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TABLE OF CONTENTS

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ALFRED N. GOLDSMITH, Ph.D.

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THE TABLE OF CONTENTS FOLLOWS ON PAGE 61

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CONTENTS

The Officers and the Committees of the Institute for 1918 will appear in a forthcoming issue.

OSCILLATING AUDION CIRCUITS*

By

L. A. Hazeltine

(Assistant Professor of Electrical Engineering, Stevens Institute of Technology, Hoboken, New Jersey)

1. Introduction and General Principles.—The purposes of this paper are to explain the general principles upon which depend the use of the audion for generating electrical oscillations and for amplifying externally impressed oscillations by "regenerative action," and to discuss in detail the action in certain circuits.[1] It will be shown that the criterion for the generation of an oscillation and for the measure of its intensity is directly determinable from the constants of the circuit and from the characteristics of the audion.

The term "audion," in accordance with common usage, is applied to an evacuated bulb having three electrodes: a *filament*, maintained at incandescence by a heating current; a *plate* or wing; and a *grid* interposed between the filament and the plate. These three electrodes are electrically connected to an external circuit, which includes a battery so arranged as to make the filament a cathode (negative) and the plate an anode (positive) for a thermionic discharge thru the bulb. The grid may be considered an auxiliary anode whose purpose is to control the main discharge to the plate by variations in its potential relative to the filament. The device may thus be properly described as a *thermionic relay*.

The physical action in the audion is briefly as follows: The incandescent filament gives off electrons (negatively charged particles) at a rate depending on its temperature. If no voltage

*Paper submitted for presentation before The Institute of Radio Engineers, September 5, 1917. Received by the Editor April 15, 1917.

[1]The author takes this opportunity to acknowledge his great indebtedness to Mr. E. H. Armstrong who has done the pioneer work with the oscillating audion. The fundamental principles and various circuits in which they are practically applied are given in Mr. Armstrong's article in the "Electrical World" for December, 1914 (volume 64, number 24, page 1149) and in his paper presented before The Institute of Radio Engineers in March, 1915 ("Proc. I. R. E.", volume 3, page 215).

[2]Langmuir, "Proc. I. R. E.", volume 3, page 261 (1915). This paper goes fully into the physical action of thermionic conduction and its application.

were present to draw away these electrons, they would be driven back to the filament by the negative potential which they produce in the surrounding space. However, the positive potential of the plate will attract the electrons and cause a continuous stream to flow from the filament to the plate, constituting a "thermionic current." Since the grid lies in the path of the electrons, its potential will have even more effect on the electron stream than will that of the plate, a positive grid potential increasing the plate current, a negative grid potential decreasing it. In bulbs having a high vacuum the relation of the plate current to the plate and grid potentials follows simple and definite laws;[2] but in ordinary bulbs, the residual gas molecules are ionized by the impact of the electrons and have an important effect on the current, causing a wide variation in the characteristics of bulbs of identical form but of accidental difference in the kind and amount of residual gas.

Independently of its use as a relay, the audion also serves as a *rectifier*, which is essential to its use as a radio detector. This rectifying property affects the oscillations only indirectly, and will not be particularly considered here.

The relay action of the audion is illustrated by its *characteristic curve*, *M N*, Figure 1, showing the relation between the plate current and the potential of the grid relative to the filament. An audion has one such characteristic curve for every combination of filament temperature and plate potential relative to the filament. The effectiveness of the audion as a relay depends primarily on the *slope* of the characteristic curve. This slope, being the quotient of a current by a voltage associated therewith, is of the dimensions of a conductance and may be called the *mutual conductance* of the grid toward the plate. For any small change in grid potential, the effective mutual conductance is the slope of the tangent to the characteristic curve at the point corresponding to the actual grid potential; while if the grid potential is varying periodically, the effective mutual conductance may be considered the slope of the secant line connecting the points corresponding to highest and lowest grid potential.[3] Evidently the mutual conductance has a maximum

[3]A more accurate expression for the effective mutual conductance in this case may be found by assuming the variations in grid potential to be sinusoidal and plotting the corresponding wave of plate current against time. The quotient of the fundamental harmonic component of the plate current by the grid potential is then the mutual conductance. This refinement hardly seems necessary in any practical case, especially as the "dynamic characteristic" of the audion may differ from the "static characteristic" and may be difficult to determine.

at the point of inflection P, Figure 1, and in general decreases with increasing amplitude of oscillation.

In the following discussion we will deal only with the *variations* in the plate current, grid potential, etc., from their mean values during the oscillation. In other words, the pulsating currents and voltages will each be resolved into a continuous

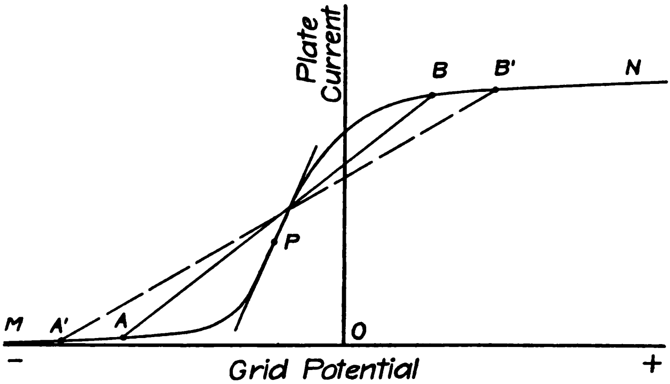


FIGURE 1

part and an alternating (or oscillating) part; and the latter will be treated by the usual methods of the alternating-current circuit. For simplicity, also, the typical circuit diagrams will show only the elements essential to the oscillating current, omitting the heating battery, plate-circuit battery, stopping condenser, telephone receivers, etc. The oscillating voltage tending to send current *from* the filament *to* the grid will, for brevity, be called the "grid voltage"; the oscillating voltage tending to send current from the plate to the filament will be called the "plate voltage"; and the oscillating current from the plate will be called the "plate current." These quantities, in their positive senses, are represented in Figure 2, their virtual (or r. m. s.) values being denoted by E_g, E_p and I_p respectively.

If at any instant of time the grid voltage is such as to make the grid negative with respect to the filament, current will tend to flow from the filament to the grid, as represented by the dotted arrow in Figure 2. But, by virtue of the relay action, current is caused to flow from the filament to the *plate*, as represented by the full arrow, and as given by the equation,

$$I_p = E_g g, \qquad (1)$$

where g is the mutual conductance of the grid toward the plate. If at the chosen instant the plate is positive with respect to the filament, there will be an *output* of power from the audion, equal to

$$P = E_p I_p = E_p E_g g. \qquad (2)$$

If the audion is to act as an oscillating-current generator, the required value of g is then given by the relation,

$$g = \frac{P}{E_g E_p}, \qquad (3)$$

where P represents the oscillating-current power supplied by the audion. The criterion for the generation of oscillating currents thru the relay action of the audion is thus as follows:

The audion must be so connected to the oscillating-current

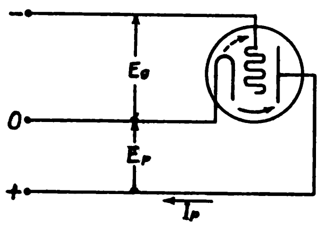


FIGURE 2

circuit that the grid and the plate are of opposite polarities with respect to the filament; and the quotient of the total oscillating-current power by the product of the grid voltage and the plate voltage (or, if these voltages are not in phase, the product of one by the in-phase component of the other) must be equal to a possible value of the mutual conductance of the audion. The smaller this quotient $\frac{P}{E_g E_p}$, the more easily will the audion oscillate, or—with a given setting of the audion—the stronger will be the oscillation[4]. If the quotient $\frac{P}{E_g E_p}$ is greater than the value of g obtainable from the audion, no free oscillation will occur; but

[4]An oscillation will be stable only if the characteristic curve is such that a greater range in grid potential corresponds to a smaller slope of the secant line (i. e., $A'B'$, Figure 1, has a smaller slope than AB). Otherwise the oscillation will increase until this condition obtains. This effect is analogous to the stability or instability of a self-excited generator, which also depends on the slope of a characteristic curve.

an externally impressed oscillation which causes the grid and plate to be of opposite polarities may be greatly amplified; this has been called by Armstrong "regenerative action."

If an externally impressed oscillation causes the grid and plate to be of the *same* polarity, its energy will be *absorbed* by the audion and it will be weakened or more rapidly damped out. The audion may thus be used to prevent or diminish undesirable oscillations.

The above discussion neglects two features which may affect the oscillations:

First, the plate current depends somewhat on the plate voltage as well as on the grid voltage, and so should be represented by ($I_p = E_g g - E_p g_p$), instead of (1). This means that in the denominator of (3), and all equations derived therefrom, we should strictly introduce the factor $\left(1 - \frac{E_p g_p}{E_g g}\right)$; but as the ratio $\frac{g_p}{g}$ is small in most designs of audion[5], this factor may usually be neglected unless $\frac{E_p}{E_g}$ is large.

Second, there will be a current in the grid circuit, due to electrons or positive ions reaching the grid. This current is usually small, and may always be neglected in high-vacuum audions when the grid is negative. In audions having considerable positive ionization, the grid current may assist in producing oscillations[6].

Equation (3) shows the plate potential and grid potential to be interchangeable; so these electrodes may be interchanged in their connection to an oscillating-current circuit. Whether connections should be made so that E_g is greater than E_p or the reverse depends on the following considerations: The value of g for given adjustments of the audion depends on E_g, being smaller

[5]Langmuir, "Proc. I. R. E.", volume 3, page 279.

[6]This action is due to the fact that within a certain range, the grid current *decreases* with increasing grid potential. The grid circuit thus has a characteristic curve like that of the electric arc and, like the arc, may be used to produce oscillations. The following explanation for the decrease in grid current with increase in grid potential (this being negative) is offered: When the potential of the grid is started from zero and made more and more negative, it will at first cause more positive ions to flow to it and so increase the current; but as the flow of electrons from the filament is diminished by the negative potential of the grid, fewer gas molecules will be ionized and hence the number of positive ions reaching the grid will ultimately fall and decrease the grid current.

Positive ionization may also cause an oscillation in the plate circuit, due to alternate breakdown and recovery of the gas. This may occur in periods of several seconds or at audio frequencies.

for higher values of E_g. Hence if the highest possible value of E_g is desired, for a given energy of oscillation (as in radio receiving), E_g should be made large relative to E_p; while if the greatest possible energy of oscillation is desired (as in radio sending), E_g should be made small relative to E_p. A limitation in the latter case is imposed by the effect of E_p on the plate current, as explained above.

The derivation of the equations of oscillation in particular cases is given in the articles immediately following. In most radio-frequency oscillating-current circuits the resistances of the branches are so small in comparison with the reactances that only the latter need be considered in finding the current distribution. For a similar reason the plate current of the audion may be neglected in comparison with the main oscillating currents. With these assumptions, all currents will be in phase with each other and may be calculated as in a direct-current circuit. The current of each branch may then be squared and multiplied by the resistance of that branch to give the power loss; and the total power loss may be substituted in equation (3): this is here called the "loss method." In certain cases where the resistances are high or the coupling between circuits very loose, or where high accuracy is desired, both resistances and reactances must be considered together. This is best done by the use of complex quantities, and is here called the "complex method" (Article 4).

2. Simple Oscillating-Current Circuits Treated By the Loss Method.—An elementary oscillating-current circuit contains a single coil connected to a single condenser, as represented by (C, L), Figure 3a. Any accidental disturbance in this circuit will set up a free oscillation whose angular frequency ω is such as to make the joint reactance zero:

$$\omega L - \frac{1}{\omega C} = 0, \tag{4}$$

or

$$\frac{1}{\omega^2} = CL. \tag{5}$$

If the loss in the effective resistance r were not supplied in some manner, this oscillation would die out according to the familiar exponential law. However, the alternation of the current in L induces a voltage in the grid coil by virtue of the mutual inductance M_g. This causes an alternating current in the plate circuit which flows against the voltage induced in the plate coil (if this is properly connected) and so supplies power to maintain the oscillation. This power is, of course, transferred to the coil L by the mutual inductance M_p.

