Electronics

Radio

Television

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ARITHMETIC FOR ELECTRONICS

ASSIGNMENT 4

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In our electronics work, problems will be encountered which will require the use of mathematics. In this assignment, and in a few others spaced throughout the training, all of the mathematics which will be needed to solve these problems and to become a competent electronics technician will be covered.

There is one point which we wish to emphasize at this time. The mathematics which is included in this training program should offer no particular difficulty to you. We will assume that you know only the very basic operations of arithmetic—addition, subtraction, multiplication, and division. Each operation will be started at this most basic point and explained thoroughly from there on. We will show you simple methods of solving problems—in other words, we will show you the easy way to do the job by applying short cuts, etc.

Bear in mind that we **do not** expect you to be a mathematician now. As a matter of fact, you **will not** be a mathematician when you complete the training program. We are going **to teach you only the math you need to become an electronics technician.** Even if you have experienced difficulty with math in school, you should be able to master all of the math in this training program, as each step will be explained to you. The proven UEL training method will accomplish this for you, just as it will give you complete understanding of electronics parts and circuits.

Before we get into the actual subject of mathematics, there are a few words of advice that might be of great value to you. For one thing, you should get in the habit of doing your work neatly, carefully, and accurately, in order to reduce the possibility of careless mistakes. Even if you know what you're doing—know the mathematical operation and know the electronics work to which you are applying it—your answer may be worthless if you make a careless mistake in arithmetic.

None of us is perfect. We all stand a chance of making a mistake in arithmetic in even the most simple problems—even when we do the work neatly and carefully. We suggest, therefore, that you develop the habit of checking every bit of your work before you accept your answer as being correct.

When you start your work on the first Laboratory Experiment in the training program, you may find the operation of soldering rather awkward. However, as you proceed in the training and thereby have the opportunity of practicing soldering as you perform the many experiments, you'll soon find that it is a very simple operation, becoming "second nature" to you. You will find that, to a great extent, the same thing is true regarding mathematics. When you first start to use a particular operation in mathematics, you may find your handling of it a little awkward. However, as you acquire **practice** in its use, you will find that you'll also become efficient in its use.

You have, of course, found this to be true in the past, both in your own experience and in the experience of others. For example, multiplication is a short cut for addition, but you had to learn multiplication tables (through practice) before the multiplication process became useful to you. The typist finds it easier to prepare a page with a typewriter than with a pen or pencil yet considerable **practice** was necessary before the typewriter became a useful tool to her. Many short cuts will be presented in connection with the mathematics in this training. Practice in their use will enable you to arrive at a simple solution to electronics problems which would be difficult without mathematics.

Definitions

In this assignment, we will deal with addition, subtraction, multiplication, and division, and we will use whole numbers, fractions, mixed numbers, and decimals. Of course, the operations of addition, subtraction, multiplication, and division are so well known to everyone as to not require any definition. However, as it may have been a number of years since you "met" the various types of numbers we will use in this assignment, it might be well for us to "reintroduce" these numbers to you. In other words, we will give you definitions of them.

Whole Number. A whole number is a number which contains **no** fractions or decimals. Examples of whole numbers are: 1, 2, 13, 99, 796, 843, etc.

Fraction. A fraction is one number over another, and is actually an indication of division. For example, 1/2 is a fraction and means divide 1 by 2. Some more examples of fractions are: 2/3, 4/5, 3/10, 99/1000, etc.

Mixed Number. A mixed number is a whole number and a fraction. For example: 1 1/2, 3 7/8, 99 47/100, etc.

Decimal. A decimal is another way of expressing a fraction. For example, the fraction 1/10 may be expressed as the decimal .1. Other examples are: .7, .99, .76543, etc.

Simple Arithmetic

Simple arithmetic, involving the processes of addition, subtraction, multiplication and division of whole numbers, is used each day in our everyday life and is familiar to all. Examples of each process will be given here as a foundation for other arithmetic.

Addition

Example 1. Add these numbers
4732, 21, 492

Solution:
```
 4732
   21
  492
 ----
 5245
```
Answer: 5245

Example 2. Add these numbers:
976, 74, 3986, 10

Solution:
```
  976
   74
 3986
   10
 ----
 5046
```
Answer: 5046

For practice, work the following problems:

1. 721 + 432 = 1153
2. 821 + 32 + 4312 + 8 = 5173
3. 976 + 73 + 99 + 127 = 1275
4. 7932 + 9732 + 2379 + 3792 = 23835

Subtraction

Example 1. From 5245, subtract 492.

Solution:	5245	Check:	4753
	—492		+492
Answer:	4753		5245

Example 2. From 99,876, subtract 11.

Solution:	99,876	Check:	99,865
	—11		+11
Answer:	99,865		99,876

All subtraction problems should be "checked" by adding the answer obtained to the amount subtracted, and checking to see if the original number is obtained. Thus, in Example 1, 4753 should be added to 492, as shown at the right of the solution of the problem. Since the answer to the check, 5245 in this example, is equal to the original number, we know that the subtraction is correct. The check for Example 2 is also shown. For practice, work and check the following problems:

1. 7852 — 623 = 7229
2. 491 — 287 = 204
3. 688 — 400 = 288
4. 9763 — 27 = 9736

Multiplication

Example 1. Multiply 77 by 61.

Solution:

```
    77
   x61
    77
  462
Answer: 4697
```

Check:

```
        77
61 / 4697
     427
      427
      427
        0
```

Example 2. Multiply 4753 by 492.

