Electronics
Radio
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TWO BASIC FORMS OF ELECTRICITY

ASSIGNMENT 13

TWO BASIC FORMS OF ELECTRICITY

Thus far, in all our discussion of current and voltage we have dealt with direct currents and voltages; that is, the kind obtained from a battery. The chemical action of a battery keeps the terminals of the batttery at a fixed potential difference, and when a load is connected to it, a steady current flows from the negative terminal of the battery through the load and back to the positive terminal of the battery. This steady current is known as direct current. (There are several abbreviations used to indicate direct current. Some of these are: D-C, D.C., DC, d-c, and d.c.)

The type of d-c current which we have been studying is shown in Figure 1. The battery has a constant emf of 100 volts. The load consists of a 20 ohm resistor. To find the current flowing in this circuit, we may apply Ohm's Law. $I = E/R = 100/20 = 5$ amperes.

Figure 1(B) is a graph of this current flow, in respect to time. The vertical axis of the graph represents current in amperes, and the horizontal axis represents time in minutes. We see from the graph, that at one minute the current is 5 amperes, at 2 minutes the current is 5 amperes, also at 3 or 4 minutes the current is 5 amperes. This current is, then, a steady, constant value.

Pulsating Direct Current

Strictly speaking, however, a direct current or voltage merely has to act in one direction and may change somewhat in magnitude (amount). It has become common practice to apply the terms **direct current** and **direct voltage** (sometimes **d-c** voltage) to currents and voltages that are practically constant, and the term **pulsating direct current** to a direct current that acts in one direction but varies somewhat in magnitude over a period of time.

As an example of pulsating direct current, consider the circuit of Figure 2(A). This circuit consists of a 100-volt battery connected to a 20 ohm resistor in series with a rheostat which can have its resistance varied from 0 ohms to 80 ohms. When the resistance of the rheostat is 0 ohms, the current from the battery will be determined by the 20 ohm resistor and will be 5 amperes. With the resistance of the rheostat completely in the circuit, the current from the battery will be determined by the resistance of the fixed resistor, (20 ohms) plus the resistance of the rheostat (80 ohms), or a total resistance of 100 ohms. One ampere of current will flow, as determined by Ohm's Law. $I = E/R = 100/100 = 1$ ampere.

Now suppose that we were to manually adjust this rheostat so that its resistance varies smoothly from 0 to 80 ohms in just one minute, then immediately begin to reduce this resistance back toward zero, again taking just one minute, then increase the resistance toward its maximum of 80 ohms, and so on. While we are doing this, let us see what is happening to the current flowing from the battery. When the rheostat is set for 0 ohms there will be 5 amperes flowing, and as we start to adjust the rheostat, this current will start to fall off, to 1 ampere, reaching this value one minute later. It will immediately begin to increase again as we decrease the resistance, reaching 5 amperes at the end of the second minute. Then it will begin to decrease to 1 ampere, and so on.

We can plot this information on a graph, putting the current on the vertical axis and time on the horizontal axis, as shown in Figure 2(B). Let us examine this graph. At 0 time, the current is 5 amperes. This decreases to 1 ampere at one minute of time, when the entire rheostat resistance is in the circuit. At 2 minutes the current is 5 amperes again since the rheostat is 0 ohms. At 3 minutes the current has again decreased to 1 ampere, etc. The graph is merely a pictorial representation of the manner in which the current varies over a period of time.

A study of Figure 2(B) will reveal several things: (1) The current does not remain constant, (2) the current never stops or reaches zero, and (3) the current never reverses its direction, but always flows in the same direction. This is a **direct current** because it always flows in the same direction, but since it varies appreciably in magnitude, it is called a **pulsating direct current** or **pulsating d-c.**

In the early days of commercial electricity, direct currents and voltages were used exclusively because nearly all the electrical power came from storage batteries, which were recharged at intervals by d-c generators. However, it soon became quite evident that it was impossible to send this d-c power over long lines without excessive losses occurring in the lines, especially with more and more electrical current being used. As you know, one formula for electrical power is, $P = I^2R$, making the power loss in the wires increase as the square of the current. Another formula for power is, $P = E \times I$. From this formula we can see, that for a given amount of power, less current will be required if the voltage is increased. To illustrate this, let us assume that 1 kilowatt, or 1000 watts, of electrical power is being used and the voltage is 100 volts. The current will be 10 amperes.

$$P = E \times I$$
$$1000 = 100 \times I$$
$$100I = 1000$$
$$I = 10 \text{ amperes}$$

Now, let us assume that 1 kilowatt of power is to be used, but that the voltage is 1000 volts. The current is 1 ampere.

$$P = E \times I$$
$$1000 = 1000 \times I$$
$$1000I = 1000$$
$$I = 1 \text{ ampere}$$

If the load, in the two preceding examples, is located at some distance from the source, so that the resistance of the lines is appreciable, say 5 ohms, there will be losses occurring in the lines. The power loss in the lines can be found by the formula, $P = I^2R$.

In the first example the power loss **in the lines** will be 500 watts as shown by the following problem:

$P = I^2R \qquad P = (10)^2 \times 5 \qquad P = 100 \times 5 = 500$ watts

In the second example the power loss **in the lines** will be only 5 watts as shown by the following problem:

$P = I^2R \qquad P = (1)^2 \times 5 \qquad P = 1 \times 5 = 5$ watts

From these examples we see that the same amount of power (1 kw in this case) can be transmitted from the source to the load with much lower losses occurring, if the voltage is high. Unfortunately, however, there is no simple way to change low d-c voltages into high d-c voltages. For this reason, a different type of electrical current and voltage, known as **alternating current** and **alternating voltage,** was developed. Alternating current is abbreviated several ways. Some of these are: A-C, AC, A.C., a-c, a.c.

Alternating Current

Alternating-current systems began commercially in the United States in 1886. Continued experimentation, investigation, and theoretical analysis have disclosed many merits of a-c systems. The outstanding advantage of the a-c system is the relative ease with which alternating voltages can be generated, and transformed in magnitude. For example, it is a very simple matter to change 100 volts a-c into 1000 volts a-c. The result is that, at the present time, approximately 95 per cent of the electrical energy consumed in the United States is generated, transmitted, and actually utilized in the form of alternating current.

In the normal a-c power distribution system, the a-c voltage is developed by huge a-c generators or dynamos, which are driven by waterpower or steam. The output voltage of these generators is high, about 13,000 volts or greater, but if the electrical energy were transmitted at this voltage, the line losses would be high due to the large amount of power handled. For this reason, this high a-c voltage is fed to large transformers. These transformers increase the voltage to a much greater value, in the order of 150,000 volts. This high a-c voltage is then transmitted over the high-tension transmission lines for distances ranging up to hundreds of miles in some cases. At this very high voltage, the current will be low for a given amount of power, so that the line losses are low.

Of course, this extremely high voltage can not be used safely in homes, so it must be lowered to some safer value, usually 110 volts before it is brought to the homes of the consumers. This is also done by transformers. Transformers perform this operation, changing low values of a-c voltage into high values of a-c voltage, or vice versa, very efficiently, and for this reason, cost very little to operate. Transformers **cannot** be used with d-c, and no other means of transforming the low values of d-c voltage into high values, in an efficient manner has been developed. For this reason a-c is used almost exclusively as mentioned previously.

In future assignments, we will learn that we must change the a-c voltage from the power line into d-c voltage for operation of electronic circuits. This is performed by circuits known as **rectifier circuits.** We shall also learn that when we apply this d-c voltage to certain types of transistor or vacuum tube circuits, these circuits will generate an a-c voltage. This type of circuit is called an **oscillator.** Each of these circuits will be studied in detail at a later date.

By definition, **alternating current (a-c) is a current that periodically changes in magnitude and direction.** Figure 3(A) shows a diagram of a circuit that can be used to produce an alternating current. When switch (S) is in position 1, point X will be 45V positive with respect to point Y. When the switch is in position 2, point X will be 45V negative with respect to point Y. Study the circuit of Figure 3(A) and visualize these results. Figure 3(B) is a graph with the voltage at point X with respect to point Y plotted on the vertical axis. The voltage (E) will have the square waveform shown on the graph if the double throw switch is alternately held in each position for one second. This is an alternating voltage (abbreviated a-c voltage). An alternating current will flow through the resistor connected between points X and Y. This current fits the definition of a-c. It **periodically changes in magnitude and direction.**

Let us study Figure 3(A) carefully to see just what happens. When the switch is in position 1, current will flow from the negative terminal of the battery to point Y, through the load resistor from point Y to point X, back to the battery. When the switch is thrown from position 1 to position 2, the current flow in this direction stops and now the current flows through the load resistor from point X to point Y. If we continue throwing the switch from position 1 to position 2 and then from 2 to 1 at one second intervals, there will be an alternating voltage across the load resistor. This is shown by the graph in Figure 3(B). The current through the load resistor will be reversing itself at one second intervals.

The a-c voltage which is generated commercially differs considerably from the a-c voltage illustrated in Figure 3(B). The most common type of a-c and the type most practical to generate is called a **sine wave** voltage.

