SECTION 2

ADVANCED PRACTICAL RADIO ENGINEERING

TECHNICAL ASSIGNMENT

TUBES AND ASSOCIATED CIRCUITS

254 F

- TABLE OF CONTENTS -

TUBES AND ASSOCIATED CIRCUITS

SCOPE OF ASSIGNMENT

The conventional negative-grid tube has produced a revolution in the fields of communication and control, because it requires negligible power input to actuate it and produces appreciable power output. As a result, it has been used to amplify almost unbelievably small amounts of power and what at first seemed to be almost unbelievably high frequencies.

However, as the art progressed, a lower limit was found to the magnitude of the input signal it could amplify, because of the inherent thermal noise associated with the resistors in its input circuit, and also because of the so-called "shot-effect" noise produced in the tube itself. These limitations have been studied by many men, and considerable information is now available, although the phenomena involved are still not completely understood.

Limitations have also been found as to the high-frequency operation of the tube. One obvious limitation is that of instability; at the higher frequencies feedback through interelectrode capacitance becomes particularly troublesome.

In the early days of the art the pentode tube provided a very satisfactory solution to this problem, but unfortunately the pentode tube is relatively noisy, and so the "quieter" triode has staged a comeback in u-h-f applications.

Since the triode is unstable as regards interelectrode capacitance feedback, a modified circuit known as the grounded-grid circuit has been devised. This circuit is useful in the range above 100 mc or so, and hence will be discussed in this assignment.

However, as one proceeds to the higher frequencies, other effects arise that seriously hamper its operation. One is the inductance of the cathode lead-in wire; this causes the negative grid to act as if a resistor were connected between it and the cathode, so that considerable input power is absorbed.

Such input loading owing to cathode lead inductance can be minimized by proper design, but it does tend to counteract the normal tendency of a negative grid not to draw current and hence input power. This is the next topic to be discussed in the assignment.

However, another type of input loading occurs at the higher frequencies that is known as "transit-time" loading. It occurs because of the fact that a finite time is required for the electrons to traverse the space between the cathode and control grid and between the latter and the plate. If the frequency is high enough, the time for one cycle, or the period, is comparable to the above transit time, and gives rise to a certain amount of input resistance and reactance.

Such input loading, as well as that due to cathode lead inductance, tends to reduce the tube gain. Transit-time loading also produces a certain amount of noise in addition to the shot effect in the tube.

The result is that at the very high frequencies, weak signals tend to be "swamped" by the various sources of noise, so that the amplifier tube fails to be of value in that even if it can still amplify somewhat at such high frequencies, the noise is so great that the signal-to-noise ratio at the output of the tube is *less* than that at its input, and hence makes the tube useless for the amplification of weak signals.

This assignment will discuss transit-time loading, and will then take up the calculation of the signal-to-noise ratio employing an r-f stage and mixer, particularly for the grounded-grid connection, to show that at a sufficiently high frequency the signal-to-noise ratio is better when the antenna directly feeds the mixer stage than when it feeds an intervening r-f stage. The student will thereby obtain a more profound insight as to the operation of ordinary negative-grid tubes at the higher frequencies.

THE GROUNDED-GRID AMPLIFIER

INTERELECTRODE CAPACITANCES.- In the very early days of vacuum-tube applications, it was found that the r-f amplifier stages were unstable and would tend to oscillate. An analysis indicated that the grid-to-plate capacitance was to blame; if the plate load impedance was inductive, the input impedance would develop a negative resistance component which could cause the preceding tuned circuit to oscillate. Hazeltine then introduced his famous neutrodyne circuit to obviate this effect. It will be of interest to examine this factor in some detail.

GRID-TO-PLATE CAPACITY.—The circuit is shown in Fig. 1. The capacity C_{gk} can be tuned out by an appropriate inductance of the source, and C_{pk} by inductance in Z_L, so that these interelectrode capacities merely replace a certain amount of capacity in the tuning circuits.

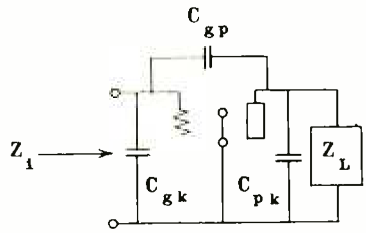


Fig. 1.—Typical amplifier circuit showing the interelectrode capacitances.

This leaves C_{gp} to be accounted for. It acts as a feedback coupling between the output (plate) and input (grid) circuits. Its effect depends upon several factors:

1) The magnitude of the output voltage across Z_L for a given magnitude of input voltage, i.e., the gain of the stage.

2) The phase of the output voltage relative to the input voltage. This in turn depends upon the nature of the impedance Z_L.

3) The operating frequency and the magnitude of C_{gp}, since the higher either is, the lower is the reactance of the coupling path.

The effect of C_{gp} is to modify the impedance Z_i seen looking into the input circuit. For all kinds of plate load impedance Z_L (inductive, capacitive, or resistive), a capacitive reactance is introduced into Z_i by the feedback through C_{gp}. Thus, an additional amount of capacity appears in parallel with C_{gk}. The total is

$$C_1 = C_{gk} + C_{gp}\ (1 + A\cos\theta) \qquad (1)$$

where A is the gain of the stage and θ is the phase angle between the input and output voltages. Both A and θ depend upon the tube and Z_L. If the latter is inductive, θ comes out positive. Since the gain A is normally greater than one and θ is usually fairly small, A cos θ is generally greater than unity, and the factor (1 + A cos θ) is therefore fairly large, which means that C_{gp} appears in the input circuit as a much larger capacity than it is in actual value. (A numerical example will be given very shortly.)

However, an even more important effect of C_{gp} is that it introduces a resistive component into Z_1 of value

$$R_{1c} = -\frac{1/\omega C_{gp}}{A\sin\theta} \qquad (2)$$

For positive values of θ (inductive loads), R_{1c} comes out to be a *negative* resistance, which means that if a tuned circuit of sufficiently high Q is connected to the input terminals, the stage will oscillate. It was this effect that led to the neutrodyne circuit and to the development of the screen grid tubes, in which the screen grid (operated at a-c ground) acted as an electrostatic shield between the grid and plate, and converted C_{gp} essentially into two capacities to ground.

As an example of the feedback effects, suppose one has a tube for which C_{gk} = 7 μμf, C_{gp} = 4 μμf, f = 10^6 c.p.s., and the gain at that frequency is 20 at an angle of +40°, i.e., the output voltage is 20 times as great as the input voltage and leads it by 40°. This indicates that Z_L is inductive. Then the total capacity is

$$C_1 = 7 + 4(1 + 20\cos 40°)$$

$$= 7 + 4(16.32) = 7 + 65.3$$

$$= 72.3\ \mu\mu f$$

Note that the effect of C_{gp} of value 4 μμf is as if 65.3 μμf were connected between the grid and the cathode!

The apparent resistance R_{1c} between the grid and ground (hence in parallel with C_1) is, by Eq. (2)

$$R_{1c} = \frac{-\dfrac{1}{2\pi 10^6 \times 4 \times 10^{-12}}}{20\sin 40°} = -3{,}090\ \Omega$$

Suppose the grid is fed from a tuned circuit of another tube, and that this circuit has an equivalent shunt resistance of 50,000 ohms. Then the total resistance across the tuned circuit is its own 50,000 ohms in parallel with the tube's (-3,090) ohms, and has the value

$$R = \frac{(50{,}000)\ (-3{,}090)}{(50{,}000 - 3{,}090)} = -3{,}290 \text{ ohms}$$

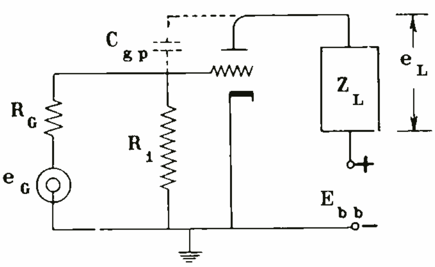
Since this is negative, it indicates *negative damping* for the tuned circuit, or oscillations.

As stated above (and in a previous assignment on screen-grid tubes), the screen grid tube was designed to eliminate or reduce to a negligible minimum C_{gp} and the above effects. However, at ultra-high frequencies the screen grid tube is not as desirable as the triode because it is relatively noisier, and one is left to deal with the triode and the plate-to-grid capacity. In such a case the tube is operated with the grid grounded, and the signal fed in between the cathode and ground.

This type of amplifier is particularly suited to be interposed between low impedance sources, such as dipole antennas, and high impedance loads, such as high Q resonant circuits. It therefore functions essentially as an impedance-transforming device, but suitable r-f transformers can be employed in conjunction with it to enable it to function primarily as an amplifier stage. Its operation will now be discussed.

THE GROUNDED-GRID CIRCUIT.—The tendency of the triode to regenerate and oscillate when employed in a conventional amplifier stage, as shown in Fig. 2(A) was shown to be potential, acts as a screen between the two electrodes and causes each to have a certain amount of capacity to it (ground) instead of to one another, thus eliminating C_{gp}. However, as indicated above, the pentode is "noisier" and loses much of its value where weak signals are to be amplified, such as at ultra-high frequencies.

To avoid regeneration, or at least to minimize it, a triode can be operated as in Fig. 2(B). Here the grid is grounded, and the cathode and plate are "high" to ground. Hence the feedback capacitance should be that between cathode and plate, C_{kp}. Since the grid is between these two elements and tends to shield one from the other, it can be expected that C_{kp} is smaller than the grid-to-cathode capacitance C_{gk},



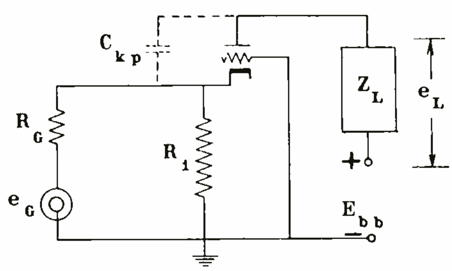


Fig. 2—Grounded-cathode and grounded-grid amplifier stages.

due to the feedback between the two electrodes that are "high" to ground, namely, the plate and grid, with feedback occurring through the plate-to-grid capacity C_{gp}. It is for that reason that the pentode tube was introduced, for the screen grid, when operated at a-c ground or the plate-to-grid capacitance C_{gp}. Hence less difficulty should be experienced operating a triode with the grid grounded than with the cathode grounded, particularly at high frequencies where the capacitive reactance is low and the feedback therefore large. This is found to be the case. The amplifier shown in Fig. 2(B) is commonly known as the grounded-grid amplifier.

There is an important advantage of the conventional cathode-grounded amplifier over the grounded-grid type, particularly at *low frequencies*. This is that the former requires negligible power from the signal source, i.e., draws no appreciable current from the source. The source is shown in Fig. 2(A) or (B) as having a generated voltage e_G and an internal resistance R_G.)

As a result, the voltage between the grid and the cathode in (A) is practically the generated voltage e_G. On the other hand, the grounded-grid amplifier of (B) has a low input impedance looking into the cathode and ground terminals. The magnitude depends upon the r_p of the tube, the load impedance Z_L, and the μ of the tube, as will be shown. Thus the grounded-grid amplifier draws appreciable current from the source, and the voltage between the cathode and ground can be markedly less than the generated voltage e_G. The result is that at the lower frequencies, there is obtained much less gain from the grounded-grid stage.

At the higher frequencies, however, either type of stage exhibits the phenomenon of input loading, whether owing to interelectrode capacitance, lead inductance, or transit-time effects. This is represented by the resistance R_1 in Fig. 2. It therefore does not make much difference which type of circuit is employed so far as gain is concerned. Furthermore, it can be shown that the signal/noise ratio is about the same for either, and this is the most important criterion, especially at u.h.f. Therefore the grounded-grid amplifier becomes the preferred form of circuit because of its greater freedom from regeneration, and so this type of amplifier stage has found favor at least in the medium u-h-f range, say from 100 to perhaps 900 mc.

INPUT IMPEDANCE. — From the foregoing it is evident that the input impedance of the grounded-grid amplifier should be an important quantity. Its value can be easily derived with the aid of Fig. 3. (Note that the d-c voltages have been omitted in this figure for clarity.) To calculate the impedance between the two terminals AB, it is convenient to assume that a generator having zero impedance is connected to the terminals. The current i that its generated voltage e produces can be calculated, and then the impedance between A and B is simply

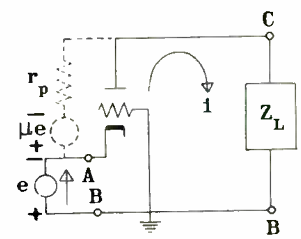


Fig. 3.—Circuit showing factors involved in deriving the input impedance of a grounded-grid amplifier.

$$Z_{AB} = e/i$$

In Fig. 3 note that the application of a voltage e between the cathode and grid (ground) produces an apparent voltage μe in the plate circuit. This is the normal tube action. Voltage μe is *in phase* with e, as shown, and thus helps e to pump current i around the circuit, i.e., current i is larger owing to the helping action of μe. The resulting impedance is consequently lower than it would otherwise be.

From the figure it is clear that

$$i = (e + \mu e)/(r_p + Z_L) = e(1 + \mu)/(r_p + Z_L) \quad (3)$$

from which

$$Z_{AB} = \frac{e}{i} = \frac{r_p + Z_L}{1 + \mu} \quad (4)$$

This means that the input impedance Z_{AB}^* depends upon the tube (μ and r_p) *and also upon the output impedance* Z_L.

As a simple example, consider a 6C4 triode. It has a μ of 19.5 and an r_p of 6250 ohms. Suppose the plate load impedance Z_L is 5000 ohms resistive. Then

$$Z_{AB} = \frac{6250 + 5000}{1 + 19.5} = 548 \text{ ohms}$$

For a grounded-cathode stage Z_{AB} would be practically infinite at the lower frequencies.

OUTPUT IMPEDANCE.—In a similar manner it can be shown that the impedance Z_{CB} which the plate load Z_L sees, looking back into the tube, is affected by the feedback relations in the tube, and depends upon the *impedance between cathode and ground.* Suppose the impedance between the cathode and ground is Z_k. This may be composed of the source impedance R_G and the input loading R_1 in parallel, Fig. 4. Then

$$Z_{CB} = r_p + (\mu + 1)Z_k \quad (5)$$

or the output impedance depends upon the tube and the impedance between cathode and ground.

As a continuation of the previous example, suppose that R_1 is 1200 ohms, and R_G is 1000 ohms, so that the input impedance between grid and cathode as viewed from the output or the internal output impedance, as it is sometimes called, is

$$Z_k = \frac{1200 \times 1000}{1200 + 1000} = 546 \text{ ohms}$$

Then the output impedance looking in at C and B will be

$$Z_{CB} = 6250 + (19.5 + 1)(546) = 6250 + 11{,}200 = 17{,}450 \text{ ohms}$$

In the conventional grounded-cathode stage the impedance looking back into the tube would be r_p, or 6,250 ohms. Owing to feed-back effects, the impedance looking back into a grounded-grid stage exceeds r_p by the quantity $(\mu + 1)Z_k$, which in this example is 11,200 ohms and in itself exceeds r_p.

IMPEDANCE MATCHING.—The minimum input impedance, as given by Eq. (4) occurs when Z_L is zero, and is then $Z_{AB} = r_p/(1 + \mu)$. The minimum output impedance, as given by Eq. (5), occurs when Z_k equals zero, and is then $Z_{CB} = r_p$. If an input transformer of the proper impedance ratio is employed between the signal source and the cathode, and an out-

*Note that Z_{AB} is the impedance looking into the grid circuit owing to the method of connection. In addition to this there is the input, R_1, which becomes important at high frequencies, and which is in parallel with Z_{AB}.

put transformer is used between the plate of the tube and the load resistance, Z_L, then it may be possible to have the source impedance

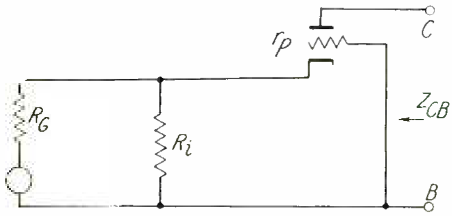


Fig. 4.—Circuit considerations involved in calculating the internal output impedance of a grounded-grid tube.

R_G matched to the input impedance Z_{AB} of the tube, *in parallel with the tube's input loading* R_1, *and at* the same time have the load Z_L matched by means of the output transformer to the tube, regardless of the minimum values of Z_{AB} and Z_{CB}. Note that matching on one side involves the impedances on the other side of the circuit.

The circuit is shown in Fig. 5. Transformer T_1 and T_2 are of the tuned type. Quarter-wave transmission lines can be used in their place. If a higher Q is desired, in

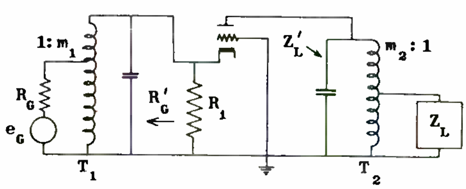


Fig. 5.—Use of matching transformers to match the input and output impedances of a grounded-grid stage to the source and load impedances, respectively.

order to narrow the band width, both impedances on either transformer can be connected to points tapped down on the transformer, as has been discussed in previous assignments. It will be assumed that the voltage ratio for T_1 is m_1, and for T_2 is m_2, as indicated in the figure.