Solution:

```
    4753
    x492
    9506
  42777
 19012
Answer: 2338476
```

Check:

```
          4753
492 / 2338476
      1968
       3704
       3444
        2607
        2460
         1476
         1476
            0
```

Each multiplication problem should be checked by dividing the answer obtained by one of the numbers originally multiplied together. If the other original number is obtained as the answer to this check, the multiplication is correct. Thus, in the check for Example 1, the answer 4697, is divided by 61, and 77 is obtained. Since 77 is the number in the example which was multiplied by 61, the multiplication is correct. If any other number, such as 76, or 77 $\frac{13}{61}$ etc., is obtained, it indicates that there is a mistake in the arithmetic. The check for Example 2 is also shown. For practice, work and check the following problems:

1. 421 x 78 = 32838
2. 9770 x 420 = 4103400
3. 623 x 796 = 495908
4. 977 x 23,784 = 23236968

Division

Example 1. Divide 99 by 11.

Solution:

```
      9 (Answer)
11 / 99
     99
      0
```

Check:

```
 11
 x9
 99
```

Example 2. Divide 26,677 by 721.

Solution:

```
          37 (Answer)
721 /26677
     2163
      5047
      5047
         0
```

Check:

```
  721
  x37
 5047
2163
26677
```

Example 3. Divide 784 by 53.

Solution:

```
      14
53 / 784
     53
     254
     212
      42
```

Remainder: 42

We have a remainder of 42. The answer may be written as 14 + 42/53 or 14 42/53. The remainder is expressed as a **fraction,** 42/53.

The entire answer, 14 42/53, is a **mixed** number, because it contains a whole number and a fraction.

All division problems should be checked by multiplying the answer by the divisor (the part **divided by** in the problem). If the dividend (the part **to be divided** in the problem) is obtained from this multiplication in the check, the arithmetic is correct. In the check for Example 1, the answer 9 is multiplied by the divisor 11, and 99 is obtained. Since 99 is the dividend in the problem, the arithmetic is correct. The check for Example 2 is also shown.

To check a division problem where the answer is a mixed number, as in Example 3, we simply multiply the whole number of the answer by the divisor, then **add** the remainder. If the dividend is obtained from this operation, the arithmetic is correct. To check Example 3 we do the following:

```
   53   (divisor)
 x 14   (whole number in answer)
 ----
  212
  53
 ----
  742
 + 42   (Remainder)
 ----
  784
```

Since 784 is the dividend, the arithmetic is correct.

In simple arithmetic it is usually best to express fractions as decimals. However, you will have to manipulate fractions when you work with electronics formulas later on. The best way to review the rules for using fractions is to practice with problems in simple arithmetic.

A fraction is made up of two quantities, a top number, or **numerator,** and a bottom number, or **denominator.** In the fraction 42/53 42 is the numerator and 53 is denominator. The answer to the division problem, Example 3, tells us we have 14 whole numbers plus a fraction. The answer is between 14 and 15. If we take one and divide it into 53 parts and take 42 of these 53 parts, we will have the correct amount to add to the 14.

The denominator tells us into how many parts we have divided the whole unit. The numerator tells us how many of these parts we have taken.

It will often be necessary to combine fractions. Addition and subtraction are opposite and the same general rules will apply for either operation.

Addition and Subtraction of Fractions

Example 1. Add 3/8 and 2/8.
Answer: 3/8 + 2/8 = 5/8.

Example 2. Add 5/13 and 6/13.
Answer: 5/13 + 6/13 = 11/13.

Example 3. Subtract 2/8 from 3/8.
Answer: 3/8 — 2/8 = 1/8.

Example 4. From 6/13 subtract 2/13.
Answer: 6/13 — 2/13 = 4/13.

Example 9. Add 1/6 and 3/8.

1/6 + 3/8 =

Using 48 as the common denominator;

8/48 + 18/48 = 26/48

In this case we could use 24 as the common denominator of the problem. Then to solve this same problem we proceed as follows:

1/6 + 3/8 =

Using 24 as the common denominator;

4/24 + 9/24 = 13/24

13/24 is equal to 26/48, so we have solved the problem and used smaller numbers than before.

You may wonder where the 24 was obtained as the common denominator in the preceeding example. It is the smallest number into which the 8 and the 6 can each be divided into evenly. In a great majority of the cases this number can be found by inspection. Another way the lowest common denominator can be found is shown by Example 10.

Example 10. Add 5/8, 7/12, and 11/18.

To find the lowest common denominator (abbreviated LCD), we proceed as follows:

(1)	2	8,	12,	18
(2)	2	4,	6,	9
(3)	2	2,	3,	9
(4)	3	1,	3,	9
(5)	3	1,	1,	3
		1,	1,	1

Explanation: To obtain the LCD of these numbers, write their denominators, 8, 12, and 18 in a line as shown and divide by the smallest number that will go into one or more of the numbers without a remainder. Thus, 2 will go into 8 four times, into 12 six times and into 18 nine times. Write the 4 under the 8, 6 under the 12 and 9 under the 18 as shown in line (2). We now divide each of these numbers in line (2) by 2, and where one of these numbers in the lines can not be divided evenly by the divisor, we bring down the number itself. Thus 2 goes into 4 twice, so we put the 2 under the 4. The number 2 goes into 6 three times, so we put the 3 under the 6, but since the 2 does not go into the 9 evenly, we bring down the 9. In this way we obtain line (3). Dividing the numbers in line (3), by 2 we obtain for line (4), 1, 3, 9. Dividing line (4) by 3, we obtain 1, 1, 3, for line (5). Again dividing by 3, we obtain 1, 1, 1, for line (6). Now we obtain the lowest common denominator by multiplying the divisors together. In this case, 2 x 2 x 2 x 3 x 3 = 72. Therefore, 72 is the lowest common denominator of these fractions. We then use the LCD to find the solution to the example. Therefore:

5/8 + 7/12 + 11/18 =

45/72 + 42/72 + 44/72 = 131/72

To apply this same method to solve Example 9, we proceed as follows:

1/6 + 3/8 =

2	6,	8
2	3,	4
2	3,	2
3	3,	1
	1,	1

LCD 2 x 2 x 2 x 3 = 24
This is how the common denominator of 24 was obtained in Example 9.

