of its principal uses is to deduce the transmitting properties of a transmitter from its receiving properties, and vice versa.

In 1923 I was asked to look into the question as to whether the theorem is valid in radio-transmission. A brief examination showed that the proof of the theorem, as given by Rayleigh, is valid for quasi-stationary systems only, and is, in fact, deduced from the equations of rigid mechanics. It proved, however, not too difficult a matter to deduce the following generalized reciprocal theorem.[1]

IIa. *Let a distribution of impressed periodic electric intensity* $F'e^{i\omega t}=F'(x,y,z)e^{i\omega t}$ *produce a corresponding distribution of current density* $u'e^{i\omega t}=u'(x,y,z)e^{i\omega t}$, *and let a second distribution* $F''e^{i\omega t}$ *produce a second distribution* $u''e^{i\omega t}$ *of current density;* then

$$\int (F'\cdot u'')dv=\int (F''\cdot u')dv$$

* Dewey decimal classification: R190. Original manuscript received by the Institute, December 27, 1928. Presented before joint meeting of the Institute and the International Union of Scientific Radiotelegraphy, American Section, at Fourth Annual Convention, Washington, D. C., May 15, 1929.

[1] A Generalization of the Reciprocal Theorem, *Bell Sys. Tech. Jour.*, July, 1924.

the volume integration being extended over all conducting and dielectric media. F and u are vectors and the expression $(F \cdot u)$ denotes their scalar product.

In 1925 Sommerfeld published the following reciprocal theorem[2]

III. *If A_1 and A_2 are two antennas located at O_1 and O_2 respectively, and have arbitrary orientations, and signals are first sent from A_1 and received by A_2 and then sent with the same average power from A_2 and received by A_1, then the intensity and phase of the electric field at the receiver A_1 will be equal to that previously produced at A_2, regardless of the electrical properties and geometry of the intervening media and the form of the antennas.*

In order to bring out the distinction between theorems IIa and III and to state the former in a form more suited for immediate application theorem IIa may be stated more concretely as follows:

II. *If an electromotive force is inserted in the transmitting branch of antenna A_1 and the current measured in the receiving branch of A_2, then an equal current (both as regards amplitude and phase) will be received in the transmitting branch of A_1 if the same electromotive force is inserted in the receiving branch of A_2.*

Just as theorem II is based on the more general theorem IIa, so the Sommerfeld-Pfrang theorem III is based on the following reciprocal theorem due to Lorentz.

IIIa. *If $E'e^{i\omega t}$, $H'e^{i\omega t}$ are the field vectors due to a periodic disturbance from a source A_1 located at O_1 and $E''e^{i\omega t}$, $H''e^{i\omega t}$ the corresponding field vectors due to a disturbance originating in A_2 from a source located at O_2, then*

$$\int_{1+2} [E'' \cdot H'] d\sigma = \int_{1+2} [E' \cdot H''] d\sigma$$

the surface integrals as indicated by the subscripts $1+2$ being taken over closed surfaces 1 and 2 surrounding the sources A_1 and A_2 respectively.

It now remains to discuss the utility and significance of these theorems in radio transmission problems and to point out the essential distinctions between them, concerning which there appears to be considerable confusion. Before doing so, however, it seems desirable, and even necessary, to discuss explicitly the restriction imposed on the

[2] *Zeit. für Hochfrequenztechnik*, 26, 93, 1925. In this paper Sommerfeld gives credit to H. Pfrang for the original statement of this theorem (München Dissentation, 1925) and at the same time points out that it is deduced from a reciprocal theorem due to Lorentz. The Sommerfeld paper is reviewed by Stuart Ballantine in the April 1928 issue of these PROCEEDINGS. In that review Ballantine mentions in passing that I have generalized Rayleigh's theorem for *electric circuits*. This would hardly seem an adequate description of the generality and scope of theorem IIa.

generality of the theorems; restrictions which, as regards the Sommerfeld-Pfrang theorem III, seriously limit its applicability to radio transmission problems.

As regards the generalized Rayleigh reciprocal theorem, the only restriction is that the current (conduction plus polarization) must be linear in the electric intensity. Otherwise the medium may vary arbitrarily from point to point. In particular the transmitting and receiving antennas are absolutely unrestricted in their geometrical form, electrical constants, and disposition with respect to other conductors and conducting bodies, such as the earth. Both theorems fail when the waves are propagated in an ionized medium in which the earth's magnetic field has an appreciable effect on the conduction currents.[3]

In my original proof of the theorem the permeability μ of the medium was restricted to unity. Very recently Pleijel[4] has stated the theorem without this restriction. This theorem is not, however, the same as mine because the applied forces are not defined in exactly the same way and do not agree exactly with the physical definition of applied forces. The question as to whether the theorem holds when $\mu \neq 1$ seems to me still open.

In Sommerfeld's paper[5] and in Ballantine's review[5] thereof, certain restrictions on theorem III are carefully pointed out. For example, the relations $D=\epsilon E$, $B=\mu H$ and $I=\sigma E$, must be linear. On the other hand, other and in practice more serious restrictions are not stated but are implicit in Sommerfeld's mathematical proof. For this reason an analysis of his proof was made, which showed that the following restrictions limit the generality of the Sommerfeld-Pfrang reciprocal theorem III:

(1) The transmitting and receiving antennas, instead of being arbitrary geometrical forms, must both behave like simple electric or magnetic dipoles, as regards their radiation fields.

(2) The transmitting and receiving antennas must both be so far removed and isolated from other conducting bodies (*including the earth*) as to make the reflected field due to induced currents and charges in such bodies negligible in the neighborhood of the antenna which is regarded as the transmitter.

[3] This fact makes the application of the Reciprocal theorems to short wave transmission somewhat doubtful but does not seriously affect their application to long waves.

[4] Ingeniörs Vetenskaps Akademien, No. 68, 1927. There are two small errors in Pleijel's proof, which, however, are easily remedied.

[5] See footnote 2.

These restrictions are so serious and so limit the applicability of the theorem in practical engineering problems as to call for the detailed analysis of the proof which follows:

Let $Ee^{i\omega t}$ and $He^{i\omega t}$ be the electric and magnetic vectors, respectively; $ue^{i\omega t}$ the current density; and let it be assumed that the linear relations

$$D=\epsilon E; \qquad B=\mu H; \qquad u=\sigma E \tag{1}$$

hold everywhere in the medium.

Now suppose that E', H' denote the electric vectors due to a source A_1 located at point O_1; and E'', H'', the corresponding field vectors due to a source A_2, located at point O_2. The Lorentz reciprocal theorem then states that

$$\text{div}\,[E' \cdot H''] = \text{div}\,[E'' \cdot H'] \tag{2}$$

the square brackets denoting vector product. It follows immediately that

$$\int_1 [E' \cdot H'']_n d\sigma_1 + \int_2 [E' \cdot H'']_n d\sigma_2 = \int_1 [E'' \cdot H']_n d\sigma_1 + \int_2 [E'' \cdot H']_n d\sigma_2 \tag{3}$$

the integrals 1 and 2 being taken over closed surfaces enclosing the sources A_1 and A_2, respectively. Subject to the linear relations (1), formulas (2) and (3), which I infer from Sommerfeld's paper are due to Lorentz, are perfectly general.

In applying these formulas, however, to deriving the reciprocal theorem III, the following approximations and restrictions are introduced:

(1) Over the surface of integration $\sum_1$ enclosing source A_1, the vectors E'', H'' are taken as *constant*. Similarly, over the surface of integration $\sum_2$ enclosing the source A_2, the vectors E', H' are taken as *constant*.

(2) In effecting the integration called for in formula (3), the vectors E', H' are assumed to be derivable from the Hertzian vector for either a simple electric or magnetic dipole; a corresponding restriction applies to the vectors E'', H'' on the surface $\sum_2$ enclosing the source A_2.

It is immediately evident that assumption (2) imposes very serious restrictions on the applicability of theorem III. These restrictions are that (1) the antennas A_1 and A_2 are simple linear oscillators, and (2) they are so far removed and isolated from other conducting bodies, (*including the earth*), that the secondary field due to currents and charges induced in such bodies is negligible in the neighborhood of the antenna which is regarded as the transmitter. In addition assumption (1) excludes the theorem from application to extended antenna arrays and the wave antenna. In view of these restrictions it seems to the writer that Sommerfeld's statement of the theorem is much too broad; in fact so broad as to be misleading.

Irrespective, however, of these restrictions, it seems to me that the generalized Rayleigh theorem II is of greater practical utility than the Sommerfeld-Pfrang theorem III. The former enables the engineer to deduce at once the transmitting properties of an antenna system from the receiving properties, and vice-versa. For example, the receiving properties of the wave antenna are quite simply deducible, to a good approximation, whereas a direct theoretical determination of its transmitting properties is extremely difficult. The generalized Rayleigh theorem furnishes an immediate answer to the problem.

On the other hand, in applying the Sommerfeld-Pfrang theorem, we require an adjustment of the two antennas for the same radiated power; a difficult matter whether by calculation or experiment. Furthermore in correlating the theorem with experimental results, the electric intensity is deducible only in terms of a detailed knowledge of the receiving properties of the antennas for reception from any direction or angle.

The generalized Rayleigh theorem says nothing explicitly about power and efficiency. For the very important case, however, where the impedances of transmitter and receiver are adjusted for maximum output amd maximum absorption of energy it can be shown that the ratio of the power output of the generator to the power absorbed by the receiver is the same for transmission in either direction. Thus, in the case of transmission between two entirely dissimilar antennas the transmission efficiency is the same in the two directions provided the terminal impedances are properly adjusted. This proposition is not immediately obvious but is very simply deducible from the Rayleigh theorem.

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THE MEASUREMENT OF DIRECT INTERELECTRODE CAPACITANCE OF VACUUM TUBES*

By

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Summary—*A method of measuring direct capacitance in the range 10^{-10} to 10^{-15} farads by a charging current at radio frequencies is described. A low resistance current indicator is shunted across one of the direct capacitances. The charging current $I=\omega EC$ is a measure of the direct capacitance provided that ωE is maintained constant. A simple circuit arrangement provides an apparatus giving reliable results measured visually by substitution with a standard.*

The method has been practically applied in the measurement of direct capacitances present in vacuum tubes providing data for design purposes and for control of product uniformity. The method is specially advantageous in its ability to measure exceedingly small direct capacitances present in screen-grid tubes.

WITH the development of more efficient electric circuits employing the vacuum tube, the determination of the magnitude of the individual or *direct* interelectrode capacitances became of engineering importance. The input impedance of a vacuum tube is calculable only when the direct capacitances, the "constants" of the tube, and the associated circuit dimensions are known.[1]

A three-electrode tube forms a simple network of three direct capacitances. Where there are n electrodes, the number of direct capacitances N is

$$N=\frac{n}{2}(n-1)$$

Maxwell,[2] in determining the direct capacitances present in a guard ring capacitor, used an absolute charging current method and an electrometer indicator. More recently, in a general discussion of direct impedances, Campbell,[3] has described a simple ammeter method. This method has been used by Hull[4] for the measurement of the extremely small direct capacitances present in screen-grid tubes. With modifications, it forms the basis of the method used by the present authors.

In order to obtain data for purposes of tube design and for purposes

* Dewey decimal classification: R220. Original manuscript received by the Institute, March 6, 1929.

[1] Nichols, *Phys. Rev.*, 13, 404–419, 1919. Miller, Bul. Bur. Std. Scientific Paper No. 351. Hartshorn, *Proc.* Phys. Soc., 39, 108, 1926.

[2] Maxwell, *Electricity and Magnetism*, 1, 350, ed. 1892.

[3] Campbell, *Bell Sys. Tech. Jour.*, 1, 18–38, 1922.

[4] Hull, *Phys. Rev.*, 27, 437, 1926.

of quality control of product uniformity, it is necessary to measure direct capacitance with a high degree of precision in a reasonably rapid manner. In consideration of the anomalous frequency behavior of the dielectric of the tube and the high resistance metal film, called "getter," on the inside walls of the glass bulb, and in consideration of leakances of a very high order but comparable with the capacitive reactance of the small direct interelectrode capacitances at audio frequencies, the optimum of accuracy of capacitance measurement of receiving tubes was found to occur with measurements obtained by a charging current method at about one-half megacycle. Practically, the effective direct capacitance at radio frequencies is principally desired although it is about five per cent lower than the capacitance measured at audio frequencies due to the above mentioned effects.

In complex capacitance networks, as exemplified by the screen-grid tube, the errors of the Schering direct capacitance bridge or the Campbell-Colpitts bridge may exceed the capacitance sought. For information regarding the Schering bridge see the reference to Hartshorn.

The charging current method is flexible in permitting extremely small capacitances in the range 10^{-10} to 10^{-15} farads to be measured with known accuracy. The method can be made simple and visual and provides easily obtained results that are reliable.

The Theory of the Method

The fundamental circuit arrangement for the charging current method is shown in Fig. 1.

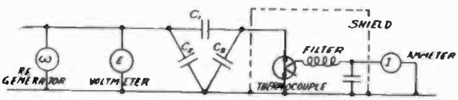


Fig. 1—Showing the Fundamental Circuit Arrangement for the Charging Current Method for Measuring Small Direct Capacitance.

The high-frequency generator supplies voltage sufficient to cause a measurable current to flow in the branch C_1 and the thermocouple. This current is not affected by C_2 if the voltage is held constant, nor by the shunting effect of C_3 and the capacitance of the leads to the shield since the reactance of these capacitances is high compared with a low resistance thermocouple. The error introduced by the shunting effect is easily calculable and by choice of thermocouple resistance this error may be made vanishingly small for capacitance dimensions encountered in vacuum tubes up to the 100-kw size. In order to have the current indicator at ground potential it is connected to the thermocouple through a filter in the manner shown.

It is evident that the precision of the measurement depends on the accuracy of the determination of the three quantities—voltage, current, and frequency. To measure the three quantities independently would require excessively refined technique and apparatus to obtain precise results. However, in practically applying the fundamental method a simple solution has been developed.

The Practical Circuit Arrangement

The application of this fundamental method requires some alteration of circuit arrangement to make it direct reading. Fig. 2 shows the circuit.

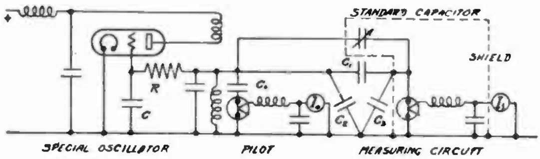


Fig. 2—Showing the Practical Circuit Arrangement for the Charging Current Method for Measuring Direct Capacitance of Receiving Tubes.

In the above figure the network at the left represents an oscillator which is so adjusted that when the frequency is lowered by the addition of capacitance across the oscillating circuit, the terminal voltage rises sufficiently to maintain constant the product ωE. This constancy is indicated by a pilot consisting of a very small fixed capacitor C_o and an indicator of its charging current I_o.

The current charging a capacitance is

$$I = \omega E C$$

Where ω = angular velocity of fundamental frequency

$$E = 0.707\sqrt{E^2_{1m} + 4E^2_{2m} + 9E^2_{3m} + \ldots}$$

in which series E_{1m}, E_{2m}, E_{3m}, etc., express the maximum value of the voltage of the fundamental frequency, the second harmonic, the third harmonic, etc., respectively. A square law symmetrical indicator of current, like the vacuum thermocouple used, obeys the law of superposition of vector quantities, very exactly in this case where the harmonics present are small compared with the fundamental.

In the above equation, if C_o is constant and the current indicator, I_o, shows a constant deflection, obviously ωE must be constant also. The use of a special oscillator is obligatory only when special accuracy is required in rapid measurement as otherwise ωE may be held constant manually. The method thus escapes from the difficulties encountered in measuring frequency and voltage independently.

A likewise simple use of a standard substitution capacitor eliminates the need of measuring the absolute value of the current and thus there is no need for calibration of the current indicator. The constancy of

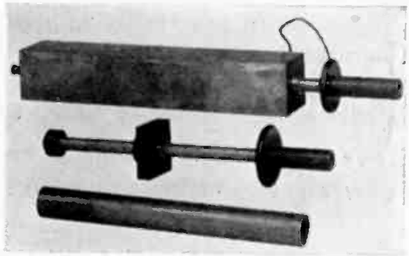


Fig. 3—Showing a Standard Capacitor Having a Capacitance Increment of 2.00 $\times 10^{-12}$ farad per inch of cylinder length.

the calibration of the standard capacitor is all that is required and the use of coaxial cylinders makes this easily possible.

Method of Measuring Three-Electrode Receiving Tubes

With the standard capacitor set at any convenient arbitrary zero, the tube, represented in the figure by C_1, C_2, and C_3, is inserted in a

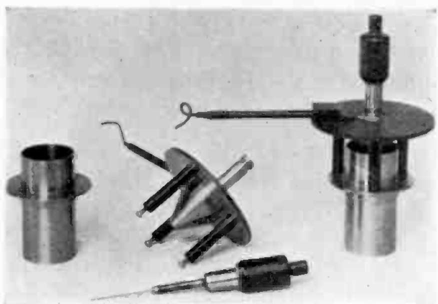


Fig. 4—Showing a Standard Capacitor Having a Capacitance Increment of 1.00×10^{-14} farad per turn of the micrometer head.

shielded socket fixture and the reading of the meter I_1 is noted. On removal of the tube, the standard capacitor is increased until I_1 reads the same as noted. Then the increment of the standard capacitor is equal to the direct capacitance of the tube, C_1 in the figure. This is true only, if meanwhile, the pilot circuit indicator I_o has shown no change in deflection.

It is feasible to provide a circuit arrangement which will allow the meter, I_1, to be scaled off in micromicrofarads, thus providing a "direct capacitance meter."

The Standard Capacitors

The standard substitution capacitors are of the calculable increment coaxial cylinder type calibrated linearly in micromicrofarads. Physicists have used this type of capacitor for many years. The calculation is given in Bul. Bur. Std. Circular No. 74, p. 236. An interesting work on the determination of errors in coaxial cylinder standards has been published.[5]

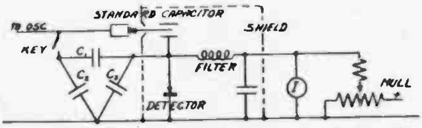


Fig. 5—Showing a Sensitive Current Indicator and Small Increment Standard Capacitor Arranged for Measurement of 0.0005 to 0.100 $\mu\mu$f of direct capacitance.

Fig. 3 shows the photograph of a standard capacitor most generally used in vacuum-tube measurements. It consists of three parts, a movable inner cylinder having a well insulated handle and connected to the oscillator by a flexible lead, a fixed outer cylinder connected to the thermocouple, and a grounded outer shield which also provides mechanical support for the two insulated cylinders.

Fig. 4 shows a photograph of a similar standard capacitor having a capacitance increment of 1.00×10^{-14} farad per turn of the micrometer head. It is especially used to measure the extremely small feed-back capacitance of the screen-grid type of tube.

Special Measurement Technique for Screen-Grid Tubes

The measurement of the screen-grid tube feed-back capacitance C_{pg} requires a more sensitive current indicator than the thermocouple. A carborundum fixed detector is used to indicate the small charging current. A filtered balanced circuit permits the use of a microammeter and provides a sensitivity of about 1 μa per 1/100 $\mu\mu$f of direct capacitance. Due to the resistance of the detector, the shunting effect of the direct capacitances is not negligible, and the tube is not removed from its measuring position but the oscillator lead is broken instead when the standard substitution is made.

The network shown in Fig. 5 represents the circuit used for the measurement of extremely small direct capacitances.

[5] M. Bedeau, *L'Onde Electrique*, 5, 613, 1926.

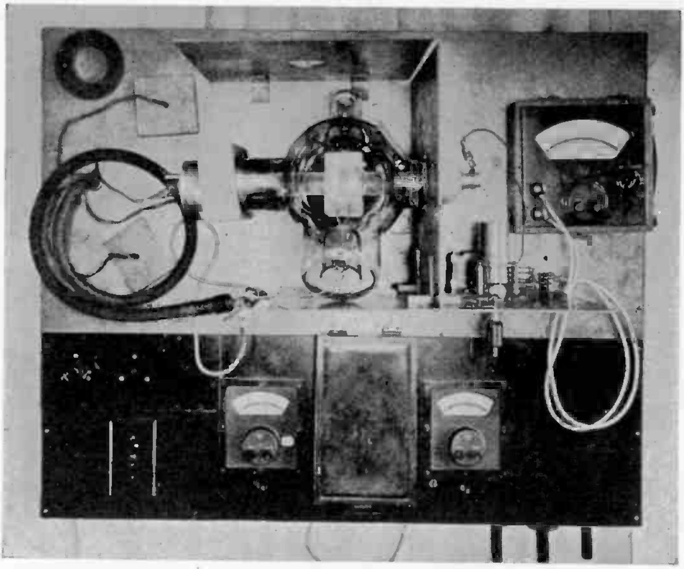


Fig. 6—Showing a Screen-grid Power Amplifier Being Measured for Plate to Control Grid Direct Capacitance.

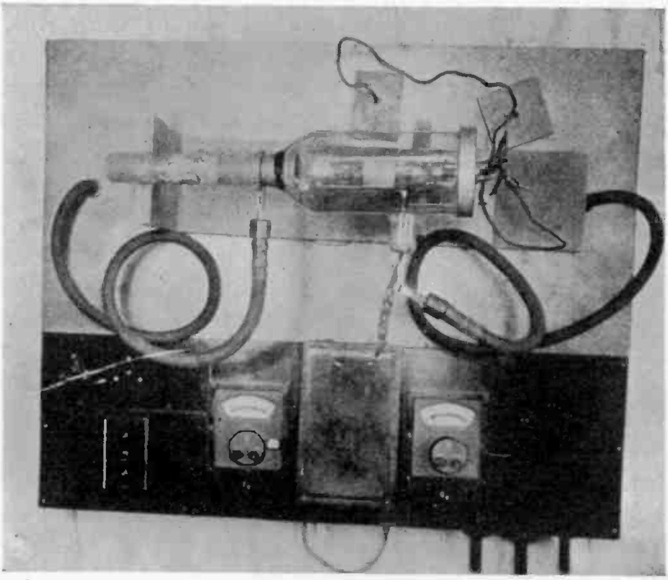


Fig. 7—Showing a Power Tube Being Measured for Direct Capacitance.

As has been pointed out by Moullin[6], an asymmetrical non-linear circuit element, like a crystal detector, is not a rigorous indicator of the current flowing in the circuit, as it is affected by wave form and does not indicate true effective current. However, the crystal has such an advantage in simplicity over any equally sensitive instrument whose indication is independent of wave form that it is used here in spite of its somewhat lesser accuracy. For this reason it is preferable in accurate measuring practice to employ the measuring circuit itself (Fig. 5) as the pilot in the following manner. Without the tube in the measuring position, let the residual capacitance of the standard capacitor and

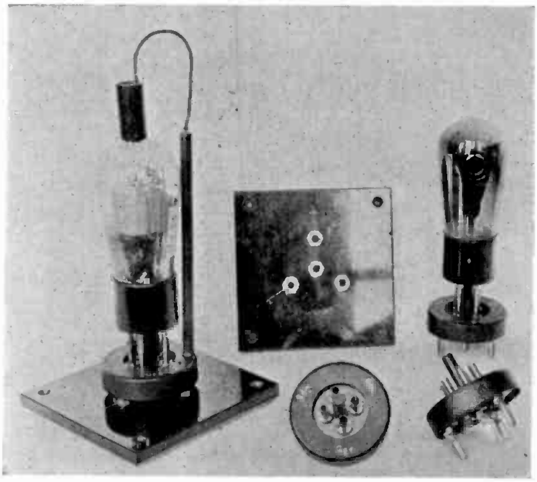


Fig. 8—Showing the Shielded Fixture Used for Measuring C_{gp}, C_{gf}, and C_{pf} by a Simple Rotation of the Fixture on the Instrument Panel.

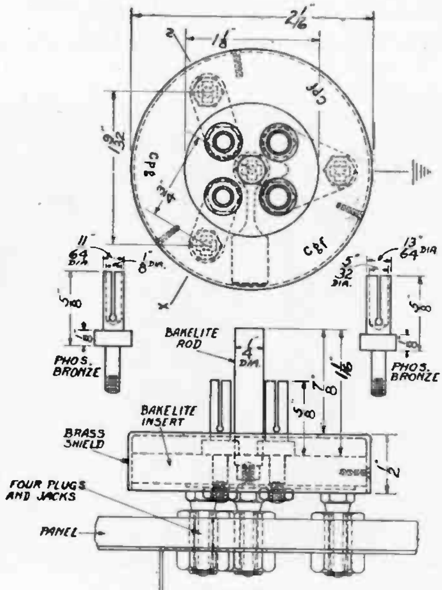
the crystal detector serve the same function as C_o and I_o in Fig. 2. If connecting the tube to be measured across the oscillator terminals results in no change in the current indicator I in Fig. 5, then it is evident that the oscillator is properly adjusted and the measurement may proceed.

Power Tube Capacitance Measurements

The general class of power tubes are measured on a large metal sheet which is the ground plane. Two shielded flexible low capacitance cables make the necessary oscillator and thermocouple connections to the elements of the tube being measured. Heavy flexible ground connection is made to the elements that are grounded and care is

[6] Moullin, *Jour.* I. E. E., 298, 1926.

taken to provide contacts to the elements that are of negligibly small resistance. A low resistance shunt is provided across the thermocouple of the measuring circuit for limiting the thermocouple current when measuring large direct capacitance. In other respects the measurement procedure is the same as for capacitances of smaller dimensions. Fig. 7 shows a photograph of a power tube on the apparatus used for general direct capacitance measurements.



SHIELDED FIXTURE
(SAME WITH UY PIN SPACING)

Fig. 9

Fixtures Used

The measurement of the direct capacitance of receiving tubes can be made independent of the leads to the tube and the external disposition of the wiring only by the use of a special shielded standardizable fixture. Ordinary sockets are out of the question due to expanding prongs and errors involving 0.1 $\mu\mu$f in stray coupling capacitance to parts of the tube being measured. A fixture devised to insure independence of the geometry of the leads and to insure reproducibility of the measurement is shown in Figs. 8 and 9. A simple rotation of the shielded fixture on the instrument panel permits the measurement of C_{pg}, C_{gf}, C_{pf}. The UX and UY types of fixtures differ only in their pin spacing.

Apparatus for Receiving Tube Capacitance Measurements

The ensemble of apparatus used for receiving tube measurements is shown in Fig. 10. One side of the apparatus is provided for the measurement of feed-back capacitance of the screen-grid type of tube

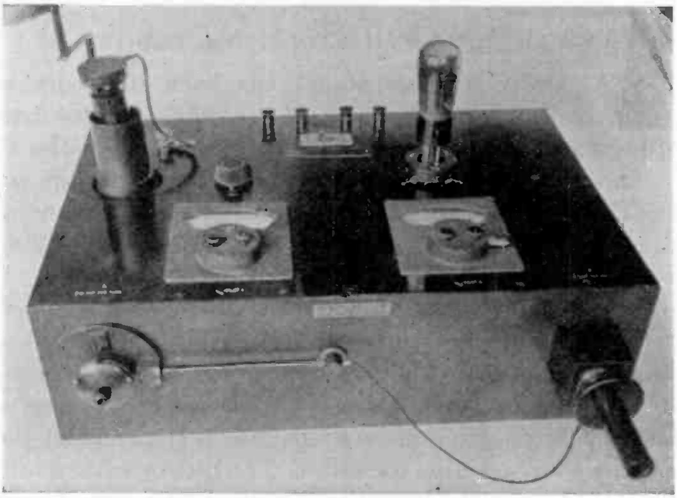


Fig. 10—Showing the Ensemble of Apparatus Used for Receiving Tube Capacitance Measurements.

and the other side for the measurement of the direct interelectrode capacitance of the ordinary standard type of receiving tube. The figure shows the handles of the standard capacitors which are operated from the front of the metal case and shows two tubes mounted in the measuring position, one a screen-grid tube in a shield cup and the other a three-electrode tube in the shielded socket fixture previously described.

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PHOTORADIO DEVELOPMENTS*

By
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General Picture Developments, 1926-1928

OUTSTANDING in this period has been the work of the American Telephone and Telegraph Company in the extension of their commercial picture network throughout the United States. A fine example of the work they are able to perform is given herewith. (Fig. 1) It is interesting to note that the telephone service has been used extensively for the transmission of printed matter—bond circulars in particular.

The other wire line method that continues to be used commercially is the Bartlane System handled between the *Daily Mirror* in London, and the Pacific and Atlantic Photo Service in this country. The method consists in reducing a picture to a perforated tape of the same sort as is used for cable messages, and then this tape is transmitted over the Western Union System across the water. The picture is then reformed from the holes punched in a duplicate tape on the other side of the Atlantic.

On December 1st, 1927, wire picture service was inaugurated between Berlin and Vienna by the Karolus-Telefunken System. Two novel features of this system must be mentioned. The first is the use of the Kerr cell. The Kerr effect has been hidden in the archives of science for forty years, and it remained for Dr. Karolus to bring it to practical use in the transmission of pictures. The Kerr cell consists of a gap between two oppositely charged electrodes with a solution of nitrobenzol covering them. Such a combination has the ability to change the speed of light passing through the cell depending upon the plane of polarization of that light. Specifically, if the light is vertically polarized with respect to the surface of the electrodes, the light train will pass through quicker when potential is applied between the electrodes; if the light is in the same plane as the electrodes it will be retarded.

To make use of this phenomenon, a nicol prism is turned to give a beam of light at 45 deg. with the electrode surfaces on entering the

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cell. A second nicol prism is then used on the side from which the light leaves the cell, but it is turned to allow only light at 90 deg. with respect to the first prism to pass. The effect of this optical system is to allow no light to pass when there is no electrostatic distortion produced in the cell. Now, however, when voltage is applied to the electrodes, the distortion will take place. A vector diagram shows that if the horizontal component is slowed down half a wavelength with respect to the vertical in passing through the cell, the emerging light will be plane polarized at 90 deg. from the direction in which it entered, and will therefore then be able to pass through the second or analyzing

Fig. 1

(original) (received)

nicol prism perfectly. This means that with the right amount of voltage applied to the electrodes, we will get a change from no light to full light, and the outfit therefore works as a simple light valve. It has the great advantage of being practically inertialess. The practical voltage range for operation is of the order of 500 to 1000 v. The slight disadvantages of the method are due to the fact that nitrobenzol is not a perfect dielectric and allows some current to flow, which causes some deterioration, and secondly the response is linear only over a restricted part of the curve, but its speed action is a tremendous asset.

The other outstanding development of this system is the photocell construction for which Dr. Schriever of the Telefunken Company is responsible. Instead of being in the more usual Christmas tree ball form, this photocell is much like a very flat doughnut. (Fig. 2) The purpose is two-fold; first to simplify the light system, and second and most important, to increase the available light change in the pick-up at the transmitter some fifty-fold.

To accomplish this, at the picture transmitter, this doughnut cell is placed directly in front of the picture transmitting cylinder. A very intense beam of light is then focused down to a very fine point through the hole of the doughnut at the picture. This fine little spot of light which seems to be a little glowing fire then reflects back on to the solid part of the glass photocell, and acts on the light sensitive

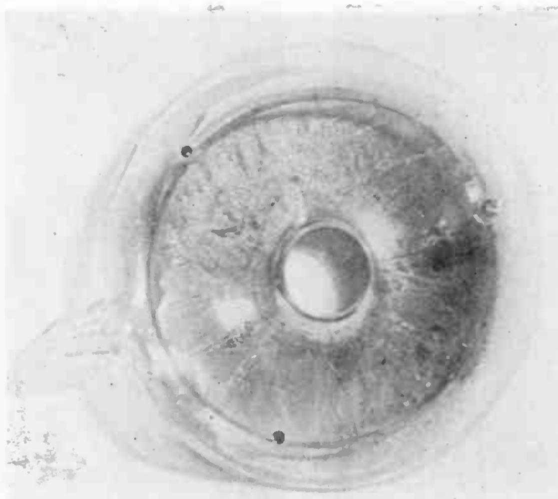


Fig. 2

chemicals placed there. Due to the fact that the active part includes such a large angle from the little spot of light, practically all the available light is used. In this sense it corresponds to having a lense aperture of FO.5, and all camera enthusiasts know what that would do in the way of speed and intensity of action.

Another entrant into the picture transmission work is the Marconi Company, with Mr. Wright heading their activities in this direction. He is working on the principle of reversing the usual method of analysis at the transmitter and receiver—he uses a stationary cylinder at each end. The picture is slid forward in a curved form along the length of the cylinder. Half way around the cylinder there is a fine slit corresponding to a line of the picture, and as the picture is moved forward, a fine light spot is rapidly revolved inside the cylinder to cross the picture from one side to the other, one line at a time through this slit, as the picture advances. He is working at quite high speeds, planning to take full advantage of the higher speed capabilities of the Marconi radio beam.

