



# SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

MAGNETIC CIRCUITS

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## MAGNETIC CIRCUITS

Before beginning the study of alternating currents one should have a good working knowledge of magnets and magnetic circuits. This is important because of the very extensive part played by magnetic circuits in radio, particularly in the design and operation of audio-frequency transformers, audio-frequency chokes, meters, power transformers, loudspeakers, etc. It will be found that magnetic circuits are analogous to electrical circuits, and can be analyzed in the same manner. The difference between the two types of circuits is mainly that of degree, and of what quantity is to be solved for.

### GENERAL THEORY

Although historically magnetism was first noticed as a property apparently unrelated to electricity, as in the case of certain natural minerals containing iron, called "lodestone", it is preferable to study magnetism today as a property of a moving electric field (current flow). It can then be shown that a permanent magnet has magnetic properties because of tiny electrical currents occurring in the atoms of the material, owing to the orbital motions of the electrons in the atom as well as the spinning of the electrons on their own axes.

**ELECTROMAGNETISM.**—In 1820 Hans Christian Oersted, a professor of Physics at the University of Copenhagen, Denmark, discovered that a magnetic field is produced by and surrounds an electric current. A simple little experiment will demonstrate this fact. Thus, suppose a

current is passed through a wire, as in Fig. 1. The direction of current flow shown is the conventional one, in order that the rules to be given conform with those given in standard texts. The actual flow of electrons is from left to right in Fig. 1.

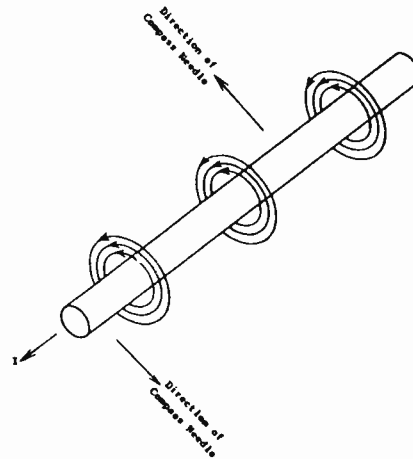


Fig. 1.—Current producing circular lines of force around a conductor.

As the current flows through the wire, certain effects will be noted in the surrounding space. For example, if a small magnet, such as a compass needle, be placed over the wire, it will align itself in a position *perpendicular* to the wire, in the direction shown, whereas if it is placed below the wire, it will arrange itself perpendicular to the wire facing in the opposite direction. If placed behind the wire, so that its dial is perpendicular, it will point downward; if in front of the wire, it will point upward. In all cases it aligns itself perpendicular to the wire.

Evidently the current produces some sort of a force in the region

that acts in a circular direction surrounding it. This is shown in Fig. 1. The circular whirls portrayed there indicate the direction in which a compass needle will be forced to indicate at every point in the region. It is to be noted, however, that there are actually no force-free spaces between the whirls shown: no matter where the compass needle is placed, a force acts upon it. Separate circles are shown merely to facilitate illustrating this effect; actually the force is distributed uniformly throughout the region.

On the other hand, the *magnitude* of the force decreases as one proceeds radially outward from the wire, so that at some distance from the wire the effect becomes inappreciable. But this tapering off of the force takes place in a smooth, uniform manner, as is stressed in the preceding paragraph.

Another simple experiment is the following. The conductor is slipped through a hole in a sheet of paper and a current passed through it (Fig. 2).

On the paper are sprinkled iron filings, and the paper gently tapped. The filings arrange themselves in circles around the paper, as shown in the figure. These filings also make visible the circular forces existing around the body; they become magnetized by a process called induction when exposed to the magnetic forces, and then arrange themselves like a series of little compass needles around the wire. In order to perform these experiments, the wire must be part of a closed circuit containing a battery or other source of electrical energy. The effects of the rest of the circuit can be minimized by making the wire

very long and running long leads perpendicularly from its ends to a battery located at some distance from the wire.

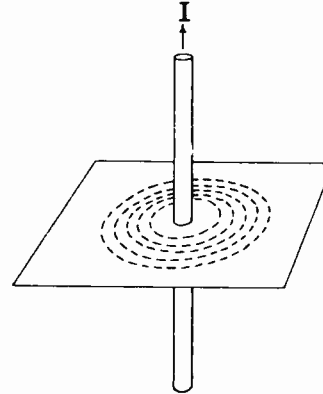


Fig. 2.—Illustration of how lines of force can be demonstrated.

*PROPERTIES OF MAGNETIC LINES OF FORCE.*—Certain properties can be attributed to magnetic lines of force that help to explain the action of magnets. These properties were developed by Faraday and Maxwell, two English physicists.

1. *Relation to Current Flow.*—The magnetic lines of force are set up by moving electric fields and always surround them in the form of closed loops. As the electric lines of force are displaced, they produce the lines of magnetic force, which appear in each region of space through which the electric lines pass.

The electric field lines normally are those emanating or terminating on an electrically charged particle. Indeed, a particle exhibits an electrical charge only because of its associated electric field. By agreement or convention, a positive charge is one from which

the field lines emanate; a negative charge is one on which the field lines from some other positive charges terminate.

If the charged body moves, then the associated field lines move, and this motion constitutes a *current flow*. Such current flow immediately produces loops of magnetic flux that enclose the moving charges. This is illustrated in Fig. 3. Some of the

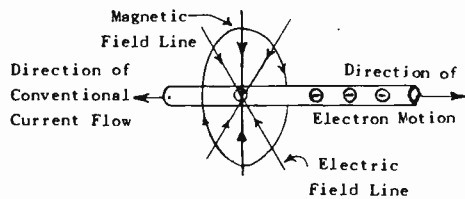


Fig. 3.—Showing the relation of fields and current flow.

free electrons of a conductor are shown. The electric field lines form radial spokes directed toward the electrons. For clarity, only some of the radial field lines for one electron are shown.

The electrons are now assumed to move or flow from left to right, owing to some source of electromotive force. This is not shown in Fig. 3, and it must also be understood that the conductor shown forms part of a *closed* circuit including the source of electromotive force.

The motion of the electrons from left to right represents a *conventional* current flow in the opposite direction, i.e., from right to left. This matter has been discussed in a previous assignment. As the radial field lines move, they set up circles or whirls of magnetic flux, as shown. These are at right angles to the electric field lines,

and also enclose the moving charges.

Ordinarily the electric field moves as a unit with its associated charge. Hence the current flow can be regarded as due to the moving field or the moving charges; it is immaterial which is considered the cause. However, it is at least theoretically possible for an electric field line to be flexed back and forth so that a portion is displaced to and fro, while the rest of the field line and the associated charge are at rest. The portion that moves to and fro represents a local current flow in that region, and will produce a magnetic field in that region. Further discussion of such special effects will be given in the lesson on radiation. Ordinarily a charge and its associated field move as a unit, so that either can be considered as producing the magnetic field by virtue of its motion.

2. *Tension in Lines.*—The second property exhibited by magnetic lines of force is that of tension: the lines act as if they were stretched rubber bands. Suppose magnetic lines extend from the face of one piece of iron to the face of another piece of iron, as shown in Fig. 4. (The complete path for each line is not shown, and may be a loop of any shape. Interest here is only in that portion of the line extending between the two pieces of iron.)

Owing to the tension of the lines, they tend to contract, and in doing so, pull the two pieces of iron toward one another. It may be said that the two pieces of iron *attract* one another; according to the concept of tension in the lines, the latter contract and pull the two pieces together.

The particular pattern or field for the lines in Fig. 4 is based on

the assumption that the left-hand piece is capable of sending out lines from its face, and that the right-hand piece is willing to accept these lines. How this situation can arise will be discussed subsequently. At this point it is of value to note that the face from

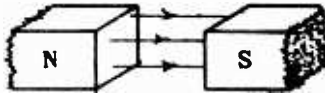


Fig. 4.—Lines of force between two magnets or pieces of iron.

which lines emerge is called a *north pole*, and the face into which the lines enter is called a *south pole*. Thus the left-hand piece of iron exhibits a north pole, as indicated in Fig. 4, and the right-hand piece of iron exhibits a south pole.

3. *Lateral Repulsion of Lines.*—Lines of magnetic force repel one another laterally (sideways). As a result they do not cross one another, but curl away from one another to avoid doing so. In Fig. 5 are shown two pieces of iron, from each of which emerges lines of force, i.e., each presents a north pole to the other. The lines curl away as shown in order to avoid crossing one another. The lines repel one another sideways and try to force each other away. This force of *repulsion* is transmitted by the lines to the two pieces of iron, and tends to force them apart.

The same effect will be noted if the two pieces of iron have south poles facing one another. In this

case the flux pattern is the same as before; one merely reverses the direction of the arrows on the flux lines. The lines still repel one another laterally and cause the two pieces of iron to repel one another. Hence a law may be stated for magnetic poles similar to that for electric charges:

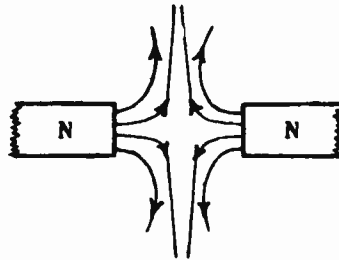


Fig. 5.—Lines of force when opposing poles are adjacent.

**Law of Magnetism.** Like poles repel, unlike poles attract.

The lateral repulsion of the lines of force, together with their internal tension, can explain the shapes assumed by the lines. For example, in Fig. 6 are shown lines of force issuing from a north to a south pole, similar to Fig. 4. Only three lines are shown in the figure, AB, CD, and EF. Line AB repels

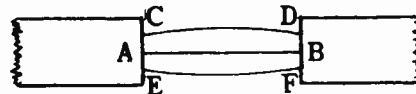


Fig. 6.—Illustration of lateral repulsion of lines of force.

lines CD and EF laterally, causing them to bow out as shown. But this increases their length and hence their tension, and this effect prevents the bowing out from becoming excessive: lines CD and EF bow out until the tension force just balances the lateral force of repulsion. On the other hand, AB remains straight because the lateral forces of CD and EF balance one another. The pattern for *all* the lines set up by a magnet follows the above rules, but is correspondingly more complicated because of the larger number of lines involved.

4. *Right-hand Rule*.—Returning now to the current flowing in a straight conductor, one can employ a simple rule. If the *right-hand* be held as in Fig. 7, so that the thumb points in the direction of the conventional current flow, then the remaining fingers will indicate the direction of the circular whirls of magnetic flux around the wire.

If it is desired to employ the true *electron* direction of flow, then use the *left-hand* in the same manner and obtain the correct results.

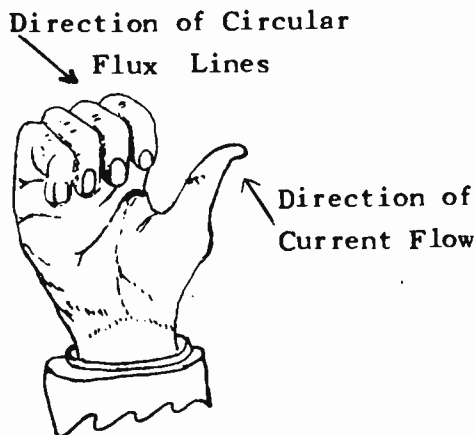


Fig. 7.—Illustration of the right-hand rule.

*REACTIONS BETWEEN ELECTRICAL CIRCUITS*.—The above rules for the magnetic lines of force can now be applied to the study of the reactions between current-carrying conductors, and electric circuits in general. Suppose one has two parallel conductors carrying currents in the same direction. These conductors may be parts of independent circuits or the parallel branches of one circuit; the magnetic effect will be the same. These two conductors are shown in cross section in Fig. 8; the current in each is assumed to be into the paper, as suggested by

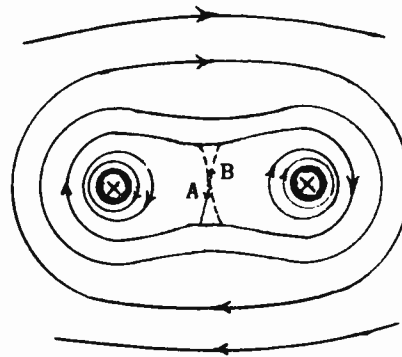


Fig. 8.—Pattern of flux lines resulting when the current flow in each of two parallel, adjacent, conductors is in the same direction.

the crosses within each circular conductor.

Close to each conductor circles of flux will be set up as shown, and in conformity with the right-hand rule just given. But, as one proceeds radially outward to the two circles of flux labelled A and B, it is noted that they would annul one another. As a result, the two combine to form one single flux line that encircles both conductors, as

shown. This is true for all lines at radial distances greater than the one above. In this way no two flux lines need cross one another.

However, these encircling loops tend to contract. In doing so, they pull the two conductors together in order that they may continue to envelope both. Hence the two conductors are attracted to one another when they carry current in the same direction.

This force is normally weak and can be withstood by ordinary rigid conductors. Under short-circuit conditions, when the current is abnormally high, the force of attraction may be very large and tend to deform the circuit configuration.

Another instance is that of a liquid metal conductor, such as that formed in an induction furnace. Different parts of any cross section of the molten metal may be regarded as separate conductors, adjacent to one another, and carrying current in the same direction. The effect here is to squeeze the metal together and cause it to break its continuous circuit. This is known as the "pinch effect."

Now consider the case of two parallel conductors carrying currents in opposite directions. This is shown in Fig. 9. The left-hand conductor has current flowing into the paper (cross) while the right-hand conductor has current flowing out of the paper (dot). Application of the right-hand rule, with due note of the other properties of the magnetic lines, yields the pattern shown. Large circles are merely suggested in the figure.

Note now that the lines cannot combine to form loops that encircle both conductors; instead, they must remain as separate groups of circles.

These two groups, however, repel one another laterally, and as a result, the conductors now repel one another.

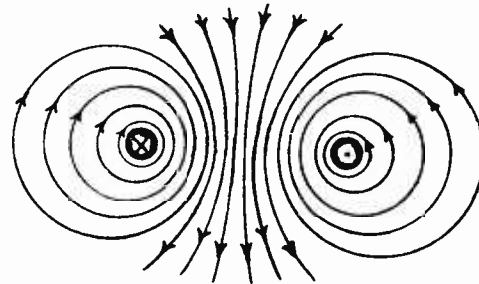


Fig. 9.—Pattern of flux lines resulting when the current flow in each of two parallel, adjacent conductors is in the opposite direction.

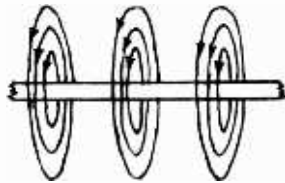
*SOLENOIDS.*—In Fig. 1 was shown the circles or whirls of flux around a straight wire. This is reproduced in Fig. 10(A). Suppose this wire is bent into a circular loop into which a generator is inserted as in Fig. 10(B). The flux lines still encircle the wire, but owing to the fact that the latter is now bent into the form of a circle, the flux lines pass through the wire loop from right to left. This is even more clearly shown in the cross-sectional view of Fig. 10(C). Note that in effect the flux issues from the left-hand face of the loop, making it a north pole, and enters the right-hand face, making this a south pole. The loop thus acts like a very short permanent magnet.

The magnetic effects can be

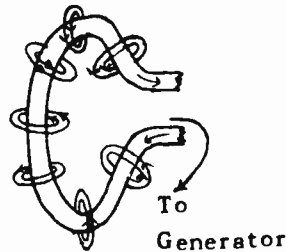


greatly intensified by causing the current to flow through a great many turns closely spaced, one next to

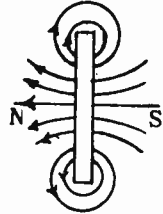
iron is introduced into the coil, as this permits more flux to be set up



(A)



(B)

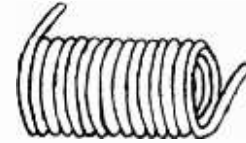


(C)

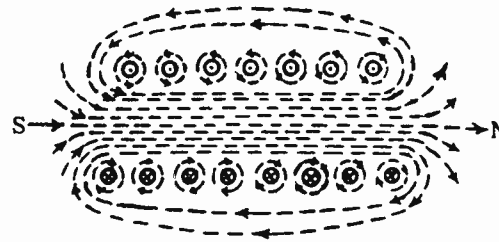
Fig. 10.—(A) Whirls of flux around a straight wire—(B) and (C) whirls around a coiled wire.

the other, like in a coil spring. The form is shown in Fig. 11(A), and in cross section in (B). It is usually called a solenoid.

