SECTION 2

ADVANCED PRACTICAL RADIO ENGINEERING

TECHNICAL ASSIGNMENT

ULTRA-HIGH FREQUENCY TECHNIQUES

PART II - WAVE GUIDES AND CAVITY RESONATORS

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ULTRA-HIGH FREQUENCY TECHNIQUES

WAVE GUIDES AND CAVITY RESONATORS

SCOPE OF ASSIGNMENT

FOREWORD.—This assignment on Ultra-High Frequency Techniques deals with wave guides, and cavity resonators. The mechanism of operation is described, and such applications as are in use at the present time are discussed. It is to be appreciated that in such a rapidly advancing art, new applications and procedures will constantly make their debut; this assignment will provide the background for understanding and applying such new developments.

FIELD THEORY

Up until now the student has mainly used the circuit concept in analyzing the action of an electrical device: its operation was discussed in terms of the flow of electrons in well-defined paths called circuits. However, an exception has already been noted in the assignment on Radiating System, in which the mutual interaction of electric and magnetic fields *in free space* has been studied in the form of an electromagnetic wave. Such a wave cannot be considered as confined to any one set of paths, but instead pervades all space.

As a result, the mathematical treatment becomes more involved, and the analysis deals primarily with the field interactions rather than with the electrons that originate these fields. Thus, it was noted previously that a moving or varying electric field produces a magnetic field in closed loops around the electric field, and that a moving or varying magnetic field produces an electric field in closed loops around the magnetic field. As a consequence, in free space each type of field can produce and sustain the other by its variation, remote from the actuating *circuit*. Similar effects can be noted in enclosed spaces remote from the actuating circuit, such as in wave guides and cavity resonators.

It is sometimes stated that the notion of circuit must be discarded when one goes to ultra-high frequencies, and that a new and different concept--that of electric and magnetic fields--must be adopted in its place.

This is not entirely so. The field concept applies to low-frequency circuits just as well as the circuit concept, and the latter can be applied to u.h.f., if desired. However, it is more convenient to use the circuit concept for l.f. work and the field concept, in general, for u.h.f.; and it is primarily convenience, rather than correctness, that dictates the choice of either.

ELECTRIC FIELD MEASUREMENT.--This can be made clear by a simple example. In Fig. 1 is shown in cross-section a two-wire line. A current flows into the paper in the left-hand conductor, and returns (out of the paper) in the right-hand conductor. Between the two conductors at the cross-section shown, there exists a potential difference, or voltage V. Suppose the left-hand conductor is the positive side of the line. Then a *unit positive* charge, when placed between the wires, such as at point A, will be repelled by the left-hand conductor and attracted by the right-hand conductor, and will move along the curved path shown as an electric field line.

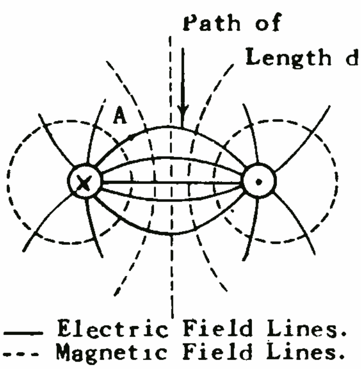


Fig. 1

The force exerted by the conductors on this unit charge is a measure of the intensity of the electric field. This intensity can also be expressed by saying that there are a certain number of electric lines passing through one square centimeter at that point in space. A third method of measuring the intensity is to divide the potential difference V by the total length of the curved path along which A moves. Suppose this length is d centimeters. Then the average electric field intensity is

$$E = \frac{V}{d} \tag{1}$$

and can be expressed as so many volts per centimeter. It will be found that whether E is measured in terms of force on a unit charge, or as so many lines per square centimeter, or as so many volts per centimeter, the values are all equivalent, as is to be expected.

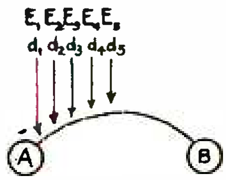


Fig. 2

Suppose it is desired to measure V indirectly by measurements of E along path d. To do so, it is necessary to break up d into very small sections d_1, d_2, d_3, etc., along each of which E is practically constant in value. Thus, along d_1, E might have the value E_1, along d_2, E may have changed to the value E_2, and so on. This is shown in Fig. 2.

Since E represents a certain number of volts per centimeter; E multiplied by the number of centimeters distance along a path represents the total voltage across the ends of the path. Consequently, $E_1 \times d_1$ is the voltage between the two ends of path d_1; $E_2 \times d_2$ is the voltage across path d_2, etc. The *total* voltage between conductors A and B is the sum of these little products, or

$$V = E_1d_1 + E_2d_2 + E_3d_3 + \dots \quad (2)$$

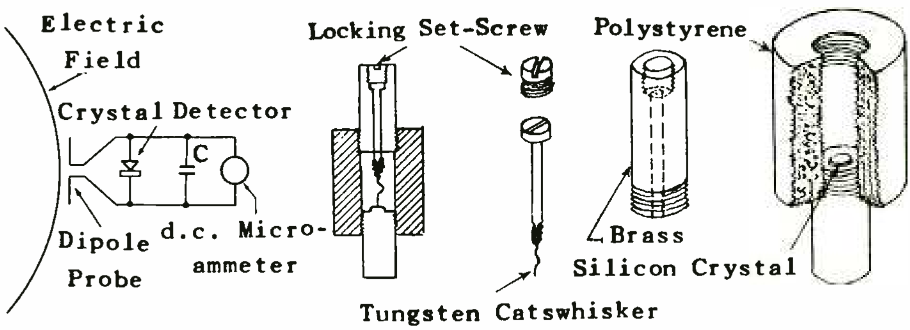
Thus the value of V can be found if a field intensity meter is employed to measure E along a path joining the conductors. It will be found that the value of V comes out to be the same regardless of the path chosen; if a longer path is chosen, E along it is weaker, and the summation of Eq. (2) contains more terms for the longer path, but each term is correspondingly smaller in value. (This holds only at low frequencies.)

Fortunately, at low frequencies, such an elaborate measurement is unnecessary. One merely has to connect a device known as a voltmeter between conductors A and B, and the summation indicated by Eq. (2) appears directly on the voltmeter scale.

At u.h.f., however, things are somewhat different. It was mentioned in the lesson on radiation that a varying magnetic field induces electric field lines in space around it; and that a varying electric field may be regarded as a displacement current, and is capable of setting up magnetic lines around it. The latter effect is appreciable only if the rate of variation--frequency--is very high.

As a result of these effects, appreciable voltages may exist between points in space remote from the actuating circuit, owing to electric field lines passing through these points. An ordinary voltmeter will not be able to measure the potential difference because the source of this voltage--free space itself--has such a high impedance that the voltmeter current may be small and the reading incorrect.

Field Detector.--In such a case, a direct field intensity meter may be employed. This is essentially a small dipole antenna or probe, connected to a thermogalvanometer, or, preferably to a crystal detector and d.c. microammeter. In Fig. 3 is shown a circuit for much a probe, and the details of



Details of Crystal Detector Set

Fig. 3

the crystal detector unit. The dipole probe is aligned with the electric field lines in its vicinity by noting the position that gives maximum reading on the d. c. microammeter (usually 0-200 microamperes). A voltage is developed across the dipole that is equal to its effective height multiplied by the field strength E. This u.h.f. voltage causes a corresponding current to flow, which can actuate a thermogalvanometer. Greater sensitivity is possible, however, if the u.h.f. current is rectified by a crystal detector, and fed to a microammeter by-passed by a small condenser.

A very satisfactory detector or rectifier is the crystal type shown in Fig. 3. It can be calibrated at sixty cycles and then used up to the highest frequencies generated without appreciable error. It thus can operate in the microwave range where ordinary vacuum tube detectors begin to fail because of transit time effects (to be described later). Moreover, it is very small and can be placed inside of a wave guide, if desired, and requires no heater, plate, or bias voltages.

The actual mechanism of detection in a crystal is not completely known, although recently Schottky has developed an analysis based on the quantum theory that seems to fit the observed phenomena fairly well. Suffice it to say that current can flow more readily from the catswhisker to the crystal than in the reverse direction, i. e., the conductivity is greater when the crystal is positive to the catswhisker, assuming a silicon carbide crystal is employed.

Representative values are 8,000 ohms in the forward direction and 40,000 ohms in the reverse direction, or about a 5 to 1 ratio of resistance. The actual values vary considerably. If a poor spot on the crystal is selected, a ratio of only 2 to 1 or less may result. By turning the screw slightly in which the tungsten catswhisker is anchored, a spot on the crystal may be found that has the desired ratio of 5 to 1 for the forward and reverse resistances. The locking set-screw is then tightened to preserve the adjustment.

An ordinary low-voltage ohmmeter may be used to measure the resistance. Voltages as high as five volts may safely be applied to the crystal, although the normal operating voltage is far less. The current flow is generally less than 1 milliampere, and is, of course, determined not only by the impressed voltage from the dipole probe, but by the total resistance of the circuit to d. c. In order to enhance the rectifying action, the d. c. load, such as the microammeter, is normally by-passed by a small condenser of a few micromicrofarads capacity. If the load resistance is made high compared to the forward resistance of the crystal, then the d. c. voltage across the load is substantially equal to the peak a. c. voltage impressed.

Current and Magnetic Field.--The magnetic field lines are closed loops. In Fig. 4 is shown such a loop (H) encircling a conductor. If one were to multiply H (measured in lines per square centimeter perpendicular to its direction) by the path length--similar to the process described for E--one would obtain a value that equals the current flow *surrounded by the loop path.*

It must be remembered that although H and E are measured in lines per square centimeter, actually there are not separate lines of E or H in space, but rather a kind of electric or magnetic *strain* in space, that is distributed through the space. The concept of lines per square centimeter is merely a convenient way of measuring this strain at any point in space: where the strain is more intense, more lines are said to pass through this region; where it is less intense, less lines are considered passing through there.

Returning to Fig. 4, note that the measurement of H times the closed path indicates the current through that path. The current thus measured is conduction current (electron flow in a conductor) plus any displacement current present; i. e., if the path encloses electric field lines that are varying, then it measures their equivalent current flow.

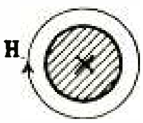


Fig. 4

In the configuration shown in Fig. 1, the electric and magnetic field lines both lie in the plane of the paper. Thus the magnetic lines do not encircle the electric lines, but instead cut through them. Hence a measurement of H around a closed path does not indicate any displacement current, but simply the conduction current in the wire. Thus any flux loop shown in Fig. 1 may be used to measure the current in the enclosed conductor.

It is evident, however, that in the case of a two-wire line, the current may be measured far more easily by means of an ammeter inserted directly in series with the conductor, than by means of a device that measures magnetic field intensity at every point of the field path, and then requires a summation process to give the value of the enclosed current. The ammeter, in short, produces this summation process directly, and its pointer directly indicates the magnitude of the current.

At ultra-high frequencies this fortunate circumstance usually fails to hold. The current does not flow only in the conductors; it may flow as a charging or displacement current between points in space. There is still a circuit, in a sense, but the paths are no longer well-defined, and the current measured by an ammeter may be but part of the total current flow. In a circular wave guide it will be shown that the current flow in the center of the pipe is solely a *displacement current*, which returns as a *conduction current* on the inner surface of the tube. Were it attempted to measure the central displacement current with an ammeter, it would be found that not only would the presence of the ammeter wires interfere with the normal behavior of the guide, but the current could not be made to flow through the ammeter wires and the ammeter: instead it would flow across their intercepting terminals and continue down the tube.

From the above, it is evident that the circuit has spread out from points forming paths, into points forming volumes of space, and the action can no longer be nicely summarized by specifying the total current as if it were concentrated in a line path.

It is therefore necessary to describe the activity at each point of space, and this is best done by specifying the electric and magnetic field intensities at each point. In proceeding from one point to the next in the space involved, it is found that these fields form certain distinctive patterns that describe the activity taking place more significantly than voltage and current flow.

Loop Detector.--In Fig. 3 was shown a probe for detecting the E (electric) field at any point, both in magnitude (reading on microammeter) and direction (orientation of probe for maximum reading at that point). If the dipole probe be replaced by a small loop of wire, measurements can be made on the H (magnetic) field instead of the E field.

The loop is turned until it encloses the maximum amount of magnetic flux passing through that small region, as shown in Fig. 5. Its plane is then clearly *perpendicular* to the direction of the flux. Maximum reading is thereupon indicated on the d. c. microammeter for that point in space. Maximum readings can similarly be found for other points in space. and thus the *relative* strength and direction of the magnetic field found throughout the region.

Either type of probe can be calibrated for absolute values by taking a reading in a field of known intensity, but ordinarily only the relative values are required, so that such calibration is unnecessary. It is also clear that the smaller the probe is, the more nearly will the readings represent *point* readings rather than average values over an area, but, unfortunately, the sensitivity of the instrument will go down as the probe size is decreased.

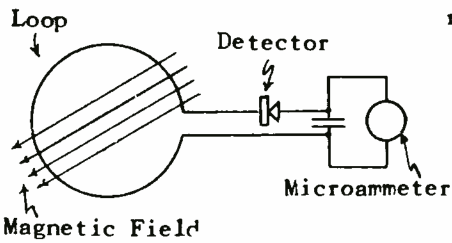


Fig. 5

In taking relative readings, it is to be noted that the crystal detector has approximately a square-law characteristic below about 25 μ amperes (*above that it is linear*). Thus if, at one point the reading on the microammeter is twice that at another point, then the field strength is not twice that at the other point, but only $\sqrt{2}$ times as great. In other words, the square root of the microammeter readings are used rather than the readings themselves.

WAVE GUIDES

General.--Experiments by Dr. G. C. Southworth, of the Bell Telephone Laboratories, and by Prof. W. L. Barrow of the Massachusetts Institute of Technology, have shown that electrical energy at sufficiently high frequencies can be transmitted by *single* conductors in the form of hollow tubes of various shapes--rectangular, circular, etc. The transmission is a kind of radiation confined within the tubes, but differs from radiation in free space in that either the electric or the magnetic field lines have components *that are in the direction of transmission (longitudinal)*, whereas, in the case of radiation in free space, both sets of lines are at right angles (transverse) to the direction of propagation.

In a wave guide, if the electric field is partly longitudinal, then the magnetic field is transverse, i. e., lies entirely in planes perpendicular to the axial direction of propagation down the pipe, and if the magnetic field is partly longitudinal, then the electric field is entirely transverse. Accordingly, the waves in hollow pipes can be classified either on the basis as to which field has a longitudinal component, or as to which field is wholly transverse. The latter system appears to have become more general, and so the terms TM (transverse magnetic) and TE (transverse electric) will be employed in this assignment.

As has been indicated, the properties of a wave guide are best studied with reference to their electric and magnetic field patterns, rather than with reference to the voltages between walls or the currents flowing in them. Nevertheless, a good physical picture can be obtained by regarding the guide as a kind of transmission line circuit, and since circuit concepts are more familiar to the student, such an analysis will be of benefit in introducing him to the properties of the hollow tube.

Circular Wave Guide.--In Fig. 6 is shown a concentric cable whose inner conductor is terminated abruptly. The electric field lines are shown extending from the inner to the outer conductor, and are due to the voltage existing between conductors. In the left-hand region, where both conductors exist coaxially, the electric lines are radial. The magnetic lines have been omitted to increase the clarity of the picture. They form concentric circles that lie in planes at right angles (transverse) to the axis of cable, and fill the space between the inner and outer conductor.

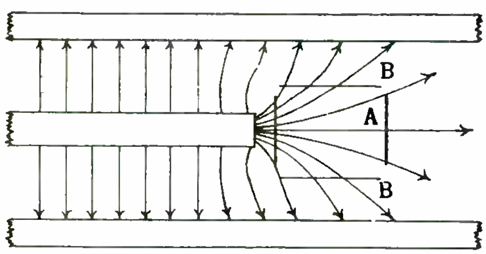


Fig. 6

To the right of the inner conductor the electric lines spread out into a fringing field that terminates on the walls of the outer conductor.

In the left-hand region one can speak of shunt capacity and leakage, and series inductance and resistance per unit length of the coaxial cable: the transmission line constants. In the right-hand region, where only the outer conductor is present, such constants are not as apparent: the circuit is not as evident. At low frequencies, the fringing lines of force may be described as an end effect that is of little consequence; i. e., the line is effectively open-circuited by the abrupt termination of the inner conductor.

At sufficiently high frequencies, however, the end effect becomes very important. It represents a capacity, one of whose plates is the end of the inner conductor, and the other the wall of the other conductor extending to the right. This capacity has a rather odd shape, but it may be resolved into two condensers of suitable size in series: one consists of two circular plates shown in cross section in Fig. 6 as A, and the other as a concentric condenser having a concentric shell B as one plate, and the inner wall of the outer conductor as the other plate.

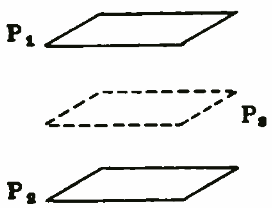


Fig. 7

It is not necessary that metallic plates actually exist at these points. For example, in Fig. 7 is shown an ordinary condenser having plates P_1 and P_2. Suppose the capacity between these plates is 1 μf. One can with equal correctness say that one has two condensers in series, each of 2 μf capacity, and composed of P_1 and P_3 (an imaginary plate) for the one, and of P_3 and P_2 for the other. By the rule for condensers in series, the total capacity is

$$C = \frac{1}{\frac{1}{2} + \frac{1}{2}} = 1\ \mu f$$

which is the same value as if P_1 and P_2 were considered alone. The capacity resides in the dielectric space; the plates are merely convenient conductive surfaces to charge that particular portion of space. Thus, 2 volts applied between P_1 and P_2 so that P_1 is positive and P_2 negative, will store the same amount of electrostatic energy in the intervening space as 1 volt applied between P_1 and P_3 (P_1 positive to P_3) and 1 volt applied between P_3 and P_2 (P_3 positive to P_2). (Of course P_3 is assumed to have negligible thickness.)

The entire space to the right may be resolved into axial condensers of the form A, and concentric or radial condensers of the form B. Such an arrangement is shown in Fig. 8 in a circuit form. The radial condensers B form shunt elements, the axial condensers A form series elements, and it is apparent that the empty space within the tube of Fig. 6, to the right of the center conductor, is beginning to assume the appearance of a transmission line.

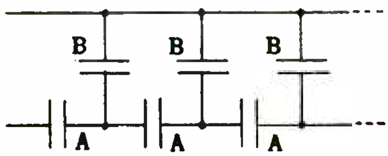


Fig. 8

At low frequencies the charging or displacement current in these condensers is negligibly small, and it may be assumed without any appreciable error that no current flows in this region, i. e., that the concentric cable is open-circuited. At high frequencies, however, the displacement current begins to be appreciable, as does also the magnetic flux set up in this region.

This flux is of the form of concentric circles lying in transverse planes in the cable, just like the flux loops in the left-hand region. Indeed, in the right-hand region the axial condensers A take over the function of the inner conductor in the left-hand region, and serve to transmit longitudinal (displacement) current down the guide. This current then returns via the outer conductor as a conduction current.

The magnetic flux set up in this region gives these portions of the circuit a certain amount of inductance per unit length. The outer conductor also has a certain amount of ohmic resistance per unit length, but this is usually small and will be ignored.

