SECTION 2

ADVANCED PRACTICAL RADIO ENGINEERING

TECHNICAL ASSIGNMENT

ULTRA-HIGH FREQUENCY TECHNIQUE

251E

- TABLE OF CONTENTS -

ULTRA-HIGH FREQUENCY TECHNIQUE

ADVANTAGES AND DISADVANTAGES OF ULTRA-HIGH FREQUENCIES

The domain of ultra-high frequencies has become an exceedingly important one at the present time, and the trend is unmistakably toward the use of more and more services in this range of the spectrum. It must not be supposed, however, that u.h.f. channels will ultimately replace the low-frequency channels: rather they will supplement the latter.

Indeed, in the case of many services, the reason for the use of u.h.f. is fundamentally that the low and intermediate frequency and the medium short-wave portions of the spectrum were practically used up, and new channels could be found only at higher frequencies. As these channels began to be studied, and even employed, advantages and also some very grave disadvantages were discovered, and research and development are continuing at an accelerated pace to minimize the difficulties that lie in the way of their utilization. The student can, with benefit, review the series of assignments on antennas, particularly where reference to u.h.f. is made.

The term "ultra-high frequencies" is often used as a general term to denote frequencies above about 30 mc. However, the F.C.C. has adopted certain designations which were given in an earlier assignment on radiation, and which will be repeated here:

FREQUENCY	WAVELENGTH	DESIGNATION
30 — 300 mc	10 — 1 meters	very high frequencies (v.h.f.)
300 — 3,000 mc	1 meter — 10 cm	ultra-high frequencies (u.h.f.)
3,000 — 30,000 mc	10 cm — 1 cm	super-high frequencies (s.h.f.)

Although these help to distinquish one range from the other, no harm will be done if in this assignment all frequencies in the above three ranges are denoted by the single term, ultra-high frequencies, and where there is a difference in behavior, the frequency range under discussion will be given numerically.

If one considers the range from 0 to 30 mc, and compares it with the range from 30 to 30,000 mc, one will find that the latter band is

$$\frac{30000 - 30}{30}$$

or nearly 1,000 times as great as the low-frequency and medium short-wave bands combined! This indicates the enormous amount of room available in the spectrum beyond the 30-mc band, and is the original, if not principal reason for going into that part of the spectrum.

There are several other advantages besides that of more room. The advent of f.m., and particularly television, has necessitated wide-band modulation and radiation. For example, consider a television carrier frequency of 51.25 mc. Suppose it is modulated with video (picture) frequencies up to 4 $^{1}/_{4}$ mc. There will appear in the form of upper and lower side bands, frequencies ranging from close to the carrier frequency to 51.25 + 4.25 = 55.5 mc (upper side band) and 51.25 - 4.25 = 47 mc (lower side band) for the highest video frequency of 4.25 mc.

A suitable vestigial side-band filter removes most of the lower side bands without appreciably affecting the upper side bands. The band of frequencies that is then involved is shown in Fig. 1. The sound carrier and side bands have

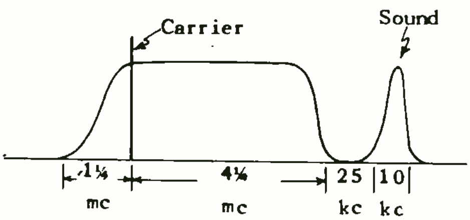


Fig. 1.—Band of frequencies for a television channel.

also been indicated. Fig. 1 shows the wide band of frequencies that has to be radiated by the transmitting antenna, picked up by the receiving antenna, and possibly amplified by an r.f. stage in the receiver.

Suppose the carrier frequency were 2 megacycles. The lowest video side band to be radiated would be $2 - 1\,{}^1/_4 = 3/4$ megacycles, and the highest video side band would be $2 + 4\,{}^1/_4 = 6\,{}^1/_4$ megacycles. The radiation resistance of the antenna would vary widely over such a range—$6\,{}^1/_4 - 3/4 = 5\,{}^1/_2$ megacycles.

Moreover, considering the antenna as a transmission line, if it were a quarter-wave at 2 mc, it would be $(3/4 \div 2) \times \lambda/4 = 3/32\,\lambda$ at .75 mc and $(6\,{}^1/_4 \div 2) \times \lambda/4 = 25/32\,\lambda$ or slightly over $3/4\,\lambda$ at $6\,{}^1/_4$ mc. Such an antenna would be practically impossible to tune *simultaneously* over such a wide range; it could not be made aperiodic except over a range of possibly 160 kc.

Consider, however, a carrier of 51.25 mc. Suppose a dipole is employed that is a half-wave at this frequency. Then at 51.25 + 4.25 = 55.5 mc, it would be $(55.5/51.25) \times \lambda/2 = .542\lambda$ or practically $\lambda/2$, and even more nearly $\lambda/2$ at the lower side-band frequency of 51.25 - 1.25 = 50 mc. The above illustrates an important advantage of u.h.f. carriers for television transmission, or any other wide-band system, such as that for pulse transmission.

Another advantage besides that of wide-band possibilities is that of accuracy of aeronautical navigation courses. This matter is discussed in detail in the Aeronautical Radio Engineering Section of the course. Suffice it to say at this point that when radio courses in the form of beam transmissions at about 100 mc are used instead of at 300 kc, particularly in mountainous regions, there is less danger of course bending and multiple courses. The C.A.A. (Civil Aeronautical Administration) is changing over from l.f. to the u.h.f. radio ranges as rapidly as possible.

A further point to note, particularly in connection with instrument landing systems, is that horizontally polarized waves, when reflected from the earth, appear to be less susceptible to variations in ground conditions as compared to vertically polarized waves. This tends to make a horizontally polarized glide path more reliable than one that is vertically polarized.

Since horizontal polarization implies a horizontal antenna, and this in turn, for appreciable low-angle radiation, means an antenna removed by several velengths from the earth's surface, it is evident that only in the u.h.f. range can horizontal polarization be feasible without the antenna having to be set at a prohibitive height above the earth's surface. Furthermore, at very high frequencies, parabolic reflectors and horn radiators are feasible, and these can radiate such narrow beams that the radiation does not necessarily have to strike and be reflected by the earth. In this case the radiation is entirely independent of ground conditions—such as snow and ice.

Besides the above advantages of u.h.f., there are other considerations that may be considered as advantages in some applications and as disadvantages in others. Thus, the range of transmission is limited to less than twice the line-of-sight for f.m. at about 88.1 mc, and not appreciably more than line-of-sight for the more exacting service of television at 50 mc and above. This means that stations separated by sufficient distances (more than twice the line-of-sight) can operate on the same frequency without objectionable interference. This tends further to conserve the number of channels required for a type of service such as broadcasting or aeronautical radio ranges.

Consider, for example, f.m. broadcasting on a frequency of 88.1 mc. If the antenna is sufficiently elevated, coverage over a radius of 90 miles is entirely feasible. This is not noticeably less than the primary coverage area for standard broadcast stations.

Another station 180 miles away can presumably employ the same frequency, and yet not interfere with the pickup of this station in its service area, particularly in view of the ability of the f.m. receiver to suppress the weaker of two signals in the presence of the stronger. However, this matter of common frequency assignment has not as yet been thoroughly studied, and modifications to the above discussion may have to be made because of variations in transmission from time to time, particularly with reference to the tropospheric wave (see assignment on antennas).

The above limitation of transmission appears to be an advantage. However, where long-distance transmission is desired, u.h.f. is out of the question, and one has to rely upon the medium short waves and upon the very low-frequency bands for this purpose, unless a chain of relay stations is possible and acceptable.

Another advantage of u.h.f. is that all components can be made relatively smaller: smaller, light-weight transmitters, receivers, and antennas. Alternatively, in the case of antennas, complicated arrays that do not require excessive space are feasible, with the result that sharp beams can be produced. This is equivalent to an increase in transmitter power, and is true even when the beam is compressed only in the vertical plane into the form of a nearly horizontal conical sheet, as for broadcasting purposes. Compare this with the difficulty of obtaining a maximum of ground wave with a minimum of sky wave in the standard broadcast band.

There are, however, some very serious difficulties encountered in attempting to employ u.h.f. One is that the circuit components become vanishingly small if one attempts to employ them in lumped form. Fortunately, alternative forms of circuit elements of the distributed type become available. For example, short-circuited and open-circuited sections of transmission lines of very high Q can be used as resonant elements, particularly in the lower frequency portion of the u.h.f. spectrum, and cavity resonators lend themselves admirably for the same purpose in the upper frequency portion of the u.h.f. spectrum. Indeed, if lumped elements, such as coils and condensers, are attempted to be used, it will be found that they inherently tend to function as distributed circuit elements, and appear as inductances or capacities depending upon the frequency at which they are operated. (Review the assignment on r.f. measurements.) Even short connecting leads function as transmission lines and accordingly must be taken into account.

As a result, all connections should be as short and direct as possible. This refers even to the internal tube connections and to the size of the tube elements. For that reason the parts, and particularly the tubes, become so small that they are incapable of dissipating the heat generated when handling large amounts of power. Thus, power output in the very high frequency portion of the u.h.f. domain is generally on the order of a few hundred watts or less, and in many cases a transmitter may have an output of but a few watts.*

To counteract this severe power limitation is the fact that the radiated energy can be concentrated into narrow beams and also picked up by highly directional receiving antennas, so that at least in point-to-point communication a few watts in the u.h.f. region may be as effective as hundreds of watts in the lower frequency portion of the spectrum. Nevertheless, the power limitations of vacuum tubes operating at ultra-high frequencies constitute one of the greatest problems in utilizing this part of the spectrum.

The first topic to be considered in detail will be that of Propagation.

U.H.F. PROPAGATION

In a previous assignment it was pointed out that frequencies above about 30 or 40 mc are normally not reflected by the ionosphere to produce skywave effects, but instead penetrate the ionosphere and are lost to the earth. At the same time the ground wave in the u.h.f. range is so rapidly attenuated as to be of no practical value for any but the very shortest distances of transmission.

Transmission is then by means of two other components of the radiation: the direct and the ground-reflected rays, i.e., the space

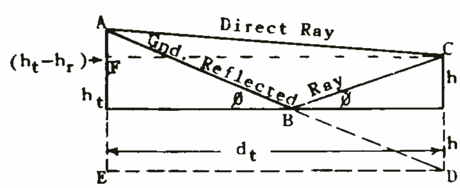


Fig. 2.—Geometry of ground-reflected and direct rays.

*In the case of pulse transmission, the instantaneous peak power may be on the order of kilowatts, or even more than a megawatt, but the average power will be a small fraction of this (depending upon th pulse width).

wave. For distances d_t large compared to the elevation h_t and h_r of the antennas, the angle ϕ, Fig. 2, is very small—less than the Brewster angle. Accordingly, in normal practice vertically polarized waves as well as horizontally polarized waves have their electric field vectors reversed in phase upon reflection, so that the ground-reflected ray tends to meet the direct ray inherently 180° out of phase and thus to cancel it.

The difference in path length between the two rays, however, produces an *additional* shift in phase for the ground-reflected ray, so that the two do not cancel, and hence a signal is produced at the receiving antenna that is the vector difference between the two rays. As h_t and h_r are increased, the difference in path lengths approaches $\lambda/2$. For this difference in path length, the phase shift is a full 180° plus 180° owing to reflection. The total is 360°, or the two rays meet in phase and maximum signal is obtained.

From Fig. 2 it is evident that the length of the ground-reflected path, which is AB + BC, is the same as the straight line distance AB + BD. Since ABD is the hypotenuse of the right-angled triangle AED; ED represents the distance d_t between antennas; and AE, the sum of the two antenna heights, $h_t + h_r$, it follows that

$$AD = \sqrt{(ED)^2 + (AE)^2}$$

$$= \sqrt{d_t^2 + (h_t + h_r)^2}$$

The length of the direct path is AC, which is the hypotenuse of triangle AFC. Hence,

$$AC = \sqrt{(FC)^2 + (AF)^2}$$

$$= \sqrt{d_t^2 + (h_t - h_r)^2}$$

The difference between the two path lengths is

$$l = AD - AC$$

$$= \sqrt{d_t^2 + (h_t + h_r)^2} - \sqrt{d_t^2 + (h_t - h_r)^2}$$

If $(h_t + h_r)$ is very small compared to d_t, as is normally the case in practice, then it can be shown that

$$l \cong \frac{2h_t h_r}{d_t}^*$$

($\cong$ means approximately equal)

*Thus, consider $(a + b)^2 = a^2 + 2ab + b^2$. If b is very small compared to a, then b^2 will be negligible compared to a^2 and 2ab, or

$$(a + b)^2 \cong a^2 + 2ab$$

From this it follows that $\sqrt{a^2 + 2ab} \cong a + b$. In the first square root expression in the text, d_t^2 corresponds to a^2, and $(h_t + h_r)^2$ to 2ab.

Hence,

$$\frac{2ab}{2a} = b = \frac{(h_t + h_r)^2}{2d_t}$$

or

$$\sqrt{d_t^2 + (h_t + h_r)^2} \cong d_t + \frac{(h_t + h_r)^2}{2d_t}$$

Similarly,

$$\sqrt{d_t^2 + (h_t - h_r)^2} \cong d_t + \frac{(h_t - h_r)^2}{2d_t}$$

Then

$$\sqrt{d_t^2 + (h_t + h_r)^2} - \sqrt{d_t^2 + (h_t - h_r)^2}$$

$$= d_t + \frac{(h_t + h_r)^2}{2d_t} - d_t - \frac{(h_t - h_r)^2}{2d_t}$$

$$= \frac{(h_t + h_r)^2 - (h_t - h_r)^2}{2d_t}$$

$$= \frac{h_t^2 + 2h_t h_r + h_r^2 - h_t^2 + 2h_t h_r - h_r^2}{2d_t}$$

$$= \frac{2h_t h_r}{d_t}$$

The difference in path length, when measured in wavelengths, is

$$\frac{l}{\lambda} = \frac{2h_t h_r}{\lambda d_t}$$

and when measured in radians is

$$\psi = 2\pi \left(\frac{l}{\lambda}\right) = \frac{4\pi h_t h_r}{\lambda d_t} \qquad (1)$$

If the value of ψ is desired in degrees, substitute 360 for 2π in the above formula.

Assume that the reflection from the earth takes place with no absorption; i.e., the ground-reflected ray is equal in strength to the direct ray. In that case, if there were no difference in path length, the vectors would cancel each other, as shown by the solid lines, E_d (direct-ray vector) and E_r (ground-reflected ray) in Fig. 3 (A). If there is a difference in path length with a corresponding angle of lag ψ, the ground-reflected vector will have a position shown by the dotted-line vector E_r'. The sum of E_r' and E_d, or the resultant field strength, can be found by subtracting $-E_r'$ from E_d. This is shown in Fig. 3 (B), where E_r' is reversed in direction to give $-E_r'$, which is then *subtracted* vectorially from E_d to give the resultant E_t. It will be observed that this is the same as adding E_r' to E_d (dotted lines) in (A) of Fig. 3.

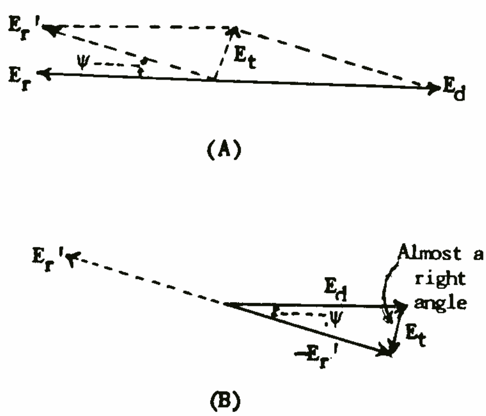


Fig. 3.—Vector relations for ground-reflected and direct rays.

From Fig. 3 (B) it can be seen that if ψ is small, the three vectors form practically a right triangle, so that

$$E_t = (-E_r')\sin\psi = E_d \sin\psi$$

$$\cong E_d\,\psi = E_d \frac{4\pi h_t h_r}{\lambda d_t} \qquad (2)$$

since E_r' and E_d are assumed equal in length (perfect reflection) and also if ψ is small, then $\sin\psi \cong \psi$. Thus, if the *direct* field strength at any point in space is known, then the *resultant* field strength—that which actually induces a voltage in the receiving antenna—can be found.

If the phase angle ψ is large, the more cumbersome but more accurate expression for the resultant of two vectors must be used, namely,

$$E_t = \sqrt{E_r^2 + E_d^2 - 2E_r E_d \cos\psi}$$

$$= E_d \sqrt{2 - 2\cos\psi} \qquad (3)$$

The intensity of the direct ray, E_d, for a half-wave dipole antenna is

$$E_d = (7\sqrt{w})/d_t \text{ volts per meter} \quad (4)$$

where w is in watts and d_t is in meters. If Eq. (4) be substituted in Eq. (2), the resultant field strength can be found:

$$E_t = (28\pi \sqrt{w}\, h_t h_r)/d_t^2\, \lambda \text{ volts/m} \quad (5)$$

This equation is satisfactory for propagation over average ground, for $(h_t + h_r)$ small compared to d_t, and for small path differences (small values of ψ, so that $\sin \psi$ approximately equals ψ in radians). Note that E_t is not the voltage, but the electric-field intensity, i.e., *volts per meter*, if λ is in meters. (h_t, h_r, and d_t must be in the same units, e.g., in meters.)

Suppose a half-wave dipole is employed as the receiving antenna. The current and voltage distributions on the antenna will be half-sine waves as shown in Fig. 4. The

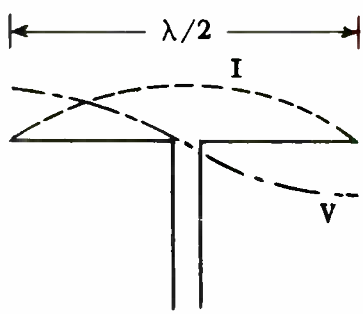


Fig. 4.—Current and voltage relations in a half-wave dipole.

average value of the half-sine wave, *as averaged over a half cycle*, is $2/\pi$. This means that if the dipole were reduced from a length of $\lambda/2$ to $(\lambda/2)(2/\pi) = \lambda/\pi$, and the current and voltage were of constant peak value throughout the length of this reduced antenna, the total voltage induced in this antenna would be the same as in the actual dipole. This finally means that the *effective length* of a half-wave dipole is λ/π instead of $\lambda/2$.

If the effective length is multiplied by the (uniform) field strength, the total induced voltage V_r is obtained. This would be equal to $E_t\, \lambda/\pi$, which becomes, upon substituting the value of E_t from Eq. (5)

$$V_r = (28 \sqrt{w}\, h_t h_r)/d_t^2 \quad (6)$$

where h_t, h_r, and d_t must be measured in the same units, such as meters.

The received power can then be calculated, under the assumption, for example, that the input impedance of the tube circuit equals the impedance of the source—here the radiation resistance of the antenna. This is 75 ohms for a half-wave dipole, hence the input impedance—as viewed by the antenna—must be be 75 ohms too. If the load impedance equals the source impedance, then half of the generated voltage V_r is consumed in the source (antenna), and only half, or $V_r/2$ appears across the load.

This voltage is across 75 ohms. Hence, the received power is

$$w_r = \left(\frac{V_r}{2}\right)^2 \Big/ 75 = \left(\frac{28 \sqrt{w}\, h_t h_r}{2 d_t^2}\right)^2 \Big/ 75$$

$$= (2.6\, w\, h_t^2 h_r^2)/d_t^4 \quad (7)$$

where h_t, h_r, and d_t are measured in the same units, such as meters. At this point it is desirable to discuss the foregoing equations in order to see more clearly where they apply and what they imply.

In the first place, Fig. 2 indicates that the same results can be expected if the transmitting and receiving antennas are interchanged. This is borne out by Eqs. (1), (2), (5), (6), and (7). In all of these h_t and h_r appear in the equation as a product, which is unchanged if the numerical values of the individual factors h_t and h_r are interchanged. In broadcast service, such as f.m. and television, Eq. (5) indicates, for example, that for a given power w that is radiated, the field strength at the receiver E_t can be increased if the height of the transmitting antenna h_t is increased, even though the receiving antenna height h_r is low. From a practical viewpoint, an elevated site for the transmitting antenna will be easier and *cheaper* to obtain than elevated sites for all the receiving antennas.

On the other hand, in point-to-point communication both sites can be selected for optimum effect. receiving antennas above the reflecting plane. In order to take into account the refracting effect of the troposphere (lower atmosphere), it is customary to plot this map with a radius that is 4/3 of the actual earth's radius. The profile should be examined carefully to determine if there is any possibility of more than one ground-reflected ray directed toward the receiving antenna. In Fig. 5 is illustrated a profile map of the region between a proposed transmitter and receiver.