With the notation indicated in Figure 3a, the grid voltage is

$$E_g = I\,\omega\,M_g; \tag{6}$$

and the plate voltage is

$$E_p = I\,\omega\,M_p. \tag{7}$$

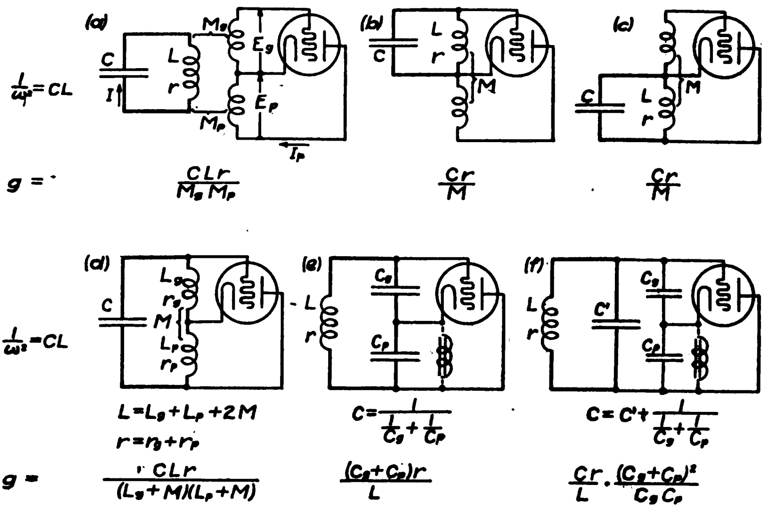


FIGURE 3

The loss in the oscillating-current circuit is

$$P = I^2 r. \tag{8}$$

Substituting in (3), the value of g necessary to maintain an oscillation is

$$g = \frac{P}{E_g E_p} = \frac{I^2 r}{I\,\omega\,M_g\,.\,I\,\omega\,M_p} = \frac{r}{\omega^2\,M_g\,M_p}. \tag{9}$$

Substituting the value of ω from (5), we have also

$$g = \frac{C\,L\,r}{M_g\,M_p}. \tag{10}$$

In the above equations r represents the effective resistance of the coil and condenser in series, and may be considered constant only when the main loss is due to the direct-current resistance of the coil. Instead of expressing the loss in terms of a series resistance r, we may express it in terms of an effective shunt conductance g', related to r by the equation,

$$r=g'\omega^2L^2=\frac{g'L}{C}. \tag{11}$$

Substituting in (10),

$$g=\frac{g'L^2}{M_g M_p}, \tag{12}$$

The conductance g' may be considered constant when the main loss is due to true leakage in the condenser. It will also be constant when the main loss is due to eddy currents in the coil; for this eddy-current loss is proportional to the square of the frequency (for a given working current and negligible skin effect), and so makes r proportional to ω^2 and thence g' constant in (11).

When the main loss is in a dielectric, various experiments have shown the power factor to be approximately constant. The corresponding effective resistance is then

$$r=p\,\omega L=p\sqrt{\frac{L}{C}}, \tag{13}$$

where p is the power factor of the capacity. Substituting in (10),

$$g=\frac{p\sqrt{CL^3}}{M_g M_p}. \tag{14}$$

Suppose that the loss is made up of three parts, that due to direct-current resistance, that due to eddy currents, and the loss in the dielectric of the coil itself. The total loss may then be expressed as

$$P=I^2r+I^2g'\frac{L}{C}+I^2\frac{C'}{C}p\sqrt{\frac{L}{C}}, \tag{15}$$

where g' represents the effective conductance due to eddy currents, and where C' represents that part of the total capacity associated with the dielectric of the coil and having the power factor p. Substituting in (3),

$$g=\frac{CLr+g'L^2+pC'\sqrt{\frac{L^3}{C}}}{M_g M_p}. \tag{16}$$

If the frequency is varied by changing C, this expression has a minimum value when

$$CLr=\frac{1}{2}pC'\sqrt{\frac{L^3}{C}}, \tag{17}$$

or, in words, when the dielectric loss is twice that due to direct-current resistance. The author has found experimentally that there is a particular frequency at which the oscillation of such a coil is the strongest, verifying in a general way the above conclusion. Evidently the effect of eddy-current loss on the strength of an oscillation is the same at all frequencies.

The circuit of Figure 3a may be modified in various ways, as indicated in Figures 3b to 3f. The main oscillating-current circuit in each case is equivalent to a single coil connected to a single condenser; so the frequency is given by equation (5), in which C is the joint capacity of all condensers and L the joint self-inductance of all coils.

Figure 3b is derived from Figure 3a by combining the main coil and the grid coil into one. The value of g is then given by putting $(M_g=L)$ and $(M_p=M)$ in equation (10):

$$g=\frac{Cr}{M}. \tag{18}$$

Similar substitutions lead to the same equation for Figure 3c.

In Figure 3d the effective mutual inductance M_g between the grid coil alone and the two coils together is (L_g+M), and similarly for M_p. Making these substitutions in (10),

$$g=\frac{CLr}{(L_g+M)\,(L_p+M)}. \tag{19}$$

Evidently the mutual inductance between the coils may be omitted.

In Figure 3e or 3f the total voltage is $I\omega L$, which is divided between the grid and plate condensers in inverse proportion to their capacities—

$$E_g=I\,\omega L\frac{C_p}{C_g+C_p} \text{ and } E_p=I\,\omega L\frac{C_g}{C_g+C_p}. \tag{20}$$

Substituting these equations with (8) and (5) in (3),

$$g=\frac{r}{\omega^2L^2\frac{C_gC_p}{(C_g+C_p)^2}}=\frac{Cr}{L}\cdot\frac{(C_g+C_p)^2}{C_gC_p}. \tag{21}$$

For Figure 3e, we may substitute for the joint capacity C in this equation the value $\frac{C_gC_p}{C_g+C_p}$, giving

$$g=\frac{(C_g+C_p)\,r}{L}. \tag{22}$$

In the circuits of Figures 3a to 3d, the two coils connected to the filament may evidently be parts of a single coil having an intermediate tap. The oscillation will occur most easily when this tap is such as to make E_g and E_p approximately equal. This will not usually be the most desirable place, however, as explained in the next to last paragraph of the preceding article. Similar remarks apply to the connection between the condensers in Figures 3e and 3f. These circuits may be further extended by subdividing the coils and condensers. The essential action will remain the same as long as the main oscillating-current circuit is electrically equivalent to a single coil connected to a single condenser; for any such circuit is characterized by the fact that it permits of an oscillation at but a single frequency.

3. Coupled Circuits Treated By the Loss Method.—Two oscillating-current circuits (C_1, L_1) and (C_2, L_2) may be coupled by having mutual inductance M_{12} between their coils, as represented by the heavy lines in Figure 4. A free oscillation in the combined circuit will have such a frequency that the resultant voltage in each separate circuit is zero:

$$I_1\left(\omega L_1-\frac{1}{\omega C_1}\right)+I_2\,\omega\,M_{12}=0; \tag{23}$$

and

$$I_2\left(\omega L_2-\frac{1}{\omega C_2}\right)+I_1\,\omega\,M_{12}=0. \tag{24}$$

Combining these equations,

$$\left(\omega L_1-\frac{1}{\omega C_1}\right)\left(\omega L_2-\frac{1}{\omega C_2}\right)-(\omega\,M_{12})^2=0; \tag{25}$$

and solving for ω,

$$\frac{1}{\omega^2}=\frac{C_1L_1+C_2L_2}{2}\pm\sqrt{\left(\frac{C_1L_1-C_2L_2}{2}\right)^2+C_1\,C_2\,M_{12}^2}. \tag{26}$$

The two circuits are said to be "in tune with each other" when their separate natural frequencies are equal, or when

$$C_1L_1=C_2L_2. \tag{27}$$

For this condition of tuning, (26) becomes

$$\frac{1}{\omega^2}=C_1L_1\pm\sqrt{C_1\,C_2\,M_{12}^2}. \tag{28}$$

Substituting the "coefficient of coupling,"

$$k_{12}=\frac{M_{12}}{\sqrt{L_1L_2}}, \tag{29}$$

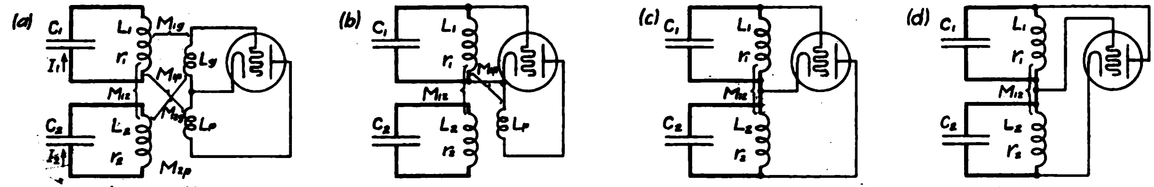
this becomes

$$\frac{1}{\omega^2}=C_1L_1\,(1\pm k_{12})=C_2L_2\,(1\pm k_{12}). \tag{30}$$

Fig.4

$$\frac{1}{\omega^2} = C_1L_1(1 \pm k_{12}) = C_2L_2(1 \pm k_{12})$$

$$k_{12}^2 = \frac{M_{12}^2}{L_1L_2}$$



$g =$

(a) $$\frac{(C_1r_1+C_2r_2)(1 \pm k_{12})}{\sqrt{L_gL_p}\,(k_{1g} \pm k_{2g})(k_{1p} \pm k_{2p})}$$

(b) $$\frac{C_1r_1+C_2r_2}{M_{1p}}$$

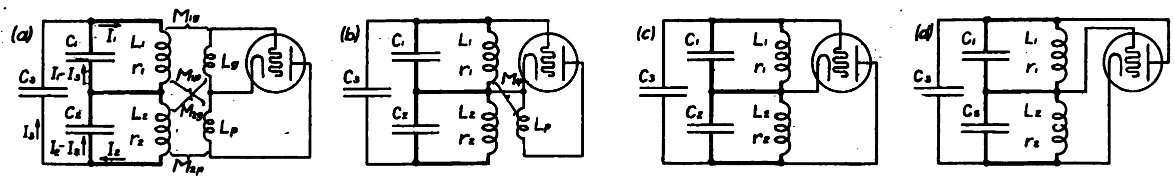
(c) $$\frac{C_1r_1+C_2r_2}{\pm\sqrt{L_1L_2}\,(1 \pm k_{12})}$$

(d) $$\frac{C_1r_1+C_2r_2}{(\mp\sqrt{L_1L_2}-L_2)(1 \pm k_{12})}$$

Fig.5

$$\frac{1}{\omega^2} = (C_1+C_3)L_1(1 \pm k) = (C_2+C_3)L_2(1 \pm k)$$

$$k^2 = \frac{C_3^2}{(C_1+C_3)(C_2+C_3)}$$



$g =$

(a) $$\frac{[(C_1+C_3)r_1+(C_2+C_3)r_2][1 \pm k]}{\sqrt{L_gL_p}\,(k_{1g} \pm k_{2g})(k_{1p} \pm k_{2p})}$$

(b) $$\frac{[(C_1+C_3)r_1+(C_2+C_3)r_2][1 \pm k]}{M_{1p}}$$

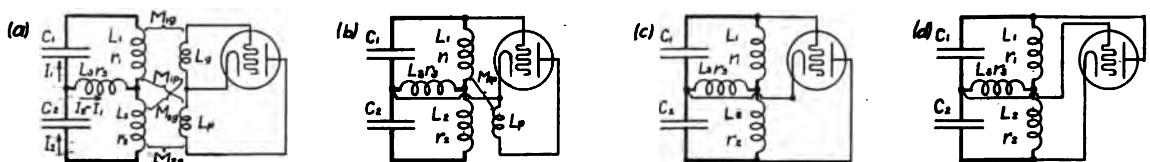
(c) $$\frac{[(C_1+C_3)r_1+(C_2+C_3)r_2][1 \pm k]}{\pm\sqrt{L_1L_2}}$$

(d) $$\frac{[(C_1+C_3)r_1+(C_2+C_3)][1 \pm k]}{\mp\sqrt{L_1L_2}-L_2}$$

Fig.6 *

$$\frac{1}{\omega^2} = C_1(L_1+L_3)(1 \pm k) = C_2(L_2+L_3)(1 \pm k)$$

$$k^2 = \frac{L_3^2}{(L_1+L_3)(L_2+L_3)}$$



$g =$

(a) $$\frac{[C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3][1 \pm k]}{\sqrt{L_gL_p}\,(k_{1g} \pm k_{2g})(k_{1p} \pm k_{2p})}$$

(b) $$\frac{C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3}{M_{1p}}$$

(c) $$\frac{C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3}{\pm\sqrt{(L_1+L_3)(L_2+L_3)}\,[1 \pm k]}$$

(d) $$\frac{C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3}{[\pm\sqrt{(L_1+L_3)(L_2+L_3)}-(L_2+L_3)][1 \pm k]}$$

*In this figure M_{1g} represents the mutual inductance between L_g and the combination of L_1 and L_3; and similarly for M_{2g}, M_{1p} and M_{2p}.