In view of the series inductance present, Fig. 8 must be modified to include it. This has been done in Fig. 9, and for comparison, has been joined to the equivalent circuit for the coaxial cable, itself, as shown. In the figure, L_1 and L_2 represent the series inductances, C_c the shunt capacity of the cable (left-hand region of Fig. 6), and A and B have the same meaning as in Fig. 8. Examination of Fig. 9 discloses that as the frequency is raised, the series impedance in the pipe, equal to $(2\pi fL_2 - 1/2\pi fA)$, decreases until at some frequency

$$f_r = \frac{1}{2\pi\sqrt{L_2 A}}$$

it is zero, i. e., L_2 and A are in series resonance.

Beyond this frequency, $2\pi f L_2$ exceeds $1/2\pi f A$, so that the *net* series impedance is a pure *inductive* reactance, and the pipe may be regarded as being made up of some particular value of series inductance and shunt capacitance, in short--it appears just like the coaxial cable, and may therefore be expected to behave similarly. This is indeed the case, *above f_r the hollow tube begins to transmit energy down the line just as it is transmitted in the coaxial cable of which it is a continuation.*

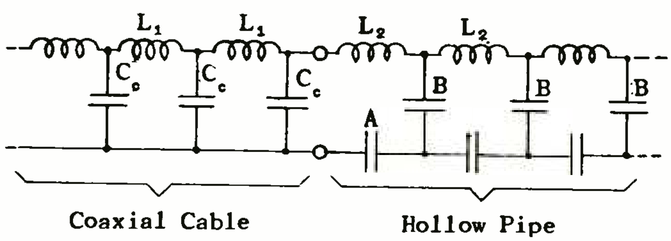
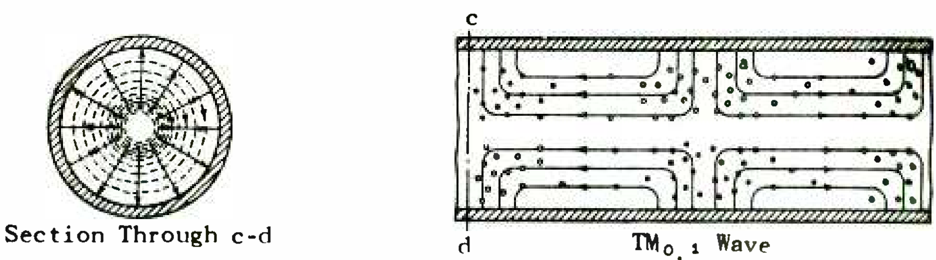


Fig. 9

Note, however, that it does not function like the cable except above a certain resonant frequency. This is called the *cut off frequency*. Below this frequency no transmission takes place, or more correctly, waves start out from the end of the inner conductor, but are rapidly attenuated within a short distance, so that no appreciable energy appears farther down the line. The attenuated waves are not converted into heat, but rather represent energy stored in the medium surrounding the end of the inner conductor during one portion of the cycle, and returned to the cable during the other part of the cycle. Such storage and return of energy is exactly that which takes place in a condenser, for example, and represents a *capacitive* end effect to the cable. *Above the cutoff frequency* the end effect becomes more than a shunting capacitive reactance, it also represents energy transmitted down the guide and hence lost to the cable. Thus it appears to the cable partly as a *resistance* termination, although not necessarily of the correct value to match the characteristic impedance of the cable.

Wave Pattern--Circular Guide.--The above analysis indicates that the magnetic field is entirely transverse (lies in planes perpendicular to the axis of the guide). On the other hand, the electric field lines pass longitudinally down the tube (condenser A effect of Fig. 6), and ultimately curl up in a radial or transverse direction to meet the inner surface of the tube (condenser B effect of Fig. 6). Thus the electric field is partly *longitudinal*, and partly *transverse*, whereas the magnetic field is *entirely transverse*. The wave can be classified as a TM (transverse magnetic) wave.

The exact field patterns cannot be obtained from the circuit analysis presented above, but can be found by solving certain equations first derived by Maxwell, and hence called *Maxwell's equations*. The resulting pattern is shown in Fig. 10 and represents conditions far down the tube, remote from the point where the central conductor was terminated. The field in the neighborhood of the central conductor is as shown in Fig. 6. But the circuit conditions have been shown to be favorable for wave propagation (Fig. 9). Hence the electric and magnetic field lines, "launched" by the central conductor, propagate themselves down the tube as shown in Fig. 10, and produce travelling positive and negative groupings, just like the current and voltage loops in an ordinary two-conductor transmission line. The distance between two such groupings, such as between two successive points where an electric field loop in Fig. 10 touches the walls of the tube, is



Courtesy of Bell System Technical Journal, Hyper-Frequency Wave Guides, April 1936.

Fig. 10. Small solid circles represent lines of force directed away from the observer. Propagation is assumed to be directed to the right and away from the observer.

known as a half-wave length. Its magnitude will be discussed farther on.

Property Of An Electric Field Line.--In analyzing the field patterns, considerable information can be gleaned from an important property of electric field lines. This property is that the lines must be perpendicular to the surface of a perfect conductor, and cannot be parallel to it.

Suppose a line E made an angle θ, Fig. 11, with such a surface. This line, being a vector, can be resolved into a component E_n normal (perpendicular) to the surface, and a component E_t tangential (parallel) to the surface. The component E_t will exert a force on the free electrons in the perfect conductor urging them to slide along it. Since the conductor has been assumed to have no resistance, a finite magnitude of E_t will cause an *infinite* current to flow (infinite transportation of free electrons). There will thus instantly occur a re-grouping of charges (electrons) that will send out a field opposite to E_t, and thus wipe it out. Hence, except for the merest instant, there cannot be a tangential component; i. e., E must meet the conducting surface perpendicular to it.

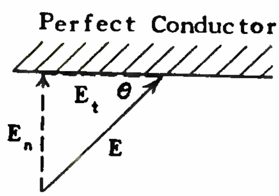


Fig. 11

The normal component E_n may try to push electrons through the wall thickness to the outside, but except for a slight displacement this effect is inappreciable since there is an open circuit in this direction. Actually, owing to inductance (skin) effect the current is confined to the inner skin of the tube, and no appreciable current flows on the outer surface. There is therefore no voltage drop along the outer surface, and consequently no *coupling* to other circuits. This is very important, for it eliminates one of the most serious difficulties in high frequency operation: that of unwanted or stray coupling between circuits.

Comparison Of Coaxial Cable And Circular Pipe.--In the circular pipe it has been seen that for the TM wave, the electric field will have a longitudinal component. In a coaxial cable, the electric field must be perpendicular to both the inner and outer conductors (Fig. 6) and hence must have a *radial* direction. But the radii lie in transverse planes, i. e., are perpendicular to the axis. Hence in a coaxial cable the electric, as well as the magnetic field, *are entirely transverse*, as indicated in Fig. 12, and the wave propagation is known as *transverse electromagnetic* (TEM). This type of propagation is not possible in a wave guide because of the absence of the center conductor and the requirement that the capacity of the inner space provide a substitute, such as condenser A of Fig. 6.

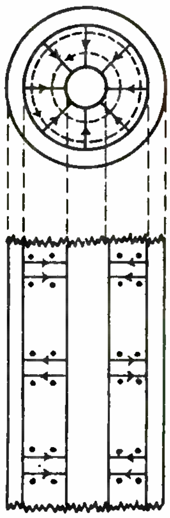


Fig. 12

The presence of a center conductor in a coaxial cable permits transmission to take place down to zero frequency (d. c.), for there is no need for a resonant frequency above which the system will have a net series inductance and shunt capacitance, as is the case for a wave guide. Thus, a coaxial cable can have a radius small compared to the wave length it is transmitting and yet be an efficient transmission system. A circular wave guide, on the other hand, must have a radius that is at least .383 of the wave length to be transmitted. Lower frequencies, having wave lengths greater than the above, will not be propagated. Hence a wave guide becomes of reasonable and economic size only at the very highest frequencies, where wave lengths are on the order of centimeters.

Rectangular Wave Guides.--Another type of wave guide that is used, possibly most often, is one having a rectangular cross section. This can be regarded, from a circuit viewpoint, as being made up of a two-wire line, shunted all along its length by pairs of parallel half-turns. See Fig. 13. The equivalent distributed circuit is as shown in Fig. 14. The elements L and C are those of the two-wire line, while L' represents the inductance of the two half-turns--one on either side--in parallel. This circuit acts like a high-pass filter similar to the one shown in Fig. 9. Thus, the cutoff frequency is that at which L' and C resonate. Above this frequency transmission takes place, and as shown in the right-hand diagram of Fig. 13, both longitudinal and transverse currents flow on the inner surfaces of the walls.

An interesting variation in this viewpoint is to consider the shunting elements as portions of short-circuited transmission lines, and the two-wire line as having only series inductance, its shunt capacity being assigned

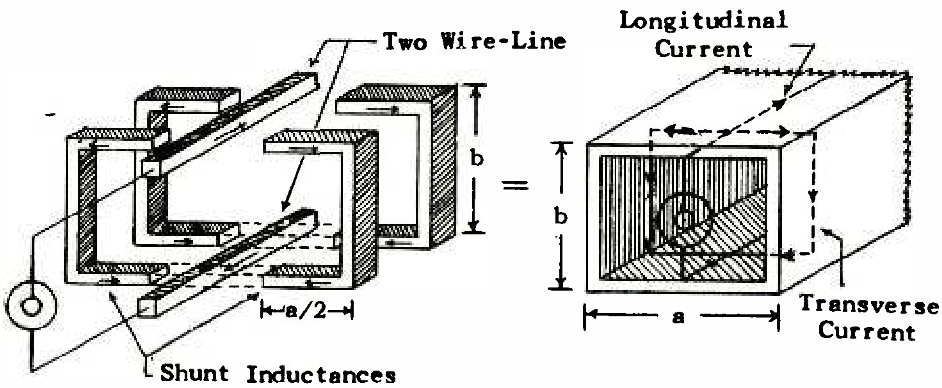


Fig. 13

to the short-circuited lines instead. The short-circuit is the side element of dimension b. Although its length is comparable to the sides of the line of length a/2, nevertheless it will be regarded as a short-circuit. At a frequency and wave length λ such that a/2 is equal to $\lambda/4$, the shorted line element is a quarter wave long, and therefore appears as an infinite impedance (resistive) at its open end. As such it has no shunting effect on the two-wire line.

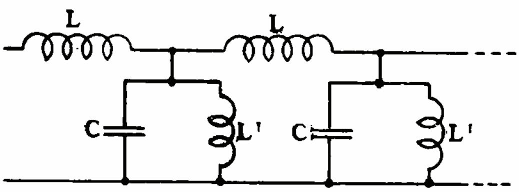


Fig. 14

Above this frequency it appears capacitive in nature, and therefore of the right kind of reactance to cooperate with the inductance of the two-wire line to form a wave transmission system. Below this frequency it is inductive in nature, and therefore unable to form a transmission system. This is illustrated in Fig. 15.

The critical or cutoff frequency is evidently that for which the pipe width a equals $\lambda/2$, so that each half is $\lambda/4$. Thus, with surprising ease there has been found the critical dimension for a rectangular pipe, namely, the width must be *at least* half a wave length.

The field distribution can also be found by this viewpoint. In a shorted quarter-wave line, the voltage varies sinusoidally from a maximum at the open end to zero at the shorted end. The electric field lines be-

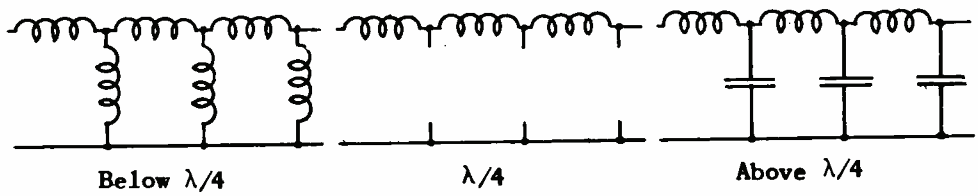


Fig. 15

tween the conductors vary in density in the same sinusoidal manner. This is shown in Fig. 16. Note that at the short-circuit end, which constitutes a side wall of the tube, the field is zero. This is correct; there can be no field lines parallel to a conductor and immediately adjacent to it. Since the lines are perpendicular to the top and bottom conductors, the only way they can avoid being parallel to the side walls is by not being there at all, i. e., by the field being zero at that point.

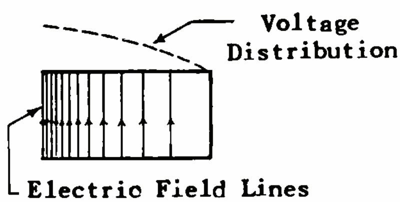


Fig. 16

The electric field also varies in sinusoidal fashion along the axis of

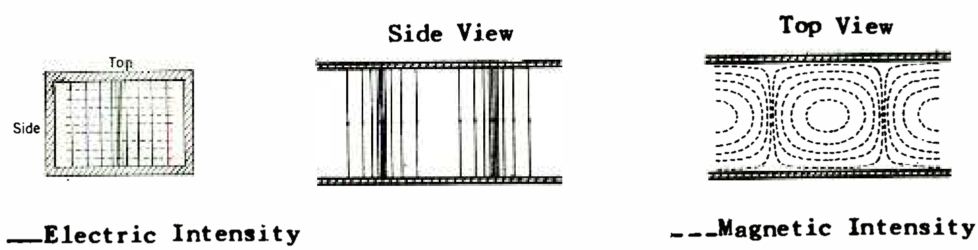


Fig. 17

the tube, owing to the wave propagation along the two-wire line. The resulting field patterns are shown in Fig. 17. Note how the electric field

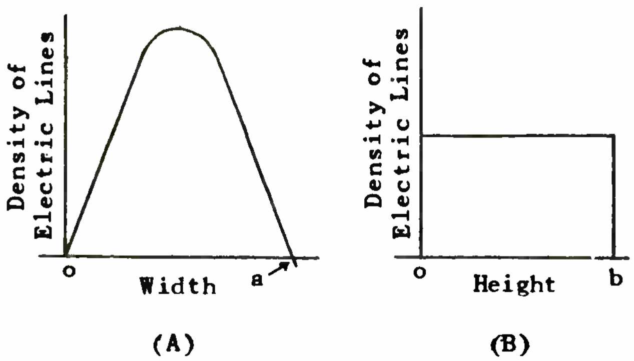


Fig. 18

lines vary across a cross-section.

If the density of lines (lines per sq. cm.) be plotted against the horizontal distance, a half-sine-wave curve is obtained, as shown in Fig. 18(A). On the other hand, along any vertical path the density is the same throughout--there is no variation in this direction. This is indicated by the horizontal line in Fig. 18(B). These facts will be employed to classify the various possible modes of propagation.

The side and top views indicate the patterns at any one instant of time. These patterns travel down the guide with an apparent velocity greater than that of light in free space. This paradox will be explained later. Note at present that the magnetic lines are partly transverse and partly longitudinal, whereas the electric lines are wholly transverse. This is therefore a TE wave.

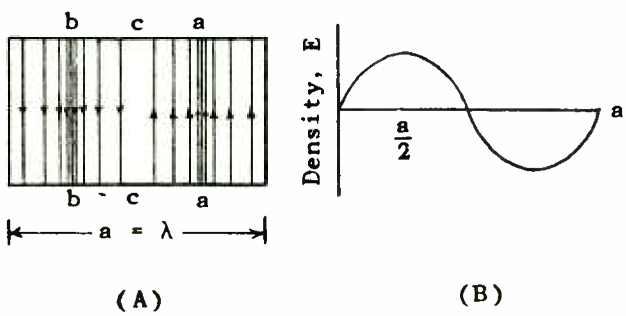


Fig. 19

Higher Modes.--The wave patterns shown for the circular and rectangular guides are not the only ones possible, however. More complicated patterns, both of the TE and TM type can be obtained, depending upon the method and frequency of excitation of the guide, and its dimensions. For example, suppose that the shorted line elements into which a rectangular guide can be decomposed, are $\lambda/2$ in length instead of $\lambda/4$. The entire width of the guide is then λ, i. e., $a = \lambda$ instead of $\lambda/2$. In this case an electric field distribution as shown in Fig. 19(A) will be obtained, and the plot of the density will be a full sine curve, shown in (B). To obtain such a distribution, the exciting source had best be located at line a-a or b-b, *or else two oppositely poled* sources at the *two* locations. On the other hand, if a single source be located at the *exact center* c-c, then only a half-sine-wave distribution would be obtained.

Mode Designation.--The previous examples indicate that the electric and magnetic field lines can assume quite complicated patterns. In attempting to classify such field configurations, one of the first steps is to recognize the existence of basic patterns or modes. A mathematical treatment (verified by experimental tests) shows that either the magnetic or electric field lines proceed for part of the distance along the axis, i.e., have axial components.

However, if the magnetic field, for example, has an axial component, then it is found that the electric field lines lie wholly in a transverse or cross-sectional plane of the wave guide. Or, if the electric field has an axial component, then the magnetic field is wholly transverse.

This indicates a method of classifying the waves. At first they were identified by the axial component; thus an E wave was one whose electric field had an axial component, and an H wave was one whose magnetic field had an axial components.

A later and present mode of designation is by means of the transverse wave: a TM wave means one whose magnetic lines of force lie wholly in transverse planes; a TE wave is one whose electric field lines lie wholly in transverse planes. Thus, the present TM wave corresponds to the originally designated E wave, and the present TE wave corresponds to the orginal H wave.

The next step is to designate the peculiarities in the patterns of the TM or TE waves. This is not so difficult a task for the rectangular wave guides where variations in the field patterns are measured by traversing two paths; one parallel to one dimension of the guide and the other parallel to the other dimension (which is perpendicular to the first.)

In the case of the circular wave guide, the two paths along which the field variations were first noted were along the radius and around a circle concentric with the circular guide wall. It was here that ambiguities arose. For example, if one proceeds along one radius, the field variations are of a certain kind; but if one proceeds along some other radius the field variations may be quite different. Similar ambiguities are noted when one circular path or another is chosen.

The final method of identifying the various modes or patterns for a *circular* wave guide has been decided by the I.R.E. on the following basis:

1. In all cases the electric field is to be the basis of designation regardless of whether the wave is a TM or TE type.

2. Instead of radial and circular paths, axial planes (planes passing through the axis of the guide) and circular cylinders, concentric with *and including the guide wall*, are to be used.

3. There are, of course, an infinite number of axial planes (corresponding to an infinite number of radii in any cross section of the guide) and an infinite number of concentric cylinders (corresponding to the infinite number of concentric circles in any cross section of the guide). In determining the particular mode or pattern, the observer has to find the particular axial planes and the particular cylinders to which the electric field is either perpendicular or else is zero, depending upon the definition of the mode (to be given below).

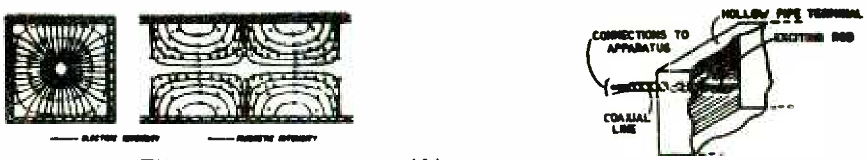
The various patterns may be classified according to the following rules:

a). Rectangular Guide:

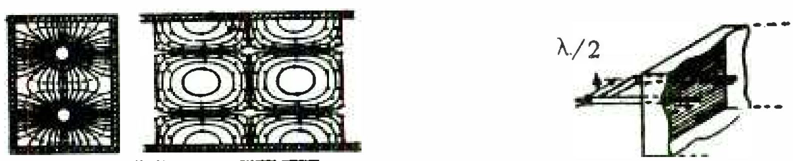
1. Count the number of half-sine waves of half-cycle variations along *the longer cross-sectional dimension*. Suppose this happens to be three.
2. Count the number of half-sine waves or half-cycles along

the shorter cross-sectional dimension. Suppose this is two.

