Book #2

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SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

LOGARITHMS

706D

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LOGARITHMS

FOREWORD

Possibly as you open your book to this assignment, you shudder and say to yourself, "More math! Why can't we get on to radio and television?" The subject of "Logarithms" doesn't sound very exciting—it probably makes you think of that very dry book, the "Table of Logarithms". But before you put it aside in disgust, think over the following fact—THE RESPONSE OF THE EYE AND THE EAR ARE LOTHARITHMIC!

This fact alone would justify a careful study of logarithms, even were it not for the fact that the use of logarithms *greatly simplifies* involved calculations in radio and television design problems—and even were it not for the fact that logarithms is the basis upon which the slide rule operates.

What does it mean that the response of the eye is logarithmic? To the television engineer this is an all important fact. It tells him, for example, that if he is illuminating a studio scene with 10,000 watts of flood lights an increase to 100,000 watts would only double the response as far as the eye is concerned, but would increase the power and fixture cost by 10 times as well as cause extreme discomfort to the cast due to the greatly increased heat.

Radio engineers think in terms of decibels (db's). In a sound system, one decibel increase in volume is the least amount of increase that the ear can just distinguish and most amplifier gain controls are variable in 2 db steps. The decibel is a logarithmic quantity since the response of the ear is logarithmic. To increase the signal output of an amplifier by 1 db the power output must be increased by about 25 per cent and for 2 db increase—which is about what the average person notices if he isn't concentrating on it—requires a power increase of about 59 per cent. This is the reason why we don't have 2 KW or 15 KW broadcast transmitters. The db signal gain of 2 KW over 1 KW, or

15 KW simply isn't worth the added cost. Thus standard transmitters come in sizes 250 watts, 1 KW, 5 KW, 50 KW. These increases produce sufficiently increased signal levels to justify the greater cost of equipment and power.

In the sound systems of television FM transmitters and in ordinary FM broadcasting, "high gain" antennas are used. Thus a directional antenna may produce from a 1 KW transmitter the signal equivalent at a given point of a 5 KW transmitter used with a non-directional antenna. This antenna would be said to have a gain of 14 db.

In addition to providing the basis for the decibel system as used in radio, logarithms provide an extremely simple method of handling certain involved calculations. In school you, of course, learned to extract the square root of a number. Possibly in your high school algebra class you learned to do "cube root"—but because it is so involved almost certainly you promptly proceeded to forget it. Extraction of roots beyond the 3rd by simple arithmetic or algebraic processes become so difficult that it is not attempted. Still by the use of logarithms the extraction of the 7th root or the 9th or any desired root is just as simple as finding the 2nd(square) root. All that is required is reference to a Table of Logarithms and a simple division.

Two facts constantly surprise engineering students who have always been afraid of mathematics: First, there are so many *practical uses* for the math that it becomes exceedingly interesting; second, the mathematical processes, when carefully studied in the proper sequence, with the non essentials eliminated, are exceedingly simple.

Most important to one who expects to derive his living from the technical fields of radio and television—THE BASIC MATHEMATICS IS A "MUST". You not only can't handle practical engineering problems without it, but also you can't understand the fundamental principals involved without it. Since your earning power all of your life will be based on the extent to which you really understand the principles of your profession, the time you spend on these subjects can be the best-paid hours you will have ever worked.

E. H. Rietzke,
President.

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LOGARITHMS

When the theory of exponents is fully understood, and the use of 10 raised to a positive or negative power to simplify the handling of large numbers and decimals is appreciated, the next logical step to further simplify practical calculations is the study and use of Logarithms. A table of logarithms, to one who does not understand its use, may seem very complex, but is in reality very easy to understand. The use of logarithms will greatly simplify the multiplication, division, raising to high powers and extracting the roots of numbers, particularly where the numbers to be handled are large or the decimals very small. The use of logarithms is very simple. The only requirement is sufficient practice to become thoroughly familiar with the tables and processes. In studying this assignment *all* examples at the end of each section should be worked before proceeding to the next section.

THEORY OF LOGARITHMS.—The basic principle of the logarithm (abbreviated LOG), is that *ANY number is equal to some number taken as a base and raised to the required power.* In the system of common logarithms the base is 10, and 10 raised to *some* power can be made to equal any *given number.* The power to which 10 is raised to equal this number may be a whole number or a decimal, or a mixed number, and might be expressed, for example, as $10^{2.54326}$. The number, 2.54326, is the logarithm, and 10 raised to the 2.54326 power is equal to some number. In this case the number is 349.35. In other words, if 10 were raised to the 2.54326 power it would equal 349.35.

From the law of exponents, 10 raised to the first power is simply 10. The exponent is 1 but this is ordinarily understood and not written. Since, however, the logarithm is the exponent of 10, then the logarithm of 10 is 1.

Continuing, 10 raised to the second power, 10^2, equals one hundred; and since the exponent is the logarithm, then the logarithm of 100 is 2. 10 raised to the third power, 10^3, equals 1,000; therefore, the logarithm of 1,000 is 3, etc.

It will be seen that if the log of 10 is 1 and the log of 100 is 2, the log of any number between 10 and 100 must lie between 1 and 2 or 1 plus some decimal. Example, Log of 17 = 1.23045 (from tables).

Likewise, any number between 100 and 1,000 will have a log equal to 2 plus a decimal. Example, Log of 170 = 2.23045 (from tables).

By the same reasoning, the log of a number between 1,000 and 10,000 is equal to 3 plus a decimal, between 10,000 and 100,000 the log will be 4 plus a decimal, etc. It would seem that considerable work would be required to find the decimal part of the logarithm, but fortunately these have been worked out and arranged in tables, some of which are accurate to as many as eight figures, and it is only necessary to learn how to use the tables.

It was shown in the previous assignment that 10 (or any other number, for that matter) raised to the zero power is equal to one. Thus, $10^0 = 2^0 = 5.6^0 = 1$. Since 10 is the base and the exponent is the log, it will be seen that the log of 1 equals zero.

One-tenth (1/10th) equals 1 divided by 10, .1 = 1/10. One-tenth may also be expressed as 10^{-1}. Also .01 may be expressed as 10^{-2}. Note that each of these decimals can be expressed as 10 raised to some negative power. Since the base is the same as in the case of whole numbers, (10), and since the exponent of 10 is the logarithm of the number, then the log of .1 = -1. The log of .01 = -2, the log of .001 = -3, etc. In accordance with the explanation above, it will be seen that for any decimal between .1 and 1 the log will be -1. plus a decimal, between .01 and .1 the log will be -2. plus a decimal, between .001 and .01 the log will be -3. plus a decimal, etc.

The whole number part of the log is called its CHARACTERISTIC number. The decimal part is called the MANTISSA. The table of logarithms expresses only the mantissa.

Example: Given a logarithm whose value is 3.25468. The *characteristic* number of this log is 3. The mantissa is .25468.

Refer now to Table 1. This table clearly demonstrates the following facts: A number between 1 and 10, that is, a *number of one* figure, will have a log composed entirely of a decimal, the value of the log being between zero and one. A number between 10 and 100, *one having two figures,* will have a log composed of 1 plus a decimal. A number between 100 and 1,000, *one having three figures,* will have a log of 2 plus a decimal, etc.

From this can be made a rule stating that, in the log of any number greater than one, the *characteristic number is one less than the number of figures in the whole part of the number.*

TABLE I

Number	Base and Exponent	Log
.0001	10^{-4}	-4
.001	10^{-3}	-3
.01	10^{-2}	-2
.1	10^{-1}	-1
1	10^{0}	0
10	10^{1}	1
100	10^{2}	2
1000	10^{3}	3

On examining a table of logarithms it is found that only the decimal part of the log (the mantissa) is given. It will also be found that *the mantissa is the same for several numbers having the same sequence of figures.* For example, the mantissas of the following numbers, 1.74, 17.4, 174, 1,740 are all .24055. (See MATHEMATICAL TABLES from Handbook of Chemistry and Physics.) But from the rule given in the preceding paragraph it will be seen that the characteristic numbers of the logs of these numbers are 0, 1, 2, and 3 respectively.

