THE BELL SYSTEM
TECHNICAL JOURNAL

VOLUME XXXI SEPTEMBER 1952 NUMBER 5

Automatic Switching for Nationwide Telephone Service

By A. B. CLARK and H. S. OSBORNE

(Manuscript received May 15, 1952)

A plan for automatic long distance switching, which will ultimately embrace the entire area of the United States and extend into Canada and perhaps Mexico, has been formulated and important steps have been taken toward its realization. The plan contemplates that when a telephone customer places a call with a long distance operator, this operator will be able to establish a connection to any desired telephone simply by playing a 10 or 11 digit code into an automatic mechanism. She will receive distinctive signals when the called telephone answers or when the telephone or the toll circuits are busy. She will completely control the establishment of the connection and will have available to her the information necessary for proper billing of the call. The plan also contemplates that telephone customers will ultimately be able to dial long distance calls themselves, wherever may be the locations of the calling and called telephones.

INTRODUCTION

Ever since the invention of the telephone 76 years ago, development work has been pressing forward both in telephone transmission and in switching. These two fields have been closely interrelated in the development of telephone service on a nationwide basis, and neither could have progressed as it has without corresponding progress in the other.

The first development of equipment for the mechanical switching of telephone lines was the local dial system to enable one customer to be connected with another in the same town. It was a natural step to develop the equipment so that operators in nearby towns could complete toll calls through this local dial equipment. This was done first by using the local equipment and then with progressive modifications making it more and more suitable for toll.

By these means through the decades of the 20's and 30's regional networks were developed for operator toll dialing, using step-by-step types of equipment, particularly in Southern California, Connecticut and Ohio. Also many short haul toll calls in metropolitan areas were handled in connection with the panel type dial equipment which was developed for automatic switching in these areas.

Also during this period the range of customer dialing in large metropolitan areas was extended, where local service is measured by message registers, through arrangements for the multiple registration of calls for which the charge was more than one local unit.

An important feature of switching development in this period was the perfecting of "common control" switching systems for large metropolitan areas endowed with a high degree of intelligence and great reliability.[1] As will be shown, still more extensive and complicated functions must be performed by the common control systems of a nationwide automatic switching system.

Also throughout this period great advance was made in the quality and stability of long distance circuits. Telephone connections, some with as many as five circuits in tandem, were being regularly established by telephone operators with satisfactory overall transmission. The limitation was in the speed and accuracy with which multiple switches could be made by operators rather than in the overall transmission characteristics.

Several factors have worked together to bring about a big expansion of long distance telephone service. These include the great growth in the numbers of telephones in service, improvements in long distance transmission, in switching, and in methods of traffic operation. Since automatic switching becomes increasingly attractive as the traffic density increases, this large growth pointed toward the desirability of further mechanizing the switching operations.

In 1943 there was cut into service in Philadelphia the first installation of the No. 4 toll crossbar system.[2] This system was designed to enable general automatic switching of toll connections in and out of large metropolitan areas and had many of the capabilities necessary for nationwide switching.

The various considerations already mentioned, coupled with the success of the No. 4 installation at Philadelphia, led to studies of the service and operating results which might be expected from a nationwide extension of automatic switching. The conclusion was reached that this would be a desirable objective of the Bell System companies and would result in a very substantial further improvement in the speed and accuracy of handling of long distance messages. Accordingly, during the next few years, a national plan was prepared and was adopted by the telephone companies.

GENERAL PLAN FOR NATIONWIDE AUTOMATIC SWITCHING

The features of this nationwide plan and the present status of its application form the subject of the three technical papers which accompany this introductory paper.[3, 4, 5] The basic requirements to be met in the development of this plan included the following:

1. It should be suitable for the nationwide extension of automatic switching both by originating toll operators and by the customers direct.

When this work was commenced it was clear that a program leading toward general nationwide operator dialing was desirable. Subsequent developments have confirmed the wisdom of making the basic plan consistent with general nationwide customer dialing as well since it now appears that a very wide extension of this form of service will become desirable.

2. The plan must provide for satisfactory overall service between any two telephones in this country and Canada.

Under manual operation satisfactory overall service was provided for by the general toll switching plan in use since about 1930. This plan is modified to recognize the far greater speed and accuracy of automatic switching compared with manual switching. This involves also modifications of transmission design standards so that the overall connections will continue to be satisfactory.

3. The system must be designed for instantaneous service, so that delays due to lack of circuits or equipment would be very infrequent. This is necessary, both from the standpoints of service and the avoidance of tieups, particularly of the automatic switching machinery.

A trunking system must therefore be devised which will most economically meet this requirement, considering overall costs of lines, switching equipment and operation.

4. Machines must be designed for use at strategic points in the network, called "control switching points", to perform automatically the various tasks required to make the overall plan operative and economical.

5. The entire plan must be such as to provide satisfactorily for growth, for flexibility to meet changing conditions and for minimum overall costs of operation.

FUNDAMENTAL PLANS FOR TOLL PLANT

Mr. Pilliod's paper, pages 832 to 850, discusses the fundamental layout of plant for nationwide operator toll dialing. This is subject to changes from time to time with further specific studies, as is the case with all far-reaching fundamental plans of this type. The additional requirements imposed by nationwide customer dialing are still under study as will be discussed a little later.

The national toll switching plan is modified so that there may be a maximum of eight toll circuits switched together to connect any two telephones compared with the previous limit of five.[6] In order to handle the entire traffic of the country, approximately 100 control switching points are necessary at which highly intelligent common control switching systems of the No. 4 crossbar type will be placed.

A very important feature of the layout is a trunking plan providing for a high degree of use of alternate routes. To design all of the toll circuit groups of the country for a no-delay service would be very expensive. However, taking advantage of the extreme rapidity of automatic switching and the ability to build into the machine capacity for using a large number of alternate routes, a trunking system has been devised in which only about one-sixth of the toll circuit groups of the country need be engineered on a very liberal basis. These are called final groups and are the groups to which the machine ultimately appeals if all of the more direct circuit groups are busy. These more direct circuit groups can then be engineered on a basis providing for high usage of the circuits, recognizing that when one group is busy the machine appeals to another and so on until as a last resort the final group is used.

In determining means for handling all of the toll messages with a relatively small number of control switching points, tremendous advantage was derived from modern transmission developments, particularly carrier systems which give a great economy from the concentration on a long distance route of large numbers of telephone circuits – numbers often running into the thousands. As a result, a considerable degree of circuitous routing and back hauling of circuits is economical if by these means the circuits can be concentrated on heavy routes. This in turn lends itself to a plan using a minimum of control switching points.

NATIONWIDE NUMBERING PLAN

In the previous use of automatic switching by toll operators, the operators were furnished with codes by means of which could be selected the various circuits necessary to reach the destination. These codes were dialed, followed by the local number of the called party. With this system, toll operators calling a given telephone from different remote cities would, in general, use different codes corresponding to the different circuit groups which they must select.

For nationwide toll dialing even by operators this system would have impossible complications, and for nationwide customer dialing it is clear that the code to be dialed must uniquely represent the office which serves the called telephone and that office only and not be dependent upon the route to be followed to reach it. In other words, it involves the development of what is called a destination type code. Another description of this code plan is to say that for toll dialing purposes each telephone in the country (and Canada) must have a distinctive telephone number different from that of every other telephone.

It is also clear that as a practical matter this number should be based upon the local telephone number of the customer prefixed by a minimum number of digits, following easily understood rules.

To bring this about has involved a very high order of planning. Such a plan has been perfected and forms currently the basis for the determination of the coding of all new telephone offices and for changes in office codes when these are necessary. The development of this is the subject of Mr. Nunn's paper.

CUSTOMER TOLL DIALING

When the customer is to dial long distance calls directly without assistance from any operator, two additional requirements are imposed beyond those necessary for nationwide operator dialing.

1. The customer normally is connected to a local central office but for the purpose of nationwide toll dialing he must be connected to the nationwide toll network. At present he does this by dialing a code such as '211" which connects him with the long distance operator. This procedure could be continued. However, since the customer must in any event dial 10 digits for the longest hauls to designate the called telephone, it is desirable if possible to cut out this preliminary step. That would mean modifying the local central office equipment so that it would receive the 10 digit numbers and transmit them on to the toll equipment. This is a simple undertaking for local central offices using the latest

type of local central office equipment, called No. 5 crossbar, which was designed with this in view.[7] For older types of equipment, the job is more difficult.

2. The switching equipment must be provided with automatic means for recording all of the information necessary for charging the call. In the case of operator dialing this is now done manually by the operator.

Great advances have been made in recent years in the development of automatic message recording equipment. In 1944 there was placed in service in California the first installation in this country of automatic ticketing equipment.[8] This equipment is associated with step-by-step local switching equipment and automatically prints for each call a ticket similar to that prepared by the operator with manual operation. In 1948 there was installed in Media, near Philadelphia, a greatly improved type of message recording equipment in which the information appears in the form of punched holes in a tape.[9] This equipment is much more economical than the earlier system and also lends itself to the automatic preparation of toll statements or bills.

The present forms of equipment have been designed to be associated with local central offices. A careful study has been made of their field of application and of the basic plan necessary to provide for a general nationwide extension of customer dialing. This indicates that there will be a large field for automatic message accounting equipment associated with the toll network and arranged to receive orders for toll messages from a number of local dial offices. This centralized AMA equipment, as it is called, is under development and an initial installation will be made next year in Washington, D. C. In this installation the range of customer dialing will be limited and certain service features will be lacking, which it is planned to add later.

The nationwide extension of customer toll dialing involves many operating problems in addition to those relating to the design of the plant. These problems involve the extent to which customers wish to dial long distance calls, requiring 10 pulls of the dial, the accuracy of dialing, the treatment of wrong numbers, provision for giving subscribers information regarding telephone numbers in distant cities, information on charges and many other questions.

Recognizing that the best way to develop these questions is a trial, arrangements were made to open such a trial last fall at Englewood, N. J. This office is equipped with a No. 5 crossbar system so that arrangements for such a trial could readily be made there. The Englewood customers are able to dial directly any of about eleven million telephones in ten metropolitan areas scattered throughout the country, including

Boston, New York, Pittsburgh, Cleveland, Chicago and San Francisco and the Bay area.

The results of this trial have been very encouraging. Subscribers are continuing to dial over 95 per cent of all the calls which can be dialed. Errors due to wrong numbers are at a minimum and other difficulties are relatively low. In so far as this trial can answer the questions, the results are all in favor of the nationwide extension of customer dialing as the development and installation of facilities suitable for this purpose make it possible to do so.

In view of the prospect of nationwide customer dialing, fundamental plan studies are now being made by the Telephone Companies throughout the country of the whole layout of plant including the distribution of centralized automatic message accounting equipments with the future general application of this method of operation. The present indication is that the number of points at which toll operating centers will be required will be greatly reduced. This will react in important ways on the design of telephone buildings, telephone equipment installations and toll circuit routes.

AUTOMATIC TOLL SWITCHING AND ACCOUNTING EQUIPMENT

All of these plans depend upon the successful development of striking innovations in toll switching and automatic message accounting equipments. The plans in turn react upon the features to be incorporated in such equipments and upon the schedule of their development. Mr. Shipley's paper, pages 860 to 882, tells about the more important features of these equipments and the problems which are involved in their development.

CONCLUSIONS

Experience with operator toll dialing shows clearly that it provides a marked improvement in toll service. This improvement will increase as progress is made toward the full application of the nationwide automatic switching plan.

The development of long distance dialing by customers is at an early stage. The results of recent trials, however, indicate that nationwide customer dialing has service advantages and will generally be received with enthusiasm by telephone users. It is anticipated, therefore, that customer dialing will rapidly expand both on a regional and on a nationwide basis.

The service advantages of nationwide automatic switching are not measured entirely by the increased speed and improved accuracy of connections. An important factor is the continued ability of the telephone system to meet the rapidly increasing demand for telephone service without making excessive demands on the available supply of labor. The development of local dial operation was absolutely necessary to handle the great growth of local telephoning which has taken place. Today, in many places, requirements for people for toll operations are very heavy and an increased amount of automatic toll switching is becoming more and more necessary to make possible handling the rapidly increasing number of long distance telephone messages.

With this development there has been a marked increase of employment. The Bell Companies today employ 244,000 operators compared with 131,000 in 1941. They have also employed many people to build and install about 300-million dollars worth of toll dialing equipment, to construct places to house it, maintain it and carry out operating rearrangements.

With respect to the future, even with the nationwide automatic switching plan in full operation and the local central offices arranged to permit customer dialing, there will still be a large amount of work for operators. They will be required to handle information and assistance traffic, person-to-person calls, collect calls and other classes of calls which do not lend themselves to customer handling, as well as any individual calls which the customers may not wish to dial themselves.

The Bell Companies have necessarily taken the lead in planning and applying these new developments. The plans, however, are all laid in such a way as to include telephone users in Independent Telephone Company offices. The Independent Companies are being kept fully informed of these plans as they develop and are participating, as the development of their own plant makes it practicable and desirable, in extending the benefits of the new forms of operation to their own customers.

This long-term development has required the very close cooperation of all parts of the Bell System – American Telephone and Telegraph Company General Department, Bell Telephone Laboratories, Western Electric Company, Long Lines and all of the Bell Operating Companies. Each installation of equipment and circuits and each operation is a part of a nationwide system and must be closely coordinated. The close interrelation and working together of the various parts of the Bell Telephone System, research and development, manufacturing, engineering and operating are necessary for the effective planning and execution of this tremendous project.

BIBLIOGRAPHY

1. F. J. Scudder and J. N. Reynolds, "Crossbar Dial Telephone Switching System," *A.I.E.E. Transactions*, **58,** pp. 179–192, 1939.
2. L. G. Abraham, A. J. Busch and F. F. Shipley, "Crossbar Toll Switching System," *A.I.E.E. Transactions*, **63,** pp. 302–309, 1944.
3. J. J. Pilliod, "Fundamental Plans for Toll Telephone Plant." Page 832 of this issue.
4. W. H. Nunn, "Nationwide Numbering Plan." Page 851 of this issue.
5. F. F. Shipley, "Automatic Toll Switching Systems." Page 860 of this issue.
6. H. S. Osborne, "A General Switching Plan for Telephone Toll Service," *A.I.E.E. Transactions*, **49,** pp. 1549–1557, 1930.
7. F. A. Korn and J. G. Ferguson, "No. 5 Crossbar Dial Telephone Switching System," *A.I.E.E. Transactions*, **69,** Part 1, pp. 244–254, 1950.
8. O. A. Friend, "Automatic Ticketing of Telephone Calls," *A.I.E.E. Transactions*, **63,** pp. 81–88, 1944.
9. John Meszar, "Fundamentals of the Automatic Telephone Message Accounting System," *A.I.E.E. Transactions*, **69,** Part 1, pp. 255–269, 1950.

Fundamental Plans for Toll Telephone Plant

By J. J. PILLIOD

(Manuscript received May 15, 1952)

This paper covers the general switching plan and fundamental plant layout proposed for handling telephone toll messages throughout the United States and Canada using automatic toll switching.

There has been rapid growth in the number of telephones and in the volume of toll traffic, particularly long haul. Toll facilities are provided under fundamental plans, an essential part of which is a toll switching plan for setting up connections quickly between any two telephones. The introduction of mechanical operation and the general improvement in the transmission performance of the communication plant over a period of years make the introduction of certain modifications in the fundamental plans possible and advantageous at this time. The important new features and the service improvements which are provided by the proposed plans are outlined in this paper. The principal types and characteristics of circuit facilities available for use in the intertoll network are also described.

GENERAL ASPECTS OF TOLL SWITCHING PROBLEMS

Switching plans providing for the systematic routing of toll telephone traffic have been employed by the communication industry for many years. These plans have contributed directly to the high quality of long distance telephone service enjoyed by the public in the United States and Canada. This generally excellent service is the result of the cooperative work of many organizations including the Bell Operating Companies, many independent connecting Companies and others in the United States as well as in adjoining countries. The techniques employed today reflect a great amount of research and engineering and improvements in manufacturing skill and in construction, maintenance and operating methods developed over a period of many years.

Throughout the United States and Canada there are approximately 20,000 different places – cities, towns, and villages – that serve as toll connecting points. The telephone offices in each of these places have access through the toll network to practically all of the 50,000,000 telephones in the United States and Canada and also to most of the telephones in the rest of the world. Currently the Bell Operating Companies are handling toll calls at an average rate of over 7,000,000 during a business day. The many millions of different connection possibilities which this number of calls involves require a definite and comprehensive switching plan.

Whenever practicable and economical direct circuits are used to handle toll message traffic between two given points. Much of the traffic in the country is handled this way. However, a substantial volume of business, about 20 per cent, is handled as a matter of economy, by switching toll circuits together. Although the volume of traffic between different points may vary over a wide range, it is nevertheless important that adequate service be provided for all possible connections. For example, there are about 110 circuits from Chicago terminating in the toll office serving Minneapolis and St. Paul. These handle about 5500 calls per day. On the other hand, only a few calls a year may be involved between some point in Western Minnesota and a point in Florida. The switching plan described in this paper is devised for the purpose of efficiently and effectively establishing connections between any two points regardless of their separation and regardless of whether traffic volume be a few calls per year or many calls per hour.

ELEMENTS OF THE PROBLEM

In order to illustrate the problem a specific example may be useful. Fig. 1 is a map of Wisconsin and Minnesota on which nearly 1200 circles indicate points at which exchange facilities may be connected to the toll network. The extent of the coverage in this area is typical of that found throughout the country.

The 150 odd larger circles represent existing offices known as "toll centers" – that is, places where operators record toll calls and perform other operations necessary to establish toll connections. These places have switching arrangements of various types depending on how they fit into the switching plan. Some may operate as control switching points in the nationwide plan as described later.

More than 1,000 smaller circles on the map represent "tributaries" – that is, towns where little or no toll operating is done. Toll connections to and from these points are completed at the toll centers which in general do the toll operating required.

In the United States and Canada as a whole, there are approximately 2,600 toll centers. The remainder of the toll connecting points—about 17,500—are tributaries.

Fig. 2 gives an idea of the variety and complexity of the network of circuit groups required to interconnect the toll centers in one area. Here each line represents a group of circuits, known as "intertoll trunks," between two toll centers. Each group may contain anywhere from one to several dozen trunks. The location of the lines on the map is unrelated to the geographical routing of the trunks, and only a part of the circuit groups are shown. To get a complete picture one should visualize that a cluster of relatively short circuit groups radiates from each toll center to its tributaries, of which there may be up to 15 or more.

Physically, the plant consists of a network of open wire lines, cables and radio systems. On these, voice frequency or carrier operation is employed in each section as required to provide the necessary intertoll trunks. The routes of the lines in Minnesota and Wisconsin are shown by Fig. 3. In this area there are no radio routes carrying telephone circuits, but a radio system between Chicago and Minneapolis is in the planning stage.

Areas like Wisconsin and Minnesota must, of course, be connected together, and Fig. 4 shows the major Bell System toll routes that accomplish this. On a map of this kind it is not possible to include anything like the detail shown in Fig. 3. One must visualize, therefore, that each state contains a network of routes generally comparable to those shown for Wisconsin and Minnesota.

This then represents the interconnection problem to be met by an orderly switching plan that will provide efficient, reliable and fast toll telephone service between any two points.

EARLIER TOLL SWITCHING PLANS

Very early in the telephone industry it became evident that: (1) There must be a plan for connecting circuits together. (2) Switching centers with suitable equipment must be established in accordance with this plan. (3) Trunks must be provided in adequate numbers to connect every place to one or more switching centers and to interconnect the switching centers. (4) All this must be done in a way that makes it possible to provide good service at reasonable cost.

As time went on, early plans crystallized into what became known as the General Toll Switching Plan. A paper presented at the summer convention of the A.I.E.E. in Toronto in 1930 by Dr. H. S. Osborne outlined the principles of this comprehensive plan for handling telephone toll traffic in the United States and Eastern Canada.[1] It involved two classes of major switching centers – Regional Centers and Primary Outlets – and some classes of less important centers. It also set up methods of designing toll trunks to give adequate transmission efficiency on all possible toll connections. In use for the last two decades this basic plan has been of great value in accommodating the tremendous growth of telephone toll business during this period.

SWITCHING PLAN FOR NATIONWIDE TOLL DIALING

The earlier general switching plan was based on manual switching and on a toll plant made up for the most part, of voice frequency circuits. The probability of operating irregularities and delays increases with the number of manual switches in tandem. Likewise, the transmission problem of operating many voice frequency trunks in tandem was so formidable that the number of intertoll trunks in tandem had to be limited to five. In practice, switching was avoided where practicable and economical.

Impact of Mechanization and Improved Transmission Facilities

On the other hand, mechanical switching is very fast and is designed to be practically free of operating irregularities. Delays can be minimized by fast switching to alternate routes. Also, in the last two decades the use of carrier has grown from a relatively minor place in the toll plant to the point where it is now commonplace.[2] Carrier provides superior transmission performance. Limitations on switching are thus greatly reduced and economies are achieved under many conditions.

In addition, mechanization of local switching systems has proceeded rapidly. With mechanized toll switching, it is becoming possible to establish many toll connections with only a single toll operator and in some cases by customer dialing, without the assistance of any operator.[3, 4]

Along with these developments has come tremendous growth in traffic. Since 1930 toll messages in the Bell Operating Companies and the Bell Telephone Company of Canada have more than trebled, growing from an annual volume of about 650 million to about 2 billion. Intertoll trunks over 25 miles in length have increased in number from about 28,000 to about 100,000. This continuing growth in traffic volume has required a large scale development of plant facilities and has permitted a more extensive use of carrier than would have been practicable with a slower rate of growth.

Consideration of these factors which offer an opportunity to improve service has led to the gradual reorientation of the fundamental plans for the intertoll trunk plant which is now under way.

The New General Toll Switching Plan

Mechanization of switching and the use of improved transmission instrumentalities permits the design of the switching plant to be controlled primarily by the balance between the costs of transmission facilities and of switching facilities.

The new general toll switching plan contemplates as many as eight intertoll trunks in tandem on the most complex connections to be established. These eight trunks can be interconnected at switching points as described later. The plan further contemplates that wherever possible, the traffic will by-pass intermediate switching points. The number of switches that can be avoided depends on the volume of traffic between the two points concerned and on the traffic load at the time the connection is established.

The proposed plan provides a systematic grouping of switching points. Under this arrangement, each ordinary Toll Center (TC) serves a cluster of nearby tributary points and has trunks to a "home" Primary Outlet (PO) which serves a cluster of toll centers. In some cases it appears practicable to utilize a simplified switching system at a PO, and in order to distinguish this type of center it has been designated a Tandem Outlet (TO). In turn, each PO or TO has trunks to a "home" Sectional Center (SC) which serves a section of the country varying in size from part of a state to all of several states depending on the density of the population. Similarly, The United States and Canada are divided into nine regions, each having a Regional Center (RC) serving as a central switching point for all sectional centers in the region. One of these RC's (St. Louis) is termed the National Center (NC). All of the higher orders of switching centers also act in the capacity of each of the lower centers. For example, any specific SC also acts as a PO and as a TC.

This arrangement is illustrated in Fig. 5, which covers approximately the same area as Fig. 1, portraying the toll connecting routes. Hibbing, Minnesota, is shown as a representative toll center with the tributaries it serves. It is in the service area of the Duluth Tandem Outlet, the approximate boundaries of which are indicated. Duluth lies in the Minneapolis "section," which includes a large portion of Minnesota, and is

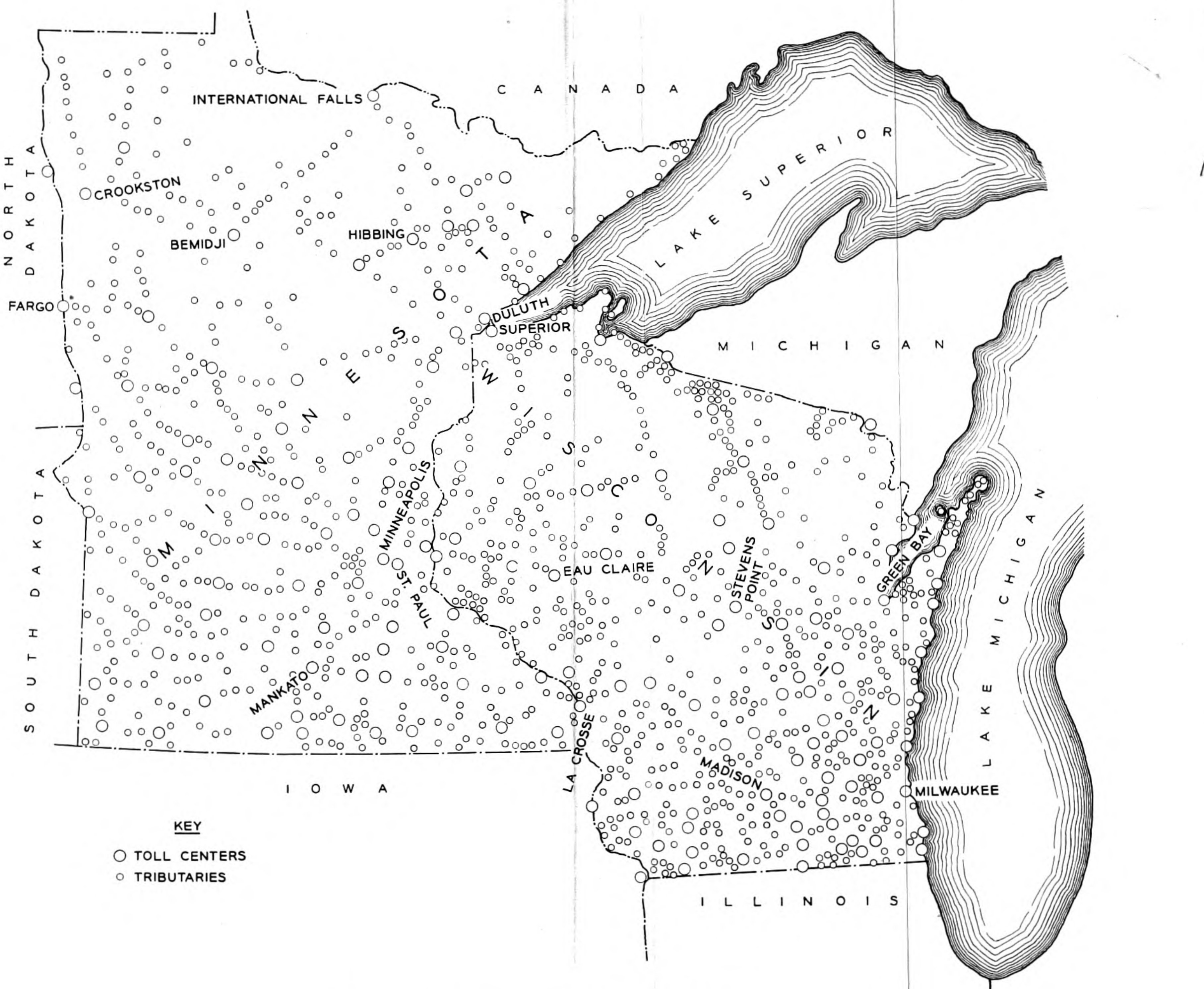


Fig. 1—Toll centers and tributaries in Minnesota and Wisconsin.

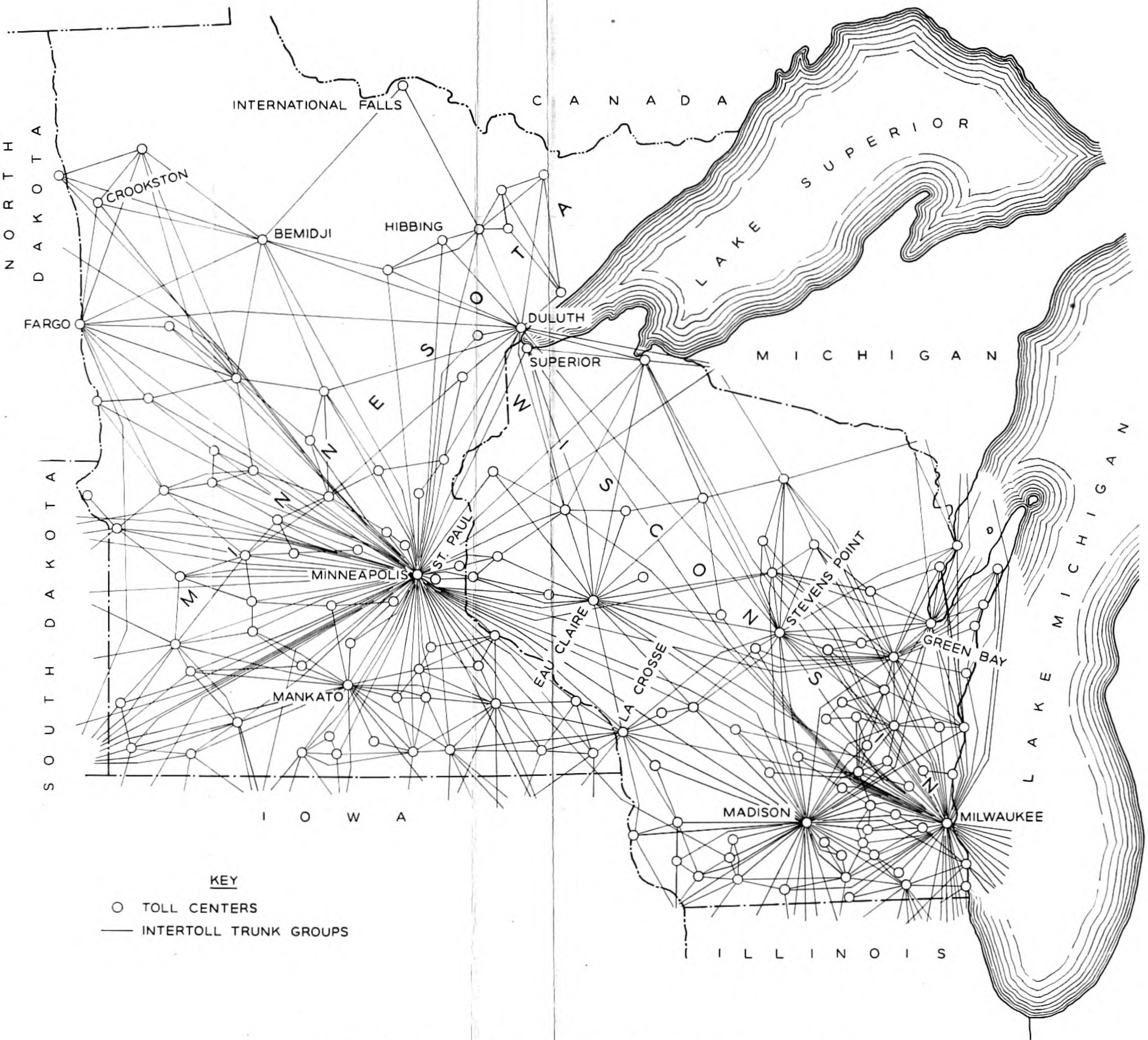


Fig. 2—Principal intertoll trunk groups in Minnesota and Wisconsin.

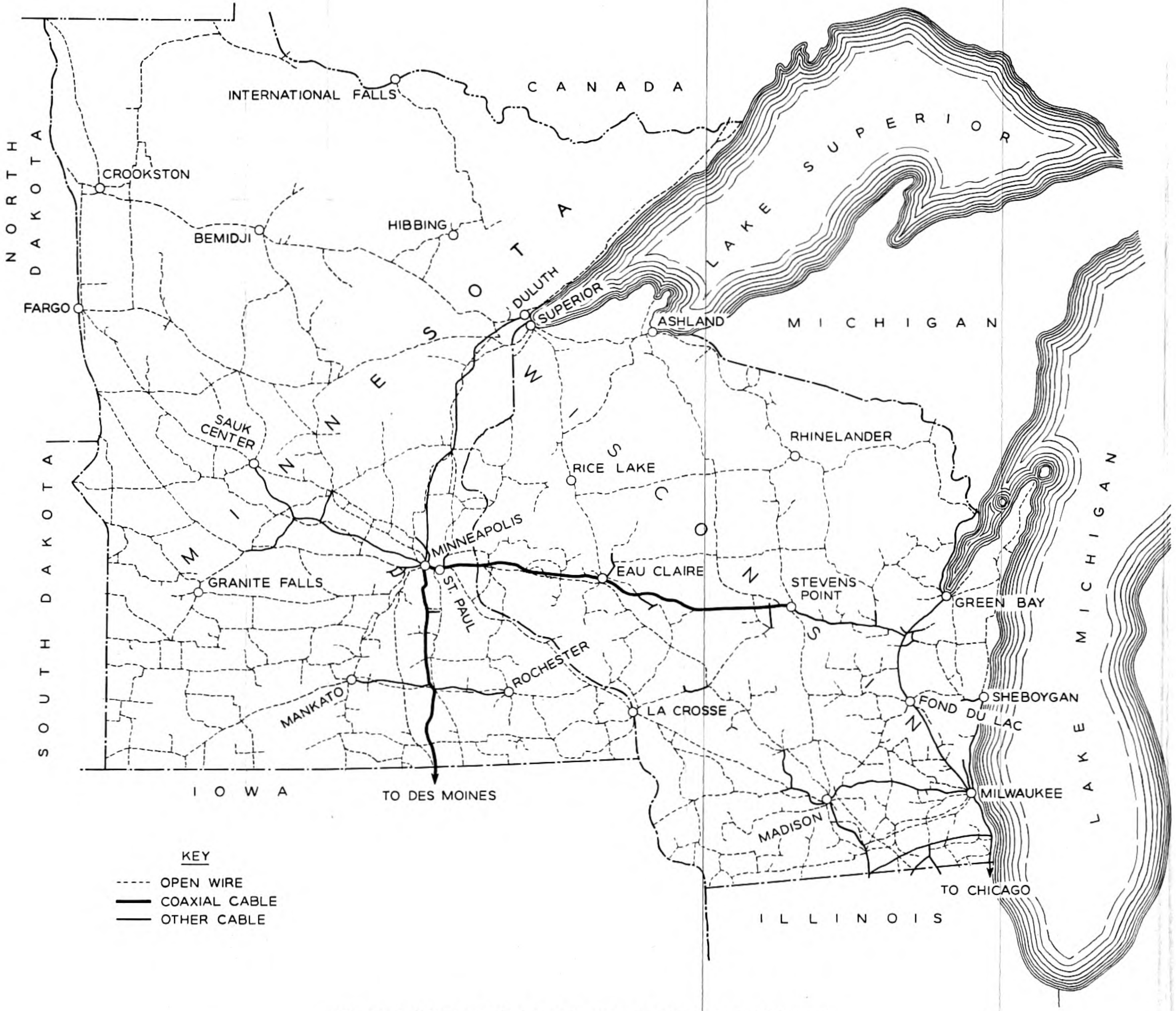


Fig. 3—Bell System telephone toll routes in Minnesota and Wisconsin.

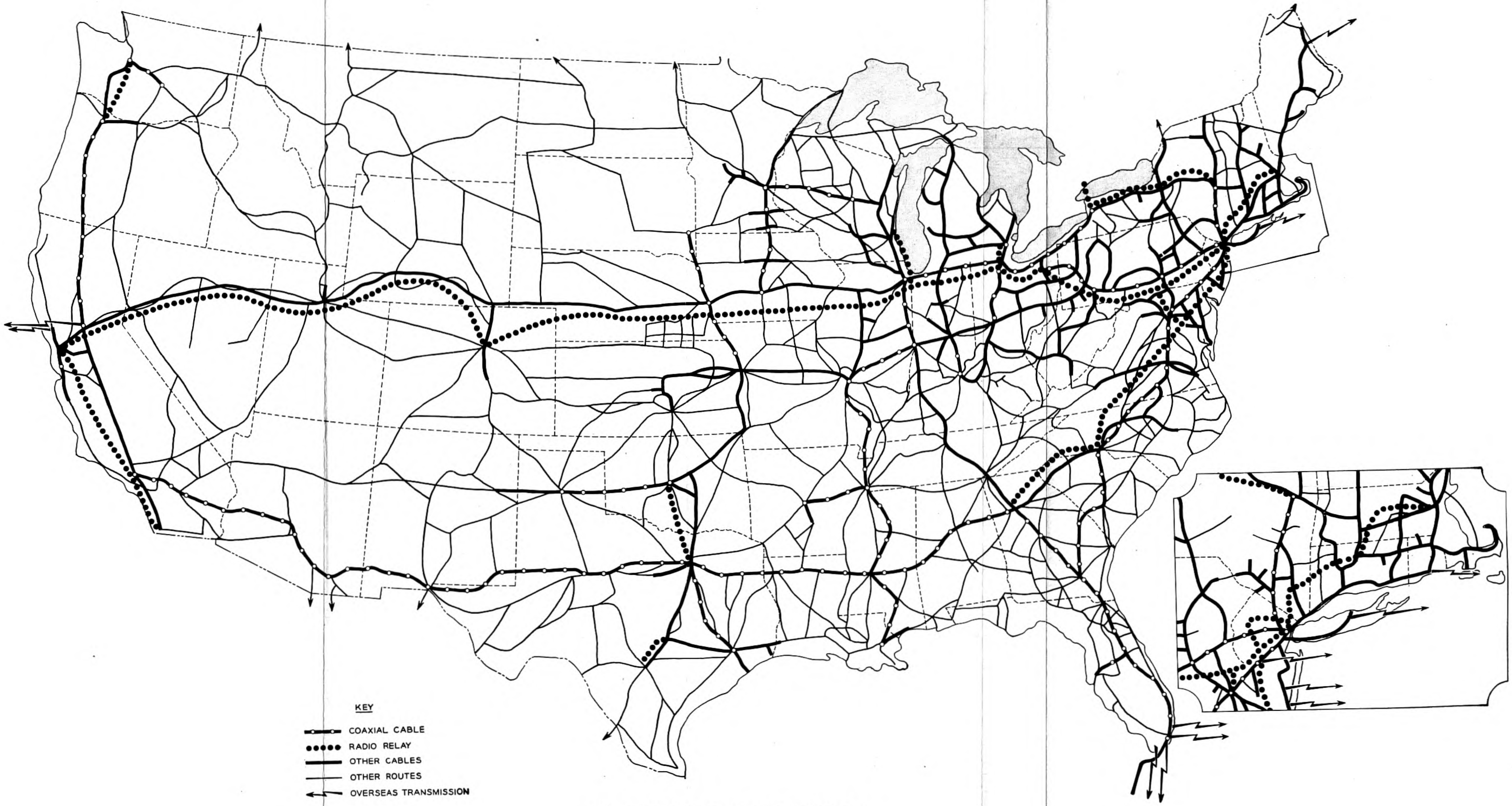


Fig. 4—Principal toll routes of the Bell System.

in turn one segment of the Chicago "region" which serves a somewhat larger area than shown by Fig. 5.

Under this arrangement, toll calls between two tributaries in the Hibbing toll center area can be completed by switching at the toll center. In a similar manner, any two points within the Duluth tandem outlet area can be served by switching at Duluth. The same treatment also applies for connections between any two points in the same sectional center area or in the same regional center area. For example, a connection from Hibbing to any point within the Chicago region (which involves more than six states as shown in Fig. 7) requires no more intertoll links than Hibbing to Duluth, Duluth to Minneapolis and Minneapolis to Chicago, and a corresponding number of links on through to another sectional center, and primary or tandem outlet to the toll center destination. Circuits between the toll center and tributaries are not referred

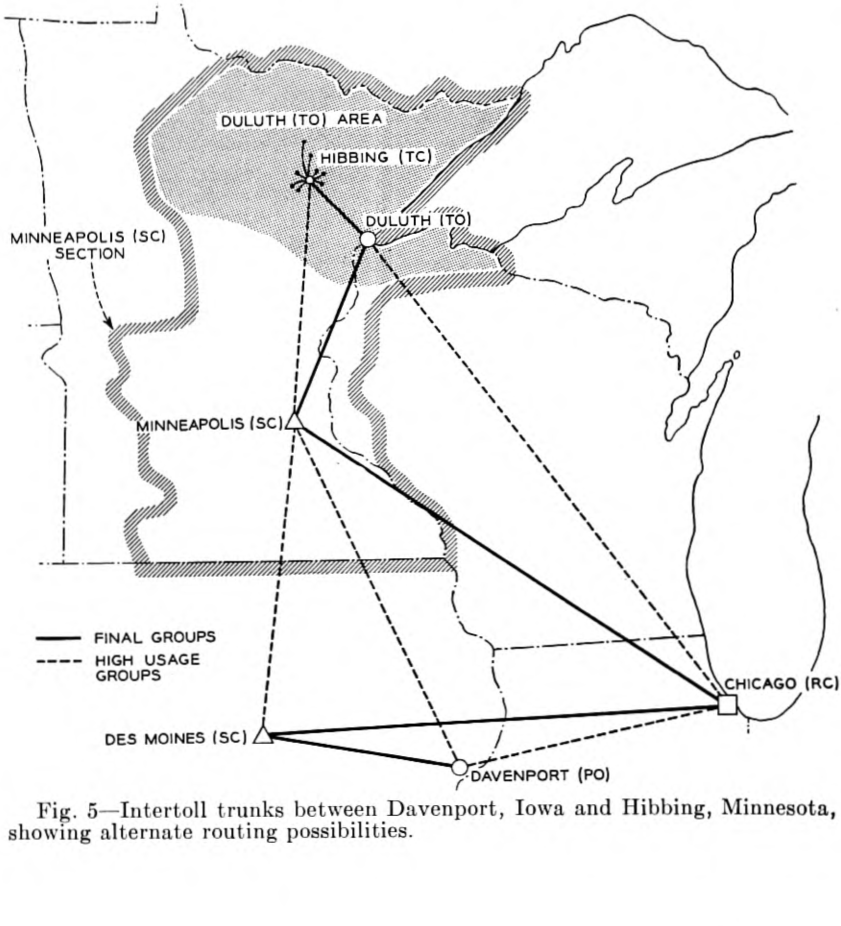


Fig. 5—Intertoll trunks between Davenport, Iowa and Hibbing, Minnesota, showing alternate routing possibilities.

to as intertoll trunks or lines but are classed as toll connecting trunks.

Where the volume of traffic warrants, direct circuits may be provided to by-pass the intermediate switching points included in the preceding example. Once such direct circuit groups have been established, it is economical and advantageous from a switching standpoint to take advantage of their existence, using routes that involve a minimum number of switches. The basic routing plan is used when the more direct circuit combinations are busy.

These routing arrangements contemplate the application of "high usage" and "final" trunk groups as an integral part of the plan. The "high usage" groups are direct groups which by-pass the higher order switching points wherever the routing of the call permits. These "high usage" groups can be engineered to carry high loads per circuit, with an adequate number of circuits in the "final" groups to take care of prac-

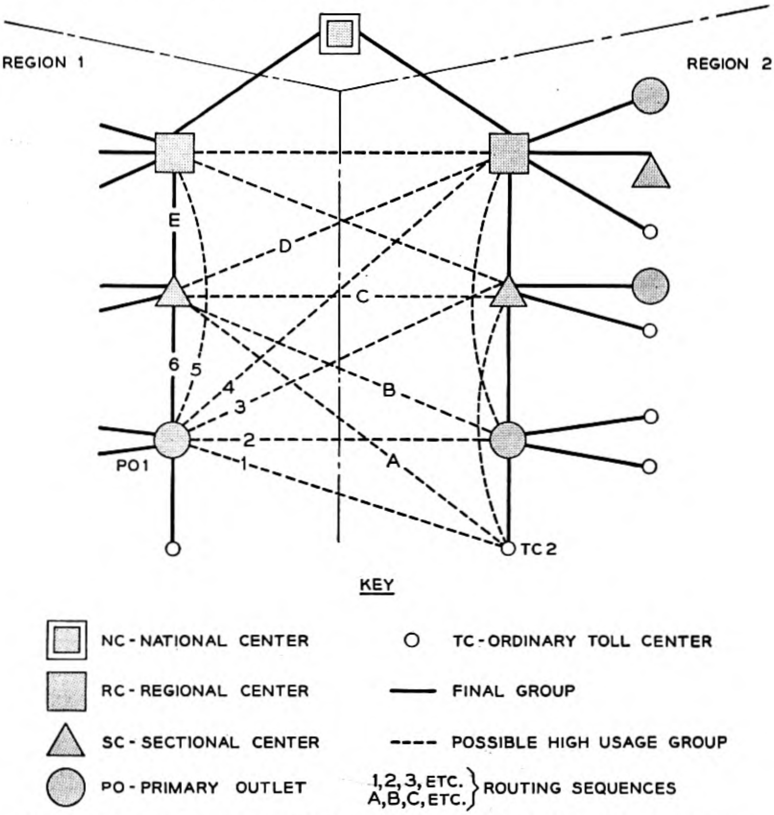


Fig. 6—Illustration of intertoll routing pattern between two regions.

Fig. 7—Tentative locations of control switching points in United States and Canada.

tically all overflows from the high usage groups during the heavy traffic periods. The "high usage" and "final" groups which could be used for routing calls between Hibbing, Minnesota and Davenport, Iowa are shown by Fig. 5.

Generalization of the Toll Switching Plan

The generalization of the arrangements discussed for the Chicago region is illustrated in Fig. 6. This shows diagrammatically all types of switching points in two regions and also indicates the relative position occupied by the National Center in the switching plan. On this chart, the solid lines represent the "final groups" of trunks, and the dotted lines represent "high usage" trunks. Examination of this chart will indicate that the mechanical switching system need perform only relatively simple toll switching operations at the toll centers. At other points the system must attempt to complete the call over the most favorable routes, in planned sequence, until the "final" route is selected.

For example, from a given primary outlet such as PO1 on a call destined for a toll center in the other region such as TC2, the switching equipment would attempt to complete the call, in sequence over the routes marked 1 to 6.

Should Route 6, which is the "final" route, be selected because all of the trunks in the "high usage" groups marked 1 to 5 were busy at the time, the switching equipment at the SC would in turn try routes marked A, B, C, etc., in attempting to complete the call. A fairly complete pattern of circuit groups is indicated in this illustration. Depending on the relative locations of the points concerned and the traffic load requirements, certain of the "high usage" groups shown may not exist. It is expected, however, that most TC's will have high usage groups to points other than their "home" PO's. Also each PO can be expected to have high usage groups to sectional centers other than its "home" SC. All regional centers will be interconnected with direct trunks, regardless of geographical location.

Control Switching Points

Because of rapid and complex switching operations required by the automatic equipment at PO's and higher order switching points, (SC's, RC's and the NC) these switching centers are called Control Switching Points (CSP's).

As covered by a companion paper,[5] the switching equipment required at the CSP's is quite complex. This equipment must have a high degree of built-in capability to perform quickly the circuit selection work associated with the alternate routing features of the switching plan. In addition, to help provide the transmission margins needed for satisfactory operation of the plan as contemplated, it must be arranged to connect circuits on a four-wire basis rather than on a two-wire basis, the latter being the arrangement used at most toll centers. The switching equipment at a CSP must not only provide for connecting one toll circuit to another; it must also perform the very important function of tying the toll networks which serve limited local areas together so that collectively they work as a smoothly functioning nationwide system. This becomes practicable when there is coordination between the design of the individual limited networks and the design of the overall system.

The location of control switching points indicated by the nationwide plan is shown in Fig. 7. This also indicates the home switching center of higher order associated with each switching point. As the number of CSP's increases, the cost of the toll circuit plant decreases because each CSP can then be located closer to the cluster of ordinary toll centers which it serves. However, because of the cost of the CSP equipment, it is necessary to weigh the cost of circuit facilities with the equipment costs in a way that will result in the minimum overall cost. Certain of the smaller Primary Outlets are being studied with the view of reclassifying them as Tandem Outlets (TO's). A Tandem Outlet occupies the same relative position in the switching plan as a Primary Outlet but is not a control switching point. The switching equipment employed is less complex than that used at control switching points and therefore provides for only limited alternate routing and does not have all the advantages of four-wire transmission.

Effects of Customer and Operator Toll Dialing

Customer dialing of short-haul toll calls has been in use, particularly in metropolitan areas, for some years. A trial of long-haul customer dialing over the intertoll trunk network and through the switching equipment provided for operator toll dialing was instituted at Englewood, New Jersey, in the Fall of 1951. The local equipment includes automatic message accounting and permits Englewood customers to dial directly to about eleven million telephones in ten metropolitan areas across the country. A trial installation of customer toll dialing, utilizing automatic message accounting equipment on a centralized basis rather than at each local office, is planned for Washington, D. C., in the Fall of 1953. Initially customers will dial toll calls within the Washington metropolitan area and to such points as Baltimore and Annapolis. The favorable results and general acceptance of the trial at Englewood indicate extensive application of customer dialing of toll calls as conditions warrant.

The general introduction of customer toll dialing as this becomes desirable will affect the number and location of ordinary toll centers since calls handled by operators may be limited to assistance calls and to person-to-person, collect and others which cannot be customer dialed. Indications are that toll operation for a number of smaller centers can be combined as the local service is converted to dial operation with operator toll dialing.

Studies now in progress indicate that the number of toll centers may be reduced by one half or more over a period of years in many areas.

Reactions on Toll Plant Layout

The expanded general toll switching plan for nationwide dialing contemplates a degree of alternate routing far in excess of that used with the former switching plan designed for manual operation. This change along with the reduction in toll centers will have a marked effect on the normal flow of many traffic items through the intertoll network. As a result the arrangement of the present intertoll trunks will be significantly modified both in number, routing and terminating points. It is necessary to take these facts into account in engineering toll plant additions so that they will lead toward an advantageous layout for future nationwide dialing as well as meet the needs of the more immediate future. Fortunately, the effect is in the direction of greater concentration of circuits in main routes so that with the new cable and radio facilities available, over-all economy and better service should result.

TYPES OF TRANSMISSION FACILITIES USED AND INCLUDED IN SWITCHING PLAN

The domestic toll network is an outgrowth of the demands of the business and the advance in communication technique over many years. At present, about 100,000 intertoll trunks over twenty-five miles in length and many thousand shorter toll trunks are in service throughout the country. They are provided generally by voice frequency or carrier frequency facilities. The choice of transmission facility on a given route is dependent on a number of factors, such as cost, length of haul, number of trunks in the cross-section, numbers of trunks to be terminated at intermediate points, the types of terrain to be transversed, storm and

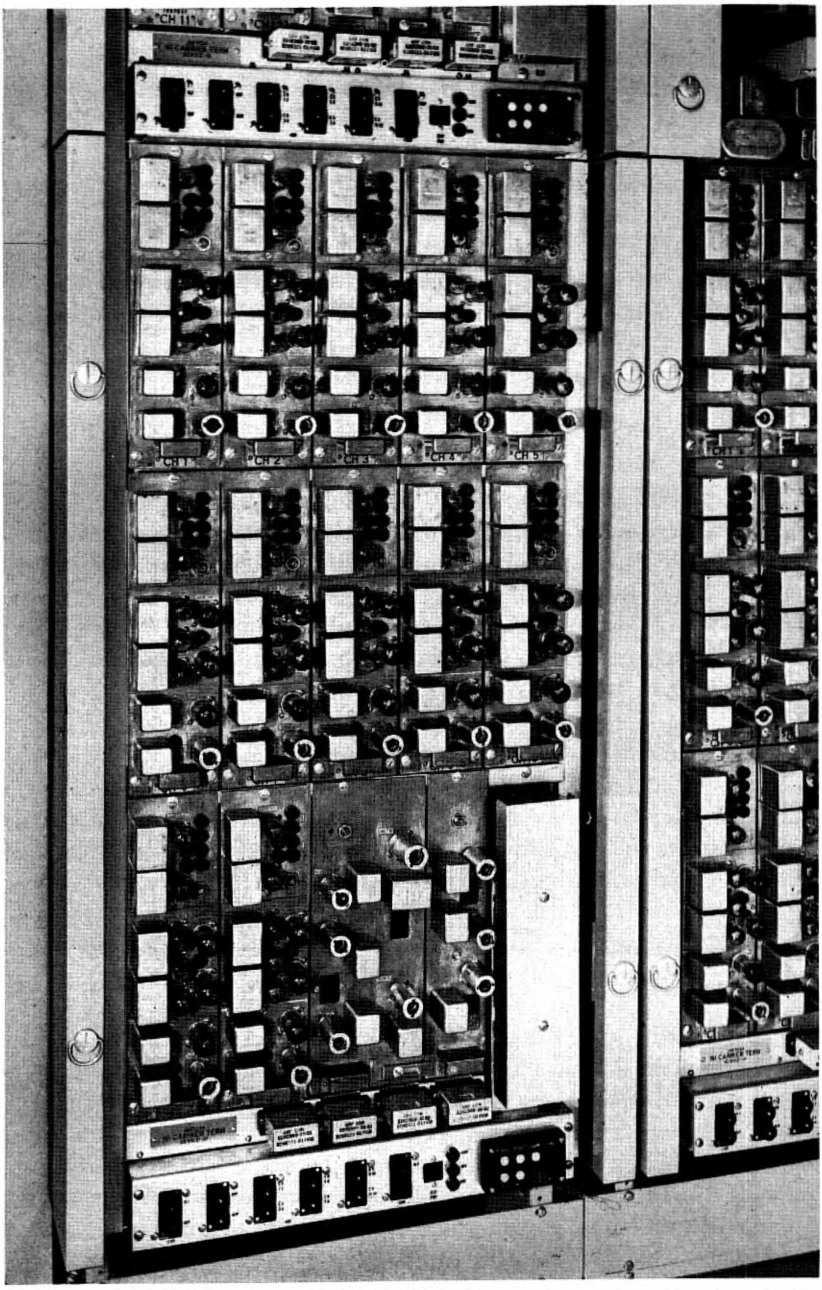


Fig. 8—Terminal equipment of type-N1 cable carrier system. Provides twelve message channels with self contained signaling equipment over two pairs of cable conductors in same sheath.

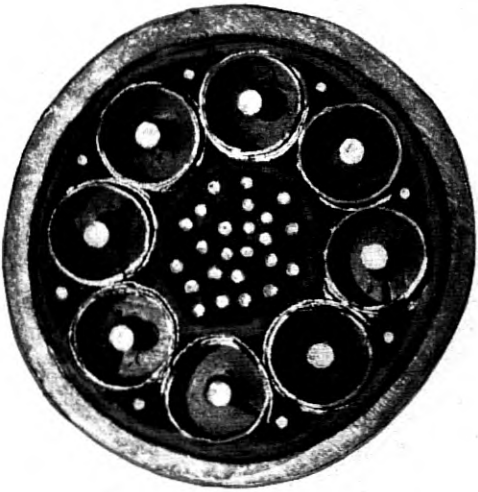


Fig. 9—Coaxial Cable. Cross section of cable containing four pairs of coaxials. Each pair can accommodate one two-way coaxial carrier system.

other conditions affecting service continuity and the transmission requirements of the circuits to be provided.

Voice frequency facilities equipped with repeaters as required are used on both open wire lines and cables. At voice frequencies it is customary to derive three trunks known as a phantom group, from two pairs of open wires or from one "quad" (two pairs) of loaded cable conductors. In general the use of voice frequency facilities is now limited to shorter circuits.

Considerations of economy and service improvement led to the introduction of carrier operation into all types of toll plant as rapidly as the state of the art permitted. This directly affects the toll switching plan from the standpoint of routing and location of switching centers.

At present, carrier systems use four broad categories of facilities: open wire, conventional paired or quadded cables, coaxial cable and radio.

Several types of open wire carrier systems permitting from one to fifteen telephone channels above the frequency band of the voice channel are now in use. In general these systems are used where trunk cross-sections are relatively small and where the terrain and weather conditions make open wire lines economical.

Cable carrier systems at present permit the operation of up to twelve telephone channels on two pairs of cable conductors. These conductors may be in one cable or divided between two separate cables, depending

FIG. 10—Microwave radio relay tower at Cotoctin Mountain, Maryland, on a New York–Washington radio route. There are 300 message circuits in service with more planned.

on the type of carrier system (Fig. 8). Coaxial cable transmission systems currently provide up to 600 telephone channels per pair of coaxials (Fig. 9). A new coaxial system, under development, is expected to produce about 1,800 telephone channels per pair of coaxials.

Most of the applications of radio for toll telephone service now contemplated, involve the use of point-to-point microwave systems. By employing channeling equipment at the terminals of these systems similar to that used for the present coaxial system, each pair of radio channels may provide up to 600 telephone channels. Several pairs of such radio channels may be operated through the same antennas (Fig. 10).

Radio systems are also useful in some cases where the number of toll trunks required is moderate, where diversity is desired or where water or other natural barriers make the provision of wire circuits difficult or impracticable.

The type of facility to be used on a particular route is sometimes affected by requirements for other services such as teletypewriter, television network facilities, program facilities, private lines and other factors.

Trend to Carrier Type Facilities and Advantages to Toll Switching Plan

About 70 per cent of the long haul toll message mileage in Bell Operating Companies is provided on carrier type facilities as contrasted with 7 per cent in 1930 (Fig. 11).

From the transmission standpoint, carrier facilities offer marked ad-

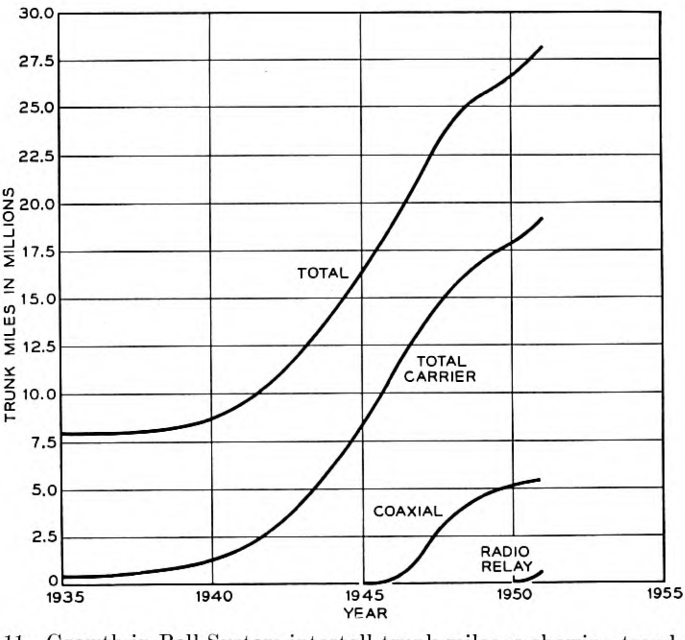


Fig. 11—Growth in Bell System intertoll trunk mileage showing trend toward more extensive use of carrier type facilities.