Owing to the concentration of the current-carrying conductor in a relatively small space, the magnetic flux lines, particularly inside of the solenoid, are much greater in number. The device is known as an electromagnet, and is particularly effective if a round bar of soft



(A)



(B)

Fig. 11.—Solenoid coil and the magnetic field around it.

for a given current and number of turns.

The flux lines, for the most part, pass through the interior of the solenoid and return via various outside paths, as indicated in Fig. 11(B). Some few lines encircle individual turns, as shown, but these are few in number, especially if the turns are spaced closely together. As may be noted from the figure, flux issues from the right-hand end of the solenoid, so that this is a north pole, and the opposite end is a south pole.

Solenoids may have an air core—as in r-f coils and chokes—(or a wooden or plastic core acting as a coil form),—or they may have an iron core, as in the case of audio and power transformers and chokes, and in loudspeakers, relays, motors, generators, etc.

**ANOTHER RIGHT-HAND RULE.**—A right-hand rule may be developed for a single turn or for a solenoid, that is similar to that given previously for the straight wire. Refer to Fig. 7. If the four fingers are held in the direction of the circular current flow, then the thumb indicates the direction of the magnetic flux through the current loop.

**INTERACTION BETWEEN SOLENOIDS.**—If two solenoids are placed so that their north poles face one another, a field pattern as shown in Fig. 12 is obtained. The pattern is exactly

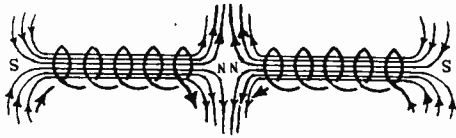


Fig. 12.—Pattern of flux lines resulting when two solenoids are placed so that their north poles face one another.

similar to that shown in Fig. 5; in fact, Fig. 12 indicates how the condition of two north poles facing each other might be obtained in practice. The two solenoids are repelled owing to the lateral repulsion between the adjacent lines.

If the current in each solenoid is reversed, the poles interchange their positions so that two south poles face each other. In this case the flux lines reverse their direction, *but their shapes remain unchanged*, so that one still obtains

a repulsion between the two solenoids.

If the current in one or the other (but not both) of the solenoids is reversed, then a north pole faces a south pole, and attraction takes place. Suppose the current in the left-hand solenoid is reversed. Then the flux pattern will be that shown in Fig. 13: The magnetic

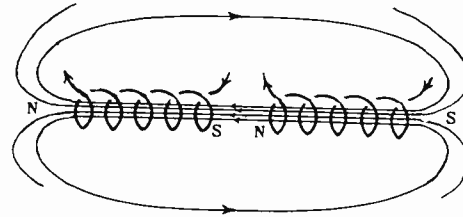


Fig. 13.—Flux pattern resulting when the current in one of the solenoids shown in Fig. 12 is reversed.

loops will pass through both solenoids in series before curving back to close on themselves.

Owing to the contraction of the lines of force, they will draw the two solenoids together, i.e., the solenoids are attracted to one another. The action is exactly similar to that shown in Fig. 4, and indicates how the condition shown in Fig. 4 could be produced, i.e., by the use of solenoids. Note further that instead of reversing the direction of the current in the left-hand solenoid, the same effect could be obtained by rewinding the solenoid in the opposite direction; that is, like a left-hand screw instead of a right-hand screw.

**PERMANENT MAGNETS.**—Either or both of the two solenoids can be replaced by permanent magnets with results identical to those described above. A permanent magnet is a piece of hardened steel or steel alloy which has been exposed to the influence of another magnet, or has been inserted into a solenoid carrying a large current for a short period of time.

Although permanent magnets are found in nature in the form of lodestones— $\text{Fe}_2\text{O}_4$ , an oxide of iron—and such a lodestone can be used to produce an artificial steel permanent magnet, nevertheless permanent magnets are commercially made today by the action of a solenoid as described above.

The atoms of steel and, to a lesser extent, of nickel and cobalt, appear to be little magnets owing to the spinning orbital electrons. (This will be discussed further later on.) When the lines of force of the solenoid pass through the steel, they tend to veer the atoms around so that they align with the solenoid's field. The atoms act like little compass needles and arrange themselves tangent to the solenoid's lines.

When the solenoid is removed, the atoms remain aligned, since the steel is hard, and the atoms with their spinning electrons continue the action started by the moving electrons in the solenoid, and maintain the magnetic field. The steel is now said to be magnetized.

The earth is a permanent magnet too. Hence any solenoid or—more simply—any permanent magnet, such as a compass needle, will align itself with the earth's field.

The end of the compass needle that points to the geographical

north pole of the earth is itself called the *magnetic north-seeking pole*, or in short, the *north pole*. It is also sometimes designated as the positive (+) pole although north pole is more commonly used. Since unlike poles attract, it is clear that if one calls the above end of the compass needle the *north pole*, then the end of the earth to which it points must be a negative magnetic pole.

Thus the earth has a *negative magnetic pole* close to its geographical north pole, and a *positive magnetic pole* close to its geographical south pole. (The geographical poles are the ends of the earth's axes upon which it spins.) The above relationships are clearly illustrated in Fig. 14.

### MAGNETIC UNITS

Magnetic devices are primarily for the purpose of establishing magnetic flux in some selected region

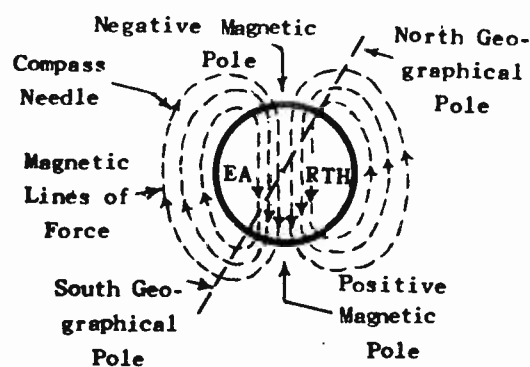


Fig. 14.—Relationship between the earth's magnetic and geographical poles.

of space. If a series of current-carrying conductors is immersed in this flux, a mechanical force is developed between the conductors and flux which will cause one to move relative to the other. This is the motor rule, and will be discussed more fully in subsequent assignments. Usually the flux is held fixed in space, and the current carrying conductors are permitted to rotate, in the case of a motor, or to vibrate, as in the case of a loudspeaker.

In another type of device, conductors are moved through the magnetic field, whereupon they generate a voltage. This is the generator rule, and will also be discussed in future assignments.

In still another case the flux between an electromagnet and a movable iron member, called an armature, causes the armature to be attracted to the electromagnet when current is passed through its windings. This arrangement represents a relay, and is used to close electrical switch contacts to operate a motor, to connect components of a broadcast system to one another, to connect two parties to a telephone line, etc.

#### GENERAL DISCUSSION OF PROBLEM.—

In all cases a certain amount of flux is required at some point, and it is the task of the design engineer to calculate a magnetic arrangement which will furnish the desired amount of flux.

The usual method of producing flux is by means of a solenoid through which current is passed. The flux is set up within the solenoid, thence through the region desired (usually an air gap) and then back to the other end of the solenoid.

The arrangement is therefore

very similar to an electrical circuit. Instead of an electromotive force (e.m.f.), one has a *magneto-motive force, m.m.f.*, generated by the solenoid. This is the actuating quantity. Instead of current, there are *lines of magnetic flux*, and instead of electrical resistance, there is magnetic resistance, called *reluctance*.

Just as for electrical circuits there has been developed Ohm's law, so in magnetic circuits there is a similar law, often called Ohm's law for magnetic circuits. It is

$$\text{Flux} = \frac{\text{mmf}}{\text{Reluctance}} \quad (1)$$

This is just as simple as Ohm's law and is handled in an identical manner.

It is the main purpose of this assignment to show how to calculate and design a magnetic circuit so that if one is given the amount of flux required, he can calculate back and find what the reluctance of the circuit will be, and also what magneto-motive force will be required. This is somewhat different from an electrical circuit problem, where one more often is given the impressed voltage, and the resistances in the circuit, and is required to find the current.

However, this is a very minor difference between electrical and magnetic circuit problems and is merely one of detail. More important differences arise, not in fundamental theory, but in the degree to which the two circuits follow ideal behavior.

In the case of the usual electrical circuit, the paths traversed by the currents are clearly defined. The insulation surrounding the con-

ductors is well-nigh perfect, and confines the current to the conductors. Only in special cases, such as when the current is required to flow through a resistance of one thousand megohms or so, is it necessary to be concerned about leakage paths through the insulation shunting this high resistance, particularly on damp days.

In the case of magnetic circuits there are no good insulators. The magnetic conductivity of different materials varies only in degree; air, brass, vacuum, copper, and the like have but a moderate conductivity or permeance, as it is called, whereas other materials such as iron, nickel, and cobalt have a permeance hundreds to thousands of times as great, provided they are not saturated (do not have too much flux set up in them).

The ratio of magnetic conductivity of the two types of materials is on the order of one thousand to one; the ratio of electrical conductivity may well be billions to one. For this reason the paths that a magnetic flux may follow are so many in number that the determination of these paths and the amount of flux set up in each path may be exceedingly difficult.

A similar problem arises in u.h.f. work, where a current may flow in part through the capacity between the two wires connected to a resistance, as well as through the resistance itself (effect of stray capacity). Another electrical problem analogous to the magnetic problem would be the following: a storage battery is immersed in the ocean (salt water); find the current flowing through the battery. It would be necessary to determine all the paths followed by the electrons through the salt water in order to

determine the total resistance presented to the battery before one could apply Ohm's law. Theoretically some current would flow from the one terminal to the other through a long looping path extending miles away from the battery. The paths of the electrons are suggested in Fig. 15, and it will be noted how similar they are to the flux paths illustrated in preceding figures.



Fig. 15.—Note the similarity between paths of electrons of a battery immersed in salt water, and between flux paths of a magnet.

Nevertheless a large class of magnetic circuit problems can be solved. These involve arrangements where the paths for the flux are fairly well defined, as in the case of iron-core circuits. Since the iron has a much higher permeance than air, it is to be expected that most of the flux will flow through the iron, and that the leakage across through the air will be small.

On the other hand, in radio receivers and transmitters air core coils, called inductances, are used. Usually no iron is used in them, or else the iron path is but a small fraction of the total magnetic path. The flux is thus set up in a medium

similar to that of the salt water for the storage battery, and the determination of the amount of flux set up, and its density at various points in space, is very difficult to determine. It requires the use of special mathematics, such as (space) vector analysis and calculus, and even then becomes prohibitively complicated except for very simple coil shapes. For these shapes, fortunately, the results can be reduced to fairly simple formulas and curves, such as the inductance charts for air-core coils. (Inductance depends upon the ability of a circuit to set up magnetic flux.)

**MAGNETOMOTIVE FORCE.**—The magnetomotive force is proportional to the current and the number of turns the current traverses hence, by a simple rule of algebra, it is proportional to their product. This product is called ampere turns, and is abbreviated to IN, although the symbol A.T. is also used. The ampere turns can be used as a measure of the magnetomotive force.

For example, suppose 3 amperes are passed through a solenoid having 100 turns. The ampere turns are  $3 \times 100 = 300$  IN.

Magnetism was first studied in connection with permanent magnets, and certain concepts and units were developed, to which it was then desired that electromagnetism conform. As a result the unit of magnetomotive force is not the ampere turn, but the *gilbert*. The two are very simply related:

One ampere turn corresponds  
to  $.4\pi$  or 1.257 gilberts

For example, in the above case of 300 IN, the magnetomotive force is  $1.257 \times 300 = 377$  gilberts.

**RELUCTANCE.**—The reluctance R, of a magnetic path or circuit is di-

rectly proportional to its length and inversely proportional to its cross section just as is the resistance of a copper wire. Stated in formula form,

$$R = \frac{L}{\mu A} \quad (2)$$

where L is the length of path in cm, A is the area in sq. cm, and  $\mu$  is the relative permeability, a quantity similar to the specific resistance (or rather conductivity) of a conductor. For air and other non-magnetic substances, such as brass, etc.,  $\mu = 1$ , but for magnetic substances such as iron,  $\mu$  may be several thousand in value. More will be said about  $\mu$  farther on in this assignment.

The unit of reluctance is that of a cube of air or vacuum, having a cross-sectional area of 1 sq. cm and a length of 1 cm, hence a volume of 1 cubic cm. This is shown in Fig. 16. The lines of force are all assumed to pass *perpendicularly* through the two faces, and to be *uniformly distributed* over the two faces.

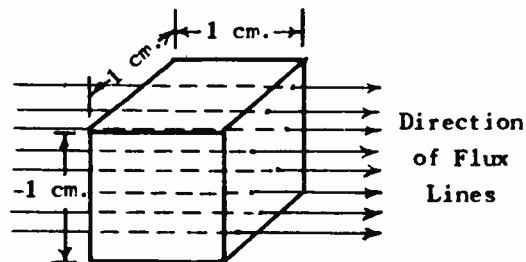


Fig. 16.—Diagram used to illustrate the unit of reluctance.

The unit of reluctance used to be called the oersted, similar to the name ohm, but an international congress in 1930 decided to use this name to represent m.m.f. per unit length, i.e., 1 oersted equals 1 gilbert per cm, so that the unit of reluctance has no special name at the present time. Karapetoff, an American professor and scientist, has suggested the name rel (from reluctivity) but this name has not as yet been officially adopted.

As an example of reluctance, consider an air space between two magnetic poles, having a length of 2 cm, and a cross section of 3 cm by 4 cm or 12 sq. cm. This is shown in Fig. 17(A). The reluctance is

$$R = \frac{2}{12} = \frac{1}{6} \text{ unit}$$

On the other hand, if the cross section is 1 cm × 2 cm = 2 sq. cm., and

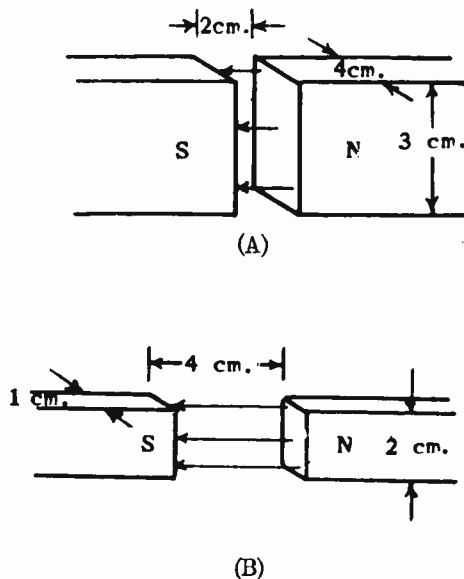


Fig. 17.—The reluctance increases inversely as the cross-sectional area, and directly as the path length.

the length of path is 4 cm., see Fig. 17(B), then the reluctance is

$$R = 4/2 = 2 \text{ units}$$

The reluctance in (B) is greater than that in (A) because

1. The path length is greater, and
2. The cross-sectional area is less.

**FACTORS DETERMINING RELUCTANCE.—**

The reluctance, as defined above, appears to be fairly simple to calculate. There are many important instances, however, where this is not so. Consider a rectangular volume of air through which flux is streaming in diverging paths, as shown in cross-section in Fig. 18. This may very well represent conditions at a region remote from an air-core coil (source of m.m.f.). The flux lines are not perpendicular to the end

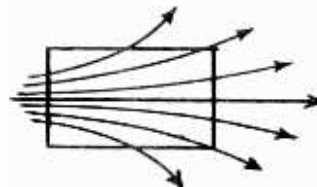


Fig. 18.—Flux lines diverging in an air path.

faces, and the distribution over these faces is by no means uniform. For such flux paths the reluctance can no longer be calculated by means of Eq. (2), and the correct value of reluctance is very difficult to ascertain: it can be calculated only by means of very advanced mathematical methods.

This situation is by no means peculiar to magnetic circuits; exactly the same difficulty would arise in attempting to calculate the resistance of a volume of salt water when the electron paths through it are not parallel to one another, as for example, in Fig. 15. Another example perhaps closer to practice is that of skin effect. At low frequencies the electrons flow through the entire volume of the conductor, and the resistance is low. At high frequencies, owing to magnetic effects the current is forced to flow mainly near the surface, so that the effective cross section is very greatly reduced, and the resistance, therefore much higher.