From Eq. (1) the phase shift ψ of the ground-reflected wave relative to the direct ray can be calculated. Note, however, that this and all succeeding equations are simplified approximations, and assume that the heights h_t and h_r are

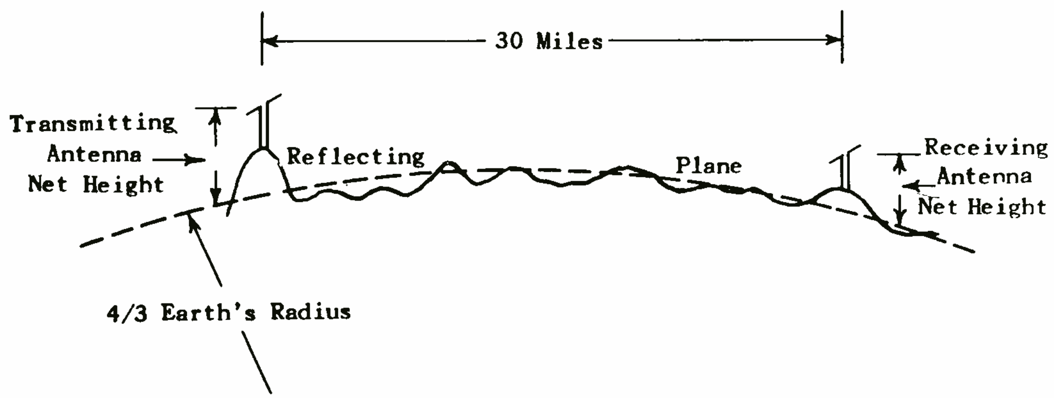


Fig. 5.—Profile map showing use of reflecting plane in calculating propagation characteristics.

Over land paths, a profile map of the intervening terrain can be drawn in order to determine the net heights of the transmitting and small compared to the distance of transmission d_t. Such assumption is permissible in most cases encountered in practice.

If the wavelength λ is sufficiently small, then for given values of h_t, h_r, and d_t, ψ may come out to be 180°. In this case the resultant field strength will be a maximum. In point-to-point communication it may be possible to choose λ, h_t, h_r, and d_t to obtain this desirable result. In other words, control of the propagation is much greater in this class of service than it is in broadcasting.

A simple example will illustrate the use of Eq. (1). Suppose h_r = 30 feet = 9.14 meters (30 × .3048); h_t = 120 feet = 36.6 m. The distance d_t will be chosen the maximum possible for line-of-sight transmission. This is given by the formula

$$d_t = 1.414\,(\sqrt{h_t} + \sqrt{h_r}) \qquad (8)$$

where d_t is in miles, and h_t and h_r are in feet.

In the above problem

$$d_t = 1.414\,(\sqrt{120} + \sqrt{30})$$

$$= 1.414\,(10.98 + 5.48)$$

$$= 23.3 \text{ miles}$$

$$= 37{,}500 \text{ meters}$$

The value of d_t is so much greater than h_t and h_r that all formulas apply. From Eq. (1)

$$\psi = \frac{4\pi \times 9.14 \times 36.6}{37{,}500\,\lambda} = \frac{.1122}{\lambda}$$

Suppose the frequency is 60 mc. Then

$$\lambda = \frac{3 \times 10^8 \text{ meters}}{60 \times 10^6} = 5 \text{ meters}$$

Substituting this value of λ in the preceding expression, we obtain

$$\psi = \frac{.1122}{5} = .0225 \text{ radians}$$

$$= 1.29° = (.0225 \times 57.3°)$$

The resultant or net field strength in terms of the direct-ray field strength E_d is given by Eq. (2) as

$$E_t = E_d \sin\psi = E_d\,\psi = .0225\,E_d$$

i.e., the net field strength is slightly over two per cent of the direct ray at 60 mc for the heights and distance chosen.

Suppose, however, the frequency was 3,000 mc, λ = 10 cm. From Eq. (1) ψ now comes out to be (.1122) ÷ (.1) = 1.122 radians or 64.3°. This indicates that the ground-reflected and direct rays are more nearly in quadrature rather than in phase opposition, so that E_t may be expected to exceed E_d. For such a large value of ψ Eq. (3) rather than Eq. (2) must be used to calculate E_t. Thus,

$$E_t = E_d\sqrt{2 - 2\cos 64.3}$$

$$= E_d\sqrt{2 - 2(.4337)} = 1.065\,E_d$$

For this frequency the net or resultant field strength is 6.5 per cent *greater* than the direct ray E_d. Furthermore, if a half-wave dipole is employed, Eq. (4) shows that for the same power w, E_d is independent of frequency.

The above simple calculation indicates the advantage of using higher frequencies in the u.h.f. region if the antenna elevations are low. On the other hand, it must not be overlooked that if the elevations are low, the line-of-sight distance, d_t, is shortened too. Hence, even at 3,000 mc high elevations may be required to cover large distances, and in such a case ψ may possibly approach 2π radians (360°), whereupon complete cancellation will occur between the two components of the space wave.

However, at these frequencies λ is so short that a minor change in the actual elevation of either antenna will change ψ from 360° to 180° or 540°, whereupon complete reinforcement instead of cancellation will be had. In particular, if the height of either antenna is increased until ψ = 540°, the line-of-sight distance d_t will be increased, too.

An example will make this clear. Let λ = 10 cm, d_t = 18.34 miles or 29,500 meters, h_r = 20.5 meters, and h_t = 72 meters. See Fig. 6, points T and D. The curvature of the earth has been exaggerated in this figure to bring out better the various antenna heights. A simple check by means of Eq. (8) will show that d_t is less than the line-of-sight distance for the value of h_t and h_r chosen. Then

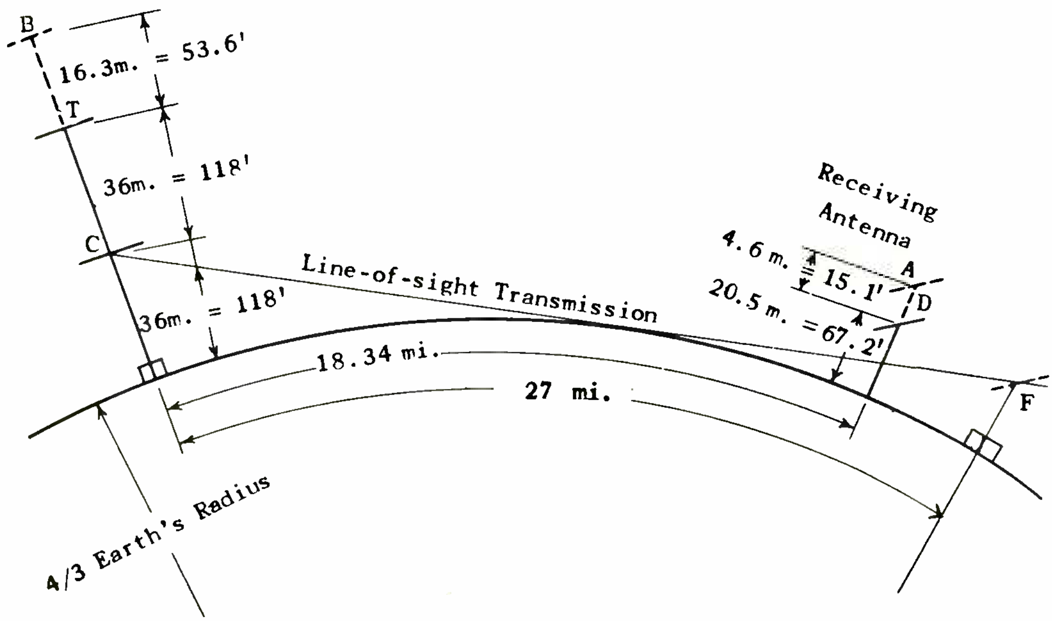


Fig. 6.—Effect of varying transmitting and/or receiving antennas on received signal.

$$\psi = \frac{4\pi \times 20.5 \times 72}{.1 \times 29{,}500} = 2\pi = 360°$$

This is the condition for complete cancellation: $E_t = 0$. To raise ψ to $3\pi = 540°$, the quantity $h_t h_r$ must be increased to $1\frac{1}{2}$ times its previous value if d_t is unchanged. This can be accomplished by increasing either h_t or h_r by the factor 1.5, or both by a smaller factor. For example, each could be increased in the same proportion by the factor $\sqrt{1.5} = 1.225$. This represents a 22.5 per cent increase in the height of each. On a percentage basis it is considerable, but in actual values it is only 25.1 - 20.5 = 4.6 meters or 15.1 feet for h_r (point A), and 88.3 - 72 = 16.3 meters or 53.6 feet for h_t (point B). In this case it might be preferable to increase only h_r by 50 per cent, i.e., from 20.5 meters (67.2 feet) to 30.8 meters (100.1 feet).

On the other hand, in the above problem d_t was less than the line-of-sight distance. Suppose h_t is reduced to one-half. ψ will be one-half its previous value or 180° instead of 360°. Reinforcement instead of cancellation will take place just as for $\psi = 540°$. The value of h_t will be 36 meters = 118 feet (point C), and h_r will remain at 20.5 meters = 67.2 feet (point D). Then the line-of-sight transmission will be

$$d_t = 1.414(\sqrt{67.2} + \sqrt{118})$$

$$= 1.414(8.2 + 10.88)$$

$$= 27 \text{ miles}$$

which exceeds the desired distance of 18.34 miles. At this maximum distance is shown an antenna, F, in dotted lines in Fig. 6. It is evident from this figure that the actual antenna D is above the line-of-sight transmission of a transmitting antenna represented by height C.

In general, at these higher frequencies, line-of-sight considerations often are the determining factor rather than the interference effects between the two components of the space wave, and once the heights are chosen to give at least line-of-sight transmission, corrections can then be made to obtain the optimum signal.

Where the antenna heights are low, and the frequency not too high, then ψ, the phase shift of the ground-reflected ray, is small. In such a case the simple approximate formula for E_t is satisfactory, or its more expanded form as given by Eq. (5). The latter is based on the use of a half-wave transmitting dipole. If a similar receiving antenna is employed, Eq. (6) gives the *voltage* induced in the antenna, and Eq. (7) the *power* received in a matched load of 75 ohms.

These two equations indicate that the induced voltage and received power are independent of the frequency and wavelength, whereas the preceding discussion indicated a wide deviation in field strength, hence, in induced and received power. The answer to this apparent contradiction is that Eqs. (6) and (7) refer only to heights so low that the phase shift ψ is small, whereas the preceding discussion involved phase shifts as high as 540°. In practice such values can occur for fairly high antennas and very high frequencies.

One further matter is of interest at this point. It is evident from Fig. 2 that if instead of a receiving antenna at point C, a reflecting object were located there, and the receiving antenna located at A adjacent to the transmitting antenna, that reception would involve transmission from A to C and thence back to A again. Thus, each path—that for the direct and that for the ground-reflected ray—is traversed twice. The angle ψ is therefore doubled, and the effect is as if $h_t h_r$ was doubled. However, the actual signal picked up at A depends upon the size and reflection coefficient of the object at C.

The above discussion was based on the use of half-wave dipoles. It is, however, more convenient to use directional arrays in the u.h.f. range. In practice, the physical size of the array is usually limited; i.e., the array occupies a certain area. This is called the aper-

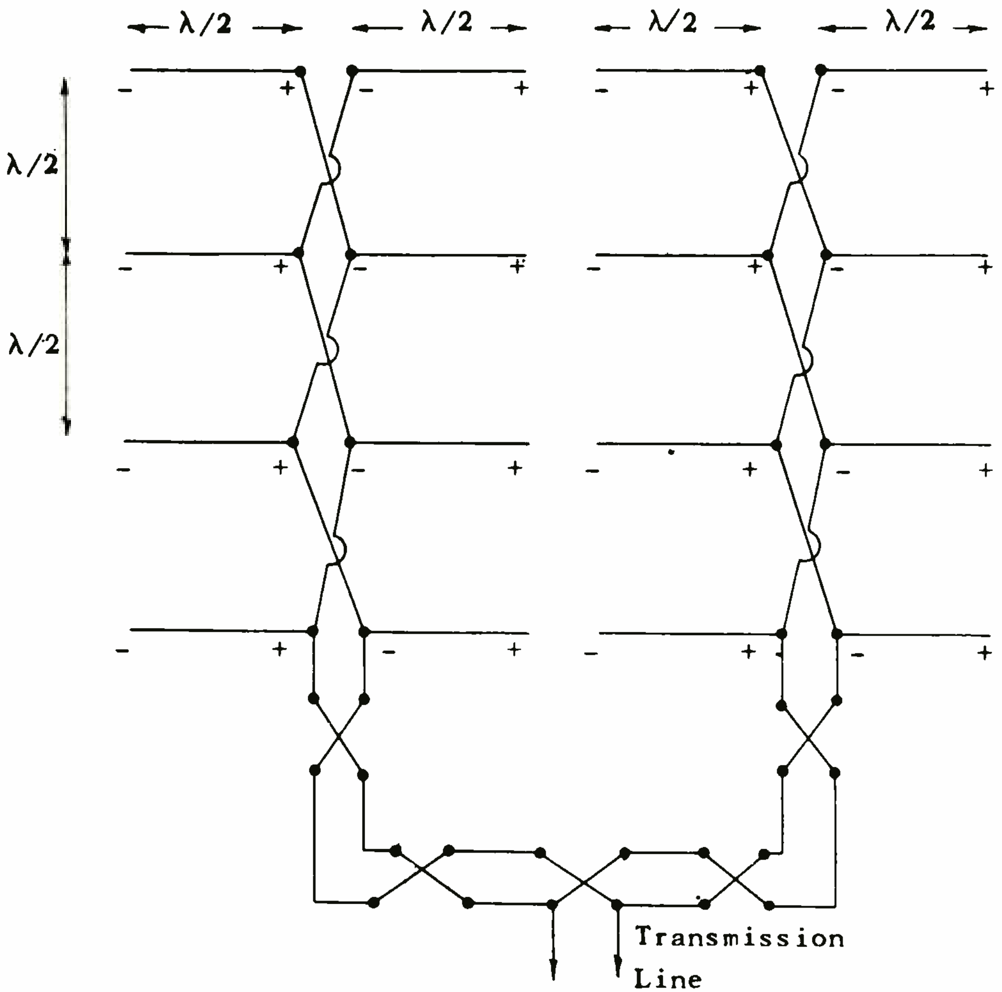


Fig. 7.—Example of an eight-element antenna array.

ture, and let it be represented as occupying S square meters.

An example of an array is shown in Fig. 7, and was discussed in a previous assignment. It transmits or receives a very narrow beam of radiation. In this array each element is $\lambda/2$ in length. In the figure, four elements are shown along the vertical, and two along the horizontal. The vertical spacing is $\lambda/2$, but the spacing between horizontal elements is very small, and can be ignored.

Hence, the horizontal width of the array is 2λ, and the vertical distance is $(3/2)\lambda$. At 100 mc, for example, λ is 3 meters, so that the array occupies a space of $2 \times 3 = 6$ meters by $3/2 \times 3 = 4^1/_2$ meters, and thus forms an aperture of $6 \times 4^1/_2 = 27$ square meters. This is the value of S for this array.

For 200 mc, the array would have to be composed of 4 times as many antennas to form the same size aperture, since each antenna would be half as long, and the spacing would be half as great. In the discussion that follows, it will be assumed that S is the same for all frequencies under consideration.

It will be recalled from an earlier assignment that energy is radiated from an antenna into space with the speed of light. Through any area in space there consequently passes a certain amount of energy, and the *rate* at which it passes through represents so much *power* through this area.

The amount of energy at any point of the area is proportional to the product of the electric and radiation fields, E and H, at that point (the Poynting vector), and since either field can be regarded as generating the other by its motion, the energy can be regarded as proportional to the square of either E or H. When all units are taken into account, the flow of energy, or power, is $E^2/120\pi$ watts per square meter, assuming that E is the same at every point (uniform field intensity). Thus, a transmitting antenna of aperture S radiates an amount of power

$$P_t = SE^2/120\pi \qquad (9)$$

and a receiving antenna of the same aperture absorbs a corresponding amount of power.

The increase of power absorbed by the area S of the directional array over that received by a half-wave dipole represents the power gain of the array over that of the dipole. Although the received power has been evaluated by Eq. (7) in terms of that radiated by a similar half-wave dipole, it will be preferable in the present case to evaluate the gain in terms of the field strength E in the region regardless as to how it is produced there.

The effective length of a half-wave dipole is λ/π, hence, the induced voltage is $(E\lambda)/\pi$. Under matched conditions half of this voltage of $(E\lambda)/(2\pi)$ appears across the 75-ohm load; so that the received power is

$$\left(\frac{E\lambda}{2\pi}\right)^2 \Big/ 75 = \frac{E^2\lambda^2}{300\,\pi^2} \text{ watts}$$

The power gain of the array over that of the dipole is then

$$G = \frac{SE^2/120\pi}{E^2\lambda^2/300\pi^2} = 2.5\pi S/\lambda^2 \quad (10)$$

The gain is the same whether the antennas are used for reception or transmission. Hence, if it is assumed that arrays of aperture S replace the dipoles at both ends of the system, the overall gain will be G^2.

Then Eq. (7) can be modified by simply multiplying it by G^2 to get the received power when arrays are used at each end. Then

$$w_r = \frac{2.6wh_t^2h_r^2}{d_t^4} G^2$$

$$= 16.2\pi^2 wS^2 \frac{h_t^2h_r^2}{d_t^4\lambda^4} \qquad (11)$$

where w, it will be recalled, is the power radiated from the transmitter, and h_t, h_r, and λ are in the same units, say, meters, and S is in similar square units, such as square meters. If the two antenna heights are chosen equal, i.e., $h_t = h_r = h$, then

$$w_r = 16.2\pi^2 wS^2 (h/d_t\lambda)^4 \qquad (12)$$

Eq. (12) indicates that the received power is proportional to the fourth power of the antenna heights, and inversely proportional to the fourth power of the wavelength and also the distance.

Hence, with fixed-aperture antennas, there is a very marked advantage in going to higher frequencies, *providing the same amount of power w is available at the higher frequencies*. This, however, is unfortunately not the case. It will be seen later that as the frequency goes up, the power output of tubes goes down.

In using Eqs. (11) and (12) the same precautions should be employed as in using the previous equations. The former are based on the same approximations, namely, that h_t and h_r are small compared to d_t, and that the phase shift ψ is small. The precautions are particularly to be employed in calculations at very high frequencies, where the antenna height may be relatively great in order to obtain a large line-of-sight distance, and consequently ψ may be very large. In such a case the net field strength at the receiving antenna should be calculated by Eq. (3) rather than by Eq. (2) for use in subsequent calculations.

For example, the voltage induced in a receiving dipole is still $E_t\lambda/\pi$, regardless of whether E_t is calculated by Eq. (2) or Eq. (3). The gain of an aperture S over a dipole is still given by Eq. (10). Consequently, the power output can still be calculated on the basis that it is $E^2\lambda^2/300\pi^2$ watts for a dipole, and $SE^2/120\pi$ watts for an aperture.

The previous analysis was based on a small phase angle ψ. As a result, as one goes up in frequency, the increase in phase shift for a given set of antenna heights and separation results in a practically proportional increase in ψ and hence in the field strength E_t. Consequently, Eq. (7) for half-wave dipole antennas indicates that the received energy is independent of the frequency because E_t increases with frequency just as rapidly as the effective length of the antenna decreases, and Eqs. (11) and (12) indicate that the received energy increases as the fourth power of the frequency (or as $1/\lambda^4$) in the case of *antenna arrays of constant aperture*.

However, as one goes up above 1,000 mc or thereabouts, the phase angle ψ becomes very appreciable for reasonable long distances of transmission, because the antenna heights must be sufficiently great to afford line-of-sight paths in accordance with Eq. (8). In this case it is usually possible to choose antenna heights such that ψ is an odd multiple of 180°, and maximum reinforcement of the direct ray is afforded by the ground-reflected ray. If such is the case for some particular frequency, then for a higher frequency cancellation instead of reinforcement will occur, and the field strength E_t will decrease. It will then be found that the received energy goes down as the frequency is increased.

In order to make a fair comparison when ψ is large, suppose that the antenna heights can be adjusted at each frequency to give line-of-sight transmission and also maximum reinforcement between the two components of the space wave. In that case the field strength at the receiver will be independent of frequency, i.e., E_t will be constant.