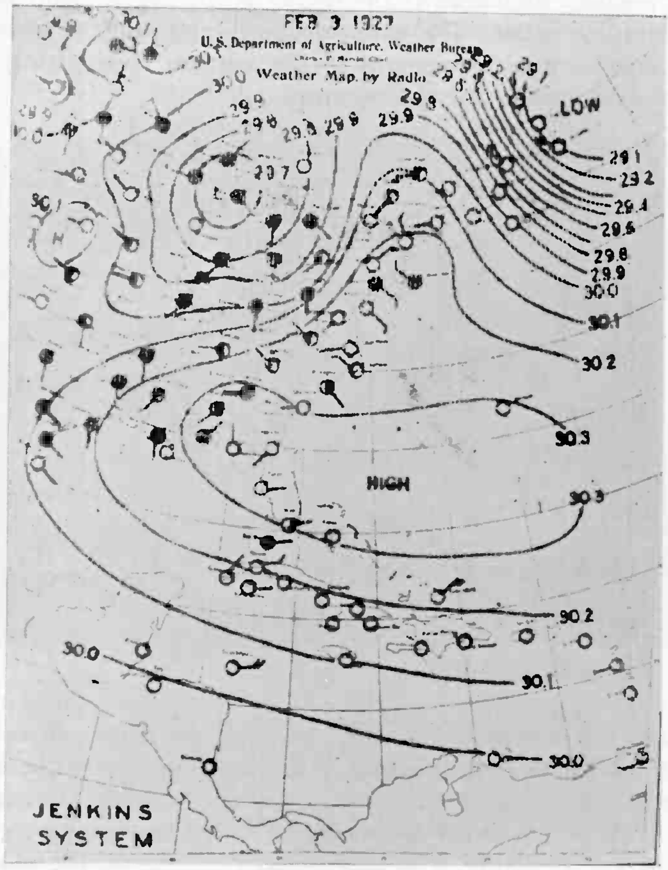


Fig. 3

Map Work

Two of the regular workers in picture radio, C. Francis Jenkins of Washington, and Dr. Max Dieckmann of Munich, Germany, have modified their equipment to simplify the recording of weather maps at sea. (See Figs. 3 and 4, respectively.)

Amateur Reception

Several workers have tried to interest the amateur in picture reception. They are—T. Thorne-Baker of London, one of the long standing workers in the field, Mr. Jenkins, and, more recently, Austin G. Cooley, in connection with *Radio Broadcast* of Garden City, Long Island. Mr. Cooley makes use of the old phonograph that may have been otherwise discarded in the amateur's home, and specializes in a "corona" discharge at the end of a fine needle point resting on photographic paper. The variations in the intensity of the incoming radio signal vary the amount of the corona, thus giving the necessary modulations for the picture.

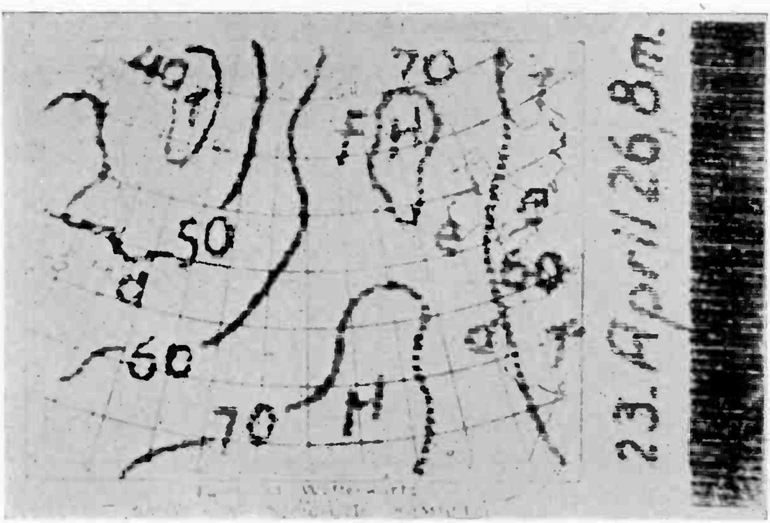


Fig. 4

Such developments are very much to the point, as surely many contributions to radio pictures will come from this stimulation; but the ease with which results will come will be with nothing like the simplicity of the broadcast reception as we know it today, where a crystal and a telephone receiver start a novice on the road to nine tube sets,—later reduced to four.

Television

Great strides have been made in the even more complicated art of television—complicated by the element of vastly greater speed. In closing this all too brief summary of the outstanding workers in these fields during the past two years, acknowledgment must again be given, as was done two years ago,[1] to the continued painstaking work of Professor A. Korn of Germany. It was he who first began to get real pictures over wire lines twenty years ago.

[1] R. H. Ranger, Proc. I. R. E., **14**, 161; April, 1926.

Photoradio Aims

As all engineers know, no particular features of their work may ever be considered as ideal solutions,—as in practical politics, it is necessary to be satisfied with compromises which conform to the general advance of the art. However, it is well to have stars on which we attempt to hang our pictures. In photoradio they have been the following:—

1. Economical operation
2. Continuous operation
3. Daylight operation
4. Visible operation
5. Finished operation

Economical operation is of course the foundation of everything worth while in the design. Naturally this economy must be in accordance with the use to which the equipment is to be put. If a larger number of sets were required, the cost of the sets themselves becomes the controlling factor as in equipment for amateurs; in photoradio to date, the prime consideration has been economy of operation—the best possible results with the least effort. Simplicity of operation is a great contributor to economy, of course, and that is where the other factors enumerated above become important. Nothing is so disastrous to production as discontinuous operation. That explains why the rotating cylinder type of equipment has been shunned in photoradio.

Daylight operation is far more satisfactory from the operator's point of view. Photography is a wonderful aid in many operations, and in much picture transmission is the obvious answer. It is certainly one of the places where compromise must be considered most carefully. The finesse of photographic recording cannot be rivalled. However, it may be said in no uncertain terms that there are many variable factors in the chemistry and physics of ordinary photography, such that its operation has become an empiric art rather than a fixed operation where definite rules will ensure a good picture in the hands of the average operator. For this reason, to add to it the further variables of a communication system increases the chances of failure tremendously. Therefore, certainly in all the original work over great radio distances, the greater certainty of visible and daylight operation has been depended upon. When the complete system so constituted has been operated sufficiently to reduce the uncertainties of the newer elements in the picture, it may well be that it will be advantageous to return to the use of photography with greater assurance. The entire matter is of course still in a state of fluctuation, working towards the best commercial method.

Furthermore, the exact requirements of photoradio on a radio circuit have brought about a tightening of the performance of that radio circuit. The ability to key a transmitter speedily and accurately is a striking example.

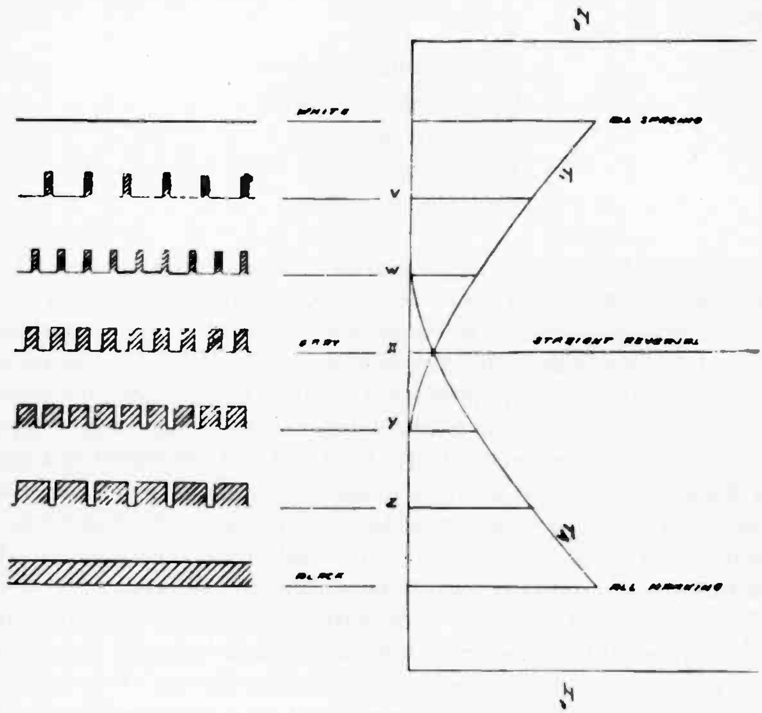


Fig. 5

It is of course advantageous that the operation be complete when the transmission has stopped. This promotes speed of delivery, which is one of the battle cries of a communication system.

Design Technique

If there is one rallying point for all of the photoradio technique, it is the use of the dot-dash method of representing picture values. This becomes at once the central feature around which all the simplification of equipment and operation is based. Variation in radio signal intensity is all too well known to require mention here. A system which is based on signal intensity to represent the picture values is placing itself under a severe handicap at the start. The dot-dash method changes light intensity variations into time variations. An isolated dot represents a light part of the picture, close dots represent gray, and heavy dots becoming solid dashes represent black. A scale showing this relationship is given. (Fig. 5) Such solid characters may be handled over involved links of wire lines to radio without material difficulty. It is only necessary that the radio transmitter trigger-on when a character is starting and trigger-off neatly at its termination. This simplification of operation has led to our realization of another important principle of design which we have called "inherent accuracy." It means the elimination of troublesome adjustment and compensating for large variables by making their effect nil on the result.

Another example of inherent accuracy came to light in the choice of gears in the driving mechanism. In some of the early apparatus, it was found that there was an occasional tendency to have errors in sequence in synchronizing—keeping the machines in step. The trouble was found to be in the fact that the gear ratios were such that they came out unevenly, such that the teeth would mesh in a given sequence for one line, and in an entirely different sequence for the next; and would then go back to the first sequence for the third line, and so on. In consequence, if the shaft carrying one of the gears was out slightly, the regular error would show up. The answer has proved to be to make the gear ratios even.

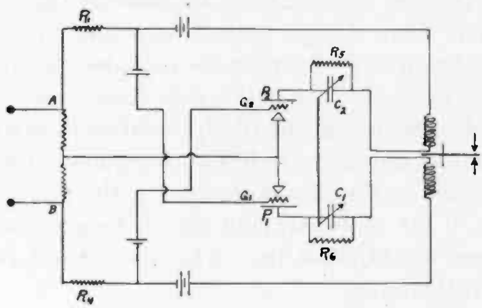
Push-Pull Relay

Everyone is familiar with the famous combination of "push-pull," but we have made use of the general principle in a unique way. Inertia is present in both mechanical and electrical devices. If the impulse to be conveyed is constant in frequency, such as a 60-cycle wave, the inertia factor is constant and therefore may be disregarded as far as obtaining correctly spaced impulses at the receiving end, but the whole idea of picture transmission is that there shall be change—change conveying thought in one form or another from the sender to the receiver.

The "reading condenser" has been a well known means for reducing the effect of electric inertia in d-c telegraphy. The time involved in producing a desired effect at a distance is determined by the time it takes to build up the current to a sufficient intensity. Very often, however, it is possible that the final current value may be much greater than that required to produce the start of the signaling effect. Under these conditions it is possible to dispense with some of this final current strength and use it to speed up the start and finish of the signal. A simple plan for this purpose is to include a condenser with a shunting resistance in series with the main signaling circuit. This will give a large current impulse on the start and finish of the signals, and only a moderate value for the steady state.

The vacuum tube gives another simple way to reduce the inertia of inductance. By tripling the voltage on the magnetic amplifiers in connection with the long-wave stations of the RCA, and introducing twice the straight resistance of the coils, practically three times the speed of keying was obtained, with consequent improvement in the picture reproduction.

As a means of improving the quality of the impulses a broad resonance has been introduced into these inertia reducers. The particular set-up is shown in Fig. 6. Double current is used in practically all



D.C. PUSH-PULL RELAY

Fig. 6

of our d-c signaling work. Let it be assumed for the moment that a spacing current from the distant point is flowing in such a direction as to make point A positive with respect to point B. Under these conditions, grid G_1 will be positive and grid G_2 negative. Under these conditions, plate P_1 will be passing current in a direction through the relay to hold it to spacing. The plate current will carry through resistance R_6 and R_4. The current through R_4 will act to make the grid G_2 even more negative. Likewise the reduction in the plate current of P_2 due to the negative value of its grid G_2 will reduce the current through R_5 and R_1 so that the grid G_1 will be even more positive. The effect of such a hook-up is that now when the current through A-B is reduced to zero, the charge on the condensers C_1 and C_2 will be such as to cause current to flow through R_1, R_2, R_3, and R_4, which will swing the grids oppositely to their previous condition and give a marking impulse. If the current from the line should still remain at zero, the charge on the condensers would quickly dissipate itself through the resistances and the inductance of the relay coils included in the plate circuits. As soon as it did, it would swing back to the original condition, and as a result a series of dots would be produced. The frequency of these dots may be set at will by proper choice of the value of the capacities C_1 and C_2 with respect to the resistances and the inductances of the relay coils.

The net effect of the action is to be ready for any change, and immediately work with it to a maximum. It accomplishes much the same effect as the famous Gulstadt relay of cable technique, but does not depend on the less reliable contacts of mechanical relays for its reversing action. The speed of the reversals of the push-pull relay is set to correspond to the fastest dots that the complete communication setup will allow. Such a push-pull relay is included both at the transmitting station and at the receiving station where the signals are finally delivered to the recording equipment.

It is hard to estimate accurately the improvement such equipment gives, due to the fact that every improvement brings out the fact that there are weaknesses elsewhere, so that it may be better said that photoradio operation throughout the past three years has been a case of gradual improvement of all the contributing factors, always hitting the weakest spots as they develop when the other factors pass them by. D. G. Ward and J. L. Finch are particularly responsible for the increased speeds that have been accomplished at the transmitting stations during this period.

Reverse Lead Screw

In order to meet the requirements of continuous operation, many plans have been proposed. Naturally the one that is most desirable is one that is continuously acting with no reversing clutches or connecting parts. On the early photoradio equipment, this was accomplished by means of a mangle rack with gear—a small ordinary gear engaging with a gear in the form of a race track. While this device has operated satisfactorily, it is not rugged and is far from inherently accurate.

In its place a reverse lead screw has been laid out. (Fig. 7) While the making of the first one was a very involved proposition, it has proved a fairly simple matter to duplicate them, as is now being regularly done in the excellent machine shop of H. O. Boehme, of New York, under the direction of Frank Kunc of our staff.

This reverse screw is cut in a solid shaft which is placed lengthwise at the base of the photoradio transmitter or receiver. A follower engages with this thread and conveys moving force to the analyzing head of the machine. The movement is uniform through 95 per cent of the travel—the remaining 5 per cent of the stroke is occupied in reversing the act. This does not mean that this entire 5 per cent is unserviceable for picture work, however; in fact the analyzing and recording is continuous up to the immediate end of the stroke when the overlapping interferes.

The curve for the reversing cut was first made in a very much enlarged form in steel as a master, and then by a pantograph arrangement this master was used to get the smaller cutting of the reversing section of the screw.

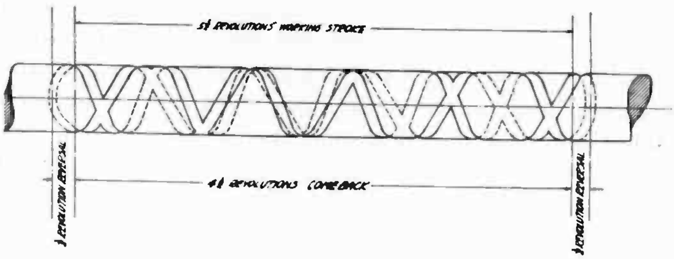


Fig. 7

Very satisfactory results from the point of service and constancy have been obtained with this screw. It is to be noted that we are still using the action of making the analyzing head work in both directions. At both transmitter and receiver, the picture making goes on while the analyzing is proceeding either from left to right or from right to left. This is inherently more difficult to do from the point of view of synchronizing, but the fact that we have done it means that we have found means of synchronizing of a fairly simple sort. If it were not for the fact that the recording is visible, however, it is certain that we would never have dared to use this back-and-forth method. As it is, we are able to correct the synchronizing very easily during the actual reception of a picture. The fact that this two-and-fro analyzing requires such rigorous synchronizing and "framing" has of course strengthened our will power in the development of synchronizing generally. Another form of speed control, which is of course the basis for synchronizing, has been developed.

Air Speed Control

Wherever radio engineers engage with problems their first solution is naturally by the use of electricity. And there is no question but that electricity is a mighty useful agent, particularly as we gain familiarity with its uses. However, with the excessive amplifications that we indulge in, we have to be very careful not to have other electric forces present which may give interference in such amplification. Speed control is a very good instance. In central office installation, there is not much difficulty in obtaining the necessary shielding from ordinary audio amplifications, but if it is desired to place a photoradio receiver directly next to a short-wave radio receiver for example, the problem becomes a little more involved. Of course one answer is to take the bull by the horns and provide the proper shielding and reduce the interference as much as possible. But it is well to inquire sometimes into other possibilities which inherently eliminate such interference.

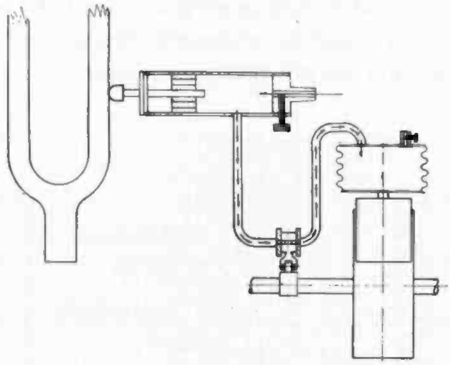


Fig. 8

As this section and the next will show, we have recently undertaken an extensive study of the uses of air. In line with this we have developed an air speed control. One form makes use of an air driven tuning fork. (Fig. 8) It is very easy to drive an air fork; all that it is necessary to do is to have an air chamber with a slightly constrained intake. This air chamber is broadly resonant to the frequency of the fork. The fork tines then have plungers on their ends which work in and out of openings in the air chamber. The action involves the filling up of the air chamber and then giving pressure to the two plungers which drive out the fork tines. In so doing they release the air pressure in the chamber quickly; the fork tines then return into the chamber, and this allows an even higher pressure to be built up in the chamber before the fork tines are again driven out to release the constrained pressure. The fork is self-starting. The frequency is modified by pressure but it is not difficult to get quite constant air pressure by the use of reduction valves, and the very fact that the frequency may be varied within narrow limits provides a means of fine adjustment. Now that we have the fork vibrating, the next question is to apply this vibration to motor speed control.

To control the motor speed, the air chamber has a small valve in it which communicates to an air brake on the motor, but the valve is opened by the motor once each revolution of the motor. If the valve is opened by the motor at a time when the pressure is at a maximum, a good bit of air will push through the valve to enter the brake. This will of course tend to reduce the motor speed below the point where the valve will open at a high pressure time. When it does so, the motor will again speed up and will find a position between the maximum and minimum air pressure periods where its speed will hold quite truly to the fork speed. Obviously there is nothing very electric about this to cause interference, and in fact an air motor could be used if necessary but it seems to be very easy to overcome brush trouble from motors of small size so that this has not proved necessary.

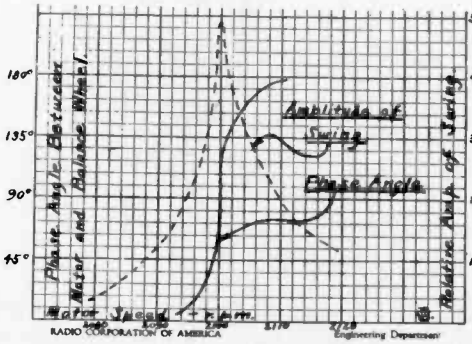


Fig. 9

Another air drive has been worked up from this by Mr. Braman and Mr. Nelson. We are all quite familiar with the use of resonance in mechanical as well as electric arrangements to effect control of one type or another. We are also quite well aware of the fact that phase differences exist, but there are not many instances where both phase and amplitude of resonance are used to effect the control. This is here accomplished. The setup consists of a cam on the motor shaft which gives the slight impulse necessary to set a vibrating arm in motion. The fact that this cam is reduced by levers to give an impulse of the order of one thousandth of an inch shows what good resonance it is possible to have in the arm. Plotting the displacement of the arm from a given reference point in the position of the motor as it revolves, Mr. Braman has developed this most interesting curve of response of the vibrator with respect to the speed of rotation of the motor. (Fig. 9) It will be appreciated that it is a combination of phase displacement and amplitude of resonance of the vibrator. It will be noted that it has a very steep portion.

To make use of this curve, air was then driven from a nozzle through a sector carried by the vibrating arm. This air was then carried on to brake the motor. The result is a very fine setting of the speed on this frequency curve of the vibrator.

Hot Air Recording

In the search for visible, daylight, finished recording, it is believed a new departure has been realized.

We are all familiar with the sensitive gas flame of Dr. Koenig of Germany. It was this which started the development of the hot air

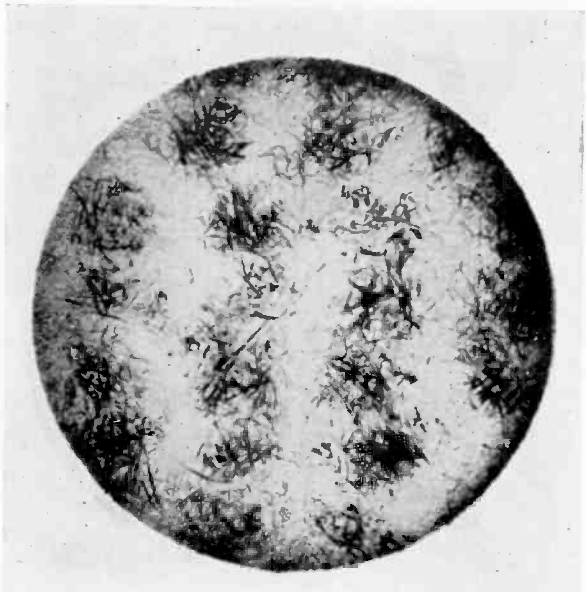


Fig. 10

recording, but as a matter of fact when the obvious attempt was made to make incoming radio signals control the sensitive flame, it was found that a much too powerful agent was at hand; it was very difficult to keep the gas from eating up everything as well as the recording paper. Then it was that Mr. Hansen worked up the use of plain hot air to do the job. Likewise, an electric spark was used, but the wear and tear on the point did not prove feasible from the operating position

By all odds the heavy part of this new development is in the sensitive paper. We are all so familiar with extreme sensitivity of photographic paper. We have tried to duplicate this where heat rather than

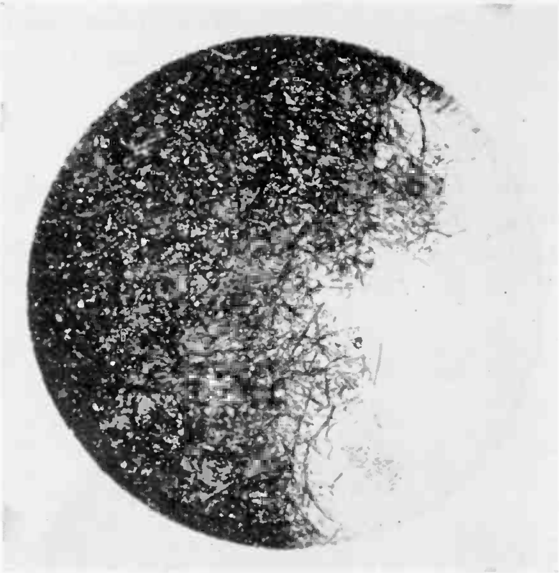


Fig. 11

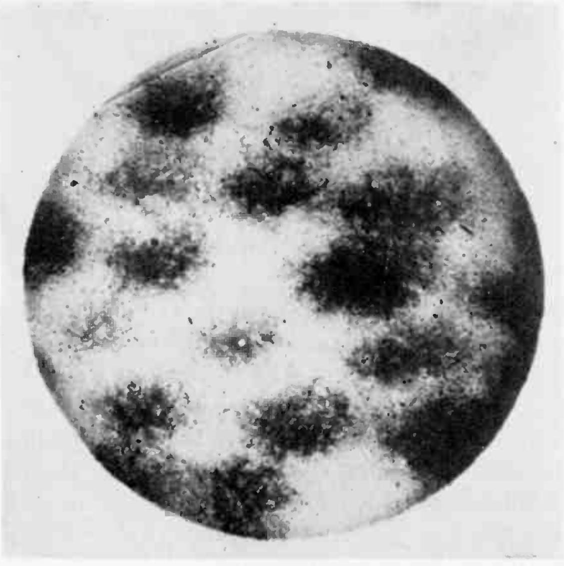


Fig. 12

light is the sensitizing agent. Likewise we have imposed the additional restriction that the product shall be finished without further treatment.

In beginning the chemical investigation of this we had the good fortune to be able to start with R. S. Bicknell as consulting chemist. He has been seconded by Mr. Morehouse who joined our staff for this specific investigation. Mr. Morehouse gives the following general outline of his problem.

Heat-Sensitive Paper

"Photoradiograms are recorded by means of a fine jet of heated air directed against heat-sensitive paper. When the incoming radio signal calls for a dot or a dash, the hot air stream is permitted to flow against the paper; when a space is required, the hot air stream is automatically prevented from striking the sensitized paper surface. This effect is accomplished by means of an electrically operated valve, the action of which is governed by the incoming radio signal.

"The character of the record left on the paper depends on the nature of the sensitizing agent, and also on the physical character of the paper base itself. The sensitizing agent in use at the present time consists of a practically colorless mixture of chemical salts, which are capable of undergoing what is known as an endothermic double decomposition reaction, with the formation of brownish black products. In other words, until heat is applied to these salts they remain in contact with each other on the surface of the paper in a comparatively inert condition, retaining their colorless appearance. But when heat is applied they absorb it rapidly, become mutually interactive, and form the black products which we see as a dot or dash on the picture.

"The importance of the physical character of the paper stock is best brought out with the aid of photomicrographs. Fig. 10 shows a few dots made by the hot air stream on an ordinary uncoated book paper made sensitive to heat. It will be noticed that the dots have a very pronounced fibrous or stringy character. The same fibrous appearance is noticeable in Fig. 11 which shows dashes made on this same type of paper. Fig. 12, on the other hand, shows the type of dot obtained with a clay-coated paper similarly sensitized. In this case the dots have a much softer and smoother appearance because of the fact that the paper fibres are completely covered by the thin layer of clay. Fig. 13 shows dashes made on the same clay-coated stock. In all four cases the chemical composition of the sensitizing agent is the same; the difference between the first two and the last two is entirely due to the difference in the physical character of the paper surface. In cases three and four the effect is very similar to that obtained by using photographic paper in the process of recording with the aid of light instead of heat."

Operation

We now come to the question of operating the equipment. Commercial picture transmission was inaugurated on May 1st, 1926, between London and New York, and likewise it was established on the West Coast between San Francisco and Honolulu. There was considerable excitement incident to the inauguration of the service, especially from London to New York. It was finally determined that one picture only to each newspaper and commercial concern would be sent in the sequence that they were received for the inception of the service. After the novelty had worn off, the photoradio service came into extensive use on a business basis.

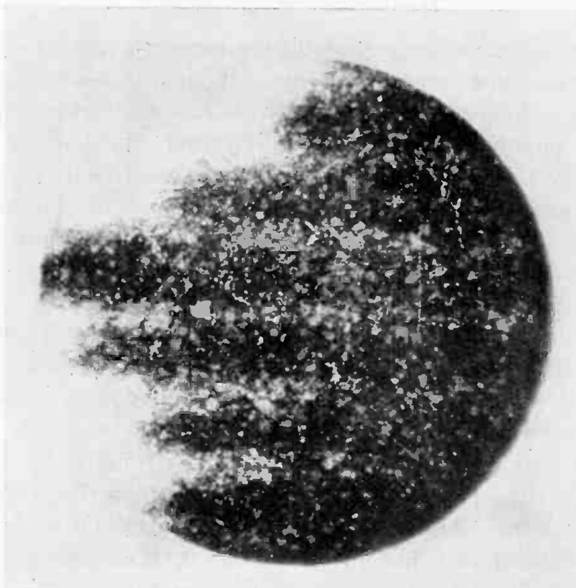


Fig. 13

Besides the news service, however, there has been a gradual but general extension of the service first into the style and fashion field and then into banking and commercial operations.

A few months ago for example, a bank came to us with the request to transmit the three signatures necessary to authorize a certain commercial transaction involving one million dollars. They advised that there were but three days remaining to get these signatures into London, and as there was no Lindbergh expedition scheduled for that particular day, it would be impossible to get these signatures across by any other means. However, we were told to take our time in the three days, and get a good facsimile of the signatures across.

We advised London of the situation and asked them to work with us until a satisfactory facsimile had been transmitted. The equipment at each end was set in motion; in twenty-five minutes (it was a half-size picture) transmission was put through once. London was asked for a report on this first transmission with the idea that the reply would indicate that it should be made darker or lighter or something else on the second trial. Back came the laconic reply "O.K.," and that was all that was necessary of the three days' grace to complete the million dollar transaction, as far as photarodio was concerned.

Department Store Demonstrations

At the request of Kauffman's of Pittsburgh we embarked on what might be termed barn storming expeditions. It might seem at first that such excursions would be far afield from a technical development, but there is little question but that these demonstrations have speeded up the development of photoradio to commercial reality in great degree. The cost of these demonstrations was met by the department stores. It therefore became imperative that a very worth while and useful service be given which would be capitalized by the department store.

The basis of these demonstrations was to bring the latest styles from Paris and London directly into the department store. The transmission to New York from Europe was easy—the problem came in relaying this on to the stores. Short waves were used for the relaying. WIZ at New Brunswick and WAQ of the Westinghouse Company at Newark were used for this relaying service; they were connected with Broad Street, New York by land lines; the picture reception comes in over land lines from Riverhead, Long Island and Belfast, Maine, to Broad Street, which therefore may be termed photoradio central. The well known fading propensity of short waves became of less importance to the photoradio signals on the dot-dash plan. Added to this, limiting was used with great success. Further demonstrations were then held at Strawbridge and Clothiers in Philadelphia, Jordan Marsh in Boston, Marshall Field in Chicago, and lately at L. S. Ayres. Further barn storming could have been developed, but it was felt that with successful termination of these demonstrations their purpose and usefulness as far as advancing the art is concerned was sufficient.

Navy Development

As the request of the Navy Department, successful transmissions were accomplished of navy manoeuvers in Honolulu direct to New York. Subsequent to this, the Navy Department has installed photoradio equipment at the Bureau at Washington. Transmission and reception has been successfuly accomplished between the photoradio station and the Radio Corporation stations. Likewise, condensed equipment has been installed on board the USS *Seattle*, later transferred to the *Texas*, when the latter became the flagship. Perhaps the most successful transmission with this equipment has been an entire page of printed matter from New York to the *Texas* when she was lying off San Diego something like three thousand miles from New York.

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GRID LOSSES IN POWER AMPLIFIERS*

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Summary—*In spite of the widespread use of radio-frequency power amplifiers, apparently no measurements of the power required to drive such amplifiers have ever been made. In this paper, a study of driving power is made at 60 cycles. It is found that the power input to the grid is proportional to the d-c grid current raised to the 1.34 power. The proportionality factor depends on the type of tube and the plate circuit conditions. With constant grid current the grid power input is found to be practically independent of grid bias voltage. A simple theory, based on an assumed relation between grid current and voltage, agrees quite well with the experimental results. Data are then given on six types of commercial air cooled transmitting tubes showing driving power, a-c driving voltage, gross power output and efficiency as functions of the d-c grid current. Finally, the effects of primary and secondary electron emission by the grid on the driving power are considered.*

1. Introduction

HIGH-VACUUM electron tubes of the transmitting type have taken on a steadily increasing significance ever since their introduction into the art of radio communication about a decade and a half ago. During this period, numerous technical investigations into the operation of such tubes as radio-frequency power amplifiers and oscillators have been published. These have dealt largely with the conditions under which optimum conversion of plate supply power to radio-frequency power is obtained. Thus the plate circuit has received careful consideration. The grid circuit, on the other hand, has been almost completely neglected. The object of this paper is to present measurements of grid circuit losses on a series of typical air cooled transmitting tubes. A knowledge of such losses is particularly useful for the design of multistage amplifiers, where the plate circuit of one tube must supply the grid circuit losses of the succeeding tube.

2. Operation of Radio-Frequency Power Amplifiers

The elementary radio-frequency power amplifier circuit is shown in Fig. 1. A voltage E_g, which generally approaches a sine wave (and is so assumed throughout this paper), is impressed upon the amplifier in series with a bias voltage E_c. Since maximum power output is desired, no particular attention is paid to the wave shape of plate

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current, a tank circuit L,C,R being relied upon to filter harmonics out of the output. This circuit is tuned to the frequency of E_g and presents a load impedance of very nearly L/RC ohms at unity power factor to the fundamental component of plate current. The impedance of this circuit for harmonics is usually quite small in comparison with L/RC, hence, in spite of badly distorted plate current wave shapes, the voltage across the tank circuit is practically sinusoidal and 180 deg. out of phase with E_g. The lower the decrement (or ratio of resistance to reactance) of the tank circuit, the more nearly a sinusoidal wave shape is obtained.

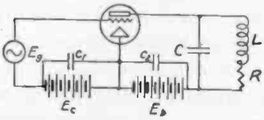

Fig. 1—Elementary Radio-Frequency Power Amplifier Circuit

Now it is well known that for high plate circuit efficiency, plate current should flow during less than half a cycle of the exciting voltage.[1,2,3,4] This is accomplished by making E_c sufficiently negative so that if E_g were reduced to zero, no plate current would flow. For proper excitation, E_g must now have an amplitude sufficient to drive the grid positive, at the instant of maximum grid potential, by an amount which varies between 50 per cent and 100 per cent of the *instantaneous* plate potential at that moment. Unless this condition obtains, an inefficient transfer of plate current results. This maximum positive grid potential may lie between 5 per cent and 15 per cent of the plate supply voltage. During the time that the grid is positive with respect to the filament, an electron flow to the grid takes place. This flow causes a twofold loss of power; first, a loss due to electron bombardment of the grid and second, a charging of the bias battery E_c. These are the losses which form the subject of this investigation. Any losses caused by the grid current in transformers, condensers, or other apparatus associated with the grid circuit will not be considered here because they are primarily a matter of circuit design.