The fact that the reluctance of a magnetic path is difficult to calculate in the case of an air-core coil makes its use in such problems of little value, but in a large class of magnetic circuits in which iron cores are involved, the calculation and use of reluctance values makes the problem very simple and straightforward. Also, magnetic circuits are generally much simpler than electrical circuits, and are relatively easy to calculate, once the reluctance has been determined.

In the case of an iron-core circuit, one has to know the permeability of the iron in order to find the reluctance by Eq. (2). Unfortunately this is not a constant quantity, but is dependent upon the amount of flux through the iron. For many approximate calculations the permeability may be regarded as constant, and where this is too inexact, magnetization curves can be used to obtain more accurate values.

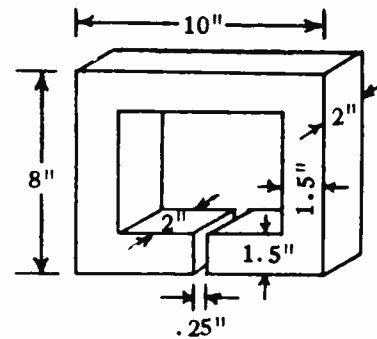
Consider the two volumes shown in Fig. 17 (A) and (B). If these contain iron instead of air, and

the permeability of the iron is 2,000, then the reluctance of the two volumes will be

$$1/6 \div 2,000 = 1/12,000 \text{ unit, and}$$

$$2 \div 2,000 = .001 \text{ unit}$$

*MAGNETIC CIRCUITS.*—The determination of the reluctance of a magnetic configuration or circuit is best explained by means of an example. Suppose one has the magnetic arrangement shown in Fig. 19. Here there is an iron path in series



Assume  $\mu = 900$   
for the iron  
 $\mu = 1$  for air

Fig. 19.—Typical magnetic circuit arrangement.

with an air gap .25" long. The reluctance of each portion must be calculated separately, and the two added together to give the total reluctance just as in electrical series circuit.

The reluctance of each path depends upon  $\mu$ , its average or mean length, and its cross section. Owing to the highly permeable iron, the



flux in it should be in lines parallel to the length and therefore perpendicular to the cross section at any point, so that Eq. (2) is applicable. In the air gap the flux lines will bow out, especially in the middle of the gap, but the effect is small, and can be taken into account by reducing the value found by Eq. (2) by a suitable correction factor. This factor is usually found from experience, but can be ignored in this problem.

To employ Eq. (2), the length of the path in the iron and the air gap must first be determined. The air gap length is practically .25 inch. The mean length in the iron can be found from Fig. 19 as follows:

The core has been redrawn in Fig. 20. The dotted center line represents the mean length of path.

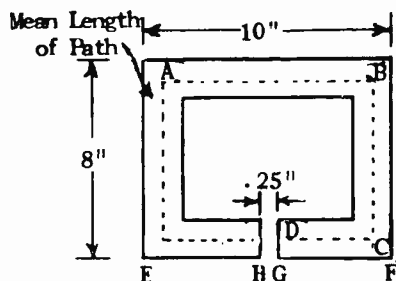


Fig. 20.—Illustration of calculating mean length of path for Fig. 19.

Since the width of the iron is 1.5", it is clear from the figure that portion AB equals 10" minus half of 1.5" at each end, or

$$AB = 10 - 2(1.5/2) = 8.5"$$

similarly

$$BC = 8 - 2(1.5/2) = 6.5"$$

The air gap is so short that it can be safely ignored, so that it will be satisfactory to take the bottom length as equal to the top length of 8.5". The total length will then be  $2(8.5) + 2(6.5) = 30"$ .

Converted to cm,  $30 \times 2.54 = 76.2$  cm. The cross-sectional area is

$$A_1 = 2 \times 1.5 = 3 \text{ sq. in.}$$

$$= 3 \times (2.54)^2 = 19.4 \text{ sq. cm}$$

This is also the cross-sectional area of the air gap. Its length is

$$L_a = .25 \times 2.54 = .635 \text{ cm}$$

The reluctance of the iron is

$$R_1 = \frac{76.2}{19.4 \times 900} = .00436 \text{ unit}$$

( $\mu = 900$  given in Fig. 19)

The reluctance of the air gap is

$$R_a = \frac{.635}{19.4 \times 1} = .0327 \text{ unit}$$

The total reluctance is

$$R_t = R_a + R_1 = .00436 + .0327 = .0371 \text{ unit}$$

Note how much greater the reluctance of the short air gap is as compared to the much longer iron path: nearly 8 times as great. This is because the permeability of the iron is 900 times as great as that of air. As a result, air gaps in electrical apparatus are made as short as possible; just sufficient to provide adequate mechanical clearance be-

tween an armature and field magnet in a motor or generator, as an example. If the air gap can be avoided, then it is gladly dispensed with, as in an ordinary transformer core.

Another example of a magnetic circuit is shown in Fig. 21. The coil developing the m.m.f. is wound on the center leg. The calculations

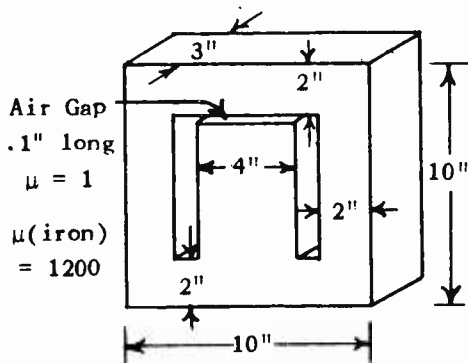


Fig. 21.—Center-leg type of magnetic circuit.

may be considerably simplified by regarding the arrangement as two magnetic circuits in parallel. From this viewpoint the core may be regarded as split in two along its center line, whereupon each half has similar elements in series, and the two parts, of equal reluctance, are in parallel. This is indicated in Fig. 22. The coil may be regarded as setting up equal m.m.f.s in each path, and this common m.m.f. is simply the product of the current through the coil times its number of turns further multiplied by 1.257 to give the m.m.f. in gilberts.

Consider either half. The mean

length of path is indicated by the dotted lines, and the calculations are exactly the same as for the previous case. Thus there is the mean

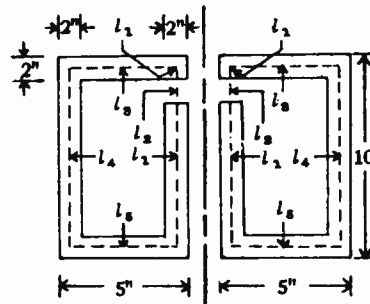


Fig. 22.—The circuit of Fig. 21 may be considered as two parallel circuits.

length  $l_1$  of the center leg, (plus the small length above  $l_2$ ) then the length  $l_2$  of the air gap, then  $l_3$  of the upper part, followed by  $l_4$ , the outside leg, and finally  $l_5$ , the lower part.

First there is the length  $l_1$  to calculate. In doing so, the air gap can be safely ignored, so that  $l_1$ , and  $l_4$  will be taken as equal. Hence calculate instead  $l_4$ , as its length is more evident from Fig. 22. (It is also clear from Fig. 22 that  $l_3 = l_5$ .) Thus

$$l_4 = l_1 = 10'' - 2(2/2) = 8'' = 20.32 \text{ cm}$$

$$l_2 = 0.1'' = .245 \text{ cm}$$

$$l_3 = l_5 = 5'' - 2(2/2) = 3'' = 7.62 \text{ cm}$$

Now the reluctances can be calculated. Let  $R_1$  be the reluctance

corresponding to  $l_1$ ;  $R_2$ , to length  $l_2$ , etc. The cross-sectional area throughout is  $3" \times 2 \times (2.54)^2 = 38.7$  sq. cm. Then

$$R_1 = R_4 = \frac{20.32}{38.7 \times 1,200} = .000437 \text{ unit}$$

( $\mu = 1,200$  given in Fig. 21)

$$R_2 = \frac{.254}{38.7} = .00656 \text{ unit}$$

$$R_3 = R_5 = \frac{7.62}{38.7 \times 1,200} = .000164 \text{ unit}$$

The total reluctance of this series path is the sum of the above, or

$$R = R_1 + R_2 + R_3 + R_4 + R_5$$

$$R = .000437 + .00656 + .000164 + .000437 + .000164 = .00776 \text{ unit}$$

However, this path is in parallel with the other path, so that the overall or total reluctance is consequently half of the above value. Thus

$$R_1 = .00776 \div 2 = .00388 \text{ unit}$$

Look back to the values for  $R_1$ ,  $R_3$ ,  $R_4$ , or  $R_5$ , which are the reluctances of the iron paths, and note how small they are compared to the reluctance of the air gap. The latter is about fifteen times as great as  $R_1$  or  $R_4$ , and about forty times as great as  $R_3$  or  $R_5$ . This is because its permeability is unity as compared to 1,200 for the iron parts of the magnetic circuit.

Another point to note from Fig. 21 is that the center leg is twice

as wide as either outer leg or top or bottom parts, i.e., it is four inches wide as compared to two inches for the other parts. The reason is that the flux of the center leg divides between the two paths around it to the left and to the right. Hence, unless the center member is made twice the area of the other members, the flux density (to be described below) will be greater than that in the other iron parts, and such greater magnetic crowding is usually undesirable. It is the normal design procedure to make the flux density in all iron parts of the circuit the same, providing the same kind of iron is used throughout, and that is why the center leg has twice the area in order to accommodate twice the flux that the other iron parts have to carry.

**FLUX,  $\phi$ .**—The object of a magnetic circuit is to establish flux in a region, such as in an air gap. The flux  $\phi$  is measured in lines of force. The unit is the maxwell; one line corresponds to one maxwell. There are several ways of expressing the magnitude of a line of flux in terms of other phenomena. It may be defined by its action on a permanent magnet of known strength, or by its effect on a current-carrying conductor of given length and having a given current flowing through it. However, until the uses of magnetism have been discussed, it is just as well to define the maxwell in terms of m.m.f. and reluctance, by means of Ohm's law for magnetism, Eq. (1). Thus, if a m.m.f. of 1 gilbert is impressed on a path having one unit of reluctance, then 1 line or maxwell will be set up in this path.

It is also correct to speak of m.m.f. drop just as one speaks of voltage drop. If there is a drop of

1 gilbert along an air path 1 cm. long and 1 sq. cm. in cross section (so that the reluctance is one unit), then there will be one maxwell established in this air path. In other words, if the m.m.f. drop along a portion of a magnetic circuit and also the reluctance of this portion are known, one can then calculate the maxwells set up in this portion of the total path.

In the problem of Fig. 21, suppose an m.m.f. of 200 gilberts is applied to the center leg. The total reluctance was calculated to be .00388 unit. Then the total flux is

$$\phi = 200 \div .00388 = 51,546 \text{ maxwells}$$

This is also the flux in the air gap. But in each half of the top or bottom members, and in the two outer legs, the flux is half of 51,546 or 25,773 maxwells, since it divides equally between these two paths.

Various calculations can be performed similar to those for an electrical circuit. For example, calculate the m.m.f. used up by the air gap. Its reluctance is .00328 units, or one-half .00656, the value computed for either half of the total air gap.

$$\text{m.m.f.} = \phi \times R =$$

$$51,546 \times .00328 = 169.07 \text{ gilberts}$$

This leaves  $200 - 169.07 = 30.93$  gilberts as the m.m.f. across each parallel branch, whose reluctance was found to be

$$\begin{aligned} R_1 + R_3 + R_4 + R_5 &= .000437 + \\ .000164 + .000437 + .000164 &= \end{aligned}$$

$$.001202 \text{ units}$$

Therefore the flux through each branch is

$$30.93 \div .001202 = 25,740 \text{ maxwells}$$

which checks the above figure quite closely.

*MAGNETIC CIRCUIT CONSIDERATIONS.*—There are some precautions to be exercised in analyzing magnetic circuits. The m.m.f. was regarded as being produced by a coil surrounding the center leg in Fig. 21. Suppose, instead, it had been wound on an outer leg, for example, the left-hand leg. In this case it would have forced flux partly through the center leg, and partly through the right-hand leg in parallel. This is illustrated in Fig. 23(A).

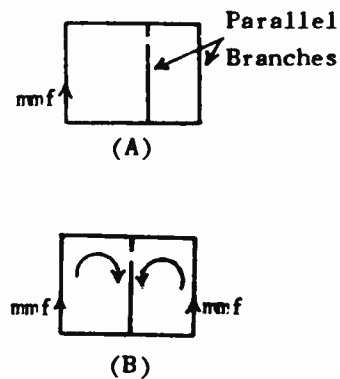


Fig. 23.—The same effect as that of Fig. 21, may be considered as two parallel circuits.

If the same flux paths as previously are desired, then *two* coils must be provided, one on *each* outer leg, as shown in Fig. 23(B). These correspond to two batteries in parallel across a resistance in an elec-

trical circuit. The two coils must each develop the same m.m.f. as a single coil on the center leg alone would produce. Hence twice as many turns are required as before, for the same amount of current. This makes the  $I^2R$  losses in the coils double that for the single coil on the center leg, so that in practice the latter method of excitation is invariably employed.

There is one important difference between an electrical and a magnetic circuit, and that is with respect to power consumption. When electrical current is drawn from a battery through a resistance, power is expended equal to  $E \times I$ . If two batteries in parallel furnish the current, then each clearly furnishes only *half the power*. On the other hand, the expenditure of power in a coil that produces an m.m.f. is merely incidental. Such power expenditure, once the flux is established, is due solely to the *ohmic resistance of the coil*. (This assumes that direct current flows in the coil. The situation is not so simple if the coil is energized by alternating current, as will be explained later.)

If the coil were wound with lead wire and cooled down to absolute zero, it would become a superconductor and have no resistance. In this case the required coil current to furnish the necessary m.m.f. could be made to flow without any external e.m.f. This means that the electrical input to the coil would be zero.

But the current-carrying coil would develop an m.m.f., and would therefore continuously maintain flux in the magnetic circuit in spite of the reluctance of the latter. This however does not represent the ex-

penditure of power and hence the magnetic circuit does not require the electrical circuit to furnish it with power.

The only power that the electrical circuit has to furnish is that consumed as heat by the coil owing to its resistance. Elimination of the resistance eliminates the need for constant electrical power input to the coil. Another example is that of the permanent magnet: it develops an m.m.f. owing to internal atomic currents, which do not encounter any resistance to their flow in the atom. Hence the permanent magnet does not require any external source of energy to maintain its m.m.f. On the other hand, nothing ordinarily is gained by employing two m.m.f.'s in parallel instead of one single m.m.f. If the sources of these are coils having resistance, then the  $I^2R$  losses will be doubled without increasing the magnetic effect.

Normally the m.m.f. is concentrated in the form of a coil located in one part of the magnetic circuit, such as one leg of the iron core. The rest of the iron circuit is then relied upon to guide the flux in the desired path. If the m.m.f. can be distributed over a greater portion of the desired path, then one can have more assurance that the flux will follow this path. This is particularly true for air paths.

For example, suppose a solenoid is bent around into the form of a doughnut (mathematically known as a toroid). This is shown in Fig. 24. Here the m.m.f. or ampere turns are distributed completely over a circular, *closed* path, as shown. The flux lines will thread their way through all the turns back on themselves and thus form close loops

within the toroid. If any flux line has an inclination to "wander" out through the turns, then the next successive turns will draw it through them, as it were, and prevent it from passing outside.

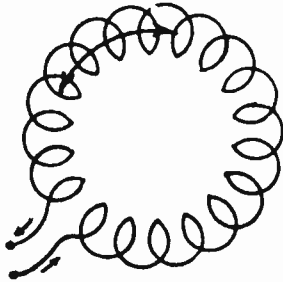


Fig. 24.—Illustration of a toroid type winding.

This is an example of a path being followed not because it has a lower reluctance than any other path, but because the m.m.f. is distributed uniformly along this path and thus makes it the preferred path for the flux. The toroidal coil has very little tendency to send flux into adjacent circuits and cause interference there. By the same token other circuits tend not to interfere with it; it is in a sense magnetically self-shielding. The main objection to such a coil is its bulk and expense to manufacture.

