SPECIALIZED
TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

ARITHMETIC

702E

ARITHMETIC

FOREWORD

Mathematics is the foundation of all engineering, and radio and television engineering is no exception. Radio engineering is one of the most precise of engineering sciences. Practically everything the radio engineer does can be planned and proved by means of mathematics. Do you want to construct an emergency fixed capacitor? You will learn later on that the area of each plate of the capacitor (often called a condenser) is a vital factor in its electrical size. The first part of Problem 5(a), Arithmetic, and Problem 9(a) of Mensuration are concerned with areas. Suppose it it necessary for you to check the height of an antenna, tower, or mast. Problem 8(a), Arithmetic, shows one way the problem may be solved. Such a solution is particularly important to the television engineer because he deals with "line-of-sight" transmission, and the strength of the signal received at a given point depends directly upon the heights of the transmitting and receiving antennas.

These examples, of course, are extremely simple, but they illustrate the point that a knowledge of practical APPLIED mathematics is necessary to the engineer and technician, even in the most fundamental processes of his profession. It is impossible to understand the operation of vacuum tubes and their associated circuits without first acquiring a knowledge of basic mathematics. THE BASIC MATHEMATICS OF RADIO AND TELEVISION IS NOT DIFFICULT BUT IT MUST BE THOROUGHLY UNDERSTOOD.

With a good working knowledge of practical mathematics as applied in radio and television engineering, the student should progress through the CREI home study course without difficulty. For this reason it is important to START your studies with a review of SIMPLE applied arithmetic and algebra.

As you progress in your course you will prepare for further electrical and radio studies by completing additional assignments in mathematics as applied to radio and television engineering. Successful completion of these assignments will enable you to complete the remaining assignments of your course.

Do not make the mistake of thinking that the study of SIMPLE arithmetic or algebra is a waste of time. Work each exercise problem, study every page and every paragraph. All of us are surprised at times to find that we are rusty in the simple applications of mathematics. Above all, be CAREFUL with your mathematical work. Check and recheck your work as well as your answers and solutions. If your method is right but your answer is wrong, you are 100 per cent wrong. The engineer is paid ONLY for correct methods AND correct answers. Be careful, precise. That is a prime requirement of the technician as well as the engineer. START NOW in your study of these assignments to CULTIVATE and increase your habits of carefulness and preciseness.

If you are one of the many students who have had a meager mathematical background or one who 'never liked mathematics', then BE SURE to study your CREI mathematical assignments thoroughly. Remember that if you have any difficulty in understanding the text matter or the problems, your instructor is ready to help you as often and as many times as you let him know about your troubles. DON'T FAIL TO CONSULT HIM WHENEVER YOU NEED HIS HELP.

Many students feel IN ADVANCE that they do not like the study of mathematics and that they are not going to like it. This feeling is usually based entirely on their previous academic school work where they were not taught the PRACTICAL APPLICATIONS of figures. Most students, when they get into applied mathematics, find the application of mathematics to technical problems fascinating. The practical approach to the mathematical work of the technician and engineer gives then a much clearer insight than they had ever believed possible. You, too, should find this to be true because this Institute, over a period of 20 years during which thousands of students have been helped to an understanding of applied mathematics, has constantly borne in mind the needs of the student with least background when revising and adding to the assignment material. The result is that the WEAKEST student can and does acquire a liking for *applied* mathematics. The student who *likes* mathematics, thoroughly enjoys the CREI treatment of this subject.

E. H. Rietzke,

President.

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ARITHMETIC

INTRODUCTION

It is assumed in writing this assignment that the student has previously obtained a satisfactory schooling in arithmetic, but because of long disassociation may require a refresher course in those subjects of greatest interest to the prospective radio engineer. The following material should be considered more in the nature of a review rather than a complete coverage of arithmetic.

In this, the first mathematical assignment of the course, it is desired to emphasize the fact that the study of mathematics is no different from the study of any other academic subject IF the student develops the proper study habits. Although mathematics is usually defined as the science of numbers, it can be more aptly called the science of clear thinking. In any mathematical problem, a concise statement of facts is given, and by means of previously agreed upon rules these facts are coordinated so as to produce a logical solution. For example, a fruit vendor sells ten apples to various customers for a total of thirty cents. What was the selling price per apple? Logic tells us that the total selling price of the apples is the product of the number of apples times the unit price per apple. Similarly, we arrive at the selling price per apple by division, the inverse of multiplication, the unit price being the selling price divided by the number sold. This method leads to a result of three cents per apple. Although the answer is logical it is not necessarily true because the vendor controls the price per apple, and it is possible that the apples were not all sold at the same price.

A second possible solution is that eight apples were sold for twenty cents and two more for ten cents which still yields a total of thirty cents for ten apples. The first solution is the most logical one, but it could be made more so if a further statement was made in the original problem that all apples were sold at the same price. In solving problems many students make errors in the solution because of failure to carefully check the exact statement of the problem. Before attempting the solution of any problem, no matter how simple, study it carefully until all the stated conditions are understood, otherwise, a modifying condition may be overlooked.

The most complicated mathematical problem is solved by means of previously agreed upon rules. For example, in arithmetic we agree on the multiplication table,

$$2 \times 2 = 4$$
$$3 \times 3 = 9$$

etc. However, in some other system of numbers (and such systems have been devised) this table may not hold, in fact, in some new system 3 × 3 might equal 4, and 2 × 2 might equal 9. Our system of numbers would then lead to a solution that apparently was in complete disagreement with some other system.

However, as long as everyone uses the same rules no confusion can result.

The rules by which problems are solved are the tools of the mathematician and to successfully master the subject the student must learn the rules as well as how to use them. Anyone can learn to recognize a hammer, saw, chisel, etc., but that does not necessarily make him a carpenter. A carpenter not only knows his tools, but HOW and WHEN to use them. He learned HOW and WHEN not by reading books on carpentry but in actual practice. Similarly, the mathematician must not only learn the rules but must also, by actual practice on exercise problems, learn when and how to use them. Any normal person can, in a short time, memorize all the mathematical rules in a textbook, but his knowledge of the subject would be incomplete unless he learns how the rules are used in a practical way.

In this and subsequent mathematical technical assignments, a considerable number of exercise problems are included to provide the necessary practice material. The study of an assignment should not be considered complete until the student has actually worked each problem and obtained an answer in close agreement with one listed on the answer sheet in the back of the assignment. Answers may vary slightly depending on the number of places to which they are carried out but extreme disagreement indicates an error in your work. If your answer does not agree with that listed, request your instructor to point out the error. It is advisable that you show all your work on the troublesome problem so the instructor can determine the particular point causing the difficulty, and can then emphasize that point in his explanation.

Only the true mathematician is interested in covering the entire field of mathematics. The radio engineer is primarily interested in those subjects of greatest use in engineering calculations. This and subsequent assignments have been written with the idea of covering the subjects of greatest importance to the student of radio. It is important to realize that every mathematical idea presented will be used somewhere later in the course, and that trouble with future assignments can be avoided only by mastering each mathematical assignment completely. When the slightest doubt exists regarding a part of the assignment, request clarification from your instructor.

The order in which subjects are presented in these assignments is based on the experience of many years, and the student is urged to follow the assignments in numerical order if the greatest benefit is to be otained.

Just as a watch is made up of individual parts none of which are particularly complicated, so is advanced radio engineering nothing more than the application of fundamental principles. The more time spent on the earlier assignments, the easier will be the more advanced ones. Master the basic principles, and as your education progresses you will find more and more complicated ways in which they can be utilized. The full-fledged radio engineer uses the same tools of the trade, the basic engineering principles, as does the embryo engineer. The only difference is that he knows more ways in which they can be used.

HOW TO STUDY MATHEMATICS

In the study of mathematics there is only one routine for success: *Memorize the rule and then learn how and when to apply it.* Throughout the course the most important rules are italicized and hence easily recognized, but the student should be on the alert to locate the minor rules that do not necessitate such strong emphasis. The minor rules are very often missed on the first study of the text, and yet they can lead to complete failure in working many of the exercise problems.

The exercise problems are selected to give the necessary practice in applying the rules. Do not work a problem with the sole idea of obtaining the correct answer. In practical work the answer is of greatest importance, but in learning a subject the HOW and WHY of a problem are of primary interest. If you know the correct method of solving a typical problem and can retain the ability to apply that method in future work, the correct answer will follow as a matter of course.

Under all circumstances avoid short cuts in solving problems. Such methods are for the experts and are most certainly a handicap in learning the applications of certain mathematical rules. Work each problem step by step, and outline the reason for each calculation. It is suggested that the reason for each step taken be written out clearly before the operation is performed. As your ability increases, the rule can be stated mentally, eliminating the necessity for writing it. Writing out the rules will tend to impress them more firmly in your memory so they can be stated clearly when future work requires their application. This method will also aid in developing your ability to analyze a problem correctly.

After the work on a problem has been completed, do not consider the solution as satisfactory until all work has been very carefully checked. This final check will often pick up minor errors that are just great enough to spoil a perfect mark on the solution. Each exercise problem illustrates a rule or method or combination of both, and the solution is not complete unless the student definitely understands the points illustrated.

As the student advances, pure mathematical problems will be replaced by more practical ones in electrical and radio engineering. Make it a point to observe the values used in these practical problems in order to develop an instinct for practical values. Quite often the radio engineer is required to solve problems based on logical assumptions, and a feeling for correct values is very helpful in making the correct assumption on the first trial.

In many cases there are several methods by which a problem may be solved. The best one is that which is most simple and direct. Careful analysis of the problem is necessary to determine the most direct method of solution.

In this assignment it will be noted that certain ideas of numbers are presented that are not usually covered in the study of arithmetic. The idea is to precondition the student for the study of algebra which follows in a later assignment. Understanding the methods by which numbers can be arranged will be of great help when literal quantities are substituted for numerical values.

If the student will conscientiously follow the suggestions outlined here the study of mathematics will be found no more difficult than any other academic subject. Your instructor is definitely interested in your progress, so request his assistance as often as necessary.

MULTIPLICATION

In multiplication two numbers are used as factors to produce a third number called the product. The following simple problem indicates the terminology used in multiplication.

2	×	4
Multiplier	**Times**	**Multiplicand**
=		8
Is Equal To		**Product**

The problem could also be written

$$4 \times 2 = 8$$

which indicates the logic of the rule that the order in which numbers are multiplied does not change the numerical value of the product.

If a certain number is the product of two numbers, then the two numbers are said to be the factors of the product. Thus, 2 and 4 are factors of the number 8 because $2 \times 4 = 8$.

A multiple of any number is the product obtained by multiplying that number by some other number:

$$3 \times 8 = 24$$

24 is a multiple of 8 and 3. Factors and multiples will be found of great importance in the study of algebra, so the student should memorize the exact meaning of each word.

The process of multiplication is usually indicated in arithmetical work by the sign "×" between the numbers, thus: $2 \times 6 = 12$ is read, "Two times six is equal to twelve." Another method of indicating multiplication is to place a dot slightly above the writing level and midway between the numbers to be multiplied, thus: $2 \cdot 7 \cdot 3 = 42$, which is read, "2 times 7 times 3 is equal to 42." The use of the elevated dot to indicate multiplication is preferable to the multiplication sign "×" to avoid confusion later in the study of algebra where the letter x is commonly used to represent an unknown quantity.

Another method of indicating multiplication is by means of parentheses, brackets, and braces. The following all mean the same: 2×4, $2 \cdot 4$, $2(4)$, $2[4]$, and $2\{4\}$.

For multiplication of terms consisting of one figure each, the multiplication table must be memorized. Multiplication of terms consisting of two or more figures consists of a series of multiplications followed by addition of all the partial products. For example,

MULTIPLY 3,546 BY 231:

3546	Multiplicand.
231	Multiplier.
3546	Multiply by 1 to obtain first partial product.
10638$\cancel{0}$	Multiply by 30 to obtain second partial product.
7092$\cancel{0}\cancel{0}$	Multiply by 200 to obtain third partial product.
819126	Adding partial products.

In actually multiplying, it is customary to omit writing the ciphers (crossed out above in the second and third partial products), but when multiplying very large numbers it is sometimes helpful to include them so as to keep all figure columns of the partial products in proper vertical alignment. This reduces the possibility of errors in adding the partial products.

A fundamental idea that will be of value later in the study of algebra can be developed from the above multiplication example. The multiplicand 3,546 is equal to 3,000 + 500 + 40 + 6. Things equal to the same thing may be substituted for each other, so 231 × 3,546 = 231(3,000 + 500 + 40 + 6). Note particularly that every term inside the parentheses must be multiplied by 231, after which the partial products are added.

231 • 3000 = 693,000
231 • 500 = 115,500
231 • 40 = 9,240
231 • 6 = 1,386
819,126

Exercises

1. 746 × 289 = ?

2. 347 × 19 × 26 = ?

3. 274 × 346 × 14 = ?

4. 27 × 198 × 200 = ?

5. 13 × 17 × 14 × 6 = ?

DIVISION

The inverse of multiplication is called division. If the product of any two numbers is given and any one of the numbers is known, the unknown factor is found by dividing the product by the known factor. However, in division different names are applied to the numbers and the result obtained. The example below shows the terms introduced in division.

50	÷	10
Dividend	**Divided By**	**Divisor**
	=	5
	Is Equal To	**Quotient**

In arithmetic when the expression "a divisor of a number" is used, it commonly means a number which will divide into the given number a certain whole number of times. Quite often, however, the divisor will not divide into the dividend an *integral* (whole) number of times. The quotient will then consist of a whole number plus a term called the *remainder*. When it is necessary to retain the remainder, it is usually written as a fraction following the quotient, the numerator of the fraction being the actual remainder from the process of division and the denominator the divisor. For example, 9 divided by 2 is equal to 4 with a remainder of 1. The quotient may then be written 4 + 1/2 or $4\,{}^1/_2$.