3. Suppose the wave in question is a TE wave, then the above sine wave cyclic variations refer to the *electric field pattern*, and the wave is designated as a $TE_{3,2}$ wave. If the wave was a TM wave, then the variations in *magnetic field density* would determine the subscripts; *whichever field is transverse* is used to determine the subscripts. Note, however, that a $TE_{2,3}$ field pattern is merely a $TE_{3,2}$ pattern turned through a right angle.



$TM_{1,1}$ (A)



$TM_{2,1}$ (B)

Courtesy of Chu and Barrow, Proc. I.R.E., Dec. 1938

Fig. 20

In the case of the rectangular wave pattern shown in Fig. 17 the variation along the shorter (vertical) dimension is zero, and along the longer (horizontal) dimension is one-half sine wave, so that this is a rectangular $TE_{1,0}$ type of wave.

Higher Order Patterns--Rectangular Guides.--Although higher order patterns are generally not desired, and have very high cutoff frequencies, nevertheless they may be inadvertently generated, and so some knowledge of them is necessary. In Fig. 20(A), is shown a $TM_{1,1}$ wave in a rectangular wave guide, together with the method of producing it, namely, by the projection of the central conductor of a coaxial cable into the center of wave guide. The latter is capped at the end as shown to reflect the radiation in that direction and cause it to flow only in the opposite direction.

In Fig. 20(B) is shown a higher transverse magnetic mode, namely, the $TM_{2,1}$. To excite this mode, two offset antennas are required, as shown, and the current in one is made to lag that of the other by 180° by interposing an additional half wave length of coaxial cable between this antenna and the source.

To excite the $TE_{1,0}$ mode of Fig. 17 the method shown in Fig. 21(A) may be used, and to excite the $TE_{2,0}$ wave indicated in Fig. 19, the method shown in Fig. 21(B) can be employed.

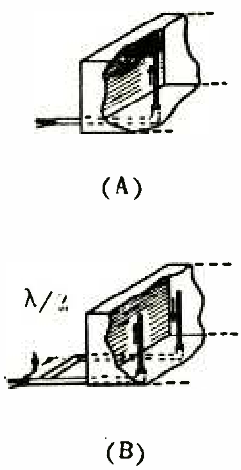
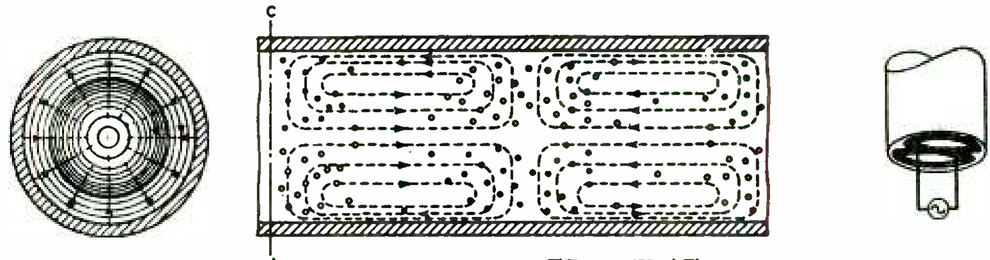


Fig. 21

(b) Circular Guide:

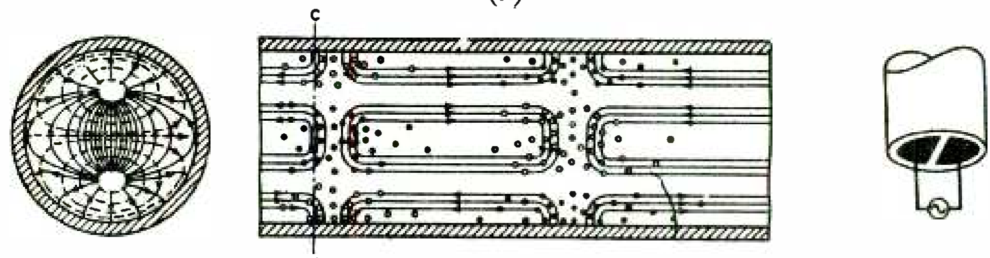
1. For a $TE_{n,m}$ wave in a circular guide, m is the number of concentric cylinders including the wave guide wall on which the electric field is zero. The values for n will be discussed below, but for the special case where the electric wave is circular in shape, n is equal to zero. Thus, the wave shown in Fig. 22(A) would be designated as the $TE_{0,1}$ mode, because the electric lines are circles, so that n = o, and there is only one cylinder, namely the guide wall itself, for which of necessity the electric field is zero, since the conducting surface shorts out any electric lines attempting to be set up on it, so that m = 1.

In a $TE_{0,2}$ mode, as shown in Fig. 22(E), we have circular lines, so that n is again equal to zero, and there are two cylinders on which the electric field is zero, so that m = 2. This is true as shown because at the periphery the conducting guide wall of course shorts out any circular



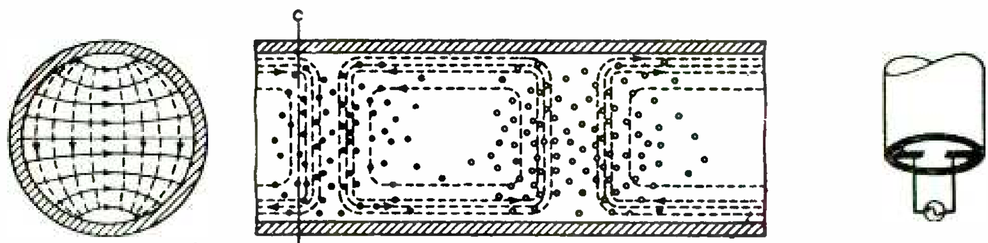
$TE_{0,1}(H_0)$ Wave

(A)



$TM_{1,1}(E_1)$ Wave

(B)

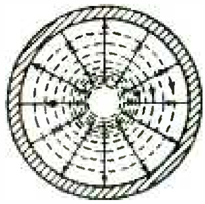


$TE_{1,1}(H_1)$ Wave

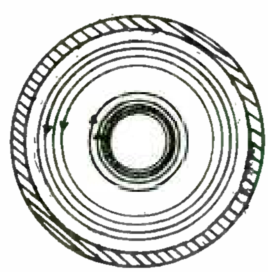
——— LINES OF ELECTRIC FORCE ----- LINES OF MAGNETIC FORCE

(C)

Approximate configuration of lines of electric and magnetic force in a typical wave guide. Small solid circles represent lines of force directed away from observer. Propagation is assumed to be directed to the right and away from the observer.



(D) $TM_{0,1}$



$TE_{0,2}$ (E)

(Courtesy BSJT and I.R.E.)

Fig. 22

electric lines coinciding with its surface; and at an intermediate distance from the center the field passes through zero as it changes from circles in a clockwise direction within this radial distance to counterclockwise circles beyond this particular radial distance.

2. When n does not equal zero for a $TE_{n,m}$ wave, the electric lines are non-circular in shape and n is the number of diameters (*axial planes*) to which the electric field is normal (i.e., perpendicular), and m is the number of *concentric Cylinders including the wave guide wall* to which the electric field is normal. Note that if n is other than zero, m is defined by the number of cylinders to which the electric field is *normal* rather than by the number of cylinders on which it is *zero.*

3. For a $TM_{n,m}$ wave in a circular guide the same rules hold as for a $TE_{n,m}$ wave. However, in a figure only magnetic lines may happen to be drawn. Remembering that the electric field lines are normal (perpendicular) to the magnetic lines, we can use the magnetic lines indirectly to indicate the electric field lines. (In connection with this, note that where the magnetic lines go to zero the electric lines also go to zero.)

For example, in Fig. 22(D) n is zero since the magnetic lines are circular in shape so that the electric lines (perpendicular to such circles) are inherently *parallel* rather than perpendicular to the radii or axial planes; m is one since the only cylinder on which the electric lines are normal to the outside wall. The mode is therefore designated as $TM_{0,1}$, and the figure is marked accordingly.

For the higher-order modes in a circular wave guide, it will be necessary to inspect the electric field pattern both in a transverse and in a longitudinal (axial) plane in order to find the value for n and m. If the electric field is not given, as is often the case for a TM mode, then the magnetic field will have to be inspected, and the direction of the electric field lines derived from such inspection on the basis that they are normal to the magnetic lines.

Consider the diagrams of Fig. 22(B). It so happens that both the electric and magnetic fields are depicted; assume, however, that only the magnetic (dotted) lines were shown. First let us find the value of n.

Refer to the left-hand diagram. If n is to be at least unity, the electric lines must be normal to at least one axial plane, whose edge would appear as a diameter of the guide. The lines must also be normal to the magnetic lines they cross. Hence some of the magnetic lines must lie in such an axial plane; i.e., at least one magnetic line in the left-hand diagram must form a diameter of the guide.

Inspection of the diagram shows that there is a horizontal magnetic line (dotted) forming a diameter of the guide. This is the only diametrical magnetic flux line, hence this is the only axial plane to which the electric field is everywhere normal. Therefore n = 1. Since the electric lines happen to be portrayed in this diagram, one can check this conclusion, and find that it is correct.

We now seek the value of m. Consider the right-hand diagram. If a cylinder can be found to which all the electric field lines in its immediate vicinity are normal, then m = 1; if two such cylinders can be found, m = 2, etc. A cylinder in the right-hand diagram would appear as two parallel lines equidistant from the guide axis. The totality of cylinders would appear as a series of such pairs of parallel lines.

It will be observed that each electric line (solid line) proceeds parallel to the guide axis for a certain distance and then curls around and proceeds radially outward to the guide wall surface. No matter what radial distance we choose within the guide, there is an electric field line there parallel to the axis. Hence there is no cylinder within the guide to which all the electric lines in its vicinity are normal; some of the lines instead lie on its surface for a portion of its length and then cross through outer cylinders.

Hence it would appear that m = o. But there is one cylinder to which the electric lines are all normal, and that is the inner surface of the wave guide wall. Since this surface counts as one cylinder, it must therefore equal unity. Therefore, the mode represented by the two diagrams of Fig. 22(B) must be a $TM_{1,1}$ wave. The student should now check the modes designated in the diagrams of Fig. 22(C), using the procedure given above.

Oblique Wave Concept.--The action of a wave guide has been analyzed on the basis that the guide can be regarded as a kind of transmission line circuit. Alternatively, its action may be studied from the basis

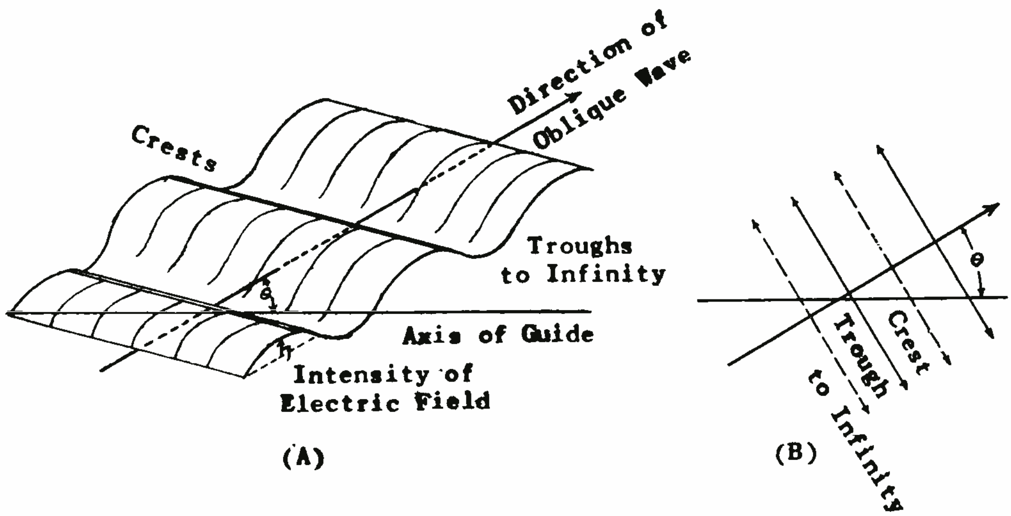


Fig. 23

of wave motion. It has been shown by Brillouin* that the wave travelling down the guide may be resolved into pairs of oblique waves travelling across the tube, just like any vector can be resolved into two or more components. This viewpoint is quite simple to apply to the $TE_{1,0}$ mode in a rectangular pipe, and yields some very interesting information as to phase and group-velocity, etc.

In Fig. 23 is shown one of these wave components, both in pictorial form (A) and in a simplified schematic form (B). The latter is a top

*Leon Brillouin, "Propagation of Electromagnetic Waves in a Tube," "Rev. Gen. Elec.," Vol. 40, P. 227, Aug. 1936.

view of the wave shown in A. Only the electric field (transverse component) has been plotted. The velocity of the wave is that of light for the medium. If the latter is air, as is normally the case, then the velocity is $c = 3 \times 10^{10}$ cm.

In Fig. 24 two such oblique waves are shown oppositely directed with respect to the axis of the guide. The double lines represent the second wave. At points E, F, G, etc. the waves meet crest to crest and reinforce one another (constructive interference) whereas at points such as

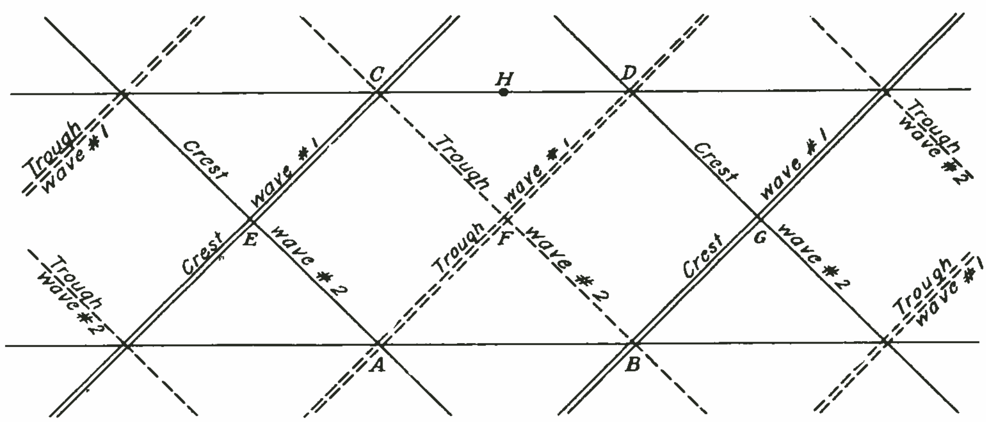


Fig. 24

A, B, C, or D the waves meet crest to trough and cancel one another (destructive interference.)

However, cancellation does not occur only at isolated points such as C and D, or A and B, but rather all along a line through C and D, and one through A and B. This is because all points on such lines are as far from the crest of one wave as they are from the trough of the other, so that they are at points of equal but opposite magnitudes of the two waves.

The result of this is that the electric field intensity is zero all along CD and along AB. Hence conducting walls can be placed at these locations without upsetting the electric field pattern, i.e., these are the proper locations for the two side walls of a rectangular guide transmitting a $TE_{1,0}$ wave. Conversely, if the walls of the guide have a given spacing, then the two oblique waves must make a certain angle θ with the axis of the guide to represent the actual, resultant wave.

The resultant wave travels axially down the guide. It has--at some instant--crests at E and G, etc., and a trough at F, etc. Its actual configuration is shown in Fig. 25. The distance between two corresponding points, such as two crests, is a wave length. The wave length λ_g is greater than that of the wave in free space, and since the frequency f of the wave is that of the oblique wave and of the actuating source, the apparent or phase velocity v_g of the wave in the guide is

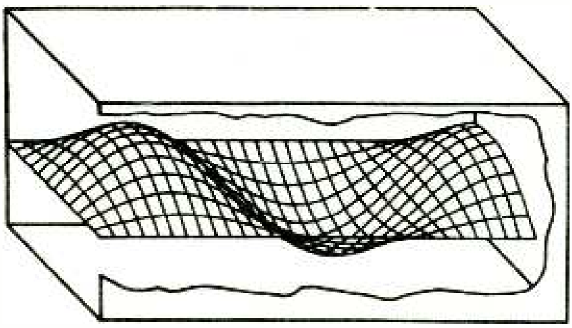


Fig. 25 *Courtesy of* Chu and Barrow: Waves in Hollow Tubes, Proc. IRE, Dec. 1938.

$$v_g = f\lambda_g$$

and is greater than that in free space.

This is in apparent contradiction to the modern theory of Relativity, which states that there cannot be any velocities exceeding that of light. The explanation is that this is an apparent velocity. When a wave is first impressed on the guide, the configuration shown in Fig. 25 is not instantly assumed at any point down the guide, but takes a finite time, depending upon the distance from the actuating source, and the velocity of the oblique waves, as modified by the fact that they move oblique to the guide. After steady-state conditions are attained, then the apparent velocity seems to be in excess of that of light, but then no account is taken of the initial transient time during which the configuration was being established.

Group and Phase Velocity.--The relationship between actual, or so-called *group* velocity, and apparent, or *phase* velocity, together with that between the wave length in a guide and in free space, can be derived by means of the oblique wave concept. Either oblique wave may be employed; indeed, either oblique wave may be regarded as the reflection of the other in the

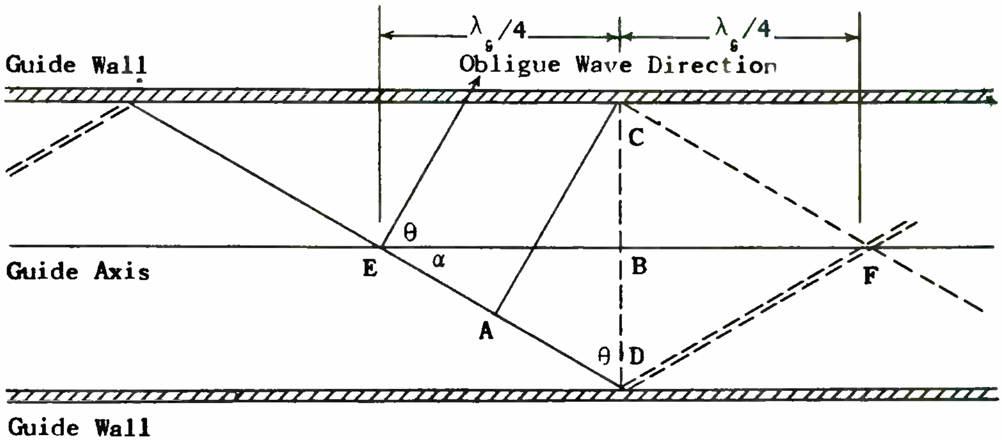


Fig. 26

walls of the guide (see Fig. 24), so that an alternative viewpoint is to regard propagation down the guide as performed by *one* oblique wave reflected back and forth in the guide as it progresses axially down the tube.

The direction of the oblique wave motion is a line perpendicular to the crests or troughs (wave fronts). Let it make an angle θ with the guide axis, Fig. 26. (This is a portion of the picture shown in Fig. 24). The angle the *wave front* makes with the axis is then clearly $\alpha = 90^\circ - \theta$, and the angle it makes with a perpendicular, CD, to the guide walls, is θ, as is clear from the geometry of the figure. Let the width of the guide, CD, be denoted by a.