The impedance R_G' is that seen looking back toward the source, and is by the rules of impedance transformation, equal to $m_1^2 R_G$. Similarly, the impedance that the plate of the tube sees is $Z_L' = m_2^2 Z_L$. The impedance between the cathode and ground is R_1 and R_G' in parallel, or

$$Z_k = \frac{R_1 m_1^2 R_G}{R_1 + m_1^2 R_G} \qquad (6)$$

Therefore, the impedance seen looking into the plate of the tube, is by Eq. (5)

$$Z_{CB} = r_p + (\mu + 1)\frac{R_1 m_1^2 R_G}{R_1 + m_1^2 R_g} \qquad (7)$$

This is to match Z_L', so that

$$Z_L' = m_2^2 Z_L = r_p + (\mu + 1)\frac{R_1 m_1^2 R_G}{R_1 + m_1^2 R_G} \qquad (8)$$

On the input side, the impedance $R_G' = m_1^2 R_G$ is to be matched to R_1 in parallel to the input impedance of the tube. The latter is, by Eq. (4)

$$Z_{AB} = (r_p + m_2^2 Z_L)/(\mu + 1) \qquad (9)$$

Hence, for impedance matching on the input side, it is necessary that—

$$R_G' = m_1^2 R_G = \frac{R_1 \; (r_p + m_2^2 Z_L)/(\mu + 1)}{R_1 + (r_p + m_2^2 Z_L)/(\mu + 1)} \qquad (10)$$

Eqs. (8) and (10) determine the voltage ratios m_1 and m_2, since all factors in these equations are given. Hence, if the two equations be solved for m_1 and m_2, there is obtained

$$m_1 = \sqrt{R_1/MR_G}$$
$$m_2 = \sqrt{Mr_p/Z_L} \qquad (11)$$

where $M = \sqrt{1 + (\mu + 1)(R_1/r_p)}$ is a quantity that appears in several of the formulas for the grounded-grid amplifier.

GAIN OF GROUNDED-GRID AMPLIFIER. The gain of a grounded-grid amplifier should be taken as the ratio of the output voltage e_L across the load Z_L to the *generated signal voltage* e_G *of the source.* This will take into account the voltage drop in the source impedance R_G owing to the input losses R_1 and the finite input impedance of the tube itself. The voltage drop owing to the latter impedance ($= [r_p + m_2^2 Z_L]/[\mu + 1]$) is properly charged to the stage. In the case of a grounded-cathode amplifier the latter impedance is infinite, and the voltage drop in the source is due to R_1, only, the input loading that occurs at high frequencies regardless of the type of stage employed.

The physical significance of the foregoing may be better appreciated by referring to Fig. 6 which is substantially the same as Fig. 5.

The generator faces a certain amount of impedance determined by R_1 and the input impedance of the stage, as transformed by the input transformer T_1. Hence the *terminal voltage* of the generator e_T is less than its generated voltage e_G owing to the voltage drop in R_G. Then e_T is stepped up to the value $m_1 e_T$ by the input transformer. It is then amplified by the tube and appears as $m_2 e_L$ at the plate of the tube. Finally it is stepped down by the output transformer to the value e_L at the load impedance Z_L. As stated previously, Z_L generally represents the input loading of the following tube.

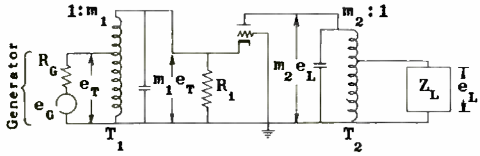


Fig. 6.—Circuit involving matching transformers for calculating the gain of a grounded-grid amplifier.

At high frequencies Z_L is fairly low, whereas the impedance looking back into the grid-grounded stage is high, as indicated in the previous numerical example. Hence the step-down transformer T_2 is justified to match the two unequal impedances.

On the input side the grounded-grid amplifier presents a low impedance, as also illustrated previously by a numerical example. Usually, however, the source impedance R_G is considerably lower, and so an input step-up transformer T_1 is justified to match the two unequal impedances at this point.

On the other hand, the tube acts as an impedance transforming device in that at its input terminals Z_L appears as $Z_L/(\mu + 1)$ and at its output terminals Z_k appears as $(\mu + 1)Z_k$. Hence sometimes, to simplify the circuit, transformers T_1 and T_2 are omitted, but it will be found that the gain of the stage is appreciably reduced.

The gain of a grounded-grid amplifier, for any value of m_1 and m_2, is

$$\alpha = \frac{(\mu + 1)\, m_1 m_2\, R_1 Z_L}{m_1^2 R_G\,[r_p M^2 + m_2^2 Z_L] + R_1\,[r_p + m_2^2 Z_L]} \quad (12)$$

If $m_1 = m_2 = 1$, and $R_1 = \infty$

$$\alpha = \frac{Z_L}{R_G + \dfrac{r_p + Z_L}{\mu + 1}}$$

which means that input current for 1 volt input is

$$\frac{1}{R_G + \dfrac{r_p + Z_L}{\mu + 1}}, \text{ and}$$

this multiplied by Z_L gives output volts, which also numerically represents the gain. If impedance matching is employed by substituting the values of m_1 and m_2 given by Eq. (11) the matched gain is obtained. This is

$$\alpha_m = \frac{(\mu + 1)\sqrt{R_1 Z_L / r_p R_G}}{2(1 + M)} \quad (13)$$

where $M = \sqrt{1 + (\mu + 1)(R_1/r_p)}$, as before.

The matched gain α_m also represents the *maximum gain* for a given tube, and for given values of R_1 and Z_L. Hence normally input and output transformers are employed. The matched condition has the further advantage that if the source is an antenna which feeds the tube through a transmission line, the latter will have minimum losses and have no detuning effect if it is matched to the tube.

NUMERICAL EXAMPLE.—A numerical example will be of value at this point. Suppose a 6J6 tube is used as an r-f grounded-grid amplifier, and that it feeds a 6J6 mixer tube. The operating frequency is 250 mc. Assume the input impedance for the mixer is 2400 ohms, and this represents Z_L to the first tube. The input impedance R_1 of the first tube will be taken as 640 ohms. Suppose a dipole antenna of 75 ohms internal resistance, R_G, be employed. Further values are $\mu = 32$, $r_p = 6{,}000$ ohms. Under matched conditions, Eq. (13) gives the gain. First M is calculated. Its value is

$$M = \sqrt{1 + (32 + 1)(640/6000)} = \sqrt{4.52}$$

$$= 2.13$$

Then

$$\alpha_m = \frac{(32 + 1)\sqrt{640 \times 2400/6000 \times 75}}{2(1 + 2.13)}$$

$$= 9.76$$

The values of m_1 and m_2 are found from Eq. (11). Thus

$$m_1 = \sqrt{640/2.13 \times 75} = \sqrt{4.01} = 2.01$$
(step-up)

$$m_2 = \sqrt{2.13 \times 6000/2400} = \sqrt{5.33}$$

$$= 2.31 \text{ (step-down)}$$

The value of $\alpha_m = 9.76$ is somewhat high, but is due to the fact that the gain (step-up) of the antenna input transformer is included in the value for α_m. This is justified in that the step-up m_1 depends upon the input loading effect of the tube circuit as well as the ordinary u.h.f. loading effect represented by R_1.

Suppose an ordinary cathode-grounded circuit had been employed. Then the only load on the input transformer would have been R_1, as shown in Fig. 7. Here it will be

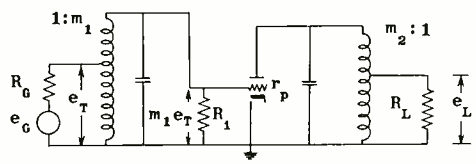


Fig. 7.—Grounded-cathode circuit used for comparison with grounded grid circuit.

assumed that R_L is matched to the output impedance of the tube, which here is simply r_p. In other words, $m_2^2 R_L = r_p$ or $m_2 = \sqrt{r_p/R_L}$. This gives maximum output voltage e_L for a given input voltage $m_1 e_T$ at the grid. The input impedance of the tube, R_1, is also assumed matched to the source impedance R_G, which gives maximum voltage $m_1 e_T = m_1 e_G/2$ at the grid of the tube.

Under these conditions the gain of the cathode-grounded stage is

$$\alpha_k = \frac{\mu\sqrt{\dfrac{R_1 Z_L}{R_G r_p}}}{4} \qquad (14)$$

Substitution of the appropriate values yields

$$\alpha_k = \frac{32}{4}\sqrt{\frac{640 \times 2400}{75 \times 6000}} = 14.80$$

Thus the cathode-grounded stage has more than 50% greater gain than the grid-grounded stage, but requires much more elaborate neutralization than the former. At higher frequencies the difference in gain is even less marked, although in either case the gain approaches zero as R_1 decreases.

COMPARISON OF AMPLIFIER STAGES. At low frequencies—such as in the standard broadcast band—the input loading effect becomes negligible; this means that R_1 and Z_L, as interpreted above, become infinite. However, the losses in the input and output transformers now become the determining factors for R_1 and Z_L. In the above analysis they were ignored because they were negligible compared to the input loading at the frequencies under consideration.

As has been mentioned in many of the previous assignments, the losses in a tuned circuit may be represented—at and around the resonant frequency—by a resistance shunting L and C. Thus the shunt resistance of the input transformer becomes R_1, and that of the output transformer becomes R_L. Specifically, the output circuit will look as shown in Fig. 8, where the output transformer actually steps-up the voltage, and R_L represents its losses, principally those in the coil. In an actual circuit, inductive rather than conductive coupling is

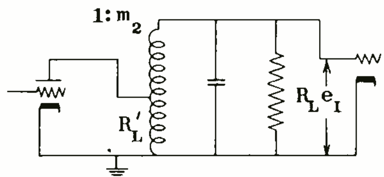


Fig. 8.—At low frequencies the output transformer has a step-up rather than a step-down ratio.

employed between the plate and the tuned coil, but the action is identical.

The grounded-cathode stage now shows its superior gain characteristics compared to the grounded-grid stage in that the tube does not present any appreciable impedance to load down the input circuit, as does the grounded-grid stage at all frequencies, and thus the step-up m_1 of the input transformer for the conventional grounded-cathode stage can be higher, as it is determined solely by the circuit losses of the transformer itself.

Thus the advantage in gain of the grounded-cathode circuit is due to the higher step-up ratio of the input transformer. At high frequencies, where input loading R_1 of the tube becomes the principal factor, the step-up ratio is determined to a great extent by R_1, and hence is not markedly different for the two types of circuit. Consequently the gain for either type is more nearly the same, and thus the grid-grounded stage becomes desirable because of its greater freedom from regeneration.

One further point will be of interest. Suppose the input and output transformers were omitted, and the antenna connected directly to the cathode, and the load Z_L directly to the plate. In such a case Eq. (12) can be employed since it gives the gain in general for such a stage. One merely has to set the turns-ratios m_1 and m_2 equal to unity. Thus

$$\alpha = \frac{(32 + 1)(640)(2400)}{75\left[(6000)(2.13)^2 + 2400\right] + (640)\left[6000 + 2400\right]} = 6.68$$

This is an appreciably lower gain than when transformers of optimum voltage ratios are employed, and indicates the value in using the transformers.

As a final point, suppose the antenna were connected directly to the output impedance Z_L through a suitable matching transformer and the tube omitted. Its voltage ratio would be $m = \sqrt{Z_L/R_G} = \sqrt{2400/75} = 5.66$. Under matched conditions half of the generated voltage e_G is consumed in the source impedance R_G, so that $e_G/2$ is available at the transformer, and $me_G/2 = e_G/2\sqrt{Z_L/R_G}$ is available at the load. The ratio of the latter voltage to e_G, or the gain, is simply $1/2\sqrt{Z_L/R_G}$. Thus, in this example, the gain would be $5.66 \div 2 = 2.83$, which is less than one-half that of the tube without transformers.

It therefore indicates that the use of a vacuum tube is justified even at ultra-high frequencies. Thus, by the use of two transformers an overall gain of 9.76 can be obtained instead of the 2.83 when a single transformer alone is employed, this makes the value of the stage apparent.

PRACTICAL CIRCUITS.—In Fig. 9 are shown three practical grounded-grid circuits for use with a 6J4 U.H.F. MINIATURE triode (courtesy RCA). These circuits are characterized by the use of a cathode bias resistor (adequately by-passed for r.f.) to limit the plate current to a safe value.

The 6J4 is a very high G_m tube (approximately 12,000 μmhos) and requires this precaution. Another point to note is that the heater circuit is "lifted" above ground by means of r-f chokes so as to be substantially at cathode potential. This prevents the heater-to-cathode capacity (equal to 2.8 μμf and shown

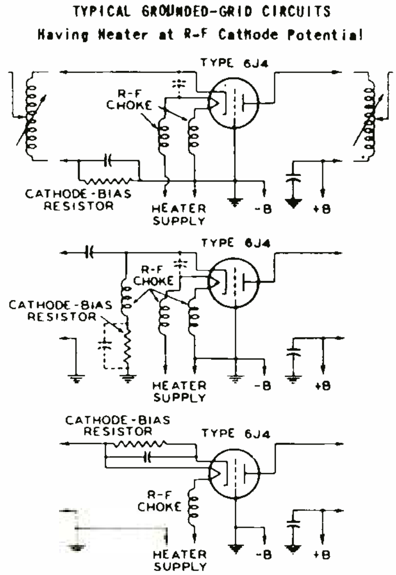


Fig. 9.—Some practical grounded-grid circuits using a 6J4 tube.

by dotted lines) from being an appreciable shunt across the cathode (input circuit). A further point is that the grid has leads brought out to three grid terminals, and all three terminals should be grounded in order to minimize grid lead inductance, as it is this inductance, rather than cathode lead inductance, that produces input loading in this type of amplifier, (in addition to transit-time loading).

An important point to note about the tube is that its plate-to-cathode capacity C_{pk} is only .24 μμf, whereas C_{gk} = 5.5 μμf and C_{pg} = 4 μμf. This indicates that regeneration should be much less when the tube is operated in the grounded-grid circuit.

INPUT LOADING

GAIN OF AMPLIFIER STAGE.—Mention has been made of the absorption of power by the input of an amplifier stage at the higher frequencies, owing to cathode-lead inductance and transit-time effects (as well as the grid-to-plate capacitance feedback at even the standard broadcast frequencies). The grounded-grid amplifier absorbs additional power at its input terminals, although this power is not wasted as heat, but instead appears as part of the output. It will now be of interest to examine the cathode-lead inductance and transit-time loading effects in greater detail.

First consider the gain of an amplifier stage operating at relatively low frequencies. Such a stage is shown in Fig. 10(A). The load impedance Z_L may be a pure resistance, inductance, etc.

The resistance R_1 represents a load effect that the grid input circuit presents to the exciting source generating voltage e_s. At relatively low frequencies, if the grid is biased negative, so that no conductive grid current flows, R_1 will be practically infinite. Hence the power delivered to it will be $e_s^2/R_1 = 0$, i.e., the tube will draw no power from the grid exciting source. There may be some capacity currents drawn from e_G owing to the interelectrode capacities, but this represents *reactive*, rather than real power, and the effect of such capacities is generally eliminated by tuning them out, as in r-f work.

Now consider the output power. First e_L must be found. By means of the Equivalent Plate Circuit Theorem, Fig. 10(A) may be replaced by Fig. 10(B), where a voltage μe_s acts in series with r_p and Z_L. The output voltage is clearly

$$e_L = \mu e_s \frac{Z_L}{r_p + Z_L} \qquad (15)$$

If the numerator and denominator of the right-hand expression are multiplied by r_p, there is obtained

$$e_L = \frac{\mu}{r_p} e_s \left(\frac{r_p Z_L}{r_p + Z_L}\right) = G_m e_s \left(\frac{r_p Z_L}{r_p + Z_L}\right) \qquad (16)$$

where $G_m = \mu/r_p$ is the transconductance of the tube. The quantity $r_pZ_L/(r_p + Z_L)$ is the expression for the parallel impedance of r_p and Z_L. Since G_m represents the change in plate current for *unit* change in grid voltage, $G_m e_s$ represents the change in plate current for a *total change* in grid voltage of e_s volts.

Hence a circuit as shown in Fig. 10(C) can be used to represent Eq. (16). Here a constant current generator furnishes current $G_m e_s$ to r_p and Z_L in parallel, and develops across them a voltage e_L. The *voltage* gain is the ratio of voltage output to voltage input, and from Eq. (16) it is clear that

$$\alpha = \frac{e_L}{e_s} = G_m \left(\frac{r_p Z_L}{r_p + Z_L}\right) \qquad (17)$$

Suppose the tube feeds the grid circuit of another similar tube. Then the latter's input resistance, also of value R_1, is in parallel with Z_L and r_p. If Z_L is a tuned circuit resonant at the frequency of e_s, it will appear as a pure resistance, call it R_L. Thus the plate load is r_p, R_L, and R_1 in parallel. At low frequencies, R_1 is practically infinite, and hence the plate load reduces essentially to r_p and R_L in parallel.

At very high frequencies, however, R_1 may drop to a very low value, such as 500 ohms or less, depending upon the frequency. (The reason for this will be discussed in the following assignment.) On the other hand, the G_m and r_p of the tube do not change markedly with frequency even in the range where R_1 begins noticeably to decrease. Furthermore, it has been shown in the previous assignment on U.H.F.

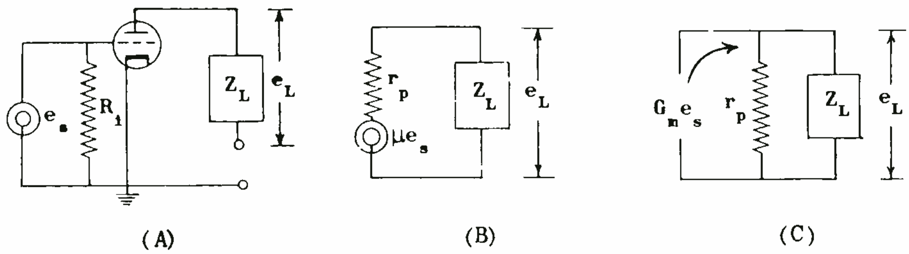


Fig. 10.—Equivalent circuits for amplifier stage.

