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TECHNICAL PAPERS AND DISCUSSIONS

EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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CONTENTS

The Officers and Committees of the Institute for 1918 will appear in a forthcoming issue.

The Institute of Radio Engineers announces with the deepest regret the death of

Commandant Camille Tissot.

M. Tissot was born in 1868, and was in succession a student at the Naval School, Officer in the Navy, Doctor of Science, Professor at the Naval School, Professor at the Institute of Electricity, Laureate of the Academy of Science at Paris, and Member of the Scientific Committee on Radio Telegraphy.

Most of the problems relating to radio communication were the object of careful studies on his part. At the same time as Mr. W. Duddell (who survived him but a few weeks), he introduced into radio telegraphic measurements the element of precision. He measured weak currents of radio frequency, using the bolometer, and was thus enabled to make quantitative measurements in radio reception. As a result of these experiments, he was able to throw much light on the laws of resonance, the types of oscillations in antennas, the damping influence of the ground, and the laws governing propagation of waves over short distances. He also devised a classic method of measuring decrements. His book, "Etude sur la resonance des systemes d'antennes dans la Telegraphie sans fil" ("Study of Resonance in Radio Antennas"), (Paris, Gauthier-Villars, 1906), set forth the results of his investigations in 1904-1905, and was a landmark in radiotelegraphic development.

His investigations were then extended in the direction of the mode of operation of the crystal and electrolytic detectors. A paper by him on "The Influence of Alternating Currents on Certain Melted Metallic Salts," was presented before The Institute of Radio Engineers in 1913, and published in Volume 2, number 1 of the "PROCEEDINGS." His researches in radio telephony, in methods of using alternating currents for the charging of condensers, in the transmission of time signals, and in other directions, merit close attention.

Two other important works were published by him: a treatise on "Electric Oscillations," which constitutes a valuable contribution on the theoretical side of this subject, and a more technical "Manual of Radio Telegraphy."

After placing himself at the entire disposal of the French Navy, he contracted on board a Mediterranean mine sweeper the illness which led to his death.

Paris, November 17, 1917. L. B.

THE DYNATRON

A Vacuum Tube Possessing Negative Electric Resistance*

By

Albert W. Hull, Ph.D.

(Research Laboratory, General Electric Company, Schenectady, New York)

1. Definition

The dynatron belongs to the kenotron family of high vacuum, hot cathode devices which the Research Laboratory has developed. Two members of this family, the kenotron rectifier and the pliotron, have already been described in this journal.[1] The fundamental characteristic of kenotrons is that their operation does not depend in any way upon the presence of gas.

In construction, the dynatron resembles the kenotron rectifier and the pliotron. In principle and operation, however, the three are fundamentally different. Each utilizes a single important principle of vacuum conduction. The kenotron rectifier utilizes the uni-directional property of the current between a hot and cold electrode in vacuum. The pliotron utilizes the space charge property of this current, which allows the current to be controlled by the electrostatic effect of a grid. The dynatron utilizes the secondary emission of electrons by a plate upon which the primary electrons fall. It is, as its name indicates, a generator of electric power, and feeds energy into any circuit to which it is connected. It is like a series generator, in that its voltage is proportional to the current thru it, but it is entirely free from the hysteresis and lag that are inherent in generators and in all devices which depend upon gaseous ionization.

2. Construction

The dynatron consists essentially of an evacuated tube containing a filament, a perforated anode and a third electrode, called the plate. The essential construction is shown in Figure 1. The plate must be situated near the anode, in such a

* Received by the Editor, January 30, 1917.

[1] "Proc. I. R. E.," September, 1915.

position that some of the electrons, set in motion by the anode voltage, will fall upon it. A battery is provided for maintaining the filament at incandescence and for maintaining the anode at a constant positive voltage of 100 volts or more, with respect to the filament. This voltage is not varied during operation, and the anode plays no part in the operation of

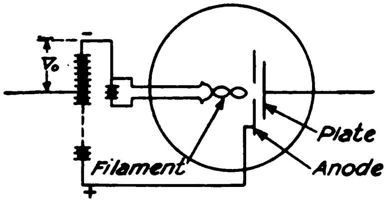

FIGURE 1

the tube except to set in motion a stream of primary electrons, and to carry away the secondary electrons from the plate; that is, to supply the power.

Figure 2 shows the construction of one of the practical types of dynatron that have been developed. The plate has been bent into the form of a cylinder (Figure 2, a) in order to utilize more fully the electron emission from the filament, and the anode has been provided with a large number of holes, instead of one. This is accomplished by using a perforated cylinder (Figure 2, b), or spiral of stout wire (Figure 2, c), or a network of fine tungsten wires (Figure 2, d). The filament is a spiral of tungsten wire (Figure 2, e). The filament may be further provided with a heavy insulated wire along its axis (Figure 2, f), or surrounded by an insulated spiral grid (Figure 2, g), making a "four member" tube, which is called a *pliodynatron*. The characteristics of the pliodynatron are discussed in Section 8.

3. Characteristics—Negative Resistance

Electrons from the filament *F* (Figure 1) are set in motion by the electric field between *F* and the anode *A*. Some of them go thru the holes in the anode and fall upon the plate *P*. If *P* is at a low potential with respect to the filament, these electrons will enter the plate and form a current of negative electricity in the external circuit. If the potential of *P* is raised, the velocity with which the electrons strike it will increase, and when this velocity becomes great enough they will, by their impact, cause the emission of secondary electrons from the plate. These secondary electrons will be attracted to the more positive anode A. The net current of electrons,

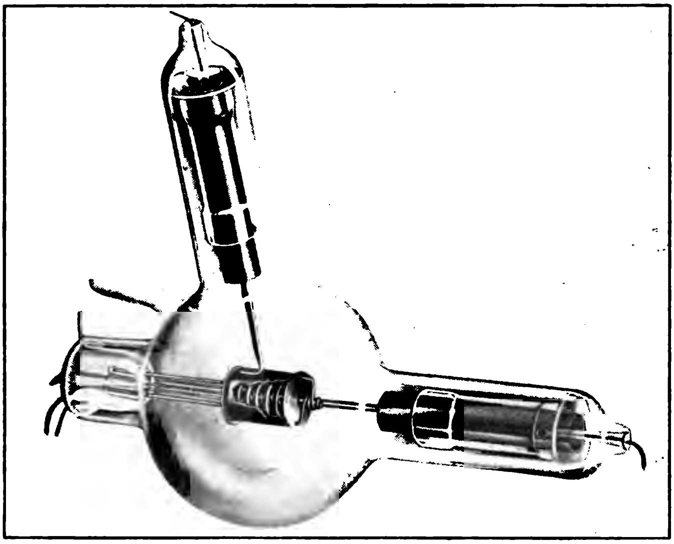

FIGURE 2—DYNATRON

received by the plate, is the difference between the number of primary electrons that strike and enter it and the number of secondary electrons which leave it. The number of primary electrons depends on the temperature of the filament and is practically independent of the voltage of the plate. The number of secondary electrons, however, increases rapidly with the voltage difference between plate and filament, and may become very much larger than the number of primary electrons; that is, each primary electron may produce several secondary electrons, as many as twenty in some cases.

The result is the characteristic voltage current relation shown in Figure 3. The abscissas represent voltages of the plate with respect to the negative end of the filament. The ordinates represent current in the plate circuit, reckoned positive for electrons passing from filament to plate, i. e., in the direction that is equivalent to positive electricity flowing from high potential to low across the vacuum. It is seen that, for low voltages, the current is very small, since only those electrons which come from the most negative end of the filament are able to reach the plate. As the voltage is increased, the current increases rapidly, and at about 25 volts, the plate is receiving the full primary current from the whole filament. For all higher voltages, this primary current remains essentially constant.

FIGURE 2, a

FIGURE 2, b

FIGURE 2, c

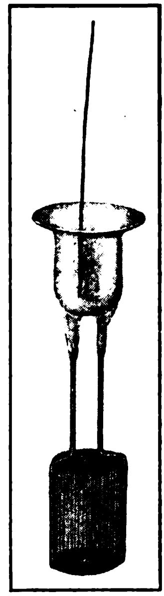

FIGURE 2, d

When the voltage is raised above 25 volts, however, the second factor becomes important. The primary electrons strike the plate with sufficient energy to cause the emission of secondary electrons, and this emission increases rapidly with the voltage, hence the *net* current to plate decreases rapidly. At 100 volts the number of secondary electrons leaving the plate is equal to the number of primary electrons entering it, so that the net current received by the plate is zero. As the voltage further increases, the number of secondary electrons becomes greater than the number of primary electrons, and the plate suffers a net loss of electrons; that is, the current is in the opposite direction to the impressed voltage. When the voltage is still further increased, a point is reached at which the anode is no longer sufficiently positive to carry away all the secondary electrons from the plate, and the current to the plate again becomes zero, and then rapidly rises to a value corresponding to the number of primary electrons.

It is evident from Figure 3 that over the range A to C, that is, between 50 and 150 volts in the case here represented, the current in the dynatron decreases almost linearly with increase

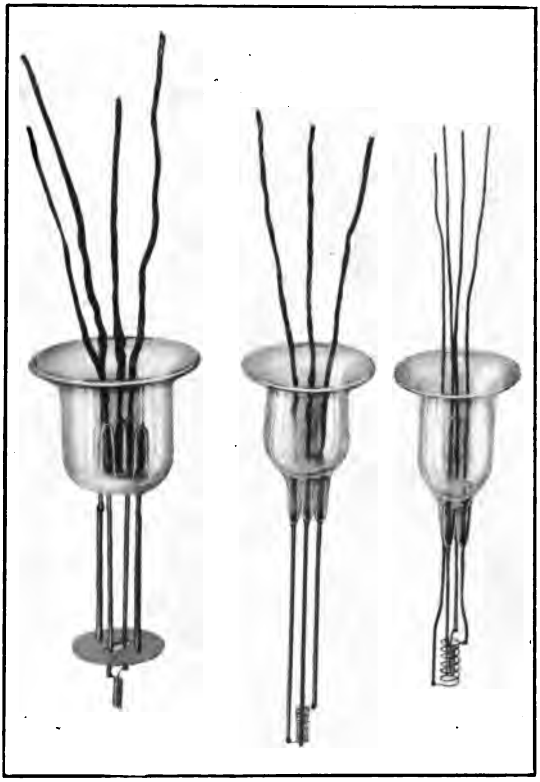

FIGURE 2, e FIGURE 2, f FIGURE 2, g

of voltage, and obeys the equation $i=\frac{E}{\bar{r}}+i_o$, where i_o and $\bar{r}$ are constants, $\bar{r}$ being negative. Since the constant i_o does not affect the variable part of the current in any of the applications for which the dynatron has been used, it is convenient to characterize the dynatron by the constant $\bar{r}$, which will be called its *negative resistance*. The justification for this name is that the behavior of the dynatron in any circuit containing resistance, capacity, inductance and electromotive force can be accurately calculated by treating the dynatron as a linear conductor with negative resistance $\bar{r}$. Examples of such calculations are given below.

The term i_0 in the above equation disappears if the dynatron is connected in series with a battery, of voltage equal to that at which the dynatron current is zero (point B, Figure 3). The combination is a *true* negative resistance, for which $i = \frac{E}{\bar{r}}$. For example, if the dynatron of Figure 1 be put, with its batteries, in a box, and two wires be brought out thru the box as terminals,

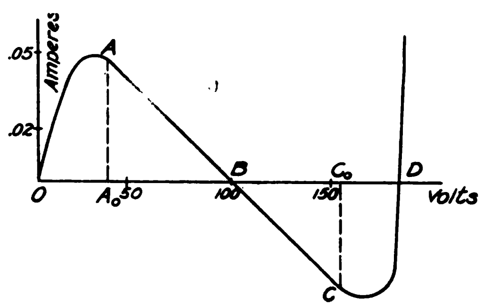

FIGURE 3

one from the plate P and one from a point V_0 of the battery corresponding to the point B of Figure 3, this "negative resistance box" would behave in all respects like a conductor with negative resistance, over the range of voltage, positive and negative, represented by BC_0 and BA_0 in Figure 3.

The magnitude of the negative resistance, which is the slope of the current voltage curve, Figure 3, and the range of voltage $A_0 - C_0$ over which it can be used, depends upon the anode voltage, the temperature of the filament, and, to some extent, on the shape and material of the electrodes. The effect of varying anode voltage alone is shown for two different types of tube in Figures 4 and 5, and the effect of varying filament temperature in Figure 6. It is seen that the effect of varying anode voltage is, in general, to shorten or lengthen the range of the negative resistance part of the curve, without changing the value of the negative resistance. A slight shift in the voltage V_0 at which the curves cross the axis is, for one tube, to the right with increasing voltage, and for the other, to the left. It is therefore to be anticipated that with proper construction, this shift could be made accurately zero, and the operation of the tube be independent of the value of anode voltage over a wide range. Varying the filament temperature, on the other hand, changes the negative resistance only, without affecting the range or the value of V_o. This affords a simple means of adjusting the negative resistance to any desired value, but at the same time imposes a condition upon the uniform operation of the tube, namely, that the temperature of the filament be kept constant.