The Sine Wave

The graph of a sine wave is shown in Figure 4. The voltage is plotted on the vertical axis and the time in seconds is plotted on the horizontal axis. This voltage is changing in magnitude and direction so it fits the definition of a-c, but its change is more gradual than that of Figure 3(B). Let us examine the graph of the a-c voltage shown in Figure 4 very carefully and see what we can learn from this graph. At 0 time, the voltage is 0. At a short time later

(1/240 second) the voltage has increased to +100 volts. Then it gradually decreases until, at 1/120 of a second, the voltage has again reached zero. Now the voltage begins to build up in the negative direction, until at 1/80 of a second, the voltage has reached a maximum value of —100 volts. In the interval of time from 1/80 to 1/60 of a second, the voltage decreases to 0 volts. This is one cycle of the a-c voltage. By definition, **a cycle is one complete succession of events.** A cycle of the seasons, for example, would be from spring to summer, summer to fall, fall to winter, and winter back to spring again. Applied to a-c, this means a voltage or current starts at one value, and goes through all of its variation and returns to that value to complete one cycle. In Figure 4, in the 1/60 of a second, from time 1/60 to 1/30 of a second on the time axis, another complete cycle occurs. Notice that each of the cycles occurs in1/60 of a second. This a-c voltage is called a 60 cycle a-c voltage, meaning 60 cycles per second. Notice that when we defined a cycle, we did not say when that cycle should begin. Going back to the seasons we could have started our cycle with fall just as easily, and in this case the complete cycle would end with the following summer. In an a-c wave, we can start our cycle anywhere on the sine wave, in which case it would end the next time a similar point on the curve appears.

Another term which is used in connection with a-c is **alternation.** An alternation is one half of a cycle. In the graph of the sine wave in Figure 4, that portion of the sine wave from 0 to 1/120 of a second is one alternation, and is called the positive alternation, since the voltage is positive during this period. The portion of the wave from 1/120 of a second to 1/60 of a second is the negative alternation.

Another term which is used in connection with a-c voltages is **frequency.** Frequency means the number of times anything happens in a given period of time. For example, if a wheel is rotating at a speed of 100 revolutions per second, its **frequency** is 100 rotations per second. Applied to an a-c voltage, the frequency represents the number of cycles which occur in one second. The frequency of the wave shown in Figure 4 is 60 cycles per second. In quite a few cases, the frequency will be called just 60 cycles. The **per second** is understood when dealing with electrical waves.

The alternating current supplied to most of the houses in this country has a frequency of 60 cycles per second. This means that the current and voltage go through 60 complete cycles (remember that this is actually 120 reversals or "alternations") in each second.

The frequency of an a-c voltage is very important. To illustrate this, consider Figure 5. The symbol at the left of Figure 5 represents a sine wave generator. This generator could be an a-c generator in a power plant or in a transistor or vacuum-tube oscillator circuit. The generator is connected to a loudspeaker. If the frequency of the a-c voltage is 60 cycles, the sound waves coming from the loudspeaker will be a very low pitched humming sound. As the frequency of the a-c voltage is increased, (the number of cycles per second becomes higher) the pitch of the note heard in the loudspeaker will become higher. At a frequency of 1000 cycles per second, the note will be a pleasant, low pitched whistle. As the frequency is increased more and more, the note will become higher and higher in pitch, until at about 20,000 cycles per second it has become so high pitched that it cannot be heard. The normal human ear can hear notes ranging from about 20 cycles per second (abbreviated 20 cps) to about 20,000 cps. These frequencies (20-20,000 cps) are called the **Audio Frequencies,** since they are audible to the human ear.

Electrical voltages, corresponding in frequency to the Audio Frequencies are called Audio Frequency voltages, or Audio Frequency signals. The abbreviation for Audio Frequency is AF.

Electrical voltages, whose frequencies are higher than 20,000 cps are called Radio Frequencies (abbreviated RF). Since these signals range in the thousands and millions of cycles, they are often expressed in kilocycles or megacycles. Radio waves range from 20kc (kilocycles) to several thousand megacycles (mc). The terms kc and mc have been used for years to designate frequency and are still widely employed. However, in 1966 a new term was adopted to stand for Cycles Per Second. This term is: **Hertz**—in honor of the early radio experimenter, Heinrich Hertz. Thus, the term 60 Hertz means the same as 60 cycles per second, 20 kHz means 20,000 cycles per second, and 30 MHz means 30 megacycles per second.

The Radio Frequency signals are generated by transistor or vacuum-tube oscillators, since the rotary generators such as used in power plants, cannot be made to produce these high frequencies. The RF signals generated by the oscillator circuit in a broadcast transmitter will be somewhere between 500 kHz and 1500 kHz, the exact frequency being specified by the Federal Communications Commission. A short-wave transmitter may have the frequency of 14,000kHz or 14 MHz. It is the fact that different radio stations use RF signals of different frequencies that makes "tuning" of a desired station possible.

The "period" of the wave is defined as the time required for one cycle to occur. For example, if the frequency of an alternating current is 60 cycles per second, each individual cycle would last for a period of one one-sixtieth of a second. We can say that the period is always equal to one divided by the frequency, and we can write this as:

$$t = \frac{1}{f}$$

where t represents the period in seconds, and f represents the frequency in cycles per second.

The Alternating Current

In our study of direct current theory we learned that potential difference, or voltage, causes a current to flow through the circuit. This also holds true for alternating current circuits, such as Figure 6(A). During the time the lower terminal of the generator oscillator is negative, an electron current will flow to the right in the lower wire, up through the resistor, and to the left in the upper wire back to the oscillator. A moment later, when the voltage of the oscillator reverses its polarity, the lower terminal will become positive (making the upper terminal negative) and the electron current will be reversed. That is, the current will flow to the right in the upper wire, down through the resistor, and to the left in the lower wire back to the oscillator.

In direct current circuits, an individual electron does not necessarily have to travel completely around the circuit. You will remember that an electron current consists of a large number of electrons slowly drifting around the circuit. Of course, if we were to wait long enough, an electron leaving the battery will return to it, but suppose that there was a switch in the circuit and we were able to close this switch for only one one-millionth of a second. An electron current would flow for this one one-millionth of a second, but this current would not last nearly long enough for an electron to leave the battery and return to it. In considering alternating current theory, it is obvious that an electron will seldom, if ever, have sufficient time to travel completely around the circuit, especially if the voltage alternations of the oscillator occur rather frequently. The flow of an alternating current in a wire may be pictured as follows: The current flow consists of an electron current just as in a direct current circuit, but these electrons are more or less confined to a particular portion of the wire. First, they slowly drift one way; then as the voltage reverses, they will drift the other way, but they will never get more than a short distance away from their original position. In other words, they "alternately" flow back and forth, forming an "alternating current".

Perhaps you are wondering whether or not an electric current that is continually reversing itself—never getting anywhere, so to speak—is of any use. Consider a paddle wheel in a stream of water, and to the paddle wheel are attached a number of millstones. We can grind grain between these stones if the stones are rubbing together. It does not matter whether the water flows continuously, turning the millstones in a certain direction, or whether the water flows first one way and then the other. Just as long as the millstones turn against each other, the grain will be ground and we will be doing work. In this same manner, electrons flowing through a resistor generate heat regardless of whether they flow steadily in one direction, or whether they reverse their direction periodically. If an alternating current flows through the filament in an electric light bulb, the bulb will be illuminated, and if the frequency of the alternating current is high enough, above 30 Hz, the bulb will appear to be giving off a steady light. Actually, the lamp filament cools off somewhat as the alternating current goes through zero, but the eye does not detect this. Alternating current can be made to run a motor just as well as a direct current. If a condition is presented wherein a-c cannot be used, it is a simple matter to change the a-c to d-c. An example of this is the voltage applied to the plate circuit of vacuum tubes, as mentioned previously.

Figure 6(B) is a graph of the voltage applied to the resistor in Figure 6(A) and the current which flows through this resistor. In the figure, notice that the points where the voltage passes through zero, and the points where the current passes through zero coincide (occur at the same instant of time), and that the voltage and current reach their maximum values at the same instant. This is true for a circuit containing resistance, but is not true if the circuit contains a coil or a capacitor, or both, as we shall learn in a later assignment. When the two curves coincide as they do in Figure 6, they are said to be "in phase"; when they do not coincide, they are said to be "out of phase".

The Characteristics of Sine Waves

Suppose we plot a sine wave voltage with vertical lines at equal time intervals, as shown in Figure 7.

The time axis is marked off in degrees instead of seconds, as in Figure 4. This is possible because a cycle represents one complete succession of events, such as one turn of a wheel. In the rotation of a wheel, we could say that one complete rotation, or cycle, was 360°, since there are 360 degrees in a circle. One half of a revolution could be represented by 180° rotation, one fourth of a rotation by 90°, etc. In a like manner, the cycle of an a-c wave may be broken into degrees.

The **maximum** value of the sine wave is indicated in Figure 7. It is the greatest value to which the current or voltage rises. The notations, "I_{max}" and "E_{max}" are used to represent maximum values in electronics. The term **peak** value is sometimes used in place of maximum value, and means the same thing.

In Figure 7, it will be apparent that if the vertical lines are drawn long enough to intersect with the sine-wave curve, each of these lines will have a different length and each will represent the voltage at some particular instant of time. The voltage at that instant of time is known as the **instantaneous value of voltage**, or the **instantaneous voltage** of the waveform. From this figure it is evident that the instantaneous voltage for a sine wave depends on the particular instant at which the voltage is measured.

Table 1 lists the instantaneous value of a sine wave, at 10° intervals, assuming that the maximum value is 1.

