Electronics
Radio
Television
Radar

UNITED ELECTRONICS LABORATORIES

LOUISVILLE KENTUCKY

REVISED 1967

POWERS AND ROOTS: POWERS OF TEN

ASSIGNMENT 7

FOREWORD

When you glanced at the title of this assignment you may have said to yourself, "I want to be an electronics technician. Why are they teaching me math?" We think this is a very good question and deserves a logical answer. After all, if a person knows just WHY he is doing something, he'll do a better job. You are eager to become a competent electronics technician, and if you can see where you will really *need* math, then it is only natural that you will apply yourself better to the task of learning it.

A limited amount of mathematics has been included in the training program because math is a very useful "tool" to an electronics technician. In other words, it will help him to do his job better, quicker, and easier.

There will be many times when you, as an electronics technician, will apply this tool—mathematics—to the problem of trouble shooting. This doesn't mean that you will necessarily stop your trouble-shooting operation, take out a pencil and paper and solve a problem. It *does* mean that you will apply the principles of mathematics to your trouble shooting. Remember, electronics deals in physical quantities. For example, *amounts of current, amounts of voltage, amounts of resistance.* Only through an understanding of basic math can you understand, in your own mind, the relationship between these physical quantities.

Let us emphasize one point. The math included in the training program—and only a very small amount is included—is for the purpose of *helping you* in your future work in the electronics field. A UEL technician is far more than a "screwdriver mechanic." The UEL technician knows what he is doing, because he understands the principles of electronics. To do this, he must also understand the principles of mathematics.

Remember, we are going to give you only the math you *need;* we're going to give it in such a way that you can learn it if you did not have it in school, or if you didn't "like it" when you were in school. All we ask is that you approach it with the proper attitude. If you will decide that since you need this math *you are going to learn it,* then *we can guarantee that you will.* In fact, we assure you that you are going to be pleasantly surprised at just how easy the math in the training program is!

ASSIGNMENT 7

POWERS AND ROOTS; POWERS OF TEN

In Assignment 4, we studied the arithmetic which will be used in solving the problems encountered in electronics. In this assignment we will deal with a slightly different form of mathematics. This is the subject of Powers and Roots.

In some electronics formulas we might be instructed to **square a number** or to **cube a number**; or we might be required to find the "square root" of a number. In this assignment we will learn how to perform these operations.

At first glance it might appear that some of the information in this assignment is difficult, but this is definitely **not** the case. There isn't a thing which you will do in this assignment which is more involved than simple arithmetic. The purpose of this assignment is to show you **simple** ways to do problems that might otherwise be difficult. In other words, this assignment gives you **simple short cuts** to use in electronics problems. Just study through this assignment **a step at a time** and you will find just how simple it is. Here we go . . .

Powers

The phrase "squaring a number" comes from the fact that if we take a number and multiply it by itself we obtain the area of a square which has that number as the length of each side. Thus, a square room that is 12 feet on each side has a floor area of $12 \times 12 = 144$ square feet.

The phrase "cubing a number" comes from the fact that if we have a cube, its volume will be equal to the length of one side multiplied by itself two times. Thus, a packing crate that is 4 feet on a side has a volume of $4 \times 4 \times 4 = 64$ cubic feet.

We all know that $12 \times 12 = 144$.

We "squared" the 12 to get 144.

Another way of writing $12 \times 12 = 144$ would be: $12^2 = 144$.

12^2 can be read either "12 squared", or "12 to the second power".

The 12 is the **base.**

The 2 is the **exponent.** It is written as a small number to the right of and slightly higher than the base.

The exponent merely tells us how many times we are to use the base in multiplication.

Thus, if we cube the number 4, we could write it as 4^3 (which means $4 \times 4 \times 4) = 64$.

Examples:
3^2 means $3 \times 3 = 9$.
4^5 means $4 \times 4 \times 4 \times 4 \times 4 = 1024$ (4^5 is read as "4 to the fifth power").

6^3 means $6 \times 6 \times 6 = 216$ (6^3 may be read as "6 cubed", or "6 to the third power").

10^2 means $10 \times 10 = 100$ (10^2 may be read as "10 squared" or "10 to the second power").

Numbers to be raised to a power may be written in parentheses, as $(4)^2$ or $(25)^3$; $(4)^2$ means the same things as 4^2, and $(25)^3$ means the same as 25^3.

If the exponent is 1, we take the base one time. In other words, $4^1 = 4$, $10^1 = 10$, $3^1 = 3$, etc. Any number then has an exponent of 1 if no other exponent is indicated. Thus $4 = 4^1$, $10 = 10^1$, $3 = 3^1$, etc.

For practice, solve the following problems:

1. $5^2 =$ 25 3. $25^2 =$ 725 5. $9^4 =$ 6561 7. $7^7 =$ 823,613

2. $7^1 =$ 7 4. $76^3 =$ 438,976 6. $10^3 =$ 1000 8. $2^9 =$ 512

Roots

The opposite of raising a number to a power is called finding the root of a number.

Thus: 3 cubed $= 3 \times 3 \times 3 = 27$. $4^2 = 16$.

The **cube root** of 27 is 3. The **square root** of 16 is 4.

We used the exponents to indicate the power to which a number is to be raised.

We use a **radical sign** ($\sqrt{\quad}$) to indicate roots.

The cube root of 27 is written $\sqrt[3]{27}$.

The square root of 16 is written $\sqrt{16}$.

Notice that for square roots it is not necessary to indicate which root is being taken. In other words, if no particular root is shown, it is understood that we mean square root.

Thus: $\sqrt{81} = 9$, because we know that $9 \times 9 = 81$.

$\sqrt{64} = 8$, because we know that $8 \times 8 = 64$.

$\sqrt{144} = 12$, because we know that $12 \times 12 = 144$.

