SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

GRAPHICAL ANALYSIS

714D

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GRAPHICAL ANALYSIS

FOREWORD

In every radio text, in almost every technical radio article in magazines, in data sheets on vacuum tubes, in instruction books on radio apparatus, one encounters "curves". A vacuum tube's operating characteristics are expressed by a set of "characteristic curves". The relationship between a condenser's capacity and dial settings is shown by means of a "calibration curve". The directional characteristics of an antenna array are shown by means of a "field strength pattern". The color response of a television iconoscope is shown by means of a "color spectrum curve".

The Chinese say, "One picture is worth a thousand words." Radio engineers believe the same thing about curves. An engineer may spend hours or days in careful measurements and calculations and collect rows of figures which possibly would require hours of laborious study to interpret. He then plots the figures on graph paper and draws in the curves—and ends up with a graphical presentation which any other informed engineer can interpret at a glance!

While curves are among the most powerful tools of the engineer, their value lies in his ability to interpret them. Thus, he must learn how to plot curves to show the facts he wishes to present. He must understand their limitations as well as their possibilities and he must learn the "tricks of the trade" so that he can draw his curves to display his data to best advantage.

Plotting and interpretation of curves form an art in which anyone who would call himself an engineer *must* become proficient. An accurately drawn curve is a work of engineering art and often is more valuable than pages of written data.

E. H. Rietzke,
President.

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TECHNICAL ASSIGNMENT

GRAPHICAL ANALYSIS

GRAPHICAL ANALYSLS

To the radio engineer the subject of graphical analysis is an important one. There are various types of calibration curves, characteristic curves of vacuum tubes, performance curves for receivers, transmitters, antennas, etc., all of which the engineer must be able to construct or read in an intelligent manner.

A graph is a pictorial representation, usually in the form of a smooth curve, showing the simultaneous relationship between any two variables. It should be understood that the word "curve" used in this assignment may refer to a "straight line"; that is, a straight line is simply a special type of curve. For example, it is customary to speak of the *straight* portion of a tube characteristic curve although the strict definition of the word "curve" does not admit the existence of a straight portion.

Curves are plotted by knowing a number of simultaneous relationships between two variables. Before plotting a calibration curve for a frequency meter, in which frequency is varied by means of a variable condenser, it is first necessary to measure the frequency of the meter at a number of pointe over the condenser dial, tabulating frequency against dial divisions.

After obtaining a number of simultaneous values it is necessary to arrange them on paper so they will show graphically the relation between the two variables *at any point* between the upper and lower limits of the plotted range. Curves are usually plotted on some form of graph paper. Paper divided similarly to that of Figure 1 is called linear cross-section paper. Special types of graph paper will be discussed later in this assignment.

RECTANGULAR COORDINATES: When a sheet of graph paper is prepared for plotting a curve it is referred to as the "plane of reference." The arrangement of the paper depends on the data to be plotted. Figure 1 shows the paper arranged for four quadrant plotting. The line XoX' is called the horizontal or X-axis. YoY' is the vertical or Y-axis. Distances measured along or parallel to the X-axis are called *abscissae* while distances measured along or parallel to the Y-axis are called *ordinates*. Thus abscissae are often called X values,

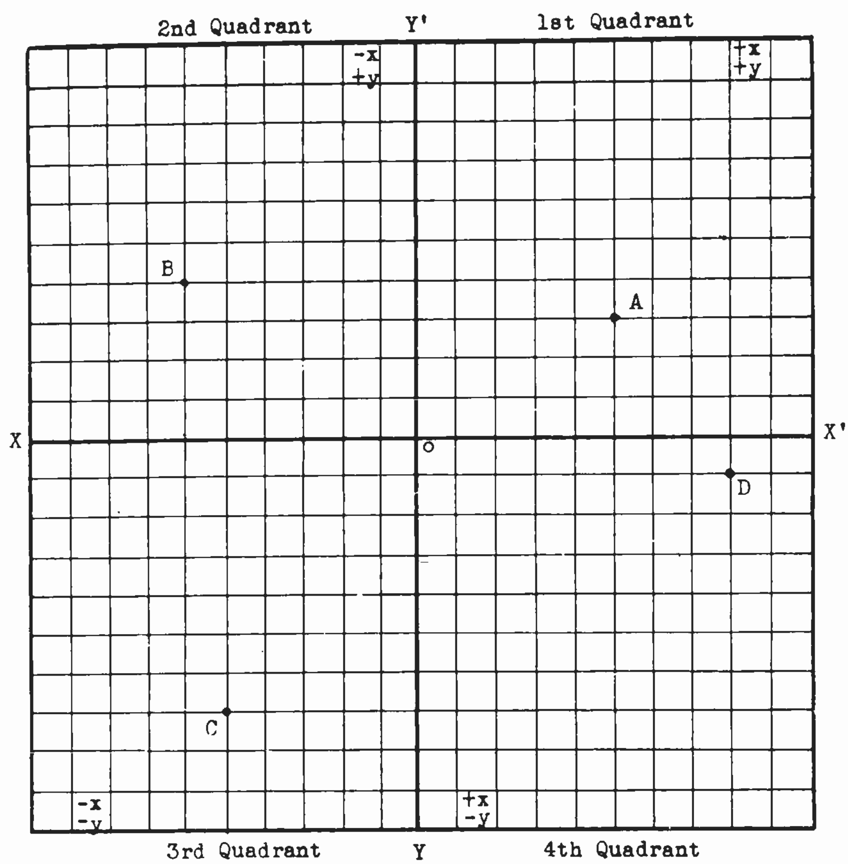


Fig. 1.

and ordinates Y values.

RULE: The abscissa of a point is the distance of the point from the Y-axis as measured along the X-axis. The ordinate of a point is the distance of the point from the X-axis as measured along the Y-axis.

The simultaneous dimensions of the related variables at any instant are classed together as the *coordinates* of the point. Locating a point on the plane of reference by means of the abscissa and ordinate of the point is called the rectangular coordinate or Cartesian system.

Referring to Figure 1, point o at the intersection of the X and Y axes is called the point of origin or pole. Any point located at this intersection has zero abscissa and ordinate. The plane of reference may consist of 1, 2, 3, or 4 quadrants depending on the data to be plotted. When a four quadrant curve is to be plotted the quadrants are numbered in counter-clockwise order as shown in Figure 1. Positive abscissae are plotted along the X-axis *to the right of point o.* Negative abscissae are plotted *to the left of point o.* Positive ordinates are plotted along the Y-axis *above point o* while negative ordinates are plotted along the same axis *downward* from point o. A point located in the first quadrant has a positive abscissa and ordinate; in the second quadrant the abscissa is negative and the ordinate positive; in the third quadrant abscissa and ordinate are both negative; in the fourth quadrant abscissa is positive and ordinate negative. For example, point A Figure 1 is 5 units to the right and 3 units upward from point o, or its abscissa is +5 and its ordinate +3. In mathematical notation this is written A (5,3) the first number in parentheses always being the abscissa of the point. The coordinates of point B are (-6,4), of point C (-5,-7) and point D(8,-1).

It is not always necessary that a four quadrant plane of reference be used, in many cases only one or two quadrants are required. If all coordinates are positive only one quadrant need be used. Two quadrants are required when one variable is always positive while the other moves through a range of positive and negative values. Figure 2 is an example of single quadrant plotting, with λ (wavelength) plotted against dial divisions. Two quadrant plotting is used in Figure 3 with I_p (plate current) plotted against E_g (grid voltage). I_p is always positive but E_g varies in a positive and negative direction from the point of origin.

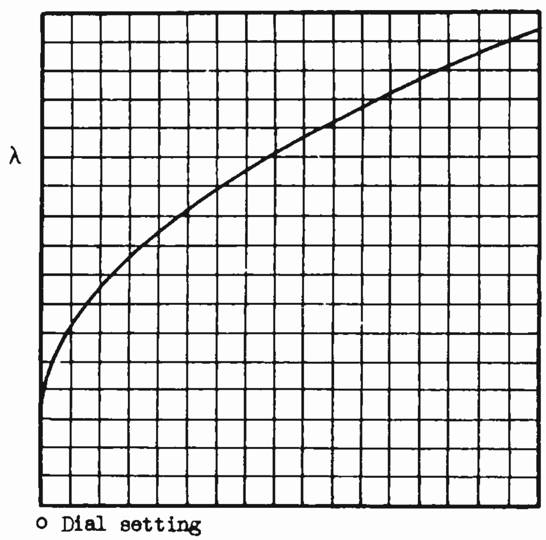


Fig. 2.

A careful inspection of Figures 2 and 3 will demonstrate one point clearly. While the amount of curvature is different at various points, yet both curves are "smooth." The variation between points is regular with no sharp irregular breaks in the curve. This is the condition usually found in nature and is to be expected in plotting a curve. Any abrupt change in the curve during plotting should be viewed with suspicion. There is a strong possibility that the measured data at that point is in error and should be verified before plotting. If only one point appears out of line it is reasonably safe to disregard that particular measurement. In the calibration curve of a frequency meter, where frequency is varied by means of a variable condenser, so long as there are no defects in the condenser, the plotted curve should be smooth. Any irregularities that do appear are almost invariably due to errors in measurement. One of the most common errors in measurement is parallax, caused by reading the dial setting at an angle other than perpendicular to the dial face. Unless a large scale vernier dial is used considerable experience is necessary before accurate readings can be taken.

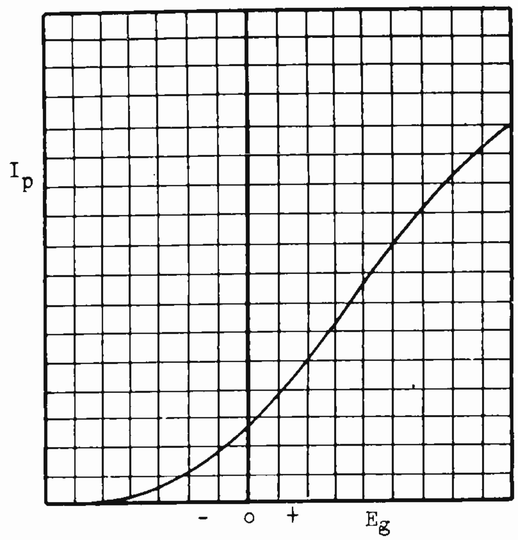


Fig. 3.

There is always the possibility that some factor may disturb the symmetry of a curve and therefore *all* abrupt changes in the curve cannot be traced to errors in measurements. In individual vacuum tube characteristic curves it is not unusual to find irregular points caused by some imperfect condition in the tube. The gas content of the tube is the principal factor in this departure from normal. The frequency versus temperature variation of a Y-cut crystal gives a rather odd type of curve in that frequency varies smoothly for several degrees change in temperature and then makes an abrupt change followed by another smooth variation. Plotted over a range of several degrees the curve appears somewhat like that shown in Figure 4.

There are certain types of measurements that are difficult to make because of adverse conditions under which the measurements are made or the difficulty in making accurate measurements with available equipment. As an illustration, when making observations of the current distribution in a vertical antenna the observer and measuring equipment are hoisted up the tower. The observer is working under adverse conditions and it is difficult to maintain constant coupling between the measuring equipment and the antenna. These factors prevent accurate observations being made but by making a large number of measurements and plotting the points as shown in Figure 5 a quite accurate curve can be obtained. No attempt is made to draw the curve through all points but rather the curve is made to represent the best possible *average* of all the points.

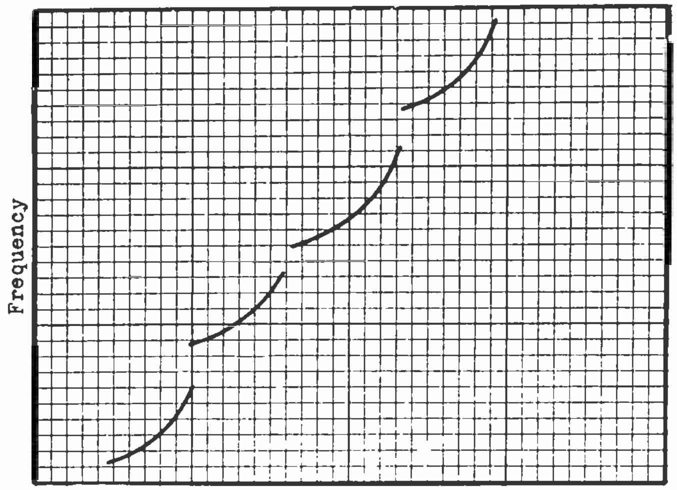


Fig. 4.

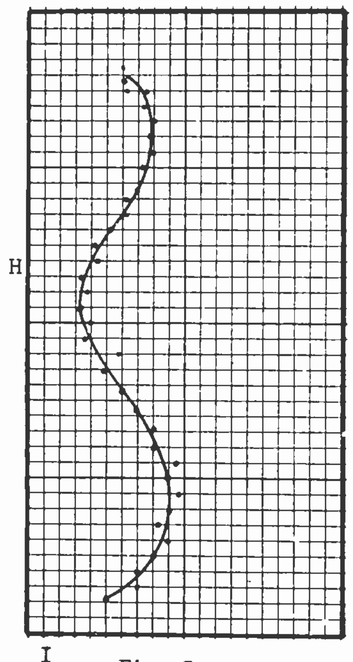


Fig. 5.

CONSTRUCTING A GRAPH: If a curve is to be read accurately it must be plotted to the proper scale. If the scales are cramped the rate of change of one variable with respect to the other may be so rapid as to make it virtually impossible to read the curve accurately. Conversely, a steep curve can be made to appear flat by cramping one scale and expanding the other. Considerable ingenuity is required in selecting the proper scales for plotting a given range of values.

Whenever one quantity, Y, varies with another quantity, X, in some definite way, Y is said to be a function of X. The graph of Y against X will show the type of function, whether it is linear, power, exponential, etc. Y is regarded as varying with X in some definite way; or, for every value of X there is a corresponding value of Y in accordance with some law or agreement. The quantity X, on which Y depends is called the *independent* variable. X is regarded as running freely through its values while Y, the *dependent* variable, must follow the variations in X. *It is standard practice* to plot the independent variable along the horizontal or X-axis and the dependent variable along the vertical or Y-axis. The independent variable is determined by inspection. For example, $P = I^2R$. P is a function of I^2 and R and if plotted against either would be the dependent variable. In the formula, $\lambda = 1884 \sqrt{LC}$, λ is a function of L, C, and the product LC. Wavelength is therefore dependent on L, C and LC. In Figure 2 frequency is dependent on the condenser setting; in Figure 3 I_p is dependent on E_g; in Figure 4 frequency is dependent on temperature. There are occasional exceptions to the rule. In Figure 5 current is a function of antenna height but it is more logical to plot height along the vertical axis than along the X-axis.

The second point in plotting a curve is the matter of scales. The larger the scales that can be used the more accurate the curve can be read. Scales should be selected to spread the curve over the greatest practical area. Before selecting scales the maximum and minimum values must be considered. If the scale is made too large the maximum values to be plotted may fall outside the limits of the plane of reference. It is good practice to select scales that will cover ranges 10 or 20 per cent greater than the range of variation to be plotted. For example, if the range of a variable is 0 to 500 use a scale not less than 550 or 600 units long. In many cases a slight extension of the curve may permit reading values beyond those actually plotted.

