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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

Also

IN THIS ISSUE

MEASUREMENTS OF THE CHARACTERISTICS OF TRANSMISSION LINES

PART II

IMPEDANCE MEASUREMENT ON TERMINATED LINE

● **POSSIBLY THE SIMPLEST METHOD** of determining the characteristic impedance is to terminate a quarter-wave section of line in a variable resistance, to measure the input resistance with the TYPE 916-A Radio-Frequency Bridge, and to adjust the terminating resistance until the input and terminating resistances are equal. The value of resistance at which the two are equal is exactly the characteristic impedance, in the ideal case of a loss-free line terminated in a pure resistance. Attenuation in the line and reactance in the termination modify the behavior slightly, in accordance with the following approximate expression:

$$R_{in} \approx R_0 \left[1 - \left(\frac{X_0 + X_T}{R_{in}} \right)^2 \right] \tag{10}$$

in which X_0 and X_T are the reactive components of the characteristic impedance and the terminating impedance, respectively. For the condition $\left(\frac{X_0 + X_T}{R_0} \right)^2 << 1$, Equation (10) may be written as

$$R_0 = R_{in} \left[1 + \frac{1}{2} \left(\frac{X_0 + X_T}{R_{in}} \right)^2 \right] \tag{11}$$

The effect of X_0[7] on Equation (11) is negligible for lines having reasonably low-loss insulation, at frequencies of the order of a megacycle. The effect of X_T[7], however, may easily make the correction term of (11) become a few per cent.

The reactive component of a decade resistance box, such as might

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[7] X_0 is capacitive in nature but Equation (11) is so written that the positive numerical value is to be used. The sign of X_T should be positive for inductive reactance in the termination and negative for capacitive reactance.

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be used for the variable termination, is almost invariably inductive in nature, at the values of resistance required for this type of measurement. It is therefore a relatively simple matter to reduce the net reactance of the termination to a negligible value by connecting in series a fixed capacitor of appropriate size.

Example of Measurement

The 117-foot length of TYPE 774 Cable was terminated in a TYPE 602 Decade-Resistance Box. The input resistance was measured on the TYPE 916-A Radio-Frequency Bridge, at 1.26 megacycles, the previously determined quarter-wave frequency. The input and terminating resistances were found to be equal for a 71.2 ohm setting of the decade box. This value, to a first approximation, corresponds to the characteristic impedance of the line.

The series reactance (X_T) of the decade box was measured directly on the bridge and found to be 10.3 ohms. X_0, the reactive component of Z_0, was known from previous measurements to be about 2.1 ohms. Inserting these values in Equation (11), the characteristic resistance is

$$\begin{aligned} R_0 &= 71.2\left[1+\left(\frac{2.1+10.3}{71.2}\right)^2\right] \\ &= 71.2\;[1+0.0152] \\ &= 72.3 \text{ ohms} \end{aligned}$$

The value of X_0 will, of course, not be known unless independent measurements are made. If it is neglected in Equation (11), the indicated value of R_0 is 72.0 ohms.

As a check on the method and on the validity of Equation (11), a capacitor having approximately 10 ohms reactance was connected in series with R_T, to reduce to a negligible value the effective value of X_T. The input and terminating resistances were then found to be equal for a setting of 72.2 ohms. The agreement with the corrected value obtained from the first measurement is excellent.

The direct-reading accuracy of the resistance dial of the TYPE 916-A is 1%. If measurements to an accuracy better than 1% are required, a correction factor for the dial reading must be determined. The correction factor may be obtained by measuring known resistors, whose values lie reasonably near the unknown value to be measured.

The data given for the two measurements just described were corrected by checking the bridge against TYPE 500 Resistors, whose resistance values are known to within a few tenths per cent at the frequency of measurement. The correction was within the nominal 1% accuracy and amounted to only a few tenths ohm.

