

LOUISVILLE

## EQUATIONS

In the previous math assignment we dealt with algebraic terms and expressions. All of the problems in that assignment were merely exercises in handling algebraic quantities. In this assignment we will apply the fundamentals which we have learned to the solution of algebraic equations. An algebraic expression containing an equal sign is called an equation. You will be able to solve some useful electronics problems at the end of this assignment.

A typical algebraic equation is:
$\mathrm{A}=14+2$
This is an equation because it has an equal sign in it. It is algebraic because it contains a letter (A) which represents some unknown quantity. The thing which we wish to know in such an equation is the numerical value of the unknown quantity. In the equation, $A=14+2$ it can be seen immediately that $\mathrm{A}=16$.

In the following equations it may not be as simple to determine the value of the unknown quantity. In each case, however, check the answer by substituting in the original equation.

$$
\begin{array}{rlrlrl}
3 \mathrm{~A} & =12 & \text { Answer: } & & \mathrm{A}=4 & \text { Check: } \\
\mathrm{A} & =10-\mathrm{A} & \text { Answer: } & \mathrm{A}=5 & \text { Check: } & 5=12 \\
2 \mathrm{R} & =9-\mathrm{R} & \text { Answer: } & \mathrm{R}=3 & \text { Check: } 2 \times 3=9-3 \\
\frac{\mathrm{C}}{2} & =\mathrm{C}-6 & \text { Answer: } & & \mathrm{C}=12 & \text { Check: }
\end{array}
$$

Part of this asignment will be devoted to a "sure-fire" method of solving for unknowns in the above types of problems.

The formulas which are used in electronics work are equations, since they all contain equal signs. You have probably mastered the three forms of Ohm's Law by this time:

$$
\mathrm{E}=\mathrm{IR} \quad \mathrm{I}=\frac{\mathrm{E}}{\mathrm{R}} \quad \mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}}
$$

Each of these formulas is an equation. You will find that it is really necessary to know only one form of Ohm's Law. If, for instance, we know that $I=E / R$, a knowledge of algebra would enable us to find the other two equations.

Handbooks and textbooks seldom give all the possible forms for electronics formulas. You will want to "solve" formulas for various unknowns.

To illustrate this, let us consider an example containing some facts which we learned in Assignment 12. We learned that three of the factors which effected the resistance of a conductor were, (1) the metal which is used to
make the wire, (2) the length of the wire, and (3), the cross-sectional area of the wire. This can be expressed by the formula:

$$
\mathrm{R}=\frac{\mathrm{oL}}{\mathrm{~A}}
$$

Q (the Greek letter "rho") is a factor which takes into account the kind of metal of which the wire is made.
$L$ is the length of the wire.
A is the cross-sectional area of the wire.
$R$ is the resistance of the wire.
If you have a piece of iron wire and have measured its length and determined its area, it is an easy matter to determine its resistance.

You look in an electrical handbook and find the value of $\varrho$ for iron. By substituting the values of $\rho, L$ and $A$ in the formula you can easily determine the resistance of the wire by simple arithmetic.

Suppose, however, you wish to use some of the same iron wire to make a 10 ohm resistor. You still know the area of the wire, A. You can still determine the value of $o$ for iron from the handbook. Now, however, you know the value of $R(10 \mathrm{ohms})$ and wish to determine $L$, the length. In other words, $L$ is the unknown quantity.

A knowledge of algebra will enable you to quickly change the formula from:

$$
\mathrm{R}=\frac{\mathrm{QL}}{\mathrm{~A}} \quad \text { to }: \quad \mathrm{L}=\frac{\mathrm{RA}}{\varrho}
$$

You can now substitute the known values of $R, A$ and $\varrho$ and determine the length of wire (L) you need by simple arithmetic. You will find that this useful process of solving formulas for different unknowns is really quite simple.

The usual method of solving a problem involving an algebraic equation is to first "solve" the equation for the desired unknown by rearranging the equation so that the desired unknown is on the left of the equal sign, and all other terms are on the right side. This is illustrated in the preceding example. The length, $L$, was the unknown quantity, the numerical value of which we wished to find. The formula was rearranged to get the unknown (L) on the left side of the equal sign. In this assignment you will learn how to do this.

## Solving Equations for Numerical Values

There is only one main point to follow in all work with equations. The equal $\operatorname{sign}(=)$ means just what it says. The terms on one side of the equal $\operatorname{sign}$ are equal to the terms on the other side. The equal sign is going to remain in our equation from start to finish. Therefore, no matter what we
do to the equation, the work will be correct provided the quantity on one side of the equal sign equals the quantity on the other side.

There is only one way we can keep both sides of an equation equal as we solve an equation. No matter what we do to one side of an equation, we have to do the same thing to the other side.

It will be useful to remember that:
a. Addition and Subtraction are opposites.
b. Multiplication and Division are opposites.
c. Powers and Roots are opposite operations.