$\sqrt[3]{27} = 3$, because $3 \times 3 \times 3 = 27$. ($\sqrt[3]{27}$ is read as the cube root of 27)

$\sqrt[4]{16} = 2$, because $2 \times 2 \times 2 \times 2 = 16$. ($\sqrt[4]{16}$ is read as the fourth root of 16)

In electronics problems you will find **very few** cases where the cube root of a number or any higher root than the second will be required, but you will find many problems in electronics work where the square root of a number will have to be determined. When you have $\sqrt{81}$, you ask yourself, "What number multiplied by itself gives 81?" The answer 9, is obvious. There are a great many cases where the roots are not obvious.

Since $\sqrt{81} = 9$, and $\sqrt{64} = 8$; $\sqrt{70}$ must be some odd decimal quantity between 8 and 9. Numbers, like 81, whose square root is a whole number, are called perfect squares. Thus, 64 is a perfect square since its square root is the whole number 8. Seventy is not a perfect square since its square root is not a whole number. Such a number is called an imperfect square.

There are three common methods of obtaining the square root of a number. One of these is by using a slide rule. You may be familiar with the operation of a slide rule; if not, do not feel concerned since the operation of a slide rule will not be required in this training program. The slide rule would tell you that the square root of 70 is approximately 8.37. Another method is to use mathematical tables. A mathematical table would tell you that the square root of 70 (to 5 significant figures) is 8.3666. You can also determine the square root of 70 (or any other number) by a "long-hand" method. A knowledge of this "long-hand" method is necessary before you can make intelligent use of the slide rule or a mathematical table. We will work out several square roots. Use these solutions as a guide when you start to work the problems at the end of the assignment. Let us first take the square root of 70. You will find that the same set of rules will apply no matter what number we work with.

EXAMPLE 1. $\sqrt{70} = ?$ or what number times itself equals 70?

$$\sqrt{70\dot{.}}$$

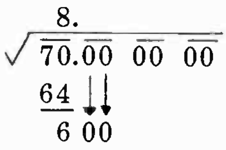
First step. Locate the decimal point in the 70 and place a decimal point directly above the decimal point in the 70. We have now located the decimal point in our answer.

$$\begin{array}{l} \qquad . \\ \sqrt{\overline{70}.\overline{00}\ \overline{00}\ \overline{00}} \end{array}$$

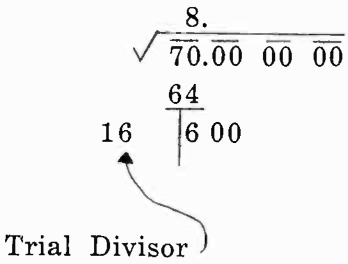
Second step. Use brackets to "pair off" the digits, or numbers. Start at the decimal and move to the left, placing the 0 and 7 under **one bracket.** Add zeros to the right of the decimal point. Start at the decimal point and move to the right, placing one bracket over each **pair** of digits. Notice that we **never** have a bracket crossing over the decimal point.

$$\begin{array}{l} \quad\ \ 8. \\ \sqrt{\overline{70}.\overline{00}\ \overline{00}\ \overline{00}} \\ \quad \underline{64} \end{array}$$

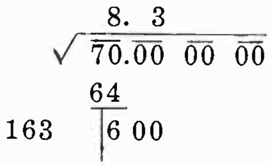
Third step. Look under the first bracket. We find the quantity 70. The largest perfect square that will fit in 70 is 8 × 8 or 64. The next largest perfect square, 9 × 9 or 81, is larger than 70. Place the 64 under the 70 and the 8 is the first digit in the answer.



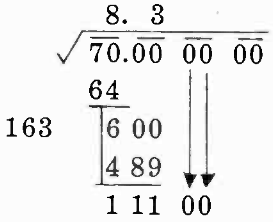
Fourth step. Subtract the 64 from the 70 and bring down **both digits** under the next bracket.



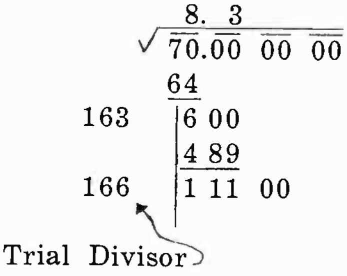
Fifth step. Obtain a trial divisor for the 600. To do this double the 8 in the answer. Place the trial divisor (2 × 8) = 16 to the left of the 600. We are going to place another figure after the 16 in a moment. We had better save a place for it. Mentally or actually cover up the last zero in the 600. This gives us 60. 16 goes into 60 three times (3 × 16 equals 48; 4 × 16 equals 64).



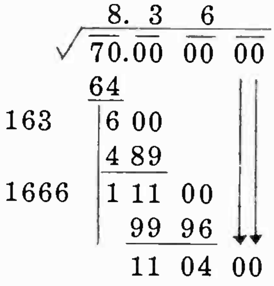
Sixth step. Place the 3 after the 16 and also enter 3 as the next digit in the answer. Notice that each bracket (pair of digits) gives us a place for one digit in our answer.



Seventh step. Enter 3 times 163 or 489 under the 600 and subtract it, leaving 111. Bring down the **next pair** of digits.



Eighth step. Obtain a trial divisor for the 11100. Simply double the 83 in the answer (2 × 83 = 166). Covering the last zero in 11100 we see that 166 goes into 1110 six times (6 × 166 = 996; 7 × 166 = 1162).



Ninth step. Enter the 6 after the 166 and also in the answer. Enter 6 times 1666 or 9996 under the 11100 and subtract. Bring down the next pair of digits.

```
           8.3    6   6
        √70.00   00  00
         64
163     | 6 00
        | 4 89
1666    |1 11   00
        |  99   96
16726      |11  04  00
           |10  03  56
              1 00  44
```

Tenth step. Our next trial divisor is 2 × 836 = 1672. Our trial divisor goes into 11040 six times. Enter the 6 after the 1672 and also as the next digit in our answer. Multiply the 16726 by 6 and subtract it from the 110400.