Examples:

Number	*Log*
1.74	0.24055
17.4	1.24055
174	2.24055
1740	3.24055

It is apparent that the mantissa indicates only the *sequence* of the figures, and the decimal point is correctly placed by the characteristic number. Using the log 2.24055, it is found from the table that the sequence of figures in the corresponding number is 1740. The characteristic number is 2. The number corresponding to the log will therefore have three figures to the left of the decimal point, 174.0. *This number is called the Antilog.*

Again referring to Table I: any number between .1 and 1 has a characteristic number of -1. Any number between .01 and .1 has a characteristic of -2, etc. A rule may now be made stating that *in the logarithm of a decimal, the characteristic number is negative and is one greater than the number of zeros immediately following the decimal point.*

Examples:

Number	*Log*
.174	-1 + .24055
.0174	-2 + .24055
.00174	-3 + .24055

As in the case of the logarithms of whole numbers, the mantissa indicates only the sequence of figures, the position of the decimal point being determined by the negative characteristic number.

In working with logarithms it must be remembered that *whether the characteristic number is positive or negative, the mantissa is always positive.* This fact is particularly important.*

Before considering the use of logarithms the student should carefully study the arrangement of a table of logarithms. The table lists the mantissa only. The mantissa indicates the sequence of figures in the corresponding number, regardless of the position of the decimal point which is placed by the characteristic number. The MATHEMATICAL TABLES from the Handbook of Chemistry and Physics is supplied with this course. The book includes a number of tables, one being that of Logarithmic Trigonometric Functions, Sines, Cosines, etc. This table is used only when handling problems involving trigonometric functions. The Table referred to in this assignment is "Five-Place Logarithms".

TO FIND THE LOGARITHM OF A GIVEN NUMBER.—See portion of log table printed in Table II. This is taken from Five-Place Logarithms, Mathematical Tables from Handbook of Chemistry and Physics.

Find the logarithm of 2,714. In the "N" column read downward to 271, the first three figures of the given number. From this point move to the right to the column headed by "4" (the fourth figure of the given number) and read 361. These are the last three figures of the mantissa. The first two, 43, are read in the "0" column so the complete mantissa of 2,714 is .43361. Note that the first two figures of all mantissas of numbers between 2,700 and 2,754 are 43. *When the last three figures of the mantissa are preceded by an asterisk (*) the first two figures are to be read in the vertical*

*Such logarithms are often written with the minus sign above the characteristic, to indicate that only this part is negative, and that the mantissa is positive. For example, -3. + .24055 is often written $\bar{3}.24055$. However, in problems where the actual value of the logarithm must be known, as in the design of certain types of attenuation pads, the former representation makes it more evident what the value of the logarithm actually is. For example: $-3. + .24055 = -(3) + (.24055) = -2.75945$. Hence, the actual value of the logarithm is the *algebraic sum* of the characteristic and the mantissa.

TABLE II

N	0	1	2	3	4	5	6	7	8	9
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232

column headed "0" on the next lower horizontal line. For example, the mantissa of 2,754 is .43996 but that of 2,755 is read .44012 because .012 is preceded by an asterisk so the first two figures are read from the next lower horizontal line.

Since the number 2,714 is between 1,000 and 10,000 the characteristic of the logarithm of 2,714 is +3 (one less than the number of figures to the left of the decimal point). Therefore the log of 2,714 is 3.43361.

Had the number been a decimal having the same sequence of figures, for example, .002714, the mantissa of the log would be found to be the same as above, that is, .43361. Since the number under discussion is a decimal and therefore less than 1, the characteristic number will be *negative* and will be *1 greater than the number of zeros following the decimal point*, or -3. The entire log will be -3 + .43361.

Exercises

Write the characteristic number of the logarithm of the following numbers:

1. 247
2. 62,458
3. 93
4. 8,472
5. .015

Find the absolute or algebraic value of the logarithm of the following numbers:

6. .0000029350
7. 293,500,000
8. .182
9. .0000029
10. 715,327,000,000

Write the complete logarithm of the following numbers:

11. .00000002
12. 15,000,000,000,000
13. 5×10^6
14. 215×10^{10}

15. 318×10^{-6}

TO FIND THE ANTILOG OF A NUMBER.—The antilog of a number is the number corresponding to a given logarithm. For example, 2 is the logarithm of 100, so 100 is the antilog of 2.

Find the antilog of 2.44188. The first step is to find the first two figures 44 of the mantissa in the "0" column of the log tables. This is found in the log tables (reprinted in part in Table II). The next step is to find the number in the columns nearest to the last three figures 188 of the mantissa. In the column headed by "6" is found 185 the closest sequence of figures to 188. The first three figures of the antilog of .44188 is 276 read in the number column N and the fourth figure 6 is found at the top of the column in which 185 appears. Thus the sequence of figures in the antilog is 2,766 and the characteristic 2 fixes the decimal point. Antilog 2.44188 = 276.6. Later in this assignment will be explained how to obtain greater accuracy in determining the antilog by the use of interpolation.

Exercises

Write the complete logarithms of the following numbers:

16. 25

17. 3,721

18. 640

19. 8

20. 256,000,000

21. 1,541

22. 6,234,000,000,000

23. 827

24. 5,432

25. 9,542

26. 8,888

27. 5,555

Find the antilog of the following logarithms:

28. -1 + .07918

29. -5 + .91645

30. -3 + .65600

31. -2 + .45788

32. 8.51799

33. 0.60206

34. 6.88638

35. -6.00000

MULTIPLICATION BY LOGARITHMS.—Multiplying like numbers having exponents is done by simply adding the exponents and affixing the resulting exponent to the common number. Since the log of any number is merely the exponent of 10 or the power to which 10 must be raised to equal the given number, it will be readily seen that if two numbers are to be multiplied and each is expressed as 10 raised to a certain power, all that is necessary in multiplying will be to add the exponents, in this case the logarithms. The resultant exponent or logarithm will then be the logarithm of the product of the two numbers. By looking up the mantissa of this log in the log table, it is a simple matter to obtain the sequence of figures in the answer and correctly place the decimal point by inspection of the characteristic number. Log (ab) = log a + log b.

Example: 249 X 68

Log 249	=	2.39620
Log 68	=	1.83251
Adding		4.22871

Log of the product of the given numbers 249 X 68.

Inspection of the table of logarithms indicates that a mantissa of .22871 corresponds to a number whose sequence of figures falls between 1,693 and 1,694. If accuracy greater than four figures is desired, interpolation may be employed, otherwise take the nearest number, in this case 1,693. (Interpolation will be taken up in the latter part of this assignment.) The sequence of figures in the antilog having been determined, all that remains is the fixing of the decimal point. The characteristic number of the logarithm 4.22871 is 4. The answer will therefore consist of 4 + 1 or 5 figures preceding the decimal point or 16,930. To check for accuracy, multiply 249 by 68. The answer obtained is 16,932. The solution by logarithms without interpolation is accurate to two parts in 16,932—a negligible error for most work.

Logarithms are particularly useful in handling very large numbers where accuracy to four or five places is sufficient.

Example:

89,463,532 X 7,648,235,196

Log 89,460,000	=	7.95163
7,648,000,000	=	9.88355
Adding		17.83518

Log of product.

(In finding the log of 89,463,532, accurate to four figures, the sequence of only the first four figures, 8,946, is considered in determining the mantissa from the table. Likewise, in the second number the mantissa is determined from the tables by considering only the first four figures, 7,648.)

