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A Laplacian Expansion for Hermitian-Laplace Functions of High Order*

By E. C. MOLINA

Among the wide variety of practical and theoretical problems confronting the telephone engineer, there is a surprisingly large number to whose solution mathematics has made notable contribution. In his kit of mathematical tools the theory of probability is a frequently used and most effective instrument. This theory of probability contains a large number of theorems, a large number of functions, which permit of application to telephony. Among these is a particular tool, a particular group of mathematical functions known as the "Hermitian Functions," each of which is identified by a number called its "order." These mathematical functions or relations have no practical utility until the variables in the equation can be assigned numerical values and the resultant numerical value of the function calculated. Tables of the numerical values of Hermitian functions of low order exist; for example, Glover's Tables of Applied Mathematics cover the ground for those of the first eight orders. But tables for the functions of higher order are still a desideratum. This paper presents an expansion by means of which the evaluation of a high order function can be readily accomplished with a considerable degree of accuracy.

The development of the expansion is prefaced by some remarks on the early history of the Hermitian functions and the relation of this history to modern theoretical physics.

I

AMONG contributions made by Laplace to the domain of pure and applied mathematics, two of great practical value are:

(*a*) His method of evaluating definite integrals [1] whose integrands involve factors raised to high powers;

(*b*) The pair of orthogonal polynomial functions [2] which he defined by the following Equations (1) and (2)

$$(1)\qquad [(2n)!\sqrt{\pi}/2^{2n}n!]U_n(u) = \int_{-\infty}^{\infty} e^{-x^2}(x - iu)^{2n}dx = 2e^{u^2}\int_0^{\infty} e^{-x^2}x^{2n}\cos(2ux)dx;$$

$$(2)\qquad [(2n+1)!\sqrt{\pi}/2^{2n}n!]U_n'(u) = i\int_{-\infty}^{\infty} e^{-x^2}(x - iu)^{2n+1}dx = 2e^{u^2}\int_0^{\infty} e^{-x^2}x^{2n+1}\sin(2ux)dx.$$

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These polynomial functions formed the coefficients in a series satisfying the partial differential equation

$$\frac{\partial U}{\partial r'} = 2U + 2u\frac{\partial U}{\partial u} + \frac{\partial^2 U}{\partial u^2}$$

to which Laplace reduced the solution of the following ball problem: [3]

Consider two urns A and B each containing n balls and suppose that of the total number of balls, $2n$, as many are white as black. Conceive that we draw simultaneously a ball from each of the urns, and that then we place in each urn the ball drawn from the other. Suppose that we repeat this operation any number, r, of times, each time shaking well the urns in order that the balls be thoroughly mixed; and let us find the probability that after the r operations the number of white balls in urn A be x.

Under the caption "The Statistical Meaning of Irreversibility" Lotka [4] has pointed out the significance of Laplace's ball problem in the modern kinetic theory of matter. Moreover, Hostinsky [5] has shown the bearing of the same problem on the theory of Brownian movements and said "In effect, the partial differential equation obtained by Laplace has been refound by Smoluchowski."

To avoid confusion with the Laplace functions which one encounters in spherical harmonic analysis, the functions defined by Equations (1) and (2) are herein designated as Hermitian-Laplace functions. Such a designation is justified by the Equations (3) and (4) derived in the next paragraph.

II

We also find in Laplace [6]

$$I_n(u)^* = \int_0^\infty e^{-x^2}x^{2n}\cos(2ux)dx = \frac{(-1)^n\sqrt{\pi}}{2^{2n+1}}\left(\frac{d^{2n}e^{-u^2}}{du^{2n}}\right),$$

$$I_n'(u) = \int_0^\infty e^{-x^2}x^{2n+1}\sin(2ux)dx = \frac{(-1)^{n+1}\sqrt{\pi}}{2^{2n+2}}\frac{(d^{2n+1}e^{-u^2})}{du^{2n+1}}.$$

Comparing these Laplacian expressions for the definite integrals $I_n(u)$ and $I_n'(u)$ with the Equations (1) and (2) we see immediately that

(3) $$U_n(u) = (-1)^n[n!/(2n)!]H_{2n}(u),$$

(4) $$U_n'(u) = (-1)^{n+1}[n!/2(2n+1)!]H_{2n+1}(u),$$

where H_{2n} and H_{2n+1} are the original Hermite polynomials [7] of order $2n$ and $2n+1$, respectively. These equations connecting the Her-

* The symbols $I_n(u)$ and $I_n'(u)$ are introduced here as convenient abbreviations for the integrals to which they are equated; these symbols do not appear in Laplace.

mite with the Laplace polynomials have been presented in an earlier paper.[8]

Appell and Feriet, Arne Fisher, T. C. Fry, H. L. Rietz and others base their definitions of the Hermite polynomials on $e^{-x^2/2}$ instead of e^{-x^2}. We shall write $A_n(u)$ for the nth polynomial as defined by these authors, reserving $H_n(u)$ to symbolize the Hermitian polynomial as defined in his paper of 1864. Thus, in what follows,

$$A_n(x) = (-1)^n e^{x^2/2}(d^n e^{-x^2/2}/dx^n), \qquad H_n(u) = (-1\sqrt{2})^n A_n(u\sqrt{}$$

III

Laplacian expansions * for the U, H, and A polynomials follow immediately from those obtainable by applying to the integrals $I_n(u)$ and $I_n'(u)$ his method of evaluating definite integrals whose integrands embrace factors raised to high powers. As will be shown in Part IV of this paper, we have

$$I_n(u)/[\sqrt{\pi}(Y\sqrt{N})^N] = [S\cos(u\sqrt{2N})] + [S'\sin(u\sqrt{2N}], \qquad N = 2n,$$

$$I_n'(u)/[\sqrt{\pi}(Y\sqrt{N})^N] = [S\sin(u\sqrt{2N})] - [S'\cos(u\sqrt{2N}], N = 2n+1,$$

where $Y = (xe^{-x^2})$ for $x = X = 1/\sqrt{2}$ and

$$S = \sum_{s=0}^{\infty}\left(\frac{-1}{4N}\right)^s [u^{-(2s+1)}K_{2s}],$$

$$S' = \left(\frac{1}{2\sqrt{N}}\right)\sum_{s=1}^{\infty}\left(\frac{-1}{4N}\right)^{(s-1)}[u^{-2s}K_{2s-1}].$$

The explicit expressions for K_0, K_2, K_4 and K_1, K_3, K_5 are given in Section V of this paper. The desired expansions are then given by the equations:

$$\begin{aligned} e^{u^2}I_n(u)/\sqrt{\pi} &= U_n(u)[(2n)!/2^{2n+1}n!] \\ &= H_{2n}(u)[(-1)^n/2^{2n+1}] \\ &= A_{2n}(u\sqrt{2})[(-1)^n/2^{n+1}], \\ e^{u^2}I_n'(u)/\sqrt{\pi} &= U_n'(u)\,[(2n+1)!/2^{2n+1}n!] \\ &= H_{2n+1}(u)[(-1)^{n+1}/2^{2n+2}] \\ &= A_{2n+1}(u\sqrt{2})[(-1)^n/2^{n+1}\sqrt{2}]. \end{aligned}$$

The numerical results shown below in Table I indicate the efficacy

* It may be of interest to compare the expansions presented in this paper with the asymptotic forms of the Hermite functions given by N. Schwid[9] and by M. Plancherel and M. Rotach.[10]

TABLE I

N	$u\sqrt{2} = x$	True Value of $A_N(x)$	$[{}_sA_N(x) - A_N(x)]/A_N(x)$		
			$s = 1$	$s = 2$	$s = 3$
9	0.1	9.32438×10^1	0.0089	0.0000	0.0000
	0.5	3.26533×10^2	0.0084	0.0001	0.0000
	1	2.80000×10^1	0.0263	− 0.0010	− 0.0002
	2	-1.90000×10^2	0.0002	0.0018	0.0001
	3	1.62000×10^3	− 0.0474	− 0.0027	0.0002
	4	-1.74680×10^4	− 0.0283	− 0.0082	− 0.0027
10	0.1	-8.98064×10^2	0.0084	− 0.0001	0.0000
	0.5	4.90439×10^1	− 0.0027	0.0013	0.0000
	1	1.21600×10^3	0.0046	0.0003	0.0000
	2	-2.62100×10^3	− 0.0147	0.0002	0.0001
	3	9.50400×10^3	− 0.0436	− 0.0044	− 0.0004
	4	-5.18090×10^4	0.0445	0.0013	− 0.0018
15	0.1	-1.98001×10^5	0.0054	0.0000	0.0000
	0.5	-5.05845×10^5	0.0052	0.0000	0.0000
	1	4.69456×10^5	0.0022	0.0002	0.0000
	2	-1.41980×10^6	− 0.0102	0.0000	0.0000
	3	4.38955×10^6	− 0.0284	− 0.0017	− 0.0001
	4	-1.85644×10^7	− 0.0338	− 0.0041	− 0.0006
20	0.1	5.90233×10^8	0.0042	0.0000	0.0000
	0.5	-4.45178×10^8	0.0035	0.0000	0.0000
	1	-1.61935×10^8	0.0046	0.0000	0.0000
	2	-1.62882×10^9	− 0.0081	0.0000	0.0000
	3	4.60718×10^9	− 0.0212	− 0.0009	0.0000
	4	8.53219×10^9	0.1241	0.0068	0.0000

of the Laplacian expansion as applied to the evaluation of $A_n(x)$, $x = u\sqrt{2}$, for values of x ranging from 0.1 to 4 and for $N = 9$, 15, 10 and 20, respectively.

Designating by ${}_sA_N(x)$ the approximate value obtained for $A_N(x)$ when one takes into account the first s terms in each of the two series S, S', the last three columns of the table show the proportional errors incurred when $s = 1$, 2 and 3, respectively. It will be noted that for $N = 10$ and $x = 4$ the second approximation is closer than the third; this situation will occasion no surprise if it is recalled that to obtain the best results the natural order of the terms may have to be altered when, for example, one expresses a term of the binomial expansion in a series of Hermite functions.

For the convenience of one who wishes to calculate $A_N(x)$ for values of N other than 9, 10, 15 and 20, there are given in Table II the values of $u^{-(m+1)}K_m$ for $m = 0, 1, 2, 3, 4, 5$ and those values of u covered by Table I.

I am indebted to Miss E. V. Wyckoff of Bell Telephone Labora-

tories, for the computations involved in the preparations of Tables I and II.

TABLE II

$u\sqrt{2}$	$u^{-1}K_0$	$u^{-3}K_2$	$u^{-5}K_4$
0.1	0.7053412	0.234229	0.034490
0.5	0.6642654	0.198970	− 0.069716
1	0.5506953	0.0936947	− 0.273541
2	0.2601300	− 0.158968	− 0.018172
3	0.07452849	− 0.121885	0.514828
4	0.01295111	0.0244632	− 0.078479
	$u^{-2}K_1$	$u^{-4}K_3$	$u^{-6}K_5$
0.1	− 0.03520828	0.00602953	0.091905
0.5	− 0.1591469	0.0450683	0.40870
1	− 0.2294564	0.150921	0.48658
2	− 0.08671002	0.255634	− 0.78050
3	0.05589638	− 0.104689	− 0.35023
4	0.04317037	− 0.162097	0.98512

IV

A simple change of variable gives

$$(5)\quad I_n(u) = (\sqrt{N})^{N+1}\int_0^\infty (e^{-x^2}x)^N \cos(x2u\sqrt{N})dx, \qquad N = 2n,$$

$$(6)\quad I_n'(u) = (\sqrt{N})^{N+1}\int_0^\infty (e^{-x^2}x)^N \sin(x2u\sqrt{N})dx, \qquad N = 2n+1.$$

Set $y(x) = e^{-x^2}x$, and note that $dy/dx = 0$ for $x = X = 1/\sqrt{2}$. Now set $Y = y(X)$,

$$[g(x)]^2 = (\log Y - \log y)/(x-X)^2$$
$$= \frac{1}{X^2}\left[1 - \frac{1}{3}\left(\frac{x-X}{X}\right) + \frac{1}{4}\left(\frac{x-X}{X}\right)^2 - \frac{1}{5}\left(\frac{x-X}{X}\right)^3 + \cdots\right],$$

$$(7),\quad t = (x-X)g(x).$$

These transformations give

$$I_n(u) = (\sqrt{N})^{N+1}Y^N\int_{-\infty}^\infty e^{-Nt^2}(dx/dt)\cos(x2u\sqrt{N})dt.$$

By (7) and the Lagrange-Laplace expansion for a function of x in powers of t we obtain

$$I_n(u) = (\sqrt{N})^{N+1}Y^N\sum_{m=0}^{\infty}\int_{-\infty}^\infty e^{-Nt^2}[t^{2m}A_{2m}/(2m)!]dt$$

or

$$I_n(u)/\sqrt{\pi}(Y\sqrt{N})^N = \sum_{m=0}^{\infty} (1/2\sqrt{N})^{2m}(A_{2m}/m!),$$

where, writing D for the differential operator d/dx,

$$A_{2m} = [D_x^{2m} g^{-(2m+1)} \cos (x2u\sqrt{N})]_{x=X}$$

or, by the Leibnitz theorem for the product of two functions,

$$A_{2m} = \sum_{r=0}^{2m} \binom{2m}{r} (2u\sqrt{N})^r \cos (u\sqrt{2N} + r\pi/2)[D_x^{2m-r} g^{-(2m+1)}]_{x=X}$$

and, therefore,

$$\frac{A_{2m}}{m!} = \cos (u\sqrt{2N}) \sum_{r=0}^{m} \binom{2m}{2r} \frac{(-1)^r(u2\sqrt{N})^{2r}}{m!} [D_x^{2m-2r} g^{-(2m+1)}]_{x=X}$$

$$- \sin (u\sqrt{2N}) \sum_{r=0}^{m-1} \binom{2m}{2r+1} \frac{(-1)^r(u2\sqrt{N})^{2r+1}}{m!} [D_x^{2m-2r-1} g^{-(2m+1)}]_{x=X},$$

on separating the even and odd terms in r. Now setting $m - r = s$ and summing with reference to s and r, instead of m and r, gives

$$\sum_{m=0}^{\infty} (1/2\sqrt{N})^{2m} A_{2m}/m!$$

$$= \cos (u\sqrt{2N}) \sum_{s=0}^{\infty} \left(\frac{1}{2\sqrt{N}}\right)^{2s} \frac{1}{(2s)!} \sum_{r=0}^{\infty} \frac{(2r+2s)!}{(2r)!} \frac{(-u^2)^r}{(r+s)!}$$

$$\times [D_x^{2s} g^{-(2r+2s+1)}]_{x=X} - \sin (u\sqrt{2N}) \sum_{s=1}^{\infty} \left(\frac{1}{2\sqrt{N}}\right)^{2s-1} \frac{1}{(2s-1)!}$$

$$\times \sum_{r=0}^{\infty} \frac{(2r+2s)!}{(2r+1)!} \frac{(-1)^r u^{2r+1}}{(r+s)!} [D_x^{2s-1} g^{-(2r+2s+1)}]_{x=X}.$$

But, writing $u/g = v$, we have

$$\sum_{r=0}^{\infty} \left(\frac{(2r+2s)!}{(2r)!}\right) \frac{(-1)^r u^{2r}}{(r+s)!} [D_x^{2s} g^{-(2r+2s+1)}]$$

$$= (-1)^s u^{-(2s+1)} \left[D_x^{2s} v^{2s+1} \sum_{r=0}^{\infty} \frac{(-1)^{r+s} v^{2r}}{(r+s)!} \frac{(2r+2s)!}{(2r)!} \right]$$

$$= (-1)^s u^{-(2s+1)} \left[D_x^{2s} v^{2s+1} D_v^{2s} \sum_{r=0}^{\infty} \frac{(-1)^{r+s} v^{2r+2s}}{(r+s)!} \right]$$

$$= (-1)^s u^{-(2s+1)} [D_x^{2s} v^{2s+1} D_v^{2s} e^{-v^2}],$$

since $D_v^{2s} v^{2m} = 0$ for m less than s.

Likewise

$$\sum_{r=0}^{\infty} \frac{(2r+2s)!}{(2r+1)!} \frac{(-1)^r u^{2r+1}}{(r+s)!} [D_x^{2s-1} g^{-(2r+2s+1)}] = (-1)^s u^{-2s} [D_x^{2s-1} v^{2s} D_v^{2s-1} e^{-v^2}].$$

Therefore, finally,

$$\frac{I_n(u)}{(Y\sqrt{N})^N \sqrt{\pi}} = \cos(u\sqrt{2N}) \sum_{s=0}^{\infty} \left(\frac{1}{2\sqrt{N}}\right)^{2s} [u^{-(2s+1)} K_{2s}](-1)^s + \sin(u\sqrt{2N}) \left(\frac{1}{2\sqrt{N}}\right) \sum_{s=1}^{\infty} \left(\frac{1}{2\sqrt{N}}\right)^{2(s-1)} [u^{-2s} K_{2s-1}](-1)^{s-1},$$

where

$$(2s)!K_{2s} = [D_x^{2s} v^{2s+1} e^{-v^2} H_{2s}(v)]_{x=X},$$
$$(2s-1)!K_{2s-1} = [D_x^{2s-1} v^{2s} e^{-v^2} H_{2s-1}(v)]_{x=X}.$$

Substituting $\sin(x2u\sqrt{N})$ for $\cos(x2u\sqrt{N})$ in the equations defining A_{2m} and then proceeding exactly as above we derive the corresponding expansion for $I_n'(u)$.

V

To obtain the values of K_{2s} and K_{2s-1} note that

$$Xg^2 = 1 - (x-X)/3X + (x-X)^2/4X^2 - (x-X)^3/5X^3 + \cdots$$

gives, for $x = X = 1/\sqrt{2}$,

$$g = \sqrt{2}, \qquad v = (1/\sqrt{2})u,$$
$$dg/dx = -1/3, \qquad dv/dx = (1/6)u,$$
$$d^2g/dx^2 = (4\sqrt{2})/9, \qquad d^2v/dx^2 = -(1/3\sqrt{2})u,$$
$$d^3g/dx^3 = -88/45, \qquad d^3v/dx^3 = (53/90)u,$$
$$d^4g/dx^4 = (824\sqrt{2})/135, \qquad d^4v/dx^4 = -(211\sqrt{2}/135)u,$$
$$d^5g/dx^5 = -28184/567, \qquad d^5v/dx^5 = (79/7)u,$$

etc.
Therefore

$$\sqrt{2}(u^{-1}K_0) = e^{-\frac{1}{2}u^2},$$
$$36\sqrt{2}(u^{-3}K_2) = e^{-\frac{1}{2}u^2}(u^6 - 6u^4 - 9u^2 + 12),$$
$$7776\sqrt{2}(u^{-5}K_4) = e^{-\frac{1}{2}u^2}(u^{12} - 12u^{10} - 183.6u^8 + 1432.8u^6 + 2889u^4 - 10368u^2 + 432), \text{ etc.}$$
$$6(u^{-2}K_1) = ue^{-\frac{1}{2}u^2}(u^2 - 3),$$

$$648(u^{-4}K_3) = ue^{-\frac{1}{2}u^2}(u^8 - 9u^6 - 59.4u^4 + 279u^2 + 54),$$

$$233280(u^{-6}K_5) = ue^{-\frac{1}{2}u^2}(u^{14} - 15u^{12} - 414u^{10} + 4494u^8 + 25152.4u^6 - 168723u^4 - 119340u^2 + 304560).$$

REFERENCES

1. "Théorie Analytique des Probabilités," Laplace, Livre I, Partie II, Chapt. I.
2, 3. Same, Livre II, Chapt. III, Art. 17.
4. "Elements of Physical Biology," Lotka.
5. "Application du Calcul des Probabilités à la Théorie du mouvement Brownien," Hostinsky, *Annales de L'Institut Henri Poincaré,* Vol. III, pp. 47 and 50.
6. "Théorie Analytique des Probabilités," Livre I, Partie II, Chapt. I, Art. 25.
7. "Sur Un Nouveau Développment En Série Des Fonctions," Hermite, *Comptes Rendus,* t LVIII, 1864.
8. "The Theory of Probability: Some Comments on Laplace's Théorie Analytique des Probabilités," Molina—*Bulletin American Mathematical Society,* 1930.
9. *Transactions of the American Mathematical Society,* Vol. 37, 1935, pp. 339–362.
10. "Commentarii Mathematici Helvetici," Vol. I, 1929, pp. 227–254.

The Relation Between Penetration and Decay in Creosoted Southern Pine Poles

By R. H. COLLEY and C. H. AMADON

Poor penetration of the non-durable sapwood is the most important factor in the decay of creosoted southern pine poles. Over 3000 such poles that had been treated with coal tar creosotes of varying types at thirteen creosoting plants in the South have been critically inspected to determine when and where decay started. The poles had been in line from five to twenty-six years under widely diverse climatic conditions in scattered localities east of the Mississippi River. Ninety-five per cent of the failures were poles in which the creosote had penetrated less than 1.8 inches and 60 percent of the sapwood thickness. No failures were found in poles that had been penetrated more than 2.1 inches and 75 per cent of the sapwood thickness. The current Bell System treating specifications require a penetration of 2.5 inches or 85 per cent of the sapwood thickness. The hazard of failure by decay during the ordinary service life of a line is reduced to a practical minimum in poles produced under these specifications.

Introduction

THE creosoted southern pine pole has been justly regarded as a long-lived unit of plant equipment. However, there have been enough instances of failure by internal decay during the first few years in line to focus attention on the poorer poles and to raise questions about the quality and probable length of service of creosoted poles in general. The data presented in this paper were obtained in the course of an investigation to determine how, when, and where decay starts in creosoted southern pine poles in line, and what proportion of the poles are decaying after different periods of service. The results of the study are of particular significance as a basis for engineering the treatment of poles in a satisfactory and economic manner.

General Conclusions About Decay in Poles in Line

In the sections of the lines that were inspected the incidence of decay was definitely correlated with the depth of penetration of the creosote and the per cent of sapwood penetrated.

When all of the 3102 inspected poles of all ages up to 26 years were taken together:

(*a*) There were 62 failures, all of which had penetration less than 2.1 inches and 75 per cent of the sapwood thickness; and the 62 failures were 2.00 per cent of the total poles inspected; and

(*b*) Of these failures 59, or 95.16 per cent, had penetration less than 1.8 inches and 60 per cent of the sapwood thickness.

All the field evidence indicates that the inspected poles, when the sapwood had been well penetrated with creosote, were practically immune to destruction by wood-destroying fungi for a long time. It is equally clear that if early failures in line and consequent replacement charges are to be reduced to a practical minimum it is essential to inspect the treated poles closely and to eliminate the poorly treated ones before they are shipped to the Telephone Companies.

The Inspected Lines

The selection of the lines to be inspected was based largely on geographical location without prior knowledge of the condition of the poles. An attempt was made to get as wide a distribution as possible. The lines were located in Florida; in the Piedmont section of North Carolina and South Carolina; in the Appalachian foothills and mountains of Tennessee, North Carolina and Virginia; in the Lake States region in Illinois, Wisconsin and Michigan; and in northern New Jersey. Sections of the chosen lines contained from 100 to 200 or more poles that had been set consecutively in one year. Old records, plus identifying marks placed on these poles when they were treated, made it possible to determine the supplier of the poles and the type of creosote used in treatment.

Method of Inspection

External decay is relatively rare in creosoted southern pine poles, so the inspection methods employed were directed particularly at finding internal decay. The latter occurs as a result of infection by water or air-borne spores that probably enter through checks or cracks

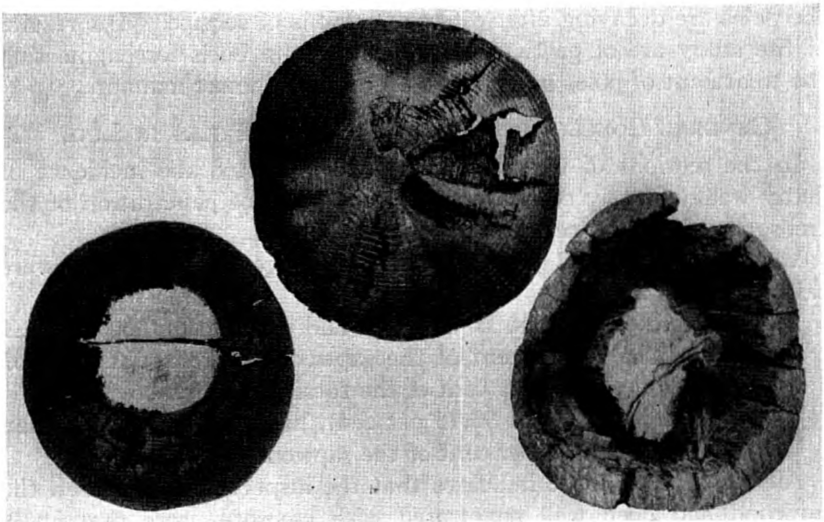

Fig. 1—Cross-sections of poles which failed because of decay that developed in the internal untreated sapwood.

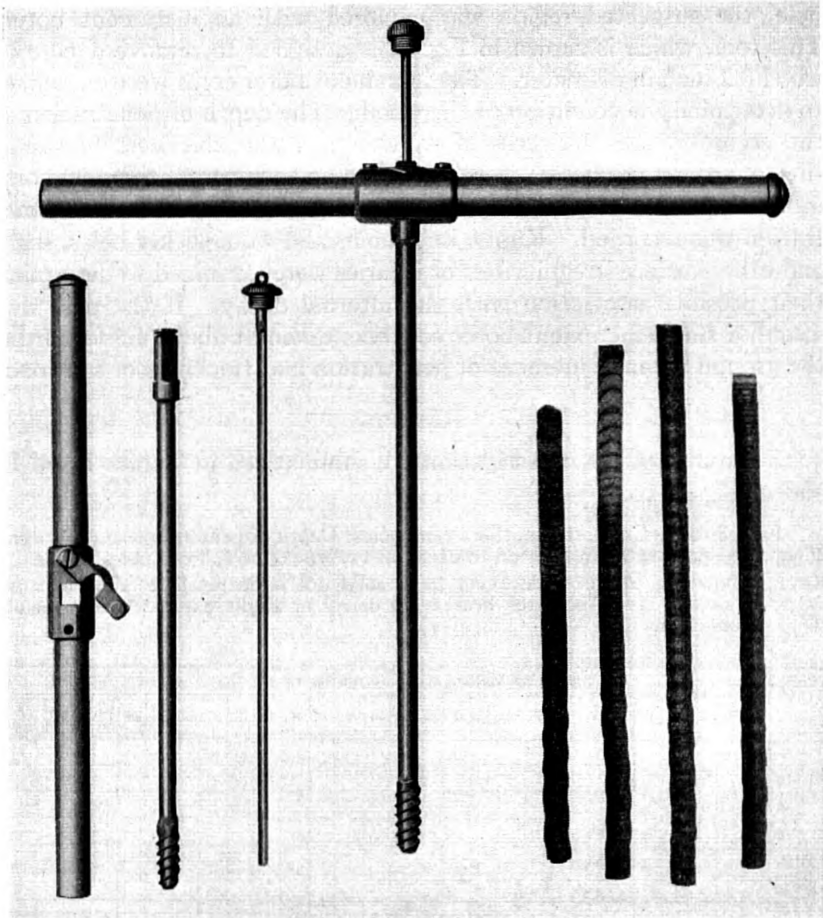

Fig. 2—The increment borer. The central figure shows the borer assembled. At the left are the extractor, the hollow boring tube, and the handle. Four increment borer cores are shown at the right.

and find favorable conditions for growth in untreated, non-durable sapwood lying beneath the treated outer layers of wood. Cross sections of poles showing internal decay of the untreated sapwood are shown in Fig 1.

A systematic inspection of 3102 poles in the selected lines was made by one of the authors, Mr. C. H. Amadon. The possible variances that might arise from different personal methods and interpretations were therefore minimized as far as practicable. Each pole was first tested by sounding with a hammer or a hatchet. When the hammer blows produced a dull lifeless tone, suggesting a hollow or decaying pole, the suspected region was explored with an increment borer. This tool, which is shown in Fig. 2, is designed to remove a core of wood 0.2 inch in diameter. The increment borer cores were examined to determine the condition of the wood. The depth of penetration of the creosote, the thickness of sapwood, or the thickness of sound shell overlying the decay, were measured on each core if there was any internal decay present. In each decaying pole the shallowest penetration was recorded. Knots, knot holes and woodpecker holes, scars and other surface irregularities or injuries were examined to determine their possible association with the internal decay. If the pole was sound, a single increment borer core was taken at about 4.5 feet from the ground for measurement of penetration and thickness of sapwood.

Results

The results of the investigation are summarized in Tables I and II and in Figs. 3 to 9.

Figs. 3 to 7—Creosoted southern pine poles: Depth of penetration and per cent of sapwood penetrated in relation to decay in twelve-pound full cell poles in line.
Key: Hollow dot indicates decaying pole; solid dot indicates pole failed because of decay; cross indicates heartwood decay or slight external decay below ground line.

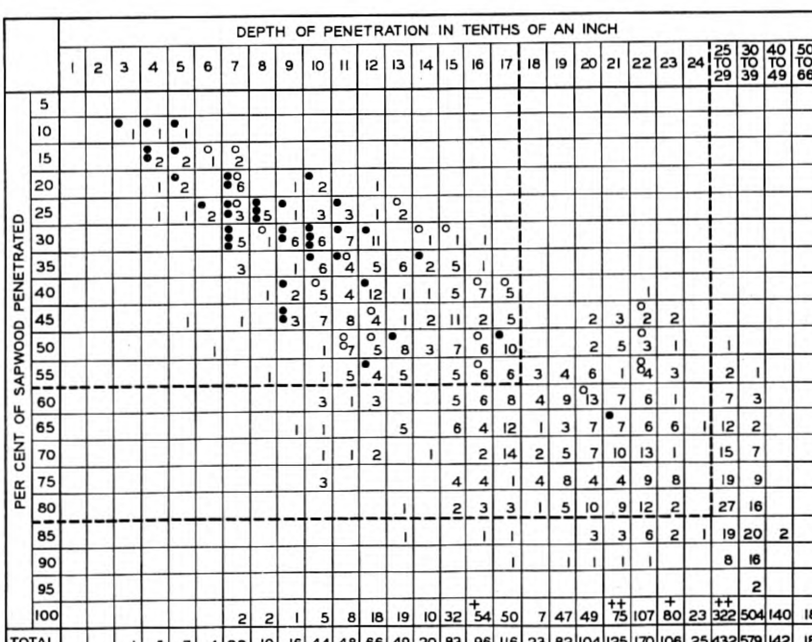

Fig. 3—2393 poles in miscellaneous lines less than ten years (average 7.7 years)

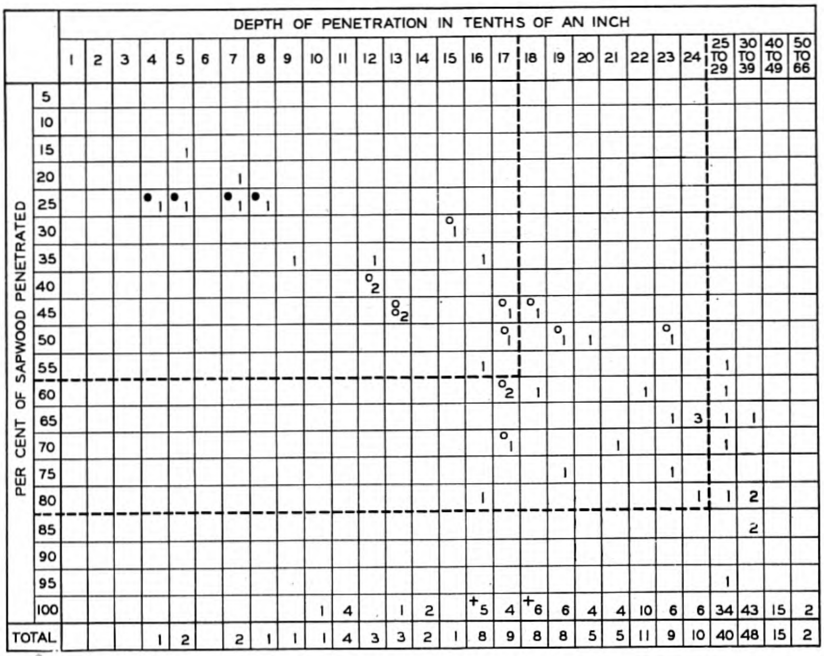

Fig. 4—199 poles in Lynchburg-Savannah line. Age 12 years.

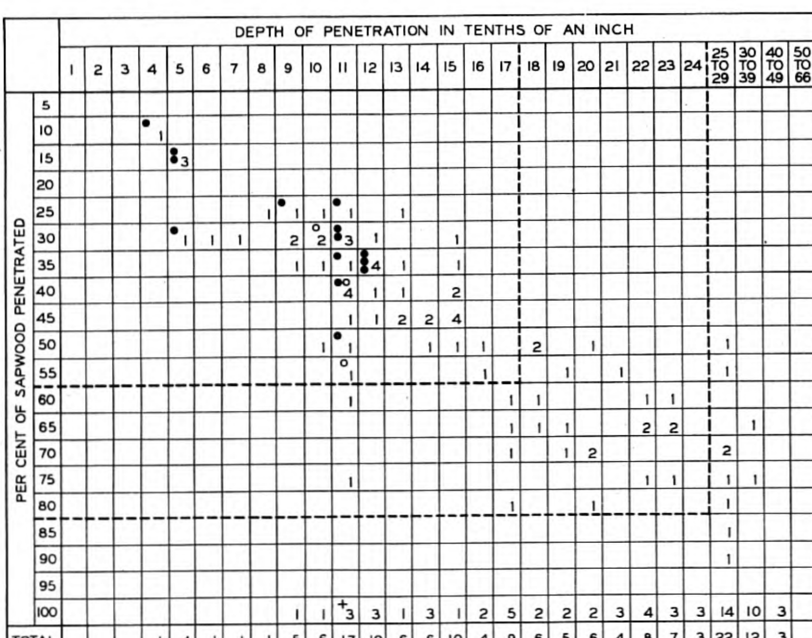

Fig. 5—157 poles in Petersburg-Denmark line. Age 15 years.

	DEPTH OF PENETRATION IN TENTHS OF AN INCH																											
PER CENT OF SAPWOOD PENETRATED	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25 TO 29	30 TO 39	40 TO 49	50 TO 66
5																												
10																												
15						o 1																						
20									1																			
25																												
30																												
35										1					1													
40																												
45												1																
50										1			1												1			
55																									1			
60															1										1			
65																												
70																1							2		1			
75																	• 1								1	1		
80																	1				1							
85																							1		1			
90																												
95																												
100									1	3	6	8	8	5	5	4	4	5	11	16	16	14	8	4	50	41	6	
TOTAL						1			2	5	6	9	9	5	7	5	6	5	11	16	17	14	11	4	56	42	6	

Fig. 6—237 poles in Jacksonville-Key West line. Age 19 years.

	DEPTH OF PENETRATION IN TENTHS OF AN INCH																											
PER CENT OF SAPWOOD PENETRATED	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25 TO 29	30 TO 39	40 TO 49	50 TO 66
5																												
10																												
15																												
20							o 1	o 1			• 2																	
25																												
30															• 1	• 1	• 2											
35									• 1	• 1		1	• 1			1	1	1	2			1	o 1					
40																				• 1		2		2				
45															1		1					1						
50																		o 1		o 1		o 1	o 3		2	1		
55																o 1				o 1		1		8 2	8 3	o 5		
60																	o 1						1		4			
65																				o 1						1		
70																										8 5		
75																										o 1		
80																				o 1					1	3	2	
85																										1	1	
90																										2	1	
95																												
100																			1	1					4	11	12	12
TOTAL							1	1	1	1	2	1	1		2	3	5	2	3	6		6	5	4	14	30	16	12

Fig. 7—116 poles in New York-Scranton line. Age 26 years.

