

EXPERIMENTER



ELECTRICAL MEASUREMENTS
TECHNIQUE AND ITS INDUSTRIAL APPLICATIONS

VOL. XI No. 8

JANUARY, 1937

THE MEASUREMENT OF MUTUAL INDUCTANCE

● **MUTUAL INDUCTANCE** is one of the most important properties of electrical circuits. By means of it the world's power is transmuted from generator voltage to transmission line voltage and back to the voltage suitable for our motors and electric lights. Yet the measurement of mutual inductance is of little importance as compared to measurements of self-inductance, capacitance, and resistance.

The major reason for this slighting of mutual inductance lies in the interesting fact that iron-core transformers are nearly perfect. The mutual inductance between the primary and secondary windings has so nearly its maximum value that no ordinary measurement of it could distinguish the minute difference. Such differences are of course of great importance and are measured by some characteristic of the transformer itself, the voltage regulation of a power transformer or its leakage reactance, and the frequency characteristic of an audio transformer.

The general theory of coupled circuits involves at one extreme the closely coupled iron-core transformers just mentioned and at the other extreme the loosely coupled tuned circuits of radio-frequency amplifiers, of which the neutrodyne receiver of Hazeltine is an historical example. Such loosely coupled circuits are now being used as band-pass filters at the intermediate frequency of heterodyne receivers.

Mutual inductance is unique in that it can exist only in the presence of self-inductance. An important measure of mutual inductance is its ratio to the geometric mean of the two self-inductances it connects, called the coefficient of coupling, k .

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad (1)$$

which may have any value from zero to unity. In loosely-coupled tuned circuits the resonance curve has a single peak for all values of coupling coefficient

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less than a so-called critical coupling.¹ A larger coupling coefficient results in a flattened top with the steepness of the sides maintained. The measurement of this small mutual inductance is of considerable importance in the adjustment of i-f filters.

Mutual inductance may be measured by using as a standard of comparison a mutual inductance, a self-inductance, or a capacitance. When the standard mutual inductance is continuously variable and of such a range that it can be made equal to the unknown mutual inductance, the Felici mutual-inductance balance is the simplest method. As

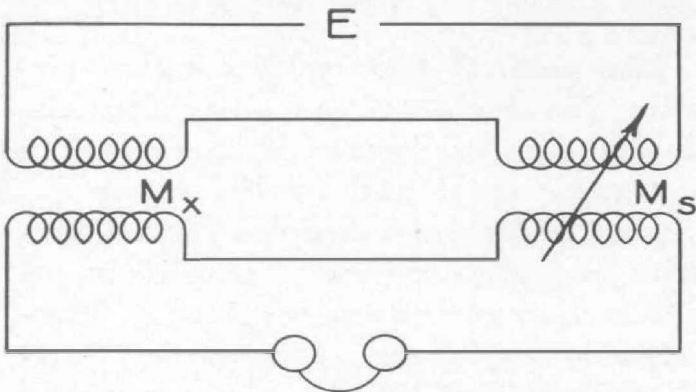


FIGURE 1. Felici mutual-inductance balance shown in Figure 1, the primaries of both mutual inductances, unknown and standard, are connected in series to a power source. Their secondaries are also connected in series to a suitable detector, head telephones, or other a-c operated meter, in such direction that their induced voltages oppose. The standard is then varied until a balance is obtained, when

$$M_x = M_s \quad (2)$$

The error in this measurement is essentially that of the standard, provided that the capacitive reactances between the coils of the mutual inductances are large compared with their mutual reactances and that the impedance of the detector is small.

¹ $k_c = \sqrt{D_1 D_2}$ or $\omega M_c = \sqrt{L_1 L_2}$ where D_1 and D_2 are the dissipation factors (reciprocal of storage factor Q) of primary and secondary.