Arithmetical division is usually indicated by the division sign "÷", but this symbol is not much used in more advanced mathematics, the slant or bar being preferable. Thus, 6 ÷ 2 can also be written

$$6/2 \text{ or } \frac{6}{2}$$

When a number is to be divided by two or more divisors it is best to perform only one division by using the product of the given divisors as a complete divisor. For example, to divide 64 by 8 and 2, the problem can be written

$$\frac{64}{2 \times 8} = \frac{64}{16} = 4$$

In multiplication the multiplicand and multiplier can be interchanged without affecting the product, but such a reversal in division between the dividend and divisor is not permissible. Thus, 4/2 = 2 but 2/4 = 1/2.

When the divisor consists of only one figure, a process called *short division* is used. The method consists of a series of individual divisions.

DIVIDE 1,583 BY 3:

$$\begin{array}{r} 3\overline{)\underline{1583}} \\ 527\frac{2}{3} \end{array}$$

This problem can be written as

$$\frac{1500}{3} + \frac{80}{3} + \frac{3}{3} = 500 + 26\frac{2}{3} + 1$$
$$= 527\tfrac{2}{3}$$

or more simply as

$$\frac{1500 + 80 + 3}{3} = 527\tfrac{2}{3}$$

Note particularly in this last form that cancellation cannot be used because each number in the dividend must be individually divided by 3 and then all the partial quotients added. In other words, cancellation cannot be used if a sum or difference is indicated in the numerator. (See page 9.)

When the divisor consists of two or more figures a long division process is used.

DIVIDE 423,617 BY 126:

$$\begin{array}{r} 3362 \\ 126\overline{)423617} \\ \underline{378} \\ 456 \\ \underline{378} \\ 781 \\ \underline{756} \\ 257 \\ \underline{252} \\ 5\rlap{\ \textit{remainder}} \end{array}$$

The first step of the division is to divide 126 into the smallest possible part of 423,617 or 423. 423 must now be divided by 126 to obtain a trial divisor. Try 4 of the number 423 and 1 of 126. Thus, 4 divided by 1 equals

4 as a trail divisor. But 4 × 126 = 504 which is greater than 423. Hence 4 is too large. Therefore try 3. Since 3 × 126 = 378 is less than 423, 3 is the correct first digit of the quotient.

The second step is to subtract 378 from 423 to obtain 45. This is not divisible by 126 so we bring down the next digit of the dividend 6, to get 456. We now find the next trial divisor in exactly the same manner as before. Thus, 456 is divisible by 126 three times, and a remainder of 78 is obtained by subtracting 378 from 456.

Repeat this process to as many places in the answer as desired. Zeros can be added to 423,617.000 as needed. Exact answers cannot be obtained in many cases and since our divisor is 126 and only has three figures, the answer 3,360, which also has three significant figures (digits to the left of the zero) would generally be considered sufficiently accurate. Accuracy of the answers is always limited by the factors used in the problem. If we used 126.325, and the dividend also had six significant figures, such as is the case here, then our answer could be carried to six significant figures. The answer (quotient) is in general limited to the same number of significant figures as the divisor or dividend, depending upon which of these has the lesser number of significant figures. In this problem the divisor 126 is the limiting factor.

The quotient of two numbers will not be affected if both the dividend and divisor are divided by the same number. For example, 2,560/64 is readily simplified by dividing both the dividend and divisor by 8, which step can be performed mentally to yield 320/8, and with another mental step the division can be completed, giving a quotient of 40. With a little practice this method can often be used to simplify division and often yields an answer much more quickly than by the operation of long division. The same method is used to simplify fractions as will be shown later.

Exercises

6. 376/14 = ?

7. 17984 ÷ 208 = ?

8. $\dfrac{3 \times 77 \times \overset{49}{98}}{2 \times 18 \times 9_{5}} = ?$

9. $\dfrac{\overset{4}{44} \times \overset{9}{81} \times \overset{6\;2}{72} \times 96}{9 \times \underset{3}{27} \times 11 \times \underset{12}{48}} = ?$ 1/2

RECIPROCALS

A reciprocal of a number is defined as 1 divided by the number. The reciprocal of 7 is 1/7, of 10 is 1/10, etc. In mathematics the number 1 is taken as unity, any number greater than 1 representing so many units; that is, the number 8 actually stands for 8 × 1 or 8/1 units. Thus, the reciprocal of any number is obtained by inverting the number.

It has been stated earlier that division is the inverse of multiplication. Any division problem can be changed to one of multiplication by multiplying the dividend by the reciprocal of the divisor. Thus, 846/27 = 846 ×

1/27. 27 is a multiple of 3 and 9, so 1/27 is equal to 1/3 × 1/9. The problem can then be written (846 × 1/3) × 1/9 or 282 × 1/9 = $31^{1}/_{3}$. Although this method is of some assistance in simplifying division problems, its greatest value will be realized in the simplification of algebraic equations.

FRACTIONS

A fraction is an indicated division. It consists of two elements, and it is customary to call the *dividend* or quantity above the line the *numerator*, and the *divisor* or quantity below the line the *denominator*. For example,

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Dividend}}{\text{Divisor}} = \frac{1}{4}$$

$$= 1 \div 4 = \text{one-fourth}$$

The denominator expresses the number of parts into which the whole is to be divided, while the numerator indicates the number of parts of the whole to be used. In the above example the whole is divided into four parts, and one part or one-quarter is to be used.

It often becomes necessary to handle fractions in the same manner as whole numbers, that is, to add, subtract, multiply, and divide them. A few simple rules are required for each process.

MULTIPLICATION OF FRACTIONS.— To multiply two or more fractions take the product of the numerators as the common numerator and the product of the denominators as the common denominator.

Examples:

1. $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$

2. $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$

3. $\frac{3}{8} \times \frac{5}{6} \times \frac{2}{3} = \frac{3 \times 5 \times 2}{8 \times 6 \times 3} = \frac{30}{144}$

   Reduce (÷2) $= \frac{15}{72}$

   Reduce (÷3) $= \frac{5}{24}$

4. $\frac{7}{8} \times \frac{3}{5} \times \frac{5}{7} = \frac{7 \times 3 \times 5}{8 \times 5 \times 7} = \frac{105}{280}$

   Reduce (÷5) $= \frac{21}{56}$

   Reduce (÷7) $= \frac{3}{8}$

In examples 3 and 4 the solutions may be simplified by means of cancellation, which is based on the rule that the numerator and denominator (dividend and divisor) may be divided by the same number without affecting the quotient. In the fourth example above, cancellation of the sevens and fives in the numerator and denominator yields a direct answer. Thus,

$$\frac{7}{8} \times \frac{3}{5} \times \frac{5}{7} = \frac{\cancel{7} \times \cancel{5} \times 3}{8 \times \cancel{5} \times \cancel{7}} = \frac{3}{8}$$

In the problem 3/8 × 2/3 cancellation can also be used.

$$\frac{3}{8} \times \frac{2}{3} = \frac{\cancel{3} \times 2}{8 \times \cancel{3}}$$

Canceling the threes gives

$$\frac{1 \times 2}{8 \times 1}$$

The 2 cancels (divides) into 8 four times so

$$\frac{1 \times \cancel{2}}{\underset{4}{\cancel{8}} \times 1} = \frac{1 \times 1}{4 \times 1} = \frac{1}{4}$$

It is important to remember in cancellation that any number, except zero, divided by itself yields a quotient of 1. At this point it is desirable to discuss the mathematical rules regarding the number zero.

Any number subtracted from itself yields a remainder of zero. Thus,

$$6 - 6 = 0$$

The product of any number and zero yields a product of zero. Thus,

$$6 \times 0 = 0 \text{ or } 0 \times 7 = 0$$

Adding zero to any number does not change the value of the number. Thus,

$$6 + 0 = 6$$

DIVISION BY ZERO IS EXCLUDED FROM ALL MATHEMATICAL OPERATIONS.

The statement is often made that dividing a number by zero gives a quotient of infinity, but infinity is a number so large it cannot be written. The correct statement is "*that as the divisor of a number approaches zero, the quotient approaches infinity.*"

DIVISION OF FRACTIONS.—In dividing fractions advantage is taken of the rule that any problem in division can be changed to one of multiplication by using the reciprocal of the divisor. The reciprocal of a fraction is the fraction inverted. Thus, the reciprocal of 3/4 is 4/3, of 1/2 is 2/1, etc.

To divide fractions or by a fraction invert the divisor and multiply.

Examples:

1. $\dfrac{1}{\frac{1}{2}} = 1 \times \dfrac{2}{1} = 2$

2. $\dfrac{3}{\frac{2}{5}} = 3 \times \dfrac{5}{2} = \dfrac{15}{2} = 7^{1}/_{2}$

3. $\dfrac{\frac{5}{4}}{\frac{3}{7}} = \dfrac{5}{4} \times \dfrac{7}{3} = \dfrac{35}{12} = 2^{11}/_{12}$

4. $\dfrac{\frac{3}{4}}{2} = \dfrac{3}{4} \times \dfrac{1}{2} = \dfrac{3}{8}$

Exercises

10. $\dfrac{2}{3} \times \dfrac{4}{5} = ?$

11. $7/8 \times 5/17 = ?$

12. $\dfrac{6}{11} \times \dfrac{2}{3} \times \dfrac{3}{5} = ?$

13. $\dfrac{15}{16} \times \dfrac{8}{3} \times \dfrac{4}{7} = ?$

14. $1/15 \times 1/12 \times 1/7 = ?$

15. $1\dfrac{3}{16} \times 1\dfrac{2}{3} \times \dfrac{7}{8} = ?$

16. $3/5 \div 5/6 = ?$

17. $7/8 \div 4/5 = ?$

18. $8/15 \div 3/4 = ?$

19. $(5/7)\,(2/3) \div 3/4 = ?$

20. $$\frac{\frac{16}{25} \times \frac{3}{2} \times \frac{8}{9}}{3\ 1/4} = ?$$

21. $$\frac{\frac{5}{4} \times \frac{2}{3}}{\frac{1}{9} \times \frac{1}{7}} = ?$$

ADDITION AND SUBTRACTION OF FRACTIONS.—The addition and subtraction of fractions is more complicated than multiplication and division. The rules to be followed in adding and subtracting fractions are:

1. Find the LCM (Least Common Multiple) of the denominators.
2. Find the equivalent fraction of each given fraction using the LCD (Least Common Denominator) as the common denominator.
3. Perform the indicated operation.
4. Reduce the final value when practical.

The lowest common multiple, abbreviated LCM, of two or more numbers, or denominators, is the smallest quantity into which each number or denominator will go a whole number of times. A common multiple of the two denominators may be found by multiplying the two denominators. If 2 and 5 are the denominators of the two fractions, the common multiple of 2 and 5 is 10 since 2 × 5 = 10. 2 and 5 are *factors* of 10. Factors of a quantity are the numbers which when multiplied together will equal the given quantity. In this case 10 is the LCM of 2 and 5. Sometimes the product of the two numbers is not the LCM. For example, with denominators of 2 and 6, 12 is the product of 2 and 6, but the LCM of 2 and 6 is 6, since 2 and 6 will divide into 6, 3 and 1 times respectively. This agrees with the definition of LCM. The LCM of denominators is called Lowest Common Denominator, abbreviated LCD.

To find the LCD of several denominators, write all the denominators in a horizontal line. Using short division, divide each denominator by any *prime number* that will divide into two or more of the given denominators. (A prime number is a quantity divisible only by itself or one, such as 2, 3, 5, 7, 11, etc.) Write the quotients and undivided numbers below the dividends, omitting unity. Using these new quotients and undivided numbers as new dividends, continue the process until there is no common prime factor in the quotients. The product of the prime numbers and the remaining undivided quotients is the LCD of the denominators.

Illustrative Examples:

1. Find the LCM of 14, 21, 36, 96.

$$\begin{array}{rrrrr}
7\,) & 14 & 21 & 36 & 96 \\
2\,) & 2 & 3 & 36 & 96 \\
 & & 3\,) & 18 & 48 \\
 & & 2\,) & 6 & 16 \\
 & & & 3 & 8
\end{array}$$

LCM = 7 × 2 × 3 × 2 × 3 × 8 = 2,016

2. Find the LCD of $\frac{9}{14}, \frac{4}{21}, \frac{5}{36}, \frac{7}{96}$

The LCD = 7 × 2 × 3 × 2 × 3 × 8
= 2,016

Same as Example 1 because the LCM of several numbers is the same as the LCD when the numbers are denominators of fractions.

3. Find the LCD of $\frac{3}{4}$, $\frac{5}{2}$, $\frac{7}{4}$.

Upon inspection, it can be seen that 4 is the LCD as each denominator of 4, 2 and 4 will divide into 4 a whole number of times.

4. Find the LCD of 2/3, 1, 2, $\frac{1}{6}$.

A whole number is a fraction with a denominator of unity (one), therefore 1 and 2 may be written as

$$\frac{1}{1} \text{ and } \frac{2}{1}$$

The denominators are now 3, 1, 1, and 6. Since 6 is the largest denominator and 3 and 1 will divide into it a whole number of times, the LCD is 6.

The equivalent fraction, with the LCD as the denominator, of a given fraction is obtained by multiplying the LCD by the given fraction and dividing the result by the LCD. This is the same as multiplying the fraction by one which does not change the value of the fraction.

Illustrative Examples:

5. Find the equivalent fraction of the following fractions:

$$\frac{9}{14} \qquad \frac{4}{21} \qquad \frac{5}{36} \qquad \frac{7}{96}$$

using LCD of the denominators.