Attenuation Measurement from Standing-Wave Ratio

The ratio of input voltage to output voltage on an open-circuited transmission line is given by

$$\begin{aligned} \frac{V_{\text{in}}}{V_{\text{out}}} &= \cosh \alpha l \cos \beta l \\ &\quad + j \sinh \alpha l \sin \beta l \end{aligned} \tag{12}$$

If the length of line, l, corresponds to an odd multiple of a quarter-wavelength, we have $\cos \beta l = 0$, $\sin \beta l = \pm 1$ and Equation (12) reduces to

$$\frac{V_{\text{in}}}{V_{\text{out}}} = \pm j \sinh \alpha l \tag{13}$$

Considering magnitude only and assuming that αl is small compared to unity, Equation (13) can in turn be written as

$$\left|\frac{V_{\text{in}}}{V_{\text{out}}}\right| \approx \alpha l \tag{14}$$

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The true condition of quarter-wave resonance is difficult to establish experimentally, but with low-loss lines no significant error is introduced if the condition of maximum voltage rise is used instead.

The condition of maximum voltage rise can be determined experimentally with a variable frequency oscillator, or signal generator, and a vacuum-tube voltmeter. Both the input and output voltages on the line will vary with frequency and the ratio of the two voltages must theoretically be taken at several frequencies to determine the maximum value. Practically, the correct frequency can be located quite accurately by adjusting for the *minimum* value of *input* voltage since the line input impedance goes through a minimum at this frequency.

Effect of Harmonics

The experimental difficulty with the voltmeter method lies in the possible serious errors that may be encountered from harmonic distortion in the voltage source. This type of error depends on the impedance of the source, as well as on the harmonic content of the voltage. At the frequency corresponding to quarter-wave resonance, the cable input impedance is extremely low so that V_0 at this frequency is small. At the second harmonic frequency, however, the input impedance is high, as the line is in approximate half-wave resonance at the double frequency. Consequently, the applied fundamental voltage is lower than on open-circuit while the second-harmonic voltage is about the same as on open-circuit. The lowered fundamental voltage is stepped up to the cable output by the resonant rise in the line and the second-harmonic voltage at the line output is essentially equal to the second-harmonic voltage at the line input. Under these circumstances the harmonic content of the input and output voltages will differ and the observed voltage ratio may be seriously in error.[8] The magnitude of the error introduced depends upon the ratio of the generator impedance to the characteristic impedance of the line, and upon the type of response of the voltmeter, that is, peak reading, r-m-s, or average.

Difficulties from harmonic distortion can, of course, be avoided by the use of a tuned voltmeter. A radio receiver may also be used, if means are available for calibrating its sensitivity.

Effect of Voltmeter Loading

The finite input resistance of the vacuum-tube voltmeter may, of course, reduce the resonant rise, and the input capacitance may shift the frequency of quarter-wave resonance. The latter effect is negligible at low frequencies. For example, the input capacitance of the TYPE 726-A Vacuum-Tube Voltmeter is approximately 6 $\mu\mu$f, while the total capacitance of a quarter-wave section of line at one megacycle may be several thousand micromicrofarads.

The effect of the resistive loading of the line by the voltmeter depends upon the ratio of the voltmeter resistance to the output resistance at the open end of the line. At a frequency of one megacycle the input resistance of the TYPE 726-A Vacuum-Tube Voltmeter is greater than one megohm. The output resistance of a typical line in quarter-wave resonance at this frequency will be very much less

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[8] An experimental observation with a 1500-ohm generator feeding a 75-ohm cable yielded a resonance rise ratio $\frac{V_{out}}{V_{in}}$ of 4, whereas the known correct value was 20.

than one megohm, and the voltmeter loading can be neglected.

Examples of Measurement

The resonant rise in the quarter-wavelength of Type 774-A Cable was measured, using a Type 805-A Standard-Signal Generator as the voltage source, and a Type 726-A Vacuum-Tube Voltmeter as the voltage indicator. The minimum ratio of V_{in} to V_{out} was found to be 0.0464 and occurred at a frequency of 1.26 megacycles. The indicated value of αl is almost 20% lower than that obtained from the quarter-wave input-resistance measurement previously described. A part of this discrepancy was traced to the effects of temperature on the attenuation constant (the voltage-rise measurement was made at an ambient temperature nearly 20° F. lower than the input-resistance measurement) and the rest is assumed to be due to the effects of harmonics on the voltage-rise measurement.