Fig. 12—Toll switchboard position with key set used for toll dialing.

vantages. They are inherently of the "four-wire" type which minimizes the number of possible singing and echo paths on a circuit. Also, the speeds of propagation over carrier systems are generally higher than over voice frequency systems thereby further minimizing the echo problem. These features are of great advantage in reducing limitations on circuit design and layouts of the general toll switching plan.

Signaling Systems

In addition to the ability to carry messages, intertoll trunks must be provided with suitable signaling facilities.[6, 7] These must provide a means of: first, attracting the attention of the distant point, either an operator or automatic equipment, to the fact that a connection is to be established; and second; in the case of dial operation, transmitting coded information in the form of pulses for establishing the connection; and third, transmitting a general class of supervisory signals including connect and disconnect signals, on and off switch hook signals, recall signals and busy signals which are essential to the efficient operation of the switching plant. The circuit design contemplated in the overall plan must take into account this requirement for transmitting signals as well as speech, to obtain accuracy and speed in setting up and taking down connections.

TRANSMISSION DESIGN ASPECTS OF CIRCUITS FOR NATIONWIDE TOLL DIALING

The more extensive use of alternate routing together with the increase in maximum possible number of trunks in tandem associated with nationwide toll dialing, tends to increase the problems of assuring adequate transmission of speech and signals on all possible connections. On the other hand, the use of four-wire switching at important points and the definiteness of the routing patterns permit more effective use of the available facilities and thus tend to simplify the problem. Extensive studies indicate that on the whole, the new toll switching plan will make feasible still further improvements in transmission. This is, of course, a desirable objective.

Transmission Design of Trunks

With dial operation, the number of trunks in tandem in a given toll connection may vary on successive calls. To avoid undesirable transmission contrasts and other adverse effects, it is important that every trunk be designed to operate as closely as possible to the theoretically correct transmission loss. The problem is complicated by the fact that the extent to which the echo, noise and crosstalk will limit the performance of an individual link is not directly proportional to the length of the circuit. In fact, the minimum loss at which a particular circuit used singly or in various built-up combinations can theoretically be operated depends on the number, length and characteristics of the other circuits connected in tandem with it. Arrangements for precisely adjusting the loss in the individual trunks for each call would be complicated. Adequate performance can be achieved however by compromise methods which provide for automatic adjustments in the loss of each trunk in accordance with the following:

1. When a trunk is switched to other intertoll trunks at both ends it is operated at the minimum loss practicable. This loss is known as "via net loss." (VNL)
2. When the trunk is switched to another intertoll trunk at one end only, the loss is increased two db.
3. When the trunk is not switched to another intertoll trunk at either

end a further loss of two db is added. This loss which is four db greater than the via net loss is known as "terminal net loss." (TNL)

The data and methods used in the derivation of the via net loss are rather complex and not within the scope of this paper.

Assignment of Facilities Among Trunks

The definite routing patterns established for the toll machine switching operation impose more severe transmission conditions on certain classes of circuits than on others. For example, a trunk in a "final" group between a TC and a PO can become involved in an eight-link connection, whereas a trunk in a "high usage" group, say, between a PO and another PO will not be involved in more than a three-link connection.

This creates a need and provides an opportunity for allocation of the available facilities among the various trunk groups in a way that will provide the best overall service. For example, to the extent practicable it is desirable to assign carrier grade facilities to trunks in "final" groups that may be involved in connections with the maximum number of links. Facilities with less favorable transmission characteristics may then be reserved for trunks in groups that are used for connections involving fewer links.

TRANSMISSION PERFORMANCE

Table I shows the approximate range of transmission losses between toll centers under the manual plan compared to ranges that appear practicable under the proposed fundamental plan, which, of course, permits more links in tandem.

Trunk Transmission Stability

It is as important that the transmission loss of a trunk used in the contemplated toll dialing network be maintained at or close to its assigned value at all times as that the assigned value be right. On multi-switched connections even a relatively small consistent excess or deficiency in the loss in the individual trunks can accumulate to overall excesses or deficiencies in loss large enough to cause difficulty – by making it hard for people to hear if the attenuation becomes too great or by creating excessive echo, crosstalk or noise if the loss becomes appreciably less than normal.

This subject has been extensively studied for the past several years and it appears that some changes in practices and the introduction of

TABLE I—APPROXIMATE RANGE OF LOSSES BETWEEN TOLL CENTERS IN DB

No. of Links in Intertoll Connection	Manual Plan	Proposed Plan
1	4–12	4–8
2	8–14	5–12
5	9–20	6–13
8	—	7–13

new methods of measuring results will lead to marked improvements. It is of some interest that one of the major factors in securing improvement appears to be the application of a statistical method of evaluating performance along somewhat the same lines as the "quality control" methods used in other fields of industry.

Since, with operator toll dialing only one operator is involved in many connections and with customer toll dialing there is no operator on the connection it is extremely important that everything be right. This is typical of the requirements of any large scale "push button" operation (Fig. 12).

CONCLUSION

The fundamental plans proposed for Telephone Toll Switching provide a basis for the progressive mechanization of toll service. The installation of suitable switching mechanisms at Control Switching Points and the provision of toll trunks utilizing the new instrumentalities will implement the toll switching plan. The plan is sufficiently flexible to adjust for changes in the telephone art as they develop. Also, the plan can fit in with the requirements of those Companies whose plants connect with the Bell operating network should they desire to arrange for operator or customer toll dialing.[8]

Average speed of service will be improved. The flexibility in plant design inherent in the new toll switching plan will increase service security and improve the utilization of the entire toll plant. In addition, adequate provision is made for the progressive introduction of customer toll dialing as this becomes practicable and desirable.

BIBLIOGRAPHY

1. H. S. Osborne, "The General Switching Plan for Telephone Toll Service," *A.I.E.E. Transactions*, **49,** pp. 1549–1557, 1930.
2. C. M. Mapes, "Carrier is King," *Bell Tel. Mag.*, **28,** pp. 191–203, Winter 1949–50.

3. J. J. Pilliod and H. L. Ryan, "Operator Toll Dialing," *Bell Tel. Mag.*, **24,** pp. 101–115, Summer 1945.
4. E. W. Baker, "Toll Dialing is Expanding Throughout the Nation," *Bell Tel. Mag.*, **30,** pp. 253–264, Winter 1951–52.
5. F. F. Shipley, "Automatic Toll Switching Systems." Page 860 of this issue.
6. C. A. Dahlbom, A. W. Horton, Jr. and D. L. Moody, "Application of Multifrequency Pulsing in Switching," *A.I.E.E. Transactions*, **68,** pp. 392–396, 1949.
7. N. A. Newell and A. Weaver, "Single-frequency Signaling System for Supervision and Dialing over Long Distance Telephone Trunks," *A.I.E.E. Transactions*, **70,** (7 pages), 1951.
8. Articles prepared by American Telephone and Telegraph Company for information of Dial Interexchange Committee of the United States Independent Telephone Association. Published in *Telephony* on dates indicated.
   a. Nationwide Operator Toll Dialing, January 12, 1946.
   b. New Toll Switching Plan for Nationwide Dialing, May 10 and 17, 1947.
   c. Nationwide Toll Dialing—Use of Tandem CDO's, July 3, 1948.

Nationwide Numbering Plan

By W. H. NUNN

(Manuscript received May 15, 1952)

In telephone language a numbering plan gives each telephone in a city, a town, or a geographical area an identity or designation different from that given any other telephone in the same area. There is a wide variation in the types of numbering arrangements in use today in the Bell System, and this paper gives the reasons for this diversity, and examples of the various numbering plans now in use. With the introduction of modern toll switching facilities and the extension of toll dialing to nationwide scope, it was realized that an improvement in the method of dialing toll calls to distant cities was essential in order to realize the maximum speed and accuracy inherent in toll dialing. A nationwide numbering plan covering the United States and Canada has been designed. Each of the more than 20,000 central offices in the two countries are to be given a distinctive designation which identifies that particular office. This designation is to consist of a regional or area code and a central office code The new switching equipment for the key points in the toll network is being designed so that any toll operator, wherever located, will use the same designation or code for reaching a given office. The combination involved in laying out these areas and the composition of the area codes are presented. A total of 152 codes are available of which approximately 90 are assigned to the present numbering plan areas. Ultimately each central office will be given a type of number consisting of an office name and five numerical digits, such as LOcust 4–5678, in which the first two letters of the office name become the two letters of the central office code. The entire program will take a considerable number of years to realize, but is one which must be accomplished in order to achieve the best results in operator toll dialing and the ultimate goal of nationwide customer toll dialing.

In telephone language a numbering plan is exactly what the name implies, a plan or system of giving each telephone in a city, a town or any geographical area an identity or designation which is different from that given every other telephone in this same area. This designation is the telephone number; it appears in the directory and in most cities on the telephone instrument itself. It is the address of the telephone in the telephone network. Just as it is essential for efficient postal and delivery service to have streets and house numbers clearly marked, it is important for good telephone service that the telephone numbering plan be such that it will be used with convenience and accuracy by the telephone customer.

A telephone number is comprised of two elements, a designation for the central office to which the telephone is connected and a number within the central office which identifies one particular telephone from all others served by that office. If there is only one central office in the city or town, the office designation is frequently omitted. A dial office is designed to serve up to 10,000 numbers with a limitation of four digits. Typical numbers are therefore MAin 2–1234, ADams–2345, 5–6789 and 3456, the office designations being MAin 2, ADams and 5 with the last four digits in all cases representing the number within the central office.

There is a wide variation in the types of numbering arrangements in use today in the Bell System. This diversity arises from the fact that telephone communities vary greatly with respect to the number of telephones served, ranging all the way from New York City with its more than three million telephones and three hundred central offices to small villages and rural communities with perhaps a few score or a few hundred telephones.

In the 1920's when the Bell System embarked upon its program of converting local offices to dial operation each exchange or city was in general an entity unto itself. Customers dialed local calls within their own city but all calls involving a toll or multi-unit charge required handling by operators for timing and ticketing. There was no advantage, therefore, in making a numbering plan for a given city more comprehensive than required to serve the telephones and central offices in that city with a suitable allowance for the expected growth. Thus there were formed a multitude of local dial communities, large and small, within which customers could dial their own calls and connections between these telephone communities were established by operators.

Over the years these basic numbering plans which were originally established for local dialing have in many of the cities proved inadequate to furnish as many office codes as later events have shown are required. This is due to a variety of causes. The station growth in many places has outstripped all expectations and the number of central offices required to serve this unprecedented demand for service consume many more office codes than the original plans provided for.

In many places local service areas were changed so that customers could call into contiguous exchanges at local rates. To enable customers to dial into these neary-by places the original numbering plans required expansion to include this increased number of offices. In addition, with the advance in the telephone art many cities introduced equipment for automatic charging on multi-unit and short haul toll calls so that customers could dial such calls directly instead of placing them with an operator for completion. In order to enable customers to dial these calls, it was necessary to expand the original city numbering plans to encompass wider and wider geographical areas.

In expanding the various types of numbering plans to serve a larger number of central offices than were originally anticipated, various expedients were resorted to. In the largest cities having three-letter office codes a numeral was substituted for the third letter thus very materially increasing the code capacity from about 325 to about 500 and making it possible to form a number of codes using the same office name. The name CANal for example, instead of serving but one office may serve a number of offices, CAnal 2, CAnal 3, CAnal 4, etc. In the medium size cities having two-letter codes, expansion meant adding a digit to the code to all or in some cases to only a part of the offices in the city.

The five-digit places were usually expanded by adding a digit to some of the numbers so that some of the telephones had five digits and others six digits in their numbers.

As a result of choosing originally a numbering plan which at the time seemed adequate and most suitable for the city involved and in many cases being forced to expand to meet changing needs, we now have in the Bell System a considerable variety of different numbering plans. These are given in Table I. The numbering plans given are all adequate to serve the present local dialing needs for the cities in which they appear.

Having reviewed the numbering plan situation as it exists today in the various cities and towns, let us turn to the problem of handling toll calls. Under ringdown operation there is an operator at the outward toll center where the call originates and another operator at the terminating or inward toll center. On built-up toll connections there are additional operators at each intermediate toll switching point. The inward toll operators, who are familiar with the numbering plans in the offices served by their particular toll center, can be relied upon to connect to the desired station even though there is uncertainty on the part of the calling customer or the outward toll operator regarding the precise pronunciation or spelling of the name of the called office or the particular form of numbering system used at the called city.

Under operator toll dialing the inward operator is replaced by dial switching equipment under the control of the outward operator; hence the outward operator has no one to rely upon but herself in completing a toll connection to a distant city. With the present method the operator dials a code for each circuit group in the connection followed by the number of the called party which may consist of any number of digits from three to seven. The operator must refer to her position bulletin or to a routing operator for the correct circuit group codes unless she happens to remember them. Where the office to be reached has central office names, the operator must rely on routing information to determine how many letters of the name are to be dialed. The great variation in the number of digits to be dialed on different calls is a source of some difficulty and confusion to the operators.

The present system of operator toll dialing by which operators use codes depending upon the routes to reach a desired destination, is a great improvement over the old manual handling methods. However, with the introduction of more modern toll switching facilities and the nationwide extension of toll dialing, it was realized that an improvement in the methods for dialing toll calls to distant cities was essential in order to realize the maximum speed and accuracy inherent in toll dialing.

These handicaps in the present toll dialing methods are to be overcome by establishing a nationwide numbering plan covering the United States and Canada by which each of the more than 20,000 central offices in the two countries is to be given a distinctive designation which identifies that particular office and that office only. This designation is to consist of

TABLE I—DIFFERENT TYPES OF NUMBERING PLANS

Place	Directory Listing	Customer Dials	Ordinarily Referred to as
Philadelphia, Pa.	LOcust 4–5678	LO 4–5678	Two-five
Los Angeles, Cal.	PArkway 2345 and REpublic 2–3456	PA 2345 and RE 2–3456	Combined two-four and two-five
Indianapolis, Ind.	MArket 6789	MA 6789	Two-four
El Paso, Texas	PRospect 2–3456 and 5–5678	PR 2–3456 and 5–5678	Combined two-five and five digit
San Diego, Cal.	Franklin 9–2345 Franklin 6789	F 9–2345 F 6789	One letter, four and five digit
Des Moines, Iowa	4–1234 and 62–2345	4–1234 and 62–2345	Combined five and six digit
Binghamton, N. Y.	2–5678	2–5678	Five digit
Manchester, Conn.	5678 and 2–2345	5678 and 2–2345	Combined four and five digit
Winchester, Va.	3456	3456	Four digit
Ayer, Mass.	629 and 2345	629 and 2345	Combined three and four digit
Jamesport, N. Y.	325	325	Three digit

two elements, a regional or area code and a central office code. Any outward toll operator, wherever located, will use that same designation in reaching that office through the dial toll switching network.

In a sense, all of the thousands of offices involved are to be treated as though they were contained in one huge multi-office city. Toll operators will use the area code and the office code in reaching an office situated outside her own numbering plan area, while on calls to points within her own numbering plan area she will dial only the number as listed for toll in the directory. In principle the method employed is to divide the two countries geographically into numbering plan areas and to give each of these areas a distinctive code. Refer to Fig. 1. Within each of these numbering plan areas each office will have a code unlike that of any other office in the same numbering plan area and also unlike any area code. Hence for toll dialing purposes each office will have an area code and central office code which will form a combination unlike that of any other central office in the two countries.

In this geographical division into numbering plan areas, border lines between states and between Canadian provinces have generally been used as numbering area boundaries. Since about 500 central offices are the maximum number which can be served in a numbering plan area, it is necessary to divide the larger and more populous states and provinces into two or more areas making, of course, due allowance for growth. New York state with the largest number of central offices is divided into six numbering plan areas; Pennsylvania, Illinois, Texas and California have four areas each. Other divided states have three or two areas depending upon the number of offices to be served. Approximately 90 areas are being provided, with 14 states and two provinces served by two or more numbering plan areas, the remaining states and provinces by one area each.

In fixing the intrastate numbering plan area boundaries of subdivided states, among other considerations effort was made to avoid cutting across heavy toll traffic routes in order to have as much of the toll traffic as possible terminating in the area in which it originated. The advantage of arranging the numbering plan areas in this manner is readily apparent since on this traffic which does not pass an area boundary the area code is not required.

Let us now consider the composition of the area codes. As indicated previously they must be of a type which will enable the switching equipment to distinguish them from the codes of central offices.

On the telephone dial plate letters are assigned only to the dial positions 2 to 9, inclusive (on some dial plates a Z appears on the 0 position but the Z is never used in a central office code), hence any office code will always avoid a 1 or a 0 in the first two places. The digits 1 and 0 can therefore be used in area codes to distinguish these from office codes. It is not practical to use them as initial digits of area codes since customers dial 0 to reach operators and the local dial equipment is arranged to ignore an initial 1 for technical reasons. A 1 or 0 in the second place, however, can be employed in an area code without conflicting with any central office codes or interfering with any existing practices. Accordingly the area codes will consist of three digits with either a 1 or a 0 as the middle digit, 516, 201, etc. A few codes of this type are now in use, leaving a practical total of 152 of these area codes available as compared to approximately 90 assigned to our present numbering plan areas. This will provide a comfortable spare for additional future numbering plan areas or possibly for reaching overseas points which may later be incorporated into the toll dialing network.

As shown in Fig. 1, states and provinces such as Montana or Alberta which are contained in a single numbering plan area will have area codes with a 0 as the middle digit to distinguish them from areas in divided states such as Texas where the middle digit will be a 1. This is to enable toll operators to differentiate between the two classifications of areas. On calls to single area states the operators will always know that every call to the state in question uses the one area code, whereas on calls to subdivided states additional information will be required to determine which of the several area codes should be employed to reach the particular destination. It is proposed to show on the operator position bulletin the codes of all single area states and the codes of all frequently called cities in multi-area states. The area codes of the less frequently called places in the multi-area will be obtained from a routing operator.

Within each numbering plan area each of the 500 or fewer offices are to be given a three-digit office code which will be different from that of any other office code in that same area. Ultimately each central office will be given a 2–5 type of number consisting of an office name and five numerical digits, such as LOcust 4–5678, illustrated for Philadelphia. In the larger cities customers will dial seven digits, LO 4–5678, on local calls to numbers in the same exchange. In many of the smaller places the customers on local calls will dial only the numerical digits, the office name being employed for toll dialing purposes only.

Considering the thousands of central offices which now have numbers other than the 2–5 type and the fact that to change existing numbering systems is a difficult and often costly procedure, it will be a number of years before this ultimate objective is realized. As a practical measure, therefore, it will be necessary during this interim period, before the central office names with the 2–5 type of number are established everywhere, to employ for operator toll dialing office codes which in many cases may not be derived from the customers' telephone number.

In dialing to a combined 2–4 and 2–5 city, for example Los Angeles, the three-digit office code for the Parkway office which has six digits in the local number, will be PAR, whereas to reach the Republic 2 office having seven digits in the local number, the office code will be RE2. To call a telephone in Winchester, Va., with only four digits in the local number, the operator will use a code consisting of numerical digits only, such as 294 which, of course, must be different from every other office code in this numbering plan area. To secure the particular office code to be used in reaching an office where the called number does not furnish complete information, the toll operator must refer to a position bulletin or the route operator. This reference work, of course, takes time and therefore imposes a delay in completing the call.

In addition to giving a distinctive three-digit code to each office within each numbering plan area, each toll center will also be given a three-digit code to enable outward operators to reach inward information, and delayed call operators at toll centers in distant cities. Calls to these operators will be routed in the same manner as calls to customers except that the operator codes will be used instead of a station number and a toll center code in place of a central office code.

The central office names now in use in the various cities in the System were chosen, generally speaking, on the basis of their suitability for customer dialing within the city itself. Many of these names are unfamiliar words to operators and customers in distant cities and the use of these names contributes materially to the operator dialing errors. This situation is gradually being corrected by using for new offices, names from a System approved list and replacing existing names which experience has shown to be particularly troublesome by names from this list.

While numbering plans are important in operator toll dialing, they play an even more essential part in the dialing of toll calls by customers. Operators can be trained to adapt their dialing procedures to the type of local numbering system encountered in the called city even though more time is consumed and more errors result than would be the case if all telephone numbers were of a uniform type. Customers, however, could not be expected to follow any plan which requires a variety of different procedures to be used in reaching different cities. Only a numbering system which is readily understandable and which customers find convenient to use and one which they can use with a very high degree of accuracy will suffice. The need for accuracy is readily apparent since with the customer's telephone being given access to the intertoll network without the intervention of an operator, a call which is misdialed can be routed to a telephone thousands of miles from the desired destination.

At present customer dialing of toll and multi-unit calls is for the most part confined to situations where the call can be completed by the use of the number as listed in the directory without any additional digits being dialed. In a few cases as from Camden, N. J. to Philadelphia and certain offices in Northern New Jersey to New York City, the code 11 is prefixed to the listed number. In the case of the current trial of customer toll dialing at Englewood, N. J., the customers are using area codes such as 415 for Oakland, California, 312 for Chicago, etc., dialing only into those cities which now have the 2–5 type of numbering.

From the Englewood experience it can be confidently predicted that this form of dialing, i.e., an area code followed by a telephone number consisting of a uniform number of digits, is one that customers will use with a reasonable degree of convenience and accuracy. The problem therefore to meet the requirements for nationwide customer toll dialing, is to establish universally for all central offices regardless of size and location a uniform pattern of numbering for toll purposes. The only form of number completely filling the needs is the 2–5 system, which is that used in the largest cities today.

Accordingly, in order to implement the program for customer dialing of toll calls on a nationwide basis, it will be necessary to place all telephone numbers on a 2–5 basis with the code of each office different from that of every other office in the same numbering plan area. Thus each of the 50,000,000 telephones in the United States and Canada will have, for toll dialing purposes, a distinct identity consisting of ten digits; a three-digit area code, an office code of two letters of an office name and a numeral, and four digits of the station number within the office. Typical numbers for toll dialing would therefore be 601–CA3–4567 or 317–MA7–6789. As with operator toll dialing, on a toll call which terminates in the same numbering plan area in which it originates, the area code will be omitted and the office code and station number—a total of seven digits will be used.

With this universal 2–5 type of number, local calls in and about the larger and medium sized exchanges will be completed by dialing the entire seven-digit number. For many of the smaller places in the more isolated sections, 5-digit or 4-digit dialing will frequently be employed where this number of digits will be adequate for all of the telephones

Fig. 1—Nationwide toll dialing areas in the United States and Canada.

in the customers' local dialing area. For these offices with five or four-digit local dialing and for offices in the larger places served by certain types of dial equipment, as they are arranged today, it will be necessary to prefix the dialing of toll calls by a transfer or directing code to permit the customer getting from the local office into the toll network.

Independent of the advantages of a universal 2–5 numbering plan for nationwide operator and customer toll dialing, the Bell System has made considerable progress in this direction over the past several years. New York and Northern New Jersey adopted 2–5 numbering in 1930 in order to take advantage of the flexibility of office code assignments and the large code capacity which this type of local numbering provides. Since World War II many cities and their environs such as Chicago, Boston, Philadelphia, San Francisco, Oakland, Pittsburgh, Milwaukee, Providence and a number of smaller cities have followed suit. Presently about 12 million telephones are in areas which have 2–5 numbering exclusively in addition to perhaps two million telephones with 2–5 numbers in mixed 2–4 and 2–5 areas. Another five million telephones are already planned for conversion to 2–5 numbers within the next several years.

The entire program will take many years to realize but it is one which must be accomplished in order to achieve the best results in operator toll dialing and make it possible for a customer at any telephone in the United States and Canada to reach a telephone anywhere in the two countries by dialing without the assistance of an operator.

Automatic Toll Switching Systems

By F. F. SHIPLEY

(Manuscript received May 12, 1952)

A new automatic toll switching system has been developed by the Bell Telephone Laboratories for use at the most important switching centers for implementing the nationwide dialing program. The job of performing the switching functions at such points is the most comprehensive ever performed by any system, requiring a high order of mechanical intelligence. The new switching system uses crossbar switches for the talking connections and fully exploits the common control principle whereby the equipment used for directing the establishment of connections through the switches is provided in pools common to the office and is used with high efficiency. To perform the complicated translating functions a new device called the card translator has been developed. It uses punched metal cards and an optical system with phototransistors. Routing changes are made by insertion of previously prepared cards in the machine. The switching system was designed with the objective of handling long distance traffic dialed by customers as well as that dialed by operators.

INTRODUCTION

This paper deals primarily with the major switching centers required for the nationwide automatic switching plan. These are called Control Switching Points (CSP's) and are supplied with switching equipment endowed with great versatility and a high order of mechanical intelligence. Mr. Pilliod's paper[1] explains how for purposes of circuit layout and routing, they are assigned different rankings as follows, starting with the lowest ranking: Primary Outlets (PO's), Sectional Centers (SC's), Regional Centers (RC's) and one National Center (NC). Substantially the same equipment is to be provided for all of these centers so that they all will have inherently the same capabilities. They will, however, differ greatly in size. In the United States and Canada, as now envisaged, there will be somewhat under 100 of these CSP's.

The system which Bell Telephone Laboratories developed for use at CSP's and which embodies all of the features required at those important switching points is based on the Toll Crossbar System[2] now in service and has been constructed by the addition of the necessary CSP features to the basic structure of that system.

FUNCTIONS OF THE CSP SWITCHING SYSTEM

The system is designed to be suitable for location in either a step-by-step or a panel-crossbar local area. In addition to the functions required for operation as a CSP, it must, of course, perform the normal toll switching functions required of any system for switching the toll traffic characteristic of the locality it serves. These may be stated very briefly.

Ordinary Toll Switching Functions

1. It accepts calls either directly from operators or from senders in distant offices. In the interest of economy it accommodates itself to the signaling language the operator's position or sender is equipped to deliver. Calls from operators may be either in the dial pulse (DP) or multi-frequency[3] (MF) form. Calls from senders will be in the MF form.

DP pulsing is the decimal type delivered directly by the dial and is at the rate of about one digit per second. MF pulsing represents a particular digit by a combination of two out of five frequencies in the voice range; it uses one of these frequencies in combination with a sixth frequency to produce a signal indicating the beginning of pulsing, and a different one of the five in combination with the sixth for an end of pulsing signal. It is transmitted from senders at the rate of about seven digits per second. Operators usually key at the rate of up to two per second.

2. The toll switching system completes calls to various types of mechanical toll and local offices and to operators, using the form of signaling dictated by economy for each call. For distant toll offices and local offices using step-by-step equipment DP will be transmitted, for other CSP's and usually for local crossbar offices MF will be transmitted and at manual toll offices an operator will be called in either automatically on seizure of the toll line or by sending a ringing signal over the line, but no pulses will be transmitted. Forms of pulsing different from either of these are used for local panel offices and for local manual offices in panel-crossbar areas.

3. It must transmit signals in one direction for initiating, holding and releasing the connection and in the opposite direction to indicate to the originating end when the called subscriber answers and hangs up. These

signals must be in a form suitable for propogation over the medium which carries them.

4. It must exercise control over the amount of amplification of voice currents introduced at the switching point so that a proper grade of transmission will be furnished.

All of these functions are performed by toll crossbar systems already in service. The features that distinguish the new system are those peculiarly characteristic of CSP operation.

CSP Functions

The following features which will be built into the equipment at Control Switching Points are commonly referred to as CSP features:

1. Storing and sending forward digits as needed.
2. Automatic alternate routing.
3. Code conversion.
4. Six-digit translation.

The first of these features is basically essential for implementation of the plan. The second produces faster service and important economies in outside plant. It also provides protection against complete interruption of service in case of failure of all circuits on particular routes. This aspect of the feature is so important that automatic alternate routing may also be considered essential. The other two features are provided for reasons of economy, and produce economies of such magnitude that they are very much worth while.

1. Storing and Sending Forward Digits as Needed

The necessity of providing this feature in CSP switching systems arises from the nature of the numbering and switching plans. The numbering plan[4] is constructed with the objective of using a minimum number of digits to give each telephone user in the country a distinctive number.

Numbers delivered to the CSP equipment are in the form ABX-XXXX if the called place is in the same numbering area as the CSP. AB represents the first two letters of any office name and X represents any numeral. If the called place is in another numbering area this set of digits will be preceded by X0X or X1X. X0X or X1X is the area code, ABX the local office code, and these are the digits used for routing purposes. Regardless of the number of switches required to complete the call, these two sets of code digits are all that will be supplied. They are universal codes in that they identify specific destinations – any place in the United States or Canada – and for a particular destination the same set of digits will be used wherever the call may originate. All CSP's must, therefore, be able to advance a call toward the same place when the same set of digits is received.

To make use of destination codes possible, each CSP must store the digits as received and pass along to the next point whatever digits may be required there for advancing the call. If the next point is a CSP not in the home numbering area of the called place, the complete ten-digit number will be sent forward. If it is a CSP in the home numbering area of the called point the area code will be dropped and the remaining seven digits will be sent forward. That CSP may in turn complete to a local office directly, dropping the office code, or through a step-by-step TO (Tandem Outlet) or TC (Ordinary Toll Center), substituting arbitrary digits for the area or office code, thereby exercising the third of the listed CSP features.

2. Automatic Alternate Routing

The system is arranged to offer the maximum number of alternate routes possible under the switching plan. As explained by Mr. Pilliod,[1] a maximum of five alternates will actually be used. This number is possible, of course, only at PO's since higher ranking CSP's have fewer CSP's above them in the final chain.

3. Code Conversion

This refers to the ability to substitute one, two or three arbitrary digits for the area code, the office code or both. It is economically important to be able to do this because it makes it possible to work with the step-by-step equipment extensively used in local offices and in toll offices in TC's or TO's without the changes in local numbering plans or rearrangements – and in some cases extra selectors – required for the step-by-step TC's or TO's to use ABX codes for routing purposes. Even though eventually all customers are listed as ABX-XXXX and TC's are arranged to use the listed number for routing the calls, this will not be accomplished for some time. Moreover, after such arrangements are in effect there will still be need for code conversion, particularly for routing calls through TO's. Many combinations of digit dropping and substitution are required to cover all possible cases.

4. Six-Digit Translation

When a CSP receives a ten-digit number it is sometimes sufficient to translate only the area code digits and sometimes necessary to translate both the area and office codes. If all points in the called area are reached by the same route out of the particular CSP concerned the area code will suffice for selection of the route. If some points are reached by one route and others by one or more different routes the office code must also be translated to determine which route should be selected.

BASIC ARRANGEMENT

In the CSP switching equipment talking connections are established through crossbar switches.[5] Incoming and outgoing toll lines and toll connecting trunks are terminated on crossbar switch frames with linkage between them to provide full access. The switches are controlled by equipment common to the office, each item of which is held only long enough to perform its task in setting up the connection.

The major items of common control equipment are senders, markers, decoders and translators. The basic functions of the senders are the same as in other common control systems, i.e., registering incoming digits and sending them out as directed. A departure from prior practice is made in the design of the marker. In other crossbar systems the marker is the principal seat of the mechanical brains. It not only controls the actual establishment of the connection but also does the translating to determine what connection should be established and what information should be passed to the sender for further disposition of the call. In this system the marker still controls the actual setting up of the connection, but it acts on instructions received from the decoder where the major portion of built-in intelligence resides.

The decoder accepts code digits from the sender, translates them, makes selection of alternate routes and gives instructions to markers and senders to enable them to carry out their assignments. To do the translating job the decoder has one, and in some cases two translators permanently associated with it and in addition has access to a common group of translators called foreign area translators which can be used by all decoders as required.

The relationship of the principal elements of the system to each other is depicted in the schematic diagram, Fig. 1.

METHODS OF OPERATION

The manner in which the various elements of the CSP system and the CSP systems at various locations cooperate to implement the nationwide switching plan may best be understood by following the progress of a call which demands the exercise of the characteristic CSP functions.

Assume an outward operator in Atlanta has received a station-to-station call for a subscriber in Monticello, Maine, whose number is ACademy 4–2345, that Monticello is a tributary of Houlton, a step-by-step TC, that Bangor is a step-by-step TO serving Houlton as its home TO and that the circuit groups provided are as indicated in Fig. 2. The dotted lines represent high usage groups and the solid lines final groups.

The Atlanta operator plugs into a tandem trunk to the toll crossbar system in Atlanta, thereby causing a sender to be attached to the trunk through the sender link frame. This causes a lamp signal to be displayed to the operator, indicating that she may key the number. She keys 207–AC4–2345 plus a start signal (signifying end of keying) into the sender and leaves the connection to handle other calls. She will give this call no further attention until the lamp associated with the cord circuit used in establishing it signifies either by flashing that the call was not completed due to a busy condition of the called line or to circuit congestion, or by going dark that the called subscriber has answered and she should start timing the call.

As soon as the area code, 207, is received by the sender it calls for a decoder and gives it the code. The decoder, by means of a self-contained translator finds that the area code is sufficient for routing purposes, that the first choice route is by way of Boston, the second New York and the final St. Louis. Without consulting other circuits it will know in which

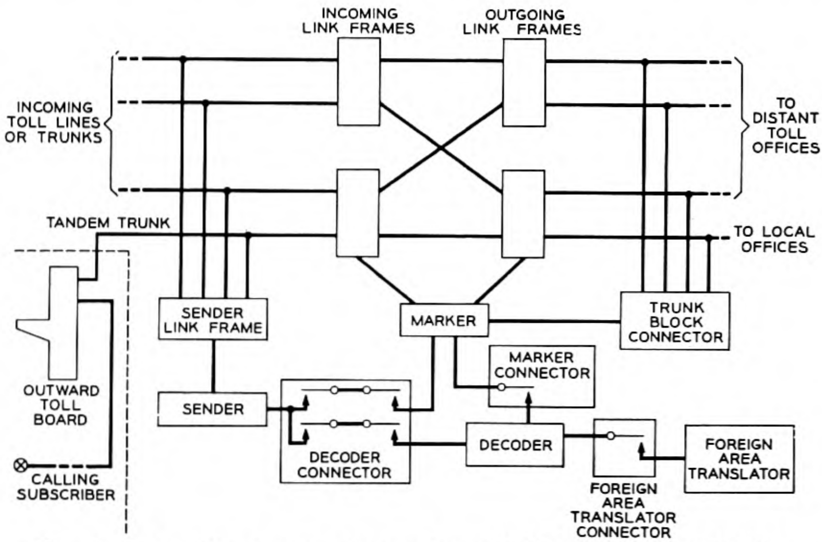


Fig. 1—Schematic diagram of crossbar switching system for CSP's.

of these groups an idle circuit may be found. Let us assume that the circuits to Boston are all busy but there are one or more idle circuits in the New York group. The decoder calls for a marker and tells it which group of leads to test, and also causes the sender to be connected to the particular marker it has selected.

The marker, following instructions from the decoder, is connected to the appropriate trunk block connector. This is one of a group of common circuits giving access to "blocks" of trunks for allowing the marker to locate an idle trunk. The marker examines the test leads of the individual toll lines to New York and as soon as it has selected an idle circuit it so informs the decoder. The decoder then tells the sender to send all

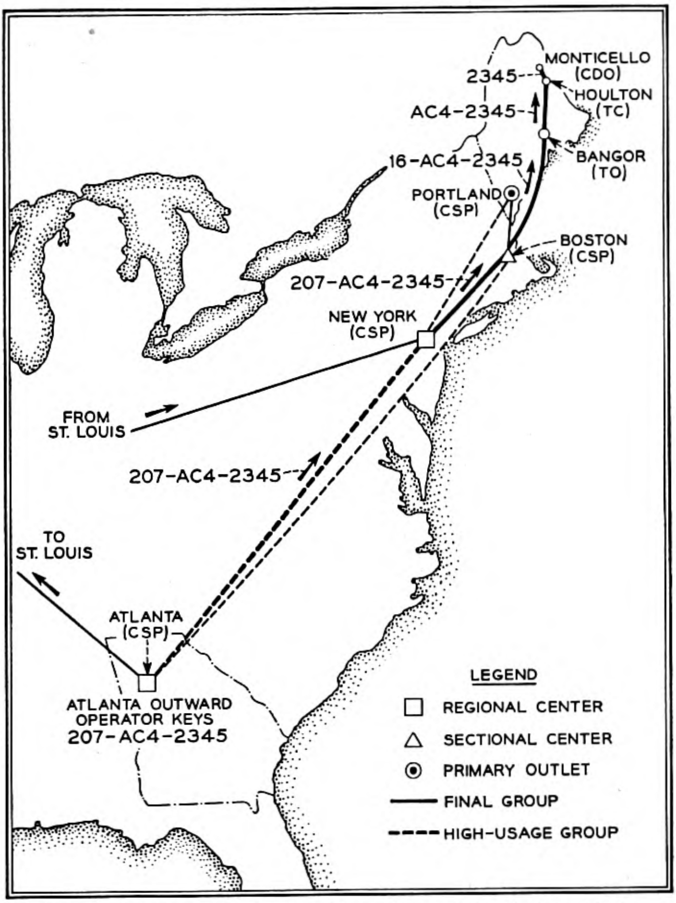


Fig. 2—Call from Atlanta, Georgia to Monticello, Maine.

digits forward by MF and leaves the connection to accept another call. This information is relayed from the decoder to the sender by way of the marker. The work time of the decoder has been in the order of a half second.

The marker determines the identity of the frames on which the incoming and outgoing circuits are located, finds an idle path between the two circuits and sets up the connection. After checking the path through the switches to be sure that there are no troubles it notifies the sender that its task has been completed and then leaves the connection. Its work time has also been in the order of a half second.

In the meantime other digits have been coming in to the sender but it does not wait for all of them to arrive before advancing the call. When the marker selected the circuit to New York a signal was immediately sent forward to summon a sender in the New York switching system. The process of attaching the sender in New York was carried on concurrently with the establishment of the connection through the switches in Atlanta.

When the New York sender is attached a signal is sent to the Atlanta sender to advise it that pulsing may proceed. It immediately sends the area code 207 to New York by MF pulsing and follows it with the remaining digits of the called number, AC4–2345, as they are received from the operator, ending with a start pulse, and then leaves the connection. All common control equipment in Atlanta is now free.

In New York, as soon as the Maine area code is received it is submitted to the decoder. Upon examination of the code the decoder finds that it is insufficient for routing purposes. New York has a direct circuit group to Portland over which traffic to some offices in Maine is routed, but other offices are reached through Bangor by way of Boston. In order to determine which route to take the decoder must know what office is desired. It, therefore, gives the sender a signal saying "come again when you have six digits" and leaves the connection. When the sixth digit arrives the sender again calls for a decoder and gives it the complete code 207–AC4.

The decoder again translates the area code, which now directs it to the foreign area translator which serves the Maine area, and submits the complete code to that translator. From the ensuing translation it learns that the route is by way of Boston and that all digits should be sent forward by MF. It then calls for a marker and releases the foreign area translator.

Subsequent operation is the same as previously described for Atlanta and the complete ten-digit number now arrives at Boston. At that point both codes are again translated since Boston also has a choice of routes to Maine, and the route to Bangor is selected. The translating equipment in Boston knows that Bangor is in the Maine area and that the area code will, therefore, not be needed. However, since Bangor is a TO having no senders, the Boston sender must pulse forward all of the digits needed to complete the call through switches in Bangor, Houlton and Monticello. It is assumed that Houlton is arranged to route the call to Monticello on receipt of the digits AC4. Numerical digits 2345 will route the call through the Monticello switches to the called customer's line. These digits are all registered in the Boston sender but the digits required to switch the call through Bangor are not and must be supplied. An arbitrary set of digits beginning with "1" can be used for this purpose since no office code begins with "1" and there will, therefore, be no conflict.

The decoder in Boston, therefore, gives the sender the proper set of arbitrary digits, say 16, to be placed ahead of the office code AC4. The sender sends forward by the DP method 16–AC4–2345 driving switches in Bangor, Houlton and Monticello to the called subscriber's line, and ringing starts automatically. The talking connection is now established and the common control equipment at all intermediate points is free.

When the called subscriber answers, the Atlanta operator's cord lamp is extinguished. When he hangs up the lamp lights to denote end of conversation. The removal of the operator's cord automatically releases the entire connection, the release of each link causing the next in line to release.

In setting up this call all of the characteristic CSP features were employed, automatic alternate routing in Atlanta, six-digit translation in New York and Boston, digit storing and variable spilling at all CSP's with substitution of arbitrary digits for the area code at Boston.

TRANSMISSION

All talking connections through the CSP system are made on a four-wire basis, that is, separate pairs of conductors are provided for transmission in the two directions. This is done in order to simplify the problem of maintaining satisfactory balance so that the loss introduced by extra links in a connection can be held to a minimum value. The importance of this feature is emphasized by the fact that the switching plan permits as many as eight intertoll trunks to be connected in tandem for the completion of a call.

The advantages of four-wire switching were fully explained in the paper[2] on the toll crossbar system now in service.

SIGNALING

In following the progress of the call from Atlanta to Monticello, Maine, it was observed that besides the transmission of information in the form of digits it was necessary to pass a number of control and supervisory signals over the toll lines. These included seizure and disconnect signals in the forward direction and switchhook supervisory signals and sender attached signals in the reverse direction. On some calls it is also necessary to send flashing signals to indicate busy lines or trunks and ringing signals in either direction when operators are called in at intermediate or terminating points to assist in establishing connections.

For the early toll dialing installations the signaling method most widely used was the composite method whereby signaling channels for the three circuits of a phantom group are derived from three of the conductors with the fourth being used for earth potential compensation. Direct current is used for signaling. This is a simple, reliable and economical method of signaling and will continue to be used on circuits where it can be applied.

Where circuits are obtained from carrier systems, however, conductors are not available in sufficient numbers for signaling channels and other methods must be employed. Since carrier is used almost exclusively on the long haul circuits it was necessary to provide a signaling system to accompany it before toll dialing could be expanded beyond networks of limited range. To meet this situation a system[6] using a frequency of 1600 cycles was developed and has been in service since 1948. Signaling is done by application and removal of the 1600-cycle signaling current. The system is used in the same manner as the composite signaling system, to carry dial pulses as well as supervisory signals when used on circuits that require it. The set of leads brought out of the signaling unit are identical in function to those brought out of the composite signaling unit so that toll line relay circuits will operate in the same manner with either type of signaling.

Since 1600 cycles is in the voice range the signaling current can be carried over the same channel that carries the speech current but the signaling circuits must, of course, be protected against false operation due to speech and precautions must likewise be taken to insure that the signaling tone does not interfere with speech. Protection against interference between signaling and speech is more difficult at 1600 cycles than at higher frequencies because there is more energy in voice currents at the lower range. That value was chosen, nevertheless, so that it would be possible to operate over the narrow band circuits that were established to relieve shortages occasioned by the war.

A new 2600-cycle system to be used only on the broader band circuits has since been developed. It is simpler and more economical than the 1600-cycle system. The older carrier systems, having been designed when practically all toll operation was by the ringdown method, made no provision for signaling since that was all done by short applications of the 1000 cycles when there was no speech on the line. Some of the new carrier systems for short haul applications are designed to provide their own signaling channel for each voice channel.

PRINCIPAL ELEMENTS OF THE CSP SYSTEM

1. Crossbar Switch Frames

Crossbar switches are used for incoming and outgoing link frames on which the trunks (both toll lines and trunks to and from local offices and switchboards) are terminated, and for sender link frames used to give trunks access to senders. These frames are similar to those in the toll crossbar systems now in service. Since they have been described in a previous paper[1] they will be passed over with only a mention of their capacity.

Each incoming or outgoing link frame normally has terminals for 300 trunks. As many frames are provided as required for the size of the office. In the smaller offices one train of switches with complete interconnection of incoming and outgoing frames is provided. In the larger offices two trains each with its own set of markers are provided. When this is done the incoming trunks are multipled to both trains and an extra build out bay is provided on the incoming frame to provide 400 terminals per frame. Since each train has a theoretical limit of 40 incoming and 40 outgoing frames the maximum size of an office is theoretically 80 of each. Practical considerations, however, such as the number of markers that can be efficiently operated in a group and the maximum size office it is feasible to operate as a single administrative unit will limit an installation to about 60 incoming and 60 outgoing frames.

The sender link frame gives 100 trunks access to 40 senders.

2. Senders

Two separate groups of incoming senders are provided, one to receive DP and the other MF pulsing. Whether the system is installed in a step-by-step or a panel-crossbar area both groups of senders will always be needed. MF will be received from senders in other CSP's and from switchboard positions. DP will be received from switchboard positions at TC's not equipped to send MF and in some cases from dialing A boards in the local area of the CSP itself.

Aside from the type of pulses received the functions of the two senders are identical. They have a capacity for receiving and sending eleven digits. They must register arbitrary digits given them by the decoder and send them out as directed. They will send out digits by either the DP or the MF method as required to control switches in distant offices, and in some installations will also send digits to an outgoing sender in the same office by the dc key pulsing method, which employs direct current in various combinations of value and polarity through a pair of conductors.

When the CSP is in a panel-crossbar area a group of outgoing senders is provided to transmit either the type of pulses required by the equipment in local panel offices or the type used to reach manual offices.

3. Markers

The marker has been stripped of its usual translating functions and performs most of its duties on instructions from the decoder. It is told what leads to test for idle circuits and where they are to be found in the trunk block connector, but having found an idle circuit it carries on the process of setting up the connection independently of the decoder. Having contact with both the incoming and outgoing trunks through connecting circuits, it determines what frames they are located on, connects itself to those frames, selects a path through them and sets up the connection.

In a single-train office one group of markers common to the office is provided. In a two-train office there is a group of markers associated with each train of switches.

4. Decoders

A single group of decoders serves the entire office whether one or two trains of switches are provided. An important element of the decoder is the translator which will be discussed separately.

The decoder contains several hundred relays. A large group is used for registering the information furnished by the translator. Others use this information to control the action of the markers and senders.

One group of decoder relays which is of particular interest is the array used for automatic selection of alternate routes. It is composed chiefly of one relay for each CSP to which the office has a direct group of toll lines. The relays are arranged in an orderly pattern simulating the pattern of the CSP network for the country as seen from the CSP concerned and are interconnected in a pattern of progression corresponding to the fixed order of alternate route selection. Group busy leads from the toll line groups are connected to the contacts of the relays in such a manner that if a group is busy the relay corresponding to the next choice route in the chain will be operated. In this way the lowest choice route having an idle circuit will be speedily selected without testing individual trunks of separate groups. The decoder learns from the translator which relay in the array to operate first and the choice of the best route available follows automatically. The principle will be readily understood by reference to the simplified sketch in Fig. 3. Contacts not shown on the relays cause the translator to select the route corresponding to the last relay operated in the chain.

5. *Translators*

The magnitude of the translating job for nationwide dialing led to the decision to develop a new translator operating on a principle radically different from that employed in other crossbar systems. In previous systems translation is done by relays. The code digits – never more than three – operate a group of relays which cause a single terminal corresponding to the code to be selected. A cross-connection is made between

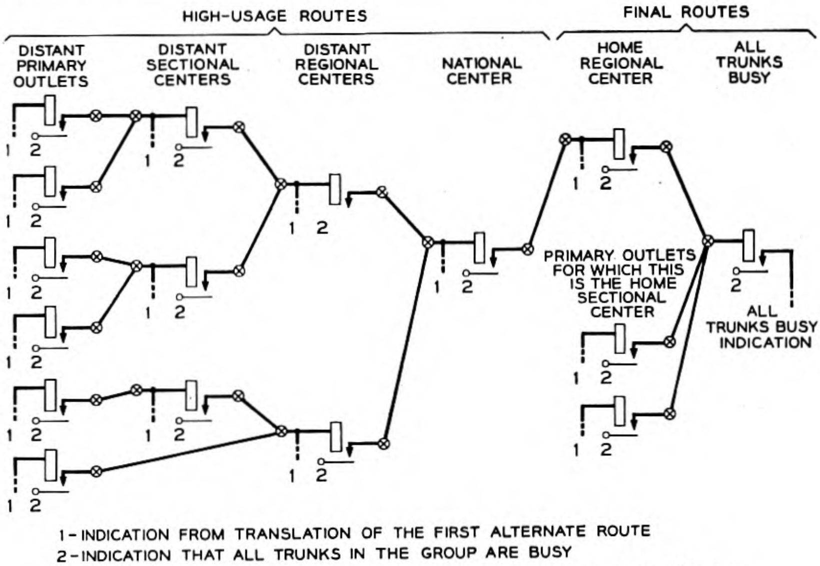


Fig. 3—Alternate route array for the decoder at a sectional center.

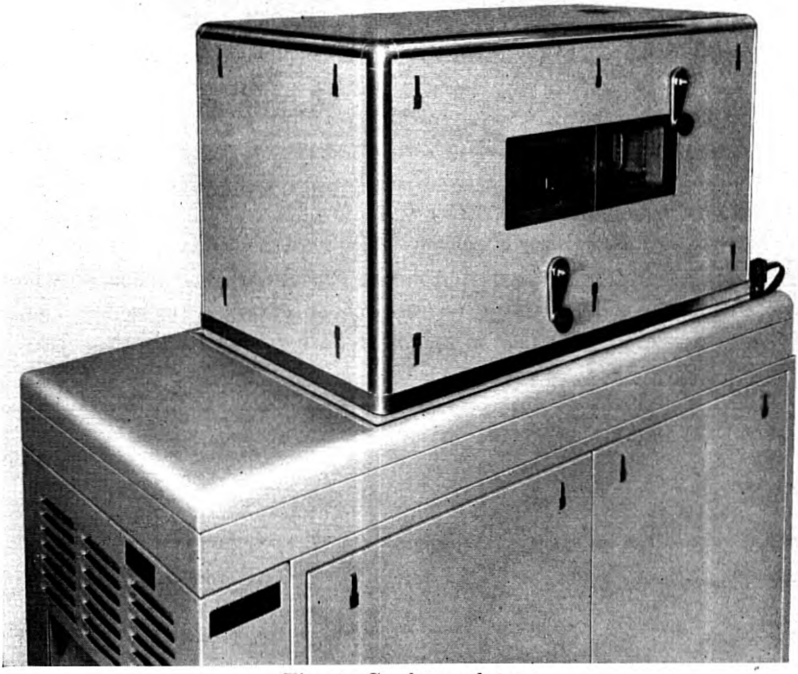


Fig. 4—Card translator.

the code point and a route relay associated with the trunk group to be selected. The route relay has a number of contacts which are cross-connected to supply the information required for proper routing of the call. When changes in routing or equipment location of trunks within the office are made it is necessary to change cross-connections.

With the nationwide dialing plan in operation routing changes or opening of new offices in one part of the country will necessitate translator changes in many offices, some of them far removed from the scene of the event that forces them to be made. The changes in any one CSP will, therefore, be frequent and to make them by running cross-connections would be cumbersome and expensive. The new translator uses punched cards instead of relays, making it possible to effect changes by the simple process of removing old cards and inserting new ones in a machine. This can be done in a very short time and not only saves labor but requires less out-of-service time for the equipment. Fig. 4 is a photograph of the machine.

A metal card about 5 by $10\frac{3}{4}$ inches is provided for each area code and also one for each office code that must be translated in a particular CSP, the cards representing destinations. The capacity of a single machine is about 1000 cards. The cards are lined up in a box as in a filing drawer, with tabs along the bottom of the card resting on select bars which run the length of the box. One-hundred and eighteen holes are punched out in all cards in fixed positions so that in the normal condition 118 tunnels are formed from one end to the other. A light source at one end of the box shines through the tunnels upon phototransistors (Fig. 5) at the other end but the phototransistors are disabled until, concurrently with the dropping of a card, voltage is applied to them.

All tabs along the bottom of the card are cut off except those which serve to identify the particular card. When a code is presented to the machine a combination of select bars corresponding to the code is

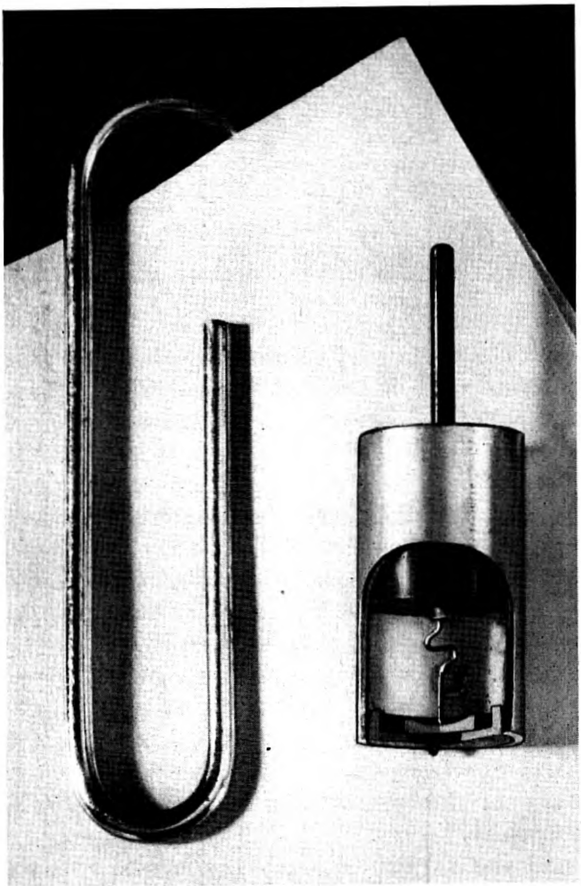


Fig. 5—Transistor.

lowered. The card having all tabs cut off except those resting on the lowered bars will drop but all other cards will remain in place. If nothing further were done the dropping of the card would cut off all light channels but on each card some holes are enlarged and through these holes the light continues to shine, energizing the corresponding phototransistors. The combination of enlarged holes furnishes all of the information needed for routing the call to the destination represented by the card.

Fig. 6 shows the functions of the various groups of tabs and holes. The designations will not appear on the actual card. Fig. 7 is a photograph of an actual card prepared for use.

a. Selecting Tabs – Input Information. The sole use of the information presented to the card translator is to enable it to select the proper card. The information presented is in the form of code digits with accompanying indications of their nature. The information is recognized by cutting off tabs along the bottom of the card in proper combinations.

The groups of tabs labeled A, B, C, D, E and F are for the six code digits. For each digit used two tabs are left since the digits are registered in the sender on a two-out-of-five basis and the leads from the sender will operate the select bars directly. If the card represents an ordinary three-digit code all tabs will be cut off except two each of the A, B and C tabs, two of the four CG tabs and perhaps either the VO or NVO tab. The CG (card group) tabs are used in combination to indicate three-digit, six-digit and alternate route card groups. The VO and NVO (via only and not via only) tabs are used when the group of toll lines over which the call will be routed is divided into one subgroup of a transmission grade suitable for only terminal traffic and another subgroup for either terminal or switched traffic. If the card represents an ordinary six-digit code two tabs will be left in each of the digit positions, and a different pair in the CG group.

b. Punch Holes – Output Information. The output information from the card translator is recognized in the decoder and marker by relays operated in the combinations set up by enlargement of associated holes in the card. The output from the phototransistors is amplified by other transistors to fire cold cathode tubes which in turn operate the relays.

The pretranslation group on the top line of Fig. 7 indicates how many digits the sender must supply for a complete translation. The term "pretranslation" implies that further translation is required. This is not always true. In many cases only the first three digits need to be translated and all information needed for routing the call is supplied by this card. In many cases the six digits of the area and office code are needed and the routing information will be on another card to be selected

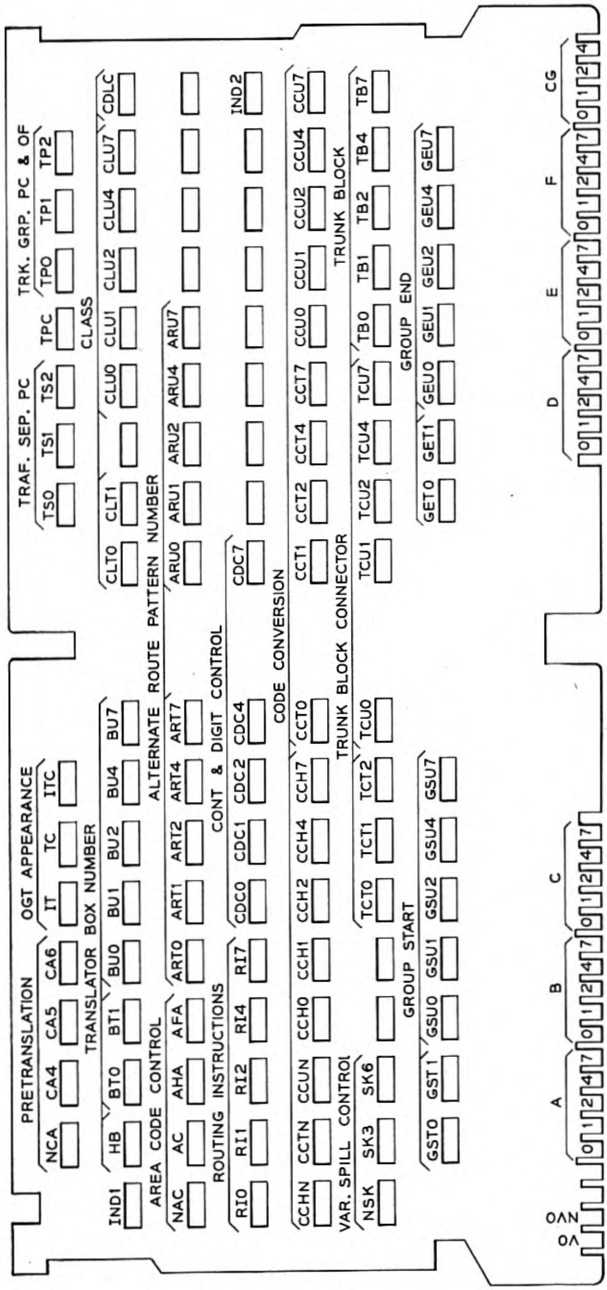


Fig. 6—Card layout.