The antilog corresponding to a mantissa of .83518 is 6,842 (Tables). The characteristic number of 17 indicates that the answer has eighteen figures, therefore add zeros to the number found in the tables until a number of eighteen figures is obtained, 684,200,000,000,000,000, or $6,842 \times 10^{14}$.

MULTIPLYING A WHOLE NUMBER BY A DECIMAL.—

Example: 648 X .00000257

Log 648	=	2.81158
Log .00000257	=	-6 + 0.40993
Adding		-3 + 0.22151

Log of product.

In this example it must be remembered that the *mantissa is always positive*, regardless of the sign of the characteristic number. Therefore, when the 1 is carried over from the mantissa in addition, it is a plus 1, and added to the plus 2 of the characteristic number of the first log, becomes plus 3. This added to minus 6 results in a characteristic number of minus 3. The sequence of figures in the antilog corresponding to the mantissa of the log -3 + .22151 is 1,665. The characteristic number -3 indicates there should be two zeros immediately following the decimal point, so the antilog becomes *.001665*.

MULTIPLYING TWO DECIMAL NUMBERS.—

.000046 x .000894

Log .000046 = -5 + .66276
Log .000894 = -4 + .95134
Adding -8 + .61410

-8 + .61410 = Log of product.

In this example, when adding, plus 1 is carried over from the mantissa. The sum of the characteristic number, both negative, is -9. But a positive 1 was carried over from the addition of the mantissas, therefore the resulting characteristic number is -9 + 1 = -8. From tables, the sequence of figures in the antilog is 4,112 which preceded by 7 zeros gives an antilog of .00000004112 or $4,112 \times 10^{-11}$.

The examples given above cover the different types of problems encountered in multiplication by logarithms. Of course if more than two numbers are to be multiplied it will be necessary to perform only one addition, adding the logs of all the numbers to be multiplied together. The most important point to remember in this type of problem is that *the mantissa is always positive and must be treated as such, regardless of the sign of the characteristic number.*

Examples: 625 x 3,720 x 4,857 x .00845 = ?

Log 625	=	2	.79588
Log 3,270	=	3	.51455
Log 4,857	=	3	.68637
Log .00845	=	-3 +	.92686
Log of product	=	7	.92366

Antilog 7.92366 = 83,880,000 or $8,388 \times 10^{4}$.

Exercises

Solve the following problems by logarithms:

36. 12 x 285
37. 3,927 x 143
38. 286 x 986,000
39. .00127 x 8,432
40. .9253 x .1876
41. .248 x 3541
42. 6,742 x 3,287 x 732,100 x .476 x .00924
43. .000325 x .0000076 x 487,000,000
44. 843 x 9 x 28 x .35 x .00000643 x 9,231
45. $523 \times 10^{-8} \times 7,265 \times 10^{-4} \times 8,531 \times 10^{10}$

DIVISION BY LOGARITHMS.—When dividing by logarithms, subtract the log of the divisor from the log of the dividend. The remainder is the log of the quotient. Log a/b = log a - log b.
Example: 8,643 ÷ 256

Log 8,643 = 3.93666
Log 256 = 2.40824
Subtracting 1.52842
1.52842 = Log of quotient.

The mantissa represents a sequence of figures of 3376. The positive 1 characteristic indicates two figures preceding the decimal point, making the antilog equal to 33.76.

As in multiplication by logs, the mantissa must always be considered as positive. This will be illustrated in the following example.

Divide: .0000243 by 962

Log .0000243 = -5 + .38561
Log 962 = 2 .98318
Subtracting -8 + .40243
-8 + .40243 = Log of quotient.

In this problem when subtracting the mantissa of .98318 from .38561 it becomes necessary to carry over one from the characteristic number -5. As this is to be added to the positive mantissa to increase its value, *the value carried over must be a plus 1.*

Subtracting a +1 from a -5 makes the actual value smaller; thus in this case the characteristic number of the log of the dividend becomes -6. To complete the subtraction, subtract 2 from -6 and obtain a resulting characteristic number of -8.

-6 + 1.38561
2 .98318
-8 + .40243

From the tables, the sequence of figures corresponding to a mantissa of .40243 is 2,526. The characteristic number -8 indicates that the decimal point must be followed by seven zeros, resulting in an answer of .00000002526 or $2{,}526 \times 10^{-11}$.

It will be noted that in all of these examples the rules for the addition and subtraction of positive and negative numbers, as studied in a previous assignment are carefully followed. No other rules are necessary.

Exercises

Solve the following problems by logarithms:

46. 476 ÷ 32

47. 8,271 ÷ 47.6

48. 35.78 ÷ 6.72

49. 853,000,000 ÷ 7,215

50. .000563 ÷ 148

51. 8,327 ÷ .00651

52. .6432 ÷ .0527

53. .0000245 ÷ .000000082

54. $(654 \times 10^{-7}) \div (4{,}865 \times 10^{-3})$

55. $(3{,}156 \times 10^{8}) \div (7{,}243 \times 10^{5})$

RAISING A NUMBER TO A HIGHER POWER.—To raise any number to a given power, multiply the log of the number by the power. Example, 347^4. By arithmetic, $347^4 = 347 \times 347 \times 347 \times 347$. $\log a^n = n \log a$.

By logs, Log 347 = 2.54033
Index of Power 4
Multiplying 10.16132
Log of power.

By reference to the log table and inspection of the characteristic number, the answer is 145×10^{8}.

Example: $.00678^5$.

In problems of this type it must be remembered the mantissa is always positive.

By logs, Log .00678 = -3 + .83123

To raise to the fifth power multiplication by 5 is necessary.

-3 + .83123
5
-15 + 4.15615

-15 + 4 = -11 so Log $.00678^5$
= -11 + .15615

The antilog of -11 + .15615 is 1433×10^{-14}.

Exercises

Solve the following problems by logarithms:

56. 634^2

57. $8,275^4$

58. 324^3

59. $.000256^5$

60. $9,683^4$

61. $.0003792^8$

62. 15^{10}

63. $.00781^3$

64. $(273 \times 10^4)^3$

65. $(7,634 \times 10^{-8})^2$

EXTRACTING THE ROOT OF A NUMBER.—To extract the root of a given number, divide the log of the number by the root to be extracted. Log $\sqrt[n]{a} = 1/n \log a$.

Example: $\sqrt[3]{3,467}$

Log 3,467 = 3.53995
Dividing $3\overline{)3.53995}$
Log of root = 1.17998
Antilog = 15.13

When raising a number to a high power or when extracting roots beyond the square root, the use of logarithms is more fully appreciated than in any other work. In fact, the only really practical way to extract roots with indices greater than 2 is by logarithms. The use of logarithms, while not essential to the correct multiplication and division of large numbers, will greatly simplify such work after the student has obtained sufficient practice to familiarize himself with the table of logarithms. When the theory of positive and negative numbers and exponents is fully understood, the ready use of the log table becomes merely a matter of practice.

Exercises

Solve the following problems by logarithms:

66. $\sqrt{824}$

67. $\sqrt[4]{3,275}$

68. $\sqrt[3]{93 \times 10^4}$

69. $\sqrt{52.6}$

70. $\sqrt[5]{82.53}$

71. $\sqrt[3]{156 \times 10^{12}}$

72. $\sqrt[6]{623.7}$

73. $\sqrt[3]{24 \times 10^4}$

74. $\sqrt[5]{25,600}$

75. $\sqrt[3]{827 \times 10^5 \times 3.26 \times .092}$

ROOT OF A DECIMAL.—An occasional case may rise when it is desired to use logarithms in extracting the root of a decimal where the root will not divide evenly into the negative characteristic. In this case it is particularly necessary to remember that, although the characteristic number is negative, the *mantissa is always positive.* A problem of this type requires a thorough understanding of positive and negative numbers. A specific example will best explain the correct procedure: To extract the 4th root of .00516.