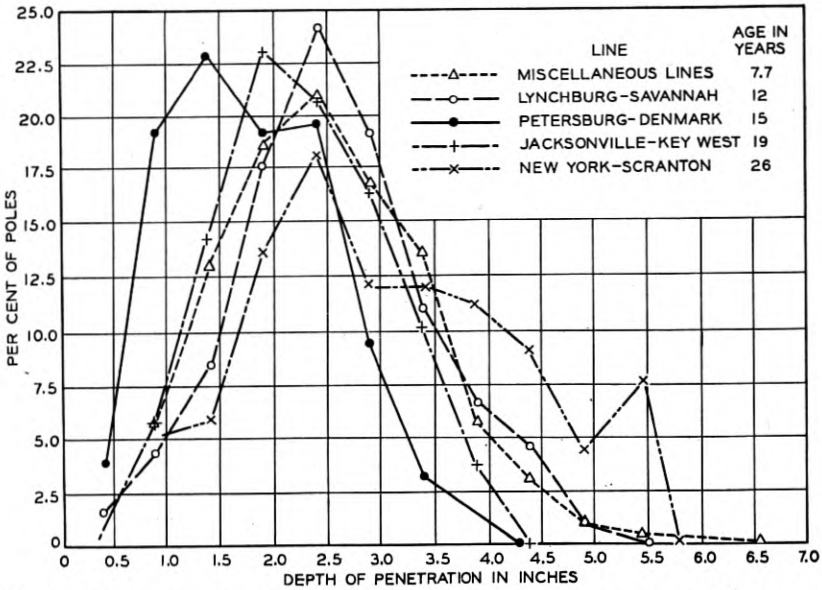

Fig. 8—Frequency curves for depth of penetration in twelve-pound full cell creosoted southern pine poles in line.

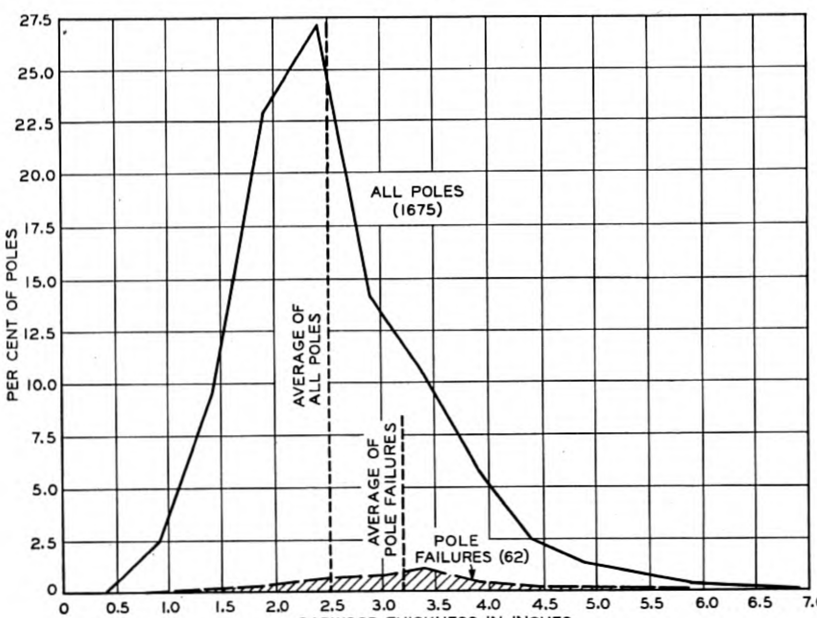

Fig. 9—Frequency curves for sapwood thickness in relation to failure from decay in twelve-pound full cell creosoted southern pine poles penetrated less than 2.5 inches.

TABLE I

CREOSOTED SOUTHERN PINE POLES: SUMMARY OF DATA ON LOCALITY, AGE, CREOSOTE, AND PENETRATION, IN RELATION TO INCIDENCE OF DECAY IN 12 POUND FULL CELL POLES IN LINE

	Years in Line	Type of Creosote (Average Residue Above 360° C.)	Number of Poles Inspected	Sound Poles				Poles with Internal Sapwood Decay			
				No.	Per Cent	Average Penetration: Inches	Average Penetration: Per Cent of Sapwood	No.	Per Cent	Average Penetration: Inches	Average Penetration: Per Cent of Sapwood
IN LINE 10 YEARS OR LESS (AVERAGE 7.7)											
FLORIDA											
West Palm Beach-Miami Line											
Group 1	8.5	11.14	85	85	100.0	2.4	84.7	—	—	—	—
" 2	8.5	12.79	27	27	100.0	2.2	83.5	—	—	—	—
" 3	8.5	16.57	137	127	92.8	2.1	84.3	10	7.2	1.1	44.5
" 4	8.5	31.12	20	19	95.0	2.4	90.5	1	5.0	0.7	33.0
	8.5		269	258	95.9	2.2	84.8	11	4.1	1.0	43.4
TENNESSEE											
Nashville-Montgomery Line											
Group 1	5.0	12.00	198	190	96.0	2.3	89.2	8	4.0	1.0	31.0
Nashville-Chattanooga Line											
Group 1	7.0	12.68	24	21	87.5	2.7	77.6	3	12.5	0.8	27.0
" 2	8.0	12.00	151	143	94.7	2.6	88.8	8	5.3	1.3	51.3
" 3	9.0	19.23	25	23	92.0	2.8	88.4	2	8.0	1.7	47.0
" 4	8.0	24.81	112	103	92.0	2.3	86.2	9	8.0	1.3	42.4
" 5	7.0	16.53	57	57	100.0	3.0	97.0	—	—	—	—
" 6	10.0	10.66	106	105	99.0	2.5	95.0	1	1.0	2.2	60.0
	8.3		475	452	95.2	2.7	90.1	23	4.8	1.3	44.6

TABLE I—*Continued*

	Years in Line	Type of Creosote (Average Residue Above 360° C.)	Number of Poles Inspected	Sound Poles				Poles with Internal Sapwood Decay			
				No.	Per Cent	Average Penetration		No.	Per Cent	Average Penetration	
						Inches	Per Cent of Sapwood			Inches	Per Cent of Sapwood
NORTH CAROLINA											
Greensboro-Selma Line											
Group 1	8.0	31.12	144	143	99.3	3.3	92.7	1	0.7	1.6	43.0
Asheville-Kings Mountain Line											
Group 1	9.0	31.12	51	51	100.0	2.1	73.9	—	—	—	—
" 2	7.0	34.07	23	22	95.7	2.9	94.3	1	4.3	0.7	20.0
	8.0		74	73	98.7	2.3	80.0	1	1.3	0.7	20.0
Asheville-Greenville Line											
Group 1	5.0	34.07	106	106	100.0	2.5	90.2	—	—	—	—
VIRGINIA											
Washington-Lynchburg Line											
Group 1	7.0	24.81	130	130	100.0	2.8	92.8	—	—	—	—
" 2	7.0	19.23	66	62	93.9	2.7	85.4	4	6.1	1.8	41.2
	7.0		196	192	98.0	2.7	90.5	4	2.0	1.8	41.2
ILLINOIS											
Crete-Watseka Line											
Group 1	9.0	12.56	199	188	94.5	2.4	79.1	11	5.5	0.9	33.2
Joliet-Ottawa Line											
Group 1	9.0	11.14	124	122	98.4	2.7	90.4	2	1.6	0.9	27.5
Wyanet-Peoria Line											
Group 1	8.0	11.14	101	101	100.0	2.5	89.5	—	—	—	—

TABLE I—*Continued*

	Years in Line	Type of Creosote (Average Residue Above 360° C.)	Number of Poles Inspected	Sound Poles				Poles with Internal Sapwood Decay			
				No.	Per Cent	Average Penetration		No.	Per Cent	Average Penetration	
						Inches	Per Cent of Sapwood			Inches	Per Cent of Sapwood
WISCONSIN											
Chicago-Minneapolis Line											
Group 1	7.0	12.91	67	66	98.5	2.6	86.1	1	1.5	0.9	42.7
" 2	7.0	12.56	32	32	100.0	2.6	84.4	—	—	—	—
" 3	9.0	31.12	65	59	90.8	2.1	79.9	6	9.2	1.0	29.4
" 4	9.0	10.66	110	109	99.1	2.3	90.6	1	0.9	1.1	34.4
" 5	9.0	16.58	23	23	100.0	2.2	95.5	—	—	—	—
	8.0		297	289	97.3	2.3	87.0	8	2.7	1.0	31.6
MICHIGAN											
Detroit-Ann Arbor Line											
Group 1	9.0	10.66	153	153	100.0	2.4	94.6	—	—	—	—
" 2	7.0	12.91	57	57	100.0	2.2	88.6	—	—	—	—
	8.4		210	210	100.0	2.3	92.9	—	—	—	—
	7.7		2393	2324	97.1	2.4	88.1	69	2.9	1.1	38.4
IN LINE 12 YEARS											
NORTH CAROLINA											
Lynchburg-Savannah Line											
Group 1	12.0	39.76	81	78	96.2	2.8	97.1	3	3.8	1.8	45.5
" 2	12.0	33.68	118	106	89.8	2.6	91.5	12	10.2	1.3	44.9
	12.0		199	184	92.5	2.7	93.8	15	7.5	1.4	45.0

TABLE I—*Continued*

	Years in Line	Type of Creosote (Average Residue Above 360° C.)	Number of Poles Inspected	Sound Poles				Poles with Internal Sapwood Decay			
				No.	Per Cent	Average Penetration		No.	Per Cent	Average Penetration	
						Inches	Per Cent of Sapwood			Inches	Per Cent of Sapwood
IN LINE 15 YEARS											
SOUTH CAROLINA Petersburg-Denmark Line Group 1	15.0	39.76	157	139	88.5	2.0	77.0	18	11.5	1.0	33.0
IN LINE 19 YEARS											
FLORIDA Jacksonville-Key West Line Group 1	19.0	31.59*	237	235	99.1	2.3	96.7	2	0.9	1.1	45.0
IN LINE 26 YEARS											
NEW JERSEY New York-Scranton Line Group 1	26.0	3.30†	116	88	75.8	3.5	77.8	28	24.2	2.0	59.8

* Residue above 315° C.
† Residue above 350° C.

TABLE II

SUMMARY OF PENETRATION AND FAILURE DATA FOR 12 POUND CREOSOTED SOUTHERN PINE POLES IN LINE

Age of Line (Years)	Number of Poles Inspected	Number and Per Cent of Poles Having Penetration Less Than								Per Cent of Poles Having 100% Sapwood Penetration	Failures * Because of Decay in Poles Having Penetration Less Than			
		1.8″ and 60%		2.5″ and 85%		3.0″ and 90%		3.5″ and 90%			1.8″ and 60%		2.5″ and 85%	
		No.	%	No.	%	No.	%	No.	%		No.	% of total poles	No.	% of total poles
7.7	2393	286	11.95	610	24.49	730	30.50	780	32.60	65.73	35	1.46	36	1.50
12.0	199	17	8.54	35	17.59	40	20.10	43	21.61	76.88	4	2.01	4	2.01
15.0	157	55	35.03	81	51.59	88	56.05	90	57.32	42.61	14	8.92	14	8.92
19.0	237	7	2.95	14	5.90	21	8.86	22	9.28	90.71	0	0.00	1	0.42
26.0	116	17	14.66	41	35.95	51	43.95	64	55.19	35.34	6	5.17	7	6.03
Total Poles	3102										59	1.90	62	2.00

* There were no failures in poles having penetration in excess of 2.1 inches and 75% of the sapwood.

Table I is a summary of the data on locality, age, creosote penetration, and incidence of decay for 3102 telephone poles in line in the eastern part of the United States. The pole groups are based on years in service and geographic settings.

Figures 3 to 7 are records of the penetration and the condition of each of the 3102 poles at the time they were inspected. Five age groups are represented, from 7.7 years to 26 years, respectively. These records are graphic illustrations of the fact that the failed poles and the decaying poles had poor penetration. Each solid dot represents a single pole failure, and each hollow dot represents a single infected pole. For example, in Fig. 3 for poles in line ten years or less, the figures and symbols in the 0.8 inch and 25 per cent block mean that there were five poles having penetration 0.8 inch and 25 per cent of the sapwood thickness; and of these five poles two were sound and three were so badly deteriorated that they had to be removed. Similarly in the 0.7 inch and 25 per cent block, two of the three poles were failures and one was infected.

The crosses in the 100 per cent line indicate poles with a little heartwood decay or with slight external decay at the ground line.

The broken lines in Figs. 3 to 7 delimit the individual data for poles having penetration less than 1.8 inches and 60 per cent of the sapwood thickness, and less than 2.5 inches and 85 per cent of the sapwood thickness, respectively. The latter is the minimum penetration called for in current American Telephone and Telegraph Company's specifications for creosoted southern pine poles.

The abscissa in each of the five figures has been warped beyond 2.4 inches in order to condense the charts to reasonable proportions for reproduction. Complete data on the range in penetration for the five age groups are shown in the form of frequency curves in Fig. 8. The 15-year sample from the Petersburg, Virginia–Denmark, South Carolina, line obviously has the poorest penetration of the five groups. It also had the highest per cent of pole failures, as shown by Table II.

Table II is a summary of the number and per cent of the 3102 poles in which the sapwood had been penetrated less than (*a*) 1.8 inches and 60 per cent, (*b*) 2.5 inches and 85 per cent, (*c*) 3 inches and 90 per cent, and (*d*) 3.5 inches and 90 per cent; and it also shows the per cent of poles with 100 per cent sapwood penetration, as well as the penetration in the poles that failed.

Table III contains typical analyses of creosote used in treating the poles.

TABLE III

TYPICAL ANALYSES OF CREOSOTES

Used during the years indicated in treatments of poles represented in Tables 1 to 3

Distillation	1906	1916	1918	1920	1925	1925	1927	1927	1929	1929
0° to 205° C.	12.0	1.86	2.41	3.88	2.98	3.39	2.53	3.47	3.24	1.55
0° to 235° C.	58.0	24.18	4.24	14.33	12.96	24.50	11.75	21.09	14.18	8.14
0° to 315° C.	91.6	68.41	31.86	44.09	43.64	72.08	46.93	66.50	45.95	55.30
0° to 360° C.	96.7*	Not recorded	44.60	59.80	64.77	88.62	67.40	86.85	64.52	82.15
Residue above 360° C.	3.3†	Not recorded	55.10	39.76	34.81	11.14	32.28	12.75	35.10	17.56
Water	Not recorded	Not recorded	1.10	2.7	1.0	1.7	1.9	1.1	1.0	0.7
Tar acids	" "	7.57	1.77	3.0	3.3	7.6	3.1	6.6	2.9	4.3
Sulphonation residue	" "	1.00	2.40	0.5	1.5	2.8	1.8	2.8	1.7	1.0
Benzol insoluble	" "	0.14	1.40	1.9	0.24	0.51	0.42	0.31	0.48	0.29
Specific gravity	1.034	1.052	1.105	1.108	1.079	1.046	1.079	1.045	1.085	1.068

* Distilling to 350° C.
† Residue above 350° C.

Discussion and Interpretation of Results

In reading the following discussion and interpretation of results it should be remembered:

(1) That the poles came from representative areas of the pine forest and from representative treating plants of the South;
(2) That all of the poles inspected were treated in accordance with the process specifications covering a full-cell treatment and calling for a net retention of at least 12 pounds of creosote per cubic foot of wood in the charge; but related studies indicate that the retention in individual poles probably varied from less than 2 to more than 20 pounds per cubic foot;
(3) That the poles were accepted if the treating process and the quantity of oil used conformed to the specifications;
(4) That there was no required inspection at the time the poles were creosoted to determine the results of treatment in terms of penetration and distribution of the creosote in the poles; and
(5) That every pole inspected in the line was the original pole placed in the respective year designated.

The evidence from the field data showed poor penetration to be by far the most important cause of fungous infection and failure by decay. As a matter of fact, the effect, if any, of geographical location or of the type of creosote used was completely masked by the penetration factor.

On account of the wide geographical distribution of lines it was expected that the effect of any definite climatic and meteorological influences on the occurrence of decay would be apparent. It might be expected, for example, that poles set along the warm, moist Florida east coast would be more vulnerable than poles located in the drier north temperate regions. However, the data in Table I for poles in line 10 years or less are not conclusive as to the effect that geographical location may have on the incidence of decay.

The creosotes used conformed as a whole, or in their most important characteristics, to the specifications in effect at the time the poles were treated; but there was a fairly wide divergence in gross chemical characteristics of the oils because of differences in the raw coal tar and in the methods of distilling the tar. Table I includes data on the kinds of creosote used, indicated by the fraction not distilling above 350° C. or 360° C. The data are taken to mean that as far as internal sapwood decay is concerned, the type of coal tar creosote, provided it is a true distillate of coal tar and that it conforms to the specifications, is less important than the thoroughness with which it is distributed throughout the non-durable sapwood of the poles.

The overall summary in Table I for the 2393 poles in line 10 years or less shows average values for penetration in sound poles and in poles with internal sapwood decay. The summary suggests the existence of the very important relationship between penetration and decay that is definitely shown in detail in Fig. 3. All of the internally decaying poles shown in this figure had penetration less than 2.3 inches and 70 per cent of the sapwood thickness. Furthermore, all except six of these decaying poles and all except one of the poles that failed had penetration less than 1.8 inches and 60 per cent of the sapwood thickness. The group defined by the latter figures may be considered as the "risk group," i.e., poles which by reason of poor penetration may become infected with wood-destroying fungi within 10 years. The 286 poles in this group in Fig. 3 were 11.9 per cent of the 2393 inspected poles that had been in line 10 years or less. The poles making up this 11.9 per cent were possible early failures, but the inspection revealed that only 63 of them, or 22.2 per cent, had actually begun to decay. Of these 63 poles with internal sapwood decay only 35, or 55.5 per cent, failed in service. The distinction between infection and failure is important. In terms of the whole 2393 poles 2.6 per cent were infected with internal decay and only 1.4 per cent failed.

The external decay at the ground line indicated in Figs. 3, 4, and 5 apparently did not exceed one half inch in depth. It was typical of the superficial rot usually found after the ground line of a pole has been raised following a few years of service. During these years the creosote in the exposed outer layers of the wood is depleted, and the favorable moisture conditions at the new ground line facilitate fungous infection of the poorly protected wood.

Another group of poles somewhat above the risk poles in quality may be defined as having penetration more than 1.8 inches and 60 per cent but less than 2.5 inches and 85 per cent of the sapwood thickness. Some of these poles are subject to infection prior to the fifteenth year. The data in Figs. 3, 4, and 5 show that decay developed in 11, or 2.98 per cent, of the poles in this group, and that only 1, or 0.27 per cent, failed in service.

The data in Figs. 6 and 7 and in Table II, show in a striking way the stability of the line when the per cent of poles having penetration greater than 2.5 inches or 85 per cent of the sapwood thickness is relatively large. Not a single pole in this group in the sample from the 19-year old line (Fig. 6) showed any indication of internal sapwood decay; and in this group in the sample from the 26-year old line (Fig. 7) decay developed in only 6, or 8.1 per cent, of the poles. Moreover, none of these decaying poles at the 26-year age had deteriorated far enough to require removal.

There is no evidence in the data warranting discrimination against thin sapwood poles because of possible extra decay hazard. The average sapwood thickness for all the inspected poles with penetration less than 2.5 inches was 2.52 inches, and the average sapwood thickness for the poles that failed was 3.19 inches. Only 34 per cent, on the average, of the sapwood thickness was penetrated in the poles that failed. When the distribution of the sapwood thicknesses of all the poles with penetration less than 2.5 inches, and the distribution of the sapwood thicknesses of the poles that failed, are plotted as in Fig. 9 there is a clear indication that serious interior decay is more likely to occur in the poorly treated thicker sapwood poles than in the thinner sapwood poles.

The results of the study of actual conditions in line provide a means for evaluating the practical effect of the current specifications for creosoting southern pine poles. The purpose of the specifications is to keep the number of well penetrated poles as high as commercial production will permit, and to eliminate practically all of the poorly penetrated poles at the source of supply. The hazard of failure by decay in poles produced under the specifications appears to be reduced to an economic minimum.

Tandem Operation in the Bell System

By F. M. BRONSON

Tandem operation is becoming of increasing importance in the Bell System. The operating and service features of the different types, and the conditions under which each type is used, are outlined. Charts are included showing, schematically, typical trunking arrangements in the various systems. The increasing use of tandem operation on traffic handled at toll boards is discussed.

THERE are 14,000,000 telephones in the Bell System, served from 6,800 central offices. Means must be provided to permit any one of these telephones to be connected to any of the others. Therefore, facilities must be provided for interconnecting all of the 6,800 central offices. Obviously it would be impracticable to provide direct circuits from each central office to all of the others; this would require $[N \times (N-1)/2]$, or more than 23 million, groups of two-way circuits, most of which would carry little or no traffic. To keep the number of circuit groups within reasonable limits and to obtain reasonable circuit efficiency, direct circuits are provided only between offices having a sufficient community of interest to justify them. Connections between the others are obtained, as required, through switching operations performed at one or more intermediate points.

The 14,000,000 telephones referred to originate 75,000,000 daily calls, the great bulk of which, of course, are local calls dialed direct by customers or handled at local manual switchboards. There are, however, about 1,500,000 short haul station-to-station toll calls which, because of the close community of interest between the cities involved, are also handled by local operators by methods essentially similar to those used on local calls. Obviously, these are largely concentrated in sections of the country having greatest population densities, such as in the New York City, Boston, and San Francisco metropolitan areas.

To facilitate the interconnection of central offices in areas having large volumes of local and short haul toll traffic, switching arrangements designed particularly for this purpose are frequently provided. These are known as tandem arrangements and, for the purpose of this paper, may be more specifically defined as facilities for the intermediate switching of traffic between central offices other than those facilities involving the use of outward, inward and through toll switchboards and of local switchboards which interconnect trunks of the ringdown signaling type. More recently, tandem arrangements have been employed in toll offices in connection with the toll lines used for long distance calls, all of which, with a few exceptions, are handled over ringdown signaling circuits.

It is the purpose of this article to describe the operating and service features of the different types of tandem arrangements employed in the Bell System and to indicate the extent to which they are used. Tandem equipment having trunks incoming from other tandem equipment, is a subtandem; some equipments operate both as tandems and subtandems.

A consideration of tandem operation may logically begin with the switching requirements of a single local exchange area. It follows from our definition that a tandem connection involves the cooperation of at least three different offices for its completion. So long as all switching operations are confined within a single office there is, therefore, no occasion for tandem connections. Neither is there any occasion for them when the number and relative locations of the various offices, call them A, B, and C, etc., within the exchange area are such that it is still practicable to handle interoffice calls over direct trunks. With increase in area and number of offices, a point is obviously reached, however, where it is no longer practicable to go, for example, from office A to office U directly, U being located in a remote division of the exchange, although it will still be feasible to go directly between offices A, B, and C, and between offices U, V, and W. Given such an extended exchange area, it will be found to contain some intermediate geographical position at which a tandem office can be profitably located with trunks extending to all the local offices and with switching facilities such that calls from A, B, and C to U, V, and W will be routed to it and will be completed by the interconnection of trunks between the tandem office and these various outlying offices. Such a tandem office would of course be a local tandem office.

Passing from the problem presented by the handling of local traffic within an exchange area, and reserving discussion for later paragraphs, it may be stated as evident that numerous other situations arise within the telephone plant for one or another type of tandem operation. These it is convenient to classify as follows:

I. Manual tandems, at which connections are made manually by plug and jack operation. These include—
   a) Manual straightforward tandems, for completing connections from manual trunks to manual trunks.
   b) Call indicator tandems, for completing connections from dial trunks to manual trunks.

c) Toll office tandems, for completing connections from manual trunks to ringdown toll circuits.

d) Straightforward toll line tandems, for completing connections from straightforward toll circuits to toll switching trunks.

e) Toll switching trunk tandems, for completing connections from manual trunks to manual toll switching trunks.

II. Dial tandems, at which the connections are made wholly by means of switch mechanisms controlled either at the tandem office or at a distant office. These include—

a) Operator tandems for completing connections from manual trunks to dial (or manual) trunks.

b) Full selector tandems, for completing connections from dial trunks to dial (or manual) trunks.

c) Trunk concentrating tandems, for automatically concentrating or collecting traffic which is to be completed over either manual or dial trunks.

Manual trunks include all types of trunks over which the order is passed orally by an operator or by a machine as in the case of call announcer trunks. Dial trunks include those over which the order is transmitted in the form of electrical impulses.

Traffic normally routed over direct straightforward trunks frequently is handled through a tandem system during the night and other hours of light traffic. This is sometimes an economical arrangement since it makes it unnecessary to provide incoming "B" operators during such hours except on positions handling the tandem completing trunks. The speed of connection at such times is substantially as fast as over direct trunks because of the number of "B" positions which it would be necessary otherwise to cover with a small number of operators. Also, a tandem system may be used as an emergency routing during periods when direct trunk groups are out of service because of cable or other failure. Frequently tandem systems are used as overflow routings for traffic normally handled over small direct trunk groups.

Table I indicates the number of the different types of tandem systems in use in the Bell System. In addition to systems of the types shown, tandem operation is obtained through the use of regular local central office equipment in a number of cities where the volume of eligible traffic is very small.

Manual Tandems

Manual Straightforward Tandems

In these tandem systems the incoming and outgoing trunks are of the straightforward type. The incoming trunks are terminated on

TABLE I

TANDEM SYSTEMS AT BELL SYSTEM TOLL CENTERS
(These Constitute Most of the Tandem Systems in Use)

Type of Tandem	No. in Use
Manual Straightforward Tandems	13
Call Indicator Tandems	5
Toll Office Tandems	27
Straightforward Toll Line Tandems	3
Toll Switching Trunk Tandems	2
Panel Sender Tandems—Total	6
—With Operators' Positions	5
Panel Office Selector Tandems	34
Step-By-Step Tandems—Total	61*
—With "B" Board Operators' Positions	5
—With Intermediate Dialing or Key Pulsing Operation	4
Trunk Concentrating Tandems—No. of Cities	9†

* 26 of these have trunks incoming from other tandems.
† With 148 groups of trunk concentrating switches.

single-ended cords on the tandem positions, and, in all but the smallest boards, the trunks are connected automatically to the tandem operator in rotating sequence, a flashing supervisory lamp associated with the trunk indicating to the tandem operator the trunk to which she is connected. When the tandem operator is in this fashion connected to a trunk which an originating operator has selected and over which she wishes to have a call completed, both operators receive momentary tone signals indicating this fact and the originating operator passes the name of the central office desired. The tandem completing trunks appear in the outgoing trunk multiple at the tandem board, usually with idle trunk indicating lamps, and the tandem operator extends the connection from her position by simply plugging the incoming trunk into an idle trunk to the office desired. Plugging into the trunk automatically signals the "B" operator at the called office. The tandem operator's telephone set may be released from the incoming trunk, either by means of a release key provided at her position for that purpose or by the act of plugging into the outgoing trunk. The release key enables the tandem operator to receive a call while establishing the connection on a previous call.

In order to distribute the load and assure the minimum of delay at the tandem board, the various groups of incoming trunks are subgrouped and the subgroups terminated on different tandem positions. In addition, on the larger trunk groups, arrangements are provided so that if the operator upon whose position a subgroup is located is busy, when one or more operators upon whose positions other subgroups terminate are idle, this is indicated to the originating operator in order that she may select a trunk to an idle tandem operator. The number of trunks handled by the various tandem operators can be varied from hour to hour by means of keys located between each group of 10 cords.

Release of the tandem trunk by the originating operator gives a disconnect signal, simultaneously at both the tandem and "B" boards, and the tandem and "B" operators then take down the connection. The tandem trunk may be reselected for a new call even before the tandem operator has taken down the cord on the previous call.

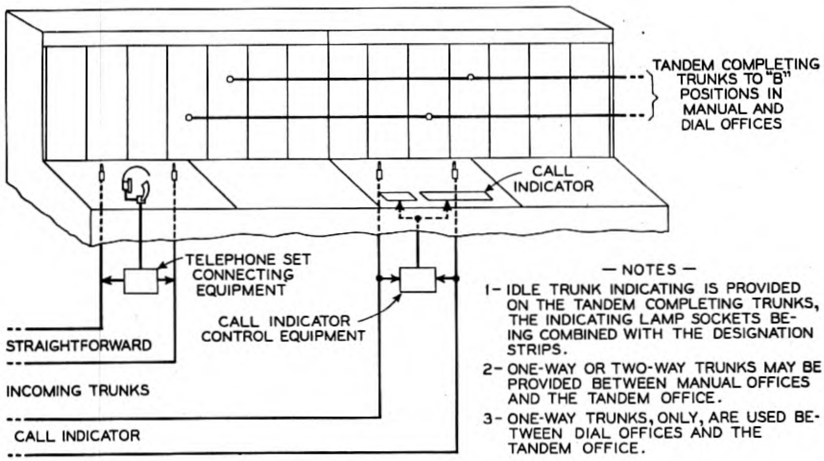

Fig. 1—Manual automatic listening straightforward and call indicator tandem arrangements.

Figure 1 shows, schematically, the circuit and equipment arrangements in the manual straightforward tandem system. Figure 2 is a photograph of the manual straightforward tandem switchboard which serves Detroit, Michigan, and surrounding communities, while Fig. 3 indicates the scope of the Detroit Tandem System.

Where the volume of traffic to be switched is too small to warrant a tandem switchboard, tandem operation frequently is obtained by routing the traffic over straightforward trunks terminating on manual "B" positions at a convenient local office and providing trunks from these positions to other central offices. The operators at these manual "B" positions, therefore, combine the functions of tandem and "B" operators.

Call Indicator Tandems

When manual offices are converted to dial, it is necessary to provide means for completing calls from dial subscribers to all offices, including manual offices, within their local dialing area. The usual arrangement with respect to manual offices to which direct trunks can be justified, is to display the number dialed by the customer on a call indicator located on a "B" position in the manual office, the operator at this position completing the connection to the called subscriber's line. Where direct trunks cannot be justified, the call indicator may be located on a manual straightforward tandem position, thus forming a "call indicator tandem." The operation at the tandem board is dissimilar in the

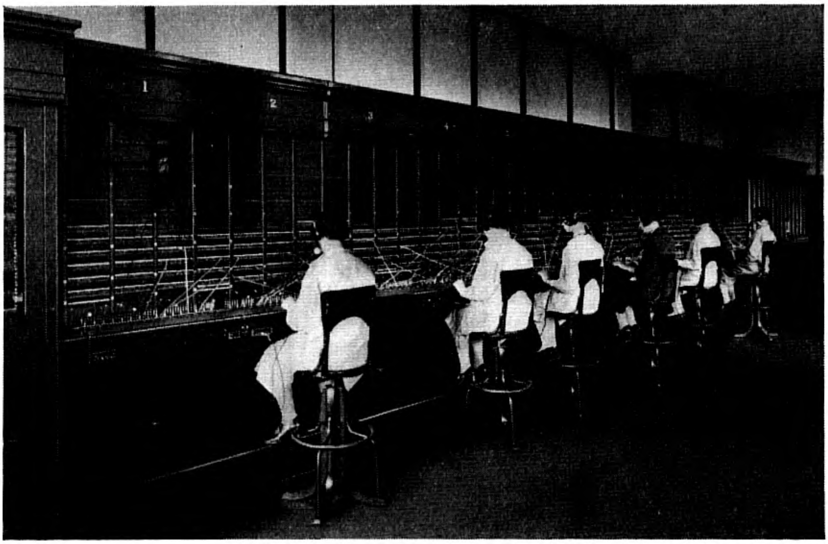

Fig. 2—Detroit manual straightforward tandem switchboard.

following respects to the manual straightforward tandem operation described above: *a*) the display of the central office code and digits of the called number, as dialed by the subscriber, take the place of the name of the central office desired passed orally by an operator; *b*) the tandem operator passes the order orally to the called office; *c*) the tandem operator is not connected automatically to the incoming trunk but, upon receiving a signal on a trunk indicating an incoming call, depresses a display key which connects her telephone circuit to the trunk and causes the number which has been dialed to be displayed on the call indicator.

The call indicator tandem arrangement is shown schematically in Fig. 1, and a photograph of a typical call indicator tandem position is shown in Fig. 4.

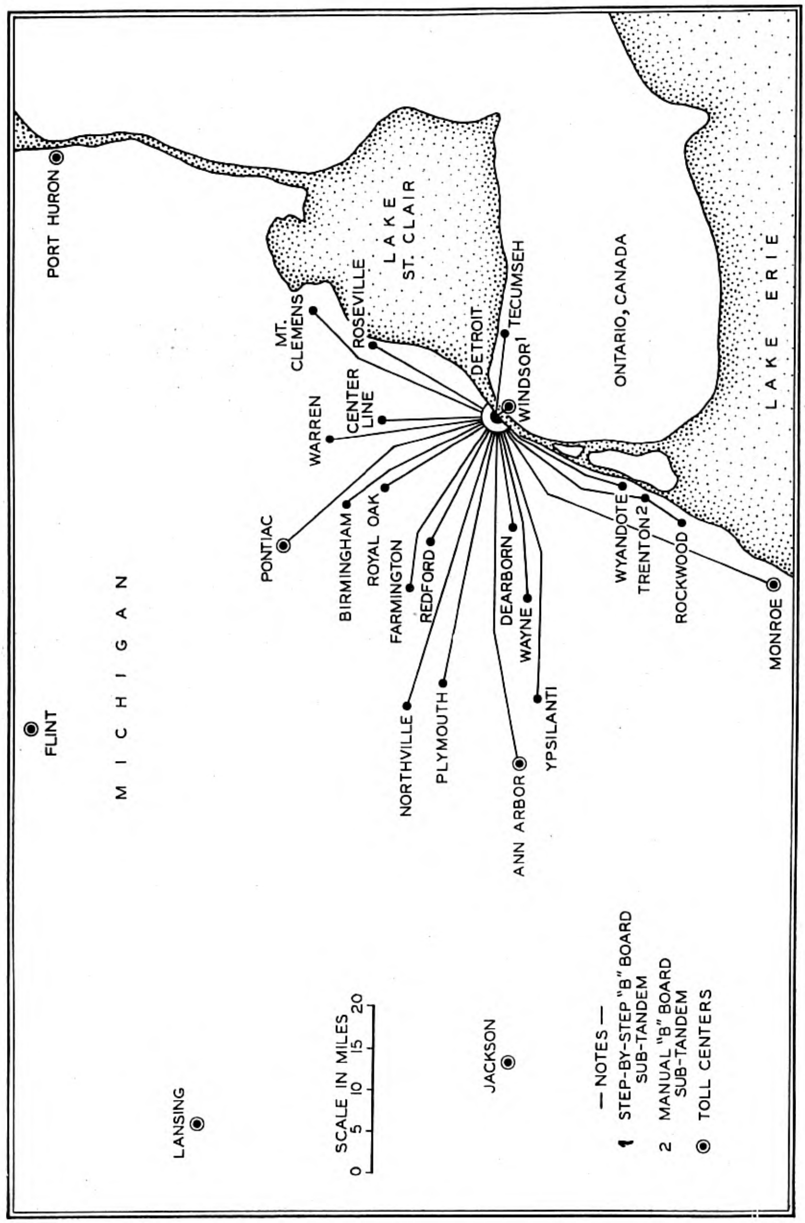

Fig. 3—Detroit tandem area.

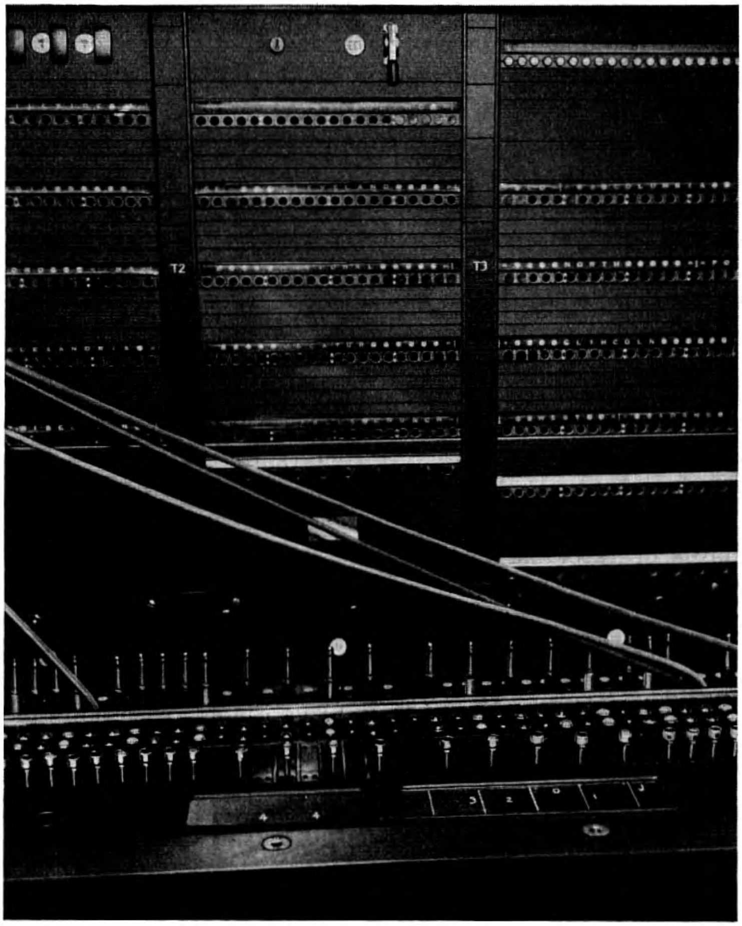

Fig. 4—Call indicator tandem position.