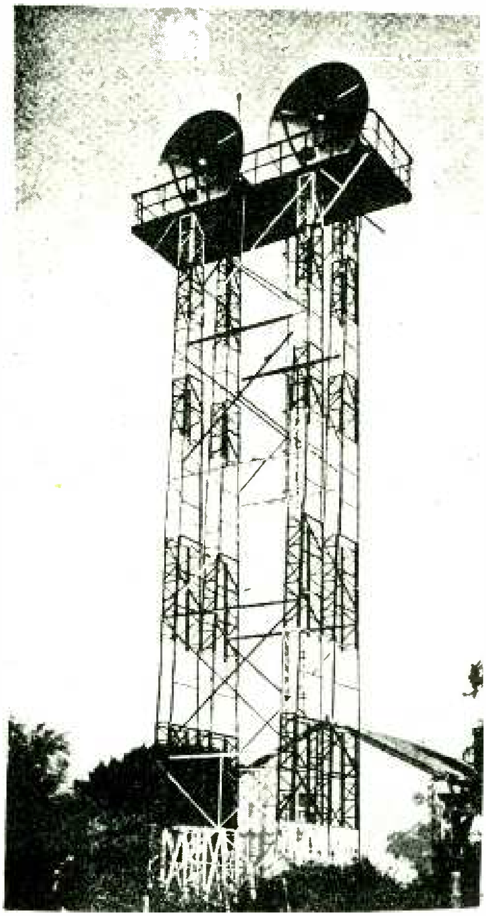
Now suppose that a half-wave receiving dipole is employed. If the frequency is doubled, the actual and also the effective length of the receiving dipole is halved, while E_t remains constant. The induced voltage, which is equal to E_t multiplied by the effective length, will therefore be halved, and the received power w_r, which is proportional to E_t^2, will therefore be one-quarter of its previous value. In the case of an aperture of area S, the received power will be constant and equal to $SE_t^2/120\pi$, instead of increasing as indicated by Eqs. (11) and (12).

In short, if maximum effect from ψ can be had at any frequency, there is a disadvantage in going to a higher frequency when a dipole receiving antenna is used, and no advantage nor disadvantage when an array of fixed aperture S is employed.

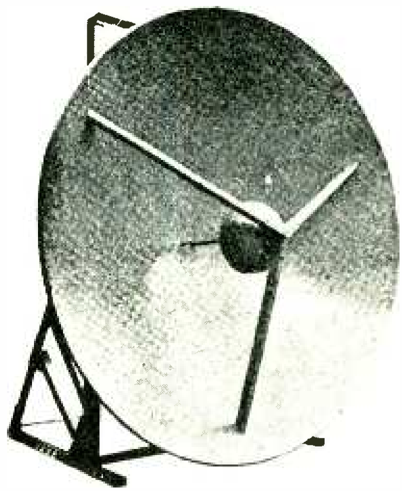
As a matter of fact, the amount of energy extracted by an antenna is from a section of the wave front that extends roughly one-quarter wavelength away from the antenna. If E_t is constant throughout this space, as assumed here, then evidently as one goes to higher frequencies, the decrease in λ will mean a decrease in energy extracted, particularly so in the case of a half-wave dipole. At very high frequencies the energy extracted will be so small as to be incapable of overriding the tube noise in the receiver. For this reason arrays, horns, and parabolic reflectors are employed, not only to obtain directional characteristics, but also to extract energy from a reasonable amount of space and thus increase the signal input. An example of a parabolic reflector is shown in Fig. 8 and was used in the hyperfrequency link across the English Channel between England and France.

A final point concerning interference should be noted. At the lower end of the u.h.f. region, atmospheric static is practically non-existent, but man-made interference is of importance. Thus, car ignition noises, and interference from diathermy machines and other electrical equipment can cause considerable trouble, particularly in the case of a high-grade service such as television.

As a general rule, it appears that the disturbing energy is mainly vertically polarized. This is due to the fact that a horizontal radiator close to the earth—such as the horizontal portions of the brush leads of a sparking motor—has its assignment on antennas.) Consequently, there is much to recommend the use of horizontal polarization for television, for the horizontal



(A)



(B)

(A) French terminal of the hyper-frequency link across the English Channel.

(B) A parabolic reflector used for the creation of a hyper-frequency.

Courtesy of Electrical Communications)

Fig. 8.—Parabolic reflectors used in u.h.f. transmission.

radiation effectively cancelled by that of its ground image, since the currents in the two flow in opposite directions. Vertical radiators, on the other hand, have a ground image whose current flow is in the same direction as the radiator, so that the effects are additive. (This has already been mentioned in the receiving antenna will be little affected by a vertically polarized wave.

Furthermore, it has already been mentioned that in the aerial navigation services employing frequencies in the order of 100 mc, horizontal polarization is favored because it appears to be much less affected by variations in the conductivity of the terrain, such as due to snow or rain.

At the very high frequencies, there is an absence of both atmospheric as well as man-made static. This is not surprising when one considers how difficult it is to generate such high frequencies. Very few pieces of electrical equipment are small enough to have natural resonances at the very high frequencies. However, tube noise is still present to as great an extent as at the lower frequencies, and in view of the small energy pickup, at least by a half-wave dipole, the tube noise becomes the important factor in determining how weak a signal can be handled. This will be discussed in greater detail farther on.

CIRCUIT ELEMENTS

TRANSMISSION LINES.—Mention has been made that as the frequency goes up, the size of the circuit elements required for resonance decreases until ultimately they become too small to be practical. It is then that transmission lines enter favorably into the picture and begin to function as circuit elements. An important feature is that the connecting wires, such as these within the tube (from the socket pin to the tube electrodes) can be incorporated with the external line to form a continuation of it, with the result that such connections are possibly an aid rather than a hindrance to the generation of u.h.f. It is advisable at this point to review the material on transmission lines that appears in the preceding assignments on antennas.

A line that is terminated in its characteristic impedance Z_o is termed a non-resonant line. The r.m.s. values of the voltage and current decrease by small amounts as one proceeds down the line, depending upon the line losses. For short lengths, the decrease is practically negligible.

When a line is not terminated in Z_o, then reflections take place at the termination, and the incident and reflected waves of voltage or current reinforce each other at some points of the line, and cancel each other at other points, thus giving rise to nodes and loops in the current and voltage.

Such effects are a maximum for complete mismatch: open- or short-circuit terminations, and for line lengths that are multiples of $\lambda/4$. For example, a line 10 cm in length is a $\lambda/4$ line at 750 mc. If open-circuited at its far end then it appears essentially as a short-circuit at its near end, that is, like a series-resonant circuit. If it is shorted at its far end, it appears as an open circuit at its near end (parallel resonant circuit).

Note the appreciable length 10 cm that acts as a series-resonant circuit at the high frequency of 750 mc. An equivalent lumped L-C circuit would have impracticably small circuit components, whereas the line is of reasonable length. Also its losses are much less, that is, its Q is higher than that of a lumped circuit.

Lines of $\lambda/2$ in length behave oppositely to those $\lambda/4$ in length, e.g., an open-circuited $\lambda/2$ line looks like an open circuit at its near end (parallel resonant). A line $3\lambda/4$ in length behaves like a $\lambda/4$ line, etc. Generally, resonant lines are $\lambda/4$, or at most $\lambda/2$ in length, because the losses go up with the length (measured in wavelengths) and the Q goes down.

Q OF A RESONANT LINE.—In ordinary circuit theory, the Q of an inductance coil, for example, is the ratio of the reactance of the coil to its resistance, or $Q = \omega L/R$.

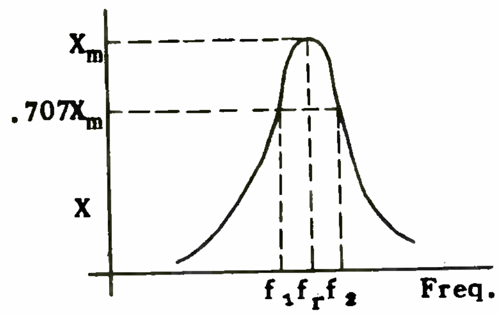


Fig. 9.—Resonance curve, showing method of measuring the Q of the resonant circuit.

For an L-C circuit it can be shown to depend upon the sharpness of the resonance curve. This is illustrated by Fig. 9.

Let f_r be the resonant frequency. Also let f_1 and f_2 be two frequencies at which the reactance is $1/\sqrt{2}$ or .707 that at resonance, denoted by X_m. For a circuit of normally high Q, f_1 is approximately as much below f_r as f_2 is above it, i.e., $(f_r - f_1) = (f_2 - f_r)$. The Q of the circuit can be shown to be

$$Q = \frac{f_r}{2(f_r - f_1)} = \frac{f_r}{2(f_2 - f_r)} \qquad (13)$$

The closer f_1 and f_2 are to f_r, the higher is the Q, and the sharper is the resonance curve.

In the case of an actual resonant line that has resistive losses, such as a $\lambda/4$ line shorted at its far end, the resonance curve does not go off to infinity, but has more nearly the shape shown in Fig. 9, i.e., it has a finite peak. Accordingly, an expression for its Q can be found on the basis of Eq. (13). Thus

$$Q = \frac{2\pi f_o Z_o}{cR} = \frac{2\pi Z_o}{R\lambda_o} \qquad (14)$$

where Z_o is the characteristic impedance of the line.
f_o is the frequency at which it is resonant as a quarter-wave line.
λ_o is the corresponding wavelength in meters.
c is the velocity of light = 3×10^8 meters per second.
R is the resistance per loop meter.

The resistance R of the line depends upon its construction, and exceeds the d.c. resistance of the conductors owing to skin effect, which forces the current to the surface of the conductors and thus increases the resistance. For a two-wire line, composed of two copper conductors each having a radius of a, in meters, see Fig. 10, the resistance is given by the formula

$$R = 8.32 \frac{\sqrt{f}}{a} \times 10^{-8} \ \Omega/\text{loop meter} \qquad (15)$$

where f is the operating frequency.

If the dimensions are given in inches,

$$R = \frac{\sqrt{f}}{a} \times 10^{-6} \ \Omega \text{ per loop foot} \qquad (15a)$$

where a is in inches.

For a coaxial line

$$R = 4.16 \times 10^{-8} \times \sqrt{f} \times \left(\frac{1}{a} + \frac{1}{b}\right) \Omega/\text{loop meter} \quad (16)$$

where a and b are in meters (see Fig. 11). Also

$$R = 5\sqrt{f}\ (1/a + 1/b) \times 10^{-7} \text{ ohms per loop ft.} \quad (16a)$$

when a and b are in inches.

It will be noted that R varies as the square root of the frequency.

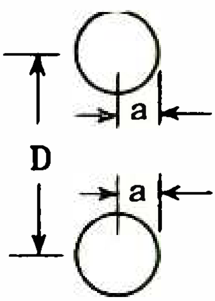


Fig. 10.—Important dimensions for a two-wire line.

Hence, substitution in Eq. (14) of the appropriate formula (15), (15a), (16) or (16a) gives rise to a $\sqrt{f}$ term in the denominator. The frequency also appears to the first

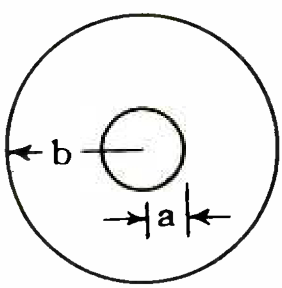


Fig. 11.—Important dimensions for a coaxial line.

power in the numerator. Dividing $\sqrt{f}$ of the denominator into f of the numerator produces $\sqrt{f}$ in the numerator. This means that Q varies as the square root of f, i.e., increases with f. Thus, if the frequency is increased *four-fold*, the Q is *doubled.*

As a result, resonant transmission lines are particularly valuable at high frequencies because their Q is much higher than that of a practical lumped circuit, and at the same time the quarter-wave line becomes sufficiently small in physical size to be practical. It will be instructive to evaluate the Q of some quarter-wave lines. For this purpose the expression for the characteristic impedance of a line must be given.

For a two-wire line this is (refer to Fig. 10)

$$Z_o = 276 \log \frac{D}{a} \text{ in ohms} \quad (17)$$

Since only the ratio of D to a is involved, Eq. (17) requires only that D and a be measured in the same units—inches or meters, etc.

For a concentric line (ignoring the effect of the dielectric spacers)

$$Z_o = 138 \log b/a \quad (18)$$

where again b and a (refer to Fig. 11) have only to be measured in the same units.

Consider now a copper two-wire quarter-wave line having the following constants:

$a = 0.5$ inch

$D = 2$ inches

$f = 300$ mc

It is desired to calculate the Q of this line. First calculate Z_o.

$$Z_o = 276 \log 2/.5 = 276 \log 4$$

$$= (276)(.6021) = 166 \text{ ohms}$$

The wavelength corresponding to 300 mc is $3 \times 10^8 \div 300 \times 10^6 = 1$ meter $= \lambda_o$. From Eq. (15a),

$$R = \frac{\sqrt{3 \times 10^8}}{.5} \times 10^{-6}$$

$$= .0346 \text{ ohm per foot}$$

$$= .0346 \times 3.28$$

$$= .1135 \text{ ohm per meter}$$

Then

$$Q = \frac{2\pi \times 166}{.1135 \times 1} = 9{,}180$$

A Q of 500 is considered very good for lumped circuits, so that it is evident that the quarter-wave line has a very high Q indeed.

A similar example can be worked out for a concentric line. Suppose this has an inner conductor of .5-inch radius too, and an outer conductor of 3.6 × .5 = 1.8 inches radius. The reason for using the factor 3.6 is that the losses of a coaxial cable are at a minimum when the ratio of outer to inner conductor is 3.6. Similarly, it will be noted above for the two-wire line that the ratio D/a is four. This is about the optimum for a two-wire line and gives the highest Q.

The characteristic impedance is

$$Z_o = 138 \log 3.6 = 138 \times .5563$$

$$= 76.8 \text{ ohms}$$

Also

$$R = 5\sqrt{3 \times 10^8}\,(1/.5 + 1/1.8) \times 10^{-7}$$

$$= .0221 \text{ ohm per loop foot}$$

$$= .0221 \times 3.28$$

$$= .0725 \text{ ohm per loop meter}$$

Then

$$Q = \frac{2\pi \times 76.8}{.0725 \times 1} = 6{,}650$$

It will be noted that the Q is actually less for the concentric line than it is for the two-wire line. This is because the concentric line has inherently a lower characteristic impedance. There are other factors, however, that must be taken into account. For example, a two-wire line radiates appreciable amounts of energy, particularly at the higher frequencies, whereas a coaxial line does not, particularly when shorted at the far end. On the other hand, a two-wire line is easier to adjust, i.e., to make changes in its spacing or in its length to vary respectively its characteristic impedance or resonant frequency.

SHORT- AND OPEN-CIRCUITED LINES.—In general, short-circuited lines are preferred to open-circuited lines because of the radiation losses from the open-end of the latter and possibility of coupling to other circuit elements. Nevertheless, open-circuited concentric lines are feasible if the outer radius is below the cutoff frequency of the equivalent wave guide (to be explained later), and the outer conductor extends a short distance beyond the inner one.

Completely to short-circuit a line, it is necessary to short circuit all of the electric field lines existing between the conductors. However, it is satisfactory merely to cap the end of a concentric cable with a copper disc that makes good contact with the outer and inner conductors, and in the case of a two-wire line, to use a disc of copper, as in Fig. 12, that is, whose

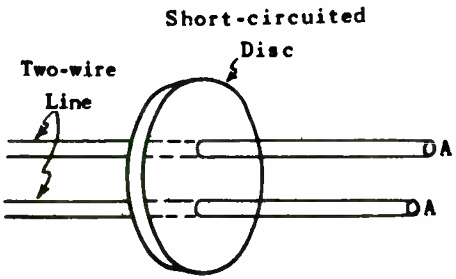


Fig. 12.—**Method of short-circuiting a two-wire line.**

diameter is three or four times the spacing between wires.

A further refinement, if a sufficient amount A-A projects beyond the short circuit, is to add another shorting disc a distance λ/4 behind the first disc. This makes the impedance of portion A-A appear as practically an open circuit at the location of the main shorting disc, and thus prevents an appreciable current from being set up in it by induction from the section to be used. This in turn prevents the unused portion from affecting the impedance of the other section.

If a shorting bar is used on a two-wire line, as in a Lecher wire system, the short-circuiting will not be as complete as for the disc shown in Fig. 12, and the Q of the line will not be as high, but the line may nevertheless be quite satisfactory for wavelength measurements, etc., where too high a Q gives too sharp a resonance peak and makes its detection difficult.

DEPTH OF PENETRATION.—Before discussing further properties of lines, it will be well to note the depth of material through which high-frequency currents flow. Consider a circular conductor as shown in Fig. 13, through which a high-frequency current is flowing. The current sets up circles of flux around the conductor as shown by the broken lines. Note that central

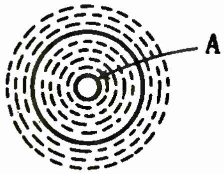


Fig. 13.—**Flux in a conductor produces skin effect.**

section A of the conductor is surrounded by more circles of flux than the surface of the conductor.

Since the inductance of a circuit, or portion thereof, is determined by the amount of flux the current through that portion can establish, it is evident that section A of the conductor will have more inductance and hence more inductive reactance than an annular (ring-like) section at the surface.

The voltage drop per unit length of the conductor must be the same for all sections since they are effectively all in parallel. Let its value be E. Then

$$E = IX_L$$

where I is the current through the section of the conductor, and X_L is the reactance of the section. If one section has a higher reactance than another section, the current through it must be less than that through the other. As a consequence, the current through the center A of the conductor is far less than that on the surface, and this effect becomes more marked as the frequency and hence difference in reactance increases.

At frequencies above one megacycle, the current flow is confined mainly to the surface, and tapers off rapidly from its maximum value there to practically zero at the center according to an exponential law. The depth of penetration is defined as the thickness of a surface shell that has the same resistance to d.c. as the entire conductor cross section has to the u.h.f. a.c. This does not mean that no current flows below this depth. However, if the actual conductor thickness is, say, four times the depth of penetration, then it is of ample thickness to carry the current with substantially the same losses as a conductor of infinite thickness.

The depth of penetration for *copper* is given by

$$S = \frac{0.0664}{\sqrt{f}} \text{ meters} = \frac{2.61}{\sqrt{f}} \text{ inches} \quad (19)$$

where f is the frequency in cycles per second. It will be instructive to employ this formula in calculating one or two examples.

1. An iron chassis for a transmitter is to be copper-plated to function as a highly conductive coating. The operating frequency of the transmitter is 1 mc. Calculate the depth of penetration and form an estimate as to the thickness of plating required.

$$S = \frac{2.61}{\sqrt{10^6}} = \frac{2.61}{10^3} = .00261 \text{ inches}$$

or 2.6 mils

(A mil is one-thousandth of an inch.) If the copper coating is about 4 × 2.6 = 10.4 mils or a little over .01 inch, very little current should flow in the iron, and hence this copper-clad chassis should be as good as an all-copper chassis. It is to be noted, however, that a plating .01 inch thick is rather heavy.

2. Suppose the operating frequency is 1,000 mc. Then

$$S = 2.61/\sqrt{10^9} = \frac{2.61}{3.16} \times 10^{-4}$$

$$= 82.6 \times 10^{-6} \text{ inches}$$

The depth of penetration is roughly 80 millionths of an inch! A coating about 4 × 80 = .32 mil would be sufficient. For this reason, a thin film of silver plate, for example, on a base metal such as iron, will function just as well as a solid silver conductor. At u.h.f. one seldom has to concern himself with the thickness of the coating. It is generally far in excess of that required by Eq. (19).

It is to be noted that some correction is necessary for the case where the bend in the metal is of a radius less than the skin depth. This will be encountered only in the case of machined, sharp corners rather than in a bent-up sheet.

FURTHER REMARKS.—The fact that the current flows near the surface of the conductor indicates that the resistance of a conductor cannot be decreased by increasing the cross-sectional area of the conductor, but rather by increasing its surface, i.e., the perimeter of the cross section, which can be hollow.

The fact that a concentric line has maximum Q when b/a = 3.6 means that if a, for example, is chosen, then b should be 3.6 a for maximum Q. If a is made ten times as great, then b will also be increased ten times, and the new line will have a resistance per unit length one-tenth as great as per Eqs. (16) and (16a). The characteristic impedance Z_o, however, depends upon the ratio b/a and not upon their actual values, so that Z_o will remain 76.8 ohms. Therefore, Eq. (14) shows that the Q of the larger line will be ten times as great. Thus, to increase the Q of the line, use larger values for a and b, and maintain their ratio at 3.6. The values for a and b must not be so great, however, that the connections thereto and any short-circuiting disc become of appreciable impedance themselves. Roughly, the diameter of the outer conductor of a concentric line, or the separation of a two-wire line, should be a fraction of a wavelength.

Another point to note is that a low-loss line that is not a multiple of $\lambda/4$ in length acts like a

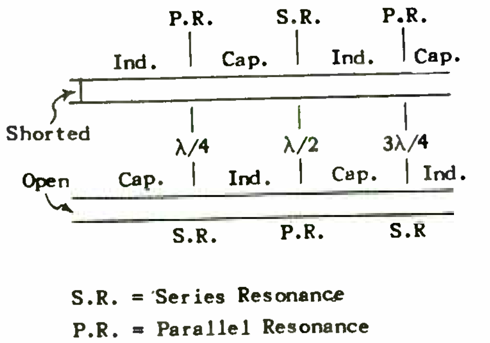


Fig. 14.—Variation in reactance with length, for an open and a shorted line.

capacitive or inductive reactance of high Q. The variations of the reactance for an open- and short-circuited line with length (or frequency) are shown in Fig. 14. Some uses of such elements will be given subsequently.