The LCD of the denominators of the above fractions was found in Example 2, therefore it is only necessary to multiply the LCD by the fraction and divide the result by the LCD. Taking the first fraction,

$$\frac{\frac{9}{14} \times \text{LCD}}{\text{LCD}} =$$

$$\frac{\frac{9}{\cancel{14}} \times \cancel{7} \times \cancel{2} \times 3 \times 2 \times 3 \times 8}{2016} =$$

Since 7 × 2 = 14, 7 × 2 may be cancelled in the LCD and 14 in the denominator of the fraction.

$$\frac{9 \times 3 \times 2 \times 3 \times 8}{2016} = \frac{1296}{2016}$$

Although 1,296/2,016 may be reduced, this is not done because the problem is to find the equivalent fraction with the LCD as the denominator.

Now taking the second fraction,

$$\frac{\frac{4}{\cancel{21}} \times \cancel{7} \times 2 \times \cancel{3} \times 2 \times 3 \times 8}{2016} =$$

7 × 3 = 21, therefore, 7 and 3 may be cancelled in the LCD and 21 in the denominator of the fraction.

$$\frac{4 \times 2 \times 2 \times 3 \times 8}{2016} = \frac{384}{2016}$$

Now taking the third fraction,

$$\frac{\frac{5}{\cancel{36}} \times 7 \times \cancel{2} \times \cancel{3} \times \cancel{2} \times \cancel{3} \times 8}{2016} =$$

$$\frac{5 \times 7 \times 8}{2016} = \frac{280}{2016}$$

Now the fourth fraction,

$$\frac{\frac{7}{\cancel{96}} \times 7 \times \cancel{2} \times 3 \times \cancel{2} \times \cancel{3} \times \cancel{8}}{2016} =$$

$$\frac{7 \times 7 \times 3}{2016} = \frac{147}{2016}$$

Therefore the equivalent fractions are:

$$\frac{9}{14} = \frac{1296}{2016} \qquad \frac{4}{21} = \frac{384}{2016}$$

$$\frac{5}{36} = \frac{280}{2016} \qquad \frac{7}{96} = \frac{147}{2016}$$

6. Find the equivalent fractions of

$$\frac{3}{4} \qquad \frac{5}{2} \qquad \frac{7}{4}$$

using the LCD found in Illustrative Example 3. LCD = 4, from Illustrative Example 3.

$$\frac{\frac{3}{\cancel{4}} \times \cancel{4}}{4} = \frac{3}{4}$$

$$\frac{\frac{5}{\cancel{2}} \times \overset{2}{\cancel{4}}}{4} = \frac{5 \times 2}{4} = \frac{10}{4}$$

$$\frac{\frac{7}{\cancel{4}} \times \cancel{4}}{4} = \frac{7}{4}$$

7. Find the equivalent fractions of

$$\frac{2}{3} \qquad 1 \qquad 2 \qquad \frac{1}{6}$$

LCD = 6

$$\frac{\frac{2}{\cancel{3}} \times \overset{2}{\cancel{6}}}{6} = \frac{4}{6}$$

$$\frac{1 \times 6}{6} = \frac{6}{6} \quad \text{or} \quad \frac{\frac{1}{1} \times 6}{6} = \frac{6}{6}$$

$$\frac{2 \times 6}{6} = \frac{12}{6}$$

$$\frac{\frac{1}{\cancel{6}} \times \cancel{6}}{6} = \frac{1}{6}$$

Observe that in Example 6 and 7, when a fraction has a denominator that is the same as the LCD, the equivalent fraction is the given fraction.

The reason for the first two rules is that fractions whose denominators are not alike *cannot* be added or subtracted until the denominators are like quantities. When adding fractions with the same denominators, add the numerators and keep the common denominator. This is equivalent to adding 5 inches and 6 inches for a total of 11 inches. The 5 and 6 are the same as the numerator of the fraction and the word inches is the same as the common denominator or LCD of the fractions. If the above had been 5 feet and 6 yards, to be added together to find the total feet, then the 6 yards must be converted to feet. This is equivalent to finding the LCM or LCD. 1 yard is equal to 3 feet, therefore 6 yards is equal to 18 feet. This is equalvent to the second rule. The sum of 5 and 18 is 23, therefore 5 feet + 6 yards = 5 feet + 18 feet = 23 feet.

The yards were converted to feet, the quantities added and the unit value of feet was kept. The question was to find the total feet, therefore the answer of 23 feet should not be reduced.

To reduce a fraction, the numerator and denominator are divided by the same quantity. If the numerator is larger than the denominator, the denominator is divided into the numerator without adding a decimal and the remainder is placed over the denominator.

Illustrative Examples:

8. $\frac{3}{4} + \frac{2}{3} + \frac{1}{6} = ?$

By inspection 12 is the LCD. It may also be found by following the method in the text.

$$\begin{array}{r|lll} 3 & 4 & 3 & 6 \\ \hline 2 & 4 & & 2 \\ \hline & 2 & & \end{array}$$

LCD = 3 × 2 × 2 = 12.

$$\frac{\frac{3}{\cancel{4}} \times 3 \times \cancel{2} \times \cancel{2}}{12} + \frac{\frac{2}{\cancel{3}} \times \cancel{3} \times 2 \times 2}{12}$$

$$+ \frac{\frac{1}{\cancel{6}} \times \cancel{3} \times \cancel{2} \times 2}{12} =$$

$$\frac{3 \times 3 + 2 \times 2 \times 2 + 1 \times 2}{12} =$$

$$\frac{9 + 8 + 2}{12} = \frac{19}{12} = 1\frac{7}{12}$$

9. $\frac{9}{14} + \frac{4}{21} + \frac{5}{36} + \frac{7}{96} = ?$

From Example 5, it was found that

$$\frac{9}{14} = \frac{1296}{2016} \qquad \frac{4}{21} = \frac{384}{2016}$$

$$\frac{5}{36} = \frac{280}{2016} \qquad \frac{7}{96} = \frac{147}{2016}$$

$$\frac{1296}{2016} + \frac{384}{2016} + \frac{280}{2016} + \frac{147}{2016} =$$

$$\frac{1296 + 384 + 280 + 147}{2016} = \frac{2107}{2016}$$

$$= 1\frac{91}{2016}$$

10. $\frac{7}{4} - \frac{3}{4} + \frac{1}{2} = ?$

LCD = 4 from inspection. Since the LCD is 4, then from Examples 6 and 7, the equivalent fraction with the LCD as the denominator is the given fraction, therefore it is only necessary to find the equivalent fraction of 1/2 using 4 as the LCD.

$$\frac{\frac{1}{\underset{1}{\cancel{2}}} \times \overset{2}{\cancel{4}}}{4} = \frac{1 \times 2}{4} = \frac{2}{4}$$

$$\frac{7}{4} - \frac{3}{4} + \frac{2}{4} = \frac{7 + 2 - 3}{4} =$$

$$\frac{9 - 3}{4} = \frac{6}{4} = 1^2/_4 = 1^1/_2$$

11. $\frac{2}{5} + 1 - \frac{2}{10} + \frac{1}{10} = ?$

LCM = 10 by inspection.

$$\frac{\frac{2}{\cancel{5}} \times \overset{2}{\cancel{10}}}{10} = \frac{2 \times 2}{10} = \frac{4}{10}$$

$$\frac{\frac{1}{1} \times 10}{10} = \frac{10}{10}$$

$$\frac{2}{5} + 1 - \frac{2}{10} + \frac{1}{10} =$$

$$\frac{4}{10} + \frac{10}{10} - \frac{2}{10} + \frac{1}{10} =$$

$$\frac{4 + 10 - 2 + 1}{10} = \frac{15 - 2}{10} =$$

$$\frac{13}{10} = 1\frac{3}{10}$$

Exercises

22. $1/3 + 1/5 = ?$

23. $15/124 + 9/27 = ?$

24. $\frac{24}{7} + \frac{6}{15} - \frac{5}{8} = ?$

25. $5/7 - 2/3 = ?$

26. $\frac{5}{6} + \frac{2}{7} + \frac{1}{5} = ?$

27. $1/2 - 3/7 + 2/3 - 1/6 = ?$

28. $\frac{18 + 1/3 - 1/9}{1/4 + 7/8 - 2/3} = ?$

29. $\frac{(3 \times 2\ 1/3 + 7/8) - 1/6}{2\ 1/2 - 1\ 2/3} =$

30. $\frac{\frac{2}{3} \times 4\frac{4}{7}}{\frac{4}{5} + \frac{11}{12}} = ?$

Find LCM of:

31. 2, 17, 22, 23, and 26.

32. 4, 7, 9, and 13.

33. 8, 12, 81, and 90.

34. 11, 88, 330, and 440.

35. 16, 17, 21, and 42.

DECIMALS

The logical step after the study of fractions is the study of decimals. A decimal expresses a fractional amount in terms of a tenth part of the unit or in tenth subdivisions such as hundredths, thousandths, ten-thousandths, etc. The "decimal point" separates a whole number and a decimal, the whole number being written to the left of the decimal point, the decimal or fractional amount to the right. Several examples of fractions expressed as decimals are shown below.

FRACTION	WRITTEN AS A DECIMAL	READ AS
$\frac{1}{10}$	.1	One-tenth.
$\frac{3}{100}$	.03	Three-hundredths.
$\frac{46}{100}$	.46	Forty-six hundreths.
$\frac{233}{10,000}$	.0233	Two hundred thirty-three ten-thousandths.

A decimal number of one figure (to right of decimal point) expresses a fraction in terms of tenths of the unit; two figures, in hundredths of the unit; three figures, in thousandths of the unit; etc. A number may consist of a whole number and a decimal such as 1.4 (read "one point four or one and four tenths"). 65.217, read "sixty-five point two one seven" is equal to sixty-five and two hundred seventeen thousandths, $65^{217}/_{1,000}$, etc.

To express a very small fraction as a decimal, for example, 4/10,000, it is written .0004, in accordance with the preceding paragraph. A decimal expressing a fraction in terms of ten thousandths must have four figures following the decimal point. The four figures are obtained by writing the numerator of the fraction (4) to the right and preceding it by the correct number of ciphers; in this case 3 ciphers. Examples of fractions expressed as decimals are given below:

37/1,000 = .037

653/100,000,000 = .00000653

16/10,000 = .0016

618/1,000 = .618

8/10 = .8

Affixing ciphers to the right of a decimal does not change its value. .2, .20, .200, .200,000, all have the same numerical value.

Inserting a cipher between the decimal point and the first figure of the decimal divides the decimal by 10. Inserting two ciphers divides by 100, etc.

Examples:

.5 = 5/10

.05 = 5/100

.005 = 5/1,000

Removing a cipher from the left of a decimal multiplies the decimal by 10. The same multiplication is accomplished by moving the decimal point one place to the right. For each figure the decimal point is moved to the right, the number is multiplied by 10.

Examples:

.0006 = 6/10,000

0.006 = 6/1,000

00.06 = 6/100

To add decimals, place the numbers to be added so that the decimal points will be directly under one another and then proceed with the addition.

Examples:

1.
$$\begin{array}{r} .42 \\ .031 \\ .652 \\ \hline 1.103 \end{array}$$

2.
$$\begin{array}{r} 1.71 \\ 2.032 \\ .517 \\ .0002 \\ \hline 4.2592 \end{array}$$

3.
$$\begin{array}{r} 176.32 \\ 2.155 \\ .002 \\ 1.71 \\ 2.003 \\ \hline 182.190 \end{array} \text{ or } 182.19$$

Similar procedure is followed in the subtraction of one decimal number from another.

Examples:

1. 2.015 - 1.76

$$\begin{array}{r} 2.015 \\ -1.76 \\ \hline .255 \end{array}$$

2. .7615 - .0012

$$\begin{array}{r} .7615 \\ -.0012 \\ \hline .7603 \end{array}$$

3. .00056 - .0002

$$\begin{array}{r} .00056 \\ -.0002 \\ \hline .00036 \end{array}$$

Exercises

36. .24 + .3 + .125 = ?

37. .827 + 2.56 + .153 + .002 = ?

38. 3.56 + 2.002 + 2.7194 = ?

39. .856 - .234 = ?

40. .125 - .013 = ?

41. 1 - .168 = ?

Multiplication of Decimals.—In the multiplication of decimals the multiplier is placed beneath the multiplicand disregarding the position of the decimal points. Multiply as with whole numbers. *In the product, point off as many decimal places as there are decimal places in both multiplier and multiplicand, beginning at the right and prefixing ciphers if necessary.*

Examples:

$$\begin{array}{rl} 3.24 & \textit{Multiplicand} \\ 1.3 & \textit{Multiplier} \\ \hline 972 & \\ 324 & \\ \hline 4.212 & \textit{Product} \end{array}$$

$$\begin{array}{r} .651 \\ .03 \\ \hline .01953 \end{array}$$

$$\begin{array}{r} 2.415 \\ .126 \\ \hline 14490 \\ 4830 \\ 2415 \\ \hline .304290 \end{array}$$

Division of Decimals.—Place the divisor to the left of the dividend as in the division of whole numbers. Place the decimal point for the quotient *directly* above the decimal point of the dividend and proceed as in the division of whole numbers.

Example:

212.56 ÷ 41

$$\begin{array}{r} 5.1843\frac{37}{41} \\ 41\overline{)212.5600} \\ \underline{205} \\ 75 \\ \underline{41} \\ 346 \\ \underline{328} \\ 180 \\ \underline{164} \\ 160 \\ \underline{123} \\ 37 \end{array}$$

If, as in the example above, the division does not come out evenly, ciphers may be added to the right of the decimal portion of the dividend as shown, and the division then carried out as many places as may be desired. It has been shown that adding ciphers to the right of a decimal does not change the value of the decimal. The further such a division is carried out, the more accurate the answer will be, but the answer can be no more accurate than the original values used. In the above example, the division has been carried out to two additional decimal places, but 5.18, or even 5.2 would be a satisfactory quotient in view of the fact that 41 has but two significant figures.