Toll Office Tandems

With the introduction of the combined line and recording method of toll board operation, it was necessary to provide means for giving each outward toll operator access to all of the toll circuits (instead of circuits to certain points only, as required by the former single ticket toll operating method) and in large toll offices where the transmission and switchboard multiple limitations prevent multipling all of the toll lines at the outward positions, toll office tandems provide such a means. Toll office tandems also are used as a means for making toll board circuits available to local operators, at both manual and dial system "A" boards in multi-operating center cities,[1] to permit the "A" board handling of station-to-station toll calls over such circuits.

Except that they are arranged for establishing connections to ringdown toll lines only and have certain additional features required thereby, the operating and service features of these toll office tandems are similar to those of the manual straightforward tandem previously described. The trunks from the toll board to the tandem positions are usually of the idle-position indicating, idle-trunk indicating, type. Trunks from "A" boards have the idle-trunk indicating feature only.

The ringdown toll lines are multipled in the tandem positions and are equipped with idle-indicating lamps. When a tandem trunk is connected to a toll line, a ring of two seconds' duration is sent automatically, but a ring-release key is provided on each tandem position to permit connection to be made to a toll line without ringing, as is necessary under certain operating conditions. Subsequent rings on the toll line may be made by the originating operator.

When ringdown toll lines appear directly in the multiple before the originating operator and she finds all of the circuits in a particular group momentarily busy, she ascertains when a circuit in the group becomes idle by observing the busy signals associated with the toll line jacks. When connections to the circuits are obtained through toll office tandem equipment, the equivalent of this arrangement for ascertaining when a circuit becomes available may be obtained by providing overflow circuits connected to jacks associated with the different circuit groups in the toll line multiple at the tandem positions. The tandem operator connects the incoming tandem trunk to the overflow circuit and the first toll line in the group to become idle causes a signal indicating this to be given over the trunk to the originating operator. Should the overflow circuit also be busy, the tandem operator connects the incoming trunk to one of a common group of circuits arranged to transmit a signal indicating this condition.

Figure 5 is a photograph of one of the two toll office tandem switchboards in the Long Lines Office in New York City.

In cities not requiring the use of tandem equipment in order to give toll board operators access to the toll lines, a somewhat different toll tandem arrangement is provided for giving "A" operators access to the toll board circuits. Under these conditions the tandem operators' positions are located in line with the toll positions, and the incoming trunks may or may not have the automatic listening feature described in connection with manual straightforward tandems. If not, the

[1] A multi-operating center city is one sufficiently large to require the local operating to be distributed between two or more buildings.

tandem operator connects herself to the trunk and gives the order tone to the originating operator by operating a key associated with the trunk. The ring-release and overflow features are not provided.

Fig. 5—Toll office tandem, No. 1—Long Lines office, New York City.

In a few cases where the volume of traffic to be handled by "A" operators over toll board circuits is very small, a form of toll office tandem operation is obtained, without the use of tandem positions of the types described above, by terminating automatic signaling trunks from the "A" boards on jacks and lamps at the outward toll board positions and having the connections between the trunks and the toll circuits made by means of the regular pairs of cords. The answering jacks are multipled at a number of the toll positions and none of the features normally associated with toll office tandem equipment are provided. The toll board operator answers on the trunk verbally and after receiving from the "A" operator the name of the place desired, establishes a connection to the toll line and rings the distant office. From this point on, the "A" operator handles the call in essentially the same manner as when regular toll tandem equipment is used.

Straightforward Toll Line Tandems

While ringdown operation is the general rule at toll boards, there are a few toll board circuit groups which are operated on a straightforward basis, notably the terminal circuits between New York and Philadelphia. These straightforward toll lines are arranged for one-way operation and terminate on single-ended cords at automatic-listening tandem positions in the Long Lines offices in New York and Philadelphia. Regular toll switching trunks are multipled at the tandem positions for reaching the various local offices on incoming calls; idle trunks are found by the tandem operators by tip test, no visual busy signals being provided. The order for connection to the called station is given to the "B" operator at the local office by the originating operator. Ringing on the toll switching trunks is controlled by equipment in the toll line circuits. Switchhook supervision from both called and calling station is received by the originating operator as in local tandem systems. The disconnect signals at the tandem board are controlled by the originating operator. The trunks from the tandem board to the local offices are of the straightforward type. In the case of dial central offices, these trunks terminate on selectors, but an operator at an associated "B" position sets up on a key set the connection to the called subscriber's line.

Toll Switching Trunk Tandems

In a number of the larger toll offices, equipment limitations prevent the multipling of toll switching trunks to all of the local offices at all of the outward positions. Each operator has direct access in the multiple at her position to trunks to the offices from which she normally receives calls. Occasionally, however, it is necessary for operators to reach subscribers connected to other local offices, and to permit this a toll switching trunk tandem is provided. The trunks incoming to the tandem positions are of the cord-ended, key-listening, straightforward type. The originating operator passes to the tandem operator the name of the local office desired.

Dial Tandems

Dial tandems receive calls from operators and, in panel areas, from subscribers also, and are of several types.

Where there is a considerable concentration of short-haul toll traffic within an area served by two or more dial tandem systems which individually serve limited areas, it is sometimes desirable to interconnect such systems and to route calls through one or more of these tandem centers, as required. An example of this is shown in Fig. 9.

Panel Tandems

Panel tandem systems employ panel selectors and are of two general types; one known as the panel sender tandem, and the other as the office selector tandem. Both are designed for use in cities employing panel type central offices, and therefore these tandems are used to serve only the larger cities and their environs.

Panel sender tandems use senders associated with the tandem equipment to control the electrical operation of the system. The completing trunks may be of the dial, call indicator, or call announcer types. The sender is a device which receives the impulses from the incoming trunk or tandem operators' positions, determines the routing for the call, and sets up the necessary electrical conditions for operating the panel type equipment in the tandem office and the associated equipment in the completing trunks.

These systems ordinarily include both operator tandem and full selector tandem equipment, the latter for traffic routed directly through the selectors from dial system "A" boards equipped with key sets. Local calls dialed directly by subscribers are also routed through full selector tandems when the volume of such calls is so small as not to warrant direct trunks between the originating and terminating offices.

While the equipment arrangements of the full selector tandem are such as to permit operators at dialing type dial system "A" boards (as distinguished from boards equipped with key sets) to dial numbers at the distant offices direct, it usually is more economical to route calls from these boards through the operator tandem positions. Manual "A" boards in panel areas usually are not equipped with either dials or key sets.

All of the trunks to a panel sender tandem office terminate on selectors, but on incoming calls, other than those from operators at switchboard positions equipped with key sets, or dialed by subscribers, the trunk automatically is connected to an idle tandem operator who sets up the called number on a key set provided with a row of keys for each letter and digit in the number. The tandem operator's position then is released automatically from the connection. On connections completed through either the operator tandem or the full selector tandem, disconnection, both at the tandem office and at the called office, is under control of the originating operator, or calling subscriber on calls dialed direct.

The use of call announcer trunks, which at present are designed for panel tandem systems only, permits tandem connections to be completed by key set or dialing operation to small outlying manual offices where, for equipment or other reasons, call indicators are not provided. At the called office call announcer trunks are similar to straightforward trunks, but are so arranged that the connection of the "B" operator's telephone circuit to the trunk causes the call announcer equipment at the tandem office to reproduce over the trunk by means of a talking film the digits of the number previously set up by the originating or tandem operator.

Trunks may also be provided from panel sender tandem offices to local central offices having step-by-step equipment, to manual tandems, and to dial tandems of either the panel or step-by-step type.

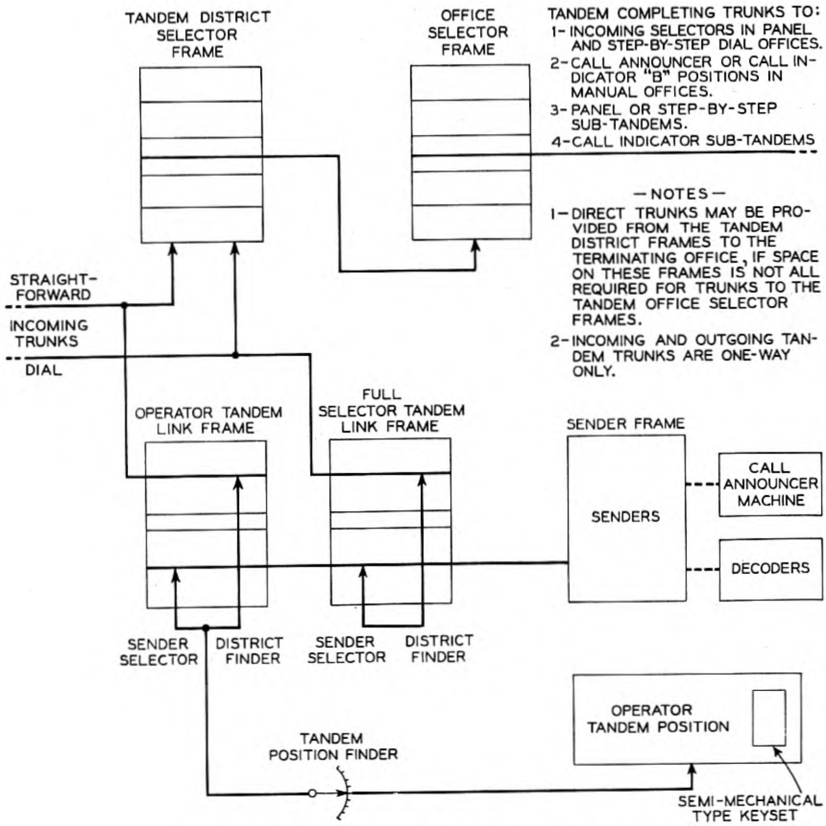

Fig. 6—Panel sender tandem system.

Figure 6 shows schematically the latest type of panel sender tandem system. Figure 7 is a photograph of the operators' positions at Suburban Tandem, one of the panel tandem systems in New York City, while Fig. 8 shows the area served by this system.

An early type of panel sender tandem system known as the semi-mechanical system is in use in New York City. It consists both of an operator tandem and a full selector tandem. The former has operators' positions equipped with key sets having a row of keys for each of the last four digits and the party line letters in subscribers' numbers, and, in addition, there are coordinate routing keys for selecting trunk groups to the various offices either inside or outside the New York City numbering plan area. In the more recent type of panel sender tandem system, trunk groups to offices outside the numbering plan area

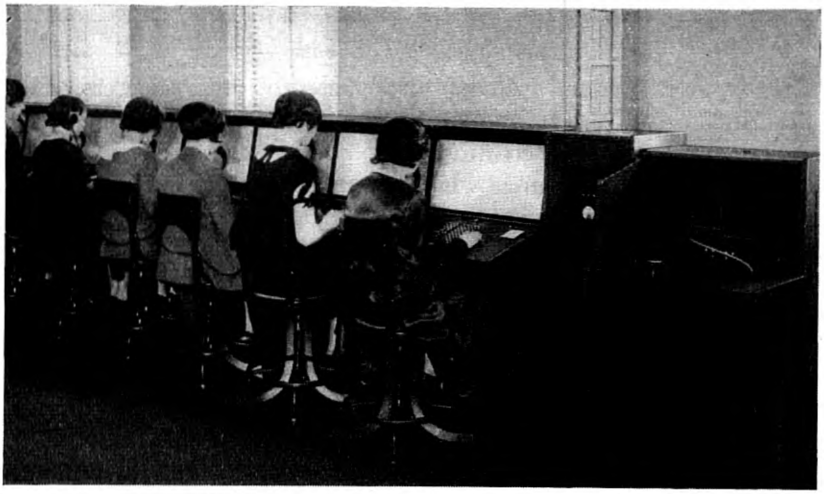

Fig. 7—Panel sender tandem operators' positions—Suburban Tandem—New York City.

are given 3-digit codes which do not conflict with office codes within the numbering plan area, and trunk selection is made by setting up these codes on the key set. The trunks to the operators' positions of the semi-mechanical tandem are on a straightforward key-listening basis and do not have the call distributing feature.

Frequently, it is found desirable in cities served by panel central office equipment to consolidate the traffic, originating in one central office building and destined to a number of central offices in one or more distant buildings, over a single group of trunks to a distant office which serves as a distributing point. This is done by placing panel office selectors at the distant point. Such equipment constitutes an office selector tandem and is arranged to complete connections incoming and outgoing over panel dial trunks only. In all cases the connections are set up by dialing on the part of a subscriber, or by dial or key set operation on the part of an operator. The operation of the office selector tandem equipment, in establishing a connection, differs from that of the full selector sender tandem in that it is controlled by the senders associated with the local central office equipment at the originating office, or by senders associated with the panel tandem equipment in the case of traffic first routed through the latter.

While panel sender tandem systems ordinarily include both operator tandem and full selector tandem equipment, Knickerbocker Tandem in New York City is entirely of the full selector type, being designed for traffic incoming over dial trunks only.

Step-by-Step Tandems

Step-by-step tandem systems employ step-by-step selectors, and, ordinarily, are of the full selector type without tandem operators' positions. Under these conditions, no senders are required at the tandem office and the pulses which select the central office to which connection is to be made are received over the tandem trunk. Connections may be made through a step-by-step tandem system to both dial and manual offices; to the latter by the use of straightforward, call indicator, or automatic-signaling ringdown tandem completing trunks. Call announcer trunks are not used. Release of the tandem trunk at the originating office automatically releases the tandem selectors and at the same time releases the selectors, or (except in the case of ringdown trunks) gives a disconnect signal, at the called office.

Figures 9 and 10 show, schematically, the step-by-step tandem system in Connecticut, over which most of the toll traffic within the state is handled. Similar systems are in use in Southern California,[1] and on a smaller scale, in other places.

In certain cases the increased trunk efficiency of step-by-step tandem operation is obtained, without the necessity of providing a tandem switching equipment, by locating some of the second selectors of local step-by-step central offices in a distant building serving two or more central offices to which calls are to be distributed. These "distant second selectors" combine all of the traffic to the terminating office over a single group of trunks. In other cases, increased trunk efficiency is obtained through the use of certain levels on the regular step-by-step selectors in a distant dial central office on which to terminate trunks to other central offices.

As stated, step-by-step tandems generally receive the controlling dial pulses over the incoming tandem trunks. When it is desired to complete connections through a step-by-step tandem system from manual offices, this may readily be done if the manual positions are equipped with dials. Where the volume of traffic on which dials could be used is

[1] For a detailed description of the design of the Los Angeles tandem system, see paper by F. D. Wheelock and E. Jacobsen, *Transactions A. I. E. E.*, Vol. 47.

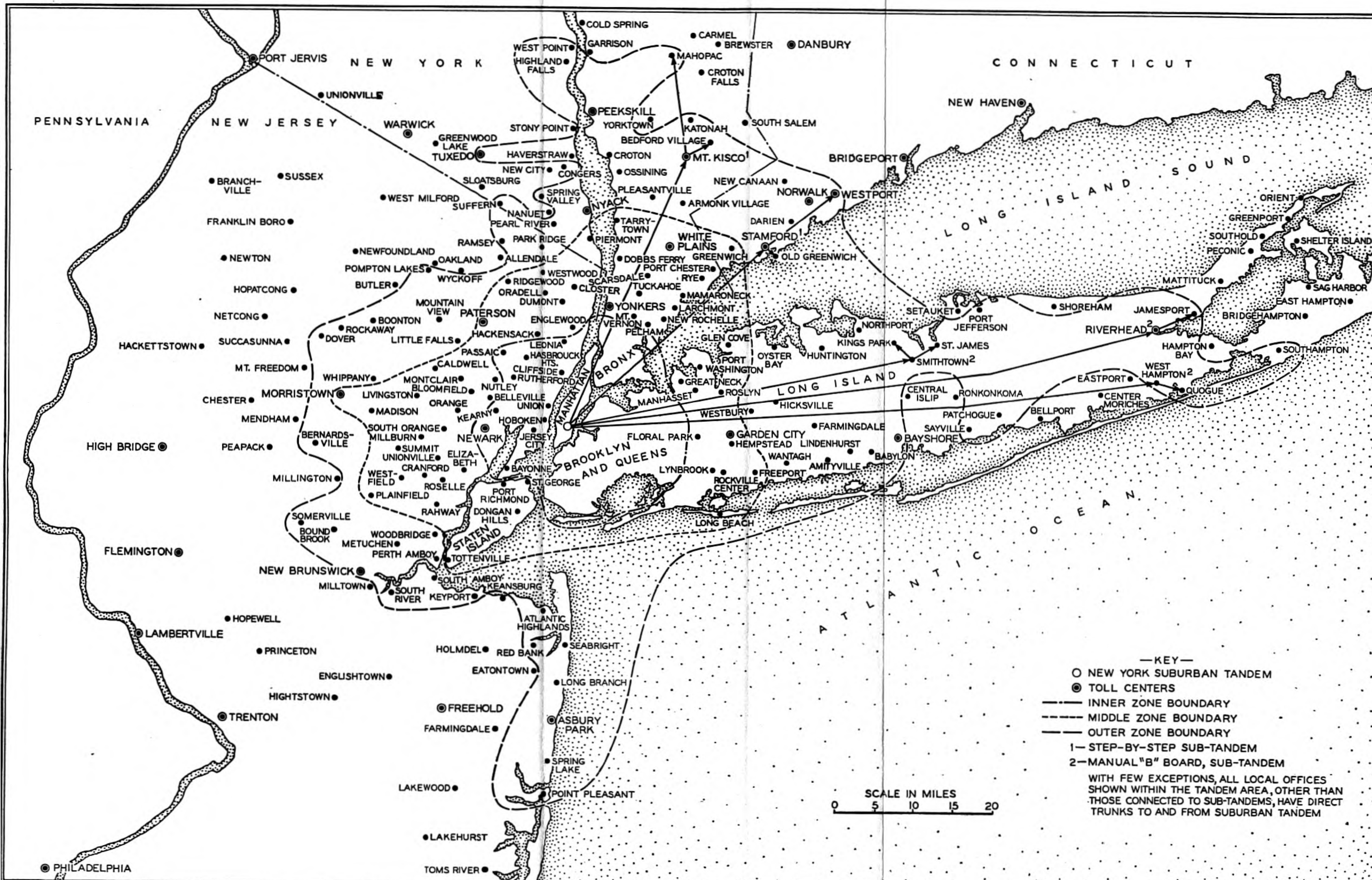

Fig. 8—New York suburban tandem area.

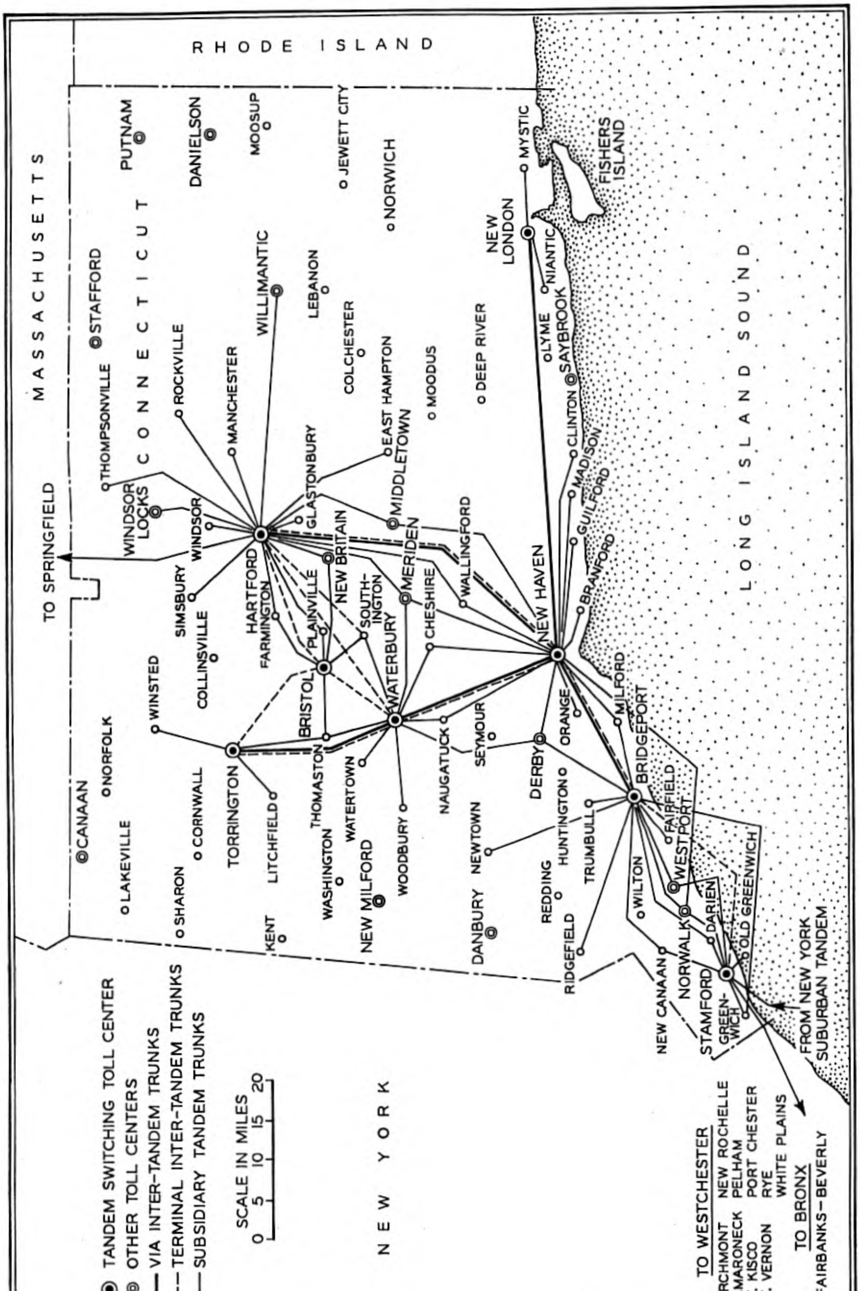

Fig. 9—Connecticut step-by-step tandem area.

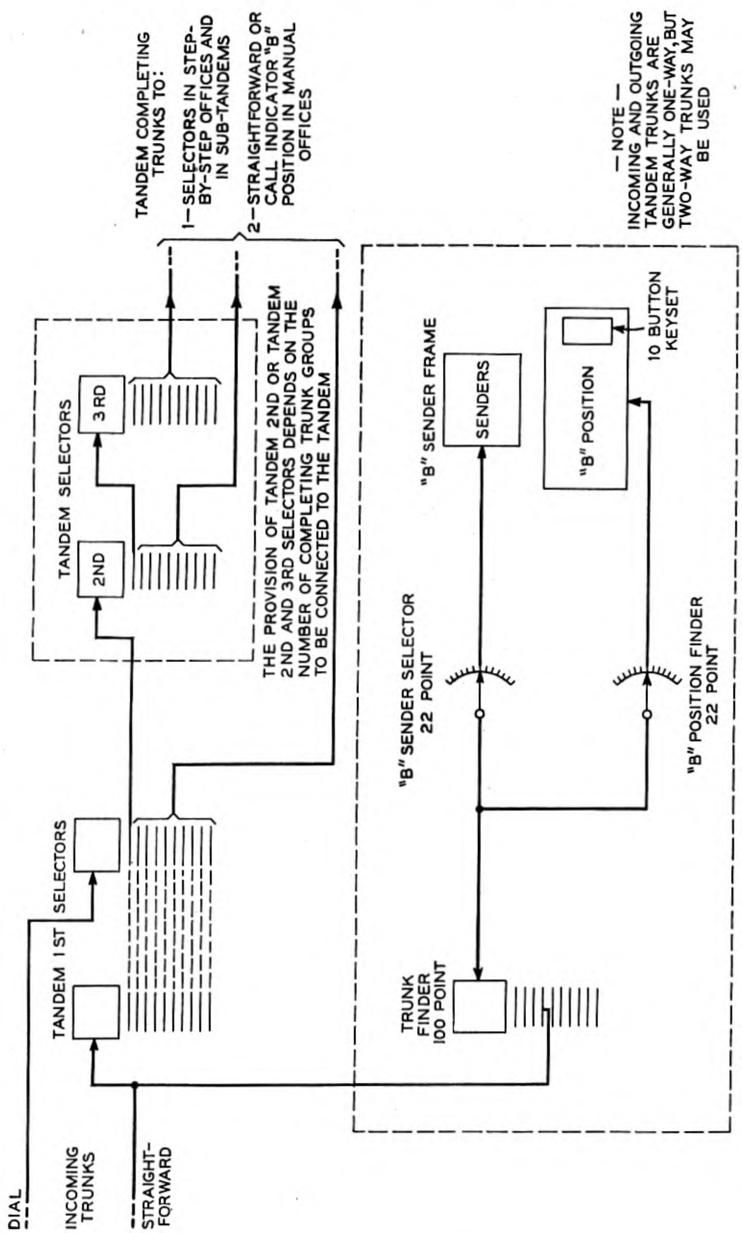

Fig. 11—Step-by-step tandem arrangement with tandem "B" position.

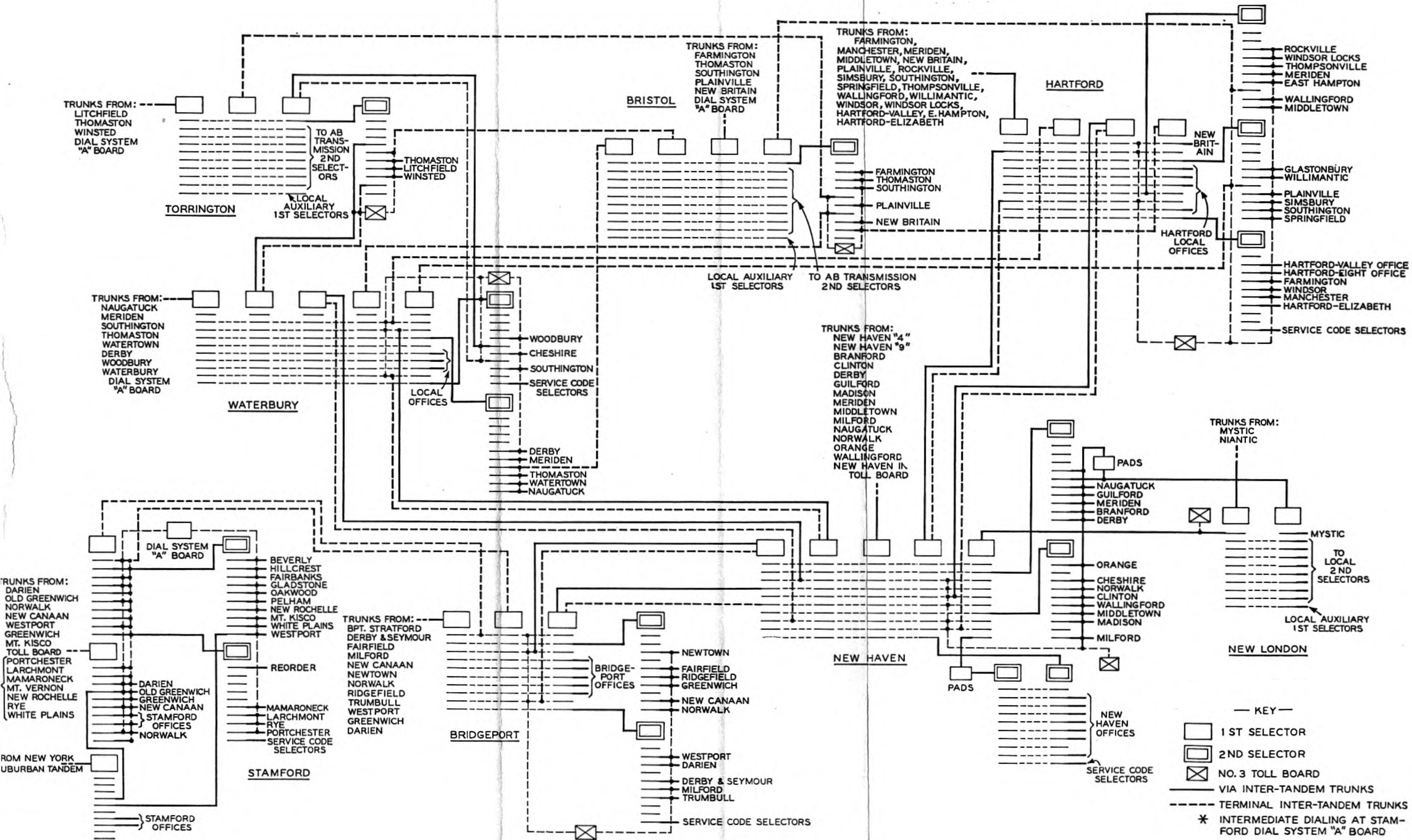

Fig. 10—Connecticut step-by-step tandem system.

relatively small, their provision may not be justified, and other means must be provided for completing connections to dial offices, whether reached over direct or tandem trunks. For tandem operation under these conditions, step-by-step "B" operators' positions, directly associated with the tandem equipment, usually are provided. The incoming trunks are of the straightforward type, and terminate on incoming selectors controlled by means of key sets on the "B" operators' positions and associated senders. An idle "B" operator is connected automatically to the incoming trunk through call distributing equipment and, upon receiving the order and setting up the required digits on her key set, her position is automatically disconnected. The final disconnection of the tandem and local office selectors is under the control of the originating operator. Except for the selectors on which the incoming trunks terminate, all of the selectors are used in common, whether controlled from the "B" positions or by pulses received over dial trunks.

The step-by-step "B" board arrangement sometimes is used as a sub-tandem in connection with manual straightforward tandem operation in a large nearby city, since it provides a convenient means for completing connections to subscribers connected to dial offices within the tandem area.

Figure 11 shows schematically the step-by-step tandem arrangement, including "B" board. Figure 12 illustrates the operators' positions.

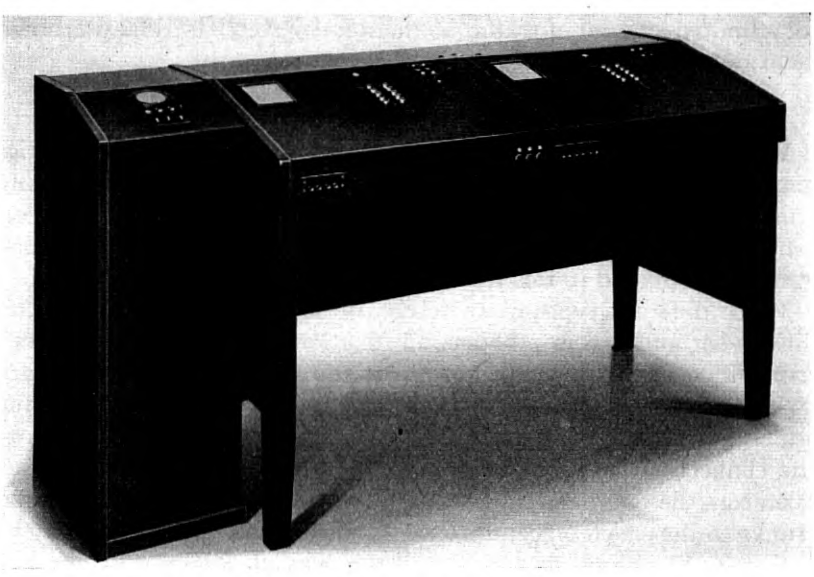

Fig. 12—Step-by-step tandem—"B" operators' positions.

It may be mentioned, in passing, that the Connecticut tandem system shown on Fig. 10 does not make use of any step-by-step "B" board equipments.

An arrangement using intermediate dialing or intermediate key pulsing circuits is used in a few cases, in lieu of the "B" board arrangement, as a sub-tandem for completing calls incoming to step-by-step central offices from a manual straightforward tandem system in a nearby large city, such as to dial subscribers in Trenton, New Jersey, from the Newark manual tandem system. Straightforward trunks from the manual tandem switchboard terminate on step-by-step selectors, and on multipled line lamps and answering jacks in the regular switchboard at the incoming end of the trunk, with an auxiliary circuit for lighting the line lamps on an incoming call. When the inward operator plugs into an answering jack in response to a lamp signal, an order tone automatically is sent back over the trunk to the originating operator, who thereupon passes the called number. On key pulsing switchboards, the inward operator sets up on her key set, the desired number, and disconnects from the trunk. On dialing boards, the inward operator dials the called number over a dialing jack associated with the trunk, using a second cord, and disconnects both cords. Release of the connection at the tandem and called offices is under the control of the originating operator.

When used in conjunction with a step-by-step tandem, the intermediate dialing or key pulsing arrangement serves the same purpose, for a limited amount of traffic, as the step-by-step "B" board arrangement described above.

Trunk Concentrating Tandems

Where small volumes of traffic to the same terminating point originate at a number of offices which are closely associated, geographically, trunk costs frequently may be reduced through the use of trunk concentrating switches. Both direct trunks and trunks to a tandem system are treated in this manner.

While different types of switches are used under the various conditions encountered in practice, all function automatically to select a trunk in a common trunk group, or the switches are permanently associated with the common trunks and operate to find the incoming trunk on which a call is waiting. No dial pulses are required to cause the connection between the trunks to be made, and to switchboard operators the outgoing trunks are practically the equivalent of direct trunks to the called office, or to the tandem office, as the case may be.

Figure 13 shows schematically the use of trunk concentrating switches for giving local operators in Philadelphia access to a common group of trunks terminating on the straightforward toll line tandem in New York City. A more extensive use on intercity traffic is in

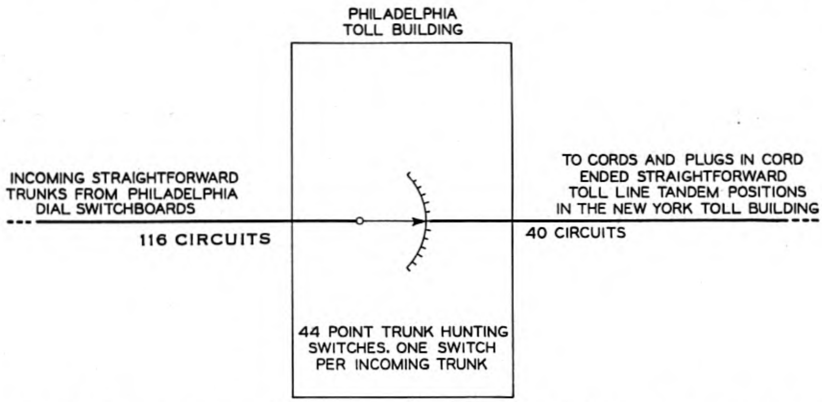

Fig. 13—Trunk hunting switch arrangement for Philadelphia to New York station-to-station "A" board toll traffic.

San Francisco and Oakland, California, chiefly for giving offices in each of these cities direct access to offices in the other city. In San Francisco, 62 incoming trunk groups containing 384 trunks are concentrated on 19 trunk groups and 239 trunks. In Oakland, 188 incoming trunk groups, containing 881 trunks, are concentrated on 25 trunk groups and 333 trunks.

Transmission Arrangements Affecting Traffic Operation

Tandem systems, unlike the general long distance system, operate within definite areas and the associated trunks usually are designed to give satisfactory transmission on connections between any two offices in the tandem area. In some of the larger systems, however, the tandem area is divided into transmission zones and the arrangements provided for traffic between the different zones may require the selection of the proper trunks or paths on the part of the operators.