3. Apparatus

Measurements at radio frequencies are rather difficult, and one of the most useful of all electrical instruments, the commercial low-frequency oscillograph, cannot be applied. Except for extremely high

[1] H. Rukop, *Jahr. d. Draht. Tel. u. Tel.*, 14, 110, 1919.
[2] J. H. Morecroft, *Jour.* A. I. E. E., 38, October, 1919.
[3] Marius Latour and H. Chireix, Proc. I. R. E., 11, 551; October, 1923.
[4] H. G. Moller, "Die Elektronenröhren," Vieweg and Sohn, 1922.

frequencies, i.e. of the order of 3×10^8 cycles, electron tubes show no time lag; consequently, for the present purpose, the same results can be obtained at commercial power frequencies as at radio frequencies. For these reasons, all experiments reported here were made at 60 cycles.

The circuit arrangement employed is shown in Fig. 2. Instead of tuning the tank circuit, L-C_2, which would have been rather difficult at the low frequency and high plate voltages used, the tank was constructed of a fixed inductance and capacitance and the frequency of the sine wave alternator varied until the plate current was a mini-

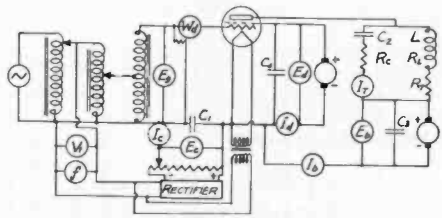

Fig. 2—Test Circuit

mum. Experiments showed that when the plate current is a minimum, the plate efficiency is maximum. The voltage at V_1 was maintained at 110 volts for supplying the frequency meter and grid bias rectifier. The grid exciting voltage was supplied by a 5-kw transformer so that the asymmetrical grid current would not distort the actual grid voltage. Oscillograms showed that the instantaneous grid voltage was for all practical purposes the sum of a pure sine wave and a constant voltage.

The tank circuit consisted of an air core inductance (L) of 0.674 ± 0.001 henries and 5.30 ohms d-c resistance, and a bank of high voltage oil condensers (C_2) whose capacity and series resistance at 60 cycles were 10.23 μf and 1.95 ohms, respectively. The constants of the condensers were determined by taking a series resonance curve. In this manner the frequency for equal inductive and capacitative reactances was found to be 60.56 cycles per second.

For varying the load on the tube, additional resistance R_t could be connected into the tank circuit. If the frequency is adjusted until $\omega^2 LC_2 = 1$, the effective resistance of the tank circuit is

$$\frac{R_c(R_1+R_t)+\dfrac{L}{C}}{R_c+R_1+R_t}$$

Since in the case under consideration $R_c = 1.95$ ohms, $R_l = 5.3$ ohms, $L/C = 65{,}900$ (ohms)^2, the first term of this expression is negligible except for values of R_t of the order of hundreds of ohms. The largest value of R_t used here was 61 ohms. Hence, the effective resistance of the tank circuit is simply L/RC ohms, where R is the total resistance of the tank circuit, or $R_l = R_l + R_c + R_t$.

The gross output of the tube is I_t^2R. For the larger values of R_t this expression gives output figures which are higher than the true output, because harmonics of the plate current flow largely through the capacity branch of the tank circuit. This was avoided by switching the I_t meter from the capacity to the inductance branch of the tank circuit (see Fig. 2). Corrections for the plate current were then necessary.

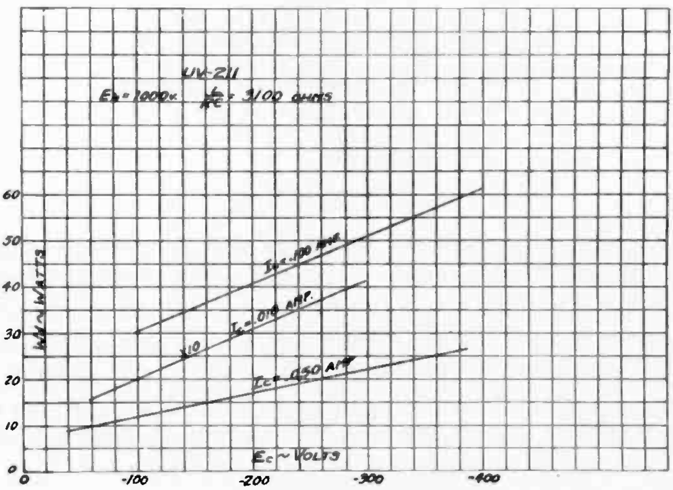

Fig. 3

The filament was supplied from the same source as the grid excitation voltage. The effect of this was to increase or decrease the grid excitation by an amount equal to about half the filament voltage. In all cases the filament voltage was adjusted to the rated value. On account of the excess emission available in thoriated filament transmitting tubes, variations in filament voltage up to ± 10 per cent have practically no effect on the output. This was convenient as it made accurate adjustment of filament voltage unnecessary.

Grid driving power was measured by a wattmeter connected to the grid circuit as shown in Fig. 2. A special meter with a current coil resistance of 0.9 ohm was selected. Low current coil resistance is essential if distortion of the grid voltage wave form is to be avoided.

The arrangement for supplying the voltage E_d in Fig. 2 was only used when four-element tubes were being tested.

4. Experimental Results

The power required to drive the grid of a power amplifier can be divided into two distinct components. With sinusoidal grid excitation as soon as $\sqrt{2}\ E_g > E_c$, grid current flows during the portion of each cycle when the grid is positive with respect to the filament. This current must overcome two voltages—the voltage from grid to filament and the bias voltage E_c. The average power loss in the bias source is simply $E_c \cdot I_c$, assuming that C_1 by passes the radio-frequency components of grid current and has negligible series resistance. If we call the average driving power W_d and the average grid loss W_g, we get

$$W_d = W_g + E_c I_c. \tag{1}$$

Now W_g is a function of E_g and E_c, but since I_c is a function of these same variables, we can eliminate E_g and consider $W_g = f(E_c, I_c)$.

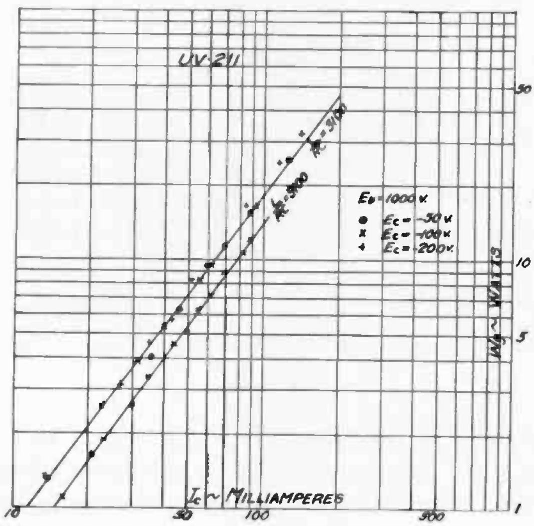

Fig. 4

The first measurement of W_d was made with a UV211 operated under "full load" plate circuit conditions. The method of determining these conditions will be described later. E_g and E_c were varied in such a manner as to keep I_c constant. The result is shown in Fig. 3. These curves show that for $I_c = 0.100$ and $I_c = 0.050$

$$\frac{\partial W_d}{\partial E_c} = I_c.$$

For $I_c = 0.010$ ampere $\partial W_d/\partial E_c = 0.91\ I_c$. However, this last value of I_c is considerably below the normal operating value for the UV211. In the normal operating region we can say $\partial W_d/\partial E_c = I_c$. From (1) it is evident that this can only be true provided $\partial W_g/\partial E_c = 0$; that is, the grid loss is independent of the grid bias for constant I_c. Equation (1) thus becomes

$$W_d = f(I_c) + E_c I_c. \tag{2}$$

This independence between W_g and E_c was also observed in the case of other tubes. It is only necessary, therefore, to show W_g as a function of I_c.

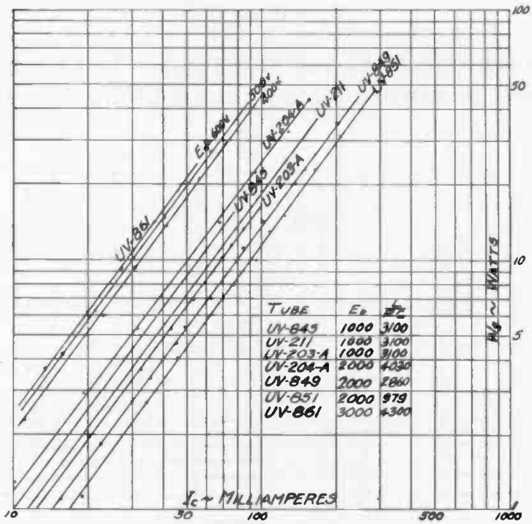

Fig. 5

The upper curve on Fig. 4 shows $W_g = f(I_c)$ for a UV211, taken with several values of E_c. The points are all on or close to one straight line. Deviations were found by repeated measurements to be due mostly to experimental error resulting from variations in supply frequency and voltages. From this curve we get

$$W_g = 0.039 I_c^{1.34} \text{ watts} \tag{3}$$

where I_c is expressed in milliamperes.

So far, full load plate circuit conditions were maintained. These are the most common conditions. The effect of a change in the plate load can be seen from the lower curve of Fig. 4 which shows W_g for a light load curve. The coefficient in (3) is now reduced to 0.03, which is a reduction of 23 per cent. It is easy to see why the grid loss should be less for light loads than for heavy loads with a given value of I_c. As the plate load resistance L/RC is increased, the minimum instantaneous plate voltage decreases steadily and may even go negative. This means that the maximum positive grid voltage may be considerably above the plate voltage. Under these conditions the grid collects most of the electrons emitted from the filament. Thus other conditions remaining the same, as the plate load resistance is increased I_c will increase. E_g must now be decreased to obtain the same value of

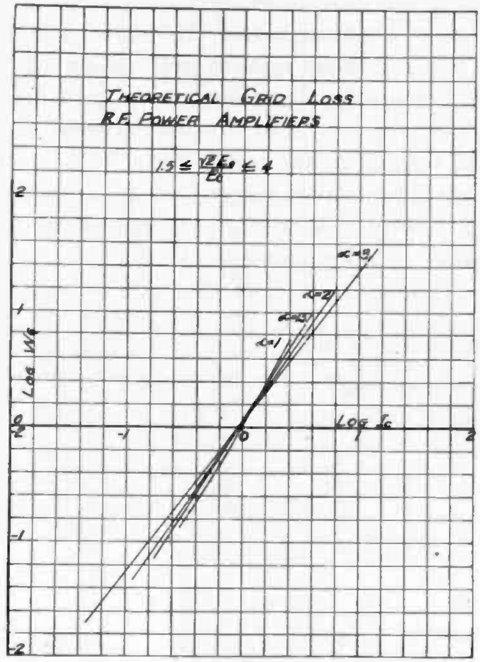

Fig. 6

I_c as at full load. After this change has been made, the wave shape of grid current is not much different from what it was originally. Consequently, since the decrease in E_g decreased the positive grid voltage, the grid loss is less.

$W_g = f(I_c)$ was now taken for a series of seven different types of transmitting tubes[5] under full load conditions. These results are shown on Fig. 5. For every tube the function $W_g = f(I_c)$ is expressible as

$$W_g = AI_c^{1.34} \text{ watts} \tag{4}$$

where I_c is again in milliamperes. The value of A is given in Table 1.

[5] Technical information on these tubes is given in Appendix III.

TABLE 1

Tube	*L/RC* ohms	*A*
UV845	3100	0.0467
UV211	3100	0.0390
UV203A	3100	0.0356
UV204A	4030	0.0531
UV849	2860	0.0293
UV851	979	0.0244
UV861 ($E_d = 500$ v)	4300	0.107

No quantitative relation connecting the coefficients A and the dimensions of the tubes could be found. However, a little consideration will show that the following qualitative statements must be true. Other conditions remaining the same, as the grid mesh is increased, (thus increasing the μ of the tube) A must decrease. This agrees with the experimental results in the case of the UV845, the UV211 and the UV203A which differ only in the grid mesh and have values of μ of 5, 12, and 25, respectively. As the μ is increased by moving the grid toward the cathode, A must decrease.

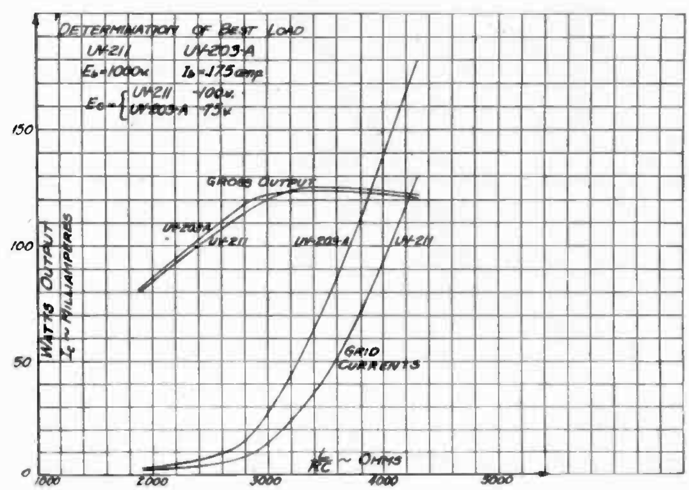

Fig. 7

5. Theory of Grid Loss

The maximum error in the exponent 1.34 was estimated to be about ±0.04. Thus the extreme values of the exponent are 1.30 and 1.38. It is now of interest to see if this exponent can be justified theoretically. To do this it is necessary to know the function connecting instantaneous grid voltage and grid current. Analysis of oscillograms showed that this function is quite complicated under normal power amplifier conditions. Among other things, it is affected by secondary electron emission, which is generally present. Assuming

$$i_g = Be_g^{\alpha} \tag{5}$$

the exponent α varied from a value 1 at the lowest grid currents to a value 4 at the highest currents.

From (5) expressions for I_c and W_g can be obtained. These are

$$I_c = \frac{B}{\pi} E_c^{\alpha} \int_0^{\cos^{-1}(E_c/\sqrt{2}E_g)} \left(\frac{\sqrt{2}E_g}{E_c} \cos x - 1 \right)^{\alpha} dx \tag{6}$$

$$W_g = \frac{B}{\pi} E_c^{\alpha+1} \int_0^{\cos^{-1}(E_c/\sqrt{2}E_g)} \left(\frac{\sqrt{2}E_g}{E_c} \cos x - 1 \right)^{\alpha+1} dx. \tag{7}$$

If E_g is eliminated from these equations, W_g can be expressed as a function of E_c and I_c. Taking $E_c = 1$, $B = \pi$, $\log W_g = f(\log I_c)$ was calculated for $\alpha = 1$, 1.5, 2, and 3 for the practical range $1.5 \leqq \sqrt{2}E_g/E_c \leqq 4$.

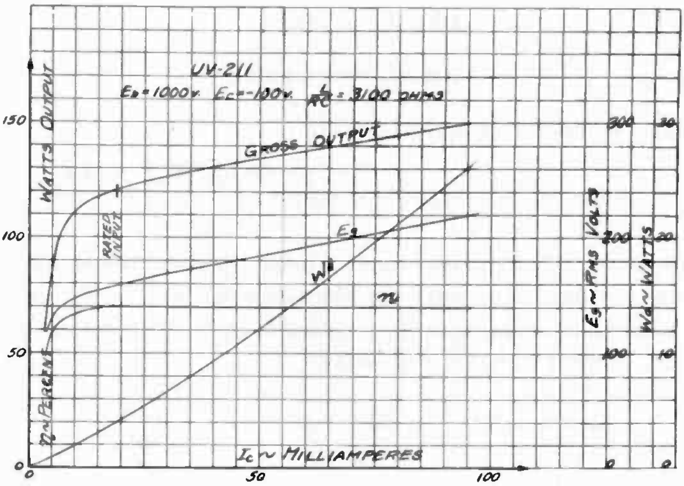

Fig. 8

These results are shown in Fig. 6. For $\alpha = 1$ a slightly curved line is obtained, but for the remaining values of α the line is straight for all practical purposes. These curves are exactly like the ones found experimentally. The slope of the experimental curves is 1.34 ± 0.04. The slope of the theoretical curves is seen from the following:

α	Slope $=\dfrac{\partial(\log W_g)}{\partial(\log I_c)}$
1	1.73
1.5	1.57
2	1.46
3	1.30
Experiment	1.34 ±0.04

Thus even though the simple relation (5) does not hold exactly, the exponent α can vary over a considerable range and still result in a type of grid power characteristic very nearly the same as the experimentally observed characteristics.

It now remains to explain the observed fact that W_g is practically independent of E_c as long as I_c is constant. The effects of variations in E_c can be seen from (6) and (7) if E_g/E_c is held constant.

$$\log I_c = \alpha \log E_c + K_1$$

$$\log W_g = (\alpha+1) \log E_c + K_2$$

Now if E_c is increased from E_c to $m\,E_c$, the corresponding increments in $\log I_c$ and $\log W_g$ are

$$\Delta \log I_c = \alpha \log m$$

$$\Delta \log W_g = (\alpha+1) \log m$$

From these equations it is clear that if

$$\text{Slope of } \left[\log W_g = f(\log I_c)\right] = \frac{\partial(\log W_g)}{\partial(\log I_c)} = \frac{\alpha+1}{\alpha} \tag{8}$$

any increase in E_c will merely move a point up along the characteristic. Thus for all values of E_c only one characteristic will be obtained. The degree with which condition (8) is satisfied by the theoretical curves of Fig. 6 can be seen from the following:

α	$\dfrac{\partial(\log W_g)}{\partial(\log I_c)}$	$\dfrac{\alpha+1}{\alpha}$
1	1.73	2
1.5	1.57	1.67
2	1.46	1.50
3	1.30	1.33

Condition (8) is thus nearly satisfied for $\alpha=2$ and $\alpha=3$. Hence, for both these values of α, W_d is practically independent of E_c, for constant I_c. Again, there is a satisfactory agreement between theory and experiment.

6. Relation between Driving Power and Plate Circuit Output

The measurements of grid losses given in the preceding section are most useful when accompanied by plate circuit output and efficiency. The purpose of this section is to give these additional data and to show how advantageous operating conditions can be chosen.

For economic reasons, transmitting tubes are usually operated under maximum rated conditions. All the tubes used here have d-c plate current and plate voltage limits for r-f power amplifier or oscil-

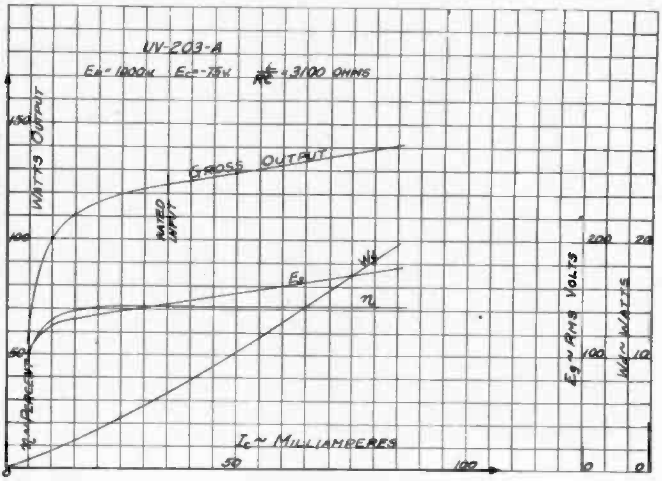

Fig. 9

lator services.[6] Thus the maximum plate input is fixed in each case. If a tube is operated at its maximum plate voltage, the input can be controlled by means of the plate load resistance L/RC, the excitation current I_c, and the grid bias E_c. The problem of finding advantageous operating conditions then consists of determining values of these three variables which will give low driving power and high plate efficiency.

It is well known that for a fixed plate current, the more negative E_c the greater the efficiency of the plate circuit. In the practical range of values of E_c the rate of increase of efficiency with E_c is quite low. Experiment shows that as E_c is made more negative, I_c must also be increased to hold constant plate current. From the preceding sections it is obvious that the driving power will therefore increase more than proportionally to E_c. The best value of E_c will therefore depend largely

[6] See Appendix III.

on the driving power available. It was sufficient for the present purpose to choose values of E_c which gave within a few per cent of the maximum full load efficiency that could be obtained with very large values of E_c.

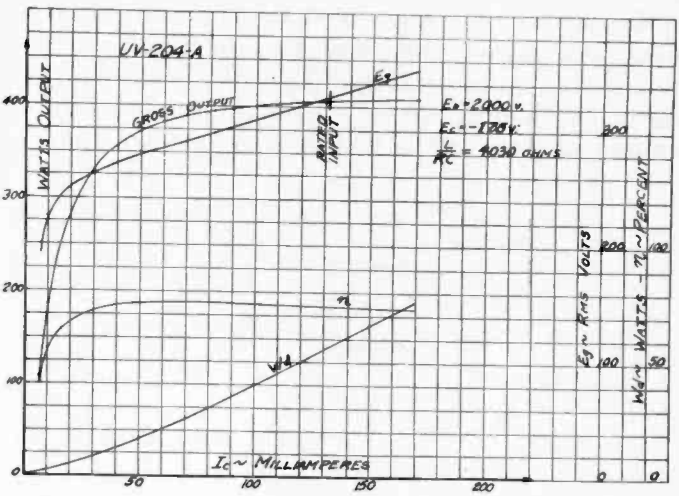

Fig. 10

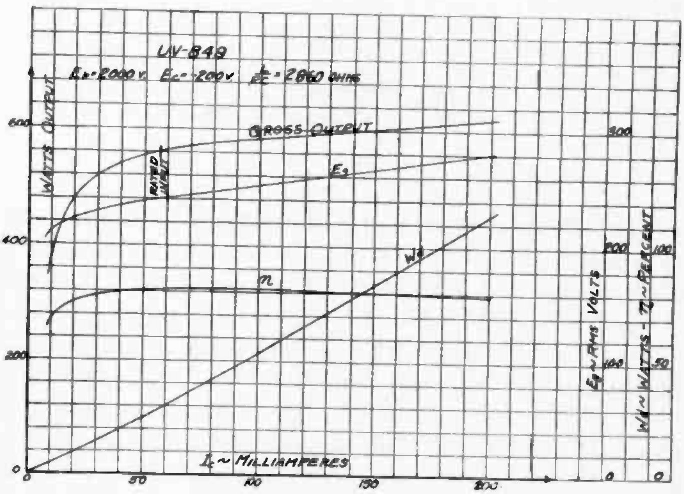

Fig. 11

Now with E_c fixed, only one value of I_c will give the desired plate input for any given value of L/RC. In this manner curves of power output against L/RC can be obtained. Two typical curves for the UV211 and UV203A are shown on Fig. 7. These curves show that maximum output can be obtained at about $L/RC=3100$ ohms. For higher values of L/RC the grid currents, and hence the driving power, increase very rapidly. The reason for the increased grid currents is as follows: When L/RC is large the minimum plate voltage is small. This produces a deep saddle in the plate current wave form, which loss in current must be made up by spreading the plate current over a large portion of each cycle (since we are holding a constant I_b). This can only be done by a large increase in E_g, so that I_c increases rapidly too. Thus for low driving power, L/RC should be chosen as low as possible. At the lower values of L/RC the output decreases rapidly because in this case the minimum plate voltage is high, which results in high plate loss and therefore a lower output. So when the optimum of output is broad, it is best to choose the lower values of L/RC.

The operating conditions chosen for several types of tubes are shown in Table II.

TABLE II

Tube type	E_b v	I_b a	E_c v	I_c ma	E_g v	L/RC ohms	Gross Output watts	η Per cent	W_d watts	$E_b/2I_b$ ohms
UV-211	1000	0.175	−100	19	160	3100	122	69.7	3.9	2860
UV-203A	1000	0.175	−75	35	145	3100	123	70.3	6.7	2860
UV-204A	2000	0.275	−175	131	328	4030	405	73.6	56.	3640
UV-849	2000	0.350	−200	57	240	2860	562	80.2	29.	3500
UV-851	2000	0.900	−200	300	274	979	1420	78.5	111.	1110
UV-861*	3000	0.350	−200	40	455	4300	665	63.5	22.5	4290

* $E_d=500$ v

The approximate value of L/RC for any case can be obtained in the following manner:

Let $K=E_t/E_b$. Then

$$\eta E_b I_b = \frac{(KE_b)^2}{\frac{L}{RC}}$$

or

$$\frac{L}{RC} = \left(\frac{K^2}{\eta}\right)\frac{E_b}{I_b}.$$

Assuming $K=0.60$, $\eta=0.70$ as average values

$$\frac{L}{RC} = \frac{E_b}{2I_b}. \tag{9}$$

This approximation is especially convenient when tubes are to be operated at unusual plate voltage and current. Values of $E_b/2I_b$ are tabulated in Table II for comparison with L/RC.

The first two tubes in Table II differ only in grid mesh, the UV211 having an amplification factor of 12 and the UV203A, 25. For the chosen conditions the UV211 requires somewhat less grid power. No absolute conclusions can be drawn from these data, however, as a slight change in L/RC can easily make the UV211 take more driving power.

The same things are true to a certain extent of the other tubes listed in Table II. The UV204A, for example, requires 56 watts driving power. If the L/RC is lowered from 4030 to 3810 ohms, the driving power is reduced to about 30 watts, with an attendant decrease in plate efficiency. In a similar manner, the driving power for the other

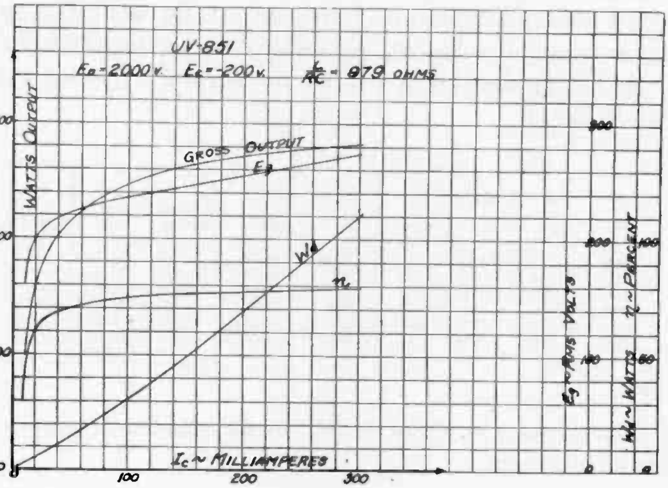

Fig. 12

tubes can be decreased. This brings out the fact which has already been explained and must be emphasized again, namely, that the operatating conditions of Table II are not the optimum conditions for all cases. These tubes can be operated very satisfactorily under conditions differing considerably from those set down in Table II. The latter are simply representative conditions which have been chosen in an endeavor to keep the driving power as low as possible without too great a loss in plate efficiency. If in any particular installation more driving power is available, it is advantageous to increase L/RC and E_c thereby increasing the plate efficiency.

Using the values of L/RC and E_c given in TableII curves of power output, efficiency, driving power and a-c grid voltage were taken as functions of I_c. These are shown in Figs. 8 to 14 inclusive. It might be argued that these curves would be more properly shown as functions of E_g, since E_g is a true independent variable. E_g is difficult to measure at high frequency and the provision for its measurement in actual installations, such as radio transmitters, is unheard of. I_c, on the other hand, is easy to measure and meters for its measurement are generally provided in practice.

There is nothing new or unusual about the curves of Figs. 8 to 14 with the exception that they show driving power. The reasons for the various forms of the curves are so obvious that they will not be recited here. These curves show clearly what happens when the excitation current (I_c) is varied, other conditions (L/RC, E_b, E_c) remaining con-

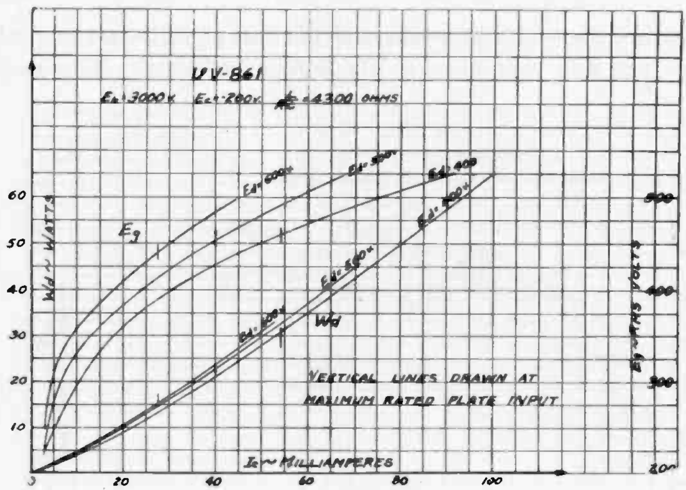

Fig. 13

stant. The output curves have a shape similar to magnetic saturation curves for iron. For this reason the value of I_c which gives an output just above the knee of the curve is sometimes called the "saturation grid current." This is generally a rather indefinite value. Furthermore, it is different for every plate load (L/RC). For large values of L/RC the saturation grid current is low and vice versa. Thus, unless L/RC is specified, the term "saturation grid current" has little significance.

Figs. 10 and 11 give an interesting comparison between the UV204A and the UV849. The latter tube is of later design and is definitely superior to the UV204A as far as excitation voltage and efficiency are concerned, when both are operated at their maximum rated plate inputs. If now the input to the UV849 were reduced to the same value as that of the UV204A by increasing L/RC its efficiency would increase while the driving power and a-c grid voltage required would decrease, thus still further increasing the superiority of the UV849.

Figs. 13 and 14 are some curves on the UV861 screen-grid tube as an r-f power amplifier. These were taken with the screen maintained at filament a-c potential by a 115-μf condenser. From these curves a comparison can be made when the tube is operating under various screen voltages, E_d. These are given in Table III.

TABLE III

$E_b = 3000$ v		$E_c = -200$ v		$I_b = 0.350$ a	
E_d	I_c	E_g	W_d	Output	η
v	ma	v	watts	watts	Per cent
400	54	460	30.5	678	64.6
500	40	453	22.5	665	63.5
600	27.5	440	15.0	632	60.5

Oscillograms[7] showed that under each of these conditions the minimum plate voltage was very close to the screen potential. This caused a

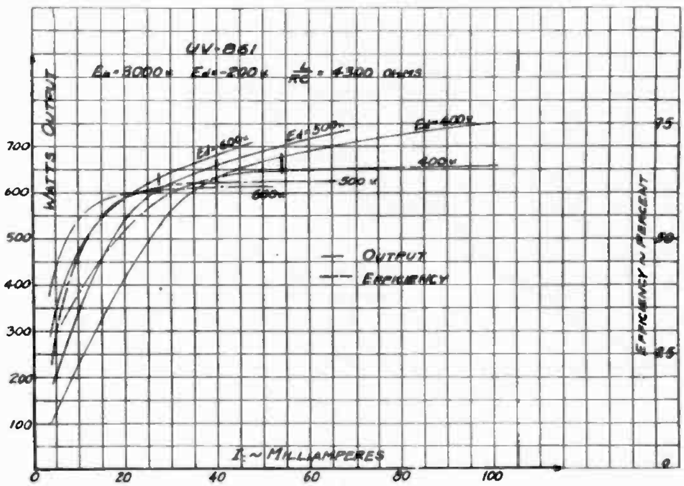

Fig. 14

deep valley in the plate current wave form. As the screen potential is lowered this valley is made less deep. Hence, for constant plate current the wave can be narrowed, thus increasing the plate efficiency. At the same time the grid must be driven more positive to get the required plate current through the screen grid. Thus for decreasing E_d, W_d increases.

7. Effect of Primary and Secondary Electron Emission from the Grid

The fact that conditions are favorable for the emission of secondary electrons by the grid, during the portion of each cycle when the

[7] See Appendix II.

grid is positive, has already been mentioned. Suppose a certain tube does not show secondary emission in the operating range of grid voltage. Let the grid current be represented by wave c in Fig. 15(a). W_d is the product of the fundamental component of this wave which is in phase with E_g, by E_g. If, as in Fig. 15(a), $e_g = \sqrt{2} E_t \sin \omega t - E_c$ then,

$$W_d = \frac{E_g \omega}{\sqrt{2}\pi} \int_0^{2\pi/\omega} i_g(t) \sin \omega t \cdot dt. \tag{10}$$

If, in some manner, the grid can be activated to emit secondary electrons, so that the wave c is changed to d, the value of the integral in (10) will obviously be diminished, thus reducing W_d, the driving power. At the same time, the plate current and power output will be increased to some extent. Therefore, the introduction of secondary emission will reduce the driving power without reduction of plate circuit output. Ample confirmation of this was found in several cases. One tube in particular, when not excited to full output, actually showed negative driving power and grid current. In other words, the grid was delivering rather than absorbing power.