In all cases, notice that when we changed to common denominators, we multiplied **both** the denominator and numerator by the **same** number. This is always necessary in order that we do not change the value of our fraction.

For practice, work the following problems:

1. 5/8 + 3/8 = 1
2. 9/16 + 3/16 = 3/4
3. 1/8 + 1/5 = 13/40
4. 3/5 — 1/6 = 13/30
5. 7/8 + 1/6 + 1/3 = 1 3/8
6. 9/16 + 7/24 + 7/32 = 1 7/96
7. 13/18 — 7/27 = 25/54
8. 1/3 + 1/4 + 1/5 + 1/6 = 19/20

Multiplication and Division of Fractions

Multiplication and Division of fractions is much easier than addition and subtraction for two reasons. In the first place, we **do not** have to bother with common denominators. In the second place, we find we can often reduce the size of the numbers we are working with, by means of **cancellation.**

In multiplication of fractions, we multiply all the numerators together to obtain the numerator of the answer, and we multiply all of the denominators together to obtain the denominator in the answer.

Example 1. Multiply 2/7 by 3/5.

Solution: 2/7 x 3/5 = 6/35

The two numerators, 2 and 3, are multiplied together to obtain the numerator (6) of the answer. The two denominators, 7 and 5, are multiplied together to obtain 35 for the denominator of the answer.

Example 2. Multiply 3/7 x 6/7.

Solution: 3/7 x 6/7 = 18/49

Example 3. Multiply together, 1/2, 2/3, and 3/4.

Solution: $1/2 \times 2/3 \times 3/4 = \dfrac{1 \times 2 \times 3}{2 \times 3 \times 4} = \dfrac{6}{24}$

Example 4. Multiply these fractions together, 3/8, 1/3, and 5/9.

Solution, $3/8 \times 1/3 \times 5/9 = \dfrac{3 \times 1 \times 5}{8 \times 3 \times 9} = \dfrac{15}{216}$

For practice, work the following problems:

1. 3/4 x 3/4 = 9/16
2. 7/8 x 19/16 = 1 5/128
3. 5/12 x 1/9 = 5/108
4. 3/4 x 5/6 x 11/12 = 55/96

Division of fractions is also a simple process. To do this we invert the divisor (turn it upside down) and change the division sign to a multiplication sign, and then multiply the fractions.

Example 5. Divide 1/2 by 1/4.

Solution: $1/2 \div 1/4 =$
$1/2 \times 4/1 = 4/2$

Note that we inverted the divisor (the 1/4 in this example), then we multiplied.

Example 6. Divide 2/3 by 3/4.

Solution: $2/3 \div 3/4 =$
$2/3 \times 4/3 = 8/9$

Example 7. Divide 7/12 by 5/8.

Solution: $7/12 \div 5/8 =$
$7/12 \times 8/5 = 56/60$

For practice, work the folowing problems:

1. $9/10 \div 3/4 =$ 1 1/5
2. $1/7 \div 5/8 =$ 8/35
3. $19/21 \div 21/37 =$ 703/441 = 1 262/441
4. $7/8 \div 2 =$ 7/16

Note: Any whole number such as 2 may be written as that number over 1, or in this case 2/1.

We mentioned previously that the multiplication and division of fractions could be simplified by using **cancellation**. Let us see how this is accomplished.

When we were finding the common denominator, we said that it was permissible to multiply both the numerator and the denominator of a fraction by the same number, and that this did not change the value of the fraction.

It is also permissible to divide **both** the numerator and denominator by the same number.

The following example will demonstrate this:

Example 1. 6/10 can be written $\frac{6 \div 2}{10 \div 2}$, now $6 \div 2 = 3$, and $10 \div 2 = 5$.

The fraction is now 3/5 which is equal to 6/10. It is found simpler to do this by the process called cancellation. To employ cancellation, examine the fraction to see if there is some number which can be divided into **both** the **numerator** and the **denominator**. In this fraction, 6/10 we see that 2 can be divided into each. Then we proceed in this manner. Without bothering to write the 2 down, we say to ourself, 2 goes into 6 three times and into 10 five times. We then cross out, or **cancel,** the 6 and put a 3 above it, and **cancel** the 10 and put a 5 below it.

$$\frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}}$$ The new fraction is 3/5.

We have reduced this fraction to its lowest terms. We can find no number which will go evenly into both 3 and 5.

Example 2. Reduce 12/60 to its lowest terms.

$$\frac{\overset{6}{\cancel{12}}}{\underset{30}{\cancel{60}}}$$ Here we divided by 2. This fraction is still not in its lowest terms, for we can still divide 2 into each of the terms. Let us do this.

$$\frac{\overset{3}{\cancel{6}}}{\underset{15}{\cancel{30}}}$$ Now we have 3/15, but this fraction is not its lowest terms since 3 and 15 can both be divided by 3. Let us do this.

$$\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}}}$$ We now have reduced this fraction to its lowest terms by cancellation.

$$\frac{\overset{1}{\overset{\cancel{3}}{\overset{\cancel{6}}{\cancel{12}}}}}{\underset{5}{\underset{\cancel{15}}{\underset{\cancel{30}}{\cancel{60}}}}}$$ All of these steps would normally be performed without re-writing the fraction each time. This is shown at the left.