The ratio of currents is, by (23),

$$\frac{I_2}{I_1}=\frac{\omega L_1-\frac{1}{\omega C_1}}{-\omega M_{12}}=\frac{L_1\left(1-\frac{1}{\omega^2 C_1 L_1}\right)}{-M_{12}}; \tag{31a}$$

and by (29) and (30), for the condition of tuning,

$$\frac{I_2}{I_1}=\frac{\mp L_1 k_{12}}{-k_{12}\sqrt{L_1 L_2}}=\pm\sqrt{\frac{L_1}{L_2}}. \tag{31b}$$

The physical significance of the double sign ($\pm$) in the above equations is that a coupled circuit has *two* natural frequencies, and so permits of two independent oscillations. Whether both oscillations will actually occur and, if so, their relative magnitudes depend on the way in which oscillation is started or maintained. The oscillation at the lower frequency has currents flowing in such directions in the two coils as to add in their magnetic effect, giving the equivalent of an increased self-inductance. The oscillation at the higher frequency has currents flowing in such directions in the two coils as to oppose in their magnetic effect, giving the equivalent of a decreased self-inductance. When the circuits are in tune, one natural frequency is thus lower and the other higher than that of each separate circuit; and the difference between the frequencies is the greater, the closer the coupling. Equation (31b) shows that the energies of oscillation in the two circuits are equal $\left(\frac{1}{2}L_1 I_1^2=\frac{1}{2}L_2 I_2^2\right)$.

If an audion is coupled to the main oscillating-current circuit, as in Figure 4a, it will be able to maintain an oscillation if the constants of the circuit are chosen so as to satisfy equation (3). We will take first the general case where the grid and plate circuits are each coupled to both oscillating-current circuits. Assuming the circuits to be in tune, the grid and plate voltages are then respectively, by (31b).

$$E_g=I_1\,\omega\,M_{1g}+I_2\,\omega\,M_{2g}=I_1\,\omega\left(M_{1g}\pm M_{2g}\sqrt{\frac{L_1}{L_2}}\right); \tag{32}$$

and

$$E_p=I_1\,\omega\,M_{1p}+I_2\,\omega\,M_{2p}=I_1\,\omega\left(M_{1p}\pm M_{2p}\sqrt{\frac{L_1}{L_2}}\right). \tag{33}$$

The power loss in the main oscillating-current circuit is

$$P=I_1^2 r_1+I_2^2 r_2=I_1^2\left(r_1+r_2\frac{L_1}{L_2}\right). \tag{34}$$

Substituting in (3),

$$g=\frac{P}{E_gE_p}=\frac{r_1+r_2\frac{L_1}{L_2}}{\omega^2\left(M_{1g}\pm M_{2g}\sqrt{\frac{L_1}{L_2}}\right)\left(M_{1p}\pm M_{2p}\sqrt{\frac{L_1}{L_2}}\right)} \tag{35}$$

$$=\frac{\frac{r_1}{L_1}+\frac{r_2}{L_2}}{\omega^2\sqrt{L_gL_p}\,(k_{1g}\pm k_{2g})\,(k_{1p}\pm k_{2p})}, \tag{36}$$

where the k's are coefficients of coupling between the main coils and the audion coils, in fashion analogous to (29). By (30),

$$g=\frac{(C_1r_1+C_2r_2)\,(1\pm k_{12})}{\sqrt{L_gL_p}\,(k_{1g}\pm k_{2g})\,(k_{1p}\pm k_{2p})}. \tag{37}$$

The equation for g when the circuits are not in tune is more complex; it is derived in the same manner, except that equations (31a) and (26) are used instead of (31b) and (30) respectively, to give the ratio of currents and the frequency.

Special modifications of Figure 4a are shown in Figures 4b to 4d, where instead of inductive coupling the audion is sometimes directly connected to the main circuit, giving the equivalent of unit coefficient of coupling. For Figure 4b, we may put in (37)

$$k_{1g}=1,\;\; k_{2g}=k_{12},\;\; k_{2p}=0,\;\; \text{and}\;\; L_g=L_1, \tag{38}$$

giving

$$g=\frac{C_1r_1+C_2r_2}{\sqrt{L_1L_p}\cdot k_{1p}}=\frac{C_1r_1+C_2r_2}{M_{1p}}. \tag{39}$$

This shows (since there is no "±" sign) that the two possible oscillations occur *with equal ease,* provided the circuits are in tune as assumed above. When the circuits are not in tune, the values of g differ for the two oscillations and only that oscillation actually occurs which corresponds to the lower value of g. The expression for g in this general case may be derived in the same way as above; but it is more instructive to view the matter physically. If the circuit (C_2, L_2) of Figure 4b is far out of tune with (C_1, L_1) and the coupling is loose, its energy of oscillation will be small, since this is all supplied from (C_1, L_1) and not directly from the audion. The oscillation then occurs approximately as if (C_2, L_2) were absent, leaving the simple circuit of Figure 3b. This oscillation is strong, since the loss in (C_2, L_2) is small. If (C_2, L_2) has a low natural frequency, its current will flow as in a short-circuited coil and will oppose the current of (C_1, L_1), thus decreasing the effective self-inductance and raising the frequency. On the other hand, if (C_2, L_2) has a high natural frequency, its current will flow as in (C_1, L_1), thus increasing the effective self-inductance and lowering the frequency. Hence if the natural frequency of (C_2, L_2) is gradually changed from a low to a high value, the strength of the oscillation will fall as the condition of tuning is approached, after which it will rise again; and, at the instant of tuning, the oscillation will suddenly change from the higher to the lower natural frequency. In practice it is found that an oscillation once started tends to maintain itself with unchanged frequency, especially if it is strong; so the change from one natural frequency to the other does not take place until after the tuning point has been passed. There is then a rapid increase in the strength of the oscillation. During the transition period both oscillations exist and produce beats. If a "stopping condenser" is placed in series with the grid circuit and the proper adjustments are made, these beats may be produced continuously if the circuits are exactly or very nearly in tune.[7]

For Figure 4c, we may put in (37)

$$k_{1g}=1,\ k_{2g}=k_{1p}=k_{12},\ k_{2p}=1,\ L_g=L_1 \text{ and } L_p=L_2, \qquad (40)$$

giving

$$g=\pm\frac{C_1 r_1+C_2 r_2}{(1\pm k_{12})\sqrt{L_1 L_2}}. \qquad (41)$$

This shows that only one of the two possible oscillations can be maintained by the audion, with a given sense of the coupling, since the other would correspond to a negative value of g. Re-

[7]This continuous production of beats may be explained briefly as follows: Suppose an audion having a stopping condenser in series with the grid is suddenly started oscillating. The oscillations will cause the grid to build up a negative charge, by the same principle of unilateral conductivity that is employed in the simple audion detector. The grid will soon become so negative that the slope of the characteristic (i. e., the value of g) will be insufficient to maintain an oscillation. The energy of the oscillation, however, prevents it from immediately ceasing; so the grid becomes even more negative while the oscillation is dying out. When the oscillation has ceased, the negative charge gradually leaks off the grid, until a point on the characteristic is reached where an oscillation can again start, after which the phenomenon is repeated. We then have an *intermittent oscillation*, the "group frequency" of which depends on the intensity with which the audion tends to oscillate and on the rate at which the negative charge can leak off the stopping condenser. Now if adjustments are made so that the group frequency is near the beat frequency of the coupled circuit, it will assume that frequency and cause continuous beats. This action has been observed at audible oscillation frequencies by the author, as well as at radio frequencies. That the intermittent oscillation and the continuous production of beats occur under similar conditions has been observed by Armstrong ("Proc. I. R. E.," volume 3, page 227); but he does not seem to have noted that one is essentially a consequence of the other.

versing the coupling changes the oscillation from one of the natural frequencies to the other. Let us call that sense of coupling "normal" which would cause the grid and plate to have opposite potentials when current flows in one coil alone [k_{12} positive in (41)]. Then we may say that normal coupling gives the stronger oscillation and the lower frequency, and as the coupling is made closer the oscillation is strengthened and the frequency falls. With "reversed coupling," on the other hand, the oscillation becomes weaker and the frequency rises as the coupling is made closer. In either case, of course, the oscillation is weakened when the coupling is made very loose, on account of the effects of the resistance on the current ratio (see the following article). If the two circuits are not in tune (as assumed above), the oscillation will be weaker. This is in contrast with Figure 4b, where the weakest oscillation occurs with the circuits in tune. The equations for this general case show, as above, that only one oscillation can occur with a given sense of coupling.

For Figure 4d, we may put in (37)

$$k_{1g}=k_{12},\ k_{2g}=1,\ L_g=L_2,\ L_p=L_1+L_2+2\,M_{12},$$

$$k_{1p}=-\frac{L_1+M_{12}}{\sqrt{L_1\,(L_1+L_2+2\,M_{12})}},\ \text{and}\ k_{2p}=-\frac{L_2+M_{12}}{\sqrt{L_2(L_1+L_2+2M_{12})}},\qquad (42)$$

giving

$$g=\frac{C_1\,r_1+C_2\,r_2}{-L_2-M_{12}\mp\sqrt{\frac{L_2}{L_1}}(L_1+M_{12})}=\frac{C_1\,r_1+C_2\,r_2}{(1\pm k_{12})(\mp\sqrt{L_1L_2}-L_2)}.\qquad (43)$$

This shows that to make g positive and thus have the possibility of an oscillation, L_1 must exceed L_2,[8], and only one oscillation can occur with a given sense of the coupling. Unlike Figure 4c, however, the oscillation will have the *higher* natural frequency with *normal* coupling, and the lower frequency with reversed coupling—"normal coupling" here indicating that a current in one coil alone would give opposite potentials to the plate and the filament. A study of this circuit will show that the possibility of an oscillation requires the voltage across L_2 to be less than that across L_1 and to *subtract* from it, the difference being

[8]Of course the same result may be attained by connecting the plate to a coil L_p coupled with L_1, instead of directly to L_1, and using such mutual inductance between them that the voltage across this coil L_p exceeds that across L_2.

the plate voltage. This differential effect causes the oscillation to occur with greater difficulty than in the preceding cases, but tends to prevent oscillation at other frequencies, which would not give the proper polarities to the plate and grid (see Article 5).

Instead of coupling two oscillating-current circuits thru the mutual inductance between their coils, they may be coupled by having capacity or self-inductance in common, as shown in Figures 5 and 6 respectively. The equations for these cases, as given on the figures for the condition of tuning, are derived similarly to those of Figure 4. The general properties of these circuits depend mainly on the mode of connecting the audion and so are essentially the same as for the corresponding circuits of Figure 4. There is this difference, however: with mutually inductive coupling, the sense of the coupling can be reversed; but not with capacity or self-inductive coupling. Thus in Figure 5c, we have the equivalent only of *normal* coupling. and consequently the lower natural frequency; while in Figure 6c, we have the equivalent only of *reversed* coupling, and consequently the higher natural frequency. Sometimes mutual inductive coupling can advantageously be combined with capacity coupling: for example, where the inherent capacity C_3 gives more coupling than desired, it may be counteracted by a reversed mutual inductance.

It may be noted that if the values of L_1 and L_2 are equal in Figure 5 (or in Figure 6), the higher-frequency oscillation will have a frequency independent of the coupling, for no current then passes thru the coupling condenser (or coil).

In Figure 5 one of the main condensers (C_1 or C_2) may be open-circuited; and in Figure 6 one of the main coils (L_1 or L_2) may be short-circuited. The main circuits of the two figures then become identical, except for notation, and consist of a series group (of coil and condenser) connected to a parallel group. With the two groups interconnected in this way, the significance of their being "in tune" is indefinite; for the equations for ω and k on Figures 5 and 6 are not the same. The coupling will be loose when the series self-inductance is high in comparison with the shunt self-inductance; and for this condition the two sets of equations are nearly in agreement.

Capacity coupling may also be accomplished by connecting a condenser in the place of the coil L_3, Figure 6. This would require a large capacity to give loose coupling and so would be less convenient than the arrangement of Figure 5. By analogy, self-inductive coupling may also be accomplished by connecting a coil in the place of the condenser, C_3, Figure 5. This would require a high self-inductance to give loose coupling, and so would be less convenient than the arrangement of Figure 6. Besides, at radio frequencies, the inherent capacity of such a high-inductance coil would usually be considerable.

If a high non-inductive resistance is substituted for the coupling condenser C_3 of Figure 5, or a low non-inductive resistance for the coupling coil L_3 of Figure 6, we have what may be called "resistance coupling." This gives a single frequency of oscillation when the circuits are in tune, which is independent of the coupling. It evidently results in greater losses and does not seem to have sufficient redeeming features.

4. Certain Circuits Treated by the Complex Method.—For the exact determination of the conditions for oscillation in an audion circuit, we may express the various independent currents as complex quantities and write a number of equations each expressing the fact that the total voltage around a closed cycle is zero. One additional equation is given by (1), Article 1, the plate current being expressed in terms of the grid voltage, which in turn may be expressed in terms of the currents and the constants of the branches connected between the grid and the filament. These equations may be combined so as to eliminate all currents, resulting in a single complex equation which may be solved for ω and g.