If you have studied algebra recently you may remember something about "transposing" or "transposition". Please observe that transposition is merely a short cut for the method recommended in this assignment. If you are familiar with transposition you may solve the problems in this assignment by that method. You will probably find the method used in this assignment less confusing and more practical.

As a guide in illustrating the problems, a simple notation will be used. The letters $\mathbf{A}, \mathbf{S}, \mathbf{M}$ and $\mathbf{D}$ will be used to indicate addition, subtraction, multiplication and division.

Thus: A : 2 means Add 2 to each side of the equation.
D : C means Divide both sides of the equation by $C$, etc.
Many examples will now be given. In each problem:

1. Follow each step carefuly.
2. Notice that in each step we do the same thing to both sides of the equation.
3. Notice the method used when numerical answers are checked by substitution.

## Problem: $\mathrm{A}+2=10 \quad$ Find A

The equation tells us what $A+2$ equals. We wish to know what $A$ equals.

On the left side of the equation we have 2 added to $A$. To get rid of the 2 on the left side do the opposite. We subtract 2 from the left side, but if we do this, we must also subtract 2 from the right side.


We have 3 subtracted from $A$. To get rid of the 3 on the left side do the opposite. Add 3 to both sides.

$$
A-3=11
$$

A:3 $\mathbf{A}-3+3=11+3$
Answer: $\quad A=14$
Check: $\quad 14-3=11$

$$
11=11
$$

Problem: $3 \mathrm{~A}=15$
Find $A$
We have A multiplied by 3 . To get rid of the 3 on the left hand side do the opposite. Divide both sides by 3 .

$$
3 \mathrm{~A}=15
$$

D : 3

$$
\frac{8 \mathrm{~A}}{\dot{\phi}}=\frac{15}{3}
$$

Answer: $\quad A=\frac{15}{3}=5$
Check: $\quad 3 \times 5=15$

$$
15=15
$$

Problem: $\frac{\mathrm{C}}{4}=7 \quad$ Find $C$
On the left side we have $C$ divided by 4 . To get rid of the 4 do the opposite. Multiply both sides by 4 .

$$
\frac{C}{4}=7
$$

M : 4

$$
\frac{\mathrm{C} \times 4}{4}=7 \times 4
$$

Answer:

$$
C=28
$$

Check:

$$
\frac{28}{4}=7
$$

$$
7=7
$$

Problem: 3A $+7 \mathrm{~A}=40$
Collect terms: $\quad 10 \mathrm{~A}=40$

D : 10

$$
\frac{10 \mathrm{~A}}{1 \emptyset}=\frac{40}{10}
$$

Answer:

$$
A=4
$$

Check: $(3 \times 4)+(7 \times 4)=40$

$$
\begin{array}{r}
12+28=40 \\
40=40
\end{array}
$$

Problem: 7A $=20+3 \mathrm{~A} \quad$ Find $A$
We have terms containing $A$ on both sides of the equal sign. We want to have all the terms containing $A$ on the left side of the equation. We have 3 A added to 20 on the right. Do the opposite. Subtract 3 A from both sides in order to group all the A's.

$$
7 A=20+3 A
$$

$\mathrm{S}: 3 \mathrm{~A} \quad 7 \mathrm{~A}-3 \mathrm{~A}=20+\not 9 \mathrm{~A}-\not 2 A$
Collect terms: $4 \mathrm{~A}=20$
D : $4 \quad \frac{4 \mathrm{~A}}{4}=\frac{20}{4}$
Answer: $\quad \mathrm{A}=5$
Check: $\quad 7 \times 5=20+(3 \times 5)$

$$
35=20+15
$$

$$
35=35
$$

Problem: $5 Y+6=4 Y-20 \quad$ Find $Y$
S : 4Y
Collect terms:

$$
5 Y+6-4 Y=4 Y-20-4 Y
$$

S : 6

$$
\mathrm{Y}+\not b-\phi=-20-6
$$

Answer:

$$
Y=-26
$$

Check:

$$
\begin{aligned}
(5 \times-26)+6 & =(4 \times-26)-20 \\
-130+6 & =-104-20 \\
-124 & =-124
\end{aligned}
$$

Problem: $\frac{6 B}{5}=4$
M :5 $\quad \frac{6 \mathrm{~B} \times \neq 6}{\not b}=4 \times 5$
Simplify :

$$
6 \mathrm{~B}=20
$$

D : 6

$$
\frac{6 \mathrm{~B}}{6}=\frac{20}{6}
$$

Answer:

$$
B=3^{1 / 3}
$$

Check:

$$
\frac{6(31 / 3)}{5}=4
$$

$$
\frac{\mathscr{b}\left(\frac{20}{b}\right)}{5}=4
$$

$$
4=4
$$

Problem: $4(\mathrm{E}-2)-6 \mathrm{E}+12=7-5(\mathrm{E}+1)+41$
Find E
Simplify: $\quad 4 \mathrm{E}-8-6 \mathrm{E}+12=7-5 \mathrm{E}-5+41$
Collect terms: $-2 \mathrm{E}+4=-5 \mathrm{E}+43$
$\mathrm{A}: 5 \mathrm{E} \quad-2 \mathrm{E}+4+5 \mathrm{E}=-5 \mathrm{E}+43+\not 5 \mathbb{Z}$
Collect terms:

$$
3 \mathrm{E}+4=43
$$

S : 4
Collect terms:

$$
3 \mathrm{E}+4-4=43-4
$$

$$
3 \mathrm{E}=39
$$

D : 3

$$
\frac{\not 8 \mathrm{E}}{\ddot{\not x}}=\frac{39}{3}
$$

Answer:

$$
\mathrm{E}=13
$$

Check:

$$
\begin{aligned}
& 4(13-2)-6 \times 13+12=7-5(13+1)+41 \\
& 4 \times 11-6 \times 13+12=7-5 \times 14+41 \\
& 44-78+12=7-70+41 \\
& -22=-22
\end{aligned}
$$

Problem: $5 \mathrm{R}-3(7+2 \mathrm{R})=\mathrm{R}+30 \quad$ Find R
Simplify: $\quad 5 R-21-6 R=R+30$
Collect terms: $\quad-\mathrm{R}-21=\mathrm{R}+30$
$\mathrm{S}: \mathrm{R} \quad-\mathrm{R}-21-\mathrm{R}=\mathrm{R}+30-\mathrm{R}$
Collect terms: $\quad-2 \mathrm{R}-21=30$
A : $21 \quad-2 \mathrm{R}-\not 2 \mathbb{A}+\not 2 \not \subset=30+21$
Collect terms: $-2 \mathrm{R}=51$

D : —2

$$
\frac{-\not \subset \mathrm{R}}{-4}=\frac{51}{-2}
$$

Answer: $R=-251 / 2$

Check:

$$
\begin{aligned}
&\left(5 \times-25^{1 / 2}\right)-3\left(7+\left[2 \times-25^{1 / 2}\right]\right)=-25^{1 / 2}+30 \\
&-1271 / 2-3(7-51)=41 / 2 \\
&-1271 / 2-3(-44)=4^{1 / 2} \\
&-127^{1 / 2}+132=4^{1 / 2} \\
& 41 / 2=4^{1 / 2}
\end{aligned}
$$

Problem: $\quad 2 R+10=4 R+8$
S : 4R
Collect terms: $\quad-2 \mathrm{R}+10=8$
S : 10
$-2 \mathrm{R}+10-1 \phi=8-10$
Collect terms: $\quad-2 \mathrm{R}=-2$
D : $-2 \quad \frac{-2 \mathrm{R}}{-2}=\frac{-2}{-2}$
Answer: $\quad R=1$
Check this answer by substituting 1 for $R$ in the original equation.

Before proceeding, go back to each of the previous problems. Cover all work with a blank sheet of paper and solve for the unknowns.

## Practice

For practice in the solution of algebraic equations, solve each of the following problems to find the numerical value of the unknown. Check each of your answers by substituting it for the unknown in the original equation.

1. $4 \mathrm{E}-1=3 \mathrm{E}+3$

Answer: $\mathrm{E}=4$
2. $5+2 \mathrm{~A}=\mathrm{A}-10$

Answer: $\mathrm{A}=-15$
3. $7 \mathrm{~B}=-14$
4. $5 \mathrm{R}+2=3+4 \mathrm{R}$
5. $21-15 \mathrm{P}=-8 \mathrm{P}-7$
6. $16 \mathrm{~B}-3=6 \mathrm{~B}+8-23 \mathrm{~B}$
7. $27-28 \mathrm{E}=19-16 \mathrm{E} \quad \frac{2}{3}$
8. $\frac{7 \mathrm{~A}}{4}=14$
9. $3(\mathrm{~A}-2)-10(\mathrm{~A}-6)=5$
10. $8 \mathrm{~A}-5(4 \mathrm{~A}+3)=-3-4(2 \mathrm{~A}-7)$ Answer: $\mathrm{A}=-10$
11. $5 \mathrm{E}-8+4 \mathrm{E}+5=7 \mathrm{E}-3-2 \mathrm{E}+5$
12. $18+5 \mathrm{R}-6-2 \mathrm{R}+\mathbf{1}+3 \mathrm{R}-25=0$ Answer: $\mathrm{R}=2$
13. $3 \mathrm{~F}-15-10 \mathrm{~F}-9+16 \mathrm{~F}-21=05$
14. $\frac{5 \mathrm{I}+3 \mathrm{I}}{4}=12$

Answer: $\mathrm{I}=6$
15. $-4 \mathrm{E}=\frac{7 \mathrm{E}-57}{3}$

## Solving Equations for Unknowns

In the following problems we will solve equations (and formulas) for different "unknowns". All we do is to move terms until the desired unknown appears by itself on one side of the equation.