If the numerical limits of the curve are very great it is often desirable to increase the accuracy by breaking up the curve into sections. In this case the most appropriate scale can be used for each section. Multiple colors can be used to distinguish each section and scale, or each section and scale can be properly labelled by name or letters, scale A being used with curve A, scale B with curve B, etc.

Care should be exercised in selecting a scale so that each small division on the paper represents an integral number of units. For example, each small division might represent 1, 2, 5, 10, 25... units. Values like 1.2, 1.7, .75, 2.6... should be avoided. Ordinary cross-section paper is usually made up of a number of major squares divided into 10 small divisions along each side. If one variable covers a range of 600 units each large square can be made equal to 100 units or one small division equals 10 units. Such a scale is easily read. If the same curve is plotted so that seven major divisions equal 600 units, then one major division equals 600/7 = 85.7+ units and one small division equals 85.7+/10 = 8.57+ units. Although the scale is made larger the odd number of units represented by each sub-division would make it very difficult to read and, unless great care is used, the larger scale may actually lead to greater errors than the smaller one.

When the scales are written in on the plane of reference the horizontal scale should ALWAYS increase toward the right and the vertical scale upwards. Never mind if this apparently makes the curve higher on the wrong side of the paper. The importance of this will be shown later. Briefly the direction of the slope indicates whether Y is a direct or inverse function of X. If Y is a direct function the curve increases toward the right, whereas a curve decreasing toward the right denotes an inverse function.

Always select a scale that will give a readable inclination. If the inclination is too great one variable changes so rapidly with respect to the other that accurate readings are out of the question. Often by cramping one scale and expanding the other an erroneous first impression is given. For example, plotting receiver gain versus frequency, if the gain scale is cramped and the frequency scale expanded the curve is flat, creating the impression that the gain is constant over a wide frequency band. Closer inspection of the scales used may actually show a large variation in gain. An inspection of Figure 2 shows clearly that this curve is more readable near the upper limits. The accuracy of the lower section of the curve could be improved by dividing the entire curve into two sections and plotting the lower section to a larger scale.

In general, the greater the number of points plotted on the graph the more accurate will be the final curve. When, after reasonable care, all the points do not plot a smooth curve, the line should be drawn so that it is the best average of the plotted points as shown in Figure 5. Sketch the curve in smoothly and lightly with pencil until it appears satisfactory. A help toward smooth drawing is to turn the paper so that the hand is on the inner or concave side of the curve. With reasonable care a satisfactory curve can be drawn without the use of any special drafting instruments. In particular avoid the use of a ruler. To make the graph a series of straight line segments implies abrupt changes in the curve quite unlike the smooth variation that should be portrayed. The only exception to the above is when all points plot a straight line in which case a ruler may be used. Even in this case the line must be straight throughout its length and not a series of line segments connecting each individual point. Rarely in plotting a straight line curve will all points fall exactly on the line.

All data pertaining to the curve should be tabulated on the plane of reference. If the graph represents a tube characteristic curve, the tabulated data should show the type of tube, fixed voltage and current adjustments, etc. If the curve is a frequency meter calibration it should show the date of calibration, type and serial number of meter and standard used. Any other information that might be of value to the user should also be included.

LOGARITHMIC CURVES: Where a very wide range of values is to be shown on a curve and where greatest attention is to be given to the values in the lower range, one or both coordinates may be plotted to a logarithmic scale. Figure 6 shows the amplification curve of a wide range amplifier with gain in decibels plotted as a function of frequency. In this curve frequencies from 100 to 100,000 cycles per second are plotted to a logarithmic scale while gain in DB is plotted to a linear scale. Both variables could be plotted to a logarithmic scale but it so happens, in this case, that the decibel itself is a logarithmic unit ($DB = 20 \log E/E_1$) so that a linear scale in DB actually represents a logarithmic scale in voltage gain.

Logarithmic scales are divided into cycles. In Figure 6 each ten divisions represents one logarithmic cycle or from 100 to 1000 cycles equals one cycle 1000 to 10,000 a second cycle, etc. When only one scale is logarithmic the paper is called semi-log. Thus paper, like that used for Figure 6, is referred to as three cycle semi-log paper. Regular log paper has both scales divided logarithmically. Two by three log paper refers to a sheet divided two cycles on the horizontal scale and three cycles vertically.

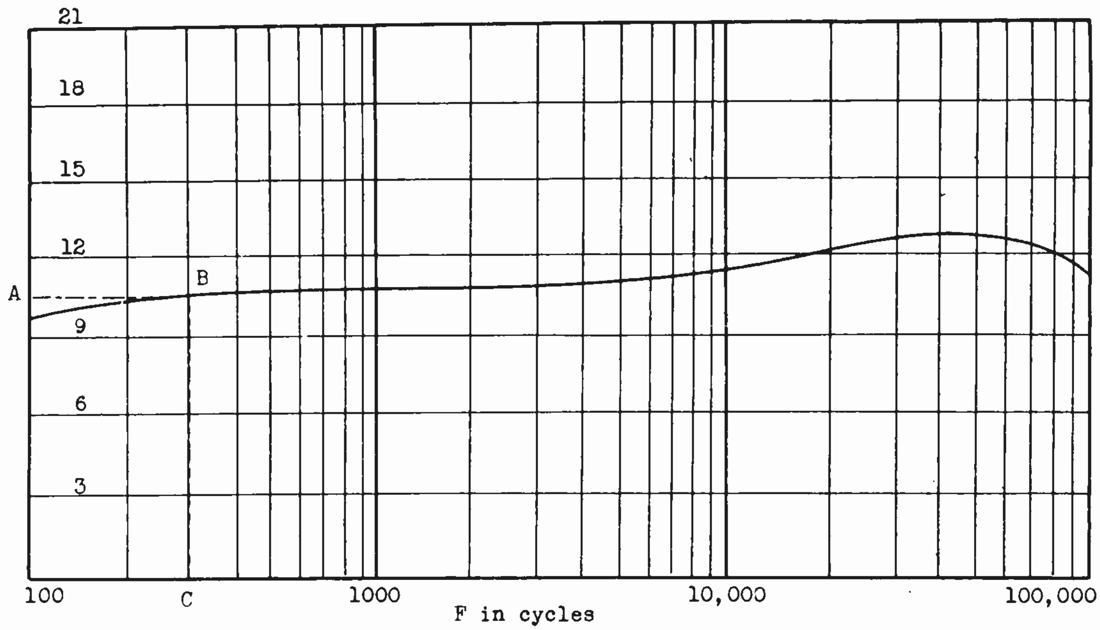


Fig. 6.

The number of cycles required for a given range of values is easily determined. Assume a range of 80 to 45,000 units. One cycle is required for values 80 to 100, another for values from 100 to 1000, a third for values 1000 to 10,000 and a fourth for the 10,000 to 100,000 range. Thus four cycles are required for this range. If the 80 to 100 range can be eliminated, then only three cycles are required. This is done in Figure 6. Gain for frequencies below 100 cycles cannot be read from this graph. Samples of log and semi-log paper are included at the end of this assignment. Study carefully the methods used in dividing the paper.

Great care is required in reading logarithmic curves. In Figure 6 each subdivision of the first cycle represents 100 cycles, in the second cycle each subdivision represents 1000 cycles, while in the third cycle each subdivision represents 10,000 cycles. Since the curve is quite flat throughout its range greatest accuracy is obtained when reading values plotted in the first cycle. Note particularly that *gain is plotted versus* cycles. If Figure 6 were plotted on ordinary cross-section paper the X scale would show the logarithm of the frequency in cycles. In other words, on ordinary cross-section paper, gain would be plotted versus the logarithm of frequency. By using log paper, gain is plotted versus frequency and it is unnecessary to look up any logarithms. The special paper does this automatically. Where X = 200 cycles the actual distance along the X axis is proportional to the log of 200. In Figure 6 the length of the dotted line BC equals 10.5 DB but the distance AB when compared to the distance between 100 and 1000 cycles on the X-axis represents the logarithm of 300.

It is apparent that dividing each main division of a cycle into 10 equal parts for purposes of interpolation would be inaccurate since each main division should be divided logarithmically. At the upper limit of the cycle dividing a major division into 10 equal parts does not introduce an error of any appreciable magnitude but this is not true for the divisions used at the begining of the cycle. In order to increase the accuracy of the curve in the earlier divisions of the cycle the paper is ruled as shown in Figure 7. The first two main divisions are divided into 20 unequal subdivisions. The next three main divisions are divided into 10 unequal subdivisions. The remainder of each cycle is divided into five subdivisions per major division. It is important to remember this variation in scale when plotting logarithmic curves. If the logarithmic scale of Figure 6 were divided as shown in Figure 7 then in the range 100 to 300 cycles each small division would represent *5 cycles*, from 300 to 600 cycles each small division equals 10 cycles and from 600 to 1000 each subdivision represents 20 cycles.

POLAR COORDINATES: As previously explained any point can be located on the plane of reference by means of its rectangular coordinates. The coordinates of point A Figure 8 are 4, 3. If the length of a straight line between point A and the point of origin is measured, the distance of A from o is established. If the angle θ between oA and oX is measured the direction of A from o is established. Therefore point A can be located by stating its distance oA from point o and *the angle θ between the horizontal base line to the*

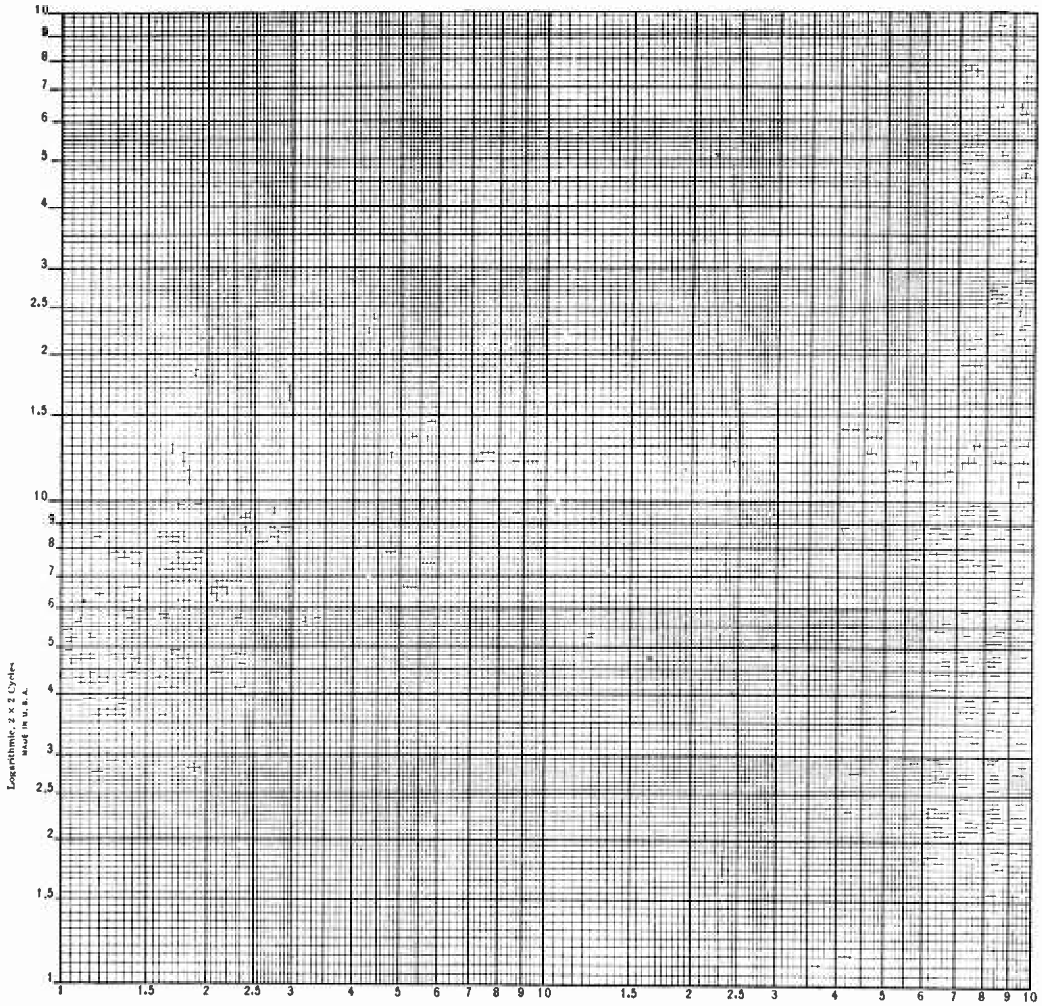


Fig. 7.

right of o. This is called the polar coordinate system. It is apparent from Figure 8 that oA is the hypotenuse of a right triangle having the ordinate of A as the side opposite θ and the abscissa as the side adjacent. Hence oA = $\sqrt{X^2 + Y^2} = \sqrt{4^2 + 3^2} = 5$. Tan θ = Y/X = 3/4 = .75, Tan^{-1} .75 = 37°. The polar coordinates of A are 5 and 37°, meaning that point A is 5 units and bears 37° from point o. θ is usually measured from oX' in a counter-clockwise direction. Angles measured from oX' in a clockwise direction are negative. Thus point A bears 37° from o in a counter-clockwise or (37 - 360) = -323° in a clockwise direction.

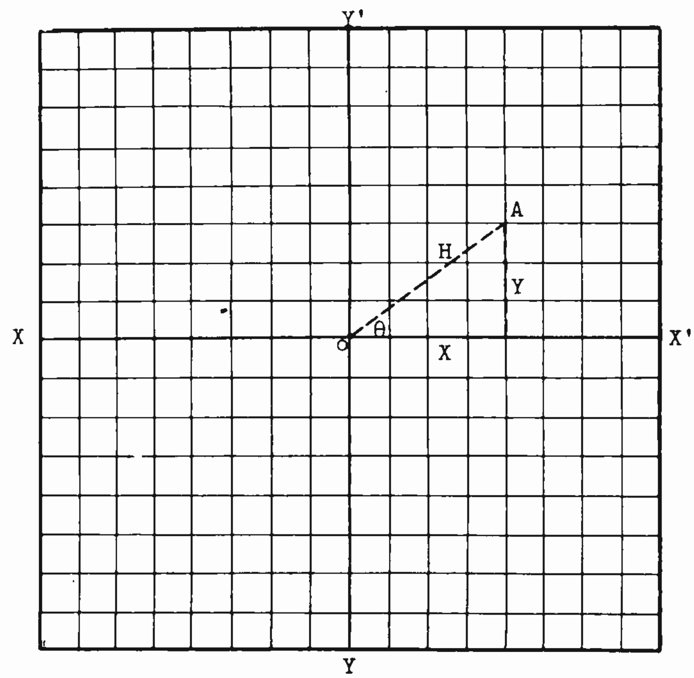


Fig. 8.