As a simple example, we have in Figure 3b:

$$I\left(r+j\,\omega L+\frac{1}{j\,\omega C}\right)+I_p\cdot j\,\omega M=0; \tag{44a}$$

and

$$I_p=-g\frac{I}{j\,\omega C}, \tag{44b}$$

where the I's represent the currents in complex form. Substituting (44b) in (44a) and dividing thru by I, we eliminate the currents and obtain

$$r+j\,\omega L+\frac{1}{j\,\omega C}-\frac{j\,\omega M g}{j\,\omega C}=0;$$

or

$$1-\omega^2 CL+j\,\omega Cr-j\,\omega Mg=0. \tag{45}$$

Separating real and imaginary parts,

$$1-\omega^2 CL=0 \qquad \text{or} \quad \frac{1}{\omega^2}=CL; \tag{46a}$$

and

$$j\,\omega Cr-j\,\omega Mg=0 \qquad \text{or} \quad g=\frac{Cr}{M}. \tag{46b}$$

These equations are identical with those given on Figure 3b, showing that in this case the loss method was exact.

Now going to coupled circuits, we have similarly in Figure 4b:

$$I_1\left(r_1+j\,\omega L_1+\frac{1}{j\,\omega C_1}\right)+I_2\cdot j\,\omega M_{12}+I_p\cdot j\,\omega M_{1p}=0; \quad (47a)$$

$$I_2\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right)+I_1\cdot j\,\omega M_{12}=0; \quad (47b)$$

and

$$I_p=-g\frac{I_1}{j\,\omega C_1}. \quad (47c)$$

Combining,

$$\left(r_1+j\omega L_1+\frac{1}{j\,\omega C_1}-\frac{j\,\omega M_{1p}\,g}{j\,\omega C_1}\right)\left(r_2+j\,\omega L_2+\frac{1}{j\,C_2\,\omega}\right)-(j\omega M_{12})^2=0.$$

Clearing of fractions and separating real and imaginary parts,

$$(1-\omega^2 C_1 L_1)\,(1-\omega^2 C_2 L_2)-(\omega C_1 r_1-\omega M_{1p}g)\,\omega C_2 r_2 -\omega^4 C_1 C_2 M_{12}^2=0; \quad (48a)$$

and $(1-\omega^2 C_1 L_1)\,C_2 r_2+(1-\omega^2 C_2 L_2)\,(C_1 r_1-M_{1p}\,g)=0.$ (48b)

If $M_{1p}g$ is eliminated, these equations lead to a cubic for ω^2, showing three possible oscillations, which in general occur with unequal ease. If the circuits are *tuned*, however, the equations are simplified. Putting

$$C_1 L_1=C_2 L_2 \quad (49)$$

in (48b) we have either

$$g=\frac{C_1 r_1+C_2 r_2}{M_{1p}}; \quad (50)$$

or else

$$\frac{1}{\omega^2}=C_1 L_1=C_2 L_2. \quad (51)$$

Substituting these values successively in (48a), we have

either $(1-\omega^2 C_2 L_2)^2+\omega^2 C_2^2 r_2^2-\omega^4 C_1 C_2 M_{12}^2=0;$ (52)

or else

$$g=\frac{\omega^2 C_1 M_{12}^2}{M_{1p}\,r_2}+\frac{C_1 r_1}{M_{1p}}=\frac{M_{12}^2}{L_1 M_{1p}\,r_2}+\frac{C_1 r_1}{M_{1p}}. \quad (53)$$

Solving (52) for ω,

$$\frac{1}{\omega^2}=C_2 L_2-\frac{C_2^2 r_2^2}{2}\pm\sqrt{-C_2 L_2\cdot C_2^2 r_2^2+\left(\frac{C_2^2 r_2^2}{2}\right)^2+C_1 C_2 M_{12}^2}. \quad (54)$$

Substituting the coefficient of coupling,

$$k_{12}=\frac{M_{12}}{\sqrt{L_1 L_2}}, \quad (55)$$

and the power factor,[9]

$$p_2 = r_2 \sqrt{\frac{C_2}{L_2}}, \tag{56}$$

the above equations reduce to the following:

$$\frac{1}{\omega^2} = C_2 L_2 \left(1 - \frac{p_2^2}{2} \pm \sqrt{k_{12}^2 - p_2^2 + \frac{p_2^4}{4}}\right) \quad \text{or} \quad C_2 L_2; \tag{57}$$

and

$$g = \frac{C_1 r_1 + C_2 r_2}{M_{1p}} \quad \text{or} \quad \frac{C_1 r_1 + C_2 r_2 \dfrac{k_{12}^2}{p_2^2}}{M_{1p}}. \tag{58}$$

These equations show that when $\left(k_{12} < \sqrt{p_2^2 - \frac{p_2^4}{4}}\right)$ only one oscillation is possible, since two values of ω^2 in (57) are then imaginary. This oscillation has the natural frequency of the separate circuits and is the stronger the weaker the coupling. When k_{12} lies between $\sqrt{p_2^2 - \frac{p_2^4}{4}}$ and the very nearly equal value p_2, three oscillations are possible; but only that at the natural frequency of the separate circuits occurs, for this gives the lowest value of g. On the other hand, when $(k_{12} > p_2)$, this oscillation gives the highest value of g; so either of the other oscillations can occur, both giving the same value for g, which is independent of the coupling. There is thus a certain critical coupling,

$$k_{12} = p_2, \tag{59}$$

below which only one oscillation can occur and above which two oscillations can occur with equal ease. The frequencies for this critical coupling are given by

$$\frac{1}{\omega^2} = C_2 L_2 \quad \text{or} \quad C_2 L_2 (1 - p_2^2.) \tag{60}$$

The difference between these two frequencies is the lowest possible beat frequency.

The conclusions of the preceding paragraph were verified experimentally by the author two years ago, the experiments[10] being first made at audible frequencies so that the change in frequency and the accompanying beats could be made directly evident in a telephone receiver. Many tests at radio frequencies

[9]See footnote on "Notation" page.

[10]These experiments were actually made with a circuit differing from Figure 4b by having the grid and the plate interchanged. As explained previously, such interchange always leads to an exactly similar set of equations.

have since confirmed these conclusions, the beat frequency then being audible and showing the change from one oscillation to another. With the values of p_2 found in well designed circuits, the minimum beat frequency in long-wave circuits is below audibility, but in short-wave circuits will often lie within the audible range.

The above results show that the equations derived by the loss method, as given on Figure 4b, are not exact for this circuit, but are approximately correct as long as the coupling is considerably closer than the critical coupling.

The complex method applied similarly to the circuit of Figure 4c gives for the condition of tuning the following exact equations:

$$\frac{1}{\omega^2}=C_1L_1\left[1+\frac{p_1p_2}{2}\pm\sqrt{k_{12}^2+p_1p_2+\frac{p_1^2p_2^2}{4}}\right]; \qquad (61)$$

and

$$g=\frac{C_1r_1+C_2r_2}{k_{12}\sqrt{L_1L_2}(1-k_{12}^2)}\left[-k_{12}^2-\frac{p_1p_2}{2}\pm\sqrt{k_{12}^2+p_1p_2+\frac{p_1^2p_2^2}{4}}\right]. \qquad (62)$$

These equations show that, as long as k_{12} is large compared with $\sqrt{p_1p_2}$, the resistances do not appreciably affect the frequency or the current ratio; so the simpler equations given on Figure 4c apply with sufficient accuracy. When the coupling is loosened, so that k_{12} is comparable with $\sqrt{p_1p_2}$, the oscillation rapidly becomes weaker. With reversed coupling there is therefore a particular value of k_{12} giving the strongest oscillations, as has been found experimentally.

5. Regenerative and Absorbing Actions.—As mentioned in Article 1, an audion may either be connected so as to supply power tending to maintain an oscillation or so as to absorb power from the oscillation. The first action is equivalent to the insertion of a *negative* resistance in some part of the circuit; the second is equivalent to the insertion of a positive resistance. The equivalent positive resistance r_{an} added to any branch (n) may be expressed as $\left(-\frac{P}{I_n^2}\right)$, where $(-P)$ is the power absorbed by the audion (or $(+P)$ is the power supplied by the audion) and I_n is the current of the branch. Hence by equation (2), Article 1,

$$r_{an}=-\frac{P}{I_n^2}=-\frac{E_gE_pg}{I_n^2}. \qquad (63)$$

Consider the circuit of Figure 4b and suppose an oscillation to be produced by a source in series with C_2. We then have

$$E_g = \frac{I_1}{\omega C_1}; \tag{64}$$

and

$$E_p = I_1 \omega M_{1p}. \tag{65}$$

Substituting in (63), the equivalent resistance added in series with L_1 is

$$r_{a1} = -\frac{E_g E_p g}{I_1^2} = -\frac{M_{1p} g}{C_1}. \tag{66}$$

Since this is negative (with normal coupling) and constant for all impressed frequencies, the audion gives constant regenerative amplification for all oscillations impressed in series with C_2. This constitutes the one serious objection to the use of this circuit in radio receiving, C_2 being the antenna capacity, for the interference is severe, especially from strays. The tendency of this circuit to oscillate equally well at its two natural frequencies is evidently a special case of the constant regenerative amplification. With reversed coupling, r_{a_1} is positive and constant for all frequencies, the audion thus opposing all oscillations.

Consider now the circuit of Figure 4c and suppose an oscillation to be produced by a source in series with C_2, as above. We then have

$$E_g = \frac{I_1}{\omega C_1}; \tag{67}$$

$$E_p = I_2 \omega L_2 + I_1 \omega M_{12}; \tag{68}$$

and

$$I_1\left(\omega L_1 - \frac{1}{\omega C_1}\right) + I_2 \omega M_{12} = 0. \tag{69}$$

Substituting in (63), the equivalent resistance in series with L_1 is

$$r_{a_1} = -\frac{E_g E_p g}{I_1^2} = -\frac{g}{\omega C_1}\left[\frac{-\omega L_2}{\omega M_{12}}\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \omega M_{12}\right] \tag{70}$$

$$= -\frac{g L_2}{\omega^2 C_1^2 M_{12}}\left[1 - \omega^2 C_1 L_1 + \omega^2 C_1 \frac{M_{12}^2}{L_2}\right] \tag{71}$$

$$= -\frac{g\sqrt{L_1 L_2}}{k_{12} C_1}\left[\frac{1}{\omega^2 C_1 L_1} - 1 + k_{12}^2\right]. \tag{72}$$

If k_{12} is positive, r_{a_1} will be negative for all frequencies lower than that given by

$$\frac{1}{\omega^2} = C_1 L_1 (1 - k_{12}^2) \tag{73}$$

and will be positive for all higher frequencies; the reverse is true if k_{12} is negative. We thus have regenerative action on one side, and absorbing action on the other side of a certain critical frequency, according to the sense of the coupling. The tendency of this circuit to oscillate at one of its two natural frequencies with normal coupling and at the other natural frequency with reversed coupling is in agreement with the above result. This circuit may be employed in radio receiving to reduce interference from frequencies either higher or lower than that to which it is tuned, but not from both at the same time.

Consider finally the circuit of Figure 4d and again suppose an oscillation to be produced by a source in series with C_2. We then have

$$E_g = -I_2\,\omega L_2 - I_1\,\omega M_{12}; \tag{74}$$

$$E_p = \frac{I_1}{\omega C_1} + I_2\,\omega L_2 + I_1\,\omega M_{12}; \tag{75}$$

and

$$I_1\left(\omega L_1 - \frac{1}{\omega C_1}\right) + I_2\,\omega M_{12} = 0. \tag{76}$$

Substituting in (63), the equivalent resistance in series with L_1 is

$$r_{a_1} = -\frac{E_g E_p g}{I_1^2} = g\left[-\frac{\omega L_2}{\omega M_{12}}\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \omega M_{12}\right] \times \left[\frac{1}{\omega C_1} - \frac{\omega L_2}{\omega M_{12}}\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \omega M_{12}\right] \tag{77}$$

$$= \frac{g L_2}{\omega^2 C_1^2 M_{12}^2}\left[1 - \omega^2 C_1 L_1 + \omega^2 C_1 \frac{M_{12}^2}{L_2}\right] \times \left[L_2 + M_{12} - \omega^2 C_1 L_1 L_2 + \omega^2 C_1 M_{12}^2\right] \tag{78}$$

$$= \frac{g L_2}{k_{12}^2 C_1}\left[\frac{1}{\omega^2 C_1 L_1} - 1 + k_{12}^2\right] \times \left[1 + k_{12}\sqrt{\frac{L_1}{L_2}} - \omega^2 C_1 L_1 (1 - k_{12}^2)\right]. \tag{79}$$

This will be positive for all frequencies except those between the following limits:

$$\frac{1}{\omega^2} = C_1 L_1 (1 - k_{12}^2); \tag{80}$$

and

$$\frac{1}{\omega^2} = \frac{C_1 L_1 (1 - k_{12}^2)}{1 + k_{12}\sqrt{\frac{L_1}{L_2}}}. \tag{81}$$

The audion thus has an absorbing effect for all frequencies outside of this narrow range. By using a high-power bulb and making suitable adjustments it is possible to increase greatly the effective resistance of the circuit for all interfering frequencies and at the same time to give regenerative action at the frequency for which it is tuned.