If we wanted to carry our answer out to more significant figures, we could add more pairs of zeros which could give us more brackets and more places in our answer. **Each** additional **bracket** would give us **one** more **place in our answer.**

If we are satisfied with four significant figures, our work is complete. Notice that we have a remainder of 10044. Tracing the path of the decimal in our 70 straight down, we see that our remainder is really .010044.

Let us check our answer.

```
       8.366
       8.366
      ------
       50196
      50196
     25098
    66928
   ---------
   69.989956
```

Adding our remainder:

```
   69.989956
     .010044
   ---------
   70.000000
```

EXAMPLE 2. What is the square root of .0000044 to four significant figures?

$$\sqrt{.00\ 00\ 04\ 40\ 00\ 00}$$

Locate the decimal point in the answer. Mark off pairs of digits with brackets, adding enough zeros to provide brackets for four significant figures.

```
 . 0   0
√.00  00  04  40  00  00
```

All we find under the first two brackets are zeros. The first two digits in our answer will be zeros.

```
 . 0   0   2
√.00  00  04  40  00  00
           4  ↓↓
              40
```

Under the next bracket we find a 4. The largest square in 4 is $(2)^2$. Enter the 2 and 4 as shown. Subtract the 4 from 4 and bring down the pair of digits under the next bracket.

```
 . 0   0   2
√.00  00  04  40  00  00
           4
   4  |       40
```

Our trial divisor is double the 2 in our answer or 4. Covering up the zero in 40 we have 4. We have 4 divided into 4 which goes once. Entering the 1 in our answer and after the 4 we place

```
      .  0   0   2   1
  √  .00 00  04  40  00  00
               4  ↓↓
  41             |40
                 |41
```

1× 41 or 41 under the 40 for subtraction. 41 is too large to be subtracted from 40. Therefore 1 is too large a number for the next digit in our answer. The next smaller number is zero.

```
      .  0   0   2   0   9
  √  .00 00  04  40  00  00
               4       ↓↓
  409           |40  00
```

In place of the 1's enter zeros in the answer and after the 4. **Bring down the next two digits.** Our trial divisor of 40 goes into 400 ten times. 9 is the largest digit we can use so place 9 as the next digit in the answer and in the divisor.

```
      .  0   0   2   0   9
  √  .00 00  04  40  00  00
               4          |
  409           |40  00   |
                |36  81   ↓↓
                  3  19  00
```

Subtract 9 × 409 from the 4000 and bring down the next pair of digits.

```
      .  0   0   2   0   9   7
  √  .00 00  04  40  00  00
               4
  409           |40  00
                |36  81
  4187            |3  19  00
                  |2  93  09
                      25  91
```

Our next trial divisor is twice 209 or 418. Our trial divisor 418 goes into 3190 seven times. Enter the 7 in the answer and after the 418. Subtract 4187 times 7 from 31900.

```
Check.          .002097     Adding the remainder:
                .002097
                 -------
                  14679         .000004397409
                 18873          .000000002591
                4194            -------------
           -------------        .000004400000
           .000004397409
```

Follow the work carefully in these next square root problems. You will soon realize that there are really very few rules to remember. Using these problems as a guide, you should be able to work out the five square root problems that are given. You will have to use square roots again in some of the "Powers of Ten" problems at the end of this assignment.

EXAMPLE 3. Find the square root of 732.8 to 4 significant figures.

```
          2   7.  0   7
       √  7  32.  80  00
          4
  4 7    |3  32
         |3  29
  5407      |3  80  00
            |3  78  49
                 1  51
```

In marking brackets to the left of the decimal you will have **one** figure by itself under a bracket if there are an odd number of digits to the left of the decimal. In this case the largest perfect square in 7 is 2 × 2 or 4. Putting the two in the first place in our answer, we proceed as in the other examples. In our first trial divisor step, 4 divided into 33 goes eight times. Since 8 × 48 = 384 (more than 332), we have to use

7. Our second trial divisor (54) was too large for 38 so the third digit in our answer is zero. When the next **pair** of digits is brought down, we find that our new trial divisor, 540, goes into 3800, 7 times. This gives 7 for the fourth figure in our answer.

```
Check.   27.07
         27.07
        ------
         18949
        18949
        5414
       ---------
       732.7849                732.7849
                              + .0151
                              ---------
                               732.8000
```

EXAMPLE 4. Find the square root of 827,564,581 to 4 significant figures.

```
          2   8   7   6   0.
        ---------------------
       √ 8  27  56  45  81.
         4
  48  |  4  27
      |  3  84
      ----------
  567    |  43  56
         |  39  69
         -----------
  5746      |  3  87  45
            |  3  44  76
            -------------
                42  69  81
```

We put a zero in our answer for our last digit since we were asked for only 4 significant figures.

Check this answer.

EXAMPLE 5. Find the square root of 58.9 to 3 significant figures.

```
          7.   6   7
        --------------
       √ 58.  90  00
         49
  146   |  9  90
        |  8  76
        ------------
  1527  |  1  14  00
        |  1  06  89
        ------------
              7  11
```

Check this answer.

EXAMPLE 6. Find the square root of .021 to 3 significant figures.

```
         . 1   4   4
        --------------
       √ .02  10  00
            1
  24    |   1  10
        |      96
        -----------
  284      |  14  00
           |  11  36
           ----------
               2  64
```

Check this answer.

Practice Problems

For practice, solve the following problems. Find the square roots to 4 significant figures.

.045716

$\sqrt{.0021}$ $\qquad$ $\sqrt{.00021}$ $\qquad$ $\sqrt{59730}$ $\qquad$ $\sqrt{2}$ $\qquad$ $\sqrt{3.75}$

Powers of Ten

In electronics work we will quite often encounter very large numbers and very small decimal fractions. For example, we will have numbers like 40 megahertz, which means 40,000,000 hertz; 1000 kilohertz, which means 1,000,000 hertz; and 10 picofarads, which means .000000000010 farads. Handling these large numbers in solving problems is very inconvenient, and leads to a large number of errors. Example 1 gives a sample of this type of problem.

