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Pseudo-Random and Random Test Signals

Using digital techniques, this precision low-frequency noise generator can synthesize repeatable, controllable, pseudo-random noise patterns as well as truly random noise.

By George C. Anderson, Brian W. Finnie and Gordon T. Roberts

ALMOST EVERY NATURAL AND MAN-MADE SYSTEM is subject to random disturbances under normal operating conditions. Consequently, it is often appropriate, and sometimes essential, to test a system with random test signals rather than with the sine waves that are so familiar to electrical engineers.

Many of the areas of application for random test signals lie outside the field of electrical engineering. Examples are biomedical phenomena, vibration, aerodynamics, and seismology. However, a growing number of electrical problems fall into this same category.

For example, it is much more appropriate to test a multi-channel telephone system with random noise simulating each speech signal, than to use a number of sine waves. The problem of communicating with deep space probes is another subject that can be adequately treated only by means of statistical techniques.

From the mathematical viewpoint, therefore, there are good reasons for using noise as a test signal. Yet, despite the fact that adequate theories have been developed, the introduction of test methods based on these theories has been delayed by a lack of suitable, convenient test equipment.

Chief among the many factors responsible for this state of affairs is that conventional noise generators employ 'natural' noise sources such as gas-discharge tubes and temperature-limited diodes. The statistics of the noise signals produced by these sources are not very stable, well-defined, or controllable. The problem is most severe at low audio and sub-audio frequencies, where much of the current interest in noise testing is focused.

To circumvent these deficiencies, the development of a new low-frequency noise generator was undertaken. The result of this development program is the instrument shown in Fig. 1. It is not a 'natural' noise source; it is a precision noise generator which synthesizes noise and noise-like (pseudo-random) signals by a controllable digital process. As a result, the characteristics of its output can be specified accurately and varied to fit the measurement situation.

This new measurement tool will realize its full potential only after people understand it and begin to see how they can use it to solve their problems. We hope to accelerate this process by describing how the new noise generator works and some of the things it can do.

Fig. 1. *A precision digital instrument, Model 3722A Noise Generator synthesizes repeated pseudo-random noise-like patterns or non-repeating random noise. Binary (two-level) and Gaussian (multi-level) outputs are generated. Amplitudes and bandwidths of outputs and lengths of pseudo-random patterns are variable.*

Specifying Noise

How can noise be specified?

Simple deterministic signals can be completely specified by a small number of parameters. For example, dc is specified by only one parameter. A step function is specified by two parameters — amplitude and time. And a sine wave is specified by three parameters—amplitude, frequency, and phase.

Random signals, on the other hand, can't be completely specified by a finite number of parameters. But we still need some way of describing them, so we resort to statistical descriptions which tell us about the average behavior of the signals.

The simplest statistic of a noise signal is its mean-square value or, equivalently, its rms value. This parameter is quite easy to measure, provided that we have an instrument with a true square-law response. We also have to carry out the averaging process over a long enough time to reduce the statistical variance of the results to an acceptably small value.

Power Density Spectrum

Another statistical description of a random signal that isn't difficult to measure is its power density spectrum. This tells us how the noise power contributed by separate frequency components of the signal is distributed over the frequency spectrum. It should have units of watts per unit bandwidth, but it is common practice in noise theory to consider (amplitude)^2 as the unit of power. For electrical signals, this gives the power density spectrum units of V^2/Hz.*

A power density spectrum is shown in Fig. 2. The total area under this curve gives the total power contained in the signal. The power contributed by all frequency components in any band, say from f_1 to f_2, is equal to the area under the power density curve between f_1 and f_2 (shaded area in Fig. 2). Power density spectra can be measured experimentally with a narrow-band, constant-bandwidth wave analyzer followed by a true square-law meter with a long averaging time.

* This inconsistency in the units of power is unacceptable to some engineers; they reconcile the difficulty by assuming a one-ohm load resistance.

Cover: Model 180A Oscilloscope *(bottom)* displays a portion of pseudo-random Gaussian noise pattern generated by Model 3722A Noise Generator *(center)*. Top instrument is a display unit from new HP Model 5400A Multi-channel Analyzer, which will be described in a future issue of the Hewlett-Packard Journal. Here the Analyzer displays the probability density function of the noise generator's Gaussian output.

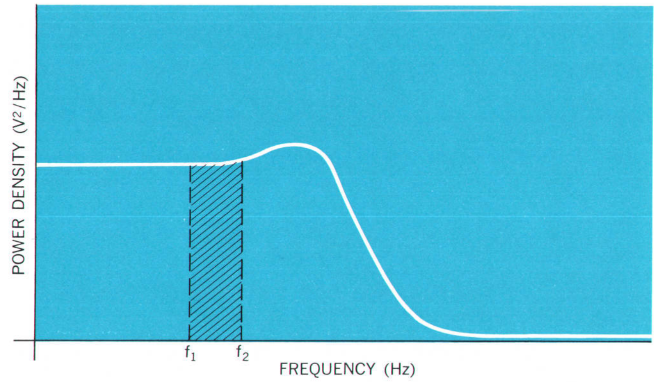


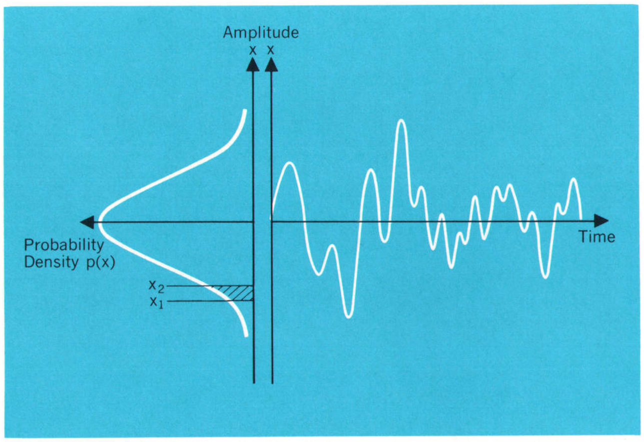
Fig. 2. *Typical power density spectrum for a random signal. Total area under curve is mean-square value of signal, usually spoken of as 'power' in noise theory. Shaded area is power in the frequency band f_1 to f_2.*

It is important to notice that the power *density* spectrum is not the same as the power spectrum. The former has units of V^2/Hz. The latter is just the square of the amplitude spectrum and has units of V^2. The power spectrum is used to describe signals which have a finite number of discrete frequency components. The amplitude or (amplitude)^2 of each component can be represented by a line of the proper length on the graph. But when the signal is a complex random waveform, the power spectrum has to have an infinite number of lines, all of zero amplitude. Thus the power spectrum shrinks to zero for a random signal. The power *density* spectrum, however, does not disappear.

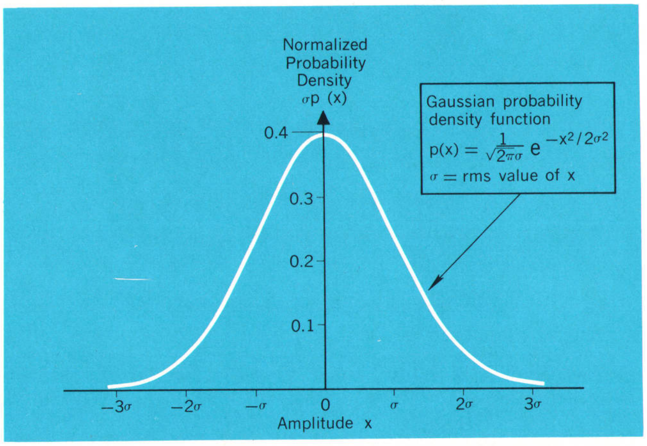
Noise which contains equal amounts of all frequencies is called 'white' noise, by analogy to white light. White noise has a power density spectrum which is simply a horizontal line representing some non-zero value of power per unit bandwidth. Truly white noise, which has infinite bandwidth and therefore infinite power, is never found in physical systems, which always have finite bandwidths. We usually call noise 'white' if it has a flat power density spectrum over the band of interest.

Probability Density Functions

The power density spectrum tells us how the energy of a signal is distributed in frequency. But it doesn't specify the signal uniquely, nor does it tell us very much about how the amplitude of the signal varies with time. That the spectrum doesn't specify the signal uniquely is a consequence of the fact that it contains no phase information. Two periodic signals, for example, have the same power spectrum if they both contain the same frequency components at the same amplitudes. But if the



(a)



(b)

Fig. 3. *Probability density function tells what proportion of time is spent by signal at various amplitudes. Shaded area in* **(a)** *is equal to proportion of time spent by signal between x_1 and x_2. Gaussian probability density function* **(b)** *is common to many natural disturbances.*

phase of just one component of one signal is shifted with respect to the phase of the corresponding component of the other, the two signals can have drastically different waveforms.

A statistic of a signal that gives waveshape information and is independent of the spectrum is the probability density function, or pdf (see Fig. 3). The pdf tells us what proportion of time, on the average, is spent by the signal at various amplitudes.

The area under a pdf between any two amplitudes x_1 and x_2 is equal to the proportion of time that the signal spends between x_1 and x_2. Equivalently, this area is the probability that the signal's amplitude at any arbitrary time will be between x_1 and x_2. The total area under a pdf is always one.

In general, the probability density function and the power spectrum or power density spectrum are two different — unrelated — properties of a signal.

Probably the most familiar pdf is the bell-shaped Gaussian curve, Fig. 3(b), which is characteristic of many naturally-occurring random disturbances. 'Gaussian' means that a curve has the shape $y = e^{-x^2}$. Probability density functions must all have areas equal to one, so a Gaussian pdf must be normalized, i.e.,

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

where σ is the rms value of the signal.

It is important not to confuse the Gaussian pdf with the output of a Gaussian filter. A Gaussian filter has an impulse response shaped like e^{-x^2} and a frequency response shaped like $e^{-\omega^2}$. The output of a Gaussian filter may indeed have a Gaussian pdf. But an arbitrary signal having a Gaussian pdf may have a power density spectrum which bears no resemblance to the frequency response curve of the Gaussian filter.

It is also important to recognize that Gaussian noise does not have to be white noise, and vice versa. *The pdf and the power density spectrum are independent.*

Correlation Functions

A statistic which is useful because it tells something about the time or phase relationship between two signals (random or not) is the cross-correlation between them. The cross-correlation function for two signals x(t) and y(t) is defined as

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)y(t+\tau)dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t-\tau)y(t)dt.$$

A block diagram of a system which performs this calculation approximately is shown in Fig. 4. One signal is multiplied by a delayed version of the other and the product is averaged. The result is a function of the delay τ. In physically realizable systems the result also depends on the averaging time T. Ideally T should be infinite, but this would mean that it would take an infinite amount of time to get an answer. Fortunately the statistical variance caused by using a finite T can usually be made acceptably small by making T fairly large.