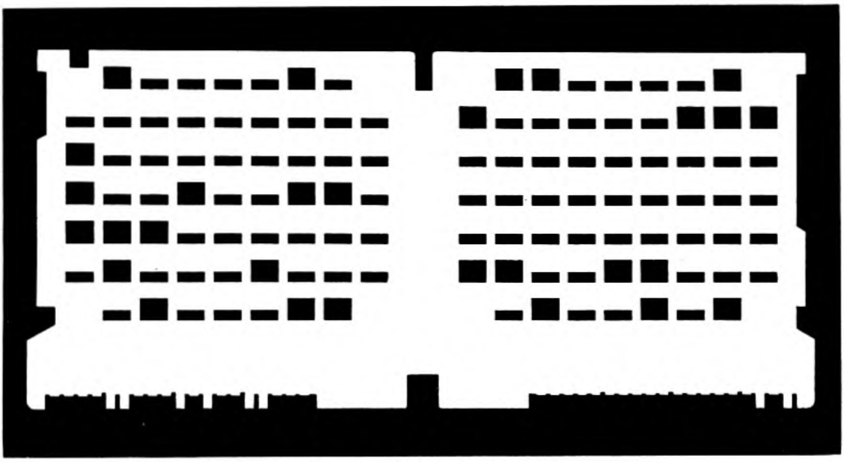


Fig. 7—Punched card.

later. For certain calls such as calls to operators only four or five digits are needed. These are treated as six-digit calls by having the sender supply the extra one or two digits to fill the complement. The NCA hole enlarged means "no come again", that is, three digits are sufficient, and translation will proceed. The other holes enlarged mean respectively "come again when you have four, five or six digits", and no further translation is done until the sender comes back a second time, probably to a different decoder, with an indication that it has the required number of digits.

The OGT appearance holes are used in a two-train office to tell which train the outgoing trunk appears on and enable the decoder to select a marker in the proper group.

The remaining holes on the top lines are for controlling operation of traffic meters.

The translator box number holes in the second line are punched on the area code cards to indicate which machine contains the individual cards for the called area when six-digit translation is required.

The IND1 hole in the second line and the IND2 hole in the fourth line are index holes and are never enlarged. Any card that drops will always cut off the light through those channels. This serves as an indication that a card has actually dropped and that the phototransistors associated with the other holes should be prepared for action. The index holes also aid in trouble detection and in proper disposition of calls where cards are deliberately omitted or where operators have dialed a blank code in error.

The class holes indicate such things as type of pulsing and nature of the signaling channel used on the trunk group out of the office.

The area code control holes in the third line are to tell the decoder what to do about dropping or spilling forward an area code registered in the sender or supplying an area code when none is registered. This information is needed primarily in connection with alternate routing.

The alternate route pattern number holes tell the decoder at what point to enter its chain of alternate route relays for the first choice alternate. Provision is made for a maximum of 100 entry points.

The holes on the fourth line are for making proper disposition of calls when no circuits are available on any routes, telling how many digits to expect on certain calls and other items of a detailed technical nature.

The code conversion holes on the fifth line supply the arbitrary digits to replace code digits on calls routed through step-by-step TO's or TC's. Provision is made for one, two or three digits as required.

The variable spill control holes in the sixth line tell whether to spill all digits received, skip the first three code digits or skip six code digits.

The remaining holes define the location on the equipment of the test leads for the trunk group over which the call will be routed.

The notches around the edges are used for proper positioning and removal of cards.

An individual card is removed from the stack by first keying the code to cause it to drop so that it may be identified. Since a card can easily be located in this manner it is unnecessary to keep cards in any ordered position in the box.

At least one translator is provided in every decoder. It contains the cards for all offices in the home numbering area of the CSP, for certain operator codes, the single three-digit card for each toll numbering area and a card for each toll line group out of the office that can be used as an alternate route. If there are other areas to which the volume of traffic is very high and for which six-digit translation is required the cards for those areas are put in a second machine in each decoder. Cards for other areas are put in foreign area translators common to the office and accessible to all decoders on a one-at-a-time basis. An emergency translator is provided to permit removal of all cards to it from any translator which may require prolonged repair work.

6. *Traffic Control Panel*

The traffic control panel is located in the operating room. The equipment in it consists of a key for each group used as an alternate route. When a particular key is operated no alternate routed traffic will be offered to the group represented by it nor to any group above it in the fixed alternate route pattern. This is done to relieve offices which are overloaded by either unforeseen or predicted traffic peaks.

MAINTENANCE

The maintenance facilities for the new CSP system are basically similar to those of the older toll crossbar system with the necessary addition of equipment to test the new features introduced. The sender test frame is, of course, obliged to test the CSP features added to the sender and the trouble indicator frame is changed to operate with the new decoders, translators and markers.

In place of the lamp trouble indicator the new trouble recorder introduced with the latest local crossbar system[7] is used. Whenever trouble is encountered it punches on a card a record of the circuits involved and of the important events that had occurred in the progress of the call, as an aid to the maintenance man in locating the trouble. A sample trouble recorder card is shown in Fig. 8.

Automatic equipment for testing the operation and transmission features of intertoll trunks has also been designed both for the older systems and for the new CSP system.

SWITCHING ASPECTS OF CUSTOMER TOLL DIALING

In the course of developing the switching system for CSP's the requirement for handling long distance traffic dialed by customers as well as that dialed by operators was kept in mind. The trial of long distance customer dialing now in progress in Englewood, N. J., confirms the soundness of the basic plan and exemplifies the principles involved in full realization of the plan. With a toll network laid out to accept a distinctive ten-digit number for any telephone in the country and route it to the proper destination, the remaining tasks to be performed are to provide for delivery of the number to the toll network from the customer's dial instead of from an operator and to provide an automatic record of the call for charging purposes.

In Englewood both tasks were quite easily performed. The Englewood local office equipment is of the most modern type[7] and includes AMA[8] facilities. When it was in the development stage the ultimate requirement for nationwide customer dialing was foreseen and provision was made for expanding the digit capacity of the switching equipment at small expense. Also the designs of the accounting center were such that corresponding changes could readily be made. In the new local office switching system, arrangements were included for sending forward the complete number, as received, to the toll office by MF pulsing. The system was also designed to be capable of automatic alternate routing and this feature is used in the trial.

Expansion of the program will, of course, demand that similar arrangements be provided for the older types of local switching systems already in service. More extensive modification will be required to make them capable of giving the customer the same service. For them, as for the most modern system, however, AMA equipment is admirable for recording the information necessary for charging for the calls.

The requirement for customer toll dialing that senders (or directors) and recording equipment be provided has a bearing on the type of equipment used at TC's and TO's. For calls handled by operators and for calls received by the customers through such offices the only disadvantage of step-by-step equipment without senders at those points is that the CSP equipment at other points must be somewhat more complicated and expensive than it would otherwise need to be. But with customer dialing, if senders and recording equipment are not provided either in the local office or in the TC or TO, the calls must be routed by the most direct means possible to a CSP where such equipment is provided. Thus some advantages that might be gained from having them at the TC or TO would be lost:

1. In some cases an indirect route to the CSP would need to be taken for the sole purpose of recording the call. For example, a call which might normally be switched from a TC through a TO to another TC would need to be connected to the CSP for making the record.

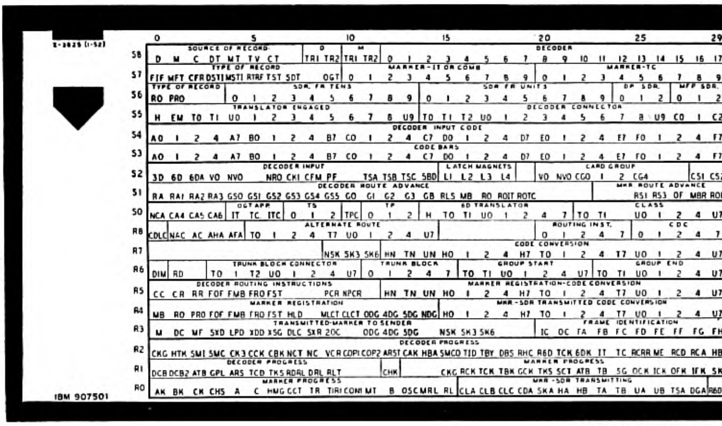


Fig. 8—Card for the new trouble recor

2. There is no operator at the TC or TO to select an alternate route and with the equipment there incapable of automatic alternate routing the economies and service protection inherent in the alternate routing procedure would be lost.

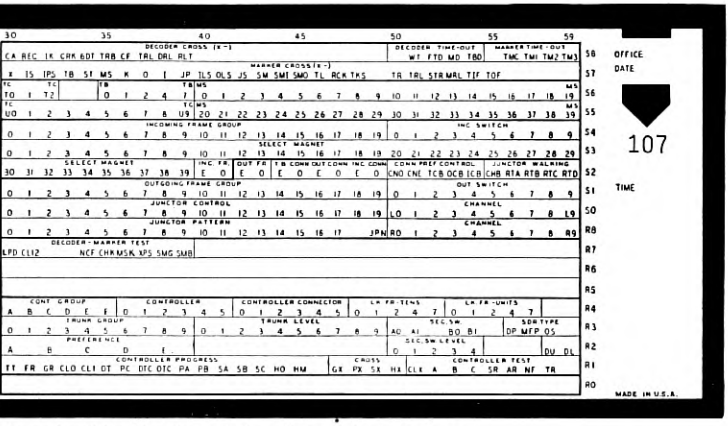
If step-by-step toll switching equipment is already provided at a TC or TO, senders (or directors) could be added, making it in effect a common control switching system. This measure would permit automatic alternate routing and the further addition of recording equipment would eliminate the indirect routings for recording purposes.

A further benefit from having common control equipment in TC's or TO's can be realized in some instances. When a customer is served by a local office that has no senders he must dial one or more directing digits (probably three digits) ahead of the seven or ten-digit number in order to get to an office where senders are provided. It is, of course, desirable to avoid this extra burden on the customer. Where the equipment in a TC or TO can be used in common for switching local and toll traffic the customers whose lines are terminated in that office will be dialing directly into senders, if the equipment uses common control, and will, therefore, benefit in that they will not have to dial directing digits.

CONCLUSION

The new system was designed to implement the nationwide switching plan, which integrates the switching network of the entire nation into a single unit. This switching job, requiring a high order of mechanical intelligence, is the most comprehensive ever performed by any system.

The skillful manipulation of code digits enables the provision of a


30 35 40 45 50 55 59
DECODER CROSS (X-) | DECODER TIME-OUT | MARKER TIME-OUT
CA REC IK CRK 6DT TRB CF TRL DRL RLT | WT FTD MD TBD | TMC TMI TM2 TM3 S8
MARKER CROSS (X-)
X IS IPS TB ST MS K O I JP ILS OLS JS SM SMI SMO TL RCK TKS TR TRL STR MRL TIF TOF S7
TC TO 1 TC T2 | TB 0 1 2 4 TB 7 | MS 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 MS 19 S6
TC U0 1 2 3 4 5 6 7 8 TC U9 | MS 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 MS 39 S5
INCOMING FRAME GROUP 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 | INC SWITCH 0 1 2 3 4 5 6 7 8 9 S4
SELECT MAGNET 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 S3
SELECT MAGNET 30 31 32 33 34 35 36 37 38 39 | INC. FR. E O | OUT FR E O | TB CONN E O | OUT CONN E O | INC CONN E O | CONN PREF CONTROL CNO CNE TCB OCB ICB | JUNCTOR WALKING CHB RTA RTB RTC RTD S2
OUTGOING FRAME GROUP 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 | OUT SWITCH 0 1 2 3 4 5 6 7 8 9 S1
JUNCTOR CONTROL 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 | CHANNEL L0 1 2 3 4 5 6 7 8 L9 S0
JUNCTOR PATTERN 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 JPN | CHANNEL R0 1 2 3 4 5 6 7 8 R9 R8
DECODER-MARKER TEST LPD CLI2 NCF CHK MSK XPS SMG SMB R7
R6
R5
CONT GROUP A B C D E F | CONTROLLER 0 1 2 3 4 5 | CONTROLLER CONNECTOR 0 1 2 3 4 5 | LK FR-TENS 0 1 2 4 7 | LK FR-UNITS 0 1 2 4 7 R4
TRUNK GROUP 0 1 2 3 4 5 6 7 8 9 | TRUNK LEVEL 0 1 2 3 4 5 6 7 8 9 | SEC.SW. A0 A1 B0 B1 | SDR TYPE DP MFP OS R3
PREFERENCE A B C D E | SEC.SW. LEVEL 0 1 2 3 4 | DU DL R2
CONTROLLER PROGRESS TT FR GR CLO CLI DT PC DTC OTC PA PB SA SB SC HO HM | CROSS GX PX SX HX | CONTROLLER TEST CLX A B C SR AR NF TR R1
R0
OFFICE
DATE
107
TIME
MADE IN U.S.A.

duced with the latest local crossbar system.

numbering plan covering the entire country with a minimum number of digits to give each customer a distinctive number. It also obviates the need for extra expense to make step-by-step toll offices satisfactory operating elements of the plan in those locations where CSP features are not essential.

The automatic and almost instantaneous selection of alternate routes makes it possible to give virtual no-delay service without greatly increasing the cost of outside plant and to make multi-switch connections at a speed comparable to that for local service.

The translating equipment simplifies administration of the plan which demands coordination of activities on a nationwide basis.

The numbering plan, the switching plan and the CSP equipment which implements them make it feasible to offer nationwide dialing service to customers without the aid of operators when automatic charging facilities and local office switching arrangements for handling the three extra digits of the national number are provided. It will be readily appreciated that so far as the CSP switching equipment is concerned it is immaterial whether the digits it receives come from an operator or from a customer. The call will be routed to its destination and supervision for charging purposes will be furnishes in the same manner in either event.

The new system represents an important step in the process of continually improving the long distance switching methods of the Bell System with consequent improvement of the service to all telephone customers in the United States and Canada.

REFERENCES

1. J. J. Pilliod, "Fundamental Plans for Toll Telephone Plant," pp. 832 of this issue.
2. L. G. Abraham, A. J. Busch and F. F. Shipley, "Crossbar Toll Switching System," *A.I.E.E. Transactions*, **63**, June Section, pp. 302–309, 1944.
3. C. A. Dahlbom, A. W. Horton, Jr. and D. L. Moody, "Application of Multifrequency Pulsing in Switching," *A.I.E.E. Transactions*, **68**, June Section, pp. 505–510, June 1949.
4. W. H. Nunn, "Nationwide Numbering Plan," pp. 851 of this issue.
5. F. J. Scudder and J. N. Reynolds, "Crossbar Dial Telephone Switching System," *A.I.E.E. Transactions*, **58**, May Section, pp. 179–192, 1939.
6. N. A. Newell and A. Weaver, "Single Frequency Signaling for Telephone Trunks," Presented at Winter General Meeting of A.I.E.E., Jan. 31, 1951.
7. F. A. Korn and J. G. Ferguson, "The Number 5 Crossbar Dial Telephone Switching System," *A.I.E.E. Tranasctions*, **69**, First Section, pp. 233–254, 1950.
8. J. Meszar, "Fundamentals of the Automatic Telephone Message Accounting System," Presented at the Winter General Meeting of A.I.E.E., Jan. 31, 1951.

Mathematical Theory of Laminated Transmission Lines—Part I

By SAMUEL P. MORGAN, JR.

A mathematical analysis is given of the low-loss, broad-band, laminated transmission lines proposed by A. M. Clogston, including both idealized parallel-plane lines and coaxial cables. Part I deals with "Clogston 1" lines, which have laminated conductors with a dielectric, chosen to provide the proper phase velocity for waves on the line, filling the space between the conductors. Part II will treat lines having an arbitrary fraction of their total volume filled with laminations and the rest with dielectric, and will be concerned in particular with "Clogston 2" lines, in which the entire propagation space is occupied by laminated material.

The electromagnetic problem is first formulated in general terms, and then specialized to yield detailed results. The major theoretical questions treated include the determination of the propagation constants and the fields of the principal mode and the higher modes in laminated transmission lines, the choice of optimum proportions for these lines, the calculation of the frequency dependence of attenuation due to the finite thickness of the laminae, the increase in loss caused by improper phase velocity (dielectric mismatch) in Clogston 1 lines and by nonuniformity of the laminated material in Clogston 2 lines, and the effects of dielectric and magnetic dissipation.

TABLE OF CONTENTS

I. Introduction 884
II. Wave Propagation Between Plane and Cylindrical Impedance Sheets . 887
III. Surface Impedance of a Laminated Boundary 896
IV. Principal Mode in Clogston 1 Lines with Infinitesimally Thin Laminae 908
V. Effect of Finite Lamina Thickness. Frequency Dependence of Attenuation in Clogston 1 Lines 921
VI. Effect of Dielectric Mismatch 931
VII. Dielectric and Magnetic Losses in Clogston 1 Lines 940
Appendix I: Bessel Function Expansions 944
Table of Symbols 946

I. INTRODUCTION

A recent theoretical paper[1] by A. M. Clogston presents the very interesting discovery that under certain conditions skin effect losses in the conductors of a transmission line at elevated frequencies can be much reduced by laminating the conducting surfaces, parallel to the direction of current flow, with alternate thin layers of conducting and insulating material. The requirements are that the thickness of each conducting layer must be considerably smaller than the skin depth in the conductor, and the phase velocity of waves on the transmission line must be held very close to a certain critical value, which depends on the relative thicknesses and the electrical properties of the conducting and insulating layers. Under these conditions the "effective skin depth" of the laminated surface is greatly increased; in other words, the eddy currents induced by a high-frequency alternating field will penetrate much farther into such a laminated structure than into a solid conductor, with consequent marked reduction of ohmic losses in the metal. The metal losses can also be made to vary much less with frequency, over a fixed band, than the ordinary skin effect losses, which are known to be very nearly proportional to the square root of frequency.

Clogston goes on to show that a laminated material composed of alternate thin conducting and insulating layers may itself be regarded as a transmission medium. For example, if the space in a coaxial cable which is ordinarily occupied by air or other dielectric be filled with a large number of coaxial cylindrical tubes which are alternately conducting and insulating, the cable will propagate various transmission modes, and under the proper circumstances some of these modes will exhibit lower attenuation constants than the transmission mode in a conventional coaxial cable of the same size at the same frequency.

Experimental verification of Clogston's theory of laminated conductors has been obtained[2] at the Bell Telephone Laboratories, and the transmission properties of a line filled with laminated material have also been measured at these Laboratories and found in reasonable agreement with theory. However experiments with structures as complex as those proposed by Clogston are by no means simple, and the experimental work on laminated conductors is still in an early, exploratory stage. Inasmuch as the experiments are necessarily time-consuming, it has been thought

---

[1] A. M. Clogston, *Proc. Inst. Radio Engrs.*, **39,** 767 (1951), and *Bell System Tech. J.*, **30,** 491 (1951). References will be to the *Bell System Technical Journal* article, although except for equation numbers the two papers are identical.

[2] H. S. Black, C. O. Mallinckrodt, and S. P. Morgan, Jr., Proc. Inst. Radio Engrs., **40,** p. 902 (1952).

desirable to carry out simultaneously as complete a theoretical treatment of Clogston-type transmission lines as possible. Clogston's original paper brought out the fundamental ideas by analysis of idealized transmission lines bounded by infinite parallel planes. The present paper considerably extends the theoretical analysis of parallel-plane systems, and also treats laminated transmission lines bounded by coaxial circular cylinders, which are of course the structures of practical engineering interest.

Part I of this paper deals with both plane and coaxial lines having laminated conductors and having the space between the conductors filled with a suitable main dielectric, which may so far as the theory is concerned also be a nonconducting magnetic material. Structures of this type are called "Clogston 1" transmission lines. Although in principle the total space may be divided between the main dielectric and the laminated stacks in any desired ratio, we suppose in Part I that the width of the main dielectric is several times the total thickness of the laminations. When this is true, the principal mode fields in the main dielectric are almost identical to the fields of the transverse electromagnetic (TEM) mode between perfectly conducting planes or cylinders. The phase velocity is controlled by the properties of the main dielectric, while the attenuation constant is determined by the surface impedances of the laminated boundaries (and the dissipation, if any, in the main dielectric). The calculation of the surface impedance of a laminated plane or cylindrical stack is reduced, using the generalized impedance concept developed by Schelkunoff, to the calculation of the input impedance of a chain of transducers with known impedance elements, the chain also being terminated in a known impedance. We are thus able to employ the language and the results of one-dimensional transmission theory to solve our three-dimensional field problem.

In the remaining sections of Part I we introduce various simplifying approximations and special assumptions into the general equations in order to obtain simple and explicit results. We first calculate the propagation constant and the field components of the principal mode under the assumption that the individual conducting laminae are extremely thin compared to the skin depth at the operating frequency, and show that the attenuation constant is substantially independent of frequency so long as this assumption is valid. We then give formulas for the reduction of the effective skin depth in the stacks and the consequent increase of attenuation with frequency when the laminae are of finite thickness. Next we investigate the effect of varying the phase velocity of the line away from the optimum value given by Clogston; and in the last section we discuss losses due to imperfect dielectrics and lossy magnetic materials.

Part II will be largely devoted to transmission lines of the so-called "Clogston 2" type, in which the entire propagation space is filled with the laminated medium, though to a lesser extent we shall also consider transmission lines having an arbitrary fraction of their total volume filled with laminations and the rest with dielectric. We shall first derive expressions for the propagation constant and the fields of the lowest Clogston 2 mode assuming infinitesimally thin laminae, so that the attenuation constant is essentially independent of frequency, and then go on to investigate the transition of the lowest Clogston 1 mode into the lowest Clogston 2 mode as the space occupied by the main dielectric is gradually filled with laminations. We shall also discuss the higher modes which can exist in Clogston 1 and Clogston 2 lines with infinitesimally thin laminae. Next the effect of finite lamina thickness on the variation of attenuation with frequency in a Clogston 2 will be investigated, and then the important question of the influence of nonuniformity of the laminated medium on the transmission properties of the line. We shall conclude with a short section on dielectric and magnetic losses.

Insofar as possible, plane and coaxial lines will be treated together throughout the paper. Since however Bessel functions are not so easy to manipulate as hyperbolic functions, there will be a few cases where explicit formulas are not yet available for the cylindrical geometry. In these cases the formulas derived for the parallel-plane geometry usually provide reasonably good approximations, or if greater accuracy is desired specific examples may be worked out numerically from the fundamental equations in cylindrical coordinates.

The purpose of the present paper is to set up a general mathematical framework for the analysis of laminated transmission lines, and to treat the major theoretical questions which arise in connection with these lines. In view of the length of the mathematical analysis, we have not devoted much space to numerical examples, although a large number of specific formulas are given which may be used to calculate the theoretical performance of almost any Clogston-type line that happens to be of interest. A considerable part of our work is directed toward evaluating the effects of deviations from the ideal Clogston structure. Both theoretical and experimental results suggest that the limitations on the ultimate applications of the Clogston cable are likely to be imposed by practical problems of manufacture. These limitations, however, depend upon engineering questions which we shall not consider here.

II. WAVE PROPAGATION BETWEEN PLANE AND CYLINDRICAL IMPEDANCE SHEETS

We shall consider waves in a homogeneous, isotropic medium of dielectric constant ϵ, permeability μ, and conductivity g (rationalized MKS units). When convenient we shall also describe the medium in terms of the secondary electromagnetic constants σ and η, defined by

$$\sigma = \sqrt{i\omega\mu(g + i\omega\epsilon)}\,, \qquad \eta = \sqrt{i\omega\mu/(g + i\omega\epsilon)}\,. \tag{1}$$

The quantity σ is called the intrinsic propagation constant and η the intrinsic impedance of the medium.

We begin by considering structures bounded by infinite planes parallel to the x-z coordinate plane, and we confine our attention to transverse magnetic waves propagating in the z-direction. We assume that the only non-vanishing component of magnetic field is H_x, and that all the fields are independent of x. Then the non-zero field components, written to indicate their dependence on the spatial coordinates, are $H_x(y, z)$, $E_y(y, z)$ and $E_z(y, z)$, the time dependence $e^{i\omega t}$ being understood throughout. The field components are shown in Fig. 1.

The field vectors are connected by Maxwell's two curl equations, which reduce in the present case to

$$\begin{aligned} \partial H_x/\partial z &= (g + i\omega\epsilon)E_y\,, \\ \partial H_x/\partial y &= -(g + i\omega\epsilon)E_z\,, \end{aligned} \tag{2}$$

and

$$\partial E_y/\partial z - \partial E_z/\partial y = i\omega\mu H_x\,. \tag{3}$$

If we eliminate E_y and E_z we get

$$\partial^2 H_x/\partial y^2 + \partial^2 H_x/\partial z^2 = \sigma^2 H_x\,, \tag{4}$$

where σ is the intrinsic propagation constant defined above. It is easy to see that (4) is satisfied by a wave function of exponential form, say

$$H_x = e^{-\kappa y - \gamma z}, \tag{5}$$

provided that the constants κ and γ are such that

$$\kappa^2 + \gamma^2 = \sigma^2. \tag{6}$$

We may regard κ and γ as the (possibly complex) propagation constants in the y- and z-directions respectively. Either may be chosen at will and the other is then determined by the condition (6). The electric field components corresponding to any particular H_x are easily obtained from equations (2).

A concept important in what follows is that of wave impedances[3] at a point. For a wave whose field components are H_x, E_y, E_z, the wave impedances looking in the positive and negative y- and z-directions at a typical point are defined to be, respectively,

$$\begin{aligned} Z_y^+ &= E_z/H_x, & Z_z^+ &= -E_y/H_x, \\ Z_y^- &= -E_z/H_x, & Z_z^- &= E_y/H_x. \end{aligned} \tag{7}$$

For waves of the type that we consider, Z_y^+ and Z_y^- are functions of y only, so that if two media having different electrical properties are separated by the plane $y = y_0$, the continuity of the tangential compo-

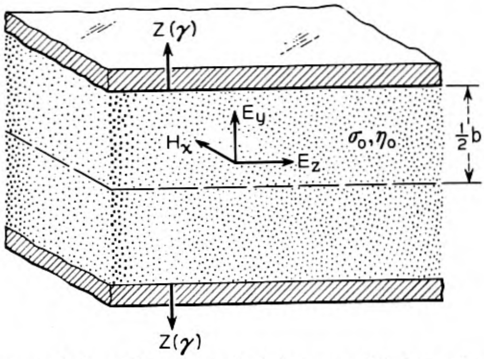


Fig. 1—Transmission line bounded by parallel impedance sheets.

nents of **E** and **H** across the boundary can be assured by merely requiring the continuity of Z_y^+ (say) at $y = y_0$. This is equivalent to the requirement that the sum of the impedances Z_y^+ and Z_y^- looking into the media on opposite sides of the boundary be zero. A similar condition holds for the impedances Z_z^+ and Z_z^- at a boundary $z = z_0$.

As an example of the use of the wave impedance concept, we shall consider the propagation of a transverse magnetic wave between parallel impedance sheets[4] which are separated by a distance b. For the moment nothing is specified about the structure of the sheets except that the normal surface impedance looking into each is $Z(\gamma)$, for a wave whose propagation constant in the z-direction is γ. The fact that in general Z will depend upon γ should be noted, since in some cases this dependence

[3] S. A. Schelkunoff, *Electromagnetic Waves*, D. van Nostrand Co., Inc., New York, 1943, pp. 249–251. Since in our problem three field components vanish identically, we need only two of the six impedances which are defined in the general case.
[4] Reference 3, pp. 484–489.

is quite important. The sheets are located at $y = \pm\frac{1}{2}b$, as shown in Fig. 1, and the space between them is filled with a medium whose electrical constants are ϵ_0, μ_0, g_0 (or σ_0, η_0, if we wish to use the derived constants).

From the symmetry of the boundary conditions it is evident that for any particular mode H_x must be either an even function or an odd function of y about the plane $y = 0$. Taking the even case first, we have

$$\begin{aligned} H_x &= \mathrm{ch}\, \kappa_0 y\, e^{-\gamma z}, \\ E_y &= -\frac{\gamma}{g_0 + i\omega\epsilon_0}\, \mathrm{ch}\, \kappa_0 y\, e^{-\gamma z}, \\ E_z &= -\frac{\kappa_0}{g_0 + i\omega\epsilon_0}\, \mathrm{sh}\, \kappa_0 y\, e^{-\gamma z}, \end{aligned} \tag{8}$$

where

$$\kappa_0^2 + \gamma^2 = \sigma_0^2 . \tag{9}$$

If we replace $g_0 + i\omega\epsilon_0$ by σ_0/η_0 and κ_0 by $(\sigma_0^2 - \gamma^2)^{\frac{1}{2}}$, the boundary condition at $y=\frac{1}{2}b$, namely

$$Z_y^+ = Z(\gamma), \tag{10}$$

becomes

$$\tfrac{1}{2}(\sigma_0^2 - \gamma^2)^{\frac{1}{2}}b \tanh \tfrac{1}{2}(\sigma_0^2 - \gamma^2)^{\frac{1}{2}}b = -\frac{\sigma_0 b}{2\eta_0}\, Z(\gamma). \tag{11}$$

Similarly, the odd case gives

$$\begin{aligned} H_x &= \mathrm{sh}\, \kappa_0 y\, e^{-\gamma z}, \\ E_y &= -\frac{\gamma}{g_0 + i\omega\epsilon_0}\, \mathrm{sh}\, \kappa_0 y\, e^{-\gamma z}, \\ E_z &= -\frac{\kappa_0}{g_0 + i\omega\epsilon_0}\, \mathrm{ch}\, \kappa_0 y\, e^{-\gamma z}; \end{aligned} \tag{12}$$

and the boundary condition becomes

$$\tfrac{1}{2}(\sigma_0^2 - \gamma^2)^{\frac{1}{2}}b \coth \tfrac{1}{2}(\sigma_0^2 - \gamma^2)^{\frac{1}{2}}b = -\frac{\sigma_0 b}{2\eta_0}\, Z(\gamma). \tag{13}$$

The transcendental equations (11) and (13) are satisfied by the propagation constants of the various even and odd modes; presumably each has an infinite number of roots, which we could find, at least in principle, if we knew the explicit form of the function $Z(\gamma)$. We shall confine ourselves here to deriving an approximate expression for the propagation constant of the principal mode (lowest even mode) when the walls are very good conductors.

If the walls were perfectly conducting we should have $Z(\gamma) = 0$, and the lowest root γ_0 of (11) would be given by

$$(\sigma_0^2 - \gamma_0^2)^{\frac{1}{2}} b = 0, \qquad \text{or} \qquad \gamma_0 = \sigma_0 . \tag{14}$$

The principal mode between perfectly conducting sheets is just an undisturbed slice of the plane TEM wave which could propagate in an unbounded medium. If $Z(\gamma_0)$ is not rigorously zero, but still so small that

$$\left| \frac{\sigma_0 b Z(\gamma_0)}{2\eta_0} \right| \ll 1, \tag{15}$$

and if $Z(\gamma)$ does not vary rapidly with γ in the neighborhood of γ_0, then the lowest root of (11) is given approximately by

$$\gamma^2 = \sigma_0^2 + 2\sigma_0 Z(\gamma_0)/\eta_0 b. \tag{16}$$

If $Z(\gamma_0)$ is so small that we have the further inequality

$$\frac{1}{2} \left| \frac{Z(\gamma_0)}{\sigma_0 b \eta_0} \right|^2 \ll 1, \tag{17}$$

then (16) yields the approximation

$$\gamma = \sigma_0 + Z(\gamma_0)/\eta_0 b, \tag{18}$$

where the second term is the first-order change in γ due to the finite impedance of the walls. If we formally set $g_0 = 0$ (this does not actually restrict us to perfect dielectrics since we could still assume ϵ_0 or μ_0 to be complex), we have

$$\sigma = i\omega\sqrt{\mu_0\epsilon_0}\,, \qquad \eta = \sqrt{\mu_0/\epsilon_0}\,. \tag{19}$$

If the medium between the sheets is lossless, the attenuation and phase constants of the principal mode become

$$\alpha = \operatorname{Re} \gamma = \operatorname{Re} Z(\gamma_0)/\eta_0 b, \tag{20}$$

$$\beta = \operatorname{Im} \gamma = \omega\sqrt{\mu_0\epsilon_0} + \operatorname{Im} Z(\gamma_0)/\eta_0 b. \tag{21}$$

Although the fields of the principal mode between perfectly conducting walls are entirely transverse to the direction of propagation, if the walls are not perfectly conducting there will also be a small longitudinal component E_z of electric field associated with this mode. The leading terms in the expressions for the field components, as obtained from equations (8), (9), and (16), are

$$\begin{aligned} H_x &\approx H_0 e^{-\gamma z}, \\ E_y &\approx -\eta_0 H_0 e^{-\gamma z}, \\ E_z &\approx \frac{2Z_0(\gamma_0)H_0 y}{b}\, e^{-\gamma z}, \end{aligned} \tag{22}$$

where H_0 is an arbitrary amplitude factor.

As an example of the use of (20) and (21), suppose that the impedance sheets in Fig. 1 are electrically thick metal walls of permeability μ_1 and (high) conductivity g_1. Then to a very good approximation at all engineering frequencies and for all ordinary dielectrics between the walls, the surface impedance is

$$Z(\gamma_0) = (1 + i)/g_1\delta_1 , \tag{23}$$

where

$$\delta_1 = \sqrt{2/\omega\mu_1 g_1} \tag{24}$$

is the skin depth in the metal. We thus obtain from (20) and (21) the familiar formulas

$$\alpha = 1/\eta_0 b g_1 \delta_1 , \tag{25}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + 1/\eta_0 b g_1 \delta_1 . \tag{26}$$

It should be noted that in practical cases the inequality (17) on which we based the approximations (20) and (21) does not hold down to the mathematical limit of zero frequency. In the present paper, however, when we speak of "low frequencies" we shall mean frequencies still high enough so that the approximations (20) and (21) for α and β are valid. Generally this will be equivalent to the assumption that the attenuation per radian is small. In our applications this assumption will usually be justified down to frequencies of the order of a few $\text{kc}\cdot\text{sec}^{-1}$.

Now let us consider transmission lines bounded by coaxial circular cylinders and confine our attention to circular transverse magnetic waves propagating in the z-direction. For these waves the fields are independent of the angle ϕ, and the only non-vanishing field components are $H_\phi(\rho, z)$, $E_\rho(\rho, z)$, and $E_z(\rho, z)$. The field components are shown in Fig. 2.

For circular transverse magnetic fields Maxwell's curl equations in a homogeneous, isoptropic medium reduce to

$$\begin{aligned} \partial H_\phi/\partial z &= -(g + i\omega\epsilon)E_\rho , \\ \partial(\rho H_\phi)/\partial\rho &= (g + i\omega\epsilon)\rho E_z , \end{aligned} \tag{27}$$

and

$$\partial E_z/\partial \rho - \partial E_\rho/\partial z = i\omega\mu H_\phi , \tag{28}$$

from which we can eliminate $E\rho$ and E_z to obtain

$$\frac{\partial^2 H\phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_\phi}{\partial \rho} - \frac{H_\phi}{\rho^2} + \frac{\partial^2 H_\phi}{\partial z^2} = \sigma^2 H_\phi. \tag{29}$$

If we assume a wave traveling in the positive z-direction with propagation constant γ and write

$$H_\phi(\rho, z) = R(\rho)e^{-\gamma z}, \tag{30}$$

we find that (29) becomes

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right) - \left(\kappa^2 + \frac{1}{\rho^2}\right)R = 0, \tag{31}$$

where κ is given by (6) as before. But (31) is just the equation satisfied by modified Bessel functions of order one and argument $\kappa\rho$, so

$$R(\rho) = AI_1(\kappa\rho) + BK_1(\kappa\rho), \tag{32}$$

where A and B are arbitrary constants. The other field components can be obtained from H_ϕ using (27); the results are

$$\begin{aligned} H_\phi &= [AI_1(\kappa\rho) + BK_1(\kappa\rho)]e^{-\gamma z}, \\ E\rho &= \frac{\gamma}{g + i\omega\epsilon}[AI_1(\kappa\rho) + BK_1(\kappa\rho)]e^{-\gamma z}, \\ E_z &= \frac{\kappa}{g + i\omega\epsilon}[AI_0(\kappa\rho) - BK_0(\kappa\rho)]e^{-\gamma z}. \end{aligned} \tag{33}$$

For cylindrical fields of the type that we are considering, the wave impedances looking in the positive and negative ρ- and z-directions at a typical point are defined to be, respectively,

$$\begin{aligned} Z_\rho^+ &= -E_z/H_\phi , & Z_z^+ &= E_\rho/H_\phi , \\ Z_\rho^- &= E_z/H_\phi , & Z_z^- &= -E_\rho/H_\phi . \end{aligned} \tag{34}$$

We shall now discuss the propagation of circular transverse magnetic waves in a homogeneous region of space whose electrical constants are ϵ_0 , μ_0 , g_0 (or σ_0 , η_0), and which is bounded by coaxial cylinders of radii ρ_1 and ρ_2 , where $\rho_2 > \rho_1$, as shown in Fig. 2. We suppose that the radial impedances looking from the main dielectric into the inner and outer cylinders are, respectively,

$$Z_\rho^-\big|_{\rho=\rho_1} = Z_1(\gamma), \qquad Z_\rho^+\big|_{\rho=\rho_2} = Z_2(\gamma). \tag{35}$$

Then from (33) and (34) the boundary conditions are

$$\begin{aligned} \eta_{0\rho} \frac{AI_0(\kappa_0\rho_1) - BK_0(\kappa_0\rho_1)}{AI_1(\kappa_0\rho_1) + BK_1(\kappa_0\rho_1)} &= Z_1(\gamma), \\ \eta_{0\rho} \frac{AI_0(\kappa_0\rho_2) - BK_0(\kappa_0\rho_2)}{AI_1(\kappa_0\rho_2) + BK_1(\kappa_0\rho_2)} &= -Z_2(\gamma), \end{aligned} \tag{36}$$

where

$$\kappa_0 = (\sigma_0^2 - \gamma^2)^{\frac{1}{2}}, \qquad \eta_{0\rho} = \frac{\kappa_0}{g_0 + i\omega\epsilon_0} = \eta_0(1 - \gamma^2/\sigma_0^2)^{\frac{1}{2}}. \tag{37}$$

If equations (36) are to be satisfied by values of A and B which are not both zero, it is easily shown that a necessary and sufficient condition is

$$\frac{\eta_{0\rho}K_0(\kappa_0\rho_1) + Z_1(\gamma)K_1(\kappa_0\rho_1)}{\eta_{0\rho}I_0(\kappa_0\rho_1) - Z_1(\gamma)I_1(\kappa_0\rho_1)} = \frac{\eta_{0\rho}K_0(\kappa_0\rho_2) - Z_2(\gamma)K_1(\kappa_0\rho_2)}{\eta_{0\rho}I_0(\kappa_0\rho_2) + Z_2(\gamma)I_1(\kappa_0\rho_2)}, \tag{38}$$

and (38) is a transcendental equation for the determination of the propagation constants of all the circular magnetic modes in the coaxial line.

As in the discussion of the parallel-plane line, we shall confine our attention to the principal mode and shall assume forthwith that the wall losses are small.[5] Since for the principal mode we expect that γ will be nearly equal to σ_0, we may write γ_0 for σ_0 and evaluate Z_1 and Z_2 at γ_0; and we may replace the modified Bessel functions in (38) by their ap-

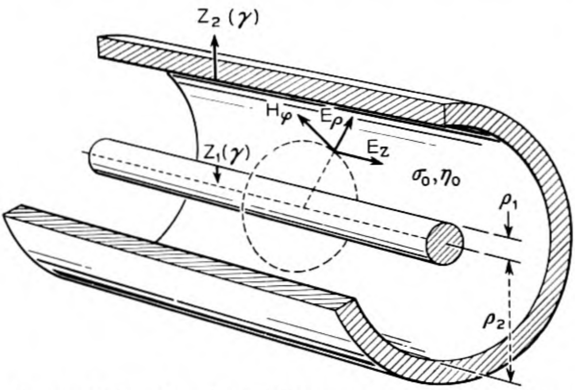


Fig. 2—Transmission line bounded by coaxial impedance cylinders.

[5] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941, pp. 551–554, gives a similar treatment of the principal mode in an ordinary coaxial cable with solid metal walls.

proximate values for small argument. From the series given in Dwight[6] 813.1, 813.2, 815.1, and 815.2, we have

$$
\begin{aligned}
I_0(x) &\approx 1, \\
I_1(x) &\approx \tfrac{1}{2}x, \\
K_0(x) &\approx -(0.5772 + \log \tfrac{1}{2}x) = -\log 0.8905x, \qquad (39) \\
K_1(x) &\approx \frac{1}{x} + \tfrac{1}{2}x \log 0.8905x,
\end{aligned}
$$

for $|x| \ll 1$, where log represents the natural logarithm. If we put these approximations into (38) and if we suppose that the wall impedances are so small that

$$| \sigma_0 \rho_1 Z_1(\gamma_0)/2\eta_0 | \ll 1, \qquad | \sigma_0 \rho_2 Z_2(\gamma_0)/2\eta_0 | \ll 1, \tag{40}$$

we obtain, after a little algebra,

$$\kappa_0^2 = \sigma_0^2 - \gamma^2 = -\frac{\sigma_0[Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2]}{\eta_0 \log (\rho_2/\rho_1)}. \tag{41}$$

Now further assuming that

$$\frac{1}{8} \left| \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{\sigma_0 \eta_0 \log (\rho_2/\rho_1)} \right|^2 \ll 1, \tag{42}$$

we get by the binomial theorem

$$\gamma = \sigma_0 + \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}. \tag{43}$$

If we formally set $g_0 = 0$, we find that the attenuation and phase constants of the principal mode in a coaxial line with low-loss walls and no dissipation in the main dielectric are

$$\alpha = \text{Re}\,\gamma = \text{Re}\,\frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}, \tag{44}$$

$$\beta = \text{Im}\,\gamma = \omega\sqrt{\mu_0 \epsilon_0} + \text{Im}\,\frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}. \tag{45}$$

As before, these approximations for α and β will ultimately break down as the frequency approaches zero, but they will certainly be valid over the frequency range in which we are interested in the present paper.

---

[6] H. B. Dwight, *Tables of Integrals and Other Mathematical Data*, Revised Edition, Macmillan, New York, 1947. We shall refer to Dwight for a number of standard series expansions.

The magnetic field lines of the principal mode will of course be circles and the electric field will be largely radial, but with a small longitudinal component unless the wall impedances are rigorously zero. The general expressions (33) for the fields may be reduced to simple approximate formulas if we use the fact that κ_0^2 is given by (41) and $\kappa_0\rho$ is small compared to unity. The ratio A/B may be obtained from either of equations (36). Introducing the approximations (39) for the Bessel functions and carrying out a little algebra, we get the following approximate expressions for the fields:

$$
\begin{aligned}
H_\phi &\approx \frac{I}{2\pi\rho}\, e^{-\gamma z}, \\
E_\rho &\approx \frac{\eta_0 I}{2\pi\rho}\, e^{-\gamma z}, \\
E_z &\approx \frac{I}{2\pi \log(\rho_2/\rho_1)} \left[\frac{Z_1(\gamma_0)}{\rho_1} \log\frac{\rho_2}{\rho} + \frac{Z_2(\gamma_0)}{\rho_2} \log\frac{\rho_1}{\rho} \right] e^{-\gamma z},
\end{aligned}
\tag{46}
$$

where the amplitude factor I is equal to the total current flowing in the inner cylinder. Incidentally we note that the above results might have been derived from more elementary arguments if we had started with the fields in a coaxial line with perfectly conducting walls and treated the effect of finite wall impedance as a small perturbation.

If we consider an ordinary coaxial cable with solid metal walls at a frequency high enough so that there is a well-developed skin effect on both conductors, then to a good approximation

$$Z_1(\gamma_0) = Z_2(\gamma_0) = (1 + i)/g_1\delta_1\,, \tag{47}$$

where g_1 and δ_1 are the conductivity and the skin thickness of the metal; and the attenuation and phase constants are given by the well-known expressions

$$\alpha = \frac{1/\rho_1 + 1/\rho_2}{2\eta_0 g_1 \delta_1 \log(\rho_2/\rho_1)}\,, \tag{48}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \frac{1/\rho_1 + 1/\rho_2}{2\eta_0 g_1 \delta_1 \log(\rho_2/\rho_1)}\,. \tag{49}$$

If necessary we may take account of dissipation in the main dielectric of either a plane or a coaxial transmission line by assigning complex values[7] to ϵ_0 and μ_0, say

---

[7] See, for example, C. G. Montgomery, *Principles of Microwave Circuits*, M. I. T. Rad. Lab. Series, **8**, McGraw-Hill, New York, 1948, pp. 365–369 and 382–385.

$$\begin{aligned} \epsilon_0 &= \epsilon_0' - i\epsilon_0'' = \epsilon_0'(1 - i \tan \phi_0), \\ \mu_0 &= \mu_0' - i\mu_0'' = \mu_0'(1 - i \tan \zeta_0), \end{aligned} \tag{50}$$

where $\tan \phi_0$ is the dielectric loss tangent and $\tan \zeta_0$ is the magnetic loss tangent (if any). Inserting (50) into (18) or (43), we find for the attenuation due to dielectric and magnetic losses,

$$\begin{aligned} \alpha_d &= \text{Re } \sigma = \text{Re } i\omega\sqrt{\mu_0'\epsilon_0'(1 - i \tan \phi_0)(1 - i \tan \zeta_0)} \\ &= \tfrac{1}{2}\omega\sqrt{\mu_0'\epsilon_0'}\,(\tan \phi_0 + \tan \zeta_0), \end{aligned} \tag{51}$$

provided that $\tan \phi_0$ and $\tan \zeta_0$ are both small compared to unity, as they will always be in practice. We shall neglect second-order effects and so regard the dielectric losses, the magnetic losses, and the wall losses as additive.

III. SURFACE IMPEDANCE OF A LAMINATED BOUNDARY

The main problem in the theory of Clogston 1 transmission lines is the computation of the surface impedance of a laminated plane or cylindrical boundary having alternate thin layers of conductor and dielectric. Portions of such laminated structures are shown schematically in Figs. 3 and 4. We shall begin with an analysis, similar to Clogston's,[8] of the plane stack. This will lead to a convenient point of view for the treatment of the mathematically more complicated coaxial stack.

Let us consider a wave with field components H_x, E_y, E_z, propagating

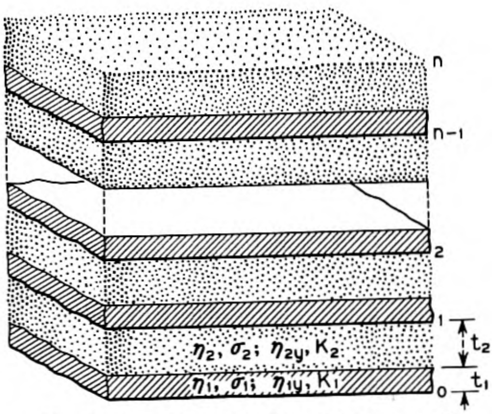


Fig. 3—Portion of laminated plane stack.

[8] Reference 1, Section III.

in a layer of homogeneous, isotropic material whose electrical constants are ϵ, μ, g (or σ, η), and which is bounded by planes perpendicular to the y-axis. Henceforth we shall always assume that the z-dependence of every field component is given by the factor $e^{-\gamma z}$, where the complex quantity γ, whose value may or may not be known a priori, is the propagation constant of the wave in the z-direction. Then the first of Maxwell's equations (2) yields

$$E_y = -[\gamma/(g + i\omega\epsilon)]H_x , \tag{52}$$

and on eliminating E_y from the other Maxwell equations, we get

$$\begin{aligned} \partial H_x/\partial y &= -(g + i\omega\epsilon)E_z , \\ \partial E_z/\partial y &= -[\kappa^2/(g + i\omega\epsilon)]H_x , \end{aligned} \tag{53}$$

where κ^2 is defined by equation (6).

Now if we formally identify H_x with "current" and E_z with "voltage", equations (53) are just the equations of a uniform one-dimensional transmission line extending in the y-direction, with series impedance $\kappa^2/(g + i\omega\epsilon)$ per unit length and shunt admittance $(g + i\omega\epsilon)$ per unit length; in other words a transmission line whose propagation constant is κ and whose characteristic impedance is η_y, where

$$\kappa = \sigma(1 - \gamma^2/\sigma^2)^{\frac{1}{2}}, \qquad \eta_y = \kappa/(g + i\omega\epsilon) = \eta(1 - \gamma^2/\sigma^2)^{\frac{1}{2}}. \tag{54}$$

Hence we can apply the whole theory of one-dimensional transmission lines with the assurance that in so doing we shall not violate the field equations. For example, if $E(0)$, $H(0)$ and $E(t)$, $H(t)$ represent the tangential field components E_z, H_x at two planes separated by a dis-

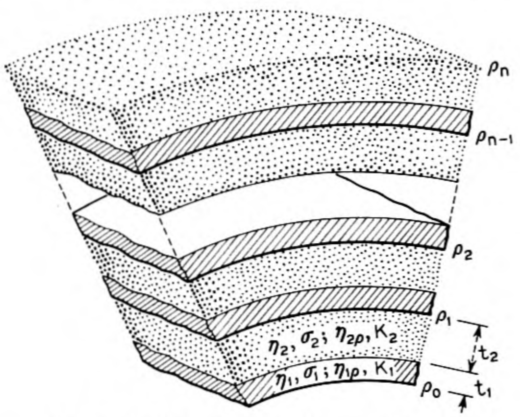


Fig. 4—Portion of laminated coaxial stack.

tance t, these fields are related by the general circuit parameter matrix of a uniform line, namely

$$\begin{pmatrix} E(0) \\ \\ H(0) \end{pmatrix} = \begin{pmatrix} \text{ch } \kappa t & \eta_y \text{ sh } \kappa t \\ \\ \dfrac{\text{sh } \kappa t}{\eta_y} & \text{ch } \kappa t \end{pmatrix} \begin{pmatrix} E(t) \\ \\ H(t) \end{pmatrix}. \tag{55}$$

We are now in a position to determine the surface impedance normal to a laminated plane structure composed of layers of which every other one has thickness t_1 and electrical constants σ_1, η_1, while the intervening layers each have thickness t_2 and electrical constants σ_2, η_2. Fig. 3 shows the cross section of such a stack in which the total number of double layers is n ($2n$ single layers), while Fig. 4 represents the corresponding coaxial stack. Ultimately we shall assume the layers of thickness t_1 to be good conductors and those of thickness t_2 to be good insulators, but these assumptions need not be brought in immediately.

If the fields in the plane stack all vary with z according to $e^{-\gamma z}$, then when we look in the direction of increasing y each double layer may be regarded as a four-terminal network formed by two sections of uniform transmission line of lengths t_1 and t_2, the propagation constants and characteristic impedances of the two sections being given respectively by

$$\begin{aligned} \kappa_1 &= \sigma_1(1 - \gamma^2/\sigma_1^2)^{\frac{1}{2}}, & \eta_{1y} &= \eta_1(1 - \gamma^2/\sigma_1^2)^{\frac{1}{2}}, \\ \kappa_2 &= \sigma_2(1 - \gamma^2/\sigma_2^2)^{\frac{1}{2}}, & \eta_{2y} &= \eta_2(1 - \gamma^2/\sigma_2^2)^{\frac{1}{2}}. \end{aligned} \tag{56}$$

The matrix of the double layer is the product of the matrices of the two single layers in the proper order. Thus if the tangential field components are E_0, H_0 at the lower surface of the first layer and E_1, H_1 at the upper surface of the second layer, we have

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix} \begin{pmatrix} E_1 \\ H_1 \end{pmatrix}, \tag{57}$$

where

$$\begin{aligned} \mathcal{A} &= \text{ch } \kappa_1 t_1 \text{ ch } \kappa_2 t_2 + \frac{\eta_{1y}}{\eta_{2y}} \text{ sh } \kappa_1 t_1 \text{ sh } \kappa_2 t_2, \\ \mathcal{B} &= \eta_{2y} \text{ ch } \kappa_1 t_1 \text{ sh } \kappa_2 t_2 + \eta_{1y} \text{ sh } \kappa_1 t_1 \text{ ch } \kappa_2 t_2, \\ \mathcal{C} &= \frac{1}{\eta_{1y}} \text{ sh } \kappa_1 t_1 \text{ ch } \kappa_2 t_2 + \frac{1}{\eta_{2y}} \text{ ch } \kappa_1 t_1 \text{ sh } \kappa_2 t_2, \\ \mathcal{D} &= \frac{\eta_{2y}}{\eta_{1y}} \text{ sh } \kappa_1 t_1 \text{ sh } \kappa_2 t_2 + \text{ch } \kappa_1 t_1 \text{ ch } \kappa_2 t_2. \end{aligned} \tag{58}$$

The stack of double layers may be regarded as a chain of iterated four-poles; such chains have an extensive literature.[9] The relation between the tangential fields E_n, H_n at the upper surface of the nth double layer and E_0, H_0 at the lower surface of the first double layer is

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = \mathbf{M}^n \begin{pmatrix} E_n \\ H_n \end{pmatrix}, \tag{59}$$

where $\mathbf{M}$ is the $\mathcal{ABCD}$-matrix appearing in equation (57). However there is a simple expression[10] for the nth power of a square matrix of order two, namely

$$\mathbf{M}^n = M^{\frac{1}{2}(n-1)} \frac{\operatorname{sh} n\Gamma}{\operatorname{sh} \Gamma} \mathbf{M} - M^{\frac{1}{2}n} \frac{\operatorname{sh} (n-1)\Gamma}{\operatorname{sh} \Gamma} \mathbf{I}, \tag{60}$$

where $\mathbf{I}$ is the unit matrix of order two, Γ is the propagation constant per section of the chain of four-poles, defined by

$$\operatorname{ch} \Gamma = (\mathcal{A} + \mathcal{D})/2M^{\frac{1}{2}}, \tag{61}$$

and M is the determinant of the matrix $\mathbf{M}$, that is,

$$M = \mathcal{A}\mathcal{D} - \mathcal{B}\mathcal{C}. \tag{62}$$

The determinant of the matrix whose elements are given by (58) is unity, as may easily be verified; but this may not be the case for all the matrices which occur in our study of cylindrical structures. M will therefore be carried explicitly in the following equations.

We now introduce the iterative impedances K_1 and K_2, defined by

$$\begin{aligned} K_1 &= \frac{(\mathcal{A} - \mathcal{D}) + \sqrt{(\mathcal{A} + \mathcal{D})^2 - 4M}}{2\mathcal{C}}, \\ K_2 &= \frac{-(\mathcal{A} - \mathcal{D}) + \sqrt{(\mathcal{A} + \mathcal{D})^2 - 4M}}{2\mathcal{C}}. \end{aligned} \tag{63}$$

K_1 is the impedance seen when we look into a semi-infinite stack of double layers if the first layer is of type 1, while K_2 is the impedance seen if the first layer is of type 2. In calculations relating to Clogston 1 lines with dissipative walls, the real parts of K_1 and K_2 will both be positive. By a straightforward procedure we may express the matrix elements $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ in terms of K_1, K_2, Γ, and M, and then transform equation

[9] See, for example, E. A. Guillemin, *Communication Networks*, **2**, Wiley, New York, 1935, pp. 161–166.

[10] F. Abelès, *Comptes Rendus*, **226**, 1872 (1948). This result was called to the author's attention by Mr. J. G. Kreer.

(60) into

$$\mathbf{M}^n = \frac{2M^{\frac{1}{2}n}}{(K_1 + K_2)} \begin{pmatrix} \frac{1}{2}(K_1e^{n\Gamma} + K_2e^{-n\Gamma}) & K_1K_2 \operatorname{sh} n\Gamma \\ \operatorname{sh} n\Gamma & \frac{1}{2}(K_1e^{-n\Gamma} + K_2e^{n\Gamma}) \end{pmatrix}. \tag{64}$$

Finally we obtain from (59) and (64) an expression for the impedance Z_0 looking into a plane stack of n double layers when the nth layer is backed by a surface whose impedance is Z_n, namely

$$Z_0 = \frac{E_0}{H_0} = \frac{\frac{1}{2}Z_n(K_1e^{n\Gamma} + K_2e^{-n\Gamma}) + K_1K_2 \operatorname{sh} n\Gamma}{Z_n \operatorname{sh} n\Gamma + \frac{1}{2}(K_1e^{-n\Gamma} + K_2e^{n\Gamma})}. \tag{65}$$

For the cylindrical geometry, matters are a good deal more complicated. If we consider waves having field components H_ϕ, E_ρ, E_z in a homogeneous, isotropic shell bounded by coaxial cylindrical surfaces, and assume a propagation factor $e^{-\gamma z}$, Maxwell's equations (27) and (28) may be written

$$E_\rho = [\gamma/(g + i\omega\epsilon)]H_\phi, \tag{66}$$

and

$$\begin{aligned} \partial(-\rho H_\phi)/\partial\rho &= -(g + i\omega\epsilon)\rho E_z, \\ \partial E_z/\partial\rho &= -[\kappa^2/(g + i\omega\epsilon)\rho](-\rho H_\phi). \end{aligned} \tag{67}$$

If desired, we might identify E_z with "voltage" and $-\rho H_\phi$ with "current" and regard equations (67) as describing a nonuniform radial transmission line, having series impedance $\kappa^2/(g + i\omega\epsilon)\rho$ per unit length and shunt admittance $(g + i\omega\epsilon)\rho$ per unit length. Since, however, in equations (34) we have already defined the radial wave impedance to be a field ratio without the extra factor of ρ, we shall carry out the analysis of the present paper directly in terms of the field components E_z and $-H_\phi$.

From the general expressions (33) for the fields in cylindrical coordinates, we can show that the matrix relation between the tangential field components E_z, $-H_\phi$ at two radii ρ_1 and ρ_2 is given by

$$\begin{pmatrix} E(\rho_1) \\ -H(\rho_1) \end{pmatrix} = \begin{pmatrix} \kappa\rho_2(K_{01}I_{12} + K_{12}I_{01}) & \eta_\rho\kappa\rho_2(K_{01}I_{02} - K_{02}I_{01}) \\ \frac{\kappa\rho_2}{\eta_\rho}(K_{11}I_{12} - K_{12}I_{11}) & \kappa\rho_2(K_{11}I_{02} + K_{02}I_{11}) \end{pmatrix} \begin{pmatrix} E(\rho_2) \\ -H(\rho_2) \end{pmatrix}, \tag{68}$$

where

$$\kappa = (\sigma^2 - \gamma^2)^{\frac{1}{2}}, \qquad \eta_\rho = \eta(1 - \gamma^2/\sigma^2)^{\frac{1}{2}}, \tag{69}$$

and we have used the abbreviations

$$I_{rs} = I_r(\kappa\rho_s), \qquad K_{rs} = K_r(\kappa\rho_s). \tag{70}$$

It may be verified that the determinant M of the square matrix appearing in (68) is simply

$$M = \rho_2/\rho_1 . \tag{71}$$

In principle equation (68) permits us to determine by matrix multiplication the relation between the tangential fields at the inner and outer surfaces of a coaxial double layer, or of a laminated stack of any number of double layers, such as is shown in Fig. 4. The difficulty is that the elements of the matrix of a single layer are not functions only of the electrical properties of the layer and its thickness, but depend in a more complicated way on the inner and outer radii separately. Whereas in the plane case we had merely to take the nth power of a single matrix, we are now faced with the problem of multiplying together n matrices, each of which differs more or less from all the others. An exact expression for the result is practically out of the question; but we can make some reasonable approximations if we assume that each individual layer is thin compared to its mean radius, so that the matrix elements do not change much from one layer to the next.