If the divisor is a decimal, when arranging the numbers for division, move the decimal point of the divisor to the right to make it a whole number, and move the decimal point of the dividend to the right *the same number of places as in the divisor.*

Example:

15.62 ÷ .0021

```
               2
          7438.──
               21
0021. )156200.
        147
         92
         84
          80
          63
          170
          168
            2
```

Note that the decimal point in the divisor and the dividend has been moved four places to the right. In the case of the dividend, it was necessary to add two zeros to complete the number. This does not change their ratio (the quotient) but simplifies the operation.

Example of Division:

.425 ÷ 23

```
          18
    .0184──
          23
23).4250
    23
    195
    184
     110
      92
      18
```

In the example above, the divisor is already a whole number, therefore, in arranging the numbers for division, the decimal point of the quotient is placed at once directly above the decimal point of the dividend. The divisor will not divide into the first figure to the right of the decimal point; a cipher is written directly above the first figure in the decimal dividend, to form the first figure of the quotient. It will also be noted that in order to obtain an accuracy of three significant figures, a cipher was added to form a fourth figure in the dividend. Additional ciphers may be added to carry the quotient out to as many decimal places as desired.

Conversion of a Common Fraction to a Decimal.—Common fractions may become somewhat difficult to handle, particularly where the addition or subtraction of several fractions having different denominators is involved. If reduced to decimals, these fractions can easily be added or subtracted. To convert a fraction to a decimal, it is only necessary to divide the numerator by the denominator.

Examples:

1. $\frac{4}{9} = \underset{.444}{\underline{9)4.000}} = .444$

2.

$$\frac{7}{8} = \begin{array}{r} 8\overline{)7.000} \\ .875 \end{array} = .875$$

3.

$$\frac{23}{127} = \begin{array}{r} .181\frac{13}{127} \\ 127\overline{)23.000} \\ \underline{127} \\ 1030 \\ \underline{1016} \\ 140 \\ \underline{127} \\ 13 \end{array} = .181$$

Suppose it is desired to add the fractions, 4/9 + 7/8 + 23/127. Having converted them to decimals it is simple. The answer is 1.5.

$$\begin{array}{r} .444 \\ .875 \\ \underline{.181} \\ 1.500 \end{array}$$

It should be noted that the answer 1.5 is not exactly correct, because in the case of two fractions, 4/9 and 23/127, the decimals were carried out to only three figures, and the division did not come out exactly even. However, *a decimal carried out to three significant figures is accurate to 1 place in 1,000,* sufficiently accurate for most purposes. Greater accuracy is easily obtained when necessary by extending the division.

Exercises

42. $.24 \div .3 = ?$

43. $.856 \div .234 = ?$

44. $.347 \times 1.628 = ?$

45. $.17 \times .00321 = ?$

46. $.827/2.56 = ?$

47. $$\frac{879 \times 1.24}{\frac{3}{4} \times .176} = ?$$

48. $$\frac{2.436 \div 12}{.00015 \div .03} = ?$$

PERCENTAGE

The term "Per cent" is commonly used in radio and electrical work to express efficiency of apparatus, proportion of the amount of power input that does useful work, etc. The term per cent is an abbreviation of the Latin words "Per Centum", meaning by the hundred or in the hundred. Suppose 100 watts of power is delivered to a transmitter, and the transmitter delivers 30 watts to the antenna. The transmitter will deliver 30/100 of its input to the load. This may be expressed otherwise by saying that "30 per cent of the input is transferred to the antenna," or that the "efficiency of the transmitter is 30 per cent."

To express any fractional amount in percentage, it is only necessary to multiply the numerator of the fraction by 100 and then divide by the denominator.

Examples:

$$\frac{3}{4} = \frac{3}{4} \times 100 = \frac{300}{4} = 75 \text{ per cent}$$

$$\frac{5}{16} = \frac{5}{16} \times 100 = \frac{500}{16} = 31.25 \text{ per cent}$$

Any decimal amount can be expressed in percentage by simply multiplying the decimal by 100.

Examples:

.15 expressed in percentage
= 15 per cent

.875 expressed in percentage
= 87.5 per cent

.013 expressed in percentage
= 1.3 per cent

Exercises

49. Express as decimals:

32/100 5/1,000 65/10,000
2,534/1,000 627/1,000,000

50. Express as decimals accurate to 3 significant figures:

2/3 5/6 8/14 25/450 125/642

51. Express as per cent:

3/4 .021 .1734 3.56 1.009

52. Express as decimals:

25% .35% 7.5% 15.8% .06%

53. The output power of a transmitter is measured and found to be 600 watts. The efficiency is known to be 45 per cent. What is the input power?

54. The input power of a transmitter is 1,350 watts, and the efficiency is 60 per cent. What is the power output?

WEIGHTS AND MEASURES

Intercourse between nations has necessitated the need for a universal and convenient system of units of measurements of one kind or another. Due to this need, there exists today two main systems, one called the English; the other the Metric. In both systems assumptions must be made in the form of certain arbitrary units. In the English system the basic units are the foot, the pound, and the second while in the Metric these become the centimeter, the gram, and the second. Quite often the systems are referred to by the abbreviation of the basic units, the English system sometimes being called the FPS (foot-pound-second) and the Metric the CGS system. The Metric system has been adopted almost universally by scientists throughout the world, while the English system finds its greatest application in commerce between the United States, Canada and Great Britain. Practically all other nations use the Metric system, and there has been considerable agitation in the English speaking countries to discard the English system for the Metric, and thus give a uniform standard of weights and measures throughout the world.

At the present time several scientific societies have adopted a new system of measurements which is an extension of the present Metric system. The new system substitutes the meter for the centimeter, and the kilogram for the gram in the CGS system. The second is the unit of time in both systems. It is claimed that the MKS system simplifies the statement and application of many scientific laws. If the student is familiar with the Metric system, there will be little difficulty recognizing the units of the MKS system.

Practically all the units in

any system are derived from a basic standard. Legally, in the United States, the unit of length is the meter; adopted by Congress in 1866. The meter is 39.37 inches in length and the foot, yard, etc., are all obtained from it. Sufficient other basic units were adopted by this same act of Congress, all metric; so, while the English system is used by commerce in the United States, the units are determined from the Metric system. The English system will be considered first; any caluclations in this system may be applied with equal correctness in the Metric system, as long as no attempt is made to mix the units of the two systems. Thus, a distance can be measured in either yards or meters, a volume can be obtained in either cubic feet or cubic meters, a weight in pounds or grams. Tables of conversion from one system to the other will be given to enable the student to make calculations in either system. The principal divisions of the English system of *linear measure* are as follows:

12 inches (in.)	=	1 foot (ft.)
3 feet	=	1 yard (yd.)
$16\frac{1}{2}$ feet	=	1 rod (rd.)
5,280 feet	=	1 mile (mi.)

Unless otherwise specified, it is customary to define the distance between two points as the shortest distance between them. This length may be expressed in any units desired. The choice will depend entirely on the magnitude of the distance, short lengths being expressed in inches, feet or yards, longer distances in rods or miles. To change from one unit to another merely requires the application of the table given above. Thus, to change 27 miles to feet requires the multiplication of 27 by 5,280 which equals 142,560 feet. To further reduce this to inches requires that the feet be multiplied by the number of inches in each foot, or 142,560 × 12 = 1,710,720 inches. To convert from inches to feet requires that the number of inches be divided by the number of inches in a foot. As an example, it is desired to convert 86,496 inches to feet. 86,496 divided by 12 equals 7,208 feet. If this is to be converted to miles, the length in feet will be divided by the number of feet in a mile. 7,208 divided by 5,280 equals 1.365+ miles. *It will be seen that a conversion from a larger to a smaller unit involves multiplication; the inverse process involves division.* This is a universal rule and must be remembered.

Solutions of typical problems involving the principle stated above follow.

1. Change 3 miles, 637 feet, 10 inches to inches.

5,280 ft. × 3 = 15,840 ft.

15,840 ft. + 637 ft. = 16,477 ft.

16,477 × 12 in. = 197,724 in.

197,724 in. + 10 in. = 197,734 in.

2. Change 11,840 feet to miles, expressing any remainder in feet.

11,840 ft. ÷ 5,280 ft.
= 2 mi and 1,280 ft.

Exercises

55. Change 6.4 miles to feet; 1,278 inches to yards; 7,562 feet to miles.

56. Reduce 28 yds., 1 foot, 9 in., to inches.

57. How many rolls of twisted pair telephone wire, each containing 2,500 feet would be required for a line 7.5 miles long?

58. Assuming 30 inches to the step, how many steps will a man take in walking 4 miles?

Square measure is used to determine the area or extent of a surface and involves two dimensions, length and width. *The dimensions must be expressed in identical units.* The area is obtained by multiplying the dimensions and the result is expressed in square units of the same kind. A table of the more commonly used square units follows:

144 square inches (sq. in.) = 1 square foot (sq. ft.)

9 square feet = 1 square yard (sq. yd.)

160 square rods (sq. rd.) = 1 acre (A)

640 acres = 1 square mile (sq. mi.)

1. Determine the number of square feet in a transmitter room 10 feet long and 8 feet wide.

Area = length × width

Area = 10 × 8 or 80 sq. ft.

2. How many square feet are there in a sheet of copper 24 feet long and 8 inches wide,

Both dimensions must be in identical units, and since the answer is to be in square feet, it will be more simple to convert the width from inches to feet before multiplying.

$$8 \text{ in.} \div 12 \text{ in.} = \frac{8}{12} = 2/3 \text{ ft.}$$

Area = 24 × 2/3 = 48/3 = 16 sq. ft.

Cubic measure is used to find *volume* and involves the product of three dimensions, *all expressed in the same units.* The results of multiplying three dimensions in feet is expressed in cubic feet; if in some other unit such as inches, then in cubic inches and similarly for other units of length.

Volume = length × width × thickness

Determine the volume of a locker 5 feet long, 24 inches wide and 1 foot deep (or thick).

$$\text{Volume} = 5 \times \frac{24}{12} \times 1 \text{ or } 10 \text{ cubic feet}$$

Of the many possible units of volume, the following are the most important:

1,728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

In the measurement of *capacity,* other units are possible and are frequently used. The measures of capacity are divided into two classes, dry measure and liquid measure. The units are different and must not be interchanged.

TABLE OF DRY MEASURE

2 pints (pt.) = 1 quart (qt.)

8 quarts = 1 peck (pk.)

4 pecks = 1 bushel (bu.)

The dry system is used in the measurement of vegetables, grain and other similar articles of commerce.

TABLE OF LIQUID MEASURE

4 gills (gi.) = 1 pint

2 pints = 1 quart

4 quarts = 1 gallon (gal.)

$31^{1}/_{2}$ gallons = 1 standard barrel (bbl.)

63 gallons = 1 hogshead(hhd.)

This system is used for measuring paint, oil, water and similar liquids of commerce. The barrel referred to above is the standard barrel, and is rarely found in practice. The capacity of a barrel is generally plainly marked, and in no case should a barrel be considered as standard unless it is definitely known to be such. The application of these tables is exactly as described for the table of lengths; when proceeding to a larger unit, division is employed; when proceeding to a smaller unit, multiply the larger by the number of smaller units equivalent to one unit of the larger.

Example:

An example of the use of volume and liquid measure might be a cooling system for a water cooled transmitter tube. If it requires 5 gallons per minute of water to cool the tube and it requires 50 times this figure for the amount of water required for cooling, then the total water needed is

$$5 \times 50 = 250 \text{ gallons}$$

Since there are 231 cu. in. to one gallon, 250 gallons corresponds to 231 × 250 = 57,750 cu. in. Furthermore, since there are 1,728 cu. in. to one cu. ft., the above 250 gallons will occupy 57,750 ÷ 1,728 = 33.42 cu. ft. If a tank were used having a square base 3 feet on a side, then the area of the base would be 9 sq. ft. In order that the tank have a volume of 33.42 cu. ft., it would have to be 33.42 ÷ 9 = 3.713 feet high.

WEIGHT MEASURE.—In the English system, there are three principal measures of weight, Avoirdupois, Troy and Apothecary. The Avoirdupois system is used in commercial dealings with most of the principal articles of commerce, the Troy system in the measurement of precious metals and stones, and the Apothecary system by pharmacists and physicians in the compounding of medicines, etc.

TABLE OF TROY WEIGHTS

24 grains = 1 pennyweight (dwt.)

20 pennyweights = 1 ounce

12 ounces = 1 pound

TABLE OF APOTHECARY WEIGHTS

20 grains = 1 scruple (sc.)

3 scruples = 1 dram

8 drams = 1 ounce

12 ounces = 1 pound

TABLE OF AVOIRDUPOIS WEIGHTS

7,000 grains (gr.) = 1 pound (lb.)

16 ounces (oz.) = 1 pound

100 pounds = 1 hundredweight (cwt.)

2,000 pounds = 1 ton (T.)

2,200 pounds = 1 long ton

The long ton is used at U.S. customs houses and occasionally by private business in transactions of bulk shipments.

Note that the three systems are connected by the fact that all are based on the grain. Also, it is customary to consider one cubic foot of pure water as weighing 62.5 pounds or 1,000 ounces. Another link between measures and capacity measures is in the fact there are 231 cubic inches in a gallon. Conversions from one unit to another are made in precisely the same way as in the other systems mentioned.

