SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

ALGEBRA

707D

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ALGEBRA

FOREWORD

The president of a certain college is said to have been approached by a prospective engineering student who said, "Dr. ---, I understand that the study of mathematics is quite difficult." The president puffed up and stated with pride, "Oh, yes—the way we teach it, it is very difficult."

We don't believe in teaching that way at CREI. We believe there are relatively few processes which cannot be broken down and explained in a simple practical manner. Many subjects have been made difficult to understand and impractical to use by the average man by the way in which they are taught, not because they are in themselves difficult. Such a subject is Algebra.

Every problem encountered in radio, from simple Ohm's Law, E = IR, to the most complex analysis of a directional antenna array, is expressed in terms of algebraic equations and solved by means of algebraic processes. *Most of these processes are basically simple.* The principal difference between the simplest problem and one which seems quite complex, is that in the latter there are a greater number of the basically simple processes to be performed and the inexperienced mathematician is tempted to give up in horror before attempting to analyze it and break it down into its simpler components.

It should be emphasized that the practical, useful, applied algebra of radio as studied in this assignment is *not* difficult. Each process should be thoroughly learned by practice before going on the the next. When the processes are learned, their application becomes simply a matter of orderly thought and reasonable care.

Don't let the term "Algebra" bluff you. Men with only a sixth grade education have successfully mastered this assignment.

E. H. Rietzke,
President.

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INTRODUCTION

The student should begin the study or review of algebra in the proper frame of mind. Some who have not previously studied algebra—and others who, looking back on their high school days, remember it as a difficult and possibly uninteresting subject—should first consider this point: *Algebra is the base of all mathematics—and without a mathematical background, the study of radio engineering principles is impossible.* Every man who has ever mastered any technical profession has first studied the basic mathematics of the profession. As a whole the average technical man is not a genius—he is simply one who has worked a little harder to master the fundamentals than has the non-technical man.

FUNDAMENTAL IDEAS.—Algebra is a very straight-forward subject—a subject of definite rules and processes which when once thoroughly understood may be manipulated in any conceivable number of combinations. Everyone uses algebra in his everyday computations but rarely thinks of it as such. All radiomen know the simple expression for Ohm's Law, E = IR and its two other forms, I = E/R and R = E/I. How many think of such transpositions of a simple equation as algebra? Still in getting from E = IR to I = E/R, algebraic division has been accomplished. Suppose one has $5.00, and then goes into a book store and purchases a $2.00 book. He then has $3.00 plus a $2.00 book; the store is minus a $2.00 book and plus $2.00 in cash. A condition of equality exists although an algebraic transposition has been made from one side of an equation to another.

Suppose that instead of the familiar voltage, current, and resistance of Ohm's Law or the concrete sum of $2.00 and a book, the terms dealt with are x, y, and z. The substitution of letters does not change the method of handling the problem because, in the final analysis, every term represented by a letter in the equation must represent some definite concrete value. However, the use of letters for concrete values makes the equation general in form; that is, x, y, z may then represent the three factors of Ohm's Law or some relationship between books and cost, etc.

A fundamental principle in the study of algebra may be expressed in one sentence—*learn to do one thing at a time.* The solution of the most complex algebraic expression always resolves into a succession of simple additions, subtractions, multiplications, divisions, raising to some power, or extraction of some root. The student who experiences difficulty with an algebraic problem can almost invariably solve the problem if figures are substituted for letters. In considering the solution of a problem, remember that the letters or symbols actually represent figures, and then use the same processes as would be used in handling numbers. For example, everyone knows that 4 times 5 equals 20. If 4 is designated by the letter B, and 5 by the letter C, then B times C equals 20. In algebraic form this would be written BC = 20. Writing the letters together simply indicates that when the proper figures are substituted for letters in the final solution, the figures will be multiplied together. Other ideas are equally simple.

In the study of radio the principal use of algebra will be to show the derivations of some of the standard electrical and radio equations in order that they may be understood and used intelligently, rather than merely memorized and taken for granted. Algebra is also used in the rearranging of such equations to find any unknown value when other values are given.

In algebra, letters and symbols are substituted in place of numerical values in order that a condition or an equation may be expressed in general form, without actually knowing the numerical value of that condition or equation. For example, it is known that the condition of resonance in an electrical circuit exists when the inductive reactance of the circuit equals the capacitive reactance. This condition may be expressed algebraically by saying that $X_L = X_c$ (X_L meaning inductive reactance, X_c meaning capacitive reactance.) The condition for resonance exists regardless of the numerical values of these terms so long as they are equal. Another familiar equation is $\lambda = 1{,}884\sqrt{LC}$. In this equation any values may be substituted for L and C, and when the problem is finally solved using any desired numerical values, the answer will be the wavelength (λ) of the circuit for the given set of conditions. A very commonly used equation is the one already mentioned expressing Ohm's Law, I = E/R. By simple algebraic processes this may be rearranged to show the value of R, R = E/I; or the value of E, E = IR.

Before proceeding with the study of algebra, it is well to tabulate some of the more commonly used signs of operation with their meanings:

WRITTEN	READ
$A + B$	A plus B
$A - B$	A minus B
$A = B$	A is equal to B
$A \neq B$	A does not equal B
$A \propto B$	A varies as B
$A > B$	A is greater than B
$A < B$	A is less than B
$A \cdot B$, or (A) (B)	A times B
$A \div B$, A/B or $\frac{A}{B}$	A divided by B
$A = \pm B$	A = plus B or A = minus B
(A + B) or [A + B]	The quantity in parentheses or in brackets
$A \cong B$ or $A \approx B$	A is approximately equal to B

THE EQUATION.—An equation is merely an expression of equality between two values, and is indicated by writing the two values separated by the sign of equality, (=). For example, R = E/I. In this case the value of R is equal to the value of E divided by I. An equation is only an equation *as long as the expressions on the two sides of the sign* *of equality are of the same value.* This rule, though simple and very evident, is the principle rule to remember in the solution of equations to find any desired value within the equation. It is permissible to change the value of one side of the equation *provided the same change is also made in the other side,* so as not to destroy the condition of equality. It is possible to add to, subtract from, multiply, divide, or change in any way *both* sides of the equation by the same amount and still have the condition of equality.

The student should memorize the following rules that apply to *any* equation.

RULE 1: The same quantity may be added to *both sides* of an equation without affecting the equality.

EXAMPLE:	$2 = 2$
Adding 1 to both sides	$2 + 1 = 2 + 1$
Therefore, if	$A = B$
Then	$A + 1 = B + 1$

RULE 2: The same quantity may be subtracted from *both sides* of an equation without affecting the equality.

EXAMPLE:	$3 = 3$
Subtracting 1 from both sides	$3 - 1 = 3 - 1$
Therefore, if	$A = B$
Then	$A - 1 = B - 1$

RULE 3: *Both sides* of an equation can be multiplied by the same number without affecting the equality.

EXAMPLE:	$2 = 2$
Multiplying both sides by 2	$2 \times 2 = 2 \times 2$
Therefore, if	$A = B$
Then	$2A = 2B$

RULE 4: *Both sides* of an equation may be divided by the same number without affecting the equality.

EXAMPLE:	$2 = 2$
Dividing both sides by 2	$\frac{2}{2} = \frac{2}{2}$
Therefore, if	$A = B$
Then	$\frac{A}{2} = \frac{B}{2}$

RULE 5: *Both sides* of an equation may be raised to the same power without affecting the equality.

EXAMPLE:	$2 = 2$
Raising both sides to the third power	$2^3 = 2^3$
Therefore, if	$A = B$
Then	$A^3 = B^3$

RULE 6: The same root may be extracted from *both sides* of an equation without affecting the equality.

EXAMPLE:	$16 = 16$
Extracting square root of both sides	$\sqrt{16} = \sqrt{16}$
Therefore, if	$A = B$
Then	$\sqrt{A} = \sqrt{B}$

A few additional rules that aid in the transposition of equations are as follows:

RULE 7: Dividing by a number is equivalent to multiplying by the reciprocal of the number. For example, dividing by 2 is equivalent to multiplying by 1/2.

RULE 8: Division by zero is

not acceptable in any equation.

RULE 9: The root of a number raised to the same power as the index of the root equals the number. For example the square root of 2 squared is 2 or $\sqrt[3]{A^3} = A^{3/3} = A$.

RULE 10: Quantities equal to the same thing are equal to each other. If A = B and B = C then A = C.

A few simple rules must be learned before an algebraic expression can be properly written.

ADDITION AND SUBTRACTION.— When adding *like* terms, prefix the common term by the number of terms which are to be added.

EXAMPLE: a + a + a may be expressed as 3a. (3 is called the coefficient of a.)

xy + xy may be expressed as 2xy. (2 is called the coefficient of xy.)

(L - N) + (L - N) + (L - N) may be expressed as 3(L - N). [3 is called the coefficient of (L - N).]

Unlike terms cannot be handled in the above manner. A + B can only be written A + B. xy + AD can only be written xy + AD. *Any expression or term not prefixed by a number is understood to have a coefficient of 1.* For example, A means 1A, xy means 1xy, etc. Thus: A + 2A = 3A. .5A + A = 1.5A.

The same rules hold true, in general, for subtraction.

EXAMPLES: 4x - 2x = 2x

x - 3x = -2x

(Subtracting three units from one unit leaves minus two units.)

(The unit in both examples is x.)

As in addition, it is not possible to subtract unlike terms and express as a common term prefixed by a numerical value; B - A can only be expressed as B - A. 4B - 2A can only be expressed as 4B - 2A. Whereas, 4B - 3B may be written as B. 3B - 5B may be expressed as -2B.

It will be noted that the rules for positive and negative numbers, as discussed in the assignment bearing that title, are followed directly in algebra. *In fact, addition, taking cognizance of the positive and negative signs of the terms, is called "Algebraic Addition".*

An example of the use of algebraic addition in a common electrical equation is found in one expression of the equation for the impedance of a circuit, $Z^2 = R^2 + X^2$. Any desired values of R (Resistance) and X (Reactance) may be used and, when the problem is completed using the desired figures, the answer will be Z^2 (impedance of the circuit squared). By extracting the square root of *both* sides of the equation (see rule 6) the impedance of the circuit may be determined. A more common form for the above expression is, $Z = \sqrt{R^2 + X^2}$. This expresses directly the impedance of the electrical circuit.

If the above circuit contained both inductive and capacitive reactance, the equation would be expressed as follows: the conditions being shown by the symbols beneath the radical, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ In this case the square of the algebraic *difference* between X_L (Inductive reactance) and X_c (Capacitive reactance) is added to the square of R (Resistance), and then the square root of the final sum is determined.

Instead of saying, "The impedance of an alternating current circuit is equal to the square root of the sum of the squares of the resistance and the reactance", this may be simply written algebraically.

$Z = \sqrt{R^2 + X^2}$ where $X^2 = (X_L - X_c)^2$

Thus algebra permits the expression of a condition or a rule in very concise form. Simple algebraic processes also make it possible to rearrange equations to find any desired values. Thus if the impedance and the resistance of the above circuit are known, and it is desired to find the reactance, the equation may be rearranged as,

$$X = \sqrt{Z^2 - R^2}$$

The methods or rules by which the rearranging of an equation is done to show varying conditions, will be discussed in detail in this assignment. In the above equation the value beneath the radical could not be further simplified—it must be written $Z^2 - R^2$ until numerical values are substituted and the problem solved for X, because the terms are not *like* terms and therefore cannot be combined in addition or subtraction with a numerical coefficient.