EXAMPLE 1. $f = \dfrac{.159}{\sqrt{.00015 \times .000000000004}} = \dfrac{.159}{\sqrt{.0000000000000006}}$

We would first have to multiply the two decimal quantities to obtain the .0000000000000006 under the radical. Then this problem can be completed, and the correct answer obtained, as long as we are careful in using the decimal point. Since so many zeros are used, a considerable amount of careful work is necessary if the decimal point is to be properly placed in the answer.

You will be dealing with these very large and very small quantities throughout your entire electronics work, so you should welcome any method that conveniently takes care of these quantities.

The **Powers of Ten** will do just that. Powers of ten are often referred to as **Engineer's Shorthand.**

In the section on Powers and Roots you found that in the expression 3^4 we have a **base** (3) and **exponent** (^{4}). The expression means $3 \times 3 \times 3 \times 3 = 81$.

Similarly:

4^3 means $4 \times 4 \times 4$, or 64

2^5 means $2 \times 2 \times 2 \times 2 \times 2$, or 32

7^2 means 7×7, or 49

10^2 means 10×10, or 100

10^4 means $10 \times 10 \times 10 \times 10$, or 10,000

We can see, then, that number 10 is easier to work with as a base than 3, 4, 2, 7, or most any other number.

Also, the number 10 ties in perfectly with our decimal system. Thus: The number 20 is **ten** times as large as the number 2, the only difference between the two numbers is the position of the decimal point with respect to the 2.

The 10's with negative exponents (—1, —2, etc.) indicate that 10 is to be taken a certain number of times in the **denominator** of a fraction whose numerator is 1. Thus, 10^{-1} means $\frac{1}{10}$; $10^{-2} = \frac{1}{10 \times 10} = \frac{1}{100}$; and $10^{-3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1000}$. Notice that 10^3 means 1000 and 10^{-3} means $\frac{1}{1000}$. Also 10^6 equals 1,000,000 and 10^{-6} equals $\frac{1}{1,000,000}$.

Multiplication with Powers of Ten

We will work out a few simple multiplication problems by arithmetic, and see what the results would be in powers of ten.

EXAMPLE 1. $100 \times 1000 = 100,000$

Let us glance at the Powers of Ten Table, and change each of the figures in the problem to powers of 10.

$$100 \times 1000 = 100,000$$
$$10^2 \times 10^3 = 10^5$$

EXAMPLE 2. $10 \times 100 = 1000$

In powers of ten, $10^1 \times 10^2 = 10^3$

EXAMPLE 3. $100 \times .001 = .1$

In powers of ten, $10^2 \times 10^{-3} = 10^{-1}$

EXAMPLE 4 $.01 \times .001 = .00001$

In powers of ten, $10^{-2} \times 10^{-3} = 10^{-5}$

Now let us examine Example 1 closely. We worked this problem out by the long method of multiplication and are sure that the answer (100,000) is correct. Now look at the powers of ten used to represent each number in the example. The number 10^2 is equal to 100; 10^3 is equal to 1000; and 10^5 is equal to 100,000. Notice that in multiplying the two numbers together, 10^2 and 10^3, we did not multiply the exponents together, but we **added the exponents.** We might write it in this manner: $10^2 \times 10^3 = 10^{2+3} = 10^5$.

This illustrates one rule for using powers of 10. **When multiplying with powers of 10, add the exponents.**

Let us check this rule in Examples 2, 3 and 4.

Example 2 could be rewritten $10^1 \times 10^2 = 10^{1+2} = 10^3$. The step of writing 10^{1+2} is unnecessary in actual work and is merely shown here for clarity.

In Example 3 we have $10^2 \times 10^{-3}$. To solve this by powers of 10 we have $10^{2-3} = 10^{-1}$. Here we add a +2 and a —3. If the addition of negative numbers confuses you, an easy thing to do is to think of the positive numbers as money you have and the negative numbers as money you owe. In this case, if you had 2 dollars and owed 3, when you pay the two dollars on your debt, you still owe one dollar. $2 - 3 = -1$.

In Example 4 we have $10^{-2} \times 10^{-3}$. This would be $10^{-2-3} = 10^{-5}$. Here we are adding two negative numbers, -2 and -3. If you owe one person two dollars and another person 3 dollars, you would owe a total of 5 dollars.

Let us solve a few multiplication problems with powers of 10.

EXAMPLE 5. $100 \times 10{,}000 = ?$

$10^2 \times 10^4$

When we look up 100 and 10,000 in our table, we may rewrite the problem: $10^2 \times 10^4$.

$10^2 \times 10^4 = 10^{2+4} = 10^6 = 1{,}000{,}000$

Adding our exponents, we obtain the answer.

Work this problem by the long method of multiplication, and check to see if the answer is the same as that obtained with powers of 10.

EXAMPLE 6. $1{,}000{,}000 \times \dfrac{1}{1{,}000} = ?$

Changing to powers of 10, $10^6 \times 10^{-3} = 10^{6-3} = 10^3 = 1{,}000$. Check this by the long method.

EXAMPLE 7. $\dfrac{1}{10{,}000} \times \dfrac{1}{1{,}000}$

$10^{-4} \times 10^{-3} = 10^{-7}$.

The answer, 10^{-7}, means $\dfrac{1}{10{,}000{,}000}$. This was not shown on our original table. This is because our table is not complete. The powers of 10 do not start at 6 and end at -6. Actually they can be continued on to any value. A very simple way to remember this is: To change a power of 10 to a number, write 1 and add as many zeros after the 1 as the exponent of the power of ten. Then, 10^1 would be 1 followed by one zero, or 10. 10^6 would be 1 followed by 6 zeros, or 1,000,000. 10^{12} would be 1 followed by twelve zeros, or 1,000,000,000,000. 10^{-7} would be $\dfrac{1}{\text{1 followed by seven zeros}}$ or $\dfrac{1}{10.000.000}$.