If y(t) = x(t) the cross-correlation function becomes the autocorrelation function of x(t), defined as

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t-\tau)x(t)dt.$$

The autocorrelation function of a signal is the Fourier transform of the power density spectrum. Hence the autocorrelation function of white noise is just a single delta-function at $\tau = 0$; this means that any two samples of the same white noise signal are uncorrelated as long as there is a nonzero time interval between them.

Since the autocorrelation function is the transform of the power density spectrum, it gives us no information that isn't contained in the spectrum. However, it is an extremely useful function and is often simpler to compute than the power density spectrum.

Pseudo-Random Noise

Noise makes a good test signal for two reasons: it is broadband, and it realistically simulates naturally-occurring disturbances. However, its randomness is not very helpful to the experimenter.

Theoretically, experiments involving random noise should be carried out over an infinite time interval so that only the average characteristics of the noise will affect the result. But every real measurement can only be made over a finite time, say T. This means that, if random noise is used as a test signal, the result of an experiment will, in general, be different from its expected value. Or, if an experiment involving random noise is repeated over and over, each repetition will yield a different result. In other words, the randomness of the noise introduces statistical variance into the results.

Variance can be reduced by extending the measurement time T. But it can never be made zero when truly random test signals are used.

What we need, obviously, is a test signal which has the good properties of random noise — i.e., broad, flat spectrum and resemblance to natural disturbances in waveform and pdf—but doesn't have the bad property—i.e., randomness. This signal should be one that introduces no statistical variance into the results, even though the measurement is made over a finite time T.

Such a signal exists. *Pseudo-random* noise is a signal which looks and acts like random noise, but is in fact periodic. This kind of noise is one of the principal products of the new noise generator.

Pseudo-random waveforms consist of completely defined patterns of selectable lengths, repeated over and over*. They have spectra and pdf's that are similar to those of random noise, but because they are synthesized, their statistics are much easier to control.

Most important is the fact that if the measurement time T is made exactly equal to the length of one pseudo-random pattern, the results of an experiment will be identical on every repetition, as long as nothing else has changed. There is no statistical variance. This means that it isn't necessary to use a long measurement time, because the reason for the long measurement time was to

* A good reference on pseudo-random signals is G. A. Korn, 'Random Process Simulation and Measurements,' New York, McGraw-Hill Book Company, 1966.

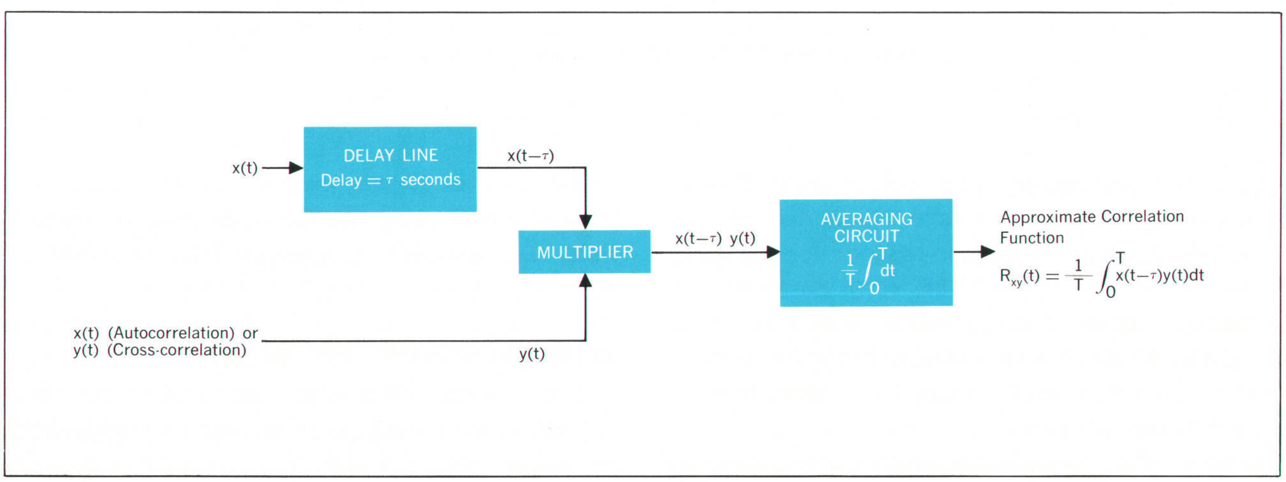


Fig. 4. *Correlation functions show time relationships between signals. They can be computed by multiplying one signal by a delayed version of the other and averaging the product.*

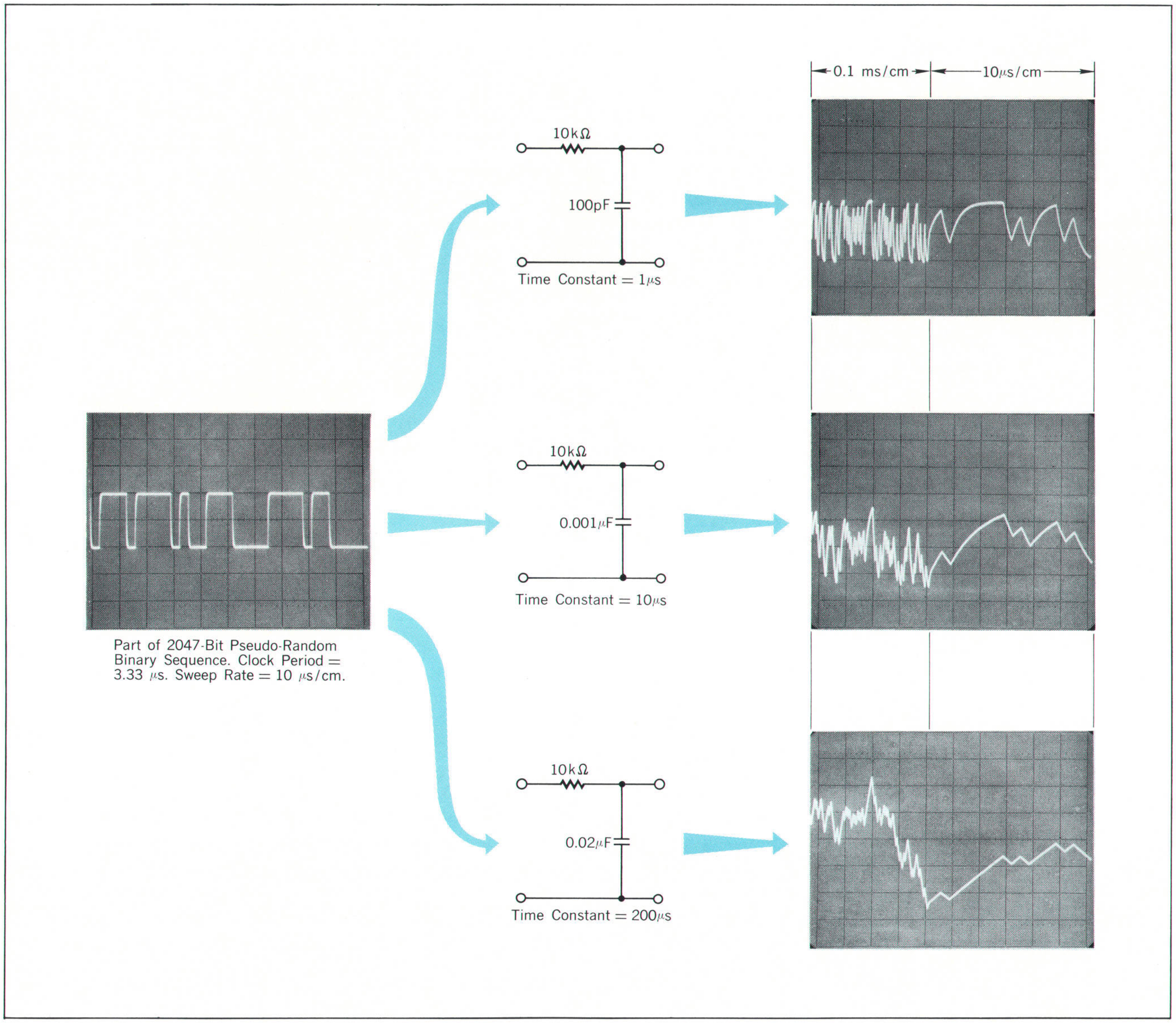


Fig. 5. *Pseudo-random or random Gaussian signals can be derived from pseudo-random or random binary signals by low-pass filtering. To give good results, filter cutoff frequency must be about 1/20 of clock frequency of binary signal.*

reduce the variance introduced by random noise. Pseudo-random noise, therefore, can save a great deal of time.

The repeatability that pseudo-random noise gives an experiment is especially valuable when parameters of the system being tested are varied, as on an analog computer. In such tests, it is important to know that changes in test results are caused by parameter manipulation and not by statistical variance.

Because measurements using pseudo-random noise are normally made over one pattern length, we lose none of the advantages of random signals by substituting pseudo-random signals, even though they are periodic. Measurements using random noise must be made in a finite time anyway, so it makes no difference whether the signal repeats or not after the measurement time is over.