If the distance of A from o is equal to H, then the ordinate, Y, of any point is equal to H Sin θ and the abscissa, X, is equal to H Cos θ. But Sin θ = Y/H and Cos θ = X/H. In the study of rectangular coordinates it was shown that in the first quadrant X and Y are positive, in the second quadrant X is negative, Y positive, in the third quadrant X and Y are negative and in the fourth quadrant X is positive and Y negative. If Sin θ = Y/H and Cos θ = X/H then in the first quadrant Sin θ and Cos θ are positive. In the second quadrant

Sin θ = +Y/H so Sin θ is positive but Cos θ = -X/H and is therefore negative. In the third quadrant with both X and Y negative Sin θ and Cos θ are negative. This explains why Sin θ and Cos θ were given signed values in the study of radians in lesson 5. *Note particularly that the sign does not apply to θ but to the sine and cosine of θ.* Attention is invited to the table in the Math tables from Handbook of Chemistry and Physics which shows the "signs and limits of the trigonometric functions" in the different quadrants under that heading.

Polar coordinates are particularly suited to the study of circular motion. In the study of the sine curve, used extensively in electrical work, the distance, H, is taken as the maximum value attained during the cycle and is constant. H being constant leaves θ the only variable. The rate at which θ is changing in degrees per second is the rate at which the vector H is turning. This is called the angular velocity of H and is represented by the Greek letter omega, ω. Since in one revolution H passes through 2π radians, if the vector rotates F times per second $\omega = 2\pi F$. From the angular velocity, ω, one can find how fast point A is moving, that is, the linear velocity of A as well as the distance traversed by A during a given time t.

If the ordinate of A always equals the instantaneous voltage, then from polar coordinates the ordinate of A at any time t must equal H Sin ωt. The polarity of the instantaneous voltage is determined from the sign of Sin θ where $\theta = \omega t$.

To simplify the plotting of polar coordinate curves it is important to assign a meaning to negative values of H. *When H is negative it is plotted in a direction opposite to that indicated by θ.* For example, the point -7, 90° is exactly opposite to the point 7, 90° and hence -7, 90° is the same as 7, 270°. *This is an important point to remember in plotting polar curves.*

The type of paper used for polar coordinate curves is shown in Figure 9. Note that angles are indicated on the plane of reference in both positive and negative direction. The OX' axis on this paper is the 0° line. The distance H is plotted in units outward from the pole. As in logarithmic curves, great care is required in reading and plotting polar curves. In the first accented circle about the pole each radial represents 10°, the next major circle is

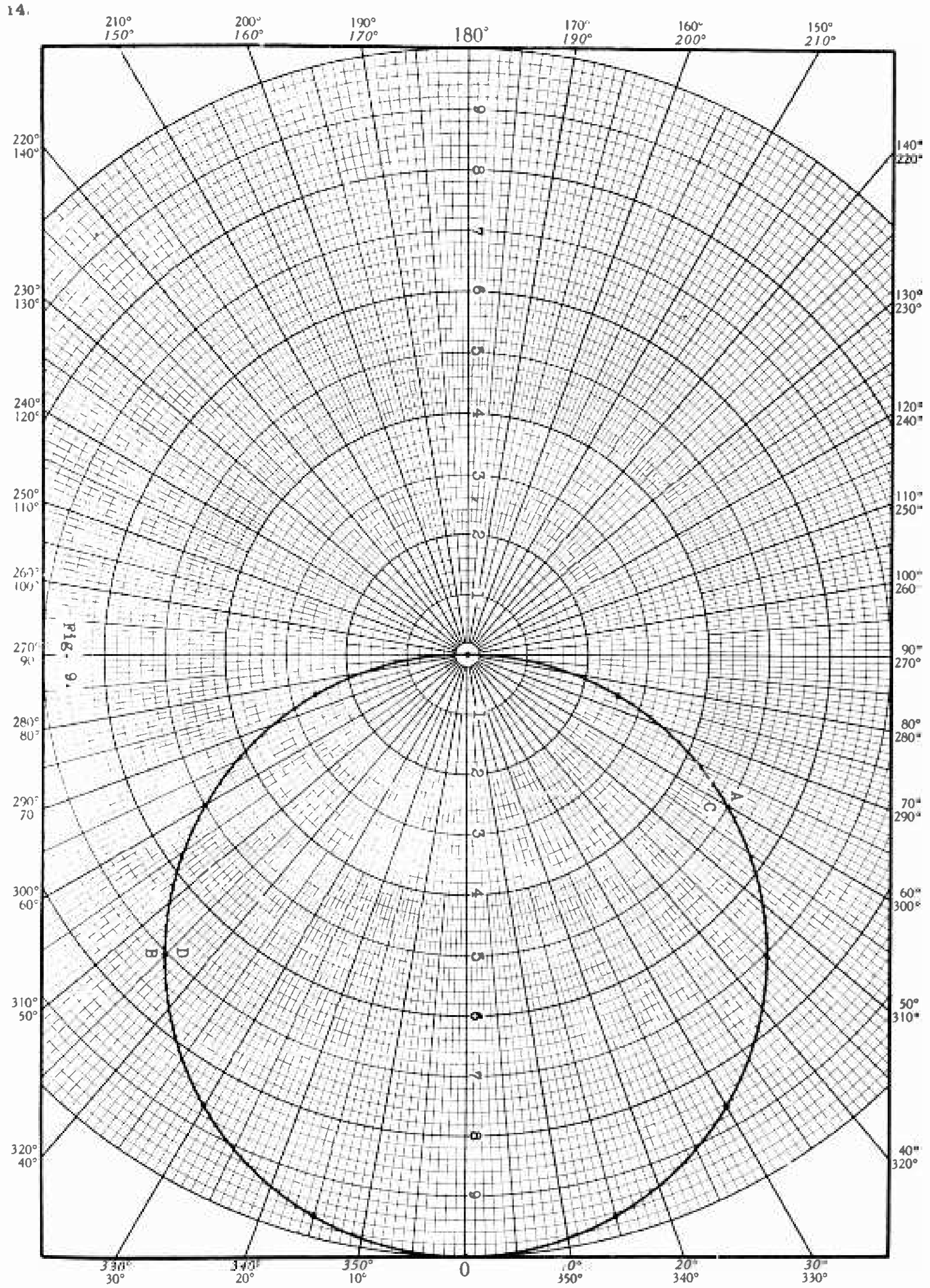

180°
0
210° 150°
200° 160°
190° 170°
170° 190°
160° 200°
150° 210°
220° 140°
230° 130°
240° 120°
250° 110°
260° 100°
270° 90°
280° 80°
290° 70
300° 60°
310° 50°
320° 40°
140° 220°
130° 230°
120° 240°
110° 250°
100° 260
90° 270°
80° 280°
70° 290°
60° 300°
50° 310°
40° 320°
330° 30°
340° 20°
350° 10°
10° 350°
20° 340°
30° 330°
A
B
C
D

Fig. 9.

divided into 5° radials, the next two circles have one radial for every 2° and all others a radial per degree. Note the distance divisions. The pole is the small + sign in the center of the circle and *the first small circle actually represents one division along the distance scale.*

As an example of how a polar curve is plotted assume that X is to be plotted as a function of θ in the equation $X = 10 \cos \theta$. To determine the points for plotting, assign various values to θ from 0° to 360° and calculate the values of X. Table I shows the calculated values of X when θ is varied from 0° to 360° in 15° steps. In determining the values of X remember that the angle used when determining the Cosine is always measured to the nearest horizontal axis as explained in an earlier assignment. The Cosine is negative in the second and third quadrants, which accounts for the negative values of X in Table I. For example, calculate X when $\theta = 150^\circ$. This gives an angle in the second quadrant so the value of X will be negative. The angle actually used to determine the Cosine is 180 - 150 or 30°. Cos 30° = .866 in the first and fourth quadrants and -.866 in the second and third quadrants. X = 10 x -.866 = -8.66 or -8.7 for plotting purposes.

TABLE I

θ	X	θ	X	θ	X	θ	X
0°	10.0	90°	0.0	180°	-10.0	270°	0.0
15	9.7	105	-2.6	195	- 9.7	285	2.6
30	8.7	120	-5.0	210	- 8.7	300	5.0
45	7.1	135	-7.1	225	- 7.1	315	7.1
60	5.0	150	-8.7	240	- 5.0	330	8.7
75	2.6	165	-9.7	255	- 2.6	345	9.7
90	0.0	180	-10.0	270	0.0	360	10.0

The first step in plotting the polar curve of this equation is to determine the proper scale. From Table I, the maximum value of X is 10 and inspection of the polar graph sheet shows there are 10 major circles counting outward from the pole. Since it is always desirable to use the largest possible scale that may be easily subdivided, then the distance from one major circle to the next, measured along any radial, can be taken as one unit of X. It is customary to lay off the scale along the 0-180° radials as shown in Figure 9. Note particularly that the scale increases in a positive direction on each side of the pole. In other words, the values measured along the 180° radial are positive just as those measured along the 0° radial are positive. *On the polar diagram there is no negative scale for plotting negative values of X.*

It is a simple matter to plot the positive values of X. For example, plot the point $X = 5.0$, $\theta = 60^\circ$. Along the 60° radial (measured counter-clockwise from 0°) measure off 5 units for X. The point is plotted as point A in Figure 9 and is located where the 60° radial intersects the fifth major circuit counting outward from the pole. Point B is plotted for $X = 7.1$, $\theta = 315^\circ$, 7.1 units outward on the 315° radial.

Plotting negative values of X offers no particular difficulty if the student does not confuse the rectangular coordinate system with the polar coordinate system. All scale values on the polar coordinate system are positive. A negative value of X acting at an angle θ must be converted to an equivalent positive value of X before plotting on the polar coordinate system. In Vector Analysis it was learned that rotating a vector 180° changed the direction and hence the sign of the vector. A negative force acting in a certain direction is equivalent to a positive force of the same magnitude acting in exactly the opposite direction. Thus if the value of X is negative for some particular value of θ then X can be made positive by the simple expedient of decreasing or increasing the given angle by 180°. For example, plot $X = -5$, $\theta = 240^\circ$. To make X positive, subtract 180° from 240° and get 60°. X is plotted 5 units outward on the 60° radial as shown by point C. Similarly, $X = -7.1$, $\theta = 135^\circ$ is plotted 7.1 units outward on the 135 + 180 or 315° radial as shown at point D.

Observe that point C coincides with point A and that point D coincides with point B. The curve of the Cosine function in the quadrants where its value is negative is found to retrace the circular curve plotted for the function in the quadrants where its value is positive.

RATE OF CHANGE: In studying a quantity that is varying continuously it is often important to know the *rate* at which it is changing, that is *how rapidly it is increasing or decreasing. A rate is defined as the ratio of the change in the dependent variable to a unit change in the independent variable.* On a graph this is represented by the amount the dependent variable increases or decreases per horizontal unit. Thus *the rate is a function of the steepness of* *the graph* and is *not* the ordinate at any point.

If the ordinate changes at a constant rate the curve must plot a straight line. Most quantities change at a varying rate. The *average rate of change* for a given interval may be found by dividing the change in the ordinate between the beginning and end of the interval by the length of the interval.

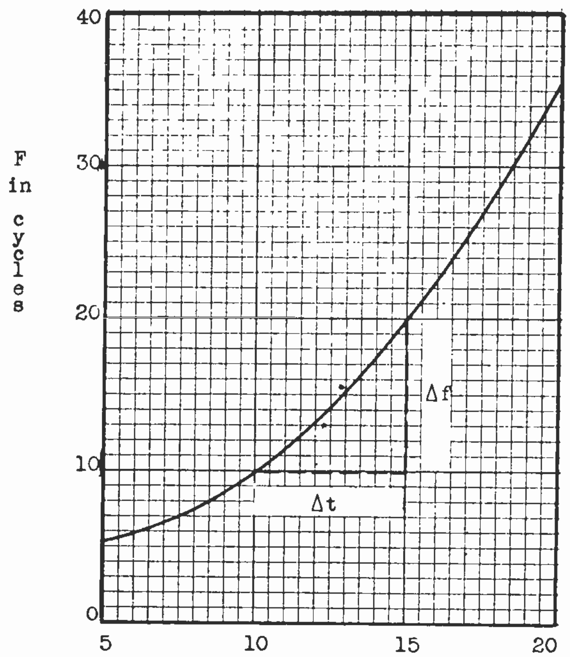


Fig. 10.

This is shown in Figure 10 where frequency is plotted as a function of temperature. To find the average change in frequency between 10 and 15 degrees, the change in t (written Δt and read "change in t") is 15 - 10 = 5 degrees. The change in frequency, Δf, from 10 to 15 degrees is 10 cycles. The average increase in f per degree increase in t is $\Delta f/\Delta t = 10/5 = 2$ cycles/degree.

When a quantity changes at a variable rate it may be necessary to determine the rate of change at some particular instant, that is the *instantaneous rate. If a straight line is drawn tangent to a curve at any point, it will forever change at the same rate as the curve at the point of tangency.* Figure 11 is the same as Figure 10. To find the instantaneous rate of F when t = 12 degrees. Erect the ordinate from t = 12 degrees until it intersects the curve at A. Draw a straight line AB any convenient length tangent to the curve at A as shown. Drop a perpendicular from any convenient point on AB to the horizontal axis. In Figure 11 when t changes from 12 to 20 degrees F varies from 13 to 27 cycles. For an 8 degree change in t, F varies 27 - 13 = 14 cycles. The instantaneous rate is $\Delta f/\Delta t = 14/8 = 1.75$ cycles/degree. Compare this with the average rate of change found for Figure 10. It must be understood that this cal-

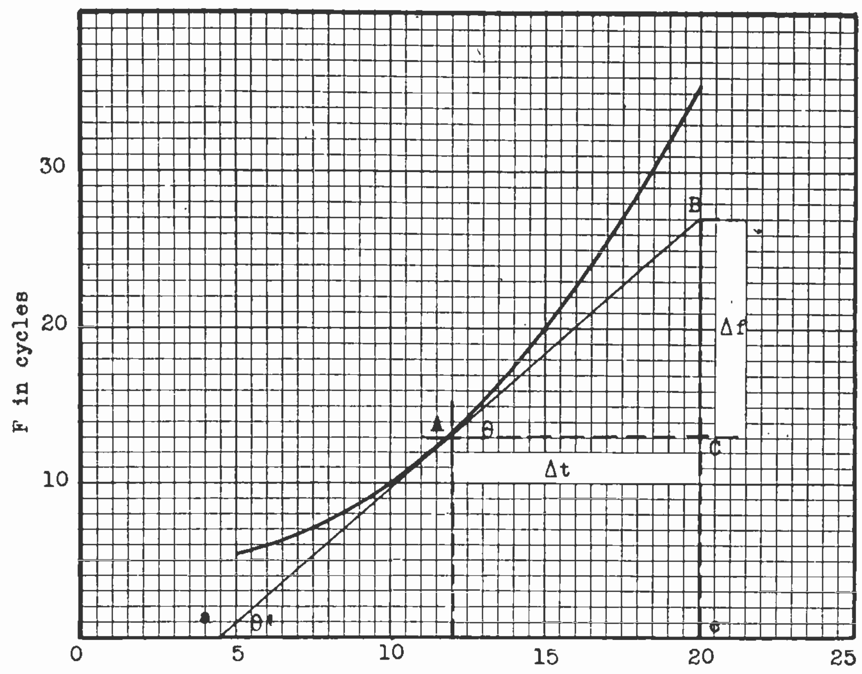


Fig. 11.

culated rate of change is *only for the instant* during which t is passing through 12°.