FLUX DENSITY B.—The flux density is the number of lines passing through a unit cross-sectional area. Thus, the density may be referred to as so many maxwells per square inch, or so many maxwells per sq. cm. One maxwell per sq. cm. is called a gauss. For example, in the pre-

vious example illustrated by Fig. 21, the flux in the air gap was found to be 51,546 maxwells. The cross sectional area of the gap is 77.4 sq. cm. Therefore the flux density is

$$B = \frac{51,546}{77.4} = 666 \text{ gauss}$$

In lines per sq. in. it is  $666 \times (2.54)^2 = 4296$  lines/sq. in.

The flux density is a very important quantity, as it represents the congestion or crowding of the flux in a material. In the case of iron, for example, the important consideration is not how much flux passes through it, but rather how much flux passes through each sq. cm. of it, that is, the flux density.

*The reluctance of the iron is variable, and depends upon the flux density. If the flux density is too high the iron saturates; its permeability goes down and its reluctance increases.* The designer must therefore proportion the area of the iron core so as to avoid too high a value of B and the saturation that would result.

MAGNETIZING FORCE H.—Mention was made previously that the m.m.f. generated by the current-carrying coil was consumed in the various parts of the magnetic circuit as m.m.f. drops, similar to voltage drops in an electrical circuit. On a unit basis, these drops can be measured as so many gilberts per cm. The unit is one gilbert per cm and is called the oersted.

Suppose one has a cube of air 1 cm. on a side, and impresses 1 gilbert between two of its faces, as indicated in Fig. 25. The cube has 1 unit of reluctance, since it has 1 sq. cm. cross-sectional area and is

## MAGNETIC UNITS

1 cm. long. Hence 1 gilbert will establish 1 maxwell in it, as shown. But 1 gilbert across 1 cm. is 1 oersted, and 1 maxwell per sq. cm is 1 gauss. Hence in air, the number of oersteds is numerically equal to the number of gauss, and in the past H was used as a measure of the flux density in the air.

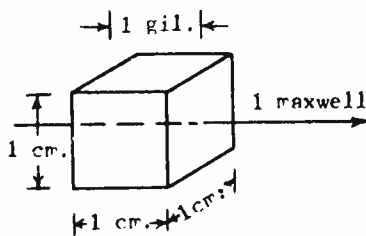


Fig. 25.—Diagram illustrating relationship between gilbert, maxwell, oersted, and gauss.

The international congress of 1930, however, decided that it should represent magnetomotive force per unit length rather than the flux density in air. This is the same as saying that while one volt produces a current flow of one ampere in a one ohm resistance, the volt and the ampere are different quantities, even though they happen to be numerically equal for a one ohm resistance.

As an example, consider Fig. 21 once again. It was found previously that 169.07 gilberts were consumed across the center air gap, which was .254 cm long. The magnetizing force in the air gap is therefore

$$H = 169.07 \div .254 = 665.6 \text{ oersteds}$$

or 665.6 gilberts for 1 cm. Actually

the air gap is only .254 cm long, but the magnetizing force is such that if it were 1 cm long, 665.6 gilberts would be required to force the required 51,546 maxwells through it. (This ignores the fact that in a 1 cm. length air gap, the flux lines would not all be parallel, and considerable leakage would occur.)

*THE B-H CURVE.*—Mention was made that the reluctance of iron and other so-called ferromagnetic substances varies with the flux density in the iron. This means that the flux will not be in direct proportion to the m.m.f., as is the case of an air path. Hence it is necessary to measure the flux for a number of different values of impressed m.m.f. and plot a curve between the two.

Since the amount of flux depends not only upon the m.m.f., but upon the length and cross section of the iron path under test, it is customary to express the relationship between the m.m.f. and flux on a per unit basis, i.e., for 1 sq. cm. of cross section and for 1 cm of length. But the flux for 1 sq. cm is the flux density B, and the m.m.f. for 1 cm length is the magnetizing force H. Hence the curve is known as the B-H curve. The actual sample under test may have many sq. cm cross section and be many cm in length, but the results are then reduced to B and H in exactly the same way as is required in the preceding exercise problems. Alternately, for practical work in this country; one can use ampere turns per inch instead of H in oersteds, and lines per sq. in. instead of lines per sq. cm. or gauss.

The m.m.f. is generally produced by passing a known current through a coil of known number of turns wound around the sample forming a closed path. Thus H may be varied in a

measurable manner. By means of a suitable device, such as a search coil and ballistic galvanometer,  $B$  can then be measured. A ballistic galvanometer has a heavy movement that does not begin to deflect until the entire current pulse has passed through its windings.

The plot of  $B$  versus  $H$ , for increasing values of  $H$  from zero, is shown in Fig. 26 and is known as the *normal curve*. For small values of  $H$  (up to  $OA$ ),  $B$  increases slowly at first. Then, as  $H$  increases from  $A$  to  $C$ ,  $B$  increases rapidly; but from  $C$  to  $D$  the increase in  $B$  is far less rapid, as saturation is beginning to be evident. This part of the curve is known as the "knee" of the characteristic.

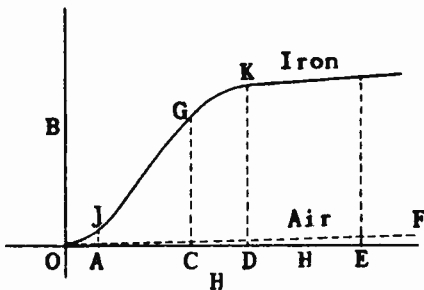


Fig. 26.—Plot of  $B$  versus  $H$  for iron, and for air.

As  $H$  increases still further from  $D$  to  $E$ ,  $B$  increases very slowly, as if it had the same reluctance as air from that point on. The iron is said to be *saturated*, it can carry no more flux on the large scale it did before, but it can still act as

an equal volume of air would function.

For d-c electromagnets, the iron is operated generally above saturation. It would thus seem to be no better than air for this large value of flux, but a moment's reflection will indicate the value of the iron as a core material even in this case. In Fig. 26 is shown the  $B$ - $H$  curve for air, namely  $OF$ . This is a straight line because the reluctance of air is a constant, there are no saturation effects, and  $B$  is therefore directly proportional to  $H$ .

Note how small  $B$  is even for a value of  $H$  equal to  $OE$ , as compared to the value of  $B$  for iron. The steep portion of the iron  $B$ - $H$  curve reduces the amount of  $H$  necessary for a required amount of  $B$ . When the iron is operated just above saturation, not only is the maximum benefit obtained from it, but small variations in  $H$  about the normal value required cause little fluctuation in  $B$ , i.e., the flux remains practically constant.

**PERMEABILITY.**—The permeability of the iron is the ratio of  $B$  to  $H$ . It thus measures the ability of  $H$  to set up  $B$  in the material. For example, in Fig. 26, if  $H = OC$ , and the corresponding value of  $B$  is  $CG$ , then at that point of the  $B$ - $H$  curve the permeability is

$$\mu = \frac{CG}{OC}$$

Numerically,  $OC$  might be 4 oersteds (gilberts/cm) and  $CG$  might be 8,200 gauss (maxwells or lines per sq. cm). Then  $\mu$  would be  $8,200 \div 4 = 2,050$ .

For  $H = OA$ ,  $\mu = JA/OA$ , and for  $H = OD$ ,  $\mu = KD/OD$ . It is clear from the figure that  $\mu$  is less for these two values of  $H$  than it is for



$H = 0C$ . Now if  $\mu$  is variable then, from Eq. (2), it is clear that the reluctance  $R$  will be variable, too.

Hence, instead of assuming  $\mu$  is fixed at some value or other depending upon the iron employed, it is more accurate to calculate magnetic circuits employing iron, directly from the B-H curves. As an example of how this is done, consider the core shown in Fig. 21 once more. This figure is repeated here for convenience.

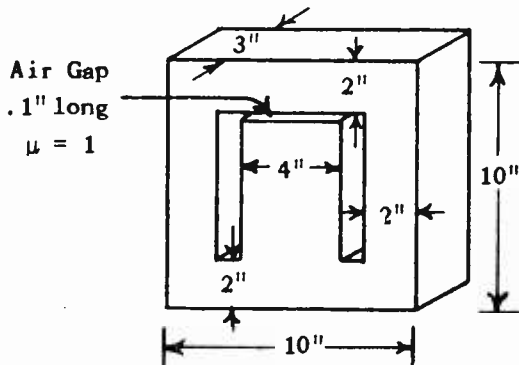


Fig. 21.—Center-leg type of magnetic circuit.

The first thing to note is that generally one knows how much total flux is required, either in a closed iron circuit, as in the case of a transformer, or in the air gap of an open iron circuit, as in the case of a field magnet of a motor or generator. In other words, the design engineer starts with the value of  $\phi$  given. He then chooses or designs a magnetic circuit that will have the least reluctance and occupy the least space possible. The next and final step is to calculate the

total m.m.f. required.

The problem is analogous to that of being given the current, to design a circuit having a minimum of resistance and requiring a minimum of voltage to furnish this current.

In the magnetic problem, suppose a flux of  $\phi = 360,000$  lines is desired. The center leg has an area of  $4 \times 3 = 12$  sq. in. Hence

$$B = 360,000 \div 12 = 30,000 \text{ lines/sq.}$$

$$\text{in.} = 30 \text{ kilolines/sq. in.}$$

First the m.m.f. for the air gap can be computed. For this purpose  $\phi$  can be used directly. The reluctance of the air gap was originally calculated to be .00328 unit, so that the m.m.f., in gilberts is

$$360,000 \times .00328 = 1,181 \text{ gilberts}$$

This problem will be worked out in American practical units, such as square inches, ampere turns, etc. Thus, 1,181 gilberts corresponds to

$$1,181 \div 1.257 = 940 \text{ ampere turns}$$

It is now necessary to calculate the number of ampere turns required for the iron parts of the circuit. In Fig. 27 are given the B-H curves for four types of steel. Note that B is in kilolines per square inch, and H is in ampere turns per inch. These are the units employed in ordinary American engineering design. The curves are given for two scale ranges of H: from 0-20 IN/inch, and from 20 to 200 IN/inch.

Observe that up to a value of  $H = 5.5$  IN/inch, silicon steel has the highest flux density and hence the highest permeability. Also observe however, that it saturates

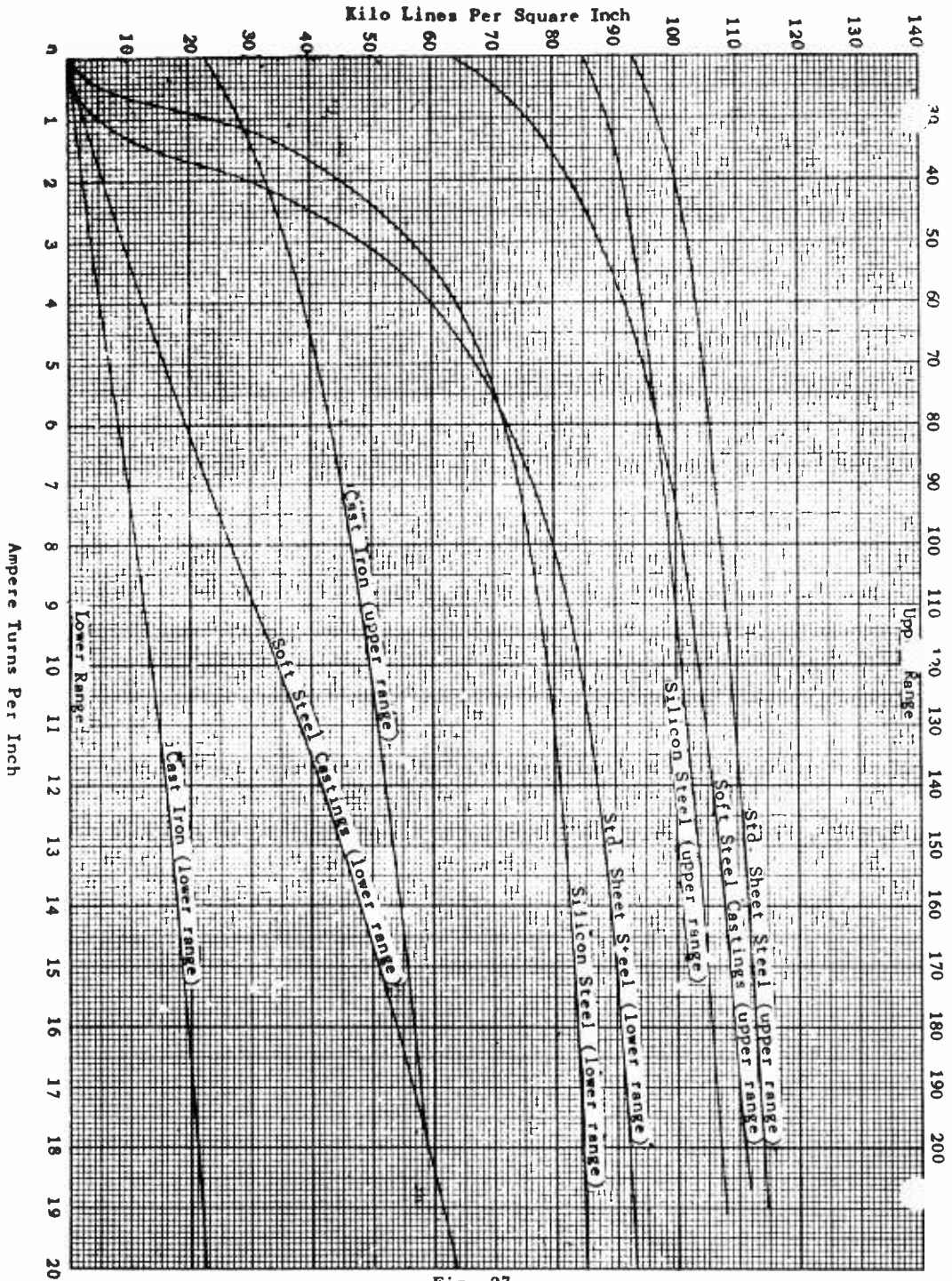


Fig. 27

Fig. 27.—B-H curves for various magnetic materials.

more readily than standard sheet steel and hence has a lower flux density for values of H greater than 5.5 IN/inch. Where operation at high flux densities and above saturation is desired as in field magnet structures of d-c motors and generators, standard sheet steel would be preferable, and even soft steel castings (for values of H greater than 8.5 IN/inch).

But for operation below saturation, as in a-c transformers, etc., silicon steel is preferable because it is more permeable, and also—as will be seen later—has a lower hysteresis loss. Suppose in this problem, however, cast steel is employed. Then for a value of flux density of 30,000 lines/sq. in., there is obtained from the graph a value of H = 8.87 IN/inch.

The length of the either iron path—Fig. 21—(ignoring the air gap), is

$$l_1 + l_3 + l_4 + l_5 =$$

$$8" + 3" + 8" + 3" = 22"$$

The total ampere turns for this path is then  $8.87 \times 22 = 195.1$  IN.

Note that the flux density was chosen as 30,000 lines/in<sup>2</sup> regardless of whether the flux was that in the center leg or in the outer legs. This is because the core may be divided—as indicated in Fig. 22—in to two parallel paths. If B = 30,000 lines/in<sup>2</sup> in the outer legs ( $l_3$ ,  $l_4$ , and  $l_5$ ) then it will also be 30,000 lines/in<sup>2</sup> in the center leg  $l_1$  because although the flux is double in the center leg, and the area is also double, and the ratio, which is the flux density, therefore comes out the same as for the outer legs. In this way all parts of the iron

circuit are worked as far as possible to the same degree of flux density. The total ampere turns required are

$$940 + 195.1 = 1135.1 \text{ ampere turns}$$

or 1,135 ampere turns

Suppose the current is to be 100 ma. Then the number of turns required is  $1,135 \div .1 = 11,350$  turns.

It is of value to note, in passing, that the permeability of the iron, when operated at 30,000 lines/sq. in., is (from Fig. 27).

$$\mu = \frac{30,000 \div (2.54)^2}{8.87 \times 1.257 \div 2.54} =$$

$$\frac{30,000}{8.87 \times 1.257 \times 2.54} =$$

$$\frac{30,000}{8.87} \times .314 = 1,060$$

Here  $30,000 \div (2.54)^2$  represents gauss, and  $8.87 \times 1.247 \div 2.54$  represents gilberts per cm.

However, when the B-H curves are employed, it is not necessary to know the value of  $\mu$  nor of R; the m.m.f. in ampere turns/inch can be obtained directly from the graph if B is known, as is generally the case in magnetic circuits.