A limited amount of secondary grid emission is desirable in r-f power amplifiers provided it can be controlled, but when the emission reaches such magnitude as to produce a reversal of grid current as the grid voltage passes a certain positive value, other disturbing factors appear. If a grid leak is used for bias, the grid may suddenly

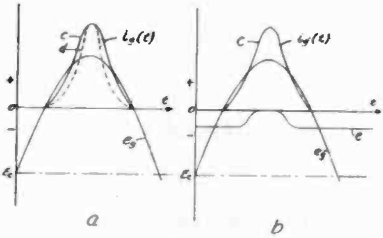

Fig. 15

assume a high positive potential which produces a destructive flow of plate current. This has for many years been termed "blocking."

Sometimes a small amount of primary (thermionic) emission from the grid is encountered. Since the grid temperature can hardly follow radio-frequency variations of E_g, the primary emission current is a continuous one provided the grid is always sufficiently negative with respect to the other electrodes to allow this current to be drawn away from the grid. From (10) it is evident that the addition of a continuous current to i_g (t) has no effect on the value of the integral and hence on W_g. If the grid potential is more positive than the plate during a fraction of a cycle, the primary emission current cannot leave the grid during this period. This is shown in Fig. 15(b), where the normal grid current and primary emission current are drawn separately as

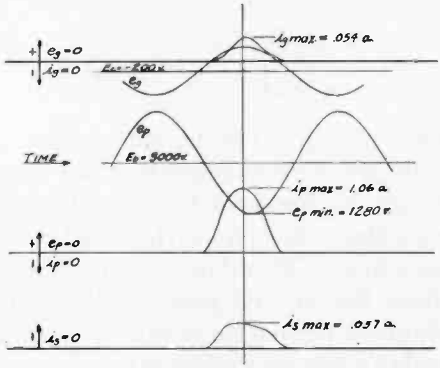

Fig. 16—UV861 under-excited.

$E_b = 3000$ v	$E_c = -200$ v
$I_b = 0.240$ a	$I_c = 10.4$ ma
Input = 720 watts	$E_g = 335$ v
Output = 352 watts	$E_d = 500$ v
Efficiency = 49 per cent	$I_d = -18.0$ ma

curves *c* and *e* respectively. Both have the same general phase. Hence, if variations in the primary emission current are taken into account, W_d will be increased by (10). Therefore, if primary grid emission is introduced into a tube, all other conditions remaining the same, either the driving power is not affected at all, or it is increased.

Appendix I

Symbols

E_b—d-c plate voltage
e_p—instantaneous plate voltage
I_b—d-c plate current
i_p—instantaneous plate current
E_t—r.m.s. voltage across tank circuit
E_g—r.m.s. grid driving voltage
e_g—instantaneous grid voltage
E_c—d-c grid bias voltage
I_c—d-c grid current or "excitation" current
i_g—instantaneous grid current
I_d—d-c screen current

i_s—instantaneous screen current
$\omega = 2\pi f$—angular velocity
f—frequency, cycles per second
L—inductance in tank curcuit
C—capacity in tank circuit
R—total resistance in tank circuit
η—plate efficiency
W_d—average grid driving power in watts
W_g—average grid loss in watts

Appendix II

Since screen-grid transmitting tubes have been developed rather recently,[8] a few oscillograms illustrating their action may not be out of place here. These oscillograms were taken in the circuit of Fig. 2, using a UV861.

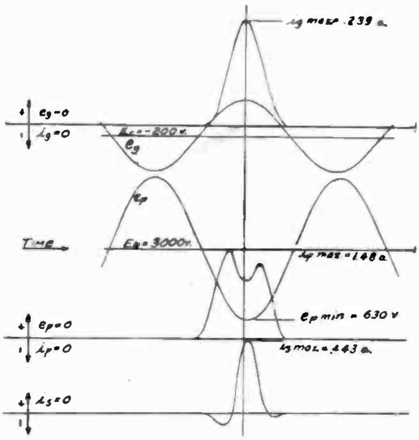

Fig. 17—UV861 fully excited.

$E_b = 3000$ v
$I_b = 0.358$ a
Input = 1074 watts
Output = 685 watts
Efficiency = 63.7 per cent

$E_c = -200$ v
$I_c = 48.0$ ma
$E_g = 496$ v
$E_d = 500$ v
$I_d = +49.0$ ma

The oscillogram of Fig. 16 was taken with the tube under-excited. As a result, the plate voltage swing was not very large, $e_{p\ min}$ being 1280 v. This caused the low efficiency shown. The screen potential was 500; this condition was favorable for an excess of secondary emission over the number of primary electrons captured by the screen and the screen current was negative as the oscillogram shows. On

[8] J. C. Warner, "Some Characteristics and Applications of Four-Electrode Tubes," Proc. I. R. E., 16, 424; April, 1928.

account of the screening effect, plate current wave shape was determined almost entirely by the positive grid voltage loop, the variation in plate voltage having very little effect. Since the positive grid voltage loop was not noticeably distorted from sinusoidal shape, the plate current looks like a section of a sine wave. The plate current leads the plate voltage slightly. This is because the frequency was always adjusted for minimum plate current. Under this condition the tank circuit has a capacitative reactance, but nevertheless, maximum efficiency is obtained.

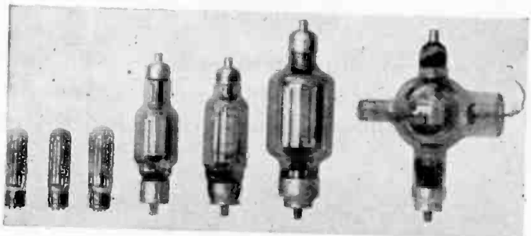

Fig. 18—Tubes Used in This Work.
Left to right: UV211, UV203A, UV845, UV204A, UV849, UV851, UV861.

Fig. 17 shows the conditions which obtained at full load and full excitation. $e_{p\,min}$ was now 630 volts, which was sufficient to pass only 1 ampere plate current. Hence, there is a saddle in the plate current wave. The plate was also robbed of current to some extent by the screen grid and control grid. As long as e_p was well above screen potential the screen current was negative, but as e_p approached the screen voltage, conditions became unfavorable for secondary emission from the screen so the screen current reversed.

Appendix III

AIR COOLED TRANSMITTING TUBES; THORIATED TUNGSTEN FILAMENTS

Filament Data	UV845	UV211	UV203A	UV204A	UV849	UV851	UV861
Volts	10.	10.	10.	11.	11.	11.	11.
Amperes	3.25	3.25	3.25	3.85	5.00	15.5	10.0
Electron Emission (amperes) Approx.	3.2	3.2	3.2	4.9	5.5	20.	11.
R-F Power Amplifier Use							
Maximum operating plate volts							
Modulated d-c	1000	1000	1000	2000	2000	2000	3000
Unmodulated d-c	1250	1250	1250	2500	2500	2500	4000
Maximum plate dissipation (watts)	100	100	100	250	400	750	400
Maximum d-c plate Current (amperes)	0.175	0.175	0.175	0.275	0.350	1.00	0.350
Output watts (Nominal)	75	75	75	250	350	1000	500

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MATHEMATICAL THEORY OF THE FOUR-ELECTRODE TUBE*

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Summary—*This paper gives the mathematics of the four-electrode tube, including in the most general case expressions for the plate and two grid currents in terms of applied voltages in the two grid circuits and the impedances of all three circuits. The results are exact, variation in the amplification factors being included. Because of the complexity of the general equations, these expressions for the currents have been developed:* I, *in terms of one grid-filament voltage and external impedances in plate and other grid circuit, assuming no voltage in latter;* II, *in terms of grid-filament voltages of both grid circuits and external impedance in plate circuit;* III, *in terms of applied voltages in both grid circuits and external impedances in plate and two grid circuits.*

The results of I *show the effect of an external impedance in the non-control grid of a "screen-grid" or "space-charge-grid" tube;* II *covers the approximate theory of a "double-function" tube;* III *gives a more exact and comprehensive theory of the tube in any use.*

THE purpose of this paper is to develop general equations for the plate and grid currents of a four-electrode tube. The work is essentially an extension to the four-electrode tube of Carson and Llewellyn's work on the three-electrode tube.[1]

Let the subscript a refer to one of the two grids, b to the other; let E_p, E_a, E_b be the total instantaneous voltages between the plate, first and second grids, and the filament, respectively; and let I_p, I_a, and I_b be corresponding currents. Then if the subscript o denote steady d.c. values obtaining before the application of any a.c. voltage to the tube, and small letters indicate changes resulting from applied a.c. voltages,

$$\begin{aligned} E_p &= E_{po}+e_p \\ E_a &= E_{ao}+e_a \\ E_b &= E_{bo}+e_b \\ I_p &= I_{po}+i_p \\ I_a &= I_{ao}+i_a \\ I_b &= I_{bo}+i_b. \end{aligned} \qquad (1)$$

* Dewey decimal classification: R132. Original manuscript received by the Institute, October 24, 1928.

[1] Carson, "A Theoretical Study of the Three Element Vacuum Tube," Proc. I R.E., 7, 187; April, 1919.

Llewellyn, "Operation of Thermionic Vacuum Tube Circuits," *Bell System Tech. Jour.* V, 433–462; July, 1926.

Since much of the interpretation of the results of this paper is similar to that of the latter paper, it will not be entered into in detail here.

The amplification factors μ_a and μ_b, the plate resistance r_p, and the mutual conductances g_a and g_b are defined as follows:

$$\mu_a = \frac{\dfrac{\partial I_p}{\partial E_a}}{\dfrac{\partial I_p}{\partial E_p}} \qquad \mu_b = \frac{\dfrac{\partial I_p}{\partial E_b}}{\dfrac{\partial I_p}{\partial E_p}} \tag{2}$$

$$\frac{1}{r_p} = \frac{\partial I_p}{\partial E_p} \tag{3}$$

whence

$$\frac{\partial I_p}{\partial E_a} = \frac{\mu_a}{r_p} = g_a \qquad \frac{\partial I_p}{\partial E_b} = \frac{\mu_b}{r_p} = g_b \tag{4}$$

Note that

$$\frac{\partial g_a}{\partial E_b} = \frac{\partial g_b}{\partial E_a}.$$

It should be recalled that a partial derivative of a function of several independent variables with respect to one of them is the ordinary derivative of the function with respect to that variable when all the other variables are held constant. Hence μ_a, as defined, is to be determined for a fixed value of E_b. In addition to the above, define six analogous quantities for the grid circuits thus:

$$\nu_a = \frac{\dfrac{\partial I_a}{\partial E_p}}{\dfrac{\partial I_a}{\partial E_a}} \qquad \nu_b = \frac{\dfrac{\partial I_b}{\partial E_p}}{\dfrac{\partial I_b}{\partial E_b}}. \tag{5}$$

$$\frac{1}{r_{ga}} = \frac{\partial I_a}{\partial E_a} \qquad \frac{1}{r_{gb}} = \frac{\partial I_b}{\partial E_b} \tag{6}$$

$$\frac{\partial I_a}{\partial E_p} = \frac{\nu_a}{r_{ga}} = g_{ga} \quad \text{and} \quad \frac{\partial I_b}{\partial E_p} = \frac{\nu_b}{r_{gb}} = g_{gb}. \tag{7}$$

The mutual conductances g_a and g_b defined by (4) and the reflex mutual conductances g_{ga} and g_{gb} defined by (7) stand for ratios μ_a/r_p etc., which often enter into the mathematical expressions. By Taylor's series,

$$i_p = P_1e_a + P_2e_b + P_3e_p + P_4e_a^2 + P_5e_b^2 + P_6e_p^2 + P_7e_ae_b + P_8e_ae_p + P_9e_be_p + \cdots \tag{8}$$

$$i_a = A_1e_a + A_2e_b + A_3e_p + A_4e_a^2 + A_5e_b^2 + A_6e_p^2 + A_7e_ae_b$$

$$+A_8e_ae_p+A_9e_be_p+\cdots \tag{9}$$

$$i_b=B_1e_a+B_2e_b+B_3e_p+B_4e_a^2+B_5e_b^2+B_6e_p^2+B_7e_ae_b$$
$$+B_8e_ae_p+B_9e_be_p+\cdots \tag{10}$$

where

$$P_1=\frac{\partial I_p}{\partial E_a}=g_a \qquad P_5=\frac{1}{2}\frac{\partial^2 I_p}{\partial E_b^2}=\frac{1}{2}\frac{\partial g_b}{\partial E_b}$$

$$P_2=\frac{\partial I_p}{\partial E_b}=g_b \qquad P_6=\frac{1}{2}\frac{\partial^2 I_p}{\partial E_p^2}=-\frac{1}{2r_p}\frac{\partial r_p}{\partial E_p}$$

$$P_3=\frac{\partial I_p}{\partial E_p}=\frac{1}{r_p} \qquad P_7=\frac{\partial^2 I_p}{\partial E_a\partial E_b}=\frac{\partial g_a}{\partial E_b}=\frac{\partial g_b}{\partial E_a}$$

$$P_4=\frac{1}{2}\frac{\partial^2 I_p}{\partial E_a^2}=\frac{1}{2}\frac{\partial g_a}{\partial E_a} \qquad P_8=\frac{\partial^2 I_p}{\partial E_p\partial E_a}=\frac{\partial g_a}{\partial E_p}$$

$$P_9=\frac{\partial^2 I_p}{\partial E_p\partial E_b}=\frac{\partial g_b}{\partial E_p}$$

$$A_1=\frac{\partial I_a}{\partial E_a}=\frac{1}{r_{ga}} \qquad A_5=\frac{1}{2}\frac{\partial^2 I_a}{\partial E_b^2}=\frac{1}{2}\frac{\partial G_a}{\partial E_b}$$

$$A_2=\frac{\partial I_a}{\partial E_b}=G_a \qquad A_6=\frac{1}{2}\frac{\partial^2 I_a}{\partial E_p^2}=\frac{1}{2}\frac{\partial g_{ga}}{\partial E_p}$$

$$A_3=\frac{\partial I_a}{\partial E_p}=g_{ga} \qquad A_7=\frac{\partial^2 I_a}{\partial E_b\partial E_a}=-\frac{1}{r_{ga}^2}\frac{\partial r_{ga}}{\partial E_b}$$

$$A_4=\frac{1}{2}\frac{\partial^2 I_a}{\partial E_a^2}=-\frac{1}{2r_{ga}^2}\frac{\partial r_{ga}}{\partial E_a} \qquad A_8=\frac{\partial^2 I_a}{\partial E_a\partial E_p}=\frac{\partial g_{ga}}{\partial E_a} \tag{11}$$

$$A_9=\frac{\partial^2 I_a}{\partial E_b\partial E_p}=\frac{\partial g_{ga}}{\partial E_b}$$

$$B_1=\frac{\partial I_b}{\partial E_a}=G_b \qquad B_5=\frac{1}{2}\frac{\partial^2 I_b}{\partial E_b^2}=-\frac{1}{2r_{gb}^2}\frac{\partial r_{gb}}{\partial E_b}$$

$$B_2=\frac{\partial I_b}{\partial E_b}=\frac{1}{r_{gb}} \qquad B_6=\frac{1}{2}\frac{\partial^2 I_b}{\partial E_p^2}=\frac{1}{2}\frac{\partial g_{gb}}{\partial E_p}$$

$$B_3=\frac{\partial I_b}{\partial E_p}=g_{gb} \qquad B_7=\frac{\partial^2 I_b}{\partial E_a\partial E_b}=-\frac{1}{r_{gb}^2}\frac{\partial r_{gb}}{\partial E_a}$$

$$B_4=\frac{1}{2}\frac{\partial^2 I_b}{\partial E_a^2}=\frac{1}{2}\frac{\partial G_b}{\partial E_a} \qquad B_8=\frac{\partial^2 I_b}{\partial E_a\partial E_p}=\frac{\partial g_{gb}}{\partial E_a}$$

$$B_9 = \frac{\partial^2 I_b}{\partial E_b \partial E_p} = \frac{\partial g_{ob}}{\partial E_b}$$

In (11), all the P's, A's and B's are constants, each derivative being evaluated at the point $E_p = E_{po}$, $E_a = E_{ao}$ and $E_b = E_{bo}$. Throughout this article this is to be understood for each derivative. The quantities G_a and G_b defined above might be termed the "cross mutual conductances."

Notation: If z is an impedance, its value at a frequency $p_m/2\pi$ may be denoted $z(p_m)$.

It will be necessary to deal with each component of a voltage or current of the form

$$e_{a1} = \frac{E_{a1}}{2} \epsilon^{j(p_1 t + \theta_1)} \tag{12}$$

(where ϵ = base nat. logs) and the corresponding impedance $z(p_1)$. The conjugate component

$$\bar{e}_{a1} = \frac{E_{a1}}{2} \epsilon^{-j(p_1 t + \theta_1)} \tag{12a}$$

has a corresponding impedance $\bar{z}(p_1)$ which is the conjugate of $z(p_1)$.

I. Effect of an Impedance in the Circuit of the Second Grid of a Four-Electrode Tube

Let us first consider a restricted problem, that in which there is an applied voltage and no external impedance in one grid circuit, say the "a" grid circuit, while there are external impedances but no applied voltages in the "b" grid circuit and the plate circuit. This would correspond to the case of a signal impressed on the "a" grid circuit, the latter having negligible external impedance, while the "b" grid circuit has an appreciable impedance.

Let the generalized impedance in the plate circuit be z and let z_b be the generalized impedance of the "b" grid circuit.

Let
$$e_a = \sum E_{an} \cos (p_n t + \theta_n)$$
$$= \sum \frac{E_{an}}{2} \left[\epsilon^{j(p_n t + \theta_n)} + \epsilon^{-j(p_n t + \theta_n)}\right] \tag{13}$$
$$i_p = \sum d_{1n} e_{an} + \sum d_{2(m+n)} (e_a^2)_{(m+n)} + \cdots \tag{13a}$$
$$i_b = \sum \beta_{1n} e_{an} + \sum \beta_{2(m+n)} (e_a^2)_{(m+n)} + \cdots \tag{13b}$$

where any term, say $\sum d_{2(m+n)} \; (e_a^2)_{(m+n)}$ represents a sum of terms, each one of the form $E\epsilon^{j[(p_m+p_n)t+(\theta_m+\theta_n)]}$ (or conjugate form) multiplied by a d coefficient.

Then $e_p = -\sum z i_p = -\sum d_{1n}(ze_a)_n - \sum d_{2(m+n)}(ze_a{}^2)_{(m+n)} + \cdots$ (14)

$$e_b = -\sum z_b i_b = -\sum \beta_{1n}(z_b e_a)_n - \sum \beta_{2(m+n)}(z_b e_a{}^2)_{(m+n)} + \cdots \quad (14a)$$

where each set of brackets () represents a voltage of the form $E\epsilon^{j(p_n t+\theta_n)}$ multiplied by the appropriate impedance $z(p_n)$. Substitute in (8) and (10) and equate coefficients of terms of like order and periodicity. Then

$$d_{1n} = \frac{\mu_a[r_{gb}+z_b(p_n)] - \mu_b r_{gb} z_b(p_n) G_b}{[r_p+z(p_n)][r_{gb}+z_b(p_n)] - \mu_b \nu_b z(p_n) z_b(p_n)} \quad (15a)$$

$$\beta_{1n} = \frac{r_{gb}G_b - d_{1n}\nu_b z(p_n)}{r_{gb}+z_b(p_n)} \quad (15b)$$

$$d_{2(m+n)} = \frac{\phi r_p[r_{gb}+z_b(p_m+p_n)] - \psi \mu_b r_{gb} z_b(p_m+p_n)}{2[r_p+z(p_m+p_n)][r_{gb}+z_b(p_m+p_n)] - \mu_b \nu_b z(p_m+p_n) z_b(p_m+p_n)} \quad (15c)$$

where

$$\phi = \frac{\partial g_a}{\partial E_a} + \frac{\partial g_b}{\partial E_b}[\beta_{1n}\beta_{1m} z_b(p_n) z_b(p_m)] - \frac{1}{r_p{}^2}\frac{\partial r_p}{\partial E_p}[d_{1n} d_{1m} z(p_m) z(p_n)]$$
$$- \frac{\partial g_a}{\partial E_b}[\beta_{1n} z_b(p_n) + \beta_{1m} z_b(p_m)] - \frac{\partial g_a}{\partial E_p}[d_{1n} z(p_n) + d_{1m} z(p_m)]$$
$$+ \frac{\partial g_b}{\partial E_p}[d_{1n}\beta_{1m} z(p_n) z_b(p_m) + d_{1m}\beta_{1n} z(p_m) z_b(p_n)]$$

and

$$\psi = \frac{\partial G_b}{\partial E_a} - \frac{1}{r_{gb}{}^2}\frac{\partial r_{gb}}{\partial E_b}[\beta_{1n}\beta_{1m} z_b(p_n) z_b(p_m)] + \frac{\partial g_{gb}}{\partial E_p}[d_{1n} d_{1m} z(p_n) z(p_m)]$$
$$+ \frac{1}{r_{gb}{}^2}\frac{\partial r_{gb}}{\partial E_a}[\beta_{1n} z_b(p_n) + \beta_{1m} z_b(p_m)] - \frac{\partial g_{gb}}{\partial E_a}[d_{1n} z(p_n) + d_{1m} z(p_m)]$$
$$+ \frac{\partial g_{gb}}{\partial E_b}[d_{1n}\beta_{1m} z(p_n) z_b(p_m) + d_{1m}\beta_{1n} z(p_m) z_b(p_n)].$$

The d and β coefficients are complex quantities, and i_p is given as a sum of complex quantities when the d's are given by (15a) and (15c).

The coefficient $d_{2(m+n)}$ represents the general coefficient of the detection or modulation terms, for it is to be understood that the term

$\sum d_{2(m+n)}\,(e_a^2)_{(m+n)}$ in (13a) represents a double sum including positive and negative values of m and n, thus giving modulation terms of the sum of the impressed periodicities (p_m+p_n), detection terms of the differences of the impressed periodicities (p_m-p_n), and including double frequency and d.c. terms as special cases. Of course for, say, a modulation term of periodicity p_m-p_n, the p_n already appearing in the equations must be replaced by $-p_n$ for a d.c. term p_n must be replaced by $-p_m$, etc.

Resistance Case. If the load in the plate circuit be a resistance r, and the impedance in the second or "b" grid circuit be a resistance r_b, then (15a) reduces to

$$d_1=\frac{\mu_a(r_b+r_{gb})-\mu_b r_b r_{gb} G_b}{(r+r_p)(r_b+r_{gb})-\mu_b\nu_b r r_b} \tag{16a}$$

and (15b) and (15c) reduce to

$$\beta_1=\frac{r_{gb}G_b(r+r_p)-\mu_a\nu_b r}{(r+r_p)(r_b+r_{gb})-\mu_b\nu_b r r_b} \tag{16b}$$

$$d_2=\frac{\begin{array}{l} r_p(r_b+r_{gb})\left[\frac{1}{2}\frac{\partial g_a}{\partial E_a}+\frac{1}{2}r_b^2\beta_1^2\frac{\partial g_b}{\partial E_b}-\frac{r^2d_1^2}{2r_p^2}\frac{\partial r_p}{\partial E_p}-r_b\beta_1\frac{\partial g_a}{\partial E_b}\right.\\ \left.-rd_1\frac{\partial g_a}{\partial E_p}+rr_bd_1\beta_1\frac{\partial g_b}{\partial E_p}\right]-\mu_b r_b r_{gb}\left[\frac{1}{2}\frac{\partial G_b}{\partial E_a}-\frac{1}{2}\frac{r_b^2\beta_1^2}{r_{gb}^2}\frac{\partial r_{gb}}{\partial E_b}\right.\\ \left.+\frac{r^2d_1^2}{2}\frac{\partial g_{gb}}{\partial E_p}+\frac{r_b\beta_1}{r_{gb}^2}\frac{\partial r_{gb}}{\partial E_a}-rd_1\frac{\partial g_{gb}}{\partial E_a}+rr_bd_1\beta_1\frac{\partial g_{gb}}{\partial E_b}\right]\end{array}}{(r+r_p)(r_b+r_{gb})-\mu_b\nu_b r r_b} \tag{16c}$$

It is evident that i_p depends in a complicated way on r_b. From d_1, conditions for maximum amplification current can be obtained by differentiation. For all cases the amplification voltage is maximum for $r=\infty$.

The coefficients reduce, when there is no second grid, to $d_1=\mu_a/r+r_p$ and

$$d_2=\frac{\left[-\mu_a^2r_p\frac{\partial r_p}{\partial E_p}+\mu_a(r_p^2-r^2)\frac{\partial \mu_a}{\partial E_p}+(r+r_p)^2\frac{\partial \mu_a}{\partial E_a}\right]}{2(r+r_p)^3} \tag{16d}$$

II. Expressions in Terms of Grid-Filament Voltages

To get the currents in terms of grid-filament voltages e_a and e_b and the external impedance z in the plate circuit, assume

$$i_p = \sum d_{1n} e_{an} + \sum d_{2h} e_{bh} + \sum d_{3(m+n)} (e_a^2)_{(m+n)} + \sum d_{4(h+k)} (e_b^2)_{(h+k)} + \sum d_{5(n+h)} (e_a e_b)_{(n+h)} + \cdots \quad (17)$$

The subscripts n and m will be used to indicate components of e_a with periodicities p_n and p_m, respectively, while h and k will be used for components e_b, with periodicities q_h and q_k. Thus a subscript $d_{5(n+h)}$ goes with a term of frequency $(p_n+q_h)/2\pi$.

$$-e_p = \sum d_{1n}(ze_a)_n + \sum d_{2h}(ze_b)_h + \sum d_{3(m+n)}(ze_a^2)_{(m+n)} + \sum d_{4(h+k)}(ze_b^2)_{(h+k)} + \sum d_{5(n+h)}(ze_a e_b)_{(n+h)} + \cdots. \quad (17a)$$

Proceeding as before,

$$d_{1n} = \frac{g_a r_p}{r_p + z(p_n)} = \frac{\mu_a}{r_p + z(p_n)} \quad (18a)$$

$$d_{2h} = \frac{g_b r_p}{r_p + z(q_h)} = \frac{\mu_b}{r_p + z(q_h)} \quad (18b)$$

$$d_{3(m+n)} = \frac{r_p \left\{ \frac{\partial g_a}{\partial E_a} [r_p + z(p_m)][r_p + z(p_n)] - g_a^2 z(p_m) z(p_n) \frac{\partial r_p}{\partial E_p} \right\} - g_a r_p^2 \frac{\partial g_a}{\partial E_p} \{ z(p_n)[r_p + z(p_m)] + z(p_m)[r_p + z(p_n)] \} }{2[r_p + z(p_m)][r_p + z(p_n)][r_p + z(p_m + p_n)]} \quad (18c)$$

Likewise

$$d_{4(h+k)} = \frac{r_p \left\{ \frac{\partial g_b}{\partial E_b} [r_p + z(q_h)][r_p + z(q_k)] - g_b^2 z(q_h) z(q_k) \frac{\partial r_p}{\partial E_p} \right\} - g_b r_p^2 \frac{\partial g_b}{\partial E_p} \{ z(q_h)[r_p + z(q_k)] + z(q_k)[r_p + z(q_h)] \} }{2[r_p + z(q_h)][r_p + z(q_k)][r_p + z(q_h + q_k)]} \quad (18d)$$

$$d_{5(n+h)} = -\frac{r_p \left\{ g_a g_b z(p_n) z(q_h) \frac{\partial r_p}{\partial E_p} + g_b r_p z(q_h)[r_p + z(p_n)] \frac{\partial g_a}{\partial E_p} + g_a r_p z(p_n)[r_p + z(q_h)] \frac{\partial g_b}{\partial E_p} - \frac{\partial g_a}{\partial E_b} [r_p + z(p_n)][r_p + z(q_h)] \right\} }{[r_p + z(p_n)][r_p + z(q_h)][r_p + z(p_n + q_h)]} \quad (18e)$$

Each d is a complex quantity, and is the coefficient of a voltage of the form $E\epsilon^{j(pt+\theta)}$, for example, $d_{3(m+n)}$ is the coefficient of $E_{am}E_{an}/2$ $\epsilon^{+j[(pm+pn)t+(\theta m+\theta n)]}$. To change this to an expression in terms of cosines, this must be combined with its conjugate

$$\bar{d}_{3(m+n)} \overline{(e_a^2)}_{(m+n)} \cdot$$

The first two terms in i_p give

$$\sum \frac{\mu_a \cos\left[p_n t+\theta_n-\tan^{-1}\left(\dfrac{x(p_n)}{r+r_p}\right)\right]}{\sqrt{(r+r_p)^2+(x(p_n))^2}} + \sum \frac{\mu_b \cos\left[q_h t+\beta_h-\tan^{-1}\left(\dfrac{x(q_h)}{r+r_p}\right)\right]}{\sqrt{(r+r_p)^2+(x(q_h))^2}} \tag{19}$$

where $jx(p_n)$ is the reactance component of $z(p_n)$ etc.

If there is negligible external impedance in the two grid circuits, so that e_a and e_b may be taken as applied voltages and no allowance must be made for impedance drops in either external grid circuit, then the expression for i_p in terms of e_a and e_b is a complete expression for the plate current in terms of the independent variables e_a and e_b. In this case, the terms in e_a, e_b, e_a^2, and e_b^2 represent the action of each grid acting as though the other were absent; the term in e_ae_b represents a joint effect of the two grids. Thus if single frequency voltages were impressed on both grids, the modulated or detected output would appear in this term.

The modulus of $d_{3(m+n)}$ is given by a fraction, the numerator of which is

$$\frac{r_p}{2}\sqrt{\left\{\frac{\partial g_a}{\partial E_a}[(r+r_p)^2-x(p_m)x(p_n)]-g_a^2\frac{\partial r_p}{\partial E_p}[r^2-x(p_m)x(p_n)] -2g_ar_p\frac{\partial g_a}{\partial E_p}[r(r+r_p)-x(p_m)x(p_n)]\right\}^2+\left\{\frac{\partial g_a}{\partial E_a}(r+r_p)[x(p_m)+x(p_n)] -rg_a^2\frac{\partial r_p}{\partial E_p}[x(p_m)+x(p_n)]-g_ar_p\frac{\partial g_a}{\partial E_p}(2r+r_p)[x(p_m)+x(p_n)]\right\}^2}$$

and the denominator of which is

$$\sqrt{[(r+r_p)^2+x(p_m+p_n)^2][(r+r_p)^2+x(p_m)^2][(r+r_p)^2+x(p_n)^2]} \tag{20}$$

The phase angle of $d_{3(m+n)}$ is the inverse tangent of the second $\{\}$ over the first $\{\}$ minus

$$\left[\tan^{-1}\frac{x(p_m+p_n)}{r+r_p}+\tan^{-1}\frac{x(p_m)}{r+r_p}+\tan^{-1}\frac{x(p_n)}{r+r_p}\right].$$

In the case of a pure resistance load, (20) reduces to

$$\frac{\frac{r_p}{2}\left[\frac{\partial g_a}{\partial E_a}(r+r_p)^2 - g_a{}^2r^2\frac{\partial r_p}{\partial E_p} - 2g_a r r_p(r+r_p)\frac{\partial g_a}{\partial E_p}\right]}{(r+r_p)^3} \tag{20a}$$

Here $g_a = \mu_a/r_p$ and if it is assumed that μ_a is constant and $\partial r_p/\partial E_a = \mu_a(\partial r_p/\partial E_p)$, then this reduces to

$$-\frac{\frac{1}{2}\mu_a{}^2 r_p \frac{\partial r_p}{\partial E_p}}{(r+r_p)^3}$$

which is the familiar coefficient for the $e_a{}^2$ term in the three-electrode tube plate current expression. (20a) and (16d) are equal.

The coefficient of $e_b{}^2(d_{4(h+k)})$ is similar to $d_{3(m+n)}$, with a, n, m, p_n and p_m replaced by b, h, k, q_h and q_k, respectively. The coefficient $d_{5(n+h)}$ is of more interest, since the $e_a e_b$ term represents the joint effect of the two grids on the plate current. The modulus of $d_{5(n+h)}$ is given by a fraction, the numerator of which is

$$r_p\sqrt{\begin{aligned}&\left\{\frac{\partial g_a}{\partial E_b}[(r+r_p)^2 - x(p_n)x(q_h)] - g_a g_b\frac{\partial r_p}{\partial E_p}[r^2 - x(p_n)x(q_h)]\right.\\ &\left.- r_p\frac{\partial(g_a g_b)}{\partial E_p}[r(r+r_p) - x(p_n)x(q_h)]\right\}^2 + \left\{\frac{\partial g_a}{\partial E_b}(r+r_p)[x(p_n)+x(q_h)]\right.\\ &-rg_a g_b\frac{\partial r_p}{\partial E_p}[x(p_m)+x(q_h)] - rr_p\frac{\partial(g_a g_b)}{\partial E_p}[x(p_n)+x(q_h)]\\ &\left.-r_p{}^2\left[x(q_h)g_b\frac{\partial g_a}{\partial E_p} + x(p_n)g_a\frac{\partial g_b}{\partial E_p}\right]\right\}^2\end{aligned}}$$

and the denominator of which is

$$\sqrt{[(r+r_p)^2 + x(p_n+q_h)^2][(r+r_p)^2 + x(p_n)^2][(r+r_p)^2 + x(q_h)^2]} \tag{21}$$

The phase angle of $d_{5(n+h)}$ is given by the inverse tangent of the second { } over the first { } minus

$$\left[\tan^{-1}\frac{x(p_n+q_h)}{r+r_p} + \tan^{-1}\frac{x(p_n)}{r+r_p} + \tan^{-1}\frac{x(q_h)}{r+r_p}\right].$$

For a pure resistance load, (21) reduces to

$$\frac{r_p\left[\frac{\partial g_a}{\partial E_b}(r+r_p)^2 - r^2 g_a g_b\frac{\partial r_p}{\partial E_p} - rr_p(r+r_p)\frac{\partial(g_a g_b)}{\partial E_p}\right]}{(r+r_p)^3} \tag{21a}$$

and if it is further assumed that μ_a and μ_b are constants, (21a) reduces to

$$-\frac{\mu_a\mu_b r_p \dfrac{\partial r_p}{\partial E_p}}{(r+r_p)^3}. \tag{21b}$$

Since $\partial g_a/\partial E_b = \partial g_b/\partial E_a$ (see (4)), equations (21) and (21a) do not actually lack symmetry in the a's and b's.

