SPECIALIZED
TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

OHM'S AND KIRCHHOFF'S LAWS AND BRIDGE CIRCUITS

Capitol Radio Engineering Institute
Washington, D. C.

709D

OHM'S AND KIRCHHOFF'S LAWS: BRIDGE CIRCUITS

FOREWORD

This assignment is, without question, one of the most important in the entire course because it deals with fundamental relationships between current, voltage and impedance. The term impedance is used advisedly instead of resistance because resistance is only a special type of impedance.

Ohm's Law, $E = IZ$, $Z = E/I$, $I = E/Z$, is the basic law of electricity and like most basic laws is remarkably simple. It is actually astounding, however, how many radiomen do not really know how to apply it to circuit analysis. Such work is taken up in great detail in this assignment. Although short and simple means have been developed for the solution of many complex circuits, a *thorough* understanding of Ohm's Law is essential to the analysis of any type of electrical circuit.

Kirchhoff's Laws extend the applications of Ohm's Law and permit the more ready solution of complex circuits and networks. Here will be used some of the special applications of algebra studied in an earlier assignment.

Bridge Circuits are found throughout radio, one of the most important applications being in measuring devices. The simplest type is the Wheatstone Bridge used for d.c. measurement of resistance. By substitution of L or C in place of R for certain elements, a bridge may be used to measure inductance or capacity. By elaborate refinements in construction and shielding, bridges may be used for measurements at high radio frequencies. Bridge circuits are used as neutralizing elements in R.F. amplifiers and as frequency stabilizing devices in R.F. oscillators, and in numerous other applications.

In this assignment only the basic principles and fundamental applications of bridges will be discussed. The various uses of the bridge circuit will be taken up in detail in later assignments. A thorough understanding of this assignment is a must.

E. H. Rietzke,
President.

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FUNDAMENTAL THEORY

It is assumed that every student at this point is more or less familiar with the most fundamental of all electrical formulas, Ohm's Law. It is believed, however, that a thorough review of the principles involved in Ohm's Law is the logical step in progressing from the study of mathematics to the use of mathematics in the more complex electrical calculations.

Ohm's Law states, in a few words, the fundamental relationship existing between the current, voltage, and impedance of a circuit. In the simpler calculations involving Ohm's Law the impedance of a circuit is usually its resistance, this being always the case in a direct current circuit. In alternating current computations the applications of this law become more complex, but fundamentally the statement of the law is correct, the word "Resistance", however, being replaced by "Impedance".

OHM'S LAW.—Ohm's Law states that *the current in a circuit varies directly as the voltage and inversely as the resistance of the circuit.*

This is ordinarily stated in the form of an equation, the three arrangements of the equation being as follows:

$I = E/R \qquad R = E/I \qquad E = IR$

When this equation is to be used in circuits that are not composed entirely of resistance, R will be replaced by Z. Z in turn will be made up of resistance and inductive or capacitive reactance, or both. The circuit may be a simple series circuit, a simple parallel circuit, or some more or less complex circuit containing both series and parallel members.

According to the statement of Ohm's Law, if the voltage and resistance are known, the current in the circuit may be calculated by dividing the voltage by the resistance in ohms, and the answer will be an expression of the current in amperes.

The other two equations expressing Ohm's Law may be solved in an equally simple manner.

SERIES CIRCUIT.—The next rule is that expressing the impedance of a series circuit. This rule states that *the impedances of a series circuit add.* If the impedances are such that they have identical effects on the phase relations of the current and voltage of the circuit, such as two or more pure resistances, two or more perfect inductances, two or more perfect capacities, or two or more combinations of the above in which the angles of lead or the angles of lag are identical, the addition of the impedances will be arithmetical. If the constants making up the impedances are not such that their effects on the phase relations of current and voltage are identical, *the addition will* be vectorial. In this discussion circuits containing only resistance will be considered, thus no vector addition will be encountered here, but it will be discussed in detail in later assignments.

A case of arithmetical addition of resistances in series is shown in Fig. 1, where R_1 equals 50 ohms and R_2 equals 50 ohms. The total resistance, the two being in

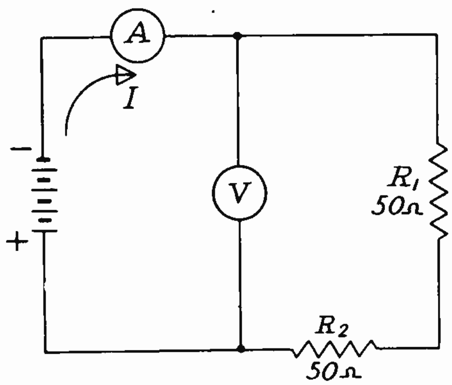


Fig. 1.--Resistances in series.

series, is equal to $R_1 + R_2$, or 100 ohms. If it is assumed that the voltage of the battery is 200 volts, as indicated by V, and Ohm's Law is applied, the current in the circuit will be indicated by the ammeter A, and will be equal to $E/R = 200/100 = 2$ amperes.

To find the voltage drop across one of the resistors, for example R_2, another statement of the equation of Ohm's Law, $E = IR$, is used. The resistance of R_2 is 50 ohms; the current through R_2 is 2 amperes; the voltage drop across this resistance is then $50 \times 2 = 100$ volts.

If the voltage of the battery is unknown but the total resistance of the circuit and the current as indicated by ammeter A are known, the voltage of the battery may be found by the same method. The battery voltage is equal to the total resistance of the circuit in ohms times the current in amperes. In this case suppose ammeter A indicates 3.6 amperes. The battery voltage will be $3.6 \times 100 = 360$ volts. (The arrow in Fig. 1 indicates the direction of electron movement.)

This relation between currents and voltages in a series circuit exists regardless of the number or the values of the resistances. The total resistance is equal to the sum of all the individual resistances, and the current is equal to the total voltage divided by the total resistance. The voltage drop across any individual resistor is equal to the value of that resistance in ohms times the current flow in amperes through the resistor.

Two cardinal points to remember when applying Ohm's Law to a series circuit are:

1. The current is the same in all parts of a series circuit.

2. If the law is applied to any *part* of the circuit, only the E, I, and R values associated *with that particular part* of the circuit should be used.

PARALLEL CIRCUIT.—The general expression of Ohm's Law for parallel circuits states:

(a) *The voltage across any number of resistances in parallel is the same across all the individual resistances.*

(b) *The current through each resistor is equal to the voltage divided by the resistance of the individual resistor.*

(c) *The current in the external circuit is equal to the sum of the currents through the individual resistances.*

The current through each branch of a parallel circuit is equal to the applied voltage divided by the resistance of that branch. A variation of the resistance of any one branch of the parallel circuit has no effect on the current in any other branch of the circuit *so long as*

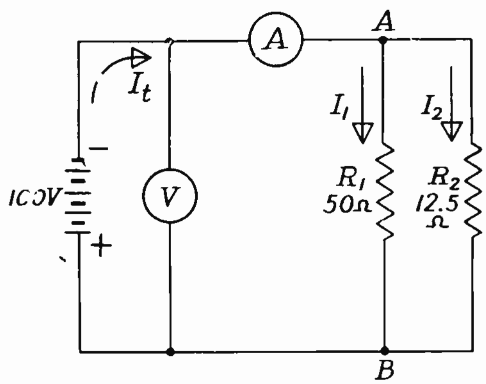


Fig. 2.--Resistances in parallel.

the voltage across the combination is held constant.

An example of the simple parallel circuit is shown in Fig. 2. Resistors R_1 and R_2 are in parallel (as indicated by the common connections A and B) across the same battery voltage. Assume that the applied potential is 100 volts. The current I_1 in R_1 is equal to 100 volts divided by the resistance of R_1 or 100/50 = 2 amperes. The current I_2 in R_2 is found in a similar manner and is equal to 100/12.5 = 8 amperes. Since the current in the external circuit is equal to the sum of the currents in the individual branches, the total battery current I_t, as indicated by A, is 2 + 8 = 10 amperes. In algebraic form $I_t = I_1 + I_2$.

Ohm's Law states that the total resistance of a circuit is equal to the applied voltage divided by the total current, so the resistance of the parallel combination must be equal to 100/10 = 10 ohms. It will be observed that the total resistance is less than the resistance of either branch. In fact, in *any* parallel circuit containing only resistance, the total resistance of the circuit will always be *less than that of any individual branch.*

The method of computing the value of the total resistance as described above may be used. This method requires the calculation of the current in each branch, the addition of all the individual currents, and the division of the sum of the currents into the known or assumed voltage. This method is extensively used in complex alternating current problems in which the currents and voltages are out of phase by various angles. These will be taken up in following assignments on parallel and series-parallel circuits.

A fundamental method of dealing with parallel circuits is developed in the following paragraphs. It is based on the fact that the resistance of any circuit is independent of the circuit voltage. Consider the circuit of Fig. 3. The resistance of R_1, R_2, and R_3 in parallel is independent of the voltage of the battery. This means that any value of E may be *assumed* to be connected across the parallel combination, and it is desired to find the total R. Assume that the battery potential is 1 volt. Then the currents I_1, I_2, and I_3 are respectively,

$$I_1 = \frac{1}{R_1}$$

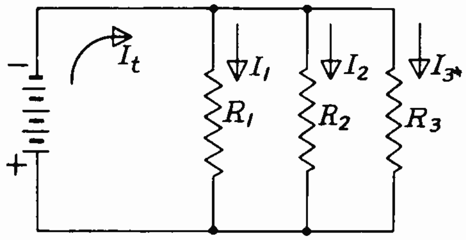


Fig. 3.--Three resistances in parallel.

$$I_2 = \frac{1}{R_2}$$

$$I_3 = \frac{1}{R_3}$$

The total current from the battery is,

$$I_t = I_1 + I_2 + I_3 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But $R_t = E/I_t$ where E = 1 volt (assumed). So

$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Therefore, *the total resistance of a parallel circuit is equal to the reciprocal of the sum of the reciprocals of the individual branches.* To solve *any* parallel combination, a.c. or d.c., where only the impedance of the branches are known, assume a voltage connected across the combination, calculate the branch currents, and add to find the total current I_t. The assumed voltage divided by the total current gives the effective impedance of the combination. The importance of this method of solution will become apparent in the study of a.c. circuits.

The reciprocal of a number is 1 divided by that number. The reciprocal of R_1 is equal to $1/R_1$. The reciprocal of resistance is called the *conductance*. The conductance of a circuit is an expression of the *ease* with which current can flow, in contrast with the resistance which is defined as the *opposition* to current flow. The symbol for conductance is G. $G = 1/R$. In the case of several resistances in parallel the total conductance will equal $G_1 + G_2 + G_3$, etc. Since $G = 1/R$, the total conductance will be: $1/R_1 + 1/R_2 + 1/R_3$, etc. The unit of conductance is the mho (ohm spelled backwards), and 1 mho is the reciprocal of 1 ohm. Another definition is: the conductance of a resistance unit in mhos is numerically equal to the *current in amperes* which would flow through the unit if 1 volt were impressed across it.

If the conductance is equal to the reciprocal of the resistance, then the resistance must be equal to the reciprocal of the conductance, 1/G. The total resistance of the circuit must be equal to the reciprocal of the total conductance.

$$R = 1/G$$

$$G = G_1 + G_2 \ldots$$

Therefore,

$$R = \frac{1}{G_1 + G_2 \ldots}$$

But

$$G_1 = 1/R_1 \text{ and } G_2 = 1/R_2$$

Therefore,

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

By simplifying the above formula (see assignment on algebra), the following is obtained:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

which means the total resistance of two resistors in parallel is equal to their product divided by their sum. The reciprocal formula is preferred when the circuit has three or more resistors in parallel, whereas the product over sum method has the advantage of simplicity when only two resistors are involved.

Using the values given in Fig. 2,

$$R = \frac{1}{\frac{1}{50} + \frac{1}{12.5}} = \frac{1}{.02 + .08}$$

$$= \frac{1}{.1} = 10 \text{ ohms}$$

or

$$R = \frac{50 \times 12.5}{50 + 12.5} = \frac{625}{62.5} = 10 \text{ ohms}$$

This agrees with the answer obtained by the current method of computation.

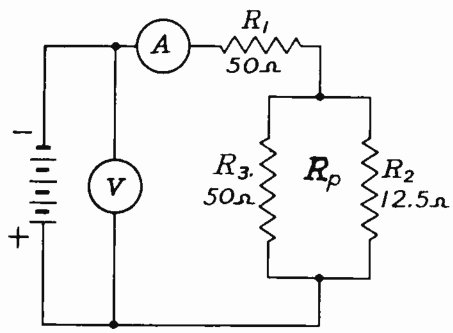
SERIES-PARALLEL COMBINATIONS.—Fig. 4(A) is an example of a series circuit in which one of the series elements consists of a parallel combination of two resistors, 12.5 and 50 ohms, respectively. The other series element (R_1) has a resistance of 50 ohms. Although this circuit contains a parallel combination of resistors, it is essentially a series circuit and must be handled as such.

It is first necessary to solve the parallel combination in order that its *equivalent series resistance* may be obtained. This is done as shown in the preceding problem.

$$R_p = \frac{50 \times 12.5}{50 + 12.5} = 10 \text{ ohms}$$

The equivalent series circuit is shown in Fig. 4(B). R_p indicates the parallel resistance of R_2 and R_3.

The total resistance is now equal to $R_1 + R_p = 50 + 10 = 60$ ohms. Assume that the battery voltage is



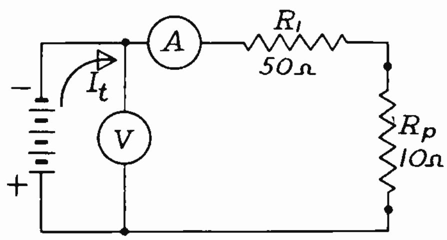


Fig. 4.--Combination of resistances in series and in parallel.

360 volts. Ammeter A will indicate the total line current I_t of $360/60 = 6$ amperes. To find the current in each branch of the par-

allel combination it is first necessary to find the voltage drop across the parallel portion of the circuit. As it is one element of the series resistance, the voltage across R_p will be equal to the product of the value of its equivalent series resistance, 10 ohms, and the total current, 6 amperes, or 60 volts.

With a common potential of 60 volts across the parallel combination, the current through R_2 will equal E_p/R_2 = 60/12.5 = 4.8 amperes. The current in R_3 will equal E_p/R_3 = 60/50 = 1.2 amperes. The total current, I_t, is 4.8 + 1.2 = 6 amperes, as previously calculated. If the parallel combination had contained more than two resistors, a similar solution would have applied.

Fig. 5 is an example of a parallel combination in which one of

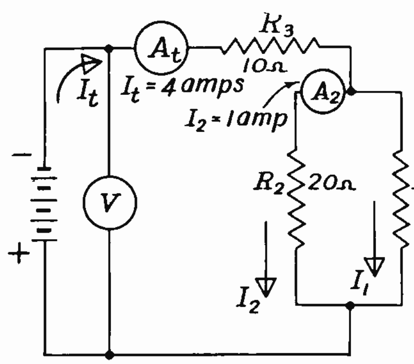


Fig. 5.--Series-parallel combination in which one of the parallel resistors and the battery voltage are unknown.

the parallel resistors and the battery voltage are unknown. The resistance of the single series element, R_3, is 10 ohms; the resistance of one parallel resistor, R_2, is 20 ohms; the series line current, I_t, is 4 amperes; and the current I_2 through the 20-ohm resistor, R_2, is 1 ampere. It is required to find the total circuit resistance, the battery voltage, the resistance of R_1, and the current through R_1.

First, solve the parallel combination. One branch of this combination (R_2) has a resistance of 20 ohms; the current through R_2 is 1 ampere; the voltage drop across the resistor is $I_2 R_2$ = 1 × 20 = 20 volts. Since the voltage across all branches of a parallel circuit are equal, the voltage across R_1 also equals 20 volts.

The total current through the circuit, as indicated by A_t, is 4 amperes. 1 ampere flows through R_2, the 20-ohm branch; the other 3 amperes *must* pass through R_1. Since the voltage across R_1 is 20 volts and the current through it is 3 amperes, R_1 must equal E_p/I_1 = 20/3 = 6.67 ohms.

The resistance of the parallel combination can now be found.

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{6.67 \times 20}{6.67 + 20} = 5 \text{ ohms}$$

The total resistance of the circuit is equal to $R_p + R_3$ = 5 + 10 = 15 ohms.

The voltage of the battery is equal to the product of the total resistance and the total line current, or $I_t R$ = 4 × 15 = 60 volts. This will be indicated by V. So far we have considered the battery as a source of voltage which is independent of the current drain and has no internal resistance. This will be clarified shortly.

Very complex combinations of resistance may be solved by the logical application of Ohm's Law and the rules governing series and parallel circuits similar to those illustrated above. In the solution of a complex circuit care must be taken to use only values of E, I, and R that apply to the particular part of the circuit under consideration.

GENERATED AND TERMINAL VOLTAGE.—At this point it will be of value to discuss a matter that is often puzzling to the student—namely, the *generated* and *terminal* voltage of an electrical source of energy. The source may be a rotating machine, a thermoelectric generator, a battery, etc. In any case the source must permit the same current to pass through it that it feeds to the external load, just as a water pump must pass the water it pumps through the rest of the system. The electrical source, like the water pump, may offer internal friction or resistance to the flow of current, or more generally, it may have internal impedance.

Thus, not only does an electrical source generate a voltage which forces the electrons around the closed circuit, but it also may develop an IR *voltage drop* owing to its internal resistance, and this voltage drop must be subtracted from the generated voltage to give the net or terminal voltage appearing across the generator's terminals, and available for forcing the current through the external part of the circuit. It is convenient to represent an actual generator having internal resistance in the form of two components as shown in Fig. 6 within the dotted line rectangle:

(a) An ideal generator (shown as a battery) having *no internal resistance*, and generating a voltage, e_G, and

(b) A resistance R_G representing the internal resistance of the actual generator. (In the general case R_G would be replaced by an impedance Z_G.)