**HYSTERESIS.**—The magnetization or B-H curve shown in Fig. 26 is obtained as H is increased from zero to some large positive value. However, suppose H is increased from zero to some moderate value, and then decreased to zero again. For increasing values the curve has a shape similar to that shown in Fig. 26, and illustrated as OC in Fig. 28.

If H is now decreased, B does

not drop as rapidly as it rose with increasing  $H$  and the normal curve  $OC$  is not retraced from right to left. Instead, the curve now follows the path  $CD$ , so that when  $H$  is reduced from  $OA$  to zero, there is still some flux set up in the material of a value  $OD$ .

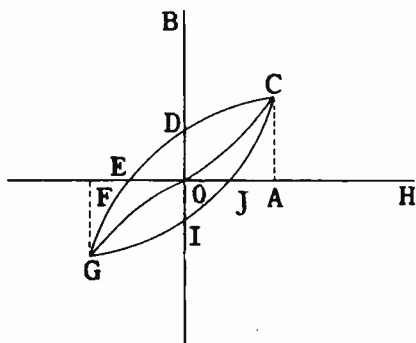


Fig. 28.—Illustrating formation of a hysteresis loop.

This is called the *residual* or *remanent* flux density, and causes the material to be a permanent magnet. For hard steels and special alloys  $OD$  can be rather large, whereas for soft iron and other special alloys  $OD$  will be very small. Hence permanent magnets are best made out of very hard materials such as tool steel, etc.

To reduce the flux density to zero it is necessary to apply an m.m.f. in the reverse direction (negative  $H$ ), such as by reversing the current in the magnetizing coil. If  $H$  is made negative to a value  $OE$ ,  $B$  will be reduced to zero. This negative value of  $H$ , namely  $OE$ , is called the *coercive force* as it re-

presents the counter or negative m.m.f. required to "coerce" the iron to lose its magnetism. Hence, the greater the coercive force the more stable will the permanent magnet be under conditions of stray demagnetizing fields, mechanical shock, etc. The greater the remanent flux density the more powerful will the permanent magnet be.

If  $H$  is increased in a negative direction from its value  $OE$  to  $OF$ , then  $B$  will increase rapidly in a *negative* direction and reach a value  $FG$  that is equal to  $AC$ , when  $OF$  equals  $OA$ .

If  $H$  is now reduced from  $OF$  to zero,  $B$  follows along the curve  $GI$ , where  $OI$  is now the remanent flux in the reverse direction. In magnitude it is equal to  $OD$ . If  $H$  is now increased in a positive direction from zero to  $OA$  once more,  $B$  follows the curve  $IJC$ , which ends up at  $C$  once more. Thereafter, the flux density  $B$  follows the closed curve  $CDEGLJC$  as  $H$  varies from  $OA$  to  $OF$ . This closed curve or loop is known as the *hysteresis* loop, and represents the lagging of  $B$  behind  $H$  as the latter alternates.

This loop has a symmetry about the origin  $O$ , as has been indicated above. Note further in this connection that  $OJ$ , the coercive force in the positive direction, is equal to  $OE$ , the coercive force in the negative direction. Another point is that if one started the magnetizing cycle in a negative direction, there would be developed the *negative branch of the normal curve*, namely  $OG$ , and the flux density for  $H = OF$  would have been  $FG$ , the same value as is obtained along curve  $CDEG$ . In other words, even though  $B$  does not follow the normal curve, upon reversing  $H$ , but lags behind it at

first, it ultimately drops rapidly enough so as finally to catch up with the other branch of the normal curve, as is clear from Fig. 28.

Suppose, after H has varied over one cycle, and one hysteresis loop has been traced out, it is thereupon increased to a higher value and then varied over this greater range. A larger hysteresis loop will be traced out. If it is then increased to a still larger value and then varied, a still larger loop will be formed, and in this way a whole family of loops can be produced.

This is illustrated in Fig. 29. Suppose the smallest loop shown has been traversed and H is now increased from OA to OC. The flux density B then increases along the normal curve from F to D. If now H is varied through the range OC to OG, the next larger loop DGED is produced. An increase of H from OC to OJ carries B up the normal curve from D to I and the variation of H from OJ to OK causes the outer loop to be produced, and so on.

Very complicated figures can be produced if H varies in an involved manner, and this is often the case if the current is of an audio nature. Some of these actions will be discussed in later assignments.

**HYSTERESIS LOSS.**—The presence of hysteresis produces a loss in the form of heat generated in the magnetic material. (Hysteresis is not present in air, brass, or similar materials, but only in such materials as iron and steel whose permeability is greater than unity.) The loss is proportional to the area of the hysteresis loop, and occurs only when the magnetizing force H is alternating in character. Thus, hysteresis is of importance in transformers, a-c iron-core chokes,

and the like. The two adverse effects of hysteresis are:

1. Increased losses and hence reduced efficiency.
2. Increased heating and therefore increased temperature rise of the device.

The loss can be calculated by a formula developed by Steinmetz, but it is generally furnished by the manufacturer for the particular material offered, and at a certain flux density. It can then be calculated for other densities on the basis that it varies approximately as the 1.6 power of B.

The loss can be furnished per cycle (once around the curve corre-

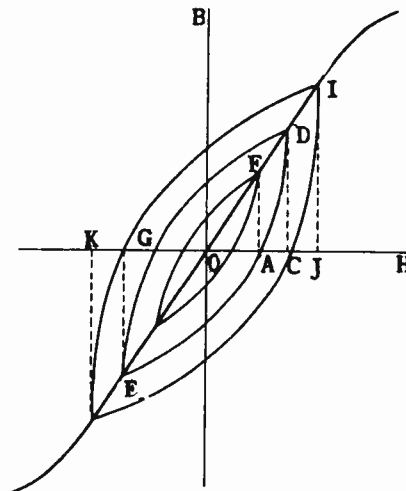


Fig. 29.—Family of hysteresis loops.

sponding to one alternation of H), or on the basis of 60 cycles per second. It also depends upon the amount of material involved, and so is given per cubic centimeter, or per cubic inch, or per pound.

The area of the hysteresis loop represents the loss for one cycle. If B is in gauss (maxwells per

sq. cm) and  $H$  is in oersteds (gilberts per cm) then 1 cubic cm is involved and the area is the loss per cycle per cu. cm. For example, from information published by the Alleghany Steel Company\* it is found that the hysteresis loss for Alleghany Transformer "C" Grade Sheet Steel is 1,815 ergs/cm<sup>3</sup>/cycle, or .658 watts/lb. for 60 cycle excitation. Measurements are at a flux density of 10,000 gausses, corresponding to 64.5 kilolines/sq. in.

The core shown in Fig. 21 contains 264 cu. in. and is operated at a flux density of 30 kilolines/sq. in. Suppose it were to operate at 25 c.p.s. The steel may be regarded as weighing 7.7 grams per cm<sup>3</sup>, or .278 lb. per cu. in. Using the latter figure, the weight of the core is found to be

$$W = 264 \times .278 = 73.4 \text{ lbs}$$

The loss for one cycle per second (for  $B = 64.5$  kilolines/in.<sup>2</sup>) is  $.658 \div 60 = .01097$  watts/lb. (For Alleghany Transformer "C" grade sheet steel.) For 25 cycle operation the loss would be  $25 \times .01097 = .274$  watts/lb. However, the flux density is 30 instead of 64.5 kilolines/sq. in. Hence the loss per pound must be reduced in the ratio of  $(30/64.5)^{1.6}$ . To evaluate, find the value of the reciprocal, and then invert. (This avoids negative logarithms.)

$$\left(\frac{64.5}{30}\right)^{1.6} = (2.15)^{1.6}$$

$$\log 2.15 = .3324$$

\*"Magnetic Core Materials Practice."

$$1.6 \times .3324 = .5318$$

$$\text{antlg } (.5318) = 3.40 =$$

$$(64.5/30)^{1.6}$$

$$1/3.40 = .294 = (30/64.5)^{1.6}$$

Hence the loss per pound at 25 c.p.s. and 30 kilolines/in.<sup>2</sup> is

$$.274 \times .294 = .0806 \text{ watts/lb}$$

and the loss for the core is  $73.4 \times .0806 = 5.92$  watts.

*TOTAL CORE LOSSES.*—There is another loss under a-c excitation produced by eddy currents in the core. It can be reduced by laminating the core. This will be discussed in a later assignment. The sum of the hysteresis and eddy current losses is called the core losses, and is also given by the manufacturer in watts/lb.

To reduce the hysteresis losses, silicon steel—steel having about 2% silicon—is normally used. Such steel also has a higher electrical resistance, so that the eddy currents are reduced and thus their losses. Note that if the resistance is doubled, the eddy currents are halved. Since the losses are  $I^2R$  in nature, they are reduced to  $1/4$  owing to  $I^2$ , and doubled owing to  $R$ , or  $1/4 \times 2 = 1/2$  as great.

In communication work, such devices as input audio transformers operate at extremely low flux densities. For these the hysteresis losses are practically negligible, and the eddy current loss is the only appreciable component of the core loss. This is true for audio transformers in general, except per-

haps at the very lowest audio frequencies. It might be supposed, on the contrary, that the losses, particularly hysteresis, might become appreciable at the very high audio frequencies, say 5,000 cycles and over. However, for a fixed impressed voltage on a transformer, the flux density varies inversely as the frequency, so that the hysteresis losses actually decrease, as the frequency is raised and the eddy current losses remain approximately constant.

One important point to note in audio transformers is that for the low magnetizing forces encountered the B-H curve shows a very low permeability. This is illustrated in Fig. 30, where the lower portion OA

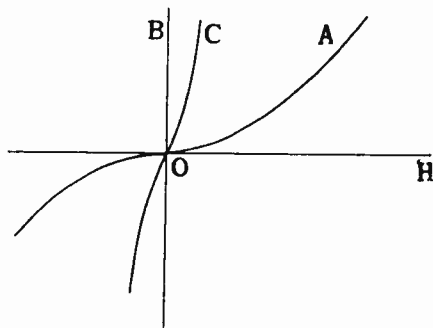


Fig. 30.—Special magnetic alloys have a much higher permeability even for low values of H than ordinary.

of the complete magnetization curve has been drawn very much magnified. It will be observed how slowly B rises at first with H. Special magnetic alloys have been developed, known as *Permalloy*, *Hipernik*, *Electric Metal*, *Mumetal*, etc. They

consist of iron and nickel (45 to 80 per cent nickel) with small amounts of chromium and molybdenum added in some cases to increase the electrical resistance and thus reduce the eddy current loss. These alloys have a very high permeability, even for very small values of H. This is illustrated by OC in Fig. 30. While these metals saturate at a lower flux density than silicon steel, they are far superior at the low levels of signal, hence magnetizing force, encountered in audio work. They also exhibit the remarkable property of having practically no hysteresis loop, hence loss, when operated below saturation.

*THEORY OF FERROMAGNETISM.*—Until a few years ago the exact nature and reason for the abnormal permeability of iron and other so-called ferromagnetic substances was not clearly understood. To-day the theory has been fairly well developed, and will be presented here very briefly.

The origin of magnetic effects is in the atom. Mention has been made in previous assignments that there are electrons in the atom that rotate in orbits around the nucleus. These orbits can be grouped into shells—thus two orbital electrons in the first shell, eight in the next with 16 as a maximum, etc. In Fig. 31 is shown an iron atom. The fourth shell is shown as a dotted circle. This is because in solid iron, the atoms are so close together that the outer shell of each becomes disrupted, and the two electrons in each become "free" electrons that can wander from atom to atom and produce electrical conductivity.

The study of the spectrum of elements has shown that the electrons not only move in orbits about the

nucleus of the atom, but also spin on their axes, very much in the same way that the earth spins on its axis as it travels about the sun.

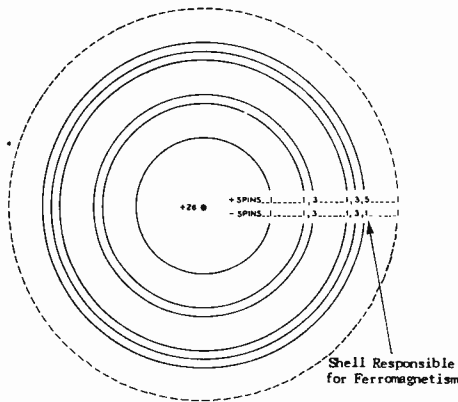


Fig. 31.—Orbits of an iron atom.

Each motion represents a current flow, and current flow produces a magnetomotive force. Where so many atoms are involved, the orbital motion of one electron is bound to be opposite to that of some other electron, so that the m.m.f.'s on the average cancel, and there is no net or resultant m.m.f. externally apparent. The same is true of all the electron spins, so that most materials do not exhibit any particular magnetic effect.

If, however, the individual little magnetomotive forces could be all aligned so that they act in a common direction, then the substance might exhibit quite strong magnetic properties. This requires a very

strong external m.m.f. corresponding to thousands of oersteds, so far as alignment of the orbits is concerned, because of the heat energy in the material at ordinary room temperatures, which vibrates the atoms so energetically that their orbits cannot ordinarily be held in alignment.

The same is true for the spin component of the orbital electrons, but here a new factor enters in that aids an external magnetizing force to align the axes of some of the orbital electrons. An examination of Fig. 31 shows that in the case of iron, there are just as many electrons spinning in one direction (called positive), as there are electrons spinning in the opposite direction (called negative), for the first two shells.

In the third shell however, it will be observed that there is an excess of four positive spins. This excess produces a net m.m.f. and makes the iron atom a little permanent magnet. Similar excess spins exist in nickel and cobalt atoms. Note that there is no friction (resistance) encountered by an electron either spinning on its axis or in travelling around its orbit, so that once these motions are established, no further electromotive force is required to maintain them.

It may therefore be expected in view of the appreciable excess spin that the iron atoms should of their own accord align themselves so that the north pole of one faces the south pole of the next, just as two compass needles will tend to align themselves. However, the heat energy in the atoms tends to disrupt even such alignment, particularly since the magnetic forces of alignment are relatively weak.

It is here that a certain elec-



trostatic force, called an "exchange interaction," comes into the picture. This force can be calculated by means of quantum mechanics, a modern and very difficult theory of atomic behavior. According to this theory, if the outer radius of the atom is 1.5 or more times the diameter of the shell in which excess spins occur, then a very strong exchange force exists which will align the spins of the various atoms in spite of the disruptive effects of heat energy. The iron, nickel, and cobalt atoms have the right ratio of radii to exhibit such strong exchange forces, and hence the spins of the electrons of the atoms become aligned to form one larger and consequently more powerful magnet.

The alignment takes place over a limited region in the material, called a "domain." Ordinarily it would seem logical to expect all the spins throughout the material, or at least each crystal of the material, to be aligned, but possibly owing to internal stresses and the like, the domain for any one direction of alignment is exceedingly small—on the order of  $10^{-8}$  or  $10^{-9}$  cu. cm.

Even so a domain will contain a million billion atoms, all aligned in some particular direction, so that the domain is saturated as far as magnetic strength is concerned. However, if the temperature (for iron) is raised to  $770^{\circ}$  C., the heat energy is so great as to overpower the "exchange interaction" and break up the domains.

Each domain lies in a certain favored direction. A true solid, such as iron, is made up of a mass of crystals. A crystal is an orderly, geometrical arrangement of atoms. In Fig. 32 is shown part of a crystal of iron. The arrangement

is that of a cube, with an atom at each corner, and one at the center. The entire crystal of iron, no matter how large, is simply a continuation

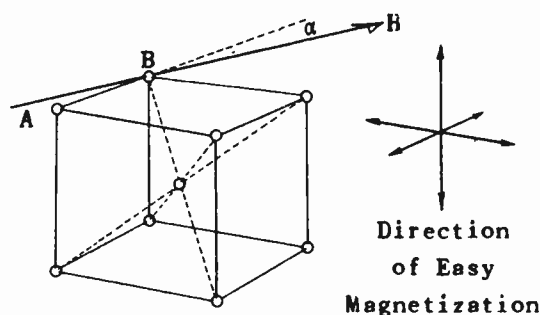


Fig. 32.—Part of a crystal of iron.

of this pattern to a larger number of atoms.

The crystals may be of various sizes in the same piece of material. Ordinarily they are too small to be seen by the naked eye, but they are nevertheless very large compared to the size of the domains within them.