NON-RESONANT LINES.—It was stated previously (as well as in an earlier assignment) that when a line is terminated in its characteristic impedance, no reflections take place at the termination, and the line behaves exactly like a line of infinite length. The *magnitude* of the current and of the voltage remains constant all along the line (if the latter has no losses) or decreases exponentially with distance from the generator end if the line has losses. There are no nodes or loops either for the current or the voltage anywhere along the line; the impedance looking into the line is Z_o, the characteristic impedance, regardless of the length of the line. It is said to be *non-resonant.*

It follows, therefore, that such a line will not appear to be reactive for lengths between even and odd multiples of $\lambda/4$ and resistive at multiples of $\lambda/4$. At radio frequencies the characteristic impedance is practically $\sqrt{L/C}$, where L and C are the inductance and capacity per unit length. This quantity—$\sqrt{L/C}$—is a pure resistance. Hence, a line so terminated will appear to have this value of resistance regardless of its length.

In a *resonant* line the current and voltage have loops and nodes (maxima and minima) at intervals of $\lambda/2$ along the line with the nodes of one $\lambda/4$ distance from the nodes of the other. The losses of the line vary as the square of the current (conductor resistance and radiation resistance) and as the square of the voltage (dielectric losses). These losses are higher in a resonant line than in a non-resonant line because of the larger current and voltage values at the loops in the former.

As a consequence, a non-resonant line is preferred when its purpose is merely to transmit power from one point to another, as it does so with a minimum of loss. A resonant line is used where one wishes to obtain reactances (capacitive or inductive) at high frequencies, or series or parallel resonance, as explained in the preceding section.

The property of a non-resonant line that its input impedance is equal to its characteristic impedance regardless of length is a valuable one. Thus, as a simple example, suppose that a dipole antenna, Fig. 15, is to be fed from a transmitter located at a distance

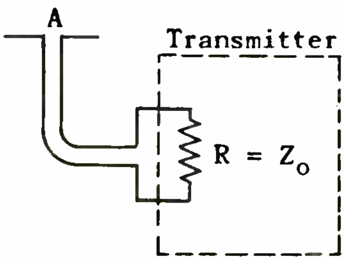


Fig. 15.—Terminating a line in its characteristic impedance makes it non-resonant regardless of its length.

from the antenna. The dipole is like a half-wave resonant line, corresponding to an open-circuited quarter-wave two-wire line, as explained in an earlier assignment. As such it should exhibit zero resistance at its center A, but owing to the losses in the antenna (principally radiation, which is desired from the antenna) it exhibits at A a resistance of 75 ohms. This is its so-called radiation resistance.

The transmission line feeding the antenna can be designed to have a characteristic impedance of 75 ohms. Thus, suppose a concentric line is employed, and a No. 10 inner conductor is used. It has a diameter of 0.1 inch. From Eq. (18),

$$Z_o = 138 \log b/a = 75$$

$$= 138 \log b/.05$$

or

$$\log b/.05 = 75/138 = .543$$

$$b/.05 = \text{anlg } .543 = 3.49$$

$$b = .05 \times 3.49 = .175''$$

This line can be of any desired length, and when the antenna is connected to it, it will be non-resonant. In short, so do the distributed capacity and inductance interact that so far as the transmitter is concerned, it sees a pure resistance of 75 ohms for *any* length of line.

On the other hand, *if the line were not designed so that its characteristic impedance were equal to the radiation resistance of the antenna*, i.e., merely a pair of wires connected between the transmitter and the antenna, then the impedance that the transmitter would see would be partly reactive, and of a value depending upon the length of the line. The result would be that not only would the line losses be increased, but the reactance appearing at the transmitter end would tend to detune the latter's tank circuit and require considerable adjustment for each value of line length.

Even when matched, however, a transmission line has losses owing to the resistance of its conductors and the dielectric losses. For the ordinary line that has mainly air for its insulation the principal loss is that of conductor resistance. Expressed in db, this is, in general, for a *terminated non-resonant line*,

$$\text{db power loss} = \frac{8.686\, Rl}{2Z_o} \qquad (20)$$

where R = the series resistance of the line *per foot*.
l = the length of the line in feet.
Z_o = the characteristic impedance of the line in ohms.

It will be recalled, however, that R is a function of the type of line and conductor size, as indicated previously by Eqs. (15), (15a), (16) and (16a).

Hence, the attenuation in db per meter for a *two-wire line* may be written

$$10 \log \frac{\text{Power out}}{\text{Power in}} = \frac{(1.3)(10^{-9})\sqrt{f}}{a \log_{10} D/a} \text{ db/meter} \qquad (21)$$

where a and D are in meters (see Fig. 10).

For a concentric line the attenuation is

$$\frac{(1.3)(10^{-9})\sqrt{f}}{\log_{10} b/a}\left(\frac{1}{a} + \frac{1}{b}\right) \text{ db/meter} \qquad (22)$$

where a and b are in meters (see Fig. 11).

As an example, consider the concentric line calculated above. What is its attenuation for 100 feet at 400 mc? From Eq. (22),

$$\text{db/meter} = \left[\frac{(1.3)(10^{-9})\sqrt{400 \times 10^6}}{\log .175/.05}\left(\frac{1}{.05 \times .0254} + \frac{1}{.175 \times .0254}\right)\right]$$

$$= .0484 \text{ db/meter}$$

$$= .0484 \div 3.28$$

$$= .01475 \text{ db/foot}$$

For 100 feet, the total db loss is .01475 × 100 = 1.475 db.

Suppose the input power P_i = 10 watts. Then the output power P_o can be found as follows:

$$10 \log P_o/P_i = -1.475$$

or

$$10 \log P_i/P_o = +1.475$$

$$P_i/P_o = \text{antilog } 1.475/10$$

$$= \text{antilog } .1475$$

$$= 1.405$$

then

$$1.405 = 10/P_o$$

$$P_o = \frac{10}{1.405} = 7.12 \text{ watts}$$

Power lost = 10 − 7.12 = 2.88 watts

It will be observed that the losses are relatively small for 100 feet of transmission.

THE LINE AS A TRANSFORMER.—The quarter-wave line has a further valuable property: it can act similar to a transformer in matching two unlike impedances. Thus, suppose an impedance Z_1 (Fig. 16) is connected to a quarter-wave line of characteristic impedance Z_o. As

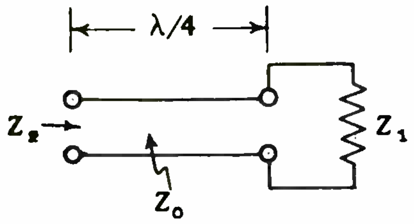


Fig. 16.—Use of a quarter-wave line to match two impedances.

viewed from the other end of the line, it appears as an impedance Z_2 of value

$$Z_2 = \frac{Z_o^2}{Z_1} \qquad (23)$$

or

$$Z_o = \sqrt{Z_1 Z_2} \qquad (24)$$

Eqs. (23) and (24) enable the engineer to build a line to match one impedance to another. For example, suppose it is desired to have a resistance of 800 ohms appear as 200 ohms. Then Z_1 = 800 ohms, and Z_2 = 200 ohms. From Eq. (24)

$$Z_o = \sqrt{800 \times 200} = 400 \text{ ohms}$$

The impedance transforming properties of a quarter-wave line are of great utility, especially when combined with some of the other properties of a line.

APPLICATIONS.—A few examples, together with calculations, will be given of the use of lines in actual circuits. In Fig. 17(A) is shown a lumped circuit type of oscillator. In Fig. 17(B) is shown a shorted quarter-wave resonant line counterpart of the former oscillator.

In the case of the line oscillator, C_1 acts as an a.c. short circuit but permits the plate d.c. potential to be different from that of the grid. It will also be noted that no r.f. choke is required, since the "B" supply connection is at a point of low r.f. potential.

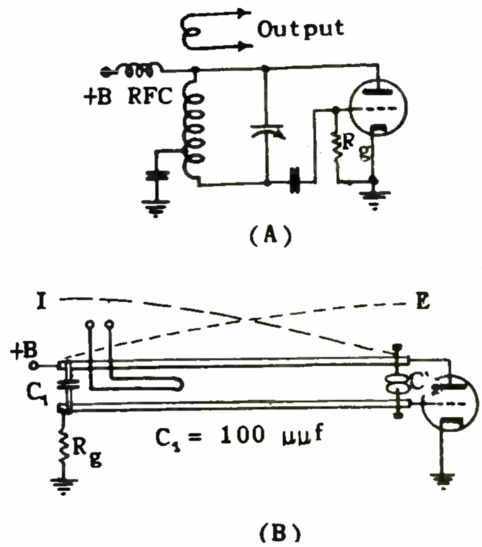


Fig. 17.—Lumped tank circuit and its transmission-line equivalent.

Indeed, at ultra-high frequencies it is rather difficult to build a lumped inductance whose distributed capacity is low enough to permit it to function as such. Note further condenser C_2. This is used to adjust the line to resonance over a narrow range of frequencies.

In addition to C_2 there are the interelectrode capacities of the tube shunting the line. These also serve to establish ground with respect to the two sides of the line. Thus, the plate-to-cathode capacity and the grid-to-cathode capacity place the two sides of the line at definite potentials with respect to the cathode and hence with respect to ground. They thus correspond to the tapped-condenser of the usual Colpitts circuit.

The use of a line instead of a lumped circuit permits elements of reasonably large size and of very high Q, to be used as has been indicated. Therefore, a line is of value for a tank circuit because its high Q or selectivity provides very good frequency stability. A quartz crystal is of value for this purpose at low frequencies, such as in the standard broadcast band, because of its exceptionally high Q. This is due to the fact that mechanical systems inherently have a high Q. But for u.h.f. stabilization, the crystal would have to be so small as to be impracticable. Fortunately, resonant lines become feasible in this range and take over the role of stabilizing the oscillator frequency very nicely. An example of their use will follow shortly.

In passing, it is to be noted that in order that the Q be high it is important that very little load be coupled to the resonant circuit.

Shortening of Lines.—It is evident that the presence of C_2 and the tube capacities require a line that serves as the inductance of the resulting tank circuit. A shorted line less than $\lambda/4$ will function as an inductance, as explained previously. The length of line required depends upon the capacity with which it is to tune, and also upon its own characteristic impedance and the operating frequency.

In Fig. 18(A) is shown a shorted line section that is to tune with a given capacitor C at some frequency $f = \omega/2\pi$. In analyzing the problem, the capacitor can be replaced by an open-circuited line less than $\lambda/4$ in length, since this has a capacitive reactance too.

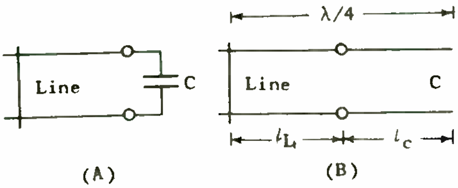


Fig. 18.—Use of a capacitor to decrease the length of a resonant line.

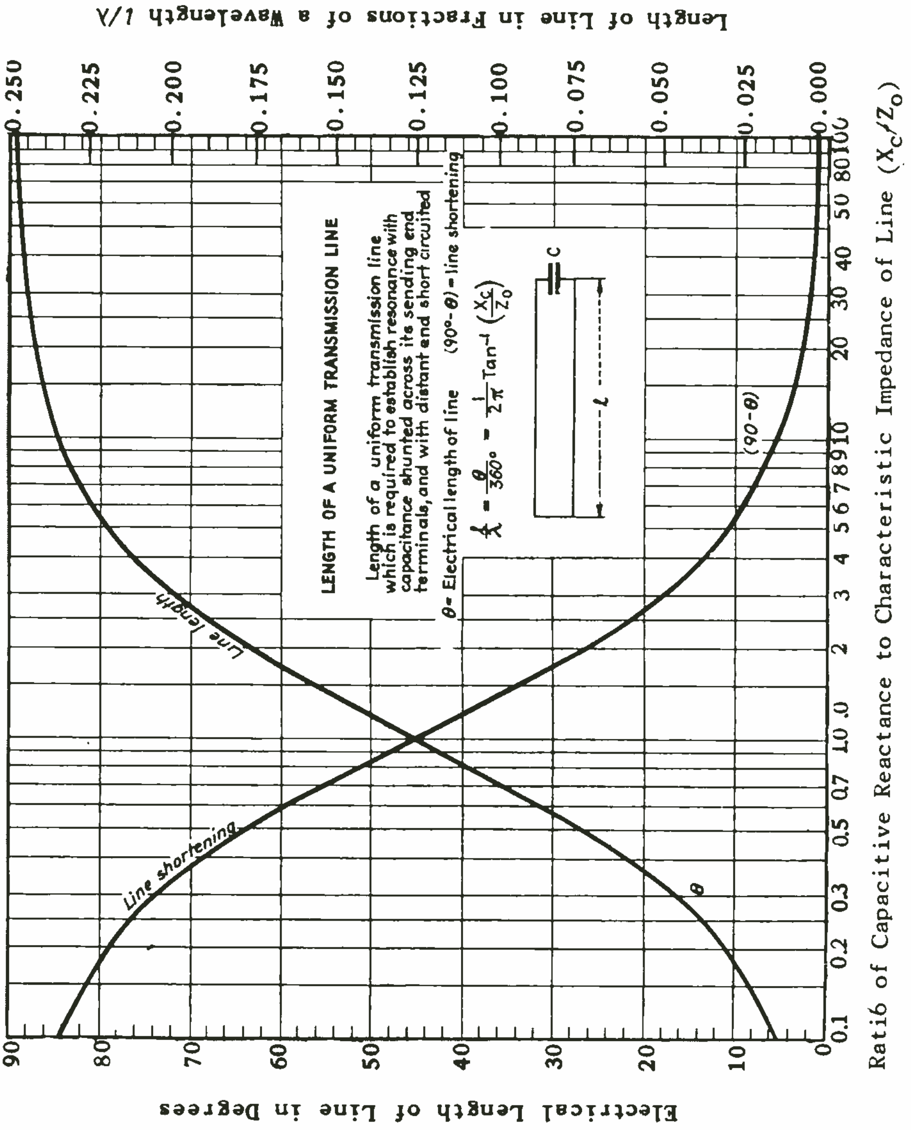
Thus, the actual line and capacitor can be replaced by an equivalent, longer line. This is shown in Fig. 18(B). The total length of the *actual* line l_L plus the *hypothetical* portion l_c that replaces C must clearly be $\lambda/4$ in length in order that parallel resonance occurs.

The length l_c is found from the following formula:

$$X_c = \frac{1}{\omega C} = \frac{Z_o}{\tan\left(\frac{2\pi l_c}{\lambda}\right)}$$

$$= Z_o \cot\left(\frac{2\pi l_c}{\lambda}\right) \qquad (25)$$

i.e., l_c is given in terms of the reactance X_c, of the condenser ($= 1/\omega C$), the characteristic impedance Z_o, and the wavelength of the line. Thus, once l_c is found, it can be subtracted from $\lambda/4$ to give the length l_L of actual line required to tune with the actual capacitor C. However, normally one is given only C and the frequency of operation. Eq. (25) indicates that Z_o must also be known before l_c can



ELECTRONICS REFERENCE SHEET

(Courtesy Electronics.)

Fig. 19.—Graphs giving the length of a shorted line that will resonate with a given capacitance across its input terminals.

be found and then l_L.

Since Z_o is not fixed by any of the requirements of the problem as it ordinarily arises, one can choose Z_o as one pleases. Usually one desires a line l_L to represent an inductance with as high a Q as possible.

It has been indicated that minimum losses and maximum Q are obtained for a concentric line if b/a = 3.6, and for a two-wire line if $D/a \cong 4$. These spacings also determine Z_o for the line. Thus, for a concentric line Z_o will be 76.8 ohms, and for a two-wire line Z_o will be 166 ohms. Therefore, these values for Z_o should normally be used in Eq. (25) to determine l_c and then, by subtraction from $\lambda/4$, l_L.

Under certain conditions, however, it may be advisable to choose a different value of Z_o. For example, if a lower value of Z_o is chosen, the required length of line will be closer to $\lambda/4$, which may be of some value at very high frequencies where λ is small. On the other hand, the greatest change of length for a given change in capacitive reactance occurs when $X_c = Z_o$. This means that the line is least critical in adjustment. However, this value of Z_o may require unusual spacing of the conductors. Hence, the values indicated previously that give maximum Q are generally preferable for Z_o.

Eq. (25) can then be solved for l_c in terms of X_c, Z_o, and λ. However, to avoid computation this equation has been plotted in the form of l_c versus X_c/Z_o as is indicated in Fig. 19. The length of the actual line that will tune with the given capacitor is then $\lambda/4 - l_c = l_L$, as stated above. This has also been plotted in Fig. 19*. In the figure the angle θ in degrees is equal to $360 \times l/\lambda$. Thus the left-hand ordinate scale gives the electrical length of the line as measured in degrees, and the right-hand scale gives the length as measured in wavelengths:

As an example of its use, suppose it is desired to tune a capacitor of 10 $\mu\mu$f with a line at 300 mc (λ = 1 meter). Suppose a concentric line of Z_o = 76.8 ohms is employed. $X_c = 1/(2\pi \times 300 \times 10^6 \times 10 \times 10^{-12})$ = 53 ohms. Then X_c/Z_o = 53/76.8 = .69. From Fig. 19 it is found that the line length is 34.5° or .0957λ. This is considerably less than .25λ, i.e., a 10-$\mu\mu$f condenser represents at 300 mc a line 55.5° long or (.25 - .0957) = .1543λ. If the concentric line were built to have a lower Z_o, such as 53 ohms (= X_c), then the line length would be $\lambda/8$ or 45° long.

Calculation of Line Impedance.—The shorted line and capacitor form a parallel resonant circuit whose impedance at the capacitor end is high. If there were no losses the impedance would be infinite. Although the capacitor can be made to have very low losses, the line will have at least a resistance loss which is fixed by the material (copper) and conductor radius and spacing as shown by Eqs. (15) or (16). These losses reduce the impedance and also the Q from infinity down to a finite value. This in turn reduces the frequency stability of the associated oscillating tube.

It would therefore appear ad-

***Taken from "Graphs for Transmission Lines," by B. Salzberg, *Electronics*, Jan. 1942; McGraw-Hill Publishing Co.**

visable to use a maximum amount of capacitance and minimum length of shorted line. This, however, is not the case. In Fig. 20 is shown

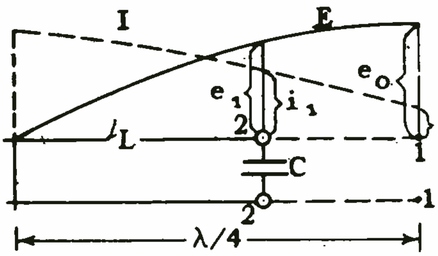


Fig. 20.—Current-voltage relations in a shorted line tuned with a capacitor to form an equivalent quarter-wave shorted line.

once again the line and associated condenser C, as well as the open-circuited line that is equivalent to C (in dotted lines). Above it are shown the (approximately) sinusoidal current I and voltage E distribution along the line.

At the open end (terminals 1-1) the current is not zero, but has a value of i_0 owing to the losses in the line, while the voltage has some peak value—call it e_0. The impedance seen at 1-1 is $Z_m = e_0/i_0$ and is finite instead of infinite in magnitude. However, at the *actual* terminals 2-2, the current is greater or i_1, and the voltage is less or e_1.

The impedance is therefore much less or $Z_1 = e_1/i_1$. This illustrates another very valuable impedance transforming property of a λ/4 line line, namely, that of a λ/4 line shorted at one end and having an impedance connected across the line at one point, while the impedance across the line is measured at another point. In this case the former impedance is that seen at the open end (of the hypothetical extention of the actual line) and is due to the losses of the line rather than due to an actual resistance across terminals 1-1. The *impedance that can be physically measured,* however, is that at terminals 2-2, at a distance from the shorted end of l_L. The ratio of these impedances is

$$\frac{Z_1}{Z_m} = \left(\frac{\sin \frac{360\, l_L}{\lambda}}{\sin 90°}\right)^2$$

$$= \sin^2 \frac{360\, l_L}{\lambda} \qquad (26)$$

Thus, as C is increased and l_L thereby shortened, Z_m increases because less l_L means less losses and a higher Q. But Z_1, *the impedance seen at the actual terminals 2-2, goes down* when calculated by Eq. (26). An involved analysis by Nergaard and Salzberg* indicates that maximum impedance is obtained when l_L is practically one-quarter wave long and C is zero. If C is not zero, then the impedance is less, and this is generally the case: one is given a certain amount of C that has to be tuned with a line.

In the case where l_L is further varied by changing the characteristic impedance Z_0 of the shorted line, it appears the impedance seen at the capacitor terminals is greater when Z_0 is chosen larger even though l_L is thereby increased, although the variation is not very marked.

*"Resonant Impedance of Transmission Lines," *Proc. I.R.E.*, Sept. 1939.

The impedance (really resistance) measured at the capacitor terminals

$$Z_1 = r$$

$$\simeq Z_o \frac{1}{k} \cdot \frac{1 - \cos 2\theta_L}{2\theta_L + \sin 2\theta_L}$$

or

$$\frac{Z_1}{Z_o} = \frac{r}{Z_o}$$

$$\simeq \frac{1}{k} \cdot \frac{1 - \cos 2\theta_L}{2\theta_L + \sin 2\theta_L} \qquad (27)$$

where θ_L is the length of the shorted line measured in radians ($= 2\pi l_L/\lambda$), and k is the dissipation factor and equals approximately ($R/2\omega L$) where R and L are the series resistance and inductance of the shorted line per unit length. The unit of length is immaterial since their ratio is involved.