It will be noticed that the negative slope of the curves in Figure 4 is less straight than those of Figure 5. This is a disadvantage where exact balancing of positive and negative resistance is desired, but for some of the purposes of radio work to be described later, it is an advantage. The degree of curvature depends upon the construction of the tube, and may be made anything that is desired.

4. Dynatron in Circuit Containing Positive Resistance
A. Series Connection. Circuit with Zero Resistance

If the dynatron is connected in series with a circuit containing positive resistance, the total resistance of the circuit is the

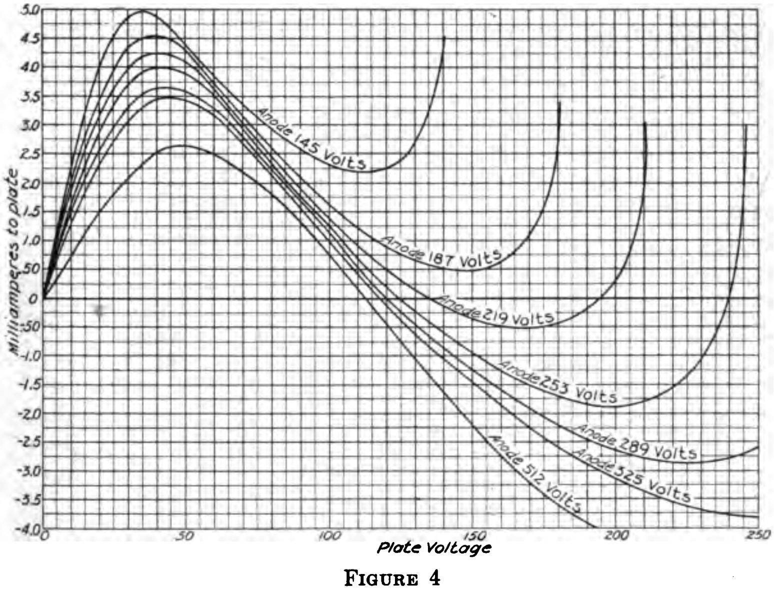

FIGURE 4

algebraic sum of the positive and negative resistances, and may be made as small as desired by making the positive and negative resistances nearly equal. Such a circuit has very interesting properties. For, while the total resistance of the circuit is very small, that of its parts, individually, is not. Hence a small change in the e.m.f. applied to the whole circuit will cause a

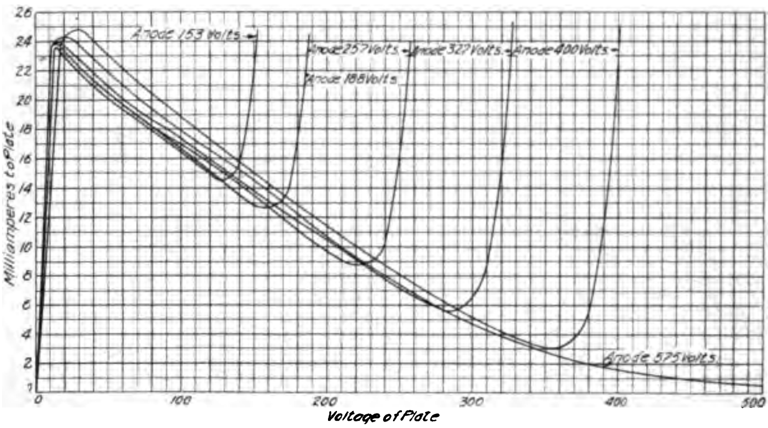

FIGURE 5

comparatively large change in current, and therefore in the *iR* drop across each part separately; i. e., the circuit acts as a voltage amplifier.

The connections are shown in Figure 7. An ohmic resistance R is connected in series with a dynatron of negative resistance $\bar{r}$, the battery terminal of the dynatron being connected at the point V_o corresponding to the voltage at which the dynatron current is zero.[2] (B, Figure 3.) If an e.m.f. E be impressed across the combination, causing a current I to flow and a voltage drop e_1 in the ohmic resistance and e_2 in the dynatron, then

[2] The amplification of *voltage changes* remains the same if the battery terminal of the dynatron is at some other point than that corresponding to the point B in Figure 3, provided it be in the range $A-C$ (Figure 3) over which the dynatron curve is straight. In that case the equations are

$$e_1 = I R$$

$$e_2 = I \bar{r} - I_o \bar{r}, \text{ where } I_o \text{ is a constant}$$

hence

$$E = I(\bar{r} + R) - I_o \bar{r}$$

$$\frac{de_1}{dE} = \frac{R}{R+\bar{r}}, \text{ that is}$$

voltage changes are amplified in the ratio $\frac{R}{R+\bar{r}}$.

$$e_1 = I R$$

$$e_2 = I \bar{r}$$

Hence $$E = I(\bar{r} + R),$$

and $$\frac{e_1}{E} = \frac{R}{\bar{r} + R}$$

is the ratio of voltage across the ohmic resistance to total voltage, that is, the voltage amplification. This can evidently be made as large as desired by making $\bar{r}$ and R nearly equal, since $\bar{r}$ is negative.

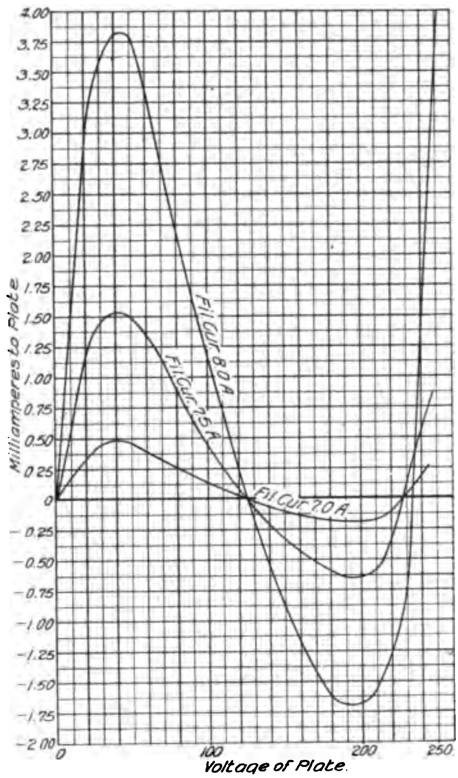

FIGURE 6

These relations may be clearly seen in the graphical representation of Figure 8, where the three curves marked e_1, e_2 and E represent the current-voltage relation in the ohmic resistance, the dynatron, and the total circuit respectively.

With constant batteries, an amplification ratio of 1000-fold can easily be maintained. For example, if R represents a high resistance galvanometer of 2,000 ohms or more, an e.m.f. of 0.01 volt impressed at the terminals of the combination will cause an e.m.f. of 10 volts across the galvanometer, with corresponding amplification of galvanometer current.

Further examples and applications of this principle to radio work are given in a later section.

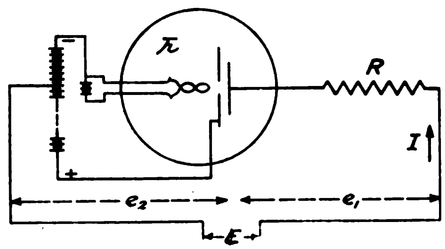

FIGURE 7

B. PARALLEL CONNECTION

If the dynatron is connected in parallel with a circuit containing positive resistance, the total conductivity of the circuit' which is the sum of the positive and negative conductivities of

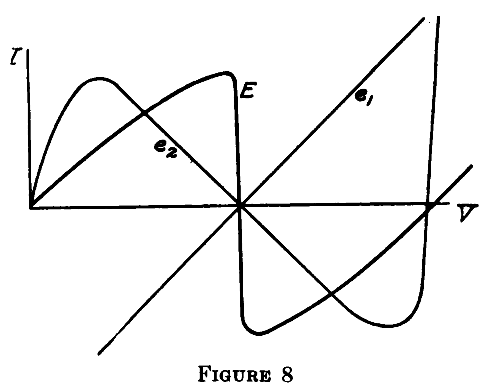

FIGURE 8

its parts, can be made very small. The circuit then acts as a current amplifier. The connections are shown in Figure 9. The total current I is the sum of the current i_1 thru the positive resistance and i_2 thru the dynatron.

Hence $$I = i_1 + i_2 = E\left(\frac{1}{\bar{r}} + \frac{1}{R}\right)$$

$$\frac{i_1}{I} = \frac{\bar{r}}{\bar{r}+R},$$ which may be made very large by making $\bar{r}$ and R nearly equal.

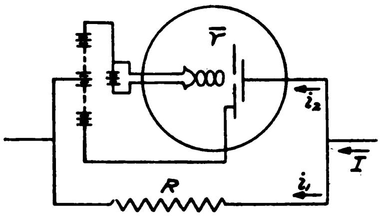

FIGURE 9

These relations are shown graphically in Figure 10, where the curves marked i_1, i_2 and I represent the current-voltage relation in the positive resistance, the dynatron, and the total circuit respectively.

The current I to be amplified may be that thru a photo-electric cell, a kenotron, or any other non-inductive device the current of which is independent of voltage.

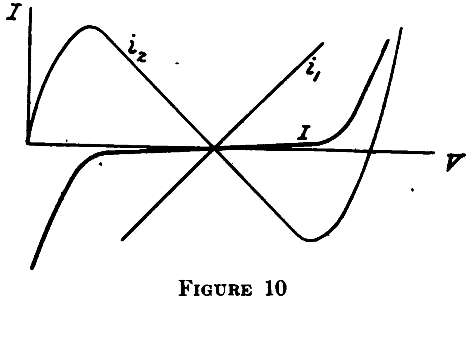

FIGURE 10

5. Dynatron in Circuit Containing Resistance, Inductance, and Capacity

If the dynatron be left open-circuited, as in Figure 1, it is unstable. This was to be expected as a necessary accompaniment of "negative resistance," and can easily be seen from the current-voltage relation in Figure 3. For when the voltage is greater than that corresponding to the point B, the plate is losing electrons, and hence becoming more positive; and the more positive it becomes, the more rapidly it loses electrons, until the point C is reached. Above C it continues to lose electrons, but more slowly, until it reaches the potential D at which it is in equilibrium. In like manner if the initial potential of the plate is less than B, it will continue to receive electrons until its potential has fallen to 0. At B the plate is in equilibrium, but the equilibrium is unstable, and if slightly disturbed, it will go to 0 or D.

The same instability occurs if the circuit of Figure 1, instead of being left open, is closed thru too high a resistance, so that the rate at which the plate receives electrons is greater than the rate at which these electrons can flow away thru the resistance. In this case the equilibrium voltages will not be D and 0, but some voltage in the range $D\ C_o$, and $O\ A_o$ respectively. This behavior may be strikingly shown by connecting a voltmeter between filament and plate, and opening the circuit. In this case the stable positions are 0 and a point just below D, and if the plate is originally at B, it will jump to either one or the other of these positions, depending on chance.

If the circuit contains inductance and capacity, as well as resistance, a similar action takes place. The plate charges up thru the vacuum, at a rate depending on the capacity and negative resistance, and discharges thru the circuit at a rate depending on the inductance and positive resistance. If the inductance is too high, the plate will receive electrons more rapidly than they can flow away thru the inductance, and will charge up to some point beyond A or C at which the rate of charge and discharge are instantaneously equal. The inertia of the inductance will then carry it backward toward B, and if the resistance is not too great it will pass thru B and oscillate continuously. Whether the circuit will oscillate continuously, or come to rest at B, or come to rest at some other voltage between 0 and D, depends on the relations between inductance, positive and negative resistance, and capacity. These relations can best be given by mathematical analysis, as follows:—

Let the dynatron, with negative resistance $\bar{r}$, be connected in series with a circuit containing inductance, L, resistance R, and capacity C, as shown in Figure 11. Then, calling the instantaneous e.m.f. across either part of the circuit E, we have:

For inductive part of circuit $I = \frac{E}{R} - \frac{L}{R}\frac{dI}{dt}$

For condenser $$I+i=-C\frac{dE}{dt}$$

For dynatron $$i=\frac{E}{\bar{r}}+i_o$$

which gives, eliminating E and i,

$$\frac{d^2I}{dt^2}+\left(\frac{R}{L}+\frac{1}{\bar{r}C}\right)\frac{dI}{dt}+\frac{1}{LC}\left(1+\frac{R}{\bar{r}}\right)I+\frac{i_o}{LC\bar{r}}=0$$

the solution of which is

$$I=\frac{i_o}{R+\bar{r}}+A\,\varepsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{1}{\bar{r}C}\right)t}\cos\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}-\frac{1}{2\bar{r}C}\right)^2}\,t-\alpha\right) \qquad (1)$$

if $$\left(\frac{R}{L}-\frac{1}{\bar{r}C}\right)^2-\frac{4}{LC}<0$$

and

$$I=-\frac{i_o}{R+\bar{r}}+A\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2\bar{r}C}\right)+\sqrt{\left(\frac{R}{2L}-\frac{1}{2\bar{r}C}\right)^2-\frac{1}{LC}}\right]t}$$

$$+B\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2\bar{r}C}\right)-\sqrt{\left(\frac{R}{2L}-\frac{1}{2\bar{r}C}\right)^2-\frac{1}{LC}}\right]t} \qquad (2)$$

if $$\left(\frac{R}{L}-\frac{1}{\bar{r}C}\right)^2-\frac{4}{LC}>0$$

where i_o, A, B, α are constants.

