SECTION 2

ADVANCED PRACTICAL RADIO ENGINEERING

RADIATING SYSTEMS AT ULTRA HIGH FREQUENCIES

253E

—TABLE OF CONTENTS—

RADIATING SYSTEMS AT ULTRA HIGH FREQUENCIES

INTRODUCTION

In the assignment on u.h.f. propagation, mention was made that the effective area of space around a receiving antenna from which energy is picked up is at most about one-quarter wavelength around the antenna. This applies particularly to a half-wave dipole. For a dipole operating in the u.h.f. range or beyond, this represents a very small amount of space.

The transmitting antenna, if non-directional, radiates energy uniformly in all directions. Hence, the amount of energy enclosed in the space indicated above will be but a small fraction of the total energy radiated, and the signal developed in the receiving antenna will be correspondingly small.

This indicates that an array having considerable size, when measured in wavelengths, is desirable. In such a case, the area of the array is practically equal to that of the area from which energy is taken. Hence, in the assignment on u.h.f. propagation, an array of aperture S square meters was considered as giving the area of space from which energy was extracted.

In this assignment on radiating systems there will be considered the properties of an array, of radiating hollow tubes, various kinds of horns, and reflectors— principally of the parabolic form. These devices promise to play an increasingly more important role as the years go by and the commercial applications of the higher and higher frequencies are utilized to a greater and greater extent.

RADIATING ARRAYS

The subject of arrays has been treated fairly completely in previous assignments on radiating systems, and these should be re-read as a preparation for this assignment. Here the remarks will be directed mainly at the properties of u.h.f. arrays, with emphasis on their special features.

At ultra-high frequencies, arrays, including reflecting surfaces many wavelengths in size, become very practical because of the short wavelengths involved, and hence the moderate size of such devices. On the other hand, difficulties begin to enter in, particularly in the microwave range, because of the ease with which transmission line feeders to the antennas can radiate, and because of the precision with which adjustments of antenna lengths and spacings have to be made.

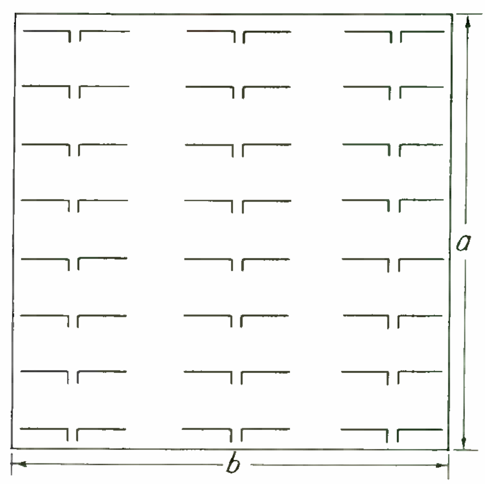


Fig. 1.— Typical antenna array.

BASIC ACTION OF AN ARRAY.—A typical array is shown in Fig. 1. The area involved is filled with small dipoles that form a two-dimensional array. The dipoles, as is evident from the figure, do not fill the entire area, but occupy only discrete portions of it, with spaces in between. The radiating currents, which flow on the surfaces of the dipoles, similarly are located in discrete lines or narrow areas in the space.

However, as a fairly good approximation to the action of an actual array, one can assume that the current is *uniformly distributed* across the entire area, and that the area radiates in its entirety. The assumed hypothetical current distribution is shown in Fig. 2. Thus,

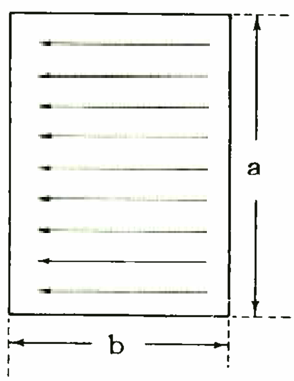


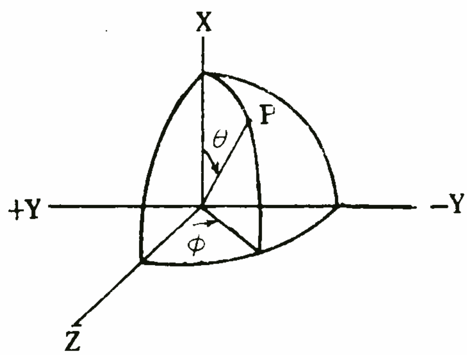
Fig. 2.—Current distribution in array of Fig. 1.

whatever point in the area is chosen, it is assumed that the r.m.s. value of the current is the same. While such a distribution departs widely from that of an actual array, many useful facts can be gleaned from this hypothetical arrangement.

The relative field strength will be

$$F = \sin\theta \left[\frac{\sin\left[(\pi a/\lambda)\cos\theta\right]}{\left[(\pi a/\lambda)\cos\theta\right]} \cdot \frac{\sin\left[(\pi b/\lambda)\sin\theta\sin\phi\right]}{\left[(\pi b/\lambda)\sin\theta\sin\phi\right]}\right]^{*} \quad (1)$$

where θ is the angle of elevation from the array to a point P in space, on a sphere surrounding the array, and ϕ is the azimuth angle, (see Fig. 3). The directional pattern shows a pair of major lobes, and an infinite number of minor lobes



(*Courtesy Microwave Transmission, by Slater*)

Fig. 3.—Space pattern of an array.

as indicated by Fig. 4. This is a cross section of the space pattern along the z-axis, the direction perpendicular to the plane in which the currents are flowing.

The angles $\text{Cos}^{-1}\,\lambda/a$ and $\pi - \text{Cos}^{-1}\,\lambda/a$ represent the two angles *with respect to the z-axis* at which the first nulls in the pattern occur. This is an approximation that is satisfactory if the dimen-

*A derivation of this formula can be found in Chap. VI, Section 34 of "Microwave Transmission" by J. C. Slater, published by McGraw Hill.

sion a of the current sheet or plane is large compared to λ.

It is clear that the angle 2λ/a, between which the major lobe

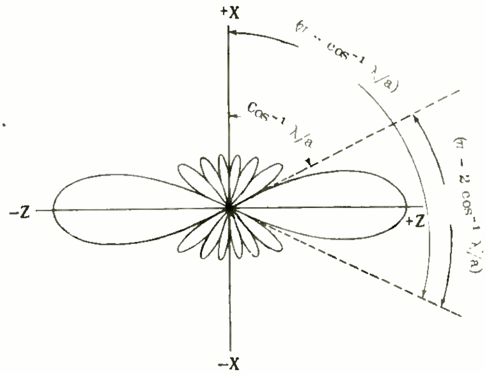


Fig. 4.—Z-axis space pattern.

is confined, is smaller if a is made larger relative to λ. A similar relation applies to the width of the major lobe in the Y direction: The greater b is compared to λ, the narrower is the lobe in this direction.

Thus, for a very narrow beam (narrow in both dimensions), the area or aperture of the current sheet must be large compared to λ^2, and more specifically, if it is desired to have a beam narrow in a given direction, the current surface must be broad in this direction. While the above conclusions have been drawn from an apparently theoretical setup—that of a plane surface, such as a thin sheet of copper, in which currents are uniformly distributed—nevertheless the same results are essentially obtained if the current is confined to rods or wire antennas situated at discrete locations in the area. Thus, the narrower the beam desired, the more antennas must be employed in the array, and the more space they must occupy.

REFLECTORS.—The lobes in the -Z direction, for example, can be suppressed by placing a reflecting array or surface on this side of the main array, and parallel to it. Such suppression cannot be achieved by any possible distribution of antennas in the plane of the main array; an auxiliary array is required. The reflecting array may be directly driven from the source, or indirectly owing to its coupling to the main array. In the latter case, it is parasitically driven. If the reflector is a metallic surface, then—as has been described in previous assignments—it may be regarded as producing an image array as far behind it as the actual array is in front of it. This is shown in Fig. 5, where, for simplicity, a two-element array is depicted. Note that the currents in the image array (arrow-heads) are opposite in direction to those in the actual array.

A driven reflector array permits the spacing between dipoles, their individual lengths, and the phase and magnitudes of the currents in them to be of any arbitrary value. As such, an exceedingly large variety of patterns may be obtained.

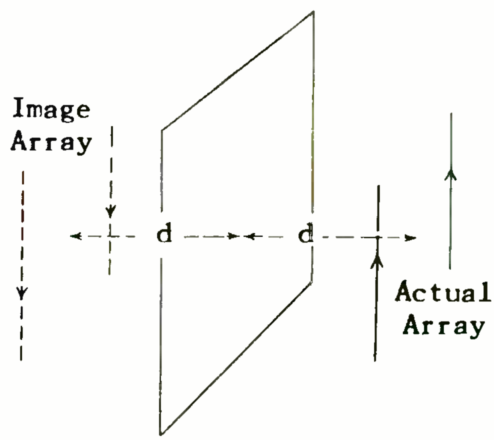


Fig. 5.—Two element array with a reflector.

For example, the reflector array may be spaced $\lambda/4$ behind the main array, and driven with an equal current whose phase is 90° ahead of that of the main array. This case has been analyzed previously with regard to two single antennas rather than two arrays. It will be recalled that the radiation was reinforced in the direction from the reflector to the main array because the delay occasioned by the separation distance $\lambda/4$ cancelled the initial 90° lead of the reflector radiation so that when this energy arrived at the position of the main array, it was just in phase with the energy radiated by the main array, and thus added directly to it. In the reverse direction—from main to reflector position with a 90° lag so that the total lag was 180° in this direction, and the waves cancelled.

The actual pattern depends upon the characteristics of the array, which in turn depends upon the characteristics of the individual elements comprising the array. These can be chosen as desired, and then the phase of the currents set as required by the nature of the pattern.

When a parasitically driven reflector is employed, there is not so much freedom in the choice of current magnitudes, phase, antenna lengths, etc. Consider, for example, two dipoles, each $\lambda/2$ in length, one directly driven from the source, and the other parasitically driven from the first. The two act as a pair of resonant circuits coupled to one another. The coupling, for the small distances involved (usually less than λ) is due partly to the inductive components, and partly to the radiation components of the electric and magnetic fields.

If the two antennas are spaced $\lambda/4$ units apart, then the voltage induced in the second by current in the first will depend upon

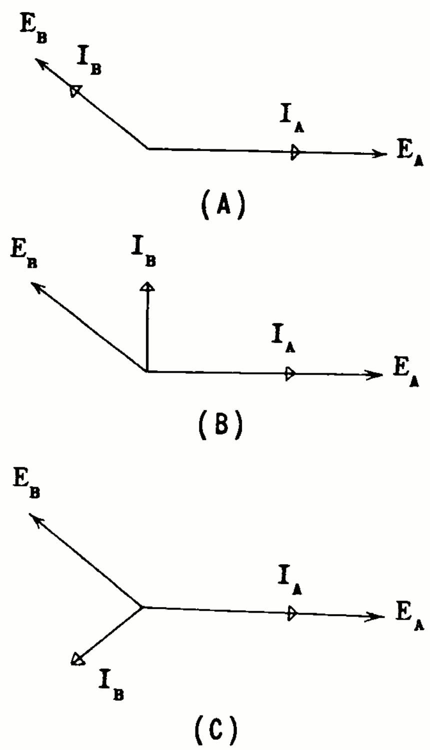


Fig. 6.—Vector relations of two antennas $\lambda/4$ apart.

the coupling factor or so-called mutual impedance between the two. The mutual impedance will in general be a complex quantity whose magnitude and phase depends upon the spacing. This, in turn, will produce a definite phase angle between the voltage induced in the second antenna and the current flowing in the first. Then, depending upon the self-impedance of the second antenna, its induced voltage will cause a current to flow of a certain magnitude and phase. The current thus depends upon the magnitude of the current in the first antenna, the mutual impedance between the two antennas, and the self-impedance of the second antenna. Hence it would appear that by varying the spacing and by tuning the second parasitically driven antenna—such as by varying its length or by inserting a non-radiating impedance in circuit with it—one can obtain the desired magnitude and phase of current in it for a given current in the first. This is not, however, the case; that is, the choice of magnitude and phase of the current in the second antenna is rather limited.

To see this, consider, in somewhat more detail, the case of the two antennas $\lambda/4$ units apart. Suppose a voltage E_A is impressed upon antenna A, and that the latter is tuned. Then its current, I_A, will be in phase with E_A, as shown in each of the three cases (A), (B), and (C) if Fig. 6. The current I_A will induce in the second antenna a voltage E_B, of a value

$$E_B = -I_A Z_{AB}$$

where Z_{AB} is a mutual impedance between the two. As stated above, this mutual impedance depends upon the dimensions of both antennas and their separation. It can be evaluated mathematically in many cases, but this is a difficult thing to do. (Curves for this quantity are given for special values of length and spacing of a broadcast antenna in the Broadcast Assignments.)

It may be measured experimentally by opening up the second antenna so that current cannot flow in it, and measuring the voltage across the break. However, antennas have transmission line characteristics, and so the break or open circuit should preferably be made at a current loop. For example, in the case of a half-wave dipole, the break should be made at the center. The voltage measured here, E_B, when divided by current I_A of the first antenna, gives the value of Z_{AB} as referred to the center of the dipole.

Each antenna has a certain amount of *self-impedance*, such as Z_A for antenna A, and Z_B for antenna B. The current that flows in antenna B is

$$I_B = \frac{E_B}{Z_B} = \frac{-I_A Z_{AB}}{Z_B}$$

Suppose Z_B were a pure resistance, i.e., antenna B is adjusted so that it is tuned and resistive in nature. Then I_B would be in phase with E_B, as shown in (A) of Fig. 6. *Note that it leads I_A by more than 90°.*

If antenna B is altered so that it has an inductive reactance instead of resistance, I_B will lag E_B as shown in (B) of Fig. 6. By suitable adjustment, I_B can be made to lead I_A by just 90°. If antenna B is a half-wave dipole, it can be made to have a desired inductive reactance X_L by increasing its length over that for a half-wave. But this increases the impedance Z_B of the antenna from a pure resistance value R to the larger value $\sqrt{R^2 + X_L^2}$. For a given value of induced voltage E_B, this reduces I_B. Hence, in the process of veering I_B around until it is just 90° ahead of I_A, *its magnitude has been decreased.*

If antenna B is shortened, its impedance becomes partly capacitive, and I_B is both reduced in amplitude and caused to lead E_B, as shown in (C) of Fig. 6. Note that the phase relation between I_A and I_B is such that I_B may be said to *lag* I_A by more than 90°.

In the case of two driven

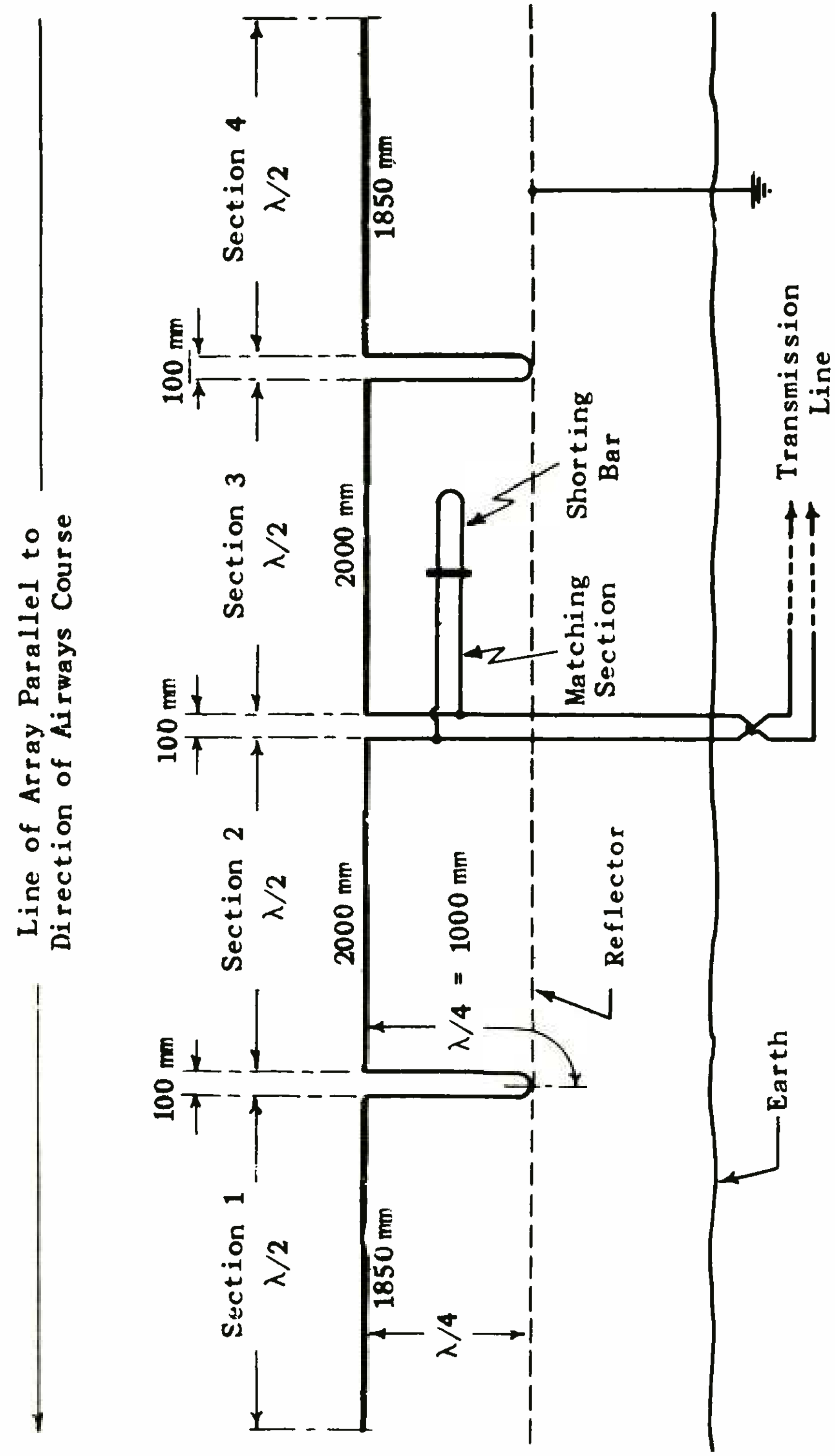


Fig. 7.—Fan marker beacon array with reflector.

antennas, it is possible to adjust I_A and I_B to any desired magnitude and phase with respect to one another. In the case just described, where one is driven and the other parasitically excited, such independent control of phase and magnitude is not possible.

If, for example, it is desired to have the parasitic antenna act as a reflector, then—as previously shown—its current must lead that of the driven antenna by 90°, and be equal in magnitude to it. This is approximated by (B), Fig. 6, but generally in the process of causing I_B to lag E_B by increasing the antenna length or similar means, I_B is appreciably reduced in amplitude, so that it cannot suppress completely radiation of the driven antenna along their line of action. Generally, by some readjustment of the spacing, Z_{AB} can be modified, as is therefore both the phase and magnitude of E_B. This may give a more optimum condition for reflector action.

A parasitically driven array is more effective as a reflector than a single antenna, and a copper sheet of area large compared to the array is generally completely effective. A variation of a copper sheet is a copper screen, whose openings are small compared to λ. In Fig. 7 is shown a wire mesh reflector used with a Fan Marker Beacon in airplane landing systems. A relatively low frequency of 75 mc is employed, so that λ is long (4 meters). Hence the mesh can be relatively coarse. In the microwave range the mesh must be much finer, such as ordinary window screening. This material can be readily shaped to various forms, such as a parabolic reflector. However, in production solid metal is just as simple to employ, and is stronger mechanically.