When the audion is oscillating, or on the verge of oscillating, we may substitute in the above equations the values of g given on Figure 4. After reduction, we then obtain for the ratio of the equivalent added resistance to the equivalent original resistance of the whole circuit referred to the coil (1):
for Figure 4b, by (66),

$$\frac{r_{a_1}}{r_1+C_2 r_2/C_1}=-1; \tag{82}$$

for Figure 4c, by (72),

$$\frac{r_{a_1}}{r_1+C_2 r_2/C_1}=-\frac{1}{k_{12}}\left[\frac{1}{\omega^2 C_1 L_1 (1+k_{12})}-1+k_{12}\right] \tag{83}$$

$$=-\frac{1}{k_{12}}\left(\frac{\omega_n^2}{\omega^2}-1\right)-1; \tag{84}$$

and for Figure 4d, by (79),

$$\frac{r_{a_1}}{r_1+C_2 r_2/C_1}=\frac{1}{k_{12}}\left[\frac{1}{\omega^2 C_1 L_1 (1-k_{12})}-1-k_{12}\right]\times$$

$$\left[1+\frac{L_2}{\sqrt{L_1 L_2}-L_2}\cdot\frac{1+k_{12}}{k_{12}}(1-\omega^2 C_1 L_1[1-k_{12}])\right] \tag{85}$$

$$=\left[\frac{1}{k_{12}}\left(\frac{\omega_n^2}{\omega^2}-1\right)-1\right]\left[1+\frac{L_2}{\sqrt{L_1 L_2}-L_2}\cdot\frac{1+k_{12}}{k_{12}}\left(1-\frac{\omega^2}{\omega_n^2}\right)\right]. \tag{86}$$

In these equations ω_n represents that natural frequency which the audion tends to maintain; in (84) it is given by

$$\frac{1}{\omega_n^2}=C_1 L_1 (1+k_{12}); \tag{87}$$

and in (86) it is given by

$$\frac{1}{\omega_n^2}=C_1 L_1 (1-k_{12}). \tag{88}$$

It will be seen that the resistance ratio reduces to (-1) at this natural frequency in (84) and (86), and has this value at all frequencies in (82). This means that the audion then adds a negative resistance equal to the original positive resistance; so the total effective resistance is zero, which is simply another way of expressing the necessary condition for a sustained oscillation.

The above three special cases of coupled circuits differ so from one another in regard to their regenerative or absorbing action at various frequencies, that it seems desirable to investigate the general circuit of Figure 4a. If the separate circuits are in tune and an oscillation is produced by a source in (C_2, L_2), the values of E_g, E_p, and I_2/I_1, are given respectively by equations (32), (33) and (31a). Substituting in (63), the equivalent resistance in series with L_1 is

$$r_{a_1} = -\frac{E_g E_p g}{I_1^2} = -\omega^2 g \left[M_{1_g} - M_{2_g}\frac{L_1}{M_{12}}\left(1-\frac{1}{\omega^2 C_1 L_1}\right)\right] \times \left[M_{1_p} - M_{2_p}\frac{L_1}{M_{12}}\left(1-\frac{1}{\omega^2 C_1 L_1}\right)\right]$$

$$= -\omega^2 g L_1 \sqrt{L_g L_p}\left[k_{1_g} - \frac{k_{2g}}{k_{12}}\left(1-\frac{1}{\omega^2 C_1 L_1}\right)\right] \times \left[k_{1_p} - \frac{k_{2p}}{k_{12}}\left(1-\frac{1}{\omega^2 C_1 L_1}\right)\right]. \tag{89}$$

When the circuit is oscillating, or on the verge of oscillating, we may substitute the value of g from (37), giving for the ratio of the added resistance to the original resistance

$$\frac{r_{a_1}}{r_1 + C_2 r_2/C_1} = -\omega^2 C_1 L_1 \left[1 \pm k_{12}\right] \frac{k_{1_g} - \frac{k_{2g}}{k_{12}}\left(1-\frac{1}{\omega^2 C_1 L_1}\right)}{k_{1g} \pm k_{2g}} \times \frac{k_{1_p} - \frac{k_{2p}}{k_{12}}\left(1-\frac{1}{\omega^2 C_1 L_1}\right)}{k_{1p} \pm k_{2p}}. \tag{90}$$

If this resistance ratio is plotted against $(1/\omega^2)$, the resulting curve will in general be a hyperbola having the resistance axis as one asymptote and crossing the frequency axis at the points,

$$\frac{1}{\omega^2} = C_1 L_1\left(1-\frac{k_{12}k_{1_g}}{k_{2g}}\right) \text{ and } \frac{1}{\omega^2} = C_1 L_1\left(1-\frac{k_{12}k_{1_p}}{k_{2p}}\right). \tag{91}$$

As these expressions are significant only when positive, we may have zero, one, or two frequencies giving zero added resistance. The three special cases treated above exemplify these three possible conditions. In each of these cases one of the natural frequencies was found to be within the range of negative added resistance, giving a possible free oscillation; but the general case, as shown by studying (89), permits the range of negative r_{a_1} to lie between, above, or below the two natural frequencies. This means that the audion may have regenerative action over a range of frequency and still be unable to maintain a free oscillation, no matter how low the resistances.

6. Application of Oscillating Audion Circuits.—The

following record of the various oscillating audion circuits that have been developed is probably incomplete and is given with the hope that it will be supplemented in the discussion and that attention will be called to any inadvertent errors.

The single oscillating-current circuits of Figure 3 are mainly employed as a basis for the more useful coupled circuits. They simply afford means for giving the grid and the plate opposite polarities for any oscillation that may be produced in the main circuit; and the choice among them is ordinarily only a matter of convenience. When the main circuit of Figure 3b is coupled to a local tuned circuit, the arrangements of Figure 4b, 5b or 6b result, according to the method of coupling; but we might equally well have used any of the other circuits of Figure 3 to couple with the local circuit, giving the same properties that have been described for Figure 4b.

Figure 3b is the basis of one of Armstrong's methods[11] for producing oscillations that has been widely used—viz., mutually inductive coupling between the plate circuit and a tuned grid circuit. What may be called the "conjugate" of this circuit, the grid and the plate being interchanged, Figure 3c, was devised by the author about two years ago and successfully used for radio receiving. It is, however, less convenient than Figure 3b, because, to give a high grid voltage and at the same time not have too strong an oscillation, the capacity C must be so high (of the order of 50 milli-microfarads) as to preclude the use of an ordinary variable air condenser. Figure 3d, employing a single tapped coil, is commonly known as the "Western Electric" circuit; without the mutual inductance between the coils, it is included among those originally given by Armstrong.[12] The circuit of Figure 3e is used in commercial radio receivers and for other purposes. Figure 3f might not be recognized as the basis of Armstrong's circuit employing an audio-frequency self-inductance (such as that of telephone receivers) in the lead from the filament to the oscillating-current circuit.[13] Here C_p represents the capacity (added or inherent) shunting the audio-frequency self-inductance; and C_g represents the inherent capacity between the grid, with the apparatus connected thereto, and the filament, with its connected apparatus. Armstrong considers the audio-frequency self-inductance in circuits of this form to be essential to the action. This, however, is not the case; for

[11]"Proc. I. R. E.", volume 3, page 219, Figure 8.

[12]Loc. cit., page 222, Figure 13.

[13]Loc. cit., Figure 12. As has been pointed out by Armstrong, this arrangement is identical with the "ultraudion" connection of de Forest.

experiments made by the author show that a high non-inductive resistance will answer the same purpose, tho requiring a higher battery voltage to give the same direct current. The real purpose of the high self-inductance is simply (as in many other audion connections) to provide a path for the direct battery current from the filament to the plate, without short-circuiting the capacity C_p which is essential to the oscillation.

Figures 4b, 5b, and 6b give examples of a class of coupled circuits in which one of the component circuits is not coupled, or only slightly coupled, with the audion. Such circuits are in general characterized by the following properties: (a) if the coupling between the component circuits is not very loose, the audion tends to produce oscillations equally well (or almost equally well) at the two natural frequencies of the combination; (b) oscillations of all frequencies impressed in the local circuit (C_2, L_2) are then amplified by regenerative action, or absorbed, according to the polarity of the connections to the audion; and (c) with very loose coupling of the local circuit (C_2, L_2), the main oscillating-current circuit (C_1, L_1) oscillates readily at its own natural frequency. The property (a) permits an audion to be used as a generator of "beats," as has been done by Armstrong.[14] If for the local circuit (C_2, L_2) is used an antenna and tuning coil, this property also makes the circuit particularly applicable for the reception of undamped waves by the "self-heterodyne" principle; for the audion may be allowed to oscillate at one natural frequency while the other natural frequency is brought to coincidence with that of the signal to be received. By making adjustments carefully, the audion may be brought to the verge of changing from one frequency to the other, under which condition enormous amplification is possible.[15] Unfortunately, this

[14]Loc. cit., Figure 17 used for this purpose is identical with Figure 6b of this paper.

[15]This method of signal amplification does not appear to have previously been brought to the attention of the Institute, tho it has been described by the author on several occasions. It should be noted that to attain the result described, the coupling between the oscillating-current circuits must be carefully adjusted so that the two possible free oscillations will give a beat note of the desired pitch. For long waves, this means a coupling coefficient between 1 and 5 per cent. Dr. Austin has used ("Proc. I. R. E.", volume 4, page 252) a so-called "sensitizing circuit" which could operate as described here; but he explains that its purpose is merely to weaken the oscillation produced by the audion. His results, of comparatively slight and constant increase in amplification, bear out his explanation for the arrangement as he used it. Armstrong, however, in the discussion of Dr. Austin's paper offers the explanation that the sensitizing circuit gives the combined circuit two natural frequencies and so permits it to be exactly in tune with the incoming oscillation while it is oscillating at another frequency, thus eliminating the reactance offered to the incoming oscillation when the circuit is slightly

condition is very critical; so a strong signal or accidental disturbance may suddenly change over the frequency of oscillation to that of the signal, which immediately disappears. Moreover, any impulse due to strays will set up simultaneous oscillations at the two natural frequencies, giving beats between them which are audible. As the audion tends to maintain both of these oscillations, a musical note results which may last a large fraction of a second and sounds like the clang of a bell. With the circuit in exact tune with the signal, this bell-like noise due to strays has just the same pitch as the signal, which it thus completely masks. In the presence of strays it is therefore necessary to operate the circuit somewhat out of tune, thus sacrificing the particular advantage of circuits of this class. The property (b) of these circuits (the tendency to amplify at *all* frequencies) is of course a disadvantage in radio receiving, as it reduces selectivity.

Circuits for heterodyne reception may be arranged so that the audion tends to produce oscillations at two frequencies, one of which oscillations does not enter the antenna circuit and so is not set up by impulses therein. This obviates the serious objection to the circuit discussed in the preceding paragraph, while retaining its advantages. However, better results are obtained when one audion is used for producing oscillations and a second audion for amplifying the signal oscillation by regenerative action.[16] The reason is as follows: When the audion is oscillating, the value of g (or mean slope of the characteristic curve, Figure 1) is such as to make the total effective resistance of the circuit equal to zero *for that oscillation.* When a second oscillation is superposed, the range of variation in grid voltage and plate current will be extended over a section of the characteristic curve the slope of which is less steep; the value of g for the second oscillation is therefore lower, and the total effective resistance for this oscillation will not be zero. If the original oscillation is very weak, so that only the practically straight portion of the characteristic curve is included, then the value of g for the second oscillation is nearly the same as for the first, and better amplification is obtained, as has been found in practice. Such a weak oscillation, however, requires very critical adjustments, and is likely to cease suddenly. When two audions are used, one may be on the verge of oscillating at the signal frequency, and will then require only a weak impressed voltage in the

out of tune. Nothing, however, was said about the tendency of the audion to oscillate at the incoming frequency.

[16]Hogan, "Proc. I. R. E.", volume 3, page 256.

antenna to produce relatively strong oscillations, which then give beats with the permanent oscillation of the other audion.

The circuit of Figure 4b, with its modifications, has been used by the author in the measurement[17] of effective resistance of coils at various frequencies. The test coil is connected to a tuning condenser, constituting the local circuit (C_2, L_2). The two component circuits are brought into tune at the desired frequency and the coupling between them is adjusted to the critical value [Article 4, equation (59)], as shown by the behavior of an ammeter[18] in the plate circuit when the tuning is varied. The local circuit is then removed and a resistance is inserted in series with L_1 until the oscillation has the same strength as before. This resistance multiplied by the ratio of self-inductances gives the effective resistance of the local circuit [compare equations (18) and (58)]. This method permits the measurement of the effective resistance (as well as the capacity) of a coil at its own natural frequency, with as much facility as at lower frequencies.

Figures 4c, 5c, and 6c give examples of a class of coupled circuits in which one component circuit is connected (or coupled) to the grid of the audion and the other to the plate. Such circuits are in general characterized by the following properties: (a) the audion tends to produce an oscillation at only one of the natural frequencies; (b) the audion gives regenerative action on one side, and absorbing action on the other side, of a certain frequency near the natural frequencies; and (c) very loose coupling may be employed before the oscillation is greatly weakened.