Techniques that high-Q resonant circuits, of high equivalent R_L at resonance, can be obtained at high frequencies by means of tuned transmission lines and cavity resonators.

Hence, at very high frequencies, the plate load becomes essentially R_1 ohms, with r_p and R_L as negligibly high shunt resistances paralleling R_1. Eq. (17) becomes

$$\alpha = G_m R_1 \qquad (18)$$

At some frequency R_1 reduces to a value such that $G_m R_1$ is unity. For example, if G_m = 2,000 μmhos, then, at a frequency where R_1 is 500 ohms, $G_m R_1$ = (.002 mhos) × (500 ohms) = 1 = α. At this frequency there is no amplification of the voltage, or $e_L = e_s$. Above this frequency R_1 decreases, and the tube actually deamplifies.

Note that at the critical frequency, the output power is e_L^2/R_1 and equals the input power e_s^2/R_1, so that one might just as well connect the source of e_s directly to the ultimate load as to connect it through an amplifier; the latter will not increase the power output over the input power. It is evident that the figure of merit of a tube is the product of its transconductance G_m and its input impedance R_1; the higher this is the better amplifier the tube will be at high frequencies.

CATHODE-LEAD INDUCTANCE. — At frequencies above about 40 mc the negatively-biased-grid tube exhibits a type of input loading that is due essentially to the inductance of the cathode lead. While this is a characteristic inherent in the tube, rather than in the associated circuit, it is an effect due to the circuit characteristic of the tube, just as are the interelectrode capacities, rather than due to an electronic behavior within the vacuum. In other words, cathode-lead inductance, like grid-to-plate capacity, is a circuit effect, and can be accentuated by artificially adding inductance *externally* to the cathode lead. On the other hand, transit-time loading is an electronic effect within the tube, and cannot be accentuated by adding a circuit element external to the tube. Hence cathode-inductance loading will be discussed in this assignment, whereas transit-time loading will be discussed in the next assignment that deals more specifically with tubes by themselves.

An analysis of the cathode-follower stage is given in the specialized Mathematics section. A simplified derivation will be given here to indicate the method of analysis. In Fig. 11 is shown a screen-grid amplifier stage. The inductance L_k

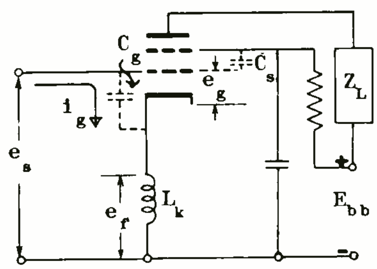


Fig. 11. — Screen-grid amplifier stage, showing inductance of cathode lead.

represents the inductance of the connection within the tube from the cathode to the tube pin, plus the socket lead up to the point where the grid and plate circuit leads connect to it. It is evident that this latter lead should be made as short as possible if the effect of cathode inductance is to be minimized.

The capacity C_g represents the capacity between the control grid and the cathode; and C_s, that between the control grid and the screen grid. Since a pentode tube is assumed, the plate load Z_L is normally small compared to the r_p of the tube and hence will not appreciably affect the flow of plate current. The a-c component of the latter is $G_m e_g$ where e_g is the actual signal voltage between the control grid and the cathode. In flowing through L_k it sets up a voltage drop e_f that opposes the impressed voltage e_s. The net voltage appearing between the control grid and the cathode is therefore not e_s, but

$$e_g = e_s - e_f$$

The value of e_f is clearly $j\omega L_k(G_m e_g)$ so that

$$e_g = e_s - j\,\omega L_k G_m e_g$$

or, transposing terms and factoring out e_g

$$e_g(1 + j\,\omega L_k G_m) = e_s$$

from which

$$e_g = e_s/(1 + j\,\omega L_k G_m) \qquad (19)$$

The current through the capacitor C_g is

$$i_g = e_g\,(j\,\omega C_g) = e_s\left(\frac{j\,\omega C_g}{1 + j\,\omega L_k G_m}\right)$$

$$= e_s\left(\frac{j\,\omega C_g}{1 + j\,\omega L_k G_m}\right)\left(\frac{1 - j\,\omega L_k G_m}{1 - j\,\omega L_k G_m}\right)$$

$$= e_s\left[j\omega\left(\frac{C_g}{1 + \omega^2 L_k^2 G_m^2}\right) + \left(\frac{\omega^2 L_k C_g G_m}{1 + \omega^2 L_k^2 G_m^2}\right)\right] \qquad (20)$$

For the usual tube L_k is very small-usually less than .1 µH, so that in the usual frequency range under consideration, $\omega^2 L_k^2 G_m$ is normally much less than unity. Hence this term can be ignored compared to one in the denominator of the expressions given in Eq. (20), so that the latter simplifies down to

$$i_g \simeq e_s(j\,\omega C_g + \omega^2 L_k C_g G_m) \qquad (21)$$

The input admittance due to the cathode inductance is

$$A_1' = i_g/e_s = j\,\omega C_g + \omega^2 L_k C_g G_m \qquad (22)$$

To this must be added the susceptance of the additional capacitor C_s, so that the total admittance is

$$A_1 = j\,\omega(C_g + C_s) + \omega^2 L_k C_g G_m \qquad (23)$$

DISCUSSION OF RESULTS.— The above expression contains two terms, one involving $j\omega$, and the other real. The first represents a capacitive *susceptance*; the second, a *conductance*. Since the *sum* is involved, this means the two are in *parallel*, and may be represented as in Fig. 12.

The effect of the cathode lead inductance is therefore to introduce a resistance across the input circuit of value $R_1 = 1/\omega^2 L_k C_g G_m$). Attention is called to the fact that R_1 is inversely proportional *to the* *square of the frequency* and the *first power of G_m*. Later on it will be shown that a relationship of precisely the same form is produced by transit-time effects. It is for that reason that the two effects were at first confused with one another. The recognition of the effect of cathode lead inductance is generally credited to M.J.O. Strutt.*

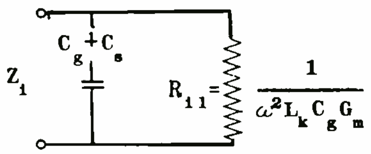


Fig. 12.—Cathode-lead inductance produces an input admittance consisting of a capacitor and resistance in parallel.

As a simple example of this effect, suppose G_m = 2,500 μmhos = .0025 mhos. C_g = 4 μμf, L_k = .08 μh, and f = 50 mc. Then

$$R_{ii} = \frac{1}{(2\pi\ 50 \times 10^6)^2\ (4 \times 10^{-12})\ (8 \times 10^{-8})\ (25 \times 10^{-4})} = 12,630 \text{ ohms}$$

Although this value is much higher than $1/G_m$ = 1/.0025 = 400 ohms, and will not reduce the gain to unity, as discussed above, it nevertheless represents an appreciable loading on the tube, particularly in view of the relatively low frequency involved, namely, 50 mc.

REDUCTION OF CATHODE LOADING EFFECTS.—Although the inductance of the cathode lead is the major factor in producing input loading, the inductance of the leads of the other electrodes is of importance too. For example, the inductance of the screen grid lead tends to increase the value of R_i and thus to reduce its effect, in proportion to the magnitude of the a-c component of the screen current, and the value of C_s. This effect is usually small, however, because the screen current is usually a small fraction of the cathode current, which includes the plate current as well.

In the case of a triode tube the control-grid-to-plate capacitance, in conjunction with the plate lead inductance, can increase R_i to a fairly large value and thus make its shunting effect of much less consequence. Further effects, of lesser consequence, are the mutual inductance effects between the various leads. The complete analysis is very involved, and does not modify unduly the simple analysis presented above.

One method of reducing the effect of the cathode lead inductance L_k is to modify the construction of the tube as shown in Fig. 13. Here a typical circuit is shown in conjunction with the tube. Essentially what has been done is to place the junction A of the grid and plate circuits inside the *tube* and *as close to the cathode sleeve* as possible. From point A *separate leads*

*See "The Causes for the Increase of the Admittances of Modern High-Frequency Amplifier Tubes on Short Waves," M.J.O. Strutt and A. Van der Ziel, Proc. I.R.E., Aug. 1938.

are brought out to separate socket connections.

The inductance of these, L_{gk} and L_{pk}, are in the grid and plate circuits, respectively, and do not produce any effect in each other's circuit. The common inductance, L_k, is that of the cathode sleeve, and is roughly equivalent to a conductor

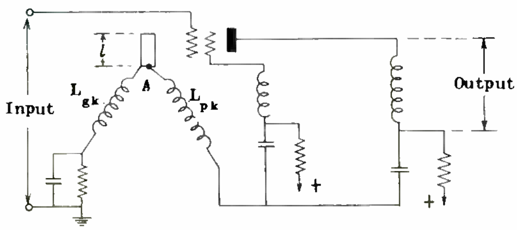


Fig. 13.—Method of reducing the cathode-lead inductance effect.

having the diameter of the cathode sleeve, and half its length, or $l/2$. Tubes so constructed are particularly well suited for u.h.f. operation. However, they are still subject to input loading owing to transit time-effects, which will be discussed in the following assignment.

NOISE

RESISTANCE NOISE.—Most radiomen are acquainted with noisy resistors, which cause trouble in receivers and similar equipment. Such resistors are defective and usually their resistance fluctuates owing to poor contact or similar fault.

However it has been found that even the most perfect resistors develop some noise. Mention of this has been made in a previous assignment. A more detailed discussion will be given here. A theoretical study by J. B. Johnson* has shown that such noise is produced by the random motion of the free electrons in the resistance. Consider a small section of such a resistance, as shown by the symbol Δl in Fig. 14.

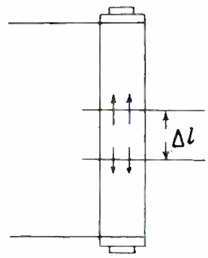


Fig. 14.—Conditions within a resistor that tend to produce thermal noise.

The free electrons in that section are darting around in all directions, very much like the molecules of a gas at room temperature. The random velocities may be resolved by the method of vector composition of forces, velocities, etc., into components across the resistor (parallel to the cross section), and components along the resistor (at right angles to the cross section).

Recall that the velocity of charged particles, such as electrons, represents current flow. Then the components along the resistance represent minute current flows out of or into the section Δl. Let the resistance of this section be ΔR. Then the minute current multiplied by ΔR, represents a small IR drop across Δl. Similar considerations

*"Thermal Agitation of Electricity in Conductors," Phys. Rev., Vol. 32, pp. 97-110, July, 1928.

hold for every other section of the resistance, and the total voltage appearing across the ends or terminals is the algebraic sum of these IR drops.

It might be reasoned that just as many electrons are flowing in one direction through Δl as are flowing in the opposite direction, so that there are two opposing currents in Δl that cancel one another. This is true as an *average condition* over any appreciable length of time, *but at any instant* more current may be flowing downward through Δl than upward through it, so that at that instant there may be a *net* current flow downward. A moment later there may be a greater current flow upward.

Hence there is a constant fluctuation in the electron flow in every portion of the resistance, and as a consequence a small but measurable a-c voltage appears at all times across the terminals of the resistance. This a-c voltage has no recurrent wave form; it varies in a perfectly random manner from instant to instant. A suggested picture of this voltage over a small period of time is given in Fig. 15.

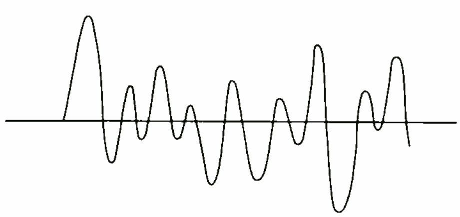


Fig. 15.— Representative random wave shape for a resistor noise voltage.

If an average value were taken, it would be found to be substantially zero, especially if the time interval were fairly long. This means that the positive areas in the curve just balance the negative areas; there is no net voltage in one direction over that in the other, i.e., no d-c component, only a-c. Hence, to measure this voltage, the r.m.s. value must be taken. This is the square root of the average of the squares of the instantaneous values, just as for a sine wave. Often the square root operation is omitted, and the noise voltage expressed as the average of the squares of the instantaneous values. Call this $e_n^{\overline{2}}$. Note how this symbol is written. The bar means the average. It is written above the exponent 2, meaning that the average of the *squares* of the instantaneous voltages is taken.

Although the voltage never repeats itself from one moment to the next the average of its instantaneous squares, when taken over a *sufficiently* long time interval, such as one second, comes out practically the same as the average for the next second. Therefore this average value, which can be measured by an a-c meter, such as the thermocouple type, is a fairly constant and reproducible quantity.

In previous assignments it was brought out that a wave that is not sinusoidal in form, but repeats its shape over and over again (is periodic in nature) may be expressed as the sum of a series of sine waves, of which that of the lowest frequency is called the fundamental, and the others are called harmonics because their frequencies come out to be integer multiples of the fundamental, such as twice the fundamental frequency (second harmonic), three times the fundamental frequency (third harmonic) etc. This series is known as the Fourier series.

When a wave shape is not periodic, but varies from moment to moment, it might appear that the above Fourier series does not apply. This is true, but fortunately another type of series similar to the above is available. It is obtained by an extension of the Fourier theorem. In this series, applicable to non-periodic waves, such as transient voltages and noise voltages, the wave can be expressed as the sum of an *infinite* number of sine waves of frequencies that differ from one another by *infinitesimal* amounts, and are therefore uniformly distributed throughout either a portion or the entire spectrum: from zero frequency (d-c) to an infinite frequency. Each sinusoidal component of this series has an infinitesimal amplitude, but since there are an infinite number of components involved, the sum comes out to be a finite quantity, the wave itself.

The difference between a periodic wave and a non-periodic wave is illustrated in Fig. 16. The periodic wave chosen is a square wave whose fundamental frequency or repetition rate is f_1. It will be observed that there are only odd harmonics present in this particular type of wave. That means that there is energy present only at certain discrete frequencies: f_1, $3f_1$, $5f_1$, etc. In between there is no energy from this wave. Another point is that the amplitude of each harmonic is finite in magnitude, although the amplitude varies inversely as the order of the harmonic for this particular wave; that is, the 3rd harmonic has an amplitude one third of the fundamental; the 5th harmonic, one fifth of the fundamental, etc.

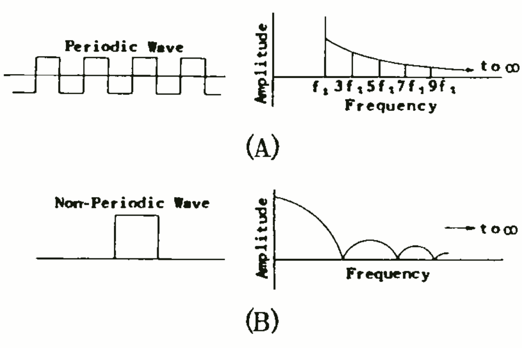


Fig. 16.—Spectra for a periodic and a non-periodic wave.

For comparison a non-periodic or transient wave is shown below the other. The wave chosen consists of only one-half cycle of the preceding wave, and is therefore non-repetitive or non-periodic. By contrast its spectrum consists of energy at practically every frequency except a certain few, as shown. The amplitudes are actually infinitesimal in magnitude, so that the scale has been infinitely magnified in order to be able to exhibit them.

In the case of the noise voltage illustrated in Fig. 15, the corresponding collection of sine wave components, or spectrum is practically a straight line as shown in Fig. 17. The significance of this is that no one component is any greater or smaller (in its infinitesimal amplitude) than any other *or the noise energy is uniformly distributed throughout the spectrum from zero to the highest frequency.*

This means that if one has a circuit that is capable of passing a band of frequencies from f_1 to f_2, in the low end of the spectrum, and another circuit that passes frequencies from f_3 to f_4, such that

$f_2 - f_1 = f_4 - f_3$, then the amount of noise energy picked up in the low-frequency circuit will be equal to that in the high-frequency circuit. Experiment confirms the theory: a u.h.f. receiver of certain band width picks up just as much noise power from a given resistance connected to its input terminals as an audio amplifier of the same band width to whose input terminals the same resistance (or one of equal value) is connected.

If the band width is doubled, twice as much noise power is transmitted because twice as many sinusoidal components, of infinitesimal but equal amplitudes, can get through. Thus the noise power is directly proportional to the frequency band. For example, if the band width is infinitely narrow, so that only one component can get through, then the noise power cor-

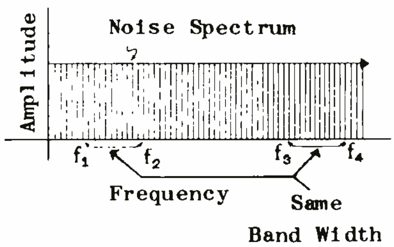


Fig. 17.—Continuous spectrum for a noise voltage.

responds to that contained in this one sinusoidal wave. Since its amplitude is infinitesimal, the corresponding noise power is infinitesimal.

The formula developed by J. B. Johnson is as follows:

$$\overline{e_n^2} = 4kTR\Delta f \qquad (24)$$

where k is a general constant of Nature discovered by Boltzmann and equal to 1.37×10^{-23} joule per degree Kelvin, T is the *absolute* temperature in degrees Kelvin (= degrees centigrade + 273°), R is the resistance in ohms, and *Δf is a symbol meaning band width, such as $f_2 - f_1$ in Fig. 17.*

Since power is proportional to the square of the voltage, the *noise power* is represented by $\overline{e_n^2}$ in Eq. (24). Thus the *noise power* is directly proportional to T, R, and Δf. The noise voltage (r.m.s.) is, then proportional to the square root of the above three quantities.