The combination is shown feeding an external load R_L, when the

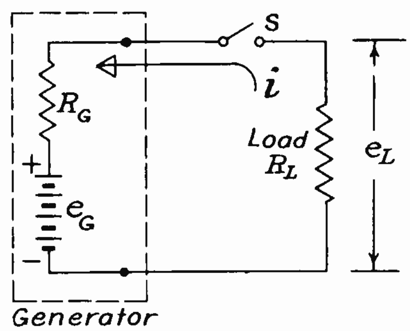


Fig. 6.--Representation of an actual generator by an ideal generator developing a generated voltage, e_G, and having an internal resistance, R_G, in series with an external load.

switch S is closed. The current i that flows produces an internal voltage drop i R_G in the generator, and an external voltage drop $e_L = i R_L$ across the load. It is clear from the figure that the sum of the voltage drops must equal the generated voltage, or

$$e_G = i R_G + i R_L$$

Factoring out i on the right-hand side, there is obtained

$$e_G = i(R_G + R_L)$$

Dividing both sides by the factor $(R_G + R_L)$, there is obtained

$$\frac{e_G}{R_G + R_L} = i$$

This shows that the current depends not only upon the generated voltage, e_G, and the external load, R_L, but also upon the internal resistance, R_G, of the generator. It is also clear from the figure, or by simple algebraic manipulations of the first equation, that

$$e_L = e_G - i\ R_G$$

This last equation states that the voltage measured across the load, R_L, and hence across the generator terminals is less than the generated voltage e_G by precisely the voltage drop $i\ R_G$ in the generator. In other words, when current is drawn from such a source by a load, a certain amount of the generated voltage is consumed within the generator, leaving less available for the external load.

As a specific example, assume $e_G = 9$ volts, $R_G = 0.3$ ohm, and $R_L = 17.7$ ohms. Then

$$i = 9/(0.3 + 17.7) = 9/18$$

$$= 0.5 \text{ ampere}$$

The voltage lost in the generator is

$$i\ R_G = (0.5)\ (0.3) = 0.15 \text{ volt}$$

and the terminal or external or load voltage is

$$e_L = e_G - i\ R_G = 9 - .15 = 8.85 \text{ volts}$$

Note that R_G is inside the generator and is not usually accessible for measurement. If the generator is a rotating machine, it can be stopped, whereupon its generated voltage ceases, and thus will not damage a resistance-measuring instrument. Suitable measuring instruments, such as a Wheatstone bridge (to be discussed later), can then be connected to the generator terminals, and the internal resistance (or impedance) measured. In the case of a battery, however, the generated voltage cannot be eliminated, and hence measurement of the internal resistance must be made in a more indirect fashion.

Similar considerations hold concerning the generated voltage e_G. This is inside the generator; the only accessible voltage is that developed at the terminals of the machine, namely, e_L. However, since

$$e_L = e_G - i\ R_G$$

if i is made zero, then

$$e_L = e_G - 0 \times R_G = e_G$$

i.e., if the switch in Fig. 6 is opened so that no current can flow through R_L, Then the terminal voltage will equal the generated voltage. Thus, the generated voltage may be called the "open-circuit" terminal voltage of the source. To measure it, a voltmeter is required that draws a negligible current, at least compared to that drawn by the normal value of load resistance R_L. This means that a high-resistance voltmeter is required, such as an electrostatic or a vacuum tube voltmeter.

Once e_G is measured by this means, the internal resistance can be determined by connecting a load to the generator, and measuring the current drawn as well as the now lower terminal voltage. Suppose that the open-circuit terminal voltage is 10 volts, and that when 2 amperes are drawn from the generator, the terminal voltage drops to 8 volts. Since

$$e_G - e_L = i\,R_G$$

then, dividing both sides by i, there is obtained

$$R_G = (e_G - e_L)/i$$

Substituting the values for the quantities on the right-hand side,

$$R_G = (10 - 8)/2 = 1 \text{ ohm}$$

In this way, by a series of measurements, both e_G and R_G can be determined.

In power work the internal resistance of the source (including the intervening distribution lines), is generally very low compared to the load resistance, in order that as little of the power be wasted in the source as possible. In this case the internal source resistance can be ignored, and this is often true of auxiliary power circuits employed in radio work. On the other hand, the actual communication circuits carrying the radio currents are more generally such that the load resistance is about equal to the internal source resistance (the so-called matching of impedances), and in this case the internal resistance can by no means be neglected.

One further point is of interest concerning the internal resistance of a storage battery. When current is drawn from the battery by an external load resistance, the internal voltage drop is *subtractive* from the generated voltage, as was shown above. On the other hand, when the battery is charged, the (electron) current flow is now in the reverse direction, as shown in Fig. 7. As a consequence, the voltage drop in R_G is now reversed and therefore *adds* to the battery's generated voltage e_G, so that the terminal voltage of the battery is

$$e_T = e_G + i\,R_G$$

The generator charging the battery must therefore deliver a voltage at the battery terminals *higher*

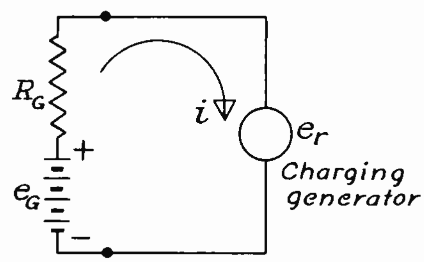


Fig. 7.—The voltage drop in R_G is in the same direction as e_G when the battery is charged.

than e_G by the i R_G drop, in order to charge the battery. The actual *generated voltage of the charging generator, however, must exceed* e_T by i R_G, where R_G is the internal resistance of the generator.

As a numerical example, suppose the battery voltage e_G is 12 volts; R_G, the internal resistance of the battery, is 0.2 ohm; and that of the charging generator, or R_G, is 0.8 ohm. Suppose a charging current of 4 amperes is desired. Then the voltage *across the battery terminals* is

$$e_T = 12 + (4)(.2) = 12.8 \text{ volts}$$

and the *generated voltage* of the charging generator is as follows:

$e_G = 12.8 + (4)(.8) = 16$ volts

TWO SIMPLE RULES.—From Ohm's Law two simple but very useful rules can be readily derived. Consider the simple series circuit

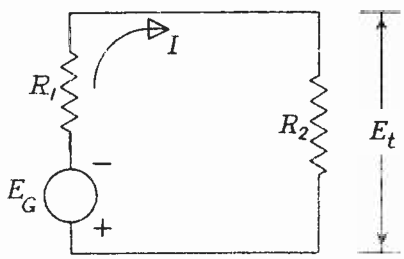


Fig. 8.--Simple series circuit used to derive relation between E_T and E_G.

shown in Fig. 8. A source generates a voltage, E_G, and R_1 may represent the internal resistance of the generator.* The problem is to find the voltage E_t developed across R_2 in terms of E_G, R_1, and R_2.

By Ohm's Law, the current that flows through the circuit is simply

$$I = E_G/(R_1 + R_2)$$

The voltage developed across R_2 is

$$E_t = I\,R_2$$

Substituting the value of I from the preceding equation, there is obtained

$$E_t = \frac{E_G}{(R_1 + R_2)} \cdot R_2 = E_G\left(\frac{R_2}{R_1 + R_2}\right)$$

This equation is the one desired; it states in words that

RULE 1: *The voltage across a resistance is equal to the total voltage multiplied by the ratio of the resistance under consideration to the total resistance of the circuit.*

As a simple example, suppose E_G = 20 volts, R_1 = 15 ohms, and R_2 = 25 ohms. Then the voltage across R_2 is

$$E_t = 20\left(\frac{25}{25 + 15}\right) = 12.5 \text{ volts}$$

A combination of series resistors is often called a voltage divider or *potentiometer* in that the voltage across part of the resistors is a fraction of the total voltage. These are often encountered in communication circuits: sometimes they are used deliberately to "divide down" the applied voltage, and sometimes their occurrence is not deliberate, but simply an accident or chance occurrence in the circuit. The student should be on the lookout for such combinations, as the simple formula given above can then be immediately applied.

A second rule is illustrated by Fig. 9. Here the applied voltage E_G causes a current I_t to flow through R_G, which may in part represent the internal resistance of the source. Current I_t then divides

*Actually R_1 may represent the internal resistance of the generator plus an additional resistor in series with it. Thus, the internal resistance may be 2 ohms, and 6 ohms may be in series with it. Then R_1 is 2 + 6 = 8 ohms. So far as R_2 is concerned, it is immaterial whether the generator's internal resistance is the full 8 ohms, or whether it is only 2 ohms and an additional 6 ohms are in series with it. In short, the generator resistance is treated in circuit analysis the same as any other resistance, although in practice it may not be directly accessible for measurement.

into two branch currents, I_1 through R_1, and I_2 through R_2. The ratio of I_1 to I_2, and of either branch current to I_t, depends upon the relative values of R_1 and R_2. To

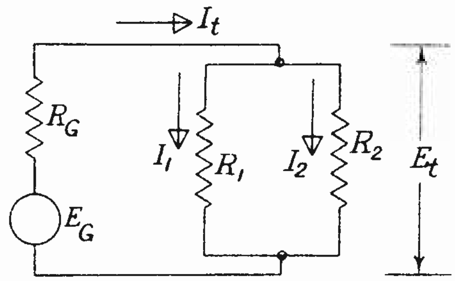


Fig. 9.--Parallel circuit from which the relation between branch and total currents can be derived.

find the value of these ratios, remember that the voltage across R_1 and R_2 is the same; denote it by E_t. Then

$$I_1 = E_t/R_1$$

$$I_2 = E_t/R_2$$

and

$$I_1 + I_2 = I_t = E_t/R_1 + E_t/R_2$$

Factoring out E_t:

$$I_t = E_t(1/R_1 + 1/R_2)$$

$$= \frac{E_t}{R_1R_2/(R_1 + R_2)}$$

which merely proves once again that the equivalent resistance for two resistances in parallel is their product over their sum.

The ratios I_1/I_2, I_1/I_t, and I_2/I_t can be now found by simply dividing the appropriate equation by the other appropriate equation, on the basis that equals divided by equals yield equal quotients. Thus, dividing the equation for I_1 by that for I_2 gives

$$I_1/I_2 = \frac{E_t/R_1}{E_t/R_2} = \frac{R_2}{R_1}$$

Note that

RULE 2: *The currents are in inverse ratio to the resistance of their branch paths; if R_2 is greater than R_1, then I_1 is greater than I_2.*

In similar manner there is obtained

$$I_1/I_t = \frac{R_1R_2/(R_1 + R_2)}{R_1} = \frac{R_2}{R_1 + R_2}$$

and

$$I_2/I_t = \frac{R_1}{R_1 + R_2}$$

An example of the use of this rule will be given further on in the section on ohmmeters. Another example is the resistance attenuation pad used to absorb a certain fraction

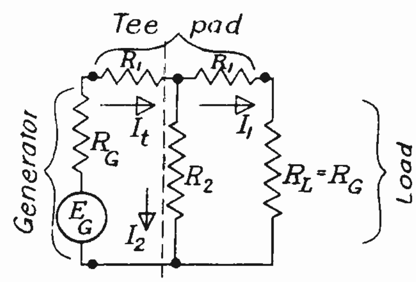


Fig. 10.--Application of current division principle to Tee-attenuation pad.

of the power delivered by a source. This is illustrated by Fig. 10. The attenuation network is called a Tee pad, from the fact that it looks like the letter T. It is interposed between the generator and the load, and the values of R_1 and R_2 are such that the generator "sees" its own internal resistance R_G when "looking into" the left-hand terminals of the pad, and the load R_L (= R_G) similarly sees its own resistance when looking into the right-hand terminals. The two are said to be matched in impedance. This feature of the pad is mentioned here merely as information; it has nothing to do with the problem at hand.

Of the total current I_t flowing from the generator, a certain amount I_2 is deliberately wasted in the pad, specifically in its shunt arm R_2, and the rest, I_1, flows through R_L. Depending upon the relative values of R_1, R_L, and R_2 the power that gets through to R_L is a fixed and known proportion of that coming from the source. The question is, "What fraction of I_t is I_1?"

This depends *in inverse proportion* to the resistance of the paths traversed by the two currents, as shown above. The paths involved are only those to the right of the dotted line. Thus, I_1 flows through R_1 and R_L in series; I_t flows through this path in parallel with R_2. The resistance of the I_1 path is therefore $(R_1 + R_L)$; that of the I_t path is

$$R_t = \frac{R_2 (R_1 + R_L)}{R_2 + R_1 + R_L}$$

Then

$$\frac{I_1}{I_t} = \frac{R_t}{R_1 + R_L} = \frac{R_2 (R_1 + R_L)}{R_2 + R_1 + R_L} \cdot \frac{1}{(R_1 + R_L)}$$

$$= \frac{R_2}{R_2 + R_1 + R_L}$$

Once the simple inverse rule of proportionality is remembered, many problems can be solved more quickly by its aid. In the above case, once the power, and hence the current division, are decided upon, the proper values of R_2 and R_1 can be found, as will be shown in a later assignment. For the moment, assume R_2 = 15 ohms, R_1 = 487 ohms, and R_L = 500 ohms. Then

$$\frac{I_1}{I_t} = \frac{15}{15 + 487 + 500} = \frac{15}{1002} = .01498$$

or I_1 is approximately 1 1/2% of I_t. As will be shown in a later assignment, the power ratio is as the square of the current ratio, or $(.01498)^2$ = .000224, which corresponds to .0224%.

APPLICATIONS TO RADIO

SERIES-DROPPING RESISTORS.—A common use of the simple series resistance in radio work is the voltage-dropping resistor used to decrease a given supply voltage to the correct value to be applied to the plate or screen grid of a vacuum tube.

Suppose that with a power supply of 1,500 volts it is necessary to furnish the plate voltage for several 50-watt radio frequency amplifiers, a 50-watt crystal-controlled oscillator, and two 250-watt amplifiers. All of these tubes are to have 1,500 volts on the plate with the exception of the crystal-controlled oscillator which requires 750 volts. The operating plate current of this tube with E_p of 750 volts will be 75 mils or .075 ampere.

Since the power source is 1,500 volts, and it is desired to use a plate voltage of 750 volts, it is necessary to expend 750 volts. This is done by placing a resistor of the correct value in series with the plate of the tube and the power supply. The correct resistance is that which, when multiplied by the current through it, will give a product equal to the desired voltage drop, in this case 750 volts. When the voltage drop and plate current are known, the resistance is computed by simple Ohm's Law, R = E/I. In this case, R = 750/.075 = 10,000 ohms.

Thus, if a resistance of 10,000 ohms is connected between the 1,500-volt terminal and the plate of the tube which is to be operated at 750 volts, when the tube circuit is so adjusted that the plate current is 75 mils, the applied plate voltage will be 750 volts. This, of course, does not affect the voltage applied to the other tubes. A resistor used in this way to reduce a voltage is usually referred to as a series-dropping resistance.

In selecting the 10,000-ohm series-dropping resistor care must be exercised to obtain a resistor capable of dissipating .075 × 750 = 56.25 watts continuously (P = IE). The power rating of a resistor is usually based on the maximum safe heat radiation with free air circulation on all sides of the resistor. A resistor required to dissipate 56.25 watts continuously should be rated at about 75 watts if mounted in a well-ventilated space, 100 watts if partly enclosed, and 150 watts if fully enclosed. Where maximum dependability is desired, a resistor rated at 3 or 4 times the dissipated power should be used.

Another familiar example of the use of a voltage dropping resistor is a circuit used for obtaining high percentage modulation in broadcast transmitters.

Without going into the theory of modulation circuits at this point it is sufficient to state, that in order to obtain high percentage modulation without overloading the modulator tube and thus introducing distortion, it is necessary that the plate voltage of the modulator tube be higher than that

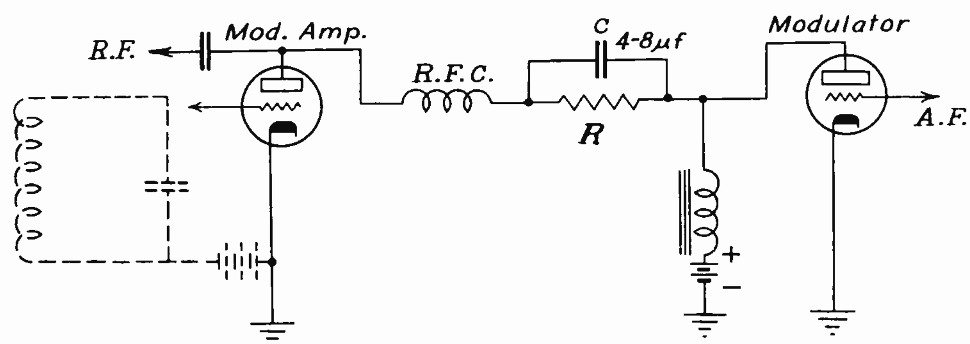


Fig. 11.--Modulator circuit.

applied to the plate of the amplifier tube being modulated. Since the plates (modulator and amplifier) are connected to the same source of power, it is apparent that the d.c. voltage applied to the amplifier tube must be decreased in some manner.

This is usually done by connecting the proper value of resistance in series between the common power supply to the modulator and amplifier and the plate of the amplifier which is being modulated. In order that this resistance will not decrease the a.c. component of voltage supplied to the plate of the modutated amplifier by the modulator, the resistor is by-passed by a large capacity, usually from 4 to 8 µf. The circuit is shown in Fig. 11.

The d.c. plate voltage for the modulated amplifier is supplied through the voltage dropping resistor (R), and the a.c. component of modulating voltage is supplied through the by-pass condenser (C).

The required series resistance is computed exactly as in the preceding example from the value of the amplifier plate current and the voltage to be lost or dissipated. Assume that the modulator tube is to operate at 10,000 volts, the modulated amplifier at 7,000 volts, and that under these conditions the normal amplifier plate current will be .5 ampere. It will be necessary to use a dropping resistance that will decrease the voltage by 10,000 - 7,000, or 3,000 volts when the current through the resistance is .5 ampere. The value of R will be 3,000/.5 = 6,000 ohms. Power rating should be IE, or .5 × 3,000 = 1,500 watts times three, or 4,500 watts for safety.

METER SHUNTS.—Almost every radio engineer finds it necessary, at some time or other, to change the scale of an ammeter or milliammeter so that it will have a different range. For example, it may be desired to use a one-milliampere meter (one in which a full-scale deflection is obtained with meter current of one milliampere) to measure several ranges of current and voltage. Suppose the meter is a Weston Model 301 having a full scale reading of 1 milliampere and a resistance of 27 ohms. It is desired to be able, by means of a switch, to obtain full scale deflection with 1 milliampere, 50 milliamperes, and 250 milliamperes.

On the first scale, 1 milliampere, the meter is used without change.

On the second scale, full deflection at 50 milliamperes, a

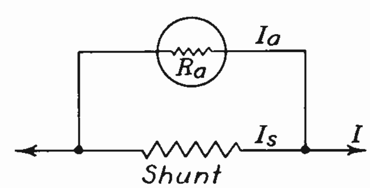


Fig. 12.--Use of ammeter shunt to increase the range of the meter.

shunt (parallel resistance) must be used. The arrangement is shown in Fig. 12. Since the meter current must be limited to 1 milliampere, and since 50 milliamperes must be measured, if the line current is I, and the meter current is I_a, then the shunt current, I_s, must be

$$I_s = I - I_a = 50 - 1 = 49 \text{ ma.}$$

Since a current through the meter for full-scale deflection is one milliampere, and the meter resistance is 27 ohms, the voltage drop across the meter or points, AB, at full scale reading is, $I_a R_a = .001 \times 27 = .027$ volt. The shunt is connected in parallel with the meter so the voltage drop across the shunt will also be .027 volt when the shunt current is 49 ma. $R_s = E/I_s = .027/.049 = .551$ ohms.