There is no set rule as to what number you should divide into both the numerator and the denominator. Fewer steps will result if the largest number possible is used. For example, in the fraction 12/60, if we had divided both parts by 12 in the first place, we would have had only one step. There are a number of ways that we could have arrived at this same answer. Some of these are illustrated below:

Dividing by 12. $$\frac{\overset{1}{\cancel{12}}}{\underset{5}{\cancel{60}}}$$

Dividing by 6 and then by 2. $$\frac{\overset{1}{\overset{\cancel{2}}{\cancel{12}}}}{\underset{5}{\underset{\cancel{10}}{\cancel{60}}}}$$

Dividing by 3 and then by 4. $$\frac{\overset{1}{\overset{\cancel{4}}{\cancel{12}}}}{\underset{5}{\underset{\cancel{20}}{\cancel{60}}}}$$

A fraction should always be reduced to its lowest terms to be in its proper form. If the answer to a problem contains a fraction, this fraction should be reduced to its lowest terms.

For practice, look back over the preceding problems on fractions and reduce each answer to its lowest terms.

Cancellation **can** be used when multiplying fractions. In division of fractions, first invert the divisor and then cancel. **Cancellation cannot** be used in addition and substraction of fractions.

Let us work a longer problem to show how much work can be saved by cancellation.

Example 1.

$\frac{625}{35} \times \frac{64}{40} \times \frac{49}{56}$ can be written as $\frac{625 \times 64 \times 49}{35 \times 40 \times 56}$

$$\frac{\overset{125}{\cancel{625}} \times \overset{8}{\cancel{64}} \times \overset{7}{\cancel{49}}}{\underset{7}{\cancel{35}} \times \underset{5}{\cancel{40}} \times \underset{8}{\cancel{56}}}$$

First: Cancel 5 into 625 and 35.

Cancel 8 into 64 and 40.

Cancel 7 into 49 and 56.

$$\frac{\overset{25}{\overset{\cancel{125}}{\cancel{625}}} \times \overset{1}{\overset{\cancel{8}}{\cancel{64}}} \times \overset{1}{\overset{\cancel{7}}{\cancel{49}}}}{\underset{1}{\underset{\cancel{7}}{\cancel{35}}} \times \underset{1}{\underset{\cancel{5}}{\cancel{40}}} \times \underset{1}{\underset{\cancel{8}}{\cancel{56}}}}$$

Next: Since we have an 8 in the numerator and the denominator, we will cancel 8 into each.

Cancel 7 into 7 and 7.

Cancel 5 into 125 and 5.

$\frac{25 \times 1 \times 1}{1 \times 1 \times 1} = \frac{25}{1} = 25$ **(answer)**

Naturally, in actual work it will not be necessary to rewrite your problem as you perform each step in cancellation.

The simplicity of this process will be further demonstrated by Example 2, 3, 4, and 5.

Example 2. Multiply 15/16 x 4/5 x 2/3.

Solution: $\frac{15}{16} \times \frac{4}{5} \times \frac{2}{3} = \frac{15 \times 4 \times 2}{16 \times 5 \times 3}$

$$\frac{\overset{1}{\overset{\cancel{3}}{\cancel{15}}} \times \overset{1}{\cancel{4}} \times \overset{1}{\cancel{2}}}{\underset{2}{\underset{\cancel{4}}{\cancel{16}}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{3}}} = \frac{1}{2}$$

Cancel 4 into 4 and 16.

Cancel 5 into 15 and 5.

Cancel 3 into 3 and 3.

Cancel 2 into 2 and 4.

Example 3. Multiply $\frac{300}{500} \times \frac{7}{8} \times \frac{45}{63}$

Solution:

$$\frac{\overset{3}{\cancel{300}} \times \overset{1}{\cancel{7}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{45}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{500}} \times 8 \times \underset{\underset{1}{\cancel{9}}}{\cancel{63}}} = \frac{3}{8}$$

Cancel 100 into 300 and 500.

Cancel 7 into 7 and 63.

Cancel 9 into 45 and 9.

Cancel 5 into 5 and 5.

Example 4. Divide 4/25 by 2/5.

Solution: $\frac{4}{25} \div \frac{2}{5} = \frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} = \frac{2}{5}$

Example 5. Divide 750/54 by 25/9.

Solution: $\frac{750}{54} \div \frac{25}{9} = \frac{\overset{\cancel{30}\ 5}{\cancel{750}}}{\underset{\underset{1}{\cancel{6}}}{\cancel{54}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{25}}} = 5$

For practice, work the following problems:

1. $\frac{9}{10} \times \frac{5}{3} =$
2. $\frac{650}{20} \times \frac{80}{15} \times \frac{75}{40} =$
3. $\frac{3}{4} \times \frac{96}{100} \times \frac{16}{27} =$
4. $\frac{77}{90} \div \frac{22}{60} =$
5. $\frac{96}{99} \times \frac{77}{27} \times \frac{18}{32} =$

Improper Fractions and Mixed Numbers

In some of the problems, we had fractions in which the numerator was larger than the denominator, $\frac{11}{10}$ for example. Such a fraction is called an improper fraction. When improper fractions appear in an answer, they should be changed to **mixed numbers.** We stated previously that a mixed number was a whole number and a fraction. The number $\frac{11}{10}$ should be changed to the mixed number $1\frac{1}{10}$. The value of the mixed number can be obtained readily by dividing the numerator by the denominator.

Example 1. Change 11/10 to a mixed number.

Solution: $10 \overline{)\,11\,}$ with quotient 1; $11 - 10 = 1$; $= 1\frac{1}{10}$

$$\begin{array}{r} 1 \\ 10\,\overline{)\,11} \\ \underline{10} \\ 1 \end{array} = 1\frac{1}{10}$$

Other examples of changing improper fractions to mixed numbers follow.