To facilitate computation of r, the quantity

$$\frac{1 - \cos 2\theta_L}{2\theta_L + \sin 2\theta_L}$$

of Eq. (27) has been plotted in Fig. 21. It is labeled there F and is plotted against θ_L in degrees.

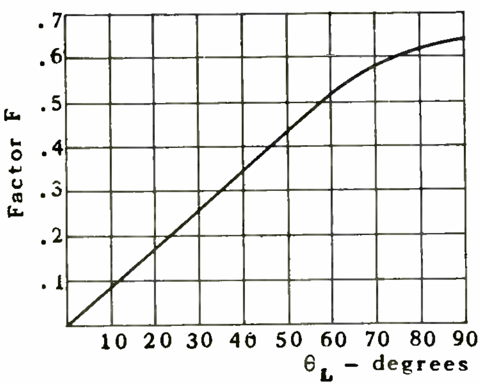


Fig. 21.—Plot of F vs. θ_L.

Its use will now be illustrated by means of the previous problem. It was found that for a 10-$\mu\mu$f condenser at 300 mc, the shorted concentric line should have an electrical length of 34.5° for Z_o = 76.8 ohms. From Fig. 21, the value of F is found to be .3. To evaluate k, R and L must first be calculated. Let a = 1/8 inch, and b = 3.6 × 1/8 = .45 inch. Then, from Eq. (16a),

$$R = 5 \times 10^{-7} \sqrt{3 \times 10^8} \left[\frac{1}{.125} + \frac{1}{.45} \right]$$

$$= .0886 \text{ ohm per foot}$$

The inductance of a concentric line is given approximately by

$$L = 0.140 \times 10^{-6} \log \frac{b}{a} \text{ h/foot} \qquad (28)$$

Hence,

$$L = 0.140 \times 10^{-6} \log 3.6$$

$$= .0779 \times 10^{-6} \text{ henry per foot}$$

Then

$$k = \frac{R}{2\omega L}$$

$$= \frac{.0886}{2 \times 2\pi \times 3 \times 10^8 \times 7.79 \times 10^{-8}}$$

$$= 3.02 \times 10^{-4}$$

The resistance measured across the capacitor is

$$r = \frac{Z_o F}{k} \qquad (29)$$

or

$$r = \frac{76.8 \times .3}{3.02 \times 10^{-4}} = 76.3 \times 10^3$$

$$= 76{,}300 \text{ ohms}$$

This is a high value for the given C and frequency, and illustrates the high Q that such a combination can have. It does not take in account the losses in the capacitor, however. These will be in parallel with the above resistance.

Line Taps.—Suppose that instead of measuring the impedance at the capacitor terminals, measurements are made at a point nearer the shorted end of the line.

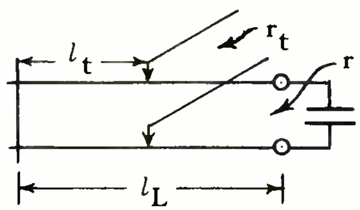


Fig. 22.—Use of taps on a shorted equivalent quarter-wave line to obtain a lower impedance.

This is shown in Fig. 22, where l_t is the length of line at which the taps are made. Since the voltage varies sinusoidally along the line, the impedance seen at any point of the line varies as the *square* of the sine of the electrical length or angle of the line. For a tuned line the impedance looking into any point of the line is a pure resistance—call it r_t. Then

$$r_t = r \frac{\sin^2 (360\, l_t/\lambda)}{\sin^2 (360\, l_L/\lambda)} = r \frac{\sin^2 \theta_t}{\sin^2 \theta_L} \qquad (30)$$

This would indicate that a lower resistance r_t, connected at point θ_t, is exactly equivalent to a higher resistance r connected at θ_L. This is only approximately true. A resistance r_t at θ_t tends to disturb the sinusoidal distribution of the voltage along the line by producing a kink at that point, unless it is very high compared to the characteristic impedance of the line and the damping in the line. At the center of the line an order of magnitude of one megohm has been indicated for r_t,* in order that the voltage distribution be not unduly disturbed.

Nevertheless, Eq. (30) can be employed with sufficient accuracy for practical purposes, particularly when the damping of the line is low and the Q is consequently high. It can serve as a first approximation even in circuits of lower Q, and if sufficient adjustment latitude is provided exact tap settings may be made subsequently by experiment.

At any rate, the impedance seen looking into the shorted line at a point nearer the shorted end is lower than that seen at a point nearer the open end. Thus, one can couple a tube to the circuit at a point where a maximum output from the tube is obtained, i.e., optimum impedance matching is possible.

In order to facilitate the computation of the tap length $\theta_t = (360\, l_t/\lambda)$, Eq. (30) has been transformed and plotted in Fig. 23. In this figure, the impedance ratio a refers to r_t/r. Since r_t can equal r only as a maximum value when $l_t = l_L$, a is a fraction whose maximum value is unity. For each line

*See R. King, "General Amplitude Relations for Transmission Lines with Unrestricted Line Parameters, Terminal Impedances, and Driving Points," *Proc. I.R.E.*, Dec. 1941.

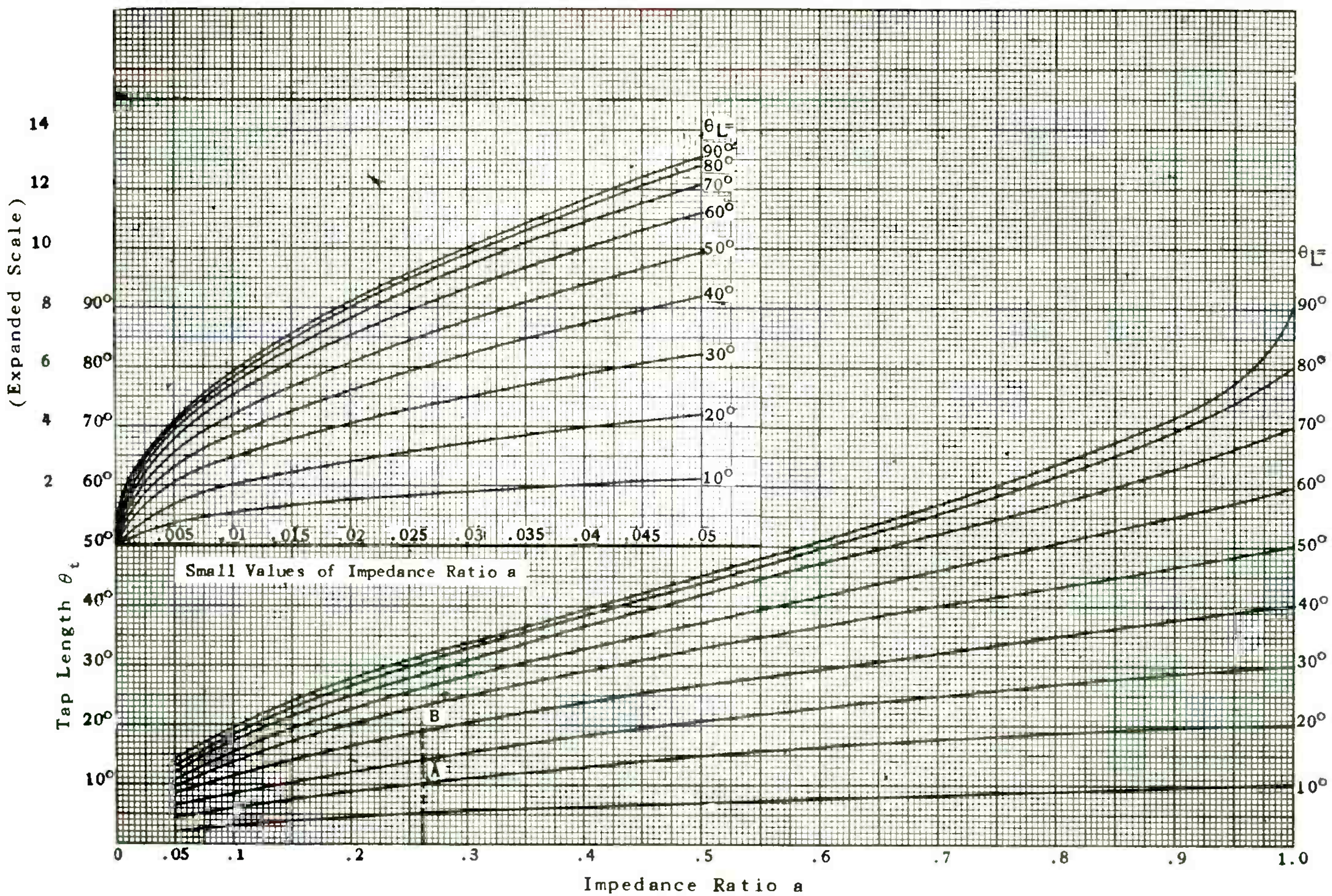


Fig. 23.—Family of curves giving relation between tap length θ_t of an equivalent $\lambda/4$ shorted line and impedance ratio a.

length in degrees θ_L, a curve has been plotted giving the ratio of

$$\frac{\sin^2\theta_t}{\sin^2\theta_L} = \frac{r_t}{r} = a$$

Thus, a family of curves for values of θ_L differing by ten degrees has been plotted. Interpolation between plotted values can then be made where necessary.

Suppose, in the previous example where f = 300 mc, it is desired to couple in the tube so that it sees an impedance of 20,000 ohms. The maximum impedance is 76,300, hence the impedance ratio a is 20,000/76,300 = .262. The line length θ_L is 34.5°. An ordinate through .262 (dotted line in Fig. 23) intersects the θ_L = 30° curve in A, and the θ_L = 40° curve in B. The tap length θ_t is 14.7° and 18.9°, respectively.

For θ_L = 34.5, θ_t, by interpolating, is

$$14.7 + \left[\frac{34.5 - 30}{40 - 30} \times (18.9 - 14.7)\right]$$

$$= 14.7 + 1.89 = 16.59°$$

If Eq. (30) is solved directly, θ comes out to be 16.85° or practically the same value as the curves.

The actual length of line up to the tap or l_t, is then very simply

$$l_t = \frac{\lambda\theta_t}{360} \qquad (31)$$

or

$$l_t = \frac{1\text{m} \times 16.85}{360} \times 3.28$$

$$= .1534 \text{ feet or } 1.841 \text{ inches}$$

Line Oscillator.—In Fig. 24 is shown another application of a concentric line to an oscillator. Here the operating frequency is low enough for lumped circuit elements to be feasible in the plate circuit, but the superior Q of the quarter-wave line makes it preferred in the grid circuit as a frequency-stabilizing element.

Of particular interest in Fig. 24 is the fact that the grid is tapped down on the line. As will be shown later, at the ultra-high frequencies vacuum tubes begin to exhibit excessive losses. Of particular importance are those developed in the grid circuit owing to transit time and cathode induc-

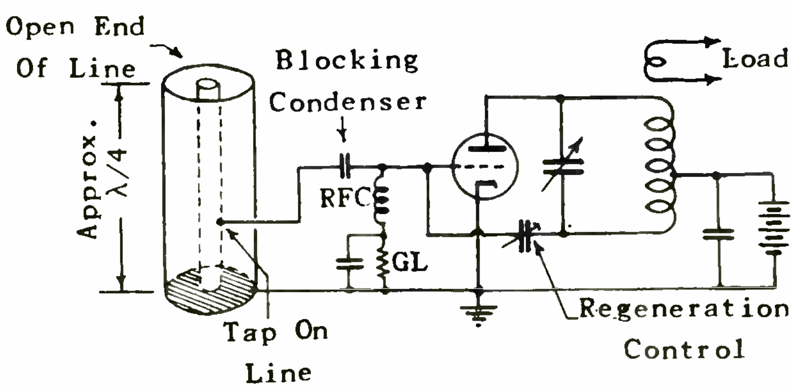


Fig. 24.—Use of a concentric line as a high-Q frequency stabilizing element in an oscillator.

tance effects. These losses occur even though the grid may be negatively biased and not permitted to draw *conductive* current.

Accordingly, the impedance looking into the grid circuit may be regarded as a resistance and capacity in parallel. If the grid were connected to the open end of the concentric line, then two things would happen. First, the grid capacity would require the line length to be shortened. Second, the resistance component would lower the impedance and hence the Q of the line.

By tapping down on the line, the resulting impedance transformation makes the equivalent capacity at the open end of the line very low. Therefore, little effect on the line length is incurred; one can assume to a fair degree of approximation that the line is one-quarter wave in length. Such an assumption simplifies the determination of the tap position for a required impedance. On the other hand, if it is desired to employ a trimmer capacitor for adjustment purposes at the open end, then the line must be shortened accordingly, and the method previously outlined must be employed.

The impedance seen at the open end of the line owing to the line's resistance can be found by using Eq. (29) where $F = 2/\pi$. In this particular case (for a quarter-wave line) Eq. (29) becomes

$$r = \frac{2Z_o^2}{Rl} = \frac{8Z_o^2}{R\lambda} \qquad (32)$$

where l is $\lambda/4$.

Then if the impedance desired to be seen at the tap is r_t, the tap position can be found by using Fig. 23. Or, alternatively, the effect of the grid resistance in damping the resonant line can be the basis for the choice of the tap position.

To illustrate the latter choice suppose a concentric line is used for which b = 7.56" (or 15.12" inner diameter of outer conductor). and a = 2.1" so that b/a = 3.6. Suppose the oscillating frequency is to be 100 mc for which λ = 3 m = 9.84 feet. The resistance per loop foot is calculated from Eq. (16a) to be

$$R = 5 \times 10^{-7} \sqrt{100 \times 10^6}\left(\frac{1}{2.1} + \frac{1}{7.56}\right)$$

$$= .00304 \text{ ohm}$$

The inductance depends solely upon b/a, and for b/a = 3.6, it has been found to be $.0779 \times 10^{-6}$ henries/ft.

$$Q = \frac{2\pi \times 76.8}{.00304 \times 9.84} = 16{,}100$$

Furthermore, the line losses look like a resistance across the *open end* of the line of value, from Eq. (32)

$$r = \frac{8(76.8)^2}{.00304 \times 9.84} = 1{,}578{,}000 \text{ ohms}$$

Now suppose the grid circuit has an input resistance of 10,000 ohms. If the grid were connected directly to the open end of the line, its 10,000 ohms would shunt the above 1,578,000 ohms and lower the Q in the same proportion as it lowers the open-end resistance. If the grid is tapped down on the line, its 10,000 ohms resistance will be reflected (transformed) to the open end of the line as a much higher impedance, and the decrease in Q will be correspondingly less.

Suppose that adequate frequency

stability is obtained even if the Q is lowered from 16,100 to 10,000. The equivalent resistance will be lowered from 1,578,000 to

$$r = 1,578,000 \times \frac{10000}{16100}$$

$$= 980,000 \text{ ohms}$$

This means that the grid resistance must be of a value r_g such that in paralleling the 1,578,000 ohms of the line, it reduces this value to 980,000 ohms. The value of r'_g must therefore be

$$r'_g = \frac{r \times r}{r - r} = \frac{1578000 \times 980000}{1578000 - 980000}$$

$$= 2,600,000 \text{ ohms}$$

Thus, 10,000 ohms must be stepped up to 2,600,000 ohms; an impedance ratio of 2,600,000 ÷ 10,000 = 260. To use Fig. 23 the reciprocal is required, or 1 ÷ 260 = .00385. The total length of line is $\theta_L = 90°$. Using this curve of Fig. 23, there is obtained for a = .00385 a value of $\theta_t = 3.7°$. This corresponds to

$$\frac{3.7}{360} \times 9.84 = .101 \text{ feet}$$

$$= 1.21 \text{ inches}$$

However, the equivalent resistance of 1,578,000 ohms at the open end of the line, which represents the line losses, appears at the grid tap as 1,578,000 ÷ 260 = 6,100 ohms. The farther down the tap is set, the lower does this resistance appear. Such a low resistance may require a rather large feedback condenser (labeled "Regenerative Control" in Fig. 24), in fact, may require an excessive amount of the tube's output.

What is desired to present in this assignment is a means of evaluating the effect of the tap setting on the tube's action. The tube is usually set up and settings experimentally determined. If the tap chosen that will permit satisfactory oscillation is too near the open-circuit end, and calculation shows that the Q of the line is thereby lowered too much, then the engineer knows how to proceed to make changes. Too high a tap setting indicates that the line losses are too high. This can be remedied by silver-plating the conductor surfaces, and particularly by increasing the conductor size, i.e., b and a, while b/a remains 3.6 in value.

The effect of each change can be calculated as described above, and it can also be determined whether one can meet a certain specification or not with the circuit components employed.

Antenna Coupling.—Another typical problem is that shown in

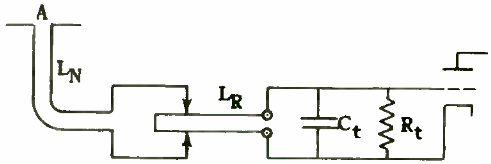


Fig. 25.—Use of a shorted, tapped matching stub between an antenna line and a vacuum tube.

Fig. 25. This represents an antenna A that is connected to a nonresonant line L_N, which in turn is connected to taps on a short-circuited stub L_R, whose open end is connected to the grid and cathode of a vacuum tube amplifier. The input impedance of the tube is

represented by the capacity C_t and resistance R_t in parallel.

Owing to the resonant properties of the line L_R, a voltage E_1 impressed at the taps is stepped up to a much higher value at the grid of the tube. At the same time the losses in L_R, together with R_t, can be made to present an impedance at the taps to L_N that is equal to its characteristic impedance. The latter therefore is non-resonant, as stated above, and merely conveys the signal picked up by the antenna over a distance of as many wavelengths as is necessary to the grid of the tube.

The resonant line L_R performs the same service at ultra-high frequencies that the conventional lumped circuit, shown in Fig. 26,

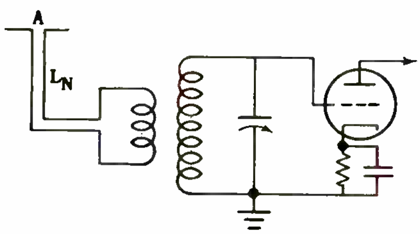


Fig. 26.—Equivalent lumped tuned circuit used to match an antenna line to a vacuum tube.

performs at lower frequencies. The performance in either case is determined by considerations somewhat different from those pertaining to oscillator operation.

In the case of the receiving antenna circuit, the Q desired is determined by the width of the side bands, and is usually on the order of magnitude of 100, instead of 10,000 or thereabouts for the oscillator tank circuit. For the lower value of Q required the line losses are normally negligible, and the Q is determined primarily by the input resistance of the associated tube. The damping effect of this resistance upon the line may be varied by tapping the grid circuit down on the line, and thus any desired value of Q may be obtained.

At the same time, it is not sufficiently accurate to assume that the voltage distribution remains sinusoidal and is unaffected by the grid tap position. Hence, a different method of attack is necessary.

Selectivity of Circuit.—The selectivity of the circuit is determined by measuring the voltage

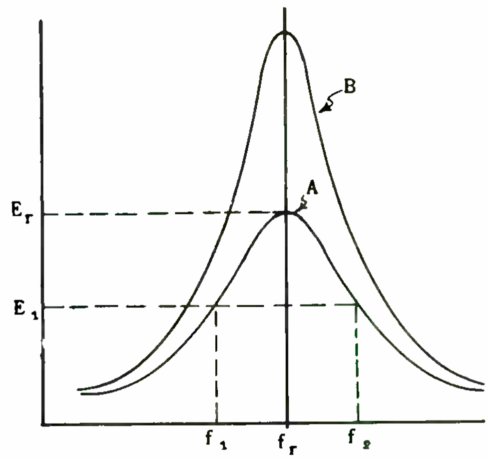


Fig. 27.—Examples of resonance curves for a low-Q and for a high-Q circuit.

at some point of the line, such as the grid tap, as the frequency is varied. A curve, such as A in Fig. 27, will be obtained. Maximum voltage E_r is obtained at the frequency f_r for which the shorted line is resonant, i.e., is one-quarter wave in length. At two other frequencies f_1 and f_2, spaced approximately by equal amounts from f_r, the voltage will have dropped to some lower value E_1. Suppose E_1 is .707 E_r. Then, as described previously,

$$\frac{f_r - f_1}{f_r} = \text{per cent frequency change off resonance}$$

$$= \frac{1}{2Q} \qquad (13a)$$

If Q is high, the per cent frequency change is small and vice versa if Q is low. Thus the Q of the circuit may be used as a measure of the selectivity of the circuit: it can indicate the sharpness of the resonance curve and thus the band width. The band width of frequencies transmitted can be arbitrarily defined as

$$2(f_r - f_1) = (f_2 - f_1)$$

This does not mean that voltages of frequency lower than f_1 or higher than f_2 are not transmitted, but that they are markedly attenuated, and hence may be presumed to have little effect upon the performance of the circuit.