This shunt can be made from ordinary enamel-covered copper wire. Allowing 1,500 CM per ampere to avoid serious variation in the shunt resistance due to temperature change, the wire diameter should be not less than $\sqrt{.049 \times 1,500}$ or approximately 9 mils. No. 30 annealed copper wire has a diameter of 10 mils and a resistance of 103.2 ohms per 1,000 feet. For a resistance of .551 ohm a total of (.551/103.2) 1,000 = 5.34 feet of No. 30 wire will be required. This could be random wound in a small flat spool and then connected directly across the meter terminals. The resistance of this shunt would vary with temperature as described in a previous assignment, but will serve for reasonably accurate current measurements, particularly since the copper coil in the meter varies in similar fashion with temperature.

A simple formula for calculating the shunt resistance for any desired meter range is

$$R_s = \frac{I_a R_a}{I_s}$$

where I_a is the meter current for full scale deflection,
R_a is the meter internal resistance,
I_s is the current in the shunt.

From Fig. 12 this formula can be seen to be identical to

$$R = \frac{E_{AB}}{I_s}$$

where

$$E_{AB} = I_s R_s = I_a R_a$$

For example, the shunt to be used for full-scale deflection with 250 ma is calculated as follows:

The meter current is still 1 milliampere, therefore,

$$I_s = I - I_a = 250 - 1 = 249 \text{ ma.}$$

$$R_s = \frac{I_a R_a}{I_s} = \frac{1 \times 27}{249} = .108+ \text{ ohms}$$

By the use of the proper shunt resistance the 1 milliampere meter could be made to give full-scale deflection with any desired current larger than that required for the meter itself. It will be seen that the shunt resistance required, even for a meter having quite high resistance, may be very small. Thus, if a high degree of accuracy is required, the resistance of the shunt must be very accurately determined and actually obtained in construction, and all the connections and leads between the shunt and the meter must be either of such low resistance as to be negligible, or else they must be taken into consideration in the calculation. For example, if an ammeter having a given low resistance is to be mounted on a switchboard several feet from its shunt, the resistance of the connecting leads may be an appreciable percentage of the meter resistance. The resistance of the connecting leads must be carefully measured, and this resistance added to R_a before calculating the value of shunt resistance. It is also essential that all connections be good and have neglegible resistance.

The milliammeter, as described above, would be used with a switching arrangement so that it could be used alone for 1 milliampere full scale, with the .551-ohm shunt for 50 milliamperes full scale, and with the .108-ohm shunt for 250 milliamperes full scale. After the shunts have been constructed and installed, the meter ranges should be checked against an accurate standard at several points on the scale for each range.

VOLTAGE MULTIPLIERS.—It may be desired to use the one-milliampere meter discussed above for a multiple range voltmeter as well as for a multiple range milliammeter. For use as a voltmeter the meter must be connected *across the line* with a large value of resistance

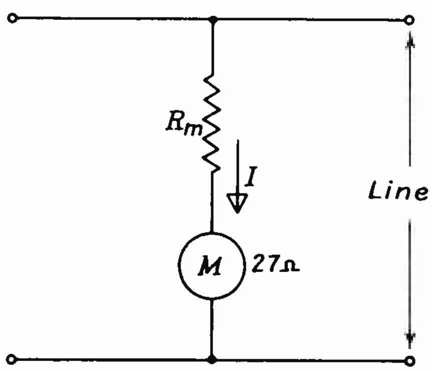


Fig. 13.--Use of series resistor to made a voltmeter from a one-milliampere meter.

in series. The connection is shown in Fig. 13. The meter to be used has a resistance R_a of 27 ohms. A multiplier resistance R_m is to be connected in series with the meter across the line. The value of this resistance must be such as to limit the current in the meter to 1 milliampere when the highest desired voltage is to be measured. Thus, if the meter is to be connected with a switching arrangement to give full scale deflection at 50 volts, 250 volts, and 1,000 volts, three separate multiplier resistances must be used. The total resistance R_t must be such as to limit the current to the full-scale value I_m when the desired full-scale value of voltage, E_m, is applied.

By Ohm's Law,

$$R_t = E_m/I_m$$

But R_t is the sum of the meter and additional series resistance, or

$$R_t = R_m + R_a$$

from which

$$R_m = R_t - R_a$$

Substituting the value for R_t found previously:

$$R_m = \frac{E_m}{I_m} - R_a$$

For 50-volt full-scale deflection,

$$R_m = \frac{50}{.001} - 27 = 50,000 - 27 \text{ ohms}$$

For 250-volt full-scale deflection,

$$R_m = \frac{250}{.001} - 27 = 250,000 - 27 \text{ ohms}$$

For 1,000-volt full-scale deflection,

$$R_m = \frac{1000}{.001} - 27 = 1,000,000 - 27 \text{ ohms}$$

It will be seen that with a meter of such low resistance and with such a low current for full-scale deflection, for the voltage ranges as shown, the meter resistance, R_a, may be neglected. If the multiplier resistance is 100 or more times greater than the meter resistance, the error introduced by neglecting R_a is less than 1 per cent. However, it must be remembered that the calibration of the voltmeter can only be as accurate as the resistance calibration. Since ordinary commercial resistances often have calibration tolerances as great as plus or minus 10 per cent (±10%), such resistances should not be used unless carefully selected by accurate measurement and known to be correct. Precision resistances manufactured within one per cent of rated value may be obtained. For high precision instruments the multiplier resistances are wire-wound non-inductively on spools to exact values of resistance.

When assembling a multi-range meter, the greatest care must be exercised after the multiplier and shunt resistors have been selected or built to keep all soldered connections and switch contacts good. Where a shunt resistance is only a small fraction of an ohm, a poorly soldered connection can easily change the total resistance by an appreciable percentage.

OHMMETERS.—An ohmmeter is a device for measuring resistance directly. The basic principle of operation is that the current in any series circuit is inversely proportional to the circuit resistance. The average ohmmeter cannot very well be classed as an instrument of precision, but where resistance measurements are to be made within the usual commercial tolerances, the ohmmeter has the advantage of saving considerable time. Where extreme accuracy of measurement is of prime importance, the Wheatstone bridge method of measuring resistance (discussed later in this assignment) is to be preferred.

Ohmmeters commonly used in commercial practice may be either of the series or shunt type, the former being the most extensively used. A simple portable series ohmmeter may be readily constructed using a 4.5-volt dry cell battery in connection with a 0-1 milliammeter. Fig. 14 shows the simple series circuit employed. To obtain full-scale deflection of the meter when the terminals are short-circuited, it is necessary for the total circuit resistance to be E/I = 4.5/.001 = 4,500 ohms. Since the meter has an internal resistance of 27 ohms, an additional resistance of 4,500 - 27 = 4,473 ohms must be inserted in the circuit. To provide for a reasonable amount of adjustment to take care of battery voltage and internal resistance

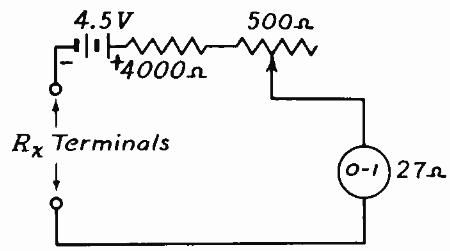


Fig. 14.--Simple series type of ohmmeter.

variations, it is desirable to have part of the total circuit resistance in adjustable form. A suitable arrangement would be a 4,000-ohm fixed resistance and a 500-ohm rheostat as shown in the figure. The adjustable resistance should not be too large, otherwise the meter may be burned out if the rheostat is set at zero resistance and the external terminals short-circuited. If this should happen in the case of Fig. 14, the current through the meter would be 4.5/4,027 = 1.11 ma. Thus, the meter would be overloaded about 11 per cent, a not too dangerous figure. However, the battery voltage can be expected to decrease and the internal resistance to increase with age and use, and the value of adjustable resistance should be such as to compensate for about a 10 per cent decrease in battery voltage. It is the variation in battery voltage that reduces the accuracy of the ohmmeter, since the meter scale is usually calibrated on a basis of exactly 4.5 volts. As the battery voltage decreases, the meter will read *high* because the meter reading varies inversely with the resistance being measured. The internal resistance of the battery increases with age and decreases the terminal voltage, but this can be compensated for by decreasing the resistance in the external circuit; specifically, the 500-ohm rheostat.

Referring again to Fig. 14, with the external terminals short-circuited the meter is adjusted to full-scale reading by means of the rheostat. When R_x is connected across the terminals, the meter reading will then be something less than full scale, the amount of deflection varying inversely with the value of R_x. The magnitude of the unknown resistance may be readily calculated from the observed meter reading by the use of simple inverse proportion.

$$I : I_a : : R : (R + R_x)$$

where I is the reading with the unknown resistor in the circuit, I_a the meter current for full-scale deflection, and R the total circuit resistance with the external terminals short-circuited.

For example, assume the meter reads .1 ma. when R_x is in the circuit. Then

$$.1 : 1 : : 4{,}500 : (4{,}500 + R_x)$$

or

$$.1/1 = 4{,}500/(4{,}500 + R_x)$$

from which

$$4{,}500 = .1(4{,}500 + R_x)$$

$$4{,}500 = 450 + .1\,R_x$$

$$R_x = 10(4{,}500 - 450)$$

$$R_x = 40{,}500 \text{ ohms}$$

The value of R_x corresponding to a number of meter readings (as, for example, in steps of .1 ma. from .1 to 1 ma.) may be calculated, and a chart or curve constructed showing the relationship between meter reading and R_x from which the unknown resistance values can be quickly determined for any meter reading. The calibration may be made directly on the meter scale, if desired, eliminating the need of a curve or chart. Special volt-ohm-milliampere scales are available for most commercial meters if it is desired to personally construct and calibrate such an instrument.

The useful resistance range of the circuit shown in Fig. 14 is approximately 200 to 300,000 ohms. If it is desired to obtain a lower range of resistance measurements, the meter and calibrating resistance may be shunted with a suitable resistance as shown in Fig. 15 by R_s.

In a multi-range instrument it is desirable to make all scales in similar units multiples or submultiples of each other. Assuming that the meter in Fig. 14 is calibrated from 200 to 300,000 ohms, it is desirable to make the next lower range 20 to 30,000 ohms, so a multiplying factor of .1 may be used. This means the total circuit current for full-scale reading of the meter must be 10 times greater or 10 ma. Since the meter reads full scale at 1 ma. then 9 ma. must flow through the shunt resistance R_s. Since R_s is connected across 4,500 ohms to carry 9 times more current, it must have only 1/9 as much resistance. 1/9 times 4,500 = 500 ohms. The effective resistance of 4,500 ohms in parallel with 500 ohms is

$$\frac{4500 \times 500}{4500 + 500} = 450 \text{ ohms}$$

Assume that the meter reading with an unknown value of resistance connected across the external terminals is again .1 ma. Then

$$I : I_a :: R : (R + R_x)$$

$$.1 : 1 :: 450 : (450 + R_x)$$

$$450 = .1(450 + R_x)$$

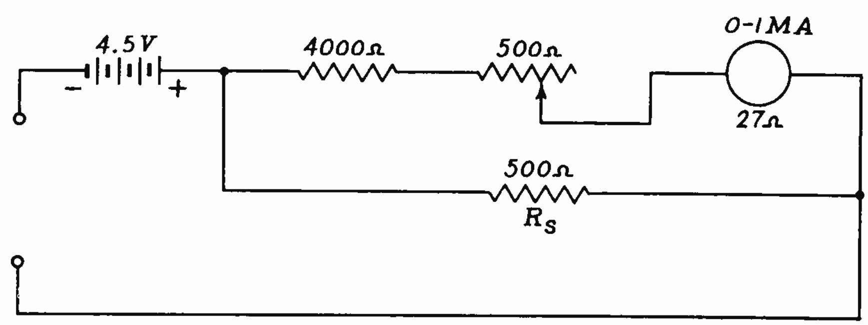


Fig. 15.--Use of shunt resistor to obtain lower range on an ohmmeter.

$$R_x = 10(450 - 45)$$

$$R_x = 4{,}050 \text{ ohms}$$

Note that R_x for Fig. 15 is just .1 that of R_x for Fig. 14 for the same meter reading. Therefore, the range of the circuit in Fig. 15 is 20 to 30,000 ohms, and if the circuits of Figs. 14 and 15 are combined with a suitable switching arrangement, the instrument will read from 20 to 300,000 ohms in two separate ranges.

A still lower range may be obtained by substituting a 45.5-

ohm resistor for the 500-ohm shunt of Fig. 15. The multiplying factor then becomes .01, and the resistance range is from 2 to 3,000 ohms. The value of current required from the battery for full-scale deflection will now be 100 ma. although only 1 ma. flows through the meter. However, the addition of this third range has several disadvantages. First, care must be exercised when measuring the resistances of devices which have a limited current-carrying capacity. For example, a receiver choke coil wound with No. 30 or No. 32 wire is incapable of carrying 100 ma. without seriously overheating even for a short period of time. Second, 100 ma. is an excessive load current for the ordinary 4.5-volt bias battery, and if this low resistance range is used to any extent, the life of the battery is short. Third, the internal resistance of the battery is negligible so far as the two upper resistance ranges are concerned, but in the low range the battery resistance may be an appreciable part of the total circuit R. This will cause the error in readings to increase very rapidly, as the battery internal resistance increases with battery age.

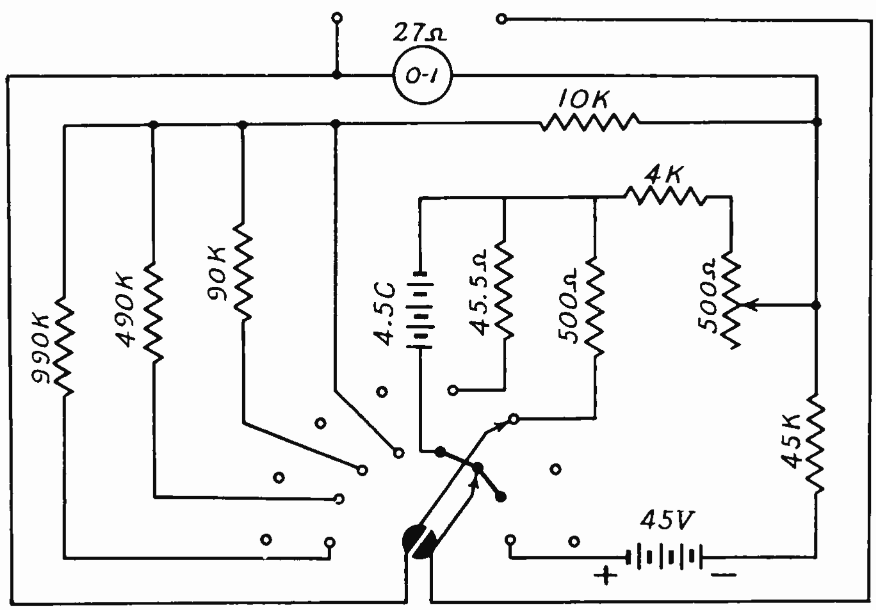


Fig. 16.--Multi-range volt-ohmmeter circuit.

If a higher range of resistance measurements is desired, the circuit of Fig. 14 can be given a multiplying factor of 10 by substituting a 45-volt battery and a 45,000-ohm resistor for the combination of 4.5 volts, 4,000-

ohm fixed resistor and 500-ohm rheostat. The rheostat will not be needed on this high range unless more than ordinary accuracy is desired. The useful range of this combination will be 2,000 to 3,000,000 ohms.

A very useful and convenient multi-range volt-ohmmeter circuit is shown in Fig. 16. This instrument will provide the essential voltage and resistance measurements and continuity tests encountered in routine radio set and circuit testing. The following voltage and ohmmeter ranges are provided.

VOLTAGE RANGES	OHMMETER RANGES
10	2 - 3,000
100	20 - 30,000
500	200 - 300,000
1,000	2,000 - 3,000,000

The shunt type of ohmmeter is best adapted to measuring low and medium values of resistance without imposing a high current drain upon the battery source or forcing an objectionably large current through the device whose resistance is being measured. A circuit for a simple single range ohmmeter of the shunt type is shown in Fig. 17. Since a 4.5-volt battery and a 1-milliampere meter are employed, the values of the fixed and adjustable resistances will be the same as for the series type previously considered. The shunt type ohmmeter derives its name from the fact that the unknown resistance is shunted across the meter as indicated by R_x. A switch is provided for opening the battery circuit when the meter is not in use to increase the useful life of the battery. The rheostat is adjusted to give full-scale deflection after closing the battery

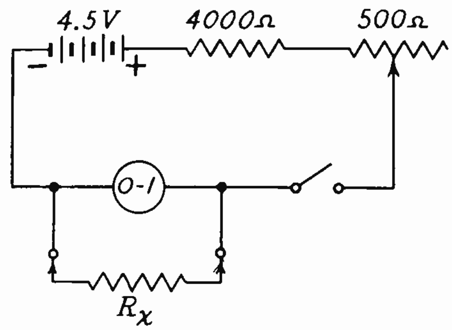


Fig. 17.--Single-range, shunt-type ohmmeter.

switch but with the external resistance not connected. When an external resistance is connected across the meter, a part of the 1 milliampere of current flowing from the battery is shunted through the

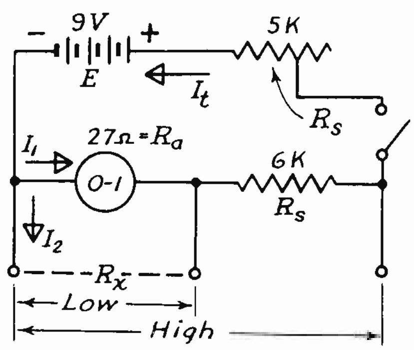


Fig. 18.--Double-range ohmmeter.

unknown resistance, and the meter deflection will be decreased by an amount depending upon the value of the shunting resistance. Note that as the value of the unknown resistance R_x is decreased, the deflection of the meter decreases, so that the meter deflection varies directly with the value of R_x, while in the series type of ohmmeter the meter deflection varies inversely with R_x.

A very useful form of double range ohmmeter is shown in Fig. 18. A 9-volt battery is employed in connection with a 1-milliampere meter so that the total resistance in the meter circuit is required to be 9/.001 = 9,000 ohms. A suitable arrangement would be to use a 6,000-ohm fixed resistor and a 5,000-ohm rheostat which is used to adjust the current to give full-scale deflection when the circuit switch is closed. An advantage of this type of circuit is the improved accuracy obtained when the battery voltage begins to fall after a period of normal use. The meter may be calibrated by the use of a number of standard resistors or a decade box. If such are not available, the calibration may be calculated by the following expression: (Using the Low Range)

$$R_x = \frac{R_s R_a/(R_s + R_a)}{(I_a/I) - 1}$$

where R_x = unknown resistance.
R_a = resistance of meter element.
R_s = resistance in the battery branch which is in parallel with the unknown resistor R_x.
I_a = full-scale reading of meter.
I = meter reading (less than I_a) when R_x is connected.