A resistor may therefore be regarded as more than a mere resistance; it is actually a source of (noise) energy as well. It can be represented as shown in Fig. 18(A)

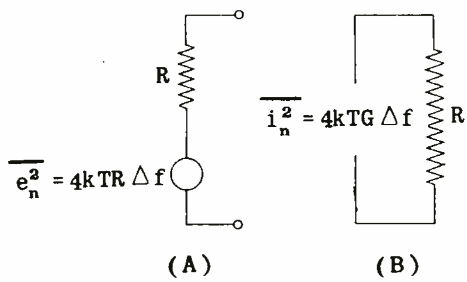


Fig. 18.—Alternative representations of a resistance noise generator.

Alternatively, it can be represented by a constant current generator, as in (B), that generates a noise current

$$\overline{i_n^2} = \frac{\overline{e_n^2}}{R^2} = \frac{4kT\Delta f}{R} = 4kTG\Delta f \qquad (25)$$

where G = 1/R and is the conductance of the resistor. The basis for Fig. 18(B) is exactly the same as that for the vacuum tube circuit shown previously in Fig. 10(C). Fig. 18 (A) is best suited for a series combination of resistors, for which the several mean-square noise *voltages* are added together, and Fig. 18(B) is best suited for a parallel combination of resistors, for which the several mean-square *currents* are added together.

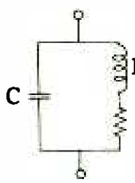


Fig. 19.—A resonant circuit can be regarded as a noise source of limited band width.

Very often, however, the resistance represents the losses of a reactive network. For example, the parallel resonant circuit shown in Fig. 19 has a certain amount of loss owing to R. Its impedance is

$$Z = \frac{(1/j\,\omega C)(R + j\,\omega L)}{R + j\,\omega L + 1/j\,\omega C}$$

$$= \frac{R + j\,\omega L}{(1 - \omega^2 LC) + j\,\omega\, CR}$$

$$\cdot \frac{(1 - \omega^2 LC) - j\,\omega CR}{(1 - \omega^2 LC) - j\,\omega CR}$$

$$= \frac{R + j(\omega L - \omega CR^2 - \omega^3 L^2 C)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

The real part of this expression,

$$R' = \frac{R}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2} \qquad (26)$$

represents a resistance in series with a reactance of value

$$X' = \frac{j(\omega L - \omega CR^2 - \omega^3 L^2 C)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

as is indicated in Fig. 20. Thus both terms vary with the frequency.

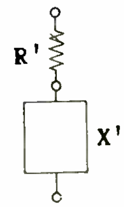


Fig. 20.—Equivalent series circuit of a reactive parallel circuit.

The variation of R′ with frequency is shown in Fig. 21. At a frequency

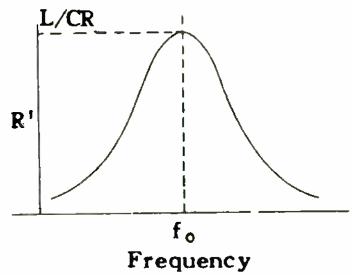


Fig. 21.—Variation of resistive component of a parallel resonant circuit with frequency.

where L and C resonate, that is, at

$$\omega_o = \frac{1}{\sqrt{LC}}$$

the quantity ω_o^2LC becomes unity,

$$\omega_o^2LC = 1$$

and the denominator reduces to $\omega_o^2C^2R^2$ which is equal to

$$\frac{C^2R^2}{LC} = \frac{CR^2}{L}$$

Therefore the equivalent resistance of the network reduces to

$$R' = \frac{R}{\frac{CR^2}{L}} = L/CR$$

This is its maximum value, as indicated in the diagram. It can also be written in terms of the Q of the coil as

$$R' = \omega_o LQ \qquad (28)$$

However, the noise power at any single frequency is infinitesimal, as has been indicated. To obtain the finite noise power over a band of frequencies, it is necessary in the above case to take account of the variation of resistance component R′ with frequency. This is best done by means of the calculus; briefly it consists in taking very narrow frequency intervals, adjacent to one another, over the given total band width. The value of R′ is essentially constant over each small frequency interval, so that the mean-square noise voltage can be computed by means of Eq. (24) or (25), as desired, for each small interval. The total noise for the entire band width is then simply the sum of the mean-square noise voltages for the various intervals. Very often however, the total bandwidth under consideration is sufficiently narrow so that R can be considered essentially constant over the band. This is very nearly the case for moderate band widths centered around the resonant frequency, as may be seen from an inspection of Fig. 21.

As an example of the magnitude of resistance noise, suppose a coil of .2μh inductance employed, with a Q of 100. Let the frequency be 100 mc, the band width 250 kc, and the room temperature 27°C. The absolute temperature is then 273 + 27 = 300°K. The equivalent noise resistance, from Eq. (28) is

$$R' = 2\pi \times 10^8 \times .2 \times 10^{-6} \times 100$$
$$= 12,570 \text{ ohms}$$

This value is assumed to hold for the entire band width of 250 kc. Then, by Eq. (24), the mean-square noise voltage is

$$\overline{e_n^2} = 4kTR\,\Delta f = 4 \times 1.37 \times 10^{-23}$$
$$\times 300 \times 12,570 \times 250 \times 10^3$$
$$= 5.17 \times 10^{-11}$$

or the r.m.s. value is

$$\sqrt{\overline{e_n^2}} = \sqrt{5.17 \times 10^{-11}}$$
$$= 7.19 \times 10^{-6} \text{ or } 7.19 \ \mu\text{volts}$$

This may seem to be a rather high value for a resistance of 12,570 ohms, and is due to the band width involved—250 kc. Nevertheless, such a band width is rather narrow for 100 mc. and is actually due to the i-f circuits following, rather than to the parallel resonant circuit under consideration.

TUBE NOISE.—As will be shown in more detail in the following assignment, plate current flows while an electron is *in transit between the cathode and the plate*, and actually ceases when the electron reaches the plate unless another electron is ready to leave the cathode at that time and proceed to the plate.

Actually, of course, there are billions of electrons proceeding to the plate. However, each electron in motion contributes its quota to the total plate current by virtue of its motion, and this contribution ceases when the electron strikes the plate.

If at that precise moment another electron began to move from the cathode to the plate, and with the same velocity as the preceding electron, then it would take over the current-producing activity of the former, and no sudden momentary decrease in plate current would occur. Such precision of timing is hardly to be expected: electrons are emitted from the cathode more or less at random, and with random initial velocities, so that random fluctuations in the plate current can be expected to occur.

Although the cause of this fluctuation, namely random emission at random velocities, is not exactly the same phenomena as the random motion of the free electrons in a resistor, the fluctuating current obtained is practically identical in form, and is therefore considered a noise current the same as that produced by thermal agitation in the resistor.

Its magnitude was first worked out for the case of a temperature-limited diode by W. Schottky,* and later developed for the case of the space-charge-limited current by D. O. North.* The difference in the resultant so-called "shot noise" is that in the temperature-limited diode the plate voltage is high enough to pull over every electron emitted by the cathode, so that the plate current is limited by its emission capability (hence temperature), whereas in the case of the space-charge-limited current the plate voltage is relatively low, and the plate current is limited by the space charge produced by the excess of electrons emitted over those pulled over to the plate. The shot noise is much less in the latter case because the cloud of electrons near the cathode acts as a kind of reservoir, absorbing the fluctuations in the emission current.

The shot noise occurs in the plate or output circuit of the tube. The noise in the signal on the other hand, appears at the grid or input terminals of the tube, and is due to the resistance component between the grid and cathode terminals. It would facilitate noise computations greatly if the two effects could be pro-rated so that they can both be regarded as occurring at the same pair of terminals, preferably the input terminals.

This is done by considering the "noisy" tube as being replaced by an ideal, noise-free tube, across whose input terminals there is connected a resistance of such value, that the noise voltage it produces, when multiplied by the G_m of the tube, produces the same noise current in *the plate circuit as is actually*

*W. Schottky, "Spontaneous Current Fluctuations in Various Conductors," *Ann. der Phys.*, Dec. 20, 1918.

*D. O. North, "Fluctuations in Space Charge Limited Currents at Moderately High Frequencies," R.C.A. Review, April, 1940.

generated in the plate circuit of the actual tube.

Suppose a tube is noisy. Then the equivalent resistance in the grid circuit will be high. Hence the merit of a tube may be judged by how *low* an equivalent resistance can be assumed to be present in its grid circuit. At this point it must be noted that this equivalent resistance does not actually exist in the grid circuit, and *therefore does not absorb power* from the signal source connected to the input of the tube. It therefore differs from input loading such as owing to cathode-lead inductance, which actually absorbs power from the source.

The equivalent resistance can be given by a relatively simple formula for the case of a negative-grid triode with an oxide-coated cathode. It is for triode amplifiers

$$R_{eq} = \frac{2.5}{G_m}$$

For triode mixers

$$R_{eq} = \frac{4}{G_c} \qquad (29)$$

A table of values for R_{eq} for various tubes is given herewith. This is taken from an article by W. A. Harris in the April 1941 issue of the RCA Review entitled "Fluctuations in Vacuum Tube Amplifiers and Input Systems." Note how the value of R_{eq} varies from 200 ohms for a 6AC7 tube to 210,000 ohms for a 6L7 pentagrid mixer or a 6SA7 frequency converter. Note that the value of R_{eq} depends — from Eq. (29) — upon the G_m of the tube when used as an amplifier, and upon the G_c (conversion transconductance) when employed as a converter.

PENTODE TUBES.— In the case of multielectrode tubes, the cathode or space current divides between the several positive electrodes. In a pentode, for example, the current divides between the screen grid and the plate. The chance of any

TABLE I

TUBE NOISE VALUES

Type	*Application*	*Voltages*			*Currents*			*Transconductance Micromhos*	*Noise-Equivalent Resistance*		*Noise-Equivalent Input Voltage (d) Microvolts*
		Plate Volts	*Screen Volts*	*Bias Volts*	*Plate ma*	*Screen ma*	*Cathode ma*		*Calculated Ohms*	*Measured Ohms*	
6SK7	Pentode Amplifier	250	100	−3	9.2	2.4	11.6	2,000	10,500	9,400–11,500	0.94
6SJ7	Pentode Amplifier	250	100	−3	3	0.8	3.8	1,650	5,800	5,800	0.70
6SG7	Pentode Amplifier	250	125	−1	11.8	4.4	16.2	4,700	3,300		0.53
6AC7/1852	Pentode Amplifier	300	150	−2	10	2.5	12.5	9,000	720	600–760	0.25
956	Pentode Amplifier	250	100	−3	5.5	1.8	7.3	1,800	9,400		0.90
1T4	Pentode Amplifier	90	45	0	2.0	0.65	2.65	750	20,000		1.3
6SA7	Frequency Converter	250	100	0	3.4	8.0	11.9	450 (c)	240,000	210,000	4.5
6K8	Frequency Converter	250	100	−3	2.5	6.0	8.5 (b)	350 (c)	290,000		4.9
1R5	Frequency Converter	90	45	0	0.8	1.8	2.75	250 (c)	170,000		3.8
6L7	Pentagrid Mixer	250	100	−3	2.4	7.1	9.5	375 (c)	255,000	210,000	4.0
6J5	Triode Amplifier	250	—	−8	9.0	—	—	2,600	960	1,250	0.28
955	Triode Amplifier	180	—	−5	4.5	—	—	2,000	1,250		0.32
6AC7/1852	Triode Amplifier	150	150	−2	—	—	12.5	11,200	220	200	0.14
6AC7/1852	Pentode Mixer	300	150	−1 (a)	5.2	1.3	6.5	3,400 (c)	2,750	3,000	0.48
6SG7	Pentode Mixer	250	125	−1 (a)	3.0	1.1	4.1	1,180 (c)	13,000		1.0
956	Pentode Mixer	250	100	−1 (a)	2.3	0.8	3.1	650 (c)	33,000		1.7
6J5	Triode Mixer	100	—	−1 (a)	2.1	—	—	620 (c)	6,500		0.74
6AC7/1852	Triode Mixer	150	150	−1 (a)	—	—	6.5	4,200 (c)	950		0.28
955	Triode Mixer	150	—	−1 (a)	2.8	—	—	660 (c)	6,100		0.72

(a) At peak of oscillator cycle.
(b) Hexode section only. Triode section takes its current from a separate part of the cathode.
(c) Conversion transconductance value.
(d) For effective bandwith of 5000 cycles.

one electron striking the screen or passing on to the plate is entirely random. Hence there is produced in the plate circuit an *additional* fluctuating current owing to this random division of the current between the two electrodes, and this constitutes an additional source of noise.

Another factor is that the G_m of a pentode tube is less than that of a similar triode tube. Recall that G_m is fundamentally the ratio of the change in cathode current produced by a change in grid voltage. However, not all of the electrons starting from the cathode reach the plate, but instead some impinge upon the screen grid in the case of a pentode tube. It is thereupon evident that the *effective* G_m, namely, the change in plate rather than cathode current for a given change in grid voltage, will be less for the pentode than for the triode tube. This means that the incoming signal will not be amplified as much by the pentode as by the triode tube, so that the signal-to-plate noise ratio will be less, and particularly so owing to the additional source of noise produced by random current division.

It must not be assumed that pentodes are not used at high frequencies, but merely that triodes are generally favored as one goes up in the spectrum.

INPUT LOADING INDUCED NOISE. At ultra-high frequencies the input grid circuit acts as if there were a resistance connected across its terminals owing to transit-time effects. Although these effects will be discussed in the following assignment, note must be taken here of the noise-producing effect of this resistance, as this may be quite appreciable. This noise is often called "induced noise."

Where the control grid is next to the cathode, and the latter is of the oxide-coated type, the induced noise is fairly simply related to the input-loading resistance effect. Thus the noise produced is as if R_1, the input loading resistance, were at 5 times room temperature. This noise may be added to the other noises on the mean-square basis. It may be noted, however, that for a grid-controlled (ordinary) tube where the control grid is next to the cathode, the space charge tends to damp out the induced noise and renders it less effective than for other types of tubes, such as the Klystron, which will be discussed in a later assignment.

The analysis of induced noise effect is still not entirely rigorous, and hence subject to question. However, the value of five times room absolute temperature is a conservative one and will, if anything, give a lower signal-to-noise ratio than is actually obtained in practice.

DISCUSSION OF AMPLIFIER STAGE. The combined effects of all the noise sources discussed above can now be studied. Usually the signal is sufficiently amplified by the first two tubes so that the noise furnished by the plate circuit of the second tube can be disregarded compared to the signal level at that point. Hence the four principal sources of noise are the thermal agitation noise of the source, that of the resistance representing the ohmic and dielectric losses of the tube's input circuit, that produced by the electronic loading,* and fi-

*This is due to transit time. Loading due to feedback, such as cathode lead inductance, does not produce noise.

nally that produced by the "shot effect" in the plate circuit of the tube, and referred back to the grid circuit as an equivalent noise resistance.

A typical case is that of an antenna feeding an r-f amplifier stage, as shown in Fig. 22(A). This was discussed in a previous assignment on transmission lines. Normally the line by itself has negligible losses compared to the input loading losses R_1 of the tube, or of the radiation resistance (plus ohmic losses) of the antenna, denoted by R_a.

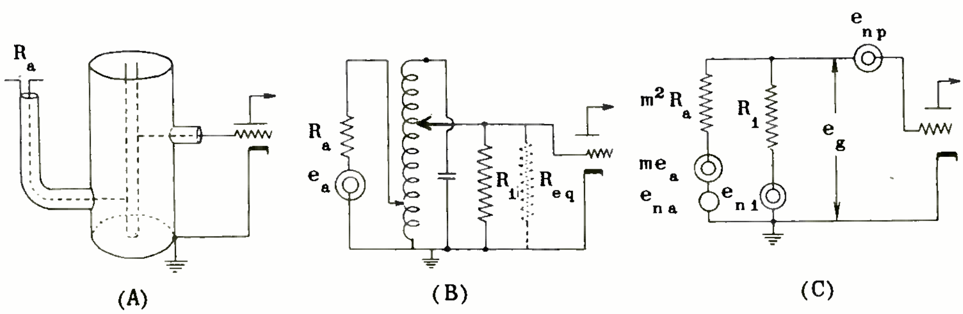


Fig. 22.—Antenna feeding an r-f stage through a resonant line transformer.

If these are sufficiently great, (R_1 sufficiently low), then if the grid were connected to the open end of the line, it would damp the line to such an extent as to lower its Q to a point where the band width would be too broad for image rejection purposes. Hence-as explained in the previous assignment the grid is tapped down on the line to a point where the Q is increased to *double* the value required for the band width.

Then the antenna tap is normally adjusted so as to match R_1. This means that the impedance transformation of the resonant line, m^2, is such that $m^2R_a = R_1$, and $R_1/m^2 = R_a$, i.e., each impedance sees its image when looking into its tap on the line. As was pointed out, the connection of the antenna to the line on a matched basis lowers the latter's Q to one-half the value obtained by connecting the tube alone, or from double the desired value down to the value needed for the required band width.

The circuit of (A) is equivalent to the lumped circuit form shown in (B). Note that the matching of the antenna resistance R_a is to the *actual* losses of the grid circuit, namely R_1. In addition a resistance R_{eq} is shown. This is a *fictitious* resistance that represents the shot noise developed in the plate circuit of the tube, and does not actually exist in the grid circuit. It is to be used in calculating the noise voltages, *but not the tap ratios of* the resonant line.

In Fig. 22(C) is shown a circuit equivalent to that in (B), but with all values referred to the grid side of the resonant line. Here the noise voltage owing to the induced effect of the input loading is shown as produced by a noise generator e_{n1}, and the noise voltage owing to the plate shot effect (or alternatively as due to R_{eq}) by e_{np}. Note that the resistance of the antenna now appears as m^2R_a, and is normally arranged to equal R_1, so that

$$m^2R_a = R_1, \text{ or } m = \sqrt{R_1/R_a} \tag{30}$$

and further, the signal voltage appears as me_a in the grid circuit.