Figure 4c is the basis of a circuit devised by the author in the fall of 1915 for receiving damped waves, C_2 being the antenna capacity.[19] About the same time this circuit, with a slightly different arrangement of the telephone receivers and battery, was brought out by Mr. F. B. Chambers and is commonly known as the "Chambers circuit." In using this circuit the author has found that the capacity of the audion frequently gives too close coupling (on the principle of Figure 5c), and this

[17]Described in a paper presented before the Radio Club of America in February, 1917, and published in "QST" for April, 1917.

[18]When the coupling is closer than the critical value, the ammeter shows a sudden drop when the oscillation changes from one of the natural frequencies to the other. When the coupling is loosened so that this drop just ceases to occur, the maximum ammeter reading is used as an index of the strength of the oscillations when the circuits are in tune.

[19]This circuit was given in a discussion before the Radio Club of America in June, 1916, and was published in "QST" for September, 1916.

may to advantage be partially neutralized by reversed coupling of the coils. The circuit of Figure 4c is not particularly suited to sustained wave reception (altho it is sometimes so used), because it must either be slightly out of tune for the received oscillation or else the received energy will tend to be absorbed by the audion.

Figure 5c is the basis of one of Armstrong's methods of producing oscillations—viz., the use of self-inductance in the plate circuit.[20] C_3 here represents the capacity between the grid and the plate of the audion, with their connected apparatus; and C_2 represents the inherent or added capacity in parallel with the plate inductance. It may be noted that, if C_2 is zero, the two component circuits may still be tuned by adjusting C_3 or L_2.

So far as the author is aware, the principle of Figure 4d, 5d, and 6d is new. If C_2 is made the antenna capacity of a radio receiving set, the audion will have a strong tendency to reduce interference, since it absorbs energy from oscillations at other frequencies than that to which it is tuned. The general circuits of Figures 4a, 5a, and 6a can give even better results in the reduction of interference from strays; for they can be arranged so as to give the best regenerative action at a frequency differing slightly from the natural frequency and can then have an absorbing action for the oscillations set up by strays at the natural frequency. Of course, the set then being slightly out of tune with the signal, some reactance for the signal oscillation will be introduced. The audion used in this way for combined regenerative and absorbing action should not also be used as a detector; for its grid is connected (or closely coupled) to the antenna circuit (C_2, L_2). A second audion (or other detector) should be connected in the usual way to the circuit (C_1, L_1), where interference is minimized in the usual manner by the loose coupling of the two tuned circuits.

The foregoing discussion makes no pretense of including all possible or useful oscillating audion circuits, even of the simpler sort. Many others have been studied by the author which exhibit different and interesting properties; but the limits of space do not allow their inclusion. It is hoped, however, that the

[20] "Proc. I. R. E.", volume 3, page 220, Figure 9, and following.

methods of treatment here described and illustrated will be of service to those working with the oscillating audion and will lead to the further development of that quite wonderful device. This is the real object of the paper.

SUMMARY: A general method is presented for investigating theoretically the conditions of oscillation in circuits supplied by an audion. This method is based on the relay action of the audion, as represented by its characteristic curve, and has for its fundamental idea the use of the slope of this curve as a physical constant of the audion, called the "mutual conductance." Two modes of applying this method are given: the simple, but approximate, "loss method" and the exact "complex method." Various particular circuits are discussed and formulas given for the natural frequency and the required value of mutual conductance to maintain an oscillation. These circuits include the elements of all the common oscillating audion circuits depending on the relay action; so it is seen that the actions in these various circuits are not distinct and independent, but are here brought under a single set of physical ideas.

The circuits chosen for detailed discussion are those which most simply illustrate the method of treatment and which exhibit the various useful properties that may be obtained with different connections of the audion. It would be impossible to include in any paper of reasonable length a discussion of, or even a reference to, all the possible oscillating-current circuits that may have practical application.

The behavior of certain circuits toward impressed oscillations (as from an antenna) at various frequencies is considered. And it is found that the audion under certain conditions tends to maintain such oscillations (regenerative action), and under other conditions tends to absorb their energies. It is also found that with certain arrangements the audion will have regenerative action only over a very short range in frequency, absorbing energy from oscillations at all other frequencies. This result affords a means of reducing interference, especially from unsustained oscillations such as are caused by strays.

The paper concludes with a brief record of oscillating audion circuits as developed by various investigators, and their practical applications.

In two appendices the author discusses respectively: the family of characteristics obtained for different plate voltages, and derives therefrom data on oscillation characteristics; and the interchangeability of the grid and plate circuits so far as oscillation production is concerned.

The following units belong to a consistent system based on the milli-ampere, the volt, and the micro-second. They give much more convenient numerical values in radio work than the ordinary units based on the ampere, volt, and second; and many of them are for this reason already in common use. Of course, all equations not involving numerical coefficients apply equally to this system or to the ordinary system.

NOTATION

SYMBOL	NAME	UNIT
C_n	Capacity in branch (n)	Milli-microfarad
E_n	Alternating voltage across branch (n)	Volt
E_g	Grid voltage, alternating part [Article 1]	Volt
E_p	Plate voltage, alternating part [Article 1]	Volt
g	Mutual conductance of grid toward plate [Article 1]	Milli-mho
g_p	Self-conductance of plate [Article 1]	Milli-mho
g'	Conductance of a branch	Milli-mho
I_n	Alternating-current in branch (n)	Milli-ampere
I_p	Plate current, alternating part [Article 1]	Milli-ampere
j	Imaginary unit, $\sqrt{-1}$	Numeric
k_{mn}	Coefficient of mutually inductive coupling, $\sqrt{\frac{M_{mn}}{L_m L_n}}$	Numeric
k	Coefficient of self-inductive or capacity coupling [Figures 5 and 6]	Numeric
L_n	Self-inductance of branch (n)	Milli-henry
M_{mn}	Mutual inductance between branches (m) and (n)	Milli-henry
ω	Angular frequency	Radian per microsecond
P	Power	Milli-watt
p_n	Power factor[21] of branch (n), $r_n \sqrt{\frac{C_n}{L_n}}$	Numeric
r_n	Resistance of branch (n)	Kil-ohm
r_{an}	Equivalent "absorbing" resistance of the audion referred to branch (n) [Article 5]	Kil-ohm

[21]This is strictly the ratio of resistance to reactance at the natural frequency, and not the ratio of resistance to impedance, which actually is the power factor. The numerical difference in any radio circuit is insignificant; and for the sake of vividness it is convenient to employ the common term "power factor."

APPENDIX I*

In Article 1 of the paper it was mentioned that the plate potential affects the oscillation, and a method of taking this into account was mentioned—viz., by means of a constant g_p expressing the quotient of a variation in the plate current by the corresponding variation in the plate potential. A more general method of treating the effect of the plate potential is given below.

In Figure 7 the full lines constitute a family of characteristic curves, such as that of Figure 1, each curve taken for a different constant plate potential, as may readily be traced experimentally by employing adjustable batteries in both grid and plate circuits.

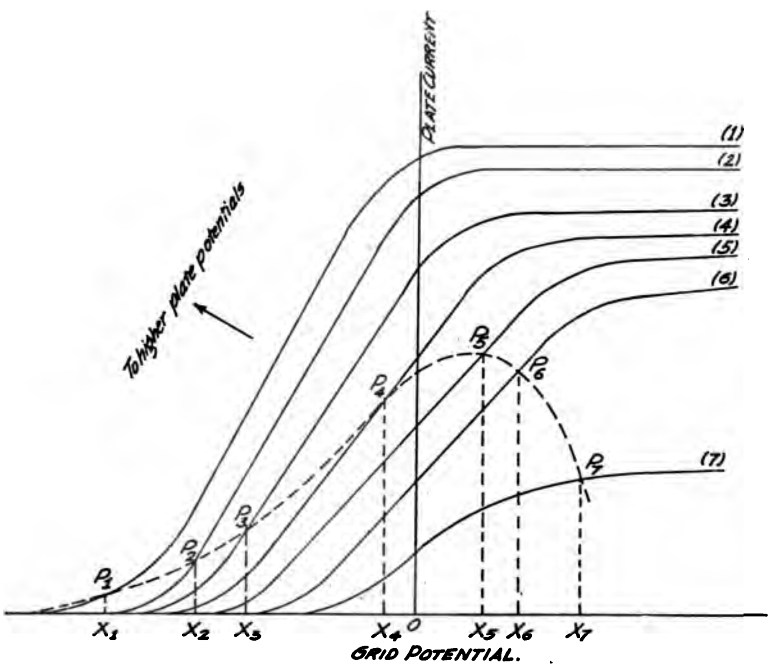


FIGURE 7

cuits. Now in any oscillating audion circuit let the ratio of the alternating plate voltage E_p to the alternating grid voltage E_g be

$$n = \frac{E_p}{E_g}.$$

* The subject matter of this appendix was presented orally at the reading of the paper itself, September 5, 1917.

These voltages being ordinarily in phase, their instantaneous values, and therefore the variations in plate and grid potentials, have the same ratio n. Hence if we start with the plate and grid potentials corresponding to the point P_1, for example, and let the plate become more negative by such an amount ΔE_p as to correspond to the curve (2), then the grid will become more positive by the amount,

$$\Delta E_g = \frac{\Delta E_p}{n}$$

which may be laid off along the axis as $X_1 X_2$ to locate the point P_2. Further points P_3, and so on, may be similarly located on successive curves, and these determine the dotted "derived characteristic" representing conditions during an oscillation. It is interesting to notice that the derived characteristics may exhibit a maximum point even where the original characteristics do not.

The derived characteristic may be used (in the same way as Figure 1) to give the value of the effective mutual conductance g under working conditions, thus including completely in the equations of the paper the effect of variations in the plate potential.

Probably the most useful application of derived characteristics is in determining the relation between the coil inductances, or between the condenser capacities, in the plate and grid circuits that will result in the greatest power output from the audion as an oscillating-current generator. Derived characteristics are drawn for various assumed values of n (and, if desired, for various value of grid battery voltage), and the output for each is roughly estimated as one-eighth the product of the range in plate current by the range in plate voltage.

APPENDIX 2

In the paper itself, it is stated that the grid and plate of an audion may always be interchanged in their connection to an oscillating-current circuit, and that the circuit equations will then remain of exactly the same form. As the only proof given for this statement was the symmetry of E_g and E_p in equation (3), it is thought desirable to append the following complete proof.

Resolving any oscillating-current network supplied by an audion into the appropriate number of independent circuits and equating to zero the sum of the voltages around each circuit, we have equations of the usual form:

$$\left.\begin{array}{l} I_1Z_{11}+I_2Z_{12}+\ldots\ldots\ldots\ldots\ldots+I_nZ_{1n}+I_pZ_{1p}=0;\\ I_1Z_{21}+I_2Z_{22}+\ldots\ldots\ldots\ldots\ldots+I_nZ_{2n}+I_pZ_{2p}=0;\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \text{and}\quad I_1Z_{n1}+I_2Z_{n2}+\ldots\ldots\ldots\ldots\ldots+I_nZ_{nn}+I_pZ_{np}=0. \end{array}\right\} \quad (1)$$

By the definition of the mutual conductance g, we have also

$$\frac{I_p}{g}=E_g=I_1Z_{g1}+I_2Z_{g2}+\ldots\ldots\ldots+I_nZ_{gn}+I_pZ_{gp}; \quad (2)$$

or
$$I_1Z_{g1}+I_2Z_{g2}+\ldots\ldots\ldots+I_nZ_{gn}+I_p\left(Z_{gp}-\frac{1}{g}\right)=0. \quad (3)$$

In these equations, the general coefficient,

$$Z_{rs}=Z_{sr}, \quad (4)$$

represents the impedance in common between the circuits r and s (conductive or mutually inductive); Z_{rr} represents the negative sum,

$$Z_{rr}=-Z_{1r}-Z_{2r}-\ldots\ldots\ldots\ldots\ldots\ldots\ldots-Z_{nr}-Z_{pr}; \quad (5)$$

the I's represent the independent circuit currents; and the subscripts g and p refer to the grid and plate circuits respectively.

Eliminating the I's from the simultaneous equations (1) and (3), there results an equation between their coefficients which is most simply expressed by the determinant:

$$\begin{vmatrix} Z_{11} & Z_{12}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{1n} & Z_{1p} \\ Z_{21} & Z_{22}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{2n} & Z_{2p} \\ \ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots \\ Z_{n1} & Z_{n2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{nn} & Z_{np} \\ Z_{g1} & Z_{g2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{gn} & Z_{gp}-\frac{1}{g} \end{vmatrix}=0. \quad (6)$$

Interchanging the two subscripts of each Z, in accordance with (4), and interchanging columns with rows, as is permissible in a determinant, we obtain

$$\begin{vmatrix} Z_{11} & Z_{12}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{1n} & Z_{1g} \\ Z_{21} & Z_{22}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{2n} & Z_{2g} \\ \ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots \\ Z_{n1} & Z_{n2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{nn} & Z_{ng} \\ Z_{p1} & Z_{p2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Z_{pn} & Z_{pg}-\frac{1}{g} \end{vmatrix}=0. \quad (7)$$

This equation differs from (6) only in having the g and p subscripts interchanged. Hence, in general, *interchanging the connections of the grid and the plate of the audion to the oscillating-current circuit does not alter the circuit equations.* It is understood, of course, that in making this interchange, the apparatus, naturally pertaining to the plate, as the plate battery, telephone receivers, and so on, is transferred with the plate, and similarly for the grid apparatus. Altho the general properties of an oscillating-current circuit are not affected by interchanging the grid and the plate, yet the strength of the oscillation may be affected; for the effective value of g will in general be changed because of the effect of the plate potential on the oscillation, as explained in Appendix 1.