One arrangement is to introduce telephone repeaters in certain of the paths between the selectors in the tandem equipment, and to route connections requiring repeater gain through these paths. Figure 14 shows schematically the Los Angeles long-haul step-by-step tandem system, in which this arrangement is used. It will be noted, for example, that connections from offices in the Metropolitan Area (Group I) to Long Beach and Santa Monica use no repeaters at the tandem office; and that connections from certain outlying offices

(Group II) are routed through one group of repeaters reached from the "O" level of the incoming 1st selectors, if destined for Long Beach and Santa Monica; and through a different group of repeaters reached from the 6th level of the incoming 1st selectors, if destined for the Norwalk-Artesia-Bellflower exchange area. The transmission gain of this second group of repeaters is higher than that of the first group.

Another arrangement is to use terminal repeaters in the tandem trunks, and to provide pads in certain of the paths between the tandem switches. The longer-haul connections are routed through the tandem switches over paths not containing pads, while terminal and other short-haul connections are routed over paths containing pads. This arrangement is indicated in connection with the New Haven tandem arrangements shown in Fig. 10, where the trunks to New London appear on the 6th level of the tandem second selectors without pads, and on the 7th level with pads.

Still another means of obtaining transmission gain is to provide a second group of trunks to the tandem office over which calls to the more distant offices are routed. For convenience to the operators, these trunks are sometimes designated as a separate tandem system as, for example, Empire Tandem in New York City, which consists of special, high-grade trunks to Suburban Tandem. In the Southern New England System, two groups of trunks are provided between certain of the tandem centers, as shown in Fig. 9, the routing code determining which group shall be used.

Speed of Operation

As might be expected, the speed with which connections can be made through tandem systems varies considerably, depending upon the type of arrangement employed. Table II indicates the relative theoretical speed of operation, in seconds, of some of the more common tandem arrangements. Direct trunks from the tandem equipment to the called office (distant city, in the case of toll office tandems) are assumed; if sub-tandems are involved, a small amount of additional time is required. Also a slight additional time is involved, in the case of step-by-step tandems, if repeaters must be dialed in.

Use of Tandem Systems in Toll Board Operation

While tandem systems have been developed primarily for local and short-haul toll station-to-station traffic handled on manual or dial "A" boards, arrangements have been provided in a number of cities which give toll board operators access to existing tandem systems in

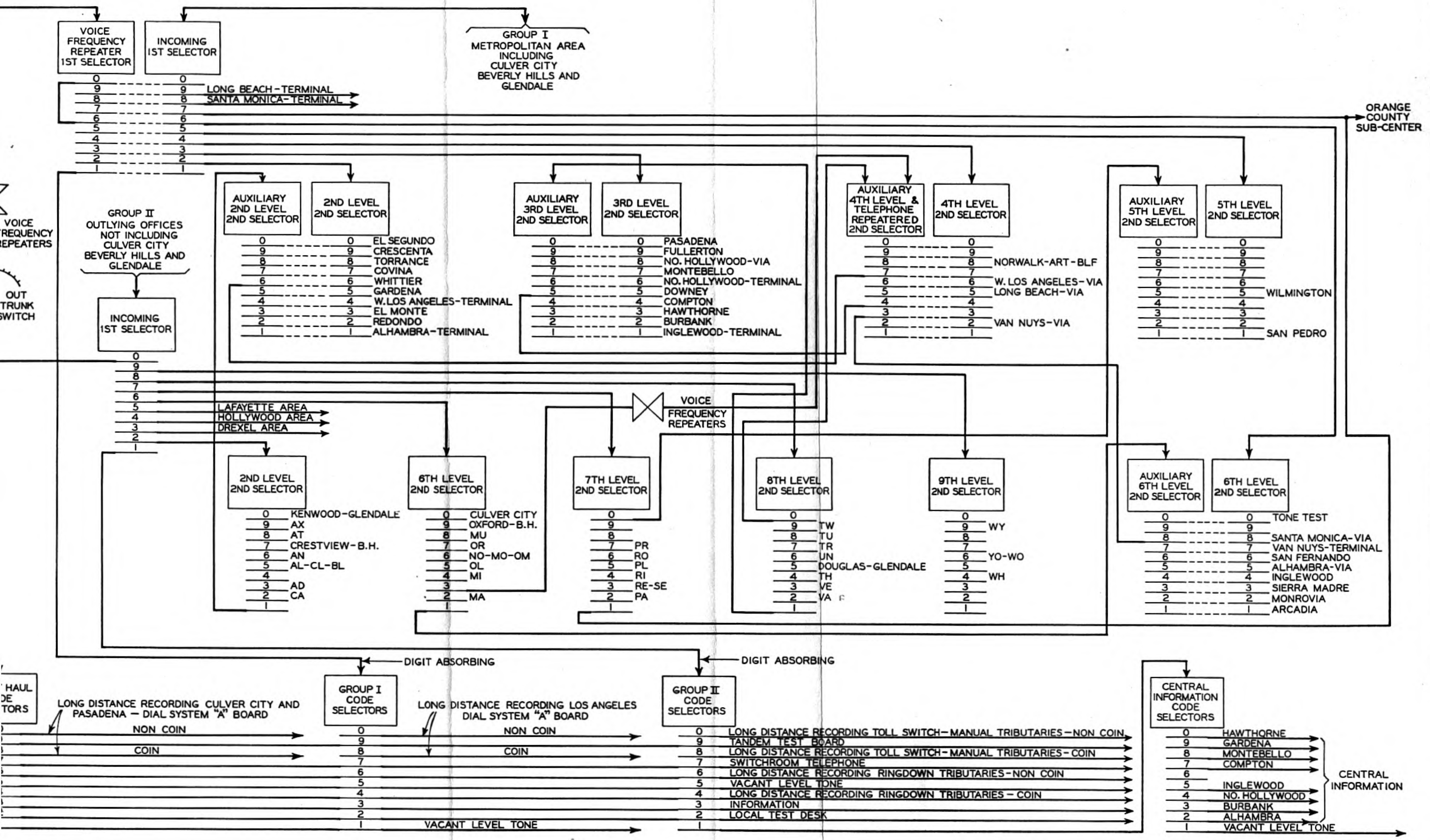

Fig. 14—Los Angeles long-haul dial tandem system.

TABLE II

RELATIVE THEORETICAL SPEED OF OPERATION OF TANDEM SYSTEMS

Type of Tandem	Types of Trunks		Speed in Seconds	
	To Tandem	From Tandem	Calls Handled By Operators*	Calls Dialed Direct By Customers†
MANUAL STRAIGHTFORWARD	Straightforward	Straightforward—to Manual "B" Pos.	25	
CALL INDICATOR	Call Indicator	Straightforward—to Manual "B" Pos.		31
TOLL OFFICE TANDEM	Straightforward	Ringdown ‡	29	
PANEL SENDER TANDEM				
Operator Tandem	Straightforward	Dial	26	
	Straightforward	Call Indicator	33	
	Straightforward	Call Announcer	34	
Full Selector Tandem				
From Operators at Boards Equipped with 10-button Keysets	Dial	Dial	24	
	Dial	Call Indicator	31	
	Dial	Call Announcer	32	
Dialed by Customers	Dial	Dial		28
	Dial	Call Indicator		35
STEP-BY-STEP				
Operator Tandem, with Step-by-Step "B" Board	Straightforward	Dial	22	
	Straightforward	Call Indicator	30	
Full Selector Tandem				
From Operators at Boards Equipped with Dials	Dial	Dial	16	
	Dial	Straightforward—to Manual "B" Pos.	20	
	Dial	Call Indicator	23	
From Operators at Boards Equipped with Keysets	Dial	Dial	17	
	Dial	Straightforward—to Manual "B" Pos.	21	
	Dial	Call Indicator	24	

* On calls handled by operators, this is the interval from receipt of signal at the switchboard to the first ring on the called line. The interval from receiver off hook at the calling station to receipt of signal, is approximately .5 second for manual stations, 3 seconds for step-by-step dial stations, and 7 seconds for panel dial stations. Both of the latter include the dialing by the customer of the "Operator" code.

† In panel areas.

‡ When completed to local multiple in toll board at called office.

order that they may complete person-to-person and other calls to points within the tandem area over tandem trunks rather than over the regular long distance circuits. Tandem systems in general make use of common battery trunks and, since some of the older types of toll switchboards either are not arranged to complete originating calls over such trunks or are not equipped with the dialing arrangements required for dialing through dial tandems, the application of this desirable arrangement is somewhat limited.

As indicated above, the only tandem operation involving ringdown toll board circuits, at present, is for the purpose of making such circuits accessible to operators at switchboard positions not having a multiple appearance of the circuits. It is the belief of the author that, with the further expansion of the long distance plant, the ringdown circuits gradually will be replaced by through supervision circuits—that is, by circuits which, like the trunks used in local and short-haul toll tandem operation, will give the originating operator switchhook supervision from the called station. These new circuits, no doubt, will be arranged for two-way and built-up circuit operation and, incoming to dial areas from toll boards equipped with dials or key sets, will be terminated directly on selectors.

Although changing conditions may suggest better arrangements, it seems probable that, from toll boards not equipped with dials or key sets, the new circuits will be operated on a straightforward basis and terminated on, or controlled at, operators' positions at the incoming end. In the larger cities these positions may have equipment and operating features quite similar to those in the panel sender tandem. In such cities, it may well be that both straightforward and ringdown incoming circuits will be terminated on switches but that the calls will be received, and both terminal and through connections set up, at the operators' positions, thus making use of the tandem type of equipment for all switching purposes in the larger cities. Also, this new inward and through toll office equipment may replace the present type of toll office tandem equipment. In smaller cities, including cities serving manual areas, the incoming circuits may be terminated on equipment having features generally similar to those in the manual straightforward system.

The gradual extension of these arrangements would eventually duplicate, in the long distance toll plant, tandem switching of the type now so extensively used on the local and short-haul toll traffic, and ultimately make the entire United States a super-tandem area.

The scope of the present tandem systems has been determined largely by economic considerations, although the desire to simplify the service to the customer in the large metropolitan areas also has been an important factor. The general introduction of the tandem type of operation on toll board circuits may affect the economic balance, and except where other factors are controlling, will tend to limit the scope of segregated tandem systems of the present type. It may well be that, eventually, in some of the smaller cities the need for a separate tandem system for the local and short-haul toll traffic will disappear altogether.

Summary

Tandem systems for local and short-haul toll traffic have been provided:

1. To reduce the number of trunk groups required in large metropolitan areas and to insure maximum efficiency on those provided; due consideration being given, of course, to a proper balance between service and costs.
2. To permit the same operating and service arrangements on short-haul toll traffic as on local traffic, thus facilitating the work of the operators and making the service faster, and easier to use by the customer.
3. To reduce the cost of handling large volumes of short-haul toll traffic through the use of toll plant designed to meet the less exacting transmission requirements, as compared with toll plant used for long distance traffic.

These systems vary in type, depending upon the types of local central office equipment which are to be interconnected.

The use of tandem operation in connection with toll board traffic is limited at the present time, but in view of the future volume of this traffic and of new arrangements which now appear feasible, further expansion of the long distance system may be along lines generally similar to those employed in the local and short-haul toll tandem systems; using, of course, operating methods and equipment arrangements which adequately meet the requirements on the longer-haul traffic. These new arrangements may involve both dialing and straightforward toll lines. They may affect the economic balance and, thereby, the scope of tandem systems provided heretofore for local and short-haul toll traffic.

References

In addition to the paper by Messrs. Wheelock and Jacobsen mentioned on Page 10, certain aspects of tandem operation are referred to in the following papers:

1. "General Engineering Problems of the Bell System," H. P. Charlesworth, *Bell Sys. Tech. Jour.*, October, 1925.

2. "Telephone System of the United States," Bancroft Gherardi and F. B. Jewett, *Bell Sys. Tech. Jour.*, January, 1930.
3. "Telephone Toll Plant in the Chicago Region," Burke Smith and G. B. West, *Jour. A. I. E. E.*, January, 1928.
4. "Telephone Trunking Plant in a Metropolitan Area," A. P. Godsho, presented at District Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa. October, 1930.
5. "The Call Announcer," W. H. Matthies, *Bell Laboratories Record*, December, 1929.
6. "Evolution of the Call-Indicator System," E. H. Clark, *Bell Laboratories Record*, December, 1929.
7. "The Manual Tandem Board," C. G. Spencer, *Bell Laboratories Record*, December, 1929.
8. "Toll Tandem Switchboard," C. E. Hokanson, *Bell Laboratories Record*, June, 1930.
9. "Key Display Type Call Indicators," S. T. Curran, *Bell Laboratories Record*, July, 1930.
10. "The Telephone Problem in the World's Largest Metropolitan Area," Kirkland A. Wilson, *Bell Tel. Quart.*, October, 1934.
11. "Dial Switching of Connecticut Toll Calls," W. F. Robb, A. M. Millard, and G. M. McPhee, *Electrical Engineering*, July, 1936.

A Non-Directional Microphone

By R. N. MARSHALL and F. F. ROMANOW

A moving coil microphone is described which responds uniformly over a wide frequency range to sound arriving from any direction. A study of diffraction, the main factor causing directivity of microphones of the pressure type, leads to the conclusion that a small spherical shape is the most desirable for a non-directional microphone. But even fulfilling this requirement in the design of the housing leaves a large directional effect. Hence an acoustic screen has been developed which diminishes diffraction to an extent necessary to make the change in response due to angle of sound incidence imperceptible to the ear. The non-directional microphone is of simple and rugged construction. Adequate precautions have been taken to prevent atmospheric changes from affecting the stability. The small size and unusual shape of the microphone contribute much to its attractive appearance.

IN many situations—such as when a microphone is used as a pick-up for large orchestras or choruses, or in sound picture studios—the sound reaching the microphone directly may be only a small part of the total. Most of the sound arrives at the microphone from directions other than normal to the plane of the diaphragm. If the microphone response differs in these various directions, the output will not truly represent the sound at the point of pick-up—and this is, of course, a form of distortion. This distortion was minimized in the Western Electric 618–A type moving coil microphone [1] by selecting the constants of the instrument so that the field response would be as uniform as possible for sound of random incidence. Still there remained a considerable change of response with the angle of sound incidence and with frequency as is shown in Fig. 2. In the non-directional microphone this variation (Fig. 3) has been greatly reduced so that it is imperceptible to the ear. Moreover, the new microphone is designed to be mounted so that its diaphragm is horizontal.* In this position the instrument is symmetrical with respect to a vertical axis through the center of the diaphragm. If a sound source is placed at some arbitrary location we may rotate the microphone around this vertical axis without changing its response. Hence the instrument is entirely non-directional with respect to the vertical axis. If the microphone is rotated around an axis in the plane of and through the center of the diaphragm a very slight residual directional effect remains and it is this one which has been plotted.

* Since the non-directional microphone is generally mounted with its diaphragm in a horizontal plane the angles of incidence have been labeled 0°, ± 30°, ± 60°, ± 90° retaining 0° as the angle of incidence for sound waves moving in the horizontal plane.

The directivity of pressure type microphones is caused by two factors: (*a*) the variation of the diffraction effect with frequency and with the angle of incidence of the sound wave, and (*b*) the decrease in pressure due to phase shift which occurs when the direction of the progressive

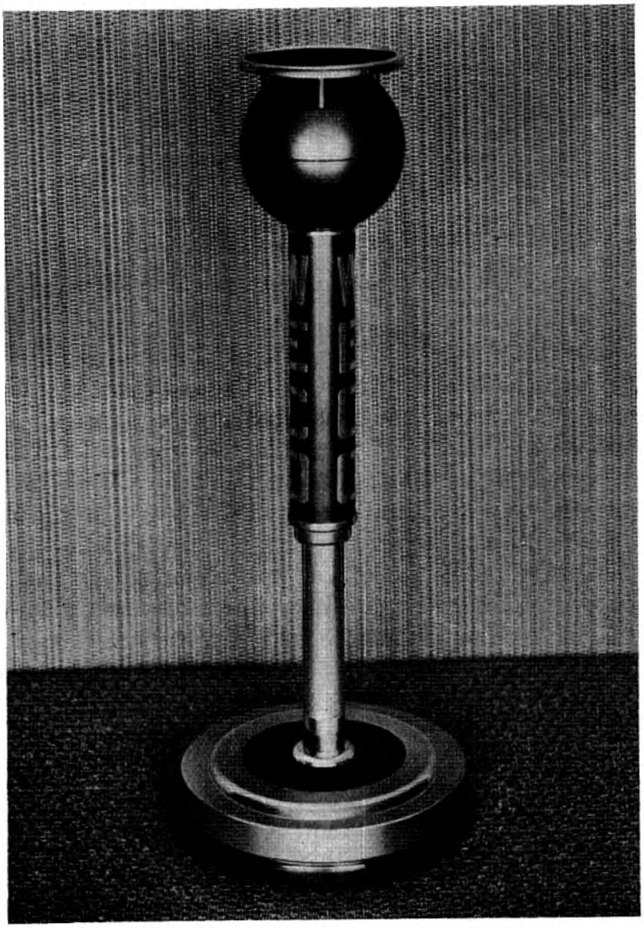

Fig. 1—630-A type moving coil microphone with deskstand.

sound wave has a component parallel to the plane of the diaphragm. Each is also a function of the dimensions of the microphone relative to the wave length of the sound, and in the instruments of the size discussed in this paper the effects become large only at frequencies above 1000 cycles. Directivity might be avoided, therefore, if the microphone could be made small enough; but calculation shows that to make the effect negligible at 10,000 cycles the instrument would have to be approximately one-half inch in diameter. While a microphone of this size could be built, it is doubtful whether an output level could be obtained which would be adequate for public address, broadcasting, and sound picture use.

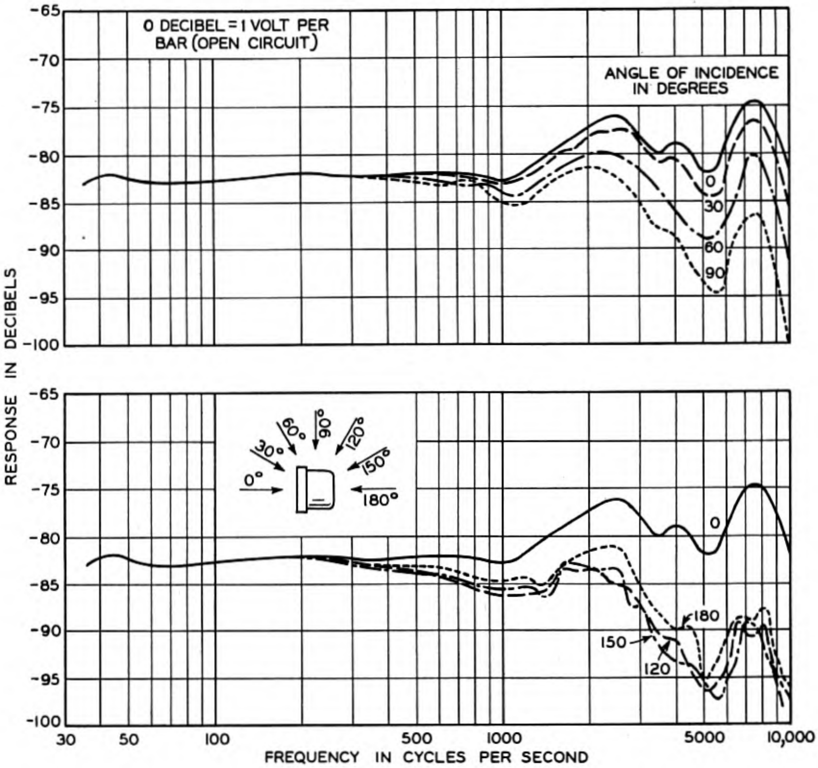

Fig. 2—Field response of the 618-A microphone showing the effect of angle of sound incidence.

Diffraction plays by far the predominant role up to a frequency where the wave-length is comparable to the diameter of the diaphragm, and the effect of phase shift may be neglected. It is principally diffraction which causes the field response of a microphone to differ from the pressure response, and because of the variation of the effect with angle of sound incidence it is impossible to correct for it by adjusting the pressure response. The only alternative is to attack the diffraction problem directly.

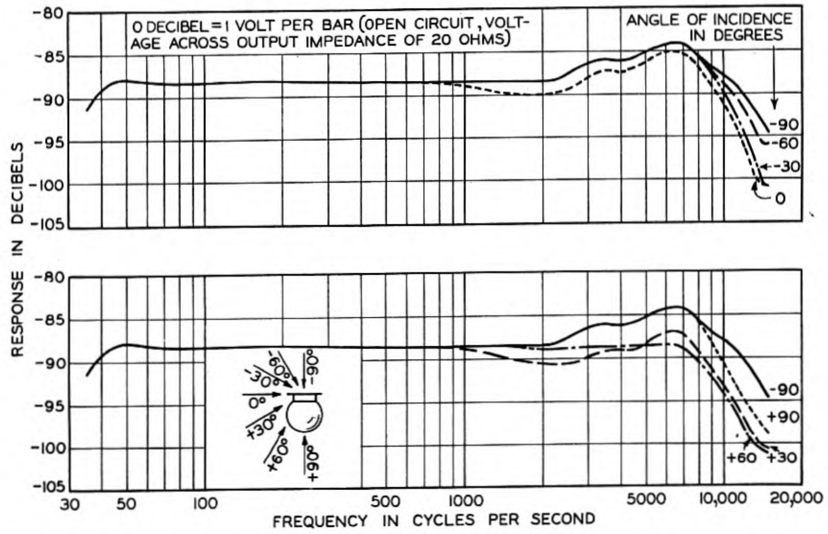

Fig. 3—Field response of a laboratory model of the new 630-A non-directional microphone for several angles of sound incidence.

Diffraction of Sound Around the Microphone

The effect of the shape of the microphone on the directional response has been brought out by study of the diffraction effect of different geometrical objects of equal diameter. The diffraction of a sphere was first treated by Rayleigh [2] and evaluated for a point on the sphere for normal incidence by S. Ballantine,[3] and for other angles of incidence by H. C. Harrison and P. B. Flanders.[4] The effect for a circular plate has been given by L. J. Sivian and H. T. O'Neil.[5] Figure 4 shows the calculated diffraction effect of the cylinder, cube, and sphere * as a function of frequency and angle of incidence in terms of the ratio of the disturbed to the undisturbed sound pressure at a point located centrally in the surface of the object. The abscissae are given as the ratio of the diameter of the object to the wave-length of sound, but the table at the bottom indicates the corresponding frequencies for diameters of 1 inch, 2 inch and 4 inch. If a microphone were built having any one of these shapes, and its diaphragm were made very small and located at the point for which the curves were computed, its response would be increased or decreased approximately in correspondence with these curves as the angle of incidence is changed from + 90° to − 90°. It will be noticed that both cylinder and cube show a marked directional

* The calculated values for the diffraction of the circular plate and cube have been taken from an unpublished work of G. G. Muller of the Bell Telephone Laboratories.

effect, made more serious by the wavy character of the response, while the variation in response for the sphere is much less and the waviness has practically disappeared.

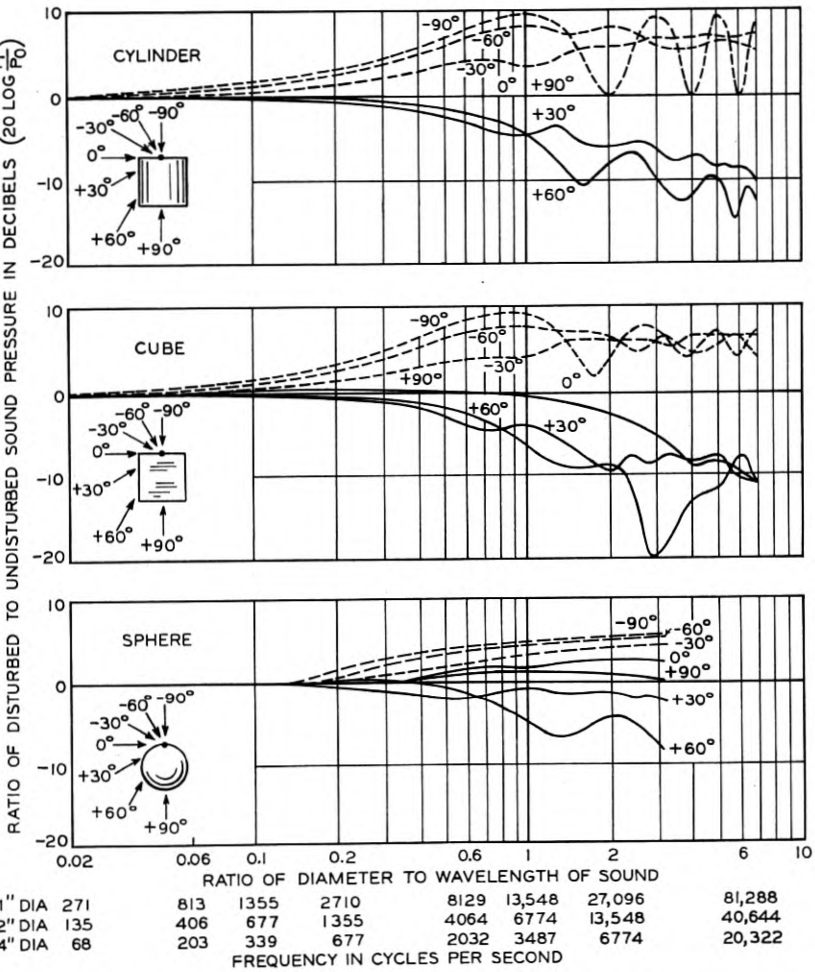

Fig. 4—Calculated diffraction effect at the center of the end form of a cylinder, cube, and sphere for sounds coming from different directions.

It appears then that a very small sphere would be the most desirable shape, but it was found impracticable to reduce the microphone housing below a 2½ inch sphere and to reduce the diaphragm diameter below 1 inch. The field response of the resultant microphone without the acoustic screen for various angles of sound incidence is shown in Fig. 5.

The directional effect, while diminished somewhat compared with that of the 618–A type (Fig. 2), is still not negligible. It was possible, however, to achieve a fairly uniform response with respect to frequency for sound of 0° incidence, that is, sound arriving in the plane of the diaphragm. Since in most instances the direct energy arrives in the horizontal plane, uniform, non-directional response for this important plane may be secured by mounting the microphone with the diaphragm in a horizontal position. Still there is a tendency for the response to be

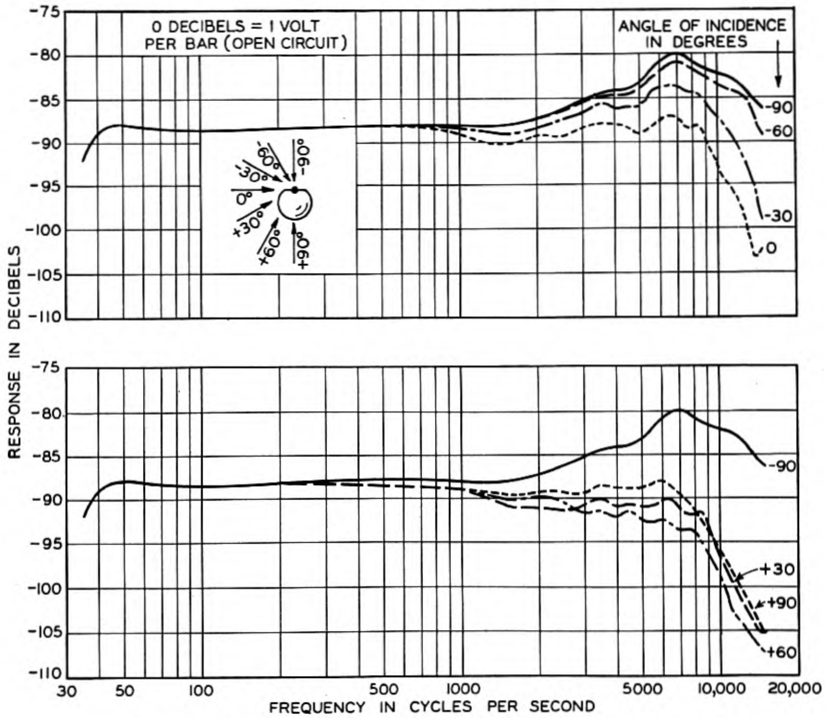

Fig. 5—Field response of a laboratory model of the 630-A non-directional microphone without screen.

too high for high-frequency sounds coming down from above, that is, directly toward the diaphragm, and too low for similar frequencies coming from angles very much below the horizontal.

To determine to what extent diffraction contributes to this residual directivity in the vertical plane let us consider the diffraction of a geometrical shape resembling that of the microphone, namely, a two and one-half inch sphere with a flat face one and one-eighth inches in diameter. We may approximate the diffraction effect for this shape by combining the known effects for a sphere and flat plate shown in Fig. 6.* Although the sphere is twice the diameter of the circular plate, it is seen that it has the larger effect only at the lower frequencies. The arrows indicate the probable effect to be taken for that of the flat-faced sphere. Although this result applies strictly only at a point at the center, measurements [5] have shown that up to 15,000 cycles for a one and one-

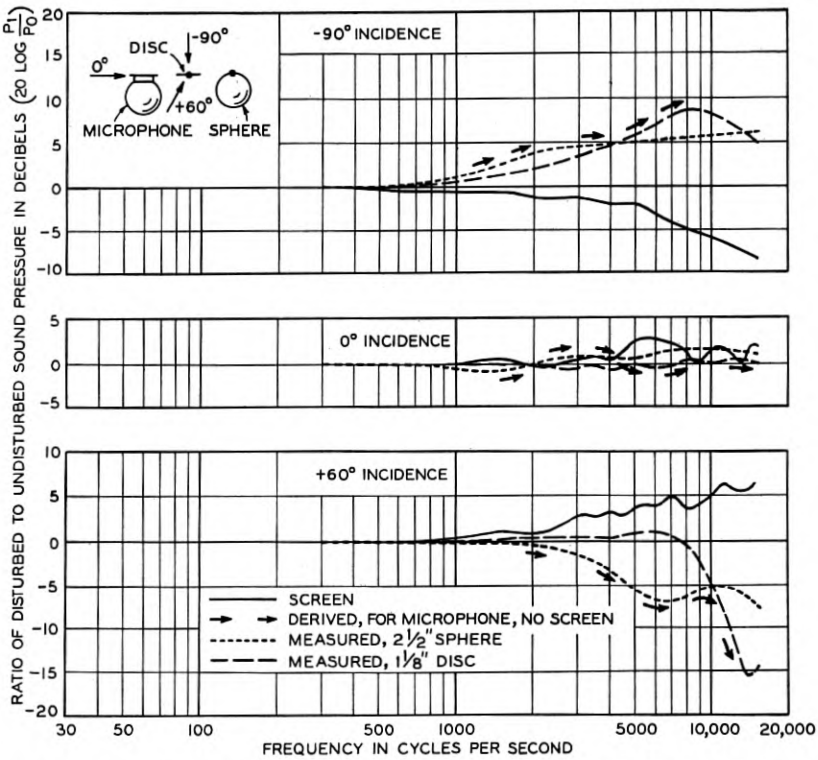

Fig. 6—Measured diffraction effect of a 2½″ sphere, 1⅛″ circular plate, and acoustic screen and derivation of diffraction effect of 630-A microphone without screen.

eighth inch circular disc there is little variation over an area comparable to the effective area of the microphone diaphragm. Hence, the diffraction effect derived in this manner is added to the computed contour pressure response (see Appendix A) to obtain the theoretical field

* The effect at + 60° incidence has been shown as more significant since the diffraction for + 90° is small. The latter effect corresponds to the optical bright spot at the center of the disc on the side away from the light source. This effect occurs over such a small area for angles very close to + 90° that it is of no practical use in this case. However, this does account for the + 90° response of microphones often being higher than the + 60° or + 30° response.

response shown in Fig. 7 where it is compared with the experimental response. The largest deviation between theoretical and experimental values is in the zero degree response at frequencies from 10,000 to 15,000 cycles. This difference is attributable to the factor mentioned earlier, namely, the decrease in effective pressure due to the phase shift of a plane sound wave traveling across the face of the diaphragm. It is of the same order of magnitude as that calculated by H. C. Harrison and P. B. Flanders [4] for a stretched circular membrane. It is concluded, therefore, that the diffraction effect indicated by arrows in Fig. 6 is representative of that of the actual microphone.

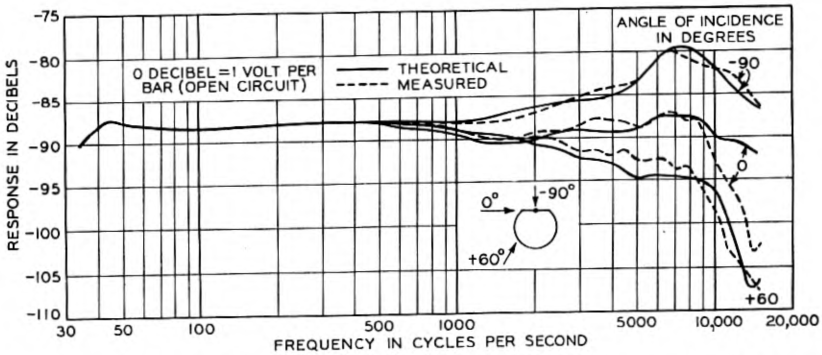

Fig. 7—Comparison of theoretical and experimental field response of a laboratory model of the No. 630-A non-directional microphone without screen.

Effect of Acoustic Screen

From the quantitative considerations of the diffraction of the spherical moving coil microphone without screen it becomes clear that if the microphone is to be made non-directional in the vertical plane also, an element must be introduced which compensates for the increase and decrease in field response due to diffraction.

The screen which was developed for this purpose is a disc 2½ inches in diameter and made of material having a very high resistance-to-mass ratio. This disc is supported approximately 1/8 inch in front of the microphone grid. The diffraction effect of this screen has been measured in terms of the effect on the face of the microphone. Figure 6 gives the effects for sound of 0°, − 90°, and + 60° incidence and compares them with those of the microphone without the screen. From these data it may be seen that the acoustic screen compensates for the microphone diffraction effect, for (1) it has least effect for sound of 0° incidence; (2) it causes a decrease in the − 90° field response; and (3) it causes an increase in the + 60° response.

The variables in a screen of this type are its diameter, impedance, and distance from the microphone grid. It is a combination diffraction and impedance screen; for part of the sound is attenuated by passing directly through the screen while the rest is diffracted. The proportion between the two is a function of the impedance of the screen and of the ratio of its diameter to the wave-length of the sound. At lower frequencies most of the sound coming from the top is bent around the screen while at higher frequencies more of it travels directly through and becomes attenuated. For sound coming from the side, the screen has little effect. When sound comes from the bottom some of it is reflected onto the face of the microphone. The acoustic screen thus makes the instrument non-directional in its response characteristics.

General Design

Besides these changes designed primarily to reduce the directional effects, extensive changes were made in the internal construction and arrangement of the microphone to make the response more uniform and to extend the frequency range. The general construction is shown in Fig. 8. The desirability of making the diaphragm as small as possible

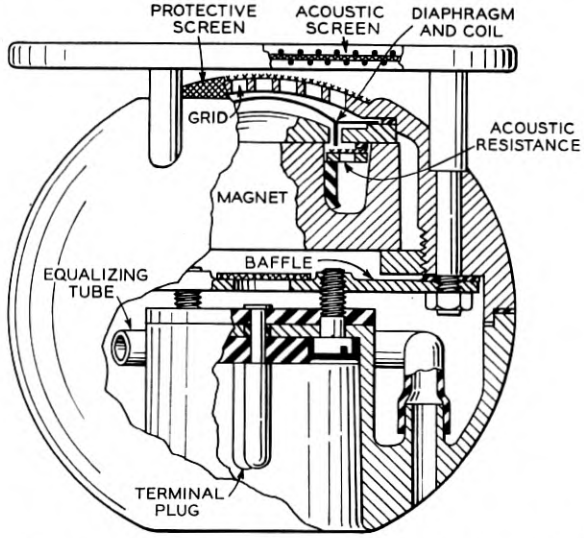

Fig. 8—Simplified cross-sectional view of the non-directional microphone.

has been pointed out in the discussion of microphone diffraction. However, decreasing the size rapidly reduces the sensitivity which is proportional to the area of the diaphragm and the flux density of the magnetic field around the coil. In order to obtain in this instrument a signal-to-noise ratio sufficiently high for all practical purposes, it was not considered advisable to use a diaphragm smaller than 1 inch in diameter. The loss in sensitivity resulting from this choice was partly offset by making the diaphragm light in weight and of very low stiffness. It is also very important that this diaphragm vibrate as a simple piston throughout the entire range; but to obtain this action over a wide range of frequencies has proved in the past to be a very difficult problem. For this new microphone, a diaphragm was developed which has a rigid spherical center and a tangentially corrugated annulus and which has in addition a high area to stiffness ratio. No evidence of vibrating in other modes is shown by this structure below 15,000 cycles. The diaphragm is cemented to a raised annulus on the outer pole-piece. The outer and inner pole pieces are of soft iron and are welded directly to the magnet which is made of high grade magnet steel. The diaphragm is damped by an acoustic resistance which is supported below the coil by a brass ring. This ring is held in place with rubber gaskets.