Binary and Gaussian Noise Generated

The most useful and most widely used pseudo-random or random test signals are of two types—pseudo-random or random binary (two-level) signals and pseudo-random or random Gaussian (multi-level) signals. The Gaussian signals are used in testing analog systems. The binary

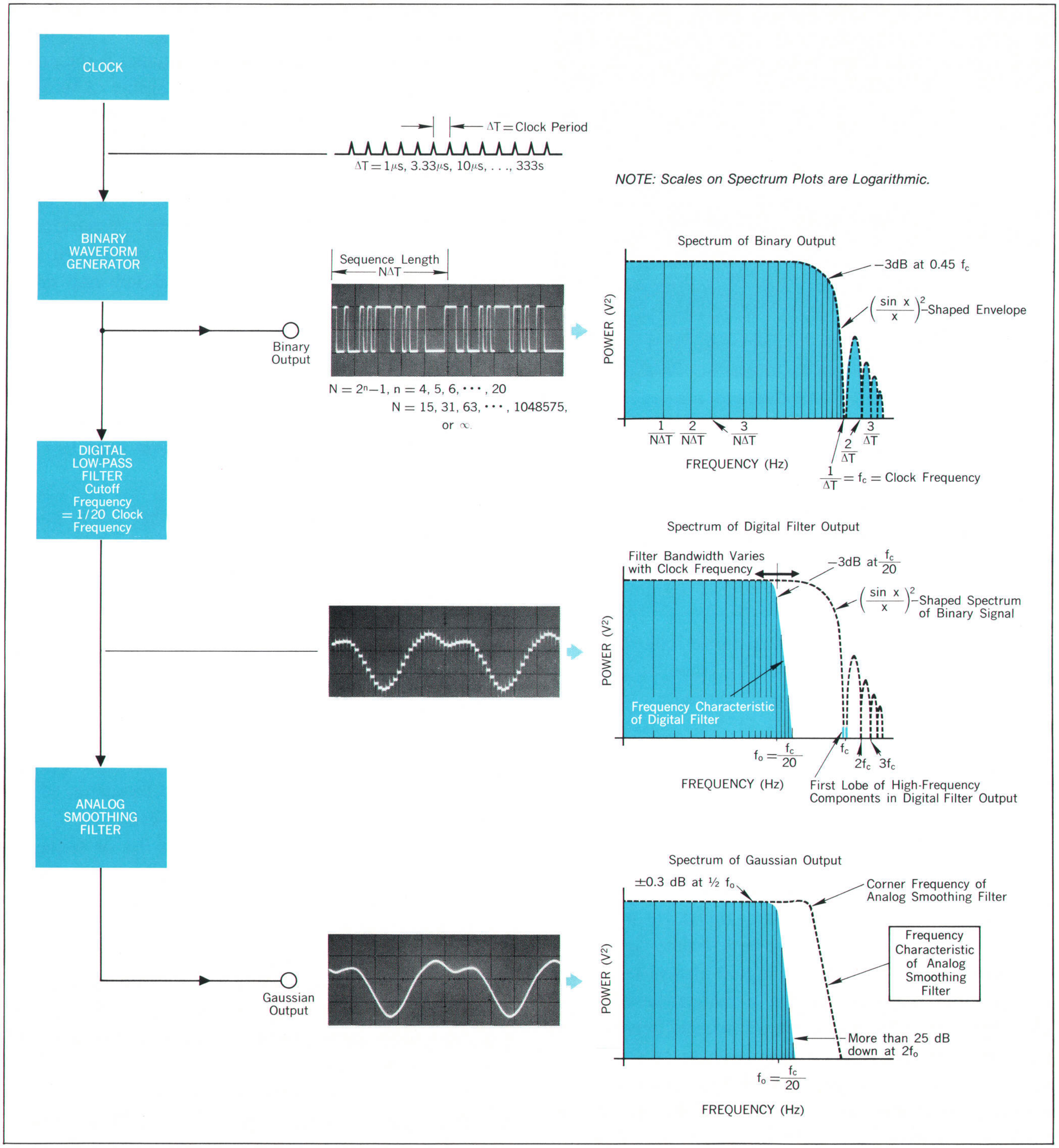


Fig. 6. *Model 3722A Noise Generator synthesizes pseudo-random or random binary signal in a digital waveform generator which is timed by a crystal-controlled clock. Clock rate and length of pseudo-random sequences are variable. Gaussian signal is derived from binary output by digital low-pass filtering. Discrete steps in digital filter output are removed by analog filter. Pseudo-random binary output of noise generator has line power spectrum having a flat envelope from dc to an upper 3 dB frequency which is selectable from 0.00135 Hz to 450 kHz. Spectrum of pseudo-random Gaussian output has flat envelope from dc to an upper 3 dB frequency which is selectable from 0.00015 Hz to 50 kHz. Random outputs have continuous power density spectra having same shapes as envelopes of spectra of pseudo-random outputs.*

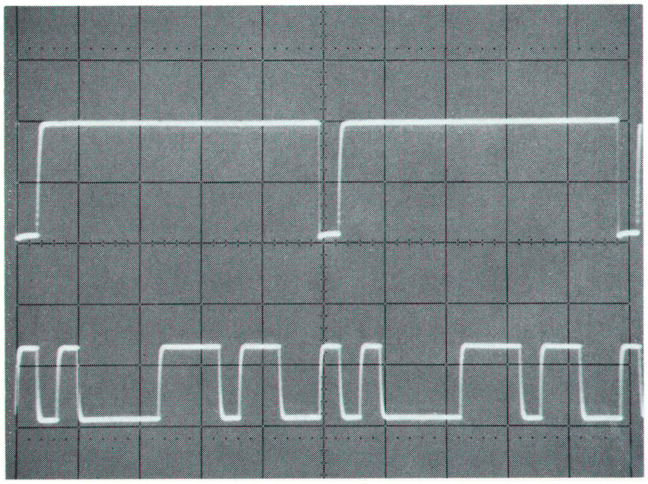


Fig. 7. *Model 3722A Noise Generator produces sync pulse (top), one clock period wide, at same point in each pseudo-random sequence.*

signals can be used in analog systems, in 'hybrid' systems—e.g., a process control system containing solenoid-operated on-off valves — or in digital systems — e.g., a PCM channel.

Although binary and Gaussian noise look quite different, it is possible to get a random Gaussian signal by sending a random binary signal through a low-pass filter (see Fig. 5).

The new noise generator produces both binary and Gaussian pseudo-random and random outputs. Using digital techniques, it synthesizes the binary waveform, then low-pass-filters the binary signal to get the Gaussian output.

Fig. 6 shows how the instrument works.

A binary waveform generator, timed by a crystal-controlled clock, synthesizes the basic binary signal. The changes of state of the binary signal always take place when a clock pulse occurs, but a change doesn't occur on every clock pulse. The clock period, and hence the interval between possible changes of state of the binary signal, is selectable from 1 μs to 333 seconds. Alternatively, the instrument may be timed by an external clock of frequency up to 1 MHz.

Depending upon the setting of a front-panel SEQUENCE LENGTH switch, the binary waveform generator produces either repetitive or non-repetitive output patterns. The repetitive, or pseudo-random patterns are periodic, but they *look* random; there is apparently a 50% probability that the binary waveform will change state on any given clock pulse. These waveforms repeat after a fixed number, N, of clock periods.

The number N of clock periods in the pseudo-random sequences is selectable from $2^4 - 1$ to $2^{20} - 1$, i.e., from 15 to 1,048,575. The length of one sequence is the product of N and the clock period, so the number of seconds in the pseudo-random sequences can be as short as 1 μs $\times$ 15 = 15 μs, or as long as 333 s $\times$ 1,048,575 = more than 11 years!

When the SEQUENCE LENGTH switch is set to its INFINITE position, the binary waveform generator is primed by a solid-state random noise source. In this condition, the binary signal is truly random and never repeats.

As Fig. 6 shows, the binary signal is one of the outputs from the noise generator. It is available at ±10 V with very low impedance, or at a selected amplitude with 600 Ω impedance. A relay-contact version of it is also available if the selected clock period is greater than 100 ms.

Spectrum of the Binary Output

A pseudo-random binary sequence has a line power spectrum, the envelope of which is a $(\sin x/x)^2$ curve, as shown in Fig. 6. Note that most of the power is contained in the first lobe, and that the nulls occur at intervals of f_c, the clock frequency. The harmonic (line) spacing is a function of sequence length and clock frequency, and is equal to f_c/N or $1/N\Delta T$ where N is the number of bits in the sequence and ΔT is the clock period.

The upper 3 dB (half-power) frequency of the binary output is 0.45 f_c. Hence, by adjusting the clock period, the operator can adjust the upper 3 dB frequency of the binary signal from 0.00135 Hz to 450 kHz.

Regardless of what clock frequency (f_c) or sequence length (N) is selected, the binary waveform always switches between the same two amplitude levels. This means that its rms value, and therefore its total power, is not changed by a change of bandwidth. Halving the bandwidth of the noise from a 'natural' noise source, on the other hand, also halves the power; this is a disadvantage when very low bandwidths are needed, since the power available becomes very small.

The power density spectrum of the purely random binary output (sequence length INFINITE) is continuous, i.e., it contains no discrete harmonics; it has the same shape as the envelope of the pseudo-random power spectrum.

Gaussian Output

The basic 'noise' produced by the noise generator is a binary waveform having a nominal bandwidth (to the half-power point) of 0.45 $\times$ clock frequency. While this is noise in the sense that is contains a multiplicity of frequency components, it is a two-level waveform bearing little resemblance — in the time domain — to naturally occurring disturbances (thermal noise, atmospheric noise, etc.). Naturally occurring noise can have a frequency content similar to that of binary noise, but it is random in amplitude, not confined to just two levels.

The noise generator provides, in addition to the basic binary signal, pseudo-random or random signals of the more familiar multi-level, or Gaussian type. 'Gaussian', in this context, means that the probability density function of the output tends to be the classical, bell-shaped curve (see Fig. 3).

As we have shown (Fig. 5), a multi-level waveform can be derived from a binary signal by conventional analog low-pass filtering. However, it takes a filter cutoff frequency that is about 1/20 of the clock frequency to give a reasonably Gaussian pdf. Since the lowest clock frequency in the new noise generator is about one cycle in five minutes, the lowest filter cutoff frequency has to be about one cycle per 100 minutes! It simply isn't practical to make analog filters with such low cutoff frequencies.

To convert the output of the binary waveform generator to a multi-level signal, we use a low-pass *digital* filter which is not subject to the same limitations as a conventional low-pass filter. The 3 dB bandwidth of the filtered signal, defined as dc to the half-power frequency, is nominally 1/20 of the clock frequency f_c.

The output of the digital filter is not a smooth signal, but a series of steps, like any waveform that has been generated digitally. These discrete steps in the multi-level output of the digital filter are removed by low-pass analog filtering (if the selected clock period is less than one second), and the resulting smooth Gaussian signal is another output of the noise generator. It is available at a fixed amplitude of 3.16 V rms with low source impedance or at a selected amplitude with 600 Ω impedance.

Fig. 6 shows a typical Gaussian output waveform from the noise generator, along with its spectrum. We will have more to say about this signal when we discuss the digital low-pass filter.

Control and Synchronization

Since pseudo-random signals are periodic, it is possible to obtain a stationary display of them on an oscilloscope, or to synchronize other equipment with them. For such purposes, the noise generator produces a sync pulse, one clock period wide, at a particular point in each pseudo-random sequence (Fig. 7).