Example 1. Change 17/2 to a mixed number.

Solution:

$$\begin{array}{r} 8 \\ 2\,\overline{)\,17} \\ \underline{16} \\ 1 \end{array} = 8\frac{1}{2}, \quad \frac{17}{2} = 8\frac{1}{2}$$

Example 2. Change 131/72 into a mixed number.

Solution:

$$\begin{array}{r} 1 \\ 72\,\overline{)\,131,} \\ \underline{72} \\ 59 \end{array} \quad \frac{131}{72} = 1\frac{59}{72}$$

Example 3. Change 413/22 to a mixed number.

Solution:

$$\begin{array}{r} 18 \\ 22\,\overline{)\,413,} \\ \underline{22} \\ 193 \\ \underline{176} \\ 17 \end{array} \quad \frac{413}{22} = 18\frac{17}{22}$$

Mixed numbers **cannot** be used conveniently in solving problems involving multiplication and division, so if a mixed number appears in a problem, it should be changed to an improper fraction before proceeding.

Example 1. Multiply 1 1/3 times 1/4.

We wish to multiply the mixed number 1 1/3 by a fraction. First we change 1 1/3 to an improper fraction. The number, 1 1/3 means one plus one third, so let us write it that way.

$$1 + \frac{1}{3}$$

$$\frac{1}{1} + \frac{1}{3}$$

$$\frac{3}{3} + \frac{1}{3} = \frac{4}{3}$$

1 may be written as $\frac{1}{1}$, so we may again re-write the mixed number. Now we have a problem of adding two fractions so we combine them by using a common denominator, 3 in this case. The improper fraction is $\frac{4}{3}$ which is equal to $1\frac{1}{3}$. The problem may easily be solved now.

Solution to Example 1: $1\frac{1}{3} \times \frac{1}{4}$

$$\frac{4}{3} \times \frac{1}{4} = \frac{\overset{1}{\not{4}}}{\underset{3}{\not{12}}} = \frac{1}{3}$$

Example 2. Change 4 3/5 to an improper fraction.

$$4\frac{3}{5} = \frac{4}{1} + \frac{3}{5} = \frac{20}{5} + \frac{3}{5} = \frac{23}{5}$$

A simpler way to write this is:

$$4\frac{3}{5} = \frac{(4 \times 5) + 3}{5} = \frac{23}{5}$$

Example 3. Change 7 9/11 to an improper fraction.

$$7\frac{9}{11} = \frac{(7 \times 11) + 9}{11} = \frac{86}{11}$$

Example 4. Change 6 7/8 to an improper fraction.

$$6\frac{7}{8} = \frac{(6 \times 8) + 7}{8} = \frac{55}{8}$$

For practice, work the problems on the following page:

Change to mixed numbers.

1. $\frac{9}{7}$ 2. $\frac{99}{62}$ 3. $\frac{163}{23}$

Change to improper fractions.

1. $9\frac{1}{3}$ 2. $7\frac{6}{8}$ 3. $99\frac{1}{9}$

In multiplication and division involving mixed numbers, first change the mixed numbers to improper fractions.

Example 1. Multiply $3\frac{5}{8}$ by $7\frac{1}{9}$.

Solution: $3\frac{5}{8} \times 7\frac{1}{9}$

$$\frac{29}{\underset{1}{\not{8}}} \times \frac{\overset{8}{\not{64}}}{9} = \frac{232}{9} = 25\frac{7}{9}$$

Example 2. Divide $4\frac{3}{5}$ by $9\frac{2}{3}$.

Solution: $4\frac{3}{5} \div 9\frac{2}{3} = \frac{23}{5} \div \frac{29}{3} = \frac{23}{5} \times \frac{3}{29} = \frac{69}{145}$

In adding and subtracting mixed numbers, we can group all the whole numbers and then group all of the fractions. This is usually the easiest way to handle this type of problem.

Example 1. $37\frac{5}{8} + 4\frac{1}{2} - 18\frac{2}{3} =$

Solution: $37 + 4 - 18 = 23$, $\frac{5}{8} + \frac{1}{2} - \frac{2}{3} = \frac{15}{24} + \frac{12}{24} - \frac{16}{24} = \frac{11}{24}$

Final answer: $23 + \frac{11}{24} = 23\frac{11}{24}$

Example 2. $21\frac{1}{4} + 8\frac{1}{2} + 6\frac{2}{3} =$

Solution: $21 + 8 + 6 = 35$, $\frac{1}{4} + \frac{1}{2} + \frac{2}{3} = \frac{3}{12} + \frac{6}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$

Final Answer: $35 + 1\frac{5}{12} = 36\frac{5}{12}$

For practice, solve the following problems:

1. $1\frac{7}{8} \times 6\frac{1}{4} =$

2. $2\frac{1}{4} \div 1\frac{1}{3} =$

3. $16\frac{1}{5} + 17\frac{1}{3} + 3\frac{1}{10} =$

4. $6\frac{1}{4} \div 2\frac{3}{5} =$

Decimals

A **decimal fraction,** commonly called a decimal, is a fraction whose denominator is 10, 100, 1000, 10,000. etc. Thus 3/10, 95/100 and 625/1000 are all decimal fractions. Since the denominator of a decimal fraction is always 10 or some power of 10, that is, since it is always 1 followed by zeros, we write a decimal fraction more compactly by omitting the denominator entirely. Thus, 3/10 is written .3, 95/100 is written .95, and 625/1000 is written .625. To distinguish the decimal fraction .3 from the whole number 3, we place a period (.) in front of the number. This period is called the **decimal point**. Any number therefore, with a decimal point in front of it, is a fraction whose numerator is the number after the decimal point, and whose denominator is a 1 followed by as many zeros as there are figures in the number to the right of the point. Thus, .1 means 1/10, .75 means 75/100, and .2754321 means 2,754,321/10,000,000. The fraction 2/100 is expressed as a decimal as .02, 2/1000 as .002, and 2/10,000 as .0002.