To get i_a and i_b in terms of e_a and e_b, assume i_a and i_b are represented by series similar to (17) with d replaced by α and β, respectively.

Evaluate coefficients as before. There results

$$\begin{aligned} \alpha_{1n} &= \frac{r_p + z(p_n)(1-\mu_a\nu_a)}{r_{ga}[r_p+z(p_n)]} \\ \alpha_{2h} &= \frac{r_pG_a + z(q_h)(G_a - \mu_b g_{ga})}{[r_p+z(p_n)]} \end{aligned} \tag{22}$$

$$\alpha_{3(m+n)} = -g_{ga}z(p_m+p_n)d_{3(m+n)} - \frac{1}{2r_{ga}^2}\frac{\partial r_{ga}}{\partial E_a}$$

$$+\frac{\mu_a^2 z(p_n)z(p_m)\dfrac{\partial g_{ga}}{\partial E_p} - \mu_a\dfrac{\partial g_{ga}}{\partial E_a}\{z(p_n)[r_p+z(p_m)]+z(p_m)[r_p+z(p_n)]\}}{2[r_p+z(p_n)][r_p+z(p_m)]}$$

$$\alpha_{4(h+k)} = -g_{ga}z(q_h+q_k)d_{4(h+k)} + \frac{1}{2}\frac{\partial G_a}{\partial E_b}$$

$$+\frac{\mu_b^2 z(q_h)z(q_k)\dfrac{\partial g_{ga}}{\partial E_p} - \mu_b\dfrac{\partial g_{ga}}{\partial E_b}\{z(q_h)[r_p+z(q_k)]+z(q_k)[r_p+z(q_h)]\}}{2[r_p+z(q_h)][r_p+z(q_k)]}$$

$$\alpha_{5(n+h)} = -g_{ga}z(p_n+q_h)d_{5(n+h)} - \frac{1}{r_{ga}^2}\frac{\partial r_{ga}}{\partial E_b}$$

$$+\frac{\dfrac{1}{2}\mu_a\mu_b z(p_n)z(q_h)\dfrac{\partial g_{ga}}{\partial E_p} - \mu_b\dfrac{\partial g_{ga}}{\partial E_a}z(q_h)[r_p+z(p_n)] - \mu_a\dfrac{\partial g_{ga}}{\partial E_b}z(p_n)[r_p+z(q_h)]}{[r_p+z(p_n)][r_p+z(q_h)]}.$$

For a pure resistance load these become

$$\alpha_1 = \frac{r_p + r(1-\mu_a\nu_a)}{r_{ga}(r+r_p)}$$

$$\alpha_2=\frac{r_pG_a+r(G_a-\mu_b g_{ga})}{r+} \tag{22a}$$

$$\alpha_3=-\frac{1}{2r_{ga}^2}\frac{\partial r_{ga}}{\partial E_a}+\frac{\mu_a{}^2r^2\frac{\partial g_{ga}}{\partial E_p}-2\mu_a r(r+r_p)\frac{\partial g_{ga}}{\partial E_a}}{2(r+r_p)^2}$$

$$-\frac{g_{ga}rr_p\left[(r+r_p)^2\frac{\partial g_a}{\partial E_a}-r^2g_a{}^2\frac{\partial r_p}{\partial E_p}-2\mu_a r(r+r_p)\frac{\partial g_a}{\partial E_p}\right]}{2(r+r_p)^3}$$

$$\alpha_4=\frac{1}{2}\frac{\partial G_a}{\partial E_b}+\frac{\mu_b{}^2r^2\frac{\partial g_{ga}}{\partial E_p}-2\mu_b r(r+r_p)\frac{\partial g_{ga}}{\partial E_b}}{2(r+r_p)^2}$$

$$-\frac{g_{ga}rr_p\left[(r+r_p)^2\frac{\partial g_b}{\partial E_b}-r^2g_b{}^2\frac{\partial r_p}{\partial E_p}-2\mu_b r(r+r_p)\frac{\partial g_b}{\partial E_p}\right]}{2(r+r_p)^3}$$

$$\alpha_5=-\frac{1}{r_{ga}^2}\frac{\partial r_{ga}}{\partial E_b}+\frac{\mu_a\mu_b r^2\frac{\partial g_{ga}}{\partial E_p}-r(r+r_p)\left(\mu_a\frac{\partial g_{ga}}{\partial E_b}+\mu_b\frac{\partial g_{ga}}{\partial E_a}\right)}{(r+r_p)^2}$$

$$-\frac{g_{ga}rr_p\left[(r+r_p)^2\frac{\partial g_a}{\partial E_b}-r^2g_ag_b\frac{\partial r_p}{\partial E_p}-rr_p(r+r_p)\frac{\partial(g_ag_b)}{\partial E_p}\right]}{(r+r_p)^3}.$$

The β coefficients are similar, β_{2h} corresponding to α_{1n}, β_{1n} to α_{2h}, $\beta_{3(m+n)}$ to $\alpha_{4(h+k)}$, $\beta_{4(h+k)}$ to $\alpha_{3(m+n)}$ and $\beta_{5(n+h)}$ to $\alpha_{5(n+h)}$.

III. Expressions in Terms of Applied Voltages

So far the plate and grid currents have been obtained in terms of the instantaneous voltages between the grids and filament (e_a, e_b). If there is an external impedance drop in either grid circuit (or both), then it is desirable to determine the currents in terms of the applied voltages e_a' and e_b'. Such a solution for i_p in terms of e_a', e_b' and the impedances should account for the action of a four-electrode tube when used as a double function tube. It should also account for the action of the tube when used as a "screen-grid" or a "space-charge-grid" tube, if current flows in the non-control grid, for only insofar as the impedance of the non-control grid circuit may be neglected may the customary equations of the three-electrode tube be used.

To return then to the original problem, let there be an external resistance r and reactance x in the plate circuit; and let r_a and r_b and x_a and x_b be corresponding quantities in the "a" and "b" grid circuits, respectively.

$$\text{Let} \quad e_a' = \sum E_{an} \cos (p_n t + \theta_n) \tag{23a}$$

be the applied voltage in the a grid circuit, and let

$$e_b' = \sum E_{bh} \cos (q_h t + \beta_h) \tag{23b}$$

be the applied voltage in the "b" grid circuit. Then

$$e_a = e_a' - \sum z_a i_a \tag{23c}$$

$$e_b = e_b' - \sum z_b i_b \tag{23d}$$

$$\text{Let } i_p = \sum h_{1n} e_{an}' + \sum h_{2h} e_{bh}' + \sum h_{3(m+n)} (e_a'^2)_{(m+n)} + \sum h_{4(h+k)} (e_b'^2)_{(h+k)} + \sum h_{5(n+h)} (e_a' e_b')_{(n+h)} + \cdots \tag{24}$$

and let i_a and i_b be similar series with h replaced by a and b respectively.

Then

$$e_a = e_a' - \sum a_{1n} (z_a e_a')_n - \sum a_{2h} (z_a e_b')_h - \sum a_{3(m+n)} (z_a e_a'^2)_{(m+n)} - \sum a_{4(h+k)} (z_a e_b'^2)_{(h+k)} - \sum a_{5(n+h)} (z_a e_a' e_b')_{(n+h)} - \cdots \tag{24a}$$

e_b is similar, and $e_p = -\sum z i_p$.

Hence

$$\left.\begin{aligned}
h_{1n} &= d_{1n} [1 - a_{1n} z_a(p_n)] - d_{2n} b_{1n} z_b(p_n) \\
h_{2h} &= d_{2h} [1 - b_{2h} z_b(q_h)] - d_{1h} a_{2h} z_a(q_h) \\
h_{3(n+m)} &= -d_{1(n+m)} z_a(p_m + p_n) a_{3(n+m)} \\
&\quad - d_{2(n+m)} z_b(p_n + p_m) b_{3(m+n)} + d_{3(m+n)} \\
&\quad [1 - a_{1n} z_a(p_n)][1 - a_{1m} z_a(p_m)] + d_{4(n+m)} \\
&\quad b_{1n} b_{1m} z_b(p_n) z_b(p_m) - d_{5(m+n)} [1 - a_{1m} z_a(p_m)] b_{1n} z_b(p_n) \\
&\quad - d_{5(n+m)} [1 - a_{1n} z_a(p_n)] b_{1m} z_b(p_m) \\
h_{5(n+h)} &= -d_{1(n+h)} a_{5(n+h)} z_a(p_n + q_h) - d_{2(n+h)} b_{5(n+h)} z_b(p_n + q_h) \\
&\quad - 2 d_{3(n+h)} [1 - a_{1n} z_a(p_n)] a_{2h} z_a(q_h) - 2 d_{4(n+h)} [1 - b_{2h} z_b(q_h)] b_{1n} z_b(p_n) \\
&\quad + d_{5(n+h)} a_{2h} b_{1n} z_a(q_h) z_b(p_n) + d_{5(n+h)} [1 - a_{1n} z_a(p_n)][1 - b_{2h} z_b(q_h)]
\end{aligned}\right\} \tag{25}$$

Here the d's have already been given in (18) and the a's are given by

$$a_{1n} = \frac{1}{\phi_n} [\alpha_{1n} + z_b(p_n)(\alpha_{1n} \beta_{2n} - \alpha_{2n} \beta_{1n})] \tag{26a}$$

$$a_{2h} = \alpha_{2h}/\phi_h \tag{26b}$$

$$a_{3(m+n)} = \frac{1}{\phi_{(n+m)}}\{\delta_{3(n+m)}[1-a_{1n}z_a(p_n)][1-a_{1m}z_a(p_m)]$$
$$+\delta_{4(n+m)}b_{1n}b_{1m}z_b(p_n)z_b(p_m) - \delta_{5(n+m)}\{[1-a_{1n}z_a(p_n)]b_{1m}z_b(p_m)$$
$$+[1-a_{1m}z_a(p_m)]b_{1n}z_b(p_n)\} \tag{26c}$$

$$a_{4(h+k)} = \frac{1}{\phi_{(h+k)}}\{\delta_{3(h+k)}a_{2h}a_{2k}z_a(q_h)z_a(q_k)$$
$$+\delta_{4(h+k)}[1-b_{2h}z_b(q_h)][1-b_{2k}z_b(q_k)] - \delta_{5(h+k)}\{[1-b_{2h}z_b(q_k)]$$
$$a_{2k}z_a(q_k)+[1-b_{2k}z_b(q_k)]a_{2h}z_a(q_k)\} \tag{26d}$$

$$a_{5(n+h)} = \frac{-1}{\phi_{(n+h)}}\{2\delta_{3(n+h)}[1-a_{1n}z_a(p_n)]a_{2h}z_a(q_h)$$
$$+2\delta_{4(n+h)}[1-b_{2h}z_b(q_h)]b_{1n}z_b(p_n)$$
$$-\delta_{5(n+h)}\{[1-a_{1n}z_a(p_n)][1-b_{2h}z_b(q_h)]$$
$$+a_{2h}b_{1n}z_a(q_h)z_b(p_n)\} \tag{26e}$$

where

$$\phi_n = [1+\beta_{2n}z_b(p_n)][1+\alpha_{1n}z_a(p_n)] - \alpha_{2n}\beta_{1n}z_a(p_n)z_b(p_n)$$
$$\delta_{3(m+n)} = \alpha_{3(m+n)} - (\beta_{3(n+m)}\alpha_{2(n+m)} - \alpha_{3(m+n)}\beta_{2(n+m)})z_b(p_m+p_n)$$
$$\delta_{4(m+n)} = \alpha_{4(m+n)} - (\beta_{4(m+n)}\alpha_{2(m+n)} - \alpha_{4(m+n)}\beta_{2(n+m)})z_b(p_m+p_n) \quad \text{etc.}$$

The corresponding b's may be written down by comparison with the a's, b_{2h} corresponding to a_{1n}, etc.

The coefficient h_{1n} may be reduced to

$$h_{1n} = \frac{d_{1n}\left[1+\frac{z_b(p_n)}{r_{gb}}\right] - d_{2n}z_b(p_n)G_b}{\left[\frac{r_{ga}+z_a(p_n)}{r_{ga}}\right]\left[\frac{r_{gb}+z_b(p_n)}{r_{gb}}\right] - G_aG_bz_a(p_n)z_b(p_n)} \cdot$$
$$-d_{1n}z(p_n)z_a(p_n)\left[g_{ga}+z_b(p_n)\left(\frac{g_{ga}}{r_{gb}} - G_ag_{gb}\right)\right]$$
$$-d_{2n}z(p_n)z_b(p_n)\left[g_{gb}+z_a(p_n)\left(\frac{g_{gb}}{r_{ga}} - G_bg_{ga}\right)\right] \tag{27a}$$

For $z_a = z_b = o$, this reduces to d_{1n} of course, while for $z_a = o$, it reduces to

$$\frac{\mu_a[r_{gb}+z_b(p_n)] - \mu_b r_{gb}z_b(p_n)G_b}{[r_{gb}+z_b(p_n)][r_p+z(p_n)] - \mu_b\nu_b z(p_n)z_b(p_n)} \quad \text{which is (15a).}$$

For $z_b = 0$, $z_a \neq 0$, which is the general three-electrode tube case, h_{1n} reduces to

$$\left(\frac{\mu_a}{r_p+z(p_n)}\right)\left[\frac{r_{ga}}{r_{ga}+z_a(p_n)\left(1-\frac{\mu_a\nu_a z(p_n)}{r_p+z(p_n)}\right)}\right] \tag{27b}$$

which is the general form of the coefficient of the amplification term of the plate current of a three-electrode tube when grid current and external grid impedance are considered. This further reduces to the customary $\mu_a/r_p+z(p_n)$ when $r_{ga} >> z_a(p_n)$.

The coefficient h_{2h} is similar.

For $z_a = z_b = 0$, $h_{3(m+n)}$ reduces to $d_{3(m+n)}$ while for $z_b = 0$, $z_a \neq 0$, the case of the three-electrode tube

$$h_{3(m+n)} = -d_{1(n+m)} z_a(p_m+p_n) a_{3(m+n)} + d_{3(m+n)}[1-a_{1n}z_a(p_n)][1-a_{1m}z_a(p_m)]. \tag{27c}$$

In this case, a_{1n} is given by

$$a_{1n} = \frac{1-\frac{\mu_a\nu_a z(p_n)}{[r_p+z(p_n)]}}{r_{ga}+z_a(p_n)\left[1-\frac{\mu_a\nu_a z(p_n)}{r_p+z(p_n)}\right]}$$

Llewellyn has used the coefficient (27c) to explain grid-leak detection in a three-electrode tube. Not all the terms depend on curvature of the plate characteristic, so that there appears in i_p, among others, a frequency $(p_m - p_n)/2\pi$, which is a detection term when p_m is a sideband and p_n the carrier periodicity, when the curvature of the plate characteristic is neglected. This detection term may be made appreciable by properly choosing z_a.

In the case of the four-electrode tube it can be seen from (25), without going into detail, that if the impedance z_b is appreciable at either radio or audio frequencies, the coefficient of the detection term will be dependent upon z_b as well as z_a when the signal (carrier and sidebands) is impressed on the "a" grid. If two signals be detected at the same time, one on each grid, there will be in addition to the usual components, others due to the term $e_a'e_b'$ which may be of the same order of magnitude as the desired detection terms.

The conditions for oscillation may be obtained from (27b) for the three-electrode tube; from (27a) for the four-electrode tube. It is desired that a current of frequency $p_n/2\pi$ appear in the plate circuit when there is no applied voltage in the grid circuits. In this case, since h_{1n} and h_{2h} will be multiplied by zero, the denominators must be zero, and the equating of the denominators to zero will give the conditions which must be fulfilled for the tube to produce oscillations. The tube may produce oscillations of two arbitrarily different frequencies, provided the denominator of h_{1n} can be made zero for one frequency, say p_n, while the denominator of h_{2h}, which is of exactly the same form as the denominator of h_{1n}, is made zero for the other frequency, say q_h. If this be achieved, then due to the term in $e_a'e_b'$ there should be a beat frequency—the tube will act as a beat-frequency oscillator.

The complexity of the coefficients does not warrant writing out the others here; for any specific application approximations can usually be made simplifying them considerably.

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DESIGN METHODS FOR SOFT MAGNETIC MATERIALS IN RADIO*

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Summary—*The present paper is concerned with a study of design methods for soft magnetic materials such as those used in radio broadcast receivers and similar apparatus. The design of such devices as interstaged transformers calls for more than choosing points on curves in a standard handbook and computing mechanical or electrical details by ordinary power transformer theory. Radio transformer design is usually complicated by the effect of passing d-c space current through the winding or by the use of an air-gap to prevent saturation of the core by unidirectional current. The procedure to follow when one of these disturbing effects is present is not very difficult, but it becomes somewhat involved if an efficient design is wanted when both are present. In this paper an attempt is made to present a workable method of taking account of these disturbing effects. The method involves the determination of the apparent and maximum obtainable permeabilities of the magnetic materials so that a proper conclusion may be arrived at by an application of simple engineering methods.*

Introduction

THE present paper is concerned with a study of design methods for soft magnetic materials such as those used in radio broadcast receivers and similar apparatus. Such a study involves the determination of the apparent and maximum obtainable permeabilities of soft magnetic materials, then showing the application of these results to definite engineering problems.

APPARENT PERMEABILITY—AIR GAP IN IRON PATH

When there is no air gap in an iron circuit, as for example in an ordinary ring sample, we can calculate the a-c permeability $\mu_{a.c.}$ from measurments of the inductance of the coil by means of the following formula:

$$\mu_{a.c.}=\frac{7.95L_{a.c.}l10^7}{k_1N^2A} \tag{1}$$

Where:

$L_{a.c.}$ = a-c inductance of reactor in henries
l = length of magnetic path in cm
k = stacking factor of core
N = number of turns of coil
A = cross sectional area of core in sq. cm.

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For direct measurements of $\mu_{a.c.}$ it is best to place two identical windings on the ring sample and measure simultaneously currents in one and voltage in the other, using an r.m.s. vacuum-tube voltmeter. The tests should best be made at 60 cycles.

As apparatus, using soft magnetic materials, usually have a superposed direct current in radio circuits, it becomes important to measure the a-c permeability for various values of d-c magnetizing forces or flux densities in the core. Accordingly, if a third winding of a few turns is wound on the ring sample and a direct current passed through it in series with an external air core inductance of high value, it is possible to calculate the d-c magnetizing force, H_o, from the following formula

$$H_o = \frac{1.256IN}{l} \tag{2}$$

where

I = current in amperes flowing in d-c winding
N = number of turns of d-c winding
l = length of iron path

Then one is able to measure the a-c permeability under various values for the d-c magnetizing force. Also, if one has before him the magnetization curves, he will be able to determine the d-c flux density for the various values of H_o used in determining the a-c permeability.

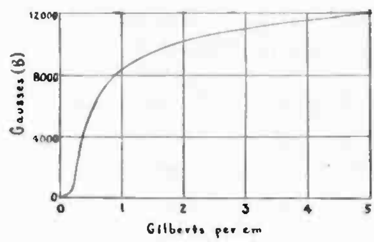

Fig. 1—Sample Soft Magnetic Alloy Magnetization Curve Used in Sample Calculations.

It will be found that the a-c permeability decreases rapidly with increasing d-c flux density. Consequently, in design work we are interested in the effective permeability of the alloys as used in actual devices. Hence, we have to consider the apparent permeability of the material where use is made of an air gap.

The introduction of the air gap in the magnetic circuit causes a decrease in the d-c flux density and a consequent rise in the apparent permeability. However, too large a gap cannot be used for it will tend to introduce too much reluctance to the a-c flux with a consequent lowering of the apparent permeability. In dealing with apparent permeability, then, there is a maximum obtainable value which corresponds to a definite condition with relation to the d-c magnetizing force, the air gap, and a-c flux density. If the air gap is reduced to zero, then the apparent and a-c permeabilities become identical. It becomes a matter of considerable importance, therefore, to study the introduction of an air gap in a magnetic device of the types we are considering, in order to obtain at a minimum cost the maximum effects desired. It is felt that this problem has not been given the attention it deserves, and it is hoped the following treatment will be useful.

Consider the magnetic circuit of a device, such as a transformer consisting of an iron path with a gap in it. Let l_1 and l_2 be the lengths of the iron and air paths, respectively. The magnetomotive force is consumed by the drop across the reluctance of the iron path and that across the air gap. Let H_1 and H_2 be the magnetic potential gradients along the iron and air paths respectively. The total magnetomotive force is given by:

$$(\text{m.m.f.}) = H_1 l_1 + H_2 l_2 \tag{3}$$

In the air gap the flux density is equal to the magnetizing force in the gap. If the former is represented by B_o, then $H_2 = B_o$, and when this result is substituted in (3), we get:

$$(\text{m.m.f.}) = H_1 l_1 + B_o l_2 \tag{4}$$

The two unknown quantities in (4) are H_1, the magnetizing force in the iron, and B_o, the flux density corresponding to this force. The relation between them is given by the magnetization curve. Equation (4) represents a straight line expressing the relation between H_1 and B_o. This straight line intersects the vertical axis of B at:

$$B_o = \frac{(\text{m.m.f.})}{l_2} \quad \text{when} \quad H_1 = 0 \tag{5}$$

and the horizontal axis of H at

$$H_1 = \frac{(\text{m.m.f.})}{l_1} \quad \text{when} \quad B_o = 0. \tag{6}$$

When we plot this straight line on the magnetization curve sheet, the former intersects the latter curve at some definite point. The coordinates (B_o and H_1) of this point satisfy both the magnetization curves and (4), and enable us to determine graphically the d-c flux density in the core under the given conditions.

If we take the derivative of H_1 with respect to B_o in (4), we get:

$$\frac{dH_1}{dB_o}=-\frac{l_2}{l_1}. \tag{7}$$

That is, the slope of the straight line represented by (4) enables us to determine the ratio of the air gap to the iron path. So, when the latter is selected, the magnitude of the air gap is determined immediately.

Considering the iron path, the a-c reluctivity is the reciprocal of the a-c permeability; that is:

$$\nu_{a.c.}=\frac{1}{\mu_{a.c.}}. \tag{8}$$

The reluctance R_1 of the iron path is:

$$R_1=\nu_{a.c.}\frac{l_1}{A} \tag{9}$$

and since the reluctivity of air is unity, the reluctance, R_2, of the air path is:

$$R_2=\frac{l_2}{A}. \tag{10}$$

Hence the reluctance of the whole path is:

$$R_1+R_2=\frac{\nu_{a.c.}l_1+l_2}{A}. \tag{11}$$

The apparent reluctivity is then obtained by dividing (11) by the total length of the path and making A equal to unity. Hence:

$$\nu_a=\frac{\nu_{a.c.}l_1}{l_1+l_2}+\frac{l_2}{l_1+l_2}. \tag{12}$$

In practical work the length of the gap is sufficiently small compared with the iron path to enable one to neglect l_2 in comparison with l_1 so that (12) becomes approximately:

$$\nu_a=\nu_{a.c.}+\frac{l_2}{l_1}. \tag{13}$$

Hence, the apparent reluctivity is equal to the a-c reluctivity plus the ratio of the air gap to the iron path. The apparent permeability is the reciprocal of the apparent reluctivity, that is,

$$\mu_a = \frac{1}{\nu_a} \tag{14}$$

or

$$\mu_a = \frac{l_1}{l_1 \nu_{a.c.} + l_2} \,. \tag{15}$$

In order, therefore, to determine the apparent permeability of a soft magnetic material, it is necessary to determine beforehand the a-c reluctivity and the ratio of the air gap to the iron path. From curves it is possible to determine the a-c permeability for the alloys under any given conditions of a-c flux density, d-c flux density, and d-c magnetizing force. The reciprocal of this gives the a-c reluctivity.

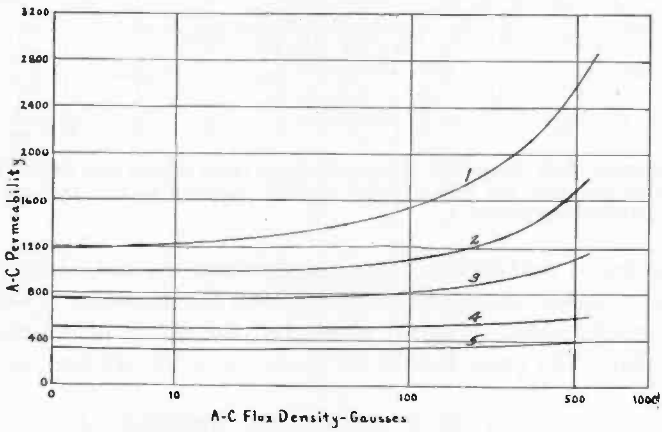

Fig. 2—Sample Soft Magnetic Alloy. A-C Permeability vs. A-C Flux Density at Various Values of D-C Flux Densities.

Curve 1—H_o (D.C.) = 0 gilberts per cm; B_o (D.C.) = 0 gausses
" 2— " = 0.5 " " " " = 5500 "
" 3— " = 1.0 " " " " = 8400 "
" 4— " = 2.0 " " " " = 10300 "
" 5— " = 4.0 " " " " = 11600 "

The ratio of the air gap to the iron path can be determined graphically by means of (7), as already discussed. When this ratio and the a-c reluctivity are determined, insertion of these values in (15) [or by use of (13) and (14)] enables one to calculate the apparent permeability under the given conditions.

Maximum Obtainable Apparent Permeability

In the beginning of this discussion we referred to the maximum obtainable apparent permeability. Ordinarily, in any particular problem we have a definite direct current flowing in a winding of a definite number of turns. This determines the d-c magnetizing force. The air gap will have its influence on the d-c and a-c flux density, as already explained, so that there is a maximum point, so to speak, where the gap can be set to secure this maximum obtainable apparent permeability under any given set of conditions.

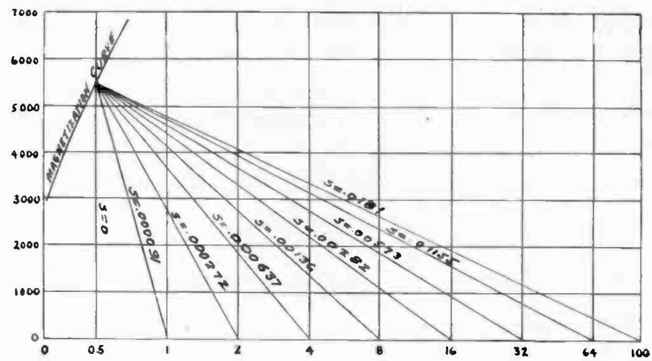

Fig. 3—Sample Soft Magnetic Alloy. Curves from which are determined the various air gaps for flux density of 5500 gausses at the various average d-c magnetizing forces.

In order to make this matter more clear, we have incorporated here the complete procedure of calculation for an alloy. The results of solutions for other alloys are given, but the details of the procedure are omitted. The procedure in all cases, however, is identical.

Calculation of Maximum Obtainable Apparent Permeability

Ordinarily we deal in practical devices with low a-c flux densities; consequently, an a-c flux density of 10 gausses has been chosen. The d-c magnetizing forces range from very small values up to perhaps 100

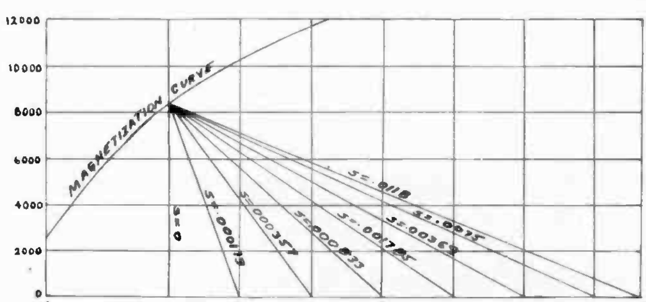

Fig. 4—Sample Soft Magnetic Alloy. Curves from which are determined the various air gaps for a flux density of 8400 gausses at the various average d-c magnetizing forces.

gilberts or more per cm for some chokes, where an ampere or so of direct current may flow in windings of a few hundred turns. Hence, the results of our calculations extend up to a d-c magnetizing force of 100 gilberts per cm.

Our curves shown in Fig. 2 on a-c permeability are for d-c flux desities of 5500, 8400, 10,300 and 11,600 gausses. In Table I the apparent permeabilities are calculated for these d-c flux densities at various average d-c magnetizing forces up to 100 gilberts per cm. In column I of the table are found the d-c magnetizing forces. In 2 is the ratio of the air gap to the iron path l_2/l_1 at a d-c flux density of 5500 gausses.

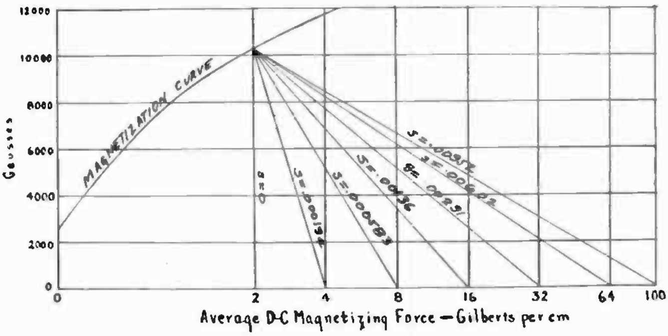

Fig. 5—Sample Soft Magnetic Alloy. Curves from which are determined the various air gaps for a flux density of 10,300 gausses at the various average d-c magnetizing forces.

These ratios are obtained by taking the slopes, in absolute value of the straight lines connecting the point, $H_0-0.5$, $B_0=5500$, on the magnetization curve to the various points on the d-c magnetizing force axis. These lines are drawn in Fig. 3. On rectangular coordinate paper the lines are straight; for convenience they are plotted on the coordinate paper with a logarithmic horizontal d-c magnetizing force axis. On this paper the lines are actually curved, so one must take the extreme ends of the lines to calculate the slopes and hence secure the ratio desired. These slopes are recorded in the 2nd, 5th, 8th, and 11th columns for the various d-c flux densities, with the corresponding curves shown in Figs. 3, 4, 5, and 6.

In accordance with (13) there must be added to these slopes the a-c reluctivity, $\nu_{a.c.}$, which is the reciprocal of the a-c permeability, $\mu_{a.c.}$, corresponding to the various d-c flux densities for an a-c flux density of 10 gausses. From curves shown in Fig. 2, the a-c permeabilities at 10 gausses a-c flux density and 5500, 8400, 10,300, and 11,600 gausses d-c flux densities are respectively 1020, 760, 520, and 325. The re-

TABLE I

CALCULATION OF APPARENT PERMEABILITY AND RATIO OF AIR GAP

H_o Gil. per cm	$B_{a.c.}=10$, $B_o=5500$ $\mu_{a.c.}=1020$, $1/\mu_{a.c.}=0.00098$			$B_{a.c.}=10$, $B_o=8400$ $\mu_{a.c.}=760$, $1/\mu_{a.c.}=0.001316$			$B_{a.c.}=10$, $B_o=10{,}300$ $\mu_{a.c.}=520$, $1/\mu_{a.c.}=0.001925$			$B_{a.c.}=10$, $B_o=11{,}600$ $\mu_{a.c.}=325$, $1/\mu_{a.c.}=0.00308$		
	ratio of gap	ν_a	μ_a	ratio of gap	ν_a	μ_a	ratio of gap	ν_a	μ_a	ratio of gap	ν_a	μ_a
0.5	0	0.00098	1020									
1.0	0.000091	0.001071	935	0	0.001316	760						
2.0	0.000272	0.001253	800	0.000119	0.001435	687	0	0.001925	520			
4.0	0.000637	0.001617	620	0.000357	0.001673	597	0.000194	0.002119	472	0	0.00308	325
8.0	0.00136	0.00234	427	0.000833	0.002149	466	0.000583	0.002508	0.400	0.000345	0.003425	292
16.0	0.00282	0.00380	263	0.001785	0.003101	323	0.001360	0.003285	305	0.001035	0.004115	243
32.0	0.00573	0.00671	149	0.00369	0.005006	200	0.00291	0.004835	207	0.002415	0.005495	182
64.0	0.01155	0.01253	79.7	0.00750	0.008816	113.5	0.00602	0.007945	126	0.00518	0.00826	121
100.	0.01810	0.01908	52.4	0.01180	0.01312	76.2	0.00952	0.01145	87.3	0.00827	0.01135	88

ciprocal of these values give the a-c reluctivities and are equal to 0.00098, 0.001316, 0.001925 and 0.00308. These values head the columns 3, 6, 9, and 12, Table I, corresponding to the zero air gap. The values for the apparent reluctivities are obtained by adding these a-c reluctivities to the ratio of the air gap to the iron path. These values for apparent reluctivities are given in columns 3, 6, 9, and 12. The reciprocals of these apparent reluctivities give the apparent permeabilities, as shown in columns 4, 7, 10, and 13.