It appears that in the case of iron the domains prefer to lie in directions parallel to the faces of the crystal rather than in any direction, i.e., their net m.m.f.'s lie along the crystal faces, as indicated by the arrows in Fig. 32. The resultant of all the m.m.f.'s is ordinarily zero, so that the iron appears unmagnetized, even though it is actually composed of countless numbers of very intense little magnets.

Now suppose a magnetizing force  $H$  is set up at some small angle  $\alpha$

to one direction of easy magnetization. If  $H$  is small (fraction of an oersted), then all that happens at first is that those domains pointing in the direction  $AB$  tend to grow at the expense of those pointing in other directions. The net effect is an increase in the flux, *not along the direction  $H$ , but along the direction  $AB$* . However, in ordinary samples of iron, there are so many crystals pointing in so many directions, that if the resultant of all the m.m.f.'s of all the crystals is taken, it will be found to lie along the direction of the external magnetizing force  $H$ . Only when a single large crystal of iron is artificially produced will it be found that the flux set up the single iron crystal is not necessarily in line with  $H$ .

The growth of some domains at the expense of others produces but a moderate amount of flux. This explains why for low values of  $H$  the permeability of iron is relatively low. But as  $H$  increases, a new phenomenon takes place. Domains that faced in directions other than  $AB$  begin to snap around to that direction. Each time a domain realigns itself there is a sudden, although small, increase in flux. If a coil be placed around the sample and connected to a high-gain amplifier system, a series of clicks will be heard in the loudspeaker as domains suddenly veer around. The magnitude of the clicks has been used to calculate the size of the domain. The phenomenon is known as the "Barkhausen effect," after its discoverer.

The larger values of  $H$  for which the domains veer around produces very much greater increases in magnetic flux, and in this range of  $H$

the permeability is much higher. Some samples of iron have shown a  $\mu$  as high as 600,000! When all the domains are aligned in a crystal direction closest to  $H$ , the "knee" of the magnetization curve has been reached.

Further increase in  $H$  now begins to veer the domains away from  $AB$  in Fig. 32 actually into the direction of  $H$ . This results in a further small increase in flux density (the knee of the magnetization curve). After this has been accomplished, the material acts like ordinary air as  $H$  is further increased. This action is summarized in Fig. 33, taken from the Bell System Technical Journal, p. 6, January 1940.

Hysteresis is apparently due to the fact that the domains prefer to "stick" in the position in which they happen to be. A certain amount of magnetizing force  $H$  is required

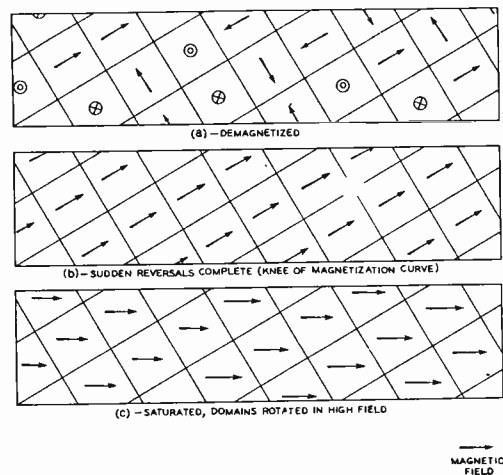


Fig. 33.—Diagram illustrating effect of an increase in  $H$  on a magnetic material.

to align them, and after they are aligned, less force is required to hold them in alignment, so that even when  $H$  is reduced to zero, there is a remanent flux that produces a permanent magnet. Note that in the case of permalloy, the domains are apparently easily pulled into alignment even for very low values of  $H$ , and that they fall out of alignment with equal ease when  $H$  is reduced to zero. Thus not only has permalloy a high permeability even for low values of  $H$ , but it also has for the same reason practically no hysteresis loss.

**PERMANENT MAGNET DESIGN.**—While hysteresis is undesirable in a-c operation, it is extremely desirable for materials to be used for permanent magnets. In Fig. 34 is shown the second quadrant portion of the  $B$ - $H$  curve, since this is pertinent

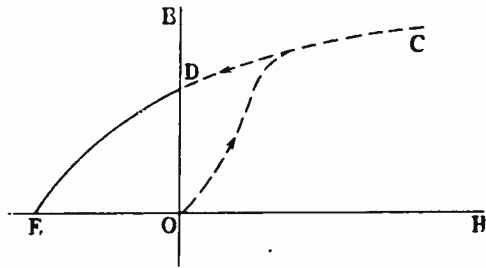


Fig. 34.—Second quadrant portion of the  $B$ - $H$  curve.

to permanent magnetism. If the steel is initially magnetized to saturation, and then the magnetizing force removed, a curve  $OD$  is obtained.

As mentioned previously,  $OD$  is

the residual or remanent flux density. In performing this test, the steel is supposed to be closed on itself or in series with a soft iron member, i.e., no air gaps are assumed to be present in its circuit.

A usual method of magnetizing a horse shoe magnet is to place it on the pole pieces of a powerful electromagnet and to magnetize it to saturation. This is shown in Fig. 35. Usually the horse shoe magnet is tapped vigorously to help its

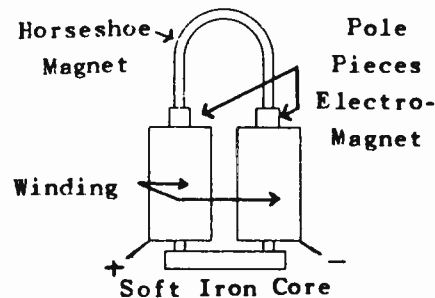


Fig. 35.—Usual method of magnetizing a horse shoe magnet.

domains align themselves. It is then slid off the pole pieces onto a bar of soft iron, called a keeper, that spans its open ends, or poles, so that no air gap is introduced. It thus acts as if it were short-circuited on itself and produces maximum residual flux, as indicated by  $OD$  in Fig. 34.

The greater the magnitude of  $OD$ , the better suited is the material for permanent magnet purposes. However, in practice it must operate under conditions where counter m.m.f.'s are present. These may be due to temperature effects, mechani-

cal vibration, or more particularly the presence of air gaps. To withstand such effects, the material must have as large a coercive force  $OE$  as possible. Hence the figure of merit (suitability) of a material as a permanent magnet is determined both by the magnitude of its residual flux density and of its coercive force.

Remember that the  $B$ - $H$  curve shown in Fig. 34 is for a cross-sectional area of 1 sq. cm (or 1 sq. in.) and for a length of 1 cm (or 1 inch). For example, for a material known as Alnico II, the residual flux density is approximately 7,200 gauss (lines/cm<sup>2</sup>) and the coercive force is 530 oersteds (gilberts/cm). If the cross-sectional area is 6 sq. cm, and the length of the magnet is 10 cm, then it furnishes on closed circuit (no air gap)  $6 \times 7,200 = 43,200$  maxwells, and has a total coercive force of  $10 \times 530 = 5,300$  gilberts.

However, practically all permanent magnets are used in conjunction with an air gap, as in a loudspeaker field or in a meter movement. The air gap has a certain reluctance depending upon its length and cross-sectional area. The effect of this reluctance is as if a counter m.m.f. or demagnetizing force were impressed on the permanent magnet. The relative magnitude of this demagnetizing force upon each centimeter of length of the steel depends upon the relative length of the steel to the air: the greater the magnet length, the less is this demagnetizing force per cm of steel.

Suppose this demagnetizing force per cm, or oersteds, is equal to  $OF$  in Fig. 36, which is similar to Fig. 34. Then the flux density will be the smaller value of  $FG$  instead of

$OD$ . By shortening the length of steel relative to the air gap, point  $F$  is moved to the left; by increasing the magnet length, it is moved to the right.

For each position of  $F$  there corresponds a certain product,  $OF \times FG$ . This is known as the energy product. For some position of  $F$  this product is a maximum, and this is the preferred position. Corresponding to it there is a certain magnet length for a given air gap in an actual design, and it will be found that for this mode of operation the volume of magnet material required is a minimum.

The maximum energy product can be found for any material simply by calculations made from its  $B$ - $H$  curve, without any reference to how operation on any part of the curve is obtained in actual practice. For some materials this product is higher than for others, and indicates the superiority of the former for permanent magnet use.

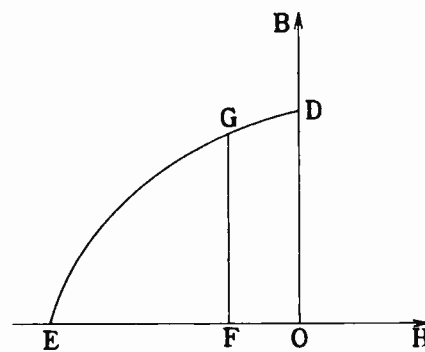


Fig. 36.—Flux density varies directly as the magnet length, assuming no change in air gap.

In the early days of the art, tool steel, hardened and tempered, was employed. Then it was found that tungsten steel was superior and it began to be used. Later on

hand, it has a much greater coercive force, so that an alnico magnet is much shorter than a cobalt magnet. In Fig. 37 are shown these characteristics for various types of al-

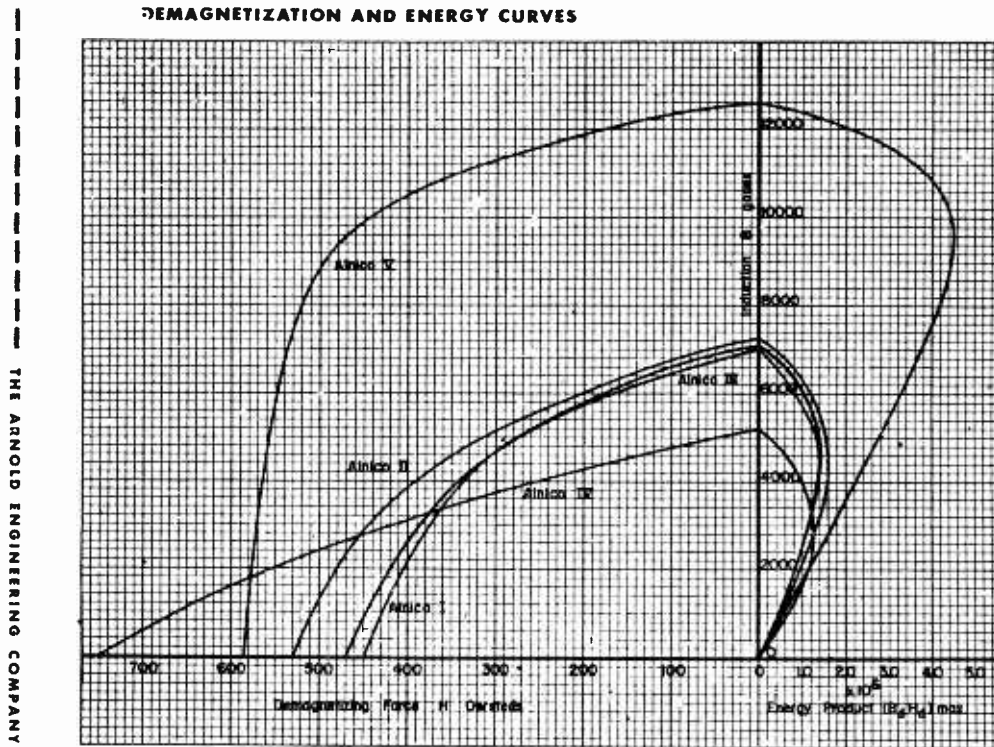


Fig. 37.—Demagnetization and energy curves for various alnico alloys.

chromium and cobalt steel were found to be even better and quite recently certain alloys, such as those containing iron, nickel, aluminum, and cobalt, and known as "Alnico," have been developed which have an exceptionally high maximum energy product—from  $1.25$  to  $4.5 \times 10^6$ , as compared to from  $.2$  to  $.92 \times 10^6$  for chromium and cobalt steels.

The residual flux density for alnico is lower than that for cobalt steel, so that in order to get a desired amount of total flux, a greater cross section is required for the alnico magnet. On the other

hand, it has a much greater coercive force of Alnico IV. This makes it particularly suitable where large demagnetizing effects are encountered, such as that owing to armature reaction in a d-c generator employing a permanent field magnet.

**STABILIZATION OF PERMANENT MAGNETS.**—In normal use, temperature variations and mechanical shock tend further to weaken the flux furnished by a permanent magnet. This is particularly undesirable in the case of a meter, as it changes its calibration.

Such decrease in flux can be artificially accelerated and the magnet thus stabilized at a somewhat lower flux density by heating the magnet for several days at a temperature in the neighborhood of the boiling point of water, or by the use of a small alternating magnetizing force, such as that produced by passing an alternating current through a coil around the magnet.

The action is best explained by means of Fig. 38. Suppose an air gap reduces the flux density from its residual value  $OC$  to the value  $AD$  by reason of the air gap corresponding to a demagnetizing force  $OA$ . Now suppose that the a-c stabilizing magnetizing force deliberately introduced, increases this counter m.m.f. from  $OA$  to  $OG$ . The flux density drops further from its value  $DA$  to  $EG$ .

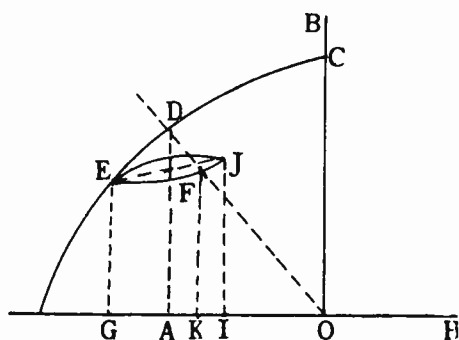


Fig. 38.—Illustrating stabilization of permanent magnets.

As the a-c magnetizing force decreases from  $G$  back to  $A$  and then to  $I$ , another hysteresis loop develops, and  $B$  continues along curve  $EFJ$  instead of along  $EDC$ . From then on a minor hysteresis loop of which

$EFJ$  is one branch, is traversed. If the a-c stabilizing force is removed, then this loop stops at  $F$ , a point on a line joining  $D$  to the origin  $O$ , and  $KF$  is the final stabilized value of flux density. (The slope of  $DO$  represents the reluctance of the air gap.)

Thereafter, any demagnetizing effect, such as vibration, temperature rise, or stray magnetic field, will operate along curve  $EFJ$  instead of  $CDE$ , and it is clear from the figure that the change in  $B$  will be much less. Moreover, if the demagnetizing effect, such as that of a stray field, is removed, the flux density will return to its stabilized value  $KF$  along curve  $EFJ$ , so that there will be no permanent change in  $B$  owing to such an effect.

As stated at the beginning of this assignment, numerous examples of the use of magnetic materials and magnetism will appear in this course. At this point one rather interesting application will be discussed, namely, the use of a varying magnetic field to deflect the electron beam in a cathode ray tube.

In television, magnetic deflection—as it is called—is generally preferred for the large picture-reproducing tubes, as well as for the camera pickup tube, such as the iconoscope. The principle upon which such deflection depends is the so-called motor rule. While this will be discussed at greater length farther on, mention of it will be made here with reference to an electron beam.

As was explained previously, a moving charge represents an electric current. In ordinary wire circuits free electrons move through the wire, and since they are (negatively)

charged particles, their motion represents a current flow. However, it is possible to have electrons moving in free space, outside of the confines of metallic conductors. This is the case in a vacuum tube (as will be discussed more thoroughly in later assignments).

In particular, in the cathode ray tube, electrons emitted from a hot electrode called the cathode, are drawn by the positive charge on a cylindrical tube or anode toward a fluorescent screen. This is illustrated in Fig. 39. Other electrodes

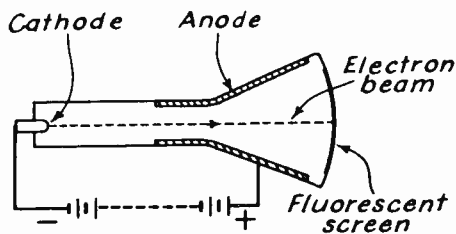


Fig. 39.—Illustration of a cathode-ray tube.

in the actual tube have been omitted in the figure for clarity.

The motion of the electrons down the tube represents a current flow, just as if they were flowing in a wire. If, now, a magnetic field is set up at right angles to their direction of flow, a force will be exerted on them at right angles both to the field and to the direction of normal motion.