It is desired to find the signal-to-noise voltage ratio, and the signal-to-noise power ratio. Note that the actual signal e_s at the grid is not me_a, but rather the fraction of it left after the voltage drop in m^2R_a is subtracted from me_a. The same is true for the two noise voltages e_{na} and e_{n1}.

The noise voltage e_{np} owing to shot effect, on the other hand, acts in full strength, as indicated in Fig. 22(C), where it is shown in *series* next to the grid of the tube, after R_1, so that neither input loading nor any other grid resistance affects its magnitude. That this is the correct viewpoint is clear from the fact that e_{np} represents a noise voltage that actually occurs in the plate circuit, and is therefore not affected, in the actual stage, by impedances in the grid circuit.*

In view of the above, the actual terminal signal and noise voltages appearing at the grid of the tube are related to the generated signal and noise voltages as follows:

$$\left.\begin{aligned}
\text{Signal voltage } e_a' &= me_a\left(\frac{R_1}{m^2R_a + R_1}\right) \\
\text{Antenna noise voltage } e_{na}' &= \frac{e_{na}R_1}{m^2R_a + R_1} = \frac{R_1\sqrt{4kT\Delta f m^2R_a}}{m^2R_a + R_1} \\
\text{Induced noise voltage } e_{n1}' &= \frac{e_{n1}m^2R_a}{m^2R_a + R_1} = \frac{m^2R_a\sqrt{4k(5T)R_1\Delta f}}{m^2R_a + R_1} \\
\text{Shot noise } e_{np} &= \sqrt{4kT\Delta f R_{eq}}
\end{aligned}\right\} \tag{31}$$

Note the factor 5T for the induced noise voltage. As discussed previously, the electronic loading R_1 generates noise as if it were at 5 times room temperature. Also note that the shot noise voltage is not

*This is true only for a cathode-grounded stage. For a grid-grounded stage it will be shown how to transform R_{eq} from its value when in series with the grid to its value when in series with the cathode.

multiplied by a fraction representing the ratio of resistances, as are the other three generated voltages.

The total noise voltage is

$$e_{nt} = \sqrt{(e'_{na})^2 + (e'_{ni})^2 + (e_{np})^2} \qquad (32)$$

The signal-to-noise voltage ratio is then e'_a/e_{nt}, and the signal-to-noise power ratio is equal to $(e'_a/e_{nt})^2$. From Eqs. (31) and (32) the ratio can be found. After much algebraic manipulation there results

$$\text{Signal/Noise} = e'_a/e_{nt} = \frac{e_a R_i}{\sqrt{4kT\Delta f\left[R_a R_i(R_i + 5m^2R_a) + \frac{R_{eq}}{m^2}(m^2R_a + R_i)^2\right]}} \qquad (33)$$

This ratio is at a maximum if the step-up ratio of the resonant line is adjusted so that

$$m^2 = \frac{R_i}{R_a}\sqrt{\frac{R_{eq}}{R_{eq} + 5R_i}} \qquad (34)$$

This is a slightly smaller step-up than that for matching R_a to R_i, as given by Eq. (30), namely

$$m^2 = R_i/R_a \qquad (30)$$

but the improvement in using Eq. (34) instead of Eq. (30) is slight, and is not desirable anyway if the antenna connects to the resonant line through a non-resonant transmission line, as this will not be terminated in its characteristic impedance if Eq. (34) is employed. Hence in general the matched value for m given in Eq. (30) is preferred.

Recall that this means that $m^2R_a = R_i$. If this be substituted in Eq. (33), there is obtained

Signal/Noise (matched) =

$$\frac{e_a}{\sqrt{4kT\Delta f\left(6 + 4\frac{R_{eq}}{R_i}\right)R_a}} \qquad (35)$$

NUMERICAL EXAMPLE.— It will be of value to use a typical case to indicate the order of magnitude that may be expected. Suppose one section of a 6J6 u.h.f. triode is to be employed at 400 mc, and that a band width of 2 mc is necessary for adequate image suppression.

Assume that the tube has a noise equivalent resistance (R_{eq}) of 700 ohms. At 400 mc, the input loading is taken as 400 ohms. This represents R_i. The (dipole) antenna resistance (mainly radiation) is 75 ohms. Hence by Eq. (30) the step-up ratio for matched conditions is

$$m = \sqrt{400/75} = 2.31$$

Note that for a 2 mc band width, which is relatively narrow at 400 mc, the Q of the resonant line will have to be high, and that therefore undoubtedly the grid, as well as the antenna, will have to be tapped down on the line. This, however, does not affect the impedance ratio m. The circuit is shown in Fig. 23.

On the other hand, suppose that such a wide band was required that it could not be quite obtained even when the grid was connected to the top (open) end of the resonant line. In that case the Q could be further lowered to give the desired band width by

1). Adding additional resistance in parallel with the grid further to load down the resonant line, or

2). Moving the antenna tap up toward the open end of the line. The latter method is preferred as it gives a more favorable signal-to-noise ratio than (1). It is known as overcoupling. Effectively, by moving the tap up, m^2 is reduced, as is also the reflected resistance m^2R_a that is in parallel with R_i across the line.

In a preceding assignment on transmission lines it was shown how to calculate the Q for a desired band width, and then the requisite damping resistance R_t to be connected across the open end of a $\lambda/4$ line to give this Q. Thus R_t can be found, R_i and R_a are known, so that finally m can be determined.

In the problem at hand, however, the value of m will be chosen for the matched condition, and — as stated above — the band width desired will be assumed to be narrow enough so that the tube and antenna both have to be tapped down on the line. Suppose it is desired to find the signal-to-noise voltage ratio when the signal picked up by the antenna is 10 μvolts. (The calculations for signal pickup by the antenna have been given in a previous assignment on Propagation.)

Since matched conditions obtain, Eq. (35) applies. Assume T = 300 °K.

$$\text{Signal/Noise(matched)} = \frac{10 \times 10^{-6}}{\sqrt{4 \times 1.37 \times 10^{-23} \times 300 \times 2 \times 10^{6}\left(6 + 4 \cdot \frac{700}{400}\right) 75}}$$

$$= \frac{10 \times 10^{-6}}{\sqrt{3.29 \times 10^{-14}\,(13)75}}$$

= 1.765 voltage ratio of signal/noise

$(1.765)^2$ = 3.11 power ratio of signal/noise

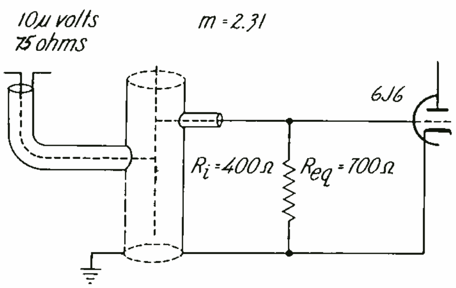


Fig. 23.—Circuit whose-signal-to-noise ratio is to be calculated.

What constitutes a satisfactory signal-to-noise ratio depends upon the nature of the signal, the reliability of the service desired, etc. No definite figures are available. A signal-to-noise voltage ratio of 1.765 is rather poor, but reception of code signals, especially CW in conjunction with a beat-frequency oscillator incorporated in the receiver, will be fairly satisfactory, especially by an experienced code operator, providing sufficient local oscillator stability can be obtained to permit such operation. For services such as television a much higher signal-to-noise ratio is desirable. On a power basis a figure of 1000 to 1 is considered good. This corresponds to the noise power being 10 log 1000 = 30 db below the signal power in level. Where the entertainment value of the program is not its principal feature, much lower signal-to-noise ratios are feasible. The proper figure will depend upon the service; this assignment shows how to calculate the signal-to-noise ratio that will be obtained in a given design and with a given signal input. A better ratio can then be obtained either by increasing the transmitter power or the directivity of either antenna.

NOISE FACTOR.—An examination of the various components that produce noise indicates that one of these, the antenna radiation resistance R_a, is associated with the source of the signal, whereas the other two, R_1 and R_{eq}, as well as any losses in the resonant line (assumed negligible in the above problem) are associated with the receiver.

In comparing various receivers on a signal/noise basis, it is desirable to eliminate the effect of the antenna itself. The mean-square noise voltage associated with the antenna is, by Eq. (24)

$$\overline{e_{na}^2} = 4kT\Delta fR_a \qquad (36)$$

The signal voltage (squared) is e_a^2. Therefore the power signal/noise ratio of the antenna is

$$(S_a/N_a)^2 = \frac{e_a^2}{4kT\Delta fR_a} \qquad (37)$$

At the output of the receiver the value of $(S/N)^2$ is reduced by the amount of noise generated in the receiver, and is equivalent to dividing $(S_a/N_a)^2$ by some factor N_e. For example, under matched conditions, the output $(S/N)^2$ is found by squaring the expression given in Eq. (35). This is

$$(S/N)^2(\text{matched}) = \left(\frac{e_a^2}{4kT\Delta fR_a}\right)\left(\frac{1}{6 + 4\frac{R_{eq}}{R_1}}\right)$$
$$= \left(\frac{S_a}{N_a}\right)^2 \cdot \left(\frac{1}{\left(6 + 4\frac{R_{eq}}{R_1}\right)}\right) \qquad (38)$$

Eq. (38) represents Eq. (37) multiplied by the factor $1/(6 + 4R_{eq}/R_1)$. Clearly the denominator of this factor is the quantity N_e. In the numerical problem worked out above this comes out to be

$$6 + \frac{4 \times 700}{400} = 13 \text{ or } N_e = 13.$$

Thus, the *input* signal/noise power ratio must be devided by 13 to get the *output* signal/noise power ratio.

The quantity N_e has been called the "excess-noise" ratio and also the " noise factor." It was suggested by Dr. D. O. North* as a means of comparing the performance of various receivers. Thus an ideal receiver that does not produce any noise would have a noise factor of unity. The noisier the receiver is, the larger is its N_e, and the smaller is the signal/noise ratio at the output compared to that inherent in the input.

Note that N_e is independent of the receiver input and signal generator impedance, of the frequency, (except as R_1 changes with frequency) and of the band width. Its definition in general is

*D. O. North, "The Absolute Sensitivity of Radio Receivers," RCA Review, Jan. 1942.

$$\frac{S/N\ (\text{ideal})}{S/N(\text{measured})} = \sqrt{N_e} \qquad (39)$$

and it therefore represents the ratio of two ratios, and has no dimensions itself, as was indicated by its independence of frequency, band width, etc. It is thus a general convenient factor expressing not the excellence, but rather the poorness of a receiver; a kind of inverse figure of merit.

The calculations just made are actually for an antenna feeding a single r-f stage. Normally, the usual superheterodyne receiver has a mixer and local oscillator as well as an r-f stage; indeed — as will be shown — there are cases where the r-f stage is preferably omitted. Hence it will be necessary to analyze the action of converters and mixers, and then in the following section to study the signal-to-noise ratio of a typical receiver in order to appreciate the various factors involved.

MIXERS AND CONVERTERS

CONVERTER CHARACTERISTICS. When a tube is employed as a converter to mix the incoming signal with that of a local oscillator and produce thereby the i-f signal, the value of the equivalent shot-noise resistance and the electronic loading is modified from the values it has as an amplifier tube.

The conversion action is due primarily to the variation in the transconductance of the tube at the oscillator frequency. In Fig. 24 is shown the relationship between the plate current and the grid voltage of a tube; the so-called transfer characteristic. Suppose a bias of E_{c1} is chosen, and then a very small alternating voltage e_s is superimposed upon it. A small alternating current i_{p1} will appear in the plate circuit, as shown. By definition, the transconductance is

$$G_{m1} = i_{p1}/e_s$$

at the operating bias E_{c1}.

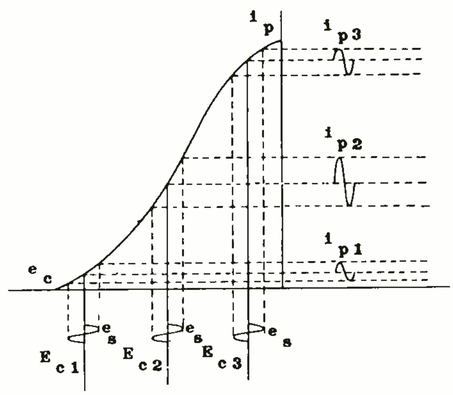


Fig. 24.— Transfer tube characteristic, showing how converter characteristic can be obtained.

If the bias is changed to a smaller value E_{c2}, the a-c plate current will be i_{p2}, and greater than i_{p1}. This indicates that the transconductance G_{m2} at this operating bias E_{c2} is larger than G_{m1}. At a still lower bias E_{c3} the plate current variation is i_{p3}. This is indicated as greater than i_{p1} but smaller than i_{p2}, so that the corresponding transconductance G_{m3} is intermediate in value between G_{m1} and G_{m2}. The transconductance is therefore a function of the grid voltage (bias) and can be plotted against it as shown in Fig. 25. The result is curve AB. Now suppose an initial bias is chosen as shown, and then the grid voltage varied at the local oscillator frequency. This is equivalent to varying the G_m at such a frequency.

The fluctuation of the transconductance at oscillator frequency causes the plate current owing to the superimposed signal voltage to fluctuate in *amplitude* at oscillator frequency, and this is equivalent to the production of the side bands,

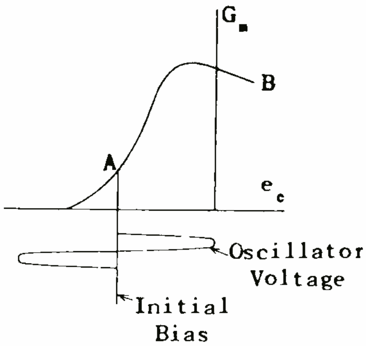


Fig. 25.—Plot of transconductance vs. grid voltage.

such as one at a frequency equal to the sum of the signal and oscillator frequencies and one at a frequency equal to the difference in the above two frequencies.

The latter difference frequency is the normally employed intermediate frequency. The load in the plate circuit of the tube is tuned to this frequency so as to present a high impedance to the i-f current, and thus develop a high i-f voltage, whereas it is arranged to be practically a short-circuit to currents of other (higher) frequencies. Thus only a voltage of i-f frequency is impressed on the grid of the following stage.

CONVERSION TRANSCONDUCTANCE.—The fact that the transconductance, in varying at the oscillator frequency, converts the input signal into one of intermediate frequency suggests that the ability of the tube to do this be measured by the i-f *current* output to the r-f *signal* input. This is also a ratio of current to a voltage, and therefore is similar to conductance and transconductance, but since the ratio is that of a current of *intermediate frequency* to a voltage of *signal frequency*, the term *conversion transconductance*, G_c, is employed.

Its value varies with the nature of the variation of the G_m of the tube as portrayed in Fig. 25, but generally it is about one-fourth of the transconductance, G_m, i.e., $G_m = 4\ G_c$. The maximum value of G_c depends upon the operating point or bias (point A in Fig. 25), and upon the magnitude of the oscillator voltage, and optimum values of these are such that the tube is cut off for slightly less than 180 degrees of the oscillator cycle, and the oscillator amplitude is such as to carry the G_m somewhat beyond its maximum value. The value of $G_c = 1/4\ G_m$ is a fairly good approximation for most purposes.

CONVERSION GAIN.—The conversion gain is calculated in exactly the same manner as amplifier gain. As was brought out earlier in this assignment, when the tube is operated as a Class A amplifier, its gain can be given as

$$\alpha_a = G_m Z_L$$

where G_m is its transconductance, and Z_L is the plate impedance.

When the tube is operated as a converter or mixer (converter refers to a combination mixer and oscillator) then the conversion gain, or ratio of i-f *output voltage* to r-f *input voltage* is

$$\alpha_c = G_c Z_L \quad (40)$$

where G_c is the conversion transconductance, and Z_L is the plate load, *tuned to the intermediate frequency.* Thus, if e_s is the r-f input voltage, then the output i-f voltage is

$$e_{i-f} = e_s G_c Z_L \quad (41)$$

INPUT LOADING.—The input loading at any given frequency depends upon the G_m of the tube, and if, in a mixer or converter, the G_m varies at the oscillator frequency, then the input loading will vary at this frequency. By taking an average of this variation, one can obtain the magnitude of this effect, R_1. **The magnitude of R_1 varies inversely as the square of the frequency, just as the** input loading for the tube operating as a Class A amplifier, so that if its value is known at any one frequency, it can easily be found for any other frequency.

CONVERTER NOISE.—The mean-square shot noise current $\overline{i^2_{pn}}$ for a converter fluctuates at the local-oscillator frequency too, so that an average value $\overline{i^2_{i-f}}$ must be found. For other than a diode mixer, this can be expressed as an equivalent noise resistance in the grid circuit by dividing $\overline{i^2_{i-f}}$ by G_c^2 to obtain the equivalent grid noise mean-square voltage, $\overline{e_n^2}$, and then obtaining the corresponding noise resistance R_{eq}. Thus

$$\overline{e_n^2} = \overline{i^2_{i-f}}/G_c^2$$

$$R_{eq} = \overline{e_n^2}/(4kT\Delta f) = \overline{i^2_{i-f}}/(4kTG_c^2\Delta f) \quad (42)$$

The value of R_{eq} for several tubes operated as mixers is given in Table 1.

DIODE MIXERS.—In the u.h.f. range multi-grid mixers and converters, like the 6L7, are not used because of their relatively poor signal/noise ratio. Instead, mixers of the pentode or triode form are used, and also diodes. The latter behave similarly to crystal detectors and mixers, but have a greater input loading owing to their greater transit time. In a crystal, the rectification appears to take place at the boundary or point of contact of the crystal and catswhisker, and the transit time in this minute distance is negligible.