TYPE 107 Variable Inductors are calibrated in such a way that the mutual inductance between their rotor and stator windings can be easily obtained from their dials calibrated in self-inductance. For a 1% full-scale accuracy of this dial calibration, the error in the mutual inductance will range from 2.5% to 10% over the portion of the scale normally used.

When the primary and secondary windings of a mutual inductance are connected in series, the self-inductance L of the pair is

$$L = L_1 + L_2 \pm 2M \quad (3)$$

The mutual inductance M may be calculated from the two self-inductances L_a and L_o obtained with the two coils aiding and opposing (using the + and - signs before $2M$).

$$M = \frac{1}{4} (L_a - L_o) \quad (4)$$

For a coefficient of coupling nearly unity where L_o is very small compared to L_a , the error in the determination of M is that of L_a itself. For smaller coupling coefficients this error increases, as always happens when the difference of two nearly equal numbers enters in any



FIGURE 2. TYPE 107 Variable Inductor. Mutual inductance at any setting is one-half the difference of the scale reading and the value of self-inductance at zero mutual inductance as entered on the nameplate

formula. For example, when $K = 0.1$ and $L_1 = L_2$, the error will be increased fivefold. The measurement of self-inductance may be made on a TYPE 650-A Impedance Bridge with an error of $2 \mu\text{h}$ or 2% and on a TYPE 667-A Inductance Bridge with an error of $0.1 \mu\text{h}$ or 0.2% . The errors which occur in inductance measurements were discussed in considerable detail in the *General Radio Experimenter* for March, 1934.¹ The TYPE 293-A Universal Bridge may also be used with resultant errors which lie between those of the other bridges mentioned. In bridges having a decade ratio arm, from the setting of which the self-inductance is calculated, such as the TYPE 667-A and TYPE 293-A Bridges, the increase in the error as the two separate bridge balances approach one another is minimized if one ratio arm is kept fixed and the change in balance taken up by minimum changes in the other. The error is then that of the change in resistance of the decade ratio arm.

Mutual inductance may be compared with a self-inductance on the Campbell mutual-inductance bridge shown in Figure 5.

$$M = \frac{AL_P - BL_N}{A + B} \quad (5)$$

The self-inductance L_P of the winding

¹R. F. Field, "The Measurement of a Small Inductance," *General Radio Experimenter*, Vol. VIII, No. 10, March, 1934.

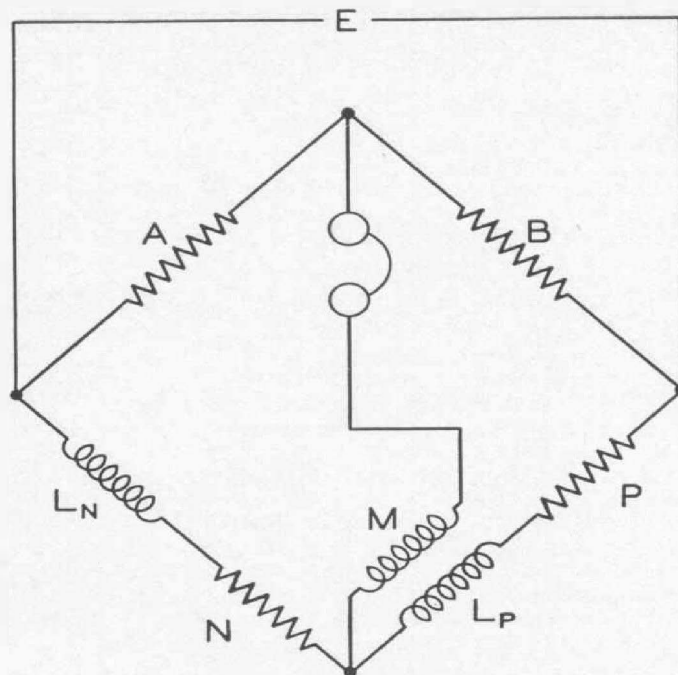


FIGURE 5. The Campbell Mutual-Inductance Bridge

connected into the bridge arm may be measured by removing the other winding from the detector circuit. Denoting these bridge readings by primes,

$$M = \frac{\frac{B'}{A'} A - B}{A + B} L_N \quad (6)$$

$$= \frac{B' - B}{A + B} L_N \quad \text{if } A = A' \quad (7)$$

Such measurements are easily made on the TYPE 667-A Inductance Bridge by connecting one winding of the mutual inductance to the unknown terminals and the other winding in series with the detector. The errors are slightly