Velocity of Propagation

The imaginary component (β) of the complex propagation constant is frequently specified in terms of v, the velocity of phase propagation. The relationship is given by

$$\beta = \frac{\omega}{v} = \frac{2\pi}{\lambda} \qquad (15)$$

The velocity of propagation can be deduced from observations on the resonant lengths of the line, using the following relationships. The ratio of the velocity on the line (v_l) to the free-space velocity (v_s) is equal to the corresponding ratio of wavelengths (λ_l and λ_s), at any given frequency.

$$\frac{v_l}{v_s} = \frac{\lambda_l}{\lambda_s} \qquad (16)$$

But λ_s, the free-space wavelength, can be expressed in terms of frequency as

$$\lambda_s = \frac{300}{f} \qquad (17)$$

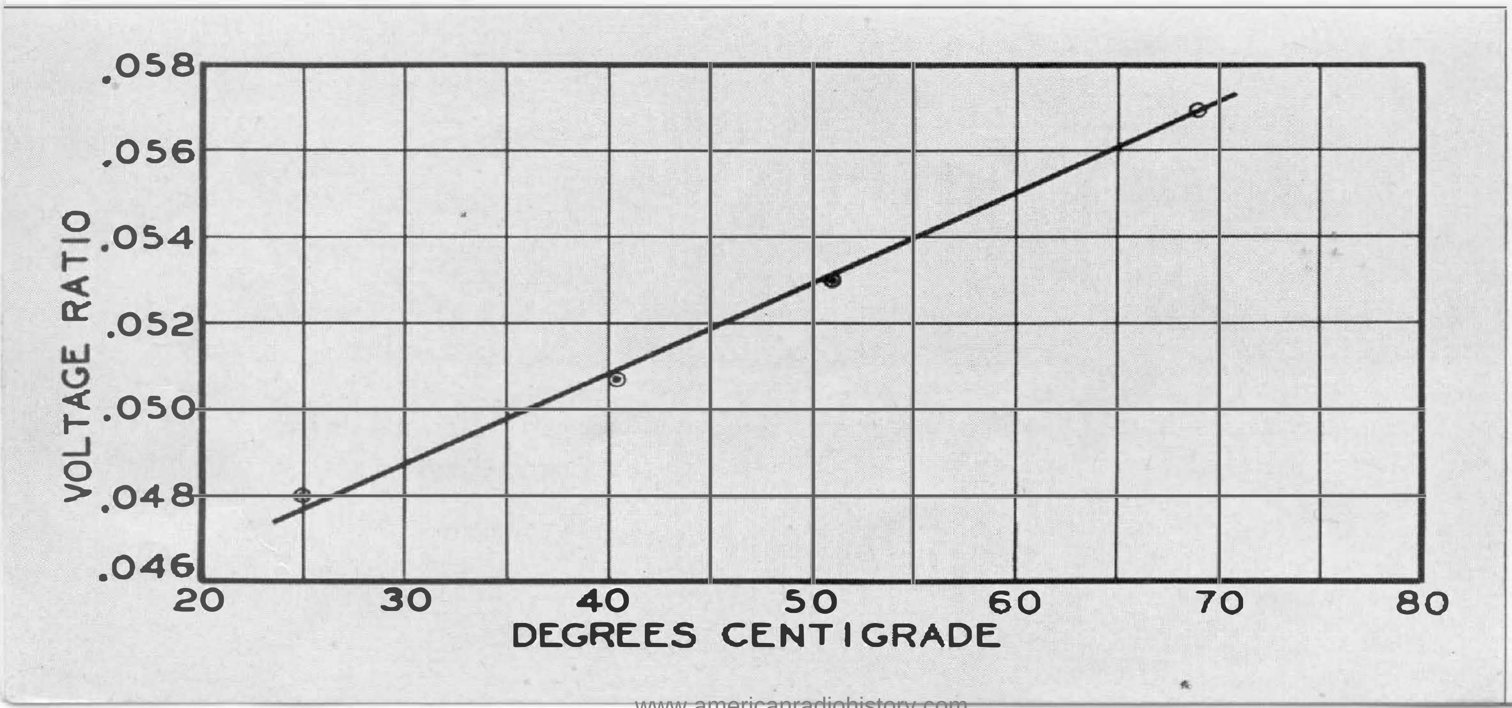
where f is in megacycles and λ_s in meters.