If the thickness t $(= \rho_2 - \rho_1)$ of a single layer is small compared to ρ_1, then the Bessel function combinations appearing in (68) may be expanded in series, as shown in Appendix I, and the circuit parameter matrix takes the following approximate form,

$$\begin{pmatrix} \left[1 + \dfrac{t}{2\rho_1}\right] \operatorname{ch} \kappa t - \dfrac{1}{2\kappa\rho_1} \operatorname{sh} \kappa t & \eta_\rho \left[1 + \dfrac{t}{2\rho_1}\right] \operatorname{sh} \kappa t \\ \dfrac{1}{\eta_\rho} \left[1 + \dfrac{t}{2\rho_1}\right] \operatorname{sh} \kappa t & \left[1 + \dfrac{t}{2\rho_1}\right] \operatorname{ch} \kappa t + \dfrac{1}{2\kappa\rho_1} \operatorname{sh} \kappa t \end{pmatrix}, \tag{72}$$

where terms of the order of t/ρ_1 represent the first-order curvature corrections. If we use the same value of ρ_1, say $\bar{\rho}$, for both parts of a double layer, then up to first order the elements of the matrix of the double layer become

$$
\begin{aligned}
\mathfrak{A} &= \left[1 + \frac{t_1 + t_2}{2\bar{\rho}}\right]\left[\text{ch } \kappa_1 t_1 \text{ ch } \kappa_2 t_2 + \frac{\eta_{1\rho}}{\eta_{2\rho}} \text{ sh } \kappa_1 t_1 \text{ sh } \kappa_2 t_2\right] \\
&\qquad - \left[\frac{1}{2\kappa_1\bar{\rho}} \text{ sh } \kappa_1 t_1 \text{ ch } \kappa_2 t_2 + \frac{1}{2\kappa_2\bar{\rho}} \text{ ch } \kappa_1 t_1 \text{ sh } \kappa_2 t_2\right], \\
\mathfrak{B} &= \left[1 + \frac{t_1 + t_2}{2\bar{\rho}}\right]\left[\eta_{2\rho} \text{ ch } \kappa_1 t_1 \text{ sh } \kappa_2 t_2 + \eta_{1\rho} \text{ sh } \kappa_1 t_1 \text{ ch } \kappa_2 t_2\right] \\
&\qquad + \left[\frac{\eta_{1\rho}}{2\kappa_2\bar{\rho}} - \frac{\eta_{2\rho}}{2\kappa_1\bar{\rho}}\right] \text{ sh } \kappa_1 t_1 \text{ sh } \kappa_2 t_2, \\
\mathfrak{C} &= \left[1 + \frac{t_1 + t_2}{2\bar{\rho}}\right]\left[\frac{1}{\eta_{1\rho}} \text{ sh } \kappa_1 t_1 \text{ ch } \kappa_2 t_2 + \frac{1}{\eta_{2\rho}} \text{ ch } \kappa_1 t_1 \text{ sh } \kappa_2 t_2\right] \\
&\qquad + \left[\frac{1}{2\eta_{2\rho}\kappa_1\bar{\rho}} - \frac{1}{2\eta_{1\rho}\kappa_2\bar{\rho}}\right] \text{ sh } \kappa_1 t_1 \text{ sh } \kappa_2 t_2, \\
\mathfrak{D} &= \left[1 + \frac{t_1 + t_2}{2\bar{\rho}}\right]\left[\frac{\eta_{2\rho}}{\eta_{1\rho}} \text{ sh } \kappa_1 t_1 \text{ sh } \kappa_2 t_2 + \text{ch } \kappa_1 t_1 \text{ ch } \kappa_2 t_2\right] \\
&\qquad + \left[\frac{1}{2\kappa_1\bar{\rho}} \text{ sh } \kappa_1 t_1 \text{ ch } \kappa_2 t_2 + \frac{1}{2\kappa_2\bar{\rho}} \text{ ch } \kappa_1 t_1 \text{ sh } \kappa_2 t_2\right].
\end{aligned} \tag{73}
$$

As in the analogous equations (58) for a plane double layer, the subscripts 1 and 2 refer to the first and second layers respectively.

If we have a stack of double layers in which all the layers of the same kind have the same thickness and same electrical constants, then the only term in (73) which varies from one double layer to the next is the mean radius $\bar{\rho}$. Depending on the circumstances, we may wish to use a single value of $\bar{\rho}$ for the whole stack, or a few different values, or even, if high-speed computing machinery is available to carry out the matrix multiplications, a different value of $\bar{\rho}$ for each double layer. The matrix of the whole stack then becomes a product of powers of as many different matrices as we have chosen values of $\bar{\rho}$. Obviously this method is better adapted to the numerical analysis of special cases than to the general theoretical treatment of a stack whose ratio of outer radius to inner radius is unspecified.

In principle we are now able to compute the normal surface impedance of any laminated plane or coaxial stack at a given frequency provided that we know the electrical constants and the thickness of each layer, the number of layers, the propagation constant γ in the z-direction, and the normal impedance Z_n of the material behind the last layer. Since the general formulas even for plane stacks are quite complicated, however, we shall introduce at this point some very good approximations which will be valid for all of the following work.

Henceforth we shall take the layers of thickness t_1 to be such good conductors that the ratio $\omega\epsilon_1/g_1$ of displacement current to conduction current is negligible in comparison with unity. For metals like copper this is an excellent approximation at even the highest engineering frequencies. Then on introducing the characteristic skin thickness δ_1, we have for the conducting layers,

$$\begin{aligned} \sigma_1 &= \sqrt{i\omega\mu_1 g_1} = (1 + i)/\delta_1 , \\ \eta_1 &= \sqrt{i\omega\mu_1/g_1} = (1 + i)/g_1\delta_1 , \end{aligned} \tag{74}$$

where

$$\delta_1 = \sqrt{2/\omega\mu_1 g_1} . \tag{75}$$

For pure copper the permeability and conductivity are

$$\begin{aligned} \mu_1 &= 1.257 \times 10^{-6} \text{ henrys}\cdot\text{meter}^{-1}, \\ g_1 &= 5.800 \times 10^{7} \text{ mhos}\cdot\text{meter}^{-1}, \end{aligned} \tag{76}$$

from which we obtain the numerical values

$$\begin{aligned} \sigma_1 &= 1.513 \times 10^{4}\ (1 + i)\sqrt{f_{\mathrm{Mc}}} \text{ meters}^{-1}, \\ \eta_1 &= 2.609 \times 10^{-4}\ (1 + i)\sqrt{f_{\mathrm{Mc}}} \text{ ohms}, \end{aligned} \tag{77}$$

and

$$\delta_1 = \frac{6.609 \times 10^{-5}}{\sqrt{f_{\mathrm{Mc}}}} \text{ meters} = \frac{2.602}{\sqrt{f_{\mathrm{Mc}}}} \text{ mils}, \tag{78}$$

where f_{Mc} is the frequency in $\mathrm{Mc}\cdot\mathrm{sec}^{-1}$. Referring to equations (56) and (69) and bearing in mind the above numerical values, we see that for the conducting layers we have

$$\begin{aligned} \kappa_1 &\approx \sigma_1 = (1 + i)/\delta_1 , \\ \eta_{1y} &= \eta_{1\rho} \approx \eta_1 = (1 + i)/g_1\delta_1 , \end{aligned} \tag{79}$$

to a very good approximation, since in our applications the quantity γ will always be of the order of $2\pi i/\lambda_v$, where the vacuum wavelength λ_v is at least a few meters, while the skin thickness δ_1 will be at most a small fraction of a centimeter.

For the insulating layers of thickness t_2 we shall set the conductivity g_2 equal to zero, so that

$$\sigma_2 = i\omega\sqrt{\mu_2\epsilon_2} , \qquad \eta_2 = \sqrt{\mu_2/\epsilon_2} . \tag{80}$$

We denote the *relative* dielectric constant and permeability by ϵ_{2r} and μ_{2r} respectively; dissipation in the insulating layers may be included if necessary by making ϵ_{2r} and/or μ_{2r} complex. In MKS units we have

$$\epsilon_2 = \epsilon_{2r}\epsilon_v, \qquad \mu_2 = \mu_{2r}\mu_v, \tag{81}$$

where the electrical constants of vacuum are

$$\begin{aligned} \epsilon_v &= 8.854 \times 10^{-12} \text{ farads}\cdot\text{meter}^{-1}, \\ \mu_v &= 1.257 \times 10^{-6} \text{ henrys}\cdot\text{meter}^{-1}. \end{aligned} \tag{82}$$

It follows that

$$\begin{aligned} \sigma_2 &= \sigma_v\sqrt{\mu_{2r}\epsilon_{2r}} = \frac{2\pi i\sqrt{\mu_{2r}\epsilon_{2r}}}{\lambda_v} = \frac{2\pi i f_{\text{Mc}}\sqrt{\mu_{2r}\epsilon_{2r}}}{299.8} \text{ meters}^{-1}, \\ \eta_2 &= \eta_v\sqrt{\mu_{2r}/\epsilon_{2r}} = 376.7\sqrt{\mu_{2r}/\epsilon_{2r}} \text{ ohms}, \end{aligned} \tag{83}$$

where as usual the subscript v refers to vacuum. It is clear that unless we deal with ferromagnetics, the quantities σ_2 and η_2 will be of roughly the same order of magnitude as σ_v and η_v. From (56) and (69) we have

$$\begin{aligned} \kappa_2 &= \sigma_2(1 - \gamma^2/\sigma_2^2)^{\frac{1}{2}}, \\ \eta_{2y} &= \eta_{2\rho} = \eta_2(1 - \gamma^2/\sigma_2^2)^{\frac{1}{2}}, \end{aligned} \tag{84}$$

where since σ_2 and γ are both of the same order of magnitude as $2\pi i/\lambda_v$, in general no further approximations can be made.

In all of what follows we shall assume that the thickness t_2 of each insulating layer is very small compared to the vacuum wavelength at the highest operating frequency; in practice t_2 will be at most a few mils and λ_v at least a few meters. Then the quantity $|\kappa_2 t_2|$, which is of the order of $2\pi t_2/\lambda_v$, will be so small that to an excellent approximation we may set $\text{sh}\,\kappa_2 t_2 = \kappa_2 t_2$ and $\text{ch}\,\kappa_2 t_2 = 1$. Using this simplification, together with the fact that $\eta_{1y} \ll \eta_{2y}$ for all frequencies which may conceivably be of interest, it is not difficult to show from (58) that the matrix elements of the plane double layer reduce to

$$\begin{aligned} \mathcal{A} &= \text{ch}\,\kappa_1 t_1, \\ \mathcal{B} &= \eta_{2y}\kappa_2 t_2\,\text{ch}\,\kappa_1 t_1 + \eta_{1y}\,\text{sh}\,\kappa_1 t_1, \\ \mathcal{C} &= \frac{1}{\eta_{1y}}\,\text{sh}\,\kappa_1 t_1, \\ \mathcal{D} &= \frac{\eta_{2y}\kappa_2 t_2}{\eta_{1y}}\,\text{sh}\,\kappa_1 t_1 + \text{ch}\,\kappa_1 t_1. \end{aligned} \tag{85}$$

The determinant of the matrix is unity, and from (61) the propagation constant per section is defined by

$$\text{ch}\,\Gamma = \frac{\eta_{2y}\kappa_2 t_2}{2\eta_{1y}}\,\text{sh}\,\kappa_1 t_1 + \text{ch}\,\kappa_1 t_1\,, \tag{86}$$

while from (63) the iterative impedances are

$$\begin{aligned} K_1 &= -\tfrac{1}{2}\eta_{2y}\kappa_2 t_2 + \sqrt{(\tfrac{1}{2}\eta_{2y}\kappa_2 t_2)^2 + \eta_{1y}\eta_{2y}\kappa_2 t_2 \coth \kappa_1 t_1 + \eta_{1y}^2}\,, \\ K_2 &= +\tfrac{1}{2}\eta_{2y}\kappa_2 t_2 + \sqrt{(\tfrac{1}{2}\eta_{2y}\kappa_2 t_2)^2 + \eta_{1y}\eta_{2y}\kappa_2 t_2 \coth \kappa_1 t_1 + \eta_{1y}^2}\,. \end{aligned} \tag{87}$$

If we make the same simplifications in the approximate expressions (73) for the matrix elements of a coaxial double layer, we obtain

$$\begin{aligned} \mathfrak{A} &= \left[1 + \frac{t_1}{2\bar{\rho}}\right] \text{ch}\,\kappa_1 t_1 - \frac{1}{2\kappa_1\bar{\rho}}\,\text{sh}\,\kappa_1 t_1\,, \\ \mathfrak{B} &= \left[1 + \frac{t_1 + t_2}{2\bar{\rho}}\right]\eta_{2\rho}\kappa_2 t_2\,\text{ch}\,\kappa_1 t_1 \\ &\qquad + \left[1 + \frac{t_1}{2\bar{\rho}} + \left(2 - \frac{\eta_{2\rho}\kappa_2}{\eta_{1\rho}\kappa_1}\right)\frac{t_2}{2\bar{\rho}}\right]\eta_{1\rho}\,\text{sh}\,\kappa_1 t_1\,, \\ \mathfrak{C} &= \left[1 + \frac{t_1}{2\bar{\rho}}\right]\frac{1}{\eta_{1\rho}}\,\text{sh}\,\kappa_1 t_1\,, \\ \mathfrak{D} &= \left[1 + \frac{t_1 + t_2}{2\bar{\rho}} + \frac{\eta_{1\rho}}{2\eta_{2\rho}\kappa_1\kappa_2 t_2\bar{\rho}}\right]\frac{\eta_{2\rho}\kappa_2 t_2}{\eta_{1\rho}}\,\text{sh}\,\kappa_1 t_1 \\ &\qquad + \left[1 + \frac{t_1 + 2t_2}{2\bar{\rho}}\right]\text{ch}\,\kappa_1 t_1\,. \end{aligned} \tag{88}$$

In the preceding equations no restrictions have been laid on the thicknesses t_1 and t_2 except the trivial requirement that t_2 shall be small compared to a wavelength. We shall now consider the limiting case in which both t_1 and t_2 are infinitesimally small. When we make this last and most drastic approximation we do not expect that the idealized structure thus obtained will show all of the features which are of interest in a physical transmission line with finite layers; but the results of the simplified analysis will be useful in some cases nevertheless. It need scarcely be pointed out that we are dealing here only with a mathematical limiting process, in which we assume that each layer, no matter how thin, always exhibits the same electrical properties as the bulk material. If this assumption be regarded as unrealistic, it may be observed that the quantity which we actually allow to tend to zero is the ratio of layer thickness to skin depth. The skin depth may be made as large as desired by lowering the frequency, so that the formulas which we derive by letting t_1 and t_2 approach zero at a finite frequency will also hold for finite thicknesses if the frequency is sufficiently low.

We shall let θ denote the fraction of the stack which is occupied by conducting material, so that

$$\theta = t_1/(t_1 + t_2), \tag{89}$$

where at present t_1 and t_2 are both infinitesimal. Then the stack may be regarded as a homogeneous, anisotropic medium, characterized by an average dielectric constant $\bar{\epsilon}$ perpendicular to the layers, an average permeability $\bar{\mu}$ parallel to the layers, and an average conductivity $\bar{g}$ parallel to the layers. Sakurai[11] has treated such an artificial anisotropic medium, and from his formulas we find that when the layers are alternately conductors and insulators, the average electrical constants are, to a very good approximation,

$$\begin{aligned} \bar{\epsilon} &= \epsilon_2/(1 - \theta), \\ \bar{\mu} &= \theta\mu_1 + (1 - \theta)\mu_2 , \\ \bar{g} &= \theta g_1 . \end{aligned} \tag{90}$$

Sakurai has also shown that the average values of the electrical constants may be used in Maxwell's equations for the average (macroscopic) fields, due regard being paid to the orientations of the field vectors with respect to the laminae.

For the plane stack, these equations read

$$\begin{aligned} \partial\bar{H}_x/\partial z &= i\omega\bar{\epsilon}\bar{E}_y , \\ \partial\bar{H}_x/\partial y &= -\bar{g}\bar{E}_z , \\ \partial\bar{E}_y/\partial z - \partial\bar{E}_z/\partial y &= i\omega\bar{\mu}\bar{H}_x , \end{aligned} \tag{91}$$

where the bars denote average values. By analysis exactly similar to that carried out at the beginning of this section for a homogeneous, isotropic medium, we may find the relation between the tangential field components E_z, H_x at the two surfaces of a stack of infinitesimally thin layers. (The bars representing average values may be omitted, since the tangential components of **E** and **H** are continuous across the boundaries of the layers.) We obtain a matrix relation analogous to (55), namely

$$\begin{pmatrix} E(0) \\ \\ H(0) \end{pmatrix} = \begin{pmatrix} \operatorname{ch} \Gamma_t s & K \operatorname{sh} \Gamma_t s \\ \\ \dfrac{1}{K} \operatorname{sh} \Gamma_t s & \operatorname{ch} \Gamma_t s \end{pmatrix} \begin{pmatrix} E(s) \\ \\ H(s) \end{pmatrix}, \tag{92}$$

[11] T. Sakurai, *J. Phys. Soc. Japan*, **5**, 394 (1950), especially Section 3.

where s is the thickness of the stack. The propagation constant Γ_ℓ per unit distance normal to the stack and the characteristic impedance K of the stack are given by

$$\Gamma_\ell = \left[\frac{i\bar{g}}{\omega\bar{\epsilon}}\,(\omega^2\bar{\mu}\bar{\epsilon} + \gamma^2)\right]^{\frac{1}{2}}, \tag{93}$$

$$K = \Gamma_\ell/\bar{g} = \left[\frac{i}{\omega\bar{\epsilon}\bar{g}}\,(\omega^2\bar{\mu}\bar{\epsilon} + \gamma^2)\right]^{\frac{1}{2}}. \tag{94}$$

Γ_ℓ and K may also be derived from equations (86) and (87) by limiting processes; we have

$$\Gamma_\ell = \lim_{t_1+t_2\to 0} \Gamma/(t_1 + t_2), \tag{95}$$

$$K = \lim_{t_1+t_2\to 0} K_1 = \lim_{t_1+t_2\to 0} K_2 . \tag{96}$$

It should perhaps be noted that terms of the order of $\omega\epsilon_1/g_1$ and $\omega\epsilon_2/g_1$ compared to unity were omitted in the expressions (90) for $\bar{\epsilon}$ and $\bar{g}$, and in the derivations of Γ_ℓ and K. Since, however, under all practical circumstances the omitted terms appear to be insignificant, we shall not take space to write out the formally more complicated results which would be obtained by keeping them.[12]

In a cylindrical stack of infinitesimal layers, the average fields satisfy

$$\begin{aligned} \partial\bar{H}_\phi/\partial z &= -i\omega\bar{\epsilon}\bar{E}_\rho , \\ \partial(\rho\bar{H}_\phi)/\partial\rho &= \bar{g}\rho\bar{E}_z , \\ \partial\bar{E}_z/\partial\rho - \partial\bar{E}_\rho/\partial z &= i\omega\bar{\mu}\bar{H}_\phi . \end{aligned} \tag{97}$$

The relation between the tangential field components E_z , $-H_\phi$ at two radii ρ_0 and ρ_n is expressed by a matrix equation analogous to (68), namely

$$\begin{pmatrix} E(\rho_0) \\ -H(\rho_0) \end{pmatrix} = \begin{pmatrix} \Gamma_\ell\rho_n(K_{00}I_{1n} + K_{1n}I_{00}) & K\Gamma_\ell\rho_n(K_{00}I_{0n} - K_{0n}I_{00}) \\ \dfrac{\Gamma_\ell\rho_n}{K}\,(K_{10}I_{1n} - K_{1n}I_{10}) & \Gamma_\ell\rho_n(K_{10}I_{0n} + K_{0n}I_{10}) \end{pmatrix} \begin{pmatrix} E(\rho_n) \\ -H(\rho_n) \end{pmatrix}, \tag{98}$$

[12] In Reference 1, equations (II–17) through (II-26) give examples of equations in which these small terms have been retained.

where

$$I_{rs} = I_r(\Gamma_t \rho_s), \qquad K_{rs} = K_r(\Gamma_t \rho_s), \tag{99}$$

and Γ_t and K are given, as in the plane case, by (93) and (94).

IV. PRINCIPAL MODE IN CLOGSTON 1 LINES WITH INFINITESIMALLY THIN LAMINAE

An idealized parallel-plane Clogston 1 transmission line is shown schematically in Fig. 5. It consists of a slab of dielectric of thickness b, with electrical constants μ_0, ϵ_0, bounded above and below by laminated stacks each of thickness s. Outside each stack there may be an insulating or a conducting sheath, of which nothing more will be assumed at present than that its normal surface impedance $Z_n(\gamma)$ is known. The total distance between the sheaths will be denoted by a, where $a = b + 2s$.

The corresponding Clogston 1 coaxial line is shown in Fig. 6. We denote the thickness of the inner and outer stacks by s_1 and s_2 respectively, while a is the radius of the inner core (if any), and b is the inner radius of the sheath around the outer stack. The inner and outer radii of the main dielectric are $\rho_1 = a + s_1$ and $\rho_2 = b - s_2$, respectively. In practice the core may be a dielectric rod and the sheath may be a conducting shield, but in the present theoretical analysis we shall merely assume that the radial impedances $Z_a(\gamma)$ and $Z_b(\gamma)$ looking into the core and the sheath are known.

In Part I of this paper we shall deal with "extreme" Clogston 1 lines, in which the space occupied by the stacks is small compared to the space occupied by the main dielectric. We may then regard the laminated boundaries as impedance sheets guiding waves whose phase velocity is

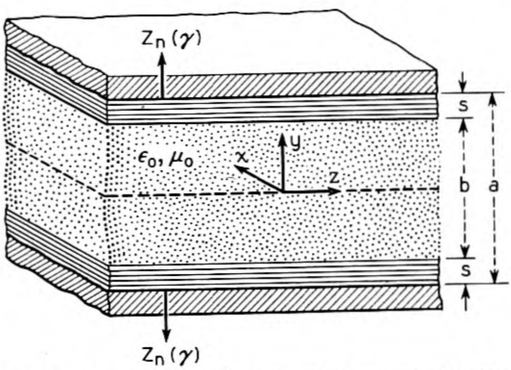


Fig. 5—Parallel-plane Clogston 1 transmission line.

determined by the properties of the main dielectric, as discussed in Section II, and we may use the intrinsic propagation constant of the main dielectric in calculating the surface impedance of the boundaries. This approximation simplifies the analysis of Clogston 1 lines a great deal. We shall treat the general case, in which an arbitrary fraction of the total space is filled with laminations, in Section IX of Part II, as a part of our study of Clogston 2 lines.

In this section we shall assume that the laminae are infinitesimally thin, so that the stacks may be completely characterized by their average properties $\bar{\epsilon}$, $\bar{\mu}$, and $\bar{g}$. The case of finite laminae will be taken up in the next section. We shall also assume throughout that dielectric and magnetic dissipation may be neglected except, as in Section VII, where the contrary is explicitly stated.

In general the current density and the other field quantities in a plane stack of infinitesimally thin layers will be linear combinations of the functions sh $\Gamma_\ell y$ and ch $\Gamma_\ell y$, where y is distance measured into the stack, and Γ_ℓ is the propagation constant per unit distance, as given by (93). The qualitative behavior of the fields in a cylindrical stack will be similar. In particular, if the stack is thick enough the current density and the fields will fall off as $e^{-\Gamma_\ell y}$, and we can define an "effective skin depth" Δ by

$$\Delta = 1/(\text{Re } \Gamma_\ell) . \tag{100}$$

Clogston's fundamental observation was that in order to minimize the

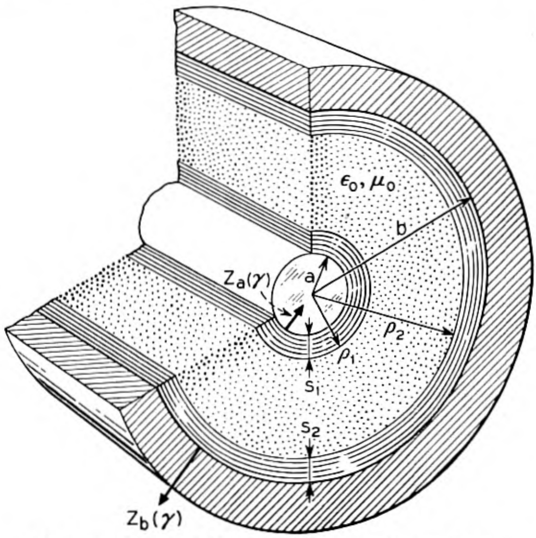


Fig. 6—Coaxial Clogston 1 transmission line.

ohmic losses in a stack carrying a fixed total current the current density should be uniform across the stack, and that we can achieve uniform current density by adjusting the $\mu_0\epsilon_0$ product of the main dielectric so as to make Γ_ℓ equal to zero. If in equation (93) we set

$$\gamma = \gamma_0 = i\omega\sqrt{\mu_0\epsilon_0}\,, \tag{101}$$

then Γ_ℓ will be zero if

$$\mu_0\epsilon_0 = \bar{\mu}\bar{\epsilon} = [\theta\mu_1 + (1 - \theta)\mu_2][\epsilon_2/(1 - \theta)]. \tag{102}$$

Equation (102) will be referred to henceforth as *Clogston's condition*. If the permeabilities of the various materials are all equal, the condition reduces to

$$\epsilon_0 = \bar{\epsilon} = \epsilon_2/(1 - \theta)\,, \tag{103}$$

which is the form employed by Clogston in Reference 1.

When Clogston's condition is satisfied, $\Gamma_\ell = 0$ and the effective skin depth of the stack is infinite;[13] that is, the current density is uniform in any stack of finite total thickness. The quantities Γ_ℓ and K vanish simultaneously, but the limiting value of their ratio is finite; and the matrix of the plane stack, as given by (92), takes the form

$$\begin{pmatrix} 1 & 0 \\ \bar{g}s & 1 \end{pmatrix}. \tag{104}$$

Accordingly we obtain, for the surface impedance $Z_0(\gamma_0)$ of the stack,

$$Z_0(\gamma_0) = \frac{1}{\bar{g}s + 1/Z_n(\gamma_0)}\,, \tag{105}$$

which is, as might have been expected, just the impedance between opposite edges of a unit square of material of conductivity $\bar{g}$ and thickness s through which the current density is uniform, in parallel with the sheath impedance $Z_n(\gamma_0)$. It follows from equations (20) and (21) of Section II that the attenuation and phase constants of the principal mode in a plane Clogston 1 line with infinitesimally thin laminae, Clogston's condition being satisfied exactly, are

[13] This statement is certainly accurate enough for all practical purposes, although an exact calculation which takes into account the small terms that were neglected in the approximate formula (93) for Γ_ℓ shows that the effective skin depth is $\lambda_0/2\pi\theta$, where λ_0 is the length of a free wave in the main dielectric. The exact result is derived by Clogston in Reference 1, equation (II-26). In practice, finite lamina thickness will restrict us to effective skin depths much smaller than this theoretical limit.

$$\alpha = \mathrm{Re}\,\frac{1}{\eta_0 b[\bar{g}s + 1/Z_n(\gamma_0)]}, \tag{106}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \mathrm{Im}\,\frac{1}{\eta_0 b[\bar{g}s + 1/Z_n(\gamma_0)]}. \tag{107}$$

In general the sheath impedance $Z_n(\gamma_0)$ will be large compared to the impedance $1/\bar{g}s$ of the stack, since even if the sheath is an electrically thick metal plate of the same material as the conducting layers, its impedance is

$$Z_n(\gamma_0) = (1 + i)/g_1\delta_1\,, \tag{108}$$

whereas θs will usually be several times the skin thickness δ_1 in the frequency range of interest. If the sheath is free space, its impedance is a fortiori much greater than $1/\bar{g}s$, since then it may be shown that

$$Z_n(\gamma_0) = -i\eta_v(\mu_{0r}\epsilon_{0r} - 1)^{\frac{1}{2}}, \tag{109}$$

where $\eta_v = 376.7$ ohms is the intrinsic impedance of free space, and μ_{0r} and ϵ_{0r} are the relative permeability and relative dielectric constant of the main dielectric. Under most circumstances, therefore, we may neglect $1/Z_n(\gamma_0)$ in comparison with $\bar{g}s$, and obtain the very simple results,

$$\alpha = 1/\eta_0 b\bar{g}s\,, \tag{110}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0}\,. \tag{111}$$

To this approximation the line exhibits neither amplitude nor phase distortion.

For a coaxial stack of infinitesimally thin layers with Clogston's condition satisfied, the stack matrix given in (98) reduces to

$$\begin{pmatrix} 1 & 0 \\ \frac{\bar{g}}{2\rho_0}\,(\rho_n^2 - \rho_0^2) & \frac{\rho_n}{\rho_0} \end{pmatrix}, \tag{112}$$

where ρ_0 and ρ_n denote the inner and outer radii of the stack. It follows from (112) that

$$\begin{aligned} \frac{Z_1(\gamma_0)}{\rho_1} &= \frac{1}{\frac{1}{2}\bar{g}(\rho_1^2 - a^2) + a/Z_a(\gamma_0)} = \frac{1}{\bar{g}s_1(a + \frac{1}{2}s_1) + a/Z_a(\gamma_0)}, \\ \frac{Z_2(\gamma_0)}{\rho_2} &= \frac{1}{\frac{1}{2}\bar{g}(b^2 - \rho_2^2) + b/Z_b(\gamma_0)} = \frac{1}{\bar{g}s_2(b - \frac{1}{2}s_2) + b/Z_b(\gamma_0)}, \end{aligned} \tag{113}$$

where $Z_1(\gamma_0)$ and $Z_2(\gamma_0)$ are the radial impedances looking into the stacks at ρ_1 and ρ_2 respectively, and $Z_a(\gamma_0)$ and $Z_b(\gamma_0)$ are the radial impedances looking into the core and the outer sheath. From equations (44) and (45) of Section II, the attenuation and phase constants of a coaxial Clogston 1 cable with infinitesimally thin layers, Clogston's condition being satisfied exactly, are

$$\alpha = \mathrm{Re}\, \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}, \tag{114}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \mathrm{Im}\, \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}, \tag{115}$$

where $Z_1(\gamma_0)/\rho_1$ and $Z_2(\gamma_0)/\rho_2$ are given by (113).

The impedances $Z_a(\gamma_0)$ and $Z_b(\gamma_0)$ may be computed if we know the structure of the core and the sheath. For a solid, homogeneous core and a homogeneous sheath of effectively infinite thickness, we have

$$Z_a(\gamma_0) = \frac{\eta\kappa}{\sigma}\frac{I_0(\kappa a)}{I_1(\kappa a)}, \qquad Z_b(\gamma_0) = \frac{\eta\kappa}{\sigma}\frac{K_0(\kappa b)}{K_1(\kappa b)}, \tag{116}$$

where

$$\kappa = \sqrt{\sigma^2 - \gamma_0^2}, \tag{117}$$

but of course the intrinsic propagation constant σ and the intrinsic impedance η need not be the same for the core and the sheath. If the sheath is of finite electrical thickness or has a laminated structure (alternate layers of copper and iron, for example, to provide effective shielding), its surface impedance may be calculated by a straightforward but longer procedure. We shall not go into this matter here, but shall merely observe that in many cases of interest $Z_a(\gamma_0)$ and $Z_b(\gamma_0)$ are so large that we may neglect the terms containing their reciprocals in (113). This means that we neglect the total conduction and displacement currents flowing in the core and the sheath, compared to the conduction currents in the stacks. Then the expressions for the attenuation and phase constants become

$$\alpha = \frac{1}{2\eta_0\bar{g} \log (\rho_2/\rho_1)}\left[\frac{1}{s_1(a + \frac{1}{2}s_1)} + \frac{1}{s_2(b - \frac{1}{2}s_2)}\right], \tag{118}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0}, \tag{119}$$

and again to this approximation there is neither amplitude nor phase distortion.

The formulas which have just been derived on the assumption of infinitesimally thin laminae approach validity for laminae of finite thickness as the frequency is reduced, provided of course that we do not go to such extremely low frequencies that the attenuation per wavelength becomes large. We shall show in the next section that the effect of finite lamina thickness is to introduce a frequency dependence into the attenuation and phase constants, in addition to the variations (if any) which arise from the frequency dependence of the core and sheath impedances.

We next write down approximate expressions for the field components in a plane Clogston 1 line with infinitesimally thin laminae. In the main dielectric we have, from equations (22) of Section II,

$$\begin{aligned} H_x &\approx H_0 e^{-\gamma z}, \\ E_y &\approx -\sqrt{\frac{\mu_0}{\epsilon_0}}\, H_0 e^{-\gamma z}, \\ E_z &\approx \frac{2Z_0(\gamma_0)H_0 y}{b}\, e^{-\gamma z}, \end{aligned} \tag{120}$$

for $-\frac{1}{2}b \leqq y \leqq \frac{1}{2}b$, where H_0 is an arbitrary amplitude factor and $Z_0(\gamma_0)$ is given by (105). In the stacks the fields are

$$\begin{aligned} H_x &\approx H_0[1 + \bar{g}Z_0(\gamma_0)(\tfrac{1}{2}b \mp y)]e^{-\gamma z}, \\ \bar{E}_y &\approx -\sqrt{\frac{\bar{\mu}}{\bar{\epsilon}}}\, H_0[1 + \bar{g}Z_0(\gamma_0)(\tfrac{1}{2}b \mp y)]e^{-\gamma z}, \\ E_z &\approx \pm Z_0(\gamma_0)H_0 e^{-\gamma z}, \end{aligned} \tag{121}$$

for $\frac{1}{2}b \leqq |y| \leqq \frac{1}{2}a$, where in cases of ambiguous sign the upper sign refers to the upper stack ($y > 0$) and the lower sign to the lower stack ($y < 0$). It should be noted that whereas the tangential field components H_x and E_z are continuous through the stack, the normal field component E_y is discontinuous at layer boundaries. From equation (52) we have, in the conducting layers,

$$E_y = -(\gamma/g_1)H_x, \tag{122}$$

while in the insulating layers,

$$E_y = -(\gamma/i\omega\epsilon_2)H_x. \tag{123}$$

To our approximation, therefore, the only contributions to the average field $\bar{E}_y$ come from the insulating layers.

The average current density $\bar{J}_z$ in either stack is uniform, being given by

$$\bar{J}_z = \bar{g}E_z = \pm\bar{g}Z_0(\gamma_0)H_0e^{-\gamma z}. \tag{124}$$

The total current per unit width carried by the stack is just $\bar{J}_z s$, where s is the thickness of the stack; there will also be small currents in the sheaths unless we assume the sheath impedance to be infinite. The potential difference between any two points y_1 and y_2 in the same transverse plane may easily be found from

$$V(y_2) - V(y_1) = -\int_{y_1}^{y_2} E_y\, dy. \tag{125}$$

For a Clogston 1 line of the proportions which we have been considering, the potential difference across the stacks will be small compared to the potential difference across the main dielectric.

In a coaxial Clogston 1 with infinitesimally thin laminae, the fields in the main dielectric are given to a good approximation by equations (46) of Section II, namely

$$\begin{aligned} H_\phi &\approx \frac{I}{2\pi\rho}\, e^{-\gamma z}, \\ E_\rho &\approx \sqrt{\frac{\mu_0}{\epsilon_0}}\, \frac{I}{2\pi\rho}\, e^{-\gamma z}, \\ E_z &\approx \frac{I}{2\pi \log(\rho_2/\rho_1)} \left[\frac{Z_1(\gamma_0)}{\rho_1} \log\frac{\rho_2}{\rho} + \frac{Z_2(\gamma_0)}{\rho_2} \log\frac{\rho_1}{\rho}\right] e^{-\gamma z}, \end{aligned} \tag{126}$$

where I is an arbitrary amplitude factor and $Z_1(\gamma_0)$ and $Z_2(\gamma_0)$ are expressed by (113). In the inner stack we have

$$\begin{aligned} H_\phi &\approx \frac{Z_1(\gamma_0)I}{2\pi\rho_1} \left[\frac{\bar{g}(\rho^2 - a^2)}{2\rho} + \frac{a}{\rho Z_a(\gamma_0)}\right] e^{-\gamma z}, \\ \bar{E}_\rho &\approx \sqrt{\frac{\bar{\mu}}{\bar{\epsilon}}}\, \frac{Z_1(\gamma_0)I}{2\pi\rho_1} \left[\frac{\bar{g}(\rho^2 - a^2)}{2\rho} + \frac{a}{\rho Z_a(\gamma_0)}\right] e^{-\gamma z}, \\ E_z &\approx \frac{Z_1(\gamma_0)I}{2\pi\rho_1}\, e^{-\gamma z}, \end{aligned} \tag{127}$$

while in the outer stack,

$$H_\phi \approx \frac{Z_2(\gamma_0)I}{2\pi\rho_2}\left[\frac{\bar{g}(b^2-\rho^2)}{2\rho}+\frac{b}{\rho Z_b(\gamma_0)}\right]e^{-\gamma z},$$
$$\bar{E}_\rho \approx \sqrt{\frac{\bar{\mu}}{\bar{\epsilon}}}\,\frac{Z_2(\gamma_0)I}{2\pi\rho_2}\left[\frac{\bar{g}(b^2-\rho^2)}{2\rho}+\frac{b}{\rho Z_b(\gamma_0)}\right]e^{-\gamma z}, \tag{128}$$
$$E_z \approx -\frac{Z_2(\gamma_0)I}{2\pi\rho_2}\,e^{-\gamma z}.$$

The average current density in either stack is uniform and is given by

$$\bar{J}_z = \bar{g}E_z\,, \tag{129}$$

though in general the current density will not be the same in the two stacks because of the difference in cross-sectional areas. The potential difference between the surface of the inner core and any other point in the same transverse plane is

$$V(\rho) - V(a) = -\int_a^\rho E_\rho\,d\rho. \tag{130}$$

If the stacks are thin compared to the thickness of the main dielectric, as we are assuming throughout Part I, then the potential difference across the stacks will be small compared to the potential difference across the main dielectric, and the characteristic impedance Z_k of the Clogston 1 cable will be approximately the same as the characteristic impedance of an ideal coaxial cable with perfect conductors of radii ρ_1 and ρ_2 and the same main dielectric, namely

$$Z_k = 60\sqrt{\frac{\mu_{0r}}{\epsilon_{0r}}}\log\frac{\rho_2}{\rho_1}\text{ ohms}. \tag{131}$$

We shall defer making any field plots for Clogston-type transmission lines until Section IX of Part II, when we shall discuss the transition from Clogston 1 to Clogston 2 as the space originally occupied by the main dielectric is gradually filled with laminations. Our present results will then appear as the limiting case in which the thickness of the stacks is small compared to the thickness of the main dielectric.

In conclusion we shall mention briefly the question of how to dispose a given amount of laminated material in a Clogston 1 coaxial cable so as to achieve the minimum attenuation constant. The whole problem of optimum proportions for Clogston cables is a complicated one of which an adequate treatment would require a separate paper in itself, with the results depending to a great extent on engineering considerations which limit the ranges of the parameters that we can vary in any practical case. Here we shall discuss only the following rather highly idealized problem:

Given a coaxial Clogston 1 with infinitesimally thin laminae, having a high-impedance core and a high-impedance sheath of fixed radius b, and in which the total thickness $s_1 + s_2$ of both stacks is a fixed constant $2s$. Assuming that $2s$ is small compared to b, what should be the radius a of the core, and how should the total stack thickness be divided between the outer and inner stacks so as to minimize the attenuation constant of the line? Finally, what should be the fraction θ of conducting material in the stacks to minimize the attenuation constant, if the electrical constants of the conducting and insulating layers are fixed, but the properties of the main dielectric are at our disposal?

If the two inequalities

$$s_1 \ll a, \qquad s_2 \ll b, \tag{132}$$

are satisfied (these restrictions will be removed in Section IX, when we discuss Clogston cables having an arbitrary fraction of their total volume filled with laminations), then equation (118) for the attenuation constant of a Clogston 1 with infinitesimally thin laminae and high-impedance boundaries becomes, approximately,

$$\alpha \approx \frac{1}{2\eta_0 \bar{g} \log (b/a)} \left[\frac{1}{as_1} + \frac{1}{bs_2} \right]. \tag{133}$$

If we write

$$s_2 = 2s - s_1 , \tag{134}$$

and vary s_1 and s_2 in accordance with this relation while holding a and b constant, it is easy to show that the expression on the right side of (133) is a minimum when

$$s_1 = \frac{2s\sqrt{b}}{\sqrt{a} + \sqrt{b}}, \qquad s_2 = \frac{2s\sqrt{a}}{\sqrt{a} + \sqrt{b}}. \tag{135}$$

These equations tell us the most efficient way to divide the stacks in a Clogston 1 when the radii of the core and the outer sheath are a and b respectively, still assuming of course that the thickness of each stack is small compared to its mean radius.

If we introduce the optimum values of s_1 and s_2 into (133), we get

$$\alpha \approx \frac{1}{2\eta_0 \bar{g}(s_1 + s_2) \log (b/a)} \left[\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} \right]^2. \tag{136}$$

If b is fixed, the last expression is a minimum, considered as a function of a, when

$$\log (b/a) = 1 + \sqrt{a/b}. \tag{137}$$

The root of this transcendental equation is

$$b/a = 4.3827, \qquad a = 0.22817b. \tag{138}$$

Substituting this value of b/a into (135), we find

$$\begin{aligned} s_1 &= 1.3535s, \\ s_2 &= 0.6465s, \\ s_1/s_2 &= 2.0935; \end{aligned} \tag{139}$$

while from (136) and (138) the minimum value of the attenuation constant is

$$\alpha \approx \frac{3.238}{\eta_0 \bar{g}(s_1 + s_2)b}. \tag{140}$$

To find the optimum value of θ, we observe that equation (118) for the attenuation constant of a Clogston 1 cable with infinitesimally thin laminae and high-impedance boundaries may be written in the form

$$\alpha = \frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{\theta g_1} f(a, b, s_1, s_2), \tag{141}$$

where the first factor depends on the electrical constants of the components of the cable, while $f(a, b, s_1, s_2)$ is a function only of the geometry. By (110) the attenuation constant of a plane Clogston 1 has the same form, only with a different dependence on the geometrical factors. Now assume that the geometrical proportions of the line are fixed, and that the electrical constants μ_1, g_1, μ_2, and ϵ_2 of the conducting and insulating layers are given, but that the constants μ_0, ϵ_0 of the main dielectric and the fraction of space θ occupied by conducting layers in the stacks are at our disposal. The $\mu_0\epsilon_0$ product of the main dielectric is to be codetermined with θ so that Clogston's condition (102) is always satisfied. Solving (102) for θ gives

$$\theta = \frac{\mu_0\epsilon_0 - \mu_2\epsilon_2}{\mu_0\epsilon_0 + (\mu_1 - \mu_2)\epsilon_2}. \tag{142}$$

Hence the first factor in the expression (141) for α may be written

$$\frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{\theta g_1} = \frac{\epsilon_0^{\frac{1}{2}}[\mu_0\epsilon_0 + (\mu_1 - \mu_2)\epsilon_2]}{g_1\mu_0^{\frac{1}{2}}[\mu_0\epsilon_0 - \mu_2\epsilon_2]}. \tag{143}$$

If we minimize the right side of (143) with respect to ϵ_0, all other quantities being held constant, by equating to zero the derivative with respect to ϵ_0 and then solving for ϵ_0, we get

$$\mu_0\epsilon_0 = \tfrac{1}{2}[(\mu_1 + 2\mu_2) + (\mu_1^2 + 8\mu_1\mu_2)^{\frac{1}{2}}]\epsilon_2 . \tag{144}$$

From (142) the value of θ is

$$\theta = \frac{\mu_1 + (\mu_1^2 + 8\mu_1\mu_2)^{\frac{1}{2}}}{3\mu_1 + (\mu_1^2 + 8\mu_1\mu_2)^{\frac{1}{2}}}, \tag{145}$$

and the corresponding attenuation constant is proportional to

$$\frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{\theta g_1} = \frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{g_1} \frac{3\mu_1 + (\mu_1^2 + 8\mu_1\mu_2)^{\frac{1}{2}}}{\mu_1 + (\mu_1^2 + 8\mu_1\mu_2)^{\frac{1}{2}}} . \tag{146}$$

It will be observed that so far we have determined only the optimum value of the product $\mu_0\epsilon_0$, and so we are still free to alter the ratio of μ_0 to ϵ_0 while holding the product of these two quantities constant. For given values of μ_1 and μ_2, we obtain the lowest attenuation constant by making ϵ_0 as small as possible and μ_0 as large as possible, subject of course to the practical restriction that ϵ_0 cannot be lower than the dielectric constant of free space. However if we permit μ_2 and μ_0 to be simultaneously increased, as by magnetic loading of both the insulating layers and the main dielectric, we find from (146) that on paper it is possible to decrease the attenuation constant without any definite limit. This observation is in accord with the fact that the attenuation constant of an ordinary coaxial cable may be decreased indefinitely, with a corresponding decrease in the velocity of propagation along the cable, if we are willing to assume an unlimited amount of lossless magnetic loading.

If $\mu_1 = \mu_2$, (144) and (145) take the form

$$\mu_0\epsilon_0 = 3\mu_2\epsilon_2 , \qquad \theta = 2/3, \tag{147}$$

from which we have the result given by Clogston:[14] If the conducting and insulating layers are infinitesimally thin and have equal permeabilities, then minimum attenuation is achieved when *the thickness of the conducting layers is twice the thickness of the insulating layers.* In this case, from (146) and (147) the attenuation is proportional to

$$\frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{\theta g_1} = \frac{3(\epsilon_0/\mu_0)^{\frac{1}{2}}}{2g_1} . \tag{148}$$

When $\mu_0 = \mu_2$, corresponding to no magnetic loading, we must take $\epsilon_0 = 3\epsilon_2$, and (148) reduces to

[14] Reference 1, pp. 513–514.

$$\frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{\theta g_1} = \frac{3\sqrt{3}\ (\epsilon_2/\mu_2)^{\frac{1}{2}}}{2g_1}, \tag{149}$$

while if we load the main dielectric so that $\mu_0 = 3\mu_2$ and we can take $\epsilon_0 = \epsilon_2$, we have

$$\frac{(\epsilon_0/\mu_0)^{\frac{1}{2}}}{\theta g_1} = \frac{\sqrt{3}\ (\epsilon_2/\mu_2)^{\frac{1}{2}}}{2g_1}, \tag{150}$$

which is just one-third of the value with no magnetic loading.

As Clogston has pointed out, if the limitation is on the total thickness of conducting material in the stacks rather than on the stack thicknesses themselves, we shall find it advantageous to use a small value of θ (a high "dilution" of conducting material) so as to make the average dielectric constant $\epsilon_2/(1 - \theta)$ of the stacks, which has to be matched by the main dielectric, as small as possible. We shall see later that the effect of finite lamina thickness is in fact to limit the total thickness of conducting material which it is useful to employ in a single stack at high frequencies, so that for physical stacks of non-magnetic layers at high frequencies the optimum value of θ is less than 2/3. Quantitative results which take into account the finite thickness of the layers will be obtained in Section XI.

To illustrate the use of some of the equations derived above by means of a numerical example, we shall compare the attenuation constant of a conventional coaxial cable with that of a Clogston 1 cable of the same size. If a and b denote the radii of the inner and outer conductors of a conventional coaxial cable, and we take $b/a = 3.5911$ to minimize the attenuation constant, then we have from equation (48) of Section II, on setting $\rho_1 = a$ and $\rho_2 = b$,

$$\alpha = \frac{1.796}{\eta_0 g_1 \delta_1 b}, \tag{151}$$

where η_0 is the intrinsic impedance of the main dielectric, which may be air. For a Clogston 1 coaxial cable with infinitesimally thin laminae, no magnetic material in the stacks ($\mu_1 = \mu_2 = \mu_v$), and the optimum proportions given by (139) and (147), we have

$$\alpha \approx \frac{4.857}{\eta_0 g_1 (s_1 + s_2) b}, \tag{152}$$

where b is the outside radius of the outer stack and η_0 is the intrinsic impedance of the main dielectric, which cannot be air in a Clogston cable. The ratio of the attenuation constant α_c of this Clogston cable to the attenuation constant α_s of an *air-filled* standard coaxial of the same size, made of the same conducting material, is

$$\frac{\alpha_c}{\alpha_s} \approx \frac{2.704\ \delta_1}{(\mu_{0r}/\epsilon_{0r})^{\frac{1}{2}}(s_1 + s_2)}, \tag{153}$$

where μ_{0r} and ϵ_{0r} refer to the main dielectric of the Clogston cable.

Since the attenuation constant of a standard coaxial cable is proportional to the square root of frequency in the range we are considering, while the attenuation constant of the ideal Clogston cable is independent of frequency in this range, there will be a crossover frequency above which the Clogston cable has a lower attenuation constant than a conventional coaxial cable of the same size. If we are dealing with copper conductors and if frequencies are measured in $\text{Mc}\cdot\text{sec}^{-1}$ and linear dimensions in mils, then from equations (78) and (153) we find that the crossover frequency is given approximately by

$$f_{\text{Mc}} \approx \frac{49.50\ (\epsilon_{0r}/\mu_{0r})}{(s_1 + s_2)^2_{\text{mils}}}. \tag{154}$$

For example, let us take an ideal Clogston 1 cable of outer diameter 0.375 inches, excluding the sheath, with no magnetic loading, and assume the following values:

$$\begin{aligned} a &= 42.8 \text{ mils} & \theta &= 2/3 \\ b &= 187.5 \text{ mils} & \epsilon_{2r} &= 2.26 \text{ (polyethylene)} \\ s_1 &= 12.69 \text{ mils} & \epsilon_{0r} &= 3\epsilon_{2r} = 6.78 \\ s_2 &= 6.06 \text{ mils} & \mu_{0r} &= \mu_{1r} = \mu_{2r} = 1 \\ s_1 + s_2 &= 18.75 \text{ mils} \end{aligned} \tag{155}$$

This cable has a lower attenuation constant than a standard air-filled coaxial of the same size at frequencies above about 1 $\text{Mc}\cdot\text{sec}^{-1}$, the approximate formula (154) yielding 0.955 $\text{Mc}\cdot\text{sec}^{-1}$ for the crossover frequency and the exact equation (118), taken in conjunction with (151), yielding 1.251 $\text{Mc}\cdot\text{sec}^{-1}$.

The reader is cautioned that the comparison given by (153) between Clogston and conventional cables is based upon certain highly idealized assumptions. In the first place we have neglected the finite thickness of the laminae, which will in fact cause the attenuation constant of a physical Clogston cable to increase with increasing frequency, and ultimately to cross over again and become higher than the attenuation constant of a conventional air-filled coaxial. We have also neglected dielectric and magnetic losses, which are likely to be directly proportional to frequency and by no means negligible at the upper end of the frequency band. In practice, too, the $\mu_0\epsilon_0$ product of the main dielectric must be held very close to the Clogston value or the benefit of the large effective skin depth is lost; and the stacks must be extremely uniform or again the depth of penetration is greatly reduced. We shall take up all these matters in later sections, and shall see that while the results just given represent ultimate limits of performance, the practical improvements which can be achieved over conventional cables depend upon the degree to which one can solve the manufacturing problems that tend to make every actual Clogston cable differ more or less from the ideal structure considered above.

V. EFFECT OF FINITE LAMINA THICKNESS. FREQUENCY DEPENDENCE OF ATTENUATION IN CLOGSTON 1 LINES

The principal effect of finite lamina thickness in a Clogston cable is to introduce a frequency dependence into the propagation constant, and in particular to cause the attenuation constant to increase, with increasing frequency, above the value which we have found for infinitesimally thin laminae (or for finite laminae at low frequencies). The increased losses are associated with the fact that the penetration depth in a laminated stack decreases with increasing frequency, even when Clogston's condition is exactly satisfied, if the laminae are of finite thickness. We shall now obtain expressions for the surface impedance of a plane laminated stack of n double layers, such as is shown in Fig. 3, when Clogston's condition is satisfied but the individual layers are of finite thickness.

We first observe that Clogston's condition (102) implies

$$\begin{aligned} \eta_{2y}\kappa_2 t_2 &= \eta_2\sigma_2(1 - \gamma_0^2/\sigma_2^2)t_2 \\ &= i\omega\mu_2\left[1 - \frac{\theta\mu_1 + (1-\theta)\mu_2}{(1-\theta)\mu_2}\right]\frac{(1-\theta)t_1}{\theta} \\ &= -i\omega\mu_1 t_1 = -\eta_1\sigma_1 t_1 \\ &\approx -\eta_{1y}\kappa_1 t_1 , \end{aligned} \tag{156}$$

where in the last step we have used the fact that in the conducting layers η_{1y} is equal to η_1 and κ_1 is equal to σ_1 to a very good approximation. We now introduce the dimensionless parameter

$$\Theta = \sigma_1 t_1 = (1 + i)t_1/\delta_1 \approx \kappa_1 t_1 , \tag{157}$$

which may be regarded as a measure of the electrical thickness of the individual conducting layers. From (86) and (156) we have, for the propagation constant per double layer,

$$\text{ch}\,\Gamma = \text{ch}\,\Theta - \tfrac{1}{2}\Theta\,\text{sh}\,\Theta, \tag{158}$$

and from (87), for the iterative impedances,

$$\begin{aligned} K_1 &= \frac{\Theta}{g_1t_1}\left[+\tfrac{1}{2}\Theta + (\tfrac{1}{4}\Theta^2 - \Theta\coth\Theta + 1)^{\frac{1}{2}}\right], \\ K_2 &= \frac{\Theta}{g_1t_1}\left[-\tfrac{1}{2}\Theta + (\tfrac{1}{4}\Theta^2 - \Theta\coth\Theta + 1)^{\frac{1}{2}}\right], \end{aligned} \tag{159}$$

since $\eta_{1y} = \kappa_1/g_1 = \Theta/g_1t_1$.

If the thickness t_1 of each conducting layer is moderately small compared to the skin depth δ_1 at the highest frequency of interest, the quantities Γ, K_1, and K_2 may conveniently be expanded in powers of Θ. The identity

$$\text{ch}\,x - 1 = 2\,\text{sh}^2\,\tfrac{1}{2}x \tag{160}$$

enables us to transform (158) into

$$\begin{aligned} \text{sh}^2\,\tfrac{1}{2}\Gamma &= \tfrac{1}{2}(\text{ch}\,\Theta - 1) - \tfrac{1}{4}\Theta\,\text{sh}\,\Theta \\ &= -\frac{\Theta^4}{48}\left[1 + \frac{\Theta^2}{15} + \frac{\Theta^4}{560} + \cdots\right], \end{aligned} \tag{161}$$

after we expand sh Θ and ch Θ by Dwight 657.1 and 657.2 and collect terms. Taking the square root by the binomial theorem gives

$$\text{sh}\,\tfrac{1}{2}\Gamma = -\frac{i}{4\sqrt{3}}\left[\Theta^2 + \frac{\Theta^4}{30} + \frac{17\Theta^6}{50400} + \cdots\right], \tag{162}$$

the negative sign being introduced because from (157) Θ^2 is a positive imaginary number and we want Re $\Gamma > 0$. Then

$$\begin{aligned} \Gamma &= 2\,\text{sh}^{-1}\left[-\frac{i}{4\sqrt{3}}\left(\Theta^2 + \frac{\Theta^4}{30} + \frac{17\Theta^6}{50400} + \cdots\right)\right] \\ &= -\frac{i}{\sqrt{3}}\left[\frac{\Theta^2}{2} + \frac{\Theta^4}{60} + \frac{\Theta^6}{525} + \cdots\right], \end{aligned} \tag{163}$$

provided that we expand the sh^{-1} function by Dwight 706. From (159) we get

$$\begin{aligned} K_1 &= \frac{1}{g_1t_1}\left[\frac{(3 - i\sqrt{3})}{6}\,\Theta^2 + \frac{i\sqrt{3}}{45}\,\Theta^4 - \frac{i\sqrt{3}}{1575}\,\Theta^6 + \cdots\right], \\ K_2 &= \frac{1}{g_1t_1}\left[-\frac{(3 + i\sqrt{3})}{6}\,\Theta^2 + \frac{i\sqrt{3}}{45}\,\Theta^4 - \frac{i\sqrt{3}}{1575}\,\Theta^6 + \cdots\right], \end{aligned} \tag{164}$$

where we have expanded coth Θ by Dwight 657.5 and chosen the sign of the square root to make Re K_1 and Re K_2 both positive.