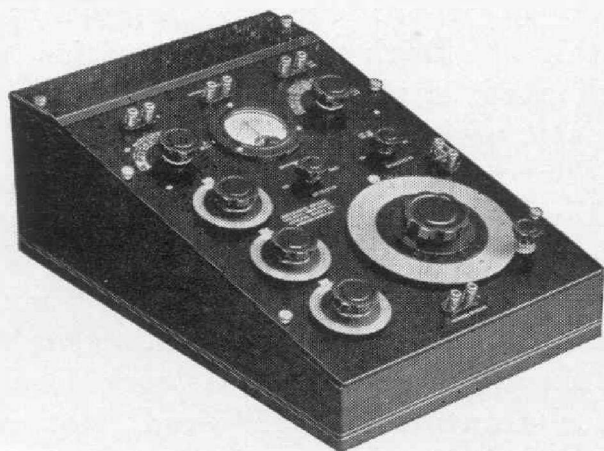


FIGURE 3. TYPE 650-A Impedance Bridge

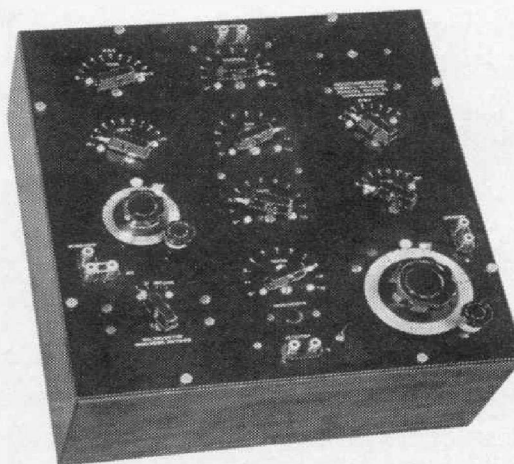


FIGURE 4. TYPE 667-A Inductance Bridge

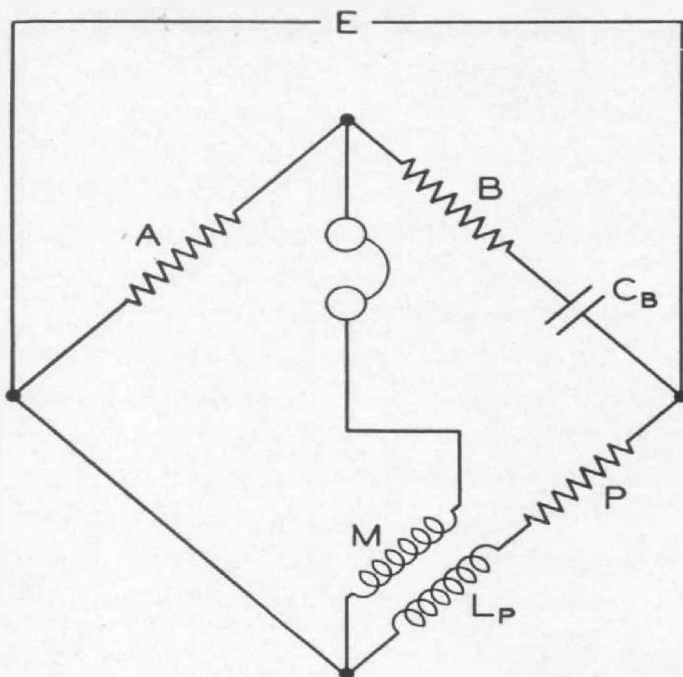


FIGURE 6. The Carey Foster Mutual-Inductance Bridge

ference between the two bridge settings is reduced by at least a factor of three and there is a greater chance of keeping both the bridge balances within a single decade setting lower than the highest one used.