The size and shape of the housing were selected with particular reference to the requirements that had to be met. The size is such that the housing fits closely over the diaphragm and thus produces little more diffractive effect than would the diaphragm itself, and the spherical form allows sufficient amount of air space behind the diaphragm, which is essential to minimize the impedance to vibration. To prevent resonance within the case an acoustic resistance baffle is provided to divide the space into two parts. A tube with its outlet at the back of the housing serves the double purpose of equalizing the inside and atmospheric pressures and of increasing the response of the instrument at low frequencies.

In the non-directional microphone the resonance in the cavity in front of the diaphragm is controlled by the design of the protective grid. Instead of being the source of an undesirable distortion, the grid and cavity have become a valuable aid in improving the response of the instrument at frequencies from 8,000 to 15,000 cycles. This grid also incorporates a screen which prevents dust and magnetic particles from collecting on the diaphragm.

Method of Measuring Field Response

The method of making the frequency-response measurements is similar in general details to the method outlined in a paper by W. C. Jones and L. W. Giles [1]. Figure 9 shows the arrangement of the room and testing apparatus. A very small, specially developed, condenser microphone was used in determining the sound field pressure. The determination of the field calibration of this reference microphone presented an interesting problem and an original solution is described in Appendix B.

At low frequencies where the wave-length of sound is very large compared to the dimensions of the microphone, the coupler shown in Figure 9 is used. At the higher frequencies a steady state sound field is set up in the damped test room and the pressure at a given point is measured by means of the small reference condenser microphone; then

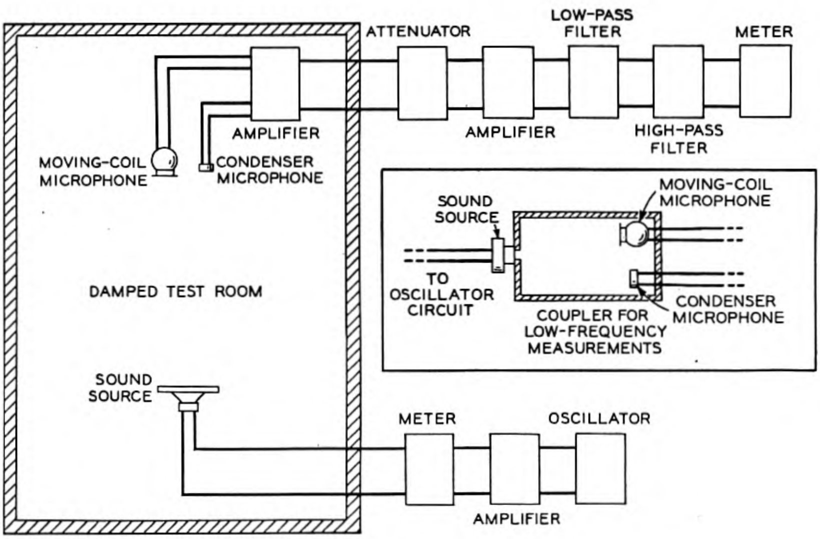

Fig. 9—Response measuring circuit.

the instrument to be tested is substituted at this point and its generated voltage recorded. The response is obtained in terms of decibels below a reference level of one volt per bar of undisturbed sound pressure.

In conclusion we wish to acknowledge our indebtedness to Mr. L. W. Giles and Mr. R. C. Miner of the Bell Telephone Laboratories who have aided us greatly in this work.

APPENDIX A

Theory

Since in most papers on moving coil microphones an equivalent circuit of this instrument is given without deriving it, it does not seem superfluous to indicate here the method of obtaining such a circuit and the calculation of its constants.

In deriving the theory of this instrument it is convenient to speak of the contour pressure response of a microphone.* We shall define the contour pressure response at a certain frequency as the generated open-circuit voltage per bar of uniform pressure over the face of the microphone. On the other hand the field response at a certain frequency is defined as the generated open-circuit voltage per bar of pressure of the undisturbed sound field. The difference between the contour pressure response and the normal incidence field response is caused by diffraction around the microphone, as explained earlier in this paper. We may approximate the normal incidence field response by adding to the contour pressure response the diffraction of the corresponding sphere and circular plate. In the considerations that follow the influence of the screen on the response will be omitted.

To obtain this contour pressure response we assume, then, a uniform pressure over the microphone face. We further require that no wave propagation shall occur within the microphone. Let a small alternating pressure be applied and consider the motion of the system when the pressure is positive. Then the air in the grid holes moves as a whole and imparts an excess pressure to the grid chamber and to the diaphragm. The central portion of the latter moves as a rigid piston. The air volume underneath the diaphragm is compressed, and some air is forced through the coil slot into a very small chamber just in front of the damping ring. We shall assume again that the air in the coil slot moves as a whole. From here the air flows through the acoustic resistance into the larger case chamber. The outside pressure instead of acting upon the diaphragm in the described manner may enter the case through the equalizing tube. On its travel through the tube it is attenuated and its phase is changed. At low frequencies this property of the tube is used to increase the response of the instrument. In the theory that follows, the tube circuit is omitted at first, but its position in the general arrangement will be shown later.

Let us use the following notation for the elements of the vibrating structure:

r_{-1} = equivalent mechanical resistance of all holes in grid,

m_{-1} = equivalent mass of all holes in grid,

n = number of holes in grid,

A_{-1} = area of all holes in grid,

* The term contour pressure response is useful when the microphone has acoustic circuits in front of its diaphragm. Pressure response is a term which has been reserved specifically for the condition where uniform pressure is applied directly to the diaphragm. (See the report of a subcommittee on fundamental sound measurements on the calibration of microphones in the journal of the *Acoustical Society of America*, Vol. 7, April, 1936, p. 301.)

A_0 = effective area of diaphragm,
m_0 = effective mass of diaphragm,
r_0 = mechanical resistance of diaphragm,
S_0 = stiffness of diaphragm,
V_1 = volume of air under diaphragm,
m_2 = equivalent mass in coil slot,
r_2 = equivalent mechanical resistance in coil slot,
A_2 = total area of coil slots,
V_3 = volume of air chamber in front of resistance ring,
m_4 = equivalent mass of air in acoustic resistance silk,
r_4 = mechanical resistance to air flow in silk,
A_4 = total area of holes in silk,
V_5 = volume of case,
r_T = equivalent mechanical resistance of tube,
m_T = equivalent mass of air in tube,
A_T = area of tube,
$\dot{x}_{-1,\,0,\,2,\,4}$ = linear velocities,
$x_{-1,\,0,\,2,\,4}$ = linear displacements,
P_0 = atmospheric pressure,
λ = ratio of specific heats for a gas,
$\dfrac{\text{Force}}{\text{Velocity}}$ = mechanical impedance (Force and Velocity are both complex quantities),
$\dfrac{\text{Force}}{\text{Displacement}}$ = stiffness coefficient.

The Lagrangian equations for a system with four independent coordinates can be written in the form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_n}\right) + \frac{\partial F}{\partial \dot{x}_n} + \frac{\partial V}{\partial x_n} = e_n(t) \qquad (n = -1, 0, 2, 4), \tag{1}$$

in which T is the kinetic energy, F is Rayleigh's dissipation function, V is the potential energy, and $e_n(t)$ is a periodic force. The kinetic energy of the system is

$$T = \tfrac{1}{2}m_{-1}\dot{x}_{-1}^2 + \tfrac{1}{2}m_0\dot{x}_0^2 + \tfrac{1}{2}m_2\dot{x}_2^2 + \tfrac{1}{2}m_4\dot{x}_4^2. \tag{2}$$

Rayleigh's dissipation function becomes

$$\mathrm{F} = \tfrac{1}{2}r_{-1}\dot{x}_{-1}^2 + \tfrac{1}{2}r_0\dot{x}_0^2 + \tfrac{1}{2}r_2\dot{x}_2^2 + \tfrac{1}{2}r_4\dot{x}_4^2, \tag{3}$$

and the potential energy takes the form

$$V = \frac{1}{2}\frac{\lambda P_0}{V_{-1}}(A_{-1}x_{-1} - A_0x_0)^2 + \frac{1}{2}S_0x_0^2 + \frac{1}{2}\frac{\lambda P_0}{V_1}(A_0x_0 - A_2x_2)^2 + \frac{1}{2}\frac{\lambda P_0}{V_3}(A_2x_2 - A_4x_4)^2 + \frac{1}{2}\frac{\lambda P_0}{V_5}(A_4x_4)^2. \quad (4)$$

Let

$$a_{-1} = \frac{\lambda P_0}{V_{-1}},$$
$$a_1 = \frac{\lambda P_0}{V_1}, \text{ etc.}$$

Differentiating the above expressions, substituting into (1), writing $\ddot{x} = j\omega\dot{x}$, $x = \frac{\dot{x}}{j\omega}$, and noting that $e_2 = e_3 = e_4 = 0$, we have

$$\left(j\omega m_{-1} + r_{-1} + \frac{a_{-1}A_{-1}^2}{j\omega}\right)\dot{x}_{-1} + \left(\frac{-a_{-1}A_{-1}A_0}{j\omega}\right)\dot{x}_0 = e_1,$$

$$-\frac{a_{-1}A_0A_{-1}}{j\omega}\dot{x}_{-1} + \left(j\omega m_0 + r_0 + \frac{a_{-1}A_0^2 + S_0 + a_1A_0^2}{j\omega}\right)\dot{x}_0 - \frac{a_1A_0A_2}{j\omega}\dot{x}_2 = 0,$$

$$-\frac{a_1A_2A_0}{j\omega}\dot{x}_0 + \left(j\omega m_2 + r_2 + \frac{a_1A_2^2 + a_3A_2^2}{j\omega}\right)\dot{x}_2 - \frac{a_3A_2A_4}{j\omega}\dot{x}_4 = 0,$$

$$-\frac{a_3A_4A_2}{j\omega}\dot{x}_2 + \left(j\omega m_4 + r_4 + \frac{a_3A_4^2 + a_5A_4^2}{j\omega}\right)\dot{x}_4 = 0.$$

If we were to draw the equivalent circuit from these equations we would find that negative stiffnesses are introduced by the different areas through which the air has to flow. In the shunt arms, however, only positive stiffnesses appear. In order to eliminate the negative stiffnesses it is customary to group the shunt stiffness with a negative stiffness and another positive stiffness into a T structure. It is simple to show that this T structure is equivalent to an ideal auto-transformer shunted by a positive stiffness. The turn ratio of the transformer is given by the ratio of two areas. Of course, we may write for the impedance looking into the high side of the auto-transformer

$$Z_H = \left(\frac{A_n}{A_m}\right)^2 Z_L,$$

where

Z_L = impedance in the low side of auto-transformer,
$A_{n,\,m}$ = areas where $A_n > A_m$.

If a voltage is in series with Z_L it must be multiplied by the ratio of A_n/A_m. If these transformations are carried out we obtain the following equations.

$$\left[j\omega m_{-1}\left(\frac{A_0}{A_{-1}}\right)^2 + r_{-1}\left(\frac{A_0}{A_{-1}}\right)^2 + \frac{a_{-1}A_0^2}{j\omega}\right]\dot{x}_{-1} - \frac{a_{-1}A_0^2}{j\omega}\dot{x}_0 = e_1\frac{A_0}{A_{-1}},$$

$$-\frac{a_{-1}A_0^2}{j\omega}\dot{x}_{-1} + \left[\frac{(a_{-1}+a_1)A_0^2 + S_0}{j\omega} + j\omega m_0 + r_0\right]\dot{x}_0 - \frac{a_1A_0^2}{j\omega}\dot{x}_2 = 0,$$

$$-\frac{a_1A_0^2}{j\omega}\dot{x}_0 + \left[\frac{(a_1+a_3)A_0^2}{j\omega} + j\omega m_2\left(\frac{A_0}{A_2}\right)^2 + r_2\left(\frac{A_0}{A_2}\right)^2\right]\dot{x}_2 - \frac{a_3A_0^2}{j\omega}\dot{x}_4 = 0,$$

$$-\frac{a_3A_0^2}{j\omega}\dot{x}_2 + \left[\frac{(a_3+a_5)A_0^2}{j\omega} + j\omega m_4\left(\frac{A_0}{A_4}\right)^2 + r_4\left(\frac{A_0}{A_4}\right)^2\right]\dot{x}_4 = 0.$$

These equations can be translated into an equivalent circuit in which the effect of the equalizing tube may be inserted as a shunt enabling the impressed force to enter the case and to reach the diaphragm after passing through numerous circuit elements.

The voltage generated in the coil due to its motion in an air gap of a permanent magnet is proportional to $\dot{x}_0$ the velocity of the coil. Hence the expression should be solved for the expression $\frac{e_1}{\dot{x}_0}\frac{(A_0)}{(A_{-1})} = Z$. If l is the length of the wire in the coil, B the flux density in the gap, then the voltage generated in the coil is

$$V = Bl\dot{x}_0$$
$$= \frac{Ble_1}{Z}\frac{(A_0)}{(A_{-1})},$$

and since $e_1 = pA_{-1}$ where p is the pressure we have finally

$$\frac{V}{p} = \frac{BlA_0}{Z}\cdot 10^{-8} \text{ volts per bar.}$$

The response in db is equal to

$$\eta = 20\log_{10}\frac{BlA_0}{Z}\cdot 10^{-8}. \tag{5}$$

This quantity when plotted against frequency constitutes the contour pressure calibration. The normal incidence field calibration is found by always adding at any frequency the larger ordinate of the curves representing the diffraction of a sphere and of a flat plate. For convenience these effects shown already in Fig. 6 are given again in Fig. 10 which also compares the theoretical with the experimental response. We observe that the theoretical field calibration is in good agreement with the experimental response.

The meaning of most constants used in evaluating (5) is evident. Some are easily calculated, while others have to be found by measurement. The resistance of the silk is found by allowing air of a certain

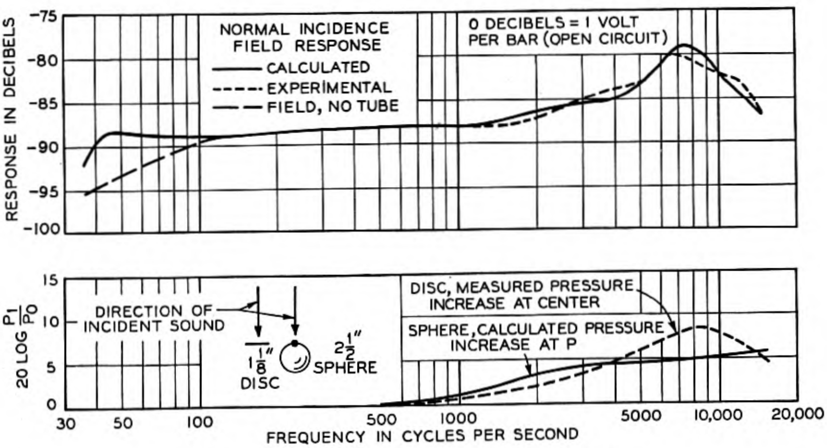

Fig. 10—Field response of 630-A microphone without screen for sound incident normally to diaphragm.

volume velocity to flow through it and by measuring at the same time the pressure drop across the resistance.[1] The mass of the silk is found by impedance measurements.[6] To find the equivalent mass of the coil slot is somewhat difficult since it consists of the mass of the slot plus a certain mass under the dome and under the outer annulus. If the separations between diaphragm and magnet structure are large the problem becomes much simpler since only the mass in the coil slot needs to be considered. The velocity in the slot varies along its width and for any point is given by

$$V = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{Z(Z - h)}{2}, \tag{6}$$

where Z is the distance from the side wall of the slot to the point in question, h is the width of the slot, $\frac{\partial p}{\partial x}$ is the pressure gradient and μ is the coefficient of viscosity of air.[7] If m is the mass per unit volume, the kinetic energy for a unit length of the slot and for a unit length along the circumference is

$$\text{K.E.} = \frac{1}{2}\int_0^h \frac{m}{\mu^2}\left[\frac{\partial p}{\partial x}\right]^2 \frac{Z^2(Z-h)^2}{4}\,dZ. \tag{7}$$

The same kinetic energy expressed in terms of the average linear velocity and the effective mass of the whole width is

$$\text{K.E.} = \frac{1}{2} m_e \frac{1}{\mu^2}\left[\frac{h}{12}\right]^2\left[\frac{\partial p}{\partial x}\right]^2. \tag{8}$$

Comparing the integrated expression of (7) with (8) we find that the ratio of the effective mass to the physical mass in the slot is $\frac{6}{5}$, that is

$m_2 = \frac{6}{5}$ mass of two slots.

Knowing the average linear velocity in the slot it is quite simple to calculate the mechanical resistance as

$$r_2 = \frac{24\,\mu\, l\pi D}{h}.$$

If the diameter of the coil is large compared to the air passages then D can be taken to be the diameter of the coil.

The constants r_0 and s_0 can be found from the location and magnitude of the resonant peak when the diaphragm is not damped by any external resistance. In making such measurements it was found that r_0 was a function of frequency. It is sufficient, however, to choose an average value because r_0 is usually small as compared to the resistance of the damping ring. m_0 is again calculated from a consideration of the kinetic energy. If the diaphragm behaves like a simple piston the dome-shaped center portion will have the same velocity at all points. For the annulus we may assume parabolic deflection. The inner region plus the effective mass of the outer annulus make up m_0.

When we consider the grid we again make the assumption that the air in the holes moves like a slug, and that the frictional losses due to the walls can be neglected. Even the impedance due to the effective mass of the slug itself is less important than its radiation impedance. Since the latter is a function of frequency it is necessary to change r_{-1} and m_{-1} for each frequency which is being considered. An account of a similar problem can be found in I. B. Crandall's "Theory of Vibrating System and Sound" and therefore will not be considered here.[8]

In order to evaluate the constants of the narrow tube used to increase the low-end response we must investigate the discriminant kr. If $|kr|$ lies between the limits $+1$ and $+10$ then the mechanical impedance for a tube of length l and area A_T is given by

$$Z = -\frac{\mu k^2 A_T l}{\left[1 - \dfrac{2}{kr}\dfrac{J_1(kr)}{J_0(kr)}\right]},^*$$

where $k = \sqrt{\dfrac{-\omega\rho i}{\mu}}$, $n = \sqrt{\dfrac{\omega\rho}{\mu}}$, r the radius of tube and ρ is the density of air. $J_1(kr)$ and $J_0(kr)$ are Bessel's functions of first and zero order respectively with complex argument. Substituting for the values of k and expressing J_0 and J_1 in terms of ber and bei functions we have [9]

$$Z = \frac{i\rho\omega A_T l}{\left[1 - \dfrac{2}{nr} \times \dfrac{\text{ber}'\, nr + i\,\text{bei}'\, nr}{-\,\text{bei}\, nr + i\,\text{ber}\, nr}\right]}.$$

If values of this impedance are plotted it will be found that the resistance and mass components vary again with frequency. It is therefore necessary to use a new value for r_T and m_T for each frequency when the response of the network is calculated.

APPENDIX B

The "pressure calibration" of the miniature condenser microphone is measured on the thermophone, but the field calibration must also be determined very carefully. The field correction to be applied to this thermophone calibration is made up of two factors, (1) the diffraction effect of the microphone and (2) the resonance of the small cavity in front of the diaphragm. The latter has been calculated carefully and checked experimentally (Fig. 11).

The condenser microphone itself was used to determine its own diffraction effect. This is possible because of the verified theoretical law giving the diffraction effect to be a function of the product of the diameter and frequency. For example, the diffraction effect that occurs at 2,000 cycles for a 6-inch disc will occur for a 1-inch disc at 12,000 cycles. Since the diffraction effect of the small condenser microphone is essentially that of a cylinder of the same diameter, it was only necessary to measure the effect of a large cylinder in a frequency

* Reference 8, p. 237.

range where the disturbance caused by the test microphone is negligible. This was accomplished by setting the microphone flush into the face of the obstacle when obtaining the disturbed sound pressure. Then by the simple law given, this diffraction effect was applied to the microphone itself and is shown in Fig. 11. The resultant normal incidence field calibration for the small condenser microphone is shown in the same figure.

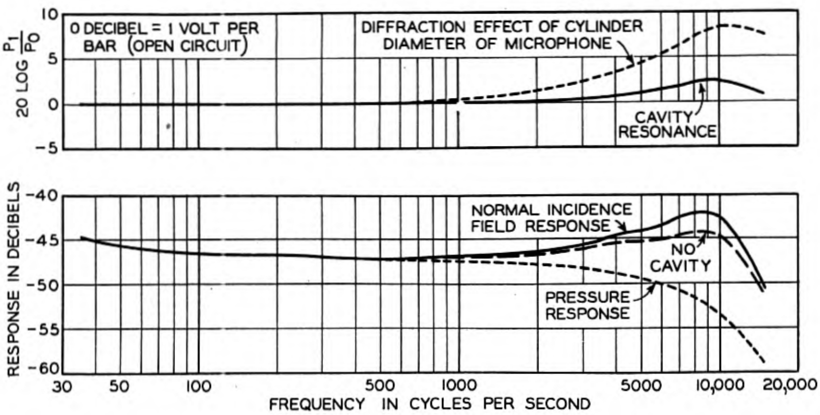

Fig. 11—Response and diffraction of miniature condenser microphone.

References

1. W. C. Jones and L. W. Giles: "A Moving Coil Microphone for High Quality Sound Reproduction," *Jour. Soc. Motion Picture Engineers*, Vol. 17, December 1931, pp. 977–993.
2. Rayleigh: "The Theory of Sound," Vol. 2, 2nd ed., Macmillan & Co., Ltd., 1896, pp. 272–284.
3. Stuart Ballantine: "Note on the Effect of Reflection by the Microphone in Sound Measurements," *Proc. I. R. E.*, Vol. 16, December 1928, pp. 1639–1644.
4. H. C. Harrison and P. B. Flanders: "An Efficient Miniature Condenser Microphone System," *Bell Sys. Tech. Jour.*, Vol. 11, July 1932, pp. 451–461.
5. L. J. Sivian and H. T. O'Neil: "Sound Diffraction by Rigid Circular Plate, Square Plate and Semi-Infinite Screen, *Jour. Acous. Soc. Amer.*, Vol. 3, April 1932, pp. 483–510.
6. P. B. Flanders: "A Method of Measuring Acoustic Impedance," *Bell Sys. Tech. Jour.*, Vol. 11, July 1932, pp. 402–410.
7. Drysdale: "Mechanical Properties of Fluids," D. Van Nostrand Co., Inc., 1924, pp. 113–116.
8. I. B. Crandall: "Theory of Vibrating Systems and Sound," 2nd ed., D. Van Nostrand Co., Inc., 1927, p. 55, pp. 143–149.
9. Dwight: "Bessel Functions for A. C. Problems," *Trans. A. I. E. E.*, Vol. 48, No. 3, July 1929, pp. 812–820.

Oscillations in Systems with Non-Linear Reactance

By R. V. L. HARTLEY

A theoretical study is presented of the properties of a condenser, one plate of which is free to vibrate, when it is included in a circuit containing a generator, the frequency of which is higher than the resonant frequency of the plate and unrelated thereto. It is shown that the plate may be maintained in oscillation at a frequency at or near its mechanical resonance, at the expense of the energy supplied by the generator, provided certain conditions are satisfied. The most favorable condition is one in which the plate is resonant at the frequency of its vibration and the electric circuit is resonant at that of the generator, and at the difference between the generator and plate frequencies, and is anti-resonant at their sum. Under these conditions the generator voltage must exceed a threshold value determined by the impedances and frequencies. This threshold voltage increases as the conditions become less favorable. Expressions are given for the values of the oscillations as functions of the voltage when the threshold is exceeded. When the sum frequency is absent, the energies dissipated at the plate and difference frequencies are in the ratio of the two frequencies.

The oscillations described represent a special case of a class of similar oscillations, all of which depend on the presence of a non-linear reactance. Another special case is a molecular model capable of reproducing the main features of the Raman effect.

Introduction

A TYPE of free oscillation has been found to occur in non-linear coupled systems, which differs from the ordinary type in that the supporting energy is drawn from an alternating sustained source, rather than from a constant source, as in the ordinary vacuum tube oscillator. The particular example of such oscillations to be described here occurs in an electric circuit containing a condenser, one plate of which is elastically supported so as to constitute a mechanically resonant system.

The possibility of such oscillations in a circuit of this kind was discovered [1] in the course of a theoretical study of the possible use of a moving plate condenser as a modulator in a carrier system. Such use was suggested by the fact that, in a condenser, the mechanical force on the plate is proportional to the square of the charge. In this study, it was assumed that a generator of alternating electromotive force of a relatively high carrier frequency was connected with the condenser terminals, and an alternating mechanical force of a relatively low frequency (corresponding to a Fourier component of a speech wave) was applied to the movable plate of the condenser. The plate was not assumed to be resonant. The non-linear relations between

[1] Hartley; *Phys. Rev.*, Vol. 33, p. 289, February, 1929.

charge and mechanical displacement then give rise to currents of the combination, or sideband frequencies. Among the properties of the system which were studied was the reaction of the plate on the mechanical "generator." This was expressed as a mechanical impedance, i.e., the complex ratio of the alternating force to the alternating velocity.

The expression for this mechanical impedance was found to include a negative resistance, which under certain conditions became equal to the positive resistance representing the remainder of the system. It was evident, therefore, that, under these conditions, oscillations of the frequencies involved could persist in the absence of any external driving force on the plate. The existence of such oscillations was first verified experimentally by Mr. E. Peterson. This and a quantitative experimental study of the phenomenon are described in an accompanying paper.[2] Oscillations of the same general type, associated with iron core coils, had been predicted much earlier by the writer and discovered independently by Mr. E. T. Burton.[3]

However, what happened once the threshold condition was passed, was not apparent from this analysis. The answer to this question was found by assuming the existence of the oscillations, computing their values, and determining under what conditions the values are real. Both methods will be employed in what follows.

Representation of the System

In the analysis it will be assumed that, except for the non-linearity associated with the electromechanical coupling, the law of superposition holds throughout. This means that all parts of the system other than the coupling may be represented by linear impedances, of the form

$$\mathcal{Z} = R + iX = Ze^{i\varphi}. \tag{1}$$

"Linear," as here used, means that the impedance is independent of the magnitudes of the oscillations.

If then the plate has an alternating velocity of magnitude V_m and phase θ_m, we represent it by $V_m e^{i\theta_m}$. The resultant of all the linear restoring forces may be represented by a force $Z_m V_m e^{i(\varphi_m+\theta_m)}$. All of the quantities involved will, in general, be functions of the frequency. Similarly a current $I_e e^{i\theta_e}$ will be accompanied by a counter electromotive force $Z_e I_e e^{i(\varphi_e+\theta_e)}$, where Z_e is the impedance of the connected electric circuit in series with that of the condenser with its movable plate at rest in the position of zero displacement.

[2] Hussey, L. W. and Wrathall, L. R.; "Oscillations in an Electromechanical System" in this issue of the *Bell Sys. Tech. Jour.*

[3] Peterson, E.; *Bell Laboratories Record*, Feb., 1929, p. 231.

To evaluate the non-linear forces, consider a parallel plate, air condenser of area, A, and normal separation, x_0, one plate of which is fixed and the other of which is free to move under the action of linear restoring forces. Let the movable plate be displaced a distance, x, in a direction to increase the separation, and let a charge, q, be put on the plates. Then it can easily be shown that the static forces tending to oppose the displacements are

$$= Sx + Kq^2, \tag{2}$$

$$e = \frac{q}{C} + 2Kxq, \tag{3}$$

where S is the stiffness of the constraints on the plate; C is the capacitance, in electrostatic units, when x is zero; and

$$K = \frac{2\pi}{A} \tag{4}$$

is a quantity which will be referred to as the constant of non-linearity.

The first terms of (2) and (3) represent components of the forces which were represented above by the mechanical and electrical impedances respectively. Hence only the last terms need be used in expressing the electromechanical coupling.

We shall assume that there is connected in series with the condenser and its associated electric impedance, a generator of negligible internal impedance, which provides an alternating electromotive force, e_g, of amplitude, E_g, and frequency, ω_g, in radians per second. The phase of this generator will arbitrarily be taken as zero.

For the first part of the analysis, we shall assume that there is an alternating force, f_m, exerted on the plate by a "mechanical generator," which has an amplitude, F_m, frequency, ω_m, and phase, ψ_m. We shall investigate the impedance offered to this force in the resulting condition of forced oscillation. In the second part, the mechanical generator will be omitted, and the free oscillations investigated. It is first necessary, however, to determine what frequencies need be considered.

Possible Frequencies

With the system just described there will be developed oscillations, the frequencies of which constitute an infinite series. It will therefore be necessary to introduce limiting assumptions. First let us consider what frequencies may be present in the system. In doing this it must be recognized that the conventional use of complex quantities is not justified when the system is non-linear. This difficulty is avoided and the advantages of the complex exponential notation are retained if we use the complete exponential expressions for the trigometric functions, since these are real.

Accordingly we shall call the electromotive force of the generator

$$e_g = \frac{E_g}{2}[e^{i\omega_g t} + e^{-i\omega_g t}]. \tag{5}$$

We shall assume that this is accompanied by an alternating current,

$$i_g = \frac{I_g}{2}[e^{i(\omega_g t+\theta_g)} + e^{-i(\omega_g t+\theta_g)}]. \tag{6}$$

We shall call the force exerted by the mechanical generator

$$f_m = \frac{F_m}{2}[e^{i(\omega_m t+\psi_m)} + e^{-i(\omega_m t+\psi_m)}], \tag{7}$$

and the accompanying alternating velocity

$$v_m = \frac{V_m}{2}[e^{i(\omega_m t+\theta_m)} + e^{-i(\omega_m t+\theta_m)}]. \tag{8}$$

When the corresponding displacements, obtained by integration of (6) and (8), are substituted in the last term of (3), the resulting electromotive force is found to consist of components of frequencies,

$$\omega_s = \omega_g + \omega_m, \tag{9}$$

$$\omega_d = \omega_g - \omega_m, \tag{10}$$

which tend to set up currents at the frequencies of the sidebands.

If such currents flow and we substitute the charges associated with them, together with that from (6), in the last term of (2), we find, in the force on the plate, components of frequency ω_m, and a variety of other frequencies including zero, i.e., a steady force. If these produce displacements which are again substituted in (2), and the process is continued, we arrive finally at the entire series of frequencies given by $m\omega_g \pm n\omega_m$, where m and n are integers.

We shall now introduce the limiting assumption that the plate is resonant at or near ω_m, and not at any other frequency. The impedance at that frequency will then be small and the response to the driving force at that frequency relatively large. At the frequencies of all the other components of the force the mechanical impedance will be relatively very high; and we will not be making a violent assumption if we say that it is so high that the velocities of response at all the other frequencies are negligible. [There may be some response to the steady force, consisting of a slight change in the position of equilibrium about which the vibrations occur. This can be taken care of by saying that the coefficients in (2) and (3), while constant for any particular condition of sustained oscillation, vary slightly with the magnitude of the oscillations.] With this assumption the frequencies of the components of the electromotive force reduce to $\omega_g \pm n\omega_m$.

If the electric circuit is resonant at any of these frequencies, we may as above neglect the currents at other frequencies. In the absence of any resonance, if the constant of non-linearity, K, is sufficiently small, the amplitudes will decrease so rapidly with increasing n that we may neglect all for which n is greater than unity. In the interests of simplicity we shall make all of these assumptions. We have then in addition to i_g and V_m the currents,

$$i_s = \frac{I_s}{2}\left[e^{i(\omega_s t+\theta_s)} + e^{-i(\omega_s t+\theta_s)}\right], \tag{11}$$

$$i_d = \frac{I_d}{2}\left[e^{i(\omega_d t+\theta_d)} + e^{-i(\omega_d t+\theta_d)}\right]. \tag{12}$$

Forced Oscillations; Impedance Solution

We wish to set up the equations of motion in terms of the applied forces, the velocities and currents at the various frequencies, and the properties of the system, as expressed in terms of its linear impedances and the constant of non-linearity, K. For each frequency, we equate whatever applied force there may be to the sum of the restoring forces due to the system. These consist of a component given by the product of the velocity or current by the impedance for the particular frequency, and other terms due to the combination of pairs of the other frequencies. To find these latter components, we integrate (6), (8), (11) and (12) with respect to time, substitute the resulting displacements in the last terms of (2) and (3), and select the components of the four significant frequencies, for insertion in their appropriate equations. Once these components are obtained, we may, since the remainder of the system is linear, safely revert to the conventional use of exponentials, so that the factor $e^{i\omega t}$ may be divided out for each equation. The final result is

$$Z_g I_g e^{i(\theta_g+\varphi_g)} + \frac{KV_m I_s}{\omega_m \omega_s} e^{i(\theta_s-\theta_m)} - \frac{KV_m I_d}{\omega_m \omega_d} e^{i(\theta_m+\theta_d)} = E_g, \tag{13}$$

$$Z_m V_m e^{i(\theta_m+\varphi_m)} + \frac{KI_g I_s}{\omega_g \omega_s} e^{i(\theta_s-\theta_g)} + \frac{KI_g I_d}{\omega_g \omega_d} e^{i(\theta_g-\theta_d)} = F_m e^{i\psi_m}, \tag{14}$$

$$Z_s I_s e^{i(\theta_s+\varphi_s)} - \frac{KI_g V_m}{\omega_g \omega_m} e^{i(\theta_g+\theta_m)} = 0, \tag{15}$$

$$Z_d I_d e^{i(\theta_d+\varphi_d)} + \frac{KI_g V_m}{\omega_g \omega_m} e^{i(\theta_g-\theta_m)} = 0. \tag{16}$$

These equations are of the second degree and so are not so simple of solution as are the linear equations of circuit theory which they formally resemble. We note, however, that if, in the last three, we assume I_g and θ_g to be constant, they become linear. We may therefore solve them as linear equations, with this assumption, provided we bear in mind that the resulting impedances will not be linear unless the oscillations are so small that the second and third terms of (13) can be neglected compared with the first.

Let us make this assumption and explore the properties of the resulting linear system represented by (14), (15) and (16). If we calculate $V_m e^{i\theta_m}$ and take the ratio $F_m e^{i\psi_m}/V_m e^{i\theta_m}$, this will be the analog of the impedance of an analogous electric circuit as measured in the mesh corresponding to vibration of the plate at frequency ω_m. This ratio, which we shall call Z_m', may be thought of as the mechanical impedance of the plate when the circuit is activated by the electrical generator. Following circuit theory, as applied to vacuum tubes, let us call Z_m' the active impedance of the plate, and Z_m the passive impedance. The value of the active impedance, when expressed in terms of resistances and reactances, is found to be

$$Z_m' = (R_m + iX_m) + \frac{K^2 I_g^2}{\omega_g^2 \omega_m \omega_s} \cdot \frac{R_s - iX_s}{Z_s^2} + \frac{K^2 I_g^2}{\omega_g^2 \omega_m \omega_s} \cdot \frac{-R_d - iX_d}{Z_d^2}. \quad (17)$$

We see that the active impedance differs from the passive impedance by two terms, each of which represents the effect of the impedance of the electric circuit at one of the side frequencies. The second term of (17), which depends on the impedance at the sum frequency, is identical in form with the impedance added to an electric circuit,[4] at a frequency, ω, by a transformer of mutual inductance, M, provided that

$$M^2\omega^2 = \frac{K^2 I_g^2}{\omega_g^2 \omega_m \omega_s}; \quad (18)$$

the impedance of the secondary circuit is equal to Z_s; and the reactances of the primary and secondary windings are included in X_m and X_s, respectively. The third term which depends on the impedance of the electric circuit at the difference frequency, is similar except that the effective resistance is negative.