Table I

Degrees	Sine	Degrees	Sine	Degrees	Sine
0	0	130	.766	260	—.985
10	.174	140	.643	270	—1.
20	.34	150	.5	280	—.985
30	.5	160	.34	290	—.94
40	.643	170	.174	300	—.866
50	.766	180	.0	310	—.766
60	.866	190	—.174	320	—.643
70	.94	200	—.34	330	—.5
80	.985	210	—.5	340	—.34
90	1.0	220	—.643	350	—.174
100	.985	230	—.766	360	0
110	.94	240	—.866		
120	.866	250	—.94		

By referring to this table, we can find the value of a sine wave, at any instant, if we know the maximum value of the sine wave. For example, if the maximum value of a sine wave of a-c is 1 volt, what is the value at 30°? By looking at the table we find that it is .5 volt. If the maximum value is any value other than one, the value of the sine wave at any instant may be found by multiplying the figures in the table by the maximum value. For example, suppose we are considering a sine wave which has a maximum value of 100 volts, and we wish to know its instantaneous value at 60°. By referring to the table, we find the value of a sine wave with a maximum of 1 volt to be .866 volt at 60°. To find the instantaneous value of this 100V maximum wave at 60° we merely multiply .866 by 100 and find that the value of the 100 volt maximum wave, at 60° is 86.6 volts. Using this same principle we could find the following:

Sine wave of 100 volts max. at 30° = 50 volts

Sine wave of 100 volts max. at 20° = 34 volts

Sine wave of 100 volts max. at 180° = 0 volts

Sine wave of 100 volts max. at 270° = —100 volts

Sine wave of 200 volts max. at 190° = —34.8 volts

Sine wave of 155 volts max. at 70° = 145.7 volts

Check the instantaneous values given in the examples above, and see if you agree with each.

Another term which is sometimes used in connection with a sine wave is **average value.** The average value of a sine wave is the average height of the curve of **one alternation** of a sine wave. Thus, if the height of all the vertical lines of an alternation in Figure 7 were measured, and the average of them taken, this average would be found to be 0.637, or 63.7% of the maximum value. We could write this as:

$E_{av} = 0.637\ E_{max}$
and $I_{av} = 0.637\ I_{max}$

To apply this formula, let us use a few examples.

Example 1. What is the average value of a sine wave with a maximum value of 100 volts?

$$E_{av} = 0.637 \times E_{max}$$
$$= 0.637 \times 100$$
$$E_{av} = 63.7 \text{ volts}$$

Example 2. What is the average value of a sine wave whose maximum value is 155 volts?

$$E_{av} = 0.637 \times E_{max}$$
$$= 0.637 \times 155$$
$$E_{av} = 98.7 \text{ volts}$$

Example 3. What is the average value of a sine wave which has a maximum of 900 volts?

$$E_{av} = 0.637 \times E_{max}$$
$$= 0.637 \times 900$$
$$E_{av} = 573.3 \text{ volts}$$

This formula can be rearranged as:

$$E_{max} = \frac{E_{av}}{0.637}$$

To find the maximum value, if the average value is known, this formula should be used.

Example 1. What is the maximum value of a sine wave which has an average value of 1000 volts?

$$E_{max} = \frac{E_{av}}{0.637}$$

$$= \frac{1000}{.637}$$

$$E_{max} = 1569.9 \text{ volts}$$

The fourth term which is **often** used when considering an a-c voltage or current is the **effective** value.

The term "average value" seems fairly obvious. Although there is nothing particularly difficult about the effective values, it cannot be said that they are obvious. The effective value of an alternating voltage or current (which you must remember is varying in magnitude at each instant) must be the same as a corresponding direct current value. If this were not true, then 1 volt of alternating voltage would not produce the same effect on a resistor as would 1 volt of direct voltage. Also, 1 ampere of alternating current would not produce the same heating effect in a given resistor as would 1 ampere of direct current, and you can see that this would never do.

The effective value of a sine wave of current or voltage is defined as that value which will produce the same heating effect in a resistor as will be produced by a given value of direct current or voltage. It can be shown mathematically that the effective value of a sine wave of current or voltage is the square root of the average of the instantaneous values squared. By applying this involved procedure, it can be found that the effective value of a sine wave is .707 times the maximum value.

Stated mathematically this is:

$E = 0.707E_{max}$ Where E is the effective voltage and

$I = 0.707\ I_{max}$ I is the effective current.

To apply these formulas let us consider a few examples.

Example 1. What is the effective value of a sine wave which has a maximum value of 200 volts?

$$E = 0.707E_{max}$$
$$= 0.707 \times 200$$
$$E = 141.4 \text{ volts}$$

This means that this sine wave of a-c voltage, which reaches a maximum of 200 volts, will do the same amount of work as a d-c voltage of 141.4 volts.

Example 2. What is the effective value of a sine wave which has a maximum value of 60 amperes?

$$I = 0.707 \times I_{max}$$
$$= 0.707 \times 60$$
$$I = 42.42 \text{ amperes.}$$

Thus we see that a sine wave of current which reaches a maximum of 60 amperes would produce as much heat in a resistor as 42.42 amperes of direct current.

Example 3. What is the effective value of a sine wave which has a maximum of 155 volts?

$$E = 0.707 \times E_{max}$$
$$E = 0.707 \times 155$$
$$E = 109.6 \text{ volts.}$$

We could rewrite this formula as:

$$E_{max} = \frac{E}{0.707} = 1.414\ E$$

$$I_{max} = \frac{I}{0.707} = 1.414\ I$$

We would use this formula to find the maximum value of a sine wave, when the effective value is known.

Example 1. What is the maximum value of a sine wave whose effective value is 6.3 volts?

$$E_{max} = 1.414 \times E$$
$$= 1.414 \times 6.3$$
$$E_{max} = 8.91 \text{ volts.}$$

Example 2. What is the maximum value of a sine wave which has an effective value of 20 amperes?

$$I_{max} = 1.414 \times I$$
$$= 1.414 \times 20$$
$$I_{max} = 28.28 \text{ amperes.}$$

Example 3. What is the maximum value of a sine wave which has an effective value of 110 volts?

$$E_{max} = 1.414 \times E$$
$$= 1.414 \times 110$$
$$E_{max} = 155.5 \text{ volts.}$$

This last example illustrates the voltage which is supplied to most homes by the electric power companies. The voltage is called 110 volts a-c. This voltage is actually a sine wave voltage with an **effective value** of 110 volts. The peak value of this voltage is 155.5 volts. All a-c voltmeters and current meters read **effective values.**

Because of the way in which they are obtained, effective values of current or voltage are frequently referred to as "root mean squared" values. This is often abbreviated "rms".

The average values of an alternating current or voltage are seldom used in ordinary electronics work. **Unless the current or voltage is specifically indicated otherwise, whenever we speak of an alternating current or voltage we mean its effective value.**

Frequency and Wavelength

The wavelength of an alternating current sine wave is the actual physical length of one cycle in space. The relation between frequency and wavelength is a simple one. The wavelength is equal to the speed at which the electric waves travel divided by the frequency in cycles. This speed is equal to 186,000 miles per second or 300,000,000 meters per second (a meter is slightly longer than a yard). To get the wavelength of the wave in meters we divide 300,000,000 by the frequency or: Wavelength in meters $= \dfrac{300 \times 10^6}{f}$.

The customary symbol for wavelength in meters is the Greek letter lambda, written λ.

Let us take several numerical examples.

Example 1. Suppose we wish to find the wavelength of a broadcast station which operates on a carrier frequency of 1000 kc. Using the formula, we have $\lambda = \frac{300{,}000{,}000}{1{,}000{,}000} = \frac{300 \times 10^6}{1 \times 10^6} = \frac{300}{1} = 300$ meters.

This tells us that the actual length in space of this radio wave is 300 meters. This is about 985 feet.

Example 2. What is the wavelength of the radio wave from a short-wave station operating on 20 megacycles?

$$\lambda = \frac{300 \times 10^6}{20 \times 10^6} = \frac{300}{20} = 15 \text{ meters.}$$

Example 3. What is the wavelength of a television station operating on 80 m.c.?

$$\lambda = \frac{300 \times 10^6}{80 \times 10^6} = \frac{300}{80} = 3.75 \text{ meters.}$$

Some radio receiver dials are marked both in frequency and in wavelength. In some countries, (particularly in Europe), the wavelength and never the frequency of the station is shown on radio dials. Even in this country, the short waves are usually referred to by wavelength rather than by frequency, as for example the 49 meter band, the 19 meter band, etc.

A few examples will show that the lower the frequency the longer the wavelength, or to say the same thing another way, the higher the frequency the shorter the wavelength. Our 60 cycle a-c power has a wavelength of 5,000,000 meters (approximately 3,000 miles) whereas the wavelength of certain television carrier frequencies is about 1 meter.

Phase Relations of Sine Waves

Perhaps you have suspected, or have been told, that alternating currents are much more difficult to understand than are direct currents. This is not true. However, alternating currents are more complex and there are more different possibilities to consider.

One factor which makes the study of a-c more complex is the matter of **phase**. This has been mentioned previously, but will be discussed in more detail now. Phase is a measure of time. It shows how one sine wave is varying in respect to another sine wave of the same frequency.

When two or more sine waves, either currents or voltages, are in phase, they pass through corresponding values at the same instant. That is, they both reach their maximum positive values at the same time, they both pass through zero at the same time. They both reach their maximum negative values at the same time, and so on. Two currents that are in phase are shown in Figure 8(A). Notice that the two waves are exactly "in-step" as far as time is concerned. They are of different amplitudes, but are in phase. (Amplitude means height of the wave, or magnitude.)