A capacitance may be used as a standard for comparison with an unknown mutual inductance on the Carey Foster mutual-inductance bridge shown in Figure 6.

$$M = APC_B \quad (8)$$

The resistance P includes that of the winding connected in that arm, which is somewhat difficult to obtain. This bridge circuit may be set up on the TYPE 293-A Universal Bridge, using as a capacitance standard a suitable TYPE 509 Mica Condenser.

— R. F. FIELD

less than for the previous case because for a given mutual inductance the dif-

NEW WAVE FILTERS

● **THERE** is a steady demand for wave filters offering a moderate amount of discrimination between the desired and undesired frequencies and having a moderate sharpness of cutoff. For some years the General Radio Company has been supplying TYPE 330 Filters for general-purpose laboratory use. For a number of uses, however, a sharper cutoff is required, and after a considerable period of supplying individually-designed filters for particular applications, the TYPE 830 Filter is offered as a stock item.

In discussing filter performance the quantities of interest are usually the sharpness of cutoff or, more briefly, the "sharpness," and the "discrimination" or difference between the insertion loss in decibels for the wanted and unwanted frequencies. For certain applications, peaks in the attenuation curve are important for the suppression of particular frequencies or harmonics.

A filter for general application, however, should be designed so that in the valleys between the peaks the filter never gives less than a certain required attenuation. From this standpoint the height of the lowest valley measures the "discrimination," and if there is more than one valley the best design will make them all of the same height. Having defined the "discrimination" as the minimum valley height, the "sharpness" can be defined as the frequency ratio in which the discrimination is obtained. Both these quantities are indicated in the typical filter characteristic of Figure 1.

For certain applications the most important consideration is sharpness of cutoff. For other applications a wide frequency interval is available for the transition from the pass band to the attenuation region, but considerable discrimination is required between frequencies in the two regions. The con-

ditions on the design which are necessary to obtain a sharp cutoff are very different from those which will give a large amount of discrimination. It turns out, in fact, that the greater the sharpness which is obtained by proportioning the elements in a particular configuration, the less will be the discrimination.

In the TYPE 830 Wave Filters the endeavor has been made to reach a compromise between sharpness and discrimination which will be satisfactory for a large number of applications. The insertion loss characteristic when the filter is working between constant resistance terminations is shown in Figure 2. It will be seen that the "discrimination" is slightly over 40 decibels and the "sharpness" is such that this value is reached at 1.5 times the cutoff for the low-pass types or .67 of the cutoff for the high-pass types.

High-pass and low-pass types can be used in tandem to provide a band-pass filter where the interval to be covered is one octave or more. The attenuation characteristic of such a combination is shown in Figure 3.