It is this negative resistance which makes possible the type of free oscillations here described. To interpret it, let us start with the small

[4] Bush, V.; "Operational Circuit Analysis," John Wiley & Sons, Inc., 1929, p. 50, Eq. (66).

applied force, f_m, acting on the plate, with the voltage of the electric generator zero. The velocity of the plate vibration is then determined by its passive impedance alone. Let us assume for the time being that the impedance, Z_s, of the electric circuit at the sum frequency is infinite, so that its effect on the active impedance of the plate disappears. Let us make φ_d equal to φ_m, and gradually increase the generator voltage. As I_g^2 increases, the negative impedance increases, the total impedance decreases and the velocity, V_m, increases. This condition is analogous with the behavior of the input impedance of a regeneratively connected amplifier when the plate current is progressively increased from zero. At a threshold value of I_g, the net impedance becomes zero and the velocity infinite. This means that a finite velocity can exist for an infinitesimal driving force, that is, the oscillations, once started, are self-sustaining, even in the absence of any sustained driving force, f_m, at the mechanical frequency.

If we make the electric impedance, Z_d, at the difference frequency infinite, all the resistances are positive; so sustained oscillations cannot occur, in a dissipative system, in the absence of current at the difference frequency. If both side frequencies are present, so that Z_s and Z_d are both finite, sustained oscillations are still possible provided the impedance at the sum frequency is not too small compared with that at the difference frequency. The presence of current at the sum frequency always increases the critical value of the current at the generator frequency.

We may also compute the active impedance of the electric circuit at the side frequencies, on the same assumption as to the constancy of the current of generator frequency as was made in deriving (17). To do this, we remove the mechanical generator, making the right member of (14) zero, and insert low measuring voltages of frequencies ω_s and ω_d in the right members of (15) and (16), in turn. In each case we compute the ratio of this voltage to the accompanying current. If we think of each frequency as being the analog of a mesh in an electric circuit, we note that the mesh corresponding to the mechanical frequency is coupled to both of the side frequencies; but the latter are not directly coupled to each other. If the mutual impedances, which depend on I_g, are small enough, we may, for a generator at the sum frequency, neglect the third term of (14), which represents the effect of the loosely coupled difference frequency mesh, compared with the first. The active impedance at the sum frequency then becomes

$$Z_s' = (R_s + iX_s) + \frac{K^2 I_g^2}{\omega_g^2 \omega_m \omega_s} \cdot \frac{R_m - iX_m}{Z_m^2}\,. \qquad (19)$$

If the third term of (14) is not neglected we must replace Z_m in (19), by the first and third terms of (17), that is, by the impedance of the mechanical frequency mesh, as modified by its coupling with the difference frequency mesh.

Similarly, when the measuring generator is of the difference frequency, we get

$$Z_d' = (R_d + iX_d) + \frac{K^2 I_g^2}{\omega_g^2 \omega_m \omega_s} \cdot \frac{-R_m - iX_m}{Z_m^2}, \tag{20}$$

where Z_m is to be replaced by the first and second terms of (17), if the second term of (14) is not neglected.

The active impedance at the difference frequency (20) contains a negative resistance similar to that which appeared at the mechanical frequency (17). In fact, if the passive impedance, Z_s, at the sum frequency is infinite, the expressions for the two active impedances are symmetrical. The active impedance at the sum frequency contains only positive resistances, except in so far as the resistance of the mechanical mesh is made negative by its coupling with the difference mesh. This serves to emphasize the fact that the presence of current of the difference frequency is essential to the oscillations, while that of current of the sum frequency tends to make their production more difficult.

Free Oscillations

In the above considerations it was assumed that the amplitudes at all of the new frequencies were small compared with that at the generator frequency. While this assumption permits us to compute the threshold conditions for the starting of free oscillations, it is violated as soon as the oscillations become appreciable. In order to find out what happens once the threshold is passed it is necessary to solve the second degree equations (13) to (16) when F_m is made zero. The presentation of this solution will be simplified by considering first the case where the sum frequency is eliminated and then the effect of its presence on the simpler solution.

The elimination of the sum frequency is accomplished by making Z_s infinite and I_s zero. This makes the second terms of (13) and (14) zero, and makes (15) indeterminate. We are left then with (13), (14), as modified, and (16). The equations for the mechanical and difference frequencies are now symmetrical. In order to solve these equations we express the exponentials in terms of sines and cosines and equate the real and imaginary parts separately. In the equations derived from (14) and (16) we transpose the second term in each equation to the right member. For each pair we divide the equation containing sines by that containing cosines and obtain a relation between the angles involved. We square each equation of a pair, and add them to obtain a relation between the magnitudes of velocities and currents, the impedances, and the frequencies. From these it follows that I_g is a constant. By means of these relations the equations derived from (13) may be reduced to a form where the only variables are E_g, V_m and θ_g, and the constant, I_g, appears only as a divisor of E_g. These equations are then squared and added to give an equation which determines V_m. The final solution takes the form

$$\varphi_m = \varphi_d, \tag{21}$$

$$I_g = \frac{\omega_g}{K}[Z_m\omega_m Z_d\omega_d]^{1/2}, \tag{22}$$

$$V_m = \frac{\omega_g}{K}\left[Z_d\omega_d Z_g\omega_g\left(-\cos(\varphi_m + \varphi_g) \pm \left\{\left(\frac{E_g}{Z_g I_g}\right)^2 - \sin^2(\varphi_m + \varphi_d)\right\}^{1/2}\right)\right]^{1/2}, \tag{23}$$

$$I_d = \left[\frac{Z_m\omega_d}{Z_d\omega_m}\right]^{1/2} V_m, \tag{24}$$

$$\theta_g = \varphi_m + \alpha \pm \frac{\pi}{2}, \tag{25}$$

where

$$\cos\alpha = \frac{Z_g I_g}{E_g}\sin(\varphi_m + \varphi_g), \tag{26}$$

and the sign in (25) is so chosen that

$$-\frac{\pi}{2} < \theta_g < \frac{\pi}{2}, \tag{27}$$

and

$$\theta_m + \theta_d = \alpha + \pi \pm \frac{\pi}{2}, \tag{28}$$

where the same sign is to be taken for $\pi/2$ as in (25).

The nature of the variation represented by (23) is shown in Fig. 1, which is taken from the accompanying experimental paper.[2] Here the amplitude, V_m/ω_m, of the plate displacement is plotted against the generator voltage, E_g, for the case of exact resonance and for one involving a slight departure from resonance.

Let us interpret these results physically. The phase angles in (21) depend only on the physical constants of the system and the frequencies of the oscillations. This equation, therefore, determines at what frequencies oscillations may occur provided the other conditions are satisfied. Thus the ratio of reactance to resistance must be the same for the plate at ω_m and for the electric circuit at ω_d. This condition is satisfied if each is resonant at its particular frequency, but resonance is not a necessary condition. All that is necessary is that there be a pair of frequencies, whose sum is equal to that of the electric generator, for which the impedances have the same phase angle. If there are an electric and a mechanical resonance such that the sum of the resonant frequencies is nearly equal to the generator frequency, and there is a marked difference in the sharpness of the two resonances, then the oscillations will fall closer to the sharper resonance. This is due to the fact that the phase angle of the impedance changes more rapidly with frequency in the neighborhood of a sharp resonance.

From (22) we see that the amplitude of the current at the generator frequency depends only on this frequency, the constants of the system, and the new frequencies. It is independent of the amplitude of the

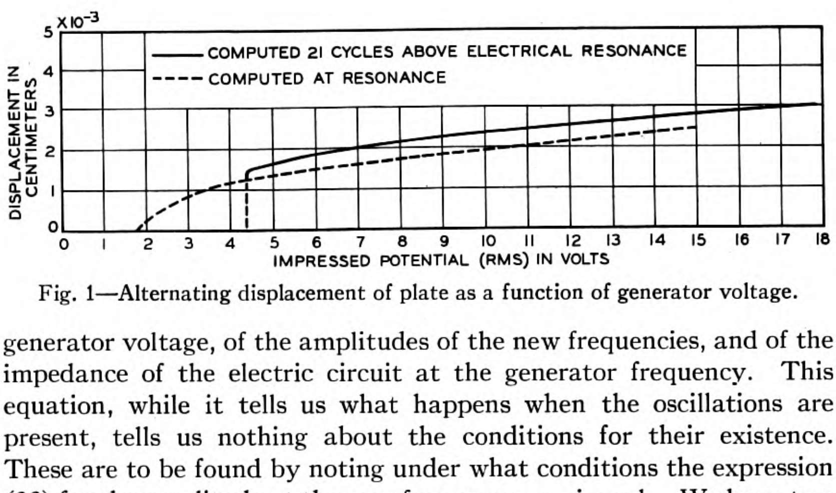

Fig. 1—Alternating displacement of plate as a function of generator voltage.

generator voltage, of the amplitudes of the new frequencies, and of the impedance of the electric circuit at the generator frequency. This equation, while it tells us what happens when the oscillations are present, tells us nothing about the conditions for their existence. These are to be found by noting under what conditions the expression (23) for the amplitude at the new frequency, ω_m, is real. We have two cases to consider which are determined by the sign of $\cos(\varphi_m + \varphi_g)$.

Assume first that this is positive, as would be the case if the plate is resonant at ω_m and there is any dissipation at ω_g, such as would be caused by resistance in the electric circuit. The first term in (23) is negative and V_m can be real only if the second term exceeds it in absolute value. This condition reduces to

$$E_g > Z_g I_g = \frac{Z_g \omega_g}{K} [Z_m \omega_m Z_d \omega_d]^{1/2}. \tag{29}$$

This shows that there is a threshold value of the generator voltage, above which the new oscillations are possible. (It is found to agree with that obtained by the negative resistance method.) Moreover, this value is that which is just necessary to maintain an electric current, of the generator frequency, in the absence of the new frequencies, with an amplitude equal to the constant amplitude that exists in the presence of the new frequencies. For values of E_g large compared with the threshold value, the amplitudes of the new frequencies increase nearly as the square root of the amplitude of the generator voltage.

In the special case of resonance at both ω_m and ω_d, Z_m and Z_d tend to be small and so from (24) the threshold voltage is correspondingly small. This therefore is a particularly favorable condition for the production of the oscillations.

The case where $\cos(\varphi_m + \varphi_g)$ is negative occurs when all of the three impedances are predominantly reactive, the reactances being all of the same sign. The first term of (23) is then positive and V_m will be real if the second term is positive, as it will be if

$$E_g > Z_g I_g |\sin(\varphi_m + \varphi_g)|. \tag{30}$$

For this case, then, the threshold amplitude of the generator voltage may be much less than that required to maintain the current at the constant amplitude, I_g, in the absence of the new frequencies.

In the extreme case where there is no dissipation and the phase angles of the impedances are all $\pm \pi/2$, the threshold voltage reduces to zero and so sustained oscillations are possible in the absence of any generator. (23) and (24) then reduce to forms symmetrical with (22). This means that for such a system the frequencies would be determined by the constants of the system and the amount of energy present, since this would limit the possible amplitudes.

There is some question as to the sign to be given to the inner radical in (23). When $\cos(\varphi_m + \varphi_g)$ is positive the plus sign must be used. When it is negative the plus sign must be used if E_g is greater than $Z_g I_g$. If E_g is between this and the threshold given by (30), either sign gives a real value for the amplitude. When the sign is negative the amplitude decreases with increasing voltage, which appears to be an unstable condition.

Regarding the phases, the condition represented by (27) is imposed because the energy flow must be from the generator to the circuit. Only the sum of the phases of the new oscillations is determined. Their individual values depend on the starting conditions, just as does the phase of a pendulum clock.

One more result may be of interest. This is the relative rates at which energy is dissipated at the two new frequencies. If P_m and P_d

are the powers corresponding to the two frequencies we have

$$\frac{P_d}{P_m} = \frac{I_d^2 Z_d \cos \varphi_d}{V_m^2 Z_m \cos \varphi_m} = \frac{\omega_d}{\omega_m}. \tag{31}$$

Thus the rate of energy dissipation is in the ratio of the frequencies.

Effect of Sum Frequency

The more general case where the sum frequency is also present calls for the solution of (13) to (16) as they stand except for F_m being zero. This may be done by substituting the values of I_s and θ_s from (15), and those of I_d and θ_d, from (16), in (13) and (14), and proceeding in a manner similar to that used above. The results take the form

$$\varphi_m = \gamma, \tag{32}$$

where

$$\tan \gamma = \frac{Z_s\omega_s \sin \varphi_d + Z_d\omega_d \sin \varphi_s}{Z_s\omega_s \cos \varphi_d - Z_d\omega_d \cos \varphi_s}, \tag{33}$$

and the signs of sin γ and cos γ are determined by the numerator and denominator of (33) respectively;

$$I_g = \frac{\omega_g}{K}\left[\frac{Z_m\omega_m Z_d\omega_d}{a}\right]^{1/2}, \tag{34}$$

where

$$a = \left[1 + \left\{\frac{Z_d\omega_d}{Z_s\omega_s}\right\}^2 - 2\frac{Z_d\omega_d}{Z_s\omega_s}\cos(\varphi_d + \varphi_s)\right]; \tag{35}$$

$$V_m = \frac{\omega_m}{K}\left[\frac{Z_d\omega_d Z_g\omega_g}{b}\left(-\cos(\delta + \varphi_g) \pm \left[\left(\frac{E_g}{Z_g I_g}\right)^2 - \sin^2(\delta + \varphi_g)\right]^{1/2}\right)\right]^{1/2}, \tag{36}$$

where

$$b = \left[1 + \left(\frac{Z_d\omega_d}{Z_s\omega_s}\right)^2 + 2\left(\frac{Z_d\omega_d}{Z_s\omega_s}\right)\cos(\varphi_d - \varphi_s)\right]^{1/2}, \tag{37}$$

and

$$\tan \delta = \frac{Z_s\omega_s \sin \varphi_d + Z_d\omega_d \sin \varphi_s}{Z_s\omega_s \cos \varphi_d + Z_d\omega_d \cos \varphi_s}; \tag{38}$$

$$I_d = \left[\frac{Z_m\omega_d}{Z_d\omega_m a}\right]^{1/2} V_m; \tag{39}$$

$$I_s = \frac{Z_d}{Z_s} I_d; \tag{40}$$

$$\theta_g = \delta + \alpha' \pm \frac{\pi}{2}, \tag{41}$$

where

$$\cos \alpha' = \frac{Z_g I_g}{E_g} \sin (\delta + \varphi_g); \tag{42}$$

$$\theta_m + \theta_d = \alpha' + \pi \pm \frac{\pi}{2} + (\delta - \varphi_d); \tag{43}$$

$$\theta_s - \theta_m = \alpha' \pm \frac{\pi}{2} + (\delta - \varphi_s), \tag{44}$$

where the sign of $\pi/2$ is again to be chosen so as to satisfy (27).

Corresponding to (21) we have (32). If the mechanical motion involves any dissipation, the mechanical resistance, $Z_m \cos \varphi_m$, must be positive, and since Z_m is positive by definition, $\cos \varphi_m$ must be positive. This means that (32) can be satisfied only if the denominator of (33) is positive. Hence oscillations can occur only if

$$\frac{Z_d \omega_d}{Z_s \omega_s} < \frac{\cos \varphi_d}{\cos \varphi_s}. \tag{45}$$

This relation can hold when Z_d, the impedance at the difference frequency, is infinite, only if φ_s is $\pm \pi/2$, that is, if there is no dissipation at the sum frequency.

To investigate the relative rates of dissipation at the sum and difference frequencies, we find the ratio of the powers P_s and P_d, associated with them.

$$\frac{P_s}{P_d} = \frac{Z_d \cos \varphi_s}{Z_s \cos \varphi_d} < \frac{\omega_s}{\omega_d}. \tag{46}$$

Thus the ratio is always less than the ratio of the frequencies and approaches it only as the limiting condition for oscillations is approached.

A discussion of all possible values of impedance and phase angle at the two side frequencies would be too involved to go into here. The special case of resonance at both frequencies is, however, of some interest since a given current is then accompanied by a maximum of dissipation. It also provides that ω_m coincides with the mechanical resonance, where Z_m is much smaller than for nearby frequencies. Since Z_m enters into the expression for the threshold force, this condition is particularly favorable for the occurrence of oscillations. When we make φ_d and φ_s zero we see from (45) that the impedances, now pure resistances, must be such that

$$\frac{Z_d \omega_d}{Z_s \omega_s} < 1. \tag{47}$$

(35) now becomes

$$a = 1 - \frac{Z_d\omega_d}{Z_s\omega_s}. \tag{48}$$

From (34) it is evident that when Z_s is finite the constant current of the generator frequency, and so also the threshold voltage, are greater than when it is infinite. Thus the presence of current of the sum frequency makes the conditions for oscillation more exacting. As we approach the limiting impedance ratio, where the powers approach the ratio of their frequencies, the threshold voltage approaches infinity, and the probability of oscillations approaches zero.

The relative powers at the difference frequency and at the mechanical frequency are now given by

$$\frac{P_d}{P_m} = \frac{\omega_d}{\omega_m\left(1 - \dfrac{Z_d\omega_d}{Z_s\omega_s}\right)}. \tag{49}$$

The presence of a finite impedance at the sum frequency increases this ratio over that of the frequencies. For the limiting condition of oscillations it approaches infinity, the amplitude at the difference frequency then becoming infinite and that at the mechanical frequency remaining finite.

From these results it appears that proportionality between power and frequency is a limiting case which occurs only under the conditions which are most and least favorable for the existence of oscillations. We should, therefore, expect to find it only under the favorable conditions where the transformation of energy is from a higher to a pair of lower frequencies.

Effect of Other Frequencies

In the interests of simplicity the above treatment was limited to the case where all but four frequencies are suppressed by high impedances. Such a limitation is not, however, essential to the production of oscillations. In fact, as many as desired of the series $m\omega_g + n\omega_m$ may be produced by the proper choice of impedances and the use of high enough voltages, provided, of course, the apparatus can withstand the stresses involved. In general, the presence of certain frequencies will be favorable to oscillations and that of others unfavorable.

Summary

By way of summary, then, it is possible to maintain a movable condenser plate in sustained oscillation by applying to the condenser an alternating electromotive force of an unrelated higher frequency, provided that the impedances of the system at these two frequencies and their various combinations satisfy certain relations, and the applied electromotive force exceeds a threshold value. When the oscillations are negligible at all frequencies except these two and their sum and difference, the most favorable condition, lowest threshold voltage, occurs when the plate vibrates at its resonant frequency, and the electric circuit is resonant at the applied frequency and at the difference frequency, and anti-resonant at the sum frequency. Once the oscillations start, the current of the applied frequency remains constant with increasing voltage. Under the most favorable conditions the rates of energy dissipation at the plate and difference frequencies are in the ratio of the frequencies.

Other Applications; Raman Effect

While in the case considered above the production of oscillations was associated with a particular type of non-linearity, the application of the principle is much more general. Here the non-linearity occurs in what might be called a mutual stiffness, serving to couple two degrees of freedom. It is not essential, however, that the non-linearity occur in a mutual impedance nor that the impedance be of the stiffness or negative reactance type. So long as the connected system is such as to provide the proper impedances, oscillations may occur in connection with any non-linear reactance.

A non-linear reactance, as here used, may be defined as any energy-storing element in which the coefficient of inertia is a function of the velocity, or that of stiffness is a function of the displacement, or any mechanical, electrical or electromechanical analog, of such an element. For a non-linear inertia, as in an iron core inductance coil, however, the power varies inversely as the frequency; instead of directly as for a non-linear stiffness.

A special case, in which one of the new frequencies is an exact submultiple of the driving frequency, has been studied by a number of workers from Rayleigh [5] down to Pedersen.[6]

Another special case may be of some interest to physicists because it provides a model of the Raman effect. The transition from the condenser to the molecular model will be made in two steps. For the first suppose that instead of making the resonant mechanical member one plate of a condenser, we attach the moving part to a point on its support by an elastic string under tension, the direction of the string

[5] Rayleigh; "Theory of Sound," *Sec. Ed.*, Vol. 1, p. 81.
[6] Pedersen; *Jr. Acous. Soc. Amer.*, Vol. VI, 4, p. 227, April, 1935.

being parallel to the direction of vibration. Suppose now that at some point along the string we apply an alternating mechanical force, acting normal to the string, through the medium of a mechanical structure, which as viewed from the string may be represented by a linear mechanical impedance. This structure prevents motion of its point of attachment to the string, in the direction of the string.

If now we analyze the forces and motions into their components in the direction, x, of the motion of the vibrating member and that, y, of the applied force, we find that, to a first approximation, the relations connecting them are identical with those used above for the condenser, provided we identify the force and velocity of the vibration in the x direction with those of the condenser plate and those of the point of attachment of the string, in the y direction, with the electromotive force and current in the electric circuit associated with the condenser. Such a structure can therefore produce oscillations of the sort described, provided the mechanical impedance of the driving structure has the proper values at the sum and difference frequencies.

Suppose now we have a molecule which we assume to be rigid with the exception of one atom, which is bound to it by a pair of electrons. Let the attached atom correspond to the plate, the relatively heavy molecule to the support and the electrons to the point of application of the driving force. Let the forces of electrostatic attraction between the electrons and the atom, and between the electrons and the center of the molecule, correspond to those due to the tense strings. Let the other static forces between the atom and the molecule correspond to the stiffness of the plate. For small displacements these forces may be assumed to vary linearly with distance, and so be capable of representation by constant coefficients of stiffness which correspond to the elasticities in the mechanical system. The applied external force is that exerted on the electrons by that component of the incident light which is normal to the line through the centers of the undisplaced particles. The mechanical impedance of the electrons for motion in the direction of the applied force corresponds to that of the structure through which the force is applied. This impedance includes the effects of any elastic constraints the rest of the molecule may exert on the electrons in this direction; of the electromagnetic mass of the electrons, which may be affected by the reactions of neighboring molecules; and of the dissipation of energy as radiation or by transfer to neighboring molecules.

Unlike other classical models of the Raman effect, this one provides for the persistence of the difference line, and the disappearance of the sum line, at low temperatures. It also provides that the intensity of the lines should depend on the probability that the force exerted by the incident radiation on the electrons of a randomly chosen molecule exceed a threshold value which is determined by the condition of its neighbors. The apparent smallness of this probability would explain the observed weakness of the Raman lines.

It would seem that this threshold, and the probability of its being exceeded, might prove helpful in interpreting the energy threshold and transition probability which are used in wave mechanics.

Acknowledgment

In conclusion, I wish to express my thanks to those of my colleagues who by discussions and suggestions have contributed to the preparation of this paper, and in particular to Mr. L. A. MacColl for some valuable suggestions on the mathematical side.

Oscillations in an Electromechanical System

By L. W. HUSSEY and L. R. WRATHALL

Experimental results are given on an oscillating electromechanical system in which, under a single frequency impressed electromotive force, mechanical vibrations are sustained at a frequency near the resonant frequency of the mechanical system and electrical oscillations at the difference between the frequency of the mechanical vibration and that of the impressed force.

The system is the one studied analytically by R. V. L. Hartley in an accompanying paper. Its performance conforms to the principal operating features predicted in his analysis.

IN AN accompanying paper [1] an analytic investigation is made of a system involving a non-linearity in the coupling between an electrical and a mechanical system. The electro-mechanical system under discussion is, in its simplest form, a condenser, with one plate sharply resonant mechanically, a generator, and an impedance, all connected in series. If the charge on the condenser is q, there will be a force on the mechanical system proportional to q^2. While the mechanical system and the electrical system involved are individually linear, there is a non-linearity in this electrostatic coupling, and hence the possibility exists of mechanical and electrical vibrations at other frequencies than the impressed frequency. On this basis the possibility of the generation of a mechanical vibration, *not at a harmonic of the impressed electromotive force*, and electrical currents at the difference between the frequency of the mechanical vibration and that of the impressed electromotive force was predicted by the analysis.

That the phenomenon discussed can occur was first verified by Mr. Eugene Peterson. A condenser microphone was given a mechanical resonance at 600 cycles per second, by cementing a small metal ball to the center of the diaphragm. An alternating electromotive force at 2200 cycles per second was impressed and the system given a series resonance at the difference frequency, 1600 cycles per second, by means of an inductance. When the impressed voltage was increased beyond a critical value mechanical vibrations suddenly built up and current of the difference frequency, larger in amplitude than the current of the impressed frequency, appeared in the electrical system. The same result was obtained using a prong of a tuning fork as the vibrating plate.

[1] "Oscillations in Systems with Non-Linear Reactance" by R. V. L. Hartley, in this issue of the *Bell Sys. Tech. Jour.*

These tests, while they exhibited the most important characteristic predicted, did not give any other check on the validity of the mathematical results because the systems differed so greatly from that assumed by Mr. Hartley. To obtain simple results in the theoretical discussion it was found necessary to make some severely restricting assumptions. The mechanical system was assumed rigid except to motion at frequencies near the resonant frequency. Similarly the electrical system was assumed to have infinite impedance to all frequency components except that impressed and the difference frequency between the impressed and the mechanical. Thus the only currents and velocities present were of the frequencies (in radians per second), ω_m (mechanical), $\omega_d = \omega_g - \omega_m$ (difference), ω_g (impressed).

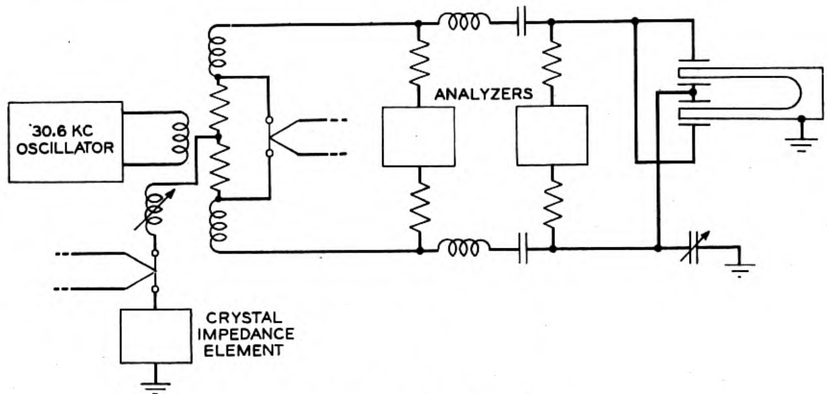

Fig. 1—Circuit diagram.

Under ordinary conditions a non-linear system, such as this one, involving two frequency components (ω_g and ω_m) would have, as modulation products, currents and velocities of all the possible combination frequencies ($r\omega_d + s\omega_m$, r, $s = 0, \pm 1, \pm 2, \cdots$). There would be dissipation of energy at each of these frequencies. These components (other than the three of interest) are the ones which must be suppressed if the system is to be a good approximation to the assumed one.

In order to satisfy the above conditions the circuit of Fig. 1 was constructed.[2] The use of the parallel system instead of a simple series circuit had several advantages. The forces on the tuning fork were so balanced that any constant displacement was avoided. Since the tuning fork had a very sharp resonance[3] the mechanical system was

[2] The tuning fork with condenser plates was designed by Mr. W. A. Marrison.
[3] The damping effect of the air was avoided by operating the fork in a vacuum.

a close approximation to the assumed one. On the electrical side the parallel system had the advantage that the current of the impressed frequency (ω_g) flowed around the outside while the sum ($\omega_g + \omega_m$) and difference ($\omega_d = \omega_g - \omega_m$) frequency components flowed through the mid-branch. The sum frequency component was the most difficult one to suppress. This could be done in the parallel system by means of a piezo-crystal impedance element,[4] tuned to ω_d without at the same time putting in a high impedance to ω_g. This piezo-crystal element had so sharp a resonance that, while its impedance at resonance (30 kilocycles per second) was only 125 ohms, it was about 60,000 ohms only 1000 cycles per second away from resonance. Thus the very high impedance to the sum frequency was obtained. There were, however, some modulation products which flowed around the outside with the impressed current. Only the impedance of a simple tuned circuit was available around this circuit so any product near in frequency to the impressed frequency would not face a very high impedance. The nearest one was $\omega_g - 2\omega_m$. This was not as completely suppressed as the other unwanted products but changing the impedance so that this component was considerably different in magnitude had no appreciable effect on the other electrical components. While this circuit gives a good approximation to the hypothetical one, the result is a highly critical system demanding very fine adjustment and a highly stable generator, since a very slight frequency change has an appreciable effect on the impedances presented by the very sharply tuned circuits.

The measurement of impressed current was made by the thermocouple across small resistors in the input transformer and checked by a current analyzer.[5] Corrections were made in the results when necessitated by the presence of the current component of frequency $\omega_g - 2\omega_m$. The corresponding voltage was obtained by measuring the current of that frequency through the large resistors R_1 by means of a current analyzer. The current of the difference frequency was measured by a thermocouple in the mid-branch. Measurement was also made of the voltage of frequency $\omega_g - 2\omega_m$ across the fork, by measuring the corresponding current through the resistances R_2. There was no simple means available for measuring the mechanical amplitude but it could be obtained from this measurement of $\omega_g - 2\omega_m$ voltage as will be explained later.

On Figs. 2 and 3 are shown curves computed from equations 28, 29 and 30 of the preceding paper for two cases, and the results of the

[4] Designed by Mr. W. P. Mason.
[5] See A. G. Landeen: "Analyzer for Complex Electric Waves," *Bell Sys. Tech. Jour.*, April, 1927.

experiment. The two cases computed are for the system operating with ω_d and ω_m exactly the frequencies of electrical and mechanical resonance and for operation with ω_d 21 cycles higher than resonance. In the latter case ω_m is determined by the requirement—noted by Mr. Hartley—that the phase angles of the impedances be equal. The actual change in ω_m is only a fraction of a cycle. The two cases are given because it was not possible, with the equipment available, to determine ω_d with sufficient accuracy to distinguish between them.

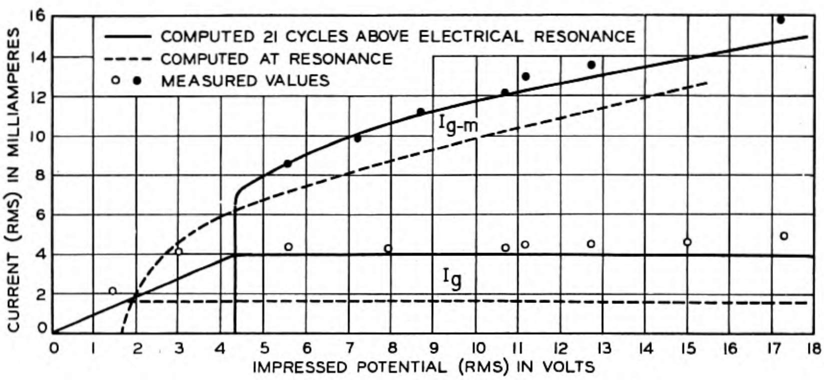

Fig. 2—Electrical current components.

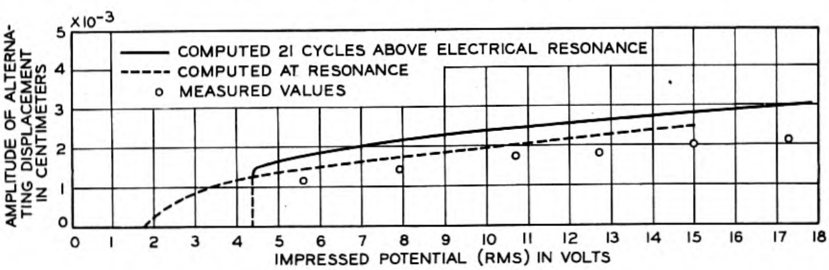

Fig. 3—Mechanical displacement.

While the frequency difference between them is only 20 cycles out of 30,000 it will be noted that, because of the critical character of the impedances, the results differ considerably in amplitudes of the components and in the threshold value.

The second case checks very closely as far as the electrical results are concerned and the mechanical results are of the same order of magnitude. The discrepancy can reasonably be laid to the inaccuracies in the indirect method of measuring the mechanical amplitude. More important is the verification of the outstanding properties predicted by the analysis. There is a threshold voltage above which the new frequency components suddenly appear and rapidly build up to large amplitudes as the voltage is increased. The current of the impressed frequency, ω_g, remains practically constant and independent of voltage above the threshold.

There remains to be described the method by which the mechanical amplitude was obtained. The current voltage relation for a condenser is

$$V = \frac{1}{C}\int i\,dt.$$

The current through the condenser involves only three frequency components in this case, so it can be written in the form

$$i = A_g \cos(\omega_g t + \varphi_g) + A_d \cos[(\omega_g - \omega_m)t + \varphi_d]. \\ + A_e \cos[(\omega_g - 2\omega_m)t + \varphi_e)]$$

and the capacity of a condenser, in e.s.u., is

$$C = \epsilon\frac{A}{S} = \frac{\epsilon A}{(S_0 + S_m \cos \omega_m t)} \text{ farads}$$

where

$$\epsilon = 8.85 \times 10^{-14} = \text{permittivity},$$

A = plate area in cm.^{2},
S_0 = constant, or average spacing cm.,
S_m = amplitude of mechanical displacement.

From these equations the amplitude of the mechanical vibration in terms of the electrical amplitudes can be determined. Neglecting phase angles the relation is

$$S_m = \frac{V_e - \dfrac{S_0 A_e}{\epsilon A(\omega_g - 2\omega_m)}}{\dfrac{A_d}{2\epsilon A(\omega_g - \omega_m)}}$$

V_e = amplitude voltage component of frequency $\omega_g - 2\omega_m$.

The neglect of the phase angles will make some inaccuracy in the results. They would be exact, in the above formula, were A_e negligibly small. The term involving A_e is a correction term necessitated by the incomplete suppression of that frequency component.

A New Type of Underground Telephone Wire

By D. A. QUARLES

A new type of telephone line is described in which a specially insulated *twin* wire is plowed into the soil. Problems of wire design, splicing and maintenance are discussed and transmission characteristics are given.

IN this day of multi-channel transmission on open-wire lines, lead-covered coaxial and multi-wire cables, and of radio and ultra-high-frequency transmission without lines at all, it behooves the development engineer concerned with line structures to be alert to advanced, even to radical, ideas. Rubber insulated telephone wire placed directly underground is a case in point.

The urge to put telephone lines underground is only a littler younger than the business itself. In large measure, this has been realized by installing lead-covered cables in underground duct systems. An alternative arrangement used more recently is spoken of as *buried cable*.[1] This is lead-covered cable, the sheath of which is protected from corrosion by successive layers of paper and jute flooded with asphalts. In addition, as a provision against mechanical injury or interference from outside electrical sources, a steel tape armoring is sometimes used. Where conditions have been favorable, the practice of burying suitably protected cables directly in the ground has been applied both to toll and to large and small exchange area cables and to one and two-pair entrance cables for underground service connections.

Because, with underground cables in conduits and with buried cables, the costs essentially limit their use to those cases where appearance is an important factor or where a considerable number of circuits can be grouped under the same sheath, these methods are not generally applicable to service on one or two circuit routes, such as those extending to remote subscribers, typical of rural distribution. Particularly in the interest of providing a lower cost type of plant and thereby making possible a more extensive development of service in rural communities, it appeared there would be a considerable incentive for the development of an inexpensive form of buried circuit. Such a circuit would obviously require that an economical means of installation be devised and even more important that the material used be serviceable for a

[1] C. W. Mier and B. D. Hull, *Bell Telephone Quarterly*, Volume 8 (October 1929).

long period under the severe moisture conditions to which it would be subjected in the ground.

Experience with the cable burial problem had led to the development of a cable laying plow, the neat operation of which in plowing cable into the ground at depths ranging up to thirty inches without trenching or backfilling in the ordinary sense has been described elsewhere.[2] The adaptation of this method to the burial of wire at an appropriate depth required that it be simplified so that it would be less expensive, and involved such considerations as the very much smaller size and tensile strength of the wire, its greater vulnerability to mechanical injury, the need for reducing and simplifying traction requirements, and the like.