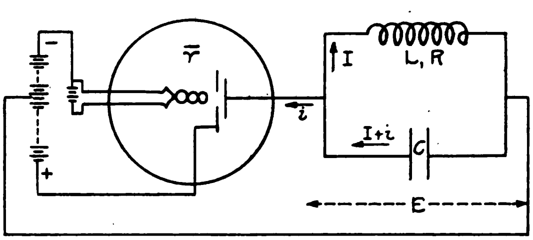

FIGURE 11

The case of most interest is the oscillatory solution, given by equation 1. This differs from the equation of a simple oscillatory circuit in that the damping factor is decreased from $\frac{R}{2L}$ to $\frac{R}{2L}-\frac{1}{2rC}$, where r represents the positive numerical value of

$\bar{r}$, and the period is increased by increasing the damping correction from $\left(\frac{R}{2L}\right)^2$ to $\left(\frac{R}{2L}+\frac{1}{2\bar{r}C}\right)^2$. It is identical in form with the equation of a circuit containing a leaky condenser, the positive leakage resistance of the condenser being replaced by the negative resistance $\bar{r}$ of the dynatron.

Two oscillatory cases are to be distinguished according as the damping factor is positive or negative. In the first case the circuit is stable, but its damping may be made as small as desired, so that an impressed oscillation will persist for a very long time. In the second case, the circuit will oscillate continuously, with an amplitude that would become infinite if the negative resistance held over an infinite range, and which is therefore limited by the length of the straight portion of the negative resistance curve.

The criterion that the circuit shall generate oscillations is that

$$\frac{R}{L}+\frac{1}{\bar{r}C}<0$$ or, if r denote the positive numerical value of $\bar{r}$,

$$Rr<\frac{L}{C} \qquad (3)$$

In order to test this relation, the inductance L in Figure 11 was made an air-core coil, and a secondary coil in series with a telephone was coupled loosely with it, in order to detect when the circuit was oscillating. With a definite value of negative resistance (determined by a separate experiment from the slope of the current-voltage curve) different capacities were introduced, and the maximum value of positive resistance was determined with which the circuit would still oscillate. The results are given in Table 1.

TABLE 1

R Ohms	r Ohms	L Henries	C Farads	Rr	L/C
75	3,000	0.689	2.90×10^{-6}	225×10^3	237×10^3
85	3,000	0.689	2.56×10^{-6}	255×10^3	269×10^3
96	3,000	0.689	2.26×10^{-6}	288×10^3	304×10^3
108	3,000	0.689	2.05×10^{-6}	324×10^3	334×10^3
126	3,000	0.689	1.75×10^{-6}	379×10^3	392×10^3
158	3,000	0.689	1.41×10^{-6}	475×10^3	487×10^3
204	3,000	0.689	1.12×10^{-6}	614×10^3	615×10^3
253	3,000	0.689	0.930×10^{-6}	760×10^3	725×10^3
78	6,520	0.689	1.27×10^{-6}	510×10^3	543×10^3
90	6,520	0.689	1.14×10^{-6}	587×10^3	602×10^3
116	6,520	0.689	0.90×10^{-6}	757×10^3	767×10^3
162	6,520	0.689	0.636×10^{-6}	$1{,}060 \times 10^3$	$1{,}080 \times 10^3$
354	6,520	0.689	0.294×10^{-6}	$2{,}310 \times 10^3$	$2{,}340 \times 10^3$
674	6,520	0.689	0.150×10^{-6}	$4{,}400 \times 10^3$	$4{,}600 \times 10^3$

According to theory, the maximum value of Rr should be very near to, but always less than $\frac{L}{C}$. It is seen that this relation is satisfied within the limits of experimental error. The values of Rr are all about 3 per cent. less than $\frac{L}{C}$, which is the limit set by the sensitiveness of the telephone with the permissible coupling.

The frequency of oscillation is given by the equation

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L} - \frac{1}{rC}\right)^2},$$

in which the bracketed term under the radical is, for most practical circuits, negligible. The range of possible frequencies which can be generated is determined by the above equation, together with the relation (3) between resistance, inductance, and capacity. The limit of radio frequency is set by the minimum value of capacity, positive resistance, and negative resistance, and can be calculated if the distributed capacity and inductance of the coils and connecting wires are known. An ordinary dynatron short-circuited by a couple of turns of heavy wire will give a frequency of about 20,000,000 cycles per second, and it is possible to go continuously from this to a frequency of less than 1 cycle per second by simply changing inductance and capacity.

The wave-form depends on the ratio of inductance to capacity and resistance. According to theory we should expect a perfect sine wave when $\frac{L}{C}$ is very nearly equal to Rr (since in this case the circuit fulfills the condition of simple harmonic motion), with increasing distortion as the ratio of $\frac{L}{C}$ to Rr increases. As this is a question of considerable importance, a series of oscillograms was taken with different ratios of $\frac{L}{C}$ to Rr. They are shown in Figure 12. The circuit is that of Figure 11, except that a secondary circuit is coupled inductively with the primary in order to show the form of the wave in a coupled circuit. In each photograph the upper curve gives the current in the coupled circuit, the middle curve the current in the primary circuit, and the lower curve a 40 cycle timing wave. Air inductance and paraffin condensers were used.

Films *A* to *D* show the effect of increasing the ratio $\frac{L}{C}$, keeping R and r constant. As $\frac{L}{C}$ increases the primary wave changes from a pure sine wave (film *A*) to a very slightly distorted wave (film *B*) and finally to a very badly distorted wave (film *D*). For comparison with curve *D*, film *E* was taken under the same conditions and the same frequency, but with a proper ratio of $\frac{L}{C}$. It is a good sine wave. It is to be noted that the oscillation in the coupled circuit is a fair sine wave, even when the primary is badly distorted.

6. Dynatron In Inductive Circuit With Impressed Periodic Electromotive Force

If a periodic e.m.f., represented by $e_0 \cos \omega t$ be impressed upon the circuit of Figure 11, the forced oscillations which it impresses upon the circuit may attain a much greater value than in a circuit containing no dynatron. This can best be seen from mathematical analysis. The equations of the circuit are:

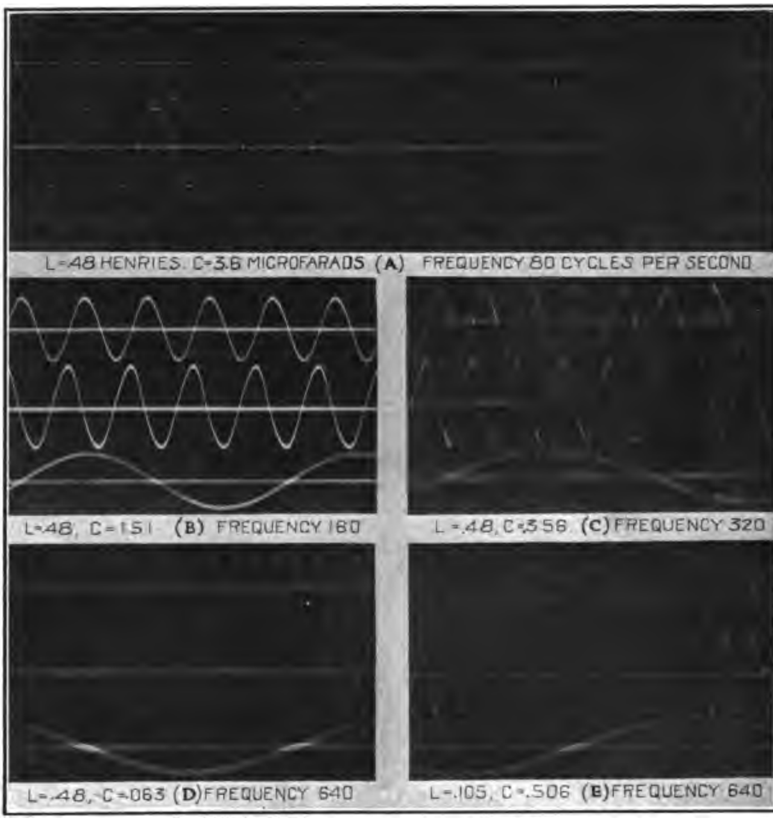

FIGURE 12—Effect of Capacity on Wave Form in Oscillating Dynatron

The middle curve in each film is the current thru the dynatron, the upper curve the current in the coupled circuit, the lower curve a 40-cycle wave for comparison

$$I\,R+L\frac{d\,I}{d\,t}=E-e_o \cos \omega\, t$$

$$i=\frac{E}{\bar{r}}+i_o$$

$$I+i=-C\frac{d\,E}{d\,t}$$

whence

$$\frac{d^2 E}{d\,t^2}+\left(\frac{R}{L}+\frac{1}{C\,\bar{r}}\right)\frac{d\,E}{d\,t}+\frac{1}{L\,C}\left(1+\frac{R}{\bar{r}}\right)E+\frac{i_o}{L\,C}\,(R+L)=\frac{e_o}{L\,C}\cos \omega\, t$$

and

$$E=-\frac{i_o(R+L)}{1+\frac{R}{\bar{r}}}+A\,\varepsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{1}{C\bar{r}}\right)t}\cos\left\{\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2}\,t-a\right\}$$

$$+\frac{e_o\cos(\omega t-\theta)}{\sqrt{\left(1+\frac{R}{\bar{r}}-LC\omega^2\right)^2+\omega^2\left(RC+\frac{L}{\bar{r}}\right)^2}} \qquad (4)$$

if

$$\frac{1}{LC}>\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2$$

or

$$E=-\frac{i_o(R+L)}{1+\frac{R}{\bar{r}}}+A\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2C\bar{r}}\right)^2+\sqrt{\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2-\frac{1}{LC}}\right]t}$$

$$+B\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2C\bar{r}}\right)^2-\sqrt{\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2-\frac{1}{LC}}\right]t}$$

$$+\frac{e_o\cos(\omega t-\theta)}{\sqrt{\left(1+\frac{R}{\bar{r}}-LC\omega^2\right)^2+\omega^2\left(RC+\frac{L}{\bar{r}}\right)^2}} \qquad (5)$$

if

$$\frac{1}{LC}<\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2$$

where A, B, a, and θ are constants, with the usual meanings.

In either case, provided $R<r$, the amplitude of the forced oscillations is

$$\frac{e_o}{\sqrt{\left(1+\frac{R}{\bar{r}}-LC\omega^2\right)^2+\omega^2\left(RC+\frac{L}{\bar{r}}\right)^2}}$$

and can be made as large as desired (since $\bar{r}$ is negative) by making

$$\left.\begin{aligned} Rr&=\frac{L}{C}\\ \text{and}\qquad \frac{R}{r}&=1-LC\omega^2\end{aligned}\right\} \qquad (6)$$

The first condition is equivalent to zero damping. The second shows that for maximum sensitiveness the frequency ω must be equal to $\sqrt{\frac{1}{LC}\left(1+\frac{R}{\bar{r}}\right)}$, which is the natural frequency of the system when its damping is zero.

It is to be noted that the sensitiveness of the system is the same whether the damping term $\frac{R}{L}+\frac{1}{C\bar{r}}$ is positive or negative. If it is positive, the natural oscillations of the system soon die out, leaving only the forced oscillation given by (4) and (5). If it is negative, the system will generate oscillations of its own of a frequency $\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2}$ slightly different from ω, in addition to the oscillations of frequency ω given by (5), and these two will produce heterodyne interference. The application of this to radio receiving is discussed below.