Equation (6), or (7), embodies the general solution of an oscillating-current circuit supplied by the audion; and may be directly employed instead of the step-by-step procedure given in Article 4 of the paper. Take, for example, the circuit of Figure 4b, and let the circuits be chosen as represented by the dotted lines in Figure 8. Equation (6) for this case becomes.

$$\begin{vmatrix} -\left(r_1+j\,\omega L_1+\frac{1}{j\,\omega C_1}\right) & j\,\omega M_{12} & j\,\omega M_{1p} \\ j\,\omega M_{12} & -\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right) & 0 \\ \frac{1}{j\,\omega C_1} & 0 & -\frac{1}{g} \end{vmatrix}=0. \quad (8)$$

Expanding,

$$\left(r_1+j\,\omega L_1+\frac{1}{j\,\omega C_1}\right)\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right)\left(-\frac{1}{g}\right) + \frac{j\,\omega M_{1p}}{j\,\omega C_1}\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right)+\frac{(j\,\omega M_{12})^2}{g}=0. \quad (9)$$

This immediately reduces to (48) of the paper.

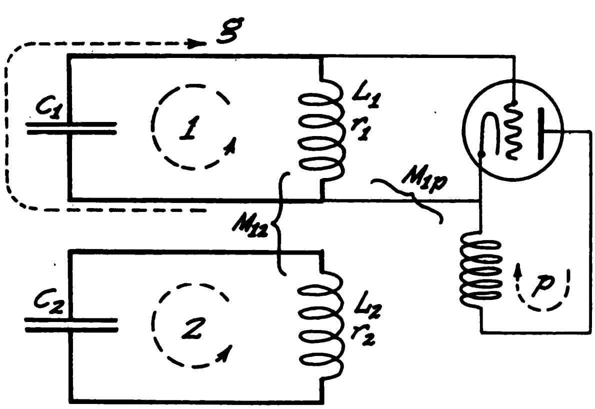


FIGURE 8

DISCUSSION

L. A. Hazeltine: Dr. de Forest calls attention to certain early discoveries of the oscillating property of the audion which do not come within the rules laid down in the paper, in that no radio-frequency apparatus was interposed between the plate and the filament. Such oscillations are due to positive ionization and not to relay action, and were not considered as coming within the scope of the paper; they were referred to, however, in Article 1 (see especially footnote 6).

Since the paper was written certain applications of oscillating-audion circuits have come to the writer's attention. A particularly interesting case is that of Meissner, who, working independently of Armstrong and along different lines, invented oscillating audion circuits at practically the same time (1913). One of Meissner's circuits is that of Figure 3a of the paper, and will be found in a letter from Meissner in the "Electrician" (London), volume 73, page 702, 1914.

A collection of oscillating audion circuits of various inventors is given by Goldsmith in his serial articles on "Radio Telephony," "Wireless Age," June and July, 1917, where further references will be found*. Arrangements especially suitable for extreme conditions and employing the pliotron oscillator, which is also a thermionic relay, have been given by White in the "General Electric Review" (for very high or very low frequencies, in the September, 1916, number; and for large currents or high voltages, in the August, 1917, number).

Logwood has recently patented a circuit for transmitting (number 1,218,195, abstracted in the "Electrical World," April, 21, 1917) which serves to confirm certain statements of the paper. This circuit is essentially that of Figure 3f; but the audio-frequency inductance is connected between the *grid* and the filament, with a shunt capacity C_g, while the capacity C_p is simply the inherent capacity of the plate. As C_p is thus smaller than C_g, the plate voltage will exceed the grid voltage, giving an increased output for transmitting, as explained in Article 1.

If proper regard is given in certain cases to the inherent capacity of the audion and other apparatus, all the various oscillating audion circuits referred to above will, in the writer's opinion, be found to be included among those considered in the paper, or to be directly derivable therefrom.

* A considerably more complete treatment of oscillating audion circuits will be found in a recent volume: Goldsmith, "Radio Telephony" (Wireless Press, 25 Elm Street, New York).

THE DETERMINATION OF THE AUDIBILITY CURRENT OF A TELEPHONE RECEIVER WITH THE AID OF THE WHEATSTONE BRIDGE*

BY

EDWARD W. WASHBURN

(PROFESSOR OF CERAMIC CHEMISTRY IN THE UNIVERSITY OF ILLINOIS)

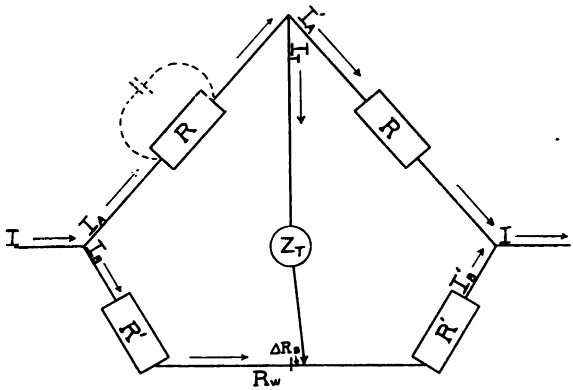
The use of the telephone receiver as a detector for small alternating currents in radio telegraphy, in many electrical measurements with the alternating current bridge[1], and for various other purposes makes it desirable to have a simple and reliable method for determining the audibility current of a telephone. By audibility current is meant that current which just suffices to produce a barely audible sound in the telephone. The curve connecting the audibility current of a telephone with the frequency employed is a valuable indication of the field of usefulness of the instrument and of its availability for the above purposes.

The usual method of determining the current-sensitivity curve of a telephone for various frequencies is to base it upon the volt-sensitivity curve of the telephone. The volt-sensitivity is usually determined with the aid of a suitable slide wire connected in shunt across a non-inductive branch of an alternating current circuit which is provided with a suitable hot-wire ammeter. [Cf. Wien, *Ann. Phys.*, **4**, 456 (1901)]. Having measured the volt-sensitivity in this way, the audibility current is then obtained by dividing the audibility voltage at each frequency by the impedance of the telephone for that frequency. Since, however, both the effective resistance and the inductive reactance of a telephone depend upon the frequency, it becomes necessary in this method to measure each of these quantities for each frequency employed. Moreover, as has been pointed out by Austin [*N. B. S. Bull.*, **5**, 155 (1908)], it is doubtful whether the values thus obtained are really applicable to the

* Received by the Editor, January 24, 1917.

[1] The importance of knowing the absolute value of the audibility current of a telephone receiver for use with an apparatus for measuring the electrical conductivity of liquids has been discussed by the writer in another place. ["Jour. Amer. Chem. Soc.," **38**, 2431 (1916).]

purpose in hand, since they are measured with the aid of currents which are very much larger than the audibility current of the telephone. In other words, both the effective resistance and the inductive reactance of the telephone are functions not only of the frequency of the current but also of its magnitude. This source of uncertainity can be avoided and the whole determination much simplified by means of a method based upon the equation of the Wheatstone bridge. While this method has



been devised by the writer primarily for the purpose of determining the audibility current of the low resistance types of telephones which are used as indicating instruments in alternating-current bridge measurements, it seemed nevertheless that it also might possibly have some advantages over the methods now in use in determining the audibility current of the high resistance types of telephone receivers used in radio work.

The Wheatstone bridge set-up is shown in the accompanying figure. The resistances R and R' must be substantially free from both inductance and capacity. For work with low resistance telephones, the Curtis type of resistance coil serves very well for this purpose. For high resistance telephones, especially for measurements at the higher frequencies, the type of film resistance devised by the writer [*Journn. Amer. Chem. Soc.*, **35,** 179 (1913)] would be preferable since even the largest units are practically free from any inductance or capacity as well as from any skin effect, even for the highest frequencies at which a telephone could be employed.

The two resistances marked R' are connected by a stretched wire of suitable resistance (R_W), provided with a scale and a sliding contact. The telephone is connected to this sliding contact and to a point between the two resistances R as indicated. The four resistances R and R' should be as compactly and symmetrically assembled as possible and the telephone cord should be surrounded by a flexible metal sheath which is earthed. A small variable air condenser is connected in shunt across whatever resistance requires it in order to complete the exact balancing of reactances, as it is very necessary, especially for high resistances and high frequencies, to obtain perfect electrical symmetry in the set-up.

The resistance of that portion of the slide wire which lies between its central point, and the position occupied by the sliding contact when the audibility current I_T is passing through the telephone, will be denoted by ΔR_B. The current I comes from a high audio frequency generator whose frequency can be controlled and kept constant at any desired value and whose wave form is practically a sine curve. The current enters the bridge network at the left and divides as indicated. In series with the bridge is placed a suitable high frequency ammeter (not shown in the figure) for indicating the magnitude of the current I. For very small currents, the most convenient ammeter to employ is a vacuum thermocouple connected to a sensitive millivoltmeter, while for larger currents a sensitive hot-wire ammeter can be employed. The lead wires which carry the current I to the bridge network should be twisted or twin wires and should be enclosed in a grounded metal sheath.

The procedure consists in adjusting the current I until the range of silence on the slide wire is found to have a convenient measurable value, say about two or three centimeters. If the telephone is being tested to determine its suitability for use as an indicating instrument with the alternating current bridge, the determination of the range of silence should be carried out in the usual manner in which the minimum is determined when working with an alternating current bridge. If, however, the telephone is one which is to be used for radio work, the dot-and-dash method would naturally be employed in determining the range of silence on the slide wire. The range of silence is obviously equal to $2\Delta R_B$ ohms. Having determined the value of $2\Delta R_B$, the value of I as indicated by the high frequency ammeter is then recorded and the calculation of I_T is carried out by means of the following equation, which can be readily derived with the aid of Kirchhoff's laws [Maxwell, *"Electricity and Magnetism,"* I, 477, 3rd Edition].

$$I_T = \frac{2\Delta R_B R I}{B(B+2R_T-2jx_T)} \tag{1}$$

where B is written in place of $R+R'+\frac{1}{2}R_W$. This equation may be put in the form

$$I_T = \frac{2\Delta R_B R I}{B} \times \frac{1}{(B+2R_T)-2jx_T} \tag{2}$$

or numerically:

$$I_T = \frac{2\Delta R_B R I}{B\sqrt{(B+2R_T)^2+4x_T^2}} \tag{3}$$

where x_T, the inductive reactance, and R_T the effective resistance, are the values which, for the frequency in question, correspond to the audibility current I_T. To distinguish these values from the values which are directly measured using currents of much greater magnitude, we will designate the former values as the *audibility reactance* and the *audibility resistance* respectively.

Now if we were to employ in this equation values for R_T and x_T determined in the usual manner; that is, for currents much greater than I_T, then each of the values so obtained would differ from the true or audibility values by some fractional amount p, and the quantity under the radical in the above equation should, therefore, be written

$$[B+2R_T(1\pm p_R)]^2+4x_T^2(1\pm p_x)^2 \tag{4}$$

Now it is evident that the fractional error in I_T which would result from the fractional errors p_R and p_x in the quantities R_T and x_T, respectively, can be made smaller than any assigned value if the quantity B is taken sufficiently large. The problem, therefore, reduces itself to the calculation of the minimum value which B may have without producing an appreciable error in the value of I_T.

Now determinations of the appearance of an audible sound in a telephone are not sufficiently reproducible, even for a given ear, to justify an attempt to measure the corresponding current with a greater degree of accuracy than, say ten per cent. There will obviously, therefore, be no object in employing a larger value for B than that which is necessary in order to insure the attainment of this degree of accuracy. The condition, therefore, which determines the minimum allowable value for B will be mathematically expressed by the relation

$$\frac{4p[BR_T+Z_T(2+p)]}{(B+2R_T)^2+4x_T^2} \not> 0.2 \tag{5}$$

where $p_R = p_x = p$. From this expression we find the desired condition, namely that

$$B \nless 2\left[-R_T(1-5p)+\sqrt{Z_T^2[5p(2+p)-1]+R_T^2(1-5p)^2}\right] \quad (6)$$

where R_T and Z_T are the *approximate* values of the effective resistance and impedance and p is the fractional amount by which each of these values differs from the true or audibility value.