The position of the driven array with respect to the reflecting surface has an important effect upon

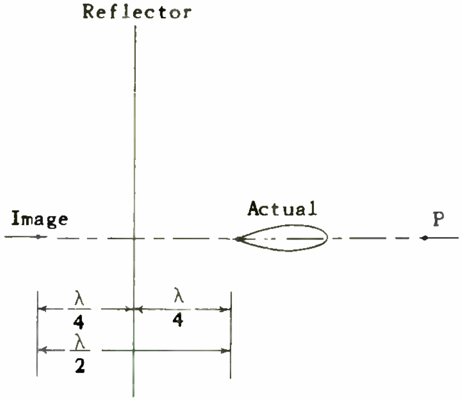


Fig. 8.—Antenna parallel to the reflector.

the intensity of radiation in the forward direction. Unlike a single parasitic antenna, a reflecting sheet acts as if an image array were as far behind it as the actual array is in front of it, and the currents in the image array must be opposite to those of the actual array (refer once again to Fig. 5). This represents a 180° phase difference between the actual and image currents, rather than a 90° phase difference. The optimum spacing between actual and image arrays is then no longer λ/4, but λ/2 for maximum radiation in the forward direction. This means that the actual array must be λ/4 units in front of the reflector. This is shown in Fig. 8. Radiation of opposite polarity, starting out from the image antenna, requires an additional time corresponding to λ/2 to reach a distant point P on the axis, as compared to the time required by the radiation from the actual array. This adds another 180° phase shift to the field of the image in addition to its initial 180° phase shift, so that a total phase

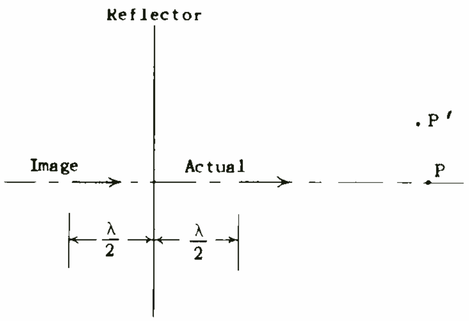


Fig. 9.—Antenna perpendicular to the reflector.

shift of 360° occurs. As a result, the radiation from the image meets that of the actual antenna at P in phase, and maximum effect is had on the axis.

If the spacing of the actual antenna to the reflector were $\lambda/2$, then the image would be λ distance away, and at P would arrive with a relative phase of 360° plus the initial 180°. This is the same as net 180° shift, so that cancellation occurs. Thus, if an antenna is moved on an axis perpendicular to the reflector, the field intensity on the axis will rise and fall as the distance of separation passes through odd and even multiples of $\lambda/4$.

One correction must be made to the above. If the antenna is perpendicular to the reflector, as in Fig. 9, then the image current is in the same direction as that in the actual antenna. (This arrangement may arise with respect to a portion of a parabolic reflector.) In this case maximum radiation is obtained at P when the actual antenna is an even multiple of $\lambda/4$ distant from the reflector, and minimum radiation is obtained at P if the distance is an odd multiple of $\lambda/4$. It is assumed that the antenna is relatively short, and measurements are made from the center of the antenna to the reflecting plane.

The above remarks were with respect to pickup on the axis. For other directions, such as at a point P′ in Fig. 9, the cancelling or reinforcing effects will not be as pronounced because the difference in path lengths for the actual and image antennas will not be the same as for P. However, if the actual system is a broad array instead of a single antenna, then the radiated beam will be very narrow, and correspond to very small angular departures from the axis. In this case, the action along the axis represents fairly well the action of the radiation in all directions of any consequence.

SUPPRESSION OF MINOR LOBES.—It has been shown previously that a large area array radiates a narrow beam, and a small area array, such as a single antenna, radiates a

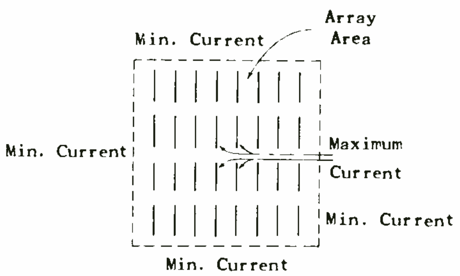


Fig. 10.—An array designed to suppress minor lobes.

broad beam. In addition, it was found that minor lobes are radiated. The reason for these is that the current is of uniform amplitude throughout an area of space, and then abruptly drops to zero over the rest of the plane containing that area.

If it is desired to suppress the minor lobes, then the current magnitude must taper off gradually to zero as one approaches the edges of the area. Theoretically the tapering off should extend to infinity in all directions, and the rate of taper should conform to a certain curve known as Gauss's Error Curve. In practice, no appreciable minor lobes will be obtained if the current magnitude tapers off to zero in a finite area. In fact, quite satisfactory results can be obtained if the current, instead of being uniformly distributed in the area, is concentrated in lines corresponding to antenna units forming the array occupying the above area, provided that the current in the antenna units becomes progressively less as one approaches the edges of the array area. This is indicated in Fig. 10. All elements near the four edges have minimum current, while the center element or elements are supplied with a maximum of current.

The disadvantage is that the edge antennas are not as effective in radiating as the center ones, so that the *effective* area of the array is smaller. This means that the *major lobe* that is radiated is *broader* than it would be if the edge antennas carried as large a current as the center ones. This in turn means that the gain of the array is decreased. Thus, in practice one must balance the factors involved in order to determine the amount of suppression of the minor lobes that is desired.

USES OF ARRAYS.—Antenna arrays become less and less useful as one goes up to very high frequencies, and fortunately, as a compensation, horns and reflectors become more and more useful. To see this, consider a driven array. Each element must be connected to the source through a feeder of some sort, usually a coaxial cable. These represent metallic bodies in the region where the radiation and electric fields are intense, and are, at the very high frequencies, of a size comparable to the small half-wave dipoles usually employed as the elements in the array. As a result, the feeders will have a marked effect in scattering the radiation and distorting the pattern, as well as introducing additional losses.

It is therefore more practical at ultra-high frequencies to drive one element directly, and have this drive the others as parasitic elements. From what has been indicated previously, the pattern will depend upon the spacing and upon the magnitude and phase of the parasitic currents. In particular, the latter depend upon spacing and also upon the length of each antenna unit. For example, if more than one driven and one parasitic antenna are employed, then there will be mutual impedances between each pair of units, and these will all have their effect upon the voltage induced in any one unit and consequently upon its current flow.

The task of adjusting such an array becomes formidable, particularly as one goes up in frequency. At higher frequencies, particularly above 3,000 mc, the dipoles become very short (about 5 cm in length or less). For adequate signal pickup the area of the array or aperture should be about the same as at lower frequencies. This means that the number of dipoles required in the array becomes greater as one goes up in frequency, as does also the difficulty in adjusting them. Also, the physical dimensions must be held to a much greater precision, which further increases the difficulty of adjustment.

As a result, arrays are better suited to the lower frequencies, such as around 100 mc. In this range they have been very successfully used for aeronautical radio purposes, such as localizer and glide paths, etc. This is covered in the assignments on Aeronautical Engineering.

DIPOLE DIMENSIONS.—It was pointed out in an earlier assignment that the greater the radius of the conductor forming a dipole, the lower is its distributed reactance. As a result, a dipole of large cross section, if tuned to some frequency (half-wave in length), will not be very much mistuned over quite a range of frequencies centering around the tuned frequency. In other words, its reactive component will rise rather slowly from zero—at resonance—as the frequency is varied from the resonant frequency. This makes such an antenna particularly valuable for receiving purposes, because its response over a range of frequencies is fairly uniform.

At ultra-high frequencies dipoles generally have relatively large cross sections, if for no other reason than that the dimensions should be compared with the wavelength at which the antenna is operated. Hence a dipole may be expected to be fairly broadly tuned in this range. However, an array of dipoles is much more critical because of the many units involved, and because the spacings themselves are critical. Hence, the ordinary directional array is best suited for single frequency operation. As will be seen, this is also true of parabolic reflectors, but the opposite is true of horns. These have an exceedingly broad response, and are particularly adapted for wide-band operation.

The rhombic antenna is very useful, at least in the lower end of the u.h.f. spectrum, because it can cover such a wide band of frequencies, and is relatively simple in structure. Its properties have been covered in detail in a previous assignment, and should be reviewed by the student at this point. An array of rhombic antennas is also possible, just as one of dipoles.

BAND WIDTH CONSIDERATIONS.—In the case of a single dipole, which is often used in the lower portion of the u.h.f. spectrum (roughly up to 500 mc), the behavior over the band of frequencies is determined not only by the width of the band and the dimensions of the dipole, but by the length of the transmission line connected to it. This has particular reference to its use as a receiving antenna.

A long line changes its impedance very rapidly with frequency when mismatched, as when terminated by a reactance instead of its (resistive) characteristic impedance Z_0. This is because its electrical *length, measured in degrees*, changes rapidly with frequency. Thus, a small change in frequency can cause its electrical length to change by

90°, corresponding to $\lambda/4$, and this corresponds to a large change in impedance, when viewed from the receiver end.

This is an important point, and will be explained in more detail in conjunction with Fig. 11. Suppose the antenna at the frequency in question looks like a capacity C, and

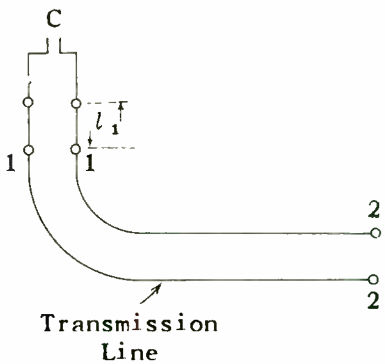


Fig. 11.—Transmission line connecting antenna to a load.

that C tunes with a length l_1 of the transmission line so that it is parallel resonant at the hypothetical terminals 1-1. This makes the impedance looking into 1-1 very high. Suppose, also, that at the frequency in question, the length of the rest of the line between 1-1 and 2-2 is an integral multiple of $\lambda/2$, such as $17(\lambda/2)$. Each half-wave portion of this length transforms the impedance at one end of it into an *equal* impedance at its other end by the properties of $\lambda/2$ sections discussed previously.

Hence ultimately the high impedance at 1-1 appears as an equally high impedance at 2-2 (ignoring line losses). Now suppose the frequency changes from the above value by a small amount, but that this is sufficient to change the line length by approximately $90° = \lambda/4$. The reactance of C will change somewhat, too.

Hypothetical terminals 1-1 will shift in position to some extent, and the distance between 1-1 and 2-2 will now be about the same number of half wavelengths as above, plus a quarter wavelength. For the right amount of frequency change this will be exactly the case. The high impedance at 1-1 will repeat down the line every half wavelength until the last $\lambda/4$ section is reached. But this will transform the high impedance into a very low impedance. Thus, as the frequency changes by small amounts, the impedance of the line will vary through wide limits, and the input to the receiver will vary correspondingly.

For a short feeder length, the band width of the dipole depends essentially on its dimensions, which determine its Q. The Q of the dipole also depends upon the receiver load coupled into it. If this load resistance is made equal to its radiation resistance (matched condition), then the Q is half of its value for zero load resistance (dipole short-circuited). The Q of a short-circuited dipole is approximately

$$Q \cong 1.4\left(\left[2.3 \log \frac{l}{a}\right] - 1\right) \qquad (2)$$

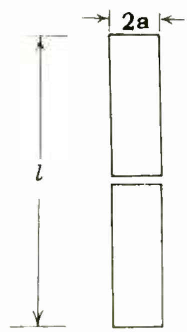


Fig. 12.—Dipole dimensions.

where l is the total length of the dipole and a is its radius, as indicated in Fig. 12. The dimensions can be in inches, cm, etc., since only the ratio is involved. When matched, the Q will be half of the above value.

The frequency range of the antenna, or total band width, may be defined, as previously, as the width corresponding to a drop in signal to .707 of its value at resonance. This is given by

$$f_{bw} = f_o/Q \qquad (3)$$

where f_o is the resonant frequency.

As an example, consider a cylindrical half-wave dipole of 1-inch diameter, operating at 300 mc. A half-wave is

$$\frac{\lambda}{2} = \frac{3 \times 10^8}{2 \times 300 \times 10^6}$$

$$= \frac{1}{2} \text{meter} \times 3.28$$

$$= 1.64 \text{ feet} = 19.68 \text{ inches}$$

By Eq. (2)

$$Q = 1.4\left(\left[2.3 \log \frac{19.68}{.5}\right] - 1\right)$$

$$= 1.4\left(\left[2.3 \times 1.5951\right] - 1\right)$$

$$= 3.74$$

The impedance looking into a dipole *exactly* $\lambda/2$ in length is 73 ohms (resistive) plus a reactance corresponding to a small capacity connected across its outer ends. To counteract this capacity end effect, the dipole is shortened somewhat so as to tune and act as a series resonant circuit having a pure resistance of 73 ohms.

If the transmission line is matched to this 73 ohms, then the Q of the antenna will be halved to $3.74 \div 2 = 1.87$. Then, by Eq. (3), the band width will be

$$f_{bw} = \frac{300 \times 10^6}{1.87} = 160.3 \text{ mc}$$

This means that at 300 ± 160.3/2 or at 380.2 mc and 219.8 mc the signal into the line will have dropped to .707 of its value at 300 mc, because the reactance of the antenna will have risen from zero to a value equal to its radiation resistance of 73 ohms. The impedance at these extremes is (see Fig. 13),

$$Z_a = 73 \pm j73$$

$$= \sqrt{2} \times 73 \angle \pm 45°$$

$$\text{or } 103.2 \angle \pm 45°$$

If a long transmission line is employed, then the band width must be reduced to considerably less than from 219.8 to 380.2 mc, as the long line will not tolerate such a change of impedance as ±j73 ohms in addition to the resistive component of

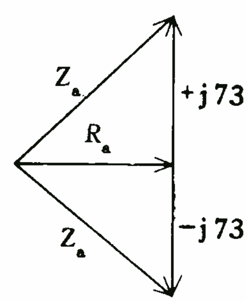


Fig. 13.—Dipole impedance vector diagram.

73 ohms. The necessary reduction in band width is somewhat involved, and probably had best be determined experimentally. The above Eqs. (2) and (3) give the maximum band width

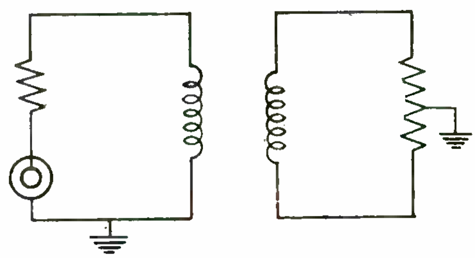


Fig. 14.—Isolation transformer circuit.

possible, i.e., are an upper limit, which cannot be approached if the connected line is long.

COUPLING METHODS.—The dipole antenna is balanced to ground, i.e., the impedance from each input terminal to ground is the same. This is a consequence of the fact that it is essentially a two-wire line opened up to afford radiation, as was discussed in an earlier assignment on radiation.

The coaxial cable usually employed to feed it at u.h.f. is, on the other hand, unbalanced to ground, since in general the outer conductor or sheath is grounded. There thus arises a difficulty in the matter of connecting the coaxial cable to the dipole.

At low frequencies, an isolation two-winding transformer, as shown in Fig. 14, can be used to connect an unbalanced source to a balanced load, or vice versa. At high frequencies the capacity coupling between the primary and secondary produce an unbalance in that the source may feed more current through one-half of the load impedance than through the other. This can be obviated by interposing an electrostatic shield between the two windings, and grounding the shield, but at u.h.f. the impedance of the ground connections raises the shield above ground potential and defeats the purpose of its use.

Fortunately there are several alternative means to couple a coaxial cable to a dipole. A common method is shown in Fig. 15. Here a "skirt" or cylinder suitably larger than the coaxial sheath, is shorted and thus also fastened to it one-quarter wavelength from the open end. Suppose the point C on the coaxial sheath is at ground potential, as is the entire length of skirt. Then, with respect to this skirt, the enclosed end of the sheath acts as an inner conductor while at the same time it acts as the outer conductor with respect to the center conductor within it.

Consider first the outer sheath and enclosing skirt. This is a $\lambda/4$ line shorted at C. If the losses are negligible, then the impedance between A and B approaches infinity. This means that if B is at ground potential, A can be other than at

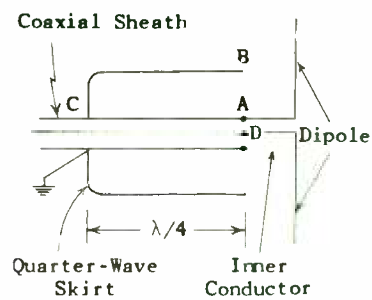


Fig. 15.—Skirt used to couple line to dipole.

ground potential, because its impedance to ground (to point B) is very high. Thus, at point C, there can be no potential between the sheath and ground, whereas at A there can be, for the high impedance between B and A will prevent such a potential from being shorted out.

At the same time there can be as large a potential as desired between A and D, a point on the inner conductor, since the 73 ohms of the dipole is between these two points. Thus, if there is a potential E between A and D, then there is an absorption of power equal to $E^2/73$, which is precisely the power fed into the dipole and radiated by the latter.

The portion of the sheath CA will tend to radiate because it is "hot" to ground. However, it is surrounded by the skirt, and thus prevented from doing so except possibly in the forward direction, i.e., to the right. The *axial* radiation of a conductor, however, is small, and moreover, such radiation is in the direction in which the dipole itself radiates, and hence not particularly undesirable.

If the skirt is omitted, then appreciable radiation from the end portion of the sheath can occur, and the radiation pattern, particularly if the dipole is one of many units in an array, will be markedly changed.

However, note that the skirt length was specified as $\lambda/4$. This will be the case only at one frequency, call it f_o. At any other frequency f, its impedance will be

$$Z = Z_o' \tan 360 \frac{l}{\lambda} \qquad (4)$$

where Z_o' is the characteristic impedance of the skirt and outer sheath, when regarded as a coaxial line, and l is the length of the skirt, and λ is the particular wavelength corresponding to the frequency f in question. Since l is a quarter wave, $\lambda_o/4$, at f_o, Eq. (4) can be written

$$Z = Z_o' \tan \frac{360}{4} \frac{\lambda_o}{\lambda}$$

$$= Z_o' \tan 90 \left(\frac{\lambda_o}{\lambda}\right)$$

$$= Z_o' \tan 90 \left(\frac{c/f_o}{c/f}\right)$$

where c is the velocity of light, or

$$Z = Z_o' \tan 90 \left(\frac{f}{f_o}\right) \qquad (5)$$

If f is greater than f_o, tan $\pi/2(f/f_o)$ is that for an angle $\pi/2(f/f_o)$, greater than $\pi/2$ radians (90°). In this case the tangent is negative, or the reactance is capacitive. If f is less than f_o, the angle is less than 90°, the tangent is positive, and the reactance is inductive.

This confirms the statement previously made that a shorted line greater than $\lambda/4$ is capacitive, and one less than $\lambda/4$ is inductive.

The dipole as normally employed, particularly at u.h.f., is symmetrically disposed with respect to ground. This means that the capacities to ground of corresponding portions of its two members are equal; in short, it is inherently balanced to ground, regardless to whether or not it is connected to ground at a center-tap. This is illustrated by Fig. 16, where the two sections are shown connected to ground through half of the radiation resistance, or 73/2 ohms each.

The transmission line, by

Thevenin's theorem, may be regarded as a generator generating a voltage e_g, and having an internal or characteristic impedance of 73 ohms,

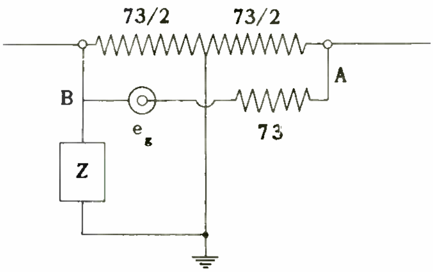


Fig. 16.—Equivalent circuit of a dipole.

to match the antenna. One end is the central conductor; suppose this connects to point A of the dipole. Then the sheath connects to B. Suppose the sheath has the impedance Z to the skirt, which is grounded. Then Z is an impedance between B and ground in Fig. 16.

If Z is much greater than 73/2 ohms—or more generally—is greater than half the radiation resistance R_a of the dipole, then it will have negligible effect in upsetting the symmetry of the configuration. Hence, it is not necessary that the skirt be $\lambda/4$ long, in which case Z is theoretically infinite. Instead, if for any frequency f different from f_o, Z does not drop to less than three times R_a, satisfactory operation will be had.

Inspection of Eq. (5) shows that for a given value of f/f_o, Z varies directly with Z_o'. Hence the greater Z_o' is made, the greater can f deviate from f_o. The characteristic impedance Z_o' is given by the ordinary formula for a coaxial cable, namely

$$Z_o' = 138 \log \frac{b}{a} \qquad (6)$$

where b and a are as shown in Fig. 17. Thus, if the band width of operation is given, $2(f_o - f)$, as well as the radiation resistance of the dipole and the coaxial cable dimensions, then the diameter of the skirt can be found.