Problem: $E=I R$
D : I $\quad \frac{\mathrm{E}}{\mathrm{I}}=\frac{\mathrm{IR}}{\square}$
Answer: $\frac{\mathrm{E}}{\mathrm{I}}=\mathrm{R} \quad$ or $\quad \mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}}$

Problem: $R=\frac{Q L}{A} \quad$ Solve for $L$
L is multiplied by $\varrho$ and divided by $A$.
We will do the opposite or divide by o and multiply by $A$.
D: 0, M:A

$$
\frac{\mathrm{RA}}{Q}=\frac{\not Q L A}{A 贝}
$$

Answer: $\frac{R \mathrm{~A}}{\varrho}=\mathrm{L} \quad$ or $\quad \mathrm{L}=\frac{\mathrm{RA}}{\varrho}$
Note: The two sides of an equation may be interchanged without changing the value of the equation.
Problem: $\quad R_{T}=R_{1}+R_{2} \quad$ Solve for $R_{1}$
$R_{2}$ is added to $R_{1}$. To separate $R_{1}$ and $R_{2}$ we will subtract $R_{2}$.
$\mathrm{S}: \mathrm{R}_{2} \quad \mathrm{R}_{\mathrm{T}}-\mathrm{R}_{2}=\mathrm{R}_{1}+\mathrm{R}_{\boldsymbol{q}}-\boldsymbol{R}_{\boldsymbol{q}}$
Answer: $\quad \mathrm{R}_{\mathrm{T}}-\mathrm{R}_{2}=\mathrm{R}_{1} \quad$ or $\mathrm{R}_{1}=\mathrm{R}_{\mathrm{T}}-\mathrm{R}_{2}$
Problem: $\quad \frac{\mathrm{ABC}}{10}=\mathrm{D} \quad$ Solve for $B$
$\mathrm{D}: \mathrm{A}, \mathrm{D}: \mathrm{C}, \mathrm{M}: 10 \quad \frac{\mathrm{ABC} \times 1 \phi}{X O A C}=\frac{\mathrm{D} \times 10}{\mathrm{AC}}$

Answer: $\quad \mathrm{B}=\frac{\mathrm{D} \times 10}{\mathrm{AC}} \quad$ or $\mathrm{B}=\frac{10 \mathrm{D}}{\mathrm{AC}}$
Problem: $\quad \mathrm{A}=\mathrm{B}+\mathrm{C}-14$ Solve for C
$\mathrm{S}: \mathrm{B}, \mathrm{A}: 14 \quad \mathrm{~A}-\mathrm{B}+14=\bar{B}+\mathrm{C}-14-\beta+14$
Answer: $\quad \mathrm{A}-\mathrm{B}+14=\mathrm{C} \quad$ or $\mathrm{C}=\mathrm{A}-\mathrm{B}+14$
Problem: $\quad A-B=C-3 \quad$ Solve for $B$
$\mathbf{S}: \mathbf{A} \quad A-\mathrm{B}-A=\mathrm{C}-3-\mathrm{A}$
This says that: $-\mathrm{B}=\mathrm{C}-3-\mathrm{A}$ but we want $+\mathbf{B}$. Change signs on both sides of the equation. This is permissible since we are doing the same thing to both sides. What we are actually doing is multiplying all terms on both sides by - 1 . We must remember to change the sign of every term on each side of the equation.

Answer: $\mathrm{B}=-\mathrm{C}+3+\mathrm{A} \quad$ or $\quad \mathrm{B}=\mathrm{A}-\mathrm{C}+3$
Problem: $\quad A-B=3 R-2 \quad$ Solve for $R$
A:2
$A-B+2=3 R-\not 2+\not 2$
D : 3
$\frac{\mathrm{A}-\mathrm{B}+2}{3}=\frac{\not 2 \mathrm{R}}{\not 2}$

Answer: $\quad \frac{\mathrm{A}-\mathrm{B}+2}{3}=\mathrm{R} \quad$ or $\quad \mathrm{R}=\frac{\mathrm{A}-\mathrm{B}+2}{3}$

Problem: $\quad B=3 \mathrm{E}-8(\mathrm{E}-6) \quad$ Solve for E
Simplify: $\quad B=3 \mathrm{E}-8 \mathrm{E}+48$
Collect terms: $\mathrm{B}=-5 \mathrm{E}+48$
S: $48 \quad \mathrm{~B}-48=-5 \mathrm{E}+48-4 \$$
D: $-5 \quad \frac{\mathrm{~B}-48}{-5}=\frac{\not-5 \mathrm{E}}{\not-7}$

Answer: $\frac{\mathrm{B}-48}{-5}=\mathrm{E}$ or $\mathrm{E}=\frac{\mathrm{B}-48}{-5}$

This can also be written $\mathrm{E}=\frac{-\mathrm{B}+48}{5}$
Problem: $\quad \frac{A}{B}=\frac{C}{D}$
Solve for C
M : D

$$
\frac{\mathrm{A} \times \mathrm{D}}{\mathrm{~B}}=\frac{\mathrm{C} \times \not{ }^{-1}}{\not \square}
$$

Answer: $\frac{A D}{B}=C$ or $C=\frac{A D}{B}$
Problem: $\quad \frac{A}{B}=\frac{C}{D}$
Solve for D
D , the term we wish to solve for, is in the denominator of a fraction. We can easily get D out of the denominator by inverting both sides of the equation.