The decimal fraction .3 is read three-tenths, .75 is read 75 hundredths, and .975 is read 975 thousandths etc.

For practice, state the following fractions as decimals:

1. $\frac{9}{10}$ = .9

2. $\frac{41}{100}$ = .41

3. $\frac{795}{1000}$ = .795

4. $\frac{29,373}{100,000}$ = .29373

Let us suppose that you are fortunate enough to have the following money in your pocket: 3 one hundred dollar bills, 7 ten dollar bills, 4 one dollar bills, 8 dimes and 6 pennies. At a moments notice you could tell someone exactly how much money you have. Let us review the rules you would instinctively use in adding up the different amounts.

```
$300.
  70.
   4.
    .8
    .06
-------
$374.86
```

In making the addition, every decimal point we used was kept in the same vertical column. This rule will always hold true for both **Addition** and **Subtraction** with decimal quantities. Also notice the position of each figure in the answer. The 3 is in the "hundreds place", the 7 is in the "tens place", and the 4 is in the ones or "units place". All these figures to the left of the decimal represent numbers of whole dollars. Those to the right of the decimal point represent decimal fractions of a dollar. The 8 (for 8 dimes) is in the "tenths place" and represents 8/10 of a dollar. The 6 is in the "hundredths place" and represents 6/100 of a dollar.

Addition and Subtraction of Decimals

Example 1. Add 963.3, 77.06, 90.234 and 17.4234.

Solution
```
 963.3
  77.06
  90.234
  17.4234.
----------
1148.0174
```
Answer: 1148.0174

Notice that the decimal points are arranged in a vertical column. The decimal point in the answer is in this same vertical column.

Example 2. Add 9.0006, 40.01, 777.777 and .000009.

Solution:
```
  9.0006
 40.01
777.777
   .000009
-----------
826.787609
```
Answer: 826.787609

Example 3. From 673.0909 subtract 423.762.

Solution:

```
  673.0909
 —423.7620
  249.3289
```

Notice that in this case we may add zeros to the right of the decimal since this is equivalent to multiplying both the numerator and denominator of the fraction by 10. The number 762/1000. is equal to 7620/10,000.

Example 4. From 9.09 subtract 4.1321.

Solution:

```
  9.0900
 —4.1321
```

Answer: 4.9579

For practice,solve the following problems:

1. Add. 976.23, 7.707, 641.0302, .000007. 1624.967207
2. Add. 7432.001, 963.1, 724.0001, 91.69. 9210.7911
3. From 879.69 subtract 432.78. 446.91
4. From 900 subtract 899.9999. .0001

Multiplication and Division of Decimals

Multiplication and division of decimals are performed in the same way as with whole numbers. The only difference is in locating the decimal point in the answer. The proper location of the decimal point is very important.

In multiplication, we locate the decimal point in the answer after we have performed the multiplication. We merely count up the number of digits, or places, we have to the right of the decimal place in the two numbers we are multiplying together, then place the decimal point that many place from the right in the answer.

Example 1.

```
 432.1
  x .07
 30.247
```

There is one place to the right of the decimal place in 432.1 and there are two places to the right of the decimal place in .07. This makes a total of three places. After multiplying the numbers together, we count three places from right to left in the answer and locate the decimal place at that point.

Example 2.

```
  7.204
  x 2.1
   7204
 14408
 15.1284
```

Example 3.

```
   .0007
   x .03
 .000021
```

Example 4.

```
  6701
   x .7
 4690.7
```

Example 5.

```
  200.001
   x.0001
 .0200001
```

In division we locate the decimal point in the answer by moving the decimal in the divisor to the right as many places as necessary to make the divisor a whole number. Then move the decimal point in the dividend to the right the same number of places. The decimal point in the answer will be immediately above the new decimal location in the dividend.

Example 1. Divide 166.296 by 4.92.

Solution: 4.92 / 166.296

492. / 16629.6

There are two decimal places in the divisor, so we move the decimal place two places to the right to make the divisor a whole number (492). Then we move the decimal place in the dividend an equal number of places to the right, making it 16629.6.

492 / 16629.6

The decimal place in the answer will be immediately above this new decimal point in the dividend. Now we divide, being sure to place each number in the answer immediately above the **last** number of the part of the dividend we are using in that step. In this example, the first 3 in the answer is placed immediately above the 2 in the dividend, since the 2 is the last number in the part of the dividend we are using in this step (1662). The next 3 in the answer goes above the 9, and the 8 in the answer goes above the 6.

```
      33.8
492 /16629.6
     1476
      1869
      1476
       3936
       3936
          0
```

The answer is 33.8. To check, multiply the answer 33.8 by the **original** divisor 4.92.

Check:

```
   33.8
   4.92
    676
  3042
 1352
166.296
```

Since the answer to the check is the original dividend, the arithmetic is correct.

Example 2. Divide 32 by .016.

```
.016 / 32

016. / 32000.

        2000.
016 / 32000
      32
       000
```

The divisor decimal point is moved three places to the right. Three zeros are added to the dividend so that the decimal point can be moved three places to the right.

Check: 2000 x .016 = 32

Example 3. Divide 96.16 by .16.

Solution: .16 /96.16

16. /9616.

```
       601.
16 / 9616
     96
      016
       16
        0
```

Check:

```
   601
   .16
  3606
   601
 96.16
```

Example 4. Divide 2468.9 by 7.23.