For example, suppose a half-wave dipole is to operate from 240 to 360 mc, and to be resonant at 300 mc. Its radiation resistance is approximately 73 ohms. The value of f/f_o depends upon whether one uses the ratio 240/300 = .8 or 360/300 = 1.2, either gives the same results in Eq. (5). Suppose we use f/f_o = .8, since this represents a more direct solution for the angle which will now be shown.

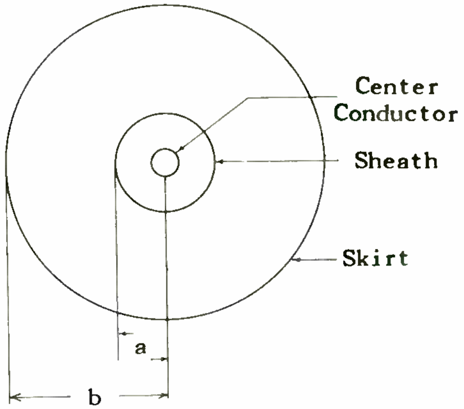


Fig. 17.—Coaxial cable dimensions using skirt.

Then, at 240 mc, Z should not be less than 3 × 73 = 219 ohms. From Eq. (5)

$$219 = Z_o' \tan 90 \times .8$$

$$= Z_o' \tan 72°$$

$$= 3.08\ Z_o'$$

$$Z_o' = \frac{219}{3.08} = 71.1 \text{ ohms}$$

Suppose a coaxial cable is employed whose outer sheath is 1/2 inch in diameter. Then, from Eq. (6)

$$71.1 = 138 \log b/.25$$

or

$$= 138 \log 4b$$

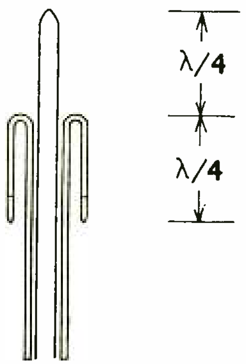
$$\log 4b = \frac{71.1}{138} = .515$$

$$4b = \text{anlg } .515 = 3.27$$

or

$$b = \frac{3.27}{4} = .817 \text{ inches}$$

or a skirt of a diameter 1.634 inches is required. This will perform satisfactorily from .8 × 300 = 240 mc, to 1.2 × 300 = 360 mc.



(Courtesy Proc. I.R.E.)

Fig. 18.—Coupling to a balanced antenna.

Another method of coupling to a balanced antenna is shown in Fig. 18. This is suitable for vertical antennas. Here the skirt is shorted at the end of the coaxial cable and hence is at ground potential, while the lower end is of high impedance to ground (the coaxial sheath). In a vertical position the extension of the center conductor acts like a grounded vertical antenna, while the skirt acts as the radiating ground image.

HOLLOW PIPE RADIATORS

As one goes up in the microwave region, one finds that ordinary antenna arrays become more and more impractical, but fortunately, hollow pipes, horns, and reflectors become more and more feasible. In the case of hollow pipes and horns, the alternating electric field lines at the open end constitute a displacement current sheet which can radiate similarly to the hypothetical current sheet discussed above. The same type of behavior is noted: if the pipe or horn has a wide dimension, then the beam in this plane is narrow, and if the dimension is narrow, the beam is correspondingly broad.

RADIATION FROM A HOLLOW PIPE.—It was probably first emphasized by Prof. W. L. Barrow that the $TE_{1,o}$ mode in a rectangular pipe is particularly suitable for radiation. The reason is not hard to see. The distribution of the electric field lines across the mouth of the tube is sinusoidal, as shown in Fig. 19. Each field line may be regarded as a filament of the displacement current. As such, it is in phase with all the other filaments, and its radiation in the *axial direction* is in phase with that of all the others. Hence, maximum radiation can be expected in the axial direction. In

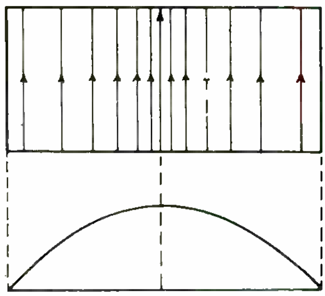


Fig. 19.—$TE_{1,0}$ mode in rectangular pipe.

directions off the axis, the contributions from various parts of the open end tend to meet out of phase, and thus to cancel one another. Thus an axial beam is formed, or the pattern is directional. This is illustrated in Fig. 20. Let CD be the width of the pipe. A top view is shown. Assume the electric field lines are vertical (coming out of the paper), and that they have a sinusoidal distribution, as indicated.

Now take two points, A and B, that are equally distant from the center, O. Consider a point P on the axis, at a considerable distance from the mouth of the pipe. It is evident from the figure that distances AP and BP are equal, and all points to the left of O can be similarly matched with points to the right of O. Since the field is in *time phase* all across the mouth, radiation from A and B start out in phase, and therefore arrive at P in phase since they traverse equal path lengths.

On the other hand, consider a point P′ off the axis. Path length AP′ is greater than BP′, so that the radiation from A arrives at P′ later than that from B, and therefore lags that of B. The two contributions

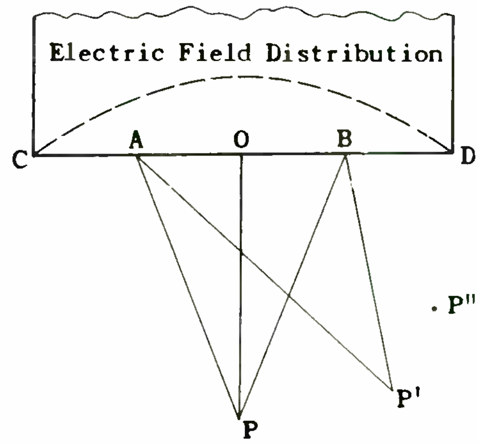


Fig. 20.—Top view of pipe showing electric field.

add vectorially to produce a resultant at P′ that is less than that at P. This is shown in Fig. 21, where the field intensities E_A and E_B, for points A and B, are represented by vectors. For point P the two add in line, producing a maximum resultant, whereas for point P′ they are out of phase and the resultant is correspondingly less.

For a point P″ considerably off the axis, the path lengths AP″ and BP″ may differ by $\lambda/2$. This depends upon the separation between A and B, the amount that P″ is off the axis, and the wavelength. In such a case the two vectors E_A and E_B are clearly 180° out of phase, and hence cancel one another.

The above discussion considered only one pair of points in the mouth area. For points farther apart,

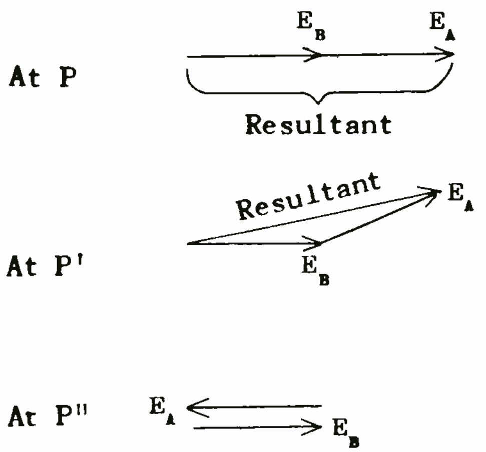


Fig. 21.—Resultant field vectors of Fig. 20.

such as C and D, the path lengths and hence time delay to P, P′, and P″ are different from those for A and B. Hence, the resultant at any of the receiving points P, P′, or P″ is different from that produced by A and B, both as regards magnitude and phase. Therefore, the *overall resultant* will depend upon the number of points involved, *and also upon the position of the receiving point.* If the individual resultants to a point P′ are less than those to a point P, then the overall resultant will probably be less. The word probably is used because while the individual resultants at P′ are smaller, their phase relationships relative to one another might be more favorable and conceivably give rise to a greater overall resultant than that at P.

The problem differs mathematically from that of an array composed of a finite number of discrete antenna elements in that in the case of the array only a finite number of resultants, each of appreciable magnitude, have to be added vectorially, whereas in the case of the open tube, an infinite number of pairs of point areas have to be considered, whose individual radiations are infinitesimal in magnitude, but whose totality of effect is a finite field strength at the receiving point. The problem becomes one for the integral calculus to solve.

The resultant pattern is generally in the form of a smooth major lobe, with few if any minor lobes. This matter will be discussed shortly. One feature that the above elementary discussion cannot explain is that there is radiation to the rear. This cannot come directly from the mouth of the tube, because such backward radiation would be inside of the tube and would be reflected by the piston at

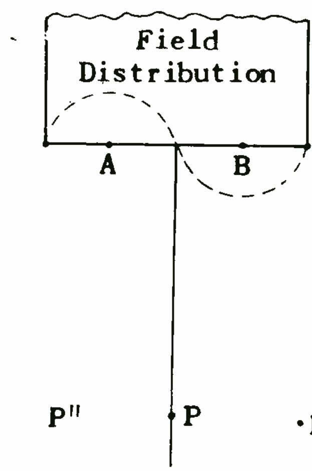


Fig. 22.—$TE_{2,0}$ mode in rectangular pipe.

the closed end of the tube and thus redirected in a forward direction once more. The explanation appears to be that the electromagnetic field at the mouth of the tube causes currents to flow on the outer surfaces of the tube, and it is these currents that can cause backward radiation. For certain dimensions, particularly in the case of a horn rather than an open tube, this backward radiation can be minimized.

THE $TE_{2,0}$ MODE.—It will be instructive to compare the radiation from an open pipe, in which the $TE_{2,0}$ mode is established, with that for the $TE_{1,0}$ mode. In Fig. 22 is shown the spatial relations in a manner similar to those of Fig. 20. For the $TE_{2,0}$ mode, however, the radiation from corresponding points A and B are 180° out of phase. For a point P, the path lengths AP and BP are equal, so that the contributions arrive at P, still 180° out of phase, and hence cancel. For certain directions off the axis, such as P′ and P″, the path length difference for A and B can be such as to add another 180° phase shift, so that the contributions are in phase.

Thus it can be seen that instead of one major lobe, there will be *two major* lobes, making equal angles to the axis, and that the axis is actually a null point. Such a pattern is seldom if ever required. It is more usual to desire a single sharp beam. From the foregoing it is evident that a $TE_{1,0}$ mode is to be used, rather than a $TE_{2,0}$ or similar mode.

There is another advantage of a $TE_{1,0}$ mode in a rectangular guide as compared to higher order modes in such a guide, such as the $TE_{1,1}$ wave shown in Fig. 23. It will be noted from the figure that the electric field lines are curved. This produces an electromagnetic wave whose

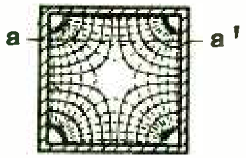


Fig. 23.—$TE_{1,1}$ mode in a wave guide.

polarization is not of one type, i.e., vertical or horizontal. The $TE_{1,0}$ wave, on the other hand, has an electric field whose lines are all straight and parallel to one another. If the actuating antenna in the tube is set in a horizontal position, and the tube height made to exceed the cutoff value, then the electric field lines will be horizontal and a horizontally polarized wave will be radiated.

As mentioned in previous assignments, horizontal polarization is generally preferred at u.h.f. because such waves are less affected by ground conditions, etc. However, if a vertically polarized wave is desired, then the actuating antenna is set vertically, and the tube width adjusted above cutoff. Usually, for the narrow beams desired, the tube dimensions normally required are in excess of those needed to operate above cutoff for the tube.

CALCULATION OF RADIATION PATTERNS.—In order to calculate the radiation pattern it is necessary to note several factors:

1. The displacement current distribution for the $TE_{1,0}$ mode is such that the radiation in the one direction *perpendicular* to the electric field lines is different from that in the other direction parallel to the field lines, i.e., the width of the radiated beam depends upon the width and the height of the tube to a different degree.

2. A pure $TE_{1\,0}$ mode is required if one desires a smooth pattern. As was mentioned in the assignment on wave guides, any off-center displacement of the actuating antenna, or asymmetry in the configuration may cause some higher mode to be produced.

This is especially possible in the case of a tube having a large cross section in order that a narrow beam be obtained. Also a short length of tube may not entirely suppress a higher order mode at the mouth even if the dimensions are below the cutoff values for the tube. This is because the higher order modes are not suppressed right at the source, but instead are rapidly damped within a short distance along the tube. If the latter is short, however, such damping may not be sufficient to prevent the higher mode from appearing to some extent at the tube mouth.

Two quantities are of value in calculating the pattern, the width of the tube and the height of the tube. The width and height of the tube can be expressed in terms of wavelengths. Thus

$$\text{width aperture} = a/\lambda$$

where a is the width of the tube in the same units as λ, such as centimeters.

Similarly, the other dimension—the height of the tube—can be expressed as

$$\text{height aperture} = b/\lambda$$

where b is the height of the tube.

Now let θ and ϕ be the angular deviations in direction from the axis, such that θ is the angular deviation in the same direction as the electric field, and ϕ is the angular deviation in a direction perpendicular to the electric field. Also let the cross-sectional dimensions of the tube be called W_θ and W_ϕ. Where W_θ represents the cross-

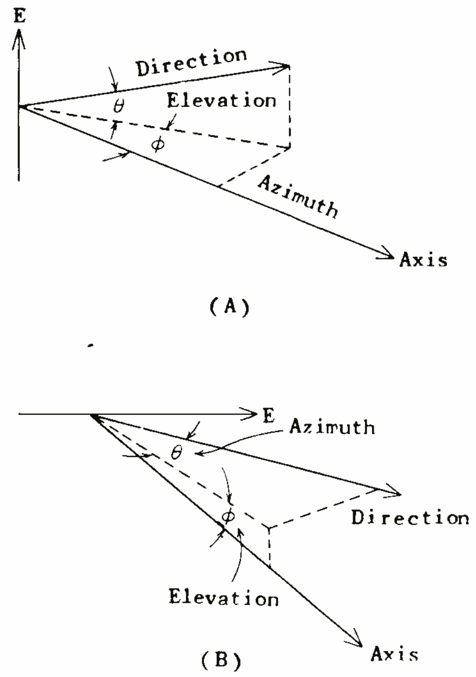


Fig. 24.—Cross-sections of space pattern.

sectional dimension (expressed in wavelengths) parallel to the electric field, and W_ϕ represents the cross-sectional dimensions (expressed in wavelengths) perpendicular to the electric field, so that $W_\phi = a/\lambda$ and $W_\theta = b/\lambda$.

The reason that θ and ϕ are defined with respect to the field direction, is that the field direction determines the width and height of the beam. Thus, if E is vertical with respect to the earth (vertical polarization), θ represents the angle of elevation with respect to the earth, and ϕ represents the azimuth angle, as is shown in Fig. 24(A). On the other hand, if horizontal polarization is employed, Fig. 24(B) applies, and θ becomes the azimuth angle, and ϕ the angle of elevation.

The pattern in the ϕ direction is given by

$$|E| \cong \text{constant} \left| \cos\phi \, \frac{\cos(\pi W_\phi \sin\phi)}{(\pi W_\phi \sin\phi)^2 - (\pi/2)^2} \right|^* \quad (7)$$

and the pattern in the θ direction is given by

$$|E| \cong \text{constant} \left| \cos^2\theta \, \frac{\sin(\pi W_\theta \sin\theta)}{\pi W_\theta \sin\theta} \right|^* \quad (8)$$

where the vertical bars represent the absolute values of the quantities involved. One point must be noted about the above two equations.

If points in space in all directions about E and at a fixed distance from it, are taken, and the field strength at each point plotted as a vector, the familiar antenna directional pattern is obtained. This is a three-dimensional space figure, and involves *all* values of ϕ and θ from 0 to 360 degrees. Usually one is interested in the shape of this pattern in a plane that contains E and the axis of the tube or in a plane containing the axis but perpendicular to E.

An inspection of Fig. 24(A) or (B) will make it clear that all points that lie in the plane containing E and the axis are such that for them ϕ is zero, and only θ can be different for each point. Eq. (8), when plotted, gives the directional pattern for these points. The resulting plane curve is the vertical pattern if E is vertical, or it is the horizontal pattern if E is horizontal.

Similarly, for points that lie in the plane containing the axis but *perpendicular* to E, θ is zero, and only ϕ can be different for each point. Eq. (7), when plotted, gives the directional pattern for these points. The resulting plane curve is the horizontal pattern if E is vertical, or it is the vertical pattern if E is horizontal.

This curve, and the preceding one, represent two mutually perpendicular cross sections of the complete space pattern. Their relation to the complete space pattern is further illustrated by Fig. 25. Usually they—that is, Eqs. (7) and

*In the numerator W_ϕ and W_θ are in degrees while in the denominator they are in radians.

(8)—are sufficient to define the beam. Sometimes only one of them is of major interest. For example, the important requirement may be that the radiation be directed mainly along the horizontal. This means that the *vertical pattern* must be narrow, and that the horizontal pattern can be fairly broad. In other cases the beam may be required to be narrow in *every* direction; in that case both horizontal and vertical patterns must be narrow.

The reason for describing the patterns in such detail is to bring out the fact that which one is vertical and which one is horizontal depends upon the orientation (direction) of E. In designing horns for example, it is necessary to have clearly in mind the relationship between the direction of E and the angles ϕ and θ, in order that the horn mouth dimensions be correctly proportioned. A careful reading of the above paragraphs will show that the relationship is very simple once it is understood.

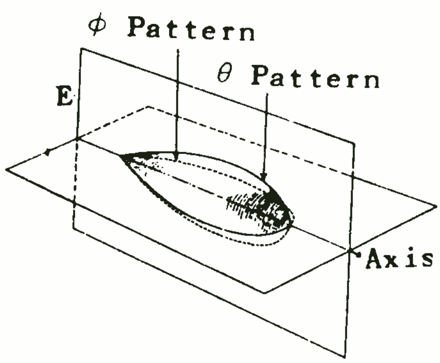


Fig. 25.—Complete space pattern.

The pattern is for points that are a distance from the antenna that is large compared to λ, a, and b. The formulas are approximate, but

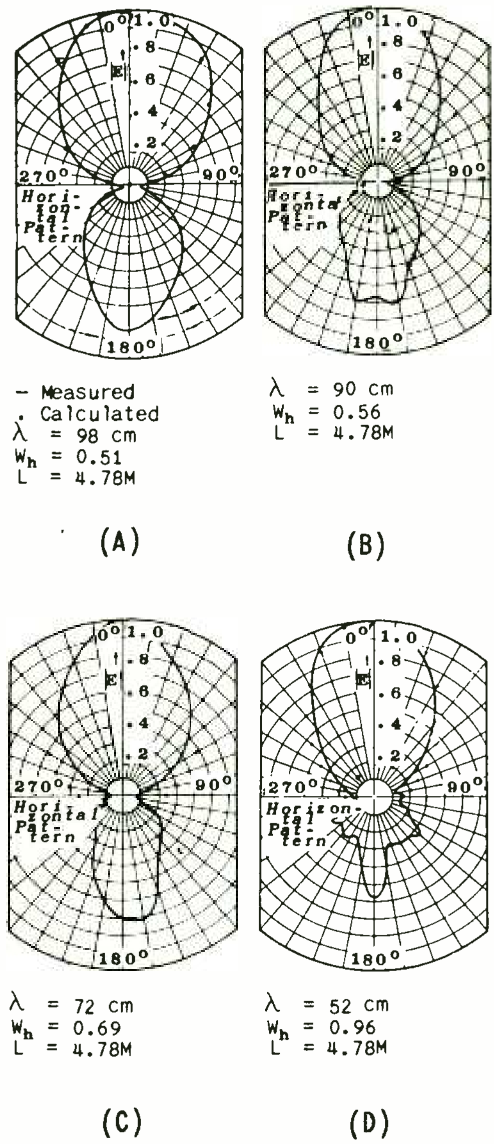


Fig. 26.—Patterns for horns at a wavelength range of λ = 52 to 98 cm, length 4.78 meters.

sufficiently close if the apertures exceed 2 or 3, i.e., if a or b are greater than 2 or 3 times λ. This is generally the case if a narrow beam is desired.

The patterns that may be expected are shown in Fig. 26.