THE METRIC SYSTEM.—The English system contains a large number of different types of units which bear no relation to one another except in a purely arbitrary manner. This difficulty is removed in the metric system, and a change from one unit to a larger or smaller unit is made by the changing of a decimal point. The unit of length is the meter, one ten millionth of the distance from the north pole to the equator; the unit of weight is the gram, the weight of one cubic centimeter of water at its greatest density; and the unit of capacity or volume is derived from the unit of length.

The unit of length is the *meter*, it being divided into smaller units according to a system of tens. The smaller units are named by prefixing to the meter, the equivalent Latin prefixes meaning ten, hundred, thousand, etc. For units larger than the meter, Greek prefixes of a like nature are used. The principal Greek prefixes are *Myria* meaning 10,000, *Kilo* meaning 1,000, *Hekto* meaning 100 and *Deka* meaning 10. The Latin prefixes for the smaller units or subdivisions of the meter are *Deci* meaning 1/10, *Centi* meaning 1/100 and *Milli* meaning 1/1,000.

METRIC TABLE OF LENGTH

10 millimeters (mm.) = 1 centimeter (cm.)

10 centimeters = 1 decimeter (dm.)

10 decimeters = 1 meter (m.)

10 meters = 1 dekameter (Dm.)

10 dekameters = 1 hektometer (Hm.)

10 hektometers = 1 kilometer (Km.)

10 kilometers = 1 myriameter (Mm.)

Measurement of Capacity.—The

Liter is the unit of capacity and is defined as a volume of a cube 1 decimeter on a side. Since the decimeter is 10 centimeters, this volume is also 1,000 cubic centimeters, abbreviated c.c. or cm^3. The liter and the cubic centimeter are the most frequently encountered capacity units.

CAPACITY TABLE

10 milliliters (ml.) = 1 centiliter (cl.)

10 centiliters = 1 deciliter (dl.)

10 deciliters = 1 liter (l.)

10 liters = 1 dekaliter (Dl.)

10 dekaliters = 1 hektoliter (Hl.)

10 hektoliters = 1 kiloliter (Kl.)

The liter is approximately equivalent to one quart liquid. The exact relationship will be stated later in a table of equivalents.

Measurement of Weight.—The unit of weight is the gram, defined as the weight of one cubic centimeter of water at its greatest density. Since this is a very small unit, in commerce it is customary to employ the kilogram and the metric ton. The table of weights commonly used is as follows:

1 Metric Ton = 1,000 Kilograms (Kg.)

1,000 grams (gm.) = 1 kilogram

1 gram = 1,000 milligrams (mg.)

The above table considers only the commonly used units, but the entire table may be arranged from the rules outlined previously. To the tables mentioned above might be added:

1 micron (μ) = .000001 meter

Certain equivalent values between the English and Metric systems are valuable and should be memorized. No attempt is made to state all the possible relationships, only those most frequently used being given:

1 inch = 2.54 centimeters

1 meter = 39.37 inches

1 gram = 15.43 grains (approx. 16 grains)

1 kilogram = 2.2046 pounds (approx. 2.2 pounds)

1 liter = 1.0567 quarts

Other equivalents to be remembered when dealing entirely with the English system are:

7,000 grains = 1 pound Avoirdupois

1 gallon = 231 cu. in. (legal)

1 bushel = 2150.4 cu. in. (legal)

1 sq. in. = 6.45 sq. cm.

1 cu. in. = 16.4 cu. cm.

The following illustrations involving the use of the Metric system and conversion between systems show the methods of working such problems.

Convert 189,579,834 centimeters (cm) to kilometers. Since the kilometer is a larger unit than the centimeter, there will be a smaller number of kilometers than centimeters. Readily remembered is the fact there are 100 cm in a meter, therefore, shifting the decimal point two places to the left will convert to meters.

189,579,834 cm = 1,895,798.34 meters

There are 1,000 meters in a kilometer by definition, so shifting the decimal point three additional places to the left will convert to Km. Therefore

1,895,798.34 meters = 1,895.79834 Km

Note the ease with which the transfer is made by the mere change in the position of the decimal point, no multiplication by some mixed number being required as in the English system.

Convert 33.65 meters to millimeters. Here the change is from a larger to a smaller unit, therefore the decimal will be shifted in the opposite direction from that of the first example. Since milli indicates one thousandth part of, the shift will be three places to the right. Hence,

33.65 meters = 33,650 millimeters

Determine the weight of 11.45 liters of water in grams. The first step will be to convert to cubic centimeters because one cu. cm of water weighs 1 gram. By definition from the table, there are 1,000 cu. centimeters in a liter. This means a shift of the decimal point to the right since the cu. cm is smaller than the liter. Therefore, 11.45 liters = 11,450 cu. cm (or cc as it is more usually written). Since one cc of water weighs 1 gram, the weight of 11,450 cc of water is 11,450 grams. This may be converted to kilograms by moving the decimal point to the left since the Kg. is greater than the gm., thus 11,450 cc = 11.45 Kg.

To convert from the Metric system to the English system, the table of equivalents is used. How many inches in 254 centimeters? Since there are 2.54 centimeters in 1 inch, 254/2.54 = 100 inches. To change from inches to centimeters is the inverse process; to change 12 inches to centimeters involves multiplication. 12 × 2.54 = 30.48 cm.

Determine the number of kilometers in 6 miles. According to the table, there are 39.37 in. in 1 meter, and 1,000 meters in a kilometer. First, convert 6 miles to inches. 1 mile = 5,280 feet. Therefore, 6 × 5,280 will give the number of feet, and multiplication of this result by 12 will give the number of inches. 6 × 5,280 × 12 = 380,160 in. Dividing by 39.37 to change the meters, 380,160/39.37 = 9,656 meters. Meters to Km. involves the movement of the decimal point three places to the left, so 9,656 meters equals 9.656 Km., which is the required answer.

How many liters are there in 650 gallons? From the table, there are 4 quarts in 1 gallon. Hence, 650 gallons = 2,600 quarts. From the table of equivalents, 1 liter = 1.0567 quarts. The liter is larger than the quart, hence there will be a smaller number of liters. This indicates division. 2,600/1.0567

= 2,460 liters.

Exercises

59. How many gallons in 53 pints; in 18 quarts? How many cu. in. in each of the above volumes?

60. An antenna is 63 meters in height, what is its height in centimeters? What is its height in feet?

61. Express the following in (a) meters and centimeters: 657 mm, 1,852 Km., 256 dm. (b) Liters: 1,576 cc, 157 Kl. (c) cu. cm: 1.65 liters, .002 Kl.

62. (a) Find the capacity in cu. cm of a container 12" × 8" × 6".

(b) How many liters of water would the container hold?

(c) How much would the water weigh in grams?

63. The atmosphere exerts a pressure of approximately 14.7 lb./in^2. What is this in grams/cm^2?

64. A man stands 5' 10" and weighs 175 pounds. What is his height in meters? His weight in kilograms?

Angular Measurement.—The ancient Babylonians are responsible for the choice of 360 degrees in a circle, believing that the earth made a revolution around the sun in 360 days. The angle formed in one day at the center of the circle thus described was called a degree, and the 360 days accounted for the 360 degrees. The degree was subdivided into minutes, and the minutes into seconds.

60 seconds = 1 minute

60 minutes = 1 degree

360 degrees = 1 circle

A different notation is used to indicate minutes and seconds of angular measurement than for time; 36 degrees, 10 minutes and 15 seconds would be written 36° 10' 15". The degree may also be divided decimally, 36° 15' being expressed as 36.25°. The decimal part arises from the fact that since there are 60 minutes in a degree, the 15 minutes would be 15/60 of a degree or .25 degree. The greatest difficulty in working with angular measurement will be encountered in adding and subtracting the angular values. Such work is not difficult if one exercises care. In adding or subtracting, begin with the smallest part first, and then proceed step by step to the largest. Two illustrations will make the process clear.

$$\begin{array}{lrrr} \textit{Add} & 36^\circ & 45' & 20'' \\ & 45^\circ & 31' & 42'' \\ \hline & 82^\circ & 17' & 2'' \end{array}$$

First, add the seconds, the result being 62 seconds. Since there are more than 60", a result in minutes and seconds is obtained, 62 seconds equals 1 minute and 2 seconds. The 2" is written in the seconds column and the minute is mentally carried over to the minute column. Adding the minute column, 45' + 31' + 1' = 77 minutes. This is more than 60 and is reduced to degrees and minutes, 1 degree 17 minutes. The 17' is written in the minutes column and the 1 degree is mentally carried over to the degree column and added to the degrees, 36° + 45° + 1° = 82°.

Subtract	75°	34'	20"
	40°	21'	45"
	35°	12'	35"

First, subtract the seconds. Since 45" is more than 20", borrow 60", or 1' from the minute column, and add the 60" to those already in the column to get 80". Subtracting the 45" leaves 35". Next subtract the minutes, remembering that one minute was borrowed from the upper minute value. Thus, 33-21 leaves 12'. Next, subtract the degrees, the total answer becoming 35° 12' 35".

In addition to the fact there are 360° in a circle and angles are measured in degrees, there are other facts about angles which should be understood. An angle is determined by the intersection of two lines. The opening between the lines is measured in degrees as shown. The intersection point of the lines is called the *vertex* of the angle. Note that the size of an angle has nothing whatever to do with the length of the sides but only with the opening of the sides. In naming an angle, the vertex is given a letter, and somewhere along the sides, generally at the ends, will be found other letters. Beginning with one end of a side, the angle is defined an angle ABC as in Fig. 1. Note that the middle letter B is at the vertex. If there can be no confusion resulting, angles are sometimes defined by the vertex letter alone. In Fig. 2, confusion would result if one referred to angle B, since there are several angles all with vertex B, but no confusion can result from the angle ABC.

When the sum of two angles, such as angles ABC and ABD is equal to 90°, the angles are said to be *complementary*. The angle DBC (Fig. 2) is a *right angle*, that is, an angle of 90° and is equal to the sum of angles ABC and ABD, hence the two angles are complementary. When the sum of two angles is 180° or two right angles, the angles are said to be *supplementary*. Such supplementary angles are ABC and ABE.

TEMPERATURE CONVERSION

In radio and television work,

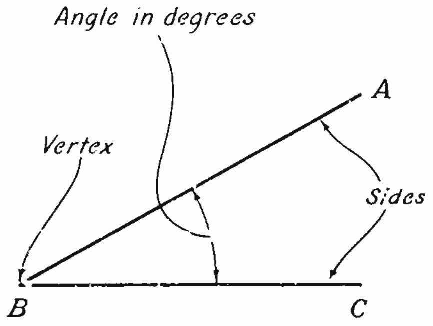

Fig. 1.—Vertex of an angle.

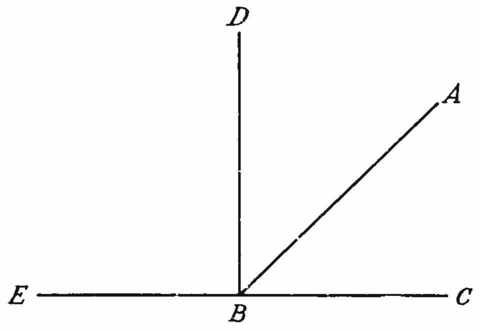

Fig. 2.—Vertex of several angles.

one deals very extensively with temperature. Crystals must be operated at certain temperatures to maintain specified frequencies; a vacuum-tube filament must be operated at a given temperature in order to secure adequate electron emission; the vacuum-tube plate must be operated below a certain temperature to prevent damage; etc. In dealing with temperature two systems or scales are encountered, the Fahrenheit as commonly used in ordinary thermometers, and the Centigrade as used in technical and scientific work.

The Fahrenheit thermometer employs an arbitrary scale in which the commonly accepted temperature standards, freezing and boiling temperatures of water, have no symmetrical relation. On the Fahrenheit scale, the freezing point of water is 32° above zero and the boiling point to 212° above zero.

The Centigrade scale is symmetrical with respect to the ordinary standards. On this scale the freezing point of water is taken as 0° and the boiling point as 100°, thus the temperature difference between the freezing and boiling points of water is covered in 100 divisions.

It is often necessary to transfer from one scale to another. The difference on the Fahrenheit scale between freezing and boiling temperatures is 180°, 212° - 32°. This same range is covered on the Centigrade scale in 100 divisions. Therefore, the conversion factor is 180/100 or 1.8. However, the starting point is not the same. 0° on a Centigrade thermometer represents 32° on a Fahrenheit scale. Thus, "*to convert from Centigrade to Fahrenheit first multiply by 1.8 then add 32°.*" Inverting this process, "*to convert from Fahrenheit to Centigrade first subtract 32° and then divide by 1.8.*"

The preceding paragraph is correct for the conversion of fixed temperatures. One often speaks of a "temperature variation" of a certain number of degrees. For example, it may be stated that the temperature varied 10° C. To convert this variation to degrees Fahrenheit, it is only necessary to multiply by the conversion factor 1.8. To convert from F to C for a variation of temperature simply divide by 1.8. When dealing with a temperature variation the factor 32° is neglected.