Solution: 7.23 /2468.9

723. / 246890.

```
        341.
723 / 246890.
      2169
       2999
       2892
        1070
         723
         347
```

Notice that when we carried out the divisor the answer did not come out even, but that there is a remainder. In this case we may add zeros at the right of the decimal place in the dividend. We may continue this process as far as we wish.

```
        341.47
723 / 246890.00
      2169
       2999
       2892
        1070
         723
         3470
         2892
          5780
          5061
           719
```

For practice, solve the following problems:

1. 6.702 x 90.6031 = 607.2219762
2. 4321.001 x .008 = 34.568008
3. 9.70101 x .000006 = .0000582060606
4. 15.1284 ÷ 2.1 = 7.204
5. 625 ÷ .125 = 5000.
6. 823.010 ÷ .0007 = 1,175,728.57142

Changing Fractions to Decimals

In solving arithmetic problems in practical electronics work, it will often be best to work out problems in fractions by means of decimals. It is an easy matter to convert any fraction to a decimal. It will be recalled that, near the first of this assignment, it was stated that a fraction was an indication of division. Thus 3/4 means three divided by four. To convert this fraction into a decimal, all we have to do is to carry out this indicated division.

Example 1. Convert 3/4 to a decimal.

Solution:

```
     .75
  4/ 3.00
     28
      20
      20
       0
```

$$\frac{3}{4} = .75$$

Example 2. Convert 1/8 to a decimal.

Solution:

```
     .125
  8 /1.000
      8
     20
     16
      40
      40
```

$$\frac{1}{8} = .125$$

Example 3. Convert 3/25 to a decimal.

Solution:

```
      .12
  25 /3.00
      2 5
       50
       50
        0
```

$$\frac{3}{25} = .12$$

Example 4. Convert 1/3 to a decimal.

Solution:

```
     .333+
  3 /1.000
      9
     10
      9
      10
       9
       1
```

$$\frac{1}{3} = .333 +$$

The + indicates that this decimal did not come out even, but had a remainder. The number .333 is accurate enough for all practical electronics work.

Example 5. Add 7/8, 3/25, 2/5.

Solution: $\frac{7}{8} = 7 \div 8 = .875$

$\frac{3}{25} = 3 \div 25 = .12$

$\frac{2}{5} = 2 \div 5 = .4$

$$\begin{array}{r} .875 \\ .12 \\ .4 \\ \hline 1.395 \end{array}$$

Answer: 1.395

Example 6. Add 17 3/4, 22 3/16, 5 3/8.

Solution: $\frac{3}{4} = 3 \div 4 = .75$

$\frac{3}{16} = 3 \div 16 = .1875$

$\frac{3}{8} = 3 \div 8 = .375$

$$\begin{array}{r} 17.75 \\ 22.1875 \\ 5.375 \\ \hline 45.3125 \end{array}$$

Answer: 45.3125

For practice solve the following problems using decimals:

1. Add 1/4, 1/5, 1/8. .25 + .2 + .125 = .575
2. Add 3/10, 5/8, 1/3. .3 + .625 + .3333 = 1.2583
3. From 3/4 subtract 1/3. .75 - .3333 = .4167

Percentage

Percentage is merely a useful way of comparing different quantities. The word percent (sometimes shown by the symbol %) means hundredth. Thus, 1 per cent means 1/100 part.

Example 1. What is 1% of $500?

Solution: $1\% = \frac{1}{100}$

$\frac{1}{100} \times 500 = \$5$ (answer)

In this example, 1% is called the "rate", and 500 is called the "base".
Example 2. What is 2% of 1000 Volts?

Solution: $2\% = \frac{2}{100}$

$\frac{2}{100} \times 1000 = 20$ Volts

As we have learned previously, any fraction with 100 as a denominator may be written as a decimal. Therefore, since 1% = 1/100, it also equals .01. A simple way to solve percentage problems is to convert the percentage to a decimal and then multiply this decimal by the base. To solve Example 1 by this method, we follow the procedure below:

What is 1% of $500?

1% = .01

$$\begin{array}{r} 500 \\ \times\ .01 \\ \hline \$5.00 \end{array}$$

Answer: $5.00

Example 3. What is 5% of 75?

Solution: $5\% = \frac{5}{100} = .05$

$$\begin{array}{r} 75 \\ \times\ .05 \\ \hline 3.75 \end{array}$$

Answer: 3.75

Example 4. A certain voltmeter is guaranteed by the manufacturer to be accurate within plus or minus 2%. If the meter is indicating 8 Volts, how much error may there be?

Solution: 2% = .02

$$\begin{array}{r} .02 \\ \text{x}\ \ 8 \\ \hline .16 \end{array}$$

Answer: .16 Volt error

Since the guarantee stated plus or minus 2% the actual voltage may be as high as 8 + .16 Volts, or 8.16 Volts; or it may be as low as 8 — .16 Volts, or 7.84 Volts.

Example 5. What is .5% of 50 Volts?

Solution: $.5\% = \frac{.5}{100} = \frac{5}{1000} = .005$

$$\begin{array}{r} 50 \\ .005 \\ \hline .250 \end{array}$$

Answer: .250 Volts

Example 6. A certain industrial Induction Heating Unit is 20% efficient. That is, its output is 20% of its input. If the input power of this equipment is one kilowatt (1000 watts), what is the output power?

Solution: 20% = .20

1000
x .20

Answer: 200.00 Watts

In electronics work a different type of percent problem may be encountered.

Example 7. What percent of 8 is 7?

Solution: 7 is ⅞ of 8, and ⅞ = .875 = 87.5%

To solve this type of problem then, we divide the 7 by the 8 and then change this decimal to percentage. To change a decimal to percentage, move decimal point two places to right. Thus .875 = 87.5%.