When two sine waves are out of phase they do not pass through corresponding values at the same time. Figure 8(B) shows two sine waves which are 90° out of phase. These waves are 90° out of phase because they pass through corresponding values 90° apart on the time axis. Notice that the sine wave of current, I_1, has reached a maximum (completed ¼ of a cycle) at the time I_2 is at zero. The current I_1, is said to be **leading** I_2 by 90°.

Figure 8(C) shows two sine wave currents which are 90° out of phase; but in this case, I_2 reaches its maximum 90° before I_1 does, so I_2 is leading I_1 by 90°. It is just as correct to say that I_1 is **lagging** I_2 by 90°.

In these examples we have seen the phase relationship of two sine waves of current. Figure 9 illustrates the phase relationship of two sine waves of voltage. In Figure 9(A) the two voltages are in phase, in Figure 9(B) E_1 is leading E_2 by 90°, and in Figure 9(C) E_1 is lagging E_2 by 90°.

In Figure 10(A) we see two sine waves 45° out of phase. E_1 is leading E_2 by 45° since E_1 is reaching its maximum 45° before E_2 reaches its maximum.

Figure 10(B) illustrates two sine waves which are 180° out of phase. These two waves go through their zero values at the same instant, but one is increasing in a positive direction while the other is increasing in a negative direction.

Figure 10(C) shows two sine waves out of phase approximately 15°. The voltage E_1 is leading E_2 by approximately 15°.

In Figure 11 we see the phase relationship of a sine wave of voltage and a sine wave of current. The voltage wave, E, leads the current wave, I, by 90° in Figure 11(A). In Figure 11(B), the current wave I leads the voltage wave E by 90°. Figure 11(C) shows the voltage wave E, leading the current I by approximately 135°.

These examples illustrate the wide variety of phase relations which will be encountered in the use of a-c in electronics circuits. You are probably wondering under what conditions these out of phase conditions occur. The answer to that question is an easy one. Any time a circuit with a-c voltage applied has either inductance (coils), or capacitance (capacitors), there will be an out of phase conditions between the voltage and the current, and between the voltages at different points in the circuit.

Figure 12(A) shows an a-c generator connected to a capacitor, and Figure 12(B) shows the phase relationship which results between the voltage and the current. The current is leading the voltage by 90°. The reason why this occurs will be discussed in the assignment on capacitors.

Figure 13(A) shows an a-c generator connected to a coil, and the phase relationship of the voltage and current are shown in Figure 13(B). Notice that in this case, the current is lagging the voltage by 90°.

In Figure 6 we have already seen the phase relationship of the voltage and current in an a-c circuit containing resistance alone.

If an a-c circuit contains a combination of resistance and capacitance, resistance and inductance, or resistance, capacitance, and inductance, a wide variety of phase relationship may result. The amount of phase difference will be determined by the value of the individual components.

The Addition of Sine Waves

In direct current circuits, we can readily find the resultant value of two voltages or currents in the same circuit since all we have to do is add their individual values. Figure 14 illustrates this. In Figure 14(A) the resultant of E_1 and E_2 is 150 volts. We find this by adding + 100 and +50. In Figure 14 (B) the two voltage sources are so connected that the two emf's are opposing each other, or "bucking". To find the resultant voltage of these two in series, we add the individual values algebraically. The number, +100 added to —50 gives +50 as an answer. The resultant voltage is 50 volts as indicated in the figure.

The resultant of two or more a-c waves can be found by adding their **instantaneous values.** This is shown graphically in Figure 15 and Figure 16.

In Figure 15 we have two sine wave generators connected in series. These two generators are delivering a-c voltages which are of the same frequency, and are in phase. The maximum value of E_1 is 100 volts, and the maximum value of E_2 is 50 volts. We wish to know the resultant value of these two voltages in series. In the graph on Figure 15, we have plotted these two voltages, and the resultant of them in series. The resultant voltage is labeled $E_1 + E_2$. To obtain this curve we add the **instantaneous values** of each wave. At zero on the time axis E_1 is 0 and E_2 is 0. Adding these two we obtain 0 for $E_1 + E_2$. At 90° on the time axis, E_1 is + 100 volts and E_2 is +50 volts. This gives us +150 volts for $E_1 + E_2$ at this point. At 180° both E_1 and E_2 are 0, so $E_1 + E_2$ is also zero. At 270°, E_1 is —100 volts, E^2 is —50 volts, so the resultant $E_1 + E_2$ is —150 volts. At 360° the resultant is again 0.

In Figure 16 we have two a-c generators, each delivering 100 volts maximum. The two voltages are 90° out of phase. (E_2 is leading E_1 by 90°). We wish to find the resultant of these two voltages. This is done graphically by plotting the two waves E_1 and E_2, and adding their **instantaneous values.** We find that the resultant of these two voltages is **not** 200 volts. The maximum of the resultant of these two voltages is only 141 volts. Furthermore the resultant voltage, $E_1 + E_2$, is out of phase with each of the original voltages. If Figure 16 is studied carefully, the reason the resultant voltage is not equal to the sum of E_1 and E_2 will be apparent. It is because these voltages are not acting together. In Figure 15 the two voltages were acting together, since they were in phase, but in Figure 16, the two voltages are out of phase and do not reach their maximum values at the same time. When E_2 is maximum, E_1 is at zero, and when E_1 is maximum E_2 is at zero. When E_1 is at 45° its instantaneous value is 70.7 volts, and at this same time the instantaneous value of E_2 is also 70.7 volts. This gives a resultant value of 141 volts at this time. If all other points are plotted it will be found that the sum of the two instantaneous values is never greater than 141 volts. The graph of the resultant of the two voltages shows that the resultant voltage reaches a maximum positive at 45°, a maximum negative at 225°, and goes through zero at 135° and 315°. The resultant wave is 45° out of phase with E_1 and E_2.

As an examination of Figures 15 and 16 shows, it is considerable trouble to combine alternating currents or voltages by plotting their instantaneous values, point by point, in this fashion.

These difficulties have led to the adoption of "**vectors**" for combining currents and voltages in alternating current circuits, since the use of vectors greatly simplifies the solution of many of the a-c problems encountered in electronics work.

Vectors

Suppose that the line I_{max} in Figure 17 is revolving counterclockwise at some constant speed. This speed could be measured easily in "degrees per second" since there are 360° in a complete circle or in one revolution of the line.

As the line I_{max} revolves, let us stop it at 30° intervals (points 2,3,4, etc. in Figure 17) and measure its height above its starting horizontal line. If this height is plotted on the vertical axis of a graph, and the horizontal axis is plotted in degrees representing the angle through which the line has turned, we would obtain the sine wave shown at the right in Figure 17.

This shows us that it is possible to develop a sine wave by a line whose length represents the magnitude of the current or voltage, and which is rotating at a rate equal to one revolution per cycle. Since it is possible to develop a sine wave, by plotting the height of the rotating line (I_{max}) above the horizontal line, it is permissible to use such a rotating line to represent a sine wave. A longer line would represent a greater current, and one which is rotating faster represents a higher frequency. In this example, I_{max} is equal to a maximum value of an alternating current. Likewise we could represent a sine wave voltage by a counterclockwise rotating line having a length E_{max}.

In Figure 17 the line I_{max} has a certain definite length. It also has an arrowhead on one end of it, indicating that it has direction. We call such a quantity, one that has magnitude (length) and direction, a "vector quantity".

Using Vectors to Show Phase Relationship

The phase relationship between two sine waves of the same frequency may be indicated by vectors. Remember that a vector is a line which is rotating one revolution for each cycle. Suppose we had two sine waves and represented each by a vector. If the frequency of the two sine waves were the same, these two vectors would be rotating at the same speed. It might be compared to the spokes on a wagon wheel. As the wheel turns, each of the spokes rotate at the same rate. The angle between the two spokes remains the same. Thus, if we are comparing two sine waves of the same frequency, their phase relationship may be indicated by the angle between the two vectors. This is shown in Figure 19(B). The vector I_1 represents a sine wave current, the vector I_2 represents another sine wave current of the same frequency. The two currents are 45° out of phase. They are both rotating at a rate of one revolution per cycle and thus the two vectors rotate "in step" just as the spokes of a wheel. The 45° angle will be maintained between these two vectors (spokes).

Remember these things concerning vectors. **1.** A vector may be used to represent a sine wave of voltage or current. **2.** A vector is considered to be rotating one revolution per cycle, in a counter-clockwise direction. **3.** The length of a vector represents the amplitude of the voltage or current. **4.** A vector has direction as indicated by the arrow. **5.** Phase relationships between two or more sine waves can be indicated by the angle between the vectors used to represent these waves.

The Addition of Vector Quantities

We represent alternating sine wave currents or voltages by vectors since it is much simpler to add together two vectors which represent the two currents or voltages, than it is to add the two sine waves, point by point. Because of this, the solution of most alternating current problems involves vector addition, so let us see how this is done.

An ordinary unit, such as an ohm, expresses only a quantity, and so we can add ohms directly. A vector, however, has both magnitude and direction, so they must be added in such a manner that these two things (magnitude and direction) are considered.

In Figure 18(A) we have shown a small portion of an electronics circuit. We have a junction where two alternating currents combine and flow in one common wire. The amount of current in two of the wires is known. The phase angle between the two currents is also known. The current in the common wire is to be determined.