— W. N. TUTTLE

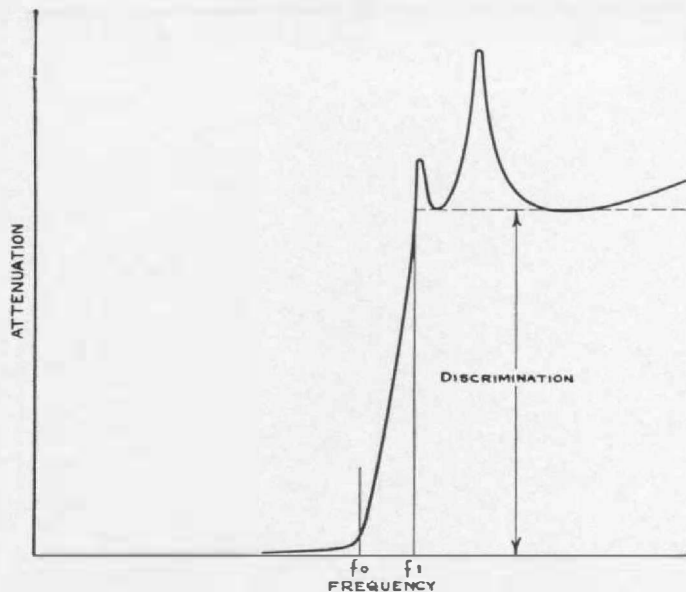


FIGURE 1. Insertion loss characteristic of a filter indicating what is meant by discrimination and sharpness. The ratio $\frac{f_0}{f_1}$ (or its reciprocal, depending on whether the filter is low-pass or high-pass) is a measure of sharpness

TYPE 830 Wave Filters are available in both high-pass and low-pass types. Stock models have cut-off frequencies of either 500 or 1000 cycles and image impedances of either 500 or 5000 ohms. Complete specifications are given on page 114 of Catalog J.

Type 830 Filters are manufactured and sold under patents of the American Telephone and Telegraph Company solely for utilization in research investigation, measurement, testing, instruction and development work in pure and applied science.

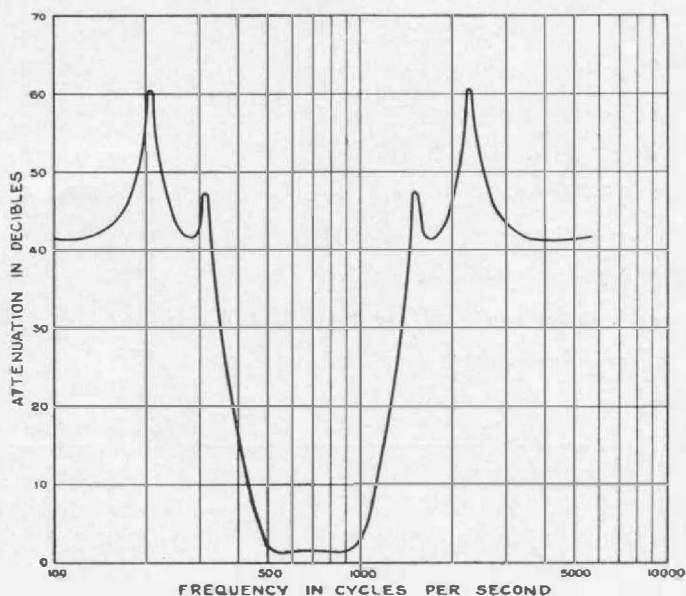
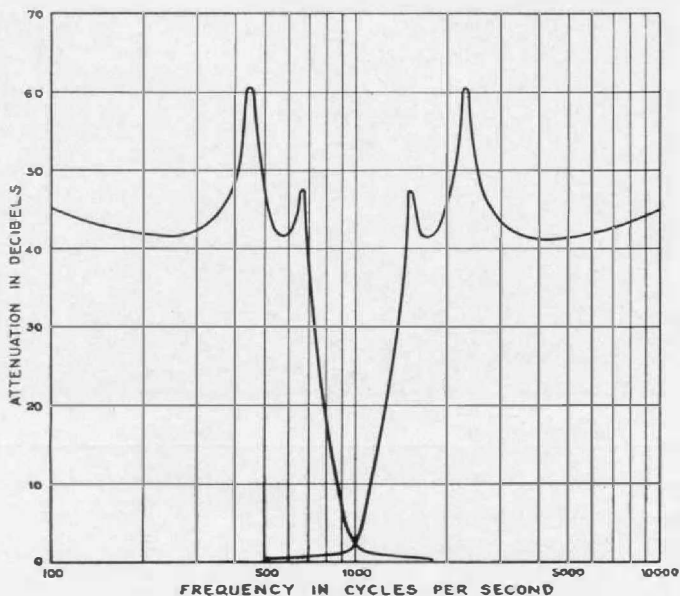
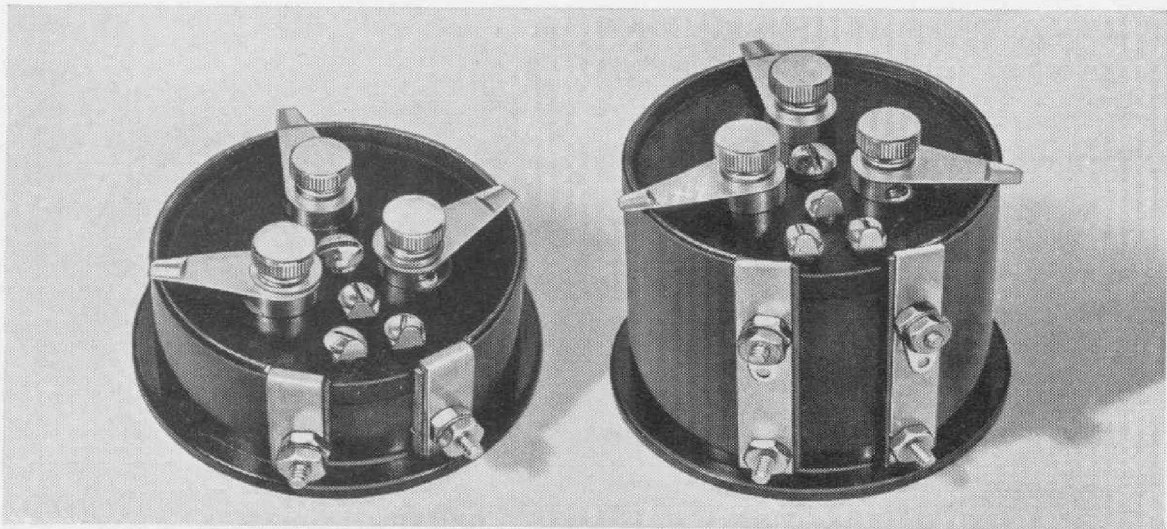