The half wave length in the guide, $\lambda_g/2$, is the axial distance between a successive crest and trough, such as EF. On the other hand, the free space half wave length $\lambda/2$ is the distance measured along the oblique direction between a crest and trough, such as AC, since the velocity of the component oblique ray is assumed to be that in free space. Therefore, from trigonometry,

$$\lambda/2 = CA = CD \sin\theta = a \sin\theta \qquad (3)$$

whereas $$\lambda_g/4 = EB = BD \tan\theta = (a/2) \tan\theta \qquad (4)$$

The value for a in Eq. (4) can be found from Eq. (3). Thus

$$\lambda_g/4 = \frac{\lambda \tan\theta}{4 \sin\theta} = \frac{\lambda}{4 \sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{\lambda}{4 \cos\theta} \qquad (5)$$

from which

$$\lambda_g = \frac{\lambda}{\cos\theta}$$

or $$\cos\theta = \lambda/\lambda_g \qquad (6)$$

Eq. (6) defines the oblique angle at which the component wave must move when it has a wave length λ, in order to produce zero electric field intensity at the walls of the guide (which would otherwise be parallel to the electric field), and to have a resultant wave length λ_g, in the guide.

But

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

and from Eq. (3)

$$\sin\theta = \frac{\lambda}{2a}$$

so that

$$\cos\theta = \sqrt{1 - (\lambda/2a)^2} \qquad (7)$$

From this and Eq. (6) there thus results

$$\lambda/\lambda_g = \sqrt{1 - (\lambda/2a)^2}$$

or,

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/2a)^2}} \tag{8}$$

Eq. (8) gives the wave length λ_g in the guide in terms of the free-space wave length λ and the guide dimension a. This formula enables one to determine the length of a guide to act as a half wave resonator, as well as for many other applications, and is therefore very useful. The free-space wave length (for air or vacuum) is the familiar formula

$$\lambda = c/f \tag{9}$$

where c is the velocity of light = 3×10^{10} cm, and f is the operating frequency in c.p.s. Eq. (8) applies only to a $TE_{1,0}$ wave in a rectangular guide. For certain other modes or shapes the values will be given farther on.

In passing, it is to be noted that if the operating frequency is lowered, and λ thereby increased until it equals 2a, then from Eq. (8), λ_g comes out to be *infinite*, and from Eq. (7), $\cos\theta = 0$ or $\theta = 90^o$. This defines the cutoff frequency, and checks the value found previously from transmission line considerations, namely, $\lambda = 2a$. At this frequency the oblique waves move *perpendicularly* to the guide axis, and thus do not progress down the tube, but merely bounce back and forth between the guide walls. There is thus no transmission, and that is why the corresponding frequency is known as the cutoff frequency.

The reason that the guide wave length λ_g is infinite is that the oblique wave is assumed to have infinite width (infinite wave front), so that as the cutoff frequency is approached, the wave direction veers around towards perpendicularity to the guide axis and walls, and the wave front towards parallelism with the axis and walls. From Fig. 26, for example, it is evident that as this occurs, the intercept of a wave front, such as ED, with the guide wall moves indefinitely to the right as the wave front pivots on E.

The actual or group velocity of the wave can be found from Fig. 27. The oblique wave velocity is that of light, or c. The velocity *along the guide axis* is clearly

$$v_a = c \cos \theta \tag{10}$$

and the greater θ becomes as cutoff is approached, the smaller is cos θ and hence v_a. At cutoff $v_a = 0$; there is no transmission down the guide.

The apparent or phase velocity is

$$v_g = \lambda_g f = \frac{\lambda f}{\sqrt{1 - (\lambda/2a)^2}} = \frac{c}{\sqrt{1 - (\lambda/2a)^2}} \tag{11}$$

since $c = \lambda f$, and λ_g can be expressed in terms of λ by Eq. (8). Here, as λ approaches 2a, its cutoff value, v_g approaches infinity, as mentioned previously. The phase velocity means the speed with which an observer would have to travel in order always to stay with any selected phase of the wave. For example, if he chose for his phase the peak (crest) of the wave, then he would have to travel apparently with infinite velocity to stay with the peak all along the guide, at cutoff. The catch is that he would first have to wait an infinite time before such a particular steady-state condition would obtain. Hence the phase velocity in itself has no importance, but is useful in determining the wave length λ_g in the

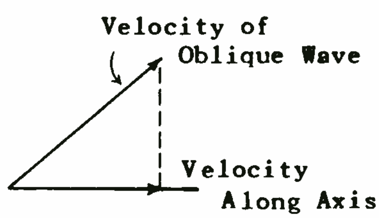


Fig. 27

guide.

The group velocity, v_a, on the other hand, has practical significance. It indicates the velocity with which a disturbance would travel down the guide. Such a disturbance may be, for example, an increase or decrease in the amplitude of the wave when it is amplitude-modulated. ***The change in the envelope of the wave*** will travel down the guide with the group velocity v_a, and at cutoff this velocity will be zero. Thus the transmission of intelligence depends upon the group velocity.

If the frequency is made very high, and λ therefore very small, Eqs. (10) and (11) both approach the common value c, since $(\lambda/2a)^2$ can be ig-

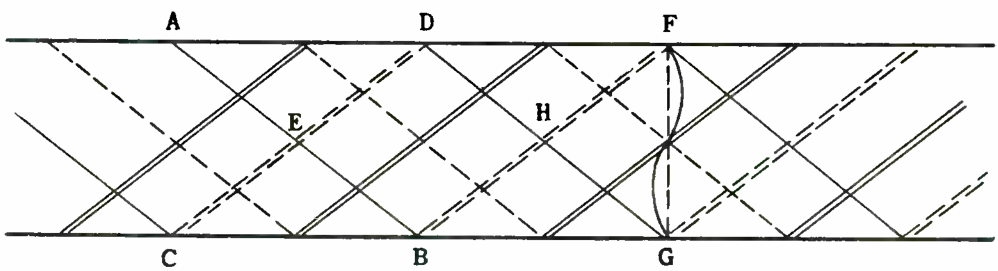


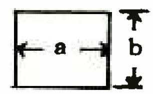
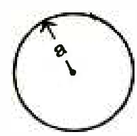
Fig. 28

nored compared to unity. This means that the guide is relatively so large that transmission in it approaches that of free space, where v_a and v_g are equal to C.

Production of Higher Order Modes.--The concept of oblique wave transmission can be employed to explain the production of higher order modes in a rectangular wave guide. In Fig. 28 is shown a diagram similar to Fig. 24, except that cancellation is considered between the crest of one wave and the *second* trough of the other wave. Thus crest AB, for example, meets its first trough CD at E, producing cancellation, and the second successive trough BF at B. Zero field is along lines ADF, EH, and CBG. If the guide walls were located at ADF and EH, a $TE_{1,0}$ wave would result, but if they are spaced twice as far apart, through ADF and CBG, then the electric field distribution will be a full sine wave--shown at FG--and this means that a $TE_{2,0}$ wave is being propagated. Similarly, higher modes can be produced.

Cutoff Frequencies.--The cutoff wave length for a $TE_{1,0}$ mode in a rectangular guide was very easily found to be twice the tube width, and from Fig. 28 it is clear that the cutoff wavelength for a $TE_{2,0}$ mode should be half that for a $TE_{1,0}$ mode, or equal to the tube width. The cutoff values for other modes and particularly for other shapes, such as circular guides, require more extensive mathematical manipulations; accordingly Table I has been prepared to furnish such information.

Table I. Cutoff Characteristics

Guide Shape	Mode	Cutoff Frequency f_o	Cutoff Wavelength λ_o	Remarks
Rectangular	$TE_{1,0}$	$c/2a$	$2a$	
Rectangular	$TM_{1,1}$	$\frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$	$\frac{2}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}$	
Circular	$TE_{0,1}$	$0.610\frac{c}{a}$	$1.64a$	
Circular	$TE_{1,1}$	$0.289\frac{c}{a}$	$3.46a$	
	$TM_{0,1}$	$0.383\frac{c}{a}$	$2.61a$	

The $TE_{1,0}$ mode in a rectangular guide is unique in that for a given operating wave length, only *the dimension perpendicular to the electric field* (call it the width) has to exceed a minimum value; the height can be as small as desired. (This does not take into account the increased loss in an actual guide when the height is decreased.) For all the other modes both dimensions must exceed certain minimum values, and these are larger than for a a $TE_{1,0}$ mode. Thus the pipe can be made of such size as to permit only the $TE_{1,0}$ mode to be transmitted, and to exclude other modes. This is usually very desirable, as the $TE_{1,0}$ mode has a minimum of losses and other desirable properties.

As an example, suppose it is desired to transmit this mode at a frequency of 4000 mc and to avoid any $TE_{2,0}$ pattern. What should be the guide dimensions?

First $\lambda = 3 \times 10^{10}/4000 \times 10^6 = 7.5$ cm. wavelength in free space. From Table I, a must be at least $\lambda/2$ or $7.5/2 = 3.75$ cm. Dimension b can have any value. For a $TE_{2,0}$ wave, it has been seen that a must be double that for a $TE_{1,0}$ mode, or 7.5 cm. in value. Thus, if a is made 5 cm., it will transmit the $TE_{1,0}$ mode and cutoff the $TE_{2,0}$ mode. It may seem unnecessary to be concerned about the presence of a $TE_{2,0}$ wave, since if the method of launching, using a single central antenna is employed (see Fig. 21(A)) then only a $TE_{1,0}$ wave should result. However, if the antenna is slightly off center or if there is some obstruction, some $TE_{2,0}$ waves may be produced, and this can be filtered out by the proper guide dimension.

If it were desired to launch a $TM_{1,1}$ wave in a rectangular guide at 4000 mc, both a and b would have to be determined, although neither is fixed by itself. Thus, choose b = 5 cm. Then, for cutoff, from Table I,

$$\lambda = 7.5 \text{ cm.} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} = \frac{2}{\sqrt{\frac{1}{a^2} + \frac{1}{25}}}$$

Squaring and transposing

$$\frac{1}{a^2} + \frac{1}{25} = \frac{4}{(7.5)^2}$$

from which

$$a = \sqrt{\frac{225}{7}} = 5.66 \text{ cm}$$

A larger value, say 6 cm, should preferably be used for a, as 5.66 cm is the cutoff value.

If it is desired to transmit a $TE_{1,1}$ wave in a circular guide at 4000 mc, then Table I shows that the minimum value of the radius a should be such that

$$7.5 = 3.46\ a$$

or

$$a = 7.5/3.46 = 2.17 \text{ cm}$$

Thus a might be in practice equal to 2.5 cm to exceed the cutoff value of 2.17 cm.

Attenuation in Wave Guides.-- The losses in a wave guide filled with air occur practically entirely in the walls as I^2R losses. They are surprisingly low, even if the walls are made of ordinary galvanized iron. The magnitude of the losses depends upon the mode, the shape of the pipe, and its dimensions in wave lengths. The attenuation depends in all cases upon the ratio of operating to cutoff frequency (f/f_o). A typical formula is that for the $TE_{1,0}$ mode in a square guide. The db attenuation per mile is

$$\alpha = \frac{K_1}{a^{3/2}} \; \frac{1/2\left(\frac{f}{f_o}\right)^{3/2} + \left(\frac{f}{f_o}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_o}\right)^2 - 1}} \text{ db per mile} \qquad (12)$$

where a is either side of the square guide, and K_1 is a constant depending upon the conductivity of the walls. If in Eq. (12), the right-hand factor involving (f/f_o) be plotted for various values of (f/f_o), a curve is obtained labelled $TE_{1,0}$ $\square$ in Fig. 29. This provides a factor F by which $K_1/a^{3/2}$ can be multiplied to give α, and saves considerable tedius calculations.

Table II gives the formulas for α in db per mile, per foot, and per meter, in terms of the appropriate factor F taken from Fig. 29, the dimension a, and the proper value of K_1, (given on the following page).

Table II. Attenuation Formulas.

Wave Mode	db/mile	db/foot	db/meter
	—Square Guide—		
$TE_{1,0}$	$\frac{K_1 F}{a^{3/2}}$	$\frac{.0001891\ K_1 F}{a^{3/2}}$	$\frac{.000621\ K_1 F}{a^{3/2}}$
$TM_{1,1}$	$\frac{1.189\ K_1 F}{a^{3/2}}$	$\frac{.000225\ K_1 F}{a^{3/2}}$	$\frac{.000738\ K_1 F}{a^{3/2}}$
	—Circular Guide—		
$TM_{0,1}$	$\frac{0.4378\ K_1 F}{a^{3/2}}$	$\frac{.0000828\ K_1 F}{a^{3/2}}$	$\frac{.000272\ K_1 F}{a^{3/2}}$
$TE_{0,1}$	$\frac{0.5520\ K_1 F}{a^{3/2}}$	$\frac{.0001042\ K_1 F}{a^{3/2}}$	$\frac{.000342\ K_1 F}{a^{3/2}}$
$TE_{1,1}$	$\frac{0.3824\ K_1 F}{a^{3/2}}$	$\frac{.0000723\ K_1 F}{a^{3/2}}$	$\frac{.000237\ K_1 F}{a^{3/2}}$

K_1 = 236.5 for copper; 313.6 for aluminum. Dimension a in centimeters.

An inspection of Fig. 29 indicates that the factors F for the various waves all have minimum values. The attenuation α shows similar minimum values, but at somewhat higher frequencies above cutoff than the curves for F. If the guide dimensions are chosen too close to the cutoff values excessive attenuation will result.

An interesting exception to the previous statement regarding a minimum value is that for the circular $TE_{0,1}$ wave. From Fig. 29 it may be noted that the F factor, hence α becomes progressively smaller as (f/f_0) is increased, i.e., as one goes up in frequency. This is surprising, as one usually expects increased losses at higher frequencies. Unfortunately, this wave mode has never been actually observed, and theoretical considerations indicate that the slightest eccentricity in the tube will change the attenuation curve into one having a minimum value and thereafter rising with frequency. A more practical wave mode is the circular $TE_{1,1}$ which has the lowest attenuation of any. A square $TE_{1,0}$ wave has the lowest attenuation of any in this shape guide. It is also to be noted that a rectangular guide has slightly less losses than a square guide but the values for a square guide can be used for it with good accuracy.

Suppose--as an example--a 4,000 mc $TE_{1,1}$ wave in a circular copper guide, of radius equal to 2.5 cm, is to be transmitted. It is desired to find the attenuation for 100 feet of the pipe. From Table I, the cutoff frequency f_0 for the circular $TE_{1,1}$ wave is

$$f_0 = (0.289 \times 3 \times 10^{10})/2.5 = 3468 \text{ mc}$$

Then (f/f_0) is 4000/3468 = 1.15. From Fig. 29, the corresponding factor F = 2.5 approximately. Then, from Table II

$$\alpha = \frac{.0000723 \times 236.5 \times 2.5}{(2.5)^{3/2}} = .0108 \text{ db per foot}$$

or, for 100 feet, the attenuation is 1.08 db.

This can be compared with a coaxial cable of the same outer conductor diameter, b. Let the ratio of the inner conductor a to the outer one be 3.6, so that a = 2.5 + 3.6 = .695 cm. Then, from the formula given previously on transmission line losses:

$$\alpha = \frac{1.3 \times 10^{-9}\sqrt{f}}{\log_{10} b/a}\left(\frac{1}{a} + \frac{1}{b}\right) \text{ db per meter} \tag{13}$$

where a and b are in meters. Substituting the above values in Eq.(13) one obtains

$$\alpha = \frac{1.3 \times 10^{-9}\sqrt{4 \times 10^{9}}}{\log_{10} 3.6}\left(\frac{1}{.00695}\right) + \left(\frac{1}{.025}\right) = .0273 \text{ db/meter}$$

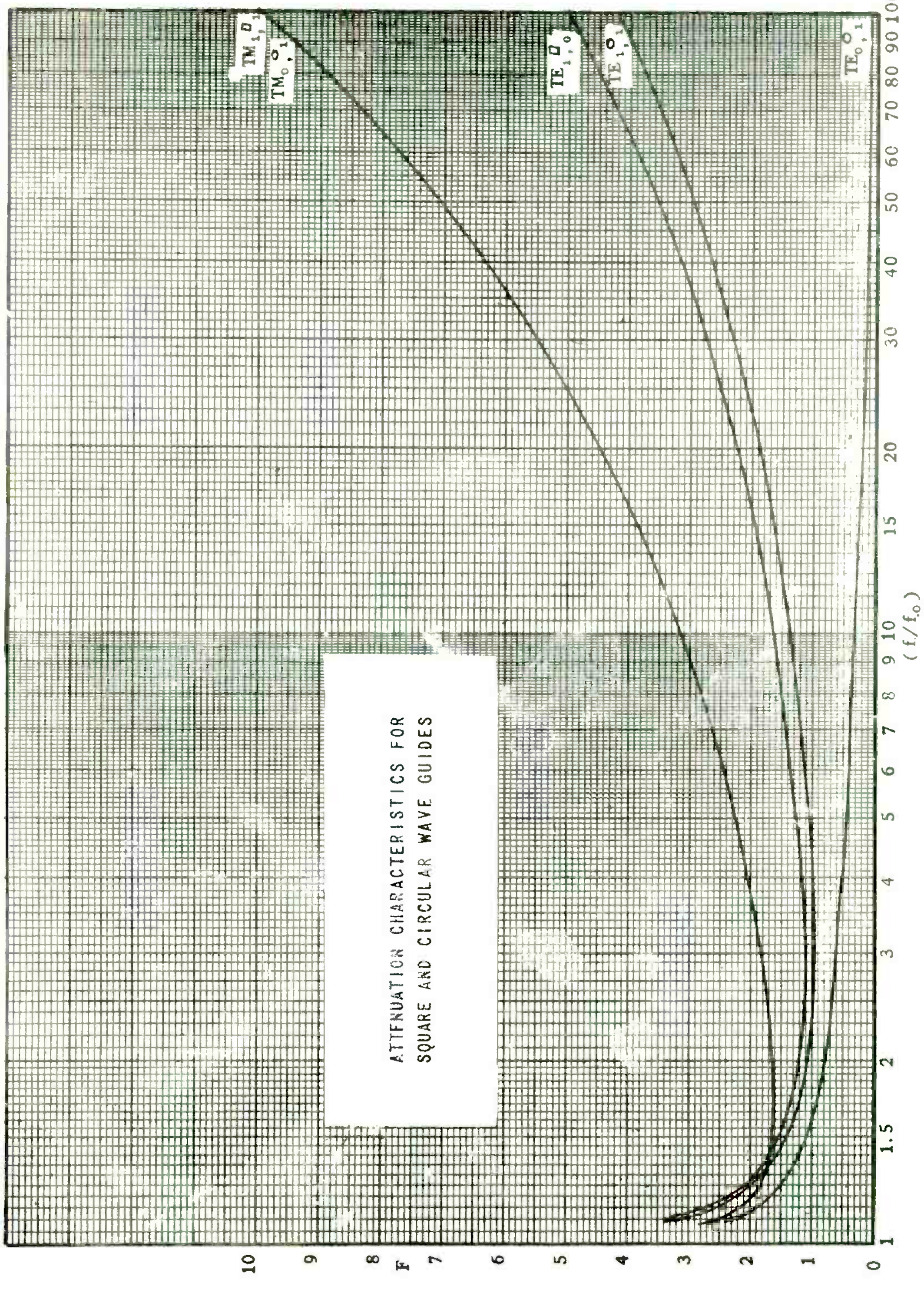

ATTENUATION CHARACTERISTICS FOR
SQUARE AND CIRCULAR WAVE GUIDES
F
(f/f₀)
TM□ 1,1
TM○ 0,1
TE□ 1,0
TE○ 1,1
TE○ 0,1

Fig. 29

$$= .0273 \div 3.28 = .0083 \text{ db/foot}$$

For 100 feet, the attenuation will be .0083 x 100 = 0.83 db. This is less than that for the corresponding wave guide in spite of the fact that the coaxial cable has an inner conductor in which there are I^2R losses.