The action of the diode is very well explained by Herold.* A brief summary will be given here. A schematic circuit is given in Fig. 26. The

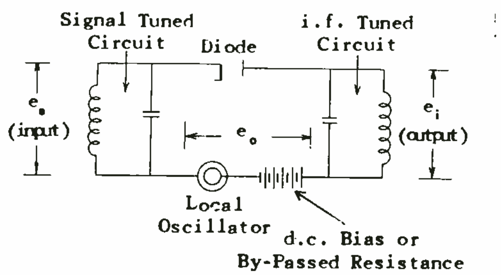


Fig. 26.—Schematic circuit diagram for a diode mixer.

signal input e_s, as well as the i-f output e_1, is assumed to be of small amplitude, as is normally the case.

*See E. W. Herold, "Some Aspects of Radio Reception at Ultra-high Frequencies," Proc. I.R.E., Oct. 1943.

The local oscillator amplitude, as well as the magnitude of the d-c bias, is relatively large, and the two are chosen to give the maximum amount of e_1 for a given e_s.

The combined voltage e_c varies the conductance (or resistance) of the diode at local-oscillator frequency, in the same way that the transconductance of a triode or pentode tube is varied. The result is *a conversion conductance* G_c, an effect exactly similar to conversion transconductance, with the result that e_s applied to the signal tuned circuit produces e_1 across the i-f tuned circuit.

There is one important difference between the diode and grid-type mixer. In the latter, the i-f voltage e_1 does not appreciably react back on the input circuit, whereas in the case of the diode it can. The i-f voltage e_1 developed across the i-f tuned circuit can be regarded as a voltage generated there. This can then react in conjunction with the local-oscillator voltage applied to the diode, owing to the latter's nonlinearity or variability of its conductance, to produce — among other things — a voltage of a frequency equal to the sum of that of e_1 and that of the local oscillator, i.e., one of signal frequency once more. Thus an additional counter voltage of signal frequency appears across the signal tuned circuit, and must be vectorially combined with the applied voltage e_s before the behavior of the diode can be analyzed.

This can all be accomplished by means of an equivalent pi-network shown in Fig. 27. Suppose the diode tube is taken, and known signal voltage e_s applied to it in series with the proper local-oscillator and d-c bias voltage. The i-f *current* i_1 is measured, as well as the *signal frequency* current i_s. Then the signal and conversion conductances can be calculated. These are respectively

$$G_o = i_s/e_s$$

and

$$G_c = i_1/e_s \qquad (43)$$

Note that there is zero i-f impedance in series with the diode when these measurements are made.

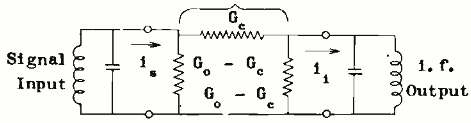


Fig. 27.—Pi-network that is equivalent to the diode mixer.

Then the pi-network shown in Fig. 27, composed of series arm G_c and shunt arms each $(G_o - G_c)$ in value, will correctly furnish the signal and i-f currents and voltages that would be measured in the actual circuit arrangement containing the diode. The significance of Fig. 27 is that the magnitude of the right-hand i-f impedance affects the impedance seen looking into the left-hand signal terminals, and should be taken into account in connecting the antenna or antenna transmission line to the circuit. Further, the output i-f voltage can never equal the input signal voltage because the diode has the characteristics of a pi attenuation pad, and thus produces a conversion loss.

Although Fig. 27 permits formulas to be developed as to the input impedance for any i-f load impedance, and as to the conversion loss, these are not of much value in the case of a crystal mixer, to which Fig. 27 applies equally well, because of the fact that G_o and G_c vary with the position of the catswhisker, as well as with time. Hence numerical computations will not be made, and merely the above general remarks used as a guide in design work and in analyzing qualitatively the behavior of the device in an actual circuit.

The diode not only reduces the signal amplitude (e_1 compared to e_s) because of its conversion loss, so that the signal/noise ratio is less when it is employed than would theoretically be the case if the i-f amplifier could be used by itself, but it also contributes some noise of its own. However, exact relationships, particularly those taking into account transit-time effects, have not as yet worked out.

HARMONIC MIXING.—It is possible to obtain an i-f voltage when the local oscillator is of a much lower frequency than the signal voltage. For example, if the signal frequency is 300 mc, and the i-f is 50 mc, then normally the local oscillator frequency is 300 - 50 = 250 mc. It is possible, however, to employ an oscillator at 125 mc. The mixer can produce a 250-mc second harmonic of this, and then beat this 250-mc second harmonic with the incoming 300-mc signal to produce the 50-mc i.f. Such operation may be called harmonic mixing.

The advantage of this is that the oscillator is required to operate at a lower frequency — usually an easier task — and further there is less danger of "pulling", i.e., of having the incoming frequency react on the local oscillator to tend to lock it at signal frequency. *Maximum* harmonic conversion transconductance is less than the conversion transconductance at fundamental frequency, and generally requires a higher oscillator output. Nevertheless, the signal/noise ratio for harmonic operation is nearly as good as at fundamental operation, thus making harmonic operation perfectly feasible.

TYPICAL RECEIVER NOISE CALCULATIONS

TRANSITION FREQUENCY — R-F VS I-F AMPLIFICATION.—At low frequencies and with strong signal inputs, such as in the standard broadcast band, r-f amplification is employed mainly to provide better image rejection and to prevent reradiation of the local-oscillator frequency from the antenna.

Although an amplifier stage has roughly about four times the gain of a converter stage because $G_m = 4\ G_c$, nevertheless a converter with its lower gain must be used somewhere along the line in the receiver. Since the i-f gain can be made at least as high as the r-f gain, no particular justification for r-f gain ahead of a converter can be had on this basis. Hence the preceding two considerations of image rejection and reradiation are the sole reasons for employing r-f gain at the lower frequencies, and therefore in cheaper receivers the r-f stage is dispensed with.

At ultra-high frequencies, for strong signal inputs, the same considerations apply except that now an additional factor arises-namely, that a tunable r-f stage is rather complicated mechanically and hence undesirable. As a consequence, immediate conversion is usually preferred, and image rejection promoted by the use of a fairly high intermediate frequency. This was discussed in a previous assignment of this series on u-h-f techniques. A high intermediate frequency also minimizes reradiation because the local-oscillator frequency in that case is sufficiently remote from the signal frequency so that the antenna is often de-tuned to the oscillator frequency and therefore radiates but little, since little antenna current flows at this frequency.

On the other hand at low signal levels the main consideration is signal/noise ratio. At frequencies below a certain value, known as a "transition" or "cross-over" frequency, r-f amplification is of value in improving the signal-to-noise ratio, and above this frequency immediate conversion is preferable.

Basically the transition frequency depends upon the band width desired, and the quality of the tubes available. It will be recalled that for a single-resonant circuit, Fig. 28, the percentage band width is

$$\frac{\Delta f}{f_c} = \frac{1}{Q} = \frac{1}{2\pi f_c CR} \qquad (44)$$

where Δf is the band width; f_c the mid-band or carrier frequency, and C and R have the significance shown in Fig. 28. By multiplying both sides of Eq. (44) by f_c, one obtains

$$\Delta f = \frac{1}{2\pi CR} \qquad (45)$$

This equation is very interesting and basic in receiver and even filter design. It states that the band width Δf is proportional to the constant $1/2\pi$, and inversely proportional to C and R, and hence their product. The same relationship is true if one employs a more complicated filter type of circuit

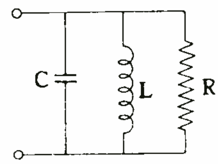


Fig. 28.— Single-resonant circuit, showing the factors that determine its band width.

instead of the simple resonant circuit of Fig. 28; the only change is from the constant $1/2\pi$ to some other value.

Thus *the band width is independent of the frequency of operation;* if the same value of C is employed in an i-f or an r-f amplifier, then, for the same value of R, the same band width is obtained, or alternatively, for a given band width, the value of R (damping) required is the same. (Of course L will have to be different, but this does not affect the required resonant impedance R for a given band width.)

Now the gain of the stage has been shown to be $G_m R$, so that if the capacity C to be employed is specified, then as Δf is increased, R must be decreased, and hence the gain goes down. Eq. (45) can be simply transformed algebraically to yield

$$R = \frac{1}{2\pi\Delta fC} \qquad (46)$$

where it is evident that as Δf increases, R decreases unless C can be decreased.

In order to obtain maximum gain possible for a given band width, C is made as small as possible; for fixed tuning, C may very well be the input capacity of the tube itself.* Once this minimum value is chosen, R and the gain are fixed by the band width required. For filter circuits, as used in television, it follows that, given a tube with a certain C, and a certain band width, the value of R and the gain $G_m R$ are the same whether the tube is employed with a *low-pass* type of filter for video amplification (approximately 30 c.p.s. to 6 mc); with a band-pass filter for i-f amplification covering 8 to 14 mc; or with some form of band-pass filter for r-f amplification covering, for example, 50-56 mc.

At the lower end of the r-f band, if Δf is fairly wide, R, from Eq. (46), must be relatively low. This normally means a low *plate load resistance* used in conjunction with the reactive elements, such as L and C for a simple resonant circuit. As the carrier frequency is increased, however, input loading of the following tube increases (its R_1 decreases) and ultimately a value of R_1 is attained sufficient *by itself* to provide the band width *without the aid of an external shunting resistance.*

For frequencies higher than the one for which R_1 is just sufficient, the value of R_1 continues to decrease, the band becomes wider than necessary, and the gain decreases unnecessarily, in a sense. Ultimately the gain reduces to unity at some high frequency and the r-f stage is clearly useless. However, even for frequencies lower than the one for which the gain is unity, the r-f stage may be useless. The reason is mainly that once R_1 is lower than that required for the band width, the use of a converter becomes desirable, because here the plate circuit operates at the lower intermediate frequency, the loading of the following tube is negligible, and thus the load impedance at i-f is greater than that at r-f. When this difference in plate load becomes great enough to overbalance the contrary difference between transconductance G_m and conversion conductance G_c, the use of a mixer as the input tube is indicated.

Actually r-f gain at a higher frequency than that suggested above, is usually justified because the equivalent shot noise of an amplifier is generally noticeably less than that of a mixer, so that the gain of the latter must exceed that of the amplifier by a considerable amount before a better signal/noise ratio is obtained in dispensing with the r-f stage.

PRACTICAL CIRCUIT CONSIDERATIONS.—The preceding discussion has been applied mainly in the past to the ordinary grounded-cathode stages. Such stages are satisfactory at i-f and low r-f frequencies, perhaps up to 200 mc or so. Special circuits have been worked out for use particularly at the lower frequencies, that yield the highest signal-to-noise ratio possible with given tubes.

For example, one such arrangement is a cathode-follower circuit fed by the antenna and in turn feeding a grounded-grid stage. It is

*This cannot occur if resonant lines are used, since these contain capacity as well as inductance.

illustrated in Fig. 29, where it is evident that dual triodes of the 6J6 type (having a common cathode) are particularly suitable. The circuit is generally known as the "cathode-coupled" type; the common cathode may include a tuned circuit as shown or not, depending upon the tuning range and band width to be covered.

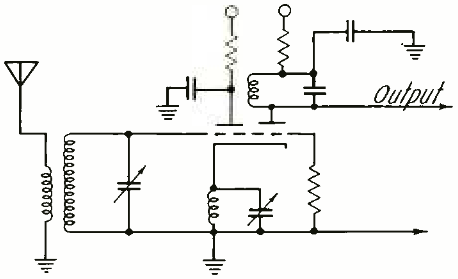


Fig. 29.—Cathode-coupled amplifier.

However, when one proceeds to frequencies of 400 mc or higher the only practical stable amplifier is that of the grounded-grid type; both the r-f amplifier and the mixer will have to employ this form of circuit. Fig. 30 illustrates a typical combination r-f amplifier and mixer that can be employed from say 400 mc to 900 mc or so, depending upon the u-h-f capabilities of the tubes involved.

Resistors R_{i1} and R_{i2} represent the transit-time loading effects in the r-f and mixer stages, respectively. Cathode bias is shown as obtained by an adequately by-passed resistor, and actually does not enter into the signal-to-noise calculations. Also observe that the local-oscillator signal can be injected into the mixer grid by grounding the grid through a resistor R_g; this resistor does not prevent the stage from otherwise acting as a grounded-grid mixer.

It will be recalled from an earlier section in this assignment that the input impedance of a grounded-grid amplifier, even at low frequencies, is relatively low even though the grid is negatively biased, and is equal to

$$Z_{AB} = \frac{r_p + Z_L}{1 + \mu} \qquad (4)$$

or in the specific case of Fig. 30, where $Z = R_{L1}$ for the first or r-f stage,

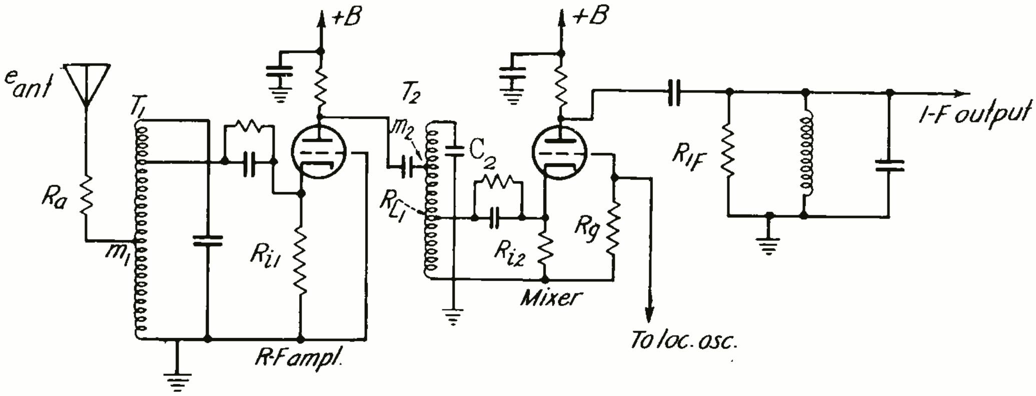


Fig. 30.—Schematic of a grounded-grid r-f stage feeding a grounded-grid mixer stage.

$$Z_{AB} = \frac{r_p + R_{L1}}{1 + \mu} \qquad (47)$$

This low input impedance has an important effect on the noise properties of the circuit, as will be shown. At high frequencies Z_{AB} is still further reduced by the transit time loading of R_{i1} or R_{i2}, as the case may be in Fig. 30. Another important effect of this low input impedance is that compared with the band width normally specified the stage has inherently a large band width, so that one may not have to be concerned with this factor.

A third point is that although R_i varies inversely as the square of the frequency, one must proceed to very high frequencies indeed before R_i becomes appreciably lower than Z_{AB} produced by the cathode feedback effect, and thereupon is able to cause the gain to drop rapidly as the frequency is further increased.

In short, the gain of a grounded-grid amplifier does not drop rapidly with frequency. This in turn means that the r-f stage does not drop out of the picture until a very high frequency is reached; and hence that the transition or crossover frequency is higher for a grounded-grid stage than it is for a grounded-cathode stage.

NOISE RESISTANCE CORRECTIONS.—Normally the equivalent grid resistance R_{eq} for the shot effect in a tube is a resistor assumed to be in series with the grid, and unaffected by the source impedance or transit-time loading. When, however, the grid is grounded and the cathode is "high", this value of resistance must be corrected because of the feedback characteristic of such a stage.

The action is as follows: The shot effect as originally derived by North referred to a constant noise current developed in the plate circuit. This effect could then be replaced by an equivalent noise resistor in series with the grid. But when the grid is grounded, analysis shows that this resistor acts as a noise voltage generator in series with the input impedance Z_{AB} of Eqs. (4) or (47) and the impedance of the signal source driving the cathode. The actual noise voltage is that appearing across Z_{AB} and is less than that appearing in series with the grid.

This is shown in Fig. 31. The

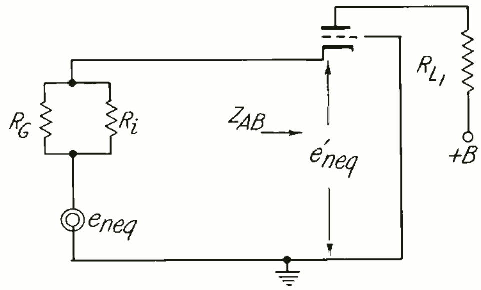


Fig. 31.—Circuit for correcting equivalent noise resistance in a grounded-grid amplifier.

voltage e_{neq} represents that generated by R_{eq} in series with the grid, and which is equivalent in effect to the shot noise actually generated in the plate circuit of the tube if the latter is operated as a grounded-cathode amplifier.

This voltage in the grounded-grid case, must however be shown in series with R_G and R_i in parallel, where R_G is the impedance of the signal source, and R_i is the transit-time loading.

The effective noise voltage is

then e'_{neq} appearing across Z_{AB}. In the case of Fig. 30, R_G represents, for example, the transformed impedance of the antenna resistance R_a (normally 75 ohms for a dipole). In other words:

$$R_G = m_1^2 R_a \tag{48}$$

where m_1 is the voltage step-up (or step-down) ratio of the matching r-f transformer connected between the antenna and the cathode of the r-f grounded-grid stage.

There may be some question as to whether R_i should be associated with R_G or with Z_{AB}. The former arrangement, portrayed in Fig. 31, appears more correct and is also more conservative in that it yields the lower signal/noise ratio.