One other scale of interest to the radio engineer is the Absolute or Kelvin scale. In this scale zero degrees is at Absolute Zero, defined in physics as that temperature at which all molecular activity ceases. As a rule absolute degrees are given in degrees Centigrade, absolute zero or 0° K = -273° C. On the Fahrenheit scale absolute zero is 459.4 degrees below zero or -491.4° F below the freezing point of water. Check points on the three scales are as follows:

	C	F	K
Boiling point of water	100°	212°	373°
Freezing point of water	0°	32°	273°
Absolute zero	-273°	-459.4°	0°

Note

$$°C = \frac{5}{9}(°F - 32) \qquad (1)$$

$$°F = \frac{9}{5}(°C) + 32 \qquad (2)$$

$$^{\circ}K = 273 + (^{\circ}C) \tag{3}$$

$$^{\circ}C = (^{\circ}K) - 273 \tag{4}$$

Examples: A reading of 24°C is how many degrees Fahrenheit?

$$^{\circ}F = \frac{9}{5}(24^{\circ}C) + 32 = 1.8(24) + 32$$

$$= 43.2 + 32 = 75.2^{\circ}F$$

-70°F is how many degrees C?

$$^{\circ}C = \frac{5}{9}(-70 -32) = \frac{5}{9}(-102)$$

$$= -56.67^{\circ}C$$

Exercises

65. (A) Express the number of degrees in the complements of the following angles: 40°, 75° 15', 25° 13' 23".

(B) Are these angles supplementary: (a) 86° 19' 21" and 93° 40' 19"? (b) 23° 42' 40" and 156° 17' 20"?

66. Change 98°F to degrees C.

67. Change 256°C to degrees F.

RATIO AND PROPORTION

Frequently one quantity is compared to another by stating the magnitude of the one in comparison to the other as a simple quotient. This quotient is the definition of a ratio. If one radio receiver has 20 resistors and another has 5 resistors of similar types, then the receivers might be compared in the ratio of 20/5, or by reducing the fraction to its lowest terms, 4/1 or 4 to 1. This comparison is meaningless unless the two quantities compared are of the same kind. *A ratio should never be expressed between unlike things such as ships and apples.* The fraction expressing the ratio is usually reduced to the smallest whole integers. The first part of the ratio is called the *antecedent* and the second part is the *consequent*.

A proportion is simply a statement that two ratios are equal. The two ratios need not consider the same kind of things; all that is necessary is that *each ratio shall consider like things* and that the two ratios be equal for a proportion. This is expressed mathematically as follows: 2/3 = 8/12. This expression may also be written 2 : 3 = 8 : 12 or 2 : 3 : 8 : 12. All three expressions are read: 2 is to 3 as 8 is to 12, or the ratio of 2 to 3 equals the ratio of 8 to 12.

Important properties in connection with ratio and proportion are:

1. The first and last terms are called the *extremes*. In the above illustration 2 and 12 are the extremes. The second and third terms are called the *means*, 3 and 8 being means in the above proportion.

2. *The product of the means equals the product of the extremes in any proportion.*

3. The product of the means divided by one extreme gives the other extreme, and the product of the extremes divided by either means gives the other mean. This is evident from the illustration

above.

Examples to show the importance of ratio and proportion will follow:

If 10 transformers cost \$80, how much will 60 cost? Since the cost is proportional to the amount, then the cost ratio will equal the amount ratio. The unknown cost will be designated by the letter C. The ratio of the amounts is 60/10 and the ratio of the cost is C/80. The ratios are equal, so C/80 = 60/10. Applying rule (3), C (one of the extremes) will equal the product of the means divided by the other extreme, or (80 × 60)/10 = \$480. This may be expressed mathematically,

$$C = \frac{80 \times 60}{10} = \$480$$

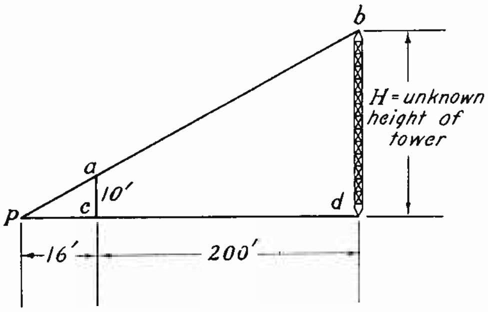

Fig. 3.—Finding the unknown height of a tower.

Ratio and proportion is particularly useful in problems dealling with the measurement of unknown distances. It is desired to determine the height of a tower. See Fig. 3. The height of the tower will be designated as H. A stick 10 ft. long is held vertically upright and, with the observer at p, is moved back and forth until the top of the stick and the top of the tower are directly in line. The distance from p to the base of the stick and from p to the base of the tower are measured. From the figure pc = 16 and pd = 216 while ac = 10. In the right triangles thus formed, a proportion can always be set up as follows: Side ac is to side pc as side bd (of the large triangle) is to side pd. Expressed matematically ac : pc :: bd : pd. Substituting the values above

$$10 : 16 :: H : 216$$

Applying rule (3)

$$H = \frac{10 \times 216}{16} = 135 \text{ ft.}$$

This fact about right triangles constructed as in the foregoing illustration should be remembered, as it has many uses in practical work of all kinds.

The type of proportion dealt with in the examples above is known as a direct proportion. That is, from the problem just illustrated, the height of the stick and the height of the tower are directly related to the distance of the stick from p and the base of the tower from p. There is another type of proportion (known as inverse proportion) where *as the related parts of the one ratio increase, the related parts of the second ratio decrease.* This may be illustrated by considering the wheel of the car. For a given car speed, as the diameter of the wheel is increased, the number of revolutions decreases. The circumferences of two different sized wheels are in direct proportion to their diameters, but the larger wheel will turn more slowly than the smaller and hence, the number of revolutions for a given time or a given distance are in inverse proportion. This may be demonstrated by a practical problem. One wheel having a diameter of 2 ft. makes 25 revolutions in rolling a certain distance. How many revolutions will a wheel 5 ft. in diameter make in rolling the same distance? Let R be the number of revolutions the larger wheel will make. The ratio of the diameters is 2 to 5, the ratio of the number of revolutions is 25 to R. *Since one ratio is the inverse of the other, one of the ratios must be inverted.* Only by so doing can the equality of the ratios be maintained. The second ratio will be inverted. The two ratios are then equated and rule (3) is applied.

$$\frac{2}{5} = \frac{R}{25}$$

$$R = \frac{25 \times 2}{5} = 10$$

This reasoning may also be applied to the determination of the sizes of gears and other similar problems in practical engineering. One gear makes 1,200 revolutions per minute and has 60 teeth, another driven by the first makes 800 RPM. How many teeth has the driven gear? The number of teeth is a function of the circumference which is a function of the diameter. Since the driven gear rotates more slowly than the driving gear, it must be the larger. An inverse relation is thus indicated, and the driven gear will have more teeth. The proportion is set up by letting N represent the number of teeth in the driven gear.

$$\frac{N}{60} = \frac{1200}{800}$$

$$N = \frac{60 \times 1200}{800} = 90$$

Exercises

Find the value of the letter in each of the following:

68. 15 : 45 :: X : 60

69. P : 7 :: 81 : 10

70. A radio beam rises 25 feet per 100 feet. If it continues to rise at the same rate, what is the rise per mile?

71. 4 men do a piece of work in 20 days; in how many days will 15 men do the work?

72. A man 6 feet in height casts a shadow 4 feet long, at the same time an antenna casts a shadow 32 feet long. How high is the antenna?

POSITIVE AND NEGATIVE NUMBERS

The positive number is defined as a number greater than zero and is the number used throughout the study of common arithmetic. Positive numbers may be added, subtracted, multiplied or divided, and the answer is always positive with one exception, namely, where a larger number is to be subtracted from a smaller. As the study of mathematics advances, the need for a method which will permit larger numbers to be subtracted from smaller ones becomes more and more necessary. For example, if a man has ten dollars and owes a total of 12 dollars, there is no way in common arithmetic in which a mathematical statement as to the financial condition of the man can be made. We can say the man owes two dollars, but since common arithmatic does not make use of negative numbers, there is no way to express the result.

Negative numbers may be easily understood if the student adopts the idea that numbers have direction. Consider a number as some force operating on a point of origin called zero. The idea is illustrated graphically in Fig. 4.

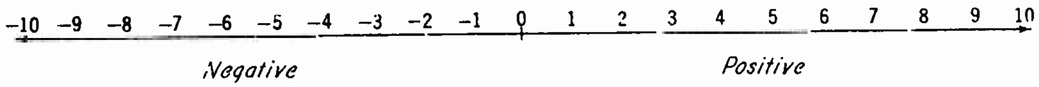

Fig. 4.—Representation of positive and negative numbers by two opposite directions in space.

It is evident that a negative number acting as a force in direct opposition to the positive number will cancel all or part of the positive number. One will counteract an equal amount of the other. If it is desired to add a -8 to plus 5, count eight units from plus 5 in a negative direction. The count stops on -3 so 5 - 8 = -3. Similarly, 8 - 3 = +5. From this it is possible to formulate a rule regarding the addition of positive and negative numbers.

When adding two numbers of unlike sign, subtract the smaller from the larger and prefix the result with the sign of the larger number.

Examples:

$$4 + (-8) = -(8 - 4) = -4$$

$$8 + (-4) = +(8 - 4) = 4$$

The rule may be extended to include the addition of a series of positive and negative numbers. In this case add all the positive and all negative numbers and then apply the rule stated above. For example:

$$8 - 9 + 17 - 22 - 4 + 26$$

$$= (8 + 17 + 26) + (-9 -22 -4)$$

$$= 51 + (-35) = 16$$

Referring to Fig. 4 it is seen that if a negative number is added to a positive number, it counteracts a certain amount of the positive number. On the other hand, if it were desired *to subtract a negative number the effect would be reversed because subtraction is the reverse of addition. Therefore, subtracting a negative number is equivalent to adding an equivalent positive amount.* It is seen that the subtraction of a negative number is the same as adding an equivalent positive number, and subtracting a positive number is the same as adding an equivalent negative number. Therefore, *when subtracting one number from another, change the sign of the subtrahend and add.* When adding remember the rule for the addition of positive and negative numbers.

Examples:

4 minus -6 = 4 + 6 = 10

-4 minus 6 = -10

-10 minus -2 = -10 + 2 = -8

10 minus 2 = 8

Multiplication and Division.— The rules for multiplying and dividing positive and negative numbers are quite similar. *When multiplying or dividing numbers having like signs the answer is always positive.* This is true if the signs are both positive or both negative.

Examples:

4 × 2 = 8   4/2 = 2

-4 × -2 = 8   -4/-2 = 2

When multiplying or dividing numbers having unlike signs, the answer is always negative.

Examples:

8 × -2 = -16   8/-2 = -4

-8 × 2 = -16

Exercises

Add:

73. 34, -88, -176, 310.

74. -210, -349, -219, -319.

75. 3,896, -4,980, 596, 6,200

76. -346.5, .298, 376.42, -772.39.

77. .475, -.242, -3.78, 2.

Subtract:

78. -18 from 72.

79. 76 from -218.

80. (-215) from -420.

81. 316 from -212.

Perform the indicated operations:

82. .988 -(1.4)

83. (2 + 8 - 42 - 16)
- (26 - 33 - 41) = ?

84. (1 - 9 + 2.37 - .999) - .476
+ (12 - 1.32) = ?

85. (2,489 - 37,842 + 28,193 +
1,906 - 21,300) = ?

86. *Subtract* (3 + 8 + 17 - 49)
from (17 - 42 + 19)

87. -426 × 128 = ?

88. -3 × -4 × 18 = ?

89. 17 × 18 × -1 = ?

90. 26 × .4 × -29 × -2 = ?

91. (26 × -4) (21 × 22) = ?

92. $-429/73 = ?$

93. $3{,}465/-105 = ?$

94. $-41{,}768/(3.5 \times -18) = ?$

95. $\dfrac{-31 \times 189 \times 12}{-1 \times 213 \times 3} = ?$

96. $\dfrac{(-4 \times -8)(-2 \times 9)}{-3 - 6 + 8 - 4} = ?$

INVOLUTION

The exponent of a number is a small figure placed to the right and slightly above the number. For example, in the numbers 2^2, 4^3 and 5^4 the small figures are exponents. The exponent of any number indicates the number of times that number is to be used as a factor. Thus, 3^2 means 3×3. The product, 9, is said to be a power of 3 and more explicitly the second power of 3 or 3 squared. $3^3 = 3 \times 3 \times 3 = 27$. The product is the third power or the cube of 3. Powers higher than the cube are expressed numerically. 4^4 is the fourth power of 4, 5^8 is the eighth power of 5, etc.

Before taking up the more basic laws of exponents, the student should realize that any number without an indicated exponent represents that number raised to the first power. Thus, $2 = 2^1$, $3 = 3^1$, or $91 = 91^1$. It should also be realized that any number represents so many units, that is, $9 = 1 \times 9$, $8 = 1 \times 8$, etc., although it is customary to omit the multiplication because multiplying any number by 1 does not change its absolute value. The absolute value of a number is its numerical value without regard to sign. The absolute value of -4 and +4 are both the same, 4. The importance of remembering that any number represents the number times 1 will become evident later in dealing with the laws of exponents.

Consider the following powers of 10:

$$10^4 = 1 \times 10 \times 10 \times 10 \times 10 = 10{,}000$$

$$10^3 = 1 \times 10 \times 10 \times 10 = 1{,}000$$

$$10^2 = 1 \times 10 \times 10 = 100$$

$$10^1 = 1 \times 10 = 10$$

$$10^0 = 1 \times 1 = 1$$

$$10^{-1} = 1 \times .1 = \frac{1}{10}$$

$$10^{-2} = 1 \times .01 = \frac{1}{100}$$

$$10^{-3} = 1 \times .001 = \frac{1}{1000}$$

$$10^{-4} = 1 \times .0001 = \frac{1}{10000}$$

Several important facts can be derived from the above table. First, *any number raised to the zero power is equal to one.* Second, *any number having a negative exponent can be written as a fraction by making the numerator 1 and the denominator the given number with the sign of the EXPONENT changed.* This can be further elaborated to state that any number can be transposed from the numerator to the denominator or vice-versa by changing *the sign of the EXPONENT of the number.*

Examples:

$$3^{-1} = \frac{1}{3^{+1}} = \frac{1}{3}$$

$$7^{0} = 1$$

$$5^{-3} = \frac{1}{5^{+3}} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$$

$$\frac{1}{15^{-2}} = 15^{+2} = 15 \times 15 = 225$$

$$\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^{2}} = \frac{1}{\frac{4}{9}} = 1 \times \frac{9}{4} = 2\,{}^{1}/_{4}$$

$$\frac{2^{-2}}{3} = \frac{1}{3 \times 2^{2}} = \frac{1}{3 \times 4} = \frac{1}{12}$$

USE OF EXPONENTS

In multiplying *like* numbers having exponents, the product is equal to the given number with an exponent equal to the sum of the exponents of all the like numbers.