Thus, $\frac{\mathrm{B}}{\mathrm{A}}=\frac{\mathrm{D}}{\mathrm{C}}$
D is now in the numerator.
$\mathbf{M}: C \quad \frac{\mathrm{~B} \times \mathrm{C}}{\mathrm{A}}=\frac{\mathrm{D} \times \varnothing}{\varnothing}$

Answer: $\frac{\mathrm{BC}}{\mathrm{A}}=\mathrm{D}$ or $\mathrm{D}=\frac{\mathrm{BC}}{\mathrm{A}}$
Problem: $\mathrm{AC}=10^{\circ} \mathrm{E}$
Solve for E
D : $10^{\infty} \frac{\mathrm{AC}}{10^{\mathrm{s}}}=\frac{\gamma \chi^{\prime} \phi \mathrm{E}}{1 \phi \phi}$
Answer: $\frac{\mathrm{AC}}{10^{5}}=\mathrm{E}$ or $\mathrm{E}=\frac{\mathrm{AC}}{10^{\mathrm{s}}}$ or $\mathrm{E}=10^{-8} \mathrm{AC}$

## Practice

Before working the problems which follow, solve each of the problems in the preceding group. Cover up the solution with a blank sheet of paper as you work each problem.

1. $R_{t}=R_{1}+R_{2}+R_{3} \quad$ Solve for $R_{2} \quad$ Answer: $R_{2}=R_{t}-R_{1}-R_{3}$
2. $\mathrm{f}=\frac{\mathrm{PN}}{120}$

Solve for $N \quad$ Answer: $N=\frac{120 f}{\mathrm{P}}$
3. $\mathrm{R}=\frac{\mathrm{KL}}{\mathrm{M}}$

Solve for L
4. $\mathrm{X}_{\mathrm{c}}=\frac{1}{2 \pi \mathrm{fC}}$

Solve for C (note: $\mathrm{X}_{\mathrm{c}}$ is one term)
5. $\frac{\mathrm{E}_{\mathrm{p}}}{\mathrm{E}_{\mathrm{s}}}=\frac{\mathrm{N}_{\mathrm{p}}}{\mathrm{N}_{\mathrm{s}}}$

Solve for $N_{p}$ Answer: $N_{p}=\frac{N_{s} E_{p}}{E_{s}}$
6. $\frac{\mathrm{I}_{\mathrm{p}}}{\mathrm{I}_{\mathrm{s}}}=\frac{\mathrm{N}_{\mathrm{s}}}{\mathrm{N}_{\mathrm{p}}}$

Solve for $\mathrm{N}_{\mathrm{p}}$
7. $\mathrm{I}_{\mathrm{p}} \mathrm{E}_{\mathrm{p}}=\mathrm{I}_{\mathrm{s}} \mathrm{E}_{\mathrm{s}}$

Solve for $E_{p}$
8. $\mathrm{Q}=\frac{\mathrm{WL}}{\mathrm{R}}$

Solve for $R$
9. $\mathrm{E}=\frac{\mathrm{BLV}}{10^{8}}$

Solve for L
10. $\mathrm{H}=\frac{4 \pi \mathrm{NI}}{\mathrm{L}}$

Solve for N
11. $\mathrm{Q}=0.24$ EIT $\quad$ Solve for $T$
12. $\mathrm{C}=0.08842 \mathrm{~K} \frac{\mathrm{~A}}{\mathrm{D}} \quad$ Solve for K
13. $\mathrm{H}=0.057 \mathrm{I}^{2} \mathrm{R}_{\mathrm{t}} \quad$ Solve for $\mathrm{R}_{\mathrm{t}}$
14. $\mathrm{P}=\mathrm{I}^{2} \mathrm{R}$
15. $\mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL}$

Solve for $R$
Solve for f

## Solving Equations by Substitution

In some cases, it is possible to solve an equation, or to change it into a desired form, by means of substitution. One fundamental rule of algebra says that "an equal quantity may be substituted for any terra in an equation, without making the sides of the equation unequal". For example, in the formula, $\mathrm{X}_{\mathrm{L}}=2 \pi f \mathrm{~L}$, we have the term $\pi$ in the right side of the equation.
$\pi$ is equal to 3.14 , so we can substitute this value in place of $\pi$ in the problem, without "unbalancing" the equation. The formula could be written:

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{L}}=2 \times 3.14 \times \mathrm{fL} \\
& \mathrm{X}_{\mathrm{L}}=6.28 \mathrm{fL}
\end{aligned}
$$

To further illustrate the use of substitution, several problems and their solution will be given. Study each carefully to observe what is done in each step, and why.