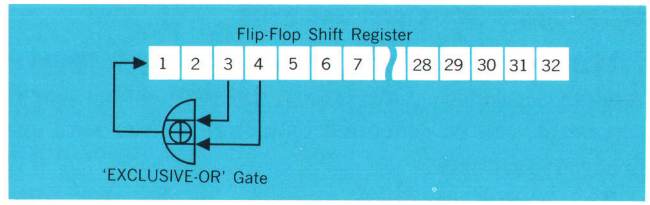


Fig. 8. *Fifteen-bit pseudo-random binary sequence is generated by four stages of shift register with feedback.*

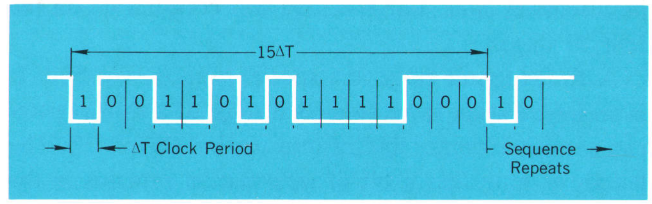


Fig. 9. *Fifteen-bit pseudo-random binary sequence generated by system of Fig. 8.*

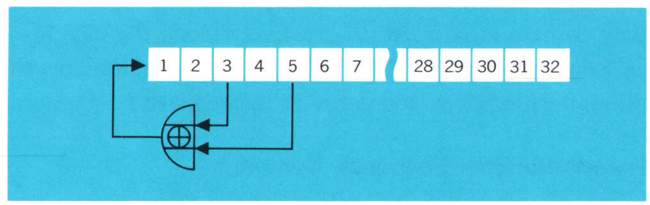


Fig. 10. *If n is number of stages involved in feedback loop, length of pseudo-random sequence is $N = 2^n - 1$ clock periods. This is a 31-bit sequence generator, i.e., $n = 5$.*

Besides the sync pulse, there is also a GATE output which can be used for controlling external equipment (e.g., a computer). Gate lengths of 1, 2, 4, or 8 pseudo-random sequences can be selected.

Another control feature is a HOLD button which, when pressed, stops the pseudo-random waveform. Subsequently pressing the RUN button restarts the waveform from the same point in the sequence that had been reached when the HOLD button was pressed. There is also a RESET button which sets the waveform generator to the '0' state and removes its supply of clock pulses. Pressing the RUN button then starts the generator by restoring the clock pulses and placing a '1' in the first stage of the waveform generator.

RUN, HOLD, and RESET can all be remotely programmed.

Shift-Register Waveform Generator

Many binary waveforms have the properties of pseudo-random sequences. One family, called maximal-length sequences, can be generated by a shift register with appropriate feedback.

The binary waveform generator in the new noise generator consists of the first 20 stages of a 32-stage shift register. These 20 stages and the last 12 stages also form part of the digital low-pass filter, which will be discussed later. For now, we will concentrate on the first 20 stages.

A shift-register stage is a special-purpose flip-flop. It is an information store, and each stage of a shift register can store one binary 'bit' of information ('0' or '1'). The length of time that a bit of information remains in the stage is equal to the time interval between two successive clock, or shift, pulses.

Individual shift-register stages are connected in cascade so that, on receipt of shift pulses, the information they contain is stepped progressively along the chain — as if on a conveyor belt. (In this case, 'information' means the pattern of ones and zeros in the register.)

Pseudo-Random Sequence Generation

When generating pseudo-random binary sequences, the shift register operates in a closed loop condition, and the input to the first stage is supplied via a feedback path from later stages of the shift register. Fig. 8 shows a simple form of pseudo-random sequence generator. In this example, only the first four of the shift-register stages are actually involved in generation of the sequence.

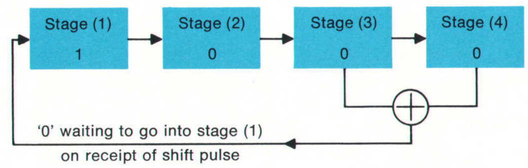
Feedback to the first stage is taken from stages 3 and 4, the outputs from which are processed in an EXCLUSIVE OR gate (otherwise known as: modulo-two adder, half adder, non-equivalence or anti-coincidence gate). This gate gives a '1' output only when its two inputs are dissimilar, according to the following truth table:

Truth Table for EXCLUSIVE OR Gate

Input 1	Input 2	Output
0	0	0
0	1	1
1	0	1
1	1	0

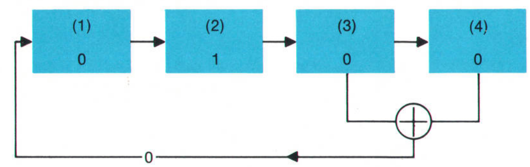
The sequence generated by the four-stage arrangement of Fig. 8 can easily be derived. For the purpose of illustration, the initial contents of the first four stages are taken, arbitrarily, to be as follows:

Before 1st shift pulse



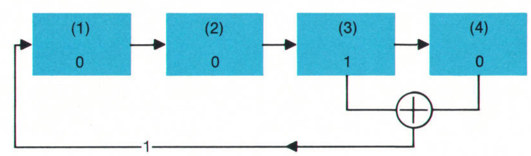
The modulo-two sum of the outputs from the last two stages is '0' (this can be written $0 \oplus 0 = 0$). At the first shift pulse, the '1' in the first stage is transferred to the second, and is replaced by the '0' in the feedback line. This gives the pattern:

After 1st pulse



Again, the modulo-two summation yields '0'. The next pattern is therefore:

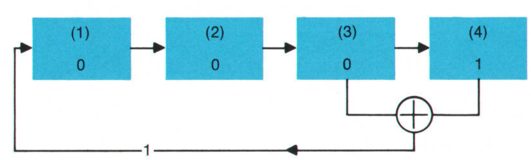
After 2nd pulse



With this pattern, the outputs from the third and fourth stages are dissimilar — so the modulo-two sum is '1'. The '1' thus placed in the feedback line will enter the first stage on arrival of the next shift pulse.

The remainder of the sequence can be worked out in a similar manner. After the 14th pulse, the register pattern is:

After 14th pulse



The fifteenth pulse restores the register to the initial state (1000), and thereafter the sequence repeats.

With the exception of 0000, the register generates the maximum number of '1' and '0' combinations possible with four stages. The all-zero condition cannot arise (if it were to occur, all stages of the shift register would remain in the '0' state, and the output would thereafter be an infinite sequence of zeros).

The pattern appearing at the output from the first stage is exactly the same as that from the second, the third and the fourth, and so on throughout the 32 stages of the shift register. There is a delay of one clock period between the pattern from one stage and the pattern from the next. The digit sequence from any of the stages is:

...... | 100110101111000 | 100110101111000 | ......

— Time →

Fig. 9 shows this sequence translated into a two-level, or 'binary' waveform ('1' is represented by the relatively

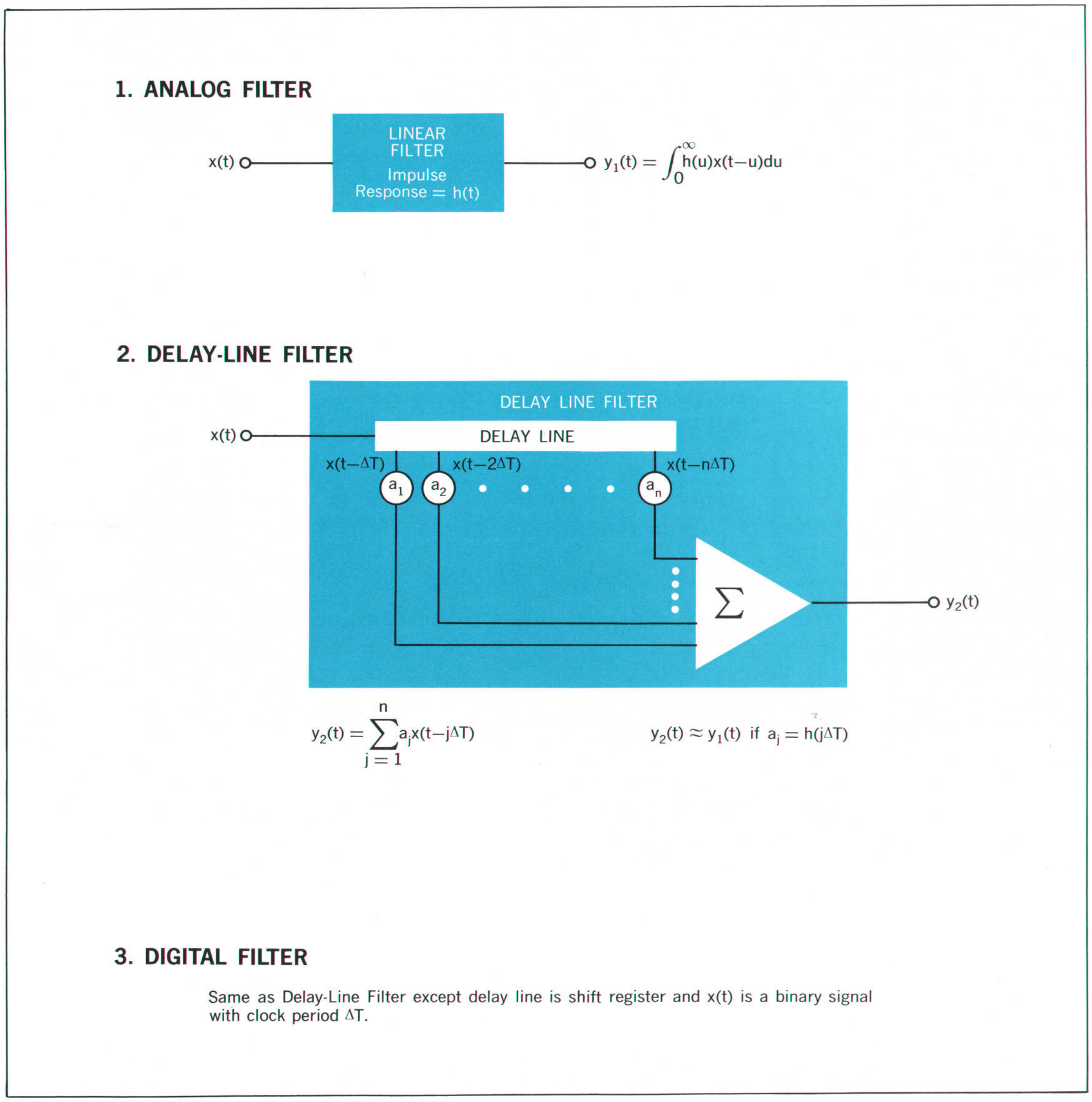


Fig. 11. *To get good Gaussian signals from binary signals, lowest cutoff frequency required of low-pass filter in Model 3722A Noise Generator is about one cycle per 100 minutes. This makes analog filter impractical, so generator uses digital approximation to ideal low-pass filter. Delay line in noise generator is 32-stage shift register and weighting networks a_j are resistors.*