FIGURE 2 (left). Characteristics of 1000-cycle, low-pass and high-pass filters
FIGURE 3 (right). The 500-cycle, high-pass and 1000-cycle, low-pass types can be combined to form a band-pass filter covering one octave

A HANDY VOLTAGE DIVIDER

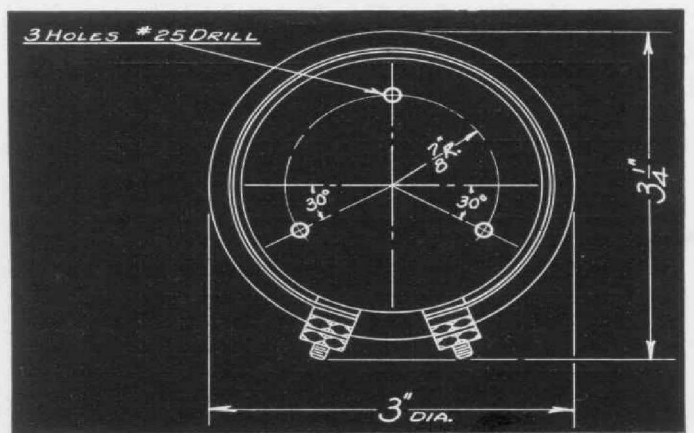


● **IN BUILDING** specialized laboratory equipment and in circuit development work, the engineer often needs an inexpensive, compact, and easily adjustable voltage divider for supplying plate and grid voltages to vacuum tubes. In several recently-developed General Radio instruments, the units shown in the accompanying photograph have been used for this purpose and have proved so useful that they are now offered for general sale.

The resistance winding and the moulded bakelite forms are identical in construction with those used in TYPE 471-A and TYPE 314-A Rheostat-Potentiometers. Three adjustable rotary contacts are provided, each covering one-third of the total resistance. Adjustments are made by loosening a thumbscrew and moving the arm to the desired position, after which the adjustment is locked by tightening the thumbscrew.