7. The Effect of a Magnetic Field

A profound change in characteristics is produced by placing the cylindrical type of dynatron shown in Figure 2 in a magnetic field parallel to the axis of the cylinder. The electrons from the filament, which in the absence of the magnetic field move in nearly straight lines to the anode and pass freely thru its holes (Figure 13 a), are constrained by the field to move in spirals, and strike the anode more or less tangentially (Figure 13 b), so that a much larger proportion are stopped by it. The

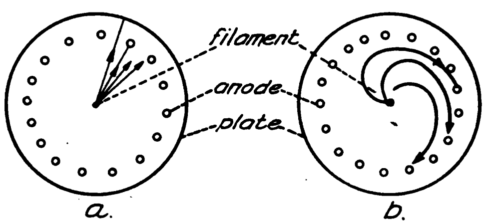

Figure 13 a   Figure 13 b

result is to diminish greatly the number of electrons reaching the plate. Superimposed upon this effect is a restraining effect of the field upon the secondary electrons which try to leave the plate, resulting in a change from negative resistance to positive resistance characteristic.

These effects are shown in Figure 14, where each curve rep-

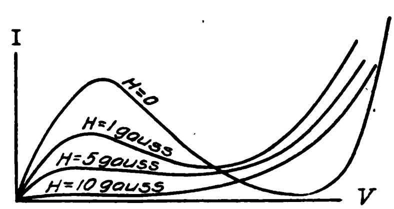

Figure 14

resents the voltage-current relation of the dynatron in a definite field. It will be seen that as the field increases the curves become lower and flatter, and soon lose their negative slope altogether. It is thus possible, by varying the magnetic field, to control the behavior of the dynatron. This method of control is especially applicable to the radiophone, as will be explained later.

8. The Pliodynatron

An electrostatic field may be used instead of a magnetic field to control the number of electrons reaching the plate. It has been shown (see Figure 6) that the effect of changing the number of electrons leaving the filament, by varying its temperature, is to change the negative resistance without affecting the other characteristics of the current voltage relation. If the temperature of the filament could be easily and rapidly changed, this would be an effective means of controlling the dynatron. The same result may be accomplished, however, by the electrostatic action of a grid close to the filament; that is, by the application of the pliotron principle. A dynatron which thus utilizes the pliotron principle is called a *pliodynatron*. Its construction is the same as that of the simple dynatron with the addition of a "control member," which may be a grid surrounding the filament (Figure 2, g) or a metal rod inside the (spiral) filament (Figure 2, f).

Its relation to the pliotron can be most clearly seen in the "plate type" of pliodynatron, a photograph of which is shown in Figure 15. It is identical in construction with the pliotron except for the addition of the perforated anode.

The characteristics of the pliodynatron can be seen from Figure 6, if for filament temperature we substitute grid potentials. The steepness of the curve increases, that is, the negative resistance decreases, with increasing grid potential. The relation is capable of more exact statement: It is known that in the pliotron, with constant anode voltage, the number of electrons leaving the filament is proportional to grid potential over a wide range, and this must be true in the pliodynatron, where the anode voltage is always constant. It may be shown, both theoretically and experimentally, that the negative resistance is inversely proportional, over a wide range, to the total number of electrons leaving the filament. The negative resistance is therefore inversely proportional to grid potential. The behavior of the pliodynatron in circuits containing resistance, inductance and capacity is therefore given by equations (1) to (6) if we replace R in these equations by $\frac{R_o}{v}$, where R_o is a constant and v the potential difference between grid and filament.

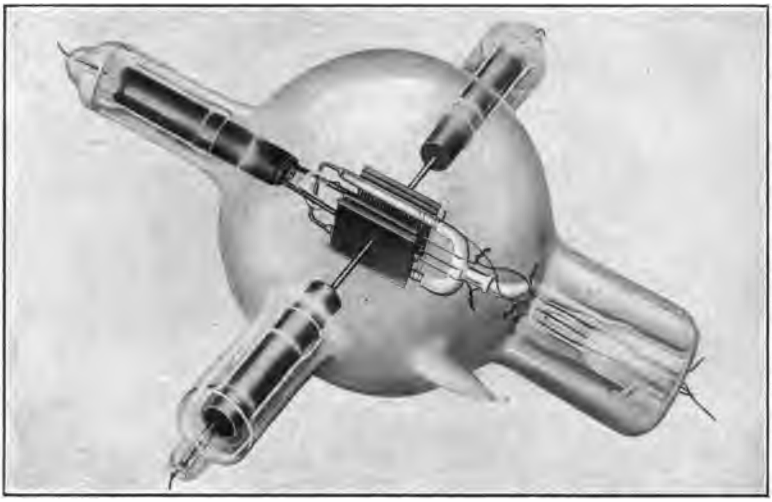

FIGURE 15—General Electric Company Pliodynatron

The negative resistance of the pliodynatron makes it a powerful amplifier. An increase of grid potential, by increasing the current thru the load in the plate circuit and hence the voltage drop over the load, lowers the voltage of the plate. In the pliotron this lowering of plate voltage tends to decrease the plate current, and thus opposes the effect of the grid. In the pliodynatron, however, a decrease in plate voltage means an increase in current, which may be very large if positive and negative resistance are nearly equal. This will be clear from Figure 16, where the curves marked v_1 and v_2 represent the current voltage relation for the grid voltages v_1 and v_2 respectively of a pliodynatron and a pliotron. If we start with an initial current of i_1, corresponding to plate voltage E_1, and raise the grid voltage from v_1 to v_2, the current tends to rise to i_2. On account of the decrease in plate voltage, however, the pliotron current will rise to some smaller value i', while the pliodynatron current will rise to a much larger value i''. The advantage to be gained in this way may be large, if the resistance in the circuit is high. For example, the maximum aperiodic voltage amplification thus far obtained with a pliotron is about 15-fold, while with a pliodynatron we have obtained 1000-fold.

A better method of representing the characteristic behavior

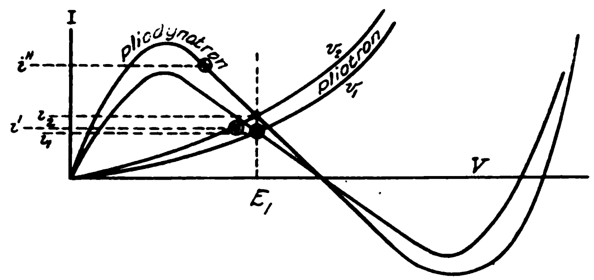

FIGURE 16

of the pliodynatron is, instead of plotting the current to the plate against plate voltage, to plot it against the total voltage across plate and series resistance, as in curve *E*, Figure 8. A series of such plots, for different grid potentials, is shown in Figure 17. The voltage plotted is now constant, being that of

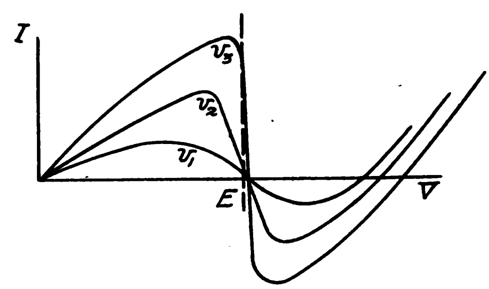

FIGURE 17

the battery, and for any given value *E* we obtain the currents corresponding to different grid potentials from the intersections of the curves with a vertical line thru *E*. If *E* is taken just to the left of the point where the curves cross the axis, the current will increase at first slowly and then very rapidly with grid potential, as shown in Figure 18. The amplification is, under these circumstances, both asymmetric and high, and the tube should constitute a good radio receiver. This is discussed more fully in Section 14 below.

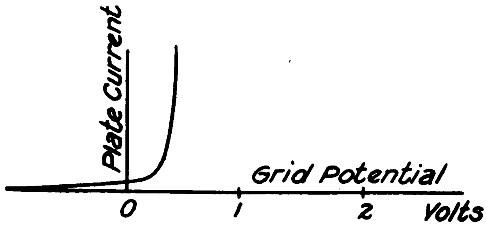

FIGURE 18

APPLICATIONS OF THE DYNATRON TO RADIO WORK

9. DYNATRON AS GENERATOR OF RADIO WAVES

It has been shown in section (5) that the dynatron always oscillates providing $Rr < \frac{L}{C}$, where R and r are the positive and negative resistance, respectively, of the circuit, L the inductance and C the capacity. The frequency of oscillation is approximately $\frac{1}{2\pi\sqrt{LC}}$, and may be given any value from 1 to 10,000,000 by changing inductance and capacity alone. It has also been shown that for low frequencies the oscillations are very nearly pure sine waves provided $\frac{L}{C}$ is not too great compared with Rr. Theory indicates that this should be true for all frequencies, and a search for harmonics at radio frequencies has verified the expectation.

The dynatron therefore satisfies all the requirements of a radio generator, and has the advantage that its operation is invariable and free from lag, and that the frequency may be given any value by changing a single inductance or capacity. Its oscillations may be controlled either by opening and closing the main circuit, or by changing any one of the four factors L, C, R, and r in accordance with the condition of oscillation given above. Its efficiency is low, probably less than 50 per cent. under best conditions. This is not, however, a serious limitation, except as regards the cost of power, since the tubes are capable of running very hot without deterioration. The maximum output at radio frequency of the tubes thus far constructed is about 100 watts, but no effort has been made to develop a high power tube.

It is generally necessary to transform the radio energy by means of a coupled circuit. In the discussion thus far the effect of such a coupled circuit on the oscillation has been neglected. The calculation for the case of inductively coupled circuits is not easy, but it may be shown experimentally that conditions similar to those derived above hold, even when the coupled circuit absorbs nearly all of the energy.

10. Pliodynatron as Radio Telephone

The simplest method of controlling the oscillations of the dynatron is to vary the negative resistance, by means of a grid around the filament, as in the pliodynatron. It has been shown in Section (8) that the negative resistance of the pliodynatron is inversely proportional to grid potential. Hence if the ratio of inductance to capacity and resistance be initially just large enough to produce oscillation (which is also the condition for producing pure sine waves), a slight decrease in grid potential will stop the oscillations.

If the negative resistance part of the pliodynatron curve, instead of being straight, is curved like that of Figure 4, the oscillations will not fall abruptly from full value to zero when the grid potential is reduced beyond the critical value, but will be gradually reduced in amplitude as the grid potential is decreased. This is exactly what is required for the radiophone and it is easy to make pliodynatrons which have this characteristic.

The connections are shown in Figure 19. The oscillating circuit is the same as Figure 11, except that the dynatron is replaced by a pliodynatron, and is coupled inductively to the antenna. A microphone M, coupled thru the transformer T to the grid circuit of the pliodynatron, serves to control the amplitude of the oscillations. A battery of a few volts, between grid and filament, keeps the grid always negative with respect to filament.

It is found that, with a proper ratio of inductance to capacity, the amplitude of the radio waves is very nearly proportional to the grid potential, and hence to the instantaneous displacement in the vocal wave. This was proved for constant grid potential by means of a hot wire ammeter in the antenna circuit, and for alternating grid potentials by impressing a sine wave on the transformer *T*, and observing the form of the rectified radio waves in a coupled circuit containing a kenotron rectifier and oscillograph.

Under these circumstances, it was found that speech transmitted to the microphone *M* and received at a station a few miles distant suffered very little more distortion than in the ordinary wire telephone. With a small tube giving about 10

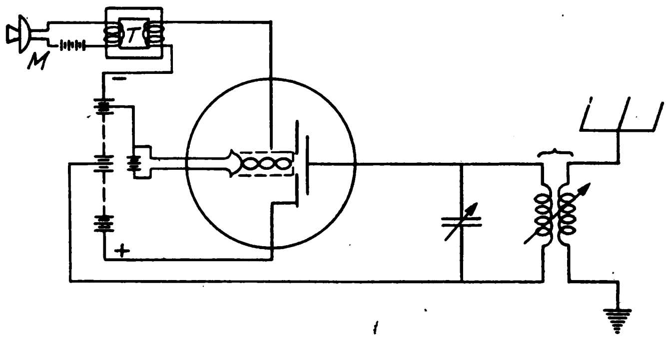

FIGURE 19

watts, it was possible to radiophone 16 miles (26 km.) with good intensity and articulation. No attempt has been made to telephone greater distances, or to develop high power pliodynatrons. The maximum output of a single tube which it has been possible to control thus far is about 60 watts.

11. MAGNETICALLY CONTROLLED DYNATRON AS RADIO TELEPHONE

Instead of controlling the negative resistance by a grid, as in the pliodynatron, we may use a magnetic field, as explained in Section (7). It is seen from Figure 14 that the change both in slope and amplitude of the negative resistance portion of the curves is a continuous function of the magnetic field strength. Hence if the magnetic field coil is connected in series with a microphone, the amplitude of the radio oscillations may be controlled by the voice, as in the pliodynatron. The energy needed to set up a magnetic field of the required strength is small, and can easily be furnished by the microphone circuit, but the impedance of the coil tends to choke out the higher voice frequencies.