Our first observation is that when the lamina thickness is finite the effective skin depth of the stack is also finite. We have, from (157) and (163),

$$\Gamma = \frac{1}{\sqrt{3}}\left[\frac{t_1^2}{\delta_1^2} + \frac{it_1^4}{15\delta_1^4} - \frac{8t_1^6}{525\delta_1^6} - \cdots\right], \tag{165}$$

and the average propagation constant per unit distance into the stack is

$$\Gamma_\ell = \frac{\Gamma}{(t_1 + t_2)} = \frac{1}{\sqrt{3}(t_1 + t_2)}\left[\frac{t_1^2}{\delta_1^2} + \frac{it_1^4}{15\delta_1^4} - \frac{8t_1^6}{525\delta_1^6} - \cdots\right]. \tag{166}$$

If as usual we define the effective skin depth Δ to be the distance, measured into an infinitely deep stack, at which the current density has fallen to $1/e$ of its value at the surface, then keeping only the first term in (166) we have

$$\Delta = \frac{1}{\operatorname{Re}\Gamma_\ell} = \frac{\sqrt{3}(t_1 + t_2)\delta_1^2}{t_1^2} = \frac{\sqrt{3}(t_1 + t_2)}{\pi\mu_1 g_1 f t_1^2}, \tag{167}$$

a result also given by Clogston.[15] The number N of double layers in one effective skin depth is

$$N = \frac{\Delta}{(t_1 + t_2)} = \frac{\sqrt{3}\delta_1^2}{t_1^2} = \frac{\sqrt{3}}{\pi\mu_1 g_1 f t_1^2}, \tag{168}$$

while the total thickness T_Δ of conducting material in these layers is

$$T_\Delta = Nt_1 = \frac{\sqrt{3}\delta_1^2}{t_1} = \frac{\sqrt{3}}{\pi\mu_1 g_1 f t_1}. \tag{169}$$

T_Δ is essentially the thickness of conducting material in each stack which is effectively carrying current; it is evident that for small values of t_1/δ_1 this effective thickness is inversely proportional to the frequency f and to the thickness t_1 of the individual conducting layers, but independent of the thckness t_2 of the insulating layers, provided that t_2 is very small compared to the length of a free wave in the insulating material.

In the general case, still assuming of course that Clogston's condition is satisfied, the surface impedance $Z_0(\gamma_0)$ of a plane Clogston stack is given by equation (65) of Section III, namely

$$Z_0(\gamma_0) = \frac{\frac{1}{2}Z_n(\gamma_0)(K_1 e^{n\Gamma} + K_2 e^{-n\Gamma}) + K_1 K_2 \operatorname{sh} n\Gamma}{Z_n(\gamma_0)\operatorname{sh} n\Gamma + \frac{1}{2}(K_1 e^{-n\Gamma} + K_2 e^{n\Gamma})}, \tag{170}$$

[15] Reference 1, equation (III-44).

where $Z_n(\gamma_0)$ is the impedance of the surface behind the stack. If $\Theta = 0$, (170) reduces to (105) of Section IV, that is,

$$Z_0(\gamma_0) = \frac{1}{\bar{g}s + 1/Z_n(\gamma_0)} = \frac{1}{g_1 T_1 + 1/Z_n(\gamma_0)}, \tag{171}$$

where T_1 is the total thickness of conducting material in the stack. If $Z_n(\gamma_0)$ is infinite, then for all values of Θ and n we have

$$Z_0(\gamma_0) = \frac{\Theta}{g_1 t_1} [\tfrac{1}{2}\Theta + (\tfrac{1}{4}\Theta^2 - \Theta \coth \Theta + 1)^{\frac{1}{2}} \coth n\Gamma]; \tag{172}$$

and if Re $n\Gamma$ is large, corresponding to a stack many effective skin depths thick, then for any $Z_n(\gamma_0)$ we have

$$Z_0(\gamma_0) = K_1 . \tag{173}$$

Once $Z_0(\gamma_0)$ has been computed for a particular frequency, the attenuation and phase constants of the plane Clogston 1 line at that frequency are given, as in Section II, by

$$\alpha = \text{Re } Z_0(\gamma_0)/\eta_0 b, \tag{174}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \text{Im } Z_0(\gamma_0)/\eta_0 b. \tag{175}$$

Explicit expressions for the surface impedance of a coaxial stack of finite layers have not been derived. However, if in a coaxial Clogston 1 the thickness of each stack is small compared to its mean radius, or if the depth of penetration given by (167) is small compared to the radius of the surface near which the currents flow, then the parallel-plane formula (170) may be used for the stack impedances $Z_1(\gamma_0)$ and $Z_2(\gamma_0)$ which are to be substituted into the equations of Section II for the attenuation and phase constants, namely

$$\alpha = \text{Re } \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}, \tag{176}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \text{Im } \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}. \tag{177}$$

If the plane approximations are regarded as insufficiently accurate, one can compute the surface impedance of a cylindrical stack by repeated multiplication of matrices similar to the one given by equations (88) of Section III. This procedure would obviously involve considerable numerical computation, but we can hardly expect that it would reveal anything qualitatively new for Clogston cables of the proportions considered in Part I.

It will be instructive to compare the impedance of a laminated plane stack with the impedance of a solid metal plate over the full frequency range from zero to very high frequencies.[16] If the stack contains n conducting layers, each of thickness t_1, and the metal plate is of thickness $T_1 = nt_1$, the impedances of the plate and of the stack will be equal at zero frequency, and also at very high frequencies where the first layer of the stack is already many skin depths thick. For simplicity we assume that both the plate and the stack are backed by infinite-impedance surfaces at all frequencies.

To orient ourselves we shall define three critical frequencies, for which respectively the thickness of the solid plate is equal to one skin depth in the metal, the thickness of the stack is equal to one "effective skin depth", and the thickness of a single conducting layer is equal to $\sqrt{3}$ skin depths in the metal. These frequencies are

$$\begin{aligned} f_1 &= 1/(\pi\mu_1 g_1 T_1^2) && (T_1 = \delta_1), \\ f_2 &= \sqrt{3}/(\pi\mu_1 g_1 t_1 T_1) = \sqrt{3}nf_1 && (T_1 = T_\Delta), \\ f_3 &= 3/(\pi\mu_1 g_1 t_1^2) = 3n^2 f_1 && (t_1 = \sqrt{3}\delta_1). \end{aligned} \tag{178}$$

The approximate forms of the surface impedance functions of the plate and the stack in the various frequency ranges are then quite simple.

In the range $0 \leqq f \leqq f_1$, the surface impedance of the solid plate is approximately constant and given by

$$Z_0(\gamma_0) \approx 1/g_1 T_1, \tag{179}$$

while in the range $f \geqq f_1$ we see approximately the surface impedance of an infinite plate,

$$Z_0(\gamma_0) \approx (1 + i)/g_1\delta_1 = (1 + i)\sqrt{\pi\mu_1 f/g_1}, \tag{180}$$

which is proportional to $\sqrt{f}$. The surface impedance of the stack is approximately constant in the range $0 \leqq f \leqq f_2$, where

$$Z_0(\gamma_0) \approx 1/g_1 T_1, \tag{181}$$

while in the range $f_2 \leqq f \leqq f_3$ it is approximately equal to the impedance K_1 of an infinitely deep stack of moderately thin layers as given by the first of equations (164), namely

$$Z_0(\gamma_0) \approx (1/\sqrt{3} + i)\pi\mu_1 t_1 f, \tag{182}$$

---

[16] In this connection see also Reference 1, Fig. 2, p. 494. Clogston compares a laminated stack with a solid plate of the same total thickness as the stack, hence a plate which contains more conducting material than the stack.

which is directly proportional to frequency (and independent of conductivity). For $f \geqq f_3$ the stack acts much like an infinitely thick solid plate, for which

$$Z_0(\gamma_0) \approx (1 + i)/g_1\delta_1 = (1 + i)\sqrt{\pi\mu_1 f/g_1}, \tag{183}$$

an impedance again proportional to $\sqrt{f}$.

The real parts of the approximate expressions for surface impedance may be plotted on log-log paper, where power-law relationships are represented by straight lines, to give quite a good idea of the way in which the stack resistance varies over the entire frequency range. To show how the exact resistance departs from the approximate formulas in the transition regions, we have calculated the resistance of a particular stack over the full frequency range from equation (172), and also the resistance of the corresponding solid plate from the formula

$$Z_0(\gamma_0) = (1 + i)\sqrt{\pi\mu_1 f/g_1} \coth [(1 + i)\sqrt{\pi\mu_1 g_1 f}T_1], \tag{184}$$

and plotted the results, together with those for an infinite plate and an infinite stack, in Fig. 7. The actual numerical values were chosen solely for ease in plotting, and are of no particular significance. It should be noted that the exact curves oscillate slightly around the asymptotic lines in the transition regions. For example, the resistance of the laminated stack is actually higher than the resistance of the solid plate at certain frequencies slightly above f_3. These oscillations appear clearly in the numerical results, but are scarcely visible on the plots because of the logarithmic compression of the upper ends of the frequency and resistance scales.

We shall next obtain an expression for the rate at which the surface impedance of a laminated stack begins to depart from its dc value as the frequency is increased. For this purpose we must expand the various factors appearing in equation (170) for $Z_0(\gamma_0)$ in powers of Θ. Using the expansions (163) and (164) which have already been derived for Γ, K_1, and K_2, it is a matter of straightforward if tedious algebra to show that:

$$e^{n\Gamma} = 1 - \frac{i\sqrt{3}n}{6}\Theta^2 - \frac{(15n^2 + i2\sqrt{3}n)}{360}\Theta^4 + \cdots, \tag{185}$$

$$e^{-n\Gamma} = 1 + \frac{i\sqrt{3}n}{6}\Theta^2 - \frac{(15n^2 - i2\sqrt{3}n)}{360}\Theta^4 + \cdots, \tag{186}$$

$$\text{sh}\, n\Gamma = -\frac{in}{2\sqrt{3}}\left[\Theta^2 + \frac{\Theta^4}{30} - \frac{(175n^2 - 48)}{12600}\Theta^6 + \cdots\right], \tag{187}$$

$$K_1e^{n\Gamma} + K_2e^{-n\Gamma} \tag{188}$$
$$= -\frac{i\Theta^2}{\sqrt{3}g_1t_1}\left[1 + \frac{(15n - 4)}{30}\Theta^2 - \frac{(175n^2 - 70n - 16)}{4200}\Theta^4 + \cdots\right],$$

$$K_1e^{-n\Gamma} + K_2e^{n\Gamma} \tag{189}$$
$$= -\frac{i\Theta^2}{\sqrt{3}g_1t_1}\left[1 - \frac{(15n + 4)}{30}\Theta^2 - \frac{(175n^2 + 70n - 16)}{4200}\Theta^4 + \cdots\right],$$

$$K_1K_2 \operatorname{sh} n\Gamma = \frac{1}{(g_1l_1)^2}\frac{in}{6\sqrt{3}}\Theta^6 + \cdots . \tag{190}$$

By substituting the above series into equation (170), we can obtain the variation of the stack impedance with frequency so long as t_1/δ_1 is sufficiently small. Although in principle there would be no difficulty in taking into account an arbitrary sheath impedance $Z_n(\gamma_0)$, for brevity we shall restrict ourselves here to the case in which the sheath impedance is so high that at all frequencies of interest the current in the sheath may be neglected. Then we have equation (191) (see next page).

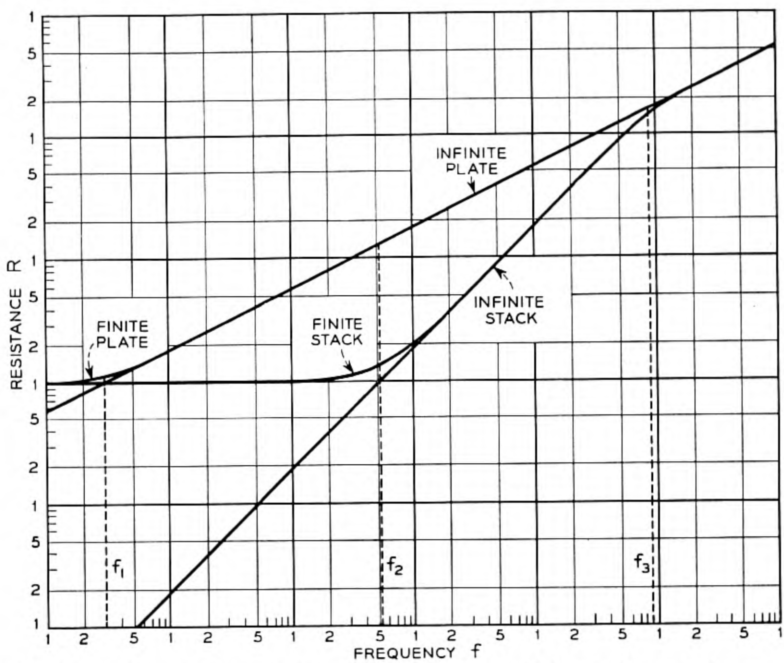


Fig. 7—Surface resistance R of solid plates and laminated stacks versus frequency f on log-log scale.

$$Z_0(\gamma_0) = \frac{K_1 e^{n\Gamma} + K_2 e^{-n\Gamma}}{2 \text{ sh } n\Gamma}, \tag{191}$$

which can be reduced to

$$\begin{aligned} Z_0(\gamma_0) &= \frac{1}{ng_1t_1}\left[1 + \frac{(3n-1)}{6}\,\Theta^2 - \frac{(5n^2-1)}{180}\,\Theta^4 + \cdots\right] \\ &\approx \frac{1}{g_1T_1}\left[1 + \frac{iT_1t_1}{\delta_1^2} + \frac{T_1^2t_1^2}{9\delta_1^4} + \cdots\right], \end{aligned} \tag{192}$$

the last expression being valid if the number of double layers is not too small ($n \geqq 5$, say). To this approximation the fractional changes in the resistance and reactance of the stack are

$$\frac{\Delta R}{R_0} = \frac{T_1^2t_1^2}{9\delta_1^4} = \frac{T_1^2t_1^2\pi^2\mu_1^2g_1^2f^2}{9}, \tag{193}$$

$$\frac{\Delta X}{R_0} = \frac{T_1t_1}{\delta_1^2} = T_1t_1\pi\mu_1g_1f, \tag{194}$$

where

$$R_0 = 1/g_1T_1 \tag{195}$$

is the dc resistance. From the exact calculations described above it appears that (193) and (194) are valid up to the neighborhood of the critical frequency

$$f_2 = \sqrt{3}/(\pi\mu_1g_1t_1T_1), \tag{196}$$

at which frequency the approximate formulas yield

$$\Delta R/R_0 = 1/3, \qquad \Delta X/R_0 = \sqrt{3}. \tag{197}$$

For $f > f_2$, however, these approximations rapidly break down.

We may now answer the question: What must be the thickness t_1 of the individual conducting layers in a plane stack which contains a given total thickness T_1 of conducting material, if at a specified top frequency f_m the resistance of the stack is not to have increased by more than a specified small fraction of its dc value? We find that the permissible value of t_1 is

$$t_1 = \frac{3}{\pi\mu_1g_1T_1f_m}\sqrt{\frac{\Delta R}{R_0}}, \tag{198}$$

and we note that this value of t_1 is inversely proportional both to f_m and to T_1. If we measure t_1 and T_1 in mils and f_m in Mc·sec^{-1}, then on putting in the numerical values of μ_1 and g_1 for copper, we have

$$(t_1)_{\text{mils}} = \frac{20.31}{(f_m)_{\text{Mc}}(T_1)_{\text{mils}}} \sqrt{\frac{\Delta R}{R_0}}. \tag{199}$$

For a plane Clogston 1 with stacks of equal thickness, the attenuation constant is given by (174), and the fractional change in attenuation with frequency is equal to the fractional change in resistance of either stack, as calculated from (193). For a coaxial Clogston 1 with stacks thin enough so that the plane approximation is valid we may also use (193), but the fractional changes in resistance will be different for the two stacks if these are of different thicknesses, and the fractional change in the attenuation constant must be calculated from equation (176). If R_{10} and R_{20} are the dc resistances "per square" of the two stacks, and ΔR_1 and ΔR_2 their increments as obtained from (193), then the fractional increase in attenuation is given approximately by

$$\frac{\Delta\alpha}{\alpha_0} \approx \frac{\Delta R_1/\rho_1 + \Delta R_2/\rho_2}{R_{10}/\rho_1 + R_{20}/\rho_2}. \tag{200}$$

For either plane or cylindrical geometry we find that if we scale up a particular Clogston line by multiplying the thicknesses of the stacks and the main dielectric by the same factor, then the low-frequency attenuation constant will be divided by the square of the scale factor. However, the permissible thickness of the individual conducting layers, if we are to have the attenuation flat to a specified degree up to a fixed frequency, is inversely proportional to the scale factor. Thus if we double the overall dimensions of the line and double the amount of conducting material in the stacks, we shall divide the low-frequency attenuation constant by four, but we shall have to make the individual layers half as thick in order to maintain the same relative increase in attenuation constant at the same top frequency f_m. In addition it is clear that if we double the top frequency while maintaining the same requirement on $\Delta\alpha/\alpha_0$ for a line of given dimensions, we shall also have to cut the thickness of the individual layers in half.

As a numerical example, let us return to the cable whose specifications were given by (155) at the end of Section IV. For this cable we have:

$$\begin{aligned} \rho_1 &= 55.49 \text{ mils} & \theta s_1 &= 8.46 \text{ mils} \\ \rho_2 &= 181.44 \text{ mils} & \theta s_2 &= 4.04 \text{ mils} \\ \rho_2/\rho_1 &= 3.270 & R_{20}/R_{10} &\approx s_1/s_2 = 2.094 \end{aligned} \tag{201}$$

If the conducting layers are copper, we find that equation (200) for the fractional increase in attenuation becomes, numerically,

$$\Delta\alpha/\alpha_0 \approx 0.121(t_1)^2_{\text{mils}} f^2_{\text{Mc}} . \tag{202}$$

If for example the copper layers are 0.1 mil thick and the polyethylene layers 0.05 mil thick, since we are assuming $\theta = 2/3$, then the attenuation constant has increased by 10 per cent of its "flat" value at a frequency of about 9.1 $\text{Mc}\cdot\text{sec}^{-1}$.

We may also ask for the upper crossover frequency, above which the Clogston cable will have a higher attenuation constant than a standard air-filled coaxial of the same size. Such a crossover frequency must exist because the dielectric loading of the Clogston cable (in our case $\epsilon_{0r} = 6.78$) introduces a factor $\sqrt{\epsilon_{0r}}$ into the asymptotic expression for the attenuation constant at extremely high frequencies when the stacks look like solid metal walls; in addition there will be slight differences due to the fact that the geometric proportions of the conventional and Clogston cables are not exactly the same.

We assume, subject to a posteriori verification, that the upper crossover frequency lies between the critical frequencies f_2 and f_3, defined by (178), for each stack. Then we have in effect infinitely deep stacks of moderately thin laminae, whose surface resistances are equal and are given by (182) to be

$$R_1 = R_2 \approx \pi\mu_1 t_1 f/\sqrt{3} = 5.79 \times 10^{-5}(t_1)_{\text{mils}} f_{\text{Mc}} \text{ ohms}. \tag{203}$$

The attenuation constants of the conventional and Clogston cables are obtained from (151) and (176) respectively, where for the conventional coaxial we set $\eta_0 = \eta_v$. After a little arithmetic we find for the upper crossover frequency in this particular case,

$$f_{\text{Mc}} \approx 2.79/(t_1)^2_{\text{mils}} . \tag{204}$$

Thus if the copper layers are 0.1 mil thick, the upper crossover frequency is about 280 $\text{Mc}\cdot\text{sec}^{-1}$, which turns out to lie well inside the interval between the critical frequencies f_2 and f_3 for both stacks.

Comparing this result with the result at the end of Section IV, we see that a 0.375-inch Clogston 1 cable with 0.1-mil copper conductors and the other specifications given by (155) is nominally better than a conventional air-filled coaxial cable of the same size in the frequency range from about 1 $\text{Mc}\cdot\text{sec}^{-1}$ to 280 $\text{Mc}\cdot\text{sec}^{-1}$. We are still neglecting the effect of failure to satisfy Clogston's condition exactly, the effect of stack nonuniformity, and dielectric losses. All of these factors will be present to a greater or less degree in any physical embodiment of a Clogston cable, and will reduce, or in extreme cases even eliminate, the frequency range over which the Clogston cable exhibits lower loss than a conventional coaxial cable.

VI. EFFECT OF DIELECTRIC MISMATCH

We may think of Clogston's relation (102) as a condition imposed on the phase velocity in a laminated transmission line to maximize the depth of eddy current penetration into the stacks. If this condition is not exactly satisfied, that is, if the $\mu_0\epsilon_0$ product of the main dielectric is not equal to the $\bar{\mu}\bar{\epsilon}$ product of the stacks, then the effective skin depth of the stacks is finite at finite frequencies and decreases with increasing frequency even in the ideal case of infinitesimally thin layers, while if the layers are of finite thickness the effective skin depth is even less than it would be with a perfectly matched main dielectric. The losses in the stacks at moderate frequencies where Clogston's penetration effect is of importance are correspondingly increased by the presence of dielectric mismatch.

For a quantitative discussion we define the amount of dielectric mismatch $\Delta(\mu_0\epsilon_0)$ by

$$\Delta(\mu_0\epsilon_0) = \mu_0\epsilon_0 - \bar{\mu}\bar{\epsilon}, \tag{205}$$

and also the dielectric mismatch parameter k by

$$k = \frac{\Delta(\mu_0\epsilon_0)}{\bar{\mu}\bar{\epsilon} - \mu_2\epsilon_2} = \frac{(1 - \theta)}{\theta} \frac{\Delta(\mu_0\epsilon_0)}{\mu_1\epsilon_2}. \tag{206}$$

In terms of k, the general expressions for Γ, K_1, and K_2 in a plane stack of finite layers take a relatively simple form. We have

$$\begin{aligned} \eta_{2y}\kappa_2 t_2 &= \eta_2\sigma_2(1 - \gamma_0^2/\sigma_2^2)t_2 \\ &= \frac{i\omega\mu_1}{\mu_1\epsilon_2}[\mu_2\epsilon_2 - \mu_0\epsilon_0]\frac{(1-\theta)t_1}{\theta} \\ &= -i\omega\mu_1(1 + k)t_1 = -(1 + k)\eta_1\sigma_1 t_1 \\ &\approx -(1 + k)\eta_{1y}\kappa_1 t_1, \end{aligned} \tag{207}$$

after a little rearrangement, where the only approximation that has been made so far is to set $\eta_{1y} \approx \eta_1$ and $\kappa_1 \approx \sigma_1$. Substituting (207) into (86) and (87) gives

$$\operatorname{ch} \Gamma = \operatorname{ch} \Theta - \tfrac{1}{2}(1 + k)\Theta \operatorname{sh} \Theta, \tag{208}$$

and

$$K_1 = \frac{\Theta}{g_1 t_1} \left[\tfrac{1}{2}(1 + k)\Theta + \sqrt{\tfrac{1}{4}(1 + k)^2\Theta^2 - (1 + k)\Theta \coth \Theta + 1}\right],$$
$$K_2 = \frac{\Theta}{g_1 t_1} \left[-\tfrac{1}{2}(1 + k)\Theta + \sqrt{\tfrac{1}{4}(1 + k)^2\Theta^2 - (1 + k)\Theta \coth \Theta + 1}\right], \tag{209}$$

where as usual

$$\Theta = \sigma_1 t_1 = (1 + i)t_1/\delta_1 \approx \kappa_1 t_1 . \tag{210}$$

If $k = 0$, equations (208) and (209) evidently reduce to (158) and (159) of the preceding section. For a stack of infinitesimally thin layers, the constants Γ_ℓ and K are given by equations (93) and (94) of Section III, namely

$$\Gamma_\ell = \left[\frac{i\bar{g}}{\omega\bar{\epsilon}} (\omega^2\bar{\mu}\bar{\epsilon} - \omega^2\mu_0\epsilon_0)\right]^{\frac{1}{2}} = (-2ik)^{\frac{1}{2}}\theta/\delta_1 , \tag{211}$$

$$K = \Gamma_\ell/\bar{g} = (-2ik)^{\frac{1}{2}}/g_1\delta_1 . \tag{212}$$

Up to this point we have set no restrictions on the magnitude of k, and we have not even assumed that k is necessarily real. Throughout the rest of this section, however, we shall assume that k is a positive or negative real number, as it must be if there is no dielectric or magnetic dissipation.

In practice both the lamina thickness and the amount of dielectric mismatch will be as small as it is feasible to make them. It will be useful, therefore, to obtain approximate expressions for Γ, K_1, and K_2 under the assumptions

$$|\,\Theta\,| \ll 1, \qquad |\,k\,| \ll 1. \tag{213}$$

Then equation (208) yields

$$\begin{aligned} \mathrm{sh}^2\, \tfrac{1}{2}\Gamma &= \tfrac{1}{2}(\mathrm{ch}\,\Theta - 1) - \tfrac{1}{4}(1 + k)\Theta\,\mathrm{sh}\,\Theta \\ &= -\frac{k}{4}\,\Theta^2 - \frac{(1 + 2k)}{48}\,\Theta^4 - \cdots . \end{aligned} \tag{214}$$

If $|\,k\,| \ll 1$ we can neglect $2k$ compared to unity in the coefficient of Θ^4, but since we have made no assumptions as to the relative magnitudes of $|\,\Theta\,|$ and $|\,k\,|$, we cannot drop either the term in $k\Theta^2$ or the term in Θ^4. If we replace $\mathrm{sh}\,\tfrac{1}{2}\Gamma$ by $\tfrac{1}{2}\Gamma$ in (214), we get

$$\begin{aligned}\Gamma &\approx [-k\Theta^2 - \Theta^4/12]^{\frac{1}{2}} \\ &= \frac{t_1}{\sqrt{3}\delta_1}[(t_1/\delta_1)^2 - 6ik]^{\frac{1}{2}} \\ &= \frac{t_1}{\sqrt{3}\delta_1}\{[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}} \\ &\qquad - i(\operatorname{sgn} k)[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} - \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}}\},\end{aligned} \tag{215}$$

where we have taken the square root of the complex quantity by Dwight 58.2, and

$$\operatorname{sgn} k = \begin{matrix} +1 \text{ if } k > 0, \\ -1 \text{ if } k < 0. \end{matrix} \tag{216}$$

Similarly, from (209),

$$\begin{aligned}K_1 &= \frac{1}{g_1t_1}\left[\frac{(1+k)}{2}\Theta^2 + \Theta\sqrt{-k - \frac{(1-2k-3k^2)}{12}\Theta^2 - \cdots}\right] \\ &\approx \frac{1}{g_1t_1}[\tfrac{1}{2}\Theta^2 + \Theta\sqrt{-k - \Theta^2/12}] \\ &= \frac{it_1}{g_1\delta_1^2} + \frac{1}{\sqrt{3}g_1\delta_1}[(t_1/\delta_1)^2 - 6ik]^{\frac{1}{2}} \\ &= \frac{1}{\sqrt{3}g_1\delta_1}\{[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}} + i\sqrt{3}t_1/\delta_1 \\ &\qquad - i(\operatorname{sgn} k)[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} - \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}}\}, \\ K_2 &\approx \frac{1}{\sqrt{3}g_1\delta_1}\{[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}} - i\sqrt{3}t_1/\delta_1 \\ &\qquad - i(\operatorname{sgn} k)[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} - \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}}\}.\end{aligned} \tag{217}$$

The effective skin depth of a stack of moderately thin layers in the presence of slight dielectric mismatch is, from (215),

$$\Delta = \frac{(t_1 + t_2)}{\operatorname{Re}\Gamma} = \frac{\sqrt{3}(t_1 + t_2)\delta_1/t_1}{[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \tfrac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}}}. \tag{218}$$

An equation essentially equivalent to this was given by Clogston, in somewhat different notation.[17] It is clear from (211) or (218) that if the layers are infinitesimally thin, we have

$$\Delta = \delta_1/\theta \mid k \mid^{\frac{1}{2}}, \tag{219}$$

and the effective skin depth in the stack is proportional to the skin

[17] Reference 1, equation (III–42).

depth δ_1 in the conducting material at the operating frequency, although if the mismatch parameter k is small, the proportionality constant multiplying δ_1 will be large. In the general case, the number of double layers in one effective skin depth is

$$N = \frac{\Delta}{t_1 + t_2} = \frac{\sqrt{3}\delta_1/t_1}{[\sqrt{\frac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \frac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}}}, \tag{220}$$

and the total thickness of conducting material in these layers is

$$T_\Delta = Nt_1 = \frac{\sqrt{3}\delta_1}{[\sqrt{\frac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \frac{1}{2}(t_1/\delta_1)^2]^{\frac{1}{2}}}. \tag{221}$$

It is instructive to plot the effective skin depth of a given stack at a fixed frequency as a function of dielectric mismatch. If

$$\Delta_0 = \sqrt{3}(t_1 + t_2)\delta_1^2/t_1^2 \tag{222}$$

denotes the effective skin thickness when there is no mismatch, then the relative skin thickness when the mismatch parameter is k is just

$$\frac{\Delta}{\Delta_0} = \frac{\sqrt{2}}{[\sqrt{1 + 36k^2\delta_1^4/t_1^4} + 1]^{\frac{1}{2}}}. \tag{223}$$

This ratio is plotted against k in Fig. 8, a universal curve being obtained by measuring k in units of $(t_1/\delta_1)^2$. It is worth noting that when $k = (t_1/\delta_1)^2$, the effective skin thickness is only 53 per cent of the skin thickness with perfect dielectric match.

The surface impedance $Z_0(\gamma_0)$ of a laminated plane stack at any frequency and with any amount of dielectric mismatch is given by equation (65),

$$Z_0(\gamma_0) = \frac{\frac{1}{2}Z_n(\gamma_0)(K_1e^{n\Gamma} + K_2e^{-n\Gamma}) + K_1K_2 \operatorname{sh} n\Gamma}{Z_n(\gamma_0) \operatorname{sh} n\Gamma + \frac{1}{2}(K_1e^{-n\Gamma} + K_2e^{n\Gamma})}. \tag{224}$$

For a stack with infinitesimally thin layers and total thickness s, the equation becomes

$$Z_0(\gamma_0) = K\frac{Z_n(\gamma_0) \operatorname{ch} \Gamma_t s + K \operatorname{sh} \Gamma_t s}{Z_n(\gamma_0) \operatorname{sh} \Gamma_t s + K \operatorname{ch} \Gamma_t s}, \tag{225}$$

where Γ_t and K are given by (211) and (212). At zero frequency,

$$Z_0(\gamma_0) = \frac{1}{\bar{g}s + 1/Z_n(\gamma_0)} = \frac{1}{g_1T_1 + 1/Z_n(\gamma_0)}, \tag{226}$$

while if $Z_n(\gamma_0)$ is infinite, in general

$$Z_0(\gamma_0) = \frac{\Theta}{g_1 t_1} \{\tfrac{1}{2}(1 + k)\Theta + [\tfrac{1}{4}(1 + k)^2\Theta^2 - (1 + k)\Theta \coth \Theta + 1]^{\frac{1}{2}} \coth n\Gamma\}, \tag{227}$$

which simplifies, for infinitesimally thin layers, to

$$Z_0(\gamma_0) = K \coth \Gamma_\ell s. \tag{228}$$

If the stack is many effective skin depths thick, we have

$$Z_0(\gamma_0) = K_1 , \tag{229}$$

while if the individual layers are infinitesimally thin,

$$Z_0(\gamma_0) = K , \tag{230}$$

where K_1 and K are given by (209) and (211), respectively.

When $Z_0(\gamma_0)$ is known, the attenuation and phase constants of the parallel-plane Clogston 1 are given as usual by

$$\alpha = \operatorname{Re} Z_0(\gamma_0)/\eta_0 b, \tag{231}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \operatorname{Im} Z_0(\gamma_0)/\eta_0 b. \tag{232}$$

For the coaxial cable we use

$$\alpha = \operatorname{Re} \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}, \tag{233}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0} + \operatorname{Im} \frac{Z_1(\gamma_0)/\rho_1 + Z_2(\gamma_0)/\rho_2}{2\eta_0 \log (\rho_2/\rho_1)}, \tag{234}$$

but the impedances of the cylindrical stacks are easy to compute only if we can employ the parallel-plane approximation for each stack. To take

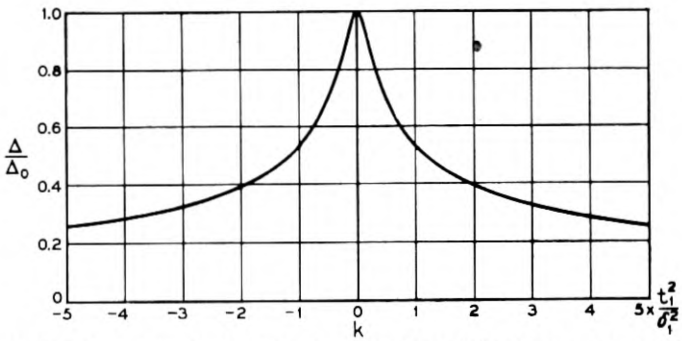


Fig. 8—Relative skin depth Δ/Δ_0 in a stack of finite layers versus dielectric mismatch parameter k, measured in units of $(t_1/\delta_1)^2$.

curvature effects into account would require a considerable amount of numerical calculation. Equation (98) of Section III provides an explicit expression for the surface impedance of a cylindrical stack of infinitesimally thin layers in the presence of dielectric mismatch, in terms of Bessel functions of complex argument; but if the layers are of finite thickness we can at present do nothing better than multiply out the matrices of the individual layers step by step.

The variation of the surface impedance of a laminated stack with frequency over the full frequency range is not quite so simple in the presence of dielectric mismatch as when Clogston's condition is exactly satisfied, but a somewhat analogous discussion may be given. As in the preceding section, we consider a plane stack of n conducting layers each of thickness t_1, where $nt_1 = T_1$, and backed by an infinite-impedance surface. When the mismatch parameter is k, the three critical frequencies are:

$$\begin{aligned} f_1 &= 1/(\pi\mu_1 g_1 T_1^2) && (T_1 = \delta_1), \\ f_2 &= \sqrt{3}/\left(\pi\mu_1 g_1 t_1 T_1 \sqrt{1 + 3n^2k^2}\right) && \\ &= \sqrt{3}nf_1/\sqrt{1 + 3n^2k^2} && (T_1 = T_\Delta), \\ f_3 &= 3/(\pi\mu_1 g_1 t_1^2) = 3n^2 f_1 && (t_1 = \sqrt{3}\delta_1). \end{aligned} \tag{235}$$

In the range $0 \leqq f \leqq f_2$, the surface impedance of the stack is approximately constant, being given by

$$Z_0(\gamma_0) \approx 1/g_1 T_1 . \tag{236}$$

In the range $f_2 \leqq f \leqq f_3$, we have

$$Z_0(\gamma_0) \approx K_1 , \tag{237}$$

where K_1 is given by (217) provided that k is small compared to unity. For infinitesimally thin layers the upper critical frequency f_3 is infinite, and we have for $f \geqq f_2$,

$$\begin{aligned} Z_0(\gamma_0) &\approx | k |^{\frac{1}{2}}(1 - i \operatorname{sgn} k)/g_1\delta_1 \\ &= (1 - i \operatorname{sgn} k)\sqrt{\pi\mu_1 | k | f/g_1} , \end{aligned} \tag{238}$$

which is proportional to $\sqrt{f}$. If the layers are of finite thickness but $k = 0$, we have the result obtained in the preceding section,

$$Z_0(\gamma_0) \approx (1/\sqrt{3} + i)\pi\mu_1 t_1 f, \tag{239}$$

which is proportional to f up to the critical frequency f_2. If neither the mismatch parameter k nor the layer thickness t_1 is zero, then the surface impedance $Z_0(\gamma_0)$ cannot be represented by a simple power of f in the range $f_2 \leqq f \leqq f_3$. At frequencies above f_3, if the layer thickness is finite, the impedance is approximately that of a solid conductor, namely

$$Z_0(\gamma_0) \approx (1 + i)/g_1\delta_1 = (1 + i)\sqrt{\pi\mu_1 f/g_1}, \tag{240}$$

which is proportional to $\sqrt{f}$.

Since in general the surface resistance depends upon the two parameters t_1/δ_1 and k, it is not possible to plot a single curve which shows the variation of resistance with frequency under all possible conditions of dielectric mismatch. However if we compare a matched stack of finite layers with a similar mismatched stack, we see that the asymptotic behavior of $Z_0(\gamma_0)$ is the same for both stacks at very low and very high frequencies. A numerical study of the exact equation for $Z_0(\gamma_0)$ shows that in the neighborhood of the critical frequency f_2, the resistance of the mismatched stack is higher than the resistance of the matched stack. (The critical frequency f_2 as defined in (235) is a function of the mismatch parameter k, but will be of the same order of magnitude for a slightly mismatched stack as for a perfectly matched stack.) The resistance of the mismatched stack exhibits relatively small fluctuations above and below the resistance of the matched stack in the neighborhood of the upper critical frequency f_3, but this region is not of as much practical interest as the region near f_2, where the stack resistance is definitely increased by the effect of dielectric mismatch.

An explicit expression for the rate at which the surface impedance of a mismatched stack begins to depart from its dc value as the frequency is increased has been worked out only for the ideal case of infinitesimally thin layers. For a plane stack of infinitesimal layers backed by an infinite-impedance surface, equation (228) gives, at moderately low frequencies,

$$\begin{aligned} Z_0(\gamma_0) &= \frac{K}{\Gamma_t s}\left[1 + \frac{(\Gamma_t s)^2}{3} - \frac{(\Gamma_t s)^4}{45} + \cdots\right] \\ &= \frac{1}{g_1 T_1}\left[1 - \frac{2ikT_1^2}{3\delta_1^2} + \frac{4k^2T_1^4}{45\delta_1^4} + \cdots\right], \end{aligned} \tag{241}$$

from which the fractional changes in resistance and reactance are

$$\frac{\Delta R}{R_0} = \frac{4k^2T_1^4}{45\delta_1^4} = \frac{4k^2\pi^2\mu_1^2 g_1^2 T_1^4 f^2}{45}, \tag{242}$$

$$\frac{\Delta X}{R_0} = -\frac{2kT_1^2}{3\delta_1^2} = -\frac{2k\pi\mu_1 g_1 T_1^2 f}{3}. \tag{243}$$

The admissible value of $| k |$, if the fractional change in resistance is not to exceed a specified value $\Delta R/R_0$ at a given top frequency f_m, is

$$| k | = \frac{3\sqrt{5}\delta_1^2}{2T_1^2}\sqrt{\frac{\Delta R}{R_0}} = \frac{3\sqrt{5}}{2\pi\mu_1 g_1 f_m T_1^2}\sqrt{\frac{\Delta R}{R_0}}, \tag{244}$$

which is inversely proportional both to f_m and to the square of the total thickness of conducting material in the stack. If we express T_1 in mils, f_m in $\text{Mc}\cdot\text{sec}^{-1}$, and assume the conducting layers to be copper, we get

$$| k | = \frac{22.71}{(f_m)_{\text{Mc}}(T_1)^2_{\text{mils}}}\sqrt{\frac{\Delta R}{R_0}}. \tag{245}$$

The variation with frequency of the surface impedance of a matched stack of finite layers at moderate frequencies (say $f \leqq f_2$) is given by equation (192) of Section V; but no simple formula has yet been derived for the surface impedance of a mismatched stack of finite layers in this frequency range. The derivation of such a formula would appear to involve nothing more than some rather formidable algebra, the difficulties centering around the fact that in the general case we can make no a priori assumptions as to the relative magnitudes of k and $(t_1/\delta_1)^2$. It is reasonable to suppose, however, that if both dielectric mismatch and finite lamina thickness contribute appreciably to $\Delta R/R_0$, the permissible values of $| k |$ and t_1 individually will be less, if we are to achieve a given flatness of the attenuation versus frequency curve, then the permissible value of either if the other factor were unimportant.

To exhibit the effect of dielectric mismatch from a slightly different point of view, we may plot the surface resistance of an infinitely deep plane stack of moderately thin layers (a finite stack several effective skin depths thick would show essentially the same behavior) at a fixed frequency, as a function of the mismatch parameter k. The surface resistance is just Re K_1, which may be obtained from (217) if k and t_1/δ_1 are assumed small compared to unity. Fig. 9 shows the dimensionless quantity

$$\text{Re } g_1\delta_1 K_1 = \frac{1}{\sqrt{3}}\left[\sqrt{\tfrac{1}{4}(t_1/\delta_1)^4 + 9k^2} + \tfrac{1}{2}(t_1/\delta_1)^2\right]^{\frac{1}{2}}, \tag{246}$$

for the three values $t_1/\delta_1 = 0$, $t_1/\delta_1 = 0.1$, and $t_1/\delta_1 = 0.2$. For an electrically thick solid conductor we have simply

$$\text{Re } g_1\delta_1 K_1 = 1; \tag{247}$$

hence to get any benefit from the laminated stack we must have Re $g_1\delta_1 K_1$ smaller than unity. Actually, if we meet Clogston's condition by

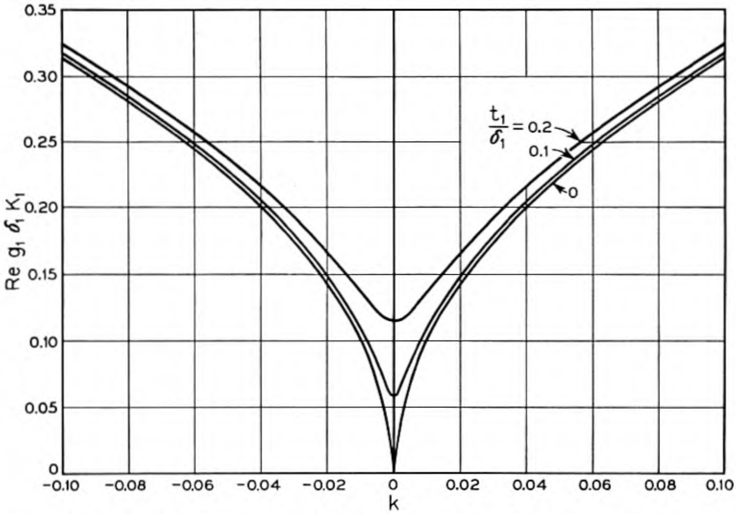


Fig. 9—Normalized stack resistance Re $g_1\delta_1 K_1$ versus dielectric mismatch parameter k, for different values of t_1/δ_1.

raising the dielectric constant and thus lowering the impedance of the main dielectric, then since the attenuation constant of the line is proportional to the ratio of stack resistance to dielectric impedance, we must have Re $g_1\delta_1 K_1$ considerably smaller than unity to obtain a lower attenuation with the Clogston line than with an ordinary air-filled line having solid metal walls.

For a plane Clogston 1 line with stacks of equal thickness, the fractional change in the attenuation constant with frequency is equal to the fractional change in the resistance of either stack, whether this change arises from the effects of finite lamina thickness or from dielectric mismatch or both. The fractional change in the attenuation constant of a coaxial Clogston 1 depends not only on the change in resistance of each stack, but also on the geometric proportions of the cable, in the manner expressed by equation (200) of Section V.

The effect of dielectric mismatch on the overall attenuation versus frequency characteristic of a Clogston cable is in general to reduce the total frequency range (in Mc$\cdot$sec^{-1}) over which the Clogston cable has a smaller attenuation constant than a conventional air-filled coaxial cable of the same size. To calculate the lower crossover frequency we may ordinarily neglect finite lamina thickness effects and use equation (241) for the stack impedances, while at the upper crossover frequency the stack impedances are very nearly equal to K_1, as given by (217).

It should be remembered that mismatch of the $\mu_0\epsilon_0$ product of the main dielectric will usually be accompanied by a change in the dielectric impedance $\sqrt{\mu_0/\epsilon_0}$. Thus under certain conditions the lower crossover frequency may even be reduced by choosing ϵ_0 slightly below the Clogston value, inasmuch as the increase in dielectric impedance may more than compensate for the increase in stack resistance at low frequencies; but it appears that this will be paid for in a steeper slope of the attenuation versus frequency curve and a consequent greater reduction of the upper crossover frequency.

It would be very useful to make a numerical study of the effects of dielectric mismatch in Clogston cables having a variety of different proportions; but in the present paper space limitations restrict us to a few observations concerning orders of magnitude. For the cable which we considered at the end of the preceding section, it turns out than an increase or decrease of 1 per cent in the value of ϵ_0 makes a change of at most a very few per cent in either crossover frequency; with a matched dielectric, we recall, these crossover frequencies were about 1 $\text{Mc}\cdot\text{sec}^{-1}$ and about 280 $\text{Mc}\cdot\text{sec}^{-1}$ respectively. However if we had designed a laminated cable with thicker stacks or thinner laminae or both, so as to increase the theoretical factor of improvement over a conventional cable in the working frequency range, we should have found that the tolerable deviation of ϵ_0 from Clogston's value, instead of being of the order of 1 per cent, was more nearly of the order of 0.1 per cent or even smaller; and the greater the improvement striven for, the more stringent the requirement of accurate dielectric match.

VII. DIELECTRIC AND MAGNETIC LOSSES IN CLOGSTON 1 LINES

Dielectric and magnetic dissipation in the main dielectric and in the stacks can be taken into account by introducing complex dielectric constants and permeabilities for the lossy materials. Thus we may write

$$\begin{aligned}
\epsilon_0 &= \epsilon_0' - i\epsilon_0'' = \epsilon_0'(1 - i\tan\phi_0),\\
\epsilon_2 &= \epsilon_2' - i\epsilon_2'' = \epsilon_2'(1 - i\tan\phi_2),\\
\mu_0 &= \mu_0' - i\mu_0'' = \mu_0'(1 - i\tan\zeta_0),\\
\mu_1 &= \mu_1' - i\mu_1'' = \mu_1'(1 - i\tan\zeta_1),\\
\mu_2 &= \mu_2' - i\mu_2'' = \mu_2'(1 - i\tan\zeta_2),
\end{aligned} \tag{248}$$

where in the most general case the loss tangents may all be different, though it will be assumed that they are all small compared to unity, so that the problem may be treated by first-order perturbation methods.

The average rate of energy dissipation per unit volume in a lossy dielectric by a harmonically varying electric field of maximum amplitude E is just $\frac{1}{2}\omega\epsilon''E^2$, since the imaginary part ϵ'' of the complex dielectric constant corresponds to a conductivity $g = \omega\epsilon''$. Similarly the average rate of energy dissipation per unit volume in a lossy magnetic material by a harmonically varying magnetic field of maximum amplitude H is $\frac{1}{2}\omega\mu''H^2$. The part of the attenuation constant which arises from dielectric and magnetic dissipation is one-half the ratio of power dissipated per unit length of line to total transmitted power, provided of course that the attenuation per wavelength is small. Since the loss tangents of the various materials are assumed small, we can use the fields found for the lossless case to calculate the transmitted and dissipated power.

If the volume occupied by currents in the stacks is small compared to the volume of the main dielectric, so that we can neglect the power flow in the stacks in the direction of wave propagation compared to the power flow in the main dielectric, then the part of the attenuation constant which is due to dielectric and magnetic dissipation is given by equation (51) of Section II, namely

$$\alpha_d = \tfrac{1}{2}\omega\sqrt{\mu_0'\epsilon_0'}(\tan\phi_0 + \tan\zeta_0) = \frac{\pi\sqrt{\mu_{0r}'\epsilon_{0r}'}}{\lambda_v}(\tan\phi_0 + \tan\zeta_0), \tag{249}$$

where λ_v is the vacuum wavelength and μ_{0r}', ϵ_{0r}' are the real parts of the relative permeability and relative dielectric constant of the main dielectric. This equation will be derived from energy considerations presently. It should be noted that the part of the attenuation constant given by (249) is directly proportional to frequency, provided that the loss tangents are independent of frequency; but it is the same for both plane and coaxial geometry and is independent of all the geometrical factors which describe the size and the relative proportions of the line.

Equation (249) will probably be sufficiently accurate for all Clogston 1 lines having the proportions (stacks thin compared to main dielectric) which we have considered in Part I. As an example wherein we also take into account the power flow in the stacks, however, we shall treat a parallel-plane line with infinitesimally thin laminae backed by high-impedance walls. Then, according to equations (120) and (121) of Section IV, the principal field components in the main dielectric are

$$\begin{aligned} H_x &\approx H_0\,, \\ E_y &\approx -\sqrt{\mu_0'/\epsilon_0'}H_0\,, \end{aligned} \tag{250}$$

and in the stacks,

$$\begin{aligned} H_x &\approx H_0(\tfrac{1}{2}a \mp y)/s, \\ \bar{E}_y &\approx -\sqrt{\bar{\mu}'/\bar{\epsilon}'}H_0(\tfrac{1}{2}a \mp y)/s, \end{aligned} \tag{251}$$

the propagation factor $e^{-\gamma z+i\omega t}$ being understood throughout. To take account of dielectric and magnetic dissipation in the stacks, we write

$$\begin{aligned} \bar{\epsilon} &= \bar{\epsilon}' - i\bar{\epsilon}'' = [\epsilon_2'/(1-\theta)] - i[\epsilon_2''/(1-\theta)], \\ \bar{\mu} &= \bar{\mu}' - i\bar{\mu}'' = [\theta\mu_1' + (1-\theta)\mu_2'] - i[\theta\mu_1'' + (1-\theta)\mu_2'']. \end{aligned} \tag{252}$$

The average power P_0 transmitted through the main dielectric is obtained by integrating the real part of the z-component of the complex Poynting vector $\frac{1}{2}\mathbf{E} \times \mathbf{H}^*$ over unit width of the line; thus

$$P_0 = \frac{1}{2}\int_{-\frac{1}{2}b}^{\frac{1}{2}b} \sqrt{\mu_0'/\epsilon_0'}H_0H_0^*\,dy = \tfrac{1}{2}\sqrt{\mu_0'/\epsilon_0'}H_0H_0^*b. \tag{253}$$

Similarly, the average power P_1 transmitted per unit width of either stack is

$$\begin{aligned} P_1 &= \frac{1}{2}\int_{\frac{1}{2}b}^{\frac{1}{2}a} [\sqrt{\bar{\mu}'/\bar{\epsilon}'}\;H_0H_0^*(\tfrac{1}{2}a - y)^2/s^2]\,dy \\ &= \tfrac{1}{6}\sqrt{\bar{\mu}'/\bar{\epsilon}'}H_0H_0^*s. \end{aligned} \tag{254}$$

The average power ΔP_0 dissipated in the main dielectric per unit length and width of the line is

$$\begin{aligned} \Delta P_0 &= \tfrac{1}{2}\omega\int_{-\frac{1}{2}b}^{\frac{1}{2}b} [\epsilon_0''E_yE_y^* + \mu_0''H_xH_x^*]\,dy \\ &= \tfrac{1}{2}\omega[\epsilon_0''(\mu_0'/\epsilon_0') + \mu_0'']H_0H_0^*b \\ &= \tfrac{1}{2}\omega\mu_0'H_0H_0^*b(\tan\phi_0 + \tan\zeta_0), \end{aligned} \tag{255}$$

while the average power ΔP_1 dissipated per unit length and width of either stack is

$$\begin{aligned} \Delta P_1 &= \tfrac{1}{2}\omega\int_{\frac{1}{2}b}^{\frac{1}{2}a} [\bar{\epsilon}''\bar{E}_y\bar{E}_y^* + \bar{\mu}''H_xH_x^*]\,dy \\ &= \tfrac{1}{6}\omega\bar{\mu}'H_0H_0^*s\,(\tan\phi_2 + \tan\bar{\zeta}), \end{aligned} \tag{256}$$

where

$$\tan\bar{\zeta} = \frac{\bar{\mu}''}{\bar{\mu}'} = \frac{\theta\mu_1'' + (1-\theta)\mu_2''}{\theta\mu_1' + (1-\theta)\mu_2'}. \tag{257}$$

The attenuation constant due to dielectric and magnetic dissipation is

$$\begin{aligned}\alpha_d &= \frac{\Delta P_0 + 2\Delta P_1}{2(P_0 + 2P_1)} \\ &= \tfrac{1}{2}\omega\sqrt{\mu_0'\epsilon_0'}\;\frac{(\tan\phi_0 + \tan\zeta_0) + (2\bar{\mu}'s/3\mu_0'b)(\tan\phi_2 + \tan\bar{\zeta})}{1 + (2s/3b)\sqrt{\bar{\mu}'\epsilon_0'/\mu_0'\bar{\epsilon}'}},\end{aligned} \tag{258}$$

which reduces to (249) if we neglect the terms in s/b. The total attenuation is the sum of the metal losses, given by equation (110), and the dielectric and magnetic losses.

For a coaxial Clogston 1 cable with infinitesimally thin laminae and high-impedance boundaries, the principal field components are given by equations (126)–(128) of Section IV. In the main dielectric we have

$$\begin{aligned}H_\phi &\approx \frac{I}{2\pi\rho}, \\ E_\rho &\approx \sqrt{\frac{\mu_0'}{\epsilon_0'}}\,\frac{I}{2\pi\rho},\end{aligned} \tag{259}$$

while in the inner stack,

$$\begin{aligned}H_\phi &\approx \frac{I(\rho^2 - a^2)}{2\pi\rho(\rho_1^2 - a^2)}, \\ \bar{E}_\rho &\approx \sqrt{\frac{\bar{\mu}'}{\bar{\epsilon}'}}\,\frac{I(\rho^2 - a^2)}{2\pi\rho(\rho_1^2 - a^2)},\end{aligned} \tag{260}$$

and in the outer stack,

$$\begin{aligned}H_\phi &\approx \frac{I(b^2 - \rho^2)}{2\pi\rho(b^2 - \rho_2^2)}, \\ \bar{E}_\rho &\approx \sqrt{\frac{\bar{\mu}'}{\bar{\epsilon}'}}\,\frac{I(b^2 - \rho^2)}{2\pi\rho(b^2 - \rho_2^2)}.\end{aligned} \tag{261}$$

The average power transmitted through the main dielectric is

$$P_0 = \frac{1}{2}\sqrt{\frac{\mu_0'}{\epsilon_0'}}\,\frac{II^*}{2\pi}\log\frac{\rho_2}{\rho_1}, \tag{262}$$

while for the average power transmitted through the inner and outer stacks it will be sufficient to replace the exact expressions by the following simple approximations,

$$P_1 \approx \frac{1}{2}\sqrt{\frac{\bar{\mu}'}{\bar{\epsilon}'}}\,\frac{II^*}{2\pi}\,\frac{s_1}{3\rho_1}, \tag{263}$$

$$P_2 \approx \frac{1}{2}\sqrt{\frac{\bar{\mu}'}{\bar{\epsilon}'}}\,\frac{II^*}{2\pi}\,\frac{s_2}{3\rho_2}. \tag{264}$$

For the average power dissipated per unit length of line in the main dielectric and the inner and outer stacks we have, respectively,

$$\Delta P_0 = \tfrac{1}{2}\omega\mu_0' \frac{II^*}{2\pi} \log \frac{\rho_2}{\rho_1} (\tan \phi_0 + \tan \zeta_0), \tag{265}$$

$$\begin{aligned} \Delta P_1 &\approx \tfrac{1}{2}\omega\bar{\mu}' \frac{II^*}{2\pi} \frac{s_1}{3\rho_1} (\tan \phi_2 + \tan \bar{\zeta}), \\ \Delta P_2 &\approx \tfrac{1}{2}\omega\bar{\mu}' \frac{II^*}{2\pi} \frac{s_2}{3\rho_2} (\tan \phi_2 + \tan \bar{\zeta}). \end{aligned} \tag{266}$$

The part of the attenuation constant which is due to dielectric and magnetic dissipation is therefore

$$\begin{aligned} \alpha_d &= \frac{\Delta P_0 + \Delta P_1 + \Delta P_2}{2(P_0 + P_1 + P_2)} \\ &= \tfrac{1}{2}\omega\sqrt{\mu_0'\epsilon_0'} \, \frac{\log \dfrac{\rho_2}{\rho_1} (\tan \phi_0 + \tan \zeta_0) + \dfrac{1}{3}\dfrac{\bar{\mu}'}{\mu_0'}\left(\dfrac{s_1}{\rho_1} + \dfrac{s_2}{\rho_2}\right)(\tan \phi_2 + \tan \bar{\zeta})}{\log \dfrac{\rho_2}{\rho_1} + \dfrac{1}{3}\sqrt{\dfrac{\bar{\mu}'\epsilon_0'}{\mu_0'\bar{\epsilon}'}}\left(\dfrac{s_1}{\rho_1} + \dfrac{s_2}{\rho_2}\right)}. \end{aligned}$$

We need scarcely point out that if the loss tangents are not small compared to unity, it may be impossible to satisfy Clogston's condition (102) very closely with a real value of θ, and the resulting mismatch may reduce the depth of penetration and increase the metal losses in the stacks. In practice, however, the loss tangents will be of the order of 0.001 or even 0.0001, and matching the imaginary parts of $\mu_0\epsilon_0$ and $\bar{\mu}\bar{\epsilon}$ will be much less of a practical problem than matching the real parts.

Appendix I

BESSEL FUNCTION EXPANSIONS

Let ρ_1 and ρ_2 be the inner and outer radii of a cylindrical shell and let the thickness t, given by

$$t = \rho_2 - \rho_1 , \tag{A1}$$

be less than ρ_1. Then, following Schelkunoff,[1] we may replace the Bessel functions appearing in equation (68) of Section III by their Taylor expansions, namely

[1] S. A. Schelkunoff, *Bell System Tech. J.*, **13**, pp. 561–562 (1934).

$$
\begin{aligned}
I_0(\kappa\rho_2) &= I_0(\kappa\rho_1 + \kappa t) = \sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} I_0^{(n)}(\kappa\rho_1), \\
K_0(\kappa\rho_2) &= K_0(\kappa\rho_1 + \kappa t) = \sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} K_0^{(n)}(\kappa\rho_1), \\
I_1(\kappa\rho_2) &= I_0'(\kappa\rho_2) = \sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} I_0^{(n+1)}(\kappa\rho_1), \\
K_1(\kappa\rho_2) &= -K_0'(\kappa\rho_2) = -\sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} K_0^{(n+1)}(\kappa\rho_1).
\end{aligned} \tag{A2}
$$

It follows that

$$
\begin{aligned}
K_0(\kappa\rho_1)I_1(\kappa\rho_2) + K_1(\kappa\rho_2)I_0(\kappa\rho_1) &= -\sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} B_{n+1}(\kappa\rho_1), \\
K_0(\kappa\rho_1)I_0(\kappa\rho_2) - K_0(\kappa\rho_2)I_0(\kappa\rho_1) &= -\sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} B_n(\kappa\rho_1), \\
K_1(\kappa\rho_1)I_1(\kappa\rho_2) - K_1(\kappa\rho_2)I_1(\kappa\rho_1) &= \sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} A_{n+1}(\kappa\rho_1), \\
K_1(\kappa\rho_1)I_0(\kappa\rho_2) + K_0(\kappa\rho_2)I_1(\kappa\rho_1) &= \sum_{n=0}^{\infty} \frac{(\kappa t)^n}{n!} A_n(\kappa\rho_1),
\end{aligned} \tag{A3}
$$

where

$$
\begin{aligned}
A_n(x) &= I_0'(x)K_0^{(n)}(x) - K_0'(x)I_0^{(n)}(x), \\
B_n(x) &= I_0(x)K_0^{(n)}(x) - K_0(x)I_0^{(n)}(x).
\end{aligned} \tag{A4}
$$

The quantities $A_n(x)$ and $B_n(x)$ turn out to be finite polynomials in $1/x$, the general expressions for the coefficients having been derived in a rather inaccessible monograph by Pleijel.[2] When x is large, however, the leading terms are quite simple. From Pleijel's analysis, or directly by substituting the asymptotic series for $I_0(x)$ and $K_0(x)$ into (A4), we find

$$
\begin{aligned}
A_{2m}(x) &= 1/x + O(1/x^3), \\
A_{2m+1}(x) &= -m/x^2 + O(1/x^4), \\
B_{2m}(x) &= m/x^2 + O(1/x^4), \\
B_{2m+1}(x) &= -1/x + O(1/x^3),
\end{aligned} \tag{A5}
$$

where m is a positive integer or zero.