It is just as simple to change numbers into powers of 10. Just use for the exponent of 10 the number of zeros after the one in the number. Thus, to change 10,000, which is 1 followed by four zeros, to powers of 10, we write 10^4. 100 is 10^2 and 1000 is 10^3. $\dfrac{1}{100}$ is 10^{-2} and $\dfrac{1}{10{,}000{,}000} = 10^{-7}$.

Practice Problems

For practice, solve the following problems using powers of ten. If in doubt, check your answer using the long method.

1. $10^4 \times 10^6$
2. $1000 \times 1{,}000{,}000$
3. $10^3 \times 10^{-2}$
4. $10{,}000 \times \frac{1}{1{,}000}$
5. $100 \times \frac{1}{100}$

Division with Powers of Ten

Let us find how to use powers of ten in division.

EXAMPLE 1. Suppose we wish to divide 1000 by 100. $\frac{1000}{100} = 10.$

We find by mathematics that the answer is 10 or 10^1. Let us write this in powers of ten.

$$\frac{10^3}{10^2} = 10^1$$

We can obtain the 10^1 by changing the sign of the exponent of the denominator (bottom number of the fraction) and then adding exponents.
In this example,

$$\frac{10^3}{10^2} = 10^{3-2} = 10^1$$

We changed the exponent 2 to a —2 since it is in the denominator.

Let us apply this method to a few more examples.

EXAMPLE 2. Divide 10,000 by 10.

$$\frac{10^4}{10^1} = 10^{4-1} = 10^3.$$ 10^3 is equal to 1000.

EXAMPLE 3. Divide 100 by .00001.

$$\frac{10^2}{10^{-5}} = 10^{2+5} = 10^7.$$ Check this problem by the long method.

EXAMPLE 4. $$\frac{10^{-2}}{10^{-3}} = 10^{-2+3} = 10^1 = 10$$

Practice Problems

For practice, solve the following problems with powers of ten.

1. $\frac{10^{6}}{10^{-5}}$ 10^{11} 2. $\frac{10^{-2}}{10^{-4}}$ 10^{2} 3. $\frac{10^{3}}{10^{6}}$ 10^{-3} 4. $\frac{10^{-4}}{10^{-12}}$ 10^{8}

Moving Powers of Ten from Top to Bottom of Fraction

It is permissible to change a power of ten from the top to the bottom, or from the bottom to the top of a fraction merely by changing the sign of the exponent. We have actually been doing this in the division problems just solved.

EXAMPLE 1. $\frac{10^{3}}{10^{2}} = \frac{10^{3\ -\ 2}}{1} = \frac{10^{1}}{1} = 10.$

This could have been written $\frac{10^{3} \times 10^{-2}}{1} = 10^{1}.$

In this case we moved the 10^2 from the denominator to the numerator of the fraction, and changed the sign of the exponent so that our power of ten is now 10^{-2}. Then we multiplied by powers of ten to obtain our answer.

This problem could be solved in the following manner:

$$\frac{10^{3}}{10^{2}} = \frac{1}{10^{2} \times 10^{-3}} = \frac{1}{10^{-1}} = \frac{1}{.1} = 10.$$

In this case we moved the 10^3 from the numerator to the denominator, and changed the exponent's sign so that we had 10^{-3}. Then we multiplied, using powers of 10.

This may seem to be a rather useless operation with the simple problem in Example 1, but will be very handy in the solution of more complex problems.

Let us apply this principle to a few more problems.

EXAMPLE 2. $\frac{10^{2}}{10^{6}} = \frac{10^{2} \times 10^{-6}}{1} = 10^{-4}$

EXAMPLE 3. $\frac{10^{2}}{10^{6}} = \frac{1}{10^{6} \times 10^{-2}} = \frac{1}{10^{4}} = \frac{10^{-4}}{1}$

EXAMPLE 4. $\frac{10^{3} \times 10^{4}}{10^{6} \times 10^{-5}} = \frac{10^{7}}{10^{1}} = \frac{10^{7} \times 10^{-1}}{1} = 10^{6}$

Changing Numbers into Powers of Ten

Any number, large or small, can be broken up into a workable figure times a power of ten.

Thus: $200 = 2 \times 100 = 2 \times 10^2$

$270000 = 27 \times 10000 = 27 \times 10^4$

or $270000 = 2.7 \times 100000 = 2.7 \times 10^5$

$3600000000 = 36 \times 10^8 = 3.6 \times 10^9$

$.2 = 2 \times .1 = 2 \times 10^{-1}$

$.003 = 3 \times .001 = 3 \times 10^{-3}$

$.00000085 = 8.5 \times .0000001 = 8.5 \times 10^{-7}$

or $.00000085 = 85 \times .00000001 = 85 \times 10^{-8}$

Check the above figures until you are satisfied that they are correct. Notice that in each case the number of the exponent tells us the number of places we have moved our decimal point. The + or — sign in front of the exponent tells us whether we have moved our decimal to the left or right.

(a) $27\ 00\ 00. = 27 \times 10^4$ (c) $.00\ 00\ 00\ 85 = 8.5 \times 10^{-7}$

(b) $.00\ 00\ 00\ 27 = 27 \times 10^{-8}$ (d) $63\ 00\ 00\ 00 = 6.3 \times 10^7$

Moving the decimal to the left gives us a positive exponent. Moving the decimal to the right gives us a negative exponent.

Practice Problems

For practice, express the following as whole numbers times a power of 10.

1. 36000 3.6×10^4 3. 930000 9.3×10^5 5. .00000081 8.1×10^{-7}

2. .0004 4×10^{-4} 4. 72100 7.21×10^4 6. .000000000043 4.3×10^{-11}

Summation

Except for a few helpful hints we have covered the subject of powers of ten. The four simple rules (which you should be careful to understand rather than memorize) are:

1. In multiplication of powers of ten, **add exponents.**

2. A power of ten can be moved from denominator to numerator of a fraction (and vice versa) providing the **sign of the exponent is changed.**

3. The numerical value of the exponent tells us how many places we have moved the decimal point.

4. The sign of the exponent tells us in what direction we have moved the decimal point.

Using Powers of Ten

Now let us use the powers of 10 to solve some more complex problems.