We now desire the maximum obtainable apparent permeabilities corresponding to the various d-c magnetizing forces shown in the first column of Table I. For example, consider the apparent permeabilities shown in the table for 8 gilberts per cm, d-c magnetizing force. The

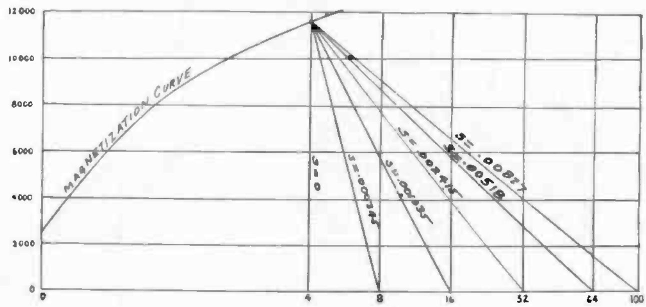

Fig. 6—Sample Soft Magnetic Alloy. Curves from which are determined the various air gaps for a flux density of 11,600 gausses at the various average d-c magnetizing forces.

values are 427, 466, 400, and 292. There is thus a maximum obtainable value of apparent permeability for 8 gilberts. The d-c flux density and the ratio of the air gap to iron path will be of some definite values to permit the attainment of this maximum value of apparent permeability.

Curves are, therefore, plotted as shown in Fig. 7. The curves are apparent permeability vs. the d-c flux density. The maximum obtainable permeability value for each curve is easily determined, and the corresponding d-c flux density is determined also. These maximum values are then plotted against the d-c magnetizing force and the resulting curve is shown in Fig. 9 curve 2.

Now, similarly, the air gap ratios shown in Table I are plotted against the apparent permeability and the air gap ratio, corresponding to the maximum obtainable apparent permeabilities, is determined graphically. See Fig. 8. These ratios corresponding to the maximum obtainable permeabilities at the various d-c magnetizing forces are plotted also in Fig. 9, curve 2.

In an exactly similar manner the curves for other soft magnetic alloys were calculated and determined graphically. The results of this work are plotted in Fig. 9, also, so that the comparative results may be seen at once.

Application of Results

As an illustrative example of the foregoing results, let us calculate the primary inductance of an audio interstage transformer having a closed core of the various alloys discussed above. Suppose it is a second stage transformer with 0.002 ampere flowing in a primary winding consisting of 4500 turns. Let the cross section be 3/4 in. or 0.562 sq. in.

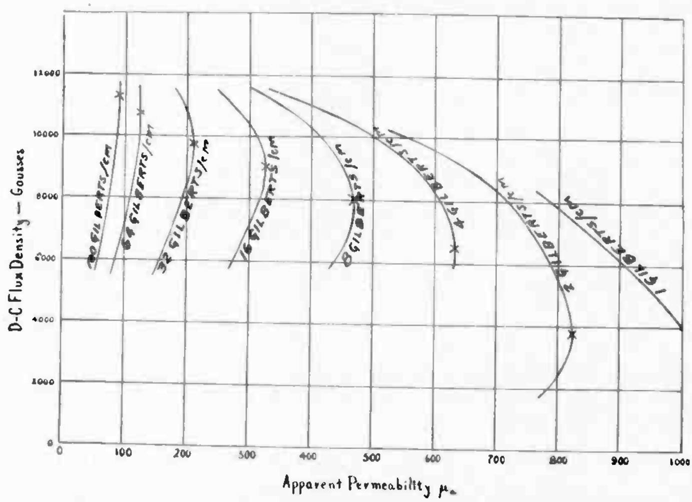

Fig. 7—Sample Alloy. Determination of maximum obtainable apparent permeability at the various d-c magnetizing forces.

Let the mean length of the iron path be 5 in. The core is to be stacked with the joints alternating side to side, so that the gap is negligible.

From (1) the a-c inductance

$$L_{a.c.} = \frac{1.256 A K_1 N^2 \mu_{a.c.}}{l} \times 10^{-8} \text{ henries.} \qquad (16)$$

The cross sectional area of the iron circuit is $0.562 \times 6.45 = 3.62$ sq. cm $= A$. $K_1 = 0.875$. $N = 4500$. $l = 5 \times 2.54 = 12.7$ cm

By means of (2) the magnetizing force in gilberts per cm is

$$H_o = \frac{1.256 \times 0.002 \times 4500}{12.7} = 0.89.$$

The a-c permeabilities for this value of d-c magnetizing force can be obtained for the various alloys by interpolating between curves nearest to this value of the magnetizing force for an a-c flux density of 10 gausses. For four different magnetic alloys these a-c permeabilities are 278, 602, 810, 1088. Substitution of these data in formula (16) gives the following results for the a-c inductance and for the reactance at 60 cycles.

Alloy	Inductance	Reactance at 60∽
1	17.7 henries	6,680 ohms
2	38.5	14,500
3	51.7	19,500
4	69.5	26,200

Since it is desirable to have as much voltage as possible across the primary of the transformer compared with that across the plate of the tube (say a 201A, or a 226), then the importance of selecting materials giving higher a-c permeabilities is apparent, if low frequencies are to be reproduced efficiently. This calls for using alloy No. 4.

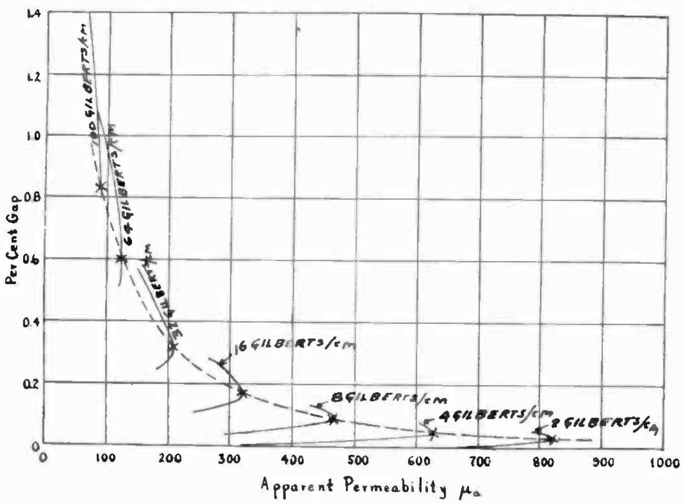

Fig. 8—Sample Alloy. Determination of air gap corresponding to maximum obtainable apparent permeability for the various d-c magnetizing forces.

As a second example let us consider an output transformer, say for a 171 tube having 0.018 ampere plate current. Let the cross sectional area of the core be 0.766 sq. in., and the mean length of the magnetic path be 6 in. Let the air gap be 0.005 in. Let us calculate the apparent inductance of the primary winding when 0.018 ampere is flowing in the primary winding and use is made of various alloys.

The length of the magnetic path is $l = 15.2$ cm

The area of the cross section is $A = 4.95$ sq. cm

The total d-c magnetomotive force is (m.m.f.) $= 1.256\, I\, N = 90.9$ gilberts.

The average d-c magnetizing force is $H_0 -$ m.m.f.$/L = 5.95$ gilberts per cm.

The problem reduces itself to applying (13) to determine the apparent reluctivity from which we can calculate the apparent permeability. This result can be substituted in the following equation to calculate the apparent inductance

$$L_a = \frac{1.256 K_1 N^2 A_1 \mu_a}{l \times 10^8}. \tag{17}$$

This equation corresponds to (16). In (13) the second term is l_2/l_1 and this is equal to 0.0008 as given by the statement of the problem. To get $\mu_{\text{a.c.}}$ and therefore $\nu_{\text{a.c.}}$, we must calculate the d-c flux density corresponding to the given conditions. This flux is obtained

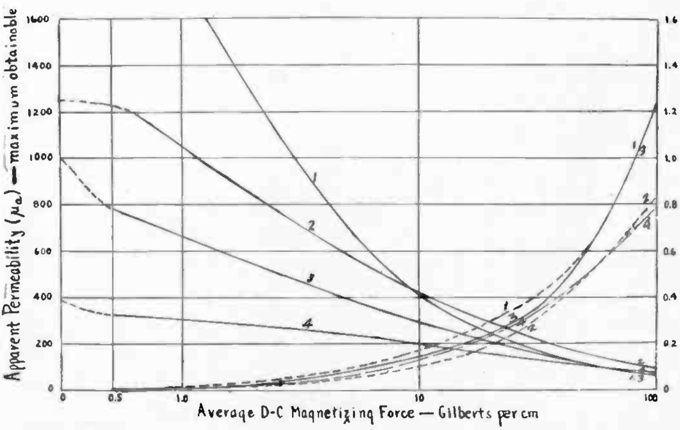

Fig. 9—Maximum obtainable apparent d-c permeability and the necessary air gap vs. d-c magnetizing force for various kinds of soft magnetic alloys.

by the intersection of the straight line, given by (4), with the magnetization curves for various alloys. The intersection of this line with the B_0 axis is, according to (5),

$$\frac{\text{m.m.m.f.}}{l_2} = \frac{90.6}{0.0127} = 7.140 \text{ gausses}. \tag{18}$$

The lines continue to the right with a negative slope corresponding to

$$\frac{l_1}{l_2} = \frac{15.2}{0.0127} = 1200 \text{ gausses.} \qquad (19)$$

per gilbert per cm.

This line intersects the magnetization curves for the various alloys at the following d-c flux densities:

Alloy	D-C Flux Density
1	6200 gausses
2	5600 "
3	6550 "
4	5950 "

Interpolating between curves for a-c permeabilities corresponding to a low a-c flux density of 10 gausses, we obtained the following a-c permeabilities corresponding to the various d-c flux densities given above. The reciprocals of these give the a-c reluctivity.

Alloy	A-C Permeability	A-C Reluctivity
1	280	0.00357
2	525	0.001905
3	925	0.001081
4	750	0.001330

The results of the calculations for the apparent permeabilities are included in the following table:

Alloy	A-C Reluctivity	Ratio of Air Gap	Apparent Reluctivity	Apparent Permeability
1	0.00357	0.0008	0.00437	229
2	0.001905	0.0008	0.00271	369
3	0.001081	0.0008	0.00188	531
4	0.001330	0.0008	0.00213	469

From this table it is evident that the highest apparent permeability is given by using alloy No. 3, which is the best material if highest amplification is desired. These illustrations indicate how the method described in the paper is applied to particular problems and how the best solution in one case may be different from that in another.

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A NEW METHOD OF DETERMINING HEIGHT OF THE KENNELLY-HEAVISIDE LAYER*

By
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Summary—*Graphic records of radio signals transmitted from high-frequency aircraft radio transmitters are observed to show periodic variations of fairly constant frequency over considerable time intervals. A theory is evolved connecting this frequency with the transmitting distance, the ground speed of the plane and the effective height of the Kennelly-Heaviside Layer. Records of such periodic fading are reproduced from which, in accordance with this theory, the height of the layer is computed. A short discussion is given of the results obtained.*

A NUMBER of measurements have been made at the Naval Research Laboratory during the past year of field strength resulting from signals transmitted by airplanes in flight. These measurements have been taken with measuring equipment supplied by courtesy of the Bell Telephone Laboratories, which makes a graphic record on a moving chart laid off to a logarithmic scale of trans-

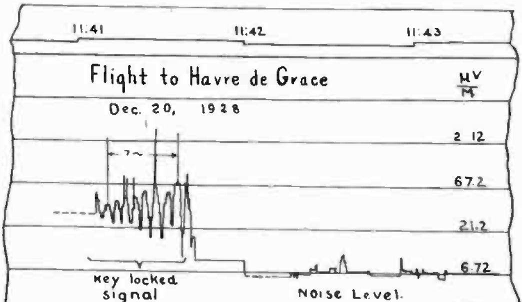

Fig. 1

mission units or TU. All of these measurements have been confined to frequencies between 2500 and 6000 kc at distances up to 130 miles (209 km). In most of them marked fading has been observed at distances beyond fifteen or twenty miles.

From data obtained during such flights which include the velocity of the plane over the ground, distance from recording station, and dis-

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tance between peaks of fading record, a determination of the effective height of the Kennelly-Heaviside layer has been attempted. The

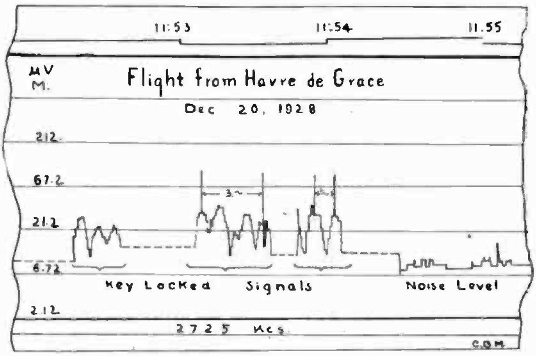

Fig. 2

method used is based upon the theory that as the plane transmitter moves from or toward the receiver, the ratio of the length of the ground path to the length of the sky path is changing, and the phase relation

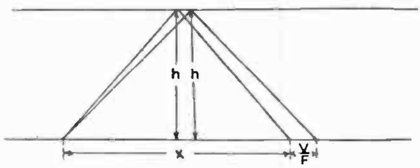

Fig. 3

between the two components is also progressively changing. When the plane has traveled the distance V/F in Fig. 3, or p–p^1 in Fig. 4, the graphic record has gone from one peak to the next and the phase of

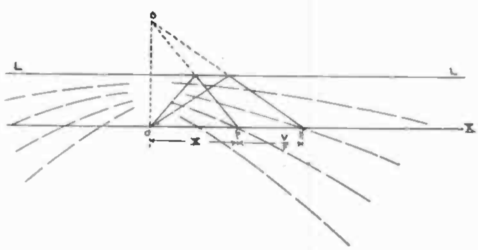

Fig. 4

one component has changed 360 deg. with respect to the other. That is, the difference between the optical lengths of the two paths has changed by a distance equal to the length of the transmitted wave.

It is assumed that we are dealing with simple reflection from a horizontal plane at the effective height of the layer.

Let V = velocity at which transmitting plane travels over the ground in a direction to or from recording station.

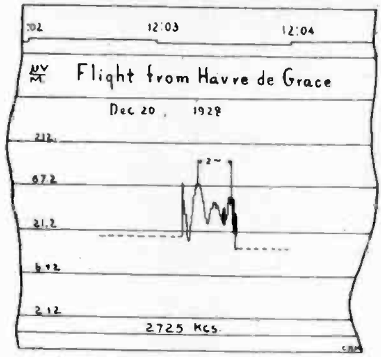

Fig. 5

τ = time between successive maxima of signal.

F = frequency of intensity variation.

X = distance from recording station.

λ = wavelength of transmission.

Then $V\tau$ (Fig. 3) $= V/F$.

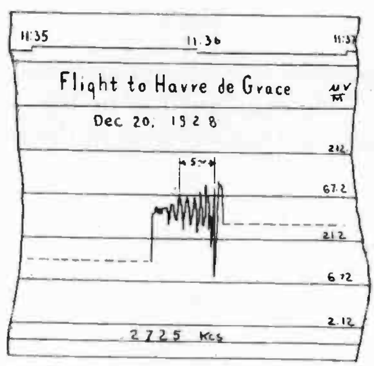

Fig. 6

From the geometry of Fig. 3

$$\sqrt{h^2+\left(\frac{x}{2}\right)^2}-\frac{x}{2}-\left[\sqrt{h^2+\left(\frac{x+V/F}{2}\right)^2}-\left(\frac{x+V/F}{2}\right)\right]=\frac{\lambda}{2} \quad (1)$$

Taking advantage of the fact that V/F is small compared to X, equation (1) reduces approximately to

$$h=\frac{x\sqrt{\lambda\left(\frac{2V}{F}-\lambda\right)}}{2\left(\frac{V}{F}-\lambda\right)}. \tag{2}$$

Amplifying Fig. 3 somewhat we have Fig. 4. Transmission from O reaches p, p', by both direct and reflected path. Recognizing the

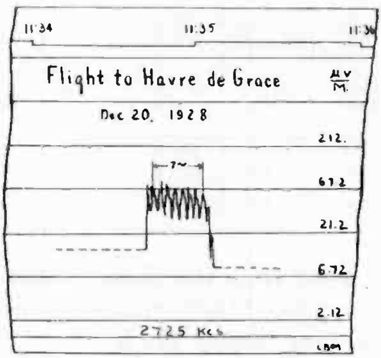

Fig. 7

similarity to well-known optical phenomena the figure is extended to indicate a similar source or image at O'. Interference patterns will

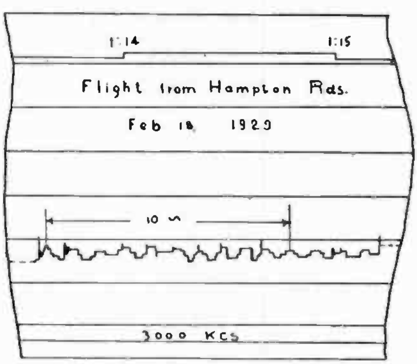

Fig. 8

then lie along hyperbolic curves with O–O' as their axis. The path of an observer along O–X will cut these lines of maximum or minimum reception.

Thus far we have considered a fixed transmitter and a moving observer, whereas in the case of our experiment the transmitter is in the moving plane and the receiver is on the ground. However, for purpose of analysis we may consider the ground station as moving

TABLE I

Date	Time	Freq. kc f	Freq. of fading cycle sec. F	Distance between minima meters $V/F\times10^{-2}$	Ground speed mi/hr. V	Distance of transmission miles X	Hgt. of K-H layer miles h	
12/20/28	11:35 11:36 11:41	2725 2725 2725	0.379 0.367 0.282	160. 165.0 215.0	135.5 135.5 135.5	48.25 50.5 60.7	73.4 71.5 53.4	*To* Havre de Grace
12/20/28	12:04 11:53	2725 2725	0.1345 0.1186	176.5 199.0	52.9 52.9	48.52 65.1	59.8 62.0	*From* Havre de Grace
2/18/29	1:19 1:14	3000 3000	0.177 0.144	200.0 246.0	79.2 79.2	96.0 103.0	83.4 70.0	*From* Hampton Roads
2/25/29	12:17 12:48 12:55 1:05	3000 3000 3000 3000	0.243 0.162 0.170 0.121	160.0 240.0 229.0 324.0	87.6 87.6 87.6 87.6	59.65 105.50 115.25 129.85	74.0 73.0 85.0 67.8	*To* Hampton Roads

relatively toward or away from the plane. With this conception the figure may correctly represent the conditions of measurement. It is significant that by this construction the intercept $p-p'$ between fading lines grows greater as the distance between transmitter and receiver increases.

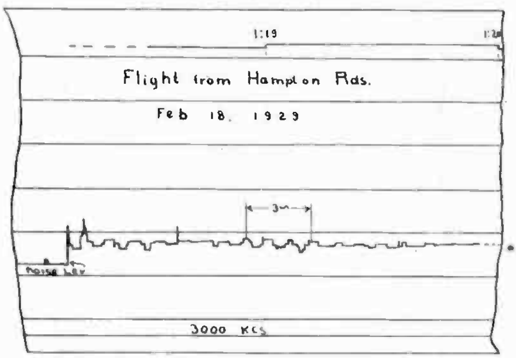

Fig. 9

A particularly good record of fading appears on sections of chart shown in Figs. 1 and 2, which were taken on December 20, 1928 between 11 A.M. and 1 P.M. at a frequency of 2725 kc. This flight was made from the U. S. Naval Air Station at Anacostia, D. C. to Havre de Grace, Md. and return over a straight course very nearly north and south.

The altitude of transmitting plane was kept as nearly constant as possible at 2000 ft. The transmission was unmodulated CW. From position reports transmitted from the plane by radio, the ground speed was estimated at 135.5 miles per hour going and 52.9 miles per hour returning. This checks approximately with ground observations of wind velocity at 40 miles per hour. Uniform velocity in each direc-

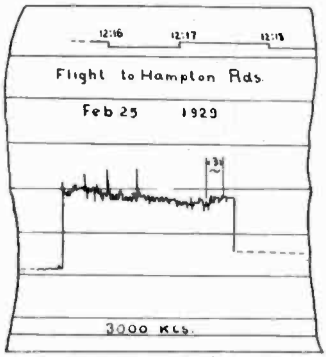

Fig. 10

tion was assumed and the position and distance of the plane for each record was computed on this basis. Frequency of fading was determined by comparing the distance between maximums on the chart with one-minute timing lines at top. The average of several cycles was taken in each case, those chosen for average being indicated on charts.

Converting to centimeters and seconds the values obtained from records 1 and 2, we have

	Outward Flight	Return Flight
X =	9,770,000 cm	10,150,000 cm
F =	0.282 cycles per sec.	0.1186 cycles per sec.
V =	6065 cm per sec.	2360 cm per sec.
λ =	11000 cm	11000 cm

Substituting these values in (2) gives the height of the reflecting layer as

Outward Flight	Return Flight
h = 54.2 miles (87.1 km)	h = 63.1 miles (101.7 km)

By a similar procedure several subsequent determinations were made as indicated by records Nos. 5 to 13 inclusive for which results are tabulated. It may be noted that in practically every case the distance between maxima of fading increases with the distance of transmission which is taken to support the theory as outlined in Fig. 4.

The results here obtained seem in fair agreement with daytime determinations by other observers. Breit and Tuve[1] at Washington, D. C., working at a wavelength of 70 m, found a height of from 90 to

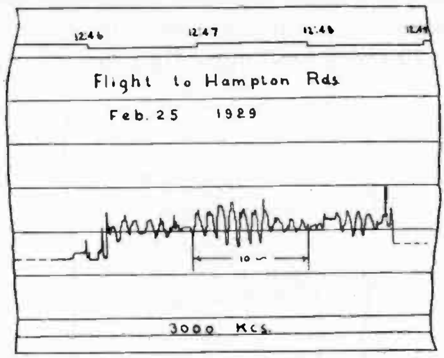

Fig. 11

220 km for the reflecting layer in daytime in summer and autumn. Taylor and Hulburt[2] calculated from observed skip distances averaged over the year for daylight conditions for waves below forty meters in length, that these waves reach heights between 120 and 240 km. Hollingsworth[3] in England finds 75 km in summer and 90 km in winter as the apparent daytime height reached by waves 6000 to 14,000 m long.

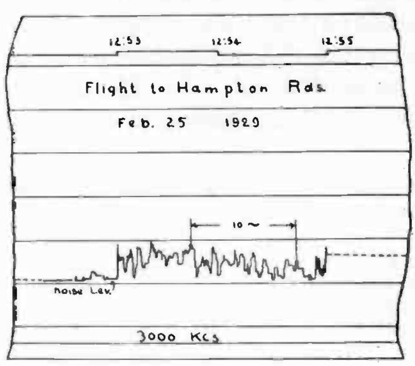

Fig. 12

Observations at night show much greater heights. Appleton[4] finds indications that 400-m waves near midnight in midwinter reach heights of 300 to 400 km.

[1] Breit and Tuve, *Phys. Rev.*, 28, 554, 1926.
[2] Taylor and Hulburt, *Phys. Rev.*, 27, 189, 1926.
[3] Hollingsworth, *Jour.* A. I. E. E., 64, 579, 1926.
[4] Appleton, *Nature*, 120, 330, 1927.

The method of this paper may be contrasted with the interference fringe or beat note method used by Appleton, Barnett, Heising and others, in that the frequency is kept constant and the distance from transmitting station is varied instead of varying the frequency and

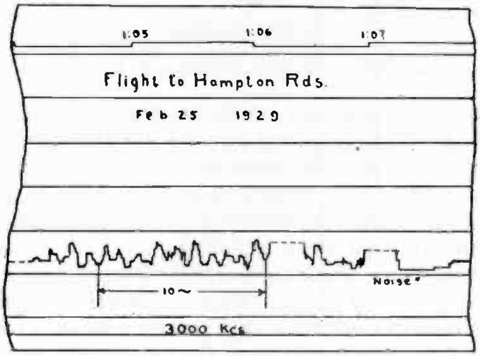

Fig. 13

observing at a constant distance from the transmitter. For this reason the observations are confined strictly to a single frequency. It is believed that these observations add one more method to those by which the effective height of the reflecting layer have been determined, and therefore contribute that much toward filling out the general theory.

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SOME OBSERVATIONS OF SHORT PERIOD RADIO FADING*

By

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Summary—*The data presented are the product of an investigation started at the beginning of 1928 with the object of studying the short-period fading of radio broadcast transmissions. Particular attention was paid to those intensity changes which take place during periods ranging from a few seconds to several minutes. Various antenna combinations were used in making simultaneous records in order to separate the effects of various causes of fading.*

The data secured partly confirm the conclusions of previous investigations, partly point to other sources of fading. Varying intensity of the indirect ray and interference between indirect and ground rays are evidenced as in earlier experiments, but rotation of the plane of polarization of the indirect ray is also shown to be a considerable factor and there are suggestions of lateral direction shifts of the indirect rays and of their arrival by multiple paths.

THE data presented in this paper are the product of an investigation started by the Bureau of Standards at the beginning of 1928 with the object of studying the short-period fading of radio broadcast transmissions. This study was directed particularly to those intensity changes in the receiving antenna which range, in period, from a few seconds to several minutes between peaks. These variations have been variously ascribed by previous investigators to one or more of the following causes: (1) intensity changes undergone by the indirect or atmospheric ray, namely, that portion of the radiation which, at night especially, travels to the upper atmosphere and is returned to earth at the receiving antenna by refraction; (2) interference, due to varying phase relationships of two or more indirect rays or of an indirect ray and the ground ray, which follows a direct path along the surface of the earth from transmitting to receiving antenna; (3) direction shifts, this expression being used to connote deviations of the indirect ray from the great-circle plane passing through transmitting and receiving points; (4) changes in the angle of incidence, that is, in the vertical plane of arrival of the indirect ray; (5) rotation of the plane of polarization of the atmospheric ray, this rotation being considered as a continuous process rather than in the optical sense of a

* Dewey decimal classification: R113. Original manuscript received by the Institute. Presented at meeting of International Scientific Radio Union, Brussels, Belgium, Sept. 13, 1928. Reprinted from forthcoming issue of Bureau of Standards Journal of Research. Publication approved by the Director of the Bureau of Standards of the Department of Commerce, Washington, D. C.

definite displacement of the plane at completion of the process. The aim of the present investigation was to separate the effects due to the above-mentioned causes with a view to determining to what extent each might be a factor in fading.

Method and Apparatus

The method consisted in selecting a particular broadcasting station and making simultaneous graphic records of its carrier wave as received by different types of antennas attached to duplicate receiving sets. The antennas used were (1) a vertical single-wire, 15 meters high, with a 2-meter horizontal lead-in; (2) a vertical coil having its turns in a great-circle plane common to transmitting and receiving points, which coil will hereafter be referred to as "at maximum," or "in the

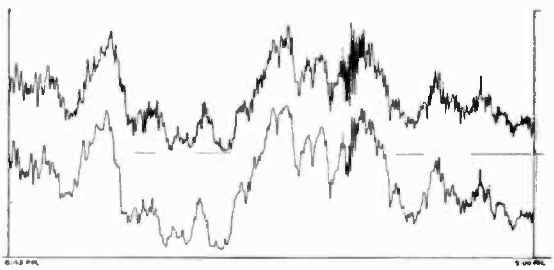

Fig. 1—WIOD (1210 kc), Miami Beach, Florida, received at Kensington, Md., 1500 km, March 15, 1928. Upper record with vertical single-wire antenna; lower record coil antenna at maximum.

maximum position;" (3) a vertical coil perpendicular to the great-circle plane, and hereafter to be designated as "at minimum," or "in the minimum position;" (4) a so-called barrage antenna, consisting of the single-wire antenna so coupled to a coil antenna in the maximum position as to introduce an out-of-phase current and thus neutralize the effect of the ground ray as well as of the vertical component of the indirect ray. The coil antennas were so balanced by a center tap arrangement as to eliminate any appreciable "antenna effect." For all tests the various antennas were located from three to five meters apart. Carefully conducted tests proved that these distances were sufficiently small so that records obtained simultaneously with similar antennas were alike, and that the distances were sufficiently large to prevent interaction between the separate receiving circuits. All observations were made at the Bureau of Standards field station located at Kensington, Maryland, 8 kilometers north of the Bureau and in a position practically free from "man-made" interference and distorting influences.

Other apparatus employed was essentially that described by G. W. Pickard[1] in 1923 and used later in the cooperative investigation conducted by the Bureau of Standards in 1925.[2] The receiving sets were superheterodynes of the same make and were capable of adjustment to practically the same amplification. The driving motors of the Shaw

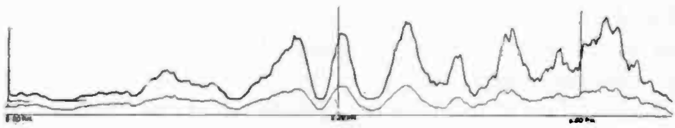

Fig. 2—WJZ (660 kc), Bound Brook, N. J., received at Kensington, Md., 300 km, May 15, 1928. Upper record, coil antenna at maximum; lower record, vertical single-wire antenna.

manual recorders were automatically synchronized every second, thus making it possible to compare two records by superposition.

Observations were concentrated largely upon two stations, but less frequent records were made on transmissions from other stations involving frequencies of 550 to 1480 kc and direct transmission paths of 13 to 1500 km. The greater number of records were made upon WJZ, Bound Brook, N. J., and upon WBAL, Baltimore, Md., because of the regularity with which each could be received, day or night, and

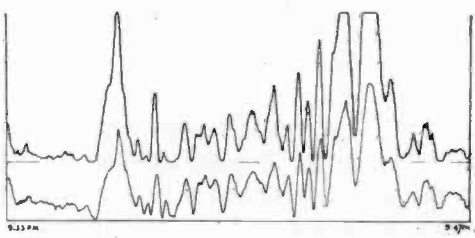

Fig. 3—WRVA (1180 kc), Richmond, Va., received at Kensington, Md., 155 km, June 1, 1928. Upper record, coil antenna at maximum; lower record, vertical single-wire antenna.

because of what appeared to be characteristic features. Many of the observations started during daylight when there was no appreciable indication of an indirect ray, and when, consequently, the most accurate bearings could be secured with the coil antennas. At this time,

[1] G. W. Pickard, "Short period variations in radio reception," Proc. I. R. E., 12, 119–158; April, 1924.

[2] J. H. Dellinger, C. B. Jolliffe, and T. Parkinson, "Cooperative measurements of radio fading in 1925," Bureau of Standards Scientific Paper No. 561, pp. 423–425.

also, the two receiving systems could be so adjusted that the ground wave produced the same deflection on the recorder galvanometers of the two receiving systems. When records were started after dark no attempt was made to have them on an accurately comparable intensity basis, the chief aim being to study the simultaneous changes on two records rather than their exact magnitudes.

The arrangement of the pairs of simultaneous records in the illustrations will be found somewhat inconsistent since the character of the records made it easier to compare them by placing the one made with a given type of antenna sometimes above, sometimes below, the record made with another type.

Results

Simultaneous Coil and Vertical Single-Wire Antenna Records. The first method of analysis in the present investigation was by means of simultaneous fading records made with coil antenna in maximum

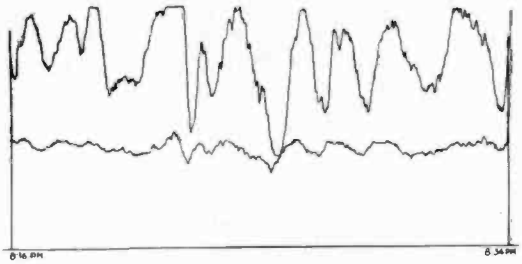

Fig. 4—WBAL (1050 kc), Baltimore, Md., received at Kensington, Md., 50 km, May 9, 1928. Upper record, coil antenna at maximum; lower record, vertical single-wire antenna.

position and with vertical single-wire antenna, respectively. Assuming both ground and reflected rays to be in the great circle plane which passes through transmitting and receiving points, certain effects were anticipated. The ground ray presumably remains normally polarized at night as during daylight; therefore, so long as there is a normally polarized component of the indirect ray any changes in the path length of the latter should result in changing phase relations between the two rays and consequently in similar and simultaneous intensity variations in the two types of antenna. Since the total electric vector in the vertical plane is effective on the coil antenna the signal strength in it should range between the sum and the difference of the two fields produced by direct and indirect rays, respectively. On the vertical single-wire antenna similar variations should be of less absolute magnitude since only the vertical component of the electric vector is effective and this component decreases with the angle of incidence. With either antenna the night intensity, for non-local stations at least, will sometimes exceed, sometimes fall below, the relatively constant daylight level, and similar variations should occur simultaneously on both records.