Great care is required in drawing the tangent line. The ruler must have the direction of the curve *at the point of tangency* and should run along the curve quite closely in both directions from the point.

Rate units are often written as fractions; thus, KC/s, ft/sec, read kilocycles per second, feet per second. Two specially important rates are velocity and acceleration. Velocity is the rate at which distance is travelled whereas acceleration is the rate at which velocity is changing. Feet per second would be the velocity of an object but feet per second gained per second would be the acceleration. The first would be written ft/sec and the second ft/sec^2 read feet per second per second.

INCLINATION AND SLOPE: The *inclination* of a curve at any point is defined as the angle θ made by a line, drawn tangent to the curve at the given point, and the horizontal axis. The *slope* of the line at any given point is the *instan-*

taneous rate at that point and is equal to Tan θ, or the tangent of the inclination. In Figure 11 if AB were extended as shown by the line aAB, then $\theta' = \theta$ since the triangles ABC and aBc are similar. Therefore the inclination at point A is equal to θ and the slope is equal to Tan $\theta = \Delta f/\Delta t$.

In the study of the vacuum tube one method of analysis employs what is called the *load line*, and the slope of the load line is made equal to $1/R_L$ where R_L is the plate load resistance. (Disregard the theory of such analysis. It will be discussed in detail in a later assignment.) In using this analysis a family of E_pI_p tube characteristic curves is drawn with E_p plotted as the independent variable. A family of such curves is shown in Figure 12.

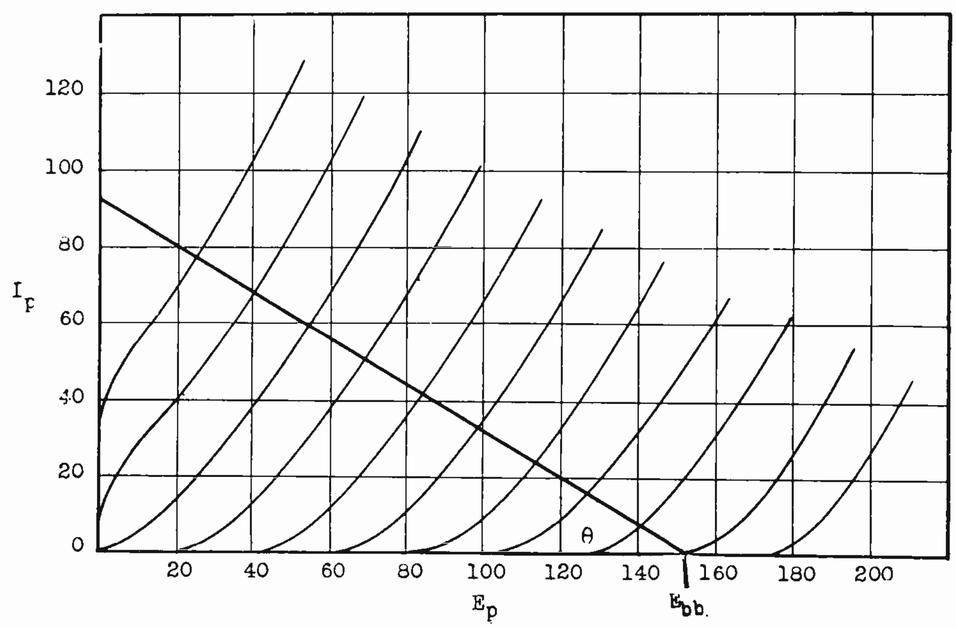


Fig. 12.

The theory of the slope of the load line is quite simple. From Ohm's Law $R = E/I$. Taking the reciprocal of both sides $1/R = I/E$. It will be seen that the load line in Figure 12 forms the hypotenuse of a right triangle with E_p as one side and I_p as the other. With reference to θ, E_p is the adjacent and I_p the opposite side. Be definition Tan θ = opposite/adjacent = I_p/E_p. But

$1/R_L = I_p/E_p$ so Tan $\theta = 1/R_L$. Cotangent of $\theta = 1/\text{Tan } \theta = R_L$.

In the equations above E, I, and R are assumed to be in units, volts, amperes and ohms respectively. It is rarely convenient in a family of E_pI_p curves to plot I_p in amperes except possibly in the very large transmitting tubes. Ordinarily I_p is plotted in milliamperes, amperes x 10^{-3}. In Figure 12 I_p is in milliamperes and E_p in volts and *both are plotted* to the same scale, that is, one milliampere on the Y-axis represents the same distance as one volt on the X-axis. When the I_p scale is changed from amperes to milliamperes it is actually expanded 10^3 times. To maintain Tan θ correct this necessitates changing the equation for the slope to Tan $\theta = 10^3/R_L$. In other words expanding the opposite side 1000 times makes Tan θ equal to 1000 I_p in MA divided by E_p in volts. This would be a practical scale for most transmitting tubes.

For example, using the family of curves in Figure 12 find the inclination of the load line when R_L = 1250 ohms.

$$\text{Slope} = \text{Tan } \theta = \frac{10^3}{R_L} = \frac{1000}{1250} = .8$$

This means that for each volt variation in E_p, I_p will vary .8 milliampere. This is true because the load line is a straight line and hence the rate of change at all points is constant. Average rate equals instantaneous rate in this case. If the slope of the load line is .8 the inclination, θ, is Tan^{-1} .8 = 38.7°.

In receiving tubes the ratio of plate current to plate voltage, I_p/E_p, is much smaller than with transmitting tubes with a correspondingly larger value of R_L. From the practical equation Tan $\theta = 10^3/R_L$; when R_L is large θ may be inconveniently small. Leaving the general form of the equation unchanged, the scale of I_p can be further enlarged so that each division on the I_p scale represents fewer milliamperes. If the I_p scale is further enlarged the numerator of the fraction $10^3/R_L$ must be multiplied by the factor of expansion just as was done when I_p was changed from amperes to milliamperes.

For example, assume 10 divisions along the I_p axis equals 25 MA while 10 divisions along the E_p axis equals 150 volts. This is 150/25 = 6 to 1 expansion of the I_p scale. Consider a case where R_L = 10,000 ohms, then

$$\text{Slope} = \text{Tan } \theta = \frac{10^3 \times 6}{10,000} = .6$$

Inclination = $\mathrm{Tan}^{-1}\ \theta = \mathrm{Tan}^{-1}\ .6 = 31^\circ$. *Note particularly that it is Tan θ that is multiplied by the scale expansion factor and not θ itself.* If the I_p scale were not expanded then,

$$\mathrm{Tan}\ \theta = \frac{10^3}{10000} = .1 \text{ and } \theta = 5.7^\circ$$

If θ is multiplied by 6 then $\theta = 6 \times 5.7^\circ = 34.2^\circ$, an error of more than 3°. This occurs because Tan θ does not vary linearly with θ. Figure 13 shows why it is necessary to expand the I_p scale. When I_p is plotted to the same scale as E_p the dotted line represents the load line. The full line shows the load line when the I_p scale is expanded six times. When it is remembered that the accuracy of the curve depends on accurate readings of the plate current, the marked advantages of the larger scale for I_p is readily apparent. The actual use of the curves and equations in analyzing vacuum tube operation will be discussed in detail in a later assignment.

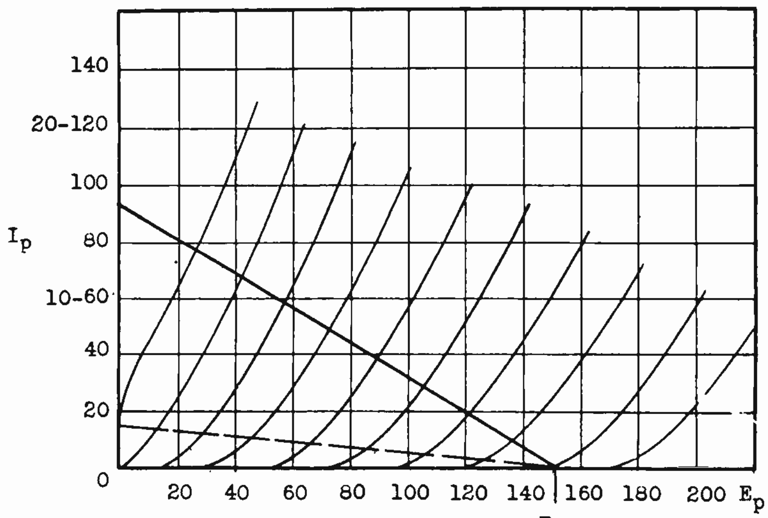


Fig. 13.

EQUATIONS AS LAWS OF VARIATION: If variation in a quantity, X, produces variation in another quantity, Y, then Y is said to be a function of X. In this definition of a function, it should be particularly noted that the *nature* of the variation of Y with changes in X has not been specified. Y merely depends on X for its value. Increasing the value of X may increase or decrease the value of Y. The rate of change of Y with respect to X, (the slope of the curve) may be very pronounced or barely perceptible. At certain values of X, an increase in X may result in a larger Y value, while at other values of X a similar increase in X may result in a reduction in the value of Y. In fact, Y may vary with X in any manner whatever. The statement, "Y is a function of X" implies only that for every value of X there is a corresponding value of Y. In other words, Y is a function of X, (written mathematically $Y = f[X]$ or $Y = F[X]$), whenever there is evidence to show that quantity Y is changing with quantity X according to some definite law of variation, however complex that law may be.

In radio and electrical work, the law of variation is frequently quite simple. For example, the resistance of a copper wire of uniform cross-section is a function of the length of the wire. If the length of the conductor is doubled, the resistance of the conductor will be twice as great. If its length is increased by four times, its resistance will also be increased by four times, etc. The resistance of the wire is said to vary directly as its length. In the shorthand of mathematics, the nature of the variation may be indicated as follows:

$R \propto L$ Where R is the resistance of the wire
L is the length of the wire
$\propto$ is a symbol of relation, read "varies as"

As written above, $R \propto L$ is merely a statement of variation and cannot be manipulated according to the rules for handling algebraic equations. The statement that the resistance varies directly as the length may be translated into an algebraic equation by the use of a multiplying constant K. Thus:

$R = kL$ Where R is the resistance of the wire
L is the length of the wire
k is the multiplying constant which makes kL actually equal to R.

The evaluation of constant k is not difficult and requires only a single measurement of L and R. Thus if the resistance of 200 feet of a conductor is carefully measured and found to be 1.3 ohms, then by substitution:

$$1.3 = k(200)$$

$$k = 1.3/200 = .0065$$

Rewriting the former equation substituting the calculated value of k gives:

$$R = .0065L$$

Where R is resistance of wire in ohms
L is the length of the wire in feet.

This formula can be used for determining the resistance of any length of copper wire having the same cross-section as the sample upon which measurements were made. For example, the resistance of 1000 feet of the wire will be:

$$R = .0065(1000) = 6.5 \text{ ohms}$$

Reference to a wire table will show that No. 18 B and S copper wire has a resistance of 6.51 ohms per 1000 feet and is therefore the size of wire upon which the measurements were made.

Many quantities do not vary directly as other quantities. For example, the frequency of vibration of a quartz crystal used for the control of frequency in broadcast transmitters is a function of the thickness of the crystal. However, doubling the thickness of the crystal does not double the frequency, but has the opposite effect, that is, the frequency of vibration is halved. If the thickness of the crystal is diminished to one-half its present value, its frequency of vibration will be doubled. If the thickness is made one-third as great, the frequency will be three times as high, etc. Evidently varying the thickness of the crystal has an opposite effect on the frequency. The frequency is said to vary inversely as the thickness of the crystal. It may also be said that the frequency varies directly as the reciprocal of the thickness. Thus:

$$f \propto \frac{1}{t}$$

Where f is the frequency of vibration
t is the thickness of the crystal

The statement of variation can again be translated into an algebraic equation by the use of a multiplying constant k:

$$f = \frac{k}{t}$$

Where f is the frequency of vibration
t is the thickness of the crystal
k is the multiplying constant which makes k/t actually equal to f.

The evaluation of k theoretically requires only a single observation of the thickness of the crystal and its corresponding frequency of vibration. In practice, many such measurements may be made and the average of the calculated k values taken for the value of the constant. The method of calculation of k can be demonstrated by a single pair of measurements: The thickness of an "X-cut" crystal is carefully measured and found to be 196.8 mils (1 mil = .001 inch). Its frequency of vibration is observed to be 574 kilocycles per second. By substitution:

$$574000 = \frac{k}{196.8}$$

$$k = 196.8 \times 574000 = 113 \times 10^6$$

Rewriting the former equation substituting the calculated value of k:

$$f = \frac{113 \times 10^6}{t}$$

where f is frequency in cycles/sec
t is thickness of crystal in mils.

The above equation is the general formula for the determination of the approximate frequency of vibration of an X-cut crystal from its thickness dimension.

In many cases a quantity depends on more than one other quantity for its value. For example, Ohm's Law states that the current in an electric circuit varies directly as the applied voltage and inversely as the resistance of the circuit. Stated algebraically:

$$I = \frac{E}{R}$$

where I is the current in amperes.
E is the voltage in volts.
R is the resistance in ohms.

The multiplying constant k does not appear in the right hand member of the equation because the units in which E and R are measured, namely the volt and the ohm, are related to each other in such a manner that the value of k becomes unity and therefore does not affect the value of E/R.

Note that E, with which I varies directly, appears in the numerator of the fraction, while R, with which I varies inversely, appears in the denominator. This is in accordance with the general principle that if a quantity Y varies directly as certain quantities and inversely as others, the relationship can be shown by an algebraic equation in which the left-hand member is Y and the right-hand member is a fraction containing all the quantities with which Y varies directly as factors in the numerator, and all the quantities with which Y varies inversely as factors in the denominator, the entire fraction being multiplied by a suitable constant to secure an actual equality of the members of the equation.

Thus if quantity Y varies directly as A and B and inversely as C and D, this fact may be stated algebraically by the equation:

$$Y = k\frac{AB}{CD}$$

where k is the multiplying constant whose value is such as to make k(AB/CD) actually equal Y.

Every formula used in radio work is in effect a statement of a law of variation. For example, the reactance of a condenser is given by the equation:

$$X_c = \frac{1}{2\pi fC}$$

where X_c is the reactance in ohms.
f is the frequency in c/s.
C is the capacity in farads.

The lack of factors in the numerator of the right-hand member indicates there are no quantities with which the reactance of the condenser varies directly. In the denominator, 2π represents a constant, while the factors f and C are quantities with which X_c varies inversely. Hence as a law of variation this equation states that the reactance of a condenser varies inversely as the frequency and inversely as the capacity. If its capacity is doubled the condenser offers half as much reactance. If the frequency is increased by three times, the reactance is reduced to one-third of its previous value, etc.