12. Dynatron as Amplifier and Detector

It has been shown in Section 6 that a small periodic electromotive force impressed upon a circuit containing a dynatron may be amplified in any desired ratio by properly adjusting the capacity and inductance of the circuit; that is, the resonant value of current or voltage in the dynatron circuit is infinite, except as it is limited by the length and straightness of the dynatron curve. The impressed oscillations may be radio oscillations in an antenna coupled with the dynatron circuit, and the amplified voltage or current be used to operate a detector. It is important to notice that the energy consumed in the detector does not decrease the amplification, since the dynatron can be adjusted just to neutralize this loss, in addition to the other losses in the oscillating circuit. The simplest examples are when the detector losses are of a pure resistance nature, as, for example, when a high resistance galvanometer, such as one of the Einthoven type, is inserted in the oscillating circuit, or an audion with leaky grid, the leakage being proportional to voltage, is connected across any part of the oscillating circuit. In these cases, equation (4) of Section 6 applies directly, the positive resistance R being the total resistance of the circuit, including galvanometer and grid. In the cases where the detector is inductively coupled to the oscillating circuit, the impedance due to the coupling is equivalent to a resistance, so that similar relations hold.

Since the amplitude of the "resonant current" in the dynatron circuit is limited by the length and straightness of the negative resistance curve, it is evident that if we operate the dynatron in a region very near one end of the curve, as at A or C, Figure 3, the current will be asymmetric, and the dynatron may itself be used as a detector. Suitable connections are shown in Figure 20, where a telephone T with condenser C' across its terminals is inserted directly in the dynatron circuit. The distributed capacity between turns of the telephone offers low resistance to radio frequencies, so that the conditions of amplification discussed above still hold. But the high inductance of the telephone will, according to condition (3) of Section 5, cause the circuit to oscillate at audio frequency, unless its resistance be very high, or a condenser C', of suitable capacity, be connected across its terminals.

The circuit shown in Figure 20 has two advantages, in addition to its high amplification, viz.:

1. The ratio of inductance to capacity may be adjusted

so that the circuit oscillates with natural frequency very near that of the radio waves, as explained in Section 6, thus producing heterodyne beats.

2. The capacity C' and negative resistance $\bar{r}$ may be so adjusted as to neutralize the resistance of the telephone for a particular audio frequency, determined by the product of C' and telephone inductance, and if this frequency be made the same as the group frequency of the incoming radio waves, the sensitiveness becomes very great.

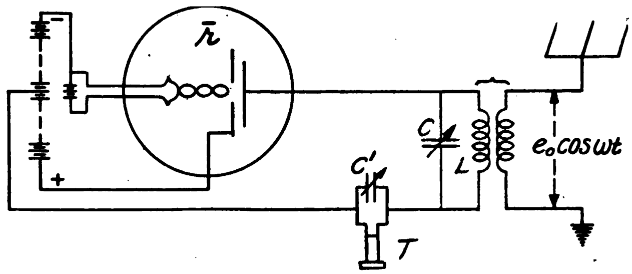

FIGURE 20

These predictions have been verified separately by experiment. In order to test the behavior of the complete circuit, it was set up as in Figure 20, and its reception of signals from a small spark set compared with that of a sensitive audion. For very weak signals the audion was the more sensitive, indicating small asymmetry in the dynatron oscillation. For medium signals, however, the dynatron response was many times stronger, and its intensity could be increased to almost any degree by adjustment of the capacity C'.

It is interesting to note that the coupling in a circuit like that of Figure 19 may be made very close without affecting the selectivity, since the condition for selectivity, viz.: a small damping factor, still holds. This is true both for the antenna coupling and that of the auxiliary detecting circuit, when one is used. The fact that sensitiveness and selectivity are independent of both resistance and coupling coefficient makes it possible to use a much more effective ratio of transformation than has hitherto been practicable.

13. USE OF DYNATRON FOR NEUTRALIZING RESISTANCE IN RADIO CIRCUITS

The negative resistance of the dynatron may be utilized to supply the energy losses of whatever nature, in any circuit, and the circuit thereupon behaves, as regards selectivity, damping, and sensitiveness to external stimuli, like a circuit having zero resistance. The amount of energy fed into the circuit by the dynatron is i^2r, where r is the negative resistance and i the current (steady value or r.m.s.) thru the dynatron. Examples of this use of the dynatron in simple circuits containing resistance, inductance and capacity have already been given in Sections 3 to 6. Two further examples will illustrate its use in circuits where the resistance characteristic is more complex.

(a) Dynatron in Plate Circuit of Pliotron for Aperiodic Amplification.

The current thru the pliotron, for constant grid voltage, increases with increasing voltage of the plate, that is, it has the characteristic of a positive resistance, which limits its amplifying power as explained in Section 8. This resistance characteristic may be neutralized by connecting a dynatron in parallel with the pliotron, as in Figure 21. Using a pliotron whose "positive

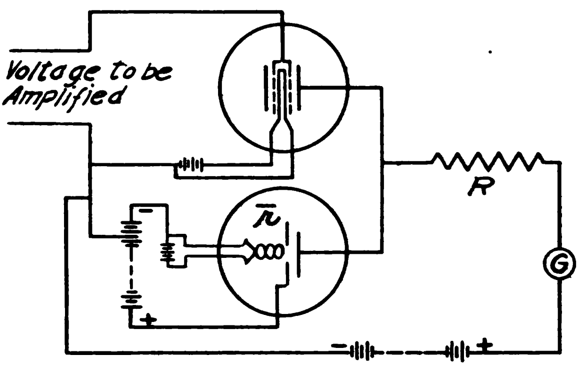

FIGURE 21

resistance" was 100,000 ohms, and a series resistance of 250,000 ohms in the circuit, we were able in this way to increase the d. c. voltage amplification from 12-fold, for the pliotron alone, to 625-fold. A further advantage in this connection is that the dynatron can be operated at such a voltage that its current is just equal and opposite to that of the pliotron, so that the total current thru the circuit is zero. This allows the use of a more sensitive measuring instrument.

(b) Dynatron in Grid Circuit of Pliotron Detector.

The increase in voltage of the grid of a pliotron detector is opposed by a leakage current which increases with voltage, as in a positive resistance, and also by the counter e. m. f. and losses in its own and the coupled antenna circuit. These losses may be neutralized by a dynatron in parallel with the grid, as in Figure 22. With this arrangement the intensity of weak signals from a spark set was increased from audibility to a roar.

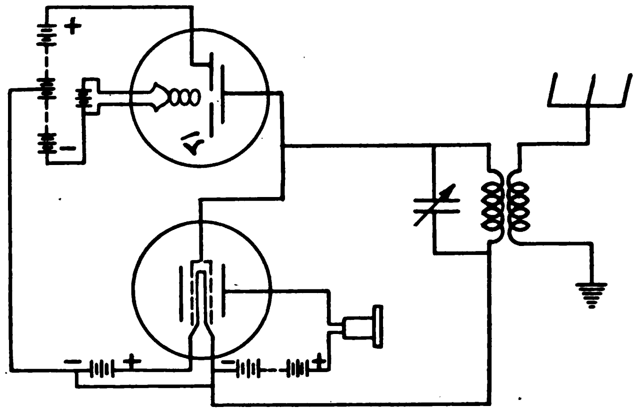
FIGURE 22

The dynatron, instead of being connected directly to the grid of the pliotron, may be in a separate circuit which is inductively coupled to any part of the grid or antenna circuit.

14. PLIODYNATRON AS AMPLIFIER AND DETECTOR

It has been shown in Section 8 that a pliodynatron in series with a suitable resistance is capable of producing an aperiodic voltage amplification of 1000-fold. To maintain this amplification requires constant batteries and continuous attention. A value of 100-fold, is, however, very easy to maintain. By connecting two pliodynatrons in series a total amplification of 10,000-fold has been obtained. With this amplification it should be possible to receive radiograms on an aperiodic antenna.

This arrangement of pliodynatron and positive resistance is equally applicable to a tuned antenna circuit. The connections are shown in Figure 23. The telephone itself furnishes sufficient resistance, and a condenser C' connected across the telephone is adjusted so that its capacity is just sufficient to keep the circuit from oscillating, according to condition 3, Section 5. With this connection, the amplification is asymmetric, i. e., different for positive and negative variation in grid potential, as shown in Figure 18. To increase the selectivity, a circuit LC, tuned for radio frequency may be included in series with the telephone, and either adjusted to the verge of oscillation, or allowed to generate oscillations for heterodyne work. The telephone should, in case of radiograms, be tuned for the

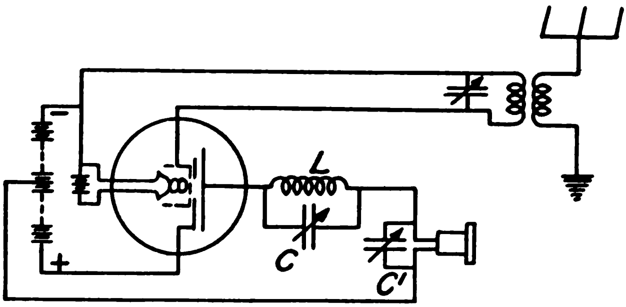

FIGURE 23

group frequency of the signals. It may then be brought to the verge of audio-oscillation by adjusting the negative resistance, and the final adjustment for radio sensitiveness be made by varying the ratio of L to C, keeping their product constant.

In the circuit of Figure 23 all the losses may be compensated, in the manner just described, except those in the grid circuit and antenna. Figure 24 shows a modification of the circuit of Figure 23 in which the grid and antenna losses also are compensated. The modification consists in connecting the grid, not to the filament, but to a properly chosen point P on a resistance R in series with the plate. The pliodynatron is then operated

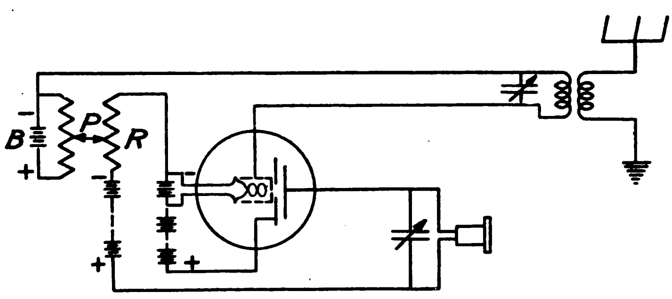

FIGURE 24

at such voltage that the current in the plate circuit is negative (between B and C, Figure 3), that is, positive electricity, or its equivalent, flows from filament to plate across the vacuum and thence thru the battery and resistance R back to filament. Raising the potential of the grid increases the current thru R, and raises the potential of P, which tends still further to increase the potential of the grid. By this mechanism, energy is fed back from the plate circuit, which may be adjusted to furnish any amount of energy desired, into the grid circuit, and by properly adjusting P, the amount of energy thus appropriated may be made just sufficient to neutralize the losses in grid and antenna, without causing oscillation. The antenna coupling should be close, and its resistance may be as large as desired. A potentiometer is shown connected across a battery B the voltage of which is equal to the normal drop in R, for keeping the grid potential constant during adjustment.

SUMMARY: A new, hot cathode, three electrode vacuum tube, the dynatron, is described. A constant, positive voltage is applied between the hot cathode and the perforated rugged anode. A supplementary anode is placed beyond the main anode, and is maintained at a lower positive potential than the main anode.

Because of secondary electronic emission from the supplementary anode, thru a certain range of applied voltages, the supplementary anode-to-filament circuit acts as a true negative resistance. Consequently the dynatron can be used as an oscillator at almost any desired audio or radio frequency or as a voltage or current amplifier. The theory of oscillation therefor is given, and experimentally verified.

The effect of magnetic fields on the value of the negative resistance is studied. The effect of inserting a true grid (thus producing a pliodynatron) is also considered. The latter device is not only an amplifier, but can readily be used as a controlled oscillator for radio telephony. In this connection, experiments are described.

The use of the dynatron as an amplifying detector and as a means for neutralizing circuit resistance is explained, as well as the similar employment of the pliodynatron. All receiver circuit losses can be compensated and selectivity retained at close coupling.

TELEPHONE RECEIVERS AND RADIO TELEGRAPHY*

By

H. O. Taylor, Ph.D.

(Thorogood–Taylor–Dubois Industrial Development and Research Corporation, Boston, Massachusetts.

A general knowledge of the characteristics and operation of the telephone receiver may be obtained by a study of the motional-impedance circle, an account of which was first published by Kennelly and Pierce in September, 1912.[1] Subsequent researches have been completed[2] and others are now being carried on by way of applying the microscope to different parts of the general proposition.

The motional-impedance of the telephone receiver at a given frequency of vibration is the difference between its impedance when the diafram is free to vibrate and its impedance when the vibration of the diafram is prevented; that is, it is the difference between the free and damped impedances of the receiver. Impedance may be measured by means of the ordinary impedance bridge shown in Figure 1. In work already done, vibratory electric current ranging in frequency from about 400∿ to about 2500∿ was supplied by the Vreeland oscillator, the potential across the bridge being kept constant at about 15 volts. The resistance and inductance of the receiver coils were balanced by adjusting the variable resistance and inductance in one of the bridge arms until silence was noted in the indicating telephone of the bridge.