In Fig. 26(A) is shown the ϕ pattern for $W_\phi = 0.51$, $\lambda = 98$ cm; in (B), for $W_\phi = 0.56$, $\lambda = 90$ cm; in (C), for $W_\phi = 0.69$, $\lambda = 72$ cm; and in (D), for $W_\phi = 0.96$, $\lambda = 52$ cm. The length of the tube is 4.78 meters. For a shorter tube of length 2.39 meters, for which $W_\phi = 0.96$, $W_\theta = 1.0$, and $\lambda = 52$ cm, the horizontal and vertical patterns have the form shown in Fig. 27(A) and (B).

It will be observed that the patterns are not as smooth in the forward direction. This is due to the shorter pipe employed. The wave could not quite establish its $TE_{1,0}$ mode in this short length, and as a consequence the field distribution was not exactly a half sinusoid across the width. As a result, irregularities are produced in the radiation pattern. Similar effects will be noted in the case of the sectoral horn. In addition, backward radiation will be observed in these figures.

The patterns have null directions, that is, the major lobe is confined between two equal angular directions to either side of the axis. Thus the width of the beam can be defined as the smallest angle, θ_m or ϕ_m, as the case may be, at which E becomes zero in Eqs. (7) and (8). For the θ pattern this is

$$\theta_m = 2\sin^{-1}\left(\frac{\lambda}{b}\right) = 2\sin^{-1}\left(\frac{1}{W_\theta}\right) \qquad (9)$$

and for the ϕ plane it is

$$\phi_m = 2\sin^{-1}\left(\frac{3\lambda}{2a}\right) = 2\sin^{-1}\left(\frac{3}{2W_\phi}\right) \qquad (10)$$

In Eqs. (9) and (10), the angles are the *total* angles containing the beam from one side of the axis to the other. It will be noted from Eqs. (9) and (10) that:

1. Just as in the case of the array, here too, the greater the aperture W_θ or W_ϕ, the narrower the beam.

2. The θ-pattern is narrower than the ϕ pattern for equal values of the apertures in the ratio of 2 : 3, i.e., $\theta_m = 2/3\ \phi_m$. This is because the field is of uniform intensity along its length, and is therefore of maximum effectiveness, whereas at right angles to its length it varies sinusoidally, and is a maximum only at the center. On the other hand, it varies more gradually towards the walls of the tube, so that the ϕ-pattern might

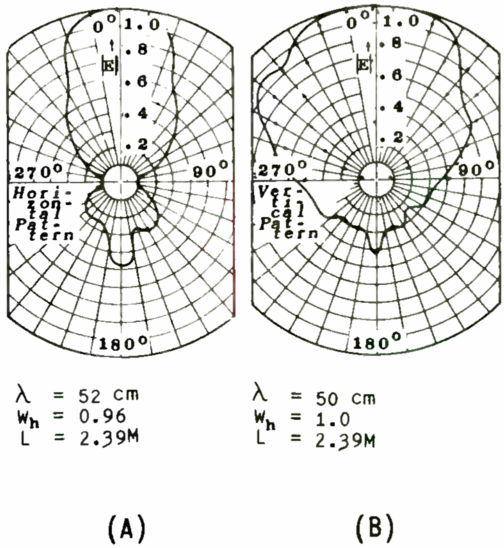


Fig. 27.—Pattern for shorter horn than in Fig. 28.

be expected to be more free of minor lobes, explained earlier in this assignment. However, experimentally it is not easy—at least in the case of a tube—to have a perfectly sinusoidal field distribution, and any irregularities in the distribution may vitiate to a great extent the above conclusion.

To aid in computing the size of the tube opening, ϕ and θ have been plotted for different apertures in Fig. 28. As an example of the design of a tube, suppose it is desired to radiate a horizontally polarized beam 7.5° wide in either

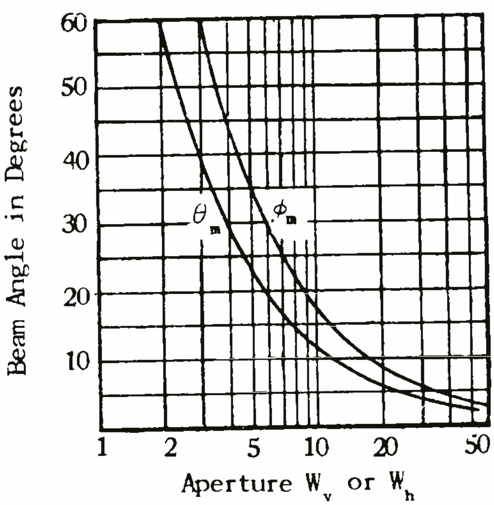


Fig. 28.—Plot of beam angle versus aperture.

dimension at 3,000 mc (λ = 10 cm). Find the dimensions of the tube.

Since the electric field is horizontal, the θ-pattern will be the horizontal pattern, and the ϕ-pattern will be the vertical one. From Eq. (10), or directly from Fig. 28,

$$\sin\frac{7.5^\circ}{2} = \frac{3}{2W_\phi} = \sin 3.75^\circ$$

$$= .0655$$

or

$$W_\phi = \frac{3}{2 \times .0655} = 22.9$$

This is the *vertical* aperture in wavelengths. The height of the tube is λW_ϕ = 10 × 22.9 cm = 90″ or 7.5 feet. The width of the tube can be found from Eq. (9), but, as mentioned previously, it is 2/3 of 90″ or 60″ = 5 feet. These dimensions are surprisingly large for such a short wave as 10 cm, but are due to the very narrow beam desired.

The length of the tube required is not clearly defined. For radiating tubes whose cross-sectional dimensions are about equal to λ, a length of about 7λ has been stated by Barrow to be sufficient to set up the required $TE_{1,0}$ mode. If one were to proportion the tube calculated above according to this rule, it would be necessary to make the length

$$l = W_\phi \times 7\lambda = 22.9 \times 7\lambda$$

$$= 22.9 \times 7 \times 10 = 1{,}603 \text{ cm}$$

or

$$631'' = 52.6 \text{ feet!}$$

This indicates one impractical feature of the tube as a radiator: for very narrow beams, tubes of large cross-sectional dimensions are required, and these in turn demand a tube of inordinate length. A further factor is that it is very difficult to excite such a large tube in only the $TE_{1,0}$ mode, as has been mentioned previously.

A horn, on the other hand, obviates these difficulties. It does not have to be as long as the tube to furnish the requisite beam, although its cross section has to be somewhat greater. Furthermore, it is actuated at the throat end, where its cross section is small, and thus higher order modes can be avoided. Accordingly, the subject of horns will be taken up next, but it must not be construed that the above formulas and their application to the tube were of no value; indeed, it will be found that most of them can be applied to the design of the horn equally well, and that so much of the theory of the radiating tube applies to the horn that the discussion on the tube forms a logical introduction to the analysis of the radiating horn.

HORNS

Horns have been used to a great extent in acoustical work for matching the high acoustic impedance of the loudspeaker unit, at its throat, to the low acoustic impedance of the auditorium, at its mouth. The usual shape is exponential, which means that the percentage increase in cross section is the same at every point of its length. As a result, it flares more and more rapidly as one approaches the mouth, because the same percentage of a larger area is a larger numerical increase in area.

RADIATING HORNS.—In radiation applications, no optimum shape of horn has been found, and it is customary to use a conical, or straight-sided horn, although other shapes have been employed. Indeed, as in acoustical work, various reflectors can in themselves be regarded as a kind of horn, and the dividing lines between horns, reflectors, and ordinary antennas are not sharply defined, as has been brought out by Schelkunoff.*

The cross section of a horn may be round, square, rectangular, etc. It may flare uniformly in all directions, or at the other extreme, two opposite sides may flare, and the other two sides may be parallel. The latter type of horn has been denoted by Barrow as a "sectoral" horn because it is like a sector cut out of a pie. It is shown in (A) of Fig. 29, while in (B) is shown a rectangular horn that flares in both dimensions. A more unusual type is shown in (C), and is called a biconical horn. Its characteristic is that it radiates uniformly in all directions in a horizontal plane, but over a small vertical angle—i.e., in the form of a sheet. It appears particularly suited to u.h.f. broadcasting, as it can cover the line-of-sight area without wasting radiation in a vertical direction.

The question of polarization has bearing on the shape of the cross section. For a horizontally polarized wave, which is of particular value in aeronautical engineering, a $TE_{0,1}$ wave is employed, since the half-sinusoidal distribution along the vertical of the horizontal electric field vector provides the polarization desired. A sectoral horn, Fig. 29(B), is particularly suited for this purpose.

Before discussing this type of horn, it is to be noted in passing that:

1. Very sharp beams can be

*Schelkunoff—"Theory of Antennas of Arbitrary Size and Shape," *Proc. I.R.E.*, Sept. 1941.

formed by horns of suitable size.

2. These beams are reasonably free from secondary beams or lobes in other directions. This is particularly important for localizer, glide path, obstacle detection, and direction-finding applications.

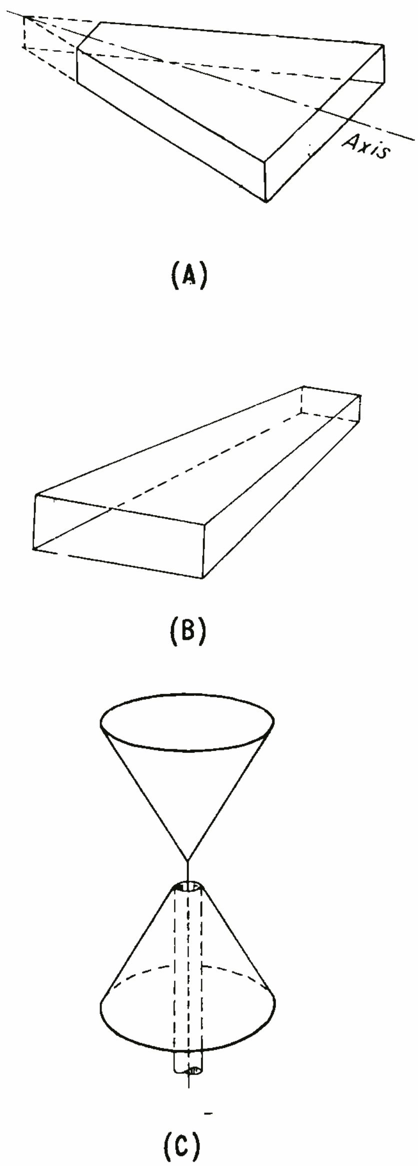


Fig. 29.—Various shapes of radiating horns.

3. The horn is relatively aperiodic as compared to arrays and parabolic reflectors: it can cover an enormous frequency range without readjustment when the frequency is changed from one value to another. For example, in the 700 mc range this may mean a *band width* of from 350 to 1,050 mc, which is enormous, and makes the horn particularly attractive for television and other broad-band services.

4. The horn is mechanically simple to construct and to adjust and operate. It may be made of galvanized iron in the smaller sizes, or of wood lined with sheet copper in the larger sizes.

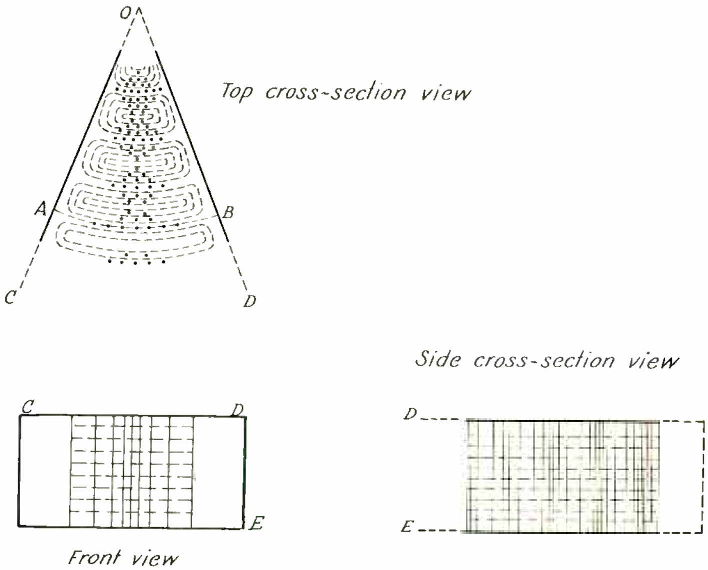
5. A disadvantage is to be noted: it is more bulky than an array or parabolic reflecting system, since it must have the same mouth area, and has a much greater length. In the microwave range, however, this may not be as great a disadvantage as at the lower frequencies.

6. Owing to its beam-forming capabilities the horn can give antenna gains as high as 25 or more. Thus for glide path and other services mentioned above, as well as point-to-point communication, the horn system is able to cover distances of 25 to 50 miles with but a few watts of power. This is important since the generation of u.h.f. power is more difficult than that at the lower frequencies.

THE SECTORAL HORN.—The sectoral horn may be regarded as a flaring hollow tube. As a result, similar wave patterns can be set up by suitable excitation, and it is the role of the engineer to choose that pattern best suited for his purpose—usually radiation.

For the horn, the radiation characteristics depend not only upon the horn mouth dimensions but also (for a sectoral horn) the dimension in which the horn flares.

The first subscript will refer to the number of half-cycle variations of the transverse electric field along a path parallel to the dimension in which the horn flares: the second subscript will refer to variations in the dimension which remains constant.



(Courtesy, Proc. I.R.E.)

Fig. 30.—$TE_{1,0}$ mode horn pattern.

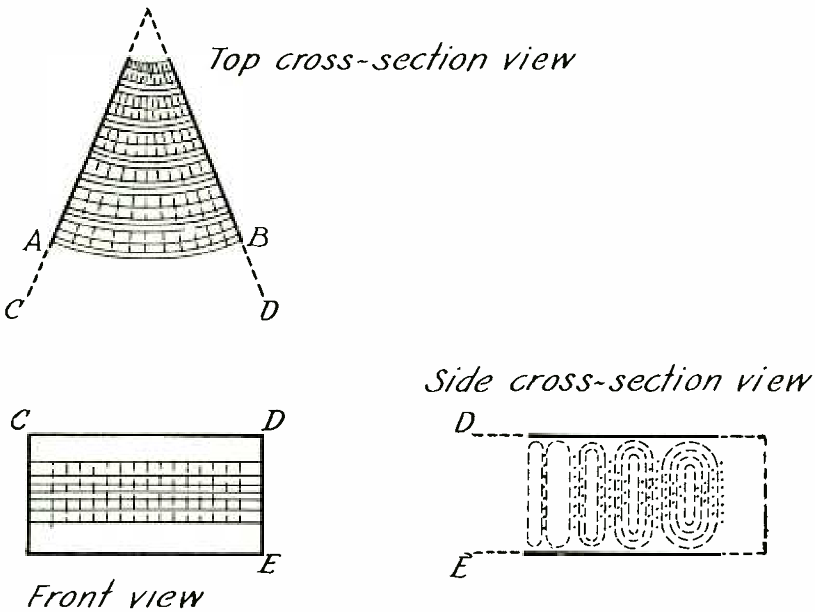
In Fig. 30, the exciting antenna is set perpendicular to the two parallel faces of the horn, whereas in Fig. 31 it is set parallel to these faces. As a result, one has a $TE_{1,0}$ mode for Fig. 30, and a $TE_{0,1}$ mode for Fig. 31. To be accurate, the pattern should be measured along an arc AB, every point of which is equidistant from a corresponding point O on the apex of the horn.

In Fig. 30, the electric field lines are straight, but are disposed, one next to the other, along, an arc AB. The density, for the $TE_{1,0}$ mode, varies from a maximum at the center, to zero at the sides. This, it will be recalled, is in conformity with the general principle mentioned in the previous assignment that electric field lines cannot be parallel to a perfectly conducting surface in its immediate vicinity. The distribution will be a half sine wave, just as in the case of the $TE_{1,0}$ mode in a rectangular wave guide. For a $TE_{2,0}$ mode, the distribution will be according to a full sine wave, and so on for higher modes of this type.

In Fig. 31, it will be noted that the electric-field lines are circular arcs such as AB. At the mouth, these arcs lie one directly below the other along a straight line CD. The density, if plotted along such a line (along a horizontal in the transverse section), would be a half sinusoid, whereas along the field line, such as AB, the density would be unvarying or constant. Hence in this case the mode is $TE_{0,1}$. Higher order modes are defined in the same manner as for the $TE_{0,1}$ type.

It might be expected that a certain minimum width, perpendicular to the electric field lines, is required to prevent cutoff, just as in the case of ordinary nontapering wave guides. The situation, however, is not quite the same for the horn as for the ordinary guide.

It was stated that in the rectangular guide, for example, the $TE_{1,0}$ mode would not be propagated if the width were equal to or less than half the free-space wavelength, or $\lambda/2$. Preferably the width should exceed this minimum figure of $\lambda/2$. In the case of a horn shown in Fig. 30, the width is constantly increasing. Starting at a given point where the width is, say $\lambda/2$ or slightly less, the wave may get through to the mouth because it is not instantly attenuated along the length, and as it proceeds toward the mouth, the attenuation decreases with increasing width. On the other hand, if the exciting antenna is too close to the apex, where the width is small, only a small amount will



(Courtesy, Proc. I.R.E.)

Fig. 31.—$TE_{0,1}$ mode horn pattern.

get through to a point along the horn where the attenuation becomes negligible. Thus it is desirable to start the horn at an appreciable distance R_c from the apex.

The excitation can be from a wave guide fitted to the horn at AB, or by means of a closed end portion

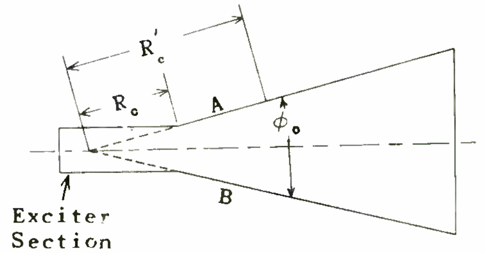


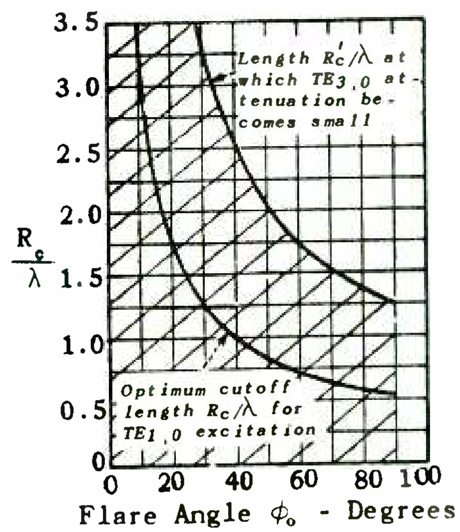
Fig. 32.—Position of exciting antenna.

of a guide called the exciter section in Fig. 32, in which is placed the exciting antenna.

The value to be used for R_c depends upon the mode, $TE_{1,0}$ or $TE_{2,0}$ or $TE_{3,0}$, etc., and upon the flare angle ϕ_0. Normally only the $TE_{1,0}$ is desired, as the $TE_{2,0}$ mode, for example, produces two split beams instead of one axial beam. Fortunately R_c can be so chosen as to permit the transmission of the $TE_{1,0}$ mode and to suppress the higher modes.

The reason is that the higher modes, at the same frequency, require a greater width for propagation, and so if the initial width is chosen just large enough to permit the transmission of the $TE_{1,0}$ mode, it will at the same time be inadequate for appreciable transmission of the higher modes. In connection with this it is to be noted that a single central antenna will inherently tend to suppress the even order modes, so that the next higher mode of immediate concern is the $TE_{3,0}$ mode rather than the $TE_{2,0}$ mode.

The $TE_{3,0}$ mode will be suppressed unless the horn is cutoff farther away from the apex. The distance from the apex at which the horn is to start depends upon how rapidly it flares, i.e., upon ϕ_0. Hence this distance, call it R_c for the $TE_{1,0}$ mode, and R'_c for the $TE_{3,0}$ mode, will depend upon the wavelength λ, and the flare angle ϕ_0. Thus these distances R_c and R'_c or preferably their values when measured in wavelengths R_c/λ and R'_c/λ can be plotted against ϕ_0. This has been done in Fig. 33. The horn should start close to a value of R_c from its apex. Then the distance $R'_c - R_c$ will indicate the region where the $TE_{3,0}$ wave is undergoing attenuation but the $TE_{1,0}$ wave is not.



(Revision of Terman's Handbook)

Fig. 33.—Curves to determine R_c/λ.