The multiplying constant in the above case is $1/2\pi$ or 1/6.28 approximately. Expressing the constant in terms of π is preferable for when so designated the accuracy of the formula is limited only by the number of significant figures to which the value of π is taken.

Whenever the value of the multiplying constant is given, the units in which the variables are measured must be specified. This is of especial importance in radio work where the variables are frequently stated in micro-units, kilo-units, milli-units etc., and where either the English or metric system may be encountered.

For example, the inductance of a single layer air core inductance is given by the equation:

$$L = .03948\frac{a^2N^2F}{b}$$

where L = inductance in microhenries
a = radius of coil in cm.
b = length of coil in cm.
N = number of turns in coil.
F = shape factor of coil.

As a law of variation, this formula states that the inductance of the coil varies directly as square of the coil radius, inversely as the length of the coil, directly as the square of the number of turns and directly as the coil shape factor. (The meaning of the terms Henries, shape factor, inductance, etc. will be fully explained in later assignments).

When calculating the actual inductance of a coil in microhenries using the coil dimensions in centimeters, the value of the multiplying constant is .03948. To secure the inductance in millihenries by direct calculation in the formula, a multiplying constant of 3.948×10^{-5} would be indicated instead of .03948. To secure the inductance in microhenries when the given dimensions are in inches, the dimensions may be converted to centimeters and substituted in the original equation above, or the dimensions in inches may be substituted directly in the formula provided a multiplying constant of .10028 is used in the place of .03948.

The above example shows the necessity of specifying the units in which the variables are to be measured, whenever formulas are recorded for future reference, or whenever scales are marked on a graph.

When one quantity is a function of another quantity, no special problem is encountered in plotting the curve on the rectangular coordinate system. The function is plotted along the Y-axis and the independent variable along the X-axis, each to the largest convenient scale.

However, if one quantity is a function of two or more variables, only one of these variables can be plotted along the X-axis. All other variables are held constant and their value must be stated on the graph.

For example, the reactance of a condenser is an inverse function of both the frequency and the capacity as shown by the equation:

$$X_c = \frac{1}{2\pi fC}$$

If reactance is to be plotted as a function of frequency, then the value of capacity is held constant and should be plainly stated on the graph. On the other hand, if reactance is to be plotted as a function of capacity, then the fixed frequency should be marked on the graph. The choice of the variable to be plotted on the X-axis is dictated by the purpose to which the graph will be devoted.

It is possible to plot the function and both of its variables simultaneously on a three-dimensional or solid graph, where an additional Z axis at right angles to both the X and Y-axis is provided for plotting the second variable. Models of such graphs have been constructed of wood, plaster of paris, or other materials but their use has been very limited because the same results can usually be secured with much less effort by letting the second variable assume a series of fixed values and plotting the corresponding curves of the function on the conventional plane of reference.

An example is given in the tube curves of Figure 13. (Ignore the load-lines.) The plate current of a vacuum tube is a function of both the grid and plate voltages. The plate voltage has been chosen as the independent variable and has been plotted along the horizontal axis. Each curve depicts the variation of plate current with plate voltage for a different value of constant grid voltage. The complete set of curves secured by letting the grid assume a series of fixed voltages is called the tube family, and permits the prediction of the operating characteristics of the tube for any combination of plate and grid voltages. The technique and theory of the load line analysis of tube operation will be discussed in a later assignment.

RELATIONS BETWEEN CURVES AND EQUATIONS: Analytic geometry deals with graphic methods of expressing algebraic equations. It is a rather odd fact that geometry was highly developed long before many of the fundamental algebraic equations were discovered. The Greeks were specialists in geometry but knew little of algebra.

Many fundamental equations used in radio may be expressed in the form of simple curves--and very often the curve itself will make certain facts stated by the equation much more readily apparent. While, within the scope of this discussion, it is impossible to show a great number of such examples, a few typical examples should clearly demonstrate the principles involved.

Figure 14 is geometric proof of the algebraic equation $(A + B)^2 = A^2 + 2AB + B^2$. Each side of the large square equals A + B and the area of the large square is obviously equal to the sum of the areas of the smaller squares. Therefore the algebraic expression $(A + B)^2$ must represent the area of a square the side of which equals A + B.

Algebraic equations may be divided into several principle classifications. An equation may express a direct relation. For example $X_L = 2\pi FL$ where X_L is inductive reactance in ohms, 2π is a constant, F is frequency in cycles per second and L is inductance in henries. This equation is said to be direct and linear. *An equation is direct when an increase in value of any factor on one side of the equation results in a direct increase in the quantity on the other side.* X_L varies directly as F and L because any increase in F or L or the product FL, will cause a proportionate increase in X_L. With L fixed, if X_L is plotted as a function of F a straight line is obtained. The same type of curve is obtained if F is fixed and X_L is plotted as a function of L. *The rate of change is constant and the equation is said to be linear.*

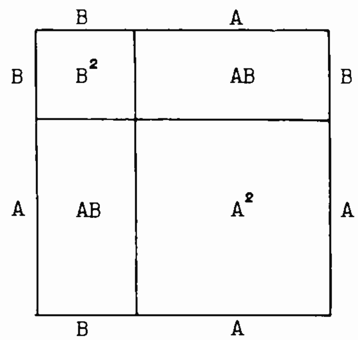


Fig. 14.

Figure 15 shows the type of curve obtained when X_L is plotted as a function of F. The equation clearly states that if F = 0 then $X_L = 0$ since 6.28 x L x 0 = 0. The equation represents the opposition to the flow of current of frequency F by a given inductance L. When F = 0 the current is pure D.C. and no opposition is offered by L.

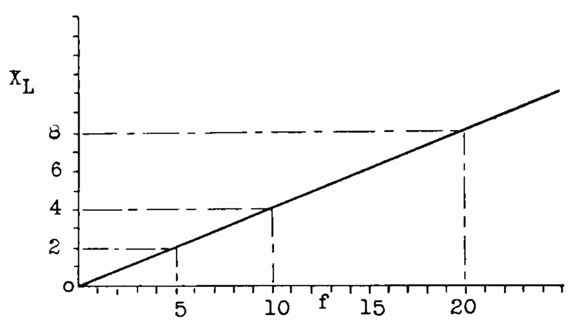


Fig. 15.

Referring to Figure 15, at 5 divisions on the X-axis assume the scale is such that $X_L = 2$ units. According to the equation if F is doubled X_L must also double, therefore at 10 divisions on the X-axis $X_L = 4$ units and at 20 divisions $X_L = 8$ units, etc. These are the points plotted in Figure 15. Drawing a curve through the plotted points results in a straight line because the rate of change is constant.

Any equation that has the form $Y = AX + B$ is said to be linear when A and B are constants and Y is plotted as a function of X. The values of the constants A and B may be positive or negative, and B may be equal to zero.

The equation $X_L = 2\pi fL$, as plotted in Figure 15, is a typical example of a linear function of the general form $Y = AX + B$. In this case X_L is the function Y and f is the independent variable X. L is not being varied, hence $2\pi L$ is the constant A. The constant B is apparently missing. Actually, the value of B is zero and the B term is therefore omitted in writing the equation of the function.

Observe in Figure 15 that the curve passes through the origin. This will always be the case with functions of the general form $Y = AX + B$ when the value of B is zero.

In many types of linear functions, the curve does not pass through the origin. Consider the case of a graph designed for the rapid conversion of temperature readings from the Centigrade to the Fahrenheit scale. In an earlier lesson the rule was given: "To convert from Centigrade to Fahrenheit first multiply by 1.8 then add 32°." If the Centigrade reading is denoted by C and the equivelent Fahrenheit reading by F, then the rule may be translated into the algebraic equation:

$$F = 1.8C + 32$$

In plotting the curve of this equation, the function F will be plotted along the vertical scale and the independent variable C along the horizontal scale. The range of variation of the independent variable to be indicated on the graph will depend on the type of work for which the graph is to be employed. Assume in this case that only temperatures between the freezing point and boiling point of water, 0° and 100° Centigrade respectively, are anticipated. The necessary range of the independent variable C plotted along the horizontal axis will therefore be from 0 to 100.

The cross-section paper of Figure 16 has twelve major divisions along the horizontal axis, each division being composed of ten minor divisions. If each major division is assigned a value of 10° C. and each minor division a value of 1° C. then a temperature range of 120° in all will be covered by the scale of the horizontal axis. By starting the scale at -10° C. and continuing to $+110^\circ$ C., the use of the graph is extended to cover temperatures slightly lower than the freezing point and slightly higher than the boiling point of water.

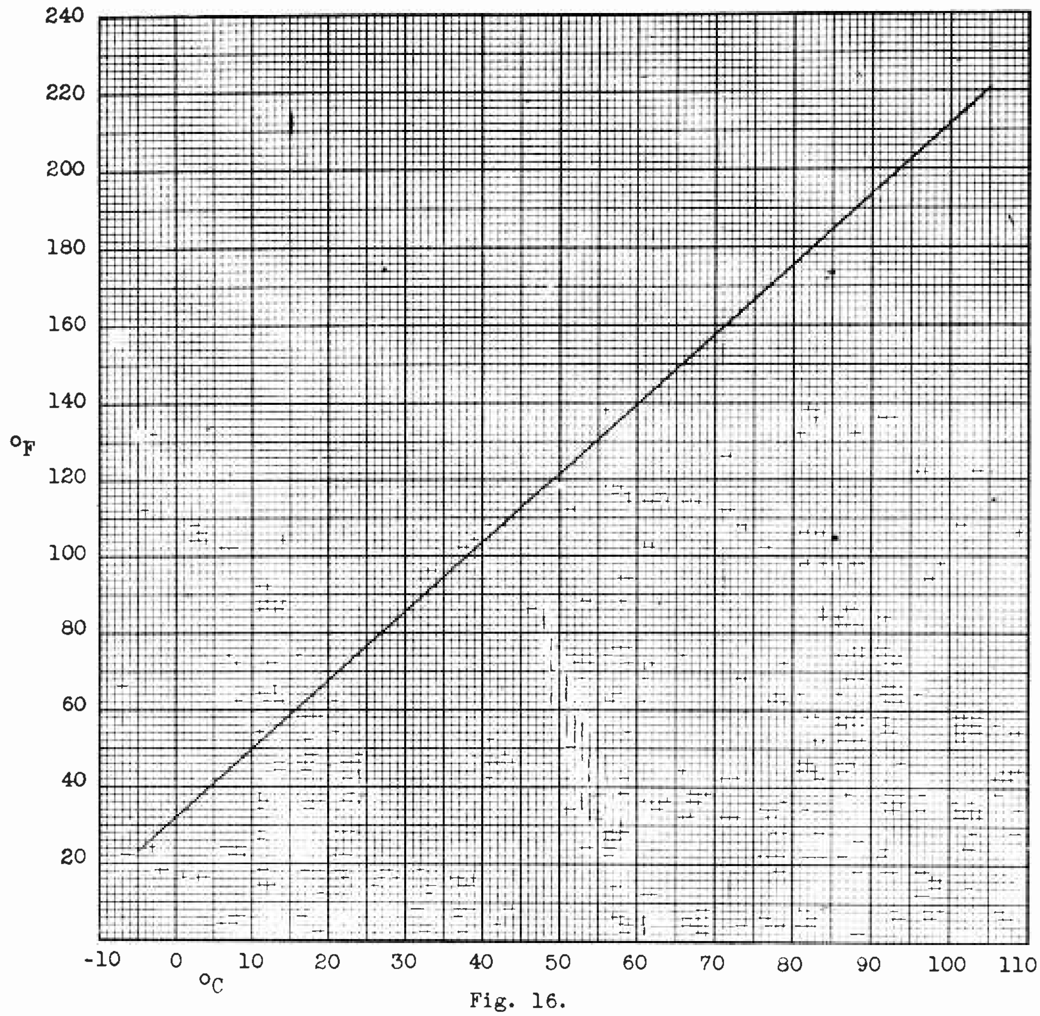


Fig. 16.

To plot the curve, assign C various values between -10 and +110 and calculate the corresponding values of F:

When C = -10, F = 1.8(-10) + 32 = -18 + 32 = +14.

When C = 0, F = 1.8(0) + 32 = +32.

When C = +10, F = 1.8(10) + 32 = +18 + 32 = +50.

When C = +20, F = 1.8(20) + 32 = 36 + 32 = +68.

The calculations of F may be continued for 10° increases in C until the upper limit of the scale has been reached. The results should be tabulated as shown in Table II for use in plotting the points.

TABLE II

C	F
-10	+14
0	+32
+10	+50
+20	+68
+30	+86
+40	+104
+50	+122
+60	+140
+70	+158
+80	+176
+90	+194
+100	+212
+110	+230

First it will be necessary to choose the scale along the vertical axis. The overall variation in F within the limits of the C variation is $230^\circ - 14^\circ$ or 216°. The graph paper of Figure 16 has twelve major divisions along the vertical axis, each containing ten minor divisions. By assigning a value of 20° Fahrenheit to each major division along the vertical axis, or 2° to each minor division, the vertical scale will cover Fahrenheit temperatures from 0 to 240°, which is adequate in the present case.

When the scales have been properly marked, the points of Table II may be plotted. It will be observed that all the points lie in a straight line, and the curve may be drawn with the aid of a ruler. A linear curve is to be expected whenever the function is of the general form $Y = AX + B$. In the present case the value of B is not zero, hence the curve does not pass through the origin.

Frequently in laboratory work, when a series of measurements is plotted, the graph is practically linear. The engineer is often desirous of discovering the mathematical relationship between a variable and the quantity upon which it depends for its value. That is, he wishes to find the equation of a given curve. When the curve is linear, the equation of the curve can be easily determined.

The equation of a linear curve is in the form $Y = AX + B$. Every point on the curve will have a pair of coordinates (X, Y) which will satisfy the equation $Y = AX + B$. Thus the two unknown constants A and B can be determined by setting up simultaneous equations for any two well separated points on the curve. The calculated values of A and B are then substituted in the equation $Y = AX + B$ to give the equation of the curve.

For example, given the curve of Figure 16, to find the equation of the curve. Note the coordinates of two well separated points on the curve, such as the points (C = 35, F = 95) and (C = 75, F = 167). The procedure follows:

$Y = AX + B$ General equation of linear function.

$Y = F, X = C$	Equivalent terms in present case.
$F = AC + B$	Equation for present case, in which A and B are to be evaluated.
$AC + B = F$	Transposing both members for convenience.
$35A + B = 95$	Condition at point (C = 35, F = 95)
$75A + B = 167$	Condition at point (C = 75, F = 167)

$-40A = -72$	By subtraction
$A = \frac{-72}{-40} = 1.8$	Solving for A
$35A + B = 95$	Bringing down one of above equations.
$B = 95 - 35A$	Transposing to solve for B.
$B = 95 - 35(1.8)$	Substituting for A.
$B = 95 - 63 = 32$	Solving for B.

If the values of A and B are now substituted in the equation $F = AC + B$, the result will be $F = 1.8C + 32$, which is the equation of the curve of Figure 16.