Note that if the voltage is attenuated to 70.7 per cent, that is, to $1/\sqrt{2}$ as compared to E_r, then the power developed across a resistance by this voltage will vary as the *square* of the voltage. Thus the power developed by E_1 in Fig. 27 is $(1/\sqrt{2})^2 = 1/2$ of the power developed by E_r across the same resistance. Expressed in db this is

$$\text{db relative attenuation} = 10 \log (1/2)$$

$$= -10 \log 2 = -3 \text{ db}$$

where the minus sign indicates attenuation

If the tube is tapped down on the line nearer the shorted end, the damping is less and the Q is higher. In such a case a resonance curve such as B, Fig. 27, may be obtained. This clearly has a smaller band width.

The importance of the band width of the antenna circuit may be very great at u.h.f. At these frequencies tubes do not amplify very well, particularly in the upper end of this spectrum. Hence, at frequencies above a so-called "cross-over point," the antenna signal is fed directly to a converter tube or crystal first detector, and mixed with a local oscillator signal to produce at once a lower, i.f. signal, which may then be selectively amplified. In short, u.h.f. receivers are often of the superheterodyne type, without benefit of an r.f. stage.

The absence of the r.f. stage tends to decrease the r.f. selectivity, and hence the attenuation of the image signal. For that reason a selective antenna circuit is important: it is the only means by which image rejection can be obtained.

Image Rejection.—It will be of value to review briefly the question of image rejection. Let f_s be the signal it is desired to pick up, and f_i the intermediate frequency to which it will be converted and then amplified by the selective i.f. amplifier. The conversion is accomplished by mixing the incoming signal with that of a local oscillator in a converter tube, and then extracting the difference beat frequency between them to amplify in the i.f. amplifier.

Thus, let the local oscillator frequency be f_o. Then $f_i = f_s - f_o$ or $f_o - f_s$, depending upon which is the higher frequency. Suppose f_s is higher than f_o. It is evident that another signal frequency f_k that is below f_o by an amount f_i will also produce in conjunction with f_o a beat frequency f_i in the converter tube, and thus be amplified by the i.f. amplifier too, *provided f_k can get through the antenna tuned circuit.* This frequency f_k is known as an *image frequency*. (Other frequencies that beat with *harmonics* of the local oscillator can also produce f_i, but such frequencies are generally too far away from f_s *to be able to pass through the antenna circuit.)*

On the other hand, if f_k is close to f_s it can pass through the antenna circuit, but will produce in the converter tube's output a beat frequency $(f_o - f_k)$ that is so much lower than f_i *that the i.f. amplifier will reject it.* Hence, the antenna circuit must be able to block frequencies f_k that are f_i cycles below f_o, where f_o is in itself f_i cycles below the signal frequency f_s. In other words, unwanted signals f_k that are 2 f_i below f_s must be rejected by the antenna circuit.

The ratio of image to signal response can be expressed in terms of the circuit Q and the band width as previously defined. The latter was stated to be the total number of cycles below and above resonance at which the response falls off to 70.7 per cent of the response at resonance. Let the number of cycles to *either* side of resonance f_r be denoted by the symbol f_b. Thus, f_b corresponds to $(f_r - f_1)$ in Eq. (13a) and is half of the total band width. Multiplying both sides of Eq. (13a) by f_r, there is obtained

$$f_b = \frac{f_r}{2Q} \qquad (13b)$$

Now let f_i be the i.f. frequency, and suppose the antenna system is tuned to the desired signal frequency, i.e., $f_r = f_s$. If the unwanted image frequency, call it f_k, is 2 f_i cycles away from f_s, then the output of the converter tube due to f_k will be of the same frequency as that due to f_s, namely, f_i; in other words, this relationship defines the image frequency. Then the ratio of the response at image frequency to that at signal frequency will be

$$\frac{\text{image response}}{\text{signal response}} \sim= \frac{f_b}{2f_i} \qquad (33)^*$$

This ratio can be reduced by reducing f_b or by increasing f_i. Physically this means that if the Q of the antenna circuit is high, then the half band width f_b is small and if, furthermore, a high intermediate frequency f_i is chosen, then the image frequency f_k will be far down on the narrow resonance curve, so that image rejection will be high.

This is clearly shown in Fig. 28. Curve A is a high Q resonant response; curve B, a low Q response. For A the half band width is f_{b1}, i.e., at this number of cycles off resonance, the response is 70.7 per cent of its value at the peak of the curve. For B the half band width is f_{b2}, which is farther away from the peak because Curve B is

***See, for example, Herold and Malter, "Radio Reception at Ultra-High Frequency"—Part I, *Proc. I.R.E.*, Aug. 1943.**

flatter than A. The unwanted image frequency f_k is shown as 2 f_i cycles off resonance (which has been set to correspond with the desired frequency f_s). As is clear from the

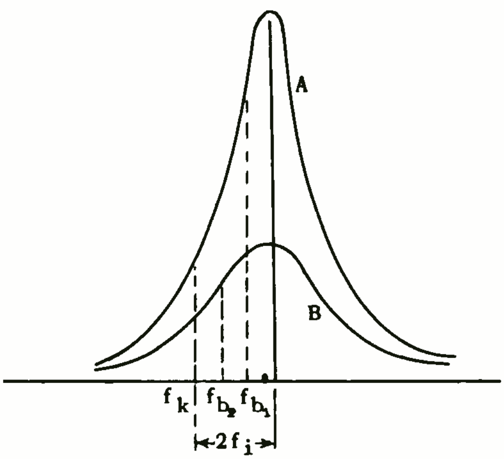


Fig. 28.—Relation between half band width and image frequency for two values of circuit Q.

figure, the difference in response between that for f_{b1} and f_k for curve A is much greater than that between the response at f_k and f_{b2}, i.e., curve A has the greater image rejection.

At the same time, note that if f_i is increased f_k moves to the left, and the image rejection is greater along either curve. It is also clear from the figure that owing to the symmetry of the resonance curves, the same reasoning applies if f_k is 2 f_i cycles to the right of the resonant frequency, i.e., the image rejection is practically the same whether the local oscillator is adjusted to operate f_i cycles above or below f_s. Eq. (33) holds under most practical conditions, such as for values of f_b and f_i that are both small compared to the signal frequency. In practice f_i is generally made 1/5 of f_s or less, which meets the above requirement.

Calculation of Q of Loaded Line.—The above can now be applied to the antenna problem discussed previously. First the Q of the resonant line must be calculated. As mentioned previously, it is assumed that the losses of the line are negligible compared to that of the tube's input, so that the latter determines the Q. When the tube is connected to the open end of the line, the input capacitance C_t shortens the line from its normal length of $\lambda/4$, for resonance to be maintained.

The procedure to be employed in calculating the Q is as follows: As has been mentioned in previous assignments, a circuit having distributed constants may be replaced by a suitable circuit having lumped constants, *at any one frequency or band of frequencies.* If the frequency band is small, and around the resonant frequency of the line

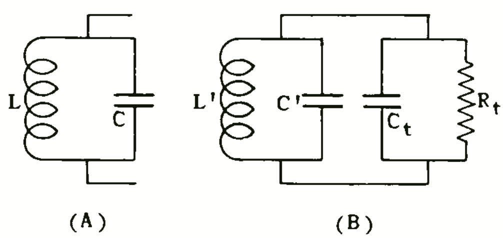


Fig. 29.—Representation of a quarter-wave line over a narrow band of frequencies by an equivalent lumped circuit.

(distributed circuit) then a simple combination of lumped inductance and capacity can be used to represent the line quite satisfactorily. For a quarter-wave line shorted at the far end, the lumped arrangement is that shown in Fig. 29(A). (This assumes the line losses are negligible.)

If a parallel combination of resistance and capacity is connected to the end of the line, the line must be shortened. A line shorted at the far end and less than $\lambda/4$ in length, has an *inductive reactance*. The proper length of it will have sufficient inductive reactance to tune with the attached capacity to the given frequency, and the connected resistance will damp the circuit and give it a certain Q.

In Fig. 29(B) is shown the reduced line equivalent, together with C_t and R_t representing the input impedance of the tube. The line equivalent has an inductance L' and a capacity C' that is less than L and C for the $\lambda/4$ line equivalent shown in Fig. 29(A). The important thing to note is that in the case of the reduced line, its equivalent lumped circuit, although having an inductive reactance, cannot be just an inductance but must contain *both inductance L' and capacity C' in order to simulate the reduced line over a band of frequencies*. The values of L' and C' are such that at any frequency under consideration the combination has a *net inductive reactance*, but it would lead to an incorrect calculation of the Q of the circuit to attempt to represent the reduced line by an inductance alone.

The reason is that the Q of the combination may be expressed in terms of the angular frequency $\omega (= 2\pi f)$, the resistance R_t and the *total* capacitance, namely, $C' + C_t$. Thus

$$Q = \omega(C' + C_t)R_t = \omega C R_t \quad (34)$$

where C is the total capacitance and equals $(C' + C_t)$.

It remains to determine the capacitance C' residing in the reduced section of line. It might be thought that if the capacity per unit length of the line is C_o, and the line length is l, that C' would simply be $C_o l$. This, however, is not the case, as can be seen from an examination of Fig. 30. The line can be regarded as

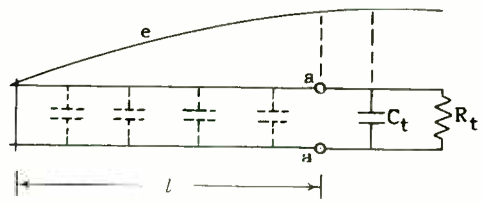


Fig. 30.—The voltage varies along a resonant line, so that each element of capacitance has voltage across it.

having an infinite number of infinitesimal capacitors shunting it along its length (suggested by the dotted line capacitors). The voltage distribution is a section of a sine wave.

It will be observed that the voltage across those capacitors near the shorted end of the line is lower than that across those near the open end of the line. Hence, the latter capacitors draw more charging current than the former, and are therefore more effective in determining the average or net capacity of the line.

The combined effects of all the infinitesimal condensers can be prorated as equivalent to a single lumped condenser C' connected to the terminals a-a, where the measurement is to be made. The method of determining this is based on the integral calculus. The result is most compactly given for the *total* capacity C, *which includes* C_t. It is

$$C = \frac{1}{2}\left(C_t + \frac{C_o l}{\sin^2 \theta}\right) \quad (35)^*$$

where θ is the line length expressed in electrical degrees instead of as l. Thus, $\theta = 360\, l/\lambda$ degrees. The distributed capacity per unit length C_o and the characteristic impedance of the line both depend upon the line dimensions. Hence, C_o can be expressed in terms of Z_o. Thus, if C_o is in $\mu\mu$f per centimeter

$$C_o = \frac{33.3}{Z_o} \quad (36)^{**}$$

for $Z_o = 76.8$ ohms, $C_o = .433\ \mu\mu\text{f/cm}$.

Then, from Eq. (34), the Q of the combination is

$$Q = \frac{\omega R_t}{2}\left(C_t + \frac{C_o l}{\sin^2 \theta}\right) \quad (37)$$

It will be instructive to apply the above to an actual problem. Suppose the operating frequency is 150 mc, $C_t = 5\ \mu\mu$f, and $R_t = 7{,}500$ ohms. (Note how low the input resistance R_t of a tube can become at u.h.f.) The reduced line length can be found from Fig. 19. Assume $Z_o = 76.8$ ohms. The capacitive reactance of C_t is $X_t = 1/[(2\pi \times 150 \times 10^6 \times 5 \times 10^{-12})] = 212$ ohms.

Then $X_t/Z_o = 212/76.8 = 2.76$. From Fig. 19, the line length θ_L comes out to be 70°, and the corresponding length is $70\lambda/360 = 70 \times 3 \times 10^{10} \div (150 \times 10^6 \times 360) = 38.9$ cm. (Note that $\lambda = c/f = 3 \times 10^{10}/150 \times 10^6$.)

Then, by Eqs. (36) and (37), the Q is

$$\frac{2\pi \times 150 \times 10^6 \times 7500}{2} \times \left(5 + \frac{.433 \times 38.9}{\sin^2 70°}\right) \times 10^{-12}$$

$$= 15 \times 75 \times \pi\left(5 + \frac{16.85}{(.9397)^2}\right) \times 10^{-3}$$

$$= 85.3$$

Location of Antenna Tap.—The 75-ohm antenna non-resonant line is connected to the resonant line at another tap, located so that the impedance seen *looking into the resonant line at that point* is 75 ohms in order to match the antenna line. The position of this tap requires extensive calculations to locate it accurately. Fortunately, in practical work it need be located only approximately and then sufficient latitude in adjustment provided to permit an exact match to be determined experimentally. An exact match is indicated by an absence of standing waves (voltage or current) on the non-resonant antenna line.

The antenna tap can be estimated by the same rule as that employed for the very high Q oscillator tank circuit, namely, by the sine-squared rule. Thus, if θ_A is the length of the resonant line up to the antenna tap, and θ_t the

*These and many of the succeeding formulas are given in the article by Herold and Malter previously mentioned.

**Refers to the same source as Eq. (35).

length up to the tube connection, and R_A is the resistance required to terminate the antenna line, then

$$\sin^2 \theta_A = \frac{R_A}{R_t} \sin^2 \theta_t$$

$$= a \sin^2 \theta_t \qquad (38)$$

where a is the impedance ratio R_A/R_t. In the problem at hand R_t = 7,500 ohms, R_A = 75 ohms, so that a = 75/7,500 = .01. Now use Fig. 23, noting that θ_L there corresponds to the value θ_t which is of interest in this problem. For θ_t (= θ_L) = 70°, θ_A is found to be about 5.4°. This corresponds to a length

$$l_A = \frac{5.4}{360} \times 2m \times 3.28$$

$$= .095 \text{ feet or } 1.14$$

while the total length of the line is

$$l_t = \frac{70}{360} \times 2 \times 3.28$$

$$= 1.275 \text{ feet or } 15.3 \text{ inches}$$

When the antenna is connected to the resonant line at the appropriate point for matched conditions, it acts to load the line down by an amount equal to that of the tube at its tap. As a result, the Q of the coupling circuit is reduced to one-half; in this case, to 85.3 ÷ 2 = 42.65. Such a final value of Q may be too low for the required image rejection desired, and more than low enough for the band width required. Consequently, a higher Q may be desired. This, as mentioned previously, can be obtained by tapping the tube down on the resonant line. The procedure to be followed is best described by continuing the discussion of the numerical problem given above.

Determination of Image Response.—The voltage ratio of the response at image frequency to that at signal frequency was given by Eq. (33) as

$$\frac{\text{image response}}{\text{signal response}} \sim= \frac{f_b}{2f_i} \qquad (33)$$

The power ratio is as the square, namely, $(f_b/2\ f_i)^2$. The db ratio is 10 times the logarithm of this ratio, or 20 log $(f_b/2\ f_i)$ = $-20 \log (2\ f_i/f_b)$. Thus

$$\text{db image attenuation}$$

$$= -20 \log (2\ f_i/f_b) \qquad (39)$$

The minus sign merely signifies that the db represents attenuation. This form gives for $2\ f_i/f_b$ a number greater than one, for which the logarithm is more readily found than for a fraction.

Suppose that in the problem the intermediate frequency is 25 mc = f_i. The half band width f_b can be found from Eq. (13b). It is

$$f_b = \frac{f_r}{2Q} = \frac{150 \times 10^6}{2 \times 42.6} = 1.76 \text{ mc}$$

$$-20 \log (2 \times 25 \times 10^6 \div 1.76 \times 10^6)$$

$$= -20 \log 28.4$$

$$= -20(1.4533)$$

$$= 29.06 \text{ db}$$

This is in itself an acceptably large value of attenuation, but at 150 mc a greater value should be possible. Values as high as 60 db and more are obtainable in the ordinary communication receivers that operate in the standard broadcast and short-wave bands, although it is true that these receivers have one or more stages of r.f. amplification involving additional selective circuits.

On the other hand, the complete band width is 2 × 1.76 mc or 3.52 mc. This is more than adequate for a communication receiver. A band width of 20 kc would be adequate, and a band width of two or three hundred kc would take care of f.m. transmission. (Due allowance must also be made for local oscillator drift.)

In such a case the Q can be increased to decrease the band width and to improve the image rejection. Suppose a 40-db image attenuation is desired. Eq. (39) can be solved for f_b to give

$$f_b = \frac{2 f_i}{\text{anlog}\left(\dfrac{\text{db image attenuation}}{20}\right)} \qquad (40)$$

from which

$$f_b = \frac{2 \times 25 \times 10^6}{\text{anlog}(40/20)} = .5 \times 10^6 \text{ c.p.s.}$$

The band width is thus $2 f_b$ = 1 mc, which is more than adequate for communication purposes.

From Eq. (13b), the Q required is

$$Q = \frac{f_r}{2 f_b} = \frac{150 \times 10^6}{2 \times .5 \times 10^6} = 150$$

This value includes the damping effect of the antenna; the Q of the resonant line and connected tube must therefore be double this, or 300. To obtain it, the tube must be tapped down the line.

The circuit is shown in Fig. 31(A) (with the antenna connection omitted). The reduction in overall line length will be less because C_t is at a point of lower voltage and consequently is less effective. The problem is to determine θ_1 so as to obtain a Q of 300 in the above problem, and also to determine θ_2 so that θ_1, θ_2 and C_t will form a resonant circuit.

The Q value is determined by

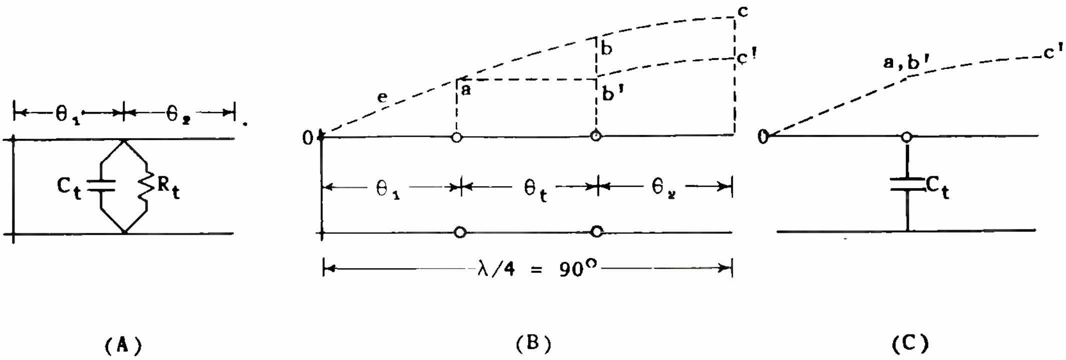


Fig. 31.—Tapping down on quarter-wave line to obtain a higher Q, and method of calculating tap point.

Eq. (34), except that now the total capacity is composed of that residing in θ_1, that in θ_2, plus C_t. It is thus necessary to determine the lumped capacities equivalent to that in θ_1 and that in θ_2. In order to do this, the voltage distribution along the tapped line must be known. The distribution will not be a single sine curve, but two segments discontinuous at the tap point. The shapes can be evaluated from the following reasoning:

From the short-circuit at the left (where the voltage is zero), the voltage rises along θ_1 in a sinusoidal manner up to C_t. The latter can be replaced by an appropriate section of line less than $\lambda/4$ in length. (It will be recalled that a line less than $\lambda/4$, open-circuited at one end, has a net capacitive reactance when measured from the other end.) Call the length of the equivalent line θ_t. On the end of this can be added θ_2, so that $\theta_1 + \theta_t + \theta_2 = 90°$. This is shown in Fig. 31(B). Across such an equivalent line the voltage can be regarded as building up smoothly in *one* sinusoidal curve from zero at the left to a maximum at the right, as shown by the smooth dotted line.

The voltage along θ_1 is oa, and along θ_2 is bc. Now if θ_t be replaced by the lumped capacitor C_t, the two sections θ_1 and θ_2 are no longer separated, and hence their voltage segments must be pieced together so that their shapes are unchanged, but so that they have a *common value* at their junction point. This means that bc must be moved downward until b is no higher than a, and thus b′c′ is obtained. The final distribution with C_t instead of θ_t is therefore as shown in Fig. 31(C). The effect of C_t has been to produce a break or discontinuity in the voltage wave, and that is why the simpler method given previously is not employed in this problem: the addition of capacity or resistance at some point along a resonant line affects the voltage distribution, unless the impedance of such a shunt is very high, or the connection is sufficiently close to a point of low voltage, such as the shorted end of the line. In this case θ_1 is very small. This was the situation for the oscillator, but in the antenna problem a more careful analysis is necessary.