If practical, you would merely insert an ammeter in the common wire to measure the combined current I_t. In studying a circuit diagram, or in a good many actual circuits, it will not be possible to insert an ammeter in the common wire to measure I_t, the sum of I_1 and I_2.

The known currents I_1 and I_2 are each 3 amperes and I_2 is known to be leading I_1 by 45 degrees. The two currents are said to be 45 degrees out of phase. We can plot the waveforms of I_1 and I_2 on the same axis and add their instantaneous values to obtain the waveform of I_t. See Figure 18(B). Notice that we are careful to plot the waveforms of I_1 and I_2, 45 degrees out of phase. The waveform of I_t has a maximum value of approximately 5.5 amps. The waveform of I_t lags I_2 by 22.5 degrees and leads I_1 by 22.5 degrees.

The only fault we can find in this solution is that it takes a lot of time and careful work.

In Figure 18(C) we have added I_1 and I_2 and determined I_t by means of vectors. You can see at a glance that the vector solution doesn't involve much work. The vector solution gives us the same answer as the more tedious addition of waveforms.

It does not take a mathematician to set up and add I_1 and I_2 using vectors. Choose a convenient scale, say ¼ inch equals 1 ampere. The lengths of the arrows indicate the amounts of each current in amperes. The vectors representing I_1 and I_2 should each be ¾ inches long since I_1 and I_2 are each 3 amperes.

First draw a line ¾ inches long as shown in Figure 19(A). Label this vector I_1. The vector representing I_2 will also be ¾ inches long. I_2 is known to be leading I_1 by 45 degrees. We will have to have a 45 degree angle between the vectors representing I_2 and I_1. To indicate that one sine wave is leading another, the leading vector is drawn on the counterclockwise side of the other vector. In Figure 19(B) we see the vector I_2 drawn on the counterclockwise side of I_1 and the angle between the two lines is 45°. Now we have drawn the vectors representing the two sine waves. The length of each vector indicates the amplitude of each sine wave, and the angle between them (45°) indicates the phase relationship between them. As mentioned previously, each of these vectors is rotating, but since their speed of rotation is equal, they will maintain the 45° angle between them. For all practical purposes, we could "stop" the rotating vectors in some convenient position and analyze them.

There are several ways of adding vectors, but the most simple method is shown in Figure 19(C). To find the resultant (the sum of the two) of I_1, and I_2, we "complete the parallelogram". To do this, from the tip of the arrow I_2, we draw a line which is parallel with I_1. This is the dotted line (a) in Figure 19(C). Then from the tip of I_1 draw a line which is parallel with I_2. The sum of the two or the resultant, then, is represented by the line drawn from the "tail" of I_1 and I_2, to the point where these two dotted lines cross. This is the solid line I_t in Figure 19(C). The angle that this line has in respect to the two other vectors indicates the phase angle, and the length of the line represents the magnitude of the current. If we were to measure the angle of I_t, in respect to I_1 and I_2, with a protractor, (a device for measuring angles), we would find that I_t leads I_1 by 22.5° and that it lags I_2 by 22.5°. Its length is 1⅜ inches. Since we have used ¼ inch to represent one ampere, the 1⅜ inch long resultant would indicate 5.5 amps. This is the same information as obtained in Figure 18(B), but is found much more simply by using vectors.

To further illustrate the use of vectors, let us consider Figure 20. In this figure we have two oscillators (a-c generators) connected in series across a resistor. One oscillator is putting out 2 volts. We call this voltage E_1.

The second oscillator is putting out 1 volt. We call this voltage E_2. The second oscillator voltage E_2, is lagging E_1 by 60 degrees. How much voltage do we have across the resistor? We could plot the wave forms of the two voltages and add the instantaneous values as shown in Figure 20(B). The easy method will involve just a few strokes of a pencil for rapid vector addition. This is shown in Figure 20(C). Draw the first oscillator voltage to a convenient scale representing 2 volts. [E_1 of Figure 20(C)]. Draw the second oscillator voltage vector half as long (1 volt) and of such direction that it indicates a lag of 60 degrees. [E_2 of Figure 20(C)]. Add the two vectors by the method shown in Figure 19(C), and the combined voltage across the resistor E_t can be quickly scaled and found to be approximately 2.65 volts. A protractor will show that E_t "lags" E_1 by about 19 degrees and "leads" E_2 by about 41 degrees.

Figure 21(A) shows the vectors for the waveforms shown in Figure 16. Study this vector diagram and see if it doesn't convey the same information as the wave shapes shown in Figure 16(B).

Figure 21(B) shows the vectors for the voltages shown in Figure 15. Notice that since the two voltages are in phase, they are laid out on the same line, "tail to head". The resultant voltage is equal to the total length of the line, or 150 volts in this case.

Figure 21(C) shows a vector representation of the voltage and current associated with a capacitor. Compare this with the waveforms shown in Figure 12.

Figure 21(D) shows a vector diagram of the voltage and current of a coil. Compare this with Figure 13(B).

These examples will serve to introduce the subject of vectors. Other applications of vectors will be made from time to time in the training program. We shall make use of this simple way of representing sine waves in the explanation of a great deal of a-c circuits.

A-C Waves, Other than Sine Waves

A-C voltages and currents which have sine-wave shapes are encountered to **a great extent** in electronics, but there are some cases where a-c voltages and currents will be found which have wave shapes differing from sine waves. In Figure 3 we have seen one of these wave shapes, that of a square wave. Figure 22 shows another wave shape that is sometimes encountered in electronics equipment, especially certain types of test equipment, and is frequently encountered in television equipment. This wave is an a-c wave, since it is periodically changing in magnitude and direction. This wave shape is called a saw-tooth wave due to its resemblance to a tooth on a saw. Another wave shape which differs from a sine wave is the audio signal which we have mentioned previously. The wave shape of a typical audio signal is shown in Figure 23. Before discussing this wave shape let us review briefly how this signal is developed.

Any vibrating body will set up sound waves in the air. For example, when a key is struck on a piano, the hammer strikes the string, setting it into a state of vibration. The vibrating string sets up sound waves in the air, by causing regions of higher than normal, and lower than normal air pressure to travel away from the string. When these sound waves strike the diaphragm of a microphone, they cause the diaphragm to vibrate. The microphone then changes these vibrations into audio signals, which are sound waves in an electrical form.

You may have wondered why different musical instruments have a different sound when playing the same note. For example, if middle C is played on a piano, and on a horn, it does not sound the same. Actually, both of these notes are of the same frequency, (the number of times per second that the vibrations are occurring), but the difference in sound is due to the difference in wave shape. As the sound waves strike the diaphragm of a microphone, the vibration of the diaphragm is directly "in step" with the sound wave. It produces audio signals which correspond to the sound waves, not only in frequency, but also in amplitude. The amplitude of the audio signal will vary in step with any irregularities in the sound waves. In this way, audio signals are not pure sine waves, but are closer to the wave form shown in Figure 23. The wave shape of an audio signal produced by the sound waves from a vibrating string may be closer to a pure sine wave than that in Figure 23, but the audio signal produced by a human voice is much more irregular than the wave shape shown in Figure 23. The characteristic sounds of different instruments and voices is due to the wave shape of the sound waves produced. As was pointed out previously, the frequency of audio signals range from 20 to approximately 20,000 cycles per second.

Harmonics

Harmonic is the term used to define some multiple of a fundamental frequency. For example, the second harmonic of a 60 cycle per second signal is 120 cycles per second, the third harmonic is 180 cycles per second, the fourth is 240 cycles per second, etc.

Figure 24 shows a fundamental and its third harmonic plotted on the same graph.

Summary

This assignment has presented a large amount of information about alternating current and voltages. To summarize, let us put some of this in the form of definitions.

Pulsating d-c—A current which is always in one direction, but which is varying in amplitude.

Alternating Current or Voltage—A current or voltage that periodically changes in magnitude and direction.

Cycle—One complete succession of events. Applied to a sine wave; from zero to a maximum, back to zero, to a maximum of opposite polarity and back to zero again. It is equal to 360 electrical degrees.

Frequency—Number of cycles per second: measured in cycles per second, or Hz.

Power Frequencies or Commercial Frequencies—The frequencies of the a-c power delivered to homes. In most localities the a-c frequency is 60 cycles per second. In some cases 25 and 50 cycles per second are used.

Audio Frequencies—The frequencies which are in the range of the human ear—approximately 20-20,000 cycles per second (20Hz — 20kHz).

Radio-Frequencies—Frequencies higher than 20,000 cycles per second.

Period—Length of time required for one cycle.

Instantaneous Value—The value of voltage or current for any given instant.

Peak Value of a Sine Wave—The maximum value of voltage or current during one cycle. It is equal to 1.414 times the effective value.

Effective or RMS Value of a Sine Wave—That value of the sine wave which will produce the same heating effect as the same amount of d-c voltage or current. Numerically it equals .707 times the maximum value of the sine wave. This is the value which is read by a-c meters.

Average Value of a Sine Wave—The average of all the instantaneous values for one alternation. It is equal to .637 times the peak value.

Harmonics—Multiples of a fundamental frequency.

Phase Relationship—A measure of the time difference in degrees of two sine waves of the same frequency, in reaching corresponding points on the same time axis.

Vectors—Rotating lines which may be used to represent sine waves. The length of each vector is determined by the magnitude of the sine wave, and the angle between vectors is determined by the amount of phase difference.