Example:

$$8^{2} \times 8^{3} \times 8 \times 8^{5} = 8^{2+3+1+5} = 8^{11}$$

Note particularly in this example that the third factor in the multiplication is 8 and although the exponent is not written, it must be included in the addition of exponents to find the exponent of the product.

Another important point to remember in the multiplication of numbers with exponents is that only the exponents of *like* numbers can be added. Thus:

$$8^{2} \times 8 \times 4^{2} \times 4^{2} = 8^{2+1} \times 4^{2+2}$$
$$= 8^{3} \times 4^{4}$$

The product is evaluated by raising 8 to the third power and multiplying it by the fourth power of 4. $8^3 = 8 \times 8 \times 8 = 512$, and $4^4 = 4 \times 4 \times 4 \times 4 = 256$. Thus $8^3 \times 4^4 = 512 \times 256 = 131,072$.

In multiplying numbers having exponents with unlike signs the rules for the addition of positive and negative numbers must be applied.

Examples:

$$9^{3} \times 9^{-2} = 9^{3-2} = 9^{1} = 9$$

$$(7^{-9} \times 7^{3})^{1/3} = (7^{-9+3})^{1/3} = (7^{-6})^{1/3}$$
$$= 7^{-6 \times 1/3} = 7^{-2}$$

The last result may be further simplified by transposing 7^{-2} below the line and changing the sign of the exponent:

$$7^{-2} = \frac{1}{7^{2}} = \frac{1}{7 \times 7} = \frac{1}{49}$$

Division of LIKE numbers having exponents is carried out by SUBTRACTING the exponent of the DIVISOR from the exponent of the dividend with due regard for the rules of subtraction of positive and negative numbers.

Examples:

$$10^{7}/10^{5} = 10^{7-5} = 10^{2} = 100$$

$$10^{3}/10^{5} = 10^{3-5} = 10^{-2} = \frac{1}{10^{2}} = \frac{1}{100}$$
$$= .01$$

$$10^{6}/10^{-8} = 10^{6-(-8)} = 10^{6+8} = 10^{14}$$
$$= 100,000,000,000,000.$$

$$10^{-2}/10^{-5} = 10^{-2-(-5)} = 10^{-2+5}$$

$$= 10^{3} = 1{,}000$$

$$10^{-3}/10^{-3} = 10^{-3-(-3)} = 10^{-3+3}$$

$$= 10^{0} = 1$$

Note particularly the last example. It bears out the rule that any number divided by itself yields a quotient of 1.

$$10^{-3}/10^{-3} = \frac{1}{1000} \div \frac{1}{1000} = \frac{.001}{.001} = 1$$

The use of exponents has its greatest application where multiplication, division, involution, and evolution are in order. When numbers having exponents are to be added or subtracted, it is best to evoluate the numbers and then add or subtract the results. Thus,

$$8^2 + 8^2 - 15^2 = 8 \times 8 + 8 \times 8 - 15 \times 15$$

$$= 64 + 64 - 225$$

$$= 128 - 225 = -97$$

ACCURACY IN MATHEMATICAL PROCESSES

The student is often faced with the problem of determining how accurate an answer is required to satisfy a given problem. For example, 7/3 = 2.3333333..... In practical work just where should this repeating decimal be discontinued? In most radio and electrical work there are definite limitations on the accuracy which may be obtained in practice. Most electrical problems are based on one or more meter readings. The average general service meter is rarely accurate to more than 2 per cent, and this accuracy is usually guaranteed by the manufacturer only at full-scale reading. The higher grade meters may be accurate to 1 per cent at full-scale reading while laboratory instruments may be accurate from .1 to .5 per cent at full scale. For example, suppose an ammeter reading 100 amperes at full scale is guaranteed accurate to 1 per cent. Then when the meter reads 100 amperes the actual current through the meter will be somewhere between 99 and 101 amperes, but the exact value between these two limits cannot be determined unless a meter with greater accuracy is used. Meters, resistors, condensers, etc., may be built to a very high standard of accuracy if desired, but the greater the accuracy the more care required in construction and hence the higher the manufacturing cost. In order to sell these components at a reasonable price a happy medium must be struck between accuarcy and manufacturing cost. The maximum accuracy obtainable in most electrical equipment is in the order of .1 per cent or 1 part in a 1,000. Therefore, if the student works all electrical and radio problems to an accuracy of 1 part in a thousand the work will contribute nothing to the original errors in the problem components.

One part in a thousand represents an accuracy to three significant figures and is easily obtained on the average 10-inch slide rule. The student should understand the meaning of significant figures. The number 24,728 is accurate to five significant figures and may be written as 24,700, if accuracy to only three significant figures is desired. However, the number .002453 is accurate to only four significant figures although there are six decimal places in the number. If the number is written .00245, it is accurate to three significant figures, but if written .002 it is only accurate to one part in ten, and if used in this way, may lead to a very large error.

All problems in this course are considered satisfactorily solved if the answer is correct to .1 per cent, and the student is urged to adopt this as standard. Greater accuracy in electrical and radio work is unnecessary and only increases the possibility of making errors without adding anything of material value to the solution.

If in working to three significant figures, the fourth figure of a number is 5 or greater, the number can be changed to three significant figures by rounding off the last figure. For example, 2,435 or 2,437 is written as 2,440; but, 2,433 would be written as 2,430. Numbers less than 5 are written to the nearest small number as just shown. A number such as 1,964,375 would be written as 1,960,000 to three significant figures.

POWERS OF TEN

When the accuracy of a problem is limited to three significant figures the use of powers of ten are often a great aid in simplifying the solution. For example,

$$2{,}432{,}895 \times 562{,}417$$

may be simplified by reducing each number to three significant figures, thus

$$2{,}430{,}000 \times 562{,}000$$

The number 2,430,000 is equal to

$$2.43 \times 1{,}000{,}000 = 2.43 \times 10^6$$

and

$$562{,}000 = 5.62 \times 100{,}000 = 5.62 \times 10^5$$

The problem reduces to:

$$2.43 \times 10^6 \times 5.62 \times 10^5$$

$$= 2.43 \times 5.62 \times 10^{6+5}$$

$$= 2.43 \times 5.62 \times 10^{11}$$

By simple multiplication $2.43 \times 5.62 = 13.6566$; which may be reduced to $13.7 = 1.37 \times 10$. Therefore:

$$2.43 \times 5.62 \times 10^{11} = 1.37 \times 10 \times 10^{11}$$

$$= 1.37 \times 10^{12}$$

If the final product is written 1,370,000,000,000, it is very bulky and difficult to handle. The powers of ten facilitate handling large numbers when the degree of accuracy in problem solutions is not too stringent.

Consider the product of .0000456 and .000045723. If three figure accuracy is satisfactory, the second decimal can be written as .0000457. .0000456 is equivalent to 456 divided by 10 seven successive times or $456/10{,}000{,}000 = 456/10^7$. Since a number can be transposed above or below the line by changing the sign of the exponent the division can be changed to a mulitplication, thus $456/10^7 = 456 \times 10^{-7}$. Similarly, .0000457 can be reduced to $457/10{,}000{,}000 = 457 \times 10^{-7}$. The original problem can now be written as $456 \times 10^{-7} \times 457 \times 10^{-7} = 4.56 \times 4.57 \times 10^{-7-7+4} = 4.56 \times 4.57$

$\times 10^{-10}$. $4.56 \times 4.57 = 20.8392$. This is approximately ($\cong$) 20.8 so, $4.56 \times 4.57 \times 10^{-10} \cong 20.8 \times 10^{-10}$. In this form the number is very easily written and handled. If the decimal form is desired, it can be written as $20.8/10^{10} = .00000000208$.

The powers of ten are a great aid in handling both very large and very small numbers with a minimum amount of effort and the student is encouraged to use them throughout his work in this course. Their use is practically mandatory when the slide rule is used as a means of calculating.

Examples:

1. $$\frac{364 \times 1961}{105 \times 637}$$

$$= \frac{3.64 \times 10^2 \times 1.961 \times 10^3}{1.05 \times 10^2 \times 6.37 \times 10^2}$$

$$= \frac{3.64 \times 1.961 \times 10^5}{1.05 \times 6.37 \times 10^4}$$

$$= \frac{3.64 \times 1.961 \times 10^{5-4}}{1.05 \times 6.37}$$

$$= \frac{7.138 \times 10}{6.689} = 10.67$$

2. $$\frac{14000}{5000} \times \frac{75000}{7000} = \frac{14 \times 75 \times 10^6}{5 \times 7 \times 10^6}$$

$$= 30 \times 10^0 = 30$$

Note numbers are rounded off to three significant figures to simplify the calculation. This is useful in slide rule calculations

Exercises

Express as powers of ten times a number between *one* and *ten* accurate to three significant figures:

97. 376542, 281795, 3661, 248129.3 7894321, 789542×10^4, 26.542×10^{-2}

98. .0003457, .0126, $.76542 \times 10^{-2}$, 3.4576×10, .3567, 380.9.

Solve:

99. $.00345 \times 342.9 = ?$

100. $.0854/26934 = ?$

101. $.002753/\left[2457 \times 10^{-6}\right] = ?$

102. $\frac{24381 \times .003246}{.0032 \times 4.561} = ?$

103. $\frac{.0065 \times .0000001 \times 30000}{3.24 \times 10^2 \times 2.1 \times 10^{-5}} = ?$

104. $\frac{10^{-7} \times 10^6 \times 10 \times 10^0}{10^2 \times 10^{-4} \times .0023} = ?$

105. $\frac{8^2 \times 3^2 \times 2^4}{6^2} = ?$

106. $3^3 + 4^2 + 2^{-3} = ?$

---

3. $$\frac{1647 \times .017 \times 97482}{.0079 \times 10^3 \times 4362} = \frac{1.647 \times 10^3 \times 1.7 \times 10^{-2} \times 9.7482 \times 10^4}{7.9 \times 10^{-3} \times 10^3 \times 4.362 \times 10^3}$$

$$\frac{1.647 \times 1.7 \times 9.75 \times 10^{7-2}}{7.9 \times 4.36 \times 10^{6-3}} = .793 \times \frac{10^5}{10^3} = .793 \times 10^2 = 79.3$$

107. $2^{-4} - 2^2 - 3^2 + 4^{-3} = ?$

108. $3^0 \times 4^0 \times 5^0 \times 6 = ?$

109. $3^0 + 4^0 + 5^0 + 6 = ?$

110. $(3 \times 10^4) + (5^2 \times 10^3) = ?$

111. $\dfrac{10^4 - 10^2 + 10^3}{10 \times 10^2} = ?$

EVOLUTION

The inverse process of "obtaining the original number from the power to which the number has been raised" is called "obtaining the root of a given power of a number" or more concisely, "evolution." While there are methods of extracting any root of a given number, the obtaining of the square root is the one most commonly encountered in ordinary work and will be the only method discussed here. The obtaining of other roots requires more involved mathematics than can be discussed at this point. Under the study of logarithms simplified methods of extracting any root of a number will be explained.

The square root of a number consists in finding one of the two equal numbers or factors which when multiplied together will equal the power. The cube root would be one of three equal factors, the fourth root, one of four equal factors, etc. To indicate a root, the radical sign $\sqrt{}$ is used, and the small number called the index of the root written in the opening of the radical sign indicates the root to be taken. If no number is written, then the square root is always meant. Thus, $\sqrt{6400}$ means the square root of 6,400, $\sqrt[3]{81}$ means the cube root of 81, and $\sqrt[5]{125}$ means that the fifth root of 125 is to be extracted.

The extraction of a square root is a simple process and the various steps should be memorized. The solution for a typical case will be shown step by step. The problem is to determine the square root of 54,321.

STEP 1. Write the number under the radical sign $\sqrt{54321.}$

STEP 2. Place the decimal point above the decimal point appearing in the number.

$$\begin{array}{r} \cdot \\ \sqrt{54321.} \end{array}$$

STEP 3. Starting at the decimal point, divide the number into groups of two figures each.

$$\begin{array}{r} \cdot \\ \sqrt{5,43,21.} \end{array}$$

STEP 4. It will be seen there is only one figure in the group on the left. This is to be treated as a whole group containing two figures. Find the largest whole number which when squared will be 5 or just smaller than 5. The largest whole number that can be squared and not exceed 5 is 2. Write this number above the first group.

$$\begin{array}{l} \;\;2 \qquad\quad \cdot \\ \sqrt{5,43,21.} \end{array}$$

STEP 5. Square the number and place it immediately below the first group, 5. 2 squared is 4.

STEP 6. Subtract and bring down the remainder

$$\begin{array}{l} \;\;2 \qquad\quad \cdot \\ \sqrt{5,43,21.} \\ \;\;\underline{4} \\ \;\;1\;43 \end{array}$$

STEP 7. Bring down the second group of figures.