If we substitute these approximations into the first of equations (A3), we obtain

[2] H. Pleijel, *Beräkning af Motstånd och Självinduktion*, K. L. Beckmans Boktryckeri, Stockholm, 1906.

$$
\begin{aligned}
&K_0(\kappa\rho_1)I_1(\kappa\rho_2) + K_1(\kappa\rho_2)I_0(\kappa\rho_1) \\
&\approx \frac{1}{\kappa\rho_1}\sum_{m=0}^{\infty}\frac{(\kappa t)^{2m}}{(2m)!} - \frac{1}{(\kappa\rho_1)^2}\sum_{m=0}^{\infty}\frac{(m+1)(\kappa t)^{2m+1}}{(2m+1)!} \\
&= \frac{1}{\kappa\rho_2}\,\mathrm{ch}\,\kappa t - \frac{1}{(\kappa\rho_1)^2}\left[\frac{1}{2}\frac{d}{dx}(x\,\mathrm{sh}\,x)\right]_{x=\kappa t} \\
&= \left[\frac{1}{\kappa\rho_1} - \frac{t}{2\kappa\rho_1^2}\right]\mathrm{ch}\,\kappa t - \frac{1}{2(\kappa\rho_1)^2}\,\mathrm{sh}\,\kappa t.
\end{aligned}
\tag{A6}
$$

The other three equations may be treated similarly. Doing so, and remembering that

$$
\rho_2/\rho_1 = 1 + t/\rho_1\,, \tag{A7}
$$

we obtain the results which were quoted in Section III, namely

$$
\begin{aligned}
\kappa\rho_2(K_{01}I_{12} + K_{12}I_{01}) &\approx \left[1 + \frac{t}{2\rho_1}\right]\mathrm{ch}\,\kappa t - \frac{1}{2\kappa\rho_1}\,\mathrm{sh}\,\kappa t, \\
\kappa\rho_2(K_{01}I_{02} - K_{02}I_{01}) &\approx \left[1 + \frac{t}{2\rho_1}\right]\mathrm{sh}\,\kappa t, \\
\kappa\rho_2(K_{11}I_{12} - K_{12}I_{11}) &\approx \left[1 + \frac{t}{2\rho_1}\right]\mathrm{sh}\,\kappa t, \\
\kappa\rho_2(K_{11}I_{02} + K_{02}I_{11}) &\approx \left[1 + \frac{t}{2\rho_1}\right]\mathrm{ch}\,\kappa t + \frac{1}{2\kappa\rho_1}\,\mathrm{sh}\,\kappa t,
\end{aligned}
\tag{A8}
$$

up to first order in t/ρ_1.

Table of Symbols

Note: Rationalized MKS units are employed throughout. The subscripts 0, 1, 2 applied to symbols representing material constants, such as ϵ, μ, g, σ, and η, have the significance that 0 refers to the main dielectric in a Clogston line, while 1 refers to the conducting layers and 2 refers to the insulating layers in the stacks. Bars denote average values. Subscripts not included in the present table are explained in the context where they are used.

$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$: Elements of the general circuit parameter matrix (Section III).

a: Distance between outer sheaths of plane Clogston line. Radius of inner core of coaxial Clogston line.

b: Thickness of main dielectric in plane Clogston line. Inner radius of outer sheath of coaxial Clogston line.

C:	A parameter related to the degree of nonuniformity in a laminated medium (Section XII).
E:	Electric field intensity; coordinate subscripts indicate components.
f:	Frequency.
g:	Electrical conductivity.
$\bar{g}$:	θg_1 ; average conductivity parallel to laminated stack.
H:	Magnetic field intensity; coordinate subscripts indicate components.
h:	$-i\kappa_0$; a transverse separation constant (Section X).
I:	Electric current.
i:	$\sqrt{-1}$.
J:	Electric current density; coordinate subscripts indicate components.
K:	Characteristic impedance of stack of infinitesimally thin laminae.
K_1 , K_2 :	Characteristic or iterative impedances of laminated stack (introduced in Section III).
k:	A parameter related to dielectric mismatch in a Clogston 1 line (Section VI).
$\mathbf{M}$:	The general circuit parameter matrix ($\mathfrak{ABCD}$-matrix).
m:	A mode number.
n:	Number of double layers in a laminated stack.
p:	A mode number.
q:	A parameter related to the propagation constant in a Clogston 2 line (Section XI).
R:	A-c resistance of a laminated stack.
r:	Ratio of attenuation constants of Clogston and conventional lines (Section XII).
s:	Thickness of a laminated stack.
s_1 , s_2 :	Thicknesses of inner and outer stacks in a coaxial Clogston 1.
T:	Total thickness of conducting material in a laminated stack (subscripts explained in context).
T_Δ :	Total thickness of conducting material in one effective skin depth.
t:	Thickness of an electrically homogeneous layer. Time.
t_1 :	Thickness of a single conducting layer.
t_2 :	Thickness of a single insulating layer.
V:	Electric potential.
w:	An abbreviation for H_y in Section XII.

X: AC reactance of a laminated stack.

x: Rectangular coordinate in the direction of magnetic field in a plane Clogston line.

y: Rectangular coordinate in the direction normal to the stacks in a plane Clogston line.

Z: Surface impedance of a plane or cylindrical boundary; ratio of tangential components of the electric and magnetic fields (subscripts explained in context).

Z_k: Characteristic impedance of a transmission line.

z: Rectangular coordinate in the direction of wave propagation.

α: Re γ; attenuation constant.

β: Im γ; phase constant.

Γ: Propagation constant per double layer normal to laminated stack.

Γ_ℓ: $\Gamma/(t_1 + t_2)$; average propagation constant per unit distance normal to laminated stack.

γ: Propagation constant in longitudinal direction.

Δ: Effective skin depth; the depth at which the current density in an infinite plane stack has fallen to $1/e$ of its value at the surface. A small change in a quantity.

δ: $\sqrt{2/\omega\mu g}$; skin thickness in a solid conductor.

ϵ: Dielectric constant (capacitivity or permittivity).

$\bar{\epsilon}$: $\epsilon_2/(1 - \theta)$; average dielectric constant measured normal to laminated stack.

ϵ_r: ϵ/ϵ_v; relative dielectric constant.

ϵ_v: Dielectric constant of vacuum; 8.854×10^{-12} farads·meter^{-1}.

ϵ', ϵ'': Real and (negative) imaginary parts of complex dielectric constant.

ζ: $\tan^{-1}(\mu''/\mu')$; phase angle of complex permeability.

η: $\sqrt{i\omega\mu/(g + i\omega\epsilon)}$; intrinsic impedance of medium.

η_v: Intrinsic impedance of vacuum; 376.7 ohms.

η_y, η_ρ: $\eta(1 - \gamma^2/\sigma^2)^{\frac{1}{2}}$; characteristic impedance looking in the y- or ρ-direction in a homogeneous medium.

Θ: $(1 + i)t_1/\delta_1$; a parameter related to the electrical thickness of a conducting layer.

θ: $t_1/(t_1 + t_2)$; fraction of stack volume filled by conducting layers.

κ: $(\sigma^2 - \gamma^2)^{\frac{1}{2}}$; transverse propagation constant in the y- or ρ-direction in a homogeneous medium.

Λ:	A parameter related to the propagation constant in a Clogston 2 (Section XII).
λ:	Wavelength.
λ_v :	Free-space wavelength.
μ:	Permeability.
$\bar{\mu}$:	$\theta\mu_1 + (1 - \theta)\mu_2$; average permeability measured parallel to laminated stack.
μ_r :	μ/μ_v ; relative permeability.
μ_v :	Permeability of vacuum; $4\pi \times 10^{-7}$ henrys·meter^{-1}.
μ', μ'':	Real and (negative) imaginary parts of complex permeability.
ξ:	$y/a + \frac{1}{2}$; normalized coordinate transverse to a plane Clogston 2 line (Section XII).
ρ:	Radial coordinate in cylindrical system.
ρ_1 , ρ_2 :	Inner and outer radii of main dielectric in coaxial Clogston line.
σ:	$\sqrt{i\omega\mu(g + i\omega\epsilon)}$; intrinsic propagation constant of medium.
ϕ:	Angular coordinate in cylindrical system. Phase angle, $\tan^{-1}(\epsilon''/\epsilon')$, of complex dielectric constant.
χ:	$-i\Gamma_t$; a transverse separation constant.
ω:	Angular frequency in radians·second^{-1}.

FUNCTION SYMBOLS

Re:	Real part.
Im:	Imaginary part.
log:	Natural logarithm.
sh:	Hyperbolic sine.
ch:	Hyperbolic cosine.
J_0 , J_1 :	Bessel functions of the first kind.
N_0 , N_1 :	Bessel (Neumann) functions of the second kind.
I_0 , I_1 :	Modified Bessel functions of the first kind.
K_0 , K_1 :	Modified Bessel functions of the second kind.

Electrical Noise In Semiconductors

By H. C. MONTGOMERY

(Manuscript received June 3, 1952)

Transistors, diodes, and single crystal filaments of germanium have common noise properties: a spectrum varying inversely with frequency, and strong dependence on the biasing current. Theoretical attempts to explain this noise are reviewed briefly. Experiments with single crystal filaments indicate that the noise resides in the behavior of the minority carrier. In one type of experiment, the correlation of noise voltages in adjacent portions of a filament is quantitatively related to the lifetime and transit time of minority carrier. In another, the effect of a magnetic field on the noise is found in accord with calculated changes in lifetime of the minority carrier.

In the development of the transistor it was recognized quite early that electrical noise in the device was considerably in excess of Johnson noise, particularly at low frequencies. Noise having a similar spectrum had been observed many years earlier in microphonic carbon contacts carrying a current, and in copper oxide rectifiers, composition resistors, and crystal diodes. Flicker noise in vacuum tubes appears to be a related phenomenon. A number of attempts have been made to determine the mechanism of production of noise of this sort, but none have been particularly successful.

In this paper we will first survey the more important characteristics of noise in germanium diodes and transistors. This will be followed by a partial hypothesis as to the nature of the noise mechanism. We will then discuss experimental work on noise in filaments of single crystal germanium carrying a dc current. These experiments strongly support the hypothesis, and in fact led to its formulation in the first place.

I. NOISE IN DIODES AND TRANSISTORS

There are many similarities in the noise phenomena found in diodes and transistors of both the point contact and junction type. It seems likely that the noise mechanism is similar in all these devices.

One of the most characteristic features of the noise in such structures is the spectrum. The spectral density (power per unit bandwidth) varies inversely as the frequency, according to the relation

$$dW = f^{-n}\, df$$

where the exponent lies between 1 and 1.5 with an average about 1.2. This type of spectrum will be referred to as a 1/f spectrum. Measurements of the spectra of silicon point contact diodes have been reported by P. H. Miller[1] for the frequency range 20 cycles to 300 kilocycles. Spectra of point contact transistors measured by the author have been reported elsewhere[2, 3] for the range 20 to 15,000 cycles. Typical spectra for *p-n* junction type diodes and transistors are shown in Fig. 1. Almost without exception, our measurements and those reported in the literature have shown the 1/f spectrum over most of the frequency range covered. There is some evidence from the related fields of flicker noise and carbon microphone noise that the 1/f spectrum may extend to frequencies well below 0.1 cycle per second. Some departures from this type of spectrum have been noted in the neighborhood of 100 kc, as shown in the curves.

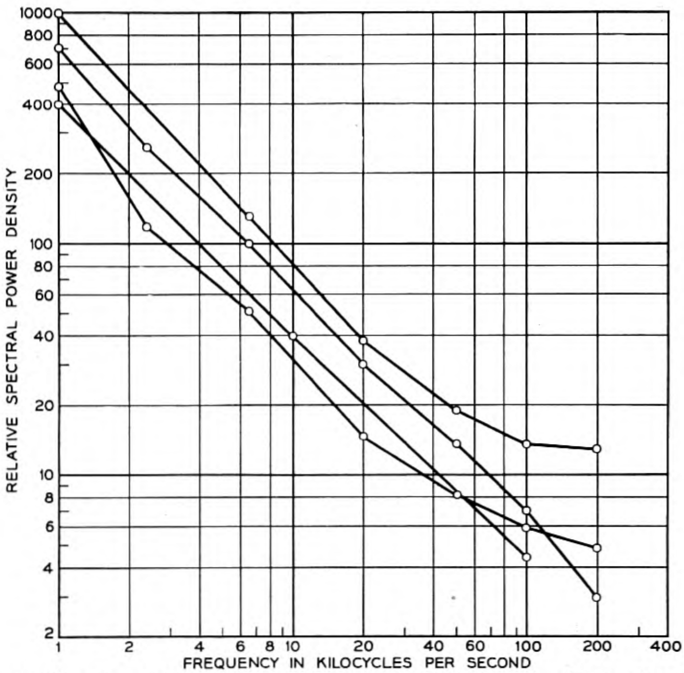


Fig. 1—The spectrum of noise in *n-p-n* transistors varies inversely with frequency.

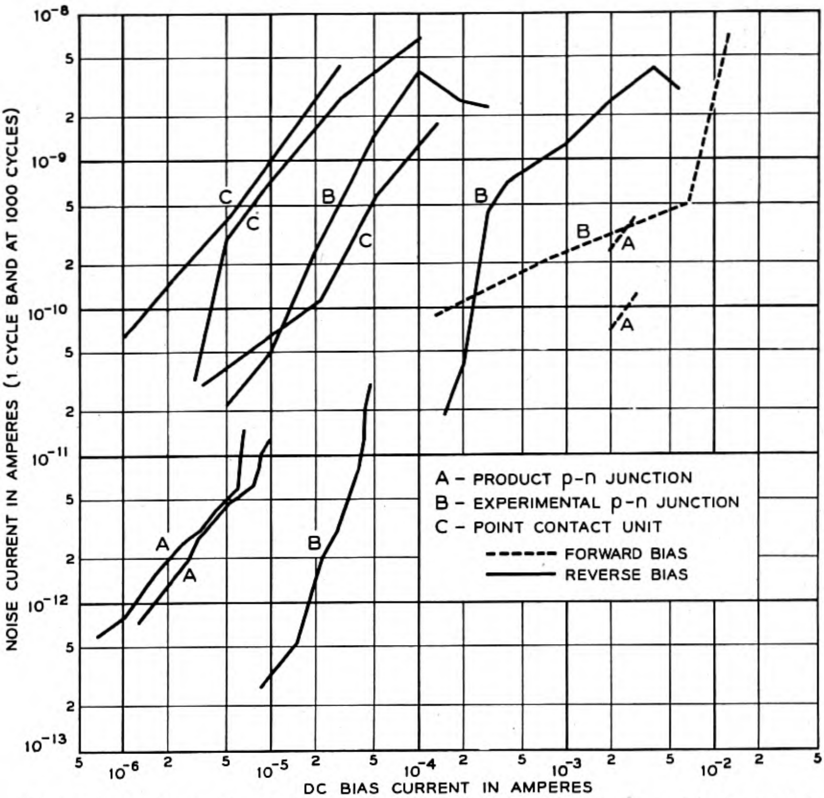


Fig. 2—The short-circuit noise current from a point contact or junction diode generally increases with dc bias current.

A second characteristic feature of noise in all semiconductor devices is that it is current dependent. In the absence of biasing current only Johnson noise is observed. When biasing current is present the noise power may be as much as three or four orders of magnitude above Johnson noise. As a general thing the noise increases as the bias is increased, although some minor exceptions to this rule are noted, usually at bias values where the slope of the current-voltage curve is changing rapidly.

To illustrate the bias-dependent behavior, the noise properties of some germanium diodes of various types are shown in Fig. 2. The short circuit noise current in a 1-cycle band at 1000 cycles is plotted as a function of dc bias current, some of the data being for forward bias, but most for reverse bias. Several curves are shown for each type of unit, and these are typical of the variations encountered. There is a general tendency for noise current to increase in proportion to bias current, but in limited regions the individual units may have slopes considerably different from unity. It would perhaps be more logical to plot current densities rather than total currents, but because of the general form of the relations this makes little difference in the overall picture, and there is some difficulty in estimating the appropriate area for the point contact units. There is an almost unlimited number of different ways of representing noise data. For example, noise current, current density, voltage, or available power may be expressed as a function of various bias parameters. Of a good many combinations tried, none gave an outstandingly simple picture of noise behavior, and the representation used in Fig. 2 is probably as good as any for an overall picture of diode noise.

The noise behavior of transistors depends on two bias parameters. Selection of the emitter current and collector voltage for the parameters usually leads to a rather simple representation. It often turns out that the noise behavior as an amplifier over the commonly used range of bias values depends largely on the collector voltage and is relatively independent of the emitter bias. Data of this sort were shown for point contact transistors in a previous reference,[3] and have been given for an n-p-n transistor by Wallace and Pietenpol.[4] A somewhat more complete family of curves is shown in Fig. 3 for a recent n-p-n transistor.

A few attempts have been made to determine the effect of tempera-

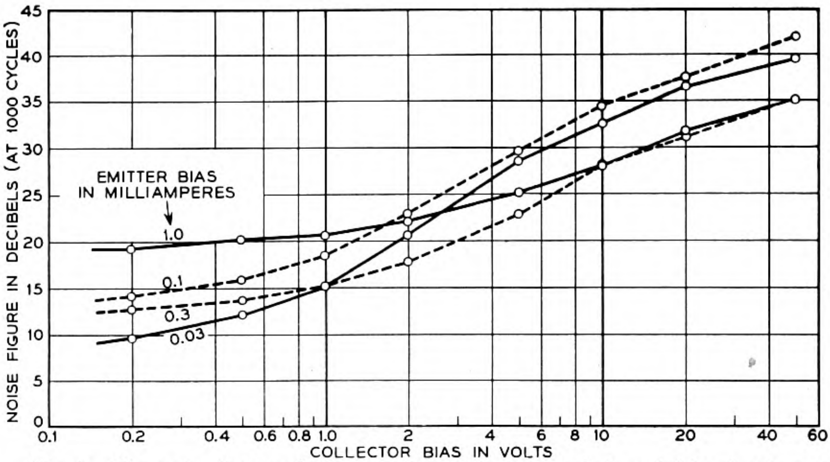


Fig. 3—The noise figure of an n-p-n transistor depends in a fairly simple way on emitter current and collector voltage.

ture on noise behavior. Such experiments have been rather unsatisfactory because the changes in impedance and gain characteristics as a function of frequency are of the same order as the changes in noise properties. This makes the interpretation ambiguous. By and large, such experiments suggest that changes in noise with temperature are rather small, perhaps of the order of the change in absolute temperature, and not at all like the exponential changes associated with a diffusion process. This observation does not necessarily rule out a diffusion-like noise process; it might indicate merely that we are not looking at the right part of the spectrum to observe exponential changes with temperature.

II. A HYPOTHESIS REGARDING THE NOISE MECHANISM

Considerable work has been done on the theory of current-dependent noise having a 1/f spectrum. Among the earliest was that of Schottky[5] in connection with flicker noise in vacuum tubes. He considered the arrival of foreign atoms on the emitting surface of the cathode as a random series of events governed by a diffusion law with a characteristic time constant, and arrived at a $1/f^2$ rather than a 1/f spectrum, and a highly temperature sensitive process. Surdin[6] pointed out that by postulating a series of decay processes with suitably distributed time constants a 1/f spectrum could be achieved. From physical arguments regarding the emission process from cathodes, Macfarlane[7] obtained a range of relaxation times and a 1/f spectrum, in a process which was highly temperature dependent. Richardson[8] gave a very general analysis of the noise properties of systems in which the conductivity was governed by a diffusion process. One conclusion was that a geometrically simple diffusion process in one, two, or three dimensions could not lead to 1/f spectrum, although by some highly specialized assumptions about the geometry of a contact surface he was able to obtain such a spectrum. DuPré,[9] in considering a hypothesis somewhat resembling that of Surdin, showed that the required range of activation energies was physically reasonable, and that the assumptions could be set up in such a way as to make the process relatively temperature independent. Several of the above authors and Van der Ziel[10] discuss the physical basis for applying flicker noise theory to the noise in semiconductors. Although this theoretical work has contributed a great deal to distinguishing between suitable and unsuitable mechanisms, there is still no specific physical theory of noise in semiconductors which can be tied in a quantitative manner to experimental results.

The experimental work described in the remainder of this paper has led to a hypothesis regarding the noise mechanism, which is by no means a complete explanation, but which may be a useful step in that direction. This hypothesis resulted largely from the experimental work, but it seems worth while to describe it first to help appreciate the significance of some of the experimental results.

It has been observed that in many semiconductor structures the noise voltage is approximately proportional to the dc bias current. This relation suggests that the noise is the result of fluctuations of the conductivity of the material, which modulate the bias current and produce a fluctuating voltage across the specimen. Such fluctuations in conductivity could result from variations in concentration of the minority carrier (holes in n-type material, electrons in p-type). The magnitude of the observed noise and the type of spectrum seem to demand that the fluctuation be coarse-grained in time to a much greater extent than could be accounted for by random statistical fluctuations of carrier density. Experiments of Haynes[11] on lifetime and transit of injected carriers in rods of germanium have occasionally indicated finite sources of minority carriers in the material. Our hypothesis is that such sources of carriers are rather generally distributed over the material (although mostly too small to be noticed in experiments of the Haynes type), and that their activity is being modified at a slow rate by some unspecified local influence in a suitable way to agree with the observed noise spectrum.

The experiments described below involving noise correlation phenomena and the effect of a magnetic field on noise point strongly to an important role for the minority carrier in the noise mechanism, and hence strongly suggest some such hypothesis as that just described.

III. NOISE IN SINGLE CRYSTAL FILAMENTS

It was found several years ago that a filament cut from single crystal germanium of high purity exhibits noise well above Johnson noise when a dc current is flowing in it. It is not clear whether this noise arises in the body of the material or on the surface, but to date no method of preparing the sample has eliminated this noise, and it is a prominent feature even at bias fields as low as 10 volts per centimeter. This noise seems to have most of the characteristics of the noise in diodes and transistors: it has the $1/f$ spectrum, is current dependent, and is quite stable with time. It has been the subject of considerable study in the hope that a better understanding of it would illuminate the whole field of semiconductor noise.

Samples, referred to as "bridges", have been cut from thin slabs of single crystal germanium, by a technique devised by W. L. Bond,[12] often of a form shown in Fig. 4. Side arms for both the current and the noise measuring electrodes have been found necessary to avoid spurious noise at the electrodes. A large inductance in the bias circuit greatly reduces the effect of any noise voltage generated at the bias electrodes. The spurious noise power from this source is seldom more than a few per cent of that being measured. It should be noted that the contact area for the noise measuring electrodes should not be on a portion of the specimen carrying bias current, otherwise spurious noise may be generated at these electrodes. Typical dimensions for the straight central filmanent of the bridge are 0.05 x 0.05 x 0.7 cm. The side arms have sandblasted surfaces to suppress holes or electrons injected at the electrodes. The central portion may be etched, sandblasted, or otherwise treated at will. The enlarged circular areas are rhodium plated to provide good contacts to each side arm.

Measurements of the noise spectrum in such bridges with several different etching treatments and with sandblasted surfaces are characterized by the 1/f spectrum over a wide frequency range.* Fairly extensive measurements have been made in the audio frequency range, and a few covering the range from 20 cycles to 1 megacycle. A typical spectrum is shown in Fig. 5.

The current dependence of the noise is shown in Fig. 6 for a number of samples, mostly n-type, one p-type, and with various resistivities. The outstanding feature is that noise voltage always increases with dc bias voltage. In many cases there is direct proportionality at the lower bias values, increasing to a square law at higher biases. There are some

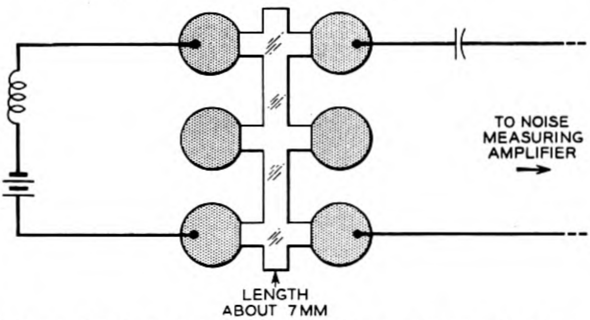


Fig. 4—Filament with side arms cut out of a single crystal of germanium.

* Departures from the 1/f spectrum at frequencies of the order of 100 kilocycles and above were first discovered by G. B. Herzog and A. Van der Ziel. See Reference 13.

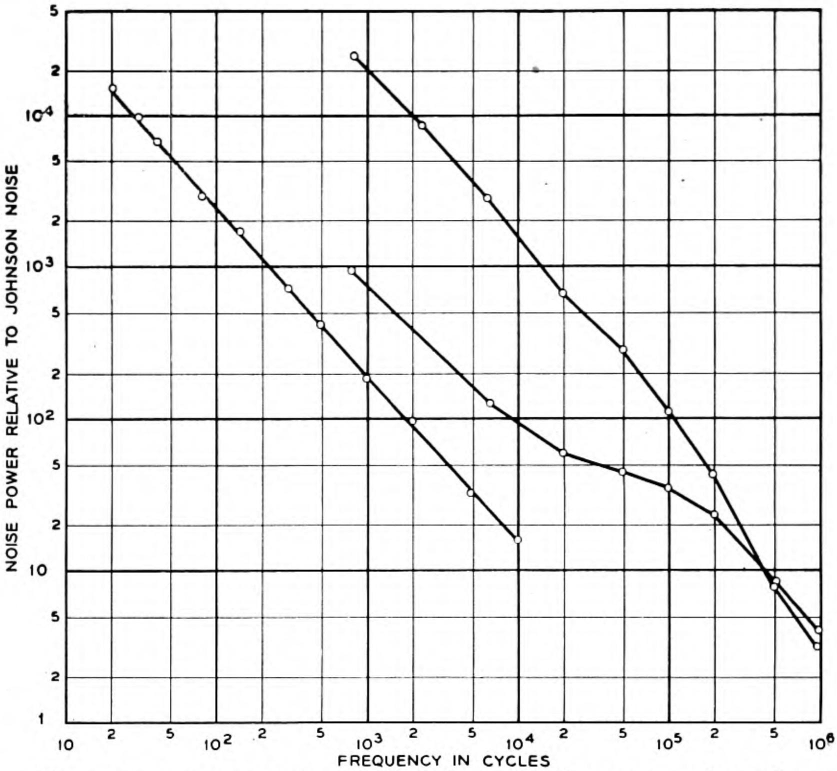


Fig. 5—Typical spectra of noise in single crystal filaments carrying a dc current.

exceptions to this trend. Also, there are large variations in the magnitude of the noise. An average unit shows a noise voltage about three times Johnson noise at a bias of 10 volts per centimeter.

The noise behavior at reduced temperatures has been investigated. Results on three different bridges are shown in Fig. 7. The open circuit noise voltage is shown as a function of temperature for constant bias voltage. Although the curves show rather large irregularities, there seems to be no general trend for noise to decrease with decreasing temperature over the range covered, from $-200°C$ to room temperature.

The surface treatment applied to a bridge may affect the noise very substantially. A sandblasted surface usually gives the lowest noise. Etching the surface may raise the noise voltage by a factor of ten or more, though the dc resistance changes only a few per cent. The technique of washing and drying the surface may have an important effect

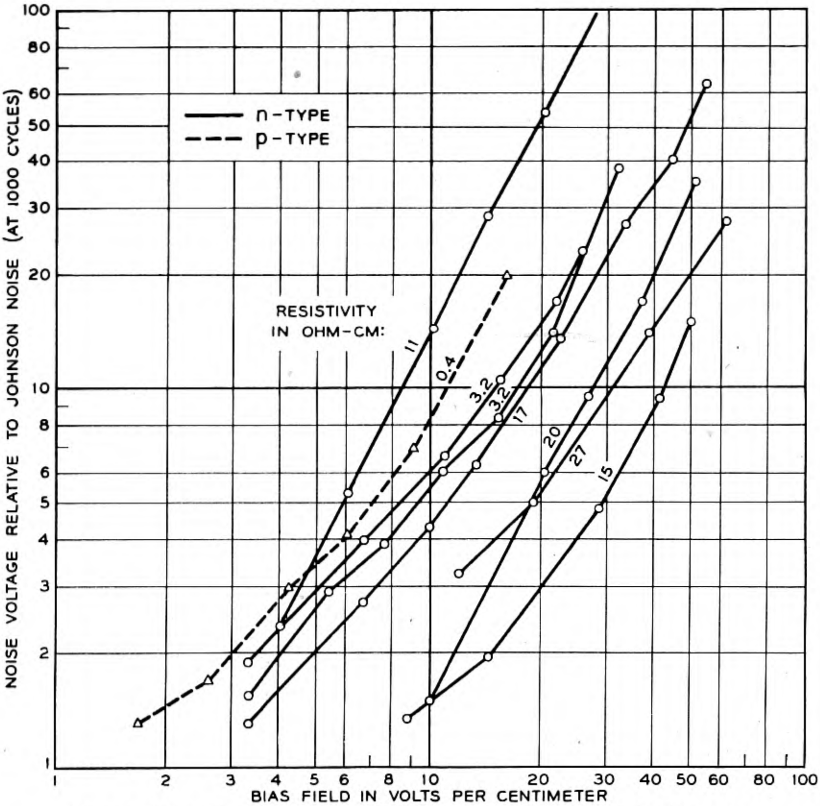


Fig. 6—Variation of noise with dc bias in single crystal filaments.

on the noise. Some of these processes also affect the lifetime of carriers in the bridge to a large extent. However, there seems to be no direct and simple relation between the two effects, since treatments have been found which change the noise by a large factor with almost no effect on lifetime, and vice versa.

Fig. 8 shows measurements of noise voltage on several dozen bridges at a uniform bias of 10 volts per centimeter, all having sandblasted surfaces, mostly of n-type but a few of p-type germanium, and with widely different values of resistivity, produced by varying impurity concentrations. There is considerable scatter in the results, but there is a fairly obvious tendency for noise voltage to increase in proportion to resistivity. Since Johnson noise also increases in proportion to resistivity in a structure of fixed dimensions, the conclusion is that with constant bias voltage the ratio of current induced noise to Johnson noise tends to be

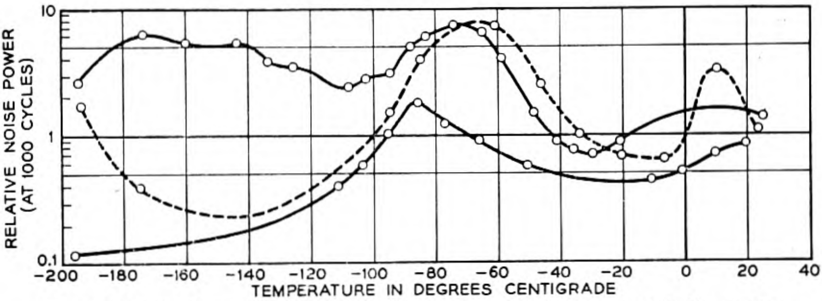


Fig. 7—Variation of noise with temperature in single crystal filaments.

independent of the resistivity of the material. From the data it also appears that there is no consistent difference between *n*- and *p*-type material.

Noise does not appear to depend on orientation of the filament with respect to the crystal axes. Filaments orientated along the 100, 110, and 111 directions and rotated in several ways about these directions showed

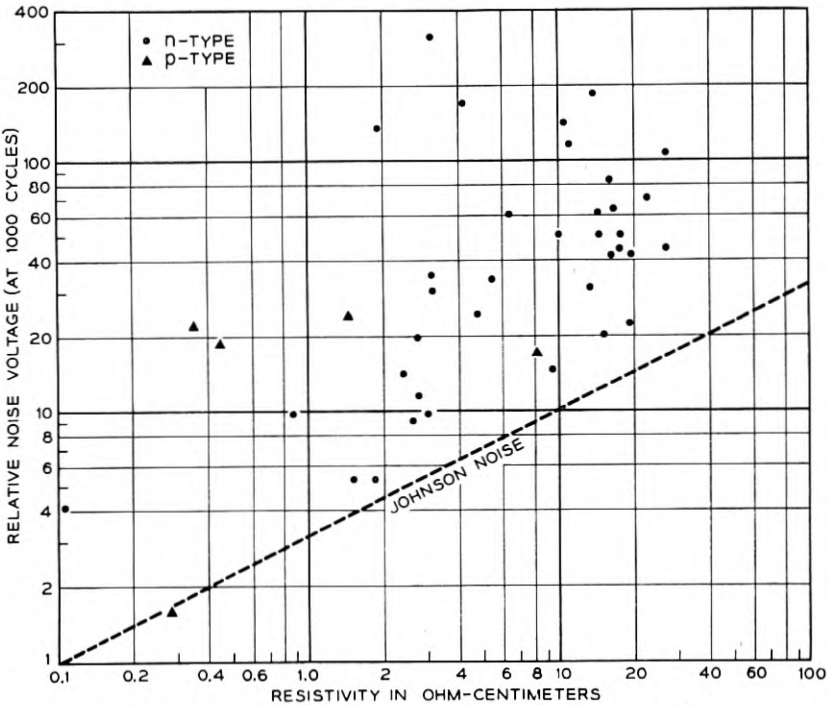


Fig. 8—Variation of noise with resistivity in single crystal filaments.

no significant differences in noise behavior. It should be noted, however, that small variations might be hidden in the large scatter in the data from undetermined causes.

IV. NOISE AND MAGNETIC FIELDS

An important role for the minority carrier in the noise mechanism was first clearly indicated in experiments on the effect of a magnetic field on noise in germanium filaments. It has been found experimentally that the noise in a single crystal filament may change by a substantial factor when the filament is subjected to a steady transverse magnetic field. The following discussion will show that this behavior is in harmony with the hypothesis of noisy injection of minority carriers, as set forth in a preceding section.*

The physical picture on which this treatment is based involves the random injection of holes into an n-type filament by hole sources which may be either in the interior or on the surface of the filament.† It is assumed that the spectrum of the noise arises from the fluctuating nature of the noise source. The effect which any source has will depend upon the lifetime of the holes which it emits. If these holes remain in the filament for a long time, they will produce more noise than if they remain in the filament for a short time. We shall be concerned with the effect of magnetic fields upon these lengths of time and shall not deal in this paper with the fluctuations of the noise sources themselves. If a transverse magnetic field is applied to an n-type germanium filament, a Hall effect voltage is set up and the holes will be deflected towards one surface of the filament. Since recombination takes place principally at the surfaces, this may cause a substantial change in the lifetime of the holes. In order to determine the effect of the magnetic field on the noise we proceed along the following lines.

(a) We assume that the observed noise is due to fluctuations in the conductivity of the filament produced by fluctuations in the hole concentration. Since these fluctuations are small, we may take the change in conducitivity to be proportional to the change in average hole den-

* The following semi-quantitative theory of the dependence of noise on magnetic field is taken with some modification from unpublished work of W. Shockley and H. Suhl, on the basis of which the calculations leading to the curves of Figs. 10 and 11 were carried out. It is hoped that this work may be published in the near future.

† To simplify the terminology, the discussion is based on n-type material with holes as minority carrier. An exactly similar argument could be made for p-type material with electrons as the minority carrier. There is some experimental evidence of the similarity of behavior of n- and p-type germanium, though most of the experimental work has been done with n-type.

sity. (b) We restrict the noise measurements to frequencies low enough so that the period is long compared to the lifetime of a hole. It is then evident that the contribution of a hole source to the noise is proportional to the fluctuating hole current generated by the source and to the average lifetime of the holes. This lifetime depends on the position of the source in the filament, the absorption properties of the surfaces and the electric and magnetic fields.* (c) We assume that the generation properties of the sources are unaffected by the magnetic field, hence, the calculation of the effect of the field on the noise reduces to a problem of calculating the change in lifetime produced by the field. (d) We neglect body recombination in comparison with surface recombination. In germanium filaments of the size usually dealt with, this approximation causes only a small error in the lifetime. (e) Individual sources (or at any rate groups of sources over regions small compared to the dimensions of the filament) will be considered to be statistically independent; therefore, the total effect on the noise can be determined by summing the squares of the contributions from individual sources. Hence we wish to evaluate the following expression:

$$\text{Change in noise power at field } H = \langle \tau^2(H) \rangle / \langle \tau^2(0) \rangle \tag{1}$$

where the symbol $\langle \rangle$ indicates an average over all the noise sources. The statments (a) to (e) represent the principal assumptions in developing the theory.

In order to calculate τ as a function of the magnetic field, H, we consider a steady state case in which a current of holes J_0 is injected into a region in which the average lifetime is τ. If the density of holes in the region is $p(x, y, z)$, the total number is

$$P = \int p(x, y, z)\, dx\, dy\, dz.$$

However, $P = J_0\tau/q$, where q is the charge carried by a hole. Therefore,

$$\tau = \frac{q}{J_0} \int p(x, y, z)\, dx\, dy\, dz \tag{2}$$

This is the method of evaluating τ which is used in the qualitative discussion which follows, and also in the calculation of the curves of Figs. 10 and 11.

* It should be pointed out that a consequence of the hole injection theory of noise in a filament is that marked frequency dispersion should occur when the frequency being studied is high enough so that a period is short compared to the lifetime of holes in the filament. However, we shall neglect this important and interesting aspect of the problem.

Three cases will be treated. In all of these it will be supposed that the width of the filament parallel to the magnetic field is relatively large, so that effects from the edges can be neglected. Also, we are concerned only with average effects over the long dimension of the filament. This permits us to deal with a one-dimensional problem. We shall consider first the case in which holes are supposed to be injected from the surfaces, and the two surfaces have equal and rather large recombination rates. In Fig. 9, part (a) shows how holes injected from each surface are distributed across the thickness in the absence of a magnetic field, and part (b) shows the distribution with a moderate field. The form of these distributions may be determined from the following arguments.

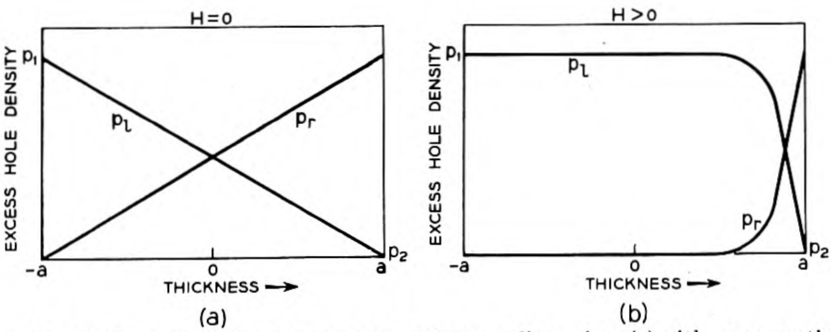


Fig. 9—Excess hole density across the thickness dimension, (a) with no magnetic field, (b) with moderate magnetic field.

If we suppose a steady hole current J_0 emitted from the left-hand surface of the filament, then a relatively high concentration p_1 of holes will appear directly in front of the surface. Some of these holes will recombine upon the surface, the rate J_1 being given by

$$J_1 = p_1 S q$$

where S is the recombination constant for the surface. The balance of the holes will diffuse through the filament to recombine upon the right surface at a rate

$$J_2 = p_2 S q$$

and we note that $J_1 + J_2 = J$. Because of the high recombination rate, p_2 will be very small; hence, J_2 will be much smaller than J_1. In the absence of a magnetic field the gradient is uniform, and the concentrations will be linear, as shown in part (a) of the figure. An identical argument applies to p_r, the concentration of holes emitted from the right-hand side.

Under the influence of a magnetic field pushing holes toward the right, the concentrations will change to those shown in part (b) of the figure. The magnetic field will pull holes through the filament and tend to prevent diffusion from right to left. For some moderate value of field, the value of J_2 is not increased enough to change J_1 appreciably, so the value of p_1 is nearly the same as with no field. At the same time the effects of diffusion are suppressed by the field so that the concentration p_1 extends nearly to the right side of the figure. By the same action, the concentration of holes emitted from the right surface drops to zero very quickly.

From the curves of Fig. 2 and relation (2), we see that the area under the density curve, and hence the lifetime of holes injected at the left is at most doubled by the magnetic field, while the lifetime of those injected at the right is reduced nearly to zero. Recalling that the noise is proportional to a summation of the square of the lifetimes, we see that the noise power is at most doubled at a suitable value of magnetic field.

Higher values of field will sweep so many holes to the right-hand surface as to substantially reduce p_1, so at very high fields the noise decreases monotonically to zero.

Thus it is seen that the noise behavior is the result of competing tendencies. On the one hand, the magnetic field helps holes escape from the surface at which they are emitted, but on the other hand it tends to push these holes against the opposite surface and thereby reduce their lifetime. The relative importance of those two tendencies depends on the surface recombination properties and the strength of the magnetic field.

Calculation of the lifetime along the lines just discussed involves solution of the continuity equation

$$D \frac{d^2p}{dx^2} - \mu_p E_H \frac{dp}{dx} = 0$$

with suitable boundary conditions. The results of such a calculation carried out by Shockley and Suhl in the work already referred to are plotted in Fig. 10.

In order to make the results independent of sample dimensions, the following parameters are used. The first parameter is proportional to the applied magnetic field, and is defined as the effective transverse potential in units of kT/q:

$$\Phi = \frac{tE_H}{kT/q} = 172tEH \times 10^{-5} \tag{3}$$

where t = thickness of the filament (cm)

H = magnetic field (oersteds)

E = applied electric field (volts per cm)

E_H = effective transverse component due to Hall effect (volts per cm)

q = unit electronic charge

kT = Boltzman's constant $\times$ absolute temperature.

The constant may be derived by noting that kT/q is 1/40 volt at room temperature, and that the effective transverse field, E_H, may be expressed as follows. (See Reference 14, Section 8.8.)

$$\begin{aligned} E_H &= \theta E = (\theta_n + \theta_p)E \\ &= (\mu_n + \mu_p)HE \times 10^{-8} \\ &= 4.3 \times 10^{-5}HE \end{aligned}$$

where θ = Hall angle

μ_n = Hall mobility for electrons (2800 cm^2/volt-sec)

μ_p = Hall mobility for holes (1500 cm^2/volt-sec).

The other dimensionless parameter is proportional to the rate of surface recombination, and is defined as the ratio of the surface recombination velocity to the diffusion velocity from the center:

$$\psi = st/2D = st/86$$

where s = recombination velocity characteristic of the surface (cm/sec)

D = diffusion constant (cm^2/sec).

The numerical constant is given for holes at room temperature. The noise changes are expressed in decibles, that is, ten times the common logarithm of the ratio of noise powers with and without the magnetic field.

A second case is that in which generation and recombination are on the surfaces, but the two surfaces have unequal absorption properties. It might be expected that rather large increases in noise would result when the magnetic field was poled to pull holes away from the surface with high absorption properties, and this turns out to be the case when the calculations are carried out. The results are shown in Fig. 11 for a

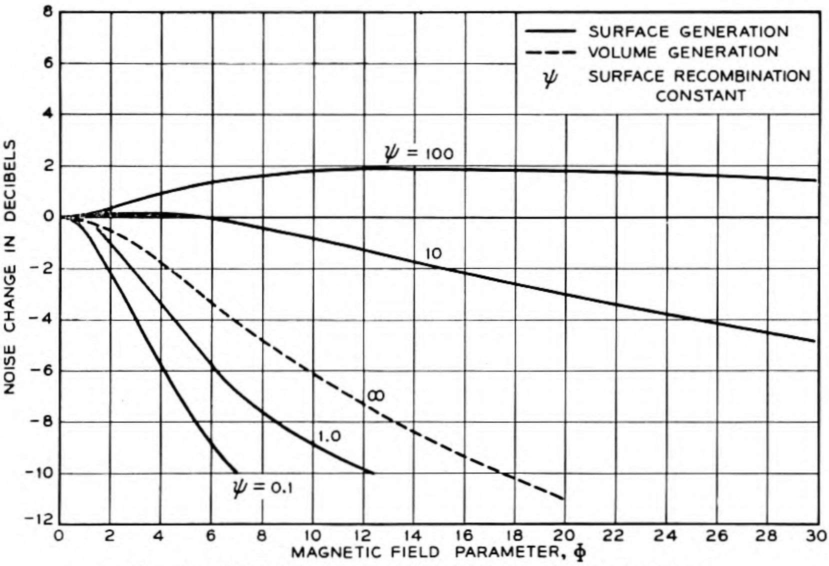


Fig. 10—Calculated magnetic effect for similar surfaces.

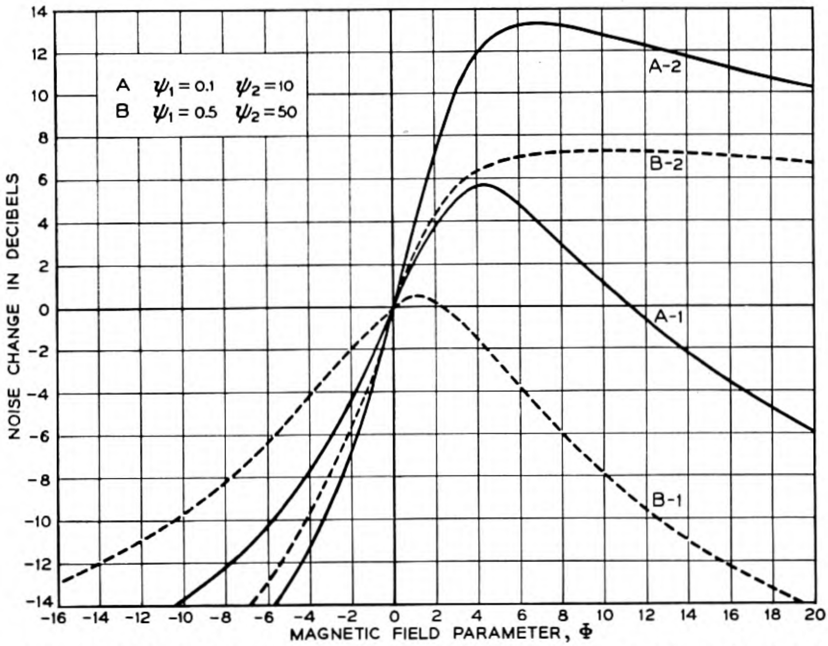


Fig. 11—Calculated magnetic effect for dissimilar surfaces. Curve 1 of each pair is for the contribution from the surface having the lower recombination constant.

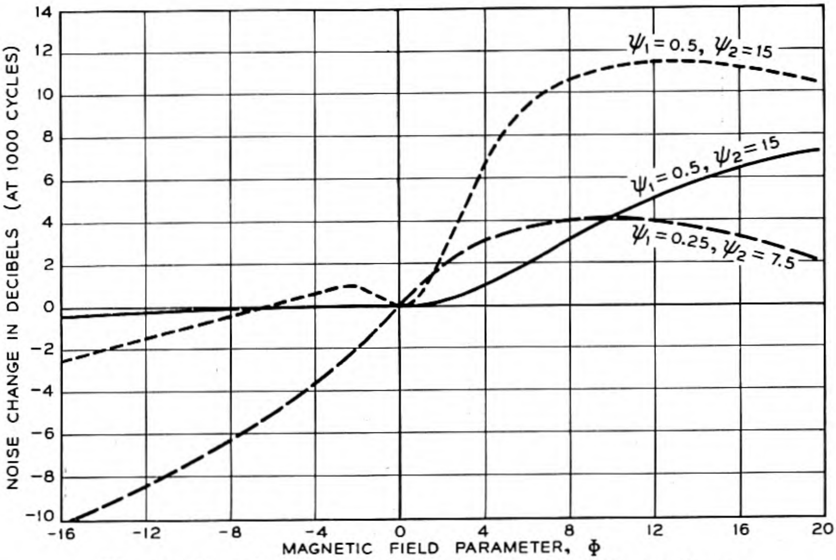


Fig. 12—Experimental magnetic effect for dissimilar surfaces.

case where the two recombination parameters are 0.1 and 10, and for a second case where the parameters are 0.5 and 50. In this figure, separate curves have been shown for noise due to holes generated on each of the two surfaces. The total noise would be gotten by adding the noise powers represented by the two curves after appropriate weighting for the contributions of the two surfaces. At present we do not see any way of determining the weighting factor.

A third case is that in which it is assumed that the noisy generation of holes is uniform throughout the body of the filament, but that recombination takes place on the surfaces only. These assumptions seem at first sight to be in contradiction to the statistical mechanical principle of detailed balancing, which states that under equilibrium conditions all processes occur with equal frequency in the forward and reverse directions. Thus it would seem that if holes are generated in the interior, we must consider recombination in the interior also. Actually this is not necessary under the non-equilibrium conditions which prevail during noise measurements. There is no necessity for the noise generated by a source and a sink for holes to be simply related to the strength of this source. Thus we may suppose there are relatively weak sources and sinks for holes in the interior, but that the hole absorption and generation of the sources is very noisy compared to the recombination and generation processes on the surfaces. If this is the state of affairs, most of the noise will be generated in the interior, but a hole generated in the interior will be much more likely to recombine on the surface. The dotted curve of Fig. 10 has been calculated assuming a uniform distribution of noise sources throughout the interior of the filament and equal and very large recombination constants for the two surfaces. It is seen that for this case the reduction of lifetime predominates, and there is a monotonic decrease in noise with increasing magnetic field.

Experimental work has given results which in most cases are in fair qualitative agreement with the calculated relations. Measurements for three filaments, each of which had one high recombination and one low recombination surface, are shown in Fig. 12. The recombination parameters, as shown on the curves, were of the order of $\psi = 10$ for one surface, and $\psi = 0.5$ for the other. The general shape of the curves is quite similar to the calculated curves of Fig. 11. The maxima are of the right order of magnitude, and occur at reasonable values of the field parameter Φ. The lack of detailed agreement between the measured and calculated curves is not surprising, because the experimental conditions did not fulfill the assumptions made for the calculations in several respects. The filaments were neither wide enough nor long enough so that edge and end effects could be overlooked. The recombination properties of the surfaces could not be measured directly, but had to be estimated from other filaments which had been similarly treated. One experimental curve shows a secondary maximum on the opposite side of the origin. This might indicate a defective portion of one surface having an anomalous recombination constant.

Experimental results are shown in Fig. 13 for four filaments, each of which had nominally equal recombination constants for the two surfaces. These may be compared with the calculated curves of Fig. 10. It will be noted that the experimental curves are not symmetrical about $\Phi = 0$. This lack of symmetry is probably due to dissymmetry in the

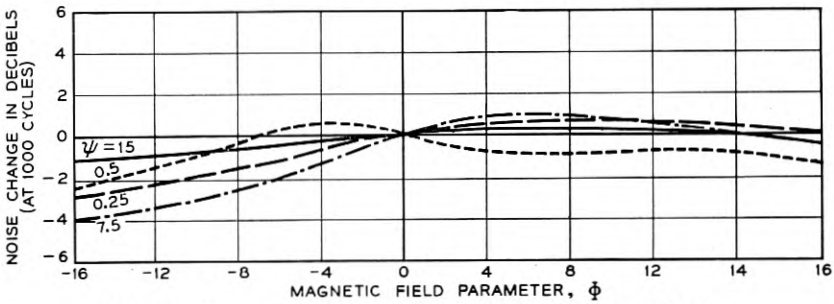


Fig. 13—Experimental magnetic effect for similar surfaces.

samples, particularly the fact that one surface of each filament was cemented to a support, which would probably change the surface recombination properties somewhat. Aside from the lack of symmetry, the behavior of the two filaments with the higher recombination constants is in reasonable agreement with the calculated curves. The filaments with the lower recombination constants are in poor agreement with calculated values, in that the noise does not fall off with increasing field nearly as fast as calculated. The cause of this behavior is not understood. These experimental curves may also be compared with the dotted curve of Fig. 10, calculated on the assumption of volume generation and surface recombination. The similarity is quite poor in all cases. The somewhat better agreement with the surface generation calculations than with the volume generation calculations is not the basis for anything more than a very tentative feeling that the experimental results support the surface generation viewpoint.

While there are many discrepancies in detail between the experimental and calculated relations between noise and magnetic field, these are at least partially understandable in terms of the differences between the experimental setup and the theoretical model. The high degree of qualitative agreement considerably strengthens the hypothesis of noisy injection of minority carriers as an important element in the noise process.

V. NOISE CORRELATION PHENOMENA

The noisy hole injection hypothesis leads one to expect certain correlation phenomena in the noise voltage observed in neighboring portions of a filament. Consider first noise measurements at a frequency so low that the transit time of a hole* along the filament is negligibly small. This might be a frequency of one kilocycle in a typical experiment. The holes have an average lifetime, from which can be determined an average life path, which is defined as the product of the lifetime by the drift velocity under the existing electric field. Noise voltage measurements across segments of the filament much shorter than a life path should be highly correlated, since nearly all the holes which make a transit of one segment will make an almost simultaneous transit of the other segment. On the other hand, noise voltages across segments much longer than a life path should show little correlation, because most of the holes appearing in the two segments are from different sources, and the sources have been assumed to be statistically independent.

* As before, the concepts apply equally well to electrons in p-type material.

A second situation arises when noise is measured at frequencies high enough so that the transit time of holes between segments is not negligible. In this case we should expect the correlation between the noise voltages to be improved by incorporating in one channel of the measuring circuit a delay equal to the transit time between segments.

In order to calculate the correlation resulting from the first situation, we set up a theoretical model based on a few simplifying assumptions: (a) The noise process may be represented by an array of noisy hole current generators which are statistically independent; (b) These generators are uniformly distributed along the filament over the segments where the noise is to be observed, and for a sufficient distance on either side to produce uniform conditions over the segments; (c) The hole currents from the generators decay exponentially with a decay constant determinable from the lifetime; (d) Measurements are made at low enough frequencies so that time of transit of holes may be neglected. We will consider later an alternative to the second assumption. The correlation coefficient between two voltages of instantaneous values v_1 and v_2 may be defined as

$$\rho_{12} = \overline{v_1 v_2}/(\overline{v_1^2} \times \overline{v_2^2})^{1/2}$$

where the bars represent time averages. To evaluate this expression, the contribution of a single generator to the noise voltage in each segment is determined by integrating over the appropriate portion of the decay curve. The total contribution from all generators to the mean voltage product and the mean squared voltages is then determined by integrating the product or square over all the generators. The details are carried out in the appendix, and lead to the solid curve of Figs. 14–16, in which the ordinates are the correlation between noise voltages in two segments of a filament and the abscissae are the ratio of life path of a hole to the segment length.

In an experiment the lifetime τ of holes remains fixed, determined chiefly by the recombination properties of the surface. Consequently the life path ℓ is proportional to the hole velocity, which is determined by the electric field, according to the relation

$$\ell = \tau \mu E$$

where E is the applied field in volts per centimeter and μ is the drift mobility of holes. Hence, by varying the biasing voltage a large range of life path values can be obtained.

The correlation is measured by carrying the noise voltages through separate amplifying channels having identical pass bands extending from 800 to 1300 cycles per second. A switching arrangement makes it possible to apply either of the output voltages or their sum or difference to a rectifier-meter combination. From the readings of the meter the correlation can be computed according to the relation

$$\rho_{12} = (S^2 - D^2)/4V_1V_2 . \tag{6}$$

V_1 and V_2 are rms values of the individual noise voltages, and S and D are the rms values of their sum and difference. The equivalence to expression (5) can be seen by noting that

$$S^2 - D^2 = \overline{(v_1 + v_2)^2} - \overline{(v_1 - v_2)^2} = 4\overline{v_1v_2} .$$

Results of correlation measurements on three bridges are shown in Figs. 14–16. In each case the calculated curve is shown for reference. The values of ℓ were calculated from decay measurements on optically injected holes, as described by J. R. Haynes,[11] using a value for mobility of 1700 cm^2/volt-sec. In Fig. 14 the agreement with the theoretical model is very good. The scatter in the points is due to fluctuations in the noise, which are quite large in the band used for these measurements. In Fig. 15 the agreement could be made quite good with a lateral shift

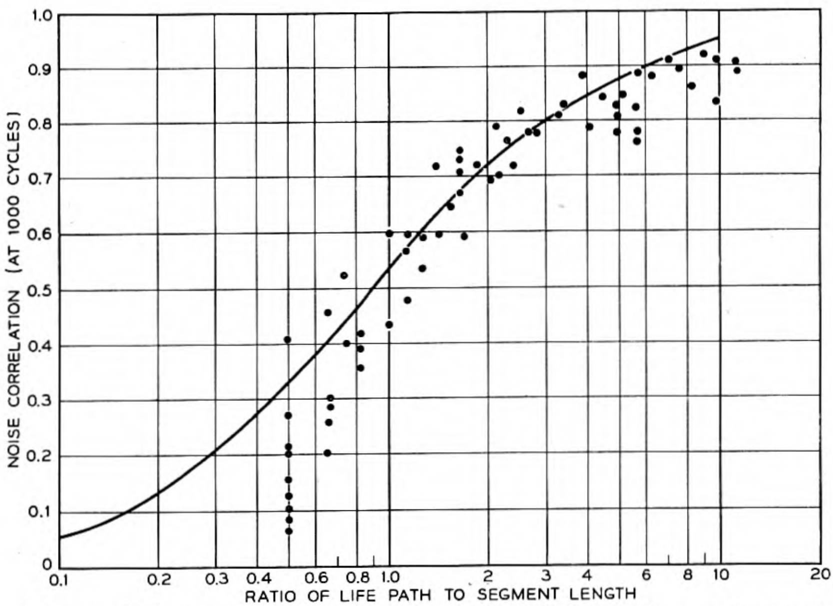


Fig. 14—Noise correlation. The solid curve is calculated, the points experimental.