The reason is that the wave guide is being operated too close to the cutoff value. Suppose the radius of the guide is increased until (f/f_o) equals 2 instead of 1.15. For f fixed at 4000 mc, this means f_o has been decreased, i.e., dimension a has been increased. Thus

$$f/f_o = 2 \text{ or } f_o = f/2 = 4000 \div 2 = 2000 \text{ mc}.$$

Then, from Table I,

$$a = \frac{.289 \times 3 \times 10^{10}}{2000 \times 10^6} = 4.34 \text{ cm}$$

instead of 2.5 cm. The factor for $(f/f_o) = 2$ is F = 1.1. Then

$$\alpha = \frac{.0000723 \times 236.5 \times 1.1}{(4.34)^{3/2}} = \frac{.0188}{9.05} = .00208 \text{ db/foot}$$

or .208 db for 100 feet. This is about one fifth of its previous value for somewhat less than twice the size of pipe. The corresponding increased size of coaxial cable would have a decreased attenuation too, but this varies *inversely* as the dimensions, as is indicated by Eq.(13). The attenuation of a coaxial cable whose outer conductor radius is 4.34 cm would therefore be

$$.83 \text{ db} \times \frac{2.5}{4.34} = .478 \text{ db}$$

or more than twice that of the circular guide.

This indicates that at 4,000 mc there is not very much to choose between a coaxial cable and a wave guide unless the latter is made rather large compared to its cutoff value. In the case of a coaxial cable, it has been mentioned that transmission is in the TEM mode, which is not possible in a wave guide because at least one field must have an axial (longitudinal) component. However, if the coaxial cable is made as large as the wave guide and hence comparable in radius to the operating wave length, it may operate in higher modes than the TEM type, and thus perform as a special kind of wave guide. In such a case one might as well omit the inner conductor, and convert the cable into a guide.

If coaxial cable operation is desired (TEM mode) then the cable must be made small in radius compared to the wave length. At low frequencies this is easy to attain, and permits a small cross section cable to transmit about as well as a huge wave guide (huge, so as to operate above its cutoff frequency). At high frequencies, such as 10,000 mc, on the other hand, the coaxial cable would have to be so small as to be impractical. Moreover, the I^2R losses--particularly on the inner conductor--and the ionization losses around the latter, when large amounts of power are to be transmitted, would be excessive. If the cable's cross section is increased to obviate this, it tends to operate as a wave guide, and may therefore just as well be made in that simpler form, since the losses can thereby be decreased.

Uses of Coaxial Cables.--The coaxial cable has its uses even at very high frequencies. As a short connecting link between two pieces of apparatus, it is often simpler to couple than a wave guide, and can also be bent around corners more readily because the fields are better guided between two conductors than by one alone. In short lengths, moreover, its losses are not excessive. Another use as a wavemeter will be described later.

Wave Filters.--Often it is desired to transmit one mode in a wave guide, such as the $TE_{1,0}$ in a rectangular guide. Unfortunately, asymmetry in coupling, or unavoidable obstructions in the guide may cause other modes to appear. The size of the guide may be such that it will not cut off these modes. In such a case certain filter structures, when inserted in the guide, will eliminate the unwanted modes without prejudice to the desired mode.

For example, at 4,000 mc a rectangular guide of square cross section, 5.5 cm on a side, will have a cutoff wave length λ_0 (from Table I), of $2 \times 5.5 = 11$ cm for the $TE_{1,0}$ mode, and $5.5 \times 1.414 = 7.78$ cm for the $TM_{1,1}$ mode. The operating wave length λ is $(3 \times 10^{10}) \div 4000 \times 10^6 = 7.5$ cm, or *shorter* than either of the above cutoff wave lengths, so that 4,000 mc is *above* the cutoff frequency for either of the above modes. Consequently either of these modes will be transmitted if generated anywhere along the guide. Assume that it is desired to transmit the $TE_{1,0}$ mode and to reject the $TM_{1,1}$ mode.

If a grating of wires be interposed in the cross section such that the wires conform to the electric field configuration for any particular mode, that mode will be partly absorbed and partly reflected back to the source and the wave passing through the grating will be rid of that mode. (Some energy will be radiated forward.) A suggested form for the grating to accomplish the stated objective is that shown in Fig. 30. The wires have the same configuration as the electric field lines of Fig. 20(A).

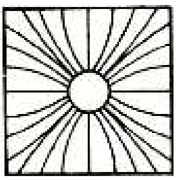


Fig. 30

A further improvement is to locate another grating one-quarter wave length after the first. The second then acts as a parasitic antenna excited by the first 90° out of phase. As shown in a lesson on antennas, the radiation from the second is 180° out of phase with that of the first in the forward direction, and in phase in the backward direction. Thus, any energy in the $TM_{1,1}$ mode radiated in a forward direction by the first grating will be neutralized by the second, and the wave finally transmitted will be practically a pure $TE_{1,0}$ type.

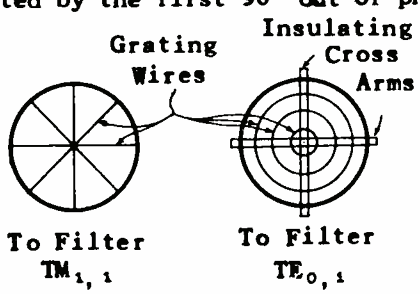


Fig. 31

Examination of Fig. 30, however, shows that the wires have vertical components to their shape. These will tend to absorb and reflect some of the $TE_{1,0}$ wave, which has a transverse vertical field. The wire gratings are perhaps best suited to filter a mode whose electric field lines are exactly perpendicular to those of the desired mode. An example of this is the circular $TE_{0,1}$ and $TM_{1,1}$ shown in Fig. 31.

Another type of filter is one consisting of sheets of metal running *longitudinally* along the guide. A simple example is shown in Fig. 32. Here a horizontal sheet H extends a short distance down the guide. Any *horizontal* electric lines are shorted out by the sheet; vertical lines are normal to its surface and hence unaffected by its presence. More important, however, is the fact that the $TM_{1,1}$ mode has a longitudinal electric field component, whereas the $TE_{1,0}$ mode has not. Hence the longitudinal dimension of H will serve to short out the longitudinal component of the $TM_{1,1}$ mode, and not affect the $TE_{1,0}$ mode. Finally, a stack of plates similar to H, and one above the other, can be employed to enhance the filtering effect.

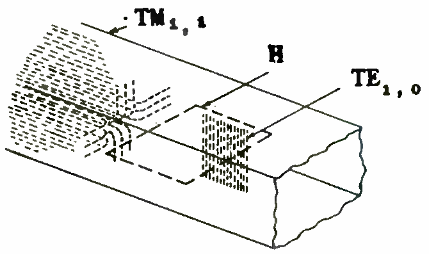


Fig. 32

Wave Converters.--Sometimes it is desired to convert one mode into another. This can be accomplished by a composite form of grating. Thus a $TE_{1,1}$ wave can be converted to a $TE_{0,1}$ wave by means of a grating partaking of the characteristics of both types of waves. This is shown in Fig. 33. One can see there the circular outline for a $TE_{0,1}$ wave and the curved vertical lines corresponding to the $TE_{1,1}$ wave. Suppose a $TE_{1,1}$ wave is initially launched. The electric lines of this wave align with the vertical portions of the grating upon reaching it, and cause short circuit currents to flow as indicated by the arrows. These currents have a circular component and are therefore capable of launching the $TE_{0,1}$ mode. Subsequent grating filters can then eliminate any remnants of the first mode. The same converter can work from either mode to the other.

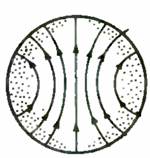


Fig. 33

One reason for using such a converter is that sometimes it is more convenient to launch one type of mode, and farther on in the system a different mode may be more desirable. For example, the $TE_{0,1}$ and the $TM_{1,1}$ modes have circular symmetry, and are therefore desirable where one part of the equipment rotates with respect to another part. In these modes, the fields are not twisted out of shape during rotation.

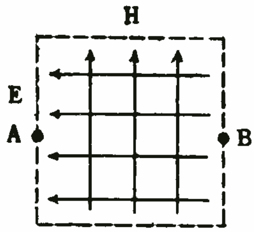


Fig. 34

Concept of Impedance.—Earlier in this assignment it was indicated how the electric field E multiplied by distance represents voltage along the path, and the magnetic field H multiplied by distance along its path represents the current enclosed by the path. By means of these two concepts, a ratio defined as the impedance of the guide can be developed.

Thus, normally, impedance is defined as the ratio of voltage to current: $Z = V/I$. Suppose in free space there is a uniform electromagnetic field being radiated, as shown in Fig. 34.

For a given area, say one meter square, the voltage V across the area is in proportion to E, and the current I to H. Hence, instead of taking the ratio of V to I, one can take the ratio of E to H. Thus, the impedance of space can be expressed as E/H. However, E is produced by time variation of H, and H by the time variation in E, as discussed in the lesson on radiation. The time variation of either field is their velocity through space with the speed of light, c. This can be expressed in terms of the dielectric constant ε and the permeability μ of the medium; thus

$$c = \sqrt{\frac{1}{\varepsilon\mu}}$$

From this there finally comes about that the impedance of a medium is

$$Z = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} \tag{14}$$

For a vacuum (also practically for air), $\mu = 4\pi \times 10^{-7}$ henries per meter, and $\varepsilon = 8.85 \times 10^{-12}$ farads per meter, so that $\sqrt{\mu/\varepsilon} = 376.6$ or 377 ohms. This is the impedance of free space per square meter.

In a wave guide similar impedances may be written. But here the fields are not uniform, and depend upon the shape and size of the guide and the wave mode. Consequently, the impedance of a guide of a certain shape differs from that of another shape of guide, and for any particular shape, the impedance depends upon the mode, and its dimensions, as measured in wave lengths.

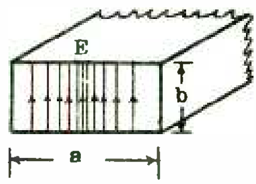


Fig. 35

For example, the impedance of an air-filled rectangular guide operating in the $TE_{1,0}$ mode is (see Fig. 35)

$$Z_{1,0} = 592 \frac{1}{\sqrt{1 - (\lambda/\lambda_0)^2}} \cdot \frac{b}{a} \tag{15}$$

The quantity λ_0 is the cutoff wave length, and is equal to 2a, so that this may be substituted for λ_0 in Eq. (15), if desired. Note that $Z_{1,0}$ increases directly with b, and inversely with a. This is to be expected, since the larger b is, the greater is the product Eb, or the voltage, and the greater a is, the greater is the *longitudinal* current. Since $Z = V/I$, it will vary directly with V, hence with b, and inversely with I, hence with a. The factor 592 includes the quantity 377, and a quantity $\pi/2$ that represents the form factor of the sinusoidal distribution of E across the guide, and hence of the *longitudinal* current flow in the guide walls.

There are two other definitions of impedance based on the ratio of power to $(\text{current})^2$, and $(\text{voltage})^2$ to power. For ordinary circuits these are all equal, but for wave guides they are not. However, the definition given by Eq.(15) will be found useful in determining the size of a diaphragm inserted *in the* guide, etc., and so will be used here.

For a $TM_{1,1}$ wave in the same guide, the impedance is

$$Z_{1,1} = \frac{94.3\ ab}{a^2 + b^2} \sqrt{1 - \left(\frac{\lambda}{\lambda_0}\right)^2} \tag{16}$$

For a $TE_{1,1}$ wave in a circular guide the impedance is given by

$$Z_{1,1} = \frac{520}{\sqrt{1 - \left(\frac{\lambda}{\lambda_0}\right)^2}} \tag{17}$$

and for a $TM_{0,1}$ wave it is

$$Z_{0,1} = 48\sqrt{1 - \left(\frac{\lambda}{\lambda_0}\right)^2} \tag{18}$$

Matching of Guides.--Since wave guides are a kind of transmission line involving but one conductor, it would be very desirable to calculate their performance in the same relatively simple manner as for ordinary lines. For example, two ordinary two-conductor lines can be connected together on the basis of matched impedances, and reflections thereby avoided.

This is not so easy to do for two wave guides. Each mode in a guide has a different impedance--as indicated above--and conditions that are right for one mode may be incorrect for another. Hence it is not possible in general to match two guides, particularly if at the point of matching some discontinuity exists that produces other modes. On the other hand, if the guides are designed to suppress other modes, impedance matching is possible.

Suppose it is desired to match two rectangular guides to one another, for a $TE_{1,0}$ wave mode propagated in them. Eq.(15) indicates that b and a can be varied and yet produce the same value of $Z_{1,0}$ Eq.(15) can be transformed into the following equation

$$\frac{\sqrt{1-\gamma^2}}{\gamma} = \frac{1184}{Z_{1,0}} \cdot \frac{b}{\lambda} \tag{19}$$

where $\gamma = \lambda/2a$, and λ is the operating wave length. Note that λ must be less than 2a in order that operation occur above cutoff for the guide. Hence γ is a fraction, and can therefore represent the cosine of some angle, call it α.* Then $\sqrt{1-\gamma^2}$ is evidently $\sin\alpha$, and $\sqrt{1-\gamma^2}/\gamma = \tan\alpha$. Hence if $\tan\alpha$ be plotted against $\cos\alpha$, for various values of α chosen arbitrarily, a curve will be obtained, as shown in Fig. 36. (Curve B is a continuation of curve A for small values of γ.) From this curve, values for the height b for the second guide can be found if its width a is given, such that the second guide matches the first one.

A numerical example will show its use. Suppose the operating frequency is 4,000 mc, for which λ = 7.5 cm, and the width of the first guide is a_1 = 5.5 cm, and the height is b_1 = 4 cm. A $TE_{1,0}$ mode is assumed. The value of γ for the first guide is

$$\gamma_1 = \frac{\lambda}{2a_1} = \frac{7.5}{2 \times 5.5} = .682$$

Then, from curve A of Fig. 36, the ordinate representing $\sqrt{1-\gamma^2}/\gamma$, for the above value of γ_1, is 1.066. Call this factor F_1.

Here b_1 and λ are known, so that $Z_{1,0}$, the impedance of the first guide, can be found. Thus b_1/λ = 4/7.5 = .533, so that

$$Z_{1,0} = \frac{1184\ b_1}{F_1\ \lambda} \tag{20}$$

or

$$Z_{1,0} = \frac{1184 \times .533}{1.066} = 593 \text{ ohms}$$

This illustrates the first use of these curves: that of finding the impedance of the guide if the dimensions and operating wave length are given.

Now suppose it is desired to match the guide to a second one whose width is a_2 = 7 cm. This second guide, to prevent reflections of the wave

* Reference to Eq. (3) and to Fig. 26 shows that γ is the sine of θ, and hence is the cosine of α, where α is the angle that the oblique wave makes with the axis. Thus γ has physical significance.

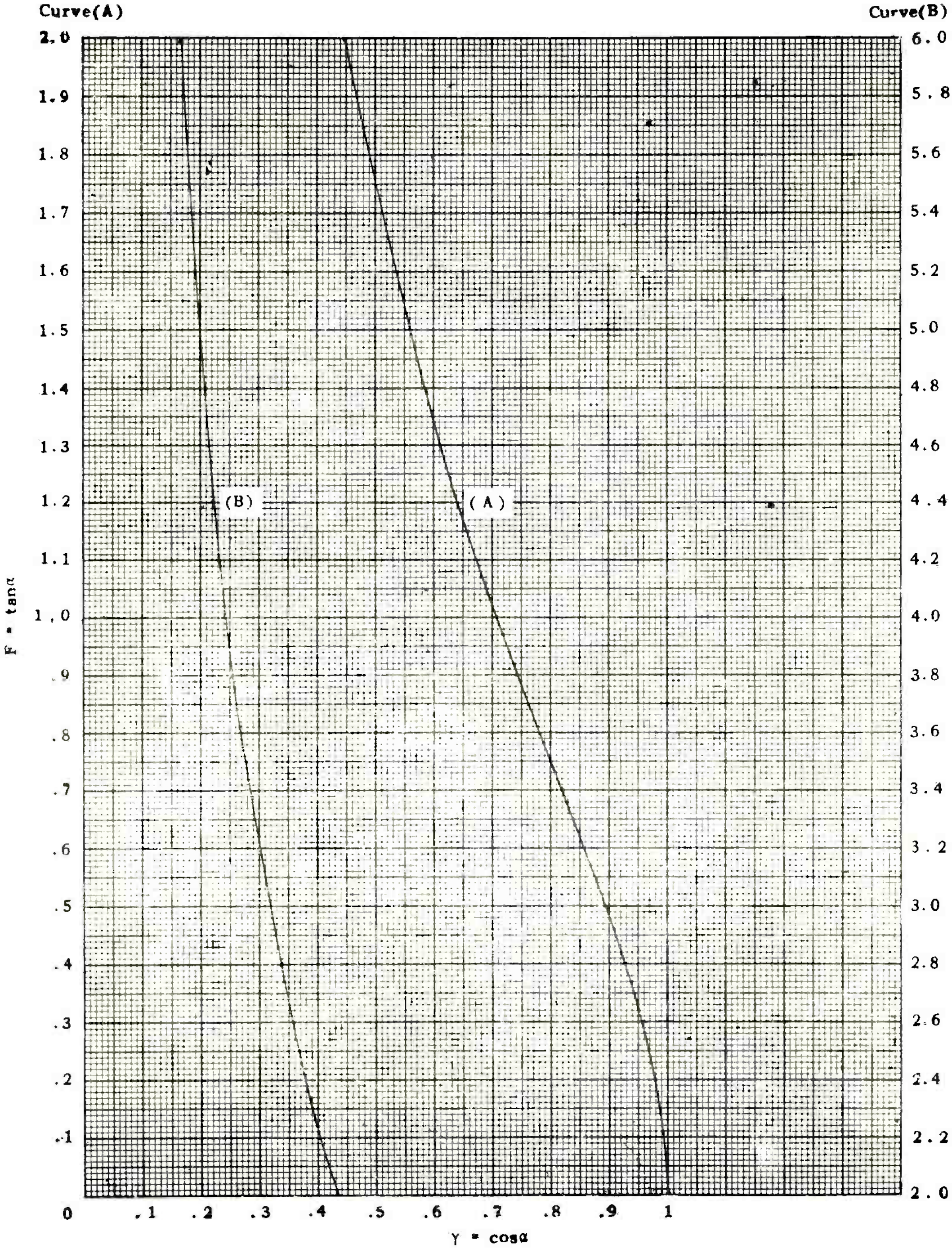

Curve(A)
Curve(B)
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
6.0
5.8
5.6
5.4
5.2
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
(B)
(A)
F = tanα
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
Y = cosα

Fig. 36

at its junction with the first, must have the **same impedance of 593 ohms.** Thus its height b_2 must be determined.