Summarizing the characteristics of linear functions: An increase in the value of the independent variable results in a direct increase in the magnitude of the function. The rate of change of the function with respect to the independent variable is constant, as attested by the fact that the graph of a linear function is a straight line. The general equation of a linear function is $Y = AX + B$, in which Y represents the function, X represents the independent variable, A is the constant rate of change of Y with respect to X, and B is the constant which determines the point where the curve crosses the Y-axis.

POWER LAWS: Many algebraic formulae can be classified under the power law group meaning that Y varies as some power of X. A very common formula is $P = I^2R$. With I constant, P, plotted as a function of R, is linear since the formula then has the form $Y = AX$, but if R is constant and P is plotted against I^2 a curve like that in Figure 17 is obtained. Assume R = 1 ohm and plot the curve for a range of current values from 1 to 4 amperes.

When: I = 1 amp. $P = 1^2 \times 1 = 1$ watt

I = 2 amp. $P = 2^2 \times 1 = 4$ watts

When : $I = 3$ amp. $P = 3^2 \times 1 = 9$ watts.

$I = 4$ amp. $P = 4^2 \times 1 = 16$ watts.

Figure 17 shows these points plotted to a convenient scale. The relation between P and I is still direct (P increases as I increases) but the curve is not linear because the instantaneous rate of change varies.

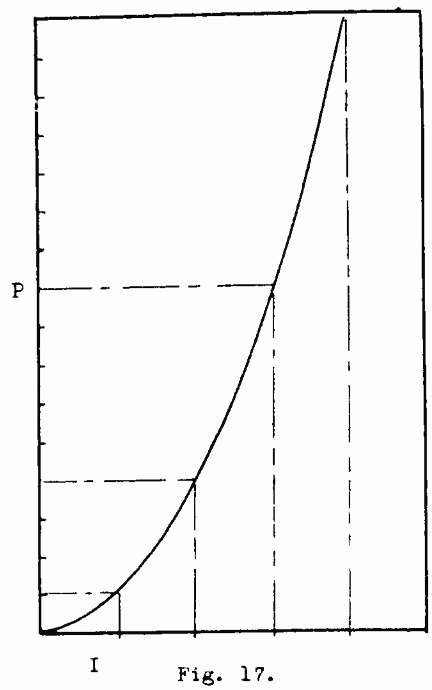


Fig. 17.

Thus P is said to be direct and linear in respect to R but only direct in respect to I. The rate of change of P is small for low values of I but the inclination increases as I increases.

It will be found that *an equation is linear only when it is of the first degree*, meaning that the variable has an exponent of one. If the exponent of the variable is two the equation is of the second degree, if the exponent is three it is a third degree equation, etc. An equation is said to follow a power law when it takes the form $Y = AX^n$ or $Y = AX^n + B$ where A and B are constants and n may have any value positive or negative, integral or fractional, except the value of unity. Some examples of common electrical equations that follow power laws are $P = E^2/R$, $\lambda = 1884\sqrt{LC}$, $F = 1/(2\pi\sqrt{LC})$, etc. These may be changed to the form $Y = AX^n$ as follows:

$$P = \frac{1}{R} \cdot E^2$$

$$\lambda = 1884(LC)^{\frac{1}{2}}$$

$$F = \frac{1}{2\pi}(LC)^{-\frac{1}{2}}$$

Note particularly the last two examples. λ varies directly as the square root of LC since increasing either L or C will increase λ. In the formula $F = 1/(2\pi\sqrt{LC})$ an increase in L or C will decrease F. It was mentioned earlier in this assignment that the X-scale should always increase from left to right and the Y-scale should increase in a vertical direction. When this is done a glance at the plotted curve will show whether Y is a direct or inverse function of X. The slope of the curve is said to be *positive* when Y increases as X increases. The slope is *negative* when the height of the curve decreases as X increases. A negative slope indicates Y is an inverse function of X. Going further, in the form $Y = AX^n$ when n is positive the slope of the curve is positive and when n is negative the slope is negative. Therefore $\lambda = 1884(LC)^{\frac{1}{2}}$ gives a positive and $F = \frac{1}{2\pi}(LC)^{-\frac{1}{2}}$ a negative slope when either λ or F are plotted as functions of L, C or product LC.

In the formula $X_c = \frac{1}{2\pi FC}$, X_c is an *inverse* function of F and C. Assume X_c is plotted as a function of F. As F approaches 0, X_c becomes very large. In mathematical notation this is written $X_c \rightarrow \infty$ as $F \rightarrow 0$ and read X_c approaches infinity as F approaches zero. At zero frequency, the condition for direct current, the condenser forms an open circuit because the opposition to current flow X_c, becomes infinitely large. With finite values of X_c and F and a convenient scale Figure 18 shows the form of curve obtained when X_c is plotted as a function of F. If $X_c = 40$, when $F = 1$ then when $F = 2$, $X_c = 20$ since doubling F will halve X_c. If $F = 4$, $X_c = 10$, $F = 8$, $X_c = 5$, $F = 16$, $X_c = 2.5$ etc. $X_c \rightarrow 0$ as $F \rightarrow \infty$.

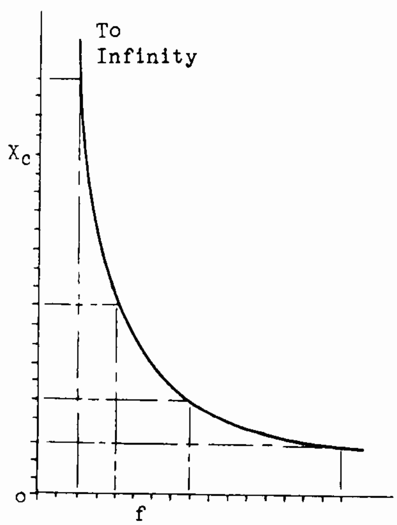


Fig. 18.

An equation consisting of an inverse proportion in the first degree does not plot a straight line as shown in Figure 18. However, in a pure capacity circuit the current varies directly as F or C so current plotted as a function of F or C will give a straight line as shown by the following equations.

$$I_c = E/X_c \text{ and } X_c = 1/2\pi FC$$

Substituting $I_c = \frac{E}{1/2\pi FC} = 2\pi FCE$

This equation is in the first degree and I varies linearly with F, C, and E.

Conversely, in a circuit containing pure inductance I varies inversely as F or L. The equation $X_L = 2\pi FL$ shows X_L is a direct and linear function of F and L, but $I_L = E/X_L$ and substituting $2\pi FL$ for X_L, $I_L = E/2\pi FL$. If I_L is plotted against F or L the curve will have a negative slope indicating I_L is an inverse function of both.

Therefore the *inverse of a linear equation plots a curve with a negative slope* whereas the reciprocal of such a curve plots a straight line.

In the curve showing X_c a function of F, the two variables always have positive values. Consider a curve for a formula in the form

$$Y = \frac{30}{1 - X}$$

Plot Y for a range of X from -5 to +7. Table III shows the values of Y for assigned X values. What happens when X approaches one?

TABLE III

X	Y
-5	5
-4	6
-3	7.5
-2	10
-1	15
0	30
1	?
2	-30
3	-15
4	-10
5	- 7.5
6	- 6
7	- 5

When X = .1, $Y = \frac{30}{1 - .1} = 33.3$

When X = .5, $Y = \frac{30}{1 - .5} = 60$

When X = .9, $Y = \frac{30}{1 - .9} = 300$

When X = .99, $Y = \frac{30}{1 - .99} = 3000$

When X = .999, $Y = \frac{30}{1 - .999} = 30,000$

The closer X approaches 1 the larger the dependent variable becomes. It is impossible to find a value for Y when X = 1 because under these conditions Y = 30/0 and the equation is said to be indefinite. In mathematics it is agreed that division by zero is impossible. To divide 30 by 0 it is necessary to find a quotient which *when multiplied by zero* gives a product of 30. This is impossible since any number times zero equals zero. Figure 19 shows the curve plotted from data on Table III. The curve will never cross the vertical axis since no value of Y can be found when X = 1. There is a tremendous break in the curve at the point where X = 1 and the function is said to be *discontinuous.*

In general whenever a function involves a fraction, if any real value of

X reduces the denominator of the fraction to zero the curve will break and the function is discontinuous.

PRACTICAL APPLICATIONS OF GRAPHS: Two expressions frequently encountered in the study and work with broadcast receivers are "Square Law" and "Linear" detectors. With amplitude modulation the carrier wave is made to vary in amplitude with the degree of modulation. The detector is an amplitude operated device, that is, the output is a function of the amplitude of the input voltage. A linear detector is one in which the plate current variations are in the same form as the grid voltage variations. I_p varies directly and linearly with E_g. Such linearity can be obtained only when the E_gI_p characteristic curve of the tube approaches a straight line. Practically this is obtained with large signal amplitude and a tube having a very sharp plate current cut-off characteristic. Such operation is illustrated in Figure 20.

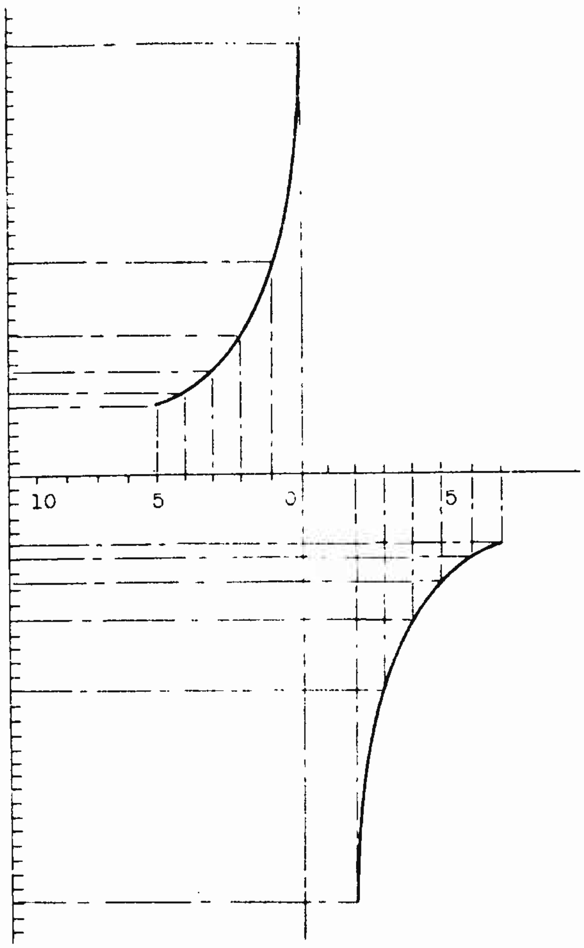


Fig. 19.

Since the portion of the E_gI_p curve over which the variations in the modulation envelope take place is essentially a straight line, the relation between E_g and I_p will be linear. The slight departure from linearity at the lower end of the curve may be neglected when E_g is large enough to operate over

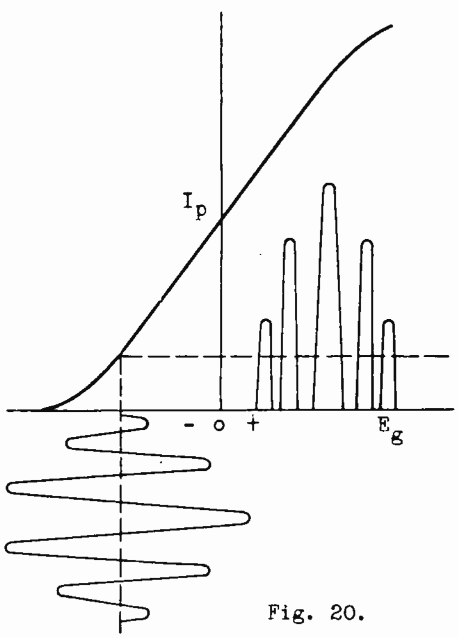


Fig. 20.

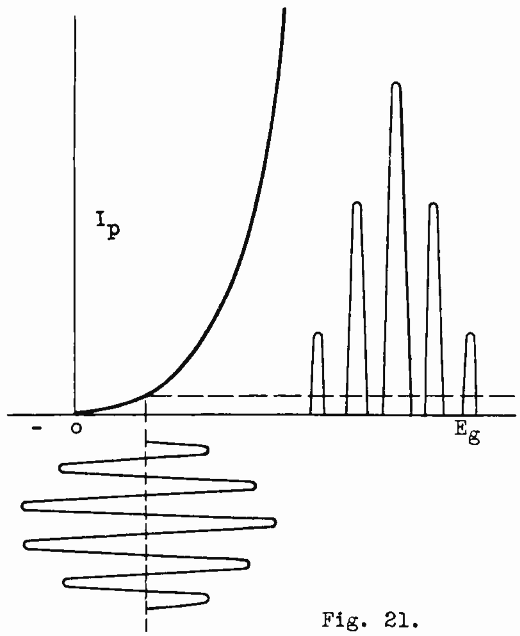


Fig. 21.

the major portion of the straight section of the curve. The rectified plate current variations will be a linear reproduction of the grid voltage variations. Since I_p varies directly and linearly with E_g the output voltage, IR, across the load resistance will vary in the same way and there will be very little distortion introduced by the detector.

A small signal detector operating on the lower *bend* of the E_gI_p curve is shown in Figure 21. Two facts should be particularly noted. First, the section of the curve along which operation takes place is not linear, and second, careful measurement will show that while peak E_g amplitude on the third positive alternation is just twice as high as the amplitude of the first alternation, the amplitude of the third I_g alternation is four times as great as the first I_g alternation.

The curve of Figure 21 closely resembles that of Figure 17 which follows a power law. Since I_g increases to four times its original value when E_g is doubled the detection follows a square law. Thus distortion is introduced by the detector because the output is not a true reproduction of the input. For maximum sensitivity the small signal detector must operate on the bend of the tube characteristic curve and the bend of the I_g or I_p curve approximates the curve of a second degree equation. The response of such a detector will vary approximately as the square of the input voltage. It is possible for this type of detector to produce as much as 50 per cent distortion on signals modulated 100 per cent, in fact, distortion is approximately equal to M/2 where M is the percentage of modulation. A square law detector has little application in broadcast receivers but is used to some extent in communication receivers where maximum sensitivity is required with a minimum number of tubes. The high sensitivity is due to the square law action since doubling the input voltage increases the output four times whereas in a linear detector doubling the input E will simply double the output voltage.

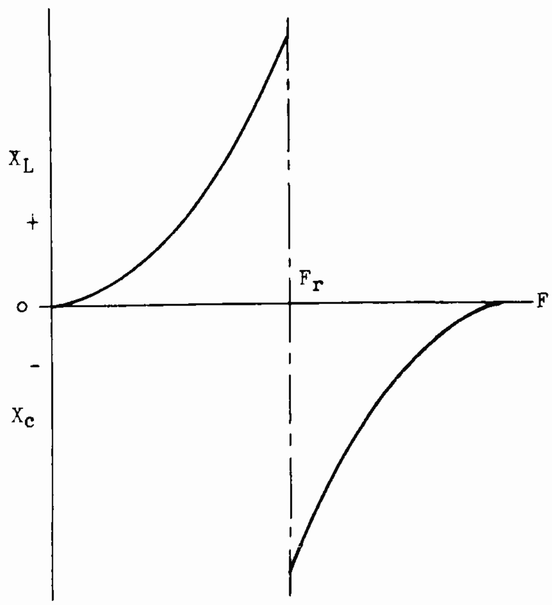


Fig. 22.