Log .00516 is -3 + .71265

To extract the 4th root, divide the log by 4,

$$4\overline{)-3 + .71265}$$

It is at once apparent that 4 will not divide evenly into the negative characteristic. However, it is not correct to merely place a decimal point below the line and divide into 37 as would be done if the entire number were positive.

It is necessary to carry over a sufficient *negative* value to make the negative characteristic evenly divisible by the root, 4. In this case -1 must be carried over from the mantissa thus making the characteristic -4.

It is an axiom of arithmetic that if the same amount is added and subtracted from a number the value of the number does not change. For example adding +5 and -5 to 10 will still give a sum of 10. The above logarithm may be written as -3 + .712650. If we add +1 and -1 to the characteristic the logarithm will not be changed.

$$\begin{array}{r} -3. + 0.71265 \\ \underline{-1. + 1.00000} \\ -4. + 1.71265 \end{array}$$

Note that -4 + 1 still gives a characteristic of -3. However, it is now a simple matter to divide both characteristic and mantissa by 4.

$$\begin{array}{r} 4\overline{)-4. + 1.71265} \\ -1. + 0.42816+ \end{array}$$

$\sqrt[4]{.00516}$ = antilog -1 + .42816 = .268

If the number is .0516 and the 4th root is to be extracted, the steps will be Log .0516 = -2 + .712650. To make the characteristic evenly divisable by 4 add +2 and -2 to the logarithm

$$\begin{array}{r} -2 + .71265 \\ \underline{-2 + 2.} \\ 4\overline{)-4 + 2.71265} \\ -1 + 0.67816+ \end{array}$$

$\sqrt[4]{.0516}$ = antilog -1 + .67816 = .4766

It is always necessary to make the negative characteristic *evenly divisible by the root* by adding a negative quantity. To maintain the equality an equal positive quantity must then be added to the mantissa.

Exercises

Solve the following problems by logarithms:

76. $\sqrt{72.3}$

77. $\sqrt{.000427}$

78. $\sqrt[4]{3,582,000}$

79. $\sqrt[5]{543.7}$

80. $\sqrt[3]{.001765}$

81. $\sqrt[3]{.000257}$

82. $\sqrt[5]{.000000075}$

83. $\sqrt[4]{.0156}$

84. $\sqrt[5]{.392}$

85. $\sqrt[4]{95.87}$

There are other methods of handling logs having negative characteristic numbers, and any method which will give the same results may of course be used. The writer greatly prefers the above method because it coordinates, in a simple and logical manner, the use of logarithms and the rules of positive and negative numbers.

INTERPOLATION.— Interpolation is used in many different kinds of work in which tables are employed in order to obtain a greater degree of accuracy than can be had from a direct reading of the table. Take the familiar case of a variable condenser, calibrated every ten divisions of the dial. The table gives the capacity of the condenser for each ten-dial divisions, and if it is desired to know the capacity at any particular setting in between two of these points it will be necessary to interpolate.

Example: The capacity at 40° is 230 µµF. At 50° it is 270 µµF. To determine the capacity at a setting of 47 degrees, there are ten divisions between 40 and 50. The capacity change between these points is 270 - 230 or 40 µµF. If the change in capacity is 40 µµF for 10°, the change per degree is 40 divided by 10 or 4 µµF. 7° is added to 40° to get a dial reading of 47°. For each degree added, add 4 µµF to the capacity. For 7 divisions, therefore, add 4 x 7 or 28 µµF to the capacity at the 40° setting, making the capacity at 47° equal to 230 + 28 or 258 µµF. (This applies only to condensers in which the capacity varies in a straight line.)

The above problem is a simple example of interpolation. The same principle will be applied to logarithms in the following example: To find the log of the number 24,573; from the log table, the mantissa of the log of 24,570 is 39,041; the mantissa of the log of 24,580 is 39,058; the difference between the two mantissas is 39,058 - 39,041 or 17. In other words, a difference of ten (24,570 to 24,580) in the number makes a difference of 17 points in the mantissas of the logs of these numbers or 1.7 for each unit added as the fifth figure of the number. The lower number is to be increased by three (24,570 to 24,573), hence, the mantissa of the lower number is increased by 1.7 x 3 or 5.1, or approximately 5. The mantissa of the log of 24,573 is equal to the mantissa of the log of 24,570 plus 5, or 39,041 + 5 = 39,046. The mantissa of the number 24,573 (.39046) is now accurate to five figures in the number instead of four which may be obtained without interpolation.

Reversing this process, suppose it is desired to find the antilog corresponding to a log of 7.39614. Neglecting, temporarily, the characteristic number, reference to the table indicates that a mantissa of 39,602 corresponds to an antilog sequence of figures of 2,489. A mantissa of 39,620 corresponds to an antilog of 2,490. The mantissa 39,614 belongs to a number somewhere between the two. The difference between the tabular mantissas is 39,620 - 39,602 or 18 points. In other words *to increase the antilog from 2,489 to 2,490 it would be necessary to increase the mantissa by 18 points.* But actually the mantissa is increased only by 39,614 - 39,602 or 12 points. The antilog instead of being increased one whole point is only increased by 12/18 of a point, or .67 point. This affixed to the right of the smaller number produces a resulting sequence of figures of 248,967. The characteristic number being seven indicates that the resulting antilog must have eight figures or 24,896,700. Interpolation for the entire eight figures could have been obtained by carrying out the division of 12 by 18 to four places instead of two as illustrated above, but interpolating beyond two figures leads to questionable accuracy because the last figure of any mantissa listed in a log table is not in itself 100 per cent accurate. It will be seen that by the use of interpolation any desired degree of accuracy in calculations may be obtained through the use of logarithms within the limits just mentioned.

When accuracy only to the fifth figure of the number is required, the process of interpolation may be simplified by the use of the proportional parts tables included along the right-hand margin of each page. These tables eliminate the operation of multiplication or division in finding the quantity to be added to the next lower tabulated value. For example, in finding the log of 24,573, after determining the difference between the next lower and the next higher tabulated mantissas, 17 points, enter the proportional parts table headed by the figure 17. Opposite 3 in the left hand column, the value 5.1 may be read in the right hand column. Thus, the tabulated mantissa for 24,570 must be increased by 5.1 points in order to obtain the mantissa for 24,573.

The proportional parts table may also be used in finding the fifth figure of an antilogarithm. In this case, the right-hand column of the table is entered with the number of points by which the *given mantissa exceeds the next lower mantissa*. The fifth figure of the antilog is then indicated in the left-hand column. For example, in finding the antilog of 7.39614, the given mantissa was found to exceed the next lower tabulated mantissa by 12 points, while the difference between the next lower and the next higher tabulated mantissas was found to be 18 points. Entering the right-hand column of the proportional parts table headed 18, the value most nearly approaching 12 is found to be 12.6. To the left of 12.6, the fifth figure of the antilog is read as 7. The sequence of figures in the antilog will therefore be 24,897. As shown before, the characteristic figure of 7 in the logarithm indicates that eight figures shall precede the decimal point in the antilog. The resulting antilog is therefore 24,897,000. A study of the "use of mathematical tables" in the Handbook of Chemistry and Physics will help to further clarify the use of the Five-Place Logarithm Tables.

USE OF LOGARITHMS IN RADIO PROBLEMS.--The radio engineer will find the use of logarithms convenient in many types of practical problems.