Preparatory to finding the damped impedance, the receiver was mounted so that its diafram was in a vertical plane. The motion of the diafram was damped by connecting its center to the end of a small horizontal brass rod to the other end of which was fixed a suspended brass cylinder weighing 1,000 grams (2.2 pounds) or more. This damping mass was so supported as not to strain the diafram or change the air gap between it and the pole pieces. Impedance observations were then made for a number of frequencies of vibration of the exciting current between 400∿ and

*Received by the Editor July 15, 1917.

[1]Bibliography 2, pages 113–151.

[2]Bibliography 4, pages 421–482; and Bibliography 5, pages 415–460.

1500∿. The curves between damped resistance and damped reactance as ordinates and frequency of vibratory current as abscissa are given in Figure 2. If the damped reactances for

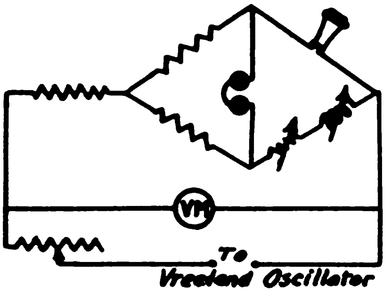

FIGURE 1

Impedance Bridge

given frequencies thruout the range are plotted as ordinates against the corresponding damped resistances as abscissas, the damped impedance curve shown in Figure 3 is obtained.

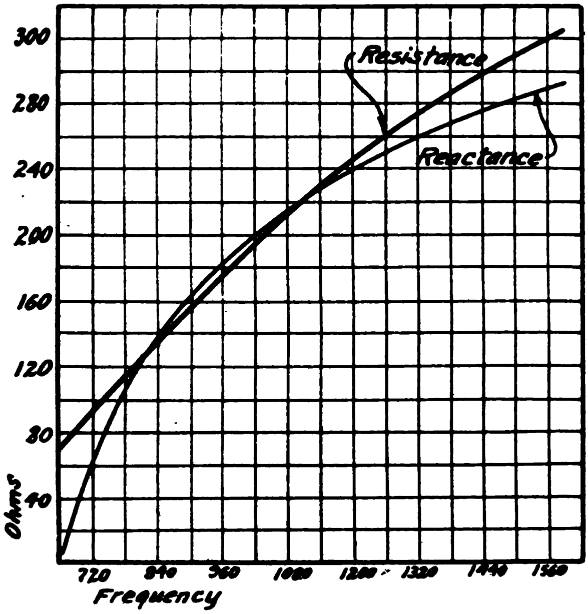

FIGURE 2

Damped Resistance and Reactance of Receiver Against Frequency

For the free impedance of the receiver, the damping mass was detached from the diafram, and impedance observations made at the same frequencies of vibration as before. The curves between free resistance and free reactance as ordinates and frequency of vibratory current as abscissa are given in Figure 4. If the free reactances for given frequencies thruout the range are plotted as ordinates against the corresponding free resistances as abscissas, the free impedance curve shown in Figure 5 is obtained.

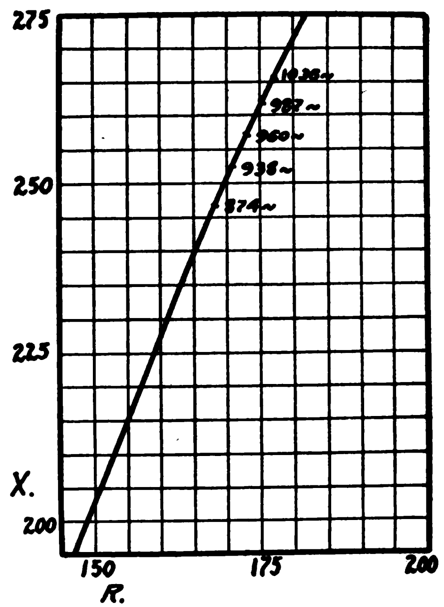

FIGURE 3
Damped Impedance

The differences between the free and damped impedances of the telephone receiver are shown in Figure 6, where the curve of free impedances is superimposed upon that of damped impedances. The straight lines joining the points of each curve corresponding to the same frequency represent the impedance at each frequency due entirely to the vibratory motion of the diafram. Each of these lines then represents in magnitude and direction the motional-impedance of the telephone recevier for the given frequency, and if drawn radially with one end at a single point as origin, the far ends of the lines will be observed to lie upon a circle. This circle, shown in Figure 7, is the motional-impedance circle of the telephone receiver.

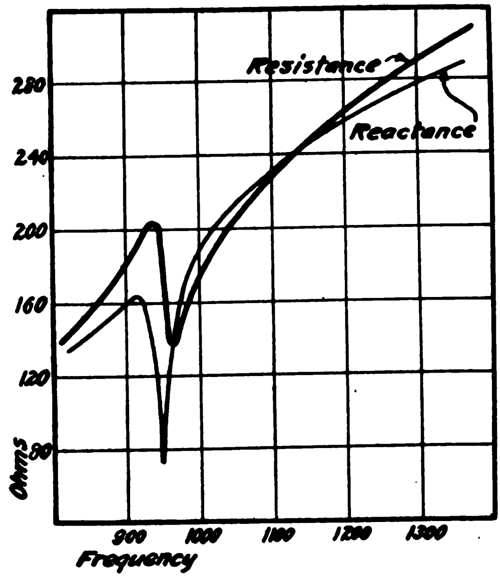

FIGURE 4

Free Resistance and Reactance of Receiver Against Frequency

As usually plotted, the difference between the free and damped reactances for a number of frequencies which include the resonance frequency of the diafram is plotted as ordinates against the difference between the free and damped resistances corresponding to the same frequencies respectively as abscissas. The points so located lie upon a circle and the chords drawn from the origin are the motional-impedances for the various frequencies. The diameter of the circle is the motional-impedance corresponding to the natural frequency of the diafram. The angle thru which this diameter is depressed below the resistance axis represents the lag of the vibration of the diafram behind the current, due to hysteresis and eddy currents and to the vibrational variation of the air gap between the diafram and pole pieces. The quadrantal points lie at the ends of the diameter drawn perpendicular to the impedance diameter and the frequencies, n_1 and n_2, corresponding to these points are used in the following equation in the computation of the damping factor, Δ, of the diafram:[3]

$$\Delta = \pi (n_2 - n_1) \qquad \text{hyp. rad./sec.} \qquad (1)$$

Since impedance has the dimensions of a velocity, and the curve of the motional-impedances of a telephone receiver is a circle, the inference is natural that the curve of the diafram velocities is also a circle. This conclusion has been found true experimentally. The equation of motion of the center of the diafram is

$$m\ddot{x} + r\dot{x} + s\,x = f = F \sin \omega t \qquad \text{dynes} \angle \qquad (2)$$

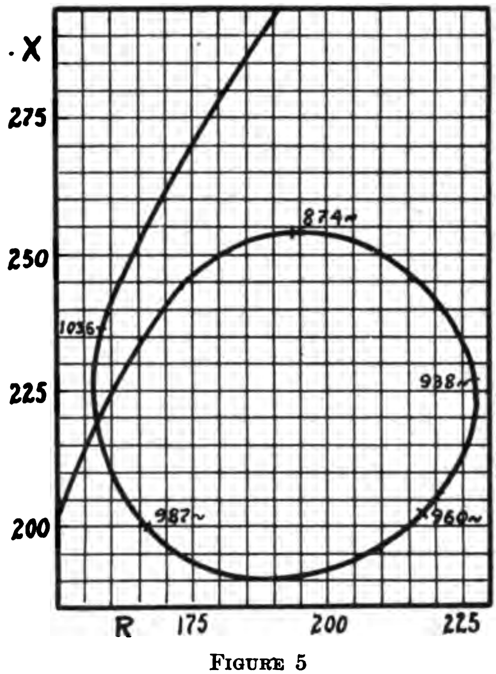

FIGURE 5
Free Impedance

where	m = equivalent mass of the diafram,	gms.
	r = mechanical resistance constant,	dynes/kine
	s = elastic constant,	dynes/cm.
	f = instantaneous pull,	dynes
	F = maximum pull at any frequency,	dynes
	x = displacement at center of diafram,	cm.

[3]For derivation, see Bibliography, 2, page 146.

The solution of this equation for velocity is

$$\dot{x} = \frac{f}{\sqrt{r^2 + \left(m\,\omega - \frac{s}{\omega}\right)^2}} \quad \text{cm./sec.} \angle \qquad (3)$$

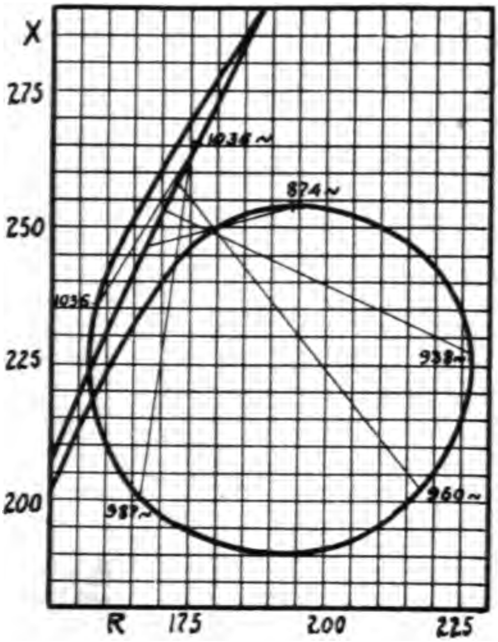

FIGURE 6
Impedance Due to Vibration of Diafram.

and the phase angle, a, between the pull, f, and the velocity, $\dot{x}$, is given by

$$tan^{-1}\,a = \frac{m\,\omega - \frac{s}{\omega}}{r} \qquad \text{numeric} \qquad (4)$$

where $\omega = 2\,\pi\,n$, $(n = \text{frequency})$.

If the velocity, $\dot{x}$, and the phase angle, a, are computed for a number of frequencies in the resonance region, and a curve is plotted in polar coordinates between a and $\dot{x}$, the points so found lie upon a circle called the velocity circle of the diafram, Figure 8. Velocity circles have been obtained experimentally by means of a diafram vibration explorer,[4] and show the same general characteristics as the motional-impedance circle.[5] Thus the immediate cause of the motional-impedance circle is shown to be the diafram velocity, and incidentally the deduction that impedance has the dimensions of velocity is upheld experimentally.

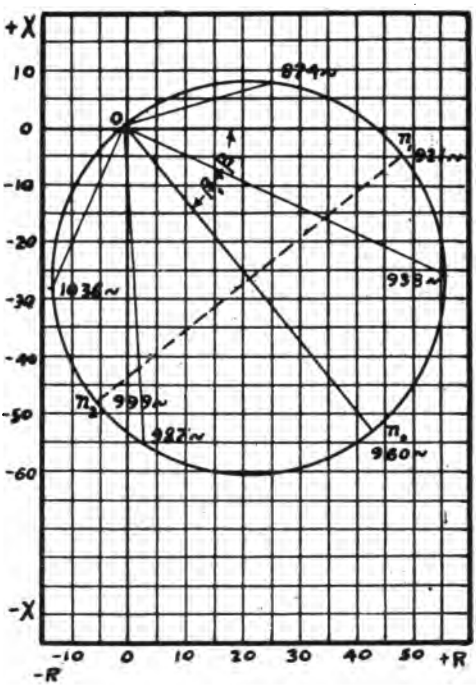

FIGURE 7
Motional-Impedance Circle

The phenomena of the vibration of the telephone diafram as shown by the velocity circle occur so rapidly that the eye cannot follow them. A similar phenomenon may however be produced slowly so that the eye can easily follow it by means of coupled torsion pendulums.[6] In a trial made, one of the coupled units consisted of a heavy bob attached to the end of a stiff wire and so fastened to the support that its frequency could

[4]Bibliography, 3, page 97.
[5]Bibliography, 4, page 451
[6]Bibliography, 5, page 439.

be varied by adjusting the length of suspension. The second coupled unit was a very small torsion pendulum suspended from the axis of the large bob. When vibrating, the steady state was soon reached when both pendulums had the frequency of the large one. As the frequency of the large pendulum was increased in steps by adjusting the length of its suspension, from a frequency less than the natural frequency of the small pendulum to one greater, the phase relation of the displacements of the two bobs passed from co-phase, thru quadrature at resonance, to phase opposition; and the amplitude of the small pendulum increased as resonance was approached, was a maximum at resonance, and then decreased as resonance was passed. The

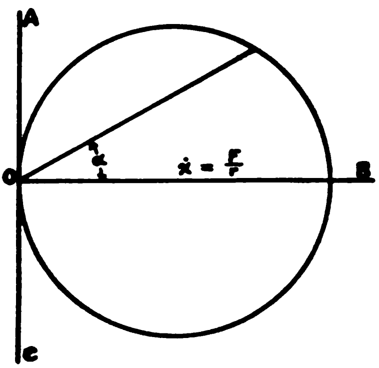

FIGURE 8
Velocity Circle of Telephone Diafram

displacement of the large pendulum corresponds to the force pulling on the diafram, and the velocity of the small pendulum corresponds to the velocity of the diafram. Vibrational velocity leads displacement by an angle of $\frac{\pi}{2}$, therefore, if the observed phase difference between the maximum velocity of the small bob and the maximum displacement of the large bob is plotted to polar coordinates against the velocity of the small bob as radius vector, an approximate circle is obtained—the velocity circle.