Problem: $3 \mathrm{~A}+7 \mathrm{~B}=30 \quad$ Given: $\mathrm{A}=\mathrm{B}$, Find A .
Solution: Since $A=B$, it can be substituted fon $B$.
Substitution: $3 \mathrm{~A}+7 \mathrm{~A}=30$
Collect terms: $10 \mathrm{~A}=30$

D : 10

$$
\frac{10 \mathrm{~A}}{1 \varnothing}=\frac{30}{10}
$$

$$
\mathrm{A}=\frac{30}{10}
$$

Answer:

$$
A=3
$$

Since $A=B$. If $A=3, B$ also equals 3 .
Check: $3(3)+7(3)=30$

$$
9+21=30
$$

Problem: $2 \mathrm{D}+3=\mathrm{E}+7$, Given: $\mathrm{D}=\mathrm{E}$, Find D .
Substitution:

$$
\begin{aligned}
2 \mathrm{D}+3 & =\mathrm{D}+7 \\
2 \mathrm{D}+3-\mathrm{D} & =\nsupseteq+7-\not 口
\end{aligned}
$$

S:D
Collect terms:
$\mathrm{D}+3=7$
S:3
$\mathrm{D}+\not \approx-\not \approx=7-3$
Answer:
$\mathrm{D}=4$
Since $D=E$
$\mathrm{E}=4$
Check:

$$
\begin{aligned}
2(4)+3 & =4+7 \\
8+3 & =4+7 \\
11 & =11
\end{aligned}
$$

Problem: $\mathrm{Y}+\mathrm{Z}=15$, Given: $2 \mathrm{Z}=\mathrm{Y}$, Find Z
Substitute: $2 \mathrm{Z}+\mathrm{Z}=15$ (since $2 \mathrm{Z}=\mathrm{Y}$, we substitute 2 Z for Y )
Collect terms: $3 Z=15$
Answer: $\quad Z=5$
To find $Y$ we go back to what was given in the problem:

$$
2 \mathrm{Z}=\mathrm{Y}
$$

Substitute: $2 \times 5=Y \quad$ (we found $Z=5$ )

$$
10=\mathrm{Y}
$$

Answer: $\quad Y=10$
Check: $10+5=15$
$15=15$

Many practical uses may be found for this substitution process. For example, let us assume that we know the power formula, $\mathrm{P}=\mathrm{E} \times \mathrm{I}$, and that we know Ohm's Laws; we wish to convert the power formula so that it is in terms of current and resistance, rather than current and voltage. Problem: $\mathrm{P}=\mathrm{E} \times \mathrm{I} \quad$ Convert to terms of current and resistance.

Ohm's Law tells us that $E=I R$. Notice that IR is equal to $E$. It may be substituted for E in the power formula, $\mathrm{P}=\mathrm{E} \times \mathrm{I}$.
Substitute: $\mathrm{P}=\mathrm{IR} \times \mathrm{I}$
Answer: $\quad P=I^{2} R$. This gives a formula for power in terms of current and resistance.

Problem: $\mathrm{P}=\mathrm{E} \times \mathrm{I} \quad$ Convert to terms of voltage and resistance.
Ohm's Law tells us that $I=\frac{E}{R}$. Since the term $\frac{E}{R}=I$, it may be substituted for $I$ in the equation, $\mathbf{P}=\mathrm{E} \times \mathrm{I}$.

Substitution: $\quad P=E \times \frac{E}{R}$.

$$
\begin{aligned}
& P=\frac{\mathrm{E} \times \mathrm{E}}{\mathrm{R}} \\
& \mathrm{P}=\frac{\mathrm{E}^{2}}{\mathrm{R}}
\end{aligned}
$$

These examples illustrate that the substitution process often aids in getting a formula in the desired form.

## Practice

For practice, work the following problems. Check your answers on the numerical problems.

1. $\mathrm{a}+\mathrm{b}=4$
2. $7 \mathrm{I}-6 \mathrm{I}+2=-\mathrm{K} \quad$ Given: $\mathrm{K}=\mathrm{I}$. Find I
3. $\mathrm{C}+2 \mathrm{D}=8 \quad$ Given: $2 \mathrm{D}=\mathrm{C}$. Find C
4. $\mathrm{P}=\mathrm{I}^{2} \mathrm{R} \quad$ Given: $\mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}}$. Find power in terms of E and I .
5. $\mathrm{P}=\frac{\mathrm{E}^{2}}{\mathrm{R}}$

Given: $\mathrm{E}=\mathrm{IR}$. Find power in terms of I and R.

## Equations Containing Powers and Roots

In an equation it is permissible to raise both sides to the same powers, or to extract the same root of both sides.