Exercises

Reduce to the simplest form:

1. $A + 2A - 5A = ?$
2. $3n - m + 2n + 4m = ?$
3. $1.5x + 2y + 4x - 6y = ?$
4. $.417R_2 + R_2 + 10 = ?$
5. $1.25R + 2R_1 + 4.85R = ?$

Add:

6. $2x + y$ and $3x + y$
7. $7x + y$, $3x + 7y$ and $2x - 4y$
8. $a + b + c$ and $a - b + c$
9. $x^2 + xy + y^2$, $x^2 - 2xy + y^2$, and $x^2 + 2xy + y^2$
10. $5x^2 + 3x + 7$, $6x^2 - 3x + 7$, and $x + 2$ and $+ 13$

Subtract:

11. 3.9a from 4.7a
12. (-3) from 7
13. 2.5x from 3x
14. $30\sqrt{A}$ from $45\sqrt{A}$
15. $(-b^2 x^2 y^2)$ from $(a^2 x^2 y^2)$

MULTIPLICATION.—In expressing the multiplication of terms, the terms to be multiplied are simply written in succession with no signs between terms.

Examples: I times R is written IR. Thus, in the equation of Ohm's Law, E = IR, E equals the *product* of I times R, expressed simply as IR. The equation for Inductive Reactance shows that X_L is equal to the *product* of 2π (6.28) times F (Frequency) times L (Inductance). This is written $X_L = 2\pi fl$.

The equation for Wavelength (λ) shows that wavelength equals the *product* of 1,884 times the square root of the product of L and C. This is written, $\lambda = 1{,}884\sqrt{LC}$. To solve this problem, first substitute the desired figures for L and C, extract the square root of the *product* of L times C, and multiply the square root by 1,884.

2N times 4PQ is written 8NPQ. (Note—the numerical coefficients are actually multiplied. The letters, being unlike, cannot be treated in such manner, and the multiplication can only be indicated by writing the letters in succession.)

Any terms written in succession, and not separated by signs of operation, in an algebraic expression are to be multiplied. Thus 15ABC means 15 times A times B times C. It must be understood that in multiplying unlike terms or letters it is only possible to *indicate* the multiplication.

When multiplying *like* terms or letters, make use of the laws of exponents. $a \cdot a \cdot a \cdot a \cdot a = a^5$. This is the same as aaaaa. Since the factors to be multiplied are identical, they may be expressed as the common factor with an exponent equal to the number of factors to be multiplied together. This also applies to the more complex terms.

Example: $(2abcd)\ (2abcd) = (2abcd)^2$

Since the exponent applies to everything within the parentheses, this could also be written $2^2a^2b^2c^2d^2$ or $4a^2b^2c^2d^2$. The first method of expression, $(2abcd)^2$ is usually to be preferred.

Another example of multiplication: $(2a^2b^3)\ (4ab^2c)$.

Multiplying, the product becomes $8a^3b^5c$. This can be clearly proved by enlarging each quantity separately and then combining: $2a^2b^3 = 2aabbb$; $4ab^2c = 4abbc$. Combining, 2aabbb4abbc. Multiplying the coefficients, combining the like letters, and affixing the proper exponents, $8a^3b^5c$. *Any number of factors combined by multiplication is considered as one term.*

Examples:

a^3b^2c is one term.

$a^2 + b - c$ are three terms.

$xyz + 2a - b^2d$ are three terms.

An exponent on the outside of parentheses refers to everything within the parentheses.

Example: $(3a^2bc)^3$

The exponent 3 on the outside of the parentheses means that everything within the parentheses is to be raised to the third power. $3^3 = 27$. a^2 becomes a^6. b becomes b^3 and c becomes c^3. The entire quantity could then be written $27a^6b^3c^3$.

An exponent immediately following any letter refers to that letter only.

Example: ab^2. The exponent refers only to b, and the quantity could be enlarged as abb. But, *if the two letters are enclosed within parentheses and the exponent placed outside the parentheses as, $(ab)^2$, then the exponent refers to both a and b,* and the quantity may be expressed as a^2b^2 and further enlarged to aabb.

When multiplying two or more terms expressed as one quantity, as (a + b) by another term Q, Q(a + b), *all the terms in the quantity to be multiplied by the multiplier Q must be multiplied individually;* thus, Q(a + b) means a + b multiplied by Q. *Both* a and b must be multiplied individually to enlarge and remove the parentheses.

Multiplicand	a + b
Multiplier	Q
Product	aQ + bQ
Therefore,	Q(a + b) = aQ + bQ

Another example: $xy(bc - d + x)$

Multiplicand	$bc - d + x$
Multiplier	xy
Product	$bcxy - dxy + x^2y$

The answer is still an expression of three terms, each of the original terms having been multiplied by xy to remove the parentheses. In most cases the expression would be left in its original form, but there are times when it is necessary to remove the parentheses in order to be able to isolate one of the terms which is enclosed, and in that case each term must be actually multiplied by xy instead of multiplication being merely indicated as was done in the original expression. It will be noted that careful attention must be given to the signs of the terms in order that the signs of the product may be correct. This will be shown below:

$$c(a + b) = ac + bc$$

$$-c(a + b) = -ac - bc$$

$$c(a - b) = ac - bc$$

$$-c(a - b) = -ac + bc$$

When multiplying terms having LIKE signs, the product is positive. When multiplying terms having UNLIKE signs, the product is negative.

When multiplying two expressions, each expression having two or more terms, the procedure is as shown in the examples following: (Note particularly that the sign of each term must be correct in each step.)

$(a + b)\ (a + b) = a + b$

	$a + b$
multiplying by a	$a^2 + ab$
multiplying by b	$ab + b^2$
adding for total product	$a^2 + 2ab + b^2$

$(a + b)\ (a - b) = a + b$

	$a - b$
multiplying by a	$a^2 + ab$
multiplying by -b	$- ab - b^2$
adding for total product	$a^2 - b^2$

(Note, ab and -ab when added equal zero.)

$(a - b)\ (a - b) = a - b$

	$a - b$
multiplying by a	$a^2 - ab$
multiplying by -b	$- ab + b^2$
adding for total product	$a^2 - 2ab + b^2$

(Note that -b times -b equals plus b^2 and that -ab plus -ab equals -2ab.)

$(a + b)\ (x - y) = a + b$

	$x - y$
multiplying by x	$ax + bx$
multiplying by -y	$- ay - by$
adding for total product	$ax + bx - ay - by$

(Note that since no terms are similar, no two can be combined in addition.)

It should be noted that in each of the above examples *extreme care must be taken to keep the signs of the terms correct.* It should also be noted that each step of the problem is extremely simple and that the problem is merely divided into a number of smaller elementary problems with the entire product being finally determined as the sum of the individual products.

Exercises

Multiplication,

16. $x \cdot x^2 \cdot x^4 = ?$

17. $(x^2y^3z)\ (x^3y^{-1}z^2) = ?$

18. $(ab^2c)\ (a^2b)\ (a^{-1}bc^4) = ?$

19. $(a^2 + ab + c) \cdot a = ?$

20. $(a^2 + ab)^2 = ?$

21. $(L + M)\ (L - M) = ?$

22. $(a - b)^3 = ?$

23. $(a + 2b - c)\ (3a - b) = ?$

24. $(a^2 + b)^2 \cdot x = ?$

25. $(a + ab)^2\ (x - y) = ?$

DIVISION.—The rules for division are the inverse of those for multiplication. When dividing *like* terms having exponents, the laws of exponents apply.

Examples: $\frac{a^3}{a^2} = a.$ This can be shown as $\frac{aaa}{aa} = a$

$$\frac{a^2}{a^{-4}} = \frac{aa}{a^{-1}a^{-1}a^{-1}a^{-1}} = aaaaaa = a^6$$

Remember $\frac{1}{a^{-1}} = a$, and $a^{-1} = \frac{1}{a}$

$$\frac{abc}{bc} = a \quad \frac{x^2yz}{x} = \frac{xxyz}{x} = xyz \quad \frac{a}{a} = 1$$

This last example is a *very important* one to remember. In the third example, $\frac{abc}{bc}$, bc is cancelled into bc. This leaves a times 1. Since multiplying by one does not change the value of a quantity, simply drop the 1 and express the answer as a. In the last example however the divisor is equal to the dividend and the answer is 1, alone. It can be seen that the 1 cannot now be dropped as the answer would then be zero, which is *not* the case.

Proof: If a = 2, then 2/2 = 1 not zero.

When dividing *unlike* terms the division can only be indicated.

Examples: $\frac{a}{b}, \frac{xv}{z}, \frac{E}{R}, \frac{1}{2\pi fC}.$

These cannot be further simplified.

Referring back to multiplication: It will be remembered that in the case of several terms as a quantity to be multiplied by another term, *each* term in the quantity must be multiplied, as $a(b - c + d) = ab - ac + ad$. A similar rule exists for division. When several terms as a quantity are to be divided by another term, *each* term in the quantity must be divided. Thus, when dividing

$$\frac{axy + a^2z - ab}{a}$$

each term in the dividend must be divided by the divisor a, which divides evenly into each and results in a quotient, $xy + az - b$.

In the case of several terms in the dividend to be divided by the divisor, wherein one or more terms of the divisor will not divide evenly, division can only be indicated.

Example: $\frac{ab + c + adx + xz}{a}$

a will divide evenly into the first and third terms but not into the second and fourth. In the case of the latter terms division can only be indicated; thus, a will divide evenly into ab and adx and will not divide evenly into c and xz. The answer is then,

$$b + dx + \frac{c + xz}{a}$$

This expression cannot be further simplified.

Exercises

Division,

26. $a^2bc \div ac = ?$

27. $xy^2 z^3 \div az^2 = ?$

28. $pq^{-4} \div q^2 = ?$

29. $(ab + ac - a^2x) \div a = ?$

30. $(x^2y^4 - 4x^2y - x^5z) \div x^2 = ?$

31. $(a^2b + 2ac - a) \div a = ?$

32. $(m^2n + mo + q) \div m = ?$

33. $(a^2b + 3ab^2 + ab) \div ab = ?$

34. $3x + 9y - 18 \div 3x = ?$

35. $18a + 16ax - 14 \div (-2a) = ?$

DIVISION BY A POLYNOMINAL: Division by a polynominal (an expression of more than one term) is best shown by example.

Example I: Divide $a^2 + 2ab + b^2$ by $a + b$. Set up the problem as follows:

Divisor Dividend Quotient
$a + b \,|\, a^2 + 2ab + b^2 \,|\, a$

STEP 1. Divide the first term (a) of the divisor into the first term (a^2) of the dividend. Thus, $a^2 \div a = a$. Set this quotient (a) at the right as shown.

STEP 2. Multiply the divisor (a + b) by (a), the first term of the quotient. This product is clearly ($a^2 + ab$); set it under like terms in the dividend, namely, $a^2 + 2ab$, as shown in heavy print at the right.

$a + b \,|\, a^2 + \mathbf{2ab} + b^2 \,|\, a$
$\mathbf{a^2 + ab}$

STEP 3. *Subtract* $a^2 + ab$ from $a^2 + 2ab$, and obtain ab. Bring down the unused terms, namely. (b^2) of the original dividend to form a new dividend ($ab + b^2$).

$a + b \,|\, a^2 + 2ab + b^2 \,|\, a$
Subtract $a^2 + ab$ ↓
$ab + b^2$ new dividend

STEP 4. Divide the first term (a) of the divisor into the *first* term (ab) of the new dividend. Thus, $ab \div a = b$. Write this result as the second term of the quotient.

$a + b \,|\, a^2 + 2ab + b^2 \,|\, a + b$
$a^2 + ab$
$ab + b^2$
second term of quotient

STEP 5. Multiply the divisor (a + b) by the second term (b) of the quotient. This gives $ab + b^2$; set this (heavy print) under the new dividend, and then subtract it from the new dividend. The remainder is clearly zero in this particular problem and means that $a^2 + 2ab + b^2$ is evenly divided by $a + b$ to give $a + b$ as the quotient. In short, $a^2 + 2ab + b^2$ is the square of $a + b$; that is, $a^2 + 2ab + b^2 = (a + b)^2$.