As an example of its use, suppose from certain considerations to be presented, it is found that a horn length ρ (to the apex) of 5λ is required, and that the flare angle ϕ_0 must be 40°. Suppose the operating frequency is 3,000 mc, then λ = 10 cm, and $\rho = 5 \times 10 = 50$ cm.

From Fig. 33, for $\phi_0 = 40°$, $R_c/\lambda = 1.0$ and $R'_c/\lambda = 2.55$. The horn should therefore start at a distance from the apex of $R_c = 1.0 \times \lambda = 10$ cm. The distance from the apex at which the $TE_{3,0}$ mode is no longer attenuated is $R'_c = 2.55\lambda = 25.5$ cm. Hence, the first 25.5 - 10 = 15.5 cm of the horn is the region where the $TE_{3,0}$ wave is attenuated. Moreover, the actual length of the horn is $\rho - R_c = 50 - 10 = 40$ cm. These values are all shown in Fig. 34.

If the $TE_{0,1}$ wave is to be transmitted, then the height, rather than the width AB in Fig. 34, is important. This is because the electric field is now parallel to the two parallel faces. As shown in Fig. 35, the electric field density must vary in a half-sinusoidal manner from the top to the bottom. Thus the width b must be at least $\lambda/2$ to permit such a distribution. On the other hand, dimension a can

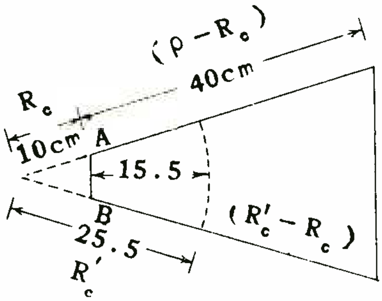


Fig. 34.—R_c for a $TE_{0,1}$ mode.

be any value, as was pointed out for a hollow rectangular tube operating in the $TE_{1,0}$ mode.

Thus, although dimension a varies along the length of the horn,

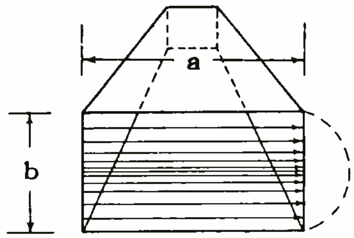
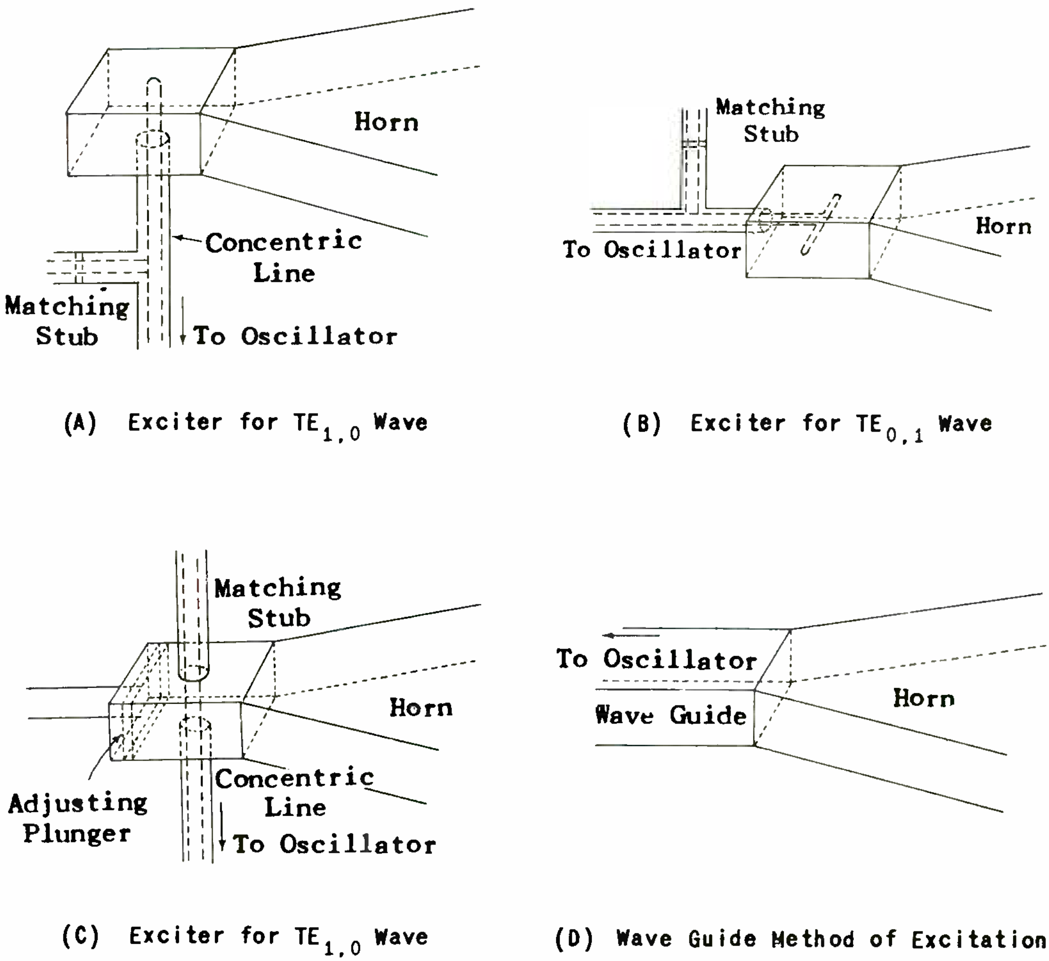


Fig. 35.—Electric field variation in $TE_{0,1}$ mode horn.

it has no essential effect upon the transmission of the $TE_{0,1}$ wave. On the other hand, dimension b, *which is constant* for the sectoral type of horn being considered, is of paramount importance. If it exceeds $\lambda/2$, the wave is transmitted, if it does not, the wave is attenuated. Similarly, the $TE_{0,3}$ mode will be transmitted if b equals or exceeds $3\lambda/2$. Hence, if it is desired to transmit the $TE_{0,1}$ wave, and to attenuate the $TE_{0,3}$ wave, dimension b must be somewhere between $\lambda/2$ and $3\lambda/2$, and preferably closer to the former value.

METHODS OF EXCITATION.—One method of exciting a horn has already been mentioned, namely, by the use of an antenna which is the projection of the inner conductor of a coaxial cable into the horn. This method is similar to that employed for excitation of a wave guide. Various forms for this purpose are shown in Fig. 36 (A), (B), and (C). Note that either a probe or dipole form of antenna can be employed to excite either the $TE_{1,0}$ or the $TE_{0,1}$ mode. In general the antenna will not present a pure resistive termination to its concentric line. Hence, a tuning stub is employed to balance out the reactive component introduced by the antenna and thus make it present a pure resistance to its line. The magnitude of this resistance (mainly that of radiation) can then be adjusted to equal the characteristic impedance of the line by moving the antenna relative to the rear closing wall of the exciter box. This can be done by providing a longitudinal slot in the bottom face, Fig. 36(A) along which the coaxial cable can be slid, or by providing a plunger or piston as the rear closing wall, as is shown in Fig. 36(C). The matching stub shown in this figure has been discussed previously in the assignment on wave guides.

In Fig. 36(D) is shown a wave



(A) Exciter for $TE_{1,0}$ Wave

(B) Exciter for $TE_{0,1}$ Wave

(C) Exciter for $TE_{1,0}$ Wave

(D) Wave Guide Method of Excitation

(Courtesy "Radio Engineers' Handbook", by Terman.)

Fig. 36.—Methods of exciting horns.

guide feed for the horn. The dimensions of the guide will be above cutoff if it is to transmit energy, hence the horn starts out with a suitably large dimension. As such it will be found to be practically a perfect termination for the guide, i.e., no reflection of energy will occur at their junction, nor at the mouth of the horn, if this is made suitably large. Thus the horn serves not only as a radiator, but also in the role of an impedance-matching device, much as a taper guide does in matching two dissimilar wave guides.

If an antenna is located in a horn too close to the apex (R_c of Fig. 32 too small) then the horn will present an inductive reactance to the antenna. This is because the attenuation of the horn in the vicinity of the antenna is due predominantly to the storage of magnetic flux; only a small portion of the energy here propagates towards the mouth of the horn. In short, attenuation, like attenuation in a wave guide operated below cutoff, is not due to actual dissipation of the electrical energy in the form of heat, but rather to its periodic storage in the system and return to the source during each a.c. cycle. This effect produces reactance: if the storage is that of magnetic flux, the reactance is inductive, if that of electric lines, the reactance is capacitive. In the horn, as just mentioned, the reactance is inductive.

RADIATION FROM A SECTORAL HORN.—As a good engineering approximation, the radiation from a sectoral horn may be considered the same as that from a wave guide with suitable qualifications. Thus, the width of the beam in either dimension is determined by the size of the horn mouth in that direction: the greater either dimension of the horn mouth, the narrower is the beam in this plane.

However, the propagation of the electromagnetic energy in the horn is that of a segment of a cylindrical wave, as indicated in Fig. 37(A), whereas that in a rectangular wave guide is essentially a plane wave. As a result, the flare of the beam radiated from a hollow tube is due

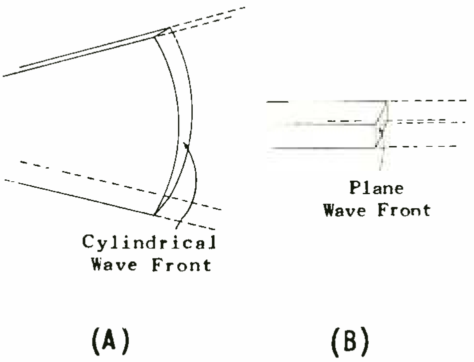


Fig. 37.—Wave propagated from horn and wave guide or tube.

solely to the interference or diffraction effects from the different elements of the mouth cross section, as was discussed for the antenna array; whereas the flare of the beam from a sectoral horn is due in part to the above effect, plus the fact that even in the horn the energy is being propagated in the form of a diverging beam of angle ϕ_0. *The beam radiated will thus never be less than* ϕ_0; it will be at the least equal to ϕ_0 only if the mouth opening is infinite. This in turn would mean a horn of infinite length.

It will be found therefore that for a given angular beam dimension, the horn mouth must be greater than that of a hollow tube, and the length such that ϕ_0 is less than the beam angle. This indicates that for a given final cross section, the hollow tube will radiate a narrower beam than will a horn. This is true, but is more than counterbalanced by the fact that a horn can be much shorter than a tube, particularly for very narrow beams (corresponding to large guide or horn cross sections), and moreover—as has been shown above—higher modes can be attenuated more readily in a horn than in a tube by the proper choice of R_c.

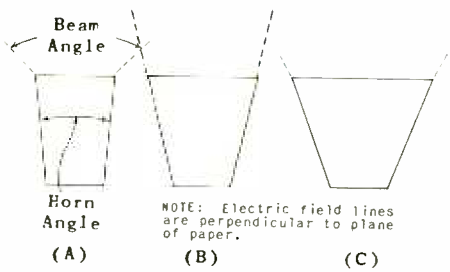


Fig. 38.—Relation of beam and flare angles.

The radiation pattern of a horn can be calculated on the assumption that the electric and magnetic field patterns at the point where the mouth is located are the same as the patterns at that point if the horn were infinite in length. (In the mathematical solution it is simpler to study the infinite horn than the finite horn, just as in the case of transmission lines.) The above assumption seems to be very close to the truth.

Radiation from the mouth of a horn does not seem to have a very marked effect upon the field within and at the mouth of the horn. Indeed, a very similar effect is noted when two horns are arranged so that they partly face each other. Very little reaction on the input of one horn is noted owing to the presence of the other. This is of particular importance where a transmitter and receiver are employed physically next to one another, and operating on the same frequency. The receiver is not necessarily overloaded, at least by the radiation of the local transmitter and thus temporarily rendered inoperative to the distant transmitter.

HORN RELATIONSHIPS.—The relationship between beam and horn flare angles can perhaps best be understood from the following considerations. In Fig. 38(A) (B), and (C) are shown three horns, all of the same length, and initial throat size, but of different flares and therefore of different mouth dimensions. The horn of (A) has the smallest flare and mouth; that of (C) has the largest flare and mouth. In (A), the mouth dimension is relatively small, so that the beam angle or flare will be large. In (C), the mouth dimension is large, so that the beam angle should be small. However, it cannot be smaller than the horn angle, and since the latter is large, the beam angle is consequently large. Actually it is slightly larger than the horn angle because of the slight additional divergence caused by a finite, although large, mouth dimension. In (B), the mouth dimension is reasonably large, so that the beam angle does not exceed the horn angle very markedly, and the horn angle is reasonably small, so that the beam angle is a minimum. This indicates that for a given size beam, there is an optimum value of mouth dimension and horn flare angle that will give the desired beam angle.

If the beam angle is given, then it is clear from the above discussion that the flare angle will be smaller than the beam angle, and for any particular larger value, the mouth area required, and hence the length of the horn, is determined. In Fig. 39 is given a family of curves showing the relation between flare angle and beam angle for various values of the length of the horn, measured in wavelengths, or P_1/λ. The solid portions are accurate, but the dotted portions are approximate, although they are of service in practical design. The curve marked 20 has been drawn in by interpolation from the other curves. It is to be noted that in this figure, the beam angle is defined as the angle between the *nulls* of the major lobe. To illustrate its use in the design of a horn, suppose a 15° beam is to be produced in the ϕ direction (see Fig. 39) at 30,000 mc. The curve in Fig. 39 closest to the point where 15° intersects the light straight line 0A is the one labeled P_1/λ = 100. The minimum value of this curve corresponds to a beam angle of approximately 14°, for which the flare angle is about 9°. For a 15° beam, the flare angle is about 7.5°. (These are all shown as light dotted lines). The mouth dimension a can be calculated by the aid of Fig. 40. From the geometry of the figure, it is clear that

$$a/2 = 100\lambda \sin 7.5°/2$$

For f = 30,000 mc. λ = 1 cm, and the horn length to the apex is therefore 100 cm = 1 meter = 3.28 feet. At

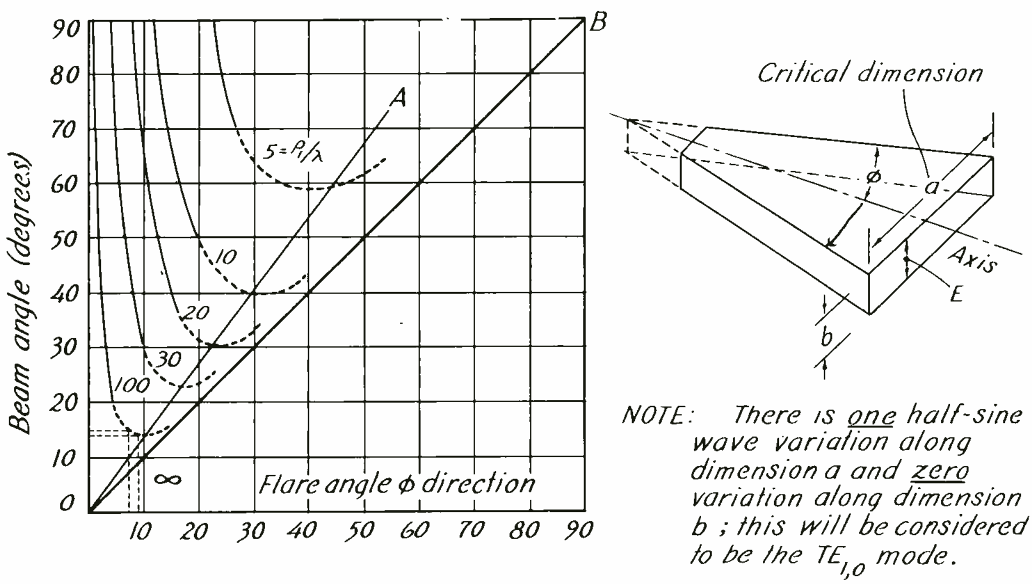


Fig. 39.—Curves to find beam or flare angle for a $TE_{1,0}$ mode. When used with $TE_{0,1}$ mode, correct flare angle with Fig. 42. (See top of page 37).

these short wavelengths, even horns that are long when measured in wavelengths, are short when measured in feet. From the above formula.

$$a = 2 \times 100 \times .0654$$
$$= 13.08 \text{ wavelengths}$$
$$= 13.08 \text{ cm} \div 2.54$$
$$= 5.14 \text{ inches}$$

(λ = 1 cm so 13.08 wavelengths = 13.08 cm)

Suppose, further, that it is desired to have a beam width of 15° also in the vertical (θ) direction. To calculate the height b, of the horn, i.e., the dimension perpendicular to the parallel faces to produce this same beam angle, it is first necessary to find the relevant cross-sectional dimension of

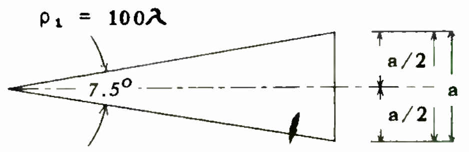


Fig. 40.—Top view of sectoral horn.

the equivalent tube that produces the same beam. Eqs. (9) and (10) can be rewritten as

$$\sin(\theta_m/2) = 1/W_\theta \qquad (9a)$$

$$\sin(\phi_m/2) = 3/2W_\phi \qquad (10a)$$

The width dimension corresponding to a for the horn is found by solving Eq. (10a) for W_ϕ. Since the beam angle is 15° in both planes, $\theta_m = \phi_m = 15°$.

$$W_\phi = \frac{3}{2 \sin 7.5°} = \frac{3}{2 \times .1305}$$
$$= 11.5 \text{ wavelengths (tube equivalent to "a" of Fig. 40)}$$

The corresponding dimension is W_ϕ = 1 × 11.5 = 11.5 cm width of tube. Similarly

$$W_\theta = 1/\sin 7.5° = 1/.1305$$
$$= 7.67 \text{ wavelengths}$$

(tube equivalent to height of Fig. 41)

and the corresponding height is λW_θ = 7.67 cm.

Note that the tube width is 11.5 cm as compared to 13.08 cm for the horn. This bears out the statement that for the same beam angle the horn must be wider than the tube.

However, in the case of the height dimension, the sectoral horn does not flare in this direction for the $TE_{1,0}$ mode, hence its width of beam in this plane is about the same as that of the tube for the same dimension. Hence the height of the horn may be taken as that of the tube, namely, 7.67 cm. The dimensions of the horn mouth are shown in Fig. 41. The shaded area represents the area of the equivalent tube that

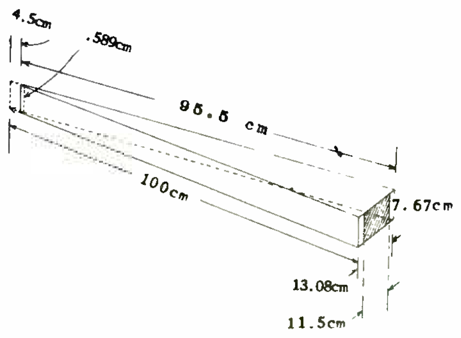


Fig. 41.—Sectoral horn of example problem.

would radiate the same size beam, *if it were made sufficiently long.*

The actual length of the horn is from the mouth up to a point sufficiently ahead of the apex. The portion cutoff is found, as before, from Fig. 33. Thus, for $\phi_0 = 7.5°$, $R_c/\lambda \cong 4.5$. The width of the throat is found similarly to that for the mouth,

$$a_t = 2(R_c) \sin (\phi_0/2)$$

$$= 2 \times 4.5 \times .0654$$

$$= .589 \text{ cm}$$

The radiation pattern can then be found by means of Eqs. (7) and (8), using the values of W_θ and W_ϕ found above. The pattern or beam is of a size determined by the smaller area of the equivalent tube, rather than the horn mouth area, as has been discussed previously. The computations are rather laborious, and are given in detail in the more specialized courses, where the shape is of importance, as in the case of a glide path for airplanes.