For example, consider the equation for the resonant frequency of a circuit,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Where f = cycles per second
2π = 6.28
L = inductance in Henries
C = capacity in Farads

Assume that L = 235 μH or .000235 H, and C = 426 μμF = 426×10^{-12} f. Substituting these values the equation becomes,

$$f = \frac{1}{6.28\sqrt{235 \times 10^{-6} \times 426 \times 10^{-12}}}$$

$$f = \frac{1}{6.28\sqrt{235 \times 426 \times 10^{-9}}}$$

$$= \frac{10^9}{6.28\sqrt{235 \times 426}}$$

Log 235 = 2.37107
Log 426 = 2.62941 Adding to multiply
2/5.00048 numbers
2.50024 Divide by 2 to
Log 6.28 = 0.79796 extract square root
3.29820 Adding to multiply by 6.28

Log 10^9 = 9.00000
3.29820
5.70180 Subtract to divide into 10^9

f = antilog 5.70180 = *503,300 cycles per second.*

After the equation was arranged algebraically in its simplest form, the actual calculations involved only very simple addition and subtraction, and one division by 2.

To find the wavelength (λ) in meters of the above circuit from the equation, in which L and C are in μ-units,

$$\lambda = 1884\sqrt{LC}$$

$$\lambda = 1884\sqrt{235 \times .000426}$$

Log 235 = 2.37107
Log .000426 = -4+.62941 Adding logs
2 /-1+.00048 to multiply
-1+.50024 Divide by 2 to extract square root—*note carefully*

Log 1,884 = 3.27508
2.77532 Adding to multiply by 1884

λ = antilog 2.77532 = *596 meters*

To find the capacitive reactance (X_c) of a .000623 μF condenser (C) at a frequency (F) of 248 kilocycles from the equation,

$$X_c = \frac{1}{2\pi FC}$$

Where F = cycles per second = KC/s. $\times$ 1,000 = 248 x 10^3.

C = capacity in farads = 623 x 10^{-12}

X_c = reactance in ohms

$2\pi = 6.28$

$$X_c = \frac{1}{6.28 \times 248 \times 10^3 \times 623 \times 10^{-12}}$$

$$X_c = \frac{1}{6.28 \times 248 \times 623 \times 10^{-9}}$$

$$= \frac{10^9}{6.28 \times 248 \times 623}$$

Log 6.28 = 0.79796
Log 248 = 2.39445
Log 623 = 2.79449
5.98690 Adding logs to multiply

Log 10^9 = 9.00000
5.98690
3.01310 Subtracting logs to divide

X_c = antilog 3.01310 = *1,031 ohms*

To find the inductive reactance (X_L) of a coil having inductance (L) of .2 Henry at a frequency (F) of 8,950 cycles per second, from the equation,

$$X_L = 2\pi FL$$

$$X_L = 6.28 \times 8,950 \times .2$$

Log 6.28 =	0.79796	
Log 8,950 =	3.95182	
Log .2 =	-1 + .30103	
	4.05081	Adding to multiply

X_L = antilog 4.05081 = *11,241 ohms*

Many problems of the above type, and more elaborate ones composed of combinations of problems as shown above, are very frequently encountered in radio calculations. They may be worked quickly, accurately, and without difficulty by the intelligent use of logarithms.

*NATURAL LOGARITHMS**.--In this assignment logarithms to the base 10 have been discussed. These are called common logarithms and were invented by Briggs. In some formulas another base is frequently used. This base is a number that cannot be expressed exactly in figures, to seven decimal places it is 2.7182818. Logarithms to this base are frequently called natural logarithms, hyperbolic logarithms or, after the inventor of the system Napier, Napierian logarithms. When the base 2.7182 is to be used it is designated by the letter "e". For example: "$\log_e$ 112."

Tables of the natural logarithms of numbers are available. However, such tables are not necessary for determining the values of the natural logs and antilogs as they may be found from an ordinary table of common logarithms by the use of a multiplying constant. To derive a formula for calculating the natural logarithm of a number from its common logarithm, designate the number by N and its natural logarithm by x. That is, let

$$\text{Log}_e\ N = x \qquad (1)$$

At the beginning of this technical assignment the logarithm of a number was defined as the exponent indicating the power to which the base of the logarithm must be raised in order to equal the given number. In this case, x is the exponent of the power to which the base e must be raised in order to equal N:

$$e^x = N \qquad (2)$$

Equation (2) is a typical example of an exponential equation. In such equations, the unknown is an exponent, and is to be evaluated in terms of the known values. Equations of this type cannot be directly solved by the methods employed in the more familiar algebraic solutions. Whenever exponential equations are encountered, it is best to convert the equation into the logarithmic form immediately. This may be done in one simple operation. From the definition of a logarithm, it is apparent that logarithms to the same base of equal numbers will be equal. Thus if Equation (2) is true, the following is also true:

$$\log_{10} e^x = \log_{10} N \qquad (3)$$

The equation is now in the logarithmic form, and may be further simplified. In discussing the use of logarithms in raising numbers to powers, it was shown that the logarithm of the power of a number can be found by multiplying the logarithm

*Natural logarithms and their base e, appear in many scientific formulas. Examples of this will be found farther on, particularly in the specialized mathematics section.

of the number by the exponent indicating the power. In the present case:

$$\log_{10} e^x = x\ (\log_{10} e) \qquad (4)$$

Since things equal to the same thing are equal to each other, Equations (3) and (4) may be combined:

$$x\ (\log_{10} e) = \log_{10} N \qquad (5)$$

To solve for x, divide through by $(\log_{10} e)$.

$$\frac{x\ (\log_{10} e)}{(\log_{10} e)} = \frac{\log_{10} N}{\log_{10} e} \qquad (6)$$

$$x = \frac{\log_{10} N}{\log_{10} e} \qquad (7)$$

Combining Equations (1) and (7):

$$\log_e N = \frac{\log_{10} N}{\log_{10} e} \qquad (8)$$

Substituting for e the value 2.7182.

$$\log_e N = \frac{\log_{10} N}{\log_{10} 2.7182} = \frac{\log_{10} N}{.4343} \qquad (9)$$

$$\log_e N = 2.3026 \log_{10} N \quad (10)$$

Multiplying both sides of Equation (9) by .4343:

$$\log_{10} N = .4343 \log_e N \quad (11)$$

Equation (10) shows that when a Napierian logarithm is designated, it may be obtained from a common logarithm by multiplying by the factor 2.3026. Thus $\log_e 276$ becomes $2.3026 \log_{10} 276$ or $2.3026 \times 2.44091 = 5.62044$.

When the antilog of a Napierian logarithm is desired, the logarithm is first converted to the common system by multiplying by .4343. The antilog of the common logarithm is then found in the usual manner from the tables. For example, to find the number whose natural logarithm is 5.90536. By the use of Equation (11), the equivalent common logarithm is found to be .4343 x 5.90536 = 2.564697. Reference to the common logarithm table shows that the antilog of 2.564697 is 367. This is the number whose natural logarithm is 5.90536.

As a second example, evaluate $\log_e 45.1$.

$$\log_e 45.1 = 2.3026 \log_{10} 45.1 \qquad (12)$$

1. By the use of arithmetic:

```
log10 45.1 = 1.65418
              x2.3026
               992508
              330836
            4962540
           330836
           3808914868
```

(Mark off 9 decimal places)

$\log_e 45.1 = 3.80891$ *Answer*

2. By the use of logarithms: (Since one of the factors to be multiplied is already a logarithm of a number, it is necessary at first to find the logarithm of the logarithm of that number.)

$\log_{10} (\log_{10} 45.1) =$
$\log_{10} 1.65418 = 0.21859$
$\log_{10} 2.3026 = \underline{0.36222}$
$\log_{10} \log_e 45.1 = 0.58081$ (Sum)
$\log_e 45.1 =$ Antilog $0.58081 = 3.809$
(Ans.)