$a + b \,|\, a^2 + 2ab + b^2 \,|\, a + b$
$a^2 + ab$
$ab + b^2$
Subtract $\mathbf{ab + b^2}$
$\mathbf{0 + 0}$ remainder

NOTE: Sometimes the divisor is set under the quotient as shown to the right. This facilitates multiplying of the two as discussed in Steps 2 and 5, but either arrangement is satisfactory, and the student can use the one that he happens to prefer. Indeed, a third variation is to set the quotient *under* the divisor, instead of above it. Obviously, these are mere details of the process.

$$
\begin{array}{rrrr|l}
\text{Dividend} & & & & \text{Quotient} \\
a^2 & +\,2ab & +\,b^2 & & a + b \\
\cline{5-5}
\underline{a^2} & \underline{+\;\;ab} & & & a + b \\
 & ab & +\,b^2 & & \text{Divisor} \\
 & \underline{ab} & \underline{+\,b^2} & & \\
 & 0 & +\,0 & & \text{Remainder}
\end{array}
$$

Example II: Divide $a^3 - b^3 - 3a^2b + 3ab^2$ by $a-b$ This example will bring out many points that did not arise in the first example. Thus, since the divisor is written $a - b$; i.e., with a preceding the b, arrange the dividend in *descending* powers of a, and then proceed as in the first example.

$$
\begin{array}{r|rrrr|l}
\text{Divisor} & \text{Dividend} & & & & \text{Quotient} \\
a-b & a^3 & -\,3a^2b & +\,3ab^2 & -\,b^3 & a^2 - \mathbf{2ab} + b^2 \\
\cline{1-1}\cline{6-6}
 & \underline{a^3} & \underline{-\;\;a^2b} & \downarrow & \downarrow & \\
 & & -\,\mathbf{2a^2b} & +\,3ab^2 & & \\
 & & \underline{-\,2a^2b} & \underline{+\,2ab^2} & & \\
 & & & ab^2 & -\,b^3 & \\
 & & & \underline{ab^2} & \underline{-\,b^3} & \\
 & & & 0 & -\,0 & \text{Remainder}
\end{array}
$$

NOTE: $-3a^2b - (-a^2b) = -3a^2b + a^2b = -2a^2b$ →

NOTE: $3ab^2 - 2ab^2 = ab^2$ →

Watch signs at every step. Consider $-2a^2b$ (shown in **heavy print**). When divided by first term (a) of divisor, it yields $-2a^2b \div a = -2ab$, as shown by heavy print in quotient. In other words, a negative number, when divided by a positive number, yields a negative number as the quotient. This follows from the general rule that division involving terms of unlike signs gives a negative sign in the quotient.

Example III: Divide $(4a^4 + 2b^4)$ by $(2a^2 - 2ab + b^2)$. The important difference in this example is that all powers of (a) except the fourth are missing in the dividend. Since terms involving such intermediate powers of a will appear in the process of division, provision should be made for them in the dividend. The latter should therefore be written as:

$$4a^4 + 0a^3b + 0a^2b^2 + 0ab^3 + 2b^4$$

Terms having zero coefficients are normally not written, but it must not be forgotten that they are really present. Hence, the above expression can be written with the terms $4a^4$ and $2b^4$ separated to leave room for the zero terms which, however, are not written in.

Upon performing the first division of $4a^4$ by $2a^2$, thus obtaining the first term of the divisor $2a^2$, and then multiplying the entire divisor by this term, there is obtained the first minuend, $4a^4 - 4a^3b + 2a^2b^2$ (shown in heavy print). When this is subtracted from the dividend, $4a^4$ yields zero when subtracted from $4a^4$. But $-4a^3b$, when subtracted from $0a^3b$ (understood), yields $0a^3b - (-4a^3b) = +4a^3b$, as shown. Similarly for $+2a^2b^2$; $0a^2b^2 - (2a^2b^2) = -2a^2b^2$.

$$
\begin{array}{l|llllll}
2a^2 - 2ab + b^2 & 4a^4 & & & & +2b^4 & \underline{|2a^2 + 2ab + b^2} \\
 & \underline{4a^4} & \underline{-4a^3b} & \underline{+2a^2b^2} & & & \\
 & 0 & +4a^3b & -2a^2b^2 & & & \\
 & & \underline{4a^3b} & \underline{-4a^2b^2} & \underline{+2ab^3} & & \\
 & & & 2a^2b^2 & -2ab^3 & +2b^4 & \\
 & & & \underline{2a^2b^2} & \underline{-2ab^3} & \underline{+b^4} & \\
 & & & & & b^4 & \text{Remainder}
\end{array}
$$

Next note that $2b^4$ of the dividend is not brought down until the third operation, when b^4 is subtracted from it. Finally, note another difference between this example and the previous two—namely, that there is a remainder, b^4. The complete answer is shown as in arithmetic:

$$2a^2 + 2ab + b^2 + \frac{b^4}{2a^2 - 2ab + b^2} \quad \text{Answer}$$

To prove an example in division, multiply the divisor by the quotient. The product should equal the dividend. Consider Example II: the divisor is $a - b$, and the quotient is $a^2 - 2ab + b^2$. Multiplying, there is obtained:

$$
\begin{array}{lrrrr}
 & a^2 & -2ab & +b^2 & \\
 & & \underline{a} & \underline{-b} & \\
\text{Multiplying by } a & a^3 & -2a^2b & +ab^2 & \\
\text{Multiplying by } -b & & \underline{-a^2b} & \underline{+2ab^2} & \underline{-b^3} \\
\text{Adding} & a^3 & -3a^2b & +3ab^2 & -b^3
\end{array}
$$

which is the original dividend

Exercises

Divide and prove:

36. $16a^2 + 32ab + 16b^2$ by $a + b$

37. $a^3 - a^2b + ab^2$ by $a - b$

38. $a^3 - b^3$ by $a - b$

39. $a^3 + b^3$ by $a + b$

40. $a^4 - b^4$ by $a^2 + b^2$

41. $x^3 + 7x^2y + 7xy^2 + 2y^3$ by $x + y$

42. $x^4 + 64$ by $x^2 + 4x + 8$

43. $1 - x - 3x^2 - x^5$ by $1 + 2x + x^2$

44. $x^5 + x^3 + x^4y + y^3 - 2xy^2 - x^3y^2$ by $x^3 + x - y$

45. $(b^3 - 125) / (b - 5)$

FACTORING.—In many cases it becomes necessary to remove a factor or term from within parentheses when that factor or term is combined with other factors or terms. The desired factor may be combined with several terms within the parentheses. It is not desired to change the total *value* of the entire expression.

Example: $(ab + bcq - 2bd)$ To factor remove b from within the parentheses, and rearrange the expression, so that its value remains unchanged.

Each of the three terms within the parentheses is composed of the product of b and one or more letters or figures. ab is the product of a times b. To remove b from ab it is necessary to divide ab by b; $ab/b = a$. The other terms must be treated in the same manner. Thus, if the entire expression is divided by b, $(ab + bcq - 2bd)/b = (a + cq - 2d)$. (It has previously been shown that if an expression composed of several terms is divided, *every* term in the expression must be divided. Thus, in the above expression b is divided into *each* term individually.)

But, $(a + cq - 2d)$ is not equal in value to $(ab + bcq - 2bd)$. It is expressly stated that b is to be removed from within the parentheses *without* changing the total value of the expression.

Since $(ab + bcq - 2bd)$ was divided by b to obtain $(a + cq - 2d)$, if $(a + cq - 2d)$ is now multiplied by b the expression will again have its original value. However, $(a + cq - 2d)$ can be multiplied by b without placing b within the parentheses. $b(a + cq - 2d) = (ab + bcq - 2bd)$. (See rules 3 and 4.)

Thus, b has been removed from within the parentheses, but without changing the total value of the expression. The entire expression now consists of the product of b times the quantity within the parentheses. Practical use of factoring in the solutions of equations will be shown later in this assignment.

Exercises

Factoring:

46. $(ab + ab^2 + ay)$ Factor for a

47. $(x^2y + aby + 3y)$ Factor for y

48. $(2mn + 15m + mn^2p)$ Factor for m

49. $(ab + 2a^2b + b)$ Factor for b

50. $(4xy + 2x + nx)$ Factor for x

SUBSTITUTION OF TERMS.—In any equation one term or combination of terms may be substituted for another, *provided* the term substituted is exactly equal to the value for which it is to be substituted.

Example: $a = \frac{2xy}{z}$

Given the equation, $\frac{2xy}{z} + q = \frac{bc}{d}$

Substituting a for its equivalent value, $a + q = \frac{bc}{d}$ (See rule 10.)

Substitution of terms is used in many cases where a complex expression must be carried through several operations in solving for an unknown. It is also extensively used to enlarge an equation after the final solution has been reached. A practical use of substitution is found in the equation which states the condition of resonance in an electrical circuit. $X_L = X_c$. It is desired to enlarge this equation. Knowing that $X_L = 2\pi fL$ and that $X_c = 1/2\pi fC$, substitute the enlarged values for X_L and X_c,

$$2\pi fL = \frac{1}{2\pi fC}$$

It is possible to express this equation in terms of L and C and still keep it in a more concise form. ω (Omega) is the symbol for $2\pi f$. Again substitute and write the equation as, $\omega L = 1/\omega C$.

Thus the condition of resonance may be expressed by any one of the three equations, depending upon the detail it is desired to show:

$$X_L = X_c, \quad \omega L = \frac{1}{\omega C} \quad \text{or} \quad 2\pi fL = \frac{1}{2\pi fC}$$

The equation which expresses the impedance of an alternating current circuit containing Resistance, Inductance and Capacity, is:

$$Z = \sqrt{R^2 + X^2}$$

But, $X = X_L - X_c$

Substituting, $Z = \sqrt{R^2 + (X_L - X_c)^2}$

SOLUTION OF EQUATIONS

In reading the assignment thus far, the student must have been struck by the fact that an equation is not only an equality, but also a statement, *in shorthand form*, of what quantities are related to what other quantities, and *in what manner*. One of the most significant advances of Mankind has been the ability to express exact relationships in the mathematical shorthand form of the equation, rather than in the more unwieldly sentence form. Indeed, the Arabs, who developed algebra, lacked a great deal of modern mathematical symbolism, and this lack hampered them greatly in attempting to solve any but the most elementary equations.

As an example of the power of modern mathematical expression, the following equation is taken from Coffin's book "Vector Analysis".

$$\overline{\nabla} \quad L = 0$$

The entire subject of classical dynamics (a branch of mechanics) is included in this remarkable formula! The catch, of course, is in the proper interpretation of the symbols $\overline{\nabla}$ and L, and this requires a knowledge of advanced mathematical methods.

Where these symbols are used often enough to cause them to be remembered, their use is an aid in unifying the subject—in permitting a central fundamental idea to enable any problem in the subject to be solved. On the other hand, there is a danger that the reader will forget what the shorthand expression or symbol stands for; i.e., the mathematical shorthand can be abbreviated to a point where it becomes meaningless. Accordingly, there is a tendency today—in certain cases—to use a combination of the ordinary sentence form and the mathematical shorthand.

As an example, in a later assignment on vacuum tubes, the formula or equation for the second harmonic distortion in an amplifier stage is expressed as follows:

% second-harmonic distortion =

$$\frac{I_{max} + I_{min} - 2I_b}{I_{max} - I_{min}}$$

Note that the left-hand side is really the beginning of a sentence, whereas the right-hand side is in the mathematical shorthand form. While one could have called the percent second-harmonic distortion H, and used H on the left-hand side, this would have required an auxiliary sentence to define H, so that it is just as brief to state the left-hand side directly in sentence form. However, in the discussion which follows, the more usual types of equations, in which symbols are used throughout, will be analyzed.

FUNDAMENTAL PRINCIPLES OF SOLUTION.—In applying the rules and examples given previously to the practical solution of equations, it will be seen that there are three practices in general use, and, of course, combinations of these practices.