From (17) and (16) we may write the ratio[9] of velocity on the line to velocity in space as

$$\frac{v_l}{v_s} = \frac{\lambda_l f}{300} \qquad (18)$$

In Equation (18) λ_l is the wavelength in meters on the line at a frequency f megacycles.

Figure 3. Plot of the variation with temperature of the observed ratio of input voltage to output voltage on the open-circuited section of a line in quarter-wave resonance.



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[9] v is usually specified as a percentage or fraction of free space velocity, rather than an absolute value.

Since it will usually be more convenient to determine the quarter-wavelength (or some odd multiple thereof) rather than the full wavelength, λ_l, let us rewrite (18) as

$$\frac{v_l}{v_s} = \frac{l}{300n} f \qquad (19)$$

where l is the length in meters, f the frequency in megacycles, and n the ratio of line length to wavelength. For the quarter-wave line, we have

$$\frac{v_l}{v_s} = \frac{lf}{75} \qquad (20)$$

or if l is in feet

$$\frac{v_l}{v_s} = \frac{lf}{246} \qquad (21)$$

The condition of quarter-wave resonance required for the above equations may be determined by bridge measurements or from voltage-rise observations. (A reaction method is frequently recommended for indicating resonance, but has the disadvantage that most commercially available oscillators or signal generators have a buffer amplifier between oscillator and output, so that no reaction on oscillator plate or grid current can be obtained.)

Characteristic Impedance from Resonant Frequency and Capacitance

The velocity is related to the inductance and capacitance (L and C) per unit length of line by the approximate expression

$$v \approx \frac{1}{\sqrt{LC}} \qquad (22)$$

which can be written as

$$v \approx \frac{1}{CZ_0} \qquad (22a)$$

From Equation (22a) the characteristic impedance can be determined if the velocity of propagation and the capacitance per unit length are known. The capacitance can be measured directly on the TYPE 821-A Twin-T Impedance-Measuring Circuit, provided a sufficiently short length is used so that resonance effects are insignificant. The length used should not exceed about 1/10 of a quarter-wavelength, if accurate results are desired. (At a line angle of 10°, the effective input capacitance differs by 1% from the static value.)

The characteristic impedance can be expressed directly in terms of the frequency of quarter-wave resonance and total line capacitance as

$$Z_0 = \frac{1}{4C_0 f} \qquad (23)$$

Example of Measurements

The capacitance of approximately six feet of the TYPE 774-A Cable was measured at 1.26 Mc (the previously determined quarter-wave resonant frequency for the 117-foot length) and found to be 134.0 $\mu\mu$f. At 1000 cycles the capacitance was measured as 143.4 $\mu\mu$f. The total capacitance of the 117-foot length at 1000 cycles was 2954 $\mu\mu$f. Consequently, the total capacitance of 117 feet at 1.26 Mc can be computed as

$$C_0 = 2954 \times \frac{134.0}{143.4} = 2760\ \mu\mu\text{f}$$

Inserting these values in Equation (23) we have

$$Z_0 = \frac{10^6}{4 \times 1.26 \times 2760} = \underline{\underline{71.9\text{ ohms}}}$$

Summary

In the table following are summarized the results obtained by the various methods of measurement described.