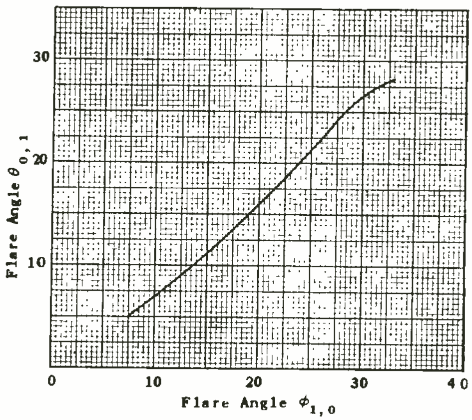
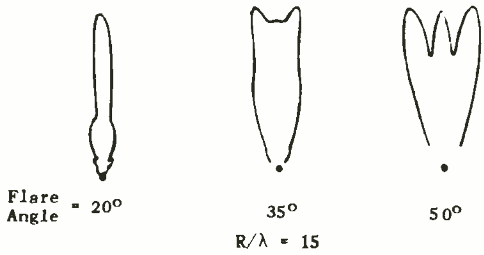


Fig. 42.—Conversion chart for flare angle in the ϕ and θ directions.

It is also to be noted that if dimension b is at most equal to λ, then the vertical pattern will have only a major lobe, whereas if it exceeds λ, some small minor lobes may



(Courtesy "Radio Engineers' Handbook," by Terman.)

Fig. 43.—Variation of beam with flare angle for $TE_{0,1}$ horn.

be present. In the problem above $b = 7.67\lambda$, so it can be expected that the pattern will not be a single lobe.

$TE_{0,1}$ MODE.—The radiation pattern for a $TE_{0,1}$ mode in a sectoral horn is approximately the same as that for a $TE_{1,0}$ mode rotated through a right angle. This is because the half-sinusoidal distribution of the electric field lines for the one is 90° away from that for the other.

If the distribution along a dimension is half-sinusoidal instead of uniform, then it will be recalled that the *effective* width is less, and the angular beam width is therefore greater. The radiation pattern can therefore be calculated if W_θ and W_ϕ are interchanged in Eqs. (7) to (10), although the above rule becomes more and more approximate as the dimensions (measured in wave lengths) become of value 2 or greater.

Hence, as an alternative, Fig. 42 has been provided. This enables the flare angle to be calculated that will give the same width of beam in the plane in which the horn flares as for the $TE_{1,0}$ mode. This is based on the approximation that for the same beam angle, the *optimum* horn length is about the same for either mode. The flare angle for the $TE_{0,1}$ mode, however, is somewhat less than that for the $TE_{1,0}$ mode, as is given by Fig. 42.

Hence, if Fig. 39 is employed first to find the flare angle and the length of the horn for a given beam width, on the preliminary assumption that a $TE_{1,0}$ mode is to be employed; then, if instead, a $TE_{0,1}$ mode is desired, the same length of horn can be employed, but the flare angle can be reduced to the value given by Fig. 42.

As an example of its use, consider the preceding problem of a horn operating on 30,000 mc and giving a beam 15° wide. For the $TE_{1,0}$ mode, Fig. 39 gave 7.5° as the flare angle, and the optimum length (to the apex) of $100\lambda = 100$ cm.

If now a $TE_{0,1}$ mode is desired instead, then the length remains 100 cm, but the flare angle, from Fig. 42, is now 5°. This in turn means that the mouth width will be

$$a_{0,1} = 2 \times 100 \sin (5/2)$$

$$= 200\ (.0436) = 8.72 \text{ cm}$$

as compared to 13.08 cm required by the $TE_{1,0}$ mode. The vertical dimension, however, will have to be approximately 50 per cent greater than before, or

$$b_{0,1} = 1.5 \times 7.67 = 11.51 \text{ cm}$$

A disadvantage of the $TE_{0,1}$ mode is that if the flare is large, the pattern tends to become very jagged. A representative example is shown in Fig. 43. For this reason the $TE_{1,0}$ mode is preferred, as a general rule, to the $TE_{0,1}$ mode, but there may be occasions where the polarization desired, and the horn dimensions required for a given beam, may be more readily met by using the latter mode. If the values given by Fig. 42 are employed, a relatively smooth pattern will be obtained, and the jagged outline avoided.

PYRAMIDAL HORNS.—A pyramidal horn flares in both dimensions. Consider such a horn with a vertical electric field, as shown in Fig. 44. With reference to the flare in the vertical direction, the field lines correspond to the $TE_{0,1}$ mode; with reference to the flare in the horizontal direction, the field lines correspond to the $TE_{1,0}$ mode. The design curves of Fig. 39 can be used in conjunction with Fig. 44 to design a pyramidal horn having optimum dimensions.

Thus, assume a 30° beam width in the horizontal direction, and a 35° beam width in the vertical direction, and $\lambda = 1$ cm. Consider now the triangle seen from the top (perpendicular to the "E" lines and including side "a"). The distribution of electric field across—for example—the throat of this triangle is sinusoidal. This is of course also the distribution along side "a" at the mouth. The dimension of the throat of this triangle has both maximum and minimum critical values. If the dimension at the throat is too small (less than $\pi/2$), the energy will be very rapidly attenuated and the horn will be incapable of being "excited." If the dimension of the throat is too large, parasitic modes may be launched; hence an optimum value for this throat dimension must be determined.

Alongside "b" the electric field is constant in intensity, hence there is no practical maximum nor minimum limit to the corresponding throat dimension as regards excitation of the horn.

It is obvious that each triangle must have the same hypotenuse (length) if they are to represent the top and side views of the same horn, hence, in all probability both throat dimensions cannot be made optimum in value. The critical triangle is the one that includes side

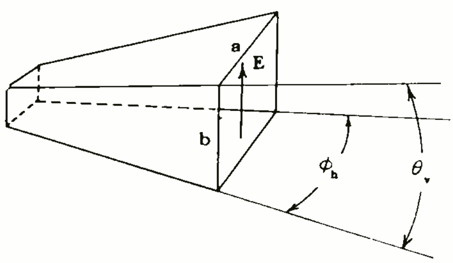


Fig. 44.—Pyramidal horn.

"a," hence this will be determined first and the other triangle made to fit.

From Fig. 39, for triangle "a" using a beam angle of 30°, the optimum ρ_1/λ = 20 and the flare angle = 22°. Side "a" then is calculated the same as in the sectoral horn by use of trigonometry:

$$a = 2 \times 20\lambda \sin 22/2$$

$$= 2 \times 20 \times .1908 = 7.632 \text{ cm}$$

Triangle "b" then has ρ_1/λ = 20 and must produce a beam angle of 35°. From Fig. 39 where 35° beam angle abscissa intersects the curve ρ_1/λ = 20, is found the ordinate representing a flare angle of 18°. This is then corrected by Fig. 42 to give the flare angle for triangle "b" = 14°.

Hence triangle "b" has a hypotenuse of 20λ and flare angle of 14°, or

$$b = 2 \times 20\lambda \times \sin 14/2$$

$$= 40 \times .1219 = 4.88 \text{ cm}$$

The throat dimensions can then be found in exactly the same way as shown in the previous problem, thus,

(use 22° for a in ϕ direction)

$$a_t = .646 \text{ cm}; \quad b_t = .415 \text{ cm}$$

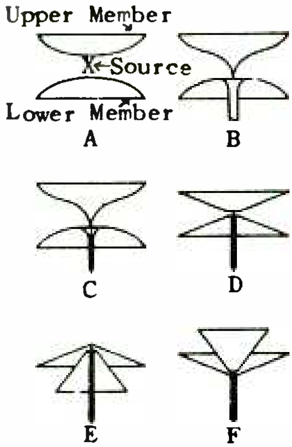
$$R_c = 1.7$$

THE BICONICAL HORN.—The biconical horn belongs to the class of antennas that are capable of radiating or absorbing energy *uniformly* in a plane. For example; if the horn is set as shown by any of the shapes in Fig. 45, then the pickup in any direction in the horizontal plane is uniform. The means of excitation may be a radiating source as located in (A); a hollow pipe, as in (B); or a coaxial feed, as shown in the remaining figures.

The shape of the two halves may be curved, as shown in (A), (B), and (C), or straight-sided, as in (D), (E), and (F). The curved types have been used in modified form for television purposes; they are a kind of vertical dipole of very large surface. They therefore have a low reactance, and are thus well suited for broad band operation, as in television.

The straight-sided shapes are

the type known as biconical, since their two elements are cones. The design curves to be furnished are those for horn (D), which tends to



(Courtesy Proc. I.R.E.)
Fig. 45.—Shape of biconical horns.

radiate mainly in a horizontal plane, although some radiation is set upward and some downward. Shape (E), on the other hand, suppresses the upward radiation and instead, concentrates it downward, while (F) suppresses the downward radiation, and tends to concentrate it in an upward direction.

Thus (E) might be preferable for land broadcasting, and (F) for aeronautical marker systems, while (D) could be used for communication with planes and land stations over a range of directions. However, if the shape (D) has a suitably large aperture or mouth, then the beam is very narrow, and essentially in a horizontal plane.

The biconical horn of shape (D) may be regarded as a sectoral horn that is rotated around a vertical axis, and thus generates the shape shown. As such, the biconical horn has properties very similar to that of the sectoral horn.

First it is necessary to note that various wave modes can be established in it. Of these, the two lowest order modes are the only ones that are normally of value for radiation purposes. These are the TEM and the $TE_{1,0}$ modes shown in Fig. 46. The TEM wave is characterized by the fact that *both* the electric and magnetic fields are *transverse*. This is similar to the fundamental mode in a coaxial cable, as mentioned previously in the assignment on wave guides. In analogous manner one can expect that there is no cutoff frequency for the TEM mode in the biconical horn, i.e., it can be excited in this mode even at the lowest frequencies. Indeed, operated in this manner it is essentially a dipole antenna of large cross section. Of course, at low frequencies its radiation may be negligible, and a Marconi or grounded type antenna may be preferred as a radiator but

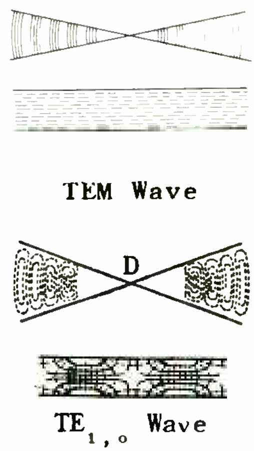


Fig. 46.—Modes in biconical horns.

as far as transmitting the wave motion along the conductors is concerned, there is no cutoff frequency for this mode.

Comparison of the electric field lines of the TEM mode with those of the $TE_{1,0}$ mode in a sectoral horn indicates that they are very similar in that in both the electric lines are perpendicular to the flaring sides. Hence, the same rules with regard to optimum flare angle apply to both types.

The $TE_{1,0}$ mode in the biconical horn compares to the $TE_{1,0}$ mode in the sectoral horn in that in both the electric field lines are parallel to the flaring sides. Here too the same rules apply to both as regards optimum dimensions.

Which mode is to be applied depends upon the polarization required. Ordinarily, the horn is operated with its axis vertical. The TEM mode then clearly gives a vertically polarized wave, and the $TE_{1,0}$ mode a horizontally polarized wave. As has been mentioned in previous assignments, horizontal polarization appears to be preferable at ultra-high frequencies, and so the $TE_{1,0}$ mode may be expected to be employed more often than the TEM mode. However, it is of value to have design data for both types of waves.

POWER GAIN.—The power gain of a biconical horn may be expressed in terms of a doublet antenna. The ratio of the power required to be fed to the doublet to that required to be fed to the biconical horn, for equal field strength in the direction of principal transmission, is the gain of the biconical horn over that of the doublet. (The doublet is explained farther on in this assignment.)

The direction of principal (maximum) transmission for a symmetrical biconical horn is the equatorial plane (Fig. 47). A

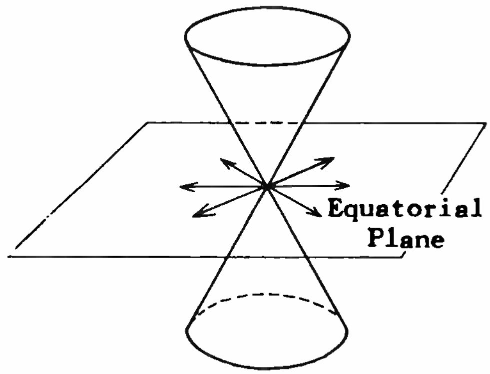
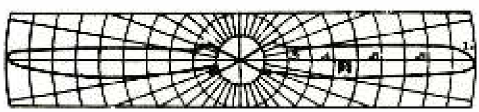


Fig. 47.—Transmission from a biconical horn.

representative radiation pattern is shown in Fig. 48. The pattern shown is a vertical cross section of the space model, i.e., it shows the directivity in a plane perpendicular to the equatorial plane. It is for a horn approximately 36 inches



(Courtesy Proc. I.R.E.)

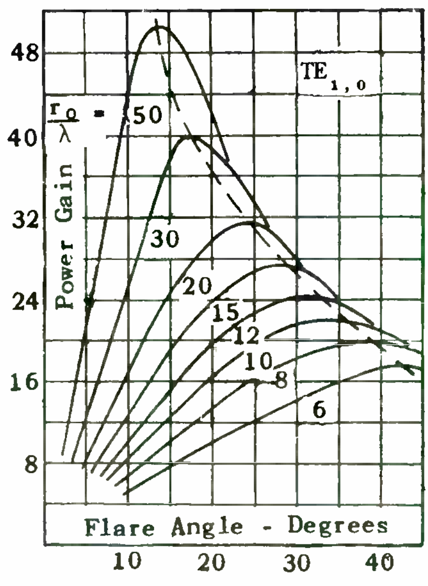
Fig. 48.—Radiation pattern of a representative biconical horn.

maximum diameter, having sides that flare at an angle of 35°, and are 5.7 λ long. The wave length was 8.3 cm.

Since a vertical doublet radiates a more nearly uniform field in such a plane, and radiates a perfectly uniform field in a horizontal plane, it is evident that a doublet would require far more power than a biconical horn in order to deliver as much field strength in a horizontal direction to some remote point. The doublet might require 40 times as much power, in which case the power gain would be 40.

This represents a very worth while saving in power for the use of the biconical horn over a doublet, both for transmitting and receiving purposes. Note that the biconical horn is *nondirectional in* the equatorial plane, so that it can cover a circular area of which it is the center i.e., broadcast radiation. A sectoral horn, on the other hand, delivers a beam that can be narrow in both vertical and horizontal planes, so that it is better suited to point-to-point communication, and also for some special aeronautical services, such as the glide path of an instrument landing system.

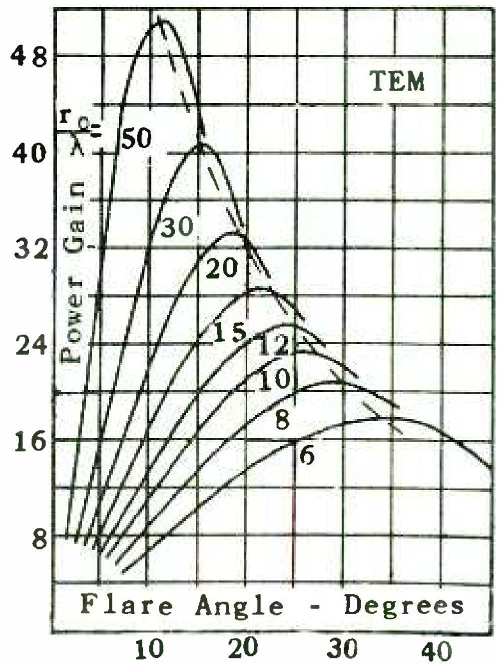
DESIGN CURVES.—In Figs. 49 and 50 are given a series of curves



Dotted curve represents shortest horns for given power gain.

(Courtesy Proc. I.R.E.)

Fig. 49.—Gain of biconical horn $TE_{1,0}$ mode.



Dotted curve represents shortest horns for given power gain.

(Courtesy Proc. I.R.E.)

Fig. 50.—Gain of biconical horn TEM mode.

that relate the gain of the horn to the length in wavelengths (r_o/λ), and the flare angle in degrees, for both modes.

As an example, suppose it is desired to use a biconical horn at 2,000 mc, to have a power gain of 20. It is desired to have a horizontal polarization, and to have a maximum radiation along the horizontal. It is desired, if possible, to keep the largest dimension of the horn in this example, within eight feet.

For maximum radiation along the horizontal the horn axis should be vertical. This is a normal position. For horizontal polarization, the $TE_{1,0}$ mode should be used, since this gives an electric field parallel to the top and bottom of the cones, hence perpendicular to the axis, which is vertical. The wavelength is $\lambda = (3 \times 10^{10}) \div (2 \times 10^{9}) = 15$ cm. The largest

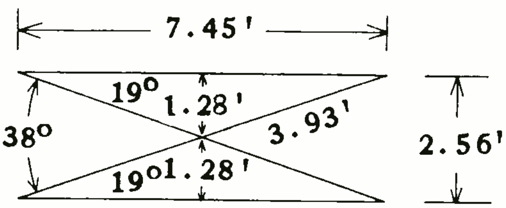


Fig. 51.—Dimensions of biconical horn for sample problem.

dimensions would usually be the diameter of the base of either cone, and this should not exceed eight feet in this example. The sides of either cone will be somewhat more than half of this, or four feet. This is r_o, i.e., r_o = 4 feet = 122 cm. Then $r_o/\lambda = 122 \div 15 = 8.13$.

Examination of Fig. 49 shows that for $r_o/\lambda = 8$, a power gain of 20 is possible with a flare angle of 38°. Then $r_o = 8 \times 15 = 120$ cm = 3.93 feet. Moreover these are

(Courtesy Proc. I.R.E.)

Fig. 52.—Horn used by Barrow.

optimum dimensions, i.e., the length of the horn will be a minimum. Hence, it appears that the specifications can be met.

Thus, referring to Fig. 51, it is clear that the height of either cone is 3.93 sin 19° = (3.93)(.3256) = 1.28 feet, or the total height is 2.56 feet. The diameter of either cone is

$$2 \times 3.93 \cos 19° = 7.86(.9455)$$

$$= 7.45 \text{ feet}$$

The two cones can be of copper,

brass, or even of galvanized iron, since the surfaces are large, the current density (amperes per square inch) is low, and therefore the I^2R losses will be low even for

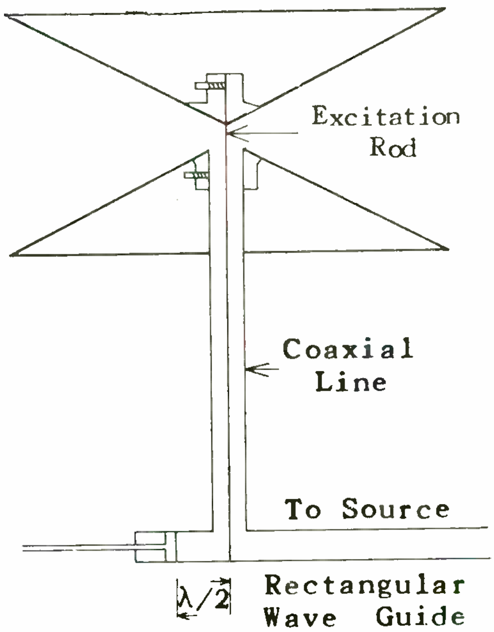


Fig. 53.—Exciting a biconical horn TEM mode.

galvanized iron. The two cones can be held apart by wooden or other insulating supports situated at their outer edges. For example, in a horn described by Barrow*, three wooden supports, situated 120° apart around the periphery, supported the two members. This, as well as the coaxial cable feed, is shown in Fig. 52. While these supports distort the horizontal field pattern

*Barrow, Chu, and Jansen: "Biconical Electromagnetic Horns," *Proc. I.R.E.*, Dec., 1939.

somewhat, the effect is not serious particularly if they are made fairly thin.

METHODS OF EXCITATION.—There are several methods of exciting a biconical horn, as has already been indicated in Fig. 45. One typical method is to use a coaxial line as a means of feeding and exciting the unit, even though the main feeder is a hollow tube.

A possible arrangement is shown in Fig. 53. This is for the TEM mode. Here the main feed from the source is a rectangular hollow tube. Between it and the biconical horn is interposed a coaxial line, as shown. The plunger in the left-end of the wave guide is adjusted to about half a wavelength from the inner conductor of the coaxial line in order to obtain matching and maximum energy transfer.

The top end of the coaxial sheath is connected to the lower cone. This acts like a cuff on the sheath, similar to the λ/4 cuff shown in Fig. 18. The inner conductor connects to the top cone. In this way a voltage difference is established between the apexes of the two cones. Or, to put it another way, the short section of inner conductor between the two cones acts as an exciting antenna or radiator.