1. Addition, subtraction, multiplication, and division performed on each side of the equation.

2. Raising a number to a given power, and extracting roots of a number.

3. Substitution of terms. (Factoring is a combination of division and multiplication).

The practical uses of these methods in the solution of equations will be shown. An equation can be compared to a balance scale. If one pound is added to or subtracted from one side of the scale, a similar operation must be performed on the other side to keep the scale in a balanced condition. If A is added to one side of an equation, A must also be added to the other side of the equation. Otherwise, the meaning of the word equation is destroyed; that is, there is no longer an *equality.*

For example,

(1) $A + B = C$

If D is added to A + B, D must also be added to C on the other side of the equation.

(2) $A + B + D = C + D$

This statement can be proved by assigning numerical values to A,B,C and D. Let A = 10, B = 5, C = 15, D = 7.

(1) $10 + 5 = 15$

$15 = 15$

(2) $10 + 5 + 7 = 15 + 7$

$22 = 22$

It is quite evident that the addition of D to only one side of the equation would not give an equality, and no balance would exist.

For example,

$A + B + D = C$

$10 + 5 + 7 = 15$ (false)

$22 \neq 15$

The solution of an equation consists of getting the desired term on one side of the equality sign, with *all* the other terms on the other side. In many equations the desired factor or term is found in several places in the equation. The solution consists of the steps necessary to place factors containing the desired term on one side of the equation and all the other terms on the other side of the equation. Such a solution might result from solving an equation such as the following for A:

$$7D(2C + 3A) = 4B + 3C$$

for which the solution is

$$A = \frac{4B + 3C - 14CD}{21D}$$

The practical use of algebra lies in the fact that when a condition is expressed algebraically, the value of the left-hand expression may be found by substituting known numerical values for the letters or symbols in the right-hand expression. Performing the indicated processes will then lead to the desired solution.

Consider the algebraic expression of Ohm's Law, used in the solution of many radio circuit problems.

$$I = \frac{E}{R}$$

The known values of E and R may be substituted in this equation to solve for I. Assume E = 100 volts, and R = 10 ohms. Then,

$$I = \frac{E}{R} = \frac{100}{10} = 10 \text{ amperes}$$

It will be seen that the unknown value I is on one side of the equation only, and that all the other values are known, thus making the solution possible. If two unknown values were present, then a complete numerical solution would not be possible.

Consider another example:

$$A = B + C \quad \textit{(Solve for B)}$$

Here, B is added to C and must be isolated to result in a solution where B is alone on one side of the equation. This can be done by subtracting C from *both* sides of the equation:

$$A - C = B + C - C$$

$$A - C = B$$

Note that

$$C - C = 0$$

leaving B alone. It is clear that if A - C = B, then B = A - C, since an equality can be reversed without affecting it.

It should now be evident that we can move any quantity from one side of the equality sign to the other side by merely changing its sign. This is true where a quantity such as BC is moved, but we cannot take B alone or C alone by merely changing the sign.

Consider next the expression

$$\frac{A}{B} = C \quad \textit{(Solve for A)}$$

First rewrite the value A/B as A(1/B). Then

$$A(1/B) = C$$

Multiply both sides by B.

$$A(B/B) = BC$$

$$A = BC$$

It is now evident that a quantity can be moved from the *denominator* of a fraction to the *numerator* on the other side of the equality sign without changing its sign or the value of the equation. The reverse is also true: a quantity can be moved from the *numerator* on one side of the equation, to the *denominator* on the other side; this can be proved by merely reversing the steps given above. (These operations represent another application of the basic rules of algebra previously discussed.)

Thus, consider

$$X = YZ$$

Solve for Y. If Z is divided into both sides of the equation, there is obtained:

$$\frac{X}{Z} = \frac{Y\cancel{Z}}{\cancel{Z}}$$

$$\frac{X}{Z} = Y$$

In many algebraic equations there is more than one term on each side of the expression. The same procedure is employed in the solution as when only one term is found. Consider the equation

$$a - b + c = d + e$$

Solve for c. Simply transpose both a and -b (note the minus sign before b) to the right-hand side of the equation where these become -a and +b, respectively, and at the same time c is left alone on the left-hand side:

$$c = d + e - a + b$$

Another point that arises very frequently in algebra is the use of parentheses in an expression. Consider the equation

$$a = b - (c - d)$$

This states that the difference (c - d) must in turn be subtracted from b, whereupon the remainder equals a. To put it in a slightly different way, d must first be subtracted from c, and the remainder in turn subtracted from b to yield a.

However, the order of addition or subtraction of a set of quantities is immaterial. If desired, c can first be subtracted from b, and then (-d) *subtracted* from the above remainder. The *subtraction* of (-d) is the same as the *addition* of (+d), so that the above expression can just as well be written as

$$a = b - c + d$$

Note that now the parentheses is not employed. Its significance in the preceding expression was to bring out the fact that the quantity to be subtracted from b was in turn the difference of two quantities: (c - d). As shown directly above, the same result is obtained by removing the parentheses, and changing the signs within the parentheses *if a minus sign precedes the parentheses.* (If a plus sign precedes the parentheses, then it can be removed without any change in sign.)

Parentheses arise in algebraic expressions because in setting up the equation it has been found convenient to group a series of terms together. As an elementary example, suppose A borrows

$1,000 from B in order to carry out some business transaction. The venture proves a failure; for the $1,000 invested, only $300 is salvaged. A therefore shows on his books

$$(1{,}000 - 300) = \$700 \text{ as a net liability}$$

Now suppose A's uncle suddenly dies and leaves him $7,500. A's financial position has suddenly changed for the better. He can now write as his assets

$$7{,}500 - (1{,}000 - 300) = 7{,}500 - 700$$
$$= \$6{,}800$$

Alternatively he can write

$$7{,}500 - 1{,}000 + 300 = \$6{,}800$$

Whether he uses the parentheses or not depends in this example upon the matter of timing: if his uncle's death occurred very shortly after his loss he would be inclined to associate the three terms together without the parentheses; if the death occurred several months later, he would already have calculated his loss as (1,000 - 300) and then, upon receiving the $7,500, would have subtracted the parenthetical expression from it; i.e., 7,500 - (1,000 - 300).

The same sort of reason, for having a parentheses occurs in other problems in algebra. A problem may arise that is sufficiently complicated to require its being broken down into parts. Suppose the answer to the first part is $(c - d)$. In tackling the second part, analysis reveals that the solution to this part requires the subtraction of the answer to the first part from the quantity b. Therefore, the solution to the second part is

$$b - (c - d) = a$$

The solution can be left in this form, or the parentheses removed, whereupon (in view of the minus sign in front of the parentheses), the solution becomes

$$a = b - c + d$$

Either form is about equally desirable. Often, however, there is an advantage in removing the parentheses in that the expression is simplified thereby. Suppose the solution to the first part of the problem had been $(c - 2b)$. Then the solution to the second part would be

$$\begin{aligned} a &= b - (c - 2b) \\ &= b - c + 2b \\ &= 3b - c \end{aligned}$$

The last expression $3b - c$ is clearly simpler than the first.

Quite frequently equations are encountered in which the same letter appears several times. For example:

$$ax + ba + ca = 10$$

Solve for a. First factor out a, which is common to each term.

$$a(x + b + c) = 10$$

To solve for a, all the other terms must be moved to the other side of the equation. Since $(x + b + c)$ occurs as a multiplier in the numerator on the left-hand side, it will appear in the denominator on the right-hand side, or

$$a = \frac{10}{x + b + c}$$

The parentheses in the denominator may be removed since no other terms appear, and there is no sign preceding the parentheses to change the signs.

One equation of the Varley Loop, which is a bridge circuit used in measuring long lines such as encountered in telephony, is

$$R_2 (L - x) = R(R_3 + x)$$

Find R_3. First expand the terms on the right-hand side by multiplying R by all the terms within the parentheses.

$$R_2 (L - x) = RR_3 + Rx$$

Second, transpose Rx from the right-hand side of the equation to the left-hand side so that RR_3 remains alone:

$$R_2 (L - x) - Rx = RR_3$$

Third, R_3 must be alone and not multiplied by R; therefore, transpose R to the left-hand side (in the denominator):

$$\frac{R_2 (L - x) - Rx}{R} = R_3$$

Another form of the same equation for the Varley Loop is,

$$\frac{R_2}{R} = \frac{R_3 + x}{L - x}$$

Solve for R_2. Transpose R to the right-hand side, and obtain

$$R_2 = \frac{R(R_3 + x)}{L - x}$$

One of the most common uses for algebra in radio is in the study of the derivation of standard equations and formulas. A commonly used equation is that expressing the resonant frequency of a tuned circuit.

$$f = \frac{1}{2\pi \sqrt{LC}}$$

The condition of resonance results when the inductive reactance of a circuit equals the capacitive reactance. This may be expressed algebraically as

$$X_L = X_C$$

Neither term, in its present form, contains f. It is necessary, therefore, to enlarge the equation and substitute equivalent values in the form of terms in which f appears. It will be shown later that

$$X_L = \omega L$$

and

$$X_C = \frac{1}{\omega C}$$

By substitution

$$\omega L = \frac{1}{\omega C}$$

Again it will be shown that

$$\omega = 2\pi f$$

so that by substitution

$$2\pi f L = \frac{1}{2\pi f C}$$

f is now contained in both sides of the equation, and it is only necessary to solve for f.

First clear of fractions by transposing $2\pi fC$ to the left-hand side:

$$(2\pi fC)(2\pi fL) = 1$$

or

$$(2\pi)^2 f^2 LC = 1$$

Here $(2\pi)(2\pi)$ is equivalent to $(2\pi)^2$. f^2 is not included in the parentheses, as it is desired to isolate f^2 in order to solve for f itself.

In order to eliminate $LC(2\pi)^2$ on the left-hand side of the equation, it is necessary to transpose it to the right-hand side whereupon there is obtained

$$f^2 = \frac{1}{LC(2\pi)^2}$$

If the square root of f^2 is extracted, f remains. However, if the square root of one side of the equation is extracted, then the square root of the other side of the equation must be extracted, too. The equation then becomes:

$$\sqrt{f^2} = f = \sqrt{\frac{1}{LC(2\pi)^2}}$$

On the right-hand side of the equation,

$$\sqrt{\frac{1}{LC(2\pi)^2}}$$

the square root of

$$\frac{1}{(2\pi)^2} = \frac{1}{2\pi}$$

It is not possible to actually extract the square root of LC. The operation can only be indicated as $\sqrt{LC}$.

Rearranging, the equation becomes:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

This equation may be transformed further to obtain the equation expressing the wavelength (λ) of a circuit in terms of L and C. (Although not previously stated it should be noted at this time that in the above equation for f, the values are *all in units*, L in Henries, C in Farads, and f in cycles per second.)

The fundamental equation for the wavelength of any circuit is $\lambda = V/f$, where V is the velocity of propagation of an electro-magnetic field through space, usually taken as 3×10^8 meters per second; f is the frequency in cycles per second; and λ is the wavelength in meters. Enlarging the fundamental equation by substituting its known equivalent values,

$$\lambda = \frac{V}{f}$$

$$V = 3 \times 10^8 \text{ meters per second}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Then

$$\lambda = \frac{3 \times 10^8}{\frac{1}{2\pi\sqrt{LC}}}$$

Looking at just the right-hand side of the equation, 3×10^8 is divided by $1/2\pi\sqrt{LC}$. Use the rule for the division of fractions—invert the divisor and multiply: $3 \times 10^8 \times 2\pi\sqrt{LC}/1$; or the entire expression becomes:

$$\lambda = 3 \times 10^8 \times \frac{2\pi\sqrt{LC}}{1}$$

or

$$\lambda = 3 \times 10^8 \times 2\pi \sqrt{LC}$$

$\pi = 3.14$, so 2π is equivalent to 6.28 or 628×10^{-2}.