Wave shape—Sine waves are the one most commonly encountered. Others which may be found in electronics circuits are square waves, saw-tooth waves, and audio signals which are irregular in shape.

In future assignments, we shall apply our knowledge of a-c to the subject of coils and capacitors, and find out how each of these circuit components react to alternating currents and voltages. We will then be in a position to study one of the most fascinating subjects in electronics—the action of coils and capacitors in combination.

"HOW TO PRONOUNCE . . ."

(Note: the accent falls on the part shown in CAPITAL letters.)

amplitude	(AMM-plih-tude)
audio	(AWE-dee-owe)
diaphragm	(DIE-uh-framm)
harmonic	(har-MONN-ic)
oscillator	(OSS-ill-aye-tor)
parallelogram	(pare-a-LELL-owe-gramm)
phase	(FAZE)
vector	(VEKK-tor)

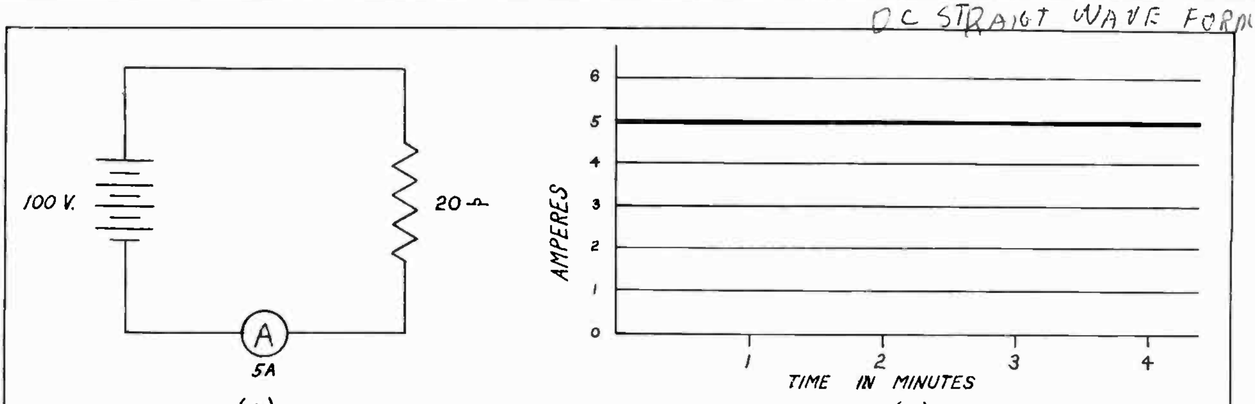
Test Questions

Be sure to number your Answer Sheet Assignment 13.

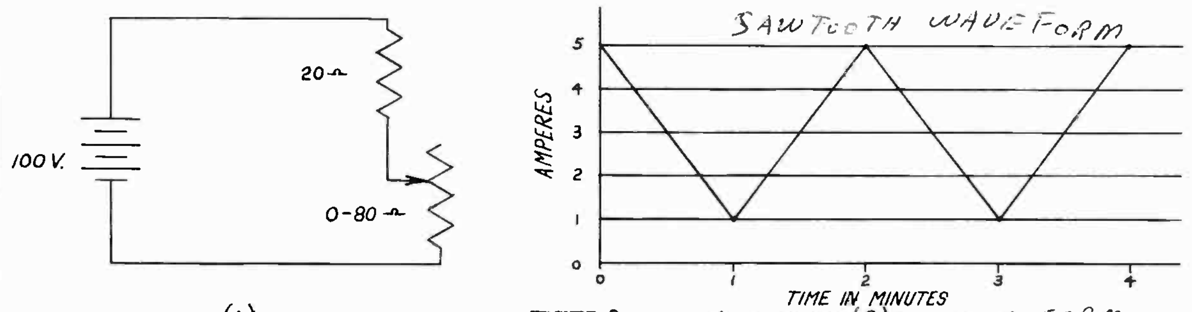
Place your Name and Associate Number on **every** Answer Sheet.

Submit your answers for this assignment immediately after you finish them. This will give you the greatest possible benefit from our personal grading service.

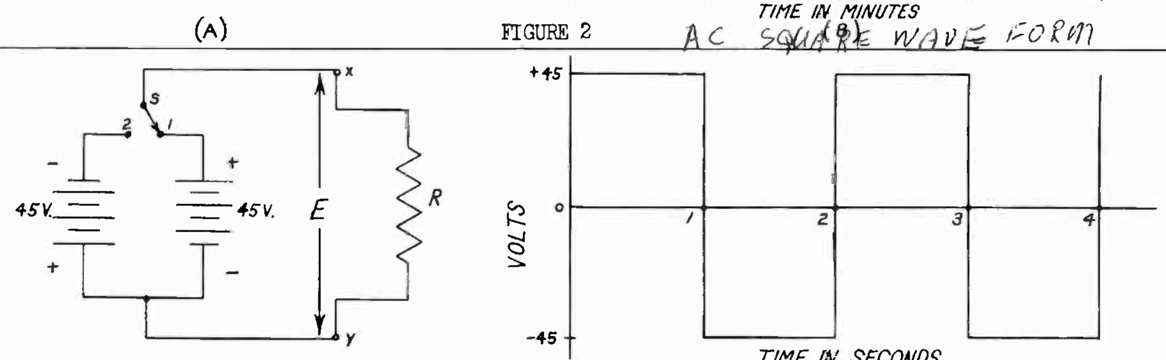
1. What is a direct current which is changing in magnitude, called?
2. What is the frequency of the a-c voltage supplied to most homes in the United States?
3. Does an a-c meter read; peak, effective, or average values of a-c?
4. If the peak value of an a-c voltage is 300 volts, what is the effective value?
5. An a-c voltage has a frequency of 10,000 cycles per second. (A) Is this called an Audio Frequency or a Radio Frequency? (B) How would you state this frequency using the term Hertz?
6. What is the frequency of the third harmonic of a 100 cycle per second a-c voltage?
7. Draw the vectors for the following:
   Two a-c voltages, each of 100 volts maximum, and 90° out of phase.
8. Use the values given in Table I on page 9 and draw a sine wave.
9. Which can be changed from a low value to a high value easier, a-c or d-c?
10. The effective value of voltage delivered to most homes is 110 volts. What is the peak value of this voltage?



(A) FIGURE 1 (B)



(A) FIGURE 2 (B)



(A) FIGURE 3 (B)