First find

$$\gamma_2 = \frac{\lambda}{2a_2} = \frac{7.5}{14} = .536$$

The corresponding ordinate is found from curve A, too, and is $1.57 = F_2$. Then

$$b_2 = \frac{F_2 \cdot \lambda \cdot Z_{1,0}}{1184} \qquad (21)$$

or

$$b_2 = \frac{1.57 \times 7.5 \times 593}{1184} = 5.94 \text{ cm}$$

The two wave guides are shown in Fig. 37, and also the direction of the electric field.

Apertures.--One can regard the larger guide in Fig. 37 as capped at its left-hand end, and an aperture cut in this end cap of a size equal to the smaller guide. More generally, an aperture or diaphragm can be inserted in a guide to produce certain desired effects.

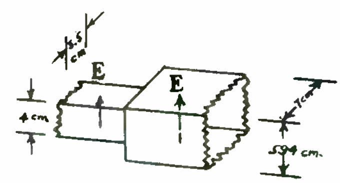


Fig. 37

Such a device is illustrated in Fig. 38. The diaphragm may be considered as a very short wave guide interposed in series between the two portions of the actual wave guide to the right and left of it. If its dimensions conform to Eq.(15), so that its impedance is the same as that of the actual guide, namely $Z_{1,0}$, then it has no effect on the wave transmission (assuming that the concentration of the power flow through it does not produce excessive I^2R losses and ionization within it). Thus, if matched to the guide, it may be regarded as an *antiresonant* circuit of very high impedance shunting the guide at that point.

On the other hand, it may purposely be designed so as not to have the relation between a and b satisfy Eq.(15). For example, it may be as wide as the guide, but narrower in height, Fig. 39(A). In this case the shunt capacity of the guide is increased at this point, as if the top and bottom walls were locally brought closer together. This produces the effect as if a small condenser were inserted at this point in parallel with the guide, and connected between the top and bottom walls.

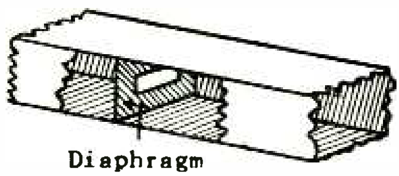


Fig. 38

On the other hand, if the constriction is in the width, as shown at (B), then the side wall currents(see arrows) are enhanced, or the diaphragm acts like a shunt inductance connected across the guide. The opening can be made adjustable, so that the magnitude of this reactance (either inductive or capacitive) can be varied. In the case of a circular guide, an iris diaphragm similar to that used on a camera lens is often employed.

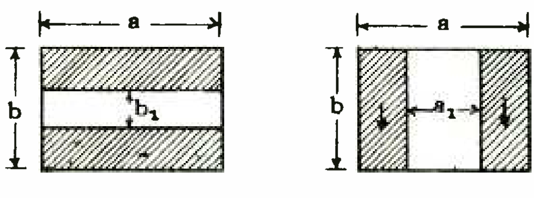


Fig. 39

The value of the reactance is given by the following equation:

$$X = \frac{Z_1 \lambda}{2\pi L\left[(Z_1/Z_0)^2 - 1\right]} \tag{22}$$

Here L is the length of the diaphragm when regarded as a very short section of a wave guide, or in other words, the *thickness* of the metal used for the diaphragm, Z_0 represents the impedance of the wave guide, and Z_1 that of the diaphragm. Both Z_0 and Z_1 can be found with the aid of Fig. 36 in the manner described previously. Also note that λ is the *operating* wave length.

If Z_1 is greater than Z_0, X will be positive, or inductive, if Z_1 is less than Z_0, X is negative or capacitive. Reference to Eq.(15) shows that if b is kept constant, but a decreased to a value a_1--as in Fig. 39(B)--then $Z_{1,0}$ increases from the guide value of Z_0 to a higher value of Z_1 for the aperture. Then $(Z_1/Z_0)^2$ is greater than one, the denominator of Eq.(22) is positive, as is inherently the numerator, so that X is positive or inductive. Thus the formulas check with the physical reasoning previously given that Fig. 39(B) will represent an inductance shunted across the guide. A similar line of reasoning shows that Fig. 39(A) will be a capacitive reactance.

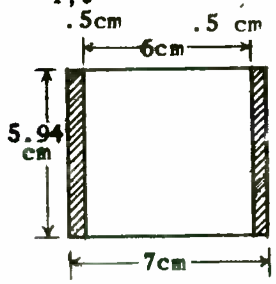


Fig. 40

As a numerical example, consider the wave guide previously discussed, of 7 cm width and 5.94 cm height, into which two vertical partitions are inserted, constricting its width to 6 cm as shown in Fig. 40. The thickness is 1 mm, and f = 4,000 mc. What is the reactance of this aperture?

The impedance Z_0 of the guide has previously been found to be 593 ohms, and $\lambda = 7.5$ cm. Then for the aperture, $Y = 7.5/(2 \times 6) = .625$. The corresponding factor--from Fig. 36--is F = 1.24. Furthermore $b/\lambda = 5.94/7.5 = .792$. Then, from Eq.(20)

$$Z_1 = \frac{1184 \times .792}{1.24} = 757 \text{ ohms}$$

Now that Z_1 and Z_0 are both known, X can be found from Eq.(22).

$$X = \frac{757 \times 7.5}{2\pi(.1)[(757/593)^2 - 1]} = 14,570 \text{ ohms inductive}$$

The equivalent inductance is

$$14,570 \div 2\pi \times 4,000 \times 10^6 = 580 \times 10^{-9} \text{ henry} = .58\mu \text{ henry}$$

Eq.(22) fails when the aperture is made so narrow that twice its width (perpendicular to the electric field of the $TE_{1,0}$ mode) is less than λ. (In the problem 2 x 6 cm is greater than λ = 7.5 cm.) This is because the analysis is based on the viewpoint that the aperture is a very short wave guide *operated above its cutoff frequency.* However, this does not mean that an aperture less than λ/2 fails to transmit energy. If its *thickness* is small compared to λ, as is practically always the case, then it will not cutoff, but instead will transmit practically all the energy to the other side of the guide, and at the same time act as a kind of reactance. its behavior under such conditions is difficult to evaluate; Eq.(22) holds only for apertures whose width exceeds λ/2.

A further refinement is to insert screws through threaded holes in the walls. If these are perpendicular to the electric field in their vicinity, then they act as a high shunt inductance; if parallel, then as a small shunt capacity. Fine adjustments can be had by screwing them in or out, as required. They can be used by themselves, or in conjunction with a fixed aperture to act as a vernier adjustment.

Taper Wave Guides.--It has been shown how to choose the dimensions of two wave guides so that they have the same characteristic impedance and can be joined together on a matched impedance basis. In the numerical problem, the dimensions of the one guide were 5.5 cm wide by 4 cm high, and the width of the second guide was chosen as 7 cm, whereupon it was found that its height must be 5.94 cm.

Suppose, however, that the dimensions of both guides are arbitrarily fixed, and that as a result the impedances of the two guides are unequal. In such a case they cannot be joined in the manner previously described, i.e., by fitting one into the other on a diaphragm basis. Instead, it has been found that a guide section that gently tapers from the smaller to the larger one can be used, and moreover, the two unlike impedances will be matched over *a wide frequency range,* unlike the diaphragm method, which matches two impedances at essentially one frequency.

In Fig. 41(A) are shown, for simplicity, two rectangular guides G_1 and G_2, having the same height, but different widths. The required taper guide

G_T will flare only in the horizontal direction. In (B) is shown a top view of the assembly. The dotted portion representing the apex of the

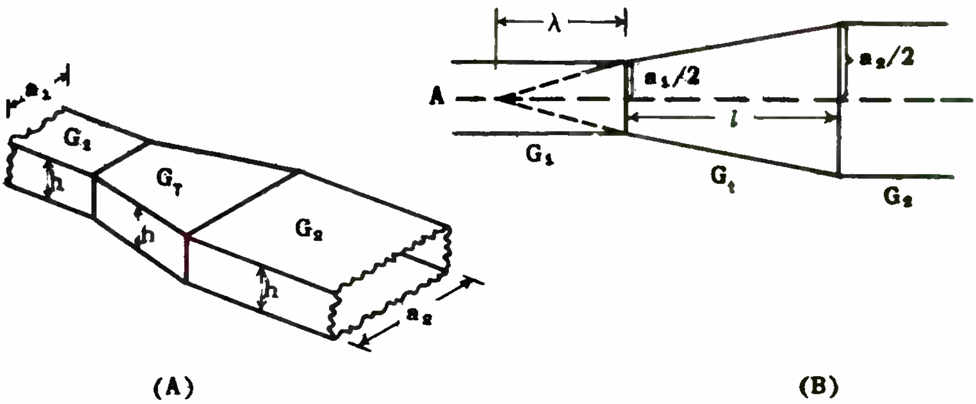


Fig. 41

taper guide G_t represents a region where the fields are considerably compressed, so that this portion would not be suitable for matching. The taper guide proper should start about a wave length from the apex or tip A. Given the two widths of the guides G_1 and G_2, the length l of a taper section that meets the above requirement is easily determined from the geometry of the figure--thus, by similar triangles

$$\frac{a_1/2}{\lambda} = \frac{a_2/2 - a_1/2}{l}$$

from which

$$l = \frac{a_2 - a_1}{a_1}\lambda \qquad (23)$$

If the heights are different, too, then the taper guide must flare in this direction, too. A formula exactly similar to Eq.(23) can be used to evaluate the length l required for two different heights h_1 and h_2. The dimensions that require the longest l then determine the *actual* length of the taper guide. In the case of a circular guide, substitute the two radii for a_2 and a_1 in Eq.(23). Finally, if space limitations demand a shorter value of l, fairly good results can be obtained even if l is cut down to as little as one-half of the value given by Eq.(23).

In passing, it is well to note that in an art as young and fluid as U.H.F. techniques, there are often a large number of methods of accomplishing the same result, such as impedance matching of two wave guides. What has been presented in this assignment are some of the simpler and probably more satisfactory methods. Even for these the mathematical analyses upon which their formulas are based, are exceedingly complex, and many methods that have been experimentally devised, have so far defied or at least discouraged a rigorous mathematical treatment. Hence, it must be remembered in employing the foregoing formulas that--as in the case of transmission lines--they represent working approximations, and that mechanical means should be provided to afford additional adjustments so that more optimum conditions can then be obtained experimentally. The formulas thus serve as a guide and help limit the amount of experimental adjustment that is subsequently necessary.

It will be noted that emphasis has been placed mainly on the $TE_{1,0}$ mode in a rectangular guide. Rectangular guides appear to be preferred to circular guides at present, particularly for any appreciable distance of transmission, because they are able to hold the electric field vectors more accurately fixed in space. Difficulty is encountered in the use of a circular guide owing to "tortuosity". The circular guide is symmetrical in cross section; hence any deformations in its shape at various points along its length--such as due to bending around a corner--tend to twist the fields. As a result, a vertical electric field at the transmitting end may come out horizontal or, at any rate, at an angle to the vertical at the receiving end, so that pickup loops or antennas will be incorrectly oriented on the assumption that the field is still vertical at this end. Such difficulties are clearly obviated by the use of a rectangular guide.

CAVITY RESONATORS

It has been shown that the single-conductor or wave guide behaves very much like the two-conductor or ordinary transmission line, particularly if each mode of the guide is considered separately. It can therefore be expected that a guide can be operated in a resonant condition very much like a short-circuited or open-circuited line, and with much the same properties This is found to be the case--with the proper qualifications.

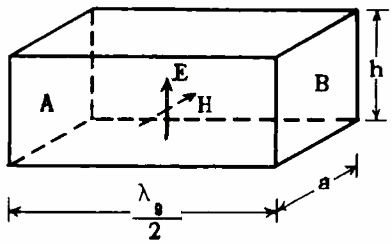


Fig. 42

A guide can be short-circuited by closing its end with a plug or piston of highly conducting material, such as copper. This piston shorts out the electric field lines in the same manner as was discussed in the previous lesson for a transmission line--specifically the coaxial cable.

It is more difficult to open-circuit a guide. This is because it is of a size comparable to the wave length of the electromagnetic energy, and thus forms a rather efficient radiator. This means that its open end acts as if it were terminated by a finite resistance--that owing to radiation of energy--rather than by an infinite resistance or open circuit. Hence, as will be shown, half-wave resonators are preferable to quarter-wave resonators.

Half-Wave Resonator.--A half-wave rectangular resonator is shown in Fig. 42. This is for the $TE_{1,0}$ mode. The direction of propagation is from left-to-right and right-to-left. The wave, upon striking either end wall, A or B, is reflected, and if the length is just right, standing waves will be produced. The proper length is $\lambda_g/2$, *where* λ_g *is the wave length in the guide, and exceeds the free-space wave length* λ, as shown previously. The The value for λ_g was given previously by Eq.(8)

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/2a)^2}} \tag{8}$$

This can be re-written, by multiplying and dividing the numerator of the right-hand side by 2a, as

$$\lambda_g = 2a \frac{(\lambda/2a)}{\sqrt{1 - (\lambda/2a)^2}} \tag{24}$$

Again it may be noted that (λ/2a) is a fraction and can therefore represent the cosine of some angle α , whereupon Eq.(24) can be written as

$$\lambda_g = 2a \frac{\cos \alpha}{\sin \alpha} = \frac{2a}{\tan \alpha}$$

This in turn means that Fig. 36 can be used to find λ_g when λ and a are given: The factor F = tan α then is used to determine λ_g as follows:

$$\lambda_g = \frac{2a}{F}$$

or Length of Resonator = $\lambda_g/2 = a/F$ (25)

Thus, suppose the operating frequency is 4000 mc, and the width of the resonator is 6 cm = a. Since the operating wave length λ is 7.5 cm, which is less than 2a = 12 cm, the resonator will be able to support the $TE_{1,0}$ mode. To find the length of the resonator, first find λ/2a = 7.5/12 = .625. From Fig. 36, the factor is F = 1.24. Then from Eq.(25), the length of the cavity resonator will be

$$\lambda_g/2 = \frac{6}{1.24} = 4.84 \text{ cm}$$

(Note that this is greater than the operating free-space half-wave length λ/2, which is 7.5 ÷ 2 = 3.75 cm.)

Some General Properties of the Cavity Resonator.--No means of establishing the wave in the cavity has been indicated in Fig. 42. Such coupling can be established by means of a small antenna, or coupling loop, or aperture. These will be discussed later.

Assume, however, that a standing wave pattern is established in the resonator, of the $TE_{1,0}$ mode. Note that as a is varied (but never made less than $\lambda/2$), λ_g varies, and hence the length of the cavity. Thus the cavity length is prescribed by its width, but the height can be any value desired, as far as the $TE_{1,0}$ mode is concerned. For other modes, and other shapes, however, all three dimensions are involved.

The half-wave resonator may be regarded as two quarter-wave resonators in series. Each quarter-wave resonator may be regarded as short-circuited at one end and open-circuited at the other end. As such its impedance looking into the open end may be regarded as infinite, similar to the action of an ordinary quarter-wave transmission line shorted at the far end.

Conversely, an open-circuit at one end of a quarter-wave tube looks like a short-circuit at its other end. Thus, the two halves of a half-wave cavity resonator present an open-circuit to one another at the center, where they are joined, when their far ends are short-circuited. Thus the effect of a quarter-wave resonator can be achieved by coupling into the center of a half-wave resonator, which, *being sealed at both ends*, has no radiation losses. The same is true for a transmission line, although the radiation losses from the open end of a quarter-wave line become excessive only at the higher frequencies. Also, as in the case of a transmission line, any integer multiple of $\lambda_g/2$, such as $3(\lambda_g/2)$, will function similarly to a half-wave resonator.

Shunt Resistance of a Cavity Resonator.--In Fig. 43 is shown the electric field distribution for a $TE_{1,0}$ mode in a rectangular cavity. Note that it is a half-sine wave along the length and along the width. The electric field, as expected, is zero at the side and end walls, since it is vertical and hence parallel to these walls.

Maximum field occurs at the center, hence the greatest voltage exists between the top and bottom walls at this point, such as between A and B of Fig. 43.

If the cavity has no losses, either in the dielectric (usually air) or in the walls, then the impedance measured between points A and B will be infinite, as may be expected from the previous remarks. If the cavity walls, for example, have losses, then the impedance seen looking into A and B will be finite, although usually very high. It will be recognized immediately that the behavior is similar to that of a quarter-wave transmission line shorted at its far end. The impedance looking into its near (open) end is infinite if it has no losses, and in the order of megohms if it has the usual losses of copper.

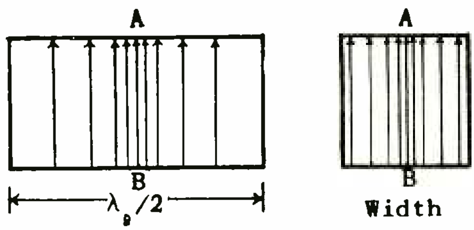


Fig. 43

As in the case of a transmission line, a cavity resonator having losses may be regarded as equivalent to a loss-less resonator shunted by a resistance connected between points A-B. The shunt resistance is of such value that it absorbs as much energy as the actual resonator dissipates in its walls. The concept of an equivalent shunt resistance is as useful here as it was shown to be in the case of transmission lines.

Q of a Cavity Resonator.--Cavity resonators behave like lumped resonant circuits of very high Q, as is evidenced by the high value of equivalent shunt resistance. The Q can be defined on the basis of the shape of the resonant curve, as was discussed in the previous lesson on transmission lines. It can also be defined on the basis of the ratio of energy stored to the energy lost per cycle. The energy is stored in the electric and magnetic fields occupying the volume of the resonator, the energy lost is that in the walls--specifically, at ultra-high frequencies, in a thin skin involving the inner surfaces of the cavity. Hence, by using a large volume, the ratio of surface to volume is less than for a small volume, and the Q is increased.

However, for most shapes the size is fixed by the resonant frequency desired. Formulas will be given farther on, but it may be noted here that for a spherical resonator, one particular resonant wave length is equal to 2.83 times the radius. (A cavity resonator can usually resonate in an infinite number of modes at correspondingly different frequencies.) The dimension of the sphere is thus comparable to the wave length, and thus reasonably large, so that the ratio of surface to volume is small, and the Q therefore large. On the other hand, a lumped circuit would be exceedingly small--if at all possible--and would have *relatively* high losses and a low Q. In the microwave range particularly, it is indeed fortunate that cavity resonators are available, as they provide resonance effects of very high Q (as much as 20,000 and more), and are in themselves structures of reasonable size: neither too large nor too small.

Cavity Resonator Shapes.--Cavity resonators may be built in various shapes--spherical, cylindical, prismatic--such as a cube, etc. Generally the shape that gives a maximum ratio of volume to area is preferred because this gives maximum Q. This means a shape that is everywhere convex, such as those mentioned above.

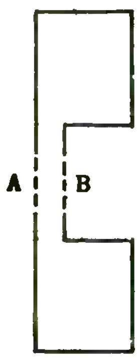


Fig. 44

On the other hand, re-entrant shapes, such as that shown in Fig. 44, are of great utility when the u.h.f. output of an electron beam is to be coupled to it. The reason is that maximum impedance occurs across distance AB. This impedance is the equivalent shunt resistance of the resonator. While the Q, and hence the resistance, is less than that for a convex shape, the resistance is developed here across a very short path AB. This means that electrons shooting through this path will encounter *a maximum of impedance in a minimum of (transit) time*, and will afford a much superior tube performance.