By substitution

$$\lambda = 3 \times 10^8 \times 628 \times 10^{-2} \sqrt{LC}$$

(Performing the indicated multiplication)

$$\lambda = 1{,}884 \times 10^6 \sqrt{LC}$$

In this equation, values of L and C are in units of the henry and the farad, respectively. In tuned radio circuits small values of L and C are generally used, and it is more convenient to evaluate them in micro-units (μh and $\mu\mu$f).

The above equation can be changed to use these values as follows:

$$\sqrt{L \times 10^{-6} \times C \times 10^{-6}}$$

$$= \sqrt{LC \times 10^{-12}}$$

L and C may now be expressed in micro-unit values as the multiplication by 10^{-12} automatically converts both L and C to unit values within the equation.

While the square root of L and C can only be expressed $\sqrt{LC}$, it is possible to actually extract the square root of 10^{-12};

$$\sqrt{10^{-12}} = 10^{-6}$$

The entire quantity then becomes $\sqrt{LC} \times 10^{-6}$. Substituting this value in the complete equation,

$$\lambda = 1{,}884 \times 10^6 \times 10^{-6} \sqrt{LC}$$

Remembering that

$$(10^6 \times 10^{-6} = 10^0 = 1)$$

it follows that

$$\lambda = 1{,}884 \sqrt{LC}$$

In this final equation the factors L and C are expressed in micro-unit values and λ in meters.

The equations for frequency and wavelength are only two of the many commonly used formulas which are taken for granted. They may be traced back to their sources, usually by some very simple equation expressing a condition, as in the above example. Tracing the equations through their various steps and forms, greatly facilitates the thorough understanding of alternating current theory, which is, of course, the basic theory of all radio frequency circuits.

Consider another electrical problem which involves practically all the algebraic processes taken up in this assignment. This equation is that of the Murray Loop. Study this only as an algebraic problem and disregard all of the electrical theory involved. Solve for X.

$$\frac{R_2}{R} = \frac{X}{L - X}$$

First, transpose R and L - X to the opposite sides of the equation and obtain:

$$R_2 (L - X) = XR$$

(Performing the indicated multiplication)

$$R_2 L - R_2 X = XR$$

Transpose $(-R_2X)$ to the right-hand side of the equation and obtain

$$R_2L = XR + R_2X$$

X is common to both terms on one side of the equation, so an X may be factored out.

$$R_2L = X(R + R_2)$$

Now transpose $(R + R_2)$ to the left-hand side:

$$\frac{R_2L}{R + R_2} = X$$

Two principal methods are used in the solution of parallel resistive circuits (see Fig. 1), each being expressed by an equation giving the total resistance.

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

or

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Either expression is correct, and both are commonly used. They are derived from a fundamental expression,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

R is the reciprocal of 1/R which is the conductance. If the reciprocal is taken of both sides, we have,

$$\frac{R}{1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

The Least Common Denominator (LCD) must be found on the right-hand side of the equation, and is seen

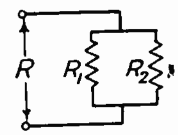

Fig. 1.—Two resistances in parallel.

to be R_1R_2. Changing the fraction by use of the LCD,

$$R = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}}$$

The fraction on the right-hand side indicates a division, which is performed as shown, by inverting the denominator and multiplying it by the numerator (which is one in this case). This is a rule from arithmetic for handling any fraction.

$$\frac{R_1 R_2}{R_2 + R_1} \times 1 = \frac{R_1 R_2}{R_2 + R_1}$$

or

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

which is the alternative form.

As an example of the use of algebra in calculating the constants of a vacuum tube, consider the following problem:

If e_g volts are applied to the grid of the vacuum tube, then (as will be developed in a future assignment) the effect is as if μe_g volts were generated in the plate circuit of the tube, where μ (Greek letter mu) represents the so-called amplification factor of the tube.

The apparent generated voltage μe_g acts in series with the so-called R_p or internal plate resistance of the tube. Suppose a load resistance R_L is connected in series with the tube, so that the total resistance in the plate circuit is the sum of the two, or $R_p + R_L$ as indicated in Fig. 2.

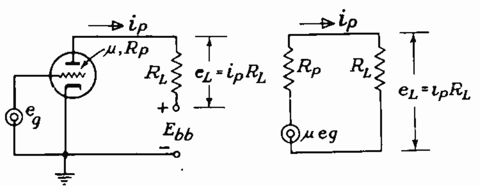

Fig. 2.—Vacuum tube stage and its equivalent circuit.

It will then be found that part of the apparent generated voltage μe_g is consumed internally in R_p, and the remainder appears externally as e_L across the load resistance R_L.

Suppose that the tube mu (μ) is known, and e_g, R_L, and e_L are also either specified or measured. The problem is to find R_p, the internal resistance of the tube. Since this is not readily accessible to direct measurement, as is R_L, it must be calculated indirectly in the manner to be shown.

The current i_p that flows through the two resistors is given by Ohm's Law (to be discussed more fully in a subsequent technical assignment) in the following equation:

$$\mu e_g = i_p(R_p + R_L)$$

This equation states that the impressed voltage μe_g is equal to the voltage drop produced by the current i_p flowing through the total circuit resistance $(R_p + R_L)$. By the same reasoning, the voltage drop e_L produced across the external resistor R_L alone is

$$e_L = i_pR_L$$

Transfer R_L to the left-hand side and obtain,

$$e_L/R_L = i_p$$

i.e., the equation has been solved for i_p. Now substitute this value, namely, e_L/R_L for i_p in the previous equation, and obtain

$$\mu e_g = \left(\frac{e_L}{R_L}\right)(R_p + R_L)$$

The last equation is now ready for further algebraic manipulation in order to separate R_p from the remaining symbols whose values are known—in short, this equation can be solved for R_p in terms of the other quantities. First, multiply both sides by R_L, the denominator of the right-hand term; i.e., transpose R_L to the left-hand side, and obtain

$$\mu e_gR_L = e_L(R_p + R_L)$$

Next perform the indicated multiplication on the right-hand side. Thus,

$$\mu e_g R_L = e_L R_p + e_L R_L$$

Transpose $e_L R_L$ to the left-hand side:

$$\mu e_g R_L - e_L R_L = e_L R_p$$

Now factor out R_L from both terms of the left-hand side:

$$R_L (\mu e_g - e_L) = e_L R_p$$

(This step is not really essential to solve for R_p).

Finally divide both sides of the equation by e_L, i.e., transpose e_L on the right-hand side to the left-hand side and obtain:

$$\frac{R_L (\mu e_g - e_L)}{e_L} = R_p$$

Thus, R_p can be determined from specified or measured values of R_L, μ, e_g, and e_L.

As an example of actual numerical values, suppose $\mu = 20$, $e_g = 3$ volts, $R_L = 20{,}000$ ohms, and e_L is measured as 40 volts. Then

$$R_p = \frac{20000(20 \times 3 - 40)}{40}$$

$$= \frac{20000(60 - 40)}{40}$$

$$= 20{,}000\left(\frac{20}{40}\right) = 10{,}000 \text{ ohms}$$

Most common equations are derived by equally simple methods. Derivations of many other equations will be shown from time to time in later assignments. A good basic foundation in algebra will be one step toward appreciating the derivations of formulas, which are ordinarily known as such through constant use without any thought being given as to how they are derived.

COMBINATION OF LOGARITHMS AND ALGEBRA.—Students frequently encounter difficutly in handling equations involving logarithms. Such an equation is one expressing the surge or characteristic impedance of a transmission line in terms of d (diameter of wire) and D (distance or spacing between the centers of the two wires making up the transmission).

The equation is $Z = 276 \log 2D/d$ (solve for D). Transposing 276 to the left-hand side:

$$\frac{Z}{276} = \log \frac{2D}{d}$$

(Refer to assignment on Logarithms)

$$\frac{2D}{d} = \text{antilog} \frac{Z}{276}$$

where the antilog is represented merely as the inverse function to the logarithm.

Transpose 2 and d from the left-hand side of the equation to the right-hand side, and obtain:

$$D = \frac{d \text{ antilog} \frac{Z}{276}}{2}$$

or the equation may be written

$$D = .5d \text{ antilog } Z/276$$

If a 600-ohm line were desired using No. 10 wire, the diameter of which is .102 inch, the spacing of the wires would be found as follows:

Substituting known values and performing the indicated operations

$$\frac{600}{276} = \log \frac{2D}{.102}$$

$$2.174 = \log \frac{2D}{.102}$$

$$\text{antilog } 2.174 \cong 149$$

(where $\cong$ means approximately equal to)

$$149 = 19.6D$$

$$D = \frac{149}{19.6} = 7.6 \text{ inches}$$

Or substitute directly in the equation

$$D = .5d \text{ antilog } \frac{Z}{276}$$

$$D = .5(.102) \text{ antilog } \frac{600}{276}$$

$$D = .051 \text{ antilog } 2.174$$

$$D = .051 \ (149)$$

$$D = 7.6 \text{ inches}$$

Thus, for a surge or characteristic impedance of 600 ohms using No. 10 wire, the two wires should be spaced 7.60 inches between centers.

RESUMÉ. —All algebra is based upon logical steps. It is necessary that one step be taken at a time in solving equations. An algebraic operation performed on one side of the equation must be performed on the other side of the equation, so that an equality continues to exist. Indicated arithmetical operations may be performed on only one side of the equation without up-setting the *balance* or equality of the equation. Whenever in doubt on the solution of symbolic equations, substitute numerical values for symbols in the original equation and in the final solution. If an identity or equality results, the solution is correct. (Numerical values must first be substituted in the original equation, and must be of such values as to make that equation true).

Example: (original equation)

$$ab = c$$

Let a = 10, b = 5, so c must equal 50 (since 5 × 10 = 50) Solve for b.

$$b = \frac{c}{a} \text{ (final solution)}$$

Substituting previous values

$$5 = \frac{50}{10}$$

5 = 5 which is an identity, so the final equation is correct. Algebra is nothing more than an abbreviated method of writing an expression. Algebraic manipulations are logical steps to a final solution for an unknown quantity. Practice will provide a good foundation for the solution of problems.

SIMULTANEOUS EQUATIONS

In certain types of electrical problems, such as in the solutions of resistance networks by means of Kirchhoff's Laws, several sets of conditions will simultaneously exist in a circuit. For example, one resistor may be common to two or more circuits, and the current through this resistor will be the resultant of the currents in the several circuits, all of which are not necessarily flowing through it in the same direction. It may be necessary to determine, at any given instant, the simultaneous currents flowing in the various parts of the circuit. To do this, two or more simultaneous sets of conditions will be determined and expressed by two or more simultaneously correct equations which contain two or more unknown terms.

The electrical theory of such circuits will be discussed in later assignments. At this point only the mathematical solutions of typical equations will be considered. One point must be thoroughly understood: If, for example, the two terms for which the calculation is to be made represent two circuit currents, these currents must be related to each other in the circuit—*and this relation must exist at some instant or under a given set of conditions*—in other words, *simultaneously*.

TWO UNKNOWNS.—Consider the following pair of simultaneous equations, to solve for I_1 and I_2.

$$2I_1 + 3I_2 = 13 \qquad (1)$$

$$5I_1 - 2I_2 = 4 \qquad (2)$$

It is necessary to find a pair of values that satisfy each of the equations. The solution can be made by elimination. One unknown can be temporarily eliminated, and the solution made for the other unknown, which is then placed in either equation to furnish the solution for the second unknown.