In most cases, R_a, will be very low compared to R_s, so that the latter shunts R_a to a negligible extent, and the parallel combination represented by the expression

$$R_s R_a/(R_s + R_a)$$

reduces to simply R_a. (Although in Fig. 18 R_s does not appear to shunt R_a, nevertheless the formula shows that the behavior of the instrument is as if it did.) If R_a is very low compared to R_s, the above formula may be simplified to

$$R_x \cong \frac{R_a}{(I_a/I) - 1}$$

The symbol $\cong$ means "approximately equal to". Note that in this simplified formula, R_s is absent, or the meter reading I, corresponding to R_x, is independent of R_s.

Suppose the meter shows a deflection of .3 ma. when an unknown resistance is shunted across the low-range terminals. The value of this unknown resistance may be obtained from the above formula. Since (reference to Fig. 18) R_a = 27 ohms, and R_s will have a value between 5,000 + 6,000 = 11,000 and 6,000 ohms, depending upon the setting of the rheostat arm, it is clear that R_s will be a negligible shunt across R_a, and hence the simplified formula may be used:

$$R_x = \frac{27}{(1/.3) - 1} = 11.6 \text{ ohms}$$

The derivation of the above formulas is an interesting application of algebra, and is not difficult, particularly if the steps are taken in systematic sequence. When R_x is not connected, the current flows through R_a and R_s in series. By adjusting R_s to the proper value, the current can be set to the full-scale value of I_a. Hence (by Ohm's Law)

$$I_a = E/(R_a + R_s)$$

where R_s, as stated above, is of such value that E causes I_a ma. to flow. Multiplying through by $(R_a + R_s)$, there is obtained

$$E = I_a(R_a + R_s)$$

When R_x is connected in parallel with R_a, the greater current I_t flows such that

$$I_t = \frac{E}{\dfrac{R_a R_x}{R_a + R_x} + R_s}$$

$$= \frac{E}{\dfrac{R_a R_x + R_s(R_a + R_x)}{R_a + R_x}}$$

$$= \frac{E(R_a + R_x)}{R_a R_x + R_s(R_a + R_x)}$$

Substitute the preceding expression for E, and obtain

$$I_t = \frac{I_a(R_a + R_s)(R_a + R_x)}{R_a R_x + R_s(R_a + R_x)}$$

Now use Rule 2 to find the fraction of the current I_t, that flows through the meter. This fraction—call it I—will be less than the previous full-scale meter current I_a. Thus, by Rule 2

$$\frac{I}{I_t} = \frac{R_a R_x/(R_a + R_x)}{R_a} = \frac{R_x}{R_a + R_x}$$

from which, multiplying through by I_t, there is obtained

$$I = \frac{I_t R_x}{R_a + R_x}$$

Substitute the expression found previously for I_t and obtain

$$I = \frac{I_a(R_a + R_s)(R_a + R_x)}{R_a R_x + R_s(R_a + R_x)} \cdot \frac{R_x}{R_a + R_x}$$

$$= \frac{I_a(R_a + R_s)R_x}{R_a R_x + R_s(R_a + R_x)}$$

It is assumed that I_a, R_a, and R_s are known, and that a value of I is now assumed. It is desired to know what value of R_x will correspond to this value of I. Thus, for every value of I read on the meter, the corresponding value of R_x can then be computed. Algebraically, the above statement means that it is desired to solve for R_x in terms of the other quantities. To do so, it is advisable to multiply out the terms in the denominator, and then re-factor so as to have R_x as a common factor instead of R_s. Thus,

$$I = \frac{I_a(R_a + R_s)R_x}{R_a R_x + R_s R_a + R_s R_x}$$

$$= \frac{I_a(R_a + R_s)R_x}{R_x(R_a + R_s) + R_s R_a}$$

Now multiply through by the denominator $R_x(R_a + R_s) + R_s R_a$:

$$I\,R_x(R_a + R_a) + I\,R_s R_a$$

$$= I_a\,R_x(R_a + R_s)$$

Transpose all terms containing R_x to the left-hand side of the equation; and all terms not containing R_x, to the right-hand side, and then factor out R_x:

$$I\,R_x(R_a + R_s) - I_a\,R_x(R_a + R_s)$$

$$= -\,I\,R_s R_a$$

$$R_x(R_a + R_s)(I - I_a) = -\,I\,R_s R_a$$

Now divide through by $(R_a + R_s)(I - I_a)$ and obtain:

$$R_x = \frac{-\,I\,R_s R_a}{(I - I_a)(R_a + R_s)}$$

Divide the numerator and denominator by -I:

$$R_x = \frac{R_s R_a}{R_a + R_s} \cdot \frac{1}{\left(-1 + \frac{I_a}{I}\right)}$$

$$= \left(\frac{R_s R_a}{R_a + R_s}\right)\left(\frac{1}{\frac{I_a}{I} - 1}\right)$$

If $R_s \gg R_a$ (read R_s much greater than R_a), this reduces--as stated previously--to

$$R_x = \frac{R_a}{\left(\frac{I_a}{I} - 1\right)}$$

This derivation points out a very important fact to the radio man. As long as R_a is very low compared to R_s, the simplified formula is satisfactory. At the same time, consider the instrument from a physical point of view: I_a is the (full-scale) meter current when R_x is not connected across the meter; and when R_x is connected to the meter, it diverts some of the battery current through itself, thus reducing the meter current to a value I less than the full-scale reading I_a. The amount by which I is less than I_a depends upon how low R_x is compared to R_a; i.e., how much of a shunt R_x is across the meter. If R_x is very high, then I will be very nearly I_a, and meter readings for various values of R_x will be crowded near the top end of the scale, so that the accuracy of the instrument will be poor.

In order to improve the accuracy when R_x is high, it is necessary to increase the meter resistance R_a to a correspondingly higher value. Readings will then be lower down on the scale (I will be appreciably less than I_a), and variations in the value of R_x will then produce greater changes in the meter readings. Hence, for the "high" scale, the meter resistance must be increased.

This is done artificially by adding a resistance in series with the meter and connecting R_x across the two in series. Refer to Fig. 18, and note that for the high scale, the terminals across which R_x is to be connected include not only R_a, the actual meter resistance, but the 6K (6,000) ohm resistor as well. The new value of meter resistance is now $R_a' = R_a + 6{,}000$ instead of R_a.

However, the value of R_s, the series resistor, is now less. For the low scale it was 6,000 ohms plus the amount of resistance of the 5,000-ohm rheostat; now it is simply the resistance of the latter, and the 6,000 ohms has now become part of the meter resistance R_a'. R_a' is now, if anything, larger than the new value of R_s, *and hence the simplified formula no longer holds*, and the previous more accurate formula must be used. This points out the fact previously cited, namely, that in using a formula, it is imperative that the engineer have a clear idea as to the range of its applicability, and does not attempt to use it where it no longer applies. The simplified formula is perfectly satisfactory for the low range; for the high range the more extended formula must be used.

To show this in actual figures, calculate the setting required of the rheostat; this will be the new value of R_m. The current determining this is I_a = 1 ma., and E = 9 volts, so that the *total* resistance was previously found to be 9,000 ohms. The meter resistance is now $R_a' = 6{,}000 + 27 = 6{,}027$ ohms. Therefore, the new value of R_s is 9,000 - 6,027 = 2,973 ohms. It is this value of R_s that must be taken in parallel with R_a':

$$\frac{(2973)\,(6027)}{6027 + 2973} = \frac{(2973)\,(6027)}{9000}$$

$$= 1{,}991 \text{ ohms}$$

Now suppose a value of R_x is connected across the "high" terminals of such value that I = 0.3 ma. Then

$$R_x = \frac{1991}{(1/.3) - 1} = 853 \text{ ohms}$$

If the more approximate formula had been used, then there would be obtained

$$R_x = \frac{R_a'}{(I_a/I) - 1} = \frac{6027}{(1/.3) - 1} = 2{,}579 \text{ ohms}$$

or more than three times the correct value of 853 ohms.

The instrument shown will have a range of approximately .5 to 1,500 ohms on the low range, and 75 to 300,000 ohms on the high range scale.

If a still higher range is desired, a simple rectifier power supply may be added to it and used as a series type of ohmmeter which will enable measurements to be made up to approximately 30 megohms, as indicated in Fig. 19. This higher range ohmmeter will be found useful in servicing modern receivers. If the output of the rectifier power is adjusted to 450 volts, the scale range will be given a multiplying factor of 10 times that of the highest range of the series ohmmeter described previously. The required resistance of the measuring circuit must now be 450/.001 = 450,000 ohms. A suitable arrangement would be to use a fixed resistance of 400,000 ohms in series with a 100,000-ohm rheostat as indicated in the diagram. The calibration of this high reading scale may be calculated as before, or obtained by the use of known standard resistances. Note from Fig. 19 how the two sources of power are connected to the same meter, yet act independently of one another.

Table I shows the internal

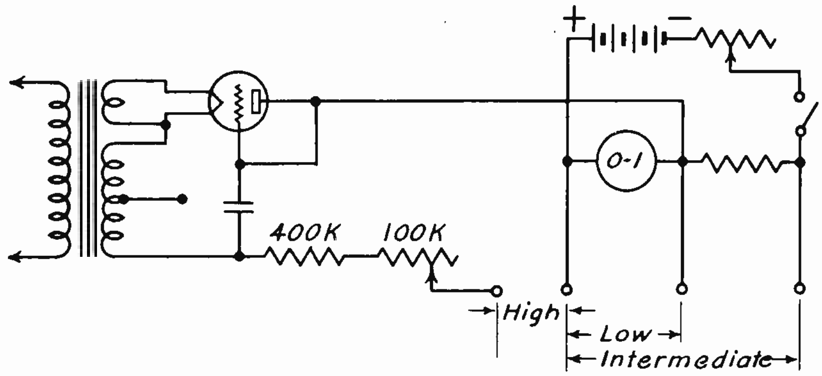


Fig. 19.--Use of a rectifier power supply to obtain a higher range on an ohmmeter.

resistance (resistance of moving element) of some of the more common types of commercial micro and milliammeters.

high-voltage plate supplies for transmitters, high-voltage rectifiers, etc.

Fig. 20 shows a typical volt-

TABLE I

CONSTANTS OF SOME COMMON METERS

RANGE IN MA.	RESISTANCE (IN OHMS) OF MOVING ELEMENT					
	WESTON 3 INCH "301"	JEWEL "88"	WESTON 2 INCH "506"	READRITE	TRIPLETT	SUPREME (WESTINGHOUSE)
.2	55	140	--	--	360	1,080
.3	55	140	--	--	--	--
.5	55	140	--	--	156	--
1.0	27	30	27	--	33	57.2
1.5	18	30	18	--	22	38.6
2	18	25	18	--	--	15.1
3	18	20	18	--	11	10.4
5	12	12	8.5	2,160	8.5	3.7
10	8.5	7	3.2	500	3.1	2.1
15	3.2	5	.15	295	2.0	1.5
20	1.5	4	--	--	--	2.5
25	1.2	3	--	85	1.2	2.0
30	1.2	2	--	--	--	1.67
50	2.0	1.5	1.0	20	.6	1.0
100	1.0	.75	.5	5.5	.3	.5
150	.66	.5	.33	3.3	.23	.33
200	.5	.37	--	1.9	.15	.25
250	.4	.3	--	--	--	.2
300	.33	.25	.16	.8	.1	.17
500	.2	.15	.1	--	.06	.1
1,000	.1	.1	--	--	.03	.05

(The Triplett Universal Meter = 100 ohms, 1 milliampere scale.)

KIRCHHOFF'S LAWS

VOLTAGE-DIVIDER CIRCUITS.—Voltage-divider circuits are in use in all types of radio apparatus. Sucl circuits are used in the plate supply of a.c. operated receivers, age-divider circuit, the principle of which is the same as in the power pack voltage-divider circuits in modern a.c. operated receivers. and in the rectifier power supply circuits of modern transmitters. The only differences will be in the number of different voltages that are desired,

At any point in a circuit, there is as much current flowing *away* from the point as there is flowing *to* it.

Kirch. 1st

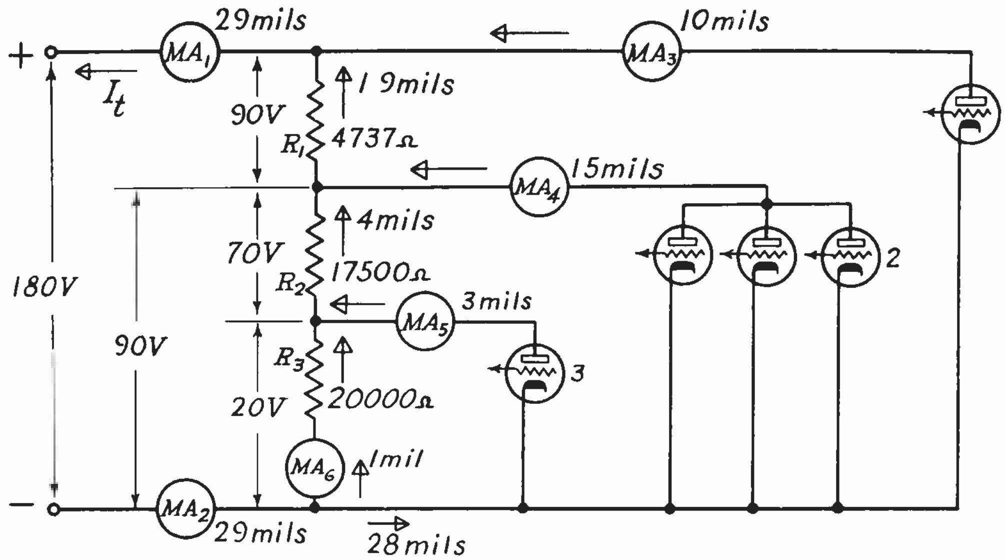


Fig. 20.--Typical bleeder-type voltage-divider circuit.

and in the values of currents, voltages, and resistances to be obtained. Note, however, that these voltage-divider circuits differ from that previously discussed under Rule 1 in that one of the resistors has a load (that of the vacuum tubes) connected in parallel with it.

The calculations by which the voltage-divider circuit constants are computed are based on Kirchhoff's Laws. These laws are merely extensions of Ohm's Law and may be so considered. They provide a means of handling problems dealing with the relation of current, resistance, and voltage in the more complex circuits.

Kirchhoff's First Law states:

Kirchhoff's Second Law states: The sum of the several IR drops around any one path of an electric circuit equals the sum of the EMF's impressed on that same path. Another way of stating this is: The sum of all the voltage drops around a circuit is equal to the applied voltage.

Kirch. 2nd

Both of these laws apply to the voltage-divider circuit. Fig. 20 shows a circuit supplying plate voltage to five receiving tubes. The total source voltage, with the normal operating load, is 180 volts. (Calculations for any combination of tubes, voltages, and currents may be made by exactly the same principles as explained in this problem.) One tube operates directly from the full voltage and requires a plate current of 10 milliamperes. Three tubes require 5 milliamperes each, or a total of 15 milliamperes at 90 volts. One tube requires 3 milliamperes at 20 volts. There is an additional current of 1 milliampere flowing through R_3 as a leakage or "bleeder" current to provide the drop across R_3. The reason for this latter current will be discussed later. Thus, a total of 29 milliamperes must be supplied by the rectifier. The proper resistances must be provided to drop the voltage to the correct value for each part of the load. The arrows indicate the electron flow in the circuit.

In computing the necessary values of resistance, the current load for each portion of the circuit must be known. The load currents with the correct voltages at each point are shown in the circuit diagram with the meters to indicate the individual currents.

The problem is to calculate the values of resistance necessary to provide the specified voltages with the currents as indicated. It will be seen that the current through Tube 1 does not flow through any of the divider resistors. The current of Tube 2 (actually 3 tubes) flows through R_1. The current of Tube 3 flows through R_1 and R_2. R_3 carries only the bleeder current necessary to provide the 20-volt difference of potential for Tube 3 from the negative side of the circuit.

To provide the 20 volts required for Tube 3, the difference of potential is obtained by inserting the correct value of resistance at R_3. The resistance required will be equal to E/I = 20/.001 = 20,000 ohms.

Next calculate the resistance of R_2. R_2 must carry the bleeder current through R_3 in addition to the load current of Tube 3. The current of Tube 3, 3 milliamperes as indicated by MA_5, and the bleeder current of 1 milliampere as indicated by MA_6, flow *into* the bottom point of R_2. According to Kirchhoff's Law, the same 4 milliamperes must flow *away* from this point or through R_2.

The power supplied to Tube 2 is to be delivered at a voltage of 90 volts, with respect to the negative end of the circuit. There is already a 20-volt drop in R_3, so 90 - 20 = 70 volts must be the drop across R_2. It will be necessary to use a resistance of the correct value to provide a 70-volt drop with a current flow of 4 milliamperes. Thus, R_2 = 70/.004 = 17,500 ohms.

To calculate the resistance of R_1 it is first necessary to calculate the current flow through it. This current will be the sum of the currents from Tube 2, Tube 3, and the bleeder current. The latter two currents, totaling 4 milliamperes, flow through R_2. The current from Tube 2 is indicated by MA_4 and is 15 milliamperes. Thus, the total current through R_1 will be 19 milliamperes.

The total applied voltage is 180 volts. The plate potential of Tube 2 is to be 90 volts. The difference of 180 - 90, or 90 volts, must be the drop across R_1. R_1 is equal to E/I = 90/.019 = 4,737 ohms. As a rule it is unnecessary to use such an exact value, and in practice a resistor of 4,500 ohms or 5,000 ohms would probably be satisfactory; this would change the voltages only slightly.

Addition will show that the total current in the line is the sum of all the currents through the various tubes plus the leakage or bleeder current. 19 milliamperes is the load to Tubes 2 and 3 plus the bleeder current. Tube 1 requires 10 milliamperes as indicated by MA_3. Thus, 29 milliamperes must flow through MA_1. This current is distributed through the various parts of the circuit, all branches coming together at the positive side of the line and the combined current of 29 mils flowing through MA_1.

Suppose one tube of the combination, Tube 2, becomes defective and the flow of current through it ceases. Assume the same current of 1 milliampere flows through R_3, and R_2 carries the current of R_3 plus 3 milliamperes from Tube 3, a total of 4 milliamperes as before. However, the current in Tube 2 is decreased by 1/3 so that this current is now only 10 milliamperes. The current through R_1 is now 4 + 10 = 14 milliamperes instead of 19 milliamperes as before. The voltage drop across R_1 = 4,737 × .014 = 66 volts.