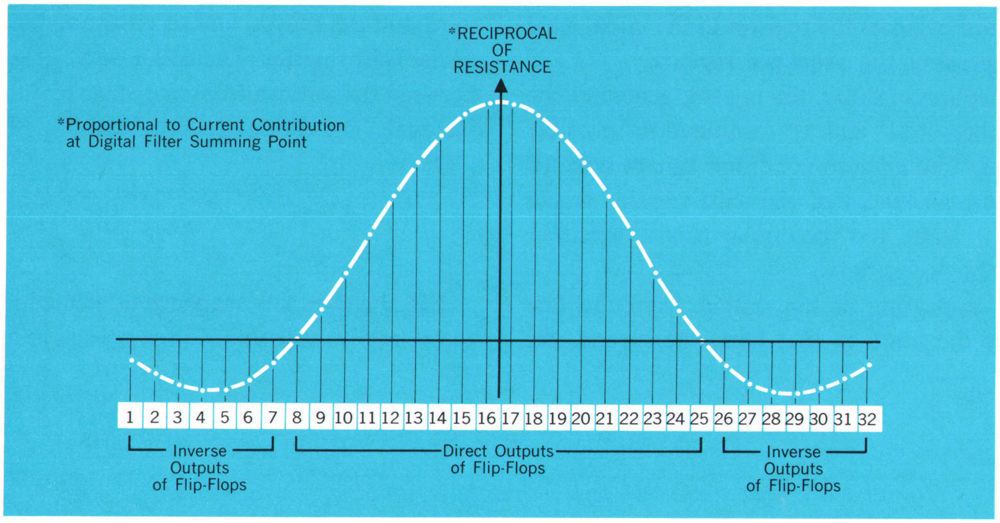


Fig. 12. *For digital filter in Model 3722A Noise Generator, outputs of 32-stage flip-flop shift register are weighted by resistors and added. Values of resistors are graded as shown to make pulse response of filter approximate (sin x/x) shape.*

negative level). This is the waveform obtained at the BINARY connector of the noise generator with the SEQUENCE LENGTH switch set to 15.

The next setting (31) of the SEQUENCE LENGTH switch selects, for modulo-two addition, the outputs from stages 3 and 5, as shown in Fig. 10. With five stages the maximum number of '1' and '0' combinations is 32 but, as before, the all-zero condition cannot occur. The resulting sequence is therefore 31 bits long.

The number of stages included in the feedback loop is increased by one at each setting of the SEQUENCE LENGTH switch. Feedback is always taken from the last of the 'active' stages, and from one or more of the preceding stages. For the 127-bit sequence, for example, feedback is taken from stage 7 (7 is the 'n' number engraved on the front panel) and also from stages 3, 4, and 5. Where more than two outputs are modulo-two added, extra EXCLUSIVE OR gates are used.

The number of bits, N, in pseudo-random sequences is always one less than the maximum number of '1' and '0' combinations possible with the selected length of register. Thus if n is the number of active stages, $N = 2^n - 1$. In the new noise generator, n is variable from 4 to 20 and N ranges between 15 and 1,048,575.

Random Operation of the Shift Register

With the SEQUENCE LENGTH switch set to INFINITE, the feedback system is disconnected and the first stage of the shift register is controlled by a semiconductor noise source, giving a truly random output signal. Just before each shift pulse, the random signal is sampled by a level detector which decides, on arrival of the shift

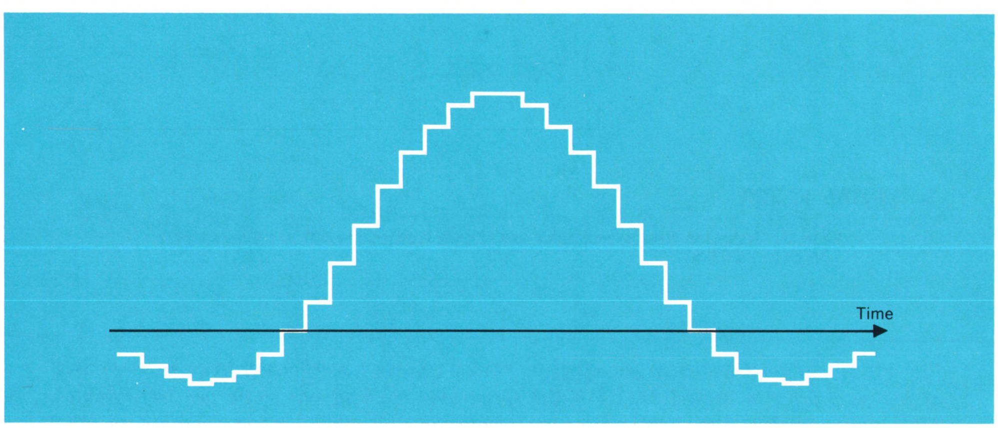


Fig. 13. *Single-pulse response of digital filter is a discrete-step approximation to (sin x/x)-shaped impulse response of ideal low-pass filter.*

pulse, whether a '1' or a '0' is to be placed in the first stage of the register. Since the random signal is non-periodic, there is no repeated pattern in the resulting series of ones and zeros from the register. The power density spectrum of the random signal is continuous, and has the same shape as the envelope of the power spectrum of the pseudo-random signal.

Digital Low-pass Filter

A linear filter having an impulse response h(t) and input x(t) has an output

$$y(t) = \int_{-\infty}^{\infty} h(u)\, x(t-u)\, du. \qquad (1)$$

A finite-sum approximation to this integral can be synthesized using a delay line. Fig. 11 shows a filter composed of a delay line, a number of multipliers or weighting networks, and a summing amplifier. The output of port j of the delay line is $x(t - j\Delta T)$ where x(t) is the input and ΔT is the delay between ports. The summing amplifier output is then

$$y(t) = \sum_{j=1}^{n} a_j x(t - j\Delta T). \qquad (2)$$

If $a_j = h(j\Delta T)$, and if n is sufficiently large, the sum, equation 2, approximates the integral, equation 1.

When x(t) is a binary signal, as it is in the new noise generator, the delay line can be a shift register. This in fact is how the noise generator's digital low-pass filter is constructed. It uses a 32-stage shift register as a delay line. The first 20 stages of the same register do double duty as the binary waveform generator, as we have already explained.

The desired frequency response of the digital filter is the rectangular response of an ideal low-pass filter. Therefore, the coefficients a_j are selected to approximate an impulse response of (sin x/x) shape — the impulse response of an ideal low-pass filter.

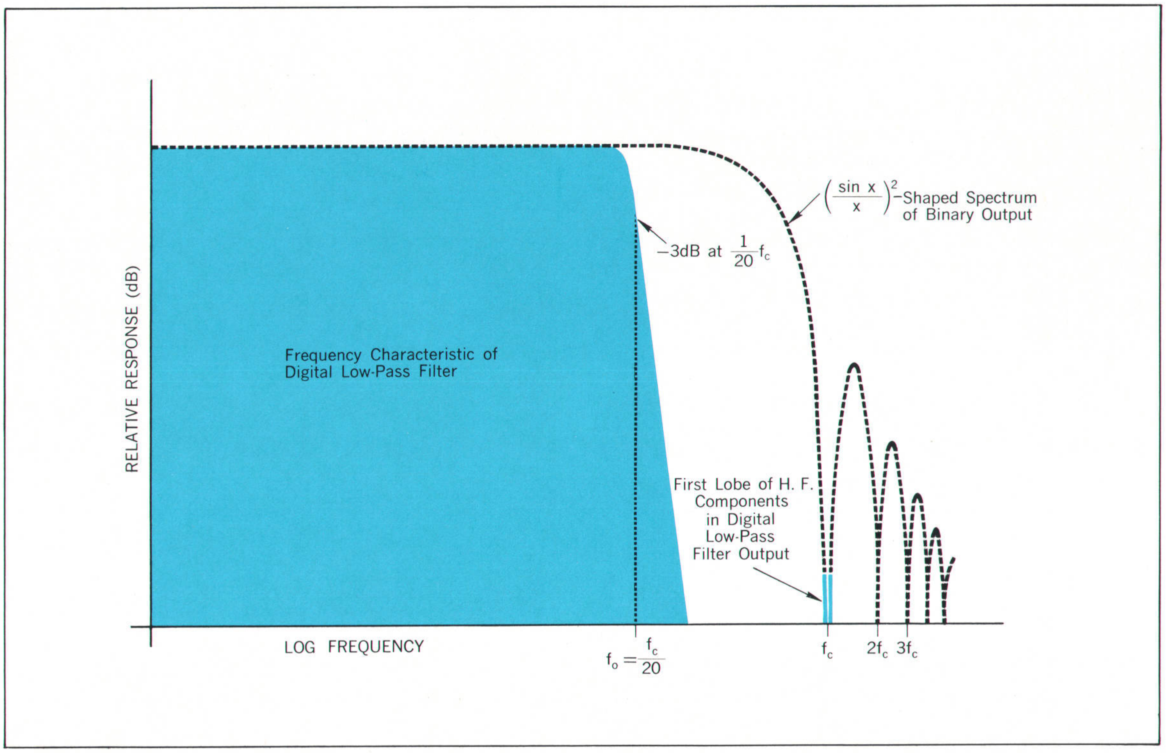


Fig. 14. *Frequency response of digital low-pass filter is nearly rectangular. Small high-frequency components are caused by steps in digital-filter output; they are subsequently removed by analog filtering.*

The weighting networks used in the noise generator are simply resistors. The resistor values are chosen such that the contributions of the outputs of successive shift-register stages to the current at the summing point are graded to follow the (sin x/x) curve, as shown in Fig. 12.