The term "Antilog...", or "$\log^{-1}$..." as it is frequently designated, (also used is anlog), can be translated "the number whose logarithm is...". The antilogarithm of 0.58081 is found by locating 58,081 in the Math Tables, and reading the significant figures 380 at the same level in the left column and 9 at the top of the right-hand column. The characteristic 0 indicates that the decimal point is to be so placed as to give a number between 1 and 10- Thus, $\log^{-1}$ 0.58081 is equal to 3.809. This value checks closely with the value of 3.80891 obtained by the use of simple multiplication; the values could be made to coincide more closely by the use of seven- or higher-place logarithm tables and by taking the value of the multiplying factor 2.3026.... to more places. To ten decimal places the value of this conversion factor is 2.3025850930. For all ordinary problems in radio and acoustical work, the value 2.3026 is sufficiently accurate.

In determining the value of the Napierian logarithm of a decimal quantity, the same methods of long-hand or logarithmic multiplication can be applied, but certain questions of procedure will arise which are not encountered in finding the Napierian logarithms of quantities greater than one.

It should be kept in mind that all mantissas listed in the logarithm tables are positive. Likewise in the standardized method of writing the logarithm of a decimal quantity, the characteristic is negative but the mantissa is positive.

As a third example, evaluate $\log_e .7854$.

$$\log_e .7854 = 2.3026 \log_{10} .7854$$

Reference to the Math Tables, indicates that the common logarithm of .7854 is $-1 + .89509$. Bearing in mind that the tabulated mantissas are positive, it is seen that in multiplying by 2.3026 we should recognize $\log_{10} .7854$ as actually being $(-1 + .89509)$. Thus:

$$\log_e .7854 = 2.3026(-1. + .89509)$$
$$= 2.3026(-.10491)$$

$$\begin{array}{r} -1.00000 \\ +0.89509 \\ \hline -\ .10491 \end{array}$$

1. By the use of arithmetic:

$$\begin{array}{r} 2.3026 \\ \times(-.10491) \\ \hline 23026 \\ 207334 \\ 92104 \\ 230260 \\ \hline 241566766 \end{array}$$

(Mark off nine decimal places and affix negative sign to product.)

$$= -.24157$$

As previously pointed out, the standard method of writing logarithms calls for a positive mantissa. Therefore, the value -0.24157 which is all negative must be converted to an equivalent value whose decimal part is positive.... If the same quantity is added to and subtracted from a number, the value of the number is not changed. It is therefore permissible to employ the artifice of adding -1 to the integral part and +1 to the fractional part of (0 - .24157), without changing its value:

$$\begin{array}{rl} -1 & +\ 1. \\ \underline{-0} & \underline{-\ .24157} \\ -1 & +\ .75843 \end{array}$$

Since the standard method of writing logarithms calls for a positive mantissa:

$\log_e .7854 = -1 + .75843$ *Answer*

An alternate method of using arithmetic lies in separately multiplying the factors:

$$\log_e .7854 = 2.3026\ (-1 + .89509)$$

$$= -2.3026 + 2.06103$$

$$= 0 - .24157$$

(using same artifice)

$$\begin{array}{rl} -1 & +\ 1. \\ \underline{0} & \underline{-\ .24157} \\ -1 & +\ .75843 = -1 + .75843 \text{ Answer} \end{array}$$

2. By the use of logarithms:

$$\log_e .7854 = 2.3026 \log_{10} .7854$$

Since $\log_{10} .7854 = -1 + .89509$ or $-.10491$, it is apparent that $\log_{10} (\log_{10} .7854)$ will be the logarithm of a negative quantity. Strictly speaking, there is no mathematically correct logarithm of a negative number: Logarithms of positive quantities less than unity (positive decimals) are negative and approach negative infinity as the decimal is made smaller and smaller, that is, as the decimal quantity approaches zero. Thus, quantities less than zero (negative quantities) have no finite logarithms. However, it is permissible to use logarithms to determine the product of the magnitudes of a positive and a negative number, realizing that a negative sign will have to be prefixed to the product of those magnitudes in order to have the correct answer.

$$\log_e .7854 = 2.3026 \log_{10} .7854$$

$$= 2.3026\ (-1 + .89509)$$

$$= 2.3026\ (-.10491)$$

$$= -(2.3026)\ (.10491)$$

$$\begin{array}{rl} \log 2.3026 = & 0 + .36222 \\ \log .10491 = & \underline{-1 + .02082} \\ & -1 + .38304 \end{array}$$

$$\log^{-1} -1 + .38304 = .24157$$

$$\log_e .7854 = -.24157$$

Since the standard manner of writing a logarithm requires that the mantissa be positive, the use of the artifice of adding -1 and $+1$ is again indicated:

$$\begin{array}{rl} -1 & +\ 1. \\ \underline{0} & \underline{-\ .24157} \\ -1 & +\ .75843 \end{array}$$

$\log_e .7854 = -1 + .78543$ *Answer*

In finding the antilogarithm of a Napierian logarithm, the suggested procedure is to convert the given Napierian logarithm into a common logarithm by means of Equation (11).

Once the common logarithm of the number is known, the number can be determined from the common log table by the usual procedure for looking up an antilogarithm.

When $\log_e N$ is positive, the solution is quite straightforward, merely involving a multiplication by .4343 by the use of arithmetic or logarithms, and finding the antilogarithm of the product in the common log table.

When $\log_e N$ has a negative characteristic, it must be kept in mind that in the standard manner of writing the logarithm the mantissa will be positive. As an illustration, consider the following:

Find the number whose Napierian logarithm is -3 + .045.

$$\log_{10} N = .4343\ (-3 + .045)$$

$$= -1.3029 + .01954$$

$$= -1.28336 \text{ (all negative)}$$

$$= -1 - .28336$$

(Employing familiar artifice for obtaining positive mantissa)

$$\begin{array}{rrr} & -1 & +\ 1. \\ & -1. & -\ \ .28336 \\ \hline = & -2 & +\ \ .71664 \end{array}$$

$$\log_{10} N = -2 + .71664 \text{ (stand. form)}$$

$$N = \log^{-1} -2 + .71664 = .052076$$

In the conversions incident to finding Napierian logarithms or antilogarithms by the use of the common logarithms, the average student will usually experience less difficulty if multiplications are performed by the methods of arithmetic rather than by the use of logarithms. As a means of checking answers, attention is invited to the tables of Napierian logarithms of numbers from .01 to 1,109, of the "Mathematical Tables from the Handbook of Chemistry and Physics". Note that the tables are not repetitive in powers of ten as in the case of common logarithms because the base of the Naperian system is 2.71828... instead of 10. Logarithms of numbers .000 to .999 have both the characteristic and mantissa negative which is the absolute or algebraic value of the logarithm. Log_e .7854 is found in the Natural Logarithm Table to lie between -0.24207 and -0.24080. Converting these to the standard manner of writing the logarithm with a positive mantissa

$$\begin{array}{rl} -1 & +\ 1. \\ 0 & -\ 0.24207 \\ \hline -1 & +\ \ .75793 \end{array} \qquad \begin{array}{rl} -1 & +\ 1. \\ 0 & -\ 0.24080 \\ \hline -1 & +\ \ .75920 \end{array}$$

proves that the calculated value of -1 + .75843 to be reasonable. Logarithms of numbers starting with one and increasing in value are given in the table with both positive characteristic and mantissa. The natural logarithm of 45.1 is 3.80888 which checks with the calculated value of 3.809. The important thing to remember when using this table for the logarithm of a number less than one is that the absolute value is given and that to write this in the standard manner, the above procedure must be followed.