Assuming the circuit of Fig. 31, the reader will see that since R_G and R_i in parallel will absorb some of the noise voltage e_{neq}, the remaining fraction e'_{neq} across Z_{AB} will be less, as was stated above. This voltage, e'_{neq}, is the *effective* voltage that produces the shot noise in a grounded-grid amplifier. It is less than e_{neq}, the voltage that produces the shot noise in a grounded-cathode amplifier, but so is the gain of the grounded-grid amplifier less than that of a cathode-grounded amplifier in the same proportion, so that the signal/noise ratio for either is about the same.

However, the lower gain of a grounded-grid r-f stage does have some effect upon the value of the stage preceding the mixer stage, and hence the calculations for this type of circuit will be carried through in their entirety, rather than the simpler calculations for a grounded-cathode system, that is not of practical importance at the higher radio frequencies.

Referring to Fig. 31 once more, the student will see that e'_{neq} is that fraction of e_{neq} that Z_{AB} is of the total impedance. In short:

$$e'_{neq} = e_{neq}\left[\frac{Z_{AB}}{\left(\dfrac{R_G\, R_i}{R_G + R_i}\right) + Z_{AB}}\right] \tag{49}$$

Just as e_{neq} corresponds to a fictitious noise resistor R_{eq}, so does e'_{neq} correspond to a fictitious noise resistor R'_{eq}. From Eq. (24), these resistors are proportional to the SQUARES of the equivalent noise voltages, so that R'_{eq} is related to R_{eq} as the square of the fraction included in brackets in Eq. (49). Hence

$$R'_{eq} = R_{eq}\left[\frac{Z_{AB}}{\left(\dfrac{R_G\, R_i}{R_G + R_i}\right) + Z_{AB}}\right]^2 \tag{50}$$

Eq. (50) gives the value of the equivalent shot-noise resistor R'_{eq} for a grounded-grid amplifier when the equivalent shot-noise resistor R_{eq} for a grounded-cathode amplifier has been furnished. Since the latter value is the one more often given, Eq. (50) enables the transformation to a grounded-grid stage to be made.

In the case of the mixer stage, the equivalent noise resistor R_{eq} must be similarly corrected. Here however some minor modifications have to be made. The impedance Z_{AB} looking into the cathode, although essentially given by Eq. (47), must be modified because the plate load impedance in this stage is ZERO AT THE RADIO FREQUENCY OF OPERATION. In other words, the resistor R_{if} shown in Fig. 30 is by-passed so that it appears to be zero at r.f. even though the by-passing is negligible at i.f.

As a result, Eq. (47) for the

mixer stage reduces to

$$Z_{AB} = \frac{r_p}{1 + \mu} \tag{51}$$

a somewhat simpler expression. This can then be substituted in Eq. (50) to obtain the mixer R'_{eq}.

Similar corrections must be made to the transit-time resistors R_{11} and R_{12} of Fig. 30. Consider, for example, R_{11}. It acts as a noise source whose generated voltage is given by Eq. (24), BUT AT A TEMPERATURE OF 5 T, where T is the room temperature, and whose internal resistance is R_1. This resistor is loaded down with the associated signal source and cathode input impedances R_G and Z_{AB}, respectively, in parallel.

The circuit is similar to Fig. 31, and is shown in Fig. 32. The effective voltage is e'_{n1}, and similar to that given in Eq. (49), it is

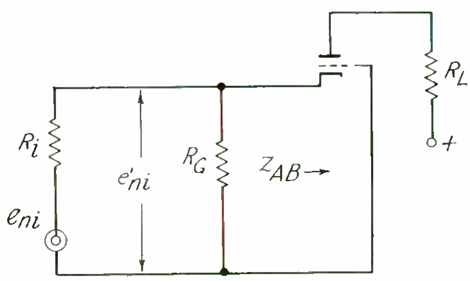


Fig. 32.—Circuit for calculating the induced noise effect of the transit-time loading R_1 in the case of a grounded-grid amplifier.

given by (52)

$$e'_{n1} = e_{n1}\left[\frac{Z_{AB}R_G/(Z_{AB} + R_G)}{Z_{AB}R_G/(Z_{AB} + R_G) + R_1}\right]$$

Then similar to Eq. (50) the equivalent noise resistance is

$$R'_1 = R_1\left[\frac{Z_{AB}R_G/(Z_{AB} + R_G)}{Z_{AB}R_G/(Z_{AB} + R_G) + R_1}\right]^2 \tag{53}$$

Thus, from the value of transit-time loading normally given for a grounded-cathode resistor, the induced noise for a grounded-grid amplifier can be calculated by Eq. (53).

METHOD OF CALCULATION.—We are in a position to calculate the signal-to-noise ratio for the combination of an antenna, r-f stage, and mixer, as shown in Fig. 30. One obvious method is to assume a certain generated antenna voltage e_a, follow it through the system as it is amplified by the r-f and mixer stages, and at the appropriate points in the circuit pick up the various noise voltages, follow them likewise through the circuit, and finally at the output calculate the signal/noise ratio.

Another method is to refer each noise source back to the antenna, and compare it with the generated voltage e_a at that point. This was the method essentially employed in a previous section of this assignment, and will likewise be followed here.

The fundamental idea is as follows: If an amplifier stage of gain A intervenes between the noise source and the antenna, the noise voltage can be divided by A to give its *equivalent* effect at the antenna. Or, the noise resistor can be divided by A^2 to give the equivalent noise resistor at the antenna.

If two stages intervene, the resistor must be divided by the product of the squares of the two gains to give its equivalent value at the antenna. Specifically, consider the i-f load resistor R_{1f} in Fig. 30. Between it and the antenna are the converter and r-f stages of gains A_c and A_{rf}, respectively. (The turns ratios of the matching transformers shown are included in A_{rf}.) Then the resistor in series with the antenna that is equivalent to R_{if} at the output of the converter is

$$R'_{if} = \frac{R_{if}}{A^2_{rf}A^2_c} \quad (54)$$

Thus there may be two steps involved at each point: The resistor, if referred to the grid, must be transformed to the cathode by formulas such as Eqs. (50) or (53). Then the transformed values must be referred to the antenna terminals by means of formulas such as Eq. (54).

After this is all done, the equivalent resistors at the antenna terminals are all added together, and the total used to give the noise voltage that is to be compared to the antenna voltage e_a in order to give the signal-to-noise ratio. This can then be compared with the signal-to-noise ratio existing in the antenna itself in order to give the noise figure for the receiver.

The mode of operation affects the signal/noise ratio, and in turn depends upon the conditions specified. For example, suppose the band width to be amplified is small. Then the load or damping resistance associated with the capacity encountered in any stage can be fairly high. In such a case it is generally safe to assume that the output transformer of the grounded-grid stage, (such as T_2 in Fig. 30), can be employed to step up the low input impedance of the following grounded-grid stage to a value to match the impedance looking back into the plate of the grounded-grid stage. This impedance is given by Eq. (5):

$$Z_{CB} = r_p + (\mu + 1)Z_k \quad (5)$$

and is therefore greater than the r_p of the tube by the factor $(\mu + 1)Z_k$.

As an example, suppose r_p = 6000 ohms and Z_{CB} comes out to be 8450 ohms, the actual damping resistance will be half of this because of the matched conditions or 8450/2 = 4225 ohms. Suppose further that the output capacity of the tubes is 5μμf. The band width that can be covered by the use of an inductance to tune with the 5μμf capacitor is given by a simple re-arrangement of Eq. (46), namely

$$\Delta f = \frac{1}{2\pi RC} \quad (55)$$

or

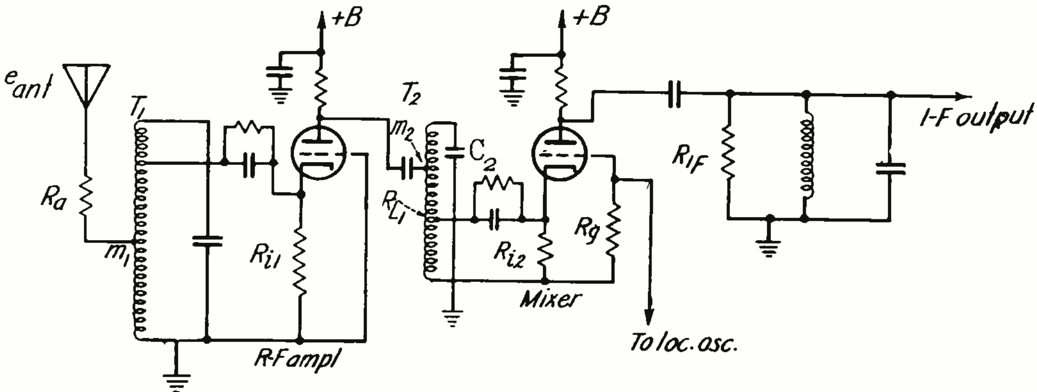


Fig. 30.— Schematic of a grounded-grid r-f stage feeding a grounded-grid mixer stage.

$$\Delta f = 1/(2\pi \times 4225 \times 5 \times 10^{-12})$$

$$= 7.54 \text{ mc.}$$

If the band width desired is less than 7.54 mc, then the use of a matching transformer to step up the actual low input resistance of the following stage to a value of 8,450 ohms is permissible and also desirable in that it furnishes maximum gain for the stage. On the other hand, if an 8 mc band width were required, then the step-up would have to be less than that for the matched condition in order to provide a lower net damping resistance.

One factor that aids in the use of a matching transformer between two grounded-grid stages is that the input capacity of the second stage is reduced by the impedance step-up characteristic of the intervening matching transformer. This is illustrated in Fig. 33.

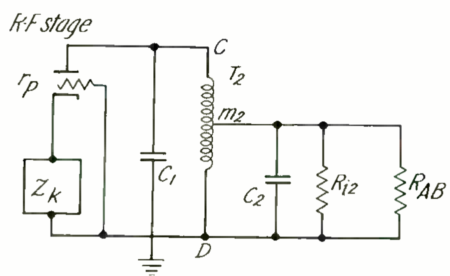


Fig. 33.— Impedance conditions in the output of a grounded-grid r-f stage feeding a grounded-grid mixer stage.

The r-f grounded-grid stage feeds in turn the mixer grounded-grid stage, which presents an impedance composed of its input capacity C_2, transit-time loading R_{12}, and feedback impedance R_{AB} owing to the fact that the cathode is "high" to ground. The output capacity of the r-f stage is denoted by C_1.

Suppose the impedance looking back into the r-f tube is 8,450 ohms, and hence the impedance looking into terminals CD must also be 8,450 ohms to match. Suppose further that R_{12} and R_{AB} are of such value that the turns ratio m_2 of transformer T_2 must be 7.59:1 step-up to terminals CD.

Assume that C_1 and C_2 are each 5μμf. Then because T_2 steps up impedances from the tap to terminals CD, C_2 will appear as a SMALLER equivalent capacitor of correspondingly HIGHER reactance at terminals CD. The apparent value of C_2 will be

$$C_2' = C_2/m_2^2 = 5 \times 10^{-12}/(7.59)^2$$

$$= 0.0868 \ \mu\mu f$$

which is practically negligible compared to the 5μμf of C_1.

In other words, the step-up transformer T_2 makes the input capacity of the following grounded-grid mixer stage of negligible effect on the band width, so that one has to concern himself only with C_1 in calculating the band width possible.

On the other hand, the mixer stage, in feeding the first i-f tube directly, has to face its own output capacity in parallel with that of the input of the first i-f tube. If each is 5μμf, then a total of 10μμf must be faced, and R_{if} (Fig. 30) will be half of that possible for the r-f stage.

The reader may therefore ask why is not a third matching transformer employed between the plate of the mixer tube and the grid of the first i-f tube? The answer is that the latter tube has a very high input resistance because it operates at a relatively low (intermediate) frequency where cathode lead inductance and transit-time loading are negligible in magnitude and moreover it is operated stably as a grounded-cathode amplifier.

Hence a high step-up rather than step-down ratio would be required in this third transformer. This step-up ratio would transform the i-f input capacitance to a much HIGHER value to the plate of the mixer tube and would therefore tend to decrease the band width. It is therefore preferable in this case to feed the i-f tube directly from the mixer.

In the case of the r-f stage feeding the grounded mixer, the r-f stage faces the low input impedance of the mixer, and if direct feed were used, LESS gain would be obtained than is achieved by the use of a matching transformer, even though the latter has a step-down turns ratio and delivers less signal voltage to the cathode of the mixer than is developed at the plate of the r-f tube. It is fortunate in such a case that the step-down required to match the *resistive* components at least renders the input capacity of the mixer negligible compared to the output of the r-f tube.

The procedure for calculating the signal/noise ratio of such a pair of stages can be summarized in a series of steps in conjunction with the block diagram shown in Fig. 34, and which corresponds to the schematic diagram shown in Fig. 30.

In Fig. 34, the antenna generates a signal voltage e_a and has an internal resistance R_a. It feeds the r-f stage through a matching transformer T_1 of turns ratio m_1.

The r-f stage in turn is matched to the grounded-grid mixer through the step-down transformer T_2 of turns ratio m_2, so that a matched load impedance R_{L1} is presented to the plate of the r-f tube. The mixer in turn feeds the i-f amplifier, the total capacity in this circuit is C_2, and for a given band width Δf, a damping resistor R_{if} is required. The r-f stage has an output capacitance of C_1; this must be checked with R_{L1} to see if matched conditions are possible in view of the band width Δf that is specified. Normally this will be the case.

The transit-time loading of the r-f stage is denoted by R_{tt1}; that of the mixer stage by R_{tt2}. The equivalent noise resistance (referred to the grid) of the r-f stage is denoted by R_{eq1}; that of the mixer stage, by R_{eq2}.

The steps in the analysis are then as follows:

1). Calculation of R'_{if} and R_{if}.

$$R'_{if} = \frac{1}{2\pi C_2 \Delta f} \tag{56}$$

Note that R'_{if} represents the parallel combination of the actual damping resistor R_{if} employed and r_{p2}, the plate resistance of the mixer tube, whose shunting effect is appreciable when a triode is employed. Hence

$$R_{if} = \frac{R'_{if}\, r_{p2}}{r_{p2} - R'_{if}} \tag{57}$$

2). Calculation of the gain A_c of the mixer or converter stage:*

*The words converter and mixer are used interchangeably in this text because only the mixing properties of a converter are of interest here.

$$A_c = G_c R'_{if} \qquad (58)$$

where G_c is the conversion transconductance of the mixer tube.

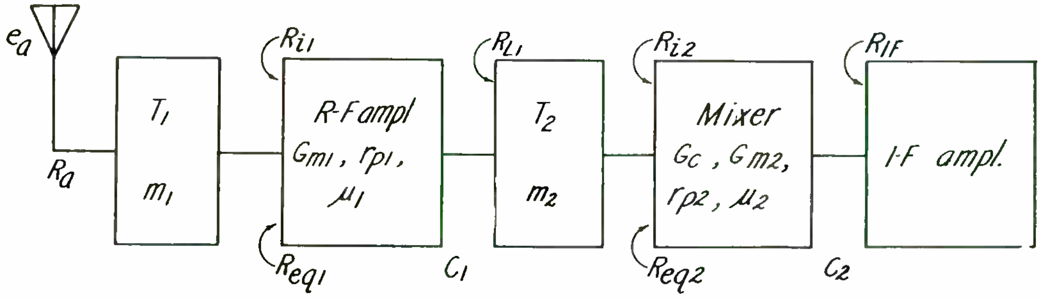


Fig. 34.—Block diagram of a grounded-grid r-f stage matched to an antenna and a grounded-grid mixer stage, which in turn feeds an i-f stage. The use of a grounded-grid mixer permits the oscillator signal to be injected in the otherwise grounded-grid circuit without "pulling" occurring.

3). Calculation of Z_L, which here represents the input impedance of the converter:

$$Z_L = \frac{R_{it2}}{1 + G_{m2} R_{it2}} \qquad (59)$$

4). Refer to Eq. (11) once more. Note that R_0 now is R_a, the antenna resistance. Thus, the step-down ratios of transformers T_1 and T_2 are respectively:

$$m_1 = \sqrt{R_{it1}/MR_a} \qquad (60)$$

$$m_2 = \sqrt{Mr_{p1}/Z_L} \qquad (61)$$

where as before

$$M = \sqrt{1 + (\mu_1 + 1)\ (R_{it1}/r_{p1})} \qquad (62)$$

5). Find R_{L1}, the load impedance presented to the r-f stage:

$$R_{L1} = m_2^2\ Z_L = Mr_{p1} \qquad (63)$$

6). Check to see if R_{L1} is not too high for the band width required:

$$\Delta f' = 1/(\pi R_{L1} C_1)\ * \qquad (64)$$

where $\Delta f'$ is then compared with the specified value Δf to see if it is greater or equal to Δf. If so, the rest of the procedure follows. Since normally this is the case, it will be so assumed, and the rest of the procedure now given.

7). Calculation of the gain A_{rf} of the r-f stage:

$$A_{rf} = \frac{(\mu+1)\sqrt{R_{it1} R_{it2}/r_{p1} R_a}\,(1+G_{m2} R_{it2})}{2(1+M)} \qquad (65)$$

The gain is from antenna to mixer cathode under *matched* conditions of m_1 and m_2, and hence includes the two transformers T_1 and T_2.