Notice in Fig. 12 that the contribution made by the first and last groups of seven resistors is required to be of the opposite polarity to that made by resistors in the central group. This can be arranged by supplying all of the weighting resistors in the central group with 'direct' outputs from the shift register, and supplying those in the outer groups with 'inverse' outputs ('direct' and 'inverse' are used here to describe the two outputs from opposite sides of a flip-flop). A '1' starting at one end of the register and being conveyed to the other, by a series of shift pulses, will generate the time waveform shown in Fig. 13.

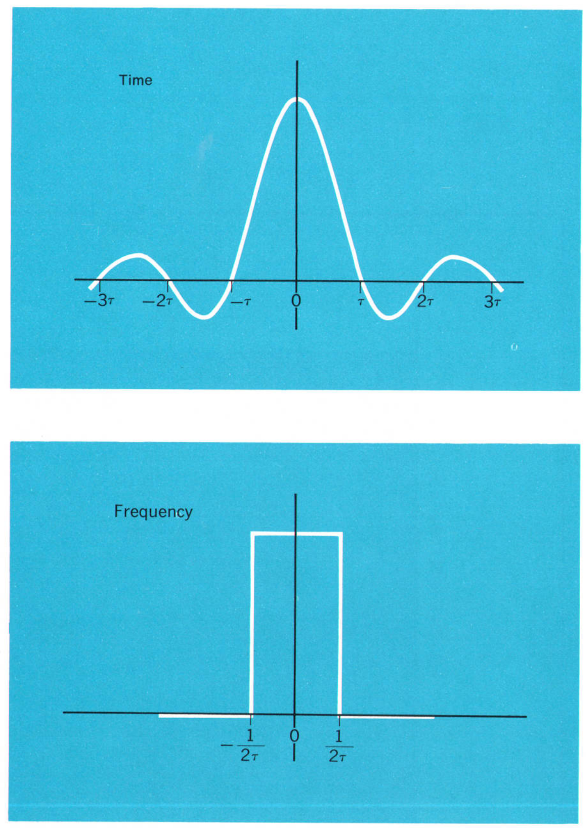
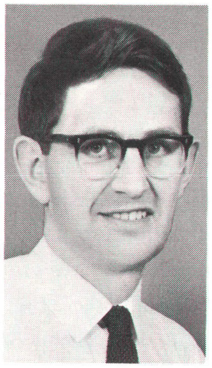
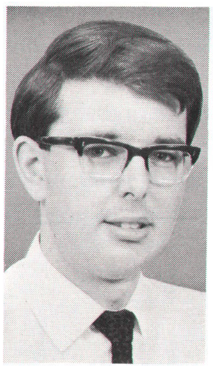


Fig. 15. *Bandwidth of ideal low-pass filter is inversely proportional to time of first null in impulse response. In noise generator, first null in digital-filter pulse response occurs at nine clock periods, so cutoff frequency is theoretically 1/18 of clock frequency. Actual response is not ideal, and has 3 dB frequency equal to 1/20 of clock frequency. Thus bandwidth can be varied simply by changing clock frequency.*



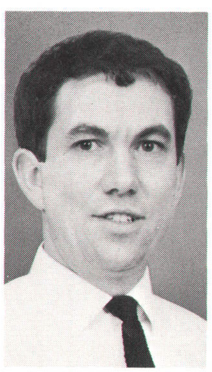
George C. Anderson

After graduating in 1954 from the Heriot-Watt University (Edinburgh), George Anderson completed a two-year graduate apprenticeship course in electrical engineering. This was followed by varied industrial work and a three-year period with the Royal Observatory, where he developed data recording systems for the Seismology Unit. George, who was the 3722A project leader, joined HP in 1966.



Brian W. Finnie

Brian received the degree of BS from Manchester University in 1962. He spent the next three years at Edinburgh University, where he worked in the research team headed by Gordon Roberts. He was concerned with an advanced system for real-time correlation, and was awarded the degree of PhD for his work in this field. Brian joined HP in 1965, and was responsible for initial design work on the 3722A. He is currently investigating a new range of instrumentation, and is working up routines for computer-aided design using the HP 2116A.



Gordon T. Roberts

In 1954 Gordon graduated from the University of Bangor (North Wales) with the degree of BS in electrical engineering. This was followed by a three year period at Manchester University, where he investigated problems of noise in non-linear systems; for this work, Gordon was awarded the degree of PhD. After five years of industrial work, a return to more academic surroundings—this time at Edinburgh University, where he lectured in control theory and headed a research team investigating the uses of noise signals in systems evaluation. Gordon has continued to work in these fields since he joined HP in 1965. He is now technical manager of Hewlett-Packard Limited in South Queensferry, Scotland.

The digital filter has an effective frequency response which approximates a rectangular spectrum (Fig. 14). Owing to the limitation on the size of the shift register, which results in truncation of the (sin x/x) curve, the corner of the spectrum is not perfectly square. There are also high-frequency components in the digital filter output spectrum. These components, caused by the abrupt changes in output level as pulses pass down the shift register, are removed by analog filtering, as described later.

Changes in clock frequency do not affect the rectangular shape of the spectrum, they simply alter the upper frequency limit. So here is a low-pass filter whose cut-off frequency automatically keeps in step with clock frequency (see Fig. 15).

Probability Density Function

The amplitude pdf of the multi-level signal is not significantly affected by the values of weighting resistor assigned to the various stages. The Gaussian nature of the pdf arises mainly from the apparent randomness of the changing pattern of ones and zeros in the register — the pdf becomes more nearly Gaussian as the sequence length, and hence the 'randomness', is increased. This is a consequence of the Central Limit Theorem of probability theory, which states that the sum of a large number of independent random variables tends to have a Gaussian pdf regardless of what the pdf's of the individual variables look like.

For sequence lengths of 8191 or more, the pdf of the multi-level signal closely approximates the Gaussian curve, and the waveform closely resembles naturally occurring noise (Fig. 16).

Fig. 17 shows the measured deviations of the noise generator's output pdf from the true Gaussian curve for sequence lengths of 8191 or greater. Worst-case deviations are less than ±0.020, which corresponds to about ±10%.

Analog Filtering

In analog computing applications, time derivatives (i.e., differentiated versions) of signals occur frequently and, whenever a signal has sharp edges, there is the danger that derivatives could cause overload. In the case of a boxcar waveform, with its very fast transit times, even the first time derivative would be a series of very large amplitude spikes, which could overload the system.

For this reason, a second-order analog filter is used to remove sharp edges from the digital-filter output waveform. As a result, neither the first nor the second time derivatives of the waveform yield sharp spikes. The pdf for both derivatives is reasonably Gaussian (see Fig. 17).

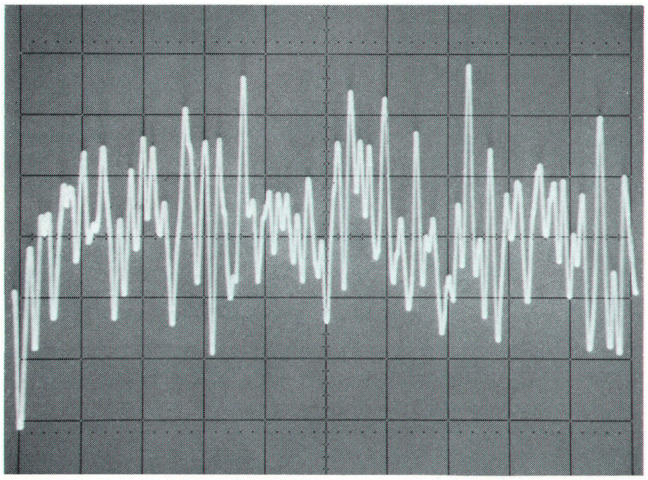


Fig. 16. *Part of 8191-bit pseudo-random Gaussian pattern. Clock period is 1 μs; bandwidth is 50 kHz.*

The analog filter cut-off frequency is selected by the CLOCK PERIOD switch, and is nominally 1/5th of the clock frequency (that is, four times the half-power frequency of the digital filter). This feature is included for all clock periods commonly of interest to analog computer users, i.e., noise bandwidth from 50 kHz to 0.15 Hz. At frequencies of 0.05 Hz and below, the analog filter cut-off remains at the same frequency as for the 0.15 Hz position.

Crest Factor of Gaussian Output

The crest factor (ratio of peak to rms values) of the Gaussian output of the noise generator is 3.75, except for the shortest sequences. This gives an excellent fit to the Gaussian curve.

The crest factor of a truly Gaussian signal is, of course, infinite, and some 'natural' noise sources have higher crest factors than 3.75. However, it is often necessary to wait a long time to be sure that one of their largest peaks has occurred. With the pseudo-random output of the noise generator, on the other hand, a definite number of the highest peaks occur in every sequence.

Acknowledgments

Major contributions to the development of the noise generator were made by Duncan Reid, Alistair MacParland, Glyn Harris, Michael Perry, and Richard Rex.