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TABLE I

Method	*Instrument*	Z_0
Open- and short-circuit impedance	TYPE 821-A Twin-T Impedance-Measuring Circuit	73.0 — j2.1 ohms
Open- and short-circuit impedance	TYPE 916-A Radio-Frequency Bridge	72.3 — j2.1 ohms
Resistance-Termination	TYPE 916-A Radio-Frequency Bridge	72.3 ohms (corrected for reactance of terminating resistance)
Resistance-Termination with capacitor in series with terminating resistance	TYPE 916-A Radio-Frequency Bridge	72.2 ohms
Capacitance and Resonant Frequency	TYPE 821-A Twin-T Impedance-Measuring Circuit, TYPE 716-A Capacitance Bridge, TYPE 805-A Standard-Signal Generator, TYPE 726-A Vacuum-Tube Voltmeter	71.9 ohms

It should be noted that in all the methods described for the determination of characteristic impedance, with the exception of the resistance-termination method, the frequency of measurement enters directly into the calculations. If the maximum accuracy of the measuring instruments is to be realized, the frequency must accordingly be known accurately, preferably to within 0.1%.

The measurements described in this article were made using oscillators and signal generators whose standard accuracy of frequency calibration is ±1%. More accurate frequency measurements were not made as it was desired to determine the consistency of results that would be obtained with standard laboratory equipment. The results obtained by the four different methods of measurement are seen to agree with a spread of less than 2%.

Uniformity

If the transmission line is not uniform in its characteristics or has discontinuities in construction, the observed value of Z_0 may depend upon the direction of propagation. For the particular section of cable on which observations were made, a difference of 1.2 ohms was noted, when the measurement was made by the open- and short-circuit impedance method. By the resistance termination method, the same value was obtained for either direction of propagation.

— IVAN G. EASTON

This is the second of a series of two articles by Mr. Easton dealing with transmission line measurements. The first appeared in the November issue of the EXPERIMENTER.

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SERVICE AND MAINTENANCE NOTES

● **LISTED BELOW** are the General Radio instruments for which Service and Maintenance Notes are now available. If you have recently acquired any of these instruments, and service notes are not in your files, write for a copy today. When requesting service notes, please give type number and serial number of instrument and approximate date of purchase. All correspondence regarding Service and Maintenance Notes should be addressed to our Service Department.

25-A	Broadcast Frequency Monitor
25-AB	Broadcast Frequency Monitor
516-C	Radio-Frequency Bridge ✓
544-B	Megohm Bridge ✓
544-P3	A-C Power Supply
561-D	Vacuum-Tube Bridge ✓
605-A,-B	Standard-Signal Generator ✓
608-A	Oscillator
614-C	Selective Amplifier
616-D	Heterodyne Frequency Meter ✓
617-C	Interpolation Oscillator
620-A	Heterodyne Frequency Meter and Calibrator ✓
625-A	Bridge
631-A,-B	Strobotac ✓
636-A	Wave Analyzer
648-A	Strobolux
650-A	Impedance Bridge
667-A	Inductance Bridge ✓
684-A	Modulated Oscillator
690-C	Piezo-Electric Oscillator
691-C	Temperature-Control Box
692-B	Multivibrators
693-B	Syncronometer
694-C	Control Panel
698-A	Duplex Multivibrator
700-A	Wide-Range Beat-Frequency Oscillator
707-A	Cathode-Ray Null Detector ✓
713-A	Beat-Frequency Oscillator ✓
713-B	Beat-Frequency Oscillator ✓
714-A	Amplifier
715-A	Direct-Current Amplifier
716-A	Capacitance Bridge ✓
716-B	Capacitance Bridge
723-A,-B,-C,-D	Vacuum-Tube Fork
726-A	Vacuum-Tube Voltmeter ✓
727-A	Vacuum-Tube Voltmeter
729-A	Megohmmeter
731-A	Modulation Monitor
731-B	Modulation Monitor
732-A	Distortion and Noise Meter ✓
732-B	Distortion and Noise Meter
733-A	Oscillator
736-A	Wave Analyzer
740-B	Capacitance Test Bridge ✓
740-BG	Capacitance Test Bridge
757-A	U-H-F Oscillator
759-A	Sound-Level Meter
759-B	Sound-Level Meter
760-A	Sound Analyzer
761-A	Vibration Meter
769-A	Square-Wave Generator
775-A	Frequency-Limit Monitor
804-A	U-H-F Signal Generator ✓
804-B	U-H-F Signal Generator ✓
805-A	Standard-Signal Generator
821-A	Twin-T Impedance-Measuring Circuit
834-B	Electronic Frequency Meter
913-A	Beat-Frequency Oscillator
—	Variacs (all types) ✓