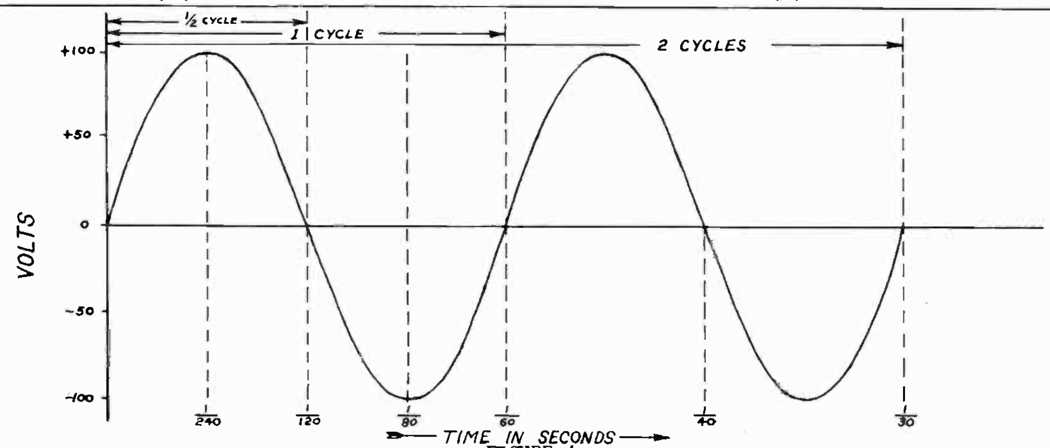


FIGURE 4

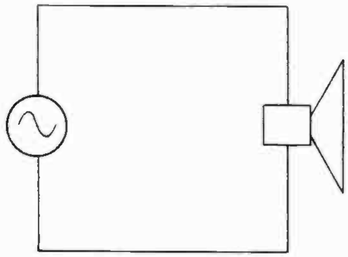


FIGURE 5

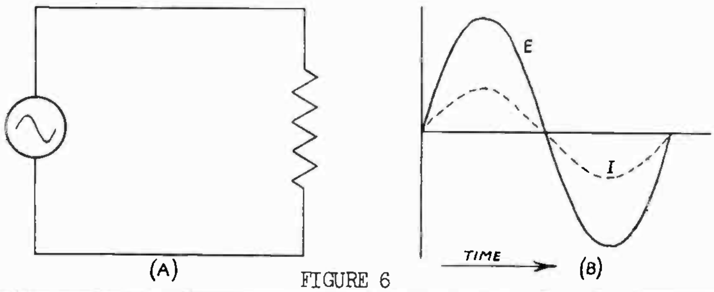


FIGURE 6

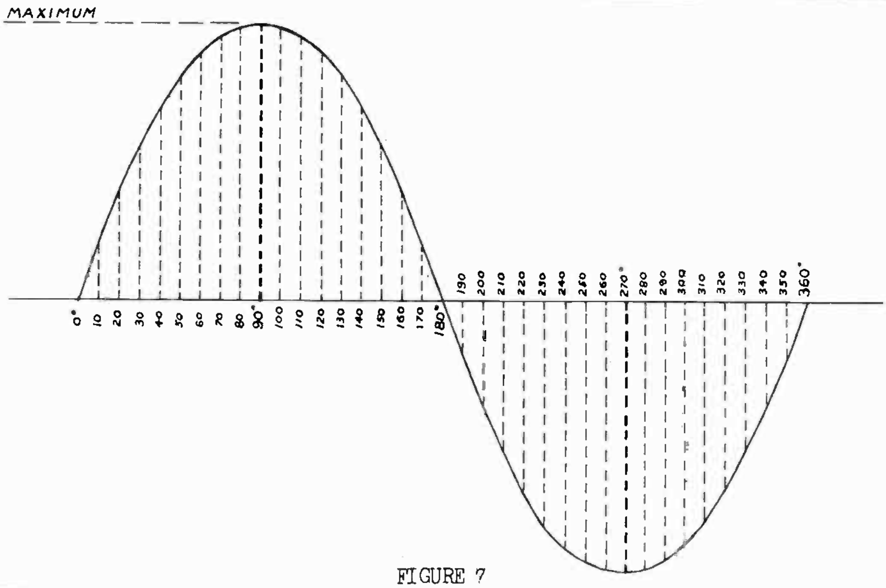


FIGURE 7

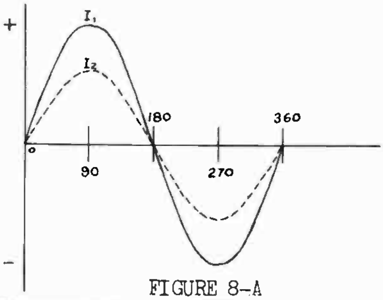


FIGURE 8–A

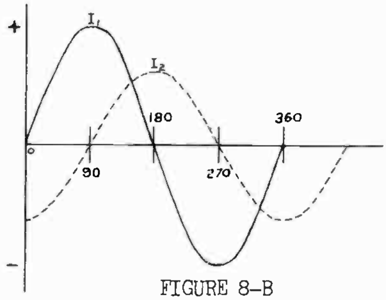


FIGURE 8–B

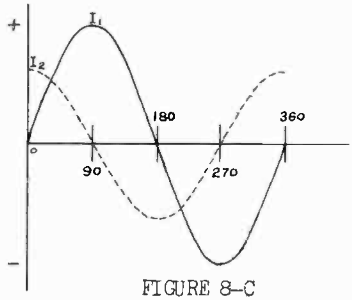


FIGURE 8–C

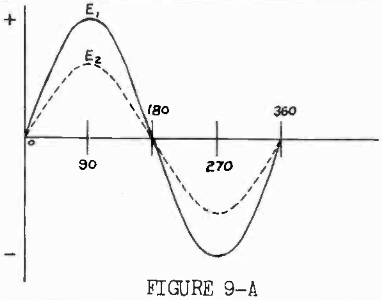


FIGURE 9–A

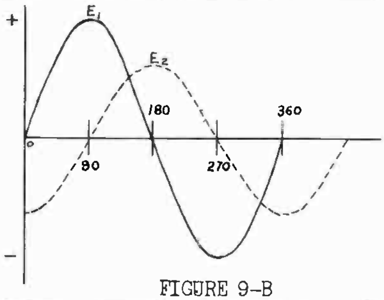


FIGURE 9–B

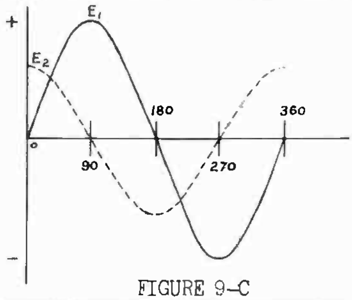


FIGURE 9–C

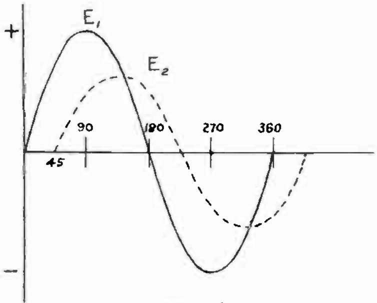


FIGURE 10-A

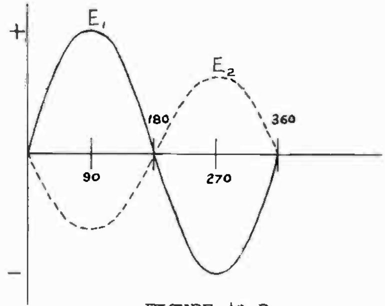


FIGURE 10-B

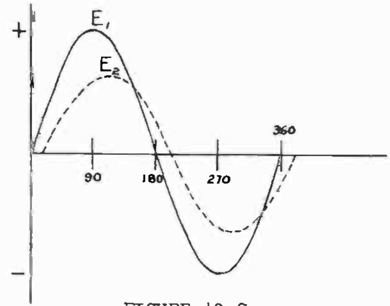


FIGURE 10-C

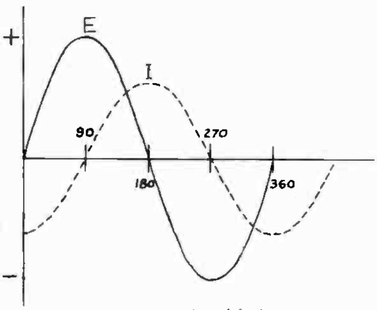


FIGURE 11-A

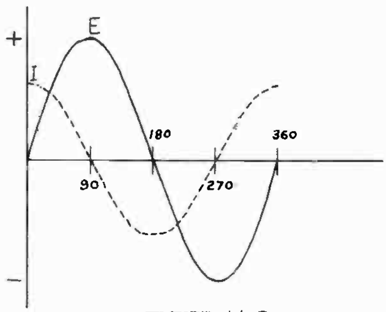


FIGURE 11-B

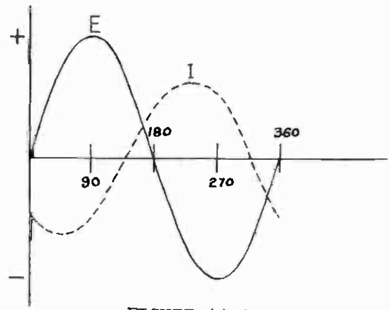


FIGURE 11-C

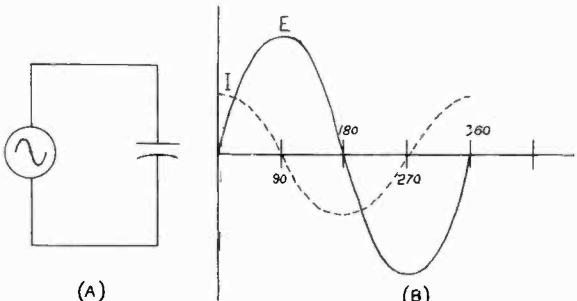