Thus, the voltage applied to the plates of Tube 2 is now 180 − 66 = 114 volts, an increase at this point of 24 volts over the 90 volts previously applied.

If it is assumed that the current through R_2 has not materially changed, the drop across this resistor is also unchanged so that the voltage applied to Tube 3 will now be 114 − 70 or 44 volts instead of 20 volts as before. This will not be quite true because with a higher voltage at Tube 3 the current through both R_3 and Tube 3 will be increased, this in turn increasing the voltage drop across both R_2 and R_1.

However, the current increase in R_3 and Tube 3 should not increase as much as the current decrease in Tube 2, so that the combined result of a defective unit in Tube 2, if the adjustments of the other tubes are unchanged, will be a voltage increase at both Tube 2 and Tube 3. To calculate exactly what these voltages will be under this condition will require a knowledge of the tube resistance under the changed operating condition. It should also be observed that with the higher voltage at Tube 2 the current from the two remaining units of Tube 2 should be greater than 10 milliamperes. Thus, the general increase of circuit voltages will not be quite so high as preliminary calculations tend to indicate.

By using a combination of the principles shown in the example above, no trouble should be experienced in solving any type of resistance problem encountered in practice. In problems involving Kirchhoff's Laws it is important that the engineer be familiar at all times with the exact value of the current in every part of the circuit and the direction in which each current flows. The sum of the individual currents *must* equal the total line current, and the total line current must be the same in each side of the line.

The following problem will indicate the proper approach to the design of a power supply voltage distribution system for a radio transmitter. The method used is somewhat different from the one used for the receiver voltage divider although both methods will give identical results. It is understood that current flow is electron flow, and electrons always

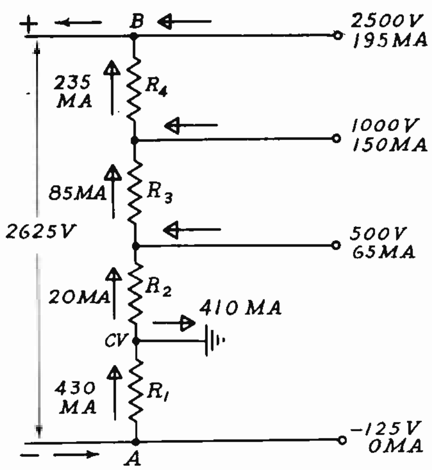


Fig. 21.--Voltage divider for a transmitter.

flow from negative to positive.

It is desired to design a voltage distribution system for a transmitter utilizing the following tubes: one Type 801 as a crystal oscillator; one Type 211 as intermediate or buffer amplifier; and one Type 806 as a power amplifier. Reference to a transmitting tube manual shows the following conditions are typical for these tubes in the services indicated:

It is desired to obtain the negative grid bias voltage for the crystal oscillator from the main power supply.

Fig. 21 shows the circuit diagram of the divider. All voltages are with respect to ground, and it should be noted that the divider is grounded at the junction of R_1 and R_2. Since electrons enter the divider at point A, and flow upward through R_1, then point A is negative to ground by the drop across R_1. Point B is 2,500 volts positive with respect to ground, so the total voltage across the divider (voltage required from the power source) is 2,500 + 125 = 2,625 volts.

Before it is possible to calculate the various resistance values, it is necessary to decide upon a value of bleeder current. No set rules can be given regarding this selection since it is mostly a matter of experience. In receiver work a bleeder value between 1 and 10 milliamperes is usually satisfactory. In selecting the bleeder current for the voltage divider of a transmitter power supply the following points should be considered. The larger the bleeder current, the more power dissipated as heat in the divider, and the larger must be the power ratings of the resistors. However, if the bleeder I is made very small, the individual resistances will be large, which means a very slight change in load current

	TYPE 801	TYPE 211	TYPE 806
Plate Voltage	500	1,000	2,500
Plate Current (Amp.)	.065	.15	.195
Grid Bias Voltage	-125	-175	-500

will cause a large change in load voltage, hence, it is desirable that the bleeder current be as large as the power rating of the voltage source and good economics will permit in order to give good voltage regulation. Another factor, often overlooked, is that proper selection of bleeder current value will allow most of the resistors in the divider to work out to standard values which are easily obtained commercially.

The total load current for the tubes is 195 + 150 + 65 = 410 ma. With this load current the power source will probably be rated at not less than 500 ma. A bleeder current approximately 5 per cent of this value would not be excessive. Selecting a value of 20 ma. will make the total load current 430 ma. Since all the electrons enter the divider at point A (no current flows from grid to filament in the crystal oscillator tube), then the total resistance in R_1 must be 125/.430 = 290 ohms. A standard value of 300 ohms would be satisfactory for R_1. Power dissipation = I^2R = $.430^2 \times 300$ = 55.5 watts. A 150- or 200-watt resistor would be used depending on how well the voltage divider compartment is ventilated.

Standard power ratings for fixed resistors above 50 watts rating are 50, 75, 100, 150, and 200 watts, and these values are *usually* based on the safe heat dissipation with free air circulation on all sides. It is customary to provide a safety factor by using 2 to 5 times the power dissipation for the power ratings of power supply resistors depending on the degree of ventilation available. The modern trend in transmitters is to provide forced air circulation in which case resistors may be operated very close to their rated maximum dissipation.

All the tube currents leave the divider at the ground point and flow to the tube cathodes, through the tubes to the plate and thence back to the divider. Therefore, the only current in R_2 is the bleeder current of 20 ma., and since the voltage drop across this resistor is 500 volts, the resistance of R_2 must be 500/.02 = 25,000 ohms $P = .02^2 \times 25{,}000$ = 10 watts. A 25 or 50-watt resistor could be used here. The plate current of the oscillator, 65 ma., joins the bleeder current at the junction of R_2 and R_3 and flows upward through R_3. Total current in R_3 = 65 + 20 = 85 ma. Drop across R_3 = 1,000 - 500 = 500 volts. R_3 = 500/.085 = 5,880 ohms. If R_3 is made 6,000 ohms, the voltage at the plate of the intermediate amplifier would be increased only about 17 volts, a negligible amount as far as tube operation is concerned. $P = .085^2 \times 6{,}000$ = 43.3 watts. A 150 or 200-watt resistor could be satisfactory. The plate current of the intermediate amplifier joins the bleeder I and plate current of the oscillator at the junction of R_3 and R_4; hence, the current in R_4 is 20 + 65 + 150 = 235 ma. Drop across R_4 = 2,500 - 1,000 = 1,500 volts. R_4 = 1,500/.235 = 6,382 ohms. A 6,500-ohm resistor could be used here. $P = .235^2 \times 6{,}500$ = 359 watts. For an adequate safety factor R_4 should be capable of dissipating from 700 to 1,000 watts. Since 200 watts is maximum standard dissipation for commercial fixed resistances, R_4 could be made up of three 200-watt, 2,000-ohm resistors in series with a 50-watt, 500-ohm unit; or six 150-watt, 1,000-ohm resistors in series with a 500-ohm, 75-watt unit; or four 1,500-ohm, 200-watt units in series with a 500-ohm, 75-watt resistor. Other combinations to satisfy the given conditions could also be used.

In considering the actual design of a resistance network to use in a new receiver or transmitter, the question may arise, "How can I know what the currents will be before the receiver is built?" Here again, as in *all* original design, the engineer *must know*, either by experience or from data sheets, what the basic factors are. In this case the problem is simple, because characteristic curves of all standard tubes may easily be obtained from the manufacturers or from published tube manuals.

In designing voltage-divider circuits it is particularly important that the resistors used are sufficiently rugged to carry the required current without overheating. This is one of the weak points in many receivers and transmitters and is apparently often overlooked by design engineers who fail to provide a sufficiently high factor of safety. On the other hand, receivers and transmitters are manufactured for sale at a profit, and the design engineer must keep this in mind in writing specifications, so that unnecessarily large units do not run up production costs.

NETWORK SOLUTIONS.—Reference to Kirchhoff's Laws has already been made, and their application to voltage divider networks demonstrated. However, the direction of current flow to a tap (junction) of a voltage divider is readily apparent from an inspection of the circuit, and simple applications of Ohm's law are possible.

In the case of more complicated circuits or networks, the directions of the current flows to any junction are not readily apparent. Fortunately, the directions do not have to be known; they can be assumed as desired. After the solution has been obtained, a negative sign in front of the numerical value obtained for any current indicates that the direction chosen was incorrect, and that the current actually flows in the opposite direction. *The numerical value, however, is not affected by a wrong choice of direction.*

Since the direction of current flow does not have to be known beforehand, it can be assumed, at the beginning of the solution, to flow in all cases in a clockwise direction around any circuit path. The result is a simple, systematic procedure, first developed by Maxwell, the great English mathematical physicist, and almost universally used today.

Before describing this method, some general remarks on circuit configurations are in order. In Fig. 22 is shown a typical circuit. It will be observed to contain three closed paths along which current may flow: path ABEFA, path BCDEB, and path ABCDEFA. Although other paths are possible, the three given are characterized by the fact that in each path *the current traverses* each resistor or other circuit element but once.

Such paths are known as *meshes*. In the circuit of Fig. 22 there are three meshes, BUT ONLY TWO—ANY

TWO—ARE NEEDED TO SOLVE FOR THE CURRENTS AND VOLTAGE DROPS IN THE CIRCUIT. The circuit is therefore, said to be a two-mesh network. For

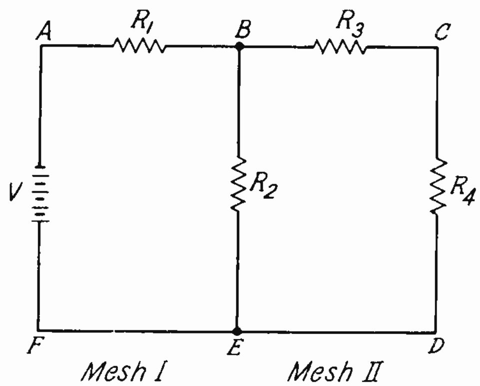


Fig. 22.—Example of a two-mesh network.

convenience, meshes ABEF and BCDE will be chosen.

Maxwell's method will now be employed to solve this circuit. As shown in Fig. 23, numerical values have been assigned to the circuit elements of Fig. 22, so that a specific problem will be obtained. Clockwise (electron) currents are

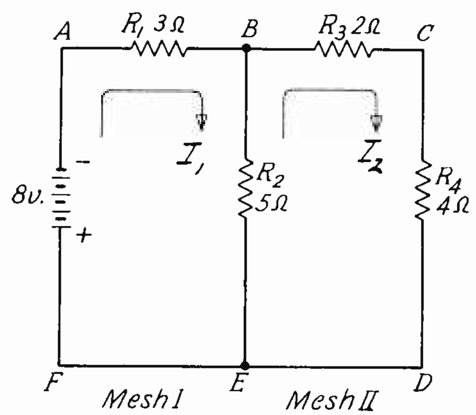


Fig. 23.—Procedure in solving for two mesh currents in the circuit of Fig. 22.

chosen for the two meshes; they are I_1 and I_2 as shown.

It should be immediately apparent from Fig. 23, that the currents I_1 and I_2 are assumed to flow independently of one another in the circuit. Moreover, I_1 or I_2 are not necessarily the actual currents in each circuit element. It is true that the current in R_1 is actually I_1, and the current in R_3 and R_4 is actually I_2. But the current in R_2 is neither I_1 nor I_2 alone, but is their *algebraic sum*.

Thus, for some of the resistors, the actual currents are those indicated; in the case of R_2, the current is actually $I_1 - I_2$ or $I_2 - I_1$. The minus sign occurs because in R_2 one current is assumed to flow in a direction opposite to that of the other; if *either* direction is taken as positive, the other must be taken as negative.

The sign to be employed with a current depends upon which mesh is under consideration. For example, in Mesh I_1 a clockwise direction is assumed positive. Hence I_1 is a positive current. Its direction through R_2 is *downward*, hence *downward* is the *positive* direction for a current in R_2. However, there also flows in R_2 the current I_2. Since this flows *upward* through R_2, its direction—as regards Mesh I—is to be taken as *negative*.

The opposite state of affairs regarding R_2 exists if the viewpoint is shifted to Mesh II. In this case, the upward direction of a current in R_2 is to be taken as *positive*, because that coincides with a *clockwise flow* in Mesh II. Accordingly, I_2 is now the *positive* current component in R_2, and I_1 is the *negative* current component.

In passing, it will be of

interest to note that this method of choosing currents automatically satisfies Kirchhoff's current law. To show this, consider each current flow separately. Take junction B and consider I_1's flow at this junction by itself. I_1 flows TOWARD B through R_1, and also flows AWAY from B through R_2, hence—so far as I_1 is concerned—as much current flows into B as flows out of it.

Next consider I_2 by itself. I_2 flows up TOWARD B through R_2, and AWAY from B through R_3. Hence—so far as I_2 is concerned—as much current flows into B as flows out of it. Since this is true individually for both current components at B_1 it is true for their sum; i.e., it is true for the *total* current at B, namely—as much current flows into B as flows out of it. Hence, the use of individual mesh currents automatically satisfies Kirchhoff's current law.

To proceed with the analysis, it is now necessary to employ Kirchhoff's voltage law. This states that the sum of the IR drops around a circuit is equal to the impressed voltage in the circuit. From Fig. 23, the impressed potential in Mesh I is +8 volts; i.e., it is so connected in the circuit as to tend to make an (electron) current flow in a *clockwise* direction. The IR drops involved are I_1R_1, I_1R_2, and I_2R_2.

NOTE PARTICULARLY THE LAST TERM I_2R_2. There may seem to be a contradiction in that it was previously implied that I_1 was the current in MESH I. What was meant by this is that if Mesh I were taken out of the rest of the circuit, then I_1 would be the current flow. When, however, Mesh I is in the complete circuit, I_2 is also involved through R_2, a resistor COMMON OR MUTUAL TO BOTH MESHES.

Before proceding to equate the IR drops to the impressed voltage, note that—with respect to Mesh I—current I_1 is taken as positive, and current I_2 is taken as negative. (This was discussed previously.) Therefore the *total* voltage drop in R_2 is $I_1R_2 - I_2R_2$. If I_1R_2 is taken as a VOLTAGE DROP, then $-I_2R_2$ must be regarded as a VOLTAGE RISE.

With the above explanatory remarks in mind, let us write the voltage equation for the first mesh. It is

$$I_1R_1 + I_1R_2 - I_2R_2 = 8$$

Factoring out I_1, there is obtained

$$I_1(R_1 + R_2) - I_2R_2 = 8$$

The reason for such factoring is that I_1 is one unknown; I_2 is the other, and the second form for the equation shows each unknown, I_1 and I_2, occurring as single terms.

The numerical values for R_1 and R_2 can be substituted to give

$$I_1(3 + 5) - I_2(5) = 8$$

or

$$8\,I_1 - 5\,I_2 = 8$$

We now proceed to Mesh II. In this mesh the impressed e.m.f. is *zero*. The fact that there is an eight-volt battery in Mesh I is of no concern here. The IR drops are now I_2R_2, I_2R_3, I_2R_4, AND IN ADDITION $-I_1R_2$. Note that with respect to Mesh II, I_2R_2 is the voltage drop, and $-I_1R_2$ is the voltage rise, because with respect to R_2, I_2 flows in the positive direction, and I_1 flows in the opposite or negative direction.

The voltage equation for the second mesh is therefore

$$- I_1R_2 + I_2R_2 + I_2R_3 + I_2R_4 = 0$$

Factoring out I_2, there is obtained

$$- I_1R_2 + I_2(R_2 + R_3 + R_4) = 0$$

When numerical values are substituted, the equation becomes

$$- 5 I_1 + 11 I_2 = 0$$

The equations for the two meshes can then be written together and solved simultaneously. Thus

$$8 I_1 - 5 I_2 = 8 \quad \text{MESH I}$$

$$- 5 I_1 + 11 I_2 = 0 \quad \text{MESH II}$$

Multiply the first by 5 and the second by 8, and add:

$$\begin{array}{r} 40 I_1 - 25 I_2 = 40 \\ - 40 I_1 + 88 I_2 = 0 \\ \hline 0 \quad + 63 I_2 = 40 \end{array}$$

$$I_2 = 40/63 = 0.635 \text{ ampere}$$

Now that I_2 is known, its value can be substituted in either of the preceding to obtain

$$8 I_1 - (5)(40/63) = 8$$

or

$$8 I_1 = 8 + 200/63$$

$$8 I_1 = 8 + 3.18 = 11.18$$

$$I_1 = 11.18/8 = 1.398 \text{ ampere}$$

Since both answers are positive, the clockwise directions assumed for I_1 and I_2 are correct. As a check on the numerical values, substitute the values for I_1 and I_2 in the second equation:

$$- 5(1.398) + 11(6.35)$$

$$= - 6.99 + 6.985 \cong 0$$

or the values check within the limits of slide-rule accuracy.

It will be instructive to specify the current in each circuit element. The current in R_1 is $I_1 = 1.398$ amperes. The current in R_3 and R_4 is $I_2 = 0.635$ ampere. The current in the mutual resistor R_2 (which is common to both meshes) is $I_1 - I_2 = 1.398 - 0.635 = 0.763$ ampere. The plus sign indicates that it has the same direction as I_1, namely *downward* through R_2. Finally, the voltage drop in any element can be readily found. For example, the voltage drop in R_2 is $(I_1 - I_2)R_2$ or

$$0.763 \times 5 = 3.815 \text{ volts}$$

Exercises

1. Solve the circuit of Fig. 22 if the impressed voltage is 12 and the polarity of the battery is reversed, and further, $R_1 = 7$ ohms, $R_2 = 10$ ohms, $R_3 = 4$ ohms, and $R_4 = 12$ ohms.

2. Solve the problem worked out in the text using, however, meshes ABEFA and ABCDEFA of Fig. 22.

3. Solve Exercise 1 using the same meshes as in Exercise 2.

A SECOND NETWORK EXAMPLE.—As a further illustration of the method of solving a network by means of

Kirchhoff's Laws, consider Fig. 24. The distinguishing feature of this

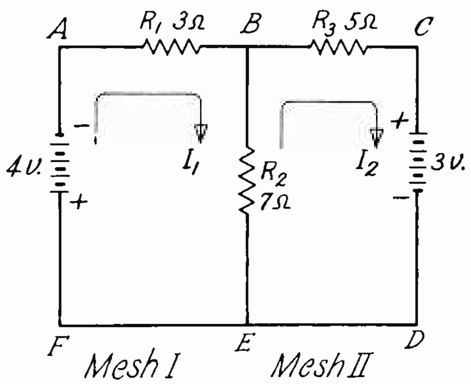


Fig. 24.—An example of a two-mesh network, having two sources of potential.

network is that there is a source of potential in each mesh.