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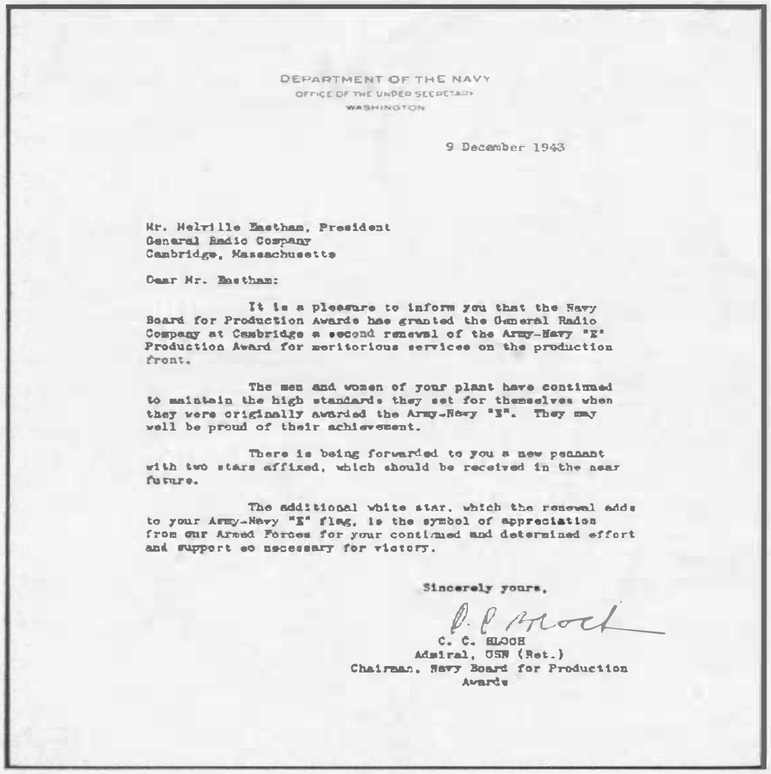
A SECOND STAR FOR OUR "E" BANNER

The accompanying letter from the Navy's Board of Production Awards speaks for itself. Needless to say, we at General Radio are proud that our efforts have won a second renewal of the "E." To all in other organizations whose co-operation has helped us to keep up our production record we express our sincere appreciation.



DEPARTMENT OF THE NAVY
OFFICE OF THE UNDER SECRETARY
WASHINGTON

9 December 1943

Mr. Melville Eastham, President
General Radio Company
Cambridge, Massachusetts

Dear Mr. Eastham:

It is a pleasure to inform you that the Navy Board for Production Awards has granted the General Radio Company at Cambridge a second renewal of the Army-Navy "E" Production Award for meritorious services on the production front.

The men and women of your plant have continued to maintain the high standards they set for themselves when they were originally awarded the Army-Navy "E". They may well be proud of their achievement.

There is being forwarded to you a new pennant with two stars affixed, which should be received in the near future.

The additional white star, which the renewal adds to your Army-Navy "E" flag, is the symbol of appreciation from our Armed Forces for your continued and determined effort and support so necessary for victory.

Sincerely yours,

C. C. Bloch

C. C. BLOCH
Admiral, USN (Ret.)
Chairman, Navy Board for Production Awards

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