STEP 8. Multiply the number above the radical by 20 and set this number down as the trial divisor.
Trial divisor = 20 × 2 = 40

```
  2     .
√5,43,21.
 4
 1 43
```

STEP 9. The 143 is now divided by 40. The division to the nearest smaller whole number is 3. This 3 is placed in the position now occupied by 0 in the trial divisor. Also this 3 is placed to the right of the 2 to make 23 in the root position. The number 43 is now multiplied by the 3 to give 129. This is placed under the 143. The problem is always to determine the largest number which when substituted for the 0 and the complete trail divisor is multiplied by this integer, will give a product smaller than the remainder. *The one proviso is that the integer must be the largest one possible and the product obtained smaller than the remainder.* In this case, the integer is 3, the number 43 and the product 129, which is just smaller than 143. If 4 had been chosen instead of 3, the number would have been 44 and product 176 which would be greater than the remainder 143, the condition necessary to avoid. This number must be found by either trial or inspection, depending on the problem

```
                              2 3   .
                             √5,43,21.
                              4
First trial divisor     40    1 43
Complete trial divisor  43    1 29
```

STEP 10. Subtract the 129 from 143.

```
      2 3   .
     √5,43,21.
      4
40    1 43
43    1 29
        14
```

STEP 11. Bring down the next two figures and annex them to the remainder of 14 and repeat steps 8, 9 and 10.

```
                               2 3  3 .
                              √5,43,21.
                               4
                               1 43
                               1 29
Trial divisor 20 × 23 = 460     14 21
Complete trial divisor = 463    13 89
                                   32
```

If the accuracy to three figures is sufficient for the answer to this problem, then 233 is to be considered as the square root of 54,321. If greater accuracy is desired, ciphers may be added in groups of two to the right of the decimal point, and the operations outlined above continued to as many places as desired. If the number is a perfect square, the answer or root will come out even and no remainder will result in the final step.

If the original number of which the square root is to be extracted is a decimal, the procedure is the same except that the groups are pointed off to the right of the decimal. In all cases *the groups are pointed off from the decimal point.* Examples of the pointing off into groups of various types of numbers are shown "in exercises 112 to 120

Exercises

112. $\sqrt{44\ 21\ 76.}$

113. $\sqrt{8\ 67\ 52.}$

114. $\sqrt{55\ 43.\ 21\ 76}$

115. $\sqrt{6\ 54\ 31.\ 98\ 27\ 54}$

116. $\sqrt{.24\ 36\ 77}$

117. $\sqrt{.00\ 00\ 05\ 68\ 12}$

118. $\sqrt{3\ 64.\ 57\ 20}$

119. $\sqrt{1.02\ 30}$

120. $\sqrt{.00\ 65\ 41}$

It will be observed that in all cases the decimal point in the answer which is placed on top of the radical, should be placed immediately above the decimal point in the number of which the square root is to be extracted. When the decimal part of the number contains an odd number of figures a zero is added as shown so that in pointing off each group will contain two figures. Extract the square root of each of the examples above.

Different methods of solution may give slightly different values but act as a check. That is,

$$.707 \cong \frac{1}{1.414}$$

one is approximately equal to the other. Since these are repeating decimals and not exact numbers, the use of one or the other factor will make a slight difference in the result. This would not be true for $.5 = 1/2$ since these values are equal.

Never write exponents in a form other than shown in the assignment. For example write

$$.00315 = 3.15 \times 10^{-3}$$

but don't write

$$3.15^{-3} \text{ or } 3.15^{10-3}$$

which is wrong.

CHECKING ARITHMETICAL PROCESSES

To check on arithmetical processes it is desirable to use some independent method. For example, if the student multiplies 6 × 3 = 18, how can he check his work? In this case he can divide 18 by 3 to see if he obtains 6 for an answer. This may appear obvious but in a problem such as 1,371 × 496 the answer would not be so easily checked by observation.

For division the reverse would be true. Thus if 18 ÷ 3 = 6, then it is easily seen that 6 × 3 should equal 18.

In the case of subtraction for example take 27 - 13 = 14. As a check add 14 + 13 = 27.

A few minutes spent in checking your work will often prevent mistakes, since the same mistake cannot very easily be made by an independent check. On the other hand, the student may repeat the same error in working a problem twice without checking the result independently.

SUMMARY

1. Memorize the rules.
2. Learn how to apply these rules.
3. Work the exercise problems until you are certain that you have mastered the method involved.
4. Make it a habit to check the answer to every problem.
5. Avoid short cuts.
6. Learn to develop an instinct for practical values used in mathematical problems.
7. Learn to analyze each problem and plan the method to be used before beginning the actual work.

ANSWERS TO EXERCISE PROBLEMS

1. 215,594
2. 171,418
3. 1,327,256
4. 1,069,200
5. 18,564
6. $26\,{}^{6}/_{7}$
7. $86\,{}^{6}/_{13}$
8. $69\,{}^{47}/_{54}$
9. 192
10. 8/15
11. 35/136
12. 12/55
13. $1\,{}^{3}/_{7}$
14. 1/1,260
15. $1\,{}^{281}/_{384}$
16. 18/25
17. $1\,{}^{3}/_{32}$
18. 32/45
19. 40/63
20. 256/975
21. 52.5
22. 8/15
23. 169/372
24. $3\,{}^{57}/_{280}$
25. 1/21
26. $1\,{}^{67}/_{210}$
27. 4/7
28. $39\,{}^{25}/_{33}$
29. $9\,{}^{1}/_{4}$
30. $1\,{}^{559}/_{721}$
31. 111,826
32. 3,276
33. 3,240
34. 1,320
35. 5,712
36. .665
37. 3.542
38. 8.2814
39. .622
40. .112
41. .832
42. .8
43. 3.658+
44. .564916
45. .0005457
46. .323+
47. 8,257+
48. 40.6
49. .32, .005, .0065
    2.534, .000627
50. .667-, .833+, .571+
    .0556-, .195-
51. 75%, 2.1%, 17.34%
    356%, 100.9%
52. .25, .0035, .075
    .158, .0006
53. 1,333 watts
54. 810 watts
55. 33,792 feet, 35.5 yards
    1.432+ miles
56. 1,029 inches
57. 15.84 rolls
58. 8,448 steps
59. $6\,{}^{5}/_{8}$ gals, 1,530 cu. in.
    $4\,{}^{1}/_{2}$ gals, 1,039.5 cu. in.
60. 6,300 cm; 206.7 ft.
61. .657 M., 65.7 cm
    1,852,000 M.,
    185,200,000 cm
    25.6 M., 2,560 cm
    1.57 L., 157,000 L.
    1,650 cc., 2,000 cc.
62. 9,446 cc., 9.446 L.
    9,446 gms
63. 1,034 gms/cm^2
64. 1.778 M., 79.38 Kg.
65. 50°, 14° 45', 64° 46' 37"
    No, Yes.
66. 36.7° C.
67. 492.8° F.
68. X = 20
69. P = 56.7
70. 1,320 ft.
71. 5.33 days
72. 48 feet
73. 80
74. -1097
75. 5,712
76. -742.172
77. -1.547
78. 90
79. -294

80. -205
81. -528
82. -.412
83. 0
84. 3.575
85. -26554
86. 15
87. -54528
88. 216
89. -306
90. 603.2
91. -48048
92. -5.876
93. -33
94. 662.98
95. 110
96. 115.2
97. 3.77×10^5, 2.82×10^5
3.66×10^3, 2.48×10^5
7.89×10^6, 7.90×10^9
2.65×10^{-1}
98. 3.46×10^{-4}, 1.26×10^{-2}
7.65×10^{-3}, 3.46×10
3.57×10^{-1}, 3.81×10^2
99. $1.18 \times 10^0 = 1.18$
100. 3.17×10^{-6}
101. $1.12 \times 10^0 = 1.12$
102. 5.43×10^3
103. 2.87×10^{-3}
104. 4.35×10^4
105. 256
106. 43.125
107. -12.92
108. 6
109. 9
110. 55,000
111. 10.9
112. 664.9
113. 294.5
114. 74.45
115. 255.796
116. .4936
117. .00238
118. 19.09
119. 1.011
120. .08087

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TECHNICAL ASSIGNMENT

ARITHMETIC

EXAMINATION

Perform the indicated operations:

1. (A) 276 × 314 × 27 =

276
314
1104
276
828
86664

86664
27
606648
173328
2,339,928

2, 339, 928 or 2,340,000

to 3 sig. figures

(B) 743261 ÷ 246 =

~~2614.8~~

246) 743261
592
1512
1476
366
246
1201

1201
984
2170
1968
202

~~2614.8 or 2610~~ to 3 sig. fig.

3021.386

2. (A) $\frac{7}{15} \times \frac{8}{22} \times \frac{4}{7} =$

$\frac{\cancel{7}}{15} \times \frac{8}{\cancel{22}_{11}} \times \frac{\cancel{4}^{2}}{\cancel{7}} = \frac{16}{165}$

(B) $\frac{4}{17} \div \frac{6}{25} =$ $\frac{4}{17} \times \frac{25}{6} = \frac{2}{17} \times \frac{25}{3} = \frac{50}{51}$

3. (A) $\frac{2}{7} \times \frac{3}{5} \times \frac{1}{4} =$ $\frac{1}{7} \times \frac{3}{5} \times \frac{1}{2} = 3/70$

ARITHMETIC

3. (B) $\frac{7}{8} - \frac{4}{5} =$ $\frac{35}{40} - \frac{32}{40} = \frac{3}{40}$

$LCD = 40$

4. Express as a decimal the sum of:

$.126 + 32\frac{1}{8} + \frac{7}{1000} + 1.57 + \frac{1}{16}$ $= .126 + 32.125 + .007 + 1.57 + .0625$

$= 33.8905$ or 33.9

.126
32.125
.007
1.57
.0625
33.8905

5. A concrete floor for a transmitter building is 42' 6" long and 27' 9" wide.

(A) Express the length in feet.

(B) Express the width in feet.

(C) What is its area in square feet?

(D) If the floor is 4" thick, how many cubic yards of concrete does it contain?

(A) 42.5 feet.
(B) 27.75 feet.
(C) Area = 42.5 x 27.75 = 1180 sq. ft. (slide rule)
(D) $1180 \times \frac{1}{3} \times \frac{1}{27} = 14.6$ cu. yds.

6. The power input of a motor is 12 horsepower and its power output is 11.5 horsepower. (1 HP = 746 watts).

(A) What is the input power expressed in *watts*?

746 x 12 = 8952 watts

746
12
1492
746
8952

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ARITHMETIC

EXAMINATION, Page 3.

6. (B) What is the output power expressed in *kilowatts?*

$$\frac{11.5 \times 746}{1000} = 8.579 \text{ kw.}$$

(C) Calculate the percent of efficiency of this motor.

$$Eff = \frac{Output}{Input} \times 100 = \frac{8579}{8952} \times 100 = 95.7\ \%$$

7. (A) Extract the square root of: 1,110,916. (Show your work.)

$$
\begin{array}{ll}
 & \phantom{\sqrt{}}1\ \ 0\ \ 5\ \ 4. \\
 & \sqrt{1\ 11\ 09\ 16.} \\
 & \underline{1} \\
20 & 0\ 11 \\
20\overset{5}{\cancel{0}} & 1109 \\
 & \underline{1025} \\
210\overset{4}{\cancel{0}} & 8416 \\
 & \underline{8416}
\end{array}
$$

1054.

(B) A given angle contains 42° 50' 36". What is its complementary angle?

$= 90° - 42° 50' 36"$

$= 89° 59' 60" - 42° 50' 36"$ = 47° 9' 24"

(C) A given angle contains 42° 50' 36". What is its supplementary angle?

179° 59' 60" − 42° 50' 36" = 137° 9' 24"

ARITHMETIC

EXAMINATION, Page 4.

7. (D) (45 - 18 - 36 + 9)(74 - 14 + 6) =

$= (54 - 54)(80 - 14)$

$= 0 \times 66 = \underline{0}$

8. (A) 67°F = 19.4 degrees Centigrade (Show all work.)

$C^\circ = \frac{5}{9}(F^\circ - 32^\circ) = \frac{5}{9}(67^\circ - 32^\circ) = \frac{5}{9}(35^\circ) = \frac{175^\circ}{9} = 19.4^\circ$

(B) 13°C = 286 degrees Kelvin (Show all work.)

$0^\circ C = 273^\circ K.$

$13^\circ C = 273 + 13 = 286^\circ K.$

9. One gear has 45 teeth and revolves at 1800 r.p.m.; another, driven by the first, revolves at 200 r.p.m.

(A) The kind of proportion existing between the number of teeth and the r.p.m. of the two gears is (direct, [✓ inverse]).

(B) How many teeth has the driven gear?

$\frac{45}{x} = \frac{200}{1800} \qquad x = 45 \times 9 = \underline{405 \text{ teeth}}$

10. $\frac{.0000065 \times 24000}{70000 \times .000025}$

(A) Express each factor in the above expression as a number

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ARITHMETIC

EXAMINATION, Page 5.

10. (A) between one and ten, times the proper power of 10.

$$\frac{(6.5 \times 10^{-6})\ (2.4 \times 10^{4})}{(7 \times 10^{4})\ \ (2.5 \times 10^{-5})}$$

(B) Express the answer of the above problem as a number between one and ten, times the proper power of 10.

$$\frac{(6.5 \times 10^{-6})\ (2.4 \times 10^{4})}{(7 \times 10^{4})\ (2.5 \times 10^{-5})} = \frac{6.5 \times 2.4 \times 10^{-2}}{7 \times 2.5 \times 10^{-1}}$$

$$= \frac{15.6 \times 10^{-2}}{17.5 \times 10^{-1}} = \frac{1.56 \times \cancel{10^{-1}}}{\cancel{1.76}\ 17.5 \times \cancel{10^{-1}}} = \frac{1.56}{17.5} = 0.089$$

$$= 8.9 \times 10^{-2}$$