If clockwise currents are chosen, as shown, it is clear that both the 4-volt and 3-volt batteries act in the same clockwise direction and are therefore both to be considered as plus.

The two-mesh equations are

$$I_1(3 + 7) - I_2(7) = 4 \quad \text{MESH I}$$

$$-I_1(7) + I_2(7 + 5) = 3 \quad \text{MESH II}$$

or

$$10\ I_1 - 7\ I_2 = 4$$

$$-7\ I_1 + 12\ I_2 = 3$$

Multiply the first equation by 7, and the second by 10, add, and obtain:

$$\begin{array}{r} 70\ I_1 - 49\ I_2 = 28 \\ -70\ I_1 + 120\ I_2 = 30 \\ \hline 0 + 71\ I_2 = 58 \end{array}$$

$$I_2 = 58/71 = 0.817 \text{ ampere}$$

Substitute in the first equation and obtain

$$10\ I_1 - (7)(.817) = 4$$

$$10\ I_1 = 4 + 5.72 = 9.72$$

$$I_1 = 0.972 \text{ ampere}$$

The plus signs indicate that both currents flow in clockwise directions. As a check, the second equation yields

$$(-7)(0.972) + (12)(0.817) = 3$$

$$-6.804 + 9.804 = 3 \quad \textit{Check}$$

A THIRD NETWORK EXAMPLE.—A third network example is shown in Fig. 25. The distinguishing feature

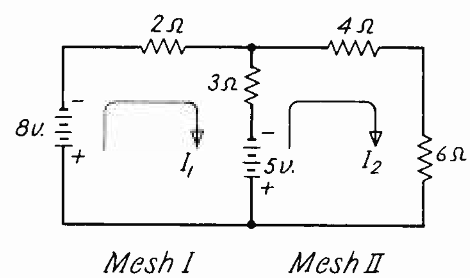


Fig. 25.—A third example of a two-mesh network.

in this example is the presence of the 5-volt battery in the mutual (3-ohm) branch of the two meshes.

This battery will appear not only in the equation for Mesh I, BUT ALSO IN THE EQUATION FOR MESH II. Furthermore, since it tends to produce a COUNTERCLOCKWISE current flow in Mesh I, it appears in the equation for this mesh with a NEGATIVE sign. On the other hand, the same battery clearly tends to produce a CLOCKWISE flow in Mesh II, hence it appears in the equation for this mesh with a POSITIVE sign.

The equations are therefore as follows:

$$I_1(2 + 3) - I_2(3) = 8 - 5 \quad \text{(MESH I)}$$

$$-I_1(3) + I_2(3 + 4 + 6) = 5 \quad \text{(MESH II)}$$

or

$$5 I_1 - 3 I_2 = 3$$

$$-3 I_1 + 13 I_2 = 5$$

Multiply the first and second equations by 3 and 5 respectively, add, and obtain:

$$\begin{array}{r} 15 I_1 - 9 I_2 = 9 \\ -15 I_1 + 65 I_2 = 25 \\ \hline 0 + 56 I_2 = 34 \end{array}$$

$$I_2 = 34/56 = 0.607 \text{ ampere}$$

Substitution of this value in the first equation yields

$$5 I_1 - (3)(.607) = 3$$

$$5 I_1 = 3 + 1.821 = 4.82$$

$$I_1 = 4.82/5 = 0.964 \text{ ampere}$$

The plus sign for each current shows it flows in a clockwise direction, as assumed.

To check, substitute the values for I_1 and I_2 in the second equation and obtain

$$(-3)(.964) + (13)(.607) = 5$$

$$-2.892 + 7.891 = 5$$

$$4.999 \cong 5 \quad \textit{Check}$$

The fact that the solutions for the currents in the problems given in the text come out positive, so that the assumed clockwise direction of flow is correct, is merely because the batteries were chosen so poled as to produce such a current flow. In the exercise problems, however, negative values of current will be obtained, which of course will merely indicate that the current actually flows counterclockwise. Whether or not this is evident from an inspection of the circuit, it is advisable for the student *always to assume the clockwise direction*, and work the problem out accordingly.

Exercises

4. Solve the circuit of Fig. 24, using as meshes ABEFA and ABCDEFA.

5. Reverse the polarity of the 5-volt battery in Fig. 25, and solve for the currents.

STAR- AND DELTA-CONNECTED NETWORKS.—Although all resistance networks are readily solved by application of Kirchhoff's Law, there are certain types of circuits that respond readily to special methods of solution. It is not within the scope of this technical assignment to discuss all of these special forms, but it is believed one will be of interest, since it has direct bearing on the solution of three phase circuits to be discussed in the assignment on polyphase systems.

The three resistances in Fig. 26(A) are said to be connected in STAR (or Y), while those in Fig. 26 (B) are said to be DELTA connected. These combinations might represent the load in a three-phase circuit with points A, B, and C each connected to one of the lines of the system. The two methods of connection are interchangeable if certain relations between the various resistances are maintained.

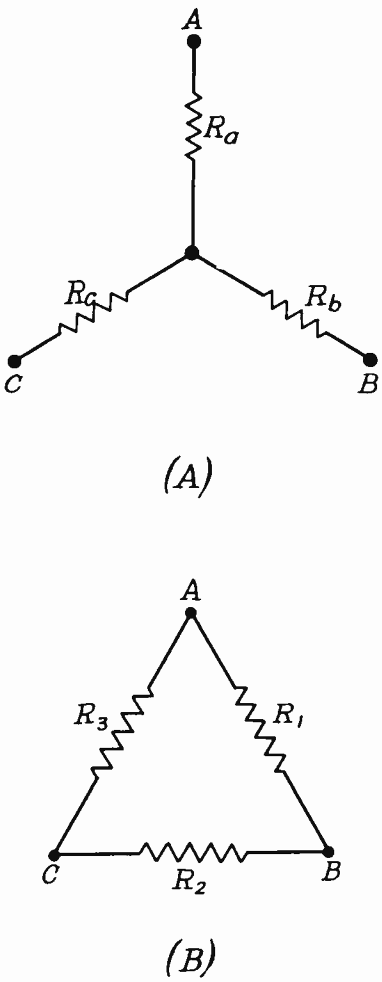


Fig. 26.--Star and delta networks.

If the delta network is to be the equivalent of the star, then the resistance R_{AB} between points A and B must be the same for each circuit. In Fig. 26(B) the resistance between points A and B is the effective resistance of the combination of R_2 in series with R_3, across R_1. Therefore,

$$R_{AB} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

$$= \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$$

$$= R_a + R_b \qquad (1)$$

Similarly, the resistance R_{BC} between points B and C must be

$$R_{BC} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_1 + R_3}}$$

$$= \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

$$= R_b + R_c \qquad (2)$$

$$R_{AC} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_1 + R_2}}$$

$$= \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$$

$$= R_a + R_c \qquad (3)$$

If a star network such as that shown in Fig. 26(A) is to be transformed to a delta network then R_a, R_b, and R_c will be known, and it will be necessary to calculate R_1, R_2, and R_3 of Fig. 26(B) from the known values of R_a, R_b, and R_c.

By transposing the preceding equations for R_{AB}, R_{AC}, and R_{BC}, it can be shown that

$$R_1 = \frac{R_aR_b + R_aR_c + R_bR_c}{R_c}$$

$$R_2 = \frac{R_aR_b + R_aR_c + R_bR_c}{R_a}$$

$$R_3 = \frac{R_aR_b + R_aR_c + R_bR_c}{R_b}$$

The derivation of such equations is rather simple as will now be shown:

Adding Eqs. (1) and (3):

$$\frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} + \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$$

$$= 2\,R_a + R_b + R_c$$

By substituting for $R_b + R_c$ above, its equal

$$\frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

the equation becomes

$$\frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} + \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$$

$$= 2\,R_a + \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

solving for R_a,

$$R_a = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

Likewise solve for R_b and R_c.

$$R_b = \frac{R_1R_2}{R_1 + R_2 + R_3}$$

and

$$R_c = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

and

$$R_a = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

$$R_aR_b = \frac{R_1^2R_2R_3}{(R_1 + R_2 + R_3)^2}$$

and

$$R_bR_c = \frac{R_1R_2^2R_3}{(R_1 + R_2 + R_3)^2}$$

and

$$R_aR_c = \frac{R_1R_2R_3^2}{(R_1 + R_2 + R_3)^2}$$

Therefore:

$$R_aR_b + R_bR_c + R_aR_c$$

$$= \frac{R_1^2R_2R_3 + R_1R_2^2R_3 + R_1R_2R_3^2}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1R_2R_3(R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$R_aR_b + R_bR_c + R_aR_c = \frac{R_1R_2R_3}{R_1 + R_2 + R_3}$$

Since:

$$R_c = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

$$R_1R_c = R_aR_b + R_bR_c + R_aR_c$$

Therefore:

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c}$$

Similarly R_2 and R_3 can be found by algebraic methods as shown for R_1. This is a good exercise for the student who is interested in algebra.

The equations for R_1, R_2, and R_3 are not difficult to remember if it is noted that all three numerators are the same, the product of the pairs of the three resistances while the denominator is, in each case, the star resistance opposite to the delta resistance which is to be calculated.

If the transformation is to be made from delta to star, the following equations derived from the first three can be used. In this case R_1, R_2, and R_3 will be known, and R_a, R_b, and R_c are to be calculated.

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Note that in this transformation the denominators are all alike being equal to the sum of the three known resistances, while the numerator is the product of the two resistances which meet at the corner to which the star is to be connected.

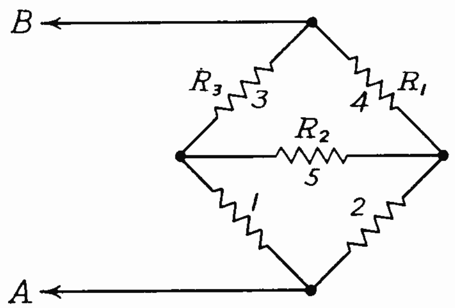


Fig. 27.--Typical bridge circuit.

This particular transformation is of use in finding the total resistance between two points that are connected through a resistance network that has the characteristics of delta or star combinations. For example, consider Fig. 27 which shows a typical bridge combination between the two points A and B. Bridge circuits will be discussed later in this technical assignment. Assume that it is desired to find the total resistance between points A and B. It is noted that the 3, 4, and 5-ohm resistances form a delta network which may be transformed to a star combination. The transformed circuit will then appear as shown in Fig. 28. Note that Fig. 28 is a simple series parallel combination of resistances which may be easily solved by Ohm's Law.

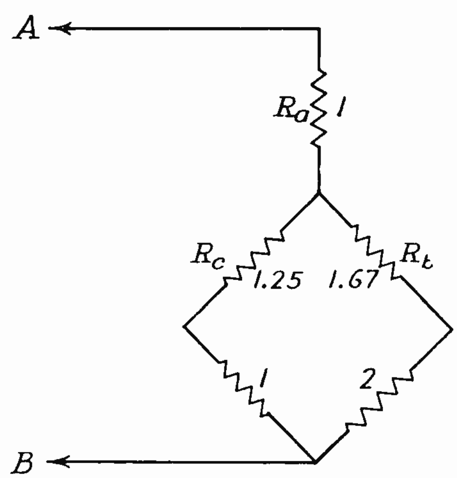


Fig. 28.--Network equivalent to the bridge circuit and obtained from it by a delta-star transformation.

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{3 \times 4}{3 + 4 + 5}$$

$$= \frac{12}{12} = 1 \text{ ohm}$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{4 \times 5}{3 + 4 + 5}$$

$$= \frac{20}{12} = 1.67 \text{ ohm}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{3 \times 5}{3 + 4 + 5}$$

$$= \frac{15}{12} = 1.25 \text{ ohm}$$

With the transformation complete the total resistance between points A and B is easily found.

$$R = 1 + \frac{1}{\frac{1}{1.25 + 1} + \frac{1}{1.67 + 2}}$$

$$= 1 + \frac{1}{.444 + .272} = 1 + \frac{1}{.716}$$

$$= 2.4 \text{ ohms}$$

It should be noted that this transformation simplifies the calculation of the *total* R between points A and B, but it is of little use if the actual currents in the various mesh curcuits of Fig. 27 are to be determined. In that case, the logical method is to solve the network by applying Kirchhoff's laws.

BRIDGE CIRCUITS.--A circuit commonly encountered in radio work, in transmitters, receivers, and in measurements, is some form of bridge circuit. There is the well-known Wheatstone bridge for the accurate measurements of resistance, and similar bridges are used for the measurement and calibration of capacities and inductances. There are neutralizing or balancing circuits in transmitters, and receivers that function as an ordinary capacity bridge in neutralizing the internal capacity of a vacuum tube, etc. Bridge circuits stand out prominently in modern radio practice, and it is important that the bridge principle be thoroughly understood.

The Wheatstone bridge, when properly designed and constructed, may be used for the accurate measurement of all values of resistance from a small fraction of an ohm to many thousands of ohms.

The Wheatstone bridge operates on the principle of voltage drops across a split circuit. Consider

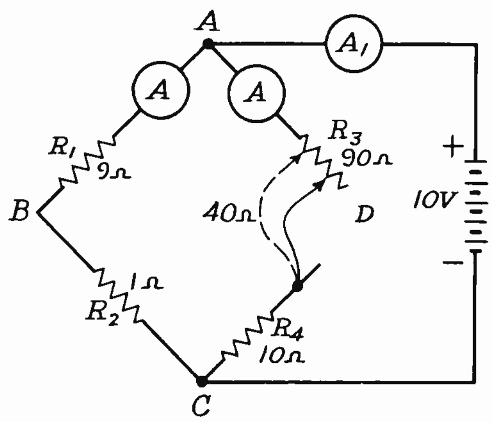


Fig. 29.--Circuit illustrating the Wheatstone bridge principle.

the circuit of Fig. 29. The circuit is made up of four resistors, R_1, R_2, R_3, and R_4. These resistances are so connected that R_1 and R_2 are in series, R_3 and R_4 are in series, and the two combinations are in parallel. The battery is connected across the circuit at points A and C. Ammeter A_1 in the supply line indicates the total current through the two branches. This current will be the sum of the currents through the other two ammeters, one of which indicates the current in branch ABC consisting of R_1 and R_2 in series, and the other, the current flow through circuit ADC consisting of R_3 and R_4.

Circuits ABC and ADC are in parallel, and the current through each is dependent *only* upon the resistance of each; i.e., the resistance of ABC has no effect on the current through ADC and vice versa. Since the two circuits are in parallel, the same voltage, 10 volts, is applied across both. The sum of the voltage drops in each circuit must therefore equal 10 volts.

The total resistance of ABC is $R_1 + R_2$ = 9 ohms + 1 ohm = 10 ohms. The current through this circuit is E/R = 10/10 = 1 ampere. The total resistance of ADC is $R_3 + R_4$ = 90 ohms + 10 ohms = 100 ohms. The current through this circuit is equal to E/R = 10/100 = .1 ampere.

Under this condition the voltage drop between A and B across R_1 is equal to $IR_1 = 1 \times 9 = 9$ volts. The voltage drop across R_2 between B and C is $IR = 1 \times 1 = 1$ volt. The sum is equal to the battery voltage. The voltage drop between A and D across R_3 is $IR_3 = .1 \times 90 = 9$ volts. The drop between D and C across R_4 is $IR_4 = .1 \times 10 =$ 1 volt. The sum of these drops also equals the battery voltage, 10 volts.

It will be seen that B is negative with respect to A by 9 volts. B is also positive with respect to C by 1 volt. Also, D is negative with respect to A by 9 volts and positive with respect to C by 1 volt.

Points B and D are both 9 volts negative with respect to A and 1 volt positive with respect to C. Since both points, B and D, are at the same difference of potential with respect to either end of the circuit, there is *no* difference of potential between B and D. Under this condition if a galvanometer is connected between points B and D, no current will flow through the galvanometer. (A galvanometer is a very sensitive current indicating device usually calibrated in divisions of deflection instead of actual current flow.)

It is apparent that in order to have this condition of no voltage between B and D, the voltage drop across R_1 must equal the drop across R_3; also the drop across R_2 must equal the drop across R_4. This does *not* mean that $R_1 = R_3$ and $R_2 = R_4$. This is not the case as shown in Fig. 29. It is only necessary that *the ratio of R_1 to R_2 be equal to the ratio of R_3 to R_4*. This latter condition is absolutely essential if there is to be no difference of potential between B and D.

For example, with R_1, R_2, and R_4 fixed, change the variable resistor R_3 to the 40-ohm position as shown by the dotted line in Fig. 29. Since circuit ABC has not been changed, point B is still 9 volts negative with respect to A, and 1 volt positive with respect to C.

Circuit ADC has been changed. R_3 = 40 ohms; R_4 = 10 ohms. The total resistance of this circuit is now $R_3 + R_4 = 40 + 10 = 50$ ohms. The current through this branch is 10 volts/50 ohms, or .2 ampere. The voltage drop across R_3 is $40 \times .2 = 8$ volts. The drop across R_4 is $10 \times .2 = 2$ volts. The total drop is still 10 volts, but the 10 volts is now made up of the sum of 8-volt and 2-volt drops instead of 9-volt and 1-volt drops. D is 8 volts negative with respect to A and 2 volts positive with respect to C.

But B is still 9 volts negative with respect to A. Since B is 9 volts negative with respect so A, and D is only 8 volts negative with respect to the same point, B must be negative with respect to D by the difference, 9 volts - 8 volts, or 1 volt. This can be checked by working from the other end of the circuit; i.e., D is 2 volts positive with respect to C, while B is only 1 volt positive with respect to C. D is therefore 1 volt positive with respect to B, and if a galvanometer is connected between B and D, a current flow will be indicated by the galvanometer.

It should particularly be noted that in this condition the ratio of R_1 to R_2 does *not* equal the ratio of R_3 to R_4. The ratio of R_1 to R_2 is 9 to 1. The ratio of R_3 to R_4 is 40 to 10 or 4 to 1. Since each combination is, in itself, a series circuit, it is apparent that the ratios of the voltage drops are the same as the ratios of the resistances (as shown previously in deriving Rule 1).

In order that the difference of potential between B and D shall be zero (as indicated by no current flow through a galvanometer connected between those points) the ratios between the voltage drops, and therefore between the resistances of the circuits ABC and ADC, must be the same. If one is 10 to 1, the other must be 10 to 1. Under that condition the circuit or bridge is said to be *balanced,* and the condition may be expressed as a relation in ratio and proportion as follows:

$$R_1 : R_2 :: R_3 : R_4$$

This is read, R_1 is to R_2 as R_3 is to R_4, and means that the ratio of R_1 to R_2 is equal to the ratio of R_3 to R_4.