Multiply Eq. (1) by 5 and Eq. (2) by 2. Write these respectively as Eq. (3) and Eq. (4). (See rule 3.)

$$\begin{array}{lrcl} [\text{Eq. (1)} \times 5] & 10I_1 + 15I_2 & = 65 & (3) \\ [\text{Eq. (2)} \times 2] & 10I_1 - 4I_2 & = 8 & (4) \\ \hline (\text{Subtract}) & 19I_2 & = 57 & (5) \end{array}$$

Eq. (5) is obtained by subtracting Eq. (4) from Eq. (3). *The factors used in multiplication were such that the coefficients of the I_1 values in the two equations became equal and hence disappeared in the subtraction.* Had these identical terms been of *opposite* sign, *addition* of the two equations would have been necessary to accomplish the elimination.

Continuing,

$$I_2 = 57/19 = 3 \qquad (6)$$

Substituting this value of I_2 in Eq. (1),

$$2I_1 + (3 \times 3) = 13 \qquad (7)$$

$$2I_1 + 9 = 13 \qquad (8)$$

$$2I_1 = 13 - 9 \qquad (9)$$

$$I_1 = \frac{13 - 9}{2} = 2 \qquad (10)$$

Check by substituting 2 for I_1 and 3 for I_2 in Eq. (2).

$$(5 \times 2) - (2 \times 3) = 4 \qquad (11)$$

$$10 - 6 = 4 \quad \text{(Check)}$$

THREE UNKNOWNS.—Consider a case where there are *three unknown terms* as represented by *three simultaneous equations*. Let these represent three currents, I_1, I_2, and I_3.

$$3I_1 + 2I_2 - I_3 = 7 \qquad (12)$$

$$4I_1 - I_2 + 5I_3 = 30 \qquad (13)$$

$$7I_1 + 3I_2 - 4I_3 = 3 \qquad (14)$$

In this problem first eliminate one term between two equations and then between another two. In this case suppose I_2 is to be eliminated first between Eq. (12) and Eq. (13). Multiply Eq. (13) by 2 to make the I_2 values equal.

Eq. (13) × 2:

$$8I_1 - 2I_2 + 10I_3 = 60 \quad (15)$$

+ Eq. (12):

$$3I_1 + 2I_2 - I_3 = 7 \quad (12)$$

$$11I_1 + 9I_3 = 67 \quad (16)$$

Next, eliminate I_2 between Eq. (13) and Eq. (14). Multiply Eq. (13) by 3 to make the I_2 values equal.

Eq. (13) × 3:

$$12I_1 - 3I_2 + 15I_3 = 90 \quad (17)$$

+ Eq. (14):

$$7I_1 + 3I_2 - 4I_3 = 3 \quad (14)$$

$$19I_1 + 11I_3 = 93 \quad (18)$$

Eqs. (16) and (18) now contain only *two* unknowns, I_1 and I_3, and are solved as in the preceding problem. Multiply Eq. (16) by 11 and Eq. (18) by 9, and subtract the former from the latter to eliminate I_3. Thus:

Eq. (18) × 9:

$$171I_1 + 99I_3 = 837 \quad (19)$$

Eq. (16) × 11:

$$121I_1 + 99I_3 = 737 \quad (20)$$

(Subtract)

$$50I_1 = 100 \quad (21)$$

$$I_1 = 2 \quad (22)$$

Substituting $I_1 = 2$ in Eq. (16)

$$(11 \times 2) + 9I_3 = 67 \quad (23)$$

$$22 + 9I_3 = 67 \quad (24)$$

$$9I_3 = 67 - 22 = 45 \quad (25)$$

$$I_3 = 45/9 = 5 \quad (26)$$

Substituting $I_1 = 2$ and $I_3 = 5$ in Eq. (13).

$$(4 \times 2) - I_2 + (5 \times 5) = 30 \quad (27)$$

$$8 - I_2 + 25 = 30 \quad (28)$$

$$-I_2 = 30 - 8 - 25 \quad (29)$$

$$I_2 = 8 + 25 - 30$$

$$= 3 \quad (30)$$

This may be proved by substituting $I_1 = 2$, $I_2 = 3$, and $I_3 = 5$ in Eq. (12).

$$(3 \times 2) + (2 \times 3) - 5 = 7 \quad (31)$$

$$6 + 6 - 5 = 7 \text{ (Check)} \quad (32)$$

When a system of equations contains more than three unknowns, procedure similar to the above may be followed, eliminating some one unknown from pairs of equations to form a system of one fewer equations in one fewer unknowns. Then progressively eliminate unknowns until only one remains. This can be solved and by substitution the other unknowns may be determined.

There are several other methods of solving simultaneous equations which may be used but which will not be discussed here. It should also be remembered that other sequences of elimination other than those shown could have been used in the problem above. These problems should be very carefully studied until the processes are thoroughly understood. This type of problem is very important to the radio engineer because it contains the basic solution of complex networks, such as voltage dividers and attenuation pads.

It should be understood, that the sign of an unknown factor may work out to be either positive or negative. This will be shown in several of the following exercise problems. Check each one as proof of your work.

Exercises

51. $5x + 2y = 39$
$2x - y = 3$

52. $2x + y = 3$
$7x + 5y = 21$

53. $7x - 2y = 11$
$x + 5y = 28$

54. $x + 4y = 35$
$2x - 3y = 26$

55. $x + 2y = 9$
$3x - 3y = 90$

56. $2x - y = 9$
$5x - 3y = 14$

57. $x + y - 8 = 0$
$y + z - 28 = 0$
$x + z - 14 = 0$

58. $5x - 2y - 2z = 12$
$x + y + z = 8$
$7x + 3y + 4z = 42$

59. $10x - y + 3z = 42$
$6x + 2y + z = 53$
$3x + 3y - z = 24$

60. $6x - 2y + 5z = 53$
$5x + 3y + 7z = 33$
$x + y + z = 5$

61. $2x + 7y + 10z = 25$
$x + y - z = 9$
$7x - 7y - 11z = 73$

62. $x + 2y + 10z = 44$
$3x + 3y + 7z = 384$
$2x + y + z = 256$

QUADRATIC EQUATIONS

DEFINITION.—Quadratic equations are those equations in which the one unknown quantity appears as a square. Pure quadratic equations contain the unknown only as a term to the second power. Affected quadratic equations contain terms in both the first and the second power of the one unknown.

An example of a pure quadratic equation is the equation, $3x^2 = 48$. The solution of a pure quadratic equation is quite simple: Divide both members by the coefficient of the term in the unknown and then extract the square root of both members:

$$3x^2 = 48$$

$$x^2 = \frac{48}{3} = 16$$

$$x = \sqrt{16} = +4 \text{ or } -4$$

also written ± 4, for brevity.

Note that there are two roots that will satisfy a quadratic equation, one positive and one negative.

An example of an affected quadratic equation is the equation

$$x^2 + 2x + 5 = 0$$

The solution of this type of equation usually involves more work than the pure quadratic equation solution, but the procedure is made quite straight-forward by the use of the formula:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

DERIVATION OF FORMULA.—First, consider the derivation and significance of the above equation, and then its application to quadratic equation solutions will be studied.

An affected quadratic equation may be manipulated in such a manner as to contain in its left member the algebraic sum of three terms, namely, a term in the unknown to the second power, a term in the unknown to the first power, and usually a numerical term. The right-hand member will then be zero. All affected quadratic equations may, therefore, be considered as having the general form, $Ax^2 + Bx + C = 0$, where x represents the unknown, A is the numerical coefficient of x^2, B is the coefficient of x, and C is the numerical term.

Thus, $Ax^2 + Bx + C = 0$ can represent the equation $x^2 + 2x + 5 = 0$ if $A = +1$, $B = +2$, and $C = +5$. Similarly, $Ax^2 + Bx + C = 0$ can be made to represent any other affected quadratic equation we might encounter, if we simply assign suitable value to A, B, and C. It follows that if $Ax^2 + Bx + C = 0$ is solved for x, then in effect any affected quadratic equation encountered in the future will have been solved in principle. Solving means to find the value or values of x that make the left-hand expression equal to zero, i.e., that satisfy the equation. The particular values of x are known as the *roots* of the equation.

To solve $Ax^2 + Bx + C = 0$, make use of a method known as "completing the square". In making this solution, observe that none of the rules for handling algebraic equations has been violated:

General equation

$$Ax^2 + Bx + C = 0$$

Transposing C to the right-hand side:

$$Ax^2 + Bx = -C$$

Multiplying both members by A,

$$A^2x^2 + ABx = -AC$$

Adding $\frac{B^2}{4}$ to both sides,

$$A^2x^2 + ABx + \frac{B^2}{4} = \frac{B^2}{4} - AC$$

Multiplying both sides by 4,

$$4A^2x^2 + 4ABx + B^2 = B^2 - 4AC$$

[The left-hand member of the last equation is a perfect square, and its square root is $2Ax + B$. This can be proved by evaluating $(2Ax + B)^2$. It will be found to equal $4A^2x^2 + 4ABx + B^2$].

Extracting the root of both members,

$$2Ax + B = \pm\sqrt{B^2 - 4AC}$$

Transposing B to the right-hand side,

$$2Ax = -B \pm \sqrt{B^2 - 4AC}$$

Dividing both members by 2A, i.e., transposing 2A to the right-hand side:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

This is the general formula for finding the value of the unknown in an affected quadratic equation. The student should memorize this formula, since it may be used to solve any quadratic equation. Some examples of its application follow:

Examples:

Ex. 1: Find x when $2x^2 + 13x + 15 = 0$.

In order that this equation may be represented by the general equation $Ax^2 + Bx + C = 0$, let A = 2 B = 13, and C = 15.

Substituting these values in the general formula:

$$x = \frac{-13 \pm \sqrt{(13)^2 - 4(2)(15)}}{2(2)}$$

$$= \frac{-13 \pm \sqrt{49}}{4} = \frac{-13 \pm 7}{4}$$

$$= \frac{-20}{4} \text{ or } \frac{-6}{4} = -5 \text{ or } -1.5$$

Before accepting the solution as correct, it should be ascertained whether or not the above roots will satisfy the original equation:

$$2x^2 + 13x + 15 = 0$$

$$2(-5)^2 + 13(-5) + 15 = 0$$

$$50 - 65 + 15 = 0 \quad \textit{(Check)}$$

$$2(-1.5)^2 - 13(-1.5) + 15 = 0$$

$$4.5 - 19.5 + 15 = 0 \quad \textit{(Check)}$$

Ex. 2: Find x when $x^2 - 2x - 15 = 0$.

In order that this equation may be represented by the equation $Ax^2 + Bx + C = 0$, the values assigned to A, B, and C *must include the sign of the term;* thus, A = +1, B = -2, C = -15, Substituting these values in the general formula:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)}$$

$$= \frac{+2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm 8}{2} = \frac{10}{2}$$

$$= 5 \text{ and } \frac{-6}{2} = -3$$

Either of these values of x will satisfy the original equation.

Ex. 3: Solve for x, when $x^2 + 2x + 5 = 0$, the example given above.

The same procedure is employed A = 1, B = 2, C = 5.

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2}$$

$$= \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$$

The quantity $\sqrt{-16}$ must be handled with care. The square root of -16 is not +4 *NOR* -*4*. Either of these numbers squared would yield +16. $\sqrt{-16}$ falls in the category of imaginary numbers. It can be simplified, however. If the quantity under the radical sign is factored, and if any factor appears twice, then that factor may be rewritten outside the radical sign as a multiplying factor to the first power. Hence,

$$\sqrt{-16} = \sqrt{(-1)(4)(4)} = 4\sqrt{-1}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4\sqrt{-1}}{2}$$

$$= -1 \pm 2\sqrt{-1}$$

Imaginary numbers will be discussed more fully in a later assignment.