The length of this radiator is of importance. In order to obtain a high radiation resistance in this antenna, it should be as long as possible. This also makes the annular throat area of the horn large, so that the wave is propagated outward with little attenuation. The situation is similar to that for a sectoral horn, where a throat area sufficient to propagate the $TE_{1,0}$ mode without undue attenuation, was desirable.

On the other hand, if the throat area is too large, higher TM (transverse magnetic) modes, such as $TM_{1,0}$, $TM_{3,0}$ etc. can be propagated through the horn, and these in turn will distort the radiation pattern from its desired shape. As a result of these opposing factors, it is in general found desirable to limit the spacing between cones to a fraction of a wavelength. The spacing may be between the two apexes, or the latter can be cut off and replaced by flat circular discs—as indicated in (D), Fig. 45, in which case the spacing is measured between these flat portions.

The $TE_{1,0}$ wave requires an exciter or antenna which is at least approximately concentric with the cone surfaces. This can be produced in either of the two ways shown in Fig. 54. In A, a small loop antenna is set up between the cones, and fed by a two-wire *balanced* line. It is desirable that the current flow as nearly equal in amplitude and phase in all parts of the loop as possible in order to generate a uniform circular electric field. Hence, the length of the loop should be noticeably less than a half-wave length. This in turn results in a small loop which has a very small radiation resistance, and hence low radiation effectiveness.

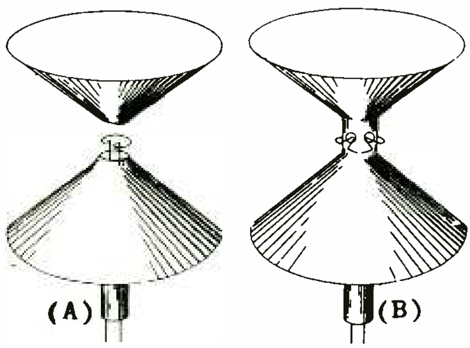


Fig. 54.—Exciting a biconical horn $TE_{1,0}$ mode.

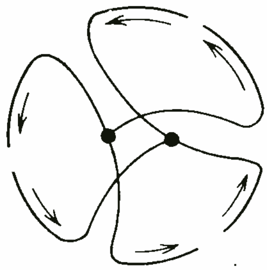


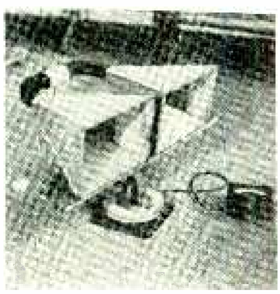
Fig. 55.—Antennas for Fig. 54(B)

In (B), Fig. 54, a series of antennas bent in a circle is employed. The diameter of the circle is preferably made an odd multiple of $\lambda/2$. The feed is arranged so that the currents flow in all antenna sections in the same circular direction. The method of connection to the two-wire line to obtain this effect is shown in Fig. 55, where a view looking down on the arrangement has been drawn. The two dots represent the two-wire line. Note further that by the use of a *balanced* two-wire line, the voltage between any portion of the ring antenna to either cone is balanced by that of another portion of opposite polarity, so that there is no tendency for electric field lines to be set up vertically between the two cones and terminating on them. This eliminates any tendency to set up the TEM mode.

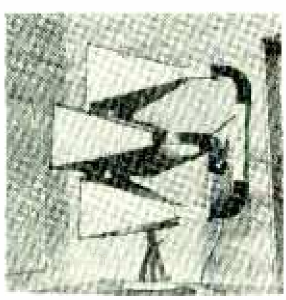
MULTIPLE UNIT HORNS.—Horns may be used in combinations or arrays in a manner similar to that employed for ordinary antennas. An advantage of a horn array over that of single horn is that the array can be shorter in length for a given sharpness of beam. There are also other intresting possibilities in the use of such arrays.

In Fig. 56 is shown a two-unit and a four-unit pyramidal horn array. These are capable of giving very sharp and smooth beams, as well as multiple beams. The horns can be used either as radiators or absorbers (receivers) of electromagnetic energy. The field patterns are the same in either case.

Suppose, for example, the horns are used for reception. As shown in Fig. 56, they are interconnected by wave guides. In the case of the two-horn array, a pickup probe or antenna is inserted in the guide, and so arranged that it can move longitudinally along the guide.



A



B

(Courtesy Proc. I.R.E.)
Fig. 56.—Multiple horn installations.

When it is located at a point equidistant between the two horns,

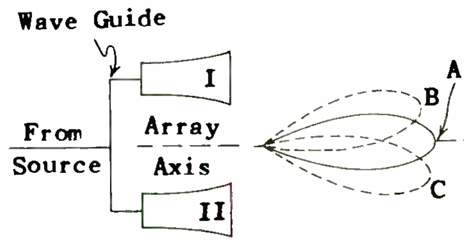


Fig. 57.—Connections to two horns.

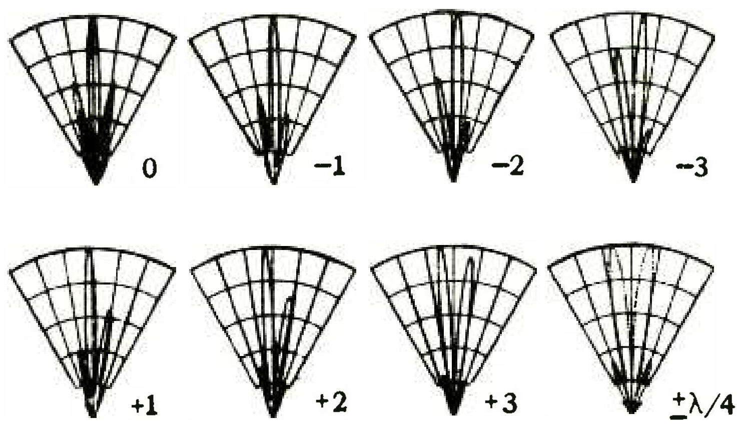
it combines their outputs in phase. Or, if the probe is fed from a source, it feeds the two horns in phase. If it is displaced towards one horn, it receives or feeds—as the case may be—energy from this horn earlier (leading phase) than that from the other horn. In this simple way energy may be fed to or received from the horns in phase, or leading to one and lagging to the other by the position of probe in the connecting wave guide.

It will be recalled that the phase velocity and wavelength within a guide are greater than that in free space. This means that a relatively large longitudinal displacement in the guide corresponds to a small phase variation. Stated in another fashion, a small phase change can be accurately produced by a not-too-critical displacement of the probe.

The significance of the above is that steerable beams can be produced by such an array with simple mechanical means. Thus, suppose two similar horns (Fig. 57) are employed. Each horn gives a beam of a certain size and shape. The two in combination give a narrower beam, as shown by A, B, or C. (Pronounced minor lobes may also be present). If the source connection is on the center of the wave guide, the two horns are fed in phase, and the beam points along their axis, as shown by pattern A. If the source connection is displaced toward horn I, its energy leads that of horn II, and the beam swings around to position C. If the connection is displaced toward horn II, the beam swings around to position B. Barrow* gives a set of patterns for two horns as shown in Fig. 58. The horns were arranged along a horizontal line (one alongside of the other), with a spacing between centers at the mouth of 4.8λ. The electric field was vertical.

The beam in a vertical plane was a single lobe—that of either horn. But in the horizontal plane, it is evident from Fig. 58 that the major lobe is accompanied by at least two prominent minor lobes, whose relative magnitudes vary with the steering (displacement of the probe in the connecting wave guide). These minor lobes were not shown (for simplicity) in Fig. 58.

The numbers 0, -1, -2, +1, +2, etc. indicate unit displacements of the probe to one side or the other of the center position. It will be noted that the main lobe swings around to one side or the other of the axis. At the same time one minor lobe increases in magnitude while the other decreases in magnitude. If it is desired to reduce these, a four-unit horn array as shown in (B) of Fig. 56 may be employed, such that each pair reinforce



(Courtesy Proc. I.R.E.)

Fig. 58.—Patterns for two horns as in Fig. 57.

*Barrow and shulman: "Multi-unit Electromagnetic Horns." Proc. I.R.E., March, 1940.

only each others major lobe. This makes its amplitude greater relative to the minor lobes, and hence reduces the importance of the latter.

An interesting case is that shown for $\pm \lambda/4$ in Fig. 58. For this displacement (corresponding to a 90° phase displacement between the energy fed to the two horns), one minor lobe has been practically completely suppressed, and the other accentuated until it is equal to the major lobe. Moreover, both are displaced by equal but opposite angles to the horn array axis.

Such a device can be used as a very sensitive direction finder. If the array is mechanically rotated so that the received wave lies along the null position between the two lobes, then the output meter reads zero. If the horns are rotated from this position by a small amount in one direction or the other, a very strong output will be obtained. Owing to the sharpness of the two lobes, a very small angular displacement will give rise to a very strong output. Barrow reports* that the resolution so obtained was greater than that which could be read or maintained by the orientation table employed, as it was some small fraction of a degree. This indicates the possibilities of a horn array for direction-finding and "obstacle" detection.

It is well to note that the pattern of each horn is practically independent of the presence of the other horn, i.e., their mutual impedance is practically zero. This simplifies the design of a horn array over that of an antenna array, where each element reacts on the other to modify its impedance, and make its action as a parasitic element difficult to predetermine, or even the phasing and feeding of the elements in a driven array. Further in the case of the two-horn array, for example, connected together by a common wave guide, each horn acts practically as a perfect match for the other, so that the pickup or exciting antenna is free from reflection effects. This makes the array capable of operating over a wide band without any special adjustment.

*See previous reference.

REFLECTORS

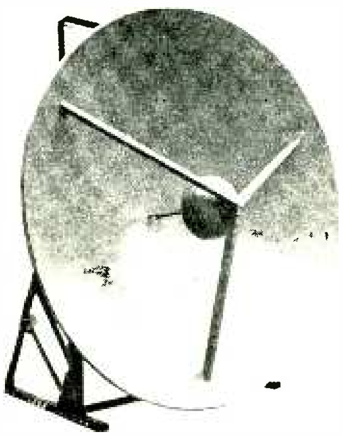
The horn has one disadvantage: that of excessive length for many applications. It will have been observed that for a given sharpness of beam, the radiating area has to be roughly of a certain size regardless of whether this area is that of a flat antenna array or the mouth of a horn. Hence, if a desired sharpness of beam can be obtained by means of an array or similar system instead of a horn, a worth-while saving in length and hence volume of space will be obtained.

A horn, on the other hand, is simple mechanically, can furnish a very smooth beam, and can operate over a broad band of frequencies. Often the latter two considerations are not very important, particularly that of band width. At the present time most of the u.h.f. and microwave services appear to be on single frequencies, owing to the mechanical difficulty, for example, of tuning a receiver employing cavity resonators, to a band of frequencies. In view of this, the wide band possibilities of the horn are not of such great importance at present, and its bulk is a factor to be considered in its use.

For single frequency operation, a tuned unit, such as an antenna array or parabolic reflecting system merits serious consideration because of its small bulk. In the *microwave range*, however, the antenna array becomes too complicated and difficult to adjust, so that instead the parabolic reflector is employed in one form or another to a great extent.

PARABOLIC REFLECTOR.—A form of parabolic reflector is shown in Fig. 59. Mathematically this is known as a paraboloid of revolution. The parabola is a plane curve which may be defined as the curve generated by a point which moves so that its distance from a fixed point, called the focus, equals its distance to a fixed line, called the directrix. These are illustrated in Fig. 60.

The moving point is P. Its distance from the directrix is the perpendicular distance PD. If



(Courtesy Electrical Communications, by Clavier)

Fig. 59. Parabolic reflector.

$$PD = PO$$

for any position of P, then P is constrained to move along a parabolic curve. This curve is derived in the Specialized Advanced Mathematics series and hence only its

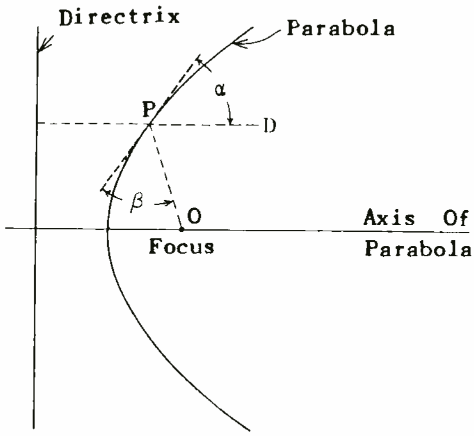


Fig. 60.—Geometry of a Parabola.

properties as pertinent to reflection will be discussed here.

If the parabola is rotated around its axis, a surface called a surface of revolution is produced. For the parabola this surface is known as a paraboloid of revolution. From its symmetry it may be expected to have the same properties in any plane containing the axis.

One important property of the parabola is that if the tangent to the curve is drawn at any point P (Fig. 60), then the angle α that this tangent makes with DP equals the angle β that it makes with PO. The significance of this is that if a source of wave energy is placed at

O, it is reflected by the parabolic surface at P, i.e., the tangent at P, such that the angle of incidence β must equal the angle of reflection α. But since α is also the angle made by PD and the tangent line, and since PD, being perpendicular to the directrix, is therefore parallel to the axis, it is clear that all rays emanating from the focal point will be reflected parallel to the axis and hence parallel to one another.

The wave front is the surface perpendicular to the individual rays. In the case of the rays proceeding radially from the focal point, this surface is spherical (incident wave). In the case of the parallel reflected rays proceeding from the surface, the wave front is a plane. Hence, a parabolic reflector converts a spherical, diverging wave from a point source into a plane wave which theoretically does not diverge. Thus a sharp directional beam may be obtained from a small non-directional source.

The above is only approximate, and becomes more so the smaller the parabola is when measured in wave lengths. Thus, if the parabola's dimensions are measured in inches, and the wavelength is on the order of a one hundred-thousandth of an inch, as in the case of light, then the above conclusions are practically correct.

On the other hand, if the wavelength is on the order of 10 cm (micro-wave range), and the diameter of the parabola is but a few wave lengths, then diffraction effects are pronounced, and the beam spreads in a manner similar to that for an antenna array or horn mouth of comparable size. In addition to this diffraction effect there is also present a diverging beam (spherical wave) radiated directly by the source in the same direction as the reflected wave. Thus the resultant wave has considerable divergence unless the source is made directional so that it radiates appreciably only in directions intercepted by the reflecting surface.

A further complication is that the source is often a half-wave dipole, whose dimensions are hardly that of a point source. As a consequence, not all parts of the dipole are at the focus, and this produces some distortion of the beam from its theoretical shape. Fortunately, however, most of the radiation from a dipole is from its central portion (where the current is a maximum) and in an axial direction,—if the dipole is set perpendicular to the axis,—so that the distortion and broadening of the beam is not as great as might otherwise be expected.

USE OF SECOND REFLECTOR.—To eliminate the forward diverging beam radiated directly by the source, a second reflector can be employed, as shown in Fig. 61. The second reflector is in the form of a spherical shield S which surrounds the dipole D, and throws its forward radiation back toward D and the parabolic reflector P, from which it is ultimately radiated in the desired plane wave form.

The shield should be as large as possible, yet should not be so large as to intercept an undue portion of the energy radiated by the reflector. Its circumferential extent should not greatly exceed points A and B, since rays from D along AD and DB will intercept the main reflector P anyway.

The shield should have a radius r, Fig. 61, of a value $\lambda/2$ or a multiple of this value. This will appear contrary to the discussion earlier in this assignment, where it was stated that for *reinforcement* of the beam on the axis, the source should be a multiple of $\lambda/4$ from the reflector, and that a distance of $\lambda/2$ produced cancellation instead of reinforcement.

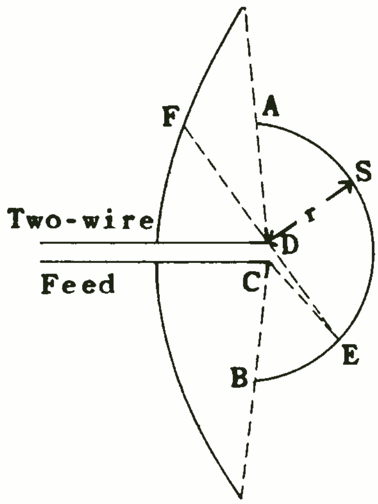


Fig. 61.—Use of two reflectors.

The difference is that here reinforcement or concellation of the axial portion of the beam is of no importance, since this part of the beam would not get past the shield, anyway. The important rays are those making an appreciable angle with the axis, such as CE. This particular ray is shown proceeding from the lower leg of the dipole with a certain phase. Upon reflection from S it undergoes an 180° phase reversal and returns as EF toward P. *Note that it now crosses the upper leg of the dipole.* If r is $\lambda/2$, then the total path length from D back to D is λ, which corresponds to 360°. This, plus the 180° phase reversal on reflection, makes the wave returning along EF, 180° out of phase with respect to how it started out. It is therefore in phase with the radiation along EF toward P from the upper leg of the dipole, and hence produces reinforcement.

Thus a value for r of $\lambda/2$ or a multiple of $\lambda/2$ enables S to return the forward diverging beam from D in the proper phase toward P to reinforce the beam proceeding directly from D to P. This analysis is based on the assumption that at most a half-wave dipole is used, and that the radiation from it is mainly from the central portion, so that rays CE and EF make a very small angle with each other. Which multiple of $\lambda/2$ to use for r depends upon the size of the parabola P; the larger P is, the greater r can be.

SIZE OF PARABOLA.—The parabolic reflector can be specified by its mouth diameter D and by its focal length f. Thus, suppose three parabolas have the same mouth diameter, but different focal lengths. In Fig. 62(A) the focal length is least, and in (C) it is greatest. Parabola (A) has the

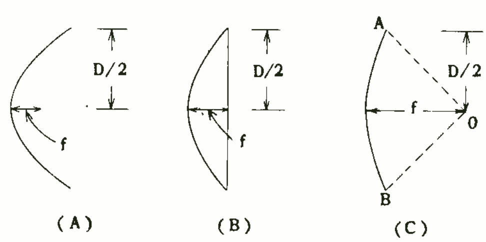


Fig. 62.—Size of reflector.

greatest curvature, and (C) has the least. Parabola (B) is of interest: its focus lies in the diametral plane. One of the properties of a parabola is that in this particular case f = D/4. Stated in another way, if the parabola is prolonged until its mouth contains the focal point, then the focal distance equals half the mouth radius.

This particular arrangement is preferred for an antenna of the doublet type, as will be explained. However, if the antenna is of a directional type, and confines its radiation to a relatively narrow beam in one direction, then a shallower parabolic reflector can be used with advantage. For example, the parabola shown in (C) has its focal point O outside of its diametral plane. If an antenna is located at O such that it radiates only within the angle AOB, then the reflector will intercept all the energy and redirect it forward in a narrower beam.

DIPOLES AND DOUBLETS.—However, if the ordinary half-wave dipole is employed, its directional characteristic is essentially as follows: In a plane perpendicular to its length (plane of the paper for a dipole projecting up through it) it is nondirectional and radiates equally well in all directions. This is shown in Fig. 63(A). In any plane containing the dipole, its pattern is as shown by the solid line curve in (B). The essential feature is that there is no radiation from the ends of the dipole, i.e., in a vertical direction in Fig. 63(B).

Another antenna often discussed, especially in theoretical derivations, is the doublet or elementary dipole. This is an extremely short dipole, with sufficient end capacity to eliminate a current node at either end. In other words, it

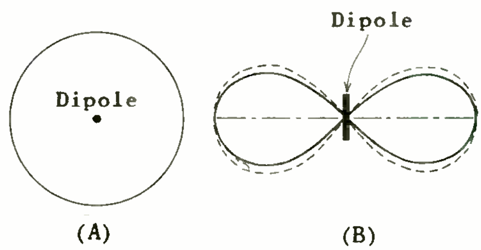
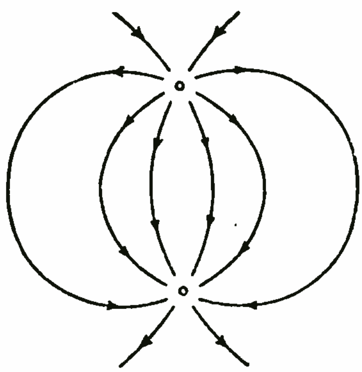


Fig. 63.—Dipole Antenna.

is a hypothetical antenna in which the current is everywhere in phase and the same in magnitude even at its ends. Its value in general is that ordinary antennas can be regarded as composed of a succession of doublets, and even a horn may be replaced by a plane containing certain special types of doublets, in order to evaluate the radiation characteristics of the actual antenna. In the case of the parabolic reflector, the action is more readily analyzed when it is assumed to be excited by a doublet then by an actual half-wave dipole. This is because a doublet is so short that it may be assumed concentrated at the focus of the parabola, so that the rays from it are essentially those coming from the focal point. Such a simplification is not possible when the antenna if of appreciable size, such as $\lambda/2$.