With C_t given, once θ_1 is chosen, θ_2 can be found as follows:

The line length θ_2 can be regarded as an equivalent capacitor C_2 that is in parallel with C_t to form a total C that tunes θ_1 to resonance. The value of C in terms of θ_1 can be found from a slight rearrangement of Eq. (25)

$$\cot\theta_1 = \omega C Z_o = \omega(C_t + C_2)Z_o$$

$$= \omega C_t Z_o + \omega C_2 Z_o \qquad (41)$$

from which

$$\omega C_2 Z_o = \cot\theta_1 - \omega C_t Z_o \qquad (41a)$$

But $\omega C_2 Z_o$ represents the effect of C_2 as a capacitor connected to $\theta_1 + \theta_t$, and can be written directly from Eq. (25) as

$$\omega C_2 Z_o = \cot(\theta_1 + \theta_t)$$

$$= \tan[90° - (\theta_1 + \theta_t)]$$

$$= \tan\theta_2$$

$$= \cot\theta_1 - \omega C_t Z_o \qquad (42)$$

since, for resonance, $\theta_1 + \theta_t + \theta_2 = 90°$, so that $\theta_2 = 90° - (\theta_1 + \theta_t)$.

Eq. (42) defines θ_2 in terms of θ_1, C_t, and Z_o, which is generally taken as 76.8 ohms for a concentric line in order to obtain a line of inherently maximum Q.

From trigonometry

$$\cos^2 \theta_2 = \frac{1}{1 + \tan^2 \theta_2} \qquad (43)$$

If this be solved in conjunction with Eq. (42), one finally obtains

$$\cos^2 \theta_2 = \frac{1}{1 + (\cot \theta_1 - \omega C_t Z_o)^2} \qquad (44)$$

The reason for presenting the steps leading to Eq. (44) was to show the student how the effect on the line length of lumped impedances, such as C_t, can be calculated. A careful rereading of the foregoing steps will indicate the care that must be exercised in analyzing a transmission line problem.

It is first necessary properly to choose θ_1. Then θ_2 is determined by Eq. (44). The capacity equivalent to θ_1 and that equivalent to θ_2 must be added to C_t to give the *total* capacity C, *as it appears at the tap point.* Then the Q is given by Eq. (34) where $C' + C_t$ is here represented by C. The value of C is calculated in the same manner as previously by means of the integral calculus, once the voltage distribution is known [Fig. 31(C)]. It is

$$C = \frac{1}{2}\left(C_t + \frac{C_o l_1}{\sin^2 \theta_1} + \frac{C_o l_2}{\cos^2 \theta_2}\right) \qquad (45)$$

where C_o is the distributed capacity per unit length, such as the centimeter, and each l is the length of the line segment which is also measured by the corresponding θ in degrees, i.e., l_1 or θ_1, and l_2 or θ_2. The dimensions for the l's should be in the same units as that for C_o, such as centimeters.

The value of Q is then, from Eq. (34), given as

$$Q = \frac{\omega R_t}{2}\left[C_t + \frac{C_o l_1}{\sin^2 \theta_1} + \frac{C_o l_2}{\cos^2 \theta_2}\right] \qquad (46)$$

Application of Eqs. (45) and (46) will reveal the following:

1. As the tube is tapped down the line θ_1 decreases and θ_2 increases. Thus, Eqs. (35) and (37) are special cases of Eqs. (45) and (46) for which θ_1 is a maximum and θ_2 is zero.

2. The actual length of line is $(\theta_1 + \theta_2)$. If C_t is zero, then $(\theta_1 + \theta_2)$ has a maximum value of 90°, i.e., the line is $\lambda/4$ in length.

3. If θ_2 is zero (C_t connected to the line—now simply θ_1—at its open end), then the shortening of the actual line owing to C_t is a maximum. This is because C_t is at a maximum voltage point and has maximum effect. Also R_t has maximum effect, so that the Q is a minimum.

4. If θ_1 is zero (C_t connected to the line—now simply θ_2—at its shorted end) then the shortening of the actual line owing to C_t is a minimum, zero, so that $\theta_2 = 90°$. This is because C_t is at a zero voltage point and has therefore zero effect. The same is true for R_t and hence the Q is a maximum. Eq. (46) indicates the Q to be infinite in such a case, but

this is merely because the line has been assumed to have no losses of itself.

From the above, it is clear that by properly choosing θ_1 for any given value of C_t and R_t, the Q can be made to have any value desired (up to a maximum determined by the line losses, and a minimum determined by R_t and C_t being connected to the open end of the line). If a lower Q should be desired, R_t would have to be artificially reduced by connecting a resistor in parallel with the input terminals of the tube. This is seldom the case as usually the opposite is the case: at u.h.f. the input resistance of the tube R_t is generally too low.

The problem of the antenna circuit design thus resolves itself into the following:

1. A certain band width of image attenuation is desired.

2. This determines the required Q of the circuit by Eq. (13b) and Eq. (39).

3. To obtain the required Q the proper tap position must be found. This means θ_1 must be determined.

Unfortunately, there is no simple equation to give θ_1 once Q, R_t, and C_t are known. A series of trials may be made, and for each value of θ_1 the Q may be calculated by means of Eq. (46) until the proper value is obtained. This is quite laborious unless the engineer has had considerable experience in such matters or else makes a lucky guess.

However, θ_1 can be determined graphically by the aid of the curves given in Fig. 32. These represent two families of curves, in which θ_1 is plotted versus θ_2 for various values of each of two parameters, V and K. The particular values of V and K in any specific problem are calculated from the data given in the problem. Then, where the curve for the particular value of V intersects that for the particular value of K, is a point whose ordinate is the value of θ_1 required, and whose abscissa is the value of θ_2 required.

The method is best explained in connection with the antenna problem under discussion. The derivation of these curves is given in the appendix. It will be recalled that the desired value of Q is 300, the operating frequency is 150 mc, C_t = 5 $\mu\mu f$, and R_t = 7,500 ohms. From these data, the quantities V and K can be calculated as follows:

$$V = \frac{1.145\, QZ_o}{R_t} - 3.6\, fC_tZ_o \qquad (47)$$

$$K = 6.28\, fC_tZ_o \qquad (48)$$

It is to be noted from the above two equations that a value for the characteristic impedance Z_o of the transmission line is to be chosen before V and K can be calculated. Any desired value for Z_o can be employed. However, the usual value for Z_o is 76.8 ohms, corresponding to a line of maximum inherent Q. If this value for Z_o be used, Eqs. (47) and (48) become

$$V = \frac{87.9\, Q}{R_t} - 276.5\, f\, C_t \qquad (49)$$

$$K = 482\, f\, C_t \qquad (50)$$

Assume Z_o = 76.8 ohms. Then,

by Eqs. (49) and (50),

$$V = \frac{(87.9)(300)}{7500} - (276.5)(150 \times 10^{6})(5 \times 10^{-12})$$

$$= 3.31$$

$$K = (482)(150 \times 10^{6})(5 \times 10^{-12})$$

$$= .362$$

The nearest V-curves in Fig. 32 are the ones labelled 3.0 and 3.5; the value 3.31 must be estimated (interpolated) between them. It is approximately 2/3 of the distance between the given curves from the 3.0 curve.

The nearest K-curves are K = .3 and K = .4. Proceed along the estimated V = 3.31 curve until about .6 of the distance between the two K-curves. The result is point A. Going across to the left, θ_1 is found to be equal to 26.7°.

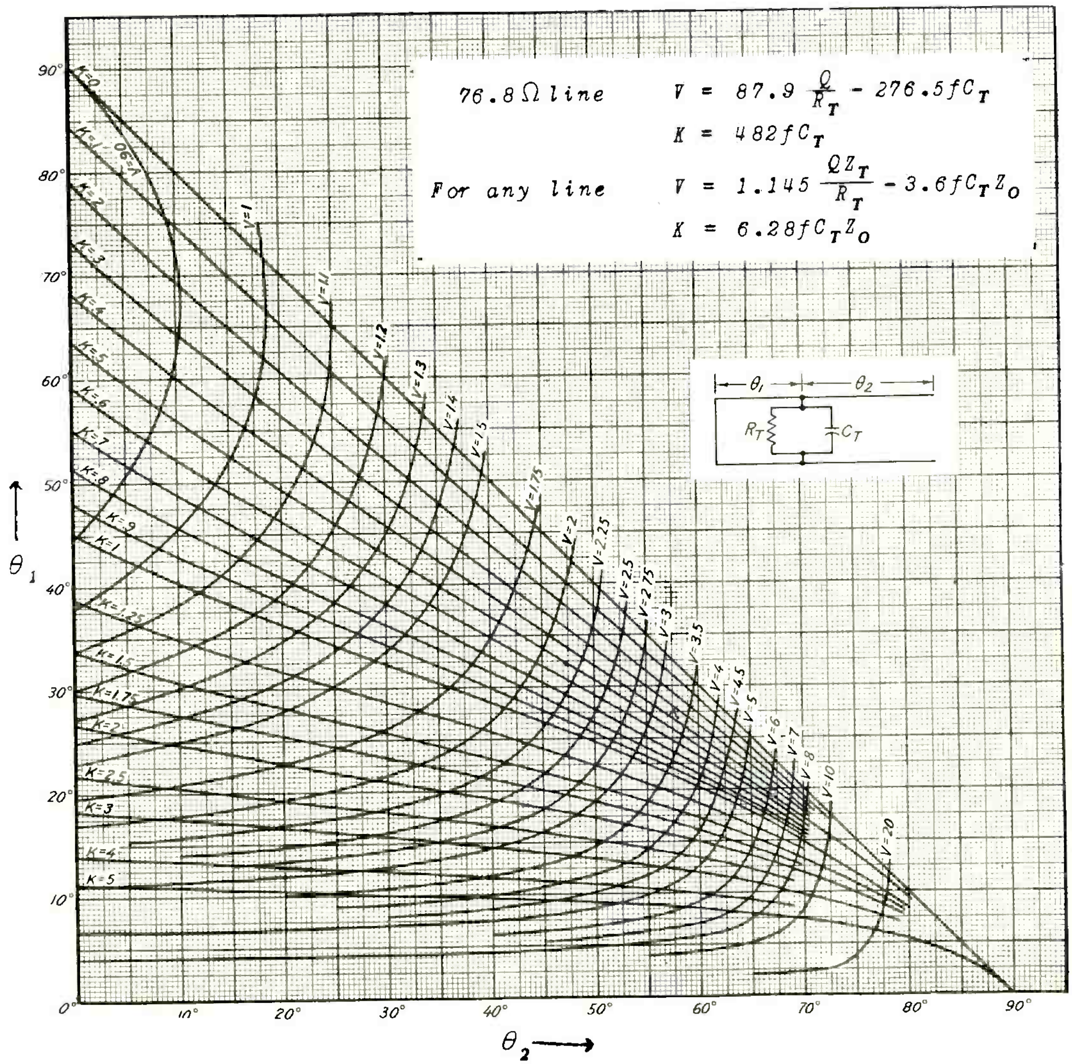


Fig. 32.—Curves giving tap position for a given load, frequency, and desired value of Q.

Going directly down from point A, θ_2 is found to be 58°.

The total length of the line (in degrees), is then 26.7° + 58° = 84.7°. It is tapped 26.7° from the shorted end. The actual linear distances can now be readily calculated:

$$l_1 = \frac{26.7 \times 200 \text{ cm}}{360}$$

$$= 14.83 \text{ cm} \div 2.54$$

$$= 5.85 \text{ inches}$$

$$l_2 = \frac{58 \times 200}{360}$$

$$= 32.2 \text{ cm} \div 2.54$$

$$= 12.68 \text{ inches}$$

$$l_1 + l_2 = 5.85 + 12.68$$

$$= 18.53$$

(Note that 200 cm is the wavelength for f = 150 mc.)

The results can now be checked by computing the Q for the above values and comparing it with the desired value of 300. From Eq. (46)

$$Q = \frac{2\pi \times 150 \times 10^6 \times 7500}{2}\left(5 + \frac{.433 \times 14.83}{\sin^2 26.7^\circ} + \frac{.433 \times 32.2}{\cos^2 58^\circ}\right) \times 10^{-12}$$

$$= \frac{2.25\pi}{2}\left(5 + \frac{6.42}{(.4493)^2} + \frac{13.96}{(.5299)^2}\right)$$

$$= \frac{2.25\pi}{2}(5 + 31.8 + 49.7) = \frac{2.25\pi \times 86.5}{2} = 305$$

This is less than 3 per cent higher than desired, and is therefore a very satisfactory value. The values for l_1 and l_2 may be used in building the line, but some adjustment means should be provided to enable an accurate setting to be made experimentally.

It is to be noted that if a value for Z_o other than 76.8 ohms had been chosen, Fig. 32 would still apply, but V and K would have to be calculated from Eqs. (47) and (48), using the particular value of Z_o chosen. The values for θ_1 and θ_2 would then be different in order to conform with the selected value of Z_o.

There remains to evaluate the antenna tap θ_A. This can be calculated with sufficient accuracy for all practical purposes by the use of Fig. 23. The impedance ratio a is 75/7,500 = .01. The given angle is θ_1 = 27°, and the desired angle θ_A can be found from Fig. 23 by interpolating between 30° and 20°. Thus, θ_A comes out to be about 2.6°, corresponding to a length of (2.6 × 200)/360 = 1.44 cm ÷ 2.54 = .567 inch. This is a rather short distance from the shorted end. Indeed, the shorting disc or bar may be of this order of magnitude (instead of—theoretically—of zero length). It may therefore be advisable to use a different method of coupling the antenna line to the resonant line, namely, by means of a loop.

Loop Coupling.—This is shown in Fig. 33. The loop is often known as a "hairpin" coupling. It is connected to the non-resonant line, and intercepts the magnetic

Fig. 33.—Method of coupling to a two-wire transmission line by means of a loop.

lines of force set up by the currents in the resonant line. In the case of a two-wire line, the loop can be mounted on a hinged member so that it can swing in and out of the plane of the transmission line and thus vary its coupling over a wide range of values.

In the case of a concentric line, it can be mounted through a hole in the casing, as shown in

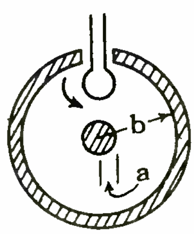


Fig. 34.—Loop coupling to a coaxial cable.

Fig. 34, and arranged to rotate (as indicated by the arrow). Maximum coupling occurs when the plane of the loop is perpendicular to the cross section of the cable, and zero coupling when it is parallel to the cross section. The loop should be preferably located near the shorted end of the line where the current is a maximum and the voltage is a minimum. This provides maximum magnetic coupling and minimum electrostatic, thus precluding stray coupling of an unpredictable nature.

A very simple rule as to the size of the loop, which applies primarily to wave guides, may be used in the case of a concentric line. This rule states that twice the ratio of the loop area A_l to the nonconductive cross-sectional area of the line, call it A_c, is equal to the square root of the impedance ratio n, where n is taken not with respect to the actual load tapped down somewhere on the resonant line, but with respect to the equivalent shunt resistance at the open end of the line, R_t'. In formula form

$$n = \left(\frac{2\,A_l}{A_c}\right)^2$$

or

$$A_l = \frac{A_c\,\sqrt{n}}{2} \qquad (51)$$

This rule can be applied to the problem under discussion. Assume a line for which a = .2 inch and b = 3.6 × .2″ = .72 inch. Since b/a = 3.6, Z_o = 76.8 ohms as originally assumed. The nonconductive cross-sectional area is clearly the difference between the circular area within the outer conductor and that of the inner conductor (see Fig. 34), or

$$A_c = \pi\left[(.72)^2 - (.2)^2\right]$$

$$= 1.5 \text{ sq. in.}$$

To determine the impedance

ratio n for this formula, remember that R_t is 7,500 ohms, but that it is equivalent to a resistance of

$$R'_t = \frac{Q}{\omega C_{\lambda/4}}$$

$$= \frac{305}{2\pi \times 150 \times 10^6 \times 10.8 \times 10^{-12}}$$

$$= 29{,}900 \text{ ohms}$$

(Note: $C_{\lambda/4} = 10.8\ \mu\mu f = \frac{C_o \lambda}{8}$)

located at the open end of an equivalent quarter-wave line. This is slightly greater than the original value of 29,500 ohms, and is due to the fact that the Q as finally determined by the tube tap setting was 305 instead of 300. Then

$$n = 75 \div 29{,}900 = .00251$$

from which, by Eq. (51)

$$a_l = \frac{1.5\sqrt{.00251}}{2} = .0376 \text{ sq.in.}$$

The radius of the loop will be

$$r_l = \sqrt{.0376/\pi} = .1093 \text{ inch}$$

or slightly over .2 inch in diameter. In practice, a 1/4-inch diameter loop could be used so as to have somewhat excess coupling. It can then be rotated so as to cut down the coupling to the value desired. The inductance of the loop can be tuned out experimentally by slight readjustment of the line length θ_2.

Voltage Step Up.—There remains to calculate the voltage step up of this grid-coupling circuit:

1. When R_t and C_t are connected to the end of the line, and
2. When they are tapped down on the line.

At first glance it might appear that the greatest voltage is impressed on the grid of the tube when it is connected to the open end of the line because this is the point of highest voltage. However, a moment's reflection will indicate that this is not the case; that the voltage step up from the antenna to the grid is the same in any case.

The reason is that when R_t is connected to a point of the line where the voltage is E_t, then the power dissipated in R_t is

$$W_t = \frac{E_t^2}{R_t}$$

At the point where the antenna is connected, R_t appears to be of a value R_A (in the problem R_A = 75 ohms). Let the antenna terminal voltage be E_A. Then the power input from the antenna is

$$W_A = \frac{E_A^2}{R_A}$$

This power must equal that actually dissipated in R_t (ignoring any slight losses in the resonant line). Thus

$$W_A = W_t = \frac{E_A^2}{R_A} = \frac{E_t^2}{R_t}$$

or

$$\sqrt{\frac{R_t}{R_A}} = \frac{E_t}{E_A}$$

$$= \text{voltage step up from antenna to grid} \qquad (52)$$

Eq. (52) indicates that the step up is independent of the actual positions of the antenna and tube connections on the resonant line and depends simply upon the square root of the two impedances involved.

Thus for a given antenna voltage, the tube voltage (under matched conditions) is the same regardless of whether it is connected to the end of the resonant line or tapped down on it. The reason is that when the tube is tapped down on the line, the Q of the latter is increased by just the right amount to compensate for the fact that the voltage at this point is less than that at the open end.

Thus, tapping down on the line does not materially change the voltage input to the tube, but does alter the Q of the resonant line so as to meet selectivity and band width requirements.

Stub Matching.—A more general method of matching any kind of impedance can be developed on the same basis as the preceding analysis. In this method the short-circuited portion of the resonant line up to the tap position is called a "matching stub." The method also assumes negligible line losses, but applies even where the connected load is inductive instead of capacitive in nature.

Suppose a pure resistance, such as the characteristic impedance Z_o of a low-loss non-resonant line, is to be matched to an impedance consisting partly of resistance R and reactance X. As was mentioned in the assignment on r.f. measurements, any impedance can be represented at any one frequency by a suitable resistance and reactance in parallel. Let the above R and X thus represent an impedance.

The matching can be accomplished in the same manner as illustrated in Fig. 35. Thus, in Fig. 35(A), L_N represents the non-resonant line to which it is desired to connect a net impedance Z_o, say of pure resistance. The actual impedance Z to be connected is partly resistive and partly reactive. By connecting it to a resonant line L_R of suitable length l_L, the reactance X may be tuned out. Then at the tap only R will be reflected as a resistance Z_o if the tap length l_t is chosen correctly. Note that only R is considered and that the line losses are ignored, which is permissible if R is not very great.

In the preceding cases, X was that of a capacitance, such as the input capacitance of a tube or a trimmer condenser. In this section

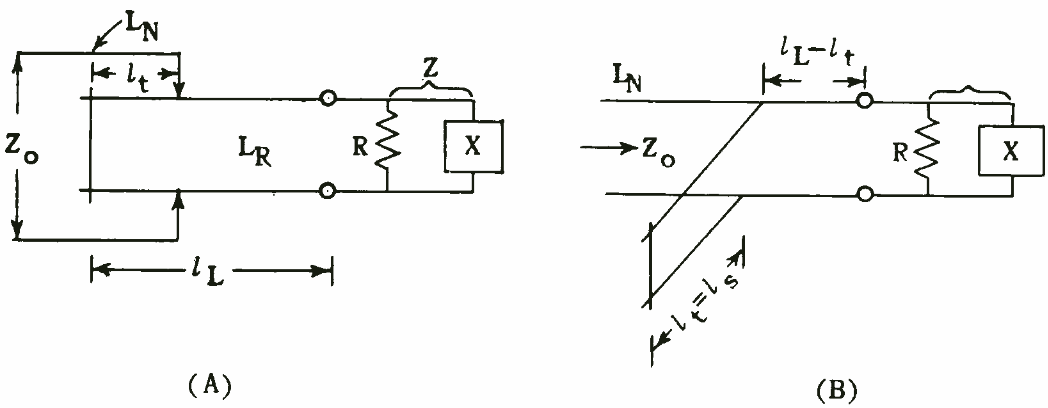


Fig. 35.—Use of a shorted stub for matching any kind of an impedance Z.