Turning now to the data and examining some fifty records involving transmission frequencies of 660 to 1480 kc and distances of 13 to 1500 km, what do we find? For stations more than about 150 km distant from the receiving antennas the results were practically as anticipated, as the typical graphic records of Figs. 1 to 3 indicate. There is a slight shifting of phase at one point in the WIOD record shown in Fig. 1 which may be real or may have been caused by changing speed of one recording drum at a time when there was difficulty

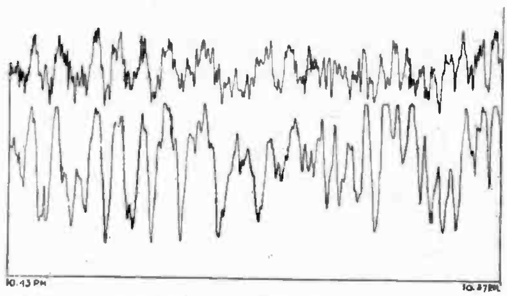

Fig. 5—WTFF (1480 kc), Mt. Vernon Hills, Va., received at Kensington, Md., 31 km, March 19, 1928. Upper record, vertical single-wire antenna; lower record, coil antenna at maximum.

with the synchronization of the two drums. Conditions prevented making another record on this station. With this single exception the intensity changes are in phase on the two antennas. An anomaly appears, however, in the records for the nearer group of stations, 13 to 53 km distant, in that the intensity changes are at times as much as 180 deg. out of phase on the two records. This holds true for variations of a few seconds duration as well as for those having periods of a minute or two. Figs. 4 to 7 show typical illustrations. Here we have the same out-of-phase effect that Appleton and Ratcliffe[3] found in simultaneous records made respectively with vertical single-wire antenna and with a loop in the maximum position, the effect of the ground ray being suppressed in the loop by means of the so-called barrage system. They accounted for the recorded differences by an interference between indirect and ground rays which was effective in

[3] E. V. Appleton and J. A. Ratcliffe, "On the nature of wireless signal variations." *Proc.* Royal Society, 115A, pp. 291–317, June 1927. See especially pp. 311–312 and Fig. 9.

the vertical antenna but was eliminated from the loop. This explanation is not applicable to the present data since with the ground ray effective in both antennas there is still a lack of synchronism in the intensity changes of the two records. Appleton and Ratcliffe, on the other hand, found the two records varying together when the ground ray was not suppressed.[4]

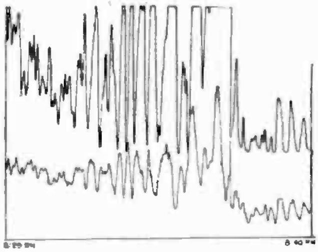

Fig. 6—WTFF (1480 kc), Mt. Vernon Hills, Va., received at Kensington, Md., 31 km, May 17, 1928. Upper record, coil antenna at maximum; lower record, vertical single-wire antenna.

A possible explanation of this anomaly may be reached by the process of elimination. Making the usual assumption that the indirect ray remains in the plane of the great circle connecting transmitter and receiver, the combined effect of ground and downcoming rays should be to cause similar changes in both vertical single-wire and coil antennas. The only exception would be when the atmospheric ray arrived

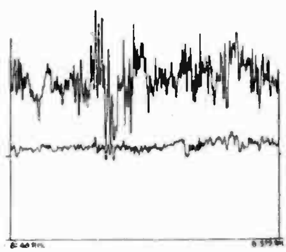

Fig. 7—WMAL, (1240 kc), Washington, D. C., received at Kensington, Md., 13 km, May 7, 1928. Upper record, coil antenna at maximum; lower record, vertical single-wire antenna.

with zero angle of incidence, in which case only the ground ray should be effective in the vertical antenna and the intensity should remain at the daylight level, while in the coil antenna the combined effect of the two rays should continue to produce fading changes. This explanation does not fit the data, since fluctuations continue on both records. It is the variation in phase relation between the two that must be ac-

[4] Reference 3, p. 300, Figs. 4 and 5.

counted for. Rotation of the plane of polarization of the incident ray also should produce like effects in the two receiving systems; therefore, by itself, this offers no solution.

The answer is presumably associated, directly or indirectly, with the nearness of the transmitting and receiving stations. For the distances concerned, 13 to 53 km, it is perfectly possible to have direction shifts of the incident ray as great as 90 deg., due to irregularities in the refracting or reflecting upper atmosphere. This could account for changes on the vertical antenna not occurring on the coil antenna, but not for the considerable time lag between similar changes produced in the two receiving systems. If, however, there should be a lateral direction shift combined with a rotation of the plane of polarization for the downcoming ray, we should expect just such differences in the pairs of records as appear. Suppose, for purposes of illustration, that the reflected ray is arriving with constant intensity and normally

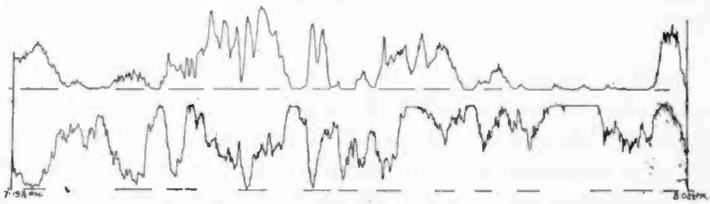

Fig. 8—WJZ (660 kc), Bound Brook, N. J., received at Kensington, Md., 300 km, April 2, 1928. Upper record, coil antenna at maximum; lower record, coil antenna at minimum position.

polarized. If, now, the lateral angle of arrival shifts until it becomes 90 deg. the signal intensity in the vertical antenna will remain constant while that in the coil antenna will decrease to a minimum, in fact, to zero, if there should be no ground wave present.

If, next, the plane of polarization rotates, the vertical component of the electric field will decrease until, with the electric field horizontal, the ray ceases to produce a current in the vertical antenna. Simultaneously, the component of the magnetic field in the vertical plane will have increased from zero to a maximum, thereby producing maximum current in the coil antenna for the given angle of incidence. Thus we have similar fading characteristics 180 deg. out of phase in the two antennas. If the direction of arrival of the incident ray gradually returns to the plane of the maximum coil antenna and the rotation of the plane of polarization continues, whether at regular or irregular speed, the phase difference between the two records will gradually decrease until it disappears. With the direction shifts more or less erratic we should have just the sort of changing phase relations shown on the records of Figs. 4 to 7. When rotation of the plane of polarization is absent we should find the two records in phase, as in the early part of Fig. 5 or the latter part of Fig. 6.

The existence of the direction shifts necessary to the above explanation requires proof. Bearings on stations were taken at night by means of a coil antenna, and revealed apparent direction shifts which were considerable. A convincing test, however, can only be made with an

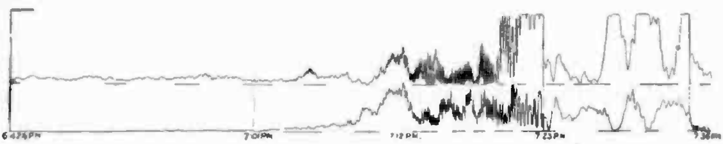

Fig. 9—WJZ (660 kc), Bound Brook, N. J., received at Kensington, Md., April 5, 1928. Local sunset at 6:36. Upper record, coil antenna at maximum; lower record, coil antenna at minimum.

antenna of the Adcock type[5] if true directions of arrival of the reflected ray are to be found. Such a test awaits the construction of further apparatus.

Interference between the indirect ray and the ground ray is evidenced on many of the records made with the coil antenna in maximum position, and to a lesser degree by those made with the vertical single-wire antenna. Wherever the intensity of one of these records falls below the daylight level the indirect ray must be to some extent out of phase

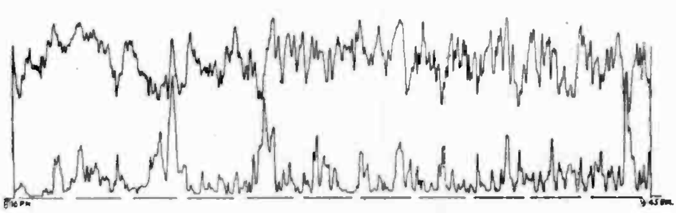

Fig. 10—WBAL (1050 kc), Baltimore, Md., received at Kensington, Md., 50 km, April 17, 1928. Upper record, coil antenna at maximum; lower record, coil antenna at minimum.

with the waves arriving along the surface of the earth. Figs. 4 to 7 give clear indications of this. Note particularly that even for stations as near as 13 km from the observing point, as shown by Fig. 7, the intensity of the downcoming ray is sometimes nearly as great as that of the ground wave, so that when the two are opposite in phase the intensity of the ground ray is largely neutralized and the record drops

[5] Improvement in means for determining the direction of a distant source of electromagnetic radiation. British Patent 130490, 1919. For actual applications see reference 14 in Appendix.

almost to zero. Presumably in these same records the peaks are due to the addition of the reception by direct and indirect paths. It will later be shown, however, that peaks may be due to other causes, whether the ground wave be present or not.

Simultaneous Fading Records with Two Coil Antennas, at Maximum and Minimum Positions, Respectively. The second method of analysis was by simultaneous fading records of the same transmission as received by two coil antennas one in the maximum and the other in the minimum position. The purpose of this arrangement was to secure some data as to fading caused by rotation of the plane of polarization of the indirect ray; for such a phenomenon, if present, should cause an increase of intensity in either coil antenna to be accompanied by a decrease in the other. So far as possible the minimum bearing was found

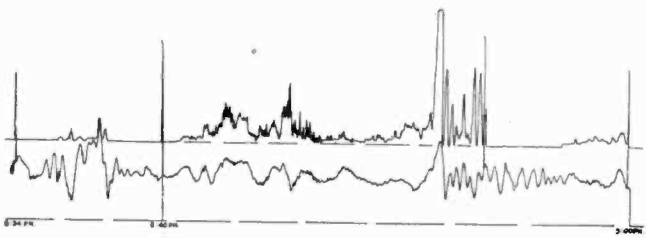

Fig. 11—WMAL (1240 kc), Washington, D. C., received at Kensington, Md., 13 km, June 5, 1928. Upper record, coil antenna at minimum; lower record, coil antenna at maximum.

for the various stations during daylight when there was no evidence of an indirect ray, and these bearings checked satisfactorily on different days. For the more distant stations, however, no ground wave could be received in daylight and it was necessary to trust to approximate bearings, which were sufficient for the project in hand.

The data secured offers considerable striking evidence of the above-mentioned phenomenon. Fig. 8[6] is a typical section cut from a pair of simultaneous records. Here it is possible to see at a glance that the two are practically 180 deg. out of phase most of the time. The minima do not always fall to zero as would be expected with the ground wave practically negligible as it is with the stage of amplification used in this case, and it therefore becomes necessary to postulate more than one indirect ray. It may be that in this record we have the effect of two rays rotating at different speeds so that their minima do not synchronize most of the time. Interference between the two is also

[6] Figs. 8 and 9 and other data indicating rotation of the plane of polarization of radio waves were shown by the author at the meeting of the American Section, U. R. S. I., in Washington, D. C., April 19, 1928.

probably present but the effect cannot be isolated in the record. Fig. 9 is another record made on reception from the same station as Fig. 8, and is typical of what occurs during the sunset period. For approximately one-half hour after sunset the weak ray from the upper atmosphere appears to remain normally polarized and so produces intensity changes only in the coil at maximum position, possibly through a combination of interference effects and varying absorption. Evidence then gradually appears that an indirect ray somewhat abnormally polarized is producing a current in the minimum coil. With no further rotation of the plane of polarization any variations in the intensity of the indirect ray may now result in parallel changes such as appear in the first large intensity peak at 7:12. Here, due probably to the combined effects of very abnormal polarization and of small angle

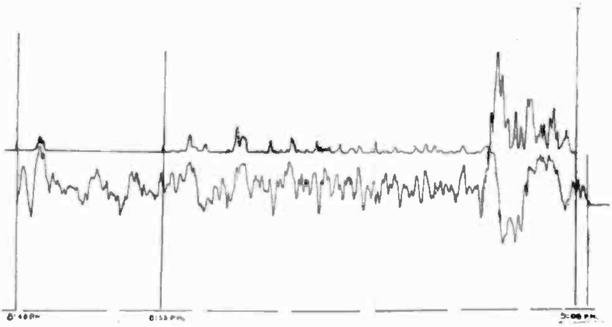

Fig. 12—WMAL (1240 kc), Washington, D. C., received at Kensington, Md., 13 km, June 6, 1928. Upper record, coil antenna at minimum; lower record, coil antenna at maximum.

of incidence accompanying increased height of the ionized layer, the intensity in the coil at minimum is practically the same as in that at maximum. From this peak on, nearly to the end of the record, the average trend of the maximum coil antenna record is 180 deg. out of phase with the minimum, indicating that the rotation of the plane of polarization of the indirect ray has gotten under way. Starting at about 7: 04 P.M. on this same record there is a superposed rapid variation having a period of from 3 to 30 seconds. This characteristic is also exactly out of phase on the two records and so suggestive of a second and relatively much weaker ray with plane of polarization rotating rapidly. Assuming the polarization to be plane, a complete rotation would have to occur in six seconds to produce the most rapid of these changes.

While these and other data seemed to be consistent with rotation of the plane of polarization of the indirect rays, there were times when the phase difference between the two records varied widely, sometimes becoming almost zero, as shown in Figs. 10 to 12. The ground ray could in no way be held responsible, for its effect had been eliminated by the coil antenna in minimum position, and yet variations in one antenna led or lagged behind similar variations in the other. In a recent article Eckersley[7] reports that records of high-frequency transmissions received simultaneously with vertical and horizontal antennas are opposite in phase. It is interesting to note that the illustration given in that paper, however, shows clearly the same sort of phase shifts between the two records as appear in the data of the present paper. To account for this phenomenon by an interference effect, at least two elliptically polarized indirect rays would be necessary, with their vertically polarized components undergoing changes in phase relationship similar to but lagging or leading those of the horizontally polarized components.

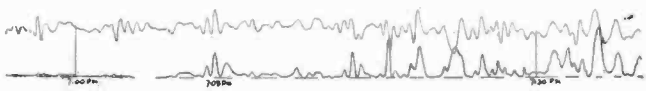

Fig. 13—WBAL (1050 kc), Baltimore, Md., received at Kensington, Md., 50 km, April 27, 1928. Local sunset at 6:58. Upper record, coil antenna at maximum; lower record, barrage minimum.

A more plausible explanation is that suggested for similar out-of-phase relations which occurred in the records made simultaneously on coil and vertical-wire antennas. If an indirect ray with rotating plane of polarization were arriving with an angle of incidence of nearly 90 deg. and the plane of its arrival should gradually shift laterally from 0 deg. to 45 deg. with respect to the maximum position of the coil antenna the records made simultaneously by coil antennas in maximum and minimum positions would gradually shift from the 180 deg. out-of-phase relationship to a nearly in-phase relationship. If the angle of incidence were considerably less than 90 deg. similar changes in phase relationships would take place as the direction of arrival shifted to the 45 deg. position, except that the two records would never come completely into phase. As the angle of incidence decreased still further the two records would be more and more out of phase. If the indirect rays always arrive by means of a single reflection the angle of incidence at the receiving end would increase with the distance between transmitting and receiving points and a direction shift of given angle would produce a greater phase shift in the pair of fading records for a distant than in the pair for a nearer station because the incident ray

[7] T. L. Eckersley, "Polarization and fading of short wireless waves," *Nature*, 121, 707; May, 1928.

for the former is more nearly horizontal. On the other hand, it should be possible for the ray from a nearer station to arrive at larger angles with the plane of the coil antenna in maximum position since lateral radiation from the distant station would have to go to improbable distances to be reflected back at large angles. Very distant transmissions, furthermore, would presumably be received almost entirely in the great circle plane passing through transmitting and receiving stations. In the case of possible multiple reflections of the same ray or of an inverted U-shaped path the situation would be altogether different. Further measurements of the apparent effective height of the ionized region according to the method of Appleton and Ratcliffe, providing a sufficient variety of distances be covered, may give some useful information, since this method should prove most accurate and consistent for the distant stations if there are actual lateral reflections present.

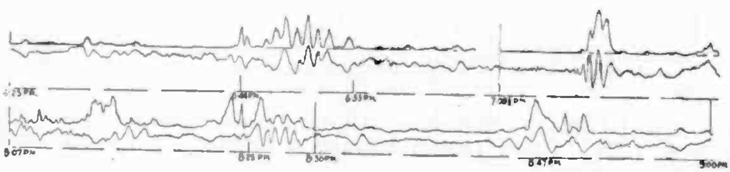

Fig. 14—WBAL (1050 kc), Baltimore, Md., received at Kensington, Md., 50 km, April 26, 1928. Local sunset at 6:57. Upper record, barrage minimum; lower record, coil antenna at maximum.

As a partial check of the possible shifts in the direction of arrival of the indirect ray, observations were made simultaneously with three receiving systems utilizing, respectively, a coil antenna in maximum position, a second in the minimum position, and a third in a vertical plane at an angle of 45 deg. with the other two. Comparisons were made vocally by the three observers as changes occurred in the indicating galvanometers, since the third recorder for securing graphic records was lacking. Observing was done at a time when the phase relation between changes in the perpendicular coils was varying. The triple observations showed that for periods of a number of minutes at a time the variations in the 45 deg. coil were more nearly in phase with those in the coil at maximum, for similar periods were more nearly in phase with changes in the coil at minimum position, and at other times were considerably out of phase with both. Such effects do not prove direction shifts to exist but are similar to what would be expected if such shifts were real.

Simultaneous Fading Records Made with Barrage Antenna and with Vertical Single-Wire or Coil Antennas. Using the barrage combination of antennas to eliminate effects due to the ground wave, and the coil antenna in the maximum position to receive both ground and indirect rays, it was hoped that differences in records would indicate the amount of fading due to interference between these two rays. With the indirect ray remaining in the plane of the great circle connecting the two stations, changes other than those due to interference should have similar effects on the two receiving systems, though the magnitude of changes should differ.

Figs. 13 and 14 show typical pairs of records. The earlier portion of the records of Fig. 13 occurs during sunset before the angle of incidence of the indirect ray has decreased sufficiently to produce more than a feeble horizontal component of the electric field. Consequently the intensity changes are quite feeble at first but increase as the ionized layer moves upward. During the first half of the records the intensity

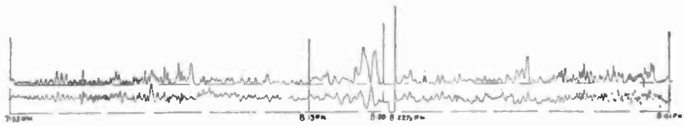

Fig. 15—WBAL (1050 kc), Baltimore, Md., received at Kensington, Md., 50 km, July 6, 1928. Upper record, barrage minimum; lower record, vertical single-wire antenna.

variations are in phase in the two antenna systems, indicating that these changes are produced farther back in the transmission path. The later portions of the records, however, show some changes in one antenna to be lagging similar changes in the other. Fig. 14 shows similar lagging effects at approximately 6:44 to 6:53, 8:25 to 8:30 and 8:47 P.M. At other times this pair of records is usually in phase or 180 deg. out. The latter condition is well explained by the fact that as the horizontal electrical component of the indirect ray increases, as shown by the barrage record, the vertical component presumably increases also, and being out of phase with the ground ray, whether partially or completely at these particular times, causes a reduction of intensity in the coil antenna set at maximum position.

Fig. 15 shows records made by a method similar to that last described except that a vertical single-wire antenna was substituted for the coil in maximum position. This method was used by Appleton and Ratcliffe.[8] Though the fading is more rapid in Fig. 15, the phase relations between the records are similar to those of Figs. 13 and 14, with less positive indications of the lagging effect because of the rapidity of variations. This effect does appear on a record[8] shown by Apple-

[8] Reference 3, pp. 311–312, and Fig. 9.

ton and Ratcliffe, however. The explanation given by them, that the difference in records is due to the interference between indirect and ground rays which can affect only one of the two receiving systems, is a possible one for all of these pairs of records. Two facts, however, cast some question upon this as a complete solution: (1) it is doubtful if interference can account for the slight phase displacements in similar portions of the two records as it may well account for the 180 deg. out-of-phase relationships; (2) similar varying phase relationships appear in records already discussed, those made with the vertical antenna and with the coil antenna in maximum position with no elimination of the ground ray. Shifts in the direction of arrival of the downcoming ray would make the latter more effective on the vertical antenna portion of the barrage system than on either coil.

If rotating plane of polarization be combined with a direction shift in this ray the phase relationships appearing in the pairs of simultane-

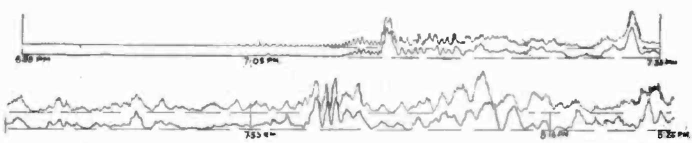

Fig. 16—WBAL (1050 kc), Baltimore, Md., received at Kensington, Md., 50 km, March 30, 1928. Local sunset at 6:30. Upper record, coil antenna at minimum; lower record, barrage minimum.

ous records become explicable. Three simultaneous records utilizing the vertical antenna, the barrage system, and the coil antenna, should throw further light on the problem.

Again, the barrage system was used simultaneously with the coil antenna in minimum position. This provided a method of eliminating results of the ground wave from both coils so that remaining effects should be due to indirect rays only and to the angles which the coils made with that indirect reception. The effect of the entire vertical electric component of this ray should, of course, be eliminated in the barrage system. Fig. 16 shows the type of record secured on transmission during the sunset period from a station only 53 km distant. The changes, both large and small, are very nearly of the same magnitude and the larger variations are usually in phase on the two records. The phase relationships of the smaller periodic variations change considerably but are most often 180 deg. out. In other words, this pair of records has much the same characteristics as appeared on the WJZ records, made with coils in maximum and minimum positions, in which the effect of the ground wave was practically absent because of great attenuation due to distance. There is in the early part of the records a superposed periodic variation suggestive of an indirect ray of small intensity with rotating plane of polarization and some shift in the lateral angle of arrival to cause changing phase relationships in the two fading records. The major variations would appear to be caused by a much stronger indirect ray of varying intensity with its plane of polarization unchanging at first but later rotating slowly, thus causing changes in one record to lag behind similar changes in the other. This rotation effect was not found in the records made by Appleton and Ratcliffe[9] with similar antennas. The intensity variations shown by their simultaneous records were so largely in phase that most of the fading could be explained by intensity variations occurring at some distance from the receiving antennas.

Short Period Fading. The rapid periodic fading previously mentioned and illustrated in Figs. 9 and 16 should have some significance since there is a considerable degree of regularity in its appearance, at least in reception from the two stations upon which observations have been most consistently made. During the cooperative investigation of fading conducted by the Bureau of Standards in 1925 this same phenomenon was noted,[10] though less consistently, in observations made at several different localities on transmissions from WGY (790 kc), Schenectady, N. Y. In the earlier case the phenomenon occurred for a few days at the spring equinox and was not found again. The newer cases were first noted at the same season, but have continued for several months with very few exceptions whenever records were made. On the WJZ records this type of fading starts from 15 to 30 minutes after the Washington sunset and lasts in the vicinity of a half hour. On the WBAL records, which were made much less frequently, the start is likely to be practically at sunset, though there is some variation both ways.

The explanation of this phenomenon suggested in the earlier paper as an interference effect between ground and reflected rays is now proved unsatisfactory by the fact that this periodic fading is present even when the ground ray is absent. As previously stated, the fact that such fading is out of phase on records made with coils at maximum and minimum positions, respectively, makes any interference explanation difficult and suggests a relatively weak indirect ray with rapid rotation of its plane of polarization. No explanation is given at this time for the rotation itself.

[9] Reference 3, p. 313, and Fig. 10.
[10] Reference 2, pp. 437–439 and Fig. 5, opposite p. 427.

Conclusions

1. Considerable fading is caused by the fact that the indirect ray from a radio broadcasting station undergoes variations of intensity before arriving at the receiving antenna. This is proved by the fact that at times similar intensity changes occur simultaneously in coil antennas at maximum and minimum positions as well as in barrage antenna and coil at minimum.

2. Much evidence of fading caused by interference between ground and indirect rays is found in records of reception from transmitting stations sufficiently near to produce a ground wave at the receiving point. The night-time intensity is often less than the daytime intensity which is due to the constant ground ray. The out-of-phase indirect ray often neutralizes the effect of the ground ray partially and sometimes completely, even for short transmission paths.

3. Direction shifts are apparently necessary to the explanation of phase displacements of less than 180 deg. between otherwise similar records made simultaneously: (1) with coil antenna at maximum and with vertical single-wire antenna; (2) with coil antenna at maximum and with barrage antenna; (3) with barrage and vertical single-wire antennas; (4) with coil antennas at maximum and minimum positions; (5) with barrage and with coil antenna at minimum.

4. No proof of fading caused by fluctuating height of the ionized layer is found, but evidence of refraction of the indirect ray from a rising layer is found in the fact that the signal intensity in the coil antenna at minimum position starts at zero in daylight and gradually increases during sunset.

5. Rotation of plane of polarization of radiation refracted from the upper atmosphere is shown to be the cause of much fading, particularly, though by no means wholly, during the sunset period. The 180 deg. out-of-phase relationship between records made simultaneously with coil antennas in maximum and minimum positions seems to demand this explanation. The cause of such rotation is not explained.

6. Refractions arriving by multiple paths are evidenced in records showing a periodic type of fading superposed on the main intensity variations and yet 180 deg. out of phase on simultaneous records made with coil antennas in maximum and minimum positions. This fact makes the calculation of the effective height of the ionized layer by means of the sine of the angle of incidence, as determined by the method of Appleton and Ratcliffe,[11] somewhat doubtful, especially when measuring on other than distant stations.

[11] Reference 3, pp. 297–305.

7. The rapid periodic type of fading noted in a previous paper as occurring in signals from one station during the sunset period has been found to be quite common in the reception from two other stations. The newer data point toward rotation of plane of polarization rather than interference as the cause.

As stated in the introduction, this investigation is far from complete. It is planned to make similar measurements over a much longer period of time to observe possible seasonal effects; to make simultaneous measurements by more than two receiving systems over a wider range of frequencies; to compare results of transmissions perpendicular to the earth's magnetic field with those parallel to it; to make direction and oscillographic observations simultaneously with the fading records; to make more quantitative measurements of these observations and compare calculated heights of the refracting layer, as determined by the angle of incidence method, on transmissions involving a wide range of distances, in order to note whether there is greater consistency in measurements on the more distant stations.

Acknowledgments

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Acknowledgment is also made of the assistance of S. A. Buckingham, R. P. Battle, and T. R. Gilliland in preparing apparatus and making the large number of records required in this work.

Appendix

Summary of Conclusions of Previous Investigations

Both Pickard, in his 1923 investigation,[12] and Appleton and Ratcliffe in their recent work[13] have given the chief credit for fading to intensity changes undergone in the path of the indirect ray. Pickard ascribes these changes to varying absorption and suggests a possible explanation, while Appleton and Ratcliffe find reason to suspect that interferences between rays travelling by multiple paths are the cause. These authors agree also in crediting interference between ground and indirect rays with a minor role in causing fading; and practically eliminate direction shifts as a factor. Appleton and Barnett,[14] as

[12] Reference 1, pp. 152–158.
[13] Reference 3, pp. 314–315.
[14] E. V. Appleton and M. A. Barnett, "On some direct evidence for downward atmospheric reflection of electric rays," *Proc.* Royal Society, A, **109**, 1925, 621–641. See specially 623 and 635–636.

well as Smith-Rose and Barfield,[15] had previously satisfied themselves that apparent direction shifts were due chiefly to elliptical polarization of the atmospheric ray, and very little, if at all, to actual changes in lateral angle with the great circle plane through transmitting and receiving points. Pickard did not discuss changing angle of incidence nor rotation of the plane of polarization of the downcoming ray in connection with his fading data. Appleton and Ratcliffe were unable to detect any evidence that these factors were affecting fading,[16] in spite of the fact that rotation of the plane of polarization of the incident ray had been accepted as explanation for apparent direction shifts. The Bureau of Standards report[17] on cooperative measurements in 1925, while accepting varying absorption as the more general cause of fading, gave considerable weight to interference effects so far as fading of nearer stations was concerned, but no data upon the other factors were then secured.

While the present paper deals only with observations in the broadcast range of frequencies and with limited methods of analysis it is well to have in mind the conclusions reached by investigations at other frequencies and by other methods. Working particularly with frequencies above 3000 kc Breit and Tuve[18] found fading occurring independently of interference between ground and indirect rays and suggested multiple reflections and flickering effect due to wavy character of surface of the ionized region. Heising[19] as well as Taylor and Young,[20] using an oscillographic method similar to that of Breit and Tuve found proof of a multiplicity of paths of different lengths. Heising found these regularly accompanied by fading and simultaneous with an apparent rising of the ionized layer. Rukop[21] in working with transmission of pictures at high frequency had earlier secured evidences of multiple paths which were accompanied by fading which, however, was not necessarily associated with the multiplicity of paths. The path length changed at times as rapidly as 50 km a second giving the Doppler effect of a change in frequency at the

[15] R. L. Smith-Rose and R. H. Barfield, "The cause and elimination of night errors in radio direction-finding," *Jour.* I. E. E., **64**, 831–837, 1926.

[16] Reference 3, pp. 312–315.

[17] Reference 2, pp. 447–448.

[18] G. Breit and M. A. Tuve, "A test of the existence of the conducting layer," *Phys. Rev.*, **28**, 554–575, September, 1926. See especially p. 564.

[19] R. A. Heising, "Experiments and observations concerning the ionized regions of the atmosphere," Proc. I. R. E., **16**, 75; January, 1928.

[20] A. Hoyt Taylor and L. C. Young, "Studies of high-frequency radio wave propagation," Proc. I. R. E., **16**, 561; May, 1928.

[21] Von H. Rukop, "Die Bildtelegraphie als Untersuchungsmethode für die Ausbreitung der kurzen Wellen," *Elektrische Nachrichten-Technik*, **3**, 316–318, 1926.

receiving system. From theoretical considerations Pedersen[22] concluded that there should be an infinite number of downcoming rays at a given point on the earth's surface.

Fading produced by interference was found by Bown, Martin and Potter[23] in an oscillographic study of the distortion in broadcast reception within a given area, and oscillographic observations by Friis[24] of reception on 18.75 megacycles proved consistent with the belief that fading is mainly caused by wave interference.

Very good evidence of real direction shifts was found by Friis[24] using a cathode-ray oscillograph to observe the phase difference of signal waves as received by antenna systems located a fraction of a wavelength apart.

Changes in angle of incidence were found by Friis[24] to be much more frequent and of greater magnitude than direction shifts of the received signals. Heising, by his observations of signal retardation, had previously computed changes in angle of incidence sufficient to indicate rapid variations in the height of the ionized layer, sometimes as great as 72 km (45 miles) in two minutes.

Rotation of the plane of polarization of the indirect ray was suggested from theoretical considerations of the earth's magnetic field and varying ionization in the upper atmosphere by Appleton and Barnett[25] and by Nichols and Schelleng,[26] who indicated that this phenomenon might be expected in transmissions largely along the line of a magnetic meridian. Evidences of abnormal polarization causing apparent direction shifts were pointed out by Eckersley,[27] and Pickard[28] found by many observations made on high frequencies with an analyzer that the plane of polarization under certain conditions rotated 90 deg. in transit. Alexanderson's investigation,[29] also on the high frequencies, seemed to indicate a continuous rotation

[22] P. O. Pedersen, "The propagation of radio waves along the surface of the earth and in the atmosphere," p. 230. Copenhagen, 1927.

[23] R. Bown, De L. K. Martin, and R. K. Potter, "Some studies in radio broadcast transmission," Proc. I. R. E., 14, 57; February, 1926.

[24] H. T. Friis, "Oscillographic observations on the direction of propagation and fading of short waves," Proc. I. R. E., 16, 658; May, 1928.

[25] E. V. Appleton and M. A. F. Barnett, "Wireless wave propagation," *Electrician*; **94**, 398, April 3, 1925.

[26] H. W. Nichols and J. C. Schelleng, "The propagation of radio waves over the earth," *Nature*, **115**, 334; March 7, 1925. Also *Bell Sys. Tech. Jour.*, **IV**, 215–234; April, 1925.

[27] T. L. Eckersley, "The effect of the Heaviside layer on the apparent direction of electromagnetic waves," *Radio Review*, **2**, 60–65, and 231–248; February and May, 1921.

[28] G. W. Pickard, "The polarization of radio waves," Proc. I. R. E., **14**, 205–212; April, 1926.

[29] E. F. W. Alexanderson, "Polarization of radio waves," *Jour.* A. I. E. E., 45, 636–640; July, 1926.

of the plane of polarization with alternate points of plane and circular polarization along the transmission path. Hollingworth[30] recently made measurements to determine the state of polarization of the indirect ray which produces apparent progressive direction shifts during the sunset period, and on the basis of several assumptions calculated that the plane of polarization of an indirect ray received from a selected transmission of 21 kc (14350 m) rotated 90 deg. during sunset and remained thus abnormally polarized during the night. Eckersley[7] working with the high frequencies, made simultaneous fading records with horizontal and vertical antennas, and reports that under some conditions the fading changes in the two records were so nearly opposite in phase as to indicate rotation of the plane of polarization of the indirect ray.

[30] J. Hollingworth, "The polarisation of radio waves," *Proc.* Royal Society, A, 119, 444–464, 1928.