EXAMPLE 1. $$\frac{.000014 \times .00016}{.00000056 \times 2000} = \frac{14 \times 10^{-6} \times 16 \times 10^{-5}}{56 \times 10^{-8} \times 2 \times 10^{3}}$$

The next thing to do is to combine the powers of 10 in the numerator and denominator. Remember 2 × 6 × 8 is the same as 8 × 6 × 2, or 6 × 2 × 8. In multiplication, it makes no difference in what order we proceed, so it is permissible for us to re-write the fraction as follows:

$$\frac{14 \times 16 \times 10^{-6} \times 10^{-5}}{56 \times 2 \times 10^{-8} \times 10^{3}}$$

$$\frac{14 \times 16 \times 10^{-11}}{56 \times 2 \times 10^{-5}}$$

Now we combine the powers of 10 in the numerator and denominator. (After some practice you will be able to do this mentally and will not have to write this step. Now we cancel.

$$\frac{\overset{2}{\cancel{14}} \times \overset{\overset{1}{\cancel{2}}}{\cancel{16}} \times 10^{-11}}{\underset{\underset{1}{\cancel{8}}}{\cancel{56}} \times \cancel{2} \times 10^{-5}} = \frac{2 \times 10^{-11}}{10^{-5}} = \frac{2 \times 10^{-11} \times 10^{+5}}{1} = 2 \times 10^{-6} = .000002$$

Apply the long method to obtain the answer to this problem to check the answer. Do you now see how the powers of 10 will save time and effort?

EXAMPLE 2. $$\frac{6000}{.00009 \times .01 \times 400 \times .00001} =$$

$$\frac{6 \times 10^{3}}{9 \times 10^{-5} \times 10^{-2} \times 4 \times 10^{2} \times 10^{-5}} =$$

$$\frac{6 \times 10^{3}}{9 \times 4 \times 10^{-5} \times 10^{-2} \times 10^{-5} \times 10^{2}} =$$

$$\frac{\overset{\overset{1}{\cancel{3}}}{\cancel{6}} \times 10^{3}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{4}} \times 10^{-10}} = \frac{10^{3}}{6 \times 10^{-10}} = \frac{10^{3} \times 10^{10}}{6} = \frac{10^{13}}{6}$$

We have our answer $\frac{10^{13}}{6}$; but we may convert it in the form of a decimal. We could of course divide 6 into 10,000,000,000,000 but this would be using big numbers again. The simplest method is to change 10^{13} to $\frac{10^1 \times 10^{12}}{6}$. Then rewrite the fraction.

$\frac{10^1 \times 10^{12}}{6}$ Now all we have to do is to divide 6 into 10 (10^1 is 10).

We do this by division and find that 6 goes into 10, 1.67 times. Our answer can be re-written to be:

1.67×10^{12}

It is a good idea to leave the answer in this form, since we are familiar enough with powers of 10 to know just what 10^{12} means. If we wanted to write our answer for someone not familiar with powers of 10 it would be 1,670,000,000,000.

EXAMPLE 3. $\frac{.000012 \times .01 \times .003}{2400 \times 40000} =$

$$\frac{12 \times 10^{-6} \times 10^{-2} \times 3 \times 10^{-3}}{24 \times 10^{2} \times 4 \times 10^{4}} =$$

$$\frac{\overset{1}{\cancel{12}} \times 3 \times 10^{-11}}{\underset{2}{\cancel{24}} \times 4 \times 10^{6}} = \frac{3 \times 10^{-11}}{8 \times 10^{6}} =$$

$$\frac{3 \times 10^{-11} \times 10^{-6}}{8} = \frac{3 \times 10^{-17}}{8} =$$

$$\frac{3 \times 10^{1} \times 10^{-18}}{8} = \frac{30 \times 10^{-18}}{8} = 3.75 \times 10^{-18}$$

Study this example carefully making sure you understand each step. Do you see why the 10^{-17} was changed to $10^{1} \times 10^{-18}$?

Practice Problems

For practice, solve the following problems. Express your answers in a number between one and ten, times a power of ten.

1. $\frac{.0000008}{.002}$ 4×10^{-4}

3. $\frac{45{,}000 \times 10^{3}}{.0005 \times .003}$ 3×10^{13}

2. $\frac{.00009}{6000}$ 1.5×10^{-8}

4. $\frac{625{,}000 \times 9800 \times 10^{-3} \times .0036}{350 \times 6.3 \times 10^{4} \times .004 \times 10^{6}}$ $= 7.5 \times 10^{-7}$

Square Roots with Powers of Ten

Powers of 10 give a very convenient method of extracting square roots. Let us use some examples to demonstrate the process.

EXAMPLE 1. $\sqrt{10{,}000} = 100$ since $100 \times 100 = 10{,}000$

Let us put this in powers of ten:

$\sqrt{10^4} = 10^2$ since $10^2 \times 10^2 = 10^4$

EXAMPLE 2. $\sqrt{.00000001} = .0001$ since $.0001 \times .0001 = .00000001$

Stated in powers of ten:

$\sqrt{10^{-8}} = 10^{-4}$ since $10^{-4} \times 10^{-4} = 10^{-8}$

These examples illustrate that to take the square root of a power of 10 we merely divide the exponent by two. In example 1 the square root of 10^4 is 10^2 since the exponent 2 is one half of 4.

In example 2, the square root of 10^{-8} is 10^{-4} since the exponent —4 is one half of —8.

EXAMPLE 3. $\sqrt{1{,}000{,}000} = \sqrt{10^6} = 10^3$

EXAMPLE 4. $\sqrt{\frac{1}{100}} = \sqrt{10^{-2}} = 10^{-1}$

Now let us use this method for numbers consisting of whole numbers and powers of 10.