The intimate connection between the motion of the diafram and the motional-impedance circle makes it possible to use the latter in the determination of the motional constants, m, r, and s, of the diafram if one of them is known.[7] These constants have

[7] Bibliography, 4, page 472.

been found in so many cases that from the experience gained it is now possible for one to estimate them to a fair degree of accuracy from measurements made on the diafram, its diameter, thickness, and mass.

The equivalent mass, m, is defined as the mass which, when concentrated at the center of the diafram, exhibits the same kinetic energy as is exhibited by the actual distributed mass of the diafram. In the ideal case, the motion of the diafram is expressed by a Bessel's equation,[8] and the distribution of vibration amplitudes over the surface is such that the equivalent mass is given by[9]

$$m = 0.183M \qquad \text{mgs.} \qquad (5)$$

where M is the active mass (that included within the clamping ring).

This amplitude equation was derived by Lord Rayleigh,[10] and is expressed by the curves given in Figure 9. If x_m is the

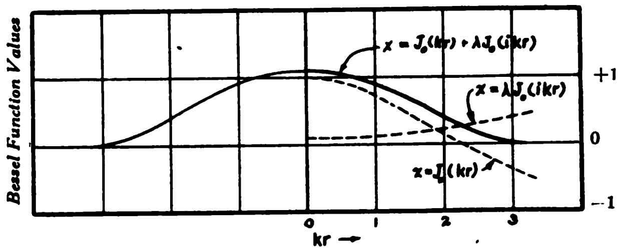

FIGURE 9
Amplitudes Along a Diameter of a Vibrating Diafram

maximum amplitude of a point at a distance r from the center of the diafram, the relation is given by

$$x_m = P[J_o(\mathrm{kr}) + \lambda J_o(i\,\mathrm{kr})] \qquad \text{cms.} \qquad (6)$$

where J_o is the Bessel's function of the zeroth order; k is a constant involving frequency of vibration, density, thickness and elasticity of the diafram; λ is a constant satisfying the boundary conditions; $i = \sqrt{-1}$; and P is the constant of amplitude ratio.

The equivalent mass as already given is found by using this

[8] Bibliography, 3, page 122.
[9] Bibliography, 3, page 131.
[10] Bibliography, 1, page 352.

equation in connection with the equation defining the equivalent mass of the diafram,[11] which is

$$m = \frac{2\pi\rho}{x_o^2}\int_o^a x_r^2\,r\,dr \qquad \text{gms.} \qquad (7)$$

where ρ = density, a = radius of diafram, x_o = maximum amplitude at the center and x_r = maximum amplitude at a distance r from the center.

In the vibration of telephone diaframs, the ideal distribution of amplitudes over the surface is departed from because of non-uniformities in density, in clamping, in force application, etc. In many cases, the amplitude at the center is reduced, thereby causing an increase in the equivalent mass; a working value for this may be taken approximately as

$$m = 0.3M \qquad \text{gms.} \qquad (8)$$

While individual cases depart from this value, it has nevertheless proved useful in preliminary work.

The elastic constant, s, of the diafram is the pull at the center in dynes required to produce a deflection of 1 cm. It is given by

$$s = (k\,t^3\,Y)/D^2 \qquad \text{dynes/cm.} \qquad (9)$$

where t = thickness of diafram, D = diameter, and Y = Young's modulus.

If the elastic constant and dimensions of one telephone diafram are known, then the elastic constant of any other telephone diafram of the same material may be found approximately by a simple proportion if its dimensions are given. In one instance, the constants of a telephone diafram were found to be

$m = 1.16$	grams
$D = 5$	cms.
$t = 0.03$	cm.
$s = 40 \times 10^6$	dynes/cm.

The natural frequency of the diafram is obtained from equation (3) when the vibrational velocity is a maximum. This occurs when

$$\omega_o^2 = \frac{s}{m} \qquad (\text{rad./sec.})^2 \qquad (10)$$

where $\omega_o = 2\pi n_o$

(n_o = the natural frequency of the diafram.)

Thus

$$n_o = \frac{1}{2\pi}\sqrt{\frac{s}{m}} \qquad \text{cycles/sec.} \qquad (11)$$

[11]Bibliography, 3, page 131.

In the case of the diafram cited, the natural frequency is

$$n = 935 \sim$$

Means are thus provided for giving an approximate idea of the value of the natural frequency of any telephone diafram when it is clamped in the usual way in a telephone receiver.

At this point it may be well to draw attention to the fact that diaframs do not always vibrate in a simple way but are often affected by non-uniformities in clamping which introduce vibrational systems superimposed upon that of the diafram. If a part of the boundary is not tightly clamped, the effect upon the vibration of the diafram has been found[12] to be the same as tho a piece of spring brass were attached to the center of the diafram, one end of the spring strip being left free to vibrate with the diafram. If the motional-impedance circle of the receiver with this spring attached to the diafram is taken, it is found that a re-entrant loop occurs at the frequency corresponding to the natural frequency of the attached spring; see Figure 10. This re-entrant loop occurs also in the velocity circle, as has been determined experimentally. If the natural frequency of this spring should be that of the diafram itself, then the amplitude at resonance, which should be the largest, would collapse to a value much smaller, the magnitude of the shrinkage depending upon the importance of the superimposed vibrational system; see Figure 11. The motional constants, m, r, and s, of this superimposed system may be computed in a way similar to that for the constants of the diafram. This superimposed vibrational system is related to the vibratory motion of the diafram somewhat as an infinite impedance loop is related to the alternating electric current circuit in which it is placed.

The mechanical resistance constant, r, of the telephone diafram is important as a regulator of the sharpness of tuning and resonant amplitude. The relation between the decrement and the resistance constant of the diafram is found from the well-known solution[13] of equation (2) when the right side is zero. This may be written:

$$\dot{x} = \frac{X \omega_o^2}{\omega_r} \varepsilon^{-\frac{r}{2m}t} \sin \omega_r t \qquad \text{cm./sec.} \angle \qquad (12)$$

$$= \frac{X \omega_o^2}{\omega_r} \varepsilon^{-\Delta t} \sin \omega_r t \qquad \text{cm./sec.} \angle \qquad (13)$$

[12]Bibliography, 5, page 434.
[13]Bibliography, 6, page 122.

where X = initial displacement; $\omega_o = 2\pi n_o$ where n_o = natural frequency for forced oscillations; and $\omega_r = 2\pi n_r$, where n_r = natural frequency for free oscillations.

Equations (12) and (13) indicate the relation between the logarithmic decrement[14] per second, Δ, and the resistance constant, r, which is

$$\Delta = \frac{r}{2m} \qquad \text{hyp. rad./sec.} \quad (14)$$

From equation (1),

$$\frac{r}{m} = 2\pi(n_2 - n_1) \qquad \text{cycles/sec.} \quad (15)$$

The range of resonance is defined as that portion of the peak of the resonance curve where the vibrational energy of the diafram is equal to or greater than one-half that at resonance;

[14] Following is a simple derivation:

$$x = x_o \varepsilon^{-\frac{r}{2m}t} \sin \omega_r t$$

where $\dot{x} = \frac{X\omega_o^2}{\omega_r}$ = the initial maximum vibrational velocity.

$$\omega_o = \sqrt{\frac{s}{m}} = \text{resonance angular velocity of forced oscillations.}$$

and

$$\omega_r = \sqrt{\frac{s}{m} - \frac{r^2}{4m^2}} = \text{resonance angular velocity of free oscillations.}$$

$$= \sqrt{\omega_o^2 - \Delta^2}$$

where $\Delta = \frac{r}{2m}$.

Let $\dot{x}_1, \dot{x}_2, \ldots$ be the values of $\dot{x}$ at times

$\frac{T}{4}, \frac{5T}{4}, \ldots$

where $T = \frac{2\pi}{\sqrt{\omega_o^2 - \Delta^2}}$ = periodic time.

These values of $\dot{x}$ will be the successive maximum velocities of the oscillatory motion in the same direction; and

$$\dot{x}_1 = \dot{x}_o \varepsilon^{-\frac{rT}{8m}}$$

$$\dot{x}_2 = \dot{x}_o \varepsilon^{-\frac{5rT}{8m}}$$

$$\cdots\cdots$$

From these equations, we have

$$\log \frac{\dot{x}_1}{\dot{x}_2} = \frac{rT}{2m}$$

This quantity, $\frac{rT}{2m}$, is the well-known logarithmic decrement. Dividing by time gives $\frac{r}{2m} = \Delta$, which is the damping factor, or logarithmic decrement per second. It is observed that the difference between the squares of the resonance angular velocities of forced and free oscillations is equal to the logarithmic decrement per second, Δ.

thus the range of resonance is the frequency interval defined by (n_2-n_1)[15]. The breadth of tuning is given by[15] $\frac{n_2-n_1}{n_o}$. By comparing this expression with equation (15), it is seen that the breadth of tuning for a given natural frequency depends upon the ratio of the mechanical resistance, r, and the equivalent mass, m, of the diafram.[16]

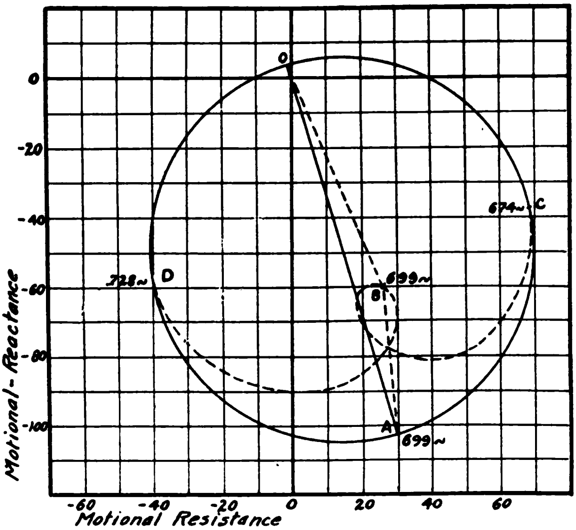

FIGURE 10

Distorted Motional-Impedance Circle Re-entrant Loop

The general effect of the thickness and radius of the diafram upon the breadth of tuning are indicated in Figures 12 and 13, respectively. The assumption will be made that r varies simply as the area of the diafram; this is a rough approximation. Referring to Figure 12, let

$$n_o=\frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

represent the natural frequency of the diafram of thickness, t; then, for a thickness, $\frac{t}{2}$, we have, from equation (9),

$$n_o'=\frac{1}{2\pi}\sqrt{\frac{s}{8}\div\frac{m}{2}}=\frac{1}{2}\times\frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

or the natural tone is lowered by one octave when the thickness

[15] Bibliography, 5, page 449.
[16] Bibliography, 7, pages 15–16.

is reduced one-half while the resonance velocity $\dot{x}=F/r$ remains the same (within our approximation).

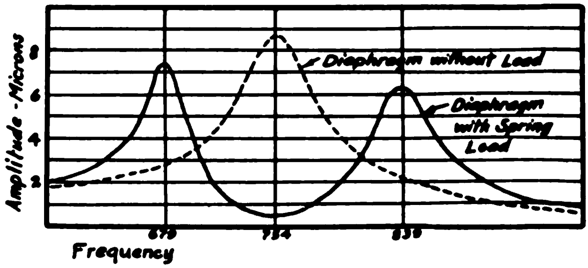

FIGURE 11

Vibration of Diafram with Spring Load

The breadth of tuning is given by

$$(n_2-n_1)=\frac{r}{2\pi m}$$

for the diafram of thickness, t; then, by making the thickness $\frac{t}{2}$, the breadth of tuning becomes

$$(n_2-n_1)=\frac{r}{2\pi\frac{m}{2}}=2\times\frac{r}{2\pi m},$$

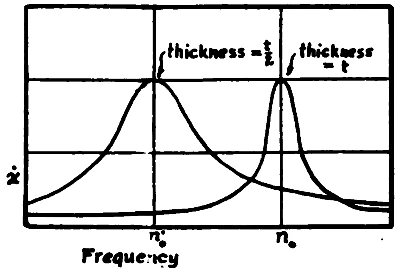

FIGURE 12

Effect of Change in Diafram Thickness on Resonance and Breadth of Tuning

or the breadth of tuning is doubled when the thickness is reduced one-half. Thus, the thinner the diafram, the broader the tuning and the lower the natural tone.