On the point of serviceability, it remained for our research chemists first to develop a rubber compound that could be relied on to maintain suitable insulating properties over a period of years under the severe moisture conditions under ground.

With these fundamentals in hand, the development engineers undertook to study the mechanical and electrical problems involved and design a wire that would have appropriate transmission and handling characteristics. In addition, they had to devise methods of splicing the wire; to adapt plow equipment to its installation; to develop loading arrangements for use on the longer lengths; to study methods of tracing the path of the wire and locating faults, etc. In short, the job was to develop buried wire as a practicable plant instrumentality.

The Insulated Wire

The wire as actually developed employs 17-gauge annealed and tin-coated copper conductors, insulated in parallel twin construction with a special rubber compound designed to withstand long water immersion without serious deterioration of the electrical properties. The wire is adapted to a continuous process of extrusion and vulcanization.[3]

While the insulation of this wire is very resistant to water absorption, it is, in common with most high grade rubber insulating compounds, quite sensitive to sunlight so that it must be carefully guarded from any unnecessary exposure to direct rays of the sun and from any extended exposure to indirect rays.

Splicing

One of the principal problems in using a wire of this kind is that of splicing, as it is quite obvious that the splice must be essentially as

[2] C. W. Nystrom, *Telephony*, Volume 98 (June 21, 1930).
[3] S. E. Brillhart, *Mechanical Engineering*, Volume 54, pages 405–9 (1932).

resistant to water absorption as the wire itself. The splice actually developed (see Fig. 1) has two features of interest: the joints in the conductor proper and the method of patching the insulation. The

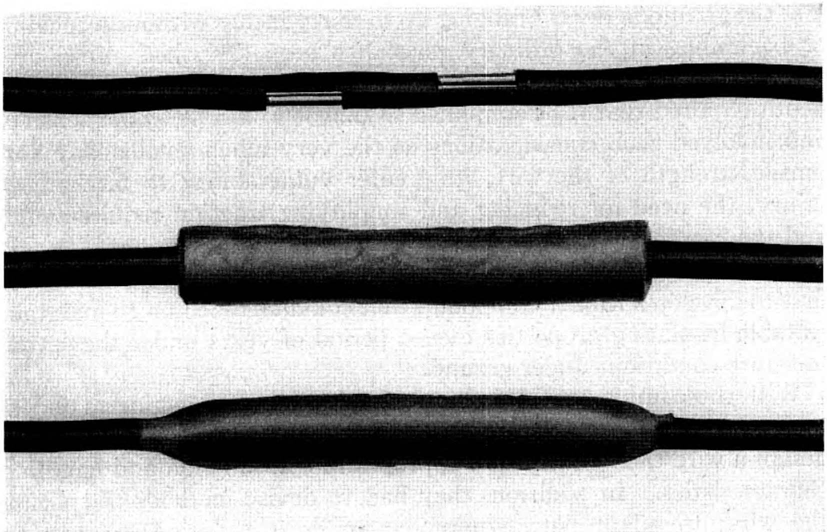

Fig. 1—Splicing buried wire. Top: pressed sleeve joints in conductors. Center: unvulcanized rubber pad in place. Bottom: after vulcanization.

conductor joint is made by pressing a cylindrical sleeve on the abutted ends of the wires to be joined, in this way producing a tight joint of high electrical efficiency and relatively immune to corrosion. The joints in the two wires are staggered and the whole encased in a pad of unvulcanized rubber which is pressed in place and vulcanized in an electrically heated mold, shown in Fig. 2. The vulcanizer is equipped with a thermostatic device to insure proper control of the temperature. This splice is intended for burial directly in the ground without other protection and tests indicate it to be the equivalent of the unspliced wire.

Electrical Properties and Loading

In cross-section, the insulated twin is an oval having a major diameter of about .33″ and a minor diameter of about .165″. The cross-section has been designed to give optimum electrical characteristics for the amounts of copper and rubber compound employed per unit length. The average mutual capacitance per thousand pair feet after seven days water immersion is about 0.022 mf. with an average loop resistance per thousand feet of about 10.2 ohms. While the trans-

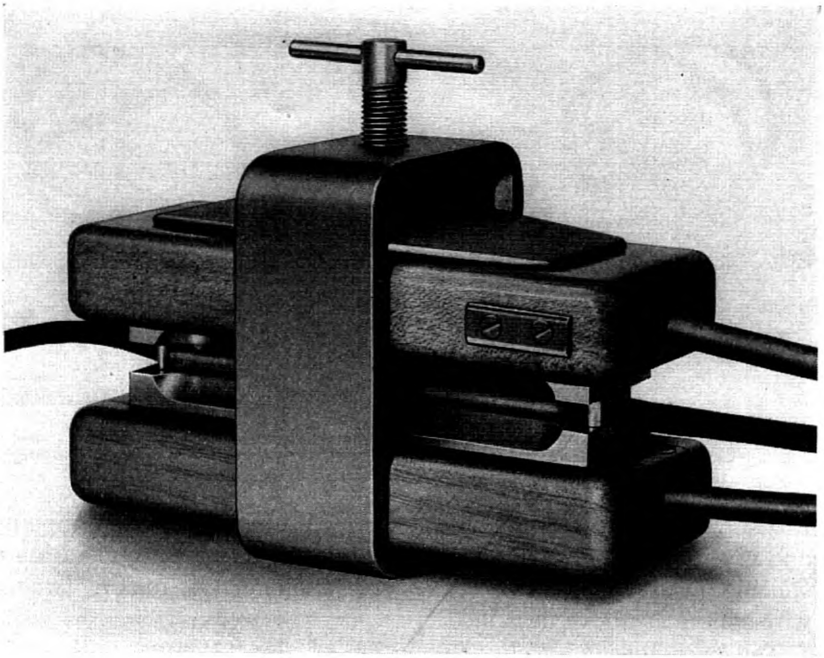

Fig. 2—Vulcanizer for buried wire splice. The buried wire is in the center and storage battery leads for heating vulcanizer are above and below on right.

mission requirements to be placed on a buried circuit will, of course, depend upon the facilities with which it is associated, it is expected that buried circuits up to about five miles in length will, in general, not require loading. Where loading is required, provision has been made for it in the form of a permalloy dust [4] core coil having 44 millihenries inductance which is individually potted with rubber insulated lead-out wires. It is intended to be spliced into the wire at 8,000-foot intervals and buried directly in the ground with the wire.

The potting arrangement for the buried wire coil has several features of interest. The loading coil is first potted in a small metal container which is vacuum impregnated with a moisture resistant compound. The lead-out wires from this container are then spliced to stub lengths of the buried wire, as shown in Fig. 3. This container is then placed in a larger sheet copper container, the rubber insulated wires being brought out through tubes soldered into the copper container and pressed down into intimate contact with the rubber insulation. The lead-out wires are taped for reinforcement at the outer ends of the

[4] G. W. Elmen, *Bell System Technical Journal*, Volume 15 (January 1936).

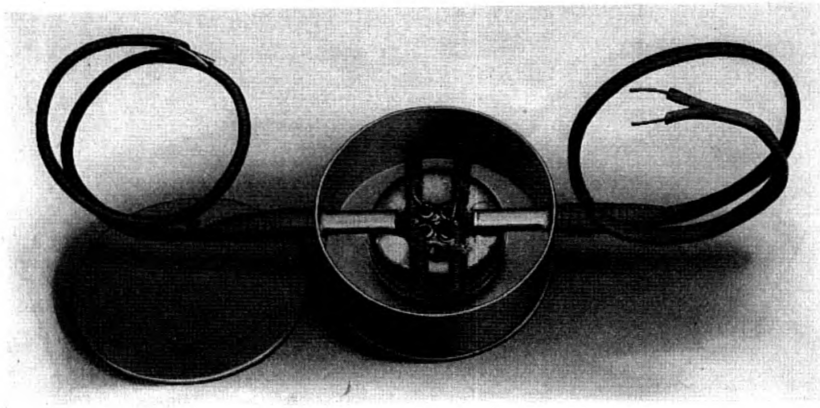

Fig. 3—Loading coil for buried wire before filling outer case, showing the splicing of the rubber covered stubs to the lead-out wires from the inner case.

tubes. This outer can is then filled with a moisture-proof compound and given a dip coating of moisture-proof enamel. The operation of splicing the loading coil into the line wire then involves making two line wire splices as above described.

The one-thousand-cycle attenuation of this 17-gauge buried wire after seven days water immersion and at 70° F. is about 1.1 db per mile for the non-loaded line and the corresponding attenuation of the loaded line is about .49 db per mile. The characteristic impedance of the non-loaded line at one thousand cycles and under the same conditions as above would be $275\overline{/40^\circ}$ and of the loaded line would be $525\overline{/8^\circ}$. The nominal cut-off frequency of the loaded circuit is 3600 cycles.

Layout of Buried Circuits

At the present time, the most promising use of buried wire in the telephone plant appears to be for rural distribution on routes requiring one or two pairs. These routes would commonly have a number of party line subscribers, each subscriber being bridged on the buried circuit. For the most part, it has been found preferable to follow the route of existing public roadways, laying the wire in the shoulder of the road. Installing the wire on right-of-way across private property is advantageous under some circumstances, however. At points where a service connection is to be made, there is the alternative of bridging a service lead across the through circuit or looping the circuit into the subscriber's house, the latter being preferred where the house is a short distance from the through route. Where a bridged connection is to be made, it has been found desirable to bring the wires up above ground for binding post cross-connection so as to provide a test point rather than splicing the bridging wire in permanently and burying the splice in the ground. A small terminal has been provided for this purpose. The buried wires are brought up into the terminal housing through a galvanized iron pipe in order to provide protection of the rubber insulation from sunlight. The terminal is mounted on any convenient pole or post or on a short stub set for the purpose.

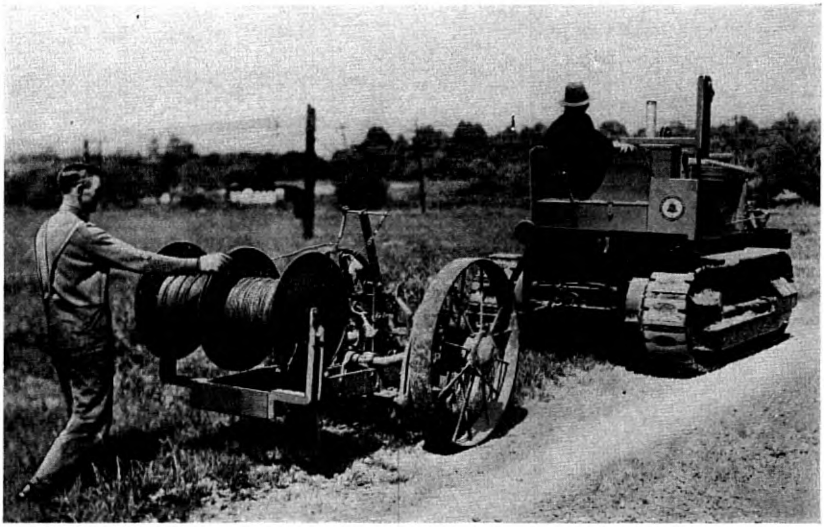

Fig. 4—Plowing-in two pairs of buried wire along roadside.

As buried wire will, in general, be associated with exposed wire circuits, it is planned to provide the same type of electrical protection at subscribers' premises for buried circuits as for drops from open wire or exposed cable circuits. It is also planned to provide protection for buried wire at junctions with open-wire lines over one-half mile in length.

Plowing-in Operations

The success of buried wire is in considerable measure dependent upon the efficiency of the equipment provided for plowing it into the ground. This problem has therefore been studied carefully with a view to reducing the traction requirements to a minimum for the desired depth of placing, so as to permit the use of readily available tractive equipment. Experiments have indicated that in a given type of soil the tractive load on the plow increases approximately as the square of the depth of setting. The choice of depth is, of course, a matter of judgment and is influenced somewhat by the local conditions. In general, it is felt that depths between 16″ and 20″ are adequate and that more shallow installations are justified only under special conditions.

The plow equipment that has been developed for this purpose is shown in operation in Fig. 4 and in more detail in Fig. 5. The plowshare is a vertical blade with a tube fastened to the back edge through

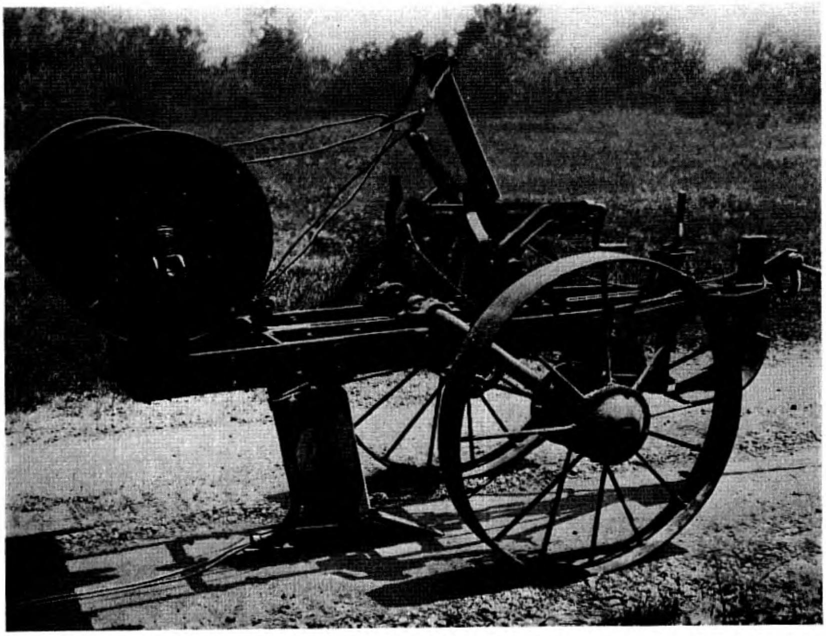

Fig. 5—Wire plow in elevated position, showing duct for wires at back of plow-share.

which one or two pairs of wires may be fed into the soil. The depth of the blade in the ground is readily adjustable to meet local conditions. It has been found that in fairly hard soil with a liberal supply of rock, an equivalent of a 40 or 50 hp caterpillar tractor is required to draw the plow. Across stretches of private right-of-way or at other locations where it is not convenient to use tractors or trucks in direct traction, however, alternative methods have been employed, such as using the winch line of a construction truck to pull the plow. The speed at which the plow may be operated is controlled largely by the number and character of obstacles encountered. Under favorable conditions, the plow may be operated at a speed of three or four miles an hour but a much lower average is to be expected under the less favorable conditions commonly found.

One consideration of some importance in installing wire of this type is the possibility of the insulation being crushed by boulders displaced by the plow, particularly where the trench with wire in place is to be rolled down or subjected to heavy traffic. This danger is, in fact, of such importance that buried wire of this type is probably not a serviceable form of construction through a terrain where nested boulders are frequently encountered.

While it is generally possible to plow across gravel highways, this method can not be used when hard-surface highways are encountered, and in such cases it becomes necessary either to use a pipe pushed under the roadway or to span the highway with open wire. Where conditions are such as to require routing the wire through or over culverts, across ditches, streams and the like, involving actual or potential exposure of the wire as by soil erosion, iron pipe or equivalent protection against mechanical injury and light will generally be required.

Interference

As in the case of other types of telephone circuits, the problem of avoiding noise and crosstalk must be considered. Where more than one pair is buried at a time, there is a crosstalk problem but experience has shown that the introduction of twists every few feet, either by twisting the wire in the process of laying or by having it pretwisted on the reels, is sufficient to reduce the crosstalk to low values.

Special care must be given the wire in manufacture to assure a good degree of balance between the capacitances of the two conductors to ground. This is important in order to avoid noise in the buried wire circuits when they are exposed to power circuits or when the connected open wire is exposed. Under severe exposure conditions, even with the best balance obtainable in manufacture, it may be necessary to resort to special balancing measures in the field to assure satisfactorily quiet circuits.

Maintenance Questions

The introduction of a new type of plant such as buried wire will naturally involve some new maintenance problems. Although records will, in general, be kept of buried wire routes, it will at times be desirable to have fairly precise methods of tracing the underground path. Experiments have indicated that this may be done with considerable precision by putting a tone current on the wire and following along the surface of the ground with an exploring coil device. The location of faults in buried wire also involves some problems which are different from those experienced with cable circuits but experiments have indicated that established methods may be adapted to this new use with an acceptable degree of precision.

Conclusion

As less than fifty miles of buried wire circuits of the type described have actually been installed and put into service, it is recognized that many problems may yet arise and that this type of plant should still be considered as in a trial stage. It is, for example, not known to what extent burrowing rodents such as gophers might cause difficulties. Soil erosion may also introduce problems not as yet clearly visualized. On the other hand, many wind, ice and tree interference troubles peculiar to open-wire construction, involving such things as broken insulators, broken poles, wires crossed or broken, etc., should be avoided by placing the wire under ground. Buried wire should also, in general, be free from lightning troubles when properly protected at junctions with open-wire lines. Considerations of this kind will be largely controlling in determining the eventual field of use for the buried type circuit. Present indications are that in any event many locations may be found where this type of construction will prove economical.

Effect of Electric Shock on the Heart *

By L. P. FERRIS, B. G. KING,† P. W. SPENCE and H. B. WILLIAMS †

AS a basis for the development of protective measures and practices, knowledge of the limits of dangerous electric shock is obviously important and this joint investigation at the College of Physicians and Surgeons of Columbia University was initiated in the hope of obtaining some of the needed data. In seeking a value of current which if exceeded would be dangerous to man, it is important to consider for different practical conditions the effects which are brought about as the current is increased. The threshold of sensation is reached at about one milliampere for a frequency of 60 cycles. Other investigators have found that at about 15 milliamperes from hand to hand the subject becomes unable to control the muscles subjected to stimulation.

Any currents that prevent voluntary control of the skeletal muscles are dangerous because their pathway through the body might include the respiratory muscles and stop breathing during the shock. If prolonged, asphyxial death would result, but the time required is a matter of minutes rather than seconds, so that opportunity may be afforded for action to release the victim. No serious or permanent after-effects are likely to appear merely from the cessation of respiration, provided it is not continued beyond the point where the victim can be resuscitated by artificial respiration.

Currents somewhat greater than those just necessary to stop respiration by action on the muscles may cause fatalities, even though the duration of such shocks is but a few seconds or less—far too short to be important from the standpoint of interruption of respiration and obviously too short to give any opportunity for rescue before the end of the shock. Death under such conditions is brought about by ventricular fibrillation, which is a disruption of normal heart action. This condition is an uncoordinated asynchronous contraction of the

* Digest of a paper published in the May 1936 issue of *Electrical Engineering* and presented at the A. I. E. E. Summer Convention, Pasadena, California, June 22–26, 1936.

† Drs. H. B. Williams and B. G. King, joint authors of this paper, are of the staff of the College of Physicians and Surgeons of Columbia University, New York. Electrical facilities of the Bell Telephone Laboratories supplemented those of the Physiology Laboratories of the Medical Center where most of the experimental work was done.

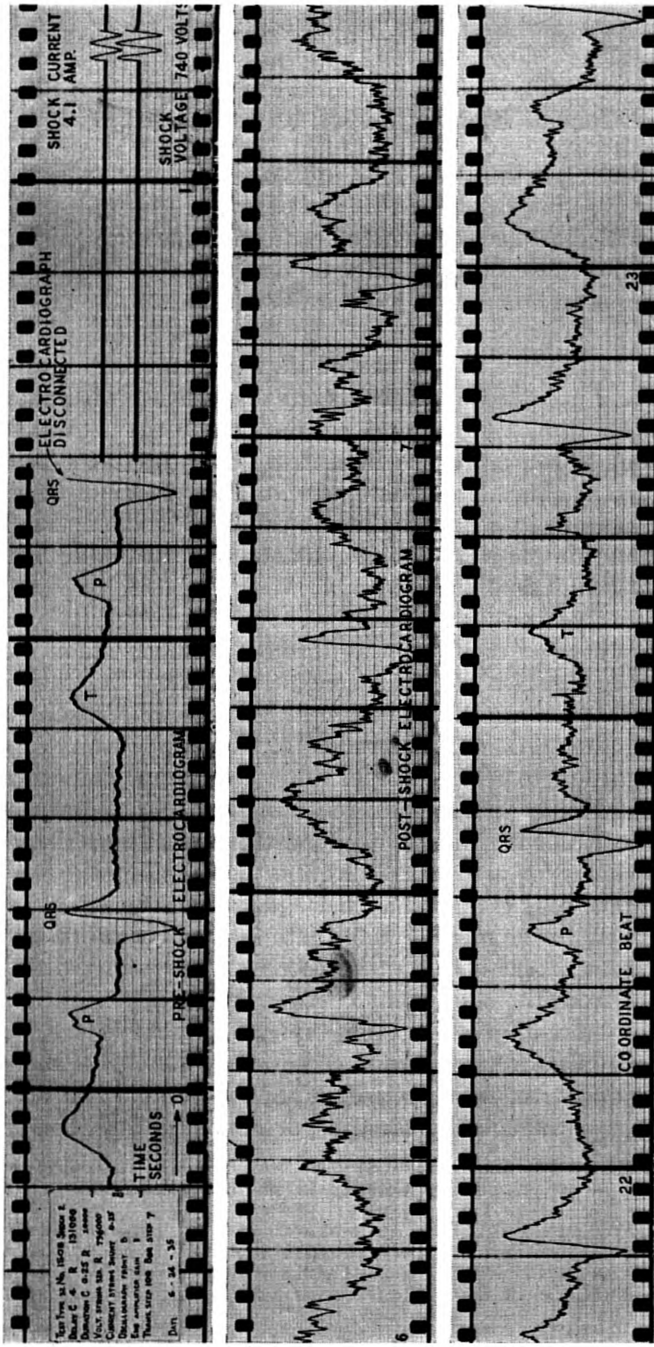

Fig. 1A—Typical records of 0.03-second shocks to sheep during insensitive phase of cardiac cycle. The electrodes were fastened to the right fore leg and left hind leg.

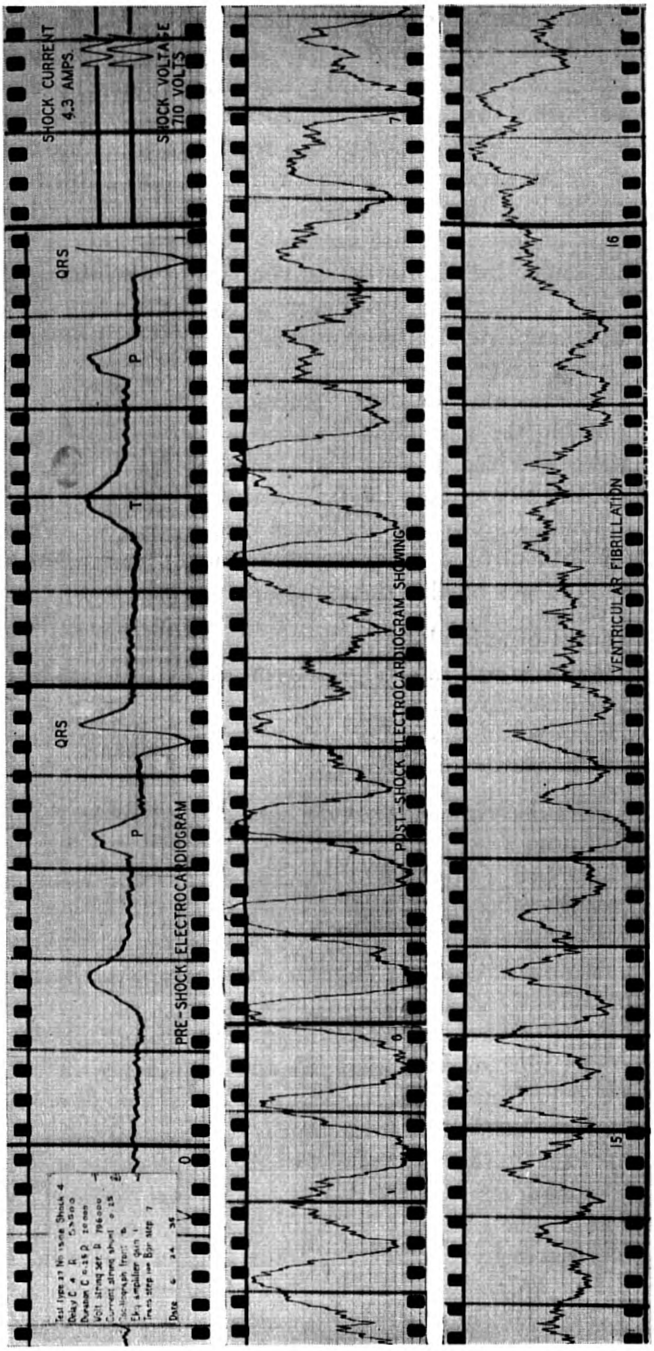

Fig. 1B—Typical records of 0.03-second shocks to sheep during sensitive phase of cardiac cycle.

ventricular muscle fibers in contrast to their normal coordinated and rhythmic contraction. It results from an abnormal stimulation rather than from damage to the heart. In the fibrillating condition, the heart seems to quiver rather than to beat; no heart sounds can be heard with a stethoscope; the pumping action of the heart ceases; failure of circulation results in an asphyxial death within a few minutes. The medical profession long has recognized that ventricular fibrillation once set up in man is unlikely to cease naturally before death. The value of current just under the threshold for ventricular fibrillation, therefore, may be taken as the maximum current to which man safely may be subjected, because regardless of rescue or after-treatment, death is liable to result from greater current.

This experimental investigation, therefore, was directed chiefly toward determining the minimum current that would initiate ventricular fibrillation and the variation of this threshold current with several factors which enter into the practical application of the results in the development of protective devices and measures. From the standpoints of both physiology and engineering, it was important to determine the influence on this threshold of:

1. Species and size of animal.
2. Path of current through the body (determined by points of contact).
3. Frequency of the current.
4. Time of occurrence of short shocks in relation to the cardiac cycle.
5. Duration of shock.

Thresholds were determined for seven species of animals: the guinea pig, rabbit, cat, dog, sheep, pig, and calf. Standard reference conditions included the use of 60-cycle alternating current for a duration of 3 seconds with electrodes on the right fore leg and left hind leg. These conditions typify those of many accidental shocks to man and are very dangerous from the point of view of ventricular fibrillation because the heart is almost directly in the current path.

Three significant records were made by an oscillograph for each shock. These are illustrated in Figs. 1A and 1B. They include electrocardiograms before and after shock and oscillograms of shock current and voltage. An electrocardiogram is a graphical record of the time variation of the voltage that is always associated with the action of the heart. The character of the variations in this voltage indicate certain facts as to the heart's condition, the electrocardiogram of a fibrillating heart being very different from that of a normal heart. The group of Fig. 1A shows a shock followed by coordinate beating. The group of Fig. 1B shows a shock which resulted in ventricular fibrillation. The character of both post-shock electrocardiograms is somewhat masked by higher frequency currents resulting from skeletal muscle activity following the shock. However, the post-shock electrocardiogram of the group 1A reveals the same typical sequence of prominent deflections that appear before shock, while that of the group 1B shows an entirely different wave shape. The appearance of this typical fibrillating wave and the absence of heart sounds following shock were taken as conclusive evidence of ventricular fibrillation.

Threshold of Fibrillation

The threshold current increases roughly with both the heart weight and body weight of the different species of animals, although if the three smaller species be considered alone this relationship does not hold, their threshold currents being practically the same despite widely different body weights. Data from tests on a number of different species are summarized in Fig. 2.

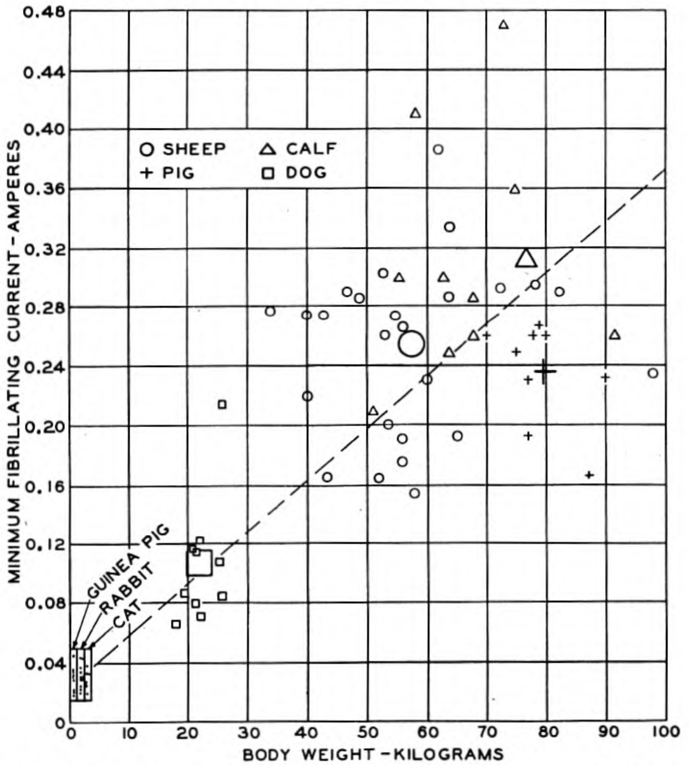

Fig. 2—Relation of minimum current causing ventricular fibrillation to body weight for different animal species. Shock duration 3 seconds. Frequency 60 cycles. Electrodes on right fore and left hind legs. (Averages shown by larger symbols.)

These results serve to indicate the probable threshold current for man under similar conditions. The average weight of an adult man is approximately 70 kilograms (154 lbs.) and his heart weight, 330 grams (.75 lbs.). The average threshold current for a body weight of 70 kilograms is 0.26 ampere and that corresponding to a heart weight of 330 grams is 0.29 ampere. Knowledge of such average currents is useful, but in the practical application of this information it is the lower limit of current causing ventricular fibrillation that must be taken into consideration. The thresholds differ widely for different individuals of the same species. The results on the whole indicate for man that currents in excess of 0.1 ampere at 60 cycles from hand to foot would be dangerous for shock durations of three seconds or more.

Effect of Frequency

To determine the effect of changing the frequency other tests were made on sheep with 25-cycle current and direct current, the general

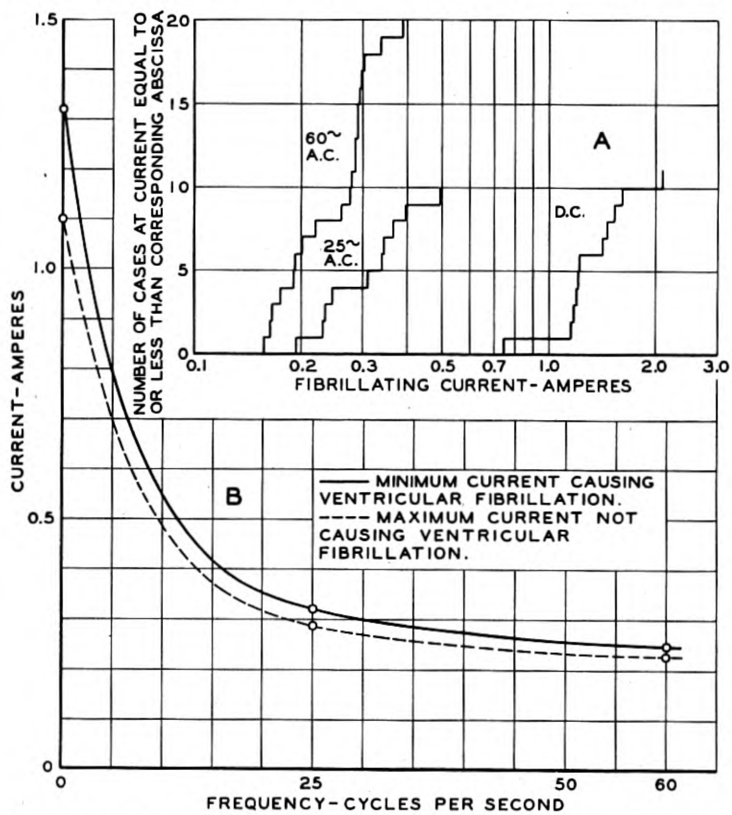

Fig. 3—Effect of frequency on threshold current causing ventricular fibrillation in sheep. Shock duration 3 seconds. Electrodes on right fore and left hind legs.

conditions remaining the same as for the 60-cycle current. The results of these tests are illustrated graphically in Fig. 3.

Susceptibility of Heart in Different Phases of Cardiac Cycle

Physiologists have established that the heart is responsive to moderate electrical stimuli during the period of relaxation (diastole) whereas such stimuli during the period of contraction (systole) do not elicit further response. During early systole (*QRS* to beginning of *T*, Fig. 1) the heart muscle is totally unresponsive, while during diastole (end of *T* to beginning of *QRS*) the ventricular muscles are responsive to stimuli. At the time of the *T* wave of the electrocardiogram, contraction starts to disintegrate and parts of the ventricular muscle will respond differently to electrical stimuli. In view of these facts and some erratic responses to shocks of three to four cycles of 60-cycle current, differences were expected in the response of the heart to very short shocks during different phases of its cycle. Special apparatus was developed for applying short shocks at predetermined parts of the cardiac cycle. The results of 913 shocks of 0.03-second duration on 132 sheep are plotted in Fig. 4, to show the position of the mid-point of each shock in the cardiac cycle, approximate shock current and whether or not fibrillation occurred. None of the shocks occurring during the period of complete contraction or complete relaxation of the heart caused ventricular fibrillation, this result appearing only for shocks falling during the period of diminishing contraction.

Of 370 shocks of 0.12 second duration applied to 38 sheep, only one shock definitely outside the partial refractory phase resulted in ventricular fibrillation. This shock began at a point in the electrocardiogram between *P* and *Q* waves at which time the ventricles are completely relaxed and resting.

Effect of Duration

In the determination of thresholds for shocks of very short durations, a third or less of the duration of one heart beat, the time of occurrence of the shocks was regulated so as always to involve the partial refractory phase, corresponding to the appearance of the *T* wave. The thresholds were found in the same way as for 3-second shocks, by applying successive shocks at intervals of 5 minutes, each at increased current until ventricular fibrillation resulted.

The effect of duration on the threshold is illustrated in Fig. 5. It is believed that the current required to initiate fibrillation would increase markedly as the duration is decreased below 0.03 second. It

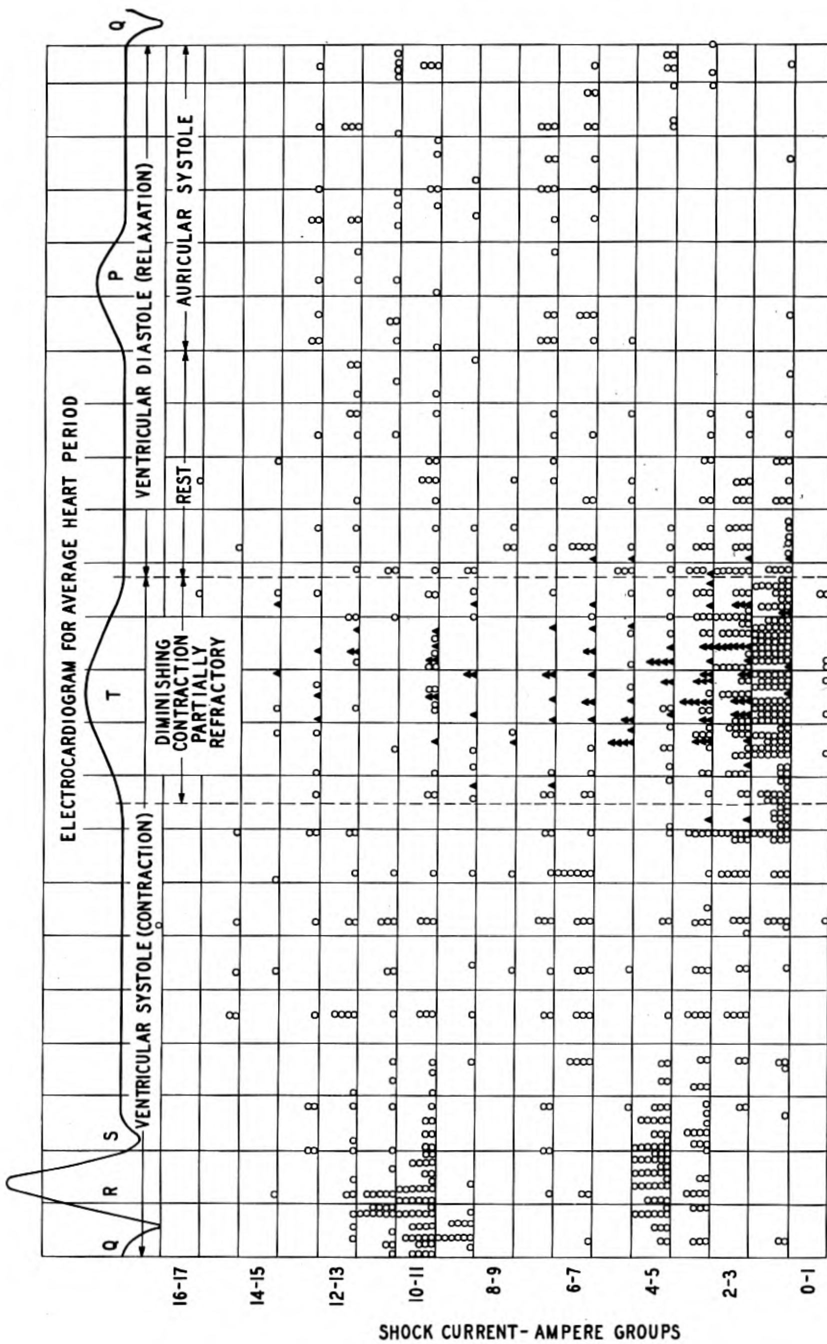

Fig. 4—Distribution in cardiac cycle and results of 913 shocks of 0.03-second duration to 132 sheep. 60 cycles. Electrodes on right fore and left hind legs. Triangle indicates fibrillation. Circle indicates coordinate beat.