This type of resonator is employed, for example, in the klystron tube for the above reason. (The action of this tube will be explained later.) To vary the resonant frequency of the cavity over a small range, one face is made flexible so as to be able to approach or recede from the other face. For greater variations in a cavity resonator, especially of the shape shown in Fig. 42, one end is made in the form of a piston, so as to be able to vary the length of the cavity.

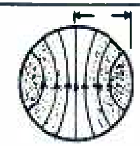
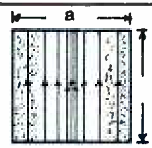
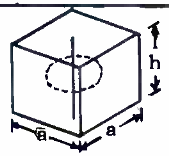
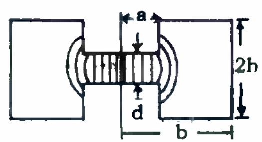
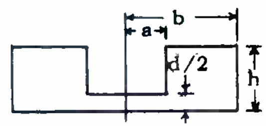
Formulas for Resonators.--Cavity resonators, like wave guides, can have various wave modes set up in them. This is in addition to the fact that they can resonate when a half-wave long, or any multiple of $\lambda/2$. In many cases the Q of a resonator when operating in a higher mode is greater than when operating in the lowest or fundamental mode, owing to a lower magnetic flux density near the walls for the higher mode. However, the difference in frequency (frequency separation) for two higher modes is small, and so the lowest or fundamental mode of operation is preferred.

In Table III are given the formulas for the resonant frequency and wave length, the Q, shunt resistance, and impedance in ohms coupled into a loop of A_l sq. cm, as well as the fundamental field configuration for which the above values apply. In the case of magnetic coupling, the loop should be placed near the outer edge where the magnetic field is greatest, and so oriented as to enclose the maximum number of magnetic lines. Variation in coupling is then obtained by rotating the loop, just as in the case of loop coupling to a transmission line discussed in a previous lesson.

Indeed, the dividing line between resonant transmission lines and cavity resonators is not at all sharply defined. For example, the re-entrant shape shown in Fig. 44 may be considered as a section of a coaxial cable, with the central conductor shorter than the outer sheath, which is capped with a circular disc on the left-hand side, and is shorted to the inner conductor on the right-hand side by an annular ring. The value of loop coupling area given in Table III can be extended to practically any shape--as a satisfactory approximation--by regarding the ratio of the impedance coupled into the loop to the shunt resistance of the resonator as roughly equal to the square of the ratio of loop area to half the cross-sectional area of the resonator.

Eq.(25) gave the *length* of a rectangular prism in terms of the *desired resonant frequency* and the *width* of the prism. (The height did not affect resonance.) In Table III, on the other hand, the formulas give the *resonant*

TABLE III

SHAPE	DIAGRAM	WAVE LENGTH CM	Q	SHUNT RESISTANCE R_{sh} IN OHMS	IMPED. COUPLED INTO TO LOOP A_l SQ. CM
Sphere		$2.28\ a$	$1.024 \frac{a}{\delta}$ $= .01255\ R_{sh}$	$81.6 \frac{a}{\delta}$ $= 79.7\ Q$	$354 \frac{A_l^2}{a^2 \delta}$
Cylinder		$2.61\ a$	$1.414 \frac{a h}{\delta (h + \frac{a}{2})}$ $= \frac{.00693\ R_{sh}\ a}{h}$	$\frac{204}{\delta} \frac{h^2}{(h + \frac{a}{2})}$ $= \frac{144.2\ Q\ h}{a}$	$\frac{204}{\delta}\left(\frac{A_l^2}{a}\right)^2 \frac{1}{(h + \frac{a}{2})}$
Prism (square)		$1.414\ a$	$\frac{.707\ a\ h}{\delta (h + \frac{a}{2})}$ $= \frac{.00776\ R_{sh}\ a}{h}$	$\frac{91.1\ h^2}{\delta (h + \frac{a}{2})}$ $= \frac{128.9\ Q\ h}{a}$	$\frac{2370}{\delta}\left(\frac{A_l}{a}\right)^2 \frac{1}{(h + \frac{a}{2})}$
Toroidal Type		$4.6\pi \left(\frac{h\ a^2}{d} \log \frac{b}{a}\right)^{1/2}$	$\frac{6.5 \frac{h}{\delta} \log \frac{b}{a}}{\frac{h}{a}(1 + \frac{a}{b}) + 2.3 \log \frac{b}{a}}$ $= \frac{.000577\ R_{sh}\ \lambda}{h \log b/a}$	$\frac{\frac{11260\ h^2}{\lambda \delta} \log^2 \frac{b}{a}}{\frac{h}{a}(1 + \frac{a}{b}) + 2.3 \log \frac{b}{a}}$ $= \frac{1732\ Q\ h}{\lambda} \log \frac{b}{a}$	- - - - - -
Toroidal Type		As Above	As Above	Half of Above	- - - - - -

or --- Magnetic Field Lines
——— Electric Field Lines
All Dimensions in Centimeters
For Copper, $\delta = 54 \times 10^{-6} \sqrt{\lambda cm}$

Adapted from Terman--"Radio Engineers' Handbook".

frequency in terms of the *dimensions*. It is to be noted that if the dimensions of an odd-shaped cavity are desired for a given resonant frequency, f_1, a cavity of any size can be built, and its resonant frequency f_2 found experimentally. Then by changing *all* dimensions in the ratio of f_1/f_2, a cavity of the desired shape will be obtained that is resonant to f_1.

To illustrate the use of Table III, assume a *square* copper prism, of dimensions a = 5 cm, h = 6 cm. The resonant wave length is 1.414 a = 1.414 x 5 = 7.07 cm. *This is the free-space wave length*, λ, and not the wave length in the guide, λ_g. The corresponding resonant frequency is

$$f = c/\lambda = 3 \times 10^{10}/7.07 = 4.24 \times 10^9 = 4240 \text{ mc}$$

The value of the loss coefficient δ is

$$\delta = 54 \times 10^{-6}\sqrt{7.07} = 143.7 \times 10^{-6}$$

The Q of the resonator is then

$$Q = \frac{.707 \times 5 \times 6}{143.7 \times 10^{-6}(6 + \frac{5}{2})} = 17{,}380$$

The shunt resistance is

$$R_{sh} = \frac{91.1\ (6)^2}{143.7 \times 10^{-6}\ (6 + \frac{5}{2})} = 2{,}690{,}000 \text{ or } 2.69 \text{ megohms}$$

Now suppose that it is desired to couple a 76.8 ohm concentric line to this resonator by means of a loop, located as described previously. From Table III, the area A_l of the loop can be found in terms of the impedance Z it is to reflect into the connected line, the cavity dimensions, and the free-space wave length λ. Thus, solving the expression in the right-hand column for the square prism in Table III, one obtains

$$A_l = \sqrt{\frac{Z\, a^2\, \delta\, (h + a/2)}{2370}} \tag{26}$$

Hence, in the problem given,

$$A_l = \sqrt{\frac{76.8\,(5)^2 \times 143.7 \times 10^{-6}(6 + 5/2)}{2370}} = 31.4 \times 10^{-3} \text{ sq. cm}$$

The radius r_l of the loop is then

$$r_l = \sqrt{A_l/\pi} = \sqrt{31.4 \times 10^{-3}/\pi} = .1 \text{ cm or } 1 \text{ mm}$$

Such a loop is very small. One twice as large could be used, (2 mm radius or 4 mm diameter), and then it could be rotated so as to reduce the coupling until the desired impedance was obtained experimentally.

Uses of Cavity Resonators.--Cavity resonators can be employed in the microwave range in the same manner as transmissions lines are employed at lower frequencies, or lumped circuits still farther down in the frequency spectrum. Some representative examples will be given at this point.

Wave Meter: The cavity resonator may be used as a wave meter. However, as the wave length to be measured approaches that of cutoff for the cavity, the wavelength adjustment in it approaches infinity, and the adjustment and readings are not in proportion to the free-space wave length that it is desired to measure.

For that reason a modification as shown in Fig. 45 is usually employed. Here a coaxial resonator is employed, with an adjustable piston to resonate it to the wave entering it from the wave guide. A small probe feeds a crystal detector and d.c. galvanometer, as shown, and maximum indication on the meter is obtained when the piston is adjusted to produce resonance in the guide.

The readings on the scale are in direct proportion to the free-space wave length because the coaxial resonator has no cutoff frequency, owing to the TEM wave set up in it. Thus non-uniformity in the scale readings is avoided. The Q of this type of resonator may be somewhat less than that of a single-conductor resonator, but in practice this is somewhat of an advantage, as too sharp a resonant point is easily passed through unnoticed.

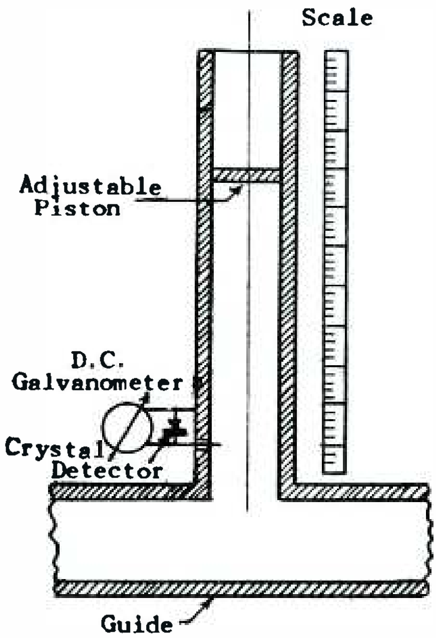


Fig. 45

In actual equipment great care must be exercised not to have too tight coupling between the detecting probe and the coaxial resonator, or between the latter and the source of energy, as otherwise multiple-peaked resonance effects can result, with consequent inaccuracy in reading.

Launching and Reception of Waves: In Fig. 21(A) was shown a method of launching a $TE_{1,0}$ wave. A further refinement is shown in Fig. 46. A coaxial cable is used to feed the wave guide, and the direction of propagation is to be to the right, as indicated. The inner conductor of the cable may be regarded as an antenna projecting into the guide. As such it has a certain amount of reactance and radiation resistance, as well as some ohmic resistance--preferably small.

By means of the coaxial piston and the upper portion of the coaxial cable, the reactance of the antenna portion may be tuned out, so that it appears as a pure resistance--essentially that owing to its radiation into the guide. Then, by adjusting the guide piston, the magnitude of this radiation resistance--*as it appears to the cable*--may be matched to the characteristic impedance of the cable.

The action depends upon the effect of the guide piston upon the radiation resistance of the dipole. This effect can be explained on the basis of an image dipole, situated as far behind the piston as the actual dipole is in front of it. This is shown in Fig. 47. The arrows indicate the direction of the various waves radiated by the actual, and by the image dipole. If the actual dipole is $\lambda/2$ in front of the piston, then the image dipole is $\lambda/2$ behind it, or a distance of λ behind the actual dipole. Its radiation is inherently 180° out of phase with that of the actual dipole. This, in conjunction with the greater path length λ, makes its radiation at some point P down the guide cancel that of the actual dipole, so that the net radiation from the latter along the guide is zero.

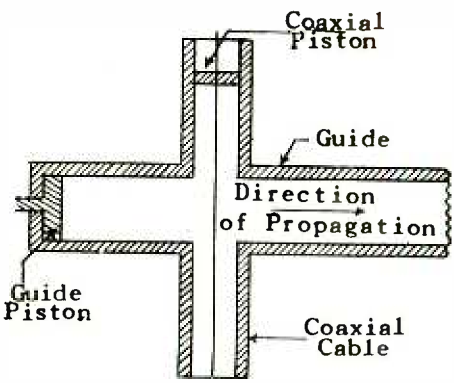


Fig. 46

If the actual dipole is $\lambda/4$ from the piston, hence $\lambda/2$ from its image, then the difference in path length adds another 180° to the 180° difference in radiation between the two, thus producing a direct additive effect or the radiation at P is doubled and a maximum. Thus the radiation of the actual dipole down the guide can be varied from zero to twice its radiation resistance in a guide extending to infinity in either direction, by adjusting the piston distance. If the characteristic impedance of the coaxial cable falls within these limits, as is usually the case, then the dipole can be matched to the coaxial cable feeding it, and all the energy coming through the matched cable is ultimately radiated down the guide to the right.

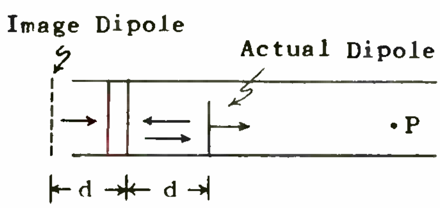


Fig. 47

The same method may be employed in receiving a wave. The wave sets up currents in the receiving antenna, and these currents are then rectified by a detector and applied to the l.f. output circuit. The detector, if of the crystal type and therefore small, can be inserted in series with the wire within the guide itself, instead of externally. It will be found that its position along the wire modifies the matching adjustments. There are many modifications possible, and one method of reception will be analyzed in greater detail farther on.

Reactive Coaxial Sections.--In Fig. 48 is shown a coaxial short-circuited stub inserted into a wave guide. By adjusting the piston, the distance h can be varied. The action is then as if a shunt impedance were connected across the guide at that point. For example, if h is less than $\lambda/4$ for the coaxial stub, the latter acts as if an inductance were shunting the guide. If $h \approx \lambda/4$, then the stub acts like an open circuit shunting the guide, whereas if h is between $\lambda/4$ and $\lambda/2$, the stub acts like a shunt capacitance. Finally, if $h \approx \lambda/2$, the stub acts like a short circuit.

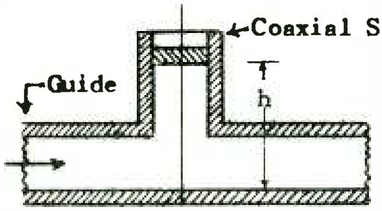


Fig. 48

In the latter case, if the wave motion is from left to right, then at the stub practically complete reflection occurs; if the wire is of copper, only about one per cent of the energy passes through to the right of it. It will be observed that the device acts like a tuned single-wire grating, and also that it acts like a diaphragm, and is possibly more readily adjusted.

As such it can be employed in a variety of ways. One example is that shown in Fig. 49. Here it is used as a matching stub between two wave guides G_1 and G_2 of different characteristic impedances. Standing waves occur in the stub and in the section of length l, but not in the rest of G_1 nor in G_2. It is stated* that the distance is not critical, and the adjustment of the piston stub for the elimination of the standing waves is determined experimentally.

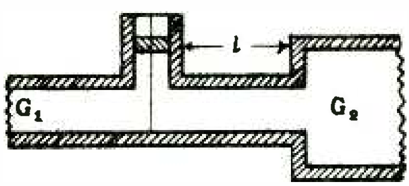


Fig. 49

A variation of the coaxial stub is that of a single-conductor or wave guide stub, capped by a piston. The advantage of the coaxial stub at the lower frequencies is that it can be

* See U. S. Patent No. 2,232,179--"Transmission of Guided Waves,"--A. P. King. Feb. 18, 1941.

made of much smaller diameter than the equivalent guide and hence be more definitely located along G_1. Also it has but one mode--the TEM mode--and its behavior is more readily controlled than that of a guide. At the upper end of the spectrum, say above 10,000 mc, however, its diameter may be too small, and possibly its losses excessive, and the wave guide stub then becomes more practical.

Multiplex Transmission.--It is of interest to note that various modes at the same frequency can be propagated simultaneously through a guide made large enough to accomodate even the highest order mode having the highest cutoff frequency. Separate launching and receiving devices will then respond separately to each individual mode. Thus, several messages can be sent along the same guide--in other words, multiplex transmissionis possible.

This can be accomplished even with one mode. Thus, if a horizontally polarized and a vertically polarized $TE_{1,0}$ mode are sent simultaneously through a guide, then a horizontal pickup antenna or probe will respond only to the horizontally polarized mode, and a vertical pickup antenna only to the vertically polarized mode, thereby accomplishing separation of the two messages.

Receiver System: In the previous assignment on transmission lines it was stated that at these high frequencies ordinary r.f. amplifier tubes fail to amplify appreciably, and so it is common practice to beat the incoming wave with that of a local oscillator and to obtain the intermediate or beat frequency through the use of a detector, such as a silicon carbide crystal. The beat frequency, being much lower, can be successfully amplified in a selective i.f. amplifier and then detected, for example, by an ordinary diode detector to furnish the audio or video output. Thus, at u.h.f., a superheterodyne receiver is used without benefit of r.f. amplification and selectivity.

It was shown in the previous assignment that r.f. selectivity is necessary to obtain image frequency attenuation, and that the only source of such selectivity is in the antenna circuit, which feeds a converter or first detector tube directly. A representative problem was worked out for an antenna and transmission line system, whereby the final element--a quarter-wave short-circuited line--furnished the requisite Q for the selectivity needed. It was shown how to couple the input circuit of the tube and also the antenna line to this quarter-wave section to obtain the above results.

A similar problem can be worked out for a wave guide system, but certain factors are different. One such factor is that the local oscillator is directly coupled into the detector system, rather than to a grid of the converter tube different from that to which the incoming signal is applied. Thus some note must be taken of the direct effect of the local oscillator on the rest of the system.

In Fig. 50 is shown a possible receiving system. For simplicity, a horn (to be described in the next assignment) is used as the antenna pickup device. This feeds a wave guide, which is connected by means of a matching diaphragm to the cavity resonator. The latter furnishes the desired selectivity for the required amount of image attenuation, or alternatively, for the desired band width, if wide band transmission is required.

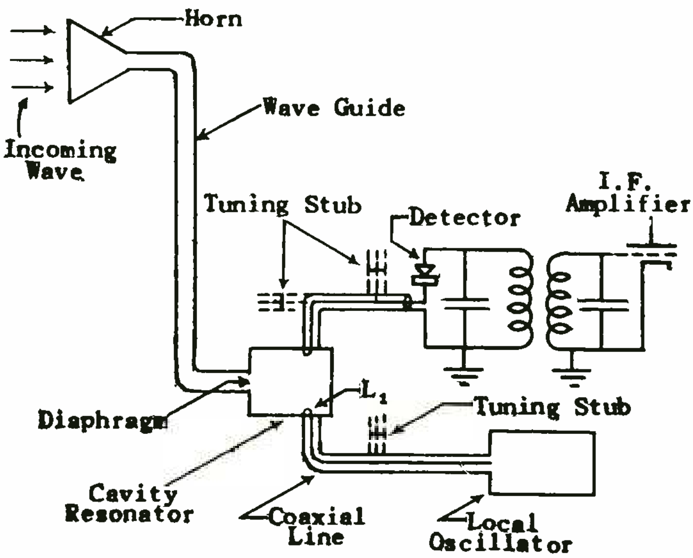


Fig. 50

A local oscillator is coupled to the resonator by means of a coupling loop. The oscillator may be a Klystron tube, which will be described in a latter assignment, and it is loosely coupled to the cavity resonator so as not to reduce unduly the Q of the latter and also so as not to be "pulled" into synchronism with the incoming wave. In the usual superheterodyne receiver such "pulling" is avoided by loosely coupling the local oscillator to the appropriate grid of the first detector tube, and also by interposing, in the tube itself, grids at fixed d.c. voltages between the signal input and the local oscillator grids of the tube, thus preventing coupling and pulling in the first detector tube itself. When a crystal first detector is employed, such decoupling is not possible, hence the local oscillator is made to generate excess power, so that, in spite of the very loose coupling necessary, the amplitude of the local oscillator signal will still be in excess of the incoming signal. A further aid to decouple the local oscillator from the cavity is to employ a coaxial line for the oscillator feed that has appreciable losses, such as by the insertion of a high-loss dielectric.