X may be either inductive or capacitive. If inductive, then if L_R is a shorted line, it must be greater than $\lambda/4$ in length in order to be capacitive and tune with X: indeed, it must be as much shorter than a $\lambda/2$ line as it would be shorter than a $\lambda/4$ line if X were capacitive instead of inductive. Thus, the line length θ_L given by Fig. 19 can be used for an inductive reactance termination provided a quarter-wave length or 90° is added to it.

On the other hand, for X inductive, L_R could be an open-circuited line in order to tune with X. In this case its line length can be had directly from Fig. 19 without the need of adding 90° to the value obtained from the figure. The remaining figures and calculations would then be the same in either case.

In Fig. 35(B) is shown the same circuit as in (A), but arranged somewhat differently. The portion from the tap to the short-circuit (or open-circuit, if that type of resonant line is employed) is shown in the form of a stub line, instead of part of the resonant line L_R. The rest of L_R has lost its identity and has been merged with the non-resonant portion L_N. This assumes that all portions have a common characteristic impedance which the preceding analysis did not assume.

The problem, as represented by Fig. 35(B), becomes one of choosing a suitable length l_s of open- or short-circuited stub, and a suitable distance $(l_L - l_t)$ of connection from the impedance end Z.

Usually the fact that Z is not equal to Z_o is made manifest by standing waves of voltage and current along the line L_N when it is connected to Z. Hence, a simplification of the preceding method is possible. First the line is connected directly to the impedance and standing waves observed. For this purpose a vacuum tube voltmeter or crystal rectifier may be employed, depending upon the frequency. This will be discussed more fully later. If the voltage standing waves are measured, then the current loops and nodes can be determined, as they are half-way between the voltage loops and nodes. In addition, the ratio of a voltage (or current) maximum to a minimum is measured.

It will be recalled that if a line is not terminated in its characteristic impedance, a combination of a forward wave and a partial reflection of this wave in the opposite direction is set up. The reflected wave, at most, equals the forward wave in amplitude. This occurs when the mismatch is complete: short or open circuit of line. For lossless line the voltage and current nodes would be zero and the loops twice the average value. For a partial mismatch the nodes are greater than zero because the reflected wave cannot completely cancel the forward wave at the nodal points along the line, and hence the ratio of maximum to minimum voltage or current indirectly indicates the departure of the termination from the value of the characteristic impedance of the line.

In Fig. 36 are presented curves that enable the length of the stub and its distance from a current minimum to be determined. The location of the stub will be within a wavelength's distance from the

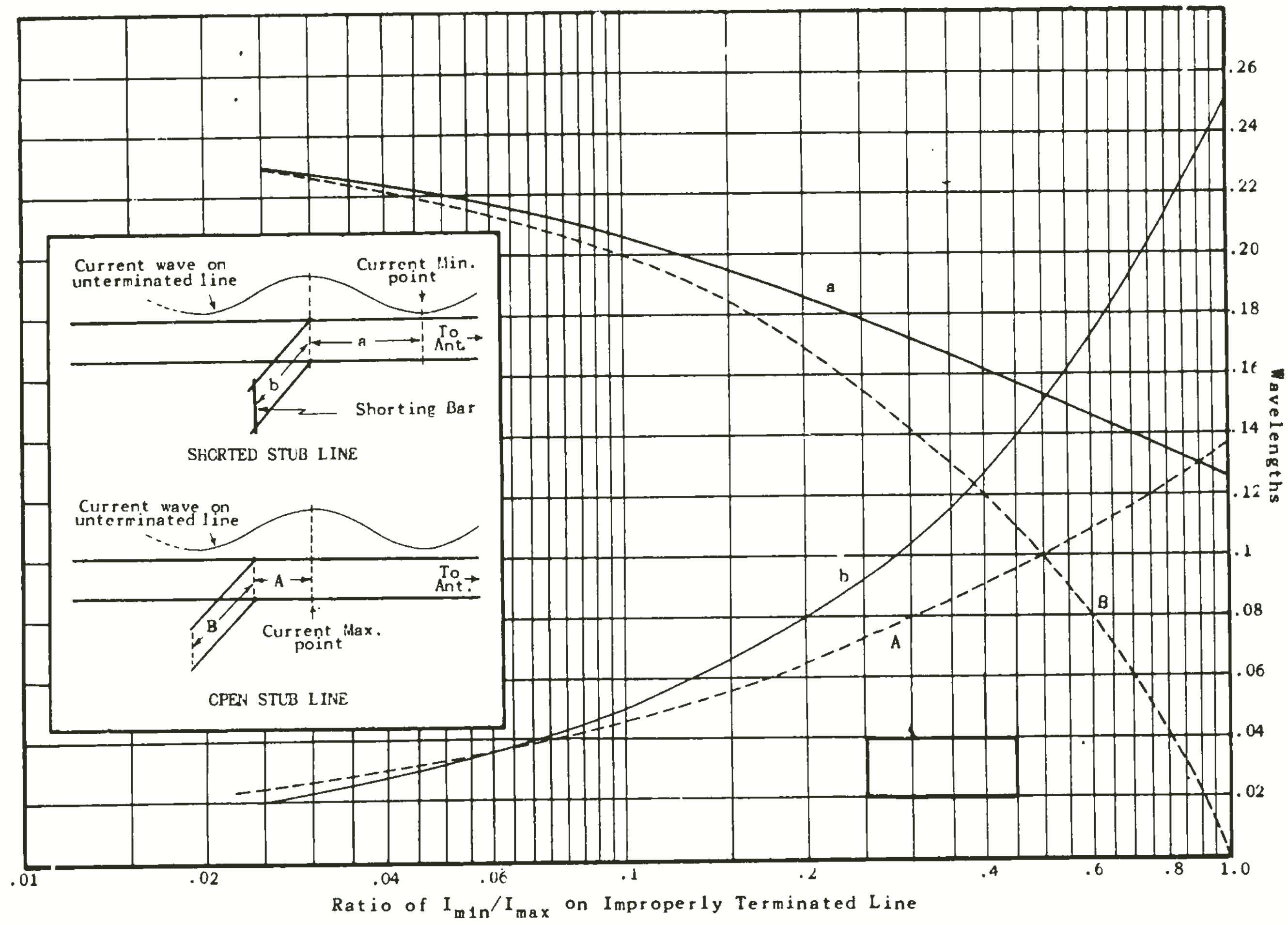


Fig. 36.—Curves giving the length of matching stub and its distance from the load impedance, in terms of the standing wave ratio.

load. It will be noted that in this method the actual value of the load Z does not have to be known directly; instead, merely its effect in producing standing waves, as determined experimentally.

To illustrate its use, suppose $E_{min}/E_{max} = I_{min}/I_{max}$ is found to be 0.3, and it is desired to use a short-circuited stub. Reference to the a and b curves shows that the required stub length is b = .104 λ, and the distance from the current minimum is a = .172 λ.

If an open-circuited stub is desired, the stub length is B = .142 λ, and the position from a current maximum is A = .08 λ. In actual practice, some slight readjustment will have to be made experimentally to these values to eliminate standing waves in the main portion of the line to the left of the stub position. Standing waves will then appear only on the stub and the portion of the line between the stub and the load.

CONCLUSION

This concludes the assignment on u.h.f. propagation and transmission lines. The properties of u.h.f. transmission were discussed first, and it was shown that instead of the ground and sky waves—important at the lower r.f. frequencies—the direct and ground-reflected waves are of paramount importance in determining propagation at frequencies above about 50 mc. Moreover, particularly at the higher frequencies, propagation is essentially over line-of-sight paths, and to obtain any appreciable coverage, high transmitting and (if possible) high receiving antennas are necessary.

The disadvantages of u.h.f. propagation are the line-of-sight limitations and the relatively small powers available, particularly at the highest frequencies; the advantage of u.h.f. propagation is the possibility of using directional arrays, including horns and reflectors, to provide sharp beams that furnish not only the equivalent of much greater power in the desired direction, but enable the energy to be beamed in the desired direction, thereby eliminating interference to other services located in other directions and operating on the same frequency.

At the higher frequencies lumped circuit elements become exceedingly small and impractical, but fortunately transmission line elements can take the place of the lumped elements. In particular, shorted and open-circuited quarter-wave lines can act as resonant circuits, and taps on such lines or portions thereof enable impedance matching to be performed. Several examples of such use are given, and representative problems are worked out to enable the student to calculate the performance of such elements

APPENDIX I

DERIVATION OF CURVES FOR DETERMINING TAP POSITION

The theory underlying the construction of the curves of Fig. 32 is comparatively simple. In Eq. (42)

$$\tan\theta_2 = \cot\theta_1 - \omega C_t Z_o \qquad (42)$$

set $\omega C_t Z_o = K$, a constant. Thus

$$\tan\theta_2 = \cot\theta_1 - K \qquad (a)$$

or

$$K = \cot\theta_1 - \tan\theta_2$$

$$= \omega C_t Z_o \qquad (b)$$

If ω, C_t, and Z_o are given in any problem, K is determined. There are then an infinite set of *pairs of values* of θ_1 and θ_2 that satisfy Eq. (b). Thus, the mere determination of K in any given problem does not uniquely determine θ_1 and θ_2.

However, two other factors must be satisfied, namely, Q and R_t. These are connected with θ_1 and θ_2 by Eq. (46):

$$Q = \frac{\omega R_t}{2}\left[C_t + \frac{C_o l_1}{\sin^2\theta_1} + \frac{C_o l_2}{\cos^2\theta_2}\right] \qquad (46)$$

where C_o is the capacitance per unit length of cable. Since

$$\left.\begin{aligned} l_1 &= \frac{\lambda\,\theta_1}{360} \\ \text{and} \quad l_2 &= \frac{\lambda\,\theta_2}{360} \end{aligned}\right\} \qquad (c)$$

these values may be substituted in Eq. (46) to obtain

$$Q = \frac{\omega R_t}{2}\left[C_t + C_o\left(\frac{\frac{\lambda}{360}\theta_1}{\sin^2\theta_1} + \frac{\frac{\lambda}{360}\theta_2}{\cos^2\theta_2}\right)\right]$$

or, multiplying out,

$$Q = \frac{\omega R_t C_t}{2} + \frac{\omega R_t C_o \lambda}{720}\left(\frac{\theta_1}{\sin^2\theta_1} + \frac{\theta_2}{\cos^2\theta_2}\right) \qquad (d)$$

Now solve for the right-hand quantities in the parenthesis:

$$\frac{\theta_1}{\sin^2\theta_1} + \frac{\theta_2}{\cos^2\theta_2} = \frac{Q - \frac{\omega R_t C_t}{2}}{\frac{\omega R_t C_o\,\lambda}{720}} = \frac{720\left(Q - \frac{\omega R_t C_t}{2}\right)}{\omega R_t C_o\,\lambda} \qquad (e)$$

Let

$$V = \frac{720\left(Q - \frac{\omega R_t C_t}{2}\right)}{\omega R_t C_o\,\lambda} \qquad (f)$$

so that

$$V = \frac{\theta_1}{\sin^2\theta_1} + \frac{\theta_2}{\cos^2\theta_2} \qquad (g)$$

If Q, ω, R_t, C_t, and C_o are all given in the problem, V can be calculated by Eq. (f). Then Eq. (g) gives a rather complicated relationship between θ_1, θ_2, and V. Just as in the case of the quantity K, for any given value of V there are an infinite set of pairs of values of θ_1, and θ_2 that satisfy Eq. (g), so that the determination of V in any given problem does not uniquely determine θ_1 and θ_2.

However, if in the problem all quantities are given to determine *both* V and K, then there are obtained *two* equations in θ_1 and θ_2, namely Eqs. (b) and (g). *Two equations* in *two unknowns* can determine the unknowns uniquely; the simultaneous solution of Eqs. (b) and (g) will give a unique value for θ_1 and for θ_2, on the assumption that V and K are determined by the data in the problem.

Unfortunately, there is no simple, algebraic method for solving Eqs. (b) and (g) because these involve trigonometric functions. It can be shown by the methods of the calculus that a trigonometric function corresponds to an algebraic equation of infinite degree, and is therefore called a transcendental function; i.e., it transcends or is beyond the ordinary algebraic equation of finite degree.

Hence, some other means of solving these equations is necessary. A practical method is a graphical solution. Various values of K are chosen, such as K = 0, 0.1, 0.2, etc. For each value of K, a value of θ_2 is chosen, substituted together with the value of K in Eq. (b), and the value of θ_1 found in conjunction with a set of trigonometric tables. (Or θ_1 can be chosen, and θ_2 determined.)

For example, for K = 0.1, if θ_2 is chosen as 5°, θ_1 is found to equal 79.4°. If θ_2 is chosen as 10°, θ_1 = 74.6°, and so on. The pairs of values of θ_1 and θ_2 are plotted, and give rise to a curve labelled K = 0.1 in Fig. 32. Similar curves for K = 0.2, etc. are plotted, and give rise to the family of K-curves shown in the figure.

A similar procedure is employed with regard to Eq. (g). Values of V = 0.9, 1.0, 1.1, 1.2, etc., are chosen. For each value of V, a value say, of θ_1 is chosen, and θ_2 solved for from Eq. (g). A whole series of pairs of values of θ_1 and θ_2 are obtained for each value of V; these are then plotted as shown in Fig. 32. A whole family of such curves are obtained for successive values of V.

If a given pair of values for θ_1 and θ_2 are to satisfy both equations, they must lie both on the given V-curve and on the given K-curve. This can only occur where the two curves intersect, since the intersection is clearly such a common point. Hence the procedure described in the text; after the values of V and K have been independently determined by the data given in the problem, the corresponding curves are chosen, and where they intersect is the point determining both θ_1 and θ_2.

It is thus apparent that the graphical method is just as easy to apply to complicated equations as to simple equations, and is therefore a valuable aid in the simultaneous solution of complicated equations. It is true that the range is limited by the number of curves drawn, and the accuracy by such mechanical factors as the accuracy of the drawing of the curves, the thickness of the lines, the scale used, and the accuracy of interpolation. However, for most practical purposes, the accuracy is sufficient for an engineering solution.

Refer once more to Eq. (f). This can be multiplied out to give

$$V = \frac{720Q - 360\,\omega R_t C_t}{\omega R_t C_o\,\lambda} \qquad \text{(h)}$$

Since the velocity of propagation on a low-loss transmission line is practically that of light: $C = 3 \times 10^{10}$ cm/sec., and since further

$$C = f\lambda$$

so that

$$\omega\lambda = 2\pi C = 6\pi \times 10^{10} \text{ cm/sec.}$$

This numerical quantity can be substituted for $\omega\lambda$ in the denominator of Eq. (h), so that there is obtained:

$$V = \frac{3820}{C_o}\,\frac{Q}{R_t} - \frac{1910}{C_o}\,\omega C_t \qquad \text{(i)}$$

where C_o is in $\mu\mu$f/cm.

From Eq. (36) in the text, we have

$$C_o = \frac{33.3}{Z_o} \qquad (36)$$

This can be substituted in Eq. (i) to yield

$$V = 114.5\,\frac{QZ_o}{R_t} - 57.3\,\omega C_t Z_o \qquad \text{(j)}$$

Note that V is now expressed in terms of Z_o instead of C_o. This is desirable, since a transmission line is more usually specified in terms of its characteristic impedance Z_o than in terms of its capacitance C_o per unit length.

A further detail (or possibly refinement) is to divide Eq. (j) by 100 to yield smaller numbers. This is usually more satisfying from a psychological viewpoint, but is of no mathematical significance. When this division is performed, there is obtained

$$V = 1.145\,\frac{QZ_o}{R_t} - 3.6\,fC_t Z_o \qquad (47)$$

which is Eq. (47) of the text. It is to be noted that f is in cycles per sec. and C_t is in *farads* in this formula.

Eq. (b) for K can be expressed in terms of f instead of ω:

$$K = 6.28\,fC_t Z_o \qquad (48)$$

which is Eq. (48) of the text. However, in calculating the points for the curves of Fig. 32, the actual values for V are employed. In labelling the curves, on the other hand, these values of V are divided by 100 to furnish values that conform with Eq. (47).

If $Z_o = 76.8$ is substituted in Eqs. (47) and (48), there are obtained

$$V = \frac{87.9\,Q}{R_t} - 276.5\,fC_t \qquad (49)$$

$$K = 482\,fC_t \qquad (50)$$

where the equation numbers here again correspond to these in the text. It will be observed that even after division by 100, numbers such as 87.9, 276.5, and 482 are obtained. Had the division not been initially performed, numbers 100 times greater would have been obtained, with the greater possibility of producing arithmetical errors.

In summary, the curves of Eq. (32) are based on the idea of separating the functions of θ_1 and θ_2 from the rest of the variables, and setting the combinations of the latter equal to V and K, respectively. Then curves are drawn in which, for each value of V and K, values of θ_1 and θ_2 are found satisfying the respective equations. The intersection of a given curve of one family with the corresponding one of the other family, gives a unique value for θ_1 and θ_2 that satisfies the initial conditions of the problem. This is so because the initial conditions determine the value of V and K to be used in choosing the proper curve of each family.

ULTRA-HIGH FREQUENCY TECHNIQUE

EXAMINATION

1. (A) What becomes of the ground wave at ultra-high frequencies?

   (B) What wave is effective at ultra-high frequencies?

2. (A) Give two reasons for the value of a high transmitting and/or receiving antenna at the low end of the u.h.f. spectrum.

   (B) Why is one of the above reasons eliminated when one gets into the microwave range?

2. *(Continued)*

3. A transmitter feeds 100 watts at a frequency of 150 mc into an antenna array at a height of 225 feet. The antenna array is similar to that shown in Fig. 7. A similar array is used for the receiver. The latter is 100 feet high, and is located at the limit of the line-of-sight distance from the transmitter.

(A) What is the distance between receiver and transmitter?

(B) What is the value of the received power?

3. (C) What is the field strength in μ-volts/meter at the receiving location?

(D) What voltage would be induced in the receiving antenna if it were a dipole instead of an array, while the transmitting conditions were unchanged?

4. A quarter-wave coaxial line, shorted at one end, has a Q of 10,000. Suppose another line exactly the same as the above, except that it is twice as long and shorted at both ends, is used. What will be its Q? Explain.

   HINT: Consider the half-wave line as consisting of two quarter-wave lines, connected to one another at their adjacent ends, and shorted at their far ends. Such two quarter-wave lines appear to be in parallel to the generator. Consider how this affects the ratio of stored to dissipated energy as compared to one quarter-wave line alone.

5. What would be the advantage of the second line in Question 4 over the quarter-wave line, as regards radiation in the microwave range if either line is driven by a generator through a coupling loop at a shorted end?

5. *(Continued)*

6. A quarter-wave resonant line oscillator is to be built to operate at 300 mc. The input resistance of the tube is 3,000 ohms. It is desired to have a net Q of 2,500, and for the line to present to the grid of the tube a resistance of 3,000 ohms. Design the line and calculate the tap position.

HINT: Note that the impedance looking into the line at the grid tap is equal to the input resistance of the tube. This means that the damping of the line is due in equal parts to the line losses and input resistance of the tube; i.e., the line Q alone is double the net Q. Hence, the line must be designed to have the proper Q. In calculating its dimensions for the desired Q, use the optimum value of Z_o for which $b/a = 3.6$. This means that either a or b has to be *found*; for example, a, then b is simple 3.6a.

6. *(Continued)*

7. (A) Calculate the depth of penetration in inches for copper of a current whose frequency is 30,000 mc.

7. (B) How would the resistances of two copper rods of equal length and solid circular cross-section, but one having four times the cross-sectional area of the other, compare at this frequency? At d.c.?

8. Calculate the attenuation of 100 feet of a two-wire line whose radii are each 2 mm, and whose separation between conductor centers is 2 cm, operated at 900 mc.

9. A communication system is to operate at 500 mc. It is desired to have an image attenuation of 40 db, and the Q of the antenna system is 200. What must be the intermediate frequency?

9. *(Continued)*

10. Given a converter tube whose input resistance is 2,300 ohms and whose input capacitance is 4 $\mu\mu f$. The operating frequency is 350 mc, and the desired half-band width is 3.5 mc. Assume a quarter-wave shorted concentric line is to be employed as the step-up device, whose inner conductor is 0.1 inch diameter, and whose outer conductor is 3.6 × 0.1 = .36 inches. The characteristic impedance of the line is therefore 76.8 ohms. The non-resonant antenna line to be connected to the above resonant line has a characteristic impedance of 75 ohms. The intermediate frequency is to be 30 mc.

(A) Calculate the Q of the resonant line, loaded with the tube and antenna, required to give the desired band width.

10. (B) Calculate the position of the grid tap θ_t in electrical degrees from the shorted end of the resonant line; also find the position in inches. First calculate V and K, then find θ_1 and θ_2.

(C) Calculate the position of the antenna tap θ_A in degrees from the end of the resonant line; also the position in inches.

(D) Calculate the actual length of the line in degrees and in inches, $\theta_1 + \theta_2$.

10. (D) continued.

(E) Calculate the image rejection in db.

(F) Check your calculations by computing the Q of the actual line using the equation (46) for this check.