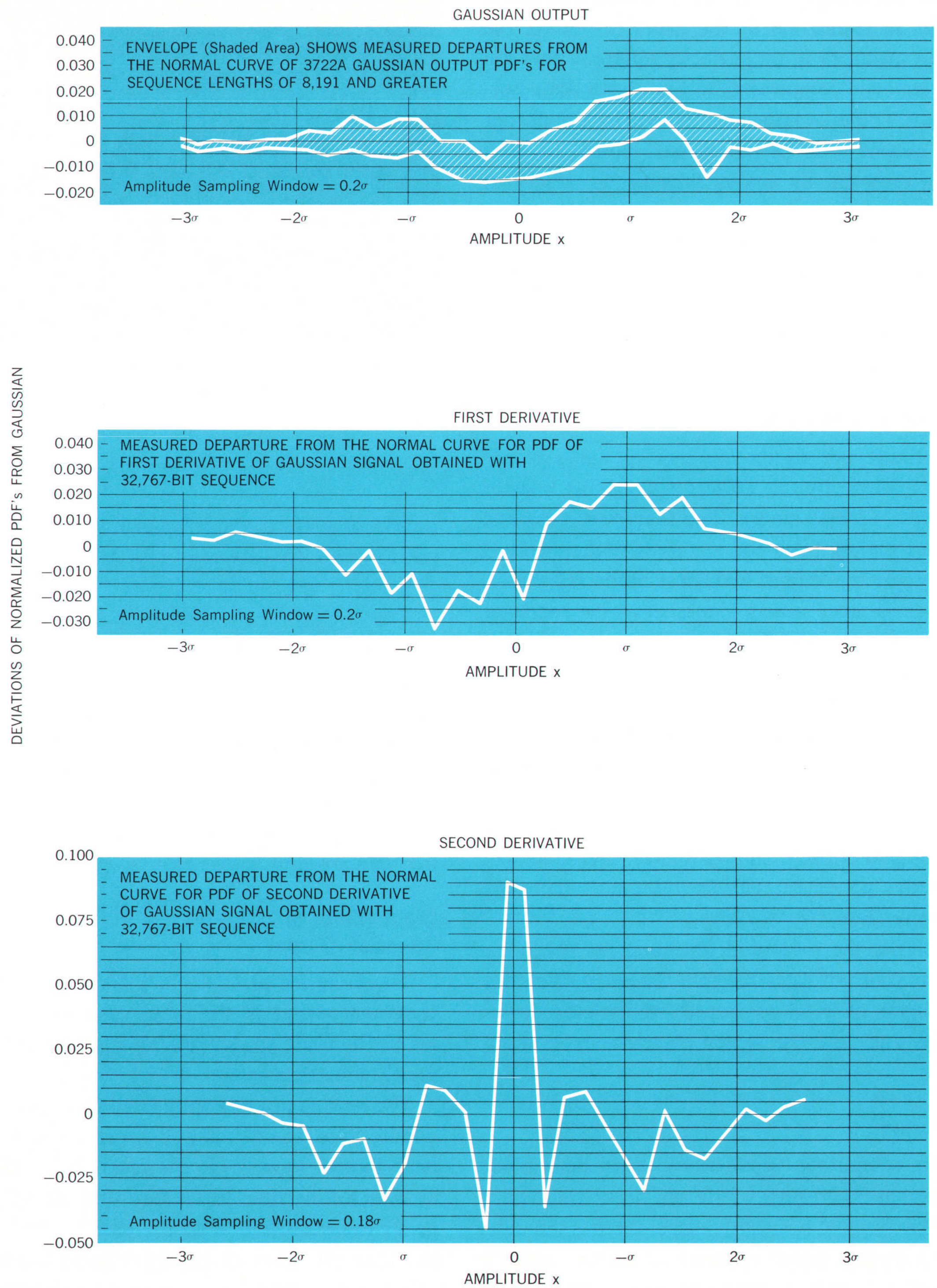


Fig. 17. *Measured deviations from true Gaussian probability density function for noise generator 'Gaussian' output are less than about ±10% for sequence lengths of 8191 and greater. First two derivatives are also reasonably Gaussian.*

SPECIFICATIONS

HP Model 3722A Noise Generator

BINARY OUTPUT (Fixed Amplitude)

Amplitude: ±10V ±1% when clock period ≥333 μs,
±3% when 1 μs <(clock period) <333 μs,
±5% when clock period = 1 μs.

Output Impedance: <5 Ω if clock period ≥333 μs,
<10 Ω if clock period ≤100 μs.

Load Impedance: 1kΩ minimum.

Rise Time: <100 ns.

Power Density

Approxmately equal to (clock period × 200) V^2/Hz, at low frequency end of spectrum.

Power Spectrum

$(\sin x/x)^2$ form: first null occurs at clock frequency and −3 dB point occurs at 0.45 x clock frequency.

GAUSSIAN OUTPUT (Fixed Amplitude)

Amplitude: 3.16 V rms ±2% when bandwidth ≥0.15 Hz,
+6% −2% if bandwidth ≤0.05 Hz.

This specification is valid only when sequence length ≥1,023.

Output Impedance: <1 Ω.

Load Impedance: 600 Ω minimum.

Zero Drift: <5 mV change in zero level in any 10°C range from 0° to +55°C.

Power Density

Approximately equal to (clock period × 200) V^2/Hz at low frequency end of spectrum.

Power Spectrum

Rectangular, low pass: nominal upper frequency f_o (−3 dB point) equal to ½₀th of clock frequency. Spectrum is flat within ±0.3 dB up to ½ f_o, and more than 25 dB down at 2 f_o.

Crest Factor: Up to 3.75, dependent on sequence length.

Probability Density Function: See error curves, page 16.

VARIABLE OUTPUT (Binary or Gaussian)

Amplitude (Open Circuit)

BINARY

4 ranges: ±1 V, ±3 V, ±3.16 V and ±10 V, with ten steps in each range, from × 0.1 to × 1.0.

GAUSSIAN

3 ranges: 1 V rms, 3 V rms and 3.16 V rms, with ten steps in each range, from × 0.1 to × 1.0.

Calibration Accuracy

Better than ±2.5%, plus tolerance on binary or Gaussian output, as selected.

Output Impedance: 600 Ω ±1%.

MAIN CONTROLS

Sequence Length Switch

First 17 positions select different pseudo-random sequence lengths: final position selects random mode of operation (INFINITE sequence length). Sequence length (N) is number of clock periods in sequence: possible values of N are 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535, 131071, 262143, 524287, 1048575. $N = 2^n - 1$, where n is in the range 4 to 20 inclusive.

CLOCK PERIOD SWITCH: Selects 18 frequencies from internal clock:

Clock period	Clock frequency	Gaussian noise bandwidth
333 s	**0.003 Hz**	**0.00015 Hz**
100 s	**0.01 Hz**	**0.0005 Hz**
33.3 s	**0.03 Hz**	**0.0015 Hz**
10 s ↓ **3.33 μs**	**0.1 Hz** ↓ **300 kHz**	**0.005 Hz** ↓ **15 kHz**
1 μs	**1 MHz**	**50 kHz**

INTERNAL CLOCK

Crystal Frequency

3 MHz nominal.

Frequency Stability

<±25 ppm over ambient temperature range 0° to +55°C.

Output

+1.5 V to +12.5 V rectangular wave, period as selected by CLOCK PERIOD switch. Maximum current at 1.5 V level, 10 mA.

EXTERNAL CLOCK

Input Frequency

1 MHz maximum, for stated specifications. Usable BINARY output (pseudo-random only) with external clock frequencies up to 1.5 MHz.

Input Level

Negative-going signal from +5 V to +3 V initiates clock pulse. Maximum input ±20 V.

Input Impedance: 1 kΩ nominal.

SECONDARY OUTPUTS

Sync

Negative-going pulse (+12.5 V to +1.5 V) occurring once per pseudo-random sequence: duration of pulse equal to selected clock period. Maximum current at 1.5 V level, 10 mA.

Gate

Gate signal indicates start and completion of selected number of pseudo-random sequences (1, 2, 4 or 8, selected by front panel control). Two outputs are provided:

1. Logic signal: output normally +12.5 V, falls to +1.5 V at start of gate interval and returns to +12.5 V at end of interval. Maximum current at 1.5 V level, 10 mA.
2. Relay changeover contacts: gate relay switching is synchronous with logic signal.

Maximum current controlled by relay: 500 mA (cont.).
Maximum voltage across relay contacts: 100 V.
Maximum load controlled by relay: 3 W (cont.).

Binary Relay

Relay changeover contacts operate in sync with binary output signal (available only when clock period ≥100 ms). Relay specification as for gate relay above.

REMOTE CONTROL

Control Inputs

Remote control inputs for RUN, HOLD, RESET and GATE RESET functions are connected to 36-way receptacle on rear panel.

Command signal (each input): dc voltage between +1.5 V and zero volts.

No-command condition: open-circuit input, or dc voltage between +5.5 V and +12.5 V.

Input impedance: 5 kΩ nominal (RUN, HOLD, RESET).
1.5 kΩ nominal (GATE RESET).

Sequence Length Indication

18 pins plus one common pin on the 36-way receptacle are used for remote signaling of selected sequence length (contact closure between common pin and any one of the 18 pins).

GENERAL

Construction: Standard 19 in. rack-width module, with tilt stand.

Ambient Temperature Range: 0° to +55°C.

Power Requirement: 115 or 230 V ±10%, 50 to 1000 Hz, 70 W.

Weight: Net 10.5 kg (23 lb), shipping 13.5 kg (30 lb).

Accessories Furnished

Detachable power cord, rack mounting kit, circuit extender board, 36-way male cable plug, operating and service manual.

Price $2,650.00

OPTION 01

Zero Moment Option

Shifts relative position of sync pulse and pseudo-random binary sequence such that first time moment of sequence, taken with respect to sync pulse, is zero (sequence shift mechanism is operative only when selected sequence length is ≤1023): option 01 also provides facility for inverting binary output signal.
ADD $50.00.

MANUFACTURING DIVISION: HEWLETT-PACKARD LTD.
South Queensferry
West Lothian, Scotland

Testing with Pseudo-Random and Random Noise

Pseudo-random noise is faster, more accurate, and more versatile than random noise in most measurement situations.

THE NEW NOISE GENERATOR described in the article beginning on page 2 is different from conventional noise sources in that it synthesizes noise by a digital process. This not only makes its output statistics more stable and controllable, but also allows it to produce pseudo-random noise as well as random noise. Pseudo-random signals are periodic signals that look random; they have the same advantages as random noise for testing, but don't have the disadvantage of randomness.

Here are some of the ways in which noise is useful as a test signal, with emphasis on the uses of pseudo-random noise.

Noise as a Broadband Test Signal

Broadband noise makes an excellent test signal for

- environmental testing. For example, the vibrations produced by a shake table with a noise input are similar to those a product will meet in service. A loudspeaker connected to a noise generator makes a useful acoustical noise source for testing microphones, materials, rooms, and so on. In fatigue testing, pseudo-random noise is helpful because it has a known number of peaks of various amplitudes; this means that test time can often be reduced, since it is not necessary to wait a long time to be sure a certain number of peaks have occurred.
- process control system evaluation. Process control systems can be tested for their responses to random fluctuations in the controlled variables, e.g., temperature, pressure, flow, concentration, etc. Pseudo-random signals are helpful here because they do not introduce statistical variance into the results. Measurements are completed in the time required for only one pseudo-random pattern. This is especially important in low-speed systems, which might have to be tied up for hours if truly random noise were used as a test signal.

Pseudo-random noise is also especially useful in

- testing large systems. As a system gets bigger, it gets harder to test on a lab bench. Eventually it must be tested under working conditions. A good example is an airplane, which in the end must be tested in flight. Pseudo-random noise can speed these tests for the same reasons given above under 'process control system evaluation'.
- limited time situations. Pseudo-random noise is better than random noise when the situation to be measured exists only for a short time — e.g., a missile during blastoff. Again, this is because measurements that use pseudo-random noise are made over only one pattern length, and no statistical variance is introduced into the results by the noise.