Problem: $\quad X^{2}=4$
Taking square root of both sides: $\sqrt{\mathrm{X}^{2}}=\sqrt{4}$ Answer: $\mathrm{X}=2$

Check: $\quad(2)^{2}=4$
Problem: $\mathrm{Y} \because+6=42 \quad$ Find Y
S:6

$$
\mathrm{Y}^{2}+\not 6-\not \subset=42-6
$$

$$
\mathrm{Y}^{2}=36
$$

Taking square root of both sides: $\sqrt{\mathrm{Y}^{2}}=\sqrt{36}$
Answer: $\quad Y=6$
Check: $\quad(6 \times 6)+6=42$

$$
\begin{aligned}
36+6 & =42 \\
42 & =42
\end{aligned}
$$

Problem: $\quad \sqrt{Y}=4 \quad$ Find $Y$
Squaring both sides: $(\sqrt{\mathrm{Y}})^{2}=(4)^{2} \quad\left[\right.$ Note $\left.:(\sqrt{\mathrm{Y}})^{2}=\mathrm{Y}\right]$
Answer: $\quad Y=16$
Check: $\quad \sqrt{16}=4$

$$
4=4
$$

Problem: $\quad \sqrt{B}-3=5$ Find $B$
$\begin{array}{ll}\mathrm{A}: 3 & \sqrt{\mathrm{~B}}=\{3+\not \equiv=5+3 \\ & \sqrt{\mathrm{B}}=8\end{array}$
Squaring both sides: $\quad(\sqrt{B})^{2}=(8)^{2}$
Answer: $\quad B=64$
Check: $\quad \sqrt{64}-3=5$

$$
\begin{array}{r}
8-3=5 \\
5=5
\end{array}
$$

Problem: $\quad \mathrm{P}=\mathrm{I}^{2} \mathrm{R}$
Find I
$\mathrm{D}: \mathrm{R} \quad \frac{\mathrm{P}}{\mathrm{R}}=\frac{\mathrm{I} 2 \mathbb{R}}{\vec{R}}$

$$
\frac{\mathrm{P}}{\mathrm{R}}=\mathrm{I}^{2}
$$

Taking square root of both sides: $\sqrt{\frac{P}{R}}=\sqrt{I^{2}}$
Answer $\sqrt{\frac{P}{R}}=I$

$$
\text { or } I=\sqrt{\frac{P}{R}}
$$

Problem: $\quad P=\frac{\mathrm{E}^{2}}{\mathrm{R}} \quad$ Find E
$M: R \quad P R=\frac{E^{2} \times R}{\not R}$
$\mathrm{PR}=\mathrm{E}^{2}$
Taking square root of both sides: $\sqrt{\mathrm{PR}}=\sqrt{\mathrm{E}^{2}}$
Answer: $\sqrt{\mathrm{PR}}=\mathrm{E} \quad$ or $\mathrm{E}=\sqrt{\mathrm{PR}}$
Problem: $\quad \mathrm{L}=\frac{1.25 \mathrm{~N}^{2} \mathrm{P}}{10^{8}} \quad$ Find N
$\mathbf{M}: 10^{s}, \mathrm{D}: 1.25 \mathrm{P} \quad \frac{10^{5} \mathrm{~L}}{1.25 \mathrm{P}}=\mathrm{N}^{2}$

Taking square root of both sides: $\sqrt{\frac{10^{3} \mathrm{~L}}{1.25 \mathrm{P}}}=\sqrt{\mathrm{N}^{2}}$

Answer: $\quad 10^{4} \sqrt{\frac{\mathrm{~L}}{1.25 \mathrm{P}}}=\mathrm{N} \quad$ or $\mathrm{N}=10^{4} \sqrt{\frac{\mathrm{~L}}{1.25 \mathrm{P}}}$

## Practice

For practice solve the following problems. Check the answers to the problems with numerical answers by substituting in the original formula.

1. $\sqrt{\mathrm{Z}}=7$
2. $\sqrt{P}+4=8$
3. $\quad \mathrm{X}^{2}=49$

$$
\text { 4. } \quad 2 \mathrm{Y}^{2}-3=15 \quad \text { 5. } \quad \mathrm{A}=\frac{\mathrm{B}^{2}}{\mathrm{C}} \quad \text { Find } \mathrm{B}
$$

## Solving Electronics Problems by Using Algebraic Equations

Up to this point in the training program you have actually been solving problems using algebraic equations, although they have been called formulas. For example, the Ohm's Law formulas are algebraic equations, and we have had no difficulty in solving Ohm's Law problems. To enable you to solve for the three unknowns, $I, E$, and $R$, you were given three Ohm's Law formulas: $\mathrm{I}=\mathrm{E} / \mathrm{R}, \mathrm{E}=\mathrm{IR}$, and $\mathrm{R}=\mathrm{E} / \mathrm{I}$. This enabled you to find whichever one of the unknowns was desired. Due to the fact that there are a great number of equations in electronics, all forms of the equation will not be given in most cases. You will have to apply your knowledge of algebra to change the
formula that is given, into a form which will enable you to solve the problem at hand. For example, you will soon encounter the formula for inductive reactance in one of your assignments. The formula is stated thus:

$$
\mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL}
$$

$\mathrm{X}_{\mathrm{L}}$ is inductive reactance, in ohms
$\pi=3.14$
$\mathrm{f}=$ Frequency of a-c wave in hertz (cycles per second)
$\mathrm{L}=$ Inductance in henries
If you were given the frequency in hertz, and the inductance in henries, it would be a very simple matter to put these values in the formula and find the inductive reactance. For example, let us assume the frequency to be 60 hertz and the inductance to be 30 henries. Substituting in the formula we have:

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL} \\
& \mathrm{X}_{\mathrm{L}}=2(3.14) \times 60 \times 30 \\
& \mathrm{X}_{\mathrm{L}}=11,304 \text { ohms }
\end{aligned}
$$

Now let us suppose that the inductive reactance is known, (22,608 ohms), the frequency is known ( 30 hertz), and the inductance in henries in unknown. The simple way to solve such a problem would be to solve the equation for $L$, and then substitute the known values in this new equation.

To solve the equation for $L$ we proceed as in the previous illustrations. $\mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL}$
D : 2, D : $\pi, \mathrm{D}: \mathrm{f} \quad \frac{\mathrm{X}_{\mathrm{L}}}{2 \pi \mathrm{f}}=\frac{\not 2 \pi f \mathrm{~L}}{\text { } 2 \nexists f}$

Answer:

$$
\frac{\mathrm{X}_{\mathrm{L}}}{2 \pi \mathrm{f}}=\mathrm{L}, \text { or } \mathrm{L}=\frac{\mathrm{X}_{\mathrm{L}}}{2 \pi \mathrm{f}}
$$

Let us now substitute the known values in this new equation.

$$
\mathrm{L}=\frac{\mathrm{X}_{\mathrm{L}}}{2 \pi \mathrm{f}}
$$

$$
\mathrm{L}=\frac{22,608}{6.28 \times 30}
$$

$\mathrm{L}=120$ henries
As another example, let us assume that we have a 25 watt resistor which has 25,000 ohms resistance. We wish to know the amount of current which this resistor can safely handle. The power formula is $\mathrm{P}=\mathrm{I}^{2} \mathrm{R}$.

Solving for $I$ we proceed as previously:

$$
\mathrm{P}=\mathrm{I}^{2} \mathrm{R}
$$

D : R

$$
\begin{aligned}
& \frac{\mathrm{P}}{\mathrm{R}}=\frac{\mathrm{I}^{2} \mathrm{R}}{\mathrm{R}} \\
& \frac{\mathrm{P}}{\mathrm{R}}=\mathrm{I}^{2}
\end{aligned}
$$

Taking square root of both sides: $\sqrt{\frac{P}{R}}=\sqrt{I^{2}}$

$$
\sqrt{\frac{\mathrm{P}}{\mathrm{R}}}=\mathrm{I} \quad \text { or } \quad \mathrm{I}=\sqrt{\frac{\mathrm{P}}{\mathrm{R}}}
$$

Now we substitute our known values in the equation.

$$
\begin{aligned}
& \mathrm{I}=\sqrt{\frac{25}{25,000}} \\
& \mathrm{I}=\sqrt{\frac{1}{1,000}}=\sqrt{.001} \\
& \mathrm{I}=.0316 \text { amperes or } 31.6 \mathrm{~mA} .
\end{aligned}
$$

These few examples illustrate the practical value of algebra to the electronics technician. You will find it to be an invaluable aid to the solution of many practical electronics problems.

## "How To Pronounce . . ."

(Note: the accent falls on the part shown in CAPITAL letters.) equation
(ee - KWAY - zhun)

## Test Questions

Be sure to number your Answer Sheet Assignment 14.
Place your Name and Associate Number on every Answer Sheet.
Submit your answers for this assignment immediately after you finish them. This will give you the greatest possible benefit from our personal grading service.
In answering these algebra problems, show all of your work. Draw a circle around your answer. Do your work neatly and legibly.

1. If one side of an equation is multiplied by a number, for example 5 , what must be done to the other side of the equation to keep both sides equal?
2. If one side of an equation is divided by a letter, for example E, what must be done to the other side of the equation to keep both sides equal?
Find the numerical value of the unknowns in the following equations:
(To make sure your answers are correct, check each one.)
3. $3 \mathrm{~A}=18$
4. $\frac{3 \mathrm{E}}{4}=6$
5. $\mathrm{Y}-6=18$
6. $\mathrm{Y}^{2}=25$

In the following problems solve for the indicated unknowns.
7. $\mathrm{A}=\frac{\mathrm{B}}{\mathrm{C}}$
8. $\mathrm{M}=\mathrm{NO}$
9. $\mathrm{R}_{\mathrm{t}}=\mathrm{R}_{1}+\mathrm{R}_{2}+\mathrm{R}_{3}+\mathrm{R}_{4}$
10. $\mathrm{I}_{\mathrm{s}} \mathrm{E}_{\mathrm{s}}=\mathrm{I}_{\mathrm{p}} \mathrm{E}_{\mathrm{p}}$

Find B.

Solve for 0
Solve for $\mathrm{R}_{2}$.
Solve for $I_{p}$.