EXAMPLE 5. $\sqrt{.0008 \times .00009} = \sqrt{8 \times 10^{-4} \times .9 \times 10^{-4}}$

$= \sqrt{7.2 \times 10^{-8}} = 2.68 \times 10^{-4}$ Note: ($\sqrt{7.2}$ is 2.68)

Note: We were careful to move our decimal point so that the power of ten under the radical was an **even** number. You can see that if we had an odd exponent under the radical sign we would have ended up with a fractional exponent. (There is nothing wrong with a fractional exponent except that they are much harder to handle in a problem.)

EXAMPLE 6.

$$\frac{1}{\sqrt{.0008 \times .00009}} = \frac{1}{\sqrt{8 \times 10^{-4} \times .9 \times 10^{-4}}}$$

$$= \frac{1}{\sqrt{7.2 \times 10^{-8}}} = \frac{1}{2.68 \times 10^{-4}} = \frac{10^4}{2.68}$$

$$= \frac{10 \times 10^3}{2.68} = 3.73 \times 10^3$$

EXAMPLE 7. $\sqrt{7 \times 10^9}$

$\sqrt{7 \times 10^9} = \sqrt{70 \times 10^8} = 8.37 \times 10^4$

(Note: Square root of 70 = 8.37)

EXAMPLE 8 $\sqrt{.07 \times .00008} = \sqrt{.7 \times 10^{-1} \times 8 \times 10^{-5}}$
$= \sqrt{5.6 \times 10^{-6}} = 2.37 \times 10^{-3}$
(2.37 is the square root of 5.6)

Practice Problems

2.683282 $\times 10^{-3}$ 21.07131 $\times 10^{6}$

For practice, use the powers of [redacted] the following problems. [redacted]

1. $\sqrt{.0009}$ 3×10^{-2} 3. 2.683282 $\sqrt{.009 \times .0008}$ 5. $\sqrt{37 \times 10^{6} \times .12 \times 10^{8}}$

2×10^{-4}

2. $\sqrt{.00000004}$ 4. $\sqrt{36 \times 10^{3} \times 4 \times 10^{7}}$ 12×10 5

Another Method

In the above examples where we had several numbers **multiplied** together inside the radical sign, we could have taken the square roots of the different numbers individually and multiplied our answers together. The same thing holds true for division inside the radical sign. Let us solve Example 8 using this method.

EXAMPLE 8. $\sqrt{.07 \times .00008} = \sqrt{7} \times \sqrt{10^{-2}} \times \sqrt{.8} \times \sqrt{10^{-4}}$
$2.65 \times 10^{-1} \times .894 \times 10^{-2} = 2.37 \times 10^{-3}$

If we have **addition** or **subtraction** inside the radical sign, the addition or subtraction **must** be performed before the square root is taken.

For example: $\sqrt{7 + 8} = \sqrt{15} = 3.87$

Addition and Subtraction with Powers of Ten

Powers of ten are particularly beneficial in the operations already explained. They are of little benefit when adding or subtracting.

In all addition or subtraction we are "tied down" when using powers of ten. Remember that when we work with decimals we were always careful to keep the decimal points in a vertical column. Since the exponent in our power of ten locates the decimal point, **the powers of ten of all numbers must be identical before they can be added or subtracted.**

EXAMPLE 1. Add 7×10^{-5}, 86×10^{-4}, 33×10^{-6}

Answer:

$$\begin{array}{r} 70 \times 10^{-6} \\ 8600 \times 10^{-6} \\ 33 \times 10^{-6} \\ \hline 8703 \times 10^{-6} \end{array}$$

(or 8.703×10^{-3})

EXAMPLE 2. Add 3.7×10^{3}, 4.3×10^{6}, 37×10^{5}

Answer:

$$\begin{array}{r} 3.7 \times 10^{3} \\ 4300. \times 10^{3} \\ 3700. \times 10^{3} \\ \hline 8003.7 \times 10^{3} \end{array}$$

(or 8.0037×10^{6})

Using Powers of Ten with Electronics Terms

You will see and use the words mega, kilo, milli, micro, nano, and pico throughout your electronics work. Powers of ten provide an easy means of dealing with these terms.

Simply remember that:

Mega means million or 1,000,000 or 10^6
Kilo means thousand or 1000 or 10^3
Milli means thousandth or .001 or 10^{-3}
Micro means millionths or .000001 or 10^{-6}
Nano means thousandth part of a millionth part or 10^{-9}
Pico means millionth part of a millionth part or .000000000001 or 10^{-12} (The term micro-micro is also used to express this same quantity)

Thus: 3 megohms $= 3 \times 10^6$ ohms.
413 kilocycles $= 413 \times 10^3$ cycles.
.3 milliamperes $= .3 \times 10^{-3}$ amperes.
8 microfarads $= 8 \times 10^{-6}$ farads.
6 nanofarads $= 6 \times 10^{-9}$ farads.
10 micro-microfarads $= 10 \times 10^{-6} \times 10^{-6} = 10 \times 10^{-12}$ farads.
10 picofarads $= 10 \times 10^{-12}$ farads.

Also 700000 ohms $= .7 \times 10^6$ ohms or .7 megohms.
4700 volts $= 4.7 \times 10^3$ volts or 4.7 kilovolts.
.0000472 amps $= 47.2 \times 10^{-6}$ amps or 47.2 microamps.

We will now apply powers of ten to solve the problem given in Example 1 on page 8 of this assignment.

This problem is:

$$f = \frac{.159}{\sqrt{.00015 \times .000000000004}}$$

$$= \frac{.159}{\sqrt{1.5 \times 10^{-4} \times 4 \times 10^{-12}}}$$

$$= \frac{.159}{\sqrt{6 \times 10^{-16}}} = \frac{.159}{2.45 \times 10^{-8}} = \frac{.159 \times 10^{8}}{2.45}$$

$$= \frac{15.9 \times 10^{6}}{2.45} = 6.5 \times 10^{6}$$

The powers of 10 will save many minutes in the solution of most electronics problems. You are strongly advised to study this assignment several times until you are completely familiar with the use of these powers of ten.