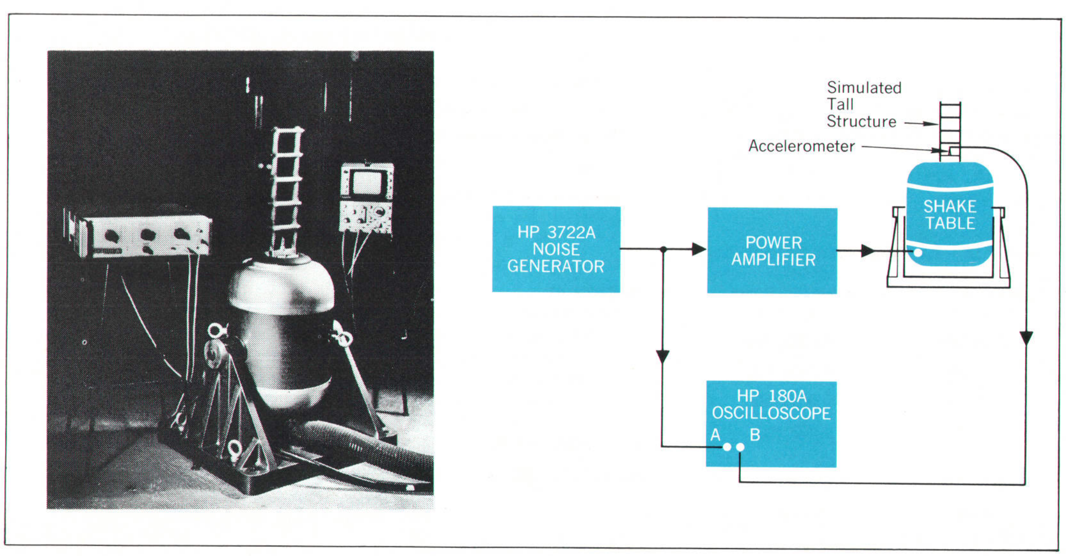


Fig. 1. *Model simulation of tall structure. Noise-driven shake table simulates ground disturbances, and accelerometer measures structure's response.*

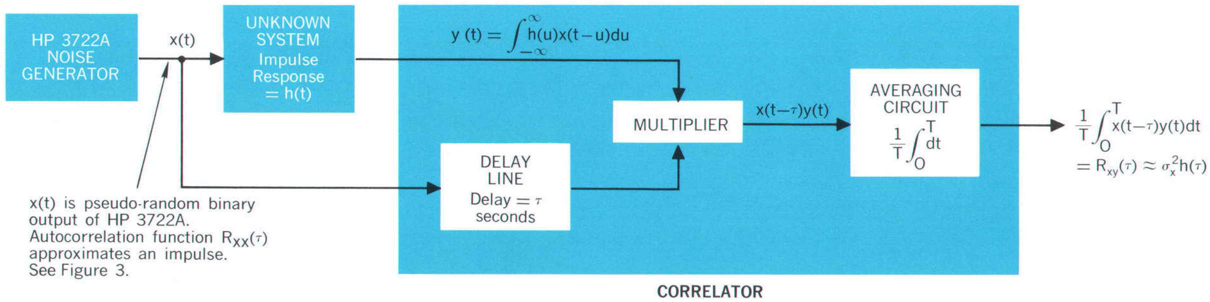


Fig. 2. *System for obtaining impulse responses with noise and correlation techniques.*

Flat Spectrum at Low Frequencies

In most of the applications of noise as a broadband test signal, the preferred shape of the power density spectrum is flat, at least through the band of interest. This is a difficult requirement for conventional 'natural' noise sources to meet, especially at low audio and sub-audio frequencies, where flicker noise, 1/f noise, hum, ambient temperature fluctuations, vibrations, and microphonics all degrade the spectrum. In addition, a noise source usually produces a small amplitude signal. If low frequencies are important, this signal must be amplified by a dc coupled amplifier, and the random drifts of such an amplifier cannot be distinguished from the low-frequency portion of the original noise signal.

Low-frequency noise, however, is a necessary product of a useful noise source. The main use of very-low-frequency noise, e.g., in the 0 to 50 Hz range, is in testing systems which have long time constants. These include such things as massive mechanical arrays, nuclear reactors, and chemical processes, where the effect of changing any parameter of the process takes a long time to become evident. When testing these systems, the lowest frequency content of the test signal must be comparable with the system time constant. This also holds true when the system is being simulated on an analog computer.

The spectrum of the binary output of the new noise generator is virtually flat from dc to an upper 3 dB frequency which can be adjusted from 0.00135 Hz to 450 kHz. The Gaussian signal has a spectrum which is flat from dc to an upper 3 dB point of 0.00015 Hz to 50 kHz.

Regardless of selected cutoff frequency, the generator's total power output is constant; in other words, when we halve the bandwidth, we don't halve the power — as occurs when the output from a conventional noise source is low-pass filtered.

Model and Computer Simulation

Control systems, buildings, ships, automobiles, aircraft, aerospace guidance systems, bridges, missiles, and a host of other complex objects can often be designed and studied most easily by simulating them in the laboratory. This can be done either by using a scale model of the object or by simulating it on an analog computer.

In either case, the new noise generator can provide realistic simulations of road roughness, air turbulence, earthquakes, storms at sea, target evasive action, controlled-variable fluctuations, and so on. Particularly useful is the pseudo-random output of the generator, which has the same effect on the model as real noise, but which can be repeated at will.

Analog computer users should find the following characteristics of the noise generator particularly helpful:

- accurately defined signals
- amplitude controls not subject to loading errors
- ability to change time scale without changing amplitude or pattern shape
- remote programming for RUN, HOLD, RESET
- gate circuits to control operations in the computer
- good autocorrelation function (see Fig. 3)
- zero-moment option (see Specifications, p. 17).

Fig. 1 shows a model simulation of a tall structure mounted on a shake table which is being excited by Gaussian noise from the new noise generator. This set-up, currently in use at Edinburgh University, provides experimental data on the behavior of tall buildings subjected to ground disturbances. The lower trace shows the acceleration of the first floor of the structure, as measured by the accelerometer mounted on the model.

Impulse Responses Without Impulses

All the information necessary to characterize a linear system completely is contained in its impulse response. Given any unknown system, then, it would be desirable to be able to find its impulse response. One way to do this would be to excite the system with an impulse or a train of impulses and observe the output with an oscilloscope.

However, impulses are dangerous; they are likely to cause overload and saturation. Of course, small impulses could be used, but if they are small enough to be safe they usually produce outputs which are so small that they are obscured by background disturbances.

One of the really interesting features of statistical techniques is that we can inject low-amplitude noise into a system and, by suitably processing the output, obtain the system impulse response, without subjecting the system to a damaging high-level test signal. This technique has two other advantages.

- The test may be performed while the system is operating 'on line'. This is possible because the intensity of the noise test signal can be low enough so it doesn't affect normal operation of the system.
- The results are largely unaffected by background disturbances in the system. This is because the results are obtained by correlation, and the disturbances aren't correlated with the test noise.

Fig. 2 shows a setup for obtaining impulse responses from noise responses. The output of the system is *cross-correlated* with the noise input; that is, the output is multiplied by a delayed version of the input and the product is averaged. The average as a function of the delay τ is the same as the impulse response of the system as a function of time *provided that the autocorrelation function of the noise input is an impulse* (i.e., the noise should be wideband compared with the system's frequency response). If the autocorrelation function of the noise isn't a true impulse, the result will be less than perfectly accurate. The accuracy of the correlator output is also affected by the correlator's averaging time.

Mathematically, the setup of Fig. 2 works as follows. If the noise is x(t), the unknown impulse response is h(t), and the response of the system to the noise is y(t), then

$$y(t) = \int_{-\infty}^{\infty} h(u)\, x(t-u)du$$

The cross-correlation function of y(t) with x(t) is defined as

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t-\tau)\, y(t)dt.$$

Substituting for y(t) gives

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} h(u)\, R_{xx}(u-\tau)du.$$

where $R_{xx}(\tau)$ is the autocorrelation function of the noise x(t). If $R_{xx}(\tau)$ is a true impulse then

$$R_{xy}(\tau) = \sigma_x^2\, h(\tau),$$

where σ_x is the rms value of the noise x(t). In other words, the unkown impulse response is proportional to the cross-correlation function of the input noise x(t) with the output y(t).

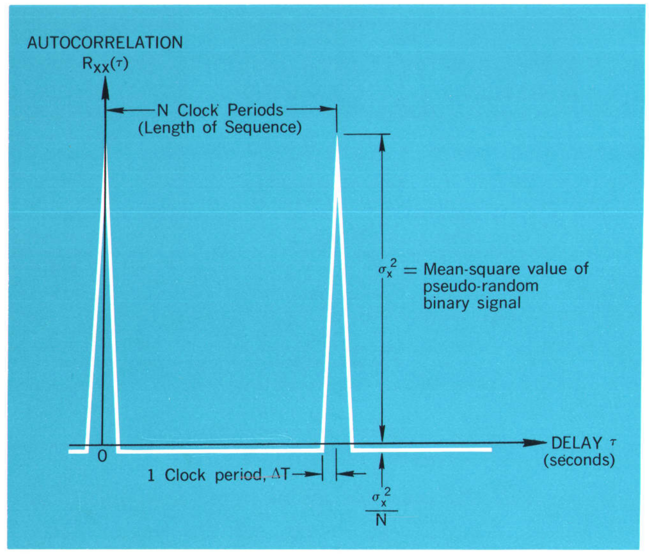


Fig. 3. *Autocorrelation function of pseudo-random binary sequence approximates an impulse.*

The binary pseudo-random noise synthesized by the new noise generator has an autocorrelation function which, while not precisely an impulse, is very close to one, as shown in Fig. 3. What's more, the averaging time T for the correlation system only needs to be as long as one period of the pseudo-random waveform, i.e., as long as one complete pseudo-random pattern. Unlike random noise, pseudo-random noise introduces no statistical variance into the results, as long as the averaging time T is exactly one pattern length.

Calibration, Research, Training

Other uses of the noise generator include

- research in communication, biomedical engineering seismology, underwater sound, PCM, etc.
- calibration of true-rms voltmeters, spectrum analyzers, and other low-frequency test equipment (e.g., the pseudo-random signal generates a comb of frequencies, useful for checking wave analyzers).
- student familiarization with random-signal theory and the behavior of systems with noise inputs.

It will be interesting to see how this list grows as the potential of controllable, repeatable noise becomes more widely realized. ■

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