This may also be written in equation form as

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

This equation provides the means of finding the value of an unknown resistance. In an expression of ratio and proportion, such as $R_1 : R_2 :: R_3 : R_4$, *the product of the means is equal to the product of the extremes.* R_1 and R_4 are the extremes. R_2 and R_3 are the means. Therefore, $R_1R_4 = R_2R_3$. This can easily be checked by the rules of algebra as applied to the equation form.

This is now in the form of a very simple algebraic equation and

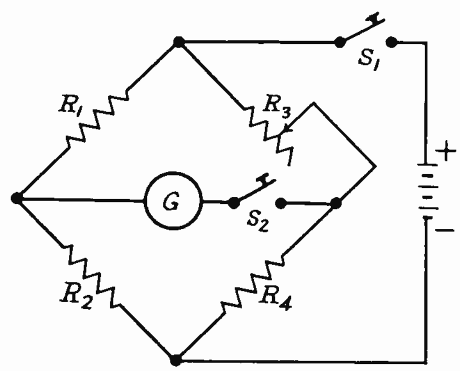


Fig. 30.--**Common form of Wheatstone bridge.**

if three values are known it is easy to find the fourth. If the values of R_1, R_2, and R_3 are known,

$$R_4 = \frac{R_2 R_3}{R_1}$$

For example, if R_1 = 100 ohms, R_2 = 10 ohms, R_3 = 760 ohms, then the equation $R_4 = R_2R_3/R_1$ becomes

$$R_4 = \frac{10 \times 760}{100} = \frac{7600}{100} = 76 \text{ ohms}$$

Fig. 30 shows a form of bridge commonly used. R_1 and R_2 are fixed resistances with either their values or their ratio known. R_3 is a *calibrated* variable resistor, usually variable in very small steps. R_4 is the unknown resistance which is to be measured and is usually referred to as R_x or simply X.

R_3 is varied until the ratio of R_3 to the unknown R_4 is equal to the ratio of R_1 to R_2. This condition will be indicated by zero reading in the galvanometer when S_1 and S_2 are closed. In this condition the bridge is said to be balanced and therefore $R_1 : R_2 :: R_3 : R_4$.

Then R_4 compares with R_3 as R_2 compares with R_1. From this may be determined the value of the unknown resistance R_4, if it is known how R_2 and R_1 compare, even though their actual values are not known. For example, if it is known that R_2 is ten times as large as R_1, then the resistance of R_4 must be ten times as great as that of R_3. *This is true regardless of the actual values of R_2 and R_1.*

This principle is employed in Wheatstone bridges. Means are provided for changing the ratio of R_2 to R_1. This switch is ordinarily a rotary dial switch with a number of contacts and is called the "Multiplier". If the multiplier is at a dial setting of 1, it indicates that the ratio of R_2 to R_1 is 1, that is, $R_2 = R_1$. Under this condition when the bridge is balanced by varying the resistance R_3, in the balanced condition $R_4 = R_3$. The resistance of R_4 is then indicated *directly* by the setting of R_3.

If the multiplier is set at 10, it means that R_2 is ten times greater than R_1, in the balanced condition R_4 then being 10 times greater than R_3. If the bridge shows a balanced condition when R_3 is set at 241 ohms, the resistance of R_4 will be 2,410 ohms.

The multiplier may also be adjusted to decimal settings. If the multiplier dial is set at .01, it means that R_2 is only 1/100th as great as R_1. If R_3 is set at 326 ohms when the bridge is balanced, then R_4 is equal to 326 × .01 or 3.26 ohms.

In most commercial bridges of this type the multiplier has the following settings: .001; .01; .1; 1.; 10; 100; 1,000. This allows the resistance as indicated on R_3 to be multiplied by any one of those values as required and permits the measurement of a very wide range of resistances.

R_3 is usually made up in the form of a decade box, i.e., in 10 steps of each unit. One switch cuts in ten steps of resistance, 1 ohm per step; another switch cuts in resistance in steps of 10 ohms; another in steps of 100 ohms, and another in steps of 1,000 ohms. Each switch may be set on any value from 0 to 9. This permits R_3 to be varied in steps of 1 ohm between 1 and 9,999 ohms, inclusive, giving a measurable range, with the use of the multiplier arm, of from

.001 ohm to 9,999,000 ohms.

Fig. 31 shows the connections for a variable resistance of this

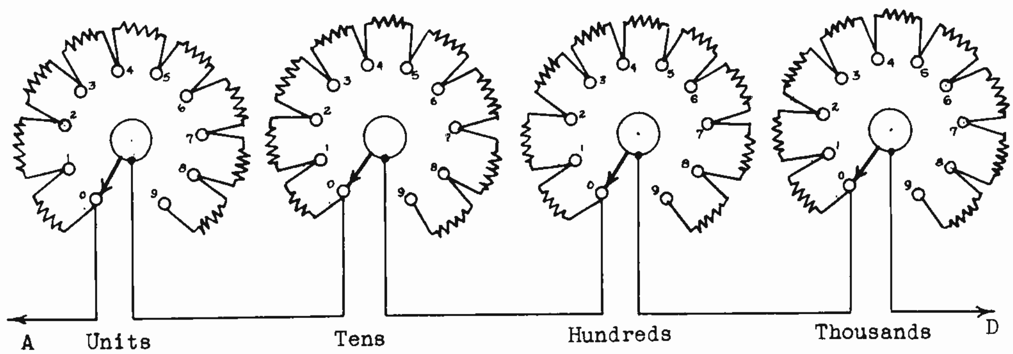


Fig. 31.--Schematic diagram of a decade resistance box.

type. One end connection A goes to point A, Fig. 29, and the end connection D goes to point D, Fig. 29. Each resistance element should be non-inductively wound with wire having as nearly as possible a zero temperature-resistance coefficient at ordinary temperatures. Switch elements and contacts must be designed so as to have negligible resistance and contact potential.

Fig. 32 shows the indicator and scale arrangement of one type of galvanometer used with a Wheatstone bridge. This galvanometer has a zero center indicating dial. If the galvanometer indicator deflects in the + direction, more resistance should be added in the variable resistor. If it deflects in the - direction, the variable resistance should be decreased. If the indicator deflects violently, it indicates that the bridge is considerably off balance. If the deflection is slow, it indicates a near balance, and from that point until the exact balance is reached the resistance should be varied in small steps.

Fig. 30 shows two switches, S_1 and S_2. On a commercial bridge

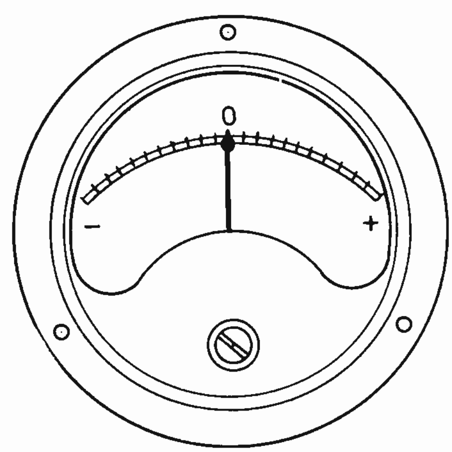


Fig. 32.--Zero-center type of galvanometer.

these are usually two small push buttons mounted close together on the face of the panel. S_1 is marked "B" for battery, and S_2 is marked "G" for galvanometer. It is necessary that both switches be closed in order to take a reading. In closing the switches, "B" should be closed first by holding that button down with one finger; then tap the switch "G" very lightly and observe the deflection of the galvanometer indicator. If the deflection is in the + direction, add resistance; if in the - direction, decrease resistance. Do this, taking a reading each time the resistance is varied, until the point is reached where the indicator deflects very slowly. Then both switches may be kept closed while the final fine adjustments are made to get the indicator to the exact zero reading. When this is done, the bridge is balanced.

Suppose at this point the resistance dials read as follows: Thousands, 8; hundreds, 4; tens, 9; units, 3. The multiplier is set at .1. The sum of all the resistance settings is 8,493 ohms. The multiplier indicates that this must be multiplied by one-tenth, therefore, the unknown resistance is 849.3 ohms. In closing the two switches G and B, the battery switch should *always be closed first and opened last*. The reason for this is that the galvanometer is a comparatively delicate instrument and may be damaged by an inductive kickback through the resistance winding if the galvanometer switch is closed when the battery circuit is closed or opened. The voltage built up across the resistance when the battery circuit is opened suddenly may be several times the battery voltage. (Even though the bridge resistors are non-inductive, the resistance being measured may have a highly inductive winding, such as the secondary of a transformer. Inductive effects and inductance will be considered at length in a subsequent tecnnical assignment early in the course.)

When measuring an unknown value of resistance, a two-volt battery or one or two dry cells or flashlight cells, one usually being sufficient, is connected to the terminals marked "Bat". The resistance to be measured is connected to the two terminals provided for that purpose; on one style of bridge these are marked "X" and "X_1". Next, set the multiplier arm on the 1 position. This position may be used if the value of resistance to be measured is between 1 and 9,999 ohms. Set the variable resistance at a low setting of a few ohms. Press the switches G and B lightly. If the indicator deflects in the "plus" direction change the variable resistance to a high value of several thousand ohms. (It is assumed that the operator has no means of knowing even the approximate resistance before making the measurement.) If the deflection is now in the "minus" direction, the resistance to be measured will lie between the two values of resistance just tried. In this case it is only necessary to vary the resistance until the balanced condition of no deflection is obtained. When the condition of near balance or slow deflection is reached, both switches may be held closed and the fine adjustment made.

If, with the multiplier set on 1 and all of the variable resistance cut in, the indicator still deflects in the "plus" direction, it is evident that the value of the unknown resistance is greater than the total value of the variable. In that case the variable resistance should be left at its highest value and the multipler changed to 10. If the indicator *now* deflects toward "minus", it indicated that the unknown resistance is between 9,999 and 99,990 ohms, and the multiplier may be left at 10 and the bridge balanced. In this condition when a balance is obtained, the resistance settings of the variable should be multiplied by 10 to find the value of the unknown. For example, if the settings total 8,743, the value of the resistance measured is 87,430 ohms. The measurement in this case would be accurate to within 10 ohms.

If the resistance is still much higher, a higher value of the multiplier setting will be necessary, and either 100 or 1,000 may be used as required.

For very small values of resistance a greater accuracy may be obtained by using one of the decimal settings of the multiplier. For example, assume a resistance of 3 and some fractional ohms resistance. The multiplier setting of 1 only permits accuracy to 1 ohm; with this multiplier setting, with the variable resistance set at 3 ohms, the indicator will deflect "plus"; when moved to 4 ohms it will deflect "minus", indicating that the resistance is between 3 and 4 ohms. Set the multiplier to .01, cut in 300 ohms and tap the switches; the indicator will deflect "plus". Set at 400 and take a reading; the indicator will deflect "minus". Then set the "hundreds" switch to 3 and vary the "tens" switch and "units" switch until a balance is found. Suppose the bridge balances at 368. The multiplier setting is .01. Therefore, the value of the resistance measured is 368 × .01 or 3.68 ohms. This measurement will be accurate to within one one-hundredth of an ohm. Accuracy to within one one-thousandth of an ohm may be obtained by the use of the .001 setting of the multiplier. In that case, suppose that with the above resistance, the "thousands" switch is set at 3, the "hundreds" switch at 6, the "tens" switch at 8, and the exact balance is obtained with the "units" switch at 2, the measured resistance will be 3,682 × .001 or 3.682 ohms; accurate to .001 ohm. For most work such extreme accuracy is not required, and usually cannot actually be obtained because of the difficulty in getting sufficiently low resistance leads and connections.

SLIDE-WIRE BRIDGE.—Fig. 33 shows the construction of a simple slide-wire bridge that can be built up with the material available in practically any radio room or shop.

R_3 is a fixed resistance which should be very accurately measured and used as a standard. The degree of accuracy with which the resistance of R_3 is known is one of the limiting factors in determining the accuracy of the bridge.

Instead of a battery and galvanometer, a source of alternating current at an audible frequency and a pair of phones are used. The power supply may be an audio frequency oscillator, a high-frequency buzzer, a step-down transformer in the 60-cycle lighting circuit, or the output from a broadcast receiver; any audio frequency source that gives a good signal in the

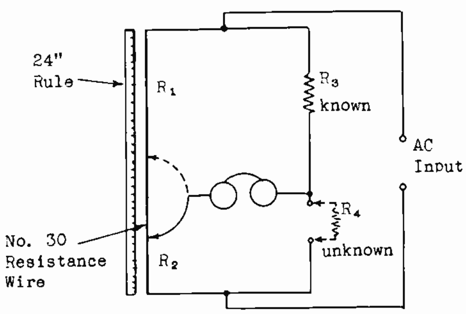


Fig. 33.—Slide-wire type of bridge.

phones may be used, although a fairly high frequency in the order of 1,000 cycles per second is to be preferred.

To balance the bridge it is simply necessary to move the contact along the wire R_1R_2 until the signal disappears or is brought to a minimum. At this point the ratio of R_1 to R_2 is equal to the ratio of R_3 to R_4. By knowing the length of R_1 and R_2 in inches and fractions of an inch, the ratio may be determined, since the resistances are in the same proportions as the lengths of the wire segments. Assume that R_1 is 18 inches, R_2 is 6 inches (so that the total length of wire is 24 inches), and the known value of R_3 is 200 ohms. Then,

$$R_1 : R_2 : : R_3 : R_4$$

$$18 : 6 : : 200 : R_4$$

(Substituting known values)

$$R_4 = \frac{1200}{18} = 66.6 \text{ ohms}$$

It will be seen that it is unnecessary to know the actual resistance of R_1 and R_2. Their *ratio* in inches, feet, centimeters, ohms, or in any convenient unit is all that is required.

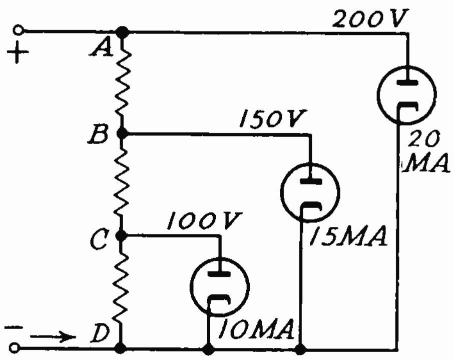
Wheatstone bridges can also be employed to measure circuit elements other than resistances. For example, capacitors and inductances can be measured, as well as combinations of these with resistors. Examples of such bridges and measurements will appear farther on in this course.

RESUMÉ

This concludes the assignment on Ohm's and Kirchhoff's Laws and Bridge Circuits. First, simple circuits were analyzed by means of Ohm's Law, and the method applied to meters and ohmmeters, as well as voltage dividers. Then more complicated circuits were studied by means of the application of Kirchhoff's Laws. More involved bleeder circuits for vacuum tube operation were analyzed by this means, and finally bridge circuits were taken up. In connection with these circuits a simple method known as the delta-star transformation was employed and found to considerably facilitate the analysis. Finally, Wheatstone Bridges were discussed, and their design and application to the measurement of resistance developed.

ADDITIONAL EXERCISES

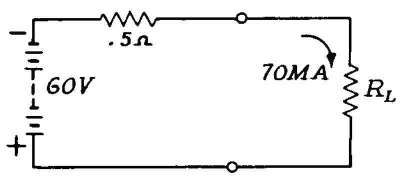
6.



Find the values of R_{AB}, R_{BC}, and R_{CD}. Find the current in the voltage divider when the tubes are disconnected.

10 MA bleeder current in R_{CD} with load connected.

7. A battery has an open-circuit voltage of 60 volts. The internal resistance is .5 ohm. The load is a five tube radio receiver with a total drain of 70 ma from the battery.



Find the terminal voltage of the battery and the equivalent resistance the receiver represents to the battery.

8. The following tubes in a radio receiver are to have their heaters connected in series across a 110-volt line: 6SK7 (r.f.), 6SA7 (converter), 6SK7 (i.f.), 6SR7 (detector-first audio), 25L6 (a.f.), 25L6 (a.f.), 25Z5 (rectifier). The heaters of these tubes are rated at 6.3 volts and 25 volts respectively at 0.3 ampere current. Find the necessary series resistor needed in series with the heaters.

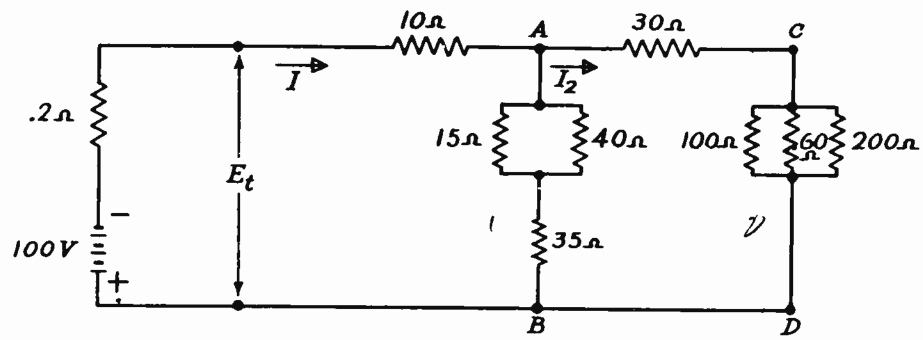
9. Calculate the resistor network and combine the groups until an equivalent series circuit is obtained.

Calculate the value of the equivalent series load resistance facing the battery (not including the internal resistance).

Find the current I supplied by the battery.

- 2 -

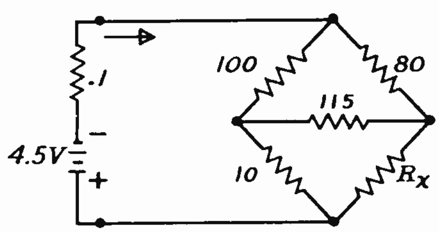
9. Find the voltage across AB and CD.



Find the current I_2.

Find the terminal voltage E_t of the battery,

10.



If the circuit shown in the figure is balanced, calculate the current in each resistance and the total current through the battery.

11. Design a voltage divider to provide the following voltages and currents from a 1,500-volt source. 1,250 volts at 100 ma, 1,000 volts at 60 ma, 600 volts at 30 ma. Specify resistance values that would be used, giving power ratings based on a safety factor of not less than double the dissipation. Assume a bleeder current of 20 ma, and choose nearest whole number for resistor value.