Example 8. A certain ultrasonic cleaner has 600 watts output and 900 watts input. What percentage of the input is the output?

$$\text{Solution: } \frac{\text{Output}}{\text{Input}} = \frac{\overset{2}{\cancel{6}}\cancel{0}\cancel{0}}{\underset{3}{\cancel{9}}\cancel{0}\cancel{0}} = \frac{2}{3} = .666 = 66.6\%$$

For practice, solve the following problems:

1. 2% of 8 Volts = .16v
2. .2% of 8 Volts = .016v
3. 10.2% of 14 Volts = 1.428v
4. 63% of 700 Watts = 441 watts
5. What % of 700 is 400? 57.142857142̅8%

Significant Figures

In pure mathematics, a number is generally considered to be exact. For example, 110 would mean 110.000, etc., for as many zeros as we wish to add after the decimal point. In electronics work this is not always the case. For example, a certain switchboard meter might read 110 volts, but a reading made with an expensive precision meter might indicate 110.2 volts. A series of precise readings might indicate the voltage to be 110.24 volts. Thus we can see that the 110 volt reading of the switchboard meter was not an **exact** reading, but was an approximate reading. The 110 volts was approximately 110 volts, and not 110.000000 volts as in pure mathematics.

Any number representing a measurement, or the amount of some quantity, expresses the accuracy of the measurement. The figures required are known as the **significant figures.**

The significant figures of any number are the figures 1, 2, 3, 4, 5, 6, 7, 8 and 9, in addition to all zeros that occur between them.

Thus, .00368 volt has 3 significant figures.
.20007 amperes has 5 significant figures.

The zeros in .20007 fall in between the 2 and 7 and are significant figures.
73.25 has 4 significant figures.
47321.4 has 6 significant figures.

The number of significant figures in a meter reading, or in the answer to a problem, is a measure of the accuracy of the reading or answer.

Thus, one man worked out a simple electrical problem and calculated that 2.36 amperes flowed through a certain resistor and another man calculated 2.3623 amperes through the same resistor. The second man evidently worked the problem out more accurately.

In general, then, the greater the number of significant figures in an answer, the more accurate is the answer. A little common sense will go a long way in working with significant figures.

If you took a reading with a voltmeter and read 47.87 volts, you have four significant figures to work with. It might be better to call your reading 47.9 volts, or even 48 volts in many cases for the following three reasons:

A. Certainly, your eye isn't so good that you can read the average meter to four significant figures.

B. You might need only two significant figures, in which case the .87 volt is a needless display of accuracy.

C. The meter might only be accurate to within ± .5 volt in which case you have no assurance that the .87 volt has much significance.

Therefore the .07 volt could not be relied on.

In electronics work, an accuracy of three significant figures is usually sufficient.

Rounding Numbers

In "rounding off" the 47.87 volts to 3 significant figures (47.9 volts) and to 2 significant figures (48 volts), we follow a simple rule.

As you drop figures when rounding off numbers, try to make the remaining significant figures show values as close as possible to the original number.

Examples.

.00737373 to four significant figures = .007374. We increased the last 3 to a 4 since the next number was a 7 (larger than 5).

375.4381 to three significant figures = 375
49.371 to three significant figures = 49.4
80,750 to two significant figures = 81,000
.0303 to one significant figure = .03
3.3333 to four significant figures = 3.333
6.6666 to four significant figures = 6.667

If the number we drop is exactly 5 we can either increase or decrease the previous number.

37.5 to two significant figures = 37 or 38
.18250 to three significant figures = .182 or .183,

but .18253 to three significant figures equals only .183 since we know that the 5 is followed by a 3, and .53 is greater than one half.

For practice, round these numbers off to three significant figures:

1. 2.7382 = 2.74
2. .00047213 = .000472
3. 9,777,700 = 9,780
4. 6.2914 = 6.29
5. .00300821 = .00301

In this assignment we have covered the basic operations of arithmetic. These are: Addition, subtraction, multiplication and division. We have applied these operations to whole numbers, mixed numbers, fractions and decimals. We have also discussed percentage and rounding of numbers.

We suggest that you "work through" this assignment several times, concentrating particularly on any part of it that you find at all strange, or unfamiliar. As suggested at the first of the assignment, **practice** on these parts until everything in the assignment is quite clear to you.

Test Questions

Be sure to number your Answer Sheet, Assignment 4.
Place your Name and Associate number on every Answer Sheet.
Send in your Answers for this assignment immediately after you finish them. This will give you the greatest possible benefit from our personal grading service.

In answering these arithmetic problems, **show all of your work. Draw a circle around your answer.** Do your work neatly and legibly.

1. Multiply 378 x 226. 85,428
2. Add 21, 728, 64, 9643. 10,456
3. Divide 777 by 21. 37
4. Divide $\frac{5}{12}$ by $\frac{1}{3}$. $\frac{5}{4}$ 1.25
5. Multiply $\frac{294}{98}$ x $\frac{126}{56}$ x $\frac{112}{4}$. 189
6. Multiply 973.01 by .0063. 6.129963
7. Divide 973.01 by .0063. 154,446
8. If a meter has an error of plus or minus 1%, how high and how low might the voltage actually be if the meter indicates 110 volts?
   111.1 - 108.9
9. Multiply $7\frac{1}{7}$ by $4\frac{3}{4}$. 33.928, $\frac{475}{14}$, $33\frac{13}{14}$
10. Convert these fractions to decimals, and add $\frac{7}{8}$, $\frac{9}{10}$, $\frac{3}{4}$, $\frac{1}{2}$.

    3.025