Exercises

Evaluate the following:

86. Log_e 5.34

87. Log_e 3050

88. Log_e 60.2

89. Log_e .6893

90. Log_e .067

91. N, if $\text{Log}_e N = 4.1744$

92. N, if $\text{Log}_e N = -1 + .9060$

93. N, if $\log_e N = 6.6187$

94. N, if $\log_e N = -2 + .2571$

95. x, if $5^x = 22$

96. Given: $M = \dfrac{10^6 \log_e \frac{x}{y}}{60\ N}$

$x = 200$

$y = 125$

$N = 50$

Find M.

COLOGARITHMS.—To divide by any number is equivalent to multiplying by the reciprocal of the number. For example:

$$\frac{2 \times 3 \times 4}{2 \times 7} = 2 \times 3 \times 4 \times \frac{1}{7} \times \frac{1}{2}$$

Therefore, any problem involving several multiplications and divisions can be reduced to one of multiplication alone by using the reciprocal of the divisors.

The logarithm of the reciprocal of a number is called its cologarithm. Adding the cologarithm of a number is equivalent to subtracting its logarithm. Thus, in performing a division by the use of logarithms, the usual process of subtracting the logarithm of the divisor may be changed to one of adding its cologarithm. The advantage of the use of cologarithms is most evident in problems where several multiplications and divisions are to be performed. By using the logarithms of the factors to be multiplied and the cologarithms of the divisors, the entire operation is reduced to one of addition.

Since the cologarithm of a number is equivalent to the logarithm of its reciprocal, the cologarithm may be found by subtracting the logarithm from the log of 1 which is zero. For example, to find the colog of 235. This is equivalent to the log of 1/235:

$$\begin{array}{lr} \log 1 & = 0.00000 \\ \log 235 & = \underline{2.37107} \\ \text{Subtracting} & ?? \end{array}$$

It is at once evident that when subtracting the mantissa of .37107, it will be necessary to carry over one from the characteristic number 0. As this is to be added to the mantissa .00000 to increase its value, it must be subtracted from the characteristic to keep the entire value of the logarithm unchanged:

$$\begin{array}{lll} \log 1 & = -1 + 1.00000 & \\ \log 235 & = \underline{+2 + \ \ .37107} & \\ \text{Subtracting} & \ \ \ -3 + \ \ .62893 & = \text{Colog } 235 \end{array}$$

A careful check of the above process will show the justification for the following rule, by means of which the colog may be written directly from the logarithm.

RULE: Change the sign of the characteristic algebraically adding -1 to it, then mentally, beginning at the left, subtract each figure of the mantissa from 9, except the last one, which is subtracted from 10.

Example: Find the colog of 7,720.

$$\log 7{,}720 = 3.88762$$

Changing sign of characteristic and adding -1:

$$-3 + (-1) = -4$$

$$\begin{array}{l} .9999(10) \\ \underline{.8876\ \ \ 2} \\ .1123\ \ \ 8 \end{array}$$

Colog 7720 = -4 + .11238

Proof: Antilog -4 + .11238 = .0001295 to four significant figures.

$$1/7720 = .000129533^{+}$$

An example of a problem solved by the use of cologs will now be shown:

$$\left(\frac{4895}{4922}\right)^4 = 4 \log 4895 - 4 \log 4922$$

$$\text{or } 4 \ (\log 4895 + \text{colog } 4922)$$

4 log 4895 = 4 (3.68975)
4 colog 4922 = 4 (6.30786 - 10)
or by our method
4 colog 4922 = 4 (-4.30786)

4 log 4895 =	14 +	0.7590
4 colog 4922 =	-16 +	1.23144
Adding	-2 +	1.99044
=	-1 +	.99044

antilog -1 + .99044 ≅ .9782 or by interpolation = .97823.

Note that the subtraction of the log of the denominator from the log of the numerator would be awkward since the first log is larger than the second log.

In many cases solving problems by logarithms will show an answer differing from that obtained by other mathematical methods. A log table showing 4 place mantissas is accurate to three significant figures without interpolation and to four figures with interpolation. Therefore, a four place mantissa will give an accuracy to .1 of 1 per cent or to .001. When logarithms are read from a 10-inch slide rule to an accuracy about one per cent can be expected. In most practical work .1 per cent is all that is necessary and, in general, slide rule accuracy to one per cent is satisfactory.

THE SLIDE RULE.—After studying the theory and application of Logarithms it is logical to consider the application of this theory to the slide rule. It has been shown how the use of logarithms simplifies multiplication and division by reducing these processed to addition and subtraction; how the raising to a power or the extraction of a root is changed to very simple multiplication and division. The slide rule is simply a mechanical device for performing the logarithmic processes.

Multiplication by logarithms is performed by adding the logs of the two or more numbers involved. The scales on a slide rule are logarithmic; that is, the lengths are marked off on the rule in proportion to the *logarithms* of the numbers and not in proportion to the numerical values of the numbers. Thus, when two numbers are multiplied by means of a slide rule the operation is performed by adding the proportionate lengths of two of the scales corresponding to the numbers in question. The total length will be the sum of the individual lengths just as the total log of the product of two numbers will be the sum of the logs of the individual numbers.

In order that the student may become familiar with the use of a slide rule, a student's slide rule is furnished with this volume. With the rule is supplied an explanation of its operation and instructions in its use in performing the various mathematical processes.

In some cases a certain arrangement of the decimal point is very desirable when handling, by means of a slide rule, large numbers and decimals which are ordinarily handled by the use of the powers of ten. For example, to multiply 4,670,000 by .0000342. This problem can be handled by converting 4,670,000 to 467×10^4, .0000342 to 342×10^{-7} and then rearranging to get

$$467 \times 342 \times 10^4 \times 10^{-7}$$

or

$$467 \times 342 \times 10^{-3}$$

Multiply 467 x 342 and then multiply by 10^{-3}. In performing this operation by the slide rule there may be confusion about where to place the decimal point in the slide rule product of 467 x 342. The problem may be simplified at this stage by converting the given numbers to 4.67×10^6 and 3.42×10^{-5}. The problem will then be written,

$$4.67 \times 3.42 \times 10^6 \times 10^{-5}$$

or

$$4.67 \times 3.42 \times 10$$

In multiplying 4.67 x 3.42 it is easily seen by inspection that the product will contain two whole figures. Place the decimal point to the right of the second figure in the slide rule product and then multiply this number by 10 to obtain the answer 15.97 x 10 = 159.7 (here four places can be read on the scale).

The slide rule product, neglecting the decimal point, is 1597. By placing the decimal point, as determined above, to the right of the second figure it becomes 15.97; then multiplying by 10 produces the final answer 159.7. With this procedure the problem of locating the decimal is greatly simplified.

To become proficient in the operation of the slide rule the student should use it at every opportunity. To retain this proficiency the engineer must continue to use the rule as much as possible. With a good rule the accuracy of the work depends almost entirely upon the care with which the settings and readings are made. Long mathematical problems can be quickly solved with a very fair degree of accuracy by means of expert handling of a good slide rule. Careless readings will introduce corresponding errors. The student is urged to use the slide rule for all future problem work. Slide rule accuracy is satisfactory for all exercise and examination problems unless otherwise indicated.

(a) The following exercise problems should be worked using the slide rule and the accuracy checked by working the same problem by logarithms.

97. 245 x 216

98. 360 x 415

99. 176 x 815

100. 762 x 583

101. 1.45 x 24.6

102. 89.5 x 247

103. 482 x 615

104. .0715 x .0025

105. 7.35 x .241

106. .615 x .0000412

107. 150000 x 27400000

108. .00036 x 682

109. 270000 x .00000515

110. 18700 x 4.67

111. .00925 x .0416

(b) Now instead of multiplying, divide the first number in each of the above exercises by the second number.

(c) Extract the square root of the first number in each exercise. (Disregard the second number in each case.)

(d) Square the second number in each exercise. (Disregard the first number in each case.)