This is illustrated in Fig. 40, which shows a cross section at the neck of the tube. The electrons

have been bunched into a narrow pencil or beam whose cross section appears as a circle in the figure; they are assumed to be issuing out of the

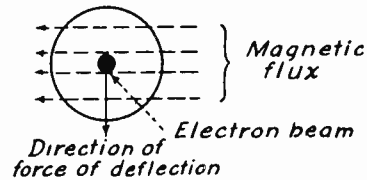


Fig. 40.—Cross-section view of cathode-ray tube.

page. The magnetic field is shown as proceeding from right to left. Under these conditions the force on the electron beam is downward, and hence perpendicular to both the flux and the direction electron motion.

The force on each electron, and therefore the total force on the beam, is proportional to the electron's velocity and to the flux density  $B$ . The greater the flux density, the greater will be the force and hence the deflection. As indicated in Fig. 41, if the flux is zero, the electrons proceed axially and strike the fluorescent screen at A. If the flux is from right to left (Fig. 40), the electrons are deflected downward and strike the screen at B. If the flux is directed from left to right, the force is upward, and the electrons strike the screen at C.

The path is altered only in the region where the flux passes through the tube; in this region the electron path or trajectory is an arc of

a circle, as can be demonstrated by a mathematical analysis. The trajectory up to this arc of a circle, and the trajectory from the arc to the screen are both straight lines tangent to the arc, as indicated in Fig. 41.

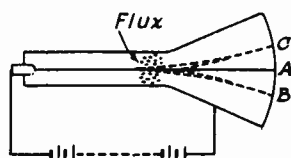


Fig. 41.—Flux paths in a cathode-ray tube.

The magnetic flux is set up by a pair of pancake coils located on either side of the neck of the tube. These provide vertical deflection. Another pair of pancake coils are set above and below the neck at this same point; these coils produce horizontal deflection. This follows from the right-angle direction of the force with respect to the direction of the magnetic field. The coil positions are indicated in Fig. 42(A).

In Fig. 42(B) are shown the flux paths for the two horizontal-deflection coils. (The vertical-deflection coils have been omitted for clarity.) The coils may be regarded as very short solenoids, and produce an m.m.f. acting through their enclosed areas. Generally the coils of each pair are connected in series. If the current traverses them in one direction, flux is set up in a down-

ward direction, as shown; if the current is reversed, the flux is set up in the reverse (upward) direction.

An alternating current will produce a corresponding alternating flux, this in turn will produce an

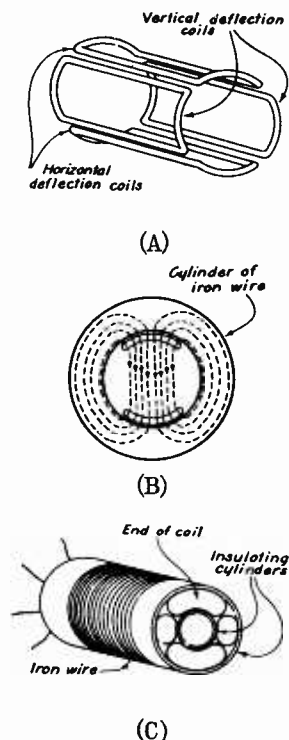


Fig. 42.—Deflections coils in a cathode-ray tube.

alternating force and hence an alternating deflection. In this way the beam may be made to sweep back and forth across the fluorescent screen in a scanning motion. Specifically, if the current has a saw-tooth wave-shape, the beam will move uniformly with time across the screen in one direction (forward stroke), and then snap back quickly on the return stroke.

It will be observed that the flux must traverse the glass walls and the enclosed space or vacuum within the tube. The permeability



of this region is of necessity the minimum value, namely unity, and hence the reluctance is high. The flux then curls around, as shown in Fig. 42(B) to form closed loops. This return path is also through air unless a ring of iron is provided to surround the coils and thus furnish a low-reluctance return path.

Such a ring or yoke of iron is usually provided. Owing to the alternating (sawtooth) character of the deflection current and flux, it is necessary to laminate the iron path in order to minimize the eddy-current loss (to be explained subsequently). The method employed is to wind iron wire around the coils, as shown in Fig. 42(C). The coils are placed within two concentric insulating cylinders of which the in-

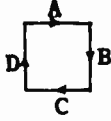
ner one just fits over the neck of the tube. The iron wire is then wound over the outer cylinder, and then a protective covering is provided. The entire assembly is then known as a deflection yoke; its use provides an interesting example of the application of magnetism to television. Other applications will appear at appropriate places in the course.

*CONCLUSION.*—This concludes the discussion of the general theory of magnetism. Numerous further applications of magnetism will be discussed at the appropriate points in subsequent assignments, and this assignment will be found to contain the basic material for the understanding of those future analyses.

## MAGNETIC CIRCUITS

### Exercise Problems.

1.



Given the current flow in the rectangular loop shown. Sketch the circles of flux and their direction, close to the wire at points A, B, C, and D.

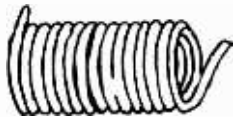
2.



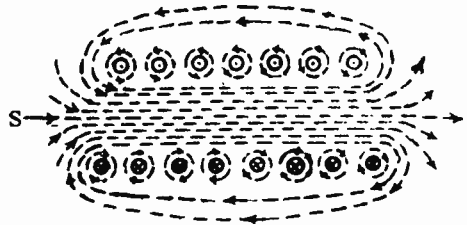
A coaxial cable consists of a conductor that is surrounded by a return conductor in the form of a tube, as shown in the accompanying cross-sectional sketch.

Assume current flows into the paper in the center conductor, and returns out of the paper in the outer tube. For regions outside of the outer tube, it acts magnetically as if its current were concentrated at the center of the cross section. Within the tube, it acts as if no current were flowing in the outer tube, i.e., it produces no magnetic field. This leaves the inner conductor to produce the field between the two, whereas for the outer region, both conductors are effective and act independently of one another.

Sketch the field in the region between the inner and outer conductors, and in the region external to the outer conductor.



(A)



(B)

Fig. 11.

3. Suppose the current enters the right-hand side of the solenoid shown in Fig. 11(A), and leaves at the left-hand end. Draw a sketch similar to Fig. 11(B) for this direction of current.

4. Consider anyone turn of a solenoid carrying a current. What forces are set up in the turn owing to the flux passing through it?

5. What forces are set up between adjacent turns of the solenoid?

MAGNETIC CIRCUITS

6. A coil in which 5 amperes flows is made up of 145 turns. Express the mmf in ampere turns and in gilberts.

7. The field coil of a dynamic loud speaker consists of 7,000 turns around the iron core. The current through the coil is 80 milliamperes. Express mmf in ampere turns and in gilberts.

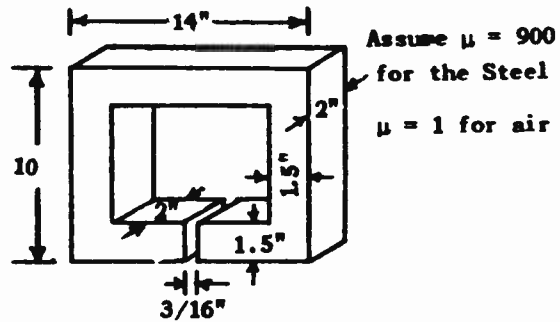
8. A magnetizing force of 740 gilberts is desired in the field coil of a dynamic speaker. A magnetizing current of 110 milliamperes is available. How many turns will be required on the coil?

9. A magnetizing force of 480 gilberts is required. The magnetizing coil contains 1500 turns. What current is required?

10. A generator field coil contains 1500 turns and requires a current of 2.4 amperes. Express the mmf in ampere turns and in gilberts.

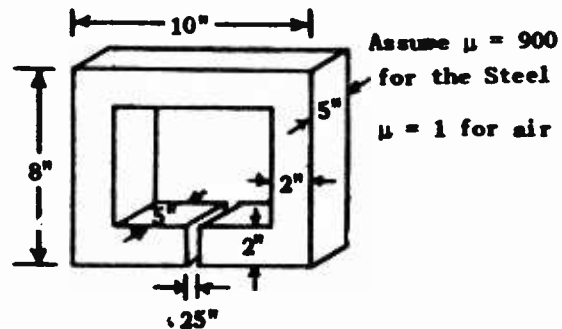
11.

Find the reluctance of the iron path, the air gap, and the total circuit.



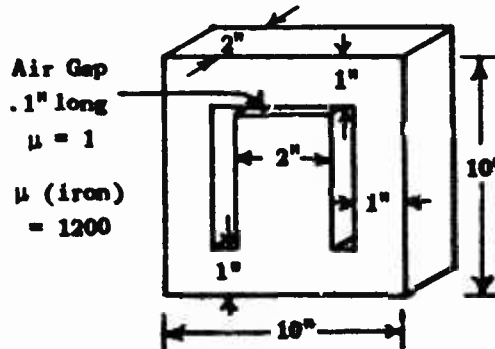
12.

Find the reluctance of the iron path, the air gap, and the total circuit.



13.

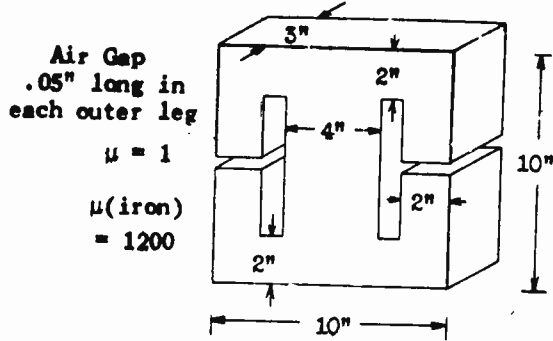
What is the reluctance of the total circuit?



MAGNETIC CIRCUITS

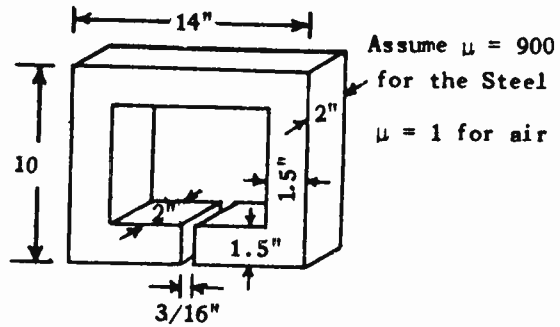
14.

Find the total reluctance.



15.

What will the flux be in the air gap, if a coil of 1000 turns is wound on the core and 200 ma current flows in the coil?

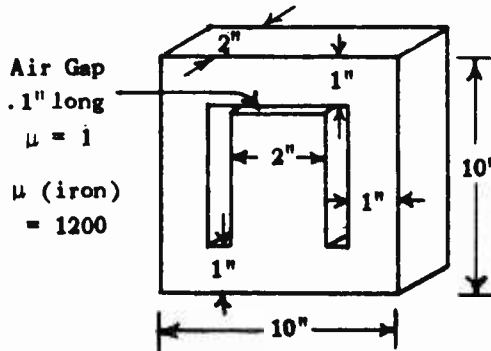


16.

What mmf is consumed across the air gap in problem 15?

17.

A coil of 5000 turns is placed on the center leg. Using a current of 100 ma, find the flux in the air gap, and in the outer legs.



18.

What is the flux density in the outside legs, in the air gap, in the center leg, of problem 17?

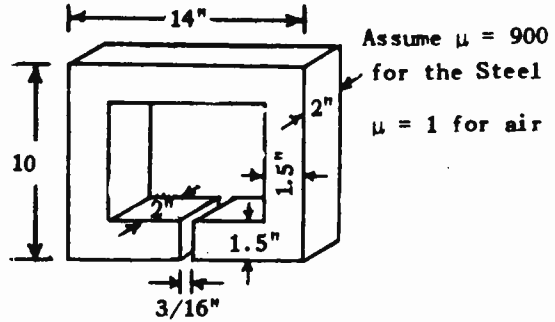
19.

A sample of silicon steel should not operate with a flux density exceeding 80,000 lines per square inch. A flux of 380,000 maxwells is required. What must be the area of the core?

MAGNETIC CIRCUITS

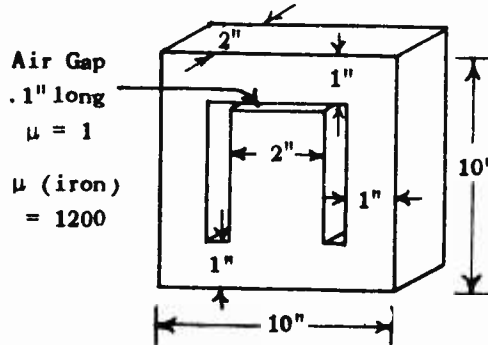
20.

Calculate  $H$  for the air gap and for the iron path. In each case take the total mmf required for that path and divide it by the length of that path. Use a coil of 1000 turns and current of 200 ma.



21.

Use the same method as problem 20 and calculate  $H$  for the air gap and for either iron path. Use a coil of 5000 turns and a current of 100 ma.



22.

Use standard sheet steel. (See pages 33-35). Find the total IN required to give  $B = 30,000$  lines per square inch. Find the number of turns needed for 100 ma current.

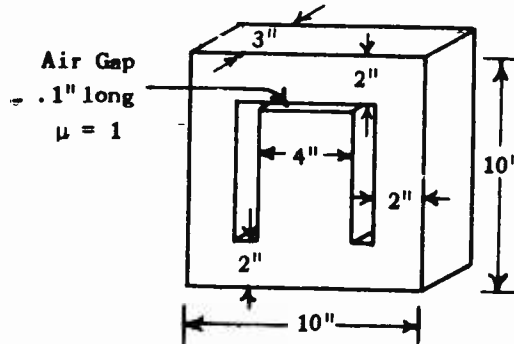


Fig. 21

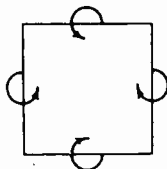
23.

Calculate the permeability of silicon steel for  $H = 2$ ,  $H = 5$ ; and  $H = 40$  IN per inch, using figure 27 of the text.

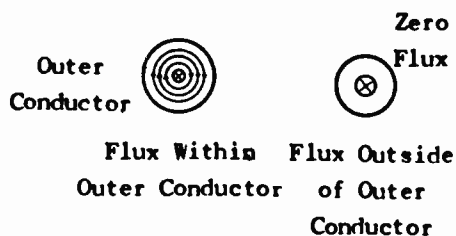
## MAGNETIC CIRCUITS

### ANSWERS TO EXERCISE PROBLEMS.

1.



2. Outside of the outer conductor, the latter acts as if it were a line conductor concentrated at the center, and having current flowing *out* of the paper. This current equals that flowing *into* the paper in the inner conductor.



Hence their magnetic effects cancel for all points outside of the outer conductor, but inside of the outer conductor the inner conductor alone produces flux up to the walls of the outer conductor as if it were all alone in free space. Hence the circles of flux shown.

3. This sketch will be exactly the same as Fig. 11 (B) except that the flux lines within the solenoid will point from right to left. This merely means that the arrow heads have to be reversed. Also the top circles representing conductor cross-sections will have crosses, and the bottom circles will have dots, since the direction of current flow has been reversed.

4. Forces tending to expand the turn owing to the lateral repulsion between the flux lines passing through it.

5. Forces of attraction because of tendency for encircling flux lines to contract.

6. 725 IN or 911 gilberts.

7. 560 IN or 704 gilberts.

8. 5,352 turns.