To prove, substitute the two possible values for x in the original equation in turn:

EQUATION	WHEN $x = -1 + 2\sqrt{-1}$	WHEN $x = -1 - 2\sqrt{-1}$
$x^2 =$	$1 - 4\sqrt{-1} - 4$	$1 + 4\sqrt{-1} - 4$
$+2x =$	$-2 + 4\sqrt{-1}$	$-2 - 4\sqrt{-1}$
$+5 =$	$+ 5$	$+ 5$
$x^2 + 2x + 5 = 0$	$4 - 4 = 0$	$4 - 4 = 0$ *(Check)*

Ex. 4:

$$x^2 - 4x = 60$$

$$x^2 - 4x - 60 = 0$$

A = 1, (x = 1x), B = -4, C = -60

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 1 \times -60)}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{256}}{2} = \frac{4 \pm 16}{2}$$

$$x = \frac{4 + 16}{2} = +10$$

$$x = \frac{4 - 16}{2} = -6$$

Note this equation can also be factored; that is, $(x - 10)(x + 6) = x^2 - 4x - 60$, when multiplied together. Taking each factor and making it equal to zero, $x - 10 = 0$, or $x = 10$; $x + 6 = 0$, or $x = -6$.

Inspection of the problem and trial and error will often enable solution of an equation to be obtained fairly easily when the values of x are not fractions.

Substitution of either of these values for x in the original equation solves the equation. A study of the above examples shows that *any quadratic* equation having one unknown letter always has two roots. In many practical problems if x works out to both a positive and negative quantity, the negative value of x may be discarded. If one root cannot be discarded by inspection, both must be considered correct.

METHOD OF FACTORING.—The use of the quadratic formula for the solution of quadratic equations has been discussed. Factoring is another method which can be used to solve quadratic equations. This method is often much simpler to use than the quadratic formula. However, the quadratic formula can always be used when factors are not readily determined.

In using the multiplication tables in arithmetic, factors are frequently employed without the user being aware of them. The product 12 is a result of several combinations of factors; namely, 6×2, 3×4, 12×1, etc. 6, 2, 3, 4, 12, and 1 are *all factors* of 12. Factors, then, are merely *quantities* which when multiplied yield the given number as their product.

In this assignment on algebra, it has been noted that the product $(A - B)(A - B) = A^2 - 2AB + B^2$. Therefore, from the above definition of factors, $(A - B)$ and $(A - B)$ are factors of $(A^2 - 2AB + B^2)$. Similarly, if $(2x + 4)$ were multiplied by $(x - 3)$, a product would result and $(x - 3)$ and $(2x + 4)$ would be the factors of that product as shown.

$$\begin{array}{r} x - 3 \\ \underline{2x + 4} \\ 2x^2 - 6x \qquad \\ \underline{+ 4x - 12} \\ 2x^2 - 2x - 12 \end{array}$$

Another example:

$$\begin{array}{r} 2x + 4 \\ \underline{x - 5} \\ 2x^2 + 4x \qquad \\ \underline{- 10x - 20} \\ 2x^2 - 6x - 20 \end{array}$$

Therefore, $(2x + 4)$ and $(x - 5)$ are the factors of the expression $2x^2 - 6x - 20$.

Factoring of equations is done by the trial and error method. Consider the equation $A^2 - B^2 = 0$. What are its factors? The factors of A^2 must be A and A, since only A times A yields A^2. Similarly, the factors of B^2 are B and B, since only B times B yields B^2. But, a minus sign prefixes the expression B^2. From the laws of positive and negative numbers, the only means of obtaining a minus sign is to multiply unlike signs or a minus times a plus. Therefore, it may be surmised that the factors of $A^2 - B^2$ are $(A + B)$ and $(A - B)$. Now to prove that this reasoning is correct if $(A + B)$ and $(A - B)$ are factors of $(A^2 - B^2)$, then the product of $(A - B)$ and $(A + B)$ must yield $(A^2 - B^2)$.

$$\begin{array}{l} A + B \\ \underline{A - B} \\ A^2 + AB \\ \underline{\quad - AB - B^2} \\ A^2 \qquad\quad - B^2 \end{array}$$

Therefore, $(A + B)$ and $(A - B)$ are factors of $(A^2 - B^2)$. Since A and B are two numbers in general, it can, therefore, be deduced that an expression involving the difference of two squares is factorable in two terms, one of which is the sum of the two numbers, and the other of which is their difference.

Next, consider the expression $(2A^2 + 3AB + B^2)$. What are its factors? In determining the factors of $2A^2$ the coefficient 2 must be considered as well as A^2. The only factors of $2A^2$ are 2A and A. $(2A \times A = 2A^2)$. The only factors of B^2 are

B and B. Therefore, the factors of $(2A^2 + 3AB + B^2)$ may possibly be $(2A + B)$ and $(A + B)$. To check this possibility, multiply $(2A + B)$ by $(A + B)$ to determine if $(2A^2 + 3AB + B^2)$ is their product.

$$\begin{array}{lll} 2A & + B & \\ A & + B & \\ \hline 2A^2 & + AB & \\ & + 2AB & + B^2 \\ \hline 2A^2 & + 3AB & + B^2 \end{array}$$

Therefore, $(2A + B)$ and $(A + B)$ are factors of $2A^2 + 3AB + B^2$.

Another expression which employs all the basic ideas that have just been discussed is $(x^2 + 7x + 12)$. What are its factors?

First, the factors of x^2 are x and x.

Second, the factors of (12) could be:

Case (1):	12×1
Case (2):	3×4
Case (3):	6×2

However, a middle term, 7x, exists in the expression, which must be satisfied by the factors as well as x^2 and 12.

Consider the first combination:

$$(x + 12)\ (x + 1)$$

To determine if $x + 12$ and $x + 1$ are factors, it is necessary to multiply these quantities.

$$\begin{array}{lll} x & + 12 & \\ x & + 1 & \\ \hline x^2 & + 12x & \\ & + x & + 12 \\ \hline x^2 & + 13x & + 12 \end{array}$$

It is evident that $(x + 12)$ and $(x + 1)$ are *not* factors of $(x^2 + 7x + 12)$. Instead, they are factors of $(x^2 + 13x + 12)$.

Consider Case (2) using the same procedure as that followed for Case (1). To determine if $(x + 3)$ and $(x + 4)$ are factors;

$$\begin{array}{lll} x & + 3 & \\ x & + 4 & \\ \hline x^2 & + 3x & \\ & + 4x & + 12 \\ \hline x^2 & + 7x & + 12 \end{array}$$

Since $(x + 3)$ times $(x + 4)$ results in $(x^2 + 7x + 12)$, then the *factors* of $(x^2 + 7x + 12)$ have been determined. It is not necessary that $(x + 6)$ and $(x + 2)$ be tried, since *there are but two factors which will satisfy a quadratic equation.*

It will be noted here the statement was made that factoring is based upon a *trial and error process*. In the above example, $(x + 12)$ and $(x + 1)$ were tried; but were eliminated as factors, since their product was not $(x^2 + 7x + 12)$. However, the product of $(x + 3)$ and $(x + 4)$ resulted in $(x^2 + 7x + 12)$, which determined the correct factors for this expression.

Note that the numbers, 3 and 4, one in each factor, when added together give 7, the coefficient of the middle term. This will always help eliminate the trials which give wrong results.

Exercises

Factor the following:

63. $A^2 + 2AB + B^2$ (A + B) (A + B)

64. $2x^2 + 3xC + C^2$ (2x + C) (x + C)

65. $x^2 - 3x - 10$ (x - 5) (x + 2)

66. $x^2 - 11x + 24$ (x - 8) (x - 3)

67. $2x^2 - 46x - 48$ 2x - 48 (x + 1)

ROOTS OF QUADRATIC EQUATIONS.—The quadratic roots of equations are solved by the quadratic formula. Quadratic roots may also be determined by factoring. It might be well to note here that the roots or values that will satisfy an equation are as many as the highest power of the unknown quantity. For example, in the expression $(x^4 + 3x^3 + 7x^2 + 8x + 10 = 0)$ there will be four roots to satisfy the equation, since 4 is the highest power to which the unknown quantity, x, is raised. There will be eight roots in the expression $(2y^8 + 4y^6 + 2y^4 + 3y^2 + 2 = 0)$, since 8 is the highest power to which the unknown quantity, *y*, is raised. In this assignment the discussion will be confined to quadratic roots only. As was previously stated, quadratic equations are those equations in which the unknown quantity appears as a square.

As was previously determined, the factors of the equation, $(x^2 + 7x + 12 = 0)$, are $(x + 3)$ and $(x + 4)$. If the polynonial $x^2 + 7x + 12$ equals zero, and also equals $(x + 3)(x + 4)$, then this product must equal zero. The product will equal zero if either factor (or both) are equal to zero. Hence, equate each factor to zero, and solve each for x.

Thus,

$x + 3 = 0$

$x + 4 = 0$ (Equating factors to zero)

Solving each, there is obtained:

$x = -3$ (Solving for x the unknown.)

and

$x = -4$

If $x = -3$, and $x = -4$, then these numerical values should satisfy the equation $(x^2 + 7x + 12 = 0)$.

Substituting the value $x = -3$ in the expression:

$$x^2 + 7x + 12 = 0$$

$$(-3)^2 + 7(-3) + 12 = 0$$

Note: $-3(-3) = +9$

$$9 - 21 + 12 = 0$$

$$21 - 21 = 0$$

Then, $0 = 0$, an identity exists, and $x = -3$ is a root of the equation.

Substitute the value $x = -4$, to determine if it is a root of the equation.

$$x^2 + 7x + 12 = 0$$

$$(-4)^2 + 7(-4) + 12 = 0$$

$$16 - 28 + 12 = 0$$

$$28 - 28 = 0$$

The numerical values for the unknown quantity, x, which satisfy that equation have been determined. This method is much more abbreviated that the quadratic formula when factors are readily attainable. However, when the equation is not factorable, then the quadratic formula must be used.

Determine the roots of the equation $(x^2 - 5x - 14 = 0)$.

An inspection of this equation indicates the obvious factors of x^2, namely, $x \cdot x$; and of -14 are -14 and + 1, or +14 and -1, or −7 and +2, or +7 and −2. Of these, −7 and +2, when added, give −5, the coefficient of the second term (−5x); hence, the factors are probably (x − 7) and (x + 2). To check, multiply them together:

$$\begin{array}{r} x - 7 \\ x + 2 \\ \hline x^2 - 7x \\ + 2x - 14 \\ \hline x^2 - 5x - 14 \end{array}$$

(Proving factors correct)

Equate the factors to zero:

$$x - 7 = 0$$
$$x + 2 = 0$$

Solve for the unknown, x:

$$x = 7$$
$$x = -2$$

The roots of the equation, ($x^2 - 5x - 14 = 0$), are ($x = 7$) and ($x = -2$). To prove that $x = 7$ and $x = -2$ are the correct numerical values for the unknown quantity, x, substitute these values into the original equation.

$$x^2 - 5x - 14 = 0$$
$$x = -2$$
$$(-2)^2 - 5(-2) - 14 = 0$$

(Note, the algebraic signs must be correctly used.)

$$4 + 10 - 14 = 0$$
$$14 - 14 = 0 \quad \textit{(Check)}$$

$$x^2 - 5x - 14 = 0$$
$$x = 7$$
$$(7)^2 - 5(7) - 14 = 0$$
$$49 - 35 - 14 = 0$$
$$49 - 49 = 0 \quad \textit{(Check)}$$

Solve for the roots of the equation ($x^2 + 2x + 5 = 0$). By trial process of factoring, the *only* factors of x^2 are x and x, and the *only* factors of 5 are 5 and 1. But,

$$\begin{array}{r} x + 5 \\ x + 1 \\ \hline x^2 + 5x \\ + x + 5 \\ \hline x^2 + 6x + 5 \end{array}$$

Which is clearly different from the given polynonial. Hence, the latter is not factorable into simple terms, and so for this particular equation the quadratic *formula* must be used.

Factoring is an easy method of obtaining the roots of an equation when that equation can be factored. When the equation cannot be factored in the ordinary sense of the word, then the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

must be used. In order for the student to become more familiar with factoring, the exercise problems below should be solved and proved.