LOGARITHMS

ANSWERS TO EXERCISE PROBLEMS

1. 2
2. 4
3. 1
4. 3
5. -2
6. -5.53239
7. 8.46761
8. -.73993
9. -5.53760
10. 11.85449
11. -8 + .30103
12. 13.17609
13. 6.69897
14. 12.33244
15. -4 + .50243
16. 1.39794
17. 3.57066
18. 2.80618
19. 0.90309
20. 8.40824
21. 3.18780
22. 12.79477
23. 2.91751
24. 3.73496
25. 3.97964
26. 3.94880
27. 3.74468
28. .12
29. .0000825
30. .004529
31. .0287
32. 3296×10^{5}
33. 4
34. 7698×10^{3}
35. 1×10^{-6}
36. 3420
37. 561,600
38. 282×10^{6}
39. 10.71
40. .1736
41. 878.2
42. 7136×10^{7}
43. 1.203
44. 4413
45. 3241×10^{5}
46. 14.87
47. 173.8
48. 5.324

ANSWERS TO EXERCISE PROBLEMS, *LOGARITHMS* **, Page 2.**

49. 118200

50. 3804×10^{-9}

51. 1279×10^{3}

52. 12.2

53. 298.8

54. 1344×10^{-8}

55. 435.7

56. 402×10^{3}

57. 4689×10^{12}

58. 3401×10^{4}

59. 11×10^{-19}

60. 8791×10^{12}

61. 4275×10^{-21}

62. 5766×10^{8}

63. 4764×10^{-10}

64. 2035×10^{16}

65. 5828×10^{-12}

66. 28,71

67. 7.565

68. 97.61

69. 7.253

70. 2.417

71. 53830

72. 2.923

73. 62.14

74. 7.615

75. 291.6

76. 8.503

77. .02066

78. 43.5

79. 3.524

80. .1208

81. .06358

82. .03759

83. .3534

84. .8292

85. 3.129

86. 1.675

87. 8.020

88. 4.0977

89. $-1 + .6279$

90. $-3 + .2969$

91. 65

92. .91

93. 749

94. .175

95. 1.92

96. $M = 156.7$

ANSWERS TO EXERCISE PROBLEMS, *LOGARITHMS* , Page 3.

	Multiplication	Division	Sq. Rt. 1st. No.	Sq. 2nd. No.
97.	52920	1.134	15.65	46660
98.	149400	.8675	18.97	172200
99.	143400	.2159	13.27	664200
100.	444200	1.307	27.6	339900
101.	35.67	.05894	1.204	605.2
102.	22100	.3623	9.461	61010
103.	296400	.7837	21.95	378200
104.	1787×10^{-7}	28.6	.2674	625×10^{-8}
105.	1.771	30.5	2.711	.05808
106.	2534×10^{-8}	14920	.7842	1697×10^{-12}
107.	411×10^{10}	547×10^{-5}	387.3	7508×10^{11}
108.	.2455	5278×10^{-10}	.01897	465100
109.	1.391	5243×10^{7}	519.6	265×10^{-13}
110.	87330	4004	136.7	21.81
111.	3848×10^{-7}	.2224	.09618	173×10^{-5}

TELEVISION TECHNICAL ASSIGNMENT

LOGARITHMS

EXAMINATION

Show all work:

1. Amplifier A amplifies (multiplies) its input voltage 576.4 times. Amplifier B amplifier its input voltage 3,968 times. If the input terminals of Amplifier B are connected to the output terminals of Amplifier A, what will be the overall amplification factor (from the input terminals of Amplifier A to the output terminals of Amplifier B)?

log 576.4 = 2.76072
log 3968 = 3.59857
6.35929

Amplification factor = 2287100. ✓

2. The input signal to an amplifier is .0004625 volt. The amplifier amplifies this signal 4,253 times. What is the magnitude of the signal at the output terminals?

log 0.0004625 = $\bar{4}$.66511
log 4,253 = 3.62870
0.29381

Output = 1.967 volts

3. (a) A condenser has a capacity of $5,629 \times 10^{-8}$ farad. An inductance connected to it has an inductance of .0002168 henry. What is the product of these two magnitudes, i.e.—the so-called LC product used in resonance calculations?

$\log 5.629 \times 10^{-5} = \bar{5}.75043$

$\log 2.168 \times 10^{-4} = \bar{4}.33606$

$\bar{8}.08649$

$LC = 1.2204 \times 10^{-8}$ ✓

(b) What is the square root of this product?

$2 \overline{)\bar{8}.08649}$ $\quad \bar{4}.04324$

$\sqrt{LC} = 1.1047 \times 10^{-4}$ ✓

4. $4,831 \div 2,873 = ?$ By interpolation solve to five significant figures.

$\log 4831 = 3.68404$

$\log 2873 = 3.45834$

0.22570

Antilog $= 1.6815$ ✓

EXAMINATION, Page 3.

5. $\frac{.00003564 \times 5843 \times 10^6}{.7603 \times 7819} =$ $\frac{3.564 \times 10^{-5} \times 5.843 \times 10^3 \times 10^6}{7.603 \times 10^{-1} \times 7.819 \times 10^3} = \frac{3.564 \times 5.843 \times 10^4}{7.603 \times 7.819 \times 10^2}$

$= \frac{3.564 \times 5.843 \times 10^2}{7.603 \times 7.819}$ $= 35.031$ ✓

log 3.564 = 0.55194 → 0.76664

log 5.843 =

1.31858

+ log 10^2 = 3.31858

Subtract 1.77413

1.54445

log 7.603 = 0.88098

log 7.819 = 0.89315

1.77413

6. A cube has a volume of .01253 cu. in. What is the length of each side?

log 0.01253 = $\bar{2}$.09795

−1 +1.

−2 .09795

3)−3 1.09795

−1 .36598

length = 0.23225 in. ✓

7. $\sqrt[5]{376.9} =$

log 376.9 = 2.5763

5)2.5763

0.51526

$\sqrt[5]{376.9} = 3.2754$

8. $\left(\dfrac{\sqrt{.0002671}}{(583.2)^3}\right)^2 =$

log 0.0002671 = $\bar{4}$.42667
divide by 2 = $\bar{2}$.21333
Subtract 8.29746
$\overline{11}$.91587

log 583.2 = 2.76582
Multiply by 3 = 8.29746

Multiply by 2 = $\overline{21}$.83174

Antilog = 6.788×10^{-21}

9. $\dfrac{.003714 \times 9764 \times 1438}{3723 \times 0.0865 \times 7946} =$

log 0.003714 = $\bar{3}$.56984
log 9764 = 3.98963
log 1438 = 3.15776
4.71723

log 3723 = 3.57089
log 0.0865 = $\bar{2}$.93702
log 7946 = 3.90015
6.40806

Sub.
4.71723
6.40806
$\bar{2}$.30917

~~Antilog = 0.02559~~
Antilog = 0.020379

EXAMINATION, Page 5.

10. In finding the insulation resistance say, of a cathode ray tube power supply insulator by the leakage method

$$R = \frac{10^6\ T}{C\ \text{Log}_e \left(\frac{E_o}{E}\right)}$$

If T = 100, C = .5, E_o = 220, and E = 100, find R.

$$R = \frac{10^6 \times 10^2}{.5 \times \log_e \left(\frac{220}{100}\right)} = \frac{10^8}{5 \times 10^{-1} \times \log_e 2.20} = \frac{10^9}{5 \log_e 2.20}$$

log 2.20 = 0.34242
log log 2.20 = $\bar{1}$.53456
log 2.3026 = 0.36222
add for log $\log_e$ 2.20 = $\bar{1}$.89678
log 5 = .69897
0.59575

log 10^9 = 9.00000

Sub. 9.00000
.59575
8.40425

= 2.5366×10^8