MAGNETIC CIRCUITS

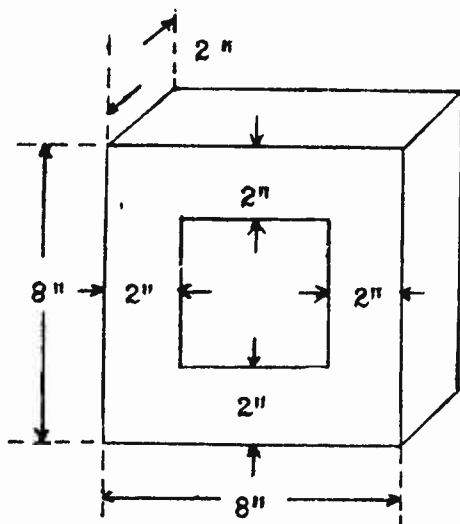
ANSWERS TO EXERCISE PROBLEMS, Page 2

9. .255 ampere.
10. 3,600 IN or 4,525 gilberts
11.  $R_1 = .00612$  unit.  
 $R_a = .0245$  unit.  
 $R_t = .0306$
12.  $R_1 = .001211$  unit.  
 $R_a = .00984$  unit  
 $R_t = .01105$  unit.
13.  $R_1 = .011983$  unit.
14.  $R_t = .002241$  unit.
15.  $\phi = 8,200$  maxwells.
16. 201 gilberts.
17. 52,450 maxwells in air gap.  
26,225 maxwells in each outer leg.
18.  $B = 2,033$  gauss in air gap and in outer legs.
19. 4.75 sq. in.
20. 422 oersteds (air gap).  
.472 oersted (iron circuit).
21. 2,033 oersteds (air gap).  
1.685 oersteds (iron path).
22. 985 ampere turns total.  
9,850 turns for current of 0.1 ampere.
23.  $\mu$  (at 2IN/inch) = 7,049.  
 $\mu$  (at 5 IN/inch) = 4,296.  
 $\mu$  (at 40 IN/inch) = 713

SOLUTION OF PROBLEM 8  
MAGNETIC CIRCUITS

The hysteresis loss of the transformer shown is .642 watts/lb for 64.5 kilolines per sq. in. and 60 c.p.s. Suppose the transformer is used at 100 cycles per second, and the flux density is 20 kilolines per sq. in. (a) Calculate the number of pounds of iron in the core of the transformer shown.

Overall volume =  $8 \times 8 \times 2 = 128$  cu. in.  
Volume of windows =  $4 \times 4 \times 2 = 32$  cu. in.  
Volume of iron core = 96 cu. in.  
Density of iron = .278 lbs. per cu. in.  
 $96 \times .278 = 26.7$  lbs. of iron in transformer



(b) What is the hysteresis loss in watts per lb. at 100 c.p.s. and 20 kilolines per square inch?

Since the hysteresis loss is directly proportional to the frequency, then,  $.642$  watt/lb  $\times 100/60 = 1.07$  watts/lb for 100 cycle operation at 64.5 kilolines per square inch.

Since hysteresis loss varies as the 1.6 power of the flux density, then the flux density of 20 kilolines will be

$$\left(\frac{20}{64.5}\right)^{1.6} = .31^{1.6}$$

$$\log .31 = -1 + .4914 = -.5086$$

$$(.31)^{1.6} = 1.6 \times -.5086 = -.81376 \text{ or } -.8138 \text{ or } -1 + .1862$$

$$\text{Antilog} (-1 + .1862) = .1535$$

Therefore the hysteresis loss at 20 kilolines per square inch is .1535 as great as for the same frequency at 64.5 kilolines per sq. inch. Then, if the loss is 1.07 watts per pound at 64.5 kilolines and 100 cycles per second, it follows that the power loss is  $.1535 \times 1.07 = .1642$  watt/lb.

(c) What is the hysteresis loss in watts for the transformer shown?

From 8 (a) there were 26.7 lbs of iron in the transformer.

From 8 (b) the iron has a hysteresis loss of .1642 watt/lb under the specified conditions.

Therefore, the total hysteresis loss of transformer shown is  $.1642 \times 26.7 = \underline{4.38}$  watts.



## MAGNETIC CIRCUITS

9. (a) Low initial permeability in an iron core is produced by the domains lying in direction close to that of the external magnetizing force  $H$ , growing at the expense of domains lying in other directions.

(b) As  $H$  is increased, the domains lying in other directions turn or swap around into directions parallel to those of the first mentioned domains (i.e. in directions of each magnetization close to that of the external magnetizing force). This produces a much greater increase in  $B$  for a given increase in  $H$  and hence a greater permeability.

(c) After this has been accomplished, further increase in  $H$  begins to turn the domains from the direction of easy magnetization closest to the direction of  $H$  into directions actually parallel to  $H$ . This produces a further increase in  $B$ , but not a very large one. When all the domains are in the direction of  $H$ , further increase of the latter merely increases  $B$  as if the permeability of the material were now merely unity.

10. Since the permanent magnet in this problem is to be used in conjunction with an air gap, the reluctance of the air gap offers the same effect as a demagnetizing force. The effect of the demagnetizing force may be compensated for by using a steel of high coercive force or by increasing the length of the magnet. In this problem the length is limited so we must choose the steel having the greatest coercive force. Fig. 37 indicates the least slope for Alnico IV on its demagnetizing slope and therefore the greatest coercive force. Although all the other steels have greater residual flux densities at nearly all points, this is no consideration here since the problem states that the area of the magnet is unlimited and will thus permit us to attain the total flux desired by increasing the area by the necessary amount.

MAGNETIC CIRCUITS

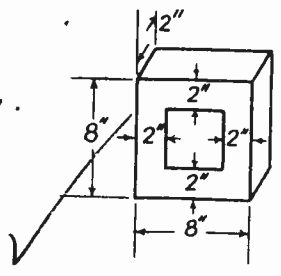
EXAMINATION, Page 7

8. The hysteresis loss of the transformer shown is .642 watts per pound for 64.5 kilolines per sq. in. and 60 cycles per second. Suppose the transformer is used at 100 cycles per second, and the flux density is 20 kilolines per sq. in.

(A) Find volume and weight of iron.  
Use .278 lbs/cu. in.

Volume =  $8 \times 8 \times 2 - (4 \times 4 \times 2) = \underline{96 \text{ cu. in.}}$

Weight =  $96 \times .278 = \underline{26.69 \text{ lbs.}}$



(B) Find loss/lb.

loss for one cycle =  $\frac{.642}{60} = .0107 \text{ watts/lb.}$

at 25 cycles loss =  $25 \times .0107 = .2875 \text{ watts/lb.}$

for 20 kilolines loss =  $.2875 \times \left(\frac{20}{64.5}\right)^{1.6} = .2875 \times \left(\frac{1}{3.22}\right)^{1.6}$

=  $.2875 \times \frac{1}{6.495} = \underline{0.044 \text{ Watts/lb.}}$

(C) Find the total loss.

Total loss =  $0.044 \times 26.69 = \underline{1.174 \text{ Watts}}$

*But you are giving some loss for transformer at 100 cycles!*

*check solution*

## MAGNETIC CIRCUITS

EXAMINATION, Page 8.

9. (a) What produces the low initial permeability in an iron core, i.e., when it is subjected to a weak magnetizing force  $H$ ?

The iron is composed of various "domains" having  $mmf$ 's in all directions so the net  $mmf$  is zero. As a low magnetizing force is applied, the domains having  $mmf$ 's in the approximate direction of the magnetizing force are aligned in that direction thus causing a moderate increase of permeability.

- (b) Why does the permeability of the core increase as  $H$  is increased?

As the magnetizing force is increased it approaches a value sufficient to snap around domains having  $mmf$ 's even in the opposite direction to  $H$  thereby greatly increasing the permeability.

- (c) What produces the "knee" in the magnetization curve?

After a certain value of  $H$  is reached all possible domains have been aligned so any increase of  $H$  has very little effect on the permeability.

MAGNETIC CIRCUITS

EXAMINATION, Page 9.

9.

- 10. It is desired to use a permanent magnet of as short a length as possible in conjunction with an air gap of a certain length for magnetic focusing in a television picture tube. Suppose there are no limitations on the cross-sectional area in this particular problem. Which of the alnico steels whose characteristics are shown in Fig. 37 is best suited for this application? Why?

If the demagnetizing force of the air gap is less than 600 Oersteds, Alnico V would be best suited due to its high energy product.

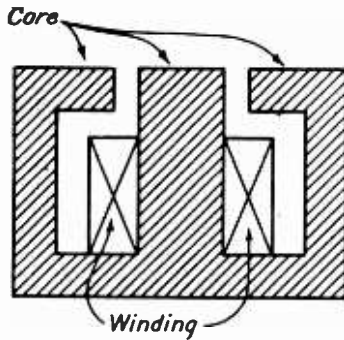
But you don't know how little or how great the demagnetizing force is going to be. With a very small magnet there will be a very great demagnetizing force.

See our discussion

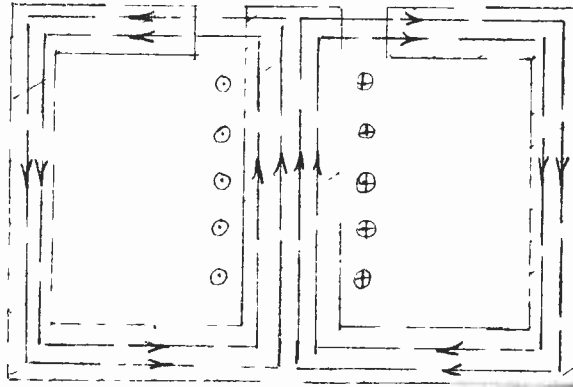
MAGNETIC CIRCUITS

EXAMINATION

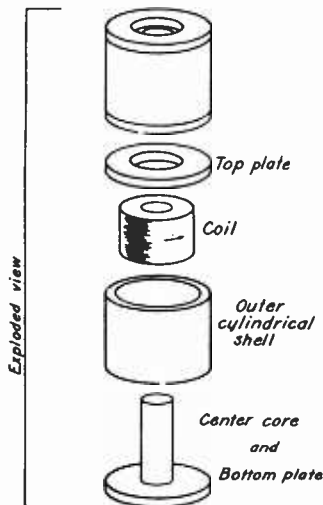
1.



A core of soft iron of a loudspeaker has the shape shown. A coil is wound around the central leg, as indicated by the two rectangles in the figure. In each turn of the coil, current flows out of the paper on the left-hand side, and into the paper on the right-hand side. Sketch the magnetic flux lines, and show their direction in each part of the core.



2.



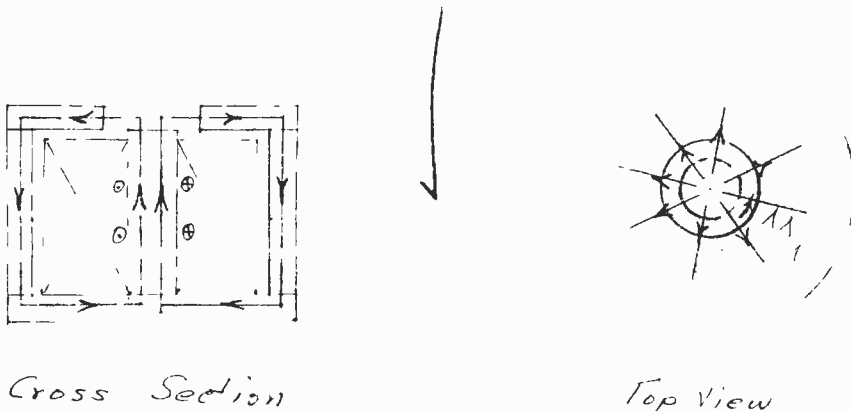
The figure represents the field magnet of a dynamic loudspeaker. The coil is wound around the center core or leg, and is inclosed by the outer cylindrical shell. The exploded view shows the details of the assembly. Current is assumed to flow in the direction shown by the arrow on the coil. Draw a cross-sectional side view and a top view of the structure, and in each view show the direction of the flux.

*HINT.*—Note that this structure is exactly what would be produced if the structure of Problem 1 were rotated on a vertical axis through the center leg.

MAGNETIC CIRCUITS

EXAMINATION, PAGE 2.

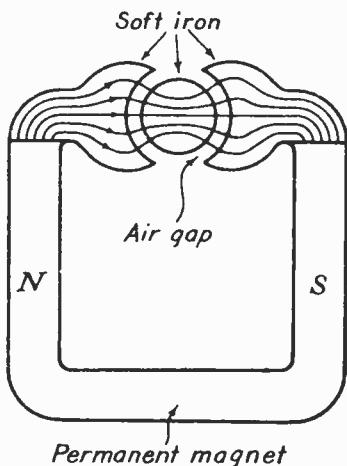
2.



Cross Section

Top View

3.



The figure represents the magnet field structure of a D'Arsonval meter movement. The air gap between each pole piece and the center soft iron cylinder is of constant length measured along any radius of the center cylinder.

Why do the field lines near the upper and lower tips of the pole pieces proceed radially inward to the center cylinder rather than horizontally across from one pole tip to the other.

NOTE: The above magnetic field pattern produces a uniform scale in this type of meter.

They proceed to the cylinder because the shortest air gap is the path to it, and they go toward the cylinder's center because all lines leave and enter iron at right angles to the surface of the iron.

OK. — However, lines of flux take the path of least reluctance. Since air gap offers less reluctance, flux

lines proceed radially inward

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3.

4. Two coils of 300 turns each are wound on the same cylindrical form so that the turns of one fit in between those of the other coil. Assume the axis of the two coils is horizontal. In coil A, one ampere flows into the paper in the top coil sides and out of the paper in the bottom coil sides. In coil B, .3 ampere flows in a direction opposite to that in A. What mmf is set up by this system in ampere turns and in gilberts?

NOTE: This illustrates the action of a differential relay.

$$\begin{aligned}
 \text{mmf} &= (300 \times 1) - (300 \times .3) \\
 &= 300 - 90 = \underline{\underline{210 \text{ IN}}} \\
 &= 210 \times 1.257 = \underline{\underline{263.97 \text{ Gilberts}}}
 \end{aligned}$$

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5. A piece of brass is placed between the pole pieces of a permanent magnet. It is .1 cm long, 5 cm wide, and 4 cm high. The flux lines are parallel to the .1 cm dimension. What is the reluctance of this piece of brass?

$$R = \frac{L}{\mu A} \quad \mu \text{ of Brass} = 1$$

$$R = \frac{.1}{1 \times 5 \times 4} = \frac{.1}{20} = \underline{\underline{0.005 \text{ units}}}$$

6. The above volume is now made up of iron for .05 cm length ( $\mu = 2,000$ ) and the remaining .05 cm consists of brass. What is the total reluctance of this volume now?

$$R = \frac{.05}{1 \times 5 \times 4} + \frac{.05}{2000 \times 5 \times 4}$$

$$= 0.0025 + 0.00000125$$

$$= \underline{\underline{0.00250125 \text{ units}}}$$



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## EXAMINATION, Page 5

7. Given a magnetic circuit composed of Silicon steel as shown, and having a winding as shown, suppose it is desired that the flux density shall be 36 kilolines/sq. in.

(A) From Fig. 27 find the IN/in.

$$IN/in \text{ for steel} = 1.45 \quad \checkmark$$

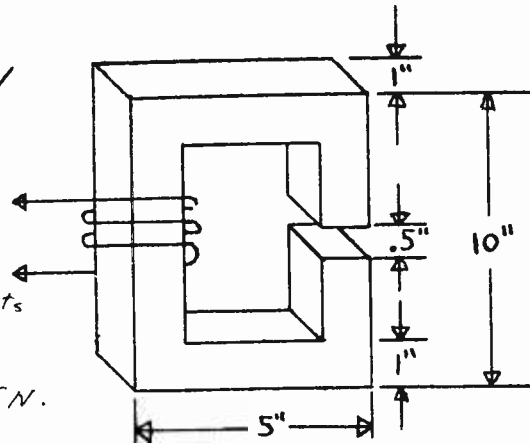
$$\bar{I}N = 1.45 \times 26 = 37.7 \text{ IN}$$

$$R(\text{air gap}) = \frac{.5 \times 2.54}{1 \times 1 \times (2.54)^2} = .196$$

$$\begin{aligned} \text{MMF of air gap} &= 36,000 \times .196 = 7056 \text{ gilberts} \\ &= \frac{7056}{1.257} = 5613 \text{ IN.} \end{aligned}$$

$$\text{Total } \bar{I}N = 5613 + 37.7 = 5651 \text{ IN.}$$

$$\text{or } \frac{5651}{26} = \underline{\underline{217 \text{ IN/in}}}$$



(B) Compute the required current which can be passed through the winding for this flux density.  $N = 6000$

$$\bar{I}N = 5651$$

$$N = 6000$$

$$\bar{I} = \frac{5651}{6000}$$

$$= 0.925 \text{ amp.} \quad \checkmark$$

(C) Assume that there is now no air gap present—compute the required current which can be passed through the winding for a flux density of 50 kilolines/sq. in.  $N = 6000$

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7. (Cont'd.)

From Fig. 27

50 kilolines/sq.in. require 2.35 IN/in.

$$\text{Total } IN = 2.35 \times 26 = 61.1 \text{ IN}$$

$$N = 6000 \text{ turns}$$

$$I = \frac{61.1}{6000} = .0102 \text{ amp.}$$

or 102 ~~amps~~ <sup>m.a.</sup>  
✓