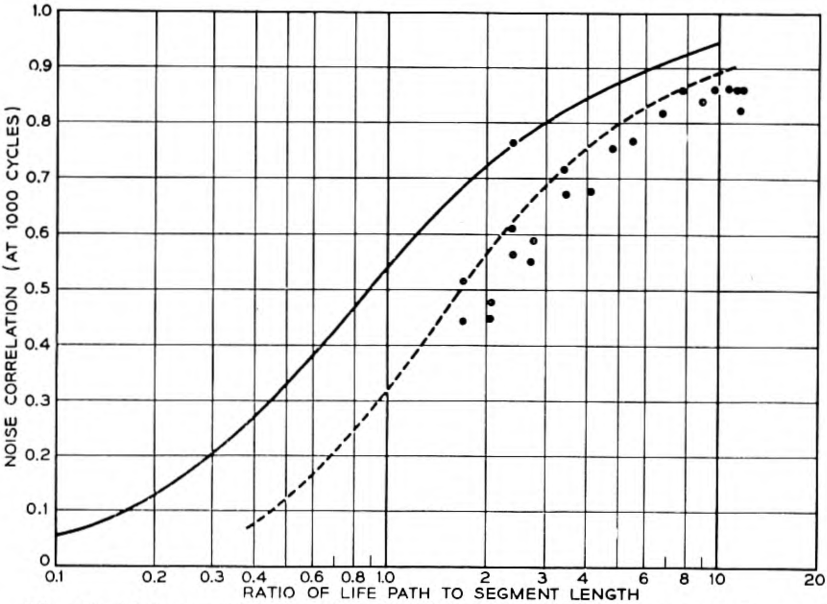


Fig. 15—Noise correlation. The dotted curve includes allowance for losses at the side arms.

by a factor of two. In Fig. 16 the form of the experimental curve seems different from that calculated. In particular, the slope is steeper, and the curve tends to level off at a correlation of about 0.8. It seems possible to explain the discrepancies between the experimental data and the calculations on the basis of two considerations which were not included in the model. (a) The pair of side arms separating the two segments of the filament serve to drain off some holes which would otherwise contribute to the correlation. The dashed curve in Fig. 15 shows the calculated effect, on the assumption that the absorption in the side arms is equivalent to an extra section of filament equal in length to half a segment. The actual distance across the side arms is only 20 per cent of a segment, but it is not hard to believe that the decay rate in this region might increase by a factor of two or three due to the reduced electric field and loss of holes down the side arms. (b) The model assumed a uniform distribution of noise sources along the filament. There is experimental evidence that the distribution may be quite spotty. This can have a substantial effect on the form of the correlation curve. For example, the dashed curve in Fig. 16 shows the curve calculated for noise sources lumped at the mid-point of each segment. Other assumed positions might shift the curve considerably along the horizontal axis.

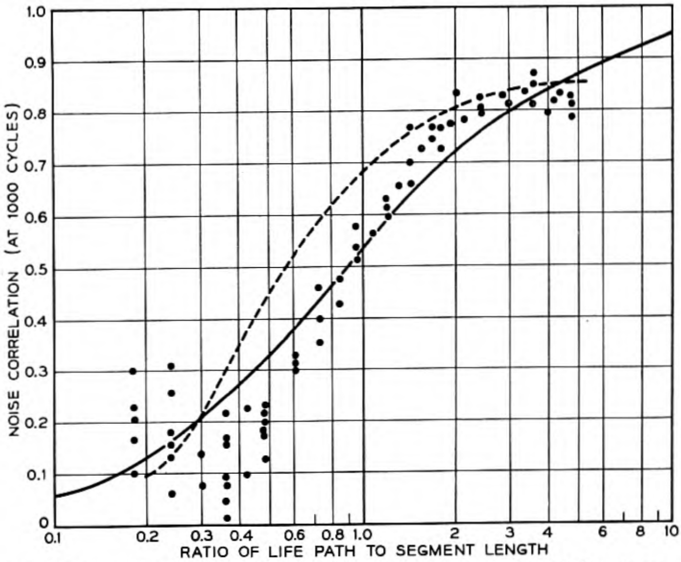


Fig. 16—Noise correlation. The dotted curve is calculated for lumped noise sources.

In view of these considerations there seems to be very satisfactory agreement between the experimental results and the model.

Another type of experiment involves noise measurements at frequencies high enough so that the transit time of a hole across a segment is an appreciable fraction of a cycle. In this case the correlation between noise voltages from adjacent segments can be improved by putting a time delay in one channel of the measuring circuit. Measurements were made by taking the noise voltages from the two segments through separate amplifying channels having identical pass bands extending from 17 to 24 kilocyles. The outputs of the two channels were put on the vertical and horizontal plates of a cathode ray oscilloscope, forming a sort of Lissajous pattern. The patterns differ from those obtained with sinusoidal voltages in that the elliptical figures are filled in solid, due to the continual variation in amplitude of the noise. A phase shifting device is included in one channel, and as the phase is shifted to give optimum correlation, the elliptical pattern narrows down and approaches a line inclined at 45°. For a quadrature phase shift, the pattern becomes circular, and in practice this setting can be determined more precisely than the in-phase setting, largely because the background noise in the circuit is less troublesome. With the phase shift for optimum correlation determined, the delay at the center of the pass band is easily calculated, and since the band is not very wide, the variation in delay over the band is not important. From the drift mobility of holes we may estimate the transit time between segments, according to the relation

$$t = L/\mu E$$

where t = transit time, seconds

L = distance between segment mid-points, cm

E = applied field, volts/cm

μ = mobility of holes, cm^2/volt-sec.

Data for a bridge of n-type germanium of resistivity about 20 ohm-cm are given in Table I. The transit distance, L, after a small correction

TABLE I

E volt/cm	Delay micro sec.	Bridge Temp. °K.	Mobility cm^2/volt-sec.	Transit Time micro sec.
10	21.1	298	1700	18.0
14	15.7	299	1690	12.9
20	11.8	301	1670	9.1
30	9.2	305	1640	6.2
40	9.3	313	1580	4.8

for reduced field across the side arm, was taken as 0.305 cm. As noted in the table, the bridge temperature rose somewhat at the higher bias values, and the assumed values of mobility have been modified according to the inverse three-halves power of the absolute temperature. The delay required for optimum correlation is shown in the second column of the table, and the calculated transit time between segments in the last column. It is seen that the two are in reasonably good agreement, especially at low fields. When the direction of the field is reversed, an equal delay is required, but in the opposite channel of the measuring circuit, as would be expected. Here, again, we have experimental evidence supporting the noisy hole injection hypothesis. The cause of the discrepancy shown in the table at higher fields is not understood. It is possible that trapping phenomena increase the transit time over that calculated from the mobility. There is some evidence for this sort of behavior in lifetime experiments, but to date there does not seem to be enough information for any estimate of magnitude of such an effect.

VI. GENERAL COMMENTS

These studies of electrical noise in semiconductors leave little doubt that the noise is closely related to the behavior of the minority carriers.

It is not yet clear whether the noise is a surface or a volume property of the material, but it is well established that the surface properties have an important connection with the magnitude of the noise. From some of the experimental work it seems likely that the generation and recombination processes are separate and have different noise properties. Because of the nonequilibrium situation, this does not violate the principle of detailed balancing. It seems probable that a more complete understanding of the generation and recombination processes and a clearer picture of the origin of noise in semiconductors may be expected to develop together.

VII. ACKNOWLEDGEMENT

The analysis leading to the theoretical relations between noise and magnetic field is the work of W. Shockley and H. Suhl, under whose direction the calculations leading to the curves of Figs. 10–11 were carried out. The continued interest of Dr. Shockley in the experimental work has been invaluable. The author is indebted to many associates for helpful discussion of certain problems, and also for the construction of many of the devices and materials which entered into the experimental work.

Appendix

Suppose that a source of holes located at a point x_0 in a filament produces a fluctuating current of holes of rms value J_1 in a specified frequency band. The hole current is swept down the filament by a field E and is assumed to decay exponentially according to the relation

$$J = J_1 e^{-(x-x_0)/\ell} \tag{1}$$

where the life path ℓ may be expressed in terms of drift velocity v, hole mobility μ, and lifetime τ

$$\ell = v\tau = \mu E \tau.$$

Assuming that the frequency of measurement is low enough to justify neglecting the hole transit time, the noise voltage due to holes from a single source is proportional to the number of holes present in the segment. This is obtained by integrating (1) over an appropriate range

$$\begin{aligned} dv &= J_1 \int_a^b e^{-(x-x_0)/\ell}\, dx \\ &= \begin{cases} K_1 e^{x_0/\ell}[e^{-a/\ell} - e^{-b/\ell}] & x_0 < a \\ K_1[1 - e^{-(b-x_0)/\ell}] & a < x_0 < b \end{cases} \end{aligned} \tag{2}$$

where K_1 is an omnibus constant which will cancel out in the final result.

Under the assumption that the sources are statistically independent, the total voltage squared is obtained by integrating the square of (2) over all the sources.

$$\overline{v_1^2} = \overline{v_2^2} = K_1 \int_{-\infty}^{a} e^{2x_0/\ell}[e^{-a/\ell} - e^{-(a+L)/\ell}]^2\, dx_0$$

$$+ K_1 \int_{a}^{a+L} [1 - e^{-(a+L-x_0)/\ell}]^2\, dx_0$$

$$= K_2\left[1 - \frac{\ell}{L} + \frac{\ell}{L}\, e^{-L/\ell}\right].$$

Similarly, the cross product of voltages in two segments, extending say from 0 to L and L to $2L$, is

$$\overline{v_1 v_2} = K_1 \int_{-\infty}^{0} e^{2x_0/\ell}[1 - e^{-L/\ell}][e^{-L/\ell} - e^{-2L/\ell}]\, dx_0$$

$$+ K_1 \int_{0}^{L} e^{x_0/\ell}[1 - e^{-(L-x_0)/\ell}][e^{-L/\ell} - e^{-2L/\ell}]\, dx_0$$

$$= K_2 \frac{\ell}{2L}\, [1 - e^{-L/\ell}]^2.$$

From the definition of the correlation coefficient

$$\rho_{12} = \overline{v_1 v_2}/(\overline{v_1^2} \times \overline{v_2^2})^{1/2} = \frac{\ell}{2L}\, \frac{(1 - e^{-L/\ell})^2}{1 - \dfrac{\ell}{L}\,(1 - e^{-L/\ell})}$$

which is the desired relation, from which the solid curves of Figs. 14–16 were calculated.

REFERENCES

1. P. H. Miller, *Proc. Inst. Radio Engrs.*, **35,** pp. 252–256 (1947).
2. J. A. Becker and J. N. Shive, *Elec. Eng.*, **68,** pp. 215–221 (1949).
3. R. M. Ryder and R. J. Kircher, *Bell System Tech. J.*, **28,** pp. 367–400 (1949).
4. R. L. Wallace, Jr., and W. J. Pietenpol, *Bell System Tech. J.*, **30,** pp. 530–563 (1951).
5. W. Shottky, *Phys. Rev.*, **28,** pp. 74–103 (1926).
6. M. Surdin, *Journal de Phys. et le Rad.*, **10,** 188–9 (1939) do **12,** pp. 777–783 (1951).
7. G. G. Macfarlane, *Proc. Phys. Soc.*, **59,** Pt. 3, 366–374 (1947) do **B63,** pp. 807–814 (1950).
8. J. M. Richardson, *Bell System Tech. J.*, **29,** pp. 117–141 (1950).
9. F. K. duPré, *Phys. Rev.*, **78,** p. 615 (1950).
10. A. Van der Ziel, *Physica*, **16,** pp. 359–372 (1950).
11. W. Shockley, G. L. Pearson, J. R. Haynes, *Bell System Tech. J.*, **28,** pp. 344–366 (1949).
12. W. L. Bond, *Phys. Rev.*, **78,** p. 646 (1950).
13. G. B. Herzog and A. Van der Ziel, *Phys. Rev.*, **84,** pp. 1249–1250 (1951).
14. W. Shockley, *Electrons and Holes in Semiconductors*, Van Nostrand (1950).

Important Design Factors Influencing Reliability of Relays

By J. R. FRY

(Manuscript received June 13, 1952)

Relays are produced by a large number of manufacturers in this country. When we survey their product, we find that there are many kinds and varieties. They differ widely as to their size, shapes and configurations. Many of these differences are dictated by the requirements of the task they must perform and by the environments under which they must work. Other differences are brought about from considerations of cost and by the design and fabrication techniques the particular manufacturer employs. However, all relays have a common objective. For whatever use they are employed, it is highly desirable that they be reliable. They are expected to function each time they are called upon without failure and over the expected life of the equipments in which they are used. This paper deals with the more important design factors which all relays have in common that greatly influence their reliability of performance. Contact spring pile-up stability and the importance of strength of screws, insulating materials with low cold flow and moisture absorption, and manufacturing procedures and controls to achieve this end are discussed. Coil construction so as to minimize the occurrence of open windings due to corrosion of the wire and breakage of the lead-out wires is dwelt upon. Contact reliability and how it is affected by the material used, its size and shape, the method of actuation, the presence of contaminating vapors, and single versus twin contacts are discussed. The degree by which magnetic materials change their magnetic properties with age and treatments for alleviating this effect are described. The importance of adequate structural design so that the relay will be rugged and remain stable so that its performance is substantially unaffected by wear, shock and vibration is stressed. Methods of test to determine how well the relay meets these objectives are described.

Although a relay is conceptually a simple device, the wide range of conditions under which relays are required to operate, the many different characteristics they must have, and the complete dependence placed upon them in many circuit applications, make them a subject of continuous study.

In the telephone industry, for example, the completion of a single call may bring into play a thousand or more relays. While their principal function is to close electrical contacts, there are many facets to the problem of doing this satisfactorily. Relays are produced by many manufacturers in this country. When we survey their product we find that there are many kinds and varieties. Shapes, sizes and configurations of relays may differ in accordance with the requirements of the tasks they must perform, and the environments under which they may work; other differences may be brought about by cost considerations and by design and fabrication techniques of the manufacturer.

All relays, however they may be used, have one common objective – they must be reliable. They are expected to function each time they are called upon, should do this without failure, and should continue to do so over the expected life of the equipment in which they are used. It is the purpose of this paper to discuss the more important factors that are common to all relays and which have considerable influence on their reliability of performance. The design considerations discussed in this paper are presented in the following order.

(1) Contact Pile-up Stability,
(2) Coil Construction,
(3) Contact Reliability,
(4) Magnetic Stability, and
(5) Structural Stability.

CONTACT SPRING PILEUP STABILITY

Stability of the contact spring pile-up contributes in a large degree to the reliability in performance of the relay. Since contact springs may be assembled into pile-ups of from two springs to a dozen or more, they must be secured so that they will not shift position during the life of the relay, even when it is subjected to relatively large changes in temperature and humidity, and to vibration and shock during shipment, installation, wiring, and under operating conditions. It is also important that the dimensional relations between the contact ends of the springs and the actuating members of the relay do not change appreciably; otherwise, changes in contact separation, contact follow, contact pressure, and operating and releasing current values may cause faulty operation of the relay.

Insulators for securing the springs should be made of materials having low cold flow and moisture absorption characteristics; in telephone relays, the better grades of phenol fibre have been found adequate. They should have generous clamping surfaces, so that when the spring pile-up is clamped under force, high pressures on the insulators are avoided – thus minimizing cold flow and keeping well below the crushing strength of the material.

Pile-up screws, clamping plates, and screw threads should be porportioned such that permanent deformation under any condition does not take place, i.e., the maximum stress does not exceed the elastic limit of the metal. It has been found advantageous to use high tensile strength steel for these parts – a tensile strength of 100,000 lbs per square inch or higher.

In the manufacture of relays, the desired pile-up tightness requires certain procedures and controls. Insulators are baked in an oven at a temperature of about 150°F for a minimum of 24 hours and assembled in the relay while in the dry condition. During assembly, prior to tightening the screws, the pile-up is pressure clamped under a hydraulic or air powered fixture to a controlled force of 1,300 lbs to 3,200 lbs, depending upon the size of the relay; while under this pressure, the screws are tightened using a controlled torque. To assure that the processes and materials are under control, the relay is then tested on a "no-go" basis in a fixture that applies a definite force on the contact springs in a direction to rotate them in the pile-up.

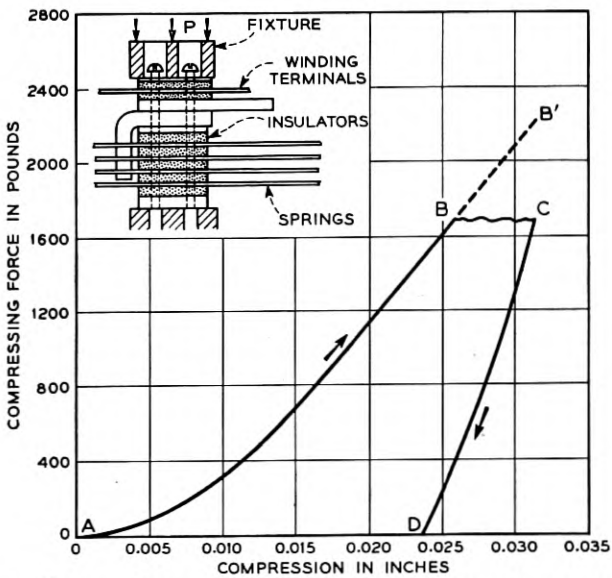


Fig. 1—Compression cycle of a relay contact spring pile-up assembly.

Pressure clamping during manufacture enables the spring pile-up to better maintain its adjustment through cycles of humidity and drying, and to prevent displacement during installation and wiring. The action of a pile-up under this compression is illustrated in Fig. 1. Curve AB represents the application of a 1,700-lb force to the pile-up by a power driven fixture prior to tightening of the pile-up screws. At the start, the relationship is not linear due to "nesting" of the parts, but a linear slope is soon reached, representing the stiffness of the pile-up without the screws. At the point B the two screws are tightened with a controlled torque; further compression takes place, indicated by the jagged line BC. An estimate of the tension put into the screws by this tightening operation can be made by extrapolating the curve AB to the point B^1, vertically above the point C. When the pressure fixture is released, the pile-up tends to expand and follows the line CD. The slope of this line represents the combined stiffness of pile-up parts and screws, which makes it stiffer than the original compression slope. When the pile-up is released, an equilibrium point is reached where the tension in the screws equals the force with which the pile-up tends to expand.

A series of measurements on a typical relay pile-up screw and clamp plate assembly is shown in Fig. 2 to illustrate the stress-strain relationship when a force is applied axially to put the screws under tension. As force is applied, Hooke's law is followed up to the point A; strain is proportional to the stress and no permanent deformation takes place. Beyond point A the elastic limit of the metal is exceeded and permanent deformation begins. When a point B is reached, somewhere below the breaking point of the screw, and the force is released, a permanent deformation results. Note that, in Fig. 2, the high strength screw will permit a higher screw tension without deformation – and its resultant looseness of the pile-up – than will the lower strength screw.

Analytical methods are available for estimating the range of screw tensions that exist during the life of the relay. By taking into account the known cold flow characteristics of the insulators with the relay in the dry state, a minimum value can be estimated. It should be of sufficient value to hold the springs securely in place. By considering the conditions that obtain in the humid state, maximum value of tension can be predicted, which should not exceed the strength characteristics of the materials used.

To determine how well the design objectives for stability are being realized, accelerated laboratory tests are made upon the relay, and from these results predictions can be made as to its performance during its life. In the telephone system, relays are generally subjected to repeated cycles of alternate humid and dry environments over the years. Humid conditions exist during the summer season, followed by a dry exposure during the winter months when the central offices are heated. Experience has shown that a relay exposed for six days to 90 per cent relative humidity at 85°F will be comparable to those observed in service in humid localities. Although the six day exposure is admittedly an accelerated test, the dimensional changes produced are approximately the same as those caused by the accumulating effect of fluctuating humidity during the entire season. Similarly, a period of six days exposure to 120°F produces the same effect of drying as that which occurs during the heating season.

By making careful measurements on an adjusted relay of such important parameters as operate current, releasing current, contact separation, contact spring tension, armature back tension, stud gaps, etc., and then subjecting the relay to repeated cycles of humid and dry conditions, repeating the measurements after each exposure and noting the changes, a good appraisal of the relay can be made. A repetition of the test will reproduce the same pattern of results as the first cycle unless permanent deformation of the materials in the relay has taken place.

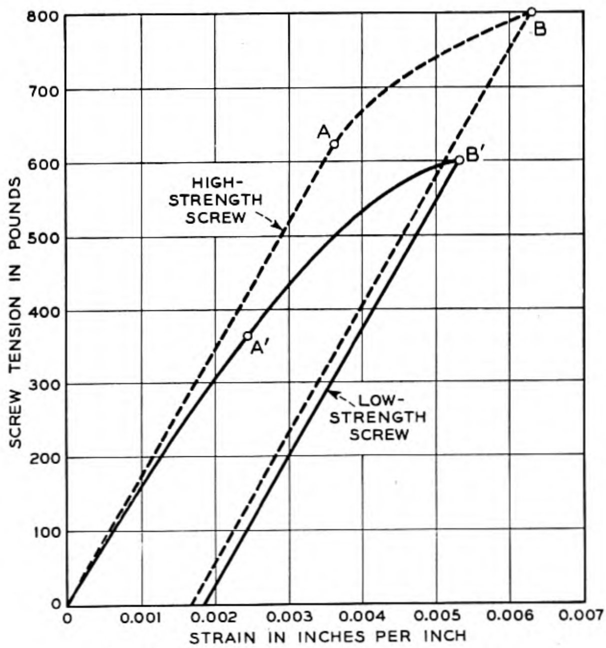


Fig. 2—Strength of relay pile-up screws.

COIL CONSTRUCTION

If, for any reason, the winding of a relay should become open during its life, usefulness of the relay ceases. One of the most prevalent causes for failure of this kind can be due to so-called corrosion of the wire. This is not corrosion in the ordinary sense of the word, but is caused by an electrolytic action within the coil when an electrical potential is applied to the winding. This action can take place only in the presence of moisture. If there are any active impurities in the insulating materials intimately associated with the copper wire, an electrolyte is formed and disintegration of the wire proceeds to the point where failure may occur. This trouble is accentuated with coils using small diameter wires, because with the smaller cross-section of copper, failure of the section will occur in a shorter period of time.

There are two methods of approach to minimize corrosion failure. One is to thoroughly dry the coils, and in this condition seal them in a potting compound, which prevents the entrance of any moisture into the coil, or enclose the coil in a hermetically sealed chamber. For relays this is an expensive and cumbersome way to overcome corrosion troubles. The second and more practical method is to use, in the construction of the coil, insulating materials that are chemically inert and free from corrosion promoting impurities.

For many years, studies were made with a view towards eliminating the occasional corrosion failures of fine wire windings which occurred under unfavorable atmospheric and circuit conditions. Although improvements were effected by the use of the better grades of phenol fibre for spoolheads and waxed varnished papers for core and winding insulation, an entirely satisfactory coil was not achieved until the use of cellulose acetate insulation was adopted. This material, in thin sheet form, can be applied to spool-wound coils, where the coils are wound individually, and the so-called "filled" coils, where a multiple number are wound simultaneously on automatic winding machines. The coils are wound on a mandrel, as many as twelve individual coils per mandrel, with a thin sheet of insulation between the layers of wire. The "stick" of coils is wound so as to leave a small separation between coils to provide insulation at the ends of the coils and to permit cutting the stick into individual coils, after which they are assembled to relay cores using conveyor assembly methods. This results in not only a more economical coil, but in a higher quality coil as well. The thin sheet of insulation between the layers of wire, generally not provided on the spool wound type of construction, eliminates the occurrence of short-circuited turns.

The degree of improvement obtained by the use of cellulose acetate over the materials formerly used is shown in Fig. 3.[1] This is the result of an accelerated corrosion test adopted as a means of obtaining data on proposed constructions, using as a basis for comparison, the performance, in this test, of the earlier spool wound construction. The latter had given satisfactory service in the field, only occasional failures occurring under the most severe circuit and atmospheric conditions. This test is made with double or triple wound coils since these represent the most

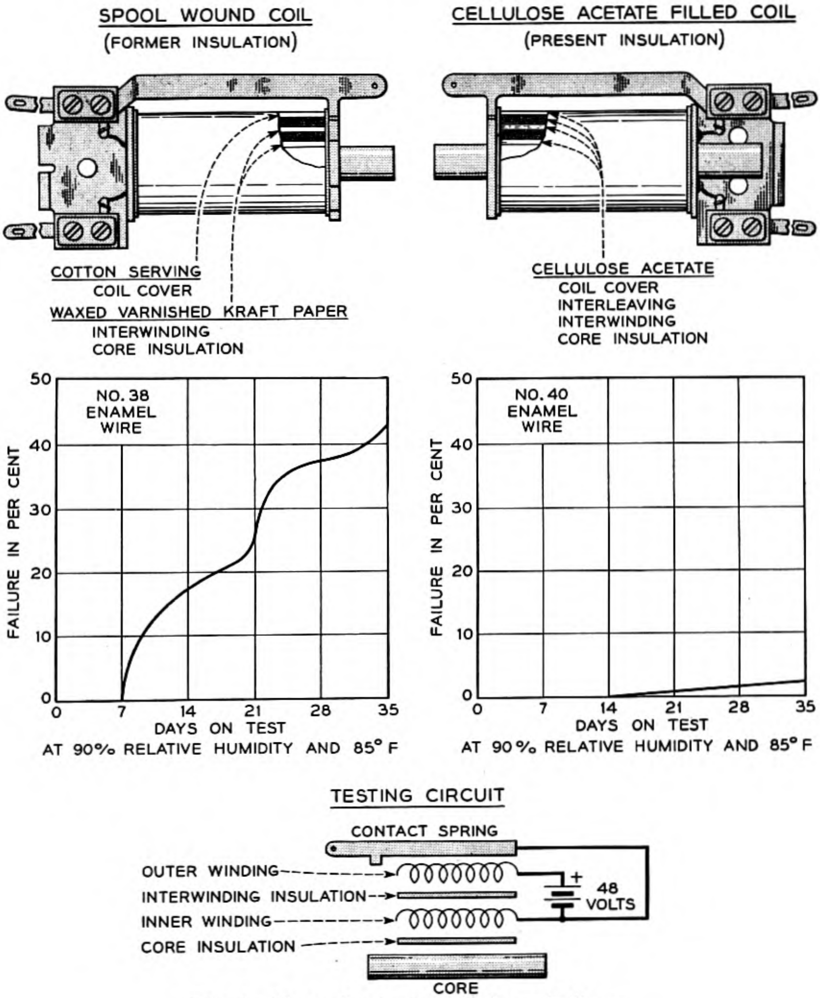


Fig. 3—Corrosion tests on relay windings.

serious conditions. A group of coils are subjected to 90 per cent relative humidity at 85°F with negative potential applied to the inner winding and positive potential to the outer winding. No current flows in the windings. Depending upon the type of apparatus in which the coil is used, other parts of the structure may be made positive or negative to simulate actual service conditions. When electrolytic action takes place, copper is always eaten away from the positive electrode. Consequently in practice where there is a choice, it is better to keep the winding negative with respect to its surroundings. During a 35 to 40 day period, continuity checks are made periodically using a Wheatstone Bridge having a battery supply of $1\frac{1}{2}$ volts in a series with 10,000 ohms. This method of test does not provide high enough voltage nor permit flow of sufficient current to establish continuity through a minute length where the wire may be corroded through, nor does it cause a reduced section of wire to burn out. In other words, this method of test does not restore continuity in a corroded through section nor does it destroy metallic continuity. Thus more consistent results are obtained than if higher voltages or currents were to be used. The marked superiority of the cellulose acetate insulated coil is apparent, and experience with its use in service has shown that corrosion failures have been eliminated.

From time to time the question arises as to how the cellulose acetate insulated coil compares with coils vacuum impregnated with a varnish and employing other types of insulation for use in equipment for the Armed Services where atmospheric conditions are more severe than those ordinarily encountered in the telephone plant. Frequently specifications for these applications require impregnation of the windings. Tests have shown that impregnation will extend the life of a coil employing inferior materials, but that corrosion will take place in a shorter period than where cellulose acetate is used throughout without impregnation.

Results of such tests are shown in Fig. 4 where the various groups of coils were kept in a humidity chamber at 95 per cent relative humidity. The temperature within the chamber was raised and lowered between limits of 85° and 150°F in cycles so as to produce severe condensation on the coils. Each cycle, plotted as abscissae represents 24 hours of exposure. The top two curves IIIA and IIIB show the failure rates of two groups of coils constructed exactly alike except that one group was impregnated while the other was not. They were cellulose acetate filled coils, but used lead-out wires insulated with commercial grades of braided cotton. The two curves IIA and IIB represent the results on two groups of spool wound coils having cellulose acetate core and interwinding insulation, but provided with vincellatate muslin covers and red-rope paper

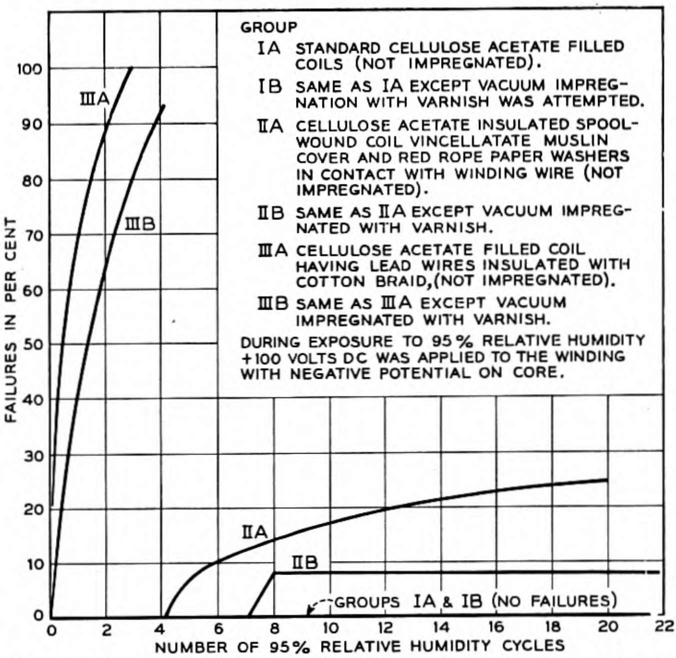


Fig. 4—Corrosion comparison of impregnated and non-impregnated relay coils.

winding washers. Likewise, one group was impregnated while the other was not. Fifth and sixth groups of coils having cellulose acetate insulation throughout with and without impregnation were exposed and there were no failures at the end of the test. This shows that the corrosive effects of impure materials can be retarded, but not overcome by resorting to impregnation. In general, impregnation of relay coils is not desirable because of the risk of contaminating the vital working surfaces of the relay.

In the normal operation of a relay when the circuit through its winding is opened, a transient voltage, which may reach hundreds of volts is generated across the winding terminals by the collapsing flux. If the insulation between the lead-out wires or between the lead-out wires and the end turns of the winding is not adequate, electrical breakdown causes arcing and repeated operation of the relay may cause ultimate disintegration of the wire and consequent failure of the relay. It is important, therefore, to design the coil so that lead-out wires under all conditions are properly spaced, and to provide adequate insulation between those portions of the winding where high voltages can exist. A test has been devised for use during manufacture which will detect an incipient failure of this kind. By pulsing the relay in its normal fashion the self-generated coil voltage on breaking the circuit can be observed on a cathode ray tube; any deviation in this voltage caused by momentary breakdown or shorted turns can be detected readily.

Another source of coil failure is lead breakage, caused principally by fatigue of the small copper wires. Copper has a low fatigue strength and if it is subjected to repeated bending strains, eventually it will break. As the relay operates and releases, impact of the armature against the core and backstop causes shock and vibration of the coil; coil construction therefore needs to be such that strains are not imposed upon the fine wire by this motion.

On spool wound coils, being individually wound, the fine winding wires can be reinforced by stranded lead-out wires for connecting to the relay terminals. Besides, the coil is generally wound tightly on the relay core. These factors, to a large extent, preclude lead breakage. With the cellulose acetate filled coil it is desirable to bring the winding wire directly to terminals on the terminal spoolhead to which the end of the coil is later bonded. Since the filled coil must slide loosely over the core for assembly reasons, it can have a small lateral motion at the non-terminal end. To eliminate this movement a "motion limiting" washer has been provided to fit snugly over the core and which is bonded to this end of the coil.[2] The washer and the way it is used is illustrated in Fig. 5. In assembly, the thin cellulose acetate faced phenol fibre washer and the non-terminal spoolhead are forced over the knurl on the core and the washer is bonded to the end of the coil. The tight fit of the washer on the knurl is the feature that prevents lateral movement of the coil. There is always some slight shrinkage of the cellulose acetate filled coil in the

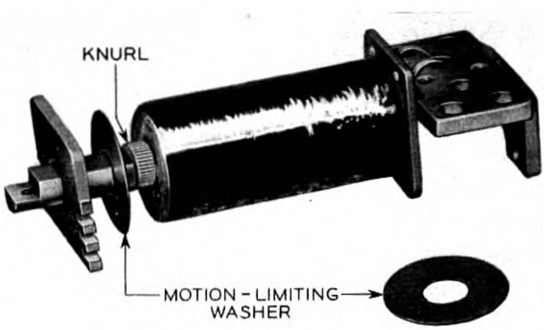


Fig. 5—Relay coil employing a motion limiting washer to prevent lead breakage.

longitudinal direction, but the washer can move with the coil, although eliminating lateral movement. Use of this washer has practically eliminated fatigue lead breakage.

CONTACT RELIABILITY

Since the opening and closing of contacts are the prime objectives of a relay, it is extremely important that the contacts themselves are made reliable. To realize these objectives, several factors should be taken into consideration. First is the contact material. While much could be said regarding the behavior of contact metals, space does not permit more than a brief treatment. The contact should maintain reasonably low resistance and under the environment in which it is used, be able to withstand the erosion. Electrical resistivity of most metals is low enough to be satisfactory from the resistance standpoint, but unfortunately, most of them develop tarnish or corrosion films when exposed to the atmosphere, thus increasing the contact resistance and rendering them unsuitable for a contact. These metals are sometimes referred to as "base" metals, and include aluminum, brass, bronze, copper, chromium, nickel and stainless steel. There is a much smaller group of metals known as "noble" or precious metals, such as platinum, palladium, gold and iridium. These are relatively free from the tendency to tarnish and will maintain low contact resistance. Alloys of these metals and certain alloys in which silver is included are widely used in the telephone plant. Pure silver is also used and is attractive because of its low cost; however, it has a tendency to form high resistance tarnish films and therefore has limitations in its use. It is employed in signaling circuits where the contact makes or breaks current. Its contact resistance remains low because the films that form on the silver are broken down or destroyed by the arc. It is not employed in circuits carrying voice currents on account of its tendency to introduce noise.

Enough metal must be provided to give satisfactory life. Each time a contact makes and breaks an electrical circuit, a small part of the metal may be lost, so that life may be considered roughly proportional to the volume of metal available for erosion. The pair of contacts must have sufficient height to provide enough contact spring clearance to allow for spring adjustment and to insure that the springs will not touch during the normal operation of the relay. At least one contact of a pair must be large enough, that is, present a sufficiently large target area, to insure full registration of the contacts with normal manufacturing variations of the position of the contacts on the springs and with the variations in alignment of the springs during assembly.[3]

In the early days of relay design, contacts were attached to the springs by riveting. In fact, many of the relays manufactured abroad today are made in this manner. In this country, during the past 30 years or more, spot welding has largely replaced the riveted construction. This was done for economy reasons. Spot welded contacts, unless carefully controlled during manufacture, may not be so reliably attached as the riveted contacts and the likelihood of the contacts dropping off during the life of the relay may be greater. It has been found in welding the millions of contacts required in the telephone system that close control is required in several factors affecting the quality of welds obtained with any given material. Important factors are cleanliness of the welding surfaces, pressure between electrodes, welding current and the time during which the current is applied. In order to insure that these factors are at all times under control, and since the consequences are rather grave when poor quality welds are produced, it has been found desirable to institute frequent inspectons of the quality of welds at each welding machine on a sampling basis. Periodically a small number of contacts are subjected to a destructive test in which the force required to shear off the contact is measured. In this manner, any deterioration in the quality of welds can be detected early, and corrective measures can be applied.

One type of failure sometimes experienced with relays is contact locking.[4] When a contact is closed by the operation of the relay it may become mechanically locked to the contact member with which it is engaged and fail to open when the relay is released. As a result of arcing as the contact closes and opens an electrical circuit there is a transfer of metal from one contact to the other. This building up and wearing away leaves both contacts roughened. If the opening and closing motion were along a perpendicular to the face of the contacts, this roughening would ordinarily have little effect. But with a slight sliding or rocking motion at the contacts after they come into engagement, small projections on one contact may lock mechanically in a cavity on the other and thus prevent the contacts from opening when they should. When contacts have locked, measurements have shown that forces in excess of 100 grams may be required to separate them.

This kind of failure can be avoided by employing an improved method of spring actuation.[5] This is illustrated in Fig. 6. At the top of the figure is a stud actuated contact spring assembly. The spring carrying the moving contact is tensioned away from the fixed contact member and exerts a force to hold the armature against the backstop when the relay is unoperated. A stud, moved by the armature, presses against the moving contact spring a short distance back of the contact to close the contact when the relay is operated. As will be noticed, the further deflection of the contact spring necessary to obtain the required contact force after closure, causes the moving contact to slide and rock slightly on the fixed contact. For the reasons previously mentioned such a contact is prone to lock when the conditions are favorable.

Now, the bottom of the figure shows a card actuated contact spring arrangement. Here a phenol fibre card is employed to operate the contact instead of a stud. The moving contact spring itself is pretensioned against the fixed contact to give the desired contact force. The card is held by two card springs that are tensioned away from the fixed contact in a slightly greater amount than the moving contact spring is tensioned toward it. As a result, when the relay is unoperated, the card holds the make contact away from the fixed contact. When the relay operates, the armature pressing against the top of the card pushes it toward the fixed contact and allows the contact to close. It is quite apparent that with this actuation the moving contact engages the fixed contact without

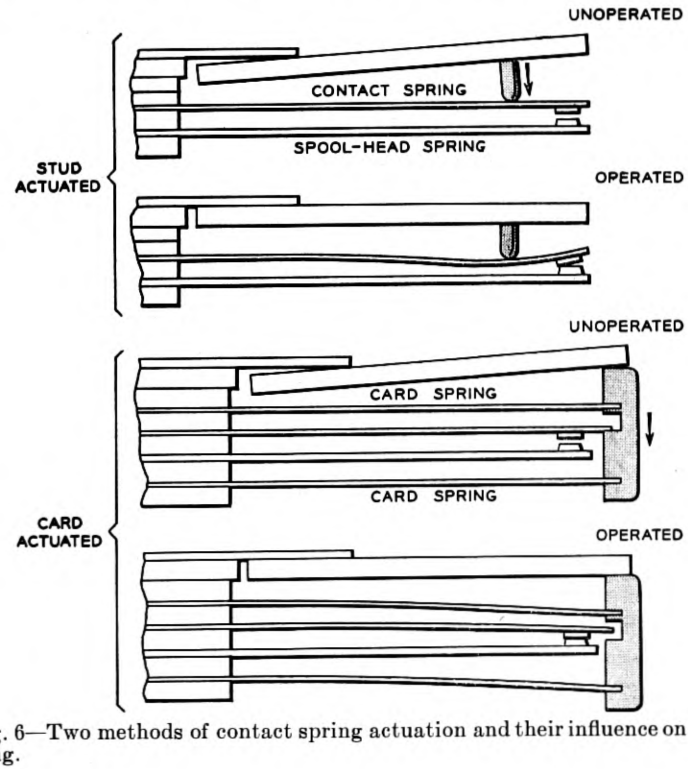


Fig. 6—Two methods of contact spring actuation and their influence on contact locking.

sliding or rocking on it and the tendency to lock is thus largely avoided. The likelihood of locking is further decreased because the restoring force is greater and is applied closer to the contact, and because of the impact of the card on the spring when the relay releases. With the contact closed there is clearance between the contact spring and the bottom of the slot in the card, and thus when the card hits the spring in opening, it is already moving and has acquired kinetic energy. This energy is available on impact to overcome any locks which may have occurred.

Recent studies have shown that erosion of electrical contacts on closure is due almost entirely to an arc occurring, in most cases, before the contacts touch.[6] When there is no arc there is no erosion. It has also been observed that the occurrence of an arc between the approaching contacts is influenced by operation in the presence of various organic vapors; for example, benzene derivatives.[7] The effect of such operation is to permit arcing on lower currents than is the case with clean contacts and results in increased erosion rates. This is true for noble metal contacts, and when so exposed they are said to have become "activated". A metal surface which has been activated by organic vapor remains active indefinitely if there is no arcing at the surfaces. With continued operation and accompanying arcing, the activating material is burned away, and the surface returns to the inactive condition, provided no contaminating vapor is present.

Some materials used in relays may give off organic vapors which can aggravate the arcing at the contacts. A series of experiments has been made by placing various materials under test in a small glass enclosure and proceeding to find if, and at what elevated temperature, vapor from the materials will give rise to arcing on "make", with contacts that are operating within the enclosure. The materials tested varied widely in their effects upon arcing at the relay contacts. In the solid organic group, they ranged from polystyrene, which produced arcing at room temperature, to teflon, which did not cause arcing until heated above 200°C.

The precise correlation between the results of these tests and the changes in erosion rates, which occur when these materials are used in the relay construction, has not yet been established. However, they may be used as an aid in the choice of such materials. Cases have come to our attention both in the laboratory and the field where the erosion rates of relay contacts operating in confined chambers were many fold those which occurred when the relays were operated in the open. This was at least partially ascribed to the presence of contaminating materials known to be present.

Another type of failure that is quite generally experienced in relay operation is "open" contacts due to small insulating particles present in the atmosphere becoming trapped between the contacts. This causes high resistance or open circuit and consequent circuit failure. Many attempts have been and are being made to reduce "open" contact troubles. Examples are filtering the air supply to the central office, enclosing the relay equipments in closed cabinets, pressurizing the enclosing cabinets, covering smaller groups of contacts by independent covers, employing twin contacts rather than single contacts, and enclosing the relay or its contacts in a hermetically sealed chamber. Even going to the extreme of completely isolating the relay from its surroundings is not a complete answer. There is always the possibility of failure by wear particles generated within the enclosure by the relay actuation.

The most widely employed method to reduce dust failures in the telephone plant is the use of twin contacts in combination with some of the above mentioned types of enclosures. If the incidence of dust failures followed the laws of probability, then elementary considerations would lead us to predict that if single contacts failed at the rate of once in 1,000 operations, the simultaneous failure of the two such contacts comprising the twin would be once in 1,000,000 operations. This is the so-called "square" law. However there are a number of reasons why this is not realized in practice and why the figure of merit for the twin contact is very much less than that indicated by the "square" law. In the first place, when foreign matter becomes lodged on a contact, it seldom falls out on the first subsequent operation, but will require a number of operations before it cleans itself; in fact, it may remain inoperative indefinitely. When this happens to one of the twin contacts, during this period of time, the twin contact is no better than the single contact. In practice, twin contacts are generally used with the same total force as the single contact, being nominally divided equally between the two contacts. This reduction in force per contact on the twin contacts is of considerable importance in reducing its effectiveness. Relay designs employing twin contacts that have been used in the past do not have complete mechanical independence of the two members to which the contacts are attached. Foreign material or protrusions under one contact can adversely influence the performance of its mate. In a new design of relay which is about to go into production, the design criterion that twin contacts to be most effective should be completely and mutually independent has been met. Laboratory tests and field experience obtained to date show a marked improvement over the former designs in regard to the incidence of open contact failures.

Tests have been made repeatedly in the laboratory for comparing the performance of twin contacts with single contacts, and arriving at a figure of merit. Data have also been collected from relays in service in the telephone plant on the basis of numbers of found open troubles on both types of contacts. As might be expected the results varied widely, with the twin contact being superior by a factor of anywhere from 3 to 100 with perhaps 10 as a reasonable figure.

MAGNETIC STABILITY

Magnetic materials in relays have been found to change in their magnetic characteristics with time and temperatures to which they are subjected in their normal usage. This effect is known as aging. The direction of the change is such as to decrease the permeability and increase the coercive force of the material. The degree of change in certain applications, such as relays in marginal and time delay circuits, may be so large as to be of serious concern.

A high grade of magnetic iron which has been extensively used in the telephone system has been found to age considerably under conditions simulating operation in the plant. Aging of iron is attributed to the precipitation of impurities such as carbon, nitrogen, and oxygen. The solubility of these elements decreases with decreasing temperature. When iron is cooled from a high temperature, impurities, such as carbides and nitrides, do not have sufficient time to precipitate completely, so a supersaturated solid solution is produced. Consequently the impurities tend to continue to precipitate slowly at low temperatures where the diffusion rate is extremely slow, and internal strains are produced which affect the magnetic properties.[8]

It has been found that if these parts are annealed in atmospheres of dry hydrogen instead of the ordinary "pot" anneal, this aging effect is greatly reduced. Not only is the aging effect reduced to where it is of no great engineering importance, but the magnetic properties of the material are improved. The maximum permeability is increased and the coercive force is decreased both by a factor of about two. The use of relays in critical applications is thus greatly enhanced.

The degree by which magnetic materials change by aging may be determined readily by laboratory tests. Long time aging effects can be simulated by baking ring samples of the material or the relays at 100°C for several hundred hours and measuring the magnetic properties of the ring specimens or the operating characteristics of the relays before and after aging. For "pot" annealed magnetic iron the effect of such aging is to decrease the maximum permeability by about 50 per cent and to approximately double the coercive force. When the iron is hydrogen annealed, the corresponding changes caused by aging will be about a 15 per cent decrease in maximum permeability and 15 to 20 per cent increase in coercive force.

The improvement in aging effect on relay performance obtained by the hydrogen treatment is illustrated in Fig. 7.[2] This was obtained on a design of relay having a closely coupled magnetic circuit for use in time delay circuits. The ordinates show the change in residual grams; hours of aging are plotted as abscissae. Residual grams represent the force with which the armature is held attracted to the core by the residual flux remaining in the magnetic circuit after the electrical circuit through its winding is opened. This force is a measure of the coercive force of the magnetic material. As was noted, the effect of aging is to increase the coercive force and hence the residual grams. For this relay, a change in residual grams will cause a change in the delay time of the relay under a given adjustment and is therefore important.

How hydrogen annealing improves the pull characteristics of a relay is shown in Fig. 8. This was taken on a relay designed for sensitive and marginal circuit applications. The curves show the grams pull, plotted as ordinates, produced on the relay armature at a given air gap by various values of ampere turns on the relay plotted as abscissae. The ability of the hydrogen treated relay to operate given loads on considerably smaller currents is obvious. This improvement is due to the higher permeabilities obtained by the hydrogen anneal.

There are other magnetic materials available for use in relays and in which the aging effect is practically non-existent or is considerably smaller than that just described. Several kinds of nickel-iron alloys known as permalloy are widely used in the telephone system where their

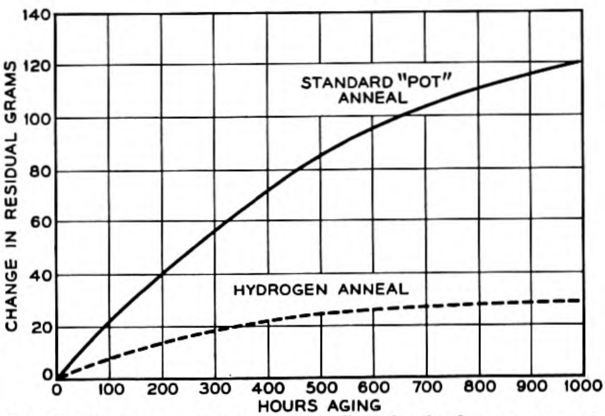


Fig. 7—Improvement in aging effect by hydrogen anneal.

excellent magnetic properties are needed in difficult applications. They are substantially non-aging. Low silicon-iron alloys are being more widely employed. They have good magnetic properties and the aging effect is small. Where the intrinsically inferior magnetic properties of low carbon steel alloys, such as SAE 1010, can be tolerated they are used. While initially they have poorer magnetic characteristics than magnetic iron, their aging effect is considerably smaller.

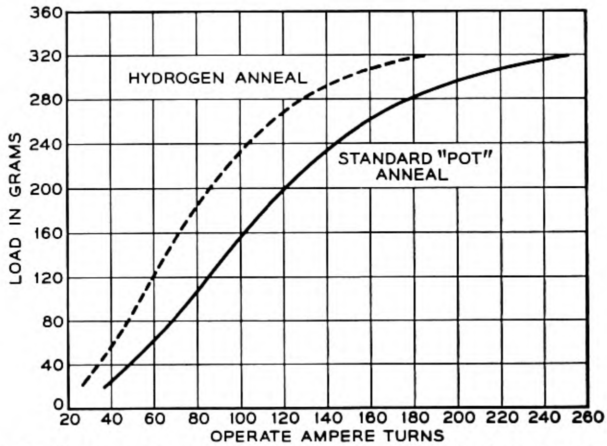


Fig. 8—Improvement in relay performance by hydrogen anneal.

STRUCTURAL STABILITY

Response of a relay depends upon the value of the magnetic force of attraction produced between the armature and core when the coil is energized, and upon the magnitude of the mechanical forces acting upon the armature. To keep the response constant during the life of the relay, it is essential that the relationships between these two forces be not changed. The force of attraction varies approximately inversely with the square of the length of the air-gap between the armature and core. Since this distance is usually small, any small change will have a relatively large effect on the pull. The core and armature, together with their associated members, should be of stable design and secured in such fashion that their dimensional relationships remain unchanged when the relay is subjected to shock, vibrations, and stresses incident to attaching the relay to its mounting. The design of the structure and the thermal coefficients of expansion of the materials used should be such that deformation does not take place when the temperature is varied throughout the operating range.

Moving parts, such as the armature and its suspension, together with the associated actuating members and springs, should move freely under all conditions without binding or friction. Since friction is inherently a variable quantity and difficult to control, it should be kept as small as possible, otherwise it will be a cause for instability. If the friction component is an appreciable part of the total load, the relay will be unsatisfactory, particularly for marginal operation. The friction part of the load on the moving system of a relay can be determined readily by an improved measuring technique which automatically plots the force required to move the armature, and its displacement, as it moves from its unoperated to its operated position. This is illustrated in Fig. 9. The top curve is the force required to move the armature and operate its associated contact springs as it moves from its back-stop to its fully operated position. The lower curve is the force acting on the armature that allows it to restore to its unoperated position. The vertical displacement between the two curves represents double the friction. Since friction always opposes motion, it adds to the force required to operate the relay and detracts from the force releasing the relay. If the ideal of no friction were obtained the two curves would coincide. When

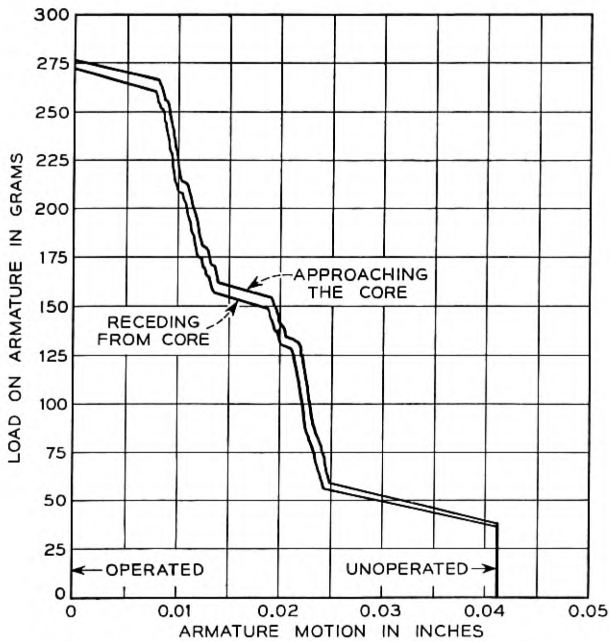


Fig. 9—Mechanical load of a relay and its friction component.

the displacement and hence the friction is large, aside from the fact that it is indicative of a rapid rate of wear, the relay would be unstable.

While the wear of the relay parts can be minimized by good design, it cannot be eliminated entirely, especially for relays required to operate a very large number of times during their life. For telephone relays the design objective is for a 40-year life. The effects of wear on performance to a great extent can ofttimes be counteracted by ingenious design. Fig. 10 is an illustration of such a case.[5] The diagram on the left shows a moving system of a relay in which the contact springs are stud actuated. The moving springs are tensioned toward the armature and exert a force tending to open the contacts. When the armature operates, the stud presses the moving springs into engagement with the stationary springs. There is no contact force when engagement is first made and further flexing of the spring is necessary to build up the contact force to the desired value when the armature reaches its fully operated position. As the contacts and studs wear, it is apparent that the contact force and consequently the load on the armature decreases rapidly. The stud wear becomes cumulative in its effect on the outside pair of springs as more springs are added to the pile-up.

The diagram to the right shows a moving system of a relay using what is called "lift-off" card actuation. The moving springs are ten-

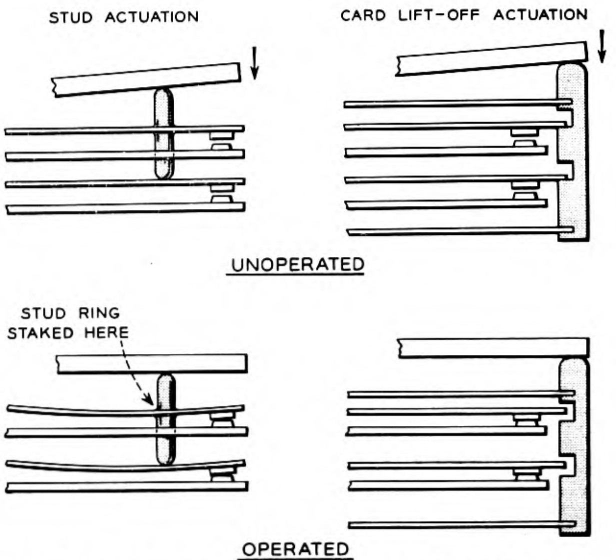


Fig. 10—Two moving systems of relays in relation to the effects of wear on their performance.

sioned, before assembly, toward the stationary spring by an amount necessary to give the desired contact force. Two supplementary springs are provided to support the card and tensioned to restore the armature and contact springs to their unoperated position. Upon operation, the motion of the card permits the contacts to close, and when engagement of the contacts occurs, the contact force reaches its predetermined value very rapidly. Further motion of the card, provided for by the width of the slot in the card, allows for wear of the contact and card without appreciably affecting the contact force or the load on the armature.

The effect of this wear on the contact force is shown in Fig. 11 for both types of actuation. For the stud actuated relay, as the contact and stud wear continues, the contact force decreases very rapidly. After 0.010 inch wear only about 6 grams remains out of an original 26 grams. This is accounted for by the fact that the combined stiffness of the moving spring in engagement with the stationary spring is 2 grams per 0.001 inch deflection. This requires 0.013 inch contact follow to establish a contact force of 26 grams when the relay is adjusted initially. For the card "lift-off" actuated relay where the moving spring had been pretensioned to give a contact force of 25 grams initially, after 0.010 inch wear of the contacts, the contact force will have decreased about 1 gram. This is because the stiffness of the moving spring is about 0.1 gram per 0.001 inch deflection. Card wear does not affect the contact force so long as it is provided for by the width of the slot in the card.

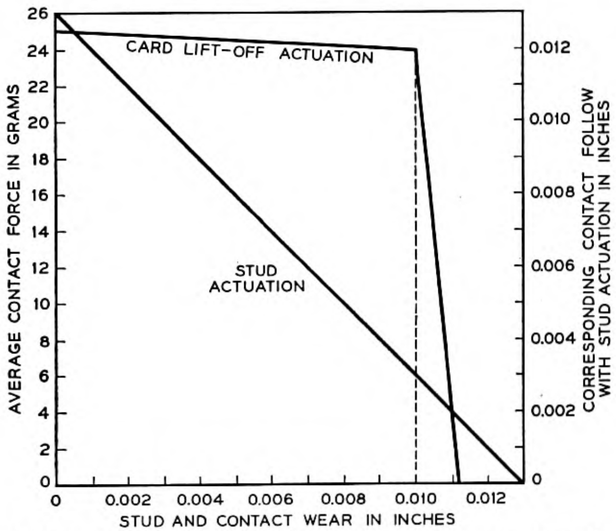


Fig. 11—Comparison of effects of wear on contact pressure of a relay.

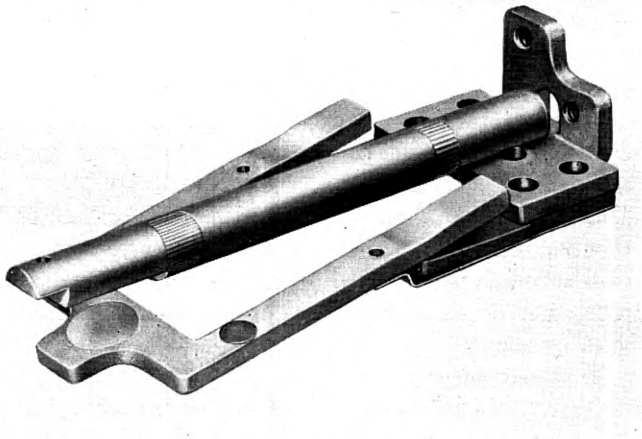


Fig. 12—Magnetic circuit of a relay having embossed pole faces.