To illustrate the use of this equation, let us consider a specific case. Suppose it is desired to determine the audibility current, at 1000 cycles, of a telephone receiver the effective resistance of which is approximately 8,000 ohms and the effective impedance of which is approximately 10,000 ohms at this frequency. Let us suppose further that there is the possibility that each of these values might differ from the true or audibility value by as much as *fifty per cent.* In other words it is desired to choose a value for B such that an error of fifty per cent. in *each* of the values R_T and Z_T shall be without appreciable influence upon the value found for I_T. Putting $p=0.5$ in equation (6) and solving, we find that

$$B \nless 76{,}000 \text{ ohms.} \quad (7)$$

If, therefore, we choose R and R' in Figure 1 each equal to say 40,000 ohms, B would then be equal to 80,000 ohms and would obviously be well above the minimum value just calculated.

A little consideration will show that this method of determining the audibility current of a telephone not only eliminates any error arising from uncertainty as to the true values of the effective resistance and the inductive reactance, but that it can even be so employed as to make it unnecessary to determine the values of these two quantities at all. That is, knowing the direct current value of the resistance of our telephone, we may then estimate or guess at the approximate values for its impedance and its effective resistance and may then take such a value for p that we can feel sure that our estimated values do not differ from the actual values by more than $\frac{p}{100}$ per cent. We then solve equation (6) for B.

Furthermore, if the condition

$$B \nless 8R_T + 4\sqrt{(Z_T^2+4R_T^2)} \quad (8)$$

is fulfilled, equation (3) reduces to the simple form

$$I_T = \frac{2\Delta R_B R I}{B^2} \quad (9)$$

That is, the terms containing R_T and Z_T are entirely negligible. Thus, for example, if one used a bridge in which a value 10,000 ohms was taken for B, this bridge could then be used to determine the audibility currents of all the ordinary types of low resistance telephones without paying any attention to either the resistance or the inductance of the telephone. That is, equation (9) could be applied directly.

As an example of the determination of the audibility current of a low resistance telephone receiver by this method the following values may be cited. The high audio frequency generator was operated at 990 cycles per second. The telephone the audibility current of which was to be measured had a direct current resistance of 170 ohms and its effective resistance at 1,000 cycles was about 220 ohms, its impedance at this frequency being about 290 ohms. The value chosen for B was 5,000 ohms and it will be noticed that this fulfills the condition laid down by equation (8) so that the value of I_T may be directly calculated from equation (9). The range of silence on the slide wire was found to be $2\Delta R_B = 0.05$ ohm and the vacuum thermocouple in series with the bridge indicated a current, $I = 0.4$ milliampere. Substituting these values in equation (9), we find the audibility current of the telephone to be 0.002 microampere at 990 cycles.

In conclusion, it may be pointed out that by combining the audibility current of a telephone, obtained in this way, with the audibility voltage obtained by the shunt method, the true or audibility impedance of the telephone can be calculated.

Department of Ceramic Engineering,
University of Illinois,
January 20, 1917.

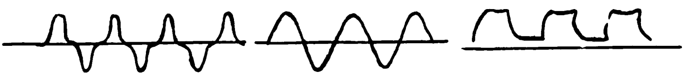
SUMMARY: A method of determining the audibility *current* of a telephone receiver, using a Wheatstone bridge with the telephone as indicator is described practically and theoretically. The method differs from the usual shunted telephone method which determines an audibility *voltage*.

A method is given whereby a desired percentage accuracy can be secured even if the telephone resistance and impedance are only very approximately known.

DISCUSSION

Charles S. Ballantine (by letter): I have read Professor Washburn's paper with very great interest, and believe that radio engineers are greatly indebted to him for his valuable contribution to a very indefinite subject. It is very evident upon examination, for instance, of the experimental data given by such observers as Austin, Fuller, and others that a certain amount of variability in results exists which cannot be wholly accounted for on the basis of the influence of the "personal equation" of the observers. This regrettable lack of agreement is probably due to some extent to a lack of precision in the definition of the fundamental unit upon which the measurement of signal strength is based, i. e., the audibility current of the telephone receiver. For this reason the method of Professor Washburn becomes of value in making such comparisons and deriving theoretical values for the currents from the shunt readings. There are several points in connection with the method described which may be mentioned here. It has been stated in certain foreign publications that as the wave form of the measuring current departs from the pure harmonic form a corresponding loss of sensitiveness in the receiver results. I have never discovered any exact quantitative data accruing from any investigation of a receiver accompanying such statements, and it is difficult to decide at once whether the results obtained by the above method would be applicable in actual practical conditions. If such is the case, it is evident that the method using the Wheatsone bridge on any current form and comparing the results with shunted telephone readings obtained with actual signaling currents of another wave form would have very little engineering value. Another point of interest is the influence of the mechanical movement of the receiver diafram upon the currents in the windings. This motion would obviously result in a reactive E. M. F. which would oscillate in several periods giving rise to transients and further complicating the functional relation between the impedance and frequency. The proposed method provides for the elimination of the variable inductance error due to the saturation of the iron with different currents but fails to take into consideration the effect of the resonance of the diafram when comparison is to be made between audibility current and shunt readings.

In order to obtain a quantitative estimate of the actual influence of these factors, an experimental investigation was made on a typical telephone receiver using Professor Washburn's method. The first group of experiments was made to determine the influence of wave form on the measurements, and for this purpose three different conditions were imposed as shown in Figure 1. Type *A* current was supplied from an alternator giving



Type A Type B Type C

FIGURE 1

currents up to 240 cycles frequency. The oscillogram taken across the bridge showed nearly 40 per cent. of third harmonic current which produced a rather sharply peaked wave form as a limiting condition. The usual harmonic current shown as type *B* was supplied from a small 50-watt laboratory harmonic set which gave currents of any frequency up to 1,200 cycles per second. The other extreme condition was supplied by a broken direct current giving an extremely flat topped wave form showing the usual exponential rise and decay. It is believed that the variation in form factor of the three types employed was sufficient to indicate any error if it existed. Unfortunately, owing to the limited range of the peaked wave alternator, investigation could not be made with this current in the neighborhood of the resonance point of the diafram but it is believed from the close agreement of the three types up to the limit of this machine at 240 cycles that extrapolation on this basis is permissible. The integrated value of the current in each case was given by a Leeds-Northrup galvanometer of the reflecting type shunted across a Siemens-Halske vacuum thermo-element.

The telephone receiver used in the investigation was one of the Western Electric Company's special receivers and showed a measured resistance to direct current of 543 ohms. The impedance at 1,000 cycles with currents of the order of 0.015 ampere was found to be about 1,054 ohms. The principal results are represented in the table and plotted graphically in Figure 2. The heavy line curve was obtained under the usual operating conditions and the depression in the curve at the point of resonance 535 cycles is very marked. The dotted curve which does not evidence the slightest trace of this effect was obtained with a paper washer placed between the core and the diafram and the cap screwed on very lightly. The effect of damping and air gap

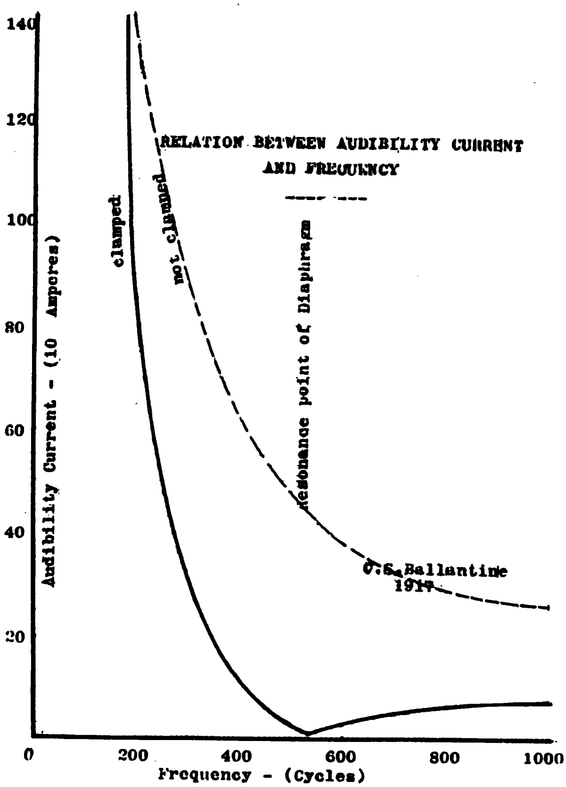


FIGURE 2

is very clearly shown by this curve. Figure 3 shows a series of values taken near the point of resonance and the slope of this curve would tend to indicate the advisability of taking ample precaution to see that the frequency of the current used in determining the audibility current by this method and that of the received signals upon which shunt readings have been obtained correspond, or the accuracy in determining the actual values of the signals in amperes is considerably lessened. With these points in mind, it seems possible by means of Professor Washburn's valuable solution of the problem to evaluate a shunted telephone reading in micro-amperes with a degree of accuracy approaching scientific precision, permitting the use of this method where the use of thermo-element and galvanometer would not be practical.

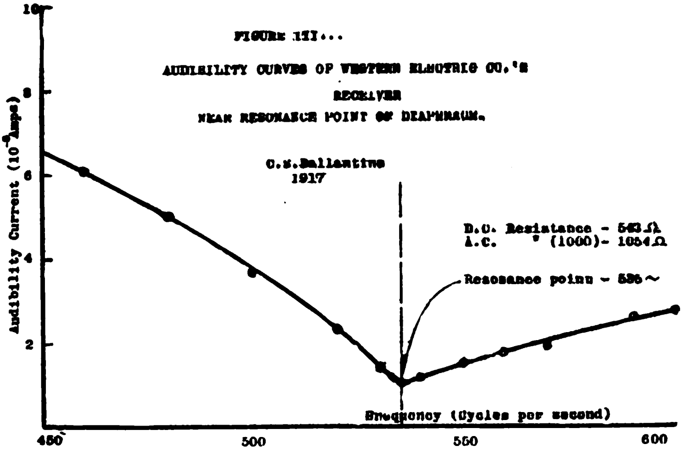


FIGURE 3

Table 1

Frequency	Current (x10)
178	99.0
200	80.1
300	30.2
410	10.1
500	3.75
520	2.32
535	1.10
600	2.75
800	6.20
1,000	7.50
1,030	7.60

H. A. Frederick: In the same way in which it appears obvious that by increasing the resistance of the bridge arms relative to the impedance of the telephone receiver, the system may be made a constant current system to any desired degree of precision and hence the audibility current be determined with any desired accuracy. So also by decreasing the resistance of the bridge arms relative to the impedance of the receiver we may obtain with any desired precision a constant voltage system. From a determination made with such a bridge may be obtained simply the audibility voltage of the telephone. The quotient of this audibility voltage by the audibility current gives the modulus of the impedance at this value of current.

In the determination of the audibility voltage there be plàced in series with the telephone receiver a variable condenser which is then adjusted so as to give the minimum audibility voltage a value of audibility voltage E' is obtained which if divided by the audibility current will give the effective resistance of the telephone receiver at this value of current, thus giving the argument of the impedance. The audibility power of course follows simply as the product of the square of the audibility current into the effective resistance just obtained. This last quantity is, of course, the quantity of real significance in the comparative determination of the efficiency of various telephone receivers.

March 13, 1917.

ADDITIONAL NOTE ON "THE COUPLED CIRCUIT BY THE METHOD OF GENERALISED ANGULAR VELOCITIES"

By

V. Bush

(Consulting Engineer, American Radio and Research Corporation, Medford, Massachusetts)

In the December, 1917, issue of the "Proceedings of The Institute of Radio Engineers," Dr. Carson gives a very interesting criticism of my summary of Wagner's proof of Heaviside's formula which appeared in the October issue. While this proof is not essential to the paper, since we have to thank Dr. Carson himself for an excellent proof of a formula which includes Heaviside's as a special case, it seems to me that Wagner's method gives promise of further application which makes discussion well worth while.

Dr. Carson bases his criticism upon the statement that the path of integration of the infinite integral (1) passes through a pole of the function. It will be noted, however, from Figures 3 and 4 of my paper that this pole is avoided by surrounding it with a small semicircle, as is, of course, the usual method of avoiding this difficulty. Perhaps I should have been more explicit in abstracting. The integral taken over the path as I showed it properly defines the function with which we have to deal.

This same remark applies to the contour integrals (2) and (4). It is unnecessary to become involved in the difficulties of evaluating an integral the path of integration of which cuts a pole of the function since we can easily avoid the pole in the manner indicated, and base the entire proof upon integrals thus defined.

It was not my intention to attempt rigor in a short abstract, for such questions can better be referred to the original paper. I believed, however, that Wagner's method would be of aid in forming a concrete conception of the quantity I have termed at the suggestion of Dr. Kennelly the *threshold impedance* of an oscillating circuit.

The use of the idea of the threshold impedance of an oscillating-current circuit will, I believe, be of considerable aid to practical men in the application to specific problems of Heaviside's formula. In much the same way in the early days of alternating currents the concept of the impedance of a circuit aided greatly in the practical application of a theory which was at that time in a somewhat similarly involved condition.