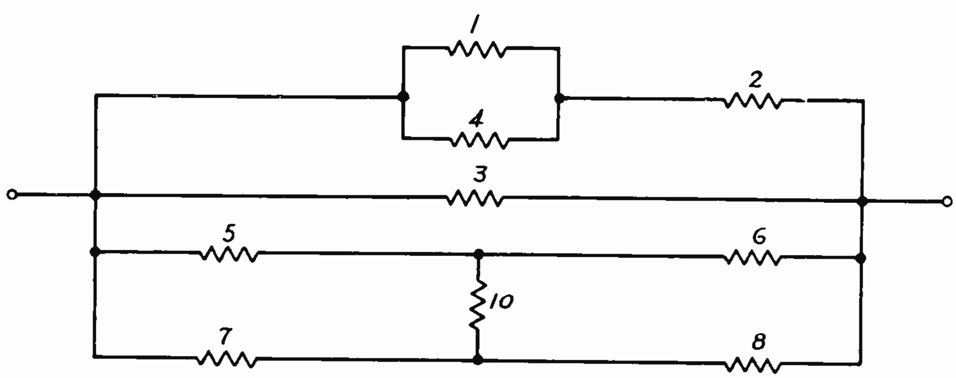
12 Reference Problem 11. For the same load requirements redesign the divider to also provide negative bias voltages of -50 and -150 volts.

\- 3 -

13. A 100-ohm, 200-watt, continuously variable rheostat will pass how much current when adjusted to 50 per cent of its total resistance without exceeding the rated power dissipation? Rated power applies to the entire unit.

14. A 250-volt voltmeter having a resistance of 1,000 ohms/volt is connected in series with an unknown resistance across a 120-volt source. If the voltmeter reads 80 volts, what is the resistance value of the unknown resistor?

15.



Find the total resistance of the circuit. (Hint: Apply delta-wye transformation to either one of two lower meshes, then combine with other lower mesh, and so on.

ANSWERS TO EXERCISES

1. I_1 = -.9123 amp.
   I_2 = -.3508 amp.
   $I_{10\Omega}$ = -.5615 amp.

2. I_1 = .763 amp.
   I_2 = .635 amp.
   $I_{3\Omega}$ = 1.398 amp.

3. I_1 = -.5615 amp.
   I_2 = -.3508 amp.
   $I_{7\Omega}$ = -.9123 amp.

4. I_1 = .155 amp.
   I_2 = .817 amp.
   $I_{7\Omega}$ = .155 amp.

ANSWERS TO EXERCISES, Page 2.

5. I_2 = .25 amp.
   I_1 = 2.75
   I_3 = $I_1 - I_2$ = 2.5

6. R_{AB} = 1,430 ohms
   R_{BC} = 2,500 ohms
   R_{CD} = 10,000 ohms
   I = 14.35 ma

7. E_t = 59.965 volts
   R_L = 856.6 ohms

8. R = 32.65 ohms

9. I = 2.74 amp.
   V_{AB} = 72 volts
   V_{CD} = 36.8 volts
   I_2 = 1.168 amp.
   E_t = 99.453 volts

10. R_x = 8 ohms
    I_{115} = 0
    I_{100} = 40.8 ma
    I_{10} = 40.8 ma
    I_{80} = 51.2 ma
    I_8 = 51.2 ma
    I_{total} = 92 ma

11. I_b = 20 ma
    R_1 = 30,000 ohms, 25 watts
    R_2 = 8,000 ohms, 50 watts
    R_3 = 2,500 ohms, 75 watts
    R_4 = 1,200 ohms, 150 watts

12. R_1 = 500 ohms, 50 watts
    R_2 = 250 ohms, 25 watts
    R_3 = 30,000 ohms, 25 watts
    R_4 = 8,000 ohms, 50 watts
    R_5 = 2,500 ohms, 75 watts
    R_6 = 500 ohms, 50 watts

13. 1.41 amp.

14. 125,000 ohms

15. 1.18 ohms

V-9174-33

TELEVISION TECHNICAL ASSIGNMENT

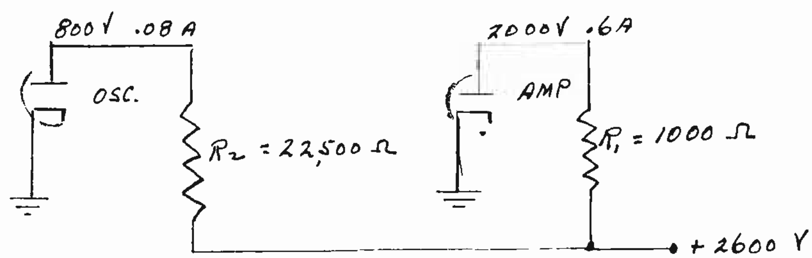
OHM'S AND KIRCHHOFF'S LAWS AND BRIDGE CIRCUITS

EXAMINATION

Show all calculations.

1. It is desired to operate an oscillator tube drawing 80 ma at 800 volts, and an amplifier tube drawing 0.6 ampere at 2,000 volts from a common plate voltage supply source of 2,600 volts. Resistors are used to reduce the voltage from 2,600 volts to that required for each tube. Two circuit arrangements are possible. Sketch a diagram of both, and calculate and show the values of the required resistors. Do not use a bleeder resistance in these circuits.

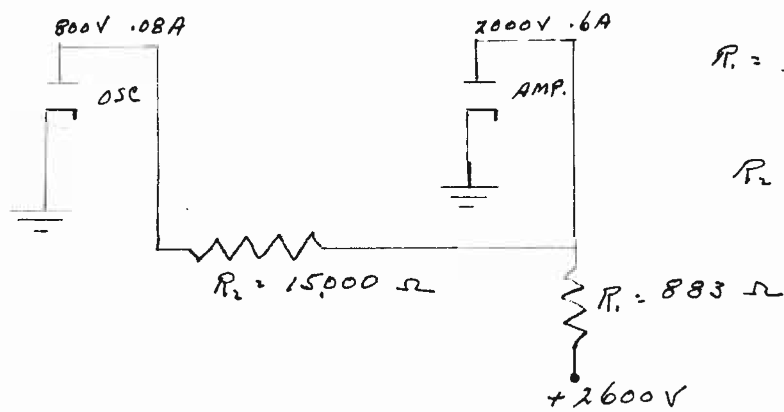
(a)



$R_1 = \frac{2600-2000}{.6} = \frac{600}{.6} = 1000\ \Omega$

$R_2 = \frac{2600-800}{.08} = \frac{1800}{.08} = 22{,}500\ \Omega$

(b)



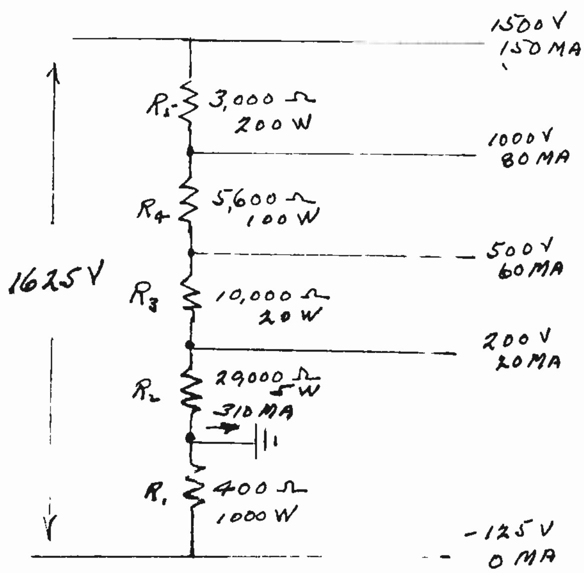
$R_1 = \frac{600}{.68} = 883\ \Omega$

$R_2 = \frac{1200}{.08} = 15000\ \Omega$

2. Design a voltage distribution circuit for a transmitter to provide the following voltages and load currents: 1,500 volts at 150 ma, 1,000 volts at 80 ma, 500 volts at 60 ma, 200 volts at 20 ma, and -125 volts at 0 ma. Bleeder current is to be 10 ma. Sketch the divider, showing the value of each resistor and the total voltage required from the source.

EXAMINATION, Page 2.

2.



$V = 1500 + 125 = 1625\ V$

$R_1 = \frac{125}{.32} = 391$ use $400\ \Omega$

$I^2R = .32^2 \times 400 = 405\ W$

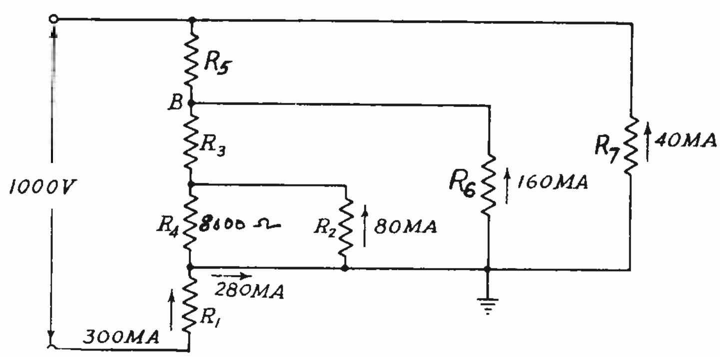
$R_2 = \frac{200}{.01} = 20,000\ \Omega \qquad .01^2 \times 20000 = 2\ W$

$R_3 = \frac{300}{.03} = 10,000\ \Omega \qquad .03^2 \times 10000 = 9\ W$

$R_4 = \frac{500}{.09} = 5555\ \Omega$ or $5600\ \Omega \qquad .09^2 \times 5600 = 45.4\ W$

$R_5 = \frac{500}{.170} = 2940$ or $3000\ \Omega \qquad .17^2 \times 3000 = 87\ W$

3. Given: the circuit shown. Find: the total resistance between tap B and ground (⏚), taking care to include all paths except the 1,000 V generator.



R_1 = 100 ohms

R_2 = 2,000 ohms

R_3 = 4,000 ohms

R_5 = 1,578 ohms

I in R_4 = 20 MA

I in R_3 = 100 MA

I in R_5 = 260 MA

V across $R_1 = 100 \times .3 = 30\ V$

$V_{R_2} = 2000 \times .08 = 160\ V = V_{R_4}$

$R_4 = \frac{160}{.02} = 8000\ \Omega$

$V_{R_3} = 4000 \times .1 = 400\ V$

$V_{R_6} = V_{R_4} + V_{R_3} = 400 + 160 = 560\ V$

$R_6 = \frac{560}{.16} = 3500\ \Omega$

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OHM'S AND KIRCHHOFF'S LAWS AND BRIDGE CIRCUITS

EXAMINATION, Page 3.

3. $V_{R_7} = 1000 - 30 = 970$ V $R_7 = \frac{970}{.04} = 24,250\ \Omega$

Branch A = $R_5 + R_7 = 24,250 + 1578 = 25,828\ \Omega$

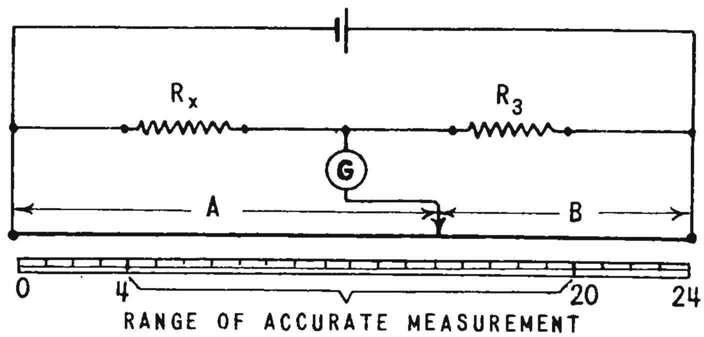
Branch B = $R_6 = 3500\ \Omega$

$R_{AB} = \frac{25,828 \times 3500}{29,328} = 3082\ \Omega$

Branch C = $R_3 + \frac{R_4 R_2}{R_4 + R_2} = 4000 + \frac{8000 \times 2000}{10000} = 4000 + 1600 = 5600\ \Omega$

$R_T = \frac{5600 \times 3082}{8682} = 1988$ ohms

4. In a slide-wire bridge using a 100-cm slide wire, if, when the circuit is balanced, R_1 = 21.2 cm, and R_3 = 200 ohms, what is the value of R_x?



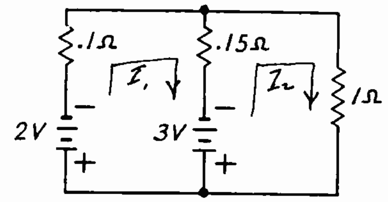
If $R_1 = 21.2$ cm is distance A then:

$R_2 = B = 100 - 21.2 = 78.8$ cm

and $\frac{R_1}{R_2} = \frac{R_x}{R_3}$ $R_x = \frac{R_1 R_3}{R_2} = \frac{21.8 \times 200}{78.8} = 55.4$ ohms

EXAMINATION, Page 4.

5.



Find the current supplied (or drawn) by each battery and the current in the load resistor. *SHOW THE CURRENTS ASSUMED IN THE DIAGRAM.*

$$I_1(.1+.15) - I_2(.15) = 2-3$$
$$-I_1(.15) + I_2(.15+1.) = 3$$

$$.25 I_1 - .15 I_2 = -1$$
$$-.15 I_1 + 1.15 I_2 = 3$$

$$.75 I_1 - .45 I_2 = -3$$
$$-.75 I_1 + 5.75 I_2 = 15$$

$$5.35 I_2 = 12 \qquad I_2 = 2.24 \text{ Amps.}$$

$$.25 I_1 - (.15 \times 2.24) = -1$$
$$.25 I_1 - .336 = -1$$
$$I_1 = \frac{.336-1}{.25} = -\frac{.664}{.25} = -2.65 \text{ Amp}$$

3 V Battery supplies 2.24 + 2.65 = 4.89 Amps

2 V Battery draws 2.65 Amps

Current in Load = 2.24 Amps

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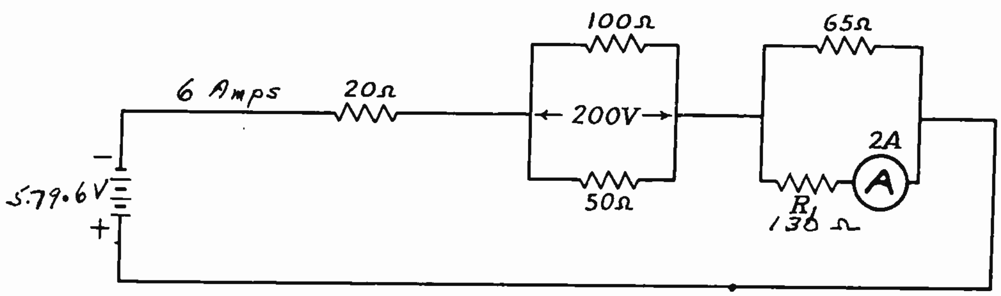
OHM'S AND KIRCHHOFF'S LAWS AND BRIDGE CIRCUITS

EXAMINATION, Page 5.

6. Find the value of R_1, total circuit resistance, total line current, and battery voltage.



I in 100Ω $= \frac{200}{100} = 2$ A

I in 50Ω $= \frac{200}{50} = 4$ A

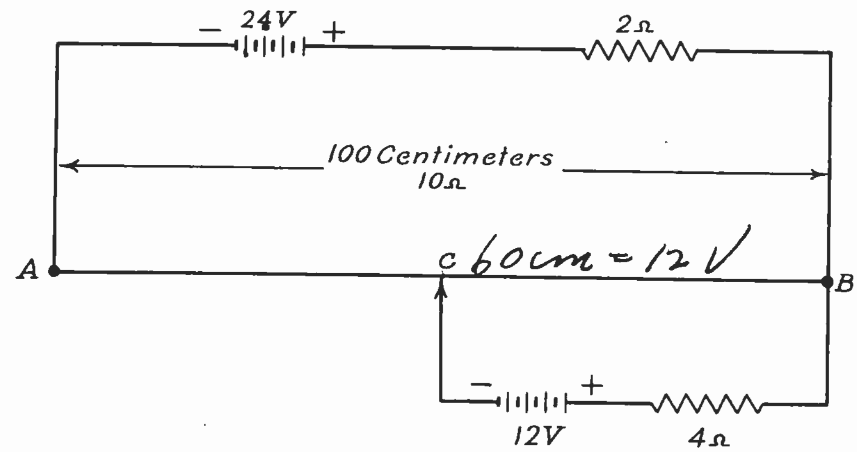
Total I = 6 Amps

$$\frac{I_{65\Omega}}{I_{R_1}} = \frac{R_1}{65} \qquad R_1 = \frac{65 \times 4}{2} = 130 \text{ ohms}$$

$$R_T = 20 + \frac{100 \times 50}{150} + \frac{65 \times 130}{195} = 20 + 33.3 + 43.3 = 96.6 \text{ ohms}$$

$$V = IR = 6 \times 96.6 = 579.6 \text{ Volts}$$

9. B, then when the lower mesh is connected to C, no current will flow through it. How far in centimeters must C be from B to obtain this condition?



24 V drop is across 2 + 10 = 12 Ω

12 V drop across 12/2 = 6 Ω = (2 + 4 Ω) = 60 cm

So $R_{BC} = 4\ \Omega$

$10\ \Omega = 100$ cm

$4\ \Omega = \frac{100 \times 4}{10} = 40$ cm.

BC = 40 cm

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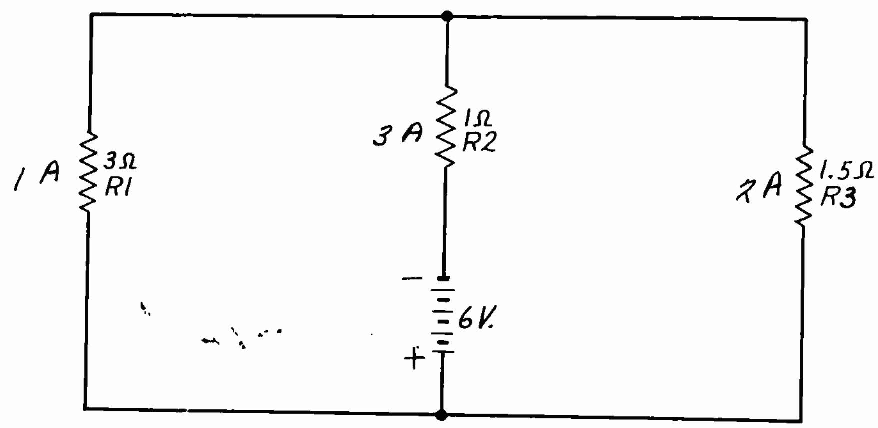
OHM'S AND KIRCHHOFF8S LAWS AND BRIDGE CIRCUITS

EXAMINATION, Page 9.

10. Find I in each resistor.



$R_T = 1. + \frac{1.5 \times 3}{4.5} = 1 + 1 = 2\ \Omega$

$I\ total = I_{R_2} = \frac{V}{R} = \frac{6}{2} = \underline{3\ Amps}$

$V\ across\ R_1\ and\ R_3 = 6 - (3 \times 1) = 3\ V$

$I_{R_1} = \frac{V}{R_1} = \frac{3}{3} = \underline{1\ amp}$

$I_{R_3} = \frac{V}{R_3} = \frac{3}{1.5} = \underline{2\ amp}$