Nevertheless, the pattern of the doublet (dotted line in Fig. 63 (B) is not very much different from that of the dipole, and many conclusions that are more easily obtained by the assumption of the use of a doublet can be carried over with a fair approximation when an actual half-wave dipole is employed.

REFLECTOR CIRCULATING CURRENTS.—One factor is that of the behavior of the reflector in conjunction with the dipole. The action of the dipole is to produce



(Courtesy "Microwave Transmission", by Slater.)

Fig. 64.—Currents in the reflector from dipole antenna.

circulating currents in the reflector, just as it would in any other metallic body, such as a parasitic antenna. These currents have roughly the shape shown in Fig. 64. As is clear from the figure, they appear to diverge from one point of the pole of the reflector, and converge to another pole.

These poles are points on the reflector in line with the ends of the actuating dipole, and are denoted as P, P in Fig. 65. In this figure it is assumed that the focal point is *inside* of the diametral plane or mouth of the reflector. Note from Fig. 64, that currents above the upper pole flow toward it, whereas currents below it flow away from it. The same is true for the lower pole.

The currents in the reflector produce radiation from it. Indeed, this is the mechanism by which the parabola acts as a reflector of electromagnetic energy impinging upon it. It is evident from Fig. 64 that the currents above the upper pole and below the lower pole act opposite to the currents between the two poles and tend to distort the reflection pattern produced by the latter.

Hence, it is inadvisable to prolong the parabolic surface beyond the poles P, P in Fig. 65. In other words, when a dipole antenna is employed, it is desirable to design the parabola so that the focal point is in the diametral plane, rather than within this plane. This then defines its mouth-radius as equal to twice its focal length.

RADIATION FROM A PARABOLIC REFLECTOR.—The radiation from a parabolic reflector is in the form of a beam, For a circular reflector,

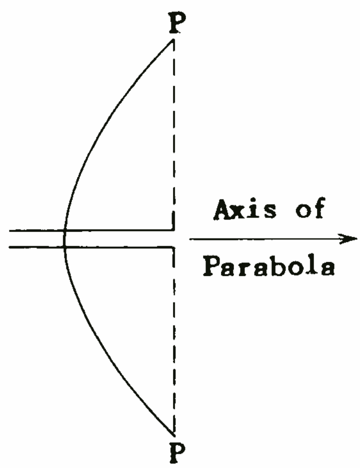


Fig. 65.—Size of Parabola.

the beam is wider in the plane containing the actuating dipole than in the plane at right angles to it. Thus, in Fig. 65, *the beam is wider in the plane of the paper than in a plane at right angles to the paper.* The reason can be seen from the dipole radiation pattern of Fig. 63.

Consider the radiation pattern of the dipole in the plane of the paper in Fig. 65. The radiation from the dipole in the axial direction is greater than that in the direction PP, as is clear from the pattern of Fig. 63(B). This means that the regions embracing P-P of the parabola (Fig. 65) are not receiving as much radiation from the dipole as the region embracing the center, and consequently are not reflecting as much energy. This reduces the effectiveness of the regions surrounding P-P, and hence makes the *effective* width of the mouth of the parabola in the plane of the paper less than the actual width.

In a plane *perpendicular* to the paper, however, the dipole radiates uniformly in all directions as may be noted from Fig. 63(A). Hence, *all* portions of the parabola in this plane are equally affected, and are therefore equally *effective*. Thus the effective width of the mouth in this plane is equal to the actual width.

From what has been said in the early part of this assignment it is clear that the greater the dimension of an array, the smaller is the beam angle in that plane. Hence, for the orientation of the dipole with respect to the parabolic reflector shown in Fig. 65, the beam angle in the plane of the paper is greater than that in a plane perpendicular to the paper.

This is summarized in Fig. 66. Here a vertical dipole is shown. The horizontal width of the beam pattern is less than the vertical width as is indicated in the figure. Actually the beam is accompanied by minor lobes because the current distribution in the reflector, at least in the horizontal plane, is fairly uniform, as is indicated by Fig. 64. The width of the major lobe, between nulls, in the horizontal plane, is given approximately by the following equation.

$$\text{(Angular beam width in degrees)} = 137.5/(D/\lambda) \qquad (12)$$

where D/λ is the mouth diameter measured in wavelengths. The width

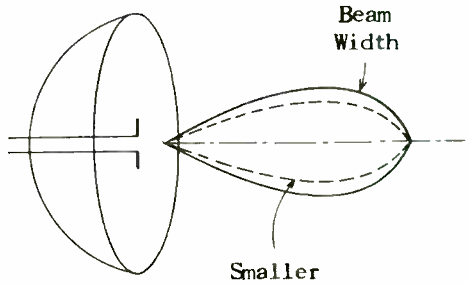


Fig. 66.—Beam widths in vertical and horizontal planes.

is for a non-directional source, which is the characteristic possessed by a vertical dipole in the horizontal plane. THE ANGULAR BEAM WIDTH IN THE VERTICAL PLANE IS APPROXIMATELY 1.25 TIMES THAT IN THE

HORIZONTAL PLANE IN THE CASE OF A DOUBLET ACTUATING ANTENNA. Since a half-wave dipole has a directional pattern in this plane somewhat narrower than that of the doublet, the vertical width of the beam when a dipole is used instead of the doublet may be expected to be less than 1.25 times the width in the horizontal plane.

REFLECTOR DESIGN.—As an example of the use of Eq. (12) in reflector design, suppose a beam having a vertical width 10° is desired at 10,000 mc (λ = 3 cm). Horizontal polarization is required. This means that the actuating dipole must be horizontal, hence Eq. (12) applies directly to its *vertical pattern*. Thus

$$10° = 137.5/D/\lambda$$

$$D/\lambda = \frac{137.5}{10} = 13.75$$

from which

$$D = 13.75\lambda = 13.75 \times 3$$

$$= 41.3 \text{ cm} = 16.2 \text{ inches}$$

From the preceding discussion it is evident that the focal length should equal D/4 or 10.3 cm = 4.05 inches. The shape of the parabola can be plotted on cross-section paper from the following equation

$$h = \frac{r^2}{4f} = \frac{r^2}{D} \qquad (13)$$

where h and r are the axial and radial distances to any point P on the parabola, as shown in Fig. 67. In the case of the above problem, f = 4.05 inches, so that Eq. (13) becomes

$$h = \frac{r^2}{16.2}$$

where h and r are both measured in inches, or

$$h = \frac{r^2}{41.3}$$

if h and r are to be measured in centimeters. The curve has been plotted in Fig. 68, by assuming

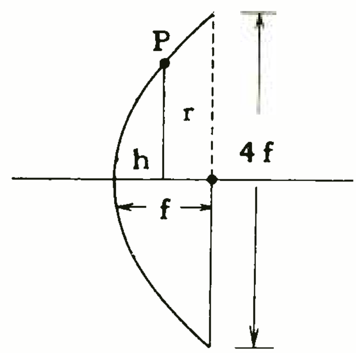


Fig. 67.—**Axial and radial distances in parabolic reflector.**

various values of r as shown in the Table accompanying the figure. In this way a template may be constructed to which the parabola may be fabricated. It may be spun out of copper or brass, or even turned out of wood and copper foil fastened to it. Indeed, for many purposes, fine mesh screen may be employed instead of foil.

It is interesting to note that the impedance of the dipole is not altered markedly by the presence of the reflector. This is apparently due to the fact that the reflector takes the radiation emitted by the dipole, and redirects it into a beam such that little of the redirected radiation impinges on the dipole. Thus very little counter voltage is induced in the dipole by the redirected radiation, and hence little reaction of the reflector upon the dipole is noted. The

TABLE	
r	h
0	0
.5"	.01544"
1.	.0617
1.5	.139
2.0	.247
2.5	.386
3.0	.556
3.5	.757
4.0	.938
5.0	1.543
6.0	2.22
7.0	3.02
8.1	4.05

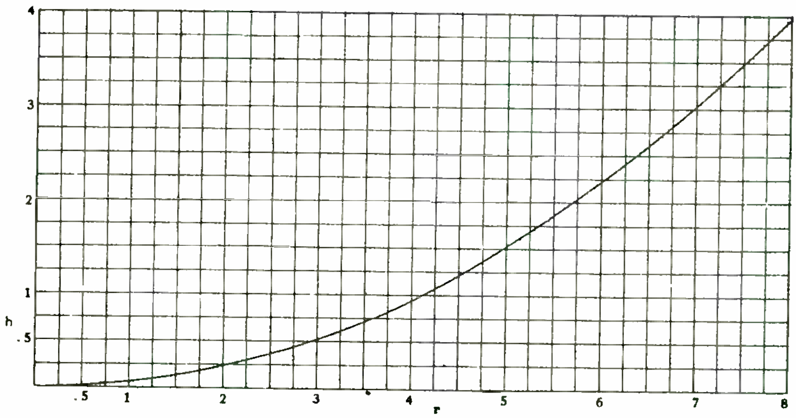


Fig. 68.—Curve of radial and axial distances.

radiation resistance of the latter remains in the neighborhood of 70 ohms.

The power gain of a paraboloidal reflector, when energized by a *doublet* antenna, is

$$\text{P.G.} = \left(\frac{2\pi}{\lambda}\,\frac{2fR^2}{4f^2 + R^2}\right)^2 \qquad (14)$$

where f is the focal length, and R is the radius of the mouth opening. The power gain (P.G.) is with reference to a doublet antenna when used alone. The gain is a maximum when f equals R/2, which is the basis of the design of the above parabola. For this optimum condition

$$\text{P.G. (maximum)} = \left(\pi\frac{R}{\lambda}\right)^2 \qquad (15)$$

In the problem just worked out, R = D/2 = 41.3/2 = 20.65 cm and λ = 3 cm. Then

$$\text{P.G.} = \left(\pi\,\frac{20.65}{3}\right)^2 = 467$$

On a db basis this is

$$10 \log \text{P.G.} = 10 \log 467$$

$$= 10(2.6693) = 26.7 \text{ db}$$

If a second reflector is employed, then theoretically the power gain will be quadrupled, or 4 × 467 = 1868, and the db gain will be increased by six, or will be 26.7 + 6 = 32.7 db. An increase of 5 db has been obtained in an actual installation.

From Fig. 63(B) it is evident that the half-wave dipole is somewhat more directional than the doublet. This means that the parabolic reflector would not be relatively as directional with respect to a half-wave dipole as it is with respect to a doublet, and hence its power gain relative to a dipole would not be as great. This is further complicated by the fact that not all parts of a dipole can be near the focal length of the reflector, particularly if the latter is small when measured in wavelengths. Hence, the power gain when a half-wave dipole is used instead of a doublet will be somewhat less than that given by Eq. (15), but probably the difference will not exceed two or three db under ordinary conditions.

CYLINDRICAL PARABOLIC REFLECTORS.—A cylinder is the surface generated by a straight line moving along a curve and always remaining parallel to a fixed line in space. If the curve along which the line moves is a parabola, then the surface generated is a parabolic cylinder. This is shown in Fig. 69. Reflectors of this shape are also employed in u.h.f. work.

The focus is a *line parallel to the surface*, rather than just a point, and the actuating antenna can be located along this focal line as shown in the figure. The antenna then radiates equally well (non-directionally) in a horizontal plane, but—as explained previously—radiates very little in directions approaching the vertical.

It is clear from this shape that the parabolic surface can extend considerably beyond the focal line without having opposing currents flow in its surface, as was

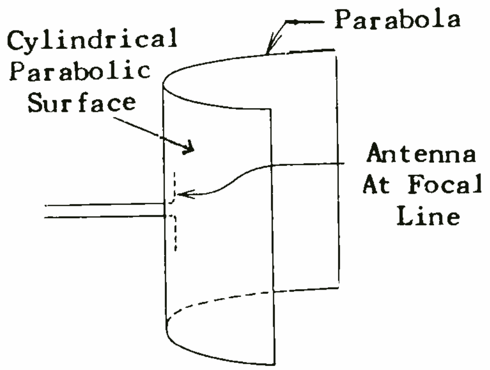


Fig. 69.—Cylindrical parabolic reflector.

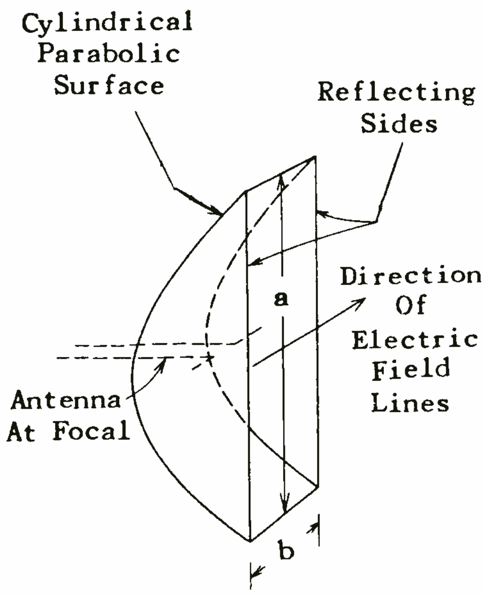


Fig. 70.—Horizontally polarized reflector and antenna.

the case for the paraboloidal reflector. The mouth can thus be beyond the position of the focal line, and thus most of the energy radiated by the antenna can be intercepted by the reflector and redirected in the form of a beam.

A further variation is shown in Fig. 70. Here the sides (top and bottom of Fig. 69) have been closed with conducting surfaces. The reflector now acts very much like a sectoral horn, in that it flares in one dimension, and is of constant width in the other. For example, in the case of Fig. 70, the directional pattern in the vertical plane is approximately that of the sectoral horn of zero flare angle excited in the $TE_{1,0}$ mode, that is, Eq. (7) for the tube and the sectoral horn apply here:

$$\left|E\right| \cong \text{constant} \left| \cos\theta \frac{\cos(\pi W\phi \sin\phi)}{(\pi W\phi \sin\phi)^2 - (\pi/2)^2} \right| \qquad (7)$$

where ϕ is the vertical angle of elevation if the reflector is set as in Fig. 70. The null angle in the vertical plane is given by Eq. (19):

$$\phi_m = 2\sin^{-1}\left(\frac{3\lambda}{2a}\right)$$

$$= 2\sin^{-1}\left(\frac{3}{2W\phi}\right) \qquad (10)$$

Note that E horizontal, i.e. horizontally polarized radiation is obtained from the reflector set as in Fig. 70.

In the horizontal plane the directional pattern is that given by Eq. (8):

$$\left|E\right| \cong \text{constant} \left| \cos^2\theta \frac{\sin(\pi W\theta \sin\theta)}{\pi W\theta \sin\theta} \right| \qquad (8)$$

and the null angle by Eq. (9):

$$\theta_m = 2\sin^{-1}\left(\frac{\lambda}{b}\right)$$

$$= 2\sin^{-1}\left(\frac{1}{W\theta}\right) \qquad (9)$$

A further refinement would be to use a second cylindrical parabolic reflector set in front of the actuating antenna similar to that shown in Fig. 61, in order to suppress the diverging forward radiation of the actuating antenna. In this case particularly the radiation and beam widths will be those given by Eqs. (7) to (10).

It will be recalled that these equations are for a hollow tube, rather than for the sectoral horn, and that the beam from the latter is somewhat wider because of the flaring sides of the horn. As a result, the reflector of Fig. 70, can give a narrower beam than the horn in the vertical plane, and as narrow a beam in the horizontal plane. At the same time its axial length will be less than that of the horn.

SUMMARY

This concludes the assignment on u.h.f. radiating systems. General considerations concerning arrays and their dimensions were discussed, and it was shown that

1. The greater a given dimension of an array, the narrower is the beam in that plane, and

2. The more gradually the current varies from a maximum at the center of the array to a minimum at the edges, the less pronounced are the minor lobes. At the same time the effective width of the array is decreased, and the major lobe is consequently broader.

Next, the radiation from hollow tubes was discussed, and it was shown how the beam dimensions depend here upon the mouth dimensions of the tube in exactly the same way as for an array.

The subject of sectoral horn followed logically that of hollow tubes. It was shown that for given mouth dimensions, the beam from a horn tends to exceed that of a hollow tube because of the flare in the horn, but that the horn length is considerably less in the case of sharp beams.

The modifications in the design procedure for pyramidal horns was next discussed. This type of horn is a kind of sectoral horn having the $TE_{0,1}$ mode in one plane, and the $TE_{1,0}$ mode in the other plane.

The biconical horn was next taken up, and the method of design developed for a given power gain. This horn seems well suited for u.h.f. broadcast purposes.

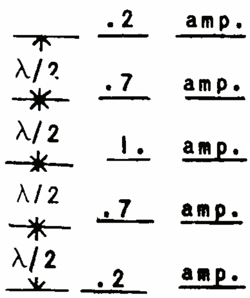
There then followed a discussion of multi-unit horn and their characteristics. These include sharp beams, steerable beams, and split beams of possible value for direction finding.

Finally, the characteristics and design of paraboloidal and cylindrical reflectors were discussed, together with a comparison of their properties and those of a horn type radiator.

RADIATING SYSTEMS AT ULTRA HIGH FREQUENCIES

EXAMINATION

1. (A) Given the antenna array shown in the figure. The currents, for simplicity, are assumed all in phase. Now suppose that the currents are all made 1 ampere in value, and are all in phase. What can you say, *in general terms*, regarding the shape and the angular width of the pattern in a *vertical* plane, for each of the two current distributions? also for minor lobes?



(B) What is the action of a reflecting sheet behind the array? *Explain.*

2. (A) What difficulties are encountered in attempting to employ a driven array in the microwave range?

(B) What difficulties are encountered in the case of a parasitic array?

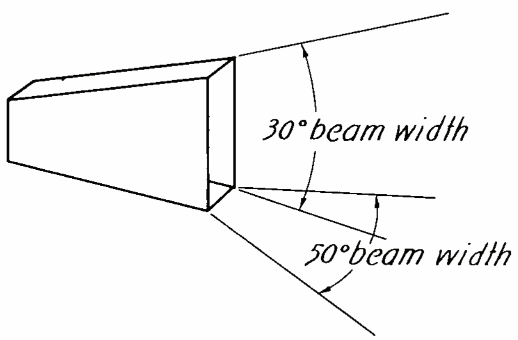
EXAMINATION, Page 3.

3. Given a vertical dipole set in front of and *parallel* to a large, plane, metallic reflector. Maximum field strength is desired along a line perpendicular to the dipole. What is the minimum distance that the dipole can be set in front of the reflector? *Explain.*

4. Given a half-wave dipole of 1/4 inch radius, operating at 500 mc through a very short transmission line. What band width will this dipole cover with a reduction of signal strength to at most 70.7% of its maximum value?

5. The above antenna is fed by a coaxial cable whose outer sheath is 5/8 inch diameter. Design a skirt for the above line which will function satisfactorily over the range of frequencies found above.

6. It is desired to design a sectoral horn operating in the $TE_{1,0}$ mode at 10,000 mc. Horizontal polarization is desired. The beam is to have a width of 30° in the vertical plane, and 50° in the horizontal plane. Calculate the horn dimensions, including the throat cross section.



EXAMINATION, Page 5.

6. *(Continued)*

7. Design a pyramidal horn that will furnish the same beam widths as in Problem 6. Horizontal polarization is desired at 10,000 mc.

8. Design a biconical horn to operate at 3,000 mc and to have a power gain of 30. Horizontal polarization is desired, and the horn axis is vertical. Use optimum design from curves.

9. How would you energize the above horn? Show sketch.

10. A paraboloidal reflector is to have a power gain of 400 at 30,000 mc (horizontal polarization). The focus lies in the plane of the mouth of the reflector. What will the beam width be in a horizontal and in the vertical plane.

NOTE: Assume a doublet antenna is used.