Another instance where the effects of mechanical variations upon its performance have been largely nullified by design, is in the design of slow release copper sleeve relays. To make most effective use of the copper sleeve, which causes the delayed action, it is desirable to provide as low a reluctance as possible of the magnetic circuit when the relay is in the operated position. Instead of providing small non-magnetic separators in the air-gap between armature and the core as is usually

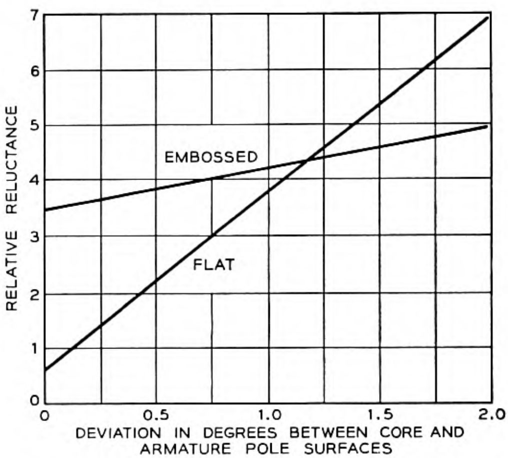


Fig. 13—Comparison of flat and embossed pole surfaces and their magnetic closed circuit reluctance with misalignment.

done with the ordinary quick-to-release relays, for slow release relays the armature is allowed to contact the core, finish to finish. When plane flat pole face surfaces are provided, it is expensive and difficult to insure in commercial practice that precise and uniform alignment of the pole face surfaces will obtain. Variations in the alignment of these two surfaces will cause variations in the closed magnetic circuit reluctance and consequently on the release time of the relay.

In Fig. 12 is shown a design where the necessity for holding the alignment of core and armature so precisely is not so great.[9] A spherical surface of rather large radius is embossed on the front end of the armature, so that with commercial variations in alignment, the armature always presents a point on the surface of a sphere for contacting the flat surface of the core. Similarly, the legs of the armature where they pivot on the front ends of the hinge bracket are likewise embossed. The results of the effects of these structural differences on the closed circuit reluctance are shown in Fig. 13 for a design with flat surfaces and one with embossed surfaces. While it is true that with perfect alignment the relay with flat surfaces will give longer release times, it is apparent that as variations in alignment occur from time to time and from relay to relay, it will have larger variations in performance than the relay with the embossed surfaces. This is a feature which has proven of great value in the manufacture of slow release relays of reasonable time precision.

REFERENCES

1. C. Schneider, "Cellulose Acetate Filled Coils," *Bell Labs. Record*, **29**, p. 514, Nov., 1951.
2. W. C. Slauson, "Improved U, UA and Y Type Relays," *Bell Labs. Record*, **29**, p. 466, Oct., 1951.
3. B. F. Runyon, "Contacts for Crossbar Apparatus," *Bell Labs. Record*, **18**, p. 278, May, 1940.
4. P. W. Swenson, "Contacts," *Bell Labs. Record*, **27**, p. 50, Feb., 1949.
5. H. M. Knapp, "The UB Relay," *Bell Labs. Record*, **27**, p. 355, Oct., 1949.
6. L. H. Germer and F. E. Haworth, *J. Appl. Phys.*, **20**, p. 1085, 1949.
7. L. H. Germer, *J. Appl. Phys.*, **22**, p. 955, 1951.
8. R. M. Bozorth, *Ferromagnetism*, D. Van Nostrand Co., Inc., 1951.
9. F. A. Zupa, "The Y-Type Relay," *Bell Labs. Record*, **16**, p. 310, May, 1938.

Impedance Bridges for the Megacycle Range

By H. T. WILHELM

(Manuscript received August 19, 1952)

This paper reviews ac bridges developed for use in the Bell System for the measurement of impedance parameters, particularly at frequencies in the megacycle range. Three recent bridges designed for measuring networks and components for coaxial systems are described.

INTRODUCTION

The need during recent years for increased accuracy of impedance measurement in the megacycle range has led to advances in the art of bridge measurement. A particular stimulus has been the development of a new coaxial system, designated L-3, for transmitting over distances up to several thousand miles a continuous frequency band extending roughly from 0.3 to 8 megacycles per second. Such a system will be capable of providing on a single coaxial unit the combination of a single television channel and as many as 600 one-way telephone channels. The large loss inherent in transmitting this wide frequency band over the cable makes it necessary to provide an amplifier about every four miles, and these amplifiers and associated networks have created difficult measurement problems.

MEASUREMENT PROBLEMS

The measurement problems arise partly from the wide frequency band, approximately thirty times the minimum frequency. This makes equalization of the system for satisfactory transmission very difficult, particularly in transmitting a television signal which covers a frequency band equivalent to about a thousand telephone channels and which must be equalized for phase as well as loss.

Even more important, however, are the problems arising from the close spacing of the amplifiers, with the result that a transcontinental circuit requires up to a thousand amplifiers in its path. Departures in individual transmission characteristics will produce cumulative errors, making it necessary to maintain close control over the manufacture and adjustment of all of these amplifiers and associated networks. This calls for networks of highly refined design and requires ancillary measurement facilities of greater precision than heretofore available at these higher frequencies.

The design of transmission networks to meet exacting requirements is a subtle art, embracing on the one hand the use of complex mathematical manipulation to produce theoretical networks having the desired loss and phase characteristics, and requiring, on the other hand, a down-to-earth knowledge of the properties of the actual components used including parasitic effects and interaction of the various elements when assembled into a network. To furnish this knowledge, to measure the component resistors, capacitors, inductors and transformers which are the building blocks of the networks, to evaluate the ever-present parasitic effects, to determine simplified circuit equivalents of the more complex components such as transformers, and to answer other questions too numerous to mention, measurements of impedance parameters – precise measurements – are required.

EXISTING BRIDGE TECHNIQUE

For measuring impedance and admittance parameters, that is R, L, C and G, suitable ac bridges, ordinarily simply designated as impedance bridges, have long held a high place in the Bell System because of their inherent reliability and precision, and their ability to cover a wide range of values. The development of many of the original bridges[1, 2, 3, 4] for frequencies above the audio range stemmed from the needs of the earlier carrier systems. With this development came also analysis of shielding technique,[5] standardization of capacitance,[6, 7] and a systematic classification of bridge methods[8] by J. G. Ferguson in 1933, in which bridges were grouped into two major types designated as ratio-arm and product-arm, respectively. Following this classification, combined impedance and admittance bridges were developed,[9, 10] utilizing a single set of bridge standards for both kinds of parameters by changing the configuration of the bridge network. There have also been special purpose bridges[11, 12, 13, 14] for use at audio and the lower carrier frequencies. More recently, coaxial impedance standards[19] having values calculable from physical dimensions have been developed.

Bridges for frequencies above one-half megacycle were used in the Bell System as early as 1919,[15] but relatively few bridges were built until the mid 1930's when new carrier systems required bridges in the megacycle range. A ratio-arm bridge[16] using external standards was developed for precise measurements up to three megacycles. Interconnection of bridge and standards using coaxial cords provided flexibility of configuration resulting in an admittance bridge for high impedances and a series-reactance bridge for medium impedances. These two bridge circuits are shown schematically in (a) and (b) of Fig. 1. A separate, self-contained Maxwell product-arm inductance bridge, shown schematically in Fig. 1c and illustrated in Fig. 2, was designed primarily for measuring low-impedance parameters up to one megacycle/sec. Inductance was measured using calibrated air capacitors, and resistance was measured by means of conductance decades employing wire-wound resistors. The bridge included a double-shielded coupling transformer and complete shielding not shown in the simplified schematic.

To show clearly the scope and inter-relation of these three bridge methods, it is helpful to plot their ranges on a Slonczewski reactance/frequency chart[17] shown in Fig. 3. In this chart, the top frequency shown for the ratio-arm bridge is three megacycles, and for the Maxwell bridge is one megacycle, as these are considered boundaries

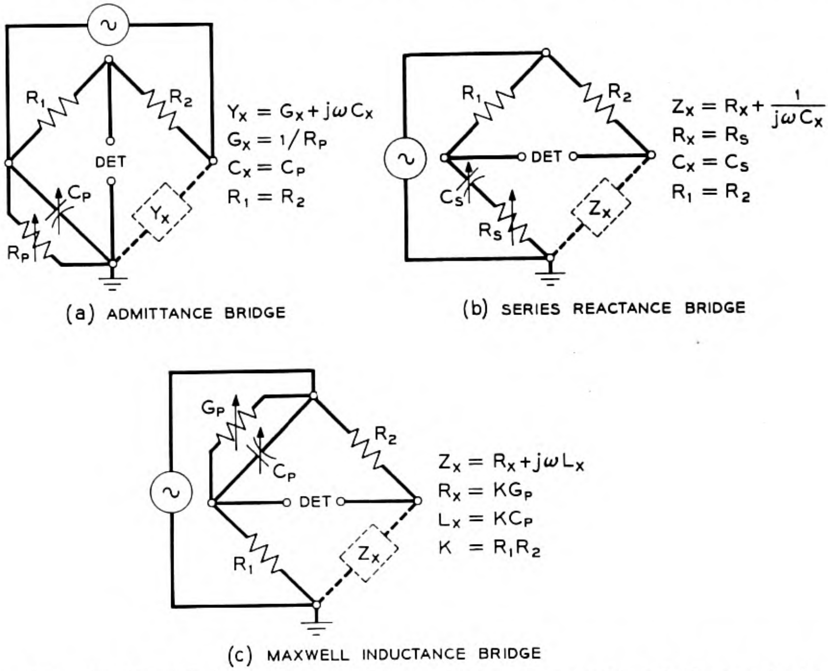


Fig. 1—Simplified schematics showing the basic circuits of three existing bridges for use at frequencies up to about three megacycles.

for their best performance, even though both bridges are useable at higher frequencies. It will be observed that while there is some overlapping of the three ranges, all three methods are necessary to obtain the impedance coverage shown. It should be emphasized that all the ranges shown cover both capacitive and inductive reactances. In the case of the admittance and series-reactance bridges, inductive impedances are measured by using a resonating capacitor, in parallel or series, respectively, with the apparatus being measured. In the Maxwell inductance bridge, capacitive impedances are measured by using a fixed resonating inductor in series with the impedance under test. A complete accuracy statement for these bridges is necessarily complex, but in general accuracies of ± 0.25 per cent for the major component

Fig. 2—One-megacycle Maxwell inductance bridge, shown schematically in Fig. 1c, designed for relay-rack mounting.

was obtained over most of the range plotted on the reactance chart. These bridges have been very successful for the purpose for which they were designed, but they are not useable up to the eight megacycles or higher required by the L3 system.

REQUIREMENTS OF BRIDGES FOR L3 SYSTEM

When the L3 system was contemplated, it was evident that new bridges would be needed. It was required to be able to measure virtually any impedance value at frequencies up to and beyond the second harmonic of the 8.4 megacycle upper limit of the system. Accordingly, a top

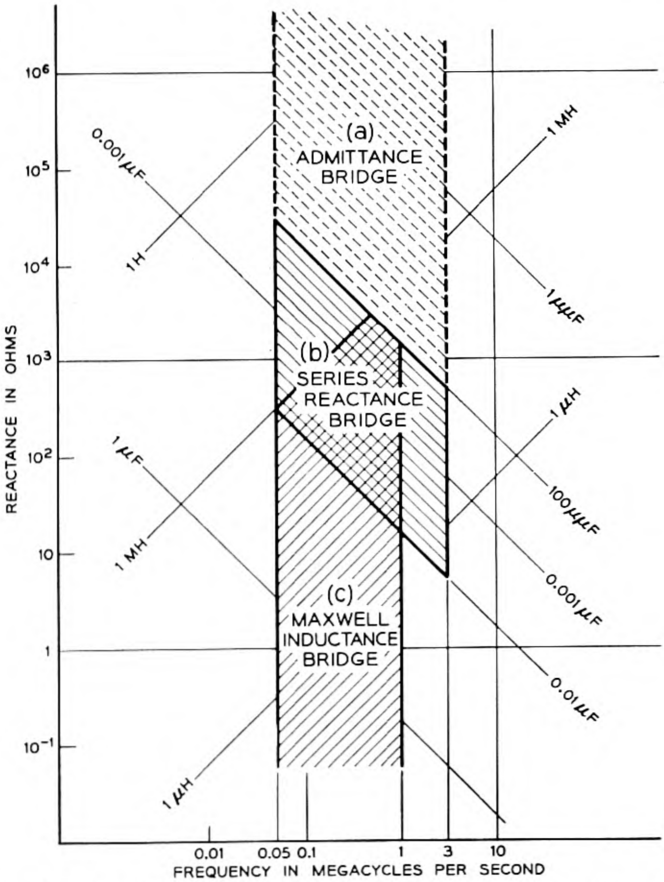


Fig. 3—Reactance/frequency chart showing the measurement range of the bridges shown in Fig. 1.

frequency of twenty megacycles was decided upon as a design objective with a basic accuracy of ±0.5 per cent for the major component. The immediate need was for a general-purpose bridge, but it was expected that special-purpose bridges having better accuracy would be required later.

GENERAL PURPOSE 20-MEGACYCLE BRIDGE

It was decided first to develop a single bridge unit which would embrace both admittance and series impedance methods, and thereby cover a reactance range from a few ohms up to nearly a megohm, as shown in Fig. 4. Such a bridge would combine the features of (a) and (b) of Fig. 1. There were numerous departures from the earlier designs, however, including the use of a series range capacitor to reduce the size of the series capacitance standard, the use of deposited carbon resistors,[18] the form and construction of both conductance and resistance standards, and especially the use of transformer-coupled inductive ratio arms.

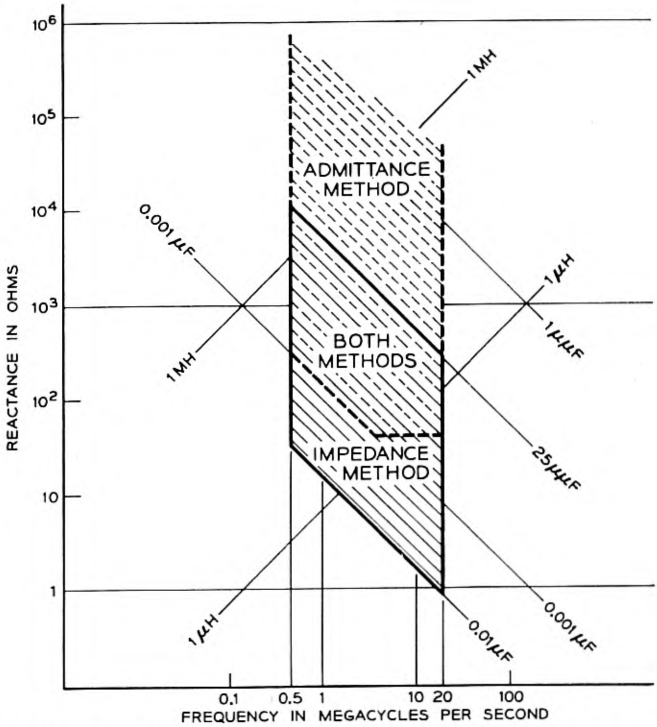


Fig. 4—Reactance/frequency chart applying to the general-purpose bridge shown in Fig. 5.

The successful use of a center-tapped transformer for ratio arms in a 465-KC direct capacitance bridge[13] indicated that the resistance ratio arms R1, R2 of Fig. 1 might be omitted if a suitable transformer could be developed for higher frequencies. The transformer group of the Laboratories succeeded in producing a transformer with a deviation from unity ratio of less than 0.1 per cent over a frequency range from 0.5 to 20 megacycles. This was made possible by precise location of the windings in fine milled grooves in the form of reversed helices, cut on a longitudinally-split brass cylinder for the inner winding, and on a surrounding phenol

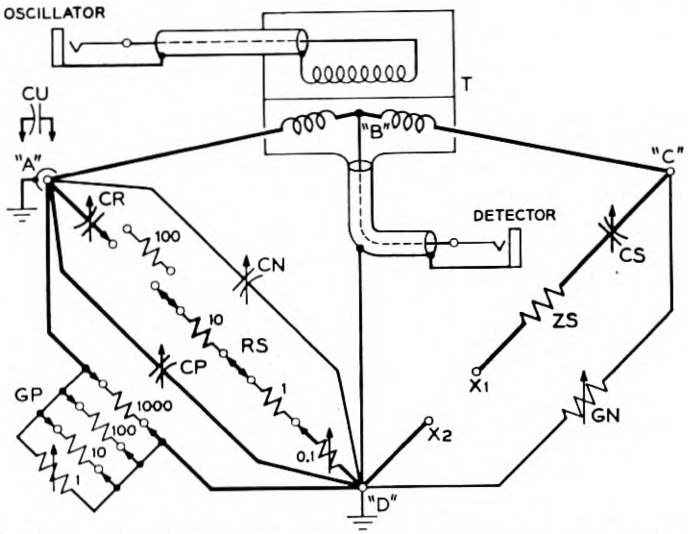


Fig. 5—Schematic of the 20-megacycle general-purpose bridge showing both the series (impedance) and parallel (admittance) bridge circuits combined in a single unit.

fibre cylinder for the bifilar outer winding which serves as the bridge ratio arms. Electrostatic shielding limits the direct capacitance between primary and secondary to less than 0.01 $\mu\mu f$. The core material is compressed powdered molybdenum permalloy. This transformer was the nucleus around which the general purpose bridge was built, and the resulting bridge is shown schematically in Fig. 5.

In Fig. 5, the letters A, B, C and D designate the four bridge corners, and T is the ratio-arm transformer already described. Apparatus to be measured by the admittance method is connected to terminals C and D, and is balanced by the calibrated capacitor CP and conductance standard GP. To use the series reactance method, CP and GP are set at minimum settings, apparatus to be measured is connected to terminals X1 and X2, and is balanced by the calibrated capacitor CS and resistance standard RS. The range capacitor CR consists of several mica capacitors for extending the range of CS, as will be described below; and ZS is merely a compensating impedance, essentially an inductive two-ohm resistor. The circuit is thus basically quite simple and avoids the use of switches or other complications which would impair performance at these high frequencies.

Capacitors CP and CS are worm-driven air capacitors with a range of about 220 $\mu\mu f$, and were specially designed for this bridge. In the case of CS, any direct conductance between rotor and stator would result in an effective series resistance which would vary both with frequency and capacitor setting, and therefore require laborious correction. This was avoided by arranging the construction so that the rotor and stator are mounted on independent insulating supports to the ground panel, thereby completely eliminating direct conductance from rotor to stator. While this results in some conductance from test terminal X1 to ground, the amount is small and its effect is negligible because of the relatively low impedance values measured. In the case of CP, on the other hand, it is important to minimize series resistance and inductance to avoid conductance and capacitance corrections which would change both with frequency and capacitor setting. This was accomplished by careful design of the rotor brush using silver contact surfaces and center-fed connections to both rotor and stator.

The conductance standard, GP, and resistance standard, RS, were designed to emphasize high-frequency performance. Deposited carbon resistors[18] on ceramic rods $\frac{1}{8}''$ in diameter and $\frac{3}{4}''$ long mounted on small decade rotors were used, so arranged that only one resistor on a rotor is in the circuit at any time, and that adjacent resistors are short-circuited by means of auxiliary shorting brushes to eliminate shunting admittance which might vary with frequency. For GP the resistance values are such that the two lower decades and the slide-wire rheostat each have a residual conductance of 333 micromhos, thereby avoiding the use of resistors exceeding 3,000 ohms in value which would be more likely to vary with frequency. The structure is designed to minimize series inductance and to maintain constant capacitance for all settings. For RS, on the other hand, it is necessary to maintain constant inductance for all settings. This was accomplished by adding small wire-loop compensating inductors in series with individual resistors in the 10-ohm and 100-ohm decades when necessary. To minimize the over-all inductance, the resistor rotors are placed very close together and are driven by gearing from the corresponding dials.

The range capacitor, CR, has already been mentioned. It consists of a rotor switch on which are mounted five uncalibrated mica capacitors which enable CS to measure both positive and negative reactance values up to 10,000 $\mu\mu f$ without additional switching. The 20 $\mu\mu f$ CR capacitor covers capacitance measurements up to 60 $\mu\mu f$; the 40 $\mu\mu f$ capacitor covers up to 150 $\mu\mu f$; the 80 $\mu\mu f$ up to 600 $\mu\mu f$; the 140 $\mu\mu f$ up to 10,000 $\mu\mu f$; and the 200 $\mu\mu f$ capacitor covers all the positive series reactance measurements. Since the CR capacitor permits the bridge to be balanced with the test leads short-circuited, the value of the effective resistance under test is simply equal to the difference between RS readings for the measurement balance and the short-circuit balance, and the reactance under test is determined from a computation of the two readings of CS.

A front view of the general purpose bridge is shown in Fig. 6. The four lower dials are for GP; above them are the four RS dials; and above them is the CR dial. The capacitors CS and CP are located adjacent to the test terminals, but are operated remotely by the dial knobs at the extreme right end of the bridge. This was done to remove the operator's hands as far as possible from the test terminals. Near the test terminals is a coaxial connector engraved A. This allows plug-in capacitors (CU in Fig. 5) to be added in parallel with CP for extending the capacitance range. Compact silvered mica capacitors in steps of 200 $\mu\mu f$ are used. Fig. 7 shows the interior of the same bridge with CP and CS in the lower foreground, GP at the left and RS in the upper right.

Fig. 6—Front view of the general-purpose bridge shown in Fig. 5. The bridge is approximately 10½ inches high and 19 inches wide.

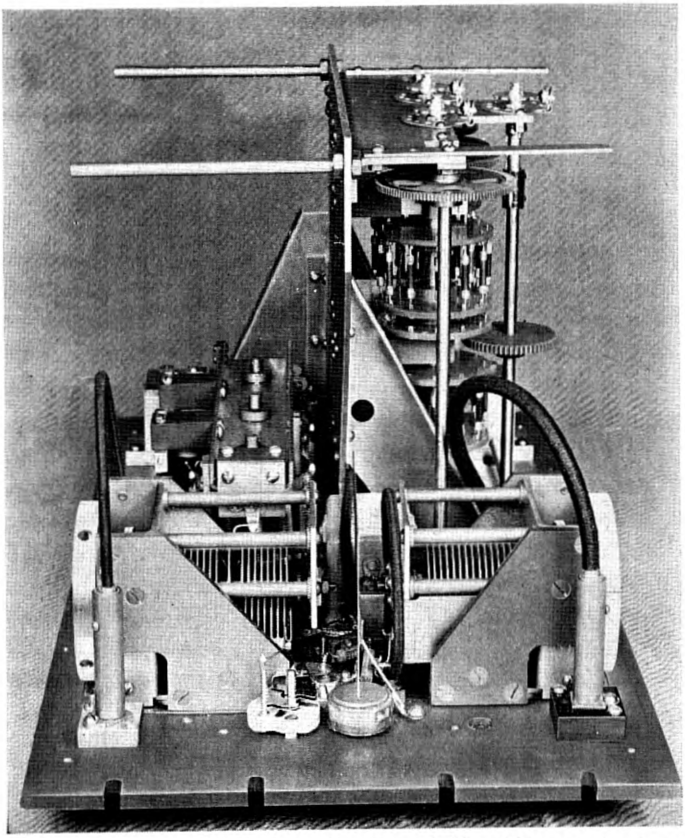


Fig. 7—Interior view of the general-purpose bridge. The panel edge shown in the foreground is the left edge of the bridge shown in Fig. 6.

FIVE-MEGACYCLE MAXWELL INDUCTANCE BRIDGE

To facilitate the measurement of low-valued inductors, there was need for a direct-reading inductance bridge inasmuch as such measurements entail considerable computation effort when using the general-purpose bridge. Accordingly, it was decided to build a five-megacycle Maxwell inductance bridge to cover a range from 0.001 microhenry up to 10 microhenries, and effective resistance values up to 11 ohms. The basic circuit is the same as the Maxwell bridge in Fig. 1, but the design embraces such refinements as glass-sealed deposited-carbon resistors for the conductance standard, and a worm-driven center-fed variable air capacitor. Special woven-wire resistors on spools of Teflon are used for the two fixed arms, and are compensated to give a constant product of practically zero

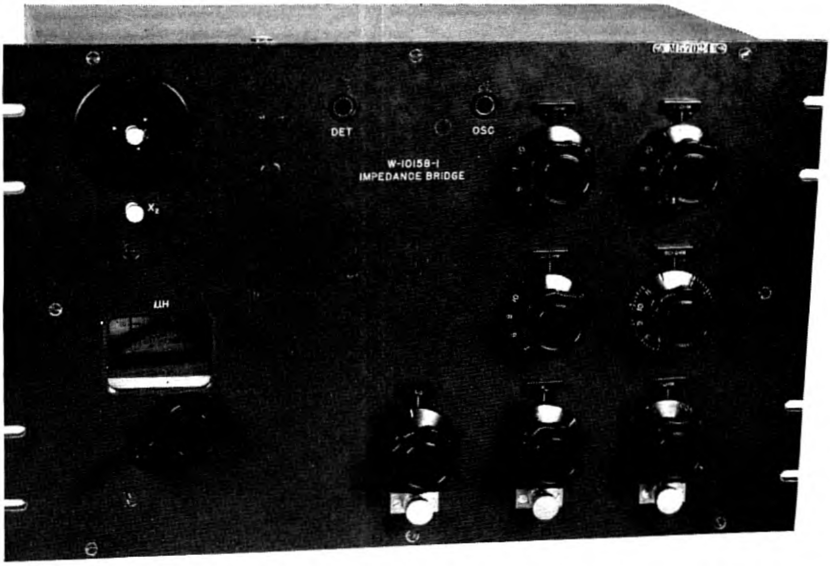


Fig. 8—The five-megacycle Maxwell inductance bridge is approximately $12\frac{1}{4}$ inches high and 19 inches wide. Test terminals are at upper left, and the three knobs at lower right are zero-balance adjusters.

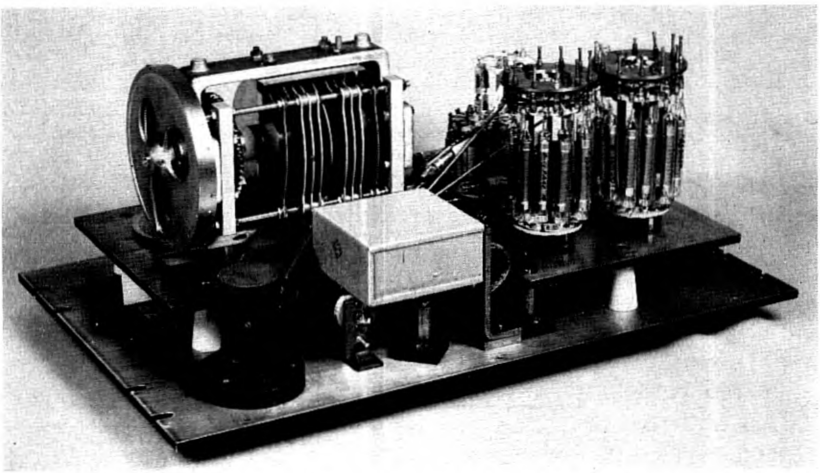


Fig. 9—Interior of bridge of Fig. 8 showing the shielding for test terminal X1 in foreground; at the left is the calibrated air capacitor; at the right are the conductance decades using glass-sealed deposited-carbon resistors.

phase angle over the entire frequency range. The result is a direct-reading bridge shown in Figs. 8 and 9 which has greatly facilitated the development of inductors in the megacycle range. The accuracy for major component varies from ±0.25 per cent at one megacycle up to ±1 per cent at five megacycles.

TEN-MEGACYCLE ADMITTANCE BRIDGE

Development of capacitors for the L3 coaxial system has required a new ten-megacycle admittance bridge. Intended especially for determining temperature coefficient and frequency characteristics of small capacitors, the bridge is capable of measuring capacitance values up to 200 $\mu\mu f$ with a precision of ± 0.01 $\mu\mu f$, and a wide range of conductance values. Unlike the other two bridges described which make grounded measurements only, this bridge is arranged for direct and balanced-to-ground measurements as well. This is accomplished by using the ratio-arm transformer already described in combination with a simple grounding circuit using a three-position key, as shown in the bridge schematic of Fig. 10.

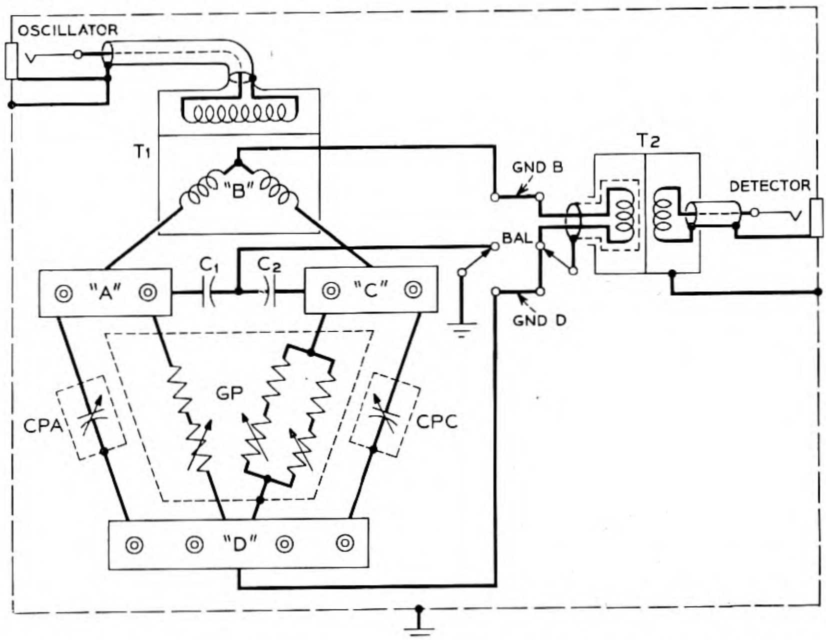


Fig. 10—Ten-megacycle admittance bridge with three-position key for shifting ground to B for measuring direct admittance, to junction of C1 and C2 for balanced admittance, and to D for grounded admittance. Unknowns may be connected from A to D or C to D.

The direct-capacitance measurements are useful in the development of low valued capacitors, and the balanced-to-ground measurements are helpful in evaluating low-admittance off-ground networks.

CONCLUSION

Bridges have been developed for the measurement of impedance and admittance parameters at megacycle frequencies with accuracies heretofore possible only at much lower frequencies. Several of the twenty-megacycle general-purpose bridges have been built and are furnishing useful measurements of networks and components. Experience with these bridges has indicated ranges for which supplementary special-purpose bridges would be desirable, and two such bridges have been built: a Maxwell bridge for low-valued inductors, and an admittance bridge for low-valued capacitors. One feature of all of these bridges not generally available in commercial measuring instruments for megacycle frequencies is the provision of standards having a range of several decades. These allow balances to be made with greater precision over a wider range of phase angles in the apparatus under test, and assure that the absolute accuracy will not be limited by readability. This added precision is very useful in comparing similar components or in measuring characteristics such as temperature coefficient.

ACKNOWLEDGMENTS

The development of impedance bridges during the past thirty years has been under the direction of J. G. Ferguson, and the work described in this article has been under the supervision of S. J. Zammataro. Their assistance in the preparation of this paper has been very helpful and is hereby acknowledged. It is a pleasure also to acknowledge the contributions of a number of the author's colleagues particularly J. E. Nielsen who was largely responsible for the twenty-megacycle general-purpose bridge, and L. E. Herborn for the five megacycle Maxwell bridge.

REFERENCES

1. W. J. Shackelton, "A Shielded Bridge for Inductive Impedance Measurements," *Bell System Tech. J.*, **6,** pp. 142–171, Jan., 1927.
2. W. J. Shackelton and J. G. Ferguson, "Electrical Measurement of Communication Apparatus," *Bell System Tech. J.*, **7,** pp. 70–89, Jan., 1928.
3. J. G. Ferguson, "Measurement of Inductance by the Shielded Owen Bridge," *Bell System Tech. J.*, **6,** pp. 375–386, July, 1927.
4. S. J. Zammataro, "Impedance Bridges," *Bell Labs. Record*, **8,** pp. 167–170, Dec., 1929.
5. J. G. Ferguson, "Shielding in High-Frequency Measurement," *Bell System Tech. J.*, **8,** pp. 560–575, Aug., 1929.

6. J. G. Ferguson and B. W. Bartlett, "The Measurement of Capacitance in Terms of Resistance and Frequency," *Bell System Tech J.*, **7**, pp. 420–437, July, 1928.
7. W. D. Voelker, "An Improved Capacitance Bridge for Precision Measurements," *Bell Labs. Record*, **20**, pp. 133–137, Jan., 1942.
8. J. G. Ferguson, "Classification of Bridge Methods for Measuring Impedances," *Bell System Tech. J.*, **12**, pp. 452–468, Oct., 1933.
9. S. J. Zammataro, "An Inductance and Capacitance Bridge," *Bell Labs. Record*, **16**, pp. 341–346, June, 1938.
10. H. T. Wilhelm, "Impedance Bridge with a Billion-to-One Range," *Bell Labs. Record*, **23**, pp. 89–92, Mar., 1945.
11. H. T. Wilhelm, "Measuring Inductance of Coils with Superimposed Direct Current," *Bell Labs. Record*, **14**, pp. 131–135, Dec. 1935.
12. H. T. Wilhelm, "A Bridge for Measuring Core Loss," *Bell Labs. Record*, **19**, 92–96, Nov. 1940.
13. C. H. Young, "Measuring Inter-Electrode Capacitances," *Bell Labs. Record*, **24**, pp. 433–438, Dec., 1946.
14. H. T. Wilhelm, "Maxwell Bridge for Measuring Loading Coils," *Bell Labs. Record*, **28**, pp. 453–457, Oct., 1950.
15. Carl Englund, "Note on Radio Frequency Measurements," *Proc. Inst. Radio Engrs.*, **8**, pp. 326–333, Aug., 1920.
16. C. H. Young, "A 5-Megacycle Impedance Bridge," *Bell Labs. Record*, **15**, pp. 261–265, Apr., 1937.
17. T. Slonczewski, "A Versatile Nomagram for Circuit Problems," *Bell Labs. Record*, **10**, pp. 71–73, Nov., 1930.
18. R. O. Grisdale, A. C. Pfister, W. van Roosbroeck, Pyrolitic Film Resistors—Carbon and Borocarbon," *Bell System Tech. J.*, **30**, pp. 271–314, Apr., 1951.
19. R. A. Kempf, "Coaxial Impedance Standards," *Bell System Tech. J.*, **30**, pp. 689–705, July, 1951.

Abstracts of Bell System Technical Papers* Not Published in This Journal

A Full Automatic Private-Line Teletypewriter Switching System. W. M. BACON[1] and G. A. LOCKE[1]. *Trans. A.I.E.E.*, **70**, Part 1, pp. 473–480, 1951. (Monograph 1837).

This paper describes a full automatic teletypewriter message switching system for use in private-line networks involving one or more switching centers and a multiplicity of local or long-distance lines, each of which may have one or more stations. This system provides fast teletypewriter communication from any station to any other station or group of stations in the network. At its point of origin a message first is perforated in tape accompanied by suitable directing and end-of-message characters, thereafter it is transmitted automatically, stored temporarily in perforated tape at a switching office, and then routed at high speed to its point or points of destination. Important features are the arrangements provided to permit efficient use of long full duplex transmission lines, the full automatic handling of multiple-address messages with only a single originating transmission, and the various guards and alarms which are provided to protect against loss of messages in case of trouble.

Operational Study of a Highway Mobile Telephone System. L. A. DORFF[1]. *Trans. A.I.E.E.*, **70**, Part 1, pp. 31–37, 1951. (Monograph 1838).

The Dynamics of the Middle Ear and Its Relation to the Acuity of Hearing. H. FLETCHER[1]. *J. Acoust. Soc. Am.*, **24**, pp. 129–131, March, 1952.

The transformer action of the middle ear as measured by Bekésy is shown to be the principal cause for the low acuity of hearing for low frequencies. Because of the very low mechanical impedance across the basilar membrane at low frequencies, large acoustical pressures in front of the ear drum produce appreciable acoustical pressures across the basilar membrane. For example, at 100 cps this pressure is thirty times and at 6000 cps it is one-tenth that created across the basilar membrane.

Diffusion of Donor and Acceptor Elements Into Germanium. C. S. FULLER[1]. *Phys. Rev.*, **86**, pp. 136–137, April 1, 1952.

* Certain of these papers are available as Bell System Monographs and may be obtained on request to the Publication Department, Bell Telephone Laboratories, Inc., 463 West Street, New York 14, N. Y. For papers available in this form, the monograph number is given in parentheses following the date of publication, and this number should be given in all requests.

[1] Bell Telephone Laboratories

A Submarine Telephone Cable with Submerged Repeaters. J. J. GILBERT[1]. *Trans. A.I.E.E.*, **70**, Part 1, pp. 564–572, 1951. (Monograph 1815).

Physical Structure and Magnetic Anisotropy of Alnico 5. Part I. R. D. HEIDENREICH[1] and E. A. NESBITT[1]. *Jl. Appl. Phys.*, **23**, pp. 352–371, March, 1952. (Monograph 1976).

It is concluded from electron metallographic results that the high coercive force and anisotropy of Alnico 5 are caused by a very finely divided precipitate produced by the permanent magnet heat treatment. This precipitate is a transition structure rich in cobalt and is face-centered cubic with a_0 = 10A and appears as rods growing along the [100] directions of the matrix crystal when no magnetic field is applied during heat treatment. The size of the precipitate rods at optimum properties is approximately 75–100A by 400A long. The spacing between rows of rods is about 200A. The rods are not distinctly resolved in the electron images unless they are grown by aging at 800°C. Their orientation and structure is clearly evident in the electron diffraction patterns at all stages of growth. The precipitate responds to a magnetic field applied during heat-treatment both by suppression of nuclei making an angle greater than about 70° with the field and by the forcing of the rods off the [100] direction into that of the field. The precipitate rods tend to scatter in direction about the field vector when the field is off the [100] but are aligned accurately when the field is along [100].

Energy of a Bloch Wall on the Band Picture. I. Spiral Approach. C. HERRING[1]. *Phys. Rev.*, **85**, pp. 1003–1011, March 15, 1952.

It is shown that the band or itinerant electron model of a solid is capable of accounting for the "exchange stiffness" which determines the properties of the transition region, known as the Bloch wall, which separates adjacent ferromagnetic domains with different directions of magnetization. In this treatment the constant spin function usually assigned to each running electron wave is replaced by a variable spin function. At each point of space the spin of a moving electron is inclined at a small velocity-dependent angle to the mean spin direction of the other electrons, and this gives rise to an exchange torque which makes the spin direction of the given electron precess as it moves through the transition region, the precession rate being just sufficient to keep it in approximate alignment with the macroscopic magnetization. Physical insight into the mechanisms involved is provided by a rigorous solution of the wall problem for a ferromagnetic free electron gas in the Slater-Fock approximation, although it is known that the free electron gas is not likely to be ferromagnetic in higher approximations. Rough upper limits to the exchange stiffness constants for actual ferromagnetic metals can be calculated without using any empirical constants other than the saturation moment and the lattice constant. The results are only a few times larger than the observed values.

Elastic and Plastic Properties of Very Small Metal Specimens. C. HERRING[1] and J. K. GALT[1]. *Phys. Rev.*, **85**, pp. 1060–1061, March 15, 1952. (Monograph 1977).

[1] Bell Telephone Laboratories

A Scanner for Rapid Measurement of Envelope Delay Distortion. L. E. HUNT[1] and W. J. ALBERSHEIM[1]. *Proc. I.R.E.*, **40**, pp. 454–459, April, 1952. (Monograph 1967).

A measuring device is described which instantaneously displays the envelope delay-frequency characteristic on a cathode-ray screen. Loop and one-way measurements of long-distance radio networks can be carried out. The frequency range extends from 60 to 80 megacycles; the limits of accuracy are 1 millimicrosecond or 2 per cent of the measured delay range. Comparison of two characteristics can be carried out by superposition of alternate scanning traces.

The device has been found useful in measuring the delay distortion of the TD-2 radio-relay system and in designing and adjusting the delay equalizers needed to correct it.

Numerical Integration Near a Singularity. E. L. KAPLAN[1]. *J. Math. Phys.*, **31**, pp. 1–28, April, 1952. (Monograph 1980).

Measurement of Diffusion in Semiconductors by a Capacitance Method. K. B. MCAFEE[1], W. SHOCKLEY[1] and M. SPARKS[1]. *Phys. Rev.*, **86**, pp. 137–138, April, 1952.

Probing the Space Charge Layer in a p-n Junction. G. L. PEARSON[1], W. T. READ[1] and W. SHOCKLEY[1]. *Phys. Rev.*, **85**, pp. 1055–1057, March 15, 1952.

Control Methods Used in a Study of the Vowels. G. E. PETERSON[1] and H. L. BARNEY[1]. *J. Acoust. Soc. Am.*, **24**, pp. 175–184, March, 1952. (Monograph 1982)

Relationships between a listener's identification of a spoken vowel and its properties as revealed from acoustic measurement of its sound wave have been a subject of study by many investigators. Both the utterance and the identification of a vowel depend upon the language and dialectal backgrounds and the vocal and auditory characteristics of the individuals concerned. The purpose of this paper is to discuss some of the control methods that have been used in the evaluation of these effects in a vowel study program at Bell Telephone Laboratories. The plan of the study, calibration of recording and measuring equipment, and methods for checking the performance of both speakers and listeners are described. The methods are illustrated from results of tests involving some 76 speakers and 70 listeners.

Current Multiplication in the Type-A Transistor. W. R. SITTNER[1]. *Proc. I.R.E.*, **40**, pp. 448–454, April, 1952. (Monograph 1969).

One of the basic phenomena exhibited by transistors is current multiplication. In transistors of the point-contact type (one of these has been called the Type-A), the mechanism giving rise to this effect has been somewhat uncertain. Four

[1] Bell Telephone Laboratories

possible mechanisms of the current multiplication process in the Type-A transistor are discussed. One of the mechanisms is based on trapping holes in the collector barrier of the semiconductor. By means of this trapping model, the effect of emitter current and temperature on the current multiplication is predicted. It is shown that these predictions are in reasonable accord with experiment. Furthermore, assuming this model to hold, the trap density and activation energy (produced by forming) may be evaluated.

Faraday Rotation of Guided Waves. H. SUHL[1] and L. R. WALKER[1]. *Phys. Rev.*, **86**, pp. 122–123, April 1, 1952.

Transistor Forming Effects in n-Type Germanium. L. B. VALDES[1]. *Proc. I.R.E.*, **40**, pp. 445–448, April, 1952. (Monograph 1969).

Some of the effects of electrical forming of the collector of an n-type germanium transistor are discussed. Evidence is presented for the existence of a region of *p*-type germanium underneath the formed electrode, together with some indication of the size of the formed region. These experiments lend support to the *p-n* hook mechanism in that they explain the observed high values of alpha in transistors. This relation is discussed.

Domain Structure of Perminvar Having a Rectangular Hysteresis Loop. H. J. WILLIAMS[1] and M. GOERTZ[1]. *Jl. Appl. Phys.*, **23**, pp. 316–323, March, 1952. (Monograph 1985).

An investigation has been made of the magnetic domain structure of Perminvar (43 per cent Ni, 34 per cent Fe, 23 per cent Co) ring specimens having rectangular hysteresis loops after heat-treatment in a magnetic field. Domain patterns obtained with colloidal magnetite showed curved domain boundaries extending completely around the rings, forming circles concentric with them. Changes in magnetization occur when an applied field causes the circular boundaries either to expand or contract so that there is a change in the relative values of clockwise and counter-clockwise flux. A nucleus of reversed magnetization was formed by making a small notch in a specimen, and this decreased the coercive force and hysteresis loss by a factor of two. It was found that in a 180° domain boundary it was possible to make the change in spin orientations, which occurs in going from one side of the boundary to the other, have either a right- or left-hand screw relation, by the application of a field of appropriate sign perpendicular to the surface. The effect of superposing an applied alternating field was also investigated, and an effective permeability of 4,000,000 was obtained.

Measuring Techniques for Broad-Band. Long-Distance Radio Relay Systems. W. J. ALBERSHEIM[1]. *Proc. I.R.E.*, **40**, pp. 548–551, May, 1952. (Monograph 1971).

Line-up and maintenance of radio relay systems require sensitive yet rapid measurements. These are obtained by scanning the systems response as functions of time, frequency, and amplitude. Parameters thus scanned include the

[1] Bell Telephone Laboratories

transient response to step functions; frequency characteristics of gain, phase, impedance and their frequency derivatives; and amplitude characteristics of output nonlinearity and of intermodulation products.

Aluminum Die Castings—The Effect of Process Variables on Their Properties. W. BABINGTON[1] and D. H. KLEPPINGER[4]. *Proc. A.S.T.M.*, **51**, pp. 169–197, 1951.

Diffusion in Alloys and the Kirkendall Effect. J. BARDEEN[1] and C. HERRING[1]. pp. 261–288 of *Imperfections in Nearly Perfect Crystals*, Wiley N. Y., 1952, 490 p. Edited by W. Shockley, J. H., Hollomon, R. Maurer and F. Seitz. Symposium held at Pocono Manor, Oct. 12–14, 1950, by Committee on Solids, National Research Council.

Lightning Protection for Fixed Radio Stations. D. W. BODLE[1]. *Tele-Tech*, **11**, pp. 58–60, 126+, June, 1952.

Common grounds, parallel conducting paths, and discharge gaps provide three important means for avoiding equipment damage from high current surges. Protection of connecting facilities must also be considered to preserve service.

Compression Tests on Lead Alloys at Extrusion Temperatures. G. M. BOUTON[1] and G. S. PHIPPS[1]. *Proc. A.S.T.M.*, v. **51,** pp. 761–770, 1951.

Load-deflection measurements made during compression tests on lead and lead-alloy cylinders at various temperatures show the effects of alloying ingredients on the force required to produce deformation. The curves also furnish clues as to changes taking place in the materials during the course of the test. The load, P, to produce definite small deformation in pure lead at various temperatures, T, are shown to follow the relationship $P = Ae^{-BT}$, where A and B are constants for the material. This is the same relationship found by others in extrusion studies. The elements added to lead were those most commonly used in the manufacture of cable sheath, namely, antimony, arsenic, bismuth, silver, tellurium, and tin. The results show that the stronger alloys now used for cable sheathing deform less readily at extrusion temperatures than pure lead or the weaker alloys.

RF Phase Control in Pulsed Magnetrons. E. E. DAVID, JR[1]. *Proc. I.R.E.*, **40**, pp. 669–685, June, 1952.

This paper describes the behavior of a magnetron oscillator started in the presence of an externally applied rf exciting signal whose frequency is not greatly different from the unperturbed steady-state frequency of the magnetron.

Effect of Prior Strain at Low Temperatures on the Properties of Some Close-Packed Metals at Room Temperature. W. C. ELLIS[1] and E. S. GREINER[1]. *J. Metals*, **4**, pp. 648–651, June, 1952. (Monograph 1966).

[1] Bell Telephone Laboratories
[4] Frankford Arsenal, Philadelphia, Pa.

The Fatigue Test as Applied to Lead Cable Sheath. G. R. GOHN[1] and W. C. ELLIS[1]. *Proc. A.S.T.M.*, **51**, pp. 721–740, 1951.

This paper discusses the more important factors affecting the design of laboratory test methods suitable for obtaining significant fatigue data from reversed bending tests on cantilever-beam specimens of lead cable sheathing alloys. Data are presented to show the effect of cycling rate, temperature, shape of specimen, alloy additions, and aging on fatigue life. The close correlation between bending fatigue tests on strip specimens and full size sections of cable is demonstrated. The fatigue data are analyzed in terms of (1) cycle life versus deflection, (2) cycle life versus strain, and (3) cycle life versus stress. Photomicrographs illustrating representative laboratory and field failures are included.

Thermal Conductivity of Germanium. A. GRIECO[1] and H. C. MONTGOMERY[1]. *Phys. Rev.*, **86**, p. 570, May 15, 1952.

Bell System Cable Sheath Problems and Designs. F. W. HORN[1] and R. B. RAMSEY[1]. *Trans. A.I.E.E.*, **70**, Part 2, pp. 1811–1816, 1951. (Monograph 1917).

Powdered Standards for Spectrochemical Analysis. E. K. JAYCOX[1]. *Applied Spectroscopy*, **6**, pp. 17–19, May, 1952. (Monograph 1978).

Engineering for Low Product Cost and High Product Quality at the Western Electric Company. A. C. JONES[3]. *Ind. Quality Control*, **8**, pp. 53–59, May, 1952.

The Approximation with Rational Functions of Prescribed Magnitude and Phase Characteristics. J. G. LINVILL[1]. References. *Proc. I.R.E.*, **40**, pp. 711–721, June, 1952.

A successive-approximations method is applied to the selection of network functions having desired magnitude and phase variation with frequency. The first approximation, the first set of pole and zero locations, can be selected on the basis of known solutions to similar problems or through use of a set of curves. In succeeding approximations the pole and zero locations are adjusted to decrease the deviation of the earlier approximations from the desired characteristics. The process adjusts the magnitude and phase characteristics simultaneously. Its flexibility permits accommodation of practical constraints not possible with other methods.

The Magnetic Structure of Alnico 5. E. A. NESBITT[1] and R. D. HEIDENREICH[1]. *Elec. Eng.*, **71**, pp. 530–534, June, 1952. (Monograph 1981).

In the investigation of Alnico 5, two problems arose. What is the mechanism which enables the alloy to respond to heat treatment in a magnetic field? What causes the alloy to have a high coercive force of 600 oersteds? The first problem has been solved and progress has been made toward solving the second.

[1] Bell Telephone Laboratories
[3] Western Electric Company

Single-Frequency Signaling System for Supervision and Dialing Over Long-Distance Telephone Trunks. N. A. NEWELL[1] and A. WEAVER[1]. *Trans. A.I.E.E.*, **70**, Part 1, pp. 489–494, 1951. (Monograph 1841).

The single-frequency signaling system for long-distance telephone trunks frees dial calls from the range and other limitations imposed by dc signaling methods. It uses alternating currents in the voice range as the signaling medium and so can be used with any trunk of any length or type of line facility which meets voice-transmission requirements. The signaling requirements, design problems, main features of the circuit and equipment arrangements, and the operation of this system are outlined in this paper. The system described is the first practical arrangement of its type satisfactorily to meet all the conditions of telephone service in the Bell Telephone System.

Experimental Information on Slip Lines. W. T. READ, JR[1]. pp. 129–151 of *Imperfections in Nearly Perfect Crystals*, Wiley, N. Y., 1952, 490 p. Edited by W. Shockley, J. H. Hollomon, R. Maurer and F. Seitz. Symposium held at Pocono Manor, Oct. 12–14, 1950, by Committee on Solids, National Research Council.

On the Geometry of Dislocations. W. T. READ, JR.[1] and W. SHOCKLEY[1]. pp. 77–94 of *Imperfections in Nearly Perfect Crystals*, Wiley, N. Y., 1952, 490 p. Edited by W. Shockley, J. H. Hollomon, R. Maurer and F. Seitz. Symposium held at Pocono Manor, Oct. 12–14, 1950, by Committee on Solids, National Research Council.

A Servo System for Heterodyne Oscillators. T. SLONCZEWSKI[1]. *Trans. A.I.E.E.*, **70**, Part 1, pp. 1070–1072, 1951. (Monograph 1883).

A constant rate of progression of frequency of a motor-driven heterodyne oscillator is obtained by comparing its output with a frequency standard. The result is fed into a servo loop which drives the motor at the proper speed. When used in connection with a level recorder a linear frequency scale is obtained which is more accurate than the static calibration of the oscillator.

Metallic Rectifiers in Telephone Power Plants. D. E. TRUCKSESS[1]. *Trans. A.I.E.E.*, **70**, Part 2, pp 1464–1467, 1951. (Monograph 1987).

Metallic rectifiers are a comparatively new means of converting power from alternating current to direct current. Most of the component apparatus used in the Telephone Systems operates with direct current while the normal power source is alternating current. Therefore a static device without expendable parts which is obtainable in small and large current capacity lends itself as a means for power conversion in telephone power plants.

[1] Bell Telephone Laboratories

Contributors to this Issue

A. B. Clark, B.E.E., University of Michigan, 1911. A. T. & T. Co., 1911–34; Bell Telephone Laboratories, 1934–. Toll Transmission Development Engineer, 1929; Toll Transmission Development Director, 1934; Director of Transmission Development, 1935; Director of Systems Development, 1940; Vice President, 1944. Bell System Chairman of Joint Subcommittee on Development and Research of the Edison Electric Institute and Bell System since 1938. Since June, 1951, Mr. Clark has been in charge of coordinating all Bell System programs at the Laboratories. During World War II he served both as a consultant to and a member of various divisions of the Office of Scientific Research and Development. In 1944 he was appointed Consultant to the Secretary of War, and in connection with this work made trips to the European and Mediterranean theaters of operation. Member of I.R.E., Tau Beta Pi, Sigma Xi, and A.A.A.S. and Fellow of A.I.E.E. and the Acoustical Society of America.

J. R. Fry, M.E., Cornell University, 1915. Western Electric Company, 1915–25. Bell Telephone Laboratories 1925–. Mr. Fry has been Assistant Switching Apparatus Engineer in the Switching Apparatus Development Department since 1946. Except for the years 1941–45, when he worked on military projects, most of Mr. Fry's Bell System service has been devoted to the design and development of electromagnetically operated switching apparatus such as relays, switches, registers, and selectors. Member of Eta Kappa Nu.

H. C. Montgomery, A.B., University of Southern California, 1929; M.A., Columbia University, 1933. Bell Telephone Laboratories, 1929–. Prior to the war, Mr. Montgomery was engaged in studies of hearing acuity and the analysis of speech sounds. His recent work in the transistor physics group has been concerned with fluctuation phenomena in semiconductors.

Samuel P. Morgan, Jr., B.S., California Institute of Technology, 1943; M.S., California Institute of Technology, 1944; Ph.D., California Institute of Technology, 1947. Bell Telephone Laboratories, 1947–. A research mathematician, Dr. Morgan specializes in electromagnetic theory. He has been particularly concerned with problems of wave guide and coaxial cable transmission. Member of the American Physical Society, Tau Beta Pi, and an associate member of Sigma Xi.

W. H. Nunn joined the Home Telephone and Telegraph Company of Los Angeles in 1915. He became Plant Staff Engineer in 1927; Traffic Engineer in 1928; General Traffic Engineer, Oregon, 1935; General Traffic Engineer, Northern California and Nevada, 1940; Traffic Operations Engineer, Pacific Telephone and Telegraph Company, 1942; and General Traffic Manager, Northern California and Nevada, 1947. In July of 1949 he transferred to the American Telephone and Telegraph Company as Traffic Facilities Engineer, and since March of this year has been Assistant Chief Engineer.

H. S. Osborne, B.S., Mass. Inst. of Technology, 1908; Eng. D., Mass. Inst. of Technology, 1910; A. T. & T., Co., 1910–. Since joining the American Telephone and Telegraph Company in 1910, Mr. Osborne has been with the company continuously: as engineer in the Transmission and Protection Department until 1914; assistant to Transmission and Protection Engineer, 1914–1920; Transmission Engineer, 1920–1939; Operating Results Engineer, 1939–1940; Plant Engineer, 1940–1942; Assistant Chief Engineer, 1942–1943; and Chief Engineer from 1943 until his retirement in August of this year. During the war Mr. Osborne was Special Consultant in the office of the Secretary of War and a member of the Telegraph Committee, War Communications Board. In addition, he is a member of the Industry Advisory Council, Federal Specifications Board; of the Industry Advisory Committee for Supply Cataloging, Munitions Board; and of the Domestic Communications Industry Advisory Committee to N.P.A. For many years he has been active in the work of the A.I.E.E.: Chairman, Standards Committee, 1923–1926; member, Committee on Communications, 1931–1934; member, Edison Medal Committee, 1936–1943 and 1947–1952; Chairman, Committee on Award of Institute Prizes, 1936–1939; Chairman, Technical Program Committee, 1936–1939; member, Publication Committee, 1936–1939; Chairman, Special Committee on Institute Activities, 1936–1937; member, Committee on Planning and Coordination, 1936–1942, 1945–1946, and 1947–1949; member, Alfred Noble Prize Committee, 1937–1942; Chairman, Finance Committee, 1939–1942; President, 1942–1943; Chairman, Executive Committee, 1942–1943; member, Board of Directors and Executive Committee, 1942–1945; member, John Fritz Medal Board of Award, 1942–1946; Chairman, Board of Trustees, A.I.E.E. Retirement System, 1944–1945; member, Hoover Medal Board of Award, 1945–1951; and Chairman, Board of Trustees of Volta Memorial Fund, 1949–. He also has been active in the American Standards Association. He was long a member of the Board of Directors, and was Chairman of the Standards Council from 1942–1945 and Vice President 1948–1951. Since 1949 he has been President of the U. S. National Committee of the International Electrotechnical Commission. He is a member of the Joint Conference Committee on Standards of the Department of Commerce and ASA, and Chairman of the U. S. N. C. Executive Council Subcommittee. He is Fellow of the American Institute of Electrical Engineers, Acoustical Society of America, American Physical Society, American Association for the Advancement of Science, and of the Institute of Radio Engineers; and is a member of the American Society for Engineering Education and of Tau Beta Pi.

J. J. Pilliod, E.E. 1908, D.E. (Hon.) 1939, Ohio Northern University; A. T. & T. Co., 1908–. From 1910 until 1943 Mr. Pilliod was associated with the Long Lines Department and the General Engineering Department of the American Telephone and Telegraph Company. From 1914 to 1918 he was Division Plant Engineer in Chicago; 1918–1920, Engineer of Transmission, New York City; 1920–1941, Engineer in charge of Long Lines Engineering Department; and 1941–1943, General Manager of the Long Lines Department. In 1943 he assumed his present position as Assistant Chief Engineer of the American Telephone and Telegraph Company. From October 1942 to April 1943 he was Chief of Signal Section, Production Division, Army Service Force. He is a Fellow of the A.I.E.E. and is a Trustee of Ohio Northern University and of Vassar College.

F. F. Shipley, B.S. in E.E., Purdue University, 1925. A. T. & T. Co., 1925–34; Bell Telephone Laboratories, 1934–. Since 1948, Mr. Shipley has been switching engineer in charge of planning large automatic switching systems, both local and toll. This includes panel, crossbar, and large step-by-step systems. Member of the A.I.E.E., Tau Beta Pi, and Eta Kappa Nu.

H. T. Wilhelm, B.S. in E.E., Cooper Union, 1927; E.E., Cooper Union, 1936. Western Electric Company, 1922–24; Bell Telephone Laboratories, 1925–. Since joining the Laboratories Mr. Wilhelm's work has been with the Transmission Apparatus Development Department, where he has designed electrical measurement apparatus and developed test methods. Member of A.I.E.E. and Tau Beta Pi.