Exercises

Determine the roots by factoring the following equations and prove by substitution:

68. $x^2 - 6x - 16 = 0$ (x − 8) (x + 2

69. $3x^2 - 27x + 54 = 0$ (x − 6) (3x − 9)

70. $9x^2 + 18x - 16 = 0$

71. $15x^2 - 8x\sqrt{A} + A = 0$

72. $x^2 + 5x + 6 = 0$

The student should carefully work all the following exercise problems before starting the examination.

73. $I = \frac{E}{R + \Delta}$ Find Δ

74. $L = L_1 + L_2 + 2M$ Find M

75. Factor $(x^3 - 1)$

76. $C = .0885\ KA/d$ Find A

77. $R = KL/M$ Find M

78. $L = \frac{1.26\ N^2\ A\mu}{10^8 d}$ Find A

79. $W = CE^2/2$ Find E

80. $K = \frac{M}{\sqrt{L_1 + L_2}}$ Find L_2

81. $P = I^2R$ Find I

82. $F = 159/\sqrt{LC}$ Find C

83. $7y^2 + 2y - 32 = 0$ Find y

84. $2A^2 + 5A = -2$ Find A

85. $9x^2 + 363 = 132x$ Find x

86. $9B = 4 - 3B^2$ Find B

87. $(x - 4)(x - 3) = 0$ Find x

88. $x^2 - 56 = x$ Find x

89. $(2y + 1)(y + 3) = y^2 - 9$ Find y

90. $\frac{x^2}{9} + \frac{x}{3} = \frac{35}{4}$ Find x

91. $B = \frac{.866(D_1 - D_2)}{6}$ Find D_2

92. $H = \sqrt{\frac{A + B}{A - B}}$ Find A

93. $IN = \frac{ABL}{.4\pi\mu}$ Find A

94. $U = \frac{1}{8\pi}(E^2 + H^2)$ Find H

95. $x = \pi D \sqrt{\frac{2F}{P}}$ Find P

96. $R = r\,[1 + a(t_1 - t_2)]$ Find a

97. $i = \frac{e}{\sqrt{R^2 + (x_L - x_c)^2}}$ Find R

98. $H = \frac{1.256\ NI}{L}$ Find N

99. $L = \frac{1}{39.5\ F^2C}$ Find C

100. $L = .002A\ \text{Log}_e \left(\frac{4A}{d} - 1\right)$

Find d in terms of logarithms to the base 10.

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ANSWERS TO EXERCISE PROBLEMS

1. $-2A$
2. $5n + 3m$
3. $5.5x - 4y$
4. $1.417R_2 + 10$
5. $6.1R + 2R_1$
6. $5x + 2y$
7. $12x + 4y$
8. $2a + 2c$ or $2(a + c)$
9. $3x^2 + xy + 3y^2$
10. $11x^2 + x + 29$
11. $.8a$
12. 10
13. $.5x$
14. $15\sqrt{A}$
15. $(axy)^2 + (bxy)^2$
16. x^7
17. $x^5y^2z^3$
18. $a^2b^4c^5$
19. $a^3 + a^2b + ac$
20. $a^4 + 2a^3b + a^2b^2$
21. $L^2 - M^2$
22. $a^3 - 3a^2b + 3ab^2 - b^3$
23. $3a^2 + 5ab - 3ac + bc - 2b^2$
24. $a^4x + 2a^2bx + b^2x$
25. $a^2x + 2a^2bx + a^2b^2x - a^2y - 2a^2by - a^2b^2y$
26. ab
27. $\frac{xy^2z}{a}$
28. pq^{-6}
29. $b + c - ax$
30. $y^4 - 4y - x^3z$
31. $ab + 2c - 1$
32. $mn + o + \frac{q}{m}$
33. $a + 3b + 1$
34. $1 + \frac{3y}{x} - \frac{6}{x}$
35. $\frac{7}{a} - 8x - 9$
36. $16a + 16b = 16(a + b)$
37. $a - ab + \frac{ab}{a - b}$
38. $a^2 + ab + b^2$
39. $a^2 - ab + b^2$
40. $a^2 - b^2$
41. $x^2 + 6xy + y^2 + \frac{y^3}{x + y}$
42. $x^2 - 4x + 8$
43. $1 - 3x + 2x^2 - x^3$
44. $x^2 + xy - y^2$
45. $b^2 + 5b + 25$
46. $a(b + b^2 + y)$
47. $y(x^2 + ab + 3)$
48. $m(2n + 15 + n^2p)$

ALGEBRA

ANSWERS TO EXERCISE PROBLEMS

49. $b(a + 2a^2 + 1)$

50. $x(4y + 2 + n)$

51. $x = 5,\ y = 7$

52. $x = -2,\ y = 7$

53. $x = 3,\ y = 5$

54. $x = 19,\ y = 4$

55. $x = 23,\ y = -7$

56. $x = 13,\ y = 17$

57. $x = -3,\ y = 11,\ z = 17$

58. $x = 4,\ y = 2,\ z = 2$

59. $x = -2,\ y = 19,\ z = 27$

60. $x = 7,\ y = -3,\ z = 1$

61. $x = 11,\ y = -1,\ z = 1$

62. $x = 100,\ y = 77,\ z = -21$

63. $A + B,\ A + B$

64. $2x + C,\ x + C$

65. $x - 5,\ x + 2$

66. $x - 8,\ x - 3$

67. $2x + 2,\ x - 24$

68. $x = -2,\ x = 8$

69. $x = 3,\ x = 6$

70. $x = 2/3,\ x = -2\ 2/3$

71. $x = \frac{\sqrt{A}}{5},\ x = \frac{\sqrt{A}}{3}$

72. $x = -2,\ x = -3$

73. $\Delta = \frac{E - IR}{I}$

74. $M = \frac{L - L_1 - L_2}{2}$

75. $(x - 1)(x^2 + x + 1)$

76. $A = \frac{Cd}{.0885K}$

77. $M = \frac{KL}{R}$

78. $A = \frac{Ld\ 10^8}{1.26\ N^2\mu}$

79. $E = \sqrt{\frac{2W}{C}}$

80. $L_2 = \left(\frac{M}{K}\right)^2 - L_1$

81. $I = \sqrt{P/R}$

82. $C = 159^2/F^2L$

83. $y = 2 \text{ or } -16/7$

84. $A = -2 \text{ or } -.5$

85. $x = 11/3 \text{ or } 11$

86. $B = .39 \text{ or } - 3.39$

87. $x = 4 \text{ or } 3$

88. $x = -7 \text{ or } 8$

89. $y = -3 \text{ or } -4$

90. $x = 7.5 \text{ or } -10.5$

91. $D_2 = \frac{.866D_1 - 6B}{.866}$

$= D_1 - 6.928B$

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ANSWERS TO EXERCISE PROBLEMS

92. $A = \frac{B(H^2 + 1)}{(H^2 - 1)}$

93. $A = \frac{.4\pi IN\mu}{BL}$

94. $H = \sqrt{8\pi U - E^2}$

95. $P = 2F\left(\frac{\pi D}{x}\right)^2$

96. $a = \frac{R - r}{r(t_1 - t_2)}$

97. $R = \frac{\sqrt{e^2 - i^2(x_L - x_c)^2}}{i}$

98. $N = \frac{HL}{1.256I}$

99. $C = \frac{1}{39.5F^2L}$

100. $d = \frac{4A}{\left(\text{Log}^{-1}\frac{L}{.0046A}\right) + 1}$

V-9174-33

TELEVISION TECHNICAL ASSIGNMENT

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EXAMINATION

Show all work:

1. A transmitter tube is x years old. State its age in algebraic symbols 3 years ago. How old will it be y years from now?

Three yrs ago = $x - 3$

y yrs from now = $x + y$

2. Simplify, (remove fractions): $\frac{x}{6} + \frac{y}{3} = \frac{7}{18}$

$\frac{3x}{18} + \frac{6y}{18} = \frac{7}{18}$ $3x + 6y = 7$

or $3x + 6y - 7 = 0$

3. The capacitive reactance of a condenser is denoted by X_c, the frequency by f, and its capacitance by C. A certain mathematical constant is denoted by 2π, which is approximately equal to 6.28^+ The relation between these quantities is expressed by $X_c = \frac{1}{2\pi fC}$ Solve for C.

$X_c = \frac{1}{6.28fC}$ $6.28fC = \frac{1}{X_c}$

$C = \frac{1}{6.28fX_c}$

4. In the formula for wavelength developed in the text $\lambda = 1,884\sqrt{LC}$ Solve for L.

$\lambda = 1,884\sqrt{LC}$

$\lambda^2 = (1,884)^2 LC$ $L = \frac{\lambda^2}{(1,884)^2 C}$

EXAMINATION, Page 2.

4. *(Continued)*

5. In the resonance formula developed in the text

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Solve for C.

$f^2 = \frac{1}{4\pi^2 LC}$ $C = \frac{1}{4\pi^2 f^2 L}$

6. Mulitply (3x + y) by (2x - 46 -3).

$3x + y$
$2x - 49$
$6x^2 + 2xy - 147x - 49y$

7. Divide $9A^6B^3 - 12A^{-1}B + 6A^2B^4$ by $3AB^2$.

$3A^5B + 2AB^2 - 4A^{-2}B^{-1}$ ← Ans.

$3AB^2 \overline{) 9A^6B^3 + 6A^2B^4 - 12A^{-1}B}$
$9A^6B^3$
$0 + 6A^2B^4$
$+ 6A^2B^4$
$0 - 12A^{-1}B$
$- 12A^{-1}B$
0

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EXAMINATION, Page 3.

7. *(Continued)*

8. An electrical network has two branches or circuits. Current I_1 flows in the first branch; current I_2 in the second. The equations for these currents are

$$\frac{I_1}{5} + \frac{I_2}{2} = 5 \text{ and } I_1 - I_2 = 4$$

Find I_1 and I_2.

(Mult. by 2) $\frac{2}{5} I_1 + I_2 = 10$

$I_1 - I_2 = 4$

$\frac{7}{5} I_1 = 14 \quad I_1 = \frac{14 \cdot 5}{7} = 10$

$I_1 - I_2 = 4$

$10 - I_2 = 4 \quad I_2 = 10 - 4 = 6$

EXAMINATION, Page 4.

9. The so-called characteristic or surge impedance of a concentric line (Such as used to feed a television or other antenna) is given by the expression

$$Z_o = 138 \log \frac{D}{d}$$

where D is the diameter of the outer conductor, or sheath and d is the diameter of the inner conductor.

If D = .6 inch, find d when Z_o = 200 ohms.

$200 = 138 \log \frac{0.6}{d}$

$\frac{200}{138} = \log \frac{0.6}{d}$

$\log 200 = 2.30103$

$\log 138 = 2.13988$

0.16115

Anti log = 1.4493

$1.4493 = \log \frac{0.6}{d}$

$28.14 = \frac{0.6}{d}$

$d = \frac{0.6}{28.14} = 0.021323$ in.

$\log .6 = \bar{1}.77815$

$\log 28.14 = 1.4493$

$\bar{2}.32885$

10. (a) Solve by the quadratic formula $x^2 - 40 = 22x$.

(b) $x^2 - 48 = 22x$ Find x by factoring.

(a) $x^2 - 22x - 40 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{22 \pm \sqrt{484 + 160}}{2}$

$\frac{22 \pm \sqrt{644}}{2}$

$\frac{22 \pm 25.4}{2}$

$x = \frac{47.4}{2}, \frac{-3.4}{2} = 23.7, -1.7.$

(b) $x^2 - 22x - 48 = 0$

$(x - 24)(x + 2) = 0$

$x - 24$

$x + 2$

$x^2 - 24x$

$+ 2x - 48$

$x^2 - 22x - 48$

$x = 24$

$x = -2$