FIGURE 12

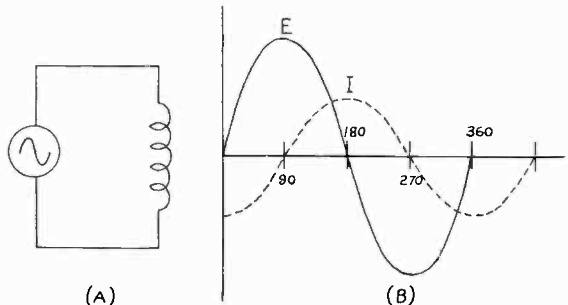


FIGURE 13

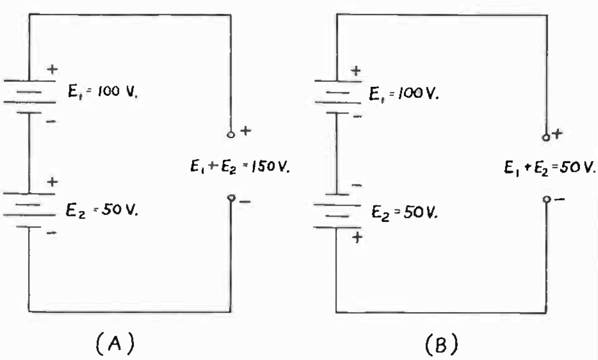


FIGURE 14

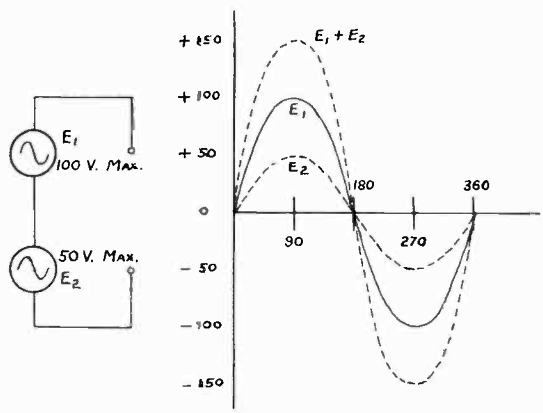


FIGURE 15

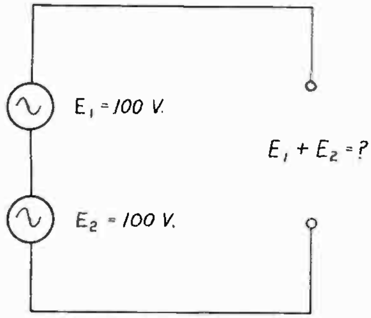


FIGURE 16-A

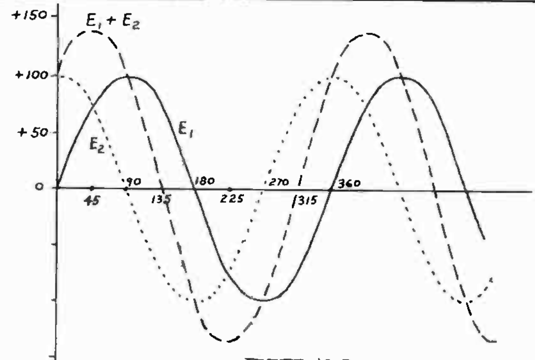


FIGURE 16-B

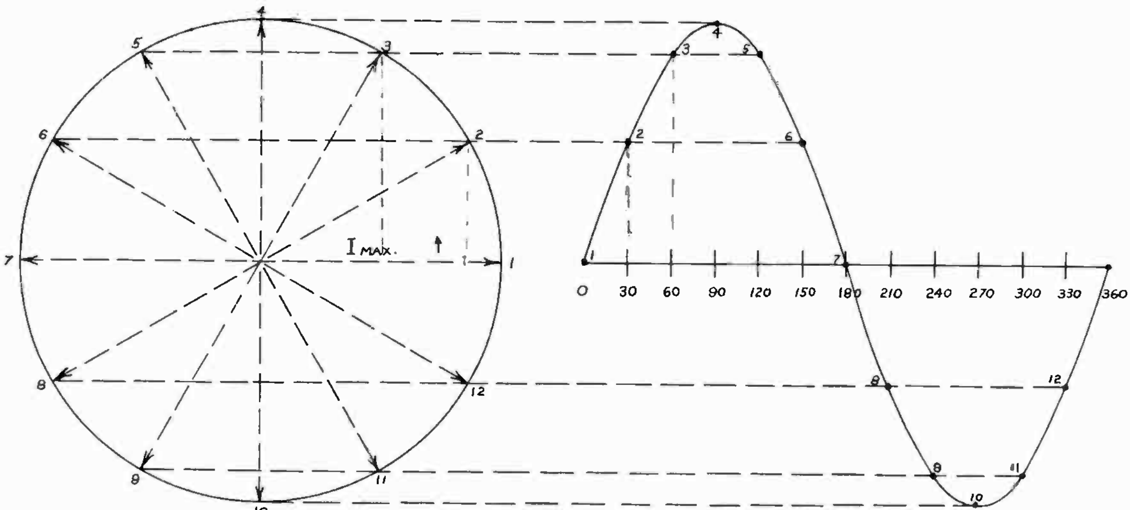


FIGURE 17

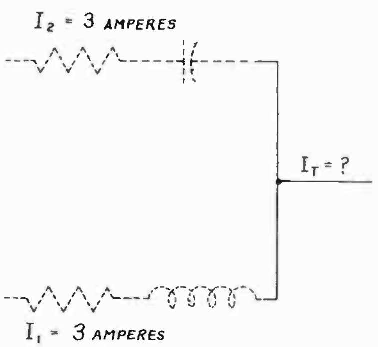


FIGURE 18-A

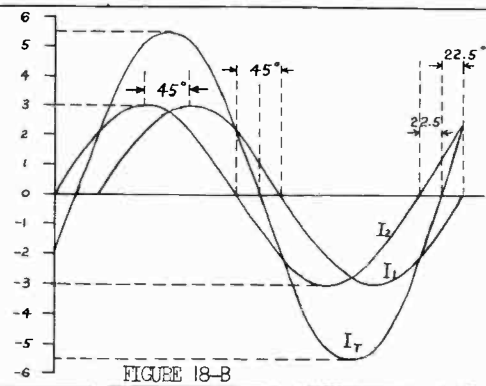


FIGURE 18-B

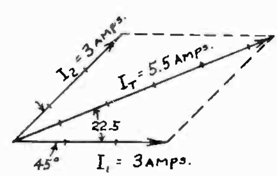


FIGURE 18-C

FIGURE 19-A

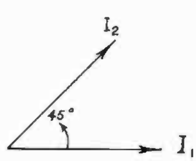


FIGURE 19-B

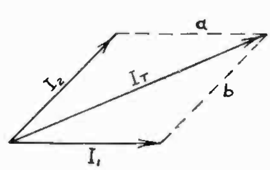


FIGURE 19-C

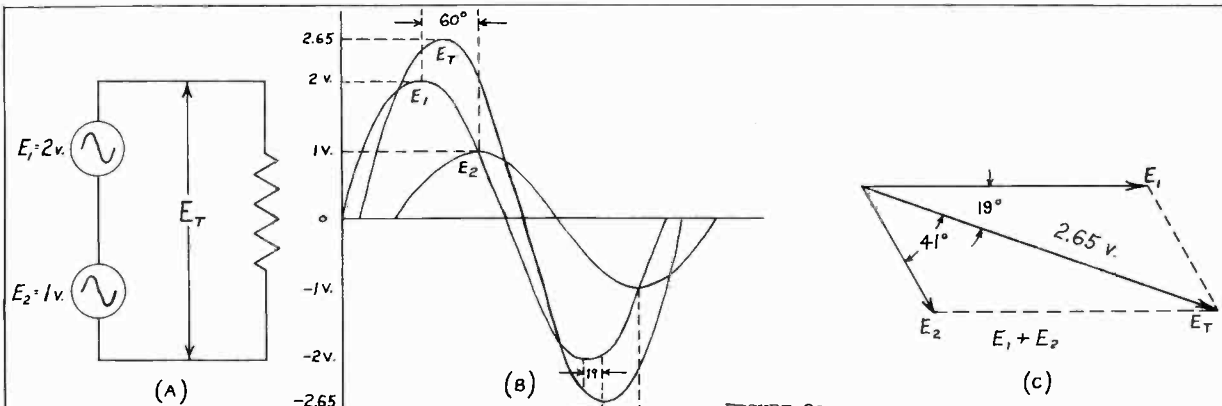


FIGURE 20

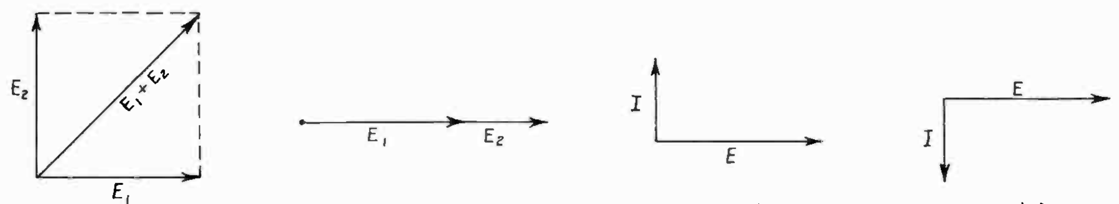


FIGURE 21

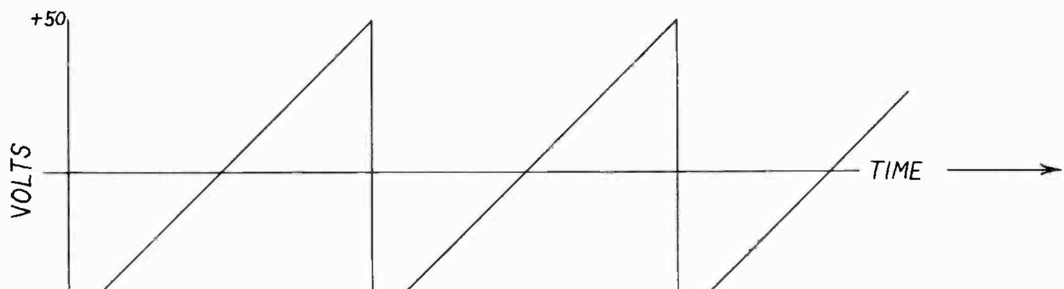


FIGURE 22

SAW TOOTH WAVE

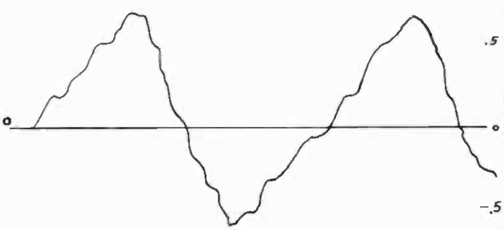


FIGURE 23

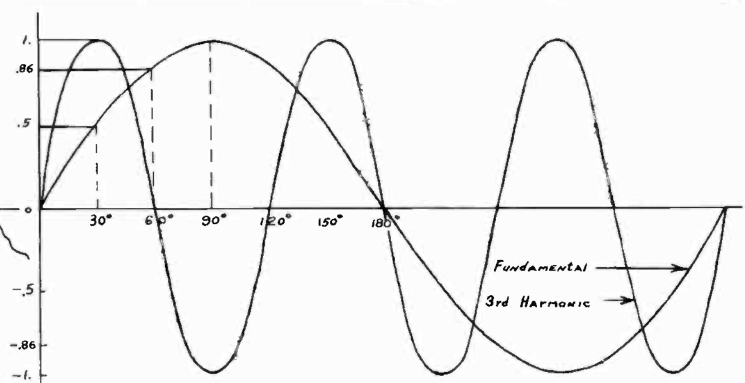


FIGURE 24