Mathematical Tables

For your convenience, we are including a mathematical table at the end of this assignment. This table gives the square, and square root of all numbers from 1 to 100.

By changing larger or smaller numbers to numbers in this range, times a power of ten, this table can be used for a great many numbers. For example, if we wished to find the square root of 700, it could be changed to 7×10^2. The table tells us that the square root of 7 is 2.6458 and we know that the square root of 10^2 is 10. The square root of 700 is then 2.6458×10 or 26.458.

To find the square root of 990,000 we change it to 99×10^4. The square root of 99, from the table is 9.9499, and of course the square root of 10^4 is 10^2 or 100. The square root of 990,000 is then 9.9499×100 or 994.99.

To find the square root of .0069 we would change it to 69×10^{-4}. The square root of 69 is 8.3066 and the square root of 10^{-4} is 10^{-2}. The square root of .0069 is then 8.3066×10^{-2} or .083066.

To find the value of 760 squared we would change it to 76×10^1. $(76)^2$ is 5776 and ten squared is 10^2 or 100. 760 squared is then $5776 \times 100 = 577{,}600$.

Practice Problems

For practice, use the table and powers of ten to solve the following problems:

1. $\sqrt{17}$ 4.1231
2. $(170)^2$ 2.89×10^4
3. $\sqrt{8700}$ 9.3274×10
4. $(19)^2$ 361
5. $\sqrt{.000043}$ 6.5574×10^{-3}
6. $\sqrt{910000}$ $9.5394\,10^2$

"HOW TO PRONOUNCE . . ."

(Note: the accent falls on the part shown in CAPITAL letters.)

exponent	(ex-POE-nent)
farad	(FAIR-add)
radical	(RADD-ee-cul)

TEST QUESTIONS

Be sure to number your Answer Sheet Assignment 7.
Place your Name and Associate number on **every** Answer Sheet.
Send in your answers for this assignment immediately after you finish them. This will give you the greatest possible benefit from our personal grading service.

In answering these mathematical problems **show all of your work. Draw a circle around your answer.**

Do your work neatly and legibly.

1. $14^2 =$ 196
2. $\sqrt{121} =$ 11 (use long-hand method)
3. $4^5 =$ 1024
4. $\sqrt{72} =$ 8.4853 (use mathematical table)
5. State in powers of 10.
   - (a) 3000 3×10^3
   - (b) .000009 9×10^{-6}
   - (c) 1 1×10^0
   - (d) 9000000 9×10^6
6. What does the expression 10^3 mean? 1000
7. Solve by using powers of 10: 10,000 × 1,000 = 1×10^7
8. $\dfrac{1}{27000 \times 3 \times 10^{-5}}$ 71

   $$\begin{array}{r} 1.408 \\ 71\overline{)100.0} \\ 71 \\ \hline 290 \\ 284 \\ \hline 600 \\ 568 \\ \hline \end{array}$$

   1.408
9. Write in powers of ten.
   - (a) 17 megohms 1.7×10^7
   - (b) 3 milliamperes 3×10^{-3}
   - (c) 72 micro-microfarads 7.2×10^{-13}
   - (d) 270 kilovolts 2.7×10^5
   - (e) 72 picofarads 7.2×10^{-13}
10. $98^2 =$ (use mathematical table) 9,604

MATHEMATICAL TABLE of SQUARES and SQUARE ROOTS

No.	Square	Sq. Root	No.	Square	Sq. Root
1	1	1.0000	51	2,601	7.1414
2	4	1.4142	52	2,704	7.2111
3	9	1.7321	53	2,809	7.2801
4	16	2.0000	54	2,916	7.3485
5	25	2.2361	55	3,025	7.4162
6	36	2.4495	56	3,136	7.4833
7	49	2.6458	57	3,249	7.5498
8	64	2.8284	58	3,364	7 6158
9	81	3.0000	59	3,481	7.6811
10	100	3.1623	60	3,600	7 7460
11	121	3.3166	61	3,721	7.8102
12	144	3.4641	62	3,844	7.8740
13	169	3.6056	63	3,969	7.9373
14	196	3.7417	64	4,096	8.0000
15	225	3.8730	65	4,225	8.0623
16	256	4.0000	66	4,356	8.1240
17	289	4.1231	67	4,489	8.1854
18	324	4.2426	68	4,624	8.2462
19	361	4.3589	69	4,761	8.3066
20	400	4.4721	70	4,900	8.3666
21	441	4.5826	71	5,041	8.4261
22	484	4.6904	72	5,184	8.4853
23	529	4.7958	73	5,329	8.5440
24	576	4.8990	74	5,476	8.6023
25	625	5.0000	75	5,625	8.6603
26	676	5.0990	76	5,776	8.7178
27	729	5.1962	77	5,929	8.7750
28	784	5.2915	78	6,084	8.8318
29	841	5.3852	79	6,241	8.8882
30	900	5.4772	80	6,400	8.9443
31	961	5.5678	81	6,561	9.0000
32	1,024	5.6569	82	6,724	9.0554
33	1,089	5.7446	83	6,889	9.1104
34	1,156	5.8310	84	7,056	9.1652
35	1,225	5.9161	85	7,225	9.2195
36	1,296	6.0000	86	7,396	9.2736
37	1,369	6.0828	87	7,569	9.3274
38	1,444	6.1644	88	7,744	9.3808
39	1,521	6.2450	89	7,921	9.4340
40	1,600	6.3246	90	8,100	9.4868
41	1,681	6,4031	91	8,281	9.5394
42	1,764	6.4807	92	8,464	9.5917
43	1,849	6.5574	93	8,649	9.6437
44	1,936	6.6332	94	8,836	9.6954
45	2,025	6.7082	95	9,025	9.7468
46	2,116	6.7823	96	9,216	9.7980
47	2,209	6.8557	97	9,409	9.8489
48	2,304	6.9282	98	9,604	9.8995
49	2,401	7.0000	99	9.801	9.9499
50	2,500	7.0711	100	10,000	10.0000