Proceedings of the Institute of Radio Engineers
Volume 17, Number 6 *June, 1929*

Discussion on
DETECTION CHARACTERISTICS OF THREE-ELEMENT VACUUM TUBES*

By
F. E. Terman and T. M. Googin

J. C. Warner:[1] The detection characteristics given in this interesting paper have been derived mainly from the static characteristics of the various tubes. This method is accurate when applied to the ordinary type of three-element tube, but in the alkali vapor tube (UX200A) the effect of the positive ion movements, which greatly influence the detector action, is not indicated by the static curves. The mechanism of rectification which takes place in the alkali vapor tube depends upon the amplitude and frequency of the impressed signal voltage. If the frequency is low, for example, 60 cycles, the rectification can be predicted from an examination of the static characteristics. But if a small radio-frequency voltage is applied, of the order of 50 millivolts, the behavior of the tube is quite different. Rectification takes place in such a way as to decrease the plate current even when no grid leak and condenser are used. The static mutual characteristic would, of course, indicate just the opposite. Also, the grid leak and condenser have little effect on the magnitude of detection, and the chief function of the grid leak seems to be in fixing a proper bias.

As the signal voltage is increased the positive ion effects reach saturation and the action then becomes much the same as in the ordinary tube. The conclusions of the paper are therefore justifiable for relatively strong signals but do not apply to the region where the UX200A shows its greatest relative sensitivity in comparison with other tubes, i.e., on weak signals.

F. E. Terman:[2] Mr. Warner is correct in his inference that the results of our paper depend upon static curves. The question with gas tubes is then as to the extent to which the detection in these tubes departs from that given by the static characteristic. I have made some rough measurements at 50 kc which indicate that on an unmodulated voltage at least 50 per cent to 75 per cent of the detector output can be determined by the static characteristic. Just what would happen at 1000 kc I do not know. It will be remembered that the gas tube is used on no commercial sets and, in fact, the Radiola battery sets have always used the 201A tube as a detector in preference to the 200A. In view of these facts I feel that our statement in regard to the 200A tube was not quantitatively justified by the data we had at hand, but that qualitatively it is not very much in error. The unimportance of grid-leak resistance in detection with the 200A is not significant as our static curves indicate that over the entire range of grid leak resistance ordinarily used the detection is equally sensitive.

* Proc. I. R. E., 17, 149; January, 1929.
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Proceedings of the Institute of Radio Engineers
Volume 17, Number 6 *June, 1929*

MONTHLY LIST OF REFERENCES TO CURRENT RADIO LITERATURE

THIS is a monthly list of references prepared by the Bureau of Standards and is intended to cover the more important papers of interest to professional radio engineers which have recently appeared in periodicals, books, etc. The number at the left of each reference classifies the reference by subject, in accordance with the scheme presented in "A Decimal Classification of Radio Subjects—An Extension of the Dewey System," Bureau of Standards Circular No. 138, a copy of which may be obtained for 10 cents from the Superintendent of Documents, Government Printing Office, Washington, D. C. The various articles listed below are not obtainable from the Government. The periodicals can be secured from their publishers and can be consulted at large public libraries.

R100. Radio Principles

R113 Störmer, C. Sur un echo d'ondes electromagnetiques courtes arrivant plusieurs seconds apres le signal emis et son explication d'apres la theorie des Auroras Borealis. (On an echo of short electromagnetic waves arriving several seconds after the main signal and explanation by means of the theory of the Aurora Borealis). *L'Onde Electrique,* **7,** pp. 531–33; Dec., 1928.

(Same work reported in abstract in *Comptes Rendus,* **187,** p. 811, Nov. 5, 1921, and in *Nature* (London), Nov. 3, 1921).

R113 Van der Pol, B. Les echos des ondes courtes et les Auroras Borealis. (Short wave echoes and the Auroras Borealis). *L'Onde Electrique,* **7,** pp. 534–37; Dec., 1928.

(Does not agree with Störmer's explanation since rays will never pass through the upper region of the Heaviside layer, but where short waves pass into the layer at a large angle they may go into electron densities where the phase velocity becomes almost infinity, that is, the group velocity almost zero. The reflected wave can then come back to earth after a considerable time has elapsed).

R113 Jelstrup, H. S. J. Essai d'explication de l'echo Störmer-Hals sur les ondes de 31.4 metres de PCJJ. (Analysis of the explanation of the echo of Störmer-Hals on waves of 31.4 meters from PCJJ). *L'Onde Electrique,* **7,** pp. 538–40; Dec., 1928.

(Attributes the long interval echoes to many reflections between the Heaviside layer and the earth and calculates the time lapse between the original signal and the echo by means of Howe's equivalent transmission line formula for the Heaviside layer).

R113 Clapp, J. K. Some experiments in short distance short wave radio transmission. Proc. I. R. E., **17,** pp. 479–93; March, 1929.

(Some experiments in short-wave radio transmission over a distance of 55 miles are described).

R113 Fassbender, H. and Kurlbaum, G. Abhängigkeit der Reichweite sehr kurzer Wellen von der Höhe des Senders über der Erde. (Dependence of the range of very short waves on the height of the transmitter above the earth). *Zeits. für Hochfrequenztechnik*, **33**, pp. 52–55; Feb., 1929.

(Experiments on range of transmission on wavelength of 3.7 m. This work was carried out on an airplane, the transmitter being on the plane and the receiver on the ground.)

R113 Gerth, F. and Scheppman, W. Untersuchungen über die Ausbreitungsvorgänge ultra-kurzer Wellen. (Experiments on the radiation of ultra-short waves). *Zeits. für Hochfrequenztechnik*, **33**, pp. 23–27; January, 1929.

(An investigation of radiation for waves shorter than 28 megacycles. It is shown that the waves spread out like light and that objects larger than a wavelength produce shadow effects. It is assumed that only direct waves play a part and calculations are given for the distance range).

R113.4 Guyot. Étude sur la propagation des ondes courtes. (Study of the propagation of short waves). *L'Onde Electrique*, **7**, pp. 509–530; Dec., 1928.

(Review of the propagation of short waves assuming that the Heaviside layer is defined by the position of the earth's surface with respect to the sun. Based on this, the height of the layer depends on the time during the 24 hours, the latitude of the station and the declination of the sun. Experimental curves are given based on simplified formulas for the calculation of the height of the layer).

R113.5 Maris, H. B. and Hulburt, E. O. Wireless telegraphy and magnetic storms. Proc. I. R. E., **17**, pp. 494–500; March, 1929.

(A recent theory of auroras and magnetic storms attributes these phenomena to the action of a flash of ultraviolet light from the sun; this flash causing an unusual ionization in the Kennelly-Heaviside layer. This theory is found to be borne out in a detailed discussion of data of high-frequency (7500 to 20000 kc) circuits of the U. S. Navy during the magnetic storms of May 28, July 7, Oct. 18 and Oct. 24, 1928).

R125.1 Smith-Rose, R. L. Radio direction finding by transmission and reception (with particular reference to its application to marine navigation). Proc. I. R. E., **17**, pp. 425–478; March, 1929.

(A resumé of the performance of apparatus employed for radio direction finding determination either by transmission or by reception is given. Investigations made in England on this subject and results obtained are reviewed. Application of direction finding to navigation and possible effect of coastal and night errors in connection therewith are discussed).

R125.6 Pistolkors, A. A. The radiation resistance of beam antennas. Proc. I. R. E., **17**, pp. 562–79; March, 1929.

(A method proposed by Brillouin for the calculation of radiation resistance applied to several types of beam antennas is described.)

R130 Kingdon, K. H. and Mott-Smith, H. M. The operation of radio receiving tube filaments on alternating currents. *General Electric Rev.*, **32**, pp. 139–148; March, 1929.

(It is shown that three disturbances may occur in amplifiers using a-c feed for the filament supply, namely, the primary potential ripple, the primary magnetic ripple and the primary temperature ripple. In this part of the paper (part I) the case of the amplifiers is treated and methods given for reducing these effects.)

R132 Brain, B. C. Output characteristics of thermionic amplifiers. *Experimental Wireless and W. Engr.* (London, **6**, pp. 119–127; March, 1929.

(Analytical treatment of the output amplifier for optimum undistorted energy transfer. It is shown that the maximum output is obtained by making the load resistance about 1.6 times the a-c resistance of the tube.)

R133 Lazaref, W. Über die Instabilität der Frequenz von Röhren generatoren und deren Stabilisierung. (On the instability of the frequency of electron tube generators and their stabilization). *Zeits. für Hochfrequenztechnik*, **33**, pp. 55–63; Feb., 1929.

(The effect of filament excitation, "B" voltage and feedback on the frequency is studied. The cause is due to the grid current which increases the decrement).

R133 Hollmann, H. E. Zusammenfassender Bericht: Die Erzeugung kürzester elektrischer Wellen mit Elektronenrohren. (Compilation on the production of ultra short waves). *Zeits. für Hochfrequenztechnik*, **33**, pp. 27–30, January; pp. 66–74, Feb., 1929.

(Lists the methods for the production of ultra short waves).

R134 Nelson, J. R. Notes on grid circuit detection. Proc. I. R. E., **17**, pp. 551–561; March, 1929.

(Shows that the value of the second derivative of grid current with respect to grid voltage the main term in grid rectification, may be found by the change in direct current in the grid circuit with any desired value of input voltage. An experimentally determined curve is given showing the detector frequency distortion of a commercial set for a 2-megohm grid leak and for a ½-megohm grid leak.)

R134.7 Aigner, F. Das Problem der ökonomischten Vielfachtransponierung. (The problem of economic multi-heterodyning). *Zeits. für Hochfrequenztechnik*, **33**, pp. 9–15, January; pp. 47–52, February, 1929.

(Methods are investigated by means of which the incoming frequency can be changed with minimum number of local oscillators. Two heterodyne oscillators suffice to produce almost any multiple frequency changes.)

R149 Kryter, R. J. Alternating current rectification as applied to radio—Part I. *QST*, **13**, pp. 33–37; Apr., 1929.

(Discussion of tube and chemical rectifiers.)

R200. Radio Measurements and Standardization

R230 Harris, S. An extension of the method of measuring inductances and capacities. Proc. I. R. E., **17**, pp. 516–20; March, 1929.

(Substitution method commonly employed for measuring small capacities is here shown to be a special case of a more general principle.)

R300. Radio Apparatus and Equipment

R334 Pike, O. W. and Spitzer, E. E. A new low power screen grid transmitting tube—UX 865. *QST*, **13**, pp. 43–45; April, 1929.

(Uses of tube as amplifier and characteristics of tube given.)

R343 Beers, G. L. and Carlson, W. L. Recent developments in superheterodyne receivers. Proc. I. R. E., **17**, pp. 501–515; March, 1929.

(Major electrical elements of a modern superheterodyne receiver discussed briefly, and practical volume control is described.)

R357 Freese, H. Beseitigung der Nebenfrequenzen beim statischen Frequenzwandler. (Elimination of side frequencies for the static frequency changer). *Zeits. für Hochfrequenztechnik*, **33**, pp. 1–8, January; pp. 41–46, Feb., 1929.

(The absorption system due to Lorenz (British patent No. 263825) and that due to Zenneck is investigated. As analyser, the cathode-ray oscillograph or an undamped wave meter was used.)

R376.3 Kyle, C. The Kyle condenser reproducer. *Radio Engineering*, **9**, pp. 26–29; March, 1929.

(Practical and theoretical discussion of an electrostatic speaker of interesting design.)

R376.3 Hector, L. G. and Kozanowski, H. N. Apparent equality of loudspeaker output at various frequencies. Proc. I. R. E., **17**, pp. 521–535; March, 1929.

(Describes a type of alternation phonometer which permits rapid switching of power at two frequencies to the same loud speaker without the distracting effects of the transients that would result from ordinary types of commutation. This result is obtained by the use of rotating condensers to provide variable capacitive reactance in the input circuit of the power amplifier that operates the speaker.)

R381 Coursey, P. R. On the capacity of dry electrolytic condensers. *Experimental Wireless and Wireless Engr.* (London), **6**, pp. 128–132; March, 1929.

(Gives a description of the wet and semi-wet (so called dry) electrolytic condensers and emphasizes that especially for the latter type in alternating current method (not vallistic method) has to be used for measuring the effective capacity.)

R386 Reed, F. Superheterodyne band pass filters. Radio Listeners' Guide and Call Book, pp. 45–47, Spring, 1929.

(Design data for band pass filters.)

R400. Radio Communication Systems

R412 Hull, R. A. Modern practice in high frequency radio telephony. *QST*, **13**, pp. 8–22; April, 1929.

(Discussion of improved method in amateur phone transmission)

R431 Tanner, R. Wm. Reducing noise in broadcast receivers. *Radio Engineering*, **9**, pp. 24–25; March, 1929.

(Use of resonance wave coil, band pass filters and power detector tube.)

R500. Applications of Radio

R520 Marriott, R. H. Electrical aids to navigation (abridgment). *Jour. A. I. E. E.*, **48**, pp. 195–199; March, 1929.

(Electrical and magnetic aids to navigation by water and air are outlined. Discussions on the earth and inductor compass, radio compass, radio beacon, submarine signals, height indicators for aircraft, etc. are given. A bibliography is included.)

R526.1 Dellinger, J. H. Directional radio. *Aeronautical World*, **2**, pp. 20–22; Feb., 1929.

(Description of Bureau of Standards work on radio beacon).

R582 Zworykin, V. Facsimile picture transmission. Proc. I. R. E., **17**, pp. 536–550; March, 1929.

(A facsimile picture transmitting system is described whose chief object is to produce a simple rugged apparatus for practical usage without requiring the attention of a skilled operator).

R800. Non-Radio Subjects

534 Does a vibrating diaphragm carry a mass of air with it? (editorial). *Exp. Wireless and W. Engr.* (London), 6, pp. 117–118; March, 1929.

(Since dealing mostly with divergent waves the velocity of air particles lags behind the pressure. Hence the force at the surface of a spherical surface will be out of phase with its velocity just as though the diaphragm had a certain mass or as if a mass of air were attached to it.)

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* Paper published in April, 1929 PROCEEDINGS.

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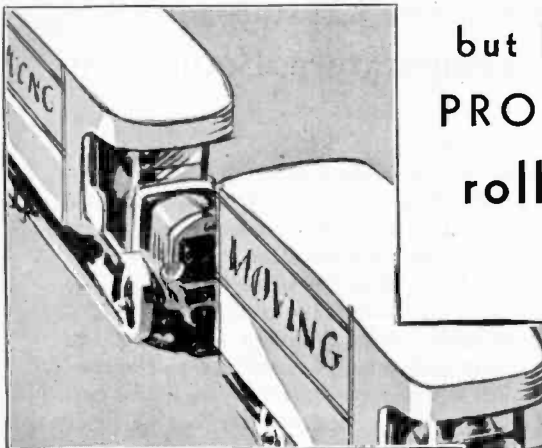

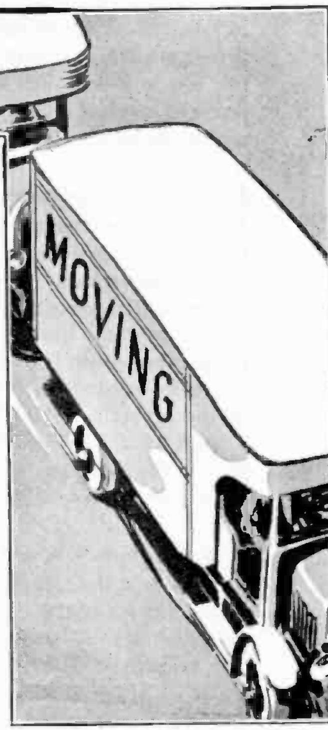

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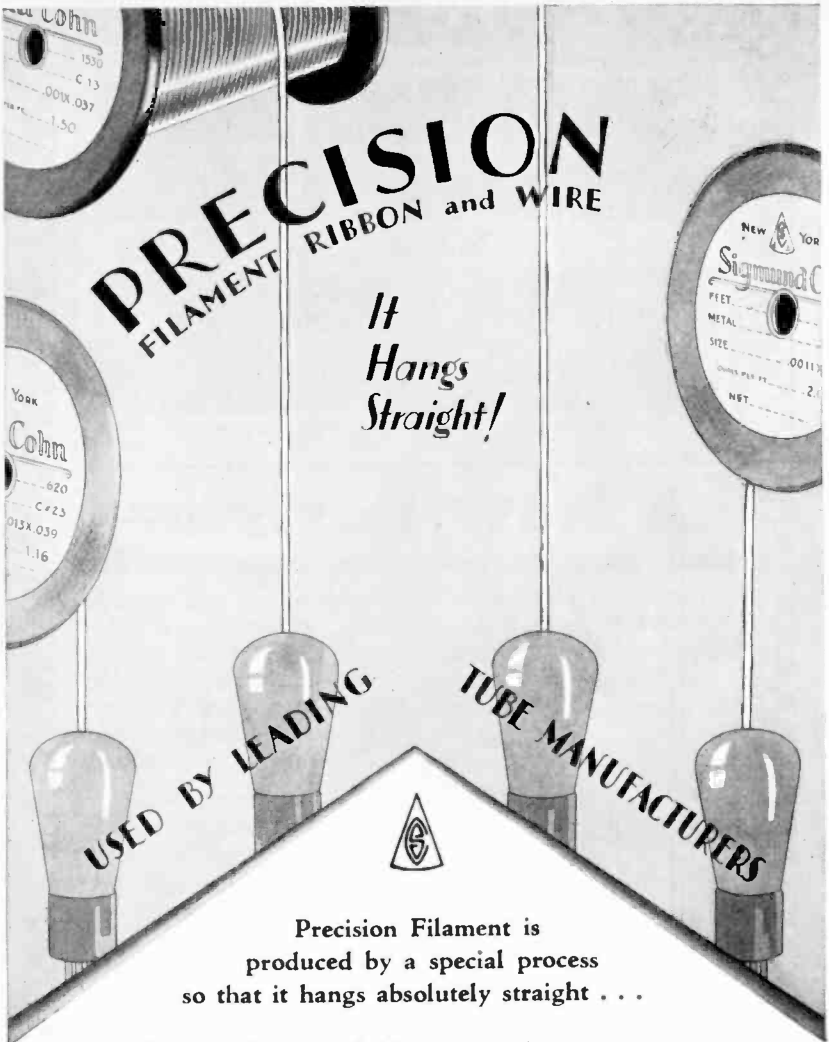

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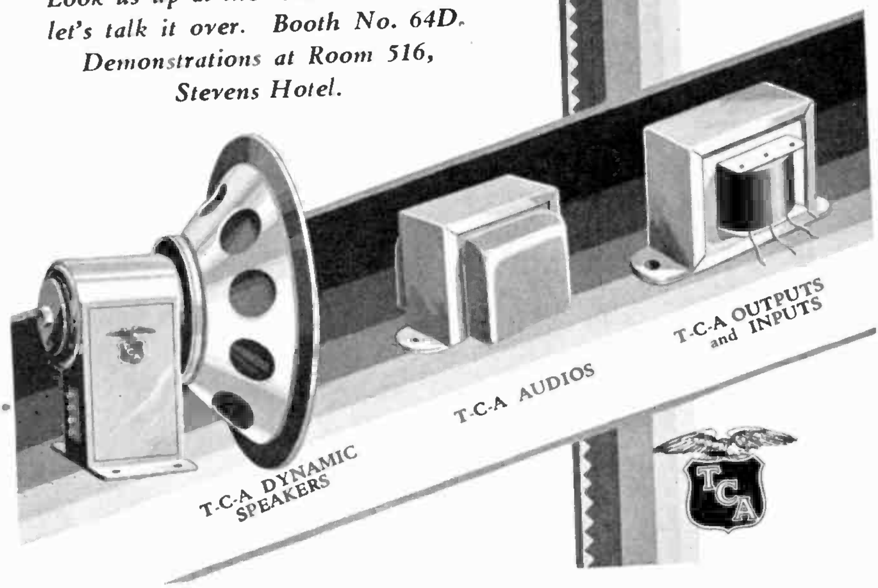

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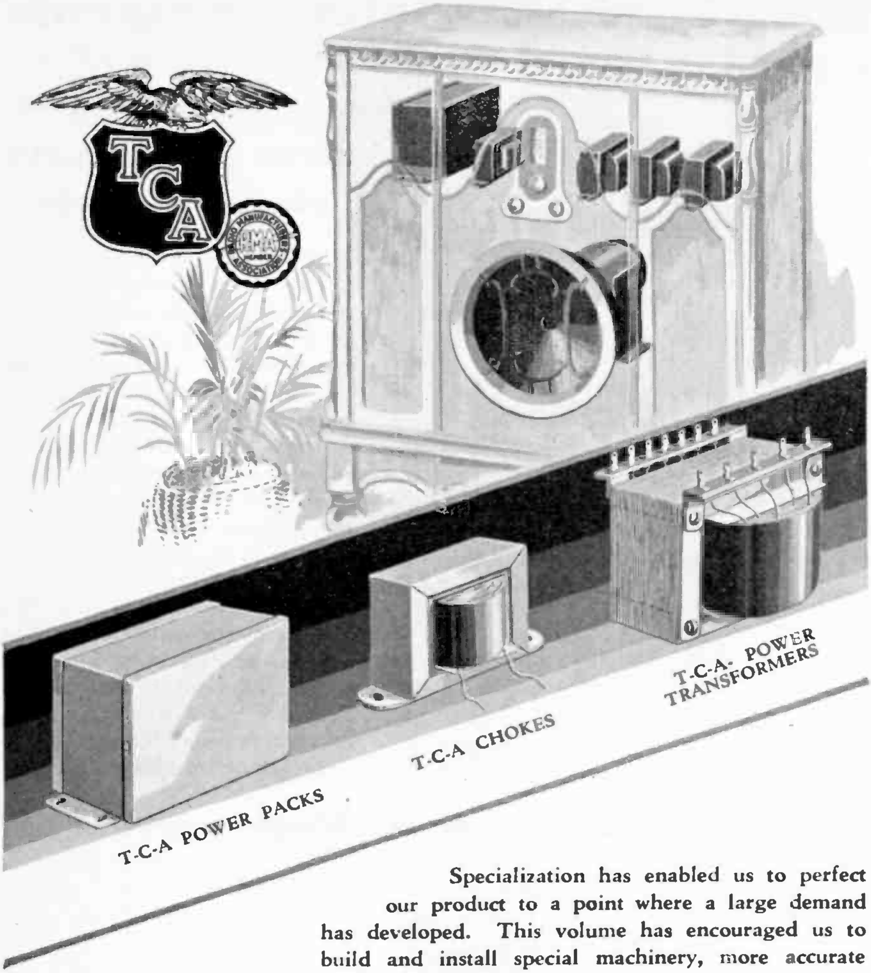
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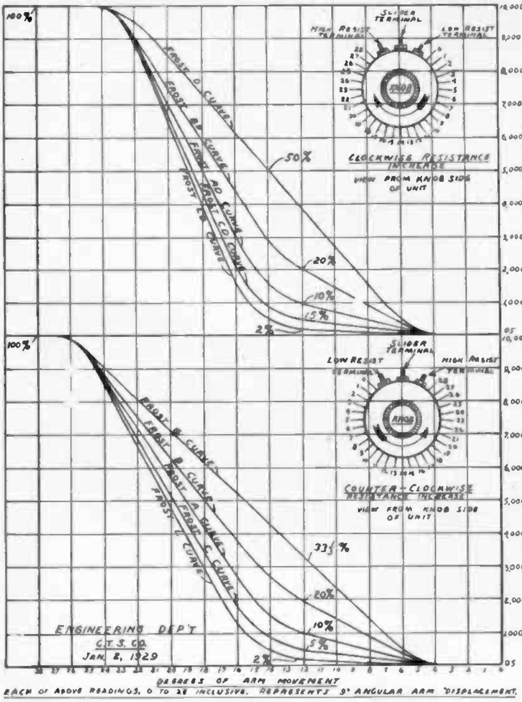

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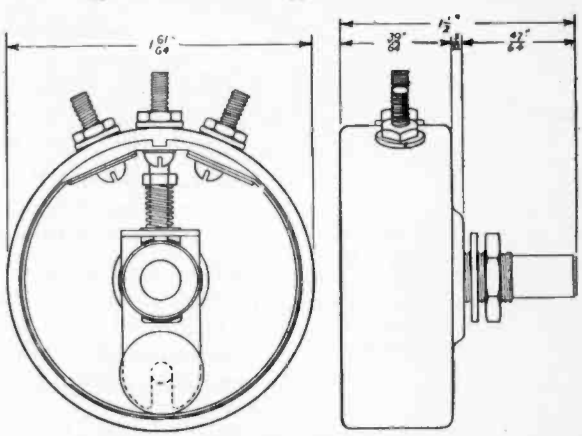
1 7/8 in.	43/64 in.	Metal
1 7/8 in.	3/4 in.	Metal
1 5/8 in.	37/64 in.	Bakelite
1 1/2 in.	43/64 in.	Metal

For other than standard units, please indicate in your request for samples any special dimensions, terminal positions, curve desired, and watt load, and in requesting tandem unit samples state maximum permissible mounting depth. Please note that bushings, threads per inch, shaft lengths and diameters illustrated are standard. Made in two or three terminal type.



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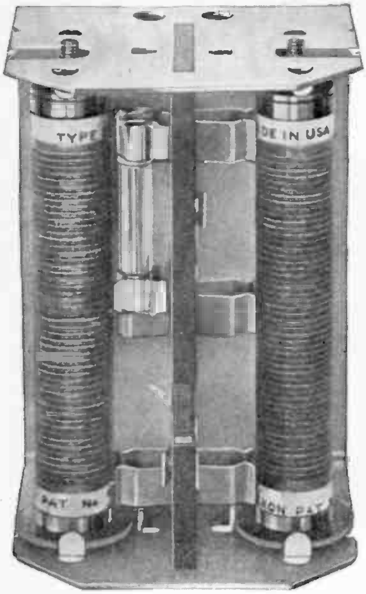

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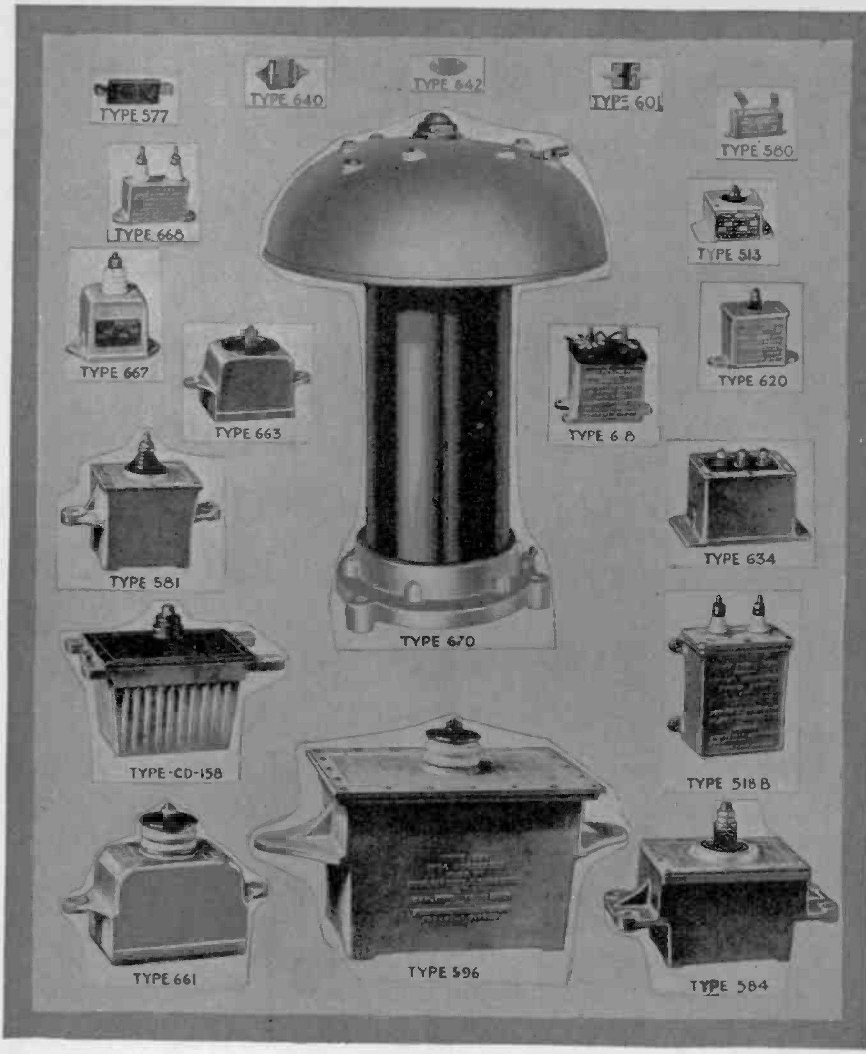

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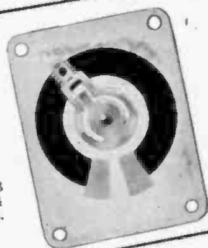

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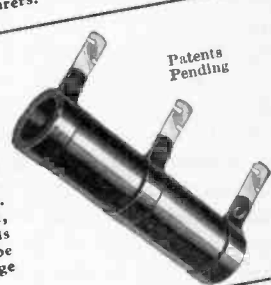

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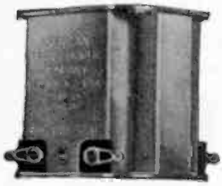

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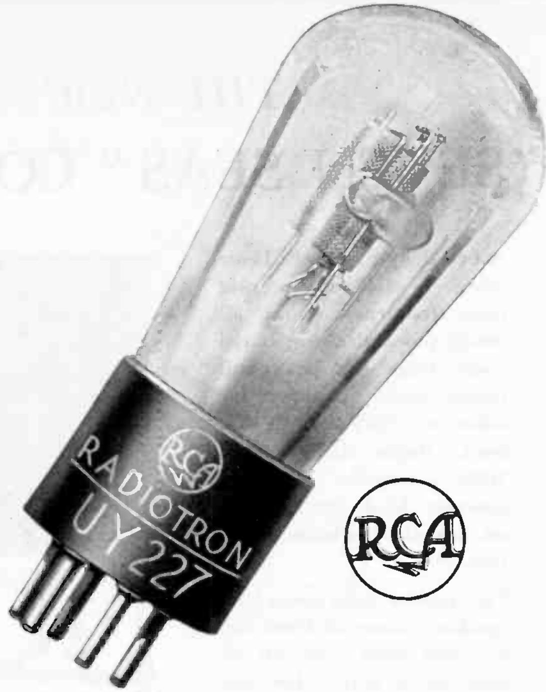
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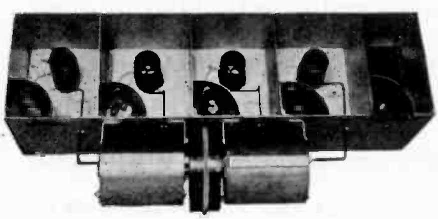

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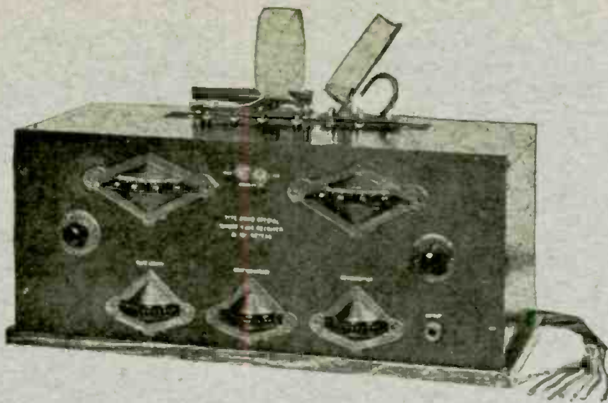

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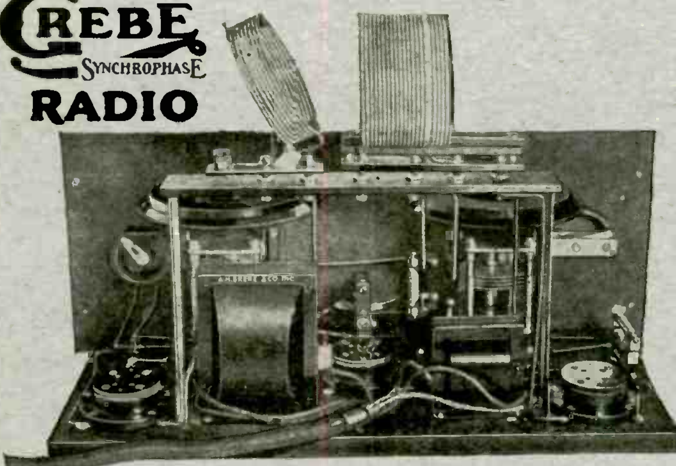

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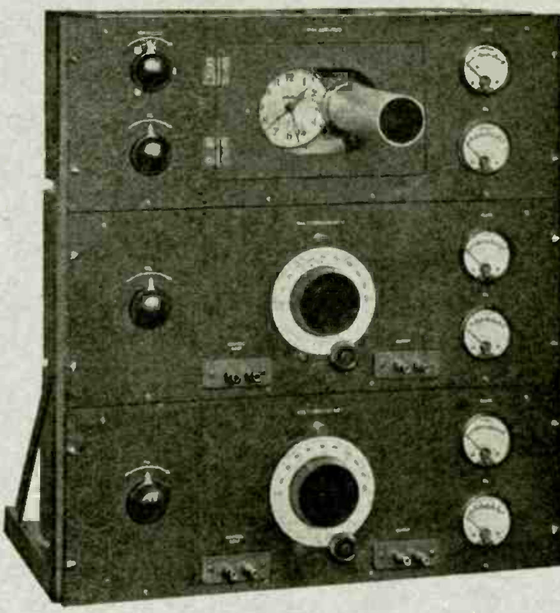

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