Figure 22 shows the impedance of an R.F. choke plotted as a function of frequency. At frequencies below resonance the choke acts as an inductance and at frequencies above resonance it acts as a capacity. The impedance of the choke is considered positive when it acts as an inductance and negative when it acts as a capacity. This curve is similar to that shown in Figure 19 and hence impedance must be a discontinuous function of frequency.

THE EXPONENTIAL FORMULA: In the formula $Y = AX^2$, Y varies according to the power law. If X is doubled Y is 2^2 or 4 times as great. Suppose X appears as an exponent as in $Y = A2^X$. Here if X = 2, Y = 4A, if X = 3 then Y = 8A, if X = 4 then Y = 16A. Note that as X is increased by 1 unit Y is doubled in every case. In the first case it is a variable raised to a constant power but in the second case it is a constant raised to a variable power where the constant must be positive and X must not equal 1. In the second case Y is said to vary exponentially or $Y = A2^X$ is an *exponential formula*. Many scientific laws are expressed by such formula and X may be either positive or negative.

THE COMPOUND INTEREST LAW: Many quantities in nature grow like a sum of money at compound interest where the interest is compounded continuously (that is at very small intervals) instead of the usual 3 or 6 months periods. The basic formula for computing compound interest is:

$$A = P\left(1 + \frac{r}{k}\right)^{kn}$$

Where A is the amount after n years, P is the principal or original amount, r is the rate of interest and k is the number of times the interest is compounded per year. If n = 1 then:

$$A = P\left(1 + \frac{r}{k}\right)^{k}$$

Assume that r = 100 per cent and P = 1 dollar.

Then $$A = \left(1 + \frac{1}{k}\right)^{k}$$

Let k = 1, 10, 100, etc., that is the interest is compounded per year 1 time, 10 times, 100 times, etc. In other words the time intervals between compounding points is made shorter and shorter. By substituting values given for k in the above formula the following values for A are obtained:

When k = 1, A = 2
When k = 10, A = 2.594
When k = 100, A = 2.704
When k = 1000, A = 2.717
When k = 10,000, A = 2.718
When k = 100,000, A = 2.7183

As k continues to increase A approaches a limit, that is, its *rate* of growth decreases and approaches the limit 2.7183... This limiting quantity for A is the result of compounding exceedingly often or continuously. The number 2.7183 should be familiar since it is the base of natural logarithms. In the original formula $A = P(1 + r/k)^{kn}$ if z is made equal to k/r then $1/z = r/k$ and $k = zr$.

Substituting $$A = P\left(1 + \frac{r}{zr}\right)^{zrn}$$

or $$A = P\left(1 + \frac{1}{z}\right)^{zrn}$$

Rearranging $$A = P\left[\left(1 + \frac{1}{z}\right)^{z}\right]^{rn}$$

As k increases without limit so must z since $k = zr$ and it was previouslv shown that $(1 + 1/k)^k$ approaches 2.7183.. or e, the base of natural logarithms, as a limit. Therefore, $A = Pe^{rn}$ which is a general formula for the amount of any principal P after n years when compounded *continuously* at any annual rate r. In the same manner any other physical quantity Q, if it grows at a constant percentage rate per unit of time (day, month, year, etc.), its value after t time units is:

$$Q = Pe^{rt}$$

In the above formula the quantity Q is growing and the sign of r is positive. If the quantity is decreasing or declining the formula becomes

$$Q = Pe^{-rt}$$

Hence the sign of the exponent r indicates whether the quantity is increasing or decreasing exponentially. The formula $Y = Pe^{rx}$ is called the compound interest law.

Any quantity represented by an exponential function or by such a function times a constant must vary in accordance with the compound interest law *provided the constant is positive and the exponent is not equal to 1.* Exponential equations and curves are quite common in electrical and radio engineering. For example the emission current of a pure tungsten filament is given for the formula

$$I_s = 60.2\, T^2 e^{-\frac{52400}{T}}$$

Where I_s is in amperes per square centimeter and T is the temperature of the filament in degrees Kelvin. (0° Kelvin = absolute zero = -273° C = -459.4° F.)

Exponential curves are usually plotted from equations in the form $Y = e^x$, $Y = Ae^x$, $Y = Ae^{kx}$, or $Y = AB^{kx}$ where A, B, and k are constants and x is allowed to assume various positive and negative values. Frequently the value of x or kx is not an integer. The procedure for determining the value of a number raised to a non-integral power involves the use of logarithms and has been described in an earlier assignment. The work is often expedited by converting the entire equation to the logarithmic form and simplifying, as follows:

Exponential Form	Logarithmic Form
$Y = e^x$	$\log_{10}Y = x(\log_{10}e) = .4343x$
$Y = Ae^x$	$\log_{10}Y = \log_{10}A + .4343x$
$Y = Ae^{kx}$	$\log_{10}Y = \log_{10}A + K(.4343X)$
$Y = AB^{kx}$	$\log_{10}Y = \log_{10}A + kx(\log_{10}B)$

The determination of the value of Y in any of the exponential equations shown above is merely a matter of substituting the values of A, B, k, and x in the corresponding logarithmic equation and finding the antilog of the right-hand member of the equation.

For example, in the equation $Y = Ae^x$, to find the value of Y when A = 7, x = 3.2:

$\log_{10}Y = \log_{10}A + .4343x$ Logarithmic form of $Y = Ae^x$

$\log_{10}Y = \log_{10}7 + .4343(3.2)$ Substituting for A and x.

$\log_{10}Y = 0.8451 + 1.3898 = 2.2349$ Simplifying.

$Y = \log^{-1}2.2349 = 171.8$

Attention is invited to the Table of Exponential Functions given in the Mathematical Tables from Handbook of Chemistry and Physics, where the numerical values of e^x and e^{-x} are listed for values of x from 0.00 to 10.0. When the value of x lies between -10 and +10, the value of e^x may be read directly from the table and the use of logarithmic equations may be avoided.

Solving for Y in the above example by the use of the Table of Exponential Functions:

When $x = 3.2$, $e^x = 24.533$ From Table

$Y = Ae^x = 7 \times 24.533 = 171.5$ By substitution

The result secured by the use of the Table of Exponential Functions agrees very closely with that obtained in the logarithmic solution shown above.

Figure 23 shows the general shape of the curve plotted for $Y = e^x$. The higher the curve goes the faster it rises never quite reaching a direction where it is rising perpendicular to the X-axis. Towards the left the curve never quite reaches the X-axis.

If X is negative the curve is reversed, that is, it starts out high and dies away but never quite reaches the X-axis.

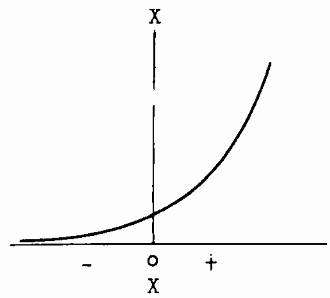


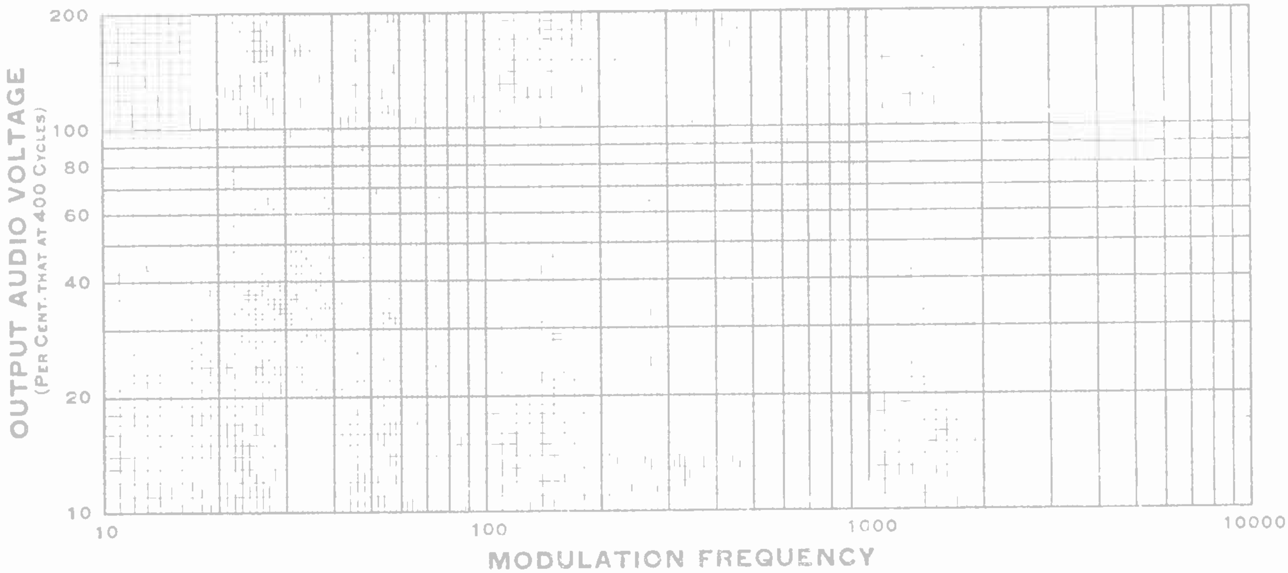
Fig. 23.

Further discussion of exponential functions and graphs will be given in future assignments as the need arises.

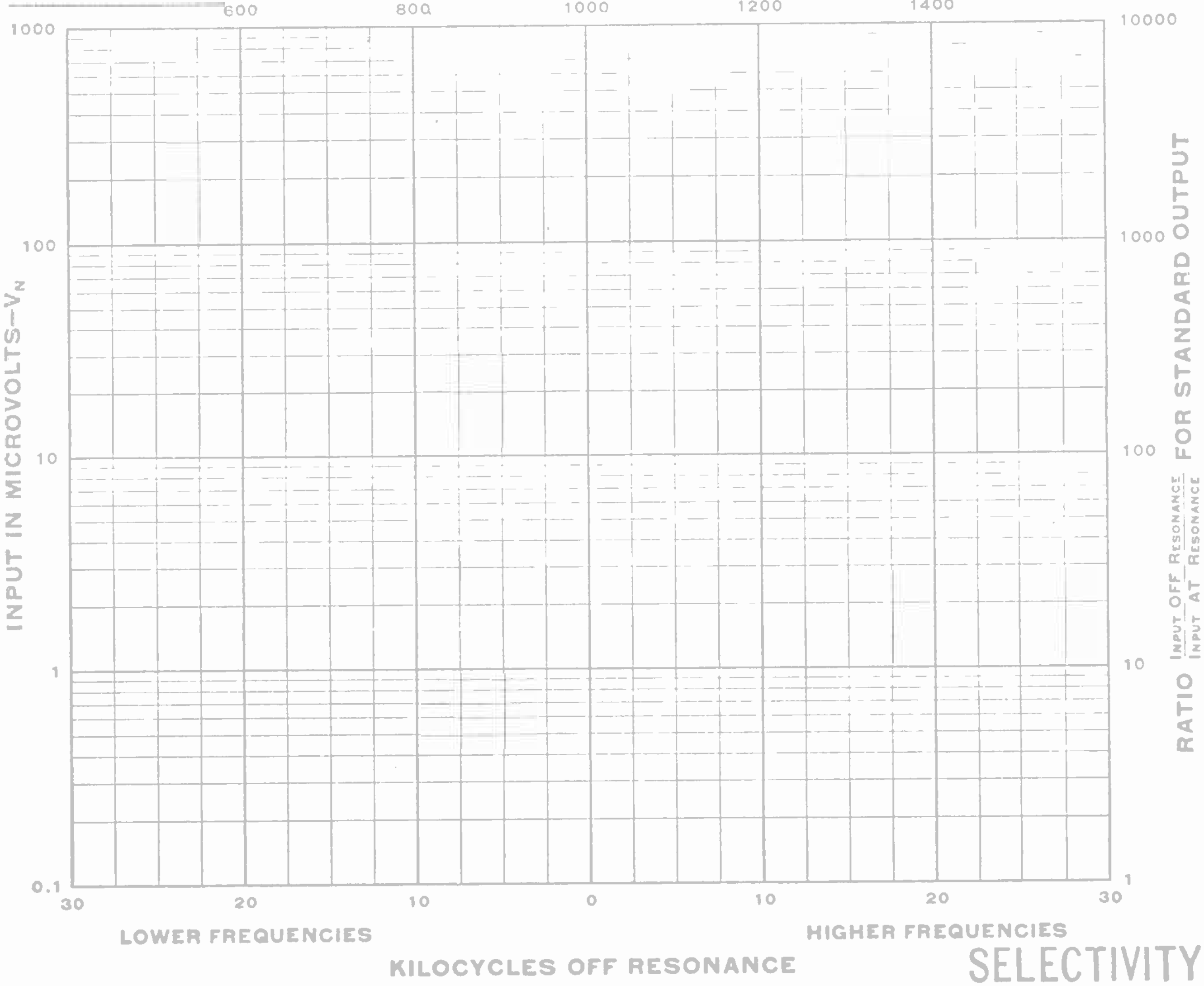
The following pages illustrate various types of graph paper in common use.

MODEL DATE

FIDELITY



SENSITIVITY



SELECTIVITY

V - 9174 - 33

TELEVISION TECHNICAL ASSIGNMENT

GRAPHICAL ANALYSIS

EXAMINATION

Show All Work:

1. You have completed the calibration of a 50 $\mu\mu F$ straight line capacity condenser having a 180 division dial, for use in a buffer stage of a television transmitter. Minimum capacity is obtained at zero divisions on the dial. The following data was obtained:

Condenser Divisions	Capacity in $\mu\mu F$
5	5.0
35	13.05
75	23.2
100	30.0
125	37.0
150	44.0

Construct a curve to show capacity as a function of dial divisions.

2. Reference Problem 1.

(a) What points appear to be in error? First two points - 5 & 13.05 uuF

(b) What are the approximate maximum and minimum capacities of the condenser? Max. approx 52.5 uuF. Min. approx. 2.5 uuF.

3. Reference Problem 1.

(a) What is the rate of change of capacity with dial divisions?

5/18

(b) Is the average rate of change equal to the instantaneous rate? If so, why? Yes - Capacity is a linear function of dial setting.

EXAMINATION, Page 2.

4. Reference Problem 1.

(a) What is the capacity at a dial setting of 140 divisions?

41.1 μμF ✓

(b) What is the dial setting for a capacity of 20 μμF?

63.5

5. The formula for the current flow in a circuit containing only inductance, L, is

$$I = E/2\pi FL, \quad E = 100\ V,\ L = .1\ Henry.$$

Plot a curve of I as a function of F from F = 20 to F = 100 cycles.

6. The power P expended in a line adjusting rheostat (for use in checking television regulated power supplies) is

$$P = I^2R. \quad R = 100\ ohms.$$

Plot P as a function of I from I = 1 ampere to I = 30 amperes on 3 by 3 cycle log paper.