SPECIFICATIONS

- Power Dissipation:** TYPE 154-A, 6 watts; TYPE 154-B, 12 watts.
- Accuracy of Resistance:** $\pm 10\%$.
- Mounting:** Standard 3-hole; see accompanying sketch.
- Dimensions:** TYPE 154-A, $3\frac{1}{8}$ inches (maximum diameter) x $1\frac{3}{4}$ inches (height). TYPE 154-B, $3\frac{1}{8}$ inches (maximum diameter) x $2\frac{7}{8}$ inches (height).
- Net Weight:** TYPE 154-A, 6 ounces; TYPE 154-B, 8 ounces.



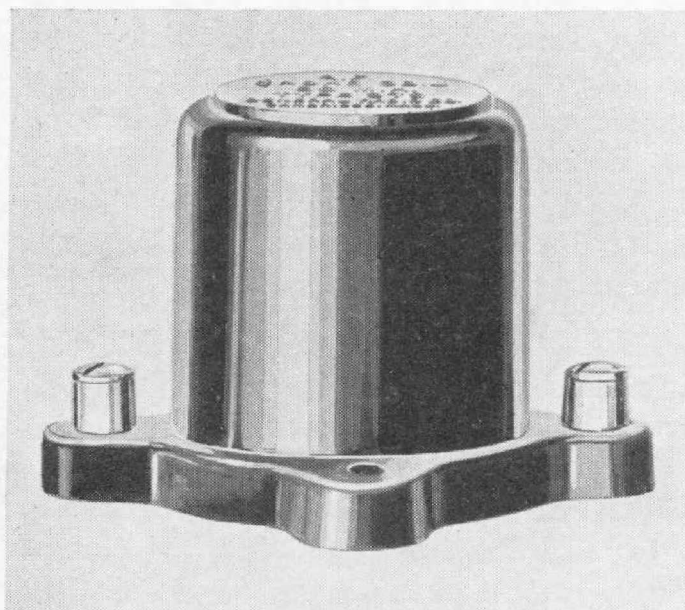
Type	Total Resistance	Code Word	Price
154-A	5,000 Ω	DIVIDARMY	\$6.50
154-A	10,000 Ω	DIVIDBOAT	6.50
154-A	20,000 Ω	DIVIDCAPE	6.50
154-B	50,000 Ω	DIVIDEYRE	8.50
154-B	100,000 Ω	DIVIDFACT	8.50
154-B	200,000 Ω	DIVIDGIRL	8.50

A W I D E - R A N G E R - F C H O K E

● **THE UTILITY** of most general-purpose radio-frequency chokes is limited to operation over a comparatively narrow range of frequencies. Ordinary methods of winding produce a high value of distributed capacitance, while separating the winding into several individual sections in series results in such a large number of resonant frequencies that at least one is usually located in the band where it is desired to work. The **TYPE 119-A Radio-Frequency Choke** has been designed to eliminate these faults.

The winding is composed of a large number of spiral-wound pies, each only one wire wide. By this method, the capacitance is much more uniformly distributed than with thicker pies, resulting in only one resonant point. The effective capacitance is extremely low, so that the choke is useful at fairly high radio frequencies.

The specifications for **TYPE 119-A Radio-Frequency Choke** are listed below. It will be noted that the capacitance of $2 \mu\mu\text{f}$ is extremely low for an

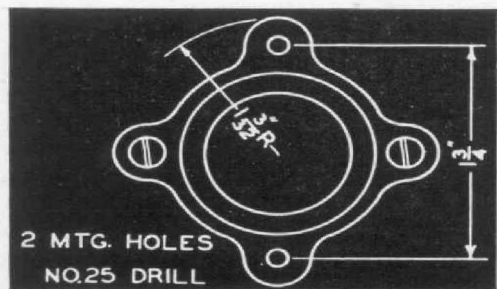


inductance of 250 mh. These constants are practically independent of frequency and no minor resonances are found up to at least 50 megacycles.