8). Calculation of various noise resistors as referred to the antenna terminals:

*This equation is lacking the number two that appears, for example, in Eq. (56). The reason is that the tube's internal resistance matches R_{L1} and therefore also has that value. Hence, the NET resistance shunting the tube's output capacity C_1 is R_{L1} and the tube's $R_p = R_{L1}$ in parallel, or $R_{L1}/2$. If $R_{L1}/2$ is substituted in Eq. (56), Eq. (64) lacking the "two" is obtained.

a). $$R'_{eq1} = \frac{R_{eq1}}{4m_1^2}\left[\frac{(\mu_1 + 1)R_{it1} + 2(r_{p1} + R_{L1})}{(\mu_1 + 1)R_{it1} + (r_{p1} + R_{L1})}\right]^2$$

$$= \frac{R_{eq1}R_a}{4R_{i1}} \frac{[(\mu_1 + 1)R_{it1} + 2(r_{p1} + R_{L1})]^2}{[(\mu_1 + 1)R_{it1} + (r_{p1} + R_{L1})]^2 [r_{p1} + R_{L1}]} \quad (66)$$

b). $$R'_{eq2} = \frac{R_{eq2}}{4A_{rf}^2}\left[\frac{2 + G_{m2}R_{it2}}{1 + G_{m2}R_{it2}}\right]^2 \quad (67)$$

c). $$R'_{i1} = \frac{5R_a(r_{p1} + R_{L1})}{4[(r_{p1} + R_{L1}) + (\mu_1 + 1)R_{it1}]} \quad (68)$$

d). $$R'_{i2} = \frac{5R_{i2}}{4A_{rf}^2 \ (1 + G_{m2}R_{it2})^2} \quad (69)$$

e). $$R''_{if} = \frac{R_{if}}{(A_{rf})^2 \ (A_c)^2} \quad (70)$$

9). The equivalent noise resistance R_{nt} at the antenna is the sum of all these plus R_a, or

$$R_{nt} = R_a + R'_{eq1} + R'_{eq2} + R'_{it1} + R'_{it2} + R''_{if} \quad (71)$$

10). The signal-to-noise ratio is then:

$$S/N = \frac{e_a}{\sqrt{4kT\Delta fR_{nt}}} \quad (72)$$

11). The noise figure of the receiver is then

$$N = \sqrt{\frac{R_{nt}}{R_a}} \quad (73)$$

Although an expression can be obtained for N that is free of R_a, it is felt that the simpler expression given is preferable.

Now suppose the antenna feeds the mixer tube directly. Then R_{eq1} and R_{it1} drop out of the formulas, and we have the antenna matched directly to the cathode of the mixer tube, as is portrayed in Fig. 35. In this case the steps in the analysis are as follows:

1). Calculation of m_2.

$$m_2 = \sqrt{\frac{R_{it2}}{R_a(1 + G_{m2}R_{it2})}} \quad (74)$$

2). Calculation of the various noise resistors as referred to the antenna terminals:

a).
$$R'_{eq2} = \frac{R_{eq2}}{4m_2^2}\left[\frac{2 + G_{m2}R_{it2}}{1 + G_{m2}R_{it2}}\right]^2 \quad (75)$$

b).
$$R'_{i2} = \frac{5R_{i2}}{4m_2^2(1 + G_{m2}R_{it2})^2} \quad (76)$$

c).
$$R''_{if} = \frac{R_{if}}{m_2^2(A_c)^2} \quad (77)$$

3). The total noise resistance referred to the antenna terminals is:

$$R_{nt} = R_a + R'_{eq2} + R'_{it2} + R''_{if} \quad (78)$$

4). The signal/noise ratio is now:

$$S'/N' = \frac{e_a}{\sqrt{4kT\,\Delta f\ R_{nt}}} \quad (79)$$

5). The noise figure is, as before:

$$N' = \sqrt{\frac{R_{nt}}{R_a}} \quad (80)$$

Calculations for S/N and S′/N′ can be made at various frequencies. Where they come out equal is the transition frequency; above this value S′/N′ will exceed S/N and the r-f stage will be undesirable.

NUMERICAL EXAMPLE.—The foregoing procedure will be better understood if it be applied to an actual example. Therefore assume that two 6J6 tubes are employed, a section of one operating as an r-f amplifier, and a section of the second operating as a mixer. The circuit is that shown schematically in Fig. 30 and in block-diagram form in Fig. 34. Where the mixer stage is fed directly from the antenna, Fig. 35 applies.

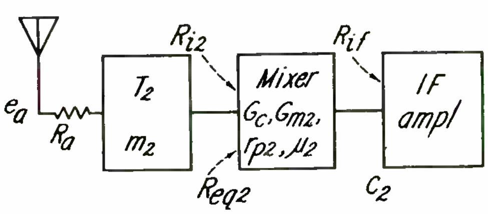


Fig. 35.—Block diagram of direct feed of the antenna to the mixer stage via matching transformer T_2.

The constants for a 6J6 tube may be taken as follows:

$r_{p1} = r_{p2} = 6{,}000$ ohms $G_c = 1400$ μmhos $R_{eq1} = 600$ ohms

$\mu_1 = \mu_2 = 32$ $R_{it1} = 600$ ohms at 400 mc $R_{eq2} = 2200$ ohms

$G_{m1} = G_{m2} = 5300$ μmhos $R_{it2} = 1600$ ohms at 400 mc $C_1 = 5 \mu\mu f$

$\Delta f = 5$mc $R_a = 75$ ohms $C_2 = 10\ \mu\mu f$

$e_a = 10\ \mu$volts

Calculations will first be made at 400 mc, in accordance with the procedure outlined in the previous section.

1. $R'_{if} = 1/(2\pi \times 10 \times 10^{-12} \times 5 \times 10^6) = 3{,}190$ ohms

$$R_{if} = \frac{3190 \times 6000}{6000 - 3190} = 6810 \text{ ohms.}$$

2. $A_c = 1400 \times 10^{-6} \times 3190 = 4.470$

3. $$Z_L = \frac{1600}{1 + 5300 \times 10^{-6} \times 1600} = 168.8 \text{ ohms.}$$

4. $M = \sqrt{1 + (32 + 1)\ (600/6000)} = 2.08$

$$m_1 = \sqrt{600/2.08 \times 75} = 1.963$$

$$m_2 = \sqrt{2.08 \times 6000/168.8} = 8.62$$

5. $R_{L1} = (8.62)^2\ (168.8) = 12{,}480$ ohms or

$2.08 \times 6000 = 12{,}480$ ohms.

6. $\Delta f' = 1/(\pi \times 12480 \times 5 \times 10^{-12}) = 5.1$ mc which exceeds the 5.0 mc specified.

7. $$A_{rf} = \frac{(32 + 1)\sqrt{600 \times 1600/6000 \times 75(1 + 5300 \times 10^{-6} \times 1600)}}{2(1 + 2.08)}$$

$= 2.55$

8. a). $$R'_{eq1} = \left(\frac{600 \times 75}{4 \times 600}\right) \frac{[33 \times 600 + 2(6000 + 12480)]^2}{[33 \times 600 + (6000+12480)]^2 [6000+12480]}$$
$= 33.4$ ohms.

b). $$R'_{eq2} = \frac{2200}{4(2.55)^2}\left[\frac{2+5300 \times 10^{-6} \times 1600}{1+5300 \times 10^{-6} \times 1600}\right]^2 = 102.8 \text{ ohms.}$$

c). $$R'_{it1} = \frac{5 \times 75 \ (6000 + 12480)}{4[(6000+12480) + (33)\ (600)]} = 45.2 \text{ ohms.}$$

d). $$R'_{it2} = \frac{5 \times 1600}{4(2.55)^2 \ (1+5300 \times 10^{-6} \times 1600)^2} = 3.42 \text{ ohms.}$$

e). $$R''_{ir} = \frac{6810}{(4.47)^2 \ (2.55)^2} = 52.4 \text{ ohms.}$$

9. $R_{nt} = 75 + 33.4 + 102.8 + 45.2 + 3.42 + 52.4 = 312.2$ ohms.

10. $$S/N = \frac{10 \times 10^{-6}}{\sqrt{4 \times 1.37 \times 10^{-23} \times 300 \times 5 \times 10^6 \times 312.2}} = 1.97$$

11. $$N = \sqrt{\frac{312.2}{75}} = \sqrt{4.16} = 2.1$$

This is now to be compared with the signal/noise ratio when the antenna feeds the mixer directly through a suitable matching transformer. Following the procedure outlined, there is obtained:

1. $$m_2 = \sqrt{\frac{1600}{75 \ (1 + 5300 \times 10^{-6} \times 1600)}} = \sqrt{2.25} = 1.501$$

2. a). $$R'_{eq2} = \frac{2200}{4\ (1.501)^2}\left[\frac{2 + 5300 \times 10^{-6} \times 1600}{1 + 5300 \times 10^{-6} \times 1600}\right]^2 = 298 \text{ ohms}$$

b). $$R'_{it2} = \frac{5 \times 1600}{4(1.501)^2(1 + 5300 \times 10^{-6} \times 1600)^2} = 9.88 \text{ ohms.}$$

2. c).

$$R'_{if} = \frac{6810}{(1.501)^2(4.47)^2} = 152 \text{ ohms}$$

3.

$$R_{nt} = 75 + 298 + 9.9 + 152 = 535.4 \text{ ohms}$$

4.

$$S'/N' = \frac{10 \times 10^{-6}}{\sqrt{4 \times 1.37 \times 10^{-23} \times 300 \times 5 \times 10^6 \times 535.4}} = 1.51$$

5.

$$N' = \sqrt{\frac{535.4}{75}} = \sqrt{7.14} = 2.67$$

Since the signal/noise ratio is lower, and the noise figure is higher, than when an r-f stage is employed, the value of including the latter is therefore evident.

Consider now the circumstances at 800 mc. First the transit-time loading resistances must be computed at this new frequency, since they vary inversely as the square of the frequency. Hence

$$R_{1t1} = 600 \times \left(\frac{400}{800}\right)^2 = 150 \text{ ohms}$$

and

$$R_{1t2} = 1600 \times \left(\frac{400}{800}\right)^2 = 400 \text{ ohms}$$

All other quantities such as $G_{m1} = G_{m2}$, R_{eq1}, R_{eq2}, etc., remain unchanged. Moreover, since the band width has not been changed, R'_{if} and R_{1f}, and therefore A_c remain fixed. The remaining quantities, however, are altered. If the computations be made in exactly the same manner as previously, the following results are obtained:

S/N = 1.226 when an r-f stage is employed.
S'/N' = 1.255 when the antenna feeds the mixer directly.

Since the latter value is somewhat higher, it indicates that 800 mc is above the crossover frequency and that the r-f stage should be eliminated, although the difference in signal/noise ratio is admittedly small.

The actual crossover frequency is found to be approximately 760 mc, at which point the two signal/noise ratios are equal to 1.296. This is a very low value, and indicates that a signal voltage e_a = 10 μvolts is hardly sufficient for satisfactory reception.

It will be observed that the crossover frequency is very high for negative-grid tubes, and great care must be taken to enable them to operate properly with the circuit elements available. As stated previously, the reason that the crossover frequency is so high, is that the gain of a grounded-grid amplifier changes rather slowly with frequency; it does not drop rapidly until its input transit-time loading drops appreciably below the electronic or feedback loading which is equal to

$$\left(\frac{r_p + R_L}{\mu + 1}\right)$$

Another factor to be noted is that the matched condition of operation was assumed here. If the band width is so great, however, that the matched value of R_{L1} is too great relative to C_1 to permit this band width to be obtained, then R_{L1} must be chosen to meet this requirement rather than that of matching the source impedance, and the formulas become considerably more involved.

Nevertheless, the underlying theory is just as straightforward as for the matched condition, and has been sufficiently explained in the text to enable a student to work out this case if he wishes. It was not deemed of sufficient importance to present here in that the matched condition meets the reasonable value of band width normally encountered, yields simpler formulas, and moreover illustrates just as well the ideas of crossover frequency, signal-to-noise ratio, and the noise figure for a receiver.

RESUME'

This concludes the assignment on the behavior of a negative-grid tube at ultra-high frequencies. The first topic to be discussed was the grounded-grid amplifier in order to permit the subsequent analysis to be made. It was shown why the grounded-grid amplifier is more stable, and how it exhibited a low input impedance. Also the method of matching the source and load impedances by means of matching transformers was discussed. It is possible to match both sides simultaneously, **PROVIDED THAT AN ADDITIONAL RESISTANCE R_{1t} REPRESENTING TRANSIT-TIME LOADING. IS PRESENT AT THE INPUT.**

Then cathode-lead-inductance loading was analyzed and its effects and their minimization discussed. Transit-time loading was also mentioned, although its analysis is deferred to the following assignment.

Following this the subject of noise was taken up. Both thermal and shot-effect noise were discussed, and the concept of equivalent noise resistance in the grid circuit developed. Sample signal-to-noise computations for a single stage were then made.

The next topic was that of triode and diode mixers. The concepts of conversion transconductance and conversion conductance were developed in order to show how the gain of a mixer stage could be calculated. Also the input loading and equivalent noise resistance magnitudes were discussed to show their similarity to the values employed in the ordinary r-f amplifier.

Next a receiving circuit involving both an r-f and mixer stage of the grounded-grid type was analyzed, and it was shown how to obtain the equivalent noise resistances for this type of circuit. Sample computations, indicating how the crossover frequency can be determined, concluded the assignment.

TUBES AND ASSOCIATED CIRCUITS

EXAMINATION

1. An amplifier stage has a signal of one volt impressed on its input grid terminals. The plate output voltage is 22 volts, at an angle of +20° to the input voltage. The grid-to-cathode capacity C_{gk} is 5 μμf, and the grid-to-plate capacity is 2 μμf.

   (a) What is the gain A of the stage?

   (b) What is the total input capacity of this stage?

   (c) What is the input reactance at 2 mc?

   (d) What is the input resistance at 2 mc?

2. (a) An amplifier stage operates at 70 mc. The tube has a G_m = 3,000 μmhos, a grid-to-cathode capacity of 3.8 μμf, and a cathode lead inductance of .06 μhenry. Find the input loading (resistance).

   (b) How can loading due to cathode lead inductance be minimized?

3. (a) What is the basic cause of noise in a perfect resistor?

   (b) Give an equivalent circuit for the resistor.

   (c) Suppose you were given two perfect 1,000 ohm resistors, and both were connected to exactly similar noise-free amplifiers of identical gain and band width. The two amplifiers were then connected to identical cathode ray oscilloscopes, whose horizontal sweeps are synchronized. Would you expect the two wave patterns to be identical during each horizontal sweep of the cathode ray beams, or to differ? Explain the reason for your answer.

4. (a) Why is the *mean* square noise voltage, $\overline{e_n^2}$ used instead of the instantaneous noise voltage itself?

   (b) The *root-mean-square* noise voltage $\sqrt{\overline{e_n^2}}$ is 10 μvolts for a resistor at a room temperature of 20°C and for a band-width of 4,000 cycles. What is the value of the resistance?

   (c) What is its *root-mean-square noise* voltage for a room temperature of 40° C and band-width of 7,000 cycles?

5. Calculate the *mean-square* noise voltage of a 10,000 ohm resistor at a room temperature of 25° C, operating over a range of frequencies from 1,000,000 to 1,050,000 c.p.s.

6. A parallel resonant circuit has a capacitor of value 200 μμf, a coil of 1 μhenry inductance and a Q of 75. The band width of interest is 30 kc, and the temperature is 27°. Calculate the *root-mean-square* noise voltage.

7. A dipole antenna of 75 ohms resistance feeds a transmission line of the same characteristic impedance terminated in a tuned circuit that acts as a matching transformer. An r-f amplifier stage is connected to its output terminals. The incoming signal has a frequency of 300 mc, and for adequate image suppression a band width of 1.5 mc is necessary. A 955 acorn tube is employed. Tests made indicate that this tube has an input loading resistance of 18,000 *ohms at* 100 *mc*. The resistance equivalent to the shot noise is given in Table 1. Use G_m = 2,000 μmhos and G_c = 660 from Table.

   (a) What is the step-up ratio of the tuned circuit between the tube and the antenna transmission line?

   (b) Calculate the signal/noise voltage and power ratios at the output of the r-f amplifier stage, disregarding any noise owing to the plate load resistor. Assume the antenna line is matched by the tube, that the room temperature is 27° C, and that the antenna pickup is 10 μvolts.

   (c) What is the noise factor for this stage?

8. Calculate the signal/noise ratio at 700 and 1000 mc: if an r-f stage is employed: if the converter is connected directly to the antenna. The r-f and converter tubes are a 6J6 having the constants given in the illustrative problem in the assignment.

9. (a) Given a miniature tube which has a G_m = 2,000 μmhos and μ = 25. It is connected as a grounded-grid amplifier, the plate load resistance is 2,000 ohms while the cathode load is 1,000 ohms = R_{1t} owing to transit time. Assume generator impedance is very large compared to R_{1t}. Calculate the impedance looking in between the cathode and ground Z_{AB}, and the impedance looking in between plate and ground Z_{CB}. Also what impedance would a signal source see looking into the tube between cathode and ground.

(b) Suppose the signal source for the grounded-grid stage in Part (a) is a dipole antenna having an internal impedance of 75 ohms. Calculate the voltage *step-up* ratio m_1 for the input transformer, and the voltage *step-down* ratio m_2 for the output transformer for matched conditions.

(c) Calculate the overall gain of the stage under the above matched conditions.

(d) Calculate the gain if the input transformer is omitted.

(e) Calculate the gain if the output transformer is omitted.

(f) Calculate the gain if both transformers are omitted.

(g) Calculate the gain of a circuit in which a single matching transformer is employed between the antenna and the plate load impedance, and the tube is omitted.

10. (a) What advantage is obtained by employing a matching transformer between the signal source and the input terminal of a grounded-grid amplifier stage, and a matching transformer between the output terminals and the plate load impedance, where this is fixed by other considerations, such as that it represents the input impedance of the following stage?

(b) Why is a grounded-grid amplifier superior to the more conventional grounded-cathode type at ultra-high frequencies?

(c) What advantage does the grounded-cathode type have over the grounded-grid type at low frequencies?