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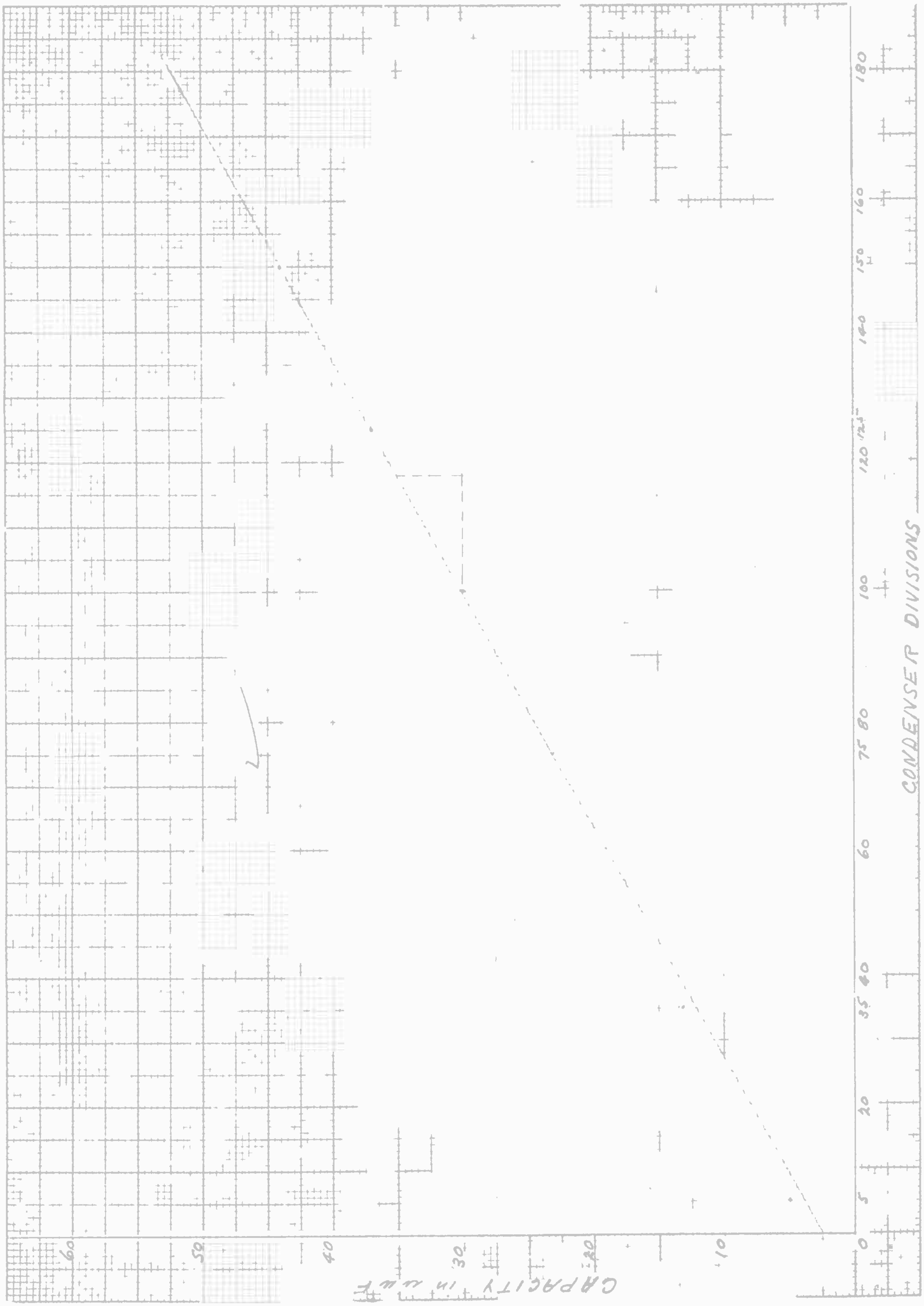
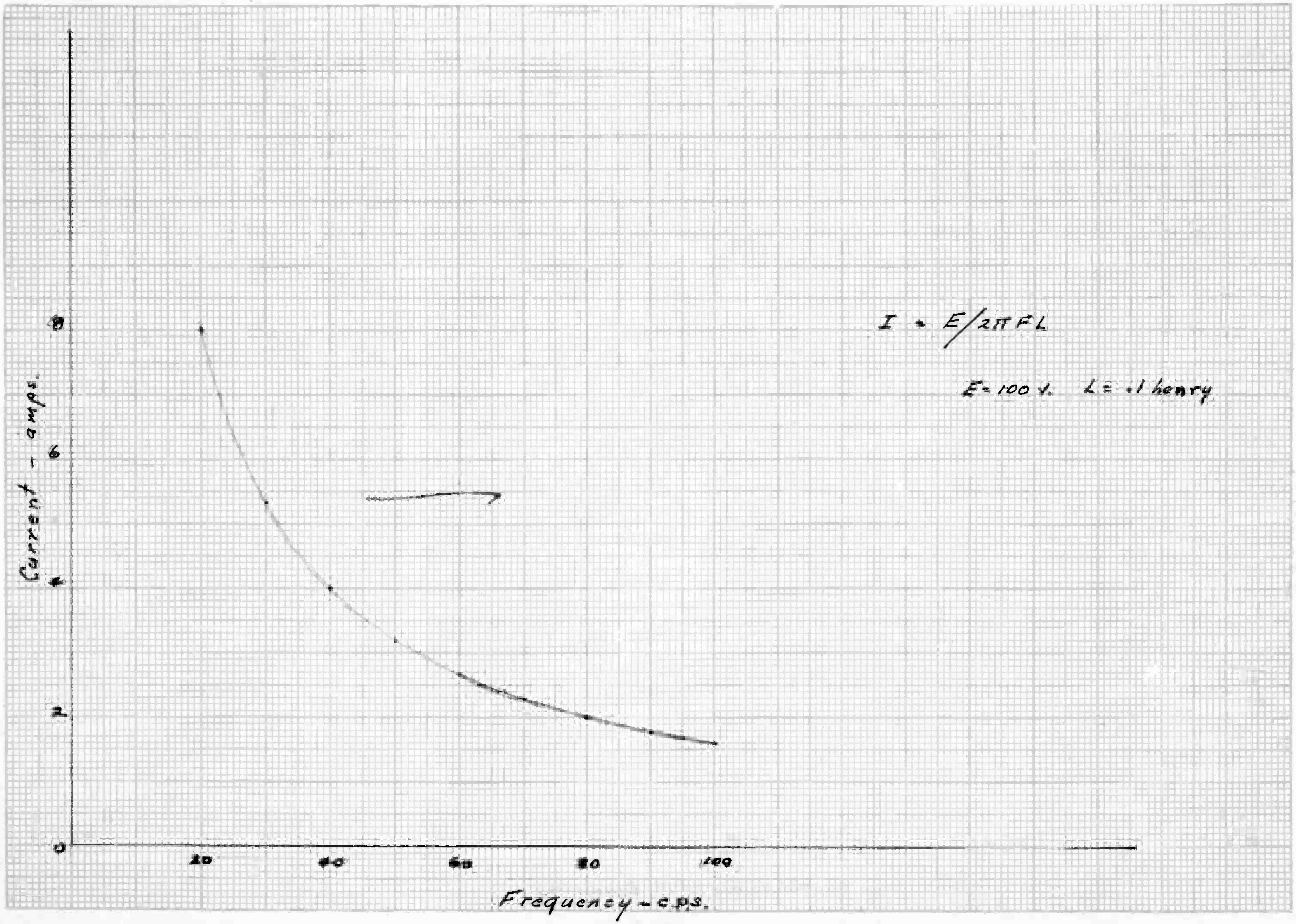

CONDENSER DIVISIONS
CAPACITY in uuf
0
5
20
35
40
60
75
80
100
120
140
150
160
180
10
20
30
40
50
60



Current - amps.
0
2
4
6
8
20
40
60
80
100
Frequency - c.p.s.
I = E/2πFL
E = 100 V. L = .1 henry

GRAPHICAL ANALYSIS

EXAMINATION, Page 3.

6. *(Continued)* $P = 100 I^2$

When $I = 1$	$P = 100$ watts		When $I = 20$	$P = 40,000$ watts	
.. $I = 2$	$P = 400$	..	.. $I = 25$	$P = 62,500$	..
.. $I = 3$	$P = 900$	..	.. $I = 30$	$P = 90,000$	..
.. $I = 4$	$P = 1600$	..			
.. $I = 5$	$P = 2500$	..			
.. $I = 6$	$P = 3600$	..			
.. $I = 7$	$P = 4900$	..			
.. $I = 8$	$P = 6400$	..			
.. $I = 9$	$P = 8100$	..			
.. $I = 10$	$P = 10,000$	..			
.. $I = 11$	$P = 12,100$	..			
.. $I = 13$	$P = 16,400$	..			
.. $I = 15$	$P = 22,500$	..			

7. Plot a load line to satisfy the following conditions:

10 divisions along X-axis equals 100 V

10 divisions along Y-axis equals 25 milliamperes, R_L = 7,500 ohms.

Let load line intersect X-axis at E_{bb} = 200 volts. Show correct angle of load line.

Expansion of I_p scale = 100/25 = 4 to 1.

Slope = $\tan\theta = \frac{10^3 \times 4}{7500} = .533$

Inclination = $\tan^{-1} .533 = 28° 4'$

8. (a) What is the slope of a line if the two points A(2,-3) and B(8,7) are on the line?

$$Slope = \frac{7+3}{8-2} = \frac{10}{6} = 1.666$$

8. (a) *(Continued)*

(b) What is the inclination of the line?

Show your results on cross-section paper.

Inclination = $\tan^{-1} 1.666 = 59° 2'$

9. Plot the curve R = 10 Sin θ, representing the directional response of a certain television antenna, by substituting values for θ from 0° to 360° in 15° steps. Use polar coordinate paper.

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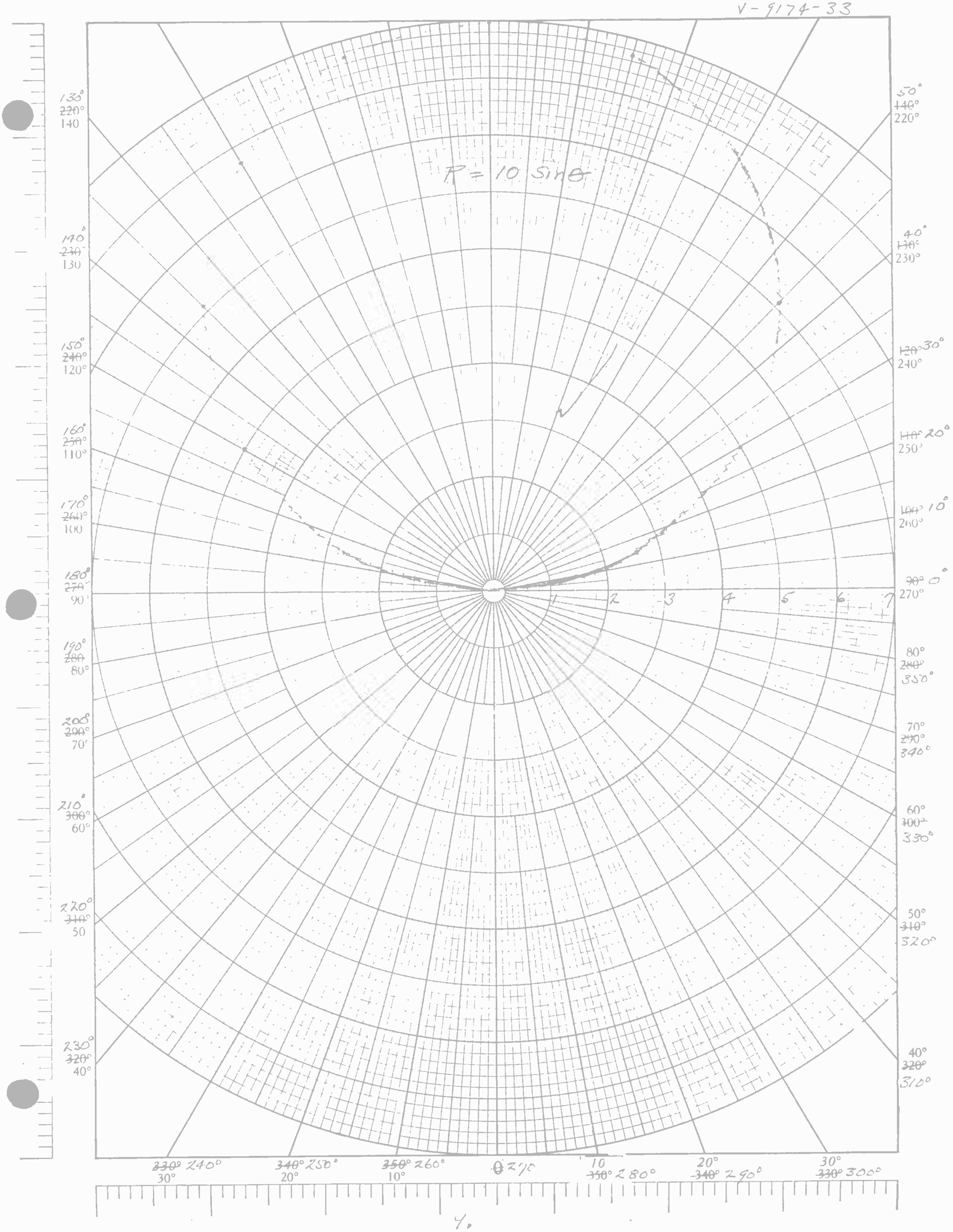
R = 10 Sin θ
1
2
3
4
5
6
7
130°
220°
140
140°
230°
130
150°
240°
120°
160°
250°
110°
170°
260°
100
180°
270°
90°
190°
280
80°
200°
290°
70°
210°
300°
60°
220°
310°
50
230°
320°
40°
50°
140°
220°
40°
130°
230°
120°
30°
240°
110°
20°
250°
100°
10°
260°
90°
0°
270°
80°
280°
350°
70°
290°
340°
60°
300°
330°
50°
310°
320°
40°
320°
310°
330°
240°
30°
340°
250°
20°
350°
260°
10°
0
270
10
350°
280°
20°
340°
290°
30°
330°
300°

9. *(Continued)*

10. After studying a series of curves made from actual measurements on a taut guy wire for supporting a television tower, the following information is derived. The fundamental frequency of the wire varies as the square root of the tension T, inversely as the radius R, inversely as the length L and inversely as the square root of the density D. By actual measurement the frequency is found to be 1 cycle/sec when R = 5, L = 50, D = 4, and T = $\pi \times 10^6$. What is the basic formula? Hint: Solve for the missing constant k.

$$F = k \frac{\sqrt{T}}{RL\sqrt{D}}$$

$$1 = k \frac{10^3\sqrt{\pi}}{5 \times 50 \times 2} \qquad k = \frac{500}{10^3\sqrt{\pi}} = \frac{1}{2\sqrt{\pi}} = \frac{1}{2 \times 1772} = .2887 \quad .282$$

$$F = .2887 \; (.282) \frac{\sqrt{T}}{RL\sqrt{D}}$$