Referring to Figure 13, let

$n_o = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$ represent the natural frequency of the diafram of diameter, D;

then, for a diameter of $\frac{D}{2}$, we have from equation (9),

$$n_o' = \frac{1}{2\pi}\sqrt{\frac{4\,s}{m/4}} = 4 \times \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

or the natural tone is raised by two octaves when the diameter

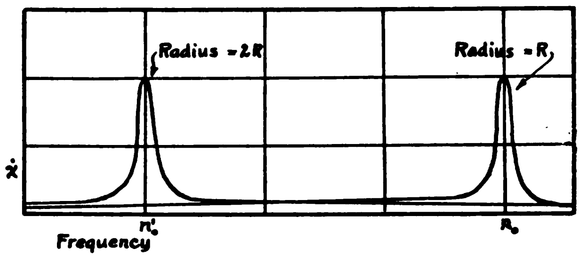

FIGURE 13

Effect of Change in Radius of Diafram on Resonance and Breadth of Tuning

is reduced one-half. The breadth of tuning is unaffected by the change in the diameter for, in this case, r is reduced to $\frac{r}{4}$, and m to $\frac{m}{4}$, thus leaving $(n_2 - n_1)$ unchanged (within our approximation). The sharpness of tuning, however, has decreased since in the expression for it, $\frac{n_o'}{n_2 - n_1}$, n_o' is four times as large as n_o.

The change of resonance velocity with change of r may be deduced from equation (3). If the maximum velocity at resonance for a diafram of diameter, D, is given by

$$\dot{x}_{max} = \frac{F}{r} \qquad \text{cm./sec.} \qquad (16)$$

then, for a diafram of diameter, $\frac{D}{2}$, elasticity remaining constant, this velocity would be given by

$$\dot{x}'_{max} = \frac{4F}{r} \qquad \text{cm./sec.} \qquad (17)$$

That is, when the diameter is reduced one-half, the resistance would be reduced one-fourth, and the velocity would be four times as great; but, by equation (9), the change in diameter increases the elastic constant by 4 and thus reduces the resonance velocity to one-fourth. The combined effects of the resistance and elastic constants on the maximum resonance velocity of the diafram thus leave this velocity unchanged for all diameters of the diafram (within our approximation). This result indicates that diafram damping and elasticity offers a field for more work.

It should be pointed out that, besides the assumption of a constant mechanical resistance, r, for various thicknesses, t, of the diafram, some other contributing causes also render the curves of Figures 12 and 13 of value only as approximations. Among these are: the increasing membraneous quality in the diafram as it becomes thinner; the change in the magnetic distribution as the diafram becomes increasingly concave; and the resulting change in the air gap which produces corresponding changes in the eddy currents present in the diafram. In practice, these curves may be more nearly realized if the air gap between the diafram and pole pieces is kept constant for all diafram thicknesses by means of the proper adjustment of the distance between the pole pieces and the plane of the clamping ring upon which the diafram rests. Since a telephone diafram is at all times subjected to the pull of the magnets, it possesses more or less the characteristics of a stretched membrane, and these characteristics increase as the thickness of the diafram decreases. The fundamental frequency of a stretched membrane depends, among other things, upon the tension in its surface, while the corresponding determining factor of a diafram is its elasticity. It is thus quite evident that a complete change in the relations which determine the natural frequency and breadth of tuning may ensue as the diafram is made increasingly thinner and the natural period becomes a function of the size of the air gap and the strength of the magnetic pull. The curves of Figure 12 are more nearly true for diaframs having a minimum thickness of about 0.02 cms.

Sensitiveness

The sensitiveness of a telephone receiver has to do with the intensity of sound received for a given impressed vibratory voltage. The pull on the diafram for a given air gap is proportional to the ampere-turns and to the flux density in the air gap due to the permanent magnet, and inversely proportional to the magnetic reluctance. The variation of these quantities is limited by considerations of space, weight, etc., but they indicate one line of attack (the electrical) in designing telephone receivers when sensitiveness requires attention.

The sound should reach the ear under conditions which will concentrate it with a minimum of loss. The intensity of sound will then vary approximately as the square of the amplitude of diafram vibration and as the area of the diafram.

If selectiveness of tuning is being carried out in the design, the acoustical resonance of the chamber between the ear and receiver cap may be advantageously considered.

The electrical, mechanical and acoustical lines of attack afford a considerable range of variation in the adjustment of the sensitiveness of the telephone receiver.

Audibility

A test of the sensitiveness of the telephone receiver is the audibility current at resonance. For the same size of diafram, length of air gap, and construction of cap, the intensity of sound received is proportional to the square of the amplitude of vibration of the diafram. Audibility in radio work, however, has come to be considered as proportional to the first power of the amplitude rather than to its square. For a given telephone receiver and at a given frequency, the amplitude of diafram vibration is proportional to the current in the coils.

The ear is not a trustworthy indicating instrument for measuring audibility but it is the one most used. This of course is a transient state of the art. When audibility measurements become vital, precision instruments, either null or deflection, will be devised from the means now waiting to be utilized.

Careful observations of the amplitude of vibration of the telephone diafram were made by Shaw[17] for various audibilities. While he found a difference in the sensitiveness of the two ears, the minimum audible amplitude at the center of a diafram on a clamping circle 5 centimeters (2 inches) in diameter may be taken as 7×10^{-8} cm. (3×10^{-8} inch). Shaw's table includes four degrees of audibility, namely: (1) minimum audibility, expectant ear; (2) minimum audibility, unexpectant ear; (3) loud; (4) overpowering; having the relation, 1 : 70 : 1400 : 7000.

Following is a table in which these measurements are applied to four telephone receivers, *A*, *B*, *C*, and *D*, which were studied by Kennelly and Affel[18] and a description of which appears in their paper.

[17] Bibliography, 8, pages 360–366.
[18] Bibliography, 4, page 449.

	A	B	C	D
Amplitude at Resonance	7.53×10^{-4} cm.	7.19×10^{-4} cm.	10.35×10^{-4} cm.	6.64×10^{-4}
Current at Resonance	0.00202 amps.	0.002 amps.	0.00204 amps.	0.00116 amps.
Audibility at Resonance	10760	10280	14780	9480
Min. Aud. Current, Expectant Ear	1.878×10^{-4} milliamps.	1.948×10^{-4} milliamps.	1.38×10^{-4} milliamps.	1.224×10^{-4} milliamps.
Min. Aud. Current, Unexpectant Ear	0.01342 milliamps.	0.01389 milliamps.	0.00985 milliamps.	0.00874 milliamps.
Current, Loud Sound	0.268 milliamps.	0.278 milliamps.	0.197 milliamps.	0.1749 milliamps.
Current, Overpowering Sound	1.342 milliamps.	1.389 milliamps.	0.985 milliamps.	0.874 milliamps.

Table of Audibility and Audibility Current of Four Telephone Receivers
Four grades of audibility are given

Receiver D, the most sensitive, was of the watch case type with a resistance of over 1,000 ohms, while the three others were the desk type, having a resistance of less than 100 ohms. The minimum audible amplitude for the expectant ear, 7×10^{-8}, would seem to be the logical starting point for a scale of audibility measurements.

Some Types of Receivers

The following four types of receivers give points of difference which are of interest:

(1) Monopolar	
(2) Bipolar	ferrotype diafram
(3) Monopolar, with boundary return	
(4) Quadrapolar	mica diafram

(1) In the monopolar receiver, the force is active at the center of the diafram, and the magnetic return is thru air. The equivalent mass of the diafram is thus relatively small (a good point), and the magnetic reluctance relatively large (a weak point).

(2) In the bipolar receiver, the force is active at points on the diafram removed from the center, and the air gaps in the magnetic path are small. The equivalent mass of the diafram is thus relatively large (a weakness), and the magnetic reluctance small (a good point).

(3) In the monopolar, with boundary return, the force is active at the center of the diafram, and the air gaps in the magnetic path are small. These points are both favorable for sensitiveness, but the return flux at the boundary is not utilized in force action as in the bipolar.

(4) In this type of receiver, the force is applied at the center of the diafram by means of a connecting rod from the vibrating armature which is suspended between the two sets of opposite magnetic poles, and the air gaps in the magnetic path are small, also the return flux is partially utilized in force action.[19]

In most telephones there is a magnetic amplifying action due to the dependence of the magnetic reluctance, in this case, upon the air gap. As the reluctance is a function, among

[19] Since writing this classification, a new telephone has appeared which constitutes the fifth class, the characteristic features being two poles of permanent magnets bridged by a soft iron solenoid core, the magnetic return path being thru a third pole which is an armature operating between the two poles. A rod connects the armature to the center of the diafram. The equivalent mass of the diafram is small, the return magnetic flux is in iron and the air gaps are small, and both air gaps are utilized in the force action. The electro-magnetic features are thus all good. The magnetic amplifying principle utilized in this telephone makes it a very sensitive receiver.

other things, of the plan of the magnetic path as well as of the precision realized in the mechanical construction of the parts, this amplification varies from one telephone to another.

This brief outline merely indicates a line of analysis which may be extended to include many other characteristics of telephone receivers.

Telephones are used for a variety of purposes, and a knowledge of the elements involved, electrical, mechanical and acoustical, make possible a design which will be the most suitable for a given purpose. In radio work, the telephone occupies a most important place, and the particular application requires a peculiar construction in which size, weight, selectivity, sensitiveness, etc., are determining factors.

SUMMARY: The motional-im·edance circle of telephone receivers is explained, the experimental methods of determining it are given, and the equation of motion of the diafram is derived. The effect of imperfect clamping of the diafram is studied.

The influence of thickness, radius, and elasticity of the diafram on its natural period are considered.

The electrical, mechanical, and acoustic methods of improving receiver sensitiveness are given. The significance of audibility measurements is next treated; together with suggested improvements in the direction of quantitative measurement of receiver response.

The characteristic advantages and disadvantages of monopolar, bipolar, and quadrapolar receivers are given, followed by a partial bibliography of receiver investigations.

TABLE OF SYMBOLS

α	Phase angle	radians
D	Diameter of diafram	cms.
Δ	Logarithmic decrement per second	hyp. rad./sec.
ϵ	Naperian logarithmic base	numeric
F	Maximum pull on diafram	dynes
f	$(=F \sin \omega t)$ Instantaneous pull on diafram	dynes $\angle$
i	$(=\sqrt{-1})$	numeric
J_o	Bessel's function of zeroth order	numeric
k	Proportionality constant	numeric
k	Constant involving diafram dimensions	cms.$^{-1}$
λ	Constant satisfying boundary conditions	numeric
M	Diafram mass within clamping ring	gms.
m	Equivalent mass of diafram	gms.
n	Frequency of vibration	cycles/sec.
n_o	Resonance frequency	cycles/sec.
n'_o	Natural frequency (size of diafram varied)	cycles/sec.
n_1	Frequency of quadrantal point below res.	cycles/sec.
n_2	Frequency of quadrantal point above res.	cycles/sec.
n_r	Natural frequency for free oscillations	cycles/sec.
ω	$(=2\pi n)$	rad./sec.
ω_o	Resonance angular velocity	rad./sec.
ω_r	Resonance angular vel., free oscil.	rad./sec.
P	Constant of amplitude ratio	cms.
π	3.1416	numeric
r	Mechanical resistance constant of diafram	dynes/kine
r	Distance from center of diafram	cms.
ρ	Density of diafram material	gms./cm.^{3}
s	Elastic constant of diafram	dynes/cm.
T	Periodic time	secs.
t	Thickness of diafram	cms.
t	Elapsed time in vibratory motion	secs.
X	Initial displacement of diafram center	cms.
x	Instantaneous displacement diafram center	cms. $\angle$
x_o	Initial amplitude	cms.
x_m	Max. amplitude r cms. from center	cms.
x_r	Amplitude at r from center	cms.

$\dot{x}$	Velocity at center of diafram	cms./sec. ∠
$\dot{x}'_{max}$	Max. vel. at res. (size of diafram varied)	cms./sec.
$\dot{x}_{max}$	Maximum velocity at resonance	cms./sec.
$\dot{x}_o$	Initial max. vibrational velocity	cms./sec.
$\ddot{x}$	Acceleration at center of diafram	cms./sec.^{2} ∠
Y	Young's modulus	dynes/cm.^{2}
∽	Cycles per second	
∠	Indicates complex quantity	

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