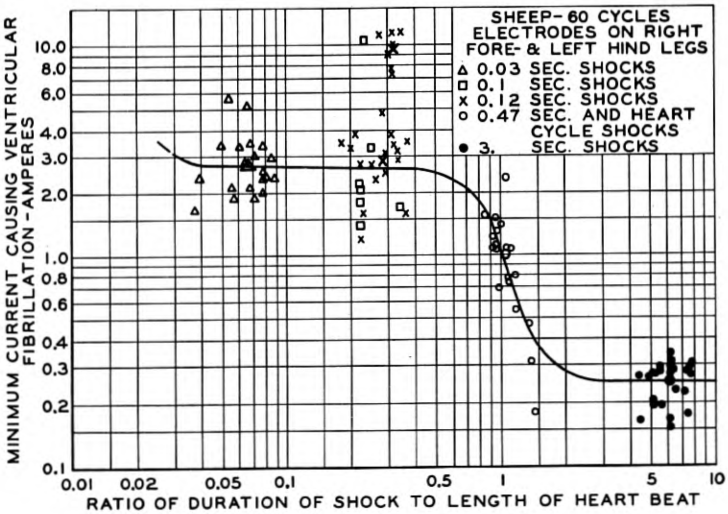

Fig. 5—Effect of duration on threshold.

is, however, not believed that increasing the duration beyond 3 seconds would reduce the average threshold current appreciably below 0.25 ampere.

Ventricular fibrillation has been found to be the only serious cardiac effect of the currents applied in these tests; however, temporary disruptions of normal cardiac activity frequently can be observed. A most common effect of electric shock is a change in heart rate. Electrocardiograms after shock frequently indicate disturbances of conduction in the heart. Premature heart beats (extrasystoles) and fibrillation of the auricles have also been observed.

The persistence of any of these conditions for more than a few minutes is rare. There is no evidence of any cardiac abnormalities or the presence of cardiac damage in electrocardiograms taken at intervals up to two months following shocks which did not immediately cause death.

Effect of High Currents

There was evidence from the work of Prevost and Battelli and some of the early results of this investigation that the stimulus of a powerful current would be less liable to cause fibrillation than a current moderately above the threshold. To test such evidence repeated short shocks of 23 to 26 amperes were given to a group of sheep in the sensitive phase of their cardiac cycle. Ten survived 5 shocks each without fibrillation, while an eleventh fibrillated on the initial shock. Each of the 10 surviving sheep was given additional similar shocks, except that the current was reduced to between 4 and 5 amperes. Five developed ventricular fibrillation on the first shock, and 3 on the second shock. A single animal survived 5 shocks, and another animal 2, without fibrillating. A comparison and combination of these results and those previously obtained definitely establish that the susceptibility of the heart to ventricular fibrillation becomes very much less when the current is increased to about 25 amperes, 10 times the average threshold value for shocks of short duration. This variation of susceptibility to fibrillation is shown graphically in Fig. 6.

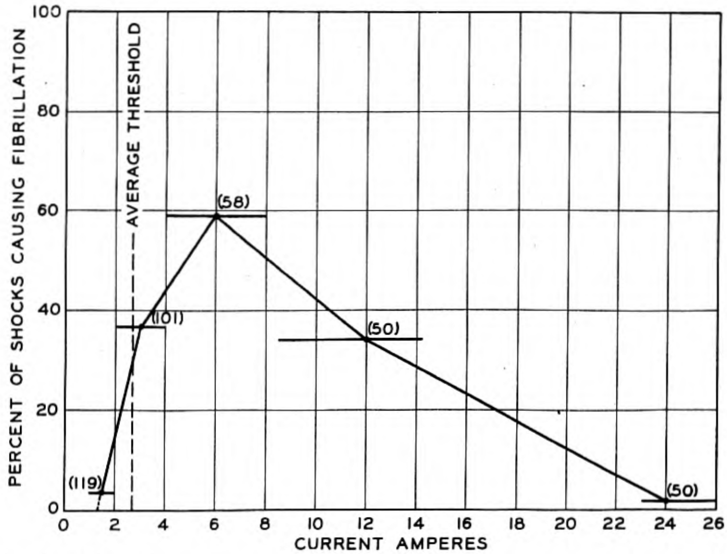

Fig. 6—Effect of current on susceptibility of sheep hearts to ventricular fibrillation. Shocks of 0.03 second, 60 cycles, applied in partial refractory period of cardiac cycle. Number of shocks and current spread indicated for each point on curve.

To determine whether such a decrease in susceptibility to fibrillation would occur also for shocks of about 3 seconds duration if the current were increased about 15 times the average threshold, 5 sheep were subjected to 4 ampere shocks of 3 seconds duration. In all cases ventricular fibrillation resulted on the initial shock, indicating either that at this duration no such decrease in susceptibility takes place with increase of current, or that 4 amperes is not a sufficiently high current to bring it about. The apparatus was not capable of giving much higher currents for this duration.

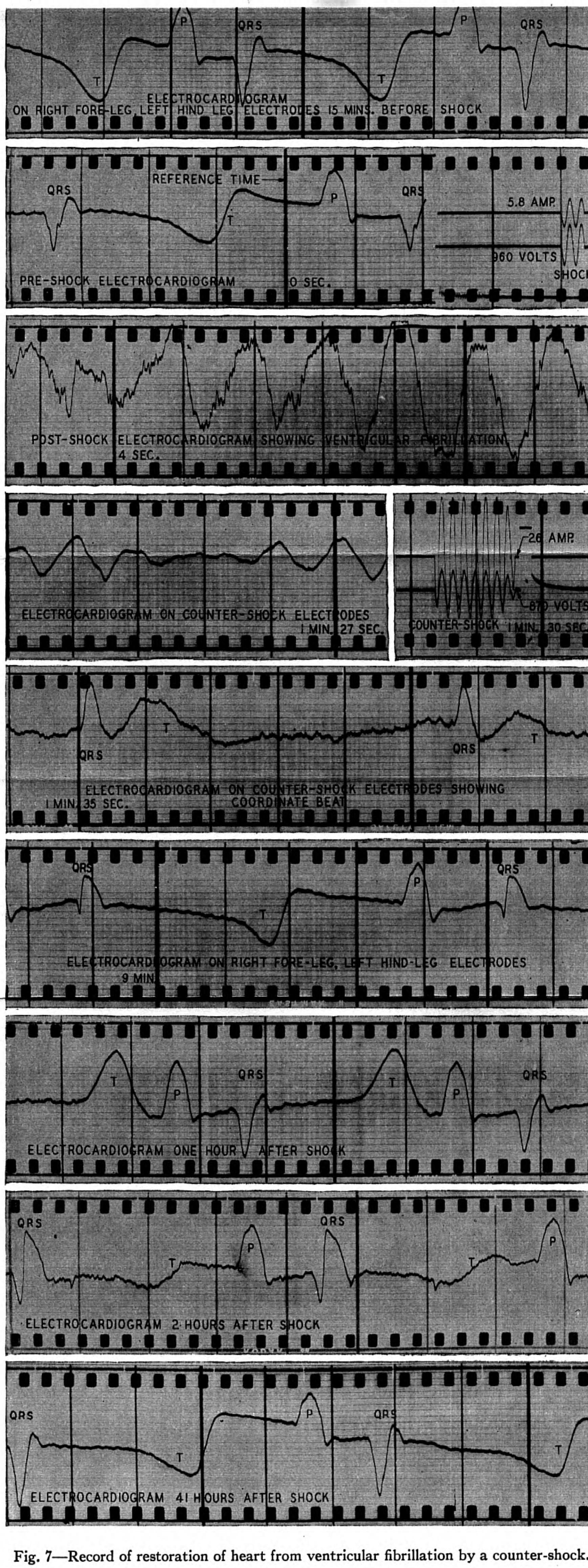

Fig. 7—Record of restoration of heart from ventricular fibrillation by a counter-shock.

Recovery of Heart from Ventricular Fibrillation

Recovery of the heart from ventricular fibrillation by the application of a short intense shock was reported first by Prevost and Battelli in 1899. Kouwenhoven, Langworthy and Hooker also have reported the recovery of dogs from fibrillation by the application of what has recently been termed a "counter-shock." The opportunity to experiment on recovery from ventricular fibrillation with large animals arose with the use of sheep and other large species in 1932. Currents up to 25 or 30 amperes were applied through large electrodes placed near the heart.

After some preliminary tests all counter-shocks were given with electrodes outside the skin on both sides of the chest so as to include the heart between them. Such counter-shocks were found to be effective in recovering coordinate heart action in about 60 per cent of the cases.

Figure 7 is an electrocardiographic record of heart action before and after a shock that caused fibrillation and at different stages after the application of a counter-shock that arrested the fibrillation and allowed the heart to resume its coordinate beating. The current and voltage of the fibrillating shock and the counter-shock are shown also to the same scale for comparison. The fibrillating shock of 6 amperes and 0.03 second duration occurred during the sensitive phase of the cardiac cycle. The counter-shock which followed 1½ minutes later was of 26 amperes for 0.1 second duration. Times marked on the different sections are referred to the beginning of the record. It may be observed that the last electrocardiogram is practically identical with the pre-shock electrocardiogram. Many sheep have been observed for periods of months after recovery from fibrillation, with no evidence of abnormalities. Several have given birth to normal lambs, and in many instances the recovered sheep have been used in subsequent tests and again recovered.

The possibilities of counter-shock have not been fully explored to determine the optimum conditions for its application, particularly as regards magnitude and character of current, its duration, and points of application. In regard to the latter, however, it would seem that some short path embracing the heart would be best. Any technique of recovery of the heart must be applied promptly so as not to permit deterioration of the brain which might result in impairing the competency of the victim if recovered. While the time limit would depend on many factors, it is a matter of minutes rather than seconds. The prompt application of artificial respiration ventilates the lungs and is believed also to maintain a small circulation of blood, sufficient to delay to give a somewhat higher threshold current. For the pathway from leg to leg, the proportion of current reaching the region of the heart is so small that fibrillation is not liable to result, even at currents of 15 amperes or more, although such currents probably would burn the victim unless the contacts were good and the shock of short duration.

c. Frequency. For shocks of one second or more in duration, the 25-cycle threshold current is about 25 per cent higher than the 60-cycle value, and the d-c. threshold current 5 times the 60-cycle value. For shock durations of a small fraction of a second this relation probably does not hold, all thresholds being expected to approach one another.

d. Time of Occurrence of Short Shocks in Relation to Cardiac Cycle. The heart is most sensitive to fibrillation for shocks occurring during the partial refractory phase of its cycle, which is about 20 per cent of the whole and which occurs simultaneously with the *T* wave of the electrocardiogram. With shocks of a duration of about 0.1 second or less, it is practically impossible to produce ventricular fibrillation, unless such shocks coincide in part at least with this sensitive phase of the cardiac cycle. The middle of the partial refractory phase is more sensitive than its beginning or end.

e. Duration of Shock. The threshold current varies inversely with shock duration but not uniformly, being most sensitive to change as the duration approaches the duration of one heart beat. Within the sensitive phase of the heart cycle the threshold fibrillating current for shock durations of about 0.1 second or less is 10 or more times the threshold for durations of one second or more. Shocks ⅓ or more of the heart cycle in duration may cause ventricular fibrillation, even though they would not extend into the sensitive phase of the cycle if the heart continued its normal beat after the initiation of the shock. The reason for this is probably the initiation of a premature heart beat which brings about a premature sensitive phase prior to the end of the shock.

5. Successive shocks have no cumulative effect on the susceptibility of the heart to fibrillation.

6. The susceptibility of the heart to fibrillation by short shocks increases with current up to several times the threshold, then diminishes, becoming very small at currents of the order of 25 amperes through the body in the vicinity of the heart. However, other serious injury may be expected from such currents when brought about by accidental contacts.

7. Fibrillation produced by an electric shock will in the majority of cases be arrested by a subsequent electric shock of high intensity and short duration through the heart, allowing the resumption of co-ordinate beating with no permanent damage.

A New High-Efficiency Power Amplifier for Modulated Waves *

By W. H. DOHERTY

THE use of increasingly higher power levels in broadcasting in the last few years has attached new importance to the matter of more efficient operation of the high-power stages in radio transmitters. The resulting reductions in cost of power, size of high-voltage transformers and rectifier, and water cooling requirements, are of particular importance in transmitters having outputs of 50 kilowatts or more.

The linear radio-frequency power amplifier, in the form in which it has been used extensively in broadcast transmission, may not be operated at a plate efficiency higher than about 33 per cent, if it is to supply the peak power required for amplifying a completely modulated wave. With this efficiency the d-c power input to a 50-kilowatt amplifier, for example, is 150 kilowatts, of which 100 kilowatts must be dissipated at the anodes of the water-cooled tubes.

This inherent weakness of the conventional linear amplifier has occasioned the development of certain other systems of amplification or modulation which permit a more economical use of power. Of these the high-level Class B modulation system [1] and the ingenious "outphasing modulation" scheme of Chireix [2] are most worthy of note.

A new form of linear power amplifier has been developed which removes the limitation of low efficiency inherent in the conventional circuit, permitting efficiencies of 60 to 65 per cent to be realized, while retaining the principal advantages associated with low-level modulation systems and linear amplifiers, namely, the absence of any high-power audio equipment, the ease of adding linear amplifiers to existing equipment to increase its power output, and the adaptability of such systems to types of transmission other than the carrier-and-double-side-band transmission most common at present.

Linear radio-frequency power amplifiers are ordinarily biased

* Digest of a paper presented before the Annual Convention of the Institute of Radio Engineers, May 11–13, 1936, at Cleveland, Ohio, and to be published later in the *Proceedings*.

[1] Chambers, Jones, Fyler, Williamson, Leach, and Hutcheson, "The WLW 500-Kilowatt Broadcast Transmitter," *Proc. I. R. E.*, Vol. 22, p. 1151, October, 1934.

[2] Chireix, "High Power Outphasing Modulation," *Proc. I. R. E.*, Vol. 23, p. 1370, November, 1935.

nearly to the cut-off point so that the plate-current pulse width is approximately a half cycle. Under these conditions the plate efficiency is proportional to the amplitude of the radio-frequency plate voltage. It is possible to obtain large outputs from tubes with radio-frequency plate voltage amplitudes of 0.85 to 0.9 of the applied d-c potential, i.e., with the plate voltage swinging down to a minimum value as low as 10 to 15 per cent of the d-c potential. The corresponding plate efficiency for the tube and its tuned circuit is approximately 67 per cent. This condition, however, prevails only at the peak output of the amplifier, and since the amplitude of the plate voltage wave, in a transmitter capable of 100 per cent modulation, is only half as great for the unmodulated condition as for the peaks of modulation, the efficiency with zero modulation in the conventional amplifier does not exceed half this peak value, or about 33 per cent. Even during complete modulation the effective efficiency over the whole audio cycle is only 50 per cent, and for the average percentage modulation of broadcast programs the all-day efficiency is only slightly greater than the efficiency for unmodulated carrier.

In order to improve this situation it is necessary to devise a system in which the amplitude of the alternating plate voltage wave is high for the unmodulated condition, and in which the increased output required for the positive swings of modulation is obtained in some other manner than by an increase in this voltage.

A simple and fundamental means is available for achieving this result. One embodiment of the scheme is illustrated in Fig. 1. Each

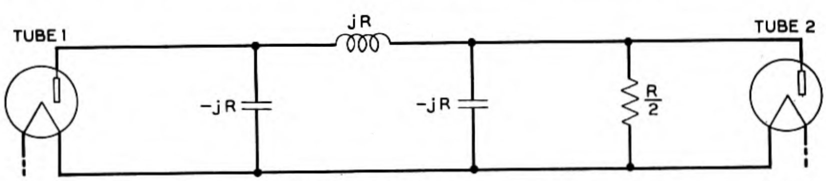

Fig. 1—Form of high-efficiency circuit.

of the two tubes shown in this figure is designed to deliver a peak power of E^2/R watts into an impedance of R ohms. The total peak output of the two tubes being $2E^2/R$ watts, the tubes are suitable for use in an amplifier whose carrier output is one-fourth of this value, or $E^2/2R$ watts. If the tubes were to be connected in parallel in a conventional amplifier circuit the load impedance used would be $R/2$ ohms, and each tube would work effectively into R ohms by virtue of the presence of the other tube. The same load impedance $R/2$ is used in the new circuit, but between the load and one of the tubes

(Tube 1) there is interposed a network which simulates a quarter-wave transmission line at the frequency at which the amplifier is operated.

It is a well-known property of quarter-wave transmission lines and their equivalent networks that their input impedance is inversely proportional to the terminating impedance. The network of Fig. 1, in particular, presents to Tube 1 an impedance of R ohms when its effective terminating impedance is also R ohms, that is, when half of the power in the load is being furnished by Tube 2; but should Tube 2 be removed from the circuit, or prevented from contributing to the output, the terminating impedance of the network would be reduced to $R/2$ ohms, with a consequent increase in the impedance presented to Tube 1 from R to $2R$ ohms. Under this condition Tube 1 could deliver the carrier power $E^2/2R$ at its maximum alternating plate voltage E and consequently at high efficiency.

The operation of the amplifier over the modulation cycle is as follows: The grids of both tubes are excited by the modulated output of the preceding amplifier stage, but for all instantaneous outputs from zero up to the carrier level Tube 2 is prevented by a high grid bias from contributing to the output, and the power is obtained entirely from Tube 1, which is working into $2R$ ohms, twice the impedance into which it is to work when delivering its peak output. In consequence, the radio-frequency plate voltage on this tube at the carrier output is nearly as high as is permissible and the efficiency is correspondingly high. Beyond this point the dynamic characteristic of Tube 1, unassisted, would flatten off very quickly because the plate voltage swing could not be appreciably increased. Tube 2, however, is permitted to come into play as the instantaneous excitation increases beyond the carrier point. In coming into play Tube 2 not only delivers power of itself, but through the action of the impedance-inverting network causes an effective lowering of the impedance into which Tube 1 works, so that Tube 1 may increase its power output without increasing its plate voltage swing, which was already a maximum at the carrier point. At the peak of a 100-per-cent modulated wave each tube is working for an instant into the impedance R most favorable to large output and delivering E^2/R watts, twice the carrier power, so that the total instantaneous output is the required value of four times the carrier power. Thus the required tube capacity is the same as in a conventional linear power amplifier.

Since it is usually desirable in power amplifiers to provide low-impedance paths for the harmonic components of the radio-frequency plate current wave, the reactive elements designated $-jR$ in a practical circuit ordinarily consist of a considerably larger capacity shunted by a coil, and in the tuning process either the coil or the condenser is adjusted so that the impedance of the combination is the required value of $-jR$ ohms. The effective shunt load $R/2$ is then usually obtained by coupling the radiating system of the transmitter to the necessary extent into the parallel circuit associated with Tube 2.

The presence of a quarter-wave network in the output circuit of the amplifier causes the plate potentials on the two tubes to be 90 degrees apart in phase. This requires that the voltages impressed on the two grids be 90 degrees apart in order that each may be opposite in phase to the related plate potential, as is necessary in any power amplifier. In addition to this phase requirement, there arises from the variation in load impedance for Tube 1 the requirement that the excitation on this tube shall rise considerably less than 100 per cent on the positive peaks of modulation. Without some limiting action on this excitation the grid current in Tube 1 would be excessive and would result in a diminution of its output at modulation peaks.

Both of these requirements concerning the input to the amplifier are satisfied by the use of the input circuit shown in Fig. 2. With the

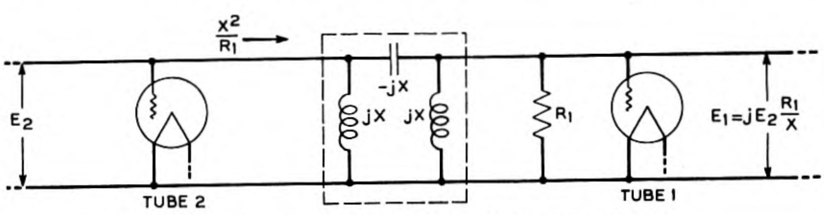

Fig. 2—Input circuit for a high-efficiency amplifier.

output of the previous stage applied directly to the grid of Tube 2, the resulting excitation E_1 on Tube 1 is proportional to the terminating resistance R_1 of the quarter-wave network. With a suitable value of R_1 the input conductance of Tube 1 arising from the flow of grid current at high excitations causes an effective lowering of R_1 which gives the desired limiting action on the excitation. At the same time the input impedance X^2/R_1 of the quarter-wave network is increased, compensating to a large extent for the shunting effect of the grid current in Tube 2, so that the previous stage is assisted in maintaining the proper excitation on the amplifier.

In a preliminary study of the behavior of an amplifier under these new conditions of operation, the results shown in Fig. 3 were obtained with a pair of small tubes. The radio-frequency plate potential of Tube 2 is the potential across the load circuit and is required to be linear with excitation. The short dotted portion halfway up on this characteristic shows the curvature that would be obtained if Tube 2 were not allowed to come into action. With proper adjustment of the bias and relative excitation on Tube 2 this effect is eliminated and the characteristic continues to rise up to the desired peak amplitude.

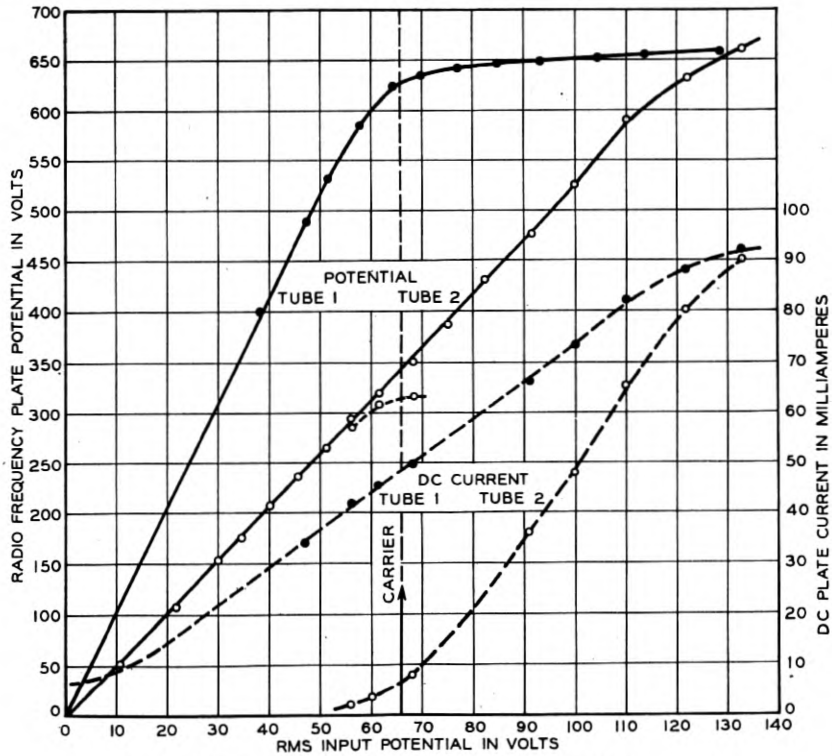

Fig. 3—Dynamic characteristics of an experimental low-power high-efficiency amplifier.

The radio-frequency plate voltage of Tube 1 is seen to be twice that of Tube 2 up to the point where curvature begins, and then to increase only slightly between the carrier output and peak output. The plate current of Tube 2 commences just before the carrier point is reached and rises twice as rapidly as the plate current of Tube 1. The equality of plate currents and radio-frequency plate potentials on the two tubes at the peak of modulation indicates that the tubes are contributing about equally to the instantaneous output at this point.

The high radio-frequency plate potential of Tube 1 at the carrier amplitude results in an efficiency of 63 per cent, and by integrating the d-c plate currents of Fig. 3 over a complete cycle of modulation it is found that the effective average efficiency at 100 per cent modulation is also 63 per cent. The d-c plate current of the amplifier therefore rises 50 per cent with full modulation, as does the output power.

The necessity for careful adjustment of the relative excitation and bias on Tube 2, to obtain a linear characteristic in the amplifier, is eliminated when the feedback principle[3] due to Black is employed. Negative feedback may be used in radio transmitters at either radio frequency or audio frequency. The resulting improvements in linearity are useful in noise reduction as well as in distortion correction.

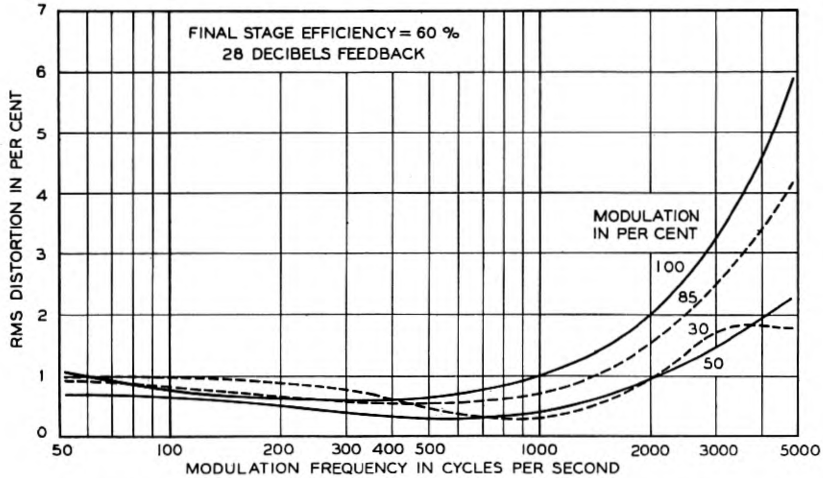

Fig. 4—Distortion measurements on a 50-kilowatt transmitter.

Figure 4 gives the results of distortion measurements on a complete 50-kilowatt transmitter built in the Laboratories and operating at a plate circuit efficiency for the final stage of 60 per cent. The use of 28 db of audio-frequency feedback, besides permitting alternating-current filament heating for all of the tubes, resulted in a distortion level less than 1 per cent at any frequency between 50 and 1000 cycles. At the higher audio frequencies the feedback is less effective because of the cumulative phase shifts in the various stages. The percentage modulation actually occurring in a broadcast program at these high frequencies, however, is so small that the distortion measured at high percentages of modulation is not of practical significance. The test, moreover, was made at the low-frequency end of the broadcast

[3] H. S. Black, "Stabilized Feedback Amplifiers," *Electrical Engineering*, January, 1934; *Bell Sys. Tech. Jour.*, January, 1934.

spectrum, where the effect is most pronounced because of the smaller band width.

The power required by this transmitter, including all auxiliary equipment, was 135 kilowatts with normal program modulation, as compared with approximately 230 kilowatts required in the usual 50-kilowatt installation.

High-efficiency operation, in addition to affording a large saving in the plate power supply, reduces the plate dissipation by a factor of three or four, with a resulting economy in the cooling system and an improvement in tube life.

The absence of any such requisites as the complicated driving stages of the Chireix system or the large audio equipment involved in high-level modulation gives the new circuit an important advantage over other high-efficiency systems in cost of apparatus and simplicity of design. The new amplifier, moreover, is operated at a plate voltage consistent with safety to the tubes, and is therefore not subject to the operating difficulties encountered at the high peak plate voltages required in high-level modulation.

Abstracts of Technical Articles from Bell System Sources.

A Review of Radio Communication in the Mobile Services.[1] CLIFFORD N. ANDERSON. Developments in radio communication in the mobile services during 1935 have been largely in the nature of gradual improvement of existing equipments and services.

In the marine field, the safety-of-life aspect is assuming increased importance. Rearrangements have been made of the frequencies and the schedules of radio beacons to avoid interference thereby making the system more effective. The improvement of radio compasses, regulations regarding motor lifeboat equipment and public address alarm systems, requirements for radio auto alarms and experimentation with collision prevention equipment are other items on which progress has been made the past year. The development in marine radiotelegraphy has been chiefly along the lines of greater application of the high frequencies. Directional antennas at the shore receiving stations have mitigated the effects of interference. Facsimile transmission of weather maps and press is being tried out. Improvements have been made in radio telephone equipments of various powers and frequency ranges for various types of marine service. A system utilizing ultra-high frequencies was put into operation at Philadelphia during the year. Three commercial stations are in operation in the two-megacycle range, the one at Seattle having been opened this year.

Radio is an important factor in the operation of modern air lines. Special mention should be made of the important role played by radio in the newly established transpacific service by the Pan American Airways. Improvements have been made during the year in airway beacons and radio compasses; airport traffic control and blind landing systems are being tried out. In addition to the beacon and communication receiver, a small five-watt transmitter has been made available for the use of itinerant flyers in communicating with airports.

The use of radiotelephony with police cars is the most important application of radio with automobiles. There are two general types of this service, both of which have expanded materially during the past year. One consists of a one-way service from police headquarters to the cars and is usually conducted on a frequency in the range of 1500 to 2500 kilocycles. The other is a two-way service generally operating in the ultra-high-frequency range of 30,000 to 40,000 kilocycles.

[1] *Proc. I. R. E.*, March, 1936.

Another phase of radio with automobiles is the use of broadcast receivers in pleasure cars. Under-the-car antennas, made necessary by introduction of all metal automobile tops, inclusion of the radio control as a part of the instrument board, and the use of circuits for reducing ignition noise are the more important features of 1935 developments.

Photons and Electrons.[2] KARL K. DARROW. In a book entitled "Biological Effects of Radiation" edited by Professor B. M. Duggar of the University of Wisconsin, Dr. Darrow contributes the first chapter of 42 pages. In descriptive and brief manner the following topics are discussed: Waves and Corpuscles; Monochromatic light and measurement of wave-length; External photoelectric effect and measurement of photon energy; Units of wave-length, wave number, frequency, and photon energy; Regions of the spectrum; Absorption of light by atoms; Continua in absorption spectra, and ionization by light; Theory of absorption lines; Terms; Absorption in X-ray region; Emission of light; X-ray emission spectra; Production of X-rays; Production of light of the optical spectrum; Scattering of light without change of frequency; Scattering of light with change of frequency; Scattering of X-rays; Transmutation of electron-pairs and photons.

Neutralizing Transformer to Protect Power Station Communication.[3] E. E. GEORGE, R. K. HONAMAN, L. L. LOCKROW, E. L. SCHWARTZ. The use of commercial telephone circuits by power companies for a wide range of communication services including not only telephone but also telemetering, remote alarms, supervisory control and pilot wire control has focused attention on the problems of protection of this type of service. Where such circuits enter power stations which are subject to rise in ground potential at times of faults, the neutralizing transformer provides a means of securing adequate protection. Circuits operated into power stations through neutralizing transformers experience no adverse effects from potential rise up to 4,000 volts r.m.s. This result is produced by causing the transformer to introduce into affected communication circuits a counter voltage to neutralize the difference in ground potential. Transformers for indoor and outdoor use have been designed. The characteristics are such that they produce substantially no adverse reaction upon the transmission over the communication circuits they protect. Trials were made in the territory of the Tennessee Electric Power Company. In five locations

[2] Chapter in book, "Biological Effects of Radiation," Vol. I, McGraw-Hill Book Company, Inc., 1936.
[3] *Elec. Engg.*, May, 1936.

in which transformers have been installed, they have prevented interruption of the circuits not only for long periods but also for short periods lasting only for the duration of a surge.

On the Preparation of Iron and Steel Specimens for Microscopic Investigations.[4] FRANCIS F. LUCAS. A given lens system has certain potential resolving powers. This potential resolving power may or may not be fully realized in practice. Even a low power objective has remarkable resolving ability and the very high aperture objectives are capable of furnishing sharp brilliant images of details measuring about two hundred atom diameters.

The author describes in this paper methods and materials for the critical preparation of iron and steel specimens. A flotation apparatus which he has developed for the preparation of abrasives is described and a typical particle size analysis of a magnesium oxide abrasive prepared by this method is given.

Some Alloys of Copper and Iron (The Tensile, Electrical and Corrosion Properties).[5] EARLE E. SCHUMACHER and ALEXANDER G. SOUDEN. Bars of copper-iron alloy 0.75 and 1.0 in. in diameter and 20 in. in length were prepared with compositions ranging from 75 Cu–25 Fe to 37.5 Cu–62.5 Fe without segregation sufficient to detect by differences in electrical and mechanical properties. These alloys were hot worked satisfactorily to 0.25 in. rod. The copper-iron alloys in the range investigated consist of a mixture of solid solutions of the constituent elements, the phasial relationships of which depend on the thermal treatment.

A few of the observations made concerning these alloys are listed below:

1. The alloys in the range investigated are of the precipitation hardening type, but do not require a drastic quenching treatment to retain a supersaturated iron-rich phase. The optimum combination of tensile and electrical properties is obtained in the 50 copper-50 iron alloy by aging at 500° C. followed by hard drawing.
2. High tensile strengths associated with desirable electrical conductivities can be developed for certain of the compositions. An alloy of 50 Cu–50 Fe, for example, can be prepared in the No. 18 AWG with an ultimate strength of 180,000 to 190,000 lbs. per sq. in. and an electrical conductivity of approximately 30 per cent.

[4] *Trans. Amer. Soc. for Metals*, March, 1936.
[5] *Metals and Alloys*, April, 1936.

3. The alloy of 50 copper-50 iron can be satisfactorily tinned commercially.
4. Within the range of alloys studied, corrosion resistance decreases with increase of iron content. Various corrosion tests indicate that these alloys might prove corrosion resistant in inland rural districts, but that they are unsuitable for use in marine atmospheres and would probably be unsatisfactory in most industrial atmospheres, particularly in regions near the sea coast.

A Study of the Electromagnetic Field in the Vicinity of a Radiator.[6] F. R. STANSEL. The complete equations for the electromagnetic field of an infinitesimal current element are given. The integration of these equations is considered for the case of a finite radiator having an empirical current distribution. Tables are included to facilitate computation and consideration is given to difference in phase of the current in various portions of the radiator.

An Analysis of Theater and Screen Illumination Data.[7] S. K. WOLF. During the past twenty years much information on theater and screen illumination has been accumulated. The significance and reliability of these data are discussed in the light of known physical factors influencing proper illumination. As a first approximation to a standard, it is suggested that the data indicate a value of 8 to 12-foot candles as representing satisfactory illumination. Variation of required illumination with screen size is analyzed, and a solution of the problem is suggested. The brightness of screen surroundings also is discussed. It is concluded that improvement in projection may be made by stricter application of existing information but that further investigations are desirable.

[6] *Proc. I. R. E.*, May, 1936.
[7] *Jour. S. M. P. E.*, May, 1936.

Contributors to this Issue

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