In addition to its function as a choke, this unit has been found extremely useful as the inductance element in high-impedance low-pass filters.

S P E C I F I C A T I O N S

- Inductance:** 250 millihenrys.
- Capacitance:** $2 \mu\mu\text{f}$.
- D-C Resistance:** 450 ohms.
- Maximum Current:** 60 milliamperes.
- Dimensions:** Base, $1\frac{3}{4} \times 1\frac{3}{4}$ inches; height, $1\frac{7}{8}$ inches, over-all.
- Mounting:** See sketch.
- Net Weight:** 3 ounces.



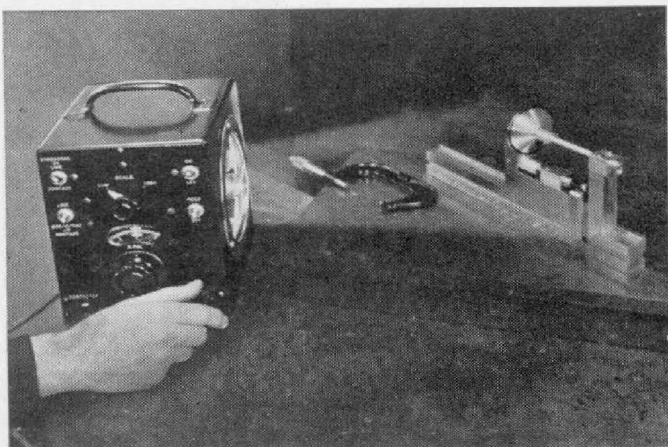
Type	Code Word	Price
119-A	IMAGE	\$1.50

MISCELLANY

● **AMONG RECENT VISITORS** to the General Radio Laboratories was Dr. Georg Keinath, Director of the Instrument Laboratories of Siemens and Halske. Dr. Keinath is the author of "Die Technik Elektrische Messgeräte," a well-known text on electrical measuring instruments.

● **THE "R.M.A. ENGINEER,"** published by the Engineering Division of the Radio Manufacturers' Association made its first appearance with the Fall Meeting of the IRE at Rochester. This publication, which will carry standardization reports and similar data, as well as technical articles, will be issued three times a year. Volume I, No. 1 is an achievement of which the board of editors may be proud.

● **PERIODICALLY** it becomes necessary to revise the *Experimenter* and catalog mailing list. Shortly after you receive this issue of the *Experimenter*, a post card "subscription" renewal form will reach you. The revised G-R mailing list will be made up from the returns received from these cards.



In order to insure your name remaining on the list, the return card should be filled in completely and mailed promptly.

● **AN INGENUOUS** solution to a difficult problem is illustrated in the accompanying photograph.

The wheel shown in the photograph is an experimental part for a new integrator now under development in the Electrical Engineering Research Laboratory at Massachusetts Institute of Technology. The problem is to measure the friction losses. Since the shaft runs in enclosed jewel bearings, it is not accessible for speed measurement. Furthermore, the power level involved is so small as to preclude the possibility of measurement by ordinary means.

The solution devised by Dr. S. H. Caldwell and Mr. J. J. Jaeger of Massachusetts Institute of Technology is simple. A General Radio Strobotac was set at a convenient value of speed, and the disc was driven by a stream of compressed air until its speed was above that of the flashing Strobotac lamp. The disc was then allowed to coast to a standstill. Whenever its speed or a multiple of it coincided with the fundamental or a subharmonic of the Strobotac flash speed, the disc appeared to stand still momentarily. Intervals between these points were timed with a stop watch, and a speed-vs.-time curve for the complete deceleration period was plotted. From this curve and the moment of inertia, the frictional loss was easily calculated.

GENERAL RADIO COMPANY

30 STATE STREET - CAMBRIDGE A, MASSACHUSETTS
 BRANCH ENGINEERING OFFICE — 90 WEST STREET, NEW YORK CITY