Such a cable not only acts as a u.h.f. attenuator and hence decoupler, but at the same time loads up the local oscillator so as to stabilize its performance. It is even possible to use a crystal controlled master oscillator, and by means of doubling and tripling buffer stages, to control and stabilize the frequency of the klystron or similar local oscillator.

The i.f. circuit containing the crystal first detector and the double-tuned i.f. stage can then be coupled to the cavity resonator in several ways, such as by means of a loop as shown in Fig. 50. The tuning stubs shown in dotted lines in Fig. 50 can be used if desired to eliminate reflection effects in the coaxial cables owing to impedance mismatch, such as at the crystal detector end of the line, or to the inductance of the coupling loops.

Image Attenuation Considerations: At ultra-high frequencies, an intermediate frequency that is roughly one-fifth of the signal frequency is often employed. This results in an intermediate frequency that is in itself so high, that it is often amplified in one or two stages, and then converted to a lower i.f. that can be amplified more readily. The result of this double conversion is greater gain and selectivity, but the objection is that two very stable local oscillators are required, as well as the additional complications attendent to so complex a mode of operation. However, high gain is possible without danger of excessive feedback and oscillation because feedback at any one frequency can occur only over a limited number of stages.

As an example, suppose the incoming frequency is 3,000 mc. An intermediate frequency of 3,000 ÷ 5 = 600 mc might be used. This could then be converted by a jump of 20 down to 30 mc, and further amplification employed. However, suppose the local oscillator can be suitably stabilized as to frequency so that it does not drift unduly. A single intermediate frequency as low as 60 mc might then be used without requiring the i.f. system to have an unduly wide band width. For example, if it had a band width of 2 mc, then the maximum drift of the local oscillator would have to be less than 2 mc in approximately 3,000 mc, or .067 per cent. While this is small, it is not too difficult when the receiver is designed to operate at a single frequency, so that the local oscillator does not have to be tuned over a range of frequencies.

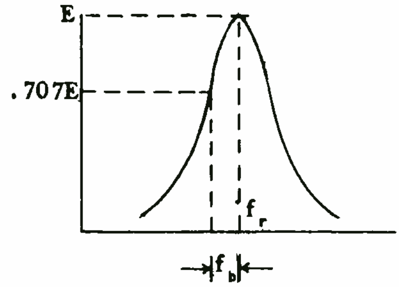


Fig. 51

Assume that the intermediate frequency is to be f_i = 120 mc, and that the image attenuation is to be 30 db. The cavity resonator causes the antenna system of Fig. 50 to have a selective response, as shown in Fig. 51. Here f_r = 3,000 mc. For a variation in frequency of f_b c.p.s., it will be noted that the response drops to 70.7 per cent of its value at resonance. As discussed in the previous lesson, f_b is considered the *half* band width of the selective circuit. As was also mentioned there,

$$f_b = \frac{2\ f_i}{\text{anlog(db image attenuation/20)}} \tag{27}$$

Hence

$$f_b = \frac{2 \times 120 \times 10^6}{\text{anlog }(30/20)} = \frac{2 \times 120 \times 10^6}{31.6} = 7.6 \text{ mc}$$

But the half band width is also related to the Q of the circuit as follows:

$$Q = f_r/2f_b \qquad (28)$$

Hence, in this problem, the fully loaded Q of the cavity resonator must be

$$Q = \frac{3,000 \times 10^6}{2 \times 7.6 \times 10^6} = 197.5 \text{ or, in round numbers, } 200$$

It will be assumed, in view of the previous remarks, that the local oscillator loads the cavity very lightly, and thus affects its Q to a negligible extent. This leaves the detector circuit and the wave guide as the sole loading on the resonator. For matched conditions the two should have equal loading (damping) effect so that either alone should produce a Q of twice 200, or 400. (This assumes that the losses of the cavity are so low that the Q is determined practically entirely by the loads coupled into the resonator, and not by its losses. Such an assumption is perfectly reasonable in view of the fact that the Q of the resonator alone can be as high as 50,000.)

Design of Detector Coupling Loop: The method of designing the coupling loop was discussed in a previous section of this lesson. In the present example, first the shunt resistance equivalent to a Q of 400 must be found, and *then the loop designed to this value of* R_{sh} *rather than to the higher value of the cavity resonator alone.*

The value of R_{sh} equivalent to a given value of Q, for a square prism, is given in Table III as

$$R_{sh} = \frac{128.9 \; Q \; h}{a}$$

There is some question as to its validity in this problem because the normal Q of the resonator is based on the assumption that the losses are small, and that the field patterns calculated for the simpler case of perfect conductivity can be used to calculate the losses and the Q. In the problem given, where the Q is to be only 400, the relatively high losses may change the pattern shapes and hence the behavior. However, a Q of 400 is high compared to the Q's obtained in low-frequency work, so that the patterns are probably not too different from those for perfect conductivity. From a practical viewpoint, the methods to be described should require but a reasonable amount of further experimental adjustment.

Before R_{sh} can be calculated by the preceding formula, the dimensions for the wave guide and cavity resonator must be chosen. Let those for the guide be: a_1(width) = 7 cm, and b_1(height) = 4 cm. For 3,000 mc, λ = 10 cm, and $2a_1 = 2 \times 7 = 14$ cm is greater than λ, so that the guide is clearly being operated above its cutoff wave length of 14 cm.

For the cavity resonator, if a square prism is chosen then, from Table III, each side must be $\lambda/1.414 = 7.07$ cm $= a_2$. Let the height h also equal 7.07 cm, i. e., the cavity is a cube. Then

$$R_{sh} = \frac{128.9 \times 400 \times 7.07}{7.07} = 51,560 \text{ ohms}$$

This must be matched to the coaxial detector cable, whose characteristic impedance Z_o will be assumed to be 76.8 ohms. The impedance ratio is

$$n = \frac{R_{sh}}{Z_o} = \frac{51,560}{76.8} = 671.3$$

The area of the loop is then given by the following formula

$$A_l = \frac{.1958 \text{ a h}}{\sqrt{n}} \qquad (28)$$

This formula differs from that given in Table III in that there the impedance coupled into a loop of specified area from a low-loss cavity resonator is given, whereas here the area of the loop is given in terms of the cavity dimensions and the impedance ratio desired, i. e., how much resistance the loop is going to couple into the cavity.

In the problem at hand,

$$A_l = \frac{.1958 \times 7.07 \times 7.07}{\sqrt{671.3}} = .377 \text{ sq. cm}$$

The radius is then

$$r = \sqrt{\frac{.377}{\pi}} = \sqrt{.12} = .347 \text{ cm}$$

It is to be noted that if the local oscillator is coupled to the cavity by a 76.8 ohm cable too, and a loop of area one-tenth or so of the above employed, then the effect of the local oscillator on the Q of the cavity will be negligible, and conversely, the effect of the cavity on the oscillator can be ignored.

Coupling of Guide to Cavity: The wave guide must be coupled to the cavity resonator so that it, by itself, will damp the latter down to a Q of 400, too. While a short section of a coaxial cable with a loop at each end could be used to couple the two together, an alternative mode will be employed, namely, a diaphragm between the two.

The aperture in the diaphragm will be designed as per Fig. 36 so as to have the same impedance as the wave guide. This will avoid an impedance mismatch between the two. At the same time, the *area* of the aperture will be such that it will damp the cavity resonator down to a Q of 400.

If the cavity were a source of energy, then a hole or aperture in it of area A_d would lower its Q from an inherently high value down to a value

$$Q' = 37 \left(\frac{a^2}{4A_d}\right)^3 \frac{h}{a} \tag{29}$$

where a and h respectively are the length or width, and the height of a square prism resonator. From this A_d can be found in terms of the resonator dimensions and the desired Q'.

$$A_d = \sqrt[3]{\frac{37a^5\,h}{64\ Q'}} \tag{30}$$

Thus, for $Q' = 400$, and $a = h = 7.07$ cm,

$$A_d = \sqrt[3]{\frac{37(7.07)^6}{64 \times 400}} = (7.07)^2 \sqrt[3]{\frac{37}{25600}} = 50\ \sqrt[3]{.001446}$$

To find the cube root of .001446 first multiply and divide by $10^3 = 1{,}000$. Then

$$\sqrt[3]{.001446} = \sqrt[3]{\frac{1.446}{10^3}} = .1\ \sqrt[3]{1.446}$$

Now take the logarithm of 1.446, divide by 3 to get the logarithm of the cube root, and find the antilog. Thus

$$\log 1.446 = .1602$$
$$1/3 \log 1.446 = .0534$$
$$\text{anlg } .0534 = 1.131$$

Hence

$$A_d = 50 \times .1 \times 1.131 = 5.65 \text{ sq. cm}$$

It now remains to determine the two dimensions a_3 and b_3 of the diaphragm aperture so that its area, a_3b_3, will be 5.65 sq. cm, and at the same time it will have the proper proportions to match the wave guide.

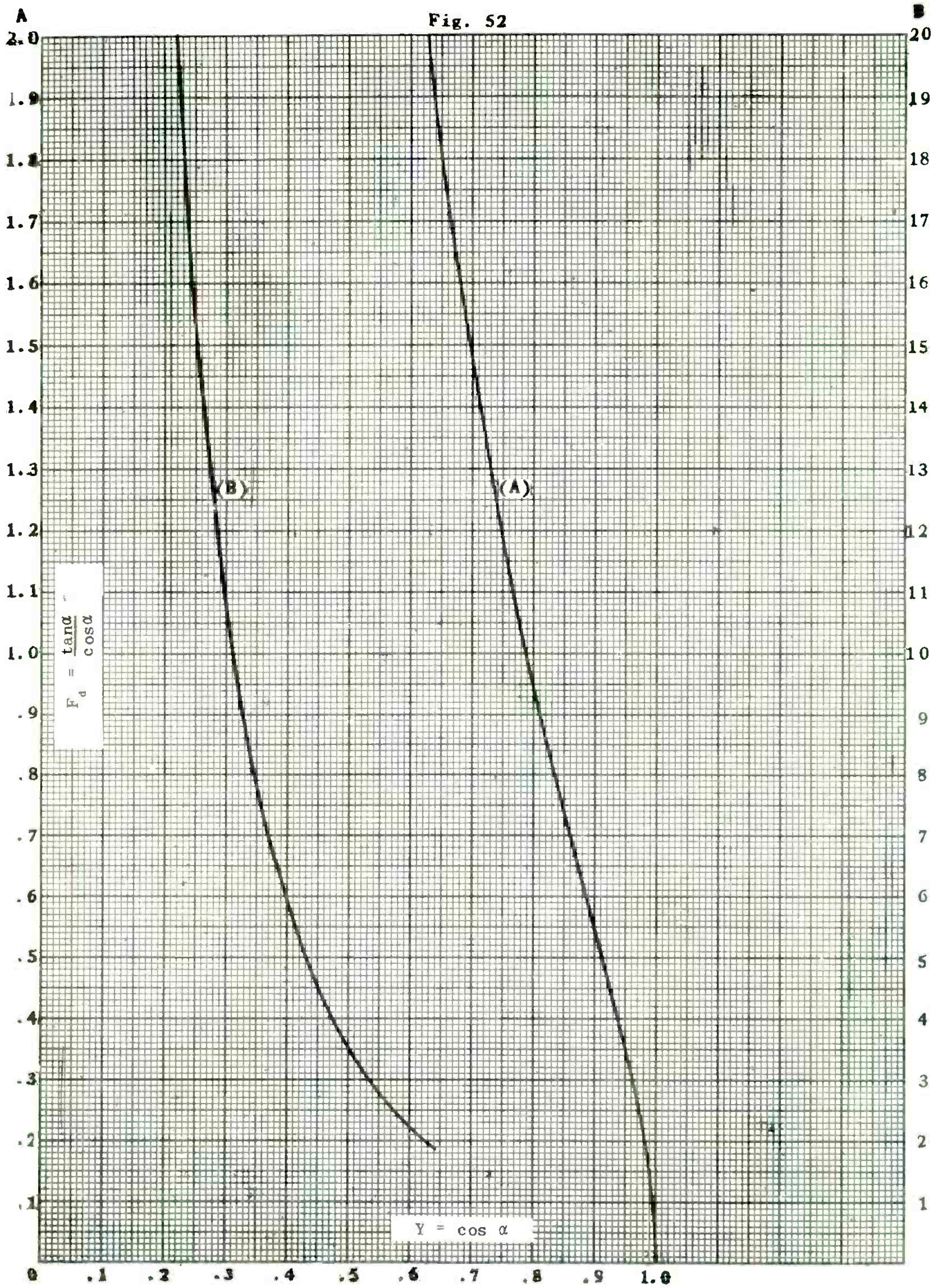
Eq.(15) gave these proportions in terms of the aperture impedance as

$$Z_{1,0} = 592 \frac{1}{\sqrt{1 - (\lambda/\lambda_0)^2}} \cdot \frac{b}{a} = 592 \frac{1}{\sqrt{1 - (\lambda/2a)^2}} \frac{b}{a} \tag{15a}$$

Multiply and divide the right-hand expression of Eq.(15a) by $(\lambda/2a)^2$ and obtain

$$Z_{1,0} = \frac{592(\frac{\lambda}{2a})^2}{\sqrt{1 - (\lambda/2a)^2}} \ \frac{4a^2b}{\lambda^2 a} = \frac{2368(\frac{\lambda}{2a})^2\ ab}{\lambda^2 \sqrt{1 - (\lambda/2a)^2}}$$

Fig. 52


A
B
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
(B)
(A)
$F_d = \frac{\tan\alpha}{\cos\alpha}$
Y = cos α
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

from which

$$\frac{\sqrt{1 - (\lambda/2a)^2}}{(\lambda/2a)^2} = \frac{2368ab}{\lambda^2 \; Z_{1,0}} = \frac{2368 \; A_d}{\lambda^2 \; Z_{1,0}}$$

As before, $\lambda/2a$ is recognized as $\cos\alpha$, whereupon the left-hand expression becomes $\tan\alpha/\cos\alpha$, so that

$$\frac{2368 \; A_d}{\lambda^2 \; Z_{1,0}} = \frac{\tan\alpha}{\cos\alpha} = F_d \qquad (31)$$

In Fig. 52 $\tan\alpha/\cos\alpha$ has been plotted, for a range of values of α, against $\cos\alpha$ in two curves; (A) for large values of $\cos\alpha$, and (B) for small values. Hence, if A_d, the aperture area, has been determined by Eq.(30), and $Z_{1,0}$ is known for the wave guide, then F_d can be calculated, and $\cos\alpha = \lambda/2a$ found from Fig. 52. Once $\lambda/2a_3$ is known, a_3 can be found, since λ is known. Then, from Fig. 36, b_3 can be found, and thus the slot dimensions have been determined.

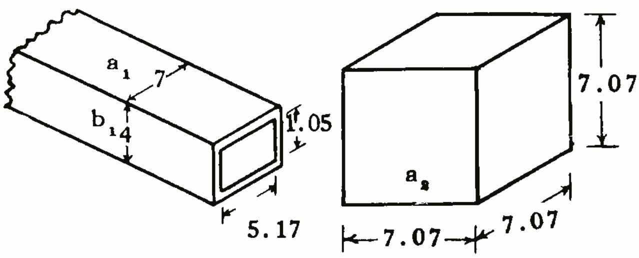


Fig. 53

First, in the numerical problem, $Z_{1,0}$ for the guide must be found with the aid of Fig. 36. Since for the guide, $\lambda/2a_1 = 10/(2 \times 7) = .714 = Y$, the factor F from Fig. 36 is found from curve (A) to be .98. Also, for the guide, $b_1/\lambda = 4/10 = .4$. Then, by Eq.(20).

$$Z_{1,0} = \frac{1184 \times .4}{.98} = 484 \text{ ohms}$$

Now F_d can be found from Eq.(31).

$$F_d = \frac{2368 \times 5.65}{(10)^2\ 484} = .275$$

This is an *ordinate* on Fig. 52, curve A. The corresponding abscissa is α (= $\cos\alpha = \lambda/2a_3$) = .968 Then

$$a_3 = \frac{\lambda}{2\gamma} = \frac{10}{2 \times .968} = 5.17 \text{ cm}$$

Also, from Fig. 36, for γ = .968, F = .256, and

$$b_3 = \frac{F\,\lambda\,Z_{1,0}}{1184} = \frac{.256 \times 10 \times 484}{1184} = 1.05 \text{ cm}$$

In Fig. 53 are shown the dimensions for the wave guide, the diaphragm aperture, and the cavity resonator.

As a check on the graphs and the calculations, the area of the aperture from the dimensions is 5.17 × 1.05 = 5.4 sq. cm, which is approximately the value found necessary to reduce the Q of the cavity to 400.

U.H.F. TECHNIQUES

EXAMINATION

1. (a) What basic difference is there in the behavior of a single-conductor (wave guide) transmission system and a two-conductor transmission system?

   (b) Why are wave guides unsuited for low-frequency transmission?

1. *(Continued)*

2. (a) What significant characteristic do the direction of the electric and magnetic fields in a wave guide have?

   (b) How is this used to classify the wave patterns in the guide?

2. *(Continued)*

3. (a) Define a TM wave mode.

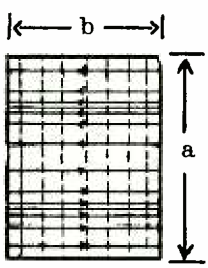
(b) (b) Define a TE wave mode.

(c) Define a TEM wave mode.

3. *(Continued)*

(d) What type of transmission system is required to transmit the TEM mode?

4.



(a) What wave mode is represented in the figure to the left?

(b) Suppose the frequency is 6,000 mc. What should be the minimum dimensions for a and b in order to have transmission?

4. *(Continued)*

5. (a) Find the cutoff frequency for a $TE_{1,1}$ mode in a circular guide of radius equal to 3 cm.

(b) What would be the cutoff frequency for a rectangular guide of *square* cross-section whose sides were each 6 cm long, for a $TM_{1,1}$ wave?

5. *(Continued)*

6. What is the attenuation of a 3,000 mc $TE_{1,0}$ wave mode in a square guide whose width and height are each 7 mc, and whose length is 50 feet?

7. (a) What is a wave filter used for?

7. *(Continued)*

(b) What is the principle upon which it is based?

(c) What is a wave converter and how does it operate?

8. Given two rectangular guides required to pass a 3,000 mc wave. One has a width of 6 cm and height of 5 cm; the other, a width and a height of 7 cm each. Design a device that will match these two guides over a range of frequencies centered around 3,000 mc and draw a picture of the system.

9. Given a cavity resonator in the form of a square prism, of height = 10 cm, required to resonate at 6,000 mc. It is to be fed by a rectangular wave guide of height = 3.5 cm, and width = 3 cm through an aperture. The resonator is further to be coupled to a detector cable of 80 ohms characteristic impedance. The overall Q required for suitable image attenuation is 300. (Note h does not equal a_2). $TE_{1,0}$ mode

   Calculate the dimensions of an aperture that matches the guide and at the same time imparts, in conjunction with the detector cable, the above Q of 300 to the cavity resonator. Sketch the guide, aperture, and cavity resonator, indicating dimensions, similar to Fig. 53.

9. *(Continued)*

10. Calculate the minimum size of the coupling loop required for the cable.